Computer algebra independent integration tests

0-Independent-test-suites/Apostol-Problems

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [175]. This is test number [1].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (175)	% 0.00 (0)
Mathematica	% 100.00 (175)	% 0.00 (0)
Maple	% 98.86 (173)	% 1.14 (2)
Maxima	% 94.86 (166)	% 5.14 (9)
Fricas	% 98.29 (172)	% 1.71 (3)
Sympy	% 90.29 (158)	% 9.71 (17)
Giac	% 96.57 (169)	% 3.43 (6)
Mupad	% 96.57 (169)	% 3.43 (6)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

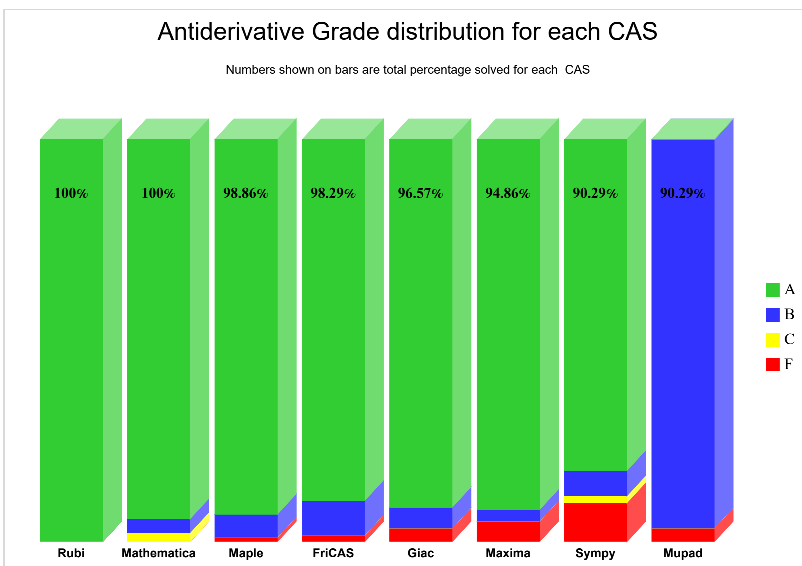
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

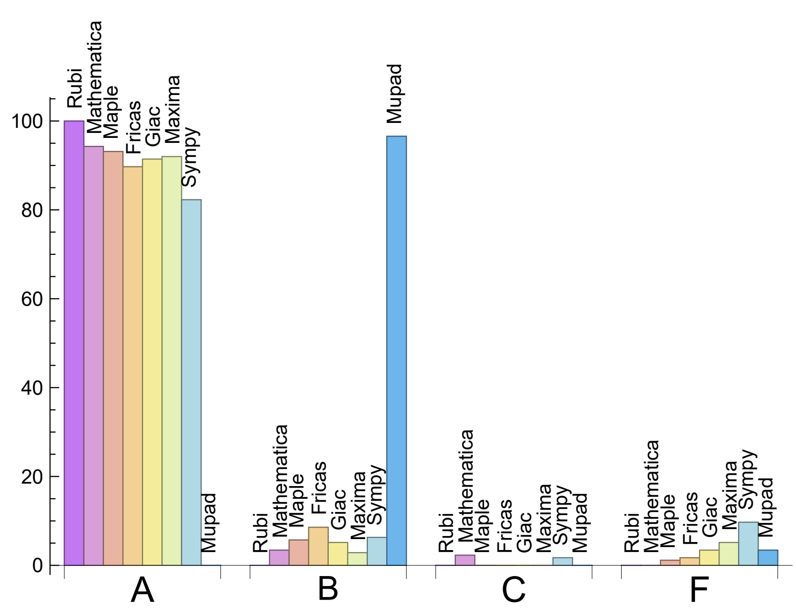
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	94.29	3.43	2.29	0.00
Maple	93.14	5.71	0.00	1.14
Maxima	92.00	2.86	0.00	5.14
Fricas	89.71	8.57	0.00	1.71
Sympy	82.29	6.29	1.71	9.71
Giac	91.43	5.14	0.00	3.43
Mupad	0.00	96.57	0.00	3.43

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	2	100.00 %	0.00 %	0.00 %
Maxima	9	66.67 %	0.00 %	33.33 %
Fricas	3	100.00 %	0.00 %	0.00 %
Sympy	17	94.12 %	5.88 %	0.00 %
Giac	6	0.00 %	0.00 %	100.00 %
Mupad	6	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

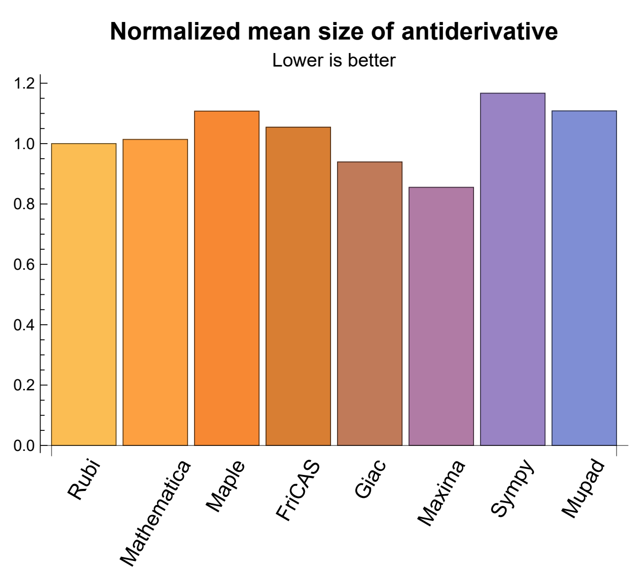
1.3 Performance

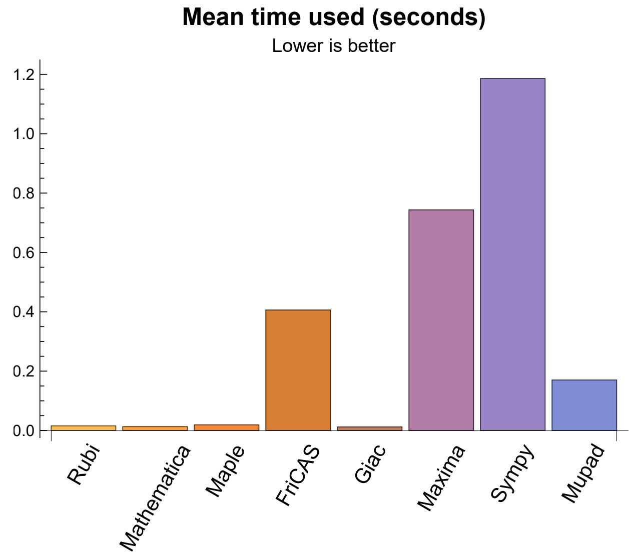
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.02	23.09	1.00	19.00	1.00
Mathematica	0.01	21.69	1.01	18.00	1.00
Maple	0.02	23.20	1.11	16.00	0.88
Maxima	0.74	18.24	0.86	14.50	0.81
Fricas	0.41	22.04	1.05	17.00	0.86
Sympy	1.19	27.87	1.17	17.00	0.84
Giac	0.01	20.35	0.94	15.00	0.83
Mupad	0.17	31.83	1.11	14.00	0.79

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {98}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
```

'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'

SageMath loading of Maxima abs_integrate was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error Exception raised: NotImplementedError

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function LeafSize is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user slelievre at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

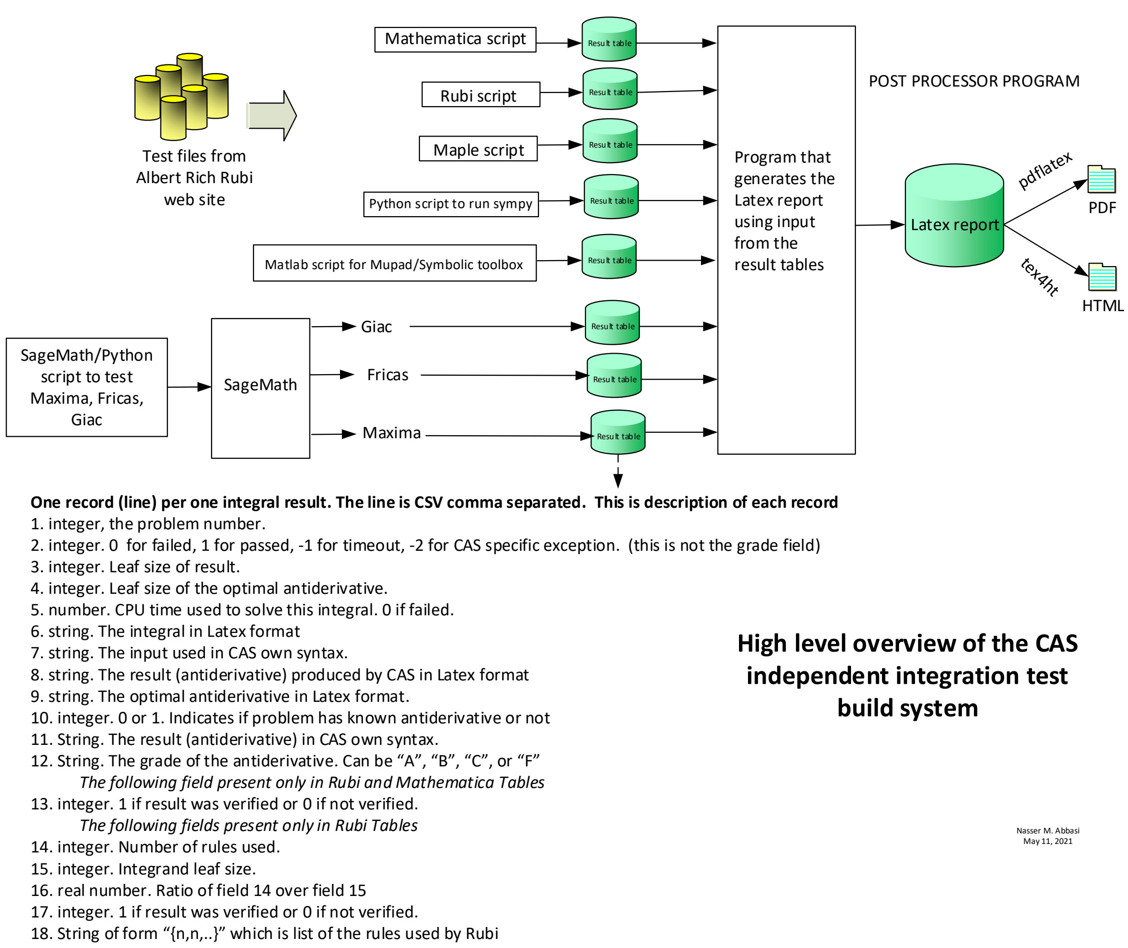
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 112, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174 }

B grade: { 50, 51, 83, 84, 105, 154 }

C grade: { 41, 98, 113, 175 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 15, 16, 18, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 49, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 171, 173, 174, 175 }

B grade: { 14, 17, 48, 50, 51, 114, 139, 158, 169, 170 }

C grade: { }

F grade: { 19, 172 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 100, 101, 102, 103, 106, 107, 108, 109, 110, 111, 112, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 170, 171, 172, 173 }

B grade: { 51, 83, 84, 113, 169 }

C grade: { }

F grade: { 19, 41, 98, 99, 104, 105, 141, 174, 175 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 46, 47, 49, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 85, 86, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 115, 116, 117, 118, 119, 120, 121, 122, 123, 125, 126, 127, 128, 129, 130, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 147, 148, 149, 150, 151, 152, 153, 154, 155, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174 }

B grade: { 16, 44, 45, 48, 50, 51, 61, 84, 88, 113, 114, 124, 131, 145, 146 }

C grade: { }

F grade: { 41, 156, 175 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 16, 18, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 43, 44, 45, 46, 49, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 84, 85, 86, 87, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 102, 106, 107, 108, 109, 110, 111, 112, 113, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 147, 148, 149, 150, 152, 157, 158, 159, 160, 161, 166, 167, 168, 169, 170, 171, 172, 175 }

B grade: { 9, 17, 42, 47, 48, 50, 51, 90, 101, 114, 144 }

C grade: { 83, 89, 156 }

F grade: { 19, 88, 103, 104, 105, 145, 146, 151, 153, 154, 155, 162, 163, 164, 165, 173, 174 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 46, 47, 48, 49, 50, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 157, 158, 159, 160, 161, 162, 163, 165, 166, 167, 168, 169, 170, 171, 173, 174 }

B grade: { 44, 45, 51, 83, 84, 113, 136, 155, 164 }

C grade: { }

F grade: { 9, 41, 62, 156, 172, 175 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 100, 101, 102, 103, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 166, 167, 168, 169, 170, 171, 172, 173, 175 }

C grade: { }

F grade: { 98, 99, 104, 105, 165, 174 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	9	8	9	9
normalized size	1	1.00	1.00	0.77	0.69	0.69	0.62	0.69	0.69
time (sec)	N/A	0.001	0.004	0.034	0.556	0.393	0.103	0.005	0.321

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	18	15	19	19	39	19	14
normalized size	1	1.00	0.67	0.56	0.70	0.70	1.44	0.70	0.52
time (sec)	N/A	0.004	0.007	0.003	0.461	0.403	1.043	0.008	0.085

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	21	18	22	22	48	22	19
normalized size	1	1.00	0.62	0.53	0.65	0.65	1.41	0.65	0.56
time (sec)	N/A	0.005	0.007	0.004	0.546	0.393	1.388	0.008	0.069

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	18	15	19	14	61	19	14
normalized size	1	1.00	0.67	0.56	0.70	0.52	2.26	0.70	0.52
time (sec)	N/A	0.005	0.006	0.003	0.501	0.409	0.952	0.009	0.057

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	22	22	12	12
normalized size	1	1.00	1.00	0.93	0.86	1.57	1.57	0.86	0.86
time (sec)	N/A	0.003	0.005	0.007	0.439	0.390	0.117	0.010	0.065

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	15	11	11	11	8	11	10
normalized size	1	1.00	1.15	0.85	0.85	0.85	0.62	0.85	0.77
time (sec)	N/A	0.006	0.002	0.145	0.497	0.429	0.068	0.006	0.158

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	16	13	15	17	92	15	12
normalized size	1	1.00	0.70	0.57	0.65	0.74	4.00	0.65	0.52
time (sec)	N/A	0.004	0.006	0.004	0.708	0.394	0.988	0.008	0.031

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	10	8	6	6
normalized size	1	1.00	1.00	0.88	0.75	1.25	1.00	0.75	0.75
time (sec)	N/A	0.013	0.005	0.019	0.489	0.413	0.067	0.009	0.418

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	18	29	0	12
normalized size	1	1.00	1.00	0.81	0.75	1.12	1.81	0.00	0.75
time (sec)	N/A	0.023	0.014	0.026	0.567	0.423	0.337	0.000	0.171

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	6	6	5	6	6
normalized size	1	1.00	1.00	1.17	1.00	1.00	0.83	1.00	1.00
time (sec)	N/A	0.018	0.015	0.066	0.698	0.408	0.415	0.006	0.043

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	12	12	6	9
normalized size	1	1.00	1.00	0.92	0.83	1.00	1.00	0.50	0.75
time (sec)	N/A	0.028	0.010	0.073	0.636	0.429	0.649	0.006	0.199

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	10	8	8
normalized size	1	1.00	1.00	0.90	0.80	0.80	1.00	0.80	0.80
time (sec)	N/A	0.016	0.022	0.009	0.520	0.408	0.331	0.007	0.171

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	9	9	7	9	9
normalized size	1	1.00	1.00	1.11	1.00	1.00	0.78	1.00	1.00
time (sec)	N/A	0.011	0.018	0.018	0.791	0.423	6.727	0.010	0.207

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	32	11	11	10	11	11
normalized size	1	1.00	1.00	2.13	0.73	0.73	0.67	0.73	0.73
time (sec)	N/A	0.003	0.004	0.010	0.457	0.396	0.274	0.008	0.354

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	16	13	15	15	34	15	12
normalized size	1	1.00	0.70	0.57	0.65	0.65	1.48	0.65	0.52
time (sec)	N/A	0.004	0.006	0.003	0.499	0.385	1.013	0.008	0.034

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	9	22	8	9	9
normalized size	1	1.00	1.00	0.91	0.82	2.00	0.73	0.82	0.82
time (sec)	N/A	0.001	0.003	0.004	0.467	0.402	0.734	0.019	0.072

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	27	11	11	27	11	11
normalized size	1	1.00	1.00	1.80	0.73	0.73	1.80	0.73	0.73
time (sec)	N/A	0.003	0.005	0.006	0.533	0.388	0.377	0.008	0.185

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	11	12	11	15
normalized size	1	1.00	1.00	0.80	0.73	0.73	0.80	0.73	1.00
time (sec)	N/A	0.026	0.059	0.031	0.717	0.414	0.350	0.031	0.239

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	37	0	0	23	0	15	47
normalized size	1	1.00	1.16	0.00	0.00	0.72	0.00	0.47	1.47
time (sec)	N/A	0.101	0.058	0.144	0.000	0.460	0.000	0.061	0.589

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	14	13	13
normalized size	1	1.00	1.00	0.82	0.76	0.76	0.82	0.76	0.76
time (sec)	N/A	0.111	0.016	0.010	0.589	0.438	0.370	0.009	0.210

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	13	13	7	12	15	12	9
normalized size	1	1.00	0.81	0.81	0.44	0.75	0.94	0.75	0.56
time (sec)	N/A	0.008	0.008	0.003	0.549	0.408	1.751	0.009	0.156

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	8	8	7	8	8
normalized size	1	1.00	1.00	1.12	1.00	1.00	0.88	1.00	1.00
time (sec)	N/A	0.009	0.002	0.015	0.524	0.409	0.183	0.007	0.064

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	15	18	15	15	17	15	15
normalized size	1	1.00	0.88	1.06	0.88	0.88	1.00	0.88	0.88
time (sec)	N/A	0.021	0.017	0.011	0.562	0.414	0.337	0.007	0.033

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	19	24	20	20	26	20	24
normalized size	1	1.00	0.83	1.04	0.87	0.87	1.13	0.87	1.04
time (sec)	N/A	0.036	0.016	0.017	0.641	0.402	0.644	0.007	0.030

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	20	25	21	21	26	21	23
normalized size	1	1.00	0.83	1.04	0.88	0.88	1.08	0.88	0.96
time (sec)	N/A	0.036	0.015	0.016	0.536	0.405	0.641	0.007	0.030

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	5	6	6
normalized size	1	1.00	1.00	0.88	0.75	0.75	0.62	0.75	0.75
time (sec)	N/A	0.007	0.001	0.003	0.627	0.403	0.063	0.006	0.019

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	18	18	14	17	24	14	18
normalized size	1	1.00	0.78	0.78	0.61	0.74	1.04	0.61	0.78
time (sec)	N/A	0.013	0.003	0.005	0.513	0.419	0.343	0.009	0.095

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	10	10	10
normalized size	1	1.00	1.00	0.79	0.71	0.71	0.71	0.71	0.71
time (sec)	N/A	0.005	0.002	0.016	0.571	0.420	0.068	0.008	0.032

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	15	11	11	11	8	11	10
normalized size	1	1.00	1.15	0.85	0.85	0.85	0.62	0.85	0.77
time (sec)	N/A	0.006	0.002	0.000	0.582	0.411	0.073	0.006	0.002

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	22	18	16	19	24	16	16
normalized size	1	1.00	0.92	0.75	0.67	0.79	1.00	0.67	0.67
time (sec)	N/A	0.009	0.002	0.125	0.494	0.417	0.068	0.006	0.036

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	23	17	17	17	17	17	17
normalized size	1	1.00	1.10	0.81	0.81	0.81	0.81	0.81	0.81
time (sec)	N/A	0.007	0.002	0.104	0.511	0.429	0.074	0.007	0.043

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	30	24	24	25	36	22	22
normalized size	1	1.00	0.88	0.71	0.71	0.74	1.06	0.65	0.65
time (sec)	N/A	0.016	0.002	0.103	0.624	0.414	0.067	0.007	0.039

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	25	19	19	36	19	19
normalized size	1	1.00	1.00	1.00	0.76	0.76	1.44	0.76	0.76
time (sec)	N/A	0.014	0.011	0.014	0.500	0.428	0.358	0.007	0.093

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	31	23	23	23	39	23	25
normalized size	1	1.00	0.94	0.70	0.70	0.70	1.18	0.70	0.76
time (sec)	N/A	0.022	0.009	0.057	0.567	0.419	0.646	0.007	0.119

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	29	37	26	29	56	26	28
normalized size	1	1.00	0.71	0.90	0.63	0.71	1.37	0.63	0.68
time (sec)	N/A	0.031	0.039	0.036	0.498	0.435	0.653	0.007	0.063

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	10	10	10
normalized size	1	1.00	1.00	0.79	0.71	0.71	0.71	0.71	0.71
time (sec)	N/A	0.006	0.002	0.016	0.517	0.430	0.064	0.006	0.026

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	15	11	9	10	8	9	9
normalized size	1	1.00	1.36	1.00	0.82	0.91	0.73	0.82	0.82
time (sec)	N/A	0.006	0.002	0.094	0.578	0.412	0.067	0.006	0.033

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	22	18	16	19	24	16	16
normalized size	1	1.00	0.92	0.75	0.67	0.79	1.00	0.67	0.67
time (sec)	N/A	0.011	0.002	0.109	0.539	0.426	0.065	0.006	0.029

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	62	69	60	60	180	50	37
normalized size	1	1.00	0.74	0.82	0.71	0.71	2.14	0.60	0.44
time (sec)	N/A	0.015	0.077	0.008	1.309	0.423	3.709	0.047	0.215

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	25	22	34	21	39	28	20
normalized size	1	1.00	0.66	0.58	0.89	0.55	1.03	0.74	0.53
time (sec)	N/A	0.013	0.008	0.006	1.427	0.416	1.242	0.007	0.029

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	34	168	0	0	31	0	301
normalized size	1	1.00	0.20	0.98	0.00	0.00	0.18	0.00	1.75
time (sec)	N/A	0.029	0.006	0.184	0.000	0.435	0.719	0.000	0.075

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	6	6	7	6	6
normalized size	1	1.00	1.00	1.17	1.00	1.00	1.17	1.00	1.00
time (sec)	N/A	0.004	0.004	0.005	1.220	0.408	0.065	0.009	0.070

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	18	13	12	12	19	12	12
normalized size	1	1.00	1.29	0.93	0.86	0.86	1.36	0.86	0.86
time (sec)	N/A	0.009	0.003	0.004	1.274	0.412	0.070	0.009	0.034

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	12	10	20	8	18	8
normalized size	1	1.00	1.00	1.50	1.25	2.50	1.00	2.25	1.00
time (sec)	N/A	0.005	0.002	0.004	1.272	0.414	0.069	0.018	0.057

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	18	14	16	48	19	34	10
normalized size	1	1.00	1.50	1.17	1.33	4.00	1.58	2.83	0.83
time (sec)	N/A	0.010	0.003	0.003	1.277	0.421	0.078	0.019	0.022

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	26	21	20	18	26	18	20
normalized size	1	1.00	1.18	0.95	0.91	0.82	1.18	0.82	0.91
time (sec)	N/A	0.011	0.014	0.021	0.536	0.417	0.197	0.013	0.103

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	9	22	9	9
normalized size	1	1.00	1.00	0.77	0.69	0.69	1.69	0.69	0.69
time (sec)	N/A	0.002	0.002	0.003	0.438	0.390	0.196	0.007	0.025

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	11	52	9	51	58	9	9
normalized size	1	1.00	0.85	4.00	0.69	3.92	4.46	0.69	0.69
time (sec)	N/A	0.001	0.002	0.002	0.464	0.350	0.063	0.008	0.097

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	16	20	19	19	15	14	14
normalized size	1	1.00	0.89	1.11	1.06	1.06	0.83	0.78	0.78
time (sec)	N/A	0.002	0.004	0.006	0.576	0.380	0.102	0.009	0.042

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	43	32	9	31	31	9	31
normalized size	1	1.00	3.91	2.91	0.82	2.82	2.82	0.82	2.82
time (sec)	N/A	0.002	0.002	0.001	0.524	0.351	0.058	0.008	0.025

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	140	107	106	106	131	106	106
normalized size	1	1.00	2.50	1.91	1.89	1.89	2.34	1.89	1.89
time (sec)	N/A	0.025	0.002	0.003	0.610	0.345	0.080	0.008	0.464

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	4	4	3	4	4
normalized size	1	1.00	1.00	1.25	1.00	1.00	0.75	1.00	1.00
time (sec)	N/A	0.008	0.009	0.005	0.608	0.407	0.646	0.008	0.096

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	46	49	37	37	60	37	41
normalized size	1	1.00	0.74	0.79	0.60	0.60	0.97	0.60	0.66
time (sec)	N/A	0.043	0.035	0.007	0.528	0.422	2.031	0.011	0.226

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	8	8	8
normalized size	1	1.00	1.00	0.90	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.007	0.003	0.003	0.531	0.418	0.325	0.006	0.050

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	12	15	12	12
normalized size	1	1.00	1.00	0.81	0.75	0.75	0.94	0.75	0.75
time (sec)	N/A	0.031	0.014	0.071	0.478	0.440	2.059	0.008	0.193

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	7	9	6
normalized size	1	1.00	1.00	0.90	0.80	0.80	0.70	0.90	0.60
time (sec)	N/A	0.001	0.001	0.000	0.543	0.390	0.059	0.005	0.074

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	12	15	15	15	12
normalized size	1	1.00	1.00	1.07	0.80	1.00	1.00	1.00	0.80
time (sec)	N/A	0.004	0.001	0.009	0.518	0.413	0.094	0.006	0.079

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	9
normalized size	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.53
time (sec)	N/A	0.003	0.001	0.003	0.502	0.389	0.090	0.007	0.032

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	17	22	22	22	17
normalized size	1	1.00	1.00	0.82	0.61	0.79	0.79	0.79	0.61
time (sec)	N/A	0.009	0.001	0.003	0.538	0.404	0.108	0.008	0.030

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	4	4	3	5	4
normalized size	1	1.00	1.00	1.25	1.00	1.00	0.75	1.25	1.00
time (sec)	N/A	0.000	0.001	0.001	0.529	0.389	0.058	0.005	0.015

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	11	3	4	3
normalized size	1	1.00	1.00	1.33	1.00	3.67	1.00	1.33	1.00
time (sec)	N/A	0.003	0.002	0.002	0.483	0.417	0.068	0.010	0.023

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	21	36	28	32	94	0	38
normalized size	1	1.00	0.75	1.29	1.00	1.14	3.36	0.00	1.36
time (sec)	N/A	0.010	0.007	0.026	0.584	0.418	0.768	0.000	0.205

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	17	22	26	22	17
normalized size	1	1.00	1.00	0.82	0.61	0.79	0.93	0.79	0.61
time (sec)	N/A	0.017	0.001	0.003	0.522	0.397	0.107	0.008	0.029

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	3	3	4	3
normalized size	1	1.00	1.00	1.33	1.00	1.00	1.00	1.33	1.00
time (sec)	N/A	0.012	0.005	0.003	0.458	0.405	0.097	0.005	0.070

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	10	10
normalized size	1	1.00	1.00	0.92	0.83	0.83	0.67	0.83	0.83
time (sec)	N/A	0.015	0.002	0.001	0.556	0.412	0.096	0.008	0.346

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	16	18	17	12	20	17	13
normalized size	1	1.00	0.70	0.78	0.74	0.52	0.87	0.74	0.57
time (sec)	N/A	0.038	0.018	0.007	0.596	0.403	5.068	0.006	0.168

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	32	23	31	37	31	23
normalized size	1	1.00	1.00	0.82	0.59	0.79	0.95	0.79	0.59
time (sec)	N/A	0.030	0.002	0.004	0.510	0.391	0.126	0.009	0.036

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	7	6	6	5	6	6
normalized size	1	1.00	1.00	0.78	0.67	0.67	0.56	0.67	0.67
time (sec)	N/A	0.012	0.002	0.005	0.488	0.407	0.089	0.005	0.076

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	12	12	11	10	11	11
normalized size	1	1.00	1.00	0.86	0.86	0.79	0.71	0.79	0.79
time (sec)	N/A	0.009	0.004	0.004	0.581	0.409	0.185	0.006	0.105

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	8	7	7	7	7	7
normalized size	1	1.00	1.00	0.80	0.70	0.70	0.70	0.70	0.70
time (sec)	N/A	0.009	0.008	0.014	0.635	0.413	0.357	0.006	0.101

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	14	14	11	13	15	11	11
normalized size	1	1.00	0.74	0.74	0.58	0.68	0.79	0.58	0.58
time (sec)	N/A	0.007	0.015	0.013	0.528	0.413	0.310	0.006	0.018

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	12	14	9	13	15	9	9
normalized size	1	1.00	0.63	0.74	0.47	0.68	0.79	0.47	0.47
time (sec)	N/A	0.007	0.010	0.015	0.505	0.401	0.323	0.006	0.019

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	12	9	9	7	9	9
normalized size	1	1.00	1.00	1.20	0.90	0.90	0.70	0.90	0.90
time (sec)	N/A	0.006	0.003	0.008	0.567	0.408	0.084	0.006	0.045

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	7	7	6	6	5	6	6
normalized size	1	1.00	0.64	0.64	0.55	0.55	0.45	0.55	0.55
time (sec)	N/A	0.005	0.001	0.002	0.570	0.399	0.085	0.005	0.018

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	11	10	9	9	7	9	9
normalized size	1	1.00	0.69	0.62	0.56	0.56	0.44	0.56	0.56
time (sec)	N/A	0.007	0.005	0.002	0.515	0.387	0.085	0.019	0.021

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	12	12	11	11	10	11	11
normalized size	1	1.00	0.63	0.63	0.58	0.58	0.53	0.58	0.58
time (sec)	N/A	0.015	0.005	0.003	0.532	0.401	0.088	0.006	0.022

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	19	19	16	16	17	16	16
normalized size	1	1.00	0.59	0.59	0.50	0.50	0.53	0.50	0.50
time (sec)	N/A	0.018	0.008	0.003	0.655	0.383	0.090	0.007	0.075

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	16	17	11	11	20	11	11
normalized size	1	1.00	0.67	0.71	0.46	0.46	0.83	0.46	0.46
time (sec)	N/A	0.008	0.007	0.009	0.543	0.403	0.201	0.006	0.024

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	16	14	13	13	12	13	13
normalized size	1	1.00	0.62	0.54	0.50	0.50	0.46	0.50	0.50
time (sec)	N/A	0.020	0.003	0.004	0.512	0.384	0.091	0.007	0.109

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	28	40	27	31	136	36	27
normalized size	1	1.00	0.68	0.98	0.66	0.76	3.32	0.88	0.66
time (sec)	N/A	0.011	0.032	0.023	0.478	0.400	1.112	0.008	0.032

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	29	41	29	33	139	38	29
normalized size	1	1.00	0.69	0.98	0.69	0.79	3.31	0.90	0.69
time (sec)	N/A	0.011	0.031	0.007	0.531	0.414	1.100	0.009	0.021

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	12	21	13
normalized size	1	1.00	1.00	0.93	0.87	0.87	0.80	1.40	0.87
time (sec)	N/A	0.003	0.002	0.004	0.540	0.417	0.212	0.012	0.105

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	64	22	35	33	17	37	21
normalized size	1	1.00	3.37	1.16	1.84	1.74	0.89	1.95	1.11
time (sec)	N/A	0.009	0.074	0.007	0.473	0.445	2.182	0.030	0.609

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	64	20	35	35	17	37	20
normalized size	1	1.00	3.76	1.18	2.06	2.06	1.00	2.18	1.18
time (sec)	N/A	0.009	0.048	0.006	0.600	0.444	2.243	0.029	0.209

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	23	23	22	23	22
normalized size	1	1.00	1.00	0.96	0.92	0.92	0.88	0.92	0.88
time (sec)	N/A	0.034	0.008	0.074	1.130	0.424	0.204	0.017	0.032

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	33	39	22	38	20
normalized size	1	1.00	1.00	0.95	1.50	1.77	1.00	1.73	0.91
time (sec)	N/A	0.016	0.002	0.005	1.225	0.453	2.021	0.021	0.021

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	6	23	19	9	14
normalized size	1	1.00	1.00	0.94	0.38	1.44	1.19	0.56	0.88
time (sec)	N/A	0.002	0.004	0.004	1.225	0.404	1.146	0.039	0.163

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	14	10	11	21	0	9	11
normalized size	1	1.00	1.40	1.00	1.10	2.10	0.00	0.90	1.10
time (sec)	N/A	0.009	0.006	0.005	1.361	0.393	0.000	0.034	0.090

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	10	20	10	10
normalized size	1	1.00	1.00	1.10	1.00	1.00	2.00	1.00	1.00
time (sec)	N/A	0.002	0.002	0.005	1.374	0.402	0.117	0.007	0.036

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	16	15	67	53	15	16
normalized size	1	1.00	1.00	0.67	0.62	2.79	2.21	0.62	0.67
time (sec)	N/A	0.004	0.005	0.007	1.490	0.393	0.139	0.007	0.101

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	17	16	16	26	16	16
normalized size	1	1.00	1.00	0.89	0.84	0.84	1.37	0.84	0.84
time (sec)	N/A	0.009	0.006	0.002	1.422	0.402	0.117	0.007	0.076

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	15	13	15	15	14
normalized size	1	1.00	1.00	0.76	0.71	0.62	0.71	0.71	0.67
time (sec)	N/A	0.008	0.003	0.003	1.299	0.423	0.251	0.010	0.023

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	30	34	33	24	32	33	24
normalized size	1	1.00	0.75	0.85	0.82	0.60	0.80	0.82	0.60
time (sec)	N/A	0.022	0.013	0.007	1.268	0.429	0.367	0.022	0.029

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	26	30	34	25	29	29	29
normalized size	1	1.00	0.74	0.86	0.97	0.71	0.83	0.83	0.83
time (sec)	N/A	0.056	0.009	0.014	1.261	0.432	0.358	0.021	0.124

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	18	17	16	14	19	16	16
normalized size	1	1.00	0.82	0.77	0.73	0.64	0.86	0.73	0.73
time (sec)	N/A	0.005	0.007	0.003	1.130	0.438	1.520	0.016	0.057

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	6
normalized size	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.75
time (sec)	N/A	0.030	0.007	0.005	0.535	0.419	1.403	0.011	0.661

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	20	18	17	31	15	17	17
normalized size	1	1.00	0.87	0.78	0.74	1.35	0.65	0.74	0.74
time (sec)	N/A	0.003	0.006	0.003	1.344	0.397	0.208	0.015	0.078

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	A	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	37	16	0	15	31	24	-1
normalized size	1	1.00	1.68	0.73	0.00	0.68	1.41	1.09	-0.05
time (sec)	N/A	0.041	0.011	0.006	0.000	0.426	18.839	0.013	0.000

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	16	0	15	31	22	-1
normalized size	1	1.00	1.00	0.80	0.00	0.75	1.55	1.10	-0.05
time (sec)	N/A	0.023	0.007	0.004	0.000	0.436	17.375	0.012	0.000

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	21	12	15	17
normalized size	1	1.00	1.00	0.84	0.79	1.11	0.63	0.79	0.89
time (sec)	N/A	0.004	0.009	0.008	1.124	0.387	0.102	0.009	0.028

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	4	3	3	15	3	3
normalized size	1	1.00	1.00	1.00	0.75	0.75	3.75	0.75	0.75
time (sec)	N/A	0.016	0.003	0.011	1.114	0.418	0.112	0.006	0.100

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	25	19	28	19	21	22
normalized size	1	1.00	1.00	0.93	0.70	1.04	0.70	0.78	0.81
time (sec)	N/A	0.018	0.020	0.009	0.508	0.428	7.745	0.010	0.136

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	83	64	49	38	0	36	49
normalized size	1	1.00	1.98	1.52	1.17	0.90	0.00	0.86	1.17
time (sec)	N/A	0.013	0.055	0.022	1.181	0.423	0.000	0.040	0.070

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	106	122	0	80	0	61	-1
normalized size	1	1.00	1.49	1.72	0.00	1.13	0.00	0.86	-0.01
time (sec)	N/A	0.022	0.167	0.017	0.000	0.422	0.000	0.030	0.000

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	72	28	0	43	0	22	-1
normalized size	1	1.00	2.25	0.88	0.00	1.34	0.00	0.69	-0.03
time (sec)	N/A	0.012	0.030	0.004	0.000	0.390	0.000	0.117	0.000

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	12	15	13
normalized size	1	1.00	1.00	0.93	0.87	0.87	0.80	1.00	0.87
time (sec)	N/A	0.004	0.004	0.006	0.434	0.381	0.105	0.007	0.051

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	13	13	14	15	13
normalized size	1	1.00	1.00	0.74	0.68	0.68	0.74	0.79	0.68
time (sec)	N/A	0.004	0.004	0.007	0.471	0.391	0.104	0.007	0.038

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	19	19	19	21	19
normalized size	1	1.00	1.00	0.87	0.83	0.83	0.83	0.91	0.83
time (sec)	N/A	0.027	0.005	0.007	0.481	0.396	0.109	0.008	0.041

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	17	17	17	20	19
normalized size	1	1.00	1.00	0.78	0.74	0.74	0.74	0.87	0.83
time (sec)	N/A	0.033	0.006	0.010	0.507	0.407	0.140	0.008	0.189

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	22	19	18	26	19	24	18
normalized size	1	1.00	0.92	0.79	0.75	1.08	0.79	1.00	0.75
time (sec)	N/A	0.014	0.011	0.010	0.454	0.407	0.118	0.009	0.050

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	29	28	28	3	29	57
normalized size	1	1.00	1.00	1.04	1.00	1.00	0.11	1.04	2.04
time (sec)	N/A	0.028	0.012	0.009	1.172	0.404	0.132	0.010	0.186

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	61	37	36	51	14	37	53
normalized size	1	1.00	1.24	0.76	0.73	1.04	0.29	0.76	1.08
time (sec)	N/A	0.067	0.032	0.013	1.296	0.399	0.170	0.011	0.101

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	24	19	39	38	39	37	21
normalized size	1	1.00	1.14	0.90	1.86	1.81	1.86	1.76	1.00
time (sec)	N/A	0.009	0.023	0.046	1.239	0.413	0.514	0.035	0.326

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	266	14	55	44	14	87
normalized size	1	1.00	1.00	16.62	0.88	3.44	2.75	0.88	5.44
time (sec)	N/A	0.045	0.031	0.105	0.621	0.400	5.396	0.009	0.157

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	9	9	9	9	8	11	9
normalized size	1	1.00	0.82	0.82	0.82	0.82	0.73	1.00	0.82
time (sec)	N/A	0.004	0.004	0.001	0.574	0.396	0.091	0.007	0.047

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	19	19	20	22	19
normalized size	1	1.00	1.00	0.87	0.83	0.83	0.87	0.96	0.83
time (sec)	N/A	0.008	0.007	0.009	0.471	0.388	0.132	0.008	0.111

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	28	21	20	27	22	22	18
normalized size	1	1.00	0.93	0.70	0.67	0.90	0.73	0.73	0.60
time (sec)	N/A	0.017	0.010	0.008	0.606	0.394	0.102	0.007	0.041

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	23	23	20	26	30
normalized size	1	1.00	1.00	0.89	0.85	0.85	0.74	0.96	1.11
time (sec)	N/A	0.031	0.006	0.009	0.504	0.382	0.133	0.008	0.095

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	24	24	20	32	17	16	15
normalized size	1	1.00	1.04	1.04	0.87	1.39	0.74	0.70	0.65
time (sec)	N/A	0.026	0.013	0.009	0.601	0.389	0.115	0.007	0.086

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	14	14	15	14
normalized size	1	1.00	1.00	0.94	0.88	0.88	0.88	0.94	0.88
time (sec)	N/A	0.017	0.005	0.006	1.121	0.383	0.097	0.008	0.044

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	13	12	12	14	12	12
normalized size	1	1.00	1.00	0.72	0.67	0.67	0.78	0.67	0.67
time (sec)	N/A	0.012	0.008	0.010	1.136	0.389	0.148	0.010	0.038

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	11	8	13	11
normalized size	1	1.00	1.00	1.09	1.00	1.00	0.73	1.18	1.00
time (sec)	N/A	0.005	0.002	0.006	0.489	0.392	0.101	0.006	0.103

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	24	32	19	29	20
normalized size	1	1.00	1.00	0.88	1.00	1.33	0.79	1.21	0.83
time (sec)	N/A	0.010	0.010	0.013	0.565	0.388	0.106	0.008	0.035

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	44	39	46	83	46	52	45
normalized size	1	1.00	0.96	0.85	1.00	1.80	1.00	1.13	0.98
time (sec)	N/A	0.013	0.021	0.012	0.521	0.404	0.200	0.009	0.088

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	16	8	11	10
normalized size	1	1.00	1.00	1.10	1.00	1.60	0.80	1.10	1.00
time (sec)	N/A	0.004	0.003	0.007	0.473	0.395	0.079	0.006	0.028

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	18	17	13	10	16	13
normalized size	1	1.00	1.00	1.06	1.00	0.76	0.59	0.94	0.76
time (sec)	N/A	0.007	0.003	0.009	0.518	0.382	0.096	0.007	0.110

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	15	14	14	17	16	14
normalized size	1	1.00	1.00	0.75	0.70	0.70	0.85	0.80	0.70
time (sec)	N/A	0.008	0.003	0.007	0.508	0.376	0.106	0.007	0.045

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	12	13	12	16	8	13	12
normalized size	1	1.00	0.75	0.81	0.75	1.00	0.50	0.81	0.75
time (sec)	N/A	0.005	0.004	0.005	0.491	0.376	0.083	0.007	0.037

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	14	17	10	14	14
normalized size	1	1.00	1.00	1.07	1.00	1.21	0.71	1.00	1.00
time (sec)	N/A	0.010	0.008	0.008	1.267	0.395	0.140	0.009	0.082

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	20	20	17
normalized size	1	1.00	1.00	0.86	0.81	0.81	0.95	0.95	0.81
time (sec)	N/A	0.018	0.005	0.007	0.626	0.401	0.135	0.008	0.063

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	27	28	23	34	20	25	17
normalized size	1	1.00	1.29	1.33	1.10	1.62	0.95	1.19	0.81
time (sec)	N/A	0.003	0.008	0.009	0.528	0.387	0.111	0.007	0.087

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	16	17	17	16
normalized size	1	1.00	1.00	0.77	0.73	0.73	0.77	0.77	0.73
time (sec)	N/A	0.011	0.003	0.005	1.233	0.383	0.095	0.008	0.159

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	14	18	8	14	10
normalized size	1	1.00	1.00	1.10	1.40	1.80	0.80	1.40	1.00
time (sec)	N/A	0.015	0.005	0.008	0.576	0.387	0.101	0.009	0.030

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	22	19	25	19	21	16
normalized size	1	1.00	1.00	0.71	0.61	0.81	0.61	0.68	0.52
time (sec)	N/A	0.009	0.002	0.009	0.488	0.376	0.110	0.006	0.040

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	16	16	15	17	24
normalized size	1	1.00	1.00	0.94	0.89	0.89	0.83	0.94	1.33
time (sec)	N/A	0.025	0.005	0.006	1.435	0.416	0.127	0.009	0.039

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	25	10	17	17	17	19	9
normalized size	1	1.00	1.92	0.77	1.31	1.31	1.31	1.46	0.69
time (sec)	N/A	0.003	0.004	0.002	1.370	0.397	0.127	0.007	0.029

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	64	58	72	95	73	72	33
normalized size	1	1.00	0.75	0.68	0.85	1.12	0.86	0.85	0.39
time (sec)	N/A	0.039	0.020	0.003	1.413	0.394	0.152	0.009	0.115

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	15	16	15	26	14	15	15
normalized size	1	1.00	0.65	0.70	0.65	1.13	0.61	0.65	0.65
time (sec)	N/A	0.007	0.009	0.006	1.222	0.403	0.108	0.010	0.081

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	41	11	11	8	11	11
normalized size	1	1.00	1.00	3.73	1.00	1.00	0.73	1.00	1.00
time (sec)	N/A	0.006	0.007	0.010	0.517	0.373	0.117	0.020	0.062

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	23	20	23	36	39	47	21
normalized size	1	1.00	0.51	0.44	0.51	0.80	0.87	1.04	0.47
time (sec)	N/A	0.024	0.030	0.060	1.272	0.417	0.486	0.010	0.100

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	31	30	0	111	110	53	28
normalized size	1	1.00	0.84	0.81	0.00	3.00	2.97	1.43	0.76
time (sec)	N/A	0.018	0.026	0.025	0.000	0.460	3.193	0.014	0.319

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	20	16	37	50	36	35	15
normalized size	1	1.00	0.36	0.29	0.66	0.89	0.64	0.62	0.27
time (sec)	N/A	0.015	0.015	0.017	1.348	0.408	0.343	0.032	0.239

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	20	16	19	23	32	40	32
normalized size	1	1.00	0.65	0.52	0.61	0.74	1.03	1.29	1.03
time (sec)	N/A	0.011	0.013	0.028	1.302	0.430	0.292	0.009	0.219

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	18	15	14	33	248	48	26
normalized size	1	1.00	0.50	0.42	0.39	0.92	6.89	1.33	0.72
time (sec)	N/A	0.038	0.035	0.037	1.228	0.441	57.584	0.015	0.208

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	15	43	0	26	15
normalized size	1	1.00	1.00	1.07	1.00	2.87	0.00	1.73	1.00
time (sec)	N/A	0.023	0.050	0.069	1.143	0.446	0.000	0.017	0.455

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	14	39	0	13	29
normalized size	1	1.00	1.00	0.82	0.82	2.29	0.00	0.76	1.71
time (sec)	N/A	0.012	0.034	0.154	0.572	0.411	0.000	0.017	0.634

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	22	25	41	11	22	25	34
normalized size	1	1.00	0.73	0.83	1.37	0.37	0.73	0.83	1.13
time (sec)	N/A	0.029	0.043	0.075	1.148	0.421	0.316	0.022	0.314

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	23	22	29	24	22	22
normalized size	1	1.00	1.00	0.79	0.76	1.00	0.83	0.76	0.76
time (sec)	N/A	0.004	0.009	0.003	1.137	0.408	0.213	0.014	0.037

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	8	11	11
normalized size	1	1.00	1.00	0.92	0.85	0.85	0.62	0.85	0.85
time (sec)	N/A	0.002	0.002	0.004	0.482	0.414	0.153	0.007	0.169

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	33	30	41	40	88	47	35
normalized size	1	1.00	0.89	0.81	1.11	1.08	2.38	1.27	0.95
time (sec)	N/A	0.020	0.009	0.005	1.198	0.397	1.381	0.012	0.209

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	31	22	25	25	0	26	21
normalized size	1	1.00	1.41	1.00	1.14	1.14	0.00	1.18	0.95
time (sec)	N/A	0.007	0.016	0.006	0.511	0.399	0.000	0.018	0.085

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	21	20	25	24	25	20
normalized size	1	1.00	1.00	0.78	0.74	0.93	0.89	0.93	0.74
time (sec)	N/A	0.003	0.009	0.005	1.274	0.391	0.212	0.009	0.088

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	21	22	27	0	27	23
normalized size	1	1.00	1.00	0.78	0.81	1.00	0.00	1.00	0.85
time (sec)	N/A	0.011	0.005	0.006	1.197	0.394	0.000	0.028	0.053

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	29	12	15	17	0	18	11
normalized size	1	1.00	2.07	0.86	1.07	1.21	0.00	1.29	0.79
time (sec)	N/A	0.003	0.006	0.003	0.554	0.395	0.000	0.024	0.171

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	68	88	59	92	0	168	73
normalized size	1	1.00	1.00	1.29	0.87	1.35	0.00	2.47	1.07
time (sec)	N/A	0.037	0.033	0.009	1.185	0.433	0.000	0.037	0.085

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	C	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	13	12	0	58	0	13
normalized size	1	1.00	1.00	1.00	0.92	0.00	4.46	0.00	1.00
time (sec)	N/A	0.015	0.002	0.004	0.592	0.421	2.439	0.000	0.031

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	15	2	2	2	2	2
normalized size	1	1.00	1.00	1.00	0.13	0.13	0.13	0.13	0.13
time (sec)	N/A	0.010	0.011	0.023	0.633	0.404	0.200	0.003	0.099

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	8	2	2	2	2	2
normalized size	1	1.00	1.00	4.00	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.010	0.005	0.006	0.660	0.398	0.737	0.004	0.007

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	9	4	4	3	4	4
normalized size	1	1.00	1.00	2.25	1.00	1.00	0.75	1.00	1.00
time (sec)	N/A	0.012	0.008	0.003	0.823	0.385	0.782	0.005	0.014

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	16	5	13	7	13	14
normalized size	1	1.00	1.00	1.45	0.45	1.18	0.64	1.18	1.27
time (sec)	N/A	0.020	0.009	0.005	0.694	0.396	0.976	0.007	0.024

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	9	13	10	18	9
normalized size	1	1.00	1.00	1.07	0.64	0.93	0.71	1.29	0.64
time (sec)	N/A	0.012	0.002	0.003	0.645	0.399	1.054	0.009	0.017

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	17	16	14	0	14	14
normalized size	1	1.00	1.00	1.13	1.07	0.93	0.00	0.93	0.93
time (sec)	N/A	0.015	0.014	0.010	0.872	0.400	0.000	0.009	0.028

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	13	10	0	10	10
normalized size	1	1.00	1.00	1.08	1.00	0.77	0.00	0.77	0.77
time (sec)	N/A	0.065	0.021	0.009	0.790	0.395	0.000	0.007	0.118

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	22	16	23	0	80	17
normalized size	1	1.00	1.00	1.16	0.84	1.21	0.00	4.21	0.89
time (sec)	N/A	0.023	0.023	0.007	0.796	0.405	0.000	0.012	0.126

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	19	0	16	-1
normalized size	1	1.00	1.00	1.06	1.00	1.06	0.00	0.89	-0.06
time (sec)	N/A	0.020	0.010	0.043	0.766	0.406	0.000	0.007	0.000

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	11	10	9	9	7	9	9
normalized size	1	1.00	0.69	0.62	0.56	0.56	0.44	0.56	0.56
time (sec)	N/A	0.009	0.005	0.002	0.476	0.398	0.081	0.006	0.017

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	16	15	14	14	12	14	14
normalized size	1	1.00	0.62	0.58	0.54	0.54	0.46	0.54	0.54
time (sec)	N/A	0.021	0.006	0.002	0.511	0.400	0.087	0.007	0.030

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	21	20	19	19	17	19	19
normalized size	1	1.00	0.58	0.56	0.53	0.53	0.47	0.53	0.53
time (sec)	N/A	0.034	0.006	0.003	0.484	0.393	0.089	0.007	0.023

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	39	111	181	60	360	77	2034
normalized size	1	1.00	0.81	2.31	3.77	1.25	7.50	1.60	42.38
time (sec)	N/A	0.034	0.134	0.130	1.296	0.439	1.000	0.046	10.613

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	9	3	2	2	3	2
normalized size	1	1.00	1.00	4.50	1.50	1.00	1.00	1.50	1.00
time (sec)	N/A	0.002	0.002	0.007	0.632	0.402	0.458	0.006	0.009

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	17	6	14	7	11	10
normalized size	1	1.00	1.00	1.70	0.60	1.40	0.70	1.10	1.00
time (sec)	N/A	0.004	0.001	0.001	0.692	0.385	0.465	0.006	0.034

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	0	22	15	24	0	22
normalized size	1	1.00	1.00	0.00	1.00	0.68	1.09	0.00	1.00
time (sec)	N/A	0.017	0.013	0.040	0.653	0.442	2.743	0.000	0.062

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	10	12	11	9	0	9	9
normalized size	1	1.00	0.83	1.00	0.92	0.75	0.00	0.75	0.75
time (sec)	N/A	0.015	0.016	0.006	0.668	0.380	0.000	0.008	0.015

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	23	0	20	0	20	-1
normalized size	1	1.00	1.00	1.05	0.00	0.91	0.00	0.91	-0.05
time (sec)	N/A	0.056	0.038	0.013	0.000	0.406	0.000	0.010	0.000

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	17	116	0	0	27	0	155
normalized size	1	1.00	0.17	1.13	0.00	0.00	0.26	0.00	1.50
time (sec)	N/A	0.008	0.002	0.133	0.000	0.415	0.625	0.000	0.285

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [83] had the largest ratio of [2.000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.00	9	0.111
2	A	2	1	1.00	11	0.091
3	A	2	1	1.00	11	0.091
4	A	2	1	1.00	11	0.091
5	A	1	1	1.00	14	0.071
6	A	2	1	1.00	4	0.250
7	A	2	1	1.00	9	0.111
8	A	2	2	1.00	7	0.286
9	A	2	2	1.00	17	0.118
10	A	2	2	1.00	9	0.222
11	A	3	3	1.00	11	0.273
12	A	3	3	1.00	16	0.188
13	A	2	2	1.00	10	0.200

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
14	A	1	1	1.00	15	0.067
15	A	2	1	1.00	9	0.111
16	A	1	1	1.00	9	0.111
17	A	1	1	1.00	15	0.067
18	A	1	1	1.00	17	0.059
19	A	3	2	1.00	20	0.100
20	A	1	1	1.00	26	0.038
21	A	2	2	1.00	20	0.100
22	A	2	2	1.00	4	0.500
23	A	3	2	1.00	6	0.333
24	A	4	2	1.00	6	0.333
25	A	4	2	1.00	6	0.333
26	A	2	2	1.00	5	0.400
27	A	3	3	1.00	6	0.500
28	A	2	2	1.00	4	0.500
29	A	2	1	1.00	4	0.250
30	A	3	2	1.00	4	0.500
31	A	2	1	1.00	4	0.250
32	A	4	2	1.00	4	0.500
33	A	2	2	1.00	6	0.333
34	A	3	3	1.00	6	0.500
35	A	4	4	1.00	8	0.500
36	A	2	2	1.00	4	0.500
37	A	2	1	1.00	4	0.250
38	A	3	2	1.00	4	0.500
39	A	5	3	1.00	13	0.231
40	A	3	2	1.00	13	0.154
41	A	2	2	1.00	13	0.154
42	A	2	2	1.00	4	0.500
43	A	3	2	1.00	4	0.500
44	A	2	2	1.00	4	0.500
45	A	3	2	1.00	4	0.500
46	A	2	2	1.00	10	0.200
47	A	1	1	1.00	11	0.091
48	A	1	1	1.00	9	0.111
49	A	1	1	1.00	13	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
50	A	1	1	1.00	11	0.091
51	A	2	1	1.00	11	0.091
52	A	2	2	1.00	8	0.250
53	A	5	3	1.00	8	0.375
54	A	1	1	1.00	10	0.100
55	A	3	2	1.00	17	0.118
56	A	1	1	1.00	7	0.143
57	A	2	2	1.00	4	0.500
58	A	1	1	1.00	4	0.250
59	A	2	2	1.00	6	0.333
60	A	1	1	1.00	5	0.200
61	A	1	1	1.00	2	0.500
62	A	1	1	1.00	8	0.125
63	A	2	2	1.00	8	0.250
64	A	2	2	1.00	8	0.250
65	A	2	2	1.00	14	0.143
66	A	3	2	1.00	14	0.143
67	A	3	2	1.00	8	0.250
68	A	1	1	1.00	9	0.111
69	A	1	1	1.00	13	0.077
70	A	2	2	1.00	9	0.222
71	A	1	1	1.00	6	0.167
72	A	1	1	1.00	6	0.167
73	A	4	4	1.00	7	0.571
74	A	2	2	1.00	5	0.400
75	A	2	2	1.00	7	0.286
76	A	3	2	1.00	7	0.286
77	A	3	2	1.00	9	0.222
78	A	3	3	1.00	7	0.429
79	A	2	2	1.00	11	0.182
80	A	1	1	1.00	10	0.100
81	A	1	1	1.00	10	0.100
82	A	2	2	1.00	2	1.000
83	A	4	4	1.00	2	2.000
84	A	4	4	1.00	2	2.000
85	A	3	3	1.00	4	0.750

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	4	4	1.00	6	0.667
87	A	2	2	1.00	13	0.154
88	A	2	2	1.00	14	0.143
89	A	1	1	1.00	9	0.111
90	A	1	1	1.00	9	0.111
91	A	2	2	1.00	10	0.200
92	A	3	3	1.00	4	0.750
93	A	4	3	1.00	6	0.500
94	A	5	5	1.00	6	0.833
95	A	4	4	1.00	6	0.667
96	A	1	3	1.00	17	0.176
97	A	2	2	1.00	11	0.182
98	A	1	1	1.00	15	0.067
99	A	1	1	1.00	14	0.071
100	A	2	2	1.00	11	0.182
101	A	2	2	1.00	13	0.154
102	A	5	6	1.00	10	0.600
103	A	3	3	1.00	15	0.200
104	A	4	4	1.00	15	0.267
105	A	3	3	1.00	15	0.200
106	A	3	2	1.00	16	0.125
107	A	3	2	1.00	16	0.125
108	A	6	4	1.00	18	0.222
109	A	3	2	1.00	23	0.087
110	A	2	1	1.00	19	0.053
111	A	5	5	1.00	18	0.278
112	A	6	5	1.00	31	0.161
113	A	2	2	1.00	7	0.286
114	A	3	2	1.00	22	0.091
115	A	2	1	1.00	16	0.062
116	A	2	1	1.00	17	0.059
117	A	2	1	1.00	12	0.083
118	A	3	2	1.00	21	0.095
119	A	2	1	1.00	20	0.050
120	A	3	3	1.00	16	0.188
121	A	4	3	1.00	16	0.188

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
122	A	2	1	1.00	11	0.091
123	A	3	2	1.00	11	0.182
124	A	2	1	1.00	16	0.062
125	A	2	1	1.00	7	0.143
126	A	5	5	1.00	9	0.556
127	A	4	3	1.00	12	0.250
128	A	3	2	1.00	14	0.143
129	A	4	4	1.00	21	0.190
130	A	3	2	1.00	18	0.111
131	A	2	2	1.00	7	0.286
132	A	3	3	1.00	11	0.273
133	A	3	2	1.00	16	0.125
134	A	3	2	1.00	11	0.182
135	A	5	4	1.00	18	0.222
136	A	3	3	1.00	7	0.429
137	A	9	6	1.00	7	0.857
138	A	3	3	1.00	14	0.214
139	A	1	1	1.00	16	0.062
140	A	3	3	1.00	12	0.250
141	A	2	2	1.00	8	0.250
142	A	2	2	1.00	8	0.250
143	A	1	1	1.00	10	0.100
144	A	3	3	1.00	13	0.231
145	A	2	1	1.00	19	0.053
146	A	1	1	1.00	11	0.091
147	A	3	3	1.00	11	0.273
148	A	2	2	1.00	11	0.182
149	A	1	1	1.00	13	0.077
150	A	4	4	1.00	15	0.267
151	A	3	3	1.00	13	0.231
152	A	2	2	1.00	9	0.222
153	A	3	3	1.00	12	0.250
154	A	2	2	1.00	9	0.222
155	A	6	6	1.00	18	0.333
156	A	2	2	1.00	8	0.250
157	A	3	3	1.00	5	0.600

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
158	A	1	1	1.00	7	0.143
159	A	1	1	1.00	9	0.111
160	A	2	2	1.00	7	0.286
161	A	2	2	1.00	5	0.400
162	A	1	1	1.00	14	0.071
163	A	2	2	1.00	14	0.143
164	A	2	2	1.00	9	0.222
165	A	2	3	1.00	8	0.375
166	A	2	2	1.00	7	0.286
167	A	3	2	1.00	9	0.222
168	A	4	2	1.00	9	0.222
169	A	1	1	1.00	21	0.048
170	A	1	1	1.00	4	0.250
171	A	2	2	1.00	4	0.500
172	A	2	2	1.00	8	0.250
173	A	1	1	1.00	11	0.091
174	A	4	2	1.00	16	0.125
175	A	1	1	1.00	9	0.111

Chapter 3

Listing of integrals

3.1 $\int \sqrt{1+2x}\,dx$

Optimal. Leaf size=13

$$\frac{1}{3}(2x+1)^{3/2}$$

```
[Out] 1/3*(1+2*x)^(3/2)
```

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$\frac{1}{3}(2x+1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + 2*x],x]
```

```
[Out] (1 + 2*x)^(3/2)/3
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \sqrt{1+2x}\,dx = \frac{1}{3}(1+2x)^{3/2}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{1}{3}(2x+1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + 2*x],x]
```

```
[Out] (1 + 2*x)^(3/2)/3
```

fricas [A] time = 0.39, size = 9, normalized size = 0.69

$$\frac{1}{3}\,(2\,x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*x + 1)^(3/2)
```

giac [A] time = 0.01, size = 9, normalized size = 0.69

$$\frac{1}{3}\,(2\,x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*(2*x + 1)^(3/2)
```

maple [A] time = 0.03, size = 10, normalized size = 0.77

$$\frac{(2x+1)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)^(1/2),x)
```

```
[Out] 1/3*(1+2*x)^(3/2)
```

maxima [A] time = 0.56, size = 9, normalized size = 0.69

$$\frac{1}{3}\,(2\,x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(2*x + 1)^(3/2)
```

mupad [B] time = 0.32, size = 9, normalized size = 0.69

$$\frac{(2\,x+1)^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 1)^(1/2),x)
```

```
[Out] (2*x + 1)^(3/2)/3
```

sympy [A] time = 0.10, size = 8, normalized size = 0.62

$$\frac{(2x+1)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**(1/2),x)
```

```
[Out] (2*x + 1)**(3/2)/3
```

3.2 $\int x\sqrt{1+3x}\,dx$

Optimal. Leaf size=27

$$\frac{2}{45}(3x+1)^{5/2}-\frac{2}{27}(3x+1)^{3/2}$$

```
[Out] -2/27*(1+3*x)^(3/2)+2/45*(1+3*x)^(5/2)
```

Rubi [A] time = 0.00, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{2}{45}(3x+1)^{5/2}-\frac{2}{27}(3x+1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[1 + 3*x],x]
[Out] (-2*(1 + 3*x)^(3/2))/27 + (2*(1 + 3*x)^(5/2))/45
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x\sqrt{1+3x}\,dx &= \int\left(-\frac{1}{3}\sqrt{1+3x}+\frac{1}{3}(1+3x)^{3/2}\right)dx\\ &= -\frac{2}{27}(1+3x)^{3/2}+\frac{2}{45}(1+3x)^{5/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 0.67

$$\frac{2}{135}(3x+1)^{3/2}(9x-2)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[1 + 3*x],x]
[Out] (2*(1 + 3*x)^(3/2)*(-2 + 9*x))/135
```

fricas [A] time = 0.40, size = 19, normalized size = 0.70

$$\frac{2}{135}\left(27x^2+3x-2\right)\sqrt{3x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+3*x)^(1/2),x, algorithm="fricas")
[Out] 2/135*(27*x^2 + 3*x - 2)*sqrt(3*x + 1)
```

giac [A] time = 0.01, size = 19, normalized size = 0.70

$$\frac{2}{45}(3x+1)^{\frac{5}{2}}-\frac{2}{27}(3x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+3*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/45*(3*x + 1)^(5/2) - 2/27*(3*x + 1)^(3/2)
```

maple [A] time = 0.00, size = 15, normalized size = 0.56

$$\frac{2\,(3x+1)^{\frac{3}{2}}\,(9x-2)}{135}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(1+3*x)^(1/2),x)
```

```
[Out] 2/135*(1+3*x)^(3/2)*(9*x-2)
```

maxima [A] time = 0.46, size = 19, normalized size = 0.70

$$\frac{2}{45}\,(3\,x+1)^{\frac{5}{2}}-\frac{2}{27}\,(3\,x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+3*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/45*(3*x + 1)^(5/2) - 2/27*(3*x + 1)^(3/2)
```

mupad [B] time = 0.09, size = 14, normalized size = 0.52

$$\frac{2\,(3\,x+1)^{3/2}\,(9\,x-2)}{135}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(3*x + 1)^(1/2),x)
```

```
[Out] (2*(3*x + 1)^(3/2)*(9*x - 2))/135
```

sympy [A] time = 1.04, size = 39, normalized size = 1.44

$$\frac{2x^2\sqrt{3x+1}}{5}+\frac{2x\sqrt{3x+1}}{45}-\frac{4\sqrt{3x+1}}{135}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+3*x)**(1/2),x)
```

```
[Out] 2*x**2*sqrt(3*x + 1)/5 + 2*x*sqrt(3*x + 1)/45 - 4*sqrt(3*x + 1)/135
```

3.3 $\int x^2\sqrt{1+x}\,dx$

Optimal. Leaf size=34

$$\frac{2}{7}(x+1)^{7/2}-\frac{4}{5}(x+1)^{5/2}+\frac{2}{3}(x+1)^{3/2}$$

```
[Out] 2/3*(1+x)^(3/2)-4/5*(1+x)^(5/2)+2/7*(1+x)^(7/2)
```

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{2}{7}(x+1)^{7/2}-\frac{4}{5}(x+1)^{5/2}+\frac{2}{3}(x+1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[1 + x],x]
[Out] (2*(1 + x)^(3/2))/3 - (4*(1 + x)^(5/2))/5 + (2*(1 + x)^(7/2))/7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{1+x}\,dx &= \int\left(\sqrt{1+x}-2(1+x)^{3/2}+(1+x)^{5/2}\right)dx\\ &= \frac{2}{3}(1+x)^{3/2}-\frac{4}{5}(1+x)^{5/2}+\frac{2}{7}(1+x)^{7/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 0.62

$$\frac{2}{105}(x+1)^{3/2}\left(15x^2-12x+8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[1 + x],x]
[Out] (2*(1 + x)^(3/2)*(8 - 12*x + 15*x^2))/105
```

fricas [A] time = 0.39, size = 22, normalized size = 0.65

$$\frac{2}{105}\left(15\,x^3+3\,x^2-4\,x+8\right)\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(1+x)^(1/2),x, algorithm="fricas")
[Out] 2/105*(15*x^3 + 3*x^2 - 4*x + 8)*sqrt(x + 1)
```

giac [A] time = 0.01, size = 22, normalized size = 0.65

$$\frac{2}{7}\,(x+1)^{\frac{7}{2}}-\frac{4}{5}\,(x+1)^{\frac{5}{2}}+\frac{2}{3}\,(x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/7*(x + 1)^(7/2) - 4/5*(x + 1)^(5/2) + 2/3*(x + 1)^(3/2)
```

maple [A] time = 0.00, size = 18, normalized size = 0.53

$$\frac{2\left(x+1\right)^{\frac{3}{2}}\left(15x^{2}-12x+8\right)}{105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(1+x)^(1/2),x)
```

```
[Out] 2/105*(1+x)^(3/2)*(15*x^2-12*x+8)
```

maxima [A] time = 0.55, size = 22, normalized size = 0.65

$$\frac{2}{7}\left(x+1\right)^{\frac{7}{2}}-\frac{4}{5}\left(x+1\right)^{\frac{5}{2}}+\frac{2}{3}\left(x+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/7*(x + 1)^(7/2) - 4/5*(x + 1)^(5/2) + 2/3*(x + 1)^(3/2)
```

mupad [B] time = 0.07, size = 19, normalized size = 0.56

$$-\frac{2\left(x+1\right)^{3/2}\left(42\,x-15\left(x+1\right)^{2}+7\right)}{105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x + 1)^(1/2),x)
```

```
[Out] -(2*(x + 1)^(3/2)*(42*x - 15*(x + 1)^2 + 7))/105
```

sympy [A] time = 1.39, size = 48, normalized size = 1.41

$$\frac{2x^{3}\sqrt{x+1}}{7}+\frac{2x^{2}\sqrt{x+1}}{35}-\frac{8x\sqrt{x+1}}{105}+\frac{16\sqrt{x+1}}{105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(1+x)**(1/2),x)
```

```
[Out] 2*x**3*sqrt(x + 1)/7 + 2*x**2*sqrt(x + 1)/35 - 8*x*sqrt(x + 1)/105 + 16*sqr
t(x + 1)/105
```

3.4 $\int \frac{x}{\sqrt{2-3x}}\,dx$

Optimal. Leaf size=27

$$\frac{2}{27}(2-3x)^{3/2}-\frac{4}{9}\sqrt{2-3x}$$

```
[Out] 2/27*(2-3*x)^(3/2)-4/9*(2-3*x)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{2}{27}(2-3x)^{3/2}-\frac{4}{9}\sqrt{2-3x}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[2 - 3*x],x]
[Out] (-4*Sqrt[2 - 3*x])/9 + (2*(2 - 3*x)^(3/2))/27
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{2-3x}}\,dx &= \int \left(\frac{2}{3\sqrt{2-3x}}-\frac{1}{3}\sqrt{2-3x}\right)dx\\ &= -\frac{4}{9}\sqrt{2-3x}+\frac{2}{27}(2-3x)^{3/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 0.67

$$-\frac{2}{27}\sqrt{2-3x}\,(3x+4)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[2 - 3*x],x]
[Out] (-2*Sqrt[2 - 3*x]*(4 + 3*x))/27
```

fricas [A] time = 0.41, size = 14, normalized size = 0.52

$$-\frac{2}{27}\,(3\,x+4)\sqrt{-3\,x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2-3*x)^(1/2),x, algorithm="fricas")
[Out] -2/27*(3*x + 4)*sqrt(-3*x + 2)
```

giac [A] time = 0.01, size = 19, normalized size = 0.70

$$\frac{2}{27}\,(-3\,x+2)^{\frac{3}{2}}-\frac{4}{9}\,\sqrt{-3\,x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2-3*x)^(1/2),x, algorithm="giac")
[Out] 2/27*(-3*x + 2)^(3/2) - 4/9*sqrt(-3*x + 2)
```

maple [A] time = 0.00, size = 15, normalized size = 0.56

$$-\frac{2(3x+4)\sqrt{-3x+2}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2-3*x)^(1/2),x)
[Out] -2/27*(3*x+4)*(2-3*x)^(1/2)
```

maxima [A] time = 0.50, size = 19, normalized size = 0.70

$$\frac{2}{27}\left(-3\,x+2\right)^{\frac{3}{2}}-\frac{4}{9}\,\sqrt{-3\,x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2-3*x)^(1/2),x, algorithm="maxima")
[Out] 2/27*(-3*x + 2)^(3/2) - 4/9*sqrt(-3*x + 2)
```

mupad [B] time = 0.06, size = 14, normalized size = 0.52

$$-\frac{2\sqrt{2-3\,x}\,\left(3\,x+4\right)}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2 - 3*x)^(1/2),x)
[Out] -(2*(2 - 3*x)^(1/2)*(3*x + 4))/27
```

sympy [A] time = 0.95, size = 61, normalized size = 2.26

$$\begin{cases} -\frac{2ix\sqrt{3x-2}}{9}-\frac{8i\sqrt{3x-2}}{27} & \text{for}\ \frac{3|x|}{2}>1 \\ -\frac{2x\sqrt{2-3x}}{9}-\frac{8\sqrt{2-3x}}{27} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2-3*x)**(1/2),x)
[Out] Piecewise((-2*I*x*sqrt(3*x - 2)/9 - 8*I*sqrt(3*x - 2)/27, 3*Abs(x)/2 > 1),
(-2*x*sqrt(2 - 3*x)/9 - 8*sqrt(2 - 3*x)/27, True))
```

3.5 $\int \frac{1+x}{(2+2x+x^2)^3} dx$

Optimal. Leaf size=14

$$-\frac{1}{4(x^2+2x+2)^2}$$

[Out] -1/4/(x^2+2*x+2)^2

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {629}

$$-\frac{1}{4(x^2+2x+2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/(2 + 2*x + x^2)^3,x]
[Out] -1/(4*(2 + 2*x + x^2)^2)
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1+x}{(2+2x+x^2)^3} dx = -\frac{1}{4(2+2x+x^2)^2}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.00

$$-\frac{1}{4(x^2+2x+2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/(2 + 2*x + x^2)^3,x]
[Out] -1/4*1/(2 + 2*x + x^2)^2
```

fricas [A] time = 0.39, size = 22, normalized size = 1.57

$$-\frac{1}{4(x^4+4x^3+8x^2+8x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2+2*x+2)^3,x, algorithm="fricas")
[Out] -1/4/(x^4 + 4*x^3 + 8*x^2 + 8*x + 4)
```

giac [A] time = 0.01, size = 12, normalized size = 0.86

$$-\frac{1}{4\left(x^2+2x+2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2+2*x+2)^3,x, algorithm="giac")
[Out] -1/4/(x^2 + 2*x + 2)^2
```

maple [A] time = 0.01, size = 13, normalized size = 0.93

$$-\frac{1}{4\left(x^2+2x+2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)/(x^2+2*x+2)^3,x)
[Out] -1/4/(x^2+2*x+2)^2
```

maxima [A] time = 0.44, size = 12, normalized size = 0.86

$$-\frac{1}{4\left(x^2+2x+2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2+2*x+2)^3,x, algorithm="maxima")
[Out] -1/4/(x^2 + 2*x + 2)^2
```

mupad [B] time = 0.06, size = 12, normalized size = 0.86

$$-\frac{1}{4\left(x^2+2x+2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/(2*x + x^2 + 2)^3,x)
[Out] -1/(4*(2*x + x^2 + 2)^2)
```

sympy [A] time = 0.12, size = 22, normalized size = 1.57

$$-\frac{1}{4x^4+16x^3+32x^2+32x+16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x**2+2*x+2)**3,x)
[Out] -1/(4*x**4 + 16*x**3 + 32*x**2 + 32*x + 16)
```

3.6 $\int \sin^3(x)\,dx$

Optimal. Leaf size=13

$$\frac{\cos^3(x)}{3} - \cos(x)$$

```
[Out] -cos(x)+1/3*cos(x)^3
```

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2633}

$$\frac{\cos^3(x)}{3} - \cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^3,x]
[Out] -Cos[x] + Cos[x]^3/3
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sin^3(x)\,dx &= -\text{Subst}\left(\int \left(1-x^2\right)\,dx, x, \cos(x)\right)\\ &= -\cos(x) + \frac{\cos^3(x)}{3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.15

$$\frac{1}{12}\cos(3x) - \frac{3\cos(x)}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^3,x]
[Out] (-3*Cos[x])/4 + Cos[3*x]/12
```

fricas [A] time = 0.43, size = 11, normalized size = 0.85

$$\frac{1}{3}\cos(x)^3 - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3,x, algorithm="fricas")
[Out] 1/3*cos(x)^3 - cos(x)
```

giac [A] time = 0.01, size = 11, normalized size = 0.85

$$\frac{1}{3}\cos(x)^3 - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3,x, algorithm="giac")
[Out] 1/3*cos(x)^3 - cos(x)
```

maple [A] time = 0.14, size = 11, normalized size = 0.85

$$-\frac{\left(\sin^2(x)+2\right)\cos(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3,x)
[Out] -1/3*(2+sin(x)^2)*cos(x)
```

maxima [A] time = 0.50, size = 11, normalized size = 0.85

$$\frac{1}{3}\cos(x)^3-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3,x, algorithm="maxima")
[Out] 1/3*cos(x)^3 - cos(x)
```

mupad [B] time = 0.16, size = 10, normalized size = 0.77

$$\frac{\cos(x)\left(\cos(x)^2-3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3,x)
[Out] (cos(x)*(cos(x)^2 - 3))/3
```

sympy [A] time = 0.07, size = 8, normalized size = 0.62

$$\frac{\cos^3(x)}{3}-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**3,x)
[Out] cos(x)**3/3 - cos(x)
```

3.7 $\int \sqrt[3]{-1+z}\, z\, dz$

Optimal. Leaf size=23

$$\frac{3}{7}(z-1)^{7/3}+\frac{3}{4}(z-1)^{4/3}$$

```
[Out] 3/4*(-1+z)^(4/3)+3/7*(-1+z)^(7/3)
```

Rubi [A] time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$\frac{3}{7}(z-1)^{7/3}+\frac{3}{4}(z-1)^{4/3}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + z)^(1/3)*z,z]
```

```
[Out] (3*(-1 + z)^(4/3))/4 + (3*(-1 + z)^(7/3))/7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{-1+z}\, z\, dz &= \int \left(\sqrt[3]{-1+z}+(-1+z)^{4/3}\right) dz\\ &= \frac{3}{4}(-1+z)^{4/3}+\frac{3}{7}(-1+z)^{7/3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 0.70

$$\frac{3}{28}(z-1)^{4/3}(4z+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + z)^(1/3)*z,z]
```

```
[Out] (3*(-1 + z)^(4/3)*(3 + 4*z))/28
```

fricas [A] time = 0.39, size = 17, normalized size = 0.74

$$\frac{3}{28}\left(4z^2-z-3\right)(z-1)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+z)^(1/3)*z,z, algorithm="fricas")
```

```
[Out] 3/28*(4*z^2 - z - 3)*(z - 1)^(1/3)
```

giac [A] time = 0.01, size = 15, normalized size = 0.65

$$\frac{3}{7}(z-1)^{\frac{7}{3}}+\frac{3}{4}(z-1)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+z)^(1/3)*z,z, algorithm="giac")
```

```
[Out] 3/7*(z - 1)^(7/3) + 3/4*(z - 1)^(4/3)
```

maple [A] time = 0.00, size = 13, normalized size = 0.57

$$\frac{3\,(z-1)^{\frac{4}{3}}\,(4z+3)}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+z)^(1/3)*z,z)
```

```
[Out] 3/28*(-1+z)^(4/3)*(4*z+3)
```

maxima [A] time = 0.71, size = 15, normalized size = 0.65

$$\frac{3}{7}\,(z-1)^{\frac{7}{3}}+\frac{3}{4}\,(z-1)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+z)^(1/3)*z,z, algorithm="maxima")
```

```
[Out] 3/7*(z - 1)^(7/3) + 3/4*(z - 1)^(4/3)
```

mupad [B] time = 0.03, size = 12, normalized size = 0.52

$$\frac{3\,(4\,z+3)\,(z-1)^{4/3}}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(z*(z - 1)^(1/3),z)
```

```
[Out] (3*(4*z + 3)*(z - 1)^(4/3))/28
```

sympy [A] time = 0.99, size = 92, normalized size = 4.00

$$\begin{cases} \frac{3z^2\sqrt[3]{z-1}}{7}-\frac{3z\sqrt[3]{z-1}}{28}-\frac{9\sqrt[3]{z-1}}{28} & \text{for } |z|>1 \\ \frac{3z^2\sqrt[3]{1-z}\,e^{\frac{i\pi}{3}}}{7}-\frac{3z\sqrt[3]{1-z}\,e^{\frac{i\pi}{3}}}{28}-\frac{9\sqrt[3]{1-z}\,e^{\frac{i\pi}{3}}}{28} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+z)**(1/3)*z,z)
```

```
[Out] Piecewise((3*z**2*(z - 1)**(1/3)/7 - 3*z*(z - 1)**(1/3)/28 - 9*(z - 1)**(1/
3)/28, Abs(z) > 1), (3*z**2*(1 - z)**(1/3)*exp(I*pi/3)/7 - 3*z*(1 - z)**(1/
3)*exp(I*pi/3)/28 - 9*(1 - z)**(1/3)*exp(I*pi/3)/28, True))
```

3.8 $\int \cot(x)\csc^2(x)\,dx$

Optimal. Leaf size=8

$$-\frac{1}{2}\csc^2(x)$$

```
[Out] -1/2*csc(x)^2
```

Rubi [A] time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2606, 30}

$$-\frac{1}{2}\csc^2(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]*Csc[x]^2,x]
```

```
[Out] -Csc[x]^2/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \cot(x)\csc^2(x)\,dx &= -\text{Subst}\left(\int x\,dx, x, \csc(x)\right)\\ &= -\frac{1}{2}\csc^2(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 8, normalized size = 1.00

$$-\frac{1}{2}\csc^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]*Csc[x]^2,x]
```

```
[Out] -1/2*Csc[x]^2
```

fricas [A] time = 0.41, size = 10, normalized size = 1.25

$$\frac{1}{2\left(\cos(x)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/sin(x)^3,x, algorithm="fricas")
```

```
[Out] 1/2/(cos(x)^2 - 1)
```

giac [A] time = 0.01, size = 6, normalized size = 0.75

$$-\frac{1}{2 \sin(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/sin(x)^3,x, algorithm="giac")
[Out] -1/2/sin(x)^2
```

maple [A] time = 0.02, size = 7, normalized size = 0.88

$$-\frac{1}{2 \sin(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/sin(x)^3,x)
[Out] -1/2/sin(x)^2
```

maxima [A] time = 0.49, size = 6, normalized size = 0.75

$$-\frac{1}{2 \sin(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/sin(x)^3,x, algorithm="maxima")
[Out] -1/2/sin(x)^2
```

mupad [B] time = 0.42, size = 6, normalized size = 0.75

$$-\frac{\cot(x)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/sin(x)^3,x)
[Out] -cot(x)^2/2
```

sympy [A] time = 0.07, size = 8, normalized size = 1.00

$$-\frac{1}{2 \sin^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/sin(x)**3,x)
[Out] -1/(2*sin(x)**2)
```

3.9 $\int \cos(2x)\sqrt{4-\sin(2x)}\,dx$

Optimal. Leaf size=16

$$-\frac{1}{3}(4-\sin(2x))^{3/2}$$

```
[Out] -1/3*(4-sin(2*x))^(3/2)
```

Rubi [A] time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2668, 32}

$$-\frac{1}{3}(4-\sin(2x))^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*x]*Sqrt[4 - Sin[2*x]],x]
```

```
[Out] -(4 - Sin[2*x])^(3/2)/3
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(2x)\sqrt{4-\sin(2x)}\,dx &= -\left(\frac{1}{2}\text{Subst}\left(\int \sqrt{4+x}\,dx, x, -\sin(2x)\right)\right)\\ &= -\frac{1}{3}(4-\sin(2x))^{3/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$-\frac{1}{3}(4-\sin(2x))^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*x]*Sqrt[4 - Sin[2*x]],x]
```

```
[Out] -1/3*(4 - Sin[2*x])^(3/2)
```

fricas [A] time = 0.42, size = 18, normalized size = 1.12

$$\frac{1}{3}\left(\sin\left(2\,x\right)-4\right)\sqrt{-\sin\left(2\,x\right)+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*(4-sin(2*x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(sin(2*x) - 4)*sqrt(-sin(2*x) + 4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*(4-sin(2*x))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: integ
rate(cos(x*2)*exp(ln(-sin(2*x)+4)/2),x)
```

maple [A] time = 0.03, size = 13, normalized size = 0.81

$$-\frac{\left(-\sin\left(2x\right)+4\right)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)*(4-sin(2*x))^(1/2),x)
```

```
[Out] -1/3*(4-sin(2*x))^(3/2)
```

maxima [A] time = 0.57, size = 12, normalized size = 0.75

$$-\frac{1}{3}\left(-\sin\left(2\,x\right)+4\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*(4-sin(2*x))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*(-sin(2*x) + 4)^(3/2)
```

mupad [B] time = 0.17, size = 12, normalized size = 0.75

$$-\frac{\left(4-\sin\left(2\,x\right)\right)^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)*(4 - sin(2*x))^(1/2),x)
```

```
[Out] -(4 - sin(2*x))^(3/2)/3
```

sympy [B] time = 0.34, size = 29, normalized size = 1.81

$$\frac{\sqrt{4-\sin\left(2x\right)}\,\sin\left(2x\right)}{3}-\frac{4\sqrt{4-\sin\left(2x\right)}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*(4-sin(2*x))**(1/2),x)
```

```
[Out] sqrt(4 - sin(2*x))*sin(2*x)/3 - 4*sqrt(4 - sin(2*x))/3
```

3.10 $\int \frac{\sin(x)}{(3+\cos(x))^2}\,dx$

Optimal. Leaf size=6

$$\frac{1}{\cos(x)+3}$$

```
[Out] 1/(3+cos(x))
```

Rubi [A] time = 0.02, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2668, 32}

$$\frac{1}{\cos(x)+3}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(3 + Cos[x])^2,x]
```

```
[Out] (3 + Cos[x])^(-1)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(x)}{(3+\cos(x))^2}\,dx &= -\text{Subst}\left(\int \frac{1}{(3+x)^2}\,dx, x, \cos(x)\right)\\ &= \frac{1}{3+\cos(x)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 6, normalized size = 1.00

$$\frac{1}{\cos(x)+3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(3 + Cos[x])^2,x]
```

```
[Out] (3 + Cos[x])^(-1)
```

fricas [A] time = 0.41, size = 6, normalized size = 1.00

$$\frac{1}{\cos(x)+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(3+cos(x))^2,x, algorithm="fricas")
```

```
[Out] 1/(cos(x) + 3)
```

giac [A] time = 0.01, size = 6, normalized size = 1.00

$$\frac{1}{\cos(x) + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(3+cos(x))^2,x, algorithm="giac")
[Out] 1/(cos(x) + 3)
```

maple [A] time = 0.07, size = 7, normalized size = 1.17

$$\frac{1}{\cos(x) + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(3+cos(x))^2,x)
[Out] 1/(3+cos(x))
```

maxima [A] time = 0.70, size = 6, normalized size = 1.00

$$\frac{1}{\cos(x) + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(3+cos(x))^2,x, algorithm="maxima")
[Out] 1/(cos(x) + 3)
```

mupad [B] time = 0.04, size = 6, normalized size = 1.00

$$\frac{1}{\cos(x) + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(cos(x) + 3)^2,x)
[Out] 1/(cos(x) + 3)
```

sympy [A] time = 0.41, size = 5, normalized size = 0.83

$$\frac{1}{\cos{\left(x \right)} + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(3+cos(x))**2,x)
[Out] 1/(cos(x) + 3)
```

3.11 $\int \frac{\sin(x)}{\sqrt{\cos^3(x)}}\,dx$

Optimal. Leaf size=12

$$\frac{2\cos(x)}{\sqrt{\cos^3(x)}}$$

```
[Out] 2*cos(x)/(cos(x)^3)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3207, 2565, 30}

$$\frac{2\cos(x)}{\sqrt{\cos^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/Sqrt[Cos[x]^3],x]
```

```
[Out] (2*Cos[x])/Sqrt[Cos[x]^3]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin(x)}{\sqrt{\cos^3(x)}}\,dx &= \frac{\cos^{\frac{3}{2}}(x)\int \frac{\sin(x)}{\cos^{\frac{3}{2}}(x)}\,dx}{\sqrt{\cos^3(x)}}\\
&= -\frac{\cos^{\frac{3}{2}}(x)\,\text{Subst}\left(\int \frac{1}{x^{3/2}}\,dx, x, \cos(x)\right)}{\sqrt{\cos^3(x)}}\\
&= \frac{2\cos(x)}{\sqrt{\cos^3(x)}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 12, normalized size = 1.00

$$\frac{2\cos(x)}{\sqrt{\cos^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/Sqrt[Cos[x]^3],x]
[Out] (2*Cos[x])/Sqrt[Cos[x]^3]
```

fricas [A] time = 0.43, size = 12, normalized size = 1.00

$$\frac{2\sqrt{\cos(x)^3}}{\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(cos(x)^3)^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(cos(x)^3)/cos(x)^2
```

giac [A] time = 0.01, size = 6, normalized size = 0.50

$$\frac{2}{\sqrt{\cos(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(cos(x)^3)^(1/2),x, algorithm="giac")
[Out] 2/sqrt(cos(x))
```

maple [A] time = 0.07, size = 11, normalized size = 0.92

$$\frac{4\cos(x)}{\sqrt{3\cos(x)+\cos(3x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(cos(x)^3)^(1/2),x)
[Out] 2*cos(x)/(cos(x)^3)^(1/2)
```

maxima [A] time = 0.64, size = 10, normalized size = 0.83

$$\frac{2\cos(x)}{\sqrt{\cos(x)^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(cos(x)^3)^(1/2),x, algorithm="maxima")
[Out] 2*cos(x)/sqrt(cos(x)^3)
```

mupad [B] time = 0.20, size = 9, normalized size = 0.75

$$\frac{2\,|\cos(x)|}{\cos(x)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(cos(x)^3)^(1/2),x)
[Out] (2*abs(cos(x)))/cos(x)^(3/2)
```

sympy [A] time = 0.65, size = 12, normalized size = 1.00

$$\frac{2\cos(x)}{\sqrt{\cos^3(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(cos(x)**3)**(1/2),x)
```

```
[Out] 2*cos(x)/sqrt(cos(x)**3)
```

3.12 $\int \frac{\sin\left(\sqrt{1+x}\right)}{\sqrt{1+x}}\,dx$

Optimal. Leaf size=10

$$-2\cos\left(\sqrt{x+1}\right)$$

```
[Out] -2*cos((1+x)^(1/2))
```

Rubi [A] time = 0.02, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3431, 15, 2638}

$$-2\cos\left(\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[Sqrt[1 + x]]/Sqrt[1 + x],x]
```

```
[Out] -2*Cos[Sqrt[1 + x]]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3431

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - (e*h)/f + (h*x^(1/n))/f)^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned} \int \frac{\sin\left(\sqrt{1+x}\right)}{\sqrt{1+x}}\,dx &= 2\,\text{Subst}\left(\int \frac{x\sin(x)}{\sqrt{x^2}}\,dx, x, \sqrt{1+x}\right) \\ &= 2\,\text{Subst}\left(\int \sin(x)\,dx, x, \sqrt{1+x}\right) \\ &= -2\cos\left(\sqrt{1+x}\right) \end{aligned}$$

Mathematica [A] time = 0.02, size = 10, normalized size = 1.00

$$-2\cos\left(\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[Sqrt[1 + x]]/Sqrt[1 + x],x]
```

```
[Out] -2*Cos[Sqrt[1 + x]]
```

fricas [A] time = 0.41, size = 8, normalized size = 0.80

$$-2 \cos\left(\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((1+x)^(1/2))/(1+x)^(1/2),x, algorithm="fricas")
[Out] -2*cos(sqrt(x + 1))
```

giac [A] time = 0.01, size = 8, normalized size = 0.80

$$-2 \cos\left(\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((1+x)^(1/2))/(1+x)^(1/2),x, algorithm="giac")
[Out] -2*cos(sqrt(x + 1))
```

maple [A] time = 0.01, size = 9, normalized size = 0.90

$$-2 \cos\left(\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin((x+1)^(1/2))/(x+1)^(1/2),x)
[Out] -2*cos((x+1)^(1/2))
```

maxima [A] time = 0.52, size = 8, normalized size = 0.80

$$-2 \cos\left(\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((1+x)^(1/2))/(1+x)^(1/2),x, algorithm="maxima")
[Out] -2*cos(sqrt(x + 1))
```

mupad [B] time = 0.17, size = 8, normalized size = 0.80

$$-2 \cos\left(\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin((x + 1)^(1/2))/(x + 1)^(1/2),x)
[Out] -2*cos((x + 1)^(1/2))
```

sympy [A] time = 0.33, size = 10, normalized size = 1.00

$$-2 \cos\left(\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((1+x)**(1/2))/(1+x)**(1/2),x)
[Out] -2*cos(sqrt(x + 1))
```

3.13 $\int x^{-1+n}\sin(x^n)\,dx$

Optimal. Leaf size=9

$$-\frac{\cos(x^n)}{n}$$

```
[Out] -cos(x^n)/n
```

Rubi [A] time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3379, 2638}

$$-\frac{\cos(x^n)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)*Sin[x^n],x]
```

```
[Out] -(Cos[x^n]/n)
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\int x^{-1+n}\sin(x^n)\,dx &= \frac{\text{Subst}\left(\int \sin(x)\,dx, x, x^n\right)}{n}\\ &= -\frac{\cos(x^n)}{n}\end{aligned}$$

Mathematica [A] time = 0.02, size = 9, normalized size = 1.00

$$-\frac{\cos(x^n)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)*Sin[x^n],x]
```

```
[Out] -(Cos[x^n]/n)
```

fricas [A] time = 0.42, size = 9, normalized size = 1.00

$$-\frac{\cos(x^n)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*sin(x^n),x, algorithm="fricas")
```

```
[Out] -cos(x^n)/n
```

giac [A] time = 0.01, size = 9, normalized size = 1.00

$$-\frac{\cos\left(x^{n}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*sin(x^n),x, algorithm="giac")
[Out] -cos(x^n)/n
```

maple [A] time = 0.02, size = 10, normalized size = 1.11

$$-\frac{\cos\left(x^{n}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+n)*sin(x^n),x)
[Out] -cos(x^n)/n
```

maxima [A] time = 0.79, size = 9, normalized size = 1.00

$$-\frac{\cos\left(x^{n}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*sin(x^n),x, algorithm="maxima")
[Out] -cos(x^n)/n
```

mupad [B] time = 0.21, size = 9, normalized size = 1.00

$$-\frac{\cos\left(x^{n}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 1)*sin(x^n),x)
[Out] -cos(x^n)/n
```

sympy [A] time = 6.73, size = 7, normalized size = 0.78

$$-\frac{\cos\left(x^{n}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*sin(x**n),x)
[Out] -cos(x**n)/n
```

3.14 $\int \frac{x^5}{\sqrt{1-x^6}}\,dx$

Optimal. Leaf size=15

$$-\frac{1}{3}\sqrt{1-x^6}$$

```
[Out] -1/3*(-x^6+1)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {261}

$$-\frac{1}{3}\sqrt{1-x^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[1 - x^6],x]
```

```
[Out] -Sqrt[1 - x^6]/3
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^5}{\sqrt{1-x^6}}\,dx = -\frac{1}{3}\sqrt{1-x^6}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$-\frac{1}{3}\sqrt{1-x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[1 - x^6],x]
```

```
[Out] -1/3*Sqrt[1 - x^6]
```

fricas [A] time = 0.40, size = 11, normalized size = 0.73

$$-\frac{1}{3}\sqrt{-x^6+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^6+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(-x^6 + 1)
```

giac [A] time = 0.01, size = 11, normalized size = 0.73

$$-\frac{1}{3}\sqrt{-x^6+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^6+1)^(1/2),x, algorithm="giac")
[Out] -1/3*sqrt(-x^6 + 1)
```

maple [B]   time = 0.01, size = 32, normalized size = 2.13

$$\frac{(x-1)(x+1)\left(x^2+x+1\right)\left(x^2-x+1\right)}{3\sqrt{-x^6+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-x^6+1)^(1/2),x)
[Out] 1/3*(-1+x)*(x+1)*(x^2+x+1)*(x^2-x+1)/(-x^6+1)^(1/2)
```

maxima [A]   time = 0.46, size = 11, normalized size = 0.73

$$-\frac{1}{3}\sqrt{-x^6+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^6+1)^(1/2),x, algorithm="maxima")
[Out] -1/3*sqrt(-x^6 + 1)
```

mupad [B]   time = 0.35, size = 11, normalized size = 0.73

$$-\frac{\sqrt{1-x^6}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(1 - x^6)^(1/2),x)
[Out] -(1 - x^6)^(1/2)/3
```

sympy [A]   time = 0.27, size = 10, normalized size = 0.67

$$-\frac{\sqrt{1-x^6}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-x**6+1)**(1/2),x)
[Out] -sqrt(1 - x**6)/3
```

3.15 $\int t\sqrt[4]{1+t}\,dt$

Optimal. Leaf size=23

$$\frac{4}{9}(t+1)^{9/4}-\frac{4}{5}(t+1)^{5/4}$$

```
[Out] -4/5*(1+t)^(5/4)+4/9*(1+t)^(9/4)
```

Rubi [A] time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$\frac{4}{9}(t+1)^{9/4}-\frac{4}{5}(t+1)^{5/4}$$

Antiderivative was successfully verified.

```
[In] Int[t*(1 + t)^(1/4),t]
```

```
[Out] (-4*(1 + t)^(5/4))/5 + (4*(1 + t)^(9/4))/9
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int t\sqrt[4]{1+t}\,dt &= \int\left(-\sqrt[4]{1+t}+(1+t)^{5/4}\right)dt\\ &= -\frac{4}{5}(1+t)^{5/4}+\frac{4}{9}(1+t)^{9/4}\end{aligned}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 0.70

$$\frac{4}{45}(t+1)^{5/4}(5t-4)$$

Antiderivative was successfully verified.

```
[In] Integrate[t*(1 + t)^(1/4),t]
```

```
[Out] (4*(1 + t)^(5/4)*(-4 + 5*t))/45
```

fricas [A] time = 0.39, size = 15, normalized size = 0.65

$$\frac{4}{45}\left(5t^2+t-4\right)(t+1)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t*(1+t)^(1/4),t, algorithm="fricas")
```

```
[Out] 4/45*(5*t^2 + t - 4)*(t + 1)^(1/4)
```

giac [A] time = 0.01, size = 15, normalized size = 0.65

$$\frac{4}{9}(t+1)^{\frac{9}{4}}-\frac{4}{5}(t+1)^{\frac{5}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t*(1+t)^(1/4),t, algorithm="giac")
```

```
[Out] 4/9*(t + 1)^(9/4) - 4/5*(t + 1)^(5/4)
```

maple [A] time = 0.00, size = 13, normalized size = 0.57

$$\frac{4 \left(t+1\right)^{\frac{5}{4}} \left(5 t-4\right)}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(t*(1+t)^(1/4),t)
```

```
[Out] 4/45*(1+t)^(5/4)*(5*t-4)
```

maxima [A] time = 0.50, size = 15, normalized size = 0.65

$$\frac{4}{9} \left(t+1\right)^{\frac{9}{4}} - \frac{4}{5} \left(t+1\right)^{\frac{5}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t*(1+t)^(1/4),t, algorithm="maxima")
```

```
[Out] 4/9*(t + 1)^(9/4) - 4/5*(t + 1)^(5/4)
```

mupad [B] time = 0.03, size = 12, normalized size = 0.52

$$\frac{4\,\left(5\,t-4\right)\,{\left(t+1\right)}^{5/4}}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(t*(t + 1)^(1/4),t)
```

```
[Out] (4*(5*t - 4)*(t + 1)^(5/4))/45
```

sympy [A] time = 1.01, size = 34, normalized size = 1.48

$$\frac{4 t^{2} \sqrt[4]{t+1}}{9} + \frac{4 t \sqrt[4]{t+1}}{45} - \frac{16 \sqrt[4]{t+1}}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t*(1+t)**(1/4),t)
```

```
[Out] 4*t**2*(t + 1)**(1/4)/9 + 4*t*(t + 1)**(1/4)/45 - 16*(t + 1)**(1/4)/45
```

3.16 $\int \frac{1}{(1+x^2)^{3/2}} \, dx$

Optimal. Leaf size=11

$$\frac{x}{\sqrt{x^2+1}}$$

```
[Out] x/(x^2+1)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {191}

$$\frac{x}{\sqrt{x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)^(-3/2),x]
```

```
[Out] x/Sqrt[1 + x^2]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{1}{(1+x^2)^{3/2}} \, dx = \frac{x}{\sqrt{1+x^2}}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$\frac{x}{\sqrt{x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)^(-3/2),x]
```

```
[Out] x/Sqrt[1 + x^2]
```

fricas [B] time = 0.40, size = 22, normalized size = 2.00

$$\frac{x^2 + \sqrt{x^2+1}\,x + 1}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] (x^2 + sqrt(x^2 + 1)*x + 1)/(x^2 + 1)
```

giac [A] time = 0.02, size = 9, normalized size = 0.82

$$\frac{x}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(3/2),x, algorithm="giac")
[Out] x/sqrt(x^2 + 1)
```

maple [A] time = 0.00, size = 10, normalized size = 0.91

$$\frac{x}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)^(3/2),x)
[Out] x/(x^2+1)^(1/2)
```

maxima [A] time = 0.47, size = 9, normalized size = 0.82

$$\frac{x}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(3/2),x, algorithm="maxima")
[Out] x/sqrt(x^2 + 1)
```

mupad [B] time = 0.07, size = 9, normalized size = 0.82

$$\frac{x}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 + 1)^(3/2),x)
[Out] x/(x^2 + 1)^(1/2)
```

sympy [A] time = 0.73, size = 8, normalized size = 0.73

$$\frac{x}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)**(3/2),x)
[Out] x/sqrt(x**2 + 1)
```

3.17 $\int x^2 \left(27+8x^3\right)^{2/3} \, dx$

Optimal. Leaf size=15

$$\frac{1}{40} \left(8x^3+27\right)^{5/3}$$

```
[Out] 1/40*(8*x^3+27)^(5/3)
```

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {261}

$$\frac{1}{40} \left(8x^3+27\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(27 + 8*x^3)^(2/3),x]
```

```
[Out] (27 + 8*x^3)^(5/3)/40
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^2 \left(27+8x^3\right)^{2/3} \, dx = \frac{1}{40} \left(27+8x^3\right)^{5/3}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{1}{40} \left(8x^3+27\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(27 + 8*x^3)^(2/3),x]
```

```
[Out] (27 + 8*x^3)^(5/3)/40
```

fricas [A] time = 0.39, size = 11, normalized size = 0.73

$$\frac{1}{40} \left(8 x^3+27\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(8*x^3+27)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/40*(8*x^3 + 27)^(5/3)
```

giac [A] time = 0.01, size = 11, normalized size = 0.73

$$\frac{1}{40} \left(8 x^3+27\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(8*x^3+27)^(2/3),x, algorithm="giac")
[Out] 1/40*(8*x^3 + 27)^(5/3)
```

maple [**B**] time = 0.01, size = 27, normalized size = 1.80

$$\frac{(2x+3)\left(4x^2-6x+9\right)\left(8x^3+27\right)^{\frac{2}{3}}}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(8*x^3+27)^(2/3),x)
[Out] 1/40*(3+2*x)*(4*x^2-6*x+9)*(8*x^3+27)^(2/3)
```

maxima [**A**] time = 0.53, size = 11, normalized size = 0.73

$$\frac{1}{40}\left(8\,x^3+27\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(8*x^3+27)^(2/3),x, algorithm="maxima")
[Out] 1/40*(8*x^3 + 27)^(5/3)
```

mupad [**B**] time = 0.19, size = 11, normalized size = 0.73

$$\frac{\left(8\,x^3+27\right)^{5/3}}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(8*x^3 + 27)^(2/3),x)
[Out] (8*x^3 + 27)^(5/3)/40
```

sympy [**B**] time = 0.38, size = 27, normalized size = 1.80

$$\frac{x^3\left(8x^3+27\right)^{\frac{2}{3}}}{5}+\frac{27\left(8x^3+27\right)^{\frac{2}{3}}}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(8*x**3+27)**(2/3),x)
[Out] x**3*(8*x**3 + 27)**(2/3)/5 + 27*(8*x**3 + 27)**(2/3)/40
```

3.18 $\int \frac{\cos(x)+\sin(x)}{\sqrt[3]{-\cos(x)+\sin(x)}}\,dx$

Optimal. Leaf size=15

$$\frac{3}{2}(\sin(x)-\cos(x))^{2/3}$$

```
[Out] 3/2*(-cos(x)+sin(x))^(2/3)
```

Rubi [A] time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {3145}

$$\frac{3}{2}(\sin(x)-\cos(x))^{2/3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x] + Sin[x])/(-Cos[x] + Sin[x])^(1/3),x]
```

```
[Out] (3*(-Cos[x] + Sin[x])^(2/3))/2
```

Rule 3145

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(n_.)*(
cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :
> Simp[((c*B - b*C)*(b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(
b^2 + c^2)), x] /; FreeQ[{b, c, d, e, B, C}, x] && NeQ[n, -1] && NeQ[b^2 +
c^2, 0] && EqQ[b*B + c*C, 0]
```

Rubi steps

$$\int \frac{\cos(x)+\sin(x)}{\sqrt[3]{-\cos(x)+\sin(x)}}\,dx=\frac{3}{2}(-\cos(x)+\sin(x))^{2/3}$$

Mathematica [A] time = 0.06, size = 15, normalized size = 1.00

$$\frac{3}{2}(\sin(x)-\cos(x))^{2/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x] + Sin[x])/(-Cos[x] + Sin[x])^(1/3),x]
```

```
[Out] (3*(-Cos[x] + Sin[x])^(2/3))/2
```

fricas [A] time = 0.41, size = 11, normalized size = 0.73

$$\frac{3}{2}\left(-\cos(x)+\sin(x)\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+sin(x))/(-cos(x)+sin(x))^(1/3),x, algorithm="fricas")
```

```
[Out] 3/2*(-cos(x) + sin(x))^(2/3)
```

giac [A] time = 0.03, size = 11, normalized size = 0.73

$$\frac{3}{2}\left(-\cos(x)+\sin(x)\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+sin(x))/(-cos(x)+sin(x))^(1/3),x, algorithm="giac")
[Out] 3/2*(-cos(x) + sin(x))^(2/3)
```

maple [A] time = 0.03, size = 12, normalized size = 0.80

$$\frac{3\left(-\cos(x)+\sin(x)\right)^{\frac{2}{3}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)+sin(x))/(-cos(x)+sin(x))^(1/3),x)
[Out] 3/2*(-cos(x)+sin(x))^(2/3)
```

maxima [A] time = 0.72, size = 11, normalized size = 0.73

$$\frac{3}{2}\left(-\cos(x)+\sin(x)\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+sin(x))/(-cos(x)+sin(x))^(1/3),x, algorithm="maxima")
[Out] 3/2*(-cos(x) + sin(x))^(2/3)
```

mupad [B] time = 0.24, size = 15, normalized size = 1.00

$$\frac{3\,2^{1/3}\left(-\cos\left(x+\frac{\pi}{4}\right)\right)^{2/3}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x) + sin(x))/(sin(x) - cos(x))^(1/3),x)
[Out] (3*2^(1/3)*(-cos(x + pi/4))^(2/3))/2
```

sympy [A] time = 0.35, size = 12, normalized size = 0.80

$$\frac{3\left(\sin\left(x\right)-\cos\left(x\right)\right)^{\frac{2}{3}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+sin(x))/(-cos(x)+sin(x))**(1/3),x)
[Out] 3*(sin(x) - cos(x))**(2/3)/2
```

3.19 $\int \frac{x}{\sqrt{1+x^2+\left(1+x^2\right)^{3/2}}}\,dx$

Optimal. Leaf size=32

$$\frac{2\sqrt{\left(x^2+1\right)\left(\sqrt{x^2+1}+1\right)}}{\sqrt{x^2+1}}$$

[Out] 2*((x^2+1)*((x^2+1)^(1/2)+1))^(1/2)/(x^2+1)^(1/2)

Rubi [A] time = 0.10, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6715, 1588}

$$\frac{2\sqrt{\left(x^2+1\right)\left(\sqrt{x^2+1}+1\right)}}{\sqrt{x^2+1}}$$

Antiderivative was successfully verified.

[In] Int[x/Sqrt[1 + x^2 + (1 + x^2)^(3/2)],x]

[Out] (2*Sqrt[(1 + x^2)*(1 + Sqrt[1 + x^2])])/Sqrt[1 + x^2]

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\sqrt{1+x^2+\left(1+x^2\right)^{3/2}}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1+x+(1+x)^{3/2}}}\,dx, x, x^2\right)\\
&= \text{Subst}\left(\int \frac{x}{\sqrt{x^2(1+x)}}\,dx, x, \sqrt{1+x^2}\right)\\
&= \frac{2\sqrt{\left(1+x^2\right)\left(1+\sqrt{1+x^2}\right)}}{\sqrt{1+x^2}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 37, normalized size = 1.16

$$\frac{2\left(x^2+\sqrt{x^2+1}+1\right)}{\sqrt{\left(x^2+1\right)\left(\sqrt{x^2+1}+1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[1 + x^2 + (1 + x^2)^(3/2)],x]
```

```
[Out] (2*(1 + x^2 + Sqrt[1 + x^2]))/Sqrt[(1 + x^2)*(1 + Sqrt[1 + x^2])]
```

fricas [A] time = 0.46, size = 23, normalized size = 0.72

$$\frac{2\sqrt{x^2 + \left(x^2+1\right)^{\frac{3}{2}} + 1}}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x^2+(x^2+1)^(3/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x^2 + (x^2 + 1)^(3/2) + 1)/sqrt(x^2 + 1)
```

giac [A] time = 0.06, size = 15, normalized size = 0.47

$$2\sqrt{\sqrt{x^2+1}+1} - 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x^2+(x^2+1)^(3/2))^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(sqrt(x^2 + 1) + 1) - 2
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^2+1+\left(x^2+1\right)^{\frac{3}{2}}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1+x^2+(x^2+1)^(3/2))^(1/2),x)
```

```
[Out] int(x/(1+x^2+(x^2+1)^(3/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^2+\left(x^2+1\right)^{\frac{3}{2}}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x^2+(x^2+1)^(3/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(x^2 + (x^2 + 1)^(3/2) + 1), x)
```

mupad [B] time = 0.59, size = 47, normalized size = 1.47

$$\frac{2\left(x^2+1\right)\sqrt{\sqrt{x^2+1}+1}}{\left(\sqrt{\sqrt{x^2+1}+1}+1\right)\sqrt{\left(x^2+1\right)^{3/2}+x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x^2 + 1)^(3/2) + x^2 + 1)^(1/2),x)
```

```
[Out] (2*(x^2 + 1)*((x^2 + 1)^(1/2) + 1)^(1/2))/((((x^2 + 1)^(1/2) + 1)^(1/2) + 1
)*((x^2 + 1)^(3/2) + x^2 + 1)^(1/2))
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{\left(x^2+1\right)\left(\sqrt{x^2+1}+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x**2+(x**2+1)**(3/2))**(1/2),x)
```

```
[Out] Integral(x/sqrt((x**2 + 1)*(sqrt(x**2 + 1) + 1)), x)
```

3.20 $\int \frac{x}{\sqrt{1+x^2}\sqrt{1+\sqrt{1+x^2}}}\,dx$

Optimal. Leaf size=17

$$2\sqrt{\sqrt{x^2+1}+1}$$

[Out] 2*((x^2+1)^(1/2)+1)^(1/2)

Rubi [A] time = 0.11, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {6686}

$$2\sqrt{\sqrt{x^2+1}+1}$$

Antiderivative was successfully verified.

[In] Int[x/(Sqrt[1 + x^2]*Sqrt[1 + Sqrt[1 + x^2]]),x]

[Out] 2*Sqrt[1 + Sqrt[1 + x^2]]

Rule 6686

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[(q*y^(m + 1))/(m + 1), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x}{\sqrt{1+x^2}\sqrt{1+\sqrt{1+x^2}}}\,dx = 2\sqrt{1+\sqrt{1+x^2}}$$

Mathematica [A] time = 0.02, size = 17, normalized size = 1.00

$$2\sqrt{\sqrt{x^2+1}+1}$$

Antiderivative was successfully verified.

[In] Integrate[x/(Sqrt[1 + x^2]*Sqrt[1 + Sqrt[1 + x^2]]),x]

[Out] 2*Sqrt[1 + Sqrt[1 + x^2]]

fricas [A] time = 0.44, size = 13, normalized size = 0.76

$$2\sqrt{\sqrt{x^2+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x^2+1)^(1/2)/((x^2+1)^(1/2)+1)^(1/2),x, algorithm="fricas")

[Out] 2*sqrt(sqrt(x^2 + 1) + 1)

giac [A] time = 0.01, size = 13, normalized size = 0.76

$$2\sqrt{\sqrt{x^2+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+1)^(1/2)/((x^2+1)^(1/2)+1)^(1/2),x, algorithm="giac")
[Out] 2*sqrt(sqrt(x^2 + 1) + 1)
```

maple [A] time = 0.01, size = 14, normalized size = 0.82

$$2\sqrt{\sqrt{x^2+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2+1)^(1/2)/((x^2+1)^(1/2)+1)^(1/2),x)
[Out] 2*((x^2+1)^(1/2)+1)^(1/2)
```

maxima [A] time = 0.59, size = 13, normalized size = 0.76

$$2\sqrt{\sqrt{x^2+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+1)^(1/2)/((x^2+1)^(1/2)+1)^(1/2),x, algorithm="maxima")
[Out] 2*sqrt(sqrt(x^2 + 1) + 1)
```

mupad [B] time = 0.21, size = 13, normalized size = 0.76

$$2\sqrt{\sqrt{x^2+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x^2 + 1)^(1/2)*((x^2 + 1)^(1/2) + 1)^(1/2)),x)
[Out] 2*((x^2 + 1)^(1/2) + 1)^(1/2)
```

sympy [A] time = 0.37, size = 14, normalized size = 0.82

$$2\sqrt{\sqrt{x^2+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2+1)**(1/2)/((x**2+1)**(1/2)+1)**(1/2),x)
[Out] 2*sqrt(sqrt(x**2 + 1) + 1)
```

3.21 $\int \frac{\sqrt[5]{1-2x+x^2}}{1-x} \, dx$

Optimal. Leaf size=16

$$-\frac{5}{2}\sqrt[5]{x^2-2x+1}$$

```
[Out] -5/2*(x^2-2*x+1)^(1/5)
```

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {643, 629}

$$-\frac{5}{2}\sqrt[5]{x^2-2x+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*x + x^2)^(1/5)/(1 - x),x]
[Out] (-5*(1 - 2*x + x^2)^(1/5))/2
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 643

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[e^(m - 1)/c^((m - 1)/2), Int[(d + e*x)*(a + b*x + c*x^2)^(p +
(m - 1)/2), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] &
&  !IntegerQ[p] && EqQ[2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[5]{1-2x+x^2}}{1-x} \, dx &= \int \frac{1-x}{\left(1-2x+x^2\right)^{4/5}} \, dx \\
&= -\frac{5}{2}\sqrt[5]{1-2x+x^2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 0.81

$$-\frac{5}{2}\sqrt[5]{(x-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*x + x^2)^(1/5)/(1 - x),x]
[Out] (-5*((-1 + x)^2)^(1/5))/2
```

fricas [A] time = 0.41, size = 12, normalized size = 0.75

$$-\frac{5}{2}\left(x^2-2x+1\right)^{\frac{1}{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2*x+1)^(1/5)/(1-x),x, algorithm="fricas")
[Out] -5/2*(x^2 - 2*x + 1)^(1/5)
```

giac [A] time = 0.01, size = 12, normalized size = 0.75

$$-\frac{5}{2}\left(x^2-2\,x+1\right)^{\frac{1}{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2*x+1)^(1/5)/(1-x),x, algorithm="giac")
[Out] -5/2*(x^2 - 2*x + 1)^(1/5)
```

maple [A] time = 0.00, size = 13, normalized size = 0.81

$$-\frac{5\left(x^2-2x+1\right)^{\frac{1}{5}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-2*x+1)^(1/5)/(1-x),x)
[Out] -5/2*(x^2-2*x+1)^(1/5)
```

maxima [A] time = 0.55, size = 7, normalized size = 0.44

$$-\frac{5}{2}\left(x-1\right)^{\frac{2}{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2*x+1)^(1/5)/(1-x),x, algorithm="maxima")
[Out] -5/2*(x - 1)^(2/5)
```

mupad [B] time = 0.16, size = 9, normalized size = 0.56

$$-\frac{5\left({\left(x-1\right)}^2\right)^{1/5}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 2*x + 1)^(1/5)/(x - 1),x)
[Out] -(5*((x - 1)^2)^(1/5))/2
```

sympy [A] time = 1.75, size = 15, normalized size = 0.94

$$-\frac{5\sqrt[5]{x^2-2x+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-2*x+1)**(1/5)/(1-x),x)
[Out] -5*(x**2 - 2*x + 1)**(1/5)/2
```

3.22 $\int x\sin(x)\,dx$

Optimal. Leaf size=8

$$\sin(x) - x\cos(x)$$

```
[Out] -x*cos(x)+sin(x)
```

Rubi [A] time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3296, 2637}

$$\sin(x) - x\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sin[x],x]
[Out] -(x*Cos[x]) + Sin[x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x\sin(x)\,dx &= -x\cos(x) + \int \cos(x)\,dx\\ &= -x\cos(x) + \sin(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$\sin(x) - x\cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sin[x],x]
[Out] -(x*Cos[x]) + Sin[x]
```

fricas [A] time = 0.41, size = 8, normalized size = 1.00

$$-x\cos(x) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x),x, algorithm="fricas")
[Out] -x*cos(x) + sin(x)
```

giac [A] time = 0.01, size = 8, normalized size = 1.00

$$-x\cos(x) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x),x, algorithm="giac")
```

```
[Out] -x*cos(x) + sin(x)
```

maple [A] time = 0.02, size = 9, normalized size = 1.12

$$-x\cos(x) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x),x)
```

```
[Out] -x*cos(x)+sin(x)
```

maxima [A] time = 0.52, size = 8, normalized size = 1.00

$$-x\cos(x) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x),x, algorithm="maxima")
```

```
[Out] -x*cos(x) + sin(x)
```

mupad [B] time = 0.06, size = 8, normalized size = 1.00

$$\sin(x) - x\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x),x)
```

```
[Out] sin(x) - x*cos(x)
```

sympy [A] time = 0.18, size = 7, normalized size = 0.88

$$-x\cos(x) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x),x)
```

```
[Out] -x*cos(x) + sin(x)
```

3.23 $\int x^2 \sin(x)\,dx$

Optimal. Leaf size=17

$$x^2(-\cos(x)) + 2x\sin(x) + 2\cos(x)$$

```
[Out] 2*cos(x)-x^2*cos(x)+2*x*sin(x)
```

Rubi [A] time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3296, 2638}

$$x^2(-\cos(x)) + 2x\sin(x) + 2\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sin[x],x]
```

```
[Out] 2*Cos[x] - x^2*Cos[x] + 2*x*Sin[x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\sin(x)\,dx &= -x^2\cos(x) + 2\int x\cos(x)\,dx\\ &= -x^2\cos(x) + 2x\sin(x) - 2\int \sin(x)\,dx\\ &= 2\cos(x) - x^2\cos(x) + 2x\sin(x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 15, normalized size = 0.88

$$2x\sin(x) - \left(x^2-2\right)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sin[x],x]
```

```
[Out] -((-2 + x^2)*Cos[x]) + 2*x*Sin[x]
```

fricas [A] time = 0.41, size = 15, normalized size = 0.88

$$-\left(x^2-2\right)\cos(x) + 2\,x\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(x),x, algorithm="fricas")
```

```
[Out] -(x^2 - 2)*cos(x) + 2*x*sin(x)
```

giac [A] time = 0.01, size = 15, normalized size = 0.88

$$-\left(x^2-2\right)\cos(x) + 2\,x\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(x),x, algorithm="giac")
```

```
[Out] -(x^2 - 2)*cos(x) + 2*x*sin(x)
```

maple [A] time = 0.01, size = 18, normalized size = 1.06

$$-x^2\cos(x) + 2x\sin(x) + 2\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(x),x)
```

```
[Out] 2*cos(x)-x^2*cos(x)+2*x*sin(x)
```

maxima [A] time = 0.56, size = 15, normalized size = 0.88

$$-\left(x^2 - 2\right)\cos(x) + 2\,x\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(x),x, algorithm="maxima")
```

```
[Out] -(x^2 - 2)*cos(x) + 2*x*sin(x)
```

mupad [B] time = 0.03, size = 15, normalized size = 0.88

$$2\,x\,\sin(x) - \cos(x)\,\left(x^2 - 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(x),x)
```

```
[Out] 2*x*sin(x) - cos(x)*(x^2 - 2)
```

sympy [A] time = 0.34, size = 17, normalized size = 1.00

$$-x^2\cos(x) + 2x\sin(x) + 2\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(x),x)
```

```
[Out] -x**2*cos(x) + 2*x*sin(x) + 2*cos(x)
```

3.24 $\int x^3 \cos(x)\, dx$

Optimal. Leaf size=23

$$x^3 \sin(x) + 3x^2 \cos(x) - 6x \sin(x) - 6 \cos(x)$$

```
[Out] -6*cos(x)+3*x^2*cos(x)-6*x*sin(x)+x^3*sin(x)
```

Rubi [A] time = 0.04, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3296, 2638}

$$x^3 \sin(x) + 3x^2 \cos(x) - 6x \sin(x) - 6 \cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cos[x],x]
```

```
[Out] -6*Cos[x] + 3*x^2*Cos[x] - 6*x*Sin[x] + x^3*Sin[x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int x^3 \cos(x)\, dx &= x^3 \sin(x) - 3 \int x^2 \sin(x)\, dx \\
&= 3x^2 \cos(x) + x^3 \sin(x) - 6 \int x \cos(x)\, dx \\
&= 3x^2 \cos(x) - 6x \sin(x) + x^3 \sin(x) + 6 \int \sin(x)\, dx \\
&= -6 \cos(x) + 3x^2 \cos(x) - 6x \sin(x) + x^3 \sin(x)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 19, normalized size = 0.83

$$x \left(x^2 - 6\right) \sin(x) + 3 \left(x^2 - 2\right) \cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Cos[x],x]
```

```
[Out] 3*(-2 + x^2)*Cos[x] + x*(-6 + x^2)*Sin[x]
```

fricas [A] time = 0.40, size = 20, normalized size = 0.87

$$3 \left(x^2 - 2\right) \cos(x) + \left(x^3 - 6\, x\right) \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(x),x, algorithm="fricas")
```

```
[Out] 3*(x^2 - 2)*cos(x) + (x^3 - 6*x)*sin(x)
```

giac [A] time = 0.01, size = 20, normalized size = 0.87

$$3\left(x^2-2\right)\cos(x)+\left(x^3-6x\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(x),x, algorithm="giac")
[Out] 3*(x^2 - 2)*cos(x) + (x^3 - 6*x)*sin(x)
```

maple [A] time = 0.02, size = 24, normalized size = 1.04

$$x^3\sin(x)+3x^2\cos(x)-6x\sin(x)-6\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cos(x),x)
[Out] -6*cos(x)+3*x^2*cos(x)-6*x*sin(x)+x^3*sin(x)
```

maxima [A] time = 0.64, size = 20, normalized size = 0.87

$$3\left(x^2-2\right)\cos(x)+\left(x^3-6x\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(x),x, algorithm="maxima")
[Out] 3*(x^2 - 2)*cos(x) + (x^3 - 6*x)*sin(x)
```

mupad [B] time = 0.03, size = 24, normalized size = 1.04

$$\cos(x)\left(3x^2-6\right)-\sin(x)\left(6x-x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cos(x),x)
[Out] cos(x)*(3*x^2 - 6) - sin(x)*(6*x - x^3)
```

sympy [A] time = 0.64, size = 26, normalized size = 1.13

$$x^3\sin\left(x\right)+3x^2\cos\left(x\right)-6x\sin\left(x\right)-6\cos\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cos(x),x)
[Out] x**3*sin(x) + 3*x**2*cos(x) - 6*x*sin(x) - 6*cos(x)
```

3.25 $\int x^3 \sin(x)\,dx$

Optimal. Leaf size=24

$$x^3(-\cos(x)) + 3x^2\sin(x) - 6\sin(x) + 6x\cos(x)$$

```
[Out] 6*x*cos(x)-x^3*cos(x)-6*sin(x)+3*x^2*sin(x)
```

Rubi [A] time = 0.04, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3296, 2637}

$$x^3(-\cos(x)) + 3x^2\sin(x) - 6\sin(x) + 6x\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sin[x],x]
[Out] 6*x*Cos[x] - x^3*Cos[x] - 6*Sin[x] + 3*x^2*Sin[x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int x^3\sin(x)\,dx &= -x^3\cos(x) + 3\int x^2\cos(x)\,dx\\
&= -x^3\cos(x) + 3x^2\sin(x) - 6\int x\sin(x)\,dx\\
&= 6x\cos(x) - x^3\cos(x) + 3x^2\sin(x) - 6\int\cos(x)\,dx\\
&= 6x\cos(x) - x^3\cos(x) - 6\sin(x) + 3x^2\sin(x)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 20, normalized size = 0.83

$$3\left(x^2-2\right)\sin(x) - x\left(x^2-6\right)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sin[x],x]
[Out] -(x*(-6 + x^2)*Cos[x]) + 3*(-2 + x^2)*Sin[x]
```

fricas [A] time = 0.41, size = 21, normalized size = 0.88

$$-\left(x^3-6\,x\right)\cos(x) + 3\left(x^2-2\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(x),x, algorithm="fricas")
```

```
[Out] -(x^3 - 6*x)*cos(x) + 3*(x^2 - 2)*sin(x)
```

giac [**A**] time = 0.01, size = 21, normalized size = 0.88

$$-\left(x^{3}-6\,x\right)\cos\left(x\right)+3\,\left(x^{2}-2\right)\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(x),x, algorithm="giac")
[Out] -(x^3 - 6*x)*cos(x) + 3*(x^2 - 2)*sin(x)
```

maple [**A**] time = 0.02, size = 25, normalized size = 1.04

$$-x^{3}\cos(x)+3x^{2}\sin(x)+6x\cos(x)-6\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sin(x),x)
[Out] 6*x*cos(x)-x^3*cos(x)-6*sin(x)+3*x^2*sin(x)
```

maxima [**A**] time = 0.54, size = 21, normalized size = 0.88

$$-\left(x^{3}-6\,x\right)\cos\left(x\right)+3\,\left(x^{2}-2\right)\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(x),x, algorithm="maxima")
[Out] -(x^3 - 6*x)*cos(x) + 3*(x^2 - 2)*sin(x)
```

mupad [**B**] time = 0.03, size = 23, normalized size = 0.96

$$\cos(x)\,\left(6\,x-x^{3}\right)+\sin(x)\,\left(3\,x^{2}-6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sin(x),x)
[Out] cos(x)*(6*x - x^3) + sin(x)*(3*x^2 - 6)
```

sympy [**A**] time = 0.64, size = 26, normalized size = 1.08

$$-x^{3}\cos\left(x\right)+3x^{2}\sin\left(x\right)+6x\cos\left(x\right)-6\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sin(x),x)
[Out] -x**3*cos(x) + 3*x**2*sin(x) + 6*x*cos(x) - 6*sin(x)
```

3.26 $\int \cos(x)\sin(x)\,dx$

Optimal. Leaf size=8

$$\frac{\sin^2(x)}{2}$$

```
[Out] 1/2*sin(x)^2
```

Rubi [A] time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2564, 30}

$$\frac{\sin^2(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sin[x],x]
```

```
[Out] Sin[x]^2/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned} \int \cos(x)\sin(x)\,dx &= \text{Subst}\left(\int x\,dx, x, \sin(x)\right)\\ &= \frac{\sin^2(x)}{2} \end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$-\frac{1}{2}\cos^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sin[x],x]
```

```
[Out] -1/2*Cos[x]^2
```

fricas [A] time = 0.40, size = 6, normalized size = 0.75

$$-\frac{1}{2}\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x),x, algorithm="fricas")
```

```
[Out] -1/2*cos(x)^2
```

giac [**A**] time = 0.01, size = 6, normalized size = 0.75

$$-\frac{1}{2}\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x),x, algorithm="giac")
[Out] -1/2*cos(x)^2
```

maple [**A**] time = 0.00, size = 7, normalized size = 0.88

$$\frac{\left(\sin^2(x)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x),x)
[Out] 1/2*sin(x)^2
```

maxima [**A**] time = 0.63, size = 6, normalized size = 0.75

$$-\frac{1}{2}\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x),x, algorithm="maxima")
[Out] -1/2*cos(x)^2
```

mupad [**B**] time = 0.02, size = 6, normalized size = 0.75

$$\frac{\sin(x)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x),x)
[Out] sin(x)^2/2
```

sympy [**A**] time = 0.06, size = 5, normalized size = 0.62

$$\frac{\sin^2(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x),x)
[Out] sin(x)**2/2
```

3.27 $\int x \cos(x) \sin(x) \, dx$

Optimal. Leaf size=23

$$-\frac{x}{4}+\frac{1}{2}x\sin^2(x)+\frac{1}{4}\sin(x)\cos(x)$$

```
[Out] -1/4*x+1/4*cos(x)*sin(x)+1/2*x*sin(x)^2
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3443, 2635, 8}

$$-\frac{x}{4}+\frac{1}{2}x\sin^2(x)+\frac{1}{4}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[x]*Sin[x],x]
```

```
[Out] -x/4 + (Cos[x]*Sin[x])/4 + (x*Sin[x]^2)/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3443

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sin[a + b*x^n]^(p + 1))/(b*n*(p + 1
)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p +
1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int x\cos(x)\sin(x)\,dx &= \frac{1}{2}x\sin^2(x)-\frac{1}{2}\int \sin^2(x)\,dx\\ &= \frac{1}{4}\cos(x)\sin(x)+\frac{1}{2}x\sin^2(x)-\frac{\int 1\,dx}{4}\\ &= -\frac{x}{4}+\frac{1}{4}\cos(x)\sin(x)+\frac{1}{2}x\sin^2(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 0.78

$$\frac{1}{8}\sin(2x)-\frac{1}{4}x\cos(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[x]*Sin[x],x]
```

```
[Out] -1/4*(x*Cos[2*x]) + Sin[2*x]/8
```

fricas [A] time = 0.42, size = 17, normalized size = 0.74

$$-\frac{1}{2}\,x\cos(x)^2+\frac{1}{4}\,\cos(x)\sin(x)+\frac{1}{4}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)*sin(x),x, algorithm="fricas")
[Out] -1/2*x*cos(x)^2 + 1/4*cos(x)*sin(x) + 1/4*x
```

giac [A] time = 0.01, size = 14, normalized size = 0.61

$$-\frac{1}{4}\,x\cos\left(2\,x\right)+\frac{1}{8}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)*sin(x),x, algorithm="giac")
[Out] -1/4*x*cos(2*x) + 1/8*sin(2*x)
```

maple [A] time = 0.00, size = 18, normalized size = 0.78

$$-\frac{x\left(\cos^2(x)\right)}{2}+\frac{\cos(x)\sin(x)}{4}+\frac{x}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x)*sin(x),x)
[Out] -1/2*x*cos(x)^2+1/4*cos(x)*sin(x)+1/4*x
```

maxima [A] time = 0.51, size = 14, normalized size = 0.61

$$-\frac{1}{4}\,x\cos\left(2\,x\right)+\frac{1}{8}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)*sin(x),x, algorithm="maxima")
[Out] -1/4*x*cos(2*x) + 1/8*sin(2*x)
```

mupad [B] time = 0.09, size = 18, normalized size = 0.78

$$\frac{\sin\left(2\,x\right)}{8}+\frac{x\,\left(2\,{\sin(x)}^2-1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x)*sin(x),x)
[Out] sin(2*x)/8 + (x*(2*sin(x)^2 - 1))/4
```

sympy [A] time = 0.34, size = 24, normalized size = 1.04

$$\frac{x\sin^2\left(x\right)}{4}-\frac{x\cos^2\left(x\right)}{4}+\frac{\sin\left(x\right)\cos\left(x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)*sin(x),x)
[Out] x*sin(x)**2/4 - x*cos(x)**2/4 + sin(x)*cos(x)/4
```

3.28 $\int \sin^2(x)\,dx$

Optimal. Leaf size=14

$$\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)$$

```
[Out] 1/2*x-1/2*cos(x)*sin(x)
```

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2635, 8}

$$\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2,x]
```

```
[Out] x/2 - (Cos[x]*Sin[x])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\begin{aligned}\int \sin^2(x)\,dx &= -\frac{1}{2}\cos(x)\sin(x)+\frac{\int 1\,dx}{2}\\ &= \frac{x}{2}-\frac{1}{2}\cos(x)\sin(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{x}{2}-\frac{1}{4}\sin(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2,x]
```

```
[Out] x/2 - Sin[2*x]/4
```

fricas [A] time = 0.42, size = 10, normalized size = 0.71

$$-\frac{1}{2}\cos(x)\sin(x)+\frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2,x, algorithm="fricas")
```

```
[Out] -1/2*cos(x)*sin(x) + 1/2*x
```

giac [A] time = 0.01, size = 10, normalized size = 0.71

$$\frac{1}{2}\,x - \frac{1}{4}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*x - 1/4*sin(2*x)
```

maple [A] time = 0.02, size = 11, normalized size = 0.79

$$-\frac{\cos(x)\sin(x)}{2} + \frac{x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2,x)
```

```
[Out] 1/2*x-1/2*cos(x)*sin(x)
```

maxima [A] time = 0.57, size = 10, normalized size = 0.71

$$\frac{1}{2}\,x - \frac{1}{4}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x - 1/4*sin(2*x)
```

mupad [B] time = 0.03, size = 10, normalized size = 0.71

$$\frac{x}{2} - \frac{\sin\left(2\,x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2,x)
```

```
[Out] x/2 - sin(2*x)/4
```

sympy [A] time = 0.07, size = 10, normalized size = 0.71

$$\frac{x}{2} - \frac{\sin\left(x\right)\cos\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2,x)
```

```
[Out] x/2 - sin(x)*cos(x)/2
```

3.29 $\int \sin^3(x)\,dx$

Optimal. Leaf size=13

$$\frac{\cos^3(x)}{3}-\cos(x)$$

```
[Out] -cos(x)+1/3*cos(x)^3
```

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2633}

$$\frac{\cos^3(x)}{3}-\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^3,x]
[Out] -Cos[x] + Cos[x]^3/3
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sin^3(x)\,dx &= -\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,\cos(x)\right)\\ &= -\cos(x)+\frac{\cos^3(x)}{3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.15

$$\frac{1}{12}\cos(3x)-\frac{3\cos(x)}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^3,x]
[Out] (-3*Cos[x])/4 + Cos[3*x]/12
```

fricas [A] time = 0.41, size = 11, normalized size = 0.85

$$\frac{1}{3}\cos(x)^3-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3,x, algorithm="fricas")
[Out] 1/3*cos(x)^3 - cos(x)
```

giac [A] time = 0.01, size = 11, normalized size = 0.85

$$\frac{1}{3}\cos(x)^3-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3,x, algorithm="giac")
```

```
[Out] 1/3*cos(x)^3 - cos(x)
```

maple [A] time = 0.00, size = 11, normalized size = 0.85

$$-\frac{\left(\sin^2(x)+2\right)\cos(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3,x)
```

```
[Out] -1/3*(sin(x)^2+2)*cos(x)
```

maxima [A] time = 0.58, size = 11, normalized size = 0.85

$$\frac{1}{3}\cos(x)^3-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3,x, algorithm="maxima")
```

```
[Out] 1/3*cos(x)^3 - cos(x)
```

mupad [B] time = 0.00, size = 10, normalized size = 0.77

$$\frac{\cos(x)\left(\cos(x)^2-3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3,x)
```

```
[Out] (cos(x)*(cos(x)^2 - 3))/3
```

sympy [A] time = 0.07, size = 8, normalized size = 0.62

$$\frac{\cos^3(x)}{3}-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**3,x)
```

```
[Out] cos(x)**3/3 - cos(x)
```

3.30 $\int \sin^4(x)\,dx$

Optimal. Leaf size=24

$$\frac{3x}{8}-\frac{1}{4}\sin^3(x)\cos(x)-\frac{3}{8}\sin(x)\cos(x)$$

```
[Out] 3/8*x-3/8*cos(x)*sin(x)-1/4*cos(x)*sin(x)^3
```

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2635, 8}

$$\frac{3x}{8}-\frac{1}{4}\sin^3(x)\cos(x)-\frac{3}{8}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^4,x]
```

```
[Out] (3*x)/8 - (3*Cos[x]*Sin[x])/8 - (Cos[x]*Sin[x]^3)/4
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\begin{aligned}\int \sin^4(x)\,dx &= -\frac{1}{4}\cos(x)\sin^3(x)+\frac{3}{4}\int \sin^2(x)\,dx\\ &= -\frac{3}{8}\cos(x)\sin(x)-\frac{1}{4}\cos(x)\sin^3(x)+\frac{3\int 1\,dx}{8}\\ &= \frac{3x}{8}-\frac{3}{8}\cos(x)\sin(x)-\frac{1}{4}\cos(x)\sin^3(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 0.92

$$\frac{3x}{8}-\frac{1}{4}\sin(2x)+\frac{1}{32}\sin(4x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^4,x]
```

```
[Out] (3*x)/8 - Sin[2*x]/4 + Sin[4*x]/32
```

fricas [A] time = 0.42, size = 19, normalized size = 0.79

$$\frac{1}{8}\left(2\,\cos(x)^3-5\,\cos(x)\right)\sin(x)+\frac{3}{8}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4,x, algorithm="fricas")
[Out] 1/8*(2*cos(x)^3 - 5*cos(x))*sin(x) + 3/8*x
```

giac [A] time = 0.01, size = 16, normalized size = 0.67

$$\frac{3}{8}x + \frac{1}{32}\sin(4x) - \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4,x, algorithm="giac")
[Out] 3/8*x + 1/32*sin(4*x) - 1/4*sin(2*x)
```

maple [A] time = 0.12, size = 18, normalized size = 0.75

$$\frac{3x}{8} - \frac{\left(\sin^3(x) + \frac{3\sin(x)}{2}\right)\cos(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^4,x)
[Out] -1/4*(sin(x)^3+3/2*sin(x))*cos(x)+3/8*x
```

maxima [A] time = 0.49, size = 16, normalized size = 0.67

$$\frac{3}{8}x + \frac{1}{32}\sin(4x) - \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4,x, algorithm="maxima")
[Out] 3/8*x + 1/32*sin(4*x) - 1/4*sin(2*x)
```

mupad [B] time = 0.04, size = 16, normalized size = 0.67

$$\frac{3x}{8} - \frac{\sin(2x)}{4} + \frac{\sin(4x)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^4,x)
[Out] (3*x)/8 - sin(2*x)/4 + sin(4*x)/32
```

sympy [A] time = 0.07, size = 24, normalized size = 1.00

$$\frac{3x}{8} - \frac{\sin^3(x)\cos(x)}{4} - \frac{3\sin(x)\cos(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**4,x)
[Out] 3*x/8 - sin(x)**3*cos(x)/4 - 3*sin(x)*cos(x)/8
```

3.31 $\int \sin^5(x)\,dx$

Optimal. Leaf size=21

$$-\frac{1}{5}\cos^5(x)+\frac{2\cos^3(x)}{3}-\cos(x)$$

```
[Out] -cos(x)+2/3*cos(x)^3-1/5*cos(x)^5
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2633}

$$-\frac{1}{5}\cos^5(x)+\frac{2\cos^3(x)}{3}-\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^5,x]
```

```
[Out] -Cos[x] + (2*Cos[x]^3)/3 - Cos[x]^5/5
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sin^5(x)\,dx &= -\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx,x,\cos(x)\right)\\ &= -\cos(x)+\frac{2\cos^3(x)}{3}-\frac{\cos^5(x)}{5}\end{aligned}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 1.10

$$-\frac{5\cos(x)}{8}+\frac{5}{48}\cos(3x)-\frac{1}{80}\cos(5x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^5,x]
```

```
[Out] (-5*Cos[x])/8 + (5*Cos[3*x])/48 - Cos[5*x]/80
```

fricas [A] time = 0.43, size = 17, normalized size = 0.81

$$-\frac{1}{5}\cos(x)^5+\frac{2}{3}\cos(x)^3-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^5,x, algorithm="fricas")
```

```
[Out] -1/5*cos(x)^5 + 2/3*cos(x)^3 - cos(x)
```

giac [A] time = 0.01, size = 17, normalized size = 0.81

$$-\frac{1}{5}\cos(x)^5+\frac{2}{3}\cos(x)^3-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^5,x, algorithm="giac")
[Out] -1/5*cos(x)^5 + 2/3*cos(x)^3 - cos(x)
```

maple [A] time = 0.10, size = 17, normalized size = 0.81

$$-\frac{\left(\sin^4(x)+\frac{4\left(\sin^2(x)\right)}{3}+\frac{8}{3}\right)\cos(x)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^5,x)
[Out] -1/5*(8/3+sin(x)^4+4/3*sin(x)^2)*cos(x)
```

maxima [A] time = 0.51, size = 17, normalized size = 0.81

$$-\frac{1}{5}\cos(x)^5+\frac{2}{3}\cos(x)^3-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^5,x, algorithm="maxima")
[Out] -1/5*cos(x)^5 + 2/3*cos(x)^3 - cos(x)
```

mupad [B] time = 0.04, size = 17, normalized size = 0.81

$$-\frac{\cos(x)^5}{5}+\frac{2\cos(x)^3}{3}-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^5,x)
[Out] (2*cos(x)^3)/3 - cos(x) - cos(x)^5/5
```

sympy [A] time = 0.07, size = 17, normalized size = 0.81

$$-\frac{\cos^5(x)}{5}+\frac{2\cos^3(x)}{3}-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**5,x)
[Out] -cos(x)**5/5 + 2*cos(x)**3/3 - cos(x)
```

3.32 $\int \sin^6(x)\,dx$

Optimal. Leaf size=34

$$\frac{5x}{16}-\frac{1}{6}\sin^5(x)\cos(x)-\frac{5}{24}\sin^3(x)\cos(x)-\frac{5}{16}\sin(x)\cos(x)$$

```
[Out] 5/16*x-5/16*cos(x)*sin(x)-5/24*cos(x)*sin(x)^3-1/6*cos(x)*sin(x)^5
```

Rubi [A] time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2635, 8}

$$\frac{5x}{16}-\frac{1}{6}\sin^5(x)\cos(x)-\frac{5}{24}\sin^3(x)\cos(x)-\frac{5}{16}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^6,x]
```

```
[Out] (5*x)/16 - (5*Cos[x]*Sin[x])/16 - (5*Cos[x]*Sin[x]^3)/24 - (Cos[x]*Sin[x]^5
)/6
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\begin{aligned}
\int \sin^6(x)\,dx &= -\frac{1}{6}\cos(x)\sin^5(x)+\frac{5}{6}\int \sin^4(x)\,dx\\
&= -\frac{5}{24}\cos(x)\sin^3(x)-\frac{1}{6}\cos(x)\sin^5(x)+\frac{5}{8}\int \sin^2(x)\,dx\\
&= -\frac{5}{16}\cos(x)\sin(x)-\frac{5}{24}\cos(x)\sin^3(x)-\frac{1}{6}\cos(x)\sin^5(x)+\frac{5\int 1\,dx}{16}\\
&= \frac{5x}{16}-\frac{5}{16}\cos(x)\sin(x)-\frac{5}{24}\cos(x)\sin^3(x)-\frac{1}{6}\cos(x)\sin^5(x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 0.88

$$\frac{5x}{16}-\frac{15}{64}\sin(2x)+\frac{3}{64}\sin(4x)-\frac{1}{192}\sin(6x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^6,x]
```

```
[Out] (5*x)/16 - (15*Sin[2*x])/64 + (3*Sin[4*x])/64 - Sin[6*x]/192
```

fricas [A] time = 0.41, size = 25, normalized size = 0.74

$$-\frac{1}{48}\left(8\cos(x)^5-26\cos(x)^3+33\cos(x)\right)\sin(x)+\frac{5}{16}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^6,x, algorithm="fricas")
```

```
[Out] -1/48*(8*cos(x)^5 - 26*cos(x)^3 + 33*cos(x))*sin(x) + 5/16*x
```

giac [A] time = 0.01, size = 22, normalized size = 0.65

$$\frac{5}{16}\,x - \frac{1}{192}\,\sin\left(6\,x\right) + \frac{3}{64}\,\sin\left(4\,x\right) - \frac{15}{64}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^6,x, algorithm="giac")
```

```
[Out] 5/16*x - 1/192*sin(6*x) + 3/64*sin(4*x) - 15/64*sin(2*x)
```

maple [A] time = 0.10, size = 24, normalized size = 0.71

$$\frac{5x}{16} - \frac{\left(\sin^5(x) + \frac{5\left(\sin^3(x)\right)}{4} + \frac{15\sin(x)}{8}\right)\cos(x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^6,x)
```

```
[Out] -1/6*(sin(x)^5+5/4*sin(x)^3+15/8*sin(x))*cos(x)+5/16*x
```

maxima [A] time = 0.62, size = 24, normalized size = 0.71

$$\frac{1}{48}\,\sin\left(2\,x\right)^3 + \frac{5}{16}\,x + \frac{3}{64}\,\sin\left(4\,x\right) - \frac{1}{4}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^6,x, algorithm="maxima")
```

```
[Out] 1/48*sin(2*x)^3 + 5/16*x + 3/64*sin(4*x) - 1/4*sin(2*x)
```

mupad [B] time = 0.04, size = 22, normalized size = 0.65

$$\frac{5\,x}{16} - \frac{15\,\sin\left(2\,x\right)}{64} + \frac{3\,\sin\left(4\,x\right)}{64} - \frac{\sin\left(6\,x\right)}{192}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^6,x)
```

```
[Out] (5*x)/16 - (15*sin(2*x))/64 + (3*sin(4*x))/64 - sin(6*x)/192
```

sympy [A] time = 0.07, size = 36, normalized size = 1.06

$$\frac{5x}{16} - \frac{\sin^5\left(x\right)\cos\left(x\right)}{6} - \frac{5\sin^3\left(x\right)\cos\left(x\right)}{24} - \frac{5\sin\left(x\right)\cos\left(x\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**6,x)
```

```
[Out] 5*x/16 - sin(x)**5*cos(x)/6 - 5*sin(x)**3*cos(x)/24 - 5*sin(x)*cos(x)/16
```

3.33 $\int x\sin^2(x)\,dx$

Optimal. Leaf size=25

$$\frac{x^2}{4}+\frac{\sin^2(x)}{4}-\frac{1}{2}x\sin(x)\cos(x)$$

```
[Out] 1/4*x^2-1/2*x*cos(x)*sin(x)+1/4*sin(x)^2
```

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3310, 30}

$$\frac{x^2}{4}+\frac{\sin^2(x)}{4}-\frac{1}{2}x\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sin[x]^2,x]
```

```
[Out] x^2/4 - (x*Cos[x]*Sin[x])/2 + Sin[x]^2/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\begin{aligned}\int x\sin^2(x)\,dx &= -\frac{1}{2}x\cos(x)\sin(x)+\frac{\sin^2(x)}{4}+\frac{\int x\,dx}{2}\\ &= \frac{x^2}{4}-\frac{1}{2}x\cos(x)\sin(x)+\frac{\sin^2(x)}{4}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{x^2}{4}-\frac{1}{4}x\sin(2x)-\frac{1}{8}\cos(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sin[x]^2,x]
```

```
[Out] x^2/4 - Cos[2*x]/8 - (x*Sin[2*x])/4
```

fricas [A] time = 0.43, size = 19, normalized size = 0.76

$$-\frac{1}{2}x\cos(x)\sin(x)+\frac{1}{4}x^2-\frac{1}{4}\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)^2,x, algorithm="fricas")
[Out] -1/2*x*cos(x)*sin(x) + 1/4*x^2 - 1/4*cos(x)^2
```

giac [A] time = 0.01, size = 19, normalized size = 0.76

$$\frac{1}{4}\,x^2 - \frac{1}{4}\,x \sin\left(2\,x\right) - \frac{1}{8}\,\cos\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)^2,x, algorithm="giac")
[Out] 1/4*x^2 - 1/4*x*sin(2*x) - 1/8*cos(2*x)
```

maple [A] time = 0.01, size = 25, normalized size = 1.00

$$-\frac{x^2}{4} + \frac{\left(\sin^2(x)\right)}{4} + \left(-\frac{\cos(x)\sin(x)}{2} + \frac{x}{2}\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x)^2,x)
[Out] x*(-1/2*cos(x)*sin(x)+1/2*x)-1/4*x^2+1/4*sin(x)^2
```

maxima [A] time = 0.50, size = 19, normalized size = 0.76

$$\frac{1}{4}\,x^2 - \frac{1}{4}\,x \sin\left(2\,x\right) - \frac{1}{8}\,\cos\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)^2,x, algorithm="maxima")
[Out] 1/4*x^2 - 1/4*x*sin(2*x) - 1/8*cos(2*x)
```

mupad [B] time = 0.09, size = 19, normalized size = 0.76

$$\frac{\sin(x)^2}{4} - \frac{x\,\sin\left(2\,x\right)}{4} + \frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x)^2,x)
[Out] sin(x)^2/4 - (x*sin(2*x))/4 + x^2/4
```

sympy [A] time = 0.36, size = 36, normalized size = 1.44

$$\frac{x^2 \sin^2\left(x\right)}{4} + \frac{x^2 \cos^2\left(x\right)}{4} - \frac{x \sin\left(x\right) \cos\left(x\right)}{2} + \frac{\sin^2\left(x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)**2,x)
[Out] x**2*sin(x)**2/4 + x**2*cos(x)**2/4 - x*sin(x)*cos(x)/2 + sin(x)**2/4
```

3.34 $\int x\sin^3(x)\,dx$

Optimal. Leaf size=33

$$\frac{\sin^3(x)}{9}+\frac{2\sin(x)}{3}-\frac{2}{3}x\cos(x)-\frac{1}{3}x\sin^2(x)\cos(x)$$

```
[Out] -2/3*x*cos(x)+2/3*sin(x)-1/3*x*cos(x)*sin(x)^2+1/9*sin(x)^3
```

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3310, 3296, 2637}

$$\frac{\sin^3(x)}{9}+\frac{2\sin(x)}{3}-\frac{2}{3}x\cos(x)-\frac{1}{3}x\sin^2(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sin[x]^3,x]
```

```
[Out] (-2*x*Cos[x])/3 + (2*Sin[x])/3 - (x*Cos[x]*Sin[x]^2)/3 + Sin[x]^3/9
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\begin{aligned}\int x\sin^3(x)\,dx&=-\frac{1}{3}x\cos(x)\sin^2(x)+\frac{\sin^3(x)}{9}+\frac{2}{3}\int x\sin(x)\,dx\\&=-\frac{2}{3}x\cos(x)-\frac{1}{3}x\cos(x)\sin^2(x)+\frac{\sin^3(x)}{9}+\frac{2}{3}\int\cos(x)\,dx\\&=-\frac{2}{3}x\cos(x)+\frac{2\sin(x)}{3}-\frac{1}{3}x\cos(x)\sin^2(x)+\frac{\sin^3(x)}{9}\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 0.94

$$\frac{3\sin(x)}{4}-\frac{1}{36}\sin(3x)-\frac{3}{4}x\cos(x)+\frac{1}{12}x\cos(3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sin[x]^3,x]
```

```
[Out] (-3*x*Cos[x])/4 + (x*Cos[3*x])/12 + (3*Sin[x])/4 - Sin[3*x]/36
```

fricas [A] time = 0.42, size = 23, normalized size = 0.70

$$\frac{1}{3}\,x\cos(x)^3 - x\cos(x) - \frac{1}{9}\left(\cos(x)^2 - 7\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)^3,x, algorithm="fricas")
[Out] 1/3*x*cos(x)^3 - x*cos(x) - 1/9*(cos(x)^2 - 7)*sin(x)
```

giac [A] time = 0.01, size = 23, normalized size = 0.70

$$\frac{1}{12}\,x\cos(3\,x) - \frac{3}{4}\,x\cos(x) - \frac{1}{36}\,\sin(3\,x) + \frac{3}{4}\,\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)^3,x, algorithm="giac")
[Out] 1/12*x*cos(3*x) - 3/4*x*cos(x) - 1/36*sin(3*x) + 3/4*sin(x)
```

maple [A] time = 0.06, size = 23, normalized size = 0.70

$$\frac{\left(\sin^3(x)\right)}{9} - \frac{\left(\sin^2(x) + 2\right)x\cos(x)}{3} + \frac{2\sin(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x)^3,x)
[Out] -1/3*x*(sin(x)^2+2)*cos(x)+1/9*sin(x)^3+2/3*sin(x)
```

maxima [A] time = 0.57, size = 23, normalized size = 0.70

$$\frac{1}{12}\,x\cos(3\,x) - \frac{3}{4}\,x\cos(x) - \frac{1}{36}\,\sin(3\,x) + \frac{3}{4}\,\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)^3,x, algorithm="maxima")
[Out] 1/12*x*cos(3*x) - 3/4*x*cos(x) - 1/36*sin(3*x) + 3/4*sin(x)
```

mupad [B] time = 0.12, size = 25, normalized size = 0.76

$$\frac{x\cos(x)^3}{3} - \frac{\sin(x)\cos(x)^2}{9} - x\,\cos(x) + \frac{7\,\sin(x)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x)^3,x)
[Out] (7*sin(x))/9 + (x*cos(x)^3)/3 - (cos(x)^2*sin(x))/9 - x*cos(x)
```

sympy [A] time = 0.65, size = 39, normalized size = 1.18

$$-x\sin^2(x)\cos(x) - \frac{2x\cos^3(x)}{3} + \frac{7\sin^3(x)}{9} + \frac{2\sin(x)\cos^2(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)**3,x)
[Out] -x*sin(x)**2*cos(x) - 2*x*cos(x)**3/3 + 7*sin(x)**3/9 + 2*sin(x)*cos(x)**2/
3
```

3.35 $\int x^2 \sin^2(x)\,dx$

Optimal. Leaf size=41

$$\frac{x^3}{6}-\frac{1}{2}x^2\sin(x)\cos(x)-\frac{x}{4}+\frac{1}{2}x\sin^2(x)+\frac{1}{4}\sin(x)\cos(x)$$

```
[Out] -1/4*x+1/6*x^3+1/4*cos(x)*sin(x)-1/2*x^2*cos(x)*sin(x)+1/2*x*sin(x)^2
```

Rubi [A] time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3311, 30, 2635, 8}

$$\frac{x^3}{6}-\frac{1}{2}x^2\sin(x)\cos(x)-\frac{x}{4}+\frac{1}{2}x\sin^2(x)+\frac{1}{4}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sin[x]^2,x]
```

```
[Out] -x/4 + x^3/6 + (Cos[x]*Sin[x])/4 - (x^2*Cos[x]*Sin[x])/2 + (x*Sin[x]^2)/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rubi steps

$$\begin{aligned}
\int x^2\sin^2(x)\,dx &= -\frac{1}{2}x^2\cos(x)\sin(x)+\frac{1}{2}x\sin^2(x)+\frac{\int x^2\,dx}{2}-\frac{1}{2}\int\sin^2(x)\,dx\\
&= \frac{x^3}{6}+\frac{1}{4}\cos(x)\sin(x)-\frac{1}{2}x^2\cos(x)\sin(x)+\frac{1}{2}x\sin^2(x)-\frac{\int 1\,dx}{4}\\
&= -\frac{x}{4}+\frac{x^3}{6}+\frac{1}{4}\cos(x)\sin(x)-\frac{1}{2}x^2\cos(x)\sin(x)+\frac{1}{2}x\sin^2(x)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 29, normalized size = 0.71

$$\frac{1}{24}\left(4x^3+\left(3-6x^2\right)\sin(2x)-6x\cos(2x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sin[x]^2,x]
```

```
[Out] (4*x^3 - 6*x*Cos[2*x] + (3 - 6*x^2)*Sin[2*x])/24
```

fricas [A] time = 0.44, size = 29, normalized size = 0.71

$$\frac{1}{6}x^3 - \frac{1}{2}x\cos(x)^2 - \frac{1}{4}\left(2x^2 - 1\right)\cos(x)\sin(x) + \frac{1}{4}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(x)^2,x, algorithm="fricas")
```

```
[Out] 1/6*x^3 - 1/2*x*cos(x)^2 - 1/4*(2*x^2 - 1)*cos(x)*sin(x) + 1/4*x
```

giac [A] time = 0.01, size = 26, normalized size = 0.63

$$\frac{1}{6}x^3 - \frac{1}{4}x\cos(2x) - \frac{1}{8}\left(2x^2 - 1\right)\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(x)^2,x, algorithm="giac")
```

```
[Out] 1/6*x^3 - 1/4*x*cos(2*x) - 1/8*(2*x^2 - 1)*sin(2*x)
```

maple [A] time = 0.04, size = 37, normalized size = 0.90

$$-\frac{x^3}{3} - \frac{x\left(\cos^2(x)\right)}{2} + \left(-\frac{\cos(x)\sin(x)}{2} + \frac{x}{2}\right)x^2 + \frac{\cos(x)\sin(x)}{4} + \frac{x}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(x)^2,x)
```

```
[Out] x^2*(-1/2*cos(x)*sin(x)+1/2*x)-1/2*x*cos(x)^2+1/4*cos(x)*sin(x)+1/4*x-1/3*x
^3
```

maxima [A] time = 0.50, size = 26, normalized size = 0.63

$$\frac{1}{6}x^3 - \frac{1}{4}x\cos(2x) - \frac{1}{8}\left(2x^2 - 1\right)\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(x)^2,x, algorithm="maxima")
```

```
[Out] 1/6*x^3 - 1/4*x*cos(2*x) - 1/8*(2*x^2 - 1)*sin(2*x)
```

mupad [B] time = 0.06, size = 28, normalized size = 0.68

$$\frac{\sin(2x)}{8} - \frac{x\cos(2x)}{4} - \frac{x^2\sin(2x)}{4} + \frac{x^3}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(x)^2,x)
```

```
[Out] sin(2*x)/8 - (x*cos(2*x))/4 - (x^2*sin(2*x))/4 + x^3/6
```

sympy [A] time = 0.65, size = 56, normalized size = 1.37

$$\frac{x^3\sin^2(x)}{6} + \frac{x^3\cos^2(x)}{6} - \frac{x^2\sin(x)\cos(x)}{2} + \frac{x\sin^2(x)}{4} - \frac{x\cos^2(x)}{4} + \frac{\sin(x)\cos(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(x)**2,x)
```

```
[Out] x**3*sin(x)**2/6 + x**3*cos(x)**2/6 - x**2*sin(x)*cos(x)/2 + x*sin(x)**2/4
- x*cos(x)**2/4 + sin(x)*cos(x)/4
```

3.36 $\int \cos^2(x)\,dx$

Optimal. Leaf size=14

$$\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)$$

```
[Out] 1/2*x+1/2*cos(x)*sin(x)
```

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2635, 8}

$$\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^2,x]
```

```
[Out] x/2 + (Cos[x]*Sin[x])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\begin{aligned}\int \cos^2(x)\,dx &= \frac{1}{2}\cos(x)\sin(x)+\frac{\int 1\,dx}{2}\\ &= \frac{x}{2}+\frac{1}{2}\cos(x)\sin(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{x}{2}+\frac{1}{4}\sin(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2,x]
```

```
[Out] x/2 + Sin[2*x]/4
```

fricas [A] time = 0.43, size = 10, normalized size = 0.71

$$\frac{1}{2}\cos(x)\sin(x)+\frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*cos(x)*sin(x) + 1/2*x
```

giac [A] time = 0.01, size = 10, normalized size = 0.71

$$\frac{1}{2}\,x + \frac{1}{4}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*x + 1/4*sin(2*x)
```

maple [A] time = 0.02, size = 11, normalized size = 0.79

$$\frac{\cos(x)\sin(x)}{2} + \frac{x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2,x)
```

```
[Out] 1/2*x+1/2*cos(x)*sin(x)
```

maxima [A] time = 0.52, size = 10, normalized size = 0.71

$$\frac{1}{2}\,x + \frac{1}{4}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x + 1/4*sin(2*x)
```

mupad [B] time = 0.03, size = 10, normalized size = 0.71

$$\frac{x}{2} + \frac{\sin\left(2\,x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2,x)
```

```
[Out] x/2 + sin(2*x)/4
```

sympy [A] time = 0.06, size = 10, normalized size = 0.71

$$\frac{x}{2} + \frac{\sin\left(x\right)\cos\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2,x)
```

```
[Out] x/2 + sin(x)*cos(x)/2
```

3.37 $\int \cos^3(x)\,dx$

Optimal. Leaf size=11

$$\sin(x)-\frac{\sin^3(x)}{3}$$

```
[Out] sin(x)-1/3*sin(x)^3
```

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2633}

$$\sin(x)-\frac{\sin^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^3,x]
```

```
[Out] Sin[x] - Sin[x]^3/3
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^3(x)\,dx &= -\operatorname{Subst}\left(\int \left(1-x^2\right)\,dx, x, -\sin(x)\right)\\ &= \sin(x)-\frac{\sin^3(x)}{3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.36

$$\frac{3\sin(x)}{4}+\frac{1}{12}\sin(3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^3,x]
```

```
[Out] (3*Sin[x])/4 + Sin[3*x]/12
```

fricas [A] time = 0.41, size = 10, normalized size = 0.91

$$\frac{1}{3}\left(\cos(x)^2+2\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3,x, algorithm="fricas")
```

```
[Out] 1/3*(cos(x)^2 + 2)*sin(x)
```

giac [A] time = 0.01, size = 9, normalized size = 0.82

$$-\frac{1}{3}\sin(x)^3+\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3,x, algorithm="giac")
```

```
[Out] -1/3*sin(x)^3 + sin(x)
```

maple [A] time = 0.09, size = 11, normalized size = 1.00

$$\frac{\left(\cos^{2}(x)+2\right)\sin(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3,x)
```

```
[Out] 1/3*(2+cos(x)^2)*sin(x)
```

maxima [A] time = 0.58, size = 9, normalized size = 0.82

$$-\frac{1}{3}\sin(x)^{3}+\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3,x, algorithm="maxima")
```

```
[Out] -1/3*sin(x)^3 + sin(x)
```

mupad [B] time = 0.03, size = 9, normalized size = 0.82

$$\sin(x)-\frac{\sin(x)^{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3,x)
```

```
[Out] sin(x) - sin(x)^3/3
```

sympy [A] time = 0.07, size = 8, normalized size = 0.73

$$-\frac{\sin^{3}(x)}{3}+\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3,x)
```

```
[Out] -sin(x)**3/3 + sin(x)
```

3.38 $\int \cos^4(x)\,dx$

Optimal. Leaf size=24

$$\frac{3x}{8}+\frac{1}{4}\sin(x)\cos^3(x)+\frac{3}{8}\sin(x)\cos(x)$$

```
[Out] 3/8*x+3/8*cos(x)*sin(x)+1/4*cos(x)^3*sin(x)
```

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2635, 8}

$$\frac{3x}{8}+\frac{1}{4}\sin(x)\cos^3(x)+\frac{3}{8}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^4,x]
```

```
[Out] (3*x)/8 + (3*Cos[x]*Sin[x])/8 + (Cos[x]^3*Sin[x])/4
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\begin{aligned}\int \cos^4(x)\,dx &= \frac{1}{4}\cos^3(x)\sin(x)+\frac{3}{4}\int \cos^2(x)\,dx\\ &= \frac{3}{8}\cos(x)\sin(x)+\frac{1}{4}\cos^3(x)\sin(x)+\frac{3\int 1\,dx}{8}\\ &= \frac{3x}{8}+\frac{3}{8}\cos(x)\sin(x)+\frac{1}{4}\cos^3(x)\sin(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 0.92

$$\frac{3x}{8}+\frac{1}{4}\sin(2x)+\frac{1}{32}\sin(4x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^4,x]
```

```
[Out] (3*x)/8 + Sin[2*x]/4 + Sin[4*x]/32
```

fricas [A] time = 0.43, size = 19, normalized size = 0.79

$$\frac{1}{8}\left(2\,\cos(x)^3+3\,\cos(x)\right)\sin(x)+\frac{3}{8}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4,x, algorithm="fricas")
[Out] 1/8*(2*cos(x)^3 + 3*cos(x))*sin(x) + 3/8*x
```

giac [A] time = 0.01, size = 16, normalized size = 0.67

$$\frac{3}{8}\,x + \frac{1}{32}\,\sin\left(4\,x\right) + \frac{1}{4}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4,x, algorithm="giac")
[Out] 3/8*x + 1/32*sin(4*x) + 1/4*sin(2*x)
```

maple [A] time = 0.11, size = 18, normalized size = 0.75

$$\frac{3x}{8} + \frac{\left(\cos^{3}(x) + \frac{3\cos(x)}{2}\right)\sin(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^4,x)
[Out] 1/4*(cos(x)^3+3/2*cos(x))*sin(x)+3/8*x
```

maxima [A] time = 0.54, size = 16, normalized size = 0.67

$$\frac{3}{8}\,x + \frac{1}{32}\,\sin\left(4\,x\right) + \frac{1}{4}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4,x, algorithm="maxima")
[Out] 3/8*x + 1/32*sin(4*x) + 1/4*sin(2*x)
```

mupad [B] time = 0.03, size = 16, normalized size = 0.67

$$\frac{3\,x}{8} + \frac{\sin\left(2\,x\right)}{4} + \frac{\sin\left(4\,x\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^4,x)
[Out] (3*x)/8 + sin(2*x)/4 + sin(4*x)/32
```

sympy [A] time = 0.06, size = 24, normalized size = 1.00

$$\frac{3x}{8} + \frac{\sin\left(x\right)\cos^{3}\left(x\right)}{4} + \frac{3\sin\left(x\right)\cos\left(x\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**4,x)
[Out] 3*x/8 + sin(x)*cos(x)**3/4 + 3*sin(x)*cos(x)/8
```

3.39 $\int \left(a^2-x^2\right)^{5/2} \, dx$

Optimal. Leaf size=84

$$\frac{5}{24} a^2 x \left(a^2-x^2\right)^{3/2}+\frac{1}{6} x \left(a^2-x^2\right)^{5/2}+\frac{5}{16} a^6 \tan ^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)+\frac{5}{16} a^4 x \sqrt{a^2-x^2}$$

```
[Out] 5/24*a^2*x*(a^2-x^2)^(3/2)+1/6*x*(a^2-x^2)^(5/2)+5/16*a^6*arctan(x/(a^2-x^2
)^(1/2))+5/16*a^4*x*(a^2-x^2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {195, 217, 203}

$$\frac{5}{24} a^2 x \left(a^2-x^2\right)^{3/2}+\frac{1}{6} x \left(a^2-x^2\right)^{5/2}+\frac{5}{16} a^6 \tan ^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)+\frac{5}{16} a^4 x \sqrt{a^2-x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 - x^2)^(5/2),x]
```

```
[Out] (5*a^4*x*Sqrt[a^2 - x^2])/16 + (5*a^2*x*(a^2 - x^2)^(3/2))/24 + (x*(a^2 - x
^2)^(5/2))/6 + (5*a^6*ArcTan[x/Sqrt[a^2 - x^2]])/16
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \left(a^2-x^2\right)^{5/2} \, dx &= \frac{1}{6} x \left(a^2-x^2\right)^{5/2}+\frac{1}{6} \left(5 a^2\right) \int \left(a^2-x^2\right)^{3/2} \, dx\\
&= \frac{5}{24} a^2 x \left(a^2-x^2\right)^{3/2}+\frac{1}{6} x \left(a^2-x^2\right)^{5/2}+\frac{1}{8} \left(5 a^4\right) \int \sqrt{a^2-x^2} \, dx\\
&= \frac{5}{16} a^4 x \sqrt{a^2-x^2}+\frac{5}{24} a^2 x \left(a^2-x^2\right)^{3/2}+\frac{1}{6} x \left(a^2-x^2\right)^{5/2}+\frac{1}{16} \left(5 a^6\right) \int \frac{1}{\sqrt{a^2-x^2}} \, dx\\
&= \frac{5}{16} a^4 x \sqrt{a^2-x^2}+\frac{5}{24} a^2 x \left(a^2-x^2\right)^{3/2}+\frac{1}{6} x \left(a^2-x^2\right)^{5/2}+\frac{1}{16} \left(5 a^6\right) \text{Subst}\left(\int \frac{1}{1+x^2} \, dx,x,\frac{}{\sqrt{}}\right.\\
&= \frac{5}{16} a^4 x \sqrt{a^2-x^2}+\frac{5}{24} a^2 x \left(a^2-x^2\right)^{3/2}+\frac{1}{6} x \left(a^2-x^2\right)^{5/2}+\frac{5}{16} a^6 \tan ^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.08, size = 62, normalized size = 0.74

$$\frac{1}{48}\sqrt{a^2-x^2}\left(33a^4x-26a^2x^3+\frac{15a^5\sin^{-1}\left(\frac{x}{a}\right)}{\sqrt{1-\frac{x^2}{a^2}}}+8x^5\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 - x^2)^(5/2),x]
```

```
[Out] (Sqrt[a^2 - x^2]*(33*a^4*x - 26*a^2*x^3 + 8*x^5 + (15*a^5*ArcSin[x/a])/Sqrt
[1 - x^2/a^2]))/48
```

fricas [A] time = 0.42, size = 60, normalized size = 0.71

$$-\frac{5}{8}a^6\arctan\left(-\frac{a-\sqrt{a^2-x^2}}{x}\right)+\frac{1}{48}\left(33a^4x-26a^2x^3+8x^5\right)\sqrt{a^2-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -5/8*a^6*arctan(-(a - sqrt(a^2 - x^2))/x) + 1/48*(33*a^4*x - 26*a^2*x^3 + 8
*x^5)*sqrt(a^2 - x^2)
```

giac [A] time = 0.05, size = 50, normalized size = 0.60

$$\frac{5}{16}a^6\arcsin\left(\frac{x}{a}\right)\mathrm{sgn}(a)+\frac{1}{48}\left(33a^4-2\left(13a^2-4x^2\right)x^2\right)\sqrt{a^2-x^2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-x^2)^(5/2),x, algorithm="giac")
```

```
[Out] 5/16*a^6*arcsin(x/a)*sgn(a) + 1/48*(33*a^4 - 2*(13*a^2 - 4*x^2)*x^2)*sqrt(a
^2 - x^2)*x
```

maple [A] time = 0.01, size = 69, normalized size = 0.82

$$\frac{5a^6\arctan\left(\frac{x}{\sqrt{a^2-x^2}}\right)}{16}+\frac{5\sqrt{a^2-x^2}\,a^4x}{16}+\frac{5\left(a^2-x^2\right)^{\frac{3}{2}}a^2x}{24}+\frac{\left(a^2-x^2\right)^{\frac{5}{2}}x}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2-x^2)^(5/2),x)
```

```
[Out] 5/24*a^2*x*(a^2-x^2)^(3/2)+1/6*x*(a^2-x^2)^(5/2)+5/16*a^6*arctan(x/(a^2-x^2
)^(1/2))+5/16*a^4*x*(a^2-x^2)^(1/2)
```

maxima [A] time = 1.31, size = 60, normalized size = 0.71

$$\frac{5}{16}a^6\arcsin\left(\frac{x}{a}\right)+\frac{5}{16}\sqrt{a^2-x^2}\,a^4x+\frac{5}{24}\left(a^2-x^2\right)^{\frac{3}{2}}a^2x+\frac{1}{6}\left(a^2-x^2\right)^{\frac{5}{2}}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 5/16*a^6*arcsin(x/a) + 5/16*sqrt(a^2 - x^2)*a^4*x + 5/24*(a^2 - x^2)^(3/2)*
a^2*x + 1/6*(a^2 - x^2)^(5/2)*x
```

mupad [B] time = 0.21, size = 37, normalized size = 0.44

$$\frac{x\left(a^2-x^2\right)^{5/2}\,{}_2\mathrm{F}_1\left(-\frac{5}{2},\frac{1}{2};\ \frac{3}{2};\ \frac{x^2}{a^2}\right)}{\left(1-\frac{x^2}{a^2}\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 - x^2)^(5/2),x)
```

```
[Out] (x*(a^2 - x^2)^(5/2)*hypergeom([-5/2, 1/2], 3/2, x^2/a^2))/(1 - x^2/a^2)^(5
/2)
```

sympy [A] time = 3.71, size = 180, normalized size = 2.14

$$\begin{cases} -\frac{5ia^6\operatorname{acosh}\left(\frac{x}{a}\right)}{16}-\frac{11ia^5x}{16\sqrt{-1+\frac{x^2}{a^2}}}+\frac{59ia^3x^3}{48\sqrt{-1+\frac{x^2}{a^2}}}-\frac{17iax^5}{24\sqrt{-1+\frac{x^2}{a^2}}}+\frac{ix^7}{6a\sqrt{-1+\frac{x^2}{a^2}}} & \text{for } \left|\frac{x^2}{a^2}\right|>1 \\ \frac{5a^6\operatorname{asin}\left(\frac{x}{a}\right)}{16}+\frac{11a^5x\sqrt{1-\frac{x^2}{a^2}}}{16}-\frac{13a^3x^3\sqrt{1-\frac{x^2}{a^2}}}{24}+\frac{ax^5\sqrt{1-\frac{x^2}{a^2}}}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2-x**2)**(5/2),x)
```

```
[Out] Piecewise((-5*I*a**6*acosh(x/a)/16 - 11*I*a**5*x/(16*sqrt(-1 + x**2/a**2))
+ 59*I*a**3*x**3/(48*sqrt(-1 + x**2/a**2)) - 17*I*a*x**5/(24*sqrt(-1 + x**2
/a**2)) + I*x**7/(6*a*sqrt(-1 + x**2/a**2)), Abs(x**2/a**2) > 1), (5*a**6*a
sin(x/a)/16 + 11*a**5*x*sqrt(1 - x**2/a**2)/16 - 13*a**3*x**3*sqrt(1 - x**2
/a**2)/24 + a*x**5*sqrt(1 - x**2/a**2)/6, True))
```

3.40 $\int \frac{x^5}{\sqrt{5+x^2}}\,dx$

Optimal. Leaf size=38

$$\frac{1}{5}\left(x^2+5\right)^{5/2}-\frac{10}{3}\left(x^2+5\right)^{3/2}+25\sqrt{x^2+5}$$

```
[Out] -10/3*(x^2+5)^(3/2)+1/5*(x^2+5)^(5/2)+25*(x^2+5)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{1}{5}\left(x^2+5\right)^{5/2}-\frac{10}{3}\left(x^2+5\right)^{3/2}+25\sqrt{x^2+5}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[5 + x^2],x]
[Out] 25*Sqrt[5 + x^2] - (10*(5 + x^2)^(3/2))/3 + (5 + x^2)^(5/2)/5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\sqrt{5+x^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{\sqrt{5+x}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{25}{\sqrt{5+x}}-10\sqrt{5+x}+(5+x)^{3/2}\right)dx,x,x^2\right)\\ &= 25\sqrt{5+x^2}-\frac{10}{3}\left(5+x^2\right)^{3/2}+\frac{1}{5}\left(5+x^2\right)^{5/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.66

$$\frac{1}{15}\sqrt{x^2+5}\left(3x^4-20x^2+200\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[5 + x^2],x]
[Out] (Sqrt[5 + x^2]*(200 - 20*x^2 + 3*x^4))/15
```

fricas [A] time = 0.42, size = 21, normalized size = 0.55

$$\frac{1}{15}\left(3\,x^4-20\,x^2+200\right)\sqrt{x^2+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^2+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*x^4 - 20*x^2 + 200)*sqrt(x^2 + 5)
```

giac [A] time = 0.01, size = 28, normalized size = 0.74

$$\frac{1}{5}\left(x^2+5\right)^{\frac{5}{2}}-\frac{10}{3}\left(x^2+5\right)^{\frac{3}{2}}+25\sqrt{x^2+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^2+5)^(1/2),x, algorithm="giac")
```

```
[Out] 1/5*(x^2 + 5)^(5/2) - 10/3*(x^2 + 5)^(3/2) + 25*sqrt(x^2 + 5)
```

maple [A] time = 0.01, size = 22, normalized size = 0.58

$$\frac{\sqrt{x^2+5}\,\left(3x^4-20x^2+200\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^2+5)^(1/2),x)
```

```
[Out] 1/15*(x^2+5)^(1/2)*(3*x^4-20*x^2+200)
```

maxima [A] time = 1.43, size = 34, normalized size = 0.89

$$\frac{1}{5}\sqrt{x^2+5}\,x^4-\frac{4}{3}\sqrt{x^2+5}\,x^2+\frac{40}{3}\sqrt{x^2+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^2+5)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(x^2 + 5)*x^4 - 4/3*sqrt(x^2 + 5)*x^2 + 40/3*sqrt(x^2 + 5)
```

mupad [B] time = 0.03, size = 20, normalized size = 0.53

$$\sqrt{x^2+5}\,\left(\frac{x^4}{5}-\frac{4\,x^2}{3}+\frac{40}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^2 + 5)^(1/2),x)
```

```
[Out] (x^2 + 5)^(1/2)*(x^4/5 - (4*x^2)/3 + 40/3)
```

sympy [A] time = 1.24, size = 39, normalized size = 1.03

$$\frac{x^4\sqrt{x^2+5}}{5}-\frac{4x^2\sqrt{x^2+5}}{3}+\frac{40\sqrt{x^2+5}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(x**2+5)**(1/2),x)
```

```
[Out] x**4*sqrt(x**2 + 5)/5 - 4*x**2*sqrt(x**2 + 5)/3 + 40*sqrt(x**2 + 5)/3
```

3.41 $\int \frac{t^3}{\sqrt{4+t^3}}\, dt$

Optimal. Leaf size=172

$$\frac{2}{5}t\sqrt{t^3+4}-\frac{8\, 2^{2/3}\sqrt{2+\sqrt{3}}\,\left(t+2^{2/3}\right)\sqrt{\frac{t^2-2^{2/3}t+2\sqrt[3]{2}}{\left(t+2^{2/3}\left(1+\sqrt{3}\right)\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{t+2^{2/3}\left(1-\sqrt{3}\right)}{t+2^{2/3}\left(1+\sqrt{3}\right)}\right),-7-4\sqrt{3}\right)}{5\sqrt[4]{3}\sqrt{\frac{t+2^{2/3}}{\left(t+2^{2/3}\left(1+\sqrt{3}\right)\right)^2}}\,\sqrt{t^3+4}}$$

```
[Out] 2/5*t*(t^3+4)^(1/2)-8/15*2^(2/3)*(2^(2/3)+t)*EllipticF((t+2^(2/3)*(1-3^(1/2
)))/(t+2^(2/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((2*2^
(1/3)-2^(2/3)*t+t^2)/(t+2^(2/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/(t^3+4)^(1/2)
/((2^(2/3)+t)/(t+2^(2/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {321, 218}

$$\frac{2}{5}t\sqrt{t^3+4}-\frac{8\, 2^{2/3}\sqrt{2+\sqrt{3}}\,\left(t+2^{2/3}\right)\sqrt{\frac{t^2-2^{2/3}t+2\sqrt[3]{2}}{\left(t+2^{2/3}\left(1+\sqrt{3}\right)\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{t+2^{2/3}\left(1-\sqrt{3}\right)}{t+2^{2/3}\left(1+\sqrt{3}\right)}\right),-7-4\sqrt{3}\right)}{5\sqrt[4]{3}\sqrt{\frac{t+2^{2/3}}{\left(t+2^{2/3}\left(1+\sqrt{3}\right)\right)^2}}\,\sqrt{t^3+4}}$$

Antiderivative was successfully verified.

```
[In] Int[t^3/Sqrt[4 + t^3],t]
```

```
[Out] (2*t*Sqrt[4 + t^3])/5 - (8*2^(2/3)*Sqrt[2 + Sqrt[3]]*(2^(2/3) + t)*Sqrt[(2*
2^(1/3) - 2^(2/3)*t + t^2)/(2^(2/3)*(1 + Sqrt[3]) + t)^2]*EllipticF[ArcSin[
(2^(2/3)*(1 - Sqrt[3]) + t)/(2^(2/3)*(1 + Sqrt[3]) + t)], -7 - 4*Sqrt[3]])/
(5*3^(1/4)*Sqrt[(2^(2/3) + t)/(2^(2/3)*(1 + Sqrt[3]) + t)^2]*Sqrt[4 + t^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{t^3}{\sqrt{4+t^3}}\,dt = \frac{2}{5}t\sqrt{4+t^3} - \frac{8}{5}\int \frac{1}{\sqrt{4+t^3}}\,dt$$

$$= \frac{2}{5}t\sqrt{4+t^3} - \frac{8\,2^{2/3}\sqrt{2+\sqrt{3}}\,\left(2^{2/3}+t\right)\sqrt{\frac{2\sqrt[3]{2}-2^{2/3}t+t^2}{\left(2^{2/3}(1+\sqrt{3})+t\right)^2}}\,F\left(\sin^{-1}\left(\frac{2^{2/3}(1-\sqrt{3})+t}{2^{2/3}(1+\sqrt{3})+t}\right)|-7-4\sqrt{3}\right)}{5\sqrt[4]{3}\sqrt{\frac{2^{2/3}+t}{\left(2^{2/3}(1+\sqrt{3})+t\right)^2}}\sqrt{4+t^3}}$$

Mathematica [C] time = 0.01, size = 34, normalized size = 0.20

$$\frac{2}{5}t\left(\sqrt{t^3+4}-2\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{t^3}{4}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[t^3/Sqrt[4 + t^3],t]
```

```
[Out] (2*t*(Sqrt[4 + t^3] - 2*Hypergeometric2F1[1/3, 1/2, 4/3, -1/4*t^3]))/5
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{t^3}{\sqrt{t^3+4}},t\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t^3/(t^3+4)^(1/2),t, algorithm="fricas")
```

```
[Out] integral(t^3/sqrt(t^3 + 4), t)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t^3/(t^3+4)^(1/2),t, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 2/5*t
*sqrt(t^3+4)+integrate(-8/5/sqrt(t^3+4),t)
```

maple [A] time = 0.18, size = 168, normalized size = 0.98

$$\frac{2\sqrt{t^3+4}\;t}{5}+\frac{8i\sqrt{3}\,2^{\frac{2}{3}}\sqrt{i\left(t-\frac{2^{\frac{2}{3}}}{2}-\frac{i\sqrt{3}\,2^{\frac{2}{3}}}{2}\right)\sqrt{3}\,2^{\frac{1}{3}}}\sqrt{\frac{t+2^{\frac{2}{3}}}{\frac{3\,2^{\frac{2}{3}}}{2}+\frac{i\sqrt{3}\,2^{\frac{2}{3}}}{2}}}\sqrt{-i\left(t-\frac{2^{\frac{2}{3}}}{2}+\frac{i\sqrt{3}\,2^{\frac{2}{3}}}{2}\right)\sqrt{3}\,2^{\frac{1}{3}}}\;\text{EllipticF}\left(\frac{\sqrt{6}\sqrt{i\left(t\right.}}{}\right.}{15\sqrt{t^3+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(t^3/(t^3+4)^(1/2),t)
```

```
[Out] 2/5*t*(t^3+4)^(1/2)+8/15*I*3^(1/2)*2^(2/3)*(I*(t-1/2*2^(2/3)-1/2*I*3^(1/2)*
2^(2/3))*3^(1/2)*2^(1/3))^(1/2)*((2^(2/3)+t)/(3/2*2^(2/3)+1/2*I*3^(1/2)*2^(
2/3)))^(1/2)*(-I*(t-1/2*2^(2/3)+1/2*I*3^(1/2)*2^(2/3))*3^(1/2)*2^(1/3))^(1/
2)/(t^3+4)^(1/2)*EllipticF(1/6*6^(1/2)*(I*(t-1/2*2^(2/3)-1/2*I*3^(1/2)*2^(2
```

```
/3))*3^(1/2)*2^(1/3))^(1/2),(I*3^(1/2)*2^(2/3)/(3/2*2^(2/3)+1/2*I*3^(1/2)*2
^(2/3)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{t^3}{\sqrt{t^3+4}}\,dt$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t^3/(t^3+4)^(1/2),t, algorithm="maxima")
```

```
[Out] integrate(t^3/sqrt(t^3 + 4), t)
```

mupad [B] time = 0.08, size = 301, normalized size = 1.75

$$\frac{2\,t\,\sqrt{t^3+4}}{5}-\frac{16\,\sqrt{-\frac{t-2^{2/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}{2^{2/3}+2^{2/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}\,\sqrt{-\frac{t+2^{2/3}\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}{2^{2/3}-2^{2/3}\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}\,\sqrt{\frac{t+2^{2/3}}{2^{2/3}+2^{2/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}\,\left(2^{2/3}+2^{2/3}\left(\frac{1}{2}+\right.\right.}{5\sqrt{t^3+\left(2^{2/3}+2^{2/3}\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-2^{2/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\right)t^2+\left(2\,2^{1/3}\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-2\,2^{1/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1}{2}\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(t^3/(t^3 + 4)^(1/2),t)
```

```
[Out] (2*t*(t^3 + 4)^(1/2))/5 - (16*(-(t - 2^(2/3)*((3^(1/2)*1i)/2 + 1/2))/(2^(2/
3) + 2^(2/3)*((3^(1/2)*1i)/2 + 1/2)))^(1/2)*(-(t + 2^(2/3)*((3^(1/2)*1i)/2
- 1/2))/(2^(2/3) - 2^(2/3)*((3^(1/2)*1i)/2 - 1/2)))^(1/2)*((t + 2^(2/3))/(2
^(2/3) + 2^(2/3)*((3^(1/2)*1i)/2 + 1/2)))^(1/2)*(2^(2/3) + 2^(2/3)*((3^(1/2
)*1i)/2 + 1/2))*ellipticF(asin(((t + 2^(2/3))/(2^(2/3) + 2^(2/3)*((3^(1/2)*
1i)/2 + 1/2)))^(1/2)), (2^(2/3) + 2^(2/3)*((3^(1/2)*1i)/2 + 1/2))/(2^(2/3)
- 2^(2/3)*((3^(1/2)*1i)/2 - 1/2))))/(5*(t^2*(2^(2/3) + 2^(2/3)*((3^(1/2)*1i
)/2 - 1/2) - 2^(2/3)*((3^(1/2)*1i)/2 + 1/2)) - 4*((3^(1/2)*1i)/2 - 1/2)*((3
^(1/2)*1i)/2 + 1/2) + t^3 - t*(2*2^(1/3)*((3^(1/2)*1i)/2 + 1/2) - 2*2^(1/3)
*((3^(1/2)*1i)/2 - 1/2) + 2*2^(1/3)*((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2
+ 1/2)))^(1/2))
```

sympy [A] time = 0.72, size = 31, normalized size = 0.18

$$\frac{t^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\right|\frac{t^3e^{i\pi}}{4}\right)}{6\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t**3/(t**3+4)**(1/2),t)
```

```
[Out] t**4*gamma(4/3)*hyper((1/2, 4/3), (7/3,), t**3*exp_polar(I*pi)/4)/(6*gamma(
7/3))
```

3.42 $\int \tan^2(x)\,dx$

Optimal. Leaf size=6

$$\tan(x) - x$$

```
[Out] -x+tan(x)
```

Rubi [A] time = 0.00, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3473, 8}

$$\tan(x) - x$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^2,x]
```

```
[Out] -x + Tan[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned} \int \tan^2(x)\,dx &= \tan(x) - \int 1\,dx \\ &= -x + \tan(x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 6, normalized size = 1.00

$$\tan(x) - x$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^2,x]
```

```
[Out] -x + Tan[x]
```

fricas [A] time = 0.41, size = 6, normalized size = 1.00

$$-x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2,x, algorithm="fricas")
```

```
[Out] -x + tan(x)
```

giac [A] time = 0.01, size = 6, normalized size = 1.00

$$-x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2,x, algorithm="giac")
[Out] -x + tan(x)
```

maple [**A**] time = 0.00, size = 7, normalized size = 1.17

$$-x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2,x)
[Out] -x+tan(x)
```

maxima [**A**] time = 1.22, size = 6, normalized size = 1.00

$$-x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2,x, algorithm="maxima")
[Out] -x + tan(x)
```

mupad [**B**] time = 0.07, size = 6, normalized size = 1.00

$$\tan(x) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2,x)
[Out] tan(x) - x
```

sympy [**B**] time = 0.07, size = 7, normalized size = 1.17

$$-x + \frac{\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**2,x)
[Out] -x + sin(x)/cos(x)
```

3.43 $\int \tan^4(x)\,dx$

Optimal. Leaf size=14

$$x+\frac{\tan^3(x)}{3}-\tan(x)$$

```
[Out] x-tan(x)+1/3*tan(x)^3
```

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3473, 8}

$$x+\frac{\tan^3(x)}{3}-\tan(x)$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^4,x]
```

```
[Out] x - Tan[x] + Tan[x]^3/3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \tan^4(x)\,dx &= \frac{\tan^3(x)}{3}-\int \tan^2(x)\,dx\\ &= -\tan(x)+\frac{\tan^3(x)}{3}+\int 1\,dx\\ &= x-\tan(x)+\frac{\tan^3(x)}{3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.29

$$x-\frac{4\tan(x)}{3}+\frac{1}{3}\tan(x)\sec^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^4,x]
```

```
[Out] x - (4*Tan[x])/3 + (Sec[x]^2*Tan[x])/3
```

fricas [A] time = 0.41, size = 12, normalized size = 0.86

$$\frac{1}{3}\tan(x)^3+x-\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^4,x, algorithm="fricas")
```

```
[Out] 1/3*tan(x)^3 + x - tan(x)
```

giac [A] time = 0.01, size = 12, normalized size = 0.86

$$\frac{1}{3}\tan(x)^3 + x - \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^4,x, algorithm="giac")
[Out] 1/3*tan(x)^3 + x - tan(x)
```

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$\frac{\left(\tan^3(x)\right)}{3} + x - \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^4,x)
[Out] x-tan(x)+1/3*tan(x)^3
```

maxima [A] time = 1.27, size = 12, normalized size = 0.86

$$\frac{1}{3}\tan(x)^3 + x - \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^4,x, algorithm="maxima")
[Out] 1/3*tan(x)^3 + x - tan(x)
```

mupad [B] time = 0.03, size = 12, normalized size = 0.86

$$\frac{\tan(x)^3}{3} - \tan(x) + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^4,x)
[Out] x - tan(x) + tan(x)^3/3
```

sympy [A] time = 0.07, size = 19, normalized size = 1.36

$$x + \frac{\sin^3(x)}{3\cos^3(x)} - \frac{\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**4,x)
[Out] x + sin(x)**3/(3*cos(x)**3) - sin(x)/cos(x)
```

3.44 $\int \cot^2(x)\,dx$

Optimal. Leaf size=8

$$-x-\cot(x)$$

```
[Out] -x-cot(x)
```

Rubi [A] time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3473, 8}

$$-x-\cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^2,x]
[Out] -x - Cot[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \cot^2(x)\,dx &= -\cot(x)-\int 1\,dx\\ &= -x-\cot(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$-x-\cot(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^2,x]
[Out] -x - Cot[x]
```

fricas [B] time = 0.41, size = 20, normalized size = 2.50

$$-\frac{x\sin\left(2\,x\right)+\cos\left(2\,x\right)+1}{\sin\left(2\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2,x, algorithm="fricas")
[Out] -(x*sin(2*x) + cos(2*x) + 1)/sin(2*x)
```

giac [B] time = 0.02, size = 18, normalized size = 2.25

$$-x-\frac{1}{2\,\tan\left(\frac{1}{2}\,x\right)}+\frac{1}{2}\,\tan\left(\frac{1}{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2,x, algorithm="giac")
```

```
[Out] -x - 1/2/tan(1/2*x) + 1/2*tan(1/2*x)
```

maple [A] time = 0.00, size = 12, normalized size = 1.50

$$-x-\cot(x)+\frac{\pi}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^2,x)
```

```
[Out] -cot(x)+1/2*Pi-x
```

maxima [A] time = 1.27, size = 10, normalized size = 1.25

$$-x-\frac{1}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2,x, algorithm="maxima")
```

```
[Out] -x - 1/tan(x)
```

mupad [B] time = 0.06, size = 8, normalized size = 1.00

$$-x-\cot(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^2,x)
```

```
[Out] - x - cot(x)
```

sympy [A] time = 0.07, size = 8, normalized size = 1.00

$$-x-\frac{\cos{(x)}}{\sin{(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**2,x)
```

```
[Out] -x - cos(x)/sin(x)
```

3.45 $\int \cot^4(x)\,dx$

Optimal. Leaf size=12

$$x-\frac{1}{3}\cot^3(x)+\cot(x)$$

```
[Out] x+cot(x)-1/3*cot(x)^3
```

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3473, 8}

$$x-\frac{1}{3}\cot^3(x)+\cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^4,x]
```

```
[Out] x + Cot[x] - Cot[x]^3/3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \cot^4(x)\,dx &= -\frac{1}{3}\cot^3(x)-\int \cot^2(x)\,dx\\ &= \cot(x)-\frac{\cot^3(x)}{3}+\int 1\,dx\\ &= x+\cot(x)-\frac{\cot^3(x)}{3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.50

$$x+\frac{4\cot(x)}{3}-\frac{1}{3}\cot(x)\csc^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^4,x]
```

```
[Out] x + (4*Cot[x])/3 - (Cot[x]*Csc[x]^2)/3
```

fricas [B] time = 0.42, size = 48, normalized size = 4.00

$$\frac{4\,\cos(2\,x)^2+3\,(x\cos(2\,x)-x)\sin(2\,x)+2\,\cos(2\,x)-2}{3\,(\cos(2\,x)-1)\sin(2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(4*cos(2*x)^2 + 3*(x*cos(2*x) - x)*sin(2*x) + 2*cos(2*x) - 2)/((cos(2*x
) - 1)*sin(2*x))
```

giac [B] time = 0.02, size = 34, normalized size = 2.83

$$\frac{1}{24}\tan\left(\frac{1}{2}x\right)^3 + x + \frac{15\tan\left(\frac{1}{2}x\right)^2 - 1}{24\tan\left(\frac{1}{2}x\right)^3} - \frac{5}{8}\tan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^4,x, algorithm="giac")
```

```
[Out] 1/24*tan(1/2*x)^3 + x + 1/24*(15*tan(1/2*x)^2 - 1)/tan(1/2*x)^3 - 5/8*tan(1
/2*x)
```

maple [A] time = 0.00, size = 14, normalized size = 1.17

$$-\frac{\left(\cot^3(x)\right)}{3} + x + \cot(x) - \frac{\pi}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^4,x)
```

```
[Out] -1/3*cot(x)^3+cot(x)-1/2*Pi+x
```

maxima [A] time = 1.28, size = 16, normalized size = 1.33

$$x + \frac{3\tan(x)^2 - 1}{3\tan(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^4,x, algorithm="maxima")
```

```
[Out] x + 1/3*(3*tan(x)^2 - 1)/tan(x)^3
```

mupad [B] time = 0.02, size = 10, normalized size = 0.83

$$-\frac{\cot(x)^3}{3} + \cot(x) + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^4,x)
```

```
[Out] x + cot(x) - cot(x)^3/3
```

sympy [A] time = 0.08, size = 19, normalized size = 1.58

$$x + \frac{\cos(x)}{\sin(x)} - \frac{\cos^3(x)}{3\sin^3(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**4,x)
```

```
[Out] x + cos(x)/sin(x) - cos(x)**3/(3*sin(x)**3)
```

3.46 $\int (2+3x)\sin(5x)\,dx$

Optimal. Leaf size=22

$$\frac{3}{25}\sin(5x)-\frac{1}{5}(3x+2)\cos(5x)$$

[Out] -1/5*(2+3*x)*cos(5*x)+3/25*sin(5*x)

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3296, 2637}

$$\frac{3}{25}\sin(5x)-\frac{1}{5}(3x+2)\cos(5x)$$

Antiderivative was successfully verified.

[In] Int[(2 + 3*x)*Sin[5*x],x]

[Out] -((2 + 3*x)*Cos[5*x])/5 + (3*Sin[5*x])/25

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int (2+3x)\sin(5x)\,dx&=-\frac{1}{5}(2+3x)\cos(5x)+\frac{3}{5}\int \cos(5x)\,dx\\&=-\frac{1}{5}(2+3x)\cos(5x)+\frac{3}{25}\sin(5x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 1.18

$$\frac{3}{25}\sin(5x)-\frac{3}{5}x\cos(5x)-\frac{2}{5}\cos(5x)$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3*x)*Sin[5*x],x]

[Out] (-2*Cos[5*x])/5 - (3*x*Cos[5*x])/5 + (3*Sin[5*x])/25

fricas [A] time = 0.42, size = 18, normalized size = 0.82

$$-\frac{1}{5}(3x+2)\cos(5x)+\frac{3}{25}\sin(5x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+3*x)*sin(5*x),x, algorithm="fricas")

[Out] -1/5*(3*x + 2)*cos(5*x) + 3/25*sin(5*x)

giac [A] time = 0.01, size = 18, normalized size = 0.82

$$-\frac{1}{5}\,(3\,x+2)\cos\left(5\,x\right)+\frac{3}{25}\,\sin\left(5\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)*sin(5*x),x, algorithm="giac")
[Out] -1/5*(3*x + 2)*cos(5*x) + 3/25*sin(5*x)
```

maple [A] time = 0.02, size = 21, normalized size = 0.95

$$-\frac{3x\cos\left(5x\right)}{5}-\frac{2\cos\left(5x\right)}{5}+\frac{3\sin\left(5x\right)}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*x)*sin(5*x),x)
[Out] -2/5*cos(5*x)+3/25*sin(5*x)-3/5*cos(5*x)*x
```

maxima [A] time = 0.54, size = 20, normalized size = 0.91

$$-\frac{3}{5}\,x\cos\left(5\,x\right)-\frac{2}{5}\,\cos\left(5\,x\right)+\frac{3}{25}\,\sin\left(5\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)*sin(5*x),x, algorithm="maxima")
[Out] -3/5*x*cos(5*x) - 2/5*cos(5*x) + 3/25*sin(5*x)
```

mupad [B] time = 0.10, size = 20, normalized size = 0.91

$$\frac{3\,\sin\left(5\,x\right)}{25}-\frac{2\,\cos\left(5\,x\right)}{5}-\frac{3\,x\,\cos\left(5\,x\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(5*x)*(3*x + 2),x)
[Out] (3*sin(5*x))/25 - (2*cos(5*x))/5 - (3*x*cos(5*x))/5
```

sympy [A] time = 0.20, size = 26, normalized size = 1.18

$$-\frac{3x\cos\left(5x\right)}{5}+\frac{3\sin\left(5x\right)}{25}-\frac{2\cos\left(5x\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)*sin(5*x),x)
[Out] -3*x*cos(5*x)/5 + 3*sin(5*x)/25 - 2*cos(5*x)/5
```

3.47 $\int x\sqrt{1+x^2}\, dx$

Optimal. Leaf size=13

$$\frac{1}{3}\left(x^2+1\right)^{3/2}$$

```
[Out] 1/3*(x^2+1)^(3/2)
```

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {261}

$$\frac{1}{3}\left(x^2+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[1 + x^2],x]
```

```
[Out] (1 + x^2)^(3/2)/3
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\sqrt{1+x^2}\, dx = \frac{1}{3}\left(1+x^2\right)^{3/2}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{1}{3}\left(x^2+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[1 + x^2],x]
```

```
[Out] (1 + x^2)^(3/2)/3
```

fricas [A] time = 0.39, size = 9, normalized size = 0.69

$$\frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(x^2 + 1)^(3/2)
```

giac [A] time = 0.01, size = 9, normalized size = 0.69

$$\frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+1)^(1/2),x, algorithm="giac")
[Out] 1/3*(x^2 + 1)^(3/2)
```

maple [A] time = 0.00, size = 10, normalized size = 0.77

$$\frac{\left(x^{2}+1\right)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2+1)^(1/2),x)
[Out] 1/3*(x^2+1)^(3/2)
```

maxima [A] time = 0.44, size = 9, normalized size = 0.69

$$\frac{1}{3}\left(x^{2}+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+1)^(1/2),x, algorithm="maxima")
[Out] 1/3*(x^2 + 1)^(3/2)
```

mupad [B] time = 0.02, size = 9, normalized size = 0.69

$$\frac{\left(x^{2}+1\right)^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2 + 1)^(1/2),x)
[Out] (x^2 + 1)^(3/2)/3
```

sympy [B] time = 0.20, size = 22, normalized size = 1.69

$$\frac{x^{2}\sqrt{x^{2}+1}}{3}+\frac{\sqrt{x^{2}+1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**2+1)**(1/2),x)
[Out] x**2*sqrt(x**2 + 1)/3 + sqrt(x**2 + 1)/3
```

3.48 $\int x\left(-1+x^2\right)^9 \, dx$

Optimal. Leaf size=13

$$\frac{1}{20}\left(1-x^2\right)^{10}$$

```
[Out] 1/20*(-x^2+1)^10
```

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {261}

$$\frac{1}{20}\left(1-x^2\right)^{10}$$

Antiderivative was successfully verified.

```
[In] Int[x*(-1 + x^2)^9,x]
[Out] (1 - x^2)^10/20
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\left(-1+x^2\right)^9 \, dx=\frac{1}{20}\left(1-x^2\right)^{10}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 0.85

$$\frac{1}{20}\left(x^2-1\right)^{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(-1 + x^2)^9,x]
[Out] (-1 + x^2)^10/20
```

fricas [B] time = 0.35, size = 51, normalized size = 3.92

$$\frac{1}{20}x^{20}-\frac{1}{2}x^{18}+\frac{9}{4}x^{16}-6x^{14}+\frac{21}{2}x^{12}-\frac{63}{5}x^{10}+\frac{21}{2}x^8-6x^6+\frac{9}{4}x^4-\frac{1}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2-1)^9,x, algorithm="fricas")
[Out] 1/20*x^20 - 1/2*x^18 + 9/4*x^16 - 6*x^14 + 21/2*x^12 - 63/5*x^10 + 21/2*x^8
 - 6*x^6 + 9/4*x^4 - 1/2*x^2
```

giac [A] time = 0.01, size = 9, normalized size = 0.69

$$\frac{1}{20}\left(x^2-1\right)^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2-1)^9,x, algorithm="giac")
```

```
[Out] 1/20*(x^2 - 1)^10
```

maple [B] time = 0.00, size = 52, normalized size = 4.00

$$\frac{1}{20}x^{20}-\frac{1}{2}x^{18}+\frac{9}{4}x^{16}-6x^{14}+\frac{21}{2}x^{12}-\frac{63}{5}x^{10}+\frac{21}{2}x^{8}-6x^{6}+\frac{9}{4}x^{4}-\frac{1}{2}x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2-1)^9,x)
```

```
[Out] 1/20*x^20-1/2*x^18+9/4*x^16-6*x^14+21/2*x^12-63/5*x^10+21/2*x^8-6*x^6+9/4*x
^4-1/2*x^2
```

maxima [A] time = 0.46, size = 9, normalized size = 0.69

$$\frac{1}{20}\left(x^{2}-1\right)^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2-1)^9,x, algorithm="maxima")
```

```
[Out] 1/20*(x^2 - 1)^10
```

mupad [B] time = 0.10, size = 9, normalized size = 0.69

$$\frac{\left(x^{2}-1\right)^{10}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2 - 1)^9,x)
```

```
[Out] (x^2 - 1)^10/20
```

sympy [B] time = 0.06, size = 58, normalized size = 4.46

$$\frac{x^{20}}{20}-\frac{x^{18}}{2}+\frac{9x^{16}}{4}-6x^{14}+\frac{21x^{12}}{2}-\frac{63x^{10}}{5}+\frac{21x^{8}}{2}-6x^{6}+\frac{9x^{4}}{4}-\frac{x^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**2-1)**9,x)
```

```
[Out] x**20/20 - x**18/2 + 9*x**16/4 - 6*x**14 + 21*x**12/2 - 63*x**10/5 + 21*x**
8/2 - 6*x**6 + 9*x**4/4 - x**2/2
```

3.49 $\int \frac{3+2x}{(7+6x)^3} dx$

Optimal. Leaf size=18

$$-\frac{(2x+3)^2}{8(6x+7)^2}$$

```
[Out] -1/8*(3+2*x)^2/(7+6*x)^2
```

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {37}

$$-\frac{(2x+3)^2}{8(6x+7)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 2*x)/(7 + 6*x)^3,x]
```

```
[Out] -(3 + 2*x)^2/(8*(7 + 6*x)^2)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{3+2x}{(7+6x)^3} dx = -\frac{(3+2x)^2}{8(7+6x)^2}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 0.89

$$-\frac{3x+4}{9(6x+7)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 2*x)/(7 + 6*x)^3,x]
```

```
[Out] -1/9*(4 + 3*x)/(7 + 6*x)^2
```

fricas [A] time = 0.38, size = 19, normalized size = 1.06

$$-\frac{3x+4}{9(36x^2+84x+49)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(7+6*x)^3,x, algorithm="fricas")
```

```
[Out] -1/9*(3*x + 4)/(36*x^2 + 84*x + 49)
```

giac [A] time = 0.01, size = 14, normalized size = 0.78

$$-\frac{3x+4}{9(6x+7)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(7+6*x)^3,x, algorithm="giac")
[Out] -1/9*(3*x + 4)/(6*x + 7)^2
```

maple [A] time = 0.01, size = 20, normalized size = 1.11

$$-\frac{1}{18\left(6x+7\right)}-\frac{1}{18\left(6x+7\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+3)/(7+6*x)^3,x)
[Out] -1/18/(7+6*x)-1/18/(7+6*x)^2
```

maxima [A] time = 0.58, size = 19, normalized size = 1.06

$$-\frac{3\,x+4}{9\,\left(36\,x^{2}+84\,x+49\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(7+6*x)^3,x, algorithm="maxima")
[Out] -1/9*(3*x + 4)/(36*x^2 + 84*x + 49)
```

mupad [B] time = 0.04, size = 14, normalized size = 0.78

$$-\frac{3\,x+4}{9\,\left(6\,x+7\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 3)/(6*x + 7)^3,x)
[Out] -(3*x + 4)/(9*(6*x + 7)^2)
```

sympy [A] time = 0.10, size = 15, normalized size = 0.83

$$\frac{-3x-4}{324x^{2}+756x+441}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(7+6*x)**3,x)
[Out] (-3*x - 4)/(324*x**2 + 756*x + 441)
```

3.50 $\int x^4 \left(1+x^5\right)^5 \, dx$

Optimal. Leaf size=11

$$\frac{1}{30} \left(x^5+1\right)^6$$

```
[Out] 1/30*(x^5+1)^6
```

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {261}

$$\frac{1}{30} \left(x^5+1\right)^6$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(1 + x^5)^5,x]
```

```
[Out] (1 + x^5)^6/30
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^4 \left(1+x^5\right)^5 \, dx = \frac{1}{30} \left(1+x^5\right)^6$$

Mathematica [B] time = 0.00, size = 43, normalized size = 3.91

$$\frac{x^{30}}{30}+\frac{x^{25}}{5}+\frac{x^{20}}{2}+\frac{2x^{15}}{3}+\frac{x^{10}}{2}+\frac{x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(1 + x^5)^5,x]
```

```
[Out] x^5/5 + x^10/2 + (2*x^15)/3 + x^20/2 + x^25/5 + x^30/30
```

fricas [B] time = 0.35, size = 31, normalized size = 2.82

$$\frac{1}{30} x^{30}+\frac{1}{5} x^{25}+\frac{1}{2} x^{20}+\frac{2}{3} x^{15}+\frac{1}{2} x^{10}+\frac{1}{5} x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(x^5+1)^5,x, algorithm="fricas")
```

```
[Out] 1/30*x^30 + 1/5*x^25 + 1/2*x^20 + 2/3*x^15 + 1/2*x^10 + 1/5*x^5
```

giac [A] time = 0.01, size = 9, normalized size = 0.82

$$\frac{1}{30} \left(x^5+1\right)^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(x^5+1)^5,x, algorithm="giac")
```

```
[Out] 1/30*(x^5 + 1)^6
```

maple [B] time = 0.00, size = 32, normalized size = 2.91

$$\frac{1}{30}x^{30} + \frac{1}{5}x^{25} + \frac{1}{2}x^{20} + \frac{2}{3}x^{15} + \frac{1}{2}x^{10} + \frac{1}{5}x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(x^5+1)^5,x)
```

```
[Out] 1/30*x^30+1/5*x^25+1/2*x^20+2/3*x^15+1/2*x^10+1/5*x^5
```

maxima [A] time = 0.52, size = 9, normalized size = 0.82

$$\frac{1}{30}\left(x^{5}+1\right)^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(x^5+1)^5,x, algorithm="maxima")
```

```
[Out] 1/30*(x^5 + 1)^6
```

mupad [B] time = 0.03, size = 31, normalized size = 2.82

$$\frac{x^{30}}{30} + \frac{x^{25}}{5} + \frac{x^{20}}{2} + \frac{2\,x^{15}}{3} + \frac{x^{10}}{2} + \frac{x^{5}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(x^5 + 1)^5,x)
```

```
[Out] x^5/5 + x^10/2 + (2*x^15)/3 + x^20/2 + x^25/5 + x^30/30
```

sympy [B] time = 0.06, size = 31, normalized size = 2.82

$$\frac{x^{30}}{30} + \frac{x^{25}}{5} + \frac{x^{20}}{2} + \frac{2x^{15}}{3} + \frac{x^{10}}{2} + \frac{x^{5}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(x**5+1)**5,x)
```

```
[Out] x**30/30 + x**25/5 + x**20/2 + 2*x**15/3 + x**10/2 + x**5/5
```

3.51 $\int (1-x)^{20}x^4\,dx$

Optimal. Leaf size=56

$$-\frac{1}{25}(1-x)^{25}+\frac{1}{6}(1-x)^{24}-\frac{6}{23}(1-x)^{23}+\frac{2}{11}(1-x)^{22}-\frac{1}{21}(1-x)^{21}$$

```
[Out] -1/21*(1-x)^21+2/11*(1-x)^22-6/23*(1-x)^23+1/6*(1-x)^24-1/25*(1-x)^25
```

Rubi [A] time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{1}{25}(1-x)^{25}+\frac{1}{6}(1-x)^{24}-\frac{6}{23}(1-x)^{23}+\frac{2}{11}(1-x)^{22}-\frac{1}{21}(1-x)^{21}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^20*x^4,x]
```

```
[Out] -(1 - x)^21/21 + (2*(1 - x)^22)/11 - (6*(1 - x)^23)/23 + (1 - x)^24/6 - (1
- x)^25/25
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (1-x)^{20}x^4\,dx &= \int \left((1-x)^{20}-4(1-x)^{21}+6(1-x)^{22}-4(1-x)^{23}+(1-x)^{24}\right)\,dx\\ &= -\frac{1}{21}(1-x)^{21}+\frac{2}{11}(1-x)^{22}-\frac{6}{23}(1-x)^{23}+\frac{1}{6}(1-x)^{24}-\frac{1}{25}(1-x)^{25}\end{aligned}$$

Mathematica [B] time = 0.00, size = 140, normalized size = 2.50

$$\frac{x^{25}}{25}-\frac{5x^{24}}{6}+\frac{190x^{23}}{23}-\frac{570x^{22}}{11}+\frac{1615x^{21}}{7}-\frac{3876x^{20}}{5}+2040x^{19}-\frac{12920x^{18}}{3}+7410x^{17}-\frac{20995x^{16}}{2}+\frac{184756x^{15}}{15}-\frac{83980x^{1}}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^20*x^4,x]
```

```
[Out] x^5/5 - (10*x^6)/3 + (190*x^7)/7 - (285*x^8)/2 + (1615*x^9)/3 - (7752*x^10)
/5 + (38760*x^11)/11 - 6460*x^12 + 9690*x^13 - (83980*x^14)/7 + (184756*x^1
5)/15 - (20995*x^16)/2 + 7410*x^17 - (12920*x^18)/3 + 2040*x^19 - (3876*x^2
0)/5 + (1615*x^21)/7 - (570*x^22)/11 + (190*x^23)/23 - (5*x^24)/6 + x^25/25
```

fricas [B] time = 0.34, size = 106, normalized size = 1.89

$$\frac{1}{25}x^{25}-\frac{5}{6}x^{24}+\frac{190}{23}x^{23}-\frac{570}{11}x^{22}+\frac{1615}{7}x^{21}-\frac{3876}{5}x^{20}+2040x^{19}-\frac{12920}{3}x^{18}+7410x^{17}-\frac{20995}{2}x^{16}+\frac{184756}{15}x^{15}-\frac{8398}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^20*x^4,x, algorithm="fricas")
```

```
[Out] 1/25*x^25 - 5/6*x^24 + 190/23*x^23 - 570/11*x^22 + 1615/7*x^21 - 3876/5*x^2
0 + 2040*x^19 - 12920/3*x^18 + 7410*x^17 - 20995/2*x^16 + 184756/15*x^15 -
83980/7*x^14 + 9690*x^13 - 6460*x^12 + 38760/11*x^11 - 7752/5*x^10 + 1615/3
*x^9 - 285/2*x^8 + 190/7*x^7 - 10/3*x^6 + 1/5*x^5
```

giac [B] time = 0.01, size = 106, normalized size = 1.89

$$\frac{1}{25}x^{25}-\frac{5}{6}x^{24}+\frac{190}{23}x^{23}-\frac{570}{11}x^{22}+\frac{1615}{7}x^{21}-\frac{3876}{5}x^{20}+2040\,x^{19}-\frac{12920}{3}x^{18}+7410\,x^{17}-\frac{20995}{2}x^{16}+\frac{184756}{15}x^{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^20*x^4,x, algorithm="giac")
```

```
[Out] 1/25*x^25 - 5/6*x^24 + 190/23*x^23 - 570/11*x^22 + 1615/7*x^21 - 3876/5*x^2
0 + 2040*x^19 - 12920/3*x^18 + 7410*x^17 - 20995/2*x^16 + 184756/15*x^15 -
83980/7*x^14 + 9690*x^13 - 6460*x^12 + 38760/11*x^11 - 7752/5*x^10 + 1615/3
*x^9 - 285/2*x^8 + 190/7*x^7 - 10/3*x^6 + 1/5*x^5
```

maple [B] time = 0.00, size = 107, normalized size = 1.91

$$\frac{1}{25}x^{25}-\frac{5}{6}x^{24}+\frac{190}{23}x^{23}-\frac{570}{11}x^{22}+\frac{1615}{7}x^{21}-\frac{3876}{5}x^{20}+2040x^{19}-\frac{12920}{3}x^{18}+7410x^{17}-\frac{20995}{2}x^{16}+\frac{184756}{15}x^{15}-\frac{83}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^20*x^4,x)
```

```
[Out] 1/25*x^25-5/6*x^24+190/23*x^23-570/11*x^22+1615/7*x^21-3876/5*x^20+2040*x^1
9-12920/3*x^18+7410*x^17-20995/2*x^16+184756/15*x^15-83980/7*x^14+9690*x^13
-6460*x^12+38760/11*x^11-7752/5*x^10+1615/3*x^9-285/2*x^8+190/7*x^7-10/3*x^
6+1/5*x^5
```

maxima [B] time = 0.61, size = 106, normalized size = 1.89

$$\frac{1}{25}x^{25}-\frac{5}{6}x^{24}+\frac{190}{23}x^{23}-\frac{570}{11}x^{22}+\frac{1615}{7}x^{21}-\frac{3876}{5}x^{20}+2040\,x^{19}-\frac{12920}{3}x^{18}+7410\,x^{17}-\frac{20995}{2}x^{16}+\frac{184756}{15}x^{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^20*x^4,x, algorithm="maxima")
```

```
[Out] 1/25*x^25 - 5/6*x^24 + 190/23*x^23 - 570/11*x^22 + 1615/7*x^21 - 3876/5*x^2
0 + 2040*x^19 - 12920/3*x^18 + 7410*x^17 - 20995/2*x^16 + 184756/15*x^15 -
83980/7*x^14 + 9690*x^13 - 6460*x^12 + 38760/11*x^11 - 7752/5*x^10 + 1615/3
*x^9 - 285/2*x^8 + 190/7*x^7 - 10/3*x^6 + 1/5*x^5
```

mupad [B] time = 0.46, size = 106, normalized size = 1.89

$$\frac{x^{25}}{25}-\frac{5\,x^{24}}{6}+\frac{190\,x^{23}}{23}-\frac{570\,x^{22}}{11}+\frac{1615\,x^{21}}{7}-\frac{3876\,x^{20}}{5}+2040\,x^{19}-\frac{12920\,x^{18}}{3}+7410\,x^{17}-\frac{20995\,x^{16}}{2}+\frac{184756\,x^{15}}{15}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(x - 1)^20,x)
```

```
[Out] x^5/5 - (10*x^6)/3 + (190*x^7)/7 - (285*x^8)/2 + (1615*x^9)/3 - (7752*x^10)
/5 + (38760*x^11)/11 - 6460*x^12 + 9690*x^13 - (83980*x^14)/7 + (184756*x^1
5)/15 - (20995*x^16)/2 + 7410*x^17 - (12920*x^18)/3 + 2040*x^19 - (3876*x^2
0)/5 + (1615*x^21)/7 - (570*x^22)/11 + (190*x^23)/23 - (5*x^24)/6 + x^25/25
```

sympy [B] time = 0.08, size = 131, normalized size = 2.34

$$\frac{x^{25}}{25}-\frac{5x^{24}}{6}+\frac{190x^{23}}{23}-\frac{570x^{22}}{11}+\frac{1615x^{21}}{7}-\frac{3876x^{20}}{5}+2040x^{19}-\frac{12920x^{18}}{3}+7410x^{17}-\frac{20995x^{16}}{2}+\frac{184756x^{15}}{15}-\frac{8398}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**20*x**4,x)
```

```
[Out] x**25/25 - 5*x**24/6 + 190*x**23/23 - 570*x**22/11 + 1615*x**21/7 - 3876*x*
*20/5 + 2040*x**19 - 12920*x**18/3 + 7410*x**17 - 20995*x**16/2 + 184756*x*
*15/15 - 83980*x**14/7 + 9690*x**13 - 6460*x**12 + 38760*x**11/11 - 7752*x*
*10/5 + 1615*x**9/3 - 285*x**8/2 + 190*x**7/7 - 10*x**6/3 + x**5/5
```

3.52 $\int \frac{\sin\left(\frac{1}{x}\right)}{x^2}\,dx$

Optimal. Leaf size=4

$$\cos\left(\frac{1}{x}\right)$$

```
[Out] cos(1/x)
```

Rubi [A] time = 0.01, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3379, 2638}

$$\cos\left(\frac{1}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x^(-1)]/x^2,x]
```

```
[Out] Cos[x^(-1)]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned} \int \frac{\sin\left(\frac{1}{x}\right)}{x^2}\,dx &= -\text{Subst}\left(\int \sin(x)\,dx, x, \frac{1}{x}\right)\\ &= \cos\left(\frac{1}{x}\right) \end{aligned}$$

Mathematica [A] time = 0.01, size = 4, normalized size = 1.00

$$\cos\left(\frac{1}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x^(-1)]/x^2,x]
```

```
[Out] Cos[x^(-1)]
```

fricas [A] time = 0.41, size = 4, normalized size = 1.00

$$\cos\left(\frac{1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/x)/x^2,x, algorithm="fricas")
[Out] cos(1/x)
```

giac [A] time = 0.01, size = 4, normalized size = 1.00

$$\cos\left(\frac{1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/x)/x^2,x, algorithm="giac")
[Out] cos(1/x)
```

maple [A] time = 0.00, size = 5, normalized size = 1.25

$$\cos\left(\frac{1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(1/x)/x^2,x)
[Out] cos(1/x)
```

maxima [A] time = 0.61, size = 4, normalized size = 1.00

$$\cos\left(\frac{1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/x)/x^2,x, algorithm="maxima")
[Out] cos(1/x)
```

mupad [B] time = 0.10, size = 4, normalized size = 1.00

$$\cos\left(\frac{1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(1/x)/x^2,x)
[Out] cos(1/x)
```

sympy [A] time = 0.65, size = 3, normalized size = 0.75

$$\cos\left(\frac{1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/x)/x**2,x)
[Out] cos(1/x)
```

3.53 $\int \sin\left(\sqrt[4]{-1+x}\right)\,dx$

Optimal. Leaf size=62

$$12\sqrt{x-1}\,\sin\left(\sqrt[4]{x-1}\right)-24\sin\left(\sqrt[4]{x-1}\right)-4(x-1)^{3/4}\cos\left(\sqrt[4]{x-1}\right)+24\sqrt[4]{x-1}\,\cos\left(\sqrt[4]{x-1}\right)$$

```
[Out] 24*(-1+x)^(1/4)*cos((-1+x)^(1/4))-4*(-1+x)^(3/4)*cos((-1+x)^(1/4))-24*sin((
-1+x)^(1/4))+12*sin((-1+x)^(1/4))*(-1+x)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3361, 3296, 2637}

$$12\sqrt{x-1}\,\sin\left(\sqrt[4]{x-1}\right)-24\sin\left(\sqrt[4]{x-1}\right)-4(x-1)^{3/4}\cos\left(\sqrt[4]{x-1}\right)+24\sqrt[4]{x-1}\,\cos\left(\sqrt[4]{x-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[(-1 + x)^(1/4)],x]
```

```
[Out] 24*(-1 + x)^(1/4)*Cos[(-1 + x)^(1/4)] - 4*(-1 + x)^(3/4)*Cos[(-1 + x)^(1/4)
] - 24*Sin[(-1 + x)^(1/4)] + 12*Sqrt[-1 + x]*Sin[(-1 + x)^(1/4)]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3361

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Sin[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rubi steps

$$\begin{aligned}
\int \sin\left(\sqrt[4]{-1+x}\right)\,dx &= 4\,\text{Subst}\left(\int x^3\sin(x)\,dx,x,\sqrt[4]{-1+x}\right)\\
&= -4(-1+x)^{3/4}\cos\left(\sqrt[4]{-1+x}\right)+12\,\text{Subst}\left(\int x^2\cos(x)\,dx,x,\sqrt[4]{-1+x}\right)\\
&= -4(-1+x)^{3/4}\cos\left(\sqrt[4]{-1+x}\right)+12\sqrt{-1+x}\,\sin\left(\sqrt[4]{-1+x}\right)-24\,\text{Subst}\left(\int x\sin(x)\,dx,\right.\\
&= 24\sqrt[4]{-1+x}\,\cos\left(\sqrt[4]{-1+x}\right)-4(-1+x)^{3/4}\cos\left(\sqrt[4]{-1+x}\right)+12\sqrt{-1+x}\,\sin\left(\sqrt[4]{-1+x}\right.\\
&= 24\sqrt[4]{-1+x}\,\cos\left(\sqrt[4]{-1+x}\right)-4(-1+x)^{3/4}\cos\left(\sqrt[4]{-1+x}\right)-24\sin\left(\sqrt[4]{-1+x}\right)+12\sqrt{-}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 46, normalized size = 0.74

$$12\left(\sqrt{x-1}-2\right)\sin\left(\sqrt[4]{x-1}\right)-4\left(\sqrt{x-1}-6\right)\sqrt[4]{x-1}\,\cos\left(\sqrt[4]{x-1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[(-1 + x)^(1/4)],x]
```

```
[Out] -4*(-6 + Sqrt[-1 + x])*(-1 + x)^(1/4)*Cos[(-1 + x)^(1/4)] + 12*(-2 + Sqrt[-
1 + x])*Sin[(-1 + x)^(1/4)]
```

fricas [A] time = 0.42, size = 37, normalized size = 0.60

$$-4\left((x-1)^{\frac{3}{4}}-6\,(x-1)^{\frac{1}{4}}\right)\cos\left((x-1)^{\frac{1}{4}}\right)+12\left(\sqrt{x-1}-2\right)\sin\left((x-1)^{\frac{1}{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((-1+x)^(1/4)),x, algorithm="fricas")
```

```
[Out] -4*((x - 1)^(3/4) - 6*(x - 1)^(1/4))*cos((x - 1)^(1/4)) + 12*(sqrt(x - 1) -
 2)*sin((x - 1)^(1/4))
```

giac [A] time = 0.01, size = 37, normalized size = 0.60

$$-4\left((x-1)^{\frac{3}{4}}-6\,(x-1)^{\frac{1}{4}}\right)\cos\left((x-1)^{\frac{1}{4}}\right)+12\left(\sqrt{x-1}-2\right)\sin\left((x-1)^{\frac{1}{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((-1+x)^(1/4)),x, algorithm="giac")
```

```
[Out] -4*((x - 1)^(3/4) - 6*(x - 1)^(1/4))*cos((x - 1)^(1/4)) + 12*(sqrt(x - 1) -
 2)*sin((x - 1)^(1/4))
```

maple [A] time = 0.01, size = 49, normalized size = 0.79

$$24\,(x-1)^{\frac{1}{4}}\cos\left((x-1)^{\frac{1}{4}}\right)-4\,(x-1)^{\frac{3}{4}}\cos\left((x-1)^{\frac{1}{4}}\right)-24\sin\left((x-1)^{\frac{1}{4}}\right)+12\sqrt{x-1}\,\sin\left((x-1)^{\frac{1}{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin((x-1)^(1/4)),x)
```

```
[Out] 24*(x-1)^(1/4)*cos((x-1)^(1/4))-4*(x-1)^(3/4)*cos((x-1)^(1/4))-24*sin((x-1)
^(1/4))+12*sin((x-1)^(1/4))*(x-1)^(1/2)
```

maxima [A] time = 0.53, size = 37, normalized size = 0.60

$$-4\left((x-1)^{\frac{3}{4}}-6\,(x-1)^{\frac{1}{4}}\right)\cos\left((x-1)^{\frac{1}{4}}\right)+12\left(\sqrt{x-1}-2\right)\sin\left((x-1)^{\frac{1}{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((-1+x)^(1/4)),x, algorithm="maxima")
```

```
[Out] -4*((x - 1)^(3/4) - 6*(x - 1)^(1/4))*cos((x - 1)^(1/4)) + 12*(sqrt(x - 1) -
 2)*sin((x - 1)^(1/4))
```

mupad [B] time = 0.23, size = 41, normalized size = 0.66

$$4\,\cos\left((x-1)^{1/4}\right)\,\left(6\,(x-1)^{1/4}-(x-1)^{3/4}\right)+4\,\sin\left((x-1)^{1/4}\right)\,\left(3\,\sqrt{x-1}-6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin((x - 1)^(1/4)),x)
```

```
[Out] 4*cos((x - 1)^(1/4))*(6*(x - 1)^(1/4) - (x - 1)^(3/4)) + 4*sin((x - 1)^(1/4
))*(3*(x - 1)^(1/2) - 6)
```

sympy [**A**] time = 2.03, size = 60, normalized size = 0.97

$$-4(x-1)^{\frac{3}{4}}\cos\left(\sqrt[4]{x-1}\right)+24\sqrt[4]{x-1}\,\cos\left(\sqrt[4]{x-1}\right)+12\sqrt{x-1}\,\sin\left(\sqrt[4]{x-1}\right)-24\sin\left(\sqrt[4]{x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((-1+x)**(1/4)),x)
```

```
[Out] -4*(x - 1)**(3/4)*cos((x - 1)**(1/4)) + 24*(x - 1)**(1/4)*cos((x - 1)**(1/4
)) + 12*sqrt(x - 1)*sin((x - 1)**(1/4)) - 24*sin((x - 1)**(1/4))
```

3.54 $\int x \cos\left(x^2\right) \sin\left(x^2\right) dx$

Optimal. Leaf size=10

$$\frac{1}{4}\sin^2\left(x^2\right)$$

[Out] 1/4*sin(x^2)^2

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3441}

$$\frac{1}{4}\sin^2\left(x^2\right)$$

Antiderivative was successfully verified.

[In] Int[x*Cos[x^2]*Sin[x^2],x]

[Out] Sin[x^2]^2/4

Rule 3441

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[
{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]
```

Rubi steps

$$\int x \cos\left(x^2\right) \sin\left(x^2\right) dx = \frac{1}{4}\sin^2\left(x^2\right)$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$-\frac{1}{4}\cos^2\left(x^2\right)$$

Antiderivative was successfully verified.

[In] Integrate[x*Cos[x^2]*Sin[x^2],x]

[Out] -1/4*Cos[x^2]^2

fricas [A] time = 0.42, size = 8, normalized size = 0.80

$$-\frac{1}{4}\cos\left(x^2\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(x^2)*sin(x^2),x, algorithm="fricas")

[Out] -1/4*cos(x^2)^2

giac [A] time = 0.01, size = 8, normalized size = 0.80

$$-\frac{1}{4}\cos\left(x^2\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x^2)*sin(x^2),x, algorithm="giac")
```

```
[Out] -1/4*cos(x^2)^2
```

maple [A] time = 0.00, size = 9, normalized size = 0.90

$$-\frac{\left(\cos^2\left(x^2\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x^2)*sin(x^2),x)
```

```
[Out] -1/4*cos(x^2)^2
```

maxima [A] time = 0.53, size = 8, normalized size = 0.80

$$-\frac{1}{4}\cos\left(x^2\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x^2)*sin(x^2),x, algorithm="maxima")
```

```
[Out] -1/4*cos(x^2)^2
```

mupad [B] time = 0.05, size = 8, normalized size = 0.80

$$\frac{\sin\left(x^2\right)^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x^2)*sin(x^2),x)
```

```
[Out] sin(x^2)^2/4
```

sympy [A] time = 0.33, size = 8, normalized size = 0.80

$$-\frac{\cos^2\left(x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x**2)*sin(x**2),x)
```

```
[Out] -cos(x**2)**2/4
```

3.55 $\int \sqrt{1+3\cos^2(x)}\,\sin(2x)\,dx$

Optimal. Leaf size=16

$$-\frac{2}{9}\left(4-3\sin^2(x)\right)^{3/2}$$

```
[Out] -2/9*(4-3*sin(x)^2)^(3/2)
```

Rubi [A] time = 0.03, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {12, 261}

$$-\frac{2}{9}\left(4-3\sin^2(x)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + 3*Cos[x]^2]*Sin[2*x],x]
```

```
[Out] (-2*(4 - 3*Sin[x]^2)^(3/2))/9
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \sqrt{1+3\cos^2(x)}\,\sin(2x)\,dx &= \text{Subst}\left(\int 2x\sqrt{4-3x^2}\,dx, x, \sin(x)\right)\\ &= 2\,\text{Subst}\left(\int x\sqrt{4-3x^2}\,dx, x, \sin(x)\right)\\ &= -\frac{2}{9}\left(4-3\sin^2(x)\right)^{3/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$-\frac{2}{9}\left(4-3\sin^2(x)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + 3*Cos[x]^2]*Sin[2*x],x]
```

```
[Out] (-2*(4 - 3*Sin[x]^2)^(3/2))/9
```

fricas [A] time = 0.44, size = 12, normalized size = 0.75

$$-\frac{2}{9}\left(3\cos(x)^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)*(1+3*cos(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/9*(3*cos(x)^2 + 1)^(3/2)
```

giac [A] time = 0.01, size = 12, normalized size = 0.75

$$-\frac{2}{9}\left(3\cos(x)^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)*(1+3*cos(x)^2)^(1/2),x, algorithm="giac")
[Out] -2/9*(3*cos(x)^2 + 1)^(3/2)
```

maple [A] time = 0.07, size = 13, normalized size = 0.81

$$-\frac{2\left(3\left(\cos^2(x)\right)+1\right)^{\frac{3}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)*(1+3*cos(x)^2)^(1/2),x)
[Out] -2/9*(1+3*cos(x)^2)^(3/2)
```

maxima [A] time = 0.48, size = 12, normalized size = 0.75

$$-\frac{2}{9}\left(3\cos(x)^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)*(1+3*cos(x)^2)^(1/2),x, algorithm="maxima")
[Out] -2/9*(3*cos(x)^2 + 1)^(3/2)
```

mupad [B] time = 0.19, size = 12, normalized size = 0.75

$$-\frac{2\left(3\cos(x)^2+1\right)^{3/2}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)*(3*cos(x)^2 + 1)^(1/2),x)
[Out] -(2*(3*cos(x)^2 + 1)^(3/2))/9
```

sympy [A] time = 2.06, size = 15, normalized size = 0.94

$$-\frac{2\left(3\cos^2(x)+1\right)^{\frac{3}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)*(1+3*cos(x)**2)**(1/2),x)
[Out] -2*(3*cos(x)**2 + 1)**(3/2)/9
```

3.56 $\int \frac{1}{2+3x} \, dx$

Optimal. Leaf size=10

$$\frac{1}{3}\log(3x+2)$$

```
[Out] 1/3*ln(2+3*x)
```

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {31}

$$\frac{1}{3}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x)^(-1),x]
```

```
[Out] Log[2 + 3*x]/3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{2+3x} \, dx = \frac{1}{3}\log(2+3x)$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$\frac{1}{3}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x)^(-1),x]
```

```
[Out] Log[2 + 3*x]/3
```

fricas [A] time = 0.39, size = 8, normalized size = 0.80

$$\frac{1}{3}\log\left(3x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x),x, algorithm="fricas")
```

```
[Out] 1/3*log(3*x + 2)
```

giac [A] time = 0.01, size = 9, normalized size = 0.90

$$\frac{1}{3}\log\left(|3x+2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x),x, algorithm="giac")
```

```
[Out] 1/3*log(abs(3*x + 2))
```

maple [A] time = 0.00, size = 9, normalized size = 0.90

$$\frac{\ln(3x+2)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2+3*x),x)
```

```
[Out] 1/3*ln(2+3*x)
```

maxima [A] time = 0.54, size = 8, normalized size = 0.80

$$\frac{1}{3}\log(3x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x),x, algorithm="maxima")
```

```
[Out] 1/3*log(3*x + 2)
```

mupad [B] time = 0.07, size = 6, normalized size = 0.60

$$\frac{\ln\left(x+\frac{2}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x + 2),x)
```

```
[Out] log(x + 2/3)/3
```

sympy [A] time = 0.06, size = 7, normalized size = 0.70

$$\frac{\log(3x+2)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x),x)
```

```
[Out] log(3*x + 2)/3
```

3.57 $\int \log^2(x)\,dx$

Optimal. Leaf size=15

$$2x + x\log^2(x) - 2x\log(x)$$

[Out] 2*x-2*x*ln(x)+x*ln(x)^2

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2296, 2295}

$$2x + x\log^2(x) - 2x\log(x)$$

Antiderivative was successfully verified.

[In] Int[Log[x]^2,x]

[Out] 2*x - 2*x*Log[x] + x*Log[x]^2

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \log^2(x)\,dx &= x\log^2(x) - 2\int \log(x)\,dx\\ &= 2x - 2x\log(x) + x\log^2(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$2x + x\log^2(x) - 2x\log(x)$$

Antiderivative was successfully verified.

[In] Integrate[Log[x]^2,x]

[Out] 2*x - 2*x*Log[x] + x*Log[x]^2

fricas [A] time = 0.41, size = 15, normalized size = 1.00

$$x\log(x)^2 - 2\,x\log(x) + 2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(x)^2,x, algorithm="fricas")

[Out] x*log(x)^2 - 2*x*log(x) + 2*x

giac [A] time = 0.01, size = 15, normalized size = 1.00

$$x\log(x)^2 - 2\,x\log(x) + 2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)^2,x, algorithm="giac")
```

```
[Out] x*log(x)^2 - 2*x*log(x) + 2*x
```

maple [A] time = 0.01, size = 16, normalized size = 1.07

$$x\ln(x)^2 - 2x\ln(x) + 2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)^2,x)
```

```
[Out] 2*x-2*x*ln(x)+x*ln(x)^2
```

maxima [A] time = 0.52, size = 12, normalized size = 0.80

$$\left(\log(x)^2 - 2\,\log(x) + 2\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)^2,x, algorithm="maxima")
```

```
[Out] (log(x)^2 - 2*log(x) + 2)*x
```

mupad [B] time = 0.08, size = 12, normalized size = 0.80

$$x\,\left(\ln(x)^2 - 2\,\ln(x) + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x)^2,x)
```

```
[Out] x*(log(x)^2 - 2*log(x) + 2)
```

sympy [A] time = 0.09, size = 15, normalized size = 1.00

$$x\log{(x)}^2 - 2x\log{(x)} + 2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)**2,x)
```

```
[Out] x*log(x)**2 - 2*x*log(x) + 2*x
```

3.58 $\int x \log(x)\, dx$

Optimal. Leaf size=17

$$\frac{1}{2}x^2\log(x) - \frac{x^2}{4}$$

[Out] -1/4*x^2+1/2*x^2*ln(x)

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2304}

$$\frac{1}{2}x^2\log(x) - \frac{x^2}{4}$$

Antiderivative was successfully verified.

[In] Int[x*Log[x],x]

[Out] -x^2/4 + (x^2*Log[x])/2

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x\log(x)\,dx = -\frac{x^2}{4} + \frac{1}{2}x^2\log(x)$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{1}{2}x^2\log(x) - \frac{x^2}{4}$$

Antiderivative was successfully verified.

[In] Integrate[x*Log[x],x]

[Out] -1/4*x^2 + (x^2*Log[x])/2

fricas [A] time = 0.39, size = 13, normalized size = 0.76

$$\frac{1}{2}\,x^2\log(x) - \frac{1}{4}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*log(x),x, algorithm="fricas")

[Out] 1/2*x^2*log(x) - 1/4*x^2

giac [A] time = 0.01, size = 13, normalized size = 0.76

$$\frac{1}{2}\,x^2\log(x) - \frac{1}{4}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*log(x),x, algorithm="giac")

```
[Out] 1/2*x^2*log(x) - 1/4*x^2
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$\frac{x^2 \ln(x)}{2} - \frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(x),x)
```

```
[Out] -1/4*x^2+1/2*x^2*ln(x)
```

maxima [A] time = 0.50, size = 13, normalized size = 0.76

$$\frac{1}{2}\, x^2 \log(x) - \frac{1}{4}\, x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*log(x) - 1/4*x^2
```

mupad [B] time = 0.03, size = 9, normalized size = 0.53

$$\frac{x^2 \left(\ln(x) - \frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(x),x)
```

```
[Out] (x^2*(log(x) - 1/2))/2
```

sympy [A] time = 0.09, size = 12, normalized size = 0.71

$$\frac{x^2 \log{(x)}}{2} - \frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(x),x)
```

```
[Out] x**2*log(x)/2 - x**2/4
```

3.59 $\int x \log^2(x)\,dx$

Optimal. Leaf size=28

$$\frac{x^2}{4}+\frac{1}{2}x^2\log^2(x)-\frac{1}{2}x^2\log(x)$$

```
[Out] 1/4*x^2-1/2*x^2*ln(x)+1/2*x^2*ln(x)^2
```

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2305, 2304}

$$\frac{x^2}{4}+\frac{1}{2}x^2\log^2(x)-\frac{1}{2}x^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[x]^2,x]
[Out] x^2/4 - (x^2*Log[x])/2 + (x^2*Log[x]^2)/2
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x\log^2(x)\,dx &= \frac{1}{2}x^2\log^2(x)-\int x\log(x)\,dx\\ &= \frac{x^2}{4}-\frac{1}{2}x^2\log(x)+\frac{1}{2}x^2\log^2(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 28, normalized size = 1.00

$$\frac{x^2}{4}+\frac{1}{2}x^2\log^2(x)-\frac{1}{2}x^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[x]^2,x]
[Out] x^2/4 - (x^2*Log[x])/2 + (x^2*Log[x]^2)/2
```

fricas [A] time = 0.40, size = 22, normalized size = 0.79

$$\frac{1}{2}x^2\log(x)^2-\frac{1}{2}x^2\log(x)+\frac{1}{4}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*x^2*log(x)^2 - 1/2*x^2*log(x) + 1/4*x^2
```

giac [A] time = 0.01, size = 22, normalized size = 0.79

$$\frac{1}{2}x^2\log(x)^2 - \frac{1}{2}x^2\log(x) + \frac{1}{4}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x)^2,x, algorithm="giac")
[Out] 1/2*x^2*log(x)^2 - 1/2*x^2*log(x) + 1/4*x^2
```

maple [A] time = 0.00, size = 23, normalized size = 0.82

$$\frac{x^2\ln(x)^2}{2} - \frac{x^2\ln(x)}{2} + \frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(x)^2,x)
[Out] 1/4*x^2-1/2*x^2*ln(x)+1/2*x^2*ln(x)^2
```

maxima [A] time = 0.54, size = 17, normalized size = 0.61

$$\frac{1}{4}\left(2\log(x)^2 - 2\log(x) + 1\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x)^2,x, algorithm="maxima")
[Out] 1/4*(2*log(x)^2 - 2*log(x) + 1)*x^2
```

mupad [B] time = 0.03, size = 17, normalized size = 0.61

$$\frac{x^2\left(2\ln(x)^2 - 2\ln(x) + 1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(x)^2,x)
[Out] (x^2*(2*log(x)^2 - 2*log(x) + 1))/4
```

sympy [A] time = 0.11, size = 22, normalized size = 0.79

$$\frac{x^2\log(x)^2}{2} - \frac{x^2\log(x)}{2} + \frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(x)**2,x)
[Out] x**2*log(x)**2/2 - x**2*log(x)/2 + x**2/4
```

3.60 $\int \frac{1}{1+t} dt$

Optimal. Leaf size=4

$$\log(t+1)$$

[Out] ln(1+t)

Rubi [A] time = 0.00, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {31}

$$\log(t+1)$$

Antiderivative was successfully verified.

[In] Int[(1 + t)^(-1),t]

[Out] Log[1 + t]

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{1+t} dt = \log(1+t)$$

Mathematica [A] time = 0.00, size = 4, normalized size = 1.00

$$\log(t+1)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + t)^(-1),t]

[Out] Log[1 + t]

fricas [A] time = 0.39, size = 4, normalized size = 1.00

$$\log(t+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+t),t, algorithm="fricas")

[Out] log(t + 1)

giac [A] time = 0.01, size = 5, normalized size = 1.25

$$\log(|t+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+t),t, algorithm="giac")

[Out] log(abs(t + 1))

maple [A] time = 0.00, size = 5, normalized size = 1.25

$$\ln(t+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(t+1),t)
```

```
[Out] ln(t+1)
```

maxima [A] time = 0.53, size = 4, normalized size = 1.00

$$\log(t+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+t),t, algorithm="maxima")
```

```
[Out] log(t + 1)
```

mupad [B] time = 0.02, size = 4, normalized size = 1.00

$$\ln(t+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(t + 1),t)
```

```
[Out] log(t + 1)
```

sympy [A] time = 0.06, size = 3, normalized size = 0.75

$$\log(t+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+t),t)
```

```
[Out] log(t + 1)
```

3.61 $\int \cot(x)\,dx$

Optimal. Leaf size=3

$$\log(\sin(x))$$

```
[Out] ln(sin(x))
```

Rubi [A] time = 0.00, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3475}

$$\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x],x]
```

```
[Out] Log[Sin[x]]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot(x)\,dx = \log(\sin(x))$$

Mathematica [A] time = 0.00, size = 3, normalized size = 1.00

$$\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x],x]
```

```
[Out] Log[Sin[x]]
```

fricas [B] time = 0.42, size = 11, normalized size = 3.67

$$\frac{1}{2}\log\left(-\frac{1}{2}\cos(2x) + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x),x, algorithm="fricas")
```

```
[Out] 1/2*log(-1/2*cos(2*x) + 1/2)
```

giac [A] time = 0.01, size = 4, normalized size = 1.33

$$\log(|\sin(x)|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x),x, algorithm="giac")
```

```
[Out] log(abs(sin(x)))
```

maple [A] time = 0.00, size = 4, normalized size = 1.33

$$\ln(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x),x)
[Out] ln(sin(x))
```

maxima [A] time = 0.48, size = 3, normalized size = 1.00

$$\log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x),x, algorithm="maxima")
[Out] log(sin(x))
```

mupad [B] time = 0.02, size = 3, normalized size = 1.00

$$\ln(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x),x)
[Out] log(sin(x))
```

sympy [A] time = 0.07, size = 3, normalized size = 1.00

$$\log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x),x)
[Out] log(sin(x))
```

3.62 $\int x^n \log(ax)\,dx$

Optimal. Leaf size=28

$$\frac{x^{n+1}\log(ax)}{n+1}-\frac{x^{n+1}}{(n+1)^2}$$

```
[Out] -x^(1+n)/(1+n)^2+x^(1+n)*ln(a*x)/(1+n)
```

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2304}

$$\frac{x^{n+1}\log(ax)}{n+1}-\frac{x^{n+1}}{(n+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^n*Log[a*x],x]
[Out] -(x^(1 + n)/(1 + n)^2) + (x^(1 + n)*Log[a*x])/(1 + n)
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^n \log(ax)\,dx=-\frac{x^{1+n}}{(1+n)^2}+\frac{x^{1+n}\log(ax)}{1+n}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 0.75

$$\frac{x^{n+1}((n+1)\log(ax)-1)}{(n+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^n*Log[a*x],x]
[Out] (x^(1 + n)*(-1 + (1 + n)*Log[a*x]))/(1 + n)^2
```

fricas [A] time = 0.42, size = 32, normalized size = 1.14

$$\frac{\left((n+1)x\log(a)+(n+1)x\log(x)-x\right)x^n}{n^2+2\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*log(a*x),x, algorithm="fricas")
[Out] ((n + 1)*x*log(a) + (n + 1)*x*log(x) - x)*x^n/(n^2 + 2*n + 1)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*log(a*x),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: x^(n+
1)/(n+1)*ln(a*x)+integrate(-x^(n+1)/(n*x+x),x)
```

maple [A] time = 0.03, size = 36, normalized size = 1.29

$$\frac{x\,\mathrm{e}^{n\ln(x)}\ln(ax)}{n+1}-\frac{x\,\mathrm{e}^{n\ln(x)}}{n^2+2n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n*ln(a*x),x)
```

```
[Out] 1/(1+n)*x*ln(a*x)*exp(n*ln(x))-1/(n^2+2*n+1)*x*exp(n*ln(x))
```

maxima [A] time = 0.58, size = 28, normalized size = 1.00

$$\frac{x^{n+1}\log(ax)}{n+1}-\frac{x^{n+1}}{(n+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*log(a*x),x, algorithm="maxima")
```

```
[Out] x^(n + 1)*log(a*x)/(n + 1) - x^(n + 1)/(n + 1)^2
```

mupad [B] time = 0.21, size = 38, normalized size = 1.36

$$\begin{cases} \frac{\ln(a\,x)^2}{2} & \text{if } n=-1 \\ \frac{x^{n+1}\left(\ln(a\,x)-\frac{1}{n+1}\right)}{n+1} & \text{if } n\neq -1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n*log(a*x),x)
```

```
[Out] piecewise(n == -1, log(a*x)^2/2, n ~= -1, (x^(n + 1)*(log(a*x) - 1/(n + 1))
)/(n + 1))
```

sympy [A] time = 0.77, size = 94, normalized size = 3.36

$$\begin{cases} \frac{nxx^n\log(a)}{n^2+2n+1}+\frac{nxx^n\log(x)}{n^2+2n+1}+\frac{xx^n\log(a)}{n^2+2n+1}+\frac{xx^n\log(x)}{n^2+2n+1}-\frac{xx^n}{n^2+2n+1} & \text{for } n\neq -1 \\ \frac{\log(ax)^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**n*ln(a*x),x)
```

```
[Out] Piecewise((n*x*x**n*log(a)/(n**2 + 2*n + 1) + n*x*x**n*log(x)/(n**2 + 2*n +
 1) + x*x**n*log(a)/(n**2 + 2*n + 1) + x*x**n*log(x)/(n**2 + 2*n + 1) - x*x
**n/(n**2 + 2*n + 1), Ne(n, -1)), (log(a*x)**2/2, True))
```

3.63 $\int x^2 \log^2(x)\, dx$

Optimal. Leaf size=28

$$\frac{2x^3}{27} + \frac{1}{3}x^3 \log^2(x) - \frac{2}{9}x^3 \log(x)$$

```
[Out] 2/27*x^3-2/9*x^3*ln(x)+1/3*x^3*ln(x)^2
```

Rubi [A] time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2305, 2304}

$$\frac{2x^3}{27} + \frac{1}{3}x^3 \log^2(x) - \frac{2}{9}x^3 \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[x]^2,x]
[Out] (2*x^3)/27 - (2*x^3*Log[x])/9 + (x^3*Log[x]^2)/3
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^2 \log^2(x)\, dx &= \frac{1}{3}x^3 \log^2(x) - \frac{2}{3}\int x^2 \log(x)\, dx \\ &= \frac{2x^3}{27} - \frac{2}{9}x^3 \log(x) + \frac{1}{3}x^3 \log^2(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 28, normalized size = 1.00

$$\frac{2x^3}{27} + \frac{1}{3}x^3 \log^2(x) - \frac{2}{9}x^3 \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[x]^2,x]
[Out] (2*x^3)/27 - (2*x^3*Log[x])/9 + (x^3*Log[x]^2)/3
```

fricas [A] time = 0.40, size = 22, normalized size = 0.79

$$\frac{1}{3}\, x^3 \log(x)^2 - \frac{2}{9}\, x^3 \log(x) + \frac{2}{27}\, x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(x)^2,x, algorithm="fricas")
```

```
[Out] 1/3*x^3*log(x)^2 - 2/9*x^3*log(x) + 2/27*x^3
```

giac [A] time = 0.01, size = 22, normalized size = 0.79

$$\frac{1}{3}\,x^{3}\log(x)^{2}-\frac{2}{9}\,x^{3}\log(x)+\frac{2}{27}\,x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(x)^2,x, algorithm="giac")
[Out] 1/3*x^3*log(x)^2 - 2/9*x^3*log(x) + 2/27*x^3
```

maple [A] time = 0.00, size = 23, normalized size = 0.82

$$\frac{x^{3}\ln(x)^{2}}{3}-\frac{2x^{3}\ln(x)}{9}+\frac{2x^{3}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(x)^2,x)
[Out] 2/27*x^3-2/9*x^3*ln(x)+1/3*x^3*ln(x)^2
```

maxima [A] time = 0.52, size = 17, normalized size = 0.61

$$\frac{1}{27}\left(9\,\log(x)^{2}-6\,\log(x)+2\right)x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(x)^2,x, algorithm="maxima")
[Out] 1/27*(9*log(x)^2 - 6*log(x) + 2)*x^3
```

mupad [B] time = 0.03, size = 17, normalized size = 0.61

$$\frac{2\,x^{3}\,\left(\frac{9\ln(x)^{2}}{2}-3\,\ln(x)+1\right)}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(x)^2,x)
[Out] (2*x^3*((9*log(x)^2)/2 - 3*log(x) + 1))/27
```

sympy [A] time = 0.11, size = 26, normalized size = 0.93

$$\frac{x^{3}\log{\left(x\right)}^{2}}{3}-\frac{2x^{3}\log{\left(x\right)}}{9}+\frac{2x^{3}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(x)**2,x)
[Out] x**3*log(x)**2/3 - 2*x**3*log(x)/9 + 2*x**3/27
```

3.64 $\int \frac{1}{x\log(x)}\,dx$

Optimal. Leaf size=3

$$\log(\log(x))$$

```
[Out] ln(ln(x))
```

Rubi [A] time = 0.01, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2302, 29}

$$\log(\log(x))$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Log[x]),x]
```

```
[Out] Log[Log[x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\log(x)}\,dx &= \text{Subst}\left(\int \frac{1}{x}\,dx, x, \log(x)\right)\\ &= \log(\log(x))\end{aligned}$$

Mathematica [A] time = 0.00, size = 3, normalized size = 1.00

$$\log(\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Log[x]),x]
```

```
[Out] Log[Log[x]]
```

fricas [A] time = 0.40, size = 3, normalized size = 1.00

$$\log\left(\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(x),x, algorithm="fricas")
```

```
[Out] log(log(x))
```

giac [A] time = 0.01, size = 4, normalized size = 1.33

$$\log\left(|\log(x)|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(x),x, algorithm="giac")
[Out] log(abs(log(x)))
```

maple [A] time = 0.00, size = 4, normalized size = 1.33

$$\ln(\ln(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/ln(x),x)
[Out] ln(ln(x))
```

maxima [A] time = 0.46, size = 3, normalized size = 1.00

$$\log\left(\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(x),x, algorithm="maxima")
[Out] log(log(x))
```

mupad [B] time = 0.07, size = 3, normalized size = 1.00

$$\ln(\ln(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*log(x)),x)
[Out] log(log(x))
```

sympy [A] time = 0.10, size = 3, normalized size = 1.00

$$\log\left(\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/ln(x),x)
[Out] log(log(x))
```

3.65 $\int \frac{\log(1-t)}{1-t}\,dt$

Optimal. Leaf size=12

$$-\frac{1}{2}\log^2(1-t)$$

```
[Out] -1/2*ln(1-t)^2
```

Rubi [A] time = 0.02, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2390, 2301}

$$-\frac{1}{2}\log^2(1-t)$$

Antiderivative was successfully verified.

```
[In] Int[Log[1 - t]/(1 - t),t]
```

```
[Out] -Log[1 - t]^2/2
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\log(1-t)}{1-t}\,dt &= -\text{Subst}\left(\int \frac{\log(t)}{t}\,dt, t, 1-t\right)\\ &= -\frac{1}{2}\log^2(1-t)\end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$-\frac{1}{2}\log^2(1-t)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[1 - t]/(1 - t),t]
```

```
[Out] -1/2*Log[1 - t]^2
```

fricas [A] time = 0.41, size = 10, normalized size = 0.83

$$-\frac{1}{2}\log\left(-t+1\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1-t)/(1-t),t, algorithm="fricas")
```

```
[Out] -1/2*log(-t + 1)^2
```

giac [A] time = 0.01, size = 10, normalized size = 0.83

$$-\frac{1}{2}\log\left(-t+1\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1-t)/(1-t),t, algorithm="giac")
[Out] -1/2*log(-t + 1)^2
```

maple [A] time = 0.00, size = 11, normalized size = 0.92

$$-\frac{\ln\left(-t+1\right)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(1-t)/(1-t),t)
[Out] -1/2*ln(1-t)^2
```

maxima [A] time = 0.56, size = 10, normalized size = 0.83

$$-\frac{1}{2}\log\left(-t+1\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1-t)/(1-t),t, algorithm="maxima")
[Out] -1/2*log(-t + 1)^2
```

mupad [B] time = 0.35, size = 10, normalized size = 0.83

$$-\frac{\ln\left(1-t\right)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-log(1 - t)/(t - 1),t)
[Out] -log(1 - t)^2/2
```

sympy [A] time = 0.10, size = 8, normalized size = 0.67

$$-\frac{\log\left(1-t\right)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(1-t)/(1-t),t)
[Out] -log(1 - t)**2/2
```

3.66 $\int \frac{\log(x)}{x\sqrt{1+\log(x)}}\,dx$

Optimal. Leaf size=23

$$\frac{2}{3}(\log(x)+1)^{3/2}-2\sqrt{\log(x)+1}$$

```
[Out] 2/3*(1+ln(x))^(3/2)-2*(1+ln(x))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2365, 43}

$$\frac{2}{3}(\log(x)+1)^{3/2}-2\sqrt{\log(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]/(x*Sqrt[1 + Log[x]]),x]
```

```
[Out] -2*Sqrt[1 + Log[x]] + (2*(1 + Log[x])^(3/2))/3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2365

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(c_.)*(x_)^(n
_.)]*(e_.))^(q_.))/(x_), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(d +
e*x)^q, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\log(x)}{x\sqrt{1+\log(x)}}\,dx &= \text{Subst}\left(\int \frac{x}{\sqrt{1+x}}\,dx,x,\log(x)\right)\\ &= \text{Subst}\left(\int \left(-\frac{1}{\sqrt{1+x}}+\sqrt{1+x}\right)dx,x,\log(x)\right)\\ &= -2\sqrt{1+\log(x)}+\frac{2}{3}(1+\log(x))^{3/2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 16, normalized size = 0.70

$$\frac{2}{3}(\log(x)-2)\sqrt{\log(x)+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]/(x*Sqrt[1 + Log[x]]),x]
```

```
[Out] (2*(-2 + Log[x])*Sqrt[1 + Log[x]])/3
```

fricas [A] time = 0.40, size = 12, normalized size = 0.52

$$\frac{2}{3}\sqrt{\log(x)+1}\left(\log(x)-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x/(1+log(x))^(1/2),x, algorithm="fricas")
[Out] 2/3*sqrt(log(x) + 1)*(log(x) - 2)
```

giac [A] time = 0.01, size = 17, normalized size = 0.74

$$\frac{2}{3}\left(\log(x)+1\right)^{\frac{3}{2}}-2\sqrt{\log(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x/(1+log(x))^(1/2),x, algorithm="giac")
[Out] 2/3*(log(x) + 1)^(3/2) - 2*sqrt(log(x) + 1)
```

maple [A] time = 0.01, size = 18, normalized size = 0.78

$$\frac{2\left(\ln(x)+1\right)^{\frac{3}{2}}}{3}-2\sqrt{\ln(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)/x/(1+ln(x))^(1/2),x)
[Out] 2/3*(1+ln(x))^(3/2)-2*(1+ln(x))^(1/2)
```

maxima [A] time = 0.60, size = 17, normalized size = 0.74

$$\frac{2}{3}\left(\log(x)+1\right)^{\frac{3}{2}}-2\sqrt{\log(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x/(1+log(x))^(1/2),x, algorithm="maxima")
[Out] 2/3*(log(x) + 1)^(3/2) - 2*sqrt(log(x) + 1)
```

mupad [B] time = 0.17, size = 13, normalized size = 0.57

$$\sqrt{\ln(x)+1}\,\left(\frac{2\,\ln(x)}{3}-\frac{4}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x)/(x*(log(x) + 1)^(1/2)),x)
[Out] (log(x) + 1)^(1/2)*((2*log(x))/3 - 4/3)
```

sympy [A] time = 5.07, size = 20, normalized size = 0.87

$$\frac{2\left(\log{(x)}+1\right)^{\frac{3}{2}}}{3}-2\sqrt{\log{(x)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)/x/(1+ln(x))**(1/2),x)
[Out] 2*(log(x) + 1)**(3/2)/3 - 2*sqrt(log(x) + 1)
```

3.67 $\int x^3 \log^3(x)\, dx$

Optimal. Leaf size=39

$$-\frac{3x^4}{128} + \frac{1}{4}x^4 \log^3(x) - \frac{3}{16}x^4 \log^2(x) + \frac{3}{32}x^4 \log(x)$$

```
[Out] -3/128*x^4+3/32*x^4*ln(x)-3/16*x^4*ln(x)^2+1/4*x^4*ln(x)^3
```

Rubi [A] time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2305, 2304}

$$-\frac{3x^4}{128} + \frac{1}{4}x^4 \log^3(x) - \frac{3}{16}x^4 \log^2(x) + \frac{3}{32}x^4 \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Log[x]^3,x]
```

```
[Out] (-3*x^4)/128 + (3*x^4*Log[x])/32 - (3*x^4*Log[x]^2)/16 + (x^4*Log[x]^3)/4
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int x^3 \log^3(x)\, dx &= \frac{1}{4}x^4 \log^3(x) - \frac{3}{4}\int x^3 \log^2(x)\, dx \\
&= -\frac{3}{16}x^4 \log^2(x) + \frac{1}{4}x^4 \log^3(x) + \frac{3}{8}\int x^3 \log(x)\, dx \\
&= -\frac{3x^4}{128} + \frac{3}{32}x^4 \log(x) - \frac{3}{16}x^4 \log^2(x) + \frac{1}{4}x^4 \log^3(x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 39, normalized size = 1.00

$$-\frac{3x^4}{128} + \frac{1}{4}x^4 \log^3(x) - \frac{3}{16}x^4 \log^2(x) + \frac{3}{32}x^4 \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Log[x]^3,x]
```

```
[Out] (-3*x^4)/128 + (3*x^4*Log[x])/32 - (3*x^4*Log[x]^2)/16 + (x^4*Log[x]^3)/4
```

fricas [A] time = 0.39, size = 31, normalized size = 0.79

$$\frac{1}{4}\,x^4 \log(x)^3 - \frac{3}{16}\,x^4 \log(x)^2 + \frac{3}{32}\,x^4 \log(x) - \frac{3}{128}\,x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(x)^3,x, algorithm="fricas")
```

```
[Out] 1/4*x^4*log(x)^3 - 3/16*x^4*log(x)^2 + 3/32*x^4*log(x) - 3/128*x^4
```

giac [A] time = 0.01, size = 31, normalized size = 0.79

$$\frac{1}{4}\,x^4\log(x)^3 - \frac{3}{16}\,x^4\log(x)^2 + \frac{3}{32}\,x^4\log(x) - \frac{3}{128}\,x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(x)^3,x, algorithm="giac")
```

```
[Out] 1/4*x^4*log(x)^3 - 3/16*x^4*log(x)^2 + 3/32*x^4*log(x) - 3/128*x^4
```

maple [A] time = 0.00, size = 32, normalized size = 0.82

$$\frac{x^4\ln(x)^3}{4} - \frac{3x^4\ln(x)^2}{16} + \frac{3x^4\ln(x)}{32} - \frac{3x^4}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(x)^3,x)
```

```
[Out] -3/128*x^4+3/32*x^4*ln(x)-3/16*x^4*ln(x)^2+1/4*x^4*ln(x)^3
```

maxima [A] time = 0.51, size = 23, normalized size = 0.59

$$\frac{1}{128}\left(32\,\log(x)^3 - 24\,\log(x)^2 + 12\,\log(x) - 3\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(x)^3,x, algorithm="maxima")
```

```
[Out] 1/128*(32*log(x)^3 - 24*log(x)^2 + 12*log(x) - 3)*x^4
```

mupad [B] time = 0.04, size = 23, normalized size = 0.59

$$\frac{3\,x^4\left(\frac{32\ln(x)^3}{3} - 8\ln(x)^2 + 4\,\ln(x) - 1\right)}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*log(x)^3,x)
```

```
[Out] (3*x^4*(4*log(x) - 8*log(x)^2 + (32*log(x)^3)/3 - 1))/128
```

sympy [A] time = 0.13, size = 37, normalized size = 0.95

$$\frac{x^4\log{(x)}^3}{4} - \frac{3x^4\log{(x)}^2}{16} + \frac{3x^4\log{(x)}}{32} - \frac{3x^4}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(x)**3,x)
```

```
[Out] x**4*log(x)**3/4 - 3*x**4*log(x)**2/16 + 3*x**4*log(x)/32 - 3*x**4/128
```

3.68 $\int e^{x^3} x^2 \, dx$

Optimal. Leaf size=9

$$\frac{e^{x^3}}{3}$$

```
[Out] 1/3*exp(x^3)
```

Rubi [A] time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2209}

$$\frac{e^{x^3}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[E^x^3*x^2,x]
```

```
[Out] E^x^3/3
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rubi steps

$$\int e^{x^3} x^2 \, dx = \frac{e^{x^3}}{3}$$

Mathematica [A] time = 0.00, size = 9, normalized size = 1.00

$$\frac{e^{x^3}}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^3*x^2,x]
```

```
[Out] E^x^3/3
```

fricas [A] time = 0.41, size = 6, normalized size = 0.67

$$\frac{1}{3} e^{\left(x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^3)*x^2,x, algorithm="fricas")
```

```
[Out] 1/3*e^(x^3)
```

giac [A] time = 0.01, size = 6, normalized size = 0.67

$$\frac{1}{3} e^{\left(x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^3)*x^2,x, algorithm="giac")
```

```
[Out] 1/3*e^(x^3)
```

maple [A] time = 0.00, size = 7, normalized size = 0.78

$$\frac{e^{x^3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^3)*x^2,x)
```

```
[Out] 1/3*exp(x^3)
```

maxima [A] time = 0.49, size = 6, normalized size = 0.67

$$\frac{1}{3}\,e^{\left(x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^3)*x^2,x, algorithm="maxima")
```

```
[Out] 1/3*e^(x^3)
```

mupad [B] time = 0.08, size = 6, normalized size = 0.67

$$\frac{e^{x^3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*exp(x^3),x)
```

```
[Out] exp(x^3)/3
```

sympy [A] time = 0.09, size = 5, normalized size = 0.56

$$\frac{e^{x^3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**3)*x**2,x)
```

```
[Out] exp(x**3)/3
```

3.69 $\int \frac{2^{\sqrt{x}}}{\sqrt{x}}\,dx$

Optimal. Leaf size=14

$$\frac{2^{\sqrt{x}+1}}{\log(2)}$$

```
[Out] 2^(1+x^(1/2))/ln(2)
```

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2209}

$$\frac{2^{\sqrt{x}+1}}{\log(2)}$$

Antiderivative was successfully verified.

```
[In] Int[2^Sqrt[x]/Sqrt[x],x]
[Out] 2^(1 + Sqrt[x])/Log[2]
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rubi steps

$$\int \frac{2^{\sqrt{x}}}{\sqrt{x}}\,dx = \frac{2^{1+\sqrt{x}}}{\log(2)}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{2^{\sqrt{x}+1}}{\log(2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[2^Sqrt[x]/Sqrt[x],x]
[Out] 2^(1 + Sqrt[x])/Log[2]
```

fricas [A] time = 0.41, size = 11, normalized size = 0.79

$$\frac{2 \cdot 2^{\left(\sqrt{x}\right)}}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2^(x^(1/2))/x^(1/2),x, algorithm="fricas")
[Out] 2*2^sqrt(x)/log(2)
```

giac [A] time = 0.01, size = 11, normalized size = 0.79

$$\frac{2 \cdot 2^{\left(\sqrt{x}\right)}}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2^(x^(1/2))/x^(1/2),x, algorithm="giac")
[Out] 2*2^sqrt(x)/log(2)
```

maple [A] time = 0.00, size = 12, normalized size = 0.86

$$\frac{2\,2^{\sqrt{x}}}{\ln(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2^(x^(1/2))/x^(1/2),x)
[Out] 2/ln(2)*2^(x^(1/2))
```

maxima [A] time = 0.58, size = 12, normalized size = 0.86

$$\frac{2^{\sqrt{x}+1}}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2^(x^(1/2))/x^(1/2),x, algorithm="maxima")
[Out] 2^(sqrt(x) + 1)/log(2)
```

mupad [B] time = 0.10, size = 11, normalized size = 0.79

$$\frac{2\,2^{\sqrt{x}}}{\ln(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2^(x^(1/2))/x^(1/2),x)
[Out] (2*2^(x^(1/2)))/log(2)
```

sympy [A] time = 0.18, size = 10, normalized size = 0.71

$$\frac{2 \cdot 2^{\sqrt{x}}}{\log{(2)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2**(x**(1/2))/x**(1/2),x)
[Out] 2*2**(sqrt(x))/log(2)
```

3.70 $\int e^{2\sin(x)}\cos(x)\,dx$

Optimal. Leaf size=10

$$\frac{1}{2}e^{2\sin(x)}$$

```
[Out] 1/2*exp(2*sin(x))
```

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4334, 2194}

$$\frac{1}{2}e^{2\sin(x)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*Sin[x])*Cos[x],x]
```

```
[Out] E^(2*Sin[x])/2
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4334

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}\int e^{2\sin(x)}\cos(x)\,dx &= \text{Subst}\left(\int e^{2x}\,dx, x, \sin(x)\right)\\ &= \frac{1}{2}e^{2\sin(x)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 10, normalized size = 1.00

$$\frac{1}{2}e^{2\sin(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*Sin[x])*Cos[x],x]
```

```
[Out] E^(2*Sin[x])/2
```

fricas [A] time = 0.41, size = 7, normalized size = 0.70

$$\frac{1}{2}e^{(2\sin(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*sin(x))*cos(x),x, algorithm="fricas")
```

```
[Out] 1/2*e^(2*sin(x))
```

giac [A] time = 0.01, size = 7, normalized size = 0.70

$$\frac{1}{2}\,e^{(2\,\sin(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*sin(x))*cos(x),x, algorithm="giac")
[Out] 1/2*e^(2*sin(x))
```

maple [A] time = 0.01, size = 8, normalized size = 0.80

$$\frac{\mathrm{e}^{2\sin(x)}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*sin(x))*cos(x),x)
[Out] 1/2*exp(2*sin(x))
```

maxima [A] time = 0.64, size = 7, normalized size = 0.70

$$\frac{1}{2}\,e^{(2\,\sin(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*sin(x))*cos(x),x, algorithm="maxima")
[Out] 1/2*e^(2*sin(x))
```

mupad [B] time = 0.10, size = 7, normalized size = 0.70

$$\frac{\mathrm{e}^{2\,\sin(x)}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*sin(x))*cos(x),x)
[Out] exp(2*sin(x))/2
```

sympy [A] time = 0.36, size = 7, normalized size = 0.70

$$\frac{e^{2\sin{(x)}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*sin(x))*cos(x),x)
[Out] exp(2*sin(x))/2
```

3.71 $\int e^x \sin(x)\,dx$

Optimal. Leaf size=19

$$\frac{1}{2}e^x\sin(x)-\frac{1}{2}e^x\cos(x)$$

```
[Out] -1/2*exp(x)*cos(x)+1/2*exp(x)*sin(x)
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4432}

$$\frac{1}{2}e^x\sin(x)-\frac{1}{2}e^x\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sin[x],x]
```

```
[Out] -(E^x*Cos[x])/2 + (E^x*Sin[x])/2
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^x\sin(x)\,dx=-\frac{1}{2}e^x\cos(x)+\frac{1}{2}e^x\sin(x)$$

Mathematica [A] time = 0.01, size = 14, normalized size = 0.74

$$\frac{1}{2}e^x(\sin(x)-\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sin[x],x]
```

```
[Out] (E^x*(-Cos[x] + Sin[x]))/2
```

fricas [A] time = 0.41, size = 13, normalized size = 0.68

$$-\frac{1}{2}\cos(x)e^x+\frac{1}{2}e^x\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(x),x, algorithm="fricas")
```

```
[Out] -1/2*cos(x)*e^x + 1/2*e^x*sin(x)
```

giac [A] time = 0.01, size = 11, normalized size = 0.58

$$-\frac{1}{2}(\cos(x)-\sin(x))e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(x),x, algorithm="giac")
```

```
[Out] -1/2*(cos(x) - sin(x))*e^x
```

maple [A] time = 0.01, size = 14, normalized size = 0.74

$$-\frac{\cos(x)e^{x}}{2}+\frac{e^{x}\sin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sin(x),x)
```

```
[Out] -1/2*exp(x)*cos(x)+1/2*exp(x)*sin(x)
```

maxima [A] time = 0.53, size = 11, normalized size = 0.58

$$-\frac{1}{2}\left(\cos(x)-\sin(x)\right)e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/2*(cos(x) - sin(x))*e^x
```

mupad [B] time = 0.02, size = 11, normalized size = 0.58

$$-\frac{e^{x}\left(\cos(x)-\sin(x)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sin(x),x)
```

```
[Out] -(exp(x)*(cos(x) - sin(x)))/2
```

sympy [A] time = 0.31, size = 15, normalized size = 0.79

$$\frac{e^{x}\sin\left(x\right)}{2}-\frac{e^{x}\cos\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(x),x)
```

```
[Out] exp(x)*sin(x)/2 - exp(x)*cos(x)/2
```

3.72 $\int e^x \cos(x)\,dx$

Optimal. Leaf size=19

$$\frac{1}{2}e^x\sin(x)+\frac{1}{2}e^x\cos(x)$$

```
[Out] 1/2*exp(x)*cos(x)+1/2*exp(x)*sin(x)
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4433}

$$\frac{1}{2}e^x\sin(x)+\frac{1}{2}e^x\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Cos[x],x]
```

```
[Out] (E^x*Cos[x])/2 + (E^x*Sin[x])/2
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^x\cos(x)\,dx=\frac{1}{2}e^x\cos(x)+\frac{1}{2}e^x\sin(x)$$

Mathematica [A] time = 0.01, size = 12, normalized size = 0.63

$$\frac{1}{2}e^x(\sin(x)+\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Cos[x],x]
```

```
[Out] (E^x*(Cos[x] + Sin[x]))/2
```

fricas [A] time = 0.40, size = 13, normalized size = 0.68

$$\frac{1}{2}\cos(x)e^x+\frac{1}{2}e^x\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(x),x, algorithm="fricas")
```

```
[Out] 1/2*cos(x)*e^x + 1/2*e^x*sin(x)
```

giac [A] time = 0.01, size = 9, normalized size = 0.47

$$\frac{1}{2}(\cos(x)+\sin(x))e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(x),x, algorithm="giac")
```

```
[Out] 1/2*(cos(x) + sin(x))*e^x
```

maple [A] time = 0.02, size = 14, normalized size = 0.74

$$\frac{\cos(x)e^x}{2} + \frac{e^x \sin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*exp(x),x)
```

```
[Out] 1/2*cos(x)*exp(x)+1/2*exp(x)*sin(x)
```

maxima [A] time = 0.51, size = 9, normalized size = 0.47

$$\frac{1}{2}(\cos(x) + \sin(x))e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(x),x, algorithm="maxima")
```

```
[Out] 1/2*(cos(x) + sin(x))*e^x
```

mupad [B] time = 0.02, size = 9, normalized size = 0.47

$$\frac{e^x\ (\cos(x) + \sin(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*cos(x),x)
```

```
[Out] (exp(x)*(cos(x) + sin(x)))/2
```

sympy [A] time = 0.32, size = 15, normalized size = 0.79

$$\frac{e^x \sin(x)}{2} + \frac{e^x \cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(x),x)
```

```
[Out] exp(x)*sin(x)/2 + exp(x)*cos(x)/2
```

3.73 $\int \frac{1}{1+e^x} dx$

Optimal. Leaf size=10

$$x - \log\left(e^x + 1\right)$$

```
[Out] x-ln(1+exp(x))
```

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {2282, 36, 29, 31}

$$x - \log\left(e^x + 1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + E^x)^(-1),x]
```

```
[Out] x - Log[1 + E^x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{1+e^x} dx &= \text{Subst}\left(\int \frac{1}{x(1+x)} dx, x, e^x\right) \\
&= \text{Subst}\left(\int \frac{1}{x} dx, x, e^x\right) - \text{Subst}\left(\int \frac{1}{1+x} dx, x, e^x\right) \\
&= x - \log\left(1+e^x\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$x - \log\left(e^x + 1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + E^x)^(-1),x]
```

```
[Out] x - Log[1 + E^x]
```

fricas [A] time = 0.41, size = 9, normalized size = 0.90

$$x - \log\left(e^{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+exp(x)),x, algorithm="fricas")
[Out] x - log(e^x + 1)
```

giac [A] time = 0.01, size = 9, normalized size = 0.90

$$x - \log\left(e^{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+exp(x)),x, algorithm="giac")
[Out] x - log(e^x + 1)
```

maple [A] time = 0.01, size = 12, normalized size = 1.20

$$-\ln\left(\mathrm{e}^{x} + 1\right) + \ln\left(\mathrm{e}^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+exp(x)),x)
[Out] -ln(1+exp(x))+ln(exp(x))
```

maxima [A] time = 0.57, size = 9, normalized size = 0.90

$$x - \log\left(e^{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+exp(x)),x, algorithm="maxima")
[Out] x - log(e^x + 1)
```

mupad [B] time = 0.04, size = 9, normalized size = 0.90

$$x - \ln\left(\mathrm{e}^{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(exp(x) + 1),x)
[Out] x - log(exp(x) + 1)
```

sympy [A] time = 0.08, size = 7, normalized size = 0.70

$$x - \log\left(e^{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+exp(x)),x)
[Out] x - log(exp(x) + 1)
```

3.74 $\int e^x x\,dx$

Optimal. Leaf size=11

$$e^x x - e^x$$

[Out] -exp(x)+exp(x)*x

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2176, 2194}

$$e^x x - e^x$$

Antiderivative was successfully verified.

[In] Int[E^x*x,x]

[Out] -E^x + E^x*x

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned} \int e^x x\,dx &= e^x x - \int e^x\,dx \\ &= -e^x + e^x x \end{aligned}$$

Mathematica [A] time = 0.00, size = 7, normalized size = 0.64

$$e^x(x-1)$$

Antiderivative was successfully verified.

[In] Integrate[E^x*x,x]

[Out] E^x*(-1 + x)

fricas [A] time = 0.40, size = 6, normalized size = 0.55

$$(x-1)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)*x,x, algorithm="fricas")

[Out] (x - 1)*e^x

giac [A] time = 0.01, size = 6, normalized size = 0.55

$$(x-1)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x,x, algorithm="giac")
[Out] (x - 1)*e^x
```

maple [A] time = 0.00, size = 7, normalized size = 0.64

$$(x-1)\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*x,x)
[Out] (x-1)*exp(x)
```

maxima [A] time = 0.57, size = 6, normalized size = 0.55

$$(x-1)e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x,x, algorithm="maxima")
[Out] (x - 1)*e^x
```

mupad [B] time = 0.02, size = 6, normalized size = 0.55

$$e^{x}\,(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*exp(x),x)
[Out] exp(x)*(x - 1)
```

sympy [A] time = 0.09, size = 5, normalized size = 0.45

$$(x-1)\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x,x)
[Out] (x - 1)*exp(x)
```

3.75 $\int e^{-x} x \, dx$

Optimal. Leaf size=16

$$-e^{-x}x - e^{-x}$$

```
[Out] -1/exp(x)-x/exp(x)
```

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2176, 2194}

$$-e^{-x}x - e^{-x}$$

Antiderivative was successfully verified.

```
[In] Int[x/E^x,x]
[Out] -E^(-x) - x/E^x
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^{-x} x \, dx &= -e^{-x}x + \int e^{-x} \, dx \\ &= -e^{-x} - e^{-x}x\end{aligned}$$

Mathematica [A] time = 0.01, size = 11, normalized size = 0.69

$$e^{-x}(-x - 1)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/E^x,x]
[Out] (-1 - x)/E^x
```

fricas [A] time = 0.39, size = 9, normalized size = 0.56

$$-(x + 1)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/exp(x),x, algorithm="fricas")
[Out] -(x + 1)*e^(-x)
```

giac [A] time = 0.02, size = 9, normalized size = 0.56

$$-(x + 1)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/exp(x),x, algorithm="giac")
```

```
[Out] -(x + 1)*e^(-x)
```

maple [A] time = 0.00, size = 10, normalized size = 0.62

$$-(x+1)\,\mathrm{e}^{-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/exp(x),x)
```

```
[Out] -(x+1)/exp(x)
```

maxima [A] time = 0.51, size = 9, normalized size = 0.56

$$-(x+1)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/exp(x),x, algorithm="maxima")
```

```
[Out] -(x + 1)*e^(-x)
```

mupad [B] time = 0.02, size = 9, normalized size = 0.56

$$-\mathrm{e}^{-x}\,(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*exp(-x),x)
```

```
[Out] -exp(-x)*(x + 1)
```

sympy [A] time = 0.09, size = 7, normalized size = 0.44

$$(-x-1)\,e^{-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/exp(x),x)
```

```
[Out] (-x - 1)*exp(-x)
```

3.76 $\int e^x x^2 \, dx$

Optimal. Leaf size=19

$$e^x x^2 - 2e^x x + 2e^x$$

```
[Out] 2*exp(x)-2*exp(x)*x+exp(x)*x^2
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2176, 2194}

$$e^x x^2 - 2e^x x + 2e^x$$

Antiderivative was successfully verified.

```
[In] Int[E^x*x^2,x]
```

```
[Out] 2*E^x - 2*E^x*x + E^x*x^2
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned} \int e^x x^2 \, dx &= e^x x^2 - 2 \int e^x x \, dx \\ &= -2e^x x + e^x x^2 + 2 \int e^x \, dx \\ &= 2e^x - 2e^x x + e^x x^2 \end{aligned}$$

Mathematica [A] time = 0.01, size = 12, normalized size = 0.63

$$e^x \left(x^2 - 2x + 2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*x^2,x]
```

```
[Out] E^x*(2 - 2*x + x^2)
```

fricas [A] time = 0.40, size = 11, normalized size = 0.58

$$\left(x^2 - 2\,x + 2\right)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x^2,x, algorithm="fricas")
```

```
[Out] (x^2 - 2*x + 2)*e^x
```

giac [A] time = 0.01, size = 11, normalized size = 0.58

$$\left(x^2 - 2\,x + 2\right)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x^2,x, algorithm="giac")
[Out] (x^2 - 2*x + 2)*e^x
```

maple [A] time = 0.00, size = 12, normalized size = 0.63

$$\left(x^2 - 2x + 2\right)\mathrm{e}^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*x^2,x)
[Out] (x^2-2*x+2)*exp(x)
```

maxima [A] time = 0.53, size = 11, normalized size = 0.58

$$\left(x^2 - 2\,x + 2\right)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x^2,x, algorithm="maxima")
[Out] (x^2 - 2*x + 2)*e^x
```

mupad [B] time = 0.02, size = 11, normalized size = 0.58

$$\mathrm{e}^x\,\left(x^2 - 2\,x + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*exp(x),x)
[Out] exp(x)*(x^2 - 2*x + 2)
```

sympy [A] time = 0.09, size = 10, normalized size = 0.53

$$\left(x^2 - 2x + 2\right)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x**2,x)
[Out] (x**2 - 2*x + 2)*exp(x)
```

3.77 $\int e^{-2x}x^2\,dx$

Optimal. Leaf size=32

$$-\frac{1}{2}e^{-2x}x^2-\frac{1}{2}e^{-2x}x-\frac{e^{-2x}}{4}$$

```
[Out] -1/4/exp(2*x)-1/2*x/exp(2*x)-1/2*x^2/exp(2*x)
```

Rubi [A] time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2176, 2194}

$$-\frac{1}{2}e^{-2x}x^2-\frac{1}{2}e^{-2x}x-\frac{e^{-2x}}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/E^(2*x),x]
```

```
[Out] -1/(4*E^(2*x)) - x/(2*E^(2*x)) - x^2/(2*E^(2*x))
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] && !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^{-2x}x^2\,dx&=-\frac{1}{2}e^{-2x}x^2+\int e^{-2x}x\,dx\\&=-\frac{1}{2}e^{-2x}x-\frac{1}{2}e^{-2x}x^2+\frac{1}{2}\int e^{-2x}\,dx\\&=-\frac{1}{4}e^{-2x}-\frac{1}{2}e^{-2x}x-\frac{1}{2}e^{-2x}x^2\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 0.59

$$-\frac{1}{4}e^{-2x}\left(2x^2+2x+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/E^(2*x),x]
```

```
[Out] -1/4*(1 + 2*x + 2*x^2)/E^(2*x)
```

fricas [A] time = 0.38, size = 16, normalized size = 0.50

$$-\frac{1}{4}\left(2\,x^2+2\,x+1\right)e^{(-2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/exp(2*x),x, algorithm="fricas")
[Out] -1/4*(2*x^2 + 2*x + 1)*e^(-2*x)
```

giac [A] time = 0.01, size = 16, normalized size = 0.50

$$-\frac{1}{4}\left(2\,x^2+2\,x+1\right)e^{(-2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/exp(2*x),x, algorithm="giac")
[Out] -1/4*(2*x^2 + 2*x + 1)*e^(-2*x)
```

maple [A] time = 0.00, size = 19, normalized size = 0.59

$$-\frac{\left(2x^2+2x+1\right)\mathrm{e}^{-2x}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/exp(2*x),x)
[Out] -1/4*(2*x^2+2*x+1)/exp(2*x)
```

maxima [A] time = 0.65, size = 16, normalized size = 0.50

$$-\frac{1}{4}\left(2\,x^2+2\,x+1\right)e^{(-2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/exp(2*x),x, algorithm="maxima")
[Out] -1/4*(2*x^2 + 2*x + 1)*e^(-2*x)
```

mupad [B] time = 0.08, size = 16, normalized size = 0.50

$$-\frac{\mathrm{e}^{-2\,x}\,\left(4\,x^2+4\,x+2\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*exp(-2*x),x)
[Out] -(exp(-2*x)*(4*x + 4*x^2 + 2))/8
```

sympy [A] time = 0.09, size = 17, normalized size = 0.53

$$\frac{\left(-2x^2-2x-1\right)e^{-2x}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/exp(2*x),x)
[Out] (-2*x**2 - 2*x - 1)*exp(-2*x)/4
```

3.78 $\int e^{\sqrt{x}}\,dx$

Optimal. Leaf size=24

$$2e^{\sqrt{x}}\sqrt{x}-2e^{\sqrt{x}}$$

[Out] -2*exp(x^(1/2))+2*exp(x^(1/2))*x^(1/2)

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2207, 2176, 2194}

$$2e^{\sqrt{x}}\sqrt{x}-2e^{\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Int[E^Sqrt[x],x]

[Out] -2*E^Sqrt[x] + 2*E^Sqrt[x]*Sqrt[x]

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2207

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> With[{k =
Denominator[n]}, Dist[k/d, Subst[Int[x^(k - 1)*F^(a + b*x^(k*n)), x], x, (c
 + d*x)^(1/k)], x]] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2/n] &&  !Inte
gerQ[n]
```

Rubi steps

$$\begin{aligned}\int e^{\sqrt{x}}\,dx&=2\,\text{Subst}\left(\int e^{x}x\,dx,x,\sqrt{x}\right)\\&=2e^{\sqrt{x}}\sqrt{x}-2\,\text{Subst}\left(\int e^{x}\,dx,x,\sqrt{x}\right)\\&=-2e^{\sqrt{x}}+2e^{\sqrt{x}}\sqrt{x}\end{aligned}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 0.67

$$2e^{\sqrt{x}}\left(\sqrt{x}-1\right)$$

Antiderivative was successfully verified.

[In] Integrate[E^Sqrt[x],x]

[Out] 2*E^Sqrt[x]*(-1 + Sqrt[x])

fricas [A] time = 0.40, size = 11, normalized size = 0.46

$$2\left(\sqrt{x}-1\right)e^{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^(1/2)),x, algorithm="fricas")
[Out] 2*(sqrt(x) - 1)*e^sqrt(x)
```

giac [A] time = 0.01, size = 11, normalized size = 0.46

$$2\left(\sqrt{x}-1\right)e^{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^(1/2)),x, algorithm="giac")
[Out] 2*(sqrt(x) - 1)*e^sqrt(x)
```

maple [A] time = 0.01, size = 17, normalized size = 0.71

$$2\sqrt{x}\,\mathrm{e}^{\sqrt{x}}-2\,\mathrm{e}^{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^(1/2)),x)
[Out] -2*exp(x^(1/2))+2*exp(x^(1/2))*x^(1/2)
```

maxima [A] time = 0.54, size = 11, normalized size = 0.46

$$2\left(\sqrt{x}-1\right)e^{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^(1/2)),x, algorithm="maxima")
[Out] 2*(sqrt(x) - 1)*e^sqrt(x)
```

mupad [B] time = 0.02, size = 11, normalized size = 0.46

$$2\,\mathrm{e}^{\sqrt{x}}\,\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^(1/2)),x)
[Out] 2*exp(x^(1/2))*(x^(1/2) - 1)
```

sympy [A] time = 0.20, size = 20, normalized size = 0.83

$$2\sqrt{x}e^{\sqrt{x}}-2e^{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**(1/2)),x)
[Out] 2*sqrt(x)*exp(sqrt(x)) - 2*exp(sqrt(x))
```

3.79 $\int e^{-x^2} x^3 \, dx$

Optimal. Leaf size=26

$$-\frac{1}{2}e^{-x^2}x^2 - \frac{e^{-x^2}}{2}$$

```
[Out] -1/2/exp(x^2)-1/2*x^2/exp(x^2)
```

Rubi [A] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2212, 2209}

$$-\frac{1}{2}e^{-x^2}x^2 - \frac{e^{-x^2}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/E^x^2,x]
```

```
[Out] -1/(2*E^x^2) - x^2/(2*E^x^2)
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rubi steps

$$\begin{aligned}\int e^{-x^2}x^3\,dx &= -\frac{1}{2}e^{-x^2}x^2 + \int e^{-x^2}x\,dx\\ &= -\frac{e^{-x^2}}{2} - \frac{1}{2}e^{-x^2}x^2\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 0.62

$$-\frac{1}{2}e^{-x^2}\left(x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/E^x^2,x]
```

```
[Out] -1/2*(1 + x^2)/E^x^2
```

fricas [A] time = 0.38, size = 13, normalized size = 0.50

$$-\frac{1}{2}\left(x^2+1\right)e^{\left(-x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/exp(x^2),x, algorithm="fricas")
[Out] -1/2*(x^2 + 1)*e^(-x^2)
```

giac [A] time = 0.01, size = 13, normalized size = 0.50

$$-\frac{1}{2}\left(x^2+1\right)e^{\left(-x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/exp(x^2),x, algorithm="giac")
[Out] -1/2*(x^2 + 1)*e^(-x^2)
```

maple [A] time = 0.00, size = 14, normalized size = 0.54

$$-\frac{\left(x^2+1\right)\mathrm{e}^{-x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/exp(x^2),x)
[Out] -1/2*(x^2+1)/exp(x^2)
```

maxima [A] time = 0.51, size = 13, normalized size = 0.50

$$-\frac{1}{2}\left(x^2+1\right)e^{\left(-x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/exp(x^2),x, algorithm="maxima")
[Out] -1/2*(x^2 + 1)*e^(-x^2)
```

mupad [B] time = 0.11, size = 13, normalized size = 0.50

$$-\frac{\mathrm{e}^{-x^2}\,\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*exp(-x^2),x)
[Out] -(exp(-x^2)*(x^2 + 1))/2
```

sympy [A] time = 0.09, size = 12, normalized size = 0.46

$$\frac{\left(-x^2-1\right)e^{-x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/exp(x**2),x)
[Out] (-x**2 - 1)*exp(-x**2)/2
```

3.80 $\int e^{ax}\cos(bx)\,dx$

Optimal. Leaf size=41

$$\frac{be^{ax}\sin(bx)}{a^2+b^2}+\frac{ae^{ax}\cos(bx)}{a^2+b^2}$$

```
[Out] a*exp(a*x)*cos(b*x)/(a^2+b^2)+b*exp(a*x)*sin(b*x)/(a^2+b^2)
```

Rubi [A] time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {4433}

$$\frac{be^{ax}\sin(bx)}{a^2+b^2}+\frac{ae^{ax}\cos(bx)}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a*x)*Cos[b*x],x]
```

```
[Out] (a*E^(a*x)*Cos[b*x])/(a^2 + b^2) + (b*E^(a*x)*Sin[b*x])/(a^2 + b^2)
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^{ax}\cos(bx)\,dx=\frac{ae^{ax}\cos(bx)}{a^2+b^2}+\frac{be^{ax}\sin(bx)}{a^2+b^2}$$

Mathematica [A] time = 0.03, size = 28, normalized size = 0.68

$$\frac{e^{ax}(a\cos(bx)+b\sin(bx))}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a*x)*Cos[b*x],x]
```

```
[Out] (E^(a*x)*(a*Cos[b*x] + b*Sin[b*x]))/(a^2 + b^2)
```

fricas [A] time = 0.40, size = 31, normalized size = 0.76

$$\frac{a\cos(bx)\,e^{(ax)}+be^{(ax)}\sin(bx)}{a^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x)*cos(b*x),x, algorithm="fricas")
```

```
[Out] (a*cos(b*x)*e^(a*x) + b*e^(a*x)*sin(b*x))/(a^2 + b^2)
```

giac [A] time = 0.01, size = 36, normalized size = 0.88

$$\left(\frac{a\cos(bx)}{a^2+b^2}+\frac{b\sin(bx)}{a^2+b^2}\right)e^{(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x)*cos(b*x),x, algorithm="giac")
```

```
[Out] (a*cos(b*x)/(a^2 + b^2) + b*sin(b*x)/(a^2 + b^2))*e^(a*x)
```

maple [A] time = 0.02, size = 40, normalized size = 0.98

$$\frac{a\cos(bx)\,\mathrm{e}^{ax}}{a^2+b^2}+\frac{b\,\mathrm{e}^{ax}\sin(bx)}{a^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a*x)*cos(b*x),x)
```

```
[Out] a*exp(a*x)*cos(b*x)/(a^2+b^2)+b*exp(a*x)*sin(b*x)/(a^2+b^2)
```

maxima [A] time = 0.48, size = 27, normalized size = 0.66

$$\frac{(a\cos(bx)+b\sin(bx))e^{(ax)}}{a^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x)*cos(b*x),x, algorithm="maxima")
```

```
[Out] (a*cos(b*x) + b*sin(b*x))*e^(a*x)/(a^2 + b^2)
```

mupad [B] time = 0.03, size = 27, normalized size = 0.66

$$\frac{\mathrm{e}^{a\,x}\,\left(a\,\cos(b\,x)+b\,\sin(b\,x)\right)}{a^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a*x)*cos(b*x),x)
```

```
[Out] (exp(a*x)*(a*cos(b*x) + b*sin(b*x)))/(a^2 + b^2)
```

sympy [A] time = 1.11, size = 136, normalized size = 3.32

$$\begin{cases} x & \text{for } a=0 \wedge b=0 \\ \frac{ixe^{-ibx}\sin(bx)}{2}+\frac{xe^{-ibx}\cos(bx)}{2}+\frac{e^{-ibx}\sin(bx)}{2b} & \text{for } a=-ib \\ -\frac{ixe^{ibx}\sin(bx)}{2}+\frac{xe^{ibx}\cos(bx)}{2}+\frac{e^{ibx}\sin(bx)}{2b} & \text{for } a=ib \\ \frac{ae^{ax}\cos(bx)}{a^2+b^2}+\frac{be^{ax}\sin(bx)}{a^2+b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x)*cos(b*x),x)
```

```
[Out] Piecewise((x, Eq(a, 0) & Eq(b, 0)), (I*x*exp(-I*b*x)*sin(b*x)/2 + x*exp(-I*
b*x)*cos(b*x)/2 + exp(-I*b*x)*sin(b*x)/(2*b), Eq(a, -I*b)), (-I*x*exp(I*b*x
)*sin(b*x)/2 + x*exp(I*b*x)*cos(b*x)/2 + exp(I*b*x)*sin(b*x)/(2*b), Eq(a, I
*b)), (a*exp(a*x)*cos(b*x)/(a**2 + b**2) + b*exp(a*x)*sin(b*x)/(a**2 + b**2
), True))
```

3.81 $\int e^{ax} \sin(bx)\,dx$

Optimal. Leaf size=42

$$\frac{ae^{ax}\sin(bx)}{a^2+b^2}-\frac{be^{ax}\cos(bx)}{a^2+b^2}$$

```
[Out] -b*exp(a*x)*cos(b*x)/(a^2+b^2)+a*exp(a*x)*sin(b*x)/(a^2+b^2)
```

Rubi [A] time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {4432}

$$\frac{ae^{ax}\sin(bx)}{a^2+b^2}-\frac{be^{ax}\cos(bx)}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a*x)*Sin[b*x],x]
```

```
[Out] -((b*E^(a*x)*Cos[b*x])/(a^2 + b^2)) + (a*E^(a*x)*Sin[b*x])/(a^2 + b^2)
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^{ax}\sin(bx)\,dx=-\frac{be^{ax}\cos(bx)}{a^2+b^2}+\frac{ae^{ax}\sin(bx)}{a^2+b^2}$$

Mathematica [A] time = 0.03, size = 29, normalized size = 0.69

$$\frac{e^{ax}(a\sin(bx)-b\cos(bx))}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a*x)*Sin[b*x],x]
```

```
[Out] (E^(a*x)*(-(b*Cos[b*x]) + a*Sin[b*x]))/(a^2 + b^2)
```

fricas [A] time = 0.41, size = 33, normalized size = 0.79

$$-\frac{b\cos(bx)\,e^{(ax)}-ae^{(ax)}\sin(bx)}{a^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x)*sin(b*x),x, algorithm="fricas")
```

```
[Out] -(b*cos(b*x)*e^(a*x) - a*e^(a*x)*sin(b*x))/(a^2 + b^2)
```

giac [A] time = 0.01, size = 38, normalized size = 0.90

$$-\left(\frac{b\cos(bx)}{a^2+b^2}-\frac{a\sin(bx)}{a^2+b^2}\right)e^{(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x)*sin(b*x),x, algorithm="giac")
[Out] -(b*cos(b*x)/(a^2 + b^2) - a*sin(b*x)/(a^2 + b^2))*e^(a*x)
```

maple [A] time = 0.01, size = 41, normalized size = 0.98

$$\frac{a\,e^{ax}\sin(bx)}{a^2+b^2}-\frac{b\cos(bx)\,e^{ax}}{a^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a*x)*sin(b*x),x)
[Out] -b*exp(a*x)*cos(b*x)/(a^2+b^2)+a*exp(a*x)*sin(b*x)/(a^2+b^2)
```

maxima [A] time = 0.53, size = 29, normalized size = 0.69

$$-\frac{(b\cos(bx)-a\sin(bx))e^{(ax)}}{a^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x)*sin(b*x),x, algorithm="maxima")
[Out] -(b*cos(b*x) - a*sin(b*x))*e^(a*x)/(a^2 + b^2)
```

mupad [B] time = 0.02, size = 29, normalized size = 0.69

$$-\frac{e^{a\,x}\,\left(b\,\cos(b\,x)-a\,\sin(b\,x)\right)}{a^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a*x)*sin(b*x),x)
[Out] -(exp(a*x)*(b*cos(b*x) - a*sin(b*x)))/(a^2 + b^2)
```

sympy [A] time = 1.10, size = 139, normalized size = 3.31

$$\begin{cases} 0 & \text{for}\ a=0 \wedge b=0 \\ \frac{xe^{-ibx}\sin(bx)}{2}-\frac{ixe^{-ibx}\cos(bx)}{2}+\frac{ie^{-ibx}\sin(bx)}{2b} & \text{for}\ a=-ib \\ \frac{xe^{ibx}\sin(bx)}{2}+\frac{ixe^{ibx}\cos(bx)}{2}-\frac{ie^{ibx}\sin(bx)}{2b} & \text{for}\ a=ib \\ \frac{ae^{ax}\sin(bx)}{a^2+b^2}-\frac{be^{ax}\cos(bx)}{a^2+b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x)*sin(b*x),x)
[Out] Piecewise((0, Eq(a, 0) & Eq(b, 0)), (x*exp(-I*b*x)*sin(b*x)/2 - I*x*exp(-I*
b*x)*cos(b*x)/2 + I*exp(-I*b*x)*sin(b*x)/(2*b), Eq(a, -I*b)), (x*exp(I*b*x)
*sin(b*x)/2 + I*x*exp(I*b*x)*cos(b*x)/2 - I*exp(I*b*x)*sin(b*x)/(2*b), Eq(a
, I*b)), (a*exp(a*x)*sin(b*x)/(a**2 + b**2) - b*exp(a*x)*cos(b*x)/(a**2 + b
**2), True))
```

3.82 $\int \cot^{-1}(x)\,dx$

Optimal. Leaf size=15

$$\frac{1}{2}\log\left(x^2+1\right)+x\cot^{-1}(x)$$

[Out] x*arccot(x)+1/2*ln(x^2+1)

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {4847, 260}

$$\frac{1}{2}\log\left(x^2+1\right)+x\cot^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[ArcCot[x],x]

[Out] x*ArcCot[x] + Log[1 + x^2]/2

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4847

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cCot[c*x])^p, x] + Dist[b*c*p, Int[(x*(a + b*ArcCot[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \cot^{-1}(x)\,dx &= x\cot^{-1}(x)+\int \frac{x}{1+x^2}\,dx\\ &= x\cot^{-1}(x)+\frac{1}{2}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{1}{2}\log\left(x^2+1\right)+x\cot^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[ArcCot[x],x]

[Out] x*ArcCot[x] + Log[1 + x^2]/2

fricas [A] time = 0.42, size = 13, normalized size = 0.87

$$x\,\mathrm{arccot}(x)+\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccot(x),x, algorithm="fricas")

[Out] x*arccot(x) + 1/2*log(x^2 + 1)

giac [A] time = 0.01, size = 21, normalized size = 1.40

$$x \arctan\left(\frac{1}{x}\right) + \frac{1}{2} \log\left(\frac{1}{x^2} + 1\right) - \frac{1}{2} \log\left(\frac{1}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(x),x, algorithm="giac")
[Out] x*arctan(1/x) + 1/2*log(1/x^2 + 1) - 1/2*log(x^(-2))
```

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$x \operatorname{arccot}(x) + \frac{\ln\left(x^2 + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(x),x)
[Out] x*arccot(x)+1/2*ln(x^2+1)
```

maxima [A] time = 0.54, size = 13, normalized size = 0.87

$$x \operatorname{arccot}(x) + \frac{1}{2} \log\left(x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(x),x, algorithm="maxima")
[Out] x*arccot(x) + 1/2*log(x^2 + 1)
```

mupad [B] time = 0.10, size = 13, normalized size = 0.87

$$\frac{\ln\left(x^2 + 1\right)}{2} + x \operatorname{acot}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(x),x)
[Out] log(x^2 + 1)/2 + x*acot(x)
```

sympy [A] time = 0.21, size = 12, normalized size = 0.80

$$x \operatorname{acot}(x) + \frac{\log\left(x^2 + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(x),x)
[Out] x*acot(x) + log(x**2 + 1)/2
```

3.83 $\int \sec^{-1}(x)\,dx$

Optimal. Leaf size=19

$$x\sec^{-1}(x)-\tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

```
[Out] x*arcsec(x)-arctanh((1-1/x^2)^(1/2))
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 2.000, Rules used = {5214, 266, 63, 206}

$$x\sec^{-1}(x)-\tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSec[x],x]
```

```
[Out] x*ArcSec[x] - ArcTanh[Sqrt[1 - x^(-2)]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5214

```
Int[ArcSec[(c_.)*(x_)], x_Symbol] :> Simp[x*ArcSec[c*x], x] - Dist[1/c, Int
[1/(x*Sqrt[1 - 1/(c^2*x^2)]), x], x] /; FreeQ[c, x]
```

Rubi steps

$$\int \sec^{-1}(x)\,dx = x\sec^{-1}(x) - \int \frac{1}{\sqrt{1-\frac{1}{x^2}}\,x}\,dx$$
$$= x\sec^{-1}(x) + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx, x, \frac{1}{x^2}\right)$$
$$= x\sec^{-1}(x) - \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-\frac{1}{x^2}}\right)$$
$$= x\sec^{-1}(x) - \tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

Mathematica [B] time = 0.07, size = 64, normalized size = 3.37

$$x\sec^{-1}(x) - \frac{\sqrt{x^2-1}\left(\log\left(\frac{x}{\sqrt{x^2-1}}+1\right)-\log\left(1-\frac{x}{\sqrt{x^2-1}}\right)\right)}{2\sqrt{1-\frac{1}{x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSec[x],x]
[Out] x*ArcSec[x] - (Sqrt[-1 + x^2]*(-Log[1 - x/Sqrt[-1 + x^2]] + Log[1 + x/Sqrt[
-1 + x^2]]))/(2*Sqrt[1 - x^(-2)]*x)
```

fricas [A] time = 0.45, size = 33, normalized size = 1.74

$$(x-2)\,\text{arcsec}(x) + 4\,\arctan\left(-x+\sqrt{x^2-1}\right) + \log\left(-x+\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x),x, algorithm="fricas")
[Out] (x - 2)*arcsec(x) + 4*arctan(-x + sqrt(x^2 - 1)) + log(-x + sqrt(x^2 - 1))
```

giac [B] time = 0.03, size = 37, normalized size = 1.95

$$x\arccos\left(\frac{1}{x}\right) - \frac{1}{2}\,\log\left(\sqrt{-\frac{1}{x^2}+1}+1\right) + \frac{1}{2}\,\log\left(-\sqrt{-\frac{1}{x^2}+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x),x, algorithm="giac")
[Out] x*arccos(1/x) - 1/2*log(sqrt(-1/x^2 + 1) + 1) + 1/2*log(-sqrt(-1/x^2 + 1) +
 1)
```

maple [A] time = 0.01, size = 22, normalized size = 1.16

$$x\,\text{arcsec}(x) - \ln\left(x+\sqrt{-\frac{1}{x^2}+1}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsec(x),x)
[Out] x*arcsec(x)-ln(x+x*(1-1/x^2)^(1/2))
```

maxima [B] time = 0.47, size = 35, normalized size = 1.84

$$x \operatorname{arcsec}(x) - \frac{1}{2} \log\left(\sqrt{-\frac{1}{x^2} + 1} + 1\right) + \frac{1}{2} \log\left(-\sqrt{-\frac{1}{x^2} + 1} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x),x, algorithm="maxima")
```

```
[Out] x*arcsec(x) - 1/2*log(sqrt(-1/x^2 + 1) + 1) + 1/2*log(-sqrt(-1/x^2 + 1) + 1
)
```

mupad [B] time = 0.61, size = 21, normalized size = 1.11

$$x \operatorname{acos}\left(\frac{1}{x}\right) - \ln\left(x + \sqrt{x^2 - 1}\right) \operatorname{sign}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(1/x),x)
```

```
[Out] x*acos(1/x) - log(x + (x^2 - 1)^(1/2))*sign(x)
```

sympy [C] time = 2.18, size = 17, normalized size = 0.89

$$x \operatorname{asec}(x) - \begin{cases} \operatorname{acosh}(x) & \text{for } \left|x^2\right| > 1 \\ -i \operatorname{asin}(x) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asec(x),x)
```

```
[Out] x*asec(x) - Piecewise((acosh(x), Abs(x**2) > 1), (-I*asin(x), True))
```

3.84 $\int \csc^{-1}(x)\,dx$

Optimal. Leaf size=17

$$\tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)+x\csc^{-1}(x)$$

```
[Out] x*arccsc(x)+arctanh((1-1/x^2)^(1/2))
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 2.000, Rules used = {5215, 266, 63, 206}

$$\tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)+x\csc^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCsc[x],x]
```

```
[Out] x*ArcCsc[x] + ArcTanh[Sqrt[1 - x^(-2)]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5215

```
Int[ArcCsc[(c_.)*(x_)], x_Symbol] :> Simp[x*ArcCsc[c*x], x] + Dist[1/c, Int
[1/(x*Sqrt[1 - 1/(c^2*x^2)]), x], x] /; FreeQ[c, x]
```

Rubi steps

$$\int \csc^{-1}(x)\,dx = x\csc^{-1}(x) + \int \frac{1}{\sqrt{1-\frac{1}{x^2}}\,x}\,dx$$
$$= x\csc^{-1}(x) - \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx, x, \frac{1}{x^2}\right)$$
$$= x\csc^{-1}(x) + \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-\frac{1}{x^2}}\right)$$
$$= x\csc^{-1}(x) + \tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

Mathematica [B] time = 0.05, size = 64, normalized size = 3.76

$$\frac{\sqrt{x^2-1}\left(\log\left(\frac{x}{\sqrt{x^2-1}}+1\right)-\log\left(1-\frac{x}{\sqrt{x^2-1}}\right)\right)}{2\sqrt{1-\frac{1}{x^2}}\,x}+x\csc^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCsc[x],x]
[Out] x*ArcCsc[x] + (Sqrt[-1 + x^2]*(-Log[1 - x/Sqrt[-1 + x^2]] + Log[1 + x/Sqrt[
-1 + x^2]]))/(2*Sqrt[1 - x^(-2)]*x)
```

fricas [B] time = 0.44, size = 35, normalized size = 2.06

$$(x-2)\,\text{arccsc}(x) - 4\,\arctan\left(-x+\sqrt{x^2-1}\right) - \log\left(-x+\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccsc(x),x, algorithm="fricas")
[Out] (x - 2)*arccsc(x) - 4*arctan(-x + sqrt(x^2 - 1)) - log(-x + sqrt(x^2 - 1))
```

giac [B] time = 0.03, size = 37, normalized size = 2.18

$$x\arcsin\left(\frac{1}{x}\right)+\frac{1}{2}\,\log\left(\sqrt{-\frac{1}{x^2}+1}+1\right)-\frac{1}{2}\,\log\left(-\sqrt{-\frac{1}{x^2}+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccsc(x),x, algorithm="giac")
[Out] x*arcsin(1/x) + 1/2*log(sqrt(-1/x^2 + 1) + 1) - 1/2*log(-sqrt(-1/x^2 + 1) +
 1)
```

maple [A] time = 0.01, size = 20, normalized size = 1.18

$$x\,\text{arccsc}(x)+\ln\left(x+\sqrt{-\frac{1}{x^2}+1}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccsc(x),x)
[Out] x*arccsc(x)+ln(x+(-1/x^2+1)^(1/2)*x)
```

maxima [B] time = 0.60, size = 35, normalized size = 2.06

$$x \operatorname{arccsc}(x) + \frac{1}{2} \log\left(\sqrt{-\frac{1}{x^2} + 1} + 1\right) - \frac{1}{2} \log\left(-\sqrt{-\frac{1}{x^2} + 1} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccsc(x),x, algorithm="maxima")
```

```
[Out] x*arccsc(x) + 1/2*log(sqrt(-1/x^2 + 1) + 1) - 1/2*log(-sqrt(-1/x^2 + 1) + 1
)
```

mupad [B] time = 0.21, size = 20, normalized size = 1.18

$$x \operatorname{asin}\left(\frac{1}{x}\right) + \ln\left(x + \sqrt{x^2 - 1}\right) \operatorname{sign}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asin(1/x),x)
```

```
[Out] x*asin(1/x) + log(x + (x^2 - 1)^(1/2))*sign(x)
```

sympy [A] time = 2.24, size = 17, normalized size = 1.00

$$x \operatorname{acsc}(x) + \begin{cases} \operatorname{acosh}(x) & \text{for } \left|x^2\right| > 1 \\ -i \operatorname{asin}(x) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acsc(x),x)
```

```
[Out] x*acsc(x) + Piecewise((acosh(x), Abs(x**2) > 1), (-I*asin(x), True))
```

3.85 $\int \sin^{-1}(x)^2 \, dx$

Optimal. Leaf size=25

$$2\sqrt{1-x^2}\,\sin^{-1}(x) - 2x + x\sin^{-1}(x)^2$$

```
[Out] -2*x+x*arcsin(x)^2+2*arcsin(x)*(-x^2+1)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {4619, 4677, 8}

$$2\sqrt{1-x^2}\,\sin^{-1}(x) - 2x + x\sin^{-1}(x)^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcSin[x]^2,x]
```

```
[Out] -2*x + 2*Sqrt[1 - x^2]*ArcSin[x] + x*ArcSin[x]^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4619

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cSin[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSin[c*x])^(n - 1))/Sqrt[1 -
 c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4677

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n)/(2*e*(p +
 1)), x] + Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1
- c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n
 - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \sin^{-1}(x)^2 \, dx &= x\sin^{-1}(x)^2 - 2\int \frac{x\sin^{-1}(x)}{\sqrt{1-x^2}} \, dx\\ &= 2\sqrt{1-x^2}\,\sin^{-1}(x) + x\sin^{-1}(x)^2 - 2\int 1 \, dx\\ &= -2x + 2\sqrt{1-x^2}\,\sin^{-1}(x) + x\sin^{-1}(x)^2\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$2\sqrt{1-x^2}\,\sin^{-1}(x) - 2x + x\sin^{-1}(x)^2$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSin[x]^2,x]
```

```
[Out] -2*x + 2*Sqrt[1 - x^2]*ArcSin[x] + x*ArcSin[x]^2
```

fricas [**A**] time = 0.42, size = 23, normalized size = 0.92

$$x \arcsin(x)^2 + 2\sqrt{-x^2+1}\,\arcsin(x) - 2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)^2,x, algorithm="fricas")
[Out] x*arcsin(x)^2 + 2*sqrt(-x^2 + 1)*arcsin(x) - 2*x
```

giac [**A**] time = 0.02, size = 23, normalized size = 0.92

$$x \arcsin(x)^2 + 2\sqrt{-x^2+1}\,\arcsin(x) - 2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)^2,x, algorithm="giac")
[Out] x*arcsin(x)^2 + 2*sqrt(-x^2 + 1)*arcsin(x) - 2*x
```

maple [**A**] time = 0.07, size = 24, normalized size = 0.96

$$x \arcsin(x)^2 - 2x + 2\sqrt{-x^2+1}\,\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsin(x)^2,x)
[Out] -2*x+x*arcsin(x)^2+2*arcsin(x)*(-x^2+1)^(1/2)
```

maxima [**A**] time = 1.13, size = 23, normalized size = 0.92

$$x \arcsin(x)^2 + 2\sqrt{-x^2+1}\,\arcsin(x) - 2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)^2,x, algorithm="maxima")
[Out] x*arcsin(x)^2 + 2*sqrt(-x^2 + 1)*arcsin(x) - 2*x
```

mupad [**B**] time = 0.03, size = 22, normalized size = 0.88

$$2\,\mathrm{asin}(x)\,\sqrt{1-x^2} + x\,\left(\mathrm{asin}(x)^2 - 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asin(x)^2,x)
[Out] 2*asin(x)*(1 - x^2)^(1/2) + x*(asin(x)^2 - 2)
```

sympy [**A**] time = 0.20, size = 22, normalized size = 0.88

$$x \,\mathrm{asin}^2(x) - 2x + 2\sqrt{1-x^2}\,\mathrm{asin}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asin(x)**2,x)
[Out] x*asin(x)**2 - 2*x + 2*sqrt(1 - x**2)*asin(x)
```

3.86 $\int \frac{\sin^{-1}(x)}{x^2}\,dx$

Optimal. Leaf size=22

$$-\tanh^{-1}\left(\sqrt{1-x^2}\right)-\frac{\sin^{-1}(x)}{x}$$

[Out] -arcsin(x)/x-arctanh((-x^2+1)^(1/2))

Rubi [A] time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {4627, 266, 63, 206}

$$-\tanh^{-1}\left(\sqrt{1-x^2}\right)-\frac{\sin^{-1}(x)}{x}$$

Antiderivative was successfully verified.

[In] Int[ArcSin[x]/x^2,x]

[Out] -(ArcSin[x]/x) - ArcTanh[Sqrt[1 - x^2]]

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4627

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcSin[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^{-1}(x)}{x^2}\,dx &= -\frac{\sin^{-1}(x)}{x}+\int \frac{1}{x\sqrt{1-x^2}}\,dx\\ &= -\frac{\sin^{-1}(x)}{x}+\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx,x,x^2\right)\\ &= -\frac{\sin^{-1}(x)}{x}-\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{1-x^2}\right)\\ &= -\frac{\sin^{-1}(x)}{x}-\tanh^{-1}\left(\sqrt{1-x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 1.00

$$-\tanh^{-1}\left(\sqrt{1-x^2}\right)-\frac{\sin^{-1}(x)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSin[x]/x^2,x]
[Out] -(ArcSin[x]/x) - ArcTanh[Sqrt[1 - x^2]]
```

fricas [A] time = 0.45, size = 39, normalized size = 1.77

$$-\frac{x\log\left(\sqrt{-x^2+1}+1\right)-x\log\left(\sqrt{-x^2+1}-1\right)+2\,\arcsin(x)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)/x^2,x, algorithm="fricas")
[Out] -1/2*(x*log(sqrt(-x^2 + 1) + 1) - x*log(sqrt(-x^2 + 1) - 1) + 2*arcsin(x))/
x
```

giac [A] time = 0.02, size = 38, normalized size = 1.73

$$-\frac{\arcsin(x)}{x}-\frac{1}{2}\log\left(\sqrt{-x^2+1}+1\right)+\frac{1}{2}\log\left(-\sqrt{-x^2+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)/x^2,x, algorithm="giac")
[Out] -arcsin(x)/x - 1/2*log(sqrt(-x^2 + 1) + 1) + 1/2*log(-sqrt(-x^2 + 1) + 1)
```

maple [A] time = 0.00, size = 21, normalized size = 0.95

$$-\text{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)-\frac{\arcsin(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsin(x)/x^2,x)
[Out] -arcsin(x)/x-arctanh(1/(-x^2+1)^(1/2))
```

maxima [A] time = 1.23, size = 33, normalized size = 1.50

$$-\frac{\arcsin(x)}{x}-\log\left(\frac{2\sqrt{-x^2+1}}{|x|}+\frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)/x^2,x, algorithm="maxima")
[Out] -arcsin(x)/x - log(2*sqrt(-x^2 + 1)/abs(x) + 2/abs(x))
```

mupad [B] time = 0.02, size = 20, normalized size = 0.91

$$-\operatorname{atanh}\left(\frac{1}{\sqrt{1-x^2}}\right)-\frac{\operatorname{asin}(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asin(x)/x^2,x)
[Out] - atanh(1/(1 - x^2)^(1/2)) - asin(x)/x
```

sympy [A] time = 2.02, size = 22, normalized size = 1.00

$$\begin{cases} -\operatorname{acosh}\left(\frac{1}{x}\right) & \text{for}\ \frac{1}{|x^2|}>1 \\ i\operatorname{asin}\left(\frac{1}{x}\right) & \text{otherwise} \end{cases}-\frac{\operatorname{asin}(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asin(x)/x**2,x)
[Out] Piecewise((-acosh(1/x), 1/Abs(x**2) > 1), (I*asin(1/x), True)) - asin(x)/x
```

3.87 $\int \frac{1}{\sqrt{a^2-x^2}}\,dx$

Optimal. Leaf size=16

$$\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

```
[Out] arctan(x/(a^2-x^2)^(1/2))
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {217, 203}

$$\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a^2 - x^2],x]
```

```
[Out] ArcTan[x/Sqrt[a^2 - x^2]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a^2-x^2}}\,dx &= \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{x}{\sqrt{a^2-x^2}}\right)\\ &= \tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a^2 - x^2],x]
```

```
[Out] ArcTan[x/Sqrt[a^2 - x^2]]
```

fricas [A] time = 0.40, size = 23, normalized size = 1.44

$$-2\arctan\left(-\frac{a-\sqrt{a^2-x^2}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2-x^2)^(1/2),x, algorithm="fricas")
[Out] -2*arctan(-(a - sqrt(a^2 - x^2))/x)
```

giac [A] time = 0.04, size = 9, normalized size = 0.56

$$\arcsin\left(\frac{x}{a}\right)\operatorname{sgn}(a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2-x^2)^(1/2),x, algorithm="giac")
[Out] arcsin(x/a)*sgn(a)
```

maple [A] time = 0.00, size = 15, normalized size = 0.94

$$\arctan\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2-x^2)^(1/2),x)
[Out] arctan(1/(a^2-x^2)^(1/2)*x)
```

maxima [A] time = 1.22, size = 6, normalized size = 0.38

$$\arcsin\left(\frac{x}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2-x^2)^(1/2),x, algorithm="maxima")
[Out] arcsin(x/a)
```

mupad [B] time = 0.16, size = 14, normalized size = 0.88

$$\operatorname{atan}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 - x^2)^(1/2),x)
[Out] atan(x/(a^2 - x^2)^(1/2))
```

sympy [A] time = 1.15, size = 19, normalized size = 1.19

$$\begin{cases} -i \operatorname{acosh}\left(\frac{x}{a}\right) & \text{for}\ \left|\frac{x^2}{a^2}\right| > 1 \\ \operatorname{asin}\left(\frac{x}{a}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2-x**2)**(1/2),x)
[Out] Piecewise((-I*acosh(x/a), Abs(x**2/a**2) > 1), (asin(x/a), True))
```

3.88 $\int \frac{1}{\sqrt{1-2x-x^2}}\,dx$

Optimal. Leaf size=10

$$\sin^{-1}\left(\frac{x+1}{\sqrt{2}}\right)$$

```
[Out] arcsin(1/2*(1+x)*2^(1/2))
```

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {619, 216}

$$\sin^{-1}\left(\frac{x+1}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 - 2*x - x^2],x]
```

```
[Out] ArcSin[(1 + x)/Sqrt[2]]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1-2x-x^2}}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{8}}}\,dx, x, -2-2x\right)}{2\sqrt{2}}\\ &= \sin^{-1}\left(\frac{1+x}{\sqrt{2}}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.40

$$-\sin^{-1}\left(\frac{-x-1}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 - 2*x - x^2],x]
```

```
[Out] -ArcSin[(-1 - x)/Sqrt[2]]
```

fricas [B] time = 0.39, size = 21, normalized size = 2.10

$$-2\,\arctan\left(\frac{\sqrt{-x^2-2\,x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-2*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*arctan((sqrt(-x^2 - 2*x + 1) - 1)/x)
```

giac [A] time = 0.03, size = 9, normalized size = 0.90

$$\arcsin\left(\frac{1}{2}\sqrt{2}(x+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-2*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] arcsin(1/2*sqrt(2)*(x + 1))
```

maple [A] time = 0.00, size = 10, normalized size = 1.00

$$\arcsin\left(\frac{(x+1)\sqrt{2}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2-2*x+1)^(1/2),x)
```

```
[Out] arcsin(1/2*(x+1)*2^(1/2))
```

maxima [A] time = 1.36, size = 11, normalized size = 1.10

$$-\arcsin\left(-\frac{1}{2}\sqrt{2}(x+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-2*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsin(-1/2*sqrt(2)*(x + 1))
```

mupad [B] time = 0.09, size = 11, normalized size = 1.10

$$\operatorname{asin}\left(\frac{\sqrt{8}\ (2x+2)}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1 - x^2 - 2*x)^(1/2),x)
```

```
[Out] asin((8^(1/2)*(2*x + 2))/8)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^2-2x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2-2*x+1)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-x**2 - 2*x + 1), x)
```

3.89 $\int \frac{1}{a^2+x^2}\,dx$

Optimal. Leaf size=10

$$\frac{\tan^{-1}\left(\frac{x}{a}\right)}{a}$$

```
[Out] arctan(x/a)/a
```

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {203}

$$\frac{\tan^{-1}\left(\frac{x}{a}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + x^2)^(-1),x]
```

```
[Out] ArcTan[x/a]/a
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{a^2+x^2}\,dx = \frac{\tan^{-1}\left(\frac{x}{a}\right)}{a}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{x}{a}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + x^2)^(-1),x]
```

```
[Out] ArcTan[x/a]/a
```

fricas [A] time = 0.40, size = 10, normalized size = 1.00

$$\frac{\arctan\left(\frac{x}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+x^2),x, algorithm="fricas")
```

```
[Out] arctan(x/a)/a
```

giac [A] time = 0.01, size = 10, normalized size = 1.00

$$\frac{\arctan\left(\frac{x}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+x^2),x, algorithm="giac")
```

```
[Out] arctan(x/a)/a
```

maple [A] time = 0.00, size = 11, normalized size = 1.10

$$\frac{\arctan\left(\frac{x}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2+x^2),x)
```

```
[Out] arctan(x/a)/a
```

maxima [A] time = 1.37, size = 10, normalized size = 1.00

$$\frac{\arctan\left(\frac{x}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+x^2),x, algorithm="maxima")
```

```
[Out] arctan(x/a)/a
```

mupad [B] time = 0.04, size = 10, normalized size = 1.00

$$\frac{\operatorname{atan}\left(\frac{x}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + x^2),x)
```

```
[Out] atan(x/a)/a
```

sympy [C] time = 0.12, size = 20, normalized size = 2.00

$$\frac{-\frac{i\log\left(-ia+x\right)}{2}+\frac{i\log\left(ia+x\right)}{2}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+x**2),x)
```

```
[Out] (-I*log(-I*a + x)/2 + I*log(I*a + x)/2)/a
```

3.90 $\int \frac{1}{a+bx^2} \, dx$

Optimal. Leaf size=24

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

```
[Out] arctan(x*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{a+bx^2} \, dx = \frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

fricas [A] time = 0.39, size = 67, normalized size = 2.79

$$\left[-\frac{\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{2\,ab}, \frac{\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a))/(a*b), sqrt(
a*b)*arctan(sqrt(a*b)*x/a)/(a*b)]
```

giac [A] time = 0.01, size = 15, normalized size = 0.62

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a),x, algorithm="giac")
```

```
[Out] arctan(b*x/sqrt(a*b))/sqrt(a*b)
```

maple [A] time = 0.01, size = 16, normalized size = 0.67

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a),x)
```

```
[Out] 1/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

maxima [A] time = 1.49, size = 15, normalized size = 0.62

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] arctan(b*x/sqrt(a*b))/sqrt(a*b)
```

mupad [B] time = 0.10, size = 16, normalized size = 0.67

$$\frac{\operatorname{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2),x)
```

```
[Out] atan((b^(1/2)*x)/a^(1/2))/(a^(1/2)*b^(1/2))
```

sympy [B] time = 0.14, size = 53, normalized size = 2.21

$$-\frac{\sqrt{-\frac{1}{ab}}\log\left(-a\sqrt{-\frac{1}{ab}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{ab}}\log\left(a\sqrt{-\frac{1}{ab}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a),x)
```

```
[Out] -sqrt(-1/(a*b))*log(-a*sqrt(-1/(a*b)) + x)/2 + sqrt(-1/(a*b))*log(a*sqrt(-1
/(a*b)) + x)/2
```

3.91 $\int \frac{1}{2-x+x^2} dx$

Optimal. Leaf size=19

$$-\frac{2\tan^{-1}\left(\frac{1-2x}{\sqrt{7}}\right)}{\sqrt{7}}$$

```
[Out] -2/7*arctan(1/7*(1-2*x)*7^(1/2))*7^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {618, 204}

$$-\frac{2\tan^{-1}\left(\frac{1-2x}{\sqrt{7}}\right)}{\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - x + x^2)^(-1),x]
[Out] (-2*ArcTan[(1 - 2*x)/Sqrt[7]])/Sqrt[7]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{2-x+x^2} dx = -\left(2\,\text{Subst}\left(\int \frac{1}{-7-x^2} dx, x, -1+2x\right)\right)$$
$$= -\frac{2\tan^{-1}\left(\frac{1-2x}{\sqrt{7}}\right)}{\sqrt{7}}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{2x-1}{\sqrt{7}}\right)}{\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - x + x^2)^(-1),x]
[Out] (2*ArcTan[(-1 + 2*x)/Sqrt[7]])/Sqrt[7]
```

fricas [A] time = 0.40, size = 16, normalized size = 0.84

$$\frac{2}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-x+2),x, algorithm="fricas")
[Out] 2/7*sqrt(7)*arctan(1/7*sqrt(7)*(2*x - 1))
```

giac [A] time = 0.01, size = 16, normalized size = 0.84

$$\frac{2}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-x+2),x, algorithm="giac")
[Out] 2/7*sqrt(7)*arctan(1/7*sqrt(7)*(2*x - 1))
```

maple [A] time = 0.00, size = 17, normalized size = 0.89

$$\frac{2\sqrt{7}\arctan\left(\frac{(2x-1)\sqrt{7}}{7}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-x+2),x)
[Out] 2/7*7^(1/2)*arctan(1/7*(2*x-1)*7^(1/2))
```

maxima [A] time = 1.42, size = 16, normalized size = 0.84

$$\frac{2}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-x+2),x, algorithm="maxima")
[Out] 2/7*sqrt(7)*arctan(1/7*sqrt(7)*(2*x - 1))
```

mupad [B] time = 0.08, size = 16, normalized size = 0.84

$$\frac{2\sqrt{7}\,\mathrm{atan}\left(\frac{\sqrt{7}\,(2x-1)}{7}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 - x + 2),x)
[Out] (2*7^(1/2)*atan((7^(1/2)*(2*x - 1))/7))/7
```

sympy [A] time = 0.12, size = 26, normalized size = 1.37

$$\frac{2\sqrt{7}\,\mathrm{atan}\left(\frac{2\sqrt{7}x}{7}-\frac{\sqrt{7}}{7}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-x+2),x)
[Out] 2*sqrt(7)*atan(2*sqrt(7)*x/7 - sqrt(7)/7)/7
```

3.92 $\int x \tan^{-1}(x)\, dx$

Optimal. Leaf size=21

$$\frac{1}{2}x^2 \tan^{-1}(x) - \frac{x}{2} + \frac{1}{2}\tan^{-1}(x)$$

```
[Out] -1/2*x+1/2*arctan(x)+1/2*x^2*arctan(x)
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {4852, 321, 203}

$$\frac{1}{2}x^2 \tan^{-1}(x) - \frac{x}{2} + \frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[x],x]
```

```
[Out] -x/2 + ArcTan[x]/2 + (x^2*ArcTan[x])/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x \tan^{-1}(x)\, dx &= \frac{1}{2}x^2 \tan^{-1}(x) - \frac{1}{2}\int \frac{x^2}{1+x^2}\, dx \\
&= -\frac{x}{2} + \frac{1}{2}x^2 \tan^{-1}(x) + \frac{1}{2}\int \frac{1}{1+x^2}\, dx \\
&= -\frac{x}{2} + \frac{1}{2}\tan^{-1}(x) + \frac{1}{2}x^2 \tan^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{1}{2}x^2 \tan^{-1}(x) - \frac{x}{2} + \frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[x],x]
[Out] -1/2*x + ArcTan[x]/2 + (x^2*ArcTan[x])/2
```

fricas [A] time = 0.42, size = 13, normalized size = 0.62

$$\frac{1}{2}\left(x^2+1\right)\arctan(x)-\frac{1}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x),x, algorithm="fricas")
[Out] 1/2*(x^2 + 1)*arctan(x) - 1/2*x
```

giac [A] time = 0.01, size = 15, normalized size = 0.71

$$\frac{1}{2}\,x^2\arctan(x)-\frac{1}{2}\,x+\frac{1}{2}\,\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x),x, algorithm="giac")
[Out] 1/2*x^2*arctan(x) - 1/2*x + 1/2*arctan(x)
```

maple [A] time = 0.00, size = 16, normalized size = 0.76

$$\frac{x^2\arctan(x)}{2}-\frac{x}{2}+\frac{\arctan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(x),x)
[Out] -1/2*x+1/2*arctan(x)+1/2*x^2*arctan(x)
```

maxima [A] time = 1.30, size = 15, normalized size = 0.71

$$\frac{1}{2}\,x^2\arctan(x)-\frac{1}{2}\,x+\frac{1}{2}\,\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x),x, algorithm="maxima")
[Out] 1/2*x^2*arctan(x) - 1/2*x + 1/2*arctan(x)
```

mupad [B] time = 0.02, size = 14, normalized size = 0.67

$$\mathrm{atan}(x)\left(\frac{x^2}{2}+\frac{1}{2}\right)-\frac{x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(x),x)
[Out] atan(x)*(x^2/2 + 1/2) - x/2
```

sympy [A] time = 0.25, size = 15, normalized size = 0.71

$$\frac{x^2\,\mathrm{atan}\,(x)}{2}-\frac{x}{2}+\frac{\mathrm{atan}\,(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(x),x)
[Out] x**2*atan(x)/2 - x/2 + atan(x)/2
```

3.93 $\int x^2 \cos^{-1}(x)\,dx$

Optimal. Leaf size=40

$$\frac{1}{3}x^3\cos^{-1}(x)+\frac{1}{9}\left(1-x^2\right)^{3/2}-\frac{\sqrt{1-x^2}}{3}$$

```
[Out] 1/9*(-x^2+1)^(3/2)+1/3*x^3*arccos(x)-1/3*(-x^2+1)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4628, 266, 43}

$$\frac{1}{3}x^3\cos^{-1}(x)+\frac{1}{9}\left(1-x^2\right)^{3/2}-\frac{\sqrt{1-x^2}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCos[x],x]
```

```
[Out] -Sqrt[1 - x^2]/3 + (1 - x^2)^(3/2)/9 + (x^3*ArcCos[x])/3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^2\cos^{-1}(x)\,dx &= \frac{1}{3}x^3\cos^{-1}(x)+\frac{1}{3}\int\frac{x^3}{\sqrt{1-x^2}}\,dx\\
&= \frac{1}{3}x^3\cos^{-1}(x)+\frac{1}{6}\operatorname{Subst}\left(\int\frac{x}{\sqrt{1-x}}\,dx,x,x^2\right)\\
&= \frac{1}{3}x^3\cos^{-1}(x)+\frac{1}{6}\operatorname{Subst}\left(\int\left(\frac{1}{\sqrt{1-x}}-\sqrt{1-x}\right)dx,x,x^2\right)\\
&= -\frac{1}{3}\sqrt{1-x^2}+\frac{1}{9}\left(1-x^2\right)^{3/2}+\frac{1}{3}x^3\cos^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.75

$$\frac{1}{3}x^3\cos^{-1}(x)-\frac{1}{9}\sqrt{1-x^2}\left(x^2+2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCos[x],x]
[Out] -1/9*(Sqrt[1 - x^2]*(2 + x^2)) + (x^3*ArcCos[x])/3
```

fricas [A] time = 0.43, size = 24, normalized size = 0.60

$$\frac{1}{3}x^3\arccos(x) - \frac{1}{9}\left(x^2+2\right)\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccos(x),x, algorithm="fricas")
[Out] 1/3*x^3*arccos(x) - 1/9*(x^2 + 2)*sqrt(-x^2 + 1)
```

giac [A] time = 0.02, size = 33, normalized size = 0.82

$$\frac{1}{3}x^3\arccos(x) - \frac{1}{9}\sqrt{-x^2+1}\,x^2 - \frac{2}{9}\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccos(x),x, algorithm="giac")
[Out] 1/3*x^3*arccos(x) - 1/9*sqrt(-x^2 + 1)*x^2 - 2/9*sqrt(-x^2 + 1)
```

maple [A] time = 0.01, size = 34, normalized size = 0.85

$$\frac{x^3\arccos(x)}{3} - \frac{\sqrt{-x^2+1}\,x^2}{9} - \frac{2\sqrt{-x^2+1}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccos(x),x)
[Out] 1/3*x^3*arccos(x)-1/9*x^2*(-x^2+1)^(1/2)-2/9*(-x^2+1)^(1/2)
```

maxima [A] time = 1.27, size = 33, normalized size = 0.82

$$\frac{1}{3}x^3\arccos(x) - \frac{1}{9}\sqrt{-x^2+1}\,x^2 - \frac{2}{9}\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccos(x),x, algorithm="maxima")
[Out] 1/3*x^3*arccos(x) - 1/9*sqrt(-x^2 + 1)*x^2 - 2/9*sqrt(-x^2 + 1)
```

mupad [B] time = 0.03, size = 24, normalized size = 0.60

$$\frac{x^3\,\mathrm{acos}(x)}{3} - \frac{\sqrt{1-x^2}\,\left(x^2+2\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acos(x),x)
[Out] (x^3*acos(x))/3 - ((1 - x^2)^(1/2)*(x^2 + 2))/9
```

sympy [A] time = 0.37, size = 32, normalized size = 0.80

$$\frac{x^3\,\mathrm{acos}\,(x)}{3} - \frac{x^2\sqrt{1-x^2}}{9} - \frac{2\sqrt{1-x^2}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acos(x),x)
```

```
[Out] x**3*acos(x)/3 - x**2*sqrt(1 - x**2)/9 - 2*sqrt(1 - x**2)/9
```

3.94 $\int x \tan^{-1}(x)^2 \, dx$

Optimal. Leaf size=35

$$\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{2}x^2\tan^{-1}(x)^2+\frac{1}{2}\tan^{-1}(x)^2-x\tan^{-1}(x)$$

```
[Out] -x*arctan(x)+1/2*arctan(x)^2+1/2*x^2*arctan(x)^2+1/2*ln(x^2+1)
```

Rubi [A] time = 0.06, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {4852, 4916, 4846, 260, 4884}

$$\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{2}x^2\tan^{-1}(x)^2+\frac{1}{2}\tan^{-1}(x)^2-x\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[x]^2,x]
```

```
[Out] -(x*ArcTan[x]) + ArcTan[x]^2/2 + (x^2*ArcTan[x]^2)/2 + Log[1 + x^2]/2
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rubi steps

$$\begin{aligned}\int x\tan^{-1}(x)^2\,dx &= \frac{1}{2}x^2\tan^{-1}(x)^2 - \int \frac{x^2\tan^{-1}(x)}{1+x^2}\,dx\\ &= \frac{1}{2}x^2\tan^{-1}(x)^2 - \int \tan^{-1}(x)\,dx + \int \frac{\tan^{-1}(x)}{1+x^2}\,dx\\ &= -x\tan^{-1}(x) + \frac{1}{2}\tan^{-1}(x)^2 + \frac{1}{2}x^2\tan^{-1}(x)^2 + \int \frac{x}{1+x^2}\,dx\\ &= -x\tan^{-1}(x) + \frac{1}{2}\tan^{-1}(x)^2 + \frac{1}{2}x^2\tan^{-1}(x)^2 + \frac{1}{2}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [**A**] time = 0.01, size = 26, normalized size = 0.74

$$\frac{1}{2}\left(\log\left(x^2+1\right) + \left(x^2+1\right)\tan^{-1}(x)^2 - 2x\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[x]^2,x]
[Out] (-2*x*ArcTan[x] + (1 + x^2)*ArcTan[x]^2 + Log[1 + x^2])/2
```

fricas [**A**] time = 0.43, size = 25, normalized size = 0.71

$$\frac{1}{2}\left(x^2+1\right)\arctan(x)^2 - x\arctan(x) + \frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x)^2,x, algorithm="fricas")
[Out] 1/2*(x^2 + 1)*arctan(x)^2 - x*arctan(x) + 1/2*log(x^2 + 1)
```

giac [**A**] time = 0.02, size = 29, normalized size = 0.83

$$\frac{1}{2}x^2\arctan(x)^2 - x\arctan(x) + \frac{1}{2}\arctan(x)^2 + \frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x)^2,x, algorithm="giac")
[Out] 1/2*x^2*arctan(x)^2 - x*arctan(x) + 1/2*arctan(x)^2 + 1/2*log(x^2 + 1)
```

maple [**A**] time = 0.01, size = 30, normalized size = 0.86

$$\frac{x^2\arctan(x)^2}{2} - x\arctan(x) + \frac{\arctan(x)^2}{2} + \frac{\ln\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(x)^2,x)
[Out] -x*arctan(x)+1/2*arctan(x)^2+1/2*x^2*arctan(x)^2+1/2*ln(x^2+1)
```

maxima [**A**] time = 1.26, size = 34, normalized size = 0.97

$$\frac{1}{2}x^2\arctan(x)^2 - (x - \arctan(x))\arctan(x) - \frac{1}{2}\arctan(x)^2 + \frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctan(x)^2 - (x - arctan(x))*arctan(x) - 1/2*arctan(x)^2 + 1/2*log
(x^2 + 1)
```

mupad [B] time = 0.12, size = 29, normalized size = 0.83

$$\frac{\ln\left(x^2+1\right)}{2}+\frac{\operatorname{atan}(x)^2}{2}+\frac{x^2\operatorname{atan}(x)^2}{2}-x\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(x)^2,x)
```

```
[Out] log(x^2 + 1)/2 + atan(x)^2/2 + (x^2*atan(x)^2)/2 - x*atan(x)
```

sympy [A] time = 0.36, size = 29, normalized size = 0.83

$$\frac{x^2\operatorname{atan}^2(x)}{2}-x\operatorname{atan}(x)+\frac{\log\left(x^2+1\right)}{2}+\frac{\operatorname{atan}^2(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(x)**2,x)
```

```
[Out] x**2*atan(x)**2/2 - x*atan(x) + log(x**2 + 1)/2 + atan(x)**2/2
```

3.95 $\int \tan^{-1}\left(\sqrt{x}\right) dx$

Optimal. Leaf size=22

$$-\sqrt{x}+x\tan^{-1}\left(\sqrt{x}\right)+\tan^{-1}\left(\sqrt{x}\right)$$

```
[Out] arctan(x^(1/2))+x*arctan(x^(1/2))-x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5027, 50, 63, 203}

$$-\sqrt{x}+x\tan^{-1}\left(\sqrt{x}\right)+\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[Sqrt[x]],x]
```

```
[Out] -Sqrt[x] + ArcTan[Sqrt[x]] + x*ArcTan[Sqrt[x]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 5027

```
Int[ArcTan[(c_.)*(x_)^(n_)], x_Symbol] :> Simp[x*ArcTan[c*x^n], x] - Dist[c
*n, Int[x^n/(1 + c^2*x^(2*n)), x], x] /; FreeQ[{c, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \tan^{-1}\left(\sqrt{x}\right) dx &= x\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{2}\int \frac{\sqrt{x}}{1+x}\,dx\\
&= -\sqrt{x}+x\tan^{-1}\left(\sqrt{x}\right)+\frac{1}{2}\int \frac{1}{\sqrt{x}(1+x)}\,dx\\
&= -\sqrt{x}+x\tan^{-1}\left(\sqrt{x}\right)+\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{x}\right)\\
&= -\sqrt{x}+\tan^{-1}\left(\sqrt{x}\right)+x\tan^{-1}\left(\sqrt{x}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 0.82

$$(x+1)\tan^{-1}\left(\sqrt{x}\right)-\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[Sqrt[x]],x]
[Out] -Sqrt[x] + (1 + x)*ArcTan[Sqrt[x]]
```

fricas [A] time = 0.44, size = 14, normalized size = 0.64

$$(x+1)\arctan\left(\sqrt{x}\right)-\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2)),x, algorithm="fricas")
[Out] (x + 1)*arctan(sqrt(x)) - sqrt(x)
```

giac [A] time = 0.02, size = 16, normalized size = 0.73

$$x\arctan\left(\sqrt{x}\right)-\sqrt{x}+\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2)),x, algorithm="giac")
[Out] x*arctan(sqrt(x)) - sqrt(x) + arctan(sqrt(x))
```

maple [A] time = 0.00, size = 17, normalized size = 0.77

$$x\arctan\left(\sqrt{x}\right)+\arctan\left(\sqrt{x}\right)-\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x^(1/2)),x)
[Out] arctan(x^(1/2))+x*arctan(x^(1/2))-x^(1/2)
```

maxima [A] time = 1.13, size = 16, normalized size = 0.73

$$x\arctan\left(\sqrt{x}\right)-\sqrt{x}+\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2)),x, algorithm="maxima")
[Out] x*arctan(sqrt(x)) - sqrt(x) + arctan(sqrt(x))
```

mupad [B] time = 0.06, size = 16, normalized size = 0.73

$$\mathrm{atan}\left(\sqrt{x}\right)+x\,\mathrm{atan}\left(\sqrt{x}\right)-\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(x^(1/2)),x)
[Out] atan(x^(1/2)) + x*atan(x^(1/2)) - x^(1/2)
```

sympy [A] time = 1.52, size = 19, normalized size = 0.86

$$-\sqrt{x}+x\,\mathrm{atan}\left(\sqrt{x}\right)+\mathrm{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x**(1/2)),x)
[Out] -sqrt(x) + x*atan(sqrt(x)) + atan(sqrt(x))
```

3.96 $\int \frac{\tan^{-1}(\sqrt{x})}{\sqrt{x}(1+x)} \, dx$

Optimal. Leaf size=8

$$\tan^{-1}\left(\sqrt{x}\right)^2$$

```
[Out] arctan(x^(1/2))^2
```

Rubi [A] time = 0.03, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {63, 203, 6686}

$$\tan^{-1}\left(\sqrt{x}\right)^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[Sqrt[x]]/(Sqrt[x]*(1 + x)),x]
```

```
[Out] ArcTan[Sqrt[x]]^2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 6686

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[(q*y^(m + 1))/(m + 1), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}\left(\sqrt{x}\right)}{\sqrt{x}(1+x)} \, dx = \tan^{-1}\left(\sqrt{x}\right)^2$$

Mathematica [A] time = 0.01, size = 8, normalized size = 1.00

$$\tan^{-1}\left(\sqrt{x}\right)^2$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[Sqrt[x]]/(Sqrt[x]*(1 + x)),x]
```

```
[Out] ArcTan[Sqrt[x]]^2
```

fricas [A] time = 0.42, size = 6, normalized size = 0.75

$$\arctan\left(\sqrt{x}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/(1+x)/x^(1/2),x, algorithm="fricas")
```

```
[Out] arctan(sqrt(x))^2
```

giac [A] time = 0.01, size = 6, normalized size = 0.75

$$\arctan\left(\sqrt{x}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/(1+x)/x^(1/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(x))^2
```

maple [A] time = 0.00, size = 7, normalized size = 0.88

$$\arctan\left(\sqrt{x}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x^(1/2))/(x+1)/x^(1/2),x)
```

```
[Out] arctan(x^(1/2))^2
```

maxima [A] time = 0.53, size = 6, normalized size = 0.75

$$\arctan\left(\sqrt{x}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/(1+x)/x^(1/2),x, algorithm="maxima")
```

```
[Out] arctan(sqrt(x))^2
```

mupad [B] time = 0.66, size = 6, normalized size = 0.75

$$\mathrm{atan}\left(\sqrt{x}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(x^(1/2))/(x^(1/2)*(x + 1)),x)
```

```
[Out] atan(x^(1/2))^2
```

sympy [A] time = 1.40, size = 7, normalized size = 0.88

$$\mathrm{atan}^2\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x**(1/2))/(1+x)/x**(1/2),x)
```

```
[Out] atan(sqrt(x))**2
```

3.97 $\int \sqrt{1-x^2}\, dx$

Optimal. Leaf size=23

$$\frac{1}{2}\sqrt{1-x^2}\,x+\frac{1}{2}\sin^{-1}(x)$$

```
[Out] 1/2*arcsin(x)+1/2*x*(-x^2+1)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {195, 216}

$$\frac{1}{2}\sqrt{1-x^2}\,x+\frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^2],x]
```

```
[Out] (x*Sqrt[1 - x^2])/2 + ArcSin[x]/2
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{1-x^2}\, dx &= \frac{1}{2}x\sqrt{1-x^2}+\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}\, dx\\ &= \frac{1}{2}x\sqrt{1-x^2}+\frac{1}{2}\sin^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 0.87

$$\frac{1}{2}\left(\sqrt{1-x^2}\,x+\sin^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^2],x]
```

```
[Out] (x*Sqrt[1 - x^2] + ArcSin[x])/2
```

fricas [A] time = 0.40, size = 31, normalized size = 1.35

$$\frac{1}{2}\sqrt{-x^2+1}\,x-\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(-x^2 + 1)*x - arctan((sqrt(-x^2 + 1) - 1)/x)
```

giac [A] time = 0.01, size = 17, normalized size = 0.74

$$\frac{1}{2}\sqrt{-x^2+1}\,x+\frac{1}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-x^2 + 1)*x + 1/2*arcsin(x)
```

maple [A] time = 0.00, size = 18, normalized size = 0.78

$$\frac{\sqrt{-x^2+1}\,x}{2}+\frac{\arcsin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^(1/2),x)
```

```
[Out] 1/2*arcsin(x)+1/2*x*(-x^2+1)^(1/2)
```

maxima [A] time = 1.34, size = 17, normalized size = 0.74

$$\frac{1}{2}\sqrt{-x^2+1}\,x+\frac{1}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-x^2 + 1)*x + 1/2*arcsin(x)
```

mupad [B] time = 0.08, size = 17, normalized size = 0.74

$$\frac{\mathrm{asin}(x)}{2}+\frac{x\,\sqrt{1-x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^2)^(1/2),x)
```

```
[Out] asin(x)/2 + (x*(1 - x^2)^(1/2))/2
```

sympy [A] time = 0.21, size = 15, normalized size = 0.65

$$\frac{x\sqrt{1-x^2}}{2}+\frac{\mathrm{asin}\,(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**(1/2),x)
```

```
[Out] x*sqrt(1 - x**2)/2 + asin(x)/2
```

3.98 $\int \frac{e^{\tan^{-1}(x)}x}{\left(1+x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=22

$$-\frac{(1-x)e^{\tan^{-1}(x)}}{2\sqrt{x^2+1}}$$

```
[Out] -1/2*exp(arctan(x))*(1-x)/(x^2+1)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {5077}

$$-\frac{(1-x)e^{\tan^{-1}(x)}}{2\sqrt{x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTan[x]*x)/(1 + x^2)^(3/2),x]
```

```
[Out] -(E^ArcTan[x]*(1 - x))/(2*Sqrt[1 + x^2])
```

Rule 5077

```
Int[(E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symb
ol] :> -Simp[((1 - a*n*x)*E^(n*ArcTan[a*x]))/(d*(n^2 + 1)*Sqrt[c + d*x^2]),
 x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c] &&  !IntegerQ[I*n]
```

Rubi steps

$$\int \frac{e^{\tan^{-1}(x)}x}{\left(1+x^2\right)^{3/2}}\,dx=-\frac{e^{\tan^{-1}(x)}(1-x)}{2\sqrt{1+x^2}}$$

Mathematica [C] time = 0.01, size = 37, normalized size = 1.68

$$\frac{1}{2}(1-ix)^{-\frac{1}{2}+\frac{i}{2}}(1+ix)^{-\frac{1}{2}-\frac{i}{2}}(x-1)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTan[x]*x)/(1 + x^2)^(3/2),x]
```

```
[Out] (-1 + x)/(2*(1 - I*x)^(1/2 - I/2)*(1 + I*x)^(1/2 + I/2))
```

fricas [A] time = 0.43, size = 15, normalized size = 0.68

$$\frac{(x-1)e^{\arctan(x)}}{2\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(x))*x/(x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(x - 1)*e^arctan(x)/sqrt(x^2 + 1)
```

giac [A] time = 0.01, size = 24, normalized size = 1.09

$$\frac{1}{2}\left(\frac{x}{\sqrt{x^2+1}}-\frac{1}{\sqrt{x^2+1}}\right)e^{\arctan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(x))*x/(x^2+1)^(3/2),x, algorithm="giac")
[Out] 1/2*(x/sqrt(x^2 + 1) - 1/sqrt(x^2 + 1))*e^arctan(x)
```

maple [A] time = 0.01, size = 16, normalized size = 0.73

$$\frac{(x-1)\,\mathrm{e}^{\arctan(x)}}{2\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arctan(x))*x/(x^2+1)^(3/2),x)
[Out] 1/2*(x-1)*exp(arctan(x))/(x^2+1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{xe^{\arctan(x)}}{\left(x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(x))*x/(x^2+1)^(3/2),x, algorithm="maxima")
[Out] integrate(x*e^arctan(x)/(x^2 + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{x\,\mathrm{e}^{\mathrm{atan}(x)}}{\left(x^2+1\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*exp(atan(x)))/(x^2 + 1)^(3/2),x)
[Out] int((x*exp(atan(x)))/(x^2 + 1)^(3/2), x)
```

sympy [A] time = 18.84, size = 31, normalized size = 1.41

$$\frac{xe^{\operatorname{atan}(x)}}{2\sqrt{x^2+1}} - \frac{e^{\operatorname{atan}(x)}}{2\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(atan(x))*x/(x**2+1)**(3/2),x)
[Out] x*exp(atan(x))/(2*sqrt(x**2 + 1)) - exp(atan(x))/(2*sqrt(x**2 + 1))
```

3.99 $\int \frac{e^{\tan^{-1}(x)}}{\left(1+x^2\right)^{3/2}} dx$

Optimal. Leaf size=20

$$\frac{(x+1)e^{\tan^{-1}(x)}}{2\sqrt{x^2+1}}$$

```
[Out] 1/2*exp(arctan(x))*(1+x)/(x^2+1)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {5069}

$$\frac{(x+1)e^{\tan^{-1}(x)}}{2\sqrt{x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTan[x]/(1 + x^2)^(3/2),x]
```

```
[Out] (E^ArcTan[x]*(1 + x))/(2*Sqrt[1 + x^2])
```

Rule 5069

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[((n + a*x)*E^(n*ArcTan[a*x]))/(a*c*(n^2 + 1)*Sqrt[c + d*x^2]), x] /; F
reeQ[{a, c, d, n}, x] && EqQ[d, a^2*c] && !IntegerQ[I*n]
```

Rubi steps

$$\int \frac{e^{\tan^{-1}(x)}}{\left(1+x^2\right)^{3/2}} dx = \frac{e^{\tan^{-1}(x)}(1+x)}{2\sqrt{1+x^2}}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.00

$$\frac{(x+1)e^{\tan^{-1}(x)}}{2\sqrt{x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTan[x]/(1 + x^2)^(3/2),x]
```

```
[Out] (E^ArcTan[x]*(1 + x))/(2*Sqrt[1 + x^2])
```

fricas [A] time = 0.44, size = 15, normalized size = 0.75

$$\frac{(x+1)e^{\arctan(x)}}{2\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(x))/(x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(x + 1)*e^arctan(x)/sqrt(x^2 + 1)
```

giac [A] time = 0.01, size = 22, normalized size = 1.10

$$\frac{1}{2}\left(\frac{x}{\sqrt{x^2+1}}+\frac{1}{\sqrt{x^2+1}}\right)e^{\arctan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(x))/(x^2+1)^(3/2),x, algorithm="giac")
[Out] 1/2*(x/sqrt(x^2 + 1) + 1/sqrt(x^2 + 1))*e^arctan(x)
```

maple [A] time = 0.00, size = 16, normalized size = 0.80

$$\frac{(x+1)\,\mathrm{e}^{\arctan(x)}}{2\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arctan(x))/(x^2+1)^(3/2),x)
[Out] 1/2*exp(arctan(x))*(x+1)/(x^2+1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{\arctan(x)}}{\left(x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(x))/(x^2+1)^(3/2),x, algorithm="maxima")
[Out] integrate(e^arctan(x)/(x^2 + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\mathrm{e}^{\mathrm{atan}(x)}}{\left(x^2+1\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(atan(x))/(x^2 + 1)^(3/2),x)
[Out] int(exp(atan(x))/(x^2 + 1)^(3/2), x)
```

sympy [A] time = 17.37, size = 31, normalized size = 1.55

$$\frac{xe^{\operatorname{atan}(x)}}{2\sqrt{x^2+1}}+\frac{e^{\operatorname{atan}(x)}}{2\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(atan(x))/(x**2+1)**(3/2),x)
[Out] x*exp(atan(x))/(2*sqrt(x**2 + 1)) + exp(atan(x))/(2*sqrt(x**2 + 1))
```

3.100 $\int \frac{x^2}{(1+x^2)^2} \, dx$

Optimal. Leaf size=19

$$\frac{1}{2}\tan^{-1}(x) - \frac{x}{2(x^2+1)}$$

[Out] -1/2*x/(x^2+1)+1/2*arctan(x)

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {288, 203}

$$\frac{1}{2}\tan^{-1}(x) - \frac{x}{2(x^2+1)}$$

Antiderivative was successfully verified.

[In] Int[x^2/(1 + x^2)^2,x]

[Out] -x/(2*(1 + x^2)) + ArcTan[x]/2

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned} \int \frac{x^2}{(1+x^2)^2} \, dx &= -\frac{x}{2(1+x^2)} + \frac{1}{2}\int \frac{1}{1+x^2} \, dx \\ &= -\frac{x}{2(1+x^2)} + \frac{1}{2}\tan^{-1}(x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{1}{2}\tan^{-1}(x) - \frac{x}{2(x^2+1)}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(1 + x^2)^2,x]

[Out] -1/2*x/(1 + x^2) + ArcTan[x]/2

fricas [A] time = 0.39, size = 21, normalized size = 1.11

$$\frac{(x^2+1)\arctan(x) - x}{2(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((x^2 + 1)*arctan(x) - x)/(x^2 + 1)
```

giac [A] time = 0.01, size = 15, normalized size = 0.79

$$-\frac{x}{2\left(x^{2}+1\right)}+\frac{1}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] -1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

maple [A] time = 0.01, size = 16, normalized size = 0.84

$$-\frac{x}{2\left(x^{2}+1\right)}+\frac{\arctan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^2+1)^2,x)
```

```
[Out] -1/2*x/(x^2+1)+1/2*arctan(x)
```

maxima [A] time = 1.12, size = 15, normalized size = 0.79

$$-\frac{x}{2\left(x^{2}+1\right)}+\frac{1}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

mupad [B] time = 0.03, size = 17, normalized size = 0.89

$$\frac{\operatorname{atan}(x)}{2}-\frac{x}{2\left(x^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^2 + 1)^2,x)
```

```
[Out] atan(x)/2 - x/(2*(x^2 + 1))
```

sympy [A] time = 0.10, size = 12, normalized size = 0.63

$$-\frac{x}{2x^{2}+2}+\frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**2+1)**2,x)
```

```
[Out] -x/(2*x**2 + 2) + atan(x)/2
```

3.101 $\int \frac{e^x}{1+e^{2x}}\,dx$

Optimal. Leaf size=4

$$\tan^{-1}\left(e^x\right)$$

```
[Out] arctan(exp(x))
```

Rubi [A] time = 0.02, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2249, 203}

$$\tan^{-1}\left(e^x\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^x/(1 + E^(2*x)),x]
```

```
[Out] ArcTan[E^x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2249

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[(d*e*Log[F])/(g*h*Lo
g[G])]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)
*(a + b*F^(c*e - (d*e*f)/g)*x^Numerator[m])^p, x], x, G^((h*(f + g*x))/Deno
minator[m])], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e,
 f, g, h, p}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{e^x}{1+e^{2x}}\,dx &= \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, e^x\right)\\ &= \tan^{-1}\left(e^x\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 4, normalized size = 1.00

$$\tan^{-1}\left(e^x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x/(1 + E^(2*x)),x]
```

```
[Out] ArcTan[E^x]
```

fricas [A] time = 0.42, size = 3, normalized size = 0.75

$$\arctan\left(e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1+exp(2*x)),x, algorithm="fricas")
```

```
[Out] arctan(e^x)
```

giac [**A**] time = 0.01, size = 3, normalized size = 0.75

$$\arctan\left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1+exp(2*x)),x, algorithm="giac")
[Out] arctan(e^x)
```

maple [**A**] time = 0.01, size = 4, normalized size = 1.00

$$\arctan\left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/(1+exp(2*x)),x)
[Out] arctan(exp(x))
```

maxima [**A**] time = 1.11, size = 3, normalized size = 0.75

$$\arctan\left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1+exp(2*x)),x, algorithm="maxima")
[Out] arctan(e^x)
```

mupad [**B**] time = 0.10, size = 3, normalized size = 0.75

$$\operatorname{atan}\left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/(exp(2*x) + 1),x)
[Out] atan(exp(x))
```

sympy [**B**] time = 0.11, size = 15, normalized size = 3.75

$$\operatorname{RootSum}\left(4z^{2}+1,\left(i\mapsto i\log\left(2i+e^{x}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1+exp(2*x)),x)
[Out] RootSum(4*_z**2 + 1, Lambda(_i, _i*log(2*_i + exp(x))))
```

3.102 $\int e^{-x} \cot^{-1}(e^x)\, dx$

Optimal. Leaf size=27

$$-x + \frac{1}{2}\log\left(e^{2x}+1\right) - e^{-x}\cot^{-1}\left(e^x\right)$$

```
[Out] -x-arccot(exp(x))/exp(x)+1/2*ln(1+exp(2*x))
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {2194, 5208, 2282, 36, 29, 31}

$$-x + \frac{1}{2}\log\left(e^{2x}+1\right) - e^{-x}\cot^{-1}\left(e^x\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[E^x]/E^x,x]
```

```
[Out] -x - ArcCot[E^x]/E^x + Log[1 + E^(2*x)]/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 5208

```
Int[((a_.) + ArcCot[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[a + b*ArcCot[u], w, x] + Dist[b, Int[SimplifyIntegrand[(w*D[u, x])/(1 + u^2), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] && InverseFunctionFreeQ[u, x] && !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcCot[u]), x]]
```

Rubi steps

$$\begin{aligned}\int e^{-x}\cot^{-1}(e^x)\,dx &= -e^{-x}\cot^{-1}(e^x)-\int\frac{1}{1+e^{2x}}\,dx\\ &= -e^{-x}\cot^{-1}(e^x)-\frac{1}{2}\,\text{Subst}\left(\int\frac{1}{x(1+x)}\,dx,x,e^{2x}\right)\\ &= -e^{-x}\cot^{-1}(e^x)-\frac{1}{2}\,\text{Subst}\left(\int\frac{1}{x}\,dx,x,e^{2x}\right)+\frac{1}{2}\,\text{Subst}\left(\int\frac{1}{1+x}\,dx,x,e^{2x}\right)\\ &= -x-e^{-x}\cot^{-1}(e^x)+\frac{1}{2}\log\left(1+e^{2x}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 27, normalized size = 1.00

$$-x+\frac{1}{2}\log\left(e^{2x}+1\right)-e^{-x}\cot^{-1}(e^x)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[E^x]/E^x,x]
[Out] -x - ArcCot[E^x]/E^x + Log[1 + E^(2*x)]/2
```

fricas [A] time = 0.43, size = 28, normalized size = 1.04

$$-\frac{1}{2}\left(2\,xe^x-e^x\log\left(e^{(2\,x)}+1\right)+2\,\operatorname{arccot}(e^x)\right)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(exp(x))/exp(x),x, algorithm="fricas")
[Out] -1/2*(2*x*e^x - e^x*log(e^(2*x) + 1) + 2*arccot(e^x))*e^(-x)
```

giac [A] time = 0.01, size = 21, normalized size = 0.78

$$-\arctan\left(e^{(-x)}\right)e^{(-x)}+\frac{1}{2}\,\log\left(e^{(-2\,x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(exp(x))/exp(x),x, algorithm="giac")
[Out] -arctan(e^(-x))*e^(-x) + 1/2*log(e^(-2*x) + 1)
```

maple [A] time = 0.01, size = 25, normalized size = 0.93

$$-\operatorname{arccot}(e^x)\,e^{-x}+\frac{\ln\left(e^{2x}+1\right)}{2}-\ln(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(exp(x))/exp(x),x)
[Out] -arccot(exp(x))/exp(x)+1/2*ln(exp(x)^2+1)-ln(exp(x))
```

maxima [A] time = 0.51, size = 19, normalized size = 0.70

$$-\operatorname{arccot}(e^x)\,e^{(-x)}+\frac{1}{2}\,\log\left(e^{(-2\,x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(exp(x))/exp(x),x, algorithm="maxima")
[Out] -arccot(e^x)*e^(-x) + 1/2*log(e^(-2*x) + 1)
```

mupad [B] time = 0.14, size = 22, normalized size = 0.81

$$\frac{\ln\left(e^{2x}+1\right)}{2}-x-\operatorname{acot}\left(e^{x}\right)\,e^{-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(exp(x))*exp(-x),x)
[Out] log(exp(2*x) + 1)/2 - x - acot(exp(x))*exp(-x)
```

sympy [A] time = 7.74, size = 19, normalized size = 0.70

$$-x+\frac{\log\left(e^{2x}+1\right)}{2}-e^{-x}\operatorname{acot}\left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(exp(x))/exp(x),x)
[Out] -x + log(exp(2*x) + 1)/2 - exp(-x)*acot(exp(x))
```

3.103 $\int \sqrt{\frac{a+x}{a-x}}\,dx$

Optimal. Leaf size=42

$$2a\tan^{-1}\left(\sqrt{\frac{a+x}{a-x}}\right)-(a-x)\sqrt{\frac{a+x}{a-x}}$$

```
[Out] 2*a*arctan(((a+x)/(a-x))^(1/2))-(a-x)*((a+x)/(a-x))^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1959, 288, 203}

$$2a\tan^{-1}\left(\sqrt{\frac{a+x}{a-x}}\right)-(a-x)\sqrt{\frac{a+x}{a-x}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(a + x)/(a - x)],x]
```

```
[Out] -((a - x)*Sqrt[(a + x)/(a - x)]) + 2*a*ArcTan[Sqrt[(a + x)/(a - x)]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1959

```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n, Subst[Int[(
x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(1/n - 1))/(b*e - d*x^q)^(1/n + 1), x],
x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x] &
& FractionQ[p] && IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\frac{a+x}{a-x}}\,dx &= (4a)\,\text{Subst}\left(\int \frac{x^2}{\left(1+x^2\right)^2}\,dx, x, \sqrt{\frac{a+x}{a-x}}\right)\\ &= -(a-x)\sqrt{\frac{a+x}{a-x}}+(2a)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{\frac{a+x}{a-x}}\right)\\ &= -(a-x)\sqrt{\frac{a+x}{a-x}}+2a\tan^{-1}\left(\sqrt{\frac{a+x}{a-x}}\right)\end{aligned}$$

Mathematica [A] time = 0.05, size = 83, normalized size = 1.98

$$\frac{\sqrt{x-a}\sqrt{\frac{a+x}{a-x}}\left(2a^{3/2}\sqrt{\frac{a+x}{a}}\sinh^{-1}\left(\frac{\sqrt{x-a}}{\sqrt{2}\sqrt{a}}\right)+\sqrt{x-a}(a+x)\right)}{a+x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(a + x)/(a - x)],x]
```

```
[Out] (Sqrt[-a + x]*Sqrt[(a + x)/(a - x)]*(Sqrt[-a + x]*(a + x) + 2*a^(3/2)*Sqrt[
(a + x)/a]*ArcSinh[Sqrt[-a + x]/(Sqrt[2]*Sqrt[a])]))/(a + x)
```

fricas [A] time = 0.42, size = 38, normalized size = 0.90

$$2\,a\arctan\left(\sqrt{\frac{a+x}{a-x}}\right)-(a-x)\sqrt{\frac{a+x}{a-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a+x)/(a-x))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*a*arctan(sqrt((a + x)/(a - x))) - (a - x)*sqrt((a + x)/(a - x))
```

giac [A] time = 0.04, size = 36, normalized size = 0.86

$$a\arcsin\left(\frac{x}{a}\right)\operatorname{sgn}(a-x)\operatorname{sgn}(a)-\sqrt{a^2-x^2}\operatorname{sgn}(a-x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a+x)/(a-x))^(1/2),x, algorithm="giac")
```

```
[Out] a*arcsin(x/a)*sgn(a - x)*sgn(a) - sqrt(a^2 - x^2)*sgn(a - x)
```

maple [A] time = 0.02, size = 64, normalized size = 1.52

$$-\frac{\sqrt{-\frac{a+x}{-a+x}}\,(-a+x)\left(a\arctan\left(\frac{x}{\sqrt{a^2-x^2}}\right)-\sqrt{a^2-x^2}\right)}{\sqrt{-(a+x)(-a+x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a+x)/(a-x))^(1/2),x)
```

```
[Out] -(-(a+x)/(-a+x))^(1/2)*(-a+x)*(a*arctan(1/(a^2-x^2)^(1/2)*x)-(a^2-x^2)^(1/2
))/(-(a+x)*(-a+x))^(1/2)
```

maxima [A] time = 1.18, size = 49, normalized size = 1.17

$$-2\,a\left(\frac{\sqrt{\frac{a+x}{a-x}}}{\frac{a+x}{a-x}+1}-\arctan\left(\sqrt{\frac{a+x}{a-x}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a+x)/(a-x))^(1/2),x, algorithm="maxima")
```

```
[Out] -2*a*(sqrt((a + x)/(a - x))/((a + x)/(a - x) + 1) - arctan(sqrt((a + x)/(a
- x))))
```

mupad [B] time = 0.07, size = 49, normalized size = 1.17

$$2\,a\,\mathrm{atan}\left(\sqrt{\frac{a+x}{a-x}}\right)-\frac{2\,a\,\sqrt{\frac{a+x}{a-x}}}{\frac{a+x}{a-x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + x)/(a - x))^(1/2),x)
```

```
[Out] 2*a*atan(((a + x)/(a - x))^(1/2)) - (2*a*((a + x)/(a - x))^(1/2))/((a + x)/
(a - x) + 1)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{a + x}{a - x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a+x)/(a-x))**(1/2),x)
```

```
[Out] Integral(sqrt((a + x)/(a - x)), x)
```

3.104 $\int \sqrt{(b-x)(-a+x)}\,dx$

Optimal. Leaf size=71

$$-\frac{1}{4}(a+b-2x)\sqrt{x(a+b)-ab-x^2}-\frac{1}{8}(a-b)^2\tan^{-1}\left(\frac{a+b-2x}{2\sqrt{x(a+b)-ab-x^2}}\right)$$

```
[Out] -1/8*(a-b)^2*arctan(1/2*(a+b-2*x)/(-a*b+(a+b)*x-x^2)^(1/2))-1/4*(a+b-2*x)*(
-a*b+(a+b)*x-x^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1981, 612, 621, 204}

$$-\frac{1}{4}(a+b-2x)\sqrt{x(a+b)-ab-x^2}-\frac{1}{8}(a-b)^2\tan^{-1}\left(\frac{a+b-2x}{2\sqrt{x(a+b)-ab-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(b - x)*(-a + x)],x]
```

```
[Out] -((a + b - 2*x)*Sqrt[-(a*b) + (a + b)*x - x^2])/4 - ((a - b)^2*ArcTan[(a +
b - 2*x)/(2*Sqrt[-(a*b) + (a + b)*x - x^2])])/8
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1981

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && Q
uadraticQ[u, x] &&  !QuadraticMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{(b-x)(-a+x)}\,dx &= \int \sqrt{-ab+(a+b)x-x^2}\,dx\\ &= -\frac{1}{4}(a+b-2x)\sqrt{-ab+(a+b)x-x^2}+\frac{1}{8}(a-b)^2\int \frac{1}{\sqrt{-ab+(a+b)x-x^2}}\,dx\\ &= -\frac{1}{4}(a+b-2x)\sqrt{-ab+(a+b)x-x^2}+\frac{1}{4}(a-b)^2\,\text{Subst}\left(\int \frac{1}{-4-x^2}\,dx, x, \frac{a+}{\sqrt{-ab+(}}\right.\\ &= -\frac{1}{4}(a+b-2x)\sqrt{-ab+(a+b)x-x^2}-\frac{1}{8}(a-b)^2\tan^{-1}\left(\frac{a+b-2x}{2\sqrt{-ab+(a+b)x-x^2}}\right)\end{aligned}$$

Mathematica [A] time = 0.17, size = 106, normalized size = 1.49

$$\frac{(a-x)\left((a-b)^{5/2}\sqrt{b-x}\sqrt{\frac{a-x}{a-b}}\sinh^{-1}\left(\frac{\sqrt{b-x}}{\sqrt{a-b}}\right)-(a-x)(b-x)(a+b-2x)\right)}{4(x-a)\sqrt{(a-x)(x-b)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(b - x)*(-a + x)],x]
```

```
[Out] ((a - x)*(-((a + b - 2*x)*(a - x)*(b - x)) + (a - b)^(5/2)*Sqrt[(a - x)/(a
- b)]*Sqrt[b - x]*ArcSinh[Sqrt[b - x]/Sqrt[a - b]]))/(4*(-a + x)*Sqrt[(a -
x)*(-b + x)])
```

fricas [A] time = 0.42, size = 80, normalized size = 1.13

$$-\frac{1}{8}\left(a^2-2\,ab+b^2\right)\arctan\left(-\frac{\sqrt{-ab+(a+b)x-x^2}\,(a+b-2\,x)}{2\left(ab-(a+b)x+x^2\right)}\right)-\frac{1}{4}\sqrt{-ab+(a+b)x-x^2}\,(a+b-2\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b-x)*(-a+x))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/8*(a^2 - 2*a*b + b^2)*arctan(-1/2*sqrt(-a*b + (a + b)*x - x^2)*(a + b -
2*x)/(a*b - (a + b)*x + x^2)) - 1/4*sqrt(-a*b + (a + b)*x - x^2)*(a + b - 2
*x)
```

giac [A] time = 0.03, size = 61, normalized size = 0.86

$$\frac{1}{8}\left(a^2-2\,ab+b^2\right)\arcsin\left(\frac{a+b-2\,x}{a-b}\right)\mathrm{sgn}\left(-a+b\right)-\frac{1}{4}\sqrt{-ab+ax+bx-x^2}\,(a+b-2\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b-x)*(-a+x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(a^2 - 2*a*b + b^2)*arcsin((a + b - 2*x)/(a - b))*sgn(-a + b) - 1/4*sqr
t(-a*b + a*x + b*x - x^2)*(a + b - 2*x)
```

maple [A] time = 0.02, size = 122, normalized size = 1.72

$$\frac{a^2\arctan\left(\frac{-\frac{a}{2}-\frac{b}{2}+x}{\sqrt{-ab-x^2+(a+b)x}}\right)}{8}-\frac{ab\arctan\left(\frac{-\frac{a}{2}-\frac{b}{2}+x}{\sqrt{-ab-x^2+(a+b)x}}\right)}{4}+\frac{b^2\arctan\left(\frac{-\frac{a}{2}-\frac{b}{2}+x}{\sqrt{-ab-x^2+(a+b)x}}\right)}{8}-\frac{(a+b-2x)\sqrt{-ab-x^2+(a}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b-x)*(-a+x))^(1/2),x)
```

```
[Out] -1/4*(a+b-2*x)*(-a*b+(a+b)*x-x^2)^(1/2)-1/4*arctan((x-1/2*b-1/2*a)/(-a*b+(a
+b)*x-x^2)^(1/2))*a*b+1/8*arctan((x-1/2*b-1/2*a)/(-a*b+(a+b)*x-x^2)^(1/2))*
a^2+1/8*arctan((x-1/2*b-1/2*a)/(-a*b+(a+b)*x-x^2)^(1/2))*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b-x)*(-a+x))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{-(a-x)(b-x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(a - x)*(b - x))^(1/2),x)
```

```
[Out] int((-(a - x)*(b - x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{(-a+x)(b-x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b-x)*(-a+x))**(1/2),x)
```

```
[Out] Integral(sqrt((-a + x)*(b - x)), x)
```

3.105 $\int \frac{1}{\sqrt{(b-x)(-a+x)}}\,dx$

Optimal. Leaf size=32

$$-\tan^{-1}\left(\frac{a+b-2x}{2\sqrt{x(a+b)-ab-x^2}}\right)$$

```
[Out] -arctan(1/2*(a+b-2*x)/(-a*b+(a+b)*x-x^2)^(1/2))
```

Rubi [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1981, 621, 204}

$$-\tan^{-1}\left(\frac{a+b-2x}{2\sqrt{x(a+b)-ab-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(b - x)*(-a + x)],x]
```

```
[Out] -ArcTan[(a + b - 2*x)/(2*Sqrt[-(a*b) + (a + b)*x - x^2])]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1981

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && Q
uadraticQ[u, x] &&  !QuadraticMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{(b-x)(-a+x)}}\,dx &= \int \frac{1}{\sqrt{-ab+(a+b)x-x^2}}\,dx\\
&= 2\,\text{Subst}\left(\int \frac{1}{-4-x^2}\,dx, x, \frac{a+b-2x}{\sqrt{-ab+(a+b)x-x^2}}\right)\\
&= -\tan^{-1}\left(\frac{a+b-2x}{2\sqrt{-ab+(a+b)x-x^2}}\right)
\end{aligned}$$

Mathematica [B] time = 0.03, size = 72, normalized size = 2.25

$$-\frac{2\sqrt{a-b}\,\sqrt{b-x}\,\sqrt{\frac{a-x}{a-b}}\,\sinh^{-1}\left(\frac{\sqrt{b-x}}{\sqrt{a-b}}\right)}{\sqrt{(a-x)(x-b)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(b - x)*(-a + x)],x]
```

```
[Out] (-2*Sqrt[a - b]*Sqrt[(a - x)/(a - b)]*Sqrt[b - x]*ArcSinh[Sqrt[b - x]/Sqrt[
a - b]])/Sqrt[(a - x)*(-b + x)]
```

fricas [A] time = 0.39, size = 43, normalized size = 1.34

$$-\arctan\left(-\frac{\sqrt{-ab+(a+b)x-x^2}\,(a+b-2x)}{2\left(ab-(a+b)x+x^2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b-x)*(-a+x))^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(-1/2*sqrt(-a*b + (a + b)*x - x^2)*(a + b - 2*x)/(a*b - (a + b)*x +
x^2))
```

giac [A] time = 0.12, size = 22, normalized size = 0.69

$$\arcsin\left(\frac{a+b-2x}{a-b}\right)\operatorname{sgn}(-a+b)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b-x)*(-a+x))^(1/2),x, algorithm="giac")
```

```
[Out] arcsin((a + b - 2*x)/(a - b))*sgn(-a + b)
```

maple [A] time = 0.00, size = 28, normalized size = 0.88

$$\arctan\left(\frac{-\frac{a}{2}-\frac{b}{2}+x}{\sqrt{-ab-x^2+(a+b)x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b-x)*(-a+x))^(1/2),x)
```

```
[Out] arctan((-1/2*a-1/2*b+x)/(-a*b-x^2+(a+b)*x)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b-x)*(-a+x))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{-(a-x)\,(b-x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-(a - x)*(b - x))^(1/2),x)
```

```
[Out] int(1/(-(a - x)*(b - x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(-a+x)(b-x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b-x)*(-a+x))**(1/2),x)
```

```
[Out] Integral(1/sqrt((-a + x)*(b - x)), x)
```

3.106 $\int \frac{3+5x}{-3+2x+x^2} dx$

Optimal. Leaf size=15

$$2\log(1-x)+3\log(x+3)$$

[Out] 2*ln(1-x)+3*ln(3+x)

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {632, 31}

$$2\log(1-x)+3\log(x+3)$$

Antiderivative was successfully verified.

[In] Int[(3 + 5*x)/(-3 + 2*x + x^2),x]

[Out] 2*Log[1 - x] + 3*Log[3 + x]

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{3+5x}{-3+2x+x^2}\,dx &= 2\int \frac{1}{-1+x}\,dx+3\int \frac{1}{3+x}\,dx\\ &= 2\log(1-x)+3\log(3+x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$2\log(1-x)+3\log(x+3)$$

Antiderivative was successfully verified.

[In] Integrate[(3 + 5*x)/(-3 + 2*x + x^2),x]

[Out] 2*Log[1 - x] + 3*Log[3 + x]

fricas [A] time = 0.38, size = 13, normalized size = 0.87

$$3\,\log\left(x+3\right)+2\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3+5*x)/(x^2+2*x-3),x, algorithm="fricas")

[Out] 3*log(x + 3) + 2*log(x - 1)

giac [A] time = 0.01, size = 15, normalized size = 1.00

$$3\,\log\left(|x+3|\right)+2\,\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+5*x)/(x^2+2*x-3),x, algorithm="giac")
```

```
[Out] 3*log(abs(x + 3)) + 2*log(abs(x - 1))
```

maple [A] time = 0.01, size = 14, normalized size = 0.93

$$2\ln(x-1)+3\ln(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+5*x)/(x^2+2*x-3),x)
```

```
[Out] 3*ln(3+x)+2*ln(x-1)
```

maxima [A] time = 0.43, size = 13, normalized size = 0.87

$$3\log(x+3)+2\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+5*x)/(x^2+2*x-3),x, algorithm="maxima")
```

```
[Out] 3*log(x + 3) + 2*log(x - 1)
```

mupad [B] time = 0.05, size = 13, normalized size = 0.87

$$2\ln(x-1)+3\ln(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + 3)/(2*x + x^2 - 3),x)
```

```
[Out] 2*log(x - 1) + 3*log(x + 3)
```

sympy [A] time = 0.11, size = 12, normalized size = 0.80

$$2\log(x-1)+3\log(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+5*x)/(x**2+2*x-3),x)
```

```
[Out] 2*log(x - 1) + 3*log(x + 3)
```

3.107 $\int \frac{5+2x}{-3+2x+x^2} dx$

Optimal. Leaf size=19

$$\frac{7}{4}\log(1-x)+\frac{1}{4}\log(x+3)$$

```
[Out] 7/4*ln(1-x)+1/4*ln(3+x)
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {632, 31}

$$\frac{7}{4}\log(1-x)+\frac{1}{4}\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 2*x)/(-3 + 2*x + x^2),x]
[Out] (7*Log[1 - x])/4 + Log[3 + x]/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{5+2x}{-3+2x+x^2}\,dx &= \frac{1}{4}\int \frac{1}{3+x}\,dx+\frac{7}{4}\int \frac{1}{-1+x}\,dx\\ &= \frac{7}{4}\log(1-x)+\frac{1}{4}\log(3+x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 1.00

$$\frac{7}{4}\log(1-x)+\frac{1}{4}\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 2*x)/(-3 + 2*x + x^2),x]
[Out] (7*Log[1 - x])/4 + Log[3 + x]/4
```

fricas [A] time = 0.39, size = 13, normalized size = 0.68

$$\frac{1}{4}\log\left(x+3\right)+\frac{7}{4}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)/(x^2+2*x-3),x, algorithm="fricas")
```

```
[Out] 1/4*log(x + 3) + 7/4*log(x - 1)
```

giac [A] time = 0.01, size = 15, normalized size = 0.79

$$\frac{1}{4}\,\log\left(|x+3|\right)+\frac{7}{4}\,\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)/(x^2+2*x-3),x, algorithm="giac")
[Out] 1/4*log(abs(x + 3)) + 7/4*log(abs(x - 1))
```

maple [A] time = 0.01, size = 14, normalized size = 0.74

$$\frac{7\ln\left(x-1\right)}{4}+\frac{\ln\left(x+3\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5+2*x)/(x^2+2*x-3),x)
[Out] 1/4*ln(x+3)+7/4*ln(x-1)
```

maxima [A] time = 0.47, size = 13, normalized size = 0.68

$$\frac{1}{4}\,\log\left(x+3\right)+\frac{7}{4}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)/(x^2+2*x-3),x, algorithm="maxima")
[Out] 1/4*log(x + 3) + 7/4*log(x - 1)
```

mupad [B] time = 0.04, size = 13, normalized size = 0.68

$$\frac{7\,\ln\left(x-1\right)}{4}+\frac{\ln\left(x+3\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 5)/(2*x + x^2 - 3),x)
[Out] (7*log(x - 1))/4 + log(x + 3)/4
```

sympy [A] time = 0.10, size = 14, normalized size = 0.74

$$\frac{7\log\left(x-1\right)}{4}+\frac{\log\left(x+3\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)/(x**2+2*x-3),x)
[Out] 7*log(x - 1)/4 + log(x + 3)/4
```

3.108 $\int \frac{3x+x^3}{-3-2x+x^2} \, dx$

Optimal. Leaf size=23

$$\frac{x^2}{2} + 2x + 9\log(3-x) + \log(x+1)$$

```
[Out] 2*x+1/2*x^2+9*ln(3-x)+ln(1+x)
```

Rubi [A] time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1593, 1628, 632, 31}

$$\frac{x^2}{2} + 2x + 9\log(3-x) + \log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(3*x + x^3)/(-3 - 2*x + x^2),x]
```

```
[Out] 2*x + x^2/2 + 9*Log[3 - x] + Log[1 + x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{3x+x^3}{-3-2x+x^2}\,dx &= \int \frac{x\left(3+x^2\right)}{-3-2x+x^2}\,dx\\ &= \int \left(2+x+\frac{2(3+5x)}{-3-2x+x^2}\right)dx\\ &= 2x+\frac{x^2}{2}+2\int \frac{3+5x}{-3-2x+x^2}\,dx\\ &= 2x+\frac{x^2}{2}+9\int \frac{1}{-3+x}\,dx+\int \frac{1}{1+x}\,dx\\ &= 2x+\frac{x^2}{2}+9\log(3-x)+\log(1+x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 1.00

$$\frac{x^2}{2}+2x+9\log(3-x)+\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*x + x^3)/(-3 - 2*x + x^2),x]
[Out] 2*x + x^2/2 + 9*Log[3 - x] + Log[1 + x]
```

fricas [A] time = 0.40, size = 19, normalized size = 0.83

$$\frac{1}{2}\,x^2+2\,x+\log\left(x+1\right)+9\,\log\left(x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+3*x)/(x^2-2*x-3),x, algorithm="fricas")
[Out] 1/2*x^2 + 2*x + log(x + 1) + 9*log(x - 3)
```

giac [A] time = 0.01, size = 21, normalized size = 0.91

$$\frac{1}{2}\,x^2+2\,x+\log\left(|x+1|\right)+9\,\log\left(|x-3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+3*x)/(x^2-2*x-3),x, algorithm="giac")
[Out] 1/2*x^2 + 2*x + log(abs(x + 1)) + 9*log(abs(x - 3))
```

maple [A] time = 0.01, size = 20, normalized size = 0.87

$$\frac{x^2}{2}+2x+9\ln\left(x-3\right)+\ln\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+3*x)/(x^2-2*x-3),x)
[Out] 1/2*x^2+2*x+9*ln(-3+x)+ln(x+1)
```

maxima [A] time = 0.48, size = 19, normalized size = 0.83

$$\frac{1}{2}\,x^2+2\,x+\log\left(x+1\right)+9\,\log\left(x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+3*x)/(x^2-2*x-3),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + 2*x + log(x + 1) + 9*log(x - 3)
```

mupad [B] time = 0.04, size = 19, normalized size = 0.83

$$2\,x+\ln\left(x+1\right)+9\,\ln\left(x-3\right)+\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*x + x^3)/(2*x - x^2 + 3),x)
```

```
[Out] 2*x + log(x + 1) + 9*log(x - 3) + x^2/2
```

sympy [A] time = 0.11, size = 19, normalized size = 0.83

$$\frac{x^2}{2}+2x+9\log\left(x-3\right)+\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+3*x)/(x**2-2*x-3),x)
```

```
[Out] x**2/2 + 2*x + 9*log(x - 3) + log(x + 1)
```

3.109 $\int \frac{-1+5x+2x^2}{-2x+x^2+x^3}\,dx$

Optimal. Leaf size=23

$$2\log(1-x)+\frac{\log(x)}{2}-\frac{1}{2}\log(x+2)$$

```
[Out] 2*ln(1-x)+1/2*ln(x)-1/2*ln(2+x)
```

Rubi [A] time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1594, 1628}

$$2\log(1-x)+\frac{\log(x)}{2}-\frac{1}{2}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 5*x + 2*x^2)/(-2*x + x^2 + x^3),x]
```

```
[Out] 2*Log[1 - x] + Log[x]/2 - Log[2 + x]/2
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+5x+2x^2}{-2x+x^2+x^3}\,dx &= \int \frac{-1+5x+2x^2}{x\left(-2+x+x^2\right)}\,dx\\ &= \int \left(\frac{2}{-1+x}+\frac{1}{2x}-\frac{1}{2(2+x)}\right)dx\\ &= 2\log(1-x)+\frac{\log(x)}{2}-\frac{1}{2}\log(2+x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$2\log(1-x)+\frac{\log(x)}{2}-\frac{1}{2}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 5*x + 2*x^2)/(-2*x + x^2 + x^3),x]
```

```
[Out] 2*Log[1 - x] + Log[x]/2 - Log[2 + x]/2
```

fricas [A] time = 0.41, size = 17, normalized size = 0.74

$$-\frac{1}{2}\,\log\left(x+2\right)+2\,\log\left(x-1\right)+\frac{1}{2}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+5*x-1)/(x^3+x^2-2*x),x, algorithm="fricas")
```

```
[Out] -1/2*log(x + 2) + 2*log(x - 1) + 1/2*log(x)
```

giac [A] time = 0.01, size = 20, normalized size = 0.87

$$-\frac{1}{2}\,\log\left(|x+2|\right)+2\,\log\left(|x-1|\right)+\frac{1}{2}\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+5*x-1)/(x^3+x^2-2*x),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(x + 2)) + 2*log(abs(x - 1)) + 1/2*log(abs(x))
```

maple [A] time = 0.01, size = 18, normalized size = 0.78

$$\frac{\ln(x)}{2}+2\ln\left(x-1\right)-\frac{\ln\left(x+2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+5*x-1)/(x^3+x^2-2*x),x)
```

```
[Out] -1/2*ln(2+x)+2*ln(x-1)+1/2*ln(x)
```

maxima [A] time = 0.51, size = 17, normalized size = 0.74

$$-\frac{1}{2}\,\log\left(x+2\right)+2\,\log\left(x-1\right)+\frac{1}{2}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+5*x-1)/(x^3+x^2-2*x),x, algorithm="maxima")
```

```
[Out] -1/2*log(x + 2) + 2*log(x - 1) + 1/2*log(x)
```

mupad [B] time = 0.19, size = 19, normalized size = 0.83

$$2\,\ln\left(x-1\right)+\mathrm{atanh}\left(\frac{135}{11\,\left(11\,x-5\right)}+\frac{16}{11}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + 2*x^2 - 1)/(x^2 - 2*x + x^3),x)
```

```
[Out] 2*log(x - 1) + atanh(135/(11*(11*x - 5)) + 16/11)
```

sympy [A] time = 0.14, size = 17, normalized size = 0.74

$$\frac{\log\left(x\right)}{2}+2\log\left(x-1\right)-\frac{\log\left(x+2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+5*x-1)/(x**3+x**2-2*x),x)
```

```
[Out] log(x)/2 + 2*log(x - 1) - log(x + 2)/2
```

3.110 $\int \frac{3+2x+x^2}{(-1+x)(1+x)^2}\,dx$

Optimal. Leaf size=24

$$\frac{1}{x+1}+\frac{3}{2}\log(1-x)-\frac{1}{2}\log(x+1)$$

```
[Out] 1/(1+x)+3/2*ln(1-x)-1/2*ln(1+x)
```

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {893}

$$\frac{1}{x+1}+\frac{3}{2}\log(1-x)-\frac{1}{2}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 2*x + x^2)/((-1 + x)*(1 + x)^2),x]
```

```
[Out] (1 + x)^(-1) + (3*Log[1 - x])/2 - Log[1 + x]/2
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\begin{aligned}\int \frac{3+2x+x^2}{(-1+x)(1+x)^2}\,dx &= \int \left(\frac{3}{2(-1+x)}-\frac{1}{(1+x)^2}-\frac{1}{2(1+x)}\right)dx\\ &= \frac{1}{1+x}+\frac{3}{2}\log(1-x)-\frac{1}{2}\log(1+x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.92

$$\frac{1}{x+1}+\frac{3}{2}\log(x-1)-\frac{1}{2}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 2*x + x^2)/((-1 + x)*(1 + x)^2),x]
```

```
[Out] (1 + x)^(-1) + (3*Log[-1 + x])/2 - Log[1 + x]/2
```

fricas [A] time = 0.41, size = 26, normalized size = 1.08

$$-\frac{(x+1)\log(x+1)-3(x+1)\log(x-1)-2}{2(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x+3)/(-1+x)/(1+x)^2,x, algorithm="fricas")
```

```
[Out] -1/2*((x + 1)*log(x + 1) - 3*(x + 1)*log(x - 1) - 2)/(x + 1)
```

giac [A] time = 0.01, size = 24, normalized size = 1.00

$$\frac{1}{x+1}+\log\left(|x+1|\right)+\frac{3}{2}\log\left(\left|-\frac{2}{x+1}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x+3)/(-1+x)/(1+x)^2,x, algorithm="giac")
[Out] 1/(x + 1) + log(abs(x + 1)) + 3/2*log(abs(-2/(x + 1) + 1))
```

maple [A] time = 0.01, size = 19, normalized size = 0.79

$$\frac{3\ln\left(x-1\right)}{2}-\frac{\ln\left(x+1\right)}{2}+\frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2*x+3)/(x-1)/(x+1)^2,x)
[Out] 1/(x+1)-1/2*ln(x+1)+3/2*ln(x-1)
```

maxima [A] time = 0.45, size = 18, normalized size = 0.75

$$\frac{1}{x+1}-\frac{1}{2}\log\left(x+1\right)+\frac{3}{2}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x+3)/(-1+x)/(1+x)^2,x, algorithm="maxima")
[Out] 1/(x + 1) - 1/2*log(x + 1) + 3/2*log(x - 1)
```

mupad [B] time = 0.05, size = 18, normalized size = 0.75

$$\frac{3\ \ln\left(x-1\right)}{2}-\frac{\ln\left(x+1\right)}{2}+\frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + x^2 + 3)/((x - 1)*(x + 1)^2),x)
[Out] (3*log(x - 1))/2 - log(x + 1)/2 + 1/(x + 1)
```

sympy [A] time = 0.12, size = 19, normalized size = 0.79

$$\frac{3\log\left(x-1\right)}{2}-\frac{\log\left(x+1\right)}{2}+\frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2*x+3)/(-1+x)/(1+x)**2,x)
[Out] 3*log(x - 1)/2 - log(x + 1)/2 + 1/(x + 1)
```

3.111 $\int \frac{-2+2x+3x^2}{-1+x^3} \, dx$

Optimal. Leaf size=28

$$\log\left(1-x^3\right)+\frac{4\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] ln(-x^3+1)+4/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1871, 1586, 618, 204, 260}

$$\log\left(1-x^3\right)+\frac{4\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + 2*x + 3*x^2)/(-1 + x^3),x]
```

```
[Out] (4*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] + Log[1 - x^3]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{-2+2x+3x^2}{-1+x^3}\,dx &= 3\int \frac{x^2}{-1+x^3}\,dx + \int \frac{-2+2x}{-1+x^3}\,dx\\ &= \log\left(1-x^3\right) + \int \frac{1}{\frac{1}{2}+\frac{x}{2}+\frac{x^2}{2}}\,dx\\ &= \log\left(1-x^3\right) - 2\,\text{Subst}\left(\int \frac{1}{-\frac{3}{4}-x^2}\,dx, x, \frac{1}{2}+x\right)\\ &= \frac{4\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} + \log\left(1-x^3\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.00

$$\log\left(1-x^3\right) + \frac{4\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + 2*x + 3*x^2)/(-1 + x^3),x]
[Out] (4*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] + Log[1 - x^3]
```

fricas [A] time = 0.40, size = 28, normalized size = 1.00

$$\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x+1)\right) + \log\left(x^2+x+1\right) + \log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2*x-2)/(x^3-1),x, algorithm="fricas")
[Out] 4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + log(x^2 + x + 1) + log(x - 1)
```

giac [A] time = 0.01, size = 29, normalized size = 1.04

$$\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x+1)\right) + \log\left(x^2+x+1\right) + \log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2*x-2)/(x^3-1),x, algorithm="giac")
[Out] 4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + log(x^2 + x + 1) + log(abs(x -
1))
```

maple [A] time = 0.01, size = 29, normalized size = 1.04

$$\frac{4\sqrt{3}\,\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{3} + \ln\left(x-1\right) + \ln\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2*x-2)/(x^3-1),x)
[Out] ln(x-1)+ln(x^2+x+1)+4/3*arctan(1/3*(2*x+1)*3^(1/2))*3^(1/2)
```

maxima [**A**] time = 1.17, size = 28, normalized size = 1.00

$$\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\log\left(x^2+x+1\right)+\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2*x-2)/(x^3-1),x, algorithm="maxima")
```

```
[Out] 4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + log(x^2 + x + 1) + log(x - 1)
```

mupad [**B**] time = 0.19, size = 57, normalized size = 2.04

$$\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}\right)+\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)+\ln(x-1)-\frac{\sqrt{3}\,\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}\right)2i}{3}+\frac{\sqrt{3}\,\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)2i}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 3*x^2 - 2)/(x^3 - 1),x)
```

```
[Out] log(x - (3^(1/2)*1i)/2 + 1/2) + log(x + (3^(1/2)*1i)/2 + 1/2) + log(x - 1)
- (3^(1/2)*log(x - (3^(1/2)*1i)/2 + 1/2)*2i)/3 + (3^(1/2)*log(x + (3^(1/2)*
1i)/2 + 1/2)*2i)/3
```

sympy [**A**] time = 0.13, size = 3, normalized size = 0.11

$$\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2*x-2)/(x**3-1),x)
```

```
[Out] log(x - 1)
```

3.112 $\int \frac{2-x+2x^2-x^3+x^4}{(-1+x)\left(2+x^2\right)^2} \, dx$

Optimal. Leaf size=49

$$\frac{1}{2\left(x^2+2\right)}+\frac{1}{3}\log\left(x^2+2\right)+\frac{1}{3}\log(1-x)-\frac{\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{3\sqrt{2}}$$

```
[Out] 1/2/(x^2+2)+1/3*ln(1-x)+1/3*ln(x^2+2)-1/6*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1647, 1629, 635, 203, 260}

$$\frac{1}{2\left(x^2+2\right)}+\frac{1}{3}\log\left(x^2+2\right)+\frac{1}{3}\log(1-x)-\frac{\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - x + 2*x^2 - x^3 + x^4)/((-1 + x)*(2 + x^2)^2),x]
```

```
[Out] 1/(2*(2 + x^2)) - ArcTan[x/Sqrt[2]]/(3*Sqrt[2]) + Log[1 - x]/3 + Log[2 + x^
2]/3
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{2-x+2x^2-x^3+x^4}{(-1+x)\left(2+x^2\right)^2}\,dx &= \frac{1}{2\left(2+x^2\right)}-\frac{1}{4}\int \frac{-4+4x-4x^2}{(-1+x)\left(2+x^2\right)}\,dx\\
&= \frac{1}{2\left(2+x^2\right)}-\frac{1}{4}\int \left(-\frac{4}{3(-1+x)}-\frac{4(-1+2x)}{3\left(2+x^2\right)}\right)dx\\
&= \frac{1}{2\left(2+x^2\right)}+\frac{1}{3}\log(1-x)+\frac{1}{3}\int \frac{-1+2x}{2+x^2}\,dx\\
&= \frac{1}{2\left(2+x^2\right)}+\frac{1}{3}\log(1-x)-\frac{1}{3}\int \frac{1}{2+x^2}\,dx+\frac{2}{3}\int \frac{x}{2+x^2}\,dx\\
&= \frac{1}{2\left(2+x^2\right)}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{3\sqrt{2}}+\frac{1}{3}\log(1-x)+\frac{1}{3}\log\left(2+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 61, normalized size = 1.24

$$\frac{1}{2\left((x-1)^2+2(x-1)+3\right)}+\frac{1}{3}\log\left((x-1)^2+2(x-1)+3\right)+\frac{1}{3}\log(x-1)-\frac{\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - x + 2*x^2 - x^3 + x^4)/((-1 + x)*(2 + x^2)^2),x]
```

```
[Out] 1/(2*(3 + 2*(-1 + x) + (-1 + x)^2)) - ArcTan[x/Sqrt[2]]/(3*Sqrt[2]) + Log[3
 + 2*(-1 + x) + (-1 + x)^2]/3 + Log[-1 + x]/3
```

fricas [A] time = 0.40, size = 51, normalized size = 1.04

$$-\frac{\sqrt{2}\left(x^2+2\right)\arctan\left(\frac{1}{2}\sqrt{2}\,x\right)-2\left(x^2+2\right)\log\left(x^2+2\right)-2\left(x^2+2\right)\log\left(x-1\right)-3}{6\left(x^2+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^3+2*x^2-x+2)/(-1+x)/(x^2+2)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(sqrt(2)*(x^2 + 2)*arctan(1/2*sqrt(2)*x) - 2*(x^2 + 2)*log(x^2 + 2) -
2*(x^2 + 2)*log(x - 1) - 3)/(x^2 + 2)
```

giac [A] time = 0.01, size = 37, normalized size = 0.76

$$-\frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\,x\right)+\frac{1}{2\left(x^2+2\right)}+\frac{1}{3}\log\left(x^2+2\right)+\frac{1}{3}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^3+2*x^2-x+2)/(-1+x)/(x^2+2)^2,x, algorithm="giac")
```

```
[Out] -1/6*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/2/(x^2 + 2) + 1/3*log(x^2 + 2) + 1/3
*log(abs(x - 1))
```

maple [A] time = 0.01, size = 37, normalized size = 0.76

$$-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\,x}{2}\right)}{6}+\frac{\ln\left(x-1\right)}{3}+\frac{\ln\left(x^2+2\right)}{3}+\frac{1}{2x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-x^3+2*x^2-x+2)/(x-1)/(x^2+2)^2,x)
```

```
[Out] 1/2/(x^2+2)+1/3*ln(x^2+2)-1/6*arctan(1/2*x*2^(1/2))*2^(1/2)+1/3*ln(x-1)
```

maxima [A] time = 1.30, size = 36, normalized size = 0.73

$$-\frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+\frac{1}{2\left(x^2+2\right)}+\frac{1}{3}\log\left(x^2+2\right)+\frac{1}{3}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^3+2*x^2-x+2)/(-1+x)/(x^2+2)^2,x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/2/(x^2 + 2) + 1/3*log(x^2 + 2) + 1/3
*log(x - 1)
```

mupad [B] time = 0.10, size = 53, normalized size = 1.08

$$\frac{\ln\left(x-1\right)}{3}+\ln\left(x-\sqrt{2}\,1\mathrm{i}\right)\left(\frac{1}{3}+\frac{\sqrt{2}\,1\mathrm{i}}{12}\right)-\ln\left(x+\sqrt{2}\,1\mathrm{i}\right)\left(-\frac{1}{3}+\frac{\sqrt{2}\,1\mathrm{i}}{12}\right)+\frac{1}{2\left(x^2+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x - x^3 + x^4 + 2)/((x^2 + 2)^2*(x - 1)),x)
```

```
[Out] log(x - 1)/3 + log(x - 2^(1/2)*1i)*((2^(1/2)*1i)/12 + 1/3) - log(x + 2^(1/2
)*1i)*((2^(1/2)*1i)/12 - 1/3) + 1/(2*(x^2 + 2))
```

sympy [A] time = 0.17, size = 14, normalized size = 0.29

$$\frac{\log\left(x-1\right)}{3}+\frac{1}{2x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-x**3+2*x**2-x+2)/(-1+x)/(x**2+2)**2,x)
```

```
[Out] log(x - 1)/3 + 1/(2*x**2 + 4)
```

3.113 $\int \frac{1}{\cos(x)+\sin(x)}\,dx$

Optimal. Leaf size=21

$$-\frac{\tanh^{-1}\left(\frac{\cos(x)-\sin(x)}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] -1/2*arctanh(1/2*(cos(x)-sin(x))*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3074, 206}

$$-\frac{\tanh^{-1}\left(\frac{\cos(x)-\sin(x)}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x] + Sin[x])^(-1),x]
```

```
[Out] -(ArcTanh[(Cos[x] - Sin[x])/Sqrt[2]]/Sqrt[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\cos(x)+\sin(x)}\,dx &= -\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \cos(x)-\sin(x)\right)\\ &= -\frac{\tanh^{-1}\left(\frac{\cos(x)-\sin(x)}{\sqrt{2}}\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [C] time = 0.02, size = 24, normalized size = 1.14

$$(-1-i)(-1)^{3/4}\tanh^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)-1}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x] + Sin[x])^(-1),x]
```

```
[Out] (-1 - I)*(-1)^(3/4)*ArcTanh[(-1 + Tan[x/2])/Sqrt[2]]
```

fricas [B] time = 0.41, size = 38, normalized size = 1.81

$$\frac{1}{4}\sqrt{2}\log\left(\frac{2\left(\sqrt{2}-\cos(x)\right)\sin(x)-2\sqrt{2}\cos(x)+3}{2\cos(x)\sin(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)+sin(x)),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log((2*(sqrt(2) - cos(x))*sin(x) - 2*sqrt(2)*cos(x) + 3)/(2*cos
(x)*sin(x) + 1))
```

giac [B] time = 0.03, size = 37, normalized size = 1.76

$$-\frac{1}{2}\,\sqrt{2}\,\log\left(\frac{\left|-2\,\sqrt{2}+2\,\tan\left(\frac{1}{2}\,x\right)-2\right|}{\left|2\,\sqrt{2}+2\,\tan\left(\frac{1}{2}\,x\right)-2\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)+sin(x)),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*log(abs(-2*sqrt(2) + 2*tan(1/2*x) - 2)/abs(2*sqrt(2) + 2*tan(1
/2*x) - 2))
```

maple [A] time = 0.05, size = 19, normalized size = 0.90

$$\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(2\tan\left(\frac{x}{2}\right)-2\right)\sqrt{2}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)+sin(x)),x)
```

```
[Out] 2^(1/2)*arctanh(1/4*(2*tan(1/2*x)-2)*2^(1/2))
```

maxima [B] time = 1.24, size = 39, normalized size = 1.86

$$-\frac{1}{2}\,\sqrt{2}\,\log\left(-\frac{\sqrt{2}-\frac{\sin(x)}{\cos(x)+1}+1}{\sqrt{2}+\frac{\sin(x)}{\cos(x)+1}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)+sin(x)),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*log(-(sqrt(2) - sin(x)/(cos(x) + 1) + 1)/(sqrt(2) + sin(x)/(co
s(x) + 1) - 1))
```

mupad [B] time = 0.33, size = 21, normalized size = 1.00

$$-\sqrt{2}\,\operatorname{atanh}\left(\frac{\sqrt{2}}{2}-\frac{\sqrt{2}\,\tan\left(\frac{x}{2}\right)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x) + sin(x)),x)
```

```
[Out] -2^(1/2)*atanh(2^(1/2)/2 - (2^(1/2)*tan(x/2))/2)
```

sympy [A] time = 0.51, size = 39, normalized size = 1.86

$$\frac{\sqrt{2}\,\log\left(\tan\left(\frac{x}{2}\right)-1+\sqrt{2}\right)}{2}-\frac{\sqrt{2}\,\log\left(\tan\left(\frac{x}{2}\right)-\sqrt{2}-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)+sin(x)),x)
```

```
[Out] sqrt(2)*log(tan(x/2) - 1 + sqrt(2))/2 - sqrt(2)*log(tan(x/2) - sqrt(2) - 1)
/2
```

3.114 $\int \frac{x}{4-x^2+\sqrt{4-x^2}}\,dx$

Optimal. Leaf size=16

$$-\log\left(\sqrt{4-x^2}+1\right)$$

```
[Out] -ln(1+(-x^2+4)^(1/2))
```

Rubi [A] time = 0.05, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2155, 31}

$$-\log\left(\sqrt{4-x^2}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/(4 - x^2 + Sqrt[4 - x^2]),x]
```

```
[Out] -Log[1 + Sqrt[4 - x^2]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2155

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{4-x^2+\sqrt{4-x^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{4+\sqrt{4-x}-x}\,dx,x,x^2\right)\\ &= -\text{Subst}\left(\int \frac{1}{1+x}\,dx,x,\sqrt{4-x^2}\right)\\ &= -\log\left(1+\sqrt{4-x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 16, normalized size = 1.00

$$-\log\left(\sqrt{4-x^2}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(4 - x^2 + Sqrt[4 - x^2]),x]
```

```
[Out] -Log[1 + Sqrt[4 - x^2]]
```

fricas [B] time = 0.40, size = 55, normalized size = 3.44

$$-\frac{1}{2}\log\left(x^2-3\right)+\frac{1}{2}\log\left(-\frac{x^2+3\sqrt{-x^2+4}-6}{x^2}\right)-\frac{1}{2}\log\left(-\frac{x^2+\sqrt{-x^2+4}-2}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4-x^2+(-x^2+4)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/2*log(x^2 - 3) + 1/2*log(-(x^2 + 3*sqrt(-x^2 + 4) - 6)/x^2) - 1/2*log(-(
x^2 + sqrt(-x^2 + 4) - 2)/x^2)
```

giac [A] time = 0.01, size = 14, normalized size = 0.88

$$-\log\left(\sqrt{-x^2+4}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4-x^2+(-x^2+4)^(1/2)),x, algorithm="giac")
```

```
[Out] -log(sqrt(-x^2 + 4) + 1)
```

maple [B] time = 0.10, size = 266, normalized size = 16.62

$$\frac{\operatorname{arctanh}\left(\frac{2-2\sqrt{3}\,(x-\sqrt{3})}{2\sqrt{-(x-\sqrt{3})^2-2\sqrt{3}\,(x-\sqrt{3})+1}}\right)}{2\left(2+\sqrt{3}\right)\left(-2+\sqrt{3}\right)}+\frac{\operatorname{arctanh}\left(\frac{2+2\sqrt{3}\,(x+\sqrt{3})}{2\sqrt{-(x+\sqrt{3})^2+2\sqrt{3}\,(x+\sqrt{3})+1}}\right)}{2\left(2+\sqrt{3}\right)\left(-2+\sqrt{3}\right)}-\frac{\ln\left(x^2-3\right)}{2}+\frac{\sqrt{-4x-(x-2)^2+}}{2\left(2+\sqrt{3}\right)\left(-2+\sqrt{\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(4-x^2+(-x^2+4)^(1/2)),x)
```

```
[Out] -1/2*ln(x^2-3)+1/2/(2+3^(1/2))/(-2+3^(1/2))*(-(-2+x)^2-4*x+8)^(1/2)+1/2/(2+
3^(1/2))/(-2+3^(1/2))*arctanh(1/2*(2+2*3^(1/2)*(x+3^(1/2)))/(-(x+3^(1/2))^2
+2*3^(1/2)*(x+3^(1/2))+1)^(1/2))-1/2/(2+3^(1/2))/(-2+3^(1/2))*(-(x+3^(1/2))
^2+2*3^(1/2)*(x+3^(1/2))+1)^(1/2)+1/2/(2+3^(1/2))/(-2+3^(1/2))*(-(x+2)^2+4*
x+8)^(1/2)+1/2/(2+3^(1/2))/(-2+3^(1/2))*arctanh(1/2*(2-2*3^(1/2)*(x-3^(1/2)
))/(-(x-3^(1/2))^2-2*3^(1/2)*(x-3^(1/2))+1)^(1/2))-1/2/(2+3^(1/2))/(-2+3^(1
/2))*(-(x-3^(1/2))^2-2*3^(1/2)*(x-3^(1/2))+1)^(1/2)
```

maxima [A] time = 0.62, size = 14, normalized size = 0.88

$$-\log\left(\sqrt{-x^2+4}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4-x^2+(-x^2+4)^(1/2)),x, algorithm="maxima")
```

```
[Out] -log(sqrt(-x^2 + 4) + 1)
```

mupad [B] time = 0.16, size = 87, normalized size = 5.44

$$-\frac{\ln\left(x-\sqrt{3}\right)}{2}-\frac{\ln\left(\frac{\sqrt{3}\,x\,1\mathrm{i}+\sqrt{4-x^2}\,1\mathrm{i}+4\mathrm{i}}{x+\sqrt{3}}\right)}{2}-\frac{\ln\left(x+\sqrt{3}\right)}{2}-\frac{\ln\left(\frac{-\sqrt{3}\,x\,1\mathrm{i}+\sqrt{4-x^2}\,1\mathrm{i}+4\mathrm{i}}{x-\sqrt{3}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((4 - x^2)^(1/2) - x^2 + 4),x)
```

```
[Out] - log(x - 3^(1/2))/2 - log((3^(1/2)*x*1i + (4 - x^2)^(1/2)*1i + 4i)/(x + 3^
(1/2)))/2 - log(x + 3^(1/2))/2 - log(((4 - x^2)^(1/2)*1i - 3^(1/2)*x*1i + 4
i)/(x - 3^(1/2)))/2
```

sympy [B] time = 5.40, size = 44, normalized size = 2.75

$$\frac{\log\left(2\sqrt{4-x^2}\right)}{2}-\frac{\log\left(2\sqrt{4-x^2}+2\right)}{2}-\frac{\log\left(x^2-\sqrt{4-x^2}-4\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4-x**2+(-x**2+4)**(1/2)),x)
```

```
[Out] log(2*sqrt(4 - x**2))/2 - log(2*sqrt(4 - x**2) + 2)/2 - log(x**2 - sqrt(4 -
 x**2) - 4)/2
```

3.115 $\int \frac{3+2x}{(-2+x)(5+x)}\,dx$

Optimal. Leaf size=11

$$\log(2-x)+\log(x+5)$$

```
[Out] ln(2-x)+ln(5+x)
```

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {72}

$$\log(2-x)+\log(x+5)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 2*x)/((-2 + x)*(5 + x)),x]
```

```
[Out] Log[2 - x] + Log[5 + x]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{3+2x}{(-2+x)(5+x)}\,dx &= \int \left(\frac{1}{-2+x}+\frac{1}{5+x}\right)dx\\ &= \log(2-x)+\log(5+x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 9, normalized size = 0.82

$$\log(x-2)+\log(x+5)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 2*x)/((-2 + x)*(5 + x)),x]
```

```
[Out] Log[-2 + x] + Log[5 + x]
```

fricas [A] time = 0.40, size = 9, normalized size = 0.82

$$\log\left(x^2+3\,x-10\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(-2+x)/(5+x),x, algorithm="fricas")
```

```
[Out] log(x^2 + 3*x - 10)
```

giac [A] time = 0.01, size = 11, normalized size = 1.00

$$\log\left(|x+5|\right)+\log\left(|x-2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(-2+x)/(5+x),x, algorithm="giac")
```

```
[Out] log(abs(x + 5)) + log(abs(x - 2))
```

maple [A] time = 0.00, size = 9, normalized size = 0.82

$$\ln((x-2)(x+5))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+3)/(x-2)/(5+x),x)
[Out] ln((x-2)*(5+x))
```

maxima [A] time = 0.57, size = 9, normalized size = 0.82

$$\log(x+5) + \log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(-2+x)/(5+x),x, algorithm="maxima")
[Out] log(x + 5) + log(x - 2)
```

mupad [B] time = 0.05, size = 9, normalized size = 0.82

$$\ln\left(x^2 + 3\,x - 10\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 3)/((x - 2)*(x + 5)),x)
[Out] log(3*x + x^2 - 10)
```

sympy [A] time = 0.09, size = 8, normalized size = 0.73

$$\log\left(x^2 + 3x - 10\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(-2+x)/(5+x),x)
[Out] log(x**2 + 3*x - 10)
```

3.116 $\int \frac{x}{(1+x)(2+x)(3+x)} \, dx$

Optimal. Leaf size=23

$$-\frac{1}{2}\log(x+1)+2\log(x+2)-\frac{3}{2}\log(x+3)$$

```
[Out] -1/2*ln(1+x)+2*ln(2+x)-3/2*ln(3+x)
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {148}

$$-\frac{1}{2}\log(x+1)+2\log(x+2)-\frac{3}{2}\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 + x)*(2 + x)*(3 + x)),x]
[Out] -Log[1 + x]/2 + 2*Log[2 + x] - (3*Log[3 + x])/2
```

Rule 148

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(
c + d*x)^n*(e + f*x)^p*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
m}, x] && (IntegersQ[m, n, p] || (IGtQ[n, 0] && IGtQ[p, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(1+x)(2+x)(3+x)}\,dx &= \int \left(-\frac{1}{2(1+x)}+\frac{2}{2+x}-\frac{3}{2(3+x)}\right)dx\\ &= -\frac{1}{2}\log(1+x)+2\log(2+x)-\frac{3}{2}\log(3+x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$-\frac{1}{2}\log(x+1)+2\log(x+2)-\frac{3}{2}\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 + x)*(2 + x)*(3 + x)),x]
[Out] -1/2*Log[1 + x] + 2*Log[2 + x] - (3*Log[3 + x])/2
```

fricas [A] time = 0.39, size = 19, normalized size = 0.83

$$-\frac{3}{2}\,\log\left(x+3\right)+2\,\log\left(x+2\right)-\frac{1}{2}\,\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(2+x)/(3+x),x, algorithm="fricas")
[Out] -3/2*log(x + 3) + 2*log(x + 2) - 1/2*log(x + 1)
```

giac [A] time = 0.01, size = 22, normalized size = 0.96

$$-\frac{3}{2}\,\log\left(|x+3|\right)+2\,\log\left(|x+2|\right)-\frac{1}{2}\,\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(2+x)/(3+x),x, algorithm="giac")
```

```
[Out] -3/2*log(abs(x + 3)) + 2*log(abs(x + 2)) - 1/2*log(abs(x + 1))
```

maple [A] time = 0.01, size = 20, normalized size = 0.87

$$-\frac{\ln\left(x+1\right)}{2}+2\ln\left(x+2\right)-\frac{3\ln\left(x+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x+1)/(x+2)/(x+3),x)
```

```
[Out] -1/2*ln(x+1)+2*ln(x+2)-3/2*ln(x+3)
```

maxima [A] time = 0.47, size = 19, normalized size = 0.83

$$-\frac{3}{2}\,\log\left(x+3\right)+2\,\log\left(x+2\right)-\frac{1}{2}\,\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(2+x)/(3+x),x, algorithm="maxima")
```

```
[Out] -3/2*log(x + 3) + 2*log(x + 2) - 1/2*log(x + 1)
```

mupad [B] time = 0.11, size = 19, normalized size = 0.83

$$2\,\ln\left(x+2\right)-\frac{\ln\left(x+1\right)}{2}-\frac{3\,\ln\left(x+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x + 1)*(x + 2)*(x + 3)),x)
```

```
[Out] 2*log(x + 2) - log(x + 1)/2 - (3*log(x + 3))/2
```

sympy [A] time = 0.13, size = 20, normalized size = 0.87

$$-\frac{\log\left(x+1\right)}{2}+2\log\left(x+2\right)-\frac{3\log\left(x+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(2+x)/(3+x),x)
```

```
[Out] -log(x + 1)/2 + 2*log(x + 2) - 3*log(x + 3)/2
```

3.117 $\int \frac{x}{2-3x+x^3} dx$

Optimal. Leaf size=30

$$\frac{1}{3(1-x)} + \frac{2}{9}\log(1-x) - \frac{2}{9}\log(x+2)$$

```
[Out] 1/3/(1-x)+2/9*ln(1-x)-2/9*ln(2+x)
```

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2074}

$$\frac{1}{3(1-x)} + \frac{2}{9}\log(1-x) - \frac{2}{9}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[x/(2 - 3*x + x^3),x]
```

```
[Out] 1/(3*(1 - x)) + (2*Log[1 - x])/9 - (2*Log[2 + x])/9
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned} \int \frac{x}{2-3x+x^3} dx &= \int \left(\frac{1}{3(-1+x)^2} + \frac{2}{9(-1+x)} - \frac{2}{9(2+x)}\right) dx \\ &= \frac{1}{3(1-x)} + \frac{2}{9}\log(1-x) - \frac{2}{9}\log(2+x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.93

$$-\frac{1}{3(x-1)} + \frac{2}{9}\log(1-x) - \frac{2}{9}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(2 - 3*x + x^3),x]
```

```
[Out] -1/3*1/(-1 + x) + (2*Log[1 - x])/9 - (2*Log[2 + x])/9
```

fricas [A] time = 0.39, size = 27, normalized size = 0.90

$$-\frac{2(x-1)\log(x+2) - 2(x-1)\log(x-1) + 3}{9(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3-3*x+2),x, algorithm="fricas")
```

```
[Out] -1/9*(2*(x - 1)*log(x + 2) - 2*(x - 1)*log(x - 1) + 3)/(x - 1)
```

giac [A] time = 0.01, size = 22, normalized size = 0.73

$$-\frac{1}{3(x-1)} - \frac{2}{9}\log(|x+2|) + \frac{2}{9}\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3-3*x+2),x, algorithm="giac")
[Out] -1/3/(x - 1) - 2/9*log(abs(x + 2)) + 2/9*log(abs(x - 1))
```

maple [A] time = 0.01, size = 21, normalized size = 0.70

$$\frac{2\ln\left(x-1\right)}{9}-\frac{2\ln\left(x+2\right)}{9}-\frac{1}{3\left(x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^3-3*x+2),x)
[Out] -2/9*ln(x+2)-1/3/(x-1)+2/9*ln(x-1)
```

maxima [A] time = 0.61, size = 20, normalized size = 0.67

$$-\frac{1}{3\left(x-1\right)}-\frac{2}{9}\log\left(x+2\right)+\frac{2}{9}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3-3*x+2),x, algorithm="maxima")
[Out] -1/3/(x - 1) - 2/9*log(x + 2) + 2/9*log(x - 1)
```

mupad [B] time = 0.04, size = 18, normalized size = 0.60

$$-\frac{4\,\mathrm{atanh}\left(\frac{2\,x}{3}+\frac{1}{3}\right)}{9}-\frac{1}{3\,\left(x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^3 - 3*x + 2),x)
[Out] - (4*atanh((2*x)/3 + 1/3))/9 - 1/(3*(x - 1))
```

sympy [A] time = 0.10, size = 22, normalized size = 0.73

$$\frac{2\log\left(x-1\right)}{9}-\frac{2\log\left(x+2\right)}{9}-\frac{1}{3x-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**3-3*x+2),x)
[Out] 2*log(x - 1)/9 - 2*log(x + 2)/9 - 1/(3*x - 3)
```

3.118 $\int \frac{-6+2x+x^4}{-2x+x^2+x^3}\,dx$

Optimal. Leaf size=27

$$\frac{x^2}{2}-x-\log(1-x)+3\log(x)+\log(x+2)$$

```
[Out] -x+1/2*x^2-ln(1-x)+3*ln(x)+ln(2+x)
```

Rubi [A] time = 0.03, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1594, 1628}

$$\frac{x^2}{2}-x-\log(1-x)+3\log(x)+\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(-6 + 2*x + x^4)/(-2*x + x^2 + x^3),x]
```

```
[Out] -x + x^2/2 - Log[1 - x] + 3*Log[x] + Log[2 + x]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{-6+2x+x^4}{-2x+x^2+x^3}\,dx &= \int \frac{-6+2x+x^4}{x\left(-2+x+x^2\right)}\,dx\\ &= \int \left(-1+\frac{1}{1-x}+\frac{3}{x}+x+\frac{1}{2+x}\right)dx\\ &= -x+\frac{x^2}{2}-\log(1-x)+3\log(x)+\log(2+x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$\frac{x^2}{2}-x-\log(1-x)+3\log(x)+\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-6 + 2*x + x^4)/(-2*x + x^2 + x^3),x]
```

```
[Out] -x + x^2/2 - Log[1 - x] + 3*Log[x] + Log[2 + x]
```

fricas [A] time = 0.38, size = 23, normalized size = 0.85

$$\frac{1}{2}x^2-x+\log(x+2)-\log(x-1)+3\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+2*x-6)/(x^3+x^2-2*x),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 - x + log(x + 2) - log(x - 1) + 3*log(x)
```

giac [A] time = 0.01, size = 26, normalized size = 0.96

$$\frac{1}{2}\,x^2 - x + \log\left(|x+2|\right) - \log\left(|x-1|\right) + 3\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+2*x-6)/(x^3+x^2-2*x),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - x + log(abs(x + 2)) - log(abs(x - 1)) + 3*log(abs(x))
```

maple [A] time = 0.01, size = 24, normalized size = 0.89

$$\frac{x^2}{2} - x + 3\ln(x) - \ln\left(x-1\right) + \ln\left(x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+2*x-6)/(x^3+x^2-2*x),x)
```

```
[Out] 1/2*x^2-x+ln(x+2)-ln(x-1)+3*ln(x)
```

maxima [A] time = 0.50, size = 23, normalized size = 0.85

$$\frac{1}{2}\,x^2 - x + \log\left(x+2\right) - \log\left(x-1\right) + 3\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+2*x-6)/(x^3+x^2-2*x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - x + log(x + 2) - log(x - 1) + 3*log(x)
```

mupad [B] time = 0.10, size = 30, normalized size = 1.11

$$3\,\ln(x) - x + \frac{x^2}{2} + \mathrm{atan}\left(\frac{192\mathrm{i}}{7\,\left(28\,x-40\right)} + \frac{9}{7}\mathrm{i}\right)\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + x^4 - 6)/(x^2 - 2*x + x^3),x)
```

```
[Out] atan(192i/(7*(28*x - 40)) + 9i/7)*2i - x + 3*log(x) + x^2/2
```

sympy [A] time = 0.13, size = 20, normalized size = 0.74

$$\frac{x^2}{2} - x + 3\log\left(x\right) - \log\left(x-1\right) + \log\left(x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+2*x-6)/(x**3+x**2-2*x),x)
```

```
[Out] x**2/2 - x + 3*log(x) - log(x - 1) + log(x + 2)
```

3.119 $\int \frac{7+8x^3}{(1+x)(1+2x)^3}\,dx$

Optimal. Leaf size=23

$$\frac{3}{2x+1}-\frac{3}{(2x+1)^2}+\log(x+1)$$

```
[Out] -3/(1+2*x)^2+3/(1+2*x)+ln(1+x)
```

Rubi [A] time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1620}

$$\frac{3}{2x+1}-\frac{3}{(2x+1)^2}+\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 8*x^3)/((1 + x)*(1 + 2*x)^3),x]
```

```
[Out] -3/(1 + 2*x)^2 + 3/(1 + 2*x) + Log[1 + x]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned}\int \frac{7+8x^3}{(1+x)(1+2x)^3}\,dx &= \int \left(\frac{1}{1+x}+\frac{12}{(1+2x)^3}-\frac{6}{(1+2x)^2}\right)dx\\ &= -\frac{3}{(1+2x)^2}+\frac{3}{1+2x}+\log(1+x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.04

$$\frac{6x+(2x+1)^2\log(x+1)}{(2x+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 8*x^3)/((1 + x)*(1 + 2*x)^3),x]
```

```
[Out] (6*x + (1 + 2*x)^2*Log[1 + x])/(1 + 2*x)^2
```

fricas [A] time = 0.39, size = 32, normalized size = 1.39

$$\frac{\left(4\,x^2+4\,x+1\right)\log\left(x+1\right)+6\,x}{4\,x^2+4\,x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^3+7)/(1+x)/(1+2*x)^3,x, algorithm="fricas")
```

```
[Out] ((4*x^2 + 4*x + 1)*log(x + 1) + 6*x)/(4*x^2 + 4*x + 1)
```

giac [A] time = 0.01, size = 16, normalized size = 0.70

$$\frac{6\,x}{\left(2\,x+1\right)^{2}}+\log\left(\left|x+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^3+7)/(1+x)/(1+2*x)^3,x, algorithm="giac")
[Out] 6*x/(2*x + 1)^2 + log(abs(x + 1))
```

maple [A] time = 0.01, size = 24, normalized size = 1.04

$$\ln\left(x+1\right)-\frac{3}{\left(2x+1\right)^{2}}+\frac{3}{2x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x^3+7)/(x+1)/(2*x+1)^3,x)
[Out] -3/(2*x+1)^2+3/(2*x+1)+ln(x+1)
```

maxima [A] time = 0.60, size = 20, normalized size = 0.87

$$\frac{6\,x}{4\,x^{2}+4\,x+1}+\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^3+7)/(1+x)/(1+2*x)^3,x, algorithm="maxima")
[Out] 6*x/(4*x^2 + 4*x + 1) + log(x + 1)
```

mupad [B] time = 0.09, size = 15, normalized size = 0.65

$$\ln\left(x+1\right)+\frac{6\,x}{\left(2\,x+1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x^3 + 7)/((2*x + 1)^3*(x + 1)),x)
[Out] log(x + 1) + (6*x)/(2*x + 1)^2
```

sympy [A] time = 0.11, size = 17, normalized size = 0.74

$$\frac{6x}{4x^{2}+4x+1}+\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x**3+7)/(1+x)/(1+2*x)**3,x)
[Out] 6*x/(4*x**2 + 4*x + 1) + log(x + 1)
```

3.120 $\int \frac{1+x+4x^2}{-1+x^3} \, dx$

Optimal. Leaf size=16

$$\log\left(x^2+x+1\right)+2\log(1-x)$$

```
[Out] 2*ln(1-x)+ln(x^2+x+1)
```

Rubi [A] time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1875, 31, 628}

$$\log\left(x^2+x+1\right)+2\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + 4*x^2)/(-1 + x^3),x]
```

```
[Out] 2*Log[1 - x] + Log[1 + x + x^2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1875

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (-(a/b))^(1/3)}, Dist[(q*(A + B
*q + C*q^2))/(3*a), Int[1/(q - x), x], x] + Dist[q/(3*a), Int[(q*(2*A - B*q
 - C*q^2) + (A + B*q - 2*C*q^2)*x)/(q^2 + q*x + x^2), x], x] /; NeQ[a*B^3 -
 b*A^3, 0] && NeQ[A + B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x,
2] && LtQ[a/b, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x+4x^2}{-1+x^3} \, dx &= -\left(\frac{1}{3}\int \frac{-3-6x}{1+x+x^2} \, dx\right)-2\int \frac{1}{1-x} \, dx\\ &= 2\log(1-x)+\log\left(1+x+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\log\left(x^2+x+1\right)+2\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + 4*x^2)/(-1 + x^3),x]
```

```
[Out] 2*Log[1 - x] + Log[1 + x + x^2]
```

fricas [A] time = 0.38, size = 14, normalized size = 0.88

$$\log\left(x^2+x+1\right)+2\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+x+1)/(x^3-1),x, algorithm="fricas")
[Out] log(x^2 + x + 1) + 2*log(x - 1)
```

giac [A] time = 0.01, size = 15, normalized size = 0.94

$$\log\left(x^2 + x + 1\right) + 2 \log\left(|x - 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+x+1)/(x^3-1),x, algorithm="giac")
[Out] log(x^2 + x + 1) + 2*log(abs(x - 1))
```

maple [A] time = 0.01, size = 15, normalized size = 0.94

$$2 \ln(x - 1) + \ln\left(x^2 + x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+x+1)/(x^3-1),x)
[Out] 2*ln(x-1)+ln(x^2+x+1)
```

maxima [A] time = 1.12, size = 14, normalized size = 0.88

$$\log\left(x^2 + x + 1\right) + 2 \log\left(x - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+x+1)/(x^3-1),x, algorithm="maxima")
[Out] log(x^2 + x + 1) + 2*log(x - 1)
```

mupad [B] time = 0.04, size = 14, normalized size = 0.88

$$\ln\left(x^2 + x + 1\right) + 2 \ln\left(x - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 4*x^2 + 1)/(x^3 - 1),x)
[Out] log(x + x^2 + 1) + 2*log(x - 1)
```

sympy [A] time = 0.10, size = 14, normalized size = 0.88

$$2 \log(x - 1) + \log\left(x^2 + x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+x+1)/(x**3-1),x)
[Out] 2*log(x - 1) + log(x**2 + x + 1)
```

3.121 $\int \frac{x^4}{4+5x^2+x^4} dx$

Optimal. Leaf size=18

$$x - \frac{8}{3}\tan^{-1}\left(\frac{x}{2}\right) + \frac{1}{3}\tan^{-1}(x)$$

```
[Out] x-8/3*arctan(1/2*x)+1/3*arctan(x)
```

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1122, 1166, 203}

$$x - \frac{8}{3}\tan^{-1}\left(\frac{x}{2}\right) + \frac{1}{3}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(4 + 5*x^2 + x^4),x]
```

```
[Out] x - (8*ArcTan[x/2])/3 + ArcTan[x]/3
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{4+5x^2+x^4}\,dx &= x - \int \frac{4+5x^2}{4+5x^2+x^4}\,dx\\ &= x + \frac{1}{3}\int \frac{1}{1+x^2}\,dx - \frac{16}{3}\int \frac{1}{4+x^2}\,dx\\ &= x - \frac{8}{3}\tan^{-1}\left(\frac{x}{2}\right) + \frac{1}{3}\tan^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$x + \frac{8}{3}\tan^{-1}\left(\frac{2}{x}\right) + \frac{1}{3}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(4 + 5*x^2 + x^4),x]
```

```
[Out] x + (8*ArcTan[2/x])/3 + ArcTan[x]/3
```

fricas [A] time = 0.39, size = 12, normalized size = 0.67

$$x - \frac{8}{3}\arctan\left(\frac{1}{2}x\right) + \frac{1}{3}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^4+5*x^2+4),x, algorithm="fricas")
```

```
[Out] x - 8/3*arctan(1/2*x) + 1/3*arctan(x)
```

giac [A] time = 0.01, size = 12, normalized size = 0.67

$$x - \frac{8}{3}\arctan\left(\frac{1}{2}x\right) + \frac{1}{3}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^4+5*x^2+4),x, algorithm="giac")
```

```
[Out] x - 8/3*arctan(1/2*x) + 1/3*arctan(x)
```

maple [A] time = 0.01, size = 13, normalized size = 0.72

$$x + \frac{\arctan(x)}{3} - \frac{8\arctan\left(\frac{x}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^4+5*x^2+4),x)
```

```
[Out] x-8/3*arctan(1/2*x)+1/3*arctan(x)
```

maxima [A] time = 1.14, size = 12, normalized size = 0.67

$$x - \frac{8}{3}\arctan\left(\frac{1}{2}x\right) + \frac{1}{3}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^4+5*x^2+4),x, algorithm="maxima")
```

```
[Out] x - 8/3*arctan(1/2*x) + 1/3*arctan(x)
```

mupad [B] time = 0.04, size = 12, normalized size = 0.67

$$x - \frac{8\,\mathrm{atan}\left(\frac{x}{2}\right)}{3} + \frac{\mathrm{atan}(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(5*x^2 + x^4 + 4),x)
```

```
[Out] x - (8*atan(x/2))/3 + atan(x)/3
```

sympy [A] time = 0.15, size = 14, normalized size = 0.78

$$x - \frac{8\,\mathrm{atan}\left(\frac{x}{2}\right)}{3} + \frac{\mathrm{atan}\,(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**4+5*x**2+4),x)
```

```
[Out] x - 8*atan(x/2)/3 + atan(x)/3
```

3.122 $\int \frac{2+x}{x+x^2} \, dx$

Optimal. Leaf size=11

$$2\log(x) - \log(x+1)$$

```
[Out] 2*ln(x)-ln(1+x)
```

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {631}

$$2\log(x) - \log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x)/(x + x^2),x]
```

```
[Out] 2*Log[x] - Log[1 + x]
```

Rule 631

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbo
l] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || Eq
Q[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{2+x}{x+x^2} \, dx &= \int \left(\frac{1}{-1-x} + \frac{2}{x}\right) dx \\ &= 2\log(x) - \log(1+x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$2\log(x) - \log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x)/(x + x^2),x]
```

```
[Out] 2*Log[x] - Log[1 + x]
```

fricas [A] time = 0.39, size = 11, normalized size = 1.00

$$-\log(x+1) + 2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2+x),x, algorithm="fricas")
```

```
[Out] -log(x + 1) + 2*log(x)
```

giac [A] time = 0.01, size = 13, normalized size = 1.18

$$-\log(|x+1|) + 2\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2+x),x, algorithm="giac")
```

```
[Out] -log(abs(x + 1)) + 2*log(abs(x))
```

maple [**A**] time = 0.01, size = 12, normalized size = 1.09

$$2\ln(x) - \ln(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+2)/(x^2+x),x)
[Out] 2*ln(x)-ln(x+1)
```

maxima [**A**] time = 0.49, size = 11, normalized size = 1.00

$$-\log(x+1) + 2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2+x),x, algorithm="maxima")
[Out] -log(x + 1) + 2*log(x)
```

mupad [**B**] time = 0.10, size = 11, normalized size = 1.00

$$2\ln(x) - \ln(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 2)/(x + x^2),x)
[Out] 2*log(x) - log(x + 1)
```

sympy [**A**] time = 0.10, size = 8, normalized size = 0.73

$$2\log(x) - \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x**2+x),x)
[Out] 2*log(x) - log(x + 1)
```

3.123 $\int \frac{1}{x(1+x^2)^2} dx$

Optimal. Leaf size=24

$$\frac{1}{2(x^2+1)} - \frac{1}{2}\log(x^2+1) + \log(x)$$

```
[Out] 1/2/(x^2+1)+ln(x)-1/2*ln(x^2+1)
```

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {266, 44}

$$\frac{1}{2(x^2+1)} - \frac{1}{2}\log(x^2+1) + \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + x^2)^2),x]
```

```
[Out] 1/(2*(1 + x^2)) + Log[x] - Log[1 + x^2]/2
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(1+x^2)^2} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(1+x)^2} dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{-1-x} + \frac{1}{x} - \frac{1}{(1+x)^2}\right) dx, x, x^2\right) \\
&= \frac{1}{2(1+x^2)} + \log(x) - \frac{1}{2}\log(1+x^2)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.00

$$\frac{1}{2(x^2+1)} - \frac{1}{2}\log(x^2+1) + \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + x^2)^2),x]
```

```
[Out] 1/(2*(1 + x^2)) + Log[x] - Log[1 + x^2]/2
```

fricas [A] time = 0.39, size = 32, normalized size = 1.33

$$-\frac{\left(x^2+1\right)\log\left(x^2+1\right)-2\left(x^2+1\right)\log(x)-1}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2+1)^2,x, algorithm="fricas")
[Out] -1/2*((x^2 + 1)*log(x^2 + 1) - 2*(x^2 + 1)*log(x) - 1)/(x^2 + 1)
```

giac [A] time = 0.01, size = 29, normalized size = 1.21

$$\frac{x^2+2}{2\left(x^2+1\right)}-\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{2}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2+1)^2,x, algorithm="giac")
[Out] 1/2*(x^2 + 2)/(x^2 + 1) - 1/2*log(x^2 + 1) + 1/2*log(x^2)
```

maple [A] time = 0.01, size = 21, normalized size = 0.88

$$\ln(x)-\frac{\ln\left(x^2+1\right)}{2}+\frac{1}{2x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^2+1)^2,x)
[Out] 1/2/(x^2+1)+ln(x)-1/2*ln(x^2+1)
```

maxima [A] time = 0.56, size = 24, normalized size = 1.00

$$\frac{1}{2\left(x^2+1\right)}-\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{2}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2+1)^2,x, algorithm="maxima")
[Out] 1/2/(x^2 + 1) - 1/2*log(x^2 + 1) + 1/2*log(x^2)
```

mupad [B] time = 0.03, size = 20, normalized size = 0.83

$$\ln(x)-\frac{\ln\left(x^2+1\right)}{2}+\frac{1}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^2 + 1)^2),x)
[Out] log(x) - log(x^2 + 1)/2 + 1/(2*(x^2 + 1))
```

sympy [A] time = 0.11, size = 19, normalized size = 0.79

$$\log(x)-\frac{\log\left(x^2+1\right)}{2}+\frac{1}{2x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**2+1)**2,x)
[Out] log(x) - log(x**2 + 1)/2 + 1/(2*x**2 + 2)
```

3.124 $\int \frac{1}{(1+x)(2+x)^2(3+x)^3}\,dx$

Optimal. Leaf size=46

$$\frac{1}{x+2}+\frac{5}{4(x+3)}+\frac{1}{4(x+3)^2}+\frac{1}{8}\log(x+1)+2\log(x+2)-\frac{17}{8}\log(x+3)$$

```
[Out] 1/(2+x)+1/4/(3+x)^2+5/4/(3+x)+1/8*ln(1+x)+2*ln(2+x)-17/8*ln(3+x)
```

Rubi [A] time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {88}

$$\frac{1}{x+2}+\frac{5}{4(x+3)}+\frac{1}{4(x+3)^2}+\frac{1}{8}\log(x+1)+2\log(x+2)-\frac{17}{8}\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x)*(2 + x)^2*(3 + x)^3),x]
```

```
[Out] (2 + x)^(-1) + 1/(4*(3 + x)^2) + 5/(4*(3 + x)) + Log[1 + x]/8 + 2*Log[2 + x
] - (17*Log[3 + x])/8
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1+x)(2+x)^2(3+x)^3}\,dx &= \int \left(\frac{1}{8(1+x)}-\frac{1}{(2+x)^2}+\frac{2}{2+x}-\frac{1}{2(3+x)^3}-\frac{5}{4(3+x)^2}-\frac{17}{8(3+x)}\right)dx\\ &= \frac{1}{2+x}+\frac{1}{4(3+x)^2}+\frac{5}{4(3+x)}+\frac{1}{8}\log(1+x)+2\log(2+x)-\frac{17}{8}\log(3+x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 44, normalized size = 0.96

$$\frac{1}{8}\left(\frac{8}{x+2}+\frac{10}{x+3}+\frac{2}{(x+3)^2}+\log(-x-1)+16\log(x+2)-17\log(x+3)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x)*(2 + x)^2*(3 + x)^3),x]
```

```
[Out] (8/(2 + x) + 2/(3 + x)^2 + 10/(3 + x) + Log[-1 - x] + 16*Log[2 + x] - 17*Lo
g[3 + x])/8
```

fricas [B] time = 0.40, size = 83, normalized size = 1.80

$$\frac{18\,x^2-17\left(x^3+8\,x^2+21\,x+18\right)\log\left(x+3\right)+16\left(x^3+8\,x^2+21\,x+18\right)\log\left(x+2\right)+\left(x^3+8\,x^2+21\,x+18\right)\mathrm{lo}}{8\left(x^3+8\,x^2+21\,x+18\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(2+x)^2/(3+x)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(18*x^2 - 17*(x^3 + 8*x^2 + 21*x + 18)*log(x + 3) + 16*(x^3 + 8*x^2 + 2
1*x + 18)*log(x + 2) + (x^3 + 8*x^2 + 21*x + 18)*log(x + 1) + 100*x + 136)/
(x^3 + 8*x^2 + 21*x + 18)
```

giac [A] time = 0.01, size = 52, normalized size = 1.13

$$\frac{1}{x+2} - \frac{\frac{7}{x+2}+6}{4\left(\frac{1}{x+2}+1\right)^2} + \frac{1}{8}\log\left(\left|-\frac{1}{x+2}+1\right|\right) - \frac{17}{8}\log\left(\left|-\frac{1}{x+2}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(2+x)^2/(3+x)^3,x, algorithm="giac")
```

```
[Out] 1/(x + 2) - 1/4*(7/(x + 2) + 6)/(1/(x + 2) + 1)^2 + 1/8*log(abs(-1/(x + 2)
+ 1)) - 17/8*log(abs(-1/(x + 2) - 1))
```

maple [A] time = 0.01, size = 39, normalized size = 0.85

$$\frac{\ln(x+1)}{8} + 2\ln(x+2) - \frac{17\ln(x+3)}{8} + \frac{1}{x+2} + \frac{1}{4(x+3)^2} + \frac{5}{4(x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+1)/(x+2)^2/(x+3)^3,x)
```

```
[Out] 1/(x+2)+1/4/(x+3)^2+5/4/(x+3)+1/8*ln(x+1)+2*ln(x+2)-17/8*ln(x+3)
```

maxima [A] time = 0.52, size = 46, normalized size = 1.00

$$\frac{9x^2+50x+68}{4\left(x^3+8x^2+21x+18\right)} - \frac{17}{8}\log(x+3) + 2\log(x+2) + \frac{1}{8}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(2+x)^2/(3+x)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(9*x^2 + 50*x + 68)/(x^3 + 8*x^2 + 21*x + 18) - 17/8*log(x + 3) + 2*log
(x + 2) + 1/8*log(x + 1)
```

mupad [B] time = 0.09, size = 45, normalized size = 0.98

$$\frac{\ln(x+1)}{8} + 2\ln(x+2) - \frac{17\ln(x+3)}{8} + \frac{\frac{9x^2}{4}+\frac{25x}{2}+17}{x^3+8x^2+21x+18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x + 1)*(x + 2)^2*(x + 3)^3),x)
```

```
[Out] log(x + 1)/8 + 2*log(x + 2) - (17*log(x + 3))/8 + ((25*x)/2 + (9*x^2)/4 + 1
7)/(21*x + 8*x^2 + x^3 + 18)
```

sympy [A] time = 0.20, size = 46, normalized size = 1.00

$$\frac{9x^2+50x+68}{4x^3+32x^2+84x+72} + \frac{\log(x+1)}{8} + 2\log(x+2) - \frac{17\log(x+3)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(2+x)**2/(3+x)**3,x)
```

```
[Out] (9*x**2 + 50*x + 68)/(4*x**3 + 32*x**2 + 84*x + 72) + log(x + 1)/8 + 2*log(
x + 2) - 17*log(x + 3)/8
```

3.125 $\int \frac{x}{(1+x)^2} \, dx$

Optimal. Leaf size=10

$$\frac{1}{x+1} + \log(x+1)$$

```
[Out] 1/(1+x)+ln(1+x)
```

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {43}

$$\frac{1}{x+1} + \log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 + x)^2,x]
```

```
[Out] (1 + x)^(-1) + Log[1 + x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{x}{(1+x)^2} \, dx &= \int \left(-\frac{1}{(1+x)^2} + \frac{1}{1+x}\right) dx \\ &= \frac{1}{1+x} + \log(1+x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$\frac{1}{x+1} + \log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 + x)^2,x]
```

```
[Out] (1 + x)^(-1) + Log[1 + x]
```

fricas [A] time = 0.39, size = 16, normalized size = 1.60

$$\frac{(x+1)\log(x+1) + 1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)^2,x, algorithm="fricas")
```

```
[Out] ((x + 1)*log(x + 1) + 1)/(x + 1)
```

giac [A] time = 0.01, size = 11, normalized size = 1.10

$$\frac{1}{x+1} + \log(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)^2,x, algorithm="giac")
```

```
[Out] 1/(x + 1) + log(abs(x + 1))
```

maple [A] time = 0.01, size = 11, normalized size = 1.10

$$\ln(x+1)+\frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x+1)^2,x)
```

```
[Out] 1/(x+1)+ln(x+1)
```

maxima [A] time = 0.47, size = 10, normalized size = 1.00

$$\frac{1}{x+1}+\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)^2,x, algorithm="maxima")
```

```
[Out] 1/(x + 1) + log(x + 1)
```

mupad [B] time = 0.03, size = 10, normalized size = 1.00

$$\ln(x+1)+\frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x + 1)^2,x)
```

```
[Out] log(x + 1) + 1/(x + 1)
```

sympy [A] time = 0.08, size = 8, normalized size = 0.80

$$\log(x+1)+\frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)**2,x)
```

```
[Out] log(x + 1) + 1/(x + 1)
```

3.126 $\int \frac{1}{-x+x^3} \, dx$

Optimal. Leaf size=17

$$\frac{1}{2}\log\left(1-x^2\right)-\log(x)$$

[Out] -ln(x)+1/2*ln(-x^2+1)

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {1593, 266, 36, 31, 29}

$$\frac{1}{2}\log\left(1-x^2\right)-\log(x)$$

Antiderivative was successfully verified.

[In] Int[(-x + x^3)^(-1),x]

[Out] -Log[x] + Log[1 - x^2]/2

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{-x+x^3} \, dx &= \int \frac{1}{x\left(-1+x^2\right)} \, dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(-1+x)x} \, dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{-1+x} \, dx,x,x^2\right)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{x} \, dx,x,x^2\right)\\
&= -\log(x)+\frac{1}{2}\log\left(1-x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{1}{2}\log\left(1-x^2\right)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x + x^3)^(-1),x]
[Out] -Log[x] + Log[1 - x^2]/2
```

fricas [A] time = 0.38, size = 13, normalized size = 0.76

$$\frac{1}{2}\log\left(x^2-1\right)-\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-x),x, algorithm="fricas")
[Out] 1/2*log(x^2 - 1) - log(x)
```

giac [A] time = 0.01, size = 16, normalized size = 0.94

$$-\frac{1}{2}\log\left(x^2\right)+\frac{1}{2}\log\left(\left|x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-x),x, algorithm="giac")
[Out] -1/2*log(x^2) + 1/2*log(abs(x^2 - 1))
```

maple [A] time = 0.01, size = 18, normalized size = 1.06

$$-\ln(x)+\frac{\ln(x-1)}{2}+\frac{\ln(x+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3-x),x)
[Out] 1/2*ln(x+1)+1/2*ln(x-1)-ln(x)
```

maxima [A] time = 0.52, size = 17, normalized size = 1.00

$$\frac{1}{2}\log(x+1)+\frac{1}{2}\log(x-1)-\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-x),x, algorithm="maxima")
[Out] 1/2*log(x + 1) + 1/2*log(x - 1) - log(x)
```

mupad [B] time = 0.11, size = 13, normalized size = 0.76

$$\frac{\ln\left(x^2-1\right)}{2}-\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x - x^3),x)
[Out] log(x^2 - 1)/2 - log(x)
```

<u>**sympy**</u> **[A]** time = 0.10, size = 10, normalized size = 0.59

$$-\log(x) + \frac{\log(x^2 - 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**3-x),x)
```

```
[Out] -log(x) + log(x**2 - 1)/2
```

3.127 $\int \frac{x^2}{-6+x+x^2} \, dx$

Optimal. Leaf size=20

$$x + \frac{4}{5}\log(2-x) - \frac{9}{5}\log(x+3)$$

```
[Out] x+4/5*ln(2-x)-9/5*ln(3+x)
```

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {703, 632, 31}

$$x + \frac{4}{5}\log(2-x) - \frac{9}{5}\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(-6 + x + x^2),x]
```

```
[Out] x + (4*Log[2 - x])/5 - (9*Log[3 + x])/5
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{-6+x+x^2} \, dx &= x + \int \frac{6-x}{-6+x+x^2} \, dx \\
&= x + \frac{4}{5}\int \frac{1}{-2+x} \, dx - \frac{9}{5}\int \frac{1}{3+x} \, dx \\
&= x + \frac{4}{5}\log(2-x) - \frac{9}{5}\log(3+x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 20, normalized size = 1.00

$$x + \frac{4}{5}\log(2-x) - \frac{9}{5}\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(-6 + x + x^2),x]
```

```
[Out] x + (4*Log[2 - x])/5 - (9*Log[3 + x])/5
```

fricas [A] time = 0.38, size = 14, normalized size = 0.70

$$x - \frac{9}{5}\log(x+3) + \frac{4}{5}\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+x-6),x, algorithm="fricas")
```

```
[Out] x - 9/5*log(x + 3) + 4/5*log(x - 2)
```

giac [A] time = 0.01, size = 16, normalized size = 0.80

$$x - \frac{9}{5}\log(|x+3|) + \frac{4}{5}\log(|x-2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+x-6),x, algorithm="giac")
```

```
[Out] x - 9/5*log(abs(x + 3)) + 4/5*log(abs(x - 2))
```

maple [A] time = 0.01, size = 15, normalized size = 0.75

$$x + \frac{4\ln(x-2)}{5} - \frac{9\ln(x+3)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^2+x-6),x)
```

```
[Out] x-9/5*ln(x+3)+4/5*ln(x-2)
```

maxima [A] time = 0.51, size = 14, normalized size = 0.70

$$x - \frac{9}{5}\log(x+3) + \frac{4}{5}\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+x-6),x, algorithm="maxima")
```

```
[Out] x - 9/5*log(x + 3) + 4/5*log(x - 2)
```

mupad [B] time = 0.04, size = 14, normalized size = 0.70

$$x + \frac{4\,\ln(x-2)}{5} - \frac{9\,\ln(x+3)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x + x^2 - 6),x)
```

```
[Out] x + (4*log(x - 2))/5 - (9*log(x + 3))/5
```

sympy [A] time = 0.11, size = 17, normalized size = 0.85

$$x + \frac{4\log(x-2)}{5} - \frac{9\log(x+3)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**2+x-6),x)
```

```
[Out] x + 4*log(x - 2)/5 - 9*log(x + 3)/5
```

3.128 $\int \frac{2+x}{4-4x+x^2}\,dx$

Optimal. Leaf size=16

$$\frac{4}{2-x}+\log(2-x)$$

```
[Out] 4/(2-x)+ln(2-x)
```

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{27, 43\}$

$$\frac{4}{2-x}+\log(2-x)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x)/(4 - 4*x + x^2),x]
```

```
[Out] 4/(2 - x) + Log[2 - x]
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{2+x}{4-4x+x^2}\,dx &= \int \frac{2+x}{(-2+x)^2}\,dx\\ &= \int \left(\frac{4}{(-2+x)^2}+\frac{1}{-2+x}\right)dx\\ &= \frac{4}{2-x}+\log(2-x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 0.75

$$\log(x-2)-\frac{4}{x-2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x)/(4 - 4*x + x^2),x]
```

```
[Out] -4/(-2 + x) + Log[-2 + x]
```

fricas [A] time = 0.38, size = 16, normalized size = 1.00

$$\frac{(x-2)\log(x-2)-4}{x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2-4*x+4),x, algorithm="fricas")
[Out] ((x - 2)*log(x - 2) - 4)/(x - 2)
```

giac [A] time = 0.01, size = 13, normalized size = 0.81

$$-\frac{4}{x-2} + \log\left(|x-2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2-4*x+4),x, algorithm="giac")
[Out] -4/(x - 2) + log(abs(x - 2))
```

maple [A] time = 0.00, size = 13, normalized size = 0.81

$$\ln\left(x-2\right) - \frac{4}{x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+2)/(x^2-4*x+4),x)
[Out] ln(x-2)-4/(x-2)
```

maxima [A] time = 0.49, size = 12, normalized size = 0.75

$$-\frac{4}{x-2} + \log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2-4*x+4),x, algorithm="maxima")
[Out] -4/(x - 2) + log(x - 2)
```

mupad [B] time = 0.04, size = 12, normalized size = 0.75

$$\ln\left(x-2\right) - \frac{4}{x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 2)/(x^2 - 4*x + 4),x)
[Out] log(x - 2) - 4/(x - 2)
```

sympy [A] time = 0.08, size = 8, normalized size = 0.50

$$\log\left(x-2\right) - \frac{4}{x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x**2-4*x+4),x)
[Out] log(x - 2) - 4/(x - 2)
```

3.129 $\int \frac{1}{(4-4x+x^2)(5-4x+x^2)} \, dx$

Optimal. Leaf size=14

$$\frac{1}{2-x} + \tan^{-1}(2-x)$$

```
[Out] 1/(2-x)-arctan(-2+x)
```

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {27, 693, 618, 204}

$$\frac{1}{2-x} + \tan^{-1}(2-x)$$

Antiderivative was successfully verified.

```
[In] Int[1/((4 - 4*x + x^2)*(5 - 4*x + x^2)),x]
```

```
[Out] (2 - x)^(-1) + ArcTan[2 - x]
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rubi steps

$$\int \frac{1}{(4-4x+x^2)(5-4x+x^2)}\,dx = \int \frac{1}{(-2+x)^2(5-4x+x^2)}\,dx$$
$$= \frac{1}{2-x} - \int \frac{1}{5-4x+x^2}\,dx$$
$$= \frac{1}{2-x} + 2\,\text{Subst}\left(\int \frac{1}{-4-x^2}\,dx, x, -4+2x\right)$$
$$= \frac{1}{2-x} + \tan^{-1}(2-x)$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.00

$$\tan^{-1}(2-x) - \frac{1}{x-2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((4 - 4*x + x^2)*(5 - 4*x + x^2)),x]
[Out] -(-2 + x)^(-1) + ArcTan[2 - x]
```

fricas [A] time = 0.40, size = 17, normalized size = 1.21

$$-\frac{(x-2)\arctan(x-2)+1}{x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-4*x+4)/(x^2-4*x+5),x, algorithm="fricas")
[Out] -((x - 2)*arctan(x - 2) + 1)/(x - 2)
```

giac [A] time = 0.01, size = 14, normalized size = 1.00

$$-\frac{1}{x-2} - \arctan(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-4*x+4)/(x^2-4*x+5),x, algorithm="giac")
[Out] -1/(x - 2) - arctan(x - 2)
```

maple [A] time = 0.01, size = 15, normalized size = 1.07

$$-\arctan(x-2) - \frac{1}{x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-4*x+4)/(x^2-4*x+5),x)
[Out] -arctan(x-2)-1/(x-2)
```

maxima [A] time = 1.27, size = 14, normalized size = 1.00

$$-\frac{1}{x-2} - \arctan(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-4*x+4)/(x^2-4*x+5),x, algorithm="maxima")
```

```
[Out] -1/(x - 2) - arctan(x - 2)
```

mupad [B] time = 0.08, size = 14, normalized size = 1.00

$$-\operatorname{atan}(x-2)-\frac{1}{x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 - 4*x + 4)*(x^2 - 4*x + 5)),x)
```

```
[Out] - atan(x - 2) - 1/(x - 2)
```

sympy [A] time = 0.14, size = 10, normalized size = 0.71

$$-\operatorname{atan}(x-2)-\frac{1}{x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-4*x+4)/(x**2-4*x+5),x)
```

```
[Out] -atan(x - 2) - 1/(x - 2)
```

3.130 $\int \frac{-3+x}{2x+3x^2+x^3}\,dx$

Optimal. Leaf size=21

$$-\frac{3\log(x)}{2}+4\log(x+1)-\frac{5}{2}\log(x+2)$$

```
[Out] -3/2*ln(x)+4*ln(1+x)-5/2*ln(2+x)
```

Rubi [A] time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1594, 800}

$$-\frac{3\log(x)}{2}+4\log(x+1)-\frac{5}{2}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + x)/(2*x + 3*x^2 + x^3),x]
```

```
[Out] (-3*Log[x])/2 + 4*Log[1 + x] - (5*Log[2 + x])/2
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$\begin{aligned}\int \frac{-3+x}{2x+3x^2+x^3}\,dx &= \int \frac{-3+x}{x\left(2+3x+x^2\right)}\,dx\\ &= \int \left(-\frac{3}{2x}+\frac{4}{1+x}-\frac{5}{2(2+x)}\right)dx\\ &= -\frac{3\log(x)}{2}+4\log(1+x)-\frac{5}{2}\log(2+x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.00

$$-\frac{3\log(x)}{2}+4\log(x+1)-\frac{5}{2}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + x)/(2*x + 3*x^2 + x^3),x]
```

```
[Out] (-3*Log[x])/2 + 4*Log[1 + x] - (5*Log[2 + x])/2
```

fricas [A] time = 0.40, size = 17, normalized size = 0.81

$$-\frac{5}{2}\log\left(x+2\right)+4\log\left(x+1\right)-\frac{3}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+x)/(x^3+3*x^2+2*x),x, algorithm="fricas")
[Out] -5/2*log(x + 2) + 4*log(x + 1) - 3/2*log(x)
```

giac [A] time = 0.01, size = 20, normalized size = 0.95

$$-\frac{5}{2}\,\log\left(|x+2|\right)+4\,\log\left(|x+1|\right)-\frac{3}{2}\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+x)/(x^3+3*x^2+2*x),x, algorithm="giac")
[Out] -5/2*log(abs(x + 2)) + 4*log(abs(x + 1)) - 3/2*log(abs(x))
```

maple [A] time = 0.01, size = 18, normalized size = 0.86

$$-\frac{3\ln(x)}{2}+4\ln\left(x+1\right)-\frac{5\ln\left(x+2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-3)/(x^3+3*x^2+2*x),x)
[Out] -3/2*ln(x)+4*ln(x+1)-5/2*ln(x+2)
```

maxima [A] time = 0.63, size = 17, normalized size = 0.81

$$-\frac{5}{2}\,\log\left(x+2\right)+4\,\log\left(x+1\right)-\frac{3}{2}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+x)/(x^3+3*x^2+2*x),x, algorithm="maxima")
[Out] -5/2*log(x + 2) + 4*log(x + 1) - 3/2*log(x)
```

mupad [B] time = 0.06, size = 17, normalized size = 0.81

$$4\,\ln\left(x+1\right)-\frac{5\,\ln\left(x+2\right)}{2}-\frac{3\,\ln(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 3)/(2*x + 3*x^2 + x^3),x)
[Out] 4*log(x + 1) - (5*log(x + 2))/2 - (3*log(x))/2
```

sympy [A] time = 0.13, size = 20, normalized size = 0.95

$$-\frac{3\log\left(x\right)}{2}+4\log\left(x+1\right)-\frac{5\log\left(x+2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+x)/(x**3+3*x**2+2*x),x)
[Out] -3*log(x)/2 + 4*log(x + 1) - 5*log(x + 2)/2
```

3.131 $\int \frac{1}{\left(-1+x^2\right)^2} \, dx$

Optimal. Leaf size=21

$$\frac{x}{2\left(1-x^2\right)}+\frac{1}{2}\tanh^{-1}(x)$$

```
[Out] 1/2*x/(-x^2+1)+1/2*arctanh(x)
```

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {199, 207}

$$\frac{x}{2\left(1-x^2\right)}+\frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^2)^(-2),x]
```

```
[Out] x/(2*(1 - x^2)) + ArcTanh[x]/2
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\left(-1+x^2\right)^2} \, dx &= \frac{x}{2\left(1-x^2\right)}-\frac{1}{2}\int \frac{1}{-1+x^2} \, dx \\ &= \frac{x}{2\left(1-x^2\right)}+\frac{1}{2}\tanh^{-1}(x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.29

$$\frac{1}{4}\left(-\frac{2x}{x^2-1}-\log(1-x)+\log(x+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)^(-2),x]
```

```
[Out] ((-2*x)/(-1 + x^2) - Log[1 - x] + Log[1 + x])/4
```

fricas [B] time = 0.39, size = 34, normalized size = 1.62

$$\frac{\left(x^2-1\right)\log\left(x+1\right)-\left(x^2-1\right)\log\left(x-1\right)-2\,x}{4\left(x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^2,x, algorithm="fricas")
[Out] 1/4*((x^2 - 1)*log(x + 1) - (x^2 - 1)*log(x - 1) - 2*x)/(x^2 - 1)
```

giac [A] time = 0.01, size = 25, normalized size = 1.19

$$-\frac{x}{2\left(x^2-1\right)}+\frac{1}{4}\log\left(|x+1|\right)-\frac{1}{4}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^2,x, algorithm="giac")
[Out] -1/2*x/(x^2 - 1) + 1/4*log(abs(x + 1)) - 1/4*log(abs(x - 1))
```

maple [A] time = 0.01, size = 28, normalized size = 1.33

$$-\frac{\ln\left(x-1\right)}{4}+\frac{\ln\left(x+1\right)}{4}-\frac{1}{4\left(x+1\right)}-\frac{1}{4\left(x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-1)^2,x)
[Out] -1/4/(x+1)+1/4*ln(x+1)-1/4/(x-1)-1/4*ln(x-1)
```

maxima [A] time = 0.53, size = 23, normalized size = 1.10

$$-\frac{x}{2\left(x^2-1\right)}+\frac{1}{4}\log\left(x+1\right)-\frac{1}{4}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^2,x, algorithm="maxima")
[Out] -1/2*x/(x^2 - 1) + 1/4*log(x + 1) - 1/4*log(x - 1)
```

mupad [B] time = 0.09, size = 17, normalized size = 0.81

$$\frac{\mathrm{atanh}(x)}{2}-\frac{x}{2\left(x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 - 1)^2,x)
[Out] atanh(x)/2 - x/(2*(x^2 - 1))
```

sympy [A] time = 0.11, size = 20, normalized size = 0.95

$$-\frac{x}{2x^2-2}-\frac{\log\left(x-1\right)}{4}+\frac{\log\left(x+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-1)**2,x)
[Out] -x/(2*x**2 - 2) - log(x - 1)/4 + log(x + 1)/4
```

3.132 $\int \frac{1+x}{-1+x^3} dx$

Optimal. Leaf size=22

$$\frac{2}{3}\log(1-x)-\frac{1}{3}\log\left(x^2+x+1\right)$$

```
[Out] 2/3*ln(1-x)-1/3*ln(x^2+x+1)
```

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1861, 31, 628}

$$\frac{2}{3}\log(1-x)-\frac{1}{3}\log\left(x^2+x+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/(-1 + x^3),x]
```

```
[Out] (2*Log[1 - x])/3 - Log[1 + x + x^2]/3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1861

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[-(a/b), 3]], s = Denominator[Rt[-(a/b), 3]]}, Dist[(r*(B*r + A*s))/
(3*a*s), Int[1/(r - s*x), x], x] - Dist[r/(3*a*s), Int[(r*(B*r - 2*A*s) - s
*(B*r + A*s)*x)/(r^2 + r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x]
&& NeQ[a*B^3 - b*A^3, 0] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x}{-1+x^3}\,dx &= \frac{1}{3}\int \frac{-1-2x}{1+x+x^2}\,dx-\frac{2}{3}\int \frac{1}{1-x}\,dx\\ &= \frac{2}{3}\log(1-x)-\frac{1}{3}\log\left(1+x+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 1.00

$$\frac{2}{3}\log(1-x)-\frac{1}{3}\log\left(x^2+x+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/(-1 + x^3),x]
```

```
[Out] (2*Log[1 - x])/3 - Log[1 + x + x^2]/3
```

fricas [A] time = 0.38, size = 16, normalized size = 0.73

$$-\frac{1}{3}\log\left(x^2+x+1\right)+\frac{2}{3}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^3-1),x, algorithm="fricas")
[Out] -1/3*log(x^2 + x + 1) + 2/3*log(x - 1)
```

giac [A] time = 0.01, size = 17, normalized size = 0.77

$$-\frac{1}{3}\log\left(x^2+x+1\right)+\frac{2}{3}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^3-1),x, algorithm="giac")
[Out] -1/3*log(x^2 + x + 1) + 2/3*log(abs(x - 1))
```

maple [A] time = 0.00, size = 17, normalized size = 0.77

$$\frac{2\ln\left(x-1\right)}{3}-\frac{\ln\left(x^2+x+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)/(x^3-1),x)
[Out] 2/3*ln(x-1)-1/3*ln(x^2+x+1)
```

maxima [A] time = 1.23, size = 16, normalized size = 0.73

$$-\frac{1}{3}\log\left(x^2+x+1\right)+\frac{2}{3}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^3-1),x, algorithm="maxima")
[Out] -1/3*log(x^2 + x + 1) + 2/3*log(x - 1)
```

mupad [B] time = 0.16, size = 16, normalized size = 0.73

$$\frac{2\ \ln\left(x-1\right)}{3}-\frac{\ln\left(x^2+x+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/(x^3 - 1),x)
[Out] (2*log(x - 1))/3 - log(x + x^2 + 1)/3
```

sympy [A] time = 0.10, size = 17, normalized size = 0.77

$$\frac{2\log\left(x-1\right)}{3}-\frac{\log\left(x^2+x+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x**3-1),x)
[Out] 2*log(x - 1)/3 - log(x**2 + x + 1)/3
```

3.133 $\int \frac{1+x^4}{x\left(1+x^2\right)^2} \, dx$

Optimal. Leaf size=10

$$\frac{1}{x^2+1}+\log(x)$$

```
[Out] 1/(x^2+1)+ln(x)
```

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1252, 894}

$$\frac{1}{x^2+1}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)/(x*(1 + x^2)^2),x]
```

```
[Out] (1 + x^2)^(-1) + Log[x]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x^4}{x\left(1+x^2\right)^2} \, dx &= \frac{1}{2} \text{Subst}\left(\int \frac{1+x^2}{x(1+x)^2} \, dx,x,x^2\right)\\
&= \frac{1}{2} \text{Subst}\left(\int \left(\frac{1}{x}-\frac{2}{(1+x)^2}\right) \, dx,x,x^2\right)\\
&= \frac{1}{1+x^2}+\log(x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 10, normalized size = 1.00

$$\frac{1}{x^2+1}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)/(x*(1 + x^2)^2),x]
```

```
[Out] (1 + x^2)^(-1) + Log[x]
```

fricas [A] time = 0.39, size = 18, normalized size = 1.80

$$\frac{\left(x^2+1\right)\log(x)+1}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/x/(x^2+1)^2,x, algorithm="fricas")
[Out] ((x^2 + 1)*log(x) + 1)/(x^2 + 1)
```

giac [A] time = 0.01, size = 14, normalized size = 1.40

$$\frac{1}{x^2+1}+\frac{1}{2}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/x/(x^2+1)^2,x, algorithm="giac")
[Out] 1/(x^2 + 1) + 1/2*log(x^2)
```

maple [A] time = 0.01, size = 11, normalized size = 1.10

$$\ln(x)+\frac{1}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)/x/(x^2+1)^2,x)
[Out] 1/(x^2+1)+ln(x)
```

maxima [A] time = 0.58, size = 14, normalized size = 1.40

$$\frac{1}{x^2+1}+\frac{1}{2}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/x/(x^2+1)^2,x, algorithm="maxima")
[Out] 1/(x^2 + 1) + 1/2*log(x^2)
```

mupad [B] time = 0.03, size = 10, normalized size = 1.00

$$\ln(x)+\frac{1}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 1)/(x*(x^2 + 1)^2),x)
[Out] log(x) + 1/(x^2 + 1)
```

sympy [A] time = 0.10, size = 8, normalized size = 0.80

$$\log\left(x\right)+\frac{1}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)/x/(x**2+1)**2,x)
[Out] log(x) + 1/(x**2 + 1)
```

3.134 $\int \frac{1}{-2x^3+x^4}\,dx$

Optimal. Leaf size=31

$$\frac{1}{4x^2}+\frac{1}{4x}+\frac{1}{8}\log(2-x)-\frac{\log(x)}{8}$$

```
[Out] 1/4/x^2+1/4/x+1/8*ln(2-x)-1/8*ln(x)
```

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1593, 44}

$$\frac{1}{4x^2}+\frac{1}{4x}+\frac{1}{8}\log(2-x)-\frac{\log(x)}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(-2*x^3 + x^4)^(-1),x]
```

```
[Out] 1/(4*x^2) + 1/(4*x) + Log[2 - x]/8 - Log[x]/8
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{-2x^3+x^4}\,dx &= \int \frac{1}{(-2+x)x^3}\,dx\\ &= \int \left(\frac{1}{8(-2+x)}-\frac{1}{2x^3}-\frac{1}{4x^2}-\frac{1}{8x}\right)dx\\ &= \frac{1}{4x^2}+\frac{1}{4x}+\frac{1}{8}\log(2-x)-\frac{\log(x)}{8}\end{aligned}$$

Mathematica [A] time = 0.00, size = 31, normalized size = 1.00

$$\frac{1}{4x^2}+\frac{1}{4x}+\frac{1}{8}\log(2-x)-\frac{\log(x)}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2*x^3 + x^4)^(-1),x]
```

```
[Out] 1/(4*x^2) + 1/(4*x) + Log[2 - x]/8 - Log[x]/8
```

fricas [A] time = 0.38, size = 25, normalized size = 0.81

$$\frac{x^2\log(x-2)-x^2\log(x)+2\,x+2}{8\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-2*x^3),x, algorithm="fricas")
[Out] 1/8*(x^2*log(x - 2) - x^2*log(x) + 2*x + 2)/x^2
```

giac [A] time = 0.01, size = 21, normalized size = 0.68

$$\frac{x+1}{4\,x^2} + \frac{1}{8}\,\log\left(|x-2|\right) - \frac{1}{8}\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-2*x^3),x, algorithm="giac")
[Out] 1/4*(x + 1)/x^2 + 1/8*log(abs(x - 2)) - 1/8*log(abs(x))
```

maple [A] time = 0.01, size = 22, normalized size = 0.71

$$-\frac{\ln(x)}{8} + \frac{\ln(x-2)}{8} + \frac{1}{4x} + \frac{1}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4-2*x^3),x)
[Out] 1/8*ln(x-2)+1/4/x^2+1/4/x-1/8*ln(x)
```

maxima [A] time = 0.49, size = 19, normalized size = 0.61

$$\frac{x+1}{4\,x^2} + \frac{1}{8}\,\log\left(x-2\right) - \frac{1}{8}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-2*x^3),x, algorithm="maxima")
[Out] 1/4*(x + 1)/x^2 + 1/8*log(x - 2) - 1/8*log(x)
```

mupad [B] time = 0.04, size = 16, normalized size = 0.52

$$\frac{\frac{x}{4}+\frac{1}{4}}{x^2} - \frac{\mathrm{atanh}\left(x-1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(2*x^3 - x^4),x)
[Out] (x/4 + 1/4)/x^2 - atanh(x - 1)/4
```

sympy [A] time = 0.11, size = 19, normalized size = 0.61

$$-\frac{\log\left(x\right)}{8} + \frac{\log\left(x-2\right)}{8} + \frac{x+1}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4-2*x**3),x)
[Out] -log(x)/8 + log(x - 2)/8 + (x + 1)/(4*x**2)
```

3.135 $\int \frac{1-x^3}{x(1+x^2)} \, dx$

Optimal. Leaf size=18

$$-\frac{1}{2}\log\left(x^2+1\right)-x+\log(x)+\tan^{-1}(x)$$

```
[Out] -x+arctan(x)+ln(x)-1/2*ln(x^2+1)
```

Rubi [A] time = 0.03, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1802, 635, 203, 260}

$$-\frac{1}{2}\log\left(x^2+1\right)-x+\log(x)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^3)/(x*(1 + x^2)),x]
```

```
[Out] -x + ArcTan[x] + Log[x] - Log[1 + x^2]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1-x^3}{x\left(1+x^2\right)} \, dx &= \int \left(-1+\frac{1}{x}+\frac{1-x}{1+x^2}\right) \, dx \\
&= -x+\log(x)+\int \frac{1-x}{1+x^2} \, dx \\
&= -x+\log(x)+\int \frac{1}{1+x^2} \, dx-\int \frac{x}{1+x^2} \, dx \\
&= -x+\tan^{-1}(x)+\log(x)-\frac{1}{2}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$-\frac{1}{2}\log\left(x^2+1\right)-x+\log(x)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^3)/(x*(1 + x^2)),x]
[Out] -x + ArcTan[x] + Log[x] - Log[1 + x^2]/2
```

fricas [A] time = 0.42, size = 16, normalized size = 0.89

$$-x+\arctan(x)-\frac{1}{2}\log\left(x^2+1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/x/(x^2+1),x, algorithm="fricas")
[Out] -x + arctan(x) - 1/2*log(x^2 + 1) + log(x)
```

giac [A] time = 0.01, size = 17, normalized size = 0.94

$$-x+\arctan(x)-\frac{1}{2}\log\left(x^2+1\right)+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/x/(x^2+1),x, algorithm="giac")
[Out] -x + arctan(x) - 1/2*log(x^2 + 1) + log(abs(x))
```

maple [A] time = 0.01, size = 17, normalized size = 0.94

$$-x+\arctan(x)+\ln(x)-\frac{\ln\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)/x/(x^2+1),x)
[Out] -x+arctan(x)+ln(x)-1/2*ln(x^2+1)
```

maxima [A] time = 1.43, size = 16, normalized size = 0.89

$$-x+\arctan(x)-\frac{1}{2}\log\left(x^2+1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/x/(x^2+1),x, algorithm="maxima")
[Out] -x + arctan(x) - 1/2*log(x^2 + 1) + log(x)
```

mupad [B] time = 0.04, size = 24, normalized size = 1.33

$$\ln(x)-x+\ln\left(x-\mathrm{i}\right)\left(-\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)+\ln\left(x+1\mathrm{i}\right)\left(-\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3 - 1)/(x*(x^2 + 1)),x)
[Out] log(x) - log(x - 1i)*(1/2 + 1i/2) - log(x + 1i)*(1/2 - 1i/2) - x
```

<u>**sympy**</u> **[A]** time = 0.13, size = 15, normalized size = 0.83

$$-x+\log\left(x\right)-\frac{\log\left(x^{2}+1\right)}{2}+\operatorname{atan}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)/x/(x**2+1),x)
```

```
[Out] -x + log(x) - log(x**2 + 1)/2 + atan(x)
```

3.136 $\int \frac{1}{-1+x^4} \, dx$

Optimal. Leaf size=13

$$-\frac{1}{2}\tan^{-1}(x)-\frac{1}{2}\tanh^{-1}(x)$$

```
[Out] -1/2*arctan(x)-1/2*arctanh(x)
```

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {212, 206, 203}

$$-\frac{1}{2}\tan^{-1}(x)-\frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^4)^(-1),x]
```

```
[Out] -ArcTan[x]/2 - ArcTanh[x]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{-1+x^4} \, dx &= -\left(\frac{1}{2}\int \frac{1}{1-x^2} \, dx\right)-\frac{1}{2}\int \frac{1}{1+x^2} \, dx\\ &= -\frac{1}{2}\tan^{-1}(x)-\frac{1}{2}\tanh^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 1.92

$$\frac{1}{4}\log(1-x)-\frac{1}{4}\log(x+1)-\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^4)^(-1),x]
```

```
[Out] -1/2*ArcTan[x] + Log[1 - x]/4 - Log[1 + x]/4
```

fricas [A] time = 0.40, size = 17, normalized size = 1.31

$$-\frac{1}{2}\arctan(x)-\frac{1}{4}\log(x+1)+\frac{1}{4}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-1),x, algorithm="fricas")
[Out] -1/2*arctan(x) - 1/4*log(x + 1) + 1/4*log(x - 1)
```

giac [B] time = 0.01, size = 19, normalized size = 1.46

$$-\frac{1}{2}\arctan(x)-\frac{1}{4}\log(|x+1|)+\frac{1}{4}\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-1),x, algorithm="giac")
[Out] -1/2*arctan(x) - 1/4*log(abs(x + 1)) + 1/4*log(abs(x - 1))
```

maple [A] time = 0.00, size = 10, normalized size = 0.77

$$-\frac{\operatorname{arctanh}(x)}{2}-\frac{\arctan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4-1),x)
[Out] -1/2*arctan(x)-1/2*arctanh(x)
```

maxima [A] time = 1.37, size = 17, normalized size = 1.31

$$-\frac{1}{2}\arctan(x)-\frac{1}{4}\log(x+1)+\frac{1}{4}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-1),x, algorithm="maxima")
[Out] -1/2*arctan(x) - 1/4*log(x + 1) + 1/4*log(x - 1)
```

mupad [B] time = 0.03, size = 9, normalized size = 0.69

$$-\frac{\operatorname{atan}(x)}{2}-\frac{\operatorname{atanh}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4 - 1),x)
[Out] - atan(x)/2 - atanh(x)/2
```

sympy [A] time = 0.13, size = 17, normalized size = 1.31

$$\frac{\log(x-1)}{4}-\frac{\log(x+1)}{4}-\frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4-1),x)
[Out] log(x - 1)/4 - log(x + 1)/4 - atan(x)/2
```

3.137 $\int \frac{1}{1+x^4}\,dx$

Optimal. Leaf size=85

$$-\frac{\log\left(x^2-\sqrt{2}\,x+1\right)}{4\sqrt{2}}+\frac{\log\left(x^2+\sqrt{2}\,x+1\right)}{4\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\,x+1\right)}{2\sqrt{2}}$$

```
[Out] 1/4*arctan(-1+x*2^(1/2))*2^(1/2)+1/4*arctan(1+x*2^(1/2))*2^(1/2)-1/8*ln(1+x
^2-x*2^(1/2))*2^(1/2)+1/8*ln(1+x^2+x*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {211, 1165, 628, 1162, 617, 204}

$$-\frac{\log\left(x^2-\sqrt{2}\,x+1\right)}{4\sqrt{2}}+\frac{\log\left(x^2+\sqrt{2}\,x+1\right)}{4\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\,x+1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)^(-1),x]
```

```
[Out] -ArcTan[1 - Sqrt[2]*x]/(2*Sqrt[2]) + ArcTan[1 + Sqrt[2]*x]/(2*Sqrt[2]) - Lo
g[1 - Sqrt[2]*x + x^2]/(4*Sqrt[2]) + Log[1 + Sqrt[2]*x + x^2]/(4*Sqrt[2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+x^4}\,dx &= \frac{1}{2}\int \frac{1-x^2}{1+x^4}\,dx + \frac{1}{2}\int \frac{1+x^2}{1+x^4}\,dx \\ &= \frac{1}{4}\int \frac{1}{1-\sqrt{2}x+x^2}\,dx + \frac{1}{4}\int \frac{1}{1+\sqrt{2}x+x^2}\,dx - \frac{\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx}{4\sqrt{2}} - \frac{\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx}{4\sqrt{2}} \\ &= -\frac{\log\left(1-\sqrt{2}x+x^2\right)}{4\sqrt{2}} + \frac{\log\left(1+\sqrt{2}x+x^2\right)}{4\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}x\right)}{2\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,\ldots\right)}{2\sqrt{\ldots}} \\ &= -\frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{2\sqrt{2}} + \frac{\tan^{-1}\left(1+\sqrt{2}x\right)}{2\sqrt{2}} - \frac{\log\left(1-\sqrt{2}x+x^2\right)}{4\sqrt{2}} + \frac{\log\left(1+\sqrt{2}x+x^2\right)}{4\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 64, normalized size = 0.75

$$\frac{-\log\left(x^2-\sqrt{2}x+1\right)+\log\left(x^2+\sqrt{2}x+1\right)-2\tan^{-1}\left(1-\sqrt{2}x\right)+2\tan^{-1}\left(\sqrt{2}x+1\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)^(-1),x]
```

```
[Out] (-2*ArcTan[1 - Sqrt[2]*x] + 2*ArcTan[1 + Sqrt[2]*x] - Log[1 - Sqrt[2]*x + x
^2] + Log[1 + Sqrt[2]*x + x^2])/(4*Sqrt[2])
```

fricas [A] time = 0.39, size = 95, normalized size = 1.12

$$-\frac{1}{2}\sqrt{2}\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2+\sqrt{2}x+1}-1\right)-\frac{1}{2}\sqrt{2}\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2-\sqrt{2}x+1}+1\right)+\frac{1}{8}\sqrt{2}\log\left(\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 + sqrt(2)*x + 1) - 1) - 1
/2*sqrt(2)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 - sqrt(2)*x + 1) + 1) + 1/8
*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/8*sqrt(2)*log(x^2 - sqrt(2)*x + 1)
```

giac [A] time = 0.01, size = 72, normalized size = 0.85

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{1}{8}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{1}{8}\sqrt{2}\log\left(x^2-\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*sqrt(2)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2))) + 1/8*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/8*sqrt(2)
*log(x^2 - sqrt(2)*x + 1)
```

maple [A] time = 0.00, size = 58, normalized size = 0.68

$$\frac{\sqrt{2}\ \arctan\left(\sqrt{2}\ x-1\right)}{4}+\frac{\sqrt{2}\ \arctan\left(\sqrt{2}\ x+1\right)}{4}+\frac{\sqrt{2}\ \ln\left(\frac{x^{2}+\sqrt{2}\ x+1}{x^{2}-\sqrt{2}\ x+1}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+1),x)
```

```
[Out] 1/4*arctan(-1+2^(1/2)*x)*2^(1/2)+1/8*2^(1/2)*ln((1+x^2+2^(1/2)*x)/(1+x^2-2^
(1/2)*x))+1/4*arctan(1+2^(1/2)*x)*2^(1/2)
```

maxima [A] time = 1.41, size = 72, normalized size = 0.85

$$\frac{1}{4}\sqrt{2}\ \arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{4}\sqrt{2}\ \arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{1}{8}\sqrt{2}\ \log\left(x^{2}+\sqrt{2}x+1\right)-\frac{1}{8}\sqrt{2}\ \log\left(x^{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*sqrt(2)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2))) + 1/8*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/8*sqrt(2)
*log(x^2 - sqrt(2)*x + 1)
```

mupad [B] time = 0.12, size = 33, normalized size = 0.39

$$\sqrt{2}\ \mathrm{atan}\left(\sqrt{2}\ x\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)+\sqrt{2}\ \mathrm{atan}\left(\sqrt{2}\ x\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{4}-\frac{1}{4}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4 + 1),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(1/4 + 1i/4) + 2^(1/2)*atan(2^(1/2)*x*
(1/2 + 1i/2))*(1/4 - 1i/4)
```

sympy [A] time = 0.15, size = 73, normalized size = 0.86

$$-\frac{\sqrt{2}\ \log\left(x^{2}-\sqrt{2}x+1\right)}{8}+\frac{\sqrt{2}\ \log\left(x^{2}+\sqrt{2}x+1\right)}{8}+\frac{\sqrt{2}\ \mathrm{atan}\left(\sqrt{2}x-1\right)}{4}+\frac{\sqrt{2}\ \mathrm{atan}\left(\sqrt{2}x+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+1),x)
```

```
[Out] -sqrt(2)*log(x**2 - sqrt(2)*x + 1)/8 + sqrt(2)*log(x**2 + sqrt(2)*x + 1)/8
+ sqrt(2)*atan(sqrt(2)*x - 1)/4 + sqrt(2)*atan(sqrt(2)*x + 1)/4
```

3.138 $\int \frac{x^2}{\left(2+2x+x^2\right)^2} \, dx$

Optimal. Leaf size=23

$$\tan^{-1}(x+1)-\frac{x(x+2)}{2\left(x^2+2x+2\right)}$$

```
[Out] -1/2*x*(2+x)/(x^2+2*x+2)+arctan(1+x)
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {722, 617, 204}

$$\tan^{-1}(x+1)-\frac{x(x+2)}{2\left(x^2+2x+2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(2 + 2*x + x^2)^2,x]
```

```
[Out] -(x*(2 + x))/(2*(2 + 2*x + x^2)) + ArcTan[1 + x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 722

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*(2*p + 3)*(c*d^2 -
b*d*e + a*e^2))/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c
*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\left(2+2x+x^2\right)^2} \, dx &= -\frac{x(2+x)}{2\left(2+2x+x^2\right)}+\int \frac{1}{2+2x+x^2} \, dx \\
&= -\frac{x(2+x)}{2\left(2+2x+x^2\right)}-\text{Subst}\left(\int \frac{1}{-1-x^2} \, dx,x,1+x\right) \\
&= -\frac{x(2+x)}{2\left(2+2x+x^2\right)}+\tan^{-1}(1+x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 0.65

$$\frac{1}{x^2+2x+2}+\tan^{-1}(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(2 + 2*x + x^2)^2,x]
[Out] (2 + 2*x + x^2)^(-1) + ArcTan[1 + x]
```

fricas [A] time = 0.40, size = 26, normalized size = 1.13

$$\frac{(x^2+2x+2)\arctan(x+1)+1}{x^2+2x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+2*x+2)^2,x, algorithm="fricas")
[Out] ((x^2 + 2*x + 2)*arctan(x + 1) + 1)/(x^2 + 2*x + 2)
```

giac [A] time = 0.01, size = 15, normalized size = 0.65

$$\frac{1}{x^2+2x+2}+\arctan(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+2*x+2)^2,x, algorithm="giac")
[Out] 1/(x^2 + 2*x + 2) + arctan(x + 1)
```

maple [A] time = 0.01, size = 16, normalized size = 0.70

$$\arctan(x+1)+\frac{1}{x^2+2x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^2+2*x+2)^2,x)
[Out] 1/(x^2+2*x+2)+arctan(x+1)
```

maxima [A] time = 1.22, size = 15, normalized size = 0.65

$$\frac{1}{x^2+2x+2}+\arctan(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+2*x+2)^2,x, algorithm="maxima")
[Out] 1/(x^2 + 2*x + 2) + arctan(x + 1)
```

mupad [B] time = 0.08, size = 15, normalized size = 0.65

$$\operatorname{atan}(x+1)+\frac{1}{x^2+2x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(2*x + x^2 + 2)^2,x)
[Out] atan(x + 1) + 1/(2*x + x^2 + 2)
```

sympy [**A**] time = 0.11, size = 14, normalized size = 0.61

$$\operatorname{atan}(x+1) + \frac{1}{x^2 + 2x + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**2+2*x+2)**2,x)
```

```
[Out] atan(x + 1) + 1/(x**2 + 2*x + 2)
```

3.139 $\int \frac{-1+4x^5}{(1+x+x^5)^2} dx$

Optimal. Leaf size=11

$$-\frac{x}{x^5+x+1}$$

[Out] -x/(x^5+x+1)

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1588}

$$-\frac{x}{x^5+x+1}$$

Antiderivative was successfully verified.

[In] Int[(-1 + 4*x^5)/(1 + x + x^5)^2,x]

[Out] -(x/(1 + x + x^5))

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{-1+4x^5}{(1+x+x^5)^2} dx = -\frac{x}{1+x+x^5}$$

Mathematica [A] time = 0.01, size = 11, normalized size = 1.00

$$-\frac{x}{x^5+x+1}$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + 4*x^5)/(1 + x + x^5)^2,x]

[Out] -(x/(1 + x + x^5))

fricas [A] time = 0.37, size = 11, normalized size = 1.00

$$-\frac{x}{x^5+x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^5-1)/(x^5+x+1)^2,x, algorithm="fricas")

[Out] -x/(x^5 + x + 1)

giac [A] time = 0.02, size = 11, normalized size = 1.00

$$-\frac{x}{x^5+x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^5-1)/(x^5+x+1)^2,x, algorithm="giac")
```

```
[Out] -x/(x^5 + x + 1)
```

maple [B] time = 0.01, size = 41, normalized size = 3.73

$$-\frac{-3x^2+5x-1}{7\left(x^3-x^2+1\right)}+\frac{-3x-1}{7x^2+7x+7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^5-1)/(x^5+x+1)^2,x)
```

```
[Out] -1/7*(-3*x^2+5*x-1)/(x^3-x^2+1)+1/7*(-3*x-1)/(x^2+x+1)
```

maxima [A] time = 0.52, size = 11, normalized size = 1.00

$$-\frac{x}{x^5+x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^5-1)/(x^5+x+1)^2,x, algorithm="maxima")
```

```
[Out] -x/(x^5 + x + 1)
```

mupad [B] time = 0.06, size = 11, normalized size = 1.00

$$-\frac{x}{x^5+x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^5 - 1)/(x + x^5 + 1)^2,x)
```

```
[Out] -x/(x + x^5 + 1)
```

sympy [A] time = 0.12, size = 8, normalized size = 0.73

$$-\frac{x}{x^5+x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**5-1)/(x**5+x+1)**2,x)
```

```
[Out] -x/(x**5 + x + 1)
```

3.140 $\int \frac{1}{5-\cos(x)+2\sin(x)}\,dx$

Optimal. Leaf size=45

$$\frac{x}{2\sqrt{5}}+\frac{\tan^{-1}\left(\frac{\sin(x)+2\cos(x)}{2\sin(x)-\cos(x)+2\sqrt{5}+5}\right)}{\sqrt{5}}$$

```
[Out] 1/10*x*5^(1/2)+1/5*arctan((2*cos(x)+sin(x))/(5-cos(x)+2*sin(x)+2*5^(1/2)))*
5^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3124, 618, 204}

$$\frac{x}{2\sqrt{5}}+\frac{\tan^{-1}\left(\frac{\sin(x)+2\cos(x)}{2\sin(x)-\cos(x)+2\sqrt{5}+5}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(5 - Cos[x] + 2*Sin[x])^(-1),x]
```

```
[Out] x/(2*Sqrt[5]) + ArcTan[(2*Cos[x] + Sin[x])/(5 + 2*Sqrt[5] - Cos[x] + 2*Sin[
x])]/Sqrt[5]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{5-\cos(x)+2\sin(x)}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{4+4x+6x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)\\ &= -\left(4\,\text{Subst}\left(\int \frac{1}{-80-x^2}\,dx, x, 4+12\tan\left(\frac{x}{2}\right)\right)\right)\\ &= \frac{x}{2\sqrt{5}}+\frac{\tan^{-1}\left(\frac{2\cos(x)+\sin(x)}{5+2\sqrt{5}-\cos(x)+2\sin(x)}\right)}{\sqrt{5}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 23, normalized size = 0.51

$$\frac{\tan^{-1}\left(\frac{3\tan\left(\frac{x}{2}\right)+1}{\sqrt{5}}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 - Cos[x] + 2*Sin[x])^(-1),x]
```

```
[Out] ArcTan[(1 + 3*Tan[x/2])/Sqrt[5]]/Sqrt[5]
```

fricas [A] time = 0.42, size = 36, normalized size = 0.80

$$\frac{1}{10}\sqrt{5}\arctan\left(-\frac{\sqrt{5}\cos(x)-2\sqrt{5}\sin(x)-\sqrt{5}}{2(2\cos(x)+\sin(x))}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5-cos(x)+2*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/10*sqrt(5)*arctan(-1/2*(sqrt(5)*cos(x) - 2*sqrt(5)*sin(x) - sqrt(5))/(2*c
os(x) + sin(x)))
```

giac [A] time = 0.01, size = 47, normalized size = 1.04

$$\frac{1}{10}\sqrt{5}\left(x+2\arctan\left(-\frac{\sqrt{5}\sin(x)-\cos(x)-3\sin(x)-1}{\sqrt{5}\cos(x)+\sqrt{5}-3\cos(x)+\sin(x)+3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5-cos(x)+2*sin(x)),x, algorithm="giac")
```

```
[Out] 1/10*sqrt(5)*(x + 2*arctan(-(sqrt(5)*sin(x) - cos(x) - 3*sin(x) - 1)/(sqrt(
5)*cos(x) + sqrt(5) - 3*cos(x) + sin(x) + 3)))
```

maple [A] time = 0.06, size = 20, normalized size = 0.44

$$\frac{\sqrt{5}\arctan\left(\frac{\left(6\tan\left(\frac{x}{2}\right)+2\right)\sqrt{5}}{10}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5-cos(x)+2*sin(x)),x)
```

```
[Out] 1/5*5^(1/2)*arctan(1/10*(6*tan(1/2*x)+2)*5^(1/2))
```

maxima [A] time = 1.27, size = 23, normalized size = 0.51

$$\frac{1}{5}\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}\left(\frac{3\sin(x)}{\cos(x)+1}+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5-cos(x)+2*sin(x)),x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(5)*arctan(1/5*sqrt(5)*(3*sin(x)/(cos(x) + 1) + 1))
```

mupad [B] time = 0.10, size = 21, normalized size = 0.47

$$\frac{\sqrt{20}\,\mathrm{atan}\left(\frac{3\sqrt{20}\tan\left(\frac{x}{2}\right)}{10}+\frac{\sqrt{20}}{10}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*sin(x) - cos(x) + 5),x)
```

```
[Out] (20^(1/2)*atan((3*20^(1/2)*tan(x/2))/10 + 20^(1/2)/10))/10
```

sympy **[A]** time = 0.49, size = 39, normalized size = 0.87

$$\frac{\sqrt{5}\left(\operatorname{atan}\left(\frac{3\sqrt{5}\tan\left(\frac{x}{2}\right)}{5}+\frac{\sqrt{5}}{5}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5-cos(x)+2*sin(x)),x)
```

```
[Out] sqrt(5)*(atan(3*sqrt(5)*tan(x/2)/5 + sqrt(5)/5) + pi*floor((x/2 - pi/2)/pi)
)/5
```

3.141 $\int \frac{1}{1+a\cos(x)}\,dx$

Optimal. Leaf size=37

$$\frac{2\tan^{-1}\left(\frac{\sqrt{1-a}\,\tan\left(\frac{x}{2}\right)}{\sqrt{a+1}}\right)}{\sqrt{1-a^2}}$$

```
[Out] 2*arctan((1-a)^(1/2)*tan(1/2*x)/(1+a)^(1/2))/(-a^2+1)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2659, 205}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{1-a}\,\tan\left(\frac{x}{2}\right)}{\sqrt{a+1}}\right)}{\sqrt{1-a^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a*Cos[x])^(-1),x]
```

```
[Out] (2*ArcTan[(Sqrt[1 - a]*Tan[x/2])/Sqrt[1 + a]])/Sqrt[1 - a^2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+a\cos(x)}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{1+a+(1-a)x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)\\ &= \frac{2\tan^{-1}\left(\frac{\sqrt{1-a}\,\tan\left(\frac{x}{2}\right)}{\sqrt{1+a}}\right)}{\sqrt{1-a^2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 31, normalized size = 0.84

$$\frac{2\tanh^{-1}\left(\frac{(a-1)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-1}}\right)}{\sqrt{a^2-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a*Cos[x])^(-1),x]
```

```
[Out] (2*ArcTanh[((-1 + a)*Tan[x/2])/Sqrt[-1 + a^2]])/Sqrt[-1 + a^2]
```

fricas [**A**] time = 0.46, size = 111, normalized size = 3.00

$$\left[\frac{\log\left(-\frac{\left(a^2-2\right)\cos(x)^2-2\sqrt{a^2-1}\,(a+\cos(x))\sin(x)-2\,a^2-2\,a\cos(x)+1}{a^2\cos(x)^2+2\,a\cos(x)+1}\right)}{2\sqrt{a^2-1}}, -\frac{\sqrt{-a^2+1}\,\arctan\left(\frac{\sqrt{-a^2+1}\,(a+\cos(x))}{\left(a^2-1\right)\sin(x)}\right)}{a^2-1}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+a*cos(x)),x, algorithm="fricas")
```

```
[Out] [1/2*log(-((a^2 - 2)*cos(x)^2 - 2*sqrt(a^2 - 1)*(a + cos(x))*sin(x) - 2*a^2
 - 2*a*cos(x) + 1)/(a^2*cos(x)^2 + 2*a*cos(x) + 1))/sqrt(a^2 - 1), -sqrt(-a
^2 + 1)*arctan(sqrt(-a^2 + 1)*(a + cos(x))/((a^2 - 1)*sin(x)))/(a^2 - 1)]
```

giac [**A**] time = 0.01, size = 53, normalized size = 1.43

$$-\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2\,a-2)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,x\right)-\tan\left(\frac{1}{2}\,x\right)}{\sqrt{-a^2+1}}\right)\right)}{\sqrt{-a^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+a*cos(x)),x, algorithm="giac")
```

```
[Out] -2*(pi*floor(1/2*x/pi + 1/2)*sgn(2*a - 2) + arctan((a*tan(1/2*x) - tan(1/2*
x))/sqrt(-a^2 + 1)))/sqrt(-a^2 + 1)
```

maple [**A**] time = 0.02, size = 30, normalized size = 0.81

$$\frac{2\,\operatorname{arctanh}\left(\frac{(a-1)\tan\left(\frac{x}{2}\right)}{\sqrt{(a+1)(a-1)}}\right)}{\sqrt{(a+1)\,(a-1)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+a*cos(x)),x)
```

```
[Out] 2/((1+a)*(a-1))^(1/2)*arctanh((a-1)*tan(1/2*x)/((1+a)*(a-1))^(1/2))
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+a*cos(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a^2-1.0>0)', see `assume?` for more
 details)Is a^2-1.0 positive or negative?
```

mupad [**B**] time = 0.32, size = 28, normalized size = 0.76

$$\frac{2\,\operatorname{atanh}\left(\frac{\tan\left(\frac{x}{2}\right)\sqrt{a-1}}{\sqrt{a+1}}\right)}{\sqrt{a-1}\,\sqrt{a+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(x) + 1),x)
```

```
[Out] (2*atanh((tan(x/2)*(a - 1)^(1/2))/(a + 1)^(1/2)))/((a - 1)^(1/2)*(a + 1)^(1
/2))
```

sympy [A] time = 3.19, size = 110, normalized size = 2.97

$$\begin{cases} \tan\left(\frac{x}{2}\right) & \text{for } a = 1 \\ -\frac{1}{\tan\left(\frac{x}{2}\right)} & \text{for } a = -1 \\ -\frac{\log\left(-\sqrt{\frac{a}{a-1}+\frac{1}{a-1}}+\tan\left(\frac{x}{2}\right)\right)}{a\sqrt{\frac{a}{a-1}+\frac{1}{a-1}}-\sqrt{\frac{a}{a-1}+\frac{1}{a-1}}} + \frac{\log\left(\sqrt{\frac{a}{a-1}+\frac{1}{a-1}}+\tan\left(\frac{x}{2}\right)\right)}{a\sqrt{\frac{a}{a-1}+\frac{1}{a-1}}-\sqrt{\frac{a}{a-1}+\frac{1}{a-1}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+a*cos(x)),x)
```

```
[Out] Piecewise((tan(x/2), Eq(a, 1)), (-1/tan(x/2), Eq(a, -1)), (-log(-sqrt(a/(a
- 1) + 1/(a - 1)) + tan(x/2))/(a*sqrt(a/(a - 1) + 1/(a - 1)) - sqrt(a/(a -
1) + 1/(a - 1))) + log(sqrt(a/(a - 1) + 1/(a - 1)) + tan(x/2))/(a*sqrt(a/(a
 - 1) + 1/(a - 1)) - sqrt(a/(a - 1) + 1/(a - 1))), True))
```

3.142 $\int \frac{1}{1+2\cos(x)}\,dx$

Optimal. Leaf size=56

$$\frac{\log\left(\sin\left(\frac{x}{2}\right)+\sqrt{3}\cos\left(\frac{x}{2}\right)\right)}{\sqrt{3}}-\frac{\log\left(\sqrt{3}\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)}{\sqrt{3}}$$

```
[Out] -1/3*ln(-sin(1/2*x)+cos(1/2*x)*3^(1/2))*3^(1/2)+1/3*ln(sin(1/2*x)+cos(1/2*x
)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2659, 206}

$$\frac{\log\left(\sin\left(\frac{x}{2}\right)+\sqrt{3}\cos\left(\frac{x}{2}\right)\right)}{\sqrt{3}}-\frac{\log\left(\sqrt{3}\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*Cos[x])^(-1),x]
```

```
[Out] -(Log[Sqrt[3]*Cos[x/2] - Sin[x/2]]/Sqrt[3]) + Log[Sqrt[3]*Cos[x/2] + Sin[x/
2]]/Sqrt[3]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+2\cos(x)}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{3-x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)\\ &= -\frac{\log\left(\sqrt{3}\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)}{\sqrt{3}}+\frac{\log\left(\sqrt{3}\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 20, normalized size = 0.36

$$\frac{2\tanh^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*Cos[x])^(-1),x]
```

```
[Out] (2*ArcTanh[Tan[x/2]/Sqrt[3]])/Sqrt[3]
```

fricas [A] time = 0.41, size = 50, normalized size = 0.89

$$\frac{1}{6}\,\sqrt{3}\,\log\left(-\frac{2\,\cos(x)^2 - 2\left(\sqrt{3}\,\cos(x) + 2\,\sqrt{3}\right)\sin(x) - 4\,\cos(x) - 7}{4\,\cos(x)^2 + 4\,\cos(x) + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*cos(x)),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log(-(2*cos(x)^2 - 2*(sqrt(3)*cos(x) + 2*sqrt(3))*sin(x) - 4*co
s(x) - 7)/(4*cos(x)^2 + 4*cos(x) + 1))
```

giac [A] time = 0.03, size = 35, normalized size = 0.62

$$-\frac{1}{3}\,\sqrt{3}\,\log\left(\frac{\left|-2\,\sqrt{3} + 2\,\tan\left(\frac{1}{2}\,x\right)\right|}{\left|2\,\sqrt{3} + 2\,\tan\left(\frac{1}{2}\,x\right)\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*cos(x)),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*log(abs(-2*sqrt(3) + 2*tan(1/2*x))/abs(2*sqrt(3) + 2*tan(1/2*x
)))
```

maple [A] time = 0.02, size = 16, normalized size = 0.29

$$\frac{2\sqrt{3}\ \operatorname{arctanh}\left(\frac{\sqrt{3}\ \tan\left(\frac{x}{2}\right)}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+2*cos(x)),x)
```

```
[Out] 2/3*3^(1/2)*arctanh(1/3*3^(1/2)*tan(1/2*x))
```

maxima [A] time = 1.35, size = 37, normalized size = 0.66

$$-\frac{1}{3}\,\sqrt{3}\,\log\left(-\frac{\sqrt{3} - \frac{\sin(x)}{\cos(x)+1}}{\sqrt{3} + \frac{\sin(x)}{\cos(x)+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*cos(x)),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*log(-(sqrt(3) - sin(x)/(cos(x) + 1))/(sqrt(3) + sin(x)/(cos(x)
 + 1)))
```

mupad [B] time = 0.24, size = 15, normalized size = 0.27

$$\frac{2\,\sqrt{3}\,\operatorname{atanh}\left(\frac{\sqrt{3}\,\tan\left(\frac{x}{2}\right)}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*cos(x) + 1),x)
```

```
[Out] (2*3^(1/2)*atanh((3^(1/2)*tan(x/2))/3))/3
```

sympy [A] time = 0.34, size = 36, normalized size = 0.64

$$-\frac{\sqrt{3}\log\left(\tan\left(\frac{x}{2}\right)-\sqrt{3}\right)}{3}+\frac{\sqrt{3}\log\left(\tan\left(\frac{x}{2}\right)+\sqrt{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*cos(x)),x)
[Out] -sqrt(3)*log(tan(x/2) - sqrt(3))/3 + sqrt(3)*log(tan(x/2) + sqrt(3))/3
```

3.143 $\int \frac{1}{1+\frac{\cos(x)}{2}} \, dx$

Optimal. Leaf size=31

$$\frac{2x}{\sqrt{3}} - \frac{4\tan^{-1}\left(\frac{\sin(x)}{\cos(x)+\sqrt{3}+2}\right)}{\sqrt{3}}$$

```
[Out] 2/3*x*3^(1/2)-4/3*arctan(sin(x)/(2+cos(x)+3^(1/2)))*3^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2657}

$$\frac{2x}{\sqrt{3}} - \frac{4\tan^{-1}\left(\frac{\sin(x)}{\cos(x)+\sqrt{3}+2}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[x]/2)^(-1),x]
```

```
[Out] (2*x)/Sqrt[3] - (4*ArcTan[Sin[x]/(2 + Sqrt[3] + Cos[x])])/Sqrt[3]
```

Rule 2657

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2*ArcTan[(b*Cos[c + d*x])/(a + q + b*S
in[c + d*x])])/(d*q), x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rubi steps

$$\int \frac{1}{1+\frac{\cos(x)}{2}} \, dx = \frac{2x}{\sqrt{3}} - \frac{4\tan^{-1}\left(\frac{\sin(x)}{2+\sqrt{3}+\cos(x)}\right)}{\sqrt{3}}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 0.65

$$\frac{4\tan^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cos[x]/2)^(-1),x]
```

```
[Out] (4*ArcTan[Tan[x/2]/Sqrt[3]])/Sqrt[3]
```

fricas [A] time = 0.43, size = 23, normalized size = 0.74

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{2\sqrt{3}\cos(x)+\sqrt{3}}{3\sin(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+1/2*cos(x)),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(3)*arctan(1/3*(2*sqrt(3)*cos(x) + sqrt(3))/sin(x))
```

giac [A] time = 0.01, size = 40, normalized size = 1.29

$$\frac{2}{3}\sqrt{3}\left(x+2\arctan\left(-\frac{\sqrt{3}\sin(x)-\sin(x)}{\sqrt{3}\cos(x)+\sqrt{3}-\cos(x)+1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+1/2*cos(x)),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(3)*(x + 2*arctan(-(sqrt(3)*sin(x) - sin(x))/(sqrt(3)*cos(x) + sqrt
(3) - cos(x) + 1)))
```

maple [A] time = 0.03, size = 16, normalized size = 0.52

$$\frac{4\sqrt{3}\arctan\left(\frac{\sqrt{3}\tan\left(\frac{x}{2}\right)}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+1/2*cos(x)),x)
```

```
[Out] 4/3*3^(1/2)*arctan(1/3*3^(1/2)*tan(1/2*x))
```

maxima [A] time = 1.30, size = 19, normalized size = 0.61

$$\frac{4}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}\sin(x)}{3(\cos(x)+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+1/2*cos(x)),x, algorithm="maxima")
```

```
[Out] 4/3*sqrt(3)*arctan(1/3*sqrt(3)*sin(x)/(cos(x) + 1))
```

mupad [B] time = 0.22, size = 32, normalized size = 1.03

$$\frac{4\sqrt{3}\left(\frac{x}{2}-\mathrm{atan}\left(\tan\left(\frac{x}{2}\right)\right)\right)}{3}+\frac{4\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\tan\left(\frac{x}{2}\right)}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)/2 + 1),x)
```

```
[Out] (4*3^(1/2)*(x/2 - atan(tan(x/2))))/3 + (4*3^(1/2)*atan((3^(1/2)*tan(x/2))/3
))/3
```

sympy [A] time = 0.29, size = 32, normalized size = 1.03

$$\frac{4\sqrt{3}\left(\mathrm{atan}\left(\frac{\sqrt{3}\tan\left(\frac{x}{2}\right)}{3}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+1/2*cos(x)),x)
```

```
[Out] 4*sqrt(3)*(atan(sqrt(3)*tan(x/2)/3) + pi*floor((x/2 - pi/2)/pi))/3
```

3.144 $\int \frac{\sin^2(x)}{1+\sin^2(x)}\,dx$

Optimal. Leaf size=36

$$-\frac{x}{\sqrt{2}}+x-\frac{\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\sin^2(x)+\sqrt{2}+1}\right)}{\sqrt{2}}$$

```
[Out] x-1/2*x*2^(1/2)-1/2*arctan(cos(x)*sin(x)/(1+sin(x)^2+2^(1/2)))*2^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3171, 3181, 203}

$$-\frac{x}{\sqrt{2}}+x-\frac{\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\sin^2(x)+\sqrt{2}+1}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2/(1 + Sin[x]^2),x]
```

```
[Out] x - x/Sqrt[2] - ArcTan[(Cos[x]*Sin[x])/(1 + Sqrt[2] + Sin[x]^2)]/Sqrt[2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3171

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/((a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[(B*x)/b, x] + Dist[(A*b - a*B)/b, Int[1/(a +
 b*Sin[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^2(x)}{1+\sin^2(x)}\,dx &= x-\int \frac{1}{1+\sin^2(x)}\,dx\\ &= x-\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx,x,\tan(x)\right)\\ &= x-\frac{x}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\sin^2(x)}\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 18, normalized size = 0.50

$$x-\frac{\tan^{-1}\left(\sqrt{2}\tan(x)\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2/(1 + Sin[x]^2),x]
```

```
[Out] x - ArcTan[Sqrt[2]*Tan[x]]/Sqrt[2]
```

fricas [A] time = 0.44, size = 33, normalized size = 0.92

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos(x)^2 - 2\sqrt{2}}{4\cos(x)\sin(x)}\right) + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(1+sin(x)^2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - 2*sqrt(2))/(cos(x)*sin(x))) +
x
```

giac [A] time = 0.01, size = 48, normalized size = 1.33

$$-\frac{1}{2}\sqrt{2}\left(x + \arctan\left(-\frac{\sqrt{2}\sin(2x) - 2\sin(2x)}{\sqrt{2}\cos(2x) + \sqrt{2} - 2\cos(2x) + 2}\right)\right) + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(1+sin(x)^2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - 2*sin(2*x))/(sqrt(2)*cos(2*x)
 + sqrt(2) - 2*cos(2*x) + 2))) + x
```

maple [A] time = 0.04, size = 15, normalized size = 0.42

$$x - \frac{\sqrt{2}\arctan\left(\sqrt{2}\tan(x)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(1+sin(x)^2),x)
```

```
[Out] -1/2*2^(1/2)*arctan(tan(x)*2^(1/2))+x
```

maxima [A] time = 1.23, size = 14, normalized size = 0.39

$$-\frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}\tan(x)\right) + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(1+sin(x)^2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*arctan(sqrt(2)*tan(x)) + x
```

mupad [B] time = 0.21, size = 26, normalized size = 0.72

$$x - \frac{\sqrt{2}\left(x - \mathrm{atan}\left(\tan(x)\right)\right)}{2} - \frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\tan(x)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(sin(x)^2 + 1),x)
```

```
[Out] x - (2^(1/2)*(x - atan(tan(x))))/2 - (2^(1/2)*atan(2^(1/2)*tan(x)))/2
```

sympy [B] time = 57.58, size = 248, normalized size = 6.89

$$\frac{31988856\sqrt{2}\,x}{31988856\sqrt{2}+45239074}+\frac{45239074x}{31988856\sqrt{2}+45239074}-\frac{77227930\sqrt{3-2\sqrt{2}}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{3-2\sqrt{2}}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{31988856\sqrt{2}+45239074}-\frac{546}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2/(1+sin(x)**2),x)
```

```
[Out] 31988856*sqrt(2)*x/(31988856*sqrt(2) + 45239074) + 45239074*x/(31988856*sqr
t(2) + 45239074) - 77227930*sqrt(3 - 2*sqrt(2))*(atan(tan(x/2)/sqrt(3 - 2*s
qrt(2))) + pi*floor((x/2 - pi/2)/pi))/(31988856*sqrt(2) + 45239074) - 54608
393*sqrt(2)*sqrt(3 - 2*sqrt(2))*(atan(tan(x/2)/sqrt(3 - 2*sqrt(2))) + pi*fl
oor((x/2 - pi/2)/pi))/(31988856*sqrt(2) + 45239074) - 13250218*sqrt(2*sqrt(
2) + 3)*(atan(tan(x/2)/sqrt(2*sqrt(2) + 3)) + pi*floor((x/2 - pi/2)/pi))/(3
1988856*sqrt(2) + 45239074) - 9369319*sqrt(2)*sqrt(2*sqrt(2) + 3)*(atan(tan
(x/2)/sqrt(2*sqrt(2) + 3)) + pi*floor((x/2 - pi/2)/pi))/(31988856*sqrt(2) +
 45239074)
```

3.145 $\int \frac{1}{b^2\cos^2(x)+a^2\sin^2(x)}\,dx$

Optimal. Leaf size=15

$$\frac{\tan^{-1}\left(\frac{a\tan(x)}{b}\right)}{ab}$$

```
[Out] arctan(a*tan(x)/b)/a/b
```

Rubi [A] time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {205}

$$\frac{\tan^{-1}\left(\frac{a\tan(x)}{b}\right)}{ab}$$

Antiderivative was successfully verified.

```
[In] Int[(b^2*Cos[x]^2 + a^2*Sin[x]^2)^(-1),x]
```

```
[Out] ArcTan[(a*Tan[x])/b]/(a*b)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{b^2\cos^2(x)+a^2\sin^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{b^2+a^2x^2}\,dx, x, \tan(x)\right)\\ &= \frac{\tan^{-1}\left(\frac{a\tan(x)}{b}\right)}{ab}\end{aligned}$$

Mathematica [A] time = 0.05, size = 15, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{a\tan(x)}{b}\right)}{ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b^2*Cos[x]^2 + a^2*Sin[x]^2)^(-1),x]
```

```
[Out] ArcTan[(a*Tan[x])/b]/(a*b)
```

fricas [B] time = 0.45, size = 43, normalized size = 2.87

$$-\frac{\arctan\left(\frac{\left(a^2+b^2\right)\cos(x)^2-a^2}{2\,ab\cos(x)\sin(x)}\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*cos(x)^2+a^2*sin(x)^2),x, algorithm="fricas")
```

```
[Out] -1/2*arctan(1/2*((a^2 + b^2)*cos(x)^2 - a^2)/(a*b*cos(x)*sin(x)))/(a*b)
```

giac [**A**] time = 0.02, size = 26, normalized size = 1.73

$$\frac{\pi \left\lfloor \frac{x}{\pi} + \frac{1}{2} \right\rfloor + \arctan\left(\frac{a \tan(x)}{b}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*cos(x)^2+a^2*sin(x)^2),x, algorithm="giac")
[Out] (pi*floor(x/pi + 1/2) + arctan(a*tan(x)/b))/(a*b)
```

maple [**A**] time = 0.07, size = 16, normalized size = 1.07

$$\frac{\arctan\left(\frac{a \tan(x)}{b}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*cos(x)^2+a^2*sin(x)^2),x)
[Out] arctan(a*tan(x)/b)/a/b
```

maxima [**A**] time = 1.14, size = 15, normalized size = 1.00

$$\frac{\arctan\left(\frac{a \tan(x)}{b}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*cos(x)^2+a^2*sin(x)^2),x, algorithm="maxima")
[Out] arctan(a*tan(x)/b)/(a*b)
```

mupad [**B**] time = 0.46, size = 15, normalized size = 1.00

$$\frac{\mathrm{atan}\left(\frac{a \tan(x)}{b}\right)}{a\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*cos(x)^2 + a^2*sin(x)^2),x)
[Out] atan((a*tan(x))/b)/(a*b)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*cos(x)**2+a**2*sin(x)**2),x)
[Out] Timed out
```

3.146 $\int \frac{1}{(b\cos(x)+a\sin(x))^2}\,dx$

Optimal. Leaf size=17

$$\frac{\sin(x)}{b(a\sin(x)+b\cos(x))}$$

```
[Out] sin(x)/b/(b*cos(x)+a*sin(x))
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3075}

$$\frac{\sin(x)}{b(a\sin(x)+b\cos(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[x] + a*Sin[x])^(-2),x]
```

```
[Out] Sin[x]/(b*(b*Cos[x] + a*Sin[x]))
```

Rule 3075

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(b\cos(x)+a\sin(x))^2}\,dx = \frac{\sin(x)}{b(b\cos(x)+a\sin(x))}$$

Mathematica [A] time = 0.03, size = 17, normalized size = 1.00

$$\frac{\sin(x)}{b(a\sin(x)+b\cos(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[x] + a*Sin[x])^(-2),x]
```

```
[Out] Sin[x]/(b*(b*Cos[x] + a*Sin[x]))
```

fricas [B] time = 0.41, size = 39, normalized size = 2.29

$$-\frac{a\cos(x)-b\sin(x)}{(a^2b+b^3)\cos(x)+(a^3+ab^2)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(x)+a*sin(x))^2,x, algorithm="fricas")
```

```
[Out] -(a*cos(x) - b*sin(x))/((a^2*b + b^3)*cos(x) + (a^3 + a*b^2)*sin(x))
```

giac [A] time = 0.02, size = 13, normalized size = 0.76

$$-\frac{1}{(a\tan(x)+b)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(x)+a*sin(x))^2,x, algorithm="giac")
[Out] -1/((a*tan(x) + b)*a)
```

maple [A] time = 0.15, size = 14, normalized size = 0.82

$$-\frac{1}{(a\tan(x)+b)\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(x)+a*sin(x))^2,x)
[Out] -1/a/(a*tan(x)+b)
```

maxima [A] time = 0.57, size = 14, normalized size = 0.82

$$-\frac{1}{a^2\tan(x)+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(x)+a*sin(x))^2,x, algorithm="maxima")
[Out] -1/(a^2*tan(x) + a*b)
```

mupad [B] time = 0.63, size = 29, normalized size = 1.71

$$\frac{2\tan\left(\frac{x}{2}\right)}{b\left(-b\tan\left(\frac{x}{2}\right)^2+2\,a\tan\left(\frac{x}{2}\right)+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(x) + a*sin(x))^2,x)
[Out] (2*tan(x/2))/(b*(b + 2*a*tan(x/2) - b*tan(x/2)^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(a\sin(x)+b\cos(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(x)+a*sin(x))**2,x)
[Out] Integral((a*sin(x) + b*cos(x))**(-2), x)
```

3.147 $\int \frac{\sin(x)}{1+\cos(x)+\sin(x)}\,dx$

Optimal. Leaf size=30

$$\frac{x}{2}-\frac{1}{2}\log\left(\tan\left(\frac{x}{2}\right)+1\right)-\frac{1}{2}\log(\sin(x)+\cos(x)+1)$$

```
[Out] 1/2*x-1/2*ln(1+cos(x)+sin(x))-1/2*ln(1+tan(1/2*x))
```

Rubi [A] time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3137, 3124, 31}

$$\frac{x}{2}-\frac{1}{2}\log\left(\tan\left(\frac{x}{2}\right)+1\right)-\frac{1}{2}\log(\sin(x)+\cos(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(1 + Cos[x] + Sin[x]),x]
```

```
[Out] x/2 - Log[1 + Cos[x] + Sin[x]]/2 - Log[1 + Tan[x/2]]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3137

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[(c*C*(d + e*x))
/(e*(b^2 + c^2)), x] + (Dist[(A*(b^2 + c^2) - a*c*C)/(b^2 + c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] - Simp[(b*C*Log[a + b*Cos[d + e*
x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x]) /; FreeQ[{a, b, c, d, e, A, C},
x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*c*C, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(x)}{1+\cos(x)+\sin(x)}\,dx &= \frac{x}{2}-\frac{1}{2}\log(1+\cos(x)+\sin(x))-\frac{1}{2}\int \frac{1}{1+\cos(x)+\sin(x)}\,dx\\ &= \frac{x}{2}-\frac{1}{2}\log(1+\cos(x)+\sin(x))-\text{Subst}\left(\int \frac{1}{2+2x}\,dx,x,\tan\left(\frac{x}{2}\right)\right)\\ &= \frac{x}{2}-\frac{1}{2}\log(1+\cos(x)+\sin(x))-\frac{1}{2}\log\left(1+\tan\left(\frac{x}{2}\right)\right)\end{aligned}$$

Mathematica [A] time = 0.04, size = 22, normalized size = 0.73

$$\frac{x}{2}-\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(1 + Cos[x] + Sin[x]),x]
[Out] x/2 - Log[Cos[x/2] + Sin[x/2]]
```

fricas [A] time = 0.42, size = 11, normalized size = 0.37

$$\frac{1}{2}\,x - \frac{1}{2}\,\log\left(\sin(x) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+cos(x)+sin(x)),x, algorithm="fricas")
[Out] 1/2*x - 1/2*log(sin(x) + 1)
```

giac [A] time = 0.02, size = 25, normalized size = 0.83

$$\frac{1}{2}\,x + \frac{1}{2}\,\log\left(\tan\left(\frac{1}{2}\,x\right)^{2} + 1\right) - \log\left(\left|\tan\left(\frac{1}{2}\,x\right) + 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+cos(x)+sin(x)),x, algorithm="giac")
[Out] 1/2*x + 1/2*log(tan(1/2*x)^2 + 1) - log(abs(tan(1/2*x) + 1))
```

maple [A] time = 0.08, size = 25, normalized size = 0.83

$$\frac{x}{2} - \ln\left(\tan\left(\frac{x}{2}\right) + 1\right) + \frac{\ln\left(\tan^{2}\left(\frac{x}{2}\right) + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(1+cos(x)+sin(x)),x)
[Out] -ln(1+tan(1/2*x))+1/2*ln(tan(1/2*x)^2+1)+1/2*x
```

maxima [A] time = 1.15, size = 41, normalized size = 1.37

$$\arctan\left(\frac{\sin(x)}{\cos(x) + 1}\right) - \log\left(\frac{\sin(x)}{\cos(x) + 1} + 1\right) + \frac{1}{2}\,\log\left(\frac{\sin(x)^{2}}{\left(\cos(x) + 1\right)^{2}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+cos(x)+sin(x)),x, algorithm="maxima")
[Out] arctan(sin(x)/(cos(x) + 1)) - log(sin(x)/(cos(x) + 1) + 1) + 1/2*log(sin(x)
^2/(cos(x) + 1)^2 + 1)
```

mupad [B] time = 0.31, size = 34, normalized size = 1.13

$$-\ln\left(\tan\left(\frac{x}{2}\right) + 1\right) + \ln\left(\tan\left(\frac{x}{2}\right) - \mathrm{i}\right)\left(\frac{1}{2} - \frac{1}{2}\mathrm{i}\right) + \ln\left(\tan\left(\frac{x}{2}\right) + 1\mathrm{i}\right)\left(\frac{1}{2} + \frac{1}{2}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(cos(x) + sin(x) + 1),x)
[Out] log(tan(x/2) - 1i)*(1/2 - 1i/2) - log(tan(x/2) + 1) + log(tan(x/2) + 1i)*(1
/2 + 1i/2)
```

sympy [A] time = 0.32, size = 22, normalized size = 0.73

$$\frac{x}{2} - \log\left(\tan\left(\frac{x}{2}\right) + 1\right) + \frac{\log\left(\tan^{2}\left(\frac{x}{2}\right) + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+cos(x)+sin(x)),x)
```

```
[Out] x/2 - log(tan(x/2) + 1) + log(tan(x/2)**2 + 1)/2
```

3.148 $\int \sqrt{3-x^2}\,dx$

Optimal. Leaf size=29

$$\frac{1}{2}\sqrt{3-x^2}\,x+\frac{3}{2}\sin^{-1}\left(\frac{x}{\sqrt{3}}\right)$$

```
[Out] 3/2*arcsin(1/3*x*3^(1/2))+1/2*x*(-x^2+3)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {195, 216}

$$\frac{1}{2}\sqrt{3-x^2}\,x+\frac{3}{2}\sin^{-1}\left(\frac{x}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - x^2],x]
[Out] (x*Sqrt[3 - x^2])/2 + (3*ArcSin[x/Sqrt[3]])/2
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{3-x^2}\,dx &= \frac{1}{2}x\sqrt{3-x^2}+\frac{3}{2}\int \frac{1}{\sqrt{3-x^2}}\,dx\\ &= \frac{1}{2}x\sqrt{3-x^2}+\frac{3}{2}\sin^{-1}\left(\frac{x}{\sqrt{3}}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.00

$$\frac{1}{2}\sqrt{3-x^2}\,x+\frac{3}{2}\sin^{-1}\left(\frac{x}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - x^2],x]
[Out] (x*Sqrt[3 - x^2])/2 + (3*ArcSin[x/Sqrt[3]])/2
```

fricas [A] time = 0.41, size = 29, normalized size = 1.00

$$\frac{1}{2}\sqrt{-x^2+3}\,x-\frac{3}{2}\arctan\left(\frac{\sqrt{-x^2+3}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(-x^2 + 3)*x - 3/2*arctan(sqrt(-x^2 + 3)/x)
```

giac [A] time = 0.01, size = 22, normalized size = 0.76

$$\frac{1}{2}\sqrt{-x^2+3}\,x+\frac{3}{2}\arcsin\left(\frac{1}{3}\sqrt{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(-x^2 + 3)*x + 3/2*arcsin(1/3*sqrt(3)*x)
```

maple [A] time = 0.00, size = 23, normalized size = 0.79

$$\frac{\sqrt{-x^2+3}\,x}{2}+\frac{3\arcsin\left(\frac{\sqrt{3}\,x}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+3)^(1/2),x)
[Out] 3/2*arcsin(1/3*x*3^(1/2))+1/2*x*(-x^2+3)^(1/2)
```

maxima [A] time = 1.14, size = 22, normalized size = 0.76

$$\frac{1}{2}\sqrt{-x^2+3}\,x+\frac{3}{2}\arcsin\left(\frac{1}{3}\sqrt{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(-x^2 + 3)*x + 3/2*arcsin(1/3*sqrt(3)*x)
```

mupad [B] time = 0.04, size = 22, normalized size = 0.76

$$\frac{3\,\mathrm{asin}\left(\frac{\sqrt{3}\,x}{3}\right)}{2}+\frac{x\,\sqrt{3-x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3 - x^2)^(1/2),x)
[Out] (3*asin((3^(1/2)*x)/3))/2 + (x*(3 - x^2)^(1/2))/2
```

sympy [A] time = 0.21, size = 24, normalized size = 0.83

$$\frac{x\sqrt{3-x^2}}{2}+\frac{3\,\mathrm{asin}\left(\frac{\sqrt{3}x}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+3)**(1/2),x)
[Out] x*sqrt(3 - x**2)/2 + 3*asin(sqrt(3)*x/3)/2
```

3.149 $\int \frac{x}{\sqrt{3-x^2}}\,dx$

Optimal. Leaf size=13

$$-\sqrt{3-x^2}$$

```
[Out] -(-x^2+3)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$-\sqrt{3-x^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[3 - x^2],x]
```

```
[Out] -Sqrt[3 - x^2]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\sqrt{3-x^2}}\,dx = -\sqrt{3-x^2}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$-\sqrt{3-x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[3 - x^2],x]
```

```
[Out] -Sqrt[3 - x^2]
```

fricas [A] time = 0.41, size = 11, normalized size = 0.85

$$-\sqrt{-x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-x^2 + 3)
```

giac [A] time = 0.01, size = 11, normalized size = 0.85

$$-\sqrt{-x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(-x^2 + 3)
```

maple [A] time = 0.00, size = 12, normalized size = 0.92

$$-\sqrt{-x^2 + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^2+3)^(1/2),x)
[Out] -(-x^2+3)^(1/2)
```

maxima [A] time = 0.48, size = 11, normalized size = 0.85

$$-\sqrt{-x^2 + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+3)^(1/2),x, algorithm="maxima")
[Out] -sqrt(-x^2 + 3)
```

mupad [B] time = 0.17, size = 11, normalized size = 0.85

$$-\sqrt{3 - x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(3 - x^2)^(1/2),x)
[Out] -(3 - x^2)^(1/2)
```

sympy [A] time = 0.15, size = 8, normalized size = 0.62

$$-\sqrt{3 - x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**2+3)**(1/2),x)
[Out] -sqrt(3 - x**2)
```

3.150 $\int \frac{\sqrt{3-x^2}}{x} \, dx$

Optimal. Leaf size=37

$$\sqrt{3-x^2}-\sqrt{3}\tanh^{-1}\left(\frac{\sqrt{3-x^2}}{\sqrt{3}}\right)$$

[Out] -arctanh(1/3*(-x^2+3)^(1/2)*3^(1/2))*3^(1/2)+(-x^2+3)^(1/2)

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 50, 63, 206}

$$\sqrt{3-x^2}-\sqrt{3}\tanh^{-1}\left(\frac{\sqrt{3-x^2}}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[3 - x^2]/x,x]

[Out] Sqrt[3 - x^2] - Sqrt[3]*ArcTanh[Sqrt[3 - x^2]/Sqrt[3]]

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{3-x^2}}{x} dx = \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{3-x}}{x} dx, x, x^2\right)$$
$$= \sqrt{3-x^2} + \frac{3}{2}\text{Subst}\left(\int \frac{1}{\sqrt{3-x}\,x} dx, x, x^2\right)$$
$$= \sqrt{3-x^2} - 3\,\text{Subst}\left(\int \frac{1}{3-x^2} dx, x, \sqrt{3-x^2}\right)$$
$$= \sqrt{3-x^2} - \sqrt{3}\tanh^{-1}\left(\frac{\sqrt{3-x^2}}{\sqrt{3}}\right)$$

Mathematica [A] time = 0.01, size = 33, normalized size = 0.89

$$\sqrt{3-x^2} - \sqrt{3}\tanh^{-1}\left(\sqrt{1-\frac{x^2}{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - x^2]/x,x]
[Out] Sqrt[3 - x^2] - Sqrt[3]*ArcTanh[Sqrt[1 - x^2/3]]
```

fricas [A] time = 0.40, size = 40, normalized size = 1.08

$$\frac{1}{2}\sqrt{3}\log\left(-\frac{x^2 + 2\sqrt{3}\sqrt{-x^2+3} - 6}{x^2}\right) + \sqrt{-x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)^(1/2)/x,x, algorithm="fricas")
[Out] 1/2*sqrt(3)*log(-(x^2 + 2*sqrt(3)*sqrt(-x^2 + 3) - 6)/x^2) + sqrt(-x^2 + 3)
```

giac [A] time = 0.01, size = 47, normalized size = 1.27

$$\frac{1}{2}\sqrt{3}\log\left(\frac{\sqrt{3} - \sqrt{-x^2+3}}{\sqrt{3} + \sqrt{-x^2+3}}\right) + \sqrt{-x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)^(1/2)/x,x, algorithm="giac")
[Out] 1/2*sqrt(3)*log((sqrt(3) - sqrt(-x^2 + 3))/(sqrt(3) + sqrt(-x^2 + 3))) + sq
rt(-x^2 + 3)
```

maple [A] time = 0.00, size = 30, normalized size = 0.81

$$-\sqrt{3}\,\operatorname{arctanh}\left(\frac{\sqrt{3}}{\sqrt{-x^2+3}}\right) + \sqrt{-x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+3)^(1/2)/x,x)
[Out] (-x^2+3)^(1/2)-3^(1/2)*arctanh(3^(1/2)/(-x^2+3)^(1/2))
```

maxima [A] time = 1.20, size = 41, normalized size = 1.11

$$-\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{-x^2+3}}{|x|} + \frac{6}{|x|}\right) + \sqrt{-x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] -sqrt(3)*log(2*sqrt(3)*sqrt(-x^2 + 3)/abs(x) + 6/abs(x)) + sqrt(-x^2 + 3)
```

mupad [B] time = 0.21, size = 35, normalized size = 0.95

$$\sqrt{3}\ \ln\left(\sqrt{\frac{3}{x^2}-1}-\sqrt{3}\ \sqrt{\frac{1}{x^2}}\right)+\sqrt{3-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3 - x^2)^(1/2)/x,x)
```

```
[Out] 3^(1/2)*log((3/x^2 - 1)^(1/2) - 3^(1/2)*(1/x^2)^(1/2)) + (3 - x^2)^(1/2)
```

sympy [A] time = 1.38, size = 88, normalized size = 2.38

$$\begin{cases} i\sqrt{x^2-3}-\sqrt{3}\log{(x)}+\frac{\sqrt{3}\log{(x^2)}}{2}+\sqrt{3}\,i\,\mathrm{asin}\left(\frac{\sqrt{3}}{x}\right) & \text{for}\ \frac{|x^2|}{3}>1\\ \sqrt{3-x^2}+\frac{\sqrt{3}\log{(x^2)}}{2}-\sqrt{3}\log{\left(\sqrt{1-\frac{x^2}{3}}+1\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+3)**(1/2)/x,x)
```

```
[Out] Piecewise((I*sqrt(x**2 - 3) - sqrt(3)*log(x) + sqrt(3)*log(x**2)/2 + sqrt(3
)*I*asin(sqrt(3)/x), Abs(x**2)/3 > 1), (sqrt(3 - x**2) + sqrt(3)*log(x**2)/
2 - sqrt(3)*log(sqrt(1 - x**2/3) + 1), True))
```

3.151 $\int \frac{\sqrt{x+x^2}}{x} dx$

Optimal. Leaf size=22

$$\sqrt{x^2+x}+\tanh^{-1}\left(\frac{x}{\sqrt{x^2+x}}\right)$$

```
[Out] arctanh(x/(x^2+x)^(1/2))+(x^2+x)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {664, 620, 206}

$$\sqrt{x^2+x}+\tanh^{-1}\left(\frac{x}{\sqrt{x^2+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x + x^2]/x,x]
```

```
[Out] Sqrt[x + x^2] + ArcTanh[x/Sqrt[x + x^2]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x+x^2}}{x} dx &= \sqrt{x+x^2}+\frac{1}{2}\int \frac{1}{\sqrt{x+x^2}} dx \\
&= \sqrt{x+x^2}+\text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \frac{x}{\sqrt{x+x^2}}\right) \\
&= \sqrt{x+x^2}+\tanh^{-1}\left(\frac{x}{\sqrt{x+x^2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 31, normalized size = 1.41

$$\sqrt{x(x+1)}\left(\frac{\sinh^{-1}\left(\sqrt{x}\right)}{\sqrt{x}\sqrt{x+1}}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x + x^2]/x,x]
```

```
[Out] Sqrt[x*(1 + x)]*(1 + ArcSinh[Sqrt[x]]/(Sqrt[x]*Sqrt[1 + x]))
```

fricas [A] time = 0.40, size = 25, normalized size = 1.14

$$\sqrt{x^2+x} - \frac{1}{2}\log\left(-2x + 2\sqrt{x^2+x} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] sqrt(x^2 + x) - 1/2*log(-2*x + 2*sqrt(x^2 + x) - 1)
```

giac [A] time = 0.02, size = 26, normalized size = 1.18

$$\sqrt{x^2+x} - \frac{1}{2}\log\left(\left|-2x + 2\sqrt{x^2+x} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x)^(1/2)/x,x, algorithm="giac")
```

```
[Out] sqrt(x^2 + x) - 1/2*log(abs(-2*x + 2*sqrt(x^2 + x) - 1))
```

maple [A] time = 0.01, size = 22, normalized size = 1.00

$$\frac{\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)}{2} + \sqrt{x^2+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x)^(1/2)/x,x)
```

```
[Out] (x^2+x)^(1/2)+1/2*ln(1/2+x+(x^2+x)^(1/2))
```

maxima [A] time = 0.51, size = 25, normalized size = 1.14

$$\sqrt{x^2+x} + \frac{1}{2}\log\left(2x + 2\sqrt{x^2+x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] sqrt(x^2 + x) + 1/2*log(2*x + 2*sqrt(x^2 + x) + 1)
```

mupad [B] time = 0.08, size = 21, normalized size = 0.95

$$\frac{\ln\left(x+\sqrt{x\,(x+1)}+\frac{1}{2}\right)}{2} + \sqrt{x^2+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2)^(1/2)/x,x)
```

```
[Out] log(x + (x*(x + 1))^(1/2) + 1/2)/2 + (x + x^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x\,(x+1)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(x*(x + 1))/x, x)
```

3.152 $\int \sqrt{5+x^2}\,dx$

Optimal. Leaf size=27

$$\frac{1}{2}\sqrt{x^2+5}\,x+\frac{5}{2}\sinh^{-1}\left(\frac{x}{\sqrt{5}}\right)$$

```
[Out] 5/2*arcsinh(1/5*x*5^(1/2))+1/2*x*(x^2+5)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {195, 215}

$$\frac{1}{2}\sqrt{x^2+5}\,x+\frac{5}{2}\sinh^{-1}\left(\frac{x}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[5 + x^2],x]
```

```
[Out] (x*Sqrt[5 + x^2])/2 + (5*ArcSinh[x/Sqrt[5]])/2
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{5+x^2}\,dx &= \frac{1}{2}x\sqrt{5+x^2}+\frac{5}{2}\int \frac{1}{\sqrt{5+x^2}}\,dx\\ &= \frac{1}{2}x\sqrt{5+x^2}+\frac{5}{2}\sinh^{-1}\left(\frac{x}{\sqrt{5}}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$\frac{1}{2}\sqrt{x^2+5}\,x+\frac{5}{2}\sinh^{-1}\left(\frac{x}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[5 + x^2],x]
```

```
[Out] (x*Sqrt[5 + x^2])/2 + (5*ArcSinh[x/Sqrt[5]])/2
```

fricas [A] time = 0.39, size = 25, normalized size = 0.93

$$\frac{1}{2}\sqrt{x^2+5}\,x-\frac{5}{2}\log\left(-x+\sqrt{x^2+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(x^2 + 5)*x - 5/2*log(-x + sqrt(x^2 + 5))
```

giac [A] time = 0.01, size = 25, normalized size = 0.93

$$\frac{1}{2}\sqrt{x^2+5}\,x-\frac{5}{2}\,\log\left(-x+\sqrt{x^2+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(x^2 + 5)*x - 5/2*log(-x + sqrt(x^2 + 5))
```

maple [A] time = 0.00, size = 21, normalized size = 0.78

$$\frac{\sqrt{x^2+5}\;x}{2}+\frac{5\,\mathrm{arcsinh}\left(\frac{\sqrt{5}\,x}{5}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+5)^(1/2),x)
[Out] 5/2*arcsinh(1/5*x*5^(1/2))+1/2*x*(x^2+5)^(1/2)
```

maxima [A] time = 1.27, size = 20, normalized size = 0.74

$$\frac{1}{2}\sqrt{x^2+5}\,x+\frac{5}{2}\,\mathrm{arsinh}\left(\frac{1}{5}\,\sqrt{5}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(x^2 + 5)*x + 5/2*arcsinh(1/5*sqrt(5)*x)
```

mupad [B] time = 0.09, size = 20, normalized size = 0.74

$$\frac{5\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x}{5}\right)}{2}+\frac{x\,\sqrt{x^2+5}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 5)^(1/2),x)
[Out] (5*asinh((5^(1/2)*x)/5))/2 + (x*(x^2 + 5)^(1/2))/2
```

sympy [A] time = 0.21, size = 24, normalized size = 0.89

$$\frac{x\sqrt{x^2+5}}{2}+\frac{5\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x}{5}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+5)**(1/2),x)
[Out] x*sqrt(x**2 + 5)/2 + 5*asinh(sqrt(5)*x/5)/2
```

3.153 $\int \frac{x}{\sqrt{1+x+x^2}} \, dx$

Optimal. Leaf size=27

$$\sqrt{x^2+x+1} - \frac{1}{2}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

```
[Out] -1/2*arcsinh(1/3*(1+2*x)*3^(1/2))+(x^2+x+1)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {640, 619, 215}

$$\sqrt{x^2+x+1} - \frac{1}{2}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[1 + x + x^2],x]
```

```
[Out] Sqrt[1 + x + x^2] - ArcSinh[(1 + 2*x)/Sqrt[3]]/2
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\sqrt{1+x+x^2}} \, dx &= \sqrt{1+x+x^2} - \frac{1}{2}\int \frac{1}{\sqrt{1+x+x^2}} \, dx \\
&= \sqrt{1+x+x^2} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{3}}} \, dx, x, 1+2x\right)}{2\sqrt{3}} \\
&= \sqrt{1+x+x^2} - \frac{1}{2}\sinh^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$\sqrt{x^2+x+1} - \frac{1}{2}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[1 + x + x^2],x]
[Out] Sqrt[1 + x + x^2] - ArcSinh[(1 + 2*x)/Sqrt[3]]/2
```

fricas [A] time = 0.39, size = 27, normalized size = 1.00

$$\sqrt{x^2+x+1}+\frac{1}{2}\log\left(-2x+2\sqrt{x^2+x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+x+1)^(1/2),x, algorithm="fricas")
[Out] sqrt(x^2 + x + 1) + 1/2*log(-2*x + 2*sqrt(x^2 + x + 1) - 1)
```

giac [A] time = 0.03, size = 27, normalized size = 1.00

$$\sqrt{x^2+x+1}+\frac{1}{2}\log\left(-2x+2\sqrt{x^2+x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+x+1)^(1/2),x, algorithm="giac")
[Out] sqrt(x^2 + x + 1) + 1/2*log(-2*x + 2*sqrt(x^2 + x + 1) - 1)
```

maple [A] time = 0.01, size = 21, normalized size = 0.78

$$-\frac{\operatorname{arcsinh}\left(\frac{2\sqrt{3}\left(x+\frac{1}{2}\right)}{3}\right)}{2}+\sqrt{x^2+x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2+x+1)^(1/2),x)
[Out] (x^2+x+1)^(1/2)-1/2*arcsinh(2/3*3^(1/2)*(x+1/2))
```

maxima [A] time = 1.20, size = 22, normalized size = 0.81

$$\sqrt{x^2+x+1}-\frac{1}{2}\operatorname{arsinh}\left(\frac{1}{3}\sqrt{3}(2x+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+x+1)^(1/2),x, algorithm="maxima")
[Out] sqrt(x^2 + x + 1) - 1/2*arcsinh(1/3*sqrt(3)*(2*x + 1))
```

mupad [B] time = 0.05, size = 23, normalized size = 0.85

$$\sqrt{x^2+x+1}-\frac{\ln\left(x+\sqrt{x^2+x+1}+\frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x + x^2 + 1)^(1/2),x)
[Out] (x + x^2 + 1)^(1/2) - log(x + (x + x^2 + 1)^(1/2) + 1/2)/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\sqrt{x^2+x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2+x+1)**(1/2),x)
```

```
[Out] Integral(x/sqrt(x**2 + x + 1), x)
```

3.154 $\int \frac{1}{\sqrt{x+x^2}}\,dx$

Optimal. Leaf size=14

$$2\tanh^{-1}\left(\frac{x}{\sqrt{x^2+x}}\right)$$

```
[Out] 2*arctanh(x/(x^2+x)^(1/2))
```

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {620, 206}

$$2\tanh^{-1}\left(\frac{x}{\sqrt{x^2+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[x + x^2],x]
```

```
[Out] 2*ArcTanh[x/Sqrt[x + x^2]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x+x^2}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x}{\sqrt{x+x^2}}\right)\\ &= 2\tanh^{-1}\left(\frac{x}{\sqrt{x+x^2}}\right)\end{aligned}$$

Mathematica [B] time = 0.01, size = 29, normalized size = 2.07

$$\frac{2\sqrt{x}\sqrt{x+1}\,\sinh^{-1}\left(\sqrt{x}\right)}{\sqrt{x(x+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[x + x^2],x]
```

```
[Out] (2*Sqrt[x]*Sqrt[1 + x]*ArcSinh[Sqrt[x]])/Sqrt[x*(1 + x)]
```

fricas [A] time = 0.40, size = 17, normalized size = 1.21

$$-\log\left(-2\,x+2\,\sqrt{x^2+x}\,-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+x)^(1/2),x, algorithm="fricas")
[Out] -log(-2*x + 2*sqrt(x^2 + x) - 1)
```

giac [A] time = 0.02, size = 18, normalized size = 1.29

$$-\log\left(\left|-2x + 2\sqrt{x^2 + x} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+x)^(1/2),x, algorithm="giac")
[Out] -log(abs(-2*x + 2*sqrt(x^2 + x) - 1))
```

maple [A] time = 0.00, size = 12, normalized size = 0.86

$$\ln\left(x + \frac{1}{2} + \sqrt{x^2 + x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+x)^(1/2),x)
[Out] ln(x+1/2+(x^2+x)^(1/2))
```

maxima [A] time = 0.55, size = 15, normalized size = 1.07

$$\log\left(2x + 2\sqrt{x^2 + x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+x)^(1/2),x, algorithm="maxima")
[Out] log(2*x + 2*sqrt(x^2 + x) + 1)
```

mupad [B] time = 0.17, size = 11, normalized size = 0.79

$$\ln\left(x + \sqrt{x\,(x+1)} + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + x^2)^(1/2),x)
[Out] log(x + (x*(x + 1))^(1/2) + 1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^2 + x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+x)**(1/2),x)
[Out] Integral(1/sqrt(x**2 + x), x)
```

3.155 $\int \frac{\sqrt{2-x-x^2}}{x^2}\,dx$

Optimal. Leaf size=68

$$-\frac{\sqrt{-x^2-x+2}}{x}+\frac{\tanh^{-1}\left(\frac{4-x}{2\sqrt{2}\sqrt{-x^2-x+2}}\right)}{2\sqrt{2}}+\sin^{-1}\left(\frac{1}{3}(-2x-1)\right)$$

```
[Out] -arcsin(1/3+2/3*x)+1/4*arctanh(1/4*(4-x)*2^(1/2)/(-x^2-x+2)^(1/2))*2^(1/2)-
(-x^2-x+2)^(1/2)/x
```

Rubi [A] time = 0.04, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {732, 843, 619, 216, 724, 206}

$$-\frac{\sqrt{-x^2-x+2}}{x}+\frac{\tanh^{-1}\left(\frac{4-x}{2\sqrt{2}\sqrt{-x^2-x+2}}\right)}{2\sqrt{2}}+\sin^{-1}\left(\frac{1}{3}(-2x-1)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - x - x^2]/x^2,x]
```

```
[Out] -(Sqrt[2 - x - x^2]/x) + ArcSin[(-1 - 2*x)/3] + ArcTanh[(4 - x)/(2*Sqrt[2]*
Sqrt[2 - x - x^2])]/(2*Sqrt[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{2-x-x^2}}{x^2}\,dx &= -\frac{\sqrt{2-x-x^2}}{x}+\frac{1}{2}\int \frac{-1-2x}{x\sqrt{2-x-x^2}}\,dx\\ &= -\frac{\sqrt{2-x-x^2}}{x}-\frac{1}{2}\int \frac{1}{x\sqrt{2-x-x^2}}\,dx-\int \frac{1}{\sqrt{2-x-x^2}}\,dx\\ &= -\frac{\sqrt{2-x-x^2}}{x}+\frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{9}}}\,dx,x,-1-2x\right)+\text{Subst}\left(\int \frac{1}{8-x^2}\,dx,x,\frac{4-x}{\sqrt{2-x-x^2}}\right)\\ &= -\frac{\sqrt{2-x-x^2}}{x}+\sin^{-1}\left(\frac{1}{3}(-1-2x)\right)+\frac{\tanh^{-1}\left(\frac{4-x}{2\sqrt{2}\sqrt{2-x-x^2}}\right)}{2\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 68, normalized size = 1.00

$$-\frac{\sqrt{-x^2-x+2}}{x}+\frac{\tanh^{-1}\left(\frac{4-x}{2\sqrt{2}\sqrt{-x^2-x+2}}\right)}{2\sqrt{2}}+\sin^{-1}\left(\frac{1}{3}(-2x-1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 - x - x^2]/x^2,x]
```

```
[Out] -(Sqrt[2 - x - x^2]/x) + ArcSin[(-1 - 2*x)/3] + ArcTanh[(4 - x)/(2*Sqrt[2]*
Sqrt[2 - x - x^2])]/(2*Sqrt[2])
```

fricas [A] time = 0.43, size = 92, normalized size = 1.35

$$\frac{\sqrt{2}\,x\log\left(-\frac{4\sqrt{2}\sqrt{-x^2-x+2}\,(x-4)+7\,x^2+16\,x-32}{x^2}\right)+8\,x\arctan\left(\frac{\sqrt{-x^2-x+2}\,(2\,x+1)}{2\,(x^2+x-2)}\right)-8\sqrt{-x^2-x+2}}{8\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-x+2)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/8*(sqrt(2)*x*log(-(4*sqrt(2)*sqrt(-x^2 - x + 2)*(x - 4) + 7*x^2 + 16*x -
32)/x^2) + 8*x*arctan(1/2*sqrt(-x^2 - x + 2)*(2*x + 1)/(x^2 + x - 2)) - 8*s
qrt(-x^2 - x + 2))/x
```

giac [B] time = 0.04, size = 168, normalized size = 2.47

$$-\frac{1}{4}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}+\frac{2\left(2\sqrt{-x^2-x+2}-3\right)}{2\,x+1}+6\right|}{\left|4\sqrt{2}+\frac{2\left(2\sqrt{-x^2-x+2}-3\right)}{2\,x+1}+6\right|}\right)+\frac{6\left(\frac{3\left(2\sqrt{-x^2-x+2}-3\right)}{2\,x+1}+1\right)}{\frac{6\left(2\sqrt{-x^2-x+2}-3\right)}{2\,x+1}+\frac{\left(2\sqrt{-x^2-x+2}-3\right)^2}{(2\,x+1)^2}+1}-\arcsin\left(\frac{2}{3}\,x+\frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-x+2)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*log(abs(-4*sqrt(2) + 2*(2*sqrt(-x^2 - x + 2) - 3)/(2*x + 1) +
6)/abs(4*sqrt(2) + 2*(2*sqrt(-x^2 - x + 2) - 3)/(2*x + 1) + 6)) + 6*(3*(2*s
qrt(-x^2 - x + 2) - 3)/(2*x + 1) + 1)/(6*(2*sqrt(-x^2 - x + 2) - 3)/(2*x +
1) + (2*sqrt(-x^2 - x + 2) - 3)^2/(2*x + 1)^2 + 1) - arcsin(2/3*x + 1/3)
```

maple [A] time = 0.01, size = 88, normalized size = 1.29

$$\frac{\sqrt{2}\ \operatorname{arctanh}\left(\frac{(-x+4)\sqrt{2}}{4\sqrt{-x^2-x+2}}\right)}{4}-\arcsin\left(\frac{2x}{3}+\frac{1}{3}\right)-\frac{\left(-x^2-x+2\right)^{\frac{3}{2}}}{2x}-\frac{\sqrt{-x^2-x+2}}{4}+\frac{(-2x-1)\sqrt{-x^2-x+2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2-x+2)^(1/2)/x^2,x)
```

```
[Out] -1/2/x*(-x^2-x+2)^(3/2)-1/4*(-x^2-x+2)^(1/2)-arcsin(1/3+2/3*x)+1/4*arctanh(
1/4*(4-x)*2^(1/2)/(-x^2-x+2)^(1/2))*2^(1/2)+1/4*(-2*x-1)*(-x^2-x+2)^(1/2)
```

maxima [A] time = 1.19, size = 59, normalized size = 0.87

$$\frac{1}{4}\sqrt{2}\log\left(\frac{2\sqrt{2}\sqrt{-x^2-x+2}}{|x|}+\frac{4}{|x|}-1\right)-\frac{\sqrt{-x^2-x+2}}{x}+\arcsin\left(-\frac{2}{3}x-\frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-x+2)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*log(2*sqrt(2)*sqrt(-x^2 - x + 2)/abs(x) + 4/abs(x) - 1) - sqrt(
-x^2 - x + 2)/x + arcsin(-2/3*x - 1/3)
```

mupad [B] time = 0.08, size = 73, normalized size = 1.07

$$\frac{\sqrt{2}\ \ln\left(\frac{2}{x}+\frac{\sqrt{2}\ \sqrt{-x^2-x+2}}{x}-\frac{1}{2}\right)}{4}-\frac{\sqrt{-x^2-x+2}}{x}+\ln\left(x\,1\mathrm{i}+\sqrt{-x^2-x+2}+\frac{1}{2}\mathrm{i}\right)1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - x^2 - x)^(1/2)/x^2,x)
```

```
[Out] log(x*1i + (2 - x^2 - x)^(1/2) + 1i/2)*1i - (2 - x^2 - x)^(1/2)/x + (2^(1/2
)*log(2/x + (2^(1/2)*(2 - x^2 - x)^(1/2))/x - 1/2))/4
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-(x-1)(x+2)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2-x+2)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 2))/x**2, x)
```

3.156 $\int \frac{\log(t)}{1+t}\,dt$

Optimal. Leaf size=13

$$\text{PolyLog}(2,-t)+\log(t)\log(t+1)$$

```
[Out] ln(t)*ln(1+t)+polylog(2,-t)
```

Rubi [A] time = 0.02, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2317, 2391}

$$\text{PolyLog}(2,-t)+\log(t)\log(t+1)$$

Antiderivative was successfully verified.

```
[In] Int[Log[t]/(1 + t),t]
```

```
[Out] Log[t]*Log[1 + t] + PolyLog[2, -t]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\log(t)}{1+t}\,dt &= \log(t)\log(1+t)-\int \frac{\log(1+t)}{t}\,dt\\ &= \log(t)\log(1+t)+\text{Li}_2(-t)\end{aligned}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\text{PolyLog}(2,-t)+\log(t)\log(t+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[t]/(1 + t),t]
```

```
[Out] Log[t]*Log[1 + t] + PolyLog[2, -t]
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log(t)}{t+1},t\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(t)/(1+t),t, algorithm="fricas")
```

```
[Out] integral(log(t)/(t + 1), t)
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(t)/(1+t),t, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: integ
rate((t+1)^-1*ln(t),t)
```

maple [**A**] time = 0.00, size = 13, normalized size = 1.00

$$\ln(t)\ln(t+1)+\operatorname{dilog}(t+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(t)/(t+1),t)
```

```
[Out] dilog(t+1)+ln(t)*ln(t+1)
```

maxima [**A**] time = 0.59, size = 12, normalized size = 0.92

$$\log(t+1)\log(t)+\operatorname{Li}_2(-t)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(t)/(1+t),t, algorithm="maxima")
```

```
[Out] log(t + 1)*log(t) + dilog(-t)
```

mupad [**B**] time = 0.03, size = 13, normalized size = 1.00

$$\operatorname{polylog}(2,-t)+\ln(t+1)\,\ln(t)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(t)/(t + 1),t)
```

```
[Out] polylog(2, -t) + log(t + 1)*log(t)
```

sympy [**C**] time = 2.44, size = 58, normalized size = 4.46

$$\begin{cases} i\pi\log(t+1)-\operatorname{Li}_2(t+1) & \text{for } |t+1|<1 \\ -i\pi\log\left(\frac{1}{t+1}\right)-\operatorname{Li}_2(t+1) & \text{for } \frac{1}{|t+1|}<1 \\ -i\pi G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix}\middle| t+1\right)+i\pi G_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0 \end{matrix}\middle| t+1\right)-\operatorname{Li}_2(t+1) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(t)/(1+t),t)
```

```
[Out] Piecewise((I*pi*log(t + 1) - polylog(2, t + 1), Abs(t + 1) < 1), (-I*pi*log
(1/(t + 1)) - polylog(2, t + 1), 1/Abs(t + 1) < 1), (-I*pi*meijerg(((), (1,
 1)), ((0, 0), ()), t + 1) + I*pi*meijerg(((1, 1), ()), ((), (0, 0)), t + 1
) - polylog(2, t + 1), True))
```

3.157 $\int \log\left(e^{\cos(x)}\right) dx$

Optimal. Leaf size=15

$$\sin(x) - x\cos(x) + x\log\left(e^{\cos(x)}\right)$$

[Out] -x*cos(x)+x*ln(exp(cos(x)))+sin(x)

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {2548, 3296, 2637}

$$\sin(x) - x\cos(x) + x\log\left(e^{\cos(x)}\right)$$

Antiderivative was successfully verified.

[In] Int[Log[E^Cos[x]],x]

[Out] -(x*Cos[x]) + x*Log[E^Cos[x]] + Sin[x]

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \log\left(e^{\cos(x)}\right) dx &= x\log\left(e^{\cos(x)}\right) + \int x\sin(x)\,dx \\
&= -x\cos(x) + x\log\left(e^{\cos(x)}\right) + \int \cos(x)\,dx \\
&= -x\cos(x) + x\log\left(e^{\cos(x)}\right) + \sin(x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$\sin(x) + x\left(\log\left(e^{\cos(x)}\right) - \cos(x)\right)$$

Antiderivative was successfully verified.

[In] Integrate[Log[E^Cos[x]],x]

[Out] x*(-Cos[x] + Log[E^Cos[x]]) + Sin[x]

fricas [A] time = 0.40, size = 2, normalized size = 0.13

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(exp(cos(x))),x, algorithm="fricas")
[Out] sin(x)
```

giac [A] time = 0.00, size = 2, normalized size = 0.13

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(exp(cos(x))),x, algorithm="giac")
[Out] sin(x)
```

maple [A] time = 0.02, size = 15, normalized size = 1.00

$$-x\cos(x) + x\ln\left(e^{\cos(x)}\right) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(exp(cos(x))),x)
[Out] -x*cos(x)+x*ln(exp(cos(x)))+sin(x)
```

maxima [A] time = 0.63, size = 2, normalized size = 0.13

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(exp(cos(x))),x, algorithm="maxima")
[Out] sin(x)
```

mupad [B] time = 0.10, size = 2, normalized size = 0.13

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(exp(cos(x))),x)
[Out] sin(x)
```

sympy [A] time = 0.20, size = 2, normalized size = 0.13

$$\sin{\left(x \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(exp(cos(x))),x)
[Out] sin(x)
```

3.158 $\int \frac{e^t}{t} \, dt$

Optimal. Leaf size=2

$$\text{ExpIntegralEi}(t)$$

```
[Out] Ei(t)
```

Rubi [A] time = 0.01, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2178}

$$\text{ExpIntegralEi}(t)$$

Antiderivative was successfully verified.

```
[In] Int[E^t/t,t]
[Out] ExpIntegralEi[t]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rubi steps

$$\int \frac{e^t}{t} \, dt = \text{Ei}(t)$$

Mathematica [A] time = 0.01, size = 2, normalized size = 1.00

$$\text{ExpIntegralEi}(t)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^t/t,t]
[Out] ExpIntegralEi[t]
```

fricas [A] time = 0.40, size = 2, normalized size = 1.00

$$\text{Ei}(t)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t)/t,t, algorithm="fricas")
[Out] Ei(t)
```

giac [A] time = 0.00, size = 2, normalized size = 1.00

$$\text{Ei}(t)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t)/t,t, algorithm="giac")
[Out] Ei(t)
```

maple [B] time = 0.01, size = 8, normalized size = 4.00

$$-\operatorname{Ei}(1,-t)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(t)/t,t)
[Out] -Ei(1,-t)
```

maxima [A] time = 0.66, size = 2, normalized size = 1.00

$$\operatorname{Ei}(t)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t)/t,t, algorithm="maxima")
[Out] Ei(t)
```

mupad [B] time = 0.01, size = 2, normalized size = 1.00

$$\operatorname{ei}(t)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(t)/t,t)
[Out] ei(t)
```

sympy [A] time = 0.74, size = 2, normalized size = 1.00

$$\operatorname{Ei}(t)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t)/t,t)
[Out] Ei(t)
```

3.159 $\int \frac{e^{at}}{t}\,dt$

Optimal. Leaf size=4

$$\text{ExpIntegralEi}(at)$$

```
[Out] Ei(a*t)
```

Rubi [A] time = 0.01, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2178}

$$\text{ExpIntegralEi}(at)$$

Antiderivative was successfully verified.

```
[In] Int[E^(a*t)/t,t]
[Out] ExpIntegralEi[a*t]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rubi steps

$$\int \frac{e^{at}}{t}\,dt = \text{Ei}(at)$$

Mathematica [A] time = 0.01, size = 4, normalized size = 1.00

$$\text{ExpIntegralEi}(at)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a*t)/t,t]
[Out] ExpIntegralEi[a*t]
```

fricas [A] time = 0.38, size = 4, normalized size = 1.00

$$\text{Ei}\,(at)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*t)/t,t, algorithm="fricas")
[Out] Ei(a*t)
```

giac [A] time = 0.00, size = 4, normalized size = 1.00

$$\text{Ei}\,(at)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*t)/t,t, algorithm="giac")
[Out] Ei(a*t)
```

maple [A] time = 0.00, size = 9, normalized size = 2.25

$$-\operatorname{Ei}(1, -at)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a*t)/t,t)
[Out] -Ei(1,-a*t)
```

maxima [A] time = 0.82, size = 4, normalized size = 1.00

$$\operatorname{Ei}(at)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*t)/t,t, algorithm="maxima")
[Out] Ei(a*t)
```

mupad [B] time = 0.01, size = 4, normalized size = 1.00

$$\operatorname{ei}(a\,t)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a*t)/t,t)
[Out] ei(a*t)
```

sympy [A] time = 0.78, size = 3, normalized size = 0.75

$$\operatorname{Ei}(at)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*t)/t,t)
[Out] Ei(a*t)
```

3.160 $\int \frac{e^t}{t^2} \, dt$

Optimal. Leaf size=11

$$\text{Ei}(t) - \frac{e^t}{t}$$

```
[Out] -exp(t)/t+Ei(t)
```

Rubi [A] time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2177, 2178}

$$\text{ExpIntegralEi}(t) - \frac{e^t}{t}$$

Antiderivative was successfully verified.

```
[In] Int[E^t/t^2,t]
```

```
[Out] -(E^t/t) + ExpIntegralEi[t]
```

Rule 2177

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_), x_Symbol] :> Simp[((c + d*x)^(m + 1)*(b*F^(g*(e + f*x)))^n)/(d*(m + 1))
, x] - Dist[(f*g*n*Log[F])/(d*(m + 1)), Int[(c + d*x)^(m + 1)*(b*F^(g*(e +
f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && Int
egerQ[2*m] &&  !$UseGamma === True
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rubi steps

$$\begin{aligned} \int \frac{e^t}{t^2} \, dt &= -\frac{e^t}{t} + \int \frac{e^t}{t} \, dt \\ &= -\frac{e^t}{t} + \text{Ei}(t) \end{aligned}$$

Mathematica [A] time = 0.01, size = 11, normalized size = 1.00

$$\text{Ei}(t) - \frac{e^t}{t}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^t/t^2,t]
```

```
[Out] -(E^t/t) + ExpIntegralEi[t]
```

fricas [A] time = 0.40, size = 13, normalized size = 1.18

$$\frac{t\text{Ei}(t) - e^t}{t}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t)/t^2,t, algorithm="fricas")
[Out] (t*Ei(t) - e^t)/t
```

giac [A] time = 0.01, size = 13, normalized size = 1.18

$$\frac{t\mathrm{Ei}(t) - e^t}{t}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t)/t^2,t, algorithm="giac")
[Out] (t*Ei(t) - e^t)/t
```

maple [A] time = 0.00, size = 16, normalized size = 1.45

$$-\mathrm{Ei}(1, -t) - \frac{\mathrm{e}^t}{t}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(t)/t^2,t)
[Out] -exp(t)/t-Ei(1,-t)
```

maxima [A] time = 0.69, size = 5, normalized size = 0.45

$$\Gamma(-1, -t)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t)/t^2,t, algorithm="maxima")
[Out] gamma(-1, -t)
```

mupad [B] time = 0.02, size = 14, normalized size = 1.27

$$-\frac{\mathrm{e}^t}{t} - \mathrm{expint}(-t)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(t)/t^2,t)
[Out] - exp(t)/t - expint(-t)
```

sympy [A] time = 0.98, size = 7, normalized size = 0.64

$$\mathrm{Ei}(t) - \frac{e^t}{t}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t)/t**2,t)
[Out] Ei(t) - exp(t)/t
```

3.161 $\int e^{\frac{1}{t}}\,dt$

Optimal. Leaf size=14

$$e^{\frac{1}{t}}t-\mathrm{Ei}\left(\frac{1}{t}\right)$$

```
[Out] exp(1/t)*t-Ei(1/t)
```

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2206, 2210}

$$e^{\frac{1}{t}}t-\mathrm{ExpIntegralEi}\left(\frac{1}{t}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^t^(-1),t]
[Out] E^t^(-1)*t - ExpIntegralEi[t^(-1)]
```

Rule 2206

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[((c + d*x)*F^(a + b*(c + d*x)^n))/d, x] - Dist[b*n*Log[F], Int[(c + d*x)^n*F^(a + b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2/n] && ILtQ[n, 0]
```

Rule 2210

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_Symbol] :> Simp[(F^a*ExpIntegralEi[b*(c + d*x)^n*Log[F]])/(f*n), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}\int e^{\frac{1}{t}}\,dt &= e^{\frac{1}{t}}t+\int \frac{e^{\frac{1}{t}}}{t}\,dt\\ &= e^{\frac{1}{t}}t-\mathrm{Ei}\left(\frac{1}{t}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$e^{\frac{1}{t}}t-\mathrm{Ei}\left(\frac{1}{t}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^t^(-1),t]
[Out] E^t^(-1)*t - ExpIntegralEi[t^(-1)]
```

fricas [A] time = 0.40, size = 13, normalized size = 0.93

$$te^{\frac{1}{t}}-\mathrm{Ei}\left(\frac{1}{t}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1/t),t, algorithm="fricas")
```

```
[Out] t*e^(1/t) - Ei(1/t)
```

giac [A] time = 0.01, size = 18, normalized size = 1.29

$$-t\left(\frac{\mathrm{Ei}\left(\frac{1}{t}\right)}{t}-e^{\frac{1}{t}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1/t),t, algorithm="giac")
```

```
[Out] -t*(Ei(1/t)/t - e^(1/t))
```

maple [A] time = 0.00, size = 15, normalized size = 1.07

$$t\,\mathrm{e}^{\frac{1}{t}}+\mathrm{Ei}\left(1,-\frac{1}{t}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(1/t),t)
```

```
[Out] exp(1/t)*t+Ei(1,-1/t)
```

maxima [A] time = 0.65, size = 9, normalized size = 0.64

$$-\Gamma\left(-1,-\frac{1}{t}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1/t),t, algorithm="maxima")
```

```
[Out] -gamma(-1, -1/t)
```

mupad [B] time = 0.02, size = 9, normalized size = 0.64

$$t\,\mathrm{expint}\left(2,-\frac{1}{t}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(1/t),t)
```

```
[Out] t*expint(2, -1/t)
```

sympy [A] time = 1.05, size = 10, normalized size = 0.71

$$te^{\frac{1}{t}}-\mathrm{Ei}\left(\frac{1}{t}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1/t),t)
```

```
[Out] t*exp(1/t) - Ei(1/t)
```

3.162 $\int \frac{e^{-t}}{-1-a+t}\,dt$

Optimal. Leaf size=15

$$e^{-a-1}\text{Ei}(a-t+1)$$

```
[Out] exp(-1-a)*Ei(1+a-t)
```

Rubi [A] time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2178}

$$e^{-a-1}\text{ExpIntegralEi}(a-t+1)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^t*(-1 - a + t)),t]
```

```
[Out] E^(-1 - a)*ExpIntegralEi[1 + a - t]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rubi steps

$$\int \frac{e^{-t}}{-1-a+t}\,dt = e^{-1-a}\text{Ei}(1+a-t)$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$e^{-a-1}\text{Ei}(a-t+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^t*(-1 - a + t)),t]
```

```
[Out] E^(-1 - a)*ExpIntegralEi[1 + a - t]
```

fricas [A] time = 0.40, size = 14, normalized size = 0.93

$$\text{Ei}\,(a-t+1)\,e^{(-a-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(t)/(-1-a+t),t, algorithm="fricas")
```

```
[Out] Ei(a - t + 1)*e^(-a - 1)
```

giac [A] time = 0.01, size = 14, normalized size = 0.93

$$\text{Ei}\,(a-t+1)\,e^{(-a-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(t)/(-1-a+t),t, algorithm="giac")
```

```
[Out] Ei(a - t + 1)*e^(-a - 1)
```

maple [**A**] time = 0.01, size = 17, normalized size = 1.13

$$-\operatorname{Ei}(1,-a+t-1)\,\mathrm{e}^{-a-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/exp(t)/(-1-a+t),t)
[Out] -exp(-1-a)*Ei(1,-1-a+t)
```

maxima [**A**] time = 0.87, size = 16, normalized size = 1.07

$$-e^{(-a-1)}E_1(-a+t-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(t)/(-1-a+t),t, algorithm="maxima")
[Out] -e^(-a - 1)*exp_integral_e(1, -a + t - 1)
```

mupad [**B**] time = 0.03, size = 14, normalized size = 0.93

$$\mathrm{e}^{-a-1}\operatorname{ei}(a-t+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-exp(-t)/(a - t + 1),t)
[Out] exp(- a - 1)*ei(a - t + 1)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{-t}}{-a+t-1}\,dt$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(t)/(-1-a+t),t)
[Out] Integral(exp(-t)/(-a + t - 1), t)
```

3.163 $\int \frac{e^{t^2} t}{1+t^2} dt$

Optimal. Leaf size=13

$$\frac{\text{Ei}\left(t^2+1\right)}{2e}$$

```
[Out] 1/2*Ei(t^2+1)/E
```

Rubi [A] time = 0.07, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6715, 2178}

$$\frac{\text{ExpIntegralEi}\left(t^2+1\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(E^t^2*t)/(1 + t^2),t]
```

```
[Out] ExpIntegralEi[1 + t^2]/(2*E)
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\begin{aligned} \int \frac{e^{t^2} t}{1+t^2} dt &= \frac{1}{2} \text{Subst}\left(\int \frac{e^t}{1+t} dt, t, t^2\right) \\ &= \frac{\text{Ei}\left(1+t^2\right)}{2e} \end{aligned}$$

Mathematica [A] time = 0.02, size = 13, normalized size = 1.00

$$\frac{\text{Ei}\left(t^2+1\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^t^2*t)/(1 + t^2),t]
```

```
[Out] ExpIntegralEi[1 + t^2]/(2*E)
```

fricas [A] time = 0.39, size = 10, normalized size = 0.77

$$\frac{1}{2} \text{Ei}\left(t^2+1\right) e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t^2)*t/(t^2+1),t, algorithm="fricas")
[Out] 1/2*Ei(t^2 + 1)*e^(-1)
```

giac [A] time = 0.01, size = 10, normalized size = 0.77

$$\frac{1}{2}\,\mathrm{Ei}\left(t^2+1\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t^2)*t/(t^2+1),t, algorithm="giac")
[Out] 1/2*Ei(t^2 + 1)*e^(-1)
```

maple [A] time = 0.01, size = 14, normalized size = 1.08

$$-\frac{\mathrm{e}^{-1}\,\mathrm{Ei}\left(1,-t^2-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(t^2)*t/(t^2+1),t)
[Out] -1/2*exp(-1)*Ei(1,-t^2-1)
```

maxima [A] time = 0.79, size = 13, normalized size = 1.00

$$-\frac{1}{2}\,e^{(-1)}E_1\left(-t^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t^2)*t/(t^2+1),t, algorithm="maxima")
[Out] -1/2*e^(-1)*exp_integral_e(1, -t^2 - 1)
```

mupad [B] time = 0.12, size = 10, normalized size = 0.77

$$\frac{\mathrm{e}^{-1}\,\mathrm{ei}\left(t^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((t*exp(t^2))/(t^2 + 1),t)
[Out] (exp(-1)*ei(t^2 + 1))/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{te^{t^2}}{t^2+1}\,dt$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t**2)*t/(t**2+1),t)
[Out] Integral(t*exp(t**2)/(t**2 + 1), t)
```

3.164 $\int \frac{e^t}{(1+t)^2}\,dt$

Optimal. Leaf size=19

$$\frac{\text{Ei}(t+1)}{e}-\frac{e^t}{t+1}$$

```
[Out] -exp(t)/(1+t)+Ei(1+t)/E
```

Rubi [A] time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2177, 2178}

$$\frac{\text{ExpIntegralEi}(t+1)}{e}-\frac{e^t}{t+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^t/(1 + t)^2,t]
```

```
[Out] -(E^t/(1 + t)) + ExpIntegralEi[1 + t]/E
```

Rule 2177

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_), x_Symbol] :> Simp[((c + d*x)^(m + 1)*(b*F^(g*(e + f*x)))^n)/(d*(m + 1))
, x] - Dist[(f*g*n*Log[F])/(d*(m + 1)), Int[(c + d*x)^(m + 1)*(b*F^(g*(e +
f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && Int
egerQ[2*m] &&  !$UseGamma === True
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rubi steps

$$\begin{aligned}\int \frac{e^t}{(1+t)^2}\,dt &= -\frac{e^t}{1+t}+\int \frac{e^t}{1+t}\,dt\\ &= -\frac{e^t}{1+t}+\frac{\text{Ei}(1+t)}{e}\end{aligned}$$

Mathematica [A] time = 0.02, size = 19, normalized size = 1.00

$$\frac{\text{Ei}(t+1)}{e}-\frac{e^t}{t+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^t/(1 + t)^2,t]
```

```
[Out] -(E^t/(1 + t)) + ExpIntegralEi[1 + t]/E
```

fricas [A] time = 0.40, size = 23, normalized size = 1.21

$$\frac{\left((t+1)\text{Ei}\,(t+1)-e^{(t+1)}\right)e^{(-1)}}{t+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t)/(1+t)^2,t, algorithm="fricas")
```

```
[Out] ((t + 1)*Ei(t + 1) - e^(t + 1))*e^(-1)/(t + 1)
```

giac [B] time = 0.01, size = 80, normalized size = 4.21

$$\frac{(t+1)\left(\frac{1}{t+1}-1\right)\mathrm{Ei}\left(-(t+1)\left(\frac{1}{t+1}-1\right)+1\right)-\mathrm{Ei}\left(-(t+1)\left(\frac{1}{t+1}-1\right)+1\right)+e^{\left(-(t+1)\left(\frac{1}{t+1}-1\right)+1\right)}}{(t+1)\left(\frac{1}{t+1}-1\right)e-e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t)/(1+t)^2,t, algorithm="giac")
```

```
[Out] ((t + 1)*(1/(t + 1) - 1)*Ei(-(t + 1)*(1/(t + 1) - 1) + 1) - Ei(-(t + 1)*(1/
(t + 1) - 1) + 1) + e^(-(t + 1)*(1/(t + 1) - 1) + 1))/((t + 1)*(1/(t + 1) -
 1)*e - e)
```

maple [A] time = 0.01, size = 22, normalized size = 1.16

$$-\mathrm{e}^{-1}\,\mathrm{Ei}\left(1,-t-1\right)-\frac{\mathrm{e}^{t}}{t+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(t)/(t+1)^2,t)
```

```
[Out] -exp(t)/(t+1)-exp(-1)*Ei(1,-t-1)
```

maxima [A] time = 0.80, size = 16, normalized size = 0.84

$$-\frac{e^{(-1)}E_2\left(-t-1\right)}{t+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t)/(1+t)^2,t, algorithm="maxima")
```

```
[Out] -e^(-1)*exp_integral_e(2, -t - 1)/(t + 1)
```

mupad [B] time = 0.13, size = 17, normalized size = 0.89

$$\mathrm{ei}\left(t+1\right)\,\mathrm{e}^{-1}-\frac{\mathrm{e}^{t}}{t+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(t)/(t + 1)^2,t)
```

```
[Out] ei(t + 1)*exp(-1) - exp(t)/(t + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{e^{t}}{(t+1)^2}\,dt$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t)/(1+t)**2,t)
```

```
[Out] Integral(exp(t)/(t + 1)**2, t)
```

3.165 $\int e^t \log(1+t)\,dt$

Optimal. Leaf size=18

$$e^t \log(t+1) - \frac{\text{Ei}(t+1)}{e}$$

```
[Out] -Ei(1+t)/E+exp(t)*ln(1+t)
```

Rubi [A] time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2194, 2554, 2178}

$$e^t \log(t+1) - \frac{\text{ExpIntegralEi}(t+1)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[E^t*Log[1 + t],t]
```

```
[Out] -(ExpIntegralEi[1 + t]/E) + E^t*Log[1 + t]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\begin{aligned}\int e^t \log(1+t)\,dt &= e^t \log(1+t) - \int \frac{e^t}{1+t}\,dt \\ &= -\frac{\text{Ei}(1+t)}{e} + e^t \log(1+t)\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$e^t \log(t+1) - \frac{\text{Ei}(t+1)}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^t*Log[1 + t],t]
```

```
[Out] -(ExpIntegralEi[1 + t]/E) + E^t*Log[1 + t]
```

fricas [A] time = 0.41, size = 19, normalized size = 1.06

$$\left(e^{(t+1)} \log(t+1) - \text{Ei}(t+1)\right) e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t)*log(1+t),t, algorithm="fricas")
[Out] (e^(t + 1)*log(t + 1) - Ei(t + 1))*e^(-1)
```

giac [A] time = 0.01, size = 16, normalized size = 0.89

$$-\text{Ei}\left(t+1\right)e^{(-1)}+e^{t}\log\left(t+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t)*log(1+t),t, algorithm="giac")
[Out] -Ei(t + 1)*e^(-1) + e^t*log(t + 1)
```

maple [A] time = 0.04, size = 19, normalized size = 1.06

$$\mathrm{e}^{t}\ln\left(t+1\right)+\mathrm{e}^{-1}\,\text{Ei}\left(1,-t-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(t)*ln(t+1),t)
[Out] exp(t)*ln(t+1)+exp(-1)*Ei(1,-t-1)
```

maxima [A] time = 0.77, size = 18, normalized size = 1.00

$$e^{(-1)}E_{1}\left(-t-1\right)+e^{t}\log\left(t+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t)*log(1+t),t, algorithm="maxima")
[Out] e^(-1)*exp_integral_e(1, -t - 1) + e^t*log(t + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int \ln\left(t+1\right)\,\mathrm{e}^{t}\,dt$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(t + 1)*exp(t),t)
[Out] int(log(t + 1)*exp(t), t)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{t}\log\left(t+1\right)dt$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t)*ln(1+t),t)
[Out] Integral(exp(t)*log(t + 1), t)
```

3.166 $\int e^{-t} t \, dt$

Optimal. Leaf size=16

$$-e^{-t}t-e^{-t}$$

```
[Out] -1/exp(t)-t/exp(t)
```

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2176, 2194}

$$-e^{-t}t-e^{-t}$$

Antiderivative was successfully verified.

```
[In] Int[t/E^t,t]
```

```
[Out] -E^(-t) - t/E^t
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned} \int e^{-t}t\,dt &= -e^{-t}t+\int e^{-t}\,dt\\ &= -e^{-t}-e^{-t}t \end{aligned}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 0.69

$$e^{-t}(-t-1)$$

Antiderivative was successfully verified.

```
[In] Integrate[t/E^t,t]
```

```
[Out] (-1 - t)/E^t
```

fricas [A] time = 0.40, size = 9, normalized size = 0.56

$$-(t+1)e^{(-t)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t/exp(t),t, algorithm="fricas")
```

```
[Out] -(t + 1)*e^(-t)
```

giac [A] time = 0.01, size = 9, normalized size = 0.56

$$-(t+1)e^{(-t)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t/exp(t),t, algorithm="giac")
[Out] -(t + 1)*e^(-t)
```

maple [A] time = 0.00, size = 10, normalized size = 0.62

$$-(t+1)\,\mathrm{e}^{-t}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(t/exp(t),t)
[Out] -(t+1)/exp(t)
```

maxima [A] time = 0.48, size = 9, normalized size = 0.56

$$-(t+1)e^{(-t)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t/exp(t),t, algorithm="maxima")
[Out] -(t + 1)*e^(-t)
```

mupad [B] time = 0.02, size = 9, normalized size = 0.56

$$-\mathrm{e}^{-t}\,(t+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(t*exp(-t),t)
[Out] -exp(-t)*(t + 1)
```

sympy [A] time = 0.08, size = 7, normalized size = 0.44

$$(-t-1)\,e^{-t}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t/exp(t),t)
[Out] (-t - 1)*exp(-t)
```

3.167 $\int e^{-t}t^2\,dt$

Optimal. Leaf size=26

$$-e^{-t}t^2-2e^{-t}t-2e^{-t}$$

```
[Out] -2/exp(t)-2*t/exp(t)-t^2/exp(t)
```

Rubi [A] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2176, 2194}

$$-e^{-t}t^2-2e^{-t}t-2e^{-t}$$

Antiderivative was successfully verified.

```
[In] Int[t^2/E^t,t]
[Out] -2/E^t - (2*t)/E^t - t^2/E^t
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^{-t}t^2\,dt &= -e^{-t}t^2+2\int e^{-t}t\,dt\\ &= -2e^{-t}t-e^{-t}t^2+2\int e^{-t}\,dt\\ &= -2e^{-t}-2e^{-t}t-e^{-t}t^2\end{aligned}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 0.62

$$e^{-t}\left(-t^2-2t-2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[t^2/E^t,t]
[Out] (-2 - 2*t - t^2)/E^t
```

fricas [A] time = 0.40, size = 14, normalized size = 0.54

$$-\left(t^2+2\,t+2\right)e^{(-t)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t^2/exp(t),t, algorithm="fricas")
[Out] -(t^2 + 2*t + 2)*e^(-t)
```

giac [A] time = 0.01, size = 14, normalized size = 0.54

$$-\left(t^2 + 2\,t + 2\right)e^{(-t)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t^2/exp(t),t, algorithm="giac")
[Out] -(t^2 + 2*t + 2)*e^(-t)
```

maple [A] time = 0.00, size = 15, normalized size = 0.58

$$-\left(t^2 + 2t + 2\right)e^{-t}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(t^2/exp(t),t)
[Out] -(t^2+2*t+2)/exp(t)
```

maxima [A] time = 0.51, size = 14, normalized size = 0.54

$$-\left(t^2 + 2\,t + 2\right)e^{(-t)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t^2/exp(t),t, algorithm="maxima")
[Out] -(t^2 + 2*t + 2)*e^(-t)
```

mupad [B] time = 0.03, size = 14, normalized size = 0.54

$$-e^{-t}\left(t^2 + 2\,t + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(t^2*exp(-t),t)
[Out] -exp(-t)*(2*t + t^2 + 2)
```

sympy [A] time = 0.09, size = 12, normalized size = 0.46

$$\left(-t^2 - 2t - 2\right)e^{-t}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t**2/exp(t),t)
[Out] (-t**2 - 2*t - 2)*exp(-t)
```

3.168 $\int e^{-t} t^3 \, dt$

Optimal. Leaf size=36

$$-e^{-t}t^3 - 3e^{-t}t^2 - 6e^{-t}t - 6e^{-t}$$

```
[Out] -6/exp(t)-6*t/exp(t)-3*t^2/exp(t)-t^3/exp(t)
```

Rubi [A] time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2176, 2194}

$$-e^{-t}t^3 - 3e^{-t}t^2 - 6e^{-t}t - 6e^{-t}$$

Antiderivative was successfully verified.

```
[In] Int[t^3/E^t,t]
```

```
[Out] -6/E^t - (6*t)/E^t - (3*t^2)/E^t - t^3/E^t
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}
\int e^{-t}t^3 \, dt &= -e^{-t}t^3 + 3\int e^{-t}t^2 \, dt \\
&= -3e^{-t}t^2 - e^{-t}t^3 + 6\int e^{-t}t \, dt \\
&= -6e^{-t}t - 3e^{-t}t^2 - e^{-t}t^3 + 6\int e^{-t} \, dt \\
&= -6e^{-t} - 6e^{-t}t - 3e^{-t}t^2 - e^{-t}t^3
\end{aligned}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 0.58

$$e^{-t}\left(-t^3 - 3t^2 - 6t - 6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[t^3/E^t,t]
```

```
[Out] (-6 - 6*t - 3*t^2 - t^3)/E^t
```

fricas [A] time = 0.39, size = 19, normalized size = 0.53

$$-\left(t^3 + 3\,t^2 + 6\,t + 6\right)e^{(-t)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t^3/exp(t),t, algorithm="fricas")
```

```
[Out] -(t^3 + 3*t^2 + 6*t + 6)*e^(-t)
```

giac [A] time = 0.01, size = 19, normalized size = 0.53

$$-\left(t^3 + 3\,t^2 + 6\,t + 6\right)e^{(-t)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t^3/exp(t),t, algorithm="giac")
[Out] -(t^3 + 3*t^2 + 6*t + 6)*e^(-t)
```

maple [A] time = 0.00, size = 20, normalized size = 0.56

$$-\left(t^3 + 3t^2 + 6t + 6\right) \mathrm{e}^{-t}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(t^3/exp(t),t)
[Out] -(t^3+3*t^2+6*t+6)/exp(t)
```

maxima [A] time = 0.48, size = 19, normalized size = 0.53

$$-\left(t^3 + 3\,t^2 + 6\,t + 6\right)e^{(-t)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t^3/exp(t),t, algorithm="maxima")
[Out] -(t^3 + 3*t^2 + 6*t + 6)*e^(-t)
```

mupad [B] time = 0.02, size = 19, normalized size = 0.53

$$-\mathrm{e}^{-t}\,\left(t^3 + 3\,t^2 + 6\,t + 6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(t^3*exp(-t),t)
[Out] -exp(-t)*(6*t + 3*t^2 + t^3 + 6)
```

sympy [A] time = 0.09, size = 17, normalized size = 0.47

$$\left(-t^3 - 3t^2 - 6t - 6\right)e^{-t}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t**3/exp(t),t)
[Out] (-t**3 - 3*t**2 - 6*t - 6)*exp(-t)
```

3.169 $\int \frac{\mathbf{b1}\cos(x)+\mathbf{a1}\sin(x)}{b\cos(x)+a\sin(x)}\,dx$

Optimal. Leaf size=48

$$\frac{x(a\text{a1}+b\text{b1})}{a^2+b^2}-\frac{(\text{a1}b-a\text{b1})\log(a\sin(x)+b\cos(x))}{a^2+b^2}$$

```
[Out] (a*a1+b*b1)*x/(a^2+b^2)-(-a*b1+a1*b)*ln(b*cos(x)+a*sin(x))/(a^2+b^2)
```

Rubi [A] time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {3133}

$$\frac{x(a\text{a1}+b\text{b1})}{a^2+b^2}-\frac{(\text{a1}b-a\text{b1})\log(a\sin(x)+b\cos(x))}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b1*Cos[x] + a1*Sin[x])/(b*Cos[x] + a*Sin[x]),x]
```

```
[Out] ((a*a1 + b*b1)*x)/(a^2 + b^2) - ((a1*b - a*b1)*Log[b*Cos[x] + a*Sin[x]])/(a
^2 + b^2)
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\int \frac{\text{b1}\cos(x)+\text{a1}\sin(x)}{b\cos(x)+a\sin(x)}\,dx=\frac{(a\text{a1}+b\text{b1})x}{a^2+b^2}-\frac{(\text{a1}b-a\text{b1})\log(b\cos(x)+a\sin(x))}{a^2+b^2}$$

Mathematica [A] time = 0.13, size = 39, normalized size = 0.81

$$\frac{x(a\text{a1}+b\text{b1})+(a\text{b1}-\text{a1}b)\log(a\sin(x)+b\cos(x))}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b1*Cos[x] + a1*Sin[x])/(b*Cos[x] + a*Sin[x]),x]
```

```
[Out] ((a*a1 + b*b1)*x + (-(a1*b) + a*b1)*Log[b*Cos[x] + a*Sin[x]])/(a^2 + b^2)
```

fricas [A] time = 0.44, size = 60, normalized size = 1.25

$$\frac{2\,(aa_1+bb_1)x-(a_1b-ab_1)\log\left(2\,ab\cos(x)\sin(x)-\left(a^2-b^2\right)\cos(x)^2+a^2\right)}{2\left(a^2+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b1*cos(x)+a1*sin(x))/(b*cos(x)+a*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(a*a1 + b*b1)*x - (a1*b - a*b1)*log(2*a*b*cos(x)*sin(x) - (a^2 - b^2
)*cos(x)^2 + a^2))/(a^2 + b^2)
```

giac [A] time = 0.05, size = 77, normalized size = 1.60

$$\frac{(aa_1 + bb_1)x}{a^2 + b^2} + \frac{(a_1b - ab_1)\log\left(\tan(x)^2 + 1\right)}{2\left(a^2 + b^2\right)} - \frac{\left(aa_1b - a^2b_1\right)\log\left(|a\tan(x) + b|\right)}{a^3 + ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b1*cos(x)+a1*sin(x))/(b*cos(x)+a*sin(x)),x, algorithm="giac")
```

```
[Out] (a*a1 + b*b1)*x/(a^2 + b^2) + 1/2*(a1*b - a*b1)*log(tan(x)^2 + 1)/(a^2 + b^
2) - (a*a1*b - a^2*b1)*log(abs(a*tan(x) + b))/(a^3 + a*b^2)
```

maple [B] time = 0.13, size = 111, normalized size = 2.31

$$\frac{aa1\arctan(\tan(x))}{a^2 + b^2} - \frac{ab1\ln\left(\tan^2(x) + 1\right)}{2\left(a^2 + b^2\right)} + \frac{ab1\ln(a\tan(x) + b)}{a^2 + b^2} + \frac{a1b\ln\left(\tan^2(x) + 1\right)}{2a^2 + 2b^2} - \frac{a1b\ln(a\tan(x) + b)}{a^2 + b^2} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b1*cos(x)+a1*sin(x))/(b*cos(x)+a*sin(x)),x)
```

```
[Out] 1/(a^2+b^2)*ln(a*tan(x)+b)*a*b1-1/(a^2+b^2)*ln(a*tan(x)+b)*a1*b-1/2/(a^2+b^
2)*ln(tan(x)^2+1)*a*b1+1/2/(a^2+b^2)*ln(tan(x)^2+1)*a1*b+1/(a^2+b^2)*arctan
(tan(x))*a*a1+1/(a^2+b^2)*arctan(tan(x))*b*b1
```

maxima [B] time = 1.30, size = 181, normalized size = 3.77

$$a_1\left(\frac{2\,a\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^2 + b^2} - \frac{b\log\left(-b - \frac{2\,a\sin(x)}{\cos(x)+1} + \frac{b\sin(x)^2}{(\cos(x)+1)^2}\right)}{a^2 + b^2} + \frac{b\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2} + 1\right)}{a^2 + b^2}\right) + b_1\left(\frac{2\,b\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^2 + b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b1*cos(x)+a1*sin(x))/(b*cos(x)+a*sin(x)),x, algorithm="maxima")
```

```
[Out] a1*(2*a*arctan(sin(x)/(cos(x) + 1))/(a^2 + b^2) - b*log(-b - 2*a*sin(x)/(co
s(x) + 1) + b*sin(x)^2/(cos(x) + 1)^2)/(a^2 + b^2) + b*log(sin(x)^2/(cos(x)
 + 1)^2 + 1)/(a^2 + b^2)) + b1*(2*b*arctan(sin(x)/(cos(x) + 1))/(a^2 + b^2)
 + a*log(-b - 2*a*sin(x)/(cos(x) + 1) + b*sin(x)^2/(cos(x) + 1)^2)/(a^2 + b
^2) - a*log(sin(x)^2/(cos(x) + 1)^2 + 1)/(a^2 + b^2))
```

mupad [B] time = 10.61, size = 2034, normalized size = 42.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b1*cos(x) + a1*sin(x))/(b*cos(x) + a*sin(x)),x)
```

```
[Out] (2*atan((tan(x/2)*(((((a*a1 + b*b1)^3*(96*a^4*b + 96*a^2*b^3))/(a^2 + b^2)^
3 + ((((a*a1 + b*b1)*(32*a^2*a1*b^2 - ((2*a*b1 - 2*a1*b)*(96*a^4*b + 96*a^2
*b^3))/(2*(a^2 + b^2)) - 64*a1*b^4 + 128*a*b^3*b1 + 32*a^3*b*b1))/(a^2 + b^
2) - ((a*a1 + b*b1)*(2*a*b1 - 2*a1*b)*(96*a^4*b + 96*a^2*b^3))/(2*(a^2 + b^
2)^2))*(2*a*b1 - 2*a1*b))/(2*(a^2 + b^2)) - ((a*a1 + b*b1)*(32*b^3*b1^2 - (
(2*a*b1 - 2*a1*b)*(32*a^2*a1*b^2 - ((2*a*b1 - 2*a1*b)*(96*a^4*b + 96*a^2*b^
3))/(2*(a^2 + b^2)) - 64*a1*b^4 + 128*a*b^3*b1 + 32*a^3*b*b1))/(2*(a^2 + b^
2)) + 64*a^2*a1^2*b - 96*a^2*b*b1^2 + 192*a*a1*b^2*b1))/(a^2 + b^2))*(a^4*a
1^2 + 4*a1^2*b^4 - 4*a^4*b1^2 - b^4*b1^2 - 13*a^2*a1^2*b^2 + 13*a^2*b^2*b1^
2 - 18*a*a1*b^3*b1 + 18*a^3*a1*b*b1))/((a^2 + b^2)^2*(a^2*a1^2 + 4*a^2*b1^2
 + 4*a1^2*b^2 + b^2*b1^2 - 6*a*a1*b*b1)^2) - ((((((a*a1 + b*b1)*(32*a^2*a1*
```

```
b^2 - ((2*a*b1 - 2*a1*b)*(96*a^4*b + 96*a^2*b^3))/(2*(a^2 + b^2)) - 64*a1*b
^4 + 128*a*b^3*b1 + 32*a^3*b*b1))/(a^2 + b^2) - ((a*a1 + b*b1)*(2*a*b1 - 2*
a1*b)*(96*a^4*b + 96*a^2*b^3))/(2*(a^2 + b^2)^2))*(a*a1 + b*b1))/(a^2 + b^2
) - 32*a1*b^2*b1^2 - 64*a1^3*b^2 + ((2*a*b1 - 2*a1*b)*(32*b^3*b1^2 - ((2*a*
b1 - 2*a1*b)*(32*a^2*a1*b^2 - ((2*a*b1 - 2*a1*b)*(96*a^4*b + 96*a^2*b^3))/(
2*(a^2 + b^2)) - 64*a1*b^4 + 128*a*b^3*b1 + 32*a^3*b*b1))/(2*(a^2 + b^2)) +
 64*a^2*a1^2*b - 96*a^2*b*b1^2 + 192*a*a1*b^2*b1))/(2*(a^2 + b^2)) + 32*a*b
*b1^3 - ((a*a1 + b*b1)^2*(2*a*b1 - 2*a1*b)*(96*a^4*b + 96*a^2*b^3))/(2*(a^2
 + b^2)^3) + 64*a*a1^2*b*b1)*(12*a*a1^2*b^3 - 6*a^3*a1^2*b - 6*a*b^3*b1^2 +
 12*a^3*b*b1^2 + 4*a^4*a1*b1 + 4*a1*b^4*b1 - 28*a^2*a1*b^2*b1))/((a^2 + b^2
)^2*(a^2*a1^2 + 4*a^2*b1^2 + 4*a1^2*b^2 + b^2*b1^2 - 6*a*a1*b*b1)^2))*(a^4
+ b^4 + 2*a^2*b^2))/(32*b^2*b1 + 32*a*a1*b) + ((a^4 + b^4 + 2*a^2*b^2)*(32*
a1^2*b^2*b1 + ((((a*a1 + b*b1)*(((2*a*b1 - 2*a1*b)*(96*a*b^4 + 96*a^3*b^2))
/(2*(a^2 + b^2)) - 32*b^4*b1 + 64*a^2*b^2*b1 - 64*a*a1*b^3 + 32*a^3*a1*b))/
(a^2 + b^2) + ((a*a1 + b*b1)*(2*a*b1 - 2*a1*b)*(96*a*b^4 + 96*a^3*b^2))/(2*
(a^2 + b^2)^2))*(a*a1 + b*b1))/(a^2 + b^2) - ((2*a*b1 - 2*a1*b)*(((2*a*b1 -
 2*a1*b)*(((2*a*b1 - 2*a1*b)*(96*a*b^4 + 96*a^3*b^2))/(2*(a^2 + b^2)) - 32*
b^4*b1 + 64*a^2*b^2*b1 - 64*a*a1*b^3 + 32*a^3*a1*b))/(2*(a^2 + b^2)) - 32*a
*a1^2*b^2 - 32*a*b^2*b1^2 + 64*a1*b^3*b1 + 64*a^2*a1*b*b1))/(2*(a^2 + b^2))
 + ((a*a1 + b*b1)^2*(2*a*b1 - 2*a1*b)*(96*a*b^4 + 96*a^3*b^2))/(2*(a^2 + b^
2)^3) - 32*a*a1*b*b1^2)*(12*a*a1^2*b^3 - 6*a^3*a1^2*b - 6*a*b^3*b1^2 + 12*a
^3*b*b1^2 + 4*a^4*a1*b1 + 4*a1*b^4*b1 - 28*a^2*a1*b^2*b1))/((32*b^2*b1 + 32
*a*a1*b)*(a^2 + b^2)^2*(a^2*a1^2 + 4*a^2*b1^2 + 4*a1^2*b^2 + b^2*b1^2 - 6*a
*a1*b*b1)^2) - ((a^4 + b^4 + 2*a^2*b^2)*(((((a*a1 + b*b1)*(((2*a*b1 - 2*a1*
b)*(96*a*b^4 + 96*a^3*b^2))/(2*(a^2 + b^2)) - 32*b^4*b1 + 64*a^2*b^2*b1 - 6
4*a*a1*b^3 + 32*a^3*a1*b))/(a^2 + b^2) + ((a*a1 + b*b1)*(2*a*b1 - 2*a1*b)*(
96*a*b^4 + 96*a^3*b^2))/(2*(a^2 + b^2)^2))*(2*a*b1 - 2*a1*b))/(2*(a^2 + b^2
)) - ((a*a1 + b*b1)^3*(96*a*b^4 + 96*a^3*b^2))/(a^2 + b^2)^3 + ((a*a1 + b*b
1)*(((2*a*b1 - 2*a1*b)*(((2*a*b1 - 2*a1*b)*(96*a*b^4 + 96*a^3*b^2))/(2*(a^2
 + b^2)) - 32*b^4*b1 + 64*a^2*b^2*b1 - 64*a*a1*b^3 + 32*a^3*a1*b))/(2*(a^2
+ b^2)) - 32*a*a1^2*b^2 - 32*a*b^2*b1^2 + 64*a1*b^3*b1 + 64*a^2*a1*b*b1))/(
a^2 + b^2))*(a^4*a1^2 + 4*a1^2*b^4 - 4*a^4*b1^2 - b^4*b1^2 - 13*a^2*a1^2*b^
2 + 13*a^2*b^2*b1^2 - 18*a*a1*b^3*b1 + 18*a^3*a1*b*b1))/((32*b^2*b1 + 32*a*
a1*b)*(a^2 + b^2)^2*(a^2*a1^2 + 4*a^2*b1^2 + 4*a1^2*b^2 + b^2*b1^2 - 6*a*a1
*b*b1)^2))*(a*a1 + b*b1))/(a^2 + b^2) - (log(1/(cos(x) + 1))*(2*a*b1 - 2*a1
*b))/(2*(a^2 + b^2)) + (log(b + 2*a*tan(x/2) - b*tan(x/2)^2)*(a*b1 - a1*b))
/(a^2 + b^2)
```

sympy [**A**] time = 1.00, size = 360, normalized size = 7.50

$$\begin{cases} \tilde{\infty}\left(-a_1\log(\cos(x)) + b_1x\right) \\ \frac{-a_1\log(\cos(x))+b_1x}{b} \\ \frac{ia_1x\sin(x)}{2b\sin(x)+2ib\cos(x)} - \frac{a_1x\cos(x)}{2b\sin(x)+2ib\cos(x)} - \frac{ia_1\cos(x)}{2b\sin(x)+2ib\cos(x)} + \frac{b_1x\sin(x)}{2b\sin(x)+2ib\cos(x)} + \frac{ib_1x\cos(x)}{2b\sin(x)+2ib\cos(x)} + \frac{b_1\cos(x)}{2b\sin(x)+2ib\cos(x)} \\ -\frac{ia_1x\sin(x)}{2b\sin(x)-2ib\cos(x)} - \frac{a_1x\cos(x)}{2b\sin(x)-2ib\cos(x)} + \frac{ia_1\cos(x)}{2b\sin(x)-2ib\cos(x)} + \frac{b_1x\sin(x)}{2b\sin(x)-2ib\cos(x)} - \frac{ib_1x\cos(x)}{2b\sin(x)-2ib\cos(x)} + \frac{b_1\cos(x)}{2b\sin(x)-2ib\cos(x)} \\ \frac{aa_1x}{a^2+b^2} + \frac{ab_1\log\left(\sin(x)+\frac{b\cos(x)}{a}\right)}{a^2+b^2} - \frac{a_1b\log\left(\sin(x)+\frac{b\cos(x)}{a}\right)}{a^2+b^2} + \frac{bb_1x}{a^2+b^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b1*cos(x)+a1*sin(x))/(b*cos(x)+a*sin(x)),x)
```

```
[Out] Piecewise((zoo*(-a1*log(cos(x)) + b1*x), Eq(a, 0) & Eq(b, 0)), ((-a1*log(co
s(x)) + b1*x)/b, Eq(a, 0)), (I*a1*x*sin(x)/(2*b*sin(x) + 2*I*b*cos(x)) - a1
*x*cos(x)/(2*b*sin(x) + 2*I*b*cos(x)) - I*a1*cos(x)/(2*b*sin(x) + 2*I*b*cos
(x)) + b1*x*sin(x)/(2*b*sin(x) + 2*I*b*cos(x)) + I*b1*x*cos(x)/(2*b*sin(x)
```

```
+ 2*I*b*cos(x)) + b1*cos(x)/(2*b*sin(x) + 2*I*b*cos(x)), Eq(a, -I*b)), (-I*
a1*x*sin(x)/(2*b*sin(x) - 2*I*b*cos(x)) - a1*x*cos(x)/(2*b*sin(x) - 2*I*b*c
os(x)) + I*a1*cos(x)/(2*b*sin(x) - 2*I*b*cos(x)) + b1*x*sin(x)/(2*b*sin(x)
- 2*I*b*cos(x)) - I*b1*x*cos(x)/(2*b*sin(x) - 2*I*b*cos(x)) + b1*cos(x)/(2*
b*sin(x) - 2*I*b*cos(x)), Eq(a, I*b)), (a*a1*x/(a**2 + b**2) + a*b1*log(sin
(x) + b*cos(x)/a)/(a**2 + b**2) - a1*b*log(sin(x) + b*cos(x)/a)/(a**2 + b**
2) + b*b1*x/(a**2 + b**2), True))
```

3.170 $\int \frac{1}{\log(t)}\,dt$

Optimal. Leaf size=2

$$\text{LogIntegral}(t)$$

```
[Out] Li(t)
```

Rubi [A] time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2298}

$$\text{LogIntegral}(t)$$

Antiderivative was successfully verified.

```
[In] Int[Log[t]^(-1),t]
```

```
[Out] LogIntegral[t]
```

Rule 2298

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rubi steps

$$\int \frac{1}{\log(t)}\,dt = \text{li}(t)$$

Mathematica [A] time = 0.00, size = 2, normalized size = 1.00

$$\text{LogIntegral}(t)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[t]^(-1),t]
```

```
[Out] LogIntegral[t]
```

fricas [A] time = 0.40, size = 2, normalized size = 1.00

$$\text{log\_integral}(t)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(t),t, algorithm="fricas")
```

```
[Out] log_integral(t)
```

giac [A] time = 0.01, size = 3, normalized size = 1.50

$$\text{Ei}\left(\log(t)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(t),t, algorithm="giac")
```

```
[Out] Ei(log(t))
```

maple [B] time = 0.01, size = 9, normalized size = 4.50

$$-\text{Ei}\left(1,-\ln(t)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/ln(t),t)
```

```
[Out] -Ei(1,-ln(t))
```

maxima [**A**] time = 0.63, size = 3, normalized size = 1.50

$$\text{Ei}\left(\log(t)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(t),t, algorithm="maxima")
```

```
[Out] Ei(log(t))
```

mupad [**B**] time = 0.01, size = 2, normalized size = 1.00

$$\text{logint}(t)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/log(t),t)
```

```
[Out] logint(t)
```

sympy [**A**] time = 0.46, size = 2, normalized size = 1.00

$$\text{li}\left(t\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/ln(t),t)
```

```
[Out] li(t)
```

3.171 $\int \frac{1}{\log^2(t)}\,dt$

Optimal. Leaf size=10

$$\text{li}(t) - \frac{t}{\log(t)}$$

```
[Out] Li(t)-t/ln(t)
```

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2297, 2298}

$$\text{LogIntegral}(t) - \frac{t}{\log(t)}$$

Antiderivative was successfully verified.

```
[In] Int[Log[t]^(-2),t]
[Out] -(t/Log[t]) + LogIntegral[t]
```

Rule 2297

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[(x*(a + b
*Log[c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2298

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\log^2(t)}\,dt &= -\frac{t}{\log(t)} + \int \frac{1}{\log(t)}\,dt \\ &= -\frac{t}{\log(t)} + \text{li}(t) \end{aligned}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$\text{li}(t) - \frac{t}{\log(t)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[t]^(-2),t]
[Out] -(t/Log[t]) + LogIntegral[t]
```

fricas [A] time = 0.39, size = 14, normalized size = 1.40

$$\frac{\log(t)\,\text{log\_integral}(t) - t}{\log(t)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(t)^2,t, algorithm="fricas")
[Out] (log(t)*log_integral(t) - t)/log(t)
```

giac [A] time = 0.01, size = 11, normalized size = 1.10

$$-\frac{t}{\log(t)} + \text{Ei}\left(\log(t)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(t)^2,t, algorithm="giac")
[Out] -t/log(t) + Ei(log(t))
```

maple [A] time = 0.00, size = 17, normalized size = 1.70

$$-\text{Ei}\left(1, -\ln(t)\right) - \frac{t}{\ln(t)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/ln(t)^2,t)
[Out] -t/ln(t)-Ei(1,-ln(t))
```

maxima [A] time = 0.69, size = 6, normalized size = 0.60

$$\Gamma\left(-1, -\log(t)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(t)^2,t, algorithm="maxima")
[Out] gamma(-1, -log(t))
```

mupad [B] time = 0.03, size = 10, normalized size = 1.00

$$\text{logint}(t) - \frac{t}{\ln(t)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/log(t)^2,t)
[Out] logint(t) - t/log(t)
```

sympy [A] time = 0.47, size = 7, normalized size = 0.70

$$-\frac{t}{\log\left(t\right)} + \text{li}\left(t\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/ln(t)**2,t)
[Out] -t/log(t) + li(t)
```

3.172 $\int \log^{-1-n}(t)\,dt$

Optimal. Leaf size=22

$$(-\log(t))^n \log^{-n}(t)(-\operatorname{Gamma}(-n,-\log(t)))$$

```
[Out] -GAMMA(-n,-ln(t))*(-ln(t))^n/(ln(t)^n)
```

Rubi [A] time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2299, 2181}

$$(-\log(t))^n \log^{-n}(t)(-\operatorname{Gamma}(-n,-\log(t)))$$

Antiderivative was successfully verified.

```
[In] Int[Log[t]^(-1 - n),t]
[Out] -((Gamma[-n, -Log[t]]*(-Log[t])^n)/Log[t]^n)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rule 2299

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned} \int \log^{-1-n}(t)\,dt &= \operatorname{Subst}\left(\int e^t t^{-1-n}\,dt, t, \log(t)\right) \\ &= -\Gamma(-n,-\log(t))(-\log(t))^n \log^{-n}(t) \end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.00

$$(-\log(t))^n \log^{-n}(t)(-\operatorname{Gamma}(-n,-\log(t)))$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[t]^(-1 - n),t]
[Out] -((Gamma[-n, -Log[t]]*(-Log[t])^n)/Log[t]^n)
```

fricas [A] time = 0.44, size = 15, normalized size = 0.68

$$\cos(\pi + \pi n)\,\Gamma\left(-n,-\log(t)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(t)^(-1-n),t, algorithm="fricas")
[Out] cos(pi + pi*n)*gamma(-n, -log(t))
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(t)^(-1-n),t, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: integ
rate(ln(t)^(-n-1)*t/t,t)
```

maple [**F**] time = 0.04, size = 0, normalized size = 0.00

$$\int \ln(t)^{-n-1}\,dt$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(t)^(-1-n),t)
```

```
[Out] int(ln(t)^(-1-n),t)
```

maxima [**A**] time = 0.65, size = 22, normalized size = 1.00

$$-\left(-\log(t)\right)^{n}\log(t)^{-n}\Gamma\left(-n,-\log(t)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(t)^(-1-n),t, algorithm="maxima")
```

```
[Out] -(-log(t))^n*log(t)^(-n)*gamma(-n, -log(t))
```

mupad [**B**] time = 0.06, size = 22, normalized size = 1.00

$$-\frac{(-\ln(t))^{n}\,\Gamma\left(-n,-\ln(t)\right)}{\ln(t)^{n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/log(t)^(n + 1),t)
```

```
[Out] -((-log(t))^n*igamma(-n, -log(t)))/log(t)^n
```

sympy [**A**] time = 2.74, size = 24, normalized size = 1.09

$$\left(-\log\left(t\right)\right)^{n+1}\log\left(t\right)^{-n-1}\Gamma\left(-n,-\log\left(t\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(t)**(-1-n),t)
```

```
[Out] (-log(t))**(n + 1)*log(t)**(-n - 1)*uppergamma(-n, -log(t))
```

3.173 $\int \frac{e^{2t}}{-1+t}\,dt$

Optimal. Leaf size=12

$$e^2\text{Ei}(-2(1-t))$$

```
[Out] exp(2)*Ei(-2+2*t)
```

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2178}

$$e^2\text{ExpIntegralEi}(-2(1-t))$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*t)/(-1 + t),t]
```

```
[Out] E^2*ExpIntegralEi[-2*(1 - t)]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rubi steps

$$\int \frac{e^{2t}}{-1+t}\,dt = e^2\text{Ei}(-2(1-t))$$

Mathematica [A] time = 0.02, size = 10, normalized size = 0.83

$$e^2\text{Ei}(2(t-1))$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*t)/(-1 + t),t]
```

```
[Out] E^2*ExpIntegralEi[2*(-1 + t)]
```

fricas [A] time = 0.38, size = 9, normalized size = 0.75

$$\text{Ei}\,(2\,t-2)\,e^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*t)/(-1+t),t, algorithm="fricas")
```

```
[Out] Ei(2*t - 2)*e^2
```

giac [A] time = 0.01, size = 9, normalized size = 0.75

$$\text{Ei}\,(2\,t-2)\,e^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*t)/(-1+t),t, algorithm="giac")
```

```
[Out] Ei(2*t - 2)*e^2
```

maple [A] time = 0.01, size = 12, normalized size = 1.00

$$-e^2\,\mathrm{Ei}\,(1,-2t+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*t)/(-1+t),t)
[Out] -exp(2)*Ei(1,-2*t+2)
```

maxima [A] time = 0.67, size = 11, normalized size = 0.92

$$-e^2E_1\,(-2\,t+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*t)/(-1+t),t, algorithm="maxima")
[Out] -e^2*exp_integral_e(1, -2*t + 2)
```

mupad [B] time = 0.02, size = 9, normalized size = 0.75

$$e^2\,\mathrm{ei}\,(2\,t-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*t)/(t - 1),t)
[Out] exp(2)*ei(2*t - 2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{2t}}{t-1}\,dt$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*t)/(-1+t),t)
[Out] Integral(exp(2*t)/(t - 1), t)
```

3.174 $\int \frac{e^{2x}}{2-3x+x^2} \, dx$

Optimal. Leaf size=22

$$e^4\text{Ei}(2x-4)-e^2\text{Ei}(2x-2)$$

```
[Out] exp(4)*Ei(-4+2*x)-exp(2)*Ei(-2+2*x)
```

Rubi [A] time = 0.06, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2268, 2178}

$$e^4\text{ExpIntegralEi}(2x-4)-e^2\text{ExpIntegralEi}(2x-2)$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*x)/(2 - 3*x + x^2),x]
```

```
[Out] E^4*ExpIntegralEi[-4 + 2*x] - E^2*ExpIntegralEi[-2 + 2*x]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2268

```
Int[(F_)^((g_.)*((d_.) + (e_.)*(x_))^(n_.))/((a_.) + (b_.)*(x_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[F^(g*(d + e*x)^n), 1/(a + b*x + c*x^
2), x], x] /; FreeQ[{F, a, b, c, d, e, g, n}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{e^{2x}}{2-3x+x^2} \, dx &= \int \left(-\frac{2e^{2x}}{4-2x}-\frac{2e^{2x}}{-2+2x}\right) dx \\ &= -\left(2\int \frac{e^{2x}}{4-2x} \, dx\right)-2\int \frac{e^{2x}}{-2+2x} \, dx \\ &= e^4\text{Ei}(-4+2x)-e^2\text{Ei}(-2+2x) \end{aligned}$$

Mathematica [A] time = 0.04, size = 22, normalized size = 1.00

$$e^4\text{Ei}(2x-4)-e^2\text{Ei}(2x-2)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*x)/(2 - 3*x + x^2),x]
```

```
[Out] E^4*ExpIntegralEi[-4 + 2*x] - E^2*ExpIntegralEi[-2 + 2*x]
```

fricas [A] time = 0.41, size = 20, normalized size = 0.91

$${\rm Ei}\left(2\,x-4\right)e^{4}-{\rm Ei}\left(2\,x-2\right)e^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(x^2-3*x+2),x, algorithm="fricas")
```

```
[Out] Ei(2*x - 4)*e^4 - Ei(2*x - 2)*e^2
```

giac [A] time = 0.01, size = 20, normalized size = 0.91

$$\mathrm{Ei}\left(2\,x-4\right)e^{4}-\mathrm{Ei}\left(2\,x-2\right)e^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(x^2-3*x+2),x, algorithm="giac")
[Out] Ei(2*x - 4)*e^4 - Ei(2*x - 2)*e^2
```

maple [A] time = 0.01, size = 23, normalized size = 1.05

$$-\mathrm{e}^{4}\,\mathrm{Ei}\left(1,-2x+4\right)+\mathrm{e}^{2}\,\mathrm{Ei}\left(1,-2x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*x)/(x^2-3*x+2),x)
[Out] -exp(4)*Ei(1,4-2*x)+exp(2)*Ei(1,-2*x+2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{(2\,x)}}{x^{2}-3\,x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(x^2-3*x+2),x, algorithm="maxima")
[Out] integrate(e^(2*x)/(x^2 - 3*x + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\mathrm{e}^{2\,x}}{x^{2}-3\,x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*x)/(x^2 - 3*x + 2),x)
[Out] int(exp(2*x)/(x^2 - 3*x + 2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{2x}}{(x-2)\,(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(x**2-3*x+2),x)
[Out] Integral(exp(2*x)/((x - 2)*(x - 1)), x)
```

3.175 $\int \frac{1}{\sqrt{1+t^3}}\,dt$

Optimal. Leaf size=103

$$\frac{2\sqrt{2+\sqrt{3}}\,(t+1)\sqrt{\frac{t^2-t+1}{\left(t+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{t-\sqrt{3}+1}{t+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{t+1}{\left(t+\sqrt{3}+1\right)^2}}\sqrt{t^3+1}}$$

```
[Out] 2/3*(1+t)*EllipticF((1+t-3^(1/2))/(1+t+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)
+1/2*2^(1/2))*((t^2-t+1)/(1+t+3^(1/2))^2)^(1/2)*3^(3/4)/(t^3+1)^(1/2)/((1+t
)/(1+t+3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {218}

$$\frac{2\sqrt{2+\sqrt{3}}\,(t+1)\sqrt{\frac{t^2-t+1}{\left(t+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{t-\sqrt{3}+1}{t+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{t+1}{\left(t+\sqrt{3}+1\right)^2}}\sqrt{t^3+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 + t^3],t]
```

```
[Out] (2*Sqrt[2 + Sqrt[3]]*(1 + t)*Sqrt[(1 - t + t^2)/(1 + Sqrt[3] + t)^2]*Ellipt
icF[ArcSin[(1 - Sqrt[3] + t)/(1 + Sqrt[3] + t)], -7 - 4*Sqrt[3]])/(3^(1/4)*
Sqrt[(1 + t)/(1 + Sqrt[3] + t)^2]*Sqrt[1 + t^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+t^3}}\,dt=\frac{2\sqrt{2+\sqrt{3}}\,(1+t)\sqrt{\frac{1-t+t^2}{\left(1+\sqrt{3}+t\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+t}{1+\sqrt{3}+t}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+t}{\left(1+\sqrt{3}+t\right)^2}}\sqrt{1+t^3}}$$

Mathematica [C] time = 0.00, size = 17, normalized size = 0.17

$$t\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-t^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 + t^3],t]
```

```
[Out] t*Hypergeometric2F1[1/3, 1/2, 4/3, -t^3]
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{t^3+1}}, t\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(t^3+1)^(1/2),t, algorithm="fricas")
```

```
[Out] integral(1/sqrt(t^3 + 1), t)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(t^3+1)^(1/2),t, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: integ
rate(1/sqrt(t^3+1),t)
```

maple [A] time = 0.13, size = 116, normalized size = 1.13

$$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{t+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{t-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{t-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\ \text{EllipticF}\left(\sqrt{\frac{t+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}, \sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{t^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(t^3+1)^(1/2),t)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((t+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((t-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((t-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(t^3+1)^(1/2)*EllipticF(((t+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{t^3+1}}\,dt$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(t^3+1)^(1/2),t, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(t^3 + 1), t)
```

mupad [B] time = 0.29, size = 155, normalized size = 1.50

$$\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{\frac{t-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{t+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-t+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\ \mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{t+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|\,-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{t^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)t-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(t^3 + 1)^(1/2),t)
```

```
[Out] ((3^(1/2)*1i + 3)*((t + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)
*((t + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - t + 1/2)/((3^(1/
2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((t + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2
)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(t^3 - t*(((3^(1/2)*1i
)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1
i)/2 + 1/2))^(1/2)
```

sympy [**A**] time = 0.63, size = 27, normalized size = 0.26

$$\frac{t\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| t^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(t**3+1)**(1/2),t)
```

```
[Out] t*gamma(1/3)*hyper((1/3, 1/2), (4/3,), t**3*exp_polar(I*pi))/(3*gamma(4/3))
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9
```

```
#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
```

```
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
    hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
    sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
```

```
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```