Computer algebra independent integration tests
1_Algebraic_functions/1.3_Miscellaneous/1.3.2Algebraicfunctions

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Contents

1 Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from Albert Rich Rubi web site.

1.1 Listing of CAS systems tested

The following systems were tested at this time.

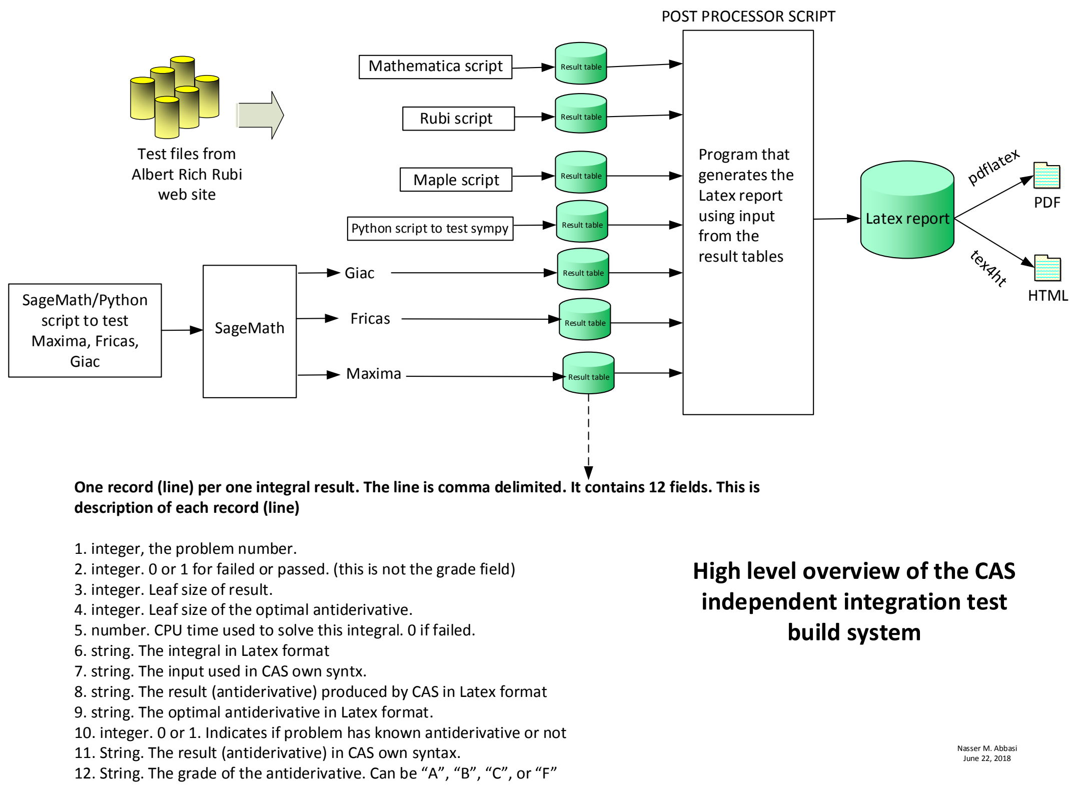
1. Mathematica 11.3 (64 bit).
2. Rubi 4.15.2 in Mathematica 11.3.
3. Rubi in Sympy (Version 1.3) under Python 3.7.0 using Anaconda distribution.
4. Maple 2018.1 (64 bit).
5. Maxima 5.41 Using Lisp ECL 16.1.2.
6. Fricas 1.3.4.
7. Sympy 1.3 under Python 3.7.0 using Anaconda distribution.
8. Giac/Xcas 1.4.9.

Maxima, Fricas and Giac/Xcas were called from inside SageMath version 8.3. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python. Rubi in Sympy was also called directly using sympy 1.3 in python.

1.2 Design of the test system

The following diagram gives a high level view of the current test build system.



1.3 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.4 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.5 Important notes about some of the results

Important note about Maxima results Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS and Giac/XCAS results There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

Important note about finding leaf size of antiderivative For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-express

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems implement a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.6 Grading of results

The table below summarizes the grading of each CAS system.

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 99.21 (878)	% 0.79 (7)
Rubi in Sympy	% 71.86 (636)	% 28.14 (249)
Mathematica	% 91.86 (813)	% 8.14 (72)
Maple	% 81.92 (725)	% 18.08 (160)
Maxima	% 36.38 (322)	% 63.62 (563)
Fricas	% 67.01 (593)	% 32.99 (292)
Sympy	% 24.29 (215)	% 75.71 (670)
Giac	% 45.88 (406)	% 54.12 (479)

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

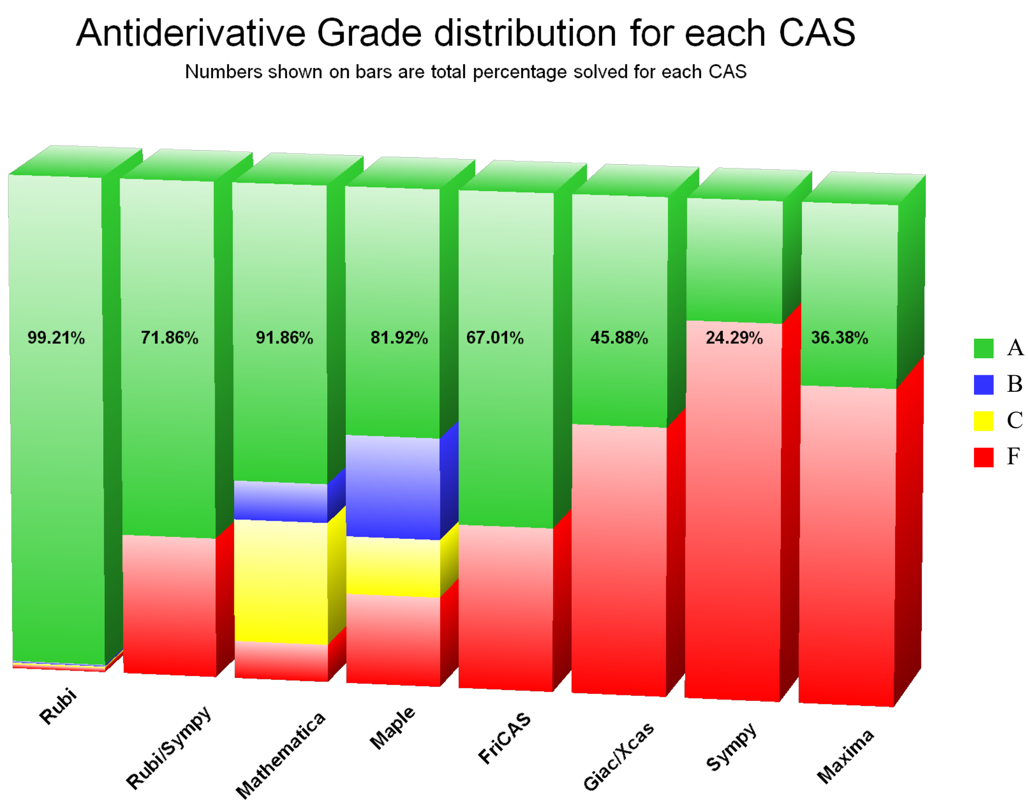
Grading is currently implemented only for for Mathematica, Rubi and Maple results. For all other CAS systems (Maxima, Fricas, Sympy, Giac, Rubi in sympy), the grading function is not yet implemented.

For these systems, a grade of A is assigned if the integrate command completes successfully and a grade of F otherwise.

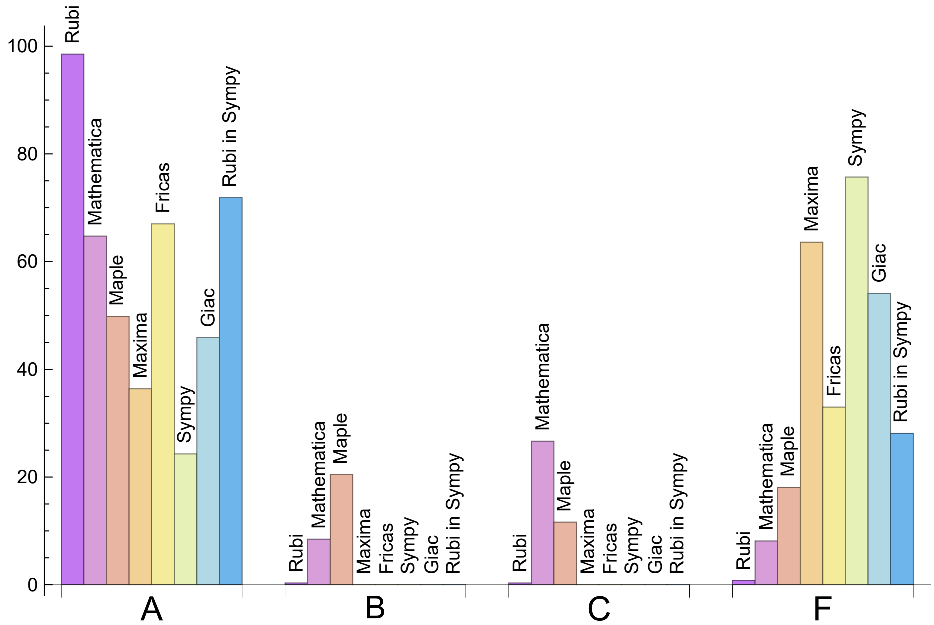
Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	98.53	0.34	0.34	0.79
Rubi in Sympy	71.86	0.	0.	28.14
Mathematica	64.75	8.47	26.67	8.14
Maple	49.83	20.45	11.64	18.08
Maxima	36.38	0.	0.	63.62
Fricas	67.01	0.	0.	32.99
Sympy	24.29	0.	0.	75.71
Giac	45.88	0.	0.	54.12

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.7 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.39	128.99	1.01	70.	1.
Rubi in Sympy	32.94	130.87	1.3	63.	0.88
Mathematica	0.79	359.78	2.66	86.	1.
Maple	0.04	1612.26	13.38	76.	1.15
Maxima	0.74	73.95	1.3	36.	1.1
Fricas	0.61	154.69	1.93	45.	1.2
Sympy	12.54	219.47	2.65	49.	0.95
Giac	0.3	90.18	1.59	47.	1.25

1.8 list of integrals that has no closed form antiderivative

{759, 760, 761, 762, 763, 764, 765, 766, 767, 768}

1.9 list of integrals not solved by each system

Not solved by Rubi {174, 455, 456, 857, 858, 879, 885}

Not solved by Rubi in Sympy {5, 6, 7, 8, 9, 18, 19, 24, 25, 26, 27, 28, 37, 38, 39, 40, 41, 46, 47, 48, 49, 50, 57, 58, 66, 67, 76, 86, 87, 88, 89, 98, 99, 100, 101, 123, 124, 125, 126, 141, 142, 143, 144, 149, 150, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 187, 188, 195, 200, 201, 204, 246, 247, 258, 259, 260, 261, 262, 263, 264, 271, 272, 273, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 299, 300, 301, 302, 303, 304, 305, 306, 311, 312, 313, 314, 318, 319, 320, 321, 322, 323, 324, 325, 326, 328, 329, 330, 331, 333, 334, 335, 336, 338, 339, 340, 341, 342, 344, 345, 346, 347, 348, 350, 351, 352, 353, 354, 356, 358, 359, 360, 361, 383, 384, 386, 391, 392, 394, 406, 407, 408, 411, 412, 413, 415, 418, 435, 446, 447, 448, 449, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 471, 472, 473, 474, 478, 479, 480, 511, 512, 513, 534, 551, 552, 556, 566, 568, 569, 571, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 653, 654, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 669, 670, 675, 680, 693, 695, 697, 721, 722, 723, 724, 725, 726, 727, 728, 729, 735, 736, 740, 744, 745, 753, 755, 756, 757, 758, 774, 775, 793, 795, 797, 802, 806, 830, 857, 858, 859, 865, 879, 881, 884, 885}

Not solved by Mathematica {18, 19, 149, 150, 172, 174, 205, 206, 207, 213, 214, 292, 299, 300, 303, 304, 311, 318, 319, 322, 323, 327, 328, 332, 333, 338, 339, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 367, 368, 381, 382, 394, 397, 398, 426, 430, 431, 432, 433, 437, 661, 662, 663, 664, 665, 666, 769, 770, 771, 772, 857, 878, 879}

Not solved by Maple {5, 6, 7, 8, 18, 19, 24, 25, 26, 27, 37, 38, 39, 40, 46, 47, 48, 49, 55, 56, 57, 58, 64, 65, 66, 67, 73, 74, 75, 76, 82, 83, 84, 85, 86, 87, 88, 89, 94, 95, 96, 97, 98, 99, 100, 101, 108, 109, 110, 111, 117, 118, 119, 120, 149, 150, 154, 158, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 174, 202, 203, 232, 292, 299, 300, 301, 302, 303, 304, 305, 306, 310, 311, 318, 319, 320, 321, 322, 323, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 401, 402, 403, 426, 427, 428, 429, 430, 431, 432, 433, 492, 493, 494, 495, 496, 530, 720, 769, 770, 771, 772, 800, 857, 858, 861, 862, 863, 870, 873, 874, 875, 876, 878, 879}

Not solved by Maxima {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 153, 154, 157, 158, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 217, 219, 220, 222, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 236, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 338, 339, 340, 341, 342, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 356, 358, 360, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 386, 387, 388, 389, 390, 394, 395, 396, 397, 398, 399, 400, 401, 402, 426, 427, 428, 429, 430, 431, 432, 433, 440, 461, 462, 463, 468, 469, 470, 475, 476, 477, 482, 483, 484, 489, 490, 491, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 525, 527, 529, 531, 532, 533, 534, 545, 546, 547, 548, 549, 550, 554, 564, 565, 567, 570, 571, 572, 584, 589, 597, 598, 599, 600, 601, 602, 603, 604, 605, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 650, 651, 655, 656, 657, 670, 671, 672, 674, 675, 676, 678, 720, 721, 722, 723, 725, 726, 729, 730, 732, 733, 734, 735, 736, 737, 739, 741, 745, 747, 750, 752, 753, 755, 756, 757, 758, 769, 770, 771, 772, 776, 777, 778, 779, 788, 796, 797, 798, 813, 814, 820, 823, 824, 825, 826, 827, 831, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 870, 871, 872, 873, 874, 875, 876, 878, 879, 880, 881, 883, 884}

Not solved by Fricas {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 82, 83, 84, 85, 86, 87, 88, 89, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 154, 158, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 217, 219, 220, 222, 225, 226, 228, 230, 231, 232, 233, 234, 235, 236, 292, 311, 327, 328, 332, 333, 338, 339, 344, 345, 349, 350, 352, 356, 360, 362, 396, 397, 398, 399, 400, 401, 402, 413, 419, 426, 427, 428, 429, 430, 431, 432, 433, 440, 452, 453, 454, 455, 456, 496, 571, 572, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 720, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 771, 772, 788, 817, 842, 845, 846, 857, 858, 862, 863, 864, 865, 870, 873, 874, 875, 876, 879}

Not solved by Sympy {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 90, 91, 92, 93, 94, 95, 96, 97, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 137, 138, 139, 140, 141, 142, 143, 144, 149, 150, 155, 156, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 177, 178, 179, 180, 181, 182, 187, 188, 193, 194, 195, 196, 197, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 220, 221, 222, 223, 224, 225, 226, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 243, 244, 245, 246, 248, 249, 250, 253, 254, 258, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 283, 287, 288, 289, 290, 292, 296, 297, 298, 299, 300, 301, 302, 303, 304, 306, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 338, 339, 340, 341, 342, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 367, 368, 373, 374, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 407, 411, 412, 413, 415, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 451, 452, 453, 454, 455, 456, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 475, 476, 477, 478, 479, 480, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 499, 500, 501, 502, 505, 506, 507, 508, 510, 511, 512, 513, 518, 519, 522, 526, 527, 529, 530, 531, 532, 533, 534, 535, 536, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 554, 556, 557, 559, 560, 561, 562, 563, 564, 565, 567, 568, 571, 572, 574, 576, 577, 578, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 651, 657, 660, 665, 666, 667, 668, 669, 670, 671, 672, 674, 675, 676, 677, 678, 682, 686, 688, 689, 691, 695, 697, 699, 701, 703, 705, 707, 708, 709, 710, 711, 713, 715, 717, 719, 720, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 750, 751, 752, 753, 754, 755, 756, 757, 758, 769, 770, 771, 772, 773, 776, 777, 778, 779, 780, 781, 783, 784, 785, 787, 788, 789, 790, 792, 793, 795, 796, 798, 800, 801, 805, 813, 814, 818, 819, 820, 826, 827, 828, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 883, 884, 885}

Not solved by Giac {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 90, 91, 92, 93, 102, 103, 104, 105, 106, 107, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 154, 158, 159, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 207, 217, 219, 220, 222, 225, 226, 228, 230, 231, 232, 233, 234, 235, 236, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 262, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 287, 289, 290, 292, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 315, 317, 318, 319, 320, 321, 322, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 365, 366, 367, 368, 371, 372, 373, 374, 375, 376, 379, 380, 381, 382, 396, 397, 398, 399, 400, 401, 402, 426, 427, 428, 429, 430, 431, 432, 433, 434, 436, 438, 440, 450, 451, 452, 453, 454, 455, 456, 468, 469, 470, 478, 479, 480, 481, 482, 483, 484, 489, 490, 491, 492, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 510, 511, 512, 513, 521, 529, 535, 536, 546, 550, 557, 558, 559, 560, 561, 562, 563, 570, 571, 572, 596, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 655, 656, 668, 673, 681, 682, 707, 708, 710, 713, 715, 717, 719, 720, 728, 729, 732, 733, 734, 735, 736, 737, 745, 769, 770, 771, 772, 778, 779, 798, 799, 800, 809, 828, 845, 846, 857, 858, 860, 861, 862, 863, 864, 865, 866, 867, 870, 871, 872, 873, 874, 875, 876, 878, 879, 880, 881}

1.10 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Rubi in Sympy {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {125, 128, 141, 142, 143, 144, 173, 187, 188, 194, 195, 200, 201, 204, 570, 618, 619, 621, 622, 624, 645, 648, 884}

Mathematica {1, 2, 3, 4, 9, 10, 11, 12, 13, 14, 15, 16, 17, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 137, 138, 139, 140, 141, 142, 143, 144, 163, 164, 165, 166, 167, 168, 169, 170, 171, 173, 179, 180, 181, 182, 202, 203, 204, 226, 228, 231, 262, 287, 289, 290, 395, 396, 399, 400, 401, 402, 594, 597, 598, 599, 608, 609, 610, 611, 612, 613, 614, 616, 617, 619, 629, 630, 635, 637, 642, 643, 645, 646, 648, 708, 710, 729, 871, 872, 880, 883}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Rubi in Sympy Verification phase not implemented yet.

2 detailed summary tables of results

2.1 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	148	139	0	0	0	0	456
normalized size	1	1.	1.02	0.96	0.	0.	0.	0.	3.14
time (sec)	N/A	0.266	0.183	0.115	0.	0.	0.	0.	142.587

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	148	143	0	0	0	0	456
normalized size	1	1.	0.92	0.89	0.	0.	0.	0.	2.85
time (sec)	N/A	0.306	0.154	0.168	0.	0.	0.	0.	144.191

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F(-2)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	146	143	0	0	0	0	427
normalized size	1	1.	0.9	0.88	0.	0.	0.	0.	2.62
time (sec)	N/A	0.283	0.184	0.076	0.	0.	0.	0.	148.569

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F(-2)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	150	139	0	0	0	0	437
normalized size	1	1.	0.96	0.89	0.	0.	0.	0.	2.8
time (sec)	N/A	0.298	0.16	0.106	0.	0.	0.	0.	144.555

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	164	0	0	0	0	0	0
normalized size	1	1.	0.59	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.558	0.295	0.12	0.	0.	0.	0.	0.

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	166	0	0	0	0	0	0
normalized size	1	1.	0.58	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.558	0.31	0.094	0.	0.	0.	0.	0.

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	167	0	0	0	0	0	0
normalized size	1	1.	0.56	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.591	0.293	0.083	0.	0.	0.	0.	0.

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	167	0	0	0	0	0	0
normalized size	1	1.	0.57	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.563	0.268	0.076	0.	0.	0.	0.	0.

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	169	495	0	0	0	0	0
normalized size	1	1.	0.68	1.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.47	0.333	0.259	0.	0.	0.	0.	0.

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	136	132	0	0	0	0	78
normalized size	1	1.	0.93	0.9	0.	0.	0.	0.	0.53
time (sec)	N/A	0.366	0.199	0.066	0.	0.	0.	0.	9.538

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	136	143	0	0	0	0	78
normalized size	1	1.	0.83	0.87	0.	0.	0.	0.	0.48
time (sec)	N/A	0.349	0.161	0.092	0.	0.	0.	0.	13.137

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	134	132	0	0	0	0	226
normalized size	1	1.	0.8	0.79	0.	0.	0.	0.	1.35
time (sec)	N/A	0.32	0.189	0.052	0.	0.	0.	0.	42.33

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	138	139	0	0	0	0	231
normalized size	1	1.	0.88	0.89	0.	0.	0.	0.	1.47
time (sec)	N/A	0.31	0.162	0.098	0.	0.	0.	0.	38.637

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	128	123	0	0	0	0	369
normalized size	1	1.	0.39	0.37	0.	0.	0.	0.	1.11
time (sec)	N/A	1.363	0.084	0.028	0.	0.	0.	0.	94.092

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	382	382	128	133	0	0	0	0	376
normalized size	1	1.	0.34	0.35	0.	0.	0.	0.	0.98
time (sec)	N/A	1.505	0.094	0.072	0.	0.	0.	0.	93.387

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	376	376	126	124	0	0	0	0	374
normalized size	1	1.	0.34	0.33	0.	0.	0.	0.	0.99
time (sec)	N/A	1.216	0.083	0.028	0.	0.	0.	0.	92.365

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	342	342	130	133	0	0	0	0	376
normalized size	1	1.	0.38	0.39	0.	0.	0.	0.	1.1
time (sec)	N/A	1.35	0.094	0.055	0.	0.	0.	0.	93.963

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F(-1)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	0	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.145	0.086	0.085	0.	0.	0.	0.	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F(-1)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	0	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.355	0.078	0.058	0.	0.	0.	0.	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	326	258	0	73	0	0	479
normalized size	1	1.	8.81	6.97	0.	1.97	0.	0.	12.95
time (sec)	N/A	0.148	0.528	0.052	0.	0.375	0.	0.	145.975

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	327	253	0	78	0	0	479
normalized size	1	1.	8.18	6.32	0.	1.95	0.	0.	11.98
time (sec)	N/A	0.173	0.496	0.057	0.	0.365	0.	0.	147.99

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	325	262	0	173	0	0	452
normalized size	1	1.	8.55	6.89	0.	4.55	0.	0.	11.89
time (sec)	N/A	0.153	0.524	0.041	0.	0.359	0.	0.	156.949

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	328	249	0	173	0	0	462
normalized size	1	1.	8.41	6.38	0.	4.44	0.	0.	11.85
time (sec)	N/A	0.159	0.482	0.049	0.	0.362	0.	0.	153.832

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	325	0	0	0	0	0	0
normalized size	1	1.	5.16	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.271	1.926	0.279	0.	0.	0.	0.	0.

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	336	0	0	0	0	0	0
normalized size	1	1.	5.17	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.292	2.062	0.27	0.	0.	0.	0.	0.

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	390	0	0	0	0	0	0
normalized size	1	1.	5.91	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.301	1.515	0.135	0.	0.	0.	0.	0.

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	375	0	0	0	0	0	0
normalized size	1	1.	5.68	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.29	1.447	0.129	0.	0.	0.	0.	0.

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	373	889	0	1	0	0	0
normalized size	1	1.	7.61	18.14	0.	0.02	0.	0.	0.
time (sec)	N/A	0.204	1.941	0.058	0.	0.392	0.	0.	0.

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	336	262	0	0	0	0	481
normalized size	1	1.	2.13	1.66	0.	0.	0.	0.	3.04
time (sec)	N/A	0.331	0.657	0.044	0.	0.	0.	0.	144.928

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	335	257	0	0	0	0	481
normalized size	1	1.	1.94	1.49	0.	0.	0.	0.	2.78
time (sec)	N/A	0.392	0.674	0.042	0.	0.	0.	0.	142.026

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	333	266	0	0	0	0	452
normalized size	1	1.	1.89	1.51	0.	0.	0.	0.	2.57
time (sec)	N/A	0.361	0.659	0.033	0.	0.	0.	0.	148.138

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	338	253	0	0	0	0	462
normalized size	1	1.	2.	1.5	0.	0.	0.	0.	2.73
time (sec)	N/A	0.372	0.677	0.034	0.	0.	0.	0.	144.832

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	340	264	0	0	0	0	483
normalized size	1	1.	2.14	1.66	0.	0.	0.	0.	3.04
time (sec)	N/A	0.375	0.71	0.038	0.	0.	0.	0.	144.083

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	340	261	0	0	0	0	483
normalized size	1	1.	1.94	1.49	0.	0.	0.	0.	2.76
time (sec)	N/A	0.425	0.697	0.039	0.	0.	0.	0.	148.262

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	338	270	0	0	0	0	454
normalized size	1	1.	1.9	1.52	0.	0.	0.	0.	2.55
time (sec)	N/A	0.379	0.682	0.036	0.	0.	0.	0.	148.028

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	342	255	0	0	0	0	464
normalized size	1	1.	2.01	1.5	0.	0.	0.	0.	2.73
time (sec)	N/A	0.376	0.712	0.035	0.	0.	0.	0.	148.946

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	336	0	0	0	0	0	0
normalized size	1	1.	1.06	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.729	2.739	0.119	0.	0.	0.	0.	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	324	324	399	0	0	0	0	0	0
normalized size	1	1.	1.23	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.793	2.205	0.113	0.	0.	0.	0.	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	333	333	400	0	0	0	0	0	0
normalized size	1	1.	1.2	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.77	2.217	0.103	0.	0.	0.	0.	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	387	0	0	0	0	0	0
normalized size	1	1.	1.18	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.697	2.138	0.103	0.	0.	0.	0.	0.

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	380	900	0	0	0	0	0
normalized size	1	1.	1.43	3.4	0.	0.	0.	0.	0.
time (sec)	N/A	0.573	2.692	0.012	0.	0.	0.	0.	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	207	258	0	0	0	0	473
normalized size	1	1.	1.43	1.78	0.	0.	0.	0.	3.26
time (sec)	N/A	0.316	0.532	0.034	0.	0.	0.	0.	137.408

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	209	253	0	0	0	0	473
normalized size	1	1.	1.31	1.58	0.	0.	0.	0.	2.96
time (sec)	N/A	0.362	0.532	0.035	0.	0.	0.	0.	139.543

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	207	262	0	0	0	0	444
normalized size	1	1.	1.27	1.61	0.	0.	0.	0.	2.72
time (sec)	N/A	0.348	0.488	0.029	0.	0.	0.	0.	143.185

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	209	249	0	0	0	0	454
normalized size	1	1.	1.34	1.6	0.	0.	0.	0.	2.91
time (sec)	N/A	0.367	0.526	0.029	0.	0.	0.	0.	144.518

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	324	0	0	0	0	0	0
normalized size	1	1.	1.18	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.617	2.081	0.11	0.	0.	0.	0.	0.

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	388	0	0	0	0	0	0
normalized size	1	1.	1.37	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.638	1.608	0.105	0.	0.	0.	0.	0.

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	389	0	0	0	0	0	0
normalized size	1	1.	1.33	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.672	1.589	0.094	0.	0.	0.	0.	0.

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	375	0	0	0	0	0	0
normalized size	1	1.	1.3	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.647	1.568	0.095	0.	0.	0.	0.	0.

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	372	892	0	0	0	0	0
normalized size	1	1.	1.51	3.63	0.	0.	0.	0.	0.
time (sec)	N/A	0.527	1.9	0.012	0.	0.	0.	0.	0.

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	265	240	0	59	0	0	371
normalized size	1	1.	11.52	10.43	0.	2.57	0.	0.	16.13
time (sec)	N/A	0.103	0.319	0.032	0.	0.341	0.	0.	98.425

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	262	240	0	63	0	0	371
normalized size	1	1.	9.7	8.89	0.	2.33	0.	0.	13.74
time (sec)	N/A	0.118	0.361	0.036	0.	0.335	0.	0.	101.795

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	260	240	0	41	0	0	369
normalized size	1	1.	10.4	9.6	0.	1.64	0.	0.	14.76
time (sec)	N/A	0.103	0.349	0.031	0.	0.354	0.	0.	90.878

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	267	240	0	43	0	0	377
normalized size	1	1.	10.68	9.6	0.	1.72	0.	0.	15.08
time (sec)	N/A	0.116	0.322	0.036	0.	0.352	0.	0.	94.908

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	A	F	F(-2)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	407	0	0	1	0	0	677
normalized size	1	1.	8.14	0.	0.	0.02	0.	0.	13.54
time (sec)	N/A	0.23	2.31	0.135	0.	0.719	0.	0.	171.741

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	370	0	0	1	0	0	677
normalized size	1	1.	7.12	0.	0.	0.02	0.	0.	13.02
time (sec)	N/A	0.237	1.245	0.151	0.	0.705	0.	0.	172.062

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	371	0	0	1	0	0	0
normalized size	1	1.	7.	0.	0.	0.02	0.	0.	0.
time (sec)	N/A	0.244	1.235	0.124	0.	0.711	0.	0.	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	410	0	0	1	0	0	0
normalized size	1	1.	7.74	0.	0.	0.02	0.	0.	0.
time (sec)	N/A	0.245	2.311	0.119	0.	0.706	0.	0.	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	295	650	0	1	0	0	549
normalized size	1	1.	6.41	14.13	0.	0.02	0.	0.	11.93
time (sec)	N/A	0.202	1.035	0.2	0.	0.389	0.	0.	144.56

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	273	246	0	0	0	0	391
normalized size	1	1.	1.96	1.77	0.	0.	0.	0.	2.81
time (sec)	N/A	0.278	0.406	0.01	0.	0.	0.	0.	93.8

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	271	246	0	0	0	0	391
normalized size	1	1.	1.77	1.61	0.	0.	0.	0.	2.56
time (sec)	N/A	0.293	0.387	0.01	0.	0.	0.	0.	95.962

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	269	246	0	0	0	0	389
normalized size	1	1.	1.72	1.58	0.	0.	0.	0.	2.49
time (sec)	N/A	0.275	0.376	0.009	0.	0.	0.	0.	94.034

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	275	246	0	0	0	0	398
normalized size	1	1.	1.83	1.64	0.	0.	0.	0.	2.65
time (sec)	N/A	0.311	0.393	0.01	0.	0.	0.	0.	65.897

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F(-2)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	419	0	0	0	0	0	745
normalized size	1	1.	1.41	0.	0.	0.	0.	0.	2.51
time (sec)	N/A	0.62	2.41	0.103	0.	0.	0.	0.	152.325

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F(-2)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	304	304	447	0	0	0	0	0	745
normalized size	1	1.	1.47	0.	0.	0.	0.	0.	2.45
time (sec)	N/A	0.609	2.462	0.101	0.	0.	0.	0.	176.072

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	448	0	0	0	0	0	0
normalized size	1	1.	1.43	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.607	2.444	0.102	0.	0.	0.	0.	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	422	0	0	0	0	0	0
normalized size	1	1.	1.36	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.647	2.38	0.094	0.	0.	0.	0.	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	384	661	0	0	0	0	588
normalized size	1	1.	1.74	2.99	0.	0.	0.	0.	2.66
time (sec)	N/A	0.55	2.006	0.012	0.	0.	0.	0.	154.518

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	193	240	0	0	0	0	379
normalized size	1	1.	1.5	1.86	0.	0.	0.	0.	2.94
time (sec)	N/A	0.249	0.433	0.01	0.	0.	0.	0.	93.534

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	195	240	0	0	0	0	379
normalized size	1	1.	1.34	1.66	0.	0.	0.	0.	2.61
time (sec)	N/A	0.277	0.452	0.009	0.	0.	0.	0.	89.156

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	193	240	0	0	0	0	376
normalized size	1	1.	1.3	1.62	0.	0.	0.	0.	2.54
time (sec)	N/A	0.258	0.435	0.008	0.	0.	0.	0.	85.672

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	195	240	0	0	0	0	386
normalized size	1	1.	1.39	1.71	0.	0.	0.	0.	2.76
time (sec)	N/A	0.287	0.436	0.009	0.	0.	0.	0.	91.33

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	407	0	0	0	0	0	687
normalized size	1	1.	1.57	0.	0.	0.	0.	0.	2.64
time (sec)	N/A	0.528	2.728	0.095	0.	0.	0.	0.	170.311

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	371	0	0	0	0	0	687
normalized size	1	1.	1.38	0.	0.	0.	0.	0.	2.56
time (sec)	N/A	0.561	1.45	0.093	0.	0.	0.	0.	173.008

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	372	0	0	0	0	0	687
normalized size	1	1.	1.34	0.	0.	0.	0.	0.	2.48
time (sec)	N/A	0.57	1.461	0.086	0.	0.	0.	0.	170.524

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	410	0	0	0	0	0	0
normalized size	1	1.	1.5	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.59	2.718	0.087	0.	0.	0.	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	295	653	0	0	0	0	561
normalized size	1	1.	1.46	3.23	0.	0.	0.	0.	2.78
time (sec)	N/A	0.517	1.17	0.011	0.	0.	0.	0.	132.163

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	267	245	0	363	0	0	134
normalized size	1	1.	6.36	5.83	0.	8.64	0.	0.	3.19
time (sec)	N/A	0.196	0.499	0.074	0.	0.364	0.	0.	29.83

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	269	243	0	366	0	0	133
normalized size	1	1.	5.85	5.28	0.	7.96	0.	0.	2.89
time (sec)	N/A	0.192	0.515	0.096	0.	0.355	0.	0.	36.41

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	267	245	0	140	0	0	76
normalized size	1	1.	6.07	5.57	0.	3.18	0.	0.	1.73
time (sec)	N/A	0.177	0.476	0.053	0.	0.356	0.	0.	20.171

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	269	247	0	143	0	0	80
normalized size	1	1.	6.11	5.61	0.	3.25	0.	0.	1.82
time (sec)	N/A	0.161	0.501	0.09	0.	0.35	0.	0.	16.953

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	322	0	0	0	0	4	250
normalized size	1	1.	4.67	0.	0.	0.	0.	0.06	3.62
time (sec)	N/A	0.361	1.054	0.227	0.	0.	0.	0.597	49.079

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	446	0	0	0	0	4	248
normalized size	1	1.	6.28	0.	0.	0.	0.	0.06	3.49
time (sec)	N/A	0.32	2.725	0.195	0.	0.	0.	0.624	56.368

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	447	0	0	0	0	4	163
normalized size	1	1.	6.21	0.	0.	0.	0.	0.06	2.26
time (sec)	N/A	0.317	2.772	0.121	0.	0.	0.	0.635	48.783

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	325	0	0	0	0	4	163
normalized size	1	1.	4.51	0.	0.	0.	0.	0.06	2.26
time (sec)	N/A	0.283	1.055	0.122	0.	0.	0.	0.602	44.573

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	A	A	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	1527	0	0	0	0	4	0
normalized size	1	1.	20.92	0.	0.	0.	0.	0.05	0.
time (sec)	N/A	0.337	8.297	0.168	0.	0.	12.29	0.603	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	A	A	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	1486	0	0	0	0	4	0
normalized size	1	1.	19.81	0.	0.	0.	0.	0.05	0.
time (sec)	N/A	0.337	8.349	0.161	0.	0.	13.705	0.613	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	A	A	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	1492	0	0	0	0	4	0
normalized size	1	1.	19.63	0.	0.	0.	0.	0.05	0.
time (sec)	N/A	0.33	8.344	0.111	0.	0.	13.514	0.606	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	A	A	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	1545	0	0	0	0	4	0
normalized size	1	1.	20.33	0.	0.	0.	0.	0.05	0.
time (sec)	N/A	0.314	8.482	0.113	0.	0.	12.985	0.6	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	269	245	0	142	0	0	78
normalized size	1	1.	6.4	5.83	0.	3.38	0.	0.	1.86
time (sec)	N/A	0.177	0.548	0.03	0.	0.313	0.	0.	16.049

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	267	247	0	144	0	0	78
normalized size	1	1.	5.8	5.37	0.	3.13	0.	0.	1.7
time (sec)	N/A	0.205	0.524	0.038	0.	0.329	0.	0.	23.305

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	265	245	0	366	0	0	133
normalized size	1	1.	6.02	5.57	0.	8.32	0.	0.	3.02
time (sec)	N/A	0.201	0.477	0.031	0.	0.317	0.	0.	33.002

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	271	243	0	369	0	0	138
normalized size	1	1.	6.16	5.52	0.	8.39	0.	0.	3.14
time (sec)	N/A	0.176	0.505	0.022	0.	0.32	0.	0.	31.107

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	320	0	0	0	0	4	162
normalized size	1	1.	4.64	0.	0.	0.	0.	0.06	2.35
time (sec)	N/A	0.327	1.031	0.164	0.	0.	0.	0.603	42.193

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	329	0	0	0	0	4	162
normalized size	1	1.	4.63	0.	0.	0.	0.	0.06	2.28
time (sec)	N/A	0.323	1.353	0.167	0.	0.	0.	0.601	50.356

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	330	0	0	0	0	4	248
normalized size	1	1.	4.58	0.	0.	0.	0.	0.06	3.44
time (sec)	N/A	0.33	1.348	0.14	0.	0.	0.	0.601	61.159

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	323	0	0	0	0	4	253
normalized size	1	1.	4.49	0.	0.	0.	0.	0.06	3.51
time (sec)	N/A	0.297	1.041	0.119	0.	0.	0.	0.615	59.654

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	A	A	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	1528	0	0	0	0	4	0
normalized size	1	1.	20.93	0.	0.	0.	0.	0.05	0.
time (sec)	N/A	0.312	7.983	0.144	0.	0.	12.659	0.61	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	A	A	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	1491	0	0	0	0	4	0
normalized size	1	1.	19.88	0.	0.	0.	0.	0.05	0.
time (sec)	N/A	0.332	8.086	0.148	0.	0.	13.636	0.605	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	A	A	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	836	0	0	0	0	4	0
normalized size	1	1.	11.	0.	0.	0.	0.	0.05	0.
time (sec)	N/A	0.333	5.598	0.114	0.	0.	13.976	0.616	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	A	A	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	1546	0	0	0	0	4	0
normalized size	1	1.	20.34	0.	0.	0.	0.	0.05	0.
time (sec)	N/A	0.322	8.036	0.13	0.	0.	13.373	0.617	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	269	245	0	0	0	0	78
normalized size	1	1.	1.86	1.69	0.	0.	0.	0.	0.54
time (sec)	N/A	0.356	0.533	0.034	0.	0.	0.	0.	13.216

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	267	245	0	0	0	0	226
normalized size	1	1.	1.84	1.69	0.	0.	0.	0.	1.56
time (sec)	N/A	0.379	0.454	0.033	0.	0.	0.	0.	41.308

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	291	260	0	0	0	0	82
normalized size	1	1.	1.68	1.5	0.	0.	0.	0.	0.47
time (sec)	N/A	0.496	0.709	0.038	0.	0.	0.	0.	13.767

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	291	264	0	0	0	0	82
normalized size	1	1.	1.56	1.41	0.	0.	0.	0.	0.44
time (sec)	N/A	0.559	0.716	0.038	0.	0.	0.	0.	16.032

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	289	262	0	0	0	0	246
normalized size	1	1.	1.52	1.38	0.	0.	0.	0.	1.29
time (sec)	N/A	0.487	0.728	0.036	0.	0.	0.	0.	45.45

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	293	258	0	0	0	0	252
normalized size	1	1.	1.6	1.41	0.	0.	0.	0.	1.38
time (sec)	N/A	0.503	0.728	0.035	0.	0.	0.	0.	44.432

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	332	332	438	0	0	0	0	4	483
normalized size	1	1.	1.32	0.	0.	0.	0.	0.01	1.45
time (sec)	N/A	1.045	3.055	0.121	0.	0.	0.	0.607	80.324

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	466	0	0	0	0	4	481
normalized size	1	1.	1.39	0.	0.	0.	0.	0.01	1.43
time (sec)	N/A	1.047	3.16	0.116	0.	0.	0.	0.59	92.826

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	345	345	467	0	0	0	0	4	177
normalized size	1	1.	1.35	0.	0.	0.	0.	0.01	0.51
time (sec)	N/A	0.969	3.15	0.104	0.	0.	0.	0.609	38.649

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	345	345	441	0	0	0	0	4	177
normalized size	1	1.	1.28	0.	0.	0.	0.	0.01	0.51
time (sec)	N/A	0.997	3.09	0.105	0.	0.	0.	0.597	38.455

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	209	255	0	0	0	0	78
normalized size	1	1.	1.54	1.88	0.	0.	0.	0.	0.57
time (sec)	N/A	0.422	0.802	0.033	0.	0.	0.	0.	10.994

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	232	257	0	0	0	0	78
normalized size	1	1.	1.53	1.69	0.	0.	0.	0.	0.51
time (sec)	N/A	0.471	1.039	0.033	0.	0.	0.	0.	13.05

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	230	255	0	0	0	0	240
normalized size	1	1.	1.4	1.55	0.	0.	0.	0.	1.46
time (sec)	N/A	0.475	0.978	0.027	0.	0.	0.	0.	41.931

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	211	253	0	0	0	0	246
normalized size	1	1.	1.35	1.62	0.	0.	0.	0.	1.58
time (sec)	N/A	0.463	0.79	0.029	0.	0.	0.	0.	40.574

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	225	255	0	0	0	0	240
normalized size	1	1.	1.53	1.73	0.	0.	0.	0.	1.63
time (sec)	N/A	0.461	0.976	0.033	0.	0.	0.	0.	41.647

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	427	0	0	0	0	0	444
normalized size	1	1.	1.54	0.	0.	0.	0.	0.	1.6
time (sec)	N/A	0.877	2.053	0.108	0.	0.	0.	0.	76.988

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	454	0	0	0	0	0	444
normalized size	1	1.	1.59	0.	0.	0.	0.	0.	1.55
time (sec)	N/A	0.915	2.301	0.107	0.	0.	0.	0.	87.777

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	282	282	455	0	0	0	0	0	168
normalized size	1	1.	1.61	0.	0.	0.	0.	0.	0.6
time (sec)	N/A	0.856	2.335	0.096	0.	0.	0.	0.	33.84

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	430	0	0	0	0	0	168
normalized size	1	1.	1.55	0.	0.	0.	0.	0.	0.6
time (sec)	N/A	0.826	2.039	0.098	0.	0.	0.	0.	32.896

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	319	319	214	275	0	0	0	0	326
normalized size	1	1.	0.67	0.86	0.	0.	0.	0.	1.02
time (sec)	N/A	2.525	1.019	0.056	0.	0.	0.	0.	155.724

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	235	264	0	0	0	0	325
normalized size	1	1.	0.71	0.8	0.	0.	0.	0.	0.98
time (sec)	N/A	2.456	1.233	0.064	0.	0.	0.	0.	165.609

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	327	327	233	273	0	0	0	0	0
normalized size	1	1.	0.71	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	1.857	1.203	0.055	0.	0.	0.	0.	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	216	266	0	0	0	0	0
normalized size	1	1.	0.67	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	2.179	1.029	0.06	0.	0.	0.	0.	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	360	360	213	275	0	0	0	0	0
normalized size	1	1.	0.59	0.76	0.	0.	0.	0.	0.
time (sec)	N/A	2.198	0.986	0.031	0.	0.	0.	0.	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	348	348	235	268	0	0	0	0	0
normalized size	1	1.	0.68	0.77	0.	0.	0.	0.	0.
time (sec)	N/A	2.111	1.255	0.031	0.	0.	0.	0.	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F(-2)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	344	344	233	277	0	0	0	0	325
normalized size	1	1.	0.68	0.81	0.	0.	0.	0.	0.94
time (sec)	N/A	1.825	1.173	0.028	0.	0.	0.	0.	154.739

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F(-1)	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	364	364	215	266	0	0	0	0	330
normalized size	1	1.	0.59	0.73	0.	0.	0.	0.	0.91
time (sec)	N/A	2.096	1.022	0.029	0.	0.	0.	0.	170.995

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	149	132	0	0	56	0	117
normalized size	1	1.	1.19	1.06	0.	0.	0.45	0.	0.94
time (sec)	N/A	0.103	0.787	0.041	0.	0.	6.849	0.	10.783

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	157	125	0	0	99	0	117
normalized size	1	1.	1.13	0.9	0.	0.	0.71	0.	0.84
time (sec)	N/A	0.113	0.818	0.049	0.	0.	8.073	0.	12.82

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	150	140	0	0	94	0	116
normalized size	1	1.	1.06	0.99	0.	0.	0.66	0.	0.82
time (sec)	N/A	0.11	1.051	0.037	0.	0.	7.924	0.	11.621

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	155	135	0	0	61	0	121
normalized size	1	1.	1.14	0.99	0.	0.	0.45	0.	0.89
time (sec)	N/A	0.111	0.951	0.042	0.	0.	6.936	0.	12.1

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	149	132	0	0	56	0	117
normalized size	1	1.	1.17	1.04	0.	0.	0.44	0.	0.92
time (sec)	N/A	0.095	0.56	0.023	0.	0.	6.917	0.	11.268

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	158	125	0	0	99	0	117
normalized size	1	1.	1.12	0.89	0.	0.	0.7	0.	0.83
time (sec)	N/A	0.12	1.483	0.023	0.	0.	8.171	0.	13.497

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	151	140	0	0	94	0	116
normalized size	1	1.	1.05	0.97	0.	0.	0.65	0.	0.81
time (sec)	N/A	0.102	1.23	0.019	0.	0.	8.255	0.	12.321

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	156	135	0	0	61	0	121
normalized size	1	1.	1.13	0.98	0.	0.	0.44	0.	0.88
time (sec)	N/A	0.11	1.314	0.019	0.	0.	6.994	0.	12.93

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	334	334	194	240	0	0	0	0	379
normalized size	1	1.	0.58	0.72	0.	0.	0.	0.	1.13
time (sec)	N/A	1.395	0.383	0.009	0.	0.	0.	0.	89.332

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	379	379	195	240	0	0	0	0	384
normalized size	1	1.	0.51	0.63	0.	0.	0.	0.	1.01
time (sec)	N/A	1.611	0.399	0.011	0.	0.	0.	0.	90.473

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	375	375	193	240	0	0	0	0	381
normalized size	1	1.	0.51	0.64	0.	0.	0.	0.	1.02
time (sec)	N/A	1.503	0.396	0.009	0.	0.	0.	0.	88.028

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	196	240	0	0	0	0	384
normalized size	1	1.	0.57	0.7	0.	0.	0.	0.	1.12
time (sec)	N/A	1.454	0.406	0.01	0.	0.	0.	0.	90.658

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	452	452	211	274	0	0	0	0	0
normalized size	1	1.	0.47	0.61	0.	0.	0.	0.	0.
time (sec)	N/A	2.847	0.928	0.011	0.	0.	0.	0.	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	F(-1)
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	476	476	233	265	0	0	0	0	0
normalized size	1	1.	0.49	0.56	0.	0.	0.	0.	0.
time (sec)	N/A	2.836	1.229	0.011	0.	0.	0.	0.	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	F(-1)
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	477	477	231	274	0	0	0	0	0
normalized size	1	1.	0.48	0.57	0.	0.	0.	0.	0.
time (sec)	N/A	2.383	1.199	0.011	0.	0.	0.	0.	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	465	465	213	265	0	0	0	0	0
normalized size	1	1.	0.46	0.57	0.	0.	0.	0.	0.
time (sec)	N/A	3.024	0.936	0.011	0.	0.	0.	0.	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	134	129	0	0	42	0	112
normalized size	1	1.	1.12	1.08	0.	0.	0.35	0.	0.93
time (sec)	N/A	0.113	0.608	0.009	0.	0.	5.348	0.	11.09

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	140	122	0	0	65	0	114
normalized size	1	1.	1.04	0.91	0.	0.	0.49	0.	0.85
time (sec)	N/A	0.121	0.603	0.008	0.	0.	5.588	0.	12.291

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	136	129	0	0	60	0	110
normalized size	1	1.	0.99	0.94	0.	0.	0.44	0.	0.8
time (sec)	N/A	0.122	0.49	0.008	0.	0.	5.499	0.	11.404

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	138	122	0	0	46	0	116
normalized size	1	1.	1.05	0.93	0.	0.	0.35	0.	0.89
time (sec)	N/A	0.126	0.624	0.008	0.	0.	5.396	0.	12.655

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	0	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.22	0.165	0.066	0.	0.	0.	0.	0.

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	0	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.428	0.205	0.061	0.	0.	0.	0.	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	204	451	342	662	6431	1238	144
normalized size	1	1.	1.27	2.82	2.14	4.14	40.19	7.74	0.9
time (sec)	N/A	0.212	0.199	0.012	0.707	0.291	51.028	0.273	40.45

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	142	283	248	470	3699	857	112
normalized size	1	1.	1.13	2.25	1.97	3.73	29.36	6.8	0.89
time (sec)	N/A	0.147	0.12	0.008	0.713	0.289	15.325	0.27	31.463

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	95	167	0	300	1904	539	83
normalized size	1	1.	1.01	1.78	0.	3.19	20.26	5.73	0.88
time (sec)	N/A	0.096	0.108	0.007	0.	0.286	9.164	0.269	24.107

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	125	0	0	0	741	0	83
normalized size	1	1.	1.26	0.	0.	0.	7.48	0.	0.84
time (sec)	N/A	0.115	0.251	0.04	0.	0.	11.085	0.	23.207

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	534	1565	811	2113	0	1	275
normalized size	1	1.	1.82	5.32	2.76	7.19	0.	0.	0.94
time (sec)	N/A	0.419	0.571	0.022	0.717	0.303	0.	0.273	87.321

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	406	1142	640	1642	0	1	228
normalized size	1	1.	1.64	4.6	2.58	6.62	0.	0.	0.92
time (sec)	N/A	0.317	0.485	0.02	0.715	0.291	0.	0.271	68.524

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	297	793	0	1206	11662	1	187
normalized size	1	1.	1.46	3.91	0.	5.94	57.45	0.	0.92
time (sec)	N/A	0.247	0.278	0.017	0.	0.289	144.844	0.272	58.148

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	420	0	0	0	4755	0	187
normalized size	1	1.	2.01	0.	0.	0.	22.75	0.	0.89
time (sec)	N/A	0.274	0.613	0.05	0.	0.	31.633	0.	57.755

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	459	459	1134	3780	1557	4811	0	0	439
normalized size	1	1.	2.47	8.24	3.39	10.48	0.	0.	0.96
time (sec)	N/A	0.683	2.011	0.043	0.733	0.331	0.	0.	144.694

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	396	396	903	2972	1287	3941	0	1	374
normalized size	1	1.	2.28	7.51	3.25	9.95	0.	0.	0.94
time (sec)	N/A	0.587	1.343	0.033	0.741	0.323	0.	0.298	121.254

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	337	337	706	2280	0	3123	0	1	316
normalized size	1	1.	2.09	6.77	0.	9.27	0.	0.	0.94
time (sec)	N/A	0.466	1.095	0.028	0.	0.319	0.	0.281	100.789

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	358	358	856	0	0	0	0	0	338
normalized size	1	1.	2.39	0.	0.	0.	0.	0.	0.94
time (sec)	N/A	0.48	1.84	0.05	0.	0.	0.	0.	100.157

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	324	324	423	0	0	0	0	0	272
normalized size	1	1.	1.31	0.	0.	0.	0.	0.	0.84
time (sec)	N/A	1.478	0.59	0.093	0.	0.	0.	0.	143.417

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	332	332	298	0	0	0	0	0	287
normalized size	1	1.	0.9	0.	0.	0.	0.	0.	0.86
time (sec)	N/A	1.603	0.692	0.083	0.	0.	0.	0.	167.999

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	142	0	0	0	0	0	243
normalized size	1	1.	0.48	0.	0.	0.	0.	0.	0.83
time (sec)	N/A	1.195	0.141	0.075	0.	0.	0.	0.	120.717

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	337	0	0	0	0	0	209
normalized size	1	1.	1.33	0.	0.	0.	0.	0.	0.83
time (sec)	N/A	0.69	0.122	0.073	0.	0.	0.	0.	73.025

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	229	0	0	0	0	0	252
normalized size	1	1.	0.8	0.	0.	0.	0.	0.	0.88
time (sec)	N/A	0.686	0.085	0.073	0.	0.	0.	0.	96.608

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	122	0	0	0	0	0	223
normalized size	1	1.	0.46	0.	0.	0.	0.	0.	0.85
time (sec)	N/A	0.472	0.051	0.073	0.	0.	0.	0.	60.838

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	377	0	0	0	0	0	241
normalized size	1	1.	1.26	0.	0.	0.	0.	0.	0.8
time (sec)	N/A	1.243	0.263	0.062	0.	0.	0.	0.	129.179

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	280	0	0	0	0	0	282
normalized size	1	1.	0.86	0.	0.	0.	0.	0.	0.87
time (sec)	N/A	1.289	0.247	0.09	0.	0.	0.	0.	163.288

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	375	0	0	0	0	0	209
normalized size	1	1.	1.48	0.	0.	0.	0.	0.	0.83
time (sec)	N/A	0.949	0.328	0.077	0.	0.	0.	0.	75.827

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	0	0	0	0	0	0	168
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.8
time (sec)	N/A	0.97	0.083	0.102	0.	0.	0.	0.	88.744

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1482	1482	820	1126	0	0	0	0	0
normalized size	1	1.	0.55	0.76	0.	0.	0.	0.	0.
time (sec)	N/A	5.746	3.575	0.142	0.	0.	0.	0.	0.

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	F	A	F	F	F	A	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	135	0	0	0	0	0	638	0	0
normalized size	1	0.	0.	0.	0.	0.	4.73	0.	0.
time (sec)	N/A	0.122	0.043	0.087	0.	0.	164.096	0.	0.

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	296	1640	0	26	0	0	0
normalized size	1	1.	18.5	102.5	0.	1.62	0.	0.	0.
time (sec)	N/A	0.109	1.362	0.097	0.	0.298	0.	0.	0.

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	280	732	0	26	0	0	0
normalized size	1	1.	14.	36.6	0.	1.3	0.	0.	0.
time (sec)	N/A	0.132	1.046	0.097	0.	0.306	0.	0.	0.

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	278	1656	0	34	0	0	0
normalized size	1	1.	15.44	92.	0.	1.89	0.	0.	0.
time (sec)	N/A	0.111	1.018	0.075	0.	0.271	0.	0.	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	298	724	0	38	0	0	0
normalized size	1	1.	16.56	40.22	0.	2.11	0.	0.	0.
time (sec)	N/A	0.123	1.187	0.092	0.	0.269	0.	0.	0.

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	424	4397	0	1	0	0	0
normalized size	1	1.	14.13	146.57	0.	0.03	0.	0.	0.
time (sec)	N/A	0.143	2.056	0.057	0.	0.291	0.	0.	0.

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	427	1908	0	1	0	0	0
normalized size	1	1.	11.24	50.21	0.	0.03	0.	0.	0.
time (sec)	N/A	0.183	2.084	0.069	0.	0.289	0.	0.	0.

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	425	4437	0	1	0	0	0
normalized size	1	1.	11.81	123.25	0.	0.03	0.	0.	0.
time (sec)	N/A	0.144	0.616	0.043	0.	0.288	0.	0.	0.

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	426	1888	0	1	0	0	0
normalized size	1	1.	13.31	59.	0.	0.03	0.	0.	0.
time (sec)	N/A	0.15	2.062	0.053	0.	0.296	0.	0.	0.

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	355	355	310	334	0	0	175	0	332
normalized size	1	1.	0.87	0.94	0.	0.	0.49	0.	0.94
time (sec)	N/A	0.503	0.724	0.048	0.	0.	10.108	0.	54.531

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	247	310	0	0	138	0	301
normalized size	1	1.	0.76	0.95	0.	0.	0.42	0.	0.92
time (sec)	N/A	0.404	0.699	0.01	0.	0.	9.741	0.	44.478

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	132	127	0	0	88	0	143
normalized size	1	1.	0.84	0.8	0.	0.	0.56	0.	0.91
time (sec)	N/A	0.181	0.553	0.006	0.	0.	8.502	0.	16.577

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	89	85	0	0	37	0	92
normalized size	1	1.	0.85	0.81	0.	0.	0.35	0.	0.88
time (sec)	N/A	0.056	0.175	0.003	0.	0.	2.108	0.	5.444

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	737	737	451	565	0	0	0	0	0
normalized size	1	1.	0.61	0.77	0.	0.	0.	0.	0.
time (sec)	N/A	1.393	1.636	0.021	0.	0.	0.	0.	0.

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1385	1385	924	402	0	0	0	0	0
normalized size	1	1.	0.67	0.29	0.	0.	0.	0.	0.
time (sec)	N/A	4.063	6.291	0.025	0.	0.	0.	0.	0.

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	240	218	0	0	141	0	272
normalized size	1	1.	0.81	0.74	0.	0.	0.48	0.	0.92
time (sec)	N/A	0.383	0.504	0.012	0.	0.	8.355	0.	42.949

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	204	197	0	0	105	0	240
normalized size	1	1.	0.78	0.75	0.	0.	0.4	0.	0.91
time (sec)	N/A	0.3	0.334	0.01	0.	0.	7.538	0.	33.249

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	107	96	0	0	61	0	109
normalized size	1	1.	0.88	0.79	0.	0.	0.5	0.	0.9
time (sec)	N/A	0.13	0.213	0.005	0.	0.	5.819	0.	13.81

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	74	70	0	0	36	0	78
normalized size	1	1.	0.84	0.8	0.	0.	0.41	0.	0.89
time (sec)	N/A	0.034	0.043	0.003	0.	0.	2.025	0.	3.916

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	413	413	200	169	0	0	0	0	367
normalized size	1	1.	0.48	0.41	0.	0.	0.	0.	0.89
time (sec)	N/A	0.602	0.434	0.008	0.	0.	0.	0.	56.331

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-1)	F	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	749	749	462	421	0	0	0	0	666
normalized size	1	1.	0.62	0.56	0.	0.	0.	0.	0.89
time (sec)	N/A	1.162	2.419	0.009	0.	0.	0.	0.	119.817

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1969	1969	884	483	0	0	0	0	0
normalized size	1	1.	0.45	0.25	0.	0.	0.	0.	0.
time (sec)	N/A	9.258	2.05	0.025	0.	0.	0.	0.	0.

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	215	261	0	0	0	0	272
normalized size	1	1.	0.72	0.88	0.	0.	0.	0.	0.91
time (sec)	N/A	0.342	0.485	0.039	0.	0.	0.	0.	37.383

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	188	239	0	0	0	0	238
normalized size	1	1.	0.7	0.89	0.	0.	0.	0.	0.88
time (sec)	N/A	0.313	0.364	0.011	0.	0.	0.	0.	34.699

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	90	115	0	0	61	0	100
normalized size	1	1.	0.79	1.01	0.	0.	0.54	0.	0.88
time (sec)	N/A	0.111	0.183	0.006	0.	0.	24.228	0.	12.695

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	102	94	0	0	36	0	94
normalized size	1	1.	0.94	0.87	0.	0.	0.33	0.	0.87
time (sec)	N/A	0.064	0.073	0.004	0.	0.	2.275	0.	5.657

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	827	827	464	496	0	0	0	0	0
normalized size	1	1.	0.56	0.6	0.	0.	0.	0.	0.
time (sec)	N/A	1.725	4.595	0.025	0.	0.	0.	0.	0.

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2413	2413	809	642	0	0	0	0	0
normalized size	1	1.	0.34	0.27	0.	0.	0.	0.	0.
time (sec)	N/A	13.525	4.066	0.038	0.	0.	0.	0.	0.

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	526	0	0	0	0	0	265
normalized size	1	1.	1.51	0.	0.	0.	0.	0.	0.76
time (sec)	N/A	1.463	0.253	0.078	0.	0.	0.	0.	86.392

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	691	0	0	0	0	0	265
normalized size	1	1.	1.98	0.	0.	0.	0.	0.	0.76
time (sec)	N/A	1.204	0.348	0.076	0.	0.	0.	0.	90.077

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1493	1493	653	1153	0	0	0	0	0
normalized size	1	1.	0.44	0.77	0.	0.	0.	0.	0.
time (sec)	N/A	15.287	8.5	0.102	0.	0.	0.	0.	0.

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	F(-2)	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	0	64	0	1	0	0	68
normalized size	1	1.	0.	0.85	0.	0.01	0.	0.	0.91
time (sec)	N/A	0.056	0.109	0.193	0.	0.396	0.	0.	11.595

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	A	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	0	57	0	1	0	92	42
normalized size	1	1.	0.	1.14	0.	0.02	0.	1.84	0.84
time (sec)	N/A	0.04	0.088	0.066	0.	0.393	0.	0.282	10.248

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	F(-2)	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	0	17	0	1	0	0	22
normalized size	1	1.	0.	0.71	0.	0.04	0.	0.	0.92
time (sec)	N/A	0.025	0.034	0.058	0.	0.358	0.	0.	8.864

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	37	55	23	0	38	22
normalized size	1	1.	1.	1.61	2.39	1.	0.	1.65	0.96
time (sec)	N/A	0.016	0.016	0.008	0.78	0.286	0.	0.274	7.016

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	30	44	68	34	0	58	44
normalized size	1	1.	0.61	0.9	1.39	0.69	0.	1.18	0.9
time (sec)	N/A	0.03	0.015	0.008	0.783	0.277	0.	0.279	8.225

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	33	28	35	39	0	39	32
normalized size	1	1.	0.89	0.76	0.95	1.05	0.	1.05	0.86
time (sec)	N/A	0.022	0.068	0.016	0.768	0.293	0.	0.261	13.96

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	33	28	35	39	0	39	32
normalized size	1	1.	0.89	0.76	0.95	1.05	0.	1.05	0.86
time (sec)	N/A	0.025	0.007	0.01	0.77	0.299	0.	0.26	14.967

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	44	38	54	55	0	57	65
normalized size	1	1.	0.62	0.54	0.76	0.77	0.	0.8	0.92
time (sec)	N/A	0.037	0.037	0.014	0.775	0.294	0.	0.264	15.073

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	0	37	57	1	0	65	42
normalized size	1	1.	0.	0.76	1.16	0.02	0.	1.33	0.86
time (sec)	N/A	0.035	0.139	0.005	0.777	0.293	0.	0.261	14.45

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	0	37	57	1	0	65	42
normalized size	1	1.	0.	0.76	1.16	0.02	0.	1.33	0.86
time (sec)	N/A	0.037	0.068	0.004	0.776	0.291	0.	0.262	15.221

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	47	43	43	1	0	51	39
normalized size	1	1.	1.07	0.98	0.98	0.02	0.	1.16	0.89
time (sec)	N/A	0.033	0.033	0.019	0.791	0.289	0.	0.265	18.392

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	32	26	26	177	0	36	36
normalized size	1	1.	0.73	0.59	0.59	4.02	0.	0.82	0.82
time (sec)	N/A	0.023	0.017	0.01	0.786	0.299	0.	0.261	8.767

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	77	76	0	0	0	0	73
normalized size	1	1.	0.93	0.92	0.	0.	0.	0.	0.88
time (sec)	N/A	0.061	0.069	0.031	0.	0.	0.	0.	10.722

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	26	74	20	26	17
normalized size	1	1.	1.	0.86	1.18	3.36	0.91	1.18	0.77
time (sec)	N/A	0.009	0.008	0.003	0.769	0.291	1.116	0.265	7.453

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	58	81	0	0	36	0	119
normalized size	1	1.	0.44	0.62	0.	0.	0.27	0.	0.91
time (sec)	N/A	0.182	0.052	0.024	0.	0.	3.484	0.	16.51

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	57	62	0	0	0	0	48
normalized size	1	1.	1.06	1.15	0.	0.	0.	0.	0.89
time (sec)	N/A	0.042	0.029	0.038	0.	0.	0.	0.	8.723

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	28	19	14	1	0	41	20
normalized size	1	1.	1.27	0.86	0.64	0.05	0.	1.86	0.91
time (sec)	N/A	0.025	0.012	0.008	0.783	0.281	0.	0.267	8.992

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	74	116	0	0	0	0	148
normalized size	1	1.	0.47	0.73	0.	0.	0.	0.	0.93
time (sec)	N/A	0.12	0.034	0.044	0.	0.	0.	0.	15.962

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	31	66	0	30	19
normalized size	1	1.	1.	0.86	1.48	3.14	0.	1.43	0.9
time (sec)	N/A	0.015	0.01	0.006	0.775	0.289	0.	0.262	7.871

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	31	38	26	0	16	22
normalized size	1	1.	1.	1.24	1.52	1.04	0.	0.64	0.88
time (sec)	N/A	0.01	0.009	0.007	0.795	0.274	0.	0.261	6.526

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	174	1521	0	0	0	0	253
normalized size	1	1.	0.6	5.21	0.	0.	0.	0.	0.87
time (sec)	N/A	0.425	0.57	0.376	0.	0.	0.	0.	19.26

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	134	270	0	0	0	0	233
normalized size	1	1.	0.52	1.04	0.	0.	0.	0.	0.9
time (sec)	N/A	0.144	0.131	0.029	0.	0.	0.	0.	15.011

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	22	321	0	1	14	47	20
normalized size	1	1.	0.96	13.96	0.	0.04	0.61	2.04	0.87
time (sec)	N/A	0.047	0.025	0.086	0.	0.326	3.653	0.264	6.193

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	106	232	0	0	0	0	100
normalized size	1	1.	0.91	2.	0.	0.	0.	0.	0.86
time (sec)	N/A	0.148	0.166	0.146	0.	0.	0.	0.	9.04

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	19	0	1	0	42	24
normalized size	1	1.	1.	0.79	0.	0.04	0.	1.75	1.
time (sec)	N/A	0.026	0.018	0.007	0.	0.285	0.	0.264	8.479

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	312	312	165	1784	0	0	0	0	282
normalized size	1	1.	0.53	5.72	0.	0.	0.	0.	0.9
time (sec)	N/A	0.445	0.462	0.096	0.	0.	0.	0.	21.17

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	146	353	0	0	0	0	255
normalized size	1	1.	0.52	1.26	0.	0.	0.	0.	0.91
time (sec)	N/A	0.174	0.397	0.032	0.	0.	0.	0.	17.468

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F(-2)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	53	0	0	0	0	0	36
normalized size	1	1.	1.43	0.	0.	0.	0.	0.	0.97
time (sec)	N/A	0.031	0.049	0.102	0.	0.	0.	0.	8.883

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F(-2)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	40	35	0	0	0	0	42
normalized size	1	1.	0.83	0.73	0.	0.	0.	0.	0.88
time (sec)	N/A	0.033	0.027	0.063	0.	0.	0.	0.	9.176

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F(-2)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	44	37	0	0	0	0	44
normalized size	1	1.	0.85	0.71	0.	0.	0.	0.	0.85
time (sec)	N/A	0.036	0.029	0.089	0.	0.	0.	0.	9.072

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	C	A	A	A	F(-2)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	80	33	30	24	0	0	0	70
normalized size	1	2.35	0.97	0.88	0.71	0.	0.	0.	2.06
time (sec)	N/A	0.065	0.059	0.037	0.844	0.	0.	0.	21.985

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	106	191	0	0	0	0	95
normalized size	1	1.	0.93	1.68	0.	0.	0.	0.	0.83
time (sec)	N/A	0.231	0.303	0.084	0.	0.	0.	0.	27.888

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	23	27	0	27	22
normalized size	1	1.	1.	1.06	1.44	1.69	0.	1.69	1.38
time (sec)	N/A	0.013	0.005	0.002	0.706	0.28	0.	0.262	2.029

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	27	43	43	0	43	41
normalized size	1	1.	1.	1.04	1.65	1.65	0.	1.65	1.58
time (sec)	N/A	0.021	0.011	0.003	0.723	0.334	0.	0.266	7.156

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	37	59	59	0	59	60
normalized size	1	1.	1.	1.03	1.64	1.64	0.	1.64	1.67
time (sec)	N/A	0.032	0.017	0.003	0.757	0.299	0.	0.265	20.952

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	140	90	0	127	0	0	121
normalized size	1	1.	0.95	0.61	0.	0.86	0.	0.	0.82
time (sec)	N/A	0.251	0.154	0.007	0.	0.296	0.	0.	27.527

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	100	66	0	95	0	0	76
normalized size	1	1.	1.05	0.69	0.	1.	0.	0.	0.8
time (sec)	N/A	0.151	0.099	0.004	0.	0.273	0.	0.	17.496

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	63	40	0	39	136	0	32
normalized size	1	1.	1.34	0.85	0.	0.83	2.89	0.	0.68
time (sec)	N/A	0.082	0.077	0.003	0.	0.275	2.468	0.	5.574

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	75	73	0	1	0	0	76
normalized size	1	1.	0.77	0.75	0.	0.01	0.	0.	0.78
time (sec)	N/A	0.197	0.077	0.008	0.	0.342	0.	0.	18.061

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	81	88	0	1	0	0	76
normalized size	1	1.	0.79	0.85	0.	0.01	0.	0.	0.74
time (sec)	N/A	0.199	0.18	0.02	0.	0.314	0.	0.	18.621

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	167	604	0	2361	0	0	197
normalized size	1	1.	0.73	2.65	0.	10.36	0.	0.	0.86
time (sec)	N/A	0.752	0.185	0.027	0.	0.309	0.	0.	61.866

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	124	431	0	1385	0	0	0
normalized size	1	1.	0.75	2.61	0.	8.39	0.	0.	0.
time (sec)	N/A	0.45	0.288	0.015	0.	0.313	0.	0.	0.

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	114	93	377	0	711	388	0	0
normalized size	1	1.81	1.48	5.98	0.	11.29	6.16	0.	0.
time (sec)	N/A	0.216	0.123	0.01	0.	0.275	3.598	0.	0.

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	140	258	0	1	0	0	119
normalized size	1	1.	1.05	1.94	0.	0.01	0.	0.	0.89
time (sec)	N/A	0.543	0.138	0.015	0.	0.308	0.	0.	46.321

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	153	274	0	1	0	0	124
normalized size	1	1.	1.09	1.94	0.	0.01	0.	0.	0.88
time (sec)	N/A	0.533	0.315	0.017	0.	0.307	0.	0.	44.242

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	375	375	138	294	0	281	0	0	342
normalized size	1	1.	0.37	0.78	0.	0.75	0.	0.	0.91
time (sec)	N/A	0.733	0.536	0.007	0.	0.279	0.	0.	78.833

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	93	222	0	215	942	0	236
normalized size	1	1.	0.36	0.85	0.	0.82	3.61	0.	0.9
time (sec)	N/A	0.505	0.398	0.004	0.	0.276	7.275	0.	53.056

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	B	A	B	F	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	151	55	146	0	143	384	0	124
normalized size	1	2.36	0.86	2.28	0.	2.23	6.	0.	1.94
time (sec)	N/A	0.213	0.252	0.004	0.	0.263	6.798	0.	25.823

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	142	181	0	1	0	0	138
normalized size	1	1.	0.9	1.15	0.	0.01	0.	0.	0.88
time (sec)	N/A	0.464	0.242	0.005	0.	0.311	0.	0.	34.081

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	223	112	252	0	1	0	0	196
normalized size	1	1.38	0.69	1.56	0.	0.01	0.	0.	1.21
time (sec)	N/A	0.565	0.512	0.004	0.	0.318	0.	0.	41.008

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	19	14	0	18	63	18	17
normalized size	1	1.	0.9	0.67	0.	0.86	3.	0.86	0.81
time (sec)	N/A	0.014	0.022	0.003	0.	0.308	1.414	0.292	1.425

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	19	14	0	18	63	18	17
normalized size	1	1.	0.9	0.67	0.	0.86	3.	0.86	0.81
time (sec)	N/A	0.014	0.023	0.002	0.	0.273	1.441	0.278	1.47

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	31	16	0	20	51	20	15
normalized size	1	1.	1.35	0.7	0.	0.87	2.22	0.87	0.65
time (sec)	N/A	0.04	0.022	0.002	0.	0.264	1.485	0.28	2.575

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	44	33	42	109	0	76	0
normalized size	1	1.	1.16	0.87	1.11	2.87	0.	2.	0.
time (sec)	N/A	0.197	0.048	0.006	0.792	0.276	0.	0.281	0.

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	55	59	46	184	194	84	0
normalized size	1	1.	1.15	1.23	0.96	3.83	4.04	1.75	0.
time (sec)	N/A	0.168	0.067	0.009	0.79	0.268	107.557	0.283	0.

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	24	24	20	78	99	36	0
normalized size	1	1.	1.26	1.26	1.05	4.11	5.21	1.89	0.
time (sec)	N/A	0.097	0.022	0.004	0.785	0.268	53.024	0.286	0.

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	33	58	23	117	42	43	0
normalized size	1	1.	1.74	3.05	1.21	6.16	2.21	2.26	0.
time (sec)	N/A	0.044	0.022	0.007	0.784	0.275	30.079	0.283	0.

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	84	51	55	105	0	0	0
normalized size	1	1.	2.62	1.59	1.72	3.28	0.	0.	0.
time (sec)	N/A	0.166	0.04	0.009	0.791	0.279	0.	0.	0.

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	35	50	32	78	0	201	0
normalized size	1	1.	1.35	1.92	1.23	3.	0.	7.73	0.
time (sec)	N/A	0.139	0.047	0.016	0.771	0.27	0.	0.299	0.

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	88	58	73	105	0	0	0
normalized size	1	1.	2.59	1.71	2.15	3.09	0.	0.	0.
time (sec)	N/A	0.168	0.056	0.017	0.776	0.294	0.	0.	0.

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	157	90	0	165	0	0	121
normalized size	1	1.	1.07	0.61	0.	1.12	0.	0.	0.82
time (sec)	N/A	0.233	0.205	0.005	0.	0.288	0.	0.	27.479

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	113	66	0	124	0	0	76
normalized size	1	1.	1.19	0.69	0.	1.31	0.	0.	0.8
time (sec)	N/A	0.182	0.107	0.004	0.	0.271	0.	0.	18.571

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	71	40	0	68	0	0	32
normalized size	1	1.	1.51	0.85	0.	1.45	0.	0.	0.68
time (sec)	N/A	0.099	0.08	0.004	0.	0.275	0.	0.	7.234

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	75	73	0	1	0	0	76
normalized size	1	1.	0.77	0.75	0.	0.01	0.	0.	0.78
time (sec)	N/A	0.15	0.062	0.005	0.	0.28	0.	0.	13.175

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	81	88	0	1	0	0	76
normalized size	1	1.	0.79	0.85	0.	0.01	0.	0.	0.74
time (sec)	N/A	0.193	0.12	0.006	0.	0.291	0.	0.	17.41

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	123	120	0	1	0	0	128
normalized size	1	1.	0.72	0.7	0.	0.01	0.	0.	0.75
time (sec)	N/A	0.247	0.183	0.015	0.	0.311	0.	0.	24.621

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	168	517	0	1	0	0	0
normalized size	1	1.	0.86	2.65	0.	0.01	0.	0.	0.
time (sec)	N/A	0.644	0.161	0.019	0.	0.325	0.	0.	0.

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	132	385	0	1	0	0	0
normalized size	1	1.	0.93	2.71	0.	0.01	0.	0.	0.
time (sec)	N/A	0.448	0.276	0.009	0.	0.295	0.	0.	0.

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	128	266	0	1	0	0	0
normalized size	1	1.	0.95	1.97	0.	0.01	0.	0.	0.
time (sec)	N/A	0.453	0.179	0.018	0.	0.318	0.	0.	0.

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	127	272	0	1	0	0	121
normalized size	1	1.	0.92	1.97	0.	0.01	0.	0.	0.88
time (sec)	N/A	0.337	0.084	0.017	0.	0.302	0.	0.	33.632

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	130	313	0	753	0	0	94
normalized size	1	1.	1.06	2.54	0.	6.12	0.	0.	0.76
time (sec)	N/A	0.453	0.213	0.017	0.	0.284	0.	0.	28.166

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	164	457	0	1432	0	0	150
normalized size	1	1.	0.94	2.63	0.	8.23	0.	0.	0.86
time (sec)	N/A	0.537	0.437	0.019	0.	0.286	0.	0.	36.183

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	114	246	0	304	0	0	253
normalized size	1	1.	0.41	0.89	0.	1.1	0.	0.	0.91
time (sec)	N/A	0.609	0.51	0.005	0.	0.267	0.	0.	58.306

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	80	172	0	225	0	0	144
normalized size	1	1.	0.49	1.06	0.	1.38	0.	0.	0.88
time (sec)	N/A	0.437	0.34	0.005	0.	0.274	0.	0.	35.166

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	127	148	0	1	0	0	131
normalized size	1	1.	0.82	0.95	0.	0.01	0.	0.	0.85
time (sec)	N/A	0.396	0.286	0.005	0.	0.28	0.	0.	32.873

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	223	143	237	0	1	0	0	199
normalized size	1	1.42	0.91	1.51	0.	0.01	0.	0.	1.27
time (sec)	N/A	0.451	0.235	0.005	0.	0.293	0.	0.	38.562

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	275	146	300	0	1	0	0	236
normalized size	1	1.68	0.89	1.83	0.	0.01	0.	0.	1.44
time (sec)	N/A	0.41	0.275	0.004	0.	0.297	0.	0.	43.372

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	63	31	138	48	51	0
normalized size	1	1.	1.	2.03	1.	4.45	1.55	1.65	0.
time (sec)	N/A	0.082	0.022	0.002	0.767	0.299	6.66	0.302	0.

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	44	33	42	109	0	76	0
normalized size	1	1.	1.16	0.87	1.11	2.87	0.	2.	0.
time (sec)	N/A	0.614	0.044	0.003	0.766	0.272	0.	0.288	0.

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	56	59	46	184	219	84	0
normalized size	1	1.	1.17	1.23	0.96	3.83	4.56	1.75	0.
time (sec)	N/A	0.567	0.059	0.002	0.764	0.274	147.376	0.292	0.

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	24	26	23	78	110	39	0
normalized size	1	1.	1.14	1.24	1.1	3.71	5.24	1.86	0.
time (sec)	N/A	0.227	0.023	0.002	0.761	0.268	72.587	0.299	0.

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	34	59	27	119	46	45	0
normalized size	1	1.	1.55	2.68	1.23	5.41	2.09	2.05	0.
time (sec)	N/A	0.093	0.023	0.002	0.769	0.277	39.257	0.292	0.

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	84	51	55	105	0	0	0
normalized size	1	1.	2.62	1.59	1.72	3.28	0.	0.	0.
time (sec)	N/A	0.381	0.041	0.003	0.769	0.266	0.	0.	0.

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	35	50	32	78	0	201	0
normalized size	1	1.	1.35	1.92	1.23	3.	0.	7.73	0.
time (sec)	N/A	0.412	0.041	0.002	0.765	0.277	0.	0.316	0.

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	F	F(-2)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	85	57	69	104	0	0	0
normalized size	1	1.	2.58	1.73	2.09	3.15	0.	0.	0.
time (sec)	N/A	0.451	0.057	0.003	0.765	0.271	0.	0.	0.

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	82	48	0	105	0	0	0
normalized size	1	1.	2.93	1.71	0.	3.75	0.	0.	0.
time (sec)	N/A	0.586	0.043	0.004	0.	0.267	0.	0.	0.

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	42	62	0	126	226	57	0
normalized size	1	1.	1.27	1.88	0.	3.82	6.85	1.73	0.
time (sec)	N/A	0.251	0.029	0.008	0.	0.308	48.769	0.311	0.

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	0	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.235	0.124	0.02	0.	0.	0.	0.	0.

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	143	175	0	217	279	220	0
normalized size	1	1.	0.82	1.	0.	1.24	1.59	1.26	0.
time (sec)	N/A	0.283	0.598	0.018	0.	0.291	25.469	0.289	0.

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	102	126	0	154	116	139	0
normalized size	1	1.	0.75	0.93	0.	1.13	0.85	1.02	0.
time (sec)	N/A	0.228	0.393	0.007	0.	0.292	8.606	0.286	0.

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	81	75	0	100	54	88	0
normalized size	1	1.	1.19	1.1	0.	1.47	0.79	1.29	0.
time (sec)	N/A	0.07	0.079	0.006	0.	0.283	7.182	0.283	0.

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	141	1325	0	252	0	0	102
normalized size	1	1.	1.21	11.32	0.	2.15	0.	0.	0.87
time (sec)	N/A	0.203	0.225	0.043	0.	0.303	0.	0.	22.418

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	248	4136	0	383	0	0	136
normalized size	1	1.	1.64	27.39	0.	2.54	0.	0.	0.9
time (sec)	N/A	0.256	0.655	0.042	0.	0.304	0.	0.	39.648

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	309	9721	0	724	0	0	178
normalized size	1	1.	1.6	50.37	0.	3.75	0.	0.	0.92
time (sec)	N/A	0.288	1.154	0.064	0.	0.366	0.	0.	46.169

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	0	0	0	1	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.477	0.631	0.043	0.	0.349	0.	0.	0.

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	0	0	0	1	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.44	0.222	0.024	0.	0.342	0.	0.	0.

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	139	0	0	1	0	0	0
normalized size	1	1.	0.73	0.	0.	0.01	0.	0.	0.
time (sec)	N/A	0.376	0.412	0.01	0.	0.337	0.	0.	0.

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	141	0	0	1	0	0	0
normalized size	1	1.	0.96	0.	0.	0.01	0.	0.	0.
time (sec)	N/A	0.242	0.672	0.02	0.	0.334	0.	0.	0.

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	0	0	0	1	0	0	0
normalized size	1	1.	0.	0.	0.	0.01	0.	0.	0.
time (sec)	N/A	0.342	0.404	0.013	0.	0.343	0.	0.	0.

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	0	0	0	1	0	0	0
normalized size	1	1.	0.	0.	0.	0.01	0.	0.	0.
time (sec)	N/A	0.422	0.722	0.013	0.	0.357	0.	0.	0.

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	34	0	0	41	42	0	0
normalized size	1	1.	0.83	0.	0.	1.	1.02	0.	0.
time (sec)	N/A	0.032	0.024	0.051	0.	0.28	0.903	0.	0.

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	57	0	0	130	0	0	0
normalized size	1	1.	0.83	0.	0.	1.88	0.	0.	0.
time (sec)	N/A	0.108	0.133	0.028	0.	0.332	0.	0.	0.

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	35	60	0	46	413	0	36
normalized size	1	1.	0.78	1.33	0.	1.02	9.18	0.	0.8
time (sec)	N/A	0.025	0.09	0.12	0.	0.295	3.913	0.	1.18

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	44	55	0	38	197	0	36
normalized size	1	1.	1.07	1.34	0.	0.93	4.8	0.	0.88
time (sec)	N/A	0.019	0.081	0.04	0.	0.291	3.866	0.	1.157

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	44	64	0	41	197	0	36
normalized size	1	1.	1.07	1.56	0.	1.	4.8	0.	0.88
time (sec)	N/A	0.02	0.082	0.022	0.	0.288	3.931	0.	1.145

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	55	0	0	95	0	0	60
normalized size	1	1.	0.83	0.	0.	1.44	0.	0.	0.91
time (sec)	N/A	0.076	0.306	0.026	0.	0.352	0.	0.	2.372

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F(-1)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	0	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.386	0.204	0.024	0.	0.	0.	0.	0.

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	255	685	0	466	0	504	0
normalized size	1	1.	0.84	2.26	0.	1.54	0.	1.66	0.
time (sec)	N/A	0.769	0.412	0.022	0.	0.321	0.	0.284	0.

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	179	409	0	296	0	302	0
normalized size	1	1.	0.76	1.73	0.	1.25	0.	1.27	0.
time (sec)	N/A	0.604	0.293	0.01	0.	0.301	0.	0.28	0.

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	120	173	0	166	0	150	0
normalized size	1	1.	1.02	1.47	0.	1.41	0.	1.27	0.
time (sec)	N/A	0.129	0.4	0.007	0.	0.289	0.	0.274	0.

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	275	4918	0	501	0	0	187
normalized size	1	1.	1.28	22.87	0.	2.33	0.	0.	0.87
time (sec)	N/A	0.449	0.551	0.075	0.	4.613	0.	0.	136.281

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	421	58067	0	1115	0	4	262
normalized size	1	1.	1.58	218.3	0.	4.19	0.	0.02	0.98
time (sec)	N/A	0.548	1.025	0.069	0.	2.436	0.	1.046	71.472

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	F	A	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	665	295147	0	2638	0	0	306
normalized size	1	1.	2.02	894.38	0.	7.99	0.	0.	0.93
time (sec)	N/A	0.701	1.541	0.205	0.	10.594	0.	0.	117.511

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	A	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	436	436	0	0	0	1	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.414	0.797	0.045	0.	0.528	0.	0.	0.

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	370	370	0	0	0	1	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.018	0.428	0.023	0.	0.497	0.	0.	0.

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	222	0	0	1	0	0	0
normalized size	1	1.	0.7	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.864	0.733	0.012	0.	0.502	0.	0.	0.

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	238	0	0	1	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.629	1.537	0.022	0.	0.499	0.	0.	0.

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	0	0	0	1	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.961	0.805	0.015	0.	0.526	0.	0.	0.

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	A	F	A	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	335	335	0	0	0	1	0	4	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.01	0.
time (sec)	N/A	1.378	1.13	0.015	0.	0.834	0.	1.414	0.

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	C	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	338	216	0	213	0	0	0
normalized size	1	1.	2.06	1.32	0.	1.3	0.	0.	0.
time (sec)	N/A	0.205	2.756	0.103	0.	0.314	0.	0.	0.

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	202	167	0	105	0	0	0
normalized size	1	1.	1.87	1.55	0.	0.97	0.	0.	0.
time (sec)	N/A	0.125	0.36	0.015	0.	0.319	0.	0.	0.

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	36	120	0	43	2147	0	0
normalized size	1	1.	0.69	2.31	0.	0.83	41.29	0.	0.
time (sec)	N/A	0.045	0.027	0.012	0.	0.324	13.182	0.	0.

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	0	0	0	0	0	0	46
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.78
time (sec)	N/A	0.123	0.04	0.033	0.	0.	0.	0.	12.277

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	0	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.114	0.042	0.025	0.	0.	0.	0.	0.

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	361	0	0	215	0	0	0
normalized size	1	1.	2.05	0.	0.	1.22	0.	0.	0.
time (sec)	N/A	0.203	3.111	0.047	0.	0.317	0.	0.	0.

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	211	0	0	107	0	0	0
normalized size	1	1.	1.82	0.	0.	0.92	0.	0.	0.
time (sec)	N/A	0.126	0.37	0.024	0.	0.335	0.	0.	0.

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	39	0	0	45	0	0	0
normalized size	1	1.	0.7	0.	0.	0.8	0.	0.	0.
time (sec)	N/A	0.045	0.024	0.024	0.	0.299	0.	0.	0.

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	0	0	0	0	0	0	46
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.73
time (sec)	N/A	0.124	0.041	0.038	0.	0.	0.	0.	12.717

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	0	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.116	0.042	0.04	0.	0.	0.	0.	0.

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	659	0	0	271	0	0	0
normalized size	1	1.	3.52	0.	0.	1.45	0.	0.	0.
time (sec)	N/A	0.224	15.851	0.025	0.	0.299	0.	0.	0.

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	355	0	0	149	0	0	0
normalized size	1	1.	2.71	0.	0.	1.14	0.	0.	0.
time (sec)	N/A	0.176	4.061	0.024	0.	0.295	0.	0.	0.

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	104	0	0	65	0	0	0
normalized size	1	1.	1.39	0.	0.	0.87	0.	0.	0.
time (sec)	N/A	0.133	0.763	0.024	0.	0.302	0.	0.	0.

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	0	20	20	311	0	12
normalized size	1	1.	1.	0.	1.18	1.18	18.29	0.	0.71
time (sec)	N/A	0.09	0.029	0.028	0.728	0.293	14.365	0.	9.211

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	0	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.12	0.048	0.025	0.	0.	0.	0.	0.

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	0	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.122	0.049	0.028	0.	0.	0.	0.	0.

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	692	0	0	275	0	0	0
normalized size	1	1.	3.44	0.	0.	1.37	0.	0.	0.
time (sec)	N/A	0.216	17.573	0.039	0.	0.303	0.	0.	0.

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	366	0	0	153	0	0	0
normalized size	1	1.	2.6	0.	0.	1.09	0.	0.	0.
time (sec)	N/A	0.18	3.19	0.04	0.	0.298	0.	0.	0.

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	112	0	0	69	0	0	0
normalized size	1	1.	1.38	0.	0.	0.85	0.	0.	0.
time (sec)	N/A	0.139	0.849	0.039	0.	0.323	0.	0.	0.

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	0	24	24	36	0	14
normalized size	1	1.	1.	0.	1.2	1.2	1.8	0.	0.7
time (sec)	N/A	0.095	0.031	0.042	0.739	0.313	10.388	0.	9.917

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	0	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.126	0.051	0.039	0.	0.	0.	0.	0.

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	0	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.125	0.05	0.039	0.	0.	0.	0.	0.

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	365	365	0	0	0	883	0	0	0
normalized size	1	1.	0.	0.	0.	2.42	0.	0.	0.
time (sec)	N/A	0.864	0.646	0.124	0.	0.332	0.	0.	0.

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	A	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	0	0	0	323	0	0	0
normalized size	1	1.	0.	0.	0.	1.35	0.	0.	0.
time (sec)	N/A	0.483	0.228	0.016	0.	0.327	0.	0.	0.

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	A	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	0	0	0	108	0	0	0
normalized size	1	1.	0.	0.	0.	1.01	0.	0.	0.
time (sec)	N/A	0.173	0.06	0.015	0.	0.346	0.	0.	0.

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	0	0	0	0	0	0	110
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.9
time (sec)	N/A	0.486	0.152	0.133	0.	0.	0.	0.	111.889

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	0	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.439	0.158	0.107	0.	0.	0.	0.	0.

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	A	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	0	0	0	108	0	0	0
normalized size	1	1.	0.	0.	0.	1.01	0.	0.	0.
time (sec)	N/A	0.235	0.083	0.014	0.	0.303	0.	0.	0.

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	0	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.766	0.106	0.105	0.	0.	0.	0.	0.

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	0	0	0	509	0	0	0
normalized size	1	1.	0.	0.	0.	1.71	0.	0.	0.
time (sec)	N/A	0.729	0.434	0.108	0.	0.308	0.	0.	0.

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	0	0	0	165	0	0	0
normalized size	1	1.	0.	0.	0.	0.96	0.	0.	0.
time (sec)	N/A	0.561	0.113	0.104	0.	0.306	0.	0.	0.

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	A	A	F(-1)	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	0	0	47	55	0	0	37
normalized size	1	1.	0.	0.	1.15	1.34	0.	0.	0.9
time (sec)	N/A	0.388	0.199	0.109	0.85	0.324	0.	0.	96.165

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	0	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.497	0.298	0.111	0.	0.	0.	0.	0.

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	A	A	F(-1)	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	0	0	47	55	0	0	37
normalized size	1	1.	0.	0.	1.15	1.34	0.	0.	0.9
time (sec)	N/A	0.668	0.123	0.111	0.851	0.313	0.	0.	109.276

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	327	327	0	0	0	312	0	0	0
normalized size	1	1.	0.	0.	0.	0.95	0.	0.	0.
time (sec)	N/A	1.052	0.123	0.114	0.	0.324	0.	0.	0.

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	0	0	51	158	0	0	0
normalized size	1	1.	0.	0.	0.55	1.7	0.	0.	0.
time (sec)	N/A	0.839	0.134	0.11	0.812	0.311	0.	0.	0.

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	0	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.945	0.225	0.115	0.	0.	0.	0.	0.

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	0	0	51	158	0	0	0
normalized size	1	1.	0.	0.	0.55	1.7	0.	0.	0.
time (sec)	N/A	1.128	0.124	0.118	0.818	0.313	0.	0.	0.

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-1)	F	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	0	352	0	0	0	0	264
normalized size	1	1.	0.	1.84	0.	0.	0.	0.	1.38
time (sec)	N/A	1.222	0.801	0.094	0.	0.	0.	0.	115.082

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	191	394	0	1	70	1	73
normalized size	1	1.	2.36	4.86	0.	0.01	0.86	0.01	0.9
time (sec)	N/A	0.114	0.188	0.07	0.	0.3	2.45	0.762	70.632

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	233	394	0	1	63	1	66
normalized size	1	1.	3.19	5.4	0.	0.01	0.86	0.01	0.9
time (sec)	N/A	0.088	0.218	0.069	0.	0.285	2.51	0.749	68.975

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	87	70	0	1	70	0	36
normalized size	1	1.	2.29	1.84	0.	0.03	1.84	0.	0.95
time (sec)	N/A	0.105	0.093	0.012	0.	0.286	3.055	0.	46.309

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	87	70	0	1	63	0	36
normalized size	1	1.	2.29	1.84	0.	0.03	1.66	0.	0.95
time (sec)	N/A	0.104	0.093	0.013	0.	0.284	3.089	0.	47.91

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	0	78	0	1	0	0	44
normalized size	1	1.	0.	2.05	0.	0.03	0.	0.	1.16
time (sec)	N/A	0.156	0.151	0.073	0.	0.323	0.	0.	69.674

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	0	72	0	1	0	0	44
normalized size	1	1.	0.	1.89	0.	0.03	0.	0.	1.16
time (sec)	N/A	0.153	0.142	0.074	0.	0.307	0.	0.	74.107

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	42	0	1	78	51	37
normalized size	1	1.	1.	1.	0.	0.02	1.86	1.21	0.88
time (sec)	N/A	0.129	0.032	0.008	0.	0.3	1.872	0.343	43.465

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	46	42	0	1	75	55	39
normalized size	1	1.	1.05	0.95	0.	0.02	1.7	1.25	0.89
time (sec)	N/A	0.135	0.036	0.004	0.	0.284	2.008	0.346	44.447

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	85	74	0	1	90	0	37
normalized size	1	1.	2.12	1.85	0.	0.02	2.25	0.	0.92
time (sec)	N/A	0.205	0.076	0.419	0.	0.286	4.235	0.	64.986

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	85	77	0	1	80	0	37
normalized size	1	1.	2.12	1.92	0.	0.02	2.	0.	0.92
time (sec)	N/A	0.206	0.083	0.377	0.	0.288	4.305	0.	68.405

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	61	42	78	0	1	0	0	37
normalized size	1	1.45	1.	1.86	0.	0.02	0.	0.	0.88
time (sec)	N/A	0.341	0.084	0.09	0.	0.3	0.	0.	106.903

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	61	42	74	0	1	0	0	37
normalized size	1	1.45	1.	1.76	0.	0.02	0.	0.	0.88
time (sec)	N/A	0.333	0.075	0.09	0.	0.298	0.	0.	116.428

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	86	74	0	1	73	0	41
normalized size	1	1.	2.15	1.85	0.	0.02	1.82	0.	1.02
time (sec)	N/A	0.142	0.077	0.013	0.	0.283	4.453	0.	48.598

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	86	74	0	1	66	0	41
normalized size	1	1.	2.15	1.85	0.	0.02	1.65	0.	1.02
time (sec)	N/A	0.145	0.074	0.013	0.	0.303	4.445	0.	50.062

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	42	0	1	78	51	37
normalized size	1	1.	1.	1.	0.	0.02	1.86	1.21	0.88
time (sec)	N/A	0.125	0.032	0.003	0.	0.312	2.465	0.271	47.968

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	46	42	0	1	75	55	39
normalized size	1	1.	1.05	0.95	0.	0.02	1.7	1.25	0.89
time (sec)	N/A	0.136	0.035	0.002	0.	0.318	2.649	0.271	49.076

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	78	0	1	0	0	37
normalized size	1	1.	1.	1.86	0.	0.02	0.	0.	0.88
time (sec)	N/A	0.349	0.088	0.055	0.	0.321	0.	0.	81.409

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	74	0	1	0	0	37
normalized size	1	1.	1.	1.76	0.	0.02	0.	0.	0.88
time (sec)	N/A	0.34	0.071	0.055	0.	0.331	0.	0.	57.814

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	0	84	0	1	0	0	63
normalized size	1	1.	0.	2.	0.	0.02	0.	0.	1.5
time (sec)	N/A	0.382	0.267	0.111	0.	0.344	0.	0.	74.639

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	0	78	0	1	0	0	63
normalized size	1	1.	0.	1.86	0.	0.02	0.	0.	1.5
time (sec)	N/A	0.374	0.281	0.108	0.	0.335	0.	0.	84.316

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	161	4947	169	315	0	209	0
normalized size	1	1.	1.2	36.92	1.26	2.35	0.	1.56	0.
time (sec)	N/A	0.619	0.335	0.086	0.707	0.354	0.	0.277	0.

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	95	3426	84	217	0	97	0
normalized size	1	1.	1.38	49.65	1.22	3.14	0.	1.41	0.
time (sec)	N/A	0.372	0.109	0.03	0.702	0.347	0.	0.278	0.

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	1941	28	142	0	30	17
normalized size	1	1.	1.	84.39	1.22	6.17	0.	1.3	0.74
time (sec)	N/A	0.136	0.017	0.026	0.697	0.306	0.	0.274	7.268

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	282	2175	0	1	0	127	0
normalized size	1	1.	3.2	24.72	0.	0.01	0.	1.44	0.
time (sec)	N/A	0.416	0.917	0.042	0.	0.374	0.	0.281	0.

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	430	2459	0	1	0	275	133
normalized size	1	1.	2.85	16.28	0.	0.01	0.	1.82	0.88
time (sec)	N/A	0.638	0.946	0.067	0.	0.921	0.	0.279	42.655

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	157	3501	0	1	0	244	128
normalized size	1	1.	1.07	23.82	0.	0.01	0.	1.66	0.87
time (sec)	N/A	0.479	0.19	0.043	0.	0.423	0.	0.286	45.568

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	83	2005	0	1	0	144	87
normalized size	1	1.	0.81	19.47	0.	0.01	0.	1.4	0.84
time (sec)	N/A	0.148	0.058	0.031	0.	0.305	0.	0.28	23.846

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	139	2289	0	1	0	285	129
normalized size	1	1.	0.87	14.31	0.	0.01	0.	1.78	0.81
time (sec)	N/A	0.506	0.171	0.044	0.	0.391	0.	0.284	50.806

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	161	1473	169	258	0	211	0
normalized size	1	1.	1.15	10.52	1.21	1.84	0.	1.51	0.
time (sec)	N/A	0.611	0.266	0.183	0.695	0.277	0.	0.276	0.

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	95	943	84	159	0	97	0
normalized size	1	1.	1.3	12.92	1.15	2.18	0.	1.33	0.
time (sec)	N/A	0.367	0.103	0.021	0.701	0.28	0.	0.272	0.

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	455	30	82	0	31	20
normalized size	1	1.	1.	17.5	1.15	3.15	0.	1.19	0.77
time (sec)	N/A	0.177	0.016	0.02	0.691	0.274	0.	0.274	8.797

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	0	636	0	1	0	127	0
normalized size	1	1.	0.	6.84	0.	0.01	0.	1.37	0.
time (sec)	N/A	0.406	0.123	0.048	0.	0.339	0.	0.277	0.

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F(-1)	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	860	863	0	1	0	275	136
normalized size	1	1.	5.58	5.6	0.	0.01	0.	1.79	0.88
time (sec)	N/A	0.618	6.77	0.053	0.	0.394	0.	0.289	42.543

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	F(-1)	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	311	311	470	1544	0	0	0	0	280
normalized size	1	1.	1.51	4.96	0.	0.	0.	0.	0.9
time (sec)	N/A	1.027	1.497	0.067	0.	0.	0.	0.	91.734

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	F(-1)	F(-1)	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	304	304	0	619	0	0	0	0	275
normalized size	1	1.	0.	2.04	0.	0.	0.	0.	0.9
time (sec)	N/A	0.73	0.148	0.071	0.	0.	0.	0.	77.517

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	F(-1)	F(-1)	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	0	619	0	0	0	0	272
normalized size	1	1.	0.	2.06	0.	0.	0.	0.	0.91
time (sec)	N/A	0.609	0.05	0.021	0.	0.	0.	0.	83.163

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	C	F	F(-1)	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	319	319	1047	3560	0	0	0	0	286
normalized size	1	1.	3.28	11.16	0.	0.	0.	0.	0.9
time (sec)	N/A	0.95	6.711	0.052	0.	0.	0.	0.	97.038

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	C	F	F(-1)	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	324	324	1044	1789	0	0	0	0	292
normalized size	1	1.	3.22	5.52	0.	0.	0.	0.	0.9
time (sec)	N/A	0.955	6.679	0.052	0.	0.	0.	0.	92.392

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	320	0	0	0	0	0	105
normalized size	1	1.	2.37	0.	0.	0.	0.	0.	0.78
time (sec)	N/A	0.216	0.896	0.014	0.	0.	0.	0.	42.494

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	373	0	0	0	0	0	126
normalized size	1	1.	2.23	0.	0.	0.	0.	0.	0.75
time (sec)	N/A	0.456	1.273	0.019	0.	0.	0.	0.	43.14

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	0	82	34	0	55	20
normalized size	1	1.	1.	0.	3.04	1.26	0.	2.04	0.74
time (sec)	N/A	0.189	0.031	0.021	0.77	0.285	0.	0.385	10.552

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	8	7	8	7
normalized size	1	1.	1.	0.88	1.	1.	0.88	1.	0.88
time (sec)	N/A	0.01	0.006	0.006	0.797	0.271	0.556	0.277	1.326

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	33	10	28	28	26	30	12
normalized size	1	1.	2.54	0.77	2.15	2.15	2.	2.31	0.92
time (sec)	N/A	0.017	0.01	0.008	0.839	0.277	1.234	0.281	2.09

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	20	26	26	22	27	0
normalized size	1	1.	1.	0.74	0.96	0.96	0.81	1.	0.
time (sec)	N/A	0.025	0.013	0.011	0.734	0.27	0.58	0.283	0.

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	92	32	32	0	32	0
normalized size	1	1.	1.	2.88	1.	1.	0.	1.	0.
time (sec)	N/A	0.03	0.013	0.037	0.695	0.268	0.	0.278	0.

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	26	26	22	26	0
normalized size	1	1.	1.	0.8	1.04	1.04	0.88	1.04	0.
time (sec)	N/A	0.024	0.01	0.008	0.745	0.271	0.556	0.281	0.

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	15	19	19	15	20	15
normalized size	1	1.	1.	0.75	0.95	0.95	0.75	1.	0.75
time (sec)	N/A	0.02	0.009	0.002	0.716	0.274	0.341	0.278	1.904

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	62	46	61	77	68	61	61
normalized size	1	1.	1.	0.74	0.98	1.24	1.1	0.98	0.98
time (sec)	N/A	0.079	0.024	0.006	0.838	0.282	2.478	0.28	5.242

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	73	173	66	66	0	66	0
normalized size	1	1.	1.	2.37	0.9	0.9	0.	0.9	0.
time (sec)	N/A	0.056	0.019	0.153	0.732	0.276	0.	0.281	0.

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	130	83	111	103	0	111	0
normalized size	1	1.	1.	0.64	0.85	0.79	0.	0.85	0.
time (sec)	N/A	0.088	0.029	0.005	0.696	0.273	0.	0.279	0.

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	F(-2)	F	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	180	175	367	0	0	188	0
normalized size	1	1.	0.9	0.88	1.84	0.	0.	0.94	0.
time (sec)	N/A	0.685	0.237	0.052	0.838	0.	0.	0.379	0.

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	8	7	8	7
normalized size	1	1.	1.	0.88	1.	1.	0.88	1.	0.88
time (sec)	N/A	0.01	0.005	0.004	0.791	0.279	1.847	0.277	1.347

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	20	20	0	20	0
normalized size	1	1.	1.	0.84	1.05	1.05	0.	1.05	0.
time (sec)	N/A	0.029	0.009	0.004	0.69	0.269	0.	0.278	0.

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	108	71	112	157	110	112	104
normalized size	1	1.	1.	0.66	1.04	1.45	1.02	1.04	0.96
time (sec)	N/A	0.149	0.046	0.007	0.808	0.284	4.888	0.28	10.477

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	83	61	81	81	0	81	75
normalized size	1	1.	1.09	0.8	1.07	1.07	0.	1.07	0.99
time (sec)	N/A	0.076	0.026	0.006	0.813	0.282	0.	0.286	3.879

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	119	76	101	96	0	101	0
normalized size	1	1.	1.	0.64	0.85	0.81	0.	0.85	0.
time (sec)	N/A	0.095	0.027	0.005	0.7	0.283	0.	0.29	0.

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	F(-2)	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	183	242	396	0	0	189	267
normalized size	1	1.	0.91	1.2	1.97	0.	0.	0.94	1.33
time (sec)	N/A	0.518	0.242	0.021	0.831	0.	0.	0.387	122.632

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	34	36	20	59	0	212	36
normalized size	1	1.	0.94	1.	0.56	1.64	0.	5.89	1.
time (sec)	N/A	0.131	0.133	0.005	0.74	0.288	0.	0.298	5.746

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	27	7	47	0	170	24
normalized size	1	1.	1.	0.93	0.24	1.62	0.	5.86	0.83
time (sec)	N/A	0.131	0.034	0.003	0.782	0.272	0.	0.286	5.264

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	27	7	45	0	76	22
normalized size	1	1.	1.	0.93	0.24	1.55	0.	2.62	0.76
time (sec)	N/A	0.092	0.029	0.003	0.759	0.267	0.	0.282	4.682

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	25	7	41	0	69	19
normalized size	1	1.	1.	1.	0.28	1.64	0.	2.76	0.76
time (sec)	N/A	0.042	0.027	0.003	0.784	0.26	0.	0.282	4.35

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	24	7	43	0	57	19
normalized size	1	1.	1.	1.	0.29	1.79	0.	2.38	0.79
time (sec)	N/A	0.13	0.027	0.003	0.757	0.262	0.	0.287	5.556

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	27	7	47	0	81	22
normalized size	1	1.	1.	0.93	0.24	1.62	0.	2.79	0.76
time (sec)	N/A	0.123	0.034	0.003	0.802	0.265	0.	0.282	5.404

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	0	0	0	0	0	0	61
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.76
time (sec)	N/A	0.149	0.085	0.132	0.	0.	0.	0.	11.366

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	134	0	0	0	121	0	105
normalized size	1	1.	0.97	0.	0.	0.	0.88	0.	0.76
time (sec)	N/A	0.283	0.131	0.048	0.	0.	16.921	0.	12.722

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	88	0	0	0	75	0	58
normalized size	1	1.	1.11	0.	0.	0.	0.95	0.	0.73
time (sec)	N/A	0.105	0.049	0.028	0.	0.	10.719	0.	6.479

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	50	0	0	0	34	0	29
normalized size	1	1.	1.25	0.	0.	0.	0.85	0.	0.72
time (sec)	N/A	0.029	0.013	0.018	0.	0.	4.796	0.	2.05

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	0	0	0	0	0	0	66
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.65
time (sec)	N/A	0.161	0.041	0.056	0.	0.	0.	0.	10.948

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	0	0	0	0	0	0	41
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.73
time (sec)	N/A	0.095	0.044	0.059	0.	0.	0.	0.	6.583

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	0	0	0	0	0	0	83
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.74
time (sec)	N/A	0.166	0.094	0.087	0.	0.	0.	0.	11.276

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	0	0	0	0	0	0	162
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.88
time (sec)	N/A	0.408	0.224	0.136	0.	0.	0.	0.	21.976

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	35	11	59	0	0	27
normalized size	1	1.	1.	1.06	0.33	1.79	0.	0.	0.82
time (sec)	N/A	0.112	0.093	0.004	0.713	0.279	0.	0.	6.004

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	31	7	55	0	53	0
normalized size	1	1.	1.	1.	0.23	1.77	0.	1.71	0.
time (sec)	N/A	0.121	0.016	0.003	0.726	0.275	0.	0.274	0.

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	42	9	55	0	0	20
normalized size	1	1.	1.	1.5	0.32	1.96	0.	0.	0.71
time (sec)	N/A	0.09	0.028	0.015	0.722	0.268	0.	0.	4.938

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	0	42	5	69	0	42	26
normalized size	1	1.	0.	1.5	0.18	2.46	0.	1.5	0.93
time (sec)	N/A	0.053	0.11	0.011	0.72	0.292	0.	0.275	5.154

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	28	9	55	0	0	22
normalized size	1	1.	1.	1.08	0.35	2.12	0.	0.	0.85
time (sec)	N/A	0.112	0.033	0.004	0.732	0.267	0.	0.	5.984

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	38	31	7	59	0	27	24
normalized size	1	1.	1.23	1.	0.23	1.9	0.	0.87	0.77
time (sec)	N/A	0.115	0.025	0.003	0.774	0.271	0.	0.275	5.781

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	406	406	540	1145	0	0	0	0	357
normalized size	1	1.	1.33	2.82	0.	0.	0.	0.	0.88
time (sec)	N/A	1.134	4.593	0.15	0.	0.	0.	0.	75.39

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	18	15	15	12	15	12
normalized size	1	1.	1.	1.2	1.	1.	0.8	1.	0.8
time (sec)	N/A	0.008	0.018	0.01	0.725	0.26	0.519	0.284	1.291

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	16	20	18	27	31	18	12
normalized size	1	1.	0.94	1.18	1.06	1.59	1.82	1.06	0.71
time (sec)	N/A	0.009	0.017	0.007	0.734	0.26	0.69	0.274	1.636

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	18	15	15	31	15	12
normalized size	1	1.	1.	1.2	1.	1.	2.07	1.	0.8
time (sec)	N/A	0.007	0.015	0.007	0.692	0.261	0.684	0.271	1.274

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	18	15	15	31	15	12
normalized size	1	1.	1.	1.2	1.	1.	2.07	1.	0.8
time (sec)	N/A	0.007	0.016	0.006	0.691	0.259	0.679	0.272	1.431

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	21	15	15	10	15	10
normalized size	1	1.	1.	1.62	1.15	1.15	0.77	1.15	0.77
time (sec)	N/A	0.007	0.013	0.004	0.692	0.273	0.341	0.27	1.919

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	45	34	45	43	39	45	0
normalized size	1	1.	1.02	0.77	1.02	0.98	0.89	1.02	0.
time (sec)	N/A	0.059	0.02	0.004	0.698	0.263	0.469	0.278	0.

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	20	16	20	20	17	22	0
normalized size	1	1.	0.95	0.76	0.95	0.95	0.81	1.05	0.
time (sec)	N/A	0.031	0.008	0.003	0.694	0.262	0.312	0.273	0.

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	34	27	35	34	27	35	0
normalized size	1	1.	1.03	0.82	1.06	1.03	0.82	1.06	0.
time (sec)	N/A	0.052	0.017	0.005	0.693	0.262	0.371	0.274	0.

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	35	35	41	39	42	51	0
normalized size	1	1.	1.06	1.06	1.24	1.18	1.27	1.55	0.
time (sec)	N/A	0.058	0.022	0.003	0.695	0.266	1.678	0.276	0.

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	17	10	13	14	14	32	0	20
normalized size	1	1.7	1.	1.3	1.4	1.4	3.2	0.	2.
time (sec)	N/A	0.141	0.019	0.024	0.687	0.309	30.625	0.	8.968

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	10	13	116	14	0	0	20
normalized size	1	1.	0.59	0.76	6.82	0.82	0.	0.	1.18
time (sec)	N/A	0.124	0.013	0.035	0.701	0.275	0.	0.	8.121

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	34	38	131	0	0	0	0
normalized size	1	1.	0.92	1.03	3.54	0.	0.	0.	0.
time (sec)	N/A	0.06	0.567	0.034	0.904	0.	0.	0.	0.

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	32	36	128	0	0	0	0
normalized size	1	1.	0.91	1.03	3.66	0.	0.	0.	0.
time (sec)	N/A	0.037	0.578	0.034	0.96	0.	0.	0.	0.

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	31	35	124	0	0	0	0
normalized size	1	1.	0.91	1.03	3.65	0.	0.	0.	0.
time (sec)	N/A	0.041	0.536	0.031	0.92	0.	0.	0.	0.

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	F	A	A	A	F(-1)	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	0	34	38	128	0	0	0	0
normalized size	1	0.	0.92	1.03	3.46	0.	0.	0.	0.
time (sec)	N/A	8.29	1.524	0.035	0.964	0.	0.	0.	0.

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	F	A	A	A	F(-1)	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	0	34	38	128	0	0	0	0
normalized size	1	0.	0.92	1.03	3.46	0.	0.	0.	0.
time (sec)	N/A	9.372	2.205	0.036	0.904	0.	0.	0.	0.

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	88	78	204	127	88	205	0
normalized size	1	1.	0.48	0.42	1.1	0.69	0.48	1.11	0.
time (sec)	N/A	0.511	0.114	0.004	0.695	0.362	2.585	0.276	0.

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	77	66	151	109	73	163	0
normalized size	1	1.	0.56	0.48	1.09	0.79	0.53	1.18	0.
time (sec)	N/A	0.358	0.097	0.003	0.707	0.289	2.439	0.274	0.

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	67	54	97	90	58	115	0
normalized size	1	1.	0.75	0.61	1.09	1.01	0.65	1.29	0.
time (sec)	N/A	0.204	0.075	0.004	0.717	0.294	2.441	0.277	0.

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	35	47	66	68	53	0
normalized size	1	1.	1.	0.85	1.15	1.61	1.66	1.29	0.
time (sec)	N/A	0.063	0.025	0.002	0.727	0.296	0.538	0.273	0.

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	62	51	0	1	129	105	0
normalized size	1	1.	1.09	0.89	0.	0.02	2.26	1.84	0.
time (sec)	N/A	0.152	0.059	0.006	0.	0.312	9.056	0.276	0.

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	63	60	0	1	196	153	68
normalized size	1	1.	1.17	1.11	0.	0.02	3.63	2.83	1.26
time (sec)	N/A	0.154	0.121	0.01	0.	0.282	19.226	0.294	10.864

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	77	81	0	1	292	170	68
normalized size	1	1.	0.96	1.01	0.	0.01	3.65	2.12	0.85
time (sec)	N/A	0.18	0.129	0.017	0.	0.279	38.476	0.294	10.615

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	232	383	362	386	0	1	306
normalized size	1	1.	0.71	1.17	1.11	1.18	0.	0.	0.94
time (sec)	N/A	0.53	0.29	0.006	0.726	0.346	0.	0.375	32.316

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	147	183	225	248	0	923	211
normalized size	1	1.	0.66	0.82	1.	1.11	0.	4.12	0.94
time (sec)	N/A	0.371	0.183	0.003	0.725	0.358	0.	0.336	22.559

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	109	94	126	139	0	460	122
normalized size	1	1.	0.82	0.71	0.95	1.05	0.	3.46	0.92
time (sec)	N/A	0.227	0.097	0.003	0.711	0.352	0.	0.303	11.82

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	55	41	58	68	0	159	48
normalized size	1	1.	0.98	0.73	1.04	1.21	0.	2.84	0.86
time (sec)	N/A	0.068	0.049	0.003	0.694	0.342	0.	0.287	4.019

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	116	221	0	262	0	0	100
normalized size	1	1.	1.	1.91	0.	2.26	0.	0.	0.86
time (sec)	N/A	0.35	0.236	0.051	0.	0.35	0.	0.	27.86

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	144	151	0	1354	0	0	117
normalized size	1	1.	1.05	1.1	0.	9.88	0.	0.	0.85
time (sec)	N/A	0.349	0.555	0.03	0.	0.366	0.	0.	23.023

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	230	2530	0	3856	0	0	228
normalized size	1	1.	1.03	11.29	0.	17.21	0.	0.	1.02
time (sec)	N/A	0.848	1.967	0.14	0.	0.474	0.	0.	81.548

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	244	394	328	308	0	533	0
normalized size	1	1.	1.06	1.71	1.43	1.34	0.	2.32	0.
time (sec)	N/A	0.522	0.696	0.008	0.707	0.284	0.	0.307	0.

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	169	235	200	186	0	320	0
normalized size	1	1.	1.12	1.56	1.32	1.23	0.	2.12	0.
time (sec)	N/A	0.328	0.351	0.007	0.705	0.294	0.	0.284	0.

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	85	116	109	96	0	177	0
normalized size	1	1.	0.94	1.29	1.21	1.07	0.	1.97	0.
time (sec)	N/A	0.179	0.084	0.006	0.699	0.29	0.	0.277	0.

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	37	87	47	45	49	68	0
normalized size	1	1.	0.9	2.12	1.15	1.1	1.2	1.66	0.
time (sec)	N/A	0.051	0.018	0.012	0.696	0.286	1.925	0.277	0.

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	61	77	0	1	0	155	73
normalized size	1	1.	0.74	0.94	0.	0.01	0.	1.89	0.89
time (sec)	N/A	0.175	0.058	0.008	0.	0.288	0.	0.284	11.103

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	144	216	0	1	0	342	119
normalized size	1	1.	1.11	1.66	0.	0.01	0.	2.63	0.92
time (sec)	N/A	0.363	0.326	0.027	0.	0.353	0.	0.287	26.343

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	228	460	0	1	0	648	187
normalized size	1	1.	1.12	2.25	0.	0.	0.	3.18	0.92
time (sec)	N/A	0.591	0.733	0.021	0.	0.873	0.	0.296	47.583

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	301	416	339	441	0	0	0
normalized size	1	1.	1.25	1.73	1.41	1.84	0.	0.	0.
time (sec)	N/A	0.575	0.452	0.014	0.698	0.279	0.	0.	0.

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	224	253	213	289	0	0	0
normalized size	1	1.	1.35	1.52	1.28	1.74	0.	0.	0.
time (sec)	N/A	0.372	0.358	0.013	0.703	0.269	0.	0.	0.

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	86	125	122	163	0	0	0
normalized size	1	1.	0.91	1.32	1.28	1.72	0.	0.	0.
time (sec)	N/A	0.199	0.123	0.012	0.693	0.267	0.	0.	0.

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	40	142	58	66	124	0	39
normalized size	1	1.	0.85	3.02	1.23	1.4	2.64	0.	0.83
time (sec)	N/A	0.064	0.029	0.027	0.702	0.277	3.437	0.	4.284

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	106	161	0	1	0	0	117
normalized size	1	1.	0.82	1.25	0.	0.01	0.	0.	0.91
time (sec)	N/A	0.262	0.282	0.013	0.	0.304	0.	0.	17.48

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	292	312	0	1	0	0	184
normalized size	1	1.	1.45	1.54	0.	0.	0.	0.	0.91
time (sec)	N/A	0.518	1.73	0.022	0.	0.605	0.	0.	40.398

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	306	390	612	0	1	0	0	275
normalized size	1	1.	1.27	2.	0.	0.	0.	0.	0.9
time (sec)	N/A	0.837	0.617	0.027	0.	1.607	0.	0.	71.683

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	324	285	383	362	312	0	1	304
normalized size	1	1.	0.88	1.18	1.12	0.96	0.	0.	0.94
time (sec)	N/A	0.533	0.486	0.004	0.694	0.338	0.	0.339	32.017

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	147	183	225	189	0	849	209
normalized size	1	1.	0.66	0.82	1.01	0.85	0.	3.82	0.94
time (sec)	N/A	0.372	0.348	0.003	0.703	0.336	0.	0.319	22.4

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	84	94	126	96	0	412	121
normalized size	1	1.	0.64	0.72	0.96	0.73	0.	3.15	0.92
time (sec)	N/A	0.227	0.09	0.003	0.699	0.331	0.	0.293	12.064

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	42	41	57	46	0	135	46
normalized size	1	1.	0.78	0.76	1.06	0.85	0.	2.5	0.85
time (sec)	N/A	0.069	0.025	0.008	0.691	0.333	0.	0.277	4.029

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	97	92	0	1003	0	0	85
normalized size	1	1.	1.	0.95	0.	10.34	0.	0.	0.88
time (sec)	N/A	0.218	0.096	0.02	0.	0.348	0.	0.	16.371

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	250	265	0	3366	0	0	138
normalized size	1	1.	1.53	1.63	0.	20.65	0.	0.	0.85
time (sec)	N/A	0.471	2.401	0.033	0.	0.387	0.	0.	41.932

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	410	834	0	5927	0	0	289
normalized size	1	1.	1.57	3.2	0.	22.71	0.	0.	1.11
time (sec)	N/A	1.013	2.838	0.117	0.	0.957	0.	0.	91.324

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	A	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	350	350	554	0	983	1912	0	0	320
normalized size	1	1.	1.58	0.	2.81	5.46	0.	0.	0.91
time (sec)	N/A	0.617	1.483	0.008	0.724	0.44	0.	0.	49.993

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	285	0	543	961	0	1	221
normalized size	1	1.	1.18	0.	2.24	3.97	0.	0.	0.91
time (sec)	N/A	0.411	0.576	0.006	0.729	0.359	0.	2.279	33.738

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	128	0	252	397	0	1	129
normalized size	1	1.	0.88	0.	1.74	2.74	0.	0.01	0.89
time (sec)	N/A	0.242	0.292	0.006	0.72	0.318	0.	0.858	17.435

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	64	0	81	109	0	836	51
normalized size	1	1.	1.03	0.	1.31	1.76	0.	13.48	0.82
time (sec)	N/A	0.083	0.053	0.007	0.707	0.298	0.	0.356	6.213

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	189	0	0	0	0	0	105
normalized size	1	1.	1.36	0.	0.	0.	0.	0.	0.76
time (sec)	N/A	0.257	0.474	0.006	0.	0.	0.	0.	11.676

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	77	70	0	1	189	0	80
normalized size	1	1.	0.83	0.75	0.	0.01	2.03	0.	0.86
time (sec)	N/A	0.171	0.097	0.009	0.	0.284	108.802	0.	7.187

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	61	54	0	1	153	0	60
normalized size	1	1.	0.87	0.77	0.	0.01	2.19	0.	0.86
time (sec)	N/A	0.133	0.06	0.005	0.	0.29	14.287	0.	5.745

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	46	40	0	1	0	0	41
normalized size	1	1.	0.94	0.82	0.	0.02	0.	0.	0.84
time (sec)	N/A	0.101	0.029	0.002	0.	0.284	0.	0.	4.673

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	0	1	0	0	27
normalized size	1	1.	1.	0.83	0.	0.03	0.	0.	0.9
time (sec)	N/A	0.077	0.023	0.007	0.	0.282	0.	0.	3.689

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	43	0	1	0	0	42
normalized size	1	1.	1.	0.83	0.	0.02	0.	0.	0.81
time (sec)	N/A	0.113	0.06	0.007	0.	0.286	0.	0.	4.928

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	66	59	0	1	0	0	63
normalized size	1	1.	0.88	0.79	0.	0.01	0.	0.	0.84
time (sec)	N/A	0.146	0.154	0.011	0.	0.288	0.	0.	6.505

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	81	86	0	1	192	0	80
normalized size	1	1.	0.8	0.85	0.	0.01	1.9	0.	0.79
time (sec)	N/A	0.178	0.103	0.011	0.	0.285	106.39	0.	8.078

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	66	65	0	1	158	0	60
normalized size	1	1.	0.87	0.86	0.	0.01	2.08	0.	0.79
time (sec)	N/A	0.14	0.067	0.005	0.	0.283	14.145	0.	6.489

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	50	46	0	1	0	0	41
normalized size	1	1.	0.94	0.87	0.	0.02	0.	0.	0.77
time (sec)	N/A	0.107	0.032	0.005	0.	0.285	0.	0.	5.223

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	0	1	0	0	26
normalized size	1	1.	1.	0.84	0.	0.03	0.	0.	0.81
time (sec)	N/A	0.081	0.026	0.007	0.	0.289	0.	0.	4.075

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	49	0	1	0	0	44
normalized size	1	1.	1.	0.88	0.	0.02	0.	0.	0.79
time (sec)	N/A	0.116	0.086	0.008	0.	0.282	0.	0.	5.537

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	70	70	0	1	0	0	63
normalized size	1	1.	0.86	0.86	0.	0.01	0.	0.	0.78
time (sec)	N/A	0.153	0.191	0.01	0.	0.284	0.	0.	7.264

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	0	1	24	28	22
normalized size	1	1.	1.	0.78	0.	0.04	1.04	1.22	0.96
time (sec)	N/A	0.022	0.013	0.006	0.	0.27	3.629	0.26	1.649

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	0	1	0	0	27
normalized size	1	1.	1.	0.83	0.	0.03	0.	0.	0.9
time (sec)	N/A	0.078	0.034	0.009	0.	0.283	0.	0.	3.69

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	32	0	1	0	0	0
normalized size	1	1.	1.	0.86	0.	0.03	0.	0.	0.
time (sec)	N/A	0.301	0.112	0.01	0.	0.283	0.	0.	0.

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	39	0	1	0	0	0
normalized size	1	1.	1.	0.89	0.	0.02	0.	0.	0.
time (sec)	N/A	0.623	0.326	0.024	0.	0.284	0.	0.	0.

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	46	0	1	0	0	0
normalized size	1	1.	1.	0.9	0.	0.02	0.	0.	0.
time (sec)	N/A	1.102	2.222	0.032	0.	0.288	0.	0.	0.

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	65	83	162	74	348	104	76
normalized size	1	1.	0.86	1.09	2.13	0.97	4.58	1.37	1.
time (sec)	N/A	0.069	0.045	0.017	0.804	0.269	32.44	0.286	4.479

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	69	90	68	216	90	60
normalized size	1	1.	1.	1.15	1.5	1.13	3.6	1.5	1.
time (sec)	N/A	0.056	0.035	0.011	0.799	0.266	18.612	0.271	3.972

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	53	55	41	58	112	77	42
normalized size	1	1.	1.2	1.25	0.93	1.32	2.55	1.75	0.95
time (sec)	N/A	0.041	0.026	0.011	0.8	0.269	10.494	0.27	2.97

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	13	22	16	8	50	8
normalized size	1	1.	1.	1.44	2.44	1.78	0.89	5.56	0.89
time (sec)	N/A	0.015	0.009	0.005	0.737	0.264	4.066	0.267	1.472

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	24	29	41	38	0	78	20
normalized size	1	1.	1.14	1.38	1.95	1.81	0.	3.71	0.95
time (sec)	N/A	0.03	0.014	0.006	0.739	0.264	0.	0.267	2.247

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	32	34	51	51	0	92	34
normalized size	1	1.	0.94	1.	1.5	1.5	0.	2.71	1.
time (sec)	N/A	0.039	0.017	0.007	0.741	0.268	0.	0.269	2.718

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	20	23	15	38	8	15	10
normalized size	1	1.	2.22	2.56	1.67	4.22	0.89	1.67	1.11
time (sec)	N/A	0.014	0.013	0.005	0.804	0.261	7.111	0.264	1.419

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	20	12	12	38	10	0	10
normalized size	1	1.	2.22	1.33	1.33	4.22	1.11	0.	1.11
time (sec)	N/A	0.014	0.009	0.005	0.814	0.26	5.917	0.	1.446

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	1059	22	90	0	22	14
normalized size	1	1.	1.	58.83	1.22	5.	0.	1.22	0.78
time (sec)	N/A	0.115	0.017	0.057	0.728	0.277	0.	0.261	4.477

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	14	70	28	43	19	22	12
normalized size	1	1.	0.88	4.38	1.75	2.69	1.19	1.38	0.75
time (sec)	N/A	0.095	0.013	0.056	0.72	0.267	0.585	0.267	4.721

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	30	27	66	49	94	51	36
normalized size	1	1.	0.68	0.61	1.5	1.11	2.14	1.16	0.82
time (sec)	N/A	0.095	0.021	0.009	0.807	0.263	2.07	0.262	6.289

Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	79	52	0	117	39	24	39
normalized size	1	1.	1.55	1.02	0.	2.29	0.76	0.47	0.76
time (sec)	N/A	0.045	0.047	0.024	0.	0.272	178.468	0.267	2.308

Problem 526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	95	78	103	109	0	41	66
normalized size	1	1.	1.32	1.08	1.43	1.51	0.	0.57	0.92
time (sec)	N/A	0.065	0.099	0.03	0.813	0.272	0.	0.269	2.425

Problem 527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	79	68	0	117	0	30	46
normalized size	1	1.	1.49	1.28	0.	2.21	0.	0.57	0.87
time (sec)	N/A	0.055	0.049	0.094	0.	0.267	0.	0.264	2.41

Problem 528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	52	58	120	72	133	186	94
normalized size	1	1.	0.43	0.48	0.99	0.6	1.1	1.54	0.78
time (sec)	N/A	0.175	0.036	0.019	0.808	0.294	54.884	0.267	12.639

Problem 529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	86	80	0	244	0	0	102
normalized size	1	1.	0.76	0.71	0.	2.16	0.	0.	0.9
time (sec)	N/A	0.151	0.084	0.085	0.	0.269	0.	0.	9.908

Problem 530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	124	0	163	116	0	63	104
normalized size	1	1.	1.17	0.	1.54	1.09	0.	0.59	0.98
time (sec)	N/A	0.119	0.125	0.066	0.808	0.275	0.	0.268	6.163

Problem 531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	49	42	0	1	0	55	39
normalized size	1	1.	0.98	0.84	0.	0.02	0.	1.1	0.78
time (sec)	N/A	0.106	0.088	0.027	0.	0.284	0.	0.267	5.563

Problem 532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	49	133	0	1	0	55	39
normalized size	1	1.	0.98	2.66	0.	0.02	0.	1.1	0.78
time (sec)	N/A	0.192	0.034	0.021	0.	0.285	0.	0.266	14.989

Problem 533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	63	56	0	1	0	74	54
normalized size	1	1.	0.93	0.82	0.	0.01	0.	1.09	0.79
time (sec)	N/A	0.102	0.18	0.016	0.	0.291	0.	0.264	6.054

Problem 534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	63	314	0	1	0	70	0
normalized size	1	1.	0.93	4.62	0.	0.01	0.	1.03	0.
time (sec)	N/A	0.97	0.089	0.029	0.	0.293	0.	0.276	0.

Problem 535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	39	44	76	53	0	0	29
normalized size	1	1.	1.15	1.29	2.24	1.56	0.	0.	0.85
time (sec)	N/A	0.053	0.025	0.015	0.769	0.504	0.	0.	3.04

Problem 536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	55	42	147	66	0	0	61
normalized size	1	1.	0.74	0.57	1.99	0.89	0.	0.	0.82
time (sec)	N/A	0.085	0.025	0.006	0.762	0.542	0.	0.	4.941

Problem 537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	18	22	14	22	14
normalized size	1	1.	1.	0.74	0.95	1.16	0.74	1.16	0.74
time (sec)	N/A	0.011	0.008	0.003	0.718	0.26	0.479	0.285	3.493

Problem 538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	77	52	90	111	182	74	53
normalized size	1	1.	1.43	0.96	1.67	2.06	3.37	1.37	0.98
time (sec)	N/A	0.136	0.046	0.008	0.811	0.282	4.124	0.304	4.286

Problem 539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	18	16	16	0	16	12
normalized size	1	1.	1.	1.29	1.14	1.14	0.	1.14	0.86
time (sec)	N/A	0.034	0.006	0.017	0.719	0.262	0.	0.286	1.257

Problem 540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	39	91	62	78	0	68	70
normalized size	1	1.	0.64	1.49	1.02	1.28	0.	1.11	1.15
time (sec)	N/A	0.08	0.026	0.011	0.809	0.27	0.	0.302	2.903

Problem 541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	93	41	51	0	41	39
normalized size	1	1.	1.	2.51	1.11	1.38	0.	1.11	1.05
time (sec)	N/A	0.066	0.021	0.018	0.802	0.264	0.	0.285	2.864

Problem 542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	38	91	61	76	0	66	70
normalized size	1	1.	0.62	1.49	1.	1.25	0.	1.08	1.15
time (sec)	N/A	0.066	0.019	0.007	0.794	0.267	0.	0.296	2.707

Problem 543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	54	28	28	0	30	26
normalized size	1	1.	1.	1.74	0.9	0.9	0.	0.97	0.84
time (sec)	N/A	0.044	0.007	0.02	0.718	0.263	0.	0.279	2.39

Problem 544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	42	101	69	84	0	73	70
normalized size	1	1.	0.65	1.55	1.06	1.29	0.	1.12	1.08
time (sec)	N/A	0.073	0.023	0.006	0.799	0.27	0.	0.294	2.554

Problem 545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	49	42	0	69	0	61	63
normalized size	1	1.	0.79	0.68	0.	1.11	0.	0.98	1.02
time (sec)	N/A	0.052	0.03	0.008	0.	0.556	0.	0.288	2.051

Problem 546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	65	55	0	82	0	0	65
normalized size	1	1.	0.87	0.73	0.	1.09	0.	0.	0.87
time (sec)	N/A	0.082	0.051	0.007	0.	0.555	0.	0.	2.684

Problem 547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	54	48	0	80	0	72	66
normalized size	1	1.	0.79	0.71	0.	1.18	0.	1.06	0.97
time (sec)	N/A	0.077	0.037	0.008	0.	0.606	0.	0.303	2.521

Problem 548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	62	60	0	99	0	92	78
normalized size	1	1.	0.78	0.75	0.	1.24	0.	1.15	0.98
time (sec)	N/A	0.079	0.055	0.009	0.	0.574	0.	0.287	2.526

Problem 549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	65	67	0	153	0	174	94
normalized size	1	1.	0.6	0.61	0.	1.4	0.	1.6	0.86
time (sec)	N/A	0.136	0.09	0.011	0.	1.327	0.	0.298	3.492

Problem 550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	45	32	0	63	0	0	37
normalized size	1	1.	0.96	0.68	0.	1.34	0.	0.	0.79
time (sec)	N/A	0.058	0.022	0.012	0.	0.476	0.	0.	2.28

Problem 551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	64	52	66	59	0	113	0
normalized size	1	1.	0.96	0.78	0.99	0.88	0.	1.69	0.
time (sec)	N/A	0.221	0.072	0.009	0.806	0.275	0.	0.284	0.

Problem 552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	70	54	69	90	0	117	0
normalized size	1	1.	0.99	0.76	0.97	1.27	0.	1.65	0.
time (sec)	N/A	0.188	0.045	0.006	0.802	0.267	0.	0.269	0.

Problem 553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	97	62	31	44	62	44
normalized size	1	1.	1.	1.87	1.19	0.6	0.85	1.19	0.85
time (sec)	N/A	0.096	0.032	0.013	0.802	0.267	3.351	0.262	7.701

Problem 554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	18	14	0	50	0	34	24
normalized size	1	1.	0.9	0.7	0.	2.5	0.	1.7	1.2
time (sec)	N/A	0.18	0.022	0.009	0.	0.484	0.	0.263	7.188

Problem 555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	34	39	39	36	41	41
normalized size	1	1.	1.	0.79	0.91	0.91	0.84	0.95	0.95
time (sec)	N/A	0.194	0.019	0.006	0.811	0.271	10.65	0.261	13.96

Problem 556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	156	90	120	103	0	120	0
normalized size	1	1.	1.34	0.78	1.03	0.89	0.	1.03	0.
time (sec)	N/A	0.22	0.201	0.007	0.718	0.263	0.	0.297	0.

Problem 557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	58	54	72	47	0	0	71
normalized size	1	1.	0.7	0.65	0.87	0.57	0.	0.	0.86
time (sec)	N/A	0.114	0.043	0.013	0.741	0.269	0.	0.	4.689

Problem 558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	62	41	54	53	216	0	54
normalized size	1	1.	0.97	0.64	0.84	0.83	3.38	0.	0.84
time (sec)	N/A	0.104	0.04	0.014	0.733	0.266	8.126	0.	4.797

Problem 559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	86	59	78	77	0	0	65
normalized size	1	1.	1.05	0.72	0.95	0.94	0.	0.	0.79
time (sec)	N/A	0.168	0.045	0.015	0.723	0.267	0.	0.	6.569

Problem 560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	58	54	72	47	0	0	71
normalized size	1	1.	0.7	0.65	0.87	0.57	0.	0.	0.86
time (sec)	N/A	0.101	0.016	0.	0.735	0.268	0.	0.	4.692

Problem 561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	135	121	162	103	0	0	165
normalized size	1	1.	0.71	0.64	0.85	0.54	0.	0.	0.87
time (sec)	N/A	0.557	0.129	0.019	0.74	0.271	0.	0.	18.108

Problem 562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	183	154	207	115	0	0	202
normalized size	1	1.	0.79	0.66	0.89	0.49	0.	0.	0.87
time (sec)	N/A	0.604	0.163	0.025	0.727	0.272	0.	0.	19.348

Problem 563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	103	107	143	84	0	0	139
normalized size	1	1.	0.64	0.67	0.89	0.52	0.	0.	0.87
time (sec)	N/A	0.428	0.106	0.012	0.727	0.275	0.	0.	16.686

Problem 564	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	16	0	47	0	34	26
normalized size	1	1.	1.	0.8	0.	2.35	0.	1.7	1.3
time (sec)	N/A	0.157	0.016	0.001	0.	0.477	0.	0.286	7.053

Problem 565	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	52	44	38	0	115	0	92	61
normalized size	1	1.18	1.	0.86	0.	2.61	0.	2.09	1.39
time (sec)	N/A	0.07	0.036	0.01	0.	0.729	0.	0.273	2.821

Problem 566	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	77	80	78	61	99	119	0
normalized size	1	1.	1.43	1.48	1.44	1.13	1.83	2.2	0.
time (sec)	N/A	0.603	0.13	0.007	0.715	0.264	11.79	0.275	0.

Problem 567	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	53	50	0	82	0	59	68
normalized size	1	1.	0.76	0.71	0.	1.17	0.	0.84	0.97
time (sec)	N/A	0.07	0.041	0.009	0.	0.977	0.	0.275	2.675

Problem 568	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	20	20	0	20	0
normalized size	1	1.	1.	0.84	1.05	1.05	0.	1.05	0.
time (sec)	N/A	0.041	0.012	0.004	0.718	0.267	0.	0.267	0.

Problem 569	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	66	50	66	66	71	68	0
normalized size	1	1.	0.86	0.65	0.86	0.86	0.92	0.88	0.
time (sec)	N/A	0.116	0.032	0.006	0.723	0.265	10.387	0.266	0.

Problem 570	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	B	A	B	F	A	A	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	224	70	443	0	582	56	0	211
normalized size	1	2.8	0.88	5.54	0.	7.28	0.7	0.	2.64
time (sec)	N/A	0.566	0.097	0.046	0.	0.286	14.345	0.	27.665

Problem 571	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	93	130	0	0	0	0	0
normalized size	1	1.	1.04	1.46	0.	0.	0.	0.	0.
time (sec)	N/A	0.47	0.019	0.017	0.	0.	0.	0.	0.

Problem 572	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F(-1)	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	73	68	0	0	0	0	78
normalized size	1	1.	1.2	1.11	0.	0.	0.	0.	1.28
time (sec)	N/A	0.889	0.03	0.021	0.	0.	0.	0.	71.783

Problem 573	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	28	36	24	26	20	7
normalized size	1	1.	1.	3.5	4.5	3.	3.25	2.5	0.88
time (sec)	N/A	0.009	0.006	0.005	0.723	0.3	3.571	0.309	1.047

Problem 574	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	32	36	36	0	30	7
normalized size	1	1.	1.	4.	4.5	4.5	0.	3.75	0.88
time (sec)	N/A	0.022	0.01	0.015	0.722	0.274	0.	0.27	1.654

Problem 575	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	42	39	66	104	60	31	17
normalized size	1	1.	1.91	1.77	3.	4.73	2.73	1.41	0.77
time (sec)	N/A	0.014	0.028	0.005	0.72	0.274	5.709	0.293	1.459

Problem 576	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	42	45	69	57	0	47	24
normalized size	1	1.	1.91	2.05	3.14	2.59	0.	2.14	1.09
time (sec)	N/A	0.017	0.006	0.005	0.713	0.273	0.	0.272	1.976

Problem 577	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	62	43	27	80	0	57	29
normalized size	1	1.	1.72	1.19	0.75	2.22	0.	1.58	0.81
time (sec)	N/A	0.044	0.046	0.022	0.798	0.296	0.	0.271	2.12

Problem 578	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	62	59	55	49	0	69	29
normalized size	1	1.	1.72	1.64	1.53	1.36	0.	1.92	0.81
time (sec)	N/A	0.056	0.014	0.02	0.797	0.284	0.	0.269	2.541

Problem 579	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	74	76	74	228	83	65	61
normalized size	1	1.	1.07	1.1	1.07	3.3	1.2	0.94	0.88
time (sec)	N/A	0.071	0.066	0.014	0.719	0.276	42.135	0.292	3.897

Problem 580	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	74	79	186	86	0	84	61
normalized size	1	1.	1.07	1.14	2.7	1.25	0.	1.22	0.88
time (sec)	N/A	0.086	0.021	0.014	0.717	0.277	0.	0.271	4.405

Problem 581	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	32	33	18	18	0	27	12
normalized size	1	1.	2.13	2.2	1.2	1.2	0.	1.8	0.8
time (sec)	N/A	0.022	0.023	0.006	0.798	0.277	0.	0.27	1.329

Problem 582	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	47	30	20	20	0	22	12
normalized size	1	1.	2.61	1.67	1.11	1.11	0.	1.22	0.67
time (sec)	N/A	0.038	0.026	0.016	0.804	0.275	0.	0.272	2.113

Problem 583	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	80	85	32	32	0	51	17
normalized size	1	1.	3.33	3.54	1.33	1.33	0.	2.12	0.71
time (sec)	N/A	0.099	0.094	0.04	0.805	0.277	0.	0.297	3.329

Problem 584	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	89	80	0	1	0	100	36
normalized size	1	1.	2.17	1.95	0.	0.02	0.	2.44	0.88
time (sec)	N/A	0.111	0.049	0.034	0.	0.279	0.	0.296	5.635

Problem 585	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	43	46	50	38	0	49	27
normalized size	1	1.	1.34	1.44	1.56	1.19	0.	1.53	0.84
time (sec)	N/A	0.031	0.028	0.005	0.799	0.274	0.	0.274	2.056

Problem 586	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	62	39	58	43	0	39	32
normalized size	1	1.	1.63	1.03	1.53	1.13	0.	1.03	0.84
time (sec)	N/A	0.037	0.035	0.006	0.793	0.272	0.	0.273	2.136

Problem 587	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	67	64	66	51	0	49	36
normalized size	1	1.	1.6	1.52	1.57	1.21	0.	1.17	0.86
time (sec)	N/A	0.036	0.085	0.023	0.804	0.279	0.	0.276	2.018

Problem 588	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	69	60	95	78	0	54	36
normalized size	1	1.	1.68	1.46	2.32	1.9	0.	1.32	0.88
time (sec)	N/A	0.048	0.049	0.017	0.714	0.275	0.	0.271	2.17

Problem 589	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	127	152	0	1	0	161	76
normalized size	1	1.	1.67	2.	0.	0.01	0.	2.12	1.
time (sec)	N/A	0.088	0.112	0.01	0.	0.292	0.	0.296	4.926

Problem 590	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	71	76	108	84	0	100	51
normalized size	1	1.	1.45	1.55	2.2	1.71	0.	2.04	1.04
time (sec)	N/A	0.037	0.068	0.015	0.803	0.286	0.	0.275	2.164

Problem 591	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	82	103	72	62	0	154	39
normalized size	1	1.	1.78	2.24	1.57	1.35	0.	3.35	0.85
time (sec)	N/A	0.079	0.064	0.031	0.802	0.278	0.	0.278	3.256

Problem 592	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	19	17	36	23	0	39	20
normalized size	1	1.	0.95	0.85	1.8	1.15	0.	1.95	1.
time (sec)	N/A	0.089	0.02	0.005	0.716	0.272	0.	0.28	3.944

Problem 593	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	17	17	22	23	0	39	14
normalized size	1	1.	0.94	0.94	1.22	1.28	0.	2.17	0.78
time (sec)	N/A	0.116	0.015	0.004	0.774	0.269	0.	0.281	6.166

Problem 594	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	101	79	139	112	0	144	48
normalized size	1	1.	1.87	1.46	2.57	2.07	0.	2.67	0.89
time (sec)	N/A	0.176	0.102	0.026	0.833	0.296	0.	0.285	6.851

Problem 595	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	17	18	15	15	0	31	8
normalized size	1	1.	1.55	1.64	1.36	1.36	0.	2.82	0.73
time (sec)	N/A	0.014	0.015	0.005	0.888	0.281	0.	0.27	1.35

Problem 596	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	41	40	19	46	0	0	26
normalized size	1	1.	1.41	1.38	0.66	1.59	0.	0.	0.9
time (sec)	N/A	0.042	0.019	0.026	0.863	0.313	0.	0.	3.711

Problem 597	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	250	551	0	379	0	387	180
normalized size	1	1.	1.39	3.06	0.	2.11	0.	2.15	1.
time (sec)	N/A	0.414	0.891	0.084	0.	0.292	0.	0.324	84.325

Problem 598	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	306	1066	0	236	0	473	264
normalized size	1	1.	1.78	6.2	0.	1.37	0.	2.75	1.53
time (sec)	N/A	0.253	1.021	0.042	0.	0.277	0.	0.31	30.41

Problem 599	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	307	307	333	6000	0	306	0	610	733
normalized size	1	1.	1.08	19.54	0.	1.	0.	1.99	2.39
time (sec)	N/A	0.445	1.099	0.062	0.	0.277	0.	0.311	92.883

Problem 600	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	74	71	0	220	0	109	53
normalized size	1	1.	1.14	1.09	0.	3.38	0.	1.68	0.82
time (sec)	N/A	0.068	0.045	0.008	0.	0.272	0.	0.277	3.973

Problem 601	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	91	118	0	317	0	193	70
normalized size	1	1.	1.1	1.42	0.	3.82	0.	2.33	0.84
time (sec)	N/A	0.076	0.079	0.026	0.	0.268	0.	0.276	4.631

Problem 602	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	111	146	0	431	0	248	85
normalized size	1	1.	1.1	1.45	0.	4.27	0.	2.46	0.84
time (sec)	N/A	0.082	0.082	0.033	0.	0.267	0.	0.274	5.18

Problem 603	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	1119	370	0	248	0	266	114
normalized size	1	1.	10.36	3.43	0.	2.3	0.	2.46	1.06
time (sec)	N/A	0.184	6.257	0.013	0.	0.282	0.	0.275	7.884

Problem 604	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	881	2407	0	154	0	355	94
normalized size	1	1.	10.13	27.67	0.	1.77	0.	4.08	1.08
time (sec)	N/A	0.156	5.199	0.108	0.	0.271	0.	0.272	3.867

Problem 605	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	914	14545	0	225	0	495	214
normalized size	1	1.	6.13	97.62	0.	1.51	0.	3.32	1.44
time (sec)	N/A	0.187	6.118	0.301	0.	0.277	0.	0.282	10.823

Problem 606	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	59	62	51	80	78	182	69	60
normalized size	1	1.4	1.48	1.21	1.9	1.86	4.33	1.64	1.43
time (sec)	N/A	0.393	0.083	0.011	0.885	0.261	2.159	0.265	23.437

Problem 607	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	59	62	51	80	78	182	69	60
normalized size	1	1.4	1.48	1.21	1.9	1.86	4.33	1.64	1.43
time (sec)	N/A	0.395	0.071	0.008	0.877	0.26	40.747	0.264	21.148

Problem 608	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	278	1050	0	0	0	0	88
normalized size	1	1.	2.73	10.29	0.	0.	0.	0.	0.86
time (sec)	N/A	0.202	0.899	0.184	0.	0.	0.	0.	22.935

Problem 609	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	256	946	0	0	0	0	58
normalized size	1	1.	4.13	15.26	0.	0.	0.	0.	0.94
time (sec)	N/A	0.155	0.886	0.039	0.	0.	0.	0.	19.939

Problem 610	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	156	200	0	0	0	0	15
normalized size	1	1.	9.18	11.76	0.	0.	0.	0.	0.88
time (sec)	N/A	0.04	0.215	0.038	0.	0.	0.	0.	10.819

Problem 611	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	261	963	0	0	0	0	63
normalized size	1	1.	3.58	13.19	0.	0.	0.	0.	0.86
time (sec)	N/A	0.16	1.351	0.051	0.	0.	0.	0.	19.962

Problem 612	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	298	1039	0	0	0	0	97
normalized size	1	1.	2.73	9.53	0.	0.	0.	0.	0.89
time (sec)	N/A	0.209	1.722	0.052	0.	0.	0.	0.	22.704

Problem 613	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	278	1050	0	0	0	0	88
normalized size	1	1.	2.73	10.29	0.	0.	0.	0.	0.86
time (sec)	N/A	0.197	1.315	0.047	0.	0.	0.	0.	15.814

Problem 614	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	256	946	0	0	0	0	58
normalized size	1	1.	4.13	15.26	0.	0.	0.	0.	0.94
time (sec)	N/A	0.151	0.939	0.039	0.	0.	0.	0.	12.434

Problem 615	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	100	200	0	0	0	0	15
normalized size	1	1.	5.88	11.76	0.	0.	0.	0.	0.88
time (sec)	N/A	0.036	0.314	0.039	0.	0.	0.	0.	3.483

Problem 616	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	298	963	0	0	0	0	63
normalized size	1	1.	4.08	13.19	0.	0.	0.	0.	0.86
time (sec)	N/A	0.155	1.275	0.045	0.	0.	0.	0.	11.985

Problem 617	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	327	1039	0	0	0	0	97
normalized size	1	1.	3.	9.53	0.	0.	0.	0.	0.89
time (sec)	N/A	0.201	1.437	0.051	0.	0.	0.	0.	14.677

Problem 618	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	730	730	10468	5229	0	0	0	0	734
normalized size	1	1.	14.34	7.16	0.	0.	0.	0.	1.01
time (sec)	N/A	1.918	6.298	0.268	0.	0.	0.	0.	157.33

Problem 619	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	622	622	5218	4890	0	0	0	0	604
normalized size	1	1.	8.39	7.86	0.	0.	0.	0.	0.97
time (sec)	N/A	1.554	6.147	0.05	0.	0.	0.	0.	127.364

Problem 620	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	822	1056	0	0	0	0	221
normalized size	1	1.	3.62	4.65	0.	0.	0.	0.	0.97
time (sec)	N/A	0.469	3.76	0.046	0.	0.	0.	0.	56.107

Problem 621	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	674	674	5276	5024	0	0	0	0	651
normalized size	1	1.	7.83	7.45	0.	0.	0.	0.	0.97
time (sec)	N/A	1.705	6.201	0.067	0.	0.	0.	0.	131.157

Problem 622	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	F	F	F	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	663	663	7543	7887	0	0	0	0	673
normalized size	1	1.	11.38	11.9	0.	0.	0.	0.	1.02
time (sec)	N/A	1.578	6.198	0.283	0.	0.	0.	0.	139.962

Problem 623	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	1065	1704	0	0	0	0	243
normalized size	1	1.	4.53	7.25	0.	0.	0.	0.	1.03
time (sec)	N/A	0.403	4.274	0.045	0.	0.	0.	0.	60.406

Problem 624	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	F	F	F	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	748	748	7629	8103	0	0	0	0	777
normalized size	1	1.	10.2	10.83	0.	0.	0.	0.	1.04
time (sec)	N/A	1.779	6.235	0.067	0.	0.	0.	0.	157.726

Problem 625	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	452	452	6287	2655	0	0	0	0	369
normalized size	1	1.	13.91	5.87	0.	0.	0.	0.	0.82
time (sec)	N/A	1.404	6.185	0.083	0.	0.	0.	0.	78.052

Problem 626	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	397	397	3470	2519	0	0	0	0	320
normalized size	1	1.	8.74	6.35	0.	0.	0.	0.	0.81
time (sec)	N/A	1.078	6.094	0.027	0.	0.	0.	0.	68.806

Problem 627	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	540	530	0	0	0	0	112
normalized size	1	1.	3.75	3.68	0.	0.	0.	0.	0.78
time (sec)	N/A	0.237	2.519	0.025	0.	0.	0.	0.	27.165

Problem 628	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	437	437	3526	2601	0	0	0	0	345
normalized size	1	1.	8.07	5.95	0.	0.	0.	0.	0.79
time (sec)	N/A	1.135	6.124	0.03	0.	0.	0.	0.	69.56

Problem 629	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	517	517	6386	2757	0	0	0	0	427
normalized size	1	1.	12.35	5.33	0.	0.	0.	0.	0.83
time (sec)	N/A	1.456	6.317	0.039	0.	0.	0.	0.	87.306

Problem 630	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	558	558	7235	2694	0	0	0	0	476
normalized size	1	1.	12.97	4.83	0.	0.	0.	0.	0.85
time (sec)	N/A	1.437	6.285	0.028	0.	0.	0.	0.	79.68

Problem 631	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	466	466	4389	2551	0	0	0	0	388
normalized size	1	1.	9.42	5.47	0.	0.	0.	0.	0.83
time (sec)	N/A	1.203	6.121	0.027	0.	0.	0.	0.	69.845

Problem 632	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	813	788	0	0	0	0	148
normalized size	1	1.	4.54	4.4	0.	0.	0.	0.	0.83
time (sec)	N/A	0.334	5.164	0.026	0.	0.	0.	0.	26.628

Problem 633	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	474	474	3593	2616	0	0	0	0	388
normalized size	1	1.	7.58	5.52	0.	0.	0.	0.	0.82
time (sec)	N/A	1.195	6.095	0.031	0.	0.	0.	0.	77.076

Problem 634	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	591	591	6452	2777	0	0	0	0	502
normalized size	1	1.	10.92	4.7	0.	0.	0.	0.	0.85
time (sec)	N/A	1.526	6.16	0.042	0.	0.	0.	0.	108.201

Problem 635	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	585	585	8500	2733	0	0	0	0	498
normalized size	1	1.	14.53	4.67	0.	0.	0.	0.	0.85
time (sec)	N/A	1.635	6.241	0.033	0.	0.	0.	0.	84.612

Problem 636	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	485	485	5647	2582	0	0	0	0	405
normalized size	1	1.	11.64	5.32	0.	0.	0.	0.	0.84
time (sec)	N/A	1.352	6.164	0.03	0.	0.	0.	0.	72.457

Problem 637	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	388	388	1247	1147	0	0	0	0	314
normalized size	1	1.	3.21	2.96	0.	0.	0.	0.	0.81
time (sec)	N/A	1.038	6.054	0.029	0.	0.	0.	0.	58.549

Problem 638	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	311	311	2941	2607	0	0	0	0	264
normalized size	1	1.	9.46	8.38	0.	0.	0.	0.	0.85
time (sec)	N/A	0.803	6.148	0.035	0.	0.	0.	0.	58.006

Problem 639	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	582	582	5812	2780	0	0	0	0	500
normalized size	1	1.	9.99	4.78	0.	0.	0.	0.	0.86
time (sec)	N/A	1.648	6.226	0.046	0.	0.	0.	0.	111.191

Problem 640	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	927	965	0	0	0	0	0
normalized size	1	1.	7.19	7.48	0.	0.	0.	0.	0.
time (sec)	N/A	0.533	0.555	1.776	0.	0.	0.	0.	0.

Problem 641	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	431	431	4865	4426	0	0	0	0	0
normalized size	1	1.	11.29	10.27	0.	0.	0.	0.	0.
time (sec)	N/A	0.997	6.065	0.03	0.	0.	0.	0.	0.

Problem 642	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	249	961	0	0	0	0	0
normalized size	1	1.	2.31	8.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.364	0.971	1.227	0.	0.	0.	0.	0.

Problem 643	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	367	367	3334	2564	0	0	0	0	0
normalized size	1	1.	9.08	6.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.71	6.051	0.028	0.	0.	0.	0.	0.

Problem 644	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	1148	1180	0	0	0	0	0
normalized size	1	1.	9.11	9.37	0.	0.	0.	0.	0.
time (sec)	N/A	0.56	0.597	1.678	0.	0.	0.	0.	0.

Problem 645	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	434	434	6019	5421	0	0	0	0	0
normalized size	1	1.	13.87	12.49	0.	0.	0.	0.	0.
time (sec)	N/A	1.012	6.071	0.03	0.	0.	0.	0.	0.

Problem 646	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	577	577	6084	5477	0	0	0	0	0
normalized size	1	1.	10.54	9.49	0.	0.	0.	0.	0.
time (sec)	N/A	1.278	6.078	0.033	0.	0.	0.	0.	0.

Problem 647	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	826	1182	0	0	0	0	0
normalized size	1	1.	6.35	9.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.428	0.202	0.911	0.	0.	0.	0.	0.

Problem 648	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	444	444	5428	5427	0	0	0	0	0
normalized size	1	1.	12.23	12.22	0.	0.	0.	0.	0.
time (sec)	N/A	0.887	6.069	0.025	0.	0.	0.	0.	0.

Problem 649	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	69	76	77	96	0	0	0
normalized size	1	1.	1.23	1.36	1.38	1.71	0.	0.	0.
time (sec)	N/A	0.409	0.104	0.025	0.844	0.284	0.	0.	0.

Problem 650	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	49	56	0	332	76	120	0
normalized size	1	1.	0.75	0.86	0.	5.11	1.17	1.85	0.
time (sec)	N/A	0.255	0.044	0.008	0.	0.281	11.862	0.273	0.

Problem 651	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	101	53	58	0	311	0	108	0
normalized size	1	1.91	1.	1.09	0.	5.87	0.	2.04	0.
time (sec)	N/A	0.365	0.049	0.007	0.	0.28	0.	0.27	0.

Problem 652	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	16	16	12	16	12
normalized size	1	1.	1.	0.93	1.14	1.14	0.86	1.14	0.86
time (sec)	N/A	0.193	0.008	0.002	0.801	0.269	0.347	0.265	11.326

Problem 653	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	46	61	58	60	58	0
normalized size	1	1.	1.	0.75	1.	0.95	0.98	0.95	0.
time (sec)	N/A	0.28	0.029	0.003	0.723	0.261	1.228	0.268	0.

Problem 654	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	114	86	115	123	116	113	0
normalized size	1	1.	1.	0.75	1.01	1.08	1.02	0.99	0.
time (sec)	N/A	0.339	0.057	0.004	0.723	0.266	1.281	0.264	0.

Problem 655	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	109	102	0	1	53	0	39
normalized size	1	1.	1.88	1.76	0.	0.02	0.91	0.	0.67
time (sec)	N/A	0.207	0.121	0.027	0.	0.283	7.309	0.	8.829

Problem 656	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	109	102	0	1	53	0	39
normalized size	1	1.	1.88	1.76	0.	0.02	0.91	0.	0.67
time (sec)	N/A	0.322	0.029	0.013	0.	0.282	28.298	0.	10.469

Problem 657	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	75	39	0	254	0	528	48
normalized size	1	1.	1.42	0.74	0.	4.79	0.	9.96	0.91
time (sec)	N/A	0.097	0.098	0.02	0.	0.296	0.	0.298	5.929

Problem 658	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	23	70	27	24	0
normalized size	1	1.	1.	0.78	1.	3.04	1.17	1.04	0.
time (sec)	N/A	0.016	0.019	0.001	0.719	0.265	0.459	0.262	0.

Problem 659	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	26	23	78	63	39	0
normalized size	1	1.	1.	1.13	1.	3.39	2.74	1.7	0.
time (sec)	N/A	0.016	0.006	0.002	0.799	0.265	3.76	0.27	0.

Problem 660	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	58	49	128	0	54	0
normalized size	1	1.	1.	1.76	1.48	3.88	0.	1.64	0.
time (sec)	N/A	0.046	0.026	0.016	0.756	0.271	0.	0.293	0.

Problem 661	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	0	35	3	3	2	42	0
normalized size	1	1.	0.	0.78	0.07	0.07	0.04	0.93	0.
time (sec)	N/A	0.253	0.092	0.006	0.921	0.265	0.199	0.264	0.

Problem 662	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	0	35	3	3	2	42	0
normalized size	1	1.	0.	0.78	0.07	0.07	0.04	0.93	0.
time (sec)	N/A	0.096	0.058	0.004	0.859	0.265	0.191	0.264	0.

Problem 663	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	0	35	3	3	2	42	0
normalized size	1	1.	0.	0.78	0.07	0.07	0.04	0.93	0.
time (sec)	N/A	0.165	0.056	0.005	0.835	0.274	0.185	0.272	0.

Problem 664	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	0	27	3	3	2	16	0
normalized size	1	1.	0.	0.6	0.07	0.07	0.04	0.36	0.
time (sec)	N/A	0.216	0.068	0.012	0.833	0.263	0.181	0.265	0.

Problem 665	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	0	41	58	8	0	8	0
normalized size	1	1.	0.	0.79	1.12	0.15	0.	0.15	0.
time (sec)	N/A	0.287	0.166	0.008	0.852	0.272	0.	0.265	0.

Problem 666	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	0	30	8	8	0	8	0
normalized size	1	1.	0.	0.58	0.15	0.15	0.	0.15	0.
time (sec)	N/A	0.214	0.086	0.014	1.166	0.27	0.	0.267	0.

Problem 667	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	51	40	11	11	0	11	0
normalized size	1	1.	0.75	0.59	0.16	0.16	0.	0.16	0.
time (sec)	N/A	0.256	0.038	0.018	1.417	0.268	0.	0.263	0.

Problem 668	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	23	32	36	68	0	0	19
normalized size	1	1.	0.74	1.03	1.16	2.19	0.	0.	0.61
time (sec)	N/A	0.058	0.053	0.013	1.049	0.266	0.	0.	2.619

Problem 669	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	23	32	34	68	0	74	0
normalized size	1	1.	0.74	1.03	1.1	2.19	0.	2.39	0.
time (sec)	N/A	0.521	0.031	0.003	0.796	0.267	0.	0.287	0.

Problem 670	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	23	81	0	68	0	92	0
normalized size	1	1.	0.74	2.61	0.	2.19	0.	2.97	0.
time (sec)	N/A	0.325	0.041	0.048	0.	0.269	0.	0.272	0.

Problem 671	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	23	81	0	68	0	92	46
normalized size	1	1.	0.74	2.61	0.	2.19	0.	2.97	1.48
time (sec)	N/A	0.3	0.029	0.004	0.	0.266	0.	0.269	4.964

Problem 672	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	23	32	0	68	0	92	15
normalized size	1	1.	0.74	1.03	0.	2.19	0.	2.97	0.48
time (sec)	N/A	1.241	0.044	0.009	0.	0.268	0.	0.286	22.256

Problem 673	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	44	73	115	59	206	0	36
normalized size	1	1.	1.02	1.7	2.67	1.37	4.79	0.	0.84
time (sec)	N/A	0.167	0.067	0.025	0.74	0.297	40.409	0.	9.52

Problem 674	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	45	0	142	0	77	49
normalized size	1	1.	1.	1.07	0.	3.38	0.	1.83	1.17
time (sec)	N/A	0.223	0.023	0.009	0.	0.27	0.	0.273	9.873

Problem 675	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	98	262	0	300	0	180	0
normalized size	1	1.	1.51	4.03	0.	4.62	0.	2.77	0.
time (sec)	N/A	0.257	0.074	0.049	0.	0.277	0.	0.278	0.

Problem 676	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	240	1407	0	1	0	724	170
normalized size	1	1.	1.23	7.22	0.	0.01	0.	3.71	0.87
time (sec)	N/A	0.469	0.437	0.045	0.	0.31	0.	0.275	39.15

Problem 677	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	29	46	65	0	46	24
normalized size	1	1.	1.	1.04	1.64	2.32	0.	1.64	0.86
time (sec)	N/A	0.055	0.039	0.04	0.782	0.286	0.	0.265	3.706

Problem 678	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	29	0	65	0	46	24
normalized size	1	1.	1.	1.04	0.	2.32	0.	1.64	0.86
time (sec)	N/A	0.072	0.021	0.037	0.	0.27	0.	0.267	5.778

Problem 679	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	24	62	17	24	17
normalized size	1	1.	1.	0.86	1.09	2.82	0.77	1.09	0.77
time (sec)	N/A	0.012	0.012	0.003	0.7	0.268	0.323	0.265	1.148

Problem 680	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	24	62	17	24	0
normalized size	1	1.	1.	0.86	1.09	2.82	0.77	1.09	0.
time (sec)	N/A	0.133	0.003	0.002	0.7	0.263	3.427	0.28	0.

Problem 681	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	12	24	15	36	0	14
normalized size	1	1.	1.	0.71	1.41	0.88	2.12	0.	0.82
time (sec)	N/A	0.085	0.021	0.013	0.947	0.305	3.903	0.	6.321

Problem 682	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	125	31	15	0	0	36
normalized size	1	1.	1.	7.35	1.82	0.88	0.	0.	2.12
time (sec)	N/A	0.072	0.016	0.123	0.975	0.302	0.	0.	37.666

Problem 683	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	7	8	26	7	8	7
normalized size	1	1.	1.	0.7	0.8	2.6	0.7	0.8	0.7
time (sec)	N/A	0.006	0.007	0.005	0.799	0.265	0.337	0.272	0.546

Problem 684	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	34	8	34	51	16	7
normalized size	1	1.	1.	3.4	0.8	3.4	5.1	1.6	0.7
time (sec)	N/A	0.014	0.007	0.01	0.809	0.266	3.855	0.268	1.502

Problem 685	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	7	8	26	7	8	7
normalized size	1	1.	1.	0.7	0.8	2.6	0.7	0.8	0.7
time (sec)	N/A	0.009	0.006	0.008	0.818	0.268	4.135	0.272	0.616

Problem 686	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	7	11	23	0	8	22
normalized size	1	1.	1.	0.58	0.92	1.92	0.	0.67	1.83
time (sec)	N/A	0.016	0.01	0.005	0.797	0.267	0.	0.269	0.702

Problem 687	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	45	31	11	23	41	18	22
normalized size	1	1.	3.75	2.58	0.92	1.92	3.42	1.5	1.83
time (sec)	N/A	0.023	0.019	0.009	0.82	0.269	3.795	0.27	1.52

Problem 688	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	45	7	11	23	0	8	22
normalized size	1	1.	3.75	0.58	0.92	1.92	0.	0.67	1.83
time (sec)	N/A	0.02	0.01	0.008	0.765	0.273	0.	0.268	0.772

Problem 689	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	11	23	0	5	24
normalized size	1	1.	1.	1.25	2.75	5.75	0.	1.25	6.
time (sec)	N/A	0.011	0.009	0.004	0.763	0.272	0.	0.268	0.705

Problem 690	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	42	29	11	23	41	18	24
normalized size	1	1.	10.5	7.25	2.75	5.75	10.25	4.5	6.
time (sec)	N/A	0.02	0.016	0.008	0.755	0.27	3.752	0.269	1.523

Problem 691	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	42	5	11	23	0	5	24
normalized size	1	1.	10.5	1.25	2.75	5.75	0.	1.25	6.
time (sec)	N/A	0.015	0.007	0.007	0.75	0.273	0.	0.268	0.774

Problem 692	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	8	9	9	8	9	8
normalized size	1	1.	1.	0.73	0.82	0.82	0.73	0.82	0.73
time (sec)	N/A	0.006	0.003	0.001	0.681	0.264	0.063	0.265	0.639

Problem 693	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	8	9	9	22	9	0
normalized size	1	1.	1.	0.73	0.82	0.82	2.	0.82	0.
time (sec)	N/A	0.022	0.001	0.002	0.675	0.261	2.912	0.264	0.

Problem 694	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	30	3	42	7	14	20
normalized size	1	1.	1.	1.11	0.11	1.56	0.26	0.52	0.74
time (sec)	N/A	0.028	0.012	0.008	0.79	0.268	2.931	0.27	0.693

Problem 695	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	30	3	42	0	14	0
normalized size	1	1.	1.	1.11	0.11	1.56	0.	0.52	0.
time (sec)	N/A	0.04	0.008	0.005	0.792	0.274	0.	0.269	0.

Problem 696	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	56	28	19	19	15	20	29
normalized size	1	1.	1.7	0.85	0.58	0.58	0.45	0.61	0.88
time (sec)	N/A	0.029	0.034	0.005	0.677	0.266	3.478	0.265	0.717

Problem 697	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	56	28	19	31	0	28	0
normalized size	1	1.	1.7	0.85	0.58	0.94	0.	0.85	0.
time (sec)	N/A	0.04	0.005	0.005	0.716	0.266	0.	0.265	0.

Problem 698	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	12	12	8	12	8
normalized size	1	1.	1.	0.91	1.09	1.09	0.73	1.09	0.73
time (sec)	N/A	0.006	0.003	0.003	0.681	0.265	0.066	0.261	0.526

Problem 699	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	23	20	16	28	0	20	8
normalized size	1	1.	2.09	1.82	1.45	2.55	0.	1.82	0.73
time (sec)	N/A	0.006	0.012	0.003	0.689	0.262	0.	0.265	1.251

Problem 700	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	9	9	7	9	7
normalized size	1	1.	1.	0.89	1.	1.	0.78	1.	0.78
time (sec)	N/A	0.004	0.002	0.003	0.683	0.259	0.065	0.265	0.523

Problem 701	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	25	22	9	31	0	18	7
normalized size	1	1.	2.78	2.44	1.	3.44	0.	2.	0.78
time (sec)	N/A	0.005	0.013	0.003	0.689	0.267	0.	0.264	1.406

Problem 702	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	12	16	10	12	10
normalized size	1	1.	1.	0.77	0.92	1.23	0.77	0.92	0.77
time (sec)	N/A	0.005	0.002	0.002	0.682	0.271	0.068	0.263	0.532

Problem 703	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	25	20	16	39	0	20	10
normalized size	1	1.	1.92	1.54	1.23	3.	0.	1.54	0.77
time (sec)	N/A	0.006	0.011	0.003	0.692	0.265	0.	0.267	1.242

Problem 704	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	8	9	9	8	9	8
normalized size	1	1.	1.	0.73	0.82	0.82	0.73	0.82	0.73
time (sec)	N/A	0.004	0.002	0.003	0.678	0.268	0.063	0.262	0.512

Problem 705	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	27	22	9	39	0	18	8
normalized size	1	1.	2.45	2.	0.82	3.55	0.	1.64	0.73
time (sec)	N/A	0.005	0.012	0.003	0.691	0.264	0.	0.265	1.404

Problem 706	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	45	67	55	78	97	53	31
normalized size	1	1.	1.29	1.91	1.57	2.23	2.77	1.51	0.89
time (sec)	N/A	0.029	0.025	0.008	0.759	0.278	5.777	0.269	2.045

Problem 707	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	79	86	55	138	0	0	31
normalized size	1	1.	2.26	2.46	1.57	3.94	0.	0.	0.89
time (sec)	N/A	0.029	0.075	0.014	0.781	0.302	0.	0.	2.955

Problem 708	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	F	F(-2)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	101	70	72	132	0	0	32
normalized size	1	1.	2.35	1.63	1.67	3.07	0.	0.	0.74
time (sec)	N/A	0.085	0.089	0.018	0.767	0.28	0.	0.	4.679

Problem 709	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	41	41	72	105	0	59	26
normalized size	1	1.	1.17	1.17	2.06	3.	0.	1.69	0.74
time (sec)	N/A	0.122	0.044	0.011	0.758	0.277	0.	0.274	5.155

Problem 710	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	A	A	F	F(-2)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	74	130	105	166	0	0	41
normalized size	1	1.	1.45	2.55	2.06	3.25	0.	0.	0.8
time (sec)	N/A	0.158	0.113	0.045	0.76	0.291	0.	0.	8.147

Problem 711	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	51	75	105	139	0	117	36
normalized size	1	1.	1.13	1.67	2.33	3.09	0.	2.6	0.8
time (sec)	N/A	0.196	0.078	0.015	0.756	0.284	0.	0.278	8.324

Problem 712	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	3	24	2	3	2
normalized size	1	1.	1.	1.5	1.5	12.	1.	1.5	1.
time (sec)	N/A	0.004	0.006	0.003	0.755	0.267	0.29	0.267	0.099

Problem 713	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	32	29	3	36	0	0	2
normalized size	1	1.	16.	14.5	1.5	18.	0.	0.	1.
time (sec)	N/A	0.005	0.017	0.017	0.778	0.27	0.	0.	1.299

Problem 714	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	3	19	2	19	2
normalized size	1	1.	1.	1.5	1.5	9.5	1.	9.5	1.
time (sec)	N/A	0.004	0.005	0.003	0.799	0.266	0.28	0.263	0.089

Problem 715	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	42	29	3	109	0	0	2
normalized size	1	1.	21.	14.5	1.5	54.5	0.	0.	1.
time (sec)	N/A	0.005	0.015	0.011	0.779	0.266	0.	0.	1.514

Problem 716	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	20	18	23	109	15	23	15
normalized size	1	1.	0.87	0.78	1.	4.74	0.65	1.	0.65
time (sec)	N/A	0.01	0.009	0.003	0.794	0.264	0.449	0.264	0.619

Problem 717	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	50	42	23	81	0	0	15
normalized size	1	1.	2.17	1.83	1.	3.52	0.	0.	0.65
time (sec)	N/A	0.011	0.045	0.011	0.839	0.269	0.	0.	1.393

Problem 718	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	18	16	20	105	15	34	15
normalized size	1	1.	0.86	0.76	0.95	5.	0.71	1.62	0.71
time (sec)	N/A	0.009	0.007	0.002	0.776	0.263	0.448	0.267	0.58

Problem 719	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	70	47	20	162	0	0	15
normalized size	1	1.	3.33	2.24	0.95	7.71	0.	0.	0.71
time (sec)	N/A	0.009	0.075	0.011	0.802	0.273	0.	0.	1.608

Problem 720	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	57	53	0	0	0	0	0	42
normalized size	1	1.16	1.08	0.	0.	0.	0.	0.	0.86
time (sec)	N/A	0.04	0.028	0.055	0.	0.	0.	0.	3.982

Problem 721	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	23	21	0	88	58	41	0
normalized size	1	1.	0.82	0.75	0.	3.14	2.07	1.46	0.
time (sec)	N/A	0.02	0.025	0.003	0.	0.266	1.467	0.267	0.

Problem 722	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	175	0	113	17	189	0
normalized size	1	1.	1.	4.73	0.	3.05	0.46	5.11	0.
time (sec)	N/A	0.092	0.021	0.047	0.	0.269	0.377	0.274	0.

Problem 723	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	74	33	0	159	27	119	0
normalized size	1	1.	1.85	0.82	0.	3.98	0.68	2.98	0.
time (sec)	N/A	0.09	0.047	0.012	0.	0.276	0.55	0.269	0.

Problem 724	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	77	111	127	90	0	158	0
normalized size	1	1.	1.43	2.06	2.35	1.67	0.	2.93	0.
time (sec)	N/A	0.258	0.042	0.019	0.767	0.281	0.	0.282	0.

Problem 725	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	77	111	0	90	0	158	0
normalized size	1	1.	1.28	1.85	0.	1.5	0.	2.63	0.
time (sec)	N/A	0.519	0.03	0.007	0.	0.274	0.	0.289	0.

Problem 726	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	41	38	0	89	0	68	0
normalized size	1	1.	0.8	0.75	0.	1.75	0.	1.33	0.
time (sec)	N/A	0.198	0.039	0.007	0.	0.273	0.	0.272	0.

Problem 727	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	41	38	73	89	0	68	0
normalized size	1	1.	0.8	0.75	1.43	1.75	0.	1.33	0.
time (sec)	N/A	0.181	0.021	0.005	0.771	0.272	0.	0.267	0.

Problem 728	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	41	51	73	86	0	0	0
normalized size	1	1.	0.8	1.	1.43	1.69	0.	0.	0.
time (sec)	N/A	0.264	0.018	0.01	0.804	0.304	0.	0.	0.

Problem 729	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	86	51	0	86	0	0	0
normalized size	1	1.	1.59	0.94	0.	1.59	0.	0.	0.
time (sec)	N/A	0.118	0.049	0.009	0.	0.276	0.	0.	0.

Problem 730	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	49	42	0	81	0	55	44
normalized size	1	1.	0.94	0.81	0.	1.56	0.	1.06	0.85
time (sec)	N/A	0.178	0.03	0.016	0.	0.273	0.	0.282	15.496

Problem 731	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	67	60	32	57	0	82	60
normalized size	1	1.	0.99	0.88	0.47	0.84	0.	1.21	0.88
time (sec)	N/A	0.335	0.071	0.04	0.817	0.28	0.	0.275	17.771

Problem 732	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	23	20	0	27	0	0	24
normalized size	1	1.	0.85	0.74	0.	1.	0.	0.	0.89
time (sec)	N/A	0.068	0.02	0.004	0.	0.262	0.	0.	1.873

Problem 733	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	12	13	0	22	0	0	235
normalized size	1	1.	0.86	0.93	0.	1.57	0.	0.	16.79
time (sec)	N/A	0.246	0.02	0.009	0.	0.274	0.	0.	50.079

Problem 734	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	0	22	0	0	15
normalized size	1	1.	1.	0.92	0.	1.83	0.	0.	1.25
time (sec)	N/A	0.111	0.012	0.006	0.	0.27	0.	0.	12.437

Problem 735	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	29	34	0	41	0	0	0
normalized size	1	1.	0.81	0.94	0.	1.14	0.	0.	0.
time (sec)	N/A	0.223	0.027	0.007	0.	0.272	0.	0.	0.

Problem 736	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	29	34	0	41	0	0	0
normalized size	1	1.	0.88	1.03	0.	1.24	0.	0.	0.
time (sec)	N/A	0.303	0.012	0.007	0.	0.277	0.	0.	0.

Problem 737	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	88	117	0	1	0	0	63
normalized size	1	1.	1.26	1.67	0.	0.01	0.	0.	0.9
time (sec)	N/A	0.163	0.08	0.072	0.	0.288	0.	0.	8.693

Problem 738	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	52	63	31	122	0	65	70
normalized size	1	1.	0.63	0.76	0.37	1.47	0.	0.78	0.84
time (sec)	N/A	0.312	0.042	0.039	0.829	0.286	0.	0.286	24.355

Problem 739	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	73	91	63	0	107	0	24	61
normalized size	1	1.55	1.94	1.34	0.	2.28	0.	0.51	1.3
time (sec)	N/A	0.784	0.04	0.028	0.	0.279	0.	0.266	35.413

Problem 740	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	70	75	31	221	0	45	0
normalized size	1	1.	0.57	0.61	0.25	1.8	0.	0.37	0.
time (sec)	N/A	0.518	0.04	0.012	0.816	0.279	0.	0.269	0.

Problem 741	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	64	62	0	169	0	116	112
normalized size	1	1.	0.48	0.47	0.	1.27	0.	0.87	0.84
time (sec)	N/A	0.195	0.084	0.024	0.	0.302	0.	0.27	9.302

Problem 742	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	44	49	47	123	0	90	76
normalized size	1	1.	0.49	0.54	0.52	1.37	0.	1.	0.84
time (sec)	N/A	0.13	0.034	0.014	0.788	0.269	0.	0.264	6.737

Problem 743	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	40	42	14	112	0	66	49
normalized size	1	1.	0.66	0.69	0.23	1.84	0.	1.08	0.8
time (sec)	N/A	0.073	0.026	0.008	0.785	0.268	0.	0.263	4.696

Problem 744	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	82	79	46	290	0	119	0
normalized size	1	1.	0.75	0.72	0.42	2.66	0.	1.09	0.
time (sec)	N/A	0.165	0.065	0.047	0.78	0.308	0.	0.307	0.

Problem 745	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	184	217	0	377	0	0	0
normalized size	1	1.	1.28	1.51	0.	2.62	0.	0.	0.
time (sec)	N/A	0.213	0.181	0.015	0.	0.282	0.	0.	0.

Problem 746	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	25	28	26	42	0	41	24
normalized size	1	1.	0.89	1.	0.93	1.5	0.	1.46	0.86
time (sec)	N/A	0.212	0.021	0.006	0.785	0.263	0.	0.267	9.253

Problem 747	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	66	60	0	1	0	96	63
normalized size	1	1.	0.88	0.8	0.	0.01	0.	1.28	0.84
time (sec)	N/A	0.238	0.05	0.017	0.	0.285	0.	0.287	7.51

Problem 748	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	26	26	26	53	38	15
normalized size	1	1.	1.	1.24	1.24	1.24	2.52	1.81	0.71
time (sec)	N/A	0.021	0.008	0.005	0.687	0.273	4.624	0.263	2.131

Problem 749	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	26	23	26	66	38	15
normalized size	1	1.	1.	1.24	1.1	1.24	3.14	1.81	0.71
time (sec)	N/A	0.031	0.013	0.003	0.7	0.303	4.698	0.272	1.954

Problem 750	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	75	64	0	1	0	88	60
normalized size	1	1.	1.03	0.88	0.	0.01	0.	1.21	0.82
time (sec)	N/A	0.238	0.067	0.01	0.	0.287	0.	0.272	8.25

Problem 751	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	32	37	62	43	0	109	39
normalized size	1	1.	0.67	0.77	1.29	0.9	0.	2.27	0.81
time (sec)	N/A	0.192	0.024	0.007	0.686	0.279	0.	0.296	6.299

Problem 752	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	99	79	0	1	0	135	99
normalized size	1	1.	0.88	0.71	0.	0.01	0.	1.21	0.88
time (sec)	N/A	0.275	0.089	0.01	0.	0.297	0.	0.272	10.483

Problem 753	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	135	236	0	1	0	239	0
normalized size	1	1.	0.53	0.93	0.	0.	0.	0.94	0.
time (sec)	N/A	0.472	0.202	0.048	0.	0.428	0.	0.28	0.

Problem 754	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	29	26	95	117	0	450	26
normalized size	1	1.	0.91	0.81	2.97	3.66	0.	14.06	0.81
time (sec)	N/A	0.031	0.027	0.005	0.707	0.291	0.	0.27	2.689

Problem 755	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	124	205	0	1	0	207	0
normalized size	1	1.	0.6	0.99	0.	0.	0.	1.	0.
time (sec)	N/A	0.156	0.112	0.01	0.	0.358	0.	0.281	0.

Problem 756	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	122	221	0	1	0	194	0
normalized size	1	1.	0.63	1.14	0.	0.01	0.	1.	0.
time (sec)	N/A	0.404	0.156	0.021	0.	0.32	0.	0.276	0.

Problem 757	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	122	215	0	1	0	250	0
normalized size	1	1.	0.58	1.03	0.	0.	0.	1.2	0.
time (sec)	N/A	0.348	0.125	0.017	0.	0.345	0.	0.278	0.

Problem 758	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	133	238	0	1	0	204	0
normalized size	1	1.	0.65	1.17	0.	0.	0.	1.	0.
time (sec)	N/A	0.424	0.155	0.018	0.	0.301	0.	0.277	0.

Problem 759	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.057	0.101	0.026	0.	0.	0.	0.	0.

Problem 760	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	0.109	0.028	0.	0.	0.	0.	0.

Problem 761	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.174	0.032	0.025	0.	0.	0.	0.	0.

Problem 762	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.19	0.033	0.025	0.	0.	0.	0.	0.

Problem 763	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	11	0	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.023	0.005	0.005	0.	0.	0.	0.	0.

Problem 764	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	11	0	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.022	0.006	0.008	0.	0.	0.	0.	0.

Problem 765	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	0.006	0.	0.	0.	0.	0.	0.

Problem 766	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.017	0.008	0.	0.	0.	0.	0.	0.

Problem 767	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.016	0.012	0.001	0.	0.	0.	0.	0.

Problem 768	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.017	0.012	0.	0.	0.	0.	0.	0.

Problem 769	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	A	F	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	0	0	0	1	0	0	44
normalized size	1	1.	0.	0.	0.	0.02	0.	0.	0.94
time (sec)	N/A	0.174	0.083	0.05	0.	1.523	0.	0.	6.27

Problem 770	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	A	F	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	0	0	0	1	0	0	44
normalized size	1	1.	0.	0.	0.	0.02	0.	0.	0.92
time (sec)	N/A	0.177	0.08	0.052	0.	1.611	0.	0.	6.421

Problem 771	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F(-1)	F	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	0	0	0	0	0	0	146
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.86
time (sec)	N/A	0.469	0.095	0.052	0.	0.	0.	0.	19.872

Problem 772	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F(-1)	F	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	0	0	0	0	0	0	231
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.86
time (sec)	N/A	0.626	0.098	0.039	0.	0.	0.	0.	26.943

Problem 773	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	28	36	34	0	36	37
normalized size	1	1.	1.	0.68	0.88	0.83	0.	0.88	0.9
time (sec)	N/A	0.071	0.02	0.006	0.755	0.289	0.	0.261	7.112

Problem 774	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	21	27	27	24	27	0
normalized size	1	1.	1.	0.81	1.04	1.04	0.92	1.04	0.
time (sec)	N/A	0.061	0.015	0.005	0.756	0.279	26.889	0.262	0.

Problem 775	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	48	41	41	39	41	0
normalized size	1	1.	1.	1.14	0.98	0.98	0.93	0.98	0.
time (sec)	N/A	0.241	0.018	0.006	0.758	0.281	18.206	0.261	0.

Problem 776	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	54	50	0	1	0	59	20
normalized size	1	1.	2.7	2.5	0.	0.05	0.	2.95	1.
time (sec)	N/A	0.029	0.042	0.015	0.	0.287	0.	0.263	2.807

Problem 777	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	64	62	0	1	0	54	20
normalized size	1	1.	3.2	3.1	0.	0.05	0.	2.7	1.
time (sec)	N/A	0.031	0.045	0.013	0.	0.288	0.	0.266	3.356

Problem 778	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	173	247	0	1	0	0	99
normalized size	1	1.	1.43	2.04	0.	0.01	0.	0.	0.82
time (sec)	N/A	0.467	0.163	0.042	0.	1.032	0.	0.	19.926

Problem 779	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	219	385	0	1	0	0	146
normalized size	1	1.	1.21	2.13	0.	0.01	0.	0.	0.81
time (sec)	N/A	0.734	0.48	0.046	0.	74.255	0.	0.	32.488

Problem 780	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	17	22	26	0	15	22
normalized size	1	1.	1.	0.65	0.85	1.	0.	0.58	0.85
time (sec)	N/A	0.015	0.016	0.005	0.71	0.267	0.	0.261	2.434

Problem 781	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	45	23	30	43	0	30	17
normalized size	1	1.	1.73	0.88	1.15	1.65	0.	1.15	0.65
time (sec)	N/A	0.031	0.028	0.009	0.786	0.265	0.	0.267	2.165

Problem 782	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	13	16	15	15	12	15	12
normalized size	1	1.	0.87	1.07	1.	1.	0.8	1.	0.8
time (sec)	N/A	0.008	0.017	0.005	0.674	0.259	0.381	0.259	1.137

Problem 783	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	19	23	38	24	0	36	17
normalized size	1	1.	0.86	1.05	1.73	1.09	0.	1.64	0.77
time (sec)	N/A	0.018	0.025	0.004	0.74	0.259	0.	0.268	1.872

Problem 784	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	37	13	12	23	0	23	10
normalized size	1	1.	3.08	1.08	1.	1.92	0.	1.92	0.83
time (sec)	N/A	0.022	0.034	0.009	0.765	0.264	0.	0.267	2.181

Problem 785	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	37	15	15	23	0	23	10
normalized size	1	1.	3.08	1.25	1.25	1.92	0.	1.92	0.83
time (sec)	N/A	0.024	0.027	0.009	0.773	0.265	0.	0.264	2.247

Problem 786	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	12	17	18	18	10	18	10
normalized size	1	1.	0.8	1.13	1.2	1.2	0.67	1.2	0.67
time (sec)	N/A	0.009	0.014	0.004	0.696	0.263	0.3	0.263	1.324

Problem 787	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	120	54	57	66	0	72	44
normalized size	1	1.	2.22	1.	1.06	1.22	0.	1.33	0.81
time (sec)	N/A	0.114	0.082	0.01	0.766	0.275	0.	0.267	6.657

Problem 788	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	F(-1)	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	57	342	0	0	0	61	49
normalized size	1	1.	0.97	5.8	0.	0.	0.	1.03	0.83
time (sec)	N/A	0.113	0.04	0.108	0.	0.	0.	1.006	6.503

Problem 789	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	41	28	38	41	0	39	51
normalized size	1	1.	0.69	0.47	0.64	0.69	0.	0.66	0.86
time (sec)	N/A	0.087	0.021	0.012	0.722	0.301	0.	0.281	4.639

Problem 790	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	51	38	62	54	0	63	83
normalized size	1	1.	0.54	0.4	0.66	0.57	0.	0.67	0.88
time (sec)	N/A	0.149	0.022	0.005	0.711	0.301	0.	0.269	7.872

Problem 791	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	20	16	19	26	24	17
normalized size	1	1.	1.	1.11	0.89	1.06	1.44	1.33	0.94
time (sec)	N/A	0.086	0.021	0.008	0.776	0.274	1.892	0.269	4.513

Problem 792	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	82	81	69	84	0	81	94
normalized size	1	1.	0.77	0.76	0.64	0.79	0.	0.76	0.88
time (sec)	N/A	0.064	0.044	0.009	0.795	0.282	0.	0.268	5.454

Problem 793	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	19	86	0	19	0
normalized size	1	1.	1.	0.96	0.76	3.44	0.	0.76	0.
time (sec)	N/A	0.031	0.038	0.006	0.707	0.277	0.	0.264	0.

Problem 794	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	34	45	57	46	45	42
normalized size	1	1.	1.	0.81	1.07	1.36	1.1	1.07	1.
time (sec)	N/A	0.063	0.017	0.006	0.788	0.273	1.902	0.263	5.45

Problem 795	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	26	34	49	0	34	0
normalized size	1	1.	1.	0.81	1.06	1.53	0.	1.06	0.
time (sec)	N/A	0.05	0.01	0.004	0.763	0.273	0.	0.262	0.

Problem 796	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	49	53	0	128	0	61	65
normalized size	1	1.	0.64	0.7	0.	1.68	0.	0.8	0.86
time (sec)	N/A	0.05	0.028	0.003	0.	0.301	0.	0.269	2.481

Problem 797	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	39	53	0	201	61	107	0
normalized size	1	1.	0.85	1.15	0.	4.37	1.33	2.33	0.
time (sec)	N/A	0.146	0.041	0.005	0.	0.277	7.151	0.267	0.

Problem 798	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	23	18	0	19	0	0	17
normalized size	1	1.	1.15	0.9	0.	0.95	0.	0.	0.85
time (sec)	N/A	0.019	0.019	0.004	0.	0.268	0.	0.	1.261

Problem 799	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	C	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	36	18	35	51	26	68	0	31
normalized size	1	1.03	0.51	1.	1.46	0.74	1.94	0.	0.89
time (sec)	N/A	0.03	0.019	0.012	0.895	0.283	3.99	0.	1.44

Problem 800	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	0	18	22	0	0	12
normalized size	1	1.	1.	0.	1.2	1.47	0.	0.	0.8
time (sec)	N/A	0.058	0.086	0.069	0.835	0.297	0.	0.	3.842

Problem 801	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	28	25	50	32	0	50	46
normalized size	1	1.	0.53	0.47	0.94	0.6	0.	0.94	0.87
time (sec)	N/A	0.05	0.014	0.006	0.691	0.273	0.	0.264	3.459

Problem 802	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	26	34	34	27	34	0
normalized size	1	1.	1.	0.84	1.1	1.1	0.87	1.1	0.
time (sec)	N/A	0.034	0.014	0.011	0.705	0.272	0.588	0.262	0.

Problem 803	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	14	12	57	8	12	8
normalized size	1	1.	1.	1.27	1.09	5.18	0.73	1.09	0.73
time (sec)	N/A	0.007	0.014	0.005	0.705	0.263	0.294	0.262	1.051

Problem 804	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	5	5	5	32	5	5
normalized size	1	1.	1.	0.83	0.83	0.83	5.33	0.83	0.83
time (sec)	N/A	0.008	0.006	0.005	0.775	0.27	3.303	0.26	1.033

Problem 805	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	17	17	22	22	0	34	14
normalized size	1	1.	0.85	0.85	1.1	1.1	0.	1.7	0.7
time (sec)	N/A	0.019	0.014	0.004	0.783	0.262	0.	0.267	1.766

Problem 806	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	12	35	15	12	15	0
normalized size	1	1.	1.	0.71	2.06	0.88	0.71	0.88	0.
time (sec)	N/A	0.009	0.004	0.002	0.707	0.263	0.278	0.262	0.

Problem 807	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	12	15	15	15	15	15
normalized size	1	1.	1.	0.63	0.79	0.79	0.79	0.79	0.79
time (sec)	N/A	0.01	0.006	0.002	0.678	0.26	1.312	0.261	1.238

Problem 808	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	28	36	34	37	36	37
normalized size	1	1.	1.	0.68	0.88	0.83	0.9	0.88	0.9
time (sec)	N/A	0.035	0.013	0.002	0.755	0.267	9.666	0.263	2.88

Problem 809	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	51	64	85	85	39	0	63
normalized size	1	1.	0.76	0.96	1.27	1.27	0.58	0.	0.94
time (sec)	N/A	0.069	0.027	0.006	0.785	0.287	3.863	0.	2.383

Problem 810	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	8	9	9	8	9	8
normalized size	1	1.	1.	0.73	0.82	0.82	0.73	0.82	0.73
time (sec)	N/A	0.006	0.001	0.	0.684	0.26	0.068	0.28	0.635

Problem 811	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	8	9	9	8	9	8
normalized size	1	1.	1.	0.73	0.82	0.82	0.73	0.82	0.73
time (sec)	N/A	0.006	0.003	0.002	0.703	0.265	0.068	0.26	0.633

Problem 812	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	46	9	9	8	9	8
normalized size	1	1.	1.	4.18	0.82	0.82	0.73	0.82	0.73
time (sec)	N/A	0.006	0.001	0.013	0.705	0.267	12.218	0.265	1.196

Problem 813	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	87	49	0	1	0	92	58
normalized size	1	1.	1.43	0.8	0.	0.02	0.	1.51	0.95
time (sec)	N/A	0.058	0.049	0.012	0.	0.284	0.	0.295	2.433

Problem 814	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	99	55	0	1	0	80	60
normalized size	1	1.	1.52	0.85	0.	0.02	0.	1.23	0.92
time (sec)	N/A	0.059	0.127	0.017	0.	0.279	0.	0.291	2.494

Problem 815	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	33	10	28	28	26	30	12
normalized size	1	1.	2.54	0.77	2.15	2.15	2.	2.31	0.92
time (sec)	N/A	0.017	0.008	0.006	0.809	0.283	1.318	0.264	1.9

Problem 816	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	33	10	28	28	26	30	12
normalized size	1	1.	2.54	0.77	2.15	2.15	2.	2.31	0.92
time (sec)	N/A	0.019	0.008	0.005	0.795	0.28	164.965	0.269	2.373

Problem 817	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	72	82	104	0	168	108	78
normalized size	1	1.	1.	1.14	1.44	0.	2.33	1.5	1.08
time (sec)	N/A	0.202	0.16	0.023	0.828	0.	4.223	0.285	4.729

Problem 818	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	23	23	20	30	0	32	32
normalized size	1	1.	0.62	0.62	0.54	0.81	0.	0.86	0.86
time (sec)	N/A	0.046	0.011	0.003	0.721	0.266	0.	0.263	3.032

Problem 819	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	37	13	12	23	0	23	10
normalized size	1	1.	3.08	1.08	1.	1.92	0.	1.92	0.83
time (sec)	N/A	0.023	0.033	0.	0.786	0.267	0.	0.268	2.198

Problem 820	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	60	67	0	89	0	69	87
normalized size	1	1.	0.63	0.71	0.	0.94	0.	0.73	0.92
time (sec)	N/A	0.111	0.048	0.005	0.	1.229	0.	0.269	6.771

Problem 821	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	28	24	31	28	184	31	29
normalized size	1	1.	0.8	0.69	0.89	0.8	5.26	0.89	0.83
time (sec)	N/A	0.026	0.013	0.004	0.692	0.269	3.626	0.26	1.197

Problem 822	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	30	28	36	30	265	43	31
normalized size	1	1.	0.81	0.76	0.97	0.81	7.16	1.16	0.84
time (sec)	N/A	0.031	0.014	0.006	0.72	0.266	3.497	0.262	1.376

Problem 823	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	26	48	0	45	32	39	27
normalized size	1	1.	0.9	1.66	0.	1.55	1.1	1.34	0.93
time (sec)	N/A	0.091	0.017	0.01	0.	0.27	6.203	0.266	4.262

Problem 824	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	26	48	0	49	87	43	26
normalized size	1	1.	1.04	1.92	0.	1.96	3.48	1.72	1.04
time (sec)	N/A	0.091	0.02	0.004	0.	0.264	8.697	0.265	5.618

Problem 825	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	0	77	56	20	15
normalized size	1	1.	1.	0.76	0.	3.67	2.67	0.95	0.71
time (sec)	N/A	0.04	0.022	0.004	0.	0.261	1.493	0.261	2.58

Problem 826	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	95	175	0	282	0	142	49
normalized size	1	1.	1.46	2.69	0.	4.34	0.	2.18	0.75
time (sec)	N/A	0.125	0.068	0.013	0.	0.269	0.	0.293	7.547

Problem 827	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	101	28	0	103	0	85	26
normalized size	1	1.	3.26	0.9	0.	3.32	0.	2.74	0.84
time (sec)	N/A	0.098	0.065	0.01	0.	0.265	0.	0.269	6.945

Problem 828	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	62	54	180	73	0	0	71
normalized size	1	1.	0.76	0.66	2.2	0.89	0.	0.	0.87
time (sec)	N/A	0.086	0.04	0.005	0.751	0.701	0.	0.	4.779

Problem 829	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	82	56	74	74	155	74	65
normalized size	1	1.	1.11	0.76	1.	1.	2.09	1.	0.88
time (sec)	N/A	0.183	0.029	0.011	0.774	0.273	3.83	0.277	6.929

Problem 830	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	125	81	108	105	223	108	0
normalized size	1	1.	1.09	0.7	0.94	0.91	1.94	0.94	0.
time (sec)	N/A	0.243	0.036	0.01	0.788	0.278	12.105	0.271	0.

Problem 831	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	51	0	27	0	5	3
normalized size	1	1.	1.	12.75	0.	6.75	0.	1.25	0.75
time (sec)	N/A	0.072	0.02	0.012	0.	0.267	0.	0.268	4.153

Problem 832	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	34	41	68	54	0	42	19
normalized size	1	1.	1.55	1.86	3.09	2.45	0.	1.91	0.86
time (sec)	N/A	0.029	0.023	0.005	0.699	0.265	0.	0.267	1.6

Problem 833	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	40	40	50	35	0	38	19
normalized size	1	1.	1.67	1.67	2.08	1.46	0.	1.58	0.79
time (sec)	N/A	0.029	0.023	0.008	0.768	0.269	0.	0.266	1.602

Problem 834	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	28	30	45	69	54	0	47	19
normalized size	1	1.17	1.25	1.88	2.88	2.25	0.	1.96	0.79
time (sec)	N/A	0.035	0.022	0.008	0.718	0.269	0.	0.268	1.909

Problem 835	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	30	60	51	51	0	51	20
normalized size	1	1.	1.25	2.5	2.12	2.12	0.	2.12	0.83
time (sec)	N/A	0.042	0.011	0.008	0.709	0.276	0.	0.274	2.636

Problem 836	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	42	45	69	57	0	47	24
normalized size	1	1.	1.91	2.05	3.14	2.59	0.	2.14	1.09
time (sec)	N/A	0.018	0.028	0.004	0.688	0.3	0.	0.272	1.884

Problem 837	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	43	44	47	34	0	47	22
normalized size	1	1.	1.48	1.52	1.62	1.17	0.	1.62	0.76
time (sec)	N/A	0.033	0.024	0.008	0.763	0.268	0.	0.27	1.74

Problem 838	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	45	28	49	47	0	34	24
normalized size	1	1.	1.36	0.85	1.48	1.42	0.	1.03	0.73
time (sec)	N/A	0.027	0.043	0.01	0.752	0.265	0.	0.261	0.953

Problem 839	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	38	7	8	22	0	8	5
normalized size	1	1.	4.75	0.88	1.	2.75	0.	1.	0.62
time (sec)	N/A	0.012	0.012	0.006	0.76	0.274	0.	0.264	0.645

Problem 840	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	11	15	15	24	0	22	10
normalized size	1	1.	0.85	1.15	1.15	1.85	0.	1.69	0.77
time (sec)	N/A	0.025	0.009	0.006	0.68	0.267	0.	0.262	1.639

Problem 841	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	38	41	54	45	0	99	20
normalized size	1	1.	1.73	1.86	2.45	2.05	0.	4.5	0.91
time (sec)	N/A	0.099	0.033	0.019	0.778	0.274	0.	0.29	6.164

Problem 842	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	39	32	15	0	14	18	42
normalized size	1	1.	1.62	1.33	0.62	0.	0.58	0.75	1.75
time (sec)	N/A	0.052	0.036	0.018	0.761	0.	0.612	0.262	2.007

Problem 843	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	84	122	0	88	0	82	56
normalized size	1	1.	1.18	1.72	0.	1.24	0.	1.15	0.79
time (sec)	N/A	0.058	0.195	0.022	0.	0.282	0.	0.265	2.01

Problem 844	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	64	28	0	35	0	30	26
normalized size	1	1.	2.	0.88	0.	1.09	0.	0.94	0.81
time (sec)	N/A	0.03	0.04	0.006	0.	0.271	0.	0.279	1.206

Problem 845	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	59	114	0	0	0	0	49
normalized size	1	1.	1.23	2.38	0.	0.	0.	0.	1.02
time (sec)	N/A	0.141	0.095	0.016	0.	0.	0.	0.	12.187

Problem 846	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	18	43	0	0	0	0	14
normalized size	1	1.	1.5	3.58	0.	0.	0.	0.	1.17
time (sec)	N/A	0.038	0.025	0.01	0.	0.	0.	0.	3.233

Problem 847	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	56	29	0	1	0	47	26
normalized size	1	1.	2.	1.04	0.	0.04	0.	1.68	0.93
time (sec)	N/A	0.022	0.029	0.005	0.	0.273	0.	0.274	1.162

Problem 848	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	56	29	0	1	0	47	26
normalized size	1	1.	2.	1.04	0.	0.04	0.	1.68	0.93
time (sec)	N/A	0.025	0.01	0.007	0.	0.272	0.	0.274	1.252

Problem 849	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	56	29	0	1	0	47	26
normalized size	1	1.	2.	1.04	0.	0.04	0.	1.68	0.93
time (sec)	N/A	0.027	0.01	0.005	0.	0.273	0.	0.275	1.247

Problem 850	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	56	29	0	1	0	47	26
normalized size	1	1.	2.	1.04	0.	0.04	0.	1.68	0.93
time (sec)	N/A	0.028	0.01	0.005	0.	0.271	0.	0.273	1.245

Problem 851	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	56	29	0	1	0	47	26
normalized size	1	1.	2.	1.04	0.	0.04	0.	1.68	0.93
time (sec)	N/A	0.026	0.01	0.006	0.	0.272	0.	0.272	1.255

Problem 852	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	56	29	0	1	0	47	26
normalized size	1	1.	2.	1.04	0.	0.04	0.	1.68	0.93
time (sec)	N/A	0.027	0.01	0.005	0.	0.283	0.	0.276	1.252

Problem 853	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	57	37	0	1	0	68	39
normalized size	1	1.	1.42	0.92	0.	0.02	0.	1.7	0.98
time (sec)	N/A	0.036	0.023	0.005	0.	0.275	0.	0.284	2.107

Problem 854	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	57	37	0	1	0	68	39
normalized size	1	1.	1.42	0.92	0.	0.02	0.	1.7	0.98
time (sec)	N/A	0.036	0.01	0.007	0.	0.275	0.	0.321	2.176

Problem 855	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	57	37	0	1	0	68	39
normalized size	1	1.	1.42	0.92	0.	0.02	0.	1.7	0.98
time (sec)	N/A	0.036	0.009	0.005	0.	0.283	0.	0.28	2.199

Problem 856	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	57	37	0	1	0	68	39
normalized size	1	1.	1.42	0.92	0.	0.02	0.	1.7	0.98
time (sec)	N/A	0.039	0.01	0.006	0.	0.274	0.	0.275	2.205

Problem 857	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	F	A	F	F	F(-1)	F	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	63	0	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.046	0.028	0.011	0.	0.	0.	0.	0.

Problem 858	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	F	B	F	F	F(-1)	F	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	0	180	0	0	0	0	0	0
normalized size	1	0.	2.73	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.229	0.703	0.031	0.	0.	0.	0.	0.

Problem 859	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	B	F	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	319	34	438	0	448	0	294	0
normalized size	1	4.09	0.44	5.62	0.	5.74	0.	3.77	0.
time (sec)	N/A	1.12	0.123	0.194	0.	0.327	0.	0.384	0.

Problem 860	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	433	753	0	1114	0	0	141
normalized size	1	1.	3.44	5.98	0.	8.84	0.	0.	1.12
time (sec)	N/A	0.355	0.689	0.143	0.	0.317	0.	0.	13.067

Problem 861	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	0	0	77	0	0	17
normalized size	1	1.	1.09	0.	0.	3.5	0.	0.	0.77
time (sec)	N/A	0.101	1.373	0.04	0.	0.787	0.	0.	4.332

Problem 862	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	50	0	0	0	0	0	34
normalized size	1	1.	1.25	0.	0.	0.	0.	0.	0.85
time (sec)	N/A	0.221	0.162	0.035	0.	0.	0.	0.	6.401

Problem 863	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	47	0	0	0	0	0	34
normalized size	1	1.	1.15	0.	0.	0.	0.	0.	0.83
time (sec)	N/A	0.24	0.152	0.036	0.	0.	0.	0.	6.468

Problem 864	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	330	1528	0	0	0	0	160
normalized size	1	1.	1.79	8.3	0.	0.	0.	0.	0.87
time (sec)	N/A	0.42	0.887	0.349	0.	0.	0.	0.	31.544

Problem 865	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	90	1036	0	0	0	0	0
normalized size	1	1.	0.69	7.91	0.	0.	0.	0.	0.
time (sec)	N/A	0.202	0.098	0.036	0.	0.	0.	0.	0.

Problem 866	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	419	514	0	1	0	0	48
normalized size	1	1.	7.76	9.52	0.	0.02	0.	0.	0.89
time (sec)	N/A	0.404	1.243	0.057	0.	12.955	0.	0.	24.684

Problem 867	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	416	517	0	1	0	0	48
normalized size	1	1.	7.85	9.75	0.	0.02	0.	0.	0.91
time (sec)	N/A	0.419	1.175	0.067	0.	12.859	0.	0.	25.782

Problem 868	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	154	69	0	262	0	221	78
normalized size	1	1.	1.83	0.82	0.	3.12	0.	2.63	0.93
time (sec)	N/A	0.268	0.186	0.031	0.	0.302	0.	0.287	24.997

Problem 869	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	17	23	9	30	36	20	15
normalized size	1	1.	0.85	1.15	0.45	1.5	1.8	1.	0.75
time (sec)	N/A	0.011	0.009	0.005	0.758	0.262	1.801	0.259	3.405

Problem 870	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F(-2)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	38	0	0	0	0	0	41
normalized size	1	1.	0.83	0.	0.	0.	0.	0.	0.89
time (sec)	N/A	0.038	0.032	0.027	0.	0.	0.	0.	4.216

Problem 871	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	13884	242984	0	1	0	0	76
normalized size	1	1.	157.77	2761.18	0.	0.01	0.	0.	0.86
time (sec)	N/A	0.417	6.324	0.181	0.	3.42	0.	0.	33.852

Problem 872	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	15147	269221	0	1	0	0	78
normalized size	1	1.	172.12	3059.33	0.	0.01	0.	0.	0.89
time (sec)	N/A	0.555	6.34	0.174	0.	3.417	0.	0.	49.368

Problem 873	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	199	0	0	0	0	0	41
normalized size	1	1.	4.33	0.	0.	0.	0.	0.	0.89
time (sec)	N/A	0.971	1.185	0.05	0.	0.	0.	0.	17.856

Problem 874	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	213	0	0	0	0	0	42
normalized size	1	1.	4.63	0.	0.	0.	0.	0.	0.91
time (sec)	N/A	0.973	1.127	0.048	0.	0.	0.	0.	17.559

Problem 875	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F(-1)	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	199	0	0	0	0	0	41
normalized size	1	1.	4.33	0.	0.	0.	0.	0.	0.89
time (sec)	N/A	1.839	0.698	0.033	0.	0.	0.	0.	25.955

Problem 876	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F(-1)	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	213	0	0	0	0	0	42
normalized size	1	1.	4.63	0.	0.	0.	0.	0.	0.91
time (sec)	N/A	1.84	0.32	0.035	0.	0.	0.	0.	26.313

Problem 877	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	75	147	127	130	0	81	17
normalized size	1	1.	3.95	7.74	6.68	6.84	0.	4.26	0.89
time (sec)	N/A	0.817	0.075	0.074	0.771	0.309	0.	0.376	17.864

Problem 878	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	A	F	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	0	0	0	489	0	0	116
normalized size	1	1.	0.	0.	0.	3.98	0.	0.	0.94
time (sec)	N/A	0.261	0.08	0.217	0.	7.256	0.	0.	16.128

Problem 879	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	F	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	103	0	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.808	0.119	0.187	0.	0.	0.	0.	0.

Problem 880	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	C	C	B	F	A	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	49	47	114	88	0	92	0	0	197
normalized size	1	0.96	2.33	1.8	0.	1.88	0.	0.	4.02
time (sec)	N/A	0.241	0.188	0.026	0.	0.322	0.	0.	72.083

Problem 881	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	383	555	0	1	0	0	0
normalized size	1	1.	4.79	6.94	0.	0.01	0.	0.	0.
time (sec)	N/A	0.701	0.77	0.059	0.	44.374	0.	0.	0.

Problem 882	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	1	0	1	0
normalized size	1	1.	1.	2.	1.	1.	0.	1.	0.
time (sec)	N/A	0.005	0.	0.001	0.695	0.243	0.065	0.274	1.681

Problem 883	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	F	A	F	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	2681	234	0	151	0	261	112
normalized size	1	1.	21.98	1.92	0.	1.24	0.	2.14	0.92
time (sec)	N/A	0.335	6.673	0.047	0.	0.261	0.	0.275	52.753

Problem 884	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	F	A	F(-1)	A	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	149	2155	234	0	151	0	261	0
normalized size	1	1.22	17.66	1.92	0.	1.24	0.	2.14	0.
time (sec)	N/A	0.775	6.562	0.082	0.	0.269	0.	0.295	0.

Problem 885	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	F	A	A	A	A	F(-1)	A	F(-2)
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	0	31	32	22	42	0	39	0
normalized size	1	0.	0.91	0.94	0.65	1.24	0.	1.15	0.
time (sec)	N/A	0.105	0.024	0.003	0.777	0.292	0.	0.255	0.

2.2 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [195] had the largest ratio of [0.8947]

Table 1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	4	1.	19	0.21
2	A	4	4	1.	23	0.174
3	A	4	4	1.	21	0.19
4	A	4	4	1.	21	0.19
5	A	4	4	1.	33	0.121
6	A	4	4	1.	35	0.114
7	A	4	4	1.	36	0.111
8	A	4	4	1.	36	0.111
9	A	4	4	1.	24	0.167
10	A	4	4	1.	20	0.2
11	A	4	4	1.	24	0.167
12	A	4	4	1.	22	0.182
13	A	4	4	1.	22	0.182
14	A	8	8	1.	15	0.533
15	A	8	8	1.	17	0.471
16	A	8	8	1.	15	0.533
17	A	8	8	1.	17	0.471
18	A	1	1	1.	25	0.04

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
19	A	3	3	1.	25	0.12
20	A	2	2	1.	28	0.071
21	A	2	2	1.	32	0.062
22	A	2	2	1.	30	0.067
23	A	2	2	1.	30	0.067
24	A	2	2	1.	53	0.038
25	A	2	2	1.	55	0.036
26	A	2	2	1.	56	0.036
27	A	2	2	1.	56	0.036
28	A	2	2	1.	30	0.067
29	A	4	4	1.	24	0.167
30	A	4	4	1.	28	0.143
31	A	4	4	1.	26	0.154
32	A	4	4	1.	26	0.154
33	A	4	4	1.	24	0.167
34	A	4	4	1.	28	0.143
35	A	4	4	1.	26	0.154
36	A	4	4	1.	26	0.154
37	A	4	4	1.	38	0.105
38	A	4	4	1.	40	0.1
39	A	4	4	1.	41	0.098
40	A	4	4	1.	41	0.098
41	A	4	4	1.	29	0.138
42	A	4	4	1.	20	0.2
43	A	4	4	1.	24	0.167
44	A	4	4	1.	22	0.182
45	A	4	4	1.	22	0.182
46	A	4	4	1.	34	0.118
47	A	4	4	1.	36	0.111
48	A	4	4	1.	37	0.108
49	A	4	4	1.	37	0.108
50	A	4	4	1.	25	0.16
51	A	2	2	1.	20	0.1
52	A	2	2	1.	22	0.091

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
53	A	2	2	1.	20	0.1
54	A	2	2	1.	22	0.091
55	A	2	2	1.	43	0.047
56	A	2	2	1.	44	0.045
57	A	2	2	1.	45	0.044
58	A	2	2	1.	46	0.043
59	A	2	2	1.	30	0.067
60	A	4	4	1.	22	0.182
61	A	4	4	1.	22	0.182
62	A	4	4	1.	20	0.2
63	A	4	4	1.	24	0.167
64	A	4	4	1.	35	0.114
65	A	4	4	1.	35	0.114
66	A	4	4	1.	36	0.111
67	A	4	4	1.	38	0.105
68	A	4	4	1.	29	0.138
69	A	4	4	1.	18	0.222
70	A	4	4	1.	18	0.222
71	A	4	4	1.	16	0.25
72	A	4	4	1.	20	0.2
73	A	4	4	1.	31	0.129
74	A	4	4	1.	31	0.129
75	A	4	4	1.	32	0.125
76	A	4	4	1.	34	0.118
77	A	4	4	1.	25	0.16
78	A	2	2	1.	30	0.067
79	A	2	2	1.	36	0.056
80	A	2	2	1.	34	0.059
81	A	2	2	1.	32	0.062
82	A	2	2	1.	58	0.034
83	A	2	2	1.	61	0.033
84	A	2	2	1.	62	0.032
85	A	2	2	1.	61	0.033
86	A	2	2	1.	52	0.038

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
87	A	2	2	1.	55	0.036
88	A	2	2	1.	56	0.036
89	A	2	2	1.	55	0.036
90	A	2	2	1.	30	0.067
91	A	2	2	1.	36	0.056
92	A	2	2	1.	34	0.059
93	A	2	2	1.	32	0.062
94	A	2	2	1.	58	0.034
95	A	2	2	1.	61	0.033
96	A	2	2	1.	62	0.032
97	A	2	2	1.	61	0.033
98	A	2	2	1.	52	0.038
99	A	2	2	1.	55	0.036
100	A	2	2	1.	56	0.036
101	A	2	2	1.	55	0.036
102	A	4	4	1.	23	0.174
103	A	4	4	1.	25	0.16
104	A	4	4	1.	25	0.16
105	A	4	4	1.	29	0.138
106	A	4	4	1.	27	0.148
107	A	4	4	1.	27	0.148
108	A	4	4	1.	42	0.095
109	A	4	4	1.	44	0.091
110	A	4	4	1.	45	0.089
111	A	4	4	1.	45	0.089
112	A	4	4	1.	21	0.19
113	A	4	4	1.	25	0.16
114	A	4	4	1.	23	0.174
115	A	4	4	1.	23	0.174
116	A	4	4	1.	23	0.174
117	A	4	4	1.	38	0.105
118	A	4	4	1.	40	0.1
119	A	4	4	1.	41	0.098
120	A	4	4	1.	41	0.098

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
121	A	6	6	1.	25	0.24
122	A	6	6	1.	29	0.207
123	A	6	6	1.	27	0.222
124	A	6	6	1.	27	0.222
125	A	6	6	1.	27	0.222
126	A	6	6	1.	31	0.194
127	A	6	6	1.	29	0.207
128	A	6	6	1.	29	0.207
129	A	5	5	1.	21	0.238
130	A	5	5	1.	25	0.2
131	A	5	5	1.	23	0.217
132	A	5	5	1.	23	0.217
133	A	5	5	1.	23	0.217
134	A	5	5	1.	27	0.185
135	A	5	5	1.	25	0.2
136	A	5	5	1.	25	0.2
137	A	8	8	1.	16	0.5
138	A	8	8	1.	18	0.444
139	A	8	8	1.	16	0.5
140	A	8	8	1.	18	0.444
141	A	8	8	1.	22	0.364
142	A	8	8	1.	24	0.333
143	A	8	8	1.	22	0.364
144	A	8	8	1.	24	0.333
145	A	6	6	1.	18	0.333
146	A	6	6	1.	20	0.3
147	A	6	6	1.	18	0.333
148	A	6	6	1.	20	0.3
149	A	1	1	1.	31	0.032
150	A	3	3	1.	30	0.1
151	A	2	1	1.	18	0.056
152	A	2	1	1.	16	0.062
153	A	2	1	1.	15	0.067
154	A	3	2	1.	18	0.111

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
155	A	2	1	1.	20	0.05
156	A	2	1	1.	18	0.056
157	A	2	1	1.	17	0.059
158	A	3	2	1.	20	0.1
159	A	2	1	1.	20	0.05
160	A	2	1	1.	18	0.056
161	A	2	1	1.	17	0.059
162	A	3	2	1.	20	0.1
163	A	7	2	1.	20	0.1
164	A	7	2	1.	20	0.1
165	A	7	2	1.	20	0.1
166	A	5	2	1.	20	0.1
167	A	5	2	1.	18	0.111
168	A	5	2	1.	17	0.118
169	A	8	3	1.	20	0.15
170	A	8	3	1.	20	0.15
171	A	5	2	1.	22	0.091
172	A	8	3	1.	20	0.15
173	A	13	12	1.	19	0.632
174	F	0	0	N/A	0	N/A
175	A	2	2	1.	27	0.074
176	A	2	2	1.	29	0.069
177	A	2	2	1.	27	0.074
178	A	2	2	1.	29	0.069
179	A	2	2	1.	31	0.065
180	A	2	2	1.	35	0.057
181	A	2	2	1.	33	0.061
182	A	2	2	1.	33	0.061
183	A	11	10	1.	19	0.526
184	A	10	9	1.	19	0.474
185	A	8	6	1.	17	0.353
186	A	2	2	1.	11	0.182
187	A	14	12	1.	19	0.632
188	A	29	14	1.	19	0.737

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
189	A	9	8	1.	19	0.421
190	A	8	7	1.	19	0.368
191	A	6	5	1.	17	0.294
192	A	1	1	1.	11	0.091
193	A	6	6	1.	19	0.316
194	A	10	9	1.	19	0.474
195	A	36	17	1.	19	0.895
196	A	4	4	1.	19	0.21
197	A	4	4	1.	19	0.21
198	A	3	3	1.	17	0.176
199	A	2	2	1.	11	0.182
200	A	13	12	1.	19	0.632
201	A	72	16	1.	19	0.842
202	A	10	3	1.	20	0.15
203	A	10	3	1.	22	0.136
204	A	14	7	1.	24	0.292
205	A	6	5	1.	19	0.263
206	A	5	5	1.	19	0.263
207	A	4	4	1.	19	0.21
208	A	2	2	1.	19	0.105
209	A	3	3	1.	19	0.158
210	A	4	4	1.	22	0.182
211	A	5	5	1.	19	0.263
212	A	6	5	1.	22	0.227
213	A	8	5	1.	33	0.152
214	A	9	6	1.	30	0.2
215	A	6	6	1.	19	0.316
216	A	3	3	1.	19	0.158
217	A	4	4	1.	19	0.21
218	A	2	2	1.	19	0.105
219	A	4	4	1.	17	0.235
220	A	3	3	1.	19	0.158
221	A	4	4	1.	19	0.21
222	A	6	6	1.	19	0.316

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
223	A	2	2	1.	19	0.105
224	A	2	2	1.	19	0.105
225	A	5	5	1.	19	0.263
226	A	4	4	1.	19	0.21
227	A	3	3	1.	17	0.176
228	A	3	3	1.	19	0.158
229	A	4	4	1.	19	0.21
230	A	6	6	1.	19	0.316
231	A	5	5	1.	19	0.263
232	A	2	2	1.	21	0.095
233	A	2	2	1.	19	0.105
234	A	2	2	1.	23	0.087
235	C	5	3	2.35	54	0.056
236	A	2	2	1.	26	0.077
237	A	2	2	1.	7	0.286
238	A	3	2	1.	15	0.133
239	A	4	2	1.	22	0.091
240	A	5	2	1.	25	0.08
241	A	5	2	1.	23	0.087
242	A	2	1	1.	21	0.048
243	A	7	4	1.	25	0.16
244	A	7	4	1.	25	0.16
245	A	9	7	1.	25	0.28
246	A	8	6	1.	23	0.261
247	A	7	5	1.81	21	0.238
248	A	9	8	1.	25	0.32
249	A	9	8	1.	25	0.32
250	A	10	2	1.	25	0.08
251	A	10	2	1.	23	0.087
252	B	6	2	2.36	21	0.095
253	A	8	4	1.	25	0.16
254	A	14	5	1.38	25	0.2
255	A	3	3	1.	15	0.2
256	A	3	3	1.	15	0.2

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
257	A	2	1	1.	17	0.059
258	A	5	3	1.	23	0.13
259	A	5	4	1.	23	0.174
260	A	3	2	1.	21	0.095
261	A	4	3	1.	19	0.158
262	A	6	5	1.	23	0.217
263	A	4	3	1.	23	0.13
264	A	6	5	1.	23	0.217
265	A	5	2	1.	25	0.08
266	A	5	2	1.	25	0.08
267	A	3	2	1.	23	0.087
268	A	8	4	1.	21	0.19
269	A	7	4	1.	25	0.16
270	A	9	5	1.	25	0.2
271	A	7	5	1.	25	0.2
272	A	6	4	1.	25	0.16
273	A	8	6	1.	23	0.261
274	A	8	6	1.	21	0.286
275	A	6	4	1.	25	0.16
276	A	7	5	1.	25	0.2
277	A	10	2	1.	25	0.08
278	A	6	2	1.	25	0.08
279	A	8	4	1.	25	0.16
280	A	14	5	1.42	23	0.217
281	A	16	5	1.68	21	0.238
282	A	4	3	1.	27	0.111
283	A	6	4	1.	42	0.095
284	A	6	5	1.	42	0.119
285	A	4	3	1.	40	0.075
286	A	5	4	1.	39	0.103
287	A	7	6	1.	42	0.143
288	A	5	4	1.	42	0.095
289	A	7	6	1.	42	0.143
290	A	15	7	1.	39	0.18

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
291	A	9	4	1.	35	0.114
292	A	4	3	1.	25	0.12
293	A	3	2	1.	25	0.08
294	A	3	2	1.	25	0.08
295	A	4	3	1.	23	0.13
296	A	3	2	1.	25	0.08
297	A	3	2	1.	25	0.08
298	A	3	2	1.	25	0.08
299	A	8	7	1.	27	0.259
300	A	8	7	1.	27	0.259
301	A	7	7	1.	27	0.259
302	A	5	5	1.	27	0.185
303	A	5	5	1.	27	0.185
304	A	6	5	1.	27	0.185
305	A	3	2	1.	17	0.118
306	A	3	2	1.	26	0.077
307	A	1	1	1.	17	0.059
308	A	1	1	1.	15	0.067
309	A	1	1	1.	15	0.067
310	A	1	1	1.	25	0.04
311	A	4	3	1.	28	0.107
312	A	3	2	1.	28	0.071
313	A	3	2	1.	28	0.071
314	A	4	3	1.	26	0.115
315	A	3	2	1.	28	0.071
316	A	3	2	1.	28	0.071
317	A	3	2	1.	28	0.071
318	A	8	7	1.	30	0.233
319	A	8	7	1.	30	0.233
320	A	7	7	1.	30	0.233
321	A	5	5	1.	30	0.167
322	A	5	5	1.	30	0.167
323	A	6	5	1.	30	0.167
324	A	3	2	1.	21	0.095

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
325	A	3	2	1.	19	0.105
326	A	3	2	1.	13	0.154
327	A	2	2	1.	21	0.095
328	A	2	2	1.	21	0.095
329	A	3	2	1.	23	0.087
330	A	3	2	1.	21	0.095
331	A	3	2	1.	15	0.133
332	A	2	2	1.	23	0.087
333	A	2	2	1.	23	0.087
334	A	3	2	1.	23	0.087
335	A	3	2	1.	23	0.087
336	A	3	2	1.	23	0.087
337	A	2	2	1.	23	0.087
338	A	2	2	1.	23	0.087
339	A	2	2	1.	23	0.087
340	A	3	2	1.	25	0.08
341	A	3	2	1.	25	0.08
342	A	3	2	1.	25	0.08
343	A	2	2	1.	25	0.08
344	A	2	2	1.	25	0.08
345	A	2	2	1.	25	0.08
346	A	4	3	1.	56	0.054
347	A	4	3	1.	54	0.056
348	A	4	3	1.	33	0.091
349	A	2	2	1.	56	0.036
350	A	3	3	1.	56	0.054
351	A	5	4	1.	33	0.121
352	A	3	3	1.	56	0.054
353	A	4	3	1.	58	0.052
354	A	4	3	1.	58	0.052
355	A	3	3	1.	58	0.052
356	A	3	3	1.	58	0.052
357	A	4	4	1.	58	0.069
358	A	5	4	1.	62	0.065

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
359	A	4	4	1.	62	0.065
360	A	4	4	1.	62	0.065
361	A	5	5	1.	60	0.083
362	A	7	6	1.	30	0.2
363	A	4	3	1.	37	0.081
364	A	4	3	1.	37	0.081
365	A	2	2	1.	37	0.054
366	A	2	2	1.	37	0.054
367	A	2	2	1.	42	0.048
368	A	2	2	1.	42	0.048
369	A	4	4	1.	30	0.133
370	A	4	4	1.	30	0.133
371	A	2	2	1.	42	0.048
372	A	2	2	1.	42	0.048
373	A	2	2	1.45	51	0.039
374	A	2	2	1.45	51	0.039
375	A	2	2	1.	40	0.05
376	A	2	2	1.	40	0.05
377	A	4	4	1.	32	0.125
378	A	4	4	1.	32	0.125
379	A	2	2	1.	51	0.039
380	A	2	2	1.	51	0.039
381	A	2	2	1.	56	0.036
382	A	2	2	1.	56	0.036
383	A	4	2	1.	29	0.069
384	A	4	2	1.	29	0.069
385	A	3	2	1.	27	0.074
386	A	7	6	1.	29	0.207
387	A	8	6	1.	29	0.207
388	A	8	7	1.	29	0.241
389	A	4	3	1.	25	0.12
390	A	7	6	1.	29	0.207
391	A	4	2	1.	29	0.069
392	A	4	2	1.	29	0.069

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
393	A	3	2	1.	29	0.069
394	A	7	6	1.	29	0.207
395	A	8	6	1.	29	0.207
396	A	10	10	1.	29	0.345
397	A	9	9	1.	27	0.333
398	A	9	9	1.	25	0.36
399	A	10	10	1.	29	0.345
400	A	10	10	1.	29	0.345
401	A	4	4	1.	25	0.16
402	A	4	4	1.	29	0.138
403	A	3	2	1.	31	0.065
404	A	3	3	1.	15	0.2
405	A	5	5	1.	15	0.333
406	A	4	3	1.	15	0.2
407	A	4	3	1.	13	0.231
408	A	4	3	1.	13	0.231
409	A	4	3	1.	15	0.2
410	A	9	9	1.	13	0.692
411	A	4	3	1.	13	0.231
412	A	4	3	1.	13	0.231
413	A	9	9	1.	15	0.6
414	A	3	3	1.	13	0.231
415	A	4	3	1.	13	0.231
416	A	13	10	1.	15	0.667
417	A	10	7	1.	19	0.368
418	A	4	3	1.	19	0.158
419	A	10	9	1.	21	0.429
420	A	3	3	1.	26	0.115
421	A	3	3	1.	26	0.115
422	A	3	3	1.	24	0.125
423	A	3	3	1.	23	0.13
424	A	3	3	1.	26	0.115
425	A	3	3	1.	26	0.115
426	A	4	3	1.	17	0.176

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
427	A	5	5	1.	17	0.294
428	A	4	4	1.	15	0.267
429	A	2	2	1.	9	0.222
430	A	5	5	1.	17	0.294
431	A	3	3	1.	17	0.176
432	A	4	4	1.	17	0.235
433	A	5	5	1.	17	0.294
434	A	3	3	1.	28	0.107
435	A	3	3	1.	28	0.107
436	A	3	3	1.	26	0.115
437	A	3	3	1.	25	0.12
438	A	3	3	1.	28	0.107
439	A	3	3	1.	28	0.107
440	A	8	6	1.	21	0.286
441	A	1	1	1.	17	0.059
442	A	1	1	1.	21	0.048
443	A	1	1	1.	17	0.059
444	A	1	1	1.	19	0.053
445	A	1	1	1.	21	0.048
446	A	4	3	1.	25	0.12
447	A	3	2	1.	17	0.118
448	A	4	3	1.	27	0.111
449	A	4	3	1.	25	0.12
450	A	5	4	1.7	22	0.182
451	A	4	3	1.	29	0.103
452	A	1	1	1.	176	0.006
453	A	1	1	1.	174	0.006
454	A	1	1	1.	164	0.006
455	F	0	0	N/A	0	N/A
456	F	0	0	N/A	0	N/A
457	A	4	3	1.	19	0.158
458	A	4	3	1.	19	0.158
459	A	4	3	1.	17	0.176
460	A	4	3	1.	15	0.2

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
461	A	7	6	1.	19	0.316
462	A	6	6	1.	19	0.316
463	A	6	6	1.	19	0.316
464	A	4	3	1.	21	0.143
465	A	4	3	1.	21	0.143
466	A	4	3	1.	19	0.158
467	A	4	3	1.	17	0.176
468	A	7	6	1.	21	0.286
469	A	8	7	1.	21	0.333
470	A	9	8	1.	21	0.381
471	A	4	3	1.	19	0.158
472	A	4	3	1.	19	0.158
473	A	4	3	1.	17	0.176
474	A	4	3	1.	15	0.2
475	A	7	6	1.	19	0.316
476	A	8	7	1.	19	0.368
477	A	9	7	1.	19	0.368
478	A	4	3	1.	19	0.158
479	A	4	3	1.	19	0.158
480	A	4	3	1.	17	0.176
481	A	4	3	1.	15	0.2
482	A	7	6	1.	19	0.316
483	A	8	7	1.	19	0.368
484	A	9	7	1.	19	0.368
485	A	4	3	1.	21	0.143
486	A	4	3	1.	21	0.143
487	A	4	3	1.	19	0.158
488	A	4	3	1.	17	0.176
489	A	6	5	1.	21	0.238
490	A	7	6	1.	21	0.286
491	A	8	6	1.	21	0.286
492	A	4	3	1.	19	0.158
493	A	4	3	1.	19	0.158
494	A	4	3	1.	17	0.176

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
495	A	4	3	1.	15	0.2
496	A	6	4	1.	19	0.21
497	A	8	6	1.	17	0.353
498	A	7	6	1.	17	0.353
499	A	6	6	1.	17	0.353
500	A	5	5	1.	17	0.294
501	A	6	6	1.	17	0.353
502	A	7	6	1.	17	0.353
503	A	8	6	1.	19	0.316
504	A	7	6	1.	19	0.316
505	A	6	6	1.	19	0.316
506	A	5	5	1.	19	0.263
507	A	6	6	1.	19	0.316
508	A	7	6	1.	19	0.316
509	A	2	2	1.	13	0.154
510	A	5	5	1.	17	0.294
511	A	6	5	1.	21	0.238
512	A	7	5	1.	25	0.2
513	A	8	5	1.	29	0.172
514	A	8	6	1.	20	0.3
515	A	7	6	1.	20	0.3
516	A	6	6	1.	18	0.333
517	A	2	2	1.	20	0.1
518	A	4	3	1.	20	0.15
519	A	4	3	1.	20	0.15
520	A	2	2	1.	18	0.111
521	A	2	2	1.	20	0.1
522	A	3	2	1.	22	0.091
523	A	3	3	1.	17	0.176
524	A	3	3	1.	23	0.13
525	A	3	3	1.	21	0.143
526	A	3	3	1.	23	0.13
527	A	3	3	1.	23	0.13
528	A	3	2	1.	22	0.091

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
529	A	5	5	1.	23	0.217
530	A	4	4	1.	25	0.16
531	A	5	4	1.	34	0.118
532	A	5	5	1.	31	0.161
533	A	6	4	1.	47	0.085
534	A	9	5	1.	58	0.086
535	A	4	4	1.	11	0.364
536	A	6	6	1.	11	0.546
537	A	2	1	1.	17	0.059
538	A	8	6	1.	13	0.462
539	A	2	1	1.	16	0.062
540	A	4	2	1.	14	0.143
541	A	5	4	1.	12	0.333
542	A	4	2	1.	13	0.154
543	A	4	2	1.	13	0.154
544	A	4	2	1.	15	0.133
545	A	5	5	1.	12	0.417
546	A	6	5	1.	14	0.357
547	A	5	4	1.	13	0.308
548	A	5	4	1.	17	0.235
549	A	5	4	1.	17	0.235
550	A	4	3	1.	13	0.231
551	A	7	5	1.	18	0.278
552	A	7	5	1.	20	0.25
553	A	8	5	1.	18	0.278
554	A	4	3	1.	26	0.115
555	A	7	5	1.	17	0.294
556	A	3	1	1.	27	0.037
557	A	5	3	1.	17	0.176
558	A	6	4	1.	17	0.235
559	A	6	4	1.	23	0.174
560	A	5	3	1.	17	0.176
561	A	6	2	1.	23	0.087
562	A	5	1	1.	25	0.04

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
563	A	5	2	1.	21	0.095
564	A	3	2	1.	23	0.087
565	A	4	3	1.18	16	0.188
566	A	3	1	1.	28	0.036
567	A	5	5	1.	16	0.312
568	A	5	4	1.	22	0.182
569	A	8	4	1.	21	0.19
570	B	16	10	2.8	20	0.5
571	A	9	6	1.	25	0.24
572	A	6	4	1.	35	0.114
573	A	2	2	1.	13	0.154
574	A	3	3	1.	15	0.2
575	A	3	3	1.	13	0.231
576	A	4	4	1.	11	0.364
577	A	3	3	1.	18	0.167
578	A	4	4	1.	17	0.235
579	A	4	4	1.	18	0.222
580	A	5	5	1.	17	0.294
581	A	2	2	1.	16	0.125
582	A	2	2	1.	21	0.095
583	A	3	3	1.	26	0.115
584	A	3	3	1.	25	0.12
585	A	3	3	1.	12	0.25
586	A	3	3	1.	15	0.2
587	A	3	3	1.	15	0.2
588	A	3	3	1.	15	0.2
589	A	3	3	1.	17	0.176
590	A	4	4	1.	15	0.267
591	A	4	4	1.	21	0.19
592	A	3	3	1.	22	0.136
593	A	4	4	1.	20	0.2
594	A	7	7	1.	20	0.35
595	A	2	2	1.	15	0.133
596	A	5	5	1.	21	0.238

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
597	A	8	6	1.	18	0.333
598	A	5	4	1.	18	0.222
599	A	6	4	1.	18	0.222
600	A	3	2	1.	16	0.125
601	A	3	2	1.	16	0.125
602	A	3	2	1.	16	0.125
603	A	10	8	1.	18	0.444
604	A	5	4	1.	18	0.222
605	A	6	5	1.	18	0.278
606	A	5	5	1.4	35	0.143
607	A	6	6	1.4	28	0.214
608	A	7	7	1.	23	0.304
609	A	6	6	1.	23	0.261
610	A	3	3	1.	23	0.13
611	A	6	6	1.	23	0.261
612	A	7	7	1.	23	0.304
613	A	7	7	1.	19	0.368
614	A	6	6	1.	19	0.316
615	A	3	3	1.	19	0.158
616	A	6	6	1.	19	0.316
617	A	7	7	1.	19	0.368
618	A	6	6	1.	31	0.194
619	A	5	5	1.	31	0.161
620	A	2	2	1.	31	0.065
621	A	5	5	1.	31	0.161
622	A	5	5	1.	34	0.147
623	A	2	2	1.	34	0.059
624	A	5	5	1.	34	0.147
625	A	8	8	1.	24	0.333
626	A	7	7	1.	24	0.292
627	A	3	3	1.	24	0.125
628	A	7	7	1.	24	0.292
629	A	8	8	1.	24	0.333
630	A	14	13	1.	26	0.5

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
631	A	12	12	1.	26	0.462
632	A	7	7	1.	26	0.269
633	A	10	10	1.	26	0.385
634	A	12	12	1.	26	0.462
635	A	15	13	1.	28	0.464
636	A	13	13	1.	28	0.464
637	A	11	11	1.	28	0.393
638	A	10	9	1.	28	0.321
639	A	13	12	1.	28	0.429
640	A	4	4	1.	19	0.21
641	A	10	10	1.	19	0.526
642	A	3	3	1.	19	0.158
643	A	9	9	1.	19	0.474
644	A	4	4	1.	24	0.167
645	A	10	10	1.	24	0.417
646	A	12	11	1.	24	0.458
647	A	4	4	1.	24	0.167
648	A	10	10	1.	24	0.417
649	A	11	10	1.	27	0.37
650	A	14	7	1.	20	0.35
651	A	12	6	1.91	21	0.286
652	A	2	1	1.	22	0.045
653	A	4	2	1.	18	0.111
654	A	4	2	1.	18	0.111
655	A	5	5	1.	23	0.217
656	A	6	6	1.	22	0.273
657	A	6	6	1.	20	0.3
658	A	3	2	1.	15	0.133
659	A	3	2	1.	21	0.095
660	A	5	4	1.	19	0.21
661	A	9	6	1.	24	0.25
662	A	9	6	1.	24	0.25
663	A	10	7	1.	21	0.333
664	A	11	8	1.	13	0.615

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
665	A	12	8	1.	25	0.32
666	A	13	9	1.	15	0.6
667	A	14	9	1.	19	0.474
668	A	2	1	1.	35	0.029
669	A	8	5	1.	37	0.135
670	A	16	8	1.	29	0.276
671	A	16	8	1.	36	0.222
672	A	31	14	1.	43	0.326
673	A	5	4	1.	21	0.19
674	A	6	4	1.	27	0.148
675	A	8	6	1.	27	0.222
676	A	4	4	1.	27	0.148
677	A	4	4	1.	20	0.2
678	A	6	6	1.	29	0.207
679	A	2	1	1.	20	0.05
680	A	3	2	1.	25	0.08
681	A	2	2	1.	28	0.071
682	A	3	3	1.	26	0.115
683	A	1	1	1.	11	0.091
684	A	2	2	1.	19	0.105
685	A	2	2	1.	15	0.133
686	A	2	2	1.	14	0.143
687	A	3	3	1.	17	0.176
688	A	3	3	1.	13	0.231
689	A	2	2	1.	14	0.143
690	A	3	3	1.	17	0.176
691	A	3	3	1.	13	0.231
692	A	1	0	1.	9	0.
693	A	4	3	1.	15	0.2
694	A	2	2	1.	13	0.154
695	A	3	3	1.	19	0.158
696	A	3	3	1.	11	0.273
697	A	4	4	1.	17	0.235
698	A	1	1	1.	9	0.111

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
699	A	2	2	1.	19	0.105
700	A	1	1	1.	7	0.143
701	A	2	2	1.	21	0.095
702	A	1	1	1.	9	0.111
703	A	2	2	1.	19	0.105
704	A	1	1	1.	7	0.143
705	A	2	2	1.	21	0.095
706	A	3	3	1.	17	0.176
707	A	4	4	1.	30	0.133
708	A	7	7	1.	20	0.35
709	A	6	6	1.	20	0.3
710	A	7	7	1.	23	0.304
711	A	6	6	1.	25	0.24
712	A	1	1	1.	11	0.091
713	A	2	2	1.	21	0.095
714	A	1	1	1.	9	0.111
715	A	2	2	1.	23	0.087
716	A	2	2	1.	11	0.182
717	A	3	3	1.	21	0.143
718	A	2	2	1.	9	0.222
719	A	3	3	1.	23	0.13
720	A	3	3	1.16	14	0.214
721	A	4	4	1.	15	0.267
722	A	7	6	1.	17	0.353
723	A	7	6	1.	17	0.353
724	A	10	8	1.	34	0.235
725	A	12	7	1.	30	0.233
726	A	5	4	1.	21	0.19
727	A	7	5	1.	23	0.217
728	A	9	7	1.	25	0.28
729	A	7	6	1.	25	0.24
730	A	4	4	1.	23	0.174
731	A	4	4	1.	28	0.143
732	A	3	3	1.	13	0.231

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
733	A	2	2	1.	24	0.083
734	A	2	2	1.	22	0.091
735	A	2	2	1.	30	0.067
736	A	3	3	1.	27	0.111
737	A	4	4	1.	23	0.174
738	A	7	5	1.	28	0.179
739	A	13	10	1.55	25	0.4
740	A	8	6	1.	29	0.207
741	A	6	6	1.	16	0.375
742	A	6	6	1.	16	0.375
743	A	4	4	1.	16	0.25
744	A	7	7	1.	16	0.438
745	A	8	8	1.	16	0.5
746	A	3	3	1.	24	0.125
747	A	4	4	1.	19	0.21
748	A	3	3	1.	17	0.176
749	A	2	2	1.	15	0.133
750	A	5	5	1.	19	0.263
751	A	3	3	1.	19	0.158
752	A	6	5	1.	19	0.263
753	A	9	5	1.	19	0.263
754	A	3	3	1.	17	0.176
755	A	8	4	1.	15	0.267
756	A	9	5	1.	19	0.263
757	A	8	5	1.	19	0.263
758	A	9	6	1.	19	0.316
759	A	0	0	0.	0	0.
760	A	0	0	0.	0	0.
761	A	0	0	0.	0	0.
762	A	0	0	0.	0	0.
763	A	0	0	0.	0	0.
764	A	0	0	0.	0	0.
765	A	0	0	0.	0	0.
766	A	0	0	0.	0	0.

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
767	A	0	0	0.	0	0.
768	A	0	0	0.	0	0.
769	A	2	2	1.	37	0.054
770	A	2	2	1.	38	0.053
771	A	5	4	1.	40	0.1
772	A	7	5	1.	40	0.125
773	A	6	5	1.	18	0.278
774	A	7	6	1.	21	0.286
775	A	8	6	1.	22	0.273
776	A	3	3	1.	21	0.143
777	A	3	3	1.	24	0.125
778	A	11	10	1.	19	0.526
779	A	10	7	1.	24	0.292
780	A	4	3	1.	19	0.158
781	A	3	3	1.	17	0.176
782	A	1	1	1.	15	0.067
783	A	1	1	1.	17	0.059
784	A	2	2	1.	17	0.118
785	A	2	2	1.	17	0.118
786	A	1	1	1.	17	0.059
787	A	6	6	1.	17	0.353
788	A	5	5	1.	11	0.454
789	A	3	3	1.	11	0.273
790	A	5	3	1.	13	0.231
791	A	2	2	1.	21	0.095
792	A	5	5	1.	16	0.312
793	A	2	2	1.	13	0.154
794	A	6	6	1.	16	0.375
795	A	5	4	1.	12	0.333
796	A	4	3	1.	18	0.167
797	A	10	6	1.	17	0.353
798	A	1	1	1.	17	0.059
799	C	3	2	1.03	27	0.074
800	A	2	2	1.	37	0.054

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
801	A	3	2	1.	17	0.118
802	A	5	4	1.	17	0.235
803	A	1	1	1.	15	0.067
804	A	3	3	1.	14	0.214
805	A	1	1	1.	17	0.059
806	A	2	1	1.	13	0.077
807	A	2	1	1.	15	0.067
808	A	7	4	1.	15	0.267
809	A	6	6	1.	15	0.4
810	A	1	0	1.	9	0.
811	A	1	0	1.	9	0.
812	A	2	1	1.	19	0.053
813	A	3	3	1.	15	0.2
814	A	3	3	1.	16	0.188
815	A	4	4	1.	15	0.267
816	A	5	5	1.	15	0.333
817	A	4	4	1.	25	0.16
818	A	2	2	1.	11	0.182
819	A	2	2	1.	17	0.118
820	A	6	6	1.	22	0.273
821	A	4	3	1.	13	0.231
822	A	4	3	1.	15	0.2
823	A	4	4	1.	19	0.21
824	A	4	4	1.	21	0.19
825	A	3	3	1.	17	0.176
826	A	7	6	1.	19	0.316
827	A	7	6	1.	19	0.316
828	A	6	6	1.	17	0.353
829	A	10	8	1.	17	0.471
830	A	11	9	1.	17	0.529
831	A	3	3	1.	22	0.136
832	A	5	5	1.	11	0.454
833	A	5	5	1.	13	0.385
834	A	5	5	1.17	11	0.454

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
835	A	5	5	1.	15	0.333
836	A	4	4	1.	11	0.364
837	A	5	5	1.	13	0.385
838	A	4	4	1.	11	0.364
839	A	3	3	1.	11	0.273
840	A	2	2	1.	11	0.182
841	A	5	5	1.	19	0.263
842	A	2	2	1.	23	0.087
843	A	4	4	1.	15	0.267
844	A	3	3	1.	15	0.2
845	A	6	6	1.	17	0.353
846	A	3	3	1.	17	0.176
847	A	2	2	1.	13	0.154
848	A	3	3	1.	11	0.273
849	A	3	3	1.	15	0.2
850	A	3	3	1.	19	0.158
851	A	3	3	1.	19	0.158
852	A	3	3	1.	19	0.158
853	A	2	2	1.	15	0.133
854	A	3	3	1.	15	0.2
855	A	3	3	1.	12	0.25
856	A	3	3	1.	16	0.188
857	F	0	0	N/A	0	N/A
858	F	0	0	N/A	0	N/A
859	B	25	12	4.09	31	0.387
860	A	5	4	1.	25	0.16
861	A	2	2	1.	27	0.074
862	A	2	2	1.	33	0.061
863	A	2	2	1.	34	0.059
864	A	7	6	1.	51	0.118
865	A	2	2	1.	49	0.041
866	A	2	2	1.	43	0.047
867	A	2	2	1.	44	0.045
868	A	9	8	1.	20	0.4

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
869	A	3	3	1.	15	0.2
870	A	3	3	1.	15	0.2
871	A	1	1	1.	52	0.019
872	A	1	1	1.	57	0.018
873	A	2	2	1.	59	0.034
874	A	2	2	1.	58	0.034
875	A	3	3	1.	58	0.052
876	A	3	3	1.	57	0.053
877	A	2	2	1.	66	0.03
878	A	9	9	1.	31	0.29
879	F	0	0	N/A	0	N/A
880	C	9	6	0.96	20	0.3
881	A	2	2	1.	46	0.043
882	A	1	0	1.	15	0.
883	A	12	9	1.	17	0.529
884	A	13	10	1.22	33	0.303
885	F	0	0	N/A	0	N/A

3 Listing of integrals

3.1 $\int \frac{1}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=145

$$\frac{2\tan^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{x^3+1}}\right)}{3\sqrt{3}}+\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

```
[Out] (2*ArcTan[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[1 + x^3]])/(3*Sqrt[3]) +
 (2*2^(1/3)*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqr
t[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)
], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*
Sqrt[1 + x^3])
```

Rubi [A] time = 0.266104, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$\frac{2\tan^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{x^3+1}}\right)}{3\sqrt{3}}+\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[1 + x^3]])/(3*Sqrt[3]) +
 (2*2^(1/3)*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqr
t[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)
], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*
Sqrt[1 + x^3])
```

Rubi in Sympy [A] time = 142.587, size = 456, normalized size = 3.14

$$\frac{2\sqrt{\frac{x^2-x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{-\sqrt{3}+2}\,(x+1)\,\mathrm{atan}\left(\frac{3^{\frac{3}{4}}\sqrt{1+\sqrt[3]{2}}\sqrt{-4\sqrt{3}+8}\sqrt{-\frac{\left(-x-1+\sqrt{3}\right)^2}{\left(x+1+\sqrt{3}\right)^2}+1}}{6\sqrt{-1+\sqrt[3]{2}}\sqrt{\frac{\left(-x-1+\sqrt{3}\right)^2}{\left(x+1+\sqrt{3}\right)^2}-4\sqrt{3}+7}}\right)}{\sqrt{\frac{x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{-1+\sqrt[3]{2}}\left(1+\sqrt[3]{2}\right)^{\frac{3}{2}}\sqrt{-4\sqrt{3}+8}\sqrt{x^3+1}} - \frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{x^2-x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{\sqrt{3}+2}\,(x+1)\,F\left(\mathrm{asin}\left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{3\sqrt{\frac{x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{x^3+1}\left(-2^{\frac{2}{3}}+1+\sqrt{3}\right)} + \frac{4\sqrt[4]{3}\sqrt{\frac{x^2-x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{-\sqrt{3}+2}\,(x+1)\left(\frac{\left(-2^{\frac{2}{3}}+1+\sqrt{3}\right)^2}{\left(-1+2^{\frac{2}{3}}+\sqrt{3}\right)^2};\mathrm{asin}\left(\frac{-x-1+\sqrt{3}}{x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{-4\sqrt{3}+7}\sqrt{x^3+1}\left(-2^{\frac{2}{3}}+1+\sqrt{3}\right)\left(-\sqrt{3}-2^{\frac{2}{3}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(2**(2/3)+x)/(x**3+1)**(1/2),x)
```

```
[Out] 2*sqrt((x**2 - x + 1)/(x + 1 + sqrt(3))**2)*sqrt(-sqrt(3) + 2)*(x
 + 1)*atan(3**(3/4)*sqrt(1 + 2**(1/3))*sqrt(-4*sqrt(3) + 8)*sqrt(
-(-x - 1 + sqrt(3))**2/(x + 1 + sqrt(3))**2 + 1)/(6*sqrt(-1 + 2**
(1/3))*sqrt((-x - 1 + sqrt(3))**2/(x + 1 + sqrt(3))**2 - 4*sqrt(3
) + 7)))/(sqrt((x + 1)/(x + 1 + sqrt(3))**2)*sqrt(-1 + 2**(1/3))*
(1 + 2**(1/3))**(3/2)*sqrt(-4*sqrt(3) + 8)*sqrt(x**3 + 1)) - 2*3*
*(3/4)*sqrt((x**2 - x + 1)/(x + 1 + sqrt(3))**2)*sqrt(sqrt(3) + 2
)*(x + 1)*elliptic_f(asin((x - sqrt(3) + 1)/(x + 1 + sqrt(3))), -
7 - 4*sqrt(3))/(3*sqrt((x + 1)/(x + 1 + sqrt(3))**2)*sqrt(x**3 +
1)*(-2**(2/3) + 1 + sqrt(3))) + 4*3**(1/4)*sqrt((x**2 - x + 1)/(x
 + 1 + sqrt(3))**2)*sqrt(-sqrt(3) + 2)*(x + 1)*elliptic_pi((-2**(
2/3) + 1 + sqrt(3))**2/(-1 + 2**(2/3) + sqrt(3))**2, asin((-x - 1
 + sqrt(3))/(x + 1 + sqrt(3))), -7 - 4*sqrt(3))/(sqrt((x + 1)/(x
+ 1 + sqrt(3))**2)*sqrt(-4*sqrt(3) + 7)*sqrt(x**3 + 1)*(-2**(2/3)
 + 1 + sqrt(3))*(-sqrt(3) - 2**(2/3) + 1))
```

Mathematica [C] time = 0.183222, size = 148, normalized size = 1.02

$$\frac{4i\sqrt{2}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\sqrt{x^2-x+1}\left(\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(1+2\,2^{2/3}-i\sqrt{3}\right)\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

```
[Out] ((4*I)*Sqrt[2]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*Sqrt[1 - x + x^2
]*EllipticPi[(2*Sqrt[3])/(I + (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sq
rt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I +
Sqrt[3])])/((1 + 2*2^(2/3) - I*Sqrt[3])*Sqrt[1 + x^3])
```

Maple [A] time = 0.115, size = 139, normalized size = 1.

$$2\,\frac{3/2-i/2\sqrt{3}}{\sqrt{x^3+1}\,\left(2^{2/3}-1\right)}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}\mathit{EllipticPi}\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},\frac{-3/2+i/2\sqrt{3}}{2^{2/3}-1},\sqrt{\frac{-3/2}{-3/2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(2^(2/3)+x)/(x^3+1)^(1/2),x)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1
/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/
(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)/(2^(2/3)-1)*EllipticPi(
((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(2^(2/3)-1
),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^3+1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^3 + 1)*(x + 2^(2/3))),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{x^3+1}\left(x+2^{\frac{2}{3}}\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^3 + 1)*(x + 2^(2/3))),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(x+1)(x^2-x+1)}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(2**(2/3)+x)/(x**3+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x + 1)*(x**2 - x + 1))*(x + 2**(2/3))), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^3+1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^3 + 1)*(x + 2^(2/3))),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

3.2 $\int \frac{1}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=160

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{3}}-\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

```
[Out] (-2*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[1 - x^3]])/(3*Sqrt[3])
- (2*2^(1/3)*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sq
rt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x
)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]
*Sqrt[1 - x^3])
```

Rubi [A] time = 0.305609, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{3}}-\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```


```
[Out] (-2*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[1 - x^3]])/(3*Sqrt[3])
- (2*2^(1/3)*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sq
rt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x
)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]
*Sqrt[1 - x^3])
```

Rubi in Sympy [A] time = 144.191, size = 456, normalized size = 2.85

$$-\frac{2\sqrt{\frac{x^2+x+1}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{-\sqrt{3}+2}\,(-x+1)\operatorname{atan}\left(\frac{3^{\frac{3}{4}}\sqrt{1+\sqrt[3]{2}}\sqrt{1-\frac{\left(x-1+\sqrt{3}\right)^2}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{-4\sqrt{3}+8}}{6\sqrt{-1+\sqrt[3]{2}}\sqrt{-4\sqrt{3}+7+\frac{\left(x-1+\sqrt{3}\right)^2}{\left(-x+1+\sqrt{3}\right)^2}}}\right)}{\sqrt{\frac{-x+1}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{-1+\sqrt[3]{2}}\left(1+\sqrt[3]{2}\right)^{\frac{3}{2}}\sqrt{-4\sqrt{3}+8}\sqrt{-x^3+1}}+\frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{x^2+x+1}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{\sqrt{3}+2}\,(-x+1)\,F\left(\operatorname{asin}\left(\frac{-x-\sqrt{3}+1}{-x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{3\sqrt{\frac{-x+1}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{-x^3+1}\left(-2^{\frac{2}{3}}+1+\sqrt{3}\right)}-\frac{4\sqrt[4]{3}\sqrt{\frac{x^2+x+1}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{-\sqrt{3}+2}\,(-x+1)\left(\frac{\left(-2^{\frac{2}{3}}+1+\sqrt{3}\right)^2}{\left(-1+2^{\frac{2}{3}}+\sqrt{3}\right)^2};\operatorname{asin}\left(\frac{x-1+\sqrt{3}}{-x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt{\frac{-x+1}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{-4\sqrt{3}+7}\sqrt{-x^3+1}\left(-2^{\frac{2}{3}}+1+\sqrt{3}\right)\left(-\sqrt{3}-2^{\frac{2}{3}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(2**(2/3)-x)/(-x**3+1)**(1/2),x)
```

```
[Out] -2*sqrt((x**2 + x + 1)/(-x + 1 + sqrt(3))**2)*sqrt(-sqrt(3) + 2)*
(-x + 1)*atan(3**(3/4)*sqrt(1 + 2**(1/3))*sqrt(1 - (x - 1 + sqrt(
3))**2/(-x + 1 + sqrt(3))**2)*sqrt(-4*sqrt(3) + 8)/(6*sqrt(-1 + 2
**(1/3))*sqrt(-4*sqrt(3) + 7 + (x - 1 + sqrt(3))**2/(-x + 1 + sqr
t(3))**2)))/(sqrt((-x + 1)/(-x + 1 + sqrt(3))**2)*sqrt(-1 + 2**(1
/3))*(1 + 2**(1/3))**(3/2)*sqrt(-4*sqrt(3) + 8)*sqrt(-x**3 + 1))
+ 2*3**(3/4)*sqrt((x**2 + x + 1)/(-x + 1 + sqrt(3))**2)*sqrt(sqrt
(3) + 2)*(-x + 1)*elliptic_f(asin((-x - sqrt(3) + 1)/(-x + 1 + sq
rt(3))), -7 - 4*sqrt(3))/(3*sqrt((-x + 1)/(-x + 1 + sqrt(3))**2)*
sqrt(-x**3 + 1)*(-2**(2/3) + 1 + sqrt(3))) - 4*3**(1/4)*sqrt((x**
2 + x + 1)/(-x + 1 + sqrt(3))**2)*sqrt(-sqrt(3) + 2)*(-x + 1)*ell
iptic_pi((-2**(2/3) + 1 + sqrt(3))**2/(-1 + 2**(2/3) + sqrt(3))**
2, asin((x - 1 + sqrt(3))/(-x + 1 + sqrt(3))), -7 - 4*sqrt(3))/(s
qrt((-x + 1)/(-x + 1 + sqrt(3))**2)*sqrt(-4*sqrt(3) + 7)*sqrt(-x*
*3 + 1)*(-2**(2/3) + 1 + sqrt(3))*(-sqrt(3) - 2**(2/3) + 1))
```

Mathematica [C] time = 0.153661, size = 148, normalized size = 0.92

$$-\frac{4i\sqrt{2}\sqrt{-\frac{i(x-1)}{\sqrt{3}+3i}}\sqrt{x^2+x+1}\left(\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(1+2\,2^{2/3}-i\sqrt{3}\right)\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```

```
[Out] ((-4*I)*Sqrt[2]*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*Sqrt[1 + x
+ x^2]*EllipticPi[(2*Sqrt[3])/(I + (2*I)*2^(2/3) + Sqrt[3]), ArcS
in[Sqrt[I + Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3
*I + Sqrt[3])])/((1 + 2*2^(2/3) - I*Sqrt[3])*Sqrt[1 - x^3])
```

Maple [A] time = 0.168, size = 143, normalized size = 0.9

$$\frac{\frac{2i}{3}\sqrt{3}}{-\frac{1}{2}+\frac{i}{2}\sqrt{3}-2^{\frac{2}{3}}}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\,EllipticPi\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\frac{i\sqrt{3}}{-\frac{1}{2}+\frac{i}{2}\sqrt{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(2^(2/3)-x)/(-x^3+1)^(1/2),x)
```

```
[Out] 2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/(-3
/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)
/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)-2^(2/3))*EllipticPi(1/3*3^(1/
2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*
3^(1/2)-2^(2/3)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\sqrt{-x^3+1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(sqrt(-x^3 + 1)*(x - 2^(2/3))),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(-x^3 + 1)*(x - 2^(2/3))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{1}{\sqrt{-x^3+1}\left(x-2^{\frac{2}{3}}\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(sqrt(-x^3 + 1)*(x - 2^(2/3))),x, algorithm="fricas")
```

```
[Out] integral(-1/(sqrt(-x^3 + 1)*(x - 2^(2/3))), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{x\sqrt{-x^3+1}-2^{\frac{2}{3}}\sqrt{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(2**(2/3)-x)/(-x**3+1)**(1/2),x)
```

```
[Out] -Integral(1/(x*sqrt(-x**3 + 1) - 2**(2/3)*sqrt(-x**3 + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\sqrt{-x^3+1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(sqrt(-x^3 + 1)*(x - 2^(2/3))),x, algorithm="giac")
```

```
[Out] integrate(-1/(sqrt(-x^3 + 1)*(x - 2^(2/3))), x)
```

3.3 $\int \frac{1}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=163

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{x^3-1}}\right)}{3\sqrt{3}}-\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

```
[Out] (-2*ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[-1 + x^3]])/(3*Sqrt[3]
) - (2*2^(1/3)*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 -
Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] -
 x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x
)^2)]*Sqrt[-1 + x^3])
```

Rubi [A] time = 0.282965, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{x^3-1}}\right)}{3\sqrt{3}}-\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[-1 + x^3]])/(3*Sqrt[3]
) - (2*2^(1/3)*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 -
Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] -
 x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x
)^2)]*Sqrt[-1 + x^3])
```

Rubi in Sympy [A] time = 148.569, size = 427, normalized size = 2.62

$$-\frac{\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(-x+1\right)\operatorname{atanh}\left(\frac{3^{\frac{3}{4}}\sqrt{1+\sqrt[3]{2}}\sqrt{\sqrt{3}+2}\sqrt{-\frac{\left(-x+1+\sqrt{3}\right)^2}{\left(x-1+\sqrt{3}\right)^2}+1}}{3\sqrt{-1+\sqrt[3]{2}}\sqrt{\frac{\left(-x+1+\sqrt{3}\right)^2}{\left(x-1+\sqrt{3}\right)^2}+4\sqrt{3}+7}}\right)}{\sqrt{\frac{x-1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{-1+\sqrt[3]{2}}\left(1+\sqrt[3]{2}\right)^{\frac{3}{2}}\sqrt{x^3-1}}+\frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{-\sqrt{3}+2}\left(-x+1\right)F\left(\operatorname{asin}\left(\frac{-x+1+\sqrt{3}}{-x-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{3\sqrt{\frac{x-1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\left(-\sqrt{3}-2^{\frac{2}{3}}+1\right)}+\frac{4\sqrt[4]{3}\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{\sqrt{3}+2}\left(-x+1\right)\Pi\left(\frac{\left(-1+2^{\frac{2}{3}}+\sqrt{3}\right)^2}{\left(-2^{\frac{2}{3}}+1+\sqrt{3}\right)^2};\operatorname{asin}\left(\frac{-x+1+\sqrt{3}}{x-1+\sqrt{3}}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt{\frac{x-1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{4\sqrt{3}+7}\sqrt{x^3-1}\left(-2^{\frac{2}{3}}+1+\sqrt{3}\right)\left(-\sqrt{3}-2^{\frac{2}{3}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(2**(2/3)-x)/(x**3-1)**(1/2),x)
```

```
[Out] -sqrt((x**2 + x + 1)/(-x - sqrt(3) + 1)**2)*(-x + 1)*atanh(3**(3/
4)*sqrt(1 + 2**(1/3))*sqrt(sqrt(3) + 2)*sqrt(-(-x + 1 + sqrt(3))*
*2/(x - 1 + sqrt(3))**2 + 1)/(3*sqrt(-1 + 2**(1/3))*sqrt((-x + 1
+ sqrt(3))**2/(x - 1 + sqrt(3))**2 + 4*sqrt(3) + 7)))/(sqrt((x -
1)/(-x - sqrt(3) + 1)**2)*sqrt(-1 + 2**(1/3))*(1 + 2**(1/3))**(3/
2)*sqrt(x**3 - 1)) + 2*3**(3/4)*sqrt((x**2 + x + 1)/(-x - sqrt(3)
 + 1)**2)*sqrt(-sqrt(3) + 2)*(-x + 1)*elliptic_f(asin((-x + 1 + s
qrt(3))/(-x - sqrt(3) + 1)), -7 + 4*sqrt(3))/(3*sqrt((x - 1)/(-x
- sqrt(3) + 1)**2)*sqrt(x**3 - 1)*(-sqrt(3) - 2**(2/3) + 1)) + 4*
3**(1/4)*sqrt((x**2 + x + 1)/(-x - sqrt(3) + 1)**2)*sqrt(sqrt(3)
+ 2)*(-x + 1)*elliptic_pi((-1 + 2**(2/3) + sqrt(3))**2/(-2**(2/3)
 + 1 + sqrt(3))**2, asin((-x + 1 + sqrt(3))/(x - 1 + sqrt(3))), -
7 + 4*sqrt(3))/(sqrt((x - 1)/(-x - sqrt(3) + 1)**2)*sqrt(4*sqrt(3
) + 7)*sqrt(x**3 - 1)*(-2**(2/3) + 1 + sqrt(3))*(-sqrt(3) - 2**(2
/3) + 1))
```

Mathematica [C] time = 0.184202, size = 146, normalized size = 0.9

$$-\frac{4i\sqrt{2}\sqrt{-\frac{i(x-1)}{\sqrt{3}+3i}}\sqrt{x^2+x+1}\,\Pi\left(\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(1+2\,2^{2/3}-i\sqrt{3}\right)\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] ((-4*I)*Sqrt[2]*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*Sqrt[1 + x
+ x^2]*EllipticPi[(2*Sqrt[3])/(I + (2*I)*2^(2/3) + Sqrt[3]), ArcS
in[Sqrt[I + Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3
*I + Sqrt[3])])/((1 + 2*2^(2/3) - I*Sqrt[3])*Sqrt[-1 + x^3])
```

Maple [A] time = 0.076, size = 143, normalized size = 0.9

$$-2\,{\frac{-3/2-i/2\sqrt{3}}{\sqrt{x^3-1}\left(-2^{2/3}+1\right)}\sqrt{{\frac{-1+x}{-3/2-i/2\sqrt{3}}}}\sqrt{{\frac{x+1/2-i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}}\sqrt{{\frac{x+1/2+i/2\sqrt{3}}{3/2+i/2\sqrt{3}}}}{\it EllipticPi} \left(\sqrt{{\frac{-1+x}{-3/2-i/2\sqrt{3}}}},{\frac{3/2+i/2\sqrt{3}}{-2^{2/3}+1}},\sqrt{}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(2^(2/3)-x)/(x^3-1)^(1/2),x)
```

```
[Out] -2*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1
/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2
))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)/(-2^(2/3)+1)*Elliptic
Pi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(-2^(2
/3)+1),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\sqrt{x^3-1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(sqrt(x^3 - 1)*(x - 2^(2/3))),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(x^3 - 1)*(x - 2^(2/3))), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(sqrt(x^3 - 1)*(x - 2^(2/3))),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{x\sqrt{x^3-1}-2^{\frac{2}{3}}\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(2**(2/3)-x)/(x**3-1)**(1/2),x)
```

```
[Out] -Integral(1/(x*sqrt(x**3 - 1) - 2**(2/3)*sqrt(x**3 - 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\sqrt{x^3-1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(sqrt(x^3 - 1)*(x - 2^(2/3))),x, algorithm="giac")
```

```
[Out] integrate(-1/(sqrt(x^3 - 1)*(x - 2^(2/3))), x)
```

3.4 $\int \frac{1}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=156

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{-x^3-1}}\right)}{3\sqrt{3}}+\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

```
[Out] (2*ArcTanh[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[-1 - x^3]])/(3*Sqrt[3])
 + (2*2^(1/3)*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 - S
qrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] +
x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)
^2)]*Sqrt[-1 - x^3])
```

Rubi [A] time = 0.298031, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{-x^3-1}}\right)}{3\sqrt{3}}+\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[-1 - x^3]])/(3*Sqrt[3])
 + (2*2^(1/3)*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 - S
qrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] +
x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)
^2)]*Sqrt[-1 - x^3])
```

Rubi in Sympy [A] time = 144.555, size = 437, normalized size = 2.8

$$\frac{\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,(x+1)\operatorname{atanh}\left(\frac{3^{\frac{3}{4}}\sqrt{1+\sqrt[3]{2}}\sqrt{1-\frac{\left(x+1+\sqrt{3}\right)^2}{\left(-x-1+\sqrt{3}\right)^2}}\sqrt{\sqrt{3}+2}}{3\sqrt{-1+\sqrt[3]{2}}\sqrt{4\sqrt{3}+7+\frac{\left(x+1+\sqrt{3}\right)^2}{\left(-x-1+\sqrt{3}\right)^2}}}\right)}{\sqrt{\frac{-x-1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-1+\sqrt[3]{2}}\left(1+\sqrt[3]{2}\right)^{\frac{3}{2}}\sqrt{-x^3-1}} - \frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-\sqrt{3}+2}\,(x+1)\,F\left(\operatorname{asin}\left(\frac{x+1+\sqrt{3}}{x-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{3\sqrt{\frac{-x-1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\left(-\sqrt{3}-2^{\frac{2}{3}}+1\right)} - \frac{4\sqrt[4]{3}\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{\sqrt{3}+2}\,(x+1)\left(\frac{\left(-1+2^{\frac{2}{3}}+\sqrt{3}\right)^2}{\left(-2^{\frac{2}{3}}+1+\sqrt{3}\right)^2};\operatorname{asin}\left(\frac{x+1+\sqrt{3}}{-x-1+\sqrt{3}}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt{\frac{-x-1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{4\sqrt{3}+7}\sqrt{-x^3-1}\left(-2^{\frac{2}{3}}+1+\sqrt{3}\right)\left(-\sqrt{3}-2^{\frac{2}{3}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(2**(2/3)+x)/(-x**3-1)**(1/2),x)
```

```
[Out] sqrt((x**2 - x + 1)/(x - sqrt(3) + 1)**2)*(x + 1)*atanh(3**(3/4)*
sqrt(1 + 2**(1/3))*sqrt(1 - (x + 1 + sqrt(3))**2/(-x - 1 + sqrt(3
))**2)*sqrt(sqrt(3) + 2)/(3*sqrt(-1 + 2**(1/3))*sqrt(4*sqrt(3) +
7 + (x + 1 + sqrt(3))**2/(-x - 1 + sqrt(3))**2)))/(sqrt((-x - 1)/
(x - sqrt(3) + 1)**2)*sqrt(-1 + 2**(1/3))*(1 + 2**(1/3))**(3/2)*s
qrt(-x**3 - 1)) - 2*3**(3/4)*sqrt((x**2 - x + 1)/(x - sqrt(3) + 1
)**2)*sqrt(-sqrt(3) + 2)*(x + 1)*elliptic_f(asin((x + 1 + sqrt(3)
)/(x - sqrt(3) + 1)), -7 + 4*sqrt(3))/(3*sqrt((-x - 1)/(x - sqrt(
3) + 1)**2)*sqrt(-x**3 - 1)*(-sqrt(3) - 2**(2/3) + 1)) - 4*3**(1/
4)*sqrt((x**2 - x + 1)/(x - sqrt(3) + 1)**2)*sqrt(sqrt(3) + 2)*(x
 + 1)*elliptic_pi((-1 + 2**(2/3) + sqrt(3))**2/(-2**(2/3) + 1 + s
qrt(3))**2, asin((x + 1 + sqrt(3))/(-x - 1 + sqrt(3))), -7 + 4*sq
rt(3))/(sqrt((-x - 1)/(x - sqrt(3) + 1)**2)*sqrt(4*sqrt(3) + 7)*s
qrt(-x**3 - 1)*(-2**(2/3) + 1 + sqrt(3))*(-sqrt(3) - 2**(2/3) + 1
))
```

Mathematica [C] time = 0.15991, size = 150, normalized size = 0.96

$$\frac{4i\sqrt{2}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\sqrt{x^2-x+1}\,\Pi\left(\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(1+2\,2^{2/3}-i\sqrt{3}\right)\sqrt{-x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] ((4*I)*Sqrt[2]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*Sqrt[1 - x + x^2
]*EllipticPi[(2*Sqrt[3])/(I + (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sq
rt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I +
Sqrt[3])])/((1 + 2*2^(2/3) - I*Sqrt[3])*Sqrt[-1 - x^3])
```

Maple [A] time = 0.106, size = 139, normalized size = 0.9

$$\frac{-\frac{2i}{3}\sqrt{3}}{\frac{1}{2}+\frac{i}{2}\sqrt{3}+2^{\frac{2}{3}}}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}EllipticPi\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\frac{i\sqrt{3}}{\frac{1}{2}+\frac{i}{2}\sqrt{3}+2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(2^(2/3)+x)/(-x^3-1)^(1/2),x)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/
2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/
(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2)+2^(2/3))*EllipticPi(1/3*3^(1/2)
*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(
1/2)+2^(2/3)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^3-1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-x^3 - 1)*(x + 2^(2/3))),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^3 - 1)*(x + 2^(2/3))), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-x^3 - 1)*(x + 2^(2/3))),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-(x+1)(x^2-x+1)}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(2**(2/3)+x)/(-x**3-1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 2**(2/3))), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^3-1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-x^3 - 1)*(x + 2^(2/3))),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-x^3 - 1)*(x + 2^(2/3))), x)
```

3.5 $\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=280

$$\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{3\sqrt{3}\sqrt{a}\sqrt[3]{b}}$$

```
[Out] (2*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) + 2^(1/3)*b^(1/3)*x))/Sqrt[a
+ b*x^3]])/(3*Sqrt[3]*Sqrt[a]*b^(1/3)) + (2*2^(1/3)*Sqrt[2 + Sqrt
[3]]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSi
n[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a^(1/3)*b^(1/3)*Sqrt[(a^(1
/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]
*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.557659, antiderivative size = 280, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$

$$\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{3\sqrt{3}\sqrt{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) + 2^(1/3)*b^(1/3)*x))/Sqrt[a
+ b*x^3]])/(3*Sqrt[3]*Sqrt[a]*b^(1/3)) + (2*2^(1/3)*Sqrt[2 + Sqrt
[3]]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSi
n[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a^(1/3)*b^(1/3)*Sqrt[(a^(1
/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]
*Sqrt[a + b*x^3])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(2**(2/3)*a**(1/3)+b**(1/3)*x)/(b*x**3+a)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.295432, size = 164, normalized size = 0.59

$$-\frac{2i\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\,\Pi\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)}{\left(\sqrt[3]{-1}+2^{2/3}\right)\sqrt[3]{b}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] ((-2*I)*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sq
rt[1 - (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(I
*Sqrt[3])/((-1)^(1/3) + 2^(2/3)), ArcSin[Sqrt[(a^(1/3) + (-1)^(2/
3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/(((-1)^(
1/3) + 2^(2/3))*b^(1/3)*Sqrt[a + b*x^3])
```

Maple [F] time = 0.12, size = 0, normalized size = 0.

$$\int 1\left(2^{\frac{2}{3}}\sqrt[3]{a}+\sqrt[3]{b}x\right)^{-1}\frac{1}{\sqrt{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

```
[Out] int(1/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^3 + a}\left(b^{\frac{1}{3}}x + 2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(b*x^3 + a)*(b^(1/3)*x + 2^(2/3)*a^(1/3))),x, algorithm="maxima
```

```
[Out] integrate(1/(sqrt(b*x^3 + a)*(b^(1/3)*x + 2^(2/3)*a^(1/3))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(b*x^3 + a)*(b^(1/3)*x + 2^(2/3)*a^(1/3))),x, algorithm="fricas
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + bx^3}\left(2^{\frac{2}{3}}\sqrt[3]{a} + \sqrt[3]{b}x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(2**(2/3)*a**(1/3)+b**(1/3)*x)/(b*x**3+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*x**3)*(2**(2/3)*a**(1/3) + b**(1/3)*x)), x
)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(b*x^3 + a)*(b^(1/3)*x + 2^(2/3)*a^(1/3))),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.6 $\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$

Optimal. Leaf size=288

$$-\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{2\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{3\sqrt{3}\sqrt{a}\sqrt[3]{b}}$$

```
[Out] (-2*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*b^(1/3)*x))/Sqrt[a
 - b*x^3]])/(3*Sqrt[3]*Sqrt[a]*b^(1/3)) - (2*2^(1/3)*Sqrt[2 + Sqr
t[3]]*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcS
in[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b
^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a^(1/3)*b^(1/3)*Sqrt[(a^(
1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2
]*Sqrt[a - b*x^3])
```

Rubi [A] time = 0.55776, antiderivative size = 288, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$

$$-\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{2\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{3\sqrt{3}\sqrt{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (-2*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*b^(1/3)*x))/Sqrt[a
 - b*x^3]])/(3*Sqrt[3]*Sqrt[a]*b^(1/3)) - (2*2^(1/3)*Sqrt[2 + Sqr
t[3]]*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcS
in[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b
^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a^(1/3)*b^(1/3)*Sqrt[(a^(
1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2
]*Sqrt[a - b*x^3])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(2**(2/3)*a**(1/3)-b**(1/3)*x)/(-b*x**3+a)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.310151, size = 166, normalized size = 0.58

$$\frac{2i\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\,\Pi\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}};\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)}{\left(\sqrt[3]{-1}+2^{2/3}\right)\sqrt[3]{b}\sqrt{a-bx^3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] ((2*I)*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqr
t[1 + (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(I*
Sqrt[3])/((-1)^(1/3) + 2^(2/3)), ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3
)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/(((-1)^(1
/3) + 2^(2/3))*b^(1/3)*Sqrt[a - b*x^3])
```

Maple [F] time = 0.094, size = 0, normalized size = 0.

$$\int 1\left(2^{\frac{2}{3}}\sqrt[3]{a}-\sqrt[3]{b}x\right)^{-1}\frac{1}{\sqrt{-bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

```
[Out] int(1/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\sqrt{-bx^3+a}\left(b^{\frac{1}{3}}x-2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/(sqrt(-b*x^3 + a)*(b^(1/3)*x - 2^(2/3)*a^(1/3))),x, algorithm="maxi
```

```
[Out] -integrate(1/(sqrt(-b*x^3 + a)*(b^(1/3)*x - 2^(2/3)*a^(1/3))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/(sqrt(-b*x^3 + a)*(b^(1/3)*x - 2^(2/3)*a^(1/3))),x, algorithm="fric
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt[3]{b}x\sqrt{a-bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(2**(2/3)*a**(1/3)-b**(1/3)*x)/(-b*x**3+a)**(1/2),x)
```

```
[Out] -Integral(1/(-2**(2/3)*a**(1/3)*sqrt(a - b*x**3) + b**(1/3)*x*sqr
t(a - b*x**3)), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/(sqrt(-b*x^3 + a)*(b^(1/3)*x - 2^(2/3)*a^(1/3))),x, algorithm="giac
```

```
[Out] Exception raised: RuntimeError
```

3.7 $\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

Optimal. Leaf size=297

$$-\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{bx^3-a}}\right)}{3\sqrt{3}\sqrt{a}\sqrt[3]{b}}$$

```
[Out] (-2*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*b^(1/3)*x))/Sqrt[
-a + b*x^3]])/(3*Sqrt[3]*Sqrt[a]*b^(1/3)) - (2*2^(1/3)*Sqrt[2 - S
qrt[3]]*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[Ar
cSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) -
 b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*a^(1/3)*b^(1/3)*Sqrt[-(
(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*
x)^2)]*Sqrt[-a + b*x^3])
```

Rubi [A] time = 0.59137, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$-\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{bx^3-a}}\right)}{3\sqrt{3}\sqrt{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*b^(1/3)*x))/Sqrt[
-a + b*x^3]])/(3*Sqrt[3]*Sqrt[a]*b^(1/3)) - (2*2^(1/3)*Sqrt[2 - S
qrt[3]]*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[Ar
cSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) -
 b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*a^(1/3)*b^(1/3)*Sqrt[-(
(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*
x)^2)]*Sqrt[-a + b*x^3])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(2**(2/3)*a**(1/3)-b**(1/3)*x)/(b*x**3-a)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.293199, size = 167, normalized size = 0.56

$$\frac{2i\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\,\Pi\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}};\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)}{\left(\sqrt[3]{-1}+2^{2/3}\right)\sqrt[3]{b}\sqrt{bx^3-a}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] ((2*I)*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqr
t[1 + (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(I*
Sqrt[3])/((-1)^(1/3) + 2^(2/3)), ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3
)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/(((-1)^(1
/3) + 2^(2/3))*b^(1/3)*Sqrt[-a + b*x^3])
```

Maple [F] time = 0.083, size = 0, normalized size = 0.

$$\int 1\left(2^{\frac{2}{3}}\sqrt[3]{a}-\sqrt[3]{b}x\right)^{-1}\frac{1}{\sqrt{bx^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

```
[Out] int(1/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\sqrt{bx^3 - a}\left(b^{\frac{1}{3}}x - 2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/(sqrt(b*x^3 - a)*(b^(1/3)*x - 2^(2/3)*a^(1/3))),x, algorithm="maxim
```

```
[Out] -integrate(1/(sqrt(b*x^3 - a)*(b^(1/3)*x - 2^(2/3)*a^(1/3))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/(sqrt(b*x^3 - a)*(b^(1/3)*x - 2^(2/3)*a^(1/3))),x, algorithm="frica
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{-a + bx^3} + \sqrt[3]{b}x\sqrt{-a + bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(2**(2/3)*a**(1/3)-b**(1/3)*x)/(b*x**3-a)**(1/2),x)
```

```
[Out] -Integral(1/(-2**(2/3)*a**(1/3)*sqrt(-a + b*x**3) + b**(1/3)*x*sq
rt(-a + b*x**3)), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/(sqrt(b*x^3 - a)*(b^(1/3)*x - 2^(2/3)*a^(1/3))),x, algorithm="giac'
```

```
[Out] Exception raised: RuntimeError
```

3.8 $\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

Optimal. Leaf size=293

$$\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{3\sqrt{3}\sqrt{a}\sqrt[3]{b}}$$

```
[Out] (2*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) + 2^(1/3)*b^(1/3)*x))/Sqrt[-
a - b*x^3]])/(3*Sqrt[3]*Sqrt[a]*b^(1/3)) + (2*2^(1/3)*Sqrt[2 - Sq
rt[3]]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[Arc
Sin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) +
b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*a^(1/3)*b^(1/3)*Sqrt[-((
a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x
)^2)]*Sqrt[-a - b*x^3])
```

Rubi [A] time = 0.562925, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{3\sqrt{3}\sqrt{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) + 2^(1/3)*b^(1/3)*x))/Sqrt[-
a - b*x^3]])/(3*Sqrt[3]*Sqrt[a]*b^(1/3)) + (2*2^(1/3)*Sqrt[2 - Sq
rt[3]]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[Arc
Sin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) +
b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*a^(1/3)*b^(1/3)*Sqrt[-((
a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x
)^2)]*Sqrt[-a - b*x^3])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(2**(2/3)*a**(1/3)+b**(1/3)*x)/(-b*x**3-a)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.267529, size = 167, normalized size = 0.57

$$-\frac{2i\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\,\Pi\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)}{\left(\sqrt[3]{-1}+2^{2/3}\right)\sqrt[3]{b}\sqrt{-a-bx^3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] ((-2*I)*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sq
rt[1 - (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(I
*Sqrt[3])/((-1)^(1/3) + 2^(2/3)), ArcSin[Sqrt[(a^(1/3) + (-1)^(2/
3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/(((-1)^(
1/3) + 2^(2/3))*b^(1/3)*Sqrt[-a - b*x^3])
```

Maple [F] time = 0.076, size = 0, normalized size = 0.

$$\int 1\left(2^{\frac{2}{3}}\sqrt[3]{a}+\sqrt[3]{b}x\right)^{-1}\frac{1}{\sqrt{-bx^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

```
[Out] int(1/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-bx^3-a}\left(b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(-b*x^3 - a)*(b^(1/3)*x + 2^(2/3)*a^(1/3))),x, algorithm="maxim
```

```
[Out] integrate(1/(sqrt(-b*x^3 - a)*(b^(1/3)*x + 2^(2/3)*a^(1/3))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(-b*x^3 - a)*(b^(1/3)*x + 2^(2/3)*a^(1/3))),x, algorithm="frica
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-a-bx^3}\left(2^{\frac{2}{3}}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(2**(2/3)*a**(1/3)+b**(1/3)*x)/(-b*x**3-a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-a - b*x**3)*(2**(2/3)*a**(1/3) + b**(1/3)*x)),
x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(-b*x^3 - a)*(b^(1/3)*x + 2^(2/3)*a^(1/3))),x, algorithm="giac'
```

```
[Out] Exception raised: RuntimeError
```

3.9 $\int \frac{1}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx$

Optimal. Leaf size=249

$$\frac{2\tan^{-1}\left(\frac{\sqrt{3}\sqrt{c}(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{3\sqrt{3}c^{3/2}d}+\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(c+2^{2/3}dx\right)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)c+2^{2/3}dx}{\left(1+\sqrt{3}\right)c+2^{2/3}dx}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}cd\sqrt{\frac{c\left(c+2^{2/3}dx\right)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\sqrt{c^3+4d^3x^3}}$$

```
[Out] (2*ArcTan[(Sqrt[3]*Sqrt[c]*(c + 2*d*x))/Sqrt[c^3 + 4*d^3*x^3]])/(
3*Sqrt[3]*c^(3/2)*d) + (2*2^(1/3)*Sqrt[2 + Sqrt[3]]*(c + 2^(2/3)*
d*x)*Sqrt[(c^2 - 2^(2/3)*c*d*x + 2*2^(1/3)*d^2*x^2)/((1 + Sqrt[3]
)*c + 2^(2/3)*d*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c + 2^(2/3)
*d*x)/((1 + Sqrt[3])*c + 2^(2/3)*d*x)], -7 - 4*Sqrt[3]])/(3*3^(1/
4)*c*d*Sqrt[(c*(c + 2^(2/3)*d*x))/((1 + Sqrt[3])*c + 2^(2/3)*d*x)
^2]*Sqrt[c^3 + 4*d^3*x^3])
```

Rubi [A] time = 0.469822, antiderivative size = 249, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$\frac{2\tan^{-1}\left(\frac{\sqrt{3}\sqrt{c}(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{3\sqrt{3}c^{3/2}d}+\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(c+2^{2/3}dx\right)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)c+2^{2/3}dx}{\left(1+\sqrt{3}\right)c+2^{2/3}dx}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}cd\sqrt{\frac{c\left(c+2^{2/3}dx\right)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\sqrt{c^3+4d^3x^3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((c + d*x)*Sqrt[c^3 + 4*d^3*x^3]),x]
```


```
[Out] (2*ArcTan[(Sqrt[3]*Sqrt[c]*(c + 2*d*x))/Sqrt[c^3 + 4*d^3*x^3]])/(
3*Sqrt[3]*c^(3/2)*d) + (2*2^(1/3)*Sqrt[2 + Sqrt[3]]*(c + 2^(2/3)*
d*x)*Sqrt[(c^2 - 2^(2/3)*c*d*x + 2*2^(1/3)*d^2*x^2)/((1 + Sqrt[3]
)*c + 2^(2/3)*d*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c + 2^(2/3)
*d*x)/((1 + Sqrt[3])*c + 2^(2/3)*d*x)], -7 - 4*Sqrt[3]])/(3*3^(1/
4)*c*d*Sqrt[(c*(c + 2^(2/3)*d*x))/((1 + Sqrt[3])*c + 2^(2/3)*d*x)
^2]*Sqrt[c^3 + 4*d^3*x^3])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(d*x+c)/(4*d**3*x**3+c**3)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.333141, size = 169, normalized size = 0.68

$$-\frac{i2^{5/6}\sqrt{\frac{\sqrt[3]{2}c+2dx}{\left(1+\sqrt[3]{-1}\right)c}}\sqrt{\frac{4d^2x^2}{c^2}-\frac{2\sqrt[3]{2}dx}{c}+2^{2/3}}\,\Pi\left(\frac{i\sqrt[3]{2}\sqrt{3}}{2+\sqrt[3]{-2}};\sin^{-1}\left(\frac{\sqrt{\frac{\sqrt[3]{2}c+2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}}{\sqrt[6]{2}}\right)|\sqrt[3]{-1}\right)}{\left(2+\sqrt[3]{-2}\right)d\sqrt{c^3+4d^3x^3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((c + d*x)*Sqrt[c^3 + 4*d^3*x^3]),x]
```

```
[Out] ((-I)*2^(5/6)*Sqrt[(2^(1/3)*c + 2*d*x)/((1 + (-1)^(1/3))*c)]*Sqrt
[2^(2/3) - (2*2^(1/3)*d*x)/c + (4*d^2*x^2)/c^2]*EllipticPi[(I*2^(
1/3)*Sqrt[3])/(2 + (-2)^(1/3)), ArcSin[Sqrt[(2^(1/3)*c + 2*(-1)^(
2/3)*d*x)/((1 + (-1)^(1/3))*c)]/2^(1/6)], (-1)^(1/3)])/((2 + (-2)
^(1/3))*d*Sqrt[c^3 + 4*d^3*x^3])
```

Maple [B] time = 0.259, size = 495, normalized size = 2.

$$2\frac{1}{d\sqrt{4\,d^3x^3+c^3}}\left(\frac{\left(1/4\,\sqrt[3]{2}-i/4\sqrt{3}\sqrt[3]{2}\right)c}{d}-\frac{\left(1/4\,\sqrt[3]{2}+i/4\sqrt{3}\sqrt[3]{2}\right)c}{d}\right)\sqrt{1\left(x-\frac{\left(1/4\,\sqrt[3]{2}+i/4\sqrt{3}\sqrt[3]{2}\right)c}{d}\right)\left(\frac{\left(1/4\,\sqrt[3]{2}-i/4\sqrt{3}\sqrt[3]{2}\right.}{d}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x)
```

```
[Out] 2/d*((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3
^(1/2)*2^(1/3))*c/d)*((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)
/((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1
/2)*2^(1/3))*c/d))^(1/2)*((x+1/2*2^(1/3)*c/d)/((1/4*2^(1/3)+1/4*I
*3^(1/2)*2^(1/3))*c/d+1/2*2^(1/3)*c/d))^(1/2)*((x-(1/4*2^(1/3)-1/
```

```
4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/
d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)/(4*d^3*x^3+c^3)
^(1/2)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+c/d)*EllipticPi((
(x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)-1/4*I*3
^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/
2),((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^
(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+c/d)
,(((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(
1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+1/2*2
^(1/3)*c/d))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{4\,d^3x^3 + c^3}(dx + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{4\,d^3x^3 + c^3}(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c + dx)\sqrt{c^3 + 4d^3x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(d*x+c)/(4*d**3*x**3+c**3)**(1/2),x)

[Out] Integral(1/((c + d*x)*sqrt(c**3 + 4*d**3*x**3)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{4\,d^3x^3 + c^3}(dx + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)),x, algorithm="giac")

[Out] integrate(1/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)), x)
```

3.10 $\int \frac{1}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=146

$$\frac{\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}+\frac{\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

```
[Out] ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]]/Sqrt[3*(3 + 2
*Sqrt[3])] + (Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + S
qrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] +
x)], -7 - 4*Sqrt[3]])/(3^(3/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*
Sqrt[1 + x^3])
```

Rubi [A] time = 0.365627, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}+\frac{\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]]/Sqrt[3*(3 + 2
*Sqrt[3])] + (Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + S
qrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] +
x)], -7 - 4*Sqrt[3]])/(3^(3/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*
Sqrt[1 + x^3])
```

Rubi in Sympy [A] time = 9.53778, size = 78, normalized size = 0.53

$$\frac{2\tilde{\infty}\sqrt{\frac{x^2-x+1}{\left(x+1+\sqrt{3}\right)^2}}\,(x+1)\,F\left(\operatorname{asin}\left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1+x+3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] 2*zoo*sqrt((x**2 - x + 1)/(x + 1 + sqrt(3))**2)*(x + 1)*elliptic_
f(asin((x - sqrt(3) + 1)/(x + 1 + sqrt(3))), -7 - 4*sqrt(3))/(sqr
t((x + 1)/(x + 1 + sqrt(3))**2)*sqrt(x**3 + 1))
```

Mathematica [C] time = 0.199474, size = 136, normalized size = 0.93

$$-\frac{4\sqrt{2}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\sqrt{x^2-x+1}\left(\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(3i+(1+2i)\sqrt{3}\right)\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (-4*Sqrt[2]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*Sqrt[1 - x + x^2]*E
llipticPi[(2*Sqrt[3])/(3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[I +
Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3]
)])/((3*I + (1 + 2*I)*Sqrt[3])*Sqrt[1 + x^3])
```

Maple [A] time = 0.066, size = 132, normalized size = 0.9

$$\frac{\left(3-i\sqrt{3}\right)\sqrt{3}}{3}\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i}{2}\sqrt{3}}}\sqrt{\frac{1}{-\frac{3}{2}-\frac{i}{2}\sqrt{3}}\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)}\sqrt{\frac{1}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)}EllipticPi\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i}{2}\sqrt{3}}},\frac{\left(-\frac{3}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1+x+3^(1/2))/(x^3+1)^(1/2),x)
```

```
[Out] 2/3*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2
-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2)
)/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*3^(1/2)*EllipticPi(((
1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2),
((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^3+1}\left(x+\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{x^3+1}\left(x+\sqrt{3}+1\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(x+1)\left(x^2-x+1\right)}\left(x+1+\sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1+x+3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x + 1)*(x**2 - x + 1))*(x + 1 + sqrt(3))), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^3+1}\left(x+\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)),x, algorithm="giac")

[Out] integrate(1/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)), x)
```

3.11 $\int \frac{1}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=164

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}-\frac{\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

```
[Out] -(ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[1 - x^3]]/Sqrt[3*(3 +
 2*Sqrt[3])]) - (Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1
+ Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3]
 - x)], -7 - 4*Sqrt[3]])/(3^(3/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^
2]*Sqrt[1 - x^3])
```

Rubi [A] time = 0.349483, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}-\frac{\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((1 + Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

```
[Out] -(ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[1 - x^3]]/Sqrt[3*(3 +
 2*Sqrt[3])]) - (Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1
+ Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3]
 - x)], -7 - 4*Sqrt[3]])/(3^(3/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^
2]*Sqrt[1 - x^3])
```

Rubi in Sympy [A] time = 13.137, size = 78, normalized size = 0.48

$$\frac{2\tilde{\infty}\sqrt{\frac{x^2+x+1}{\left(-x+1+\sqrt{3}\right)^2}}(-x+1)F\left(\operatorname{asin}\left(\frac{-x-\sqrt{3}+1}{-x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt{\frac{-x+1}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{-x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1-x+3**(1/2))/(-x**3+1)**(1/2),x)
```

```
[Out] 2*zoo*sqrt((x**2 + x + 1)/(-x + 1 + sqrt(3))**2)*(-x + 1)*ellipti
c_f(asin((-x - sqrt(3) + 1)/(-x + 1 + sqrt(3))), -7 - 4*sqrt(3))/
(sqrt((-x + 1)/(-x + 1 + sqrt(3))**2)*sqrt(-x**3 + 1))
```

Mathematica [C] time = 0.160601, size = 136, normalized size = 0.83

$$\frac{4\sqrt{2}\sqrt{-\frac{i(x-1)}{\sqrt{3}+3i}}\sqrt{x^2+x+1}\left(\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(3i+(1+2i)\sqrt{3}\right)\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((1 + Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (4*Sqrt[2]*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*Sqrt[1 + x + x^2
]*EllipticPi[(2*Sqrt[3])/(3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[I
 + Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt
[3])])/((3*I + (1 + 2*I)*Sqrt[3])*Sqrt[1 - x^3])
```

Maple [A] time = 0.092, size = 143, normalized size = 0.9

$$\frac{\frac{2i}{3}\sqrt{3}}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}-\sqrt{3}}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\mathit{EllipticPi}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\frac{i\sqrt{}}{-\frac{3}{2}+\frac{i}{2}\sqrt{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1-x+3^(1/2))/(-x^3+1)^(1/2),x)
```

```
[Out] 2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/(-3
/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)
/(-x^3+1)^(1/2)/(-3/2+1/2*I*3^(1/2)-3^(1/2))*EllipticPi(1/3*3^(1/
2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-3/2+1/2*I*
3^(1/2)-3^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{1}{\sqrt{-x^3+1}\left(x-\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(sqrt(-x^3 + 1)*(x - sqrt(3) - 1)),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(-x^3 + 1)*(x - sqrt(3) - 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{1}{\sqrt{-x^3+1}\left(x-\sqrt{3}-1\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(sqrt(-x^3 + 1)*(x - sqrt(3) - 1)),x, algorithm="fricas")
```

```
[Out] integral(-1/(sqrt(-x^3 + 1)*(x - sqrt(3) - 1)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{x\sqrt{-x^3+1}-\sqrt{3}\sqrt{-x^3+1}-\sqrt{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1-x+3**(1/2))/(-x**3+1)**(1/2),x)
```

```
[Out] -Integral(1/(x*sqrt(-x**3 + 1) - sqrt(3)*sqrt(-x**3 + 1) - sqrt(-
x**3 + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\sqrt{-x^3+1}\left(x-\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(sqrt(-x^3 + 1)*(x - sqrt(3) - 1)),x, algorithm="giac")
```

```
[Out] integrate(-1/(sqrt(-x^3 + 1)*(x - sqrt(3) - 1)), x)
```

3.12 $\int \frac{1}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=167

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{x^3-1}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}-\frac{\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

```
[Out] -(ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[-1 + x^3]]/Sqrt[3*(3
 + 2*Sqrt[3])]) - (Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(
1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[
3] - x)], -7 + 4*Sqrt[3]])/(3^(3/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] -
 x)^2)]*Sqrt[-1 + x^3])
```

Rubi [A] time = 0.319768, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{x^3-1}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}-\frac{\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((1 + Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] -(ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[-1 + x^3]]/Sqrt[3*(3
 + 2*Sqrt[3])]) - (Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(
1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[
3] - x)], -7 + 4*Sqrt[3]])/(3^(3/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] -
 x)^2)]*Sqrt[-1 + x^3])
```

Rubi in Sympy [A] time = 42.3303, size = 226, normalized size = 1.35

$$-\frac{\sqrt[4]{3}\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,(-x+1)\,\mathrm{atanh}\left(\frac{\left(\sqrt{3}+2\right)\sqrt{-\frac{\left(-x+1+\sqrt{3}\right)^2}{\left(x-1+\sqrt{3}\right)^2}+1}}{\sqrt{\frac{\left(-x+1+\sqrt{3}\right)^2}{\left(x-1+\sqrt{3}\right)^2}+4\sqrt{3}+7}}\right)}{3\sqrt{\frac{x-1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{\sqrt{3}+2}\sqrt{x^3-1}}-\frac{\sqrt[4]{3}\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{-\sqrt{3}+2}\,(-x+1)\,F\left(\mathrm{asin}\left(\frac{-x+1+\sqrt{3}}{-x-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{3\sqrt{\frac{x-1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1-x+3**(1/2))/(x**3-1)**(1/2),x)
```

```
[Out] -3**(1/4)*sqrt((x**2 + x + 1)/(-x - sqrt(3) + 1)**2)*(-x + 1)*ata
nh((sqrt(3) + 2)*sqrt(-(-x + 1 + sqrt(3))**2/(x - 1 + sqrt(3))**2
 + 1)/sqrt((-x + 1 + sqrt(3))**2/(x - 1 + sqrt(3))**2 + 4*sqrt(3)
 + 7))/(3*sqrt((x - 1)/(-x - sqrt(3) + 1)**2)*sqrt(sqrt(3) + 2)*s
qrt(x**3 - 1)) - 3**(1/4)*sqrt((x**2 + x + 1)/(-x - sqrt(3) + 1)*
*2)*sqrt(-sqrt(3) + 2)*(-x + 1)*elliptic_f(asin((-x + 1 + sqrt(3)
)/(-x - sqrt(3) + 1)), -7 + 4*sqrt(3))/(3*sqrt((x - 1)/(-x - sqrt
(3) + 1)**2)*sqrt(x**3 - 1))
```

Mathematica [C] time = 0.18891, size = 134, normalized size = 0.8

$$\frac{4\sqrt{2}\sqrt{-\frac{i(x-1)}{\sqrt{3}+3i}}\sqrt{x^2+x+1}\left(\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(3i+(1+2i)\sqrt{3}\right)\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((1 + Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (4*Sqrt[2]*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*Sqrt[1 + x + x^2
]*EllipticPi[(2*Sqrt[3])/(3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[I
 + Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt
[3])])/((3*I + (1 + 2*I)*Sqrt[3])*Sqrt[-1 + x^3])
```

Maple [A] time = 0.052, size = 132, normalized size = 0.8

$$\frac{\left(-3-i\sqrt{3}\right)\sqrt{3}}{3}\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i}{2}\sqrt{3}}}\sqrt{\frac{1}{\frac{3}{2}-\frac{i}{2}\sqrt{3}}\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)}\sqrt{\frac{1}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)}EllipticPi\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i}{2}\sqrt{3}}},-\frac{\left(\frac{3}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{}}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1-x+3^(1/2))/(x^3-1)^(1/2),x)
```

```
[Out] 2/3*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+
1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/
2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*3^(1/2)*EllipticPi((
(-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(3/2+1/2*I*3^(1/2))*3^(1/
2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\sqrt{x^3-1}\left(x-\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(sqrt(x^3 - 1)*(x - sqrt(3) - 1)),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(x^3 - 1)*(x - sqrt(3) - 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{1}{\sqrt{x^3-1}\left(x-\sqrt{3}-1\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(sqrt(x^3 - 1)*(x - sqrt(3) - 1)),x, algorithm="fricas")
```

```
[Out] integral(-1/(sqrt(x^3 - 1)*(x - sqrt(3) - 1)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{x\sqrt{x^3-1}-\sqrt{3}\sqrt{x^3-1}-\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1-x+3**(1/2))/(x**3-1)**(1/2),x)
```

```
[Out] -Integral(1/(x*sqrt(x**3 - 1) - sqrt(3)*sqrt(x**3 - 1) - sqrt(x**
3 - 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\sqrt{x^3-1}\left(x-\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(sqrt(x^3 - 1)*(x - sqrt(3) - 1)),x, algorithm="giac")
```

```
[Out] integrate(-1/(sqrt(x^3 - 1)*(x - sqrt(3) - 1)), x)
```

3.13 $\int \frac{1}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=157

$$\frac{\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(x+1)}{\sqrt{-x^3-1}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}+\frac{\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

```
[Out] ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[-1 - x^3]]/Sqrt[3*(3 +
 2*Sqrt[3])] + (Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 -
 Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3]
+ x)], -7 + 4*Sqrt[3]])/(3^(3/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)
^2)]*Sqrt[-1 - x^3])
```

Rubi [A] time = 0.310324, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(x+1)}{\sqrt{-x^3-1}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}+\frac{\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((1 + Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[-1 - x^3]]/Sqrt[3*(3 +
 2*Sqrt[3])] + (Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 -
 Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3]
+ x)], -7 + 4*Sqrt[3]])/(3^(3/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)
^2)]*Sqrt[-1 - x^3])
```

Rubi in Sympy [A] time = 38.6366, size = 231, normalized size = 1.47

$$\frac{\sqrt[4]{3}\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}(x+1)\operatorname{atanh}\left(\frac{\sqrt{1-\frac{\left(x+1+\sqrt{3}\right)^2}{\left(-x-1+\sqrt{3}\right)^2}}\left(\sqrt{3}+2\right)}{\sqrt{4\sqrt{3}+7+\frac{\left(x+1+\sqrt{3}\right)^2}{\left(-x-1+\sqrt{3}\right)^2}}}\right)}{3\sqrt{\frac{-x-1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{\sqrt{3}+2}\sqrt{-x^3-1}}+\frac{\sqrt[4]{3}\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-\sqrt{3}+2}\,(x+1)\,F\left(\operatorname{asin}\left(\frac{x+1+\sqrt{3}}{x-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{3\sqrt{\frac{-x-1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1+x+3**(1/2))/(-x**3-1)**(1/2),x)
```

```
[Out] 3**(1/4)*sqrt((x**2 - x + 1)/(x - sqrt(3) + 1)**2)*(x + 1)*atanh(
sqrt(1 - (x + 1 + sqrt(3))**2/(-x - 1 + sqrt(3))**2)*(sqrt(3) + 2
)/sqrt(4*sqrt(3) + 7 + (x + 1 + sqrt(3))**2/(-x - 1 + sqrt(3))**2
))/(3*sqrt((-x - 1)/(x - sqrt(3) + 1)**2)*sqrt(sqrt(3) + 2)*sqrt(
-x**3 - 1)) + 3**(1/4)*sqrt((x**2 - x + 1)/(x - sqrt(3) + 1)**2)*
sqrt(-sqrt(3) + 2)*(x + 1)*elliptic_f(asin((x + 1 + sqrt(3))/(x -
 sqrt(3) + 1)), -7 + 4*sqrt(3))/(3*sqrt((-x - 1)/(x - sqrt(3) + 1
)**2)*sqrt(-x**3 - 1))
```

Mathematica [C] time = 0.16232, size = 138, normalized size = 0.88

$$-\frac{4\sqrt{2}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\sqrt{x^2-x+1}\,\Pi\left(\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)\middle|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(3i+(1+2i)\sqrt{3}\right)\sqrt{-x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((1 + Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (-4*Sqrt[2]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*Sqrt[1 - x + x^2]*E
llipticPi[(2*Sqrt[3])/(3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[I +
Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3]
)])/((3*I + (1 + 2*I)*Sqrt[3])*Sqrt[-1 - x^3])
```

Maple [A] time = 0.098, size = 139, normalized size = 0.9

$$\frac{-\frac{2i}{3}\sqrt{3}}{\frac{3}{2}+\frac{i}{2}\sqrt{3}+\sqrt{3}}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}EllipticPi\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i}{2}\sqrt{3}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1+x+3^(1/2))/(-x^3-1)^(1/2),x)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/
2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/
(-x^3-1)^(1/2)/(3/2+1/2*I*3^(1/2)+3^(1/2))*EllipticPi(1/3*3^(1/2)
*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(3/2+1/2*I*3^(
1/2)+3^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^3-1}\left(x+\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(-x^3 - 1)*(x + sqrt(3) + 1)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^3 - 1)*(x + sqrt(3) + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{-x^3-1}\left(x+\sqrt{3}+1\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(-x^3 - 1)*(x + sqrt(3) + 1)),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(-x^3 - 1)*(x + sqrt(3) + 1)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-(x+1)(x^2-x+1)}\left(x+1+\sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1+x+3**(1/2))/(-x**3-1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 1 + sqrt(3))), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^3-1}\left(x+\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-x^3 - 1)*(x + sqrt(3) + 1)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-x^3 - 1)*(x + sqrt(3) + 1)), x)
```

3.14 $\int \frac{1}{(3+x)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=331

$$\frac{(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\tan^{-1}\left(\frac{\sqrt{\frac{13}{2}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}}{\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}}\right)}{\sqrt{26}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}+\frac{2\sqrt{26+15\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}+\frac{4\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(97-56\sqrt{3};-\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

```
[Out] ((1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*ArcTan[(Sqrt[13/
2]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2])/Sqrt[(1 - x + x^2)/(1 + Sqr
t[3] + x)^2]])/(Sqrt[26]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1
 + x^3]) + (2*Sqrt[26 + 15*Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1
 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3
] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)
^2]*Sqrt[1 + x^3]) + (4*3^(1/4)*(1 + x)*Sqrt[(1 - x + x^2)/(1 + S
qrt[3] + x)^2]*EllipticPi[97 - 56*Sqrt[3], -ArcSin[(1 - Sqrt[3] +
 x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[2 - Sqrt[3]]*Sqrt[
(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rubi [A] time = 1.36334, antiderivative size = 331, normalized size of antiderivative = 1., number of

steps used = 8, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533

$$\frac{(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\tan^{-1}\left(\frac{\sqrt{\frac{13}{2}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}}{\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}}\right)}{\sqrt{26}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}} + \frac{2\sqrt{26+15\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}} + \frac{4\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(97-56\sqrt{3};-\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((3 + x)*Sqrt[1 + x^3]),x]
```

```
[Out] ((1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*ArcTan[(Sqrt[13/
2]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2])/Sqrt[(1 - x + x^2)/(1 + Sqr
t[3] + x)^2]])/(Sqrt[26]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1
 + x^3]) + (2*Sqrt[26 + 15*Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1
 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3
] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)
^2]*Sqrt[1 + x^3]) + (4*3^(1/4)*(1 + x)*Sqrt[(1 - x + x^2)/(1 + S
qrt[3] + x)^2]*EllipticPi[97 - 56*Sqrt[3], -ArcSin[(1 - Sqrt[3] +
 x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[2 - Sqrt[3]]*Sqrt[
(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rubi in Sympy [A] time = 94.0925, size = 369, normalized size = 1.11

$$\frac{\sqrt{26}\sqrt{\frac{x^2-x+1}{\left(x+1+\sqrt{3}\right)^2}}\,(x+1)\operatorname{atan}\left(\frac{\sqrt{26}\cdot 3^{\frac{3}{4}}\sqrt{-\sqrt{3}+2}\sqrt{-\frac{\left(-x-1+\sqrt{3}\right)^2}{\left(x+1+\sqrt{3}\right)^2}+1}}{6\sqrt{\frac{\left(-x-1+\sqrt{3}\right)^2}{\left(x+1+\sqrt{3}\right)^2}-4\sqrt{3}+7}}\right)}{26\sqrt{\frac{x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{x^3+1}} + \frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{x^2-x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{\sqrt{3}+2}\,(x+1)\,F\left(\operatorname{asin}\left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{3\sqrt{\frac{x+1}{\left(x+1+\sqrt{3}\right)^2}}\left(-\sqrt{3}+2\right)\sqrt{x^3+1}} + \frac{4\sqrt[4]{3}\sqrt{\frac{x^2-x+1}{\left(x+1+\sqrt{3}\right)^2}}\,(x+1)\,\Pi\left(\frac{\left(-2+\sqrt{3}\right)^2}{\left(\sqrt{3}+2\right)^2};\operatorname{asin}\left(\frac{-x-1+\sqrt{3}}{x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{-4\sqrt{3}+7}\sqrt{-\sqrt{3}+2}\left(\sqrt{3}+2\right)\sqrt{x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(3+x)/(x**3+1)**(1/2),x)
```

```
[Out] sqrt(26)*sqrt((x**2 - x + 1)/(x + 1 + sqrt(3))**2)*(x + 1)*atan(s
qrt(26)*3**(3/4)*sqrt(-sqrt(3) + 2)*sqrt(-(-x - 1 + sqrt(3))**2/(
x + 1 + sqrt(3))**2 + 1)/(6*sqrt((-x - 1 + sqrt(3))**2/(x + 1 + s
qrt(3))**2 - 4*sqrt(3) + 7)))/(26*sqrt((x + 1)/(x + 1 + sqrt(3))*
*2)*sqrt(x**3 + 1)) + 2*3**(3/4)*sqrt((x**2 - x + 1)/(x + 1 + sqr
t(3))**2)*sqrt(sqrt(3) + 2)*(x + 1)*elliptic_f(asin((x - sqrt(3)
+ 1)/(x + 1 + sqrt(3))), -7 - 4*sqrt(3))/(3*sqrt((x + 1)/(x + 1 +
 sqrt(3))**2)*(-sqrt(3) + 2)*sqrt(x**3 + 1)) + 4*3**(1/4)*sqrt((x
**2 - x + 1)/(x + 1 + sqrt(3))**2)*(x + 1)*elliptic_pi((-2 + sqrt
(3))**2/(sqrt(3) + 2)**2, asin((-x - 1 + sqrt(3))/(x + 1 + sqrt(3
))), -7 - 4*sqrt(3))/(sqrt((x + 1)/(x + 1 + sqrt(3))**2)*sqrt(-4*
sqrt(3) + 7)*sqrt(-sqrt(3) + 2)*(sqrt(3) + 2)*sqrt(x**3 + 1))
```

Mathematica [C] time = 0.0838458, size = 128, normalized size = 0.39

$$-\frac{4\sqrt{2}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\sqrt{x^2-x+1}\,\Pi\left(\frac{2\sqrt{3}}{7i+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(\sqrt{3}+7i\right)\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((3 + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (-4*Sqrt[2]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*Sqrt[1 - x + x^2]*E
llipticPi[(2*Sqrt[3])/(7*I + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] -
(2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])])/((7*I
+ Sqrt[3])*Sqrt[1 + x^3])
```

Maple [A] time = 0.028, size = 123, normalized size = 0.4

$$(\frac{3}{2}-\frac{i}{2}\sqrt{3})\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i}{2}\sqrt{3}}}\sqrt{\frac{1}{-\frac{3}{2}-\frac{i}{2}\sqrt{3}}\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)}\sqrt{\frac{1}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)}\,EllipticPi\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i}{2}\sqrt{3}}},-\frac{3}{4}+\frac{i}{4}\sqrt{3},\sqrt{\frac{-\frac{3}{2}+}{-\frac{3}{2}-}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(3+x)/(x^3+1)^(1/2),x)
```

```
[Out] (3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2
*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-
3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticPi(((1+x)/(3/2-1/
2*I*3^(1/2)))^(1/2),-3/4+1/4*I*3^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/
2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^3+1}(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^3 + 1)*(x + 3)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^3 + 1)*(x + 3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{x^3+1}(x+3)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^3 + 1)*(x + 3)),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(x^3 + 1)*(x + 3)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(x+1)(x^2-x+1)}\,(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3+x)/(x**3+1)**(1/2),x)

[Out] Integral(1/(sqrt((x + 1)*(x**2 - x + 1))*(x + 3)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^3+1}(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^3 + 1)*(x + 3)),x, algorithm="giac")

[Out] integrate(1/(sqrt(x^3 + 1)*(x + 3)), x)
```

3.15 $\int \frac{1}{(3+x)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=382

$$-\frac{(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\tanh^{-1}\left(\frac{\sqrt{7}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}}{2\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}}\right)}{2\sqrt{7}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}+\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\Pi\left(\frac{1}{169}\left(553+304\sqrt{3}\right);-\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{13\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

```
[Out] -((1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*ArcTanh[(Sqrt[7
]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2])/(2*Sqrt[(1 + x + x^2)/(1 + S
qrt[3] - x)^2])])/(2*Sqrt[7]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sq
rt[1 - x^3]) - (2*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1
 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3
] - x)], -7 - 4*Sqrt[3]])/(3^(1/4)*(4 + Sqrt[3])*Sqrt[(1 - x)/(1
+ Sqrt[3] - x)^2]*Sqrt[1 - x^3]) + (4*3^(1/4)*Sqrt[2 + Sqrt[3]]*(
1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticPi[(553 +
304*Sqrt[3])/169, -ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -
7 - 4*Sqrt[3]])/(13*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^
3])
```

Rubi [A] time = 1.50548, antiderivative size = 382, normalized size of antiderivative = 1., number of

steps used = 8, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471

$$-\frac{(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\tanh^{-1}\left(\frac{\sqrt{7}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}}{2\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}}\right)}{2\sqrt{7}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}} - \frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}} + \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(\frac{1}{169}\left(553+304\sqrt{3}\right);-\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{13\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((3 + x)*Sqrt[1 - x^3]),x]
```

```
[Out] -((1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*ArcTanh[(Sqrt[7
]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2])/(2*Sqrt[(1 + x + x^2)/(1 + S
qrt[3] - x)^2])])/(2*Sqrt[7]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sq
rt[1 - x^3]) - (2*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1
 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3
] - x)], -7 - 4*Sqrt[3]])/(3^(1/4)*(4 + Sqrt[3])*Sqrt[(1 - x)/(1
+ Sqrt[3] - x)^2]*Sqrt[1 - x^3]) + (4*3^(1/4)*Sqrt[2 + Sqrt[3]]*(
1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticPi[(553 +
304*Sqrt[3])/169, -ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -
7 - 4*Sqrt[3]])/(13*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^
3])
```

Rubi in Sympy [A] time = 93.387, size = 376, normalized size = 0.98

$$-\frac{\sqrt{7}\sqrt{\frac{x^2+x+1}{\left(-x+1+\sqrt{3}\right)^2}}\,(-x+1)\,\mathrm{atanh}\left(\frac{3^{\frac{3}{4}}\sqrt{7}\sqrt{1-\frac{\left(x-1+\sqrt{3}\right)^2}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{-\sqrt{3}+2}}{6\sqrt{-4\sqrt{3}+7+\frac{\left(x-1+\sqrt{3}\right)^2}{\left(-x+1+\sqrt{3}\right)^2}}}\right)}{14\sqrt{\frac{-x+1}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{-x^3+1}} - \frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{x^2+x+1}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{\sqrt{3}+2}\,(-x+1)\,F\left(\mathrm{asin}\left(\frac{-x-\sqrt{3}+1}{-x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{3\sqrt{\frac{-x+1}{\left(-x+1+\sqrt{3}\right)^2}}\left(\sqrt{3}+4\right)\sqrt{-x^3+1}} + \frac{4\sqrt[4]{3}\sqrt{\frac{x^2+x+1}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{-\sqrt{3}+2}\,(-x+1)\left(\frac{\left(\sqrt{3}+4\right)^2}{\left(-4+\sqrt{3}\right)^2};\mathrm{asin}\left(\frac{x-1+\sqrt{3}}{-x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt{\frac{-x+1}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{-4\sqrt{3}+7}\left(-\sqrt{3}+4\right)\left(\sqrt{3}+4\right)\sqrt{-x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(3+x)/(-x**3+1)**(1/2),x)
```

```
[Out] -sqrt(7)*sqrt((x**2 + x + 1)/(-x + 1 + sqrt(3))**2)*(-x + 1)*atan
h(3**(3/4)*sqrt(7)*sqrt(1 - (x - 1 + sqrt(3))**2/(-x + 1 + sqrt(3
))**2)*sqrt(-sqrt(3) + 2)/(6*sqrt(-4*sqrt(3) + 7 + (x - 1 + sqrt(
3))**2/(-x + 1 + sqrt(3))**2)))/(14*sqrt((-x + 1)/(-x + 1 + sqrt(
3))**2)*sqrt(-x**3 + 1)) - 2*3**(3/4)*sqrt((x**2 + x + 1)/(-x + 1
 + sqrt(3))**2)*sqrt(sqrt(3) + 2)*(-x + 1)*elliptic_f(asin((-x -
sqrt(3) + 1)/(-x + 1 + sqrt(3))), -7 - 4*sqrt(3))/(3*sqrt((-x + 1
)/(-x + 1 + sqrt(3))**2)*(sqrt(3) + 4)*sqrt(-x**3 + 1)) + 4*3**(1
/4)*sqrt((x**2 + x + 1)/(-x + 1 + sqrt(3))**2)*sqrt(-sqrt(3) + 2)
*(-x + 1)*elliptic_pi((sqrt(3) + 4)**2/(-4 + sqrt(3))**2, asin((x
 - 1 + sqrt(3))/(-x + 1 + sqrt(3))), -7 - 4*sqrt(3))/(sqrt((-x +
1)/(-x + 1 + sqrt(3))**2)*sqrt(-4*sqrt(3) + 7)*(-sqrt(3) + 4)*(sq
rt(3) + 4)*sqrt(-x**3 + 1))
```

Mathematica [C] time = 0.0937006, size = 128, normalized size = 0.34

$$-\frac{4\sqrt{2}\sqrt{\frac{i(x-1)}{\sqrt{3}-3i}}\sqrt{x^2+x+1}\,\Pi\left(\frac{2\sqrt{3}}{5i+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}-i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right)}{\left(\sqrt{3}+5i\right)\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((3 + x)*Sqrt[1 - x^3]),x]
```

```
[Out] (-4*Sqrt[2]*Sqrt[(I*(-1 + x))/(-3*I + Sqrt[3])]*Sqrt[1 + x + x^2]
*EllipticPi[(2*Sqrt[3])/(5*I + Sqrt[3]), ArcSin[Sqrt[-I + Sqrt[3]
 - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])])/((
5*I + Sqrt[3])*Sqrt[1 - x^3])
```

Maple [A] time = 0.072, size = 133, normalized size = 0.4

$$\frac{-\frac{2i}{3}\sqrt{3}}{\frac{5}{2}+\frac{i}{2}\sqrt{3}}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{3}EllipticPi\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\frac{i\sqrt{3}}{\frac{5}{2}+\frac{i}{2}\sqrt{3}},\sqrt{\frac{i}{-\frac{3}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(3+x)/(-x^3+1)^(1/2),x)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/(-
3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2
)/(-x^3+1)^(1/2)/(5/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x
+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(5/2+1/2*I*3^(1/2)),
(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^3+1}(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-x^3 + 1)*(x + 3)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^3 + 1)*(x + 3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{-x^3+1}(x+3)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-x^3 + 1)*(x + 3)),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(-x^3 + 1)*(x + 3)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-(x-1)(x^2+x+1)}(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3+x)/(-x**3+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(x - 1)*(x**2 + x + 1))*(x + 3)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^3+1}(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-x^3 + 1)*(x + 3)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-x^3 + 1)*(x + 3)), x)
```

3.16 $\int \frac{1}{(3+x)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=376

$$-\frac{(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\tanh^{-1}\left(\frac{\sqrt{7}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}}{2\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}}\right)}{2\sqrt{7}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{2\sqrt{62-35\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{13\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}+\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\Pi\left(\frac{1}{169}\left(553+304\sqrt{3}\right);-\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{13\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

```
[Out] -((1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*ArcTanh[(Sqrt[7
]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2])/(2*Sqrt[(1 + x + x^2)/(1 + S
qrt[3] - x)^2])])/(2*Sqrt[7]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sq
rt[-1 + x^3]) - (2*Sqrt[62 - 35*Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^
2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - S
qrt[3] - x)], -7 + 4*Sqrt[3]])/(13*3^(1/4)*Sqrt[-((1 - x)/(1 - Sq
rt[3] - x)^2)]*Sqrt[-1 + x^3]) + (4*3^(1/4)*Sqrt[2 + Sqrt[3]]*(1
- x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticPi[(553 + 30
4*Sqrt[3])/169, -ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7
- 4*Sqrt[3]])/(13*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[-1 + x^3
])
```

Rubi [A] time = 1.21572, antiderivative size = 376, normalized size of antiderivative = 1., number of

steps used = 8, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$

$$-\frac{(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\tanh^{-1}\left(\frac{\sqrt{7}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}}{2\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}}\right)}{2\sqrt{7}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{2\sqrt{62-35\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{13\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}+\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(\frac{1}{169}\left(553+304\sqrt{3}\right);-\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{13\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((3 + x)*Sqrt[-1 + x^3]),x]
```

```
[Out] -((1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*ArcTanh[(Sqrt[7
]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2])/(2*Sqrt[(1 + x + x^2)/(1 + S
qrt[3] - x)^2])])/(2*Sqrt[7]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sq
rt[-1 + x^3]) - (2*Sqrt[62 - 35*Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^
2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - S
qrt[3] - x)], -7 + 4*Sqrt[3]])/(13*3^(1/4)*Sqrt[-((1 - x)/(1 - Sq
rt[3] - x)^2)]*Sqrt[-1 + x^3]) + (4*3^(1/4)*Sqrt[2 + Sqrt[3]]*(1
- x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticPi[(553 + 30
4*Sqrt[3])/169, -ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7
- 4*Sqrt[3]])/(13*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[-1 + x^3
])
```

Rubi in Sympy [A] time = 92.3652, size = 374, normalized size = 0.99

$$-\frac{\sqrt{7}\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}(-x+1)\operatorname{atan}\left(\frac{3^{\frac{3}{4}}\sqrt{7}\sqrt{\sqrt{3}+2}\sqrt{-\frac{\left(-x+1+\sqrt{3}\right)^2}{\left(x-1+\sqrt{3}\right)^2}+1}}{6\sqrt{\frac{\left(-x+1+\sqrt{3}\right)^2}{\left(x-1+\sqrt{3}\right)^2}+4\sqrt{3}+7}}\right)}{14\sqrt{\frac{x-1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}} - \frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{-\sqrt{3}+2}(-x+1)F\left(\operatorname{asin}\left(\frac{-x+1+\sqrt{3}}{-x-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{3\sqrt{\frac{x-1}{\left(-x-\sqrt{3}+1\right)^2}}\left(-\sqrt{3}+4\right)\sqrt{x^3-1}} - \frac{4\sqrt[4]{3}\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{\sqrt{3}+2}(-x+1)\left(\frac{\left(-4+\sqrt{3}\right)^2}{\left(\sqrt{3}+4\right)^2};\operatorname{asin}\left(\frac{-x+1+\sqrt{3}}{x-1+\sqrt{3}}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt{\frac{x-1}{\left(-x-\sqrt{3}+1\right)^2}}\left(-\sqrt{3}+4\right)\left(\sqrt{3}+4\right)\sqrt{4\sqrt{3}+7}\sqrt{x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(3+x)/(x**3-1)**(1/2),x)
```

```
[Out] -sqrt(7)*sqrt((x**2 + x + 1)/(-x - sqrt(3) + 1)**2)*(-x + 1)*atan
(3**(3/4)*sqrt(7)*sqrt(sqrt(3) + 2)*sqrt(-(-x + 1 + sqrt(3))**2/(
x - 1 + sqrt(3))**2 + 1)/(6*sqrt((-x + 1 + sqrt(3))**2/(x - 1 + s
qrt(3))**2 + 4*sqrt(3) + 7)))/(14*sqrt((x - 1)/(-x - sqrt(3) + 1)
**2)*sqrt(x**3 - 1)) - 2*3**(3/4)*sqrt((x**2 + x + 1)/(-x - sqrt(
3) + 1)**2)*sqrt(-sqrt(3) + 2)*(-x + 1)*elliptic_f(asin((-x + 1 +
 sqrt(3))/(-x - sqrt(3) + 1)), -7 + 4*sqrt(3))/(3*sqrt((x - 1)/(-
x - sqrt(3) + 1)**2)*(-sqrt(3) + 4)*sqrt(x**3 - 1)) - 4*3**(1/4)*
sqrt((x**2 + x + 1)/(-x - sqrt(3) + 1)**2)*sqrt(sqrt(3) + 2)*(-x
+ 1)*elliptic_pi((-4 + sqrt(3))**2/(sqrt(3) + 4)**2, asin((-x + 1
 + sqrt(3))/(x - 1 + sqrt(3))), -7 + 4*sqrt(3))/(sqrt((x - 1)/(-x
 - sqrt(3) + 1)**2)*(-sqrt(3) + 4)*(sqrt(3) + 4)*sqrt(4*sqrt(3) +
 7)*sqrt(x**3 - 1))
```

Mathematica [C] time = 0.0832225, size = 126, normalized size = 0.34

$$-\frac{4\sqrt{2}\sqrt{\frac{i(x-1)}{\sqrt{3}-3i}}\sqrt{x^2+x+1}\left(\frac{2\sqrt{3}}{5i+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}-i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right)}{\left(\sqrt{3}+5i\right)\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((3 + x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (-4*Sqrt[2]*Sqrt[(I*(-1 + x))/(-3*I + Sqrt[3])]*Sqrt[1 + x + x^2]
*EllipticPi[(2*Sqrt[3])/(5*I + Sqrt[3]), ArcSin[Sqrt[-I + Sqrt[3]
 - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])])/((
5*I + Sqrt[3])*Sqrt[-1 + x^3])
```

Maple [A] time = 0.028, size = 124, normalized size = 0.3

$$\frac{-\frac{3}{2}-\frac{i}{2}\sqrt{3}}{2}\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i}{2}\sqrt{3}}}\sqrt{\frac{1}{\frac{3}{2}-\frac{i}{2}\sqrt{3}}\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)}\sqrt{\frac{1}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)}EllipticPi\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i}{2}\sqrt{3}}},\frac{3}{8}+\frac{i}{8}\sqrt{3},\sqrt{\frac{\frac{3}{2}+\frac{i}{2}\sqrt{3}}{\frac{3}{2}-\frac{i}{2}\sqrt{3}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(3+x)/(x^3-1)^(1/2),x)
```

```
[Out] 1/2*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+
1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/
2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticPi(((-1+x)/(
-3/2-1/2*I*3^(1/2)))^(1/2),3/8+1/8*I*3^(1/2),((3/2+1/2*I*3^(1/2))
/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^3-1}(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^3 - 1)*(x + 3)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^3 - 1)*(x + 3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{x^3-1}(x+3)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^3 - 1)*(x + 3)),x, algorithm="fricas")

[Out] integral(1/(sqrt(x^3 - 1)*(x + 3)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(x-1)(x^2+x+1)}(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3+x)/(x**3-1)**(1/2),x)

[Out] Integral(1/(sqrt((x - 1)*(x**2 + x + 1))*(x + 3)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^3-1}(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^3 - 1)*(x + 3)),x, algorithm="giac")

[Out] integrate(1/(sqrt(x^3 - 1)*(x + 3)), x)
```

3.17 $\int \frac{1}{(3+x)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=342

$$\frac{(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\tan^{-1}\left(\frac{\sqrt{\frac{13}{2}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}}{\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}}\right)}{\sqrt{26}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}+\frac{2(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}+\frac{4\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(97-56\sqrt{3};-\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

```
[Out] ((1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*ArcTan[(Sqrt[13/
2]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2])/Sqrt[(1 - x + x^2)/(1 + Sqr
t[3] + x)^2]])/(Sqrt[26]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[-
1 - x^3]) + (2*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*El
lipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[
3]])/(3^(1/4)*Sqrt[2 - Sqrt[3]]*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^
2)]*Sqrt[-1 - x^3]) + (4*3^(1/4)*(1 + x)*Sqrt[(1 - x + x^2)/(1 +
Sqrt[3] + x)^2]*EllipticPi[97 - 56*Sqrt[3], -ArcSin[(1 - Sqrt[3]
+ x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[2 - Sqrt[3]]*Sqrt
[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[-1 - x^3])
```

Rubi [A] time = 1.35035, antiderivative size = 342, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471

$$\frac{(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\tan^{-1}\left(\frac{\sqrt{\frac{13}{2}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}}{\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}}\right)}{\sqrt{26}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}+\frac{2(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}+\frac{4\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(97-56\sqrt{3};-\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] ((1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*ArcTan[(Sqrt[13/
2]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2])/Sqrt[(1 - x + x^2)/(1 + Sqr
t[3] + x)^2]])/(Sqrt[26]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[-
1 - x^3]) + (2*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*El
lipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[
3]])/(3^(1/4)*Sqrt[2 - Sqrt[3]]*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^
2)]*Sqrt[-1 - x^3]) + (4*3^(1/4)*(1 + x)*Sqrt[(1 - x + x^2)/(1 +
Sqrt[3] + x)^2]*EllipticPi[97 - 56*Sqrt[3], -ArcSin[(1 - Sqrt[3]
+ x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[2 - Sqrt[3]]*Sqrt
[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[-1 - x^3])
```

Rubi in Sympy [A] time = 93.9631, size = 376, normalized size = 1.1

$$\frac{\sqrt{26}\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,(x+1)\operatorname{atanh}\left(\frac{\sqrt{26}\cdot 3^{\frac{3}{4}}\sqrt{1-\frac{\left(x+1+\sqrt{3}\right)^2}{\left(-x-1+\sqrt{3}\right)^2}}\sqrt{\sqrt{3}+2}}{6\sqrt{4\sqrt{3}+7+\frac{\left(x+1+\sqrt{3}\right)^2}{\left(-x-1+\sqrt{3}\right)^2}}}\right)}{26\sqrt{\frac{-x-1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}} + \frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-\sqrt{3}+2}\,(x+1)\,F\left(\operatorname{asin}\left(\frac{x+1+\sqrt{3}}{x-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{3\sqrt{\frac{-x-1}{\left(x-\sqrt{3}+1\right)^2}}\left(\sqrt{3}+2\right)\sqrt{-x^3-1}} - \frac{4\sqrt[4]{3}\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,(x+1)\,\Pi\left(\frac{\left(\sqrt{3}+2\right)^2}{\left(-2+\sqrt{3}\right)^2};\operatorname{asin}\left(\frac{x+1+\sqrt{3}}{-x-1+\sqrt{3}}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt{\frac{-x-1}{\left(x-\sqrt{3}+1\right)^2}}\left(-\sqrt{3}+2\right)\sqrt{\sqrt{3}+2}\sqrt{4\sqrt{3}+7}\sqrt{-x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(3+x)/(-x**3-1)**(1/2),x)
```

```
[Out] sqrt(26)*sqrt((x**2 - x + 1)/(x - sqrt(3) + 1)**2)*(x + 1)*atanh(
sqrt(26)*3**(3/4)*sqrt(1 - (x + 1 + sqrt(3))**2/(-x - 1 + sqrt(3)
)**2)*sqrt(sqrt(3) + 2)/(6*sqrt(4*sqrt(3) + 7 + (x + 1 + sqrt(3))
**2/(-x - 1 + sqrt(3))**2)))/(26*sqrt((-x - 1)/(x - sqrt(3) + 1)*
*2)*sqrt(-x**3 - 1)) + 2*3**(3/4)*sqrt((x**2 - x + 1)/(x - sqrt(3
) + 1)**2)*sqrt(-sqrt(3) + 2)*(x + 1)*elliptic_f(asin((x + 1 + sq
rt(3))/(x - sqrt(3) + 1)), -7 + 4*sqrt(3))/(3*sqrt((-x - 1)/(x -
sqrt(3) + 1)**2)*(sqrt(3) + 2)*sqrt(-x**3 - 1)) - 4*3**(1/4)*sqrt
((x**2 - x + 1)/(x - sqrt(3) + 1)**2)*(x + 1)*elliptic_pi((sqrt(3
) + 2)**2/(-2 + sqrt(3))**2, asin((x + 1 + sqrt(3))/(-x - 1 + sqr
t(3))), -7 + 4*sqrt(3))/(sqrt((-x - 1)/(x - sqrt(3) + 1)**2)*(-sq
rt(3) + 2)*sqrt(sqrt(3) + 2)*sqrt(4*sqrt(3) + 7)*sqrt(-x**3 - 1))
```

Mathematica [C] time = 0.0939093, size = 130, normalized size = 0.38

$$-\frac{4\sqrt{2}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\sqrt{x^2-x+1}\,\Pi\left(\frac{2\sqrt{3}}{7i+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(\sqrt{3}+7i\right)\sqrt{-x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((3 + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (-4*Sqrt[2]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*Sqrt[1 - x + x^2]*E
llipticPi[(2*Sqrt[3])/(7*I + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] -
(2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])])/((7*I
+ Sqrt[3])*Sqrt[-1 - x^3])
```

Maple [A] time = 0.055, size = 133, normalized size = 0.4

$$\frac{-\frac{2i}{3}\sqrt{3}}{\frac{7}{2}+\frac{i}{2}\sqrt{3}}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\mathit{EllipticPi}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\frac{i\sqrt{3}}{\frac{7}{2}+\frac{i}{2}\sqrt{3}},\sqrt{\frac{i\sqrt{}}{\frac{3}{2}+\frac{i}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(3+x)/(-x^3-1)^(1/2),x)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/
2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/
(-x^3-1)^(1/2)/(7/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1
/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(7/2+1/2*I*3^(1/2)),(I
*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^3-1}(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-x^3 - 1)*(x + 3)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^3 - 1)*(x + 3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{-x^3-1}(x+3)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-x^3 - 1)*(x + 3)),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(-x^3 - 1)*(x + 3)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-(x+1)(x^2-x+1)}(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3+x)/(-x**3-1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 3)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^3-1}(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-x^3 - 1)*(x + 3)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-x^3 - 1)*(x + 3)), x)
```

3.18 $\int \frac{1}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx$

Optimal. Leaf size=139

$$-\frac{3\log\left(2^{2/3}d\sqrt[3]{d^3x^3-c^3}+d(c-dx)\right)}{4\sqrt[3]{2}cd}+\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{\sqrt[3]{2}(c-dx)}{\sqrt[3]{d^3x^3-c^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}cd}+\frac{\log\left((c-dx)(c+dx)^2\right)}{4\sqrt[3]{2}cd}$$

```
[Out] (Sqrt[3]*ArcTan[(1 - (2^(1/3)*(c - d*x))/(-c^3 + d^3*x^3)^(1/3))/
Sqrt[3]])/(2*2^(1/3)*c*d) + Log[(c - d*x)*(c + d*x)^2]/(4*2^(1/3)
*c*d) - (3*Log[d*(c - d*x) + 2^(2/3)*d*(-c^3 + d^3*x^3)^(1/3)])/(
4*2^(1/3)*c*d)
```

Rubi [A] time = 0.145454, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04

$$-\frac{3\log\left(2^{2/3}d\sqrt[3]{d^3x^3-c^3}+d(c-dx)\right)}{4\sqrt[3]{2}cd}+\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{\sqrt[3]{2}(c-dx)}{\sqrt[3]{d^3x^3-c^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}cd}+\frac{\log\left((c-dx)(c+dx)^2\right)}{4\sqrt[3]{2}cd}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((c + d*x)*(-c^3 + d^3*x^3)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 - (2^(1/3)*(c - d*x))/(-c^3 + d^3*x^3)^(1/3))/
Sqrt[3]])/(2*2^(1/3)*c*d) + Log[(c - d*x)*(c + d*x)^2]/(4*2^(1/3)
*c*d) - (3*Log[d*(c - d*x) + 2^(2/3)*d*(-c^3 + d^3*x^3)^(1/3)])/(
4*2^(1/3)*c*d)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(d*x+c)/(d**3*x**3-c**3)**(1/3),x)
```

```
[Out] Integral(1/((c + d*x)*(-c**3 + d**3*x**3)**(1/3)), x)
```

Mathematica [A] time = 0.0855609, size = 0, normalized size = 0.

$$\int \frac{1}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[1/((c + d*x)*(-c^3 + d^3*x^3)^(1/3)),x]
```

```
[Out] Integrate[1/((c + d*x)*(-c^3 + d^3*x^3)^(1/3)), x]
```

Maple [F] time = 0.085, size = 0, normalized size = 0.

$$\int \frac{1}{dx+c}\frac{1}{\sqrt[3]{d^3x^3-c^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(d*x+c)/(d^3*x^3-c^3)^(1/3),x)
```

```
[Out] int(1/(d*x+c)/(d^3*x^3-c^3)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d^3x^3-c^3)^{\frac{1}{3}}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((d^3*x^3 - c^3)^(1/3)*(d*x + c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((d^3*x^3 - c^3)^(1/3)*(d*x + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((d^3*x^3 - c^3)^(1/3)*(d*x + c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{(-c+dx)\left(c^{2}+cdx+d^{2}x^{2}\right)}\,(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(d*x+c)/(d**3*x**3-c**3)**(1/3),x)
```

```
[Out] Integral(1/(((-c + d*x)*(c**2 + c*d*x + d**2*x**2))**(1/3)*(c + d
*x)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d^{3}x^{3}-c^{3}\right)^{\frac{1}{3}}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((d^3*x^3 - c^3)^(1/3)*(d*x + c)),x, algorithm="giac")
```

```
[Out] integrate(1/((d^3*x^3 - c^3)^(1/3)*(d*x + c)), x)
```

3.19 $\int \frac{1}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx$

Optimal. Leaf size=186

$$-\frac{\log\left(\sqrt[3]{2c^3+d^3x^3}-dx\right)}{4cd}+\frac{3\log\left(d(2c+dx)-d\sqrt[3]{2c^3+d^3x^3}\right)}{4cd}+\frac{\tan^{-1}\left(\frac{\frac{2dx}{\sqrt[3]{2c^3+d^3x^3}}+1}{\sqrt{3}}\right)}{2\sqrt{3}cd}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2(2c+dx)}{\sqrt[3]{2c^3+d^3x^3}}+1}{\sqrt{3}}\right)}{2cd}-\frac{\log(c+dx)}{2cd}$$

```
[Out] ArcTan[(1 + (2*d*x)/(2*c^3 + d^3*x^3)^(1/3))/Sqrt[3]]/(2*Sqrt[3]*
c*d) - (Sqrt[3]*ArcTan[(1 + (2*(2*c + d*x))/(2*c^3 + d^3*x^3)^(1/
3))/Sqrt[3]])/(2*c*d) - Log[c + d*x]/(2*c*d) - Log[-(d*x) + (2*c^
3 + d^3*x^3)^(1/3)]/(4*c*d) + (3*Log[d*(2*c + d*x) - d*(2*c^3 + d
^3*x^3)^(1/3)])/(4*c*d)
```

Rubi [A] time = 0.354867, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$

$$-\frac{\log\left(\sqrt[3]{2c^3+d^3x^3}-dx\right)}{4cd}+\frac{3\log\left(d(2c+dx)-d\sqrt[3]{2c^3+d^3x^3}\right)}{4cd}+\frac{\tan^{-1}\left(\frac{\frac{2dx}{\sqrt[3]{2c^3+d^3x^3}}+1}{\sqrt{3}}\right)}{2\sqrt{3}cd}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2(2c+dx)}{\sqrt[3]{2c^3+d^3x^3}}+1}{\sqrt{3}}\right)}{2cd}-\frac{\log(c+dx)}{2cd}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((c + d*x)*(2*c^3 + d^3*x^3)^(1/3)),x]
```


```
[Out] ArcTan[(1 + (2*d*x)/(2*c^3 + d^3*x^3)^(1/3))/Sqrt[3]]/(2*Sqrt[3]*
c*d) - (Sqrt[3]*ArcTan[(1 + (2*(2*c + d*x))/(2*c^3 + d^3*x^3)^(1/
3))/Sqrt[3]])/(2*c*d) - Log[c + d*x]/(2*c*d) - Log[-(d*x) + (2*c^
3 + d^3*x^3)^(1/3)]/(4*c*d) + (3*Log[d*(2*c + d*x) - d*(2*c^3 + d
^3*x^3)^(1/3)])/(4*c*d)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(d*x+c)/(d**3*x**3+2*c**3)**(1/3),x)
```

```
[Out] Integral(1/((c + d*x)*(2*c**3 + d**3*x**3)**(1/3)), x)
```

Mathematica [A] time = 0.0778992, size = 0, normalized size = 0.

$$\int \frac{1}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[1/((c + d*x)*(2*c^3 + d^3*x^3)^(1/3)),x]
```

```
[Out] Integrate[1/((c + d*x)*(2*c^3 + d^3*x^3)^(1/3)), x]
```

Maple [F] time = 0.058, size = 0, normalized size = 0.

$$\int \frac{1}{dx+c}\frac{1}{\sqrt[3]{d^3x^3+2\,c^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x)
```

```
[Out] int(1/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d^3x^3+2\,c^3)^{\frac{1}{3}}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((d^3*x^3 + 2*c^3)^(1/3)*(d*x + c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((d^3*x^3 + 2*c^3)^(1/3)*(d*x + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((d^3*x^3 + 2*c^3)^(1/3)*(d*x + c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c + dx)\sqrt[3]{2c^3 + d^3x^3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(d*x+c)/(d**3*x**3+2*c**3)**(1/3),x)
```

```
[Out] Integral(1/((c + d*x)*(2*c**3 + d**3*x**3)**(1/3)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d^3x^3 + 2\,c^3)^{\frac{1}{3}}(dx + c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((d^3*x^3 + 2*c^3)^(1/3)*(d*x + c)),x, algorithm="giac")
```

```
[Out] integrate(1/((d^3*x^3 + 2*c^3)^(1/3)*(d*x + c)), x)
```

3.20 $\int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=37

$$\frac{2\ 2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{x^3+1}}\right)}{\sqrt{3}}$$

```
[Out] (2*2^(2/3)*ArcTan[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[1 + x^3]])/Sqrt[
3]
```

Rubi [A] time = 0.147565, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071

$$\frac{2\ 2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{x^3+1}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(2^(2/3) - 2*x)/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*2^(2/3)*ArcTan[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[1 + x^3]])/Sqrt[
3]
```

Rubi in Sympy [A] time = 145.975, size = 479, normalized size = 12.95

$$\frac{6\cdot 2^{\frac{2}{3}}\sqrt{\frac{x^2-x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{-\sqrt{3}+2}\,(x+1)\operatorname{atan}\left(\frac{3^{\frac{3}{4}}\sqrt{1+\sqrt[3]{2}}\sqrt{-4\sqrt{3}+8}\sqrt{-\frac{\left(-x-1+\sqrt{3}\right)^2}{\left(x+1+\sqrt{3}\right)^2}+1}}{6\sqrt{-1+\sqrt[3]{2}}\sqrt{\frac{\left(-x-1+\sqrt{3}\right)^2}{\left(x+1+\sqrt{3}\right)^2}-4\sqrt{3}+7}}\right)}{\sqrt{\frac{x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{-1+\sqrt[3]{2}}\left(1+\sqrt[3]{2}\right)^{\frac{3}{2}}\sqrt{-4\sqrt{3}+8}\sqrt{x^3+1}} - \frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{x^2-x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{\sqrt{3}+2}\,(x+1)\left(2^{\frac{2}{3}}+2+2\sqrt{3}\right)F\left(\operatorname{asin}\left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{3\sqrt{\frac{x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{x^3+1}\left(-2^{\frac{2}{3}}+1+\sqrt{3}\right)} + \frac{12\cdot 2^{\frac{2}{3}}\sqrt[4]{3}\sqrt{\frac{x^2-x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{-\sqrt{3}+2}\,(x+1)\left(\frac{\left(-2^{\frac{2}{3}}+1+\sqrt{3}\right)^2}{\left(-1+2^{\frac{2}{3}}+\sqrt{3}\right)^2};\operatorname{asin}\left(\frac{-x-1+\sqrt{3}}{x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{-4\sqrt{3}+7}\sqrt{x^3+1}\left(-2^{\frac{2}{3}}+1+\sqrt{3}\right)\left(-\sqrt{3}-2^{\frac{2}{3}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2**(2/3)-2*x)/(2**(2/3)+x)/(x**3+1)**(1/2),x)
```

```
[Out] 6*2**(2/3)*sqrt((x**2 - x + 1)/(x + 1 + sqrt(3))**2)*sqrt(-sqrt(3
) + 2)*(x + 1)*atan(3**(3/4)*sqrt(1 + 2**(1/3))*sqrt(-4*sqrt(3) +
 8)*sqrt(-(-x - 1 + sqrt(3))**2/(x + 1 + sqrt(3))**2 + 1)/(6*sqrt
(-1 + 2**(1/3))*sqrt((-x - 1 + sqrt(3))**2/(x + 1 + sqrt(3))**2 -
 4*sqrt(3) + 7)))/(sqrt((x + 1)/(x + 1 + sqrt(3))**2)*sqrt(-1 + 2
**(1/3))*(1 + 2**(1/3))**(3/2)*sqrt(-4*sqrt(3) + 8)*sqrt(x**3 + 1
)) - 2*3**(3/4)*sqrt((x**2 - x + 1)/(x + 1 + sqrt(3))**2)*sqrt(sq
rt(3) + 2)*(x + 1)*(2**(2/3) + 2 + 2*sqrt(3))*elliptic_f(asin((x
- sqrt(3) + 1)/(x + 1 + sqrt(3))), -7 - 4*sqrt(3))/(3*sqrt((x + 1
)/(x + 1 + sqrt(3))**2)*sqrt(x**3 + 1)*(-2**(2/3) + 1 + sqrt(3)))
 + 12*2**(2/3)*3**(1/4)*sqrt((x**2 - x + 1)/(x + 1 + sqrt(3))**2)
*sqrt(-sqrt(3) + 2)*(x + 1)*elliptic_pi((-2**(2/3) + 1 + sqrt(3))
**2/(-1 + 2**(2/3) + sqrt(3))**2, asin((-x - 1 + sqrt(3))/(x + 1
+ sqrt(3))), -7 - 4*sqrt(3))/(sqrt((x + 1)/(x + 1 + sqrt(3))**2)*
sqrt(-4*sqrt(3) + 7)*sqrt(x**3 + 1)*(-2**(2/3) + 1 + sqrt(3))*(-s
qrt(3) - 2**(2/3) + 1))
```

Mathematica [C] time = 0.52833, size = 326, normalized size = 8.81

$$-\frac{4\sqrt[6]{2}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\left(\sqrt{2ix+\sqrt{3}-i}\left(\left(\left(-3i\sqrt[3]{2}+4\sqrt{3}+\sqrt[3]{2}\sqrt{3}\right)x+\sqrt[3]{2}\sqrt{3}-2\sqrt{3}+3i\sqrt[3]{2}+6i\right)F\left(\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)\middle|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)-6i\sqrt{}\right.\right.}{\sqrt{3}\left(1+2\,2^{2/3}-i\sqrt{3}\right)\sqrt{-2ix+\sqrt{3}+i}\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(2^(2/3) - 2*x)/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (-4*2^(1/6)*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(Sqrt[-I + Sqrt[3]
+ (2*I)*x]*(6*I + (3*I)*2^(1/3) - 2*Sqrt[3] + 2^(1/3)*Sqrt[3] + (
(-3*I)*2^(1/3) + 4*Sqrt[3] + 2^(1/3)*Sqrt[3])*x)*EllipticF[ArcSin
[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I
 + Sqrt[3])] - (6*I)*Sqrt[3]*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 -
 x + x^2]*EllipticPi[(2*Sqrt[3])/(I + (2*I)*2^(2/3) + Sqrt[3]), A
rcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])
/(3*I + Sqrt[3])]))/(Sqrt[3]*(1 + 2*2^(2/3) - I*Sqrt[3])*Sqrt[I +
 Sqrt[3] - (2*I)*x]*Sqrt[1 + x^3])
```

Maple [C] time = 0.052, size = 258, normalized size = 7.

$$-4\,\frac{3/2-i/2\sqrt{3}}{\sqrt{x^3+1}}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}\,\mathit{EllipticF}\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},\sqrt{\frac{-3/2+i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\right)$$
$$+6\,\frac{2^{2/3}\left(3/2-i/2\sqrt{3}\right)}{\sqrt{x^3+1}\left(2^{2/3}-1\right)}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}\,\mathit{EllipticPi}\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},\frac{-3/2+i/2\sqrt{3}}{2^{2/3}-1},\sqrt{\frac{-3}{-3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2^(2/3)-2*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x)
```

```
[Out] -4*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-
1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))
/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-
1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))
^(1/2))+6*2^(2/3)*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))
^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2
+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)/(2^(2/3
)-1)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(
1/2))/(2^(2/3)-1),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/
2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{2\,x-2^{\frac{2}{3}}}{\sqrt{x^3+1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(2*x - 2^(2/3))/(sqrt(x^3 + 1)*(x + 2^(2/3))),x, algorithm="maxima")
```

```
[Out] -integrate((2*x - 2^(2/3))/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

Fricas [A]   time = 0.374634, size = 73, normalized size = 1.97

$$\frac{1}{3}\sqrt{6}2^{\frac{1}{6}}\arctan\left(-\frac{\sqrt{6}2^{\frac{5}{6}}\left(x^3-3\cdot 2^{\frac{2}{3}}x^2-6\cdot 2^{\frac{1}{3}}x-2\right)}{12\sqrt{x^3+1}\left(2^{\frac{2}{3}}x+2^{\frac{1}{3}}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(2*x - 2^(2/3))/(sqrt(x^3 + 1)*(x + 2^(2/3))),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(6)*2^(1/6)*arctan(-1/12*sqrt(6)*2^(5/6)*(x^3 - 3*2^(2/3)
*x^2 - 6*2^(1/3)*x - 2)/(sqrt(x^3 + 1)*(2^(2/3)*x + 2^(1/3))))
```

Sympy [F]   time = 0., size = 0, normalized size = 0.

$$-\int\left(-\frac{2^{\frac{2}{3}}}{x\sqrt{x^3+1}+2^{\frac{2}{3}}\sqrt{x^3+1}}\right)dx-\int\frac{2x}{x\sqrt{x^3+1}+2^{\frac{2}{3}}\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2**(2/3)-2*x)/(2**(2/3)+x)/(x**3+1)**(1/2),x)
```

```
[Out] -Integral(-2**(2/3)/(x*sqrt(x**3 + 1) + 2**(2/3)*sqrt(x**3 + 1)),
 x) - Integral(2*x/(x*sqrt(x**3 + 1) + 2**(2/3)*sqrt(x**3 + 1)),
x)
```

GIAC/XCAS [F]   time = 0., size = 0, normalized size = 0.

$$\int -\frac{2\,x-2^{\frac{2}{3}}}{\sqrt{x^3+1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(2*x - 2^(2/3))/(sqrt(x^3 + 1)*(x + 2^(2/3))),x, algorithm="giac")
```

```
[Out] integrate(-(2*x - 2^(2/3))/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

3.21 $\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=40

$$-\frac{2\ 2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{\sqrt{3}}$$

```
[Out] (-2*2^(2/3)*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[1 - x^3]])/Sqrt
[3]
```

Rubi [A] time = 0.172656, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062

$$-\frac{2\ 2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(2^(2/3) + 2*x)/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```


```
[Out] (-2*2^(2/3)*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[1 - x^3]])/Sqrt
[3]
```

Rubi in Sympy [A] time = 147.99, size = 479, normalized size = 11.98

$$-\frac{6\cdot 2^{\frac{2}{3}}\sqrt{\frac{x^2+x+1}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{-\sqrt{3}+2}\,(-x+1)\operatorname{atan}\left(\frac{3^{\frac{3}{4}}\sqrt{1+\sqrt[3]{2}}\sqrt{1-\frac{\left(x-1+\sqrt{3}\right)^2}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{-4\sqrt{3}+8}}{6\sqrt{-1+\sqrt[3]{2}}\sqrt{-4\sqrt{3}+7+\frac{\left(x-1+\sqrt{3}\right)^2}{\left(-x+1+\sqrt{3}\right)^2}}}\right)}{\sqrt{\frac{-x+1}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{-1+\sqrt[3]{2}}\left(1+\sqrt[3]{2}\right)^{\frac{3}{2}}\sqrt{-4\sqrt{3}+8}\sqrt{-x^3+1}}+\frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{x^2+x+1}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{\sqrt{3}+2}\,(-x+1)\left(2^{\frac{2}{3}}+2+2\sqrt{3}\right)F\left(\operatorname{asin}\left(\frac{-x-\sqrt{3}+1}{-x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{3\sqrt{\frac{-x+1}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{-x^3+1}\left(-2^{\frac{2}{3}}+1+\sqrt{3}\right)}-\frac{12\cdot 2^{\frac{2}{3}}\sqrt[4]{3}\sqrt{\frac{x^2+x+1}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{-\sqrt{3}+2}\,(-x+1)\,\Pi\left(\frac{\left(-2^{\frac{2}{3}}+1+\sqrt{3}\right)^2}{\left(-1+2^{\frac{2}{3}}+\sqrt{3}\right)^2};\operatorname{asin}\left(\frac{x-1+\sqrt{3}}{-x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt{\frac{-x+1}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{-4\sqrt{3}+7}\sqrt{-x^3+1}\left(-2^{\frac{2}{3}}+1+\sqrt{3}\right)\left(-\sqrt{3}-2^{\frac{2}{3}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2**(2/3)+2*x)/(2**(2/3)-x)/(-x**3+1)**(1/2),x)
```

```
[Out] -6*2**(2/3)*sqrt((x**2 + x + 1)/(-x + 1 + sqrt(3))**2)*sqrt(-sqrt
(3) + 2)*(-x + 1)*atan(3**(3/4)*sqrt(1 + 2**(1/3))*sqrt(1 - (x -
1 + sqrt(3))**2/(-x + 1 + sqrt(3))**2)*sqrt(-4*sqrt(3) + 8)/(6*sq
rt(-1 + 2**(1/3))*sqrt(-4*sqrt(3) + 7 + (x - 1 + sqrt(3))**2/(-x
+ 1 + sqrt(3))**2)))/(sqrt((-x + 1)/(-x + 1 + sqrt(3))**2)*sqrt(-
1 + 2**(1/3))*(1 + 2**(1/3))**(3/2)*sqrt(-4*sqrt(3) + 8)*sqrt(-x*
*3 + 1)) + 2*3**(3/4)*sqrt((x**2 + x + 1)/(-x + 1 + sqrt(3))**2)*
sqrt(sqrt(3) + 2)*(-x + 1)*(2**(2/3) + 2 + 2*sqrt(3))*elliptic_f(
asin((-x - sqrt(3) + 1)/(-x + 1 + sqrt(3))), -7 - 4*sqrt(3))/(3*s
qrt((-x + 1)/(-x + 1 + sqrt(3))**2)*sqrt(-x**3 + 1)*(-2**(2/3) +
1 + sqrt(3))) - 12*2**(2/3)*3**(1/4)*sqrt((x**2 + x + 1)/(-x + 1
+ sqrt(3))**2)*sqrt(-sqrt(3) + 2)*(-x + 1)*elliptic_pi((-2**(2/3)
 + 1 + sqrt(3))**2/(-1 + 2**(2/3) + sqrt(3))**2, asin((x - 1 + sq
rt(3))/(-x + 1 + sqrt(3))), -7 - 4*sqrt(3))/(sqrt((-x + 1)/(-x +
1 + sqrt(3))**2)*sqrt(-4*sqrt(3) + 7)*sqrt(-x**3 + 1)*(-2**(2/3)
+ 1 + sqrt(3))*(-sqrt(3) - 2**(2/3) + 1))
```

Mathematica [C] time = 0.495861, size = 327, normalized size = 8.18

$$-\frac{4\sqrt[6]{2}\sqrt{-\frac{i(x-1)}{\sqrt{3}+3i}}\left(6i\sqrt{3}\sqrt{2ix+\sqrt{3}+i}\sqrt{x^2+x+1}\,\Pi\left(\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)\middle|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)+\sqrt{-2ix+\sqrt{3}-i}\left(\left(-3i\sqrt[3]{2}+4\sqrt{3}+\right.\right.\right.}{\sqrt{3}\left(1+2\,2^{2/3}-i\sqrt{3}\right)\sqrt{2ix+\sqrt{3}+i}\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(2^(2/3) + 2*x)/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (-4*2^(1/6)*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*(Sqrt[-I + Sqrt
[3] - (2*I)*x]*(-6*I - (3*I)*2^(1/3) + 2*Sqrt[3] - 2^(1/3)*Sqrt[3
] + ((-3*I)*2^(1/3) + 4*Sqrt[3] + 2^(1/3)*Sqrt[3])*x)*EllipticF[A
rcSin[Sqrt[I + Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])
/(3*I + Sqrt[3])] + (6*I)*Sqrt[3]*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqr
t[1 + x + x^2]*EllipticPi[(2*Sqrt[3])/(I + (2*I)*2^(2/3) + Sqrt[3
]), ArcSin[Sqrt[I + Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqr
t[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(1 + 2*2^(2/3) - I*Sqrt[3])*Sqr
t[I + Sqrt[3] + (2*I)*x]*Sqrt[1 - x^3])
```

Maple [C] time = 0.057, size = 253, normalized size = 6.3

$$\frac{4i}{3}\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\mathit{EllipticF}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)\frac{1}{\sqrt{-x^3}}$$
$$+\frac{2i2^{\frac{2}{3}}\sqrt{3}}{-\frac{1}{2}+\frac{i}{2}\sqrt{3}-2^{\frac{2}{3}}}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\mathit{EllipticPi}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\frac{i\sqrt{}}{-\frac{1}{2}+\frac{i}{2}\sqrt{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2^(2/3)+2*x)/(2^(2/3)-x)/(-x^3+1)^(1/2),x)
```

```
[Out] 4/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/(-3
/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)
/(-x^3+1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^
(1/2))^(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))+2*I*2^(2/3)*
3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/(-3/2+1/2
*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3
+1)^(1/2)/(-1/2+1/2*I*3^(1/2)-2^(2/3))*EllipticPi(1/3*3^(1/2)*(I*
(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2
)-2^(2/3)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{2\,x + 2^{\frac{2}{3}}}{\sqrt{-x^3+1}\left(x - 2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(2*x + 2^(2/3))/(sqrt(-x^3 + 1)*(x - 2^(2/3))),x, algorithm="maxima")
```

```
[Out] -integrate((2*x + 2^(2/3))/(sqrt(-x^3 + 1)*(x - 2^(2/3))), x)
```

Fricas [A] time = 0.364757, size = 78, normalized size = 1.95

$$\frac{1}{3}\sqrt{6}2^{\frac{1}{6}}\arctan\left(\frac{\sqrt{6}2^{\frac{5}{6}}\left(x^3+3\cdot 2^{\frac{2}{3}}x^2-6\cdot 2^{\frac{1}{3}}x+2\right)}{12\sqrt{-x^3+1}\left(2^{\frac{2}{3}}x-2^{\frac{1}{3}}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(2*x + 2^(2/3))/(sqrt(-x^3 + 1)*(x - 2^(2/3))),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(6)*2^(1/6)*arctan(1/12*sqrt(6)*2^(5/6)*(x^3 + 3*2^(2/3)*
x^2 - 6*2^(1/3)*x + 2)/(sqrt(-x^3 + 1)*(2^(2/3)*x - 2^(1/3))))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{2^{\frac{2}{3}}}{x\sqrt{-x^3+1}-2^{\frac{2}{3}}\sqrt{-x^3+1}}\,dx-\int\frac{2x}{x\sqrt{-x^3+1}-2^{\frac{2}{3}}\sqrt{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2**(2/3)+2*x)/(2**(2/3)-x)/(-x**3+1)**(1/2),x)
```

```
[Out] -Integral(2**(2/3)/(x*sqrt(-x**3 + 1) - 2**(2/3)*sqrt(-x**3 + 1))
, x) - Integral(2*x/(x*sqrt(-x**3 + 1) - 2**(2/3)*sqrt(-x**3 + 1)
), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int-\frac{2\,x+2^{\frac{2}{3}}}{\sqrt{-x^3+1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(2*x + 2^(2/3))/(sqrt(-x^3 + 1)*(x - 2^(2/3))),x, algorithm="giac")
```

```
[Out] integrate(-(2*x + 2^(2/3))/(sqrt(-x^3 + 1)*(x - 2^(2/3))), x)
```

3.22 $\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=38

$$-\frac{2\ 2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{x^3-1}}\right)}{\sqrt{3}}$$

```
[Out] (-2*2^(2/3)*ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[-1 + x^3]])/Sq
rt[3]
```

Rubi [A] time = 0.152818, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$-\frac{2\ 2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{x^3-1}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(2^(2/3) + 2*x)/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (-2*2^(2/3)*ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[-1 + x^3]])/Sq
rt[3]
```

Rubi in Sympy [A] time = 156.949, size = 452, normalized size = 11.89

$$-\frac{3\cdot 2^{\frac{2}{3}}\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,(-x+1)\operatorname{atanh}\left(\frac{3^{\frac{3}{4}}\sqrt{1+\sqrt[3]{2}}\sqrt{\sqrt{3}+2}\sqrt{-\frac{\left(-x+1+\sqrt{3}\right)^2}{\left(x-1+\sqrt{3}\right)^2}+1}}{3\sqrt{-1+\sqrt[3]{2}}\sqrt{\frac{\left(-x+1+\sqrt{3}\right)^2}{\left(x-1+\sqrt{3}\right)^2}+4\sqrt{3}+7}}\right)}{\sqrt{\frac{x-1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{-1+\sqrt[3]{2}}\left(1+\sqrt[3]{2}\right)^{\frac{3}{2}}\sqrt{x^3-1}}$$

$$+\frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{-\sqrt{3}+2}\,(-x+1)\left(-2\sqrt{3}+2^{\frac{2}{3}}+2\right)F\left(\operatorname{asin}\left(\frac{-x+1+\sqrt{3}}{-x-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{3\sqrt{\frac{x-1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\left(-\sqrt{3}-2^{\frac{2}{3}}+1\right)}$$

$$+\frac{12\cdot 2^{\frac{2}{3}}\sqrt[4]{3}\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{\sqrt{3}+2}\,(-x+1)\left(\frac{\left(-1+2^{\frac{2}{3}}+\sqrt{3}\right)^2}{\left(-2^{\frac{2}{3}}+1+\sqrt{3}\right)^2};\operatorname{asin}\left(\frac{-x+1+\sqrt{3}}{x-1+\sqrt{3}}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt{\frac{x-1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{4\sqrt{3}+7}\sqrt{x^3-1}\left(-2^{\frac{2}{3}}+1+\sqrt{3}\right)\left(-\sqrt{3}-2^{\frac{2}{3}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2**(2/3)+2*x)/(2**(2/3)-x)/(x**3-1)**(1/2),x)
```

```
[Out] -3*2**(2/3)*sqrt((x**2 + x + 1)/(-x - sqrt(3) + 1)**2)*(-x + 1)*a
tanh(3**(3/4)*sqrt(1 + 2**(1/3))*sqrt(sqrt(3) + 2)*sqrt(-(-x + 1
+ sqrt(3))**2/(x - 1 + sqrt(3))**2 + 1)/(3*sqrt(-1 + 2**(1/3))*sq
rt((-x + 1 + sqrt(3))**2/(x - 1 + sqrt(3))**2 + 4*sqrt(3) + 7)))/
(sqrt((x - 1)/(-x - sqrt(3) + 1)**2)*sqrt(-1 + 2**(1/3))*(1 + 2**
(1/3))**(3/2)*sqrt(x**3 - 1)) + 2*3**(3/4)*sqrt((x**2 + x + 1)/(-
x - sqrt(3) + 1)**2)*sqrt(-sqrt(3) + 2)*(-x + 1)*(-2*sqrt(3) + 2*
*(2/3) + 2)*elliptic_f(asin((-x + 1 + sqrt(3))/(-x - sqrt(3) + 1)
), -7 + 4*sqrt(3))/(3*sqrt((x - 1)/(-x - sqrt(3) + 1)**2)*sqrt(x*
*3 - 1)*(-sqrt(3) - 2**(2/3) + 1)) + 12*2**(2/3)*3**(1/4)*sqrt((x
**2 + x + 1)/(-x - sqrt(3) + 1)**2)*sqrt(sqrt(3) + 2)*(-x + 1)*el
liptic_pi((-1 + 2**(2/3) + sqrt(3))**2/(-2**(2/3) + 1 + sqrt(3))*
*2, asin((-x + 1 + sqrt(3))/(x - 1 + sqrt(3))), -7 + 4*sqrt(3))/(
sqrt((x - 1)/(-x - sqrt(3) + 1)**2)*sqrt(4*sqrt(3) + 7)*sqrt(x**3
 - 1)*(-2**(2/3) + 1 + sqrt(3))*(-sqrt(3) - 2**(2/3) + 1))
```

Mathematica [C] time = 0.524202, size = 325, normalized size = 8.55

$$-\frac{4\sqrt[6]{2}\sqrt{-\frac{i(x-1)}{\sqrt{3}+3i}}\left(6i\sqrt{3}\sqrt{2ix+\sqrt{3}+i}\sqrt{x^2+x+1}\left(\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)+\sqrt{-2ix+\sqrt{3}-i}\left(\left(-3i\sqrt[3]{2}+4\sqrt{3}+\right.\right.\right.}{\sqrt{3}\left(1+2\,2^{2/3}-i\sqrt{3}\right)\sqrt{2ix+\sqrt{3}+i}\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(2^(2/3) + 2*x)/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (-4*2^(1/6)*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*(Sqrt[-I + Sqrt
[3] - (2*I)*x]*(-6*I - (3*I)*2^(1/3) + 2*Sqrt[3] - 2^(1/3)*Sqrt[3
] + ((-3*I)*2^(1/3) + 4*Sqrt[3] + 2^(1/3)*Sqrt[3])*x)*EllipticF[A
rcSin[Sqrt[I + Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])
/(3*I + Sqrt[3])] + (6*I)*Sqrt[3]*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqr
t[1 + x + x^2]*EllipticPi[(2*Sqrt[3])/(I + (2*I)*2^(2/3) + Sqrt[3
]), ArcSin[Sqrt[I + Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqr
t[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(1 + 2*2^(2/3) - I*Sqrt[3])*Sqr
t[I + Sqrt[3] + (2*I)*x]*Sqrt[-1 + x^3])
```

Maple [C] time = 0.041, size = 262, normalized size = 6.9

$$-4\,{\frac {-3/2-i/2\sqrt {3}}{\sqrt {{x}^{3}-1}}\sqrt {{\frac {-1+x}{-3/2-i/2\sqrt {3}}}}\sqrt {{\frac {x+1/2-i/2\sqrt {3}}{3/2-i/2\sqrt {3}}}}\sqrt {{\frac {x+1/2+i/2\sqrt {3}}{3/2+i/2\sqrt {3}}}}{\it EllipticF} \left(\sqrt {{\frac {-1+x}{-3/2-i/2\sqrt {3}}}},\sqrt {{\frac {3/2+i/2\sqrt {3}}{3/2-i/2\sqrt {3}}}} \right) }-6\,{\frac {{2}^{2/3} \left(-3/2-i/2\sqrt {3} \right) }{\sqrt {{x}^{3}-1} \left(-{2}^{2/3}+1 \right) }\sqrt {{\frac {-1+x}{-3/2-i/2\sqrt {3}}}}\sqrt {{\frac {x+1/2-i/2\sqrt {3}}{3/2-i/2\sqrt {3}}}}\sqrt {{\frac {x+1/2+i/2\sqrt {3}}{3/2+i/2\sqrt {3}}}}{\it EllipticPi} \left(\sqrt {{\frac {-1+x}{-3/2-i/2\sqrt {3}}}},{\frac {3/2+i/2\sqrt {3}}{-{2}^{2/3}+1}},\sqrt{ \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2^(2/3)+2*x)/(2^(2/3)-x)/(x^3-1)^(1/2),x)
```

```
[Out] -4*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1
/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2
))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((-1+x)/(-3
/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2))
)^(1/2))-6*2^(2/3)*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/
2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+
1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)/(-2^(
2/3)+1)*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I
*3^(1/2))/(-2^(2/3)+1),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^
(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{2\,x + 2^{\frac{2}{3}}}{\sqrt{x^{3} - 1}\left(x - 2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(2*x + 2^(2/3))/(sqrt(x^3 - 1)*(x - 2^(2/3))),x, algorithm="maxima")
```

```
[Out] -integrate((2*x + 2^(2/3))/(sqrt(x^3 - 1)*(x - 2^(2/3))), x)
```

Fricas [A] time = 0.358671, size = 173, normalized size = 4.55

$$\frac{1}{6}\sqrt{6}2^{\frac{1}{6}}\log\left(-\frac{x^6-18\cdot 2^{\frac{1}{3}}x^4-56x^3-2\sqrt{6}2^{\frac{1}{6}}\sqrt{x^3-1}\left(18x^2-2^{\frac{2}{3}}\left(x^4+8x\right)-2^{\frac{1}{3}}\left(5x^3-2\right)\right)+18\cdot 2^{\frac{2}{3}}\left(x^5+2x^2\right)-8}{x^6-80x^3-6\cdot 2^{\frac{2}{3}}(x^5-10x^2)+6\cdot 2^{\frac{1}{3}}(5x^4-8x)+16}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(2*x + 2^(2/3))/(sqrt(x^3 - 1)*(x - 2^(2/3))),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(6)*2^(1/6)*log(-(x^6 - 18*2^(1/3)*x^4 - 56*x^3 - 2*sqrt(
6)*2^(1/6)*sqrt(x^3 - 1)*(18*x^2 - 2^(2/3)*(x^4 + 8*x) - 2^(1/3)*
(5*x^3 - 2)) + 18*2^(2/3)*(x^5 + 2*x^2) - 8)/(x^6 - 80*x^3 - 6*2^
(2/3)*(x^5 - 10*x^2) + 6*2^(1/3)*(5*x^4 - 8*x) + 16))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{2^{\frac{2}{3}}}{x\sqrt{x^3-1}-2^{\frac{2}{3}}\sqrt{x^3-1}}\,dx-\int\frac{2x}{x\sqrt{x^3-1}-2^{\frac{2}{3}}\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2**(2/3)+2*x)/(2**(2/3)-x)/(x**3-1)**(1/2),x)
```

```
[Out] -Integral(2**(2/3)/(x*sqrt(x**3 - 1) - 2**(2/3)*sqrt(x**3 - 1)),
x) - Integral(2*x/(x*sqrt(x**3 - 1) - 2**(2/3)*sqrt(x**3 - 1)), x
)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int-\frac{2x+2^{\frac{2}{3}}}{\sqrt{x^3-1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(2*x + 2^(2/3))/(sqrt(x^3 - 1)*(x - 2^(2/3))),x, algorithm="giac")

[Out] integrate(-(2*x + 2^(2/3))/(sqrt(x^3 - 1)*(x - 2^(2/3))), x)
```

3.23 $\int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=39

$$\frac{2\ 2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{-x^3-1}}\right)}{\sqrt{3}}$$

```
[Out] (2*2^(2/3)*ArcTanh[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[-1 - x^3]])/Sqr
t[3]
```

Rubi [A] time = 0.158851, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$\frac{2\ 2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{-x^3-1}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(2^(2/3) - 2*x)/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*2^(2/3)*ArcTanh[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[-1 - x^3]])/Sqr
t[3]
```

Rubi in Sympy [A] time = 153.832, size = 462, normalized size = 11.85

$$\frac{3\cdot 2^{\frac{2}{3}}\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,(x+1)\operatorname{atanh}\left(\frac{3^{\frac{3}{4}}\sqrt{1+\sqrt[3]{2}}\sqrt{1-\frac{\left(x+1+\sqrt{3}\right)^2}{\left(-x-1+\sqrt{3}\right)^2}}\sqrt{\sqrt{3}+2}}{3\sqrt{-1+\sqrt[3]{2}}\sqrt{4\sqrt{3}+7+\frac{\left(x+1+\sqrt{3}\right)^2}{\left(-x-1+\sqrt{3}\right)^2}}}\right)}{\sqrt{\frac{-x-1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-1+\sqrt[3]{2}}\left(1+\sqrt[3]{2}\right)^{\frac{3}{2}}\sqrt{-x^3-1}} - \frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-\sqrt{3}+2}\,(x+1)\left(-2\sqrt{3}+2^{\frac{2}{3}}+2\right)F\left(\operatorname{asin}\left(\frac{x+1+\sqrt{3}}{x-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{3\sqrt{\frac{-x-1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\left(-\sqrt{3}-2^{\frac{2}{3}}+1\right)} - \frac{12\cdot 2^{\frac{2}{3}}\sqrt[4]{3}\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{\sqrt{3}+2}\,(x+1)\,\Pi\left(\frac{\left(-1+2^{\frac{2}{3}}+\sqrt{3}\right)^2}{\left(-2^{\frac{2}{3}}+1+\sqrt{3}\right)^2};\operatorname{asin}\left(\frac{x+1+\sqrt{3}}{-x-1+\sqrt{3}}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt{\frac{-x-1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{4\sqrt{3}+7}\sqrt{-x^3-1}\left(-2^{\frac{2}{3}}+1+\sqrt{3}\right)\left(-\sqrt{3}-2^{\frac{2}{3}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2**(2/3)-2*x)/(2**(2/3)+x)/(-x**3-1)**(1/2),x)
```

```
[Out] 3*2**(2/3)*sqrt((x**2 - x + 1)/(x - sqrt(3) + 1)**2)*(x + 1)*atan
h(3**(3/4)*sqrt(1 + 2**(1/3))*sqrt(1 - (x + 1 + sqrt(3))**2/(-x -
 1 + sqrt(3))**2)*sqrt(sqrt(3) + 2)/(3*sqrt(-1 + 2**(1/3))*sqrt(4
*sqrt(3) + 7 + (x + 1 + sqrt(3))**2/(-x - 1 + sqrt(3))**2)))/(sqr
t((-x - 1)/(x - sqrt(3) + 1)**2)*sqrt(-1 + 2**(1/3))*(1 + 2**(1/3
))**(3/2)*sqrt(-x**3 - 1)) - 2*3**(3/4)*sqrt((x**2 - x + 1)/(x -
sqrt(3) + 1)**2)*sqrt(-sqrt(3) + 2)*(x + 1)*(-2*sqrt(3) + 2**(2/3
) + 2)*elliptic_f(asin((x + 1 + sqrt(3))/(x - sqrt(3) + 1)), -7 +
 4*sqrt(3))/(3*sqrt((-x - 1)/(x - sqrt(3) + 1)**2)*sqrt(-x**3 - 1
)*(-sqrt(3) - 2**(2/3) + 1)) - 12*2**(2/3)*3**(1/4)*sqrt((x**2 -
x + 1)/(x - sqrt(3) + 1)**2)*sqrt(sqrt(3) + 2)*(x + 1)*elliptic_p
i((-1 + 2**(2/3) + sqrt(3))**2/(-2**(2/3) + 1 + sqrt(3))**2, asin
((x + 1 + sqrt(3))/(-x - 1 + sqrt(3))), -7 + 4*sqrt(3))/(sqrt((-x
 - 1)/(x - sqrt(3) + 1)**2)*sqrt(4*sqrt(3) + 7)*sqrt(-x**3 - 1)*(
-2**(2/3) + 1 + sqrt(3))*(-sqrt(3) - 2**(2/3) + 1))
```

Mathematica [C] time = 0.482406, size = 328, normalized size = 8.41

$$-\frac{4\sqrt[6]{2}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\left(\sqrt{2ix+\sqrt{3}-i}\left(\left(-3i\sqrt[3]{2}+4\sqrt{3}+\sqrt[3]{2}\sqrt{3}\right)x+\sqrt[3]{2}\sqrt{3}-2\sqrt{3}+3i\sqrt[3]{2}+6i\right)F\left(\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)-6i\sqrt{}\right.}{\sqrt{3}\left(1+2\,2^{2/3}-i\sqrt{3}\right)\sqrt{-2ix+\sqrt{3}+i}\sqrt{-x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(2^(2/3) - 2*x)/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (-4*2^(1/6)*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(Sqrt[-I + Sqrt[3]
+ (2*I)*x]*(6*I + (3*I)*2^(1/3) - 2*Sqrt[3] + 2^(1/3)*Sqrt[3] + (
(-3*I)*2^(1/3) + 4*Sqrt[3] + 2^(1/3)*Sqrt[3])*x)*EllipticF[ArcSin
[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I
 + Sqrt[3])] - (6*I)*Sqrt[3]*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 -
 x + x^2]*EllipticPi[(2*Sqrt[3])/(I + (2*I)*2^(2/3) + Sqrt[3]), A
rcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])
/(3*I + Sqrt[3])]))/(Sqrt[3]*(1 + 2*2^(2/3) - I*Sqrt[3])*Sqrt[I +
 Sqrt[3] - (2*I)*x]*Sqrt[-1 - x^3])
```

Maple [C] time = 0.049, size = 249, normalized size = 6.4

$$\frac{4i}{3}\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\,EllipticF\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)\frac{1}{\sqrt{-x^3-}}$$
$$-\frac{2\,i2^{\frac{2}{3}}\sqrt{3}}{\frac{1}{2}+\frac{i}{2}\sqrt{3}+2^{\frac{2}{3}}}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\,EllipticPi\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\frac{i\sqrt{3}}{\frac{1}{2}+\frac{i}{2}\sqrt{3}+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2^(2/3)-2*x)/(2^(2/3)+x)/(-x^3-1)^(1/2),x)
```

```
[Out] 4/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2
+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(
-x^3-1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1
/2))^(1/2),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))-2*I*2^(2/3)*3^(
1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(
1/2)/(1/2+1/2*I*3^(1/2)+2^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2
-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+2^(2/
3)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{2\,x-2^{\frac{2}{3}}}{\sqrt{-x^3-1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(2*x - 2^(2/3))/(sqrt(-x^3 - 1)*(x + 2^(2/3))),x, algorithm="maxima")
```

```
[Out] -integrate((2*x - 2^(2/3))/(sqrt(-x^3 - 1)*(x + 2^(2/3))), x)
```

Fricas [A] time = 0.36217, size = 173, normalized size = 4.44

$$\frac{1}{6}\sqrt{6}2^{\frac{1}{6}}\log\left(\frac{x^6-18\cdot 2^{\frac{1}{3}}x^4+56\,x^3+2\sqrt{6}2^{\frac{1}{6}}\sqrt{-x^3-1}\left(18\,x^2-2^{\frac{2}{3}}\left(x^4-8\,x\right)+2^{\frac{1}{3}}\left(5\,x^3+2\right)\right)-18\cdot 2^{\frac{2}{3}}\left(x^5-2\,x^2\right)-8}{x^6+80\,x^3+6\cdot 2^{\frac{2}{3}}(x^5+10\,x^2)+6\cdot 2^{\frac{1}{3}}(5\,x^4+8\,x)+16}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(2*x - 2^(2/3))/(sqrt(-x^3 - 1)*(x + 2^(2/3))),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(6)*2^(1/6)*log((x^6 - 18*2^(1/3)*x^4 + 56*x^3 + 2*sqrt(6
)*2^(1/6)*sqrt(-x^3 - 1)*(18*x^2 - 2^(2/3)*(x^4 - 8*x) + 2^(1/3)*
(5*x^3 + 2)) - 18*2^(2/3)*(x^5 - 2*x^2) - 8)/(x^6 + 80*x^3 + 6*2^
(2/3)*(x^5 + 10*x^2) + 6*2^(1/3)*(5*x^4 + 8*x) + 16))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\left(-\frac{2^{\frac{2}{3}}}{x\sqrt{-x^3-1}+2^{\frac{2}{3}}\sqrt{-x^3-1}}\right)dx-\int\frac{2x}{x\sqrt{-x^3-1}+2^{\frac{2}{3}}\sqrt{-x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2**(2/3)-2*x)/(2**(2/3)+x)/(-x**3-1)**(1/2),x)
```

```
[Out] -Integral(-2**(2/3)/(x*sqrt(-x**3 - 1) + 2**(2/3)*sqrt(-x**3 - 1)
), x) - Integral(2*x/(x*sqrt(-x**3 - 1) + 2**(2/3)*sqrt(-x**3 - 1
)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{2\,x-2^{\frac{2}{3}}}{\sqrt{-x^3-1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(2*x - 2^(2/3))/(sqrt(-x^3 - 1)*(x + 2^(2/3))),x, algorithm="giac")

[Out] integrate(-(2*x - 2^(2/3))/(sqrt(-x^3 - 1)*(x + 2^(2/3))), x)
```

3.24 $\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=63

$$\frac{2\ 2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

```
[Out] (2*2^(2/3)*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) + 2^(1/3)*b^(1/3)*x))
/Sqrt[a + b*x^3]])/(Sqrt[3]*a^(1/6)*b^(1/3))
```

Rubi [A] time = 0.271289, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 53, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038

$$\frac{2\ 2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(2^(2/3)*a^(1/3) - 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^
```


```
[Out] (2*2^(2/3)*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) + 2^(1/3)*b^(1/3)*x))
/Sqrt[a + b*x^3]])/(Sqrt[3]*a^(1/6)*b^(1/3))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((2**(2/3)*a**(1/3)-2*b**(1/3)*x)/(2**(2/3)*a**(1/3)+b**(1/3)*x)
```

```
[Out] Timed out
```

Mathematica [C] time = 1.92623, size = 325, normalized size = 5.16

$$\frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{2\sqrt[4]{3}\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\sqrt[6]{-1}-\frac{i\sqrt[3]{b}x}{\sqrt[3]{a}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}}-\frac{3\sqrt[3]{-1}2^{2/3}\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\left(\cdots\right.}{\sqrt[3]{-1}+2^{2\cdots}}\right.}{\sqrt{3}\sqrt[3]{b}\sqrt{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2^(2/3)*a^(1/3) - 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[a
```

```
[Out] (2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((2*3^(
1/4)*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(-1)^(1/6) - (I*b^(1/3
)*x)/a^(1/3)]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)
*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/Sqrt[(a^(1/3) + (-
1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))] - (3*(-1)^(1/3)*2
^(2/3)*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[1 - (b^(1/3)*x)/a^(1/3) + (b
^(2/3)*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)
), ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))
*a^(1/3))]], (-1)^(1/3)])/((-1)^(1/3) + 2^(2/3))))/(Sqrt[3]*b^(1/
3)*Sqrt[a + b*x^3])
```

Maple [F] time = 0.279, size = 0, normalized size = 0.

$$\int 1\left(2^{\frac{2}{3}}\sqrt[3]{a}-2\sqrt[3]{b}x\right)\left(2^{\frac{2}{3}}\sqrt[3]{a}+\sqrt[3]{b}x\right)^{-1}\frac{1}{\sqrt{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),
```

```
[Out] int((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*
x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{2\,b^{\frac{1}{3}}x-2^{\frac{2}{3}}a^{\frac{1}{3}}}{\sqrt{bx^3+a}\left(b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(2*b^(1/3)*x - 2^(2/3)*a^(1/3))/(sqrt(b*x^3 + a)*(b^(1/3)*x + 2^(2/3)
```

```
[Out] -integrate((2*b^(1/3)*x - 2^(2/3)*a^(1/3))/(sqrt(b*x^3 + a)*(b^(1
/3)*x + 2^(2/3)*a^(1/3))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(2*b^(1/3)*x - 2^(2/3)*a^(1/3))/(sqrt(b*x^3 + a)*(b^(1/3)*x + 2^(2/3)
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\left(-\frac{2^{\frac{2}{3}}\sqrt[3]{a}}{2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{a+bx^3}+\sqrt[3]{b}x\sqrt{a+bx^3}}\right)dx-\int\frac{2\sqrt[3]{b}x}{2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{a+bx^3}+\sqrt[3]{b}x\sqrt{a+bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2**(2/3)*a**(1/3)-2*b**(1/3)*x)/(2**(2/3)*a**(1/3)+b**(1/3)*x)/(b*x
```

```
[Out] -Integral(-2**(2/3)*a**(1/3)/(2**(2/3)*a**(1/3)*sqrt(a + b*x**3)
+ b**(1/3)*x*sqrt(a + b*x**3)), x) - Integral(2*b**(1/3)*x/(2**(2
/3)*a**(1/3)*sqrt(a + b*x**3) + b**(1/3)*x*sqrt(a + b*x**3)), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(2*b^(1/3)*x - 2^(2/3)*a^(1/3))/(sqrt(b*x^3 + a)*(b^(1/3)*x + 2^(2/3)
```

```
[Out] Exception raised: TypeError
```

3.25 $\int \frac{2^{2/3} \sqrt[3]{a}+2 \sqrt[3]{b} x}{\left(2^{2/3} \sqrt[3]{a}-\sqrt[3]{b} x\right) \sqrt{a-b x^3}} \, dx$

Optimal. Leaf size=65

$$-\frac{2\ 2^{2/3} \tan^{-1}\left(\frac{\sqrt{3} \sqrt[6]{a} \left(\sqrt[3]{a}-\sqrt[3]{2} \sqrt[3]{b} x\right)}{\sqrt{a-b x^3}}\right)}{\sqrt{3} \sqrt[6]{a} \sqrt[3]{b}}$$

```
[Out] (-2*2^(2/3)*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*b^(1/3)*x)
)/Sqrt[a - b*x^3]])/(Sqrt[3]*a^(1/6)*b^(1/3))
```

Rubi [A] time = 0.292361, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 55, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036

$$-\frac{2\ 2^{2/3} \tan^{-1}\left(\frac{\sqrt{3} \sqrt[6]{a} \left(\sqrt[3]{a}-\sqrt[3]{2} \sqrt[3]{b} x\right)}{\sqrt{a-b x^3}}\right)}{\sqrt{3} \sqrt[6]{a} \sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(2^(2/3)*a^(1/3) + 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^
```

```
[Out] (-2*2^(2/3)*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*b^(1/3)*x)
)/Sqrt[a - b*x^3]])/(Sqrt[3]*a^(1/6)*b^(1/3))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((2**(2/3)*a**(1/3)+2*b**(1/3)*x)/(2**(2/3)*a**(1/3)-
b**(1/3)*x)/(-b*x**3+a)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 2.06184, size = 336, normalized size = 5.17

$$\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(-\frac{2\left(\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}F\left(\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}}+\frac{\sqrt[3]{-1}2^{2/3}\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}\sqrt{\frac{3b^{2/3}x^2}{a^{2/3}}+\frac{3\sqrt[3]{b}}{\sqrt[3]{a}}}}{}\right)}{\sqrt[3]{b}\sqrt{a-bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2^(2/3)*a^(1/3) + 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[a
```

```
[Out] (2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((-2*((
-1)^(1/3)*a^(1/3) + b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) + (-1)^(
1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqr
t[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]],
(-1)^(1/3)])/Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/
3))*a^(1/3))] + ((-1)^(1/3)*2^(2/3)*(1 + (-1)^(1/3))*a^(1/3)*Sqrt
[3 + (3*b^(1/3)*x)/a^(1/3) + (3*b^(2/3)*x^2)/a^(2/3)]*EllipticPi[
(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)), ArcSin[Sqrt[(a^(1/3) - (-1)^(
2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/((-1)^
(1/3) + 2^(2/3))))/(b^(1/3)*Sqrt[a - b*x^3])
```

Maple [F] time = 0.27, size = 0, normalized size = 0.

$$\int 1\left(2^{\frac{2}{3}}\sqrt[3]{a}+2\sqrt[3]{b}x\right)\left(2^{\frac{2}{3}}\sqrt[3]{a}-\sqrt[3]{b}x\right)^{-1}\frac{1}{\sqrt{-bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-
b*x^3+a)^(1/2),x)
```

```
[Out] int((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b
*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{2\,b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}}{\sqrt{-bx^3+a}\left(b^{\frac{1}{3}}x-2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(2*b^(1/3)*x + 2^(2/3)*a^(1/3))/(sqrt(-b*x^3 + a)*(b^(1/3)*x - 2^(2/3
```

```
[Out] -integrate((2*b^(1/3)*x + 2^(2/3)*a^(1/3))/(sqrt(-b*x^3 + a)*(b^(
1/3)*x - 2^(2/3)*a^(1/3))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(2*b^(1/3)*x + 2^(2/3)*a^(1/3))/(sqrt(-b*x^3 + a)*(b^(1/3)*x - 2^(2/3
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{2^{\frac{2}{3}}\sqrt[3]{a}}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt[3]{b}x\sqrt{a-bx^3}}\,dx-\int \frac{2\sqrt[3]{b}x}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt[3]{b}x\sqrt{a-bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2**(2/3)*a**(1/3)+2*b**(1/3)*x)/(2**(2/3)*a**(1/3)-b**(1/3)*x)/(-
b*x**3+a)**(1/2),x)
```

```
[Out] -Integral(2**(2/3)*a**(1/3)/(-2**(2/3)*a**(1/3)*sqrt(a - b*x**3)
+ b**(1/3)*x*sqrt(a - b*x**3)), x) - Integral(2*b**(1/3)*x/(-2**(
2/3)*a**(1/3)*sqrt(a - b*x**3) + b**(1/3)*x*sqrt(a - b*x**3)), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(2*b^(1/3)*x + 2^(2/3)*a^(1/3))/(sqrt(-b*x^3 + a)*(b^(1/3)*x - 2^(2/3
```

```
[Out] Exception raised: TypeError
```

3.26 $\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

Optimal. Leaf size=66

$$-\frac{2\ 2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{bx^3-a}}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

```
[Out] (-2*2^(2/3)*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*b^(1/3)*x
))/Sqrt[-a + b*x^3]])/(Sqrt[3]*a^(1/6)*b^(1/3))
```

Rubi [A] time = 0.301321, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 56, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$

$$-\frac{2\ 2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{bx^3-a}}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(2^(2/3)*a^(1/3) + 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[-
a + b*x^3]),x]
```

```
[Out] (-2*2^(2/3)*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*b^(1/3)*x
))/Sqrt[-a + b*x^3]])/(Sqrt[3]*a^(1/6)*b^(1/3))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((2**(2/3)*a**(1/3)+2*b**(1/3)*x)/(2**(2/3)*a**(1/3)-
b**(1/3)*x)/(b*x**3-a)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 1.51534, size = 390, normalized size = 5.91

$$-\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(2\left(\sqrt[3]{-1}+2^{2/3}\right)\left(\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}F\left(\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)-\sqrt[3]{-1}2^{2/3}\sqrt{\cdots}\right.}{\left(\sqrt[3]{-1}+2^{2/3}\right)\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2^(2/3)*a^(1/3) + 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[-
a + b*x^3]),x]
```

```
[Out] (-2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(2*((-
1)^(1/3) + 2^(2/3))*((-1)^(1/3)*a^(1/3) + b^(1/3)*x)*Sqrt[((-1)^(
1/3)*(a^(1/3) + (-1)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))
]*EllipticF[ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-
1)^(1/3))*a^(1/3))]], (-1)^(1/3)] - (-1)^(1/3)*2^(2/3)*Sqrt[3]*(1
 + (-1)^(1/3))*a^(1/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1
+ (-1)^(1/3))*a^(1/3))]*Sqrt[1 + (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x
^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)), ArcSi
n[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3)
)]], (-1)^(1/3)]))/(((-1)^(1/3) + 2^(2/3))*b^(1/3)*Sqrt[(a^(1/3)
- (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[-a + b*x
^3])
```

Maple [F] time = 0.135, size = 0, normalized size = 0.

$$\int 1\left(2^{\frac{2}{3}}\sqrt[3]{a}+2\sqrt[3]{b}x\right)\left(2^{\frac{2}{3}}\sqrt[3]{a}-\sqrt[3]{b}x\right)^{-1}\frac{1}{\sqrt{bx^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-
a)^(1/2),x)
```

```
[Out] int((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*
x^3-a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{2\,b^{\frac{1}{3}}x + 2^{\frac{2}{3}}a^{\frac{1}{3}}}{\sqrt{bx^3 - a}\left(b^{\frac{1}{3}}x - 2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(2*b^(1/3)*x + 2^(2/3)*a^(1/3))/(sqrt(b*x^3 - a)*(b^(1/3)*x - 2^(2/3)
```

```
[Out] -integrate((2*b^(1/3)*x + 2^(2/3)*a^(1/3))/(sqrt(b*x^3 - a)*(b^(1
/3)*x - 2^(2/3)*a^(1/3))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(2*b^(1/3)*x + 2^(2/3)*a^(1/3))/(sqrt(b*x^3 - a)*(b^(1/3)*x - 2^(2/3)
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{2^{\frac{2}{3}}\sqrt[3]{a}}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{-a + bx^3} + \sqrt[3]{b}x\sqrt{-a + bx^3}}\,dx - \int \frac{2\sqrt[3]{b}x}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{-a + bx^3} + \sqrt[3]{b}x\sqrt{-a + bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2**(2/3)*a**(1/3)+2*b**(1/3)*x)/(2**(2/3)*a**(1/3)-b**(1/3)*x)/(b*x
a)**(1/2),x)
```

```
[Out] -Integral(2**(2/3)*a**(1/3)/(-2**(2/3)*a**(1/3)*sqrt(-a + b*x**3)
 + b**(1/3)*x*sqrt(-a + b*x**3)), x) - Integral(2*b**(1/3)*x/(-2*
*(2/3)*a**(1/3)*sqrt(-a + b*x**3) + b**(1/3)*x*sqrt(-a + b*x**3))
, x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(2*b^(1/3)*x + 2^(2/3)*a^(1/3))/(sqrt(b*x^3 - a)*(b^(1/3)*x - 2^(2/3)
```

```
[Out] Exception raised: TypeError
```

3.27 $\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

Optimal. Leaf size=66

$$\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

```
[Out] (2*2^(2/3)*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) + 2^(1/3)*b^(1/3)*x)
)/Sqrt[-a - b*x^3]])/(Sqrt[3]*a^(1/6)*b^(1/3))
```

Rubi [A] time = 0.289981, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 56, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036

$$\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(2^(2/3)*a^(1/3) - 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[-
a - b*x^3]),x]
```

```
[Out] (2*2^(2/3)*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) + 2^(1/3)*b^(1/3)*x)
)/Sqrt[-a - b*x^3]])/(Sqrt[3]*a^(1/6)*b^(1/3))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((2**(2/3)*a**(1/3)-2*b**(1/3)*x)/(2**(2/3)*a**(1/3)+b**(1/3)*x)
b*x**3-a)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 1.44697, size = 375, normalized size = 5.68

$$-\frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\sqrt[3]{-1}2^{2/3}\sqrt{3}\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-\right.\right.}{\left(\sqrt[3]{-1}+2^{2/3}\right)\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{-a-bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2^(2/3)*a^(1/3) - 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[-
a - b*x^3]),x]
```

```
[Out] (-2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((-2*(
(-1)^(1/3) + 2^(2/3))*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(-1)^
(1/6) - (I*b^(1/3)*x)/a^(1/3)]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (
-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/3
^(1/4) + (-1)^(1/3)*2^(2/3)*Sqrt[3]*(1 + (-1)^(1/3))*a^(1/3)*Sqrt
[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqr
t[1 - (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(I*
Sqrt[3])/((-1)^(1/3) + 2^(2/3)), ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3
)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)]))/(((-1)^(
1/3) + 2^(2/3))*b^(1/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1
 + (-1)^(1/3))*a^(1/3))]*Sqrt[-a - b*x^3])
```

Maple [F] time = 0.129, size = 0, normalized size = 0.

$$\int 1\left(2^{\frac{2}{3}}\sqrt[3]{a}-2\sqrt[3]{b}x\right)\left(2^{\frac{2}{3}}\sqrt[3]{a}+\sqrt[3]{b}x\right)^{-1}\frac{1}{\sqrt{-bx^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-
b*x^3-a)^(1/2),x)
```

```
[Out] int((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b
*x^3-a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{2\, b^{\frac{1}{3}} x - 2^{\frac{2}{3}} a^{\frac{1}{3}}}{\sqrt{-b x^3 - a} \left(b^{\frac{1}{3}} x + 2^{\frac{2}{3}} a^{\frac{1}{3}}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(2*b^(1/3)*x - 2^(2/3)*a^(1/3))/(sqrt(-b*x^3 - a)*(b^(1/3)*x + 2^(2/3
```

```
[Out] -integrate((2*b^(1/3)*x - 2^(2/3)*a^(1/3))/(sqrt(-b*x^3 - a)*(b^(
1/3)*x + 2^(2/3)*a^(1/3))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(2*b^(1/3)*x - 2^(2/3)*a^(1/3))/(sqrt(-b*x^3 - a)*(b^(1/3)*x + 2^(2/3
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \left(-\frac{2^{\frac{2}{3}} \sqrt[3]{a}}{2^{\frac{2}{3}} \sqrt[3]{a} \sqrt{-a - b x^3} + \sqrt[3]{b} x \sqrt{-a - b x^3}}\right) dx - \int \frac{2 \sqrt[3]{b} x}{2^{\frac{2}{3}} \sqrt[3]{a} \sqrt{-a - b x^3} + \sqrt[3]{b} x \sqrt{-a - b x^3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2**(2/3)*a**(1/3)-2*b**(1/3)*x)/(2**(2/3)*a**(1/3)+b**(1/3)*x)/(-
b*x**3-a)**(1/2),x)
```

```
[Out] -Integral(-2**(2/3)*a**(1/3)/(2**(2/3)*a**(1/3)*sqrt(-a - b*x**3)
 + b**(1/3)*x*sqrt(-a - b*x**3)), x) - Integral(2*b**(1/3)*x/(2**
(2/3)*a**(1/3)*sqrt(-a - b*x**3) + b**(1/3)*x*sqrt(-a - b*x**3)),
 x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(2*b^(1/3)*x - 2^(2/3)*a^(1/3))/(sqrt(-b*x^3 - a)*(b^(1/3)*x + 2^(2/3
```

```
[Out] Exception raised: TypeError
```

3.28 $\int \frac{c-2dx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx$

Optimal. Leaf size=49

$$\frac{2\tan^{-1}\left(\frac{\sqrt{3}\sqrt{c}(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{\sqrt{3}\sqrt{c}d}$$

```
[Out] (2*ArcTan[(Sqrt[3]*Sqrt[c]*(c + 2*d*x))/Sqrt[c^3 + 4*d^3*x^3]])/(
Sqrt[3]*Sqrt[c]*d)
```

Rubi [A] time = 0.20442, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$\frac{2\tan^{-1}\left(\frac{\sqrt{3}\sqrt{c}(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{\sqrt{3}\sqrt{c}d}$$

Antiderivative was successfully verified.

```
[In]  Int[(c - 2*d*x)/((c + d*x)*Sqrt[c^3 + 4*d^3*x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[3]*Sqrt[c]*(c + 2*d*x))/Sqrt[c^3 + 4*d^3*x^3]])/(
Sqrt[3]*Sqrt[c]*d)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-2*d*x+c)/(d*x+c)/(4*d**3*x**3+c**3)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 1.9414, size = 373, normalized size = 7.61

$$\frac{\sqrt[6]{2}\sqrt{\frac{\sqrt[3]{2}c+2dx}{\left(1+\sqrt[3]{-1}\right)c}}\left(2\sqrt{\frac{\sqrt[3]{-2}c-2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}\left(\sqrt[3]{-1}\left(2+\sqrt[3]{-2}\right)c-2\left(\sqrt[3]{-1}+2^{2/3}\right)dx\right)F\left(\sin^{-1}\left(\frac{\sqrt{\frac{\sqrt[3]{2}c+2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}}{\sqrt[6]{2}}\right)\middle|\sqrt[3]{-1}\right)-\sqrt[3]{-1}2^{2/3}}{\left(2+\sqrt[3]{-2}\right)d\sqrt{\frac{\sqrt[3]{2}c+2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}\sqrt{c^3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(c - 2*d*x)/((c + d*x)*Sqrt[c^3 + 4*d^3*x^3]),x]
```

```
[Out] (2^(1/6)*Sqrt[(2^(1/3)*c + 2*d*x)/((1 + (-1)^(1/3))*c)]*(2*Sqrt[(
(-2)^(1/3)*c - 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]*((-1)^(1/3
)*(2 + (-2)^(1/3))*c - 2*((-1)^(1/3) + 2^(2/3))*d*x)*EllipticF[Ar
cSin[Sqrt[(2^(1/3)*c + 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]/2^
(1/6)], (-1)^(1/3)] - (-1)^(1/3)*2^(2/3)*Sqrt[3]*(1 + (-1)^(1/3))
*c*Sqrt[(2^(1/3)*c + 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]*Sqrt
[2^(2/3) - (2*2^(1/3)*d*x)/c + (4*d^2*x^2)/c^2]*EllipticPi[(I*2^(
1/3)*Sqrt[3])/(2 + (-2)^(1/3)), ArcSin[Sqrt[(2^(1/3)*c + 2*(-1)^(
2/3)*d*x)/((1 + (-1)^(1/3))*c)]/2^(1/6)], (-1)^(1/3)]))/((2 + (-2
)^(1/3))*d*Sqrt[(2^(1/3)*c + 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*
c)]*Sqrt[c^3 + 4*d^3*x^3])
```

Maple [C] time = 0.058, size = 889, normalized size = 18.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-2*d*x+c)/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x)
```

```
[Out] -4*((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^
(1/2)*2^(1/3))*c/d)*((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/
((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/
2)*2^(1/3))*c/d))^(1/2)*((x+1/2*2^(1/3)*c/d)/((1/4*2^(1/3)+1/4*I*
3^(1/2)*2^(1/3))*c/d+1/2*2^(1/3)*c/d))^(1/2)*((x-(1/4*2^(1/3)-1/4
*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d
-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)/(4*d^3*x^3+c^3)^
(1/2)*EllipticF(((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/
4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2
^(1/3))*c/d))^(1/2),(((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d-(1/
4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)
```

```
*2^(1/3))*c/d+1/2*2^(1/3)*c/d))^(1/2))+6*c/d*((1/4*2^(1/3)-1/4*I*
3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)*((x
-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)-1/4*I*3^(
1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)
*((x+1/2*2^(1/3)*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+1/
2*2^(1/3)*c/d))^(1/2)*((x-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d
)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(
1/2)*2^(1/3))*c/d))^(1/2)/(4*d^3*x^3+c^3)^(1/2)/((1/4*2^(1/3)+1/4
*I*3^(1/2)*2^(1/3))*c/d+c/d)*EllipticPi(((x-(1/4*2^(1/3)+1/4*I*3^
(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4
*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2),((1/4*2^(1/3)+1/4*I*3
^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/
4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+c/d),(((1/4*2^(1/3)+1/4*I*3^
(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4
*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+1/2*2^(1/3)*c/d))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{2\,dx - c}{\sqrt{4\,d^3x^3 + c^3}(dx + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(2*d*x - c)/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)),x, algorithm="maxima")
```

```
[Out] -integrate((2*d*x - c)/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)), x)
```

Fricas [A] time = 0.392157, size = 1, normalized size = 0.02

$$\left[\frac{\sqrt{3}\sqrt{-\frac{1}{c}}\log\left(\frac{2\,d^6x^6-36\,cd^5x^5-18\,c^2d^4x^4+28\,c^3d^3x^3+18\,c^4d^2x^2-c^6-\sqrt{3}\left(4\,cd^4x^4-10\,c^2d^3x^3-18\,c^3d^2x^2-8\,c^4dx-c^5\right)\sqrt{4\,d^3x^3+c^3}\sqrt{-\frac{1}{c}}}{d^6x^6+6\,cd^5x^5+15\,c^2d^4x^4+20\,c^3d^3x^3+15\,c^4d^2x^2+6\,c^5dx+c^6}\right)}{6\,d},\right.$$
$$\left.-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,d^3x^3-6\,cd^2x^2-6\,c^2dx-c^3\right)\sqrt{c}}{3\sqrt{4\,d^3x^3+c^3}\left(2\,cdx+c^2\right)}\right)}{3\sqrt{c}d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(2*d*x - c)/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)),x, algorithm="fricas")
```

```
[Out] [1/6*sqrt(3)*sqrt(-1/c)*log((2*d^6*x^6 - 36*c*d^5*x^5 - 18*c^2*d^
4*x^4 + 28*c^3*d^3*x^3 + 18*c^4*d^2*x^2 - c^6 - sqrt(3)*(4*c*d^4*
x^4 - 10*c^2*d^3*x^3 - 18*c^3*d^2*x^2 - 8*c^4*d*x - c^5)*sqrt(4*d
^3*x^3 + c^3)*sqrt(-1/c))/(d^6*x^6 + 6*c*d^5*x^5 + 15*c^2*d^4*x^4
 + 20*c^3*d^3*x^3 + 15*c^4*d^2*x^2 + 6*c^5*d*x + c^6))/d, -1/3*sq
rt(3)*arctan(1/3*sqrt(3)*(2*d^3*x^3 - 6*c*d^2*x^2 - 6*c^2*d*x - c
^3)*sqrt(c)/(sqrt(4*d^3*x^3 + c^3)*(2*c*d*x + c^2)))/(sqrt(c)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\left(-\frac{c}{c\sqrt{c^3+4d^3x^3}+dx\sqrt{c^3+4d^3x^3}}\right)dx-\int\frac{2dx}{c\sqrt{c^3+4d^3x^3}+dx\sqrt{c^3+4d^3x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-2*d*x+c)/(d*x+c)/(4*d**3*x**3+c**3)**(1/2),x)
```

```
[Out] -Integral(-c/(c*sqrt(c**3 + 4*d**3*x**3) + d*x*sqrt(c**3 + 4*d**3
*x**3)), x) - Integral(2*d*x/(c*sqrt(c**3 + 4*d**3*x**3) + d*x*sq
rt(c**3 + 4*d**3*x**3)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int-\frac{2\,dx-c}{\sqrt{4\,d^3x^3+c^3}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(2*d*x - c)/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)),x, algorithm="giac")
```

```
[Out] integrate(-(2*d*x - c)/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)), x)
```

3.29 $\int \frac{2+3x}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=158

$$\frac{2\left(2-3\ 2^{2/3}\right)\tan^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{x^3+1}}\right)}{3\sqrt{3}}+\frac{2\left(3+2\sqrt[3]{2}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

```
[Out] (2*(2 - 3*2^(2/3))*ArcTan[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[1 + x^3]
])/(3*Sqrt[3]) + (2*(3 + 2*2^(1/3))*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqr
t[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3
] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 +
x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rubi [A] time = 0.331028, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$\frac{2\left(2-3\ 2^{2/3}\right)\tan^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{x^3+1}}\right)}{3\sqrt{3}}+\frac{2\left(3+2\sqrt[3]{2}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + 3*x)/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```


```
[Out] (2*(2 - 3*2^(2/3))*ArcTan[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[1 + x^3]
])/(3*Sqrt[3]) + (2*(3 + 2*2^(1/3))*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqr
t[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3
] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 +
x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rubi in Sympy [A] time = 144.928, size = 481, normalized size = 3.04

$$\frac{2\sqrt{\frac{x^2-x+1}{\left(x+1+\sqrt{3}\right)^2}}\left(-3\cdot 2^{\frac{2}{3}}+2\right)\sqrt{-\sqrt{3}+2}\,(x+1)\operatorname{atan}\left(\frac{3^{\frac{3}{4}}\sqrt{1+\sqrt[3]{2}}\sqrt{-4\sqrt{3}+8}\sqrt{-\frac{\left(-x-1+\sqrt{3}\right)^2}{\left(x+1+\sqrt{3}\right)^2}+1}}{6\sqrt{-1+\sqrt[3]{2}}\sqrt{\frac{\left(-x-1+\sqrt{3}\right)^2}{\left(x+1+\sqrt{3}\right)^2}-4\sqrt{3}+7}}\right)}{\sqrt{\frac{x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{-1+\sqrt[3]{2}}\left(1+\sqrt[3]{2}\right)^{\frac{3}{2}}\sqrt{-4\sqrt{3}+8}\sqrt{x^3+1}} + \frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{x^2-x+1}{\left(x+1+\sqrt{3}\right)^2}}\left(1+3\sqrt{3}\right)\sqrt{\sqrt{3}+2}\,(x+1)\,F\left(\operatorname{asin}\left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{3\sqrt{\frac{x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{x^3+1}\left(-2^{\frac{2}{3}}+1+\sqrt{3}\right)} + \frac{4\sqrt[4]{3}\sqrt{\frac{x^2-x+1}{\left(x+1+\sqrt{3}\right)^2}}\left(-3\cdot 2^{\frac{2}{3}}+2\right)\sqrt{-\sqrt{3}+2}\,(x+1)\left(\frac{\left(-2^{\frac{2}{3}}+1+\sqrt{3}\right)^2}{\left(-1+2^{\frac{2}{3}}+\sqrt{3}\right)^2};\operatorname{asin}\left(\frac{-x-1+\sqrt{3}}{x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{-4\sqrt{3}+7}\sqrt{x^3+1}\left(-2^{\frac{2}{3}}+1+\sqrt{3}\right)\left(-\sqrt{3}-2^{\frac{2}{3}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2+3*x)/(2**(2/3)+x)/(x**3+1)**(1/2),x)
```

```
[Out] 2*sqrt((x**2 - x + 1)/(x + 1 + sqrt(3))**2)*(-3*2**(2/3) + 2)*sqr
t(-sqrt(3) + 2)*(x + 1)*atan(3**(3/4)*sqrt(1 + 2**(1/3))*sqrt(-4*
sqrt(3) + 8)*sqrt(-(-x - 1 + sqrt(3))**2/(x + 1 + sqrt(3))**2 + 1
)/(6*sqrt(-1 + 2**(1/3))*sqrt((-x - 1 + sqrt(3))**2/(x + 1 + sqrt
(3))**2 - 4*sqrt(3) + 7)))/(sqrt((x + 1)/(x + 1 + sqrt(3))**2)*sq
rt(-1 + 2**(1/3))*(1 + 2**(1/3))**(3/2)*sqrt(-4*sqrt(3) + 8)*sqrt
(x**3 + 1)) + 2*3**(3/4)*sqrt((x**2 - x + 1)/(x + 1 + sqrt(3))**2
)*(1 + 3*sqrt(3))*sqrt(sqrt(3) + 2)*(x + 1)*elliptic_f(asin((x -
sqrt(3) + 1)/(x + 1 + sqrt(3))), -7 - 4*sqrt(3))/(3*sqrt((x + 1)/
(x + 1 + sqrt(3))**2)*sqrt(x**3 + 1)*(-2**(2/3) + 1 + sqrt(3))) +
 4*3**(1/4)*sqrt((x**2 - x + 1)/(x + 1 + sqrt(3))**2)*(-3*2**(2/3
) + 2)*sqrt(-sqrt(3) + 2)*(x + 1)*elliptic_pi((-2**(2/3) + 1 + sq
rt(3))**2/(-1 + 2**(2/3) + sqrt(3))**2, asin((-x - 1 + sqrt(3))/(
x + 1 + sqrt(3))), -7 - 4*sqrt(3))/(sqrt((x + 1)/(x + 1 + sqrt(3)
)**2)*sqrt(-4*sqrt(3) + 7)*sqrt(x**3 + 1)*(-2**(2/3) + 1 + sqrt(3
))*(-sqrt(3) - 2**(2/3) + 1))
```

Mathematica [C] time = 0.656551, size = 336, normalized size = 2.13

$$\frac{2\sqrt[6]{2}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\left(3\sqrt{2ix+\sqrt{3}-i}\left(\left(3\sqrt[3]{2}+4i\sqrt{3}+i\sqrt[3]{2}\sqrt{3}\right)x+i\sqrt[3]{2}\sqrt{3}-2i\sqrt{3}-3\sqrt[3]{2}-6\right)F\left(\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)-4\sqrt{3}\cdots}{\sqrt{3}\left(i+2i2^{2/3}+\sqrt{3}\right)\sqrt{-2ix+\sqrt{3}+i}\sqrt{x^3+1\cdots}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(2 + 3*x)/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*2^(1/6)*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(3*Sqrt[-I + Sqrt[3]
 + (2*I)*x]*(-6 - 3*2^(1/3) - (2*I)*Sqrt[3] + I*2^(1/3)*Sqrt[3] +
 (3*2^(1/3) + (4*I)*Sqrt[3] + I*2^(1/3)*Sqrt[3])*x)*EllipticF[Arc
Sin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(
3*I + Sqrt[3])] - 4*Sqrt[3]*(-3 + 2^(1/3))*Sqrt[I + Sqrt[3] - (2*
I)*x]*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt[3])/(I + (2*I)*2^(2/3)
 + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))
], (2*Sqrt[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(I + (2*I)*2^(2/3) + S
qrt[3])*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 + x^3])
```

Maple [B] time = 0.044, size = 262, normalized size = 1.7

$$2\,\frac{\left(2-3\,2^{2/3}\right)\left(3/2-i/2\sqrt{3}\right)}{\sqrt{x^3+1}\left(2^{2/3}-1\right)}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}\mathit{EllipticPi}\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},\frac{-3/2+i/2\sqrt{3}}{2^{2/3}-1}\right.$$

$$+6\,\frac{3/2-i/2\sqrt{3}}{\sqrt{x^3+1}}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}\mathit{EllipticF}\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},\sqrt{\frac{-3/2+i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2+3*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x)
```

```
[Out] 2*(2-3*2^(2/3))*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(
1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1
/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)/(2^(2/3)-
1)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/
2))/(2^(2/3)-1),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)
)+6*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2
-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2)
)/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2
-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2))
)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3\,x+2}{\sqrt{x^3+1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x + 2)/(sqrt(x^3 + 1)*(x + 2^(2/3))),x, algorithm="maxima")
```

```
[Out] integrate((3*x + 2)/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{3\,x+2}{\sqrt{x^3+1}\left(x+2^{\frac{2}{3}}\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x + 2)/(sqrt(x^3 + 1)*(x + 2^(2/3))),x, algorithm="fricas")
```

```
[Out] integral((3*x + 2)/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3x+2}{\sqrt{(x+1)\,(x^2-x+1)}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2+3*x)/(2**(2/3)+x)/(x**3+1)**(1/2),x)
```

```
[Out] Integral((3*x + 2)/(sqrt((x + 1)*(x**2 - x + 1))*(x + 2**(2/3))),
 x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3\,x+2}{\sqrt{x^3+1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x + 2)/(sqrt(x^3 + 1)*(x + 2^(2/3))),x, algorithm="giac")
```

```
[Out] integrate((3*x + 2)/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

3.30 $\int \frac{2+3x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=173

$$\frac{2\left(3-2\sqrt[3]{2}\right)\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2\left(2+3\,2^{2/3}\right)\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{3}}$$

```
[Out] (-2*(2 + 3*2^(2/3))*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[1 - x^3
]])/(3*Sqrt[3]) + (2*(3 - 2*2^(1/3))*Sqrt[2 + Sqrt[3]]*(1 - x)*Sq
rt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[
3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 -
 x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rubi [A] time = 0.392474, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{2\left(3-2\sqrt[3]{2}\right)\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2\left(2+3\,2^{2/3}\right)\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + 3*x)/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (-2*(2 + 3*2^(2/3))*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[1 - x^3
]])/(3*Sqrt[3]) + (2*(3 - 2*2^(1/3))*Sqrt[2 + Sqrt[3]]*(1 - x)*Sq
rt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[
3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 -
 x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rubi in Sympy [A] time = 142.026, size = 481, normalized size = 2.78

$$-\frac{2\sqrt{\frac{x^2+x+1}{\left(-x+1+\sqrt{3}\right)^2}}\left(2+3\cdot 2^{\frac{2}{3}}\right)\sqrt{-\sqrt{3}+2}\,(-x+1)\operatorname{atan}\left(\frac{3^{\frac{3}{4}}\sqrt{1+\sqrt[3]{2}}\sqrt{1-\frac{\left(x-1+\sqrt{3}\right)^2}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{-4\sqrt{3}+8}}{6\sqrt{-1+\sqrt[3]{2}}\sqrt{-4\sqrt{3}+7+\frac{\left(x-1+\sqrt{3}\right)^2}{\left(-x+1+\sqrt{3}\right)^2}}}\right)}{\sqrt{\frac{-x+1}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{-1+\sqrt[3]{2}}\left(1+\sqrt[3]{2}\right)^{\frac{3}{2}}\sqrt{-4\sqrt{3}+8}\sqrt{-x^3+1}}+\frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{x^2+x+1}{\left(-x+1+\sqrt{3}\right)^2}}\left(5+3\sqrt{3}\right)\sqrt{\sqrt{3}+2}\,(-x+1)\,F\left(\operatorname{asin}\left(\frac{-x-\sqrt{3}+1}{-x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{3\sqrt{\frac{-x+1}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{-x^3+1}\left(-2^{\frac{2}{3}}+1+\sqrt{3}\right)}-\frac{4\sqrt[4]{3}\sqrt{\frac{x^2+x+1}{\left(-x+1+\sqrt{3}\right)^2}}\left(2+3\cdot 2^{\frac{2}{3}}\right)\sqrt{-\sqrt{3}+2}\,(-x+1)\left(\frac{\left(-2^{\frac{2}{3}}+1+\sqrt{3}\right)^2}{\left(-1+2^{\frac{2}{3}}+\sqrt{3}\right)^2};\operatorname{asin}\left(\frac{x-1+\sqrt{3}}{-x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt{\frac{-x+1}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{-4\sqrt{3}+7}\sqrt{-x^3+1}\left(-2^{\frac{2}{3}}+1+\sqrt{3}\right)\left(-\sqrt{3}-2^{\frac{2}{3}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2+3*x)/(2**(2/3)-x)/(-x**3+1)**(1/2),x)
```

```
[Out] -2*sqrt((x**2 + x + 1)/(-x + 1 + sqrt(3))**2)*(2 + 3*2**(2/3))*sq
rt(-sqrt(3) + 2)*(-x + 1)*atan(3**(3/4)*sqrt(1 + 2**(1/3))*sqrt(1
 - (x - 1 + sqrt(3))**2/(-x + 1 + sqrt(3))**2)*sqrt(-4*sqrt(3) +
8)/(6*sqrt(-1 + 2**(1/3))*sqrt(-4*sqrt(3) + 7 + (x - 1 + sqrt(3))
**2/(-x + 1 + sqrt(3))**2)))/(sqrt((-x + 1)/(-x + 1 + sqrt(3))**2
)*sqrt(-1 + 2**(1/3))*(1 + 2**(1/3))**(3/2)*sqrt(-4*sqrt(3) + 8)*
sqrt(-x**3 + 1)) + 2*3**(3/4)*sqrt((x**2 + x + 1)/(-x + 1 + sqrt(
3))**2)*(5 + 3*sqrt(3))*sqrt(sqrt(3) + 2)*(-x + 1)*elliptic_f(asi
n((-x - sqrt(3) + 1)/(-x + 1 + sqrt(3))), -7 - 4*sqrt(3))/(3*sqrt
((-x + 1)/(-x + 1 + sqrt(3))**2)*sqrt(-x**3 + 1)*(-2**(2/3) + 1 +
 sqrt(3))) - 4*3**(1/4)*sqrt((x**2 + x + 1)/(-x + 1 + sqrt(3))**2
)*(2 + 3*2**(2/3))*sqrt(-sqrt(3) + 2)*(-x + 1)*elliptic_pi((-2**(
2/3) + 1 + sqrt(3))**2/(-1 + 2**(2/3) + sqrt(3))**2, asin((x - 1
+ sqrt(3))/(-x + 1 + sqrt(3))), -7 - 4*sqrt(3))/(sqrt((-x + 1)/(-
x + 1 + sqrt(3))**2)*sqrt(-4*sqrt(3) + 7)*sqrt(-x**3 + 1)*(-2**(2
/3) + 1 + sqrt(3))*(-sqrt(3) - 2**(2/3) + 1))
```

Mathematica [C] time = 0.674258, size = 335, normalized size = 1.94

$$\frac{2\sqrt[6]{2}\sqrt{-\frac{i(x-1)}{\sqrt{3}+3i}}\left(4\sqrt{3}\left(3+\sqrt[3]{2}\right)\sqrt{2ix+\sqrt{3}+i}\sqrt{x^2+x+1}\left(\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)-3i\sqrt{-2ix+\sqrt{3}-i}\left(\left(-3i\sqrt{}\right.\right.\right.}{\sqrt{3}\left(i+2i2^{2/3}+\sqrt{3}\right)\sqrt{2ix+\sqrt{3}+i}\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(2 + 3*x)/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*2^(1/6)*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*((-3*I)*Sqrt[-I
+ Sqrt[3] - (2*I)*x]*(-6*I - (3*I)*2^(1/3) + 2*Sqrt[3] - 2^(1/3)*
Sqrt[3] + ((-3*I)*2^(1/3) + 4*Sqrt[3] + 2^(1/3)*Sqrt[3])*x)*Ellip
ticF[ArcSin[Sqrt[I + Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sq
rt[3])/(3*I + Sqrt[3])] + 4*Sqrt[3]*(3 + 2^(1/3))*Sqrt[I + Sqrt[3
] + (2*I)*x]*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt[3])/(I + (2*I)*
2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] + (2*I)*x]/(Sqrt[2]*3
^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(I + (2*I)*2^(2
/3) + Sqrt[3])*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[1 - x^3])
```

Maple [A] time = 0.042, size = 257, normalized size = 1.5

$$\frac{-\frac{2i}{3}\left(-2-3\,2^{2/3}\right)\sqrt{3}}{-\frac{1}{2}+\frac{i}{2}\sqrt{3}-2^{\frac{2}{3}}}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}EllipticPi\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\frac{}{-\frac{1}{2}}\right.$$
$$+2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}EllipticF\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)\frac{1}{\sqrt{-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2+3*x)/(2^(2/3)-x)/(-x^3+1)^(1/2),x)
```

```
[Out] -2/3*I*(-2-3*2^(2/3))*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(
1/2)*((-1+x)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2)
)*3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)-2^(2/3))*Elli
pticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1
/2)/(-1/2+1/2*I*3^(1/2)-2^(2/3)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))
^(1/2))+2*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+
x)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))
^(1/2)/(-x^3+1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/
2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{3\,x+2}{\sqrt{-x^3+1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(3*x + 2)/(sqrt(-x^3 + 1)*(x - 2^(2/3))),x, algorithm="maxima")
```

```
[Out] -integrate((3*x + 2)/(sqrt(-x^3 + 1)*(x - 2^(2/3))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{3\,x+2}{\sqrt{-x^3+1}\left(x-2^{\frac{2}{3}}\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(3*x + 2)/(sqrt(-x^3 + 1)*(x - 2^(2/3))),x, algorithm="fricas")
```

```
[Out] integral(-(3*x + 2)/(sqrt(-x^3 + 1)*(x - 2^(2/3))), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{3x}{x\sqrt{-x^3+1}-2^{\frac{2}{3}}\sqrt{-x^3+1}}\,dx-\int\frac{2}{x\sqrt{-x^3+1}-2^{\frac{2}{3}}\sqrt{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2+3*x)/(2**(2/3)-x)/(-x**3+1)**(1/2),x)
```

```
[Out] -Integral(3*x/(x*sqrt(-x**3 + 1) - 2**(2/3)*sqrt(-x**3 + 1)), x)
- Integral(2/(x*sqrt(-x**3 + 1) - 2**(2/3)*sqrt(-x**3 + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int-\frac{3\,x+2}{\sqrt{-x^3+1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(3*x + 2)/(sqrt(-x^3 + 1)*(x - 2^(2/3))),x, algorithm="giac")
```

```
[Out] integrate(-(3*x + 2)/(sqrt(-x^3 + 1)*(x - 2^(2/3))), x)
```

3.31 $\int \frac{2+3x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=176

$$\frac{2\left(3-2\sqrt[3]{2}\right)\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}} - \frac{2\left(2+3\ 2^{2/3}\right)\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{x^3-1}}\right)}{3\sqrt{3}}$$

```
[Out] (-2*(2 + 3*2^(2/3))*ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[-1 + x
^3]])/(3*Sqrt[3]) + (2*(3 - 2*2^(1/3))*Sqrt[2 - Sqrt[3]]*(1 - x)*
Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqr
t[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-(
(1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Rubi [A] time = 0.360584, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{2\left(3-2\sqrt[3]{2}\right)\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}} - \frac{2\left(2+3\ 2^{2/3}\right)\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{x^3-1}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + 3*x)/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```


```
[Out] (-2*(2 + 3*2^(2/3))*ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[-1 + x
^3]])/(3*Sqrt[3]) + (2*(3 - 2*2^(1/3))*Sqrt[2 - Sqrt[3]]*(1 - x)*
Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqr
t[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-(
(1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Rubi in Sympy [A] time = 148.138, size = 452, normalized size = 2.57

$$-\frac{\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(2+3\cdot 2^{\frac{2}{3}}\right)(-x+1)\operatorname{atanh}\left(\frac{3^{\frac{3}{4}}\sqrt{1+\sqrt[3]{2}}\sqrt{\sqrt{3}+2}\sqrt{-\frac{\left(-x+1+\sqrt{3}\right)^2}{\left(x-1+\sqrt{3}\right)^2}+1}}{3\sqrt{-1+\sqrt[3]{2}}\sqrt{\frac{\left(-x+1+\sqrt{3}\right)^2}{\left(x-1+\sqrt{3}\right)^2}+4\sqrt{3}+7}}\right)}{\sqrt{\frac{x-1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{-1+\sqrt[3]{2}}\left(1+\sqrt[3]{2}\right)^{\frac{3}{2}}\sqrt{x^3-1}}$$
$$+\frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(-3\sqrt{3}+5\right)\sqrt{-\sqrt{3}+2}\,(-x+1)\,F\left(\operatorname{asin}\left(\frac{-x+1+\sqrt{3}}{-x-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{3\sqrt{\frac{x-1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\left(-\sqrt{3}-2^{\frac{2}{3}}+1\right)}$$
$$+\frac{4\sqrt[4]{3}\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(2+3\cdot 2^{\frac{2}{3}}\right)\sqrt{\sqrt{3}+2}\,(-x+1)\left(\frac{\left(-1+2^{\frac{2}{3}}+\sqrt{3}\right)^2}{\left(-2^{\frac{2}{3}}+1+\sqrt{3}\right)^2};\operatorname{asin}\left(\frac{-x+1+\sqrt{3}}{x-1+\sqrt{3}}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt{\frac{x-1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{4\sqrt{3}+7}\sqrt{x^3-1}\left(-2^{\frac{2}{3}}+1+\sqrt{3}\right)\left(-\sqrt{3}-2^{\frac{2}{3}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2+3*x)/(2**(2/3)-x)/(x**3-1)**(1/2),x)
```

```
[Out] -sqrt((x**2 + x + 1)/(-x - sqrt(3) + 1)**2)*(2 + 3*2**(2/3))*(-x
+ 1)*atanh(3**(3/4)*sqrt(1 + 2**(1/3))*sqrt(sqrt(3) + 2)*sqrt(-(-
x + 1 + sqrt(3))**2/(x - 1 + sqrt(3))**2 + 1)/(3*sqrt(-1 + 2**(1/
3))*sqrt((-x + 1 + sqrt(3))**2/(x - 1 + sqrt(3))**2 + 4*sqrt(3) +
 7)))/(sqrt((x - 1)/(-x - sqrt(3) + 1)**2)*sqrt(-1 + 2**(1/3))*(1
 + 2**(1/3))**(3/2)*sqrt(x**3 - 1)) + 2*3**(3/4)*sqrt((x**2 + x +
 1)/(-x - sqrt(3) + 1)**2)*(-3*sqrt(3) + 5)*sqrt(-sqrt(3) + 2)*(-
x + 1)*elliptic_f(asin((-x + 1 + sqrt(3))/(-x - sqrt(3) + 1)), -7
 + 4*sqrt(3))/(3*sqrt((x - 1)/(-x - sqrt(3) + 1)**2)*sqrt(x**3 -
1)*(-sqrt(3) - 2**(2/3) + 1)) + 4*3**(1/4)*sqrt((x**2 + x + 1)/(-
x - sqrt(3) + 1)**2)*(2 + 3*2**(2/3))*sqrt(sqrt(3) + 2)*(-x + 1)*
elliptic_pi((-1 + 2**(2/3) + sqrt(3))**2/(-2**(2/3) + 1 + sqrt(3)
)**2, asin((-x + 1 + sqrt(3))/(x - 1 + sqrt(3))), -7 + 4*sqrt(3))
/(sqrt((x - 1)/(-x - sqrt(3) + 1)**2)*sqrt(4*sqrt(3) + 7)*sqrt(x*
*3 - 1)*(-2**(2/3) + 1 + sqrt(3))*(-sqrt(3) - 2**(2/3) + 1))
```

Mathematica [C] time = 0.658621, size = 333, normalized size = 1.89

$$\frac{2\sqrt[6]{2}\sqrt{-\frac{i(x-1)}{\sqrt{3}+3i}}\left(4\sqrt{3}\left(3+\sqrt[3]{2}\right)\sqrt{2ix+\sqrt{3}+i}\sqrt{x^2+x+1}\left(\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)-3i\sqrt{-2ix+\sqrt{3}-i}\left(\left(-3i\sqrt[3]{\cdots}\right.\right.\right.}{\sqrt{3}\left(i+2i2^{2/3}+\sqrt{3}\right)\sqrt{2ix+\sqrt{3}+i}\sqrt{x^3-1\cdots}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(2 + 3*x)/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*2^(1/6)*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*((-3*I)*Sqrt[-I
+ Sqrt[3] - (2*I)*x]*(-6*I - (3*I)*2^(1/3) + 2*Sqrt[3] - 2^(1/3)*
Sqrt[3] + ((-3*I)*2^(1/3) + 4*Sqrt[3] + 2^(1/3)*Sqrt[3])*x)*Ellip
ticF[ArcSin[Sqrt[I + Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sq
rt[3])/(3*I + Sqrt[3])] + 4*Sqrt[3]*(3 + 2^(1/3))*Sqrt[I + Sqrt[3
] + (2*I)*x]*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt[3])/(I + (2*I)*
2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] + (2*I)*x]/(Sqrt[2]*3
^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(I + (2*I)*2^(2
/3) + Sqrt[3])*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[-1 + x^3])
```

Maple [A] time = 0.033, size = 266, normalized size = 1.5

$$2\,{\frac { \left(-2-3\,{2}^{2/3} \right) \left(-3/2-i/2\sqrt {3} \right) }{\sqrt {{x}^{3}-1} \left(-{2}^{2/3}+1 \right) }\sqrt {{\frac {-1+x}{-3/2-i/2\sqrt {3}}}}\sqrt {{\frac {x+1/2-i/2\sqrt {3}}{3/2-i/2\sqrt {3}}}}\sqrt {{\frac {x+1/2+i/2\sqrt {3}}{3/2+i/2\sqrt {3}}}}{\it EllipticPi} \left(\sqrt {{\frac {-1+x}{-3/2-i/2\sqrt {3}}}},{\frac {3/2+i/ \ldots}{-{2}^{2/3} \ldots}} \right.}$$

$$-6\,{\frac {-3/2-i/2\sqrt {3}}{\sqrt {{x}^{3}-1}}\sqrt {{\frac {-1+x}{-3/2-i/2\sqrt {3}}}}\sqrt {{\frac {x+1/2-i/2\sqrt {3}}{3/2-i/2\sqrt {3}}}}\sqrt {{\frac {x+1/2+i/2\sqrt {3}}{3/2+i/2\sqrt {3}}}}{\it EllipticF} \left(\sqrt {{\frac {-1+x}{-3/2-i/2\sqrt {3}}}},\sqrt {{\frac {3/2+i/2\sqrt {3}}{3/2-i/2\sqrt {3}}}} \right) }$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2+3*x)/(2^(2/3)-x)/(x^3-1)^(1/2),x)
```

```
[Out] 2*(-2-3*2^(2/3))*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)
))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/
2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)/(-2^(2/
3)+1)*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3
^(1/2))/(-2^(2/3)+1),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1
/2))-6*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*(
(x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^
(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((-1+x)
/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1
/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{3\,x + 2}{\sqrt{x^{3} - 1}{\left(x - 2^{\frac{2}{3}}\right)}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(3*x + 2)/(sqrt(x^3 - 1)*(x - 2^(2/3))),x, algorithm="maxima")
```

```
[Out] -integrate((3*x + 2)/(sqrt(x^3 - 1)*(x - 2^(2/3))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{3\,x+2}{\sqrt{x^3-1}\left(x-2^{\frac{2}{3}}\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(3*x + 2)/(sqrt(x^3 - 1)*(x - 2^(2/3))),x, algorithm="fricas")
```

```
[Out] integral(-(3*x + 2)/(sqrt(x^3 - 1)*(x - 2^(2/3))), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{3x}{x\sqrt{x^3-1} - 2^{\frac{2}{3}}\sqrt{x^3-1}}\,dx - \int \frac{2}{x\sqrt{x^3-1} - 2^{\frac{2}{3}}\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2+3*x)/(2**(2/3)-x)/(x**3-1)**(1/2),x)
```

```
[Out] -Integral(3*x/(x*sqrt(x**3 - 1) - 2**(2/3)*sqrt(x**3 - 1)), x) -
Integral(2/(x*sqrt(x**3 - 1) - 2**(2/3)*sqrt(x**3 - 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{3\,x+2}{\sqrt{x^3-1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(3*x + 2)/(sqrt(x^3 - 1)*(x - 2^(2/3))),x, algorithm="giac")
```

```
[Out] integrate(-(3*x + 2)/(sqrt(x^3 - 1)*(x - 2^(2/3))), x)
```

3.32 $\int \frac{2+3x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=169

$$\frac{2\left(2-3\ 2^{2/3}\right)\tanh^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{-x^3-1}}\right)}{3\sqrt{3}}+\frac{2\left(3+2\sqrt[3]{2}\right)\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

```
[Out] (2*(2 - 3*2^(2/3))*ArcTanh[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[-1 - x^
3]])/(3*Sqrt[3]) + (2*(3 + 2*2^(1/3))*Sqrt[2 - Sqrt[3]]*(1 + x)*S
qrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt
[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-((
1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```

Rubi [A] time = 0.372021, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{2\left(2-3\ 2^{2/3}\right)\tanh^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{-x^3-1}}\right)}{3\sqrt{3}}+\frac{2\left(3+2\sqrt[3]{2}\right)\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + 3*x)/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```


```
[Out] (2*(2 - 3*2^(2/3))*ArcTanh[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[-1 - x^
3]])/(3*Sqrt[3]) + (2*(3 + 2*2^(1/3))*Sqrt[2 - Sqrt[3]]*(1 + x)*S
qrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt
[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-((
1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```

Rubi in Sympy [A] time = 144.832, size = 462, normalized size = 2.73

$$\frac{\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(-3\cdot 2^{\frac{2}{3}}+2\right)(x+1)\operatorname{atanh}\left(\frac{3^{\frac{3}{4}}\sqrt{1+\sqrt[3]{2}}\sqrt{1-\frac{\left(x+1+\sqrt{3}\right)^2}{\left(-x-1+\sqrt{3}\right)^2}}\sqrt{\sqrt{3}+2}}{3\sqrt{-1+\sqrt[3]{2}}\sqrt{4\sqrt{3}+7+\frac{\left(x+1+\sqrt{3}\right)^2}{\left(-x-1+\sqrt{3}\right)^2}}}\right)}{\sqrt{\frac{-x-1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-1+\sqrt[3]{2}}\left(1+\sqrt[3]{2}\right)^{\frac{3}{2}}\sqrt{-x^3-1}}+\frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(-3\sqrt{3}+1\right)\sqrt{-\sqrt{3}+2}\,(x+1)\,F\left(\operatorname{asin}\left(\frac{x+1+\sqrt{3}}{x-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{3\sqrt{\frac{-x-1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\left(-\sqrt{3}-2^{\frac{2}{3}}+1\right)}-\frac{4\sqrt[4]{3}\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(-3\cdot 2^{\frac{2}{3}}+2\right)\sqrt{\sqrt{3}+2}\,(x+1)\left(\frac{\left(-1+2^{\frac{2}{3}}+\sqrt{3}\right)^2}{\left(-2^{\frac{2}{3}}+1+\sqrt{3}\right)^2};\operatorname{asin}\left(\frac{x+1+\sqrt{3}}{-x-1+\sqrt{3}}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt{\frac{-x-1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{4\sqrt{3}+7}\sqrt{-x^3-1}\left(-2^{\frac{2}{3}}+1+\sqrt{3}\right)\left(-\sqrt{3}-2^{\frac{2}{3}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2+3*x)/(2**(2/3)+x)/(-x**3-1)**(1/2),x)
```

```
[Out] sqrt((x**2 - x + 1)/(x - sqrt(3) + 1)**2)*(-3*2**(2/3) + 2)*(x +
1)*atanh(3**(3/4)*sqrt(1 + 2**(1/3))*sqrt(1 - (x + 1 + sqrt(3))**
2/(-x - 1 + sqrt(3))**2)*sqrt(sqrt(3) + 2)/(3*sqrt(-1 + 2**(1/3))
*sqrt(4*sqrt(3) + 7 + (x + 1 + sqrt(3))**2/(-x - 1 + sqrt(3))**2)
))/(sqrt((-x - 1)/(x - sqrt(3) + 1)**2)*sqrt(-1 + 2**(1/3))*(1 +
2**(1/3))**(3/2)*sqrt(-x**3 - 1)) + 2*3**(3/4)*sqrt((x**2 - x + 1
)/(x - sqrt(3) + 1)**2)*(-3*sqrt(3) + 1)*sqrt(-sqrt(3) + 2)*(x +
1)*elliptic_f(asin((x + 1 + sqrt(3))/(x - sqrt(3) + 1)), -7 + 4*s
qrt(3))/(3*sqrt((-x - 1)/(x - sqrt(3) + 1)**2)*sqrt(-x**3 - 1)*(-
sqrt(3) - 2**(2/3) + 1)) - 4*3**(1/4)*sqrt((x**2 - x + 1)/(x - sq
rt(3) + 1)**2)*(-3*2**(2/3) + 2)*sqrt(sqrt(3) + 2)*(x + 1)*ellipt
ic_pi((-1 + 2**(2/3) + sqrt(3))**2/(-2**(2/3) + 1 + sqrt(3))**2,
asin((x + 1 + sqrt(3))/(-x - 1 + sqrt(3))), -7 + 4*sqrt(3))/(sqrt
((-x - 1)/(x - sqrt(3) + 1)**2)*sqrt(4*sqrt(3) + 7)*sqrt(-x**3 -
1)*(-2**(2/3) + 1 + sqrt(3))*(-sqrt(3) - 2**(2/3) + 1))
```

Mathematica [C] time = 0.677004, size = 338, normalized size = 2.

$$\frac{2\sqrt[6]{2}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\left(3\sqrt{2ix+\sqrt{3}-i}\left(\left(3\sqrt[3]{2}+4i\sqrt{3}+i\sqrt[3]{2}\sqrt{3}\right)x+i\sqrt[3]{2}\sqrt{3}-2i\sqrt{3}-3\sqrt[3]{2}-6\right)F\left(\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)-4\sqrt{3}}{\sqrt{3}\left(i+2i2^{2/3}+\sqrt{3}\right)\sqrt{-2ix+\sqrt{3}+i}\sqrt{-x^3-}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(2 + 3*x)/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*2^(1/6)*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(3*Sqrt[-I + Sqrt[3]
 + (2*I)*x]*(-6 - 3*2^(1/3) - (2*I)*Sqrt[3] + I*2^(1/3)*Sqrt[3] +
 (3*2^(1/3) + (4*I)*Sqrt[3] + I*2^(1/3)*Sqrt[3])*x)*EllipticF[Arc
Sin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(
3*I + Sqrt[3])] - 4*Sqrt[3]*(-3 + 2^(1/3))*Sqrt[I + Sqrt[3] - (2*
I)*x]*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt[3])/(I + (2*I)*2^(2/3)
 + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))
], (2*Sqrt[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(I + (2*I)*2^(2/3) + S
qrt[3])*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[-1 - x^3])
```

Maple [A] time = 0.034, size = 253, normalized size = 1.5

$$\frac{-\frac{2i}{3}\left(2-3\,2^{2/3}\right)\sqrt{3}}{\frac{1}{2}+\frac{i}{2}\sqrt{3}+2^{\frac{2}{3}}}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\,EllipticPi\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\frac{i}{\frac{1}{2}+\frac{i}{2}}\right.$$
$$-2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\,EllipticF\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)\frac{1}{\sqrt{-x^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2+3*x)/(2^(2/3)+x)/(-x^3-1)^(1/2),x)
```

```
[Out] -2/3*I*(2-3*2^(2/3))*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1
/2)*((1+x)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3
^(1/2))^(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2)+2^(2/3))*Elliptic
Pi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/
(1/2+1/2*I*3^(1/2)+2^(2/3)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2)
)-2*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2
+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(
-x^3-1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1
/2))^(1/2),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3\,x+2}{\sqrt{-x^3-1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x + 2)/(sqrt(-x^3 - 1)*(x + 2^(2/3))),x, algorithm="maxima")
```

```
[Out] integrate((3*x + 2)/(sqrt(-x^3 - 1)*(x + 2^(2/3))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{3\,x+2}{\sqrt{-x^3-1}\left(x+2^{\frac{2}{3}}\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x + 2)/(sqrt(-x^3 - 1)*(x + 2^(2/3))),x, algorithm="fricas")
```

```
[Out] integral((3*x + 2)/(sqrt(-x^3 - 1)*(x + 2^(2/3))), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3x+2}{\sqrt{-(x+1)(x^2-x+1)}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2+3*x)/(2**(2/3)+x)/(-x**3-1)**(1/2),x)
```

```
[Out] Integral((3*x + 2)/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 2**(2/3)))
, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3\,x+2}{\sqrt{-x^3-1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x + 2)/(sqrt(-x^3 - 1)*(x + 2^(2/3))),x, algorithm="giac")
```

```
[Out] integrate((3*x + 2)/(sqrt(-x^3 - 1)*(x + 2^(2/3))), x)
```

3.33 $\int \frac{e+fx}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=159

$$\frac{2\left(e-2^{2/3}f\right)\tan^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{x^3+1}}\right)}{3\sqrt{3}}+\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\sqrt[3]{2}e+f\right)F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

```
[Out] (2*(e - 2^(2/3)*f)*ArcTan[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[1 + x^3]
])/(3*Sqrt[3]) + (2*Sqrt[2 + Sqrt[3]]*(2^(1/3)*e + f)*(1 + x)*Sqr
t[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3
] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 +
x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rubi [A] time = 0.375366, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$\frac{2\left(e-2^{2/3}f\right)\tan^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{x^3+1}}\right)}{3\sqrt{3}}+\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\sqrt[3]{2}e+f\right)F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(e + f*x)/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*(e - 2^(2/3)*f)*ArcTan[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[1 + x^3]
])/(3*Sqrt[3]) + (2*Sqrt[2 + Sqrt[3]]*(2^(1/3)*e + f)*(1 + x)*Sqr
t[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3
] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 +
x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rubi in Sympy [A] time = 144.083, size = 483, normalized size = 3.04

$$\frac{2\sqrt{\frac{x^2-x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{-\sqrt{3}+2}\left(e-2^{\frac{2}{3}}f\right)(x+1)\operatorname{atan}\left(\frac{3^{\frac{3}{4}}\sqrt{1+\sqrt[3]{2}}\sqrt{-4\sqrt{3}+8}\sqrt{-\frac{\left(-x-1+\sqrt{3}\right)^2}{\left(x+1+\sqrt{3}\right)^2}+1}}{6\sqrt{-1+\sqrt[3]{2}}\sqrt{\frac{\left(-x-1+\sqrt{3}\right)^2}{\left(x+1+\sqrt{3}\right)^2}-4\sqrt{3}+7}}\right)}{\sqrt{\frac{x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{-1+\sqrt[3]{2}}\left(1+\sqrt[3]{2}\right)^{\frac{3}{2}}\sqrt{-4\sqrt{3}+8}\sqrt{x^3+1}}$$
$$+\frac{4\sqrt[4]{3}\sqrt{\frac{x^2-x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{-\sqrt{3}+2}\left(e-2^{\frac{2}{3}}f\right)(x+1)\left(\frac{\left(-2^{\frac{2}{3}}+1+\sqrt{3}\right)^2}{\left(-1+2^{\frac{2}{3}}+\sqrt{3}\right)^2};\operatorname{asin}\left(\frac{-x-1+\sqrt{3}}{x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{-4\sqrt{3}+7}\sqrt{x^3+1}\left(-2^{\frac{2}{3}}+1+\sqrt{3}\right)\left(-\sqrt{3}-2^{\frac{2}{3}}+1\right)}$$
$$-\frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{x^2-x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{\sqrt{3}+2}\left(e-f\left(1+\sqrt{3}\right)\right)(x+1)F\left(\operatorname{asin}\left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{3\sqrt{\frac{x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{x^3+1}\left(-2^{\frac{2}{3}}+1+\sqrt{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x+e)/(2**(2/3)+x)/(x**3+1)**(1/2),x)
```

```
[Out] 2*sqrt((x**2 - x + 1)/(x + 1 + sqrt(3))**2)*sqrt(-sqrt(3) + 2)*(e
 - 2**(2/3)*f)*(x + 1)*atan(3**(3/4)*sqrt(1 + 2**(1/3))*sqrt(-4*s
qrt(3) + 8)*sqrt(-(-x - 1 + sqrt(3))**2/(x + 1 + sqrt(3))**2 + 1)
/(6*sqrt(-1 + 2**(1/3))*sqrt((-x - 1 + sqrt(3))**2/(x + 1 + sqrt(
3))**2 - 4*sqrt(3) + 7)))/(sqrt((x + 1)/(x + 1 + sqrt(3))**2)*sqr
t(-1 + 2**(1/3))*(1 + 2**(1/3))**(3/2)*sqrt(-4*sqrt(3) + 8)*sqrt(
x**3 + 1)) + 4*3**(1/4)*sqrt((x**2 - x + 1)/(x + 1 + sqrt(3))**2)
*sqrt(-sqrt(3) + 2)*(e - 2**(2/3)*f)*(x + 1)*elliptic_pi((-2**(2/
3) + 1 + sqrt(3))**2/(-1 + 2**(2/3) + sqrt(3))**2, asin((-x - 1 +
 sqrt(3))/(x + 1 + sqrt(3))), -7 - 4*sqrt(3))/(sqrt((x + 1)/(x +
1 + sqrt(3))**2)*sqrt(-4*sqrt(3) + 7)*sqrt(x**3 + 1)*(-2**(2/3) +
 1 + sqrt(3))*(-sqrt(3) - 2**(2/3) + 1)) - 2*3**(3/4)*sqrt((x**2
- x + 1)/(x + 1 + sqrt(3))**2)*sqrt(sqrt(3) + 2)*(e - f*(1 + sqrt
(3)))*(x + 1)*elliptic_f(asin((x - sqrt(3) + 1)/(x + 1 + sqrt(3))
), -7 - 4*sqrt(3))/(3*sqrt((x + 1)/(x + 1 + sqrt(3))**2)*sqrt(x**
3 + 1)*(-2**(2/3) + 1 + sqrt(3)))
```

Mathematica [C] time = 0.710208, size = 340, normalized size = 2.14

$$\frac{2\sqrt[6]{2}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\left(f\sqrt{2ix+\sqrt{3}-i}\left(\left(3\sqrt[3]{2}+4i\sqrt{3}+i\sqrt[3]{2}\sqrt{3}\right)x+i\sqrt[3]{2}\sqrt{3}-2i\sqrt{3}-3\sqrt[3]{2}-6\right)F\left(\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)\middle|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)-2\sqrt{3}}{\sqrt{3}\left(i+2i2^{2/3}+\sqrt{3}\right)\sqrt{-2ix+\sqrt{3}+i}\sqrt{x^3+}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(e + f*x)/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*2^(1/6)*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(f*Sqrt[-I + Sqrt[3]
 + (2*I)*x]*(-6 - 3*2^(1/3) - (2*I)*Sqrt[3] + I*2^(1/3)*Sqrt[3] +
 (3*2^(1/3) + (4*I)*Sqrt[3] + I*2^(1/3)*Sqrt[3])*x)*EllipticF[Arc
Sin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(
3*I + Sqrt[3])] - 2*Sqrt[3]*(2^(1/3)*e - 2*f)*Sqrt[I + Sqrt[3] -
(2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt[3])/(I + (2*I)*2^(2
/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/
4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(I + (2*I)*2^(2/3)
+ Sqrt[3])*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 + x^3])
```

Maple [B] time = 0.038, size = 264, normalized size = 1.7

$$2\,{\frac {f \left(3/2-i/2\sqrt {3} \right) }{\sqrt {{x}^{3}+1}}\sqrt {{\frac {1+x}{3/2-i/2\sqrt {3}}}}\sqrt {{\frac {x-1/2-i/2\sqrt {3}}{-3/2-i/2\sqrt {3}}}}\sqrt {{\frac {x-1/2+i/2\sqrt {3}}{-3/2+i/2\sqrt {3}}}}{\it EllipticF} \left(\sqrt {{\frac {1+x}{3/2-i/2\sqrt {3}}}},\sqrt {{\frac {-3/2+i/2\sqrt {3}}{-3/2-i/2\sqrt {3}}}} \right) }$$
$$+2\,{\frac { \left(e-{2}^{2/3}f \right) \left(3/2-i/2\sqrt {3} \right) }{\sqrt {{x}^{3}+1} \left({2}^{2/3}-1 \right) }\sqrt {{\frac {1+x}{3/2-i/2\sqrt {3}}}}\sqrt {{\frac {x-1/2-i/2\sqrt {3}}{-3/2-i/2\sqrt {3}}}}\sqrt {{\frac {x-1/2+i/2\sqrt {3}}{-3/2+i/2\sqrt {3}}}}{\it EllipticPi} \left(\sqrt {{\frac {1+x}{3/2-i/2\sqrt {3}}}},{\frac {-3/2+i/2\sqrt {3}}{{2}^{2/3}-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x+e)/(2^(2/3)+x)/(x^3+1)^(1/2),x)
```

```
[Out] 2*f*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2
-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2)
)/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2
-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2))
)^(1/2))+2*(e-2^(2/3)*f)*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^
(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*
((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)/
(2^(2/3)-1)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/
2*I*3^(1/2))/(2^(2/3)-1),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2
)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\sqrt{x^3+1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x + e)/(sqrt(x^3 + 1)*(x + 2^(2/3))),x, algorithm="maxima")

[Out] integrate((f*x + e)/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x + e)/(sqrt(x^3 + 1)*(x + 2^(2/3))),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e + fx}{\sqrt{(x + 1)(x^2 - x + 1)}\left(x + 2^{\frac{2}{3}}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2**(2/3)+x)/(x**3+1)**(1/2),x)

[Out] Integral((e + f*x)/(sqrt((x + 1)*(x**2 - x + 1))*(x + 2**(2/3))),
 x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx + e}{\sqrt{x^3 + 1}\left(x + 2^{\frac{2}{3}}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x + e)/(sqrt(x^3 + 1)*(x + 2^(2/3))),x, algorithm="giac")

[Out] integrate((f*x + e)/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

3.34 $\int \frac{e+fx}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=175

$$-\frac{2\left(e+2^{2/3}f\right)\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{3}}-\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(\sqrt[3]{2}e-f\right)F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

```
[Out] (-2*(e + 2^(2/3)*f)*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[1 - x^3
]])/(3*Sqrt[3]) - (2*Sqrt[2 + Sqrt[3]]*(2^(1/3)*e - f)*(1 - x)*Sq
rt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[
3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 -
 x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rubi [A] time = 0.424549, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$-\frac{2\left(e+2^{2/3}f\right)\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{3}}-\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(\sqrt[3]{2}e-f\right)F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(e + f*x)/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```


```
[Out] (-2*(e + 2^(2/3)*f)*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[1 - x^3
]])/(3*Sqrt[3]) - (2*Sqrt[2 + Sqrt[3]]*(2^(1/3)*e - f)*(1 - x)*Sq
rt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[
3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 -
 x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rubi in Sympy [A] time = 148.262, size = 483, normalized size = 2.76

$$-\frac{2\sqrt{\frac{x^2+x+1}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{-\sqrt{3}+2}\left(e+2^{\frac{2}{3}}f\right)(-x+1)\operatorname{atan}\left(\frac{3^{\frac{3}{4}}\sqrt{1+\sqrt[3]{2}}\sqrt{1-\frac{\left(x-1+\sqrt{3}\right)^2}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{-4\sqrt{3}+8}}{6\sqrt{-1+\sqrt[3]{2}}\sqrt{-4\sqrt{3}+7+\frac{\left(x-1+\sqrt{3}\right)^2}{\left(-x+1+\sqrt{3}\right)^2}}}\right)}{\sqrt{\frac{-x+1}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{-1+\sqrt[3]{2}}\left(1+\sqrt[3]{2}\right)^{\frac{3}{2}}\sqrt{-4\sqrt{3}+8}\sqrt{-x^3+1}}$$

$$-\frac{4\sqrt[4]{3}\sqrt{\frac{x^2+x+1}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{-\sqrt{3}+2}\left(e+2^{\frac{2}{3}}f\right)(-x+1)\left(\frac{\left(-2^{\frac{2}{3}}+1+\sqrt{3}\right)^2}{\left(-1+2^{\frac{2}{3}}+\sqrt{3}\right)^2};\operatorname{asin}\left(\frac{x-1+\sqrt{3}}{-x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt{\frac{-x+1}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{-4\sqrt{3}+7}\sqrt{-x^3+1}\left(-2^{\frac{2}{3}}+1+\sqrt{3}\right)\left(-\sqrt{3}-2^{\frac{2}{3}}+1\right)}$$

$$+\frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{x^2+x+1}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{\sqrt{3}+2}(-x+1)\left(e+f+\sqrt{3}f\right)F\left(\operatorname{asin}\left(\frac{-x-\sqrt{3}+1}{-x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{3\sqrt{\frac{-x+1}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{-x^3+1}\left(-2^{\frac{2}{3}}+1+\sqrt{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x+e)/(2**(2/3)-x)/(-x**3+1)**(1/2),x)
```

```
[Out] -2*sqrt((x**2 + x + 1)/(-x + 1 + sqrt(3))**2)*sqrt(-sqrt(3) + 2)*
(e + 2**(2/3)*f)*(-x + 1)*atan(3**(3/4)*sqrt(1 + 2**(1/3))*sqrt(1
 - (x - 1 + sqrt(3))**2/(-x + 1 + sqrt(3))**2)*sqrt(-4*sqrt(3) +
8)/(6*sqrt(-1 + 2**(1/3))*sqrt(-4*sqrt(3) + 7 + (x - 1 + sqrt(3))
**2/(-x + 1 + sqrt(3))**2)))/(sqrt((-x + 1)/(-x + 1 + sqrt(3))**2
)*sqrt(-1 + 2**(1/3))*(1 + 2**(1/3))**(3/2)*sqrt(-4*sqrt(3) + 8)*
sqrt(-x**3 + 1)) - 4*3**(1/4)*sqrt((x**2 + x + 1)/(-x + 1 + sqrt(
3))**2)*sqrt(-sqrt(3) + 2)*(e + 2**(2/3)*f)*(-x + 1)*elliptic_pi(
(-2**(2/3) + 1 + sqrt(3))**2/(-1 + 2**(2/3) + sqrt(3))**2, asin((
x - 1 + sqrt(3))/(-x + 1 + sqrt(3))), -7 - 4*sqrt(3))/(sqrt((-x +
 1)/(-x + 1 + sqrt(3))**2)*sqrt(-4*sqrt(3) + 7)*sqrt(-x**3 + 1)*(
-2**(2/3) + 1 + sqrt(3))*(-sqrt(3) - 2**(2/3) + 1)) + 2*3**(3/4)*
sqrt((x**2 + x + 1)/(-x + 1 + sqrt(3))**2)*sqrt(sqrt(3) + 2)*(-x
+ 1)*(e + f + sqrt(3)*f)*elliptic_f(asin((-x - sqrt(3) + 1)/(-x +
 1 + sqrt(3))), -7 - 4*sqrt(3))/(3*sqrt((-x + 1)/(-x + 1 + sqrt(3
))**2)*sqrt(-x**3 + 1)*(-2**(2/3) + 1 + sqrt(3)))
```

Mathematica [C] time = 0.697047, size = 340, normalized size = 1.94

$$\frac{2\sqrt[6]{2}\sqrt{-\frac{i(x-1)}{\sqrt{3}+3i}}\left(2\sqrt{3}\sqrt{2ix+\sqrt{3}+i}\sqrt{x^2+x+1}\left(\sqrt[3]{2}e+2f\right)\left(\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)-if\sqrt{-2ix+\sqrt{3}-i}\left(\left(-3\right.\right.\right.}{\sqrt{3}\left(i+2i2^{2/3}+\sqrt{3}\right)\sqrt{2ix+\sqrt{3}+i}\sqrt{1-x}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(e + f*x)/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*2^(1/6)*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*((-I)*f*Sqrt[-I
+ Sqrt[3] - (2*I)*x]*(-6*I - (3*I)*2^(1/3) + 2*Sqrt[3] - 2^(1/3)*
Sqrt[3] + ((-3*I)*2^(1/3) + 4*Sqrt[3] + 2^(1/3)*Sqrt[3])*x)*Ellip
ticF[ArcSin[Sqrt[I + Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sq
rt[3])/(3*I + Sqrt[3])] + 2*Sqrt[3]*(2^(1/3)*e + 2*f)*Sqrt[I + Sq
rt[3] + (2*I)*x]*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt[3])/(I + (2
*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] + (2*I)*x]/(Sqrt[
2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(I + (2*I)*
2^(2/3) + Sqrt[3])*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[1 - x^3])
```

Maple [A] time = 0.039, size = 261, normalized size = 1.5

$$\frac{-\frac{2i}{3}\left(-e-2^{\frac{2}{3}}f\right)\sqrt{3}}{-\frac{1}{2}+\frac{i}{2}\sqrt{3}-2^{\frac{2}{3}}}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}EllipticPi\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\frac{}{-\frac{1}{2}+}\right.$$
$$+\frac{2i}{3}f\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}EllipticF\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)\frac{}{\sqrt{-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x+e)/(2^(2/3)-x)/(-x^3+1)^(1/2),x)
```

```
[Out] -2/3*I*(-e-2^(2/3)*f)*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(
1/2)*((-1+x)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2)
)*3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)-2^(2/3))*Elli
pticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1
/2)/(-1/2+1/2*I*3^(1/2)-2^(2/3)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))
^(1/2))+2/3*I*f*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(
(-1+x)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1
/2))^(1/2)/(-x^3+1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3
^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{fx+e}{\sqrt{-x^3+1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(f*x + e)/(sqrt(-x^3 + 1)*(x - 2^(2/3))),x, algorithm="maxima")
```

```
[Out] -integrate((f*x + e)/(sqrt(-x^3 + 1)*(x - 2^(2/3))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(f*x + e)/(sqrt(-x^3 + 1)*(x - 2^(2/3))),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{e}{x\sqrt{-x^3+1} - 2^{\frac{2}{3}}\sqrt{-x^3+1}}\,dx - \int \frac{fx}{x\sqrt{-x^3+1} - 2^{\frac{2}{3}}\sqrt{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2**(2/3)-x)/(-x**3+1)**(1/2),x)
```

```
[Out] -Integral(e/(x*sqrt(-x**3 + 1) - 2**(2/3)*sqrt(-x**3 + 1)), x) -
Integral(f*x/(x*sqrt(-x**3 + 1) - 2**(2/3)*sqrt(-x**3 + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{fx+e}{\sqrt{-x^3+1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(f*x + e)/(sqrt(-x^3 + 1)*(x - 2^(2/3))),x, algorithm="giac")
```

```
[Out] integrate(-(f*x + e)/(sqrt(-x^3 + 1)*(x - 2^(2/3))), x)
```

3.35 $\int \frac{e+fx}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=178

$$-\frac{2\left(e+2^{2/3}f\right)\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{x^3-1}}\right)}{3\sqrt{3}}-\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(\sqrt[3]{2}e-f\right)F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

```
[Out] (-2*(e + 2^(2/3)*f)*ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[-1 + x
^3]])/(3*Sqrt[3]) - (2*Sqrt[2 - Sqrt[3]]*(2^(1/3)*e - f)*(1 - x)*
Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqr
t[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-(
(1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Rubi [A] time = 0.378559, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{2\left(e+2^{2/3}f\right)\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{x^3-1}}\right)}{3\sqrt{3}}-\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(\sqrt[3]{2}e-f\right)F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(e + f*x)/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (-2*(e + 2^(2/3)*f)*ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[-1 + x
^3]])/(3*Sqrt[3]) - (2*Sqrt[2 - Sqrt[3]]*(2^(1/3)*e - f)*(1 - x)*
Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqr
t[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-(
(1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Rubi in Sympy [A] time = 148.028, size = 454, normalized size = 2.55

$$-\frac{\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(e+2^{\frac{2}{3}}f\right)(-x+1)\operatorname{atanh}\left(\frac{3^{\frac{3}{4}}\sqrt{1+\sqrt[3]{2}}\sqrt{\sqrt{3}+2}\sqrt{-\frac{\left(-x+1+\sqrt{3}\right)^2}{\left(x-1+\sqrt{3}\right)^2}+1}}{3\sqrt{-1+\sqrt[3]{2}}\sqrt{\frac{\left(-x+1+\sqrt{3}\right)^2}{\left(x-1+\sqrt{3}\right)^2}+4\sqrt{3}+7}}\right)}{\sqrt{\frac{x-1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{-1+\sqrt[3]{2}}\left(1+\sqrt[3]{2}\right)^{\frac{3}{2}}\sqrt{x^3-1}}+\frac{4\sqrt[4]{3}\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{\sqrt{3}+2}\left(e+2^{\frac{2}{3}}f\right)(-x+1)\,\Pi\left(\frac{\left(-1+2^{\frac{2}{3}}+\sqrt{3}\right)^2}{\left(-2^{\frac{2}{3}}+1+\sqrt{3}\right)^2};\operatorname{asin}\left(\frac{-x+1+\sqrt{3}}{x-1+\sqrt{3}}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt{\frac{x-1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{4\sqrt{3}+7}\sqrt{x^3-1}\left(-2^{\frac{2}{3}}+1+\sqrt{3}\right)\left(-\sqrt{3}-2^{\frac{2}{3}}+1\right)}+\frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{-\sqrt{3}+2}(-x+1)\left(e-\sqrt{3}f+f\right)F\left(\operatorname{asin}\left(\frac{-x+1+\sqrt{3}}{-x-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{3\sqrt{\frac{x-1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\left(-\sqrt{3}-2^{\frac{2}{3}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x+e)/(2**(2/3)-x)/(x**3-1)**(1/2),x)
```

```
[Out] -sqrt((x**2 + x + 1)/(-x - sqrt(3) + 1)**2)*(e + 2**(2/3)*f)*(-x
+ 1)*atanh(3**(3/4)*sqrt(1 + 2**(1/3))*sqrt(sqrt(3) + 2)*sqrt(-(-
x + 1 + sqrt(3))**2/(x - 1 + sqrt(3))**2 + 1)/(3*sqrt(-1 + 2**(1/
3))*sqrt((-x + 1 + sqrt(3))**2/(x - 1 + sqrt(3))**2 + 4*sqrt(3) +
 7)))/(sqrt((x - 1)/(-x - sqrt(3) + 1)**2)*sqrt(-1 + 2**(1/3))*(1
 + 2**(1/3))**(3/2)*sqrt(x**3 - 1)) + 4*3**(1/4)*sqrt((x**2 + x +
 1)/(-x - sqrt(3) + 1)**2)*sqrt(sqrt(3) + 2)*(e + 2**(2/3)*f)*(-x
 + 1)*elliptic_pi((-1 + 2**(2/3) + sqrt(3))**2/(-2**(2/3) + 1 + s
qrt(3))**2, asin((-x + 1 + sqrt(3))/(x - 1 + sqrt(3))), -7 + 4*sq
rt(3))/(sqrt((x - 1)/(-x - sqrt(3) + 1)**2)*sqrt(4*sqrt(3) + 7)*s
qrt(x**3 - 1)*(-2**(2/3) + 1 + sqrt(3))*(-sqrt(3) - 2**(2/3) + 1)
) + 2*3**(3/4)*sqrt((x**2 + x + 1)/(-x - sqrt(3) + 1)**2)*sqrt(-s
qrt(3) + 2)*(-x + 1)*(e - sqrt(3)*f + f)*elliptic_f(asin((-x + 1
+ sqrt(3))/(-x - sqrt(3) + 1)), -7 + 4*sqrt(3))/(3*sqrt((x - 1)/(
-x - sqrt(3) + 1)**2)*sqrt(x**3 - 1)*(-sqrt(3) - 2**(2/3) + 1))
```

Mathematica [C] time = 0.682271, size = 338, normalized size = 1.9

$$\frac{2\sqrt[6]{2}\sqrt{-\frac{i(x-1)}{\sqrt{3}+3i}}\left(2\sqrt{3}\sqrt{2ix+\sqrt{3}+i}\sqrt{x^2+x+1}\left(\sqrt[3]{2}e+2f\right)\Pi\left(\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)-if\sqrt{-2ix+\sqrt{3}-i}\left(\left(-3\right.\right.}{\sqrt{3}\left(i+2i2^{2/3}+\sqrt{3}\right)\sqrt{2ix+\sqrt{3}+i}\sqrt{x^3-}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(e + f*x)/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*2^(1/6)*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*((-I)*f*Sqrt[-I
+ Sqrt[3] - (2*I)*x]*(-6*I - (3*I)*2^(1/3) + 2*Sqrt[3] - 2^(1/3)*
Sqrt[3] + ((-3*I)*2^(1/3) + 4*Sqrt[3] + 2^(1/3)*Sqrt[3])*x)*Ellip
ticF[ArcSin[Sqrt[I + Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sq
rt[3])/(3*I + Sqrt[3])] + 2*Sqrt[3]*(2^(1/3)*e + 2*f)*Sqrt[I + Sq
rt[3] + (2*I)*x]*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt[3])/(I + (2
*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] + (2*I)*x]/(Sqrt[
2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(I + (2*I)*
2^(2/3) + Sqrt[3])*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[-1 + x^3])
```

Maple [A] time = 0.036, size = 270, normalized size = 1.5

$$2\,\frac{\left(-e-2^{2/3}f\right)\left(-3/2-i/2\sqrt{3}\right)}{\sqrt{x^3-1}\left(-2^{2/3}+1\right)}\sqrt{\frac{-1+x}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2-i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2+i/2\sqrt{3}}{3/2+i/2\sqrt{3}}}\mathit{EllipticPi}\left(\sqrt{\frac{-1+x}{-3/2-i/2\sqrt{3}}},\frac{3/2+i/}{-2^{2/3}}\right.$$
$$-2\,\frac{f\left(-3/2-i/2\sqrt{3}\right)}{\sqrt{x^3-1}}\sqrt{\frac{-1+x}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2-i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2+i/2\sqrt{3}}{3/2+i/2\sqrt{3}}}\mathit{EllipticF}\left(\sqrt{\frac{-1+x}{-3/2-i/2\sqrt{3}}},\sqrt{\frac{3/2+i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x+e)/(2^(2/3)-x)/(x^3-1)^(1/2),x)
```

```
[Out] 2*(-e-2^(2/3)*f)*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)
))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/
2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)/(-2^(2/
3)+1)*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3
^(1/2))/(-2^(2/3)+1),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1
/2))-2*f*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)
*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*
3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((-1+
x)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^
(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{fx+e}{\sqrt{x^3-1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(f*x + e)/(sqrt(x^3 - 1)*(x - 2^(2/3))),x, algorithm="maxima")
```

```
[Out] -integrate((f*x + e)/(sqrt(x^3 - 1)*(x - 2^(2/3))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(f*x + e)/(sqrt(x^3 - 1)*(x - 2^(2/3))),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{e}{x\sqrt{x^3-1}-2^{\frac{2}{3}}\sqrt{x^3-1}}\,dx-\int \frac{fx}{x\sqrt{x^3-1}-2^{\frac{2}{3}}\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x+e)/(2**(2/3)-x)/(x**3-1)**(1/2),x)
```

```
[Out] -Integral(e/(x*sqrt(x**3 - 1) - 2**(2/3)*sqrt(x**3 - 1)), x) - In
tegral(f*x/(x*sqrt(x**3 - 1) - 2**(2/3)*sqrt(x**3 - 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{fx+e}{\sqrt{x^3-1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(f*x + e)/(sqrt(x^3 - 1)*(x - 2^(2/3))),x, algorithm="giac")
```

```
[Out] integrate(-(f*x + e)/(sqrt(x^3 - 1)*(x - 2^(2/3))), x)
```

3.36 $\int \frac{e+fx}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=170

$$\frac{2\left(e-2^{2/3}f\right)\tanh^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{-x^3-1}}\right)}{3\sqrt{3}}+\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(\sqrt[3]{2}e+f\right)F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

```
[Out] (2*(e - 2^(2/3)*f)*ArcTanh[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[-1 - x^
3]])/(3*Sqrt[3]) + (2*Sqrt[2 - Sqrt[3]]*(2^(1/3)*e + f)*(1 + x)*S
qrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt
[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-((
1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```

Rubi [A] time = 0.37564, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{2\left(e-2^{2/3}f\right)\tanh^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{-x^3-1}}\right)}{3\sqrt{3}}+\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(\sqrt[3]{2}e+f\right)F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(e + f*x)/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```


```
[Out] (2*(e - 2^(2/3)*f)*ArcTanh[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[-1 - x^
3]])/(3*Sqrt[3]) + (2*Sqrt[2 - Sqrt[3]]*(2^(1/3)*e + f)*(1 + x)*S
qrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt
[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-((
1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```

Rubi in Sympy [A] time = 148.946, size = 464, normalized size = 2.73

$$\frac{\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(e-2^{\frac{2}{3}}f\right)(x+1)\operatorname{atanh}\left(\frac{3^{\frac{3}{4}}\sqrt{1+\sqrt[3]{2}}\sqrt{1-\frac{\left(x+1+\sqrt{3}\right)^2}{\left(-x-1+\sqrt{3}\right)^2}}\sqrt{\sqrt{3}+2}}{3\sqrt{-1+\sqrt[3]{2}}\sqrt{4\sqrt{3}+7+\frac{\left(x+1+\sqrt{3}\right)^2}{\left(-x-1+\sqrt{3}\right)^2}}}\right)}{\sqrt{\frac{-x-1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-1+\sqrt[3]{2}}\left(1+\sqrt[3]{2}\right)^{\frac{3}{2}}\sqrt{-x^3-1}}$$
$$-\frac{4\sqrt[4]{3}\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{\sqrt{3}+2}\left(e-2^{\frac{2}{3}}f\right)(x+1)\left(\frac{\left(-1+2^{\frac{2}{3}}+\sqrt{3}\right)^2}{\left(-2^{\frac{2}{3}}+1+\sqrt{3}\right)^2};\operatorname{asin}\left(\frac{x+1+\sqrt{3}}{-x-1+\sqrt{3}}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt{\frac{-x-1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{4\sqrt{3}+7}\sqrt{-x^3-1}\left(-2^{\frac{2}{3}}+1+\sqrt{3}\right)\left(-\sqrt{3}-2^{\frac{2}{3}}+1\right)}$$
$$-\frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-\sqrt{3}+2}\,(x+1)\left(e-f+\sqrt{3}f\right)F\left(\operatorname{asin}\left(\frac{x+1+\sqrt{3}}{x-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{3\sqrt{\frac{-x-1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\left(-\sqrt{3}-2^{\frac{2}{3}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x+e)/(2**(2/3)+x)/(-x**3-1)**(1/2),x)
```

```
[Out] sqrt((x**2 - x + 1)/(x - sqrt(3) + 1)**2)*(e - 2**(2/3)*f)*(x + 1
)*atanh(3**(3/4)*sqrt(1 + 2**(1/3))*sqrt(1 - (x + 1 + sqrt(3))**2
/(-x - 1 + sqrt(3))**2)*sqrt(sqrt(3) + 2)/(3*sqrt(-1 + 2**(1/3))*
sqrt(4*sqrt(3) + 7 + (x + 1 + sqrt(3))**2/(-x - 1 + sqrt(3))**2))
)/(sqrt((-x - 1)/(x - sqrt(3) + 1)**2)*sqrt(-1 + 2**(1/3))*(1 + 2
**(1/3))**(3/2)*sqrt(-x**3 - 1)) - 4*3**(1/4)*sqrt((x**2 - x + 1)
/(x - sqrt(3) + 1)**2)*sqrt(sqrt(3) + 2)*(e - 2**(2/3)*f)*(x + 1)
*elliptic_pi((-1 + 2**(2/3) + sqrt(3))**2/(-2**(2/3) + 1 + sqrt(3
))**2, asin((x + 1 + sqrt(3))/(-x - 1 + sqrt(3))), -7 + 4*sqrt(3)
)/(sqrt((-x - 1)/(x - sqrt(3) + 1)**2)*sqrt(4*sqrt(3) + 7)*sqrt(-
x**3 - 1)*(-2**(2/3) + 1 + sqrt(3))*(-sqrt(3) - 2**(2/3) + 1)) -
2*3**(3/4)*sqrt((x**2 - x + 1)/(x - sqrt(3) + 1)**2)*sqrt(-sqrt(3
) + 2)*(x + 1)*(e - f + sqrt(3)*f)*elliptic_f(asin((x + 1 + sqrt(
3))/(x - sqrt(3) + 1)), -7 + 4*sqrt(3))/(3*sqrt((-x - 1)/(x - sqr
t(3) + 1)**2)*sqrt(-x**3 - 1)*(-sqrt(3) - 2**(2/3) + 1))
```

Mathematica [C] time = 0.711946, size = 342, normalized size = 2.01

$$\frac{2\sqrt[6]{2}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\left(f\sqrt{2ix+\sqrt{3}-i}\left(\left(3\sqrt[3]{2}+4i\sqrt{3}+i\sqrt[3]{2}\sqrt{3}\right)x+i\sqrt[3]{2}\sqrt{3}-2i\sqrt{3}-3\sqrt[3]{2}-6\right)F\left(\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)-2\sqrt{3}}{\sqrt{3}\left(i+2i2^{2/3}+\sqrt{3}\right)\sqrt{-2ix+\sqrt{3}+i}\sqrt{-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(e + f*x)/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*2^(1/6)*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(f*Sqrt[-I + Sqrt[3]
 + (2*I)*x]*(-6 - 3*2^(1/3) - (2*I)*Sqrt[3] + I*2^(1/3)*Sqrt[3] +
 (3*2^(1/3) + (4*I)*Sqrt[3] + I*2^(1/3)*Sqrt[3])*x)*EllipticF[Arc
Sin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(
3*I + Sqrt[3])] - 2*Sqrt[3]*(2^(1/3)*e - 2*f)*Sqrt[I + Sqrt[3] -
(2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt[3])/(I + (2*I)*2^(2
/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/
4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(I + (2*I)*2^(2/3)
+ Sqrt[3])*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[-1 - x^3])
```

Maple [A] time = 0.035, size = 255, normalized size = 1.5

$$-\frac{2i}{3}f\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\,EllipticF\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)\frac{1}{\sqrt{-x}}$$
$$-\frac{\frac{2i}{3}\left(e-2^{\frac{2}{3}}f\right)\sqrt{3}}{\frac{1}{2}+\frac{i}{2}\sqrt{3}+2^{\frac{2}{3}}}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\,EllipticPi\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\frac{i\sqrt{}}{\frac{1}{2}+\frac{i}{2}\sqrt{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x+e)/(2^(2/3)+x)/(-x^3-1)^(1/2),x)
```

```
[Out] -2/3*I*f*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(
3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2
)/(-x^3-1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3
^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))-2/3*I*(e-2^(
2/3)*f)*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3
/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)
/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2)+2^(2/3))*EllipticPi(1/3*3^(1/2
)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^
(1/2)+2^(2/3)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\sqrt{-x^3-1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x + e)/(sqrt(-x^3 - 1)*(x + 2^(2/3))),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(-x^3 - 1)*(x + 2^(2/3))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x + e)/(sqrt(-x^3 - 1)*(x + 2^(2/3))),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e + fx}{\sqrt{-(x+1)(x^2-x+1)}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x+e)/(2**(2/3)+x)/(-x**3-1)**(1/2),x)
```

```
[Out] Integral((e + f*x)/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 2**(2/3)))
, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx + e}{\sqrt{-x^3-1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x + e)/(sqrt(-x^3 - 1)*(x + 2^(2/3))),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)/(sqrt(-x^3 - 1)*(x + 2^(2/3))), x)
```

3.37 $\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=316

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{a}f+\sqrt[3]{2}\sqrt[3]{b}e\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2\left(\sqrt[3]{b}e-2^{2/3}\sqrt[3]{a}f\right)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{3\sqrt{3}\sqrt{a}b^{2/3}}$$

```
[Out] (2*(b^(1/3)*e - 2^(2/3)*a^(1/3)*f)*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/
3) + 2^(1/3)*b^(1/3)*x))/Sqrt[a + b*x^3]])/(3*Sqrt[3]*Sqrt[a]*b^(
2/3)) + (2*Sqrt[2 + Sqrt[3]]*(2^(1/3)*b^(1/3)*e + a^(1/3)*f)*(a^(
1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - S
qrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)]
, -7 - 4*Sqrt[3]])/(3*3^(1/4)*a^(1/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1
/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a +
 b*x^3])
```

Rubi [A] time = 0.729293, antiderivative size = 316, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{a}f+\sqrt[3]{2}\sqrt[3]{b}e\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2\left(\sqrt[3]{b}e-2^{2/3}\sqrt[3]{a}f\right)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{3\sqrt{3}\sqrt{a}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(e + f*x)/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*(b^(1/3)*e - 2^(2/3)*a^(1/3)*f)*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/
3) + 2^(1/3)*b^(1/3)*x))/Sqrt[a + b*x^3]])/(3*Sqrt[3]*Sqrt[a]*b^(
2/3)) + (2*Sqrt[2 + Sqrt[3]]*(2^(1/3)*b^(1/3)*e + a^(1/3)*f)*(a^(
1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - S
qrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)]
, -7 - 4*Sqrt[3]])/(3*3^(1/4)*a^(1/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1
/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a +
 b*x^3])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((f*x+e)/(2**(2/3)*a**(1/3)+b**(1/3)*x)/(b*x**3+a)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 2.73935, size = 336, normalized size = 1.06

$$\frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\left(2^{2/3}\sqrt[3]{a}f-\sqrt[3]{b}e\right)\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)}{\sqrt[3]{-1}+2^{2/3}}-\frac{\sqrt[4]{3}f\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\cdots}}{\cdots}\right)}{\sqrt{3}b^{2/3}\sqrt{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(-((3^(
1/4)*f*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(-1)^(1/6) - (I*b^(1
/3)*x)/a^(1/3)]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/
3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/Sqrt[(a^(1/3) +
(-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]) + ((-1)^(1/3)*
(1 + (-1)^(1/3))*(-(b^(1/3)*e) + 2^(2/3)*a^(1/3)*f)*Sqrt[1 - (b^(
1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/(
```

```
(-1)^(1/3) + 2^(2/3)), ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*
x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/((-1)^(1/3) + 2^(2/
3))))/(Sqrt[3]*b^(2/3)*Sqrt[a + b*x^3])
```

Maple [F] time = 0.119, size = 0, normalized size = 0.

$$\int (fx+e)\left(2^{\frac{2}{3}}\sqrt[3]{a}+\sqrt[3]{b}x\right)^{-1}\frac{1}{\sqrt{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x+e)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

```
[Out] int((f*x+e)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\sqrt{bx^3+a}\left(b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x + e)/(sqrt(b*x^3 + a)*(b^(1/3)*x + 2^(2/3)*a^(1/3))),x, algorithm
```

```
[Out] integrate((f*x + e)/(sqrt(b*x^3 + a)*(b^(1/3)*x + 2^(2/3)*a^(1/3)
)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x + e)/(sqrt(b*x^3 + a)*(b^(1/3)*x + 2^(2/3)*a^(1/3))),x, algorithm
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e+fx}{\sqrt{a+bx^3}\left(2^{\frac{2}{3}}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2**(2/3)*a**(1/3)+b**(1/3)*x)/(b*x**3+a)**(1/2),x)
```

```
[Out] Integral((e + f*x)/(sqrt(a + b*x**3)*(2**(2/3)*a**(1/3) + b**(1/3
)*x)), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x + e)/(sqrt(b*x^3 + a)*(b^(1/3)*x + 2^(2/3)*a^(1/3))),x, algorithm
```

```
[Out] Exception raised: TypeError
```

3.38 $\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$

Optimal. Leaf size=324

$$-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{2}\sqrt[3]{b}e-\sqrt[3]{a}f\right)F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{2\left(2^{2/3}\sqrt[3]{a}f+\sqrt[3]{b}e\right)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{3\sqrt{3}\sqrt{a}b^{2/3}}$$

```
[Out] (-2*(b^(1/3)*e + 2^(2/3)*a^(1/3)*f)*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1
/3) - 2^(1/3)*b^(1/3)*x))/Sqrt[a - b*x^3]])/(3*Sqrt[3]*Sqrt[a]*b^
(2/3)) - (2*Sqrt[2 + Sqrt[3]]*(2^(1/3)*b^(1/3)*e - a^(1/3)*f)*(a^
(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^
2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 -
Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)
], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a^(1/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(
1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a
- b*x^3])
```

Rubi [A] time = 0.792921, antiderivative size = 324, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{2}\sqrt[3]{b}e-\sqrt[3]{a}f\right)F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{2\left(2^{2/3}\sqrt[3]{a}f+\sqrt[3]{b}e\right)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{3\sqrt{3}\sqrt{a}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(e + f*x)/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (-2*(b^(1/3)*e + 2^(2/3)*a^(1/3)*f)*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1
/3) - 2^(1/3)*b^(1/3)*x))/Sqrt[a - b*x^3]])/(3*Sqrt[3]*Sqrt[a]*b^
(2/3)) - (2*Sqrt[2 + Sqrt[3]]*(2^(1/3)*b^(1/3)*e - a^(1/3)*f)*(a^
(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^
2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 -
Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)
], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a^(1/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(
1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a
- b*x^3])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((f*x+e)/(2**(2/3)*a**(1/3)-b**(1/3)*x)/(-b*x**3+a)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 2.20532, size = 399, normalized size = 1.23

$$\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(-\left(\sqrt[3]{-1}+2^{2/3}\right)f\left(\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}F\left(\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)+\frac{\sqrt[3]{-1}\left(1+\sqrt[3]{\right.}}{}\right.}{\left(\sqrt[3]{-1}+2^{2/3}\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{a-}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(e + f*x)/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(-(((-1
)^(1/3) + 2^(2/3))*f*((-1)^(1/3)*a^(1/3) + b^(1/3)*x)*Sqrt[((-1)^
(1/3)*(a^(1/3) + (-1)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3)
)]*EllipticF[ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (
-1)^(1/3))*a^(1/3))]], (-1)^(1/3)]) + ((-1)^(1/3)*(1 + (-1)^(1/3)
```

```
)*(b^(1/3)*e + 2^(2/3)*a^(1/3)*f)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1
/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[1 + (b^(1/3)*x)/a^(1/3) +
 (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2
/3)), ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/
3))*a^(1/3))]], (-1)^(1/3)])/Sqrt[3]))/(((-1)^(1/3) + 2^(2/3))*b^
(2/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(
1/3))]*Sqrt[a - b*x^3])
```

Maple [F] time = 0.113, size = 0, normalized size = 0.

$$\int (fx+e)\left(2^{\frac{2}{3}}\sqrt[3]{a}-\sqrt[3]{b}x\right)^{-1}\frac{1}{\sqrt{-bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x+e)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

```
[Out] int((f*x+e)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{fx+e}{\sqrt{-bx^3+a}\left(b^{\frac{1}{3}}x-2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(f*x + e)/(sqrt(-b*x^3 + a)*(b^(1/3)*x - 2^(2/3)*a^(1/3))),x, algorit
```

```
[Out] -integrate((f*x + e)/(sqrt(-b*x^3 + a)*(b^(1/3)*x - 2^(2/3)*a^(1/
3))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(f*x + e)/(sqrt(-b*x^3 + a)*(b^(1/3)*x - 2^(2/3)*a^(1/3))),x, algorit
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{e}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt[3]{b}x\sqrt{a-bx^3}}\,dx-\int \frac{fx}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt[3]{b}x\sqrt{a-bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2**(2/3)*a**(1/3)-b**(1/3)*x)/(-b*x**3+a)**(1/2),x)
```

```
[Out] -Integral(e/(-2**(2/3)*a**(1/3)*sqrt(a - b*x**3) + b**(1/3)*x*sqr
t(a - b*x**3)), x) - Integral(f*x/(-2**(2/3)*a**(1/3)*sqrt(a - b*
x**3) + b**(1/3)*x*sqrt(a - b*x**3)), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(f*x + e)/(sqrt(-b*x^3 + a)*(b^(1/3)*x - 2^(2/3)*a^(1/3))),x, algorit
```

```
[Out] Exception raised: TypeError
```

3.39 $\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

Optimal. Leaf size=333

$$-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{2}\sqrt[3]{b}e-\sqrt[3]{a}f\right)F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}-\frac{2\left(2^{2/3}\sqrt[3]{a}f+\sqrt[3]{b}e\right)\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{bx^3-a}}\right)}{3\sqrt{3}\sqrt{a}b^{2/3}}$$

```
[Out] (-2*(b^(1/3)*e + 2^(2/3)*a^(1/3)*f)*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(
1/3) - 2^(1/3)*b^(1/3)*x))/Sqrt[-a + b*x^3]])/(3*Sqrt[3]*Sqrt[a]*
b^(2/3)) - (2*Sqrt[2 - Sqrt[3]]*(2^(1/3)*b^(1/3)*e - a^(1/3)*f)*(
a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*
x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1
+ Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*
x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*a^(1/3)*b^(2/3)*Sqrt[-((a^(1/3)*
(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sq
rt[-a + b*x^3])
```

Rubi [A] time = 0.769635, antiderivative size = 333, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$

$$-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{2}\sqrt[3]{b}e-\sqrt[3]{a}f\right)F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}-\frac{2\left(2^{2/3}\sqrt[3]{a}f+\sqrt[3]{b}e\right)\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{bx^3-a}}\right)}{3\sqrt{3}\sqrt{a}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(e + f*x)/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (-2*(b^(1/3)*e + 2^(2/3)*a^(1/3)*f)*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(
1/3) - 2^(1/3)*b^(1/3)*x))/Sqrt[-a + b*x^3]])/(3*Sqrt[3]*Sqrt[a]*
b^(2/3)) - (2*Sqrt[2 - Sqrt[3]]*(2^(1/3)*b^(1/3)*e - a^(1/3)*f)*(
a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*
x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1
+ Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*
x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*a^(1/3)*b^(2/3)*Sqrt[-((a^(1/3)*
(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sq
rt[-a + b*x^3])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((f*x+e)/(2**(2/3)*a**(1/3)-b**(1/3)*x)/(b*x**3-a)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 2.21713, size = 400, normalized size = 1.2

$$\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(-\left(\sqrt[3]{-1}+2^{2/3}\right) f \left(\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}F\left(\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)+\frac{\sqrt[3]{-1}\left(1+\sqrt[3]{\cdots}\right.}{\cdots}\right.}{\left(\sqrt[3]{-1}+2^{2/3}\right) b^{2/3}\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{bx\cdots}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(-(((-1
)^(1/3) + 2^(2/3))*f*((-1)^(1/3)*a^(1/3) + b^(1/3)*x)*Sqrt[((-1)^
(1/3)*(a^(1/3) + (-1)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3)
)]*EllipticF[ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (
-1)^(1/3))*a^(1/3))]], (-1)^(1/3)]) + ((-1)^(1/3)*(1 + (-1)^(1/3)
```

```
)*(b^(1/3)*e + 2^(2/3)*a^(1/3)*f)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1
/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[1 + (b^(1/3)*x)/a^(1/3) +
 (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2
/3)), ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/
3))*a^(1/3))]], (-1)^(1/3)])/Sqrt[3]))/(((-1)^(1/3) + 2^(2/3))*b^
(2/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(
1/3))]*Sqrt[-a + b*x^3])
```

Maple [F] time = 0.103, size = 0, normalized size = 0.

$$\int (fx+e)\left(2^{\frac{2}{3}}\sqrt[3]{a}-\sqrt[3]{b}x\right)^{-1}\frac{1}{\sqrt{bx^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x+e)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

```
[Out] int((f*x+e)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{fx+e}{\sqrt{bx^3-a}\left(b^{\frac{1}{3}}x-2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(f*x + e)/(sqrt(b*x^3 - a)*(b^(1/3)*x - 2^(2/3)*a^(1/3))),x, algorithm
```

```
[Out] -integrate((f*x + e)/(sqrt(b*x^3 - a)*(b^(1/3)*x - 2^(2/3)*a^(1/3
))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(f*x + e)/(sqrt(b*x^3 - a)*(b^(1/3)*x - 2^(2/3)*a^(1/3))),x, algorithm
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{e}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt[3]{b}x\sqrt{-a+bx^3}}\,dx-\int \frac{fx}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt[3]{b}x\sqrt{-a+bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2**(2/3)*a**(1/3)-b**(1/3)*x)/(b*x**3-a)**(1/2),x)
```

```
[Out] -Integral(e/(-2**(2/3)*a**(1/3)*sqrt(-a + b*x**3) + b**(1/3)*x*sq
rt(-a + b*x**3)), x) - Integral(f*x/(-2**(2/3)*a**(1/3)*sqrt(-a +
 b*x**3) + b**(1/3)*x*sqrt(-a + b*x**3)), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(f*x + e)/(sqrt(b*x^3 - a)*(b^(1/3)*x - 2^(2/3)*a^(1/3))),x, algorithm
```

```
[Out] Exception raised: TypeError
```

3.40 $\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

Optimal. Leaf size=329

$$\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{a}f+\sqrt[3]{2}\sqrt[3]{b}e\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}+\frac{2\left(\sqrt[3]{b}e-2^{2/3}\sqrt[3]{a}f\right)\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{3\sqrt{3}\sqrt{a}b^{2/3}}$$

```
[Out] (2*(b^(1/3)*e - 2^(2/3)*a^(1/3)*f)*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1
/3) + 2^(1/3)*b^(1/3)*x))/Sqrt[-a - b*x^3]])/(3*Sqrt[3]*Sqrt[a]*b
^(2/3)) + (2*Sqrt[2 - Sqrt[3]]*(2^(1/3)*b^(1/3)*e + a^(1/3)*f)*(a
^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x
)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*a^(1/3)*b^(2/3)*Sqrt[-((a^(1/3)*(
a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqr
t[-a - b*x^3])
```

Rubi [A] time = 0.696955, antiderivative size = 329, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$

$$\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{a}f+\sqrt[3]{2}\sqrt[3]{b}e\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}+\frac{2\left(\sqrt[3]{b}e-2^{2/3}\sqrt[3]{a}f\right)\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{3\sqrt{3}\sqrt{a}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(e + f*x)/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (2*(b^(1/3)*e - 2^(2/3)*a^(1/3)*f)*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1
/3) + 2^(1/3)*b^(1/3)*x))/Sqrt[-a - b*x^3]])/(3*Sqrt[3]*Sqrt[a]*b
^(2/3)) + (2*Sqrt[2 - Sqrt[3]]*(2^(1/3)*b^(1/3)*e + a^(1/3)*f)*(a
^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x
)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*a^(1/3)*b^(2/3)*Sqrt[-((a^(1/3)*(
a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqr
t[-a - b*x^3])
```

Rubi in Sympy **[F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((f*x+e)/(2**(2/3)*a**(1/3)+b**(1/3)*x)/(-b*x**3-a)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica **[C]** time = 2.13796, size = 387, normalized size = 1.18

$$\frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\left(2^{2/3}\sqrt[3]{a}f-\sqrt[3]{b}e\right)\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{3}}-\frac{\left(\sqrt[3]{-1}\cdots\right.}{\cdots}\cdots\right)}{\left(\sqrt[3]{-1}+2^{2/3}\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{-a-bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(-((((-
1)^(1/3) + 2^(2/3))*f*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(-1)^
(1/6) - (I*b^(1/3)*x)/a^(1/3)]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (
-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/3
^(1/4)) + ((-1)^(1/3)*(1 + (-1)^(1/3))*(-(b^(1/3)*e) + 2^(2/3)*a^
```

```
(1/3)*f)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*
a^(1/3))]*Sqrt[1 - (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*E
llipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)), ArcSin[Sqrt[(a^(1/3
) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3
)])/Sqrt[3]))/(((-1)^(1/3) + 2^(2/3))*b^(2/3)*Sqrt[(a^(1/3) + (-1
)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[-a - b*x^3])
```

Maple [F] time = 0.103, size = 0, normalized size = 0.

$$\int (fx+e)\left(2^{\frac{2}{3}}\sqrt[3]{a}+\sqrt[3]{b}x\right)^{-1}\frac{1}{\sqrt{-bx^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x+e)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

```
[Out] int((f*x+e)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\sqrt{-bx^3-a}\left(b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x + e)/(sqrt(-b*x^3 - a)*(b^(1/3)*x + 2^(2/3)*a^(1/3))),x, algorithm
```

```
[Out] integrate((f*x + e)/(sqrt(-b*x^3 - a)*(b^(1/3)*x + 2^(2/3)*a^(1/3
))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x + e)/(sqrt(-b*x^3 - a)*(b^(1/3)*x + 2^(2/3)*a^(1/3))),x, algorithm
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e + fx}{\sqrt{-a - bx^3}\left(2^{\frac{2}{3}}\sqrt[3]{a} + \sqrt[3]{b}x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2**(2/3)*a**(1/3)+b**(1/3)*x)/(-b*x**3-a)**(1/2),x)
```

```
[Out] Integral((e + f*x)/(sqrt(-a - b*x**3)*(2**(2/3)*a**(1/3) + b**(1/
3)*x)), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x + e)/(sqrt(-b*x^3 - a)*(b^(1/3)*x + 2^(2/3)*a^(1/3))),x, algorithm
```

```
[Out] Exception raised: TypeError
```

3.41 $\int \frac{e+fx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx$

Optimal. Leaf size=265

$$\frac{2(de-cf)\tan^{-1}\left(\frac{\sqrt{3}\sqrt{c}(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{3\sqrt{3}c^{3/2}d^2}+\frac{\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(c+2^{2/3}dx\right)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}(cf+2de)F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)c+2^{2/3}dx}{\left(1+\sqrt{3}\right)c+2^{2/3}dx}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}cd^2\sqrt{\frac{c\left(c+2^{2/3}dx\right)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\sqrt{c^3+4d^3x^3}}$$

```
[Out] (2*(d*e - c*f)*ArcTan[(Sqrt[3]*Sqrt[c]*(c + 2*d*x))/Sqrt[c^3 + 4*
d^3*x^3]])/(3*Sqrt[3]*c^(3/2)*d^2) + (2^(1/3)*Sqrt[2 + Sqrt[3]]*(
2*d*e + c*f)*(c + 2^(2/3)*d*x)*Sqrt[(c^2 - 2^(2/3)*c*d*x + 2*2^(1
/3)*d^2*x^2)/((1 + Sqrt[3])*c + 2^(2/3)*d*x)^2]*EllipticF[ArcSin[
((1 - Sqrt[3])*c + 2^(2/3)*d*x)/((1 + Sqrt[3])*c + 2^(2/3)*d*x)],
 -7 - 4*Sqrt[3]])/(3*3^(1/4)*c*d^2*Sqrt[(c*(c + 2^(2/3)*d*x))/((1
 + Sqrt[3])*c + 2^(2/3)*d*x)^2]*Sqrt[c^3 + 4*d^3*x^3])
```

Rubi [A] time = 0.572892, antiderivative size = 265, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138

$$\frac{2(de-cf)\tan^{-1}\left(\frac{\sqrt{3}\sqrt{c}(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{3\sqrt{3}c^{3/2}d^2}+\frac{\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(c+2^{2/3}dx\right)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}(cf+2de)F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)c+2^{2/3}dx}{\left(1+\sqrt{3}\right)c+2^{2/3}dx}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}cd^2\sqrt{\frac{c\left(c+2^{2/3}dx\right)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\sqrt{c^3+4d^3x^3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(e + f*x)/((c + d*x)*Sqrt[c^3 + 4*d^3*x^3]),x]
```

```
[Out] (2*(d*e - c*f)*ArcTan[(Sqrt[3]*Sqrt[c]*(c + 2*d*x))/Sqrt[c^3 + 4*
d^3*x^3]])/(3*Sqrt[3]*c^(3/2)*d^2) + (2^(1/3)*Sqrt[2 + Sqrt[3]]*(
2*d*e + c*f)*(c + 2^(2/3)*d*x)*Sqrt[(c^2 - 2^(2/3)*c*d*x + 2*2^(1
/3)*d^2*x^2)/((1 + Sqrt[3])*c + 2^(2/3)*d*x)^2]*EllipticF[ArcSin[
((1 - Sqrt[3])*c + 2^(2/3)*d*x)/((1 + Sqrt[3])*c + 2^(2/3)*d*x)],
 -7 - 4*Sqrt[3]])/(3*3^(1/4)*c*d^2*Sqrt[(c*(c + 2^(2/3)*d*x))/((1
 + Sqrt[3])*c + 2^(2/3)*d*x)^2]*Sqrt[c^3 + 4*d^3*x^3])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x+e)/(d*x+c)/(4*d**3*x**3+c**3)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 2.69194, size = 380, normalized size = 1.43

$$\frac{\sqrt[6]{2}\sqrt{\frac{\sqrt[3]{2}c+2dx}{\left(1+\sqrt[3]{-1}\right)c}}\left(-f\sqrt{\frac{\sqrt[3]{-2}c-2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}\left(\sqrt[3]{-1}\left(2+\sqrt[3]{-2}\right)c-2\left(\sqrt[3]{-1}+2^{2/3}\right)dx\right)F\left(\sin^{-1}\left(\frac{\sqrt{\frac{\sqrt[3]{2}c+2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}}{\sqrt[6]{2}}\right)|\sqrt[3]{-1}\right)+\frac{\sqrt[3]{-1}2^{2\cdots}}{\cdots}\right)}{\left(2+\sqrt[3]{-2}\right)d^2\sqrt{\frac{\sqrt[3]{2}c+2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}\sqrt{c^3+4d^3\cdots}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(e + f*x)/((c + d*x)*Sqrt[c^3 + 4*d^3*x^3]),x]
```

```
[Out] (2^(1/6)*Sqrt[(2^(1/3)*c + 2*d*x)/((1 + (-1)^(1/3))*c)]*(-(f*Sqrt
[((-2)^(1/3)*c - 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]*((-1)^(1
/3)*(2 + (-2)^(1/3))*c - 2*((-1)^(1/3) + 2^(2/3))*d*x)*EllipticF[
ArcSin[Sqrt[(2^(1/3)*c + 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]/
2^(1/6)], (-1)^(1/3)]) + ((-1)^(1/3)*2^(2/3)*(1 + (-1)^(1/3))*(-(
d*e) + c*f)*Sqrt[(2^(1/3)*c + 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))
*c)]*Sqrt[2^(2/3) - (2*2^(1/3)*d*x)/c + (4*d^2*x^2)/c^2]*Elliptic
Pi[(I*2^(1/3)*Sqrt[3])/(2 + (-2)^(1/3)), ArcSin[Sqrt[(2^(1/3)*c +
 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]/2^(1/6)], (-1)^(1/3)])/S
qrt[3]))/((2 + (-2)^(1/3))*d^2*Sqrt[(2^(1/3)*c + 2*(-1)^(2/3)*d*x
)/((1 + (-1)^(1/3))*c)]*Sqrt[c^3 + 4*d^3*x^3])
```

Maple [B] time = 0.012, size = 900, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x+e)/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x)
```

```
[Out] 2/d*f*((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I
*3^(1/2)*2^(1/3))*c/d)*((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/
d)/((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^
(1/2)*2^(1/3))*c/d))^(1/2)*((x+1/2*2^(1/3)*c/d)/((1/4*2^(1/3)+1/4
*I*3^(1/2)*2^(1/3))*c/d+1/2*2^(1/3)*c/d))^(1/2)*((x-(1/4*2^(1/3)-
1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*
c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)/(4*d^3*x^3+c^
3)^(1/2)*EllipticF(((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/(
(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2
)*2^(1/3))*c/d))^(1/2),(((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d-
(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/4*I*3^(1
/2)*2^(1/3))*c/d+1/2*2^(1/3)*c/d))^(1/2))+2*(-c*f+d*e)/d^2*((1/4*
2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(
1/3))*c/d)*((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(
1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3
))*c/d))^(1/2)*((x+1/2*2^(1/3)*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2
^(1/3))*c/d+1/2*2^(1/3)*c/d))^(1/2)*((x-(1/4*2^(1/3)-1/4*I*3^(1/2
)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(
1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)/(4*d^3*x^3+c^3)^(1/2)/((1
/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+c/d)*EllipticPi(((x-(1/4*2^
(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(
1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2),((1/4*2
^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1
/3))*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+c/d),(((1/4*2^
(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/
3))*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+1/2*2^(1/3)*c/d
))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\sqrt{4\,d^3x^3+c^3}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x + e)/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{fx+e}{\sqrt{4\,d^3x^3+c^3}(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x + e)/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e+fx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(4*d**3*x**3+c**3)**(1/2),x)
```

```
[Out] Integral((e + f*x)/((c + d*x)*sqrt(c**3 + 4*d**3*x**3)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\sqrt{4\,d^3x^3+c^3}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x + e)/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)), x)
```

3.42 $\int \frac{x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=145

$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{2\,2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{x^3+1}}\right)}{3\sqrt{3}}$$

```
[Out] (-2*2^(2/3)*ArcTan[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[1 + x^3]])/(3*S
qrt[3]) + (2*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sq
rt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x
)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]
*Sqrt[1 + x^3])
```

Rubi [A] time = 0.315631, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{2\,2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{x^3+1}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*2^(2/3)*ArcTan[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[1 + x^3]])/(3*S
qrt[3]) + (2*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sq
rt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x
)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]
*Sqrt[1 + x^3])
```

Rubi in Sympy [A] time = 137.408, size = 473, normalized size = 3.26

$$-\frac{2\cdot 2^{\frac{2}{3}}\sqrt{\frac{x^2-x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{-\sqrt{3}+2}\,(x+1)\,\mathrm{atan}\left(\frac{3^{\frac{3}{4}}\sqrt{1+\sqrt[3]{2}}\sqrt{-4\sqrt{3}+8}\sqrt{-\frac{\left(-x-1+\sqrt{3}\right)^2}{\left(x+1+\sqrt{3}\right)^2}+1}}{6\sqrt{-1+\sqrt[3]{2}}\sqrt{\frac{\left(-x-1+\sqrt{3}\right)^2}{\left(x+1+\sqrt{3}\right)^2}-4\sqrt{3}+7}}\right)}{\sqrt{\frac{x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{-1+\sqrt[3]{2}}\left(1+\sqrt[3]{2}\right)^{\frac{3}{2}}\sqrt{-4\sqrt{3}+8}\sqrt{x^3+1}}+\frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{x^2-x+1}{\left(x+1+\sqrt{3}\right)^2}}\left(1+\sqrt{3}\right)\sqrt{\sqrt{3}+2}\,(x+1)\,F\left(\mathrm{asin}\left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{3\sqrt{\frac{x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{x^3+1}\left(-2^{\frac{2}{3}}+1+\sqrt{3}\right)}-\frac{4\cdot 2^{\frac{2}{3}}\sqrt[4]{3}\sqrt{\frac{x^2-x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{-\sqrt{3}+2}\,(x+1)\,\Pi\left(\frac{\left(-2^{\frac{2}{3}}+1+\sqrt{3}\right)^2}{\left(-1+2^{\frac{2}{3}}+\sqrt{3}\right)^2};\mathrm{asin}\left(\frac{-x-1+\sqrt{3}}{x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{-4\sqrt{3}+7}\sqrt{x^3+1}\left(-2^{\frac{2}{3}}+1+\sqrt{3}\right)\left(-\sqrt{3}-2^{\frac{2}{3}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(2**(2/3)+x)/(x**3+1)**(1/2),x)
```

```
[Out] -2*2**(2/3)*sqrt((x**2 - x + 1)/(x + 1 + sqrt(3))**2)*sqrt(-sqrt(
3) + 2)*(x + 1)*atan(3**(3/4)*sqrt(1 + 2**(1/3))*sqrt(-4*sqrt(3)
+ 8)*sqrt(-(-x - 1 + sqrt(3))**2/(x + 1 + sqrt(3))**2 + 1)/(6*sqr
t(-1 + 2**(1/3))*sqrt((-x - 1 + sqrt(3))**2/(x + 1 + sqrt(3))**2
- 4*sqrt(3) + 7)))/(sqrt((x + 1)/(x + 1 + sqrt(3))**2)*sqrt(-1 +
2**(1/3))*(1 + 2**(1/3))**(3/2)*sqrt(-4*sqrt(3) + 8)*sqrt(x**3 +
1)) + 2*3**(3/4)*sqrt((x**2 - x + 1)/(x + 1 + sqrt(3))**2)*(1 + s
qrt(3))*sqrt(sqrt(3) + 2)*(x + 1)*elliptic_f(asin((x - sqrt(3) +
1)/(x + 1 + sqrt(3))), -7 - 4*sqrt(3))/(3*sqrt((x + 1)/(x + 1 + s
qrt(3))**2)*sqrt(x**3 + 1)*(-2**(2/3) + 1 + sqrt(3))) - 4*2**(2/3
)*3**(1/4)*sqrt((x**2 - x + 1)/(x + 1 + sqrt(3))**2)*sqrt(-sqrt(3
) + 2)*(x + 1)*elliptic_pi((-2**(2/3) + 1 + sqrt(3))**2/(-1 + 2**
(2/3) + sqrt(3))**2, asin((-x - 1 + sqrt(3))/(x + 1 + sqrt(3))),
-7 - 4*sqrt(3))/(sqrt((x + 1)/(x + 1 + sqrt(3))**2)*sqrt(-4*sqrt(
3) + 7)*sqrt(x**3 + 1)*(-2**(2/3) + 1 + sqrt(3))*(-sqrt(3) - 2**(
2/3) + 1))
```

Mathematica [C] time = 0.532158, size = 207, normalized size = 1.43

$$\frac{2\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\left(-\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}}+\frac{i2^{2/3}\sqrt{x^2-x+1}\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt[3]{-1}+2^{2/3}}\right)}{\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) - x)*Sqrt[((-1)
^(1/3) - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1
 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^
(2/3)*x)/(1 + (-1)^(1/3))]) + (I*2^(2/3)*Sqrt[1 - x + x^2]*Ellipt
icPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)), ArcSin[Sqrt[(1 + (-1)^(2
/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/((-1)^(1/3) + 2^(2/3))))/
Sqrt[1 + x^3]
```

Maple [B] time = 0.034, size = 258, normalized size = 1.8

$$2\,\frac{3/2-i/2\sqrt{3}}{\sqrt{x^3+1}}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}\,EllipticF\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},\sqrt{\frac{-3/2+i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\right)$$
$$-2\,\frac{2^{2/3}\left(3/2-i/2\sqrt{3}\right)}{\sqrt{x^3+1}\left(2^{2/3}-1\right)}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}\,EllipticPi\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},\frac{-3/2+i/2\sqrt{3}}{2^{2/3}-1},\sqrt{-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(2^(2/3)+x)/(x^3+1)^(1/2),x)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1
/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/
(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1
/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^
(1/2))-2*2^(2/3)*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^
(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+
1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)/(2^(2/3)
-1)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1
/2))/(2^(2/3)-1),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2
))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x^3+1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(x^3 + 1)*(x + 2^(2/3))),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x}{\sqrt{x^3+1}\left(x+2^{\frac{2}{3}}\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(x^3 + 1)*(x + 2^(2/3))),x, algorithm="fricas")
```

```
[Out] integral(x/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{(x+1)(x^2-x+1)}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(2**(2/3)+x)/(x**3+1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt((x + 1)*(x**2 - x + 1))*(x + 2**(2/3))), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x^3+1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(x^3 + 1)*(x + 2^(2/3))),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

3.43 $\int \frac{x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=160

$$\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2\,2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{3}}$$

```
[Out] (-2*2^(2/3)*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[1 - x^3]])/(3*S
qrt[3]) + (2*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sq
rt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x
)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]
*Sqrt[1 - x^3])
```

Rubi [A] time = 0.362124, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2\,2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (-2*2^(2/3)*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[1 - x^3]])/(3*S
qrt[3]) + (2*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sq
rt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x
)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]
*Sqrt[1 - x^3])
```

Rubi in Sympy [A] time = 139.543, size = 473, normalized size = 2.96

$$-\frac{2\cdot 2^{\frac{2}{3}}\sqrt{\frac{x^2+x+1}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{-\sqrt{3}+2}\,(-x+1)\,\mathrm{atan}\left(\frac{3^{\frac{3}{4}}\sqrt{1+\sqrt[3]{2}}\sqrt{1-\frac{\left(x-1+\sqrt{3}\right)^2}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{-4\sqrt{3}+8}}{6\sqrt{-1+\sqrt[3]{2}}\sqrt{-4\sqrt{3}+7+\frac{\left(x-1+\sqrt{3}\right)^2}{\left(-x+1+\sqrt{3}\right)^2}}}\right)}{\sqrt{\frac{-x+1}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{-1+\sqrt[3]{2}}\left(1+\sqrt[3]{2}\right)^{\frac{3}{2}}\sqrt{-4\sqrt{3}+8}\sqrt{-x^3+1}}+\frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{x^2+x+1}{\left(-x+1+\sqrt{3}\right)^2}}\left(1+\sqrt{3}\right)\sqrt{\sqrt{3}+2}\,(-x+1)\,F\left(\mathrm{asin}\left(\frac{-x-\sqrt{3}+1}{-x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{3\sqrt{\frac{-x+1}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{-x^3+1}\left(-2^{\frac{2}{3}}+1+\sqrt{3}\right)}-\frac{4\cdot 2^{\frac{2}{3}}\sqrt[4]{3}\sqrt{\frac{x^2+x+1}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{-\sqrt{3}+2}\,(-x+1)\,\Pi\left(\frac{\left(-2^{\frac{2}{3}}+1+\sqrt{3}\right)^2}{\left(-1+2^{\frac{2}{3}}+\sqrt{3}\right)^2};\mathrm{asin}\left(\frac{x-1+\sqrt{3}}{-x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt{\frac{-x+1}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{-4\sqrt{3}+7}\sqrt{-x^3+1}\left(-2^{\frac{2}{3}}+1+\sqrt{3}\right)\left(-\sqrt{3}-2^{\frac{2}{3}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(2**(2/3)-x)/(-x**3+1)**(1/2),x)
```

```
[Out] -2*2**(2/3)*sqrt((x**2 + x + 1)/(-x + 1 + sqrt(3))**2)*sqrt(-sqrt
(3) + 2)*(-x + 1)*atan(3**(3/4)*sqrt(1 + 2**(1/3))*sqrt(1 - (x -
1 + sqrt(3))**2/(-x + 1 + sqrt(3))**2)*sqrt(-4*sqrt(3) + 8)/(6*sq
rt(-1 + 2**(1/3))*sqrt(-4*sqrt(3) + 7 + (x - 1 + sqrt(3))**2/(-x
+ 1 + sqrt(3))**2)))/(sqrt((-x + 1)/(-x + 1 + sqrt(3))**2)*sqrt(-
1 + 2**(1/3))*(1 + 2**(1/3))**(3/2)*sqrt(-4*sqrt(3) + 8)*sqrt(-x*
*3 + 1)) + 2*3**(3/4)*sqrt((x**2 + x + 1)/(-x + 1 + sqrt(3))**2)*
(1 + sqrt(3))*sqrt(sqrt(3) + 2)*(-x + 1)*elliptic_f(asin((-x - sq
rt(3) + 1)/(-x + 1 + sqrt(3))), -7 - 4*sqrt(3))/(3*sqrt((-x + 1)/
(-x + 1 + sqrt(3))**2)*sqrt(-x**3 + 1)*(-2**(2/3) + 1 + sqrt(3)))
 - 4*2**(2/3)*3**(1/4)*sqrt((x**2 + x + 1)/(-x + 1 + sqrt(3))**2)
*sqrt(-sqrt(3) + 2)*(-x + 1)*elliptic_pi((-2**(2/3) + 1 + sqrt(3)
)**2/(-1 + 2**(2/3) + sqrt(3))**2, asin((x - 1 + sqrt(3))/(-x + 1
 + sqrt(3))), -7 - 4*sqrt(3))/(sqrt((-x + 1)/(-x + 1 + sqrt(3))**
2)*sqrt(-4*sqrt(3) + 7)*sqrt(-x**3 + 1)*(-2**(2/3) + 1 + sqrt(3))
*(-sqrt(3) - 2**(2/3) + 1))
```

Mathematica [C] time = 0.532433, size = 209, normalized size = 1.31

$$\frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(-\frac{\left(x+\sqrt[3]{-1}\right)\sqrt{\frac{(-1)^{2/3}x+\sqrt[3]{-1}}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{i2^{2/3}\sqrt{x^2+x+1}\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt[3]{-1}+2^{2/3}}\right)}{\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) + x)*Sqrt[((-1)
^(1/3) + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1
 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^
(2/3)*x)/(1 + (-1)^(1/3))]) + (I*2^(2/3)*Sqrt[1 + x + x^2]*Ellipt
icPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)), ArcSin[Sqrt[(1 - (-1)^(2
/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/((-1)^(1/3) + 2^(2/3))))/
Sqrt[1 - x^3]
```

Maple [A] time = 0.035, size = 253, normalized size = 1.6

$$\frac{2i}{3}\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\,EllipticF\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)\frac{1}{\sqrt{-x^3}}$$
$$+\frac{\frac{2i}{3}2^{\frac{2}{3}}\sqrt{3}}{-\frac{1}{2}+\frac{i}{2}\sqrt{3}-2^{\frac{2}{3}}}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\,EllipticPi\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\frac{i\sqrt{}}{-\frac{1}{2}+\frac{i}{2}\sqrt{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(2^(2/3)-x)/(-x^3+1)^(1/2),x)
```

```
[Out] 2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/(-3
/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)
/(-x^3+1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^
(1/2))^(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))+2/3*I*2^(2/3
)*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/(-3/2+1
/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x
^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)-2^(2/3))*EllipticPi(1/3*3^(1/2)*(
I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1
/2)-2^(2/3)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{\sqrt{-x^3+1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x/(sqrt(-x^3 + 1)*(x - 2^(2/3))),x, algorithm="maxima")
```

```
[Out] -integrate(x/(sqrt(-x^3 + 1)*(x - 2^(2/3))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{x}{\sqrt{-x^3+1}\left(x-2^{\frac{2}{3}}\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x/(sqrt(-x^3 + 1)*(x - 2^(2/3))),x, algorithm="fricas")
```

```
[Out] integral(-x/(sqrt(-x^3 + 1)*(x - 2^(2/3))), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{x\sqrt{-x^3+1}-2^{\frac{2}{3}}\sqrt{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(2**(2/3)-x)/(-x**3+1)**(1/2),x)
```

```
[Out] -Integral(x/(x*sqrt(-x**3 + 1) - 2**(2/3)*sqrt(-x**3 + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x}{\sqrt{-x^3+1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x/(sqrt(-x^3 + 1)*(x - 2^(2/3))),x, algorithm="giac")

[Out] integrate(-x/(sqrt(-x^3 + 1)*(x - 2^(2/3))), x)
```

3.44 $\int \frac{x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=163

$$\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{x^3-1}}\right)}{3\sqrt{3}}$$

```
[Out] (-2*2^(2/3)*ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[-1 + x^3]])/(3
*Sqrt[3]) + (2*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 -
Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] -
 x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x
)^2)]*Sqrt[-1 + x^3])
```

Rubi [A] time = 0.348498, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{x^3-1}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (-2*2^(2/3)*ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[-1 + x^3]])/(3
*Sqrt[3]) + (2*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 -
Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] -
 x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x
)^2)]*Sqrt[-1 + x^3])
```

Rubi in Sympy [A] time = 143.185, size = 444, normalized size = 2.72

$$-\frac{2^{\frac{2}{3}}\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,(-x+1)\operatorname{atanh}\left(\frac{3^{\frac{3}{4}}\sqrt{1+\sqrt[3]{2}}\sqrt{\sqrt{3}+2}\sqrt{-\frac{\left(-x+1+\sqrt{3}\right)^2}{\left(x-1+\sqrt{3}\right)^2}+1}}{3\sqrt{-1+\sqrt[3]{2}}\sqrt{\frac{\left(-x+1+\sqrt{3}\right)^2}{\left(x-1+\sqrt{3}\right)^2}+4\sqrt{3}+7}}\right)}{\sqrt{\frac{x-1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{-1+\sqrt[3]{2}}\left(1+\sqrt[3]{2}\right)^{\frac{3}{2}}\sqrt{x^3-1}}+\frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(-\sqrt{3}+1\right)\sqrt{-\sqrt{3}+2}\,(-x+1)\,F\left(\operatorname{asin}\left(\frac{-x+1+\sqrt{3}}{-x-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{3\sqrt{\frac{x-1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\left(-\sqrt{3}-2^{\frac{2}{3}}+1\right)}+\frac{4\cdot 2^{\frac{2}{3}}\sqrt[4]{3}\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{\sqrt{3}+2}\,(-x+1)\,\Pi\left(\frac{\left(-1+2^{\frac{2}{3}}+\sqrt{3}\right)^2}{\left(-2^{\frac{2}{3}}+1+\sqrt{3}\right)^2};\operatorname{asin}\left(\frac{-x+1+\sqrt{3}}{x-1+\sqrt{3}}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt{\frac{x-1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{4\sqrt{3}+7}\sqrt{x^3-1}\left(-2^{\frac{2}{3}}+1+\sqrt{3}\right)\left(-\sqrt{3}-2^{\frac{2}{3}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(2**(2/3)-x)/(x**3-1)**(1/2),x)
```

```
[Out] -2**(2/3)*sqrt((x**2 + x + 1)/(-x - sqrt(3) + 1)**2)*(-x + 1)*ata
nh(3**(3/4)*sqrt(1 + 2**(1/3))*sqrt(sqrt(3) + 2)*sqrt(-(-x + 1 +
sqrt(3))**2/(x - 1 + sqrt(3))**2 + 1)/(3*sqrt(-1 + 2**(1/3))*sqrt
((-x + 1 + sqrt(3))**2/(x - 1 + sqrt(3))**2 + 4*sqrt(3) + 7)))/(s
qrt((x - 1)/(-x - sqrt(3) + 1)**2)*sqrt(-1 + 2**(1/3))*(1 + 2**(1
/3))**(3/2)*sqrt(x**3 - 1)) + 2*3**(3/4)*sqrt((x**2 + x + 1)/(-x
- sqrt(3) + 1)**2)*(-sqrt(3) + 1)*sqrt(-sqrt(3) + 2)*(-x + 1)*ell
iptic_f(asin((-x + 1 + sqrt(3))/(-x - sqrt(3) + 1)), -7 + 4*sqrt(
3))/(3*sqrt((x - 1)/(-x - sqrt(3) + 1)**2)*sqrt(x**3 - 1)*(-sqrt(
3) - 2**(2/3) + 1)) + 4*2**(2/3)*3**(1/4)*sqrt((x**2 + x + 1)/(-x
 - sqrt(3) + 1)**2)*sqrt(sqrt(3) + 2)*(-x + 1)*elliptic_pi((-1 +
2**(2/3) + sqrt(3))**2/(-2**(2/3) + 1 + sqrt(3))**2, asin((-x + 1
 + sqrt(3))/(x - 1 + sqrt(3))), -7 + 4*sqrt(3))/(sqrt((x - 1)/(-x
 - sqrt(3) + 1)**2)*sqrt(4*sqrt(3) + 7)*sqrt(x**3 - 1)*(-2**(2/3)
 + 1 + sqrt(3))*(-sqrt(3) - 2**(2/3) + 1))
```

Mathematica [C] time = 0.488098, size = 207, normalized size = 1.27

$$\frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(-\frac{\left(x+\sqrt[3]{-1}\right)\sqrt{\frac{(-1)^{2/3}x+\sqrt[3]{-1}}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{i2^{2/3}\sqrt{x^2+x+1}\,\Pi\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt[3]{-1}+2^{2/3}}\right)}{\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) + x)*Sqrt[((-1)
^(1/3) + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1
 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^
(2/3)*x)/(1 + (-1)^(1/3))]) + (I*2^(2/3)*Sqrt[1 + x + x^2]*Ellipt
icPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)), ArcSin[Sqrt[(1 - (-1)^(2
/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/((-1)^(1/3) + 2^(2/3))))/
Sqrt[-1 + x^3]
```

Maple [B] time = 0.029, size = 262, normalized size = 1.6

$$-2\,\frac{-3/2-i/2\sqrt{3}}{\sqrt{x^3-1}}\sqrt{\frac{-1+x}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2-i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2+i/2\sqrt{3}}{3/2+i/2\sqrt{3}}}\mathit{EllipticF}\left(\sqrt{\frac{-1+x}{-3/2-i/2\sqrt{3}}},\sqrt{\frac{3/2+i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\right)$$
$$-2\,\frac{2^{2/3}\left(-3/2-i/2\sqrt{3}\right)}{\sqrt{x^3-1}\left(-2^{2/3}+1\right)}\sqrt{\frac{-1+x}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2-i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2+i/2\sqrt{3}}{3/2+i/2\sqrt{3}}}\mathit{EllipticPi}\left(\sqrt{\frac{-1+x}{-3/2-i/2\sqrt{3}}},\frac{3/2+i/2\sqrt{3}}{-2^{2/3}+1},\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(2^(2/3)-x)/(x^3-1)^(1/2),x)
```

```
[Out] -2*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1
/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2
))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((-1+x)/(-3
/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2))
)^(1/2))-2*2^(2/3)*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/
2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+
1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)/(-2^(
2/3)+1)*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I
*3^(1/2))/(-2^(2/3)+1),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^
(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{\sqrt{x^3-1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x/(sqrt(x^3 - 1)*(x - 2^(2/3))),x, algorithm="maxima")

[Out] -integrate(x/(sqrt(x^3 - 1)*(x - 2^(2/3))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{x}{\sqrt{x^3-1}\left(x-2^{\frac{2}{3}}\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x/(sqrt(x^3 - 1)*(x - 2^(2/3))),x, algorithm="fricas")

[Out] integral(-x/(sqrt(x^3 - 1)*(x - 2^(2/3))), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{x\sqrt{x^3-1}-2^{\frac{2}{3}}\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(2**(2/3)-x)/(x**3-1)**(1/2),x)

[Out] -Integral(x/(x*sqrt(x**3 - 1) - 2**(2/3)*sqrt(x**3 - 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x}{\sqrt{x^3-1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x/(sqrt(x^3 - 1)*(x - 2^(2/3))),x, algorithm="giac")

[Out] integrate(-x/(sqrt(x^3 - 1)*(x - 2^(2/3))), x)
```

3.45 $\int \frac{x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=156

$$\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{-x^3-1}}\right)}{3\sqrt{3}}$$

```
[Out] (-2*2^(2/3)*ArcTanh[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[-1 - x^3]])/(3
*Sqrt[3]) + (2*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 -
Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] +
 x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x
)^2)]*Sqrt[-1 - x^3])
```

Rubi [A] time = 0.366649, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{-x^3-1}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (-2*2^(2/3)*ArcTanh[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[-1 - x^3]])/(3
*Sqrt[3]) + (2*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 -
Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] +
 x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x
)^2)]*Sqrt[-1 - x^3])
```

Rubi in Sympy [A] time = 144.518, size = 454, normalized size = 2.91

$$-\frac{2^{\frac{2}{3}}\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,(x+1)\operatorname{atanh}\left(\frac{3^{\frac{3}{4}}\sqrt{1+\sqrt[3]{2}}\sqrt{1-\frac{\left(x+1+\sqrt{3}\right)^2}{\left(-x-1+\sqrt{3}\right)^2}}\sqrt{\sqrt{3}+2}}{3\sqrt{-1+\sqrt[3]{2}}\sqrt{4\sqrt{3}+7+\frac{\left(x+1+\sqrt{3}\right)^2}{\left(-x-1+\sqrt{3}\right)^2}}}\right)}{\sqrt{\frac{-x-1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-1+\sqrt[3]{2}}\left(1+\sqrt[3]{2}\right)^{\frac{3}{2}}\sqrt{-x^3-1}}+\frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(-\sqrt{3}+1\right)\sqrt{-\sqrt{3}+2}\,(x+1)\,F\left(\operatorname{asin}\left(\frac{x+1+\sqrt{3}}{x-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{3\sqrt{\frac{-x-1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\left(-\sqrt{3}-2^{\frac{2}{3}}+1\right)}+\frac{4\cdot 2^{\frac{2}{3}}\sqrt[4]{3}\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{\sqrt{3}+2}\,(x+1)\left(\frac{\left(-1+2^{\frac{2}{3}}+\sqrt{3}\right)^2}{\left(-2^{\frac{2}{3}}+1+\sqrt{3}\right)^2};\operatorname{asin}\left(\frac{x+1+\sqrt{3}}{-x-1+\sqrt{3}}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt{\frac{-x-1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{4\sqrt{3}+7}\sqrt{-x^3-1}\left(-2^{\frac{2}{3}}+1+\sqrt{3}\right)\left(-\sqrt{3}-2^{\frac{2}{3}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(2**(2/3)+x)/(-x**3-1)**(1/2),x)
```

```
[Out] -2**(2/3)*sqrt((x**2 - x + 1)/(x - sqrt(3) + 1)**2)*(x + 1)*atanh
(3**(3/4)*sqrt(1 + 2**(1/3))*sqrt(1 - (x + 1 + sqrt(3))**2/(-x -
1 + sqrt(3))**2)*sqrt(sqrt(3) + 2)/(3*sqrt(-1 + 2**(1/3))*sqrt(4*
sqrt(3) + 7 + (x + 1 + sqrt(3))**2/(-x - 1 + sqrt(3))**2)))/(sqrt
((-x - 1)/(x - sqrt(3) + 1)**2)*sqrt(-1 + 2**(1/3))*(1 + 2**(1/3)
)**(3/2)*sqrt(-x**3 - 1)) + 2*3**(3/4)*sqrt((x**2 - x + 1)/(x - s
qrt(3) + 1)**2)*(-sqrt(3) + 1)*sqrt(-sqrt(3) + 2)*(x + 1)*ellipti
c_f(asin((x + 1 + sqrt(3))/(x - sqrt(3) + 1)), -7 + 4*sqrt(3))/(3
*sqrt((-x - 1)/(x - sqrt(3) + 1)**2)*sqrt(-x**3 - 1)*(-sqrt(3) -
2**(2/3) + 1)) + 4*2**(2/3)*3**(1/4)*sqrt((x**2 - x + 1)/(x - sqr
t(3) + 1)**2)*sqrt(sqrt(3) + 2)*(x + 1)*elliptic_pi((-1 + 2**(2/3
) + sqrt(3))**2/(-2**(2/3) + 1 + sqrt(3))**2, asin((x + 1 + sqrt(
3))/(-x - 1 + sqrt(3))), -7 + 4*sqrt(3))/(sqrt((-x - 1)/(x - sqrt
(3) + 1)**2)*sqrt(4*sqrt(3) + 7)*sqrt(-x**3 - 1)*(-2**(2/3) + 1 +
 sqrt(3))*(-sqrt(3) - 2**(2/3) + 1))
```

Mathematica [C] time = 0.526008, size = 209, normalized size = 1.34

$$\frac{2\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\left(-\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}}+\frac{i2^{2/3}\sqrt{x^2-x+1}\,\Pi\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt[3]{-1}+2^{2/3}}\right)}{\sqrt{-x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) - x)*Sqrt[((-1)
^(1/3) - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1
 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^
(2/3)*x)/(1 + (-1)^(1/3))]) + (I*2^(2/3)*Sqrt[1 - x + x^2]*Ellipt
icPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)), ArcSin[Sqrt[(1 + (-1)^(2
/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/((-1)^(1/3) + 2^(2/3))))/
Sqrt[-1 - x^3]
```

Maple [A] time = 0.029, size = 249, normalized size = 1.6

$$-\frac{2i}{3}\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\mathit{EllipticF}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)\frac{1}{\sqrt{-x^3}}$$
$$+\frac{\frac{2i}{3}2^{\frac{2}{3}}\sqrt{3}}{\frac{1}{2}+\frac{i}{2}\sqrt{3}+2^{\frac{2}{3}}}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\mathit{EllipticPi}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\frac{i\sqrt{3}}{\frac{1}{2}+\frac{i}{2}\sqrt{3}+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(2^(2/3)+x)/(-x^3-1)^(1/2),x)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/
2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/
(-x^3-1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(
1/2))^(1/2),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))+2/3*I*2^(2/3)*
3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I
*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1
)^(1/2)/(1/2+1/2*I*3^(1/2)+2^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x-
1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+2^
(2/3)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-x^3-1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(-x^3 - 1)*(x + 2^(2/3))),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(-x^3 - 1)*(x + 2^(2/3))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x}{\sqrt{-x^3-1}\left(x+2^{\frac{2}{3}}\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(-x^3 - 1)*(x + 2^(2/3))),x, algorithm="fricas")
```

```
[Out] integral(x/(sqrt(-x^3 - 1)*(x + 2^(2/3))), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{\sqrt{-\left(x+1\right)\left(x^2-x+1\right)}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(2**(2/3)+x)/(-x**3-1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 2**(2/3))), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{\sqrt{-x^3-1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(-x^3 - 1)*(x + 2^(2/3))),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(-x^3 - 1)*(x + 2^(2/3))), x)
```

3.46 $\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=275

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{2\,2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{3\sqrt{3}\sqrt[6]{a}b^{2/3}}$$

```
[Out] (-2*2^(2/3)*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) + 2^(1/3)*b^(1/3)*x)
)/Sqrt[a + b*x^3]])/(3*Sqrt[3]*a^(1/6)*b^(2/3)) + (2*Sqrt[2 + Sqr
t[3]]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcS
in[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^
(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a
 + b*x^3])
```

Rubi [A] time = 0.6173, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{2\,2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{3\sqrt{3}\sqrt[6]{a}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (-2*2^(2/3)*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) + 2^(1/3)*b^(1/3)*x)
)/Sqrt[a + b*x^3]])/(3*Sqrt[3]*a^(1/6)*b^(2/3)) + (2*Sqrt[2 + Sqr
t[3]]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcS
in[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^
(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a
 + b*x^3])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x/(2**(2/3)*a**(1/3)+b**(1/3)*x)/(b*x**3+a)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 2.0808, size = 324, normalized size = 1.18

$$\frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{\sqrt[3]{-1}2^{2/3}\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\,\Pi\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt[3]{-1}+2^{2/3}}-\frac{\sqrt[4]{3}\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\sqrt[6]{-1}-\frac{i\sqrt[3]{\cdots}}{\sqrt[3]{\cdots}}}\cdots}{\sqrt{\frac{\sqrt[3]{a}+\cdots}{\left(1+\sqrt[3]{\cdots}\right.}}}\cdots\right)}{\sqrt{3}b^{2/3}\sqrt{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(-((3^(
1/4)*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(-1)^(1/6) - (I*b^(1/3
)*x)/a^(1/3)]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)
*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/Sqrt[(a^(1/3) + (-
1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]) + ((-1)^(1/3)*2^
(2/3)*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[1 - (b^(1/3)*x)/a^(1/3) + (b^
(2/3)*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3))
, ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*
a^(1/3))]], (-1)^(1/3)])/((-1)^(1/3) + 2^(2/3))))/(Sqrt[3]*b^(2/3
)*Sqrt[a + b*x^3])
```

Maple [F] time = 0.11, size = 0, normalized size = 0.

$$\int x \left(2^{\frac{2}{3}} \sqrt[3]{a} + \sqrt[3]{b}x\right)^{-1} \frac{1}{\sqrt{bx^3 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

```
[Out] int(x/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{bx^3 + a}\left(b^{\frac{1}{3}}x + 2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(sqrt(b*x^3 + a)*(b^(1/3)*x + 2^(2/3)*a^(1/3))),x, algorithm="maxima
```

```
[Out] integrate(x/(sqrt(b*x^3 + a)*(b^(1/3)*x + 2^(2/3)*a^(1/3))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(sqrt(b*x^3 + a)*(b^(1/3)*x + 2^(2/3)*a^(1/3))),x, algorithm="fricas
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a + bx^3}\left(2^{\frac{2}{3}} \sqrt[3]{a} + \sqrt[3]{b}x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(2**(2/3)*a**(1/3)+b**(1/3)*x)/(b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(a + b*x**3)*(2**(2/3)*a**(1/3) + b**(1/3)*x)), x
)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{bx^3 + a}\left(b^{\frac{1}{3}}x + 2^{\frac{2}{3}}a^{\frac{1}{3}}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(b*x^3 + a)*(b^(1/3)*x + 2^(2/3)*a^(1/3))),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(b*x^3 + a)*(b^(1/3)*x + 2^(2/3)*a^(1/3))), x)
```

3.47 $\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$

Optimal. Leaf size=283

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}} - \frac{2\,2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{3\sqrt{3}\sqrt[6]{a}b^{2/3}}$$

```
[Out] (-2*2^(2/3)*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*b^(1/3)*x)
)/Sqrt[a - b*x^3]])/(3*Sqrt[3]*a^(1/6)*b^(2/3)) + (2*Sqrt[2 + Sqr
t[3]]*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcS
in[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b
^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^
(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a
 - b*x^3])
```

Rubi [A] time = 0.637884, antiderivative size = 283, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}} - \frac{2\,2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{3\sqrt{3}\sqrt[6]{a}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (-2*2^(2/3)*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*b^(1/3)*x)
)/Sqrt[a - b*x^3]])/(3*Sqrt[3]*a^(1/6)*b^(2/3)) + (2*Sqrt[2 + Sqr
t[3]]*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcS
in[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b
^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^
(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a
 - b*x^3])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x/(2**(2/3)*a**(1/3)-b**(1/3)*x)/(-b*x**3+a)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 1.6076, size = 388, normalized size = 1.37

$$-\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\left(\sqrt[3]{-1}+2^{2/3}\right)\left(\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}F\left(\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)-\frac{\sqrt[3]{-1}2^{2/3}\left(1+\cdots\right.}{\cdots}\right.}{\left(\sqrt[3]{-1}+2^{2/3}\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{a-bx^{3}}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (-2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(((-1)
^(1/3) + 2^(2/3))*((-1)^(1/3)*a^(1/3) + b^(1/3)*x)*Sqrt[((-1)^(1/
3)*(a^(1/3) + (-1)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*
EllipticF[ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)
^(1/3))*a^(1/3))]], (-1)^(1/3)] - ((-1)^(1/3)*2^(2/3)*(1 + (-1)^(
1/3))*a^(1/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1
/3))*a^(1/3))]*Sqrt[1 + (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/
3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)), ArcSin[Sqrt[(a
^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)
```

```
^(1/3)])/Sqrt[3]))/(((-1)^(1/3) + 2^(2/3))*b^(2/3)*Sqrt[(a^(1/3)
- (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a - b*x^
3])
```

Maple [F] time = 0.105, size = 0, normalized size = 0.

$$\int x\left(2^{\frac{2}{3}}\sqrt[3]{a}-\sqrt[3]{b}x\right)^{-1}\frac{1}{\sqrt{-bx^{3}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

```
[Out] int(x/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{\sqrt{-bx^{3}+a}\left(b^{\frac{1}{3}}x-2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x/(sqrt(-b*x^3 + a)*(b^(1/3)*x - 2^(2/3)*a^(1/3))),x, algorithm="maxi
```

```
[Out] -integrate(x/(sqrt(-b*x^3 + a)*(b^(1/3)*x - 2^(2/3)*a^(1/3))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x/(sqrt(-b*x^3 + a)*(b^(1/3)*x - 2^(2/3)*a^(1/3))),x, algorithm="fric
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt[3]{b}x\sqrt{a-bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(2**(2/3)*a**(1/3)-b**(1/3)*x)/(-b*x**3+a)**(1/2),x)
```

```
[Out] -Integral(x/(-2**(2/3)*a**(1/3)*sqrt(a - b*x**3) + b**(1/3)*x*sqr
t(a - b*x**3)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x}{\sqrt{-bx^3+a}\left(b^{\frac{1}{3}}x-2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x/(sqrt(-b*x^3 + a)*(b^(1/3)*x - 2^(2/3)*a^(1/3))),x, algorithm="giac
```

```
[Out] integrate(-x/(sqrt(-b*x^3 + a)*(b^(1/3)*x - 2^(2/3)*a^(1/3))), x)
```

3.48 $\displaystyle\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

Optimal. Leaf size=292

$$\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}-\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{bx^3-a}}\right)}{3\sqrt{3}\sqrt[6]{a}b^{2/3}}$$

```
[Out] (-2*2^(2/3)*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*b^(1/3)*x
))/Sqrt[-a + b*x^3]])/(3*Sqrt[3]*a^(1/6)*b^(2/3)) + (2*Sqrt[2 - S
qrt[3]]*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[Ar
cSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) -
 b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*b^(2/3)*Sqrt[-((a^(1/3)
*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*S
qrt[-a + b*x^3])
```

Rubi [A] time = 0.671625, antiderivative size = 292, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108

$$\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}-\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{bx^3-a}}\right)}{3\sqrt{3}\sqrt[6]{a}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (-2*2^(2/3)*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*b^(1/3)*x
))/Sqrt[-a + b*x^3]])/(3*Sqrt[3]*a^(1/6)*b^(2/3)) + (2*Sqrt[2 - S
qrt[3]]*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[Ar
cSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) -
 b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*b^(2/3)*Sqrt[-((a^(1/3)
*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*S
qrt[-a + b*x^3])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x/(2**(2/3)*a**(1/3)-b**(1/3)*x)/(b*x**3-a)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 1.58894, size = 389, normalized size = 1.33

$$-\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\left(\sqrt[3]{-1}+2^{2/3}\right)\left(\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}F\left(\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)-\frac{\sqrt[3]{-1}2^{2/3}\left(1+\right.}{}\right.}{\left(\sqrt[3]{-1}+2^{2/3}\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{bx^3-}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (-2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(((-1)
^(1/3) + 2^(2/3))*((-1)^(1/3)*a^(1/3) + b^(1/3)*x)*Sqrt[((-1)^(1/
3)*(a^(1/3) + (-1)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*
EllipticF[ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)
^(1/3))*a^(1/3))]], (-1)^(1/3)] - ((-1)^(1/3)*2^(2/3)*(1 + (-1)^(
1/3))*a^(1/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1
/3))*a^(1/3))]*Sqrt[1 + (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/
3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)), ArcSin[Sqrt[(a
^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)
```

```
^(1/3)])/Sqrt[3]))/(((-1)^(1/3) + 2^(2/3))*b^(2/3)*Sqrt[(a^(1/3)
- (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[-a + b*x
^3])
```

Maple [F] time = 0.094, size = 0, normalized size = 0.

$$\int x \left(2^{\frac{2}{3}} \sqrt[3]{a} - \sqrt[3]{b} x\right)^{-1} \frac{1}{\sqrt{b x^3 - a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

```
[Out] int(x/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{\sqrt{b x^3 - a}\left(b^{\frac{1}{3}} x - 2^{\frac{2}{3}} a^{\frac{1}{3}}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x/(sqrt(b*x^3 - a)*(b^(1/3)*x - 2^(2/3)*a^(1/3))),x, algorithm="maxim
```

```
[Out] -integrate(x/(sqrt(b*x^3 - a)*(b^(1/3)*x - 2^(2/3)*a^(1/3))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x/(sqrt(b*x^3 - a)*(b^(1/3)*x - 2^(2/3)*a^(1/3))),x, algorithm="frica
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt[3]{b}x\sqrt{-a+bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(2**(2/3)*a**(1/3)-b**(1/3)*x)/(b*x**3-a)**(1/2),x)
```

```
[Out] -Integral(x/(-2**(2/3)*a**(1/3)*sqrt(-a + b*x**3) + b**(1/3)*x*sq
rt(-a + b*x**3)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x}{\sqrt{bx^3-a}\left(b^{\frac{1}{3}}x-2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x/(sqrt(b*x^3 - a)*(b^(1/3)*x - 2^(2/3)*a^(1/3))),x, algorithm="giac"
```

```
[Out] integrate(-x/(sqrt(b*x^3 - a)*(b^(1/3)*x - 2^(2/3)*a^(1/3))), x)
```

3.49 $\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

Optimal. Leaf size=288

$$\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}} - \frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{3\sqrt{3}\sqrt[6]{a}b^{2/3}}$$

```
[Out] (-2*2^(2/3)*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) + 2^(1/3)*b^(1/3)*x
))/Sqrt[-a - b*x^3]])/(3*Sqrt[3]*a^(1/6)*b^(2/3)) + (2*Sqrt[2 - S
qrt[3]]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[Ar
cSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) +
 b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*b^(2/3)*Sqrt[-((a^(1/3)
*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*S
qrt[-a - b*x^3])
```

Rubi [A] time = 0.647497, antiderivative size = 288, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108

$$\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}} - \frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{3\sqrt{3}\sqrt[6]{a}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (-2*2^(2/3)*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) + 2^(1/3)*b^(1/3)*x
))/Sqrt[-a - b*x^3]])/(3*Sqrt[3]*a^(1/6)*b^(2/3)) + (2*Sqrt[2 - S
qrt[3]]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[Ar
cSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) +
 b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*b^(2/3)*Sqrt[-((a^(1/3)
*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*S
qrt[-a - b*x^3])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x/(2**(2/3)*a**(1/3)+b**(1/3)*x)/(-b*x**3-a)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 1.56785, size = 375, normalized size = 1.3

$$\frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{\sqrt[3]{-1}2^{2/3}\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{3}}-\frac{\left(\sqrt[3]{-1}+2^{2/3}\right)\left(\sqrt[3]{\cdots}\right.}{\cdots}\right)}{\left(\sqrt[3]{-1}+2^{2/3}\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{-a-bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(-((((-
1)^(1/3) + 2^(2/3))*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(-1)^(1
/6) - (I*b^(1/3)*x)/a^(1/3)]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1
)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/3^(
1/4)) + ((-1)^(1/3)*2^(2/3)*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[(a^(1/3
) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[1 - (b
^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])
/((-1)^(1/3) + 2^(2/3)), ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3
)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/Sqrt[3]))/(((-1)^
```

```
(1/3) + 2^(2/3))*b^(2/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((
1 + (-1)^(1/3))*a^(1/3))]*Sqrt[-a - b*x^3])
```

Maple [F] time = 0.095, size = 0, normalized size = 0.

$$\int x\left(2^{\frac{2}{3}}\sqrt[3]{a}+\sqrt[3]{bx}\right)^{-1}\frac{1}{\sqrt{-bx^{3}-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x)

[Out] int(x/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{\sqrt{-bx^{3}-a}\left(b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(sqrt(-b*x^3 - a)*(b^(1/3)*x + 2^(2/3)*a^(1/3))),x, algorithm="maxim

[Out] integrate(x/(sqrt(-b*x^3 - a)*(b^(1/3)*x + 2^(2/3)*a^(1/3))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(sqrt(-b*x^3 - a)*(b^(1/3)*x + 2^(2/3)*a^(1/3))),x, algorithm="frica

[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-a-bx^3}\left(2^{\frac{2}{3}}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(2**(2/3)*a**(1/3)+b**(1/3)*x)/(-b*x**3-a)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-a - b*x**3)*(2**(2/3)*a**(1/3) + b**(1/3)*x)),
x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-bx^3-a}\left(b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(-b*x^3 - a)*(b^(1/3)*x + 2^(2/3)*a^(1/3))),x, algorithm="giac"
```

```
[Out] integrate(x/(sqrt(-b*x^3 - a)*(b^(1/3)*x + 2^(2/3)*a^(1/3))), x)
```

3.50 $\int \frac{x}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx$

Optimal. Leaf size=246

$$\frac{\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(c+2^{2/3}dx\right)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)c+2^{2/3}dx}{\left(1+\sqrt{3}\right)c+2^{2/3}dx}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}d^2\sqrt{\frac{c\left(c+2^{2/3}dx\right)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\sqrt{c^3+4d^3x^3}}-\frac{2\tan^{-1}\left(\frac{\sqrt{3}\sqrt{c}(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{3\sqrt{3}\sqrt{c}d^2}$$

```
[Out] (-2*ArcTan[(Sqrt[3]*Sqrt[c]*(c + 2*d*x))/Sqrt[c^3 + 4*d^3*x^3]])/
(3*Sqrt[3]*Sqrt[c]*d^2) + (2^(1/3)*Sqrt[2 + Sqrt[3]]*(c + 2^(2/3)
*d*x)*Sqrt[(c^2 - 2^(2/3)*c*d*x + 2*2^(1/3)*d^2*x^2)/((1 + Sqrt[3
])*c + 2^(2/3)*d*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c + 2^(2/3
)*d*x)/((1 + Sqrt[3])*c + 2^(2/3)*d*x)], -7 - 4*Sqrt[3]])/(3*3^(1
/4)*d^2*Sqrt[(c*(c + 2^(2/3)*d*x))/((1 + Sqrt[3])*c + 2^(2/3)*d*x
)^2]*Sqrt[c^3 + 4*d^3*x^3])
```

Rubi [A] time = 0.527489, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16

$$\frac{\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(c+2^{2/3}dx\right)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)c+2^{2/3}dx}{\left(1+\sqrt{3}\right)c+2^{2/3}dx}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}d^2\sqrt{\frac{c\left(c+2^{2/3}dx\right)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\sqrt{c^3+4d^3x^3}}-\frac{2\tan^{-1}\left(\frac{\sqrt{3}\sqrt{c}(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{3\sqrt{3}\sqrt{c}d^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x/((c + d*x)*Sqrt[c^3 + 4*d^3*x^3]),x]
```

```
[Out] (-2*ArcTan[(Sqrt[3]*Sqrt[c]*(c + 2*d*x))/Sqrt[c^3 + 4*d^3*x^3]])/
(3*Sqrt[3]*Sqrt[c]*d^2) + (2^(1/3)*Sqrt[2 + Sqrt[3]]*(c + 2^(2/3)
*d*x)*Sqrt[(c^2 - 2^(2/3)*c*d*x + 2*2^(1/3)*d^2*x^2)/((1 + Sqrt[3
])*c + 2^(2/3)*d*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c + 2^(2/3
)*d*x)/((1 + Sqrt[3])*c + 2^(2/3)*d*x)], -7 - 4*Sqrt[3]])/(3*3^(1
/4)*d^2*Sqrt[(c*(c + 2^(2/3)*d*x))/((1 + Sqrt[3])*c + 2^(2/3)*d*x
)^2]*Sqrt[c^3 + 4*d^3*x^3])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(d*x+c)/(4*d**3*x**3+c**3)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 1.89997, size = 372, normalized size = 1.51

$$\frac{\sqrt[6]{2}\sqrt{\frac{\sqrt[3]{2}c+2dx}{\left(1+\sqrt[3]{-1}\right)c}}\left(-\sqrt{\frac{\sqrt[3]{-2}c-2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}\left(\sqrt[3]{-1}\left(2+\sqrt[3]{-2}\right)c-2\left(\sqrt[3]{-1}+2^{2/3}\right)dx\right)F\left(\sin^{-1}\left(\frac{\sqrt{\frac{\sqrt[3]{2}c+2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}}{\sqrt[6]{2}}\right)\middle|\sqrt[3]{-1}\right)+\frac{\sqrt[3]{-1}2^{2/3}}{}\right.}{\left(2+\sqrt[3]{-2}\right)d^2\sqrt{\frac{\sqrt[3]{2}c+2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}\sqrt{c^3+4d^3x^3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x/((c + d*x)*Sqrt[c^3 + 4*d^3*x^3]),x]
```

```
[Out] (2^(1/6)*Sqrt[(2^(1/3)*c + 2*d*x)/((1 + (-1)^(1/3))*c)]*(-(Sqrt[(
(-2)^(1/3)*c - 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]*((-1)^(1/3
)*(2 + (-2)^(1/3))*c - 2*((-1)^(1/3) + 2^(2/3))*d*x)*EllipticF[Ar
cSin[Sqrt[(2^(1/3)*c + 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]/2^
(1/6)], (-1)^(1/3)]) + ((-1)^(1/3)*2^(2/3)*(1 + (-1)^(1/3))*c*Sqr
t[(2^(1/3)*c + 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]*Sqrt[2^(2/
3) - (2*2^(1/3)*d*x)/c + (4*d^2*x^2)/c^2]*EllipticPi[(I*2^(1/3)*S
qrt[3])/(2 + (-2)^(1/3)), ArcSin[Sqrt[(2^(1/3)*c + 2*(-1)^(2/3)*d
*x)/((1 + (-1)^(1/3))*c)]/2^(1/6)], (-1)^(1/3)])/Sqrt[3]))/((2 +
(-2)^(1/3))*d^2*Sqrt[(2^(1/3)*c + 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1
/3))*c)]*Sqrt[c^3 + 4*d^3*x^3])
```

Maple [B] time = 0.012, size = 892, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x)
```

```
[Out] 2/d*((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3
^(1/2)*2^(1/3))*c/d)*((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)
/((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1
/2)*2^(1/3))*c/d))^(1/2)*((x+1/2*2^(1/3)*c/d)/((1/4*2^(1/3)+1/4*I
*3^(1/2)*2^(1/3))*c/d+1/2*2^(1/3)*c/d))^(1/2)*((x-(1/4*2^(1/3)-1/
4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/
d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)/(4*d^3*x^3+c^3)
^(1/2)*EllipticF(((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1
/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*
2^(1/3))*c/d))^(1/2),(((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d-(1
/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2
)*2^(1/3))*c/d+1/2*2^(1/3)*c/d))^(1/2))-2*c/d^2*((1/4*2^(1/3)-1/4
*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)*
((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)-1/4*I*
3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^(1
/2)*((x+1/2*2^(1/3)*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d
+1/2*2^(1/3)*c/d))^(1/2)*((x-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*
c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*
3^(1/2)*2^(1/3))*c/d))^(1/2)/(4*d^3*x^3+c^3)^(1/2)/((1/4*2^(1/3)+
1/4*I*3^(1/2)*2^(1/3))*c/d+c/d)*EllipticPi(((x-(1/4*2^(1/3)+1/4*I
*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(
1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2),((1/4*2^(1/3)+1/4*
I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/(
(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+c/d),(((1/4*2^(1/3)+1/4*I
*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((
1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+1/2*2^(1/3)*c/d))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{4\,d^3x^3 + c^3}(dx + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x}{\sqrt{4\,d^3x^3 + c^3}(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)),x, algorithm="fricas")
```

```
[Out] integral(x/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(d*x+c)/(4*d**3*x**3+c**3)**(1/2),x)
```

```
[Out] Integral(x/((c + d*x)*sqrt(c**3 + 4*d**3*x**3)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{4\,d^3x^3+c^3}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)), x)
```

3.51 $\int \frac{1+x}{(2-x)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=23

$$\frac{2}{3}\tanh^{-1}\left(\frac{(x+1)^2}{3\sqrt{x^3+1}}\right)$$

```
[Out] (2*ArcTanh[(1 + x)^2/(3*Sqrt[1 + x^3])])/3
```

Rubi [A] time = 0.103326, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$\frac{2}{3}\tanh^{-1}\left(\frac{(x+1)^2}{3\sqrt{x^3+1}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x)/((2 - x)*Sqrt[1 + x^3]),x]

[Out] (2*ArcTanh[(1 + x)^2/(3*Sqrt[1 + x^3])])/3
```

Rubi in Sympy [A] time = 98.4245, size = 371, normalized size = 16.13

$$\frac{3\sqrt{\frac{x^2-x+1}{\left(x+1+\sqrt{3}\right)^2}}\left(\frac{\sqrt{3}}{3}+1\right)(x+1)\operatorname{atanh}\left(\frac{3^{\frac{3}{4}}\sqrt{-\sqrt{3}+2}\sqrt{-\frac{\left(-x-1+\sqrt{3}\right)^2}{\left(x+1+\sqrt{3}\right)^2}+1}}{3\sqrt{\frac{\left(-x-1+\sqrt{3}\right)^2}{\left(x+1+\sqrt{3}\right)^2}-4\sqrt{3}+7}}\right)}{\sqrt{\frac{x+1}{\left(x+1+\sqrt{3}\right)^2}}\left(\sqrt{3}+3\right)\sqrt{x^3+1}} - \frac{2\sqrt[4]{3}\sqrt{\frac{x^2-x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{\sqrt{3}+2}\,(x+1)\,F\left(\operatorname{asin}\left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+1+\sqrt{3}\right)^2}}\left(\sqrt{3}+3\right)\sqrt{x^3+1}} - \frac{12\sqrt[4]{3}\sqrt{\frac{x^2-x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{-\sqrt{3}+2}\,(x+1)\left(4\sqrt{3}+7;\operatorname{asin}\left(\frac{-x-1+\sqrt{3}}{x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{-4\sqrt{3}+7}\left(-\sqrt{3}+3\right)\left(\sqrt{3}+3\right)\sqrt{x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+x)/(2-x)/(x**3+1)**(1/2),x)
```

```
[Out] 3*sqrt((x**2 - x + 1)/(x + 1 + sqrt(3))**2)*(sqrt(3)/3 + 1)*(x +
1)*atanh(3**(3/4)*sqrt(-sqrt(3) + 2)*sqrt(-(-x - 1 + sqrt(3))**2/
(x + 1 + sqrt(3))**2 + 1)/(3*sqrt((-x - 1 + sqrt(3))**2/(x + 1 +
sqrt(3))**2 - 4*sqrt(3) + 7)))/(sqrt((x + 1)/(x + 1 + sqrt(3))**2
)*(sqrt(3) + 3)*sqrt(x**3 + 1)) - 2*3**(1/4)*sqrt((x**2 - x + 1)/
(x + 1 + sqrt(3))**2)*sqrt(sqrt(3) + 2)*(x + 1)*elliptic_f(asin((
x - sqrt(3) + 1)/(x + 1 + sqrt(3))), -7 - 4*sqrt(3))/(sqrt((x + 1
)/(x + 1 + sqrt(3))**2)*(sqrt(3) + 3)*sqrt(x**3 + 1)) - 12*3**(1/
4)*sqrt((x**2 - x + 1)/(x + 1 + sqrt(3))**2)*sqrt(-sqrt(3) + 2)*(
x + 1)*elliptic_pi(4*sqrt(3) + 7, asin((-x - 1 + sqrt(3))/(x + 1
+ sqrt(3))), -7 - 4*sqrt(3))/(sqrt((x + 1)/(x + 1 + sqrt(3))**2)*
sqrt(-4*sqrt(3) + 7)*(-sqrt(3) + 3)*(sqrt(3) + 3)*sqrt(x**3 + 1))
```

Mathematica [C]   time = 0.319002, size = 265, normalized size = 11.52

$$\frac{2\sqrt{6}\sqrt{-\frac{i(x+1)}{\sqrt{3}-3i}}\left(2\sqrt{3}\sqrt{2ix+\sqrt{3}-i}\sqrt{x^2-x+1}\left(\frac{2\sqrt{3}}{3i+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{2ix+\sqrt{3}-i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right)-i\sqrt{-2ix+\sqrt{3}+i}\left(\left(\sqrt{3}-i\right)x-\sqrt{3}-i\right)}{\left(\sqrt{3}+3i\right)\sqrt{2ix+\sqrt{3}-i}\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(1 + x)/((2 - x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[6]*Sqrt[((-I)*(1 + x))/(-3*I + Sqrt[3])]*((-I)*Sqrt[I + S
qrt[3] - (2*I)*x]*(-I - Sqrt[3] + (-I + Sqrt[3])*x)*EllipticF[Arc
Sin[Sqrt[-I + Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/
(-3*I + Sqrt[3])] + 2*Sqrt[3]*Sqrt[-I + Sqrt[3] + (2*I)*x]*Sqrt[1
 - x + x^2]*EllipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[-
I + Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-3*I + Sq
rt[3])]))/((3*I + Sqrt[3])*Sqrt[-I + Sqrt[3] + (2*I)*x]*Sqrt[1 +
x^3])
```

Maple [C]   time = 0.032, size = 240, normalized size = 10.4

$$-2\,\frac{3/2-i/2\sqrt{3}}{\sqrt{x^3+1}}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}EllipticF\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},\sqrt{\frac{-3/2+i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\right)$$

$$+2\,\frac{3/2-i/2\sqrt{3}}{\sqrt{x^3+1}}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}EllipticPi\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},1/2-i/6\sqrt{3},\sqrt{\frac{-3/2+i/2}{-3/2-i/2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+x)/(2-x)/(x^3+1)^(1/2),x)
```

```
[Out] -2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-
1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))
/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-
1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))
^(1/2))+2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*(
(x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3
^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticPi(((1+
x)/(3/2-1/2*I*3^(1/2)))^(1/2),1/2-1/6*I*3^(1/2),((-3/2+1/2*I*3^(1
/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x+1}{\sqrt{x^3+1}(x-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x + 1)/(sqrt(x^3 + 1)*(x - 2)),x, algorithm="maxima")
```

```
[Out] -integrate((x + 1)/(sqrt(x^3 + 1)*(x - 2)), x)
```

Fricas [A] time = 0.34117, size = 59, normalized size = 2.57

$$\frac{1}{3}\log\left(\frac{x^3+12\,x^2+6\,\sqrt{x^3+1}(x+1)-6\,x+10}{x^3-6\,x^2+12\,x-8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x + 1)/(sqrt(x^3 + 1)*(x - 2)),x, algorithm="fricas")
```

```
[Out] 1/3*log((x^3 + 12*x^2 + 6*sqrt(x^3 + 1)*(x + 1) - 6*x + 10)/(x^3
- 6*x^2 + 12*x - 8))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{x\sqrt{x^3+1}-2\sqrt{x^3+1}}\,dx-\int \frac{1}{x\sqrt{x^3+1}-2\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+x)/(2-x)/(x**3+1)**(1/2),x)
```

```
[Out] -Integral(x/(x*sqrt(x**3 + 1) - 2*sqrt(x**3 + 1)), x) - Integral(
1/(x*sqrt(x**3 + 1) - 2*sqrt(x**3 + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x+1}{\sqrt{x^3+1}(x-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x + 1)/(sqrt(x^3 + 1)*(x - 2)),x, algorithm="giac")
```

```
[Out] integrate(-(x + 1)/(sqrt(x^3 + 1)*(x - 2)), x)
```

3.52 $\int \frac{1-x}{(2+x)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=27

$$-\frac{2}{3}\tanh^{-1}\left(\frac{(1-x)^2}{3\sqrt{1-x^3}}\right)$$

```
[Out] (-2*ArcTanh[(1 - x)^2/(3*Sqrt[1 - x^3])])/3
```

Rubi [A] time = 0.117911, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$-\frac{2}{3}\tanh^{-1}\left(\frac{(1-x)^2}{3\sqrt{1-x^3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - x)/((2 + x)*Sqrt[1 - x^3]),x]

[Out] (-2*ArcTanh[(1 - x)^2/(3*Sqrt[1 - x^3])])/3
```

Rubi in Sympy [A] time = 101.795, size = 371, normalized size = 13.74

$$-\frac{3\sqrt{\frac{x^2+x+1}{\left(-x+1+\sqrt{3}\right)^2}}\left(\frac{\sqrt{3}}{3}+1\right)(-x+1)\operatorname{atanh}\left(\frac{3^{\frac{3}{4}}\sqrt{1-\frac{\left(x-1+\sqrt{3}\right)^2}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{-\sqrt{3}+2}}{3\sqrt{-4\sqrt{3}+7+\frac{\left(x-1+\sqrt{3}\right)^2}{\left(-x+1+\sqrt{3}\right)^2}}}\right)}{\sqrt{\frac{-x+1}{\left(-x+1+\sqrt{3}\right)^2}}\left(\sqrt{3}+3\right)\sqrt{-x^3+1}}$$
$$+\frac{2\sqrt[4]{3}\sqrt{\frac{x^2+x+1}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{\sqrt{3}+2}\,(-x+1)\,F\left(\operatorname{asin}\left(\frac{-x-\sqrt{3}+1}{-x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt{\frac{-x+1}{\left(-x+1+\sqrt{3}\right)^2}}\left(\sqrt{3}+3\right)\sqrt{-x^3+1}}$$
$$+\frac{12\sqrt[4]{3}\sqrt{\frac{x^2+x+1}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{-\sqrt{3}+2}\,(-x+1)\left(4\sqrt{3}+7;\operatorname{asin}\left(\frac{x-1+\sqrt{3}}{-x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt{\frac{-x+1}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{-4\sqrt{3}+7}\left(-\sqrt{3}+3\right)\left(\sqrt{3}+3\right)\sqrt{-x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1-x)/(2+x)/(-x**3+1)**(1/2),x)
```

```
[Out] -3*sqrt((x**2 + x + 1)/(-x + 1 + sqrt(3))**2)*(sqrt(3)/3 + 1)*(-x
 + 1)*atanh(3**(3/4)*sqrt(1 - (x - 1 + sqrt(3))**2/(-x + 1 + sqrt
(3))**2)*sqrt(-sqrt(3) + 2)/(3*sqrt(-4*sqrt(3) + 7 + (x - 1 + sqr
t(3))**2/(-x + 1 + sqrt(3))**2)))/(sqrt((-x + 1)/(-x + 1 + sqrt(3
))**2)*(sqrt(3) + 3)*sqrt(-x**3 + 1)) + 2*3**(1/4)*sqrt((x**2 + x
 + 1)/(-x + 1 + sqrt(3))**2)*sqrt(sqrt(3) + 2)*(-x + 1)*elliptic_
f(asin((-x - sqrt(3) + 1)/(-x + 1 + sqrt(3))), -7 - 4*sqrt(3))/(s
qrt((-x + 1)/(-x + 1 + sqrt(3))**2)*(sqrt(3) + 3)*sqrt(-x**3 + 1)
) + 12*3**(1/4)*sqrt((x**2 + x + 1)/(-x + 1 + sqrt(3))**2)*sqrt(-
sqrt(3) + 2)*(-x + 1)*elliptic_pi(4*sqrt(3) + 7, asin((x - 1 + sq
rt(3))/(-x + 1 + sqrt(3))), -7 - 4*sqrt(3))/(sqrt((-x + 1)/(-x +
1 + sqrt(3))**2)*sqrt(-4*sqrt(3) + 7)*(-sqrt(3) + 3)*(sqrt(3) + 3
)*sqrt(-x**3 + 1))
```

Mathematica [C] time = 0.360765, size = 262, normalized size = 9.7

$$-\frac{2\sqrt{6}\sqrt{\frac{i(x-1)}{\sqrt{3}-3i}}\left(2\sqrt{3}\sqrt{-2ix+\sqrt{3}-i}\sqrt{x^2+x+1}\left(\frac{2\sqrt{3}}{3i+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}-i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right)+\sqrt{2ix+\sqrt{3}+i}\left(i\sqrt{3}x+x+i\sqrt{3}-1\right)\right.}{\left(\sqrt{3}+3i\right)\sqrt{-2ix+\sqrt{3}-i}\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(1 - x)/((2 + x)*Sqrt[1 - x^3]),x]
```

```
[Out] (-2*Sqrt[6]*Sqrt[(I*(-1 + x))/(-3*I + Sqrt[3])]*(Sqrt[I + Sqrt[3]
 + (2*I)*x]*(-1 + I*Sqrt[3] + x + I*Sqrt[3]*x)*EllipticF[ArcSin[S
qrt[-I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-3*I
 + Sqrt[3])] + 2*Sqrt[3]*Sqrt[-I + Sqrt[3] - (2*I)*x]*Sqrt[1 + x
+ x^2]*EllipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[-I + S
qrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3]
)]))/((3*I + Sqrt[3])*Sqrt[-I + Sqrt[3] - (2*I)*x]*Sqrt[1 - x^3])
```

Maple [C] time = 0.036, size = 240, normalized size = 8.9

$$\frac{2i}{3}\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}EllipticF\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)\frac{1}{\sqrt{-x^3}}$$

$$-\frac{2i\sqrt{3}}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}EllipticPi\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i}{2}\sqrt{3}},\sqrt{\frac{}{-}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1-x)/(2+x)/(-x^3+1)^(1/2),x)
```

```
[Out] 2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/(-3
/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)
/(-x^3+1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^
(1/2))^(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))-2*I*3^(1/2)*
(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/(-3/2+1/2*I*3^(1/
2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2
)/(3/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1
/2))*3^(1/2))^(1/2),I*3^(1/2)/(3/2+1/2*I*3^(1/2)),(I*3^(1/2)/(-3/
2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x-1}{\sqrt{-x^3+1}(x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x - 1)/(sqrt(-x^3 + 1)*(x + 2)),x, algorithm="maxima")
```

```
[Out] -integrate((x - 1)/(sqrt(-x^3 + 1)*(x + 2)), x)
```

Fricas [A] time = 0.335244, size = 63, normalized size = 2.33

$$\frac{1}{3}\,\log\left(-\frac{x^3-12\,x^2-6\,\sqrt{-x^3+1}(x-1)-6\,x-10}{x^3+6\,x^2+12\,x+8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x - 1)/(sqrt(-x^3 + 1)*(x + 2)),x, algorithm="fricas")
```

```
[Out] 1/3*log(-(x^3 - 12*x^2 - 6*sqrt(-x^3 + 1)*(x - 1) - 6*x - 10)/(x^
3 + 6*x^2 + 12*x + 8))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{x\sqrt{-x^3+1}+2\sqrt{-x^3+1}}\,dx-\int \left(-\frac{1}{x\sqrt{-x^3+1}+2\sqrt{-x^3+1}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1-x)/(2+x)/(-x**3+1)**(1/2),x)
```

```
[Out] -Integral(x/(x*sqrt(-x**3 + 1) + 2*sqrt(-x**3 + 1)), x) - Integra
l(-1/(x*sqrt(-x**3 + 1) + 2*sqrt(-x**3 + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x-1}{\sqrt{-x^3+1}(x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x - 1)/(sqrt(-x^3 + 1)*(x + 2)),x, algorithm="giac")
```

```
[Out] integrate(-(x - 1)/(sqrt(-x^3 + 1)*(x + 2)), x)
```

3.53 $\int \frac{1-x}{(2+x)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=25

$$-\frac{2}{3}\tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{x^3-1}}\right)$$

```
[Out] (-2*ArcTan[(1 - x)^2/(3*Sqrt[-1 + x^3])])/3
```

Rubi [A] time = 0.103321, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$-\frac{2}{3}\tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{x^3-1}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - x)/((2 + x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (-2*ArcTan[(1 - x)^2/(3*Sqrt[-1 + x^3])])/3
```

Rubi in Sympy [A] time = 90.8784, size = 369, normalized size = 14.76

$$-\frac{3\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(-\frac{\sqrt{3}}{3}+1\right)(-x+1)\operatorname{atan}\left(\frac{3^{\frac{3}{4}}\sqrt{\sqrt{3}+2}\sqrt{-\frac{\left(-x+1+\sqrt{3}\right)^2}{\left(x-1+\sqrt{3}\right)^2}+1}}{3\sqrt{\frac{\left(-x+1+\sqrt{3}\right)^2}{\left(x-1+\sqrt{3}\right)^2}+4\sqrt{3}+7}}\right)}{\sqrt{\frac{x-1}{\left(-x-\sqrt{3}+1\right)^2}}\left(-\sqrt{3}+3\right)\sqrt{x^3-1}} - \frac{2\sqrt[4]{3}\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{-\sqrt{3}+2}\,(-x+1)\,F\left(\operatorname{asin}\left(\frac{-x+1+\sqrt{3}}{-x-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt{\frac{x-1}{\left(-x-\sqrt{3}+1\right)^2}}\left(-\sqrt{3}+3\right)\sqrt{x^3-1}} - \frac{12\sqrt[4]{3}\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{\sqrt{3}+2}\,(-x+1)\left(-4\sqrt{3}+7;\operatorname{asin}\left(\frac{-x+1+\sqrt{3}}{x-1+\sqrt{3}}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt{\frac{x-1}{\left(-x-\sqrt{3}+1\right)^2}}\left(-\sqrt{3}+3\right)\left(\sqrt{3}+3\right)\sqrt{4\sqrt{3}+7}\sqrt{x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1-x)/(2+x)/(x**3-1)**(1/2),x)
```

```
[Out] -3*sqrt((x**2 + x + 1)/(-x - sqrt(3) + 1)**2)*(-sqrt(3)/3 + 1)*(-
x + 1)*atan(3**(3/4)*sqrt(sqrt(3) + 2)*sqrt(-(-x + 1 + sqrt(3))**
2/(x - 1 + sqrt(3))**2 + 1)/(3*sqrt((-x + 1 + sqrt(3))**2/(x - 1
+ sqrt(3))**2 + 4*sqrt(3) + 7)))/(sqrt((x - 1)/(-x - sqrt(3) + 1)
**2)*(-sqrt(3) + 3)*sqrt(x**3 - 1)) - 2*3**(1/4)*sqrt((x**2 + x +
 1)/(-x - sqrt(3) + 1)**2)*sqrt(-sqrt(3) + 2)*(-x + 1)*elliptic_f
(asin((-x + 1 + sqrt(3))/(-x - sqrt(3) + 1)), -7 + 4*sqrt(3))/(sq
rt((x - 1)/(-x - sqrt(3) + 1)**2)*(-sqrt(3) + 3)*sqrt(x**3 - 1))
- 12*3**(1/4)*sqrt((x**2 + x + 1)/(-x - sqrt(3) + 1)**2)*sqrt(sqr
t(3) + 2)*(-x + 1)*elliptic_pi(-4*sqrt(3) + 7, asin((-x + 1 + sqr
t(3))/(x - 1 + sqrt(3))), -7 + 4*sqrt(3))/(sqrt((x - 1)/(-x - sqr
t(3) + 1)**2)*(-sqrt(3) + 3)*(sqrt(3) + 3)*sqrt(4*sqrt(3) + 7)*sq
rt(x**3 - 1))
```

Mathematica [C] time = 0.349488, size = 260, normalized size = 10.4

$$-\frac{2\sqrt{6}\sqrt{\frac{i(x-1)}{\sqrt{3}-3i}}\left(2\sqrt{3}\sqrt{-2ix+\sqrt{3}-i}\sqrt{x^2+x+1}\left(\frac{2\sqrt{3}}{3i+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}-i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right)+\sqrt{2ix+\sqrt{3}+i}\left(i\sqrt{3}x+x+i\sqrt{3}-1\right)}{\left(\sqrt{3}+3i\right)\sqrt{-2ix+\sqrt{3}-i}\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(1 - x)/((2 + x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (-2*Sqrt[6]*Sqrt[(I*(-1 + x))/(-3*I + Sqrt[3])]*(Sqrt[I + Sqrt[3]
 + (2*I)*x]*(-1 + I*Sqrt[3] + x + I*Sqrt[3]*x)*EllipticF[ArcSin[S
qrt[-I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-3*I
 + Sqrt[3])] + 2*Sqrt[3]*Sqrt[-I + Sqrt[3] - (2*I)*x]*Sqrt[1 + x
+ x^2]*EllipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[-I + S
qrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3]
)]))/((3*I + Sqrt[3])*Sqrt[-I + Sqrt[3] - (2*I)*x]*Sqrt[-1 + x^3]
)
```

Maple [C] time = 0.031, size = 240, normalized size = 9.6

$$-2\,{\frac{-3/2-i/2\sqrt{3}}{\sqrt{x^3-1}}\sqrt{\frac{-1+x}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2-i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2+i/2\sqrt{3}}{3/2+i/2\sqrt{3}}}{\it EllipticF}\left(\sqrt{\frac{-1+x}{-3/2-i/2\sqrt{3}}},\sqrt{\frac{3/2+i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\right)}$$
$$+2\,{\frac{-3/2-i/2\sqrt{3}}{\sqrt{x^3-1}}\sqrt{\frac{-1+x}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2-i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2+i/2\sqrt{3}}{3/2+i/2\sqrt{3}}}{\it EllipticPi}\left(\sqrt{\frac{-1+x}{-3/2-i/2\sqrt{3}}},i/6\sqrt{3}+1/2,\sqrt{\frac{3/2+i}{3/2-i}}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1-x)/(2+x)/(x^3-1)^(1/2),x)
```

```
[Out] -2*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1
/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2
))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((-1+x)/(-3
/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2))
)^(1/2))+2*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/
2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*
I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticPi(((
-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),1/6*I*3^(1/2)+1/2,((3/2+1/2*I*3
^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x-1}{\sqrt{x^3-1}(x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x - 1)/(sqrt(x^3 - 1)*(x + 2)),x, algorithm="maxima")
```

```
[Out] -integrate((x - 1)/(sqrt(x^3 - 1)*(x + 2)), x)
```

Fricas [A] time = 0.354341, size = 41, normalized size = 1.64

$$-\frac{1}{3}\arctan\left(\frac{x^3-12\,x^2-6\,x-10}{6\,\sqrt{x^3-1}(x-1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x - 1)/(sqrt(x^3 - 1)*(x + 2)),x, algorithm="fricas")
```

```
[Out] -1/3*arctan(1/6*(x^3 - 12*x^2 - 6*x - 10)/(sqrt(x^3 - 1)*(x - 1))
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{x\sqrt{x^3-1}+2\sqrt{x^3-1}}\,dx-\int\left(-\frac{1}{x\sqrt{x^3-1}+2\sqrt{x^3-1}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1-x)/(2+x)/(x**3-1)**(1/2),x)
```

```
[Out] -Integral(x/(x*sqrt(x**3 - 1) + 2*sqrt(x**3 - 1)), x) - Integral(
-1/(x*sqrt(x**3 - 1) + 2*sqrt(x**3 - 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x-1}{\sqrt{x^3-1}(x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x - 1)/(sqrt(x^3 - 1)*(x + 2)),x, algorithm="giac")
```

```
[Out] integrate(-(x - 1)/(sqrt(x^3 - 1)*(x + 2)), x)
```

3.54 $\int \frac{1+x}{(2-x)\sqrt{-1-x^3}} \, dx$

Optimal. Leaf size=25

$$\frac{2}{3}\tan^{-1}\left(\frac{(x+1)^2}{3\sqrt{-x^3-1}}\right)$$

```
[Out] (2*ArcTan[(1 + x)^2/(3*Sqrt[-1 - x^3])])/3
```

Rubi [A] time = 0.116235, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{2}{3}\tan^{-1}\left(\frac{(x+1)^2}{3\sqrt{-x^3-1}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x)/((2 - x)*Sqrt[-1 - x^3]),x]

[Out] (2*ArcTan[(1 + x)^2/(3*Sqrt[-1 - x^3])])/3
```

Rubi in Sympy [A] time = 94.9076, size = 377, normalized size = 15.08

$$\frac{3\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(-\frac{\sqrt{3}}{3}+1\right)(x+1)\operatorname{atan}\left(\frac{3^{\frac{3}{4}}\sqrt{1-\frac{\left(x+1+\sqrt{3}\right)^2}{\left(-x-1+\sqrt{3}\right)^2}}\sqrt{\sqrt{3}+2}}{3\sqrt{4\sqrt{3}+7+\frac{\left(x+1+\sqrt{3}\right)^2}{\left(-x-1+\sqrt{3}\right)^2}}}\right)}{\sqrt{\frac{-x-1}{\left(x-\sqrt{3}+1\right)^2}}\left(-\sqrt{3}+3\right)\sqrt{-x^3-1}} + \frac{2\sqrt[4]{3}\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-\sqrt{3}+2}\,(x+1)\,F\left(\operatorname{asin}\left(\frac{x+1+\sqrt{3}}{x-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt{\frac{-x-1}{\left(x-\sqrt{3}+1\right)^2}}\left(-\sqrt{3}+3\right)\sqrt{-x^3-1}} + \frac{12\sqrt[4]{3}\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{\sqrt{3}+2}\,(x+1)\left(-4\sqrt{3}+7;\operatorname{asin}\left(\frac{x+1+\sqrt{3}}{-x-1+\sqrt{3}}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt{\frac{-x-1}{\left(x-\sqrt{3}+1\right)^2}}\left(-\sqrt{3}+3\right)\left(\sqrt{3}+3\right)\sqrt{4\sqrt{3}+7}\sqrt{-x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+x)/(2-x)/(-x**3-1)**(1/2),x)
```

```
[Out] 3*sqrt((x**2 - x + 1)/(x - sqrt(3) + 1)**2)*(-sqrt(3)/3 + 1)*(x +
 1)*atan(3**(3/4)*sqrt(1 - (x + 1 + sqrt(3))**2/(-x - 1 + sqrt(3)
)**2)*sqrt(sqrt(3) + 2)/(3*sqrt(4*sqrt(3) + 7 + (x + 1 + sqrt(3))
**2/(-x - 1 + sqrt(3))**2)))/(sqrt((-x - 1)/(x - sqrt(3) + 1)**2)
*(-sqrt(3) + 3)*sqrt(-x**3 - 1)) + 2*3**(1/4)*sqrt((x**2 - x + 1)
/(x - sqrt(3) + 1)**2)*sqrt(-sqrt(3) + 2)*(x + 1)*elliptic_f(asin
((x + 1 + sqrt(3))/(x - sqrt(3) + 1)), -7 + 4*sqrt(3))/(sqrt((-x
- 1)/(x - sqrt(3) + 1)**2)*(-sqrt(3) + 3)*sqrt(-x**3 - 1)) + 12*3
**(1/4)*sqrt((x**2 - x + 1)/(x - sqrt(3) + 1)**2)*sqrt(sqrt(3) +
2)*(x + 1)*elliptic_pi(-4*sqrt(3) + 7, asin((x + 1 + sqrt(3))/(-x
 - 1 + sqrt(3))), -7 + 4*sqrt(3))/(sqrt((-x - 1)/(x - sqrt(3) + 1
)**2)*(-sqrt(3) + 3)*(sqrt(3) + 3)*sqrt(4*sqrt(3) + 7)*sqrt(-x**3
 - 1))
```

Mathematica [C] time = 0.321923, size = 267, normalized size = 10.68

$$\frac{2\sqrt{6}\sqrt{-\frac{i(x+1)}{\sqrt{3}-3i}}\left(2\sqrt{3}\sqrt{2ix+\sqrt{3}-i}\sqrt{x^2-x+1}\,\Pi\left(\frac{2\sqrt{3}}{3i+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{2ix+\sqrt{3}-i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right)-i\sqrt{-2ix+\sqrt{3}+i}\left(\left(\sqrt{3}-i\right)x-\sqrt{3}-i\right)\cdots}{\left(\sqrt{3}+3i\right)\sqrt{2ix+\sqrt{3}-i}\sqrt{-x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(1 + x)/((2 - x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*Sqrt[6]*Sqrt[((-I)*(1 + x))/(-3*I + Sqrt[3])]*((-I)*Sqrt[I + S
qrt[3] - (2*I)*x]*(-I - Sqrt[3] + (-I + Sqrt[3])*x)*EllipticF[Arc
Sin[Sqrt[-I + Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/
(-3*I + Sqrt[3])] + 2*Sqrt[3]*Sqrt[-I + Sqrt[3] + (2*I)*x]*Sqrt[1
 - x + x^2]*EllipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[-
I + Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-3*I + Sq
rt[3])]))/((3*I + Sqrt[3])*Sqrt[-I + Sqrt[3] + (2*I)*x]*Sqrt[-1 -
 x^3])
```

Maple [C] time = 0.036, size = 240, normalized size = 9.6

$$\frac{2i}{3}\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}EllipticF\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)\frac{1}{\sqrt{-x^3-\cdots}}$$

$$+\frac{2i\sqrt{3}}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}EllipticPi\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}},\sqrt{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+x)/(2-x)/(-x^3-1)^(1/2),x)
```

```
[Out] 2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2
+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(
-x^3-1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1
/2))^(1/2),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))+2*I*3^(1/2)*(I*
(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^(1/2)))^
(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)/(-3
/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))
*3^(1/2))^(1/2),I*3^(1/2)/(-3/2+1/2*I*3^(1/2)),(I*3^(1/2)/(3/2+1/
2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x+1}{\sqrt{-x^3-1}(x-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x + 1)/(sqrt(-x^3 - 1)*(x - 2)),x, algorithm="maxima")
```

```
[Out] -integrate((x + 1)/(sqrt(-x^3 - 1)*(x - 2)), x)
```

Fricas [A] time = 0.351834, size = 43, normalized size = 1.72

$$\frac{1}{3}\,\arctan\left(\frac{x^3+12\,x^2-6\,x+10}{6\,\sqrt{-x^3-1}(x+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x + 1)/(sqrt(-x^3 - 1)*(x - 2)),x, algorithm="fricas")
```

```
[Out] 1/3*arctan(1/6*(x^3 + 12*x^2 - 6*x + 10)/(sqrt(-x^3 - 1)*(x + 1))
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{x\sqrt{-x^3-1}-2\sqrt{-x^3-1}}\,dx-\int \frac{1}{x\sqrt{-x^3-1}-2\sqrt{-x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+x)/(2-x)/(-x**3-1)**(1/2),x)
```

```
[Out] -Integral(x/(x*sqrt(-x**3 - 1) - 2*sqrt(-x**3 - 1)), x) - Integra
l(1/(x*sqrt(-x**3 - 1) - 2*sqrt(-x**3 - 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x+1}{\sqrt{-x^3-1}(x-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x + 1)/(sqrt(-x^3 - 1)*(x - 2)),x, algorithm="giac")
```

```
[Out] integrate(-(x + 1)/(sqrt(-x^3 - 1)*(x - 2)), x)
```

3.55 $\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=50

$$\frac{2\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a+bx^3}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

```
[Out] (2*ArcTanh[(a^(1/3) + b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[a + b*x^3])])/
(3*a^(1/6)*b^(1/3))
```

Rubi [A] time = 0.229679, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.047

$$\frac{2\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a+bx^3}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^(1/3) + b^(1/3)*x)/((2*a^(1/3) - b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*ArcTanh[(a^(1/3) + b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[a + b*x^3])])/
(3*a^(1/6)*b^(1/3))
```

Rubi in Sympy [A] time = 171.741, size = 677, normalized size = 13.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((a**(1/3)+b**(1/3)*x)/(2*a**(1/3)-b**(1/3)*x)/(b*x**3+a)**(1/2)
```

```
[Out] -2*3**(1/4)*sqrt((a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)
/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(sqrt(3) + 2)*(a**
(1/3) + b**(1/3)*x)*elliptic_f(asin((-a**(1/3)*(-1 + sqrt(3)) + b
```

```
**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)), -7 - 4*sqrt(3)
)/(b**(1/3)*sqrt(a**(1/3)*(a**(1/3) + b**(1/3)*x)/(a**(1/3)*(1 +
sqrt(3)) + b**(1/3)*x)**2)*(sqrt(3) + 3)*sqrt(a + b*x**3)) + 3**(
3/4)*sqrt(a**(2/3)*(1 - b**(1/3)*x/a**(1/3) + b**(2/3)*x**2/a**(2
/3))/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(3 + 2*sqrt(3)
)*sqrt(-sqrt(3) + 2)*(a**(1/3) + b**(1/3)*x)*atanh(sqrt(-(a**(1/3
)*(-1 + sqrt(3)) - b**(1/3)*x)**2/(a**(1/3)*(1 + sqrt(3)) + b**(1
/3)*x)**2 + 1)/(sqrt(3 + 2*sqrt(3))*sqrt((a**(1/3)*(-1 + sqrt(3))
 - b**(1/3)*x)**2/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2 - 4*sq
rt(3) + 7)))/(3*b**(1/3)*sqrt(a**(1/3)*(a**(1/3) + b**(1/3)*x)/(a
**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(a + b*x**3)) - 12*3*
*(1/4)*sqrt(a**(2/3)*(1 - b**(1/3)*x/a**(1/3) + b**(2/3)*x**2/a**
(2/3))/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(-sqrt(3) +
2)*(a**(1/3) + b**(1/3)*x)*elliptic_pi(4*sqrt(3) + 7, asin((a**(1
/3)*(-1 + sqrt(3)) - b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/
3)*x)), -7 - 4*sqrt(3))/(b**(1/3)*sqrt(a**(1/3)*(a**(1/3) + b**(1
/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(-4*sqrt(3)
+ 7)*(-sqrt(3) + 3)*(sqrt(3) + 3)*sqrt(a + b*x**3))
```

Mathematica [C] time = 2.31001, size = 407, normalized size = 8.14

$$\frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(3i\sqrt[3]{a}\sqrt{\frac{\left(\sqrt{3}+i\right)\sqrt[3]{b}x-2i\sqrt[3]{a}}{\left(\sqrt{3}-3i\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\left(\frac{2\sqrt{3}}{3i+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{\left(i+\sqrt{3}\right)\sqrt[3]{b}x-2i\sqrt[3]{a}}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)|\frac{1}{2}\left(1+i\sqrt{3}\right)\right)-\frac{\sqrt[4]{3}\left(\left(\sqrt{3}+i\right)\cdots\right.}{\cdots}\right)}{\left(\sqrt[3]{-1}-2\right)\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a^(1/3) + b^(1/3)*x)/((2*a^(1/3) - b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(-(3^(1
/4)*((I + Sqrt[3])*a^(1/3) - (-I + Sqrt[3])*b^(1/3)*x)*Sqrt[I + S
qrt[3] - ((2*I)*b^(1/3)*x)/a^(1/3)]*EllipticF[ArcSin[Sqrt[((-2*I)
*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]],
 (1 + I*Sqrt[3])/2])/(2*Sqrt[2]) + (3*I)*a^(1/3)*Sqrt[((-2*I)*a^(
1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*Sqrt[
1 - (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sq
rt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3
])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/
((-2 + (-1)^(1/3))*b^(1/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/
((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3])
```

Maple [F] time = 0.135, size = 0, normalized size = 0.

$$\int 1 \left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)^{-1}\frac{1}{\sqrt{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

```
[Out] int((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}}{\sqrt{bx^3+a}\left(b^{\frac{1}{3}}x-2\,a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(b^(1/3)*x + a^(1/3))/(sqrt(b*x^3 + a)*(b^(1/3)*x - 2*a^(1/3))),x, al
```

```
[Out] -integrate((b^(1/3)*x + a^(1/3))/(sqrt(b*x^3 + a)*(b^(1/3)*x - 2*
a^(1/3))), x)
```

Fricas [A] time = 0.718822, size = 1, normalized size = 0.02

$$\left[\frac{1}{6}\,a^{\frac{1}{3}}\sqrt{\frac{1}{ab^{\frac{2}{3}}}}\log\left(\frac{\left(b^2x^6-88\,abx^3+136\,a^2\right)a^{\frac{2}{3}}b^{\frac{2}{3}}+12\left(6\,a^2b^{\frac{5}{3}}x^2+\left(13\,ab^2x^3+10\,a^2b\right)a^{\frac{2}{3}}+\left(ab^2x^4+4\,a^2bx\right)a^{\frac{1}{3}}b^{\frac{1}{3}}\right)\sqrt{bx^3+}}{(b^2x^6-160\,abx^3+64\,a^2)a^{\frac{2}{3}}b^{\frac{2}{3}}+12\,(5\,ab^2x^4-16\,a^2bx)a^{\frac{1}{3}}-12}\right.\right.$$

$$\left.-\frac{1}{3}\,a^{\frac{1}{3}}\sqrt{-\frac{1}{ab^{\frac{2}{3}}}}\arctan\left(-\frac{12\,a^{\frac{2}{3}}bx^2-6\,ab^{\frac{2}{3}}x+\left(bx^3+10\,a\right)a^{\frac{1}{3}}b^{\frac{1}{3}}}{6\sqrt{bx^3+a}\left(abx+a^{\frac{4}{3}}b^{\frac{2}{3}}\right)\sqrt{-\frac{1}{ab^{\frac{2}{3}}}}}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(b^(1/3)*x + a^(1/3))/(sqrt(b*x^3 + a)*(b^(1/3)*x - 2*a^(1/3))),x, al
```

```
[Out] [1/6*a^(1/3)*sqrt(1/(a*b^(2/3)))*log(((b^2*x^6 - 88*a*b*x^3 + 136
*a^2)*a^(2/3)*b^(2/3) + 12*(6*a^2*b^(5/3)*x^2 + (13*a*b^2*x^3 + 1
0*a^2*b)*a^(2/3) + (a*b^2*x^4 + 4*a^2*b*x)*a^(1/3)*b^(1/3))*sqrt(
```

```
b*x^3 + a)*sqrt(1/(a*b^(2/3))) + 12*(17*a*b^2*x^4 - 4*a^2*b*x)*a^
(1/3) + 12*(5*a*b^2*x^5 + 26*a^2*b*x^2)*b^(1/3))/((b^2*x^6 - 160*
a*b*x^3 + 64*a^2)*a^(2/3)*b^(2/3) + 12*(5*a*b^2*x^4 - 16*a^2*b*x)
*a^(1/3) - 12*(a*b^2*x^5 - 20*a^2*b*x^2)*b^(1/3))), -1/3*a^(1/3)*
sqrt(-1/(a*b^(2/3)))*arctan(-1/6*(12*a^(2/3)*b*x^2 - 6*a*b^(2/3)*
x + (b*x^3 + 10*a)*a^(1/3)*b^(1/3))/(sqrt(b*x^3 + a)*(a*b*x + a^(
4/3)*b^(2/3))*sqrt(-1/(a*b^(2/3)))))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt[3]{a}}{-2\sqrt[3]{a}\sqrt{a+bx^3}+\sqrt[3]{b}x\sqrt{a+bx^3}}\,dx-\int \frac{\sqrt[3]{b}x}{-2\sqrt[3]{a}\sqrt{a+bx^3}+\sqrt[3]{b}x\sqrt{a+bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**(1/3)+b**(1/3)*x)/(2*a**(1/3)-b**(1/3)*x)/(b*x**3+a)**(1/2),x)
```

```
[Out] -Integral(a**(1/3)/(-2*a**(1/3)*sqrt(a + b*x**3) + b**(1/3)*x*sqr
t(a + b*x**3)), x) - Integral(b**(1/3)*x/(-2*a**(1/3)*sqrt(a + b*
x**3) + b**(1/3)*x*sqrt(a + b*x**3)), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(b^(1/3)*x + a^(1/3))/(sqrt(b*x^3 + a)*(b^(1/3)*x - 2*a^(1/3))),x, al
```

```
[Out] Exception raised: TypeError
```

3.56 $\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$

Optimal. Leaf size=52

$$-\frac{2\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a-bx^3}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

```
[Out] (-2*ArcTanh[(a^(1/3) - b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[a - b*x^3])])
/(3*a^(1/6)*b^(1/3))
```

Rubi [A] time = 0.236811, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045

$$-\frac{2\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a-bx^3}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^(1/3) - b^(1/3)*x)/((2*a^(1/3) + b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (-2*ArcTanh[(a^(1/3) - b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[a - b*x^3])])
/(3*a^(1/6)*b^(1/3))
```

Rubi in Sympy [A] time = 172.062, size = 677, normalized size = 13.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((a**(1/3)-b**(1/3)*x)/(2*a**(1/3)+b**(1/3)*x)/(-b*x**3+a)**(1/2
```

```
[Out] -2*3**(1/4)*sqrt((a**(2/3) + a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)
/(a**(1/3)*(1 + sqrt(3)) - b**(1/3)*x)**2)*sqrt(sqrt(3) + 2)*(a**
(1/3) - b**(1/3)*x)*elliptic_f(asin((a**(1/3)*(-1 + sqrt(3)) + b*
```

```
*(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) - b**(1/3)*x)), -7 - 4*sqrt(3))
/(b**(1/3)*sqrt(a**(1/3)*(a**(1/3) - b**(1/3)*x)/(a**(1/3)*(1 + s
qrt(3)) - b**(1/3)*x)**2)*(sqrt(3) + 3)*sqrt(a - b*x**3)) - 3**(3
/4)*sqrt(a**(2/3)*(1 + b**(1/3)*x/a**(1/3) + b**(2/3)*x**2/a**(2/
3))/(a**(1/3)*(1 + sqrt(3)) - b**(1/3)*x)**2)*sqrt(3 + 2*sqrt(3))
*sqrt(-sqrt(3) + 2)*(a**(1/3) - b**(1/3)*x)*atanh(sqrt(-(a**(1/3)
*(-1 + sqrt(3)) + b**(1/3)*x)**2/(a**(1/3)*(1 + sqrt(3)) - b**(1/
3)*x)**2 + 1)/(sqrt(3 + 2*sqrt(3))*sqrt((a**(1/3)*(-1 + sqrt(3))
+ b**(1/3)*x)**2/(a**(1/3)*(1 + sqrt(3)) - b**(1/3)*x)**2 - 4*sqr
t(3) + 7)))/(3*b**(1/3)*sqrt(a**(1/3)*(a**(1/3) - b**(1/3)*x)/(a*
*(1/3)*(1 + sqrt(3)) - b**(1/3)*x)**2)*sqrt(a - b*x**3)) + 12*3**
(1/4)*sqrt(a**(2/3)*(1 + b**(1/3)*x/a**(1/3) + b**(2/3)*x**2/a**(
2/3))/(a**(1/3)*(1 + sqrt(3)) - b**(1/3)*x)**2)*sqrt(-sqrt(3) + 2
)*(a**(1/3) - b**(1/3)*x)*elliptic_pi(4*sqrt(3) + 7, asin((a**(1/
3)*(-1 + sqrt(3)) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) - b**(1/3
)*x)), -7 - 4*sqrt(3))/(b**(1/3)*sqrt(a**(1/3)*(a**(1/3) - b**(1/
3)*x)/(a**(1/3)*(1 + sqrt(3)) - b**(1/3)*x)**2)*sqrt(-4*sqrt(3) +
 7)*(-sqrt(3) + 3)*(sqrt(3) + 3)*sqrt(a - b*x**3))
```

Mathematica [C] time = 1.24544, size = 370, normalized size = 7.12

$$-\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\left(\sqrt[3]{-1}-2\right)\left(\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}F\left(\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)\bigg|\sqrt[3]{-1}\right)+\sqrt[3]{-1}\sqrt{3}\left(1+\sqrt[3]{\cdots}\right.\right.}{\left(\sqrt[3]{-1}-2\right)\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{a-b\cdots}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^(1/3) - b^(1/3)*x)/((2*a^(1/3) + b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (-2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((-2 +
 (-1)^(1/3))*((-1)^(1/3)*a^(1/3) + b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a
^(1/3) + (-1)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*Ellip
ticF[ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3
))*a^(1/3))]], (-1)^(1/3)] + (-1)^(1/3)*Sqrt[3]*(1 + (-1)^(1/3))*
a^(1/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a
^(1/3))]*Sqrt[1 + (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*El
lipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[(a^(1/3) - (-1)
^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)]))/((-
2 + (-1)^(1/3))*b^(1/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1
 + (-1)^(1/3))*a^(1/3))]*Sqrt[a - b*x^3])
```

Maple [F] time = 0.151, size = 0, normalized size = 0.

$$\int 1 \left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)^{-1}\frac{1}{\sqrt{-bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

```
[Out] int((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{b^{\frac{1}{3}}x-a^{\frac{1}{3}}}{\sqrt{-bx^3+a}\left(b^{\frac{1}{3}}x+2\,a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(b^(1/3)*x - a^(1/3))/(sqrt(-b*x^3 + a)*(b^(1/3)*x + 2*a^(1/3))),x, a
```

```
[Out] -integrate((b^(1/3)*x - a^(1/3))/(sqrt(-b*x^3 + a)*(b^(1/3)*x + 2
*a^(1/3))), x)
```

Fricas [A] time = 0.704831, size = 1, normalized size = 0.02

$$\left[\frac{1}{6}\,a^{\frac{1}{3}}\sqrt{\frac{1}{ab^{\frac{2}{3}}}}\log\left(\frac{\left(b^2x^6+88\,abx^3+136\,a^2\right)a^{\frac{2}{3}}b^{\frac{2}{3}}-12\left(6\sqrt{-bx^3+a}a^2b^{\frac{5}{3}}x^2-\left(13\,ab^2x^3-10\,a^2b\right)\sqrt{-bx^3+a}a^{\frac{2}{3}}+\left(ab^2x^4-\right.\right.}{(b^2x^6+160\,abx^3+64\,a^2)a^{\frac{2}{3}}b^{\frac{2}{3}}+12\,(5\,ab^2x^4+16\,a}\right.\right.$$

$$\left.-\frac{1}{3}\,a^{\frac{1}{3}}\sqrt{-\frac{1}{ab^{\frac{2}{3}}}}\arctan\left(-\frac{12\,a^{\frac{2}{3}}bx^2+6\,ab^{\frac{2}{3}}x-\left(bx^3-10\,a\right)a^{\frac{1}{3}}b^{\frac{1}{3}}}{6\left(\sqrt{-bx^3+a}abx-\sqrt{-bx^3+a}a^{\frac{4}{3}}b^{\frac{2}{3}}\right)\sqrt{-\frac{1}{ab^{\frac{2}{3}}}}}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(b^(1/3)*x - a^(1/3))/(sqrt(-b*x^3 + a)*(b^(1/3)*x + 2*a^(1/3))),x, a
```

```
[Out] [1/6*a^(1/3)*sqrt(1/(a*b^(2/3)))*log(((b^2*x^6 + 88*a*b*x^3 + 136
*a^2)*a^(2/3)*b^(2/3) - 12*(6*sqrt(-b*x^3 + a)*a^2*b^(5/3)*x^2 -
(13*a*b^2*x^3 - 10*a^2*b)*sqrt(-b*x^3 + a)*a^(2/3) + (a*b^2*x^4 -
```

```
 4*a^2*b*x)*sqrt(-b*x^3 + a)*a^(1/3)*b^(1/3))*sqrt(1/(a*b^(2/3)))
 + 12*(17*a*b^2*x^4 + 4*a^2*b*x)*a^(1/3) - 12*(5*a*b^2*x^5 - 26*a
^2*b*x^2)*b^(1/3))/((b^2*x^6 + 160*a*b*x^3 + 64*a^2)*a^(2/3)*b^(2
/3) + 12*(5*a*b^2*x^4 + 16*a^2*b*x)*a^(1/3) + 12*(a*b^2*x^5 + 20*
a^2*b*x^2)*b^(1/3))), -1/3*a^(1/3)*sqrt(-1/(a*b^(2/3)))*arctan(-1
/6*(12*a^(2/3)*b*x^2 + 6*a*b^(2/3)*x - (b*x^3 - 10*a)*a^(1/3)*b^(
1/3))/((sqrt(-b*x^3 + a)*a*b*x - sqrt(-b*x^3 + a)*a^(4/3)*b^(2/3)
)*sqrt(-1/(a*b^(2/3)))))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\left(-\frac{\sqrt[3]{a}}{2\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt[3]{b}x\sqrt{a-bx^3}}\right)dx-\int\frac{\sqrt[3]{b}x}{2\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt[3]{b}x\sqrt{a-bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**(1/3)-b**(1/3)*x)/(2*a**(1/3)+b**(1/3)*x)/(-b*x**3+a)**(1/2),x)
```

```
[Out] -Integral(-a**(1/3)/(2*a**(1/3)*sqrt(a - b*x**3) + b**(1/3)*x*sqr
t(a - b*x**3)), x) - Integral(b**(1/3)*x/(2*a**(1/3)*sqrt(a - b*x
**3) + b**(1/3)*x*sqrt(a - b*x**3)), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(b^(1/3)*x - a^(1/3))/(sqrt(-b*x^3 + a)*(b^(1/3)*x + 2*a^(1/3))),x, a
```

```
[Out] Exception raised: TypeError
```

3.57 $\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

Optimal. Leaf size=53

$$-\frac{2\tan^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{bx^3-a}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

```
[Out] (-2*ArcTan[(a^(1/3) - b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[-a + b*x^3])])
/(3*a^(1/6)*b^(1/3))
```

Rubi [A] time = 0.243995, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.044$

$$-\frac{2\tan^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{bx^3-a}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^(1/3) - b^(1/3)*x)/((2*a^(1/3) + b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (-2*ArcTan[(a^(1/3) - b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[-a + b*x^3])])
/(3*a^(1/6)*b^(1/3))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((a**(1/3)-b**(1/3)*x)/(2*a**(1/3)+b**(1/3)*x)/(b*x**3-
a)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 1.23532, size = 371, normalized size = 7.

$$-\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\left(\sqrt[3]{-1}-2\right)\left(\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}F\left(\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)+\sqrt[3]{-1}\sqrt{3}\left(1+\sqrt[3]{\cdots}\right.\right.}{\left(\sqrt[3]{-1}-2\right)\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{bx^3-\cdots}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^(1/3) - b^(1/3)*x)/((2*a^(1/3) + b^(1/3)*x)*Sqrt[-
a + b*x^3]),x]
```

```
[Out] (-2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((-2 +
 (-1)^(1/3))*((-1)^(1/3)*a^(1/3) + b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a
^(1/3) + (-1)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*Ellip
ticF[ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3
))*a^(1/3))]], (-1)^(1/3)] + (-1)^(1/3)*Sqrt[3]*(1 + (-1)^(1/3))*
a^(1/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a
^(1/3))]*Sqrt[1 + (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*El
lipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[(a^(1/3) - (-1)
^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)]))/((-
2 + (-1)^(1/3))*b^(1/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1
 + (-1)^(1/3))*a^(1/3))]*Sqrt[-a + b*x^3])
```

Maple [F] time = 0.124, size = 0, normalized size = 0.

$$\int 1\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)^{-1}\frac{1}{\sqrt{bx^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

```
[Out] int((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{b^{\frac{1}{3}}x-a^{\frac{1}{3}}}{\sqrt{bx^3-a}\left(b^{\frac{1}{3}}x+2\,a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(b^(1/3)*x - a^(1/3))/(sqrt(b*x^3 - a)*(b^(1/3)*x + 2*a^(1/3))),x, al
```

```
[Out] -integrate((b^(1/3)*x - a^(1/3))/(sqrt(b*x^3 - a)*(b^(1/3)*x + 2*
a^(1/3))), x)
```

Fricas [A] time = 0.711295, size = 1, normalized size = 0.02

$$\left[\frac{1}{6}\,a^{\frac{1}{3}}\sqrt{-\frac{1}{ab^{\frac{2}{3}}}}\log\left(\frac{\left(b^2x^6+88\,abx^3+136\,a^2\right)a^{\frac{2}{3}}b^{\frac{2}{3}}-12\left(6\,a^2b^{\frac{5}{3}}x^2-\left(13\,ab^2x^3-10\,a^2b\right)a^{\frac{2}{3}}+\left(ab^2x^4-4\,a^2bx\right)a^{\frac{1}{3}}b^{\frac{1}{3}}\right)\sqrt{bx^3}}{(b^2x^6+160\,abx^3+64\,a^2)a^{\frac{2}{3}}b^{\frac{2}{3}}+12\,(5\,ab^2x^4+16\,a^2bx)a^{\frac{1}{3}}+}\right.\right.$$

$$\left.-\frac{1}{3}\,a^{\frac{1}{3}}\sqrt{\frac{1}{ab^{\frac{2}{3}}}}\arctan\left(-\frac{12\,a^{\frac{2}{3}}bx^2+6\,ab^{\frac{2}{3}}x-\left(bx^3-10\,a\right)a^{\frac{1}{3}}b^{\frac{1}{3}}}{6\,\sqrt{bx^3-a}\left(abx-a^{\frac{4}{3}}b^{\frac{2}{3}}\right)\sqrt{\frac{1}{ab^{\frac{2}{3}}}}}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(b^(1/3)*x - a^(1/3))/(sqrt(b*x^3 - a)*(b^(1/3)*x + 2*a^(1/3))),x, al
```

```
[Out] [1/6*a^(1/3)*sqrt(-1/(a*b^(2/3)))*log(((b^2*x^6 + 88*a*b*x^3 + 13
6*a^2)*a^(2/3)*b^(2/3) - 12*(6*a^2*b^(5/3)*x^2 - (13*a*b^2*x^3 -
10*a^2*b)*a^(2/3) + (a*b^2*x^4 - 4*a^2*b*x)*a^(1/3)*b^(1/3))*sqrt
(b*x^3 - a)*sqrt(-1/(a*b^(2/3))) + 12*(17*a*b^2*x^4 + 4*a^2*b*x)*
a^(1/3) - 12*(5*a*b^2*x^5 - 26*a^2*b*x^2)*b^(1/3))/((b^2*x^6 + 16
0*a*b*x^3 + 64*a^2)*a^(2/3)*b^(2/3) + 12*(5*a*b^2*x^4 + 16*a^2*b*
x)*a^(1/3) + 12*(a*b^2*x^5 + 20*a^2*b*x^2)*b^(1/3))), -1/3*a^(1/3
)*sqrt(1/(a*b^(2/3)))*arctan(-1/6*(12*a^(2/3)*b*x^2 + 6*a*b^(2/3)
*x - (b*x^3 - 10*a)*a^(1/3)*b^(1/3))/(sqrt(b*x^3 - a)*(a*b*x - a^
(4/3)*b^(2/3))*sqrt(1/(a*b^(2/3)))))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\left(-\frac{\sqrt[3]{a}}{2\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt[3]{b}x\sqrt{-a+bx^3}}\right)dx-\int\frac{\sqrt[3]{b}x}{2\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt[3]{b}x\sqrt{-a+bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]    integrate((a**(1/3)-b**(1/3)*x)/(2*a**(1/3)+b**(1/3)*x)/(b*x**3-
a)**(1/2),x)
```

```
[Out] -Integral(-a**(1/3)/(2*a**(1/3)*sqrt(-a + b*x**3) + b**(1/3)*x*sq
rt(-a + b*x**3)), x) - Integral(b**(1/3)*x/(2*a**(1/3)*sqrt(-a +
b*x**3) + b**(1/3)*x*sqrt(-a + b*x**3)), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(b^(1/3)*x - a^(1/3))/(sqrt(b*x^3 - a)*(b^(1/3)*x + 2*a^(1/3))),x, al
```

```
[Out] Exception raised: TypeError
```

3.58 $\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

Optimal. Leaf size=53

$$\frac{2\tan^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{-a-bx^3}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

```
[Out] (2*ArcTan[(a^(1/3) + b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[-a - b*x^3])])/
(3*a^(1/6)*b^(1/3))
```

Rubi [A] time = 0.244809, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043

$$\frac{2\tan^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{-a-bx^3}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^(1/3) + b^(1/3)*x)/((2*a^(1/3) - b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (2*ArcTan[(a^(1/3) + b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[-a - b*x^3])])/
(3*a^(1/6)*b^(1/3))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((a**(1/3)+b**(1/3)*x)/(2*a**(1/3)-b**(1/3)*x)/(-b*x**3-
a)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 2.31099, size = 410, normalized size = 7.74

$$\frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(3i\sqrt[3]{a}\sqrt{\frac{\left(\sqrt{3}+i\right)\sqrt[3]{b}x-2i\sqrt[3]{a}}{\left(\sqrt{3}-3i\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\left(\frac{2\sqrt{3}}{3i+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{\left(i+\sqrt{3}\right)\sqrt[3]{b}x-2i\sqrt[3]{a}}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)|\frac{1}{2}\left(1+i\sqrt{3}\right)\right)-\frac{\sqrt[4]{3}\left(\left(\sqrt{3}+i\right)\right.}{}\right.}{\left(\sqrt[3]{-1}-2\right)\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{-a-bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a^(1/3) + b^(1/3)*x)/((2*a^(1/3) - b^(1/3)*x)*Sqrt[-
a - b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(-(3^(1
/4)*((I + Sqrt[3])*a^(1/3) - (-I + Sqrt[3])*b^(1/3)*x)*Sqrt[I + S
qrt[3] - ((2*I)*b^(1/3)*x)/a^(1/3)]*EllipticF[ArcSin[Sqrt[((-2*I)
*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]],
 (1 + I*Sqrt[3])/2])/(2*Sqrt[2]) + (3*I)*a^(1/3)*Sqrt[((-2*I)*a^(
1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*Sqrt[
1 - (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sq
rt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3
])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/
((-2 + (-1)^(1/3))*b^(1/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/
((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[-a - b*x^3])
```

Maple [F] time = 0.119, size = 0, normalized size = 0.

$$\int 1\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)^{-1}\frac{1}{\sqrt{-bx^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

```
[Out] int((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}}{\sqrt{-bx^3 - a}\left(b^{\frac{1}{3}}x - 2\,a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(b^(1/3)*x + a^(1/3))/(sqrt(-b*x^3 - a)*(b^(1/3)*x - 2*a^(1/3))),x, a
```

```
[Out] -integrate((b^(1/3)*x + a^(1/3))/(sqrt(-b*x^3 - a)*(b^(1/3)*x - 2
*a^(1/3))), x)
```

Fricas [A] time = 0.706053, size = 1, normalized size = 0.02

$$\left[\frac{1}{6}\,a^{\frac{1}{3}}\sqrt{-\frac{1}{ab^{\frac{2}{3}}}}\log\left(\frac{\left(b^2x^6 - 88\,abx^3 + 136\,a^2\right)a^{\frac{2}{3}}b^{\frac{2}{3}} + 12\left(6\sqrt{-bx^3 - a}a^2b^{\frac{5}{3}}x^2 + \left(13\,ab^2x^3 + 10\,a^2b\right)\sqrt{-bx^3 - a}a^{\frac{2}{3}} + \left(ab^2x^4\right.\right.}{(b^2x^6 - 160\,abx^3 + 64\,a^2)a^{\frac{2}{3}}b^{\frac{2}{3}} + 12\,(5\,ab^2x^4 - 1}\right.\right.$$
$$\left.\left.-\frac{1}{3}\,a^{\frac{1}{3}}\sqrt{\frac{1}{ab^{\frac{2}{3}}}}\arctan\left(-\frac{12\,a^{\frac{2}{3}}bx^2 - 6\,ab^{\frac{2}{3}}x + \left(bx^3 + 10\,a\right)a^{\frac{1}{3}}b^{\frac{1}{3}}}{6\left(\sqrt{-bx^3 - a}abx + \sqrt{-bx^3 - a}a^{\frac{4}{3}}b^{\frac{2}{3}}\right)\sqrt{\frac{1}{ab^{\frac{2}{3}}}}}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(b^(1/3)*x + a^(1/3))/(sqrt(-b*x^3 - a)*(b^(1/3)*x - 2*a^(1/3))),x, a
```

```
[Out] [1/6*a^(1/3)*sqrt(-1/(a*b^(2/3)))*log(((b^2*x^6 - 88*a*b*x^3 + 13
6*a^2)*a^(2/3)*b^(2/3) + 12*(6*sqrt(-b*x^3 - a)*a^2*b^(5/3)*x^2 +
 (13*a*b^2*x^3 + 10*a^2*b)*sqrt(-b*x^3 - a)*a^(2/3) + (a*b^2*x^4
+ 4*a^2*b*x)*sqrt(-b*x^3 - a)*a^(1/3)*b^(1/3))*sqrt(-1/(a*b^(2/3)
)) + 12*(17*a*b^2*x^4 - 4*a^2*b*x)*a^(1/3) + 12*(5*a*b^2*x^5 + 26
*a^2*b*x^2)*b^(1/3))/((b^2*x^6 - 160*a*b*x^3 + 64*a^2)*a^(2/3)*b^
(2/3) + 12*(5*a*b^2*x^4 - 16*a^2*b*x)*a^(1/3) - 12*(a*b^2*x^5 - 2
0*a^2*b*x^2)*b^(1/3))), -1/3*a^(1/3)*sqrt(1/(a*b^(2/3)))*arctan(-
1/6*(12*a^(2/3)*b*x^2 - 6*a*b^(2/3)*x + (b*x^3 + 10*a)*a^(1/3)*b^
(1/3))/((sqrt(-b*x^3 - a)*a*b*x + sqrt(-b*x^3 - a)*a^(4/3)*b^(2/3
))*sqrt(1/(a*b^(2/3)))))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt[3]{a}}{-2\sqrt[3]{a}\sqrt{-a - bx^3} + \sqrt[3]{b}x\sqrt{-a - bx^3}}\,dx - \int \frac{\sqrt[3]{b}x}{-2\sqrt[3]{a}\sqrt{-a - bx^3} + \sqrt[3]{b}x\sqrt{-a - bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a**(1/3)+b**(1/3)*x)/(2*a**(1/3)-b**(1/3)*x)/(-b*x**3-
a)**(1/2),x)
```

```
[Out] -Integral(a**(1/3)/(-2*a**(1/3)*sqrt(-a - b*x**3) + b**(1/3)*x*sq
rt(-a - b*x**3)), x) - Integral(b**(1/3)*x/(-2*a**(1/3)*sqrt(-a -
 b*x**3) + b**(1/3)*x*sqrt(-a - b*x**3)), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(b^(1/3)*x + a^(1/3))/(sqrt(-b*x^3 - a)*(b^(1/3)*x - 2*a^(1/3))),x, a
```

```
[Out] Exception raised: TypeError
```

3.59 $\int \frac{c-2dx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx$

Optimal. Leaf size=46

$$-\frac{2\tanh^{-1}\left(\frac{(c-2dx)^2}{3\sqrt{c}\sqrt{c^3-8d^3x^3}}\right)}{3\sqrt{c}d}$$

```
[Out] (-2*ArcTanh[(c - 2*d*x)^2/(3*Sqrt[c]*Sqrt[c^3 - 8*d^3*x^3])])/(3*
Sqrt[c]*d)
```

Rubi [A] time = 0.202288, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$-\frac{2\tanh^{-1}\left(\frac{(c-2dx)^2}{3\sqrt{c}\sqrt{c^3-8d^3x^3}}\right)}{3\sqrt{c}d}$$

Antiderivative was successfully verified.

```
[In]  Int[(c - 2*d*x)/((c + d*x)*Sqrt[c^3 - 8*d^3*x^3]),x]
```

```
[Out] (-2*ArcTanh[(c - 2*d*x)^2/(3*Sqrt[c]*Sqrt[c^3 - 8*d^3*x^3])])/(3*
Sqrt[c]*d)
```

Rubi in Sympy [A] time = 144.56, size = 549, normalized size = 11.93

$$-\frac{2\sqrt[4]{3}\sqrt{\frac{c^2+2cdx+4d^2x^2}{\left(c\left(1+\sqrt{3}\right)-2dx\right)^2}}\sqrt{\sqrt{3}+2}\,(c-2dx)\,F\left(\operatorname{asin}\left(-\frac{-c\left(-1+\sqrt{3}\right)-2dx}{c\left(1+\sqrt{3}\right)-2dx}\right)\middle|-7-4\sqrt{3}\right)}{d\sqrt{\frac{c(c-2dx)}{\left(c\left(1+\sqrt{3}\right)-2dx\right)^2}}\left(\sqrt{3}+3\right)\sqrt{c^3-8d^3x^3}}$$
$$-\frac{3^{\frac{3}{4}}\sqrt{\frac{c^2\left(1+\frac{2dx}{c}+\frac{4d^2x^2}{c^2}\right)}{\left(c\left(1+\sqrt{3}\right)-2dx\right)^2}}\sqrt{3+2\sqrt{3}}\sqrt{-\sqrt{3}+2}\,(c-2dx)\operatorname{atanh}\left(\frac{\sqrt{-\frac{\left(c\left(-1+\sqrt{3}\right)+2dx\right)^2}{\left(c\left(1+\sqrt{3}\right)-2dx\right)^2}+1}}{\sqrt{3+2\sqrt{3}}\sqrt{\frac{\left(c\left(-1+\sqrt{3}\right)+2dx\right)^2}{\left(c\left(1+\sqrt{3}\right)-2dx\right)^2}-4\sqrt{3}+7}}\right)}{3d\sqrt{\frac{c(c-2dx)}{\left(c\left(1+\sqrt{3}\right)-2dx\right)^2}}\sqrt{c^3-8d^3x^3}}$$
$$+\frac{12\sqrt[4]{3}\sqrt{\frac{c^2\left(1+\frac{2dx}{c}+\frac{4d^2x^2}{c^2}\right)}{\left(c\left(1+\sqrt{3}\right)-2dx\right)^2}}\sqrt{-\sqrt{3}+2}\,(c-2dx)\left(4\sqrt{3}+7;\operatorname{asin}\left(\frac{c\left(-1+\sqrt{3}\right)+2dx}{c\left(1+\sqrt{3}\right)-2dx}\right)\middle|-7-4\sqrt{3}\right)}{d\sqrt{\frac{c(c-2dx)}{\left(c\left(1+\sqrt{3}\right)-2dx\right)^2}}\sqrt{-4\sqrt{3}+7}\left(-\sqrt{3}+3\right)\left(\sqrt{3}+3\right)\sqrt{c^3-8d^3x^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-2*d*x+c)/(d*x+c)/(-8*d**3*x**3+c**3)**(1/2),x)
```

```
[Out] -2*3**(1/4)*sqrt((c**2 + 2*c*d*x + 4*d**2*x**2)/(c*(1 + sqrt(3))
- 2*d*x)**2)*sqrt(sqrt(3) + 2)*(c - 2*d*x)*elliptic_f(asin(-(-c*(
-1 + sqrt(3)) - 2*d*x)/(c*(1 + sqrt(3)) - 2*d*x)), -7 - 4*sqrt(3)
)/(d*sqrt(c*(c - 2*d*x)/(c*(1 + sqrt(3)) - 2*d*x)**2)*(sqrt(3) +
3)*sqrt(c**3 - 8*d**3*x**3)) - 3**(3/4)*sqrt(c**2*(1 + 2*d*x/c +
4*d**2*x**2/c**2)/(c*(1 + sqrt(3)) - 2*d*x)**2)*sqrt(3 + 2*sqrt(3
))*sqrt(-sqrt(3) + 2)*(c - 2*d*x)*atanh(sqrt(-(c*(-1 + sqrt(3)) +
 2*d*x)**2/(c*(1 + sqrt(3)) - 2*d*x)**2 + 1)/(sqrt(3 + 2*sqrt(3))
*sqrt((c*(-1 + sqrt(3)) + 2*d*x)**2/(c*(1 + sqrt(3)) - 2*d*x)**2
- 4*sqrt(3) + 7)))/(3*d*sqrt(c*(c - 2*d*x)/(c*(1 + sqrt(3)) - 2*d
*x)**2)*sqrt(c**3 - 8*d**3*x**3)) + 12*3**(1/4)*sqrt(c**2*(1 + 2*
d*x/c + 4*d**2*x**2/c**2)/(c*(1 + sqrt(3)) - 2*d*x)**2)*sqrt(-sqr
t(3) + 2)*(c - 2*d*x)*elliptic_pi(4*sqrt(3) + 7, asin((c*(-1 + sq
rt(3)) + 2*d*x)/(c*(1 + sqrt(3)) - 2*d*x)), -7 - 4*sqrt(3))/(d*sq
rt(c*(c - 2*d*x)/(c*(1 + sqrt(3)) - 2*d*x)**2)*sqrt(-4*sqrt(3) +
7)*(-sqrt(3) + 3)*(sqrt(3) + 3)*sqrt(c**3 - 8*d**3*x**3))
```

Mathematica [C] time = 1.03501, size = 295, normalized size = 6.41

$$-\frac{2\sqrt{\frac{c-2dx}{\left(1+\sqrt[3]{-1}\right)c}}\left(\left(\sqrt[3]{-1}-2\right)\left(\sqrt[3]{-1}c+2dx\right)\sqrt{\frac{\sqrt[3]{-1}\left(c+2\sqrt[3]{-1}dx\right)}{\left(1+\sqrt[3]{-1}\right)c}}F\left(\sin^{-1}\left(\sqrt{\frac{c-2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}\right)\middle|\sqrt[3]{-1}\right)+\sqrt[3]{-1}\sqrt{3}\left(1+\sqrt[3]{-1}\right)c\sqrt{\frac{c-2\cdots}{\left(1+\cdots\right.}}\cdots\right)}{\left(\sqrt[3]{-1}-2\right)d\sqrt{\frac{c-2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}\sqrt{c^3-8d^3x^3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(c - 2*d*x)/((c + d*x)*Sqrt[c^3 - 8*d^3*x^3]),x]
```

```
[Out] (-2*Sqrt[(c - 2*d*x)/((1 + (-1)^(1/3))*c)]*((-2 + (-1)^(1/3))*((-
1)^(1/3)*c + 2*d*x)*Sqrt[((-1)^(1/3)*(c + 2*(-1)^(1/3)*d*x))/((1
+ (-1)^(1/3))*c)]*EllipticF[ArcSin[Sqrt[(c - 2*(-1)^(2/3)*d*x)/((
1 + (-1)^(1/3))*c)]], (-1)^(1/3)] + (-1)^(1/3)*Sqrt[3]*(1 + (-1)^
(1/3))*c*Sqrt[(c - 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]*Sqrt[(
c^2 + 2*c*d*x + 4*d^2*x^2)/c^2]*EllipticPi[(2*Sqrt[3])/(3*I + Sqr
t[3]), ArcSin[Sqrt[(c - 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]],
 (-1)^(1/3)]))/((-2 + (-1)^(1/3))*d*Sqrt[(c - 2*(-1)^(2/3)*d*x)/(
(1 + (-1)^(1/3))*c)]*Sqrt[c^3 - 8*d^3*x^3])
```

Maple [C] time = 0.2, size = 650, normalized size = 14.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-2*d*x+c)/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x)
```

```
[Out] -4*(1/2*(-1/2+1/2*I*3^(1/2))*c/d-1/2*(-1/2-1/2*I*3^(1/2))*c/d)*((
x-1/2*(-1/2-1/2*I*3^(1/2))*c/d)/(1/2*(-1/2+1/2*I*3^(1/2))*c/d-1/2
*(-1/2-1/2*I*3^(1/2))*c/d))^(1/2)*((x-1/2*c/d)/(1/2*(-1/2-1/2*I*3
^(1/2))*c/d-1/2*c/d))^(1/2)*((x-1/2*(-1/2+1/2*I*3^(1/2))*c/d)/(1/
2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*(-1/2+1/2*I*3^(1/2))*c/d))^(1/2)/(
-8*d^3*x^3+c^3)^(1/2)*EllipticF(((x-1/2*(-1/2-1/2*I*3^(1/2))*c/d)
/(1/2*(-1/2+1/2*I*3^(1/2))*c/d-1/2*(-1/2-1/2*I*3^(1/2))*c/d))^(1/
2),((1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*(-1/2+1/2*I*3^(1/2))*c/d)/(
1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d))^(1/2))+6*c/d*(1/2*(-1/2+1/
2*I*3^(1/2))*c/d-1/2*(-1/2-1/2*I*3^(1/2))*c/d)*((x-1/2*(-1/2-1/2*
I*3^(1/2))*c/d)/(1/2*(-1/2+1/2*I*3^(1/2))*c/d-1/2*(-1/2-1/2*I*3^(
1/2))*c/d))^(1/2)*((x-1/2*c/d)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*
c/d))^(1/2)*((x-1/2*(-1/2+1/2*I*3^(1/2))*c/d)/(1/2*(-1/2-1/2*I*3^
(1/2))*c/d-1/2*(-1/2+1/2*I*3^(1/2))*c/d))^(1/2)/(-8*d^3*x^3+c^3)^
(1/2)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d+c/d)*EllipticPi(((x-1/2*(-1/2
-1/2*I*3^(1/2))*c/d)/(1/2*(-1/2+1/2*I*3^(1/2))*c/d-1/2*(-1/2-1/2*
I*3^(1/2))*c/d))^(1/2),(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*(-1/2+1/
2*I*3^(1/2))*c/d)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d+c/d),((1/2*(-1/2-
1/2*I*3^(1/2))*c/d-1/2*(-1/2+1/2*I*3^(1/2))*c/d)/(1/2*(-1/2-1/2*I
*3^(1/2))*c/d-1/2*c/d))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{2\,dx - c}{\sqrt{-8\,d^3x^3 + c^3}(dx + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(2*d*x - c)/(sqrt(-8*d^3*x^3 + c^3)*(d*x + c)),x, algorithm="maxima")
```

```
[Out] -integrate((2*d*x - c)/(sqrt(-8*d^3*x^3 + c^3)*(d*x + c)), x)
```

Fricas [A] time = 0.388651, size = 1, normalized size = 0.02

$$\left[\frac{\log\left(-\frac{3\left(8\,cd^4x^4-52\,c^2d^3x^3+12\,c^3d^2x^2-4\,c^4dx+5\,c^5\right)\sqrt{-8\,d^3x^3+c^3}-\left(8\,d^6x^6-240\,cd^5x^5+408\,c^2d^4x^4+88\,c^3d^3x^3+156\,c^4d^2x^2+12\,c^5dx+17\,c^6\right)\sqrt{c}}{d^6x^6+6\,cd^5x^5+15\,c^2d^4x^4+20\,c^3d^3x^3+15\,c^4d^2x^2+6\,c^5dx+c^6}\right)}{6\,\sqrt{c}d},\right.$$
$$\left.-\frac{\arctan\left(\frac{\left(4\,d^3x^3-24\,cd^2x^2-6\,c^2dx-5\,c^3\right)\sqrt{-c}}{3\,\sqrt{-8\,d^3x^3+c^3}\left(2\,cdx-c^2\right)}\right)}{3\,\sqrt{-c}d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(2*d*x - c)/(sqrt(-8*d^3*x^3 + c^3)*(d*x + c)),x, algorithm="fricas")
```

```
[Out] [1/6*log(-(3*(8*c*d^4*x^4 - 52*c^2*d^3*x^3 + 12*c^3*d^2*x^2 - 4*c
^4*d*x + 5*c^5)*sqrt(-8*d^3*x^3 + c^3) - (8*d^6*x^6 - 240*c*d^5*x
^5 + 408*c^2*d^4*x^4 + 88*c^3*d^3*x^3 + 156*c^4*d^2*x^2 + 12*c^5*
d*x + 17*c^6)*sqrt(c))/(d^6*x^6 + 6*c*d^5*x^5 + 15*c^2*d^4*x^4 +
20*c^3*d^3*x^3 + 15*c^4*d^2*x^2 + 6*c^5*d*x + c^6))/(sqrt(c)*d),
-1/3*arctan(1/3*(4*d^3*x^3 - 24*c*d^2*x^2 - 6*c^2*d*x - 5*c^3)*sq
rt(-c)/(sqrt(-8*d^3*x^3 + c^3)*(2*c*d*x - c^2)))/(sqrt(-c)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\left(-\frac{c}{c\sqrt{c^3-8d^3x^3}+dx\sqrt{c^3-8d^3x^3}}\right)dx-\int\frac{2dx}{c\sqrt{c^3-8d^3x^3}+dx\sqrt{c^3-8d^3x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*d*x+c)/(d*x+c)/(-8*d**3*x**3+c**3)**(1/2),x)
```

```
[Out] -Integral(-c/(c*sqrt(c**3 - 8*d**3*x**3) + d*x*sqrt(c**3 - 8*d**3
*x**3)), x) - Integral(2*d*x/(c*sqrt(c**3 - 8*d**3*x**3) + d*x*sq
rt(c**3 - 8*d**3*x**3)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int-\frac{2\,dx-c}{\sqrt{-8\,d^3x^3+c^3}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(2*d*x - c)/(sqrt(-8*d^3*x^3 + c^3)*(d*x + c)),x, algorithm="giac")

[Out] integrate(-(2*d*x - c)/(sqrt(-8*d^3*x^3 + c^3)*(d*x + c)), x)
```

3.60 $\int \frac{e+fx}{(2-x)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=139

$$\frac{2}{9}(e+2f)\tanh^{-1}\left(\frac{(x+1)^2}{3\sqrt{x^3+1}}\right)+\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(e-f)F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

```
[Out] (2*(e + 2*f)*ArcTanh[(1 + x)^2/(3*Sqrt[1 + x^3])])/9 + (2*Sqrt[2
+ Sqrt[3]]*(e - f)*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2
]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*S
qrt[3]])/(3*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^
3])
```

Rubi [A] time = 0.277653, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\frac{2}{9}(e+2f)\tanh^{-1}\left(\frac{(x+1)^2}{3\sqrt{x^3+1}}\right)+\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(e-f)F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(e + f*x)/((2 - x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*(e + 2*f)*ArcTanh[(1 + x)^2/(3*Sqrt[1 + x^3])])/9 + (2*Sqrt[2
+ Sqrt[3]]*(e - f)*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2
]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*S
qrt[3]])/(3*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^
3])
```

Rubi in Sympy [A] time = 93.7997, size = 391, normalized size = 2.81

$$\frac{\sqrt{\frac{x^2-x+1}{\left(x+1+\sqrt{3}\right)^2}}\left(\frac{\sqrt{3}}{3}+1\right)(e+2f)(x+1)\operatorname{atanh}\left(\frac{3^{\frac{3}{4}}\sqrt{-\sqrt{3}+2}\sqrt{-\frac{\left(-x-1+\sqrt{3}\right)^2}{\left(x+1+\sqrt{3}\right)^2}+1}}{3\sqrt{\frac{\left(-x-1+\sqrt{3}\right)^2}{\left(x+1+\sqrt{3}\right)^2}-4\sqrt{3}+7}}\right)}{\sqrt{\frac{x+1}{\left(x+1+\sqrt{3}\right)^2}}\left(\sqrt{3}+3\right)\sqrt{x^3+1}} - \frac{4\sqrt[4]{3}\sqrt{\frac{x^2-x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{-\sqrt{3}+2}\,(e+2f)(x+1)\left(4\sqrt{3}+7;\operatorname{asin}\left(\frac{-x-1+\sqrt{3}}{x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{-4\sqrt{3}+7}\left(-\sqrt{3}+3\right)\left(\sqrt{3}+3\right)\sqrt{x^3+1}} + \frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{x^2-x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{\sqrt{3}+2}\left(e-f\left(1+\sqrt{3}\right)\right)(x+1)F\left(\operatorname{asin}\left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{3\sqrt{\frac{x+1}{\left(x+1+\sqrt{3}\right)^2}}\left(\sqrt{3}+3\right)\sqrt{x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x+e)/(2-x)/(x**3+1)**(1/2),x)
```

```
[Out] sqrt((x**2 - x + 1)/(x + 1 + sqrt(3))**2)*(sqrt(3)/3 + 1)*(e + 2*
f)*(x + 1)*atanh(3**(3/4)*sqrt(-sqrt(3) + 2)*sqrt(-(-x - 1 + sqrt
(3))**2/(x + 1 + sqrt(3))**2 + 1)/(3*sqrt((-x - 1 + sqrt(3))**2/(
x + 1 + sqrt(3))**2 - 4*sqrt(3) + 7)))/(sqrt((x + 1)/(x + 1 + sqr
t(3))**2)*(sqrt(3) + 3)*sqrt(x**3 + 1)) - 4*3**(1/4)*sqrt((x**2 -
 x + 1)/(x + 1 + sqrt(3))**2)*sqrt(-sqrt(3) + 2)*(e + 2*f)*(x + 1
)*elliptic_pi(4*sqrt(3) + 7, asin((-x - 1 + sqrt(3))/(x + 1 + sqr
t(3))), -7 - 4*sqrt(3))/(sqrt((x + 1)/(x + 1 + sqrt(3))**2)*sqrt(
-4*sqrt(3) + 7)*(-sqrt(3) + 3)*(sqrt(3) + 3)*sqrt(x**3 + 1)) + 2*
3**(3/4)*sqrt((x**2 - x + 1)/(x + 1 + sqrt(3))**2)*sqrt(sqrt(3) +
 2)*(e - f*(1 + sqrt(3)))*(x + 1)*elliptic_f(asin((x - sqrt(3) +
1)/(x + 1 + sqrt(3))), -7 - 4*sqrt(3))/(3*sqrt((x + 1)/(x + 1 + s
qrt(3))**2)*(sqrt(3) + 3)*sqrt(x**3 + 1))
```

Mathematica [C] time = 0.405898, size = 273, normalized size = 1.96

$$\frac{2\sqrt{\frac{2}{3}}\sqrt{-\frac{i(x+1)}{\sqrt{3}-3i}}\left(2\sqrt{3}\sqrt{2ix+\sqrt{3}-i}\sqrt{x^2-x+1}(e+2f)\left(\frac{2\sqrt{3}}{3i+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{2ix+\sqrt{3}-i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right)-3if\sqrt{-2ix+\sqrt{3}+i}\left(\left(\sqrt{3}-i\right)\right.\right.}{\left(\sqrt{3}+3i\right)\sqrt{2ix+\sqrt{3}-i}\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(e + f*x)/((2 - x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[2/3]*Sqrt[((-I)*(1 + x))/(-3*I + Sqrt[3])]*((-3*I)*f*Sqrt
[I + Sqrt[3] - (2*I)*x]*(-I - Sqrt[3] + (-I + Sqrt[3])*x)*Ellipti
cF[ArcSin[Sqrt[-I + Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqr
t[3])/(-3*I + Sqrt[3])] + 2*Sqrt[3]*(e + 2*f)*Sqrt[-I + Sqrt[3] +
 (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]
), ArcSin[Sqrt[-I + Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqr
t[3])/(-3*I + Sqrt[3])]))/((3*I + Sqrt[3])*Sqrt[-I + Sqrt[3] + (2
*I)*x]*Sqrt[1 + x^3])
```

Maple [B] time = 0.01, size = 246, normalized size = 1.8

$$-2\frac{f\left(3/2-i/2\sqrt{3}\right)}{\sqrt{x^3+1}}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}EllipticF\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},\sqrt{\frac{-3/2+i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\right)$$
$$+\frac{(2e+4f)\left(\frac{3}{2}-\frac{i}{2}\sqrt{3}\right)}{3}\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i}{2}\sqrt{3}}}\sqrt{\frac{1}{-\frac{3}{2}-\frac{i}{2}\sqrt{3}}\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)}\sqrt{\frac{1}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)}EllipticPi\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i}{2}\sqrt{3}}},\frac{1}{2}-\frac{i}{6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x+e)/(2-x)/(x^3+1)^(1/2),x)
```

```
[Out] -2*f*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/
2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2
))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/
2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)
))^(1/2))+2/3*(e+2*f)*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/
2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x
-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*Ell
ipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),1/2-1/6*I*3^(1/2),((-3/
2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{fx+e}{\sqrt{x^3+1}(x-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(f*x + e)/(sqrt(x^3 + 1)*(x - 2)),x, algorithm="maxima")
```

```
[Out] -integrate((f*x + e)/(sqrt(x^3 + 1)*(x - 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{fx+e}{\sqrt{x^3+1}(x-2)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(f*x + e)/(sqrt(x^3 + 1)*(x - 2)),x, algorithm="fricas")
```

```
[Out] integral(-(f*x + e)/(sqrt(x^3 + 1)*(x - 2)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{e}{x\sqrt{x^3+1}-2\sqrt{x^3+1}}\,dx-\int \frac{fx}{x\sqrt{x^3+1}-2\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x+e)/(2-x)/(x**3+1)**(1/2),x)
```

```
[Out] -Integral(e/(x*sqrt(x**3 + 1) - 2*sqrt(x**3 + 1)), x) - Integral(
f*x/(x*sqrt(x**3 + 1) - 2*sqrt(x**3 + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{fx+e}{\sqrt{x^3+1}(x-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(f*x + e)/(sqrt(x^3 + 1)*(x - 2)),x, algorithm="giac")
```

```
[Out] integrate(-(f*x + e)/(sqrt(x^3 + 1)*(x - 2)), x)
```

3.61 $\int \frac{e+fx}{(2+x)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=153

$$-\frac{2}{9}(e-2f)\tanh^{-1}\left(\frac{(1-x)^2}{3\sqrt{1-x^3}}\right)-\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}(e+f)F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

```
[Out] (-2*(e - 2*f)*ArcTanh[(1 - x)^2/(3*Sqrt[1 - x^3])])/9 - (2*Sqrt[2
 + Sqrt[3]]*(e + f)*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^
2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*
Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x
^3])
```

Rubi [A] time = 0.293024, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$-\frac{2}{9}(e-2f)\tanh^{-1}\left(\frac{(1-x)^2}{3\sqrt{1-x^3}}\right)-\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}(e+f)F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(e + f*x)/((2 + x)*Sqrt[1 - x^3]),x]
```

```
[Out] (-2*(e - 2*f)*ArcTanh[(1 - x)^2/(3*Sqrt[1 - x^3])])/9 - (2*Sqrt[2
 + Sqrt[3]]*(e + f)*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^
2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*
Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x
^3])
```

Rubi in Sympy [A] time = 95.9624, size = 391, normalized size = 2.56

$$-\frac{\sqrt{\frac{x^2+x+1}{\left(-x+1+\sqrt{3}\right)^2}}\left(\frac{\sqrt{3}}{3}+1\right)(e-2f)(-x+1)\operatorname{atanh}\left(\frac{3^{\frac{3}{4}}\sqrt{1-\frac{\left(x-1+\sqrt{3}\right)^2}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{-\sqrt{3}+2}}{3\sqrt{-4\sqrt{3}+7+\frac{\left(x-1+\sqrt{3}\right)^2}{\left(-x+1+\sqrt{3}\right)^2}}}\right)}{\sqrt{\frac{-x+1}{\left(-x+1+\sqrt{3}\right)^2}}\left(\sqrt{3}+3\right)\sqrt{-x^3+1}} + \frac{4\sqrt[4]{3}\sqrt{\frac{x^2+x+1}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{-\sqrt{3}+2}\,(e-2f)(-x+1)\left(4\sqrt{3}+7;\operatorname{asin}\left(\frac{x-1+\sqrt{3}}{-x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt{\frac{-x+1}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{-4\sqrt{3}+7}\left(-\sqrt{3}+3\right)\left(\sqrt{3}+3\right)\sqrt{-x^3+1}} - \frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{x^2+x+1}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{\sqrt{3}+2}\,(-x+1)\left(e+f+\sqrt{3}f\right)F\left(\operatorname{asin}\left(\frac{-x-\sqrt{3}+1}{-x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{3\sqrt{\frac{-x+1}{\left(-x+1+\sqrt{3}\right)^2}}\left(\sqrt{3}+3\right)\sqrt{-x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x+e)/(2+x)/(-x**3+1)**(1/2),x)
```

```
[Out] -sqrt((x**2 + x + 1)/(-x + 1 + sqrt(3))**2)*(sqrt(3)/3 + 1)*(e -
2*f)*(-x + 1)*atanh(3**(3/4)*sqrt(1 - (x - 1 + sqrt(3))**2/(-x +
1 + sqrt(3))**2)*sqrt(-sqrt(3) + 2)/(3*sqrt(-4*sqrt(3) + 7 + (x -
 1 + sqrt(3))**2/(-x + 1 + sqrt(3))**2)))/(sqrt((-x + 1)/(-x + 1
+ sqrt(3))**2)*(sqrt(3) + 3)*sqrt(-x**3 + 1)) + 4*3**(1/4)*sqrt((
x**2 + x + 1)/(-x + 1 + sqrt(3))**2)*sqrt(-sqrt(3) + 2)*(e - 2*f)
*(-x + 1)*elliptic_pi(4*sqrt(3) + 7, asin((x - 1 + sqrt(3))/(-x +
 1 + sqrt(3))), -7 - 4*sqrt(3))/(sqrt((-x + 1)/(-x + 1 + sqrt(3))
**2)*sqrt(-4*sqrt(3) + 7)*(-sqrt(3) + 3)*(sqrt(3) + 3)*sqrt(-x**3
 + 1)) - 2*3**(3/4)*sqrt((x**2 + x + 1)/(-x + 1 + sqrt(3))**2)*sq
rt(sqrt(3) + 2)*(-x + 1)*(e + f + sqrt(3)*f)*elliptic_f(asin((-x
- sqrt(3) + 1)/(-x + 1 + sqrt(3))), -7 - 4*sqrt(3))/(3*sqrt((-x +
 1)/(-x + 1 + sqrt(3))**2)*(sqrt(3) + 3)*sqrt(-x**3 + 1))
```

Mathematica [C] time = 0.386612, size = 271, normalized size = 1.77

$$\frac{2\sqrt{\frac{2}{3}}\sqrt{\frac{i(x-1)}{\sqrt{3}-3i}}\left(3f\sqrt{2ix+\sqrt{3}+i}\left(i\sqrt{3}x+x+i\sqrt{3}-1\right)F\left(\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}-i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right)-2\sqrt{3}\sqrt{-2ix+\sqrt{3}-i}\sqrt{x^2+x+1}(e -}{\left(\sqrt{3}+3i\right)\sqrt{-2ix+\sqrt{3}-i}\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(e + f*x)/((2 + x)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*Sqrt[2/3]*Sqrt[(I*(-1 + x))/(-3*I + Sqrt[3])]*(3*f*Sqrt[I + Sq
rt[3] + (2*I)*x]*(-1 + I*Sqrt[3] + x + I*Sqrt[3]*x)*EllipticF[Arc
Sin[Sqrt[-I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/
(-3*I + Sqrt[3])] - 2*Sqrt[3]*(e - 2*f)*Sqrt[-I + Sqrt[3] - (2*I)
*x]*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), Arc
Sin[Sqrt[-I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/
(-3*I + Sqrt[3])]))/((3*I + Sqrt[3])*Sqrt[-I + Sqrt[3] - (2*I)*x]
*Sqrt[1 - x^3])
```

Maple [A] time = 0.01, size = 246, normalized size = 1.6

$$-\frac{2i}{3} f\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}EllipticF\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)$$
$$-\frac{\frac{2i}{3}(e-2f)\sqrt{3}}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}EllipticPi\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x+e)/(2+x)/(-x^3+1)^(1/2),x)
```

```
[Out] -2/3*I*f*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/
(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1
/2)/(-x^3+1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))
*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))-2/3*I*(e-
2*f)*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/(-3/
2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/
(-x^3+1)^(1/2)/(3/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1
/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(3/2+1/2*I*3^(1/2)),(I
*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\sqrt{-x^3+1}(x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x + e)/(sqrt(-x^3 + 1)*(x + 2)),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(-x^3 + 1)*(x + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{fx+e}{\sqrt{-x^3+1}(x+2)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x + e)/(sqrt(-x^3 + 1)*(x + 2)),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)/(sqrt(-x^3 + 1)*(x + 2)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e+fx}{\sqrt{-(x-1)(x^2+x+1)}(x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x+e)/(2+x)/(-x**3+1)**(1/2),x)
```

```
[Out] Integral((e + f*x)/(sqrt(-(x - 1)*(x**2 + x + 1))*(x + 2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\sqrt{-x^3+1}(x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x + e)/(sqrt(-x^3 + 1)*(x + 2)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)/(sqrt(-x^3 + 1)*(x + 2)), x)
```

3.62 $\int \frac{e+fx}{(2+x)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=156

$$-\frac{2}{9}(e-2f)\tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{x^3-1}}\right)-\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}(e+f)F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

```
[Out] (-2*(e - 2*f)*ArcTan[(1 - x)^2/(3*Sqrt[-1 + x^3])])/9 - (2*Sqrt[2
 - Sqrt[3]]*(e + f)*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^
2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*
Sqrt[3]])/(3*3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1
 + x^3])
```

Rubi [A] time = 0.275472, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\frac{2}{9}(e-2f)\tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{x^3-1}}\right)-\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}(e+f)F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(e + f*x)/((2 + x)*Sqrt[-1 + x^3]),x]
```


```
[Out] (-2*(e - 2*f)*ArcTan[(1 - x)^2/(3*Sqrt[-1 + x^3])])/9 - (2*Sqrt[2
 - Sqrt[3]]*(e + f)*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^
2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*
Sqrt[3]])/(3*3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1
 + x^3])
```

Rubi in Sympy [A] time = 94.0336, size = 389, normalized size = 2.49

$$-\frac{\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(-\frac{\sqrt{3}}{3}+1\right)(e-2f)(-x+1)\operatorname{atan}\left(\frac{3^{\frac{3}{4}}\sqrt{\sqrt{3}+2}\sqrt{-\frac{\left(-x+1+\sqrt{3}\right)^2}{\left(x-1+\sqrt{3}\right)^2}+1}}{3\sqrt{\frac{\left(-x+1+\sqrt{3}\right)^2}{\left(x-1+\sqrt{3}\right)^2}+4\sqrt{3}+7}}\right)}{\sqrt{\frac{x-1}{\left(-x-\sqrt{3}+1\right)^2}}\left(-\sqrt{3}+3\right)\sqrt{x^3-1}}-\frac{4\sqrt[4]{3}\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{\sqrt{3}+2}\,(e-2f)(-x+1)\left(-4\sqrt{3}+7;\operatorname{asin}\left(\frac{-x+1+\sqrt{3}}{x-1+\sqrt{3}}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt{\frac{x-1}{\left(-x-\sqrt{3}+1\right)^2}}\left(-\sqrt{3}+3\right)\left(\sqrt{3}+3\right)\sqrt{4\sqrt{3}+7}\sqrt{x^3-1}}-\frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{-\sqrt{3}+2}\,(-x+1)\left(e-\sqrt{3}f+f\right)F\left(\operatorname{asin}\left(\frac{-x+1+\sqrt{3}}{-x-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{3\sqrt{\frac{x-1}{\left(-x-\sqrt{3}+1\right)^2}}\left(-\sqrt{3}+3\right)\sqrt{x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x+e)/(2+x)/(x**3-1)**(1/2),x)
```

```
[Out] -sqrt((x**2 + x + 1)/(-x - sqrt(3) + 1)**2)*(-sqrt(3)/3 + 1)*(e -
 2*f)*(-x + 1)*atan(3**(3/4)*sqrt(sqrt(3) + 2)*sqrt(-(-x + 1 + sq
rt(3))**2/(x - 1 + sqrt(3))**2 + 1)/(3*sqrt((-x + 1 + sqrt(3))**2
/(x - 1 + sqrt(3))**2 + 4*sqrt(3) + 7)))/(sqrt((x - 1)/(-x - sqrt
(3) + 1)**2)*(-sqrt(3) + 3)*sqrt(x**3 - 1)) - 4*3**(1/4)*sqrt((x*
*2 + x + 1)/(-x - sqrt(3) + 1)**2)*sqrt(sqrt(3) + 2)*(e - 2*f)*(-
x + 1)*elliptic_pi(-4*sqrt(3) + 7, asin((-x + 1 + sqrt(3))/(x - 1
 + sqrt(3))), -7 + 4*sqrt(3))/(sqrt((x - 1)/(-x - sqrt(3) + 1)**2
)*(-sqrt(3) + 3)*(sqrt(3) + 3)*sqrt(4*sqrt(3) + 7)*sqrt(x**3 - 1)
) - 2*3**(3/4)*sqrt((x**2 + x + 1)/(-x - sqrt(3) + 1)**2)*sqrt(-s
qrt(3) + 2)*(-x + 1)*(e - sqrt(3)*f + f)*elliptic_f(asin((-x + 1
+ sqrt(3))/(-x - sqrt(3) + 1)), -7 + 4*sqrt(3))/(3*sqrt((x - 1)/(
-x - sqrt(3) + 1)**2)*(-sqrt(3) + 3)*sqrt(x**3 - 1))
```

Mathematica [C] time = 0.376116, size = 269, normalized size = 1.72

$$\frac{2\sqrt{\frac{2}{3}}\sqrt{\frac{i(x-1)}{\sqrt{3}-3i}}\left(3f\sqrt{2ix+\sqrt{3}+i}\left(i\sqrt{3}x+x+i\sqrt{3}-1\right)F\left(\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}-i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right)-2\sqrt{3}\sqrt{-2ix+\sqrt{3}-i}\sqrt{x^2+x+1}(e-}{\left(\sqrt{3}+3i\right)\sqrt{-2ix+\sqrt{3}-i}\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(e + f*x)/((2 + x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*Sqrt[2/3]*Sqrt[(I*(-1 + x))/(-3*I + Sqrt[3])]*(3*f*Sqrt[I + Sq
rt[3] + (2*I)*x]*(-1 + I*Sqrt[3] + x + I*Sqrt[3]*x)*EllipticF[Arc
Sin[Sqrt[-I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/
(-3*I + Sqrt[3])] - 2*Sqrt[3]*(e - 2*f)*Sqrt[-I + Sqrt[3] - (2*I)
*x]*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), Arc
Sin[Sqrt[-I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/
(-3*I + Sqrt[3])]))/((3*I + Sqrt[3])*Sqrt[-I + Sqrt[3] - (2*I)*x]
*Sqrt[-1 + x^3])
```

Maple [A]   time = 0.009, size = 246, normalized size = 1.6

$$2\,\frac{f\left(-3/2-i/2\sqrt{3}\right)}{\sqrt{x^3-1}}\sqrt{\frac{-1+x}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2-i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2+i/2\sqrt{3}}{3/2+i/2\sqrt{3}}}\,EllipticF\left(\sqrt{\frac{-1+x}{-3/2-i/2\sqrt{3}}},\sqrt{\frac{3/2+i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\right)$$

$$+\frac{(2\,e-4\,f)\left(-\frac{3}{2}-\frac{i}{2}\sqrt{3}\right)}{3}\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i}{2}\sqrt{3}}}\sqrt{\frac{1}{\frac{3}{2}-\frac{i}{2}\sqrt{3}}\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)}\sqrt{\frac{1}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)}\,EllipticPi\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i}{2}\sqrt{3}}},\frac{i}{6}\sqrt{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x+e)/(2+x)/(x^3-1)^(1/2),x)
```

```
[Out] 2*f*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+
1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/
2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((-1+x)/(-
3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)
))^(1/2))+2/3*(e-2*f)*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^
(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*(
(x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*El
lipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),1/6*I*3^(1/2)+1/2,((
3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\sqrt{x^3-1}(x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x + e)/(sqrt(x^3 - 1)*(x + 2)),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(x^3 - 1)*(x + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{fx+e}{\sqrt{x^3-1}(x+2)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x + e)/(sqrt(x^3 - 1)*(x + 2)),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)/(sqrt(x^3 - 1)*(x + 2)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{e+fx}{\sqrt{(x-1)\left(x^2+x+1\right)}\,(x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x+e)/(2+x)/(x**3-1)**(1/2),x)
```

```
[Out] Integral((e + f*x)/(sqrt((x - 1)*(x**2 + x + 1))*(x + 2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{fx+e}{\sqrt{x^3-1}(x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x + e)/(sqrt(x^3 - 1)*(x + 2)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)/(sqrt(x^3 - 1)*(x + 2)), x)
```

3.63 $\int \frac{e+fx}{(2-x)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=150

$$\frac{2}{9}(e+2f)\tan^{-1}\left(\frac{(x+1)^2}{3\sqrt{-x^3-1}}\right)+\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}(e-f)F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

```
[Out] (2*(e + 2*f)*ArcTan[(1 + x)^2/(3*Sqrt[-1 - x^3])])/9 + (2*Sqrt[2
- Sqrt[3]]*(e - f)*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2
]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*S
qrt[3]])/(3*3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1
- x^3])
```

Rubi [A] time = 0.310743, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{2}{9}(e+2f)\tan^{-1}\left(\frac{(x+1)^2}{3\sqrt{-x^3-1}}\right)+\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}(e-f)F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(e + f*x)/((2 - x)*Sqrt[-1 - x^3]),x]
```


```
[Out] (2*(e + 2*f)*ArcTan[(1 + x)^2/(3*Sqrt[-1 - x^3])])/9 + (2*Sqrt[2
- Sqrt[3]]*(e - f)*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2
]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*S
qrt[3]])/(3*3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1
- x^3])
```

Rubi in Sympy [A] time = 65.8967, size = 398, normalized size = 2.65

$$\frac{\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(-\frac{\sqrt{3}}{3}+1\right)(e+2f)(x+1)\operatorname{atan}\left(\frac{3^{\frac{3}{4}}\sqrt{1-\frac{\left(x+1+\sqrt{3}\right)^2}{\left(-x-1+\sqrt{3}\right)^2}}\sqrt{\sqrt{3}+2}}{3\sqrt{4\sqrt{3}+7+\frac{\left(x+1+\sqrt{3}\right)^2}{\left(-x-1+\sqrt{3}\right)^2}}}\right)}{\sqrt{\frac{-x-1}{\left(x-\sqrt{3}+1\right)^2}}\left(-\sqrt{3}+3\right)\sqrt{-x^3-1}} + \frac{4\sqrt[4]{3}\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{\sqrt{3}+2}\,(e+2f)(x+1)\left(-4\sqrt{3}+7;\operatorname{asin}\left(\frac{x+1+\sqrt{3}}{-x-1+\sqrt{3}}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt{\frac{-x-1}{\left(x-\sqrt{3}+1\right)^2}}\left(-\sqrt{3}+3\right)\left(\sqrt{3}+3\right)\sqrt{4\sqrt{3}+7}\sqrt{-x^3-1}} + \frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-\sqrt{3}+2}\,(x+1)\left(e-f+\sqrt{3}f\right)F\left(\operatorname{asin}\left(\frac{x+1+\sqrt{3}}{x-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{3\sqrt{\frac{-x-1}{\left(x-\sqrt{3}+1\right)^2}}\left(-\sqrt{3}+3\right)\sqrt{-x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x+e)/(2-x)/(-x**3-1)**(1/2),x)
```

```
[Out] sqrt((x**2 - x + 1)/(x - sqrt(3) + 1)**2)*(-sqrt(3)/3 + 1)*(e + 2
*f)*(x + 1)*atan(3**(3/4)*sqrt(1 - (x + 1 + sqrt(3))**2/(-x - 1 +
 sqrt(3))**2)*sqrt(sqrt(3) + 2)/(3*sqrt(4*sqrt(3) + 7 + (x + 1 +
sqrt(3))**2/(-x - 1 + sqrt(3))**2)))/(sqrt((-x - 1)/(x - sqrt(3)
+ 1)**2)*(-sqrt(3) + 3)*sqrt(-x**3 - 1)) + 4*3**(1/4)*sqrt((x**2
- x + 1)/(x - sqrt(3) + 1)**2)*sqrt(sqrt(3) + 2)*(e + 2*f)*(x + 1
)*elliptic_pi(-4*sqrt(3) + 7, asin((x + 1 + sqrt(3))/(-x - 1 + sq
rt(3))), -7 + 4*sqrt(3))/(sqrt((-x - 1)/(x - sqrt(3) + 1)**2)*(-s
qrt(3) + 3)*(sqrt(3) + 3)*sqrt(4*sqrt(3) + 7)*sqrt(-x**3 - 1)) +
2*3**(3/4)*sqrt((x**2 - x + 1)/(x - sqrt(3) + 1)**2)*sqrt(-sqrt(3
) + 2)*(x + 1)*(e - f + sqrt(3)*f)*elliptic_f(asin((x + 1 + sqrt(
3))/(x - sqrt(3) + 1)), -7 + 4*sqrt(3))/(3*sqrt((-x - 1)/(x - sqr
t(3) + 1)**2)*(-sqrt(3) + 3)*sqrt(-x**3 - 1))
```

Mathematica [C] time = 0.393274, size = 275, normalized size = 1.83

$$\frac{2\sqrt{\frac{2}{3}}\sqrt{-\frac{i(x+1)}{\sqrt{3}-3i}}\left(2\sqrt{3}\sqrt{2ix+\sqrt{3}-i}\sqrt{x^2-x+1}(e+2f)\left(\frac{2\sqrt{3}}{3i+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{2ix+\sqrt{3}-i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right)-3if\sqrt{-2ix+\sqrt{3}+i}\left(\left(\sqrt{3}-i\right)\right.\right.}{\left(\sqrt{3}+3i\right)\sqrt{2ix+\sqrt{3}-i}\sqrt{-x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(e + f*x)/((2 - x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*Sqrt[2/3]*Sqrt[((-I)*(1 + x))/(-3*I + Sqrt[3])]*((-3*I)*f*Sqrt
[I + Sqrt[3] - (2*I)*x]*(-I - Sqrt[3] + (-I + Sqrt[3])*x)*Ellipti
cF[ArcSin[Sqrt[-I + Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqr
t[3])/(-3*I + Sqrt[3])] + 2*Sqrt[3]*(e + 2*f)*Sqrt[-I + Sqrt[3] +
 (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]
), ArcSin[Sqrt[-I + Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqr
t[3])/(-3*I + Sqrt[3])]))/((3*I + Sqrt[3])*Sqrt[-I + Sqrt[3] + (2
*I)*x]*Sqrt[-1 - x^3])
```

Maple [A] time = 0.01, size = 246, normalized size = 1.6

$$\frac{2i}{3}f\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}EllipticF\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)\frac{1}{\sqrt{-x^3}}$$

$$+\frac{\frac{2i}{3}(e+2f)\sqrt{3}}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}EllipticPi\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x+e)/(2-x)/(-x^3-1)^(1/2),x)
```

```
[Out] 2/3*I*f*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3
/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)
/(-x^3-1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^
(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))+2/3*I*(e+2*f)
*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*
I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-
1)^(1/2)/(-3/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/
2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-3/2+1/2*I*3^(1/2)),(I*3^(
1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{fx+e}{\sqrt{-x^3-1}(x-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(f*x + e)/(sqrt(-x^3 - 1)*(x - 2)),x, algorithm="maxima")
```

```
[Out] -integrate((f*x + e)/(sqrt(-x^3 - 1)*(x - 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{fx+e}{\sqrt{-x^3-1}(x-2)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(f*x + e)/(sqrt(-x^3 - 1)*(x - 2)),x, algorithm="fricas")
```

```
[Out] integral(-(f*x + e)/(sqrt(-x^3 - 1)*(x - 2)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{e}{x\sqrt{-x^3-1}-2\sqrt{-x^3-1}}\,dx-\int\frac{fx}{x\sqrt{-x^3-1}-2\sqrt{-x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2-x)/(-x**3-1)**(1/2),x)
```

```
[Out] -Integral(e/(x*sqrt(-x**3 - 1) - 2*sqrt(-x**3 - 1)), x) - Integra
l(f*x/(x*sqrt(-x**3 - 1) - 2*sqrt(-x**3 - 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int-\frac{fx+e}{\sqrt{-x^3-1}(x-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(f*x + e)/(sqrt(-x^3 - 1)*(x - 2)),x, algorithm="giac")
```

```
[Out] integrate(-(f*x + e)/(sqrt(-x^3 - 1)*(x - 2)), x)
```

3.64 $\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=297

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2\left(2\sqrt[3]{a}f+\sqrt[3]{b}e\right)\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a+bx^3}}\right)}{9\sqrt{a}b^{2/3}}$$

```
[Out] (2*(b^(1/3)*e + 2*a^(1/3)*f)*ArcTanh[(a^(1/3) + b^(1/3)*x)^2/(3*a
^(1/6)*Sqrt[a + b*x^3])])/(9*Sqrt[a]*b^(2/3)) + (2*Sqrt[2 + Sqrt[
3]]*(b^(1/3)*e - a^(1/3)*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a^(
1/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.619602, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2\left(2\sqrt[3]{a}f+\sqrt[3]{b}e\right)\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a+bx^3}}\right)}{9\sqrt{a}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(e + f*x)/((2*a^(1/3) - b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*(b^(1/3)*e + 2*a^(1/3)*f)*ArcTanh[(a^(1/3) + b^(1/3)*x)^2/(3*a
^(1/6)*Sqrt[a + b*x^3])])/(9*Sqrt[a]*b^(2/3)) + (2*Sqrt[2 + Sqrt[
3]]*(b^(1/3)*e - a^(1/3)*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a^(
1/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi in Sympy [A] time = 152.325, size = 745, normalized size = 2.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((f*x+e)/(2*a**(1/3)-b**(1/3)*x)/(b*x**3+a)**(1/2),x)
```

```
[Out] 2*3**(3/4)*sqrt((a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/
(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(sqrt(3) + 2)*(a**(
1/3) + b**(1/3)*x)*(-a**(1/3)*f*(1 + sqrt(3)) + b**(1/3)*e)*ellip
tic_f(asin((-a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)/(a**(1/3)*(1 +
 sqrt(3)) + b**(1/3)*x)), -7 - 4*sqrt(3))/(3*a**(1/3)*b**(2/3)*sq
rt(a**(1/3)*(a**(1/3) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**
(1/3)*x)**2)*(sqrt(3) + 3)*sqrt(a + b*x**3)) + 3**(3/4)*sqrt(a**(
2/3)*(1 - b**(1/3)*x/a**(1/3) + b**(2/3)*x**2/a**(2/3))/(a**(1/3)
*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(3 + 2*sqrt(3))*sqrt(-sqrt(3
) + 2)*(a**(1/3) + b**(1/3)*x)*(2*a**(1/3)*f + b**(1/3)*e)*atanh(
sqrt(-(a**(1/3)*(-1 + sqrt(3)) - b**(1/3)*x)**2/(a**(1/3)*(1 + sq
rt(3)) + b**(1/3)*x)**2 + 1)/(sqrt(3 + 2*sqrt(3))*sqrt((a**(1/3)*
(-1 + sqrt(3)) - b**(1/3)*x)**2/(a**(1/3)*(1 + sqrt(3)) + b**(1/3
)*x)**2 - 4*sqrt(3) + 7)))/(9*a**(1/3)*b**(2/3)*sqrt(a**(1/3)*(a*
*(1/3) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sq
rt(a + b*x**3)) - 4*3**(1/4)*sqrt(a**(2/3)*(1 - b**(1/3)*x/a**(1/
3) + b**(2/3)*x**2/a**(2/3))/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x
)**2)*sqrt(-sqrt(3) + 2)*(a**(1/3) + b**(1/3)*x)*(2*a**(1/3)*f +
b**(1/3)*e)*elliptic_pi(4*sqrt(3) + 7, asin((a**(1/3)*(-1 + sqrt(
3)) - b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)), -7 - 4*
sqrt(3))/(a**(1/3)*b**(2/3)*sqrt(a**(1/3)*(a**(1/3) + b**(1/3)*x)
/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(-4*sqrt(3) + 7)*(
-sqrt(3) + 3)*(sqrt(3) + 3)*sqrt(a + b*x**3))
```

Mathematica [C] time = 2.40962, size = 419, normalized size = 1.41

$$\frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(i\sqrt{\frac{\left(\sqrt{3}+i\right)\sqrt[3]{b}x-2i\sqrt[3]{a}}{\left(\sqrt{3}-3i\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\left(2\sqrt[3]{a}f+\sqrt[3]{b}e\right)\left(\frac{2\sqrt{3}}{3i+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{\left(i+\sqrt{3}\right)\sqrt[3]{b}x-2i\sqrt[3]{a}}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)|\frac{1}{2}\left(1+i\sqrt{3}\right)\right)\right.}{\left(\sqrt[3]{-1}-2\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(e + f*x)/((2*a^(1/3) - b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(-(3^(1
/4)*f*((I + Sqrt[3])*a^(1/3) - (-I + Sqrt[3])*b^(1/3)*x)*Sqrt[I +
 Sqrt[3] - ((2*I)*b^(1/3)*x)/a^(1/3)]*EllipticF[ArcSin[Sqrt[((-2*
I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]
], (1 + I*Sqrt[3])/2])/(2*Sqrt[2]) + I*(b^(1/3)*e + 2*a^(1/3)*f)*
Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])
*a^(1/3))]*Sqrt[1 - (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*
EllipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[((-2*I)*a^(1/
3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]], (1 +
I*Sqrt[3])/2]))/((-2 + (-1)^(1/3))*b^(2/3)*Sqrt[(a^(1/3) + (-1)^(
2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3])
```

Maple [F] time = 0.103, size = 0, normalized size = 0.

$$\int (fx+e)\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)^{-1}\frac{1}{\sqrt{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x+e)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

```
[Out] int((f*x+e)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{fx+e}{\sqrt{bx^3+a}\left(b^{\frac{1}{3}}x-2\,a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(f*x + e)/(sqrt(b*x^3 + a)*(b^(1/3)*x - 2*a^(1/3))),x, algorithm="max
```

```
[Out] -integrate((f*x + e)/(sqrt(b*x^3 + a)*(b^(1/3)*x - 2*a^(1/3))), x
)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(f*x + e)/(sqrt(b*x^3 + a)*(b^(1/3)*x - 2*a^(1/3))),x, algorithm="fri
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{e}{-2\sqrt[3]{a}\sqrt{a+bx^3}+\sqrt[3]{b}x\sqrt{a+bx^3}}\,dx-\int \frac{fx}{-2\sqrt[3]{a}\sqrt{a+bx^3}+\sqrt[3]{b}x\sqrt{a+bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x+e)/(2*a**(1/3)-b**(1/3)*x)/(b*x**3+a)**(1/2),x)
```

```
[Out] -Integral(e/(-2*a**(1/3)*sqrt(a + b*x**3) + b**(1/3)*x*sqrt(a + b
*x**3)), x) - Integral(f*x/(-2*a**(1/3)*sqrt(a + b*x**3) + b**(1/
3)*x*sqrt(a + b*x**3)), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(f*x + e)/(sqrt(b*x^3 + a)*(b^(1/3)*x - 2*a^(1/3))),x, algorithm="gia
```

```
[Out] Exception raised: TypeError
```

3.65 $\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$

Optimal. Leaf size=304

$$-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{a}f+\sqrt[3]{b}e\right)F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{2\left(\sqrt[3]{b}e-2\sqrt[3]{a}f\right)\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a-bx^3}}\right)}{9\sqrt{a}b^{2/3}}$$

```
[Out] (-2*(b^(1/3)*e - 2*a^(1/3)*f)*ArcTanh[(a^(1/3) - b^(1/3)*x)^2/(3*
a^(1/6)*Sqrt[a - b*x^3])])/(9*Sqrt[a]*b^(2/3)) - (2*Sqrt[2 + Sqrt
[3]]*(b^(1/3)*e + a^(1/3)*f)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3)
+ a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/
3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1
 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a^
(1/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])
*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

Rubi [A] time = 0.609196, antiderivative size = 304, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$

$$-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{a}f+\sqrt[3]{b}e\right)F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{2\left(\sqrt[3]{b}e-2\sqrt[3]{a}f\right)\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a-bx^3}}\right)}{9\sqrt{a}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(e + f*x)/((2*a^(1/3) + b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (-2*(b^(1/3)*e - 2*a^(1/3)*f)*ArcTanh[(a^(1/3) - b^(1/3)*x)^2/(3*
a^(1/6)*Sqrt[a - b*x^3])])/(9*Sqrt[a]*b^(2/3)) - (2*Sqrt[2 + Sqrt
[3]]*(b^(1/3)*e + a^(1/3)*f)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3)
+ a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/
3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1
 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a^
(1/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])
*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

Rubi in Sympy [A] time = 176.072, size = 745, normalized size = 2.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((f*x+e)/(2*a**(1/3)+b**(1/3)*x)/(-b*x**3+a)**(1/2),x)
```

```
[Out] 2*3**(3/4)*sqrt((a**(2/3) + a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/
(a**(1/3)*(1 + sqrt(3)) - b**(1/3)*x)**2)*sqrt(sqrt(3) + 2)*(a**(
1/3) - b**(1/3)*x)*(a**(1/3)*f*(1 + sqrt(3)) + b**(1/3)*e)*ellipt
ic_f(asin((a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)/(a**(1/3)*(1 + s
qrt(3)) - b**(1/3)*x)), -7 - 4*sqrt(3))/(3*a**(1/3)*b**(2/3)*sqrt
(a**(1/3)*(a**(1/3) - b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) - b**(1
/3)*x)**2)*(sqrt(3) + 3)*sqrt(a - b*x**3)) + 3**(3/4)*sqrt(a**(2/
3)*(1 + b**(1/3)*x/a**(1/3) + b**(2/3)*x**2/a**(2/3))/(a**(1/3)*(
1 + sqrt(3)) - b**(1/3)*x)**2)*sqrt(3 + 2*sqrt(3))*sqrt(-sqrt(3)
+ 2)*(a**(1/3) - b**(1/3)*x)*(2*a**(1/3)*f - b**(1/3)*e)*atanh(sq
rt(-(a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)**2/(a**(1/3)*(1 + sqrt
(3)) - b**(1/3)*x)**2 + 1)/(sqrt(3 + 2*sqrt(3))*sqrt((a**(1/3)*(-
1 + sqrt(3)) + b**(1/3)*x)**2/(a**(1/3)*(1 + sqrt(3)) - b**(1/3)*
x)**2 - 4*sqrt(3) + 7)))/(9*a**(1/3)*b**(2/3)*sqrt(a**(1/3)*(a**(
1/3) - b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) - b**(1/3)*x)**2)*sqrt
(a - b*x**3)) - 4*3**(1/4)*sqrt(a**(2/3)*(1 + b**(1/3)*x/a**(1/3)
 + b**(2/3)*x**2/a**(2/3))/(a**(1/3)*(1 + sqrt(3)) - b**(1/3)*x)*
*2)*sqrt(-sqrt(3) + 2)*(a**(1/3) - b**(1/3)*x)*(2*a**(1/3)*f - b*
*(1/3)*e)*elliptic_pi(4*sqrt(3) + 7, asin((a**(1/3)*(-1 + sqrt(3)
) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) - b**(1/3)*x)), -7 - 4*sq
rt(3))/(a**(1/3)*b**(2/3)*sqrt(a**(1/3)*(a**(1/3) - b**(1/3)*x)/(
a**(1/3)*(1 + sqrt(3)) - b**(1/3)*x)**2)*sqrt(-4*sqrt(3) + 7)*(-s
qrt(3) + 3)*(sqrt(3) + 3)*sqrt(a - b*x**3))
```

Mathematica [C] time = 2.462, size = 447, normalized size = 1.47

$$\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(-i\sqrt{-\frac{i\left(2\sqrt[3]{a}+\left(1-i\sqrt{3}\right)\sqrt[3]{b}x\right)}{\left(\sqrt{3}-3i\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\left(\sqrt[3]{b}e-2\sqrt[3]{a}f\right)\left(\frac{2\sqrt{3}}{3i+\sqrt{3}};\sin^{-1}\left(\sqrt{-\frac{i\left(\left(1-i\sqrt{3}\right)\sqrt[3]{b}x+2\sqrt[3]{a}\right)}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)|\frac{1}{2}\left(\right.\right.\right.}{\left(\sqrt[3]{-1}-2\right)b^{2/3}\sqrt{\cdots}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(e + f*x)/((2*a^(1/3) + b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((-I/2)
*f*Sqrt[((-I + Sqrt[3])*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I
 + Sqrt[3])*a^(1/3))]*((-3*I + Sqrt[3])*a^(1/3) - (3*I + Sqrt[3])
*b^(1/3)*x)*EllipticF[ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[
3])*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]
- I*(b^(1/3)*e - 2*a^(1/3)*f)*Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt
[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1/3))]*Sqrt[1 + (b^(1/3)*x)
/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(3*I + S
qrt[3]), ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x
))/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/((-2 + (-1)
^(1/3))*b^(2/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^
(1/3))*a^(1/3))]*Sqrt[a - b*x^3])
```

Maple [F] time = 0.101, size = 0, normalized size = 0.

$$\int (fx+e)\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)^{-1}\frac{1}{\sqrt{-bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x+e)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

```
[Out] int((f*x+e)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\sqrt{-bx^3+a}\left(b^{\frac{1}{3}}x+2\,a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x + e)/(sqrt(-b*x^3 + a)*(b^(1/3)*x + 2*a^(1/3))),x, algorithm="max
```

```
[Out] integrate((f*x + e)/(sqrt(-b*x^3 + a)*(b^(1/3)*x + 2*a^(1/3))), x
)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x + e)/(sqrt(-b*x^3 + a)*(b^(1/3)*x + 2*a^(1/3))),x, algorithm="fri
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e + fx}{\left(2\sqrt[3]{a} + \sqrt[3]{b}x\right)\sqrt{a - bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2*a**(1/3)+b**(1/3)*x)/(-b*x**3+a)**(1/2),x)
```

```
[Out] Integral((e + f*x)/((2*a**(1/3) + b**(1/3)*x)*sqrt(a - b*x**3)),
x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x + e)/(sqrt(-b*x^3 + a)*(b^(1/3)*x + 2*a^(1/3))),x, algorithm="gia
```

```
[Out] Exception raised: TypeError
```

3.66 $\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

Optimal. Leaf size=313

$$-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{a}f+\sqrt[3]{b}e\right)F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}-\frac{2\left(\sqrt[3]{b}e-2\sqrt[3]{a}f\right)\tan^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{bx^3-a}}\right)}{9\sqrt{a}b^{2/3}}$$

```
[Out] (-2*(b^(1/3)*e - 2*a^(1/3)*f)*ArcTan[(a^(1/3) - b^(1/3)*x)^2/(3*a
^(1/6)*Sqrt[-a + b*x^3])])/(9*Sqrt[a]*b^(2/3)) - (2*Sqrt[2 - Sqrt
[3]]*(b^(1/3)*e + a^(1/3)*f)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3)
+ a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/
3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1
 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*a^
(1/3)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3
])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

Rubi [A] time = 0.60734, antiderivative size = 313, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{a}f+\sqrt[3]{b}e\right)F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}-\frac{2\left(\sqrt[3]{b}e-2\sqrt[3]{a}f\right)\tan^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{bx^3-a}}\right)}{9\sqrt{a}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(e + f*x)/((2*a^(1/3) + b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (-2*(b^(1/3)*e - 2*a^(1/3)*f)*ArcTan[(a^(1/3) - b^(1/3)*x)^2/(3*a
^(1/6)*Sqrt[-a + b*x^3])])/(9*Sqrt[a]*b^(2/3)) - (2*Sqrt[2 - Sqrt
[3]]*(b^(1/3)*e + a^(1/3)*f)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3)
+ a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/
3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1
 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*a^
(1/3)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3
])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((f*x+e)/(2*a**(1/3)+b**(1/3)*x)/(b*x**3-a)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 2.44402, size = 448, normalized size = 1.43

$$\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(-i\sqrt{-\frac{i\left(2\sqrt[3]{a}+\left(1-i\sqrt{3}\right)\sqrt[3]{b}x\right)}{\left(\sqrt{3}-3i\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\left(\sqrt[3]{b}e-2\sqrt[3]{a}f\right)\left(\frac{2\sqrt{3}}{3i+\sqrt{3}};\sin^{-1}\left(\sqrt{-\frac{i\left(\left(1-i\sqrt{3}\right)\sqrt[3]{b}x+2\sqrt[3]{a}\right)}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)|\frac{1}{2}\left(\cdots\right.\right.\right.}{\left(\sqrt[3]{-1}-2\right)b^{2/3}\sqrt{\frac{\sqrt[3]{\cdots}}{\left(\cdots\right)}}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(e + f*x)/((2*a^(1/3) + b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((-I/2)
*f*Sqrt[((-I + Sqrt[3])*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I
 + Sqrt[3])*a^(1/3))]*((-3*I + Sqrt[3])*a^(1/3) - (3*I + Sqrt[3])
*b^(1/3)*x)*EllipticF[ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[
3])*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]
- I*(b^(1/3)*e - 2*a^(1/3)*f)*Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt
[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1/3))]*Sqrt[1 + (b^(1/3)*x)
/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(3*I + S
qrt[3]), ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x
))/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/((-2 + (-1)
^(1/3))*b^(2/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^
```

```
(1/3))*a^(1/3))]*Sqrt[-a + b*x^3])
```

Maple [F] time = 0.102, size = 0, normalized size = 0.

$$\int (fx+e)\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)^{-1}\frac{1}{\sqrt{bx^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x+e)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x)

[Out] int((f*x+e)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\sqrt{bx^3-a}\left(b^{\frac{1}{3}}x+2\,a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x + e)/(sqrt(b*x^3 - a)*(b^(1/3)*x + 2*a^(1/3))),x, algorithm="maxi

[Out] integrate((f*x + e)/(sqrt(b*x^3 - a)*(b^(1/3)*x + 2*a^(1/3))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x + e)/(sqrt(b*x^3 - a)*(b^(1/3)*x + 2*a^(1/3))),x, algorithm="fric

[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x+e)/(2*a**(1/3)+b**(1/3)*x)/(b*x**3-a)**(1/2),x)
```

```
[Out] Integral((e + f*x)/((2*a**(1/3) + b**(1/3)*x)*sqrt(-a + b*x**3)),
 x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x + e)/(sqrt(b*x^3 - a)*(b^(1/3)*x + 2*a^(1/3))),x, algorithm="giac
```

```
[Out] Exception raised: TypeError
```

3.67 $\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

Optimal. Leaf size=310

$$\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}+\frac{2\left(2\sqrt[3]{a}f+\sqrt[3]{b}e\right)\tan^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{-a-bx^3}}\right)}{9\sqrt{a}b^{2/3}}$$

```
[Out] (2*(b^(1/3)*e + 2*a^(1/3)*f)*ArcTan[(a^(1/3) + b^(1/3)*x)^2/(3*a^
(1/6)*Sqrt[-a - b*x^3])])/(9*Sqrt[a]*b^(2/3)) + (2*Sqrt[2 - Sqrt[
3]]*(b^(1/3)*e - a^(1/3)*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3
)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1
- Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*a^(
1/3)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3]
)*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

Rubi [A] time = 0.646939, antiderivative size = 310, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}+\frac{2\left(2\sqrt[3]{a}f+\sqrt[3]{b}e\right)\tan^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{-a-bx^3}}\right)}{9\sqrt{a}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(e + f*x)/((2*a^(1/3) - b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (2*(b^(1/3)*e + 2*a^(1/3)*f)*ArcTan[(a^(1/3) + b^(1/3)*x)^2/(3*a^
(1/6)*Sqrt[-a - b*x^3])])/(9*Sqrt[a]*b^(2/3)) + (2*Sqrt[2 - Sqrt[
3]]*(b^(1/3)*e - a^(1/3)*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3
)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1
- Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*a^(
1/3)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3]
)*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((f*x+e)/(2*a**(1/3)-b**(1/3)*x)/(-b*x**3-a)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 2.38011, size = 422, normalized size = 1.36

$$\frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(i\sqrt{\frac{\left(\sqrt{3}+i\right)\sqrt[3]{b}x-2i\sqrt[3]{a}}{\left(\sqrt{3}-3i\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\left(2\sqrt[3]{a}f+\sqrt[3]{b}e\right)\left(\frac{2\sqrt{3}}{3i+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{\left(i+\sqrt{3}\right)\sqrt[3]{b}x-2i\sqrt[3]{a}}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)|\frac{1}{2}\left(1+i\sqrt{3}\right)\right)\right.}{\left(\sqrt[3]{-1}-2\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{-a-bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(e + f*x)/((2*a^(1/3) - b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(-(3^(1
/4)*f*((I + Sqrt[3])*a^(1/3) - (-I + Sqrt[3])*b^(1/3)*x)*Sqrt[I +
 Sqrt[3] - ((2*I)*b^(1/3)*x)/a^(1/3)]*EllipticF[ArcSin[Sqrt[((-2*
I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]
], (1 + I*Sqrt[3])/2])/(2*Sqrt[2]) + I*(b^(1/3)*e + 2*a^(1/3)*f)*
Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])
*a^(1/3))]*Sqrt[1 - (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*
EllipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[((-2*I)*a^(1/
3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]], (1 +
```

```
I*Sqrt[3])/2]))/((-2 + (-1)^(1/3))*b^(2/3)*Sqrt[(a^(1/3) + (-1)^(
2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[-a - b*x^3])
```

Maple [F] time = 0.094, size = 0, normalized size = 0.

$$\int (fx+e)\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)^{-1}\frac{1}{\sqrt{-bx^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x+e)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x)

[Out] int((f*x+e)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{fx+e}{\sqrt{-bx^3-a}\left(b^{\frac{1}{3}}x-2\,a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(f*x + e)/(sqrt(-b*x^3 - a)*(b^(1/3)*x - 2*a^(1/3))),x, algorithm="ma

[Out] -integrate((f*x + e)/(sqrt(-b*x^3 - a)*(b^(1/3)*x - 2*a^(1/3))),
x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(f*x + e)/(sqrt(-b*x^3 - a)*(b^(1/3)*x - 2*a^(1/3))),x, algorithm="fr

[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{e}{-2\sqrt[3]{a}\sqrt{-a-bx^3}+\sqrt[3]{b}x\sqrt{-a-bx^3}}\,dx-\int \frac{fx}{-2\sqrt[3]{a}\sqrt{-a-bx^3}+\sqrt[3]{b}x\sqrt{-a-bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x+e)/(2*a**(1/3)-b**(1/3)*x)/(-b*x**3-a)**(1/2),x)
```

```
[Out] -Integral(e/(-2*a**(1/3)*sqrt(-a - b*x**3) + b**(1/3)*x*sqrt(-a -
 b*x**3)), x) - Integral(f*x/(-2*a**(1/3)*sqrt(-a - b*x**3) + b**
(1/3)*x*sqrt(-a - b*x**3)), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(f*x + e)/(sqrt(-b*x^3 - a)*(b^(1/3)*x - 2*a^(1/3))),x, algorithm="gi
```

```
[Out] Exception raised: TypeError
```

3.68 $\int \frac{e+fx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx$

Optimal. Leaf size=221

$$-\frac{2(de-cf)\tanh^{-1}\left(\frac{(c-2dx)^2}{3\sqrt{c}\sqrt{c^3-8d^3x^3}}\right)}{9c^{3/2}d^2}-\frac{\sqrt{2+\sqrt{3}}(c-2dx)\sqrt{\frac{c^2+2cdx+4d^2x^2}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}(cf+2de)F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)c-2dx}{\left(1+\sqrt{3}\right)c-2dx}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}cd^2\sqrt{\frac{c(c-2dx)}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\sqrt{c^3-8d^3x^3}}$$

```
[Out] (-2*(d*e - c*f)*ArcTanh[(c - 2*d*x)^2/(3*Sqrt[c]*Sqrt[c^3 - 8*d^3
*x^3])])/(9*c^(3/2)*d^2) - (Sqrt[2 + Sqrt[3]]*(2*d*e + c*f)*(c -
2*d*x)*Sqrt[(c^2 + 2*c*d*x + 4*d^2*x^2)/((1 + Sqrt[3])*c - 2*d*x)
^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c - 2*d*x)/((1 + Sqrt[3])*c -
 2*d*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*c*d^2*Sqrt[(c*(c - 2*d*x))/
((1 + Sqrt[3])*c - 2*d*x)^2]*Sqrt[c^3 - 8*d^3*x^3])
```

Rubi [A] time = 0.550436, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138

$$-\frac{2(de-cf)\tanh^{-1}\left(\frac{(c-2dx)^2}{3\sqrt{c}\sqrt{c^3-8d^3x^3}}\right)}{9c^{3/2}d^2}-\frac{\sqrt{2+\sqrt{3}}(c-2dx)\sqrt{\frac{c^2+2cdx+4d^2x^2}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}(cf+2de)F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)c-2dx}{\left(1+\sqrt{3}\right)c-2dx}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}cd^2\sqrt{\frac{c(c-2dx)}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\sqrt{c^3-8d^3x^3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(e + f*x)/((c + d*x)*Sqrt[c^3 - 8*d^3*x^3]),x]
```


```
[Out] (-2*(d*e - c*f)*ArcTanh[(c - 2*d*x)^2/(3*Sqrt[c]*Sqrt[c^3 - 8*d^3
*x^3])])/(9*c^(3/2)*d^2) - (Sqrt[2 + Sqrt[3]]*(2*d*e + c*f)*(c -
2*d*x)*Sqrt[(c^2 + 2*c*d*x + 4*d^2*x^2)/((1 + Sqrt[3])*c - 2*d*x)
^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c - 2*d*x)/((1 + Sqrt[3])*c -
 2*d*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*c*d^2*Sqrt[(c*(c - 2*d*x))/
((1 + Sqrt[3])*c - 2*d*x)^2]*Sqrt[c^3 - 8*d^3*x^3])
```

Rubi in Sympy [A] time = 154.518, size = 588, normalized size = 2.66

$$\frac{3^{\frac{3}{4}}\sqrt{\frac{c^2+2cdx+4d^2x^2}{\left(c\left(1+\sqrt{3}\right)-2dx\right)^2}}\sqrt{\sqrt{3}+2}\,(c-2dx)\left(cf\left(1+\sqrt{3}\right)+2de\right)F\left(\operatorname{asin}\left(-\frac{-c\left(-1+\sqrt{3}\right)-2dx}{c\left(1+\sqrt{3}\right)-2dx}\right)\middle|-7-4\sqrt{3}\right)}{3cd^2\sqrt{\frac{c(c-2dx)}{\left(c\left(1+\sqrt{3}\right)-2dx\right)^2}}\left(\sqrt{3}+3\right)\sqrt{c^3-8d^3x^3}}+\frac{3^{\frac{3}{4}}\sqrt{\frac{c^2\left(1+\frac{2dx}{c}+\frac{4d^2x^2}{c^2}\right)}{\left(c\left(1+\sqrt{3}\right)-2dx\right)^2}}\sqrt{3+2\sqrt{3}}\sqrt{-\sqrt{3}+2}\,(c-2dx)(cf-de)\operatorname{atanh}\left(\frac{\sqrt{-\frac{\left(c\left(-1+\sqrt{3}\right)+2dx\right)^2}{\left(c\left(1+\sqrt{3}\right)-2dx\right)^2}+1}}{\sqrt{3+2\sqrt{3}}\sqrt{\frac{\left(c\left(-1+\sqrt{3}\right)+2dx\right)^2}{\left(c\left(1+\sqrt{3}\right)-2dx\right)^2}-4\sqrt{3}+7}}\right)}{9cd^2\sqrt{\frac{c(c-2dx)}{\left(c\left(1+\sqrt{3}\right)-2dx\right)^2}}\sqrt{c^3-8d^3x^3}}-\frac{4\sqrt[4]{3}\sqrt{\frac{c^2\left(1+\frac{2dx}{c}+\frac{4d^2x^2}{c^2}\right)}{\left(c\left(1+\sqrt{3}\right)-2dx\right)^2}}\sqrt{-\sqrt{3}+2}\,(c-2dx)(cf-de)\left(4\sqrt{3}+7;\operatorname{asin}\left(\frac{c\left(-1+\sqrt{3}\right)+2dx}{c\left(1+\sqrt{3}\right)-2dx}\right)\middle|-7-4\sqrt{3}\right)}{cd^2\sqrt{\frac{c(c-2dx)}{\left(c\left(1+\sqrt{3}\right)-2dx\right)^2}}\sqrt{-4\sqrt{3}+7}\left(-\sqrt{3}+3\right)\left(\sqrt{3}+3\right)\sqrt{c^3-8d^3x^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x+e)/(d*x+c)/(-8*d**3*x**3+c**3)**(1/2),x)
```

```
[Out] 3**(3/4)*sqrt((c**2 + 2*c*d*x + 4*d**2*x**2)/(c*(1 + sqrt(3)) - 2
*d*x)**2)*sqrt(sqrt(3) + 2)*(c - 2*d*x)*(c*f*(1 + sqrt(3)) + 2*d*
e)*elliptic_f(asin(-(-c*(-1 + sqrt(3)) - 2*d*x)/(c*(1 + sqrt(3))
- 2*d*x)), -7 - 4*sqrt(3))/(3*c*d**2*sqrt(c*(c - 2*d*x)/(c*(1 + s
qrt(3)) - 2*d*x)**2)*(sqrt(3) + 3)*sqrt(c**3 - 8*d**3*x**3)) + 3*
*(3/4)*sqrt(c**2*(1 + 2*d*x/c + 4*d**2*x**2/c**2)/(c*(1 + sqrt(3)
) - 2*d*x)**2)*sqrt(3 + 2*sqrt(3))*sqrt(-sqrt(3) + 2)*(c - 2*d*x)
*(c*f - d*e)*atanh(sqrt(-(c*(-1 + sqrt(3)) + 2*d*x)**2/(c*(1 + sq
rt(3)) - 2*d*x)**2 + 1)/(sqrt(3 + 2*sqrt(3))*sqrt((c*(-1 + sqrt(3
)) + 2*d*x)**2/(c*(1 + sqrt(3)) - 2*d*x)**2 - 4*sqrt(3) + 7)))/(9
*c*d**2*sqrt(c*(c - 2*d*x)/(c*(1 + sqrt(3)) - 2*d*x)**2)*sqrt(c**
3 - 8*d**3*x**3)) - 4*3**(1/4)*sqrt(c**2*(1 + 2*d*x/c + 4*d**2*x*
*2/c**2)/(c*(1 + sqrt(3)) - 2*d*x)**2)*sqrt(-sqrt(3) + 2)*(c - 2*
d*x)*(c*f - d*e)*elliptic_pi(4*sqrt(3) + 7, asin((c*(-1 + sqrt(3)
) + 2*d*x)/(c*(1 + sqrt(3)) - 2*d*x)), -7 - 4*sqrt(3))/(c*d**2*sq
rt(c*(c - 2*d*x)/(c*(1 + sqrt(3)) - 2*d*x)**2)*sqrt(-4*sqrt(3) +
7)*(-sqrt(3) + 3)*(sqrt(3) + 3)*sqrt(c**3 - 8*d**3*x**3))
```

Mathematica [C] time = 2.00568, size = 384, normalized size = 1.74

$$-\frac{i\sqrt{\frac{c-2dx}{\left(1+\sqrt[3]{-1}\right)c}}\left(4\sqrt{2}\sqrt{\frac{ic+\sqrt{3}dx+idx}{-\sqrt{3}c+3ic}}\sqrt{\frac{c^2+2cdx+4d^2x^2}{c^2}}(de-cf)\left(\frac{2\sqrt{3}}{3i+\sqrt{3}};\sin^{-1}\left(\sqrt{2}\sqrt{\frac{ic+\sqrt{3}dx+idx}{3ic-\sqrt{3}c}}\right)|\frac{1}{2}\left(1+i\sqrt{3}\right)\right)+f\sqrt{\frac{\left(\sqrt{3}-i\right)c+2}{\left(\sqrt{3}-\right.}}\right.}{2\left(\sqrt[3]{-1}-2\right)d^2\sqrt{\frac{c-2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}\sqrt{c^3-8d}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(e + f*x)/((c + d*x)*Sqrt[c^3 - 8*d^3*x^3]),x]
```

```
[Out] ((-I/2)*Sqrt[(c - 2*d*x)/((1 + (-1)^(1/3))*c)]*(f*Sqrt[((-I + Sqr
t[3])*c + 2*(I + Sqrt[3])*d*x)/((-3*I + Sqrt[3])*c)]*((-3*I + Sqr
t[3])*c - 2*(3*I + Sqrt[3])*d*x)*EllipticF[ArcSin[Sqrt[2]*Sqrt[(I
*c + I*d*x + Sqrt[3]*d*x)/((3*I)*c - Sqrt[3]*c)]], (1 + I*Sqrt[3]
)/2] + 4*Sqrt[2]*(d*e - c*f)*Sqrt[(I*c + I*d*x + Sqrt[3]*d*x)/((3
*I)*c - Sqrt[3]*c)]*Sqrt[(c^2 + 2*c*d*x + 4*d^2*x^2)/c^2]*Ellipti
cPi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[2]*Sqrt[(I*c + I*d*x
 + Sqrt[3]*d*x)/((3*I)*c - Sqrt[3]*c)]], (1 + I*Sqrt[3])/2]))/((-
2 + (-1)^(1/3))*d^2*Sqrt[(c - 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))
*c)]*Sqrt[c^3 - 8*d^3*x^3])
```

Maple [B] time = 0.012, size = 661, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x+e)/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x)
```

```
[Out] 2/d*f*(1/2*(-1/2+1/2*I*3^(1/2))*c/d-1/2*(-1/2-1/2*I*3^(1/2))*c/d)
*((x-1/2*(-1/2-1/2*I*3^(1/2))*c/d)/(1/2*(-1/2+1/2*I*3^(1/2))*c/d-
1/2*(-1/2-1/2*I*3^(1/2))*c/d))^(1/2)*((x-1/2*c/d)/(1/2*(-1/2-1/2*
I*3^(1/2))*c/d-1/2*c/d))^(1/2)*((x-1/2*(-1/2+1/2*I*3^(1/2))*c/d)/
(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*(-1/2+1/2*I*3^(1/2))*c/d))^(1/2
)/(-8*d^3*x^3+c^3)^(1/2)*EllipticF(((x-1/2*(-1/2-1/2*I*3^(1/2))*c
/d)/(1/2*(-1/2+1/2*I*3^(1/2))*c/d-1/2*(-1/2-1/2*I*3^(1/2))*c/d))^
(1/2),((1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*(-1/2+1/2*I*3^(1/2))*c/d
)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d))^(1/2))+2*(-c*f+d*e)/d^2
*(1/2*(-1/2+1/2*I*3^(1/2))*c/d-1/2*(-1/2-1/2*I*3^(1/2))*c/d)*((x-
1/2*(-1/2-1/2*I*3^(1/2))*c/d)/(1/2*(-1/2+1/2*I*3^(1/2))*c/d-1/2*(
-1/2-1/2*I*3^(1/2))*c/d))^(1/2)*((x-1/2*c/d)/(1/2*(-1/2-1/2*I*3^(
1/2))*c/d-1/2*c/d))^(1/2)*((x-1/2*(-1/2+1/2*I*3^(1/2))*c/d)/(1/2*
(-1/2-1/2*I*3^(1/2))*c/d-1/2*(-1/2+1/2*I*3^(1/2))*c/d))^(1/2)/(-8
*d^3*x^3+c^3)^(1/2)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d+c/d)*EllipticPi
```

```
(((x-1/2*(-1/2-1/2*I*3^(1/2))*c/d)/(1/2*(-1/2+1/2*I*3^(1/2))*c/d-
1/2*(-1/2-1/2*I*3^(1/2))*c/d))^(1/2),(1/2*(-1/2-1/2*I*3^(1/2))*c/
d-1/2*(-1/2+1/2*I*3^(1/2))*c/d)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d+c/d
),((1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*(-1/2+1/2*I*3^(1/2))*c/d)/(1
/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\sqrt{-8\,d^3x^3+c^3}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x + e)/(sqrt(-8*d^3*x^3 + c^3)*(d*x + c)),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(-8*d^3*x^3 + c^3)*(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{fx+e}{\sqrt{-8\,d^3x^3+c^3}(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x + e)/(sqrt(-8*d^3*x^3 + c^3)*(d*x + c)),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)/(sqrt(-8*d^3*x^3 + c^3)*(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e+fx}{\sqrt{-(-c+2dx)(c^2+2cdx+4d^2x^2)}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x+e)/(d*x+c)/(-8*d**3*x**3+c**3)**(1/2),x)
```

```
[Out] Integral((e + f*x)/(sqrt(-(-c + 2*d*x)*(c**2 + 2*c*d*x + 4*d**2*x
**2))*(c + d*x)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\sqrt{-8\,d^3x^3+c^3}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x + e)/(sqrt(-8*d^3*x^3 + c^3)*(d*x + c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)/(sqrt(-8*d^3*x^3 + c^3)*(d*x + c)), x)
```

3.69 $\int \frac{x}{(2-x)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=129

$$\frac{4}{9}\tanh^{-1}\left(\frac{(x+1)^2}{3\sqrt{x^3+1}}\right)-\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

```
[Out] (4*ArcTanh[(1 + x)^2/(3*Sqrt[1 + x^3])])/9 - (2*Sqrt[2 + Sqrt[3]]
*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin
[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3*3^(1/4
)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rubi [A] time = 0.249085, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\frac{4}{9}\tanh^{-1}\left(\frac{(x+1)^2}{3\sqrt{x^3+1}}\right)-\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/((2 - x)*Sqrt[1 + x^3]),x]
```

```
[Out] (4*ArcTanh[(1 + x)^2/(3*Sqrt[1 + x^3])])/9 - (2*Sqrt[2 + Sqrt[3]]
*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin
[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3*3^(1/4
)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rubi in Sympy [A] time = 93.5344, size = 379, normalized size = 2.94

$$\frac{2\sqrt{\frac{x^2-x+1}{\left(x+1+\sqrt{3}\right)^2}}\left(\frac{\sqrt{3}}{3}+1\right)(x+1)\operatorname{atanh}\left(\frac{3^{\frac{3}{4}}\sqrt{-\sqrt{3}+2}\sqrt{-\frac{\left(-x-1+\sqrt{3}\right)^2}{\left(x+1+\sqrt{3}\right)^2}+1}}{3\sqrt{\frac{\left(-x-1+\sqrt{3}\right)^2}{\left(x+1+\sqrt{3}\right)^2}-4\sqrt{3}+7}}\right)}{\sqrt{\frac{x+1}{\left(x+1+\sqrt{3}\right)^2}}\left(\sqrt{3}+3\right)\sqrt{x^3+1}} - \frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{x^2-x+1}{\left(x+1+\sqrt{3}\right)^2}}\left(1+\sqrt{3}\right)\sqrt{\sqrt{3}+2}\,(x+1)\,F\left(\operatorname{asin}\left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{3\sqrt{\frac{x+1}{\left(x+1+\sqrt{3}\right)^2}}\left(\sqrt{3}+3\right)\sqrt{x^3+1}} - \frac{8\sqrt[4]{3}\sqrt{\frac{x^2-x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{-\sqrt{3}+2}\,(x+1)\left(4\sqrt{3}+7;\operatorname{asin}\left(\frac{-x-1+\sqrt{3}}{x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{-4\sqrt{3}+7}\left(-\sqrt{3}+3\right)\left(\sqrt{3}+3\right)\sqrt{x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(2-x)/(x**3+1)**(1/2),x)
```

```
[Out] 2*sqrt((x**2 - x + 1)/(x + 1 + sqrt(3))**2)*(sqrt(3)/3 + 1)*(x +
1)*atanh(3**(3/4)*sqrt(-sqrt(3) + 2)*sqrt(-(-x - 1 + sqrt(3))**2/
(x + 1 + sqrt(3))**2 + 1)/(3*sqrt((-x - 1 + sqrt(3))**2/(x + 1 +
sqrt(3))**2 - 4*sqrt(3) + 7)))/(sqrt((x + 1)/(x + 1 + sqrt(3))**2
)*(sqrt(3) + 3)*sqrt(x**3 + 1)) - 2*3**(3/4)*sqrt((x**2 - x + 1)/
(x + 1 + sqrt(3))**2)*(1 + sqrt(3))*sqrt(sqrt(3) + 2)*(x + 1)*ell
iptic_f(asin((x - sqrt(3) + 1)/(x + 1 + sqrt(3))), -7 - 4*sqrt(3)
)/(3*sqrt((x + 1)/(x + 1 + sqrt(3))**2)*(sqrt(3) + 3)*sqrt(x**3 +
 1)) - 8*3**(1/4)*sqrt((x**2 - x + 1)/(x + 1 + sqrt(3))**2)*sqrt(
-sqrt(3) + 2)*(x + 1)*elliptic_pi(4*sqrt(3) + 7, asin((-x - 1 + s
qrt(3))/(x + 1 + sqrt(3))), -7 - 4*sqrt(3))/(sqrt((x + 1)/(x + 1
+ sqrt(3))**2)*sqrt(-4*sqrt(3) + 7)*(-sqrt(3) + 3)*(sqrt(3) + 3)*
sqrt(x**3 + 1))
```

Mathematica [C] time = 0.43332, size = 193, normalized size = 1.5

$$\frac{2\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\left(\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}}+\frac{2i\sqrt{x^2-x+1}\left(\frac{2\sqrt{3}}{3i+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt[3]{-1}-2}\right)}{\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x/((2 - x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*((((-1)^(1/3) - x)*Sqrt[((-1)^(
1/3) - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 +
 (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2
/3)*x)/(1 + (-1)^(1/3))] + ((2*I)*Sqrt[1 - x + x^2]*EllipticPi[(2
*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-
1)^(1/3))]], (-1)^(1/3)])/(-2 + (-1)^(1/3))))/Sqrt[1 + x^3]
```

Maple [B]   time = 0.01, size = 240, normalized size = 1.9

$$-2\,\frac{3/2-i/2\sqrt{3}}{\sqrt{x^3+1}}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}\mathit{EllipticF}\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},\sqrt{\frac{-3/2+i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\right)$$

$$+\frac{6-2\,i\sqrt{3}}{3}\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i}{2}\sqrt{3}}}\sqrt{\frac{1}{-\frac{3}{2}-\frac{i}{2}\sqrt{3}}\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)}\sqrt{\frac{1}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)}\mathit{EllipticPi}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i}{2}\sqrt{3}}},\frac{1}{2}-\frac{i}{6}\sqrt{3},\sqrt{\frac{-\frac{3}{2}+}{-\frac{3}{2}-}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(2-x)/(x^3+1)^(1/2),x)
```

```
[Out] -2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-
1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))
/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-
1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))
^(1/2))+4/3*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)
*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I
*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticPi(((
1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),1/2-1/6*I*3^(1/2),((-3/2+1/2*I*3^
(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]   time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{\sqrt{x^3+1}(x-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x/(sqrt(x^3 + 1)*(x - 2)),x, algorithm="maxima")
```

```
[Out] -integrate(x/(sqrt(x^3 + 1)*(x - 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{x}{\sqrt{x^3+1}(x-2)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x/(sqrt(x^3 + 1)*(x - 2)),x, algorithm="fricas")
```

```
[Out] integral(-x/(sqrt(x^3 + 1)*(x - 2)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{x\sqrt{x^3+1}-2\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(2-x)/(x**3+1)**(1/2),x)
```

```
[Out] -Integral(x/(x*sqrt(x**3 + 1) - 2*sqrt(x**3 + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x}{\sqrt{x^3+1}(x-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x/(sqrt(x^3 + 1)*(x - 2)),x, algorithm="giac")
```

```
[Out] integrate(-x/(sqrt(x^3 + 1)*(x - 2)), x)
```

3.70 $\int \frac{x}{(2+x)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=145

$$\frac{4}{9}\tanh^{-1}\left(\frac{(1-x)^2}{3\sqrt{1-x^3}}\right)-\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

```
[Out] (4*ArcTanh[(1 - x)^2/(3*Sqrt[1 - x^3])])/9 - (2*Sqrt[2 + Sqrt[3]]
*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin
[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3*3^(1/4
)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rubi [A] time = 0.277236, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$\frac{4}{9}\tanh^{-1}\left(\frac{(1-x)^2}{3\sqrt{1-x^3}}\right)-\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/((2 + x)*Sqrt[1 - x^3]),x]
```

```
[Out] (4*ArcTanh[(1 - x)^2/(3*Sqrt[1 - x^3])])/9 - (2*Sqrt[2 + Sqrt[3]]
*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin
[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3*3^(1/4
)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rubi in Sympy [A] time = 89.1562, size = 379, normalized size = 2.61

$$\frac{2\sqrt{\frac{x^2+x+1}{\left(-x+1+\sqrt{3}\right)^2}}\left(\frac{\sqrt{3}}{3}+1\right)(-x+1)\operatorname{atanh}\left(\frac{3^{\frac{3}{4}}\sqrt{1-\frac{\left(x-1+\sqrt{3}\right)^2}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{-\sqrt{3}+2}}{3\sqrt{-4\sqrt{3}+7+\frac{\left(x-1+\sqrt{3}\right)^2}{\left(-x+1+\sqrt{3}\right)^2}}}\right)}{\sqrt{\frac{-x+1}{\left(-x+1+\sqrt{3}\right)^2}}\left(\sqrt{3}+3\right)\sqrt{-x^3+1}} - \frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{x^2+x+1}{\left(-x+1+\sqrt{3}\right)^2}}\left(1+\sqrt{3}\right)\sqrt{\sqrt{3}+2}\,(-x+1)\,F\left(\operatorname{asin}\left(\frac{-x-\sqrt{3}+1}{-x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{3\sqrt{\frac{-x+1}{\left(-x+1+\sqrt{3}\right)^2}}\left(\sqrt{3}+3\right)\sqrt{-x^3+1}} - \frac{8\sqrt[4]{3}\sqrt{\frac{x^2+x+1}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{-\sqrt{3}+2}\,(-x+1)\left(4\sqrt{3}+7;\operatorname{asin}\left(\frac{x-1+\sqrt{3}}{-x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt{\frac{-x+1}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{-4\sqrt{3}+7}\left(-\sqrt{3}+3\right)\left(\sqrt{3}+3\right)\sqrt{-x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(2+x)/(-x**3+1)**(1/2),x)
```

```
[Out] 2*sqrt((x**2 + x + 1)/(-x + 1 + sqrt(3))**2)*(sqrt(3)/3 + 1)*(-x
+ 1)*atanh(3**(3/4)*sqrt(1 - (x - 1 + sqrt(3))**2/(-x + 1 + sqrt(
3))**2)*sqrt(-sqrt(3) + 2)/(3*sqrt(-4*sqrt(3) + 7 + (x - 1 + sqrt
(3))**2/(-x + 1 + sqrt(3))**2)))/(sqrt((-x + 1)/(-x + 1 + sqrt(3)
)**2)*(sqrt(3) + 3)*sqrt(-x**3 + 1)) - 2*3**(3/4)*sqrt((x**2 + x
+ 1)/(-x + 1 + sqrt(3))**2)*(1 + sqrt(3))*sqrt(sqrt(3) + 2)*(-x +
 1)*elliptic_f(asin((-x - sqrt(3) + 1)/(-x + 1 + sqrt(3))), -7 -
4*sqrt(3))/(3*sqrt((-x + 1)/(-x + 1 + sqrt(3))**2)*(sqrt(3) + 3)*
sqrt(-x**3 + 1)) - 8*3**(1/4)*sqrt((x**2 + x + 1)/(-x + 1 + sqrt(
3))**2)*sqrt(-sqrt(3) + 2)*(-x + 1)*elliptic_pi(4*sqrt(3) + 7, as
in((x - 1 + sqrt(3))/(-x + 1 + sqrt(3))), -7 - 4*sqrt(3))/(sqrt((
-x + 1)/(-x + 1 + sqrt(3))**2)*sqrt(-4*sqrt(3) + 7)*(-sqrt(3) + 3
)*(sqrt(3) + 3)*sqrt(-x**3 + 1))
```

Mathematica [C] time = 0.451624, size = 195, normalized size = 1.34

$$\frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(\frac{\left(x+\sqrt[3]{-1}\right)\sqrt{\frac{(-1)^{2/3}x+\sqrt[3]{-1}}{1+\sqrt[3]{-1}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{2i\sqrt{x^2+x+1}\left(\frac{2\sqrt{3}}{3i+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt[3]{-1}-2}\right)}{\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x/((2 + x)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((((-1)^(1/3) + x)*Sqrt[((-1)^(
1/3) + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 -
 (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2
/3)*x)/(1 + (-1)^(1/3))] + ((2*I)*Sqrt[1 + x + x^2]*EllipticPi[(2
*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-
1)^(1/3))]], (-1)^(1/3)])/(-2 + (-1)^(1/3))))/Sqrt[1 - x^3]
```

Maple [A] time = 0.009, size = 240, normalized size = 1.7

$$-\frac{2i}{3}\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}EllipticF\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)\frac{}{\sqrt{-}}$$
$$+\frac{\frac{4i}{3}\sqrt{3}}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}EllipticPi\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i}{2}\sqrt{3}},\sqrt{\frac{}{-\frac{3}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(2+x)/(-x^3+1)^(1/2),x)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/(-
3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2
)/(-x^3+1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3
^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))+4/3*I*3^(1/
2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/(-3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(
1/2)/(3/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3
^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(3/2+1/2*I*3^(1/2)),(I*3^(1/2)/(
-3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-x^3+1}(x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(-x^3 + 1)*(x + 2)),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(-x^3 + 1)*(x + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x}{\sqrt{-x^3+1}(x+2)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(-x^3 + 1)*(x + 2)),x, algorithm="fricas")
```

```
[Out] integral(x/(sqrt(-x^3 + 1)*(x + 2)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-(x-1)(x^2+x+1)}(x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(2+x)/(-x**3+1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-(x - 1)*(x**2 + x + 1))*(x + 2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-x^3+1}(x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(-x^3 + 1)*(x + 2)),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(-x^3 + 1)*(x + 2)), x)
```

3.71 $\int \frac{x}{(2+x)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=148

$$\frac{4}{9}\tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{x^3-1}}\right)-\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

```
[Out] (4*ArcTan[(1 - x)^2/(3*Sqrt[-1 + x^3])])/9 - (2*Sqrt[2 - Sqrt[3]]
*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin
[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3*3^(1/4
)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Rubi [A] time = 0.258307, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{4}{9}\tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{x^3-1}}\right)-\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/((2 + x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (4*ArcTan[(1 - x)^2/(3*Sqrt[-1 + x^3])])/9 - (2*Sqrt[2 - Sqrt[3]]
*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin
[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3*3^(1/4
)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Rubi in Sympy [A] time = 85.6724, size = 376, normalized size = 2.54

$$\frac{2\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(-\frac{\sqrt{3}}{3}+1\right)(-x+1)\operatorname{atan}\left(\frac{3^{\frac{3}{4}}\sqrt{\sqrt{3}+2}\sqrt{-\frac{\left(-x+1+\sqrt{3}\right)^2}{\left(x-1+\sqrt{3}\right)^2}+1}}{3\sqrt{\frac{\left(-x+1+\sqrt{3}\right)^2}{\left(x-1+\sqrt{3}\right)^2}+4\sqrt{3}+7}}\right)}{\sqrt{\frac{x-1}{\left(-x-\sqrt{3}+1\right)^2}}\left(-\sqrt{3}+3\right)\sqrt{x^3-1}} - \frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(-\sqrt{3}+1\right)\sqrt{-\sqrt{3}+2}\,(-x+1)\,F\left(\operatorname{asin}\left(\frac{-x+1+\sqrt{3}}{-x-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{3\sqrt{\frac{x-1}{\left(-x-\sqrt{3}+1\right)^2}}\left(-\sqrt{3}+3\right)\sqrt{x^3-1}} + \frac{8\sqrt[4]{3}\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{\sqrt{3}+2}\,(-x+1)\left(-4\sqrt{3}+7;\operatorname{asin}\left(\frac{-x+1+\sqrt{3}}{x-1+\sqrt{3}}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt{\frac{x-1}{\left(-x-\sqrt{3}+1\right)^2}}\left(-\sqrt{3}+3\right)\left(\sqrt{3}+3\right)\sqrt{4\sqrt{3}+7}\sqrt{x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(2+x)/(x**3-1)**(1/2),x)
```

```
[Out] 2*sqrt((x**2 + x + 1)/(-x - sqrt(3) + 1)**2)*(-sqrt(3)/3 + 1)*(-x
 + 1)*atan(3**(3/4)*sqrt(sqrt(3) + 2)*sqrt(-(-x + 1 + sqrt(3))**2
/(x - 1 + sqrt(3))**2 + 1)/(3*sqrt((-x + 1 + sqrt(3))**2/(x - 1 +
 sqrt(3))**2 + 4*sqrt(3) + 7)))/(sqrt((x - 1)/(-x - sqrt(3) + 1)*
*2)*(-sqrt(3) + 3)*sqrt(x**3 - 1)) - 2*3**(3/4)*sqrt((x**2 + x +
1)/(-x - sqrt(3) + 1)**2)*(-sqrt(3) + 1)*sqrt(-sqrt(3) + 2)*(-x +
 1)*elliptic_f(asin((-x + 1 + sqrt(3))/(-x - sqrt(3) + 1)), -7 +
4*sqrt(3))/(3*sqrt((x - 1)/(-x - sqrt(3) + 1)**2)*(-sqrt(3) + 3)*
sqrt(x**3 - 1)) + 8*3**(1/4)*sqrt((x**2 + x + 1)/(-x - sqrt(3) +
1)**2)*sqrt(sqrt(3) + 2)*(-x + 1)*elliptic_pi(-4*sqrt(3) + 7, asi
n((-x + 1 + sqrt(3))/(x - 1 + sqrt(3))), -7 + 4*sqrt(3))/(sqrt((x
 - 1)/(-x - sqrt(3) + 1)**2)*(-sqrt(3) + 3)*(sqrt(3) + 3)*sqrt(4*
sqrt(3) + 7)*sqrt(x**3 - 1))
```

Mathematica [C] time = 0.434571, size = 193, normalized size = 1.3

$$\frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(\frac{\left(x+\sqrt[3]{-1}\right)\sqrt{\frac{(-1)^{2/3}x+\sqrt[3]{-1}}{1+\sqrt[3]{-1}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{2i\sqrt{x^2+x+1}\left(\frac{2\sqrt{3}}{3i+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt[3]{-1}-2}\right)}{\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x/((2 + x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((((-1)^(1/3) + x)*Sqrt[((-1)^(
1/3) + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 -
 (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2
/3)*x)/(1 + (-1)^(1/3))] + ((2*I)*Sqrt[1 + x + x^2]*EllipticPi[(2
*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-
1)^(1/3))]], (-1)^(1/3)])/(-2 + (-1)^(1/3))))/Sqrt[-1 + x^3]
```

Maple [B] time = 0.008, size = 240, normalized size = 1.6

$$2\,\frac{-3/2-i/2\sqrt{3}}{\sqrt{x^3-1}}\sqrt{\frac{-1+x}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2-i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2+i/2\sqrt{3}}{3/2+i/2\sqrt{3}}}\mathit{EllipticF}\left(\sqrt{\frac{-1+x}{-3/2-i/2\sqrt{3}}},\sqrt{\frac{3/2+i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\right)$$
$$-\frac{-6-2\,i\sqrt{3}}{3}\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i}{2}\sqrt{3}}}\sqrt{\frac{1}{\frac{3}{2}-\frac{i}{2}\sqrt{3}}\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)}\sqrt{\frac{1}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)}\mathit{EllipticPi}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i}{2}\sqrt{3}}},\frac{i}{6}\sqrt{3}+\frac{1}{2},\sqrt{\frac{\frac{3}{2}+}{\frac{3}{2}-}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(2+x)/(x^3-1)^(1/2),x)
```

```
[Out] 2*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/
2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2)
)/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((-1+x)/(-3/
2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))
^(1/2))-4/3*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1
/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2
*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticPi((
(-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),1/6*I*3^(1/2)+1/2,((3/2+1/2*I*
3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x^3-1}(x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(x^3 - 1)*(x + 2)),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(x^3 - 1)*(x + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x}{\sqrt{x^3-1}(x+2)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(x^3 - 1)*(x + 2)),x, algorithm="fricas")
```

```
[Out] integral(x/(sqrt(x^3 - 1)*(x + 2)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{(x-1)\left(x^2+x+1\right)}\,(x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(2+x)/(x**3-1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt((x - 1)*(x**2 + x + 1))*(x + 2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x^3-1}(x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(x^3 - 1)*(x + 2)),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(x^3 - 1)*(x + 2)), x)
```

3.72 $\int \frac{x}{(2-x)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=140

$$\frac{4}{9}\tan^{-1}\left(\frac{(x+1)^2}{3\sqrt{-x^3-1}}\right)-\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

```
[Out] (4*ArcTan[(1 + x)^2/(3*Sqrt[-1 - x^3])])/9 - (2*Sqrt[2 - Sqrt[3]]
*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin
[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3*3^(1/4
)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```

Rubi [A] time = 0.287441, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{4}{9}\tan^{-1}\left(\frac{(x+1)^2}{3\sqrt{-x^3-1}}\right)-\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/((2 - x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (4*ArcTan[(1 + x)^2/(3*Sqrt[-1 - x^3])])/9 - (2*Sqrt[2 - Sqrt[3]]
*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin
[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3*3^(1/4
)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```

Rubi in Sympy [A] time = 91.3302, size = 386, normalized size = 2.76

$$\frac{2\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(-\frac{\sqrt{3}}{3}+1\right)(x+1)\operatorname{atan}\left(\frac{3^{\frac{3}{4}}\sqrt{1-\frac{\left(x+1+\sqrt{3}\right)^2}{\left(-x-1+\sqrt{3}\right)^2}}\sqrt{\sqrt{3}+2}}{3\sqrt{4\sqrt{3}+7+\frac{\left(x+1+\sqrt{3}\right)^2}{\left(-x-1+\sqrt{3}\right)^2}}}\right)}{\sqrt{\frac{-x-1}{\left(x-\sqrt{3}+1\right)^2}}\left(-\sqrt{3}+3\right)\sqrt{-x^3-1}} - \frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(-\sqrt{3}+1\right)\sqrt{-\sqrt{3}+2}\,(x+1)\,F\left(\operatorname{asin}\left(\frac{x+1+\sqrt{3}}{x-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{3\sqrt{\frac{-x-1}{\left(x-\sqrt{3}+1\right)^2}}\left(-\sqrt{3}+3\right)\sqrt{-x^3-1}} + \frac{8\sqrt[4]{3}\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{\sqrt{3}+2}\,(x+1)\left(-4\sqrt{3}+7;\operatorname{asin}\left(\frac{x+1+\sqrt{3}}{-x-1+\sqrt{3}}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt{\frac{-x-1}{\left(x-\sqrt{3}+1\right)^2}}\left(-\sqrt{3}+3\right)\left(\sqrt{3}+3\right)\sqrt{4\sqrt{3}+7}\sqrt{-x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(2-x)/(-x**3-1)**(1/2),x)
```

```
[Out] 2*sqrt((x**2 - x + 1)/(x - sqrt(3) + 1)**2)*(-sqrt(3)/3 + 1)*(x +
 1)*atan(3**(3/4)*sqrt(1 - (x + 1 + sqrt(3))**2/(-x - 1 + sqrt(3)
)**2)*sqrt(sqrt(3) + 2)/(3*sqrt(4*sqrt(3) + 7 + (x + 1 + sqrt(3))
**2/(-x - 1 + sqrt(3))**2)))/(sqrt((-x - 1)/(x - sqrt(3) + 1)**2)
*(-sqrt(3) + 3)*sqrt(-x**3 - 1)) - 2*3**(3/4)*sqrt((x**2 - x + 1)
/(x - sqrt(3) + 1)**2)*(-sqrt(3) + 1)*sqrt(-sqrt(3) + 2)*(x + 1)*
elliptic_f(asin((x + 1 + sqrt(3))/(x - sqrt(3) + 1)), -7 + 4*sqrt
(3))/(3*sqrt((-x - 1)/(x - sqrt(3) + 1)**2)*(-sqrt(3) + 3)*sqrt(-
x**3 - 1)) + 8*3**(1/4)*sqrt((x**2 - x + 1)/(x - sqrt(3) + 1)**2)
*sqrt(sqrt(3) + 2)*(x + 1)*elliptic_pi(-4*sqrt(3) + 7, asin((x +
1 + sqrt(3))/(-x - 1 + sqrt(3))), -7 + 4*sqrt(3))/(sqrt((-x - 1)/
(x - sqrt(3) + 1)**2)*(-sqrt(3) + 3)*(sqrt(3) + 3)*sqrt(4*sqrt(3)
 + 7)*sqrt(-x**3 - 1))
```

Mathematica [C] time = 0.436444, size = 195, normalized size = 1.39

$$\frac{2\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\left(\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}}+\frac{2i\sqrt{x^2-x+1}\left(\frac{2\sqrt{3}}{3i+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt[3]{-1}-2}\right)}{\sqrt{-x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x/((2 - x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*((((-1)^(1/3) - x)*Sqrt[((-1)^(
1/3) - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 +
 (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2
/3)*x)/(1 + (-1)^(1/3))] + ((2*I)*Sqrt[1 - x + x^2]*EllipticPi[(2
*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-
1)^(1/3))]], (-1)^(1/3)])/(-2 + (-1)^(1/3))))/Sqrt[-1 - x^3]
```

Maple [B] time = 0.009, size = 240, normalized size = 1.7

$$\frac{2i}{3}\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}EllipticF\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)\frac{1}{\sqrt{-x^3-}}$$

$$+\frac{\frac{4i}{3}\sqrt{3}}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}EllipticPi\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}},\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(2-x)/(-x^3-1)^(1/2),x)
```

```
[Out] 2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2
+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(
-x^3-1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1
/2))^(1/2),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))+4/3*I*3^(1/2)*(
I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^(1/2))
)^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)/(
-3/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2
))*3^(1/2))^(1/2),I*3^(1/2)/(-3/2+1/2*I*3^(1/2)),(I*3^(1/2)/(3/2+
1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{\sqrt{-x^3-1}(x-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x/(sqrt(-x^3 - 1)*(x - 2)),x, algorithm="maxima")
```

```
[Out] -integrate(x/(sqrt(-x^3 - 1)*(x - 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{x}{\sqrt{-x^3-1}(x-2)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x/(sqrt(-x^3 - 1)*(x - 2)),x, algorithm="fricas")
```

```
[Out] integral(-x/(sqrt(-x^3 - 1)*(x - 2)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{x\sqrt{-x^3-1} - 2\sqrt{-x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(2-x)/(-x**3-1)**(1/2),x)
```

```
[Out] -Integral(x/(x*sqrt(-x**3 - 1) - 2*sqrt(-x**3 - 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x}{\sqrt{-x^3-1}(x-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x/(sqrt(-x^3 - 1)*(x - 2)),x, algorithm="giac")
```

```
[Out] integrate(-x/(sqrt(-x^3 - 1)*(x - 2)), x)
```

3.73 $\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=260

$$\frac{4\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a+bx^3}}\right)}{9\sqrt[6]{a}b^{2/3}}-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] (4*ArcTanh[(a^(1/3) + b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[a + b*x^3])])/
(9*a^(1/6)*b^(2/3)) - (2*Sqrt[2 + Sqrt[3]]*(a^(1/3) + b^(1/3)*x)*
Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a
^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) +
b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])
/(3*3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.52817, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129

$$\frac{4\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a+bx^3}}\right)}{9\sqrt[6]{a}b^{2/3}}-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/((2*a^(1/3) - b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (4*ArcTanh[(a^(1/3) + b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[a + b*x^3])])/
(9*a^(1/6)*b^(2/3)) - (2*Sqrt[2 + Sqrt[3]]*(a^(1/3) + b^(1/3)*x)*
Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a
^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) +
b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])
/(3*3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi in Sympy [A] time = 170.311, size = 687, normalized size = 2.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(2*a**(1/3)-b**(1/3)*x)/(b*x**3+a)**(1/2),x)
```

```
[Out] -2*3**(3/4)*sqrt((a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)
/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*(1 + sqrt(3))*sqrt(sqr
t(3) + 2)*(a**(1/3) + b**(1/3)*x)*elliptic_f(asin((-a**(1/3)*(-1
+ sqrt(3)) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)),
-7 - 4*sqrt(3))/(3*b**(2/3)*sqrt(a**(1/3)*(a**(1/3) + b**(1/3)*x)
/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*(sqrt(3) + 3)*sqrt(a +
 b*x**3)) + 2*3**(3/4)*sqrt(a**(2/3)*(1 - b**(1/3)*x/a**(1/3) + b
**(2/3)*x**2/a**(2/3))/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*
sqrt(3 + 2*sqrt(3))*sqrt(-sqrt(3) + 2)*(a**(1/3) + b**(1/3)*x)*at
anh(sqrt(-(a**(1/3)*(-1 + sqrt(3)) - b**(1/3)*x)**2/(a**(1/3)*(1
+ sqrt(3)) + b**(1/3)*x)**2 + 1)/(sqrt(3 + 2*sqrt(3))*sqrt((a**(1
/3)*(-1 + sqrt(3)) - b**(1/3)*x)**2/(a**(1/3)*(1 + sqrt(3)) + b**
(1/3)*x)**2 - 4*sqrt(3) + 7)))/(9*b**(2/3)*sqrt(a**(1/3)*(a**(1/3
) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(a
+ b*x**3)) - 8*3**(1/4)*sqrt(a**(2/3)*(1 - b**(1/3)*x/a**(1/3) +
b**(2/3)*x**2/a**(2/3))/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)
*sqrt(-sqrt(3) + 2)*(a**(1/3) + b**(1/3)*x)*elliptic_pi(4*sqrt(3)
 + 7, asin((a**(1/3)*(-1 + sqrt(3)) - b**(1/3)*x)/(a**(1/3)*(1 +
sqrt(3)) + b**(1/3)*x)), -7 - 4*sqrt(3))/(b**(2/3)*sqrt(a**(1/3)*
(a**(1/3) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)
*sqrt(-4*sqrt(3) + 7)*(-sqrt(3) + 3)*(sqrt(3) + 3)*sqrt(a + b*x**
3))
```

Mathematica [C] time = 2.72808, size = 407, normalized size = 1.57

$$\frac{\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(8i\sqrt[3]{a}\sqrt{\frac{\left(\sqrt{3}+i\right)\sqrt[3]{b}x-2i\sqrt[3]{a}}{\left(\sqrt{3}-3i\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\left(\frac{2\sqrt{3}}{3i+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{\left(i+\sqrt{3}\right)\sqrt[3]{b}x-2i\sqrt[3]{a}}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)|\frac{1}{2}\left(1+i\sqrt{3}\right)\right)-\sqrt{2}\sqrt[4]{3}\left(\left(\sqrt{\ldots}\right.\right.\right.}{2\left(\sqrt[3]{-1}-2\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{a\ldots}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x/((2*a^(1/3) - b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(-(Sqrt[2
]*3^(1/4)*((I + Sqrt[3])*a^(1/3) - (-I + Sqrt[3])*b^(1/3)*x)*Sqrt
```

```
[I + Sqrt[3] - ((2*I)*b^(1/3)*x)/a^(1/3)]*EllipticF[ArcSin[Sqrt[(
(-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/
3))]], (1 + I*Sqrt[3])/2]) + (8*I)*a^(1/3)*Sqrt[((-2*I)*a^(1/3) +
 (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*Sqrt[1 - (b
^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])
/(3*I + Sqrt[3]), ArcSin[Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(
1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/(2*(-2
 + (-1)^(1/3))*b^(2/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1
+ (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3])
```

Maple [F] time = 0.095, size = 0, normalized size = 0.

$$\int x \left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)^{-1}\frac{1}{\sqrt{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

```
[Out] int(x/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{\sqrt{bx^3+a}\left(b^{\frac{1}{3}}x-2\,a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x/(sqrt(b*x^3 + a)*(b^(1/3)*x - 2*a^(1/3))),x, algorithm="maxima")
```

```
[Out] -integrate(x/(sqrt(b*x^3 + a)*(b^(1/3)*x - 2*a^(1/3))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x/(sqrt(b*x^3 + a)*(b^(1/3)*x - 2*a^(1/3))),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{-2\sqrt[3]{a}\sqrt{a+bx^3}+\sqrt[3]{b}x\sqrt{a+bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(2*a**(1/3)-b**(1/3)*x)/(b*x**3+a)**(1/2),x)
```

```
[Out] -Integral(x/(-2*a**(1/3)*sqrt(a + b*x**3) + b**(1/3)*x*sqrt(a + b
*x**3)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x}{\sqrt{bx^3+a}\left(b^{\frac{1}{3}}x-2\,a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x/(sqrt(b*x^3 + a)*(b^(1/3)*x - 2*a^(1/3))),x, algorithm="giac")
```

```
[Out] integrate(-x/(sqrt(b*x^3 + a)*(b^(1/3)*x - 2*a^(1/3))), x)
```

3.74 $\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$

Optimal. Leaf size=268

$$\frac{4\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a-bx^3}}\right)}{9\sqrt[6]{a}b^{2/3}}-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

```
[Out] (4*ArcTanh[(a^(1/3) - b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[a - b*x^3])])/
(9*a^(1/6)*b^(2/3)) - (2*Sqrt[2 + Sqrt[3]]*(a^(1/3) - b^(1/3)*x)*
Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a
^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) -
b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])
/(3*3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sq
rt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

Rubi [A] time = 0.560592, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$

$$\frac{4\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a-bx^3}}\right)}{9\sqrt[6]{a}b^{2/3}}-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/((2*a^(1/3) + b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (4*ArcTanh[(a^(1/3) - b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[a - b*x^3])])/
(9*a^(1/6)*b^(2/3)) - (2*Sqrt[2 + Sqrt[3]]*(a^(1/3) - b^(1/3)*x)*
Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a
^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) -
b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])
/(3*3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sq
rt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

Rubi in Sympy [A] time = 173.008, size = 687, normalized size = 2.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(2*a**(1/3)+b**(1/3)*x)/(-b*x**3+a)**(1/2),x)
```

```
[Out] 2*3**(3/4)*sqrt((a**(2/3) + a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/
(a**(1/3)*(1 + sqrt(3)) - b**(1/3)*x)**2)*(1 + sqrt(3))*sqrt(sqrt
(3) + 2)*(a**(1/3) - b**(1/3)*x)*elliptic_f(asin((a**(1/3)*(-1 +
sqrt(3)) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) - b**(1/3)*x)), -7
 - 4*sqrt(3))/(3*b**(2/3)*sqrt(a**(1/3)*(a**(1/3) - b**(1/3)*x)/(
a**(1/3)*(1 + sqrt(3)) - b**(1/3)*x)**2)*(sqrt(3) + 3)*sqrt(a - b
*x**3)) + 2*3**(3/4)*sqrt(a**(2/3)*(1 + b**(1/3)*x/a**(1/3) + b**
(2/3)*x**2/a**(2/3))/(a**(1/3)*(1 + sqrt(3)) - b**(1/3)*x)**2)*sq
rt(3 + 2*sqrt(3))*sqrt(-sqrt(3) + 2)*(a**(1/3) - b**(1/3)*x)*atan
h(sqrt(-(a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)**2/(a**(1/3)*(1 +
sqrt(3)) - b**(1/3)*x)**2 + 1)/(sqrt(3 + 2*sqrt(3))*sqrt((a**(1/3
)*(-1 + sqrt(3)) + b**(1/3)*x)**2/(a**(1/3)*(1 + sqrt(3)) - b**(1
/3)*x)**2 - 4*sqrt(3) + 7)))/(9*b**(2/3)*sqrt(a**(1/3)*(a**(1/3)
- b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) - b**(1/3)*x)**2)*sqrt(a -
b*x**3)) - 8*3**(1/4)*sqrt(a**(2/3)*(1 + b**(1/3)*x/a**(1/3) + b*
*(2/3)*x**2/a**(2/3))/(a**(1/3)*(1 + sqrt(3)) - b**(1/3)*x)**2)*s
qrt(-sqrt(3) + 2)*(a**(1/3) - b**(1/3)*x)*elliptic_pi(4*sqrt(3) +
 7, asin((a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)/(a**(1/3)*(1 + sq
rt(3)) - b**(1/3)*x)), -7 - 4*sqrt(3))/(b**(2/3)*sqrt(a**(1/3)*(a
**(1/3) - b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) - b**(1/3)*x)**2)*s
qrt(-4*sqrt(3) + 7)*(-sqrt(3) + 3)*(sqrt(3) + 3)*sqrt(a - b*x**3)
)
```

Mathematica [C] time = 1.45023, size = 371, normalized size = 1.38

$$\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\left(\sqrt[3]{-1}-2\right)\left(\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}F\left(\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)+\frac{2\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\sqrt[3]{\cdots}}{\cdots}\right.}{\left(\sqrt[3]{-1}-2\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{a-bx^3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/((2*a^(1/3) + b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((-2 +
(-1)^(1/3))*((-1)^(1/3)*a^(1/3) + b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^
(1/3) + (-1)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*Ellipt
icF[ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3)
)*a^(1/3))]], (-1)^(1/3)] + (2*(-1)^(1/3)*(1 + (-1)^(1/3))*a^(1/3
)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3)
)]*Sqrt[1 + (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*Elliptic
Pi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)
*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/Sqrt[3]))/
((-2 + (-1)^(1/3))*b^(2/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/
((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a - b*x^3])
```

Maple [F] time = 0.093, size = 0, normalized size = 0.

$$\int x\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)^{-1}\frac{1}{\sqrt{-bx^{3}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

```
[Out] int(x/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-bx^{3}+a}\left(b^{\frac{1}{3}}x+2\,a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(-b*x^3 + a)*(b^(1/3)*x + 2*a^(1/3))),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(-b*x^3 + a)*(b^(1/3)*x + 2*a^(1/3))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(-b*x^3 + a)*(b^(1/3)*x + 2*a^(1/3))),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(2*a**(1/3)+b**(1/3)*x)/(-b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x/((2*a**(1/3) + b**(1/3)*x)*sqrt(a - b*x**3)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-bx^3+a}\left(b^{\frac{1}{3}}x+2\,a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(-b*x^3 + a)*(b^(1/3)*x + 2*a^(1/3))),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(-b*x^3 + a)*(b^(1/3)*x + 2*a^(1/3))), x)
```

3.75 $\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

Optimal. Leaf size=277

$$\frac{4\tan^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{bx^3-a}}\right)}{9\sqrt[6]{a}b^{2/3}}-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$

```
[Out] (4*ArcTan[(a^(1/3) - b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[-a + b*x^3])])/
(9*a^(1/6)*b^(2/3)) - (2*Sqrt[2 - Sqrt[3]]*(a^(1/3) - b^(1/3)*x)*
Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a
^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) -
b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])
/(3*3^(1/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 -
Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

Rubi [A] time = 0.569637, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$\frac{4\tan^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{bx^3-a}}\right)}{9\sqrt[6]{a}b^{2/3}}-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/((2*a^(1/3) + b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```


```
[Out] (4*ArcTan[(a^(1/3) - b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[-a + b*x^3])])/
(9*a^(1/6)*b^(2/3)) - (2*Sqrt[2 - Sqrt[3]]*(a^(1/3) - b^(1/3)*x)*
Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a
^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) -
b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])
/(3*3^(1/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 -
Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

Rubi in Sympy [A] time = 170.524, size = 687, normalized size = 2.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(2*a**(1/3)+b**(1/3)*x)/(b*x**3-a)**(1/2),x)
```

```
[Out] -2*3**(3/4)*sqrt((a**(2/3) + a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)
/(a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)**2)*(-sqrt(3) + 1)*sqrt(-
sqrt(3) + 2)*(a**(1/3) - b**(1/3)*x)*elliptic_f(asin((a**(1/3)*(1
 + sqrt(3)) - b**(1/3)*x)/(-a**(1/3)*(-1 + sqrt(3)) - b**(1/3)*x)
), -7 + 4*sqrt(3))/(3*b**(2/3)*sqrt(-a**(1/3)*(a**(1/3) - b**(1/3
)*x)/(a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)**2)*(-sqrt(3) + 3)*sq
rt(-a + b*x**3)) + 2*3**(3/4)*sqrt(a**(2/3)*(1 + b**(1/3)*x/a**(1
/3) + b**(2/3)*x**2/a**(2/3))/(a**(1/3)*(-1 + sqrt(3)) + b**(1/3)
*x)**2)*sqrt(-3 + 2*sqrt(3))*sqrt(sqrt(3) + 2)*(a**(1/3) - b**(1/
3)*x)*atan(sqrt(1 - (a**(1/3)*(1 + sqrt(3)) - b**(1/3)*x)**2/(a**
(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)**2)/(sqrt(-3 + 2*sqrt(3))*sqrt
(4*sqrt(3) + 7 + (a**(1/3)*(1 + sqrt(3)) - b**(1/3)*x)**2/(a**(1/
3)*(-1 + sqrt(3)) + b**(1/3)*x)**2)))/(9*b**(2/3)*sqrt(a**(1/3)*(
-a**(1/3) + b**(1/3)*x)/(a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)**2
)*sqrt(-a + b*x**3)) + 8*3**(1/4)*sqrt(a**(2/3)*(1 + b**(1/3)*x/a
**(1/3) + b**(2/3)*x**2/a**(2/3))/(a**(1/3)*(-1 + sqrt(3)) + b**(
1/3)*x)**2)*sqrt(sqrt(3) + 2)*(a**(1/3) - b**(1/3)*x)*elliptic_pi
(-4*sqrt(3) + 7, asin((a**(1/3)*(1 + sqrt(3)) - b**(1/3)*x)/(a**(
1/3)*(-1 + sqrt(3)) + b**(1/3)*x)), -7 + 4*sqrt(3))/(b**(2/3)*sqr
t(a**(1/3)*(-a**(1/3) + b**(1/3)*x)/(a**(1/3)*(-1 + sqrt(3)) + b*
*(1/3)*x)**2)*(-sqrt(3) + 3)*(sqrt(3) + 3)*sqrt(4*sqrt(3) + 7)*sq
rt(-a + b*x**3))
```

Mathematica [C] time = 1.46143, size = 372, normalized size = 1.34

$$\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\left(\sqrt[3]{-1}-2\right)\left(\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}F\left(\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)\middle|\sqrt[3]{-1}\right)+\frac{2\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\sqrt[3]{\cdots}}{\cdots}\right.}{\left(\sqrt[3]{-1}-2\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{bx^3-a}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/((2*a^(1/3) + b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((-2 +
(-1)^(1/3))*((-1)^(1/3)*a^(1/3) + b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^
(1/3) + (-1)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*Ellipt
icF[ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3)
)*a^(1/3))]], (-1)^(1/3)] + (2*(-1)^(1/3)*(1 + (-1)^(1/3))*a^(1/3
)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3)
)]*Sqrt[1 + (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*Elliptic
Pi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)
*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/Sqrt[3]))/
((-2 + (-1)^(1/3))*b^(2/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/
((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[-a + b*x^3])
```

Maple [F] time = 0.086, size = 0, normalized size = 0.

$$\int x \left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)^{-1}\frac{1}{\sqrt{bx^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

```
[Out] int(x/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{bx^3-a}\left(b^{\frac{1}{3}}x+2\,a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(b*x^3 - a)*(b^(1/3)*x + 2*a^(1/3))),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(b*x^3 - a)*(b^(1/3)*x + 2*a^(1/3))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(b*x^3 - a)*(b^(1/3)*x + 2*a^(1/3))),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(2*a**(1/3)+b**(1/3)*x)/(b*x**3-a)**(1/2),x)
```

```
[Out] Integral(x/((2*a**(1/3) + b**(1/3)*x)*sqrt(-a + b*x**3)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{bx^3-a}\left(b^{\frac{1}{3}}x+2\,a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(b*x^3 - a)*(b^(1/3)*x + 2*a^(1/3))),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(b*x^3 - a)*(b^(1/3)*x + 2*a^(1/3))), x)
```

3.76 $\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

Optimal. Leaf size=273

$$\frac{4\tan^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{-a-bx^3}}\right)}{9\sqrt[6]{a}b^{2/3}}-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

```
[Out] (4*ArcTan[(a^(1/3) + b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[-a - b*x^3])])/
(9*a^(1/6)*b^(2/3)) - (2*Sqrt[2 - Sqrt[3]]*(a^(1/3) + b^(1/3)*x)*
Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a
^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])
/(3*3^(1/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 -
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

Rubi [A] time = 0.589524, antiderivative size = 273, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118

$$\frac{4\tan^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{-a-bx^3}}\right)}{9\sqrt[6]{a}b^{2/3}}-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/((2*a^(1/3) - b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```


```
[Out] (4*ArcTan[(a^(1/3) + b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[-a - b*x^3])])/
(9*a^(1/6)*b^(2/3)) - (2*Sqrt[2 - Sqrt[3]]*(a^(1/3) + b^(1/3)*x)*
Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a
^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])
/(3*3^(1/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 -
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(2*a**(1/3)-b**(1/3)*x)/(-b*x**3-a)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 2.71778, size = 410, normalized size = 1.5

$$\frac{\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(8i\sqrt[3]{a}\sqrt{\frac{\left(\sqrt{3}+i\right)\sqrt[3]{b}x-2i\sqrt[3]{a}}{\left(\sqrt{3}-3i\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\left(\frac{2\sqrt{3}}{3i+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{\left(i+\sqrt{3}\right)\sqrt[3]{b}x-2i\sqrt[3]{a}}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)|\frac{1}{2}\left(1+i\sqrt{3}\right)\right)-\sqrt{2}\sqrt[4]{3}\left(\left(\sqrt{\cdots}\right.\right.\right.}{2\left(\sqrt[3]{-1}-2\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{-\cdots}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x/((2*a^(1/3) - b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(-(Sqrt[2
]*3^(1/4)*((I + Sqrt[3])*a^(1/3) - (-I + Sqrt[3])*b^(1/3)*x)*Sqrt
[I + Sqrt[3] - ((2*I)*b^(1/3)*x)/a^(1/3)]*EllipticF[ArcSin[Sqrt[(
(-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/
3))]], (1 + I*Sqrt[3])/2]) + (8*I)*a^(1/3)*Sqrt[((-2*I)*a^(1/3) +
 (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*Sqrt[1 - (b
^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])
/(3*I + Sqrt[3]), ArcSin[Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(
1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/(2*(-2
 + (-1)^(1/3))*b^(2/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1
+ (-1)^(1/3))*a^(1/3))]*Sqrt[-a - b*x^3])
```

Maple [F] time = 0.087, size = 0, normalized size = 0.

$$\int x\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)^{-1}\frac{1}{\sqrt{-bx^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x)

[Out] int(x/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{\sqrt{-bx^3-a}\left(b^{\frac{1}{3}}x-2\,a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x/(sqrt(-b*x^3 - a)*(b^(1/3)*x - 2*a^(1/3))),x, algorithm="maxima")

[Out] -integrate(x/(sqrt(-b*x^3 - a)*(b^(1/3)*x - 2*a^(1/3))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x/(sqrt(-b*x^3 - a)*(b^(1/3)*x - 2*a^(1/3))),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{-2\sqrt[3]{a}\sqrt{-a-bx^3}+\sqrt[3]{b}x\sqrt{-a-bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(2*a**(1/3)-b**(1/3)*x)/(-b*x**3-a)**(1/2),x)

[Out] -Integral(x/(-2*a**(1/3)*sqrt(-a - b*x**3) + b**(1/3)*x*sqrt(-a -
 b*x**3)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x}{\sqrt{-bx^3 - a}\left(b^{\frac{1}{3}}x - 2\,a^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x/(sqrt(-b*x^3 - a)*(b^(1/3)*x - 2*a^(1/3))),x, algorithm="giac")
```

```
[Out] integrate(-x/(sqrt(-b*x^3 - a)*(b^(1/3)*x - 2*a^(1/3))), x)
```

3.77 $\int \frac{x}{(c+dx)\sqrt{c^3-8d^3x^3}} \, dx$

Optimal. Leaf size=202

$$\frac{2\tanh^{-1}\left(\frac{(c-2dx)^2}{3\sqrt{c}\sqrt{c^3-8d^3x^3}}\right)}{9\sqrt{c}d^2}-\frac{\sqrt{2+\sqrt{3}}(c-2dx)\sqrt{\frac{c^2+2cdx+4d^2x^2}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)c-2dx}{\left(1+\sqrt{3}\right)c-2dx}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}d^2\sqrt{\frac{c(c-2dx)}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\sqrt{c^3-8d^3x^3}}$$

```
[Out] (2*ArcTanh[(c - 2*d*x)^2/(3*Sqrt[c]*Sqrt[c^3 - 8*d^3*x^3])])/(9*S
qrt[c]*d^2) - (Sqrt[2 + Sqrt[3]]*(c - 2*d*x)*Sqrt[(c^2 + 2*c*d*x
+ 4*d^2*x^2)/((1 + Sqrt[3])*c - 2*d*x)^2]*EllipticF[ArcSin[((1 -
Sqrt[3])*c - 2*d*x)/((1 + Sqrt[3])*c - 2*d*x)], -7 - 4*Sqrt[3]])/
(3*3^(1/4)*d^2*Sqrt[(c*(c - 2*d*x))/((1 + Sqrt[3])*c - 2*d*x)^2]*
Sqrt[c^3 - 8*d^3*x^3])
```

Rubi [A] time = 0.517249, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$

$$\frac{2\tanh^{-1}\left(\frac{(c-2dx)^2}{3\sqrt{c}\sqrt{c^3-8d^3x^3}}\right)}{9\sqrt{c}d^2}-\frac{\sqrt{2+\sqrt{3}}(c-2dx)\sqrt{\frac{c^2+2cdx+4d^2x^2}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)c-2dx}{\left(1+\sqrt{3}\right)c-2dx}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}d^2\sqrt{\frac{c(c-2dx)}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\sqrt{c^3-8d^3x^3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/((c + d*x)*Sqrt[c^3 - 8*d^3*x^3]),x]
```

```
[Out] (2*ArcTanh[(c - 2*d*x)^2/(3*Sqrt[c]*Sqrt[c^3 - 8*d^3*x^3])])/(9*S
qrt[c]*d^2) - (Sqrt[2 + Sqrt[3]]*(c - 2*d*x)*Sqrt[(c^2 + 2*c*d*x
+ 4*d^2*x^2)/((1 + Sqrt[3])*c - 2*d*x)^2]*EllipticF[ArcSin[((1 -
Sqrt[3])*c - 2*d*x)/((1 + Sqrt[3])*c - 2*d*x)], -7 - 4*Sqrt[3]])/
(3*3^(1/4)*d^2*Sqrt[(c*(c - 2*d*x))/((1 + Sqrt[3])*c - 2*d*x)^2]*
Sqrt[c^3 - 8*d^3*x^3])
```

Rubi in Sympy [A] time = 132.163, size = 561, normalized size = 2.78

$$\frac{3^{\frac{3}{4}}\sqrt{\frac{c^2+2cdx+4d^2x^2}{\left(c\left(1+\sqrt{3}\right)-2dx\right)^2}}\left(1+\sqrt{3}\right)\sqrt{\sqrt{3}+2}\,(c-2dx)\,F\left(\operatorname{asin}\left(-\frac{-c\left(-1+\sqrt{3}\right)-2dx}{c\left(1+\sqrt{3}\right)-2dx}\right)\middle|-7-4\sqrt{3}\right)}{3d^2\sqrt{\frac{c(c-2dx)}{\left(c\left(1+\sqrt{3}\right)-2dx\right)^2}}\left(\sqrt{3}+3\right)\sqrt{c^3-8d^3x^3}} + \frac{3^{\frac{3}{4}}\sqrt{\frac{c^2\left(1+\frac{2dx}{c}+\frac{4d^2x^2}{c^2}\right)}{\left(c\left(1+\sqrt{3}\right)-2dx\right)^2}}\sqrt{3+2\sqrt{3}}\sqrt{-\sqrt{3}+2}\,(c-2dx)\operatorname{atanh}\left(\frac{\sqrt{-\frac{\left(c\left(-1+\sqrt{3}\right)+2dx\right)^2}{\left(c\left(1+\sqrt{3}\right)-2dx\right)^2}+1}}{\sqrt{3+2\sqrt{3}}\sqrt{\frac{\left(c\left(-1+\sqrt{3}\right)+2dx\right)^2}{\left(c\left(1+\sqrt{3}\right)-2dx\right)^2}-4\sqrt{3}+7}}\right)}{9d^2\sqrt{\frac{c(c-2dx)}{\left(c\left(1+\sqrt{3}\right)-2dx\right)^2}}\sqrt{c^3-8d^3x^3}} - \frac{4\sqrt[4]{3}\sqrt{\frac{c^2\left(1+\frac{2dx}{c}+\frac{4d^2x^2}{c^2}\right)}{\left(c\left(1+\sqrt{3}\right)-2dx\right)^2}}\sqrt{-\sqrt{3}+2}\,(c-2dx)\left(4\sqrt{3}+7;\operatorname{asin}\left(\frac{c\left(-1+\sqrt{3}\right)+2dx}{c\left(1+\sqrt{3}\right)-2dx}\right)\middle|-7-4\sqrt{3}\right)}{d^2\sqrt{\frac{c(c-2dx)}{\left(c\left(1+\sqrt{3}\right)-2dx\right)^2}}\sqrt{-4\sqrt{3}+7}\left(-\sqrt{3}+3\right)\left(\sqrt{3}+3\right)\sqrt{c^3-8d^3x^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(d*x+c)/(-8*d**3*x**3+c**3)**(1/2),x)
```

```
[Out] 3**(3/4)*sqrt((c**2 + 2*c*d*x + 4*d**2*x**2)/(c*(1 + sqrt(3)) - 2
*d*x)**2)*(1 + sqrt(3))*sqrt(sqrt(3) + 2)*(c - 2*d*x)*elliptic_f(
asin(-(-c*(-1 + sqrt(3)) - 2*d*x)/(c*(1 + sqrt(3)) - 2*d*x)), -7
- 4*sqrt(3))/(3*d**2*sqrt(c*(c - 2*d*x)/(c*(1 + sqrt(3)) - 2*d*x)
**2)*(sqrt(3) + 3)*sqrt(c**3 - 8*d**3*x**3)) + 3**(3/4)*sqrt(c**2
*(1 + 2*d*x/c + 4*d**2*x**2/c**2)/(c*(1 + sqrt(3)) - 2*d*x)**2)*s
qrt(3 + 2*sqrt(3))*sqrt(-sqrt(3) + 2)*(c - 2*d*x)*atanh(sqrt(-(c*
(-1 + sqrt(3)) + 2*d*x)**2/(c*(1 + sqrt(3)) - 2*d*x)**2 + 1)/(sqr
t(3 + 2*sqrt(3))*sqrt((c*(-1 + sqrt(3)) + 2*d*x)**2/(c*(1 + sqrt(
3)) - 2*d*x)**2 - 4*sqrt(3) + 7)))/(9*d**2*sqrt(c*(c - 2*d*x)/(c*
(1 + sqrt(3)) - 2*d*x)**2)*sqrt(c**3 - 8*d**3*x**3)) - 4*3**(1/4)
*sqrt(c**2*(1 + 2*d*x/c + 4*d**2*x**2/c**2)/(c*(1 + sqrt(3)) - 2*
d*x)**2)*sqrt(-sqrt(3) + 2)*(c - 2*d*x)*elliptic_pi(4*sqrt(3) + 7
, asin((c*(-1 + sqrt(3)) + 2*d*x)/(c*(1 + sqrt(3)) - 2*d*x)), -7
- 4*sqrt(3))/(d**2*sqrt(c*(c - 2*d*x)/(c*(1 + sqrt(3)) - 2*d*x)**
2)*sqrt(-4*sqrt(3) + 7)*(-sqrt(3) + 3)*(sqrt(3) + 3)*sqrt(c**3 -
8*d**3*x**3))
```

Mathematica [C] time = 1.1701, size = 295, normalized size = 1.46

$$\frac{\sqrt{\frac{c-2dx}{\left(1+\sqrt[3]{-1}\right)c}}\left(\left(\sqrt[3]{-1}-2\right)\left(\sqrt[3]{-1}c+2dx\right)\sqrt{\frac{\sqrt[3]{-1}\left(c+2\sqrt[3]{-1}dx\right)}{\left(1+\sqrt[3]{-1}\right)c}}F\left(\sin^{-1}\left(\sqrt{\frac{c-2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}\right)\middle|\sqrt[3]{-1}\right)+\frac{2\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)c\sqrt{\frac{c-2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}\sqrt{\cdots}}{\cdots}\right)}{\left(\sqrt[3]{-1}-2\right)d^2\sqrt{\frac{c-2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}\sqrt{c^3-8d^3x^3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x/((c + d*x)*Sqrt[c^3 - 8*d^3*x^3]),x]
```

```
[Out] (Sqrt[(c - 2*d*x)/((1 + (-1)^(1/3))*c)]*((-2 + (-1)^(1/3))*((-1)^
(1/3)*c + 2*d*x)*Sqrt[((-1)^(1/3)*(c + 2*(-1)^(1/3)*d*x))/((1 + (
-1)^(1/3))*c)]*EllipticF[ArcSin[Sqrt[(c - 2*(-1)^(2/3)*d*x)/((1 +
 (-1)^(1/3))*c)]], (-1)^(1/3)] + (2*(-1)^(1/3)*(1 + (-1)^(1/3))*c
*Sqrt[(c - 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]*Sqrt[(c^2 + 2*
c*d*x + 4*d^2*x^2)/c^2]*EllipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), A
rcSin[Sqrt[(c - 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]], (-1)^(1
/3)])/Sqrt[3]))/((-2 + (-1)^(1/3))*d^2*Sqrt[(c - 2*(-1)^(2/3)*d*x
)/((1 + (-1)^(1/3))*c)]*Sqrt[c^3 - 8*d^3*x^3])
```

Maple [B] time = 0.011, size = 653, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x)
```

```
[Out] 2/d*(1/2*(-1/2+1/2*I*3^(1/2))*c/d-1/2*(-1/2-1/2*I*3^(1/2))*c/d)*(
(x-1/2*(-1/2-1/2*I*3^(1/2))*c/d)/(1/2*(-1/2+1/2*I*3^(1/2))*c/d-1/
2*(-1/2-1/2*I*3^(1/2))*c/d))^(1/2)*((x-1/2*c/d)/(1/2*(-1/2-1/2*I*
3^(1/2))*c/d-1/2*c/d))^(1/2)*((x-1/2*(-1/2+1/2*I*3^(1/2))*c/d)/(1
/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*(-1/2+1/2*I*3^(1/2))*c/d))^(1/2)/
(-8*d^3*x^3+c^3)^(1/2)*EllipticF(((x-1/2*(-1/2-1/2*I*3^(1/2))*c/d
)/(1/2*(-1/2+1/2*I*3^(1/2))*c/d-1/2*(-1/2-1/2*I*3^(1/2))*c/d))^(1
/2),((1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*(-1/2+1/2*I*3^(1/2))*c/d)/
(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d))^(1/2))-2*c/d^2*(1/2*(-1/2
+1/2*I*3^(1/2))*c/d-1/2*(-1/2-1/2*I*3^(1/2))*c/d)*((x-1/2*(-1/2-1
/2*I*3^(1/2))*c/d)/(1/2*(-1/2+1/2*I*3^(1/2))*c/d-1/2*(-1/2-1/2*I*
3^(1/2))*c/d))^(1/2)*((x-1/2*c/d)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1
/2*c/d))^(1/2)*((x-1/2*(-1/2+1/2*I*3^(1/2))*c/d)/(1/2*(-1/2-1/2*I
```

```
*3^(1/2))*c/d-1/2*(-1/2+1/2*I*3^(1/2))*c/d))^(1/2)/(-8*d^3*x^3+c^
3)^(1/2)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d+c/d)*EllipticPi(((x-1/2*(-
1/2-1/2*I*3^(1/2))*c/d)/(1/2*(-1/2+1/2*I*3^(1/2))*c/d-1/2*(-1/2-1
/2*I*3^(1/2))*c/d))^(1/2),(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*(-1/2
+1/2*I*3^(1/2))*c/d)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d+c/d),((1/2*(-1
/2-1/2*I*3^(1/2))*c/d-1/2*(-1/2+1/2*I*3^(1/2))*c/d)/(1/2*(-1/2-1/
2*I*3^(1/2))*c/d-1/2*c/d))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-8\,d^3x^3+c^3}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(sqrt(-8*d^3*x^3 + c^3)*(d*x + c)),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(-8*d^3*x^3 + c^3)*(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x}{\sqrt{-8\,d^3x^3+c^3}(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(sqrt(-8*d^3*x^3 + c^3)*(d*x + c)),x, algorithm="fricas")
```

```
[Out] integral(x/(sqrt(-8*d^3*x^3 + c^3)*(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-(-c+2dx)(c^2+2cdx+4d^2x^2)}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(d*x+c)/(-8*d**3*x**3+c**3)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-(-c + 2*d*x)*(c**2 + 2*c*d*x + 4*d**2*x**2))*(c
 + d*x)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-8\,d^3x^3+c^3}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(-8*d^3*x^3 + c^3)*(d*x + c)),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(-8*d^3*x^3 + c^3)*(d*x + c)), x)
```

3.78 $\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=42

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{2\sqrt{3}-3}}$$

```
[Out] (-2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]])/Sqrt[-
3 + 2*Sqrt[3]]
```

Rubi [A] time = 0.196289, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{2\sqrt{3}-3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]])/Sqrt[-
3 + 2*Sqrt[3]]
```

Rubi in Sympy [A] time = 29.8299, size = 134, normalized size = 3.19

$$-\frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{x^2-x+1}{\left(x+1+\sqrt{3}\right)^2}}\,(x+1)\,\operatorname{atanh}\left(\frac{\left(-\sqrt{3}+2\right)\sqrt{-\frac{\left(-x-1+\sqrt{3}\right)^2}{\left(x+1+\sqrt{3}\right)^2}+1}}{\sqrt{\frac{\left(-x-1+\sqrt{3}\right)^2}{\left(x+1+\sqrt{3}\right)^2}-4\sqrt{3}+7}}\right)}{3\sqrt{\frac{x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{-\sqrt{3}+2}\sqrt{x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+x+3**(1/2))/(1+x-3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] -2*3**(3/4)*sqrt((x**2 - x + 1)/(x + 1 + sqrt(3))**2)*(x + 1)*ata
nh((-sqrt(3) + 2)*sqrt(-(-x - 1 + sqrt(3))**2/(x + 1 + sqrt(3))**
```

```
2 + 1)/sqrt((-x - 1 + sqrt(3))**2/(x + 1 + sqrt(3))**2 - 4*sqrt(3
) + 7))/(3*sqrt((x + 1)/(x + 1 + sqrt(3))**2)*sqrt(-sqrt(3) + 2)*
sqrt(x**3 + 1))
```

Mathematica [C] time = 0.499341, size = 267, normalized size = 6.36

$$-\frac{2\sqrt{6}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\left(4i\sqrt{-2ix+\sqrt{3}+i}\sqrt{x^2-x+1}\left(\frac{2i\sqrt{3}}{-3+(2+i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)+\sqrt{2ix+\sqrt{3}-i}\left(\left(\sqrt{3}+(-2-i)\right)x-\right.\right.}{\left(-3+(2+i)\sqrt{3}\right)\sqrt{-2ix+\sqrt{3}+i}\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*Sqrt[6]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(Sqrt[-I + Sqrt[3]
+ (2*I)*x]*((1 + 2*I) - I*Sqrt[3] + ((-2 - I) + Sqrt[3])*x)*Ellip
ticF[ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sq
rt[3])/(3*I + Sqrt[3])] + (4*I)*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[
1 - x + x^2]*EllipticPi[((2*I)*Sqrt[3])/(-3 + (2 + I)*Sqrt[3]), A
rcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])
/(3*I + Sqrt[3])]))/((-3 + (2 + I)*Sqrt[3])*Sqrt[I + Sqrt[3] - (2
*I)*x]*Sqrt[1 + x^3])
```

Maple [C] time = 0.074, size = 245, normalized size = 5.8

$$2\,\frac{3/2-i/2\sqrt{3}}{\sqrt{x^3+1}}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}EllipticF\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},\sqrt{\frac{-3/2+i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\right)$$
$$-4\,\frac{3/2-i/2\sqrt{3}}{\sqrt{x^3+1}}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}EllipticPi\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},-1/3\left(-3/2+i/2\sqrt{3}\right)\sqrt{3},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+x+3^(1/2))/(1+x-3^(1/2))/(x^3+1)^(1/2),x)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1
/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/
(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1
/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^
(1/2))-4*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((
x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^
(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticPi(((1+x
```

```
)/(3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2),((
-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+\sqrt{3}+1}{\sqrt{x^3+1}\left(x-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x + sqrt(3) + 1)/(sqrt(x^3 + 1)*(x - sqrt(3) + 1)),x, algorithm="maxima
```

```
[Out] integrate((x + sqrt(3) + 1)/(sqrt(x^3 + 1)*(x - sqrt(3) + 1)), x)
```

Fricas [A] time = 0.364487, size = 363, normalized size = 8.64

$$\frac{1}{6}\sqrt{3}\sqrt{2\sqrt{3}+3}\log\left(\frac{6322680\,x^8-13553256\,x^7+26133432\,x^6-63422352\,x^5+113743056\,x^4-136435776\,x^3+102727296\,\ldots}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x + sqrt(3) + 1)/(sqrt(x^3 + 1)*(x - sqrt(3) + 1)),x, algorithm="fricas
```

```
[Out] 1/6*sqrt(3)*sqrt(2*sqrt(3) + 3)*log((6322680*x^8 - 13553256*x^7 +
 26133432*x^6 - 63422352*x^5 + 113743056*x^4 - 136435776*x^3 + 10
2727296*x^2 - 4*(1694157*x^6 - 5868732*x^5 + 10586298*x^4 - 12840
912*x^3 + 9886740*x^2 + 2*sqrt(3)*(489061*x^6 - 1694157*x^5 + 305
6001*x^4 - 3706852*x^3 + 2854056*x^2 - 1198884*x + 205636) - 4153
056*x + 712344)*sqrt(x^3 + 1)*sqrt(2*sqrt(3) + 3) + sqrt(3)*(3650
401*x^8 - 7824976*x^7 + 15088144*x^6 - 36616912*x^5 + 65669584*x^
4 - 78771232*x^3 + 59309632*x^2 - 24558208*x + 4193392) - 4253606
4*x + 7263168)/(6322680*x^8 - 37028184*x^7 + 94872792*x^6 - 13890
3408*x^5 + 127105440*x^4 - 74438112*x^3 + 27246240*x^2 + sqrt(3)*
(3650401*x^8 - 21378232*x^7 + 54774832*x^6 - 80195920*x^5 + 73384
360*x^4 - 42976864*x^3 + 15730624*x^2 - 3290176*x + 301072) - 569
8752*x + 521472))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1+\sqrt{3}}{\sqrt{(x+1)(x^2-x+1)}\left(x-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+x+3**(1/2))/(1+x-3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] Integral((x + 1 + sqrt(3))/(sqrt((x + 1)*(x**2 - x + 1))*(x - sqr
t(3) + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+\sqrt{3}+1}{\sqrt{x^3+1}\left(x-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x + sqrt(3) + 1)/(sqrt(x^3 + 1)*(x - sqrt(3) + 1)),x, algorithm="giac")
```

```
[Out] integrate((x + sqrt(3) + 1)/(sqrt(x^3 + 1)*(x - sqrt(3) + 1)), x)
```

3.79 $\int \frac{1+\sqrt{3}-x}{\left(1-\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=46

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{2\sqrt{3}-3}}$$

```
[Out] (2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*(1 - x))/Sqrt[1 - x^3]])/Sqrt[-3
 + 2*Sqrt[3]]
```

Rubi [A] time = 0.19175, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{2\sqrt{3}-3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + Sqrt[3] - x)/((1 - Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*(1 - x))/Sqrt[1 - x^3]])/Sqrt[-3
 + 2*Sqrt[3]]
```

Rubi in Sympy [A] time = 36.4102, size = 133, normalized size = 2.89

$$\frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{x^2+x+1}{\left(-x+1+\sqrt{3}\right)^2}}\,(-x+1)\,\operatorname{atanh}\left(\frac{\sqrt{1-\frac{\left(x-1+\sqrt{3}\right)^2}{\left(-x+1+\sqrt{3}\right)^2}}\left(-\sqrt{3}+2\right)}{\sqrt{-4\sqrt{3}+7+\frac{\left(x-1+\sqrt{3}\right)^2}{\left(-x+1+\sqrt{3}\right)^2}}}\right)}{3\sqrt{\frac{-x+1}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{-\sqrt{3}+2}\sqrt{-x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1-x+3**(1/2))/(1-x-3**(1/2))/(-x**3+1)**(1/2),x)
```

```
[Out] 2*3**(3/4)*sqrt((x**2 + x + 1)/(-x + 1 + sqrt(3))**2)*(-x + 1)*at
anh(sqrt(1 - (x - 1 + sqrt(3))**2/(-x + 1 + sqrt(3))**2)*(-sqrt(3
```

```
) + 2)/sqrt(-4*sqrt(3) + 7 + (x - 1 + sqrt(3))**2/(-x + 1 + sqrt(
3))**2))/(3*sqrt((-x + 1)/(-x + 1 + sqrt(3))**2)*sqrt(-sqrt(3) +
2)*sqrt(-x**3 + 1))
```

Mathematica [C] time = 0.514982, size = 269, normalized size = 5.85

$$\frac{2\sqrt{6}\sqrt{\frac{i(x-1)}{\sqrt{3}-3i}}\left(4\sqrt{-2ix+\sqrt{3}-i}\sqrt{x^2+x+1}\left(\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}-i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right)+\sqrt{2ix+\sqrt{3}+i}\left(\left((1+2i)-i\sqrt{3}\right)x-\right.\right.}{\left((1+2i)\sqrt{3}-3i\right)\sqrt{-2ix+\sqrt{3}-i}\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(1 + Sqrt[3] - x)/((1 - Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*Sqrt[6]*Sqrt[(I*(-1 + x))/(-3*I + Sqrt[3])]*(Sqrt[I + Sqrt[3]
+ (2*I)*x]*((2 + I) - Sqrt[3] + ((1 + 2*I) - I*Sqrt[3])*x)*Ellipt
icF[ArcSin[Sqrt[-I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sq
rt[3])/(-3*I + Sqrt[3])] + 4*Sqrt[-I + Sqrt[3] - (2*I)*x]*Sqrt[1
+ x + x^2]*EllipticPi[(2*Sqrt[3])/(-3*I + (1 + 2*I)*Sqrt[3]), Arc
Sin[Sqrt[-I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/
(-3*I + Sqrt[3])]))/((-3*I + (1 + 2*I)*Sqrt[3])*Sqrt[-I + Sqrt[3]
 - (2*I)*x]*Sqrt[1 - x^3])
```

Maple [C] time = 0.096, size = 243, normalized size = 5.3

$$-\frac{2i}{3}\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}EllipticF\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)\frac{}{\sqrt{-}}$$

$$+\frac{4i}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}+\sqrt{3}}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}EllipticPi\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\frac{i}{-\frac{3}{2}+\frac{i}{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1-x+3^(1/2))/(1-x-3^(1/2))/(-x^3+1)^(1/2),x)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/(-
3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2
)/(-x^3+1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3
^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))+4*I*(I*(x+1
/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/(-3/2+1/2*I*3^(1/2)))^(1
/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(-3/2
+1/2*I*3^(1/2)+3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^
```

```
(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-3/2+1/2*I*3^(1/2)+3^(1/2)),(I*3
^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-\sqrt{3}-1}{\sqrt{-x^3+1}\left(x+\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x - sqrt(3) - 1)/(sqrt(-x^3 + 1)*(x + sqrt(3) - 1)),x, algorithm="maxim

[Out] integrate((x - sqrt(3) - 1)/(sqrt(-x^3 + 1)*(x + sqrt(3) - 1)), x
)
```

Fricas [A] time = 0.355209, size = 366, normalized size = 7.96

$$\frac{1}{6}\sqrt{3}\sqrt{2\sqrt{3}+3}\log\left(\frac{6322680\,x^8+13553256\,x^7+26133432\,x^6+63422352\,x^5+113743056\,x^4+136435776\,x^3+102727296\,x}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x - sqrt(3) - 1)/(sqrt(-x^3 + 1)*(x + sqrt(3) - 1)),x, algorithm="frica

[Out] 1/6*sqrt(3)*sqrt(2*sqrt(3) + 3)*log((6322680*x^8 + 13553256*x^7 +
 26133432*x^6 + 63422352*x^5 + 113743056*x^4 + 136435776*x^3 + 10
2727296*x^2 + 4*(1694157*x^6 + 5868732*x^5 + 10586298*x^4 + 12840
912*x^3 + 9886740*x^2 + 2*sqrt(3)*(489061*x^6 + 1694157*x^5 + 305
6001*x^4 + 3706852*x^3 + 2854056*x^2 + 1198884*x + 205636) + 4153
056*x + 712344)*sqrt(-x^3 + 1)*sqrt(2*sqrt(3) + 3) + sqrt(3)*(365
0401*x^8 + 7824976*x^7 + 15088144*x^6 + 36616912*x^5 + 65669584*x
^4 + 78771232*x^3 + 59309632*x^2 + 24558208*x + 4193392) + 425360
64*x + 7263168)/(6322680*x^8 + 37028184*x^7 + 94872792*x^6 + 1389
03408*x^5 + 127105440*x^4 + 74438112*x^3 + 27246240*x^2 + sqrt(3)
*(3650401*x^8 + 21378232*x^7 + 54774832*x^6 + 80195920*x^5 + 7338
4360*x^4 + 42976864*x^3 + 15730624*x^2 + 3290176*x + 301072) + 56
98752*x + 521472))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-\sqrt{3}-1}{\sqrt{-(x-1)\,(x^2+x+1)}\left(x-1+\sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1-x+3**(1/2))/(1-x-3**(1/2))/(-x**3+1)**(1/2),x)
```

```
[Out] Integral((x - sqrt(3) - 1)/(sqrt(-(x - 1)*(x**2 + x + 1))*(x - 1
+ sqrt(3))), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-\sqrt{3}-1}{\sqrt{-x^3+1}\left(x+\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x - sqrt(3) - 1)/(sqrt(-x^3 + 1)*(x + sqrt(3) - 1)),x, algorithm="giac"
```

```
[Out] integrate((x - sqrt(3) - 1)/(sqrt(-x^3 + 1)*(x + sqrt(3) - 1)), x
)
```

3.80 $\int \frac{1+\sqrt{3}-x}{\left(1-\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=44

$$\frac{2\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}(1-x)}{\sqrt{x^3-1}}\right)}{\sqrt{2\sqrt{3}-3}}$$

```
[Out] (2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*(1 - x))/Sqrt[-1 + x^3]])/Sqrt[-3
 + 2*Sqrt[3]]
```

Rubi [A] time = 0.177109, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059

$$\frac{2\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}(1-x)}{\sqrt{x^3-1}}\right)}{\sqrt{2\sqrt{3}-3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + Sqrt[3] - x)/((1 - Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*(1 - x))/Sqrt[-1 + x^3]])/Sqrt[-3
 + 2*Sqrt[3]]
```

Rubi in Sympy [A] time = 20.1712, size = 76, normalized size = 1.73

$$\frac{2\tilde{\infty}\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}(-x+1)\,F\left(\operatorname{asin}\left(\frac{-x+1+\sqrt{3}}{-x-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt{\frac{x-1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1-x+3**(1/2))/(1-x-3**(1/2))/(x**3-1)**(1/2),x)
```

```
[Out] 2*zoo*sqrt((x**2 + x + 1)/(-x - sqrt(3) + 1)**2)*(-x + 1)*ellipti
c_f(asin((-x + 1 + sqrt(3))/(-x - sqrt(3) + 1)), -7 + 4*sqrt(3))/
(sqrt((x - 1)/(-x - sqrt(3) + 1)**2)*sqrt(x**3 - 1))
```

Mathematica [C] time = 0.475992, size = 267, normalized size = 6.07

$$\frac{2\sqrt{6}\sqrt{\frac{i(x-1)}{\sqrt{3}-3i}}\left(4\sqrt{-2ix+\sqrt{3}-i}\sqrt{x^2+x+1}\left(\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}-i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right)+\sqrt{2ix+\sqrt{3}+i}\left(\left((1+2i)-i\sqrt{3}\right)x-\right.\right.}{\left((1+2i)\sqrt{3}-3i\right)\sqrt{-2ix+\sqrt{3}-i}\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(1 + Sqrt[3] - x)/((1 - Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*Sqrt[6]*Sqrt[(I*(-1 + x))/(-3*I + Sqrt[3])]*(Sqrt[I + Sqrt[3]
+ (2*I)*x]*((2 + I) - Sqrt[3] + ((1 + 2*I) - I*Sqrt[3])*x)*Ellipt
icF[ArcSin[Sqrt[-I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sq
rt[3])/(-3*I + Sqrt[3])] + 4*Sqrt[-I + Sqrt[3] - (2*I)*x]*Sqrt[1
+ x + x^2]*EllipticPi[(2*Sqrt[3])/(-3*I + (1 + 2*I)*Sqrt[3]), Arc
Sin[Sqrt[-I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/
(-3*I + Sqrt[3])]))/((-3*I + (1 + 2*I)*Sqrt[3])*Sqrt[-I + Sqrt[3]
 - (2*I)*x]*Sqrt[-1 + x^3])
```

Maple [C] time = 0.053, size = 245, normalized size = 5.6

$$2\,\frac{-3/2-i/2\sqrt{3}}{\sqrt{x^3-1}}\sqrt{\frac{-1+x}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2-i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2+i/2\sqrt{3}}{3/2+i/2\sqrt{3}}}\mathit{EllipticF}\left(\sqrt{\frac{-1+x}{-3/2-i/2\sqrt{3}}},\sqrt{\frac{3/2+i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\right)$$
$$-4\,\frac{-3/2-i/2\sqrt{3}}{\sqrt{x^3-1}}\sqrt{\frac{-1+x}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2-i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2+i/2\sqrt{3}}{3/2+i/2\sqrt{3}}}\mathit{EllipticPi}\left(\sqrt{\frac{-1+x}{-3/2-i/2\sqrt{3}}},1/3\,\left(3/2+i/2\sqrt{3}\right)\sqrt{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1-x+3^(1/2))/(1-x-3^(1/2))/(x^3-1)^(1/2),x)
```

```
[Out] 2*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/
2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2)
)/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((-1+x)/(-3/
2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))
^(1/2))-4*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2
)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I
*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticPi(((-
1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),1/3*(3/2+1/2*I*3^(1/2))*3^(1/2),
((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-\sqrt{3}-1}{\sqrt{x^3-1}\left(x+\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x - sqrt(3) - 1)/(sqrt(x^3 - 1)*(x + sqrt(3) - 1)),x, algorithm="maxima
```

```
[Out] integrate((x - sqrt(3) - 1)/(sqrt(x^3 - 1)*(x + sqrt(3) - 1)), x)
```

Fricas [A] time = 0.355566, size = 140, normalized size = 3.18

$$\frac{1}{3}\sqrt{3}\sqrt{2\sqrt{3}+3}\arctan\left(\frac{2340\,x^4+4680\,x^3+6516\,x^2+\sqrt{3}\left(1351\,x^4+2702\,x^3+3762\,x^2+3284\,x+1060\right)+5688\,x+1836}{2\sqrt{x^3-1}\left(627\,x^2+2\sqrt{3}(181\,x^2+265\,x+97)+918\,x+336\right)\sqrt{2\sqrt{3}+3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x - sqrt(3) - 1)/(sqrt(x^3 - 1)*(x + sqrt(3) - 1)),x, algorithm="fricas
```

```
[Out] 1/3*sqrt(3)*sqrt(2*sqrt(3) + 3)*arctan(1/2*(2340*x^4 + 4680*x^3 +
 6516*x^2 + sqrt(3)*(1351*x^4 + 2702*x^3 + 3762*x^2 + 3284*x + 10
60) + 5688*x + 1836)/(sqrt(x^3 - 1)*(627*x^2 + 2*sqrt(3)*(181*x^2
 + 265*x + 97) + 918*x + 336)*sqrt(2*sqrt(3) + 3)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-\sqrt{3}-1}{\sqrt{(x-1)\left(x^2+x+1\right)}\left(x-1+\sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1-x+3**(1/2))/(1-x-3**(1/2))/(x**3-1)**(1/2),x)
```

```
[Out] Integral((x - sqrt(3) - 1)/(sqrt((x - 1)*(x**2 + x + 1))*(x - 1 +
 sqrt(3))), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-\sqrt{3}-1}{\sqrt{x^3-1}\left(x+\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x - sqrt(3) - 1)/(sqrt(x^3 - 1)*(x + sqrt(3) - 1)),x, algorithm="giac")
```

```
[Out] integrate((x - sqrt(3) - 1)/(sqrt(x^3 - 1)*(x + sqrt(3) - 1)), x)
```

3.81 $\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=44

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}(x+1)}{\sqrt{-x^3-1}}\right)}{\sqrt{2\sqrt{3}-3}}$$

```
[Out] (-2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*(1 + x))/Sqrt[-1 - x^3]])/Sqrt[-
3 + 2*Sqrt[3]]
```

Rubi [A] time = 0.161418, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}(x+1)}{\sqrt{-x^3-1}}\right)}{\sqrt{2\sqrt{3}-3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (-2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*(1 + x))/Sqrt[-1 - x^3]])/Sqrt[-
3 + 2*Sqrt[3]]
```

Rubi in Sympy [A] time = 16.9534, size = 80, normalized size = 1.82

$$\frac{2\tilde{\infty}\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,(x+1)\,F\left(\operatorname{asin}\left(\frac{x+1+\sqrt{3}}{x-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt{\frac{-x-1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+x+3**(1/2))/(1+x-3**(1/2))/(-x**3-1)**(1/2),x)
```

```
[Out] 2*zoo*sqrt((x**2 - x + 1)/(x - sqrt(3) + 1)**2)*(x + 1)*elliptic_
f(asin((x + 1 + sqrt(3))/(x - sqrt(3) + 1)), -7 + 4*sqrt(3))/(sqr
t((-x - 1)/(x - sqrt(3) + 1)**2)*sqrt(-x**3 - 1))
```

Mathematica [C] time = 0.500513, size = 269, normalized size = 6.11

$$-\frac{2\sqrt{6}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\left(4i\sqrt{-2ix+\sqrt{3}+i}\sqrt{x^2-x+1}\left(\frac{2i\sqrt{3}}{-3+(2+i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)+\sqrt{2ix+\sqrt{3}-i}\left(\left(\sqrt{3}+(-2-i)\right)x-\right.\right.}{\left(-3+(2+i)\sqrt{3}\right)\sqrt{-2ix+\sqrt{3}+i}\sqrt{-x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (-2*Sqrt[6]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(Sqrt[-I + Sqrt[3]
+ (2*I)*x]*((1 + 2*I) - I*Sqrt[3] + ((-2 - I) + Sqrt[3])*x)*Ellip
ticF[ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sq
rt[3])/(3*I + Sqrt[3])] + (4*I)*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[
1 - x + x^2]*EllipticPi[((2*I)*Sqrt[3])/(-3 + (2 + I)*Sqrt[3]), A
rcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])
/(3*I + Sqrt[3])]))/((-3 + (2 + I)*Sqrt[3])*Sqrt[I + Sqrt[3] - (2
*I)*x]*Sqrt[-1 - x^3])
```

Maple [C] time = 0.09, size = 247, normalized size = 5.6

$$-\frac{2i}{3}\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\mathit{EllipticF}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)\frac{1}{\sqrt{-x^3}}$$

$$-\frac{4i}{\frac{3}{2}+\frac{i}{2}\sqrt{3}-\sqrt{3}}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\mathit{EllipticPi}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i}{2}\sqrt{3}-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+x+3^(1/2))/(1+x-3^(1/2))/(-x^3-1)^(1/2),x)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/
2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/
(-x^3-1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(
1/2))^(1/2),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))-4*I*(I*(x-1/2-
1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^(1/2)))^(1/2)*(
-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)/(3/2+1/2*I
*3^(1/2)-3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))
*3^(1/2))^(1/2),I*3^(1/2)/(3/2+1/2*I*3^(1/2)-3^(1/2)),(I*3^(1/2)/
(3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+\sqrt{3}+1}{\sqrt{-x^3-1}\left(x-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x + sqrt(3) + 1)/(sqrt(-x^3 - 1)*(x - sqrt(3) + 1)),x, algorithm="maxim
```

```
[Out] integrate((x + sqrt(3) + 1)/(sqrt(-x^3 - 1)*(x - sqrt(3) + 1)), x
)
```

Fricas [A] time = 0.349794, size = 143, normalized size = 3.25

$$-\frac{1}{3}\sqrt{3}\sqrt{2\sqrt{3}+3}\arctan\left(\frac{2340\,x^4-4680\,x^3+6516\,x^2+\sqrt{3}\left(1351\,x^4-2702\,x^3+3762\,x^2-3284\,x+1060\right)-5688\,x+1836}{2\sqrt{-x^3-1}\left(627\,x^2+2\sqrt{3}(181\,x^2-265\,x+97)-918\,x+336\right)\sqrt{2\sqrt{3}+3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x + sqrt(3) + 1)/(sqrt(-x^3 - 1)*(x - sqrt(3) + 1)),x, algorithm="frica
```

```
[Out] -1/3*sqrt(3)*sqrt(2*sqrt(3) + 3)*arctan(1/2*(2340*x^4 - 4680*x^3
+ 6516*x^2 + sqrt(3)*(1351*x^4 - 2702*x^3 + 3762*x^2 - 3284*x + 1
060) - 5688*x + 1836)/(sqrt(-x^3 - 1)*(627*x^2 + 2*sqrt(3)*(181*x
^2 - 265*x + 97) - 918*x + 336)*sqrt(2*sqrt(3) + 3)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1+\sqrt{3}}{\sqrt{-(x+1)(x^2-x+1)}\left(x-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+x+3**(1/2))/(1+x-3**(1/2))/(-x**3-1)**(1/2),x)
```

```
[Out] Integral((x + 1 + sqrt(3))/(sqrt(-(x + 1)*(x**2 - x + 1))*(x - sq
rt(3) + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x + \sqrt{3} + 1}{\sqrt{-x^3 - 1}\left(x - \sqrt{3} + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sqrt(3) + 1)/(sqrt(-x^3 - 1)*(x - sqrt(3) + 1)),x, algorithm="giac'
```

```
[Out] integrate((x + sqrt(3) + 1)/(sqrt(-x^3 - 1)*(x - sqrt(3) + 1)), x
)
```

3.82 $\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=69

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\sqrt[3]{b}}$$

```
[Out] (-2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) + b^(1/3)*x))/
Sqrt[a + b*x^3]])/(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rubi [A] time = 0.360771, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 58, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt
```

```
[Out] (-2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) + b^(1/3)*x))/
Sqrt[a + b*x^3]])/(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rubi in Sympy [A] time = 49.0786, size = 250, normalized size = 3.62

$$-\frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{a^{\frac{2}{3}}\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+\frac{b^{\frac{2}{3}}x^2}{a^{\frac{2}{3}}}\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\operatorname{atanh}\left(\frac{\left(-\sqrt{3}+2\right)\sqrt{-\frac{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)-\sqrt[3]{b}x\right)^2}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}+1}}{\sqrt{\frac{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)-\sqrt[3]{b}x\right)^2}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}-4\sqrt{3}+7}}\right)}{3\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{-\sqrt{3}+2}\sqrt{a+bx^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((b**(1/3)*x+a**(1/3)*(1+3**(1/2)))/(b**(1/3)*x+a**(1/3)*(1-
3**(1/2)))/(b*x**3+a)**(1/2),x)
```

```
[Out] -2*3**(3/4)*sqrt(a**(2/3)*(1 - b**(1/3)*x/a**(1/3) + b**(2/3)*x**
2/a**(2/3))/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*(a**(1/3) +
 b**(1/3)*x)*atanh((-sqrt(3) + 2)*sqrt(-(a**(1/3)*(-1 + sqrt(3))
- b**(1/3)*x)**2/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2 + 1)/sq
rt((a**(1/3)*(-1 + sqrt(3)) - b**(1/3)*x)**2/(a**(1/3)*(1 + sqrt(
3)) + b**(1/3)*x)**2 - 4*sqrt(3) + 7))/(3*b**(1/3)*sqrt(a**(1/3)*
(a**(1/3) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)
*sqrt(-sqrt(3) + 2)*sqrt(a + b*x**3))
```

Mathematica [C] time = 1.05369, size = 322, normalized size = 4.67

$$\frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{4(-1)^{5/6}\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\left(\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)}{\left((1+2i)\sqrt{3}-3i\right)\sqrt[3]{b}}-\frac{\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\sqrt[6]{-1}-\frac{i\sqrt[3]{b}x}{\sqrt[3]{a}}}F\left(\right.}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}+(}{\left(1+\sqrt[3]{}\right.}}}\right.}{\sqrt{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x
```

```
[Out] (2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(-((((-
1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(
1/3)]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1
+ (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/(3^(1/4)*b^(1/3)*Sqrt[(a^(
1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))])) + (4*(
-1)^(5/6)*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[1 - (b^(1/3)*x)/a^(1/3) +
 (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(-3*I + (1 + 2*I)*
Sqrt[3]), ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)
^(1/3))*a^(1/3))]], (-1)^(1/3)])/((-3*I + (1 + 2*I)*Sqrt[3])*b^(1
/3))))/Sqrt[a + b*x^3]
```

Maple [F] time = 0.227, size = 0, normalized size = 0.

$$\int 1\left(\sqrt[3]{b}x+\sqrt[3]{a}\left(1+\sqrt{3}\right)\right)\left(\sqrt[3]{b}x+\sqrt[3]{a}\left(-\sqrt{3}+1\right)\right)^{-1}\frac{1}{\sqrt{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(-3^(1/2)+1))/(b*x^3+a)
```

```
[Out] int((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(-3^(1/2)+
1))/(b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}\left(\sqrt{3}+1\right)}{\sqrt{bx^3+a}\left(b^{\frac{1}{3}}x - a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) + 1))/(sqrt(b*x^3 + a)*(b^(1/3)*x - a^(1/
```

```
[Out] integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) + 1))/(sqrt(b*x^3 + a)*(b
^(1/3)*x - a^(1/3)*(sqrt(3) - 1))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) + 1))/(sqrt(b*x^3 + a)*(b^(1/3)*x - a^(1/
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{a} + \sqrt{3}\sqrt[3]{a} + \sqrt[3]{b}x}{\sqrt{a+bx^3}\left(-\sqrt{3}\sqrt[3]{a} + \sqrt[3]{a} + \sqrt[3]{b}x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**(1/3)*x+a**(1/3)*(1+3**(1/2)))/(b**(1/3)*x+a**(1/3)*(1-
3**(1/2)))/(b*x**3+a)**(1/2),x)
```

```
[Out] Integral((a**(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)/(sqrt(a + b*x
**3)*(-sqrt(3)*a**(1/3) + a**(1/3) + b**(1/3)*x)), x)
```

GIAC/XCAS [A] time = 0.597072, size = 4, normalized size = 0.06

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) + 1))/(sqrt(b*x^3 + a)*(b^(1/3)*x - a^(1/
```

```
[Out] sage0*x
```

3.83 $\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$

Optimal. Leaf size=71

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\sqrt[3]{b}}$$

```
[Out] (2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) - b^(1/3)*x))/S
qrt[a - b*x^3]])/(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rubi [A] time = 0.320115, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 61, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)*Sqrt
```

```
[Out] (2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) - b^(1/3)*x))/S
qrt[a - b*x^3]])/(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rubi in Sympy [A] time = 56.3675, size = 248, normalized size = 3.49

$$\frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{a^{\frac{2}{3}}\left(1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+\frac{b^{\frac{2}{3}}x^2}{a^{\frac{2}{3}}}\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\operatorname{atanh}\left(\frac{\left(-\sqrt{3}+2\right)\sqrt{-\frac{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{b}x\right)^2}+1}}{\sqrt{\frac{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{b}x\right)^2}-4\sqrt{3}+7}}\right)}{3\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{b}x\right)^2}}\sqrt{-\sqrt{3}+2}\sqrt{a-bx^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-b**(1/3)*x+a**(1/3)*(1+3**(1/2)))/(-b**(1/3)*x+a**(1/3)*(1-
3**(1/2)))/(-b*x**3+a)**(1/2),x)
```

```
[Out] 2*3**(3/4)*sqrt(a**(2/3)*(1 + b**(1/3)*x/a**(1/3) + b**(2/3)*x**2
/a**(2/3))/(a**(1/3)*(1 + sqrt(3)) - b**(1/3)*x)**2)*(a**(1/3) -
b**(1/3)*x)*atanh((-sqrt(3) + 2)*sqrt(-(a**(1/3)*(-1 + sqrt(3)) +
 b**(1/3)*x)**2/(a**(1/3)*(1 + sqrt(3)) - b**(1/3)*x)**2 + 1)/sqr
t((a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)**2/(a**(1/3)*(1 + sqrt(3
)) - b**(1/3)*x)**2 - 4*sqrt(3) + 7))/(3*b**(1/3)*sqrt(a**(1/3)*(
a**(1/3) - b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) - b**(1/3)*x)**2)*
sqrt(-sqrt(3) + 2)*sqrt(a - b*x**3))
```

Mathematica [C] time = 2.72539, size = 446, normalized size = 6.28

$$\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(4\sqrt{3}\sqrt[3]{a}\sqrt{-\frac{2i\sqrt[3]{a}+\left(\sqrt{3}+i\right)\sqrt[3]{b}x}{\left(\sqrt{3}-3i\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\left(\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\sqrt{-\frac{i\left(\left(1-i\sqrt{3}\right)\sqrt[3]{b}x+2\sqrt[3]{a}\right)}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)|\frac{1}{2}\left(1+i\sqrt{3}\right)\right.\right.}{\left((1+2i)\sqrt{3}-3i\right)\sqrt[3]{b}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x
```

```
[Out] (2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(Sqrt[(
(-I + Sqrt[3])*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3
])*a^(1/3))]*((-3 + (2 + I)*Sqrt[3])*a^(1/3) + (-3*I + (1 + 2*I)*
Sqrt[3])*b^(1/3)*x)*EllipticF[ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 -
 I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt
[3])/2] + 4*Sqrt[3]*a^(1/3)*Sqrt[-(((2*I)*a^(1/3) + (I + Sqrt[3])
*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3)))]*Sqrt[1 + (b^(1/3)*x)/a^(
1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(-3*I + (1 +
 2*I)*Sqrt[3]), ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^
(1/3)*x))/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/((-3
*I + (1 + 2*I)*Sqrt[3])*b^(1/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3
)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a - b*x^3])
```

Maple [F] time = 0.195, size = 0, normalized size = 0.

$$\int 1\left(-\sqrt[3]{b}x+\sqrt[3]{a}\left(1+\sqrt{3}\right)\right)\left(-\sqrt[3]{b}x+\sqrt[3]{a}\left(-\sqrt{3}+1\right)\right)^{-1}\frac{1}{\sqrt{-bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(-3^(1/2)+1))/(-
b*x^3+a)^(1/2),x)
```

```
[Out] int((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(-3^(1/2
)+1))/(-b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b^{\frac{1}{3}}x - a^{\frac{1}{3}}\left(\sqrt{3}+1\right)}{\sqrt{-bx^3+a}\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) + 1))/(sqrt(-b*x^3 + a)*(b^(1/3)*x + a^(1
```

```
[Out] integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) + 1))/(sqrt(-b*x^3 + a)*(
b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) + 1))/(sqrt(-b*x^3 + a)*(b^(1/3)*x + a^(1
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{-\sqrt{3}\sqrt[3]{a} - \sqrt[3]{a} + \sqrt[3]{b}x}{\sqrt{a-bx^3}\left(-\sqrt[3]{a} + \sqrt{3}\sqrt[3]{a} + \sqrt[3]{b}x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b**(1/3)*x+a**(1/3)*(1+3**(1/2)))/(-b**(1/3)*x+a**(1/3)*(1-
3**(1/2)))/(-b*x**3+a)**(1/2),x)
```

```
[Out] Integral((-sqrt(3)*a**(1/3) - a**(1/3) + b**(1/3)*x)/(sqrt(a - b*
x**3)*(-a**(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)), x)
```

GIAC/XCAS [A] time = 0.624308, size = 4, normalized size = 0.06

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) + 1))/(sqrt(-b*x^3 + a)*(b^(1/3)*x + a^(1
```

```
[Out] sage0*x
```

3.84 $\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

Optimal. Leaf size=72

$$\frac{2\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{bx^3-a}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\sqrt[3]{b}}$$

```
[Out] (2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) - b^(1/3)*x))/Sq
rt[-a + b*x^3]])/(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rubi [A] time = 0.317444, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 62, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032

$$\frac{2\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{bx^3-a}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)*Sqrt
a + b*x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) - b^(1/3)*x))/Sq
rt[-a + b*x^3]])/(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rubi in Sympy [A] time = 48.7829, size = 163, normalized size = 2.26

$$\frac{2\tilde{\infty}\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)F\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{b}x}{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)-\sqrt[3]{b}x}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-b**(1/3)*x+a**(1/3)*(1+3**(1/2)))/(-b**(1/3)*x+a**(1/3)*(1-
3**(1/2)))/(b*x**3-a)**(1/2),x)
```

```
[Out] 2*zoo*sqrt((a**(2/3) + a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(a**(
1/3)*(-1 + sqrt(3)) + b**(1/3)*x)**2)*(a**(1/3) - b**(1/3)*x)*ell
iptic_f(asin((a**(1/3)*(1 + sqrt(3)) - b**(1/3)*x)/(-a**(1/3)*(-1
 + sqrt(3)) - b**(1/3)*x)), -7 + 4*sqrt(3))/(sqrt(-a**(1/3)*(a**(
1/3) - b**(1/3)*x)/(a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)**2)*sqr
t(-a + b*x**3))
```

Mathematica [C] time = 2.77214, size = 447, normalized size = 6.21

$$\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(4\sqrt{3}\sqrt[3]{a}\sqrt{-\frac{2i\sqrt[3]{a}+\left(\sqrt{3}+i\right)\sqrt[3]{b}x}{\left(\sqrt{3}-3i\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\left(\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\sqrt{-\frac{i\left(\left(1-i\sqrt{3}\right)\sqrt[3]{b}x+2\sqrt[3]{a}\right)}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)|\frac{1}{2}\left(1+i\sqrt{3}\right)\right.\right.}{\left((1+2i)\sqrt{3}-3i\right)\sqrt[3]{b}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x
a + b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(Sqrt[(
(-I + Sqrt[3])*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3
])*a^(1/3))]*((-3 + (2 + I)*Sqrt[3])*a^(1/3) + (-3*I + (1 + 2*I)*
Sqrt[3])*b^(1/3)*x)*EllipticF[ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 -
 I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt
[3])/2] + 4*Sqrt[3]*a^(1/3)*Sqrt[-(((2*I)*a^(1/3) + (I + Sqrt[3])
*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3)))]*Sqrt[1 + (b^(1/3)*x)/a^(
1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(-3*I + (1 +
 2*I)*Sqrt[3]), ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^
(1/3)*x))/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/((-3
*I + (1 + 2*I)*Sqrt[3])*b^(1/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3
)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[-a + b*x^3])
```

Maple [F] time = 0.121, size = 0, normalized size = 0.

$$\int 1\left(-\sqrt[3]{b}x+\sqrt[3]{a}\left(1+\sqrt{3}\right)\right)\left(-\sqrt[3]{b}x+\sqrt[3]{a}\left(-\sqrt{3}+1\right)\right)^{-1}\frac{1}{\sqrt{bx^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(-3^(1/2)+1))/(b*x^3-
a)^(1/2),x)
```

```
[Out] int((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(-3^(1/2
)+1))/(b*x^3-a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b^{\frac{1}{3}}x - a^{\frac{1}{3}}\left(\sqrt{3}+1\right)}{\sqrt{bx^3-a}\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) + 1))/(sqrt(b*x^3 - a)*(b^(1/3)*x + a^(1/
```

```
[Out] integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) + 1))/(sqrt(b*x^3 - a)*(b
^(1/3)*x + a^(1/3)*(sqrt(3) - 1))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) + 1))/(sqrt(b*x^3 - a)*(b^(1/3)*x + a^(1/
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{-\sqrt{3}\sqrt[3]{a} - \sqrt[3]{a} + \sqrt[3]{b}x}{\sqrt{-a+bx^3}\left(-\sqrt[3]{a} + \sqrt{3}\sqrt[3]{a} + \sqrt[3]{b}x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b**(1/3)*x+a**(1/3)*(1+3**(1/2)))/(-b**(1/3)*x+a**(1/3)*(1-
3**(1/2)))/(b*x**3-a)**(1/2),x)
```

```
[Out] Integral((-sqrt(3)*a**(1/3) - a**(1/3) + b**(1/3)*x)/(sqrt(-a + b
*x**3)*(-a**(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)), x)
```

GIAC/XCAS [A] time = 0.635201, size = 4, normalized size = 0.06

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) + 1))/(sqrt(b*x^3 - a)*(b^(1/3)*x + a^(1/
```

```
[Out] sage0*x
```

3.85 $\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

Optimal. Leaf size=72

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\sqrt[3]{b}}$$

```
[Out] (-2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) + b^(1/3)*x))/S
qrt[-a - b*x^3]])/(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rubi [A] time = 0.282709, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 61, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt
a - b*x^3]),x]
```

```
[Out] (-2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) + b^(1/3)*x))/S
qrt[-a - b*x^3]])/(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rubi in Sympy [A] time = 44.5732, size = 163, normalized size = 2.26

$$\frac{2\tilde{\infty}\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)F\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((b**(1/3)*x+a**(1/3)*(1+3**(1/2)))/(b**(1/3)*x+a**(1/3)*(1-
3**(1/2)))/(-b*x**3-a)**(1/2),x)
```

```
[Out] 2*zoo*sqrt((a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(-a**
(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)**2)*(a**(1/3) + b**(1/3)*x)*el
liptic_f(asin((a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)/(-a**(1/3)*(-
1 + sqrt(3)) + b**(1/3)*x)), -7 + 4*sqrt(3))/(sqrt(-a**(1/3)*(a**
(1/3) + b**(1/3)*x)/(-a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)**2)*s
qrt(-a - b*x**3))
```

Mathematica [C] time = 1.05462, size = 325, normalized size = 4.51

$$\frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{4(-1)^{5/6}\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\,\Pi\left(\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)}{\left((1+2i)\sqrt{3}-3i\right)\sqrt[3]{b}}-\frac{\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\sqrt[6]{-1}-\frac{i\sqrt[3]{b}x}{\sqrt[3]{a}}}\,F\left(\ldots\right)}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}+\ldots}{\left(1+\ldots\right)}}}\right)}{\sqrt{-a-bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x
a - b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(-((((-
1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(
1/3)]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1
+ (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/(3^(1/4)*b^(1/3)*Sqrt[(a^(
1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))])) + (4*(
-1)^(5/6)*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[1 - (b^(1/3)*x)/a^(1/3) +
 (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(-3*I + (1 + 2*I)*
Sqrt[3]), ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)
^(1/3))*a^(1/3))]], (-1)^(1/3)])/((-3*I + (1 + 2*I)*Sqrt[3])*b^(1
/3))))/Sqrt[-a - b*x^3]
```

Maple [F] time = 0.122, size = 0, normalized size = 0.

$$\int 1\left(\sqrt[3]{b}x+\sqrt[3]{a}\left(1+\sqrt{3}\right)\right)\left(\sqrt[3]{b}x+\sqrt[3]{a}\left(-\sqrt{3}+1\right)\right)^{-1}\frac{1}{\sqrt{-bx^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(-3^(1/2)+1))/(-
b*x^3-a)^(1/2),x)
```

```
[Out] int((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(-3^(1/2)+
1))/(-b*x^3-a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}\left(\sqrt{3} + 1\right)}{\sqrt{-bx^3 - a}\left(b^{\frac{1}{3}}x - a^{\frac{1}{3}}\left(\sqrt{3} - 1\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) + 1))/(sqrt(-b*x^3 - a)*(b^(1/3)*x - a^(1
```

```
[Out] integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) + 1))/(sqrt(-b*x^3 - a)*(
b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) + 1))/(sqrt(-b*x^3 - a)*(b^(1/3)*x - a^(1
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{a} + \sqrt{3}\sqrt[3]{a} + \sqrt[3]{b}x}{\sqrt{-a - bx^3}\left(-\sqrt{3}\sqrt[3]{a} + \sqrt[3]{a} + \sqrt[3]{b}x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**(1/3)*x+a**(1/3)*(1+3**(1/2)))/(b**(1/3)*x+a**(1/3)*(1-
3**(1/2)))/(-b*x**3-a)**(1/2),x)
```

```
[Out] Integral((a**(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)/(sqrt(-a - b*
x**3)*(-sqrt(3)*a**(1/3) + a**(1/3) + b**(1/3)*x)), x)
```

GIAC/XCAS [A] time = 0.602032, size = 4, normalized size = 0.06

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) + 1))/(sqrt(-b*x^3 - a)*(b^(1/3)*x - a^(1
```

```
[Out] sage0*x
```

3.86 $\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=73

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt{a}\left(x\sqrt[3]{\frac{b}{a}}+1\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

```
[Out] (-2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(1 + (b/a)^(1/3)*x))/Sq
rt[a + b*x^3]])/(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Rubi [A] time = 0.3368, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 52, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt{a}\left(x\sqrt[3]{\frac{b}{a}}+1\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] + (b/a)^(1/3)*x)/((1 - Sqrt[3] + (b/a)^(1/3)*x)*Sqrt[a + b*x^3])
```


```
[Out] (-2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(1 + (b/a)^(1/3)*x))/Sq
rt[a + b*x^3]])/(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((1+(b/a)**(1/3)*x+3**(1/2))/(1+(b/a)**(1/3)*x-3**(1/2))/(b*x**3
```

```
[Out] Timed out
```

Mathematica [C] time = 8.29686, size = 1527, normalized size = 20.92

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + Sqrt[3] + (b/a)^(1/3)*x)/((1 - Sqrt[3] + (b/a)^(1/3)*x)*Sqrt[a + b
```

```
[Out] (32*(26 - 15*Sqrt[3])*a^2*x*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/
a), (b*x^3)/(-10*a + 6*Sqrt[3]*a)])/((-5 + 3*Sqrt[3])*(2*(-5 + 3*
Sqrt[3])*a - b*x^3)*Sqrt[a + b*x^3]*(8*(-5 + 3*Sqrt[3])*a*AppellF
1[1/3, 1/2, 1, 4/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))
] + 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((b*x^3)/a), -((b*x^3)/(
10*a - 6*Sqrt[3]*a))] + (5 - 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3
, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))]))) - (32*Sqrt[3]
*(26 - 15*Sqrt[3])*a^2*x*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a),
 (b*x^3)/(-10*a + 6*Sqrt[3]*a)])/((-5 + 3*Sqrt[3])*(2*(-5 + 3*Sqr
t[3])*a - b*x^3)*Sqrt[a + b*x^3]*(8*(-5 + 3*Sqrt[3])*a*AppellF1[1
/3, 1/2, 1, 4/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))] +
 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((b*x^3)/a), -((b*x^3)/(10*
a - 6*Sqrt[3]*a))] + (5 - 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, -
((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))]))) - (60*(26 - 15*S
qrt[3])*a^2*(b/a)^(1/3)*x^2*AppellF1[2/3, 1/2, 1, 5/3, -((b*x^3)/
a), (b*x^3)/(-10*a + 6*Sqrt[3]*a)])/((-5 + 3*Sqrt[3])*(2*(-5 + 3*
Sqrt[3])*a - b*x^3)*Sqrt[a + b*x^3]*(10*(-5 + 3*Sqrt[3])*a*Appell
F1[2/3, 1/2, 1, 5/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a)
)] + 3*b*x^3*(AppellF1[5/3, 1/2, 2, 8/3, -((b*x^3)/a), -((b*x^3)/
(10*a - 6*Sqrt[3]*a))] + (5 - 3*Sqrt[3])*AppellF1[5/3, 3/2, 1, 8/
3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))]))) + (20*Sqrt[3
]*(26 - 15*Sqrt[3])*a^2*(b/a)^(1/3)*x^2*AppellF1[2/3, 1/2, 1, 5/3
, -((b*x^3)/a), (b*x^3)/(-10*a + 6*Sqrt[3]*a)])/((-5 + 3*Sqrt[3])
*(2*(-5 + 3*Sqrt[3])*a - b*x^3)*Sqrt[a + b*x^3]*(10*(-5 + 3*Sqrt[
3])*a*AppellF1[2/3, 1/2, 1, 5/3, -((b*x^3)/a), -((b*x^3)/(10*a -
6*Sqrt[3]*a))] + 3*b*x^3*(AppellF1[5/3, 1/2, 2, 8/3, -((b*x^3)/a)
, -((b*x^3)/(10*a - 6*Sqrt[3]*a))] + (5 - 3*Sqrt[3])*AppellF1[5/3
, 3/2, 1, 8/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))])))
- (16*(26 - 15*Sqrt[3])*a^2*(b/a)^(2/3)*x^3*AppellF1[1, 1/2, 1, 2
, -((b*x^3)/a), (b*x^3)/(-10*a + 6*Sqrt[3]*a)])/(Sqrt[3]*(-5 + 3*
Sqrt[3])*(2*(-5 + 3*Sqrt[3])*a - b*x^3)*Sqrt[a + b*x^3]*(4*(-5 +
3*Sqrt[3])*a*AppellF1[1, 1/2, 1, 2, -((b*x^3)/a), -((b*x^3)/(10*a
 - 6*Sqrt[3]*a))] + b*x^3*(AppellF1[2, 1/2, 2, 3, -((b*x^3)/a), -
((b*x^3)/(10*a - 6*Sqrt[3]*a))] + (5 - 3*Sqrt[3])*AppellF1[2, 3/2
, 1, 3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))]))) - (7*(2
6 - 15*Sqrt[3])*a*b*x^4*AppellF1[4/3, 1/2, 1, 7/3, -((b*x^3)/a),
(b*x^3)/(-10*a + 6*Sqrt[3]*a)])/((-5 + 3*Sqrt[3])*(2*(-5 + 3*Sqrt
```

```
[3])*a - b*x^3)*Sqrt[a + b*x^3]*(14*(-5 + 3*Sqrt[3])*a*AppellF1[4
/3, 1/2, 1, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))] +
 3*b*x^3*(AppellF1[7/3, 1/2, 2, 10/3, -((b*x^3)/a), -((b*x^3)/(10
*a - 6*Sqrt[3]*a))] + (5 - 3*Sqrt[3])*AppellF1[7/3, 3/2, 1, 10/3,
 -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))])))
```

Maple [F] time = 0.168, size = 0, normalized size = 0.

$$\int 1\left(1+\sqrt[3]{\frac{b}{a}}x+\sqrt{3}\right)\left(1+\sqrt[3]{\frac{b}{a}}x-\sqrt{3}\right)^{-1}\frac{1}{\sqrt{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+(b/a)^(1/3)*x+3^(1/2))/(1+(b/a)^(1/3)*x-3^(1/2))/(b*x^3+a)^(1/2),x)
```

```
[Out] int((1+(b/a)^(1/3)*x+3^(1/2))/(1+(b/a)^(1/3)*x-3^(1/2))/(b*x^3+a)
^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\sqrt{3}+1}{\sqrt{bx^3+a}\left(x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(b/a)^(1/3) + sqrt(3) + 1)/(sqrt(b*x^3 + a)*(x*(b/a)^(1/3) - sqrt(3)
```

```
[Out] integrate((x*(b/a)^(1/3) + sqrt(3) + 1)/(sqrt(b*x^3 + a)*(x*(b/a)
^(1/3) - sqrt(3) + 1)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(b/a)^(1/3) + sqrt(3) + 1)/(sqrt(b*x^3 + a)*(x*(b/a)^(1/3) - sqrt(3)
```

```
[Out] Timed out
```

Sympy [A] time = 12.2898, size = 0, normalized size = 0.

$$\text{NaN}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)**(1/3)*x+3**(1/2))/(1+(b/a)**(1/3)*x-3**(1/2))/(b*x**3+a)**
```

```
[Out] nan
```

GIAC/XCAS [A] time = 0.602901, size = 4, normalized size = 0.05

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(b/a)^(1/3) + sqrt(3) + 1)/(sqrt(b*x^3 + a)*(x*(b/a)^(1/3) - sqrt(3)
```

```
[Out] sage0*x
```

3.87 $\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a-bx^3}}\,dx$

Optimal. Leaf size=75

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt{a}\left(1-x\sqrt[3]{\frac{b}{a}}\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

```
[Out] (2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(1 - (b/a)^(1/3)*x))/Sqr
t[a - b*x^3]])/(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Rubi [A] time = 0.337313, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 55, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt{a}\left(1-x\sqrt[3]{\frac{b}{a}}\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] - (b/a)^(1/3)*x)/((1 - Sqrt[3] - (b/a)^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(1 - (b/a)^(1/3)*x))/Sqr
t[a - b*x^3]])/(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((1-(b/a)**(1/3)*x+3**(1/2))/(1-(b/a)**(1/3)*x-3**(1/2))/(-
b*x**3+a)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 8.34861, size = 1486, normalized size = 19.81

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + Sqrt[3] - (b/a)^(1/3)*x)/((1 - Sqrt[3] - (b/a)^(1/3)*x)*Sqrt[a - b
```

```
[Out] (32*(26 - 15*Sqrt[3])*a^2*x*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a,
 (b*x^3)/(10*a - 6*Sqrt[3]*a)])/((-5 + 3*Sqrt[3])*Sqrt[a - b*x^3]
*(2*(-5 + 3*Sqrt[3])*a + b*x^3)*(8*(-5 + 3*Sqrt[3])*a*AppellF1[1/
3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)] - 3*b*x^
3*(AppellF1[4/3, 1/2, 2, 7/3, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3
]*a)] + (5 - 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, (b*x^3)/a, (b*
x^3)/(10*a - 6*Sqrt[3]*a)]))) - (32*Sqrt[3]*(26 - 15*Sqrt[3])*a^2
*x*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3
]*a)])/((-5 + 3*Sqrt[3])*Sqrt[a - b*x^3]*(2*(-5 + 3*Sqrt[3])*a +
b*x^3)*(8*(-5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a
, (b*x^3)/(10*a - 6*Sqrt[3]*a)] - 3*b*x^3*(AppellF1[4/3, 1/2, 2,
7/3, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)] + (5 - 3*Sqrt[3])*A
ppellF1[4/3, 3/2, 1, 7/3, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)
]))) + (60*(26 - 15*Sqrt[3])*a^2*(b/a)^(1/3)*x^2*AppellF1[2/3, 1/
2, 1, 5/3, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)])/((-5 + 3*Sqr
t[3])*Sqrt[a - b*x^3]*(2*(-5 + 3*Sqrt[3])*a + b*x^3)*(10*(-5 + 3*
Sqrt[3])*a*AppellF1[2/3, 1/2, 1, 5/3, (b*x^3)/a, (b*x^3)/(10*a -
6*Sqrt[3]*a)] - 3*b*x^3*(AppellF1[5/3, 1/2, 2, 8/3, (b*x^3)/a, (b
*x^3)/(10*a - 6*Sqrt[3]*a)] + (5 - 3*Sqrt[3])*AppellF1[5/3, 3/2,
1, 8/3, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)]))) - (20*Sqrt[3]
*(26 - 15*Sqrt[3])*a^2*(b/a)^(1/3)*x^2*AppellF1[2/3, 1/2, 1, 5/3,
 (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)])/((-5 + 3*Sqrt[3])*Sqrt
[a - b*x^3]*(2*(-5 + 3*Sqrt[3])*a + b*x^3)*(10*(-5 + 3*Sqrt[3])*a
*AppellF1[2/3, 1/2, 1, 5/3, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3]*
a)] - 3*b*x^3*(AppellF1[5/3, 1/2, 2, 8/3, (b*x^3)/a, (b*x^3)/(10*
a - 6*Sqrt[3]*a)] + (5 - 3*Sqrt[3])*AppellF1[5/3, 3/2, 1, 8/3, (b
*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)]))) - (16*(26 - 15*Sqrt[3])
*a^2*(b/a)^(2/3)*x^3*AppellF1[1, 1/2, 1, 2, (b*x^3)/a, (b*x^3)/(1
0*a - 6*Sqrt[3]*a)])/(Sqrt[3]*(-5 + 3*Sqrt[3])*Sqrt[a - b*x^3]*(2
*(-5 + 3*Sqrt[3])*a + b*x^3)*(4*(-5 + 3*Sqrt[3])*a*AppellF1[1, 1/
2, 1, 2, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)] - b*x^3*(Appell
F1[2, 1/2, 2, 3, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)] + (5 -
3*Sqrt[3])*AppellF1[2, 3/2, 1, 3, (b*x^3)/a, (b*x^3)/(10*a - 6*Sq
rt[3]*a)]))) + (7*(26 - 15*Sqrt[3])*a*b*x^4*AppellF1[4/3, 1/2, 1,
 7/3, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)])/((-5 + 3*Sqrt[3])
*Sqrt[a - b*x^3]*(2*(-5 + 3*Sqrt[3])*a + b*x^3)*(14*(-5 + 3*Sqrt[
```

```
3])*a*AppellF1[4/3, 1/2, 1, 7/3, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqr
t[3]*a)] - 3*b*x^3*(AppellF1[7/3, 1/2, 2, 10/3, (b*x^3)/a, (b*x^3
)/(10*a - 6*Sqrt[3]*a)] + (5 - 3*Sqrt[3])*AppellF1[7/3, 3/2, 1, 1
0/3, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)])))
```

Maple [F] time = 0.161, size = 0, normalized size = 0.

$$\int 1\left(1-\sqrt[3]{\frac{b}{a}}x+\sqrt{3}\right)\left(1-\sqrt[3]{\frac{b}{a}}x-\sqrt{3}\right)^{-1}\frac{1}{\sqrt{-bx^{3}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-(b/a)^(1/3)*x+3^(1/2))/(1-(b/a)^(1/3)*x-3^(1/2))/(-b*x^3+a)^(1/2),x)
```

```
[Out] int((1-(b/a)^(1/3)*x+3^(1/2))/(1-(b/a)^(1/3)*x-3^(1/2))/(-b*x^3+a
)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\sqrt{3}-1}{\sqrt{-bx^{3}+a}\left(x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(b/a)^(1/3) - sqrt(3) - 1)/(sqrt(-b*x^3 + a)*(x*(b/a)^(1/3) + sqrt(3
```

```
[Out] integrate((x*(b/a)^(1/3) - sqrt(3) - 1)/(sqrt(-b*x^3 + a)*(x*(b/a
)^(1/3) + sqrt(3) - 1)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(b/a)^(1/3) - sqrt(3) - 1)/(sqrt(-b*x^3 + a)*(x*(b/a)^(1/3) + sqrt(3
```

```
[Out] Timed out
```

Sympy [A] time = 13.7046, size = 0, normalized size = 0.

$$\text{NaN}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)**(1/3)*x+3**(1/2))/(1-(b/a)**(1/3)*x-3**(1/2))/(-
b*x**3+a)**(1/2),x)
```

```
[Out] nan
```

GIAC/XCAS [A] time = 0.613065, size = 4, normalized size = 0.05

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(b/a)^(1/3) - sqrt(3) - 1)/(sqrt(-b*x^3 + a)*(x*(b/a)^(1/3) + sqrt(3
```

```
[Out] sage0*x
```

3.88 $\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a+bx^3}}\,dx$

Optimal. Leaf size=76

$$\frac{2\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt{a}\left(1-x\sqrt[3]{\frac{b}{a}}\right)}{\sqrt{bx^3-a}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

```
[Out] (2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(1 - (b/a)^(1/3)*x))/Sqrt
[-a + b*x^3]])/(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Rubi [A] time = 0.329729, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 56, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036

$$\frac{2\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt{a}\left(1-x\sqrt[3]{\frac{b}{a}}\right)}{\sqrt{bx^3-a}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] - (b/a)^(1/3)*x)/((1 - Sqrt[3] - (b/a)^(1/3)*x)*Sqrt[-
a + b*x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(1 - (b/a)^(1/3)*x))/Sqrt
[-a + b*x^3]])/(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((1-(b/a)**(1/3)*x+3**(1/2))/(1-(b/a)**(1/3)*x-3**(1/2))/(b*x**3
a)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 8.34384, size = 1492, normalized size = 19.63

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + Sqrt[3] - (b/a)^(1/3)*x)/((1 - Sqrt[3] - (b/a)^(1/3)*x)*Sqrt[-
a + b*x^3]),x]
```

```
[Out] (32*(26 - 15*Sqrt[3])*a^2*x*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a,
 (b*x^3)/(10*a - 6*Sqrt[3]*a)])/((-5 + 3*Sqrt[3])*Sqrt[-a + b*x^3
]*(2*(-5 + 3*Sqrt[3])*a + b*x^3)*(8*(-5 + 3*Sqrt[3])*a*AppellF1[1
/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)] - 3*b*x
^3*(AppellF1[4/3, 1/2, 2, 7/3, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[
3]*a)] + (5 - 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, (b*x^3)/a, (b
*x^3)/(10*a - 6*Sqrt[3]*a)]))) - (32*Sqrt[3]*(26 - 15*Sqrt[3])*a^
2*x*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[
3]*a)])/((-5 + 3*Sqrt[3])*Sqrt[-a + b*x^3]*(2*(-5 + 3*Sqrt[3])*a
+ b*x^3)*(8*(-5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)
/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)] - 3*b*x^3*(AppellF1[4/3, 1/2, 2
, 7/3, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)] + (5 - 3*Sqrt[3])
*AppellF1[4/3, 3/2, 1, 7/3, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3]*
a)]))) + (60*(26 - 15*Sqrt[3])*a^2*(b/a)^(1/3)*x^2*AppellF1[2/3,
1/2, 1, 5/3, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)])/((-5 + 3*S
qrt[3])*Sqrt[-a + b*x^3]*(2*(-5 + 3*Sqrt[3])*a + b*x^3)*(10*(-5 +
 3*Sqrt[3])*a*AppellF1[2/3, 1/2, 1, 5/3, (b*x^3)/a, (b*x^3)/(10*a
 - 6*Sqrt[3]*a)] - 3*b*x^3*(AppellF1[5/3, 1/2, 2, 8/3, (b*x^3)/a,
 (b*x^3)/(10*a - 6*Sqrt[3]*a)] + (5 - 3*Sqrt[3])*AppellF1[5/3, 3/
2, 1, 8/3, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)]))) - (20*Sqrt
[3]*(26 - 15*Sqrt[3])*a^2*(b/a)^(1/3)*x^2*AppellF1[2/3, 1/2, 1, 5
/3, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)])/((-5 + 3*Sqrt[3])*S
qrt[-a + b*x^3]*(2*(-5 + 3*Sqrt[3])*a + b*x^3)*(10*(-5 + 3*Sqrt[3
])*a*AppellF1[2/3, 1/2, 1, 5/3, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt
[3]*a)] - 3*b*x^3*(AppellF1[5/3, 1/2, 2, 8/3, (b*x^3)/a, (b*x^3)/
(10*a - 6*Sqrt[3]*a)] + (5 - 3*Sqrt[3])*AppellF1[5/3, 3/2, 1, 8/3
, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)]))) - (16*(26 - 15*Sqrt
[3])*a^2*(b/a)^(2/3)*x^3*AppellF1[1, 1/2, 1, 2, (b*x^3)/a, (b*x^3
)/(10*a - 6*Sqrt[3]*a)])/(Sqrt[3]*(-5 + 3*Sqrt[3])*Sqrt[-a + b*x^
3]*(2*(-5 + 3*Sqrt[3])*a + b*x^3)*(4*(-5 + 3*Sqrt[3])*a*AppellF1[
1, 1/2, 1, 2, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)] - b*x^3*(A
ppellF1[2, 1/2, 2, 3, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)] +
(5 - 3*Sqrt[3])*AppellF1[2, 3/2, 1, 3, (b*x^3)/a, (b*x^3)/(10*a -
 6*Sqrt[3]*a)]))) + (7*(26 - 15*Sqrt[3])*a*b*x^4*AppellF1[4/3, 1/
2, 1, 7/3, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)])/((-5 + 3*Sqr
```

```
t[3])*Sqrt[-a + b*x^3]*(2*(-5 + 3*Sqrt[3])*a + b*x^3)*(14*(-5 + 3
*Sqrt[3])*a*AppellF1[4/3, 1/2, 1, 7/3, (b*x^3)/a, (b*x^3)/(10*a -
 6*Sqrt[3]*a)] - 3*b*x^3*(AppellF1[7/3, 1/2, 2, 10/3, (b*x^3)/a,
(b*x^3)/(10*a - 6*Sqrt[3]*a)] + (5 - 3*Sqrt[3])*AppellF1[7/3, 3/2
, 1, 10/3, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)])))
```

Maple [F] time = 0.111, size = 0, normalized size = 0.

$$\int 1\left(1-\sqrt[3]{\frac{b}{a}}x+\sqrt{3}\right)\left(1-\sqrt[3]{\frac{b}{a}}x-\sqrt{3}\right)^{-1}\frac{1}{\sqrt{bx^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-(b/a)^(1/3)*x+3^(1/2))/(1-(b/a)^(1/3)*x-3^(1/2))/(b*x^3-a)^(1/2),x)
```

```
[Out] int((1-(b/a)^(1/3)*x+3^(1/2))/(1-(b/a)^(1/3)*x-3^(1/2))/(b*x^3-a)
^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\sqrt{3}-1}{\sqrt{bx^3-a}\left(x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(b/a)^(1/3) - sqrt(3) - 1)/(sqrt(b*x^3 - a)*(x*(b/a)^(1/3) + sqrt(3)
```

```
[Out] integrate((x*(b/a)^(1/3) - sqrt(3) - 1)/(sqrt(b*x^3 - a)*(x*(b/a)
^(1/3) + sqrt(3) - 1)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(b/a)^(1/3) - sqrt(3) - 1)/(sqrt(b*x^3 - a)*(x*(b/a)^(1/3) + sqrt(3)
```

```
[Out] Timed out
```

Sympy [A] time = 13.5137, size = 0, normalized size = 0.

$$\text{NaN}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)**(1/3)*x+3**(1/2))/(1-(b/a)**(1/3)*x-3**(1/2))/(b*x**3-
a)**(1/2),x)
```

```
[Out] nan
```

GIAC/XCAS [A] time = 0.606431, size = 4, normalized size = 0.05

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(b/a)^(1/3) - sqrt(3) - 1)/(sqrt(b*x^3 - a)*(x*(b/a)^(1/3) + sqrt(3)
```

```
[Out] sage0*x
```

3.89 $\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a-bx^3}}\,dx$

Optimal. Leaf size=76

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt{a}\left(x\sqrt[3]{\frac{b}{a}}+1\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

```
[Out] (-2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(1 + (b/a)^(1/3)*x))/Sqr
t[-a - b*x^3]])/(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Rubi [A] time = 0.313967, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 55, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt{a}\left(x\sqrt[3]{\frac{b}{a}}+1\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] + (b/a)^(1/3)*x)/((1 - Sqrt[3] + (b/a)^(1/3)*x)*Sqrt[-
a - b*x^3]),x]
```

```
[Out] (-2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(1 + (b/a)^(1/3)*x))/Sqr
t[-a - b*x^3]])/(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((1+(b/a)**(1/3)*x+3**(1/2))/(1+(b/a)**(1/3)*x-3**(1/2))/(-
b*x**3-a)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 8.48211, size = 1545, normalized size = 20.33

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + Sqrt[3] + (b/a)^(1/3)*x)/((1 - Sqrt[3] + (b/a)^(1/3)*x)*Sqrt[-
a - b*x^3]),x]
```

```
[Out] (32*(26 - 15*Sqrt[3])*a^2*x*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/
a), (b*x^3)/(-10*a + 6*Sqrt[3]*a)])/((-5 + 3*Sqrt[3])*Sqrt[-a - b
*x^3]*(2*(-5 + 3*Sqrt[3])*a - b*x^3)*(8*(-5 + 3*Sqrt[3])*a*Appell
F1[1/3, 1/2, 1, 4/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a)
)] + 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((b*x^3)/a), -((b*x^3)/
(10*a - 6*Sqrt[3]*a))] + (5 - 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/
3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))]))) - (32*Sqrt[3
]*(26 - 15*Sqrt[3])*a^2*x*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a)
, (b*x^3)/(-10*a + 6*Sqrt[3]*a)])/((-5 + 3*Sqrt[3])*Sqrt[-a - b*x
^3]*(2*(-5 + 3*Sqrt[3])*a - b*x^3)*(8*(-5 + 3*Sqrt[3])*a*AppellF1
[1/3, 1/2, 1, 4/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))]
 + 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((b*x^3)/a), -((b*x^3)/(1
0*a - 6*Sqrt[3]*a))] + (5 - 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3,
 -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))]))) - (60*(26 - 15
*Sqrt[3])*a^2*(b/a)^(1/3)*x^2*AppellF1[2/3, 1/2, 1, 5/3, -((b*x^3
)/a), (b*x^3)/(-10*a + 6*Sqrt[3]*a)])/((-5 + 3*Sqrt[3])*Sqrt[-a -
 b*x^3]*(2*(-5 + 3*Sqrt[3])*a - b*x^3)*(10*(-5 + 3*Sqrt[3])*a*App
ellF1[2/3, 1/2, 1, 5/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]
*a))] + 3*b*x^3*(AppellF1[5/3, 1/2, 2, 8/3, -((b*x^3)/a), -((b*x^
3)/(10*a - 6*Sqrt[3]*a))] + (5 - 3*Sqrt[3])*AppellF1[5/3, 3/2, 1,
 8/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))]))) + (20*Sqr
t[3]*(26 - 15*Sqrt[3])*a^2*(b/a)^(1/3)*x^2*AppellF1[2/3, 1/2, 1,
5/3, -((b*x^3)/a), (b*x^3)/(-10*a + 6*Sqrt[3]*a)])/((-5 + 3*Sqrt[
3])*Sqrt[-a - b*x^3]*(2*(-5 + 3*Sqrt[3])*a - b*x^3)*(10*(-5 + 3*S
qrt[3])*a*AppellF1[2/3, 1/2, 1, 5/3, -((b*x^3)/a), -((b*x^3)/(10*
a - 6*Sqrt[3]*a))] + 3*b*x^3*(AppellF1[5/3, 1/2, 2, 8/3, -((b*x^3
)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))] + (5 - 3*Sqrt[3])*AppellF1
[5/3, 3/2, 1, 8/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))]
))) - (16*(26 - 15*Sqrt[3])*a^2*(b/a)^(2/3)*x^3*AppellF1[1, 1/2,
1, 2, -((b*x^3)/a), (b*x^3)/(-10*a + 6*Sqrt[3]*a)])/(Sqrt[3]*(-5
+ 3*Sqrt[3])*Sqrt[-a - b*x^3]*(2*(-5 + 3*Sqrt[3])*a - b*x^3)*(4*(
-5 + 3*Sqrt[3])*a*AppellF1[1, 1/2, 1, 2, -((b*x^3)/a), -((b*x^3)/
(10*a - 6*Sqrt[3]*a))] + b*x^3*(AppellF1[2, 1/2, 2, 3, -((b*x^3)/
a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))] + (5 - 3*Sqrt[3])*AppellF1[2
, 3/2, 1, 3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))]))) -
```

```
(7*(26 - 15*Sqrt[3])*a*b*x^4*AppellF1[4/3, 1/2, 1, 7/3, -((b*x^3)
/a), (b*x^3)/(-10*a + 6*Sqrt[3]*a)])/((-5 + 3*Sqrt[3])*Sqrt[-a -
b*x^3]*(2*(-5 + 3*Sqrt[3])*a - b*x^3)*(14*(-5 + 3*Sqrt[3])*a*Appe
llF1[4/3, 1/2, 1, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*
a))] + 3*b*x^3*(AppellF1[7/3, 1/2, 2, 10/3, -((b*x^3)/a), -((b*x^
3)/(10*a - 6*Sqrt[3]*a))] + (5 - 3*Sqrt[3])*AppellF1[7/3, 3/2, 1,
 10/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))])))
```

Maple [F] time = 0.113, size = 0, normalized size = 0.

$$\int 1\left(1+\sqrt[3]{\frac{b}{a}}x+\sqrt{3}\right)\left(1+\sqrt[3]{\frac{b}{a}}x-\sqrt{3}\right)^{-1}\frac{1}{\sqrt{-bx^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]    int((1+(b/a)^(1/3)*x+3^(1/2))/(1+(b/a)^(1/3)*x-3^(1/2))/(-b*x^3-
a)^(1/2),x)
```

```
[Out] int((1+(b/a)^(1/3)*x+3^(1/2))/(1+(b/a)^(1/3)*x-3^(1/2))/(-b*x^3-a
)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\sqrt{3}+1}{\sqrt{-bx^3-a}\left(x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(b/a)^(1/3) + sqrt(3) + 1)/(sqrt(-b*x^3 - a)*(x*(b/a)^(1/3) - sqrt(3
```

```
[Out] integrate((x*(b/a)^(1/3) + sqrt(3) + 1)/(sqrt(-b*x^3 - a)*(x*(b/a
)^(1/3) - sqrt(3) + 1)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(b/a)^(1/3) + sqrt(3) + 1)/(sqrt(-b*x^3 - a)*(x*(b/a)^(1/3) - sqrt(3
```

```
[Out] Timed out
```

Sympy [A] time = 12.9848, size = 0, normalized size = 0.

$$\mathrm{NaN}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)**(1/3)*x+3**(1/2))/(1+(b/a)**(1/3)*x-3**(1/2))/(-
b*x**3-a)**(1/2),x)
```

```
[Out] nan
```

GIAC/XCAS [A] time = 0.599709, size = 4, normalized size = 0.05

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(b/a)^(1/3) + sqrt(3) + 1)/(sqrt(-b*x^3 - a)*(x*(b/a)^(1/3) - sqrt(3
```

```
[Out] sage0*x
```

3.90 $\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=42

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{3+2\sqrt{3}}}$$

```
[Out] (-2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]])/Sqrt[3 +
 2*Sqrt[3]]
```

Rubi [A] time = 0.176908, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{3+2\sqrt{3}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]])/Sqrt[3 +
 2*Sqrt[3]]
```

Rubi in Sympy [A] time = 16.0491, size = 78, normalized size = 1.86

$$\frac{2\tilde{\infty}\sqrt{\frac{x^2-x+1}{\left(x+1+\sqrt{3}\right)^2}}\,(x+1)\,F\left(\operatorname{asin}\left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+x-3**(1/2))/(1+x+3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] 2*zoo*sqrt((x**2 - x + 1)/(x + 1 + sqrt(3))**2)*(x + 1)*elliptic_
f(asin((x - sqrt(3) + 1)/(x + 1 + sqrt(3))), -7 - 4*sqrt(3))/(sqr
t((x + 1)/(x + 1 + sqrt(3))**2)*sqrt(x**3 + 1))
```

Mathematica [C] time = 0.54838, size = 269, normalized size = 6.4

$$\frac{2\sqrt{6}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\left(4\sqrt{-2ix+\sqrt{3}+i}\sqrt{x^2-x+1}\left(\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)+\sqrt{2ix+\sqrt{3}-i}\left(\left((1+2i)+i\sqrt{3}\right)x-\sqrt{\right.\right.}{\left(3i+(1+2i)\sqrt{3}\right)\sqrt{-2ix+\sqrt{3}+i}\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[6]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(Sqrt[-I + Sqrt[3] +
 (2*I)*x]*((-2 - I) - Sqrt[3] + ((1 + 2*I) + I*Sqrt[3])*x)*Ellipt
icF[ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqr
t[3])/(3*I + Sqrt[3])] + 4*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 - x
 + x^2]*EllipticPi[(2*Sqrt[3])/(3*I + (1 + 2*I)*Sqrt[3]), ArcSin[
Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I
+ Sqrt[3])]))/((3*I + (1 + 2*I)*Sqrt[3])*Sqrt[I + Sqrt[3] - (2*I)
*x]*Sqrt[1 + x^3])
```

Maple [C] time = 0.03, size = 245, normalized size = 5.8

$$2\,\frac{3/2-i/2\sqrt{3}}{\sqrt{x^3+1}}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}\mathit{EllipticF}\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},\sqrt{\frac{-3/2+i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\right)$$
$$-4\,\frac{3/2-i/2\sqrt{3}}{\sqrt{x^3+1}}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}\mathit{EllipticPi}\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},1/3\,\left(-3/2+i/2\sqrt{3}\right)\sqrt{3},\sqrt{\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+x-3^(1/2))/(1+x+3^(1/2))/(x^3+1)^(1/2),x)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1
/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/
(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1
/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^
(1/2))-4*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((
x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^
(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticPi(((1+x
)/(3/2-1/2*I*3^(1/2)))^(1/2),1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2),((-
3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}\left(x+\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x - sqrt(3) + 1)/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)),x, algorithm="maxima
```

```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)), x)
```

Fricas [A] time = 0.312916, size = 142, normalized size = 3.38

$$\frac{1}{3}\sqrt{3}\sqrt{2\sqrt{3}-3}\arctan\left(\frac{2340\,x^4-4680\,x^3+6516\,x^2-\sqrt{3}\left(1351\,x^4-2702\,x^3+3762\,x^2-3284\,x+1060\right)-5688\,x+1836}{2\sqrt{x^3+1}\left(627\,x^2-2\sqrt{3}(181\,x^2-265\,x+97)-918\,x+336\right)\sqrt{2\sqrt{3}-3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x - sqrt(3) + 1)/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)),x, algorithm="fricas
```

```
[Out] 1/3*sqrt(3)*sqrt(2*sqrt(3) - 3)*arctan(1/2*(2340*x^4 - 4680*x^3 +
 6516*x^2 - sqrt(3)*(1351*x^4 - 2702*x^3 + 3762*x^2 - 3284*x + 10
60) - 5688*x + 1836)/(sqrt(x^3 + 1)*(627*x^2 - 2*sqrt(3)*(181*x^2
 - 265*x + 97) - 918*x + 336)*sqrt(2*sqrt(3) - 3)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-\sqrt{3}+1}{\sqrt{(x+1)\,(x^2-x+1)}\left(x+1+\sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+x-3**(1/2))/(1+x+3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] Integral((x - sqrt(3) + 1)/(sqrt((x + 1)*(x**2 - x + 1))*(x + 1 +
 sqrt(3))), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}\left(x+\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x - sqrt(3) + 1)/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)),x, algorithm="giac")
```

```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)), x)
```

3.91 $\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=46

$$\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{3+2\sqrt{3}}}$$

```
[Out] (2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[1 - x^3]])/Sqrt[3 + 
2*Sqrt[3]]
```

Rubi [A] time = 0.204831, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$

$$\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{3+2\sqrt{3}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - Sqrt[3] - x)/((1 + Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[1 - x^3]])/Sqrt[3 + 
2*Sqrt[3]]
```

Rubi in Sympy [A] time = 23.3051, size = 78, normalized size = 1.7

$$\frac{2\tilde{\infty}\sqrt{\frac{x^2+x+1}{\left(-x+1+\sqrt{3}\right)^2}}(-x+1)\,F\left(\operatorname{asin}\left(\frac{-x-\sqrt{3}+1}{-x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt{\frac{-x+1}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{-x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1-x-3**(1/2))/(1-x+3**(1/2))/(-x**3+1)**(1/2),x)
```

```
[Out] 2*zoo*sqrt((x**2 + x + 1)/(-x + 1 + sqrt(3))**2)*(-x + 1)*ellipti
c_f(asin((-x - sqrt(3) + 1)/(-x + 1 + sqrt(3))), -7 - 4*sqrt(3))/
(sqrt((-x + 1)/(-x + 1 + sqrt(3))**2)*sqrt(-x**3 + 1))
```

Mathematica [C] time = 0.52371, size = 267, normalized size = 5.8

$$\frac{2\sqrt{6}\sqrt{\frac{i(x-1)}{\sqrt{3}-3i}}\left(\sqrt{2ix+\sqrt{3}+i}\left(\left(\left(\sqrt{3}+(2+i)\right)x+i\sqrt{3}+(1+2i)\right)F\left(\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}-i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right)-4i\sqrt{-2ix+\sqrt{3}-i}\sqrt{x^2+x}\right.\right.}{\left(3+(2+i)\sqrt{3}\right)\sqrt{-2ix+\sqrt{3}-i}\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(1 - Sqrt[3] - x)/((1 + Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*Sqrt[6]*Sqrt[(I*(-1 + x))/(-3*I + Sqrt[3])]*(Sqrt[I + Sqrt[3]
+ (2*I)*x]*((1 + 2*I) + I*Sqrt[3] + ((2 + I) + Sqrt[3])*x)*Ellipt
icF[ArcSin[Sqrt[-I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sq
rt[3])/(-3*I + Sqrt[3])] - (4*I)*Sqrt[-I + Sqrt[3] - (2*I)*x]*Sqr
t[1 + x + x^2]*EllipticPi[((2*I)*Sqrt[3])/(3 + (2 + I)*Sqrt[3]),
ArcSin[Sqrt[-I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3
])/(-3*I + Sqrt[3])]))/((3 + (2 + I)*Sqrt[3])*Sqrt[-I + Sqrt[3] -
 (2*I)*x]*Sqrt[1 - x^3])
```

Maple [C] time = 0.038, size = 247, normalized size = 5.4

$$-\frac{2i}{3}\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}EllipticF\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)\frac{1}{\sqrt{-x^3+1}}$$
$$-\frac{4i}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}-\sqrt{3}}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}EllipticPi\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}-\sqrt{3}},\ldots\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1-x-3^(1/2))/(1-x+3^(1/2))/(-x^3+1)^(1/2),x)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/(-
3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2
)/(-x^3+1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3
^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))-4*I*(I*(x+1
/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/(-3/2+1/2*I*3^(1/2)))^(1
/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(-3/2
+1/2*I*3^(1/2)-3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^
(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-3/2+1/2*I*3^(1/2)-3^(1/2)),(I*3
^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+\sqrt{3}-1}{\sqrt{-x^3+1}\left(x-\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x + sqrt(3) - 1)/(sqrt(-x^3 + 1)*(x - sqrt(3) - 1)),x, algorithm="maxim
```

```
[Out] integrate((x + sqrt(3) - 1)/(sqrt(-x^3 + 1)*(x - sqrt(3) - 1)), x
)
```

Fricas [A] time = 0.328811, size = 144, normalized size = 3.13

$$-\frac{1}{3}\sqrt{3}\sqrt{2\sqrt{3}-3}\arctan\left(\frac{2340\,x^4+4680\,x^3+6516\,x^2-\sqrt{3}\left(1351\,x^4+2702\,x^3+3762\,x^2+3284\,x+1060\right)+5688\,x+1836}{2\sqrt{-x^3+1}\left(627\,x^2-2\sqrt{3}(181\,x^2+265\,x+97)+918\,x+336\right)\sqrt{2\sqrt{3}-3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x + sqrt(3) - 1)/(sqrt(-x^3 + 1)*(x - sqrt(3) - 1)),x, algorithm="frica
```

```
[Out] -1/3*sqrt(3)*sqrt(2*sqrt(3) - 3)*arctan(1/2*(2340*x^4 + 4680*x^3
+ 6516*x^2 - sqrt(3)*(1351*x^4 + 2702*x^3 + 3762*x^2 + 3284*x + 1
060) + 5688*x + 1836)/(sqrt(-x^3 + 1)*(627*x^2 - 2*sqrt(3)*(181*x
^2 + 265*x + 97) + 918*x + 336)*sqrt(2*sqrt(3) - 3)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-1+\sqrt{3}}{\sqrt{-(x-1)(x^2+x+1)}\left(x-\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1-x-3**(1/2))/(1-x+3**(1/2))/(-x**3+1)**(1/2),x)
```

```
[Out] Integral((x - 1 + sqrt(3))/(sqrt(-(x - 1)*(x**2 + x + 1))*(x - sq
rt(3) - 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+\sqrt{3}-1}{\sqrt{-x^3+1}\left(x-\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x + sqrt(3) - 1)/(sqrt(-x^3 + 1)*(x - sqrt(3) - 1)),x, algorithm="giac'
```

```
[Out] integrate((x + sqrt(3) - 1)/(sqrt(-x^3 + 1)*(x - sqrt(3) - 1)), x
)
```

3.92 $\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=44

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{x^3-1}}\right)}{\sqrt{3+2\sqrt{3}}}$$

```
[Out] (2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[-1 + x^3]])/Sqrt[3
+ 2*Sqrt[3]]
```

Rubi [A] time = 0.201202, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{x^3-1}}\right)}{\sqrt{3+2\sqrt{3}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - Sqrt[3] - x)/((1 + Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[-1 + x^3]])/Sqrt[3
+ 2*Sqrt[3]]
```

Rubi in Sympy [A] time = 33.0019, size = 133, normalized size = 3.02

$$\frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,(-x+1)\operatorname{atanh}\left(\frac{\left(\sqrt{3}+2\right)\sqrt{-\frac{\left(-x+1+\sqrt{3}\right)^2}{\left(x-1+\sqrt{3}\right)^2}+1}}{\sqrt{\frac{\left(-x+1+\sqrt{3}\right)^2}{\left(x-1+\sqrt{3}\right)^2}+4\sqrt{3}+7}}\right)}{3\sqrt{\frac{x-1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{\sqrt{3}+2}\sqrt{x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1-x-3**(1/2))/(1-x+3**(1/2))/(x**3-1)**(1/2),x)
```

```
[Out] 2*3**(3/4)*sqrt((x**2 + x + 1)/(-x - sqrt(3) + 1)**2)*(-x + 1)*at
anh((sqrt(3) + 2)*sqrt(-(-x + 1 + sqrt(3))**2/(x - 1 + sqrt(3))**
```

```
2 + 1)/sqrt((-x + 1 + sqrt(3))**2/(x - 1 + sqrt(3))**2 + 4*sqrt(3
) + 7))/(3*sqrt((x - 1)/(-x - sqrt(3) + 1)**2)*sqrt(sqrt(3) + 2)*
sqrt(x**3 - 1))
```

Mathematica [C] time = 0.477289, size = 265, normalized size = 6.02

$$\frac{2\sqrt{6}\sqrt{\frac{i(x-1)}{\sqrt{3}-3i}}\left(\sqrt{2ix+\sqrt{3}+i}\left(\left(\sqrt{3}+(2+i)\right)x+i\sqrt{3}+(1+2i)\right)F\left(\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}-i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right)-4i\sqrt{-2ix+\sqrt{3}-i}\sqrt{x^2+x}\right.}{\left(3+(2+i)\sqrt{3}\right)\sqrt{-2ix+\sqrt{3}-i}\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(1 - Sqrt[3] - x)/((1 + Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*Sqrt[6]*Sqrt[(I*(-1 + x))/(-3*I + Sqrt[3])]*(Sqrt[I + Sqrt[3]
+ (2*I)*x]*((1 + 2*I) + I*Sqrt[3] + ((2 + I) + Sqrt[3])*x)*Ellipt
icF[ArcSin[Sqrt[-I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sq
rt[3])/(-3*I + Sqrt[3])] - (4*I)*Sqrt[-I + Sqrt[3] - (2*I)*x]*Sqr
t[1 + x + x^2]*EllipticPi[((2*I)*Sqrt[3])/(3 + (2 + I)*Sqrt[3]),
ArcSin[Sqrt[-I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3
])/(-3*I + Sqrt[3])]))/((3 + (2 + I)*Sqrt[3])*Sqrt[-I + Sqrt[3] -
 (2*I)*x]*Sqrt[-1 + x^3])
```

Maple [C] time = 0.031, size = 245, normalized size = 5.6

$$2\,\frac{-3/2-i/2\sqrt{3}}{\sqrt{x^3-1}}\sqrt{\frac{-1+x}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2-i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2+i/2\sqrt{3}}{3/2+i/2\sqrt{3}}}\mathit{EllipticF}\left(\sqrt{\frac{-1+x}{-3/2-i/2\sqrt{3}}},\sqrt{\frac{3/2+i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\right)$$
$$-4\,\frac{-3/2-i/2\sqrt{3}}{\sqrt{x^3-1}}\sqrt{\frac{-1+x}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2-i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2+i/2\sqrt{3}}{3/2+i/2\sqrt{3}}}\mathit{EllipticPi}\left(\sqrt{\frac{-1+x}{-3/2-i/2\sqrt{3}}},-1/3\,\left(3/2+i/2\sqrt{3}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1-x-3^(1/2))/(1-x+3^(1/2))/(x^3-1)^(1/2),x)
```

```
[Out] 2*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/
2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2)
)/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((-1+x)/(-3/
2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))
^(1/2))-4*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2
)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I
*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticPi(((-
```

```
1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(3/2+1/2*I*3^(1/2))*3^(1/2)
,((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+\sqrt{3}-1}{\sqrt{x^3-1}\left(x-\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x + sqrt(3) - 1)/(sqrt(x^3 - 1)*(x - sqrt(3) - 1)),x, algorithm="maxima
```

```
[Out] integrate((x + sqrt(3) - 1)/(sqrt(x^3 - 1)*(x - sqrt(3) - 1)), x)
```

Fricas [A] time = 0.317479, size = 366, normalized size = 8.32

$$\frac{1}{6}\sqrt{3}\sqrt{2\sqrt{3}-3}\log\left(\frac{6322680\,x^8+13553256\,x^7+26133432\,x^6+63422352\,x^5+113743056\,x^4+136435776\,x^3+102727296\,x}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x + sqrt(3) - 1)/(sqrt(x^3 - 1)*(x - sqrt(3) - 1)),x, algorithm="fricas
```

```
[Out] 1/6*sqrt(3)*sqrt(2*sqrt(3) - 3)*log((6322680*x^8 + 13553256*x^7 +
 26133432*x^6 + 63422352*x^5 + 113743056*x^4 + 136435776*x^3 + 10
2727296*x^2 - 4*(1694157*x^6 + 5868732*x^5 + 10586298*x^4 + 12840
912*x^3 + 9886740*x^2 - 2*sqrt(3)*(489061*x^6 + 1694157*x^5 + 305
6001*x^4 + 3706852*x^3 + 2854056*x^2 + 1198884*x + 205636) + 4153
056*x + 712344)*sqrt(x^3 - 1)*sqrt(2*sqrt(3) - 3) - sqrt(3)*(3650
401*x^8 + 7824976*x^7 + 15088144*x^6 + 36616912*x^5 + 65669584*x^
4 + 78771232*x^3 + 59309632*x^2 + 24558208*x + 4193392) + 4253606
4*x + 7263168)/(6322680*x^8 + 37028184*x^7 + 94872792*x^6 + 13890
3408*x^5 + 127105440*x^4 + 74438112*x^3 + 27246240*x^2 - sqrt(3)*
(3650401*x^8 + 21378232*x^7 + 54774832*x^6 + 80195920*x^5 + 73384
360*x^4 + 42976864*x^3 + 15730624*x^2 + 3290176*x + 301072) + 569
8752*x + 521472))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-1+\sqrt{3}}{\sqrt{(x-1)(x^2+x+1)}\left(x-\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1-x-3**(1/2))/(1-x+3**(1/2))/(x**3-1)**(1/2),x)
```

```
[Out] Integral((x - 1 + sqrt(3))/(sqrt((x - 1)*(x**2 + x + 1))*(x - sqr
t(3) - 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+\sqrt{3}-1}{\sqrt{x^3-1}\left(x-\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x + sqrt(3) - 1)/(sqrt(x^3 - 1)*(x - sqrt(3) - 1)),x, algorithm="giac")
```

```
[Out] integrate((x + sqrt(3) - 1)/(sqrt(x^3 - 1)*(x - sqrt(3) - 1)), x)
```

3.93 $\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=44

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(x+1)}{\sqrt{-x^3-1}}\right)}{\sqrt{3+2\sqrt{3}}}$$

```
[Out] (-2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[-1 - x^3]])/Sqrt[3
 + 2*Sqrt[3]]
```

Rubi [A] time = 0.175824, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(x+1)}{\sqrt{-x^3-1}}\right)}{\sqrt{3+2\sqrt{3}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[-1 - x^3]])/Sqrt[3
 + 2*Sqrt[3]]
```

Rubi in Sympy [A] time = 31.1072, size = 138, normalized size = 3.14

$$-\frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,(x+1)\,\operatorname{atanh}\left(\frac{\sqrt{1-\frac{\left(x+1+\sqrt{3}\right)^2}{\left(-x-1+\sqrt{3}\right)^2}}\left(\sqrt{3}+2\right)}{\sqrt{4\sqrt{3}+7+\frac{\left(x+1+\sqrt{3}\right)^2}{\left(-x-1+\sqrt{3}\right)^2}}}\right)}{3\sqrt{\frac{-x-1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{\sqrt{3}+2}\sqrt{-x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+x-3**(1/2))/(1+x+3**(1/2))/(-x**3-1)**(1/2),x)
```

```
[Out] -2*3**(3/4)*sqrt((x**2 - x + 1)/(x - sqrt(3) + 1)**2)*(x + 1)*ata
nh(sqrt(1 - (x + 1 + sqrt(3))**2/(-x - 1 + sqrt(3))**2)*(sqrt(3)
```

```
+ 2)/sqrt(4*sqrt(3) + 7 + (x + 1 + sqrt(3))**2/(-x - 1 + sqrt(3))
**2))/(3*sqrt((-x - 1)/(x - sqrt(3) + 1)**2)*sqrt(sqrt(3) + 2)*sq
rt(-x**3 - 1))
```

Mathematica [C] time = 0.50524, size = 271, normalized size = 6.16

$$\frac{2\sqrt{6}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\left(4\sqrt{-2ix+\sqrt{3}+i}\sqrt{x^2-x+1}\left(\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)+\sqrt{2ix+\sqrt{3}-i}\left(\left((1+2i)+i\sqrt{3}\right)x-\sqrt{\right.\right.}{\left(3i+(1+2i)\sqrt{3}\right)\sqrt{-2ix+\sqrt{3}+i}\sqrt{-x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*Sqrt[6]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(Sqrt[-I + Sqrt[3] +
 (2*I)*x]*((-2 - I) - Sqrt[3] + ((1 + 2*I) + I*Sqrt[3])*x)*Ellipt
icF[ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqr
t[3])/(3*I + Sqrt[3])] + 4*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 - x
 + x^2]*EllipticPi[(2*Sqrt[3])/(3*I + (1 + 2*I)*Sqrt[3]), ArcSin[
Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I
+ Sqrt[3])]))/((3*I + (1 + 2*I)*Sqrt[3])*Sqrt[I + Sqrt[3] - (2*I)
*x]*Sqrt[-1 - x^3])
```

Maple [C] time = 0.022, size = 243, normalized size = 5.5

$$-\frac{2\,i}{3}\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\mathit{EllipticF}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)\frac{1}{\sqrt{-x^3}}$$

$$+\frac{4\,i}{\frac{3}{2}+\frac{i}{2}\sqrt{3}+\sqrt{3}}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\mathit{EllipticPi}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i}{2}\sqrt{3}+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+x-3^(1/2))/(1+x+3^(1/2))/(-x^3-1)^(1/2),x)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/
2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/
(-x^3-1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(
1/2))^(1/2),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))+4*I*(I*(x-1/2-
1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^(1/2)))^(1/2)*(
-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)/(3/2+1/2*I
*3^(1/2)+3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))
```

```
*3^(1/2))^(1/2),I*3^(1/2)/(3/2+1/2*I*3^(1/2)+3^(1/2)),(I*3^(1/2)/
(3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x - \sqrt{3} + 1}{\sqrt{-x^3 - 1}\left(x + \sqrt{3} + 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x - sqrt(3) + 1)/(sqrt(-x^3 - 1)*(x + sqrt(3) + 1)),x, algorithm="maxim
```

```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(-x^3 - 1)*(x + sqrt(3) + 1)), x
)
```

Fricas [A] time = 0.319938, size = 369, normalized size = 8.39

$$\frac{1}{6}\sqrt{3}\sqrt{2\sqrt{3} - 3}\log\left(\frac{6322680\,x^8 - 13553256\,x^7 + 26133432\,x^6 - 63422352\,x^5 + 113743056\,x^4 - 136435776\,x^3 + 102727296}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x - sqrt(3) + 1)/(sqrt(-x^3 - 1)*(x + sqrt(3) + 1)),x, algorithm="frica
```

```
[Out] 1/6*sqrt(3)*sqrt(2*sqrt(3) - 3)*log((6322680*x^8 - 13553256*x^7 +
 26133432*x^6 - 63422352*x^5 + 113743056*x^4 - 136435776*x^3 + 10
2727296*x^2 + 4*(1694157*x^6 - 5868732*x^5 + 10586298*x^4 - 12840
912*x^3 + 9886740*x^2 - 2*sqrt(3)*(489061*x^6 - 1694157*x^5 + 305
6001*x^4 - 3706852*x^3 + 2854056*x^2 - 1198884*x + 205636) - 4153
056*x + 712344)*sqrt(-x^3 - 1)*sqrt(2*sqrt(3) - 3) - sqrt(3)*(365
0401*x^8 - 7824976*x^7 + 15088144*x^6 - 36616912*x^5 + 65669584*x
^4 - 78771232*x^3 + 59309632*x^2 - 24558208*x + 4193392) - 425360
64*x + 7263168)/(6322680*x^8 - 37028184*x^7 + 94872792*x^6 - 1389
03408*x^5 + 127105440*x^4 - 74438112*x^3 + 27246240*x^2 - sqrt(3)
*(3650401*x^8 - 21378232*x^7 + 54774832*x^6 - 80195920*x^5 + 7338
4360*x^4 - 42976864*x^3 + 15730624*x^2 - 3290176*x + 301072) - 56
98752*x + 521472))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x - \sqrt{3} + 1}{\sqrt{-(x+1)(x^2 - x + 1)}\left(x + 1 + \sqrt{3}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+x-3**(1/2))/(1+x+3**(1/2))/(-x**3-1)**(1/2),x)
```

```
[Out] Integral((x - sqrt(3) + 1)/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 1
+ sqrt(3))), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x - \sqrt{3} + 1}{\sqrt{-x^3 - 1}\left(x + \sqrt{3} + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x - sqrt(3) + 1)/(sqrt(-x^3 - 1)*(x + sqrt(3) + 1)),x, algorithm="giac'
```

```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(-x^3 - 1)*(x + sqrt(3) + 1)), x
)
```

3.94 $\int \frac{\left(1-\sqrt{3}\right) \sqrt[3]{a}+\sqrt[3]{b} x}{\left(\left(1+\sqrt{3}\right) \sqrt[3]{a}+\sqrt[3]{b} x\right) \sqrt{a+b x^3}} \, dx$

Optimal. Leaf size=69

$$-\frac{2 \tan^{-1}\left(\frac{\sqrt{3+2 \sqrt{3}} \sqrt[6]{a} \left(\sqrt[3]{a}+\sqrt[3]{b} x\right)}{\sqrt{a+b x^3}}\right)}{\sqrt{3+2 \sqrt{3}} \sqrt[6]{a} \sqrt[3]{b}}$$

```
[Out] (-2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) + b^(1/3)*x))/Sq
rt[a + b*x^3]])/(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rubi [A] time = 0.326516, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 58, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034

$$-\frac{2 \tan^{-1}\left(\frac{\sqrt{3+2 \sqrt{3}} \sqrt[6]{a} \left(\sqrt[3]{a}+\sqrt[3]{b} x\right)}{\sqrt{a+b x^3}}\right)}{\sqrt{3+2 \sqrt{3}} \sqrt[6]{a} \sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt
```

```
[Out] (-2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) + b^(1/3)*x))/Sq
rt[a + b*x^3]])/(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rubi in Sympy [A] time = 42.1929, size = 162, normalized size = 2.35

$$\frac{2 \tilde{\infty} \sqrt{\frac{a^{\frac{2}{3}} - \sqrt[3]{a} \sqrt[3]{b} x + b^{\frac{2}{3}} x^{2}}{\left(\sqrt[3]{a} \left(1+\sqrt{3}\right) + \sqrt[3]{b} x\right)^{2}}} \left(\sqrt[3]{a} + \sqrt[3]{b} x\right) F\left(\operatorname{asin}\left(\frac{- \sqrt[3]{a} \left(-1+\sqrt{3}\right) + \sqrt[3]{b} x}{\sqrt[3]{a} \left(1+\sqrt{3}\right) + \sqrt[3]{b} x}\right)\middle| -7 - 4 \sqrt{3}\right)}{\sqrt{\frac{\sqrt[3]{a} \left(\sqrt[3]{a} + \sqrt[3]{b} x\right)}{\left(\sqrt[3]{a} \left(1+\sqrt{3}\right) + \sqrt[3]{b} x\right)^{2}}} \sqrt{a + b x^{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(b**(1/3)*x+a**(1/3)*(1+3**(
```

```
[Out] 2*zoo*sqrt((a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(a**(
1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*(a**(1/3) + b**(1/3)*x)*elli
ptic_f(asin((-a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)/(a**(1/3)*(1
+ sqrt(3)) + b**(1/3)*x)), -7 - 4*sqrt(3))/(sqrt(a**(1/3)*(a**(1/
3) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(a
 + b*x**3))
```

Mathematica [C] time = 1.03132, size = 320, normalized size = 4.64

$$\frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{4\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\,\Pi\left(\frac{2i\sqrt{3}}{3+(2+i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)}{\left(3+(2+i)\sqrt{3}\right)\sqrt[3]{b}}-\frac{\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\sqrt[6]{-1}-\frac{i\sqrt[3]{b}x}{\sqrt[3]{a}}}\,F\left(\sin^{-}\right.}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/}}{\left(1+\sqrt[3]{-1}\right.}}}\right.}{\sqrt{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
```

```
[Out] (2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(-((((-
1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(
1/3)]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1
+ (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/(3^(1/4)*b^(1/3)*Sqrt[(a^(
1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))])) + (4*(
-1)^(1/3)*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[1 - (b^(1/3)*x)/a^(1/3) +
 (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[((2*I)*Sqrt[3])/(3 + (2 + I)*S
qrt[3]), ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^
(1/3))*a^(1/3))]], (-1)^(1/3)])/((3 + (2 + I)*Sqrt[3])*b^(1/3))))
/Sqrt[a + b*x^3]
```

Maple [F] time = 0.164, size = 0, normalized size = 0.

$$\int 1\left(\sqrt[3]{b}x+\sqrt[3]{a}\left(-\sqrt{3}+1\right)\right)\left(\sqrt[3]{b}x+\sqrt[3]{a}\left(1+\sqrt{3}\right)\right)^{-1}\frac{1}{\sqrt{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^(1/3)*x+a^(1/3)*(-3^(1/2)+1))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^3+a)
```

```
[Out] int((b^(1/3)*x+a^(1/3)*(-3^(1/2)+1))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2
)))/(b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b^{\frac{1}{3}}x - a^{\frac{1}{3}}\left(\sqrt{3} - 1\right)}{\sqrt{bx^3 + a}\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}\left(\sqrt{3} + 1\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))/(sqrt(b*x^3 + a)*(b^(1/3)*x + a^(1/
```

```
[Out] integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))/(sqrt(b*x^3 + a)*(b
^(1/3)*x + a^(1/3)*(sqrt(3) + 1))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))/(sqrt(b*x^3 + a)*(b^(1/3)*x + a^(1/
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{-\sqrt{3}\sqrt[3]{a} + \sqrt[3]{a} + \sqrt[3]{b}x}{\sqrt{a + bx^3}\left(\sqrt[3]{a} + \sqrt{3}\sqrt[3]{a} + \sqrt[3]{b}x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(b**(1/3)*x+a**(1/3)*(1+3**(1/2))
```

```
[Out] Integral((-sqrt(3)*a**(1/3) + a**(1/3) + b**(1/3)*x)/(sqrt(a + b*
x**3)*(a**(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)), x)
```

GIAC/XCAS [A] time = 0.603278, size = 4, normalized size = 0.06

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))/(sqrt(b*x^3 + a)*(b^(1/3)*x + a^(1/
```

```
[Out] sage0*x
```

3.95 $\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$

Optimal. Leaf size=71

$$\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

```
[Out] (2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) - b^(1/3)*x))/Sqr
t[a - b*x^3]])/(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rubi [A] time = 0.323023, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 61, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033

$$\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)*Sqrt
```

```
[Out] (2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) - b^(1/3)*x))/Sqr
t[a - b*x^3]])/(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rubi in Sympy [A] time = 50.3561, size = 162, normalized size = 2.28

$$\frac{2\tilde{\infty}\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)F\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(-b**(1/3)*x+a**(1/3)*(1+3*
b*x**3+a)**(1/2),x)
```

```
[Out] 2*zoo*sqrt((a**(2/3) + a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(a**(
1/3)*(1 + sqrt(3)) - b**(1/3)*x)**2)*(a**(1/3) - b**(1/3)*x)*elli
ptic_f(asin((a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)/(a**(1/3)*(1 +
 sqrt(3)) - b**(1/3)*x)), -7 - 4*sqrt(3))/(sqrt(a**(1/3)*(a**(1/3
) - b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) - b**(1/3)*x)**2)*sqrt(a
- b*x**3))
```

Mathematica [C] time = 1.35331, size = 329, normalized size = 4.63

$$\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{\left(\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}F\left(\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}}-\frac{4\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\left(\frac{}{3+}\right.}{3+(2+i}\right.}{\sqrt[3]{b}\sqrt{a-bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x
```

```
[Out] (2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((((-1)
^(1/3)*a^(1/3) + b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) + (-1)^(1/3
)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(
a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1
)^(1/3)])/Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))
*a^(1/3))] - (4*(-1)^(1/3)*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[1 + (b^(
1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[((2*I)*Sqrt[3
])/(3 + (2 + I)*Sqrt[3]), ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/
3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/(3 + (2 + I)*Sqr
t[3])))/(b^(1/3)*Sqrt[a - b*x^3])
```

Maple [F] time = 0.167, size = 0, normalized size = 0.

$$\int 1\left(-\sqrt[3]{b}x+\sqrt[3]{a}\left(-\sqrt{3}+1\right)\right)\left(-\sqrt[3]{b}x+\sqrt[3]{a}\left(1+\sqrt{3}\right)\right)^{-1}\frac{1}{\sqrt{-bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b^(1/3)*x+a^(1/3)*(-3^(1/2)+1))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-
b*x^3+a)^(1/2),x)
```

```
[Out] int((-b^(1/3)*x+a^(1/3)*(-3^(1/2)+1))/(-b^(1/3)*x+a^(1/3)*(1+3^(1
/2)))/(-b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}\left(\sqrt{3}-1\right)}{\sqrt{-bx^3+a}\left(b^{\frac{1}{3}}x - a^{\frac{1}{3}}\left(\sqrt{3}+1\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))/(sqrt(-b*x^3 + a)*(b^(1/3)*x - a^(1
```

```
[Out] integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))/(sqrt(-b*x^3 + a)*(
b^(1/3)*x - a^(1/3)*(sqrt(3) + 1))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))/(sqrt(-b*x^3 + a)*(b^(1/3)*x - a^(1
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{-\sqrt[3]{a} + \sqrt{3}\sqrt[3]{a} + \sqrt[3]{b}x}{\sqrt{a-bx^3}\left(-\sqrt{3}\sqrt[3]{a} - \sqrt[3]{a} + \sqrt[3]{b}x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(-b**(1/3)*x+a**(1/3)*(1+3**(1/2
b*x**3+a)**(1/2),x)
```

```
[Out] Integral((-a**(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)/(sqrt(a - b*
x**3)*(-sqrt(3)*a**(1/3) - a**(1/3) + b**(1/3)*x)), x)
```

GIAC/XCAS [A] time = 0.601401, size = 4, normalized size = 0.06

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))/(sqrt(-b*x^3 + a)*(b^(1/3)*x - a^(1
```

```
[Out] sage0*x
```

3.96 $\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

Optimal. Leaf size=72

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{bx^3-a}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

```
[Out] (2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) - b^(1/3)*x))/Sq
rt[-a + b*x^3]])/(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rubi [A] time = 0.329929, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 62, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{bx^3-a}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)*Sqrt
a + b*x^3]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) - b^(1/3)*x))/Sq
rt[-a + b*x^3]])/(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rubi in Sympy [A] time = 61.1588, size = 248, normalized size = 3.44

$$\frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{a^{\frac{2}{3}}\left(1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+\frac{b^{\frac{2}{3}}x^2}{a^{\frac{2}{3}}}\right)}{\left(\sqrt[3]{a}\left(-\sqrt{3}+1\right)-\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\operatorname{atanh}\left(\frac{\sqrt{1-\frac{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{b}x\right)^2}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}\left(\sqrt{3}+2\right)}}{\sqrt{4\sqrt{3}+7+\frac{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{b}x\right)^2}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}}\right)}{3\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(-\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(-\sqrt{3}+1\right)-\sqrt[3]{b}x\right)^2}}\sqrt{\sqrt{3}+2}\sqrt{-a+bx^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(-b**(1/3)*x+a**(1/3)*(1+3*
a)**(1/2),x)
```

```
[Out] 2*3**(3/4)*sqrt(a**(2/3)*(1 + b**(1/3)*x/a**(1/3) + b**(2/3)*x**2
/a**(2/3))/(a**(1/3)*(-sqrt(3) + 1) - b**(1/3)*x)**2)*(a**(1/3) -
 b**(1/3)*x)*atanh(sqrt(1 - (a**(1/3)*(1 + sqrt(3)) - b**(1/3)*x)
**2/(a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)**2)*(sqrt(3) + 2)/sqrt
(4*sqrt(3) + 7 + (a**(1/3)*(1 + sqrt(3)) - b**(1/3)*x)**2/(a**(1/
3)*(-1 + sqrt(3)) + b**(1/3)*x)**2))/(3*b**(1/3)*sqrt(a**(1/3)*(-
a**(1/3) + b**(1/3)*x)/(a**(1/3)*(-sqrt(3) + 1) - b**(1/3)*x)**2)
*sqrt(sqrt(3) + 2)*sqrt(-a + b*x**3))
```

Mathematica [C] time = 1.3479, size = 330, normalized size = 4.58

$$\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{\left(\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}F\left(\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}}-\frac{4\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\cdots}{3+(2+i\cdots}\cdots\right)}{\sqrt[3]{b}\sqrt{bx^3-a}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x
a + b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((((-1)
^(1/3)*a^(1/3) + b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) + (-1)^(1/3
)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(
a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1
)^(1/3)])/Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))
*a^(1/3))] - (4*(-1)^(1/3)*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[1 + (b^(
1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[((2*I)*Sqrt[3
])/(3 + (2 + I)*Sqrt[3]), ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/
3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/(3 + (2 + I)*Sqr
t[3])))/(b^(1/3)*Sqrt[-a + b*x^3])
```

Maple [F] time = 0.14, size = 0, normalized size = 0.

$$\int 1\left(-\sqrt[3]{b}x+\sqrt[3]{a}\left(-\sqrt{3}+1\right)\right)\left(-\sqrt[3]{b}x+\sqrt[3]{a}\left(1+\sqrt{3}\right)\right)^{-1}\frac{1}{\sqrt{bx^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b^(1/3)*x+a^(1/3)*(-3^(1/2)+1))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^3-
a)^(1/2),x)
```

```
[Out] int((-b^(1/3)*x+a^(1/3)*(-3^(1/2)+1))/(-b^(1/3)*x+a^(1/3)*(1+3^(1
/2)))/(b*x^3-a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}\left(\sqrt{3} - 1\right)}{\sqrt{bx^3 - a}\left(b^{\frac{1}{3}}x - a^{\frac{1}{3}}\left(\sqrt{3} + 1\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))/(sqrt(b*x^3 - a)*(b^(1/3)*x - a^(1/
```

```
[Out] integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))/(sqrt(b*x^3 - a)*(b
^(1/3)*x - a^(1/3)*(sqrt(3) + 1))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))/(sqrt(b*x^3 - a)*(b^(1/3)*x - a^(1/
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{-\sqrt[3]{a} + \sqrt{3}\sqrt[3]{a} + \sqrt[3]{b}x}{\sqrt{-a + bx^3}\left(-\sqrt{3}\sqrt[3]{a} - \sqrt[3]{a} + \sqrt[3]{b}x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(-b**(1/3)*x+a**(1/3)*(1+3**(1/2
a)**(1/2),x)
```

```
[Out] Integral((-a**(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)/(sqrt(-a + b
*x**3)*(-sqrt(3)*a**(1/3) - a**(1/3) + b**(1/3)*x)), x)
```

GIAC/XCAS [A] time = 0.601029, size = 4, normalized size = 0.06

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))/(sqrt(b*x^3 - a)*(b^(1/3)*x - a^(1/
```

```
[Out] sage0*x
```

3.97 $\int \frac{\left(1-\sqrt{3}\right) \sqrt[3]{a}+\sqrt[3]{b} x}{\left(\left(1+\sqrt{3}\right) \sqrt[3]{a}+\sqrt[3]{b} x\right) \sqrt{-a-b x^3}} \, dx$

Optimal. Leaf size=72

$$-\frac{2 \tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

```
[Out] (-2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) + b^(1/3)*x))/S
qrt[-a - b*x^3]])/(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rubi [A] time = 0.297422, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 61, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033

$$-\frac{2 \tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[-
a - b*x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) + b^(1/3)*x))/S
qrt[-a - b*x^3]])/(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rubi in Sympy [A] time = 59.654, size = 253, normalized size = 3.51

$$-\frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{a^{\frac{2}{3}}\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+\frac{b^{\frac{2}{3}}x^2}{a^{\frac{2}{3}}}\right)}{\left(\sqrt[3]{a}\left(-\sqrt{3}+1\right)+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\operatorname{atanh}\left(\frac{\sqrt{1-\frac{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)-\sqrt[3]{b}x\right)^2}}\left(\sqrt{3}+2\right)}{\sqrt{4\sqrt{3}+7+\frac{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)-\sqrt[3]{b}x\right)^2}}}\right)}{3\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(-\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(-\sqrt{3}+1\right)+\sqrt[3]{b}x\right)^2}}\sqrt{\sqrt{3}+2}\sqrt{-a-bx^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(b**(1/3)*x+a**(1/3)*(1+3**(
b*x**3-a)**(1/2),x)
```

```
[Out] -2*3**(3/4)*sqrt(a**(2/3)*(1 - b**(1/3)*x/a**(1/3) + b**(2/3)*x**
2/a**(2/3))/(a**(1/3)*(-sqrt(3) + 1) + b**(1/3)*x)**2)*(a**(1/3)
+ b**(1/3)*x)*atanh(sqrt(1 - (a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x
)**2/(a**(1/3)*(-1 + sqrt(3)) - b**(1/3)*x)**2)*(sqrt(3) + 2)/sqr
t(4*sqrt(3) + 7 + (a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2/(a**(1
/3)*(-1 + sqrt(3)) - b**(1/3)*x)**2))/(3*b**(1/3)*sqrt(a**(1/3)*(
-a**(1/3) - b**(1/3)*x)/(a**(1/3)*(-sqrt(3) + 1) + b**(1/3)*x)**2
)*sqrt(sqrt(3) + 2)*sqrt(-a - b*x**3))
```

Mathematica [C] time = 1.04142, size = 323, normalized size = 4.49

$$\frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{4\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\,\Pi\left(\frac{2i\sqrt{3}}{3+(2+i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)}{\left(3+(2+i)\sqrt{3}\right)\sqrt[3]{b}}-\frac{\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\sqrt[6]{-1}-\frac{i\sqrt[3]{b}x}{\sqrt[3]{a}}}F\left(\sin^{-}\cdots\right.}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/\cdots}}{\left(1+\sqrt[3]{-1}\cdots\right.}}}\cdots\right)}{\sqrt{-a-bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
a - b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(-((((-
1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(
1/3)]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1
+ (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/(3^(1/4)*b^(1/3)*Sqrt[(a^(
1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))])) + (4*(
-1)^(1/3)*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[1 - (b^(1/3)*x)/a^(1/3) +
 (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[((2*I)*Sqrt[3])/(3 + (2 + I)*S
qrt[3]), ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^
(1/3))*a^(1/3))]], (-1)^(1/3)])/((3 + (2 + I)*Sqrt[3])*b^(1/3))))
/Sqrt[-a - b*x^3]
```

Maple [F] time = 0.119, size = 0, normalized size = 0.

$$\int 1\left(\sqrt[3]{b}x+\sqrt[3]{a}\left(-\sqrt{3}+1\right)\right)\left(\sqrt[3]{b}x+\sqrt[3]{a}\left(1+\sqrt{3}\right)\right)^{-1}\frac{1}{\sqrt{-bx^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^(1/3)*x+a^(1/3)*(-3^(1/2)+1))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-
b*x^3-a)^(1/2),x)
```

```
[Out] int((b^(1/3)*x+a^(1/3)*(-3^(1/2)+1))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2
)))/(-b*x^3-a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b^{\frac{1}{3}}x - a^{\frac{1}{3}}\left(\sqrt{3} - 1\right)}{\sqrt{-bx^3 - a}\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}\left(\sqrt{3} + 1\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))/(sqrt(-b*x^3 - a)*(b^(1/3)*x + a^(1
```

```
[Out] integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))/(sqrt(-b*x^3 - a)*(
b^(1/3)*x + a^(1/3)*(sqrt(3) + 1))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))/(sqrt(-b*x^3 - a)*(b^(1/3)*x + a^(1
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{-\sqrt{3}\sqrt[3]{a} + \sqrt[3]{a} + \sqrt[3]{b}x}{\sqrt{-a - bx^3}\left(\sqrt[3]{a} + \sqrt{3}\sqrt[3]{a} + \sqrt[3]{b}x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(b**(1/3)*x+a**(1/3)*(1+3**(1/2))
b*x**3-a)**(1/2),x)
```

```
[Out] Integral((-sqrt(3)*a**(1/3) + a**(1/3) + b**(1/3)*x)/(sqrt(-a - b
*x**3)*(a**(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)), x)
```

GIAC/XCAS [A] time = 0.615311, size = 4, normalized size = 0.06

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))/(sqrt(-b*x^3 - a)*(b^(1/3)*x + a^(1
```

```
[Out] sage0*x
```

3.98 $\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=73

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{a}\left(x\sqrt[3]{\frac{b}{a}}+1\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

```
[Out] (-2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(1 + (b/a)^(1/3)*x))/Sqrt
[a + b*x^3]])/(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Rubi [A] time = 0.31246, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 52, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{a}\left(x\sqrt[3]{\frac{b}{a}}+1\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] + (b/a)^(1/3)*x)/((1 + Sqrt[3] + (b/a)^(1/3)*x)*Sqrt[a + b*x^3])
```

```
[Out] (-2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(1 + (b/a)^(1/3)*x))/Sqrt
[a + b*x^3]])/(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((1+(b/a)**(1/3)*x-3**(1/2))/(1+(b/a)**(1/3)*x+3**(1/2))/(b*x**3
```

```
[Out] Timed out
```

Mathematica [C] time = 7.98269, size = 1528, normalized size = 20.93

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sqrt[3] + (b/a)^(1/3)*x)/((1 + Sqrt[3] + (b/a)^(1/3)*x)*Sqrt[a + b
```

```
[Out] (-32*(26 + 15*Sqrt[3])*a^2*x*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)
/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))])/((5 + 3*Sqrt[3])*Sqrt[a +
b*x^3]*(2*(5 + 3*Sqrt[3])*a + b*x^3)*(8*(5 + 3*Sqrt[3])*a*AppellF
1[1/3, 1/2, 1, 4/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))
] - 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((b*x^3)/a), -((b*x^3)/(
10*a + 6*Sqrt[3]*a))] + (5 + 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3
, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))]))) - (32*Sqrt[3]
*(26 + 15*Sqrt[3])*a^2*x*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a),
 -((b*x^3)/(10*a + 6*Sqrt[3]*a))])/((5 + 3*Sqrt[3])*Sqrt[a + b*x^
3]*(2*(5 + 3*Sqrt[3])*a + b*x^3)*(8*(5 + 3*Sqrt[3])*a*AppellF1[1/
3, 1/2, 1, 4/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] -
3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a
 + 6*Sqrt[3]*a))] + (5 + 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, -(
(b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))]))) + (60*(26 + 15*Sq
rt[3])*a^2*(b/a)^(1/3)*x^2*AppellF1[2/3, 1/2, 1, 5/3, -((b*x^3)/a
), -((b*x^3)/(10*a + 6*Sqrt[3]*a))])/((5 + 3*Sqrt[3])*Sqrt[a + b*
x^3]*(2*(5 + 3*Sqrt[3])*a + b*x^3)*(10*(5 + 3*Sqrt[3])*a*AppellF1
[2/3, 1/2, 1, 5/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))]
 - 3*b*x^3*(AppellF1[5/3, 1/2, 2, 8/3, -((b*x^3)/a), -((b*x^3)/(1
0*a + 6*Sqrt[3]*a))] + (5 + 3*Sqrt[3])*AppellF1[5/3, 3/2, 1, 8/3,
 -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))]))) + (20*Sqrt[3]*
(26 + 15*Sqrt[3])*a^2*(b/a)^(1/3)*x^2*AppellF1[2/3, 1/2, 1, 5/3,
-((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))])/((5 + 3*Sqrt[3])*
Sqrt[a + b*x^3]*(2*(5 + 3*Sqrt[3])*a + b*x^3)*(10*(5 + 3*Sqrt[3])
*a*AppellF1[2/3, 1/2, 1, 5/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*S
qrt[3]*a))] - 3*b*x^3*(AppellF1[5/3, 1/2, 2, 8/3, -((b*x^3)/a), -
((b*x^3)/(10*a + 6*Sqrt[3]*a))] + (5 + 3*Sqrt[3])*AppellF1[5/3, 3
/2, 1, 8/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))]))) - (
16*(26 + 15*Sqrt[3])*a^2*(b/a)^(2/3)*x^3*AppellF1[1, 1/2, 1, 2, -
((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))])/(Sqrt[3]*(5 + 3*Sq
rt[3])*Sqrt[a + b*x^3]*(2*(5 + 3*Sqrt[3])*a + b*x^3)*(4*(5 + 3*Sq
rt[3])*a*AppellF1[1, 1/2, 1, 2, -((b*x^3)/a), -((b*x^3)/(10*a + 6
*Sqrt[3]*a))] - b*x^3*(AppellF1[2, 1/2, 2, 3, -((b*x^3)/a), -((b*
x^3)/(10*a + 6*Sqrt[3]*a))] + (5 + 3*Sqrt[3])*AppellF1[2, 3/2, 1,
 3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))]))) + (7*(26 +
15*Sqrt[3])*a*b*x^4*AppellF1[4/3, 1/2, 1, 7/3, -((b*x^3)/a), -((b
*x^3)/(10*a + 6*Sqrt[3]*a))])/((5 + 3*Sqrt[3])*Sqrt[a + b*x^3]*(2
```

```
*(5 + 3*Sqrt[3])*a + b*x^3)*(14*(5 + 3*Sqrt[3])*a*AppellF1[4/3, 1
/2, 1, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] - 3*b*
x^3*(AppellF1[7/3, 1/2, 2, 10/3, -((b*x^3)/a), -((b*x^3)/(10*a +
6*Sqrt[3]*a))] + (5 + 3*Sqrt[3])*AppellF1[7/3, 3/2, 1, 10/3, -((b
*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))])))
```

Maple [F] time = 0.144, size = 0, normalized size = 0.

$$\int 1\left(1+\sqrt[3]{\frac{b}{a}}x-\sqrt{3}\right)\left(1+\sqrt[3]{\frac{b}{a}}x+\sqrt{3}\right)^{-1}\frac{1}{\sqrt{bx^{3}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+(b/a)^(1/3)*x-3^(1/2))/(1+(b/a)^(1/3)*x+3^(1/2))/(b*x^3+a)^(1/2),x)
```

```
[Out] int((1+(b/a)^(1/3)*x-3^(1/2))/(1+(b/a)^(1/3)*x+3^(1/2))/(b*x^3+a)
^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\sqrt{3}+1}{\sqrt{bx^{3}+a}\left(x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(b/a)^(1/3) - sqrt(3) + 1)/(sqrt(b*x^3 + a)*(x*(b/a)^(1/3) + sqrt(3)
```

```
[Out] integrate((x*(b/a)^(1/3) - sqrt(3) + 1)/(sqrt(b*x^3 + a)*(x*(b/a)
^(1/3) + sqrt(3) + 1)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(b/a)^(1/3) - sqrt(3) + 1)/(sqrt(b*x^3 + a)*(x*(b/a)^(1/3) + sqrt(3)
```

```
[Out] Timed out
```

Sympy [A] time = 12.659, size = 0, normalized size = 0.

$$\text{NaN}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)**(1/3)*x-3**(1/2))/(1+(b/a)**(1/3)*x+3**(1/2))/(b*x**3+a)**
```

```
[Out] nan
```

GIAC/XCAS [A] time = 0.609565, size = 4, normalized size = 0.05

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(b/a)^(1/3) - sqrt(3) + 1)/(sqrt(b*x^3 + a)*(x*(b/a)^(1/3) + sqrt(3)
```

```
[Out] sage0*x
```

3.99 $\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a-bx^3}}\,dx$

Optimal. Leaf size=75

$$\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{a}\left(1-x\sqrt[3]{\frac{b}{a}}\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

```
[Out] (2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(1 - (b/a)^(1/3)*x))/Sqrt[
a - b*x^3]])/(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Rubi [A] time = 0.331808, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 55, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036

$$\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{a}\left(1-x\sqrt[3]{\frac{b}{a}}\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] - (b/a)^(1/3)*x)/((1 + Sqrt[3] - (b/a)^(1/3)*x)*Sqrt[a - b*x^3])
```

```
[Out] (2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(1 - (b/a)^(1/3)*x))/Sqrt[
a - b*x^3]])/(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((1-(b/a)**(1/3)*x-3**(1/2))/(1-(b/a)**(1/3)*x+3**(1/2))/(-
b*x**3+a)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 8.08572, size = 1491, normalized size = 19.88

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sqrt[3] - (b/a)^(1/3)*x)/((1 + Sqrt[3] - (b/a)^(1/3)*x)*Sqrt[a - b
```

```
[Out] (-32*(26 + 15*Sqrt[3])*a^2*x*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a
, (b*x^3)/(10*a + 6*Sqrt[3]*a)])/((5 + 3*Sqrt[3])*Sqrt[a - b*x^3]
*(2*(5 + 3*Sqrt[3])*a - b*x^3)*(8*(5 + 3*Sqrt[3])*a*AppellF1[1/3,
 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a)] + 3*b*x^3*
(AppellF1[4/3, 1/2, 2, 7/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*
a)] + (5 + 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, (b*x^3)/a, (b*x^
3)/(10*a + 6*Sqrt[3]*a)]))) - (32*Sqrt[3]*(26 + 15*Sqrt[3])*a^2*x
*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*
a)])/((5 + 3*Sqrt[3])*Sqrt[a - b*x^3]*(2*(5 + 3*Sqrt[3])*a - b*x^
3)*(8*(5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*
x^3)/(10*a + 6*Sqrt[3]*a)] + 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3,
(b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a)] + (5 + 3*Sqrt[3])*Appell
F1[4/3, 3/2, 1, 7/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a)])))
- (60*(26 + 15*Sqrt[3])*a^2*(b/a)^(1/3)*x^2*AppellF1[2/3, 1/2, 1,
 5/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a)])/((5 + 3*Sqrt[3])*
Sqrt[a - b*x^3]*(2*(5 + 3*Sqrt[3])*a - b*x^3)*(10*(5 + 3*Sqrt[3])
*a*AppellF1[2/3, 1/2, 1, 5/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3
]*a)] + 3*b*x^3*(AppellF1[5/3, 1/2, 2, 8/3, (b*x^3)/a, (b*x^3)/(1
0*a + 6*Sqrt[3]*a)] + (5 + 3*Sqrt[3])*AppellF1[5/3, 3/2, 1, 8/3,
(b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a)]))) - (20*Sqrt[3]*(26 + 1
5*Sqrt[3])*a^2*(b/a)^(1/3)*x^2*AppellF1[2/3, 1/2, 1, 5/3, (b*x^3)
/a, (b*x^3)/(10*a + 6*Sqrt[3]*a)])/((5 + 3*Sqrt[3])*Sqrt[a - b*x^
3]*(2*(5 + 3*Sqrt[3])*a - b*x^3)*(10*(5 + 3*Sqrt[3])*a*AppellF1[2
/3, 1/2, 1, 5/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a)] + 3*b*x
^3*(AppellF1[5/3, 1/2, 2, 8/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[
3]*a)] + (5 + 3*Sqrt[3])*AppellF1[5/3, 3/2, 1, 8/3, (b*x^3)/a, (b
*x^3)/(10*a + 6*Sqrt[3]*a)]))) - (16*(26 + 15*Sqrt[3])*a^2*(b/a)^
(2/3)*x^3*AppellF1[1, 1/2, 1, 2, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqr
t[3]*a)])/(Sqrt[3]*(5 + 3*Sqrt[3])*Sqrt[a - b*x^3]*(2*(5 + 3*Sqrt
[3])*a - b*x^3)*(4*(5 + 3*Sqrt[3])*a*AppellF1[1, 1/2, 1, 2, (b*x^
3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a)] + b*x^3*(AppellF1[2, 1/2, 2,
3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a)] + (5 + 3*Sqrt[3])*App
ellF1[2, 3/2, 1, 3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a)]))) -
 (7*(26 + 15*Sqrt[3])*a*b*x^4*AppellF1[4/3, 1/2, 1, 7/3, (b*x^3)/
a, (b*x^3)/(10*a + 6*Sqrt[3]*a)])/((5 + 3*Sqrt[3])*Sqrt[a - b*x^3
]*(2*(5 + 3*Sqrt[3])*a - b*x^3)*(14*(5 + 3*Sqrt[3])*a*AppellF1[4/
```

```
3, 1/2, 1, 7/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a)] + 3*b*x^
3*(AppellF1[7/3, 1/2, 2, 10/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[
3]*a)] + (5 + 3*Sqrt[3])*AppellF1[7/3, 3/2, 1, 10/3, (b*x^3)/a, (
b*x^3)/(10*a + 6*Sqrt[3]*a)])))
```

Maple [F] time = 0.148, size = 0, normalized size = 0.

$$\int 1\left(1-\sqrt[3]{\frac{b}{a}}x-\sqrt{3}\right)\left(1-\sqrt[3]{\frac{b}{a}}x+\sqrt{3}\right)^{-1}\frac{1}{\sqrt{-bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-(b/a)^(1/3)*x-3^(1/2))/(1-(b/a)^(1/3)*x+3^(1/2))/(-b*x^3+a)^(1/2),x)
```

```
[Out] int((1-(b/a)^(1/3)*x-3^(1/2))/(1-(b/a)^(1/3)*x+3^(1/2))/(-b*x^3+a
)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\sqrt{3}-1}{\sqrt{-bx^3+a}\left(x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(b/a)^(1/3) + sqrt(3) - 1)/(sqrt(-b*x^3 + a)*(x*(b/a)^(1/3) - sqrt(3
```

```
[Out] integrate((x*(b/a)^(1/3) + sqrt(3) - 1)/(sqrt(-b*x^3 + a)*(x*(b/a
)^(1/3) - sqrt(3) - 1)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(b/a)^(1/3) + sqrt(3) - 1)/(sqrt(-b*x^3 + a)*(x*(b/a)^(1/3) - sqrt(3
```

```
[Out] Timed out
```

Sympy [A] time = 13.636, size = 0, normalized size = 0.

$$\text{NaN}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)**(1/3)*x-3**(1/2))/(1-(b/a)**(1/3)*x+3**(1/2))/(-
b*x**3+a)**(1/2),x)
```

```
[Out] nan
```

GIAC/XCAS [A] time = 0.605362, size = 4, normalized size = 0.05

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(b/a)^(1/3) + sqrt(3) - 1)/(sqrt(-b*x^3 + a)*(x*(b/a)^(1/3) - sqrt(3
```

```
[Out] sage0*x
```

3.100 $\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a+bx^3}}\,dx$

Optimal. Leaf size=76

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{a}\left(1-x\sqrt[3]{\frac{b}{a}}\right)}{\sqrt{bx^3-a}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

```
[Out] (2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(1 - (b/a)^(1/3)*x))/Sqrt
[-a + b*x^3]])/(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Rubi [A] time = 0.33305, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 56, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{a}\left(1-x\sqrt[3]{\frac{b}{a}}\right)}{\sqrt{bx^3-a}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] - (b/a)^(1/3)*x)/((1 + Sqrt[3] - (b/a)^(1/3)*x)*Sqrt[-
a + b*x^3]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(1 - (b/a)^(1/3)*x))/Sqrt
[-a + b*x^3]])/(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((1-(b/a)**(1/3)*x-3**(1/2))/(1-(b/a)**(1/3)*x+3**(1/2))/(b*x**3
a)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 5.59776, size = 836, normalized size = 11.

$$\frac{\left(26+15\sqrt{3}\right)ax\left(x\left(x\left(-\frac{16\sqrt{3}a\left(\frac{b}{a}\right)^{2/3}F_1\left(1;\frac{1}{2},1;2;\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a+10a}\right)}{b\left(F_1\left(2;\frac{1}{2},2;3;\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a+10a}\right)+\left(5+3\sqrt{3}\right)F_1\left(2;\frac{3}{2},1;3;\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a+10a}\right)\right)x^3+4\left(5+3\sqrt{3}\right)aF_1\left(1;\frac{1}{2},1;2;\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a+10a}\right)}-\frac{}{3b\left(F_1\left(\frac{7}{3};\frac{1}{2},2;\right.\right.}\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sqrt[3] - (b/a)^(1/3)*x)/((1 + Sqrt[3] - (b/a)^(1/3)*x)*Sqrt[-
a + b*x^3]),x]
```

```
[Out] ((26 + 15*Sqrt[3])*a*x*((-96*(1 + Sqrt[3])*a*AppellF1[1/3, 1/2, 1
, 4/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a)])/(8*(5 + 3*Sqrt[3
])*a*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt
[3]*a)] + 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, (b*x^3)/a, (b*x^3)/
(10*a + 6*Sqrt[3]*a)] + (5 + 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3
, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a)])) + x*((-60*(3 + Sqrt[
3])*a*(b/a)^(1/3)*AppellF1[2/3, 1/2, 1, 5/3, (b*x^3)/a, (b*x^3)/(
10*a + 6*Sqrt[3]*a)])/(10*(5 + 3*Sqrt[3])*a*AppellF1[2/3, 1/2, 1,
 5/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a)] + 3*b*x^3*(AppellF
1[5/3, 1/2, 2, 8/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a)] + (5
 + 3*Sqrt[3])*AppellF1[5/3, 3/2, 1, 8/3, (b*x^3)/a, (b*x^3)/(10*a
 + 6*Sqrt[3]*a)])) + x*((-16*Sqrt[3]*a*(b/a)^(2/3)*AppellF1[1, 1/
2, 1, 2, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a)])/(4*(5 + 3*Sqrt
[3])*a*AppellF1[1, 1/2, 1, 2, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3
]*a)] + b*x^3*(AppellF1[2, 1/2, 2, 3, (b*x^3)/a, (b*x^3)/(10*a +
6*Sqrt[3]*a)] + (5 + 3*Sqrt[3])*AppellF1[2, 3/2, 1, 3, (b*x^3)/a,
 (b*x^3)/(10*a + 6*Sqrt[3]*a)])) - (21*b*x*AppellF1[4/3, 1/2, 1,
7/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a)])/(14*(5 + 3*Sqrt[3]
)*a*AppellF1[4/3, 1/2, 1, 7/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[
3]*a)] + 3*b*x^3*(AppellF1[7/3, 1/2, 2, 10/3, (b*x^3)/a, (b*x^3)/
(10*a + 6*Sqrt[3]*a)] + (5 + 3*Sqrt[3])*AppellF1[7/3, 3/2, 1, 10/
3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a)]))))))/(3*(5 + 3*Sqrt[
3])*(2*(5 + 3*Sqrt[3])*a - b*x^3)*Sqrt[-a + b*x^3])
```

Maple [F] time = 0.114, size = 0, normalized size = 0.

$$\int 1\left(1-\sqrt[3]{\frac{b}{a}}\, x-\sqrt{3}\right)\left(1-\sqrt[3]{\frac{b}{a}}\, x+\sqrt{3}\right)^{-1}\frac{1}{\sqrt{b x^{3}-a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-(b/a)^(1/3)*x-3^(1/2))/(1-(b/a)^(1/3)*x+3^(1/2))/(b*x^3-a)^(1/2),x)
```

```
[Out] int((1-(b/a)^(1/3)*x-3^(1/2))/(1-(b/a)^(1/3)*x+3^(1/2))/(b*x^3-a)
^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\sqrt{3}-1}{\sqrt{b x^{3}-a}\left(x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\sqrt{3}-1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(b/a)^(1/3) + sqrt(3) - 1)/(sqrt(b*x^3 - a)*(x*(b/a)^(1/3) - sqrt(3)
```

```
[Out] integrate((x*(b/a)^(1/3) + sqrt(3) - 1)/(sqrt(b*x^3 - a)*(x*(b/a)
^(1/3) - sqrt(3) - 1)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(b/a)^(1/3) + sqrt(3) - 1)/(sqrt(b*x^3 - a)*(x*(b/a)^(1/3) - sqrt(3)
```

```
[Out] Timed out
```

Sympy [A] time = 13.9764, size = 0, normalized size = 0.

NaN

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)**(1/3)*x-3**(1/2))/(1-(b/a)**(1/3)*x+3**(1/2))/(b*x**3-
a)**(1/2),x)
```

```
[Out] nan
```

GIAC/XCAS [A] time = 0.616158, size = 4, normalized size = 0.05

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(b/a)^(1/3) + sqrt(3) - 1)/(sqrt(b*x^3 - a)*(x*(b/a)^(1/3) - sqrt(3)
```

```
[Out] sage0*x
```

3.101 $\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a-bx^3}}\,dx$

Optimal. Leaf size=76

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{a}\left(x\sqrt[3]{\frac{b}{a}}+1\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

```
[Out] (-2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(1 + (b/a)^(1/3)*x))/Sqr
t[-a - b*x^3]])/(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Rubi [A] time = 0.321871, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 55, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{a}\left(x\sqrt[3]{\frac{b}{a}}+1\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] + (b/a)^(1/3)*x)/((1 + Sqrt[3] + (b/a)^(1/3)*x)*Sqrt[-
a - b*x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(1 + (b/a)^(1/3)*x))/Sqr
t[-a - b*x^3]])/(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((1+(b/a)**(1/3)*x-3**(1/2))/(1+(b/a)**(1/3)*x+3**(1/2))/(-
b*x**3-a)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 8.03579, size = 1546, normalized size = 20.34

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sqrt[3] + (b/a)^(1/3)*x)/((1 + Sqrt[3] + (b/a)^(1/3)*x)*Sqrt[-
a - b*x^3]),x]
```

```
[Out] (-32*(26 + 15*Sqrt[3])*a^2*x*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)
/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))])/((5 + 3*Sqrt[3])*Sqrt[-a -
 b*x^3]*(2*(5 + 3*Sqrt[3])*a + b*x^3)*(8*(5 + 3*Sqrt[3])*a*Appell
F1[1/3, 1/2, 1, 4/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a)
)] - 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((b*x^3)/a), -((b*x^3)/
(10*a + 6*Sqrt[3]*a))] + (5 + 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/
3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))]))) - (32*Sqrt[3
]*(26 + 15*Sqrt[3])*a^2*x*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a)
, -((b*x^3)/(10*a + 6*Sqrt[3]*a))])/((5 + 3*Sqrt[3])*Sqrt[-a - b*
x^3]*(2*(5 + 3*Sqrt[3])*a + b*x^3)*(8*(5 + 3*Sqrt[3])*a*AppellF1[
1/3, 1/2, 1, 4/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))]
- 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((b*x^3)/a), -((b*x^3)/(10
*a + 6*Sqrt[3]*a))] + (5 + 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3,
-((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))]))) + (60*(26 + 15*
Sqrt[3])*a^2*(b/a)^(1/3)*x^2*AppellF1[2/3, 1/2, 1, 5/3, -((b*x^3)
/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))])/((5 + 3*Sqrt[3])*Sqrt[-a -
 b*x^3]*(2*(5 + 3*Sqrt[3])*a + b*x^3)*(10*(5 + 3*Sqrt[3])*a*Appel
lF1[2/3, 1/2, 1, 5/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a
))] - 3*b*x^3*(AppellF1[5/3, 1/2, 2, 8/3, -((b*x^3)/a), -((b*x^3)
/(10*a + 6*Sqrt[3]*a))] + (5 + 3*Sqrt[3])*AppellF1[5/3, 3/2, 1, 8
/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))]))) + (20*Sqrt[
3]*(26 + 15*Sqrt[3])*a^2*(b/a)^(1/3)*x^2*AppellF1[2/3, 1/2, 1, 5/
3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))])/((5 + 3*Sqrt[3
])*Sqrt[-a - b*x^3]*(2*(5 + 3*Sqrt[3])*a + b*x^3)*(10*(5 + 3*Sqrt
[3])*a*AppellF1[2/3, 1/2, 1, 5/3, -((b*x^3)/a), -((b*x^3)/(10*a +
 6*Sqrt[3]*a))] - 3*b*x^3*(AppellF1[5/3, 1/2, 2, 8/3, -((b*x^3)/a
), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] + (5 + 3*Sqrt[3])*AppellF1[5/
3, 3/2, 1, 8/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))])))
 - (16*(26 + 15*Sqrt[3])*a^2*(b/a)^(2/3)*x^3*AppellF1[1, 1/2, 1,
2, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))])/(Sqrt[3]*(5 +
3*Sqrt[3])*Sqrt[-a - b*x^3]*(2*(5 + 3*Sqrt[3])*a + b*x^3)*(4*(5 +
 3*Sqrt[3])*a*AppellF1[1, 1/2, 1, 2, -((b*x^3)/a), -((b*x^3)/(10*
a + 6*Sqrt[3]*a))] - b*x^3*(AppellF1[2, 1/2, 2, 3, -((b*x^3)/a),
-((b*x^3)/(10*a + 6*Sqrt[3]*a))] + (5 + 3*Sqrt[3])*AppellF1[2, 3/
2, 1, 3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))]))) + (7*(
```

```
26 + 15*Sqrt[3])*a*b*x^4*AppellF1[4/3, 1/2, 1, 7/3, -((b*x^3)/a),
 -((b*x^3)/(10*a + 6*Sqrt[3]*a))])/((5 + 3*Sqrt[3])*Sqrt[-a - b*x
^3]*(2*(5 + 3*Sqrt[3])*a + b*x^3)*(14*(5 + 3*Sqrt[3])*a*AppellF1[
4/3, 1/2, 1, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))]
- 3*b*x^3*(AppellF1[7/3, 1/2, 2, 10/3, -((b*x^3)/a), -((b*x^3)/(1
0*a + 6*Sqrt[3]*a))] + (5 + 3*Sqrt[3])*AppellF1[7/3, 3/2, 1, 10/3
, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))])))
```

Maple [F] time = 0.13, size = 0, normalized size = 0.

$$\int 1\left(1+\sqrt[3]{\frac{b}{a}}x-\sqrt{3}\right)\left(1+\sqrt[3]{\frac{b}{a}}x+\sqrt{3}\right)^{-1}\frac{1}{\sqrt{-bx^{3}-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]    int((1+(b/a)^(1/3)*x-3^(1/2))/(1+(b/a)^(1/3)*x+3^(1/2))/(-b*x^3-
a)^(1/2),x)
```

```
[Out] int((1+(b/a)^(1/3)*x-3^(1/2))/(1+(b/a)^(1/3)*x+3^(1/2))/(-b*x^3-a
)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\sqrt{3}+1}{\sqrt{-bx^{3}-a}\left(x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(b/a)^(1/3) - sqrt(3) + 1)/(sqrt(-b*x^3 - a)*(x*(b/a)^(1/3) + sqrt(3
```

```
[Out] integrate((x*(b/a)^(1/3) - sqrt(3) + 1)/(sqrt(-b*x^3 - a)*(x*(b/a
)^(1/3) + sqrt(3) + 1)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(b/a)^(1/3) - sqrt(3) + 1)/(sqrt(-b*x^3 - a)*(x*(b/a)^(1/3) + sqrt(3
```

```
[Out] Timed out
```

Sympy [A] time = 13.3735, size = 0, normalized size = 0.

$$\text{NaN}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)**(1/3)*x-3**(1/2))/(1+(b/a)**(1/3)*x+3**(1/2))/(-
b*x**3-a)**(1/2),x)
```

```
[Out] nan
```

GIAC/XCAS [A] time = 0.617284, size = 4, normalized size = 0.05

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(b/a)^(1/3) - sqrt(3) + 1)/(sqrt(-b*x^3 - a)*(x*(b/a)^(1/3) + sqrt(3
```

```
[Out] sage0*x
```

3.102 $\int \frac{1+x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=145

$$\frac{\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{3+2\sqrt{3}}}$$

```
[Out] -(ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]]/Sqrt[3 + 2*
Sqrt[3]]) + (Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sq
rt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x
)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*S
qrt[1 + x^3])
```

Rubi [A] time = 0.356485, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$

$$\frac{\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{3+2\sqrt{3}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x)/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```


```
[Out] -(ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]]/Sqrt[3 + 2*
Sqrt[3]]) + (Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sq
rt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x
)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*S
qrt[1 + x^3])
```

Rubi in Sympy [A] time = 13.2164, size = 78, normalized size = 0.54

$$\frac{2\tilde{\infty}\sqrt{\frac{x^2-x+1}{\left(x+1+\sqrt{3}\right)^2}}\,(x+1)\,F\left(\operatorname{asin}\left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+x)/(1+x+3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] 2*zoo*sqrt((x**2 - x + 1)/(x + 1 + sqrt(3))**2)*(x + 1)*elliptic_
f(asin((x - sqrt(3) + 1)/(x + 1 + sqrt(3))), -7 - 4*sqrt(3))/(sqr
t((x + 1)/(x + 1 + sqrt(3))**2)*sqrt(x**3 + 1))
```

Mathematica [C] time = 0.532937, size = 269, normalized size = 1.86

$$\frac{2\sqrt{6}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\left(2\sqrt{-2ix+\sqrt{3}+i}\sqrt{x^2-x+1}\left(\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)+\sqrt{2ix+\sqrt{3}-i}\left(\left((1+2i)+i\sqrt{3}\right)x-\sqrt{}\right.\right.}{\left(3i+(1+2i)\sqrt{3}\right)\sqrt{-2ix+\sqrt{3}+i}\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(1 + x)/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[6]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(Sqrt[-I + Sqrt[3] +
 (2*I)*x]*((-2 - I) - Sqrt[3] + ((1 + 2*I) + I*Sqrt[3])*x)*Ellipt
icF[ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqr
t[3])/(3*I + Sqrt[3])] + 2*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 - x
 + x^2]*EllipticPi[(2*Sqrt[3])/(3*I + (1 + 2*I)*Sqrt[3]), ArcSin[
Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I
+ Sqrt[3])]))/((3*I + (1 + 2*I)*Sqrt[3])*Sqrt[I + Sqrt[3] - (2*I)
*x]*Sqrt[1 + x^3])
```

Maple [B] time = 0.034, size = 245, normalized size = 1.7

$$2\,\frac{3/2-i/2\sqrt{3}}{\sqrt{x^3+1}}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}EllipticF\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},\sqrt{\frac{-3/2+i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\right)$$
$$-2\,\frac{3/2-i/2\sqrt{3}}{\sqrt{x^3+1}}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}EllipticPi\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},1/3\,\left(-3/2+i/2\sqrt{3}\right)\sqrt{3},\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+x)/(1+x+3^(1/2))/(x^3+1)^(1/2),x)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1
/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/
(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1
/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^
```

```
(1/2))-2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((
x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^
(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticPi(((1+x
)/(3/2-1/2*I*3^(1/2)))^(1/2),1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2),((-
3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1}{\sqrt{x^3+1}\left(x+\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x + 1)/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x+1}{\sqrt{x^3+1}\left(x+\sqrt{3}+1\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x + 1)/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)),x, algorithm="fricas")
```

```
[Out] integral((x + 1)/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1}{\sqrt{(x+1)\left(x^2-x+1\right)}\left(x+1+\sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(1+x+3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] Integral((x + 1)/(sqrt((x + 1)*(x**2 - x + 1))*(x + 1 + sqrt(3)))
, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1}{\sqrt{x^3+1}\left(x+\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x + 1)/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)),x, algorithm="giac")
```

```
[Out] integrate((x + 1)/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)), x)
```

3.103 $\int \frac{1+x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=145

$$\frac{\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{2\sqrt{3}-3}}$$

```
[Out] -(ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]]/Sqrt[-3 +
 2*Sqrt[3]]) + (Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 +
 Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3]
+ x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2
]*Sqrt[1 + x^3])
```

Rubi [A] time = 0.378841, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$

$$\frac{\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{2\sqrt{3}-3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x)/((1 - Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] -(ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]]/Sqrt[-3 +
 2*Sqrt[3]]) + (Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 +
 Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3]
+ x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2
]*Sqrt[1 + x^3])
```

Rubi in Sympy [A] time = 41.3084, size = 226, normalized size = 1.56

$$-\frac{3^{\frac{3}{4}}\sqrt{\frac{x^2-x+1}{\left(x+1+\sqrt{3}\right)^2}}(x+1)\operatorname{atanh}\left(\frac{\left(-\sqrt{3}+2\right)\sqrt{-\frac{\left(-x-1+\sqrt{3}\right)^2}{\left(x+1+\sqrt{3}\right)^2}+1}}{\sqrt{\frac{\left(-x-1+\sqrt{3}\right)^2}{\left(x+1+\sqrt{3}\right)^2}-4\sqrt{3}+7}}\right)}{3\sqrt{\frac{x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{-\sqrt{3}+2}\sqrt{x^3+1}}+\frac{3^{\frac{3}{4}}\sqrt{\frac{x^2-x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{\sqrt{3}+2}(x+1)F\left(\operatorname{asin}\left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{3\sqrt{\frac{x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+x)/(1+x-3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] -3**(3/4)*sqrt((x**2 - x + 1)/(x + 1 + sqrt(3))**2)*(x + 1)*atanh
((-sqrt(3) + 2)*sqrt(-(-x - 1 + sqrt(3))**2/(x + 1 + sqrt(3))**2
+ 1)/sqrt((-x - 1 + sqrt(3))**2/(x + 1 + sqrt(3))**2 - 4*sqrt(3)
+ 7))/(3*sqrt((x + 1)/(x + 1 + sqrt(3))**2)*sqrt(-sqrt(3) + 2)*sq
rt(x**3 + 1)) + 3**(3/4)*sqrt((x**2 - x + 1)/(x + 1 + sqrt(3))**2
)*sqrt(sqrt(3) + 2)*(x + 1)*elliptic_f(asin((x - sqrt(3) + 1)/(x
+ 1 + sqrt(3))), -7 - 4*sqrt(3))/(3*sqrt((x + 1)/(x + 1 + sqrt(3)
)**2)*sqrt(x**3 + 1))
```

Mathematica [C] time = 0.454259, size = 267, normalized size = 1.84

$$-\frac{2\sqrt{6}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\left(2i\sqrt{-2ix+\sqrt{3}+i}\sqrt{x^2-x+1}\left(\frac{2i\sqrt{3}}{-3+(2+i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)+\sqrt{2ix+\sqrt{3}-i}\left(\left(\sqrt{3}+(-2-i)\right)x-\right.\right.}{\left(-3+(2+i)\sqrt{3}\right)\sqrt{-2ix+\sqrt{3}+i}\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(1 + x)/((1 - Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*Sqrt[6]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(Sqrt[-I + Sqrt[3]
+ (2*I)*x]*((1 + 2*I) - I*Sqrt[3] + ((-2 - I) + Sqrt[3])*x)*Ellip
ticF[ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sq
rt[3])/(3*I + Sqrt[3])] + (2*I)*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[
1 - x + x^2]*EllipticPi[((2*I)*Sqrt[3])/(-3 + (2 + I)*Sqrt[3]), A
rcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])
/(3*I + Sqrt[3])]))/((-3 + (2 + I)*Sqrt[3])*Sqrt[I + Sqrt[3] - (2
```

```
*I)*x]*Sqrt[1 + x^3])
```

Maple [B] time = 0.033, size = 245, normalized size = 1.7

$$2\frac{3/2-i/2\sqrt{3}}{\sqrt{x^3+1}}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}EllipticF\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},\sqrt{\frac{-3/2+i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\right)$$
$$-2\frac{3/2-i/2\sqrt{3}}{\sqrt{x^3+1}}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}EllipticPi\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},-1/3\left(-3/2+i/2\sqrt{3}\right)\sqrt{3},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+x)/(1+x-3^(1/2))/(x^3+1)^(1/2),x)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1
/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/
(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1
/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^
(1/2))-2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((
x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^
(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticPi(((1+x
)/(3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2),((
-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1}{\sqrt{x^3+1}\left(x-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x + 1)/(sqrt(x^3 + 1)*(x - sqrt(3) + 1)),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)/(sqrt(x^3 + 1)*(x - sqrt(3) + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x+1}{\sqrt{x^3+1}\left(x-\sqrt{3}+1\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 1)/(sqrt(x^3 + 1)*(x - sqrt(3) + 1)),x, algorithm="fricas")
```

```
[Out] integral((x + 1)/(sqrt(x^3 + 1)*(x - sqrt(3) + 1)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1}{\sqrt{(x+1)\left(x^2-x+1\right)}\left(x-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+x)/(1+x-3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] Integral((x + 1)/(sqrt((x + 1)*(x**2 - x + 1))*(x - sqrt(3) + 1))
, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1}{\sqrt{x^3+1}\left(x-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 1)/(sqrt(x^3 + 1)*(x - sqrt(3) + 1)),x, algorithm="giac")
```

```
[Out] integrate((x + 1)/(sqrt(x^3 + 1)*(x - sqrt(3) + 1)), x)
```

3.104 $\int \frac{e+fx}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=173

$$\frac{\left(e-\sqrt{3}f-f\right)\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}+\frac{\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(e-\left(1-\sqrt{3}\right)f\right)F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

```
[Out] ((e - f - Sqrt[3]*f)*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1
+ x^3]])/Sqrt[3*(3 + 2*Sqrt[3])] + (Sqrt[2 + Sqrt[3]]*(e - (1 - S
qrt[3])*f)*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Ellipt
icF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])
/(3^(3/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rubi [A] time = 0.496004, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$

$$\frac{\left(e-\left(1+\sqrt{3}\right)f\right)\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}+\frac{\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(e-\left(1-\sqrt{3}\right)f\right)F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(e + f*x)/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] ((e - (1 + Sqrt[3])*f)*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[
1 + x^3]])/Sqrt[3*(3 + 2*Sqrt[3])] + (Sqrt[2 + Sqrt[3]]*(e - (1 -
 Sqrt[3])*f)*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Elli
pticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]
])/(3^(3/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rubi in Sympy [A] time = 13.7666, size = 82, normalized size = 0.47

$$\frac{2\tilde{\infty}\sqrt{\frac{x^2-x+1}{\left(x+1+\sqrt{3}\right)^2}}\,(e+f)\,(x+1)\,F\left(\operatorname{asin}\left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x+e)/(1+x+3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] 2*zoo*sqrt((x**2 - x + 1)/(x + 1 + sqrt(3))**2)*(e + f)*(x + 1)*e
lliptic_f(asin((x - sqrt(3) + 1)/(x + 1 + sqrt(3))), -7 - 4*sqrt(
3))/(sqrt((x + 1)/(x + 1 + sqrt(3))**2)*sqrt(x**3 + 1))
```

Mathematica [C] time = 0.709146, size = 291, normalized size = 1.68

$$\frac{2\sqrt{\frac{2}{3}}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\left(2\sqrt{-2ix+\sqrt{3}+i}\sqrt{x^2-x+1}\left(\left(3+\sqrt{3}\right)f-\sqrt{3}e\right)\left(\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)+3f\sqrt{2ix+\sqrt{3}-}\right.}{\left(3i+(1+2i)\sqrt{3}\right)\sqrt{-2ix+\sqrt{3}+i}\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(e + f*x)/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[2/3]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(3*f*Sqrt[-I + Sqr
t[3] + (2*I)*x]*((-2 - I) - Sqrt[3] + ((1 + 2*I) + I*Sqrt[3])*x)*
EllipticF[ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))],
(2*Sqrt[3])/(3*I + Sqrt[3])] + 2*(-(Sqrt[3]*e) + (3 + Sqrt[3])*f)
*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt
[3])/(3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x
]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/((3*I + (1 +
 2*I)*Sqrt[3])*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 + x^3])
```

Maple [A] time = 0.038, size = 260, normalized size = 1.5

$$2\,\frac{f\left(3/2-i/2\sqrt{3}\right)}{\sqrt{x^3+1}}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}\,EllipticF\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},\sqrt{\frac{-3/2+i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\right)$$
$$+\frac{\left(2\,e-2\,f-2\,f\sqrt{3}\right)\left(\frac{3}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}{3}\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i}{2}\sqrt{3}}}\sqrt{\frac{1}{-\frac{3}{2}-\frac{i}{2}\sqrt{3}}\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)}\sqrt{\frac{1}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)}\,EllipticPi\left(\sqrt{\frac{1}{\frac{3}{2}-}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x+e)/(1+x+3^(1/2))/(x^3+1)^(1/2),x)
```

```
[Out] 2*f*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2
-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2)
)/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2
-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2))
)^(1/2))+2/3*(e-f-f*3^(1/2))*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*
I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1
/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1
/2)*3^(1/2)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),1/3*(-3/
2+1/2*I*3^(1/2))*3^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2
)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\sqrt{x^3+1}\left(x+\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x + e)/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{fx+e}{\sqrt{x^3+1}\left(x+\sqrt{3}+1\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x + e)/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e + fx}{\sqrt{(x+1)(x^2 - x + 1)}\left(x + 1 + \sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x+e)/(1+x+3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] Integral((e + f*x)/(sqrt((x + 1)*(x**2 - x + 1))*(x + 1 + sqrt(3)
)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx + e}{\sqrt{x^3 + 1}\left(x + \sqrt{3} + 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x + e)/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)), x)
```

3.105 $\int \frac{e+fx}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=187

$$-\frac{\left(e+\sqrt{3}f+f\right)\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}-\frac{\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(e+\left(1-\sqrt{3}\right)f\right)F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

```
[Out] -(((e + f + Sqrt[3]*f)*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[
1 - x^3]])/Sqrt[3*(3 + 2*Sqrt[3])]) - (Sqrt[2 + Sqrt[3]]*(e + (1
- Sqrt[3])*f)*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*Ell
ipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3
]])/(3^(3/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rubi [A] time = 0.559314, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138

$$-\frac{\left(e+\sqrt{3}f+f\right)\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}-\frac{\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(e-\sqrt{3}f+f\right)F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(e + f*x)/((1 + Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

```
[Out] -(((e + f + Sqrt[3]*f)*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[
1 - x^3]])/Sqrt[3*(3 + 2*Sqrt[3])]) - (Sqrt[2 + Sqrt[3]]*(e + f -
 Sqrt[3]*f)*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*Ellip
ticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]]
)/(3^(3/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rubi in Sympy [A] time = 16.0324, size = 82, normalized size = 0.44

$$\frac{2\tilde{\infty}\sqrt{\frac{x^2+x+1}{\left(-x+1+\sqrt{3}\right)^2}}\,(e+f)(-x+1)\,F\left(\operatorname{asin}\left(\frac{-x-\sqrt{3}+1}{-x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt{\frac{-x+1}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{-x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x+e)/(1-x+3**(1/2))/(-x**3+1)**(1/2),x)
```

```
[Out] 2*zoo*sqrt((x**2 + x + 1)/(-x + 1 + sqrt(3))**2)*(e + f)*(-x + 1)
*elliptic_f(asin((-x - sqrt(3) + 1)/(-x + 1 + sqrt(3))), -7 - 4*s
qrt(3))/(sqrt((-x + 1)/(-x + 1 + sqrt(3))**2)*sqrt(-x**3 + 1))
```

Mathematica [C] time = 0.71619, size = 291, normalized size = 1.56

$$\frac{2\sqrt{\frac{2}{3}}\sqrt{-\frac{i(x-1)}{\sqrt{3}+3i}}\left(2\sqrt{2ix+\sqrt{3}+i}\sqrt{x^2+x+1}\left(\sqrt{3}e+\left(3+\sqrt{3}\right)f\right)\left(\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)-3if\sqrt{-2ix+\sqrt{3}-}\right.}{\left(3i+(1+2i)\sqrt{3}\right)\sqrt{2ix+\sqrt{3}+i}\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(e + f*x)/((1 + Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*Sqrt[2/3]*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*((-3*I)*f*Sqrt
[-I + Sqrt[3] - (2*I)*x]*((-I)*((2 + I) + Sqrt[3]) + ((2 - I) + S
qrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] + (2*I)*x]/(Sqrt[2]*
3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + 2*(Sqrt[3]*e + (3 + Sqr
t[3])*f)*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[1 + x + x^2]*EllipticPi
[(2*Sqrt[3])/(3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] +
 (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/((3*
I + (1 + 2*I)*Sqrt[3])*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[1 - x^3])
```

Maple [A] time = 0.038, size = 264, normalized size = 1.4

$$\frac{-\frac{2i}{3}\left(-e-f-f\sqrt{3}\right)\sqrt{3}}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}-\sqrt{3}}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\,\sqrt{3}EllipticPi\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\right.$$

$$+\frac{2i}{3}f\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\,\sqrt{3}EllipticF\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)\frac{}{\sqrt{-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x+e)/(1-x+3^(1/2))/(-x^3+1)^(1/2),x)
```

```
[Out] -2/3*I*(-e-f-f*3^(1/2))*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))
^(1/2)*((-1+x)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/
2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(-3/2+1/2*I*3^(1/2)-3^(1/2))*El
lipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^
(1/2)/(-3/2+1/2*I*3^(1/2)-3^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)
))^(1/2))+2/3*I*f*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)
*((-1+x)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^
(1/2))^(1/2)/(-x^3+1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I
*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{fx+e}{\sqrt{-x^3+1}\left(x-\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(f*x + e)/(sqrt(-x^3 + 1)*(x - sqrt(3) - 1)),x, algorithm="maxima")
```

```
[Out] -integrate((f*x + e)/(sqrt(-x^3 + 1)*(x - sqrt(3) - 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{fx+e}{\sqrt{-x^3+1}\left(x-\sqrt{3}-1\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(f*x + e)/(sqrt(-x^3 + 1)*(x - sqrt(3) - 1)),x, algorithm="fricas")
```

```
[Out] integral(-(f*x + e)/(sqrt(-x^3 + 1)*(x - sqrt(3) - 1)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{e}{x\sqrt{-x^3+1}-\sqrt{3}\sqrt{-x^3+1}-\sqrt{-x^3+1}}\,dx-\int \frac{fx}{x\sqrt{-x^3+1}-\sqrt{3}\sqrt{-x^3+1}-\sqrt{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x+e)/(1-x+3**(1/2))/(-x**3+1)**(1/2),x)
```

```
[Out] -Integral(e/(x*sqrt(-x**3 + 1) - sqrt(3)*sqrt(-x**3 + 1) - sqrt(-
x**3 + 1)), x) - Integral(f*x/(x*sqrt(-x**3 + 1) - sqrt(3)*sqrt(-
x**3 + 1) - sqrt(-x**3 + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{fx+e}{\sqrt{-x^3+1}\left(x-\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(f*x + e)/(sqrt(-x^3 + 1)*(x - sqrt(3) - 1)),x, algorithm="giac")
```

```
[Out] integrate(-(f*x + e)/(sqrt(-x^3 + 1)*(x - sqrt(3) - 1)), x)
```

3.106 $\int \frac{e+fx}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=190

$$-\frac{\left(e+\sqrt{3}f+f\right)\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{x^3-1}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}-\frac{\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(e+\left(1-\sqrt{3}\right)f\right)F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

```
[Out] -(((e + f + Sqrt[3]*f)*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt
[-1 + x^3]])/Sqrt[3*(3 + 2*Sqrt[3])]) - (Sqrt[2 - Sqrt[3]]*(e + (
1 - Sqrt[3])*f)*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*E
llipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt
[3]])/(3^(3/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3
])
```

Rubi [A] time = 0.487386, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$

$$-\frac{\left(e+\sqrt{3}f+f\right)\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{x^3-1}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}-\frac{\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(e-\sqrt{3}f+f\right)F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(e + f*x)/((1 + Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] -(((e + f + Sqrt[3]*f)*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt
[-1 + x^3]])/Sqrt[3*(3 + 2*Sqrt[3])]) - (Sqrt[2 - Sqrt[3]]*(e + f
 - Sqrt[3]*f)*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*Ell
ipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3
]])/(3^(3/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Rubi in Sympy [A] time = 45.45, size = 246, normalized size = 1.29

$$-\frac{\sqrt[4]{3}\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(-x+1\right)\left(e+f+\sqrt{3}f\right)\operatorname{atanh}\left(\frac{\left(\sqrt{3}+2\right)\sqrt{-\frac{\left(-x+1+\sqrt{3}\right)^2}{\left(x-1+\sqrt{3}\right)^2}+1}}{\sqrt{\frac{\left(-x+1+\sqrt{3}\right)^2}{\left(x-1+\sqrt{3}\right)^2}+4\sqrt{3}+7}}\right)}{3\sqrt{\frac{x-1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{\sqrt{3}+2}\sqrt{x^3-1}} - \frac{\sqrt[4]{3}\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{-\sqrt{3}+2}\left(-x+1\right)\left(e-\sqrt{3}f+f\right)F\left(\operatorname{asin}\left(\frac{-x+1+\sqrt{3}}{-x-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{3\sqrt{\frac{x-1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x+e)/(1-x+3**(1/2))/(x**3-1)**(1/2),x)
```

```
[Out] -3**(1/4)*sqrt((x**2 + x + 1)/(-x - sqrt(3) + 1)**2)*(-x + 1)*(e
+ f + sqrt(3)*f)*atanh((sqrt(3) + 2)*sqrt(-(-x + 1 + sqrt(3))**2/
(x - 1 + sqrt(3))**2 + 1)/sqrt((-x + 1 + sqrt(3))**2/(x - 1 + sqr
t(3))**2 + 4*sqrt(3) + 7))/(3*sqrt((x - 1)/(-x - sqrt(3) + 1)**2)
*sqrt(sqrt(3) + 2)*sqrt(x**3 - 1)) - 3**(1/4)*sqrt((x**2 + x + 1)
/(-x - sqrt(3) + 1)**2)*sqrt(-sqrt(3) + 2)*(-x + 1)*(e - sqrt(3)*
f + f)*elliptic_f(asin((-x + 1 + sqrt(3))/(-x - sqrt(3) + 1)), -7
 + 4*sqrt(3))/(3*sqrt((x - 1)/(-x - sqrt(3) + 1)**2)*sqrt(x**3 -
1))
```

Mathematica [C] time = 0.728272, size = 289, normalized size = 1.52

$$\frac{2\sqrt{\frac{2}{3}}\sqrt{-\frac{i(x-1)}{\sqrt{3}+3i}}\left(2\sqrt{2ix+\sqrt{3}+i}\sqrt{x^2+x+1}\left(\sqrt{3}e+\left(3+\sqrt{3}\right)f\right)\left(\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)-3if\sqrt{-2ix+\sqrt{3}-}\right.}{\left(3i+(1+2i)\sqrt{3}\right)\sqrt{2ix+\sqrt{3}+i}\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(e + f*x)/((1 + Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*Sqrt[2/3]*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*((-3*I)*f*Sqrt
[-I + Sqrt[3] - (2*I)*x]*((-I)*((2 + I) + Sqrt[3]) + ((2 - I) + S
qrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] + (2*I)*x]/(Sqrt[2]*
3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + 2*(Sqrt[3]*e + (3 + Sqr
t[3])*f)*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[1 + x + x^2]*EllipticPi
```

```
[(2*Sqrt[3])/(3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] +
 (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/((3*
I + (1 + 2*I)*Sqrt[3])*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[-1 + x^3]
)
```

Maple [A] time = 0.036, size = 262, normalized size = 1.4

$$-\frac{\left(-2\,e-2\,f-2\,f\sqrt{3}\right)\left(-\frac{3}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}{3}\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i}{2}\sqrt{3}}}\sqrt{\frac{1}{\frac{3}{2}-\frac{i}{2}\sqrt{3}}\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)}\sqrt{\frac{1}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)}\mathit{EllipticPi}\left(\sqrt{-\right.$$

$$-2\,\frac{f\left(-3/2-i/2\sqrt{3}\right)}{\sqrt{x^3-1}}\sqrt{\frac{-1+x}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2-i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2+i/2\sqrt{3}}{3/2+i/2\sqrt{3}}}\mathit{EllipticF}\left(\sqrt{\frac{-1+x}{-3/2-i/2\sqrt{3}}},\sqrt{\frac{3/2+i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x+e)/(1-x+3^(1/2))/(x^3-1)^(1/2),x)
```

```
[Out] -2/3*(-e-f-f*3^(1/2))*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^
(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*(
(x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*3^
(1/2)*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(3/2+1/
2*I*3^(1/2))*3^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1
/2))-2*f*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)
*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*
3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((-1+
x)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^
(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{fx+e}{\sqrt{x^3-1}\left(x-\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(f*x + e)/(sqrt(x^3 - 1)*(x - sqrt(3) - 1)),x, algorithm="maxima")
```

```
[Out] -integrate((f*x + e)/(sqrt(x^3 - 1)*(x - sqrt(3) - 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{fx+e}{\sqrt{x^3-1}\left(x-\sqrt{3}-1\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(f*x + e)/(sqrt(x^3 - 1)*(x - sqrt(3) - 1)),x, algorithm="fricas")
```

```
[Out] integral(-(f*x + e)/(sqrt(x^3 - 1)*(x - sqrt(3) - 1)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{e}{x\sqrt{x^3-1}-\sqrt{3}\sqrt{x^3-1}-\sqrt{x^3-1}}\,dx-\int\frac{fx}{x\sqrt{x^3-1}-\sqrt{3}\sqrt{x^3-1}-\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x+e)/(1-x+3**(1/2))/(x**3-1)**(1/2),x)
```

```
[Out] -Integral(e/(x*sqrt(x**3 - 1) - sqrt(3)*sqrt(x**3 - 1) - sqrt(x**
3 - 1)), x) - Integral(f*x/(x*sqrt(x**3 - 1) - sqrt(3)*sqrt(x**3
- 1) - sqrt(x**3 - 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int-\frac{fx+e}{\sqrt{x^3-1}\left(x-\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(f*x + e)/(sqrt(x^3 - 1)*(x - sqrt(3) - 1)),x, algorithm="giac")
```

```
[Out] integrate(-(f*x + e)/(sqrt(x^3 - 1)*(x - sqrt(3) - 1)), x)
```

3.107 $\int \frac{e+fx}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=183

$$\frac{\left(e-\left(1+\sqrt{3}\right)f\right)\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(x+1)}{\sqrt{-x^3-1}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}+\frac{\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(e-\left(1-\sqrt{3}\right)f\right)F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

```
[Out] ((e - (1 + Sqrt[3])*f)*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt
[-1 - x^3]])/Sqrt[3*(3 + 2*Sqrt[3])] + (Sqrt[2 - Sqrt[3]]*(e - (1
 - Sqrt[3])*f)*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*El
lipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[
3]])/(3^(3/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3]
)
```

Rubi [A] time = 0.503347, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$

$$\frac{\left(e-\left(1+\sqrt{3}\right)f\right)\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(x+1)}{\sqrt{-x^3-1}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}+\frac{\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(e-\left(1-\sqrt{3}\right)f\right)F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(e + f*x)/((1 + Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```


```
[Out] ((e - (1 + Sqrt[3])*f)*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt
[-1 - x^3]])/Sqrt[3*(3 + 2*Sqrt[3])] + (Sqrt[2 - Sqrt[3]]*(e - (1
 - Sqrt[3])*f)*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*El
lipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[
3]])/(3^(3/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3]
)
```

Rubi in Sympy [A] time = 44.4323, size = 252, normalized size = 1.38

$$\frac{\sqrt[4]{3}\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(e-f\left(1+\sqrt{3}\right)\right)(x+1)\operatorname{atanh}\left(\frac{\sqrt{1-\frac{\left(x+1+\sqrt{3}\right)^2}{\left(-x-1+\sqrt{3}\right)^2}}\left(\sqrt{3}+2\right)}{\sqrt{4\sqrt{3}+7+\frac{\left(x+1+\sqrt{3}\right)^2}{\left(-x-1+\sqrt{3}\right)^2}}}\right)}{3\sqrt{\frac{-x-1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{\sqrt{3}+2}\sqrt{-x^3-1}}+\frac{\sqrt[4]{3}\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-\sqrt{3}+2}\,(x+1)\left(e-f+\sqrt{3}f\right)F\left(\operatorname{asin}\left(\frac{x+1+\sqrt{3}}{x-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{3\sqrt{\frac{-x-1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x+e)/(1+x+3**(1/2))/(-x**3-1)**(1/2),x)
```

```
[Out] 3**(1/4)*sqrt((x**2 - x + 1)/(x - sqrt(3) + 1)**2)*(e - f*(1 + sq
rt(3)))*(x + 1)*atanh(sqrt(1 - (x + 1 + sqrt(3))**2/(-x - 1 + sqr
t(3))**2)*(sqrt(3) + 2)/sqrt(4*sqrt(3) + 7 + (x + 1 + sqrt(3))**2
/(-x - 1 + sqrt(3))**2))/(3*sqrt((-x - 1)/(x - sqrt(3) + 1)**2)*s
qrt(sqrt(3) + 2)*sqrt(-x**3 - 1)) + 3**(1/4)*sqrt((x**2 - x + 1)/
(x - sqrt(3) + 1)**2)*sqrt(-sqrt(3) + 2)*(x + 1)*(e - f + sqrt(3)
*f)*elliptic_f(asin((x + 1 + sqrt(3))/(x - sqrt(3) + 1)), -7 + 4*
sqrt(3))/(3*sqrt((-x - 1)/(x - sqrt(3) + 1)**2)*sqrt(-x**3 - 1))
```

Mathematica [C] time = 0.727635, size = 293, normalized size = 1.6

$$\frac{2\sqrt{\frac{2}{3}}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\left(2\sqrt{-2ix+\sqrt{3}+i}\sqrt{x^2-x+1}\left(\left(3+\sqrt{3}\right)f-\sqrt{3}e\right)\left(\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)\middle|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)+3f\sqrt{2ix+\sqrt{3}-}\right.}{\left(3i+(1+2i)\sqrt{3}\right)\sqrt{-2ix+\sqrt{3}+i}\sqrt{-x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(e + f*x)/((1 + Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*Sqrt[2/3]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(3*f*Sqrt[-I + Sqr
t[3] + (2*I)*x]*((-2 - I) - Sqrt[3] + ((1 + 2*I) + I*Sqrt[3])*x)*
EllipticF[ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))],
(2*Sqrt[3])/(3*I + Sqrt[3])] + 2*(-(Sqrt[3]*e) + (3 + Sqrt[3])*f)
*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt
[3])/(3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x
```

```
]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/((3*I + (1 +
 2*I)*Sqrt[3])*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[-1 - x^3])
```

Maple [A] time = 0.035, size = 258, normalized size = 1.4

$$-\frac{2i}{3}f\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}EllipticF\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)\frac{}{\sqrt{-x}}$$
$$-\frac{\frac{2i}{3}\left(e-f-f\sqrt{3}\right)\sqrt{3}}{\frac{3}{2}+\frac{i}{2}\sqrt{3}+\sqrt{3}}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}EllipticPi\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\frac{}{\frac{3}{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x+e)/(1+x+3^(1/2))/(-x^3-1)^(1/2),x)
```

```
[Out] -2/3*I*f*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(
3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2
)/(-x^3-1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3
^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))-2/3*I*(e-f-f
*3^(1/2))*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/
(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/
2)/(-x^3-1)^(1/2)/(3/2+1/2*I*3^(1/2)+3^(1/2))*EllipticPi(1/3*3^(1
/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(3/2+1/2*I*
3^(1/2)+3^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\sqrt{-x^3-1}\left(x+\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x + e)/(sqrt(-x^3 - 1)*(x + sqrt(3) + 1)),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(-x^3 - 1)*(x + sqrt(3) + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{fx+e}{\sqrt{-x^3-1}\left(x+\sqrt{3}+1\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x + e)/(sqrt(-x^3 - 1)*(x + sqrt(3) + 1)),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)/(sqrt(-x^3 - 1)*(x + sqrt(3) + 1)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e+fx}{\sqrt{-(x+1)(x^2-x+1)}\left(x+1+\sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x+e)/(1+x+3**(1/2))/(-x**3-1)**(1/2),x)
```

```
[Out] Integral((e + f*x)/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 1 + sqrt(3
))), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\sqrt{-x^3-1}\left(x+\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x + e)/(sqrt(-x^3 - 1)*(x + sqrt(3) + 1)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)/(sqrt(-x^3 - 1)*(x + sqrt(3) + 1)), x)
```

3.108 $\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=332

$$-\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}e-\left(1+\sqrt{3}\right)\sqrt[3]{a}f\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{\left(\sqrt[3]{b}e-\left(1-\sqrt{3}\right)\sqrt[3]{a}f\right)\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{3\left(2\sqrt{3}-3\right)}\sqrt{a}b^{2/3}}$$

```
[Out] -(((b^(1/3)*e - (1 - Sqrt[3])*a^(1/3)*f)*ArcTanh[(Sqrt[-3 + 2*Sqr
t[3]]*a^(1/6)*(a^(1/3) + b^(1/3)*x))/Sqrt[a + b*x^3]])/(Sqrt[3*(-
3 + 2*Sqrt[3])]*Sqrt[a]*b^(2/3))) - (Sqrt[2 + Sqrt[3]]*(b^(1/3)*e
 - (1 + Sqrt[3])*a^(1/3)*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(3/4)*a^(1/
3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 1.0453, antiderivative size = 332, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$

$$-\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}e-\left(1+\sqrt{3}\right)\sqrt[3]{a}f\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{\left(\sqrt[3]{b}e-\left(1-\sqrt{3}\right)\sqrt[3]{a}f\right)\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{3\left(2\sqrt{3}-3\right)}\sqrt{a}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] -(((b^(1/3)*e - (1 - Sqrt[3])*a^(1/3)*f)*ArcTanh[(Sqrt[-3 + 2*Sqr
t[3]]*a^(1/6)*(a^(1/3) + b^(1/3)*x))/Sqrt[a + b*x^3]])/(Sqrt[3*(-
3 + 2*Sqrt[3])]*Sqrt[a]*b^(2/3))) - (Sqrt[2 + Sqrt[3]]*(b^(1/3)*e
 - (1 + Sqrt[3])*a^(1/3)*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(3/4)*a^(1/
3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi in Sympy [A] time = 80.3245, size = 483, normalized size = 1.45

$$-\frac{\sqrt[4]{3}\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^{2}}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^{2}}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-\sqrt[3]{a}f\left(1+\sqrt{3}\right)+\sqrt[3]{b}e\right)F\left(\operatorname{asin}\left(\frac{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{3\sqrt[3]{a}b^{\frac{2}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^{2}}}\sqrt{a+bx^{3}}}-\frac{\sqrt[4]{3}\sqrt{\frac{a^{\frac{2}{3}}\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+\frac{b^{\frac{2}{3}}x^{2}}{a^{\frac{2}{3}}}\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^{2}}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-\sqrt[3]{a}f\left(-\sqrt{3}+1\right)+\sqrt[3]{b}e\right)\operatorname{atanh}\left(\frac{\left(-\sqrt{3}+2\right)\sqrt{-\frac{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)-\sqrt[3]{b}x\right)^{2}}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^{2}}+1}}{\sqrt{\frac{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)-\sqrt[3]{b}x\right)^{2}}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^{2}}-4\sqrt{3}+7}}\right)}{3\sqrt[3]{a}b^{\frac{2}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^{2}}}\sqrt{-\sqrt{3}+2}\sqrt{a+bx^{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((f*x+e)/(b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(b*x**3+a)**(1/2),x)
```

```
[Out] -3**(1/4)*sqrt((a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(
a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(sqrt(3) + 2)*(a**(1
/3) + b**(1/3)*x)*(-a**(1/3)*f*(1 + sqrt(3)) + b**(1/3)*e)*ellipt
ic_f(asin((-a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)/(a**(1/3)*(1 +
sqrt(3)) + b**(1/3)*x)), -7 - 4*sqrt(3))/(3*a**(1/3)*b**(2/3)*sqr
t(a**(1/3)*(a**(1/3) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(
1/3)*x)**2)*sqrt(a + b*x**3)) - 3**(1/4)*sqrt(a**(2/3)*(1 - b**(1
/3)*x/a**(1/3) + b**(2/3)*x**2/a**(2/3))/(a**(1/3)*(1 + sqrt(3))
+ b**(1/3)*x)**2)*(a**(1/3) + b**(1/3)*x)*(-a**(1/3)*f*(-sqrt(3)
+ 1) + b**(1/3)*e)*atanh((-sqrt(3) + 2)*sqrt(-(a**(1/3)*(-1 + sqr
```

```
t(3)) - b**(1/3)*x)**2/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2 +
 1)/sqrt((a**(1/3)*(-1 + sqrt(3)) - b**(1/3)*x)**2/(a**(1/3)*(1 +
 sqrt(3)) + b**(1/3)*x)**2 - 4*sqrt(3) + 7))/(3*a**(1/3)*b**(2/3)
*sqrt(a**(1/3)*(a**(1/3) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) +
b**(1/3)*x)**2)*sqrt(-sqrt(3) + 2)*sqrt(a + b*x**3))
```

Mathematica [C] time = 3.05474, size = 438, normalized size = 1.32

$$-\frac{4\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(i\sqrt{\frac{\left(\sqrt{3}+i\right)\sqrt[3]{b}x-2i\sqrt[3]{a}}{\left(\sqrt{3}-3i\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\left(\left(\sqrt{3}-1\right)\sqrt[3]{a}f+\sqrt[3]{b}e\right)\left(\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{\left(i+\sqrt{3}\right)\sqrt[3]{b}x-2i\sqrt[3]{a}}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right.\right.\right.}{\left(3-(2-i)\sqrt{3}\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/}}{\left(1+\sqrt[3]{-1}\right.}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (-4*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(((-I/
2)*3^(1/4)*f*(((-2 - I) + Sqrt[3])*a^(1/3) + ((1 + 2*I) - I*Sqrt[
3])*b^(1/3)*x)*Sqrt[I + Sqrt[3] - ((2*I)*b^(1/3)*x)/a^(1/3)]*Elli
pticF[ArcSin[Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3
*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2])/Sqrt[2] + I*(b^(1/3
)*e + (-1 + Sqrt[3])*a^(1/3)*f)*Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[
3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*Sqrt[1 - (b^(1/3)*x)/a
^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(-3*I + (1
 + 2*I)*Sqrt[3]), ArcSin[Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(
1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/((3 -
(2 - I)*Sqrt[3])*b^(2/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((
1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3])
```

Maple [F] time = 0.121, size = 0, normalized size = 0.

$$\int (fx+e)\left(\sqrt[3]{b}x+\sqrt[3]{a}\left(-\sqrt{3}+1\right)\right)^{-1}\frac{1}{\sqrt{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(b^(1/3)*x+a^(1/3)*(-3^(1/2)+1))/(b*x^3+a)^(1/2),x)
```

```
[Out] int((f*x+e)/(b^(1/3)*x+a^(1/3)*(-3^(1/2)+1))/(b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\sqrt{bx^3+a}\left(b^{\frac{1}{3}}x-a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x + e)/(sqrt(b*x^3 + a)*(b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))),x, algo
```

```
[Out] integrate((f*x + e)/(sqrt(b*x^3 + a)*(b^(1/3)*x - a^(1/3)*(sqrt(3
) - 1))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x + e)/(sqrt(b*x^3 + a)*(b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))),x, algo
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e+fx}{\sqrt{a+bx^3}\left(-\sqrt{3}\sqrt[3]{a}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(b*x**3+a)**(1/2),x)
```

```
[Out] Integral((e + f*x)/(sqrt(a + b*x**3)*(-sqrt(3)*a**(1/3) + a**(1/3
) + b**(1/3)*x)), x)
```

<u>GIAC/XCAS</u> [A] time = 0.607311, size = 4, normalized size = 0.01

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x + e)/(sqrt(b*x^3 + a)*(b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))),x, algo
```

```
[Out] sage0*x
```

3.109 $\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$

Optimal. Leaf size=336

$$\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}f+\sqrt[3]{b}e\right)F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}+\frac{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}f+\sqrt[3]{b}e\right)\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{3\left(2\sqrt{3}-3\right)}\sqrt{a}b^{2/3}}$$

```
[Out] ((b^(1/3)*e + (1 - Sqrt[3])*a^(1/3)*f)*ArcTanh[(Sqrt[-3 + 2*Sqrt[
3]]*a^(1/6)*(a^(1/3) - b^(1/3)*x))/Sqrt[a - b*x^3]])/(Sqrt[3*(-3
+ 2*Sqrt[3])]*Sqrt[a]*b^(2/3)) + (Sqrt[2 + Sqrt[3]]*(b^(1/3)*e +
(1 + Sqrt[3])*a^(1/3)*f)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x
)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + S
qrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(3/4)*a^(1/3)*
b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/
3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

Rubi [A] time = 1.04724, antiderivative size = 336, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}f+\sqrt[3]{b}e\right)F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}+\frac{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}f+\sqrt[3]{b}e\right)\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{3\left(2\sqrt{3}-3\right)}\sqrt{a}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/(((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] ((b^(1/3)*e + (1 - Sqrt[3])*a^(1/3)*f)*ArcTanh[(Sqrt[-3 + 2*Sqrt[
3]]*a^(1/6)*(a^(1/3) - b^(1/3)*x))/Sqrt[a - b*x^3]])/(Sqrt[3*(-3
+ 2*Sqrt[3])]*Sqrt[a]*b^(2/3)) + (Sqrt[2 + Sqrt[3]]*(b^(1/3)*e +
(1 + Sqrt[3])*a^(1/3)*f)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x
)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + S
qrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(3/4)*a^(1/3)*
b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/
3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

Rubi in Sympy [A] time = 92.8255, size = 481, normalized size = 1.43

$$-\frac{\sqrt[4]{3}\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{b}x\right)^2}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\left(\sqrt[3]{a}f\left(1+\sqrt{3}\right)+\sqrt[3]{b}e\right)F\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{3\sqrt[3]{a}b^{\frac{2}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}+\frac{\sqrt[4]{3}\sqrt{\frac{a^{\frac{2}{3}}\left(1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+\frac{b^{\frac{2}{3}}x^2}{a^{\frac{2}{3}}}\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\left(\sqrt[3]{a}f\left(-\sqrt{3}+1\right)+\sqrt[3]{b}e\right)\operatorname{atanh}\left(\frac{\left(-\sqrt{3}+2\right)\sqrt{-\frac{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{b}x\right)^2}+1}}{\sqrt{\frac{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{b}x\right)^2}-4\sqrt{3}+7}}\right)}{3\sqrt[3]{a}b^{\frac{2}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{b}x\right)^2}}\sqrt{-\sqrt{3}+2}\sqrt{a-bx^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((f*x+e)/(-b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(-b*x**3+a)**(1/2),
```

```
[Out] -3**(1/4)*sqrt((a**(2/3) + a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(
a**(1/3)*(1 + sqrt(3)) - b**(1/3)*x)**2)*sqrt(sqrt(3) + 2)*(a**(1
/3) - b**(1/3)*x)*(a**(1/3)*f*(1 + sqrt(3)) + b**(1/3)*e)*ellipti
c_f(asin((a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)/(a**(1/3)*(1 + sq
rt(3)) - b**(1/3)*x)), -7 - 4*sqrt(3))/(3*a**(1/3)*b**(2/3)*sqrt(
a**(1/3)*(a**(1/3) - b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) - b**(1/
3)*x)**2)*sqrt(a - b*x**3)) + 3**(1/4)*sqrt(a**(2/3)*(1 + b**(1/3
)*x/a**(1/3) + b**(2/3)*x**2/a**(2/3))/(a**(1/3)*(1 + sqrt(3)) -
b**(1/3)*x)**2)*(a**(1/3) - b**(1/3)*x)*(a**(1/3)*f*(-sqrt(3) + 1
) + b**(1/3)*e)*atanh((-sqrt(3) + 2)*sqrt(-(a**(1/3)*(-1 + sqrt(3
```

```
)) + b**(1/3)*x)**2/(a**(1/3)*(1 + sqrt(3)) - b**(1/3)*x)**2 + 1)
/sqrt((a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)**2/(a**(1/3)*(1 + sq
rt(3)) - b**(1/3)*x)**2 - 4*sqrt(3) + 7))/(3*a**(1/3)*b**(2/3)*sq
rt(a**(1/3)*(a**(1/3) - b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) - b**
(1/3)*x)**2)*sqrt(-sqrt(3) + 2)*sqrt(a - b*x**3))
```

Mathematica [C] time = 3.15974, size = 466, normalized size = 1.39

$$-\frac{4\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{1}{2}f\left(i\left(-3+(2+i)\sqrt{3}\right)\sqrt[3]{a}+\left(3-(2-i)\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{\frac{\left(\sqrt{3}-i\right)\sqrt[3]{a}+\left(\sqrt{3}+i\right)\sqrt[3]{b}x}{\left(\sqrt{3}-3i\right)\sqrt[3]{a}}}F\left(\sin^{-1}\left(\sqrt{-\frac{i\left(\left(1-i\sqrt{3}\right)\sqrt[3]{b}x+2\cdots\right.}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}\cdots}}\right.\right.\right.}{\left(3-(2-\cdots\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(e + f*x)/(((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (-4*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((f*(I
*(-3 + (2 + I)*Sqrt[3])*a^(1/3) + (3 - (2 - I)*Sqrt[3])*b^(1/3)*x
)*Sqrt[((-I + Sqrt[3])*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I
+ Sqrt[3])*a^(1/3))]*EllipticF[ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1
- I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqr
t[3])/2])/2 - I*(b^(1/3)*e - (-1 + Sqrt[3])*a^(1/3)*f)*Sqrt[((-I)
*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1/
3))]*Sqrt[1 + (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*Ellipt
icPi[(2*Sqrt[3])/(-3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[((-I)*(2
*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1/3))
]], (1 + I*Sqrt[3])/2]))/((3 - (2 - I)*Sqrt[3])*b^(2/3)*Sqrt[(a^(
1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a -
 b*x^3])
```

Maple [F] time = 0.116, size = 0, normalized size = 0.

$$\int (fx+e)\left(-\sqrt[3]{b}x+\sqrt[3]{a}\left(-\sqrt{3}+1\right)\right)^{-1}\frac{1}{\sqrt{-bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x+e)/(-b^(1/3)*x+a^(1/3)*(-3^(1/2)+1))/(-b*x^3+a)^(1/2),x)
```

```
[Out] int((f*x+e)/(-b^(1/3)*x+a^(1/3)*(-3^(1/2)+1))/(-b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{fx+e}{\sqrt{-bx^3+a}\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(f*x + e)/(sqrt(-b*x^3 + a)*(b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))),x, al
```

```
[Out] -integrate((f*x + e)/(sqrt(-b*x^3 + a)*(b^(1/3)*x + a^(1/3)*(sqrt
(3) - 1))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(f*x + e)/(sqrt(-b*x^3 + a)*(b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))),x, al
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{e}{-\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt{3}\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt[3]{b}x\sqrt{a-bx^3}}\,dx - \int \frac{fx}{-\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt{3}\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt[3]{b}x\sqrt{a-bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(-b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(-b*x**3+a)**(1/2),x)
```

```
[Out] -Integral(e/(-a**(1/3)*sqrt(a - b*x**3) + sqrt(3)*a**(1/3)*sqrt(a
 - b*x**3) + b**(1/3)*x*sqrt(a - b*x**3)), x) - Integral(f*x/(-a*
*(1/3)*sqrt(a - b*x**3) + sqrt(3)*a**(1/3)*sqrt(a - b*x**3) + b**
(1/3)*x*sqrt(a - b*x**3)), x)
```

GIAC/XCAS [A] time = 0.589654, size = 4, normalized size = 0.01

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(f*x + e)/(sqrt(-b*x^3 + a)*(b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))),x, al
```

```
[Out] sage0*x
```

3.110 $\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

Optimal. Leaf size=345

$$\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}f+\sqrt[3]{b}e\right)F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}+\frac{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}f+\sqrt[3]{b}e\right)\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{bx^3-a}}\right)}{\sqrt{3\left(2\sqrt{3}-3\right)}\sqrt{a}b^{2/3}}$$

```
[Out] ((b^(1/3)*e + (1 - Sqrt[3])*a^(1/3)*f)*ArcTan[(Sqrt[-3 + 2*Sqrt[3
]]*a^(1/6)*(a^(1/3) - b^(1/3)*x))/Sqrt[-a + b*x^3]])/(Sqrt[3*(-3
+ 2*Sqrt[3])]*Sqrt[a]*b^(2/3)) + (Sqrt[2 - Sqrt[3]]*(b^(1/3)*e +
(1 + Sqrt[3])*a^(1/3)*f)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x
)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - S
qrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3^(3/4)*a^(1/3)*
b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(
1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

Rubi [A] time = 0.968881, antiderivative size = 345, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.089

$$\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}f+\sqrt[3]{b}e\right)F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}+\frac{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}f+\sqrt[3]{b}e\right)\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{bx^3-a}}\right)}{\sqrt{3\left(2\sqrt{3}-3\right)}\sqrt{a}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/(((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] ((b^(1/3)*e + (1 - Sqrt[3])*a^(1/3)*f)*ArcTan[(Sqrt[-3 + 2*Sqrt[3
]]*a^(1/6)*(a^(1/3) - b^(1/3)*x))/Sqrt[-a + b*x^3]])/(Sqrt[3*(-3
+ 2*Sqrt[3])]*Sqrt[a]*b^(2/3)) + (Sqrt[2 - Sqrt[3]]*(b^(1/3)*e +
(1 + Sqrt[3])*a^(1/3)*f)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x
)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - S
qrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3^(3/4)*a^(1/3)*
b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(
1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

Rubi in Sympy [A] time = 38.6492, size = 177, normalized size = 0.51

$$\frac{2\tilde{\infty}\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^{2}}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^{2}}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\left(\frac{f}{\sqrt[3]{b}}+\frac{e}{\sqrt[3]{a}}\right)F\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{b}x}{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)-\sqrt[3]{b}x}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^{2}}}\sqrt{-a+bx^{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((f*x+e)/(-b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(b*x**3-
a)**(1/2),x)
```

```
[Out] 2*zoo*sqrt((a**(2/3) + a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(a**(
1/3)*(-1 + sqrt(3)) + b**(1/3)*x)**2)*(a**(1/3) - b**(1/3)*x)*(f/
b**(1/3) + e/a**(1/3))*elliptic_f(asin((a**(1/3)*(1 + sqrt(3)) -
b**(1/3)*x)/(-a**(1/3)*(-1 + sqrt(3)) - b**(1/3)*x)), -7 + 4*sqrt
(3))/(sqrt(-a**(1/3)*(a**(1/3) - b**(1/3)*x)/(a**(1/3)*(-1 + sqrt
(3)) + b**(1/3)*x)**2)*sqrt(-a + b*x**3))
```

Mathematica [C] time = 3.15035, size = 467, normalized size = 1.35

$$-\frac{4\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{1}{2}f\left(i\left(-3+(2+i)\sqrt{3}\right)\sqrt[3]{a}+\left(3-(2-i)\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{\frac{\left(\sqrt{3}-i\right)\sqrt[3]{a}+\left(\sqrt{3}+i\right)\sqrt[3]{b}x}{\left(\sqrt{3}-3i\right)\sqrt[3]{a}}}F\left(\sin^{-1}\left(\sqrt{-\frac{i\left(\left(1-i\sqrt{3}\right)\sqrt[3]{b}x+2\right.}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right.\right.\right.}{\left(3-(2-\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(e + f*x)/(((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)*Sqrt[-
a + b*x^3]),x]
```

```
[Out] (-4*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((f*(I
*(-3 + (2 + I)*Sqrt[3])*a^(1/3) + (3 - (2 - I)*Sqrt[3])*b^(1/3)*x
)*Sqrt[((-I + Sqrt[3])*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I
+ Sqrt[3])*a^(1/3))]*EllipticF[ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1
- I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqr
t[3])/2])/2 - I*(b^(1/3)*e - (-1 + Sqrt[3])*a^(1/3)*f)*Sqrt[((-I)
*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1/
3))]*Sqrt[1 + (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*Ellipt
icPi[(2*Sqrt[3])/(-3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[((-I)*(2
*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1/3))
]], (1 + I*Sqrt[3])/2]))/((3 - (2 - I)*Sqrt[3])*b^(2/3)*Sqrt[(a^(
1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[-a
+ b*x^3])
```

Maple [F] time = 0.104, size = 0, normalized size = 0.

$$\int (fx+e)\left(-\sqrt[3]{b}x+\sqrt[3]{a}\left(-\sqrt{3}+1\right)\right)^{-1}\frac{1}{\sqrt{bx^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x+e)/(-b^(1/3)*x+a^(1/3)*(-3^(1/2)+1))/(b*x^3-a)^(1/2),x)
```

```
[Out] int((f*x+e)/(-b^(1/3)*x+a^(1/3)*(-3^(1/2)+1))/(b*x^3-a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{fx+e}{\sqrt{bx^3-a}\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(f*x + e)/(sqrt(b*x^3 - a)*(b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))),x, alg
```

```
[Out] -integrate((f*x + e)/(sqrt(b*x^3 - a)*(b^(1/3)*x + a^(1/3)*(sqrt(
3) - 1))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(f*x + e)/(sqrt(b*x^3 - a)*(b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))),x, alg
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{e}{-\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt{3}\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt[3]{b}x\sqrt{-a+bx^3}}\,dx -\int \frac{fx}{-\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt{3}\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt[3]{b}x\sqrt{-a+bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(-b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(b*x**3-a)**(1/2),x)
```

```
[Out] -Integral(e/(-a**(1/3)*sqrt(-a + b*x**3) + sqrt(3)*a**(1/3)*sqrt(
-a + b*x**3) + b**(1/3)*x*sqrt(-a + b*x**3)), x) - Integral(f*x/(
-a**(1/3)*sqrt(-a + b*x**3) + sqrt(3)*a**(1/3)*sqrt(-a + b*x**3)
+ b**(1/3)*x*sqrt(-a + b*x**3)), x)
```

GIAC/XCAS [A] time = 0.608705, size = 4, normalized size = 0.01

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(f*x + e)/(sqrt(b*x^3 - a)*(b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))),x, alg
```

```
[Out] sage0*x
```

3.111 $\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

Optimal. Leaf size=345

$$-\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}e-\left(1+\sqrt{3}\right)\sqrt[3]{a}f\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}-\frac{\left(\sqrt[3]{b}e-\left(1-\sqrt{3}\right)\sqrt[3]{a}f\right)\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3\left(2\sqrt{3}-3\right)}\sqrt{a}b^{2/3}}$$

```
[Out] -(((b^(1/3)*e - (1 - Sqrt[3])*a^(1/3)*f)*ArcTan[(Sqrt[-3 + 2*Sqrt
[3]]*a^(1/6)*(a^(1/3) + b^(1/3)*x))/Sqrt[-a - b*x^3]])/(Sqrt[3*(-
3 + 2*Sqrt[3])]*Sqrt[a]*b^(2/3))) - (Sqrt[2 - Sqrt[3]]*(b^(1/3)*e
 - (1 + Sqrt[3])*a^(1/3)*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3
)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1
- Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3^(3/4)*a^(1/
3)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*
a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

Rubi [A] time = 0.996539, antiderivative size = 345, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.089$

$$-\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}e-\left(1+\sqrt{3}\right)\sqrt[3]{a}f\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}-\frac{\left(\sqrt[3]{b}e-\left(1-\sqrt{3}\right)\sqrt[3]{a}f\right)\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3\left(2\sqrt{3}-3\right)}\sqrt{a}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(e + f*x)/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] -(((b^(1/3)*e - (1 - Sqrt[3])*a^(1/3)*f)*ArcTan[(Sqrt[-3 + 2*Sqrt
[3]]*a^(1/6)*(a^(1/3) + b^(1/3)*x))/Sqrt[-a - b*x^3]])/(Sqrt[3*(-
3 + 2*Sqrt[3])]*Sqrt[a]*b^(2/3))) - (Sqrt[2 - Sqrt[3]]*(b^(1/3)*e
 - (1 + Sqrt[3])*a^(1/3)*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3
)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1
- Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3^(3/4)*a^(1/
3)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*
a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

Rubi in Sympy [A] time = 38.4549, size = 177, normalized size = 0.51

$$\frac{2\tilde{\infty}\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^{2}}{\left(-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^{2}}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\frac{f}{\sqrt[3]{b}}+\frac{e}{\sqrt[3]{a}}\right)F\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^{2}}}\sqrt{-a-bx^{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x+e)/(b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(-b*x**3-
a)**(1/2),x)
```

```
[Out] 2*zoo*sqrt((a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(-a**
(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)**2)*(a**(1/3) + b**(1/3)*x)*(f
/b**(1/3) + e/a**(1/3))*elliptic_f(asin((a**(1/3)*(1 + sqrt(3)) +
 b**(1/3)*x)/(-a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)), -7 + 4*sqr
t(3))/(sqrt(-a**(1/3)*(a**(1/3) + b**(1/3)*x)/(-a**(1/3)*(-1 + sq
rt(3)) + b**(1/3)*x)**2)*sqrt(-a - b*x**3))
```

Mathematica [C] time = 3.08967, size = 441, normalized size = 1.28

$$-\frac{4\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(i\sqrt{\frac{\left(\sqrt{3}+i\right)\sqrt[3]{b}x-2i\sqrt[3]{a}}{\left(\sqrt{3}-3i\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^{2}}{a^{2/3}}-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\left(\left(\sqrt{3}-1\right)\sqrt[3]{a}f+\sqrt[3]{b}e\right)\left(\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{\left(i+\sqrt{3}\right)\sqrt[3]{b}x-2i\sqrt[3]{a}}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right.\right.\right.}{\left(3-(2-i)\sqrt{3}\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}}{\left(1+\sqrt[3]{-1}\right)}}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(e + f*x)/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[-
a - b*x^3]),x]
```

```
[Out] (-4*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(((-I/
2)*3^(1/4)*f*(((-2 - I) + Sqrt[3])*a^(1/3) + ((1 + 2*I) - I*Sqrt[
3])*b^(1/3)*x)*Sqrt[I + Sqrt[3] - ((2*I)*b^(1/3)*x)/a^(1/3)]*Elli
pticF[ArcSin[Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3
*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2])/Sqrt[2] + I*(b^(1/3
)*e + (-1 + Sqrt[3])*a^(1/3)*f)*Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[
3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*Sqrt[1 - (b^(1/3)*x)/a
^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(-3*I + (1
 + 2*I)*Sqrt[3]), ArcSin[Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(
1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/((3 -
(2 - I)*Sqrt[3])*b^(2/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((
1 + (-1)^(1/3))*a^(1/3))]*Sqrt[-a - b*x^3])
```

Maple [F] time = 0.105, size = 0, normalized size = 0.

$$\int (fx+e)\left(\sqrt[3]{b}x+\sqrt[3]{a}\left(-\sqrt{3}+1\right)\right)^{-1}\frac{1}{\sqrt{-bx^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x+e)/(b^(1/3)*x+a^(1/3)*(-3^(1/2)+1))/(-b*x^3-a)^(1/2),x)
```

```
[Out] int((f*x+e)/(b^(1/3)*x+a^(1/3)*(-3^(1/2)+1))/(-b*x^3-a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\sqrt{-bx^3-a}\left(b^{\frac{1}{3}}x-a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x + e)/(sqrt(-b*x^3 - a)*(b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))),x, alg
```

```
[Out] integrate((f*x + e)/(sqrt(-b*x^3 - a)*(b^(1/3)*x - a^(1/3)*(sqrt(
3) - 1))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x + e)/(sqrt(-b*x^3 - a)*(b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))),x, alg
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e + fx}{\sqrt{-a - bx^3}\left(-\sqrt{3}\sqrt[3]{a} + \sqrt[3]{a} + \sqrt[3]{b}x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(-b*x**3-a)**(1/2),x)
```

```
[Out] Integral((e + f*x)/(sqrt(-a - b*x**3)*(-sqrt(3)*a**(1/3) + a**(1/
3) + b**(1/3)*x)), x)
```

GIAC/XCAS [A] time = 0.596571, size = 4, normalized size = 0.01

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x + e)/(sqrt(-b*x^3 - a)*(b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))),x, alg
```

```
[Out] sage0*x
```

3.112 $\int \frac{x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=136

$$\frac{\sqrt{2}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(x+1)}{\sqrt{x^3+1}}\right)}{3^{3/4}}$$

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]])/3
^(3/4)) + (Sqrt[2]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2
]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*S
qrt[3]])/(3^(3/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]
)
```

Rubi [A] time = 0.421525, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$

$$\frac{\sqrt{2}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(x+1)}{\sqrt{x^3+1}}\right)}{3^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]])/3
^(3/4)) + (Sqrt[2]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2
]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*S
qrt[3]])/(3^(3/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]
)
```

Rubi in Sympy [A] time = 10.9936, size = 78, normalized size = 0.57

$$\frac{2\tilde{\infty}\sqrt{\frac{x^2-x+1}{\left(x+1+\sqrt{3}\right)^2}}\,(x+1)\,F\left(\operatorname{asin}\left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(1+x+3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] 2*zoo*sqrt((x**2 - x + 1)/(x + 1 + sqrt(3))**2)*(x + 1)*elliptic_
f(asin((x - sqrt(3) + 1)/(x + 1 + sqrt(3))), -7 - 4*sqrt(3))/(sqr
t((x + 1)/(x + 1 + sqrt(3))**2)*sqrt(x**3 + 1))
```

Mathematica [C] time = 0.802288, size = 209, normalized size = 1.54

$$\frac{2\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\left(-\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}}+\frac{2i\left(1+\sqrt{3}\right)\sqrt{x^2-x+1}\,\Pi\left(\frac{2i\sqrt{3}}{3+(2+i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{3+(2+i)\sqrt{3}}\right)}{\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) - x)*Sqrt[((-1)
^(1/3) - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1
 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^
(2/3)*x)/(1 + (-1)^(1/3))]) + ((2*I)*(1 + Sqrt[3])*Sqrt[1 - x + x
^2]*EllipticPi[((2*I)*Sqrt[3])/(3 + (2 + I)*Sqrt[3]), ArcSin[Sqrt
[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(3 + (2 + I)
*Sqrt[3])))/Sqrt[1 + x^3]
```

Maple [B] time = 0.033, size = 255, normalized size = 1.9

$$2\,\frac{3/2-i/2\sqrt{3}}{\sqrt{x^3+1}}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}\mathit{EllipticF}\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},\sqrt{\frac{-3/2+i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\right)$$
$$+\frac{\left(-2-2\sqrt{3}\right)\left(\frac{3}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}{3}\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i}{2}\sqrt{3}}}\sqrt{\frac{1}{-\frac{3}{2}-\frac{i}{2}\sqrt{3}}\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)}\sqrt{\frac{1}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)}\mathit{EllipticPi}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i}{2}\sqrt{3}}},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(1+x+3^(1/2))/(x^3+1)^(1/2),x)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1
/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/
(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1
```

```
/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^
(1/2))+2/3*(-1-3^(1/2))*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(
1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*(
(x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*3
^(1/2)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),1/3*(-3/2+1/2
*I*3^(1/2))*3^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(
1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x^3+1}\left(x+\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x}{\sqrt{x^3+1}\left(x+\sqrt{3}+1\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)),x, algorithm="fricas")
```

```
[Out] integral(x/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{(x+1)\left(x^2-x+1\right)}\left(x+1+\sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(1+x+3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt((x + 1)*(x**2 - x + 1))*(x + 1 + sqrt(3))), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x^3+1}\left(x+\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)), x)
```

3.113 $\int \frac{x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=152

$$\frac{\sqrt{2}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{1-x^3}}\right)}{3^{3/4}}$$

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[1 - x^3]])/3
^(3/4)) + (Sqrt[2]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2
]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*S
qrt[3]])/(3^(3/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3]
)
```

Rubi [A] time = 0.471326, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16

$$\frac{\sqrt{2}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{1-x^3}}\right)}{3^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/((1 + Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[1 - x^3]])/3
^(3/4)) + (Sqrt[2]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2
]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*S
qrt[3]])/(3^(3/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3]
)
```

Rubi in Sympy [A] time = 13.0498, size = 78, normalized size = 0.51

$$\frac{2\tilde{\infty}\sqrt{\frac{x^2+x+1}{\left(-x+1+\sqrt{3}\right)^2}}\,(-x+1)\,F\left(\operatorname{asin}\left(\frac{-x-\sqrt{3}+1}{-x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt{\frac{-x+1}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{-x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(1-x+3**(1/2))/(-x**3+1)**(1/2),x)
```

```
[Out] 2*zoo*sqrt((x**2 + x + 1)/(-x + 1 + sqrt(3))**2)*(-x + 1)*ellipti
c_f(asin((-x - sqrt(3) + 1)/(-x + 1 + sqrt(3))), -7 - 4*sqrt(3))/
(sqrt((-x + 1)/(-x + 1 + sqrt(3))**2)*sqrt(-x**3 + 1))
```

Mathematica [C] time = 1.03893, size = 232, normalized size = 1.53

$$\frac{2i\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(2\left(1+\sqrt{3}\right)\sqrt{x^2+x+1}\,\Pi\left(\frac{2i\sqrt{3}}{3+(2+i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)+\frac{i\sqrt{\frac{(-1)^{2/3}x+\sqrt[3]{-1}}{1+\sqrt[3]{-1}}}\left(\left(3+(2+i)\sqrt{3}\right)x+(1+2i)\sqrt{3}+3i\right)F\left(\sin^{-1}\left(\sqrt{\cdots\right.\right.}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}\right.}{\left(3+(2+i)\sqrt{3}\right)\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x/((1 + Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

```
[Out] ((2*I)*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((I*Sqrt[((-1)^(1/3) + (-1)
^(2/3)*x)/(1 + (-1)^(1/3))]*(3*I + (1 + 2*I)*Sqrt[3] + (3 + (2 +
I)*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)
^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]
 + 2*(1 + Sqrt[3])*Sqrt[1 + x + x^2]*EllipticPi[((2*I)*Sqrt[3])/(
3 + (2 + I)*Sqrt[3]), ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1
/3))]], (-1)^(1/3)]))/((3 + (2 + I)*Sqrt[3])*Sqrt[1 - x^3])
```

Maple [B] time = 0.033, size = 257, normalized size = 1.7

$$-\frac{\frac{2i}{3}\left(-1-\sqrt{3}\right)\sqrt{3}}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}-\sqrt{3}}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}EllipticPi\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\frac{\cdots}{-\frac{3}{2}+\cdots}\right.$$

$$+\frac{2i}{3}\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}EllipticF\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)\frac{\cdots}{\sqrt{-x\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(1-x+3^(1/2))/(-x^3+1)^(1/2),x)
```

```
[Out] -2/3*I*(-1-3^(1/2))*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/
2)*((-1+x)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*
```

```
3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(-3/2+1/2*I*3^(1/2)-3^(1/2))*Ellipt
icPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2
)/(-3/2+1/2*I*3^(1/2)-3^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(
1/2))+2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+
x)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))
^(1/2)/(-x^3+1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/
2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{\sqrt{-x^3+1}\left(x-\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x/(sqrt(-x^3 + 1)*(x - sqrt(3) - 1)),x, algorithm="maxima")
```

```
[Out] -integrate(x/(sqrt(-x^3 + 1)*(x - sqrt(3) - 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{x}{\sqrt{-x^3+1}\left(x-\sqrt{3}-1\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x/(sqrt(-x^3 + 1)*(x - sqrt(3) - 1)),x, algorithm="fricas")
```

```
[Out] integral(-x/(sqrt(-x^3 + 1)*(x - sqrt(3) - 1)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{x\sqrt{-x^3+1}-\sqrt{3}\sqrt{-x^3+1}-\sqrt{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(1-x+3**(1/2))/(-x**3+1)**(1/2),x)
```

```
[Out] -Integral(x/(x*sqrt(-x**3 + 1) - sqrt(3)*sqrt(-x**3 + 1) - sqrt(-
x**3 + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x}{\sqrt{-x^3+1}\left(x-\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x/(sqrt(-x^3 + 1)*(x - sqrt(3) - 1)),x, algorithm="giac")
```

```
[Out] integrate(-x/(sqrt(-x^3 + 1)*(x - sqrt(3) - 1)), x)
```

3.114 $\int \frac{x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=164

$$\frac{2\sqrt{\frac{7}{6}-\frac{2}{\sqrt{3}}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{x^3-1}}\right)}{3^{3/4}}$$

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[-1 + x^3]])
/3^(3/4)) + (2*Sqrt[7/6 - 2/Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(
1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[
3] - x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] -
 x)^2)]*Sqrt[-1 + x^3])
```

Rubi [A] time = 0.474831, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174

$$\frac{\sqrt{2\left(7-4\sqrt{3}\right)}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{x^3-1}}\right)}{3^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/((1 + Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[-1 + x^3]])
/3^(3/4)) + (Sqrt[2*(7 - 4*Sqrt[3])]*(1 - x)*Sqrt[(1 + x + x^2)/(
1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[
3] - x)], -7 + 4*Sqrt[3]])/(3^(3/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] -
 x)^2)]*Sqrt[-1 + x^3])
```

Rubi in Sympy [A] time = 41.9314, size = 240, normalized size = 1.46

$$-\frac{\sqrt[4]{3}\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(1+\sqrt{3}\right)(-x+1)\operatorname{atanh}\left(\frac{\left(\sqrt{3}+2\right)\sqrt{-\frac{\left(-x+1+\sqrt{3}\right)^2}{\left(x-1+\sqrt{3}\right)^2}+1}}{\sqrt{\frac{\left(-x+1+\sqrt{3}\right)^2}{\left(x-1+\sqrt{3}\right)^2}+4\sqrt{3}+7}}\right)}{3\sqrt{\frac{x-1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{\sqrt{3}+2}\sqrt{x^3-1}}-\frac{\sqrt[4]{3}\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(-\sqrt{3}+1\right)\sqrt{-\sqrt{3}+2}\,(-x+1)F\left(\operatorname{asin}\left(\frac{-x+1+\sqrt{3}}{-x-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{3\sqrt{\frac{x-1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(1-x+3**(1/2))/(x**3-1)**(1/2),x)
```

```
[Out] -3**(1/4)*sqrt((x**2 + x + 1)/(-x - sqrt(3) + 1)**2)*(1 + sqrt(3)
)*(-x + 1)*atanh((sqrt(3) + 2)*sqrt(-(-x + 1 + sqrt(3))**2/(x - 1
 + sqrt(3))**2 + 1)/sqrt((-x + 1 + sqrt(3))**2/(x - 1 + sqrt(3))*
*2 + 4*sqrt(3) + 7))/(3*sqrt((x - 1)/(-x - sqrt(3) + 1)**2)*sqrt(
sqrt(3) + 2)*sqrt(x**3 - 1)) - 3**(1/4)*sqrt((x**2 + x + 1)/(-x -
 sqrt(3) + 1)**2)*(-sqrt(3) + 1)*sqrt(-sqrt(3) + 2)*(-x + 1)*elli
ptic_f(asin((-x + 1 + sqrt(3))/(-x - sqrt(3) + 1)), -7 + 4*sqrt(3
))/(3*sqrt((x - 1)/(-x - sqrt(3) + 1)**2)*sqrt(x**3 - 1))
```

Mathematica [C] time = 0.977659, size = 230, normalized size = 1.4

$$\frac{2i\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(2\left(1+\sqrt{3}\right)\sqrt{x^2+x+1}\,\Pi\left(\frac{2i\sqrt{3}}{3+(2+i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)+\frac{i\sqrt{\frac{(-1)^{2/3}x+\sqrt[3]{-1}}{1+\sqrt[3]{-1}}}\left(\left(3+(2+i)\sqrt{3}\right)x+(1+2i)\sqrt{3}+3i\right)F\left(\sin^{-1}\left(\sqrt{\cdots}\right.\right.}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}\right.}{\left(3+(2+i)\sqrt{3}\right)\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x/((1 + Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] ((2*I)*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((I*Sqrt[((-1)^(1/3) + (-1)
^(2/3)*x)/(1 + (-1)^(1/3))]*(3*I + (1 + 2*I)*Sqrt[3] + (3 + (2 +
I)*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)
^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]
 + 2*(1 + Sqrt[3])*Sqrt[1 + x + x^2]*EllipticPi[((2*I)*Sqrt[3])/(
```

```
3 + (2 + I)*Sqrt[3]), ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1
/3))]], (-1)^(1/3)]))/((3 + (2 + I)*Sqrt[3])*Sqrt[-1 + x^3])
```

Maple [A] time = 0.027, size = 255, normalized size = 1.6

$$-\frac{\left(-2-2\sqrt{3}\right)\left(-\frac{3}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}{3}\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i}{2}\sqrt{3}}}\sqrt{\frac{1}{\frac{3}{2}-\frac{i}{2}\sqrt{3}}\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)}\sqrt{\frac{1}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)}\mathit{EllipticPi}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i}{2}\sqrt{3}}}\right.$$

$$-2\,\frac{-3/2-i/2\sqrt{3}}{\sqrt{x^3-1}}\sqrt{\frac{-1+x}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2-i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2+i/2\sqrt{3}}{3/2+i/2\sqrt{3}}}\mathit{EllipticF}\left(\sqrt{\frac{-1+x}{-3/2-i/2\sqrt{3}}},\sqrt{\frac{3/2+i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(1-x+3^(1/2))/(x^3-1)^(1/2),x)
```

```
[Out] -2/3*(-1-3^(1/2))*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2
)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1
/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*3^(1/2
)*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(3/2+1/2*I*
3^(1/2))*3^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
-2*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1
/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2
))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((-1+x)/(-3
/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2))
)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x}{\sqrt{x^3-1}\left(x-\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x/(sqrt(x^3 - 1)*(x - sqrt(3) - 1)),x, algorithm="maxima")
```

```
[Out] -integrate(x/(sqrt(x^3 - 1)*(x - sqrt(3) - 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{x}{\sqrt{x^3-1}\left(x-\sqrt{3}-1\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x/(sqrt(x^3 - 1)*(x - sqrt(3) - 1)),x, algorithm="fricas")
```

```
[Out] integral(-x/(sqrt(x^3 - 1)*(x - sqrt(3) - 1)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{x\sqrt{x^3-1}-\sqrt{3}\sqrt{x^3-1}-\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1-x+3**(1/2))/(x**3-1)**(1/2),x)
```

```
[Out] -Integral(x/(x*sqrt(x**3 - 1) - sqrt(3)*sqrt(x**3 - 1) - sqrt(x**
3 - 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x}{\sqrt{x^3-1}\left(x-\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x/(sqrt(x^3 - 1)*(x - sqrt(3) - 1)),x, algorithm="giac")
```

```
[Out] integrate(-x/(sqrt(x^3 - 1)*(x - sqrt(3) - 1)), x)
```

3.115 $\int \frac{x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=156

$$\frac{2\sqrt{\frac{7}{6}-\frac{2}{\sqrt{3}}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(x+1)}{\sqrt{-x^3-1}}\right)}{3^{3/4}}$$

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[-1 - x^3]])
/3^(3/4)) + (2*Sqrt[7/6 - 2/Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(
1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[
3] + x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] +
 x)^2)]*Sqrt[-1 - x^3])
```

Rubi [A] time = 0.462739, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$

$$\frac{\sqrt{2\left(7-4\sqrt{3}\right)}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(x+1)}{\sqrt{-x^3-1}}\right)}{3^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/((1 + Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[-1 - x^3]])
/3^(3/4)) + (Sqrt[2*(7 - 4*Sqrt[3])]*(1 + x)*Sqrt[(1 - x + x^2)/(
1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[
3] + x)], -7 + 4*Sqrt[3]])/(3^(3/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] +
 x)^2)]*Sqrt[-1 - x^3])
```

Rubi in Sympy [A] time = 40.5739, size = 246, normalized size = 1.58

$$-\frac{\sqrt[4]{3}\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(1+\sqrt{3}\right)(x+1)\operatorname{atanh}\left(\frac{\sqrt{1-\frac{\left(x+1+\sqrt{3}\right)^2}{\left(-x-1+\sqrt{3}\right)^2}}\left(\sqrt{3}+2\right)}{\sqrt{4\sqrt{3}+7+\frac{\left(x+1+\sqrt{3}\right)^2}{\left(-x-1+\sqrt{3}\right)^2}}}\right)}{3\sqrt{\frac{-x-1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{\sqrt{3}+2}\sqrt{-x^3-1}}-\frac{\sqrt[4]{3}\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(-\sqrt{3}+1\right)\sqrt{-\sqrt{3}+2}\,(x+1)\,F\left(\operatorname{asin}\left(\frac{x+1+\sqrt{3}}{x-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{3\sqrt{\frac{-x-1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(1+x+3**(1/2))/(-x**3-1)**(1/2),x)
```

```
[Out] -3**(1/4)*sqrt((x**2 - x + 1)/(x - sqrt(3) + 1)**2)*(1 + sqrt(3))
*(x + 1)*atanh(sqrt(1 - (x + 1 + sqrt(3))**2/(-x - 1 + sqrt(3))**
2)*(sqrt(3) + 2)/sqrt(4*sqrt(3) + 7 + (x + 1 + sqrt(3))**2/(-x -
1 + sqrt(3))**2))/(3*sqrt((-x - 1)/(x - sqrt(3) + 1)**2)*sqrt(sqr
t(3) + 2)*sqrt(-x**3 - 1)) - 3**(1/4)*sqrt((x**2 - x + 1)/(x - sq
rt(3) + 1)**2)*(-sqrt(3) + 1)*sqrt(-sqrt(3) + 2)*(x + 1)*elliptic
_f(asin((x + 1 + sqrt(3))/(x - sqrt(3) + 1)), -7 + 4*sqrt(3))/(3*
sqrt((-x - 1)/(x - sqrt(3) + 1)**2)*sqrt(-x**3 - 1))
```

Mathematica [C] time = 0.789643, size = 211, normalized size = 1.35

$$\frac{2\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\left(-\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}}+\frac{2i\left(1+\sqrt{3}\right)\sqrt{x^2-x+1}\,\Pi\left(\frac{2i\sqrt{3}}{3+(2+i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{3+(2+i)\sqrt{3}}\right)}{\sqrt{-x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x/((1 + Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) - x)*Sqrt[((-1)
^(1/3) - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1
 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^
(2/3)*x)/(1 + (-1)^(1/3))]) + ((2*I)*(1 + Sqrt[3])*Sqrt[1 - x + x
^2]*EllipticPi[((2*I)*Sqrt[3])/(3 + (2 + I)*Sqrt[3]), ArcSin[Sqrt
```

```
[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(3 + (2 + I)
*Sqrt[3])))/Sqrt[-1 - x^3]
```

Maple [A] time = 0.029, size = 253, normalized size = 1.6

$$-\frac{2i}{3}\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\, EllipticF\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)\frac{1}{\sqrt{-x^3}}$$
$$-\frac{\frac{2i}{3}\left(-1-\sqrt{3}\right)\sqrt{3}}{\frac{3}{2}+\frac{i}{2}\sqrt{3}+\sqrt{3}}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\, EllipticPi\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\frac{i}{\frac{3}{2}+\frac{i}{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(1+x+3^(1/2))/(-x^3-1)^(1/2),x)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/
2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/
(-x^3-1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(
1/2))^(1/2),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))-2/3*I*(-1-3^(1
/2))*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+
1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-
x^3-1)^(1/2)/(3/2+1/2*I*3^(1/2)+3^(1/2))*EllipticPi(1/3*3^(1/2)*(
I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(3/2+1/2*I*3^(1/
2)+3^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-x^3-1}\left(x+\sqrt{3}+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(sqrt(-x^3 - 1)*(x + sqrt(3) + 1)),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(-x^3 - 1)*(x + sqrt(3) + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x}{\sqrt{-x^3-1}\left(x+\sqrt{3}+1\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(-x^3 - 1)*(x + sqrt(3) + 1)),x, algorithm="fricas")
```

```
[Out] integral(x/(sqrt(-x^3 - 1)*(x + sqrt(3) + 1)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-(x+1)(x^2-x+1)}\left(x+1+\sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(1+x+3**(1/2))/(-x**3-1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 1 + sqrt(3))), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-x^3-1}\left(x+\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(-x^3 - 1)*(x + sqrt(3) + 1)),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(-x^3 - 1)*(x + sqrt(3) + 1)), x)
```

3.116 $\int \frac{x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=147

$$\frac{2\sqrt{\frac{7}{6}+\frac{2}{\sqrt{3}}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}(x+1)}{\sqrt{x^3+1}}\right)}{3^{3/4}}$$

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]])
/3^(3/4)) + (2*Sqrt[7/6 + 2/Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(
1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[
3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x
)^2]*Sqrt[1 + x^3])
```

Rubi [A] time = 0.461211, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174

$$\frac{\sqrt{2\left(7+4\sqrt{3}\right)}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}(x+1)}{\sqrt{x^3+1}}\right)}{3^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/((1 - Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]])
/3^(3/4)) + (Sqrt[2*(7 + 4*Sqrt[3])]*(1 + x)*Sqrt[(1 - x + x^2)/(
1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[
3] + x)], -7 - 4*Sqrt[3]])/(3^(3/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x
)^2]*Sqrt[1 + x^3])
```

Rubi in Sympy [A] time = 41.6472, size = 240, normalized size = 1.63

$$\frac{\sqrt[4]{3}\sqrt{\frac{x^2-x+1}{\left(x+1+\sqrt{3}\right)^2}}\left(-\sqrt{3}+1\right)(x+1)\operatorname{atanh}\left(\frac{\left(-\sqrt{3}+2\right)\sqrt{-\frac{\left(-x-1+\sqrt{3}\right)^2}{\left(x+1+\sqrt{3}\right)^2}+1}}{\sqrt{\frac{\left(-x-1+\sqrt{3}\right)^2}{\left(x+1+\sqrt{3}\right)^2}-4\sqrt{3}+7}}\right)}{3\sqrt{\frac{x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{-\sqrt{3}+2}\sqrt{x^3+1}}+\frac{\sqrt[4]{3}\sqrt{\frac{x^2-x+1}{\left(x+1+\sqrt{3}\right)^2}}\left(1+\sqrt{3}\right)\sqrt{\sqrt{3}+2}(x+1)F\left(\operatorname{asin}\left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{3\sqrt{\frac{x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(1+x-3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] 3**(1/4)*sqrt((x**2 - x + 1)/(x + 1 + sqrt(3))**2)*(-sqrt(3) + 1)
*(x + 1)*atanh((-sqrt(3) + 2)*sqrt(-(-x - 1 + sqrt(3))**2/(x + 1
+ sqrt(3))**2 + 1)/sqrt((-x - 1 + sqrt(3))**2/(x + 1 + sqrt(3))**
2 - 4*sqrt(3) + 7))/(3*sqrt((x + 1)/(x + 1 + sqrt(3))**2)*sqrt(-s
qrt(3) + 2)*sqrt(x**3 + 1)) + 3**(1/4)*sqrt((x**2 - x + 1)/(x + 1
 + sqrt(3))**2)*(1 + sqrt(3))*sqrt(sqrt(3) + 2)*(x + 1)*elliptic_
f(asin((x - sqrt(3) + 1)/(x + 1 + sqrt(3))), -7 - 4*sqrt(3))/(3*s
qrt((x + 1)/(x + 1 + sqrt(3))**2)*sqrt(x**3 + 1))
```

Mathematica [C] time = 0.97551, size = 225, normalized size = 1.53

$$\frac{2\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\left(\frac{\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\left(\left(\left(1+2i\right)\sqrt{3}-3i\right)x-\left(2+i\right)\sqrt{3}+3\right)F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}}-2\left(\sqrt{3}-1\right)\sqrt{x^2-x+1}\left(\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\sqrt{\right.\right.\right.}{\left((1+2i)\sqrt{3}-3i\right)\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x/((1 - Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*((Sqrt[((-1)^(1/3) - (-1)^(2/3)
*x)/(1 + (-1)^(1/3))]*(3 - (2 + I)*Sqrt[3] + (-3*I + (1 + 2*I)*Sq
rt[3])*x)*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3
))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))] - 2*
(-1 + Sqrt[3])*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt[3])/(-3*I + (
```

```
1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3)
)]], (-1)^(1/3)]))/((-3*I + (1 + 2*I)*Sqrt[3])*Sqrt[1 + x^3])
```

Maple [B] time = 0.033, size = 255, normalized size = 1.7

$$\frac{\left(-2\sqrt{3}+2\right)\left(\frac{3}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}{3}\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i}{2}\sqrt{3}}}\sqrt{\frac{1}{-\frac{3}{2}-\frac{i}{2}\sqrt{3}}\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)}\sqrt{\frac{1}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)}EllipticPi\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i}{2}\sqrt{3}}},-\right.$$

$$\left.+2\frac{3/2-i/2\sqrt{3}}{\sqrt{x^3+1}}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}EllipticF\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},\sqrt{\frac{-3/2+i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(1+x-3^(1/2))/(x^3+1)^(1/2),x)
```

```
[Out] 2/3*(-3^(1/2)+1)*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^
(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+
1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*3^(1/2)*
EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(-3/2+1/2*I*3^(
1/2))*3^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+
2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1
/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/
(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1
/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^
(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x^3+1}\left(x-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(x^3 + 1)*(x - sqrt(3) + 1)),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(x^3 + 1)*(x - sqrt(3) + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x}{\sqrt{x^3+1}\left(x-\sqrt{3}+1\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(x^3 + 1)*(x - sqrt(3) + 1)),x, algorithm="fricas")
```

```
[Out] integral(x/(sqrt(x^3 + 1)*(x - sqrt(3) + 1)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{(x+1)(x^2-x+1)}\left(x-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(1+x-3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt((x + 1)*(x**2 - x + 1))*(x - sqrt(3) + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x^3+1}\left(x-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(x^3 + 1)*(x - sqrt(3) + 1)),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(x^3 + 1)*(x - sqrt(3) + 1)), x)
```

3.117 $\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=278

$$\frac{2\sqrt{\frac{7}{6}+\frac{2}{\sqrt{3}}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{3^{3/4}\sqrt[6]{a}b^{2/3}}$$

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) + b^(1/
3)*x))/Sqrt[a + b*x^3]])/(3^(3/4)*a^(1/6)*b^(2/3))) + (2*Sqrt[7/6
 + 2/Sqrt[3]]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipt
icF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(
1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/
3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*
Sqrt[a + b*x^3])
```

Rubi [A] time = 0.877481, antiderivative size = 278, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$\frac{\sqrt{2\left(7+4\sqrt{3}\right)}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3^{3/4}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{3^{3/4}\sqrt[6]{a}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) + b^(1/
3)*x))/Sqrt[a + b*x^3]])/(3^(3/4)*a^(1/6)*b^(2/3))) + (Sqrt[2*(7
+ 4*Sqrt[3])]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipt
icF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(
1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(3/4)*b^(2/3)*Sqrt[(a^(1/
3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*
Sqrt[a + b*x^3])
```

Rubi in Sympy [A] time = 76.9876, size = 444, normalized size = 1.6

$$\frac{\sqrt[4]{3}\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^{2}}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^{2}}}\left(1+\sqrt{3}\right)\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)F\left(\operatorname{asin}\left(\frac{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{3b^{\frac{2}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^{2}}}\sqrt{a+bx^{3}}}+\frac{\sqrt[4]{3}\sqrt{\frac{a^{\frac{2}{3}}\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+\frac{b^{\frac{2}{3}}x^{2}}{a^{\frac{2}{3}}}\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^{2}}}\left(-\sqrt{3}+1\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\operatorname{atanh}\left(\frac{\left(-\sqrt{3}+2\right)\sqrt{-\frac{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)-\sqrt[3]{b}x\right)^{2}}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^{2}}+1}}{\sqrt{\frac{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)-\sqrt[3]{b}x\right)^{2}}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^{2}}-4\sqrt{3}+7}}\right)}{3b^{\frac{2}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^{2}}}\sqrt{-\sqrt{3}+2}\sqrt{a+bx^{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x/(b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(b*x**3+a)**(1/2),x)
```

```
[Out] 3**(1/4)*sqrt((a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(a
**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*(1 + sqrt(3))*sqrt(sqrt(3
) + 2)*(a**(1/3) + b**(1/3)*x)*elliptic_f(asin((-a**(1/3)*(-1 + s
qrt(3)) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)), -7
- 4*sqrt(3))/(3*b**(2/3)*sqrt(a**(1/3)*(a**(1/3) + b**(1/3)*x)/(a
**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*sqrt(a + b*x**3)) + 3**(1
/4)*sqrt(a**(2/3)*(1 - b**(1/3)*x/a**(1/3) + b**(2/3)*x**2/a**(2/
3))/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2)*(-sqrt(3) + 1)*(a**
(1/3) + b**(1/3)*x)*atanh((-sqrt(3) + 2)*sqrt(-(a**(1/3)*(-1 + sq
rt(3)) - b**(1/3)*x)**2/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)**2
+ 1)/sqrt((a**(1/3)*(-1 + sqrt(3)) - b**(1/3)*x)**2/(a**(1/3)*(1
+ sqrt(3)) + b**(1/3)*x)**2 - 4*sqrt(3) + 7))/(3*b**(2/3)*sqrt(a*
*(1/3)*(a**(1/3) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) + b**(1/3)
*x)**2)*sqrt(-sqrt(3) + 2)*sqrt(a + b*x**3))
```

Mathematica [C] time = 2.05345, size = 427, normalized size = 1.54

$$-\frac{4\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(i\left(\sqrt{3}-1\right)\sqrt[3]{a}\sqrt{\frac{\left(\sqrt{3}+i\right)\sqrt[3]{b}x-2i\sqrt[3]{a}}{\left(\sqrt{3}-3i\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\left(\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{\left(i+\sqrt{3}\right)\sqrt[3]{b}x-2i\sqrt[3]{a}}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)|\frac{1}{2}\left(1+i\sqrt{\ldots}\right.\right.\right.}{\left(3-(2-i)\sqrt{3}\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\ldots}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (-4*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(((-I/
2)*3^(1/4)*(((-2 - I) + Sqrt[3])*a^(1/3) + ((1 + 2*I) - I*Sqrt[3]
)*b^(1/3)*x)*Sqrt[I + Sqrt[3] - ((2*I)*b^(1/3)*x)/a^(1/3)]*Ellipt
icF[ArcSin[Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I
 + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2])/Sqrt[2] + I*(-1 + Sqr
t[3])*a^(1/3)*Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-
3*I + Sqrt[3])*a^(1/3))]*Sqrt[1 - (b^(1/3)*x)/a^(1/3) + (b^(2/3)*
x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(-3*I + (1 + 2*I)*Sqrt[3]),
ArcSin[Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + S
qrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/((3 - (2 - I)*Sqrt[3])*b
^(2/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^
(1/3))]*Sqrt[a + b*x^3])
```

Maple [F] time = 0.108, size = 0, normalized size = 0.

$$\int x\left(\sqrt[3]{b}x+\sqrt[3]{a}\left(-\sqrt{3}+1\right)\right)^{-1}\frac{1}{\sqrt{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(b^(1/3)*x+a^(1/3)*(-3^(1/2)+1))/(b*x^3+a)^(1/2),x)
```

```
[Out] int(x/(b^(1/3)*x+a^(1/3)*(-3^(1/2)+1))/(b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{bx^3+a}\left(b^{\frac{1}{3}}x-a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(sqrt(b*x^3 + a)*(b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))),x, algorithm="
```

```
[Out] integrate(x/(sqrt(b*x^3 + a)*(b^(1/3)*x - a^(1/3)*(sqrt(3) - 1)))
, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x}{\sqrt{bx^3+a}\left(b^{\frac{1}{3}}x-a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(sqrt(b*x^3 + a)*(b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))),x, algorithm="
```

```
[Out] integral(x/(sqrt(b*x^3 + a)*(b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))),
 x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a+bx^3}\left(-\sqrt{3}\sqrt[3]{a}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(a + b*x**3)*(-sqrt(3)*a**(1/3) + a**(1/3) + b**(
1/3)*x)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{bx^3+a}\left(b^{\frac{1}{3}}x-a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(sqrt(b*x^3 + a)*(b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))),x, algorithm="
```

```
[Out] integrate(x/(sqrt(b*x^3 + a)*(b^(1/3)*x - a^(1/3)*(sqrt(3) - 1)))
, x)
```

3.118 $\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$

Optimal. Leaf size=286

$$\frac{2\sqrt{\frac{7}{6}+\frac{2}{\sqrt{3}}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{3^{3/4}\sqrt[6]{a}b^{2/3}}$$

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) - b^(1/
3)*x))/Sqrt[a - b*x^3]])/(3^(3/4)*a^(1/6)*b^(2/3))) + (2*Sqrt[7/6
 + 2/Sqrt[3]]*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Ellipt
icF[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(
1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/
3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*
Sqrt[a - b*x^3])
```

Rubi [A] time = 0.914981, antiderivative size = 286, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$\frac{\sqrt{2\left(7+4\sqrt{3}\right)}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{3^{3/4}\sqrt[6]{a}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) - b^(1/
3)*x))/Sqrt[a - b*x^3]])/(3^(3/4)*a^(1/6)*b^(2/3))) + (Sqrt[2*(7
+ 4*Sqrt[3])]*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Ellipt
icF[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(
1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(3/4)*b^(2/3)*Sqrt[(a^(1/
3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*
Sqrt[a - b*x^3])
```

Rubi in Sympy [A] time = 87.7773, size = 444, normalized size = 1.55

$$-\frac{\sqrt[4]{3}\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^{2}}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{b}x\right)^{2}}}\left(1+\sqrt{3}\right)\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)F\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}{\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{3b^{\frac{2}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{b}x\right)^{2}}}\sqrt{a-bx^{3}}}+\frac{\sqrt[4]{3}\sqrt{\frac{a^{\frac{2}{3}}\left(1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+\frac{b^{\frac{2}{3}}x^{2}}{a^{\frac{2}{3}}}\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{b}x\right)^{2}}}\left(-\sqrt{3}+1\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\operatorname{atanh}\left(\frac{\left(-\sqrt{3}+2\right)\sqrt{-\frac{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^{2}}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{b}x\right)^{2}}+1}}{\sqrt{\frac{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^{2}}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{b}x\right)^{2}}-4\sqrt{3}+7}}\right)}{3b^{\frac{2}{3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{b}x\right)^{2}}}\sqrt{-\sqrt{3}+2}\sqrt{a-bx^{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x/(-b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(-b*x**3+a)**(1/2),x)
```

```
[Out] -3**(1/4)*sqrt((a**(2/3) + a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(
a**(1/3)*(1 + sqrt(3)) - b**(1/3)*x)**2)*(1 + sqrt(3))*sqrt(sqrt(
3) + 2)*(a**(1/3) - b**(1/3)*x)*elliptic_f(asin((a**(1/3)*(-1 + s
qrt(3)) + b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) - b**(1/3)*x)), -7
- 4*sqrt(3))/(3*b**(2/3)*sqrt(a**(1/3)*(a**(1/3) - b**(1/3)*x)/(a
**(1/3)*(1 + sqrt(3)) - b**(1/3)*x)**2)*sqrt(a - b*x**3)) + 3**(1
/4)*sqrt(a**(2/3)*(1 + b**(1/3)*x/a**(1/3) + b**(2/3)*x**2/a**(2/
3))/(a**(1/3)*(1 + sqrt(3)) - b**(1/3)*x)**2)*(-sqrt(3) + 1)*(a**
(1/3) - b**(1/3)*x)*atanh((-sqrt(3) + 2)*sqrt(-(a**(1/3)*(-1 + sq
rt(3)) + b**(1/3)*x)**2/(a**(1/3)*(1 + sqrt(3)) - b**(1/3)*x)**2
+ 1)/sqrt((a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)**2/(a**(1/3)*(1
+ sqrt(3)) - b**(1/3)*x)**2 - 4*sqrt(3) + 7))/(3*b**(2/3)*sqrt(a*
*(1/3)*(a**(1/3) - b**(1/3)*x)/(a**(1/3)*(1 + sqrt(3)) - b**(1/3)
*x)**2)*sqrt(-sqrt(3) + 2)*sqrt(a - b*x**3))
```

Mathematica [C] time = 2.30114, size = 454, normalized size = 1.59

$$-\frac{4\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(i\left(\sqrt{3}-1\right)\sqrt[3]{a}\sqrt{-\frac{i\left(2\sqrt[3]{a}+\left(1-i\sqrt{3}\right)\sqrt[3]{b}x\right)}{\left(\sqrt{3}-3i\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\left(\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\sqrt{-\frac{i\left(\left(1-i\sqrt{3}\right)\sqrt[3]{b}x+2\sqrt[3]{a}\right)}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)\right|}{\left(3-(2-i)\sqrt{3}\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x/(((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (-4*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(((I*(
-3 + (2 + I)*Sqrt[3])*a^(1/3) + (3 - (2 - I)*Sqrt[3])*b^(1/3)*x)*
Sqrt[((-I + Sqrt[3])*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I +
Sqrt[3])*a^(1/3))]*EllipticF[ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 -
I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[
3])/2])/2 + I*(-1 + Sqrt[3])*a^(1/3)*Sqrt[((-I)*(2*a^(1/3) + (1 -
 I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1/3))]*Sqrt[1 + (b^(
1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(
-3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*
Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3]
)/2]))/((3 - (2 - I)*Sqrt[3])*b^(2/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*
b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a - b*x^3])
```

Maple [F] time = 0.107, size = 0, normalized size = 0.

$$\int x\left(-\sqrt[3]{b}x+\sqrt[3]{a}\left(-\sqrt{3}+1\right)\right)^{-1}\frac{1}{\sqrt{-bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(-b^(1/3)*x+a^(1/3)*(-3^(1/2)+1))/(-b*x^3+a)^(1/2),x)
```

```
[Out] int(x/(-b^(1/3)*x+a^(1/3)*(-3^(1/2)+1))/(-b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{\sqrt{-bx^3+a}\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x/(sqrt(-b*x^3 + a)*(b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))),x, algorithm
```

```
[Out] -integrate(x/(sqrt(-b*x^3 + a)*(b^(1/3)*x + a^(1/3)*(sqrt(3) - 1)
)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{x}{\sqrt{-bx^3+a}\,b^{\frac{1}{3}}x+\sqrt{-bx^3+a}\,a^{\frac{1}{3}}\left(\sqrt{3}-1\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x/(sqrt(-b*x^3 + a)*(b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))),x, algorithm
```

```
[Out] integral(-x/(sqrt(-b*x^3 + a)*b^(1/3)*x + sqrt(-b*x^3 + a)*a^(1/3
)*(sqrt(3) - 1)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{-\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt{3}\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt[3]{b}x\sqrt{a-bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(-b*x**3+a)**(1/2),x)
```

```
[Out] -Integral(x/(-a**(1/3)*sqrt(a - b*x**3) + sqrt(3)*a**(1/3)*sqrt(a
 - b*x**3) + b**(1/3)*x*sqrt(a - b*x**3)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x}{\sqrt{-bx^3+a}\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x/(sqrt(-b*x^3 + a)*(b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))),x, algorithm
```

```
[Out] integrate(-x/(sqrt(-b*x^3 + a)*(b^(1/3)*x + a^(1/3)*(sqrt(3) - 1)
)), x)
```

3.119 $\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

Optimal. Leaf size=282

$$\frac{\sqrt{2}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)\middle|-7+4\sqrt{3}\right)}{3^{3/4}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{bx^3-a}}\right)}{3^{3/4}\sqrt[6]{a}b^{2/3}}$$

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) - b^(1/3
)*x))/Sqrt[-a + b*x^3]])/(3^(3/4)*a^(1/6)*b^(2/3))) + (Sqrt[2]*(a
^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 +
 Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x
)], -7 + 4*Sqrt[3]])/(3^(3/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) -
b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x
^3])
```

Rubi [A] time = 0.855942, antiderivative size = 282, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$

$$\frac{\sqrt{2}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)\middle|-7+4\sqrt{3}\right)}{3^{3/4}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{bx^3-a}}\right)}{3^{3/4}\sqrt[6]{a}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) - b^(1/3
)*x))/Sqrt[-a + b*x^3]])/(3^(3/4)*a^(1/6)*b^(2/3))) + (Sqrt[2]*(a
^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 +
 Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x
)], -7 + 4*Sqrt[3]])/(3^(3/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) -
b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x
^3])
```

Rubi in Sympy [A] time = 33.8402, size = 168, normalized size = 0.6

$$\frac{2\tilde{\infty}\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)F\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{b}x}{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)-\sqrt[3]{b}x}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x/(-b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(b*x**3-a)**(1/2),x)
```

```
[Out] 2*zoo*sqrt((a**(2/3) + a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(a**(
1/3)*(-1 + sqrt(3)) + b**(1/3)*x)**2)*(a**(1/3) - b**(1/3)*x)*ell
iptic_f(asin((a**(1/3)*(1 + sqrt(3)) - b**(1/3)*x)/(-a**(1/3)*(-1
 + sqrt(3)) - b**(1/3)*x)), -7 + 4*sqrt(3))/(b**(1/3)*sqrt(-a**(1
/3)*(a**(1/3) - b**(1/3)*x)/(a**(1/3)*(-1 + sqrt(3)) + b**(1/3)*x
)**2)*sqrt(-a + b*x**3))
```

Mathematica [C] time = 2.33455, size = 455, normalized size = 1.61

$$-\frac{4\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(i\left(\sqrt{3}-1\right)\sqrt[3]{a}\sqrt{-\frac{i\left(2\sqrt[3]{a}+\left(1-i\sqrt{3}\right)\sqrt[3]{b}x\right)}{\left(\sqrt{3}-3i\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\left(\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\sqrt{-\frac{i\left(\left(1-i\sqrt{3}\right)\sqrt[3]{b}x+2\sqrt[3]{a}\right)}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)\right|\right.}{\left(3-(2-i)\sqrt{3}\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (-4*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(((I*(
-3 + (2 + I)*Sqrt[3])*a^(1/3) + (3 - (2 - I)*Sqrt[3])*b^(1/3)*x)*
Sqrt[((-I + Sqrt[3])*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I +
Sqrt[3])*a^(1/3))]*EllipticF[ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 -
I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[
3])/2])/2 + I*(-1 + Sqrt[3])*a^(1/3)*Sqrt[((-I)*(2*a^(1/3) + (1 -
 I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1/3))]*Sqrt[1 + (b^(
1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(
-3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*
Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3]
)/2]))/((3 - (2 - I)*Sqrt[3])*b^(2/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*
b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[-a + b*x^3])
```

Maple [F] time = 0.096, size = 0, normalized size = 0.

$$\int x\left(-\sqrt[3]{b}x+\sqrt[3]{a}\left(-\sqrt{3}+1\right)\right)^{-1}\frac{1}{\sqrt{bx^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(-b^(1/3)*x+a^(1/3)*(-3^(1/2)+1))/(b*x^3-a)^(1/2),x)
```

```
[Out] int(x/(-b^(1/3)*x+a^(1/3)*(-3^(1/2)+1))/(b*x^3-a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x}{\sqrt{bx^3-a}\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x/(sqrt(b*x^3 - a)*(b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))),x, algorithm=
```

```
[Out] -integrate(x/(sqrt(b*x^3 - a)*(b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{x}{\sqrt{bx^3-a}\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x/(sqrt(b*x^3 - a)*(b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))),x, algorithm=
```

```
[Out] integral(-x/(sqrt(b*x^3 - a)*(b^(1/3)*x + a^(1/3)*(sqrt(3) - 1)))
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{-\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt{3}\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt[3]{b}x\sqrt{-a+bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(-b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(b*x**3-a)**(1/2),x)
```

```
[Out] -Integral(x/(-a**(1/3)*sqrt(-a + b*x**3) + sqrt(3)*a**(1/3)*sqrt(
-a + b*x**3) + b**(1/3)*x*sqrt(-a + b*x**3)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x}{\sqrt{bx^3-a}\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x/(sqrt(b*x^3 - a)*(b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))),x, algorithm=
```

```
[Out] integrate(-x/(sqrt(b*x^3 - a)*(b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))
), x)
```

3.120 $\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

Optimal. Leaf size=278

$$\frac{\sqrt{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{3^{3/4}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{3^{3/4}\sqrt[6]{a}b^{2/3}}$$

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) + b^(1/3
)*x))/Sqrt[-a - b*x^3]])/(3^(3/4)*a^(1/6)*b^(2/3))) + (Sqrt[2]*(a
^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x
)], -7 + 4*Sqrt[3]])/(3^(3/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) +
b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x
^3])
```

Rubi [A] time = 0.825604, antiderivative size = 278, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.098

$$\frac{\sqrt{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{3^{3/4}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{3^{3/4}\sqrt[6]{a}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) + b^(1/3
)*x))/Sqrt[-a - b*x^3]])/(3^(3/4)*a^(1/6)*b^(2/3))) + (Sqrt[2]*(a
^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x
)], -7 + 4*Sqrt[3]])/(3^(3/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) +
b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x
^3])
```

Rubi in Sympy [A] time = 32.8959, size = 168, normalized size = 0.6

$$\frac{2\tilde{\infty}\sqrt{\frac{a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2}{\left(-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)F\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\left(1+\sqrt{3}\right)+\sqrt[3]{b}x}{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(-\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x/(b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(-b*x**3-a)**(1/2),x)
```

```
[Out] 2*zoo*sqrt((a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(-a**
(1/3)*(-1 + sqrt(3)) + b**(1/3)*x)**2)*(a**(1/3) + b**(1/3)*x)*el
liptic_f(asin((a**(1/3)*(1 + sqrt(3)) + b**(1/3)*x)/(-a**(1/3)*(-
1 + sqrt(3)) + b**(1/3)*x)), -7 + 4*sqrt(3))/(b**(1/3)*sqrt(-a**(
1/3)*(a**(1/3) + b**(1/3)*x)/(-a**(1/3)*(-1 + sqrt(3)) + b**(1/3)
*x)**2)*sqrt(-a - b*x**3))
```

Mathematica [C] time = 2.03881, size = 430, normalized size = 1.55

$$-\frac{4\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(i\left(\sqrt{3}-1\right)\sqrt[3]{a}\sqrt{\frac{\left(\sqrt{3}+i\right)\sqrt[3]{b}x-2i\sqrt[3]{a}}{\left(\sqrt{3}-3i\right)\sqrt[3]{a}}}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\left(\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{\left(i+\sqrt{3}\right)\sqrt[3]{b}x-2i\sqrt[3]{a}}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)\middle|\frac{1}{2}\left(1+i\sqrt{\vphantom{3}}\right.\right.\right.}{\left(3-(2-i)\sqrt{3}\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{\vphantom{3}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (-4*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(((-I/
2)*3^(1/4)*(((-2 - I) + Sqrt[3])*a^(1/3) + ((1 + 2*I) - I*Sqrt[3]
)*b^(1/3)*x)*Sqrt[I + Sqrt[3] - ((2*I)*b^(1/3)*x)/a^(1/3)]*Ellipt
icF[ArcSin[Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I
 + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2])/Sqrt[2] + I*(-1 + Sqr
t[3])*a^(1/3)*Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-
3*I + Sqrt[3])*a^(1/3))]*Sqrt[1 - (b^(1/3)*x)/a^(1/3) + (b^(2/3)*
x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(-3*I + (1 + 2*I)*Sqrt[3]),
ArcSin[Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + S
qrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/((3 - (2 - I)*Sqrt[3])*b
^(2/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^
(1/3))]*Sqrt[-a - b*x^3])
```

Maple [F] time = 0.098, size = 0, normalized size = 0.

$$\int x\left(\sqrt[3]{b}x+\sqrt[3]{a}\left(-\sqrt{3}+1\right)\right)^{-1}\frac{1}{\sqrt{-bx^{3}-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(b^(1/3)*x+a^(1/3)*(-3^(1/2)+1))/(-b*x^3-a)^(1/2),x)
```

```
[Out] int(x/(b^(1/3)*x+a^(1/3)*(-3^(1/2)+1))/(-b*x^3-a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-bx^{3}-a}\left(b^{\frac{1}{3}}x-a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(-b*x^3 - a)*(b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))),x, algorithm=
```

```
[Out] integrate(x/(sqrt(-b*x^3 - a)*(b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x}{\sqrt{-bx^{3}-a}b^{\frac{1}{3}}x-\sqrt{-bx^{3}-a}a^{\frac{1}{3}}\left(\sqrt{3}-1\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(sqrt(-b*x^3 - a)*(b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))),x, algorithm=
```

```
[Out] integral(x/(sqrt(-b*x^3 - a)*b^(1/3)*x - sqrt(-b*x^3 - a)*a^(1/3)
*(sqrt(3) - 1)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-a - bx^3}\left(-\sqrt{3}\sqrt[3]{a} + \sqrt[3]{a} + \sqrt[3]{b}x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(-b*x**3-a)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-a - b*x**3)*(-sqrt(3)*a**(1/3) + a**(1/3) + b**
(1/3)*x)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-bx^3 - a}\left(b^{\frac{1}{3}}x - a^{\frac{1}{3}}\left(\sqrt{3} - 1\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(sqrt(-b*x^3 - a)*(b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))),x, algorithm=
```

```
[Out] integrate(x/(sqrt(-b*x^3 - a)*(b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))
), x)
```

3.121 $\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=319

$$-\frac{(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(c-\left(1+\sqrt{3}\right)d\right)\tan^{-1}\left(\frac{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{c^2+cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{c-d}}\right)}{\sqrt{d}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}\sqrt{c-d}\sqrt{c^2+cd+d^2}}-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\Pi\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)^2}{\left(c-\left(1-\sqrt{3}\right)d\right)^2};-\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}\left(c-\left(1-\sqrt{3}\right)d\right)}$$

```
[Out] -(((c - (1 + Sqrt[3])*d)*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3]
+ x)^2]*ArcTan[(Sqrt[c^2 + c*d + d^2]*Sqrt[(1 + x)/(1 + Sqrt[3] +
 x)^2])/(Sqrt[c - d]*Sqrt[d]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)
^2])])/(Sqrt[c - d]*Sqrt[d]*Sqrt[c^2 + c*d + d^2]*Sqrt[(1 + x)/(1
 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])) - (4*3^(1/4)*Sqrt[2 + Sqrt[3]]
*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticPi[(c -
(1 + Sqrt[3])*d)^2/(c - (1 - Sqrt[3])*d)^2, -ArcSin[(1 - Sqrt[3]
+ x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/((c - (1 - Sqrt[3])*d)*
Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rubi [A] time = 2.52528, antiderivative size = 319, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$

$$-\frac{(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(c-\left(1+\sqrt{3}\right)d\right)\tan^{-1}\left(\frac{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{c^2+cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{c-d}}\right)}{\sqrt{d}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}\sqrt{c-d}\sqrt{c^2+cd+d^2}}-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\Pi\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)^2}{\left(c-\left(1-\sqrt{3}\right)d\right)^2};-\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}\left(c-\left(1-\sqrt{3}\right)d\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + Sqrt[3] + x)/((c + d*x)*Sqrt[1 + x^3]),x]
```

```
[Out] -(((c - (1 + Sqrt[3])*d)*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3]
+ x)^2]*ArcTan[(Sqrt[c^2 + c*d + d^2]*Sqrt[(1 + x)/(1 + Sqrt[3] +
 x)^2])/(Sqrt[c - d]*Sqrt[d]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)
^2])])/(Sqrt[c - d]*Sqrt[d]*Sqrt[c^2 + c*d + d^2]*Sqrt[(1 + x)/(1
 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])) - (4*3^(1/4)*Sqrt[2 + Sqrt[3]]
*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticPi[(c -
(1 + Sqrt[3])*d)^2/(c - (1 - Sqrt[3])*d)^2, -ArcSin[(1 - Sqrt[3]
+ x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/((c - (1 - Sqrt[3])*d)*
Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rubi in Sympy [A] time = 155.724, size = 326, normalized size = 1.02

$$-\frac{4\sqrt[4]{3}\sqrt{\frac{x^2-x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{-\sqrt{3}+2}\,(x+1)\,\Pi\left(\frac{\left(-c+d+\sqrt{3}d\right)^2}{\left(c-d+\sqrt{3}d\right)^2};\operatorname{asin}\left(\frac{-x-1+\sqrt{3}}{x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{-4\sqrt{3}+7}\sqrt{x^3+1}\left(c-d+\sqrt{3}d\right)}-\frac{\sqrt{\frac{x^2-x+1}{\left(x+1+\sqrt{3}\right)^2}}\left(c-d\left(1+\sqrt{3}\right)\right)(x+1)\operatorname{atan}\left(\frac{3^{\frac{3}{4}}\sqrt{-\sqrt{3}+2}\sqrt{-\frac{\left(-x-1+\sqrt{3}\right)^2}{\left(x+1+\sqrt{3}\right)^2}+1}\sqrt{c^2+cd+d^2}}{3\sqrt{d}\sqrt{c-d}\sqrt{\frac{\left(-x-1+\sqrt{3}\right)^2}{\left(x+1+\sqrt{3}\right)^2}-4\sqrt{3}+7}}\right)}{\sqrt{d}\sqrt{\frac{x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{c-d}\sqrt{x^3+1}\sqrt{c^2+cd+d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+x+3**(1/2))/(d*x+c)/(x**3+1)**(1/2),x)
```

```
[Out] -4*3**(1/4)*sqrt((x**2 - x + 1)/(x + 1 + sqrt(3))**2)*sqrt(-sqrt(
3) + 2)*(x + 1)*elliptic_pi((-c + d + sqrt(3)*d)**2/(c - d + sqrt
(3)*d)**2, asin((-x - 1 + sqrt(3))/(x + 1 + sqrt(3))), -7 - 4*sqr
t(3))/(sqrt((x + 1)/(x + 1 + sqrt(3))**2)*sqrt(-4*sqrt(3) + 7)*sq
rt(x**3 + 1)*(c - d + sqrt(3)*d)) - sqrt((x**2 - x + 1)/(x + 1 +
sqrt(3))**2)*(c - d*(1 + sqrt(3)))*(x + 1)*atan(3**(3/4)*sqrt(-sq
rt(3) + 2)*sqrt(-(-x - 1 + sqrt(3))**2/(x + 1 + sqrt(3))**2 + 1)*
sqrt(c**2 + c*d + d**2)/(3*sqrt(d)*sqrt(c - d)*sqrt((-x - 1 + sqr
t(3))**2/(x + 1 + sqrt(3))**2 - 4*sqrt(3) + 7)))/(sqrt(d)*sqrt((x
 + 1)/(x + 1 + sqrt(3))**2)*sqrt(c - d)*sqrt(x**3 + 1)*sqrt(c**2
+ c*d + d**2))
```

Mathematica [C] time = 1.01913, size = 214, normalized size = 0.67

$$\frac{2\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\left(-\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}}+\frac{i\sqrt{x^2-x+1}\left(c-\left(1+\sqrt{3}\right)d\right)\left(\frac{i\sqrt{3}d}{c+\sqrt[3]{-1}d};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{c+\sqrt[3]{-1}d}\right)}{d\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(1 + Sqrt[3] + x)/((c + d*x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) - x)*Sqrt[((-1)
^(1/3) - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1
 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^
(2/3)*x)/(1 + (-1)^(1/3))]) + (I*(c - (1 + Sqrt[3])*d)*Sqrt[1 - x
 + x^2]*EllipticPi[(I*Sqrt[3]*d)/(c + (-1)^(1/3)*d), ArcSin[Sqrt[
(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(c + (-1)^(1/
3)*d)))/(d*Sqrt[1 + x^3])
```

Maple [A] time = 0.056, size = 275, normalized size = 0.9

$$2\,\frac{3/2-i/2\sqrt{3}}{d\sqrt{x^3+1}}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}\,EllipticF\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},\sqrt{\frac{-3/2+i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\right)$$
$$+2\,\frac{\left(d\sqrt{3}-c+d\right)\left(3/2-i/2\sqrt{3}\right)}{d^2\sqrt{x^3+1}}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}\,EllipticPi\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},(-3/2+i\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+x+3^(1/2))/(d*x+c)/(x^3+1)^(1/2),x)
```

```
[Out] 2/d*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2
-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2)
)/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2
-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2))
)^(1/2))+2*(d*3^(1/2)-c+d)/d^2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/
2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^
(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^
(1/2)/(-1+c/d)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2
+1/2*I*3^(1/2))/(-1+c/d),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2
)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+\sqrt{3}+1}{\sqrt{x^3+1}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x + sqrt(3) + 1)/(sqrt(x^3 + 1)*(d*x + c)),x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(3) + 1)/(sqrt(x^3 + 1)*(d*x + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x + sqrt(3) + 1)/(sqrt(x^3 + 1)*(d*x + c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1+\sqrt{3}}{\sqrt{(x+1)(x^2-x+1)}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3**(1/2))/(d*x+c)/(x**3+1)**(1/2),x)
```

```
[Out] Integral((x + 1 + sqrt(3))/(sqrt((x + 1)*(x**2 - x + 1))*(c + d*x
)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+\sqrt{3}+1}{\sqrt{x^3+1}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sqrt(3) + 1)/(sqrt(x^3 + 1)*(d*x + c)),x, algorithm="giac")

[Out] integrate((x + sqrt(3) + 1)/(sqrt(x^3 + 1)*(d*x + c)), x)
```

3.122 $\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=331

$$\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,\Pi\left(\frac{\left(c+\sqrt{3}d+d\right)^2}{\left(c-\sqrt{3}d+d\right)^2};-\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\left(c-\sqrt{3}d+d\right)} - \frac{(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(c+\sqrt{3}d+d\right)\tanh^{-1}\left(\frac{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{c^2-cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{c+d}}\right)}{\sqrt{d}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\sqrt{c+d}\sqrt{c^2-cd+d^2}}$$

```
[Out] -(((c + d + Sqrt[3]*d)*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] -
x)^2]*ArcTanh[(Sqrt[c^2 - c*d + d^2]*Sqrt[(1 - x)/(1 + Sqrt[3] -
x)^2])/(Sqrt[d]*Sqrt[c + d]*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^
2])])/(Sqrt[d]*Sqrt[c + d]*Sqrt[c^2 - c*d + d^2]*Sqrt[(1 - x)/(1
+ Sqrt[3] - x)^2]*Sqrt[1 - x^3])) + (4*3^(1/4)*Sqrt[2 + Sqrt[3]]*
(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticPi[(c + d
 + Sqrt[3]*d)^2/(c + d - Sqrt[3]*d)^2, -ArcSin[(1 - Sqrt[3] - x)/
(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/((c + d - Sqrt[3]*d)*Sqrt[(1
 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rubi [A] time = 2.45629, antiderivative size = 331, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$

$$\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,\Pi\left(\frac{\left(c+\sqrt{3}d+d\right)^2}{\left(c-\sqrt{3}d+d\right)^2};-\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\left(c-\sqrt{3}d+d\right)} - \frac{(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(c+\sqrt{3}d+d\right)\tanh^{-1}\left(\frac{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{c^2-cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{c+d}}\right)}{\sqrt{d}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\sqrt{c+d}\sqrt{c^2-cd+d^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + Sqrt[3] - x)/((c + d*x)*Sqrt[1 - x^3]),x]
```

```
[Out] -(((c + d + Sqrt[3]*d)*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] -
x)^2]*ArcTanh[(Sqrt[c^2 - c*d + d^2]*Sqrt[(1 - x)/(1 + Sqrt[3] -
x)^2])/(Sqrt[d]*Sqrt[c + d]*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^
2])])/(Sqrt[d]*Sqrt[c + d]*Sqrt[c^2 - c*d + d^2]*Sqrt[(1 - x)/(1
+ Sqrt[3] - x)^2]*Sqrt[1 - x^3])) + (4*3^(1/4)*Sqrt[2 + Sqrt[3]]*
(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticPi[(c + d
 + Sqrt[3]*d)^2/(c + d - Sqrt[3]*d)^2, -ArcSin[(1 - Sqrt[3] - x)/
(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/((c + d - Sqrt[3]*d)*Sqrt[(1
 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rubi in Sympy [A] time = 165.609, size = 325, normalized size = 0.98

$$\frac{4\sqrt[4]{3}\sqrt{\frac{x^2+x+1}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{-\sqrt{3}+2}\left(-x+1\right)\left(\frac{\left(c+d+\sqrt{3}d\right)^2}{\left(c-\sqrt{3}d+d\right)^2};\operatorname{asin}\left(\frac{x-1+\sqrt{3}}{-x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt{\frac{-x+1}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{-4\sqrt{3}+7}\sqrt{-x^3+1}\left(c-\sqrt{3}d+d\right)} - \frac{\sqrt{\frac{x^2+x+1}{\left(-x+1+\sqrt{3}\right)^2}}\left(-x+1\right)\left(c+d+\sqrt{3}d\right)\operatorname{atanh}\left(\frac{3^{\frac{3}{4}}\sqrt{1-\frac{\left(x-1+\sqrt{3}\right)^2}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{-\sqrt{3}+2}\sqrt{c^2-cd+d^2}}{3\sqrt{d}\sqrt{c+d}\sqrt{-4\sqrt{3}+7+\frac{\left(x-1+\sqrt{3}\right)^2}{\left(-x+1+\sqrt{3}\right)^2}}}\right)}{\sqrt{d}\sqrt{\frac{-x+1}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{c+d}\sqrt{-x^3+1}\sqrt{c^2-cd+d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1-x+3**(1/2))/(d*x+c)/(-x**3+1)**(1/2),x)
```

```
[Out] 4*3**(1/4)*sqrt((x**2 + x + 1)/(-x + 1 + sqrt(3))**2)*sqrt(-sqrt(
3) + 2)*(-x + 1)*elliptic_pi((c + d + sqrt(3)*d)**2/(c - sqrt(3)*
d + d)**2, asin((x - 1 + sqrt(3))/(-x + 1 + sqrt(3))), -7 - 4*sqr
t(3))/(sqrt((-x + 1)/(-x + 1 + sqrt(3))**2)*sqrt(-4*sqrt(3) + 7)*
sqrt(-x**3 + 1)*(c - sqrt(3)*d + d)) - sqrt((x**2 + x + 1)/(-x +
1 + sqrt(3))**2)*(-x + 1)*(c + d + sqrt(3)*d)*atanh(3**(3/4)*sqrt
(1 - (x - 1 + sqrt(3))**2/(-x + 1 + sqrt(3))**2)*sqrt(-sqrt(3) +
2)*sqrt(c**2 - c*d + d**2)/(3*sqrt(d)*sqrt(c + d)*sqrt(-4*sqrt(3)
 + 7 + (x - 1 + sqrt(3))**2/(-x + 1 + sqrt(3))**2)))/(sqrt(d)*sqr
t((-x + 1)/(-x + 1 + sqrt(3))**2)*sqrt(c + d)*sqrt(-x**3 + 1)*sqr
t(c**2 - c*d + d**2))
```

Mathematica [C] time = 1.2329, size = 235, normalized size = 0.71

$$\frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(-\frac{3\left(x+\sqrt[3]{-1}\right)\sqrt{\frac{(-1)^{2/3}x+\sqrt[3]{-1}}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\sqrt{x^2+x+1}\left(\sqrt{3}c+\left(3+\sqrt{3}\right)d\right)\left(\frac{i\sqrt{3}d}{\sqrt[3]{-1}d-c};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)\right.}{c-\sqrt[3]{-1}d}\right.}{3d\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(1 + Sqrt[3] - x)/((c + d*x)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((-3*((-1)^(1/3) + x)*Sqrt[((-1
)^(1/3) + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(
1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)
^(2/3)*x)/(1 + (-1)^(1/3))] + ((-1)^(1/3)*(1 + (-1)^(1/3))*(Sqrt[
3]*c + (3 + Sqrt[3])*d)*Sqrt[1 + x + x^2]*EllipticPi[(I*Sqrt[3]*d
)/(-c + (-1)^(1/3)*d), ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(
1/3))]], (-1)^(1/3)])/(c - (-1)^(1/3)*d)))/(3*d*Sqrt[1 - x^3])
```

Maple [A] time = 0.064, size = 264, normalized size = 0.8

$$\frac{-\frac{2i}{3}\left(c+d+d\sqrt{3}\right)\sqrt{3}}{d^2}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\mathit{EllipticPi}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},i\sqrt{\vphantom{1}}\right.$$

$$+\frac{\frac{2i}{3}\sqrt{3}}{d}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\mathit{EllipticF}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)\frac{}{\sqrt{-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1-x+3^(1/2))/(d*x+c)/(-x^3+1)^(1/2),x)
```

```
[Out] -2/3*I*(c+d+d*3^(1/2))/d^2*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/
2))^(1/2)*((-1+x)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^
(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)+c/d)*Ell
ipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(
1/2)/(-1/2+1/2*I*3^(1/2)+c/d),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1
/2))+2/3*I/d*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1
+x)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2)
)^(1/2)/(-x^3+1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1
/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x-\sqrt{3}-1}{\sqrt{-x^3+1}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x - sqrt(3) - 1)/(sqrt(-x^3 + 1)*(d*x + c)),x, algorithm="maxima")
```

```
[Out] -integrate((x - sqrt(3) - 1)/(sqrt(-x^3 + 1)*(d*x + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x - sqrt(3) - 1)/(sqrt(-x^3 + 1)*(d*x + c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \left(-\frac{\sqrt{3}}{c\sqrt{-x^3+1}+dx\sqrt{-x^3+1}}\right)dx-\int \frac{x}{c\sqrt{-x^3+1}+dx\sqrt{-x^3+1}}\,dx-\int \left(-\frac{1}{c\sqrt{-x^3+1}+dx\sqrt{-x^3+1}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1-x+3**(1/2))/(d*x+c)/(-x**3+1)**(1/2),x)
```

```
[Out] -Integral(-sqrt(3)/(c*sqrt(-x**3 + 1) + d*x*sqrt(-x**3 + 1)), x)
- Integral(x/(c*sqrt(-x**3 + 1) + d*x*sqrt(-x**3 + 1)), x) - Inte
gral(-1/(c*sqrt(-x**3 + 1) + d*x*sqrt(-x**3 + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x-\sqrt{3}-1}{\sqrt{-x^3+1}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x - sqrt(3) - 1)/(sqrt(-x^3 + 1)*(d*x + c)),x, algorithm="giac")
```

```
[Out] integrate(-(x - sqrt(3) - 1)/(sqrt(-x^3 + 1)*(d*x + c)), x)
```

3.123 $\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=327

$$\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,\Pi\left(\frac{\left(c+\sqrt{3}d+d\right)^2}{\left(c-\sqrt{3}d+d\right)^2};-\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\left(c-\sqrt{3}d+d\right)} - \frac{(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(c+\sqrt{3}d+d\right)\tanh^{-1}\left(\frac{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{c^2-cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{c+d}}\right)}{\sqrt{d}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\sqrt{c+d}\sqrt{c^2-cd+d^2}}$$

```
[Out] -(((c + d + Sqrt[3]*d)*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] -
x)^2]*ArcTanh[(Sqrt[c^2 - c*d + d^2]*Sqrt[(1 - x)/(1 + Sqrt[3] -
x)^2])/(Sqrt[d]*Sqrt[c + d]*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^
2])])/(Sqrt[d]*Sqrt[c + d]*Sqrt[c^2 - c*d + d^2]*Sqrt[(1 - x)/(1
+ Sqrt[3] - x)^2]*Sqrt[-1 + x^3])) + (4*3^(1/4)*Sqrt[2 + Sqrt[3]]
*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticPi[(c +
d + Sqrt[3]*d)^2/(c + d - Sqrt[3]*d)^2, -ArcSin[(1 - Sqrt[3] - x)
/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/((c + d - Sqrt[3]*d)*Sqrt[(
1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[-1 + x^3])
```

Rubi [A] time = 1.85727, antiderivative size = 327, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,\Pi\left(\frac{\left(c+\sqrt{3}d+d\right)^2}{\left(c-\sqrt{3}d+d\right)^2};-\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\left(c-\sqrt{3}d+d\right)} - \frac{(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(c+\sqrt{3}d+d\right)\tanh^{-1}\left(\frac{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{c^2-cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{c+d}}\right)}{\sqrt{d}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\sqrt{c+d}\sqrt{c^2-cd+d^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + Sqrt[3] - x)/((c + d*x)*Sqrt[-1 + x^3]),x]
```

```
[Out] -(((c + d + Sqrt[3]*d)*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] -
x)^2]*ArcTanh[(Sqrt[c^2 - c*d + d^2]*Sqrt[(1 - x)/(1 + Sqrt[3] -
x)^2])/(Sqrt[d]*Sqrt[c + d]*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^
2])])/(Sqrt[d]*Sqrt[c + d]*Sqrt[c^2 - c*d + d^2]*Sqrt[(1 - x)/(1
+ Sqrt[3] - x)^2]*Sqrt[-1 + x^3])) + (4*3^(1/4)*Sqrt[2 + Sqrt[3]]
*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticPi[(c +
d + Sqrt[3]*d)^2/(c + d - Sqrt[3]*d)^2, -ArcSin[(1 - Sqrt[3] - x)
/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/((c + d - Sqrt[3]*d)*Sqrt[(
1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[-1 + x^3])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1-x+3**(1/2))/(d*x+c)/(x**3-1)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 1.20336, size = 233, normalized size = 0.71

$$\frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(-\frac{3\left(x+\sqrt[3]{-1}\right)\sqrt{\frac{(-1)^{2/3}x+\sqrt[3]{-1}}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\sqrt{x^2+x+1}\left(\sqrt{3}c+\left(3+\sqrt{3}\right)d\right)\left(\frac{i\sqrt{3}d}{\sqrt[3]{-1}d-c};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right.\right.}{c-\sqrt[3]{-1}d}\right.}{3d\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(1 + Sqrt[3] - x)/((c + d*x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((-3*((-1)^(1/3) + x)*Sqrt[((-1
)^(1/3) + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(
1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)
^(2/3)*x)/(1 + (-1)^(1/3))] + ((-1)^(1/3)*(1 + (-1)^(1/3))*(Sqrt[
3]*c + (3 + Sqrt[3])*d)*Sqrt[1 + x + x^2]*EllipticPi[(I*Sqrt[3]*d
)/(-c + (-1)^(1/3)*d), ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(
1/3))]], (-1)^(1/3)])/(c - (-1)^(1/3)*d)))/(3*d*Sqrt[-1 + x^3])
```

Maple [A] time = 0.055, size = 273, normalized size = 0.8

$$2\frac{\left(c+d+d\sqrt{3}\right)\left(-3/2-i/2\sqrt{3}\right)}{d^2\sqrt{x^3-1}}\sqrt{\frac{-1+x}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2-i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2+i/2\sqrt{3}}{3/2+i/2\sqrt{3}}}\mathit{EllipticPi}\left(\sqrt{\frac{-1+x}{-3/2-i/2\sqrt{3}}},(3/2+\right.$$

$$-2\frac{-3/2-i/2\sqrt{3}}{d\sqrt{x^3-1}}\sqrt{\frac{-1+x}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2-i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2+i/2\sqrt{3}}{3/2+i/2\sqrt{3}}}\mathit{EllipticF}\left(\sqrt{\frac{-1+x}{-3/2-i/2\sqrt{3}}},\sqrt{\frac{3/2+i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1-x+3^(1/2))/(d*x+c)/(x^3-1)^(1/2),x)
```

```
[Out] 2*(c+d+d*3^(1/2))/d^2*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^
(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*(
(x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)/(1
+c/d)*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3
^(1/2))/(1+c/d),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-
2/d*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+
1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/
2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((-1+x)/(-
3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)
))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x-\sqrt{3}-1}{\sqrt{x^3-1}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x - sqrt(3) - 1)/(sqrt(x^3 - 1)*(d*x + c)),x, algorithm="maxima")
```

```
[Out] -integrate((x - sqrt(3) - 1)/(sqrt(x^3 - 1)*(d*x + c)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x - sqrt(3) - 1)/(sqrt(x^3 - 1)*(d*x + c)),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\left(-\frac{\sqrt{3}}{c\sqrt{x^3-1}+dx\sqrt{x^3-1}}\right)dx-\int\frac{x}{c\sqrt{x^3-1}+dx\sqrt{x^3-1}}\,dx-\int\left(-\frac{1}{c\sqrt{x^3-1}+dx\sqrt{x^3-1}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1-x+3**(1/2))/(d*x+c)/(x**3-1)**(1/2),x)
```

```
[Out] -Integral(-sqrt(3)/(c*sqrt(x**3 - 1) + d*x*sqrt(x**3 - 1)), x) -
Integral(x/(c*sqrt(x**3 - 1) + d*x*sqrt(x**3 - 1)), x) - Integral
(-1/(c*sqrt(x**3 - 1) + d*x*sqrt(x**3 - 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int-\frac{x-\sqrt{3}-1}{\sqrt{x^3-1}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x - sqrt(3) - 1)/(sqrt(x^3 - 1)*(d*x + c)),x, algorithm="giac")
```

```
[Out] integrate(-(x - sqrt(3) - 1)/(sqrt(x^3 - 1)*(d*x + c)), x)
```

3.124 $\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=323

$$-\frac{(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(c-\left(1+\sqrt{3}\right)d\right)\tan^{-1}\left(\frac{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{c^2+cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{c-d}}\right)}{\sqrt{d}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\sqrt{c-d}\sqrt{c^2+cd+d^2}}-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\Pi\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)^2}{\left(c-\left(1-\sqrt{3}\right)d\right)^2};-\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\left(c-\left(1-\sqrt{3}\right)d\right)}$$

```
[Out] -(((c - (1 + Sqrt[3])*d)*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3]
+ x)^2]*ArcTan[(Sqrt[c^2 + c*d + d^2]*Sqrt[(1 + x)/(1 + Sqrt[3] +
 x)^2])/(Sqrt[c - d]*Sqrt[d]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)
^2])])/(Sqrt[c - d]*Sqrt[d]*Sqrt[c^2 + c*d + d^2]*Sqrt[(1 + x)/(1
 + Sqrt[3] + x)^2]*Sqrt[-1 - x^3])) - (4*3^(1/4)*Sqrt[2 + Sqrt[3]
]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticPi[(c -
 (1 + Sqrt[3])*d)^2/(c - (1 - Sqrt[3])*d)^2, -ArcSin[(1 - Sqrt[3]
 + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/((c - (1 - Sqrt[3])*d)
*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[-1 - x^3])
```

Rubi [A] time = 2.17888, antiderivative size = 323, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$-\frac{(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(c-\left(1+\sqrt{3}\right)d\right)\tan^{-1}\left(\frac{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{c^2+cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{c-d}}\right)}{\sqrt{d}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\sqrt{c-d}\sqrt{c^2+cd+d^2}}-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\Pi\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)^2}{\left(c-\left(1-\sqrt{3}\right)d\right)^2};-\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\left(c-\left(1-\sqrt{3}\right)d\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + Sqrt[3] + x)/((c + d*x)*Sqrt[-1 - x^3]),x]
```

```
[Out] -(((c - (1 + Sqrt[3])*d)*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3]
+ x)^2]*ArcTan[(Sqrt[c^2 + c*d + d^2]*Sqrt[(1 + x)/(1 + Sqrt[3] +
 x)^2])/(Sqrt[c - d]*Sqrt[d]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)
^2])])/(Sqrt[c - d]*Sqrt[d]*Sqrt[c^2 + c*d + d^2]*Sqrt[(1 + x)/(1
 + Sqrt[3] + x)^2]*Sqrt[-1 - x^3])) - (4*3^(1/4)*Sqrt[2 + Sqrt[3]
]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticPi[(c -
 (1 + Sqrt[3])*d)^2/(c - (1 - Sqrt[3])*d)^2, -ArcSin[(1 - Sqrt[3]
 + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/((c - (1 - Sqrt[3])*d)
*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[-1 - x^3])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+x+3**(1/2))/(d*x+c)/(-x**3-1)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 1.02937, size = 216, normalized size = 0.67

$$\frac{2\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\left(-\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}}+\frac{i\sqrt{x^2-x+1}\left(c-\left(1+\sqrt{3}\right)d\right)\Pi\left(\frac{i\sqrt{3}d}{c+\sqrt[3]{-1}d};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{c+\sqrt[3]{-1}d}\right)}{d\sqrt{-x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(1 + Sqrt[3] + x)/((c + d*x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) - x)*Sqrt[((-1)
^(1/3) - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1
 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^
(2/3)*x)/(1 + (-1)^(1/3))]) + (I*(c - (1 + Sqrt[3])*d)*Sqrt[1 - x
 + x^2]*EllipticPi[(I*Sqrt[3]*d)/(c + (-1)^(1/3)*d), ArcSin[Sqrt[
(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(c + (-1)^(1/
3)*d)))/(d*Sqrt[-1 - x^3])
```

Maple [A] time = 0.06, size = 266, normalized size = 0.8

$$\frac{-\frac{2i}{3}\sqrt{3}}{d}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\,EllipticF\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)\frac{1}{\sqrt{-x^3}}$$
$$-\frac{\frac{2i}{3}\left(d\sqrt{3}-c+d\right)\sqrt{3}}{d^2}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\,EllipticPi\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},i\sqrt{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+x+3^(1/2))/(d*x+c)/(-x^3-1)^(1/2),x)
```

```
[Out] -2/3*I/d*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(
3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2
)/(-x^3-1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3
^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))-2/3*I*(d*3^(
1/2)-c+d)/d^2*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1
+x)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))
^(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2)+c/d)*EllipticPi(1/3*3^(1
/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*
3^(1/2)+c/d),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+\sqrt{3}+1}{\sqrt{-x^3-1}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sqrt(3) + 1)/(sqrt(-x^3 - 1)*(d*x + c)),x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(3) + 1)/(sqrt(-x^3 - 1)*(d*x + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sqrt(3) + 1)/(sqrt(-x^3 - 1)*(d*x + c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1+\sqrt{3}}{\sqrt{-(x+1)(x^2-x+1)}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+x+3**(1/2))/(d*x+c)/(-x**3-1)**(1/2),x)
```

```
[Out] Integral((x + 1 + sqrt(3))/(sqrt(-(x + 1)*(x**2 - x + 1))*(c + d*
x)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+\sqrt{3}+1}{\sqrt{-x^3-1}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sqrt(3) + 1)/(sqrt(-x^3 - 1)*(d*x + c)),x, algorithm="giac")
```

```
[Out] integrate((x + sqrt(3) + 1)/(sqrt(-x^3 - 1)*(d*x + c)), x)
```

3.125 $\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=360

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(\frac{\left(c-\left(1-\sqrt{3}\right)d\right)^2}{\left(c-\left(1+\sqrt{3}\right)d\right)^2};-\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{x^3+1}\left(c-\sqrt{3}d-d\right)}-\frac{(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(c-\left(1-\sqrt{3}\right)d\right)\tanh^{-1}\left(\frac{2\sqrt{2+\sqrt{3}}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{c^2+cd+d^2}}{\sqrt{d}\sqrt{\frac{\left(x+\sqrt{3}+1\right)^2}{\left(x-\sqrt{3}+1\right)^2}+4\sqrt{3}+7}\sqrt{c-d}}\right)}{\sqrt{d}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{x^3+1}\sqrt{c-d}\sqrt{c^2+cd+d^2}}$$

```
[Out] -(((c - (1 - Sqrt[3])*d)*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3]
+ x)^2]*ArcTanh[(2*Sqrt[2 + Sqrt[3]]*Sqrt[c^2 + c*d + d^2]*Sqrt[-
((1 + x)/(1 - Sqrt[3] + x)^2)])/(Sqrt[c - d]*Sqrt[d]*Sqrt[7 + 4*S
qrt[3] + (1 + Sqrt[3] + x)^2/(1 - Sqrt[3] + x)^2])])/(Sqrt[c - d]
*Sqrt[d]*Sqrt[c^2 + c*d + d^2]*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2
)]*Sqrt[1 + x^3])) + (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1
 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticPi[(c - (1 - Sqrt[3])*d)
^2/(c - (1 + Sqrt[3])*d)^2, -ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3
] + x)], -7 + 4*Sqrt[3]])/((c - d - Sqrt[3]*d)*Sqrt[-((1 + x)/(1
- Sqrt[3] + x)^2)]*Sqrt[1 + x^3])
```

Rubi [A] time = 2.19756, antiderivative size = 360, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(\frac{\left(c-\left(1-\sqrt{3}\right)d\right)^2}{\left(c-\left(1+\sqrt{3}\right)d\right)^2};-\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{x^3+1}\left(c-\sqrt{3}d-d\right)}-\frac{(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(c-\left(1-\sqrt{3}\right)d\right)\tanh^{-1}\left(\frac{2\sqrt{2+\sqrt{3}}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{c^2+cd+d^2}}{\sqrt{d}\sqrt{\frac{\left(x+\sqrt{3}+1\right)^2}{\left(x-\sqrt{3}+1\right)^2}+4\sqrt{3}+7}\sqrt{c-d}}\right)}{\sqrt{d}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{x^3+1}\sqrt{c-d}\sqrt{c^2+cd+d^2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[(1 - Sqrt[3] + x)/((c + d*x)*Sqrt[1 + x^3]),x]
```

```
[Out] -(((c - (1 - Sqrt[3])*d)*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3]
+ x)^2]*ArcTanh[(2*Sqrt[2 + Sqrt[3]]*Sqrt[c^2 + c*d + d^2]*Sqrt[-
((1 + x)/(1 - Sqrt[3] + x)^2)])/(Sqrt[c - d]*Sqrt[d]*Sqrt[7 + 4*S
qrt[3] + (1 + Sqrt[3] + x)^2/(1 - Sqrt[3] + x)^2])])/(Sqrt[c - d]
*Sqrt[d]*Sqrt[c^2 + c*d + d^2]*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2
)]*Sqrt[1 + x^3])) + (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1
 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticPi[(c - (1 - Sqrt[3])*d)
^2/(c - (1 + Sqrt[3])*d)^2, -ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3
] + x)], -7 + 4*Sqrt[3]])/((c - d - Sqrt[3]*d)*Sqrt[-((1 + x)/(1
- Sqrt[3] + x)^2)]*Sqrt[1 + x^3])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+x-3**(1/2))/(d*x+c)/(x**3+1)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.98588, size = 213, normalized size = 0.59

$$\frac{2\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\left(-\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}}+\frac{i\sqrt{x^2-x+1}\left(c+\left(\sqrt{3}-1\right)d\right)\Pi\left(\frac{i\sqrt{3}d}{c+\sqrt[3]{-1}d};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{c+\sqrt[3]{-1}d}\right)}{d\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(1 - Sqrt[3] + x)/((c + d*x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) - x)*Sqrt[((-1)
^(1/3) - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1
 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^
(2/3)*x)/(1 + (-1)^(1/3))]) + (I*(c + (-1 + Sqrt[3])*d)*Sqrt[1 -
x + x^2]*EllipticPi[(I*Sqrt[3]*d)/(c + (-1)^(1/3)*d), ArcSin[Sqrt
[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(c + (-1)^(1
/3)*d)))/(d*Sqrt[1 + x^3])
```

Maple [A] time = 0.031, size = 275, normalized size = 0.8

$$-2\,\frac{\left(d\sqrt{3}+c-d\right)\left(3/2-i/2\sqrt{3}\right)}{d^{2}\sqrt{x^{3}+1}}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}\,EllipticPi\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},(-3/2+i\right.$$
$$+2\,\frac{3/2-i/2\sqrt{3}}{d\sqrt{x^{3}+1}}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}\,EllipticF\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},\sqrt{\frac{-3/2+i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+x-3^(1/2))/(d*x+c)/(x^3+1)^(1/2),x)
```

```
[Out] -2*(d*3^(1/2)+c-d)/d^2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1
/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((
x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)/(-
1+c/d)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3
^(1/2))/(-1+c/d),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2
))+2/d*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-
1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1
/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(
3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/
2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-\sqrt{3}+1}{\sqrt{x^{3}+1}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - sqrt(3) + 1)/(sqrt(x^3 + 1)*(d*x + c)),x, algorithm="maxima")
```

```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(x^3 + 1)*(d*x + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x - sqrt(3) + 1)/(sqrt(x^3 + 1)*(d*x + c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-\sqrt{3}+1}{\sqrt{(x+1)\,(x^2-x+1)}\,(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3**(1/2))/(d*x+c)/(x**3+1)**(1/2),x)
```

```
[Out] Integral((x - sqrt(3) + 1)/(sqrt((x + 1)*(x**2 - x + 1))*(c + d*x
)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x - sqrt(3) + 1)/(sqrt(x^3 + 1)*(d*x + c)),x, algorithm="giac")
```

```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(x^3 + 1)*(d*x + c)), x)
```

3.126 $\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=348

$$-\frac{(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(c-\sqrt{3}d+d\right)\tan^{-1}\left(\frac{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{c^2-cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{c+d}}\right)}{\sqrt{d}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\sqrt{c+d}\sqrt{c^2-cd+d^2}}-\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,\Pi\left(\frac{\left(c-\sqrt{3}d+d\right)^2}{\left(c+\sqrt{3}d+d\right)^2};-\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\left(c+\sqrt{3}d+d\right)}$$

```
[Out] -(((c + d - Sqrt[3]*d)*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] -
x)^2]*ArcTan[(Sqrt[c^2 - c*d + d^2]*Sqrt[-((1 - x)/(1 - Sqrt[3] -
 x)^2)])/(Sqrt[d]*Sqrt[c + d]*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x
)^2])])/(Sqrt[d]*Sqrt[c + d]*Sqrt[c^2 - c*d + d^2]*Sqrt[-((1 - x)
/(1 - Sqrt[3] - x)^2)]*Sqrt[1 - x^3])) - (4*3^(1/4)*Sqrt[2 - Sqrt
[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticPi[(
c + d - Sqrt[3]*d)^2/(c + d + Sqrt[3]*d)^2, -ArcSin[(1 + Sqrt[3]
- x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/((c + d + Sqrt[3]*d)*Sq
rt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[1 - x^3])
```

Rubi [A] time = 2.11093, antiderivative size = 348, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$

$$-\frac{(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(c-\sqrt{3}d+d\right)\tan^{-1}\left(\frac{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{c^2-cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{c+d}}\right)}{\sqrt{d}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\sqrt{c+d}\sqrt{c^2-cd+d^2}}-\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,\Pi\left(\frac{\left(c-\sqrt{3}d+d\right)^2}{\left(c+\sqrt{3}d+d\right)^2};-\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\left(c+\sqrt{3}d+d\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - Sqrt[3] - x)/((c + d*x)*Sqrt[1 - x^3]),x]
```

```
[Out] -(((c + d - Sqrt[3]*d)*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] -
x)^2]*ArcTan[(Sqrt[c^2 - c*d + d^2]*Sqrt[-((1 - x)/(1 - Sqrt[3] -
 x)^2)])/(Sqrt[d]*Sqrt[c + d]*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x
)^2])])/(Sqrt[d]*Sqrt[c + d]*Sqrt[c^2 - c*d + d^2]*Sqrt[-((1 - x)
/(1 - Sqrt[3] - x)^2)]*Sqrt[1 - x^3])) - (4*3^(1/4)*Sqrt[2 - Sqrt
[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticPi[(
c + d - Sqrt[3]*d)^2/(c + d + Sqrt[3]*d)^2, -ArcSin[(1 + Sqrt[3]
- x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/((c + d + Sqrt[3]*d)*Sq
rt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[1 - x^3])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1-x-3**(1/2))/(d*x+c)/(-x**3+1)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 1.25461, size = 235, normalized size = 0.68

$$\frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(-\frac{3\left(x+\sqrt[3]{-1}\right)\sqrt{\frac{(-1)^{2/3}x+\sqrt[3]{-1}}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\sqrt{x^2+x+1}\left(\sqrt{3}c+\left(\sqrt{3}-3\right)d\right)\left(\frac{i\sqrt{3}d}{\sqrt[3]{-1}d-c};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)\right.}{c-\sqrt[3]{-1}d}\right.}{3d\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(1 - Sqrt[3] - x)/((c + d*x)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((-3*((-1)^(1/3) + x)*Sqrt[((-1
)^(1/3) + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(
1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)
^(2/3)*x)/(1 + (-1)^(1/3))] + ((-1)^(1/3)*(1 + (-1)^(1/3))*(Sqrt[
3]*c + (-3 + Sqrt[3])*d)*Sqrt[1 + x + x^2]*EllipticPi[(I*Sqrt[3]*
d)/(-c + (-1)^(1/3)*d), ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^
(1/3))]], (-1)^(1/3)])/(c - (-1)^(1/3)*d)))/(3*d*Sqrt[1 - x^3])
```

Maple [A] time = 0.031, size = 268, normalized size = 0.8

$$\frac{\frac{2i}{3}\sqrt{3}}{d}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}EllipticF\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)\frac{1}{\sqrt{-x^3}}$$
$$+\frac{\frac{2i}{3}\left(d\sqrt{3}-c-d\right)\sqrt{3}}{d^2}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}EllipticPi\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},i\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1-x-3^(1/2))/(d*x+c)/(-x^3+1)^(1/2),x)
```

```
[Out] 2/3*I/d*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/(
-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/
2)/(-x^3+1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*
3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))+2/3*I*(d*3
^(1/2)-c-d)/d^2*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(
(-1+x)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1
/2))^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)+c/d)*EllipticPi(1/3
*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+
1/2*I*3^(1/2)+c/d),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x+\sqrt{3}-1}{\sqrt{-x^3+1}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x + sqrt(3) - 1)/(sqrt(-x^3 + 1)*(d*x + c)),x, algorithm="maxima")
```

```
[Out] -integrate((x + sqrt(3) - 1)/(sqrt(-x^3 + 1)*(d*x + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x + sqrt(3) - 1)/(sqrt(-x^3 + 1)*(d*x + c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{3}}{c\sqrt{-x^3+1}+dx\sqrt{-x^3+1}}\,dx-\int \frac{x}{c\sqrt{-x^3+1}+dx\sqrt{-x^3+1}}\,dx-\int \left(-\frac{1}{c\sqrt{-x^3+1}+dx\sqrt{-x^3+1}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1-x-3**(1/2))/(d*x+c)/(-x**3+1)**(1/2),x)
```

```
[Out] -Integral(sqrt(3)/(c*sqrt(-x**3 + 1) + d*x*sqrt(-x**3 + 1)), x) -
 Integral(x/(c*sqrt(-x**3 + 1) + d*x*sqrt(-x**3 + 1)), x) - Integ
ral(-1/(c*sqrt(-x**3 + 1) + d*x*sqrt(-x**3 + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x+\sqrt{3}-1}{\sqrt{-x^3+1}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x + sqrt(3) - 1)/(sqrt(-x^3 + 1)*(d*x + c)),x, algorithm="giac")
```

```
[Out] integrate(-(x + sqrt(3) - 1)/(sqrt(-x^3 + 1)*(d*x + c)), x)
```

3.127 $\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=344

$$-\frac{(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(c-\sqrt{3}d+d\right)\tan^{-1}\left(\frac{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{c^2-cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{c+d}}\right)}{\sqrt{d}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\sqrt{c+d}\sqrt{c^2-cd+d^2}}-\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\Pi\left(\frac{\left(c-\sqrt{3}d+d\right)^2}{\left(c+\sqrt{3}d+d\right)^2};-\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\left(c+\sqrt{3}d+d\right)}$$

```
[Out] -(((c + d - Sqrt[3]*d)*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] -
x)^2]*ArcTan[(Sqrt[c^2 - c*d + d^2]*Sqrt[-((1 - x)/(1 - Sqrt[3] -
 x)^2)])/(Sqrt[d]*Sqrt[c + d]*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x
)^2])])/(Sqrt[d]*Sqrt[c + d]*Sqrt[c^2 - c*d + d^2]*Sqrt[-((1 - x)
/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])) - (4*3^(1/4)*Sqrt[2 - Sqr
t[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticPi[
(c + d - Sqrt[3]*d)^2/(c + d + Sqrt[3]*d)^2, -ArcSin[(1 + Sqrt[3]
 - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/((c + d + Sqrt[3]*d)*S
qrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Rubi [A] time = 1.8248, antiderivative size = 344, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$

$$-\frac{(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(c-\sqrt{3}d+d\right)\tan^{-1}\left(\frac{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{c^2-cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{c+d}}\right)}{\sqrt{d}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\sqrt{c+d}\sqrt{c^2-cd+d^2}}-\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\Pi\left(\frac{\left(c-\sqrt{3}d+d\right)^2}{\left(c+\sqrt{3}d+d\right)^2};-\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\left(c+\sqrt{3}d+d\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - Sqrt[3] - x)/((c + d*x)*Sqrt[-1 + x^3]),x]
```

```
[Out] -(((c + d - Sqrt[3]*d)*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] -
x)^2]*ArcTan[(Sqrt[c^2 - c*d + d^2]*Sqrt[-((1 - x)/(1 - Sqrt[3] -
 x)^2)])/(Sqrt[d]*Sqrt[c + d]*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x
)^2])])/(Sqrt[d]*Sqrt[c + d]*Sqrt[c^2 - c*d + d^2]*Sqrt[-((1 - x)
/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])) - (4*3^(1/4)*Sqrt[2 - Sqr
t[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticPi[
(c + d - Sqrt[3]*d)^2/(c + d + Sqrt[3]*d)^2, -ArcSin[(1 + Sqrt[3]
 - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/((c + d + Sqrt[3]*d)*S
qrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Rubi in Sympy [A] time = 154.739, size = 325, normalized size = 0.94

$$-\frac{4\sqrt[4]{3}\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{\sqrt{3}+2}\,(-x+1)\left(\frac{\left(c-\sqrt{3}d+d\right)^2}{\left(c+d+\sqrt{3}d\right)^2};\operatorname{asin}\left(\frac{-x+1+\sqrt{3}}{x-1+\sqrt{3}}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt{\frac{x-1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{4\sqrt{3}+7}\sqrt{x^3-1}\left(c+d+\sqrt{3}d\right)} - \frac{\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,(-x+1)\left(c-\sqrt{3}d+d\right)\operatorname{atan}\left(\frac{3^{\frac{3}{4}}\sqrt{\sqrt{3}+2}\sqrt{-\frac{\left(-x+1+\sqrt{3}\right)^2}{\left(x-1+\sqrt{3}\right)^2}+1}\sqrt{c^2-cd+d^2}}{3\sqrt{d}\sqrt{c+d}\sqrt{\frac{\left(-x+1+\sqrt{3}\right)^2}{\left(x-1+\sqrt{3}\right)^2}+4\sqrt{3}+7}}\right)}{\sqrt{d}\sqrt{\frac{x-1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{c+d}\sqrt{x^3-1}\sqrt{c^2-cd+d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1-x-3**(1/2))/(d*x+c)/(x**3-1)**(1/2),x)
```

```
[Out] -4*3**(1/4)*sqrt((x**2 + x + 1)/(-x - sqrt(3) + 1)**2)*sqrt(sqrt(
3) + 2)*(-x + 1)*elliptic_pi((c - sqrt(3)*d + d)**2/(c + d + sqrt
(3)*d)**2, asin((-x + 1 + sqrt(3))/(x - 1 + sqrt(3))), -7 + 4*sqr
t(3))/(sqrt((x - 1)/(-x - sqrt(3) + 1)**2)*sqrt(4*sqrt(3) + 7)*sq
rt(x**3 - 1)*(c + d + sqrt(3)*d)) - sqrt((x**2 + x + 1)/(-x - sqr
t(3) + 1)**2)*(-x + 1)*(c - sqrt(3)*d + d)*atan(3**(3/4)*sqrt(sqr
t(3) + 2)*sqrt(-(-x + 1 + sqrt(3))**2/(x - 1 + sqrt(3))**2 + 1)*s
qrt(c**2 - c*d + d**2)/(3*sqrt(d)*sqrt(c + d)*sqrt((-x + 1 + sqrt
(3))**2/(x - 1 + sqrt(3))**2 + 4*sqrt(3) + 7)))/(sqrt(d)*sqrt((x
- 1)/(-x - sqrt(3) + 1)**2)*sqrt(c + d)*sqrt(x**3 - 1)*sqrt(c**2
- c*d + d**2))
```

Mathematica [C] time = 1.17288, size = 233, normalized size = 0.68

$$\frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(-\frac{3\left(x+\sqrt[3]{-1}\right)\sqrt{\frac{(-1)^{2/3}x+\sqrt[3]{-1}}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\sqrt{x^2+x+1}\left(\sqrt{3}c+\left(\sqrt{3}-3\right)d\right)\left(\frac{i\sqrt{3}d}{\sqrt[3]{-1}d-c};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right.\right.}{c-\sqrt[3]{-1}d}\right.}{3d\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(1 - Sqrt[3] - x)/((c + d*x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((-3*((-1)^(1/3) + x)*Sqrt[((-1
)^(1/3) + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(
1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)
^(2/3)*x)/(1 + (-1)^(1/3))] + ((-1)^(1/3)*(1 + (-1)^(1/3))*(Sqrt[
3]*c + (-3 + Sqrt[3])*d)*Sqrt[1 + x + x^2]*EllipticPi[(I*Sqrt[3]*
d)/(-c + (-1)^(1/3)*d), ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^
(1/3))]], (-1)^(1/3)])/(c - (-1)^(1/3)*d)))/(3*d*Sqrt[-1 + x^3])
```

Maple [A] time = 0.028, size = 277, normalized size = 0.8

$$-2\,\frac{-3/2-i/2\sqrt{3}}{d\sqrt{x^3-1}}\sqrt{\frac{-1+x}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2-i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2+i/2\sqrt{3}}{3/2+i/2\sqrt{3}}}\mathit{EllipticF}\left(\sqrt{\frac{-1+x}{-3/2-i/2\sqrt{3}}},\sqrt{\frac{3/2+i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\right)$$
$$-2\,\frac{\left(d\sqrt{3}-c-d\right)\left(-3/2-i/2\sqrt{3}\right)}{d^2\sqrt{x^3-1}}\sqrt{\frac{-1+x}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2-i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2+i/2\sqrt{3}}{3/2+i/2\sqrt{3}}}\mathit{EllipticPi}\left(\sqrt{\frac{-1+x}{-3/2-i/2\sqrt{3}}},(3/2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1-x-3^(1/2))/(d*x+c)/(x^3-1)^(1/2),x)
```

```
[Out] -2/d*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x
+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1
/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((-1+x)/(
-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2
)))^(1/2))-2*(d*3^(1/2)-c-d)/d^2*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3
/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2
)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-
1)^(1/2)/(1+c/d)*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),(
3/2+1/2*I*3^(1/2))/(1+c/d),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2
)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x+\sqrt{3}-1}{\sqrt{x^3-1}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x + sqrt(3) - 1)/(sqrt(x^3 - 1)*(d*x + c)),x, algorithm="maxima")
```

```
[Out] -integrate((x + sqrt(3) - 1)/(sqrt(x^3 - 1)*(d*x + c)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x + sqrt(3) - 1)/(sqrt(x^3 - 1)*(d*x + c)),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{3}}{c\sqrt{x^3-1}+dx\sqrt{x^3-1}}\,dx-\int \frac{x}{c\sqrt{x^3-1}+dx\sqrt{x^3-1}}\,dx-\int \left(-\frac{1}{c\sqrt{x^3-1}+dx\sqrt{x^3-1}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1-x-3**(1/2))/(d*x+c)/(x**3-1)**(1/2),x)
```

```
[Out] -Integral(sqrt(3)/(c*sqrt(x**3 - 1) + d*x*sqrt(x**3 - 1)), x) - I
ntegral(x/(c*sqrt(x**3 - 1) + d*x*sqrt(x**3 - 1)), x) - Integral(
-1/(c*sqrt(x**3 - 1) + d*x*sqrt(x**3 - 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x+\sqrt{3}-1}{\sqrt{x^3-1}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x + sqrt(3) - 1)/(sqrt(x^3 - 1)*(d*x + c)),x, algorithm="giac")
```

```
[Out] integrate(-(x + sqrt(3) - 1)/(sqrt(x^3 - 1)*(d*x + c)), x)
```

3.128 $\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=364

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(\frac{\left(c-\left(1-\sqrt{3}\right)d\right)^2}{\left(c-\left(1+\sqrt{3}\right)d\right)^2};-\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\left(c-\sqrt{3}d-d\right)} - \frac{(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(c-\left(1-\sqrt{3}\right)d\right)\tanh^{-1}\left(\frac{2\sqrt{2+\sqrt{3}}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{c^2+cd+d^2}}{\sqrt{d}\sqrt{\frac{\left(x+\sqrt{3}+1\right)^2}{\left(x-\sqrt{3}+1\right)^2}+4\sqrt{3}+7}\sqrt{c-d}}\right)}{\sqrt{d}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\sqrt{c-d}\sqrt{c^2+cd+d^2}}$$

```
[Out] -(((c - (1 - Sqrt[3])*d)*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3]
+ x)^2]*ArcTanh[(2*Sqrt[2 + Sqrt[3]]*Sqrt[c^2 + c*d + d^2]*Sqrt[-
((1 + x)/(1 - Sqrt[3] + x)^2)])/(Sqrt[c - d]*Sqrt[d]*Sqrt[7 + 4*S
qrt[3] + (1 + Sqrt[3] + x)^2/(1 - Sqrt[3] + x)^2])])/(Sqrt[c - d]
*Sqrt[d]*Sqrt[c^2 + c*d + d^2]*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2
)]*Sqrt[-1 - x^3])) + (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(
1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticPi[(c - (1 - Sqrt[3])*d
)^2/(c - (1 + Sqrt[3])*d)^2, -ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[
3] + x)], -7 + 4*Sqrt[3]])/((c - d - Sqrt[3]*d)*Sqrt[-((1 + x)/(1
 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```

Rubi [A] time = 2.09602, antiderivative size = 364, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(\frac{\left(c-\left(1-\sqrt{3}\right)d\right)^2}{\left(c-\left(1+\sqrt{3}\right)d\right)^2};-\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\left(c-\sqrt{3}d-d\right)} - \frac{(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(c-\left(1-\sqrt{3}\right)d\right)\tanh^{-1}\left(\frac{2\sqrt{2+\sqrt{3}}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{c^2+cd+d^2}}{\sqrt{d}\sqrt{\frac{\left(x+\sqrt{3}+1\right)^2}{\left(x-\sqrt{3}+1\right)^2}+4\sqrt{3}+7}\sqrt{c-d}}\right)}{\sqrt{d}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\sqrt{c-d}\sqrt{c^2+cd+d^2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[(1 - Sqrt[3] + x)/((c + d*x)*Sqrt[-1 - x^3]),x]
```

```
[Out] -(((c - (1 - Sqrt[3])*d)*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3]
+ x)^2]*ArcTanh[(2*Sqrt[2 + Sqrt[3]]*Sqrt[c^2 + c*d + d^2]*Sqrt[-
((1 + x)/(1 - Sqrt[3] + x)^2)])/(Sqrt[c - d]*Sqrt[d]*Sqrt[7 + 4*S
qrt[3] + (1 + Sqrt[3] + x)^2/(1 - Sqrt[3] + x)^2])])/(Sqrt[c - d]
*Sqrt[d]*Sqrt[c^2 + c*d + d^2]*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2
)]*Sqrt[-1 - x^3])) + (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(
1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticPi[(c - (1 - Sqrt[3])*d
)^2/(c - (1 + Sqrt[3])*d)^2, -ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[
3] + x)], -7 + 4*Sqrt[3]])/((c - d - Sqrt[3]*d)*Sqrt[-((1 + x)/(1
 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```

Rubi in Sympy [A] time = 170.995, size = 330, normalized size = 0.91

$$\frac{4\sqrt[4]{3}\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{\sqrt{3}+2}\,(x+1)\,\Pi\left(\frac{\left(c-d+\sqrt{3}d\right)^2}{\left(-c+d+\sqrt{3}d\right)^2};\operatorname{asin}\left(\frac{x+1+\sqrt{3}}{-x-1+\sqrt{3}}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt{\frac{-x-1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{4\sqrt{3}+7}\left(c-d\left(1+\sqrt{3}\right)\right)\sqrt{-x^3-1}} - \frac{\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,(x+1)\left(c-d+\sqrt{3}d\right)\operatorname{atanh}\left(\frac{3^{\frac{3}{4}}\sqrt{1-\frac{\left(x+1+\sqrt{3}\right)^2}{\left(-x-1+\sqrt{3}\right)^2}}\sqrt{\sqrt{3}+2}\sqrt{c^2+cd+d^2}}{3\sqrt{d}\sqrt{c-d}\sqrt{4\sqrt{3}+7+\frac{\left(x+1+\sqrt{3}\right)^2}{\left(-x-1+\sqrt{3}\right)^2}}}\right)}{\sqrt{d}\sqrt{\frac{-x-1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{c-d}\sqrt{-x^3-1}\sqrt{c^2+cd+d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+x-3**(1/2))/(d*x+c)/(-x**3-1)**(1/2),x)
```

```
[Out] 4*3**(1/4)*sqrt((x**2 - x + 1)/(x - sqrt(3) + 1)**2)*sqrt(sqrt(3)
 + 2)*(x + 1)*elliptic_pi((c - d + sqrt(3)*d)**2/(-c + d + sqrt(3
)*d)**2, asin((x + 1 + sqrt(3))/(-x - 1 + sqrt(3))), -7 + 4*sqrt(
3))/(sqrt((-x - 1)/(x - sqrt(3) + 1)**2)*sqrt(4*sqrt(3) + 7)*(c -
 d*(1 + sqrt(3)))*sqrt(-x**3 - 1)) - sqrt((x**2 - x + 1)/(x - sqr
t(3) + 1)**2)*(x + 1)*(c - d + sqrt(3)*d)*atanh(3**(3/4)*sqrt(1 -
 (x + 1 + sqrt(3))**2/(-x - 1 + sqrt(3))**2)*sqrt(sqrt(3) + 2)*sq
rt(c**2 + c*d + d**2)/(3*sqrt(d)*sqrt(c - d)*sqrt(4*sqrt(3) + 7 +
 (x + 1 + sqrt(3))**2/(-x - 1 + sqrt(3))**2)))/(sqrt(d)*sqrt((-x
- 1)/(x - sqrt(3) + 1)**2)*sqrt(c - d)*sqrt(-x**3 - 1)*sqrt(c**2
+ c*d + d**2))
```

Mathematica [C] time = 1.0221, size = 215, normalized size = 0.59

$$\frac{2\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\left(-\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}}+\frac{i\sqrt{x^2-x+1}\left(c+\left(\sqrt{3}-1\right)d\right)\Pi\left(\frac{i\sqrt{3}d}{c+\sqrt[3]{-1}d};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{c+\sqrt[3]{-1}d}\right)}{d\sqrt{-x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(1 - Sqrt[3] + x)/((c + d*x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) - x)*Sqrt[((-1)
^(1/3) - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1
 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^
(2/3)*x)/(1 + (-1)^(1/3))]) + (I*(c + (-1 + Sqrt[3])*d)*Sqrt[1 -
x + x^2]*EllipticPi[(I*Sqrt[3]*d)/(c + (-1)^(1/3)*d), ArcSin[Sqrt
[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(c + (-1)^(1
/3)*d)))/(d*Sqrt[-1 - x^3])
```

Maple [A] time = 0.029, size = 266, normalized size = 0.7

$$\frac{\frac{2i}{3}\left(d\sqrt{3}+c-d\right)\sqrt{3}}{d^2}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\,EllipticPi\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},i\sqrt{3}\left(\right.\right.$$

$$-\frac{\frac{2i}{3}\sqrt{3}}{d}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\,EllipticF\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)\frac{1}{\sqrt{-x^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+x-3^(1/2))/(d*x+c)/(-x^3-1)^(1/2),x)
```

```
[Out] 2/3*I*(d*3^(1/2)+c-d)/d^2*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2
))^(1/2)*((1+x)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/
2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2)+c/d)*Ellipti
cPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)
/(1/2+1/2*I*3^(1/2)+c/d),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))-2
/3*I/d*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/
2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/
(-x^3-1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(
1/2))^(1/2),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x - \sqrt{3} + 1}{\sqrt{-x^3 - 1}(dx + c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x - sqrt(3) + 1)/(sqrt(-x^3 - 1)*(d*x + c)),x, algorithm="maxima")
```

```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(-x^3 - 1)*(d*x + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x - sqrt(3) + 1)/(sqrt(-x^3 - 1)*(d*x + c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x - \sqrt{3} + 1}{\sqrt{-(x + 1)(x^2 - x + 1)}\,(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+x-3**(1/2))/(d*x+c)/(-x**3-1)**(1/2),x)
```

```
[Out] Integral((x - sqrt(3) + 1)/(sqrt(-(x + 1)*(x**2 - x + 1))*(c + d*
x)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x - \sqrt{3} + 1}{\sqrt{-x^3 - 1}(dx + c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x - sqrt(3) + 1)/(sqrt(-x^3 - 1)*(d*x + c)),x, algorithm="giac")

[Out] integrate((x - sqrt(3) + 1)/(sqrt(-x^3 - 1)*(d*x + c)), x)
```

3.129 $\int \frac{1+\sqrt{3}+x}{x\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=125

$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{2}{3}\left(1+\sqrt{3}\right)\tanh^{-1}\left(\sqrt{x^3+1}\right)$$

```
[Out] (-2*(1 + Sqrt[3])*ArcTanh[Sqrt[1 + x^3]])/3 + (2*Sqrt[2 + Sqrt[3]
]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSi
n[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)
*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rubi [A] time = 0.102632, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238

$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{2}{3}\left(1+\sqrt{3}\right)\tanh^{-1}\left(\sqrt{x^3+1}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + Sqrt[3] + x)/(x*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*(1 + Sqrt[3])*ArcTanh[Sqrt[1 + x^3]])/3 + (2*Sqrt[2 + Sqrt[3]
]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSi
n[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)
*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rubi in Sympy [A] time = 10.7831, size = 117, normalized size = 0.94

$$-\left(\frac{2}{3}+\frac{2\sqrt{3}}{3}\right)\operatorname{atanh}\left(\sqrt{x^3+1}\right)+\frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{x^2-x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{\sqrt{3}+2}\,(x+1)\,F\left(\operatorname{asin}\left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{3\sqrt{\frac{x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+x+3**(1/2))/x/(x**3+1)**(1/2),x)
```

```
[Out] -(2/3 + 2*sqrt(3)/3)*atanh(sqrt(x**3 + 1)) + 2*3**(3/4)*sqrt((x**
2 - x + 1)/(x + 1 + sqrt(3))**2)*sqrt(sqrt(3) + 2)*(x + 1)*ellipt
ic_f(asin((x - sqrt(3) + 1)/(x + 1 + sqrt(3))), -7 - 4*sqrt(3))/(
3*sqrt((x + 1)/(x + 1 + sqrt(3))**2)*sqrt(x**3 + 1))
```

Mathematica [A] time = 0.787178, size = 149, normalized size = 1.19

$$-\frac{2\tanh^{-1}\left(\sqrt{x^3+1}\right)}{\sqrt{3}}-\frac{2}{3}\tanh^{-1}\left(\sqrt{x^3+1}\right)-\frac{2\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\sqrt{-\frac{(-1)^{2/3}\left(x+(-1)^{2/3}\right)}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + Sqrt[3] + x)/(x*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[1 + x^3]])/3 - (2*ArcTanh[Sqrt[1 + x^3]])/Sqrt[3
] - (2*((-1)^(1/3) - x)*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*Sqrt[-(((-
1)^(2/3)*((-1)^(2/3) + x))/(1 + (-1)^(1/3)))]*EllipticF[ArcSin[Sq
rt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(Sqrt[(1 +
 (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*Sqrt[1 + x^3])
```

Maple [A] time = 0.041, size = 132, normalized size = 1.1

$$2\,\frac{3/2-i/2\sqrt{3}}{\sqrt{x^3+1}}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}\mathit{EllipticF}\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},\sqrt{\frac{-3/2+i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\right)-\frac{2+2\sqrt{3}}{3}\mathit{Artanh}\left(\sqrt{x^3+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+x+3^(1/2))/x/(x^3+1)^(1/2),x)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1
/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/
(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1
```

```
/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^
(1/2))-2/3*arctanh((x^3+1)^(1/2))*(1+3^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+\sqrt{3}+1}{\sqrt{x^3+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sqrt(3) + 1)/(sqrt(x^3 + 1)*x),x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(3) + 1)/(sqrt(x^3 + 1)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x+\sqrt{3}+1}{\sqrt{x^3+1}x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sqrt(3) + 1)/(sqrt(x^3 + 1)*x),x, algorithm="fricas")
```

```
[Out] integral((x + sqrt(3) + 1)/(sqrt(x^3 + 1)*x), x)
```

Sympy [A] time = 6.84871, size = 56, normalized size = 0.45

$$\frac{x\left(\frac{1}{3}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right|x^3e^{i\pi}\right)}{3\left(\frac{4}{3}\right)}-\frac{2\sqrt{3}\operatorname{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}-\frac{2\operatorname{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+x+3**(1/2))/x/(x**3+1)**(1/2),x)
```

```
[Out] x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*g
amma(4/3)) - 2*sqrt(3)*asinh(x**(-3/2))/3 - 2*asinh(x**(-3/2))/3
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x + \sqrt{3} + 1}{\sqrt{x^3 + 1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sqrt(3) + 1)/(sqrt(x^3 + 1)*x),x, algorithm="giac")

[Out] integrate((x + sqrt(3) + 1)/(sqrt(x^3 + 1)*x), x)
```

3.130 $\int \frac{1+\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=139

$$\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2}{3}\left(1+\sqrt{3}\right)\tanh^{-1}\left(\sqrt{1-x^3}\right)$$

```
[Out] (-2*(1 + Sqrt[3])*ArcTanh[Sqrt[1 - x^3]])/3 + (2*Sqrt[2 + Sqrt[3]
]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSi
n[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3^(1/4)
*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rubi [A] time = 0.112637, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2}{3}\left(1+\sqrt{3}\right)\tanh^{-1}\left(\sqrt{1-x^3}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + Sqrt[3] - x)/(x*Sqrt[1 - x^3]),x]
```

```
[Out] (-2*(1 + Sqrt[3])*ArcTanh[Sqrt[1 - x^3]])/3 + (2*Sqrt[2 + Sqrt[3]
]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSi
n[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3^(1/4)
*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rubi in Sympy [A] time = 12.8204, size = 117, normalized size = 0.84

$$-\left(\frac{2}{3}+\frac{2\sqrt{3}}{3}\right)\operatorname{atanh}\left(\sqrt{-x^3+1}\right)+\frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{x^2+x+1}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{\sqrt{3}+2}\,(-x+1)\,F\left(\operatorname{asin}\left(\frac{-x-\sqrt{3}+1}{-x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{3\sqrt{\frac{-x+1}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{-x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1-x+3**(1/2))/x/(-x**3+1)**(1/2),x)
```

```
[Out] -(2/3 + 2*sqrt(3)/3)*atanh(sqrt(-x**3 + 1)) + 2*3**(3/4)*sqrt((x*
*2 + x + 1)/(-x + 1 + sqrt(3))**2)*sqrt(sqrt(3) + 2)*(-x + 1)*ell
iptic_f(asin((-x - sqrt(3) + 1)/(-x + 1 + sqrt(3))), -7 - 4*sqrt(
3))/(3*sqrt((-x + 1)/(-x + 1 + sqrt(3))**2)*sqrt(-x**3 + 1))
```

Mathematica [A] time = 0.817931, size = 157, normalized size = 1.13

$$-\frac{2\tanh^{-1}\left(\sqrt{1-x^3}\right)}{\sqrt{3}}-\frac{2}{3}\tanh^{-1}\left(\sqrt{1-x^3}\right)-\frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(x+\sqrt[3]{-1}\right)\sqrt{\frac{(-1)^{2/3}x+\sqrt[3]{-1}}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\sqrt{1-x^3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + Sqrt[3] - x)/(x*Sqrt[1 - x^3]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[1 - x^3]])/3 - (2*ArcTanh[Sqrt[1 - x^3]])/Sqrt[3
] - (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((-1)^(1/3) + x)*Sqrt[((-1)
^(1/3) + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1
 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(Sqrt[(1 - (-1)
^(2/3)*x)/(1 + (-1)^(1/3))]*Sqrt[1 - x^3])
```

Maple [A] time = 0.049, size = 125, normalized size = 0.9

$$-\frac{2+2\sqrt{3}}{3}Artanh\left(\sqrt{-x^3+1}\right)+\frac{2i}{3}\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}EllipticF\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)\frac{}{\sqrt{-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1-x+3^(1/2))/x/(-x^3+1)^(1/2),x)
```

```
[Out] -2/3*arctanh((-x^3+1)^(1/2))*(1+3^(1/2))+2/3*I*3^(1/2)*(I*(x+1/2-
1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/(-3/2+1/2*I*3^(1/2)))^(1/2)
*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Elliptic
```

```
F(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/
(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x-\sqrt{3}-1}{\sqrt{-x^3+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x - sqrt(3) - 1)/(sqrt(-x^3 + 1)*x),x, algorithm="maxima")
```

```
[Out] -integrate((x - sqrt(3) - 1)/(sqrt(-x^3 + 1)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{x-\sqrt{3}-1}{\sqrt{-x^3+1}x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x - sqrt(3) - 1)/(sqrt(-x^3 + 1)*x),x, algorithm="fricas")
```

```
[Out] integral(-(x - sqrt(3) - 1)/(sqrt(-x^3 + 1)*x), x)
```

Sympy [A] time = 8.07263, size = 99, normalized size = 0.71

$$-\frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\begin{cases}-\frac{2\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for } \left|\frac{1}{x^3}\right|>1\\ \frac{2i\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise}\end{cases}+\sqrt{3}\left(\begin{cases}-\frac{2\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for } \left|\frac{1}{x^3}\right|>1\\ \frac{2i\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1-x+3**(1/2))/x/(-x**3+1)**(1/2),x)
```

```
[Out] -x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(2*I*pi))/(
3*gamma(4/3)) + Piecewise((-2*acosh(x**(-3/2))/3, Abs(x**(-3)) >
```

```
1), (2*I*asin(x**(-3/2))/3, True)) + sqrt(3)*Piecewise((-2*acosh(
x**(-3/2))/3, Abs(x**(-3)) > 1), (2*I*asin(x**(-3/2))/3, True))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x-\sqrt{3}-1}{\sqrt{-x^3+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x - sqrt(3) - 1)/(sqrt(-x^3 + 1)*x),x, algorithm="giac")
```

```
[Out] integrate(-(x - sqrt(3) - 1)/(sqrt(-x^3 + 1)*x), x)
```

3.131 $\int \frac{1+\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=142

$$\frac{2}{3}\left(1+\sqrt{3}\right)\tan^{-1}\left(\sqrt{x^3-1}\right)+\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

```
[Out] (2*(1 + Sqrt[3])*ArcTan[Sqrt[-1 + x^3]])/3 + (2*Sqrt[2 - Sqrt[3]]
*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin
[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3^(1/4)*
Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Rubi [A] time = 0.10978, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217

$$\frac{2}{3}\left(1+\sqrt{3}\right)\tan^{-1}\left(\sqrt{x^3-1}\right)+\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + Sqrt[3] - x)/(x*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*(1 + Sqrt[3])*ArcTan[Sqrt[-1 + x^3]])/3 + (2*Sqrt[2 - Sqrt[3]]
*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin
[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3^(1/4)*
Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Rubi in Sympy [A] time = 11.6212, size = 116, normalized size = 0.82

$$\left(\frac{2}{3}+\frac{2\sqrt{3}}{3}\right)\operatorname{atan}\left(\sqrt{x^3-1}\right)+\frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{-\sqrt{3}+2}\,(-x+1)\,F\left(\operatorname{asin}\left(\frac{-x+1+\sqrt{3}}{-x-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{3\sqrt{\frac{x-1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1-x+3**(1/2))/x/(x**3-1)**(1/2),x)
```

```
[Out] (2/3 + 2*sqrt(3)/3)*atan(sqrt(x**3 - 1)) + 2*3**(3/4)*sqrt((x**2
+ x + 1)/(-x - sqrt(3) + 1)**2)*sqrt(-sqrt(3) + 2)*(-x + 1)*ellip
tic_f(asin((-x + 1 + sqrt(3))/(-x - sqrt(3) + 1)), -7 + 4*sqrt(3)
)/(3*sqrt((x - 1)/(-x - sqrt(3) + 1)**2)*sqrt(x**3 - 1))
```

Mathematica [A] time = 1.05054, size = 150, normalized size = 1.06

$$\frac{2}{3}\left(\sqrt{3}\tan^{-1}\left(\sqrt{x^3-1}\right)+\tan^{-1}\left(\sqrt{x^3-1}\right)\right.$$
$$\left.-\frac{3\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(x+\sqrt[3]{-1}\right)\sqrt{\frac{(-1)^{2/3}x+\sqrt[3]{-1}}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\sqrt{x^3-1}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + Sqrt[3] - x)/(x*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*(ArcTan[Sqrt[-1 + x^3]] + Sqrt[3]*ArcTan[Sqrt[-1 + x^3]] - (3*
Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((-1)^(1/3) + x)*Sqrt[((-1)^(1/3)
+ (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)
^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(Sqrt[(1 - (-1)^(2/3)*
x)/(1 + (-1)^(1/3))]*Sqrt[-1 + x^3])))/3
```

Maple [A] time = 0.037, size = 140, normalized size = 1.

$$\frac{2\sqrt{3}}{3}\arctan\left(\sqrt{x^3-1}\right)+\frac{2}{3}\arctan\left(\sqrt{x^3-1}\right)$$
$$-2\,\frac{-3/2-i/2\sqrt{3}}{\sqrt{x^3-1}}\sqrt{\frac{-1+x}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2-i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2+i/2\sqrt{3}}{3/2+i/2\sqrt{3}}}\mathit{EllipticF}\left(\sqrt{\frac{-1+x}{-3/2-i/2\sqrt{3}}},\sqrt{\frac{3/2+i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1-x+3^(1/2))/x/(x^3-1)^(1/2),x)
```

```
[Out] 2/3*arctan((x^3-1)^(1/2))*3^(1/2)+2/3*arctan((x^3-1)^(1/2))-2*(-3
/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2
*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/
2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((-1+x)/(-3/2-1/2
*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2
))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x-\sqrt{3}-1}{\sqrt{x^3-1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x - sqrt(3) - 1)/(sqrt(x^3 - 1)*x),x, algorithm="maxima")
```

```
[Out] -integrate((x - sqrt(3) - 1)/(sqrt(x^3 - 1)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{x-\sqrt{3}-1}{\sqrt{x^3-1}x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x - sqrt(3) - 1)/(sqrt(x^3 - 1)*x),x, algorithm="fricas")
```

```
[Out] integral(-(x - sqrt(3) - 1)/(sqrt(x^3 - 1)*x), x)
```

Sympy [A] time = 7.92413, size = 94, normalized size = 0.66

$$\frac{ix\,\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|x^3\right)}{3\,\Gamma\left(\frac{4}{3}\right)}+\begin{cases}\frac{2i\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for}\ \left|\frac{1}{x^3}\right|>1\\-\frac{2\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise}\end{cases}+\sqrt{3}\left(\begin{cases}\frac{2i\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for}\ \left|\frac{1}{x^3}\right|>1\\-\frac{2\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3**(1/2))/x/(x**3-1)**(1/2),x)
```

```
[Out] I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma(4/3)) + P
iecewise((2*I*acosh(x**(-3/2))/3, Abs(x**(-3)) > 1), (-2*asin(x**
(-3/2))/3, True)) + sqrt(3)*Piecewise((2*I*acosh(x**(-3/2))/3, Ab
s(x**(-3)) > 1), (-2*asin(x**(-3/2))/3, True))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x-\sqrt{3}-1}{\sqrt{x^3-1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x - sqrt(3) - 1)/(sqrt(x^3 - 1)*x),x, algorithm="giac")
```

```
[Out] integrate(-(x - sqrt(3) - 1)/(sqrt(x^3 - 1)*x), x)
```

3.132 $\int \frac{1+\sqrt{3}+x}{x\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=136

$$\frac{2}{3}\left(1+\sqrt{3}\right)\tan^{-1}\left(\sqrt{-x^3-1}\right)+\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

```
[Out] (2*(1 + Sqrt[3])*ArcTan[Sqrt[-1 - x^3]])/3 + (2*Sqrt[2 - Sqrt[3]]
*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin
[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3^(1/4)*
Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```

Rubi [A] time = 0.111464, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217

$$\frac{2}{3}\left(1+\sqrt{3}\right)\tan^{-1}\left(\sqrt{-x^3-1}\right)+\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + Sqrt[3] + x)/(x*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*(1 + Sqrt[3])*ArcTan[Sqrt[-1 - x^3]])/3 + (2*Sqrt[2 - Sqrt[3]]
*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin
[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3^(1/4)*
Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```

Rubi in Sympy [A] time = 12.1001, size = 121, normalized size = 0.89

$$\left(\frac{2}{3}+\frac{2\sqrt{3}}{3}\right)\operatorname{atan}\left(\sqrt{-x^3-1}\right)+\frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-\sqrt{3}+2}\,(x+1)\,F\left(\operatorname{asin}\left(\frac{x+1+\sqrt{3}}{x-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{3\sqrt{\frac{-x-1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+x+3**(1/2))/x/(-x**3-1)**(1/2),x)
```

```
[Out] (2/3 + 2*sqrt(3)/3)*atan(sqrt(-x**3 - 1)) + 2*3**(3/4)*sqrt((x**2
 - x + 1)/(x - sqrt(3) + 1)**2)*sqrt(-sqrt(3) + 2)*(x + 1)*ellipt
ic_f(asin((x + 1 + sqrt(3))/(x - sqrt(3) + 1)), -7 + 4*sqrt(3))/(
3*sqrt((-x - 1)/(x - sqrt(3) + 1)**2)*sqrt(-x**3 - 1))
```

Mathematica [A] time = 0.950968, size = 155, normalized size = 1.14

$$\frac{2}{3}\left(\sqrt{3}\tan^{-1}\left(\sqrt{-x^3-1}\right)+\tan^{-1}\left(\sqrt{-x^3-1}\right)-\frac{3\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\sqrt{-x^3-1}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + Sqrt[3] + x)/(x*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*(ArcTan[Sqrt[-1 - x^3]] + Sqrt[3]*ArcTan[Sqrt[-1 - x^3]] - (3*
((-1)^(1/3) - x)*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*Sqrt[((-1)^(1/3)
- (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)
^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(Sqrt[(1 + (-1)^(2/3)*
x)/(1 + (-1)^(1/3))]*Sqrt[-1 - x^3])))/3
```

Maple [A] time = 0.042, size = 135, normalized size = 1.

$$-\frac{2i}{3}\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}EllipticF\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)\frac{1}{\sqrt{-x^3}}$$
$$+\frac{2\sqrt{3}}{3}\arctan\left(\sqrt{-x^3-1}\right)+\frac{2}{3}\arctan\left(\sqrt{-x^3-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+x+3^(1/2))/x/(-x^3-1)^(1/2),x)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/
2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/
(-x^3-1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(
1/2))^(1/2),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))+2/3*arctan((-x
^3-1)^(1/2))*3^(1/2)+2/3*arctan((-x^3-1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+\sqrt{3}+1}{\sqrt{-x^3-1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x + sqrt(3) + 1)/(sqrt(-x^3 - 1)*x),x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(3) + 1)/(sqrt(-x^3 - 1)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x+\sqrt{3}+1}{\sqrt{-x^3-1}x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x + sqrt(3) + 1)/(sqrt(-x^3 - 1)*x),x, algorithm="fricas")
```

```
[Out] integral((x + sqrt(3) + 1)/(sqrt(-x^3 - 1)*x), x)
```

Sympy [A] time = 6.93559, size = 61, normalized size = 0.45

$$-\frac{ix\,\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\,\Gamma\left(\frac{4}{3}\right)}+\frac{2i\,\text{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}+\frac{2\sqrt{3}i\,\text{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+x+3**(1/2))/x/(-x**3-1)**(1/2),x)
```

```
[Out] -I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(
3*gamma(4/3)) + 2*I*asinh(x**(-3/2))/3 + 2*sqrt(3)*I*asinh(x**(-3
/2))/3
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+\sqrt{3}+1}{\sqrt{-x^3-1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sqrt(3) + 1)/(sqrt(-x^3 - 1)*x),x, algorithm="giac")
```

```
[Out] integrate((x + sqrt(3) + 1)/(sqrt(-x^3 - 1)*x), x)
```

3.133 $\int \frac{1-\sqrt{3}+x}{x\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=127

$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{2}{3}\left(1-\sqrt{3}\right)\tanh^{-1}\left(\sqrt{x^3+1}\right)$$

```
[Out] (-2*(1 - Sqrt[3])*ArcTanh[Sqrt[1 + x^3]])/3 + (2*Sqrt[2 + Sqrt[3]
]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSi
n[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)
*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rubi [A] time = 0.0953674, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$

$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{2}{3}\left(1-\sqrt{3}\right)\tanh^{-1}\left(\sqrt{x^3+1}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - Sqrt[3] + x)/(x*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*(1 - Sqrt[3])*ArcTanh[Sqrt[1 + x^3]])/3 + (2*Sqrt[2 + Sqrt[3]
]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSi
n[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)
*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rubi in Sympy [A] time = 11.2682, size = 117, normalized size = 0.92

$$-\left(-\frac{2\sqrt{3}}{3}+\frac{2}{3}\right)\operatorname{atanh}\left(\sqrt{x^3+1}\right)+\frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{x^2-x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{\sqrt{3}+2}\,(x+1)\,F\left(\operatorname{asin}\left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{3\sqrt{\frac{x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+x-3**(1/2))/x/(x**3+1)**(1/2),x)
```

```
[Out] -(-2*sqrt(3)/3 + 2/3)*atanh(sqrt(x**3 + 1)) + 2*3**(3/4)*sqrt((x*
*2 - x + 1)/(x + 1 + sqrt(3))**2)*sqrt(sqrt(3) + 2)*(x + 1)*ellip
tic_f(asin((x - sqrt(3) + 1)/(x + 1 + sqrt(3))), -7 - 4*sqrt(3))/
(3*sqrt((x + 1)/(x + 1 + sqrt(3))**2)*sqrt(x**3 + 1))
```

Mathematica [A] time = 0.559999, size = 149, normalized size = 1.17

$$\frac{2\tanh^{-1}\left(\sqrt{x^3+1}\right)}{\sqrt{3}}-\frac{2}{3}\tanh^{-1}\left(\sqrt{x^3+1}\right)-\frac{2\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\sqrt{-\frac{(-1)^{2/3}\left(x+(-1)^{2/3}\right)}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - Sqrt[3] + x)/(x*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[1 + x^3]])/3 + (2*ArcTanh[Sqrt[1 + x^3]])/Sqrt[3
] - (2*((-1)^(1/3) - x)*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*Sqrt[-(((-
1)^(2/3)*((-1)^(2/3) + x))/(1 + (-1)^(1/3)))]*EllipticF[ArcSin[Sq
rt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(Sqrt[(1 +
 (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*Sqrt[1 + x^3])
```

Maple [A] time = 0.023, size = 132, normalized size = 1.

$$\frac{-2+2\sqrt{3}}{3}Artanh\left(\sqrt{x^3+1}\right)$$
$$+2\,\frac{3/2-i/2\sqrt{3}}{\sqrt{x^3+1}}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}EllipticF\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},\sqrt{\frac{-3/2+i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+x-3^(1/2))/x/(x^3+1)^(1/2),x)
```

```
[Out] 2/3*(3^(1/2)-1)*arctanh((x^3+1)^(1/2))+2*(3/2-1/2*I*3^(1/2))*((1+
x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*
3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2
```

```
)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/
2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - sqrt(3) + 1)/(sqrt(x^3 + 1)*x),x, algorithm="maxima")
```

```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(x^3 + 1)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x-\sqrt{3}+1}{\sqrt{x^3+1}x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - sqrt(3) + 1)/(sqrt(x^3 + 1)*x),x, algorithm="fricas")
```

```
[Out] integral((x - sqrt(3) + 1)/(sqrt(x^3 + 1)*x), x)
```

Sympy [A] time = 6.91746, size = 56, normalized size = 0.44

$$\frac{x\,\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\,\Gamma\left(\frac{4}{3}\right)}-\frac{2\operatorname{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}+\frac{2\sqrt{3}\operatorname{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+x-3**(1/2))/x/(x**3+1)**(1/2),x)
```

```
[Out] x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*g
amma(4/3)) - 2*asinh(x**(-3/2))/3 + 2*sqrt(3)*asinh(x**(-3/2))/3
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - sqrt(3) + 1)/(sqrt(x^3 + 1)*x),x, algorithm="giac")
```

```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(x^3 + 1)*x), x)
```

3.134 $\int \frac{1-\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=141

$$\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2}{3}\left(1-\sqrt{3}\right)\tanh^{-1}\left(\sqrt{1-x^3}\right)$$

```
[Out] (-2*(1 - Sqrt[3])*ArcTanh[Sqrt[1 - x^3]])/3 + (2*Sqrt[2 + Sqrt[3]
]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSi
n[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3^(1/4)
*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rubi [A] time = 0.119913, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185

$$\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2}{3}\left(1-\sqrt{3}\right)\tanh^{-1}\left(\sqrt{1-x^3}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - Sqrt[3] - x)/(x*Sqrt[1 - x^3]),x]
```

```
[Out] (-2*(1 - Sqrt[3])*ArcTanh[Sqrt[1 - x^3]])/3 + (2*Sqrt[2 + Sqrt[3]
]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSi
n[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3^(1/4)
*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rubi in Sympy [A] time = 13.4973, size = 117, normalized size = 0.83

$$-\left(-\frac{2\sqrt{3}}{3}+\frac{2}{3}\right)\operatorname{atanh}\left(\sqrt{-x^3+1}\right)+\frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{x^2+x+1}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{\sqrt{3}+2}\,(-x+1)\,F\left(\operatorname{asin}\left(\frac{-x-\sqrt{3}+1}{-x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{3\sqrt{\frac{-x+1}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{-x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1-x-3**(1/2))/x/(-x**3+1)**(1/2),x)
```

```
[Out] -(-2*sqrt(3)/3 + 2/3)*atanh(sqrt(-x**3 + 1)) + 2*3**(3/4)*sqrt((x
**2 + x + 1)/(-x + 1 + sqrt(3))**2)*sqrt(sqrt(3) + 2)*(-x + 1)*el
liptic_f(asin((-x - sqrt(3) + 1)/(-x + 1 + sqrt(3))), -7 - 4*sqrt
(3))/(3*sqrt((-x + 1)/(-x + 1 + sqrt(3))**2)*sqrt(-x**3 + 1))
```

Mathematica [A] time = 1.48324, size = 158, normalized size = 1.12

$$\frac{2}{3}\left(\sqrt{3}\tanh^{-1}\left(\sqrt{1-x^3}\right)-\tanh^{-1}\left(\sqrt{1-x^3}\right)-\frac{3\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(x+\sqrt[3]{-1}\right)\sqrt{\frac{(-1)^{2/3}x+\sqrt[3]{-1}}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\sqrt{1-x^3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - Sqrt[3] - x)/(x*Sqrt[1 - x^3]),x]
```

```
[Out] (2*(-ArcTanh[Sqrt[1 - x^3]] + Sqrt[3]*ArcTanh[Sqrt[1 - x^3]] - (3
*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((-1)^(1/3) + x)*Sqrt[((-1)^(1/3)
 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1
)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(Sqrt[(1 - (-1)^(2/3)
*x)/(1 + (-1)^(1/3))]*Sqrt[1 - x^3])))/3
```

Maple [A] time = 0.023, size = 125, normalized size = 0.9

$$\frac{2i}{3}\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}EllipticF\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)\frac{1}{\sqrt{-x^3}}$$
$$+\frac{-2+2\sqrt{3}}{3}Artanh\left(\sqrt{-x^3+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1-x-3^(1/2))/x/(-x^3+1)^(1/2),x)
```

```
[Out] 2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/(-3
/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)
/(-x^3+1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^
(1/2))^(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))+2/3*(3^(1/2)
-1)*arctanh((-x^3+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x+\sqrt{3}-1}{\sqrt{-x^3+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x + sqrt(3) - 1)/(sqrt(-x^3 + 1)*x),x, algorithm="maxima")
```

```
[Out] -integrate((x + sqrt(3) - 1)/(sqrt(-x^3 + 1)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{x+\sqrt{3}-1}{\sqrt{-x^3+1}x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x + sqrt(3) - 1)/(sqrt(-x^3 + 1)*x),x, algorithm="fricas")
```

```
[Out] integral(-(x + sqrt(3) - 1)/(sqrt(-x^3 + 1)*x), x)
```

Sympy [A] time = 8.17113, size = 99, normalized size = 0.7

$$-\frac{x\,\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^3e^{2i\pi}\right)}{3\,\Gamma\left(\frac{4}{3}\right)}-\sqrt{3}\left(\begin{cases}-\frac{2\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for } \left|\frac{1}{x^3}\right|>1\\ \frac{2i\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise}\end{cases}\right)+\begin{cases}-\frac{2\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for } \left|\frac{1}{x^3}\right|>1\\ \frac{2i\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3**(1/2))/x/(-x**3+1)**(1/2),x)
```

```
[Out] -x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(2*I*pi))/(
3*gamma(4/3)) - sqrt(3)*Piecewise((-2*acosh(x**(-3/2))/3, Abs(x**
(-3)) > 1), (2*I*asin(x**(-3/2))/3, True)) + Piecewise((-2*acosh(
x**(-3/2))/3, Abs(x**(-3)) > 1), (2*I*asin(x**(-3/2))/3, True))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x+\sqrt{3}-1}{\sqrt{-x^3+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x + sqrt(3) - 1)/(sqrt(-x^3 + 1)*x),x, algorithm="giac")
```

```
[Out] integrate(-(x + sqrt(3) - 1)/(sqrt(-x^3 + 1)*x), x)
```

3.135 $\int \frac{1-\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=144

$$\frac{2}{3}\left(1-\sqrt{3}\right)\tan^{-1}\left(\sqrt{x^3-1}\right)+\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

```
[Out] (2*(1 - Sqrt[3])*ArcTan[Sqrt[-1 + x^3]])/3 + (2*Sqrt[2 - Sqrt[3]]
*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin
[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3^(1/4)*
Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Rubi [A] time = 0.101898, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{2}{3}\left(1-\sqrt{3}\right)\tan^{-1}\left(\sqrt{x^3-1}\right)+\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - Sqrt[3] - x)/(x*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*(1 - Sqrt[3])*ArcTan[Sqrt[-1 + x^3]])/3 + (2*Sqrt[2 - Sqrt[3]]
*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin
[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3^(1/4)*
Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Rubi in Sympy [A] time = 12.321, size = 116, normalized size = 0.81

$$\left(-\frac{2\sqrt{3}}{3}+\frac{2}{3}\right)\operatorname{atan}\left(\sqrt{x^3-1}\right)+\frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{-\sqrt{3}+2}\,(-x+1)\,F\left(\operatorname{asin}\left(\frac{-x+1+\sqrt{3}}{-x-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{3\sqrt{\frac{x-1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1-x-3**(1/2))/x/(x**3-1)**(1/2),x)
```

```
[Out] (-2*sqrt(3)/3 + 2/3)*atan(sqrt(x**3 - 1)) + 2*3**(3/4)*sqrt((x**2
 + x + 1)/(-x - sqrt(3) + 1)**2)*sqrt(-sqrt(3) + 2)*(-x + 1)*elli
ptic_f(asin((-x + 1 + sqrt(3))/(-x - sqrt(3) + 1)), -7 + 4*sqrt(3
))/(3*sqrt((x - 1)/(-x - sqrt(3) + 1)**2)*sqrt(x**3 - 1))
```

Mathematica [A] time = 1.22984, size = 151, normalized size = 1.05

$$\frac{2}{3}\left(-\sqrt{3}\tan^{-1}\left(\sqrt{x^3-1}\right)+\tan^{-1}\left(\sqrt{x^3-1}\right)-\frac{3\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(x+\sqrt[3]{-1}\right)\sqrt{\frac{(-1)^{2/3}x+\sqrt[3]{-1}}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\sqrt{x^3-1}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - Sqrt[3] - x)/(x*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*(ArcTan[Sqrt[-1 + x^3]] - Sqrt[3]*ArcTan[Sqrt[-1 + x^3]] - (3*
Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((-1)^(1/3) + x)*Sqrt[((-1)^(1/3)
+ (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)
^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(Sqrt[(1 - (-1)^(2/3)*
x)/(1 + (-1)^(1/3))]*Sqrt[-1 + x^3])))/3
```

Maple [A] time = 0.019, size = 140, normalized size = 1.

$$-2\,\frac{-3/2-i/2\sqrt{3}}{\sqrt{x^3-1}}\sqrt{\frac{-1+x}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2-i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2+i/2\sqrt{3}}{3/2+i/2\sqrt{3}}}EllipticF\left(\sqrt{\frac{-1+x}{-3/2-i/2\sqrt{3}}},\sqrt{\frac{3/2+i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\right)-\frac{2\sqrt{3}}{3}\arctan\left(\sqrt{x^3-1}\right)+\frac{2}{3}\arctan\left(\sqrt{x^3-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1-x-3^(1/2))/x/(x^3-1)^(1/2),x)
```

```
[Out] -2*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1
/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2
))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((-1+x)/(-3
/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2))
)^(1/2))-2/3*arctan((x^3-1)^(1/2))*3^(1/2)+2/3*arctan((x^3-1)^(1/
2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x+\sqrt{3}-1}{\sqrt{x^3-1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x + sqrt(3) - 1)/(sqrt(x^3 - 1)*x),x, algorithm="maxima")
```

```
[Out] -integrate((x + sqrt(3) - 1)/(sqrt(x^3 - 1)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{x+\sqrt{3}-1}{\sqrt{x^3-1}x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x + sqrt(3) - 1)/(sqrt(x^3 - 1)*x),x, algorithm="fricas")
```

```
[Out] integral(-(x + sqrt(3) - 1)/(sqrt(x^3 - 1)*x), x)
```

Sympy [A] time = 8.25493, size = 94, normalized size = 0.65

$$\frac{ix\,\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|x^3\right)}{3\,\Gamma\left(\frac{4}{3}\right)}-\sqrt{3}\left(\begin{cases}\frac{2i\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for } \left|\frac{1}{x^3}\right|>1\\-\frac{2\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise}\end{cases}\right)+\begin{cases}\frac{2i\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for } \left|\frac{1}{x^3}\right|>1\\-\frac{2\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3**(1/2))/x/(x**3-1)**(1/2),x)
```

```
[Out] I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma(4/3)) - s
qrt(3)*Piecewise((2*I*acosh(x**(-3/2))/3, Abs(x**(-3)) > 1), (-2*
asin(x**(-3/2))/3, True)) + Piecewise((2*I*acosh(x**(-3/2))/3, Ab
s(x**(-3)) > 1), (-2*asin(x**(-3/2))/3, True))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x+\sqrt{3}-1}{\sqrt{x^3-1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x + sqrt(3) - 1)/(sqrt(x^3 - 1)*x),x, algorithm="giac")
```

```
[Out] integrate(-(x + sqrt(3) - 1)/(sqrt(x^3 - 1)*x), x)
```

3.136 $\int \frac{1-\sqrt{3}+x}{x\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=138

$$\frac{2}{3}\left(1-\sqrt{3}\right)\tan^{-1}\left(\sqrt{-x^3-1}\right)+\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

```
[Out] (2*(1 - Sqrt[3])*ArcTan[Sqrt[-1 - x^3]])/3 + (2*Sqrt[2 - Sqrt[3]]
*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin
[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3^(1/4)*
Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```

Rubi [A] time = 0.11038, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{2}{3}\left(1-\sqrt{3}\right)\tan^{-1}\left(\sqrt{-x^3-1}\right)+\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - Sqrt[3] + x)/(x*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*(1 - Sqrt[3])*ArcTan[Sqrt[-1 - x^3]])/3 + (2*Sqrt[2 - Sqrt[3]]
*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin
[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3^(1/4)*
Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```

Rubi in Sympy [A] time = 12.9296, size = 121, normalized size = 0.88

$$\left(-\frac{2\sqrt{3}}{3}+\frac{2}{3}\right)\operatorname{atan}\left(\sqrt{-x^3-1}\right)+\frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-\sqrt{3}+2}\,(x+1)\,F\left(\operatorname{asin}\left(\frac{x+1+\sqrt{3}}{x-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{3\sqrt{\frac{-x-1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+x-3**(1/2))/x/(-x**3-1)**(1/2),x)
```

```
[Out] (-2*sqrt(3)/3 + 2/3)*atan(sqrt(-x**3 - 1)) + 2*3**(3/4)*sqrt((x**
2 - x + 1)/(x - sqrt(3) + 1)**2)*sqrt(-sqrt(3) + 2)*(x + 1)*ellip
tic_f(asin((x + 1 + sqrt(3))/(x - sqrt(3) + 1)), -7 + 4*sqrt(3))/
(3*sqrt((-x - 1)/(x - sqrt(3) + 1)**2)*sqrt(-x**3 - 1))
```

Mathematica [A] time = 1.31409, size = 156, normalized size = 1.13

$$\frac{2}{3}\left(-\sqrt{3}\tan^{-1}\left(\sqrt{-x^3-1}\right)+\tan^{-1}\left(\sqrt{-x^3-1}\right)-\frac{3\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\sqrt{-x^3-1}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - Sqrt[3] + x)/(x*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*(ArcTan[Sqrt[-1 - x^3]] - Sqrt[3]*ArcTan[Sqrt[-1 - x^3]] - (3*
((-1)^(1/3) - x)*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*Sqrt[((-1)^(1/3)
- (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)
^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(Sqrt[(1 + (-1)^(2/3)*
x)/(1 + (-1)^(1/3))]*Sqrt[-1 - x^3])))/3
```

Maple [A] time = 0.019, size = 135, normalized size = 1.

$$-\frac{2\sqrt{3}}{3}\arctan\left(\sqrt{-x^3-1}\right)+\frac{2}{3}\arctan\left(\sqrt{-x^3-1}\right)$$
$$-\frac{2i}{3}\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\,EllipticF\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)\frac{1}{\sqrt{-x^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+x-3^(1/2))/x/(-x^3-1)^(1/2),x)
```

```
[Out] -2/3*arctan((-x^3-1)^(1/2))*3^(1/2)+2/3*arctan((-x^3-1)^(1/2))-2/
3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1
/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x
^3-1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2
))^(1/2),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x - \sqrt{3} + 1}{\sqrt{-x^3 - 1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x - sqrt(3) + 1)/(sqrt(-x^3 - 1)*x),x, algorithm="maxima")
```

```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(-x^3 - 1)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x - \sqrt{3} + 1}{\sqrt{-x^3 - 1}x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x - sqrt(3) + 1)/(sqrt(-x^3 - 1)*x),x, algorithm="fricas")
```

```
[Out] integral((x - sqrt(3) + 1)/(sqrt(-x^3 - 1)*x), x)
```

Sympy [A] time = 6.99394, size = 61, normalized size = 0.44

$$-\frac{ix\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\left(\frac{4}{3}\right)} - \frac{2\sqrt{3}i\operatorname{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} + \frac{2i\operatorname{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+x-3**(1/2))/x/(-x**3-1)**(1/2),x)
```

```
[Out] -I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(
3*gamma(4/3)) - 2*sqrt(3)*I*asinh(x**(-3/2))/3 + 2*I*asinh(x**(-3
/2))/3
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-\sqrt{3}+1}{\sqrt{-x^3-1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - sqrt(3) + 1)/(sqrt(-x^3 - 1)*x),x, algorithm="giac")
```

```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(-x^3 - 1)*x), x)
```

3.137 $\int \frac{x}{(3+x)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=334

$$-\frac{3(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\tan^{-1}\left(\frac{\sqrt{\frac{13}{2}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}}{\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}}\right)}{\sqrt{26}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{2\sqrt{2\left(97+56\sqrt{3}\right)}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{12\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(97-56\sqrt{3};-\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

```
[Out] (-3*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*ArcTan[(Sqrt[
13/2]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2])/Sqrt[(1 - x + x^2)/(1 +
Sqrt[3] + x)^2]])/(Sqrt[26]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqr
t[1 + x^3]) - (2*Sqrt[2*(97 + 56*Sqrt[3])]*(1 + x)*Sqrt[(1 - x +
x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 +
 Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[
3] + x)^2]*Sqrt[1 + x^3]) - (12*3^(1/4)*(1 + x)*Sqrt[(1 - x + x^2
)/(1 + Sqrt[3] + x)^2]*EllipticPi[97 - 56*Sqrt[3], -ArcSin[(1 - S
qrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[2 - Sqrt[3
]]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rubi [A] time = 1.39459, antiderivative size = 334, normalized size of antiderivative = 1., number of

steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$-\frac{3(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\tan^{-1}\left(\frac{\sqrt{\frac{13}{2}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}}{\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}}\right)}{\sqrt{26}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{2\sqrt{2\left(97+56\sqrt{3}\right)}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{12\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(97-56\sqrt{3};-\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/((3 + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (-3*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*ArcTan[(Sqrt[
13/2]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2])/Sqrt[(1 - x + x^2)/(1 +
Sqrt[3] + x)^2]])/(Sqrt[26]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqr
t[1 + x^3]) - (2*Sqrt[2*(97 + 56*Sqrt[3])]*(1 + x)*Sqrt[(1 - x +
x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 +
 Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[
3] + x)^2]*Sqrt[1 + x^3]) - (12*3^(1/4)*(1 + x)*Sqrt[(1 - x + x^2
)/(1 + Sqrt[3] + x)^2]*EllipticPi[97 - 56*Sqrt[3], -ArcSin[(1 - S
qrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[2 - Sqrt[3
]]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rubi in Sympy [A] time = 89.3319, size = 379, normalized size = 1.13

$$-\frac{3\sqrt{26}\sqrt{\frac{x^2-x+1}{\left(x+1+\sqrt{3}\right)^2}}\,(x+1)\,\mathrm{atan}\left(\frac{\sqrt{26}\cdot 3^{\frac{3}{4}}\sqrt{-\sqrt{3}+2}\sqrt{-\frac{\left(-x-1+\sqrt{3}\right)^2}{\left(x+1+\sqrt{3}\right)^2}+1}}{6\sqrt{\frac{\left(-x-1+\sqrt{3}\right)^2}{\left(x+1+\sqrt{3}\right)^2}-4\sqrt{3}+7}}\right)}{26\sqrt{\frac{x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{x^3+1}}-\frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{x^2-x+1}{\left(x+1+\sqrt{3}\right)^2}}\left(1+\sqrt{3}\right)\sqrt{\sqrt{3}+2}\,(x+1)\,F\left(\mathrm{asin}\left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{3\sqrt{\frac{x+1}{\left(x+1+\sqrt{3}\right)^2}}\left(-\sqrt{3}+2\right)\sqrt{x^3+1}}-\frac{12\sqrt[4]{3}\sqrt{\frac{x^2-x+1}{\left(x+1+\sqrt{3}\right)^2}}\,(x+1)\left(\frac{\left(-2+\sqrt{3}\right)^2}{\left(\sqrt{3}+2\right)^2};\mathrm{asin}\left(\frac{-x-1+\sqrt{3}}{x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{-4\sqrt{3}+7}\sqrt{-\sqrt{3}+2}\left(\sqrt{3}+2\right)\sqrt{x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(3+x)/(x**3+1)**(1/2),x)
```

```
[Out] -3*sqrt(26)*sqrt((x**2 - x + 1)/(x + 1 + sqrt(3))**2)*(x + 1)*ata
n(sqrt(26)*3**(3/4)*sqrt(-sqrt(3) + 2)*sqrt(-(-x - 1 + sqrt(3))**
2/(x + 1 + sqrt(3))**2 + 1)/(6*sqrt((-x - 1 + sqrt(3))**2/(x + 1
+ sqrt(3))**2 - 4*sqrt(3) + 7)))/(26*sqrt((x + 1)/(x + 1 + sqrt(3
))**2)*sqrt(x**3 + 1)) - 2*3**(3/4)*sqrt((x**2 - x + 1)/(x + 1 +
sqrt(3))**2)*(1 + sqrt(3))*sqrt(sqrt(3) + 2)*(x + 1)*elliptic_f(a
sin((x - sqrt(3) + 1)/(x + 1 + sqrt(3))), -7 - 4*sqrt(3))/(3*sqrt
((x + 1)/(x + 1 + sqrt(3))**2)*(-sqrt(3) + 2)*sqrt(x**3 + 1)) - 1
2*3**(1/4)*sqrt((x**2 - x + 1)/(x + 1 + sqrt(3))**2)*(x + 1)*elli
ptic_pi((-2 + sqrt(3))**2/(sqrt(3) + 2)**2, asin((-x - 1 + sqrt(3
))/(x + 1 + sqrt(3))), -7 - 4*sqrt(3))/(sqrt((x + 1)/(x + 1 + sqr
t(3))**2)*sqrt(-4*sqrt(3) + 7)*sqrt(-sqrt(3) + 2)*(sqrt(3) + 2)*s
qrt(x**3 + 1))
```

Mathematica [C] time = 0.382523, size = 194, normalized size = 0.58

$$\frac{2\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\left(-\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}}+\frac{3i\sqrt{x^2-x+1}\left(\frac{i\sqrt{3}}{3+\sqrt[3]{-1}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{3+\sqrt[3]{-1}}\right)}{\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x/((3 + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) - x)*Sqrt[((-1)
^(1/3) - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1
 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^
(2/3)*x)/(1 + (-1)^(1/3))]) + ((3*I)*Sqrt[1 - x + x^2]*EllipticPi
[(I*Sqrt[3])/(3 + (-1)^(1/3)), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1
+ (-1)^(1/3))]], (-1)^(1/3)])/(3 + (-1)^(1/3))))/Sqrt[1 + x^3]
```

Maple [A]   time = 0.009, size = 240, normalized size = 0.7

$$2\,{\frac {3/2-i/2\sqrt {3}}{\sqrt {{x}^{3}+1}}\sqrt {{\frac {1+x}{3/2-i/2\sqrt {3}}}}\sqrt {{\frac {x-1/2-i/2\sqrt {3}}{-3/2-i/2\sqrt {3}}}}\sqrt {{\frac {x-1/2+i/2\sqrt {3}}{-3/2+i/2\sqrt {3}}}}{\it EllipticF} \left(\sqrt {{\frac {1+x}{3/2-i/2\sqrt {3}}}},\sqrt {{\frac {-3/2+i/2\sqrt {3}}{-3/2-i/2\sqrt {3}}}} \right) }$$
$$-3\,{\frac {3/2-i/2\sqrt {3}}{\sqrt {{x}^{3}+1}}\sqrt {{\frac {1+x}{3/2-i/2\sqrt {3}}}}\sqrt {{\frac {x-1/2-i/2\sqrt {3}}{-3/2-i/2\sqrt {3}}}}\sqrt {{\frac {x-1/2+i/2\sqrt {3}}{-3/2+i/2\sqrt {3}}}}{\it EllipticPi} \left(\sqrt {{\frac {1+x}{3/2-i/2\sqrt {3}}}},-3/4+i/4\sqrt {3},\sqrt {{\frac {-3/2+i}{-3/2-i}}}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(3+x)/(x^3+1)^(1/2),x)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1
/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/
(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1
/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^
(1/2))-3*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((
x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^
(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticPi(((1+x
)/(3/2-1/2*I*3^(1/2)))^(1/2),-3/4+1/4*I*3^(1/2),((-3/2+1/2*I*3^(1
/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x^3+1}(x+3)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(x^3 + 1)*(x + 3)),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(x^3 + 1)*(x + 3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x}{\sqrt{x^3+1}(x+3)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(x^3 + 1)*(x + 3)),x, algorithm="fricas")
```

```
[Out] integral(x/(sqrt(x^3 + 1)*(x + 3)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{(x+1)(x^2-x+1)}\,(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(3+x)/(x**3+1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt((x + 1)*(x**2 - x + 1))*(x + 3)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x^3+1}(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(x^3 + 1)*(x + 3)),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(x^3 + 1)*(x + 3)), x)
```

3.138 $\int \frac{x}{(3+x)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=379

$$\frac{3(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\tanh^{-1}\left(\frac{\sqrt{7}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}}{2\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}}\right)}{2\sqrt{7}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}} - \frac{2\sqrt{2\left(37+20\sqrt{3}\right)}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{13\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}} - \frac{12\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\Pi\left(\frac{1}{169}\left(553+304\sqrt{3}\right);-\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{13\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

```
[Out] (3*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*ArcTanh[(Sqrt[
7]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2])/(2*Sqrt[(1 + x + x^2)/(1 +
Sqrt[3] - x)^2])]])/(2*Sqrt[7]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*S
qrt[1 - x^3]) - (2*Sqrt[2*(37 + 20*Sqrt[3])]*(1 - x)*Sqrt[(1 + x
+ x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1
 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(13*3^(1/4)*Sqrt[(1 - x)/(1 +
Sqrt[3] - x)^2]*Sqrt[1 - x^3]) - (12*3^(1/4)*Sqrt[2 + Sqrt[3]]*(1
 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticPi[(553 + 3
04*Sqrt[3])/169, -ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7
 - 4*Sqrt[3]])/(13*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3
])
```

Rubi [A] time = 1.61058, antiderivative size = 379, normalized size of antiderivative = 1., number of

steps used = 8, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$

$$\frac{3(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\tanh^{-1}\left(\frac{\sqrt{7}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}}{2\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}}\right)}{2\sqrt{7}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}} - \frac{2\sqrt{2\left(37+20\sqrt{3}\right)}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{13\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}} - \frac{12\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(\frac{1}{169}\left(553+304\sqrt{3}\right);-\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{13\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/((3 + x)*Sqrt[1 - x^3]),x]
```

```
[Out] (3*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*ArcTanh[(Sqrt[
7]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2])/(2*Sqrt[(1 + x + x^2)/(1 +
Sqrt[3] - x)^2])])/(2*Sqrt[7]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*S
qrt[1 - x^3]) - (2*Sqrt[2*(37 + 20*Sqrt[3])]*(1 - x)*Sqrt[(1 + x
+ x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1
 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(13*3^(1/4)*Sqrt[(1 - x)/(1 +
Sqrt[3] - x)^2]*Sqrt[1 - x^3]) - (12*3^(1/4)*Sqrt[2 + Sqrt[3]]*(1
 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticPi[(553 + 3
04*Sqrt[3])/169, -ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7
 - 4*Sqrt[3]])/(13*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3
])
```

Rubi in Sympy [A] time = 90.4734, size = 384, normalized size = 1.01

$$\frac{3\sqrt{7}\sqrt{\frac{x^2+x+1}{\left(-x+1+\sqrt{3}\right)^2}}\left(-x+1\right)\operatorname{atanh}\left(\frac{3^{\frac{3}{4}}\sqrt{7}\sqrt{1-\frac{\left(x-1+\sqrt{3}\right)^2}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{-\sqrt{3}+2}}{6\sqrt{-4\sqrt{3}+7+\frac{\left(x-1+\sqrt{3}\right)^2}{\left(-x+1+\sqrt{3}\right)^2}}}\right)}{14\sqrt{\frac{-x+1}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{-x^3+1}} - \frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{x^2+x+1}{\left(-x+1+\sqrt{3}\right)^2}}\left(1+\sqrt{3}\right)\sqrt{\sqrt{3}+2}\left(-x+1\right)F\left(\operatorname{asin}\left(\frac{-x-\sqrt{3}+1}{-x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{3\sqrt{\frac{-x+1}{\left(-x+1+\sqrt{3}\right)^2}}\left(\sqrt{3}+4\right)\sqrt{-x^3+1}} - \frac{12\sqrt[4]{3}\sqrt{\frac{x^2+x+1}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{-\sqrt{3}+2}\left(-x+1\right)\Pi\left(\frac{\left(\sqrt{3}+4\right)^2}{\left(-4+\sqrt{3}\right)^2};\operatorname{asin}\left(\frac{x-1+\sqrt{3}}{-x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt{\frac{-x+1}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{-4\sqrt{3}+7}\left(-\sqrt{3}+4\right)\left(\sqrt{3}+4\right)\sqrt{-x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(3+x)/(-x**3+1)**(1/2),x)
```

```
[Out] 3*sqrt(7)*sqrt((x**2 + x + 1)/(-x + 1 + sqrt(3))**2)*(-x + 1)*ata
nh(3**(3/4)*sqrt(7)*sqrt(1 - (x - 1 + sqrt(3))**2/(-x + 1 + sqrt(
3))**2)*sqrt(-sqrt(3) + 2)/(6*sqrt(-4*sqrt(3) + 7 + (x - 1 + sqrt
(3))**2/(-x + 1 + sqrt(3))**2)))/(14*sqrt((-x + 1)/(-x + 1 + sqrt
(3))**2)*sqrt(-x**3 + 1)) - 2*3**(3/4)*sqrt((x**2 + x + 1)/(-x +
1 + sqrt(3))**2)*(1 + sqrt(3))*sqrt(sqrt(3) + 2)*(-x + 1)*ellipti
c_f(asin((-x - sqrt(3) + 1)/(-x + 1 + sqrt(3))), -7 - 4*sqrt(3))/
(3*sqrt((-x + 1)/(-x + 1 + sqrt(3))**2)*(sqrt(3) + 4)*sqrt(-x**3
+ 1)) - 12*3**(1/4)*sqrt((x**2 + x + 1)/(-x + 1 + sqrt(3))**2)*sq
rt(-sqrt(3) + 2)*(-x + 1)*elliptic_pi((sqrt(3) + 4)**2/(-4 + sqrt
(3))**2, asin((x - 1 + sqrt(3))/(-x + 1 + sqrt(3))), -7 - 4*sqrt(
3))/(sqrt((-x + 1)/(-x + 1 + sqrt(3))**2)*sqrt(-4*sqrt(3) + 7)*(-
sqrt(3) + 4)*(sqrt(3) + 4)*sqrt(-x**3 + 1))
```

Mathematica [C] time = 0.399102, size = 195, normalized size = 0.51

$$\frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(\frac{\left(x+\sqrt[3]{-1}\right)\sqrt{\frac{(-1)^{2/3}x+\sqrt[3]{-1}}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{3i\sqrt{x^2+x+1}\,\Pi\left(\frac{2\sqrt{3}}{5i+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt[3]{-1}-3}\right)}{\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x/((3 + x)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((((-1)^(1/3) + x)*Sqrt[((-1)^(
1/3) + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 -
 (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2
/3)*x)/(1 + (-1)^(1/3))] + ((3*I)*Sqrt[1 + x + x^2]*EllipticPi[(2
*Sqrt[3])/(5*I + Sqrt[3]), ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-
1)^(1/3))]], (-1)^(1/3)])/(-3 + (-1)^(1/3))))/Sqrt[1 - x^3]
```

Maple [A] time = 0.011, size = 240, normalized size = 0.6

$$-\frac{2i}{3}\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}EllipticF\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)\frac{}{\sqrt{-}}$$
$$+\frac{2i\sqrt{3}}{\frac{5}{2}+\frac{i}{2}\sqrt{3}}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}EllipticPi\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\frac{i\sqrt{3}}{\frac{5}{2}+\frac{i}{2}\sqrt{3}},\sqrt{\frac{}{-\frac{3}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(3+x)/(-x^3+1)^(1/2),x)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/(-
3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2
)/(-x^3+1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3
^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))+2*I*3^(1/2)
*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/(-3/2+1/2*I*3^(1
/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/
2)/(5/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(
1/2))*3^(1/2))^(1/2),I*3^(1/2)/(5/2+1/2*I*3^(1/2)),(I*3^(1/2)/(-3
/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-x^3+1}(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(-x^3 + 1)*(x + 3)),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(-x^3 + 1)*(x + 3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x}{\sqrt{-x^3+1}(x+3)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(-x^3 + 1)*(x + 3)),x, algorithm="fricas")

[Out] integral(x/(sqrt(-x^3 + 1)*(x + 3)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-(x-1)(x^2+x+1)}(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(3+x)/(-x**3+1)**(1/2),x)

[Out] Integral(x/(sqrt(-(x - 1)*(x**2 + x + 1))*(x + 3)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-x^3+1}(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(-x^3 + 1)*(x + 3)),x, algorithm="giac")

[Out] integrate(x/(sqrt(-x^3 + 1)*(x + 3)), x)
```

3.139 $\int \frac{x}{(3+x)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=375

$$\frac{3(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\tanh^{-1}\left(\frac{\sqrt{7}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}}{2\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}}\right)}{2\sqrt{7}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}} - \frac{2\sqrt{2}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}} - \frac{12\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\Pi\left(\frac{1}{169}\left(553+304\sqrt{3}\right);-\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{13\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

```
[Out] (3*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*ArcTanh[(Sqrt[
7]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2])/(2*Sqrt[(1 + x + x^2)/(1 +
Sqrt[3] - x)^2])]])/(2*Sqrt[7]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*S
qrt[-1 + x^3]) - (2*Sqrt[2]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[
3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)],
 -7 + 4*Sqrt[3]])/(3^(1/4)*(4 + Sqrt[3])*Sqrt[-((1 - x)/(1 - Sqrt
[3] - x)^2)]*Sqrt[-1 + x^3]) - (12*3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 -
 x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticPi[(553 + 304
*Sqrt[3])/169, -ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 -
 4*Sqrt[3]])/(13*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[-1 + x^3]
)
```

Rubi [A] time = 1.50346, antiderivative size = 375, normalized size of antiderivative = 1., number of

steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$\frac{3(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\tanh^{-1}\left(\frac{\sqrt{7}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}}{2\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}}\right)}{2\sqrt{7}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}} - \frac{2\sqrt{2}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}} - \frac{12\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(\frac{1}{169}\left(553+304\sqrt{3}\right);-\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{13\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/((3 + x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (3*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*ArcTanh[(Sqrt[
7]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2])/(2*Sqrt[(1 + x + x^2)/(1 +
Sqrt[3] - x)^2])])/(2*Sqrt[7]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*S
qrt[-1 + x^3]) - (2*Sqrt[2]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[
3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)],
 -7 + 4*Sqrt[3]])/(3^(1/4)*(4 + Sqrt[3])*Sqrt[-((1 - x)/(1 - Sqrt
[3] - x)^2)]*Sqrt[-1 + x^3]) - (12*3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 -
 x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticPi[(553 + 304
*Sqrt[3])/169, -ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 -
 4*Sqrt[3]])/(13*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[-1 + x^3]
)
```

Rubi in Sympy [A] time = 88.0276, size = 381, normalized size = 1.02

$$\frac{3\sqrt{7}\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(-x+1\right)\operatorname{atan}\left(\frac{3^{\frac{3}{4}}\sqrt{7}\sqrt{\sqrt{3}+2}\sqrt{-\frac{\left(-x+1+\sqrt{3}\right)^2}{\left(x-1+\sqrt{3}\right)^2}+1}}{6\sqrt{\frac{\left(-x+1+\sqrt{3}\right)^2}{\left(x-1+\sqrt{3}\right)^2}+4\sqrt{3}+7}}\right)}{14\sqrt{\frac{x-1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}} - \frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(-\sqrt{3}+1\right)\sqrt{-\sqrt{3}+2}\left(-x+1\right)F\left(\operatorname{asin}\left(\frac{-x+1+\sqrt{3}}{-x-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{3\sqrt{\frac{x-1}{\left(-x-\sqrt{3}+1\right)^2}}\left(-\sqrt{3}+4\right)\sqrt{x^3-1}} + \frac{12\sqrt[4]{3}\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{\sqrt{3}+2}\left(-x+1\right)\Pi\left(\frac{\left(-4+\sqrt{3}\right)^2}{\left(\sqrt{3}+4\right)^2};\operatorname{asin}\left(\frac{-x+1+\sqrt{3}}{x-1+\sqrt{3}}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt{\frac{x-1}{\left(-x-\sqrt{3}+1\right)^2}}\left(-\sqrt{3}+4\right)\left(\sqrt{3}+4\right)\sqrt{4\sqrt{3}+7}\sqrt{x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(3+x)/(x**3-1)**(1/2),x)
```

```
[Out] 3*sqrt(7)*sqrt((x**2 + x + 1)/(-x - sqrt(3) + 1)**2)*(-x + 1)*ata
n(3**(3/4)*sqrt(7)*sqrt(sqrt(3) + 2)*sqrt(-(-x + 1 + sqrt(3))**2/
(x - 1 + sqrt(3))**2 + 1)/(6*sqrt((-x + 1 + sqrt(3))**2/(x - 1 +
sqrt(3))**2 + 4*sqrt(3) + 7)))/(14*sqrt((x - 1)/(-x - sqrt(3) + 1
)**2)*sqrt(x**3 - 1)) - 2*3**(3/4)*sqrt((x**2 + x + 1)/(-x - sqrt
(3) + 1)**2)*(-sqrt(3) + 1)*sqrt(-sqrt(3) + 2)*(-x + 1)*elliptic_
f(asin((-x + 1 + sqrt(3))/(-x - sqrt(3) + 1)), -7 + 4*sqrt(3))/(3
*sqrt((x - 1)/(-x - sqrt(3) + 1)**2)*(-sqrt(3) + 4)*sqrt(x**3 - 1
)) + 12*3**(1/4)*sqrt((x**2 + x + 1)/(-x - sqrt(3) + 1)**2)*sqrt(
sqrt(3) + 2)*(-x + 1)*elliptic_pi((-4 + sqrt(3))**2/(sqrt(3) + 4)
**2, asin((-x + 1 + sqrt(3))/(x - 1 + sqrt(3))), -7 + 4*sqrt(3))/
(sqrt((x - 1)/(-x - sqrt(3) + 1)**2)*(-sqrt(3) + 4)*(sqrt(3) + 4)
*sqrt(4*sqrt(3) + 7)*sqrt(x**3 - 1))
```

Mathematica [C] time = 0.396384, size = 193, normalized size = 0.51

$$\frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(\frac{\left(x+\sqrt[3]{-1}\right)\sqrt{\frac{(-1)^{2/3}x+\sqrt[3]{-1}}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{3i\sqrt{x^2+x+1}\,\Pi\left(\frac{2\sqrt{3}}{5i+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt[3]{-1}-3}\right)}{\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x/((3 + x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((((-1)^(1/3) + x)*Sqrt[((-1)^(
1/3) + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 -
 (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2
/3)*x)/(1 + (-1)^(1/3))] + ((3*I)*Sqrt[1 + x + x^2]*EllipticPi[(2
*Sqrt[3])/(5*I + Sqrt[3]), ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-
1)^(1/3))]], (-1)^(1/3)])/(-3 + (-1)^(1/3))))/Sqrt[-1 + x^3]
```

Maple [A] time = 0.009, size = 240, normalized size = 0.6

$$2\,\frac{-3/2-i/2\sqrt{3}}{\sqrt{x^3-1}}\sqrt{\frac{-1+x}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2-i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2+i/2\sqrt{3}}{3/2+i/2\sqrt{3}}}\,EllipticF\left(\sqrt{\frac{-1+x}{-3/2-i/2\sqrt{3}}},\sqrt{\frac{3/2+i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\right)$$
$$-\frac{-\frac{9}{2}-\frac{3i}{2}\sqrt{3}}{2}\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i}{2}\sqrt{3}}}\sqrt{\frac{1}{\frac{3}{2}-\frac{i}{2}\sqrt{3}}\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)}\sqrt{\frac{1}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)}\,EllipticPi\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i}{2}\sqrt{3}}},\frac{3}{8}+\frac{i}{8}\sqrt{3},\sqrt{\frac{\frac{3}{2}+}{\frac{3}{2}-}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(3+x)/(x^3-1)^(1/2),x)
```

```
[Out] 2*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/
2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2)
)/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((-1+x)/(-3/
2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))
^(1/2))-3/2*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1
/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2
*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticPi((
(-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),3/8+1/8*I*3^(1/2),((3/2+1/2*I*
3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x^3-1}(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(x^3 - 1)*(x + 3)),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(x^3 - 1)*(x + 3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x}{\sqrt{x^3-1}(x+3)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(x^3 - 1)*(x + 3)),x, algorithm="fricas")
```

```
[Out] integral(x/(sqrt(x^3 - 1)*(x + 3)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{(x-1)(x^2+x+1)}(x+3)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(3+x)/(x**3-1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt((x - 1)*(x**2 + x + 1))*(x + 3)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x^3-1}(x+3)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(x^3 - 1)*(x + 3)),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(x^3 - 1)*(x + 3)), x)
```

3.140 $\int \frac{x}{(3+x)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=343

$$-\frac{3(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\tan^{-1}\left(\frac{\sqrt{\frac{13}{2}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}}{\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}}\right)}{\sqrt{26}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{2\sqrt{14+8\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{12\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(97-56\sqrt{3};-\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

```
[Out] (-3*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*ArcTan[(Sqrt[
13/2]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2])/Sqrt[(1 - x + x^2)/(1 +
Sqrt[3] + x)^2]])/(Sqrt[26]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqr
t[-1 - x^3]) - (2*Sqrt[14 + 8*Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)
/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqr
t[3] + x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3]
 + x)^2)]*Sqrt[-1 - x^3]) - (12*3^(1/4)*(1 + x)*Sqrt[(1 - x + x^2
)/(1 + Sqrt[3] + x)^2]*EllipticPi[97 - 56*Sqrt[3], -ArcSin[(1 - S
qrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[2 - Sqrt[3
]]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[-1 - x^3])
```

Rubi [A] time = 1.45368, antiderivative size = 343, normalized size of antiderivative = 1., number of

steps used = 8, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$

$$-\frac{3(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\tan^{-1}\left(\frac{\sqrt{\frac{13}{2}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}}{\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}}\right)}{\sqrt{26}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{2\sqrt{2\left(7+4\sqrt{3}\right)}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{12\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(97-56\sqrt{3};-\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/((3 + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (-3*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*ArcTan[(Sqrt[
13/2]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2])/Sqrt[(1 - x + x^2)/(1 +
Sqrt[3] + x)^2]])/(Sqrt[26]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqr
t[-1 - x^3]) - (2*Sqrt[2*(7 + 4*Sqrt[3])]*(1 + x)*Sqrt[(1 - x + x
^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 -
Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt
[3] + x)^2)]*Sqrt[-1 - x^3]) - (12*3^(1/4)*(1 + x)*Sqrt[(1 - x +
x^2)/(1 + Sqrt[3] + x)^2]*EllipticPi[97 - 56*Sqrt[3], -ArcSin[(1
- Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[2 - Sqr
t[3]]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[-1 - x^3])
```

Rubi in Sympy [A] time = 90.658, size = 384, normalized size = 1.12

$$-\frac{3\sqrt{26}\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,(x+1)\operatorname{atanh}\left(\frac{\sqrt{26}\cdot 3^{\frac{3}{4}}\sqrt{1-\frac{\left(x+1+\sqrt{3}\right)^2}{\left(-x-1+\sqrt{3}\right)^2}}\sqrt{\sqrt{3}+2}}{6\sqrt{4\sqrt{3}+7+\frac{\left(x+1+\sqrt{3}\right)^2}{\left(-x-1+\sqrt{3}\right)^2}}}\right)}{26\sqrt{\frac{-x-1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(-\sqrt{3}+1\right)\sqrt{-\sqrt{3}+2}\,(x+1)\,F\left(\operatorname{asin}\left(\frac{x+1+\sqrt{3}}{x-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{3\sqrt{\frac{-x-1}{\left(x-\sqrt{3}+1\right)^2}}\left(\sqrt{3}+2\right)\sqrt{-x^3-1}}+\frac{12\sqrt[4]{3}\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,(x+1)\left(\frac{\left(\sqrt{3}+2\right)^2}{\left(-2+\sqrt{3}\right)^2};\operatorname{asin}\left(\frac{x+1+\sqrt{3}}{-x-1+\sqrt{3}}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt{\frac{-x-1}{\left(x-\sqrt{3}+1\right)^2}}\left(-\sqrt{3}+2\right)\sqrt{\sqrt{3}+2}\sqrt{4\sqrt{3}+7}\sqrt{-x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(3+x)/(-x**3-1)**(1/2),x)
```

```
[Out] -3*sqrt(26)*sqrt((x**2 - x + 1)/(x - sqrt(3) + 1)**2)*(x + 1)*ata
nh(sqrt(26)*3**(3/4)*sqrt(1 - (x + 1 + sqrt(3))**2/(-x - 1 + sqrt
(3))**2)*sqrt(sqrt(3) + 2)/(6*sqrt(4*sqrt(3) + 7 + (x + 1 + sqrt(
3))**2/(-x - 1 + sqrt(3))**2)))/(26*sqrt((-x - 1)/(x - sqrt(3) +
1)**2)*sqrt(-x**3 - 1)) - 2*3**(3/4)*sqrt((x**2 - x + 1)/(x - sqr
t(3) + 1)**2)*(-sqrt(3) + 1)*sqrt(-sqrt(3) + 2)*(x + 1)*elliptic_
f(asin((x + 1 + sqrt(3))/(x - sqrt(3) + 1)), -7 + 4*sqrt(3))/(3*s
qrt((-x - 1)/(x - sqrt(3) + 1)**2)*(sqrt(3) + 2)*sqrt(-x**3 - 1))
 + 12*3**(1/4)*sqrt((x**2 - x + 1)/(x - sqrt(3) + 1)**2)*(x + 1)*
elliptic_pi((sqrt(3) + 2)**2/(-2 + sqrt(3))**2, asin((x + 1 + sqr
t(3))/(-x - 1 + sqrt(3))), -7 + 4*sqrt(3))/(sqrt((-x - 1)/(x - sq
rt(3) + 1)**2)*(-sqrt(3) + 2)*sqrt(sqrt(3) + 2)*sqrt(4*sqrt(3) +
7)*sqrt(-x**3 - 1))
```

Mathematica [C] time = 0.406004, size = 196, normalized size = 0.57

$$\frac{2\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\left(-\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}}+\frac{3i\sqrt{x^2-x+1}\,\Pi\left(\frac{i\sqrt{3}}{3+\sqrt[3]{-1}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{3+\sqrt[3]{-1}}\right)}{\sqrt{-x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x/((3 + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) - x)*Sqrt[((-1)
^(1/3) - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1
 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^
(2/3)*x)/(1 + (-1)^(1/3))]) + ((3*I)*Sqrt[1 - x + x^2]*EllipticPi
[(I*Sqrt[3])/(3 + (-1)^(1/3)), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1
+ (-1)^(1/3))]], (-1)^(1/3)])/(3 + (-1)^(1/3))))/Sqrt[-1 - x^3]
```

Maple [A] time = 0.01, size = 240, normalized size = 0.7

$$-\frac{2i}{3}\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\mathit{EllipticF}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)\frac{1}{\sqrt{-x^3}}$$

$$+\frac{2i\sqrt{3}}{\frac{7}{2}+\frac{i}{2}\sqrt{3}}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\mathit{EllipticPi}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\frac{i\sqrt{3}}{\frac{7}{2}+\frac{i}{2}\sqrt{3}},\sqrt{\frac{i}{\frac{3}{2}+}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(3+x)/(-x^3-1)^(1/2),x)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/
2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/
(-x^3-1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(
1/2))^(1/2),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))+2*I*3^(1/2)*(I
*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^(1/2)))
^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)/(7
/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))
*3^(1/2))^(1/2),I*3^(1/2)/(7/2+1/2*I*3^(1/2)),(I*3^(1/2)/(3/2+1/2
*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-x^3-1}(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(-x^3 - 1)*(x + 3)),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(-x^3 - 1)*(x + 3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x}{\sqrt{-x^3-1}(x+3)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(-x^3 - 1)*(x + 3)),x, algorithm="fricas")

[Out] integral(x/(sqrt(-x^3 - 1)*(x + 3)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-(x+1)(x^2-x+1)}(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(3+x)/(-x**3-1)**(1/2),x)

[Out] Integral(x/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 3)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-x^3-1}(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(-x^3 - 1)*(x + 3)),x, algorithm="giac")

[Out] integrate(x/(sqrt(-x^3 - 1)*(x + 3)), x)
```

3.141 $\int \frac{e+fx}{(c+dx)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=452

$$\frac{(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(de-cf)\tan^{-1}\left(\frac{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{c^2+cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{c-d}}\right)}{\sqrt{d}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}\sqrt{c-d}\sqrt{c^2+cd+d^2}}+\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(de-cf)\Pi\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)^2}{\left(c-\left(1-\sqrt{3}\right)d\right)^2};-\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}\,(c^2-2cd-2d^2)}+\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(e-\sqrt{3}f-f\right)F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}\left(c-\sqrt{3}d-d\right)}$$

```
[Out] ((d*e - c*f)*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*ArcT
an[(Sqrt[c^2 + c*d + d^2]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2])/(Sqr
t[c - d]*Sqrt[d]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2])])/(Sqrt
[c - d]*Sqrt[d]*Sqrt[c^2 + c*d + d^2]*Sqrt[(1 + x)/(1 + Sqrt[3] +
 x)^2]*Sqrt[1 + x^3]) + (2*Sqrt[2 + Sqrt[3]]*(e - f - Sqrt[3]*f)*
(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[
(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*(
c - d - Sqrt[3]*d)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3
]) + (4*3^(1/4)*Sqrt[2 + Sqrt[3]]*(d*e - c*f)*(1 + x)*Sqrt[(1 - x
 + x^2)/(1 + Sqrt[3] + x)^2]*EllipticPi[(c - (1 + Sqrt[3])*d)^2/(
c - (1 - Sqrt[3])*d)^2, -ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] +
x)], -7 - 4*Sqrt[3]])/((c^2 - 2*c*d - 2*d^2)*Sqrt[(1 + x)/(1 + Sq
rt[3] + x)^2]*Sqrt[1 + x^3])
```

Rubi [A] time = 2.84697, antiderivative size = 452, normalized size of antiderivative = 1., number of

steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$

$$\frac{(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(de-cf)\tan^{-1}\left(\frac{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{c^2+cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{c-d}}\right)}{\sqrt{d}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}\sqrt{c-d}\sqrt{c^2+cd+d^2}} + \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(de-cf)\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)^2}{\left(c-\left(1-\sqrt{3}\right)d\right)^2};-\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}\left(c^2-2cd-2d^2\right)} + \frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(e-\sqrt{3}f-f\right)F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}\left(c-\sqrt{3}d-d\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[(e + f*x)/((c + d*x)*Sqrt[1 + x^3]),x]
```

```
[Out] ((d*e - c*f)*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*ArcT
an[(Sqrt[c^2 + c*d + d^2]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2])/(Sqr
t[c - d]*Sqrt[d]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2])])/(Sqrt
[c - d]*Sqrt[d]*Sqrt[c^2 + c*d + d^2]*Sqrt[(1 + x)/(1 + Sqrt[3] +
 x)^2]*Sqrt[1 + x^3]) + (2*Sqrt[2 + Sqrt[3]]*(e - f - Sqrt[3]*f)*
(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[
(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*(
c - d - Sqrt[3]*d)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3
]) + (4*3^(1/4)*Sqrt[2 + Sqrt[3]]*(d*e - c*f)*(1 + x)*Sqrt[(1 - x
 + x^2)/(1 + Sqrt[3] + x)^2]*EllipticPi[(c - (1 + Sqrt[3])*d)^2/(
c - (1 - Sqrt[3])*d)^2, -ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] +
x)], -7 - 4*Sqrt[3]])/((c^2 - 2*c*d - 2*d^2)*Sqrt[(1 + x)/(1 + Sq
rt[3] + x)^2]*Sqrt[1 + x^3])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x+e)/(d*x+c)/(x**3+1)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.927848, size = 211, normalized size = 0.47

$$\frac{2\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\left(-\frac{f\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}}+\frac{i\sqrt{x^2-x+1}(cf-de)\Pi\left(\frac{i\sqrt{3}d}{c+\sqrt[3]{-1}d};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{c+\sqrt[3]{-1}d}\right)}{d\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(e + f*x)/((c + d*x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((f*((-1)^(1/3) - x)*Sqrt[((-
1)^(1/3) - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[
(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1
)^(2/3)*x)/(1 + (-1)^(1/3))]) + (I*(-(d*e) + c*f)*Sqrt[1 - x + x^
2]*EllipticPi[(I*Sqrt[3]*d)/(c + (-1)^(1/3)*d), ArcSin[Sqrt[(1 +
(-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(c + (-1)^(1/3)*d)
))/(d*Sqrt[1 + x^3])
```

Maple [A] time = 0.011, size = 274, normalized size = 0.6

$$2\,\frac{f\left(3/2-i/2\sqrt{3}\right)}{d\sqrt{x^3+1}}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}EllipticF\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},\sqrt{\frac{-3/2+i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\right)$$
$$+2\,\frac{(-cf+de)\left(3/2-i/2\sqrt{3}\right)}{d^2\sqrt{x^3+1}}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}EllipticPi\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},(-3/2+i/2\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x+e)/(d*x+c)/(x^3+1)^(1/2),x)
```

```
[Out] 2/d*f*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1
/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/
2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3
/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2
)))^(1/2))+2*(-c*f+d*e)/d^2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I
*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/
2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/
```

```
2)/(-1+c/d)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/
2*I*3^(1/2))/(-1+c/d),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))
^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\sqrt{x^3+1}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x + e)/(sqrt(x^3 + 1)*(d*x + c)),x, algorithm="maxima")

[Out] integrate((f*x + e)/(sqrt(x^3 + 1)*(d*x + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x + e)/(sqrt(x^3 + 1)*(d*x + c)),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e+fx}{\sqrt{(x+1)(x^2-x+1)}\,(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x+e)/(d*x+c)/(x**3+1)**(1/2),x)

[Out] Integral((e + f*x)/(sqrt((x + 1)*(x**2 - x + 1))*(c + d*x)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx + e}{\sqrt{x^3 + 1}(dx + c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x + e)/(sqrt(x^3 + 1)*(d*x + c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)/(sqrt(x^3 + 1)*(d*x + c)), x)
```

3.142 $\int \frac{e+fx}{(c+dx)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=476

$$-\frac{(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}(de-cf)\tanh^{-1}\left(\frac{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{c^2-cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{c+d}}\right)}{\sqrt{d}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\sqrt{c+d}\sqrt{c^2-cd+d^2}}+\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}(de-cf)\,\Pi\left(\frac{\left(c+\sqrt{3}d+d\right)^2}{\left(c-\sqrt{3}d+d\right)^2};-\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\left(c^2+2cd-2d^2\right)}-\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(e+\sqrt{3}f+f\right)F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\left(c+\sqrt{3}d+d\right)}$$

```
[Out] -(((d*e - c*f)*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*Ar
cTanh[(Sqrt[c^2 - c*d + d^2]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2])/(
Sqrt[d]*Sqrt[c + d]*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2])])/(S
qrt[d]*Sqrt[c + d]*Sqrt[c^2 - c*d + d^2]*Sqrt[(1 - x)/(1 + Sqrt[3
] - x)^2]*Sqrt[1 - x^3])) - (2*Sqrt[2 + Sqrt[3]]*(e + f + Sqrt[3]
*f)*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[Arc
Sin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3^(1/
4)*(c + d + Sqrt[3]*d)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 -
 x^3]) + (4*3^(1/4)*Sqrt[2 + Sqrt[3]]*(d*e - c*f)*(1 - x)*Sqrt[(1
 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticPi[(c + d + Sqrt[3]*d)^2
/(c + d - Sqrt[3]*d)^2, -ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] -
x)], -7 - 4*Sqrt[3]])/((c^2 + 2*c*d - 2*d^2)*Sqrt[(1 - x)/(1 + Sq
rt[3] - x)^2]*Sqrt[1 - x^3])
```

Rubi [A] time = 2.83603, antiderivative size = 476, normalized size of antiderivative = 1., number of

steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$-\frac{(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}(de-cf)\tanh^{-1}\left(\frac{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{c^2-cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{c+d}}\right)}{\sqrt{d}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\sqrt{c+d}\sqrt{c^2-cd+d^2}}$$
$$+\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}(de-cf)\left(\frac{\left(c+\sqrt{3}d+d\right)^2}{\left(c-\sqrt{3}d+d\right)^2};-\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\left(c^2+2cd-2d^2\right)}$$
$$-\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(e+\sqrt{3}f+f\right)F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\left(c+\sqrt{3}d+d\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[(e + f*x)/((c + d*x)*Sqrt[1 - x^3]),x]
```

```
[Out] -(((d*e - c*f)*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*Ar
cTanh[(Sqrt[c^2 - c*d + d^2]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2])/(
Sqrt[d]*Sqrt[c + d]*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2])])/(S
qrt[d]*Sqrt[c + d]*Sqrt[c^2 - c*d + d^2]*Sqrt[(1 - x)/(1 + Sqrt[3
] - x)^2]*Sqrt[1 - x^3])) - (2*Sqrt[2 + Sqrt[3]]*(e + f + Sqrt[3]
*f)*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[Arc
Sin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3^(1/
4)*(c + d + Sqrt[3]*d)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 -
 x^3]) + (4*3^(1/4)*Sqrt[2 + Sqrt[3]]*(d*e - c*f)*(1 - x)*Sqrt[(1
 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticPi[(c + d + Sqrt[3]*d)^2
/(c + d - Sqrt[3]*d)^2, -ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] -
x)], -7 - 4*Sqrt[3]])/((c^2 + 2*c*d - 2*d^2)*Sqrt[(1 - x)/(1 + Sq
rt[3] - x)^2]*Sqrt[1 - x^3])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x+e)/(d*x+c)/(-x**3+1)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 1.22881, size = 233, normalized size = 0.49

$$\frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(\frac{3f\left(x+\sqrt[3]{-1}\right)\sqrt{\frac{(-1)^{2/3}x+\sqrt[3]{-1}}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{\sqrt[3]{-1}\sqrt{3}\left(1+\sqrt[3]{-1}\right)\sqrt{x^2+x+1}(cf-de)\left(\frac{i\sqrt{3}d}{\sqrt[3]{-1}d-c};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right.}{\sqrt[3]{-1}d-c}\right.}{3d\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(e + f*x)/((c + d*x)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((3*f*((-1)^(1/3) + x)*Sqrt[((-
1)^(1/3) + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[
(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1
)^(2/3)*x)/(1 + (-1)^(1/3))] + ((-1)^(1/3)*Sqrt[3]*(1 + (-1)^(1/3
))*(-(d*e) + c*f)*Sqrt[1 + x + x^2]*EllipticPi[(I*Sqrt[3]*d)/(-c
+ (-1)^(1/3)*d), ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]
], (-1)^(1/3)])/(-c + (-1)^(1/3)*d)))/(3*d*Sqrt[1 - x^3])
```

Maple [A] time = 0.011, size = 265, normalized size = 0.6

$$-\frac{\frac{2i}{3}f\sqrt{3}}{d}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}EllipticF\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)\frac{}{\sqrt{}}$$
$$-\frac{\frac{2i}{3}(-cf+de)\sqrt{3}}{d^2}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}EllipticPi\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},i\sqrt{3}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x+e)/(d*x+c)/(-x^3+1)^(1/2),x)
```

```
[Out] -2/3*I/d*f*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x
)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^
(1/2)/(-x^3+1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2
))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))-2/3*I*(
-c*f+d*e)/d^2*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-
1+x)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2
))^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)+c/d)*EllipticPi(1/3*3
^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/
```

```
2*I*3^(1/2)+c/d),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\sqrt{-x^3+1}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x + e)/(sqrt(-x^3 + 1)*(d*x + c)),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(-x^3 + 1)*(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{fx+e}{\sqrt{-x^3+1}(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x + e)/(sqrt(-x^3 + 1)*(d*x + c)),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)/(sqrt(-x^3 + 1)*(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e+fx}{\sqrt{-(x-1)(x^2+x+1)}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x+e)/(d*x+c)/(-x**3+1)**(1/2),x)
```

```
[Out] Integral((e + f*x)/(sqrt(-(x - 1)*(x**2 + x + 1))*(c + d*x)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\sqrt{-x^3+1}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x + e)/(sqrt(-x^3 + 1)*(d*x + c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)/(sqrt(-x^3 + 1)*(d*x + c)), x)
```

3.143 $\int \frac{e+fx}{(c+dx)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=477

$$-\frac{(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}(de-cf)\tanh^{-1}\left(\frac{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{c^2-cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{c+d}}\right)}{\sqrt{d}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\sqrt{c+d}\sqrt{c^2-cd+d^2}}+\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}(de-cf)\Pi\left(\frac{\left(c+\sqrt{3}d+d\right)^2}{\left(c-\sqrt{3}d+d\right)^2};-\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\,(c^2+2cd-2d^2)}-\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(e+\sqrt{3}f+f\right)F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\left(c+\sqrt{3}d+d\right)}$$

```
[Out] -(((d*e - c*f)*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*Ar
cTanh[(Sqrt[c^2 - c*d + d^2]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2])/(
Sqrt[d]*Sqrt[c + d]*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2])])/(S
qrt[d]*Sqrt[c + d]*Sqrt[c^2 - c*d + d^2]*Sqrt[(1 - x)/(1 + Sqrt[3
] - x)^2]*Sqrt[-1 + x^3])) - (2*Sqrt[2 - Sqrt[3]]*(e + f + Sqrt[3
]*f)*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[Ar
cSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3^(1
/4)*(c + d + Sqrt[3]*d)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt
[-1 + x^3]) + (4*3^(1/4)*Sqrt[2 + Sqrt[3]]*(d*e - c*f)*(1 - x)*Sq
rt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticPi[(c + d + Sqrt[3]
*d)^2/(c + d - Sqrt[3]*d)^2, -ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[
3] - x)], -7 - 4*Sqrt[3]])/((c^2 + 2*c*d - 2*d^2)*Sqrt[(1 - x)/(1
 + Sqrt[3] - x)^2]*Sqrt[-1 + x^3])
```

Rubi [A] time = 2.38285, antiderivative size = 477, normalized size of antiderivative = 1., number of

steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$

$$-\frac{(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}(de-cf)\tanh^{-1}\left(\frac{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{c^2-cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{c+d}}\right)}{\sqrt{d}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\sqrt{c+d}\sqrt{c^2-cd+d^2}}+\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}(de-cf)\Pi\left(\frac{\left(c+\sqrt{3}d+d\right)^2}{\left(c-\sqrt{3}d+d\right)^2};-\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\left(c^2+2cd-2d^2\right)}-\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(e+\sqrt{3}f+f\right)F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\left(c+\sqrt{3}d+d\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[(e + f*x)/((c + d*x)*Sqrt[-1 + x^3]),x]
```

```
[Out] -(((d*e - c*f)*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*Ar
cTanh[(Sqrt[c^2 - c*d + d^2]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2])/(
Sqrt[d]*Sqrt[c + d]*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2])])/(S
qrt[d]*Sqrt[c + d]*Sqrt[c^2 - c*d + d^2]*Sqrt[(1 - x)/(1 + Sqrt[3
] - x)^2]*Sqrt[-1 + x^3])) - (2*Sqrt[2 - Sqrt[3]]*(e + f + Sqrt[3
]*f)*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[Ar
cSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3^(1
/4)*(c + d + Sqrt[3]*d)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt
[-1 + x^3]) + (4*3^(1/4)*Sqrt[2 + Sqrt[3]]*(d*e - c*f)*(1 - x)*Sq
rt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticPi[(c + d + Sqrt[3]
*d)^2/(c + d - Sqrt[3]*d)^2, -ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[
3] - x)], -7 - 4*Sqrt[3]])/((c^2 + 2*c*d - 2*d^2)*Sqrt[(1 - x)/(1
 + Sqrt[3] - x)^2]*Sqrt[-1 + x^3])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x+e)/(d*x+c)/(x**3-1)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 1.1988, size = 231, normalized size = 0.48

$$\frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(\frac{3f\left(x+\sqrt[3]{-1}\right)\sqrt{\frac{(-1)^{2/3}x+\sqrt[3]{-1}}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{\sqrt[3]{-1}\sqrt{3}\left(1+\sqrt[3]{-1}\right)\sqrt{x^2+x+1}(cf-de)\left(\frac{i\sqrt{3}d}{\sqrt[3]{-1}d-c};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right.}{\sqrt[3]{-1}d-c}}{3d\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(e + f*x)/((c + d*x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((3*f*((-1)^(1/3) + x)*Sqrt[((-
1)^(1/3) + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[
(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1
)^(2/3)*x)/(1 + (-1)^(1/3))] + ((-1)^(1/3)*Sqrt[3]*(1 + (-1)^(1/3
))*(-(d*e) + c*f)*Sqrt[1 + x + x^2]*EllipticPi[(I*Sqrt[3]*d)/(-c
+ (-1)^(1/3)*d), ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]
], (-1)^(1/3)])/(-c + (-1)^(1/3)*d)))/(3*d*Sqrt[-1 + x^3])
```

Maple [A] time = 0.011, size = 274, normalized size = 0.6

$$2\,\frac{f\left(-3/2-i/2\sqrt{3}\right)}{d\sqrt{x^3-1}}\sqrt{\frac{-1+x}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2-i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2+i/2\sqrt{3}}{3/2+i/2\sqrt{3}}}\mathit{EllipticF}\left(\sqrt{\frac{-1+x}{-3/2-i/2\sqrt{3}}},\sqrt{\frac{3/2+i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\right)$$
$$+2\,\frac{(-cf+de)\left(-3/2-i/2\sqrt{3}\right)}{d^2\sqrt{x^3-1}}\sqrt{\frac{-1+x}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2-i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2+i/2\sqrt{3}}{3/2+i/2\sqrt{3}}}\mathit{EllipticPi}\left(\sqrt{\frac{-1+x}{-3/2-i/2\sqrt{3}}},(3/2+i/2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x+e)/(d*x+c)/(x^3-1)^(1/2),x)
```

```
[Out] 2/d*f*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((
x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(
1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((-1+x)/
(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/
2)))^(1/2))+2*(-c*f+d*e)/d^2*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1
/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^
(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(
```

```
1/2)/(1+c/d)*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+
1/2*I*3^(1/2))/(1+c/d),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^
(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\sqrt{x^3-1}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x + e)/(sqrt(x^3 - 1)*(d*x + c)),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(x^3 - 1)*(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{fx+e}{\sqrt{x^3-1}(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x + e)/(sqrt(x^3 - 1)*(d*x + c)),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)/(sqrt(x^3 - 1)*(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e+fx}{\sqrt{(x-1)(x^2+x+1)}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x+e)/(d*x+c)/(x**3-1)**(1/2),x)
```

```
[Out] Integral((e + f*x)/(sqrt((x - 1)*(x**2 + x + 1))*(c + d*x)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\sqrt{x^3-1}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x + e)/(sqrt(x^3 - 1)*(d*x + c)),x, algorithm="giac")

[Out] integrate((f*x + e)/(sqrt(x^3 - 1)*(d*x + c)), x)
```

3.144 $\int \frac{e+fx}{(c+dx)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=465

$$\frac{(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(de-cf)\tan^{-1}\left(\frac{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{c^2+cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{c-d}}\right)}{\sqrt{d}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\sqrt{c-d}\sqrt{c^2+cd+d^2}}+\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(de-cf)\Pi\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)^2}{\left(c-\left(1-\sqrt{3}\right)d\right)^2};-\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\left(c^2-2cd-2d^2\right)}+\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(e-\sqrt{3}f-f\right)F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\left(c-\sqrt{3}d-d\right)}$$

```
[Out] ((d*e - c*f)*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*ArcT
an[(Sqrt[c^2 + c*d + d^2]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2])/(Sqr
t[c - d]*Sqrt[d]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2])])/(Sqrt
[c - d]*Sqrt[d]*Sqrt[c^2 + c*d + d^2]*Sqrt[(1 + x)/(1 + Sqrt[3] +
 x)^2]*Sqrt[-1 - x^3]) + (2*Sqrt[2 - Sqrt[3]]*(e - f - Sqrt[3]*f)
*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin
[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3^(1/4)*
(c - d - Sqrt[3]*d)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1
- x^3]) + (4*3^(1/4)*Sqrt[2 + Sqrt[3]]*(d*e - c*f)*(1 + x)*Sqrt[(
1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticPi[(c - (1 + Sqrt[3])*d
)^2/(c - (1 - Sqrt[3])*d)^2, -ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[
3] + x)], -7 - 4*Sqrt[3]])/((c^2 - 2*c*d - 2*d^2)*Sqrt[(1 + x)/(1
 + Sqrt[3] + x)^2]*Sqrt[-1 - x^3])
```

Rubi [A] time = 3.02387, antiderivative size = 465, normalized size of antiderivative = 1., number of

steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(de-cf)\tan^{-1}\left(\frac{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{c^2+cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{c-d}}\right)}{\sqrt{d}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\sqrt{c-d}\sqrt{c^2+cd+d^2}} + \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(de-cf)\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)^2}{\left(c-\left(1-\sqrt{3}\right)d\right)^2};-\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\,(c^2-2cd-2d^2)} + \frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(e-\sqrt{3}f-f\right)F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\left(c-\sqrt{3}d-d\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[(e + f*x)/((c + d*x)*Sqrt[-1 - x^3]),x]
```

```
[Out] ((d*e - c*f)*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*ArcT
an[(Sqrt[c^2 + c*d + d^2]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2])/(Sqr
t[c - d]*Sqrt[d]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2])])/(Sqrt
[c - d]*Sqrt[d]*Sqrt[c^2 + c*d + d^2]*Sqrt[(1 + x)/(1 + Sqrt[3] +
 x)^2]*Sqrt[-1 - x^3]) + (2*Sqrt[2 - Sqrt[3]]*(e - f - Sqrt[3]*f)
*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin
[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3^(1/4)*
(c - d - Sqrt[3]*d)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1
- x^3]) + (4*3^(1/4)*Sqrt[2 + Sqrt[3]]*(d*e - c*f)*(1 + x)*Sqrt[(
1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticPi[(c - (1 + Sqrt[3])*d
)^2/(c - (1 - Sqrt[3])*d)^2, -ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[
3] + x)], -7 - 4*Sqrt[3]])/((c^2 - 2*c*d - 2*d^2)*Sqrt[(1 + x)/(1
 + Sqrt[3] + x)^2]*Sqrt[-1 - x^3])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x+e)/(d*x+c)/(-x**3-1)**(1/2),x)
```

[Out] Timed out

Mathematica [C] time = 0.935545, size = 213, normalized size = 0.46

$$\frac{2\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\left(-\frac{f\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}}+\frac{i\sqrt{x^2-x+1}(cf-de)\left(\frac{i\sqrt{3}d}{c+\sqrt[3]{-1}d};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{c+\sqrt[3]{-1}d}\right)}{d\sqrt{-x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(e + f*x)/((c + d*x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((f*((-1)^(1/3) - x)*Sqrt[((-
1)^(1/3) - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[
(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1
)^(2/3)*x)/(1 + (-1)^(1/3))]) + (I*(-(d*e) + c*f)*Sqrt[1 - x + x^
2]*EllipticPi[(I*Sqrt[3]*d)/(c + (-1)^(1/3)*d), ArcSin[Sqrt[(1 +
(-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(c + (-1)^(1/3)*d)
))/(d*Sqrt[-1 - x^3])
```

Maple [A] time = 0.011, size = 265, normalized size = 0.6

$$\frac{-\frac{2i}{3}f\sqrt{3}}{d}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}EllipticF\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)\frac{}{\sqrt{-x}}$$

$$-\frac{\frac{2i}{3}(-cf+de)\sqrt{3}}{d^2}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}EllipticPi\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},i\sqrt{3}\left(\frac{1}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x+e)/(d*x+c)/(-x^3-1)^(1/2),x)
```

```
[Out] -2/3*I/d*f*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)
/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1
/2)/(-x^3-1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))
*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))-2/3*I*(-c*
f+d*e)/d^2*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)
/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1
/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2)+c/d)*EllipticPi(1/3*3^(1/2)
*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(
```

```
1/2)+c/d),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\sqrt{-x^3-1}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x + e)/(sqrt(-x^3 - 1)*(d*x + c)),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(-x^3 - 1)*(d*x + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x + e)/(sqrt(-x^3 - 1)*(d*x + c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e+fx}{\sqrt{-(x+1)(x^2-x+1)}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(-x**3-1)**(1/2),x)
```

```
[Out] Integral((e + f*x)/(sqrt(-(x + 1)*(x**2 - x + 1))*(c + d*x)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\sqrt{-x^3-1}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x + e)/(sqrt(-x^3 - 1)*(d*x + c)),x, algorithm="giac")

[Out] integrate((f*x + e)/(sqrt(-x^3 - 1)*(d*x + c)), x)
```

3.145 $\int \frac{e+fx}{x\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=120

$$\frac{2\sqrt{2+\sqrt{3}}f(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{2}{3}e\tanh^{-1}\left(\sqrt{x^3+1}\right)$$

```
[Out] (-2*e*ArcTanh[Sqrt[1 + x^3]])/3 + (2*Sqrt[2 + Sqrt[3]]*f*(1 + x)*
Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqr
t[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 +
 x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rubi [A] time = 0.112841, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{2\sqrt{2+\sqrt{3}}f(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{2}{3}e\tanh^{-1}\left(\sqrt{x^3+1}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(e + f*x)/(x*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*e*ArcTanh[Sqrt[1 + x^3]])/3 + (2*Sqrt[2 + Sqrt[3]]*f*(1 + x)*
Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqr
t[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 +
 x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rubi in Sympy [A] time = 11.0902, size = 112, normalized size = 0.93

$$-\frac{2e\,\mathrm{atanh}\left(\sqrt{x^3+1}\right)}{3}+\frac{2\cdot 3^{\frac{3}{4}}f\sqrt{\frac{x^2-x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{\sqrt{3}+2}\,(x+1)\,F\left(\mathrm{asin}\left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{3\sqrt{\frac{x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x+e)/x/(x**3+1)**(1/2),x)
```

```
[Out] -2*e*atanh(sqrt(x**3 + 1))/3 + 2*3**(3/4)*f*sqrt((x**2 - x + 1)/(
x + 1 + sqrt(3))**2)*sqrt(sqrt(3) + 2)*(x + 1)*elliptic_f(asin((x
 - sqrt(3) + 1)/(x + 1 + sqrt(3))), -7 - 4*sqrt(3))/(3*sqrt((x +
1)/(x + 1 + sqrt(3))**2)*sqrt(x**3 + 1))
```

Mathematica [A] time = 0.608211, size = 134, normalized size = 1.12

$$-\frac{2}{3}e\tanh^{-1}\left(\sqrt{x^3+1}\right)-\frac{2f\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\sqrt{-\frac{(-1)^{2/3}\left(x+(-1)^{2/3}\right)}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(e + f*x)/(x*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*e*ArcTanh[Sqrt[1 + x^3]])/3 - (2*f*((-1)^(1/3) - x)*Sqrt[(1 +
 x)/(1 + (-1)^(1/3))]*Sqrt[-(((-1)^(2/3)*((-1)^(2/3) + x))/(1 + (
-1)^(1/3)))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(
1/3))]], (-1)^(1/3)])/(Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*
Sqrt[1 + x^3])
```

Maple [A] time = 0.009, size = 129, normalized size = 1.1

$$2\,\frac{f\left(3/2-i/2\sqrt{3}\right)}{\sqrt{x^3+1}}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}EllipticF\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},\sqrt{\frac{-3/2+i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\right)-\frac{2\,e}{3}Artanh\left(\sqrt{x^3+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x+e)/x/(x^3+1)^(1/2),x)
```

```
[Out] 2*f*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2
-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2)
)/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2
-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2))
)^(1/2))-2/3*e*arctanh((x^3+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx + e}{\sqrt{x^3 + 1}x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x + e)/(sqrt(x^3 + 1)*x),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(x^3 + 1)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{fx + e}{\sqrt{x^3 + 1}x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x + e)/(sqrt(x^3 + 1)*x),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)/(sqrt(x^3 + 1)*x), x)
```

Sympy [A] time = 5.34849, size = 42, normalized size = 0.35

$$-\frac{2e \operatorname{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} + \frac{fx\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix}\frac{1}{3}, \frac{1}{2} \\ \frac{4}{3}\end{matrix}\middle| x^3 e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x+e)/x/(x**3+1)**(1/2),x)
```

```
[Out] -2*e*asinh(x**(-3/2))/3 + f*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,)
, x**3*exp_polar(I*pi))/(3*gamma(4/3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx + e}{\sqrt{x^3 + 1}x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x + e)/(sqrt(x^3 + 1)*x),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)/(sqrt(x^3 + 1)*x), x)
```

3.146 $\int \frac{e+fx}{x\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=134

$$-\frac{2}{3}e\tanh^{-1}\left(\sqrt{1-x^3}\right)-\frac{2\sqrt{2+\sqrt{3}}f(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

```
[Out] (-2*e*ArcTanh[Sqrt[1 - x^3]])/3 - (2*Sqrt[2 + Sqrt[3]]*f*(1 - x)*
Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqr
t[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 -
 x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rubi [A] time = 0.120678, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$-\frac{2}{3}e\tanh^{-1}\left(\sqrt{1-x^3}\right)-\frac{2\sqrt{2+\sqrt{3}}f(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(e + f*x)/(x*Sqrt[1 - x^3]),x]
```

```
[Out] (-2*e*ArcTanh[Sqrt[1 - x^3]])/3 - (2*Sqrt[2 + Sqrt[3]]*f*(1 - x)*
Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqr
t[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 -
 x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rubi in Sympy [A] time = 12.2907, size = 114, normalized size = 0.85

$$-\frac{2e\,\mathrm{atanh}\left(\sqrt{-x^3+1}\right)}{3}-\frac{2\cdot 3^{\frac{3}{4}}f\sqrt{\frac{x^2+x+1}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{\sqrt{3}+2}\,(-x+1)\,F\left(\mathrm{asin}\left(\frac{-x-\sqrt{3}+1}{-x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{3\sqrt{\frac{-x+1}{\left(-x+1+\sqrt{3}\right)^2}}\sqrt{-x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x+e)/x/(-x**3+1)**(1/2),x)
```

```
[Out] -2*e*atanh(sqrt(-x**3 + 1))/3 - 2*3**(3/4)*f*sqrt((x**2 + x + 1)/
(-x + 1 + sqrt(3))**2)*sqrt(sqrt(3) + 2)*(-x + 1)*elliptic_f(asin
((-x - sqrt(3) + 1)/(-x + 1 + sqrt(3))), -7 - 4*sqrt(3))/(3*sqrt(
(-x + 1)/(-x + 1 + sqrt(3))**2)*sqrt(-x**3 + 1))
```

Mathematica [A] time = 0.602799, size = 140, normalized size = 1.04

$$\frac{2f\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(x+\sqrt[3]{-1}\right)\sqrt{\frac{(-1)^{2/3}x+\sqrt[3]{-1}}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\sqrt{1-x^3}}-\frac{2}{3}e\tanh^{-1}\left(\sqrt{1-x^3}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(e + f*x)/(x*Sqrt[1 - x^3]),x]
```

```
[Out] (-2*e*ArcTanh[Sqrt[1 - x^3]])/3 + (2*f*Sqrt[(1 - x)/(1 + (-1)^(1/
3))]*((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) + (-1)^(2/3)*x)/(1 + (-1)^
(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))
]], (-1)^(1/3)])/(Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*Sqrt[
1 - x^3])
```

Maple [A] time = 0.008, size = 122, normalized size = 0.9

$$-\frac{2i}{3}f\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\,EllipticF\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)$$
$$-\frac{2e}{3}Artanh\left(\sqrt{-x^3+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x+e)/x/(-x^3+1)^(1/2),x)
```

```
[Out] -2/3*I*f*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/
(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1
/2)/(-x^3+1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))
*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))-2/3*e*arc
tanh((-x^3+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\sqrt{-x^3+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x + e)/(sqrt(-x^3 + 1)*x),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(-x^3 + 1)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{fx+e}{\sqrt{-x^3+1}x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x + e)/(sqrt(-x^3 + 1)*x),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)/(sqrt(-x^3 + 1)*x), x)
```

Sympy [A] time = 5.58772, size = 65, normalized size = 0.49

$$e\left(\begin{cases} -\frac{2\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for } \left|\frac{1}{x^3}\right| > 1 \\ \frac{2i\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise} \end{cases}\right) + \frac{fx\,\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3e^{2i\pi}\right)}{3\,\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x+e)/x/(-x**3+1)**(1/2),x)
```

```
[Out] e*Piecewise((-2*acosh(x**(-3/2))/3, Abs(x**(-3)) > 1), (2*I*asin(
x**(-3/2))/3, True)) + f*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x
**3*exp_polar(2*I*pi))/(3*gamma(4/3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\sqrt{-x^3+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x + e)/(sqrt(-x^3 + 1)*x),x, algorithm="giac")

[Out] integrate((f*x + e)/(sqrt(-x^3 + 1)*x), x)
```

3.147 $\int \frac{e+fx}{x\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=137

$$\frac{2}{3}e\tan^{-1}\left(\sqrt{x^3-1}\right)-\frac{2\sqrt{2-\sqrt{3}}f(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

```
[Out] (2*e*ArcTan[Sqrt[-1 + x^3]])/3 - (2*Sqrt[2 - Sqrt[3]]*f*(1 - x)*S
qrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt
[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1
- x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Rubi [A] time = 0.121515, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{2}{3}e\tan^{-1}\left(\sqrt{x^3-1}\right)-\frac{2\sqrt{2-\sqrt{3}}f(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(e + f*x)/(x*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*e*ArcTan[Sqrt[-1 + x^3]])/3 - (2*Sqrt[2 - Sqrt[3]]*f*(1 - x)*S
qrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt
[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1
- x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Rubi in Sympy [A] time = 11.4041, size = 110, normalized size = 0.8

$$\frac{2e\,\mathrm{atan}\left(\sqrt{x^3-1}\right)}{3}-\frac{2\cdot 3^{\frac{3}{4}}f\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{-\sqrt{3}+2}\,(-x+1)\,F\left(\mathrm{asin}\left(\frac{-x+1+\sqrt{3}}{-x-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{3\sqrt{\frac{x-1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x+e)/x/(x**3-1)**(1/2),x)
```

```
[Out] 2*e*atan(sqrt(x**3 - 1))/3 - 2*3**(3/4)*f*sqrt((x**2 + x + 1)/(-x
 - sqrt(3) + 1)**2)*sqrt(-sqrt(3) + 2)*(-x + 1)*elliptic_f(asin((
-x + 1 + sqrt(3))/(-x - sqrt(3) + 1)), -7 + 4*sqrt(3))/(3*sqrt((x
 - 1)/(-x - sqrt(3) + 1)**2)*sqrt(x**3 - 1))
```

Mathematica [A] time = 0.490426, size = 136, normalized size = 0.99

$$\frac{2}{3}e\tan^{-1}\left(\sqrt{x^3-1}\right)+\frac{2f\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(x+\sqrt[3]{-1}\right)\sqrt{\frac{(-1)^{2/3}x+\sqrt[3]{-1}}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(e + f*x)/(x*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*e*ArcTan[Sqrt[-1 + x^3]])/3 + (2*f*Sqrt[(1 - x)/(1 + (-1)^(1/3
))]*((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) + (-1)^(2/3)*x)/(1 + (-1)^(
1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]
], (-1)^(1/3)])/(Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*Sqrt[-
1 + x^3])
```

Maple [A] time = 0.008, size = 129, normalized size = 0.9

$$2\,\frac{f\left(-3/2-i/2\sqrt{3}\right)}{\sqrt{x^3-1}}\sqrt{\frac{-1+x}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2-i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2+i/2\sqrt{3}}{3/2+i/2\sqrt{3}}}EllipticF\left(\sqrt{\frac{-1+x}{-3/2-i/2\sqrt{3}}},\sqrt{\frac{3/2+i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\right)+\frac{2\,e}{3}\arctan\left(\sqrt{x^3-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x+e)/x/(x^3-1)^(1/2),x)
```

```
[Out] 2*f*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+
1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/
2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((-1+x)/(-
3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)
))^(1/2))+2/3*e*arctan((x^3-1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\sqrt{x^3-1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x + e)/(sqrt(x^3 - 1)*x),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(x^3 - 1)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{fx+e}{\sqrt{x^3-1}x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x + e)/(sqrt(x^3 - 1)*x),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)/(sqrt(x^3 - 1)*x), x)
```

Sympy [A] time = 5.49916, size = 60, normalized size = 0.44

$$e\left(\begin{cases} \frac{2i\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for}\ \left|\frac{1}{x^3}\right|>1 \\ -\frac{2\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise} \end{cases}\right) - \frac{ifx\,\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^3\right)}{3\,\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x+e)/x/(x**3-1)**(1/2),x)
```

```
[Out] e*Piecewise((2*I*acosh(x**(-3/2))/3, Abs(x**(-3)) > 1), (-2*asin(
x**(-3/2))/3, True)) - I*f*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,),
 x**3)/(3*gamma(4/3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx + e}{\sqrt{x^3 - 1}x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x + e)/(sqrt(x^3 - 1)*x),x, algorithm="giac")

[Out] integrate((f*x + e)/(sqrt(x^3 - 1)*x), x)
```

3.148 $\int \frac{e+fx}{x\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=131

$$\frac{2}{3}e\tan^{-1}\left(\sqrt{-x^3-1}\right)+\frac{2\sqrt{2-\sqrt{3}}f(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

```
[Out] (2*e*ArcTan[Sqrt[-1 - x^3]])/3 + (2*Sqrt[2 - Sqrt[3]]*f*(1 + x)*S
qrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt
[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1
+ x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```

Rubi [A] time = 0.126055, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$\frac{2}{3}e\tan^{-1}\left(\sqrt{-x^3-1}\right)+\frac{2\sqrt{2-\sqrt{3}}f(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(e + f*x)/(x*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*e*ArcTan[Sqrt[-1 - x^3]])/3 + (2*Sqrt[2 - Sqrt[3]]*f*(1 + x)*S
qrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt
[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1
+ x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```

Rubi in Sympy [A] time = 12.6552, size = 116, normalized size = 0.89

$$\frac{2e\,\mathrm{atan}\left(\sqrt{-x^3-1}\right)}{3}+\frac{2\cdot 3^{\frac{3}{4}}f\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-\sqrt{3}+2}\,(x+1)\,F\left(\mathrm{asin}\left(\frac{x+1+\sqrt{3}}{x-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{3\sqrt{\frac{-x-1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x+e)/x/(-x**3-1)**(1/2),x)
```

```
[Out] 2*e*atan(sqrt(-x**3 - 1))/3 + 2*3**(3/4)*f*sqrt((x**2 - x + 1)/(x
 - sqrt(3) + 1)**2)*sqrt(-sqrt(3) + 2)*(x + 1)*elliptic_f(asin((x
 + 1 + sqrt(3))/(x - sqrt(3) + 1)), -7 + 4*sqrt(3))/(3*sqrt((-x -
 1)/(x - sqrt(3) + 1)**2)*sqrt(-x**3 - 1))
```

Mathematica [A] time = 0.623961, size = 138, normalized size = 1.05

$$\frac{2}{3}e\tan^{-1}\left(\sqrt{-x^3-1}\right)-\frac{2f\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\sqrt{-\frac{(-1)^{2/3}\left(x+(-1)^{2/3}\right)}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\sqrt{-x^3-1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(e + f*x)/(x*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*e*ArcTan[Sqrt[-1 - x^3]])/3 - (2*f*((-1)^(1/3) - x)*Sqrt[(1 +
x)/(1 + (-1)^(1/3))]*Sqrt[-(((-1)^(2/3)*((-1)^(2/3) + x))/(1 + (-
1)^(1/3)))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1
/3))]], (-1)^(1/3)])/(Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*S
qrt[-1 - x^3])
```

Maple [A] time = 0.008, size = 122, normalized size = 0.9

$$-\frac{2i}{3}f\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}EllipticF\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\right)\frac{[illegible]}{\sqrt{-x[illegible]}}$$
$$+\frac{2e}{3}\arctan\left(\sqrt{-x^3-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x+e)/x/(-x^3-1)^(1/2),x)
```

```
[Out] -2/3*I*f*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(
3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2
)/(-x^3-1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3
^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))+2/3*e*arctan
((-x^3-1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx + e}{\sqrt{-x^3 - 1}x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x + e)/(sqrt(-x^3 - 1)*x),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(-x^3 - 1)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{fx + e}{\sqrt{-x^3 - 1}x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x + e)/(sqrt(-x^3 - 1)*x),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)/(sqrt(-x^3 - 1)*x), x)
```

Sympy [A] time = 5.39559, size = 46, normalized size = 0.35

$$\frac{2ie \operatorname{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} - \frac{ifx\,\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\left.\begin{matrix}\frac{1}{3}, \frac{1}{2}\\ \frac{4}{3}\end{matrix}\right| x^3 e^{i\pi}\right)}{3\,\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x+e)/x/(-x**3-1)**(1/2),x)
```

```
[Out] 2*I*e*asinh(x**(-3/2))/3 - I*f*x*gamma(1/3)*hyper((1/3, 1/2), (4/
3,), x**3*exp_polar(I*pi))/(3*gamma(4/3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx + e}{\sqrt{-x^3 - 1}x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x + e)/(sqrt(-x^3 - 1)*x),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)/(sqrt(-x^3 - 1)*x), x)
```

3.149 $\int \frac{c-dx}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx$

Optimal. Leaf size=95

$$\frac{3\log\left(d(2c+dx)-d\sqrt[3]{2c^3+d^3x^3}\right)}{2d}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2(2c+dx)}{\sqrt[3]{2c^3+d^3x^3}}+1}{\sqrt{3}}\right)}{d}-\frac{\log(c+dx)}{d}$$

```
[Out] -((Sqrt[3]*ArcTan[(1 + (2*(2*c + d*x))/(2*c^3 + d^3*x^3)^(1/3))/S
qrt[3]])/d) - Log[c + d*x]/d + (3*Log[d*(2*c + d*x) - d*(2*c^3 +
d^3*x^3)^(1/3)])/(2*d)
```

Rubi [A] time = 0.219754, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032

$$\frac{3\log\left(d(2c+dx)-d\sqrt[3]{2c^3+d^3x^3}\right)}{2d}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2(2c+dx)}{\sqrt[3]{2c^3+d^3x^3}}+1}{\sqrt{3}}\right)}{d}-\frac{\log(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In]  Int[(c - d*x)/((c + d*x)*(2*c^3 + d^3*x^3)^(1/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(1 + (2*(2*c + d*x))/(2*c^3 + d^3*x^3)^(1/3))/S
qrt[3]])/d) - Log[c + d*x]/d + (3*Log[d*(2*c + d*x) - d*(2*c^3 +
d^3*x^3)^(1/3)])/(2*d)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-d*x+c)/(d*x+c)/(d**3*x**3+2*c**3)**(1/3),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.164976, size = 0, normalized size = 0.

$$\int \frac{c - dx}{(c + dx)\sqrt[3]{2c^3 + d^3x^3}}\, dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(c - d*x)/((c + d*x)*(2*c^3 + d^3*x^3)^(1/3)),x]
```

```
[Out] Integrate[(c - d*x)/((c + d*x)*(2*c^3 + d^3*x^3)^(1/3)), x]
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int \frac{-dx + c}{dx + c} \frac{1}{\sqrt[3]{d^3x^3 + 2\,c^3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-d*x+c)/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x)
```

```
[Out] int((-d*x+c)/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{dx - c}{(d^3x^3 + 2\,c^3)^{\frac{1}{3}}(dx + c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(d*x - c)/((d^3*x^3 + 2*c^3)^(1/3)*(d*x + c)),x, algorithm="maxima")
```

```
[Out] -integrate((d*x - c)/((d^3*x^3 + 2*c^3)^(1/3)*(d*x + c)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(d*x - c)/((d^3*x^3 + 2*c^3)^(1/3)*(d*x + c)),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\left(-\frac{c}{c\sqrt[3]{2c^3+d^3x^3}+dx\sqrt[3]{2c^3+d^3x^3}}\right)dx-\int\frac{dx}{c\sqrt[3]{2c^3+d^3x^3}+dx\sqrt[3]{2c^3+d^3x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x+c)/(d*x+c)/(d**3*x**3+2*c**3)**(1/3),x)
```

```
[Out] -Integral(-c/(c*(2*c**3 + d**3*x**3)**(1/3) + d*x*(2*c**3 + d**3*
x**3)**(1/3)), x) - Integral(d*x/(c*(2*c**3 + d**3*x**3)**(1/3) +
 d*x*(2*c**3 + d**3*x**3)**(1/3)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int-\frac{dx-c}{(d^3x^3+2\,c^3)^{\frac{1}{3}}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(d*x - c)/((d^3*x^3 + 2*c^3)^(1/3)*(d*x + c)),x, algorithm="giac")
```

```
[Out] integrate(-(d*x - c)/((d^3*x^3 + 2*c^3)^(1/3)*(d*x + c)), x)
```

3.150 $\int \frac{e+fx}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx$

Optimal. Leaf size=234

$$-\frac{3(de-cf)\log\left(2^{2/3}d\sqrt[3]{d^3x^3-c^3}+d(c-dx)\right)}{4\sqrt[3]{2}cd^2}+\frac{\sqrt{3}(de-cf)\tan^{-1}\left(\frac{1-\frac{\sqrt[3]{2}(c-dx)}{\sqrt[3]{d^3x^3-c^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}cd^2}-\frac{f\log\left(\sqrt[3]{d^3x^3-c^3}-dx\right)}{2d^2}+\frac{f\tan^{-1}\left(\frac{\frac{2dx}{\sqrt[3]{d^3x^3-c^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}d^2}+\frac{(de-cf)\log\left((c-dx)(c+dx)^2\right)}{4\sqrt[3]{2}cd^2}$$

```
[Out] (f*ArcTan[(1 + (2*d*x)/(-c^3 + d^3*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]
*d^2) + (Sqrt[3]*(d*e - c*f)*ArcTan[(1 - (2^(1/3)*(c - d*x))/(-c^
3 + d^3*x^3)^(1/3))/Sqrt[3]])/(2*2^(1/3)*c*d^2) + ((d*e - c*f)*Lo
g[(c - d*x)*(c + d*x)^2])/(4*2^(1/3)*c*d^2) - (f*Log[-(d*x) + (-c
^3 + d^3*x^3)^(1/3)])/(2*d^2) - (3*(d*e - c*f)*Log[d*(c - d*x) +
2^(2/3)*d*(-c^3 + d^3*x^3)^(1/3)])/(4*2^(1/3)*c*d^2)
```

Rubi [A] time = 0.428321, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$-\frac{3(de-cf)\log\left(2^{2/3}d\sqrt[3]{d^3x^3-c^3}+d(c-dx)\right)}{4\sqrt[3]{2}cd^2}+\frac{\sqrt{3}(de-cf)\tan^{-1}\left(\frac{1-\frac{\sqrt[3]{2}(c-dx)}{\sqrt[3]{d^3x^3-c^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}cd^2}-\frac{f\log\left(\sqrt[3]{d^3x^3-c^3}-dx\right)}{2d^2}+\frac{f\tan^{-1}\left(\frac{\frac{2dx}{\sqrt[3]{d^3x^3-c^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}d^2}+\frac{(de-cf)\log\left((c-dx)(c+dx)^2\right)}{4\sqrt[3]{2}cd^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(e + f*x)/((c + d*x)*(-c^3 + d^3*x^3)^(1/3)),x]
```

```
[Out] (f*ArcTan[(1 + (2*d*x)/(-c^3 + d^3*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]
*d^2) + (Sqrt[3]*(d*e - c*f)*ArcTan[(1 - (2^(1/3)*(c - d*x))/(-c^
3 + d^3*x^3)^(1/3))/Sqrt[3]])/(2*2^(1/3)*c*d^2) + ((d*e - c*f)*Lo
g[(c - d*x)*(c + d*x)^2])/(4*2^(1/3)*c*d^2) - (f*Log[-(d*x) + (-c
^3 + d^3*x^3)^(1/3)])/(2*d^2) - (3*(d*e - c*f)*Log[d*(c - d*x) +
```

```
2^(2/3)*d*(-c^3 + d^3*x^3)^(1/3)])/(4*2^(1/3)*c*d^2)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x+e)/(d*x+c)/(d**3*x**3-c**3)**(1/3),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.204517, size = 0, normalized size = 0.

$$\int \frac{e+fx}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(e + f*x)/((c + d*x)*(-c^3 + d^3*x^3)^(1/3)),x]
```

```
[Out] Integrate[(e + f*x)/((c + d*x)*(-c^3 + d^3*x^3)^(1/3)), x]
```

Maple [F] time = 0.061, size = 0, normalized size = 0.

$$\int \frac{fx+e}{dx+c}\frac{1}{\sqrt[3]{d^3x^3-c^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x+e)/(d*x+c)/(d^3*x^3-c^3)^(1/3),x)
```

```
[Out] int((f*x+e)/(d*x+c)/(d^3*x^3-c^3)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{(d^3x^3-c^3)^{\frac{1}{3}}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x + e)/((d^3*x^3 - c^3)^(1/3)*(d*x + c)),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)/((d^3*x^3 - c^3)^(1/3)*(d*x + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x + e)/((d^3*x^3 - c^3)^(1/3)*(d*x + c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e + fx}{\sqrt[3]{(-c + dx)\left(c^{2} + cdx + d^{2}x^{2}\right)}\,(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x+e)/(d*x+c)/(d**3*x**3-c**3)**(1/3),x)
```

```
[Out] Integral((e + f*x)/(((-c + d*x)*(c**2 + c*d*x + d**2*x**2))**(1/3
)*(c + d*x)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx + e}{\left(d^{3}x^{3} - c^{3}\right)^{\frac{1}{3}}(dx + c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x + e)/((d^3*x^3 - c^3)^(1/3)*(d*x + c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)/((d^3*x^3 - c^3)^(1/3)*(d*x + c)), x)
```

3.151 $\int x^2(a+bx)^n\left(c+dx^3\right)\,dx$

Optimal. Leaf size=160

$$-\frac{a\left(2b^3c-5a^3d\right)(a+bx)^{n+2}}{b^6(n+2)}+\frac{\left(b^3c-10a^3d\right)(a+bx)^{n+3}}{b^6(n+3)}+\frac{10a^2d(a+bx)^{n+4}}{b^6(n+4)}+\frac{a^2\left(b^3c-a^3d\right)(a+bx)^{n+1}}{b^6(n+1)}-\frac{5ad(a+bx)^{n+5}}{b^6(n+5)}+\frac{d(a+bx)^{n+6}}{b^6(n+6)}$$

```
[Out] (a^2*(b^3*c - a^3*d)*(a + b*x)^(1 + n))/(b^6*(1 + n)) - (a*(2*b^3
*c - 5*a^3*d)*(a + b*x)^(2 + n))/(b^6*(2 + n)) + ((b^3*c - 10*a^3
*d)*(a + b*x)^(3 + n))/(b^6*(3 + n)) + (10*a^2*d*(a + b*x)^(4 + n
))/(b^6*(4 + n)) - (5*a*d*(a + b*x)^(5 + n))/(b^6*(5 + n)) + (d*(
a + b*x)^(6 + n))/(b^6*(6 + n))
```

Rubi [A] time = 0.212059, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056

$$-\frac{a\left(2b^3c-5a^3d\right)(a+bx)^{n+2}}{b^6(n+2)}+\frac{\left(b^3c-10a^3d\right)(a+bx)^{n+3}}{b^6(n+3)}+\frac{10a^2d(a+bx)^{n+4}}{b^6(n+4)}+\frac{a^2\left(b^3c-a^3d\right)(a+bx)^{n+1}}{b^6(n+1)}-\frac{5ad(a+bx)^{n+5}}{b^6(n+5)}+\frac{d(a+bx)^{n+6}}{b^6(n+6)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(a + b*x)^n*(c + d*x^3),x]
```

```
[Out] (a^2*(b^3*c - a^3*d)*(a + b*x)^(1 + n))/(b^6*(1 + n)) - (a*(2*b^3
*c - 5*a^3*d)*(a + b*x)^(2 + n))/(b^6*(2 + n)) + ((b^3*c - 10*a^3
*d)*(a + b*x)^(3 + n))/(b^6*(3 + n)) + (10*a^2*d*(a + b*x)^(4 + n
))/(b^6*(4 + n)) - (5*a*d*(a + b*x)^(5 + n))/(b^6*(5 + n)) + (d*(
a + b*x)^(6 + n))/(b^6*(6 + n))
```

Rubi in Sympy [A] time = 40.4505, size = 144, normalized size = 0.9

$$\frac{10a^2d\,(a+bx)^{n+4}}{b^6\,(n+4)}-\frac{a^2\,(a+bx)^{n+1}\left(a^3d-b^3c\right)}{b^6\,(n+1)}-\frac{5ad\,(a+bx)^{n+5}}{b^6\,(n+5)}+\frac{a\,(a+bx)^{n+2}\left(5a^3d-2b^3c\right)}{b^6\,(n+2)}+\frac{d\,(a+bx)^{n+6}}{b^6\,(n+6)}-\frac{(a+bx)^{n+3}\left(10a^3d-b^3c\right)}{b^6\,(n+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(b*x+a)**n*(d*x**3+c),x)
```

```
[Out] 10*a**2*d*(a + b*x)**(n + 4)/(b**6*(n + 4)) - a**2*(a + b*x)**(n
+ 1)*(a**3*d - b**3*c)/(b**6*(n + 1)) - 5*a*d*(a + b*x)**(n + 5)/
(b**6*(n + 5)) + a*(a + b*x)**(n + 2)*(5*a**3*d - 2*b**3*c)/(b**6
*(n + 2)) + d*(a + b*x)**(n + 6)/(b**6*(n + 6)) - (a + b*x)**(n +
 3)*(10*a**3*d - b**3*c)/(b**6*(n + 3))
```

Mathematica [A] time = 0.199155, size = 204, normalized size = 1.27

$$\frac{(a+bx)^{n+1}\left(-120a^5d+120a^4bd(n+1)x-60a^3b^2d\left(n^2+3n+2\right)x^2+2a^2b^3\left(c\left(n^3+15n^2+74n+120\right)+10d\left(n^3+6n^2+1\right.\right.\right.}{b^6(n+1)(n+}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(a + b*x)^n*(c + d*x^3),x]
```

```
[Out] ((a + b*x)^(1 + n)*(-120*a^5*d + 120*a^4*b*d*(1 + n)*x - 60*a^3*b
^2*d*(2 + 3*n + n^2)*x^2 + b^5*(40 + 78*n + 49*n^2 + 12*n^3 + n^4
)*x^2*(c*(6 + n) + d*(3 + n)*x^3) - a*b^4*(4 + 5*n + n^2)*x*(2*c*
(30 + 11*n + n^2) + 5*d*(6 + 5*n + n^2)*x^3) + 2*a^2*b^3*(c*(120
+ 74*n + 15*n^2 + n^3) + 10*d*(6 + 11*n + 6*n^2 + n^3)*x^3)))/(b^
6*(1 + n)*(2 + n)*(3 + n)*(4 + n)*(5 + n)*(6 + n))
```

Maple [B] time = 0.012, size = 451, normalized size = 2.8

$$-\frac{(bx+a)^{1+n}\left(-b^5dn^5x^5-15\,b^5dn^4x^5+5\,ab^4dn^4x^4-85\,b^5dn^3x^5+50\,ab^4dn^3x^4-b^5cn^5x^2-225\,b^5dn^2x^5-20\,a^2b^3dn^3x^3+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(b*x+a)^n*(d*x^3+c),x)
```

```
[Out] -(b*x+a)^(1+n)*(-b^5*d*n^5*x^5-15*b^5*d*n^4*x^5+5*a*b^4*d*n^4*x^4
-85*b^5*d*n^3*x^5+50*a*b^4*d*n^3*x^4-b^5*c*n^5*x^2-225*b^5*d*n^2*
x^5-20*a^2*b^3*d*n^3*x^3+175*a*b^4*d*n^2*x^4-18*b^5*c*n^4*x^2-274
*b^5*d*n*x^5-120*a^2*b^3*d*n^2*x^3+2*a*b^4*c*n^4*x+250*a*b^4*d*n*
x^4-121*b^5*c*n^3*x^2-120*b^5*d*x^5+60*a^3*b^2*d*n^2*x^2-220*a^2*
b^3*d*n*x^3+32*a*b^4*c*n^3*x+120*a*b^4*d*x^4-372*b^5*c*n^2*x^2+18
0*a^3*b^2*d*n*x^2-2*a^2*b^3*c*n^3-120*a^2*b^3*d*x^3+178*a*b^4*c*n
^2*x-508*b^5*c*n*x^2-120*a^4*b*d*n*x+120*a^3*b^2*d*x^2-30*a^2*b^3
*c*n^2+388*a*b^4*c*n*x-240*b^5*c*x^2-120*a^4*b*d*x-148*a^2*b^3*c*
n+240*a*b^4*c*x+120*a^5*d-240*a^2*b^3*c)/b^6/(n^6+21*n^5+175*n^4+
```

```
735*n^3+1624*n^2+1764*n+720)
```

Maxima [A] time = 0.706897, size = 342, normalized size = 2.14

$$\frac{\left((n^2+3n+2)b^3x^3+(n^2+n)ab^2x^2-2a^2bnx+2a^3\right)(bx+a)^n c}{(n^3+6n^2+11n+6)b^3} + \frac{\left((n^5+15n^4+85n^3+225n^2+274n+120)b^6x^6+(n^5+10n^4+35n^3+50n^2+24n)ab^5x^5-5(n^4+6n^3+11n^2+6n)a^2b\cdots\right.}{(n^6+21n^5+175n^4+735n^3+1624n^2+1764\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)*(b*x + a)^n*x^2,x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*a*b^2*x^2 - 2*a^2*b*n*x + 2*
a^3)*(b*x + a)^n*c/((n^3 + 6*n^2 + 11*n + 6)*b^3) + ((n^5 + 15*n^
4 + 85*n^3 + 225*n^2 + 274*n + 120)*b^6*x^6 + (n^5 + 10*n^4 + 35*
n^3 + 50*n^2 + 24*n)*a*b^5*x^5 - 5*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a
^2*b^4*x^4 + 20*(n^3 + 3*n^2 + 2*n)*a^3*b^3*x^3 - 60*(n^2 + n)*a^
4*b^2*x^2 + 120*a^5*b*n*x - 120*a^6)*(b*x + a)^n*d/((n^6 + 21*n^5
 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*b^6)
```

Fricas [A] time = 0.2909, size = 662, normalized size = 4.14

$$\frac{\left(2a^3b^3cn^3+30a^3b^3cn^2+148a^3b^3cn+240a^3b^3c-120a^6d+\left(b^6dn^5+15b^6dn^4+85b^6dn^3+225b^6dn^2+274b^6dn+120b^6\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)*(b*x + a)^n*x^2,x, algorithm="fricas")
```

```
[Out] (2*a^3*b^3*c*n^3 + 30*a^3*b^3*c*n^2 + 148*a^3*b^3*c*n + 240*a^3*b
^3*c - 120*a^6*d + (b^6*d*n^5 + 15*b^6*d*n^4 + 85*b^6*d*n^3 + 225
*b^6*d*n^2 + 274*b^6*d*n + 120*b^6*d)*x^6 + (a*b^5*d*n^5 + 10*a*b
^5*d*n^4 + 35*a*b^5*d*n^3 + 50*a*b^5*d*n^2 + 24*a*b^5*d*n)*x^5 -
5*(a^2*b^4*d*n^4 + 6*a^2*b^4*d*n^3 + 11*a^2*b^4*d*n^2 + 6*a^2*b^4
*d*n)*x^4 + (b^6*c*n^5 + 18*b^6*c*n^4 + 240*b^6*c + (121*b^6*c +
20*a^3*b^3*d)*n^3 + 12*(31*b^6*c + 5*a^3*b^3*d)*n^2 + 4*(127*b^6*
c + 10*a^3*b^3*d)*n)*x^3 + (a*b^5*c*n^5 + 16*a*b^5*c*n^4 + 89*a*b
^5*c*n^3 + 2*(97*a*b^5*c - 30*a^4*b^2*d)*n^2 + 60*(2*a*b^5*c - a^
4*b^2*d)*n)*x^2 - 2*(a^2*b^4*c*n^4 + 15*a^2*b^4*c*n^3 + 74*a^2*b^
4*c*n^2 + 60*(2*a^2*b^4*c - a^5*b*d)*n)*x)*(b*x + a)^n/(b^6*n^6 +
 21*b^6*n^5 + 175*b^6*n^4 + 735*b^6*n^3 + 1624*b^6*n^2 + 1764*b^6
*n + 720*b^6)
```

Sympy [A] time = 51.0278, size = 6431, normalized size = 40.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(b*x+a)**n*(d*x**3+c),x)
```

```
[Out] Piecewise((a**n*(c*x**3/3 + d*x**6/6), Eq(b, 0)), (60*a**8*d*log(
a/b + x)/(60*a**8*b**6 + 300*a**7*b**7*x + 600*a**6*b**8*x**2 + 6
00*a**5*b**9*x**3 + 300*a**4*b**10*x**4 + 60*a**3*b**11*x**5) + 2
7*a**8*d/(60*a**8*b**6 + 300*a**7*b**7*x + 600*a**6*b**8*x**2 + 6
00*a**5*b**9*x**3 + 300*a**4*b**10*x**4 + 60*a**3*b**11*x**5) + 3
00*a**7*b*d*x*log(a/b + x)/(60*a**8*b**6 + 300*a**7*b**7*x + 600*
a**6*b**8*x**2 + 600*a**5*b**9*x**3 + 300*a**4*b**10*x**4 + 60*a*
*3*b**11*x**5) + 75*a**7*b*d*x/(60*a**8*b**6 + 300*a**7*b**7*x +
600*a**6*b**8*x**2 + 600*a**5*b**9*x**3 + 300*a**4*b**10*x**4 + 6
0*a**3*b**11*x**5) + 600*a**6*b**2*d*x**2*log(a/b + x)/(60*a**8*b
**6 + 300*a**7*b**7*x + 600*a**6*b**8*x**2 + 600*a**5*b**9*x**3 +
 300*a**4*b**10*x**4 + 60*a**3*b**11*x**5) + 600*a**5*b**3*d*x**3
*log(a/b + x)/(60*a**8*b**6 + 300*a**7*b**7*x + 600*a**6*b**8*x**
2 + 600*a**5*b**9*x**3 + 300*a**4*b**10*x**4 + 60*a**3*b**11*x**5
) - 200*a**5*b**3*d*x**3/(60*a**8*b**6 + 300*a**7*b**7*x + 600*a*
*6*b**8*x**2 + 600*a**5*b**9*x**3 + 300*a**4*b**10*x**4 + 60*a**3
*b**11*x**5) + 300*a**4*b**4*d*x**4*log(a/b + x)/(60*a**8*b**6 +
300*a**7*b**7*x + 600*a**6*b**8*x**2 + 600*a**5*b**9*x**3 + 300*a
**4*b**10*x**4 + 60*a**3*b**11*x**5) - 250*a**4*b**4*d*x**4/(60*a
**8*b**6 + 300*a**7*b**7*x + 600*a**6*b**8*x**2 + 600*a**5*b**9*x
**3 + 300*a**4*b**10*x**4 + 60*a**3*b**11*x**5) + 60*a**3*b**5*d*
x**5*log(a/b + x)/(60*a**8*b**6 + 300*a**7*b**7*x + 600*a**6*b**8
*x**2 + 600*a**5*b**9*x**3 + 300*a**4*b**10*x**4 + 60*a**3*b**11*
x**5) - 110*a**3*b**5*d*x**5/(60*a**8*b**6 + 300*a**7*b**7*x + 60
0*a**6*b**8*x**2 + 600*a**5*b**9*x**3 + 300*a**4*b**10*x**4 + 60*
a**3*b**11*x**5) + 20*a**2*b**6*c*x**3/(60*a**8*b**6 + 300*a**7*b
**7*x + 600*a**6*b**8*x**2 + 600*a**5*b**9*x**3 + 300*a**4*b**10*
x**4 + 60*a**3*b**11*x**5) + 10*a*b**7*c*x**4/(60*a**8*b**6 + 300
*a**7*b**7*x + 600*a**6*b**8*x**2 + 600*a**5*b**9*x**3 + 300*a**4
*b**10*x**4 + 60*a**3*b**11*x**5) + 2*b**8*c*x**5/(60*a**8*b**6 +
 300*a**7*b**7*x + 600*a**6*b**8*x**2 + 600*a**5*b**9*x**3 + 300*
a**4*b**10*x**4 + 60*a**3*b**11*x**5), Eq(n, -6)), (-60*a**7*d*lo
g(a/b + x)/(12*a**6*b**6 + 48*a**5*b**7*x + 72*a**4*b**8*x**2 + 4
8*a**3*b**9*x**3 + 12*a**2*b**10*x**4) - 35*a**7*d/(12*a**6*b**6
+ 48*a**5*b**7*x + 72*a**4*b**8*x**2 + 48*a**3*b**9*x**3 + 12*a**
2*b**10*x**4) - 240*a**6*b*d*x*log(a/b + x)/(12*a**6*b**6 + 48*a*
*5*b**7*x + 72*a**4*b**8*x**2 + 48*a**3*b**9*x**3 + 12*a**2*b**10
*x**4) - 80*a**6*b*d*x/(12*a**6*b**6 + 48*a**5*b**7*x + 72*a**4*b
**8*x**2 + 48*a**3*b**9*x**3 + 12*a**2*b**10*x**4) - 360*a**5*b**
2*d*x**2*log(a/b + x)/(12*a**6*b**6 + 48*a**5*b**7*x + 72*a**4*b*
*8*x**2 + 48*a**3*b**9*x**3 + 12*a**2*b**10*x**4) - 240*a**4*b**3
*d*x**3*log(a/b + x)/(12*a**6*b**6 + 48*a**5*b**7*x + 72*a**4*b**
```

```
8*x**2 + 48*a**3*b**9*x**3 + 12*a**2*b**10*x**4) + 120*a**4*b**3*
d*x**3/(12*a**6*b**6 + 48*a**5*b**7*x + 72*a**4*b**8*x**2 + 48*a*
*3*b**9*x**3 + 12*a**2*b**10*x**4) - 60*a**3*b**4*d*x**4*log(a/b
+ x)/(12*a**6*b**6 + 48*a**5*b**7*x + 72*a**4*b**8*x**2 + 48*a**3
*b**9*x**3 + 12*a**2*b**10*x**4) + 90*a**3*b**4*d*x**4/(12*a**6*b
**6 + 48*a**5*b**7*x + 72*a**4*b**8*x**2 + 48*a**3*b**9*x**3 + 12
*a**2*b**10*x**4) + 12*a**2*b**5*d*x**5/(12*a**6*b**6 + 48*a**5*b
**7*x + 72*a**4*b**8*x**2 + 48*a**3*b**9*x**3 + 12*a**2*b**10*x**
4) + 4*a*b**6*c*x**3/(12*a**6*b**6 + 48*a**5*b**7*x + 72*a**4*b**
8*x**2 + 48*a**3*b**9*x**3 + 12*a**2*b**10*x**4) + b**7*c*x**4/(1
2*a**6*b**6 + 48*a**5*b**7*x + 72*a**4*b**8*x**2 + 48*a**3*b**9*x
**3 + 12*a**2*b**10*x**4), Eq(n, -5)), (60*a**6*d*log(a/b + x)/(6
*a**4*b**6 + 18*a**3*b**7*x + 18*a**2*b**8*x**2 + 6*a*b**9*x**3)
+ 50*a**6*d/(6*a**4*b**6 + 18*a**3*b**7*x + 18*a**2*b**8*x**2 + 6
*a*b**9*x**3) + 180*a**5*b*d*x*log(a/b + x)/(6*a**4*b**6 + 18*a**
3*b**7*x + 18*a**2*b**8*x**2 + 6*a*b**9*x**3) + 90*a**5*b*d*x/(6*
a**4*b**6 + 18*a**3*b**7*x + 18*a**2*b**8*x**2 + 6*a*b**9*x**3) +
 180*a**4*b**2*d*x**2*log(a/b + x)/(6*a**4*b**6 + 18*a**3*b**7*x
+ 18*a**2*b**8*x**2 + 6*a*b**9*x**3) + 60*a**3*b**3*d*x**3*log(a/
b + x)/(6*a**4*b**6 + 18*a**3*b**7*x + 18*a**2*b**8*x**2 + 6*a*b*
*9*x**3) - 60*a**3*b**3*d*x**3/(6*a**4*b**6 + 18*a**3*b**7*x + 18
*a**2*b**8*x**2 + 6*a*b**9*x**3) - 15*a**2*b**4*d*x**4/(6*a**4*b*
*6 + 18*a**3*b**7*x + 18*a**2*b**8*x**2 + 6*a*b**9*x**3) + 3*a*b*
*5*d*x**5/(6*a**4*b**6 + 18*a**3*b**7*x + 18*a**2*b**8*x**2 + 6*a
*b**9*x**3) + 2*b**6*c*x**3/(6*a**4*b**6 + 18*a**3*b**7*x + 18*a*
*2*b**8*x**2 + 6*a*b**9*x**3), Eq(n, -4)), (-60*a**5*d*log(a/b +
x)/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 90*a**5*d/(6*a**2*
b**6 + 12*a*b**7*x + 6*b**8*x**2) - 120*a**4*b*d*x*log(a/b + x)/(
6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 120*a**4*b*d*x/(6*a**2
*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 60*a**3*b**2*d*x**2*log(a/b
+ x)/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) + 6*a**2*b**3*c*lo
g(a/b + x)/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) + 9*a**2*b**
3*c/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) + 20*a**2*b**3*d*x*
*3/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) + 12*a*b**4*c*x*log(
a/b + x)/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) + 12*a*b**4*c*
x/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 5*a*b**4*d*x**4/(6*
a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) + 6*b**5*c*x**2*log(a/b +
x)/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) + 2*b**5*d*x**5/(6*a
**2*b**6 + 12*a*b**7*x + 6*b**8*x**2), Eq(n, -3)), (60*a**5*d*log
(a/b + x)/(12*a*b**6 + 12*b**7*x) + 60*a**5*d/(12*a*b**6 + 12*b**
7*x) + 60*a**4*b*d*x*log(a/b + x)/(12*a*b**6 + 12*b**7*x) - 30*a*
*3*b**2*d*x**2/(12*a*b**6 + 12*b**7*x) - 24*a**2*b**3*c*log(a/b +
 x)/(12*a*b**6 + 12*b**7*x) - 24*a**2*b**3*c/(12*a*b**6 + 12*b**7
*x) + 10*a**2*b**3*d*x**3/(12*a*b**6 + 12*b**7*x) - 24*a*b**4*c*x
*log(a/b + x)/(12*a*b**6 + 12*b**7*x) - 5*a*b**4*d*x**4/(12*a*b**
6 + 12*b**7*x) + 12*b**5*c*x**2/(12*a*b**6 + 12*b**7*x) + 3*b**5*
d*x**5/(12*a*b**6 + 12*b**7*x), Eq(n, -2)), (-a**5*d*log(a/b + x)
/b**6 + a**4*d*x/b**5 - a**3*d*x**2/(2*b**4) + a**2*c*log(a/b + x
)/b**3 + a**2*d*x**3/(3*b**3) - a*c*x/b**2 - a*d*x**4/(4*b**2) +
c*x**2/(2*b) + d*x**5/(5*b), Eq(n, -1)), (-120*a**6*d*(a + b*x)**
n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 162
4*b**6*n**2 + 1764*b**6*n + 720*b**6) + 120*a**5*b*d*n*x*(a + b*x
)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 +
1624*b**6*n**2 + 1764*b**6*n + 720*b**6) - 60*a**4*b**2*d*n**2*x*
```

```
*2*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b
**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) - 60*a**4*b**
2*d*n*x**2*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4
 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 2*a
**3*b**3*c*n**3*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6
*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6)
+ 30*a**3*b**3*c*n**2*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 17
5*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*
b**6) + 148*a**3*b**3*c*n*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5
+ 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n +
720*b**6) + 240*a**3*b**3*c*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**
5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n
+ 720*b**6) + 20*a**3*b**3*d*n**3*x**3*(a + b*x)**n/(b**6*n**6 +
21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1
764*b**6*n + 720*b**6) + 60*a**3*b**3*d*n**2*x**3*(a + b*x)**n/(b
**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b*
*6*n**2 + 1764*b**6*n + 720*b**6) + 40*a**3*b**3*d*n*x**3*(a + b*
x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 +
 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) - 2*a**2*b**4*c*n**4*x*
(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6
*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) - 30*a**2*b**4*c
*n**3*x*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 +
735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) - 148*a*
*2*b**4*c*n**2*x*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**
6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6)
 - 240*a**2*b**4*c*n*x*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 1
75*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720
*b**6) - 5*a**2*b**4*d*n**4*x**4*(a + b*x)**n/(b**6*n**6 + 21*b**
6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b*
*6*n + 720*b**6) - 30*a**2*b**4*d*n**3*x**4*(a + b*x)**n/(b**6*n*
*6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**
2 + 1764*b**6*n + 720*b**6) - 55*a**2*b**4*d*n**2*x**4*(a + b*x)*
*n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 16
24*b**6*n**2 + 1764*b**6*n + 720*b**6) - 30*a**2*b**4*d*n*x**4*(a
 + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n
**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + a*b**5*c*n**5*x*
*2*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b
**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 16*a*b**5*c
*n**4*x**2*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4
 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 89*
a*b**5*c*n**3*x**2*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b
**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**
6) + 194*a*b**5*c*n**2*x**2*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**
5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n
+ 720*b**6) + 120*a*b**5*c*n*x**2*(a + b*x)**n/(b**6*n**6 + 21*b*
*6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b
**6*n + 720*b**6) + a*b**5*d*n**5*x**5*(a + b*x)**n/(b**6*n**6 +
21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1
764*b**6*n + 720*b**6) + 10*a*b**5*d*n**4*x**5*(a + b*x)**n/(b**6
*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*
n**2 + 1764*b**6*n + 720*b**6) + 35*a*b**5*d*n**3*x**5*(a + b*x)*
*n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 16
24*b**6*n**2 + 1764*b**6*n + 720*b**6) + 50*a*b**5*d*n**2*x**5*(a
 + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n
```

```
**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 24*a*b**5*d*n*x*
*5*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b
**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + b**6*c*n**5
*x**3*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 73
5*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 18*b**6*
c*n**4*x**3*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**
4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 12
1*b**6*c*n**3*x**3*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b
**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**
6) + 372*b**6*c*n**2*x**3*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5
+ 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n +
720*b**6) + 508*b**6*c*n*x**3*(a + b*x)**n/(b**6*n**6 + 21*b**6*n
**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*
n + 720*b**6) + 240*b**6*c*x**3*(a + b*x)**n/(b**6*n**6 + 21*b**6
*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**
6*n + 720*b**6) + b**6*d*n**5*x**6*(a + b*x)**n/(b**6*n**6 + 21*b
**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*
b**6*n + 720*b**6) + 15*b**6*d*n**4*x**6*(a + b*x)**n/(b**6*n**6
+ 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 +
 1764*b**6*n + 720*b**6) + 85*b**6*d*n**3*x**6*(a + b*x)**n/(b**6
*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*
n**2 + 1764*b**6*n + 720*b**6) + 225*b**6*d*n**2*x**6*(a + b*x)**
n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 162
4*b**6*n**2 + 1764*b**6*n + 720*b**6) + 274*b**6*d*n*x**6*(a + b*
x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 +
 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 120*b**6*d*x**6*(a +
b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3
 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6), True))
```

GIAC/XCAS [A] time = 0.273277, size = 1238, normalized size = 7.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)*(b*x + a)^n*x^2,x, algorithm="giac")
```

```
[Out] (b^6*d*n^5*x^6*e^(n*ln(b*x + a)) + a*b^5*d*n^5*x^5*e^(n*ln(b*x +
a)) + 15*b^6*d*n^4*x^6*e^(n*ln(b*x + a)) + 10*a*b^5*d*n^4*x^5*e^(
n*ln(b*x + a)) + 85*b^6*d*n^3*x^6*e^(n*ln(b*x + a)) + b^6*c*n^5*x
^3*e^(n*ln(b*x + a)) - 5*a^2*b^4*d*n^4*x^4*e^(n*ln(b*x + a)) + 35
*a*b^5*d*n^3*x^5*e^(n*ln(b*x + a)) + 225*b^6*d*n^2*x^6*e^(n*ln(b*
x + a)) + a*b^5*c*n^5*x^2*e^(n*ln(b*x + a)) + 18*b^6*c*n^4*x^3*e^
(n*ln(b*x + a)) - 30*a^2*b^4*d*n^3*x^4*e^(n*ln(b*x + a)) + 50*a*b
^5*d*n^2*x^5*e^(n*ln(b*x + a)) + 274*b^6*d*n*x^6*e^(n*ln(b*x + a)
) + 16*a*b^5*c*n^4*x^2*e^(n*ln(b*x + a)) + 121*b^6*c*n^3*x^3*e^(n
*ln(b*x + a)) + 20*a^3*b^3*d*n^3*x^3*e^(n*ln(b*x + a)) - 55*a^2*b
^4*d*n^2*x^4*e^(n*ln(b*x + a)) + 24*a*b^5*d*n*x^5*e^(n*ln(b*x + a
)) + 120*b^6*d*x^6*e^(n*ln(b*x + a)) - 2*a^2*b^4*c*n^4*x*e^(n*ln(
b*x + a)) + 89*a*b^5*c*n^3*x^2*e^(n*ln(b*x + a)) + 372*b^6*c*n^2*
```

```
x^3*e^(n*ln(b*x + a)) + 60*a^3*b^3*d*n^2*x^3*e^(n*ln(b*x + a)) -
30*a^2*b^4*d*n*x^4*e^(n*ln(b*x + a)) - 30*a^2*b^4*c*n^3*x*e^(n*ln
(b*x + a)) + 194*a*b^5*c*n^2*x^2*e^(n*ln(b*x + a)) - 60*a^4*b^2*d
*n^2*x^2*e^(n*ln(b*x + a)) + 508*b^6*c*n*x^3*e^(n*ln(b*x + a)) +
40*a^3*b^3*d*n*x^3*e^(n*ln(b*x + a)) + 2*a^3*b^3*c*n^3*e^(n*ln(b*
x + a)) - 148*a^2*b^4*c*n^2*x*e^(n*ln(b*x + a)) + 120*a*b^5*c*n*x
^2*e^(n*ln(b*x + a)) - 60*a^4*b^2*d*n*x^2*e^(n*ln(b*x + a)) + 240
*b^6*c*x^3*e^(n*ln(b*x + a)) + 30*a^3*b^3*c*n^2*e^(n*ln(b*x + a))
 - 240*a^2*b^4*c*n*x*e^(n*ln(b*x + a)) + 120*a^5*b*d*n*x*e^(n*ln(
b*x + a)) + 148*a^3*b^3*c*n*e^(n*ln(b*x + a)) + 240*a^3*b^3*c*e^(
n*ln(b*x + a)) - 120*a^6*d*e^(n*ln(b*x + a)))/(b^6*n^6 + 21*b^6*n
^5 + 175*b^6*n^4 + 735*b^6*n^3 + 1624*b^6*n^2 + 1764*b^6*n + 720*
b^6)
```

3.152 $\int x(a+bx)^n \left(c+dx^3\right)\, dx$

Optimal. Leaf size=126

$$-\frac{a\left(b^3c-a^3d\right)(a+bx)^{n+1}}{b^5(n+1)}+\frac{\left(b^3c-4a^3d\right)(a+bx)^{n+2}}{b^5(n+2)}+\frac{6a^2d(a+bx)^{n+3}}{b^5(n+3)}-\frac{4ad(a+bx)^{n+4}}{b^5(n+4)}+\frac{d(a+bx)^{n+5}}{b^5(n+5)}$$

```
[Out] -((a*(b^3*c - a^3*d)*(a + b*x)^(1 + n))/(b^5*(1 + n))) + ((b^3*c
- 4*a^3*d)*(a + b*x)^(2 + n))/(b^5*(2 + n)) + (6*a^2*d*(a + b*x)^
(3 + n))/(b^5*(3 + n)) - (4*a*d*(a + b*x)^(4 + n))/(b^5*(4 + n))
+ (d*(a + b*x)^(5 + n))/(b^5*(5 + n))
```

Rubi [A] time = 0.147343, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$

$$-\frac{a\left(b^3c-a^3d\right)(a+bx)^{n+1}}{b^5(n+1)}+\frac{\left(b^3c-4a^3d\right)(a+bx)^{n+2}}{b^5(n+2)}+\frac{6a^2d(a+bx)^{n+3}}{b^5(n+3)}-\frac{4ad(a+bx)^{n+4}}{b^5(n+4)}+\frac{d(a+bx)^{n+5}}{b^5(n+5)}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(a + b*x)^n*(c + d*x^3),x]
```

```
[Out] -((a*(b^3*c - a^3*d)*(a + b*x)^(1 + n))/(b^5*(1 + n))) + ((b^3*c
- 4*a^3*d)*(a + b*x)^(2 + n))/(b^5*(2 + n)) + (6*a^2*d*(a + b*x)^
(3 + n))/(b^5*(3 + n)) - (4*a*d*(a + b*x)^(4 + n))/(b^5*(4 + n))
+ (d*(a + b*x)^(5 + n))/(b^5*(5 + n))
```

Rubi in Sympy [A] time = 31.4633, size = 112, normalized size = 0.89

$$\frac{6a^2d\,(a+bx)^{n+3}}{b^5\,(n+3)}-\frac{4ad\,(a+bx)^{n+4}}{b^5\,(n+4)}+\frac{a\,(a+bx)^{n+1}\left(a^3d-b^3c\right)}{b^5\,(n+1)}+\frac{d\,(a+bx)^{n+5}}{b^5\,(n+5)}-\frac{(a+bx)^{n+2}\left(4a^3d-b^3c\right)}{b^5\,(n+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(b*x+a)**n*(d*x**3+c),x)
```

```
[Out] 6*a**2*d*(a + b*x)**(n + 3)/(b**5*(n + 3)) - 4*a*d*(a + b*x)**(n
+ 4)/(b**5*(n + 4)) + a*(a + b*x)**(n + 1)*(a**3*d - b**3*c)/(b**
5*(n + 1)) + d*(a + b*x)**(n + 5)/(b**5*(n + 5)) - (a + b*x)**(n
+ 2)*(4*a**3*d - b**3*c)/(b**5*(n + 2))
```

Mathematica [A] time = 0.119842, size = 142, normalized size = 1.13

$$\frac{(a+bx)^{n+1}\left(24a^4d-24a^3bd(n+1)x+12a^2b^2d\left(n^2+3n+2\right)x^2-ab^3(n+3)\left(c\left(n^2+9n+20\right)+4d\left(n^2+3n+2\right)x^3\right)+b^4\left(\right.}{b^5(n+1)(n+2)(n+3)(n+4)(n+5)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(a + b*x)^n*(c + d*x^3),x]
```

```
[Out] ((a + b*x)^(1 + n)*(24*a^4*d - 24*a^3*b*d*(1 + n)*x + 12*a^2*b^2*
d*(2 + 3*n + n^2)*x^2 + b^4*(12 + 19*n + 8*n^2 + n^3)*x*(c*(5 + n
) + d*(2 + n)*x^3) - a*b^3*(3 + n)*(c*(20 + 9*n + n^2) + 4*d*(2 +
 3*n + n^2)*x^3)))/(b^5*(1 + n)*(2 + n)*(3 + n)*(4 + n)*(5 + n))
```

Maple [B] time = 0.008, size = 283, normalized size = 2.3

$$\frac{(bx+a)^{1+n}\left(b^4dn^4x^4+10\,b^4dn^3x^4-4\,ab^3dn^3x^3+35\,b^4dn^2x^4-24\,ab^3dn^2x^3+b^4cn^4x+50\,b^4dnx^4+12\,a^2b^2dn^2x^2-44\,ab^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(b*x+a)^n*(d*x^3+c),x)
```

```
[Out] (b*x+a)^(1+n)*(b^4*d*n^4*x^4+10*b^4*d*n^3*x^4-4*a*b^3*d*n^3*x^3+3
5*b^4*d*n^2*x^4-24*a*b^3*d*n^2*x^3+b^4*c*n^4*x+50*b^4*d*n*x^4+12*
a^2*b^2*d*n^2*x^2-44*a*b^3*d*n*x^3+13*b^4*c*n^3*x+24*b^4*d*x^4+36
*a^2*b^2*d*n*x^2-a*b^3*c*n^3-24*a*b^3*d*x^3+59*b^4*c*n^2*x-24*a^3
*b*d*n*x+24*a^2*b^2*d*x^2-12*a*b^3*c*n^2+107*b^4*c*n*x-24*a^3*b*d
*x-47*a*b^3*c*n+60*b^4*c*x+24*a^4*d-60*a*b^3*c)/b^5/(n^5+15*n^4+8
5*n^3+225*n^2+274*n+120)
```

Maxima [A] time = 0.712591, size = 248, normalized size = 1.97

$$\frac{\left(b^2(n+1)x^2+abnx-a^2\right)(bx+a)^n c}{(n^2+3\,n+2)b^2}+\frac{\left(\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)b^5x^5+\left(n^4+6\,n^3+11\,n^2+6\,n\right)ab^4x^4-4\left(n^3+3\,n^2+2\,n\right)a^2b^3x^3+12\left(n^2+n\right)a^3b^2x^2-2\right.}{(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120)b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)*(b*x + a)^n*x,x, algorithm="maxima")
```

```
[Out] (b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n*c/((n^2 + 3*n + 2)*
b^2) + ((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^5*x^5 + (n^4 + 6*n^
3 + 11*n^2 + 6*n)*a*b^4*x^4 - 4*(n^3 + 3*n^2 + 2*n)*a^2*b^3*x^3 +
 12*(n^2 + n)*a^3*b^2*x^2 - 24*a^4*b*n*x + 24*a^5)*(b*x + a)^n*d/
((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*b^5)
```

Fricas [A] time = 0.28876, size = 470, normalized size = 3.73

$$-\frac{\left(a^2b^3cn^3 + 12\,a^2b^3cn^2 + 47\,a^2b^3cn + 60\,a^2b^3c - 24\,a^5d - \left(b^5dn^4 + 10\,b^5dn^3 + 35\,b^5dn^2 + 50\,b^5dn + 24\,b^5d\right)x^5 - \left(ab^4dn^4 + \right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)*(b*x + a)^n*x,x, algorithm="fricas")
```

```
[Out] -(a^2*b^3*c*n^3 + 12*a^2*b^3*c*n^2 + 47*a^2*b^3*c*n + 60*a^2*b^3*
c - 24*a^5*d - (b^5*d*n^4 + 10*b^5*d*n^3 + 35*b^5*d*n^2 + 50*b^5*
d*n + 24*b^5*d)*x^5 - (a*b^4*d*n^4 + 6*a*b^4*d*n^3 + 11*a*b^4*d*n
^2 + 6*a*b^4*d*n)*x^4 + 4*(a^2*b^3*d*n^3 + 3*a^2*b^3*d*n^2 + 2*a^
2*b^3*d*n)*x^3 - (b^5*c*n^4 + 13*b^5*c*n^3 + 60*b^5*c + (59*b^5*c
 + 12*a^3*b^2*d)*n^2 + (107*b^5*c + 12*a^3*b^2*d)*n)*x^2 - (a*b^4
*c*n^4 + 12*a*b^4*c*n^3 + 47*a*b^4*c*n^2 + 12*(5*a*b^4*c - 2*a^4*
b*d)*n)*x)*(b*x + a)^n/(b^5*n^5 + 15*b^5*n^4 + 85*b^5*n^3 + 225*b
^5*n^2 + 274*b^5*n + 120*b^5)
```

Sympy [A] time = 15.3252, size = 3699, normalized size = 29.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(b*x+a)**n*(d*x**3+c),x)
```

```
[Out] Piecewise((a**n*(c*x**2/2 + d*x**5/5), Eq(b, 0)), (12*a**4*d*log(
a/b + x)/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*
a*b**8*x**3 + 12*b**9*x**4) + 7*a**4*d/(12*a**4*b**5 + 48*a**3*b*
*6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) + 48*a*
*3*b*d*x*log(a/b + x)/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b*
*7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) + 16*a**3*b*d*x/(12*a**4
*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*
b**9*x**4) + 72*a**2*b**2*d*x**2*log(a/b + x)/(12*a**4*b**5 + 48*
a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4)
- a*b**3*c/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 4
8*a*b**8*x**3 + 12*b**9*x**4) + 48*a*b**3*d*x**3*log(a/b + x)/(12
*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3
```

```
+ 12*b**9*x**4) - 24*a*b**3*d*x**3/(12*a**4*b**5 + 48*a**3*b**6*x
 + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) - 4*b**4*c*
x/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*
x**3 + 12*b**9*x**4) + 12*b**4*d*x**4*log(a/b + x)/(12*a**4*b**5
+ 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x
**4) - 18*b**4*d*x**4/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b*
*7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4), Eq(n, -5)), (-24*a**4*d
*log(a/b + x)/(6*a**3*b**5 + 18*a**2*b**6*x + 18*a*b**7*x**2 + 6*
b**8*x**3) - 20*a**4*d/(6*a**3*b**5 + 18*a**2*b**6*x + 18*a*b**7*
x**2 + 6*b**8*x**3) - 72*a**3*b*d*x*log(a/b + x)/(6*a**3*b**5 + 1
8*a**2*b**6*x + 18*a*b**7*x**2 + 6*b**8*x**3) - 36*a**3*b*d*x/(6*
a**3*b**5 + 18*a**2*b**6*x + 18*a*b**7*x**2 + 6*b**8*x**3) - 72*a
**2*b**2*d*x**2*log(a/b + x)/(6*a**3*b**5 + 18*a**2*b**6*x + 18*a
*b**7*x**2 + 6*b**8*x**3) - a*b**3*c/(6*a**3*b**5 + 18*a**2*b**6*
x + 18*a*b**7*x**2 + 6*b**8*x**3) - 24*a*b**3*d*x**3*log(a/b + x)
/(6*a**3*b**5 + 18*a**2*b**6*x + 18*a*b**7*x**2 + 6*b**8*x**3) +
24*a*b**3*d*x**3/(6*a**3*b**5 + 18*a**2*b**6*x + 18*a*b**7*x**2 +
 6*b**8*x**3) - 3*b**4*c*x/(6*a**3*b**5 + 18*a**2*b**6*x + 18*a*b
**7*x**2 + 6*b**8*x**3) + 6*b**4*d*x**4/(6*a**3*b**5 + 18*a**2*b*
*6*x + 18*a*b**7*x**2 + 6*b**8*x**3), Eq(n, -4)), (12*a**4*d*log(
a/b + x)/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 18*a**4*d/(2*
a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 24*a**3*b*d*x*log(a/b + x
)/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 24*a**3*b*d*x/(2*a**
2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 12*a**2*b**2*d*x**2*log(a/b
+ x)/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) - a*b**3*c/(2*a**2*
b**5 + 4*a*b**6*x + 2*b**7*x**2) - 4*a*b**3*d*x**3/(2*a**2*b**5 +
 4*a*b**6*x + 2*b**7*x**2) - 2*b**4*c*x/(2*a**2*b**5 + 4*a*b**6*x
 + 2*b**7*x**2) + b**4*d*x**4/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*
x**2), Eq(n, -3)), (-12*a**4*d*log(a/b + x)/(3*a*b**5 + 3*b**6*x)
 - 12*a**4*d/(3*a*b**5 + 3*b**6*x) - 12*a**3*b*d*x*log(a/b + x)/(
3*a*b**5 + 3*b**6*x) + 6*a**2*b**2*d*x**2/(3*a*b**5 + 3*b**6*x) +
 3*a*b**3*c*log(a/b + x)/(3*a*b**5 + 3*b**6*x) + 3*a*b**3*c/(3*a*
b**5 + 3*b**6*x) - 2*a*b**3*d*x**3/(3*a*b**5 + 3*b**6*x) + 3*b**4
*c*x*log(a/b + x)/(3*a*b**5 + 3*b**6*x) + b**4*d*x**4/(3*a*b**5 +
 3*b**6*x), Eq(n, -2)), (a**4*d*log(a/b + x)/b**5 - a**3*d*x/b**4
 + a**2*d*x**2/(2*b**3) - a*c*log(a/b + x)/b**2 - a*d*x**3/(3*b**
2) + c*x/b + d*x**4/(4*b), Eq(n, -1)), (24*a**5*d*(a + b*x)**n/(b
**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5
*n + 120*b**5) - 24*a**4*b*d*n*x*(a + b*x)**n/(b**5*n**5 + 15*b**
5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) +
12*a**3*b**2*d*n**2*x**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 +
 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 12*a**3*
b**2*d*n*x**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n*
*3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - a**2*b**3*c*n**3*(a
 + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n*
*2 + 274*b**5*n + 120*b**5) - 12*a**2*b**3*c*n**2*(a + b*x)**n/(b
**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5
*n + 120*b**5) - 47*a**2*b**3*c*n*(a + b*x)**n/(b**5*n**5 + 15*b*
*5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) -
 60*a**2*b**3*c*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*
n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 4*a**2*b**3*d*n**
3*x**3*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 22
5*b**5*n**2 + 274*b**5*n + 120*b**5) - 12*a**2*b**3*d*n**2*x**3*(
a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n
```

```
**2 + 274*b**5*n + 120*b**5) - 8*a**2*b**3*d*n*x**3*(a + b*x)**n/
(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b*
*5*n + 120*b**5) + a*b**4*c*n**4*x*(a + b*x)**n/(b**5*n**5 + 15*b
**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5)
+ 12*a*b**4*c*n**3*x*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*
b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 47*a*b**4*c*
n**2*x*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 22
5*b**5*n**2 + 274*b**5*n + 120*b**5) + 60*a*b**4*c*n*x*(a + b*x)*
*n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274
*b**5*n + 120*b**5) + a*b**4*d*n**4*x**4*(a + b*x)**n/(b**5*n**5
+ 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*
b**5) + 6*a*b**4*d*n**3*x**4*(a + b*x)**n/(b**5*n**5 + 15*b**5*n*
*4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 11*a
*b**4*d*n**2*x**4*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**
5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 6*a*b**4*d*n*x*
*4*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b*
*5*n**2 + 274*b**5*n + 120*b**5) + b**5*c*n**4*x**2*(a + b*x)**n/
(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b*
*5*n + 120*b**5) + 13*b**5*c*n**3*x**2*(a + b*x)**n/(b**5*n**5 +
15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b*
*5) + 59*b**5*c*n**2*x**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4
+ 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 107*b**
5*c*n*x**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3
+ 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 60*b**5*c*x**2*(a + b*
x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 +
274*b**5*n + 120*b**5) + b**5*d*n**4*x**5*(a + b*x)**n/(b**5*n**5
 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120
*b**5) + 10*b**5*d*n**3*x**5*(a + b*x)**n/(b**5*n**5 + 15*b**5*n*
*4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 35*b
**5*d*n**2*x**5*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*
n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 50*b**5*d*n*x**5*
(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*
n**2 + 274*b**5*n + 120*b**5) + 24*b**5*d*x**5*(a + b*x)**n/(b**5
*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n
+ 120*b**5), True))
```

GIAC/XCAS [A] time = 0.270256, size = 857, normalized size = 6.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)*(b*x + a)^n*x,x, algorithm="giac")
```

```
[Out] (b^5*d*n^4*x^5*e^(n*ln(b*x + a)) + a*b^4*d*n^4*x^4*e^(n*ln(b*x +
a)) + 10*b^5*d*n^3*x^5*e^(n*ln(b*x + a)) + 6*a*b^4*d*n^3*x^4*e^(n
*ln(b*x + a)) + 35*b^5*d*n^2*x^5*e^(n*ln(b*x + a)) + b^5*c*n^4*x^
2*e^(n*ln(b*x + a)) - 4*a^2*b^3*d*n^3*x^3*e^(n*ln(b*x + a)) + 11*
a*b^4*d*n^2*x^4*e^(n*ln(b*x + a)) + 50*b^5*d*n*x^5*e^(n*ln(b*x +
a)) + a*b^4*c*n^4*x*e^(n*ln(b*x + a)) + 13*b^5*c*n^3*x^2*e^(n*ln(
```

```
b*x + a)) - 12*a^2*b^3*d*n^2*x^3*e^(n*ln(b*x + a)) + 6*a*b^4*d*n*
x^4*e^(n*ln(b*x + a)) + 24*b^5*d*x^5*e^(n*ln(b*x + a)) + 12*a*b^4
*c*n^3*x*e^(n*ln(b*x + a)) + 59*b^5*c*n^2*x^2*e^(n*ln(b*x + a)) +
 12*a^3*b^2*d*n^2*x^2*e^(n*ln(b*x + a)) - 8*a^2*b^3*d*n*x^3*e^(n*
ln(b*x + a)) - a^2*b^3*c*n^3*e^(n*ln(b*x + a)) + 47*a*b^4*c*n^2*x
*e^(n*ln(b*x + a)) + 107*b^5*c*n*x^2*e^(n*ln(b*x + a)) + 12*a^3*b
^2*d*n*x^2*e^(n*ln(b*x + a)) - 12*a^2*b^3*c*n^2*e^(n*ln(b*x + a))
 + 60*a*b^4*c*n*x*e^(n*ln(b*x + a)) - 24*a^4*b*d*n*x*e^(n*ln(b*x
+ a)) + 60*b^5*c*x^2*e^(n*ln(b*x + a)) - 47*a^2*b^3*c*n*e^(n*ln(b
*x + a)) - 60*a^2*b^3*c*e^(n*ln(b*x + a)) + 24*a^5*d*e^(n*ln(b*x
+ a)))/(b^5*n^5 + 15*b^5*n^4 + 85*b^5*n^3 + 225*b^5*n^2 + 274*b^5
*n + 120*b^5)
```

3.153 $\int (a+bx)^n \left(c+dx^3\right) \, dx$

Optimal. Leaf size=94

$$\frac{\left(b^3c-a^3d\right)(a+bx)^{n+1}}{b^4(n+1)}+\frac{3a^2d(a+bx)^{n+2}}{b^4(n+2)}-\frac{3ad(a+bx)^{n+3}}{b^4(n+3)}+\frac{d(a+bx)^{n+4}}{b^4(n+4)}$$

```
[Out] ((b^3*c - a^3*d)*(a + b*x)^(1 + n))/(b^4*(1 + n)) + (3*a^2*d*(a +
 b*x)^(2 + n))/(b^4*(2 + n)) - (3*a*d*(a + b*x)^(3 + n))/(b^4*(3
+ n)) + (d*(a + b*x)^(4 + n))/(b^4*(4 + n))
```

Rubi [A] time = 0.0960538, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$\frac{\left(b^3c-a^3d\right)(a+bx)^{n+1}}{b^4(n+1)}+\frac{3a^2d(a+bx)^{n+2}}{b^4(n+2)}-\frac{3ad(a+bx)^{n+3}}{b^4(n+3)}+\frac{d(a+bx)^{n+4}}{b^4(n+4)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x)^n*(c + d*x^3),x]
```

```
[Out] ((b^3*c - a^3*d)*(a + b*x)^(1 + n))/(b^4*(1 + n)) + (3*a^2*d*(a +
 b*x)^(2 + n))/(b^4*(2 + n)) - (3*a*d*(a + b*x)^(3 + n))/(b^4*(3
+ n)) + (d*(a + b*x)^(4 + n))/(b^4*(4 + n))
```

Rubi in Sympy [A] time = 24.1069, size = 83, normalized size = 0.88

$$\frac{3a^2d\,(a+bx)^{n+2}}{b^4\,(n+2)}-\frac{3ad\,(a+bx)^{n+3}}{b^4\,(n+3)}+\frac{d\,(a+bx)^{n+4}}{b^4\,(n+4)}-\frac{(a+bx)^{n+1}\left(a^3d-b^3c\right)}{b^4\,(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x+a)**n*(d*x**3+c),x)
```

```
[Out] 3*a**2*d*(a + b*x)**(n + 2)/(b**4*(n + 2)) - 3*a*d*(a + b*x)**(n
+ 3)/(b**4*(n + 3)) + d*(a + b*x)**(n + 4)/(b**4*(n + 4)) - (a +
b*x)**(n + 1)*(a**3*d - b**3*c)/(b**4*(n + 1))
```

Mathematica [A] time = 0.108475, size = 95, normalized size = 1.01

$$\frac{(a+bx)^{n+1}\left(-6a^3d+6a^2bd(n+1)x-3ab^2d\left(n^2+3n+2\right)x^2+b^3\left(n^2+5n+6\right)\left(c(n+4)+d(n+1)x^3\right)\right)}{b^4(n+1)(n+2)(n+3)(n+4)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x)^n*(c + d*x^3),x]
```

```
[Out] ((a + b*x)^(1 + n)*(-6*a^3*d + 6*a^2*b*d*(1 + n)*x - 3*a*b^2*d*(2
 + 3*n + n^2)*x^2 + b^3*(6 + 5*n + n^2)*(c*(4 + n) + d*(1 + n)*x^
3)))/(b^4*(1 + n)*(2 + n)*(3 + n)*(4 + n))
```

Maple [A] time = 0.007, size = 167, normalized size = 1.8

$$-\frac{(bx+a)^{1+n}\left(-b^3dn^3x^3-6\,b^3dn^2x^3+3\,ab^2dn^2x^2-11\,b^3dnx^3+9\,ab^2dnx^2-b^3cn^3-6\,dx^3b^3-6\,a^2bdnx+6\,adx^2b^2-9\,b\right.}{b^4\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x+a)^n*(d*x^3+c),x)
```

```
[Out] -(b*x+a)^(1+n)*(-b^3*d*n^3*x^3-6*b^3*d*n^2*x^3+3*a*b^2*d*n^2*x^2-
11*b^3*d*n*x^3+9*a*b^2*d*n*x^2-b^3*c*n^3-6*b^3*d*x^3-6*a^2*b*d*n*
x+6*a*b^2*d*x^2-9*b^3*c*n^2-6*a^2*b*d*x-26*b^3*c*n+6*a^3*d-24*b^3
*c)/b^4/(n^4+10*n^3+35*n^2+50*n+24)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)*(b*x + a)^n,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.285765, size = 300, normalized size = 3.19

$$\frac{\left(ab^3cn^3+9\,ab^3cn^2+26\,ab^3cn+24\,ab^3c-6\,a^4d+\left(b^4dn^3+6\,b^4dn^2+11\,b^4dn+6\,b^4d\right)x^4+\left(ab^3dn^3+3\,ab^3dn^2+2\,ab^3dn\right)x\right.}{b^4n^4+10\,b^4n^3+35\,b^4n^2+50\,b^4n+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)*(b*x + a)^n,x, algorithm="fricas")
```

```
[Out] (a*b^3*c*n^3 + 9*a*b^3*c*n^2 + 26*a*b^3*c*n + 24*a*b^3*c - 6*a^4*
d + (b^4*d*n^3 + 6*b^4*d*n^2 + 11*b^4*d*n + 6*b^4*d)*x^4 + (a*b^3
*d*n^3 + 3*a*b^3*d*n^2 + 2*a*b^3*d*n)*x^3 - 3*(a^2*b^2*d*n^2 + a^
2*b^2*d*n)*x^2 + (b^4*c*n^3 + 9*b^4*c*n^2 + 24*b^4*c + 2*(13*b^4*
c + 3*a^3*b*d)*n)*x)*(b*x + a)^n/(b^4*n^4 + 10*b^4*n^3 + 35*b^4*n
^2 + 50*b^4*n + 24*b^4)
```

Sympy [A] time = 9.16395, size = 1904, normalized size = 20.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x+a)**n*(d*x**3+c),x)
```

```
[Out] Piecewise((a**n*(c*x + d*x**4/4), Eq(b, 0)), (6*a**3*d*log(a/b +
x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3)
+ 5*a**3*d/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**
7*x**3) + 18*a**2*b*d*x*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*
x + 18*a*b**6*x**2 + 6*b**7*x**3) + 9*a**2*b*d*x/(6*a**3*b**4 + 1
8*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 18*a*b**2*d*x**2*
log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b
**7*x**3) - 2*b**3*c/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x*
*2 + 6*b**7*x**3) + 6*b**3*d*x**3*log(a/b + x)/(6*a**3*b**4 + 18*
a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - 6*b**3*d*x**3/(6*a*
*3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3), Eq(n, -
4)), (-6*a**3*d*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x
**2) - 9*a**3*d/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 12*a**
2*b*d*x*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 1
2*a**2*b*d*x/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 6*a*b**2*
d*x**2*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - b*
*3*c/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 2*b**3*d*x**3/(2*
a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2), Eq(n, -3)), (6*a**3*d*log(
a/b + x)/(2*a*b**4 + 2*b**5*x) + 6*a**3*d/(2*a*b**4 + 2*b**5*x) +
 6*a**2*b*d*x*log(a/b + x)/(2*a*b**4 + 2*b**5*x) - 3*a*b**2*d*x**
2/(2*a*b**4 + 2*b**5*x) - 2*b**3*c/(2*a*b**4 + 2*b**5*x) + b**3*d
*x**3/(2*a*b**4 + 2*b**5*x), Eq(n, -2)), (-a**3*d*log(a/b + x)/b*
*4 + a**2*d*x/b**3 - a*d*x**2/(2*b**2) + c*log(a/b + x)/b + d*x**
3/(3*b), Eq(n, -1)), (-6*a**4*d*(a + b*x)**n/(b**4*n**4 + 10*b**4
*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 6*a**3*b*d*n*x*(a +
 b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 2
4*b**4) - 3*a**2*b**2*d*n**2*x**2*(a + b*x)**n/(b**4*n**4 + 10*b*
*4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) - 3*a**2*b**2*d*n*x
**2*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b*
```

```
*4*n + 24*b**4) + a*b**3*c*n**3*(a + b*x)**n/(b**4*n**4 + 10*b**4
*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 9*a*b**3*c*n**2*(a
+ b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n +
24*b**4) + 26*a*b**3*c*n*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 +
 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 24*a*b**3*c*(a + b*x)**n/(
b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) +
a*b**3*d*n**3*x**3*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b*
*4*n**2 + 50*b**4*n + 24*b**4) + 3*a*b**3*d*n**2*x**3*(a + b*x)**
n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4)
 + 2*a*b**3*d*n*x**3*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*
b**4*n**2 + 50*b**4*n + 24*b**4) + b**4*c*n**3*x*(a + b*x)**n/(b*
*4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 9*
b**4*c*n**2*x*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n*
*2 + 50*b**4*n + 24*b**4) + 26*b**4*c*n*x*(a + b*x)**n/(b**4*n**4
 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 24*b**4*c
*x*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**
4*n + 24*b**4) + b**4*d*n**3*x**4*(a + b*x)**n/(b**4*n**4 + 10*b*
*4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 6*b**4*d*n**2*x**
4*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4
*n + 24*b**4) + 11*b**4*d*n*x**4*(a + b*x)**n/(b**4*n**4 + 10*b**
4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 6*b**4*d*x**4*(a +
 b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 2
4*b**4), True))
```

GIAC/XCAS [A] time = 0.268955, size = 539, normalized size = 5.73

$$\frac{b^4dn^3x^4e^{(n\ln(bx+a))} + ab^3dn^3x^3e^{(n\ln(bx+a))} + 6\,b^4dn^2x^4e^{(n\ln(bx+a))} + 3\,ab^3dn^2x^3e^{(n\ln(bx+a))} + 11\,b^4dnx^4e^{(n\ln(bx+a))} + b^4cn^3xe^{(n}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)*(b*x + a)^n,x, algorithm="giac")
```

```
[Out] (b^4*d*n^3*x^4*e^(n*ln(b*x + a)) + a*b^3*d*n^3*x^3*e^(n*ln(b*x +
a)) + 6*b^4*d*n^2*x^4*e^(n*ln(b*x + a)) + 3*a*b^3*d*n^2*x^3*e^(n*
ln(b*x + a)) + 11*b^4*d*n*x^4*e^(n*ln(b*x + a)) + b^4*c*n^3*x*e^(
n*ln(b*x + a)) - 3*a^2*b^2*d*n^2*x^2*e^(n*ln(b*x + a)) + 2*a*b^3*
d*n*x^3*e^(n*ln(b*x + a)) + 6*b^4*d*x^4*e^(n*ln(b*x + a)) + a*b^3
*c*n^3*e^(n*ln(b*x + a)) + 9*b^4*c*n^2*x*e^(n*ln(b*x + a)) - 3*a^
2*b^2*d*n*x^2*e^(n*ln(b*x + a)) + 9*a*b^3*c*n^2*e^(n*ln(b*x + a))
 + 26*b^4*c*n*x*e^(n*ln(b*x + a)) + 6*a^3*b*d*n*x*e^(n*ln(b*x + a
)) + 26*a*b^3*c*n*e^(n*ln(b*x + a)) + 24*b^4*c*x*e^(n*ln(b*x + a)
) + 24*a*b^3*c*e^(n*ln(b*x + a)) - 6*a^4*d*e^(n*ln(b*x + a)))/(b^
4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^4)
```

3.154 $\int \frac{(a+bx)^n(c+dx^3)}{x} \, dx$

Optimal. Leaf size=99

$$\frac{a^2d(a+bx)^{n+1}}{b^3(n+1)} - \frac{2ad(a+bx)^{n+2}}{b^3(n+2)} + \frac{d(a+bx)^{n+3}}{b^3(n+3)} - \frac{c(a+bx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{bx}{a}+1\right)}{a(n+1)}$$

```
[Out] (a^2*d*(a + b*x)^(1 + n))/(b^3*(1 + n)) - (2*a*d*(a + b*x)^(2 + n
))/(b^3*(2 + n)) + (d*(a + b*x)^(3 + n))/(b^3*(3 + n)) - (c*(a +
b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])/(a*
(1 + n))
```

Rubi [A] time = 0.115322, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$\frac{a^2d(a+bx)^{n+1}}{b^3(n+1)} - \frac{2ad(a+bx)^{n+2}}{b^3(n+2)} + \frac{d(a+bx)^{n+3}}{b^3(n+3)} - \frac{c(a+bx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{bx}{a}+1\right)}{a(n+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x)^n*(c + d*x^3))/x,x]
```

```
[Out] (a^2*d*(a + b*x)^(1 + n))/(b^3*(1 + n)) - (2*a*d*(a + b*x)^(2 + n
))/(b^3*(2 + n)) + (d*(a + b*x)^(3 + n))/(b^3*(3 + n)) - (c*(a +
b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])/(a*
(1 + n))
```

Rubi in Sympy [A] time = 23.2067, size = 83, normalized size = 0.84

$$\frac{a^2d\,(a+bx)^{n+1}}{b^3\,(n+1)} - \frac{2ad\,(a+bx)^{n+2}}{b^3\,(n+2)} + \frac{d\,(a+bx)^{n+3}}{b^3\,(n+3)} - \frac{c\,(a+bx)^{n+1}\,{}_2F_1\left(\begin{matrix}1,n+1\\n+2\end{matrix}\middle|1+\frac{bx}{a}\right)}{a\,(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x+a)**n*(d*x**3+c)/x,x)
```

```
[Out] a**2*d*(a + b*x)**(n + 1)/(b**3*(n + 1)) - 2*a*d*(a + b*x)**(n +
2)/(b**3*(n + 2)) + d*(a + b*x)**(n + 3)/(b**3*(n + 3)) - c*(a +
```

```
b*x)**(n + 1)*hyper((1, n + 1), (n + 2,), 1 + b*x/a)/(a*(n + 1))
```

Mathematica [A] time = 0.251355, size = 125, normalized size = 1.26

$$(a+bx)^n\left(\frac{d\left(a^3\left(2-2\left(\frac{bx}{a}+1\right)^{-n}\right)-2a^2bnx+ab^2n(n+1)x^2+b^3\left(n^2+3n+2\right)x^3\right)}{b^3\left(n^3+6n^2+11n+6\right)}+\frac{c\left(\frac{a}{bx}+1\right)^{-n}{}_2F_1\left(-n,-n;1-n;-\frac{a}{bx}\right)}{n}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x)^n*(c + d*x^3))/x,x]
```

```
[Out] (a + b*x)^n*((d*(-2*a^2*b*n*x + a*b^2*n*(1 + n)*x^2 + b^3*(2 + 3*
n + n^2)*x^3 + a^3*(2 - 2/(1 + (b*x)/a)^n)))/(b^3*(6 + 11*n + 6*n
^2 + n^3)) + (c*Hypergeometric2F1[-n, -n, 1 - n, -(a/(b*x))])/(n*
(1 + a/(b*x))^n))
```

Maple [F] time = 0.04, size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n\left(dx^3+c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x+a)^n*(d*x^3+c)/x,x)
```

```
[Out] int((b*x+a)^n*(d*x^3+c)/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^3+c\right)(bx+a)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)*(b*x + a)^n/x,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)*(b*x + a)^n/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(dx^3+c)(bx+a)^n}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)*(b*x + a)^n/x,x, algorithm="fricas")
```

```
[Out] integral((d*x^3 + c)*(b*x + a)^n/x, x)
```

Sympy [A] time = 11.0848, size = 741, normalized size = 7.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x+a)**n*(d*x**3+c)/x,x)
```

```
[Out] -b**n*c*n*(a/b + x)**n*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n + 1)
/gamma(n + 2) - b**n*c*(a/b + x)**n*lerchphi(1 + b*x/a, 1, n + 1)
*gamma(n + 1)/gamma(n + 2) + d*Piecewise((a**n*x**3/3, Eq(b, 0)),
 (2*a**2*log(a/b + x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) +
3*a**2/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 4*a*b*x*log(a/b
 + x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 4*a*b*x/(2*a**2*
b**3 + 4*a*b**4*x + 2*b**5*x**2) + 2*b**2*x**2*log(a/b + x)/(2*a*
*2*b**3 + 4*a*b**4*x + 2*b**5*x**2), Eq(n, -3)), (-2*a**2*log(a/b
 + x)/(a*b**3 + b**4*x) - 2*a**2/(a*b**3 + b**4*x) - 2*a*b*x*log(
a/b + x)/(a*b**3 + b**4*x) + b**2*x**2/(a*b**3 + b**4*x), Eq(n, -
2)), (a**2*log(a/b + x)/b**3 - a*x/b**2 + x**2/(2*b), Eq(n, -1)),
 (2*a**3*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b*
*3) - 2*a**2*b*n*x*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**
3*n + 6*b**3) + a*b**2*n**2*x**2*(a + b*x)**n/(b**3*n**3 + 6*b**3
*n**2 + 11*b**3*n + 6*b**3) + a*b**2*n*x**2*(a + b*x)**n/(b**3*n*
*3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + b**3*n**2*x**3*(a + b*x)
**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + 3*b**3*n*x**
3*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + 2
*b**3*x**3*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*
b**3), True)) - b*b**n*c*n*x*(a/b + x)**n*lerchphi(1 + b*x/a, 1,
n + 1)*gamma(n + 1)/(a*gamma(n + 2)) - b*b**n*c*x*(a/b + x)**n*le
rchphi(1 + b*x/a, 1, n + 1)*gamma(n + 1)/(a*gamma(n + 2))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^3 + c)(bx + a)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)*(b*x + a)^n/x,x, algorithm="giac")

[Out] integrate((d*x^3 + c)*(b*x + a)^n/x, x)
```

3.155 $\int x^2(a+bx)^n \left(c+dx^3\right)^2 \, dx$

Optimal. Leaf size=294

$$-\frac{2a\left(b^3c-4a^3d\right)\left(b^3c-a^3d\right)(a+bx)^{n+2}}{b^9(n+2)}-\frac{10ad\left(b^3c-7a^3d\right)(a+bx)^{n+5}}{b^9(n+5)}+\frac{2d\left(b^3c-28a^3d\right)(a+bx)^{n+6}}{b^9(n+6)}+\frac{28a^2d^2(a+bx)^{n+7}}{b^9(n+7)}+\frac{\left(28a^6d^2-20a^3b^3cd+b^6c^2\right)(a+bx)^{n+3}}{b^9(n+3)}+\frac{a^2\left(b^3c-a^3d\right)^2(a+bx)^{n+1}}{b^9(n+1)}+\frac{4a^2d\left(5b^3c-14a^3d\right)(a+bx)^{n+4}}{b^9(n+4)}-\frac{8ad^2(a+bx)^{n+8}}{b^9(n+8)}+\frac{d^2(a+bx)^{n+9}}{b^9(n+9)}$$

```
[Out] (a^2*(b^3*c - a^3*d)^2*(a + b*x)^(1 + n))/(b^9*(1 + n)) - (2*a*(b
^3*c - 4*a^3*d)*(b^3*c - a^3*d)*(a + b*x)^(2 + n))/(b^9*(2 + n))
+ ((b^6*c^2 - 20*a^3*b^3*c*d + 28*a^6*d^2)*(a + b*x)^(3 + n))/(b^
9*(3 + n)) + (4*a^2*d*(5*b^3*c - 14*a^3*d)*(a + b*x)^(4 + n))/(b^
9*(4 + n)) - (10*a*d*(b^3*c - 7*a^3*d)*(a + b*x)^(5 + n))/(b^9*(5
 + n)) + (2*d*(b^3*c - 28*a^3*d)*(a + b*x)^(6 + n))/(b^9*(6 + n))
 + (28*a^2*d^2*(a + b*x)^(7 + n))/(b^9*(7 + n)) - (8*a*d^2*(a + b
*x)^(8 + n))/(b^9*(8 + n)) + (d^2*(a + b*x)^(9 + n))/(b^9*(9 + n)
)
```

Rubi [A] time = 0.418832, antiderivative size = 294, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.05$

$$-\frac{2a\left(b^3c-4a^3d\right)\left(b^3c-a^3d\right)(a+bx)^{n+2}}{b^9(n+2)}-\frac{10ad\left(b^3c-7a^3d\right)(a+bx)^{n+5}}{b^9(n+5)}+\frac{2d\left(b^3c-28a^3d\right)(a+bx)^{n+6}}{b^9(n+6)}+\frac{28a^2d^2(a+bx)^{n+7}}{b^9(n+7)}+\frac{\left(28a^6d^2-20a^3b^3cd+b^6c^2\right)(a+bx)^{n+3}}{b^9(n+3)}+\frac{a^2\left(b^3c-a^3d\right)^2(a+bx)^{n+1}}{b^9(n+1)}+\frac{4a^2d\left(5b^3c-14a^3d\right)(a+bx)^{n+4}}{b^9(n+4)}-\frac{8ad^2(a+bx)^{n+8}}{b^9(n+8)}+\frac{d^2(a+bx)^{n+9}}{b^9(n+9)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(a + b*x)^n*(c + d*x^3)^2,x]
```


```
[Out] (a^2*(b^3*c - a^3*d)^2*(a + b*x)^(1 + n))/(b^9*(1 + n)) - (2*a*(b
^3*c - 4*a^3*d)*(b^3*c - a^3*d)*(a + b*x)^(2 + n))/(b^9*(2 + n))
+ ((b^6*c^2 - 20*a^3*b^3*c*d + 28*a^6*d^2)*(a + b*x)^(3 + n))/(b^
9*(3 + n)) + (4*a^2*d*(5*b^3*c - 14*a^3*d)*(a + b*x)^(4 + n))/(b^
9*(4 + n)) - (10*a*d*(b^3*c - 7*a^3*d)*(a + b*x)^(5 + n))/(b^9*(5
 + n)) + (2*d*(b^3*c - 28*a^3*d)*(a + b*x)^(6 + n))/(b^9*(6 + n))
 + (28*a^2*d^2*(a + b*x)^(7 + n))/(b^9*(7 + n)) - (8*a*d^2*(a + b
*x)^(8 + n))/(b^9*(8 + n)) + (d^2*(a + b*x)^(9 + n))/(b^9*(9 + n)
)
```

Rubi in Sympy [A] time = 87.3212, size = 275, normalized size = 0.94

$$\frac{28a^2d^2(a+bx)^{n+7}}{b^9(n+7)}-\frac{4a^2d(a+bx)^{n+4}\left(14a^3d-5b^3c\right)}{b^9(n+4)}+\frac{a^2(a+bx)^{n+1}\left(a^3d-b^3c\right)^2}{b^9(n+1)}-\frac{8ad^2(a+bx)^{n+8}}{b^9(n+8)}+\frac{10ad(a+bx)^{n+5}\left(7a^3d-b^3c\right)}{b^9(n+5)}-\frac{2a(a+bx)^{n+2}\left(a^3d-b^3c\right)\left(4a^3d-b^3c\right)}{b^9(n+2)}+\frac{d^2(a+bx)^{n+9}}{b^9(n+9)}-\frac{2d(a+bx)^{n+6}\left(28a^3d-b^3c\right)}{b^9(n+6)}+\frac{(a+bx)^{n+3}\left(28a^6d^2-20a^3b^3cd+b^6c^2\right)}{b^9(n+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(b*x+a)**n*(d*x**3+c)**2,x)
```

```
[Out] 28*a**2*d**2*(a + b*x)**(n + 7)/(b**9*(n + 7)) - 4*a**2*d*(a + b*
x)**(n + 4)*(14*a**3*d - 5*b**3*c)/(b**9*(n + 4)) + a**2*(a + b*x
)**(n + 1)*(a**3*d - b**3*c)**2/(b**9*(n + 1)) - 8*a*d**2*(a + b*
x)**(n + 8)/(b**9*(n + 8)) + 10*a*d*(a + b*x)**(n + 5)*(7*a**3*d
- b**3*c)/(b**9*(n + 5)) - 2*a*(a + b*x)**(n + 2)*(a**3*d - b**3*
c)*(4*a**3*d - b**3*c)/(b**9*(n + 2)) + d**2*(a + b*x)**(n + 9)/(
b**9*(n + 9)) - 2*d*(a + b*x)**(n + 6)*(28*a**3*d - b**3*c)/(b**9
*(n + 6)) + (a + b*x)**(n + 3)*(28*a**6*d**2 - 20*a**3*b**3*c*d +
 b**6*c**2)/(b**9*(n + 3))
```

Mathematica [A] time = 0.571289, size = 534, normalized size = 1.82

$$\frac{(a+bx)^{n+1}\left(40320a^8d^2-40320a^7bd^2(n+1)x+20160a^6b^2d^2\left(n^2+3n+2\right)x^2-240a^5b^3d\left(c\left(n^3+24n^2+191n+504\right)+28d\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(a + b*x)^n*(c + d*x^3)^2,x]
```

```
[Out] ((a + b*x)^(1 + n)*(40320*a^8*d^2 - 40320*a^7*b*d^2*(1 + n)*x + 2
0160*a^6*b^2*d^2*(2 + 3*n + n^2)*x^2 - 240*a^5*b^3*d*(c*(504 + 19
1*n + 24*n^2 + n^3) + 28*d*(6 + 11*n + 6*n^2 + n^3)*x^3) + 240*a^
4*b^4*d*(1 + n)*x*(c*(504 + 191*n + 24*n^2 + n^3) + 7*d*(24 + 26*
n + 9*n^2 + n^3)*x^3) - 24*a^3*b^5*d*(2 + 3*n + n^2)*x^2*(5*c*(50
4 + 191*n + 24*n^2 + n^3) + 14*d*(60 + 47*n + 12*n^2 + n^3)*x^3)
+ b^8*(2240 + 4968*n + 3954*n^2 + 1485*n^3 + 285*n^4 + 27*n^5 + n
^6)*x^2*(c^2*(54 + 15*n + n^2) + 2*c*d*(27 + 12*n + n^2)*x^3 + d^
2*(18 + 9*n + n^2)*x^6) - 2*a*b^7*(28 + 39*n + 12*n^2 + n^3)*x*(c
^2*(2160 + 1302*n + 289*n^2 + 28*n^3 + n^4) + 5*c*d*(432 + 462*n
+ 163*n^2 + 22*n^3 + n^4)*x^3 + 4*d^2*(180 + 216*n + 91*n^2 + 16*
n^3 + n^4)*x^6) + 2*a^2*b^6*(c^2*(60480 + 60216*n + 24574*n^2 + 5
```

```
265*n^3 + 625*n^4 + 39*n^5 + n^6) + 20*c*d*(3024 + 6690*n + 5269*
n^2 + 1920*n^3 + 346*n^4 + 30*n^5 + n^6)*x^3 + 28*d^2*(720 + 1764
*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6)*x^6)))/(b^9*(1
+ n)*(2 + n)*(3 + n)*(4 + n)*(5 + n)*(6 + n)*(7 + n)*(8 + n)*(9 +
 n))
```

Maple [B] time = 0.022, size = 1565, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^2*(b*x+a)^n*(d*x^3+c)^2,x)
```

```
[Out] (b*x+a)^(1+n)*(b^8*d^2*n^8*x^8+36*b^8*d^2*n^7*x^8-8*a*b^7*d^2*n^7
*x^7+546*b^8*d^2*n^6*x^8-224*a*b^7*d^2*n^6*x^7+2*b^8*c*d*n^8*x^5+
4536*b^8*d^2*n^5*x^8+56*a^2*b^6*d^2*n^6*x^6-2576*a*b^7*d^2*n^5*x^
7+78*b^8*c*d*n^7*x^5+22449*b^8*d^2*n^4*x^8+1176*a^2*b^6*d^2*n^5*x
^6-10*a*b^7*c*d*n^7*x^4-15680*a*b^7*d^2*n^4*x^7+1272*b^8*c*d*n^6*
x^5+67284*b^8*d^2*n^3*x^8-336*a^3*b^5*d^2*n^5*x^5+9800*a^2*b^6*d^
2*n^4*x^6-340*a*b^7*c*d*n^6*x^4-54152*a*b^7*d^2*n^3*x^7+b^8*c^2*n
^8*x^2+11268*b^8*c*d*n^5*x^5+118124*b^8*d^2*n^2*x^8-5040*a^3*b^5*
d^2*n^4*x^5+40*a^2*b^6*c*d*n^6*x^3+41160*a^2*b^6*d^2*n^3*x^6-4660
*a*b^7*c*d*n^5*x^4-105056*a*b^7*d^2*n^2*x^7+42*b^8*c^2*n^7*x^2+58
938*b^8*c*d*n^4*x^5+109584*b^8*d^2*n*x^8+1680*a^4*b^4*d^2*n^4*x^4
-28560*a^3*b^5*d^2*n^3*x^5+1200*a^2*b^6*c*d*n^5*x^3+90944*a^2*b^6
*d^2*n^2*x^6-2*a*b^7*c^2*n^7*x-33040*a*b^7*c*d*n^4*x^4-104544*a*b
^7*d^2*n*x^7+744*b^8*c^2*n^6*x^2+185022*b^8*c*d*n^3*x^5+40320*b^8
*d^2*x^8+16800*a^4*b^4*d^2*n^3*x^4-120*a^3*b^5*c*d*n^5*x^2-75600*
a^3*b^5*d^2*n^2*x^5+13840*a^2*b^6*c*d*n^4*x^3+98784*a^2*b^6*d^2*n
*x^6-80*a*b^7*c^2*n^6*x-129490*a*b^7*c*d*n^3*x^4-40320*a*b^7*d^2*
x^7+7218*b^8*c^2*n^5*x^2+337228*b^8*c*d*n^2*x^5-6720*a^5*b^3*d^2*
n^3*x^3+58800*a^4*b^4*d^2*n^2*x^4-3240*a^3*b^5*c*d*n^4*x^2-92064*
a^3*b^5*d^2*n*x^5+2*a^2*b^6*c^2*n^6+76800*a^2*b^6*c*d*n^3*x^3+403
20*a^2*b^6*d^2*x^6-1328*a*b^7*c^2*n^5*x-277660*a*b^7*c*d*n^2*x^4+
41619*b^8*c^2*n^4*x^2+322032*b^8*c*d*n*x^5-40320*a^5*b^3*d^2*n^2*
x^3+240*a^4*b^4*c*d*n^4*x+84000*a^4*b^4*d^2*n*x^4-31800*a^3*b^5*c
*d*n^3*x^2-40320*a^3*b^5*d^2*x^5+78*a^2*b^6*c^2*n^5+210760*a^2*b^
6*c*d*n^2*x^3-11780*a*b^7*c^2*n^4*x-297840*a*b^7*c*d*n*x^4+144468
*b^8*c^2*n^3*x^2+120960*b^8*c*d*x^5+20160*a^6*b^2*d^2*n^2*x^2-739
20*a^5*b^3*d^2*n*x^3+6000*a^4*b^4*c*d*n^3*x+40320*a^4*b^4*d^2*x^4
-135000*a^3*b^5*c*d*n^2*x^2+1250*a^2*b^6*c^2*n^4+267600*a^2*b^6*c
*d*n*x^3-59678*a*b^7*c^2*n^3*x-120960*a*b^7*c*d*x^4+290276*b^8*c^
2*n^2*x^2+60480*a^6*b^2*d^2*n*x^2-240*a^5*b^3*c*d*n^3-40320*a^5*b
^3*d^2*x^3+51600*a^4*b^4*c*d*n^2*x-227280*a^3*b^5*c*d*n*x^2+10530
*a^2*b^6*c^2*n^3+120960*a^2*b^6*c*d*x^3-169580*a*b^7*c^2*n^2*x+30
1872*b^8*c^2*n*x^2-40320*a^7*b*d^2*n*x+40320*a^6*b^2*d^2*x^2-5760
*a^5*b^3*c*d*n^2+166800*a^4*b^4*c*d*n*x-120960*a^3*b^5*c*d*x^2+49
148*a^2*b^6*c^2*n^2-241392*a*b^7*c^2*n*x+120960*b^8*c^2*x^2-40320
```

```
*a^7*b*d^2*x-45840*a^5*b^3*c*d*n+120960*a^4*b^4*c*d*x+120432*a^2*
b^6*c^2*n-120960*a*b^7*c^2*x+40320*a^8*d^2-120960*a^5*b^3*c*d+120
960*a^2*b^6*c^2)/b^9/(n^9+45*n^8+870*n^7+9450*n^6+63273*n^5+26932
5*n^4+723680*n^3+1172700*n^2+1026576*n+362880)
```

Maxima [A] time = 0.716648, size = 811, normalized size = 2.76

$$\frac{\left(\left(n^2+3\,n+2\right)b^3x^3+\left(n^2+n\right)ab^2x^2-2\,a^2bnx+2\,a^3\right)(bx+a)^n c^2}{(n^3+6\,n^2+11\,n+6)b^3}+\frac{2\,\left(\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)b^6x^6+\left(n^5+10\,n^4+35\,n^3+50\,n^2+24\,n\right)ab^5x^5-5\,\left(n^4+6\,n^3+11\,n^2+6\,n\right)a}{(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+176}+\frac{\left(\left(n^8+36\,n^7+546\,n^6+4536\,n^5+22449\,n^4+67284\,n^3+118124\,n^2+109584\,n+40320\right)b^9x^9+\left(n^8+28\,n^7+322\,n^6+1960\,\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)^2*(b*x + a)^n*x^2,x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*a*b^2*x^2 - 2*a^2*b*n*x + 2*
a^3)*(b*x + a)^n*c^2/((n^3 + 6*n^2 + 11*n + 6)*b^3) + 2*((n^5 + 1
5*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*b^6*x^6 + (n^5 + 10*n^4 +
 35*n^3 + 50*n^2 + 24*n)*a*b^5*x^5 - 5*(n^4 + 6*n^3 + 11*n^2 + 6*
n)*a^2*b^4*x^4 + 20*(n^3 + 3*n^2 + 2*n)*a^3*b^3*x^3 - 60*(n^2 + n
)*a^4*b^2*x^2 + 120*a^5*b*n*x - 120*a^6)*(b*x + a)^n*c*d/((n^6 +
21*n^5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*b^6) + ((n^
8 + 36*n^7 + 546*n^6 + 4536*n^5 + 22449*n^4 + 67284*n^3 + 118124*
n^2 + 109584*n + 40320)*b^9*x^9 + (n^8 + 28*n^7 + 322*n^6 + 1960*
n^5 + 6769*n^4 + 13132*n^3 + 13068*n^2 + 5040*n)*a*b^8*x^8 - 8*(n
^7 + 21*n^6 + 175*n^5 + 735*n^4 + 1624*n^3 + 1764*n^2 + 720*n)*a^
2*b^7*x^7 + 56*(n^6 + 15*n^5 + 85*n^4 + 225*n^3 + 274*n^2 + 120*n
)*a^3*b^6*x^6 - 336*(n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a^4*b
^5*x^5 + 1680*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^5*b^4*x^4 - 6720*(n^
3 + 3*n^2 + 2*n)*a^6*b^3*x^3 + 20160*(n^2 + n)*a^7*b^2*x^2 - 4032
0*a^8*b*n*x + 40320*a^9)*(b*x + a)^n*d^2/((n^9 + 45*n^8 + 870*n^7
 + 9450*n^6 + 63273*n^5 + 269325*n^4 + 723680*n^3 + 1172700*n^2 +
 1026576*n + 362880)*b^9)
```

Fricas [A] time = 0.303252, size = 2113, normalized size = 7.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)^2*(b*x + a)^n*x^2,x, algorithm="fricas")
```

```
[Out] (2*a^3*b^6*c^2*n^6 + 78*a^3*b^6*c^2*n^5 + 1250*a^3*b^6*c^2*n^4 +
120960*a^3*b^6*c^2 - 120960*a^6*b^3*c*d + 40320*a^9*d^2 + (b^9*d^
2*n^8 + 36*b^9*d^2*n^7 + 546*b^9*d^2*n^6 + 4536*b^9*d^2*n^5 + 224
49*b^9*d^2*n^4 + 67284*b^9*d^2*n^3 + 118124*b^9*d^2*n^2 + 109584*
b^9*d^2*n + 40320*b^9*d^2)*x^9 + (a*b^8*d^2*n^8 + 28*a*b^8*d^2*n^
7 + 322*a*b^8*d^2*n^6 + 1960*a*b^8*d^2*n^5 + 6769*a*b^8*d^2*n^4 +
 13132*a*b^8*d^2*n^3 + 13068*a*b^8*d^2*n^2 + 5040*a*b^8*d^2*n)*x^
8 - 8*(a^2*b^7*d^2*n^7 + 21*a^2*b^7*d^2*n^6 + 175*a^2*b^7*d^2*n^5
 + 735*a^2*b^7*d^2*n^4 + 1624*a^2*b^7*d^2*n^3 + 1764*a^2*b^7*d^2*
n^2 + 720*a^2*b^7*d^2*n)*x^7 + 2*(b^9*c*d*n^8 + 39*b^9*c*d*n^7 +
60480*b^9*c*d + 4*(159*b^9*c*d + 7*a^3*b^6*d^2)*n^6 + 6*(939*b^9*
c*d + 70*a^3*b^6*d^2)*n^5 + (29469*b^9*c*d + 2380*a^3*b^6*d^2)*n^
4 + 9*(10279*b^9*c*d + 700*a^3*b^6*d^2)*n^3 + 2*(84307*b^9*c*d +
3836*a^3*b^6*d^2)*n^2 + 24*(6709*b^9*c*d + 140*a^3*b^6*d^2)*n)*x^
6 + 2*(a*b^8*c*d*n^8 + 34*a*b^8*c*d*n^7 + 466*a*b^8*c*d*n^6 + 56*
(59*a*b^8*c*d - 3*a^4*b^5*d^2)*n^5 + (12949*a*b^8*c*d - 1680*a^4*
b^5*d^2)*n^4 + 2*(13883*a*b^8*c*d - 2940*a^4*b^5*d^2)*n^3 + 24*(1
241*a*b^8*c*d - 350*a^4*b^5*d^2)*n^2 + 4032*(3*a*b^8*c*d - a^4*b^
5*d^2)*n)*x^5 - 10*(a^2*b^7*c*d*n^7 + 30*a^2*b^7*c*d*n^6 + 346*a^
2*b^7*c*d*n^5 + 24*(80*a^2*b^7*c*d - 7*a^5*b^4*d^2)*n^4 + (5269*a
^2*b^7*c*d - 1008*a^5*b^4*d^2)*n^3 + 6*(1115*a^2*b^7*c*d - 308*a^
5*b^4*d^2)*n^2 + 1008*(3*a^2*b^7*c*d - a^5*b^4*d^2)*n)*x^4 + 30*(
351*a^3*b^6*c^2 - 8*a^6*b^3*c*d)*n^3 + (b^9*c^2*n^8 + 42*b^9*c^2*
n^7 + 120960*b^9*c^2 + 8*(93*b^9*c^2 + 5*a^3*b^6*c*d)*n^6 + 18*(4
01*b^9*c^2 + 60*a^3*b^6*c*d)*n^5 + (41619*b^9*c^2 + 10600*a^3*b^6
*c*d)*n^4 + 12*(12039*b^9*c^2 + 3750*a^3*b^6*c*d - 560*a^6*b^3*d^
2)*n^3 + 4*(72569*b^9*c^2 + 18940*a^3*b^6*c*d - 5040*a^6*b^3*d^2)
*n^2 + 48*(6289*b^9*c^2 + 840*a^3*b^6*c*d - 280*a^6*b^3*d^2)*n)*x
^3 + 4*(12287*a^3*b^6*c^2 - 1440*a^6*b^3*c*d)*n^2 + (a*b^8*c^2*n^
8 + 40*a*b^8*c^2*n^7 + 664*a*b^8*c^2*n^6 + 10*(589*a*b^8*c^2 - 12
*a^4*b^5*c*d)*n^5 + (29839*a*b^8*c^2 - 3000*a^4*b^5*c*d)*n^4 + 10
*(8479*a*b^8*c^2 - 2580*a^4*b^5*c*d)*n^3 + 24*(5029*a*b^8*c^2 - 3
475*a^4*b^5*c*d + 840*a^7*b^2*d^2)*n^2 + 20160*(3*a*b^8*c^2 - 3*a
^4*b^5*c*d + a^7*b^2*d^2)*n)*x^2 + 48*(2509*a^3*b^6*c^2 - 955*a^6
*b^3*c*d)*n - 2*(a^2*b^7*c^2*n^7 + 39*a^2*b^7*c^2*n^6 + 625*a^2*b
^7*c^2*n^5 + 15*(351*a^2*b^7*c^2 - 8*a^5*b^4*c*d)*n^4 + 2*(12287*
a^2*b^7*c^2 - 1440*a^5*b^4*c*d)*n^3 + 24*(2509*a^2*b^7*c^2 - 955*
a^5*b^4*c*d)*n^2 + 20160*(3*a^2*b^7*c^2 - 3*a^5*b^4*c*d + a^8*b*d
^2)*n)*x)*(b*x + a)^n/(b^9*n^9 + 45*b^9*n^8 + 870*b^9*n^7 + 9450*
b^9*n^6 + 63273*b^9*n^5 + 269325*b^9*n^4 + 723680*b^9*n^3 + 11727
00*b^9*n^2 + 1026576*b^9*n + 362880*b^9)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(b*x+a)**n*(d*x**3+c)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.273453, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)^2*(b*x + a)^n*x^2,x, algorithm="giac")
```

```
[Out] Done
```

3.156 $\int x(a+bx)^n \left(c+dx^3\right)^2 \, dx$

Optimal. Leaf size=248

$$-\frac{a\left(b^3c-a^3d\right)^2(a+bx)^{n+1}}{b^8(n+1)}+\frac{\left(b^3c-7a^3d\right)\left(b^3c-a^3d\right)(a+bx)^{n+2}}{b^8(n+2)}-\frac{ad\left(8b^3c-35a^3d\right)(a+bx)^{n+4}}{b^8(n+4)}+\frac{d\left(2b^3c-35a^3d\right)(a+bx)^{n+5}}{b^8(n+5)}+\frac{21a^2d^2(a+bx)^{n+6}}{b^8(n+6)}+\frac{3a^2d\left(4b^3c-7a^3d\right)(a+bx)^{n+3}}{b^8(n+3)}-\frac{7ad^2(a+bx)^{n+7}}{b^8(n+7)}+\frac{d^2(a+bx)^{n+8}}{b^8(n+8)}$$

```
[Out] -((a*(b^3*c - a^3*d)^2*(a + b*x)^(1 + n))/(b^8*(1 + n))) + ((b^3*
c - 7*a^3*d)*(b^3*c - a^3*d)*(a + b*x)^(2 + n))/(b^8*(2 + n)) + (
3*a^2*d*(4*b^3*c - 7*a^3*d)*(a + b*x)^(3 + n))/(b^8*(3 + n)) - (a
*d*(8*b^3*c - 35*a^3*d)*(a + b*x)^(4 + n))/(b^8*(4 + n)) + (d*(2*
b^3*c - 35*a^3*d)*(a + b*x)^(5 + n))/(b^8*(5 + n)) + (21*a^2*d^2*
(a + b*x)^(6 + n))/(b^8*(6 + n)) - (7*a*d^2*(a + b*x)^(7 + n))/(b
^8*(7 + n)) + (d^2*(a + b*x)^(8 + n))/(b^8*(8 + n))
```

Rubi [A] time = 0.317461, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$

$$-\frac{a\left(b^3c-a^3d\right)^2(a+bx)^{n+1}}{b^8(n+1)}+\frac{\left(b^3c-7a^3d\right)\left(b^3c-a^3d\right)(a+bx)^{n+2}}{b^8(n+2)}-\frac{ad\left(8b^3c-35a^3d\right)(a+bx)^{n+4}}{b^8(n+4)}+\frac{d\left(2b^3c-35a^3d\right)(a+bx)^{n+5}}{b^8(n+5)}+\frac{21a^2d^2(a+bx)^{n+6}}{b^8(n+6)}+\frac{3a^2d\left(4b^3c-7a^3d\right)(a+bx)^{n+3}}{b^8(n+3)}-\frac{7ad^2(a+bx)^{n+7}}{b^8(n+7)}+\frac{d^2(a+bx)^{n+8}}{b^8(n+8)}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(a + b*x)^n*(c + d*x^3)^2,x]
```


```
[Out] -((a*(b^3*c - a^3*d)^2*(a + b*x)^(1 + n))/(b^8*(1 + n))) + ((b^3*
c - 7*a^3*d)*(b^3*c - a^3*d)*(a + b*x)^(2 + n))/(b^8*(2 + n)) + (
3*a^2*d*(4*b^3*c - 7*a^3*d)*(a + b*x)^(3 + n))/(b^8*(3 + n)) - (a
*d*(8*b^3*c - 35*a^3*d)*(a + b*x)^(4 + n))/(b^8*(4 + n)) + (d*(2*
b^3*c - 35*a^3*d)*(a + b*x)^(5 + n))/(b^8*(5 + n)) + (21*a^2*d^2*
(a + b*x)^(6 + n))/(b^8*(6 + n)) - (7*a*d^2*(a + b*x)^(7 + n))/(b
^8*(7 + n)) + (d^2*(a + b*x)^(8 + n))/(b^8*(8 + n))
```

Rubi in Sympy [A] time = 68.5241, size = 228, normalized size = 0.92

$$\frac{21a^2d^2(a+bx)^{n+6}}{b^8(n+6)}-\frac{3a^2d(a+bx)^{n+3}\left(7a^3d-4b^3c\right)}{b^8(n+3)}-\frac{7ad^2(a+bx)^{n+7}}{b^8(n+7)}+\frac{ad(a+bx)^{n+4}\left(35a^3d-8b^3c\right)}{b^8(n+4)}-\frac{a(a+bx)^{n+1}\left(a^3d-b^3c\right)^2}{b^8(n+1)}+\frac{d^2(a+bx)^{n+8}}{b^8(n+8)}-\frac{d(a+bx)^{n+5}\left(35a^3d-2b^3c\right)}{b^8(n+5)}+\frac{(a+bx)^{n+2}\left(a^3d-b^3c\right)\left(7a^3d-b^3c\right)}{b^8(n+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(b*x+a)**n*(d*x**3+c)**2,x)
```

```
[Out] 21*a**2*d**2*(a + b*x)**(n + 6)/(b**8*(n + 6)) - 3*a**2*d*(a + b*
x)**(n + 3)*(7*a**3*d - 4*b**3*c)/(b**8*(n + 3)) - 7*a*d**2*(a +
b*x)**(n + 7)/(b**8*(n + 7)) + a*d*(a + b*x)**(n + 4)*(35*a**3*d
- 8*b**3*c)/(b**8*(n + 4)) - a*(a + b*x)**(n + 1)*(a**3*d - b**3*
c)**2/(b**8*(n + 1)) + d**2*(a + b*x)**(n + 8)/(b**8*(n + 8)) - d
*(a + b*x)**(n + 5)*(35*a**3*d - 2*b**3*c)/(b**8*(n + 5)) + (a +
b*x)**(n + 2)*(a**3*d - b**3*c)*(7*a**3*d - b**3*c)/(b**8*(n + 2)
)
```

Mathematica [A] time = 0.485125, size = 406, normalized size = 1.64

$$\frac{(a+bx)^{n+1}\left(-5040a^7d^2+5040a^6bd^2(n+1)x-2520a^5b^2d^2\left(n^2+3n+2\right)x^2+24a^4b^3d\left(2c\left(n^3+21n^2+146n+336\right)+35d\left(\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(a + b*x)^n*(c + d*x^3)^2,x]
```

```
[Out] ((a + b*x)^(1 + n)*(-5040*a^7*d^2 + 5040*a^6*b*d^2*(1 + n)*x - 25
20*a^5*b^2*d^2*(2 + 3*n + n^2)*x^2 + 24*a^4*b^3*d*(2*c*(336 + 146
*n + 21*n^2 + n^3) + 35*d*(6 + 11*n + 6*n^2 + n^3)*x^3) - 6*a^3*b
^4*d*(1 + n)*x*(8*c*(336 + 146*n + 21*n^2 + n^3) + 35*d*(24 + 26*
n + 9*n^2 + n^3)*x^3) + 6*a^2*b^5*d*(2 + 3*n + n^2)*x^2*(4*c*(336
 + 146*n + 21*n^2 + n^3) + 7*d*(60 + 47*n + 12*n^2 + n^3)*x^3) +
b^7*(504 + 954*n + 595*n^2 + 165*n^3 + 21*n^4 + n^5)*x*(c^2*(40 +
 13*n + n^2) + 2*c*d*(16 + 10*n + n^2)*x^3 + d^2*(10 + 7*n + n^2)
*x^6) - a*b^6*(18 + 9*n + n^2)*(c^2*(1120 + 804*n + 211*n^2 + 24*
n^3 + n^4) + 8*c*d*(112 + 198*n + 103*n^2 + 18*n^3 + n^4)*x^3 + 7
*d^2*(40 + 78*n + 49*n^2 + 12*n^3 + n^4)*x^6)))/(b^8*(1 + n)*(2 +
 n)*(3 + n)*(4 + n)*(5 + n)*(6 + n)*(7 + n)*(8 + n))
```

Maple [B] time = 0.02, size = 1142, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(b*x+a)^n*(d*x^3+c)^2,x)
```

```
[Out] -(b*x+a)^(1+n)*(-b^7*d^2*n^7*x^7-28*b^7*d^2*n^6*x^7+7*a*b^6*d^2*n
^6*x^6-322*b^7*d^2*n^5*x^7+147*a*b^6*d^2*n^5*x^6-2*b^7*c*d*n^7*x^
4-1960*b^7*d^2*n^4*x^7-42*a^2*b^5*d^2*n^5*x^5+1225*a*b^6*d^2*n^4*
x^6-62*b^7*c*d*n^6*x^4-6769*b^7*d^2*n^3*x^7-630*a^2*b^5*d^2*n^4*x
^5+8*a*b^6*c*d*n^6*x^3+5145*a*b^6*d^2*n^3*x^6-782*b^7*c*d*n^5*x^4
-13132*b^7*d^2*n^2*x^7+210*a^3*b^4*d^2*n^4*x^4-3570*a^2*b^5*d^2*n
^3*x^5+216*a*b^6*c*d*n^5*x^3+11368*a*b^6*d^2*n^2*x^6-b^7*c^2*n^7*
x-5162*b^7*c*d*n^4*x^4-13068*b^7*d^2*n*x^7+2100*a^3*b^4*d^2*n^3*x
^4-24*a^2*b^5*c*d*n^5*x^2-9450*a^2*b^5*d^2*n^2*x^5+2264*a*b^6*c*d
*n^4*x^3+12348*a*b^6*d^2*n*x^6-34*b^7*c^2*n^6*x-19088*b^7*c*d*n^3
*x^4-5040*b^7*d^2*x^7-840*a^4*b^3*d^2*n^3*x^3+7350*a^3*b^4*d^2*n^
2*x^4-576*a^2*b^5*c*d*n^4*x^2-11508*a^2*b^5*d^2*n*x^5+a*b^6*c^2*n
^6+11592*a*b^6*c*d*n^3*x^3+5040*a*b^6*d^2*x^6-478*b^7*c^2*n^5*x-3
9128*b^7*c*d*n^2*x^4-5040*a^4*b^3*d^2*n^2*x^3+48*a^3*b^4*c*d*n^4*
x+10500*a^3*b^4*d^2*n*x^4-5064*a^2*b^5*c*d*n^3*x^2-5040*a^2*b^5*d
^2*x^5+33*a*b^6*c^2*n^5+29984*a*b^6*c*d*n^2*x^3-3580*b^7*c^2*n^4*
x-40608*b^7*c*d*n*x^4+2520*a^5*b^2*d^2*n^2*x^2-9240*a^4*b^3*d^2*n
*x^3+1056*a^3*b^4*c*d*n^3*x+5040*a^3*b^4*d^2*x^4-19584*a^2*b^5*c*
d*n^2*x^2+445*a*b^6*c^2*n^4+36576*a*b^6*c*d*n*x^3-15289*b^7*c^2*n
^3*x-16128*b^7*c*d*x^4+7560*a^5*b^2*d^2*n*x^2-48*a^4*b^3*c*d*n^3-
5040*a^4*b^3*d^2*x^3+8016*a^3*b^4*c*d*n^2*x-31200*a^2*b^5*c*d*n*x
^2+3135*a*b^6*c^2*n^3+16128*a*b^6*c*d*x^3-36706*b^7*c^2*n^2*x-504
0*a^6*b*d^2*n*x+5040*a^5*b^2*d^2*x^2-1008*a^4*b^3*c*d*n^2+23136*a
^3*b^4*c*d*n*x-16128*a^2*b^5*c*d*x^2+12154*a*b^6*c^2*n^2-44712*b^
7*c^2*n*x-5040*a^6*b*d^2*x-7008*a^4*b^3*c*d*n+16128*a^3*b^4*c*d*x
+24552*a*b^6*c^2*n-20160*b^7*c^2*x+5040*a^7*d^2-16128*a^4*b^3*c*d
+20160*a*b^6*c^2)/b^8/(n^8+36*n^7+546*n^6+4536*n^5+22449*n^4+6728
4*n^3+118124*n^2+109584*n+40320)
```

Maxima [A] time = 0.715469, size = 640, normalized size = 2.58

$$\frac{\left(b^2(n+1)x^2+abnx-a^2\right)(bx+a)^nc^2}{(n^2+3n+2)b^2}+\frac{2\left(\left(n^4+10n^3+35n^2+50n+24\right)b^5x^5+\left(n^4+6n^3+11n^2+6n\right)ab^4x^4-4\left(n^3+3n^2+2n\right)a^2b^3x^3+12\left(n^2+n\right)a^3b^2x^2-\right.}{(n^5+15n^4+85n^3+225n^2+274n+120)b^5}+\frac{\left(\left(n^7+28n^6+322n^5+1960n^4+6769n^3+13132n^2+13068n+5040\right)b^8x^8+\left(n^7+21n^6+175n^5+735n^4+1624n^3+17\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)^2*(b*x + a)^n*x,x, algorithm="maxima")
```

```
[Out] (b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n*c^2/((n^2 + 3*n + 2
)*b^2) + 2*((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^5*x^5 + (n^4 +
6*n^3 + 11*n^2 + 6*n)*a*b^4*x^4 - 4*(n^3 + 3*n^2 + 2*n)*a^2*b^3*x
^3 + 12*(n^2 + n)*a^3*b^2*x^2 - 24*a^4*b*n*x + 24*a^5)*(b*x + a)^
n*c*d/((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*b^5) + ((n
^7 + 28*n^6 + 322*n^5 + 1960*n^4 + 6769*n^3 + 13132*n^2 + 13068*n
 + 5040)*b^8*x^8 + (n^7 + 21*n^6 + 175*n^5 + 735*n^4 + 1624*n^3 +
 1764*n^2 + 720*n)*a*b^7*x^7 - 7*(n^6 + 15*n^5 + 85*n^4 + 225*n^3
 + 274*n^2 + 120*n)*a^2*b^6*x^6 + 42*(n^5 + 10*n^4 + 35*n^3 + 50*
n^2 + 24*n)*a^3*b^5*x^5 - 210*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^4*b^
4*x^4 + 840*(n^3 + 3*n^2 + 2*n)*a^5*b^3*x^3 - 2520*(n^2 + n)*a^6*
b^2*x^2 + 5040*a^7*b*n*x - 5040*a^8)*(b*x + a)^n*d^2/((n^8 + 36*n
^7 + 546*n^6 + 4536*n^5 + 22449*n^4 + 67284*n^3 + 118124*n^2 + 10
9584*n + 40320)*b^8)
```

Fricas [A] time = 0.2914, size = 1642, normalized size = 6.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)^2*(b*x + a)^n*x,x, algorithm="fricas")
```

```
[Out] -(a^2*b^6*c^2*n^6 + 33*a^2*b^6*c^2*n^5 + 445*a^2*b^6*c^2*n^4 + 20
160*a^2*b^6*c^2 - 16128*a^5*b^3*c*d + 5040*a^8*d^2 - (b^8*d^2*n^7
 + 28*b^8*d^2*n^6 + 322*b^8*d^2*n^5 + 1960*b^8*d^2*n^4 + 6769*b^8
*d^2*n^3 + 13132*b^8*d^2*n^2 + 13068*b^8*d^2*n + 5040*b^8*d^2)*x^
8 - (a*b^7*d^2*n^7 + 21*a*b^7*d^2*n^6 + 175*a*b^7*d^2*n^5 + 735*a
*b^7*d^2*n^4 + 1624*a*b^7*d^2*n^3 + 1764*a*b^7*d^2*n^2 + 720*a*b^
7*d^2*n)*x^7 + 7*(a^2*b^6*d^2*n^6 + 15*a^2*b^6*d^2*n^5 + 85*a^2*b
^6*d^2*n^4 + 225*a^2*b^6*d^2*n^3 + 274*a^2*b^6*d^2*n^2 + 120*a^2*
b^6*d^2*n)*x^6 - 2*(b^8*c*d*n^7 + 31*b^8*c*d*n^6 + 8064*b^8*c*d +
 (391*b^8*c*d + 21*a^3*b^5*d^2)*n^5 + (2581*b^8*c*d + 210*a^3*b^5
*d^2)*n^4 + (9544*b^8*c*d + 735*a^3*b^5*d^2)*n^3 + 2*(9782*b^8*c*
d + 525*a^3*b^5*d^2)*n^2 + 72*(282*b^8*c*d + 7*a^3*b^5*d^2)*n)*x^
5 - 2*(a*b^7*c*d*n^7 + 27*a*b^7*c*d*n^6 + 283*a*b^7*c*d*n^5 + 21*
(69*a*b^7*c*d - 5*a^4*b^4*d^2)*n^4 + 2*(1874*a*b^7*c*d - 315*a^4*
b^4*d^2)*n^3 + 3*(1524*a*b^7*c*d - 385*a^4*b^4*d^2)*n^2 + 126*(16
*a*b^7*c*d - 5*a^4*b^4*d^2)*n)*x^4 + 3*(1045*a^2*b^6*c^2 - 16*a^5
*b^3*c*d)*n^3 + 8*(a^2*b^6*c*d*n^6 + 24*a^2*b^6*c*d*n^5 + 211*a^2
*b^6*c*d*n^4 + 3*(272*a^2*b^6*c*d - 35*a^5*b^3*d^2)*n^3 + 5*(260*
a^2*b^6*c*d - 63*a^5*b^3*d^2)*n^2 + 42*(16*a^2*b^6*c*d - 5*a^5*b^
3*d^2)*n)*x^3 + 2*(6077*a^2*b^6*c^2 - 504*a^5*b^3*c*d)*n^2 - (b^8
*c^2*n^7 + 34*b^8*c^2*n^6 + 20160*b^8*c^2 + 2*(239*b^8*c^2 + 12*a
^3*b^5*c*d)*n^5 + 4*(895*b^8*c^2 + 132*a^3*b^5*c*d)*n^4 + (15289*
b^8*c^2 + 4008*a^3*b^5*c*d)*n^3 + 2*(18353*b^8*c^2 + 5784*a^3*b^5
*c*d - 1260*a^6*b^2*d^2)*n^2 + 72*(621*b^8*c^2 + 112*a^3*b^5*c*d
```

```
- 35*a^6*b^2*d^2)*n)*x^2 + 24*(1023*a^2*b^6*c^2 - 292*a^5*b^3*c*d
)*n - (a*b^7*c^2*n^7 + 33*a*b^7*c^2*n^6 + 445*a*b^7*c^2*n^5 + 3*(
1045*a*b^7*c^2 - 16*a^4*b^4*c*d)*n^4 + 2*(6077*a*b^7*c^2 - 504*a^
4*b^4*c*d)*n^3 + 24*(1023*a*b^7*c^2 - 292*a^4*b^4*c*d)*n^2 + 1008
*(20*a*b^7*c^2 - 16*a^4*b^4*c*d + 5*a^7*b*d^2)*n)*x)*(b*x + a)^n/
(b^8*n^8 + 36*b^8*n^7 + 546*b^8*n^6 + 4536*b^8*n^5 + 22449*b^8*n^
4 + 67284*b^8*n^3 + 118124*b^8*n^2 + 109584*b^8*n + 40320*b^8)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(b*x+a)**n*(d*x**3+c)**2,x)

[Out] Timed out
```

GIAC/XCAS [A] time = 0.271241, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)^2*(b*x + a)^n*x,x, algorithm="giac")

[Out] Done
```

3.157 $\int (a+bx)^n \left(c+dx^3\right)^2 \, dx$

Optimal. Leaf size=203

$$\frac{\left(b^3c-a^3d\right)^2(a+bx)^{n+1}}{b^7(n+1)}-\frac{3ad\left(2b^3c-5a^3d\right)(a+bx)^{n+3}}{b^7(n+3)}+\frac{2d\left(b^3c-10a^3d\right)(a+bx)^{n+4}}{b^7(n+4)}+\frac{15a^2d^2(a+bx)^{n+5}}{b^7(n+5)}+\frac{6a^2d\left(b^3c-a^3d\right)(a+bx)^{n+2}}{b^7(n+2)}-\frac{6ad^2(a+bx)^{n+6}}{b^7(n+6)}+\frac{d^2(a+bx)^{n+7}}{b^7(n+7)}$$

```
[Out] ((b^3*c - a^3*d)^2*(a + b*x)^(1 + n))/(b^7*(1 + n)) + (6*a^2*d*(b
^3*c - a^3*d)*(a + b*x)^(2 + n))/(b^7*(2 + n)) - (3*a*d*(2*b^3*c
- 5*a^3*d)*(a + b*x)^(3 + n))/(b^7*(3 + n)) + (2*d*(b^3*c - 10*a^
3*d)*(a + b*x)^(4 + n))/(b^7*(4 + n)) + (15*a^2*d^2*(a + b*x)^(5
+ n))/(b^7*(5 + n)) - (6*a*d^2*(a + b*x)^(6 + n))/(b^7*(6 + n)) +
 (d^2*(a + b*x)^(7 + n))/(b^7*(7 + n))
```

Rubi [A] time = 0.246871, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$

$$\frac{\left(b^3c-a^3d\right)^2(a+bx)^{n+1}}{b^7(n+1)}-\frac{3ad\left(2b^3c-5a^3d\right)(a+bx)^{n+3}}{b^7(n+3)}+\frac{2d\left(b^3c-10a^3d\right)(a+bx)^{n+4}}{b^7(n+4)}+\frac{15a^2d^2(a+bx)^{n+5}}{b^7(n+5)}+\frac{6a^2d\left(b^3c-a^3d\right)(a+bx)^{n+2}}{b^7(n+2)}-\frac{6ad^2(a+bx)^{n+6}}{b^7(n+6)}+\frac{d^2(a+bx)^{n+7}}{b^7(n+7)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x)^n*(c + d*x^3)^2,x]
```

```
[Out] ((b^3*c - a^3*d)^2*(a + b*x)^(1 + n))/(b^7*(1 + n)) + (6*a^2*d*(b
^3*c - a^3*d)*(a + b*x)^(2 + n))/(b^7*(2 + n)) - (3*a*d*(2*b^3*c
- 5*a^3*d)*(a + b*x)^(3 + n))/(b^7*(3 + n)) + (2*d*(b^3*c - 10*a^
3*d)*(a + b*x)^(4 + n))/(b^7*(4 + n)) + (15*a^2*d^2*(a + b*x)^(5
+ n))/(b^7*(5 + n)) - (6*a*d^2*(a + b*x)^(6 + n))/(b^7*(6 + n)) +
 (d^2*(a + b*x)^(7 + n))/(b^7*(7 + n))
```

Rubi in Sympy [A] time = 58.1476, size = 187, normalized size = 0.92

$$\frac{15a^2d^2(a+bx)^{n+5}}{b^7(n+5)}-\frac{6a^2d(a+bx)^{n+2}\left(a^3d-b^3c\right)}{b^7(n+2)}-\frac{6ad^2(a+bx)^{n+6}}{b^7(n+6)}+\frac{3ad(a+bx)^{n+3}\left(5a^3d-2b^3c\right)}{b^7(n+3)}+\frac{d^2(a+bx)^{n+7}}{b^7(n+7)}-\frac{2d(a+bx)^{n+4}\left(10a^3d-b^3c\right)}{b^7(n+4)}+\frac{(a+bx)^{n+1}\left(a^3d-b^3c\right)^2}{b^7(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x+a)**n*(d*x**3+c)**2,x)
```

```
[Out] 15*a**2*d**2*(a + b*x)**(n + 5)/(b**7*(n + 5)) - 6*a**2*d*(a + b*
x)**(n + 2)*(a**3*d - b**3*c)/(b**7*(n + 2)) - 6*a*d**2*(a + b*x)
**(n + 6)/(b**7*(n + 6)) + 3*a*d*(a + b*x)**(n + 3)*(5*a**3*d - 2
*b**3*c)/(b**7*(n + 3)) + d**2*(a + b*x)**(n + 7)/(b**7*(n + 7))
- 2*d*(a + b*x)**(n + 4)*(10*a**3*d - b**3*c)/(b**7*(n + 4)) + (a
 + b*x)**(n + 1)*(a**3*d - b**3*c)**2/(b**7*(n + 1))
```

Mathematica [A] time = 0.277576, size = 297, normalized size = 1.46

$$\frac{(a+bx)^{n+1}\left(720a^6d^2-720a^5bd^2(n+1)x+360a^4b^2d^2\left(n^2+3n+2\right)x^2-12a^3b^3d\left(c\left(n^3+18n^2+107n+210\right)+10d\left(n^3+6\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x)^n*(c + d*x^3)^2,x]
```

```
[Out] ((a + b*x)^(1 + n)*(720*a^6*d^2 - 720*a^5*b*d^2*(1 + n)*x + 360*a
^4*b^2*d^2*(2 + 3*n + n^2)*x^2 - 6*a*b^5*d*(10 + 17*n + 8*n^2 + n
^3)*x^2*(c*(42 + 13*n + n^2) + d*(12 + 7*n + n^2)*x^3) - 12*a^3*b
^3*d*(c*(210 + 107*n + 18*n^2 + n^3) + 10*d*(6 + 11*n + 6*n^2 + n
^3)*x^3) + 6*a^2*b^4*d*(1 + n)*x*(2*c*(210 + 107*n + 18*n^2 + n^3
) + 5*d*(24 + 26*n + 9*n^2 + n^3)*x^3) + b^6*(180 + 216*n + 91*n^
2 + 16*n^3 + n^4)*(c^2*(28 + 11*n + n^2) + 2*c*d*(7 + 8*n + n^2)*
x^3 + d^2*(4 + 5*n + n^2)*x^6)))/(b^7*(1 + n)*(2 + n)*(3 + n)*(4
+ n)*(5 + n)*(6 + n)*(7 + n))
```

Maple [B] time = 0.017, size = 793, normalized size = 3.9

$$\frac{(bx+a)^{1+n}\left(b^6d^2n^6x^6+21\,b^6d^2n^5x^6-6\,ab^5d^2n^5x^5+175\,b^6d^2n^4x^6-90\,ab^5d^2n^4x^5+2\,b^6cdn^6x^3+735\,b^6d^2n^3x^6+30\,a^2b^4d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x+a)^n*(d*x^3+c)^2,x)
```

```
[Out] (b*x+a)^(1+n)*(b^6*d^2*n^6*x^6+21*b^6*d^2*n^5*x^6-6*a*b^5*d^2*n^5
*x^5+175*b^6*d^2*n^4*x^6-90*a*b^5*d^2*n^4*x^5+2*b^6*c*d*n^6*x^3+7
35*b^6*d^2*n^3*x^6+30*a^2*b^4*d^2*n^4*x^4-510*a*b^5*d^2*n^3*x^5+4
8*b^6*c*d*n^5*x^3+1624*b^6*d^2*n^2*x^6+300*a^2*b^4*d^2*n^3*x^4-6*
```

```
a*b^5*c*d*n^5*x^2-1350*a*b^5*d^2*n^2*x^5+452*b^6*c*d*n^4*x^3+1764
*b^6*d^2*n*x^6-120*a^3*b^3*d^2*n^3*x^3+1050*a^2*b^4*d^2*n^2*x^4-1
26*a*b^5*c*d*n^4*x^2-1644*a*b^5*d^2*n*x^5+b^6*c^2*n^6+2112*b^6*c*
d*n^3*x^3+720*b^6*d^2*x^6-720*a^3*b^3*d^2*n^2*x^3+12*a^2*b^4*c*d*
n^4*x+1500*a^2*b^4*d^2*n*x^4-978*a*b^5*c*d*n^3*x^2-720*a*b^5*d^2*
x^5+27*b^6*c^2*n^5+5090*b^6*c*d*n^2*x^3+360*a^4*b^2*d^2*n^2*x^2-1
320*a^3*b^3*d^2*n*x^3+228*a^2*b^4*c*d*n^3*x+720*a^2*b^4*d^2*x^4-3
402*a*b^5*c*d*n^2*x^2+295*b^6*c^2*n^4+5904*b^6*c*d*n*x^3+1080*a^4
*b^2*d^2*n*x^2-12*a^3*b^3*c*d*n^3-720*a^3*b^3*d^2*x^3+1500*a^2*b^
4*c*d*n^2*x-5064*a*b^5*c*d*n*x^2+1665*b^6*c^2*n^3+2520*b^6*c*d*x^
3-720*a^5*b*d^2*n*x+720*a^4*b^2*d^2*x^2-216*a^3*b^3*c*d*n^2+3804*
a^2*b^4*c*d*n*x-2520*a*b^5*c*d*x^2+5104*b^6*c^2*n^2-720*a^5*b*d^2
*x-1284*a^3*b^3*c*d*n+2520*a^2*b^4*c*d*x+8028*b^6*c^2*n+720*a^6*d
^2-2520*a^3*b^3*c*d+5040*b^6*c^2)/b^7/(n^7+28*n^6+322*n^5+1960*n^
4+6769*n^3+13132*n^2+13068*n+5040)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)^2*(b*x + a)^n,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.28925, size = 1206, normalized size = 5.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)^2*(b*x + a)^n,x, algorithm="fricas")
```

```
[Out] (a*b^6*c^2*n^6 + 27*a*b^6*c^2*n^5 + 295*a*b^6*c^2*n^4 + 5040*a*b^
6*c^2 - 2520*a^4*b^3*c*d + 720*a^7*d^2 + (b^7*d^2*n^6 + 21*b^7*d^
2*n^5 + 175*b^7*d^2*n^4 + 735*b^7*d^2*n^3 + 1624*b^7*d^2*n^2 + 17
64*b^7*d^2*n + 720*b^7*d^2)*x^7 + (a*b^6*d^2*n^6 + 15*a*b^6*d^2*n
^5 + 85*a*b^6*d^2*n^4 + 225*a*b^6*d^2*n^3 + 274*a*b^6*d^2*n^2 + 1
20*a*b^6*d^2*n)*x^6 - 6*(a^2*b^5*d^2*n^5 + 10*a^2*b^5*d^2*n^4 + 3
5*a^2*b^5*d^2*n^3 + 50*a^2*b^5*d^2*n^2 + 24*a^2*b^5*d^2*n)*x^5 +
2*(b^7*c*d*n^6 + 24*b^7*c*d*n^5 + 1260*b^7*c*d + (226*b^7*c*d + 1
5*a^3*b^4*d^2)*n^4 + 6*(176*b^7*c*d + 15*a^3*b^4*d^2)*n^3 + 5*(50
9*b^7*c*d + 33*a^3*b^4*d^2)*n^2 + 18*(164*b^7*c*d + 5*a^3*b^4*d^2
)*n)*x^4 + 3*(555*a*b^6*c^2 - 4*a^4*b^3*c*d)*n^3 + 2*(a*b^6*c*d*n
```

```
^6 + 21*a*b^6*c*d*n^5 + 163*a*b^6*c*d*n^4 + 3*(189*a*b^6*c*d - 20
*a^4*b^3*d^2)*n^3 + 4*(211*a*b^6*c*d - 45*a^4*b^3*d^2)*n^2 + 60*(
7*a*b^6*c*d - 2*a^4*b^3*d^2)*n)*x^3 + 8*(638*a*b^6*c^2 - 27*a^4*b
^3*c*d)*n^2 - 6*(a^2*b^5*c*d*n^5 + 19*a^2*b^5*c*d*n^4 + 125*a^2*b
^5*c*d*n^3 + (317*a^2*b^5*c*d - 60*a^5*b^2*d^2)*n^2 + 30*(7*a^2*b
^5*c*d - 2*a^5*b^2*d^2)*n)*x^2 + 12*(669*a*b^6*c^2 - 107*a^4*b^3*
c*d)*n + (b^7*c^2*n^6 + 27*b^7*c^2*n^5 + 5040*b^7*c^2 + (295*b^7*
c^2 + 12*a^3*b^4*c*d)*n^4 + 9*(185*b^7*c^2 + 24*a^3*b^4*c*d)*n^3
+ 4*(1276*b^7*c^2 + 321*a^3*b^4*c*d)*n^2 + 36*(223*b^7*c^2 + 70*a
^3*b^4*c*d - 20*a^6*b*d^2)*n)*x)*(b*x + a)^n/(b^7*n^7 + 28*b^7*n^
6 + 322*b^7*n^5 + 1960*b^7*n^4 + 6769*b^7*n^3 + 13132*b^7*n^2 + 1
3068*b^7*n + 5040*b^7)
```

Sympy [A] time = 144.844, size = 11662, normalized size = 57.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x+a)**n*(d*x**3+c)**2,x)
```

```
[Out] Piecewise((a**n*(c**2*x + c*d*x**4/2 + d**2*x**7/7), Eq(b, 0)), (
60*a**9*d**2*log(a/b + x)/(60*a**9*b**7 + 360*a**8*b**8*x + 900*a
**7*b**9*x**2 + 1200*a**6*b**10*x**3 + 900*a**5*b**11*x**4 + 360*
a**4*b**12*x**5 + 60*a**3*b**13*x**6) + 22*a**9*d**2/(60*a**9*b**
7 + 360*a**8*b**8*x + 900*a**7*b**9*x**2 + 1200*a**6*b**10*x**3 +
 900*a**5*b**11*x**4 + 360*a**4*b**12*x**5 + 60*a**3*b**13*x**6)
+ 360*a**8*b*d**2*x*log(a/b + x)/(60*a**9*b**7 + 360*a**8*b**8*x
+ 900*a**7*b**9*x**2 + 1200*a**6*b**10*x**3 + 900*a**5*b**11*x**4
 + 360*a**4*b**12*x**5 + 60*a**3*b**13*x**6) + 72*a**8*b*d**2*x/(
60*a**9*b**7 + 360*a**8*b**8*x + 900*a**7*b**9*x**2 + 1200*a**6*b
**10*x**3 + 900*a**5*b**11*x**4 + 360*a**4*b**12*x**5 + 60*a**3*b
**13*x**6) + 900*a**7*b**2*d**2*x**2*log(a/b + x)/(60*a**9*b**7 +
 360*a**8*b**8*x + 900*a**7*b**9*x**2 + 1200*a**6*b**10*x**3 + 90
0*a**5*b**11*x**4 + 360*a**4*b**12*x**5 + 60*a**3*b**13*x**6) + 1
200*a**6*b**3*d**2*x**3*log(a/b + x)/(60*a**9*b**7 + 360*a**8*b**
8*x + 900*a**7*b**9*x**2 + 1200*a**6*b**10*x**3 + 900*a**5*b**11*
x**4 + 360*a**4*b**12*x**5 + 60*a**3*b**13*x**6) - 300*a**6*b**3*
d**2*x**3/(60*a**9*b**7 + 360*a**8*b**8*x + 900*a**7*b**9*x**2 +
1200*a**6*b**10*x**3 + 900*a**5*b**11*x**4 + 360*a**4*b**12*x**5
+ 60*a**3*b**13*x**6) + 900*a**5*b**4*d**2*x**4*log(a/b + x)/(60*
a**9*b**7 + 360*a**8*b**8*x + 900*a**7*b**9*x**2 + 1200*a**6*b**1
0*x**3 + 900*a**5*b**11*x**4 + 360*a**4*b**12*x**5 + 60*a**3*b**1
3*x**6) - 525*a**5*b**4*d**2*x**4/(60*a**9*b**7 + 360*a**8*b**8*x
 + 900*a**7*b**9*x**2 + 1200*a**6*b**10*x**3 + 900*a**5*b**11*x**
4 + 360*a**4*b**12*x**5 + 60*a**3*b**13*x**6) + 360*a**4*b**5*d**
2*x**5*log(a/b + x)/(60*a**9*b**7 + 360*a**8*b**8*x + 900*a**7*b*
*9*x**2 + 1200*a**6*b**10*x**3 + 900*a**5*b**11*x**4 + 360*a**4*b
**12*x**5 + 60*a**3*b**13*x**6) - 390*a**4*b**5*d**2*x**5/(60*a**
```

```
9*b**7 + 360*a**8*b**8*x + 900*a**7*b**9*x**2 + 1200*a**6*b**10*x
**3 + 900*a**5*b**11*x**4 + 360*a**4*b**12*x**5 + 60*a**3*b**13*x
**6) - 10*a**3*b**6*c**2/(60*a**9*b**7 + 360*a**8*b**8*x + 900*a*
*7*b**9*x**2 + 1200*a**6*b**10*x**3 + 900*a**5*b**11*x**4 + 360*a
**4*b**12*x**5 + 60*a**3*b**13*x**6) + 60*a**3*b**6*d**2*x**6*log
(a/b + x)/(60*a**9*b**7 + 360*a**8*b**8*x + 900*a**7*b**9*x**2 +
1200*a**6*b**10*x**3 + 900*a**5*b**11*x**4 + 360*a**4*b**12*x**5
+ 60*a**3*b**13*x**6) - 125*a**3*b**6*d**2*x**6/(60*a**9*b**7 + 3
60*a**8*b**8*x + 900*a**7*b**9*x**2 + 1200*a**6*b**10*x**3 + 900*
a**5*b**11*x**4 + 360*a**4*b**12*x**5 + 60*a**3*b**13*x**6) + 30*
a**2*b**7*c*d*x**4/(60*a**9*b**7 + 360*a**8*b**8*x + 900*a**7*b**
9*x**2 + 1200*a**6*b**10*x**3 + 900*a**5*b**11*x**4 + 360*a**4*b*
*12*x**5 + 60*a**3*b**13*x**6) + 12*a*b**8*c*d*x**5/(60*a**9*b**7
 + 360*a**8*b**8*x + 900*a**7*b**9*x**2 + 1200*a**6*b**10*x**3 +
900*a**5*b**11*x**4 + 360*a**4*b**12*x**5 + 60*a**3*b**13*x**6) +
 2*b**9*c*d*x**6/(60*a**9*b**7 + 360*a**8*b**8*x + 900*a**7*b**9*
x**2 + 1200*a**6*b**10*x**3 + 900*a**5*b**11*x**4 + 360*a**4*b**1
2*x**5 + 60*a**3*b**13*x**6), Eq(n, -7)), (-60*a**8*d**2*log(a/b
+ x)/(10*a**7*b**7 + 50*a**6*b**8*x + 100*a**5*b**9*x**2 + 100*a*
*4*b**10*x**3 + 50*a**3*b**11*x**4 + 10*a**2*b**12*x**5) - 27*a**
8*d**2/(10*a**7*b**7 + 50*a**6*b**8*x + 100*a**5*b**9*x**2 + 100*
a**4*b**10*x**3 + 50*a**3*b**11*x**4 + 10*a**2*b**12*x**5) - 300*
a**7*b*d**2*x*log(a/b + x)/(10*a**7*b**7 + 50*a**6*b**8*x + 100*a
**5*b**9*x**2 + 100*a**4*b**10*x**3 + 50*a**3*b**11*x**4 + 10*a**
2*b**12*x**5) - 75*a**7*b*d**2*x/(10*a**7*b**7 + 50*a**6*b**8*x +
 100*a**5*b**9*x**2 + 100*a**4*b**10*x**3 + 50*a**3*b**11*x**4 +
10*a**2*b**12*x**5) - 600*a**6*b**2*d**2*x**2*log(a/b + x)/(10*a*
*7*b**7 + 50*a**6*b**8*x + 100*a**5*b**9*x**2 + 100*a**4*b**10*x*
*3 + 50*a**3*b**11*x**4 + 10*a**2*b**12*x**5) - 600*a**5*b**3*d**
2*x**3*log(a/b + x)/(10*a**7*b**7 + 50*a**6*b**8*x + 100*a**5*b**
9*x**2 + 100*a**4*b**10*x**3 + 50*a**3*b**11*x**4 + 10*a**2*b**12
*x**5) + 200*a**5*b**3*d**2*x**3/(10*a**7*b**7 + 50*a**6*b**8*x +
 100*a**5*b**9*x**2 + 100*a**4*b**10*x**3 + 50*a**3*b**11*x**4 +
10*a**2*b**12*x**5) - 300*a**4*b**4*d**2*x**4*log(a/b + x)/(10*a*
*7*b**7 + 50*a**6*b**8*x + 100*a**5*b**9*x**2 + 100*a**4*b**10*x*
*3 + 50*a**3*b**11*x**4 + 10*a**2*b**12*x**5) + 250*a**4*b**4*d**
2*x**4/(10*a**7*b**7 + 50*a**6*b**8*x + 100*a**5*b**9*x**2 + 100*
a**4*b**10*x**3 + 50*a**3*b**11*x**4 + 10*a**2*b**12*x**5) - 60*a
**3*b**5*d**2*x**5*log(a/b + x)/(10*a**7*b**7 + 50*a**6*b**8*x +
100*a**5*b**9*x**2 + 100*a**4*b**10*x**3 + 50*a**3*b**11*x**4 + 1
0*a**2*b**12*x**5) + 110*a**3*b**5*d**2*x**5/(10*a**7*b**7 + 50*a
**6*b**8*x + 100*a**5*b**9*x**2 + 100*a**4*b**10*x**3 + 50*a**3*b
**11*x**4 + 10*a**2*b**12*x**5) - 2*a**2*b**6*c**2/(10*a**7*b**7
+ 50*a**6*b**8*x + 100*a**5*b**9*x**2 + 100*a**4*b**10*x**3 + 50*
a**3*b**11*x**4 + 10*a**2*b**12*x**5) + 10*a**2*b**6*d**2*x**6/(1
0*a**7*b**7 + 50*a**6*b**8*x + 100*a**5*b**9*x**2 + 100*a**4*b**1
0*x**3 + 50*a**3*b**11*x**4 + 10*a**2*b**12*x**5) + 5*a*b**7*c*d*
x**4/(10*a**7*b**7 + 50*a**6*b**8*x + 100*a**5*b**9*x**2 + 100*a*
*4*b**10*x**3 + 50*a**3*b**11*x**4 + 10*a**2*b**12*x**5) + b**8*c
*d*x**5/(10*a**7*b**7 + 50*a**6*b**8*x + 100*a**5*b**9*x**2 + 100
*a**4*b**10*x**3 + 50*a**3*b**11*x**4 + 10*a**2*b**12*x**5), Eq(n
, -6)), (60*a**7*d**2*log(a/b + x)/(4*a**5*b**7 + 16*a**4*b**8*x
+ 24*a**3*b**9*x**2 + 16*a**2*b**10*x**3 + 4*a*b**11*x**4) + 35*a
**7*d**2/(4*a**5*b**7 + 16*a**4*b**8*x + 24*a**3*b**9*x**2 + 16*a
```

```
**2*b**10*x**3 + 4*a*b**11*x**4) + 240*a**6*b*d**2*x*log(a/b + x)
/(4*a**5*b**7 + 16*a**4*b**8*x + 24*a**3*b**9*x**2 + 16*a**2*b**1
0*x**3 + 4*a*b**11*x**4) + 80*a**6*b*d**2*x/(4*a**5*b**7 + 16*a**
4*b**8*x + 24*a**3*b**9*x**2 + 16*a**2*b**10*x**3 + 4*a*b**11*x**
4) + 360*a**5*b**2*d**2*x**2*log(a/b + x)/(4*a**5*b**7 + 16*a**4*
b**8*x + 24*a**3*b**9*x**2 + 16*a**2*b**10*x**3 + 4*a*b**11*x**4)
 + 240*a**4*b**3*d**2*x**3*log(a/b + x)/(4*a**5*b**7 + 16*a**4*b*
*8*x + 24*a**3*b**9*x**2 + 16*a**2*b**10*x**3 + 4*a*b**11*x**4) -
 120*a**4*b**3*d**2*x**3/(4*a**5*b**7 + 16*a**4*b**8*x + 24*a**3*
b**9*x**2 + 16*a**2*b**10*x**3 + 4*a*b**11*x**4) + 60*a**3*b**4*d
**2*x**4*log(a/b + x)/(4*a**5*b**7 + 16*a**4*b**8*x + 24*a**3*b**
9*x**2 + 16*a**2*b**10*x**3 + 4*a*b**11*x**4) - 90*a**3*b**4*d**2
*x**4/(4*a**5*b**7 + 16*a**4*b**8*x + 24*a**3*b**9*x**2 + 16*a**2
*b**10*x**3 + 4*a*b**11*x**4) - 12*a**2*b**5*d**2*x**5/(4*a**5*b*
*7 + 16*a**4*b**8*x + 24*a**3*b**9*x**2 + 16*a**2*b**10*x**3 + 4*
a*b**11*x**4) - a*b**6*c**2/(4*a**5*b**7 + 16*a**4*b**8*x + 24*a*
*3*b**9*x**2 + 16*a**2*b**10*x**3 + 4*a*b**11*x**4) + 2*a*b**6*d*
*2*x**6/(4*a**5*b**7 + 16*a**4*b**8*x + 24*a**3*b**9*x**2 + 16*a*
*2*b**10*x**3 + 4*a*b**11*x**4) + 2*b**7*c*d*x**4/(4*a**5*b**7 +
16*a**4*b**8*x + 24*a**3*b**9*x**2 + 16*a**2*b**10*x**3 + 4*a*b**
11*x**4), Eq(n, -5)), (-60*a**6*d**2*log(a/b + x)/(3*a**3*b**7 +
9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) - 50*a**6*d**2/(3*a
**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) - 180*a*
*5*b*d**2*x*log(a/b + x)/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*
x**2 + 3*b**10*x**3) - 90*a**5*b*d**2*x/(3*a**3*b**7 + 9*a**2*b**
8*x + 9*a*b**9*x**2 + 3*b**10*x**3) - 180*a**4*b**2*d**2*x**2*log
(a/b + x)/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*
x**3) + 6*a**3*b**3*c*d*log(a/b + x)/(3*a**3*b**7 + 9*a**2*b**8*x
 + 9*a*b**9*x**2 + 3*b**10*x**3) + 5*a**3*b**3*c*d/(3*a**3*b**7 +
 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) - 60*a**3*b**3*d**
2*x**3*log(a/b + x)/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2
+ 3*b**10*x**3) + 60*a**3*b**3*d**2*x**3/(3*a**3*b**7 + 9*a**2*b*
*8*x + 9*a*b**9*x**2 + 3*b**10*x**3) + 18*a**2*b**4*c*d*x*log(a/b
 + x)/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3
) + 9*a**2*b**4*c*d*x/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**
2 + 3*b**10*x**3) + 15*a**2*b**4*d**2*x**4/(3*a**3*b**7 + 9*a**2*
b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) + 18*a*b**5*c*d*x**2*log(a
/b + x)/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x*
*3) - 3*a*b**5*d**2*x**5/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*
x**2 + 3*b**10*x**3) - b**6*c**2/(3*a**3*b**7 + 9*a**2*b**8*x + 9
*a*b**9*x**2 + 3*b**10*x**3) + 6*b**6*c*d*x**3*log(a/b + x)/(3*a*
*3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x**3) - 6*b**6*
c*d*x**3/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**2 + 3*b**10*x
**3) + b**6*d**2*x**6/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**
2 + 3*b**10*x**3), Eq(n, -4)), (60*a**6*d**2*log(a/b + x)/(4*a**2
*b**7 + 8*a*b**8*x + 4*b**9*x**2) + 90*a**6*d**2/(4*a**2*b**7 + 8
*a*b**8*x + 4*b**9*x**2) + 120*a**5*b*d**2*x*log(a/b + x)/(4*a**2
*b**7 + 8*a*b**8*x + 4*b**9*x**2) + 120*a**5*b*d**2*x/(4*a**2*b**
7 + 8*a*b**8*x + 4*b**9*x**2) + 60*a**4*b**2*d**2*x**2*log(a/b +
x)/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) - 24*a**3*b**3*c*d*lo
g(a/b + x)/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) - 36*a**3*b**
3*c*d/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) - 20*a**3*b**3*d**
2*x**3/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) - 48*a**2*b**4*c*
d*x*log(a/b + x)/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) - 48*a*
```

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*2*b**4*c*d*x/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) + 5*a**2*b
**4*d**2*x**4/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) - 24*a*b**
5*c*d*x**2*log(a/b + x)/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2)
 - 2*a*b**5*d**2*x**5/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) - 2
*b**6*c**2/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) + 8*b**6*c*d*
x**3/(4*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2) + b**6*d**2*x**6/(4
*a**2*b**7 + 8*a*b**8*x + 4*b**9*x**2), Eq(n, -3)), (-60*a**6*d**
2*log(a/b + x)/(10*a*b**7 + 10*b**8*x) - 60*a**6*d**2/(10*a*b**7
 + 10*b**8*x) - 60*a**5*b*d**2*x*log(a/b + x)/(10*a*b**7 + 10*b**8
*x) + 30*a**4*b**2*d**2*x**2/(10*a*b**7 + 10*b**8*x) + 60*a**3*b*
*3*c*d*log(a/b + x)/(10*a*b**7 + 10*b**8*x) + 60*a**3*b**3*c*d/(1
0*a*b**7 + 10*b**8*x) - 10*a**3*b**3*d**2*x**3/(10*a*b**7 + 10*b*
*8*x) + 60*a**2*b**4*c*d*x*log(a/b + x)/(10*a*b**7 + 10*b**8*x) +
 5*a**2*b**4*d**2*x**4/(10*a*b**7 + 10*b**8*x) - 30*a*b**5*c*d*x*
*2/(10*a*b**7 + 10*b**8*x) - 3*a*b**5*d**2*x**5/(10*a*b**7 + 10*b
**8*x) - 10*b**6*c**2/(10*a*b**7 + 10*b**8*x) + 10*b**6*c*d*x**3/
(10*a*b**7 + 10*b**8*x) + 2*b**6*d**2*x**6/(10*a*b**7 + 10*b**8*x
), Eq(n, -2)), (a**6*d**2*log(a/b + x)/b**7 - a**5*d**2*x/b**6 +
a**4*d**2*x**2/(2*b**5) - 2*a**3*c*d*log(a/b + x)/b**4 - a**3*d**
2*x**3/(3*b**4) + 2*a**2*c*d*x/b**3 + a**2*d**2*x**4/(4*b**3) - a
*c*d*x**2/b**2 - a*d**2*x**5/(5*b**2) + c**2*log(a/b + x)/b + 2*c
*d*x**3/(3*b) + d**2*x**6/(6*b), Eq(n, -1)), (720*a**7*d**2*(a +
b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**
4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7)
 - 720*a**6*b*d**2*n*x*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 32
2*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 +
 13068*b**7*n + 5040*b**7) + 360*a**5*b**2*d**2*n**2*x**2*(a + b*
x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4
 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) +
360*a**5*b**2*d**2*n*x**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6
 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n*
*2 + 13068*b**7*n + 5040*b**7) - 12*a**4*b**3*c*d*n**3*(a + b*x)*
*n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6
769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 216
*a**4*b**3*c*d*n**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*
b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 1
3068*b**7*n + 5040*b**7) - 1284*a**4*b**3*c*d*n*(a + b*x)**n/(b**
7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**
7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 2520*a**4*
b**3*c*d*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 +
 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n
 + 5040*b**7) - 120*a**4*b**3*d**2*n**3*x**3*(a + b*x)**n/(b**7*n
**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n
**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 360*a**4*b**3
*d**2*n**2*x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7
*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068
*b**7*n + 5040*b**7) - 240*a**4*b**3*d**2*n*x**3*(a + b*x)**n/(b*
*7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b*
*7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 12*a**3*b
**4*c*d*n**4*x*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*
n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*
b**7*n + 5040*b**7) + 216*a**3*b**4*c*d*n**3*x*(a + b*x)**n/(b**7
*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7
*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 1284*a**3*b
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**4*c*d*n**2*x*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*
n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*
b**7*n + 5040*b**7) + 2520*a**3*b**4*c*d*n*x*(a + b*x)**n/(b**7*n
**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n
**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 30*a**3*b**4*
d**2*n**4*x**4*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*
n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*
b**7*n + 5040*b**7) + 180*a**3*b**4*d**2*n**3*x**4*(a + b*x)**n/(
b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*
b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 330*a**
3*b**4*d**2*n**2*x**4*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 32
2*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 +
 13068*b**7*n + 5040*b**7) + 180*a**3*b**4*d**2*n*x**4*(a + b*x)*
*n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6
769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 6*a
**2*b**5*c*d*n**5*x**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 3
22*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2
+ 13068*b**7*n + 5040*b**7) - 114*a**2*b**5*c*d*n**4*x**2*(a + b*
x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4
+ 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) -
750*a**2*b**5*c*d*n**3*x**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**
6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*
n**2 + 13068*b**7*n + 5040*b**7) - 1902*a**2*b**5*c*d*n**2*x**2*(
a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7
*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b*
*7) - 1260*a**2*b**5*c*d*n*x**2*(a + b*x)**n/(b**7*n**7 + 28*b**7
*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b
**7*n**2 + 13068*b**7*n + 5040*b**7) - 6*a**2*b**5*d**2*n**5*x**5
*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b*
*7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*
b**7) - 60*a**2*b**5*d**2*n**4*x**5*(a + b*x)**n/(b**7*n**7 + 28*
b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 131
32*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 210*a**2*b**5*d**2*n**
3*x**5*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1
960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n +
 5040*b**7) - 300*a**2*b**5*d**2*n**2*x**5*(a + b*x)**n/(b**7*n**
7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**
3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 144*a**2*b**5*d
**2*n*x**5*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5
 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7
*n + 5040*b**7) + a*b**6*c**2*n**6*(a + b*x)**n/(b**7*n**7 + 28*b
**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 1313
2*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 27*a*b**6*c**2*n**5*(a
+ b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n
**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7
) + 295*a*b**6*c**2*n**4*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 +
 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**
2 + 13068*b**7*n + 5040*b**7) + 1665*a*b**6*c**2*n**3*(a + b*x)**
n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 67
69*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 5104
*a*b**6*c**2*n**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b*
*7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 130
68*b**7*n + 5040*b**7) + 8028*a*b**6*c**2*n*(a + b*x)**n/(b**7*n*
*7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n*
```

*3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 5040*a*b**6*c*
2(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*
b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 504
0*b**7) + 2*a*b**6*c*d*n**6*x**3*(a + b*x)**n/(b**7*n**7 + 28*b**
7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*
b**7*n**2 + 13068*b**7*n + 5040*b**7) + 42*a*b**6*c*d*n**5*x**3*(
a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7
*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b*
*7) + 326*a*b**6*c*d*n**4*x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*
n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b*
*7*n**2 + 13068*b**7*n + 5040*b**7) + 1134*a*b**6*c*d*n**3*x**3*(
a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7
*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b*
*7) + 1688*a*b**6*c*d*n**2*x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7
*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b
7*n2 + 13068*b**7*n + 5040*b**7) + 840*a*b**6*c*d*n*x**3*(a +
 b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n*
*4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7)
 + a*b**6*d**2*n**6*x**6*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 +
 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**
2 + 13068*b**7*n + 5040*b**7) + 15*a*b**6*d**2*n**5*x**6*(a + b*x
)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 +
 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 8
5*a*b**6*d**2*n**4*x**6*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 +
322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2
 + 13068*b**7*n + 5040*b**7) + 225*a*b**6*d**2*n**3*x**6*(a + b*x
)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 +
 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 2
74*a*b**6*d**2*n**2*x**6*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 +
 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**
2 + 13068*b**7*n + 5040*b**7) + 120*a*b**6*d**2*n*x**6*(a + b*x)*
*n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6
769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + b**
7*c**2*n**6*x*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n
5 + 1960*b7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b
7*n + 5040*b7) + 27*b**7*c**2*n**5*x*(a + b*x)**n/(b**7*n**7
+ 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3
+ 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 295*b**7*c**2*n**
4*x*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960
*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 50
40*b**7) + 1665*b**7*c**2*n**3*x*(a + b*x)**n/(b**7*n**7 + 28*b**
7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*
b**7*n**2 + 13068*b**7*n + 5040*b**7) + 5104*b**7*c**2*n**2*x*(a
+ b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n
4 + 6769*b7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7
) + 8028*b**7*c**2*n*x*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 3
22*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2
+ 13068*b**7*n + 5040*b**7) + 5040*b**7*c**2*x*(a + b*x)**n/(b**7
*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7
*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 2*b**7*c*d*
n**6*x**4*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5
+ 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*
n + 5040*b**7) + 48*b**7*c*d*n**5*x**4*(a + b*x)**n/(b**7*n**7 +
28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 +

```
13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 452*b**7*c*d*n**4*x
**4*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960
*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 50
40*b**7) + 2112*b**7*c*d*n**3*x**4*(a + b*x)**n/(b**7*n**7 + 28*b
**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 1313
2*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 5090*b**7*c*d*n**2*x**4
*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b*
*7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*
b**7) + 5904*b**7*c*d*n*x**4*(a + b*x)**n/(b**7*n**7 + 28*b**7*n*
*6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7
*n**2 + 13068*b**7*n + 5040*b**7) + 2520*b**7*c*d*x**4*(a + b*x)*
*n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6
769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + b**
7*d**2*n**6*x**7*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**
7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 1306
8*b**7*n + 5040*b**7) + 21*b**7*d**2*n**5*x**7*(a + b*x)**n/(b**7
*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7
*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 175*b**7*d*
*2*n**4*x**7*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n*
*5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b*
*7*n + 5040*b**7) + 735*b**7*d**2*n**3*x**7*(a + b*x)**n/(b**7*n*
*7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n*
*3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 1624*b**7*d**2
*n**2*x**7*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5
 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7
*n + 5040*b**7) + 1764*b**7*d**2*n*x**7*(a + b*x)**n/(b**7*n**7 +
 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 +
 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 720*b**7*d**2*x**7
*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b*
*7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*
b**7), True))
```

GIAC/XCAS [A] time = 0.271899, size = 1, normalized size = 0.

$$\text{Done}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)^2*(b*x + a)^n,x, algorithm="giac")
```

```
[Out] Done
```

3.158 $\int \frac{(a+bx)^n(c+dx^3)^2}{x} dx$

Optimal. Leaf size=209

$$-\frac{ad\left(4b^3c-5a^3d\right)(a+bx)^{n+2}}{b^6(n+2)}+\frac{2d\left(b^3c-5a^3d\right)(a+bx)^{n+3}}{b^6(n+3)}+\frac{10a^2d^2(a+bx)^{n+4}}{b^6(n+4)}+\frac{a^2d\left(2b^3c-a^3d\right)(a+bx)^{n+1}}{b^6(n+1)}-\frac{5ad^2(a+bx)^{n+5}}{b^6(n+5)}+\frac{d^2(a+bx)^{n+6}}{b^6(n+6)}-\frac{c^2(a+bx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{bx}{a}+1\right)}{a(n+1)}$$

```
[Out] (a^2*d*(2*b^3*c - a^3*d)*(a + b*x)^(1 + n))/(b^6*(1 + n)) - (a*d*
(4*b^3*c - 5*a^3*d)*(a + b*x)^(2 + n))/(b^6*(2 + n)) + (2*d*(b^3*
c - 5*a^3*d)*(a + b*x)^(3 + n))/(b^6*(3 + n)) + (10*a^2*d^2*(a +
b*x)^(4 + n))/(b^6*(4 + n)) - (5*a*d^2*(a + b*x)^(5 + n))/(b^6*(5
 + n)) + (d^2*(a + b*x)^(6 + n))/(b^6*(6 + n)) - (c^2*(a + b*x)^(
1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])/(a*(1 + n
))
```

Rubi [A] time = 0.274231, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$-\frac{ad\left(4b^3c-5a^3d\right)(a+bx)^{n+2}}{b^6(n+2)}+\frac{2d\left(b^3c-5a^3d\right)(a+bx)^{n+3}}{b^6(n+3)}+\frac{10a^2d^2(a+bx)^{n+4}}{b^6(n+4)}+\frac{a^2d\left(2b^3c-a^3d\right)(a+bx)^{n+1}}{b^6(n+1)}-\frac{5ad^2(a+bx)^{n+5}}{b^6(n+5)}+\frac{d^2(a+bx)^{n+6}}{b^6(n+6)}-\frac{c^2(a+bx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{bx}{a}+1\right)}{a(n+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x)^n*(c + d*x^3)^2)/x,x]
```


```
[Out] (a^2*d*(2*b^3*c - a^3*d)*(a + b*x)^(1 + n))/(b^6*(1 + n)) - (a*d*
(4*b^3*c - 5*a^3*d)*(a + b*x)^(2 + n))/(b^6*(2 + n)) + (2*d*(b^3*
c - 5*a^3*d)*(a + b*x)^(3 + n))/(b^6*(3 + n)) + (10*a^2*d^2*(a +
b*x)^(4 + n))/(b^6*(4 + n)) - (5*a*d^2*(a + b*x)^(5 + n))/(b^6*(5
 + n)) + (d^2*(a + b*x)^(6 + n))/(b^6*(6 + n)) - (c^2*(a + b*x)^(
1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])/(a*(1 + n
))
```

Rubi in Sympy [A] time = 57.7551, size = 187, normalized size = 0.89

$$\frac{10a^2d^2(a+bx)^{n+4}}{b^6(n+4)} - \frac{a^2d(a+bx)^{n+1}\left(a^3d-2b^3c\right)}{b^6(n+1)} - \frac{5ad^2(a+bx)^{n+5}}{b^6(n+5)} + \frac{ad(a+bx)^{n+2}\left(5a^3d-4b^3c\right)}{b^6(n+2)} + \frac{d^2(a+bx)^{n+6}}{b^6(n+6)} - \frac{2d(a+bx)^{n+3}\left(5a^3d-b^3c\right)}{b^6(n+3)} - \frac{c^2(a+bx)^{n+1}\,{}_2F_1\left(\begin{matrix}1,n+1\\n+2\end{matrix}\middle|1+\frac{bx}{a}\right)}{a(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x+a)**n*(d*x**3+c)**2/x,x)
```

```
[Out] 10*a**2*d**2*(a + b*x)**(n + 4)/(b**6*(n + 4)) - a**2*d*(a + b*x)
**(n + 1)*(a**3*d - 2*b**3*c)/(b**6*(n + 1)) - 5*a*d**2*(a + b*x)
**(n + 5)/(b**6*(n + 5)) + a*d*(a + b*x)**(n + 2)*(5*a**3*d - 4*b
**3*c)/(b**6*(n + 2)) + d**2*(a + b*x)**(n + 6)/(b**6*(n + 6)) -
2*d*(a + b*x)**(n + 3)*(5*a**3*d - b**3*c)/(b**6*(n + 3)) - c**2*
(a + b*x)**(n + 1)*hyper((1, n + 1), (n + 2,), 1 + b*x/a)/(a*(n +
 1))
```

Mathematica [B] time = 0.613162, size = 420, normalized size = 2.01

$$(a+bx)^n\left(\frac{2cd\left(\frac{bx}{a}+1\right)^{-n}\left(2a^3\left(\left(\frac{bx}{a}+1\right)^n-1\right)-2a^2bnx\left(\frac{bx}{a}+1\right)^n+b^3\left(n^2+3n+2\right)x^3\left(\frac{bx}{a}+1\right)^n+ab^2n(n+1)x^2\left(\frac{bx}{a}+1\right.\right.}{b^3(n+1)(n+2)(n+3)}\right.$$
$$+\frac{d^2\left(\frac{bx}{a}+1\right)^{-n}\left(-120a^6\left(\left(\frac{bx}{a}+1\right)^n-1\right)+120a^5bnx\left(\frac{bx}{a}+1\right)^n-60a^4b^2n(n+1)x^2\left(\frac{bx}{a}+1\right)^n+20a^3b^3n\left(n^2+3n+2\right)x^3\right.}{}$$
$$\left.+\frac{c^2\left(\frac{a}{bx}+1\right)^{-n}{}_2F_1\left(-n,-n;1-n;-\frac{a}{bx}\right)}{n}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x)^n*(c + d*x^3)^2)/x,x]
```

```
[Out] (a + b*x)^n*((2*c*d*(-2*a^2*b*n*x*(1 + (b*x)/a)^n + a*b^2*n*(1 +
n)*x^2*(1 + (b*x)/a)^n + b^3*(2 + 3*n + n^2)*x^3*(1 + (b*x)/a)^n
+ 2*a^3*(-1 + (1 + (b*x)/a)^n)))/(b^3*(1 + n)*(2 + n)*(3 + n)*(1
+ (b*x)/a)^n) + (d^2*(120*a^5*b*n*x*(1 + (b*x)/a)^n - 60*a^4*b^2*
n*(1 + n)*x^2*(1 + (b*x)/a)^n + 20*a^3*b^3*n*(2 + 3*n + n^2)*x^3*
(1 + (b*x)/a)^n - 5*a^2*b^4*n*(6 + 11*n + 6*n^2 + n^3)*x^4*(1 + (
b*x)/a)^n + a*b^5*n*(24 + 50*n + 35*n^2 + 10*n^3 + n^4)*x^5*(1 +
```

```
(b*x)/a)^n + b^6*(120 + 274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5)*
x^6*(1 + (b*x)/a)^n - 120*a^6*(-1 + (1 + (b*x)/a)^n)))/(b^6*(1 +
n)*(2 + n)*(3 + n)*(4 + n)*(5 + n)*(6 + n)*(1 + (b*x)/a)^n) + (c^
2*Hypergeometric2F1[-n, -n, 1 - n, -(a/(b*x))])/(n*(1 + a/(b*x))^
n))
```

Maple [F] time = 0.05, size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n\left(dx^3+c\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x+a)^n*(d*x^3+c)^2/x,x)

[Out] int((b*x+a)^n*(d*x^3+c)^2/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^3+c\right)^2(bx+a)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)^2*(b*x + a)^n/x,x, algorithm="maxima")

[Out] integrate((d*x^3 + c)^2*(b*x + a)^n/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(d^2x^6+2\,cdx^3+c^2\right)(bx+a)^n}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)^2*(b*x + a)^n/x,x, algorithm="fricas")

[Out] integral((d^2*x^6 + 2*c*d*x^3 + c^2)*(b*x + a)^n/x, x)
```

Sympy [A] time = 31.6333, size = 4755, normalized size = 22.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x+a)**n*(d*x**3+c)**2/x,x)
```

```
[Out] -b**n*c**2*n*(a/b + x)**n*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n +
 1)/gamma(n + 2) - b**n*c**2*(a/b + x)**n*lerchphi(1 + b*x/a, 1,
n + 1)*gamma(n + 1)/gamma(n + 2) + 2*c*d*Piecewise((a**n*x**3/3,
Eq(b, 0)), (2*a**2*log(a/b + x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**
5*x**2) + 3*a**2/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 4*a*b
*x*log(a/b + x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 4*a*b*
x/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 2*b**2*x**2*log(a/b
+ x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2), Eq(n, -3)), (-2*a*
*2*log(a/b + x)/(a*b**3 + b**4*x) - 2*a**2/(a*b**3 + b**4*x) - 2*
a*b*x*log(a/b + x)/(a*b**3 + b**4*x) + b**2*x**2/(a*b**3 + b**4*x
), Eq(n, -2)), (a**2*log(a/b + x)/b**3 - a*x/b**2 + x**2/(2*b), E
q(n, -1)), (2*a**3*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**
3*n + 6*b**3) - 2*a**2*b*n*x*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**
2 + 11*b**3*n + 6*b**3) + a*b**2*n**2*x**2*(a + b*x)**n/(b**3*n**
3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + a*b**2*n*x**2*(a + b*x)**
n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + b**3*n**2*x**3
*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + 3*
b**3*n*x**3*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6
*b**3) + 2*b**3*x**3*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b
**3*n + 6*b**3), True)) + d**2*Piecewise((a**n*x**6/6, Eq(b, 0)),
 (60*a**5*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3
*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**
5) + 27*a**5/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2
 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 300*a
**4*b*x*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b
**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5)
 + 75*a**4*b*x/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x*
*2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 600
*a**3*b**2*x**2*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 60
0*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**
11*x**5) + 600*a**2*b**3*x**3*log(a/b + x)/(60*a**5*b**6 + 300*a*
*4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10
*x**4 + 60*b**11*x**5) - 200*a**2*b**3*x**3/(60*a**5*b**6 + 300*a
**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**1
0*x**4 + 60*b**11*x**5) + 300*a*b**4*x**4*log(a/b + x)/(60*a**5*b
**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 +
 300*a*b**10*x**4 + 60*b**11*x**5) - 250*a*b**4*x**4/(60*a**5*b**
6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 3
00*a*b**10*x**4 + 60*b**11*x**5) + 60*b**5*x**5*log(a/b + x)/(60*
a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*
x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) - 110*b**5*x**5/(60*a**5
*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3
```

```
 + 300*a*b**10*x**4 + 60*b**11*x**5), Eq(n, -6)), (-60*a**5*log(a
/b + x)/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a
*b**9*x**3 + 12*b**10*x**4) - 35*a**5/(12*a**4*b**6 + 48*a**3*b**
7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 240*a
**4*b*x*log(a/b + x)/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**
8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 80*a**4*b*x/(12*a**4*b
**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b*
*10*x**4) - 360*a**3*b**2*x**2*log(a/b + x)/(12*a**4*b**6 + 48*a*
*3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) -
 240*a**2*b**3*x**3*log(a/b + x)/(12*a**4*b**6 + 48*a**3*b**7*x +
 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) + 120*a**2*b
**3*x**3/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*
a*b**9*x**3 + 12*b**10*x**4) - 60*a*b**4*x**4*log(a/b + x)/(12*a*
*4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 1
2*b**10*x**4) + 90*a*b**4*x**4/(12*a**4*b**6 + 48*a**3*b**7*x + 7
2*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) + 12*b**5*x**5
/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x
**3 + 12*b**10*x**4), Eq(n, -5)), (60*a**5*log(a/b + x)/(6*a**3*b
**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 50*a**5/(6
*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 180
*a**4*b*x*log(a/b + x)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*
x**2 + 6*b**9*x**3) + 90*a**4*b*x/(6*a**3*b**6 + 18*a**2*b**7*x +
 18*a*b**8*x**2 + 6*b**9*x**3) + 180*a**3*b**2*x**2*log(a/b + x)/
(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 6
0*a**2*b**3*x**3*log(a/b + x)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*
a*b**8*x**2 + 6*b**9*x**3) - 60*a**2*b**3*x**3/(6*a**3*b**6 + 18*
a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) - 15*a*b**4*x**4/(6*a
**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 3*b**
5*x**5/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x*
*3), Eq(n, -4)), (-60*a**5*log(a/b + x)/(6*a**2*b**6 + 12*a*b**7*
x + 6*b**8*x**2) - 90*a**5/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x*
*2) - 120*a**4*b*x*log(a/b + x)/(6*a**2*b**6 + 12*a*b**7*x + 6*b*
*8*x**2) - 120*a**4*b*x/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2)
 - 60*a**3*b**2*x**2*log(a/b + x)/(6*a**2*b**6 + 12*a*b**7*x + 6*
b**8*x**2) + 20*a**2*b**3*x**3/(6*a**2*b**6 + 12*a*b**7*x + 6*b**
8*x**2) - 5*a*b**4*x**4/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2)
 + 2*b**5*x**5/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2), Eq(n, -
3)), (60*a**5*log(a/b + x)/(12*a*b**6 + 12*b**7*x) + 60*a**5/(12*
a*b**6 + 12*b**7*x) + 60*a**4*b*x*log(a/b + x)/(12*a*b**6 + 12*b*
*7*x) - 30*a**3*b**2*x**2/(12*a*b**6 + 12*b**7*x) + 10*a**2*b**3*
x**3/(12*a*b**6 + 12*b**7*x) - 5*a*b**4*x**4/(12*a*b**6 + 12*b**7
*x) + 3*b**5*x**5/(12*a*b**6 + 12*b**7*x), Eq(n, -2)), (-a**5*log
(a/b + x)/b**6 + a**4*x/b**5 - a**3*x**2/(2*b**4) + a**2*x**3/(3*
b**3) - a*x**4/(4*b**2) + x**5/(5*b), Eq(n, -1)), (-120*a**6*(a +
 b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**
3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 120*a**5*b*n*x*(a
+ b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n*
*3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) - 60*a**4*b**2*n**2
*x**2*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 73
5*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) - 60*a**4*
b**2*n*x**2*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**
4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 20
*a**3*b**3*n**3*x**3*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175
*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b
```

```
**6) + 60*a**3*b**3*n**2*x**3*(a + b*x)**n/(b**6*n**6 + 21*b**6*n
**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*
n + 720*b**6) + 40*a**3*b**3*n*x**3*(a + b*x)**n/(b**6*n**6 + 21*
b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764
*b**6*n + 720*b**6) - 5*a**2*b**4*n**4*x**4*(a + b*x)**n/(b**6*n*
*6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**
2 + 1764*b**6*n + 720*b**6) - 30*a**2*b**4*n**3*x**4*(a + b*x)**n
/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624
*b**6*n**2 + 1764*b**6*n + 720*b**6) - 55*a**2*b**4*n**2*x**4*(a
+ b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n*
*3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) - 30*a**2*b**4*n*x*
*4*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b
**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + a*b**5*n**5
*x**5*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 73
5*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 10*a*b**
5*n**4*x**5*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**
4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 35
*a*b**5*n**3*x**5*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b*
*6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6
) + 50*a*b**5*n**2*x**5*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 +
175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 72
0*b**6) + 24*a*b**5*n*x**5*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5
 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n +
 720*b**6) + b**6*n**5*x**6*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**
5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n
+ 720*b**6) + 15*b**6*n**4*x**6*(a + b*x)**n/(b**6*n**6 + 21*b**6
*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**
6*n + 720*b**6) + 85*b**6*n**3*x**6*(a + b*x)**n/(b**6*n**6 + 21*
b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764
*b**6*n + 720*b**6) + 225*b**6*n**2*x**6*(a + b*x)**n/(b**6*n**6
+ 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 +
 1764*b**6*n + 720*b**6) + 274*b**6*n*x**6*(a + b*x)**n/(b**6*n**
6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2
 + 1764*b**6*n + 720*b**6) + 120*b**6*x**6*(a + b*x)**n/(b**6*n**
6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2
 + 1764*b**6*n + 720*b**6), True)) - b*b**n*c**2*n*x*(a/b + x)**n
*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n + 1)/(a*gamma(n + 2)) - b*
b**n*c**2*x*(a/b + x)**n*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n +
1)/(a*gamma(n + 2))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^3+c\right)^2 (bx+a)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)^2*(b*x + a)^n/x,x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^2*(b*x + a)^n/x, x)
```

3.159 $\int x^2(a+bx)^n \left(c+dx^3\right)^3 \, dx$

Optimal. Leaf size=459

$$-\frac{6ad^2\left(4b^3c-55a^3d\right)(a+bx)^{n+8}}{b^{12}(n+8)}+\frac{3d^2\left(b^3c-55a^3d\right)(a+bx)^{n+9}}{b^{12}(n+9)}-\frac{a\left(2b^3c-11a^3d\right)\left(b^3c-a^3d\right)^2(a+bx)^{n+2}}{b^{12}(n+2)}+\frac{55a^2d^3(a+bx)^{n+10}}{b^{12}(n+10)}+\frac{\left(b^3c-a^3d\right)\left(55a^6d^2-29a^3b^3cd+b^6c^2\right)(a+bx)^{n+3}}{b^{12}(n+3)}-\frac{15ad\left(22a^6d^2-14a^3b^3cd+b^6c^2\right)(a+bx)^{n+5}}{b^{12}(n+5)}+\frac{3d\left(154a^6d^2-56a^3b^3cd+b^6c^2\right)(a+bx)^{n+6}}{b^{12}(n+6)}+\frac{42a^2d^2\left(2b^3c-11a^3d\right)(a+bx)^{n+7}}{b^{12}(n+7)}+\frac{a^2\left(b^3c-a^3d\right)^3(a+bx)^{n+1}}{b^{12}(n+1)}+\frac{3a^2d\left(55a^6d^2-56a^3b^3cd+10b^6c^2\right)(a+bx)^{n+4}}{b^{12}(n+4)}-\frac{11ad^3(a+bx)^{n+11}}{b^{12}(n+11)}+\frac{d^3(a+bx)^{n+12}}{b^{12}(n+12)}$$

```
[Out] (a^2*(b^3*c - a^3*d)^3*(a + b*x)^(1 + n))/(b^12*(1 + n)) - (a*(2*
b^3*c - 11*a^3*d)*(b^3*c - a^3*d)^2*(a + b*x)^(2 + n))/(b^12*(2 +
 n)) + ((b^3*c - a^3*d)*(b^6*c^2 - 29*a^3*b^3*c*d + 55*a^6*d^2)*(
a + b*x)^(3 + n))/(b^12*(3 + n)) + (3*a^2*d*(10*b^6*c^2 - 56*a^3*
b^3*c*d + 55*a^6*d^2)*(a + b*x)^(4 + n))/(b^12*(4 + n)) - (15*a*d
*(b^6*c^2 - 14*a^3*b^3*c*d + 22*a^6*d^2)*(a + b*x)^(5 + n))/(b^12
*(5 + n)) + (3*d*(b^6*c^2 - 56*a^3*b^3*c*d + 154*a^6*d^2)*(a + b*
x)^(6 + n))/(b^12*(6 + n)) + (42*a^2*d^2*(2*b^3*c - 11*a^3*d)*(a
+ b*x)^(7 + n))/(b^12*(7 + n)) - (6*a*d^2*(4*b^3*c - 55*a^3*d)*(a
 + b*x)^(8 + n))/(b^12*(8 + n)) + (3*d^2*(b^3*c - 55*a^3*d)*(a +
b*x)^(9 + n))/(b^12*(9 + n)) + (55*a^2*d^3*(a + b*x)^(10 + n))/(b
^12*(10 + n)) - (11*a*d^3*(a + b*x)^(11 + n))/(b^12*(11 + n)) + (
d^3*(a + b*x)^(12 + n))/(b^12*(12 + n))
```

Rubi [A] time = 0.683146, antiderivative size = 459, normalized size of antiderivative = 1., number

of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.05$

$$-\frac{6ad^2\left(4b^3c-55a^3d\right)(a+bx)^{n+8}}{b^{12}(n+8)}+\frac{3d^2\left(b^3c-55a^3d\right)(a+bx)^{n+9}}{b^{12}(n+9)}-\frac{a\left(2b^3c-11a^3d\right)\left(b^3c-a^3d\right)^2(a+bx)^{n+2}}{b^{12}(n+2)}+\frac{55a^2d^3(a+bx)^{n+10}}{b^{12}(n+10)}+\frac{\left(b^3c-a^3d\right)\left(55a^6d^2-29a^3b^3cd+b^6c^2\right)(a+bx)^{n+3}}{b^{12}(n+3)}-\frac{15ad\left(22a^6d^2-14a^3b^3cd+b^6c^2\right)(a+bx)^{n+5}}{b^{12}(n+5)}+\frac{3d\left(154a^6d^2-56a^3b^3cd+b^6c^2\right)(a+bx)^{n+6}}{b^{12}(n+6)}+\frac{42a^2d^2\left(2b^3c-11a^3d\right)(a+bx)^{n+7}}{b^{12}(n+7)}+\frac{a^2\left(b^3c-a^3d\right)^3(a+bx)^{n+1}}{b^{12}(n+1)}+\frac{3a^2d\left(55a^6d^2-56a^3b^3cd+10b^6c^2\right)(a+bx)^{n+4}}{b^{12}(n+4)}-\frac{11ad^3(a+bx)^{n+11}}{b^{12}(n+11)}+\frac{d^3(a+bx)^{n+12}}{b^{12}(n+12)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(a + b*x)^n*(c + d*x^3)^3,x]
```

```
[Out] (a^2*(b^3*c - a^3*d)^3*(a + b*x)^(1 + n))/(b^12*(1 + n)) - (a*(2*
b^3*c - 11*a^3*d)*(b^3*c - a^3*d)^2*(a + b*x)^(2 + n))/(b^12*(2 +
 n)) + ((b^3*c - a^3*d)*(b^6*c^2 - 29*a^3*b^3*c*d + 55*a^6*d^2)*(
a + b*x)^(3 + n))/(b^12*(3 + n)) + (3*a^2*d*(10*b^6*c^2 - 56*a^3*
b^3*c*d + 55*a^6*d^2)*(a + b*x)^(4 + n))/(b^12*(4 + n)) - (15*a*d
*(b^6*c^2 - 14*a^3*b^3*c*d + 22*a^6*d^2)*(a + b*x)^(5 + n))/(b^12
*(5 + n)) + (3*d*(b^6*c^2 - 56*a^3*b^3*c*d + 154*a^6*d^2)*(a + b*
x)^(6 + n))/(b^12*(6 + n)) + (42*a^2*d^2*(2*b^3*c - 11*a^3*d)*(a
+ b*x)^(7 + n))/(b^12*(7 + n)) - (6*a*d^2*(4*b^3*c - 55*a^3*d)*(a
 + b*x)^(8 + n))/(b^12*(8 + n)) + (3*d^2*(b^3*c - 55*a^3*d)*(a +
b*x)^(9 + n))/(b^12*(9 + n)) + (55*a^2*d^3*(a + b*x)^(10 + n))/(b
^12*(10 + n)) - (11*a*d^3*(a + b*x)^(11 + n))/(b^12*(11 + n)) + (
d^3*(a + b*x)^(12 + n))/(b^12*(12 + n))
```

Rubi in Sympy [A] time = 144.694, size = 439, normalized size = 0.96

$$\frac{55a^2d^3(a+bx)^{n+10}}{b^{12}(n+10)}-\frac{42a^2d^2(a+bx)^{n+7}\left(11a^3d-2b^3c\right)}{b^{12}(n+7)}+\frac{3a^2d(a+bx)^{n+4}\left(55a^6d^2-56a^3b^3cd+10b^6c^2\right)}{b^{12}(n+4)}-\frac{a^2(a+bx)^{n+1}\left(a^3d-b^3c\right)^3}{b^{12}(n+1)}-\frac{11ad^3(a+bx)^{n+11}}{b^{12}(n+11)}+\frac{6ad^2(a+bx)^{n+8}\left(55a^3d-4b^3c\right)}{b^{12}(n+8)}-\frac{15ad(a+bx)^{n+5}\left(22a^6d^2-14a^3b^3cd+b^6c^2\right)}{b^{12}(n+5)}+\frac{a(a+bx)^{n+2}\left(a^3d-b^3c\right)^2\left(11a^3d-2b^3c\right)}{b^{12}(n+2)}+\frac{d^3(a+bx)^{n+12}}{b^{12}(n+12)}-\frac{3d^2(a+bx)^{n+9}\left(55a^3d-b^3c\right)}{b^{12}(n+9)}+\frac{3d(a+bx)^{n+6}\left(154a^6d^2-56a^3b^3cd+b^6c^2\right)}{b^{12}(n+6)}-\frac{(a+bx)^{n+3}\left(a^3d-b^3c\right)\left(55a^6d^2-29a^3b^3cd+b^6c^2\right)}{b^{12}(n+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(b*x+a)**n*(d*x**3+c)**3,x)
```

```
[Out] 55*a**2*d**3*(a + b*x)**(n + 10)/(b**12*(n + 10)) - 42*a**2*d**2*
(a + b*x)**(n + 7)*(11*a**3*d - 2*b**3*c)/(b**12*(n + 7)) + 3*a**
2*d*(a + b*x)**(n + 4)*(55*a**6*d**2 - 56*a**3*b**3*c*d + 10*b**6
*c**2)/(b**12*(n + 4)) - a**2*(a + b*x)**(n + 1)*(a**3*d - b**3*c
)**3/(b**12*(n + 1)) - 11*a*d**3*(a + b*x)**(n + 11)/(b**12*(n +
11)) + 6*a*d**2*(a + b*x)**(n + 8)*(55*a**3*d - 4*b**3*c)/(b**12*
(n + 8)) - 15*a*d*(a + b*x)**(n + 5)*(22*a**6*d**2 - 14*a**3*b**3
*c*d + b**6*c**2)/(b**12*(n + 5)) + a*(a + b*x)**(n + 2)*(a**3*d
- b**3*c)**2*(11*a**3*d - 2*b**3*c)/(b**12*(n + 2)) + d**3*(a + b
*x)**(n + 12)/(b**12*(n + 12)) - 3*d**2*(a + b*x)**(n + 9)*(55*a*
*3*d - b**3*c)/(b**12*(n + 9)) + 3*d*(a + b*x)**(n + 6)*(154*a**6
*d**2 - 56*a**3*b**3*c*d + b**6*c**2)/(b**12*(n + 6)) - (a + b*x)
**(n + 3)*(a**3*d - b**3*c)*(55*a**6*d**2 - 29*a**3*b**3*c*d + b*
*6*c**2)/(b**12*(n + 3))
```

Mathematica [B] time = 2.0107, size = 1134, normalized size = 2.47

$$(a+bx)^{n+1}\left(-39916800d^3a^{11}+39916800bd^3(n+1)xa^{10}-19958400b^2d^3\left(n^2+3n+2\right)x^2a^9+120960b^3d^2\left(55d\left(n^3+6n^2+\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(a + b*x)^n*(c + d*x^3)^3,x]
```

```
[Out] ((a + b*x)^(1 + n)*(-39916800*a^11*d^3 + 39916800*a^10*b*d^3*(1 +
 n)*x - 19958400*a^9*b^2*d^3*(2 + 3*n + n^2)*x^2 + 120960*a^8*b^3
*d^2*(c*(1320 + 362*n + 33*n^2 + n^3) + 55*d*(6 + 11*n + 6*n^2 +
n^3)*x^3) - 30240*a^7*b^4*d^2*(1 + n)*x*(4*c*(1320 + 362*n + 33*n
^2 + n^3) + 55*d*(24 + 26*n + 9*n^2 + n^3)*x^3) + 30240*a^6*b^5*d
^2*(2 + 3*n + n^2)*x^2*(2*c*(1320 + 362*n + 33*n^2 + n^3) + 11*d*
(60 + 47*n + 12*n^2 + n^3)*x^3) - 360*a^5*b^6*d*(c^2*(665280 + 43
4568*n + 117454*n^2 + 16815*n^3 + 1345*n^4 + 57*n^5 + n^6) + 56*c
*d*(7920 + 16692*n + 12100*n^2 + 3861*n^3 + 571*n^4 + 39*n^5 + n^
6)*x^3 + 154*d^2*(720 + 1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 2
1*n^5 + n^6)*x^6) + 360*a^4*b^7*d*(1 + n)*x*(c^2*(665280 + 434568
*n + 117454*n^2 + 16815*n^3 + 1345*n^4 + 57*n^5 + n^6) + 14*c*d*(
31680 + 43008*n + 22084*n^2 + 5460*n^3 + 685*n^4 + 42*n^5 + n^6)*
x^3 + 22*d^2*(5040 + 8028*n + 5104*n^2 + 1665*n^3 + 295*n^4 + 27*
n^5 + n^6)*x^6) - 18*a^3*b^8*d*(2 + 3*n + n^2)*x^2*(10*c^2*(66528
0 + 434568*n + 117454*n^2 + 16815*n^3 + 1345*n^4 + 57*n^5 + n^6)
+ 56*c*d*(79200 + 83760*n + 34834*n^2 + 7275*n^3 + 805*n^4 + 45*n
^5 + n^6)*x^3 + 55*d^2*(20160 + 24552*n + 12154*n^2 + 3135*n^3 +
445*n^4 + 33*n^5 + n^6)*x^6) + b^11*(246400 + 593520*n + 541508*n
^2 + 251352*n^3 + 66489*n^4 + 10440*n^5 + 962*n^6 + 48*n^7 + n^8)
*x^2*(c^3*(648 + 234*n + 27*n^2 + n^3) + 3*c^2*d*(324 + 171*n + 2
4*n^2 + n^3)*x^3 + 3*c*d^2*(216 + 126*n + 21*n^2 + n^3)*x^6 + d^3
*(162 + 99*n + 18*n^2 + n^3)*x^9) - a*b^10*(280 + 418*n + 159*n^2
 + 22*n^3 + n^4)*x*(2*c^3*(285120 + 221544*n + 70254*n^2 + 11645*
n^3 + 1065*n^4 + 51*n^5 + n^6) + 15*c^2*d*(57024 + 70920*n + 3257
4*n^2 + 7115*n^3 + 801*n^4 + 45*n^5 + n^6)*x^3 + 24*c*d^2*(23760
+ 32652*n + 17160*n^2 + 4421*n^3 + 591*n^4 + 39*n^5 + n^6)*x^6 +
11*d^3*(12960 + 18612*n + 10404*n^2 + 2915*n^3 + 435*n^4 + 33*n^5
 + n^6)*x^9) + 2*a^2*b^9*(c^3*(79833600 + 101378880*n + 56231712*
n^2 + 17893196*n^3 + 3602088*n^4 + 476049*n^5 + 41328*n^6 + 2274*
n^7 + 72*n^8 + n^9) + 30*c^2*d*(3991680 + 9925488*n + 9476652*n^2
 + 4665572*n^3 + 1332327*n^4 + 233481*n^5 + 25518*n^6 + 1698*n^7
+ 63*n^8 + n^9)*x^3 + 84*c*d^2*(950400 + 2589120*n + 2806008*n^2
+ 1617020*n^3 + 552426*n^4 + 116949*n^5 + 15432*n^6 + 1230*n^7 +
54*n^8 + n^9)*x^6 + 55*d^3*(362880 + 1026576*n + 1172700*n^2 + 72
3680*n^3 + 269325*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 +
 n^9)*x^9)))/(b^12*(1 + n)*(2 + n)*(3 + n)*(4 + n)*(5 + n)*(6 + n
)*(7 + n)*(8 + n)*(9 + n)*(10 + n)*(11 + n)*(12 + n))
```

Maple [B] time = 0.043, size = 3780, normalized size = 8.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(b*x+a)^n*(d*x^3+c)^3,x)
```

```
[Out] -(b*x+a)^(1+n)*(-b^11*d^3*n^11*x^11-66*b^11*d^3*n^10*x^11+11*a*b^
10*d^3*n^10*x^10-1925*b^11*d^3*n^9*x^11+605*a*b^10*d^3*n^9*x^10-3
```

*b^11*c*d^2*n^11*x^8-32670*b^11*d^3*n^8*x^11-110*a^2*b^9*d^3*n^9*
x^9+14520*a*b^10*d^3*n^8*x^10-207*b^11*c*d^2*n^10*x^8-357423*b^11
*d^3*n^7*x^11-4950*a^2*b^9*d^3*n^8*x^9+24*a*b^10*c*d^2*n^10*x^7+1
99650*a*b^10*d^3*n^7*x^10-6288*b^11*c*d^2*n^9*x^8-2637558*b^11*d^
3*n^6*x^11+990*a^3*b^8*d^3*n^8*x^8-95700*a^2*b^9*d^3*n^7*x^9+1464
*a*b^10*c*d^2*n^9*x^7+1735503*a*b^10*d^3*n^6*x^10-3*b^11*c^2*d*n^
11*x^5-110718*b^11*c*d^2*n^8*x^8-13339535*b^11*d^3*n^5*x^11+35640
*a^3*b^8*d^3*n^7*x^8-168*a^2*b^9*c*d^2*n^9*x^6-1039500*a^2*b^9*d^
3*n^6*x^9+38592*a*b^10*c*d^2*n^8*x^7+9922605*a*b^10*d^3*n^5*x^10-
216*b^11*c^2*d*n^10*x^5-1251927*b^11*c*d^2*n^7*x^8-45995730*b^11*
d^3*n^4*x^11-7920*a^4*b^7*d^3*n^7*x^7+540540*a^3*b^8*d^3*n^6*x^8-
9072*a^2*b^9*c*d^2*n^8*x^6-6960030*a^2*b^9*d^3*n^5*x^9+15*a*b^10*
c^2*d*n^10*x^4+577008*a*b^10*c*d^2*n^7*x^7+37586230*a*b^10*d^3*n^
4*x^10-6855*b^11*c^2*d*n^9*x^5-9512559*b^11*c*d^2*n^6*x^8-1052580
76*b^11*d^3*n^3*x^11-221760*a^4*b^7*d^3*n^6*x^7+1008*a^3*b^8*c*d^
2*n^8*x^5+4490640*a^3*b^8*d^3*n^5*x^8-206640*a^2*b^9*c*d^2*n^7*x^
6-29625750*a^2*b^9*d^3*n^4*x^9+1005*a*b^10*c^2*d*n^9*x^4+5399352*
a*b^10*c*d^2*n^6*x^7+92504500*a*b^10*d^3*n^3*x^10-b^11*c^3*n^11*x
^2-126180*b^11*c^2*d*n^8*x^5-49357662*b^11*c*d^2*n^5*x^8-15091797
6*b^11*d^3*n^2*x^11+55440*a^5*b^6*d^3*n^6*x^6-2550240*a^4*b^7*d^3
*n^5*x^7+48384*a^3*b^8*c*d^2*n^7*x^5+22224510*a^3*b^8*d^3*n^4*x^8
-60*a^2*b^9*c^2*d*n^9*x^3-2592576*a^2*b^9*c*d^2*n^6*x^6-79604800*
a^2*b^9*d^3*n^3*x^9+29250*a*b^10*c^2*d*n^8*x^4+32905656*a*b^10*c*
d^2*n^5*x^7+140289336*a*b^10*d^3*n^2*x^10-75*b^11*c^3*n^10*x^2-14
91309*b^11*c^2*d*n^7*x^5-173991492*b^11*c*d^2*n^4*x^8-120543840*b
^11*d^3*n*x^11+1164240*a^5*b^6*d^3*n^5*x^6-5040*a^4*b^7*c*d^2*n^7
*x^4-15523200*a^4*b^7*d^3*n^4*x^7+949536*a^3*b^8*c*d^2*n^6*x^5+66
611160*a^3*b^8*d^3*n^3*x^8-3780*a^2*b^9*c^2*d*n^8*x^3-19647432*a^
2*b^9*c*d^2*n^5*x^6-128997000*a^2*b^9*d^3*n^2*x^9+2*a*b^10*c^3*n^
10*x+484650*a*b^10*c^2*d*n^7*x^4+131616048*a*b^10*c*d^2*n^4*x^7+1
16915040*a*b^10*d^3*n*x^10-2492*b^11*c^3*n^9*x^2-11832048*b^11*c^
2*d*n^6*x^5-405697080*b^11*c*d^2*n^3*x^8-39916800*b^11*d^3*x^11-3
32640*a^6*b^5*d^3*n^5*x^5+9702000*a^5*b^6*d^3*n^4*x^6-216720*a^4*
b^7*c*d^2*n^6*x^4-53610480*a^4*b^7*d^3*n^3*x^7+180*a^3*b^8*c^2*d*
n^8*x^2+9858240*a^3*b^8*c*d^2*n^5*x^5+116942760*a^3*b^8*d^3*n^2*x
^8-101880*a^2*b^9*c^2*d*n^7*x^3-92807568*a^2*b^9*c*d^2*n^4*x^6-11
2923360*a^2*b^9*d^3*n*x^9+146*a*b^10*c^3*n^9*x+5033295*a*b^10*c^2
*d*n^6*x^4+339003552*a*b^10*c*d^2*n^3*x^7+39916800*a*b^10*d^3*x^1
0-48294*b^11*c^3*n^8*x^2-63978405*b^11*c^2*d*n^5*x^5-590770944*b^
11*c*d^2*n^2*x^8-4989600*a^6*b^5*d^3*n^4*x^5+20160*a^5*b^6*c*d^2*
n^6*x^3+40748400*a^5*b^6*d^3*n^3*x^6-3664080*a^4*b^7*c*d^2*n^5*x^
4-104005440*a^4*b^7*d^3*n^2*x^7+10800*a^3*b^8*c^2*d*n^7*x^2+58735
152*a^3*b^8*c*d^2*n^4*x^5+108488160*a^3*b^8*d^3*n*x^8-2*a^2*b^9*c
^3*n^9-1531080*a^2*b^9*c^2*d*n^6*x^3-271659360*a^2*b^9*c*d^2*n^3*
x^6-39916800*a^2*b^9*d^3*x^9+4692*a*b^10*c^3*n^8*x+33993765*a*b^1
0*c^2*d*n^5*x^4+533548224*a*b^10*c*d^2*n^2*x^7-604581*b^11*c^3*n^
7*x^2-234340020*b^11*c^2*d*n^4*x^5-477740160*b^11*c*d^2*n*x^8+166
3200*a^7*b^4*d^3*n^4*x^4-28274400*a^6*b^5*d^3*n^3*x^5+786240*a^5*
b^6*c*d^2*n^5*x^3+90034560*a^5*b^6*d^3*n^2*x^6-360*a^4*b^7*c^2*d*
n^7*x-30970800*a^4*b^7*c*d^2*n^4*x^4-103498560*a^4*b^7*d^3*n*x^7+
273240*a^3*b^8*c^2*d*n^6*x^2+204434496*a^3*b^8*c*d^2*n^3*x^5+3991
6800*a^3*b^8*d^3*x^8-144*a^2*b^9*c^3*n^8-14008860*a^2*b^9*c^2*d*n
^5*x^3-471409344*a^2*b^9*c*d^2*n^2*x^6+87204*a*b^10*c^3*n^7*x+149
923200*a*b^10*c^2*d*n^4*x^4+457781760*a*b^10*c*d^2*n*x^7-5112891*

```
b^11*c^3*n^6*x^2-565580388*b^11*c^2*d*n^3*x^5-159667200*b^11*c*d^
2*x^8+16632000*a^7*b^4*d^3*n^3*x^4-60480*a^6*b^5*c*d^2*n^5*x^2-74
844000*a^6*b^5*d^3*n^2*x^5+11511360*a^5*b^6*c*d^2*n^4*x^3+9779616
0*a^5*b^6*d^3*n*x^6-20880*a^4*b^7*c^2*d*n^6*x-138821760*a^4*b^7*c
*d^2*n^3*x^4-39916800*a^4*b^7*d^3*x^7+3773520*a^3*b^8*c^2*d*n^5*x
^2+403349184*a^3*b^8*c*d^2*n^2*x^5-4548*a^2*b^9*c^3*n^7-79939620*
a^2*b^9*c^2*d*n^4*x^3-434972160*a^2*b^9*c*d^2*n*x^6+1034754*a*b^1
0*c^3*n^6*x+422084100*a*b^10*c^2*d*n^3*x^4+159667200*a*b^10*c*d^2
*x^7-29651558*b^11*c^3*n^5*x^2-848562336*b^11*c^2*d*n^2*x^5-66528
00*a^8*b^3*d^3*n^3*x^3+58212000*a^7*b^4*d^3*n^2*x^4-2177280*a^6*b
^5*c*d^2*n^4*x^2-91143360*a^6*b^5*d^3*n*x^5+360*a^5*b^6*c^2*d*n^6
+77837760*a^5*b^6*c*d^2*n^3*x^3+39916800*a^5*b^6*d^3*x^6-504720*a
^4*b^7*c^2*d*n^5*x-328063680*a^4*b^7*c*d^2*n^2*x^4+30706020*a^3*b
^8*c^2*d*n^4*x^2+408360960*a^3*b^8*c*d^2*n*x^5-82656*a^2*b^9*c^3*
n^6-279934320*a^2*b^9*c^2*d*n^3*x^3-159667200*a^2*b^9*c*d^2*x^6+8
156274*a*b^10*c^3*n^5*x+717481440*a*b^10*c^2*d*n^2*x^4-117115476*
b^11*c^3*n^4*x^2-703304640*b^11*c^2*d*n*x^5-39916800*a^8*b^3*d^3*
n^2*x^3+120960*a^7*b^4*c*d^2*n^4*x+83160000*a^7*b^4*d^3*n*x^4-280
02240*a^6*b^5*c*d^2*n^3*x^2-39916800*a^6*b^5*d^3*x^5+20520*a^5*b^
6*c^2*d*n^5+243936000*a^5*b^6*c*d^2*n^2*x^3-6537600*a^4*b^7*c^2*d
*n^4*x-376427520*a^4*b^7*c*d^2*n*x^4+147700800*a^3*b^8*c^2*d*n^3*
x^2+159667200*a^3*b^8*c*d^2*x^5-952098*a^2*b^9*c^3*n^5-568599120*
a^2*b^9*c^2*d*n^2*x^3+42990568*a*b^10*c^3*n^4*x+655404480*a*b^10*
c^2*d*n*x^4-305860408*b^11*c^3*n^3*x^2-239500800*b^11*c^2*d*x^5+1
9958400*a^9*b^2*d^3*n^2*x^2-73180800*a^8*b^3*d^3*n*x^3+4112640*a^
7*b^4*c*d^2*n^3*x+39916800*a^7*b^4*d^3*x^4-149506560*a^6*b^5*c*d^
2*n^2*x^2+484200*a^5*b^6*c^2*d*n^4+336510720*a^5*b^6*c*d^2*n*x^3-
48336840*a^4*b^7*c^2*d*n^3*x-159667200*a^4*b^7*c*d^2*x^4+39670056
0*a^3*b^8*c^2*d*n^2*x^2-7204176*a^2*b^9*c^3*n^4-595529280*a^2*b^9
*c^2*d*n*x^3+148249816*a*b^10*c^3*n^3*x+239500800*a*b^10*c^2*d*x^
4-496433664*b^11*c^3*n^2*x^2+59875200*a^9*b^2*d^3*n*x^2-120960*a^
8*b^3*c*d^2*n^3-39916800*a^8*b^3*d^3*x^3+47779200*a^7*b^4*c*d^2*n
^2*x-283288320*a^6*b^5*c*d^2*n*x^2+6053400*a^5*b^6*c^2*d*n^3+1596
67200*a^5*b^6*c*d^2*x^3-198727920*a^4*b^7*c^2*d*n^2*x+515695680*a
^3*b^8*c^2*d*n*x^2-35786392*a^2*b^9*c^3*n^3-239500800*a^2*b^9*c^2
*d*x^3+315221184*a*b^10*c^3*n^2*x-442258560*b^11*c^3*n*x^2-399168
00*a^10*b*d^3*n*x+39916800*a^9*b^2*d^3*x^2-3991680*a^8*b^3*c*d^2*
n^2+203454720*a^7*b^4*c*d^2*n*x-159667200*a^6*b^5*c*d^2*x^2+42283
440*a^5*b^6*c^2*d*n^2-395945280*a^4*b^7*c^2*d*n*x+239500800*a^3*b
^8*c^2*d*x^2-112463424*a^2*b^9*c^3*n^2+362424960*a*b^10*c^3*n*x-1
59667200*b^11*c^3*x^2-39916800*a^10*b*d^3*x-43787520*a^8*b^3*c*d^
2*n+159667200*a^7*b^4*c*d^2*x+156444480*a^5*b^6*c^2*d*n-239500800
*a^4*b^7*c^2*d*x-202757760*a^2*b^9*c^3*n+159667200*a*b^10*c^3*x+3
9916800*a^11*d^3-159667200*a^8*b^3*c*d^2+239500800*a^5*b^6*c^2*d-
159667200*a^2*b^9*c^3)/b^12/(n^12+78*n^11+2717*n^10+55770*n^9+749
463*n^8+6926634*n^7+44990231*n^6+206070150*n^5+657206836*n^4+1414
014888*n^3+1931559552*n^2+1486442880*n+479001600)
```

Maxima [A] time = 0.73345, size = 1557, normalized size = 3.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)^3*(b*x + a)^n*x^2,x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*a*b^2*x^2 - 2*a^2*b*n*x + 2*
a^3)*(b*x + a)^n*c^3/((n^3 + 6*n^2 + 11*n + 6)*b^3) + 3*((n^5 + 1
5*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*b^6*x^6 + (n^5 + 10*n^4 +
 35*n^3 + 50*n^2 + 24*n)*a*b^5*x^5 - 5*(n^4 + 6*n^3 + 11*n^2 + 6*
n)*a^2*b^4*x^4 + 20*(n^3 + 3*n^2 + 2*n)*a^3*b^3*x^3 - 60*(n^2 + n
)*a^4*b^2*x^2 + 120*a^5*b*n*x - 120*a^6)*(b*x + a)^n*c^2*d/((n^6
+ 21*n^5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*b^6) + 3*
((n^8 + 36*n^7 + 546*n^6 + 4536*n^5 + 22449*n^4 + 67284*n^3 + 118
124*n^2 + 109584*n + 40320)*b^9*x^9 + (n^8 + 28*n^7 + 322*n^6 + 1
960*n^5 + 6769*n^4 + 13132*n^3 + 13068*n^2 + 5040*n)*a*b^8*x^8 -
8*(n^7 + 21*n^6 + 175*n^5 + 735*n^4 + 1624*n^3 + 1764*n^2 + 720*n
)*a^2*b^7*x^7 + 56*(n^6 + 15*n^5 + 85*n^4 + 225*n^3 + 274*n^2 + 1
20*n)*a^3*b^6*x^6 - 336*(n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a
^4*b^5*x^5 + 1680*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^5*b^4*x^4 - 6720
*(n^3 + 3*n^2 + 2*n)*a^6*b^3*x^3 + 20160*(n^2 + n)*a^7*b^2*x^2 -
40320*a^8*b*n*x + 40320*a^9)*(b*x + a)^n*c*d^2/((n^9 + 45*n^8 + 8
70*n^7 + 9450*n^6 + 63273*n^5 + 269325*n^4 + 723680*n^3 + 1172700
*n^2 + 1026576*n + 362880)*b^9) + ((n^11 + 66*n^10 + 1925*n^9 + 3
2670*n^8 + 357423*n^7 + 2637558*n^6 + 13339535*n^5 + 45995730*n^4
 + 105258076*n^3 + 150917976*n^2 + 120543840*n + 39916800)*b^12*x
^12 + (n^11 + 55*n^10 + 1320*n^9 + 18150*n^8 + 157773*n^7 + 90205
5*n^6 + 3416930*n^5 + 8409500*n^4 + 12753576*n^3 + 10628640*n^2 +
 3628800*n)*a*b^11*x^11 - 11*(n^10 + 45*n^9 + 870*n^8 + 9450*n^7
+ 63273*n^6 + 269325*n^5 + 723680*n^4 + 1172700*n^3 + 1026576*n^2
 + 362880*n)*a^2*b^10*x^10 + 110*(n^9 + 36*n^8 + 546*n^7 + 4536*n
^6 + 22449*n^5 + 67284*n^4 + 118124*n^3 + 109584*n^2 + 40320*n)*a
^3*b^9*x^9 - 990*(n^8 + 28*n^7 + 322*n^6 + 1960*n^5 + 6769*n^4 +
13132*n^3 + 13068*n^2 + 5040*n)*a^4*b^8*x^8 + 7920*(n^7 + 21*n^6
+ 175*n^5 + 735*n^4 + 1624*n^3 + 1764*n^2 + 720*n)*a^5*b^7*x^7 -
55440*(n^6 + 15*n^5 + 85*n^4 + 225*n^3 + 274*n^2 + 120*n)*a^6*b^6
*x^6 + 332640*(n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a^7*b^5*x^5
 - 1663200*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^8*b^4*x^4 + 6652800*(n^
3 + 3*n^2 + 2*n)*a^9*b^3*x^3 - 19958400*(n^2 + n)*a^10*b^2*x^2 +
39916800*a^11*b*n*x - 39916800*a^12)*(b*x + a)^n*d^3/((n^12 + 78*
n^11 + 2717*n^10 + 55770*n^9 + 749463*n^8 + 6926634*n^7 + 4499023
1*n^6 + 206070150*n^5 + 657206836*n^4 + 1414014888*n^3 + 19315595
52*n^2 + 1486442880*n + 479001600)*b^12)
```

Fricas [A] time = 0.330684, size = 4811, normalized size = 10.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)^3*(b*x + a)^n*x^2,x, algorithm="fricas")
```

```
[Out] (2*a^3*b^9*c^3*n^9 + 144*a^3*b^9*c^3*n^8 + 4548*a^3*b^9*c^3*n^7 +
 159667200*a^3*b^9*c^3 - 239500800*a^6*b^6*c^2*d + 159667200*a^9*
b^3*c*d^2 - 39916800*a^12*d^3 + (b^12*d^3*n^11 + 66*b^12*d^3*n^10
 + 1925*b^12*d^3*n^9 + 32670*b^12*d^3*n^8 + 357423*b^12*d^3*n^7 +
 2637558*b^12*d^3*n^6 + 13339535*b^12*d^3*n^5 + 45995730*b^12*d^3
*n^4 + 105258076*b^12*d^3*n^3 + 150917976*b^12*d^3*n^2 + 12054384
0*b^12*d^3*n + 39916800*b^12*d^3)*x^12 + (a*b^11*d^3*n^11 + 55*a*
b^11*d^3*n^10 + 1320*a*b^11*d^3*n^9 + 18150*a*b^11*d^3*n^8 + 1577
73*a*b^11*d^3*n^7 + 902055*a*b^11*d^3*n^6 + 3416930*a*b^11*d^3*n^
5 + 8409500*a*b^11*d^3*n^4 + 12753576*a*b^11*d^3*n^3 + 10628640*a
*b^11*d^3*n^2 + 3628800*a*b^11*d^3*n)*x^11 - 11*(a^2*b^10*d^3*n^1
0 + 45*a^2*b^10*d^3*n^9 + 870*a^2*b^10*d^3*n^8 + 9450*a^2*b^10*d^
3*n^7 + 63273*a^2*b^10*d^3*n^6 + 269325*a^2*b^10*d^3*n^5 + 723680
*a^2*b^10*d^3*n^4 + 1172700*a^2*b^10*d^3*n^3 + 1026576*a^2*b^10*d
^3*n^2 + 362880*a^2*b^10*d^3*n)*x^10 + (3*b^12*c*d^2*n^11 + 207*b
^12*c*d^2*n^10 + 159667200*b^12*c*d^2 + 2*(3144*b^12*c*d^2 + 55*a
^3*b^9*d^3)*n^9 + 18*(6151*b^12*c*d^2 + 220*a^3*b^9*d^3)*n^8 + 3*
(417309*b^12*c*d^2 + 20020*a^3*b^9*d^3)*n^7 + 567*(16777*b^12*c*d
^2 + 880*a^3*b^9*d^3)*n^6 + 6*(8226277*b^12*c*d^2 + 411565*a^3*b^
9*d^3)*n^5 + 36*(4833097*b^12*c*d^2 + 205590*a^3*b^9*d^3)*n^4 + 4
0*(10142427*b^12*c*d^2 + 324841*a^3*b^9*d^3)*n^3 + 288*(2051288*b
^12*c*d^2 + 41855*a^3*b^9*d^3)*n^2 + 5760*(82941*b^12*c*d^2 + 770
*a^3*b^9*d^3)*n)*x^9 + 3*(a*b^11*c*d^2*n^11 + 61*a*b^11*c*d^2*n^1
0 + 1608*a*b^11*c*d^2*n^9 + 6*(4007*a*b^11*c*d^2 - 55*a^4*b^8*d^3
)*n^8 + 21*(10713*a*b^11*c*d^2 - 440*a^4*b^8*d^3)*n^7 + 21*(65289
*a*b^11*c*d^2 - 5060*a^4*b^8*d^3)*n^6 + 2*(2742001*a*b^11*c*d^2 -
 323400*a^4*b^8*d^3)*n^5 + 2*(7062574*a*b^11*c*d^2 - 1116885*a^4*
b^8*d^3)*n^4 + 264*(84209*a*b^11*c*d^2 - 16415*a^4*b^8*d^3)*n^3 +
 360*(52984*a*b^11*c*d^2 - 11979*a^4*b^8*d^3)*n^2 + 1663200*(4*a*
b^11*c*d^2 - a^4*b^8*d^3)*n)*x^8 - 24*(a^2*b^10*c*d^2*n^10 + 54*a
^2*b^10*c*d^2*n^9 + 1230*a^2*b^10*c*d^2*n^8 + 6*(2572*a^2*b^10*c*
d^2 - 55*a^5*b^7*d^3)*n^7 + 21*(5569*a^2*b^10*c*d^2 - 330*a^5*b^7
*d^3)*n^6 + 42*(13153*a^2*b^10*c*d^2 - 1375*a^5*b^7*d^3)*n^5 + 10
*(161702*a^2*b^10*c*d^2 - 24255*a^5*b^7*d^3)*n^4 + 24*(116917*a^2
*b^10*c*d^2 - 22330*a^5*b^7*d^3)*n^3 + 360*(7192*a^2*b^10*c*d^2 -
 1617*a^5*b^7*d^3)*n^2 + 237600*(4*a^2*b^10*c*d^2 - a^5*b^7*d^3)*
n)*x^7 + 72*(1148*a^3*b^9*c^3 - 5*a^6*b^6*c^2*d)*n^6 + 3*(b^12*c^
2*d*n^11 + 72*b^12*c^2*d*n^10 + 79833600*b^12*c^2*d + (2285*b^12*
c^2*d + 56*a^3*b^9*c*d^2)*n^9 + 12*(3505*b^12*c^2*d + 224*a^3*b^9
*c*d^2)*n^8 + 3*(165701*b^12*c^2*d + 17584*a^3*b^9*c*d^2)*n^7 + 4
8*(82167*b^12*c^2*d + 11410*a^3*b^9*c*d^2 - 385*a^6*b^6*d^3)*n^6
+ (21326135*b^12*c^2*d + 3263064*a^3*b^9*c*d^2 - 277200*a^6*b^6*d
^3)*n^5 + 12*(6509445*b^12*c^2*d + 946456*a^3*b^9*c*d^2 - 130900*
a^6*b^6*d^3)*n^4 + 4*(47131699*b^12*c^2*d + 5602072*a^3*b^9*c*d^2
 - 1039500*a^6*b^6*d^3)*n^3 + 96*(2946397*b^12*c^2*d + 236320*a^3
*b^9*c*d^2 - 52745*a^6*b^6*d^3)*n^2 + 2880*(81401*b^12*c^2*d + 30
80*a^3*b^9*c*d^2 - 770*a^6*b^6*d^3)*n)*x^6 + 6*(158683*a^3*b^9*c^
3 - 3420*a^6*b^6*c^2*d)*n^5 + 3*(a*b^11*c^2*d*n^11 + 67*a*b^11*c^
2*d*n^10 + 1950*a*b^11*c^2*d*n^9 + 6*(5385*a*b^11*c^2*d - 56*a^4*
b^8*c*d^2)*n^8 + 3*(111851*a*b^11*c^2*d - 4816*a^4*b^8*c*d^2)*n^7
 + 3*(755417*a*b^11*c^2*d - 81424*a^4*b^8*c*d^2)*n^6 + 560*(17848
*a*b^11*c^2*d - 3687*a^4*b^8*c*d^2 + 198*a^7*b^5*d^3)*n^5 + 4*(70
```

```
34735*a*b^11*c^2*d - 2313696*a^4*b^8*c*d^2 + 277200*a^7*b^5*d^3)*
n^4 + 96*(498251*a*b^11*c^2*d - 227822*a^4*b^8*c*d^2 + 40425*a^7*
b^5*d^3)*n^3 + 576*(75857*a*b^11*c^2*d - 43568*a^4*b^8*c*d^2 + 96
25*a^7*b^5*d^3)*n^2 + 2661120*(6*a*b^11*c^2*d - 4*a^4*b^8*c*d^2 +
 a^7*b^5*d^3)*n)*x^5 + 72*(100058*a^3*b^9*c^3 - 6725*a^6*b^6*c^2*
d)*n^4 - 15*(a^2*b^10*c^2*d*n^10 + 63*a^2*b^10*c^2*d*n^9 + 1698*a
^2*b^10*c^2*d*n^8 + 6*(4253*a^2*b^10*c^2*d - 56*a^5*b^7*c*d^2)*n^
7 + 3*(77827*a^2*b^10*c^2*d - 4368*a^5*b^7*c*d^2)*n^6 + 3*(444109
*a^2*b^10*c^2*d - 63952*a^5*b^7*c*d^2)*n^5 + 4*(1166393*a^2*b^10*
c^2*d - 324324*a^5*b^7*c*d^2 + 27720*a^8*b^4*d^3)*n^4 + 12*(78972
1*a^2*b^10*c^2*d - 338800*a^5*b^7*c*d^2 + 55440*a^8*b^4*d^3)*n^3
+ 144*(68927*a^2*b^10*c^2*d - 38948*a^5*b^7*c*d^2 + 8470*a^8*b^4*
d^3)*n^2 + 665280*(6*a^2*b^10*c^2*d - 4*a^5*b^7*c*d^2 + a^8*b^4*d
^3)*n)*x^4 + 8*(4473299*a^3*b^9*c^3 - 756675*a^6*b^6*c^2*d + 1512
0*a^9*b^3*c*d^2)*n^3 + (b^12*c^3*n^11 + 75*b^12*c^3*n^10 + 159667
200*b^12*c^3 + 4*(623*b^12*c^3 + 15*a^3*b^9*c^2*d)*n^9 + 18*(2683
*b^12*c^3 + 200*a^3*b^9*c^2*d)*n^8 + 3*(201527*b^12*c^3 + 30360*a
^3*b^9*c^2*d)*n^7 + 9*(568099*b^12*c^3 + 139760*a^3*b^9*c^2*d - 2
240*a^6*b^6*c*d^2)*n^6 + 2*(14825779*b^12*c^3 + 5117670*a^3*b^9*c
^2*d - 362880*a^6*b^6*c*d^2)*n^5 + 12*(9759623*b^12*c^3 + 4102800
*a^3*b^9*c^2*d - 777840*a^6*b^6*c*d^2)*n^4 + 8*(38232551*b^12*c^3
 + 16529190*a^3*b^9*c^2*d - 6229440*a^6*b^6*c*d^2 + 831600*a^9*b^
3*d^3)*n^3 + 576*(861864*b^12*c^3 + 298435*a^3*b^9*c^2*d - 163940
*a^6*b^6*c*d^2 + 34650*a^9*b^3*d^3)*n^2 + 5760*(76781*b^12*c^3 +
13860*a^3*b^9*c^2*d - 9240*a^6*b^6*c*d^2 + 2310*a^9*b^3*d^3)*n)*x
^3 + 144*(780996*a^3*b^9*c^3 - 293635*a^6*b^6*c^2*d + 27720*a^9*b
^3*c*d^2)*n^2 + (a*b^11*c^3*n^11 + 73*a*b^11*c^3*n^10 + 2346*a*b^
11*c^3*n^9 + 6*(7267*a*b^11*c^3 - 30*a^4*b^8*c^2*d)*n^8 + 3*(1724
59*a*b^11*c^3 - 3480*a^4*b^8*c^2*d)*n^7 + 3*(1359379*a*b^11*c^3 -
 84120*a^4*b^8*c^2*d)*n^6 + 4*(5373821*a*b^11*c^3 - 817200*a^4*b^
8*c^2*d + 15120*a^7*b^5*c*d^2)*n^5 + 4*(18531227*a*b^11*c^3 - 604
2105*a^4*b^8*c^2*d + 514080*a^7*b^5*c*d^2)*n^4 + 72*(2189036*a*b^
11*c^3 - 1380055*a^4*b^8*c^2*d + 331800*a^7*b^5*c*d^2)*n^3 + 1440
*(125842*a*b^11*c^3 - 137481*a^4*b^8*c^2*d + 70644*a^7*b^5*c*d^2
- 13860*a^10*b^2*d^3)*n^2 + 19958400*(4*a*b^11*c^3 - 6*a^4*b^8*c^
2*d + 4*a^7*b^5*c*d^2 - a^10*b^2*d^3)*n)*x^2 + 2880*(70402*a^3*b^
9*c^3 - 54321*a^6*b^6*c^2*d + 15204*a^9*b^3*c*d^2)*n - 2*(a^2*b^1
0*c^3*n^10 + 72*a^2*b^10*c^3*n^9 + 2274*a^2*b^10*c^3*n^8 + 36*(11
48*a^2*b^10*c^3 - 5*a^5*b^7*c^2*d)*n^7 + 3*(158683*a^2*b^10*c^3 -
 3420*a^5*b^7*c^2*d)*n^6 + 36*(100058*a^2*b^10*c^3 - 6725*a^5*b^7
*c^2*d)*n^5 + 4*(4473299*a^2*b^10*c^3 - 756675*a^5*b^7*c^2*d + 15
120*a^8*b^4*c*d^2)*n^4 + 72*(780996*a^2*b^10*c^3 - 293635*a^5*b^7
*c^2*d + 27720*a^8*b^4*c*d^2)*n^3 + 1440*(70402*a^2*b^10*c^3 - 54
321*a^5*b^7*c^2*d + 15204*a^8*b^4*c*d^2)*n^2 + 19958400*(4*a^2*b^
10*c^3 - 6*a^5*b^7*c^2*d + 4*a^8*b^4*c*d^2 - a^11*b*d^3)*n)*x)*(b
*x + a)^n/(b^12*n^12 + 78*b^12*n^11 + 2717*b^12*n^10 + 55770*b^12
*n^9 + 749463*b^12*n^8 + 6926634*b^12*n^7 + 44990231*b^12*n^6 + 2
06070150*b^12*n^5 + 657206836*b^12*n^4 + 1414014888*b^12*n^3 + 19
31559552*b^12*n^2 + 1486442880*b^12*n + 479001600*b^12)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(b*x+a)**n*(d*x**3+c)**3,x)

[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)^3*(b*x + a)^n*x^2,x, algorithm="giac")

[Out] Exception raised: TypeError
```

3.160 $\int x(a+bx)^n \left(c+dx^3\right)^3 \, dx$

Optimal. Leaf size=396

$$-\frac{21ad^2\left(b^3c-10a^3d\right)(a+bx)^{n+7}}{b^{11}(n+7)}+\frac{3d^2\left(b^3c-40a^3d\right)(a+bx)^{n+8}}{b^{11}(n+8)}-\frac{a\left(b^3c-a^3d\right)^3(a+bx)^{n+1}}{b^{11}(n+1)}+\frac{\left(b^3c-10a^3d\right)\left(b^3c-a^3d\right)^2(a+bx)^{n+2}}{b^{11}(n+2)}+\frac{45a^2d^3(a+bx)^{n+9}}{b^{11}(n+9)}-\frac{3ad\left(40a^6d^2-35a^3b^3cd+4b^6c^2\right)(a+bx)^{n+4}}{b^{11}(n+4)}+\frac{3d\left(70a^6d^2-35a^3b^3cd+b^6c^2\right)(a+bx)^{n+5}}{b^{11}(n+5)}+\frac{63a^2d^2\left(b^3c-4a^3d\right)(a+bx)^{n+6}}{b^{11}(n+6)}+\frac{9a^2d\left(2b^3c-5a^3d\right)\left(b^3c-a^3d\right)(a+bx)^{n+3}}{b^{11}(n+3)}-\frac{10ad^3(a+bx)^{n+10}}{b^{11}(n+10)}+\frac{d^3(a+bx)^{n+11}}{b^{11}(n+11)}$$

```
[Out] -((a*(b^3*c - a^3*d)^3*(a + b*x)^(1 + n))/(b^11*(1 + n))) + ((b^3
*c - 10*a^3*d)*(b^3*c - a^3*d)^2*(a + b*x)^(2 + n))/(b^11*(2 + n)
) + (9*a^2*d*(2*b^3*c - 5*a^3*d)*(b^3*c - a^3*d)*(a + b*x)^(3 + n
))/(b^11*(3 + n)) - (3*a*d*(4*b^6*c^2 - 35*a^3*b^3*c*d + 40*a^6*d
^2)*(a + b*x)^(4 + n))/(b^11*(4 + n)) + (3*d*(b^6*c^2 - 35*a^3*b^
3*c*d + 70*a^6*d^2)*(a + b*x)^(5 + n))/(b^11*(5 + n)) + (63*a^2*d
^2*(b^3*c - 4*a^3*d)*(a + b*x)^(6 + n))/(b^11*(6 + n)) - (21*a*d^
2*(b^3*c - 10*a^3*d)*(a + b*x)^(7 + n))/(b^11*(7 + n)) + (3*d^2*(
b^3*c - 40*a^3*d)*(a + b*x)^(8 + n))/(b^11*(8 + n)) + (45*a^2*d^3
*(a + b*x)^(9 + n))/(b^11*(9 + n)) - (10*a*d^3*(a + b*x)^(10 + n)
)/(b^11*(10 + n)) + (d^3*(a + b*x)^(11 + n))/(b^11*(11 + n))
```

Rubi [A] time = 0.587117, antiderivative size = 396, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$

$$-\frac{21ad^2\left(b^3c-10a^3d\right)(a+bx)^{n+7}}{b^{11}(n+7)}+\frac{3d^2\left(b^3c-40a^3d\right)(a+bx)^{n+8}}{b^{11}(n+8)}-\frac{a\left(b^3c-a^3d\right)^3(a+bx)^{n+1}}{b^{11}(n+1)}+\frac{\left(b^3c-10a^3d\right)\left(b^3c-a^3d\right)^2(a+bx)^{n+2}}{b^{11}(n+2)}+\frac{45a^2d^3(a+bx)^{n+9}}{b^{11}(n+9)}-\frac{3ad\left(40a^6d^2-35a^3b^3cd+4b^6c^2\right)(a+bx)^{n+4}}{b^{11}(n+4)}+\frac{3d\left(70a^6d^2-35a^3b^3cd+b^6c^2\right)(a+bx)^{n+5}}{b^{11}(n+5)}+\frac{63a^2d^2\left(b^3c-4a^3d\right)(a+bx)^{n+6}}{b^{11}(n+6)}+\frac{9a^2d\left(2b^3c-5a^3d\right)\left(b^3c-a^3d\right)(a+bx)^{n+3}}{b^{11}(n+3)}-\frac{10ad^3(a+bx)^{n+10}}{b^{11}(n+10)}+\frac{d^3(a+bx)^{n+11}}{b^{11}(n+11)}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(a + b*x)^n*(c + d*x^3)^3,x]
```

```
[Out] -((a*(b^3*c - a^3*d)^3*(a + b*x)^(1 + n))/(b^11*(1 + n))) + ((b^3
*c - 10*a^3*d)*(b^3*c - a^3*d)^2*(a + b*x)^(2 + n))/(b^11*(2 + n)
) + (9*a^2*d*(2*b^3*c - 5*a^3*d)*(b^3*c - a^3*d)*(a + b*x)^(3 + n
))/(b^11*(3 + n)) - (3*a*d*(4*b^6*c^2 - 35*a^3*b^3*c*d + 40*a^6*d
^2)*(a + b*x)^(4 + n))/(b^11*(4 + n)) + (3*d*(b^6*c^2 - 35*a^3*b^
3*c*d + 70*a^6*d^2)*(a + b*x)^(5 + n))/(b^11*(5 + n)) + (63*a^2*d
^2*(b^3*c - 4*a^3*d)*(a + b*x)^(6 + n))/(b^11*(6 + n)) - (21*a*d^
2*(b^3*c - 10*a^3*d)*(a + b*x)^(7 + n))/(b^11*(7 + n)) + (3*d^2*(
b^3*c - 40*a^3*d)*(a + b*x)^(8 + n))/(b^11*(8 + n)) + (45*a^2*d^3
*(a + b*x)^(9 + n))/(b^11*(9 + n)) - (10*a*d^3*(a + b*x)^(10 + n)
)/(b^11*(10 + n)) + (d^3*(a + b*x)^(11 + n))/(b^11*(11 + n))
```

Rubi in Sympy [A] time = 121.254, size = 374, normalized size = 0.94

$$\frac{45a^2d^3(a+bx)^{n+9}}{b^{11}(n+9)} - \frac{63a^2d^2(a+bx)^{n+6}\left(4a^3d-b^3c\right)}{b^{11}(n+6)} + \frac{9a^2d(a+bx)^{n+3}\left(a^3d-b^3c\right)\left(5a^3d-2b^3c\right)}{b^{11}(n+3)} - \frac{10ad^3(a+bx)^{n+10}}{b^{11}(n+10)} + \frac{21ad^2(a+bx)^{n+7}\left(10a^3d-b^3c\right)}{b^{11}(n+7)} - \frac{3ad(a+bx)^{n+4}\left(40a^6d^2-35a^3b^3cd+4b^6c^2\right)}{b^{11}(n+4)} + \frac{a(a+bx)^{n+1}\left(a^3d-b^3c\right)^3}{b^{11}(n+1)} + \frac{d^3(a+bx)^{n+11}}{b^{11}(n+11)} - \frac{3d^2(a+bx)^{n+8}\left(40a^3d-b^3c\right)}{b^{11}(n+8)} + \frac{3d(a+bx)^{n+5}\left(70a^6d^2-35a^3b^3cd+b^6c^2\right)}{b^{11}(n+5)} - \frac{(a+bx)^{n+2}\left(a^3d-b^3c\right)^2\left(10a^3d-b^3c\right)}{b^{11}(n+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(b*x+a)**n*(d*x**3+c)**3,x)
```

```
[Out] 45*a**2*d**3*(a + b*x)**(n + 9)/(b**11*(n + 9)) - 63*a**2*d**2*(a
 + b*x)**(n + 6)*(4*a**3*d - b**3*c)/(b**11*(n + 6)) + 9*a**2*d*(
a + b*x)**(n + 3)*(a**3*d - b**3*c)*(5*a**3*d - 2*b**3*c)/(b**11*
(n + 3)) - 10*a*d**3*(a + b*x)**(n + 10)/(b**11*(n + 10)) + 21*a*
d**2*(a + b*x)**(n + 7)*(10*a**3*d - b**3*c)/(b**11*(n + 7)) - 3*
a*d*(a + b*x)**(n + 4)*(40*a**6*d**2 - 35*a**3*b**3*c*d + 4*b**6*
c**2)/(b**11*(n + 4)) + a*(a + b*x)**(n + 1)*(a**3*d - b**3*c)**3
/(b**11*(n + 1)) + d**3*(a + b*x)**(n + 11)/(b**11*(n + 11)) - 3*
d**2*(a + b*x)**(n + 8)*(40*a**3*d - b**3*c)/(b**11*(n + 8)) + 3*
d*(a + b*x)**(n + 5)*(70*a**6*d**2 - 35*a**3*b**3*c*d + b**6*c**2
)/(b**11*(n + 5)) - (a + b*x)**(n + 2)*(a**3*d - b**3*c)**2*(10*a
**3*d - b**3*c)/(b**11*(n + 2))
```

Mathematica [B] time = 1.34314, size = 903, normalized size = 2.28

$$(a+bx)^{n+1}\left(3628800d^3a^{10}-3628800bd^3(n+1)xa^9+1814400b^2d^3\left(n^2+3n+2\right)x^2a^8-15120b^3d^2\left(40d\left(n^3+6n^2+11n+6\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(a + b*x)^n*(c + d*x^3)^3,x]
```

```
[Out] ((a + b*x)^(1 + n)*(3628800*a^10*d^3 - 3628800*a^9*b*d^3*(1 + n)*
x + 1814400*a^8*b^2*d^3*(2 + 3*n + n^2)*x^2 - 15120*a^7*b^3*d^2*(
c*(990 + 299*n + 30*n^2 + n^3) + 40*d*(6 + 11*n + 6*n^2 + n^3)*x^
3) + 15120*a^6*b^4*d^2*(1 + n)*x*(c*(990 + 299*n + 30*n^2 + n^3)
+ 10*d*(24 + 26*n + 9*n^2 + n^3)*x^3) - 7560*a^5*b^5*d^2*(2 + 3*n
 + n^2)*x^2*(c*(990 + 299*n + 30*n^2 + n^3) + 4*d*(60 + 47*n + 12
*n^2 + n^3)*x^3) + 72*a^4*b^6*d*(c^2*(332640 + 245004*n + 74524*n
^2 + 11985*n^3 + 1075*n^4 + 51*n^5 + n^6) + 35*c*d*(5940 + 12684*
n + 9409*n^2 + 3120*n^3 + 490*n^4 + 36*n^5 + n^6)*x^3 + 70*d^2*(7
20 + 1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6)*x^6) -
 18*a^3*b^7*d*(1 + n)*x*(4*c^2*(332640 + 245004*n + 74524*n^2 + 1
1985*n^3 + 1075*n^4 + 51*n^5 + n^6) + 35*c*d*(23760 + 32916*n + 1
7404*n^2 + 4485*n^3 + 595*n^4 + 39*n^5 + n^6)*x^3 + 40*d^2*(5040
+ 8028*n + 5104*n^2 + 1665*n^3 + 295*n^4 + 27*n^5 + n^6)*x^6) + 1
8*a^2*b^8*d*(2 + 3*n + n^2)*x^2*(2*c^2*(332640 + 245004*n + 74524
*n^2 + 11985*n^3 + 1075*n^4 + 51*n^5 + n^6) + 7*c*d*(59400 + 6447
0*n + 27733*n^2 + 6048*n^3 + 706*n^4 + 42*n^5 + n^6)*x^3 + 5*d^2*
(20160 + 24552*n + 12154*n^2 + 3135*n^3 + 445*n^4 + 33*n^5 + n^6)
*x^6) + b^10*(45360 + 95436*n + 72180*n^2 + 27109*n^3 + 5620*n^4
+ 654*n^5 + 40*n^6 + n^7)*x*(c^3*(440 + 183*n + 24*n^2 + n^3) + 3
*c^2*d*(176 + 126*n + 21*n^2 + n^3)*x^3 + 3*c*d^2*(110 + 87*n + 1
8*n^2 + n^3)*x^6 + d^3*(80 + 66*n + 15*n^2 + n^3)*x^9) - a*b^9*(1
62 + 99*n + 18*n^2 + n^3)*(c^3*(123200 + 111960*n + 41214*n^2 + 7
875*n^3 + 825*n^4 + 45*n^5 + n^6) + 12*c^2*d*(12320 + 24132*n + 1
5600*n^2 + 4341*n^3 + 591*n^4 + 39*n^5 + n^6)*x^3 + 21*c*d^2*(440
0 + 9420*n + 7068*n^2 + 2427*n^3 + 411*n^4 + 33*n^5 + n^6)*x^6 +
10*d^3*(2240 + 4968*n + 3954*n^2 + 1485*n^3 + 285*n^4 + 27*n^5 +
n^6)*x^9)))/(b^11*(1 + n)*(2 + n)*(3 + n)*(4 + n)*(5 + n)*(6 + n)
*(7 + n)*(8 + n)*(9 + n)*(10 + n)*(11 + n))
```

Maple [B] time = 0.033, size = 2972, normalized size = 7.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(b*x+a)^n*(d*x^3+c)^3,x)
```

```
[Out] (b*x+a)^(1+n)*(b^10*d^3*n^10*x^10+55*b^10*d^3*n^9*x^10-10*a*b^9*d
^3*n^9*x^9+1320*b^10*d^3*n^8*x^10-450*a*b^9*d^3*n^8*x^9+3*b^10*c*
d^2*n^10*x^7+18150*b^10*d^3*n^7*x^10+90*a^2*b^8*d^3*n^8*x^8-8700*
a*b^9*d^3*n^7*x^9+174*b^10*c*d^2*n^9*x^7+157773*b^10*d^3*n^6*x^10
+3240*a^2*b^8*d^3*n^7*x^8-21*a*b^9*c*d^2*n^9*x^6-94500*a*b^9*d^3*
n^6*x^9+4383*b^10*c*d^2*n^8*x^7+902055*b^10*d^3*n^5*x^10-720*a^3*
b^7*d^3*n^7*x^7+49140*a^2*b^8*d^3*n^6*x^8-1071*a*b^9*c*d^2*n^8*x^
6-632730*a*b^9*d^3*n^5*x^9+3*b^10*c^2*d*n^10*x^4+62946*b^10*c*d^2
*n^7*x^7+3416930*b^10*d^3*n^4*x^10-20160*a^3*b^7*d^3*n^6*x^7+126*
a^2*b^8*c*d^2*n^8*x^5+408240*a^2*b^8*d^3*n^5*x^8-23184*a*b^9*c*d^
2*n^7*x^6-2693250*a*b^9*d^3*n^4*x^9+183*b^10*c^2*d*n^9*x^4+568701
*b^10*c*d^2*n^6*x^7+8409500*b^10*d^3*n^3*x^10+5040*a^4*b^6*d^3*n^
6*x^6-231840*a^3*b^7*d^3*n^5*x^7+5670*a^2*b^8*c*d^2*n^7*x^5+20204
10*a^2*b^8*d^3*n^4*x^8-12*a*b^9*c^2*d*n^9*x^3-278334*a*b^9*c*d^2*
n^6*x^6-7236800*a*b^9*d^3*n^3*x^9+4860*b^10*c^2*d*n^8*x^4+3363066
*b^10*c*d^2*n^5*x^7+12753576*b^10*d^3*n^2*x^10+105840*a^4*b^6*d^3
*n^5*x^6-630*a^3*b^7*c*d^2*n^7*x^4-1411200*a^3*b^7*d^3*n^4*x^7+10
5084*a^2*b^8*c*d^2*n^6*x^5+6055560*a^2*b^8*d^3*n^3*x^8-684*a*b^9*
c^2*d*n^8*x^3-2032569*a*b^9*c*d^2*n^5*x^6-11727000*a*b^9*d^3*n^2*
x^9+b^10*c^3*n^10*x+73710*b^10*c^2*d*n^7*x^4+13114077*b^10*c*d^2*
n^4*x^7+10628640*b^10*d^3*n*x^10-30240*a^5*b^5*d^3*n^5*x^5+882000
*a^4*b^6*d^3*n^4*x^6-25200*a^3*b^7*c*d^2*n^6*x^4-4873680*a^3*b^7*
d^3*n^3*x^7+36*a^2*b^8*c^2*d*n^8*x^2+1039500*a^2*b^8*c*d^2*n^5*x^
5+10631160*a^2*b^8*d^3*n^2*x^8-16704*a*b^9*c^2*d*n^7*x^3-9313479*
a*b^9*c*d^2*n^4*x^6-10265760*a*b^9*d^3*n*x^9+64*b^10*c^3*n^9*x+70
3719*b^10*c^2*d*n^6*x^4+33074574*b^10*c*d^2*n^3*x^7+3628800*b^10*
d^3*x^10-453600*a^5*b^5*d^3*n^4*x^5+2520*a^4*b^6*c*d^2*n^6*x^3+37
04400*a^4*b^6*d^3*n^3*x^6-399420*a^3*b^7*c*d^2*n^5*x^4-9455040*a^
3*b^7*d^3*n^2*x^7+1944*a^2*b^8*c^2*d*n^7*x^2+5958414*a^2*b^8*c*d^
2*n^4*x^5+9862560*a^2*b^8*d^3*n*x^8-a*b^9*c^3*n^9-228024*a*b^9*c^
2*d*n^6*x^3-26604186*a*b^9*c*d^2*n^3*x^6-3628800*a*b^9*d^3*x^9+17
97*b^10*c^3*n^8*x+4394079*b^10*c^2*d*n^5*x^4+51177636*b^10*c*d^2*
n^2*x^7+151200*a^6*b^4*d^3*n^4*x^4-2570400*a^5*b^5*d^3*n^3*x^5+90
720*a^4*b^6*c*d^2*n^5*x^3+8184960*a^4*b^6*d^3*n^2*x^6-72*a^3*b^7*
c^2*d*n^7*x-3200400*a^3*b^7*c*d^2*n^4*x^4-9408960*a^3*b^7*d^3*n*x
^7+44280*a^2*b^8*c^2*d*n^6*x^2+20130390*a^2*b^8*c*d^2*n^3*x^5+362
8800*a^2*b^8*d^3*x^8-63*a*b^9*c^3*n^8-1902780*a*b^9*c^2*d*n^5*x^3
-45292716*a*b^9*c*d^2*n^2*x^6+29076*b^10*c^3*n^7*x+18048210*b^10*
c^2*d*n^4*x^4+43332840*b^10*c*d^2*n*x^7+1512000*a^6*b^4*d^3*n^3*x
^4-7560*a^5*b^5*c*d^2*n^5*x^2-6804000*a^5*b^5*d^3*n^2*x^5+1234800
*a^4*b^6*c*d^2*n^4*x^3+8890560*a^4*b^6*d^3*n*x^6-3744*a^3*b^7*c^2
*d*n^6*x-13790070*a^3*b^7*c*d^2*n^3*x^4-3628800*a^3*b^7*d^3*x^7+5
51232*a^2*b^8*c^2*d*n^5*x^2+38842776*a^2*b^8*c*d^2*n^2*x^5-1734*a
*b^9*c^3*n^7-9965196*a*b^9*c^2*d*n^4*x^3-41194440*a*b^9*c*d^2*n*x
^6+299271*b^10*c^3*n^6*x+47746140*b^10*c^2*d*n^3*x^4+14968800*b^1
0*c*d^2*x^7-604800*a^7*b^3*d^3*n^3*x^3+5292000*a^6*b^4*d^3*n^2*x^
4-249480*a^5*b^5*c*d^2*n^4*x^2-8285760*a^5*b^5*d^3*n*x^5+72*a^4*b
^6*c^2*d*n^6+7862400*a^4*b^6*c*d^2*n^3*x^3+3628800*a^4*b^6*d^3*x^
6-81072*a^3*b^7*c^2*d*n^5*x-31701600*a^3*b^7*c*d^2*n^2*x^4+405464
4*a^2*b^8*c^2*d*n^4*x^2+38699640*a^2*b^8*c*d^2*n*x^5-27342*a*b^9*
c^3*n^6-32332056*a*b^9*c^2*d*n^3*x^3-14968800*a*b^9*c*d^2*x^6+203
9016*b^10*c^3*n^5*x+77043528*b^10*c^2*d*n^2*x^4-3628800*a^7*b^3*d
^3*n^2*x^3+15120*a^6*b^4*c*d^2*n^4*x+7560000*a^6*b^4*d^3*n*x^4-29
55960*a^5*b^5*c*d^2*n^3*x^2-3628800*a^5*b^5*d^3*x^5+3672*a^4*b^6*
```

```
c^2*d*n^5+23710680*a^4*b^6*c*d^2*n^2*x^3-940320*a^3*b^7*c^2*d*n^4
*x-35705880*a^3*b^7*c*d^2*n*x^4+17731656*a^2*b^8*c^2*d*n^3*x^2+14
968800*a^2*b^8*c*d^2*x^5-271929*a*b^9*c^3*n^5-61656336*a*b^9*c^2*
d*n^2*x^3+9261503*b^10*c^3*n^4*x+67536288*b^10*c^2*d*n*x^4+181440
0*a^8*b^2*d^3*n^2*x^2-6652800*a^7*b^3*d^3*n*x^3+468720*a^6*b^4*c*
d^2*n^3*x+3628800*a^6*b^4*d^3*x^4-14719320*a^5*b^5*c*d^2*n^2*x^2+
77400*a^4*b^6*c^2*d*n^4+31963680*a^4*b^6*c*d^2*n*x^3-6228648*a^3*
b^7*c^2*d*n^3*x-14968800*a^3*b^7*c*d^2*x^4+43801200*a^2*b^8*c^2*d
*n^2*x^2-1767087*a*b^9*c^3*n^4-61548768*a*b^9*c^2*d*n*x^3+2747272
4*b^10*c^3*n^3*x+23950080*b^10*c^2*d*x^4+5443200*a^8*b^2*d^3*n*x^
2-15120*a^7*b^3*c*d^2*n^3-3628800*a^7*b^3*d^3*x^3+4974480*a^6*b^4
*c*d^2*n^2*x-26974080*a^5*b^5*c*d^2*n*x^2+862920*a^4*b^6*c^2*d*n^
3+14968800*a^4*b^6*c*d^2*x^3-23006016*a^3*b^7*c^2*d*n^2*x+5356540
8*a^2*b^8*c^2*d*n*x^2-7494416*a*b^9*c^3*n^3-23950080*a*b^9*c^2*d*
x^3+50312628*b^10*c^3*n^2*x-3628800*a^9*b*d^3*n*x+3628800*a^8*b^2
*d^3*x^2-453600*a^7*b^3*c*d^2*n^2+19489680*a^6*b^4*c*d^2*n*x-1496
8800*a^5*b^5*c*d^2*x^2+5365728*a^4*b^6*c^2*d*n^2-41590368*a^3*b^7
*c^2*d*n*x+23950080*a^2*b^8*c^2*d*x^2-19978308*a*b^9*c^3*n^2+5029
2720*b^10*c^3*n*x-3628800*a^9*b*d^3*x-4520880*a^7*b^3*c*d^2*n+149
68800*a^6*b^4*c*d^2*x+17640288*a^4*b^6*c^2*d*n-23950080*a^3*b^7*c
^2*d*x-30334320*a*b^9*c^3*n+19958400*b^10*c^3*x+3628800*a^10*d^3-
14968800*a^7*b^3*c*d^2+23950080*a^4*b^6*c^2*d-19958400*a*b^9*c^3)
/b^11/(n^11+66*n^10+1925*n^9+32670*n^8+357423*n^7+2637558*n^6+133
39535*n^5+45995730*n^4+105258076*n^3+150917976*n^2+120543840*n+39
916800)
```

Maxima [A] time = 0.740581, size = 1287, normalized size = 3.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)^3*(b*x + a)^n*x,x, algorithm="maxima")
```

```
[Out] (b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n*c^3/((n^2 + 3*n + 2
)*b^2) + 3*((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^5*x^5 + (n^4 +
6*n^3 + 11*n^2 + 6*n)*a*b^4*x^4 - 4*(n^3 + 3*n^2 + 2*n)*a^2*b^3*x
^3 + 12*(n^2 + n)*a^3*b^2*x^2 - 24*a^4*b*n*x + 24*a^5)*(b*x + a)^
n*c^2*d/((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*b^5) + 3
*((n^7 + 28*n^6 + 322*n^5 + 1960*n^4 + 6769*n^3 + 13132*n^2 + 130
68*n + 5040)*b^8*x^8 + (n^7 + 21*n^6 + 175*n^5 + 735*n^4 + 1624*n
^3 + 1764*n^2 + 720*n)*a*b^7*x^7 - 7*(n^6 + 15*n^5 + 85*n^4 + 225
*n^3 + 274*n^2 + 120*n)*a^2*b^6*x^6 + 42*(n^5 + 10*n^4 + 35*n^3 +
 50*n^2 + 24*n)*a^3*b^5*x^5 - 210*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^
4*b^4*x^4 + 840*(n^3 + 3*n^2 + 2*n)*a^5*b^3*x^3 - 2520*(n^2 + n)*
a^6*b^2*x^2 + 5040*a^7*b*n*x - 5040*a^8)*(b*x + a)^n*c*d^2/((n^8
+ 36*n^7 + 546*n^6 + 4536*n^5 + 22449*n^4 + 67284*n^3 + 118124*n^
2 + 109584*n + 40320)*b^8) + ((n^10 + 55*n^9 + 1320*n^8 + 18150*n
^7 + 157773*n^6 + 902055*n^5 + 3416930*n^4 + 8409500*n^3 + 127535
76*n^2 + 10628640*n + 3628800)*b^11*x^11 + (n^10 + 45*n^9 + 870*n
```

```
^8 + 9450*n^7 + 63273*n^6 + 269325*n^5 + 723680*n^4 + 1172700*n^3
 + 1026576*n^2 + 362880*n)*a*b^10*x^10 - 10*(n^9 + 36*n^8 + 546*n
^7 + 4536*n^6 + 22449*n^5 + 67284*n^4 + 118124*n^3 + 109584*n^2 +
 40320*n)*a^2*b^9*x^9 + 90*(n^8 + 28*n^7 + 322*n^6 + 1960*n^5 + 6
769*n^4 + 13132*n^3 + 13068*n^2 + 5040*n)*a^3*b^8*x^8 - 720*(n^7
+ 21*n^6 + 175*n^5 + 735*n^4 + 1624*n^3 + 1764*n^2 + 720*n)*a^4*b
^7*x^7 + 5040*(n^6 + 15*n^5 + 85*n^4 + 225*n^3 + 274*n^2 + 120*n)
*a^5*b^6*x^6 - 30240*(n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a^6*
b^5*x^5 + 151200*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^7*b^4*x^4 - 60480
0*(n^3 + 3*n^2 + 2*n)*a^8*b^3*x^3 + 1814400*(n^2 + n)*a^9*b^2*x^2
 - 3628800*a^10*b*n*x + 3628800*a^11)*(b*x + a)^n*d^3/((n^11 + 66
*n^10 + 1925*n^9 + 32670*n^8 + 357423*n^7 + 2637558*n^6 + 1333953
5*n^5 + 45995730*n^4 + 105258076*n^3 + 150917976*n^2 + 120543840*
n + 39916800)*b^11)
```

Fricas [A] time = 0.323419, size = 3941, normalized size = 9.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)^3*(b*x + a)^n*x,x, algorithm="fricas")
```

```
[Out] -(a^2*b^9*c^3*n^9 + 63*a^2*b^9*c^3*n^8 + 1734*a^2*b^9*c^3*n^7 + 1
9958400*a^2*b^9*c^3 - 23950080*a^5*b^6*c^2*d + 14968800*a^8*b^3*c
*d^2 - 3628800*a^11*d^3 - (b^11*d^3*n^10 + 55*b^11*d^3*n^9 + 1320
*b^11*d^3*n^8 + 18150*b^11*d^3*n^7 + 157773*b^11*d^3*n^6 + 902055
*b^11*d^3*n^5 + 3416930*b^11*d^3*n^4 + 8409500*b^11*d^3*n^3 + 127
53576*b^11*d^3*n^2 + 10628640*b^11*d^3*n + 3628800*b^11*d^3)*x^11
 - (a*b^10*d^3*n^10 + 45*a*b^10*d^3*n^9 + 870*a*b^10*d^3*n^8 + 94
50*a*b^10*d^3*n^7 + 63273*a*b^10*d^3*n^6 + 269325*a*b^10*d^3*n^5
+ 723680*a*b^10*d^3*n^4 + 1172700*a*b^10*d^3*n^3 + 1026576*a*b^10
*d^3*n^2 + 362880*a*b^10*d^3*n)*x^10 + 10*(a^2*b^9*d^3*n^9 + 36*a
^2*b^9*d^3*n^8 + 546*a^2*b^9*d^3*n^7 + 4536*a^2*b^9*d^3*n^6 + 224
49*a^2*b^9*d^3*n^5 + 67284*a^2*b^9*d^3*n^4 + 118124*a^2*b^9*d^3*n
^3 + 109584*a^2*b^9*d^3*n^2 + 40320*a^2*b^9*d^3*n)*x^9 - 3*(b^11*
c*d^2*n^10 + 58*b^11*c*d^2*n^9 + 4989600*b^11*c*d^2 + 3*(487*b^11
*c*d^2 + 10*a^3*b^8*d^3)*n^8 + 6*(3497*b^11*c*d^2 + 140*a^3*b^8*d
^3)*n^7 + 21*(9027*b^11*c*d^2 + 460*a^3*b^8*d^3)*n^6 + 294*(3813*
b^11*c*d^2 + 200*a^3*b^8*d^3)*n^5 + (4371359*b^11*c*d^2 + 203070*
a^3*b^8*d^3)*n^4 + 2*(5512429*b^11*c*d^2 + 196980*a^3*b^8*d^3)*n^
3 + 36*(473867*b^11*c*d^2 + 10890*a^3*b^8*d^3)*n^2 + 360*(40123*b
^11*c*d^2 + 420*a^3*b^8*d^3)*n)*x^8 - 3*(a*b^10*c*d^2*n^10 + 51*a
*b^10*c*d^2*n^9 + 1104*a*b^10*c*d^2*n^8 + 6*(2209*a*b^10*c*d^2 -
40*a^4*b^7*d^3)*n^7 + 21*(4609*a*b^10*c*d^2 - 240*a^4*b^7*d^3)*n^
6 + 21*(21119*a*b^10*c*d^2 - 2000*a^4*b^7*d^3)*n^5 + 2*(633433*a*
b^10*c*d^2 - 88200*a^4*b^7*d^3)*n^4 + 12*(179733*a*b^10*c*d^2 - 3
2480*a^4*b^7*d^3)*n^3 + 360*(5449*a*b^10*c*d^2 - 1176*a^4*b^7*d^3
)*n^2 + 21600*(33*a*b^10*c*d^2 - 8*a^4*b^7*d^3)*n)*x^7 + 18*(1519
*a^2*b^9*c^3 - 4*a^5*b^6*c^2*d)*n^6 + 21*(a^2*b^9*c*d^2*n^9 + 45*
```

```
a^2*b^9*c*d^2*n^8 + 834*a^2*b^9*c*d^2*n^7 + 30*(275*a^2*b^9*c*d^2
 - 8*a^5*b^6*d^3)*n^6 + 3*(15763*a^2*b^9*c*d^2 - 1200*a^5*b^6*d^3
)*n^5 + 15*(10651*a^2*b^9*c*d^2 - 1360*a^5*b^6*d^3)*n^4 + 4*(7706
9*a^2*b^9*c*d^2 - 13500*a^5*b^6*d^3)*n^3 + 60*(5119*a^2*b^9*c*d^2
 - 1096*a^5*b^6*d^3)*n^2 + 3600*(33*a^2*b^9*c*d^2 - 8*a^5*b^6*d^3
)*n)*x^6 + 3*(90643*a^2*b^9*c^3 - 1224*a^5*b^6*c^2*d)*n^5 - 3*(b^
11*c^2*d*n^10 + 61*b^11*c^2*d*n^9 + 7983360*b^11*c^2*d + 6*(270*b
^11*c^2*d + 7*a^3*b^8*c*d^2)*n^8 + 210*(117*b^11*c^2*d + 8*a^3*b^
8*c*d^2)*n^7 + 3*(78191*b^11*c^2*d + 8876*a^3*b^8*c*d^2)*n^6 + 3*
(488231*b^11*c^2*d + 71120*a^3*b^8*c*d^2 - 3360*a^6*b^5*d^3)*n^5
+ 2*(3008035*b^11*c^2*d + 459669*a^3*b^8*c*d^2 - 50400*a^6*b^5*d^
3)*n^4 + 20*(795769*b^11*c^2*d + 105672*a^3*b^8*c*d^2 - 17640*a^6
*b^5*d^3)*n^3 + 72*(356683*b^11*c^2*d + 33061*a^3*b^8*c*d^2 - 700
0*a^6*b^5*d^3)*n^2 + 288*(78167*b^11*c^2*d + 3465*a^3*b^8*c*d^2 -
 840*a^6*b^5*d^3)*n)*x^5 + 9*(196343*a^2*b^9*c^3 - 8600*a^5*b^6*c
^2*d)*n^4 - 3*(a*b^10*c^2*d*n^10 + 57*a*b^10*c^2*d*n^9 + 1392*a*b
^10*c^2*d*n^8 + 6*(3167*a*b^10*c^2*d - 35*a^4*b^7*c*d^2)*n^7 + 15
*(10571*a*b^10*c^2*d - 504*a^4*b^7*c*d^2)*n^6 + 3*(276811*a*b^10*
c^2*d - 34300*a^4*b^7*c*d^2)*n^5 + 2*(1347169*a*b^10*c^2*d - 3276
00*a^4*b^7*c*d^2 + 25200*a^7*b^4*d^3)*n^4 + 42*(122334*a*b^10*c^2
*d - 47045*a^4*b^7*c*d^2 + 7200*a^7*b^4*d^3)*n^3 + 72*(71237*a*b^
10*c^2*d - 36995*a^4*b^7*c*d^2 + 7700*a^7*b^4*d^3)*n^2 + 7560*(26
4*a*b^10*c^2*d - 165*a^4*b^7*c*d^2 + 40*a^7*b^4*d^3)*n)*x^4 + 8*(
936802*a^2*b^9*c^3 - 107865*a^5*b^6*c^2*d + 1890*a^8*b^3*c*d^2)*n
^3 + 12*(a^2*b^9*c^2*d*n^9 + 54*a^2*b^9*c^2*d*n^8 + 1230*a^2*b^9*
c^2*d*n^7 + 6*(2552*a^2*b^9*c^2*d - 35*a^5*b^6*c*d^2)*n^6 + 33*(3
413*a^2*b^9*c^2*d - 210*a^5*b^6*c*d^2)*n^5 + 6*(82091*a^2*b^9*c^2
*d - 13685*a^5*b^6*c*d^2)*n^4 + 10*(121670*a^2*b^9*c^2*d - 40887*
a^5*b^6*c*d^2 + 5040*a^8*b^3*d^3)*n^3 + 24*(61997*a^2*b^9*c^2*d -
 31220*a^5*b^6*c*d^2 + 6300*a^8*b^3*d^3)*n^2 + 2520*(264*a^2*b^9*
c^2*d - 165*a^5*b^6*c*d^2 + 40*a^8*b^3*d^3)*n)*x^3 + 36*(554953*a
^2*b^9*c^3 - 149048*a^5*b^6*c^2*d + 12600*a^8*b^3*c*d^2)*n^2 - (b
^11*c^3*n^10 + 64*b^11*c^3*n^9 + 19958400*b^11*c^3 + 3*(599*b^11*
c^3 + 12*a^3*b^8*c^2*d)*n^8 + 12*(2423*b^11*c^3 + 156*a^3*b^8*c^2
*d)*n^7 + 3*(99757*b^11*c^3 + 13512*a^3*b^8*c^2*d)*n^6 + 24*(8495
9*b^11*c^3 + 19590*a^3*b^8*c^2*d - 315*a^6*b^5*c*d^2)*n^5 + (9261
503*b^11*c^3 + 3114324*a^3*b^8*c^2*d - 234360*a^6*b^5*c*d^2)*n^4
+ 4*(6868181*b^11*c^3 + 2875752*a^3*b^8*c^2*d - 621810*a^6*b^5*c*
d^2)*n^3 + 36*(1397573*b^11*c^3 + 577644*a^3*b^8*c^2*d - 270690*a
^6*b^5*c*d^2 + 50400*a^9*b^2*d^3)*n^2 + 720*(69851*b^11*c^3 + 166
32*a^3*b^8*c^2*d - 10395*a^6*b^5*c*d^2 + 2520*a^9*b^2*d^3)*n)*x^2
 + 144*(210655*a^2*b^9*c^3 - 122502*a^5*b^6*c^2*d + 31395*a^8*b^3
*c*d^2)*n - (a*b^10*c^3*n^10 + 63*a*b^10*c^3*n^9 + 1734*a*b^10*c^
3*n^8 + 18*(1519*a*b^10*c^3 - 4*a^4*b^7*c^2*d)*n^7 + 3*(90643*a*b
^10*c^3 - 1224*a^4*b^7*c^2*d)*n^6 + 9*(196343*a*b^10*c^3 - 8600*a
^4*b^7*c^2*d)*n^5 + 8*(936802*a*b^10*c^3 - 107865*a^4*b^7*c^2*d +
 1890*a^7*b^4*c*d^2)*n^4 + 36*(554953*a*b^10*c^3 - 149048*a^4*b^7
*c^2*d + 12600*a^7*b^4*c*d^2)*n^3 + 144*(210655*a*b^10*c^3 - 1225
02*a^4*b^7*c^2*d + 31395*a^7*b^4*c*d^2)*n^2 + 90720*(220*a*b^10*c
^3 - 264*a^4*b^7*c^2*d + 165*a^7*b^4*c*d^2 - 40*a^10*b*d^3)*n)*x)
*(b*x + a)^n/(b^11*n^11 + 66*b^11*n^10 + 1925*b^11*n^9 + 32670*b^
11*n^8 + 357423*b^11*n^7 + 2637558*b^11*n^6 + 13339535*b^11*n^5 +
 45995730*b^11*n^4 + 105258076*b^11*n^3 + 150917976*b^11*n^2 + 12
0543840*b^11*n + 39916800*b^11)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(b*x+a)**n*(d*x**3+c)**3,x)

[Out] Timed out
```

GIAC/XCAS [A] time = 0.29768, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)^3*(b*x + a)^n*x,x, algorithm="giac")

[Out] Done
```

3.161 $\int (a+bx)^n \left(c+dx^3\right)^3 \, dx$

Optimal. Leaf size=337

$$-\frac{18ad^2\left(b^3c-7a^3d\right)(a+bx)^{n+6}}{b^{10}(n+6)}+\frac{3d^2\left(b^3c-28a^3d\right)(a+bx)^{n+7}}{b^{10}(n+7)}+\frac{\left(b^3c-a^3d\right)^3(a+bx)^{n+1}}{b^{10}(n+1)}-\frac{9ad\left(b^3c-4a^3d\right)\left(b^3c-a^3d\right)(a+bx)^{n+3}}{b^{10}(n+3)}+\frac{36a^2d^3(a+bx)^{n+8}}{b^{10}(n+8)}+\frac{3d\left(28a^6d^2-20a^3b^3cd+b^6c^2\right)(a+bx)^{n+4}}{b^{10}(n+4)}+\frac{9a^2d^2\left(5b^3c-14a^3d\right)(a+bx)^{n+5}}{b^{10}(n+5)}+\frac{9a^2d\left(b^3c-a^3d\right)^2(a+bx)^{n+2}}{b^{10}(n+2)}-\frac{9ad^3(a+bx)^{n+9}}{b^{10}(n+9)}+\frac{d^3(a+bx)^{n+10}}{b^{10}(n+10)}$$

```
[Out] ((b^3*c - a^3*d)^3*(a + b*x)^(1 + n))/(b^10*(1 + n)) + (9*a^2*d*(
b^3*c - a^3*d)^2*(a + b*x)^(2 + n))/(b^10*(2 + n)) - (9*a*d*(b^3*
c - 4*a^3*d)*(b^3*c - a^3*d)*(a + b*x)^(3 + n))/(b^10*(3 + n)) +
(3*d*(b^6*c^2 - 20*a^3*b^3*c*d + 28*a^6*d^2)*(a + b*x)^(4 + n))/(
b^10*(4 + n)) + (9*a^2*d^2*(5*b^3*c - 14*a^3*d)*(a + b*x)^(5 + n)
)/(b^10*(5 + n)) - (18*a*d^2*(b^3*c - 7*a^3*d)*(a + b*x)^(6 + n))
/(b^10*(6 + n)) + (3*d^2*(b^3*c - 28*a^3*d)*(a + b*x)^(7 + n))/(b
^10*(7 + n)) + (36*a^2*d^3*(a + b*x)^(8 + n))/(b^10*(8 + n)) - (9
*a*d^3*(a + b*x)^(9 + n))/(b^10*(9 + n)) + (d^3*(a + b*x)^(10 + n
))/(b^10*(10 + n))
```

Rubi [A] time = 0.466382, antiderivative size = 337, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059

$$-\frac{18ad^2\left(b^3c-7a^3d\right)(a+bx)^{n+6}}{b^{10}(n+6)}+\frac{3d^2\left(b^3c-28a^3d\right)(a+bx)^{n+7}}{b^{10}(n+7)}+\frac{\left(b^3c-a^3d\right)^3(a+bx)^{n+1}}{b^{10}(n+1)}-\frac{9ad\left(b^3c-4a^3d\right)\left(b^3c-a^3d\right)(a+bx)^{n+3}}{b^{10}(n+3)}+\frac{36a^2d^3(a+bx)^{n+8}}{b^{10}(n+8)}+\frac{3d\left(28a^6d^2-20a^3b^3cd+b^6c^2\right)(a+bx)^{n+4}}{b^{10}(n+4)}+\frac{9a^2d^2\left(5b^3c-14a^3d\right)(a+bx)^{n+5}}{b^{10}(n+5)}+\frac{9a^2d\left(b^3c-a^3d\right)^2(a+bx)^{n+2}}{b^{10}(n+2)}-\frac{9ad^3(a+bx)^{n+9}}{b^{10}(n+9)}+\frac{d^3(a+bx)^{n+10}}{b^{10}(n+10)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x)^n*(c + d*x^3)^3,x]
```


```
[Out] ((b^3*c - a^3*d)^3*(a + b*x)^(1 + n))/(b^10*(1 + n)) + (9*a^2*d*(
b^3*c - a^3*d)^2*(a + b*x)^(2 + n))/(b^10*(2 + n)) - (9*a*d*(b^3*
c - 4*a^3*d)*(b^3*c - a^3*d)*(a + b*x)^(3 + n))/(b^10*(3 + n)) +
```

```
(3*d*(b^6*c^2 - 20*a^3*b^3*c*d + 28*a^6*d^2)*(a + b*x)^(4 + n))/(
b^10*(4 + n)) + (9*a^2*d^2*(5*b^3*c - 14*a^3*d)*(a + b*x)^(5 + n)
)/(b^10*(5 + n)) - (18*a*d^2*(b^3*c - 7*a^3*d)*(a + b*x)^(6 + n))
/(b^10*(6 + n)) + (3*d^2*(b^3*c - 28*a^3*d)*(a + b*x)^(7 + n))/(b
^10*(7 + n)) + (36*a^2*d^3*(a + b*x)^(8 + n))/(b^10*(8 + n)) - (9
*a*d^3*(a + b*x)^(9 + n))/(b^10*(9 + n)) + (d^3*(a + b*x)^(10 + n
))/(b^10*(10 + n))
```

Rubi in Sympy [A] time = 100.789, size = 316, normalized size = 0.94

$$\frac{36a^2d^3(a+bx)^{n+8}}{b^{10}(n+8)}-\frac{9a^2d^2(a+bx)^{n+5}\left(14a^3d-5b^3c\right)}{b^{10}(n+5)}+\frac{9a^2d(a+bx)^{n+2}\left(a^3d-b^3c\right)^2}{b^{10}(n+2)}-\frac{9ad^3(a+bx)^{n+9}}{b^{10}(n+9)}+\frac{18ad^2(a+bx)^{n+6}\left(7a^3d-b^3c\right)}{b^{10}(n+6)}-\frac{9ad(a+bx)^{n+3}\left(a^3d-b^3c\right)\left(4a^3d-b^3c\right)}{b^{10}(n+3)}+\frac{d^3(a+bx)^{n+10}}{b^{10}(n+10)}-\frac{3d^2(a+bx)^{n+7}\left(28a^3d-b^3c\right)}{b^{10}(n+7)}+\frac{3d(a+bx)^{n+4}\left(28a^6d^2-20a^3b^3cd+b^6c^2\right)}{b^{10}(n+4)}-\frac{(a+bx)^{n+1}\left(a^3d-b^3c\right)^3}{b^{10}(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x+a)**n*(d*x**3+c)**3,x)
```

```
[Out] 36*a**2*d**3*(a + b*x)**(n + 8)/(b**10*(n + 8)) - 9*a**2*d**2*(a
+ b*x)**(n + 5)*(14*a**3*d - 5*b**3*c)/(b**10*(n + 5)) + 9*a**2*d
*(a + b*x)**(n + 2)*(a**3*d - b**3*c)**2/(b**10*(n + 2)) - 9*a*d*
*3*(a + b*x)**(n + 9)/(b**10*(n + 9)) + 18*a*d**2*(a + b*x)**(n +
 6)*(7*a**3*d - b**3*c)/(b**10*(n + 6)) - 9*a*d*(a + b*x)**(n + 3
)*(a**3*d - b**3*c)*(4*a**3*d - b**3*c)/(b**10*(n + 3)) + d**3*(a
 + b*x)**(n + 10)/(b**10*(n + 10)) - 3*d**2*(a + b*x)**(n + 7)*(2
8*a**3*d - b**3*c)/(b**10*(n + 7)) + 3*d*(a + b*x)**(n + 4)*(28*a
**6*d**2 - 20*a**3*b**3*c*d + b**6*c**2)/(b**10*(n + 4)) - (a + b
*x)**(n + 1)*(a**3*d - b**3*c)**3/(b**10*(n + 1))
```

Mathematica [B] time = 1.09458, size = 706, normalized size = 2.09

$$\frac{(a+bx)^{n+1}\left(-362880a^9d^3+362880a^8bd^3(n+1)x-181440a^7b^2d^3\left(n^2+3n+2\right)x^2+2160a^6b^3d^2\left(c\left(n^3+27n^2+242n+720\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x)^n*(c + d*x^3)^3,x]
```

```
[Out] ((a + b*x)^(1 + n)*(-362880*a^9*d^3 + 362880*a^8*b*d^3*(1 + n)*x
- 181440*a^7*b^2*d^3*(2 + 3*n + n^2)*x^2 + 2160*a^6*b^3*d^2*(c*(7
20 + 242*n + 27*n^2 + n^3) + 28*d*(6 + 11*n + 6*n^2 + n^3)*x^3) -
 2160*a^5*b^4*d^2*(1 + n)*x*(c*(720 + 242*n + 27*n^2 + n^3) + 7*d
*(24 + 26*n + 9*n^2 + n^3)*x^3) + 216*a^4*b^5*d^2*(2 + 3*n + n^2)
*x^2*(5*c*(720 + 242*n + 27*n^2 + n^3) + 14*d*(60 + 47*n + 12*n^2
 + n^3)*x^3) - 9*a*b^8*d*(80 + 146*n + 81*n^2 + 16*n^3 + n^4)*x^2
*(c^2*(3780 + 1968*n + 379*n^2 + 32*n^3 + n^4) + 2*c*d*(1080 + 85
8*n + 235*n^2 + 26*n^3 + n^4)*x^3 + d^2*(504 + 450*n + 145*n^2 +
20*n^3 + n^4)*x^6) - 18*a^3*b^6*d*(c^2*(151200 + 127860*n + 44524
*n^2 + 8175*n^3 + 835*n^4 + 45*n^5 + n^6) + 20*c*d*(4320 + 9372*n
 + 7144*n^2 + 2475*n^3 + 415*n^4 + 33*n^5 + n^6)*x^3 + 28*d^2*(72
0 + 1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6)*x^6) +
18*a^2*b^7*d*(1 + n)*x*(c^2*(151200 + 127860*n + 44524*n^2 + 8175
*n^3 + 835*n^4 + 45*n^5 + n^6) + 5*c*d*(17280 + 24528*n + 13420*n
^2 + 3624*n^3 + 511*n^4 + 36*n^5 + n^6)*x^3 + 4*d^2*(5040 + 8028*
n + 5104*n^2 + 1665*n^3 + 295*n^4 + 27*n^5 + n^6)*x^6) + b^9*(129
60 + 18612*n + 10404*n^2 + 2915*n^3 + 435*n^4 + 33*n^5 + n^6)*(c^
3*(280 + 138*n + 21*n^2 + n^3) + 3*c^2*d*(70 + 87*n + 18*n^2 + n^
3)*x^3 + 3*c*d^2*(40 + 54*n + 15*n^2 + n^3)*x^6 + d^3*(28 + 39*n
+ 12*n^2 + n^3)*x^9)))/(b^10*(1 + n)*(2 + n)*(3 + n)*(4 + n)*(5 +
 n)*(6 + n)*(7 + n)*(8 + n)*(9 + n)*(10 + n))
```

Maple [B] time = 0.028, size = 2280, normalized size = 6.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x+a)^n*(d*x^3+c)^3,x)
```

```
[Out] -(b*x+a)^(1+n)*(-b^9*d^3*n^9*x^9-45*b^9*d^3*n^8*x^9+9*a*b^8*d^3*n
^8*x^8-870*b^9*d^3*n^7*x^9+324*a*b^8*d^3*n^7*x^8-3*b^9*c*d^2*n^9*
x^6-9450*b^9*d^3*n^6*x^9-72*a^2*b^7*d^3*n^7*x^7+4914*a*b^8*d^3*n^
6*x^8-144*b^9*c*d^2*n^8*x^6-63273*b^9*d^3*n^5*x^9-2016*a^2*b^7*d^
3*n^6*x^7+18*a*b^8*c*d^2*n^8*x^5+40824*a*b^8*d^3*n^5*x^8-2952*b^9
*c*d^2*n^7*x^6-269325*b^9*d^3*n^4*x^9+504*a^3*b^6*d^3*n^6*x^6-231
84*a^2*b^7*d^3*n^5*x^7+756*a*b^8*c*d^2*n^7*x^5+202041*a*b^8*d^3*n
^4*x^8-3*b^9*c^2*d*n^9*x^3-33786*b^9*c*d^2*n^6*x^6-723680*b^9*d^3
*n^3*x^9+10584*a^3*b^6*d^3*n^5*x^6-90*a^2*b^7*c*d^2*n^7*x^4-14112
0*a^2*b^7*d^3*n^4*x^7+13176*a*b^8*c*d^2*n^6*x^5+605556*a*b^8*d^3*
n^3*x^8-153*b^9*c^2*d*n^8*x^3-236817*b^9*c*d^2*n^5*x^6-1172700*b^
9*d^3*n^2*x^9-3024*a^4*b^5*d^3*n^5*x^5+88200*a^3*b^6*d^3*n^4*x^6-
3330*a^2*b^7*c*d^2*n^6*x^4-487368*a^2*b^7*d^3*n^3*x^7+9*a*b^8*c^2
*d*n^8*x^2+123660*a*b^8*c*d^2*n^5*x^5+1063116*a*b^8*d^3*n^2*x^8-3
348*b^9*c^2*d*n^7*x^3-1048446*b^9*c*d^2*n^4*x^6-1026576*b^9*d^3*n
*x^9-45360*a^4*b^5*d^3*n^4*x^5+360*a^3*b^6*c*d^2*n^6*x^3+370440*a
^3*b^6*d^3*n^3*x^6-49230*a^2*b^7*c*d^2*n^5*x^4-945504*a^2*b^7*d^3
*n^2*x^7+432*a*b^8*c^2*d*n^7*x^2+678942*a*b^8*c*d^2*n^4*x^5+98625
```

```
6*a*b^8*d^3*n*x^8-b^9*c^3*n^9-41058*b^9*c^2*d*n^6*x^3-2911668*b^9
*c*d^2*n^3*x^6-362880*b^9*d^3*x^9+15120*a^5*b^4*d^3*n^4*x^4-25704
0*a^4*b^5*d^3*n^3*x^5+11880*a^3*b^6*c*d^2*n^5*x^3+818496*a^3*b^6*
d^3*n^2*x^6-18*a^2*b^7*c^2*d*n^7*x-372150*a^2*b^7*c*d^2*n^4*x^4-9
40896*a^2*b^7*d^3*n*x^7+8748*a*b^8*c^2*d*n^6*x^2+2217024*a*b^8*c*
d^2*n^3*x^5+362880*a*b^8*d^3*x^8-54*b^9*c^3*n^8-309087*b^9*c^2*d*
n^5*x^3-4846824*b^9*c*d^2*n^2*x^6+151200*a^5*b^4*d^3*n^3*x^4-1080
*a^4*b^5*c*d^2*n^5*x^2-680400*a^4*b^5*d^3*n^2*x^5+149400*a^3*b^6*
c*d^2*n^4*x^3+889056*a^3*b^6*d^3*n*x^6-828*a^2*b^7*c^2*d*n^6*x-15
33960*a^2*b^7*c*d^2*n^3*x^4-362880*a^2*b^7*d^3*x^7+96930*a*b^8*c^
2*d*n^5*x^2+4167864*a*b^8*c*d^2*n^2*x^5-1266*b^9*c^3*n^7-1469817*
b^9*c^2*d*n^4*x^3-4332960*b^9*c*d^2*n*x^6-60480*a^6*b^3*d^3*n^3*x
^3+529200*a^5*b^4*d^3*n^2*x^4-32400*a^4*b^5*c*d^2*n^4*x^2-828576*
a^4*b^5*d^3*n*x^5+18*a^3*b^6*c^2*d*n^6+891000*a^3*b^6*c*d^2*n^3*x
^3+362880*a^3*b^6*d^3*x^6-15840*a^2*b^7*c^2*d*n^5*x-3415320*a^2*b
^7*c*d^2*n^2*x^4+636471*a*b^8*c^2*d*n^4*x^2+4073760*a*b^8*c*d^2*n
*x^5-16884*b^9*c^3*n^6-4371522*b^9*c^2*d*n^3*x^3-1555200*b^9*c*d^
2*x^6-362880*a^6*b^3*d^3*n^2*x^3+2160*a^5*b^4*c*d^2*n^4*x+756000*
a^5*b^4*d^3*n*x^4-351000*a^4*b^5*c*d^2*n^3*x^2-362880*a^4*b^5*d^3
*x^5+810*a^3*b^6*c^2*d*n^5+2571840*a^3*b^6*c*d^2*n^2*x^3-162180*a
^2*b^7*c^2*d*n^4*x-3762720*a^2*b^7*c*d^2*n*x^4+2500038*a*b^8*c^2*
d*n^3*x^2+1555200*a*b^8*c*d^2*x^5-140889*b^9*c^3*n^5-7742412*b^9*
c^2*d*n^2*x^3+181440*a^7*b^2*d^3*n^2*x^2-665280*a^6*b^3*d^3*n*x^3
+60480*a^5*b^4*c*d^2*n^3*x+362880*a^5*b^4*d^3*x^4-1620000*a^4*b^5
*c*d^2*n^2*x^2+15030*a^3*b^6*c^2*d*n^4+3373920*a^3*b^6*c*d^2*n*x^
3-948582*a^2*b^7*c^2*d*n^3*x-1555200*a^2*b^7*c*d^2*x^4+5614452*a*
b^8*c^2*d*n^2*x^2-761166*b^9*c^3*n^4-7291080*b^9*c^2*d*n*x^3+5443
20*a^7*b^2*d^3*n*x^2-2160*a^6*b^3*c*d^2*n^3-362880*a^6*b^3*d^3*x^
3+581040*a^5*b^4*c*d^2*n^2*x-2855520*a^4*b^5*c*d^2*n*x^2+147150*a
^3*b^6*c^2*d*n^3+1555200*a^3*b^6*c*d^2*x^3-3102912*a^2*b^7*c^2*d*
n^2*x+6383880*a*b^8*c^2*d*n*x^2-2655764*b^9*c^3*n^3-2721600*b^9*c
^2*d*x^3-362880*a^8*b*d^3*n*x+362880*a^7*b^2*d^3*x^2-58320*a^6*b^
3*c*d^2*n^2+2077920*a^5*b^4*c*d^2*n*x-1555200*a^4*b^5*c*d^2*x^2+8
01432*a^3*b^6*c^2*d*n^2-5023080*a^2*b^7*c^2*d*n*x+2721600*a*b^8*c
^2*d*x^2-5753736*b^9*c^3*n^2-362880*a^8*b*d^3*x-522720*a^6*b^3*c*
d^2*n+1555200*a^5*b^4*c*d^2*x+2301480*a^3*b^6*c^2*d*n-2721600*a^2
*b^7*c^2*d*x-6999840*b^9*c^3*n+362880*a^9*d^3-1555200*a^6*b^3*c*d
^2+2721600*a^3*b^6*c^2*d-3628800*b^9*c^3)/b^10/(n^10+55*n^9+1320*
n^8+18150*n^7+157773*n^6+902055*n^5+3416930*n^4+8409500*n^3+12753
576*n^2+10628640*n+3628800)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)^3*(b*x + a)^n,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.319317, size = 3123, normalized size = 9.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)^3*(b*x + a)^n,x, algorithm="fricas")
```

```
[Out] (a*b^9*c^3*n^9 + 54*a*b^9*c^3*n^8 + 1266*a*b^9*c^3*n^7 + 3628800*
a*b^9*c^3 - 2721600*a^4*b^6*c^2*d + 1555200*a^7*b^3*c*d^2 - 36288
0*a^10*d^3 + (b^10*d^3*n^9 + 45*b^10*d^3*n^8 + 870*b^10*d^3*n^7 +
 9450*b^10*d^3*n^6 + 63273*b^10*d^3*n^5 + 269325*b^10*d^3*n^4 + 7
23680*b^10*d^3*n^3 + 1172700*b^10*d^3*n^2 + 1026576*b^10*d^3*n +
362880*b^10*d^3)*x^10 + (a*b^9*d^3*n^9 + 36*a*b^9*d^3*n^8 + 546*a
*b^9*d^3*n^7 + 4536*a*b^9*d^3*n^6 + 22449*a*b^9*d^3*n^5 + 67284*a
*b^9*d^3*n^4 + 118124*a*b^9*d^3*n^3 + 109584*a*b^9*d^3*n^2 + 4032
0*a*b^9*d^3*n)*x^9 - 9*(a^2*b^8*d^3*n^8 + 28*a^2*b^8*d^3*n^7 + 32
2*a^2*b^8*d^3*n^6 + 1960*a^2*b^8*d^3*n^5 + 6769*a^2*b^8*d^3*n^4 +
 13132*a^2*b^8*d^3*n^3 + 13068*a^2*b^8*d^3*n^2 + 5040*a^2*b^8*d^3
*n)*x^8 + 3*(b^10*c*d^2*n^9 + 48*b^10*c*d^2*n^8 + 518400*b^10*c*d
^2 + 24*(41*b^10*c*d^2 + a^3*b^7*d^3)*n^7 + 6*(1877*b^10*c*d^2 +
84*a^3*b^7*d^3)*n^6 + 21*(3759*b^10*c*d^2 + 200*a^3*b^7*d^3)*n^5
+ 42*(8321*b^10*c*d^2 + 420*a^3*b^7*d^3)*n^4 + 4*(242639*b^10*c*d
^2 + 9744*a^3*b^7*d^3)*n^3 + 72*(22439*b^10*c*d^2 + 588*a^3*b^7*d
^3)*n^2 + 1440*(1003*b^10*c*d^2 + 12*a^3*b^7*d^3)*n)*x^7 + 18*(93
8*a*b^9*c^3 - a^4*b^6*c^2*d)*n^6 + 3*(a*b^9*c*d^2*n^9 + 42*a*b^9*
c*d^2*n^8 + 732*a*b^9*c*d^2*n^7 + 6*(1145*a*b^9*c*d^2 - 28*a^4*b^
6*d^3)*n^6 + 9*(4191*a*b^9*c*d^2 - 280*a^4*b^6*d^3)*n^5 + 24*(513
2*a*b^9*c*d^2 - 595*a^4*b^6*d^3)*n^4 + 4*(57887*a*b^9*c*d^2 - 945
0*a^4*b^6*d^3)*n^3 + 48*(4715*a*b^9*c*d^2 - 959*a^4*b^6*d^3)*n^2
+ 2880*(30*a*b^9*c*d^2 - 7*a^4*b^6*d^3)*n)*x^6 + 3*(46963*a*b^9*c
^3 - 270*a^4*b^6*c^2*d)*n^5 - 18*(a^2*b^8*c*d^2*n^8 + 37*a^2*b^8*
c*d^2*n^7 + 547*a^2*b^8*c*d^2*n^6 + (4135*a^2*b^8*c*d^2 - 168*a^5
*b^5*d^3)*n^5 + 4*(4261*a^2*b^8*c*d^2 - 420*a^5*b^5*d^3)*n^4 + 4*
(9487*a^2*b^8*c*d^2 - 1470*a^5*b^5*d^3)*n^3 + 48*(871*a^2*b^8*c*d
^2 - 175*a^5*b^5*d^3)*n^2 + 576*(30*a^2*b^8*c*d^2 - 7*a^5*b^5*d^3
)*n)*x^5 + 18*(42287*a*b^9*c^3 - 835*a^4*b^6*c^2*d)*n^4 + 3*(b^10
*c^2*d*n^9 + 51*b^10*c^2*d*n^8 + 907200*b^10*c^2*d + 6*(186*b^10*
c^2*d + 5*a^3*b^7*c*d^2)*n^7 + 6*(2281*b^10*c^2*d + 165*a^3*b^7*c
*d^2)*n^6 + 3*(34343*b^10*c^2*d + 4150*a^3*b^7*c*d^2)*n^5 + 3*(16
3313*b^10*c^2*d + 24750*a^3*b^7*c*d^2 - 1680*a^6*b^4*d^3)*n^4 + 2
*(728587*b^10*c^2*d + 107160*a^3*b^7*c*d^2 - 15120*a^6*b^4*d^3)*n
^3 + 36*(71689*b^10*c^2*d + 7810*a^3*b^7*c*d^2 - 1540*a^6*b^4*d^3
)*n^2 + 360*(6751*b^10*c^2*d + 360*a^3*b^7*c*d^2 - 84*a^6*b^4*d^3
)*n)*x^4 + 2*(1327882*a*b^9*c^3 - 73575*a^4*b^6*c^2*d + 1080*a^7*
b^3*c*d^2)*n^3 + 3*(a*b^9*c^2*d*n^9 + 48*a*b^9*c^2*d*n^8 + 972*a*
b^9*c^2*d*n^7 + 30*(359*a*b^9*c^2*d - 4*a^4*b^6*c*d^2)*n^6 + 3*(2
3573*a*b^9*c^2*d - 1200*a^4*b^6*c*d^2)*n^5 + 6*(46297*a*b^9*c^2*d
 - 6500*a^4*b^6*c*d^2)*n^4 + 4*(155957*a*b^9*c^2*d - 45000*a^4*b^
```

```
6*c*d^2 + 5040*a^7*b^3*d^3)*n^3 + 120*(5911*a*b^9*c^2*d - 2644*a^
4*b^6*c*d^2 + 504*a^7*b^3*d^3)*n^2 + 2880*(105*a*b^9*c^2*d - 60*a
^4*b^6*c*d^2 + 14*a^7*b^3*d^3)*n)*x^3 + 72*(79913*a*b^9*c^3 - 111
31*a^4*b^6*c^2*d + 810*a^7*b^3*c*d^2)*n^2 - 9*(a^2*b^8*c^2*d*n^8
+ 46*a^2*b^8*c^2*d*n^7 + 880*a^2*b^8*c^2*d*n^6 + 10*(901*a^2*b^8*
c^2*d - 12*a^5*b^5*c*d^2)*n^5 + (52699*a^2*b^8*c^2*d - 3360*a^5*b
^5*c*d^2)*n^4 + 8*(21548*a^2*b^8*c^2*d - 4035*a^5*b^5*c*d^2)*n^3
+ 60*(4651*a^2*b^8*c^2*d - 1924*a^5*b^5*c*d^2 + 336*a^8*b^2*d^3)*
n^2 + 1440*(105*a^2*b^8*c^2*d - 60*a^5*b^5*c*d^2 + 14*a^8*b^2*d^3
)*n)*x^2 + 360*(19444*a*b^9*c^3 - 6393*a^4*b^6*c^2*d + 1452*a^7*b
^3*c*d^2)*n + (b^10*c^3*n^9 + 54*b^10*c^3*n^8 + 3628800*b^10*c^3
+ 6*(211*b^10*c^3 + 3*a^3*b^7*c^2*d)*n^7 + 18*(938*b^10*c^3 + 45*
a^3*b^7*c^2*d)*n^6 + 3*(46963*b^10*c^3 + 5010*a^3*b^7*c^2*d)*n^5
+ 18*(42287*b^10*c^3 + 8175*a^3*b^7*c^2*d - 120*a^6*b^4*c*d^2)*n^
4 + 4*(663941*b^10*c^3 + 200358*a^3*b^7*c^2*d - 14580*a^6*b^4*c*d
^2)*n^3 + 72*(79913*b^10*c^3 + 31965*a^3*b^7*c^2*d - 7260*a^6*b^4
*c*d^2)*n^2 + 1440*(4861*b^10*c^3 + 1890*a^3*b^7*c^2*d - 1080*a^6
*b^4*c*d^2 + 252*a^9*b*d^3)*n)*x)*(b*x + a)^n/(b^10*n^10 + 55*b^1
0*n^9 + 1320*b^10*n^8 + 18150*b^10*n^7 + 157773*b^10*n^6 + 902055
*b^10*n^5 + 3416930*b^10*n^4 + 8409500*b^10*n^3 + 12753576*b^10*n
^2 + 10628640*b^10*n + 3628800*b^10)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x+a)**n*(d*x**3+c)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.281307, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)^3*(b*x + a)^n,x, algorithm="giac")
```

```
[Out] Done
```

3.162 $\int \frac{(a+bx)^n (c+dx^3)^3}{x} \, dx$

Optimal. Leaf size=358

$$-\frac{5ad^2(3b^3c-14a^3d)(a+bx)^{n+5}}{b^9(n+5)}+\frac{d^2(3b^3c-56a^3d)(a+bx)^{n+6}}{b^9(n+6)}+\frac{28a^2d^3(a+bx)^{n+7}}{b^9(n+7)}-\frac{ad(8a^6d^2-15a^3b^3cd+6b^6c^2)(a+bx)^{n+2}}{b^9(n+2)}+\frac{d(28a^6d^2-30a^3b^3cd+3b^6c^2)(a+bx)^{n+3}}{b^9(n+3)}+\frac{2a^2d^2(15b^3c-28a^3d)(a+bx)^{n+4}}{b^9(n+4)}+\frac{a^2d(a^6d^2-3a^3b^3cd+3b^6c^2)(a+bx)^{n+1}}{b^9(n+1)}-\frac{8ad^3(a+bx)^{n+8}}{b^9(n+8)}+\frac{d^3(a+bx)^{n+9}}{b^9(n+9)}-\frac{c^3(a+bx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{bx}{a}+1\right)}{a(n+1)}$$

```
[Out] (a^2*d*(3*b^6*c^2 - 3*a^3*b^3*c*d + a^6*d^2)*(a + b*x)^(1 + n))/(
b^9*(1 + n)) - (a*d*(6*b^6*c^2 - 15*a^3*b^3*c*d + 8*a^6*d^2)*(a +
 b*x)^(2 + n))/(b^9*(2 + n)) + (d*(3*b^6*c^2 - 30*a^3*b^3*c*d + 2
8*a^6*d^2)*(a + b*x)^(3 + n))/(b^9*(3 + n)) + (2*a^2*d^2*(15*b^3*
c - 28*a^3*d)*(a + b*x)^(4 + n))/(b^9*(4 + n)) - (5*a*d^2*(3*b^3*
c - 14*a^3*d)*(a + b*x)^(5 + n))/(b^9*(5 + n)) + (d^2*(3*b^3*c -
56*a^3*d)*(a + b*x)^(6 + n))/(b^9*(6 + n)) + (28*a^2*d^3*(a + b*x
)^(7 + n))/(b^9*(7 + n)) - (8*a*d^3*(a + b*x)^(8 + n))/(b^9*(8 +
n)) + (d^3*(a + b*x)^(9 + n))/(b^9*(9 + n)) - (c^3*(a + b*x)^(1 +
 n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])/(a*(1 + n))
```

Rubi [A] time = 0.479962, antiderivative size = 358, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$-\frac{5ad^2(3b^3c-14a^3d)(a+bx)^{n+5}}{b^9(n+5)}+\frac{d^2(3b^3c-56a^3d)(a+bx)^{n+6}}{b^9(n+6)}+\frac{28a^2d^3(a+bx)^{n+7}}{b^9(n+7)}-\frac{ad(8a^6d^2-15a^3b^3cd+6b^6c^2)(a+bx)^{n+2}}{b^9(n+2)}+\frac{d(28a^6d^2-30a^3b^3cd+3b^6c^2)(a+bx)^{n+3}}{b^9(n+3)}+\frac{2a^2d^2(15b^3c-28a^3d)(a+bx)^{n+4}}{b^9(n+4)}+\frac{a^2d(a^6d^2-3a^3b^3cd+3b^6c^2)(a+bx)^{n+1}}{b^9(n+1)}-\frac{8ad^3(a+bx)^{n+8}}{b^9(n+8)}+\frac{d^3(a+bx)^{n+9}}{b^9(n+9)}-\frac{c^3(a+bx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{bx}{a}+1\right)}{a(n+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x)^n*(c + d*x^3)^3)/x,x]
```

```
[Out] (a^2*d*(3*b^6*c^2 - 3*a^3*b^3*c*d + a^6*d^2)*(a + b*x)^(1 + n))/(
b^9*(1 + n)) - (a*d*(6*b^6*c^2 - 15*a^3*b^3*c*d + 8*a^6*d^2)*(a +
```

```
b*x)^(2 + n))/(b^9*(2 + n)) + (d*(3*b^6*c^2 - 30*a^3*b^3*c*d + 2
8*a^6*d^2)*(a + b*x)^(3 + n))/(b^9*(3 + n)) + (2*a^2*d^2*(15*b^3*
c - 28*a^3*d)*(a + b*x)^(4 + n))/(b^9*(4 + n)) - (5*a*d^2*(3*b^3*
c - 14*a^3*d)*(a + b*x)^(5 + n))/(b^9*(5 + n)) + (d^2*(3*b^3*c -
56*a^3*d)*(a + b*x)^(6 + n))/(b^9*(6 + n)) + (28*a^2*d^3*(a + b*x
)^(7 + n))/(b^9*(7 + n)) - (8*a*d^3*(a + b*x)^(8 + n))/(b^9*(8 +
n)) + (d^3*(a + b*x)^(9 + n))/(b^9*(9 + n)) - (c^3*(a + b*x)^(1 +
 n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])/(a*(1 + n))
```

Rubi in Sympy [A] time = 100.157, size = 338, normalized size = 0.94

$$\frac{28a^2d^3(a+bx)^{n+7}}{b^9(n+7)} - \frac{2a^2d^2(a+bx)^{n+4}(28a^3d-15b^3c)}{b^9(n+4)} + \frac{a^2d(a+bx)^{n+1}(a^6d^2-3a^3b^3cd+3b^6c^2)}{b^9(n+1)} - \frac{8ad^3(a+bx)^{n+8}}{b^9(n+8)} + \frac{5ad^2(a+bx)^{n+5}(14a^3d-3b^3c)}{b^9(n+5)} - \frac{ad(a+bx)^{n+2}(8a^6d^2-15a^3b^3cd+6b^6c^2)}{b^9(n+2)} + \frac{d^3(a+bx)^{n+9}}{b^9(n+9)} - \frac{d^2(a+bx)^{n+6}(56a^3d-3b^3c)}{b^9(n+6)} + \frac{d(a+bx)^{n+3}(28a^6d^2-30a^3b^3cd+3b^6c^2)}{b^9(n+3)} - \frac{c^3(a+bx)^{n+1}\,{}_2F_1\left(\begin{matrix}1,n+1\\n+2\end{matrix}\middle|1+\frac{bx}{a}\right)}{a(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x+a)**n*(d*x**3+c)**3/x,x)
```

```
[Out] 28*a**2*d**3*(a + b*x)**(n + 7)/(b**9*(n + 7)) - 2*a**2*d**2*(a +
 b*x)**(n + 4)*(28*a**3*d - 15*b**3*c)/(b**9*(n + 4)) + a**2*d*(a
 + b*x)**(n + 1)*(a**6*d**2 - 3*a**3*b**3*c*d + 3*b**6*c**2)/(b**
9*(n + 1)) - 8*a*d**3*(a + b*x)**(n + 8)/(b**9*(n + 8)) + 5*a*d**
2*(a + b*x)**(n + 5)*(14*a**3*d - 3*b**3*c)/(b**9*(n + 5)) - a*d*
(a + b*x)**(n + 2)*(8*a**6*d**2 - 15*a**3*b**3*c*d + 6*b**6*c**2)
/(b**9*(n + 2)) + d**3*(a + b*x)**(n + 9)/(b**9*(n + 9)) - d**2*(
a + b*x)**(n + 6)*(56*a**3*d - 3*b**3*c)/(b**9*(n + 6)) + d*(a +
b*x)**(n + 3)*(28*a**6*d**2 - 30*a**3*b**3*c*d + 3*b**6*c**2)/(b*
*9*(n + 3)) - c**3*(a + b*x)**(n + 1)*hyper((1, n + 1), (n + 2,),
 1 + b*x/a)/(a*(n + 1))
```

Mathematica [B] time = 1.84004, size = 856, normalized size = 2.39

$$(a+bx)^n\left(\frac{c^3\,{}_2F_1\left(-n,-n;1-n;-\frac{a}{bx}\right)\left(\frac{a}{bx}+1\right)^{-n}}{n}+\frac{3c^2d\left(\frac{bx}{a}+1\right)^{-n}\left(b^3\left(n^2+3n+2\right)x^3\left(\frac{bx}{a}+1\right)^n+ab^2n(n+1)x^2\left(\frac{bx}{a}+1\right)^n-2a^2bnx\left(\frac{bx}{a}+1\right)^n+2a^3\left(\left(\frac{bx}{a}+1\right)^n-1\right)\right)}{b^3(n+1)(n+2)(n+3)}+\frac{3cd^2\left(\frac{bx}{a}+1\right)^{-n}\left(b^6\left(n^5+15n^4+85n^3+225n^2+274n+120\right)x^6\left(\frac{bx}{a}+1\right)^n+ab^5n\left(n^4+10n^3+35n^2+50n+24\right)x^5\left(\frac{bx}{a}+\right.\right.}{}+\frac{d^3\left(\frac{bx}{a}+1\right)^{-n}\left(b^9\left(n^8+36n^7+546n^6+4536n^5+22449n^4+67284n^3+118124n^2+109584n+40320\right)x^9\left(\frac{bx}{a}+1\right)^n+ab^8n\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x)^n*(c + d*x^3)^3)/x,x]
```

```
[Out] (a + b*x)^n*((3*c^2*d*(-2*a^2*b*n*x*(1 + (b*x)/a)^n + a*b^2*n*(1
+ n)*x^2*(1 + (b*x)/a)^n + b^3*(2 + 3*n + n^2)*x^3*(1 + (b*x)/a)^
n + 2*a^3*(-1 + (1 + (b*x)/a)^n)))/(b^3*(1 + n)*(2 + n)*(3 + n)*(
1 + (b*x)/a)^n) + (3*c*d^2*(120*a^5*b*n*x*(1 + (b*x)/a)^n - 60*a^
4*b^2*n*(1 + n)*x^2*(1 + (b*x)/a)^n + 20*a^3*b^3*n*(2 + 3*n + n^2
)*x^3*(1 + (b*x)/a)^n - 5*a^2*b^4*n*(6 + 11*n + 6*n^2 + n^3)*x^4*
(1 + (b*x)/a)^n + a*b^5*n*(24 + 50*n + 35*n^2 + 10*n^3 + n^4)*x^5
*(1 + (b*x)/a)^n + b^6*(120 + 274*n + 225*n^2 + 85*n^3 + 15*n^4 +
 n^5)*x^6*(1 + (b*x)/a)^n - 120*a^6*(-1 + (1 + (b*x)/a)^n)))/(b^6
*(1 + n)*(2 + n)*(3 + n)*(4 + n)*(5 + n)*(6 + n)*(1 + (b*x)/a)^n)
 + (d^3*(-40320*a^8*b*n*x*(1 + (b*x)/a)^n + 20160*a^7*b^2*n*(1 +
n)*x^2*(1 + (b*x)/a)^n - 6720*a^6*b^3*n*(2 + 3*n + n^2)*x^3*(1 +
(b*x)/a)^n + 1680*a^5*b^4*n*(6 + 11*n + 6*n^2 + n^3)*x^4*(1 + (b*
x)/a)^n - 336*a^4*b^5*n*(24 + 50*n + 35*n^2 + 10*n^3 + n^4)*x^5*(
1 + (b*x)/a)^n + 56*a^3*b^6*n*(120 + 274*n + 225*n^2 + 85*n^3 + 1
5*n^4 + n^5)*x^6*(1 + (b*x)/a)^n - 8*a^2*b^7*n*(720 + 1764*n + 16
24*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6)*x^7*(1 + (b*x)/a)^n +
a*b^8*n*(5040 + 13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n
^5 + 28*n^6 + n^7)*x^8*(1 + (b*x)/a)^n + b^9*(40320 + 109584*n +
118124*n^2 + 67284*n^3 + 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7
+ n^8)*x^9*(1 + (b*x)/a)^n + 40320*a^9*(-1 + (1 + (b*x)/a)^n)))/(
b^9*(1 + n)*(2 + n)*(3 + n)*(4 + n)*(5 + n)*(6 + n)*(7 + n)*(8 +
n)*(9 + n)*(1 + (b*x)/a)^n) + (c^3*Hypergeometric2F1[-n, -n, 1 -
n, -(a/(b*x))])/(n*(1 + a/(b*x))^n))
```

Maple [F] time = 0.05, size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n\left(dx^3+c\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x+a)^n*(d*x^3+c)^3/x,x)
```

```
[Out] int((b*x+a)^n*(d*x^3+c)^3/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^3+c)^3(bx+a)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)^3*(b*x + a)^n/x,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^3*(b*x + a)^n/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(d^3x^9+3\,cd^2x^6+3\,c^2dx^3+c^3)(bx+a)^n}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)^3*(b*x + a)^n/x,x, algorithm="fricas")
```

```
[Out] integral((d^3*x^9 + 3*c*d^2*x^6 + 3*c^2*d*x^3 + c^3)*(b*x + a)^n/
x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x+a)**n*(d*x**3+c)**3/x,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^3+c)^3(bx+a)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)^3*(b*x + a)^n/x,x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^3*(b*x + a)^n/x, x)
```

3.163 $\int \frac{x^5(e+fx)^n}{a+bx^3}\,dx$

Optimal. Leaf size=324

$$\frac{a(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b^{5/3}(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)}+\frac{a(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f}\right)}{3b^{5/3}(n+1)\left(\sqrt[3]{-1}\sqrt[3]{a}f+\sqrt[3]{b}e\right)}+\frac{a(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-(-1)^{2/3}\sqrt[3]{a}f}\right)}{3b^{5/3}(n+1)\left(\sqrt[3]{b}e-(-1)^{2/3}\sqrt[3]{a}f\right)}+\frac{e^2(e+fx)^{n+1}}{bf^3(n+1)}-\frac{2e(e+fx)^{n+2}}{bf^3(n+2)}+\frac{(e+fx)^{n+3}}{bf^3(n+3)}$$

```
[Out] (e^2*(e + f*x)^(1 + n))/(b*f^3*(1 + n)) - (2*e*(e + f*x)^(2 + n))
/(b*f^3*(2 + n)) + (e + f*x)^(3 + n)/(b*f^3*(3 + n)) + (a*(e + f*
x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))
/(b^(1/3)*e - a^(1/3)*f)])/(3*b^(5/3)*(b^(1/3)*e - a^(1/3)*f)*(1
+ n)) + (a*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (
b^(1/3)*(e + f*x))/(b^(1/3)*e + (-1)^(1/3)*a^(1/3)*f)])/(3*b^(5/3
)*(b^(1/3)*e + (-1)^(1/3)*a^(1/3)*f)*(1 + n)) + (a*(e + f*x)^(1 +
 n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/(b^(1/
3)*e - (-1)^(2/3)*a^(1/3)*f)])/(3*b^(5/3)*(b^(1/3)*e - (-1)^(2/3)
*a^(1/3)*f)*(1 + n))
```

Rubi [A] time = 1.47782, antiderivative size = 324, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$\frac{a(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b^{5/3}(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)}+\frac{a(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f}\right)}{3b^{5/3}(n+1)\left(\sqrt[3]{-1}\sqrt[3]{a}f+\sqrt[3]{b}e\right)}+\frac{a(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-(-1)^{2/3}\sqrt[3]{a}f}\right)}{3b^{5/3}(n+1)\left(\sqrt[3]{b}e-(-1)^{2/3}\sqrt[3]{a}f\right)}+\frac{e^2(e+fx)^{n+1}}{bf^3(n+1)}-\frac{2e(e+fx)^{n+2}}{bf^3(n+2)}+\frac{(e+fx)^{n+3}}{bf^3(n+3)}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^5*(e + f*x)^n)/(a + b*x^3),x]
```


```
[Out] (e^2*(e + f*x)^(1 + n))/(b*f^3*(1 + n)) - (2*e*(e + f*x)^(2 + n))
/(b*f^3*(2 + n)) + (e + f*x)^(3 + n)/(b*f^3*(3 + n)) + (a*(e + f*
x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))
/(b^(1/3)*e - a^(1/3)*f)])/(3*b^(5/3)*(b^(1/3)*e - a^(1/3)*f)*(1
+ n)) + (a*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (
```

```
b^(1/3)*(e + f*x))/(b^(1/3)*e + (-1)^(1/3)*a^(1/3)*f)])/(3*b^(5/3
)*(b^(1/3)*e + (-1)^(1/3)*a^(1/3)*f)*(1 + n)) + (a*(e + f*x)^(1 +
 n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/(b^(1/
3)*e - (-1)^(2/3)*a^(1/3)*f)])/(3*b^(5/3)*(b^(1/3)*e - (-1)^(2/3)
*a^(1/3)*f)*(1 + n))
```

Rubi in Sympy [A] time = 143.417, size = 272, normalized size = 0.84

$$\frac{a(e+fx)^{n+1}\,{}_2F_1\left(\begin{matrix}1,n+1\\n+2\end{matrix}\middle|\frac{\sqrt[3]{b}(e+fx)}{-(-1)^{\frac{2}{3}}\sqrt[3]{a}f+\sqrt[3]{b}e}\right)}{3b^{\frac{5}{3}}(n+1)\left(-(-1)^{\frac{2}{3}}\sqrt[3]{a}f+\sqrt[3]{b}e\right)}+\frac{a(e+fx)^{n+1}\,{}_2F_1\left(\begin{matrix}1,n+1\\n+2\end{matrix}\middle|\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{-1}\sqrt[3]{a}f+\sqrt[3]{b}e}\right)}{3b^{\frac{5}{3}}(n+1)\left(\sqrt[3]{-1}\sqrt[3]{a}f+\sqrt[3]{b}e\right)}$$
$$-\frac{a(e+fx)^{n+1}\,{}_2F_1\left(\begin{matrix}1,n+1\\n+2\end{matrix}\middle|\frac{\sqrt[3]{b}(e+fx)}{-\sqrt[3]{a}f+\sqrt[3]{b}e}\right)}{3b^{\frac{5}{3}}(n+1)\left(\sqrt[3]{a}f-\sqrt[3]{b}e\right)}+\frac{e^2(e+fx)^{n+1}}{bf^3(n+1)}-\frac{2e(e+fx)^{n+2}}{bf^3(n+2)}+\frac{(e+fx)^{n+3}}{bf^3(n+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*(f*x+e)**n/(b*x**3+a),x)
```

```
[Out] a*(e + f*x)**(n + 1)*hyper((1, n + 1), (n + 2,), b**(1/3)*(e + f*
x)/(-(-1)**(2/3)*a**(1/3)*f + b**(1/3)*e))/(3*b**(5/3)*(n + 1)*(-
(-1)**(2/3)*a**(1/3)*f + b**(1/3)*e)) + a*(e + f*x)**(n + 1)*hype
r((1, n + 1), (n + 2,), b**(1/3)*(e + f*x)/((-1)**(1/3)*a**(1/3)*
f + b**(1/3)*e))/(3*b**(5/3)*(n + 1)*((-1)**(1/3)*a**(1/3)*f + b*
*(1/3)*e)) - a*(e + f*x)**(n + 1)*hyper((1, n + 1), (n + 2,), b**
(1/3)*(e + f*x)/(-a**(1/3)*f + b**(1/3)*e))/(3*b**(5/3)*(n + 1)*(
a**(1/3)*f - b**(1/3)*e)) + e**2*(e + f*x)**(n + 1)/(b*f**3*(n +
1)) - 2*e*(e + f*x)**(n + 2)/(b*f**3*(n + 2)) + (e + f*x)**(n + 3
)/(b*f**3*(n + 3))
```

Mathematica [C] time = 0.590475, size = 423, normalized size = 1.31

$$(e+fx)^n\left(\frac{3\left(e^3\left(2-2\left(\frac{fx}{e}+1\right)^{-n}\right)-2e^2fnx+ef^2n(n+1)x^2+f^3\left(n^2+3n+2\right)x^3\right)}{n^3+6n^2+11n+6}-\frac{af^3\left(e^2\text{RootSum}\left[-\#1^3b+3\#1^2be-3\#1be^2-af^3+be^3\&,\frac{\left(\frac{e+fx}{-\#1+e+fx}\right)^{-n}\,{}_2\ldots}{\#1\ldots}\right.\right.}{\ldots}\right.$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(x^5*(e + f*x)^n)/(a + b*x^3),x]
```

```
[Out] ((e + f*x)^n*((3*(-2*e^2*f*n*x + e*f^2*n*(1 + n)*x^2 + f^3*(2 + 3
*n + n^2)*x^3 + e^3*(2 - 2/(1 + (f*x)/e)^n)))/(6 + 11*n + 6*n^2 +
 n^3) - (a*f^3*(e^2*RootSum[b*e^3 - a*f^3 - 3*b*e^2*#1 + 3*b*e*#1
^2 - b*#1^3 & , Hypergeometric2F1[-n, -n, 1 - n, -(#1/(e + f*x -
#1))]/(((e + f*x)/(e + f*x - #1))^n*(e^2 - 2*e*#1 + #1^2)) & ] -
2*e*RootSum[b*e^3 - a*f^3 - 3*b*e^2*#1 + 3*b*e*#1^2 - b*#1^3 & ,
(Hypergeometric2F1[-n, -n, 1 - n, -(#1/(e + f*x - #1))]*#1)/(((e
+ f*x)/(e + f*x - #1))^n*(e^2 - 2*e*#1 + #1^2)) & ] + RootSum[b*e
^3 - a*f^3 - 3*b*e^2*#1 + 3*b*e*#1^2 - b*#1^3 & , (Hypergeometric
2F1[-n, -n, 1 - n, -(#1/(e + f*x - #1))]*#1^2)/(((e + f*x)/(e + f
*x - #1))^n*(e^2 - 2*e*#1 + #1^2)) & ]))/(b*n)))/(3*b*f^3)
```

Maple [F] time = 0.093, size = 0, normalized size = 0.

$$\int \frac{x^5 (fx+e)^n}{bx^3+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(f*x+e)^n/(b*x^3+a),x)
```

```
[Out] int(x^5*(f*x+e)^n/(b*x^3+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^n x^5}{bx^3+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x + e)^n*x^5/(b*x^3 + a),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^n*x^5/(b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(fx+e)^n x^5}{bx^3+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x + e)^n*x^5/(b*x^3 + a),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n*x^5/(b*x^3 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*(f*x+e)**n/(b*x**3+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^n x^5}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x + e)^n*x^5/(b*x^3 + a),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^n*x^5/(b*x^3 + a), x)
```

3.164 $\int \frac{x^4(e+fx)^n}{a+bx^3}\,dx$

Optimal. Leaf size=332

$$-\frac{a^{2/3}(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b^{4/3}(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)}+\frac{\sqrt[3]{-1}a^{2/3}(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b^{4/3}(n+1)\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)}+\frac{(-1)^{2/3}a^{2/3}(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{-1}\sqrt[3]{b}(e+fx)}{\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f}\right)}{3b^{4/3}(n+1)\left(\sqrt[3]{a}f+\sqrt[3]{-1}\sqrt[3]{b}e\right)}-\frac{e(e+fx)^{n+1}}{bf^2(n+1)}+\frac{(e+fx)^{n+2}}{bf^2(n+2)}$$

```
[Out] -((e*(e + f*x)^(1 + n))/(b*f^2*(1 + n))) + (e + f*x)^(2 + n)/(b*f
^2*(2 + n)) - (a^(2/3)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 +
 n, 2 + n, (b^(1/3)*(e + f*x))/(b^(1/3)*e - a^(1/3)*f)])/(3*b^(4/
3)*(b^(1/3)*e - a^(1/3)*f)*(1 + n)) + ((-1)^(1/3)*a^(2/3)*(e + f*
x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(2/3)*b^(1/3)
*(e + f*x))/((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)])/(3*b^(4/3)*((-1)
^(2/3)*b^(1/3)*e - a^(1/3)*f)*(1 + n)) + ((-1)^(2/3)*a^(2/3)*(e +
 f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(1/3)*b^(1
/3)*(e + f*x))/((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)])/(3*b^(4/3)*((
-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)*(1 + n))
```

Rubi [A] time = 1.60278, antiderivative size = 332, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$-\frac{a^{2/3}(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b^{4/3}(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)}+\frac{\sqrt[3]{-1}a^{2/3}(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b^{4/3}(n+1)\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)}+\frac{(-1)^{2/3}a^{2/3}(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{-1}\sqrt[3]{b}(e+fx)}{\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f}\right)}{3b^{4/3}(n+1)\left(\sqrt[3]{a}f+\sqrt[3]{-1}\sqrt[3]{b}e\right)}-\frac{e(e+fx)^{n+1}}{bf^2(n+1)}+\frac{(e+fx)^{n+2}}{bf^2(n+2)}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(e + f*x)^n)/(a + b*x^3),x]
```

```
[Out] -((e*(e + f*x)^(1 + n))/(b*f^2*(1 + n))) + (e + f*x)^(2 + n)/(b*f
^2*(2 + n)) - (a^(2/3)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 +
 n, 2 + n, (b^(1/3)*(e + f*x))/(b^(1/3)*e - a^(1/3)*f)])/(3*b^(4/
3)*(b^(1/3)*e - a^(1/3)*f)*(1 + n)) + ((-1)^(1/3)*a^(2/3)*(e + f*
x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(2/3)*b^(1/3)
*(e + f*x))/((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)])/(3*b^(4/3)*((-1)
^(2/3)*b^(1/3)*e - a^(1/3)*f)*(1 + n)) + ((-1)^(2/3)*a^(2/3)*(e +
 f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(1/3)*b^(1
/3)*(e + f*x))/((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)])/(3*b^(4/3)*((
-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)*(1 + n))
```

Rubi in Sympy [A] time = 167.999, size = 287, normalized size = 0.86

$$-\frac{\sqrt[3]{-1}a^{\frac{2}{3}}\left(e+fx\right)^{n+1}{}_2F_1\left(\begin{matrix}1,n+1\\n+2\end{matrix}\middle|\frac{(-1)^{\frac{2}{3}}\sqrt[3]{b}(e+fx)}{-\sqrt[3]{a}f+(-1)^{\frac{2}{3}}\sqrt[3]{b}e}\right)}{3b^{\frac{4}{3}}\left(n+1\right)\left(\sqrt[3]{a}f-(-1)^{\frac{2}{3}}\sqrt[3]{b}e\right)}+\frac{(-1)^{\frac{2}{3}}a^{\frac{2}{3}}\left(e+fx\right)^{n+1}{}_2F_1\left(\begin{matrix}1,n+1\\n+2\end{matrix}\middle|\frac{\sqrt[3]{-1}\sqrt[3]{b}(e+fx)}{\sqrt[3]{a}f+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{3b^{\frac{4}{3}}\left(n+1\right)\left(\sqrt[3]{a}f+\sqrt[3]{-1}\sqrt[3]{b}e\right)}+\frac{a^{\frac{2}{3}}\left(e+fx\right)^{n+1}{}_2F_1\left(\begin{matrix}1,n+1\\n+2\end{matrix}\middle|\frac{\sqrt[3]{b}(e+fx)}{-\sqrt[3]{a}f+\sqrt[3]{b}e}\right)}{3b^{\frac{4}{3}}\left(n+1\right)\left(\sqrt[3]{a}f-\sqrt[3]{b}e\right)}-\frac{e\left(e+fx\right)^{n+1}}{bf^2\left(n+1\right)}+\frac{\left(e+fx\right)^{n+2}}{bf^2\left(n+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(f*x+e)**n/(b*x**3+a),x)
```

```
[Out] -(-1)**(1/3)*a**(2/3)*(e + f*x)**(n + 1)*hyper((1, n + 1), (n + 2
,), (-1)**(2/3)*b**(1/3)*(e + f*x)/(-a**(1/3)*f + (-1)**(2/3)*b**
(1/3)*e))/(3*b**(4/3)*(n + 1)*(a**(1/3)*f - (-1)**(2/3)*b**(1/3)*
e)) + (-1)**(2/3)*a**(2/3)*(e + f*x)**(n + 1)*hyper((1, n + 1), (
n + 2,), (-1)**(1/3)*b**(1/3)*(e + f*x)/(a**(1/3)*f + (-1)**(1/3)
*b**(1/3)*e))/(3*b**(4/3)*(n + 1)*(a**(1/3)*f + (-1)**(1/3)*b**(1
/3)*e)) + a**(2/3)*(e + f*x)**(n + 1)*hyper((1, n + 1), (n + 2,),
 b**(1/3)*(e + f*x)/(-a**(1/3)*f + b**(1/3)*e))/(3*b**(4/3)*(n +
1)*(a**(1/3)*f - b**(1/3)*e)) - e*(e + f*x)**(n + 1)/(b*f**2*(n +
 1)) + (e + f*x)**(n + 2)/(b*f**2*(n + 2))
```

Mathematica [C] time = 0.691891, size = 298, normalized size = 0.9

$$\frac{(e+fx)^n\left(\frac{aef^3\text{RootSum}\left[-\#1^3b+3\#1^2be-3\#1be^2-af^3+be^3\&,\frac{\left(\frac{e+fx}{-\#1+e+fx}\right)^{-n}{}_2F_1\left(-n,-n;1-n;-\frac{\#1}{e+fx-\#1}\right)}{\#1^2-2\#1e+e^2}\&\right]}{bn}-\frac{af^3\text{RootSum}\left[-\#1^3b+3\#1^2be-3\#\right.}{}\right.}{3bf^2}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(x^4*(e + f*x)^n)/(a + b*x^3),x]
```

```
[Out] ((e + f*x)^n*((-3*(-(e*f*n*x) - f^2*(1 + n)*x^2 + e^2*(1 - (1 + (
f*x)/e)^(-n))))/(2 + 3*n + n^2) + (a*e*f^3*RootSum[b*e^3 - a*f^3
- 3*b*e^2*#1 + 3*b*e*#1^2 - b*#1^3 & , Hypergeometric2F1[-n, -n,
1 - n, -(#1/(e + f*x - #1))]/(((e + f*x)/(e + f*x - #1))^n*(e^2 -
 2*e*#1 + #1^2)) & ])/(b*n) - (a*f^3*RootSum[b*e^3 - a*f^3 - 3*b*
e^2*#1 + 3*b*e*#1^2 - b*#1^3 & , (Hypergeometric2F1[-n, -n, 1 - n
, -(#1/(e + f*x - #1))]*#1)/(((e + f*x)/(e + f*x - #1))^n*(e^2 -
2*e*#1 + #1^2)) & ])/(b*n)))/(3*b*f^2)
```

Maple [F] time = 0.083, size = 0, normalized size = 0.

$$\int \frac{x^4 (fx+e)^n}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(f*x+e)^n/(b*x^3+a),x)
```

```
[Out] int(x^4*(f*x+e)^n/(b*x^3+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^n x^4}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x + e)^n*x^4/(b*x^3 + a),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^n*x^4/(b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(fx+e)^n x^4}{bx^3+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x + e)^n*x^4/(b*x^3 + a),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n*x^4/(b*x^3 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(f*x+e)**n/(b*x**3+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^n x^4}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x + e)^n*x^4/(b*x^3 + a),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^n*x^4/(b*x^3 + a), x)
```

3.165 $\int \frac{x^3(e+fx)^n}{a+bx^3}\,dx$

Optimal. Leaf size=293

$$\frac{\sqrt[3]{a}(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)}+\frac{\sqrt[3]{a}(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b(n+1)\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)}-\frac{\sqrt[3]{a}(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{-1}\sqrt[3]{b}(e+fx)}{\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f}\right)}{3b(n+1)\left(\sqrt[3]{a}f+\sqrt[3]{-1}\sqrt[3]{b}e\right)}+\frac{(e+fx)^{n+1}}{bf(n+1)}$$

```
[Out] (e + f*x)^(1 + n)/(b*f*(1 + n)) + (a^(1/3)*(e + f*x)^(1 + n)*Hype
rgeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/(b^(1/3)*e - a
^(1/3)*f)])/(3*b*(b^(1/3)*e - a^(1/3)*f)*(1 + n)) + (a^(1/3)*(e +
 f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(2/3)*b^(1
/3)*(e + f*x))/((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)])/(3*b*((-1)^(2
/3)*b^(1/3)*e - a^(1/3)*f)*(1 + n)) - (a^(1/3)*(e + f*x)^(1 + n)*
Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(1/3)*b^(1/3)*(e + f*x))
/((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)])/(3*b*((-1)^(1/3)*b^(1/3)*e
+ a^(1/3)*f)*(1 + n))
```

Rubi [A] time = 1.1952, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{\sqrt[3]{a}(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)}+\frac{\sqrt[3]{a}(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b(n+1)\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)}-\frac{\sqrt[3]{a}(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{-1}\sqrt[3]{b}(e+fx)}{\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f}\right)}{3b(n+1)\left(\sqrt[3]{a}f+\sqrt[3]{-1}\sqrt[3]{b}e\right)}+\frac{(e+fx)^{n+1}}{bf(n+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(e + f*x)^n)/(a + b*x^3),x]
```

```
[Out] (e + f*x)^(1 + n)/(b*f*(1 + n)) + (a^(1/3)*(e + f*x)^(1 + n)*Hype
rgeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/(b^(1/3)*e - a
^(1/3)*f)])/(3*b*(b^(1/3)*e - a^(1/3)*f)*(1 + n)) + (a^(1/3)*(e +
 f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(2/3)*b^(1
/3)*(e + f*x))/((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)])/(3*b*((-1)^(2
/3)*b^(1/3)*e - a^(1/3)*f)*(1 + n)) - (a^(1/3)*(e + f*x)^(1 + n)*
```

```
Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(1/3)*b^(1/3)*(e + f*x))
/((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)])/(3*b*((-1)^(1/3)*b^(1/3)*e
+ a^(1/3)*f)*(1 + n))
```

Rubi in Sympy [A] time = 120.717, size = 243, normalized size = 0.83

$$-\frac{\sqrt[3]{a}\,(e+fx)^{n+1}\,{}_2F_1\left(\begin{matrix}1,n+1\\ n+2\end{matrix}\middle|\frac{(-1)^{\frac{2}{3}}\sqrt[3]{b}(e+fx)}{-\sqrt[3]{a}f+(-1)^{\frac{2}{3}}\sqrt[3]{b}e}\right)}{3b\,(n+1)\left(\sqrt[3]{a}f-(-1)^{\frac{2}{3}}\sqrt[3]{b}e\right)}-\frac{\sqrt[3]{a}\,(e+fx)^{n+1}\,{}_2F_1\left(\begin{matrix}1,n+1\\ n+2\end{matrix}\middle|\frac{\sqrt[3]{-1}\sqrt[3]{b}(e+fx)}{\sqrt[3]{a}f+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{3b\,(n+1)\left(\sqrt[3]{a}f+\sqrt[3]{-1}\sqrt[3]{b}e\right)}-\frac{\sqrt[3]{a}\,(e+fx)^{n+1}\,{}_2F_1\left(\begin{matrix}1,n+1\\ n+2\end{matrix}\middle|\frac{\sqrt[3]{b}(e+fx)}{-\sqrt[3]{a}f+\sqrt[3]{b}e}\right)}{3b\,(n+1)\left(\sqrt[3]{a}f-\sqrt[3]{b}e\right)}+\frac{(e+fx)^{n+1}}{bf\,(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(f*x+e)**n/(b*x**3+a),x)
```

```
[Out] -a**(1/3)*(e + f*x)**(n + 1)*hyper((1, n + 1), (n + 2,), (-1)**(2
/3)*b**(1/3)*(e + f*x)/(-a**(1/3)*f + (-1)**(2/3)*b**(1/3)*e))/(3
*b*(n + 1)*(a**(1/3)*f - (-1)**(2/3)*b**(1/3)*e)) - a**(1/3)*(e +
 f*x)**(n + 1)*hyper((1, n + 1), (n + 2,), (-1)**(1/3)*b**(1/3)*(
e + f*x)/(a**(1/3)*f + (-1)**(1/3)*b**(1/3)*e))/(3*b*(n + 1)*(a**
(1/3)*f + (-1)**(1/3)*b**(1/3)*e)) - a**(1/3)*(e + f*x)**(n + 1)*
hyper((1, n + 1), (n + 2,), b**(1/3)*(e + f*x)/(-a**(1/3)*f + b**
(1/3)*e))/(3*b*(n + 1)*(a**(1/3)*f - b**(1/3)*e)) + (e + f*x)**(n
 + 1)/(b*f*(n + 1))
```

Mathematica [C] time = 0.141482, size = 142, normalized size = 0.48

$$\frac{(e+fx)^n\left(\frac{3b(e+fx)}{n+1}-\frac{af^3\text{RootSum}\left[-\#1^3b+3\#1^2be-3\#1be^2-af^3+be^3\&,\frac{\left(\frac{e+fx}{-\#1+e+fx}\right)^{-n}{}_2F_1\left(-n,-n;1-n;-\frac{\#1}{e+fx-\#1}\right)}{\#1^2-2\#1e+e^2}\&\right]}{n}\right)}{3b^2f}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(x^3*(e + f*x)^n)/(a + b*x^3),x]
```

```
[Out] ((e + f*x)^n*((3*b*(e + f*x))/(1 + n) - (a*f^3*RootSum[b*e^3 - a*
f^3 - 3*b*e^2*#1 + 3*b*e*#1^2 - b*#1^3 & , Hypergeometric2F1[-n,
-n, 1 - n, -(#1/(e + f*x - #1))]/(((e + f*x)/(e + f*x - #1))^n*(e
^2 - 2*e*#1 + #1^2)) & ])/n))/(3*b^2*f)
```

Maple [F] time = 0.075, size = 0, normalized size = 0.

$$\int \frac{x^3 (fx+e)^n}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(f*x+e)^n/(b*x^3+a),x)
```

```
[Out] int(x^3*(f*x+e)^n/(b*x^3+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^n x^3}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x + e)^n*x^3/(b*x^3 + a),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^n*x^3/(b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(fx+e)^n x^3}{bx^3+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x + e)^n*x^3/(b*x^3 + a),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n*x^3/(b*x^3 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(f*x+e)**n/(b*x**3+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^n x^3}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x + e)^n*x^3/(b*x^3 + a),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^n*x^3/(b*x^3 + a), x)
```

3.166 $\int \frac{x^2(e+fx)^n}{a+bx^3}\,dx$

Optimal. Leaf size=253

$$-\frac{(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b^{2/3}(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)}-\frac{(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f}\right)}{3b^{2/3}(n+1)\left(\sqrt[3]{-1}\sqrt[3]{a}f+\sqrt[3]{b}e\right)}-\frac{(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-(-1)^{2/3}\sqrt[3]{a}f}\right)}{3b^{2/3}(n+1)\left(\sqrt[3]{b}e-(-1)^{2/3}\sqrt[3]{a}f\right)}$$

```
[Out] -((e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(
e + f*x))/(b^(1/3)*e - a^(1/3)*f)])/(3*b^(2/3)*(b^(1/3)*e - a^(1/
3)*f)*(1 + n)) - ((e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2
 + n, (b^(1/3)*(e + f*x))/(b^(1/3)*e + (-1)^(1/3)*a^(1/3)*f)])/(3
*b^(2/3)*(b^(1/3)*e + (-1)^(1/3)*a^(1/3)*f)*(1 + n)) - ((e + f*x)
^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/(
b^(1/3)*e - (-1)^(2/3)*a^(1/3)*f)])/(3*b^(2/3)*(b^(1/3)*e - (-1)^
(2/3)*a^(1/3)*f)*(1 + n))
```

Rubi [A] time = 0.689617, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$-\frac{(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b^{2/3}(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)}-\frac{(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f}\right)}{3b^{2/3}(n+1)\left(\sqrt[3]{-1}\sqrt[3]{a}f+\sqrt[3]{b}e\right)}-\frac{(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-(-1)^{2/3}\sqrt[3]{a}f}\right)}{3b^{2/3}(n+1)\left(\sqrt[3]{b}e-(-1)^{2/3}\sqrt[3]{a}f\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(e + f*x)^n)/(a + b*x^3),x]
```

```
[Out] -((e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(
e + f*x))/(b^(1/3)*e - a^(1/3)*f)])/(3*b^(2/3)*(b^(1/3)*e - a^(1/
3)*f)*(1 + n)) - ((e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2
 + n, (b^(1/3)*(e + f*x))/(b^(1/3)*e + (-1)^(1/3)*a^(1/3)*f)])/(3
*b^(2/3)*(b^(1/3)*e + (-1)^(1/3)*a^(1/3)*f)*(1 + n)) - ((e + f*x)
^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/(
b^(1/3)*e - (-1)^(2/3)*a^(1/3)*f)])/(3*b^(2/3)*(b^(1/3)*e - (-1)^
```

```
(2/3)*a^(1/3)*f)*(1 + n))
```

Rubi in Sympy [A] time = 73.0247, size = 209, normalized size = 0.83

$$-\frac{(e+fx)^{n+1}\,{}_2F_1\left(\begin{matrix}1,n+1\\n+2\end{matrix}\middle|\frac{\sqrt[3]{b}(e+fx)}{-(-1)^{\frac{2}{3}}\sqrt[3]{a}f+\sqrt[3]{b}e}\right)}{3b^{\frac{2}{3}}(n+1)\left(-(-1)^{\frac{2}{3}}\sqrt[3]{a}f+\sqrt[3]{b}e\right)}-\frac{(e+fx)^{n+1}\,{}_2F_1\left(\begin{matrix}1,n+1\\n+2\end{matrix}\middle|\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{-1}\sqrt[3]{a}f+\sqrt[3]{b}e}\right)}{3b^{\frac{2}{3}}(n+1)\left(\sqrt[3]{-1}\sqrt[3]{a}f+\sqrt[3]{b}e\right)}+\frac{(e+fx)^{n+1}\,{}_2F_1\left(\begin{matrix}1,n+1\\n+2\end{matrix}\middle|\frac{\sqrt[3]{b}(e+fx)}{-\sqrt[3]{a}f+\sqrt[3]{b}e}\right)}{3b^{\frac{2}{3}}(n+1)\left(\sqrt[3]{a}f-\sqrt[3]{b}e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(f*x+e)**n/(b*x**3+a),x)
```

```
[Out] -(e + f*x)**(n + 1)*hyper((1, n + 1), (n + 2,), b**(1/3)*(e + f*x
)/(-(-1)**(2/3)*a**(1/3)*f + b**(1/3)*e))/(3*b**(2/3)*(n + 1)*(-(
-1)**(2/3)*a**(1/3)*f + b**(1/3)*e)) - (e + f*x)**(n + 1)*hyper((
1, n + 1), (n + 2,), b**(1/3)*(e + f*x)/((-1)**(1/3)*a**(1/3)*f +
 b**(1/3)*e))/(3*b**(2/3)*(n + 1)*((-1)**(1/3)*a**(1/3)*f + b**(1
/3)*e)) + (e + f*x)**(n + 1)*hyper((1, n + 1), (n + 2,), b**(1/3)
*(e + f*x)/(-a**(1/3)*f + b**(1/3)*e))/(3*b**(2/3)*(n + 1)*(a**(1
/3)*f - b**(1/3)*e))
```

Mathematica [C] time = 0.122362, size = 337, normalized size = 1.33

$$\frac{(e+fx)^n\left(e^2\text{RootSum}\left[-\#1^3b+3\#1^2be-3\#1be^2-af^3+be^3\&,\frac{\left(\frac{e+fx}{-\#1+e+fx}\right)^{-n}{}_2F_1\left(-n,-n;1-n;-\frac{\#1}{e+fx-\#1}\right)}{\#1^2-2\#1e+e^2}\&\right]-2e\text{RootSum}\left[-\#1\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(x^2*(e + f*x)^n)/(a + b*x^3),x]
```

```
[Out] ((e + f*x)^n*(e^2*RootSum[b*e^3 - a*f^3 - 3*b*e^2*#1 + 3*b*e*#1^2
 - b*#1^3 & , Hypergeometric2F1[-n, -n, 1 - n, -(#1/(e + f*x - #1
))]/(((e + f*x)/(e + f*x - #1))^n*(e^2 - 2*e*#1 + #1^2)) & ] - 2*
e*RootSum[b*e^3 - a*f^3 - 3*b*e^2*#1 + 3*b*e*#1^2 - b*#1^3 & , (H
ypergeometric2F1[-n, -n, 1 - n, -(#1/(e + f*x - #1))]*#1)/(((e +
f*x)/(e + f*x - #1))^n*(e^2 - 2*e*#1 + #1^2)) & ] + RootSum[b*e^3
```

```
 - a*f^3 - 3*b*e^2*#1 + 3*b*e*#1^2 - b*#1^3 & , (Hypergeometric2F
1[-n, -n, 1 - n, -(#1/(e + f*x - #1))]*#1^2)/(((e + f*x)/(e + f*x
 - #1))^n*(e^2 - 2*e*#1 + #1^2)) & ]))/(3*b*n)
```

Maple [F] time = 0.073, size = 0, normalized size = 0.

$$\int \frac{x^2 (fx+e)^n}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(f*x+e)^n/(b*x^3+a),x)
```

```
[Out] int(x^2*(f*x+e)^n/(b*x^3+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^n x^2}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x + e)^n*x^2/(b*x^3 + a),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^n*x^2/(b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(fx+e)^n x^2}{bx^3+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x + e)^n*x^2/(b*x^3 + a),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n*x^2/(b*x^3 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(f*x+e)**n/(b*x**3+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^n x^2}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x + e)^n*x^2/(b*x^3 + a),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^n*x^2/(b*x^3 + a), x)
```

3.167 $\int \frac{x(e+fx)^n}{a+bx^3} dx$

Optimal. Leaf size=288

$$\frac{(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3\sqrt[3]{a}\sqrt[3]{b}(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)}-\frac{\sqrt[3]{-1}(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3\sqrt[3]{a}\sqrt[3]{b}(n+1)\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)}-\frac{(-1)^{2/3}(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{-1}\sqrt[3]{b}(e+fx)}{\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f}\right)}{3\sqrt[3]{a}\sqrt[3]{b}(n+1)\left(\sqrt[3]{a}f+\sqrt[3]{-1}\sqrt[3]{b}e\right)}$$

```
[Out] ((e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e
 + f*x))/(b^(1/3)*e - a^(1/3)*f)])/(3*a^(1/3)*b^(1/3)*(b^(1/3)*e
- a^(1/3)*f)*(1 + n)) - ((-1)^(1/3)*(e + f*x)^(1 + n)*Hypergeomet
ric2F1[1, 1 + n, 2 + n, ((-1)^(2/3)*b^(1/3)*(e + f*x))/((-1)^(2/3
)*b^(1/3)*e - a^(1/3)*f)])/(3*a^(1/3)*b^(1/3)*((-1)^(2/3)*b^(1/3)
*e - a^(1/3)*f)*(1 + n)) - ((-1)^(2/3)*(e + f*x)^(1 + n)*Hypergeo
metric2F1[1, 1 + n, 2 + n, ((-1)^(1/3)*b^(1/3)*(e + f*x))/((-1)^(
1/3)*b^(1/3)*e + a^(1/3)*f)])/(3*a^(1/3)*b^(1/3)*((-1)^(1/3)*b^(1
/3)*e + a^(1/3)*f)*(1 + n))
```

Rubi [A] time = 0.685589, antiderivative size = 288, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\frac{(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3\sqrt[3]{a}\sqrt[3]{b}(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)}-\frac{\sqrt[3]{-1}(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3\sqrt[3]{a}\sqrt[3]{b}(n+1)\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)}-\frac{(-1)^{2/3}(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{-1}\sqrt[3]{b}(e+fx)}{\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f}\right)}{3\sqrt[3]{a}\sqrt[3]{b}(n+1)\left(\sqrt[3]{a}f+\sqrt[3]{-1}\sqrt[3]{b}e\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(e + f*x)^n)/(a + b*x^3),x]
```

```
[Out] ((e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e
 + f*x))/(b^(1/3)*e - a^(1/3)*f)])/(3*a^(1/3)*b^(1/3)*(b^(1/3)*e
- a^(1/3)*f)*(1 + n)) - ((-1)^(1/3)*(e + f*x)^(1 + n)*Hypergeomet
ric2F1[1, 1 + n, 2 + n, ((-1)^(2/3)*b^(1/3)*(e + f*x))/((-1)^(2/3
)*b^(1/3)*e - a^(1/3)*f)])/(3*a^(1/3)*b^(1/3)*((-1)^(2/3)*b^(1/3)
*e - a^(1/3)*f)*(1 + n)) - ((-1)^(2/3)*(e + f*x)^(1 + n)*Hypergeo
```

```
metric2F1[1, 1 + n, 2 + n, ((-1)^(1/3)*b^(1/3)*(e + f*x))/((-1)^(
1/3)*b^(1/3)*e + a^(1/3)*f)])/(3*a^(1/3)*b^(1/3)*((-1)^(1/3)*b^(1
/3)*e + a^(1/3)*f)*(1 + n))
```

Rubi in Sympy [A] time = 96.6077, size = 252, normalized size = 0.88

$$\frac{\sqrt[3]{-1}\,(e+fx)^{n+1}\,{}_2F_1\left(\begin{matrix}1,n+1\\n+2\end{matrix}\middle|\frac{(-1)^{\frac{2}{3}}\sqrt[3]{b}(e+fx)}{-\sqrt[3]{a}f+(-1)^{\frac{2}{3}}\sqrt[3]{b}e}\right)}{3\sqrt[3]{a}\sqrt[3]{b}\,(n+1)\left(\sqrt[3]{a}f-(-1)^{\frac{2}{3}}\sqrt[3]{b}e\right)} - \frac{(-1)^{\frac{2}{3}}\,(e+fx)^{n+1}\,{}_2F_1\left(\begin{matrix}1,n+1\\n+2\end{matrix}\middle|\frac{\sqrt[3]{-1}\sqrt[3]{b}(e+fx)}{\sqrt[3]{a}f+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{3\sqrt[3]{a}\sqrt[3]{b}\,(n+1)\left(\sqrt[3]{a}f+\sqrt[3]{-1}\sqrt[3]{b}e\right)} - \frac{(e+fx)^{n+1}\,{}_2F_1\left(\begin{matrix}1,n+1\\n+2\end{matrix}\middle|\frac{\sqrt[3]{b}(e+fx)}{-\sqrt[3]{a}f+\sqrt[3]{b}e}\right)}{3\sqrt[3]{a}\sqrt[3]{b}\,(n+1)\left(\sqrt[3]{a}f-\sqrt[3]{b}e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(f*x+e)**n/(b*x**3+a),x)
```

```
[Out] (-1)**(1/3)*(e + f*x)**(n + 1)*hyper((1, n + 1), (n + 2,), (-1)**
(2/3)*b**(1/3)*(e + f*x)/(-a**(1/3)*f + (-1)**(2/3)*b**(1/3)*e))/
(3*a**(1/3)*b**(1/3)*(n + 1)*(a**(1/3)*f - (-1)**(2/3)*b**(1/3)*e
)) - (-1)**(2/3)*(e + f*x)**(n + 1)*hyper((1, n + 1), (n + 2,), (
-1)**(1/3)*b**(1/3)*(e + f*x)/(a**(1/3)*f + (-1)**(1/3)*b**(1/3)*
e))/(3*a**(1/3)*b**(1/3)*(n + 1)*(a**(1/3)*f + (-1)**(1/3)*b**(1/
3)*e)) - (e + f*x)**(n + 1)*hyper((1, n + 1), (n + 2,), b**(1/3)*
(e + f*x)/(-a**(1/3)*f + b**(1/3)*e))/(3*a**(1/3)*b**(1/3)*(n + 1
)*(a**(1/3)*f - b**(1/3)*e))
```

Mathematica [C] time = 0.084591, size = 229, normalized size = 0.8

$$-\frac{f(e+fx)^n\left(e\text{RootSum}\left[-\#1^3b+3\#1^2be-3\#1be^2-af^3+be^3\&,\frac{\left(\frac{e+fx}{-\#1+e+fx}\right)^{-n}{}_2F_1\left(-n,-n;1-n;-\frac{\#1}{e+fx-\#1}\right)}{\#1^2-2\#1e+e^2}\&\right]-\text{RootSum}\left[-\#\right.\right.}{3bn}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(x*(e + f*x)^n)/(a + b*x^3),x]
```

```
[Out] -(f*(e + f*x)^n*(e*RootSum[b*e^3 - a*f^3 - 3*b*e^2*#1 + 3*b*e*#1^
2 - b*#1^3 & , Hypergeometric2F1[-n, -n, 1 - n, -(#1/(e + f*x - #
1))]/(((e + f*x)/(e + f*x - #1))^n*(e^2 - 2*e*#1 + #1^2)) & ] - R
```

```
ootSum[b*e^3 - a*f^3 - 3*b*e^2*#1 + 3*b*e*#1^2 - b*#1^3 & , (Hype
rgeometric2F1[-n, -n, 1 - n, -(#1/(e + f*x - #1))]*#1)/(((e + f*x
)/(e + f*x - #1))^n*(e^2 - 2*e*#1 + #1^2)) & ]))/(3*b*n)
```

Maple [F] time = 0.073, size = 0, normalized size = 0.

$$\int \frac{x\,(fx+e)^n}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(f*x+e)^n/(b*x^3+a),x)
```

```
[Out] int(x*(f*x+e)^n/(b*x^3+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^n x}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x + e)^n*x/(b*x^3 + a),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^n*x/(b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(fx+e)^n x}{bx^3+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x + e)^n*x/(b*x^3 + a),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n*x/(b*x^3 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(f*x+e)**n/(b*x**3+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^n x}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x + e)^n*x/(b*x^3 + a),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^n*x/(b*x^3 + a), x)
```

3.168 $\int \frac{(e+fx)^n}{a+bx^3}\,dx$

Optimal. Leaf size=263

$$-\frac{(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a^{2/3}(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)}-\frac{(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a^{2/3}(n+1)\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)}+\frac{(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{-1}\sqrt[3]{b}(e+fx)}{\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f}\right)}{3a^{2/3}(n+1)\left(\sqrt[3]{a}f+\sqrt[3]{-1}\sqrt[3]{b}e\right)}$$

```
[Out] -((e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(
e + f*x))/(b^(1/3)*e - a^(1/3)*f)])/(3*a^(2/3)*(b^(1/3)*e - a^(1/
3)*f)*(1 + n)) - ((e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2
 + n, ((-1)^(2/3)*b^(1/3)*(e + f*x))/((-1)^(2/3)*b^(1/3)*e - a^(1
/3)*f)])/(3*a^(2/3)*((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)*(1 + n)) +
 ((e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(1/3
)*b^(1/3)*(e + f*x))/((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)])/(3*a^(2
/3)*((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)*(1 + n))
```

Rubi [A] time = 0.471876, antiderivative size = 263, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118

$$-\frac{(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a^{2/3}(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)}-\frac{(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a^{2/3}(n+1)\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)}+\frac{(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{-1}\sqrt[3]{b}(e+fx)}{\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f}\right)}{3a^{2/3}(n+1)\left(\sqrt[3]{a}f+\sqrt[3]{-1}\sqrt[3]{b}e\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(e + f*x)^n/(a + b*x^3),x]
```

```
[Out] -((e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(
e + f*x))/(b^(1/3)*e - a^(1/3)*f)])/(3*a^(2/3)*(b^(1/3)*e - a^(1/
3)*f)*(1 + n)) - ((e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2
 + n, ((-1)^(2/3)*b^(1/3)*(e + f*x))/((-1)^(2/3)*b^(1/3)*e - a^(1
/3)*f)])/(3*a^(2/3)*((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)*(1 + n)) +
 ((e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(1/3
)*b^(1/3)*(e + f*x))/((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)])/(3*a^(2
```

```
/3)*((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)*(1 + n))
```

Rubi in Sympy [A] time = 60.8382, size = 223, normalized size = 0.85

$$\frac{(e+fx)^{n+1}\,{}_2F_1\left(\begin{matrix}1,n+1\\n+2\end{matrix}\middle|\frac{(-1)^{\frac{2}{3}}\sqrt[3]{b}(e+fx)}{-\sqrt[3]{a}f+(-1)^{\frac{2}{3}}\sqrt[3]{b}e}\right)}{3a^{\frac{2}{3}}(n+1)\left(\sqrt[3]{a}f-(-1)^{\frac{2}{3}}\sqrt[3]{b}e\right)}+\frac{(e+fx)^{n+1}\,{}_2F_1\left(\begin{matrix}1,n+1\\n+2\end{matrix}\middle|\frac{\sqrt[3]{-1}\sqrt[3]{b}(e+fx)}{\sqrt[3]{a}f+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{3a^{\frac{2}{3}}(n+1)\left(\sqrt[3]{a}f+\sqrt[3]{-1}\sqrt[3]{b}e\right)}+\frac{(e+fx)^{n+1}\,{}_2F_1\left(\begin{matrix}1,n+1\\n+2\end{matrix}\middle|\frac{\sqrt[3]{b}(e+fx)}{-\sqrt[3]{a}f+\sqrt[3]{b}e}\right)}{3a^{\frac{2}{3}}(n+1)\left(\sqrt[3]{a}f-\sqrt[3]{b}e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x+e)**n/(b*x**3+a),x)
```

```
[Out] (e + f*x)**(n + 1)*hyper((1, n + 1), (n + 2,), (-1)**(2/3)*b**(1/
3)*(e + f*x)/(-a**(1/3)*f + (-1)**(2/3)*b**(1/3)*e))/(3*a**(2/3)*
(n + 1)*(a**(1/3)*f - (-1)**(2/3)*b**(1/3)*e)) + (e + f*x)**(n +
1)*hyper((1, n + 1), (n + 2,), (-1)**(1/3)*b**(1/3)*(e + f*x)/(a*
*(1/3)*f + (-1)**(1/3)*b**(1/3)*e))/(3*a**(2/3)*(n + 1)*(a**(1/3)
*f + (-1)**(1/3)*b**(1/3)*e)) + (e + f*x)**(n + 1)*hyper((1, n +
1), (n + 2,), b**(1/3)*(e + f*x)/(-a**(1/3)*f + b**(1/3)*e))/(3*a
**(2/3)*(n + 1)*(a**(1/3)*f - b**(1/3)*e))
```

Mathematica [C] time = 0.051229, size = 122, normalized size = 0.46

$$\frac{f^2(e+fx)^n\text{RootSum}\left[-\#1^3b+3\#1^2be-3\#1be^2-af^3+be^3\&,\frac{\left(\frac{e+fx}{-\#1+e+fx}\right)^{-n}{}_2F_1\left(-n,-n;1-n;-\frac{\#1}{e+fx-\#1}\right)}{\#1^2-2\#1e+e^2}\&\right]}{3bn}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(e + f*x)^n/(a + b*x^3),x]
```

```
[Out] (f^2*(e + f*x)^n*RootSum[b*e^3 - a*f^3 - 3*b*e^2*#1 + 3*b*e*#1^2
- b*#1^3 & , Hypergeometric2F1[-n, -n, 1 - n, -(#1/(e + f*x - #1)
)]/(((e + f*x)/(e + f*x - #1))^n*(e^2 - 2*e*#1 + #1^2)) & ])/(3*b
*n)
```

Maple [F] time = 0.073, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^n}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x+e)^n/(b*x^3+a),x)
```

```
[Out] int((f*x+e)^n/(b*x^3+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^n}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x + e)^n/(b*x^3 + a),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^n/(b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(fx+e)^n}{bx^3+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x + e)^n/(b*x^3 + a),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n/(b*x^3 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x+e)**n/(b*x**3+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^n}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x + e)^n/(b*x^3 + a),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^n/(b*x^3 + a), x)
```

3.169 $\int \frac{(e+fx)^n}{x(a+bx^3)}\,dx$

Optimal. Leaf size=300

$$\frac{\sqrt[3]{b}(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)}+\frac{\sqrt[3]{b}(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f}\right)}{3a(n+1)\left(\sqrt[3]{-1}\sqrt[3]{a}f+\sqrt[3]{b}e\right)}+\frac{\sqrt[3]{b}(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-(-1)^{2/3}\sqrt[3]{a}f}\right)}{3a(n+1)\left(\sqrt[3]{b}e-(-1)^{2/3}\sqrt[3]{a}f\right)}-\frac{(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{fx}{e}+1\right)}{ae(n+1)}$$

```
[Out] (b^(1/3)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^
(1/3)*(e + f*x))/(b^(1/3)*e - a^(1/3)*f)])/(3*a*(b^(1/3)*e - a^(1
/3)*f)*(1 + n)) + (b^(1/3)*(e + f*x)^(1 + n)*Hypergeometric2F1[1,
 1 + n, 2 + n, (b^(1/3)*(e + f*x))/(b^(1/3)*e + (-1)^(1/3)*a^(1/3
)*f)])/(3*a*(b^(1/3)*e + (-1)^(1/3)*a^(1/3)*f)*(1 + n)) + (b^(1/3
)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(
e + f*x))/(b^(1/3)*e - (-1)^(2/3)*a^(1/3)*f)])/(3*a*(b^(1/3)*e -
(-1)^(2/3)*a^(1/3)*f)*(1 + n)) - ((e + f*x)^(1 + n)*Hypergeometri
c2F1[1, 1 + n, 2 + n, 1 + (f*x)/e])/(a*e*(1 + n))
```

Rubi [A] time = 1.24279, antiderivative size = 300, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$

$$\frac{\sqrt[3]{b}(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)}+\frac{\sqrt[3]{b}(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f}\right)}{3a(n+1)\left(\sqrt[3]{-1}\sqrt[3]{a}f+\sqrt[3]{b}e\right)}+\frac{\sqrt[3]{b}(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-(-1)^{2/3}\sqrt[3]{a}f}\right)}{3a(n+1)\left(\sqrt[3]{b}e-(-1)^{2/3}\sqrt[3]{a}f\right)}-\frac{(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{fx}{e}+1\right)}{ae(n+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(e + f*x)^n/(x*(a + b*x^3)),x]
```


```
[Out] (b^(1/3)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^
(1/3)*(e + f*x))/(b^(1/3)*e - a^(1/3)*f)])/(3*a*(b^(1/3)*e - a^(1
/3)*f)*(1 + n)) + (b^(1/3)*(e + f*x)^(1 + n)*Hypergeometric2F1[1,
 1 + n, 2 + n, (b^(1/3)*(e + f*x))/(b^(1/3)*e + (-1)^(1/3)*a^(1/3
)*f)])/(3*a*(b^(1/3)*e + (-1)^(1/3)*a^(1/3)*f)*(1 + n)) + (b^(1/3
)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(
```

```
e + f*x))/(b^(1/3)*e - (-1)^(2/3)*a^(1/3)*f)])/(3*a*(b^(1/3)*e -
(-1)^(2/3)*a^(1/3)*f)*(1 + n)) - ((e + f*x)^(1 + n)*Hypergeometri
c2F1[1, 1 + n, 2 + n, 1 + (f*x)/e])/(a*e*(1 + n))
```

Rubi in Sympy [A] time = 129.179, size = 241, normalized size = 0.8

$$\frac{\sqrt[3]{b}\,(e+fx)^{n+1}\,{}_2F_1\left(\begin{matrix}1,n+1\\n+2\end{matrix}\middle|\frac{\sqrt[3]{b}(e+fx)}{-(-1)^{\frac{2}{3}}\sqrt[3]{a}f+\sqrt[3]{b}e}\right)}{3a(n+1)\left(-(-1)^{\frac{2}{3}}\sqrt[3]{a}f+\sqrt[3]{b}e\right)}+\frac{\sqrt[3]{b}\,(e+fx)^{n+1}\,{}_2F_1\left(\begin{matrix}1,n+1\\n+2\end{matrix}\middle|\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{-1}\sqrt[3]{a}f+\sqrt[3]{b}e}\right)}{3a(n+1)\left(\sqrt[3]{-1}\sqrt[3]{a}f+\sqrt[3]{b}e\right)}-\frac{\sqrt[3]{b}\,(e+fx)^{n+1}\,{}_2F_1\left(\begin{matrix}1,n+1\\n+2\end{matrix}\middle|\frac{\sqrt[3]{b}(e+fx)}{-\sqrt[3]{a}f+\sqrt[3]{b}e}\right)}{3a(n+1)\left(\sqrt[3]{a}f-\sqrt[3]{b}e\right)}-\frac{(e+fx)^{n+1}\,{}_2F_1\left(\begin{matrix}1,n+1\\n+2\end{matrix}\middle|1+\frac{fx}{e}\right)}{ae(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x+e)**n/x/(b*x**3+a),x)
```

```
[Out] b**(1/3)*(e + f*x)**(n + 1)*hyper((1, n + 1), (n + 2,), b**(1/3)*
(e + f*x)/(-(-1)**(2/3)*a**(1/3)*f + b**(1/3)*e))/(3*a*(n + 1)*(-
(-1)**(2/3)*a**(1/3)*f + b**(1/3)*e)) + b**(1/3)*(e + f*x)**(n +
1)*hyper((1, n + 1), (n + 2,), b**(1/3)*(e + f*x)/((-1)**(1/3)*a*
*(1/3)*f + b**(1/3)*e))/(3*a*(n + 1)*((-1)**(1/3)*a**(1/3)*f + b*
*(1/3)*e)) - b**(1/3)*(e + f*x)**(n + 1)*hyper((1, n + 1), (n + 2
,), b**(1/3)*(e + f*x)/(-a**(1/3)*f + b**(1/3)*e))/(3*a*(n + 1)*(
a**(1/3)*f - b**(1/3)*e)) - (e + f*x)**(n + 1)*hyper((1, n + 1),
(n + 2,), 1 + f*x/e)/(a*e*(n + 1))
```

Mathematica [C] time = 0.263433, size = 377, normalized size = 1.26

$$\frac{(e+fx)^n\left(-e^2\text{RootSum}\left[-\#1^3b+3\#1^2be-3\#1be^2-af^3+be^3\&,\frac{\left(\frac{e+fx}{-\#1+e+fx}\right)^{-n}{}_2F_1\left(-n,-n;1-n;-\frac{\#1}{e+fx-\#1}\right)}{\#1^2-2\#1e+e^2}\&\right]+2e\text{RootSum}\left[-\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(e + f*x)^n/(x*(a + b*x^3)),x]
```

```
[Out] ((e + f*x)^n*((3*Hypergeometric2F1[-n, -n, 1 - n, -(e/(f*x))])/(1
 + e/(f*x))^n - e^2*RootSum[b*e^3 - a*f^3 - 3*b*e^2*#1 + 3*b*e*#1
^2 - b*#1^3 & , Hypergeometric2F1[-n, -n, 1 - n, -(#1/(e + f*x -
```

```
#1))]/(((e + f*x)/(e + f*x - #1))^n*(e^2 - 2*e*#1 + #1^2)) & ] +
2*e*RootSum[b*e^3 - a*f^3 - 3*b*e^2*#1 + 3*b*e*#1^2 - b*#1^3 & ,
(Hypergeometric2F1[-n, -n, 1 - n, -(#1/(e + f*x - #1))]*#1)/(((e
+ f*x)/(e + f*x - #1))^n*(e^2 - 2*e*#1 + #1^2)) & ] - RootSum[b*e
^3 - a*f^3 - 3*b*e^2*#1 + 3*b*e*#1^2 - b*#1^3 & , (Hypergeometric
2F1[-n, -n, 1 - n, -(#1/(e + f*x - #1))]*#1^2)/(((e + f*x)/(e + f
*x - #1))^n*(e^2 - 2*e*#1 + #1^2)) & ]))/(3*a*n)
```

Maple [F] time = 0.062, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^n}{x\,(bx^3+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x+e)^n/x/(b*x^3+a),x)
```

```
[Out] int((f*x+e)^n/x/(b*x^3+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^n}{(bx^3+a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x + e)^n/((b*x^3 + a)*x),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^n/((b*x^3 + a)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{(fx+e)^n}{bx^4+ax}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x + e)^n/((b*x^3 + a)*x),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n/(b*x^4 + a*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x+e)**n/x/(b*x**3+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^n}{(bx^3+a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x + e)^n/((b*x^3 + a)*x),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^n/((b*x^3 + a)*x), x)
```

3.170 $\int \frac{(e+fx)^n}{x^2(a+bx^3)}\,dx$

Optimal. Leaf size=326

$$-\frac{b^{2/3}(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a^{4/3}(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)}+\frac{\sqrt[3]{-1}b^{2/3}(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a^{4/3}(n+1)\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)}+\frac{(-1)^{2/3}b^{2/3}(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{-1}\sqrt[3]{b}(e+fx)}{\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f}\right)}{3a^{4/3}(n+1)\left(\sqrt[3]{a}f+\sqrt[3]{-1}\sqrt[3]{b}e\right)}+\frac{f(e+fx)^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{fx}{e}+1\right)}{ae^2(n+1)}$$

```
[Out] -(b^(2/3)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b
^(1/3)*(e + f*x))/(b^(1/3)*e - a^(1/3)*f)])/(3*a^(4/3)*(b^(1/3)*e
 - a^(1/3)*f)*(1 + n)) + ((-1)^(1/3)*b^(2/3)*(e + f*x)^(1 + n)*Hy
pergeometric2F1[1, 1 + n, 2 + n, ((-1)^(2/3)*b^(1/3)*(e + f*x))/(
(-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)])/(3*a^(4/3)*((-1)^(2/3)*b^(1/3
)*e - a^(1/3)*f)*(1 + n)) + ((-1)^(2/3)*b^(2/3)*(e + f*x)^(1 + n)
*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(1/3)*b^(1/3)*(e + f*x)
)/((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)])/(3*a^(4/3)*((-1)^(1/3)*b^(
1/3)*e + a^(1/3)*f)*(1 + n)) + (f*(e + f*x)^(1 + n)*Hypergeometri
c2F1[2, 1 + n, 2 + n, 1 + (f*x)/e])/(a*e^2*(1 + n))
```

Rubi [A] time = 1.28852, antiderivative size = 326, normalized size of antiderivative = 1., number of

steps used = 8, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$

$$-\frac{b^{2/3}(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a^{4/3}(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)}+\frac{\sqrt[3]{-1}b^{2/3}(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a^{4/3}(n+1)\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)}+\frac{(-1)^{2/3}b^{2/3}(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{-1}\sqrt[3]{b}(e+fx)}{\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f}\right)}{3a^{4/3}(n+1)\left(\sqrt[3]{a}f+\sqrt[3]{-1}\sqrt[3]{b}e\right)}+\frac{f(e+fx)^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{fx}{e}+1\right)}{ae^2(n+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(e + f*x)^n/(x^2*(a + b*x^3)),x]
```

```
[Out] -(b^(2/3)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b
^(1/3)*(e + f*x))/(b^(1/3)*e - a^(1/3)*f)])/(3*a^(4/3)*(b^(1/3)*e
 - a^(1/3)*f)*(1 + n)) + ((-1)^(1/3)*b^(2/3)*(e + f*x)^(1 + n)*Hy
pergeometric2F1[1, 1 + n, 2 + n, ((-1)^(2/3)*b^(1/3)*(e + f*x))/(
(-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)])/(3*a^(4/3)*((-1)^(2/3)*b^(1/3
)*e - a^(1/3)*f)*(1 + n)) + ((-1)^(2/3)*b^(2/3)*(e + f*x)^(1 + n)
*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(1/3)*b^(1/3)*(e + f*x)
)/((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)])/(3*a^(4/3)*((-1)^(1/3)*b^(
1/3)*e + a^(1/3)*f)*(1 + n)) + (f*(e + f*x)^(1 + n)*Hypergeometri
c2F1[2, 1 + n, 2 + n, 1 + (f*x)/e])/(a*e^2*(1 + n))
```

Rubi in Sympy [A] time = 163.288, size = 282, normalized size = 0.87

$$\frac{f\left(e+fx\right)^{n+1}{}_2F_1\left(\begin{matrix}2,n+1\\n+2\end{matrix}\middle|1+\frac{fx}{e}\right)}{ae^2\left(n+1\right)}-\frac{\sqrt[3]{-1}b^{\frac{2}{3}}\left(e+fx\right)^{n+1}{}_2F_1\left(\begin{matrix}1,n+1\\n+2\end{matrix}\middle|\frac{(-1)^{\frac{2}{3}}\sqrt[3]{b}(e+fx)}{-\sqrt[3]{a}f+(-1)^{\frac{2}{3}}\sqrt[3]{b}e}\right)}{3a^{\frac{4}{3}}\left(n+1\right)\left(\sqrt[3]{a}f-(-1)^{\frac{2}{3}}\sqrt[3]{b}e\right)}+\frac{(-1)^{\frac{2}{3}}b^{\frac{2}{3}}\left(e+fx\right)^{n+1}{}_2F_1\left(\begin{matrix}1,n+1\\n+2\end{matrix}\middle|\frac{\sqrt[3]{-1}\sqrt[3]{b}(e+fx)}{\sqrt[3]{a}f+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{3a^{\frac{4}{3}}\left(n+1\right)\left(\sqrt[3]{a}f+\sqrt[3]{-1}\sqrt[3]{b}e\right)}+\frac{b^{\frac{2}{3}}\left(e+fx\right)^{n+1}{}_2F_1\left(\begin{matrix}1,n+1\\n+2\end{matrix}\middle|\frac{\sqrt[3]{b}(e+fx)}{-\sqrt[3]{a}f+\sqrt[3]{b}e}\right)}{3a^{\frac{4}{3}}\left(n+1\right)\left(\sqrt[3]{a}f-\sqrt[3]{b}e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x+e)**n/x**2/(b*x**3+a),x)
```

```
[Out] f*(e + f*x)**(n + 1)*hyper((2, n + 1), (n + 2,), 1 + f*x/e)/(a*e*
*2*(n + 1)) - (-1)**(1/3)*b**(2/3)*(e + f*x)**(n + 1)*hyper((1, n
 + 1), (n + 2,), (-1)**(2/3)*b**(1/3)*(e + f*x)/(-a**(1/3)*f + (-
1)**(2/3)*b**(1/3)*e))/(3*a**(4/3)*(n + 1)*(a**(1/3)*f - (-1)**(2
/3)*b**(1/3)*e)) + (-1)**(2/3)*b**(2/3)*(e + f*x)**(n + 1)*hyper(
(1, n + 1), (n + 2,), (-1)**(1/3)*b**(1/3)*(e + f*x)/(a**(1/3)*f
+ (-1)**(1/3)*b**(1/3)*e))/(3*a**(4/3)*(n + 1)*(a**(1/3)*f + (-1)
**(1/3)*b**(1/3)*e)) + b**(2/3)*(e + f*x)**(n + 1)*hyper((1, n +
1), (n + 2,), b**(1/3)*(e + f*x)/(-a**(1/3)*f + b**(1/3)*e))/(3*a
**(4/3)*(n + 1)*(a**(1/3)*f - b**(1/3)*e))
```

Mathematica [C] time = 0.247376, size = 280, normalized size = 0.86

$$\frac{(e+fx)^n\left(\frac{ef\text{RootSum}\left[-\#1^3b+3\#1^2be-3\#1be^2-af^3+be^3\&,\frac{\left(\frac{e+fx}{-\#1+e+fx}\right)^{-n}{}_2F_1\left(-n,-n;1-n;-\frac{\#1}{e+fx-\#1}\right)}{\#1^2-2\#1e+e^2}\&\right]}{n}-\frac{f\text{RootSum}\left[-\#1^3b+3\#1^2be-3\#1be^2\right.}{}\right.}{3a}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(e + f*x)^n/(x^2*(a + b*x^3)),x]
```

```
[Out] ((e + f*x)^n*((3*Hypergeometric2F1[1 - n, -n, 2 - n, -(e/(f*x))])
/((-1 + n)*(1 + e/(f*x))^n*x) + (e*f*RootSum[b*e^3 - a*f^3 - 3*b*
e^2*#1 + 3*b*e*#1^2 - b*#1^3 & , Hypergeometric2F1[-n, -n, 1 - n,
 -(#1/(e + f*x - #1))]/(((e + f*x)/(e + f*x - #1))^n*(e^2 - 2*e*#
1 + #1^2)) & ])/n - (f*RootSum[b*e^3 - a*f^3 - 3*b*e^2*#1 + 3*b*e
*#1^2 - b*#1^3 & , (Hypergeometric2F1[-n, -n, 1 - n, -(#1/(e + f*
x - #1))]*#1)/(((e + f*x)/(e + f*x - #1))^n*(e^2 - 2*e*#1 + #1^2)
) & ])/n))/(3*a)
```

Maple [F] time = 0.09, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^n}{x^2\left(bx^3+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x+e)^n/x^2/(b*x^3+a),x)
```

```
[Out] int((f*x+e)^n/x^2/(b*x^3+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^n}{(bx^3+a)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x + e)^n/((b*x^3 + a)*x^2),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^n/((b*x^3 + a)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(fx+e)^n}{bx^5+ax^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x + e)^n/((b*x^3 + a)*x^2),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n/(b*x^5 + a*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x+e)**n/x**2/(b*x**3+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^n}{(bx^3+a)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x + e)^n/((b*x^3 + a)*x^2),x, algorithm="giac")

[Out] integrate((f*x + e)^n/((b*x^3 + a)*x^2), x)
```

3.171 $\int \frac{x^2(c+dx)^{1+n}}{a+bx^3}\,dx$

Optimal. Leaf size=253

$$-\frac{(c+dx)^{n+2}\,{}_2F_1\left(1,n+2;n+3;\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3b^{2/3}(n+2)\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)}-\frac{(c+dx)^{n+2}\,{}_2F_1\left(1,n+2;n+3;\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)}{3b^{2/3}(n+2)\left(\sqrt[3]{-1}\sqrt[3]{a}d+\sqrt[3]{b}c\right)}-\frac{(c+dx)^{n+2}\,{}_2F_1\left(1,n+2;n+3;\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)}{3b^{2/3}(n+2)\left(\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d\right)}$$

```
[Out] -((c + d*x)^(2 + n)*Hypergeometric2F1[1, 2 + n, 3 + n, (b^(1/3)*(
c + d*x))/(b^(1/3)*c - a^(1/3)*d)])/(3*b^(2/3)*(b^(1/3)*c - a^(1/
3)*d)*(2 + n)) - ((c + d*x)^(2 + n)*Hypergeometric2F1[1, 2 + n, 3
 + n, (b^(1/3)*(c + d*x))/(b^(1/3)*c + (-1)^(1/3)*a^(1/3)*d)])/(3
*b^(2/3)*(b^(1/3)*c + (-1)^(1/3)*a^(1/3)*d)*(2 + n)) - ((c + d*x)
^(2 + n)*Hypergeometric2F1[1, 2 + n, 3 + n, (b^(1/3)*(c + d*x))/(
b^(1/3)*c - (-1)^(2/3)*a^(1/3)*d)])/(3*b^(2/3)*(b^(1/3)*c - (-1)^
(2/3)*a^(1/3)*d)*(2 + n))
```

Rubi [A] time = 0.949081, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$-\frac{(c+dx)^{n+2}\,{}_2F_1\left(1,n+2;n+3;\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3b^{2/3}(n+2)\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)}-\frac{(c+dx)^{n+2}\,{}_2F_1\left(1,n+2;n+3;\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)}{3b^{2/3}(n+2)\left(\sqrt[3]{-1}\sqrt[3]{a}d+\sqrt[3]{b}c\right)}-\frac{(c+dx)^{n+2}\,{}_2F_1\left(1,n+2;n+3;\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)}{3b^{2/3}(n+2)\left(\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(c + d*x)^(1 + n))/(a + b*x^3),x]
```

```
[Out] -((c + d*x)^(2 + n)*Hypergeometric2F1[1, 2 + n, 3 + n, (b^(1/3)*(
c + d*x))/(b^(1/3)*c - a^(1/3)*d)])/(3*b^(2/3)*(b^(1/3)*c - a^(1/
3)*d)*(2 + n)) - ((c + d*x)^(2 + n)*Hypergeometric2F1[1, 2 + n, 3
 + n, (b^(1/3)*(c + d*x))/(b^(1/3)*c + (-1)^(1/3)*a^(1/3)*d)])/(3
*b^(2/3)*(b^(1/3)*c + (-1)^(1/3)*a^(1/3)*d)*(2 + n)) - ((c + d*x)
^(2 + n)*Hypergeometric2F1[1, 2 + n, 3 + n, (b^(1/3)*(c + d*x))/(
b^(1/3)*c - (-1)^(2/3)*a^(1/3)*d)])/(3*b^(2/3)*(b^(1/3)*c - (-1)^
```

```
(2/3)*a^(1/3)*d)*(2 + n))
```

Rubi in Sympy [A] time = 75.8272, size = 209, normalized size = 0.83

$$-\frac{(c+dx)^{n+2}\,{}_2F_1\left(\begin{matrix}1,n+2\\ n+3\end{matrix}\middle|\frac{\sqrt[3]{b}(c+dx)}{-(-1)^{\frac{2}{3}}\sqrt[3]{a}d+\sqrt[3]{b}c}\right)}{3b^{\frac{2}{3}}(n+2)\left(-(-1)^{\frac{2}{3}}\sqrt[3]{a}d+\sqrt[3]{b}c\right)}-\frac{(c+dx)^{n+2}\,{}_2F_1\left(\begin{matrix}1,n+2\\ n+3\end{matrix}\middle|\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{-1}\sqrt[3]{a}d+\sqrt[3]{b}c}\right)}{3b^{\frac{2}{3}}(n+2)\left(\sqrt[3]{-1}\sqrt[3]{a}d+\sqrt[3]{b}c\right)}+\frac{(c+dx)^{n+2}\,{}_2F_1\left(\begin{matrix}1,n+2\\ n+3\end{matrix}\middle|\frac{\sqrt[3]{b}(c+dx)}{-\sqrt[3]{a}d+\sqrt[3]{b}c}\right)}{3b^{\frac{2}{3}}(n+2)\left(\sqrt[3]{a}d-\sqrt[3]{b}c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(d*x+c)**(1+n)/(b*x**3+a),x)
```

```
[Out] -(c + d*x)**(n + 2)*hyper((1, n + 2), (n + 3,), b**(1/3)*(c + d*x
)/(-(-1)**(2/3)*a**(1/3)*d + b**(1/3)*c))/(3*b**(2/3)*(n + 2)*(-(
-1)**(2/3)*a**(1/3)*d + b**(1/3)*c)) - (c + d*x)**(n + 2)*hyper((
1, n + 2), (n + 3,), b**(1/3)*(c + d*x)/((-1)**(1/3)*a**(1/3)*d +
 b**(1/3)*c))/(3*b**(2/3)*(n + 2)*((-1)**(1/3)*a**(1/3)*d + b**(1
/3)*c)) + (c + d*x)**(n + 2)*hyper((1, n + 2), (n + 3,), b**(1/3)
*(c + d*x)/(-a**(1/3)*d + b**(1/3)*c))/(3*b**(2/3)*(n + 2)*(a**(1
/3)*d - b**(1/3)*c))
```

Mathematica [C] time = 0.327526, size = 375, normalized size = 1.48

$$\frac{(c+dx)^n\left((n+1)\left(bc^3-ad^3\right)\text{RootSum}\left[-\#1^3b+3\#1^2bc-3\#1bc^2-ad^3+bc^3\&,\frac{\left(\frac{c+dx}{-\#1+c+dx}\right)^{-n}{}_2F_1\left(-n,-n;1-n;-\frac{\#1}{c+dx-\#1}\right)}{\#1^2-2\#1c+c^2}\&\right]+\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(x^2*(c + d*x)^(1 + n))/(a + b*x^3),x]
```

```
[Out] ((c + d*x)^n*((b*c^3 - a*d^3)*(1 + n)*RootSum[b*c^3 - a*d^3 - 3*b
*c^2*#1 + 3*b*c*#1^2 - b*#1^3 & , Hypergeometric2F1[-n, -n, 1 - n
, -(#1/(c + d*x - #1))]/(((c + d*x)/(c + d*x - #1))^n*(c^2 - 2*c*
#1 + #1^2)) & ] + b*(3*n*(c + d*x) - 2*c^2*(1 + n)*RootSum[b*c^3
- a*d^3 - 3*b*c^2*#1 + 3*b*c*#1^2 - b*#1^3 & , (Hypergeometric2F1
[-n, -n, 1 - n, -(#1/(c + d*x - #1))]*#1)/(((c + d*x)/(c + d*x -
```

```
#1))^n*(c^2 - 2*c*#1 + #1^2)) & ] + c*(1 + n)*RootSum[b*c^3 - a*d
^3 - 3*b*c^2*#1 + 3*b*c*#1^2 - b*#1^3 & , (Hypergeometric2F1[-n,
-n, 1 - n, -(#1/(c + d*x - #1))]*#1^2)/(((c + d*x)/(c + d*x - #1)
)^n*(c^2 - 2*c*#1 + #1^2)) & ]))/(3*b^2*n*(1 + n))
```

Maple [F] time = 0.077, size = 0, normalized size = 0.

$$\int \frac{x^2 (dx+c)^{1+n}}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(d*x+c)^(1+n)/(b*x^3+a),x)

[Out] int(x^2*(d*x+c)^(1+n)/(b*x^3+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{n+1}x^2}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)^(n + 1)*x^2/(b*x^3 + a),x, algorithm="maxima")

[Out] integrate((d*x + c)^(n + 1)*x^2/(b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx+c)^{n+1}x^2}{bx^3+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)^(n + 1)*x^2/(b*x^3 + a),x, algorithm="fricas")

[Out] integral((d*x + c)^(n + 1)*x^2/(b*x^3 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(d*x+c)**(1+n)/(b*x**3+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{n+1}x^2}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)^(n + 1)*x^2/(b*x^3 + a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(n + 1)*x^2/(b*x^3 + a), x)
```

3.172 $\int \frac{x^m(e+fx)^n}{a+bx^3}\,dx$

Optimal. Leaf size=211

$$\frac{x^{m+1}(e+fx)^n\left(\frac{fx}{e}+1\right)^{-n}F_1\left(m+1;-n,1;m+2;-\frac{fx}{e},-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a(m+1)}+\frac{x^{m+1}(e+fx)^n\left(\frac{fx}{e}+1\right)^{-n}F_1\left(m+1;-n,1;m+2;-\frac{fx}{e},\frac{\sqrt[3]{-1}\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a(m+1)}+\frac{x^{m+1}(e+fx)^n\left(\frac{fx}{e}+1\right)^{-n}F_1\left(m+1;-n,1;m+2;-\frac{fx}{e},-\frac{(-1)^{2/3}\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a(m+1)}$$

```
[Out] (x^(1 + m)*(e + f*x)^n*AppellF1[1 + m, -n, 1, 2 + m, -((f*x)/e),
-((b^(1/3)*x)/a^(1/3))])/(3*a*(1 + m)*(1 + (f*x)/e)^n) + (x^(1 +
m)*(e + f*x)^n*AppellF1[1 + m, -n, 1, 2 + m, -((f*x)/e), ((-1)^(1
/3)*b^(1/3)*x)/a^(1/3)])/(3*a*(1 + m)*(1 + (f*x)/e)^n) + (x^(1 +
m)*(e + f*x)^n*AppellF1[1 + m, -n, 1, 2 + m, -((f*x)/e), -(((-1)^
(2/3)*b^(1/3)*x)/a^(1/3))])/(3*a*(1 + m)*(1 + (f*x)/e)^n)
```

Rubi [A] time = 0.969815, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15

$$\frac{x^{m+1}(e+fx)^n\left(\frac{fx}{e}+1\right)^{-n}F_1\left(m+1;-n,1;m+2;-\frac{fx}{e},-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a(m+1)}+\frac{x^{m+1}(e+fx)^n\left(\frac{fx}{e}+1\right)^{-n}F_1\left(m+1;-n,1;m+2;-\frac{fx}{e},\frac{\sqrt[3]{-1}\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a(m+1)}+\frac{x^{m+1}(e+fx)^n\left(\frac{fx}{e}+1\right)^{-n}F_1\left(m+1;-n,1;m+2;-\frac{fx}{e},-\frac{(-1)^{2/3}\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a(m+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^m*(e + f*x)^n)/(a + b*x^3),x]
```

```
[Out] (x^(1 + m)*(e + f*x)^n*AppellF1[1 + m, -n, 1, 2 + m, -((f*x)/e),
-((b^(1/3)*x)/a^(1/3))])/(3*a*(1 + m)*(1 + (f*x)/e)^n) + (x^(1 +
m)*(e + f*x)^n*AppellF1[1 + m, -n, 1, 2 + m, -((f*x)/e), ((-1)^(1
/3)*b^(1/3)*x)/a^(1/3)])/(3*a*(1 + m)*(1 + (f*x)/e)^n) + (x^(1 +
m)*(e + f*x)^n*AppellF1[1 + m, -n, 1, 2 + m, -((f*x)/e), -(((-1)^
```

```
(2/3)*b^(1/3)*x)/a^(1/3))])/(3*a*(1 + m)*(1 + (f*x)/e)^n)
```

Rubi in Sympy [A] time = 88.7438, size = 168, normalized size = 0.8

$$\frac{x^{m+1}\left(1+\frac{fx}{e}\right)^{-n}(e+fx)^n \operatorname{appellf}_1\left(m+1,1,-n,m+2,-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}},-\frac{fx}{e}\right)}{3a(m+1)} + \frac{x^{m+1}\left(1+\frac{fx}{e}\right)^{-n}(e+fx)^n \operatorname{appellf}_1\left(m+1,1,-n,m+2,\frac{\sqrt[3]{-1}\sqrt[3]{b}x}{\sqrt[3]{a}},-\frac{fx}{e}\right)}{3a(m+1)} + \frac{x^{m+1}\left(1+\frac{fx}{e}\right)^{-n}(e+fx)^n \operatorname{appellf}_1\left(m+1,1,-n,m+2,-\frac{(-1)^{\frac{2}{3}}\sqrt[3]{b}x}{\sqrt[3]{a}},-\frac{fx}{e}\right)}{3a(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**m*(f*x+e)**n/(b*x**3+a),x)
```

```
[Out] x**(m + 1)*(1 + f*x/e)**(-n)*(e + f*x)**n*appellf1(m + 1, 1, -n,
m + 2, -b**(1/3)*x/a**(1/3), -f*x/e)/(3*a*(m + 1)) + x**(m + 1)*(
1 + f*x/e)**(-n)*(e + f*x)**n*appellf1(m + 1, 1, -n, m + 2, (-1)*
*(1/3)*b**(1/3)*x/a**(1/3), -f*x/e)/(3*a*(m + 1)) + x**(m + 1)*(1
 + f*x/e)**(-n)*(e + f*x)**n*appellf1(m + 1, 1, -n, m + 2, -(-1)*
*(2/3)*b**(1/3)*x/a**(1/3), -f*x/e)/(3*a*(m + 1))
```

Mathematica [A] time = 0.0833978, size = 0, normalized size = 0.

$$\int \frac{x^m(e+fx)^n}{a+bx^3}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(x^m*(e + f*x)^n)/(a + b*x^3),x]
```

```
[Out] Integrate[(x^m*(e + f*x)^n)/(a + b*x^3), x]
```

Maple [F] time = 0.102, size = 0, normalized size = 0.

$$\int \frac{x^m\,(fx+e)^n}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^m*(f*x+e)^n/(b*x^3+a),x)

[Out] int(x^m*(f*x+e)^n/(b*x^3+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^n x^m}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x + e)^n*x^m/(b*x^3 + a),x, algorithm="maxima")

[Out] integrate((f*x + e)^n*x^m/(b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(fx+e)^n x^m}{bx^3+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x + e)^n*x^m/(b*x^3 + a),x, algorithm="fricas")

[Out] integral((f*x + e)^n*x^m/(b*x^3 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**m*(f*x+e)**n/(b*x**3+a),x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^n x^m}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x + e)^n*x^m/(b*x^3 + a),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^n*x^m/(b*x^3 + a), x)
```

3.173 $\int \frac{\sqrt{c+dx^3}}{a+bx} \, dx$

Optimal. Leaf size=1482

result too large to display

```
[Out] (2*Sqrt[c + d*x^3])/(3*b) - (2*a*d^(1/3)*Sqrt[c + d*x^3])/(b^2*((
1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (c^(1/6)*Sqrt[b*c^(1/3) - a*
d^(1/3)]*Sqrt[b^2*c^(2/3) + a*b*c^(1/3)*d^(1/3) + a^2*d^(2/3)]*(c
^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3)*(1 - (d^(1/3)*x)/c^(1/3) + (d^(
2/3)*x^2)/c^(2/3)))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*ArcTan
h[(Sqrt[2 - Sqrt[3]]*Sqrt[b^2*c^(2/3) + a*b*c^(1/3)*d^(1/3) + a^2
*d^(2/3)]*Sqrt[1 - ((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)^2/((1 + Sq
rt[3])*c^(1/3) + d^(1/3)*x)^2])/(3^(1/4)*Sqrt[b]*c^(1/6)*Sqrt[b*c
^(1/3) - a*d^(1/3)]*Sqrt[7 - 4*Sqrt[3] + ((1 - Sqrt[3])*c^(1/3) +
 d^(1/3)*x)^2/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2])])/(b^(5/2)*
Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(
1/3)*x)^2]*Sqrt[c + d*x^3]) + (3^(1/4)*Sqrt[2 - Sqrt[3]]*a*c^(1/3
)*d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x
 + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[
ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3)
 + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^2*Sqrt[(c^(1/3)*(c^(1/3) + d^
(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
 + (2*Sqrt[2 + Sqrt[3]]*a*((1 - Sqrt[3])*b*c^(1/3) + a*d^(1/3))*d
^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x +
d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[Arc
Sin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) +
d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^3*Sqrt[(c^(1/3)*(c^(1/3)
 + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*
x^3]) - (2*Sqrt[2 + Sqrt[3]]*(b^3*c - a^3*d)*(c^(1/3) + d^(1/3)*x
)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])
*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3)
+ d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]
])/(3^(1/4)*b^3*((1 + Sqrt[3])*b*c^(1/3) - a*d^(1/3))*Sqrt[(c^(1/
3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*
Sqrt[c + d*x^3]) - (4*3^(1/4)*Sqrt[2 + Sqrt[3]]*c^(1/3)*(b^3*c -
a^3*d)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3)*(1 - (d^(1/3)*x)/c^(1/
3) + (d^(2/3)*x^2)/c^(2/3)))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^
2]*EllipticPi[((1 + Sqrt[3])*b*c^(1/3) - a*d^(1/3))^2/((1 - Sqrt[
3])*b*c^(1/3) - a*d^(1/3))^2, -ArcSin[((1 - Sqrt[3])*c^(1/3) + d^
(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(
b^2*(2*b^2*c^(2/3) + 2*a*b*c^(1/3)*d^(1/3) - a^2*d^(2/3))*Sqrt[(c
^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)
^2]*Sqrt[c + d*x^3])
```

Rubi [A] time = 5.74643, antiderivative size = 1482, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 12, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$

result too large to display

Warning: Unable to verify antiderivative.

```
[In]  Int[Sqrt[c + d*x^3]/(a + b*x),x]
```

```
[Out] (2*Sqrt[c + d*x^3])/(3*b) - (2*a*d^(1/3)*Sqrt[c + d*x^3])/(b^2*((
1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (c^(1/6)*Sqrt[b*c^(1/3) - a*
d^(1/3)]*Sqrt[b^2*c^(2/3) + a*b*c^(1/3)*d^(1/3) + a^2*d^(2/3)]*(c
^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3)*(1 - (d^(1/3)*x)/c^(1/3) + (d^(
2/3)*x^2)/c^(2/3)))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*ArcTan
h[(Sqrt[2 - Sqrt[3]]*Sqrt[b^2*c^(2/3) + a*b*c^(1/3)*d^(1/3) + a^2
*d^(2/3)]*Sqrt[1 - ((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)^2/((1 + Sq
rt[3])*c^(1/3) + d^(1/3)*x)^2])/(3^(1/4)*Sqrt[b]*c^(1/6)*Sqrt[b*c
^(1/3) - a*d^(1/3)]*Sqrt[7 - 4*Sqrt[3] + ((1 - Sqrt[3])*c^(1/3) +
 d^(1/3)*x)^2/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2])])/(b^(5/2)*
Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(
1/3)*x)^2]*Sqrt[c + d*x^3]) + (3^(1/4)*Sqrt[2 - Sqrt[3]]*a*c^(1/3
)*d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x
 + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[
ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3)
 + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^2*Sqrt[(c^(1/3)*(c^(1/3) + d^
(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
 + (2*Sqrt[2 + Sqrt[3]]*a*((1 - Sqrt[3])*b*c^(1/3) + a*d^(1/3))*d
^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x +
d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[Arc
Sin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) +
d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^3*Sqrt[(c^(1/3)*(c^(1/3)
 + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*
x^3]) - (2*Sqrt[2 + Sqrt[3]]*(b^3*c - a^3*d)*(c^(1/3) + d^(1/3)*x
)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])
*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3)
+ d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]
])/(3^(1/4)*b^3*((1 + Sqrt[3])*b*c^(1/3) - a*d^(1/3))*Sqrt[(c^(1/
3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*
Sqrt[c + d*x^3]) - (4*3^(1/4)*Sqrt[2 + Sqrt[3]]*c^(1/3)*(b^3*c -
a^3*d)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3)*(1 - (d^(1/3)*x)/c^(1/
3) + (d^(2/3)*x^2)/c^(2/3)))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^
2]*EllipticPi[((1 + Sqrt[3])*b*c^(1/3) - a*d^(1/3))^2/((1 - Sqrt[
3])*b*c^(1/3) - a*d^(1/3))^2, -ArcSin[((1 - Sqrt[3])*c^(1/3) + d^
(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(
b^2*(2*b^2*c^(2/3) + 2*a*b*c^(1/3)*d^(1/3) - a^2*d^(2/3))*Sqrt[(c
^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)
^2]*Sqrt[c + d*x^3])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c + dx^3}}{a + bx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**3+c)**(1/2)/(b*x+a),x)
```

```
[Out] Integral(sqrt(c + d*x**3)/(a + b*x), x)
```

Mathematica [C] time = 3.5751, size = 820, normalized size = 0.55

$$2\left(\frac{\sqrt[3]{-1}\sqrt{3}\left(1+\sqrt[3]{-1}\right)\sqrt[3]{c}\,d\sqrt{\frac{\sqrt[3]{d}x+\sqrt[3]{c}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{c}}}\sqrt{\frac{d^{2/3}x^2}{c^{2/3}}-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}+1}\left(\frac{i\sqrt{3}b\sqrt[3]{c}}{\sqrt[3]{d}a+\sqrt[3]{-1}b\sqrt[3]{c}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{d}x+\sqrt[3]{c}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{c}}}\right)|\sqrt[3]{-1}\right)a^3}{b^2\left(\sqrt[3]{d}a+\sqrt[3]{-1}b\sqrt[3]{c}\right)}-\frac{3^{3/4}d^{2/3}\left(\sqrt[3]{-1}\sqrt[3]{c}-\sqrt[3]{d}x\right)\sqrt{\ldots}}{\ldots}\right.$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[Sqrt[c + d*x^3]/(a + b*x),x]
```

```
[Out] (2*(c + d*x^3 - (3^(3/4)*a^2*d^(2/3)*((-1)^(1/3)*c^(1/3) - d^(1/3
)*x)*Sqrt[(c^(1/3) + d^(1/3)*x)/((1 + (-1)^(1/3))*c^(1/3))]*Sqrt[
(-1)^(1/6) - (I*d^(1/3)*x)/c^(1/3)]*EllipticF[ArcSin[Sqrt[(c^(1/3
) + (-1)^(2/3)*d^(1/3)*x)/((1 + (-1)^(1/3))*c^(1/3))]], (-1)^(1/3
)])/(b^2*Sqrt[(c^(1/3) + (-1)^(2/3)*d^(1/3)*x)/((1 + (-1)^(1/3))*
c^(1/3))]) + (3^(3/4)*a*c^(1/3)*d^(1/3)*((-1)^(1/3)*c^(1/3) - d^(
1/3)*x)*Sqrt[I + Sqrt[3] - ((2*I)*d^(1/3)*x)/c^(1/3)]*Sqrt[(I*(1
+ (d^(1/3)*x)/c^(1/3)))/(3*I + Sqrt[3])]*((-1 + (-1)^(2/3))*Ellip
ticE[ArcSin[Sqrt[(-1)^(1/6) - (I*d^(1/3)*x)/c^(1/3)]/3^(1/4)], (-
1)^(1/3)/(-1 + (-1)^(1/3))] + EllipticF[ArcSin[Sqrt[(-1)^(1/6) -
(I*d^(1/3)*x)/c^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))]))/
(b*Sqrt[(c^(1/3) + (-1)^(2/3)*d^(1/3)*x)/((1 + (-1)^(1/3))*c^(1/3
))]) - ((3*I)*b*c^(4/3)*Sqrt[(c^(1/3) + d^(1/3)*x)/((1 + (-1)^(1/
3))*c^(1/3))]*Sqrt[1 - (d^(1/3)*x)/c^(1/3) + (d^(2/3)*x^2)/c^(2/3
)]*EllipticPi[(I*Sqrt[3]*b*c^(1/3))/((-1)^(1/3)*b*c^(1/3) + a*d^(
1/3)), ArcSin[Sqrt[(c^(1/3) + (-1)^(2/3)*d^(1/3)*x)/((1 + (-1)^(1
/3))*c^(1/3))]], (-1)^(1/3)])/((-1)^(1/3)*b*c^(1/3) + a*d^(1/3))
+ ((-1)^(1/3)*Sqrt[3]*(1 + (-1)^(1/3))*a^3*c^(1/3)*d*Sqrt[(c^(1/3
) + d^(1/3)*x)/((1 + (-1)^(1/3))*c^(1/3))]*Sqrt[1 - (d^(1/3)*x)/c
^(1/3) + (d^(2/3)*x^2)/c^(2/3)]*EllipticPi[(I*Sqrt[3]*b*c^(1/3))/
((-1)^(1/3)*b*c^(1/3) + a*d^(1/3)), ArcSin[Sqrt[(c^(1/3) + (-1)^(
2/3)*d^(1/3)*x)/((1 + (-1)^(1/3))*c^(1/3))]], (-1)^(1/3)])/(b^2*(
(-1)^(1/3)*b*c^(1/3) + a*d^(1/3)))))/(3*b*Sqrt[c + d*x^3])
```

Maple [A] time = 0.142, size = 1126, normalized size = 0.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^3+c)^(1/2)/(b*x+a),x)
```

```
[Out] 2/3*(d*x^3+c)^(1/2)/b-2/3*I*a^2/b^3*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+
1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-
c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3
)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1
/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/
2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/
3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2
),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2
)/d*(-c*d^2)^(1/3)))^(1/2))+2/3*I*a/b^2*3^(1/2)*(-c*d^2)^(1/3)*(I
*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*
d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^
(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2
)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))
^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-
c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/
2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*
3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(
I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)
*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*
d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+2/3*I*(a^3*d-
b^3*c)/b^4*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/
2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x
-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)/(-1/
2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)+a/b)*EllipticPi
(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),I*3^(1/2)/d*(-c*d^2)^(1/3)
/(-1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)+a/b),(I*3^
(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^3 + c}}{bx + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^3 + c)/(b*x + a),x, algorithm="maxima")

[Out] integrate(sqrt(d*x^3 + c)/(b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx^3 + c}}{bx + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^3 + c)/(b*x + a),x, algorithm="fricas")

[Out] integral(sqrt(d*x^3 + c)/(b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c + dx^3}}{a + bx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**3+c)**(1/2)/(b*x+a),x)

[Out] Integral(sqrt(c + d*x**3)/(a + b*x), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^3 + c}}{bx + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^3 + c)/(b*x + a),x, algorithm="giac")

[Out] integrate(sqrt(d*x^3 + c)/(b*x + a), x)
```

3.174 $\int \frac{(d^3+e^3x^3)^p}{d+ex}\,dx$

Optimal. Leaf size=135

$$\frac{(d^3+e^3x^3)^p\left(1+\frac{2(d+ex)}{\left(-3+i\sqrt{3}\right)d}\right)^{-p}\left(1-\frac{2(d+ex)}{\left(3+i\sqrt{3}\right)d}\right)^{-p}F_1\left(p;-p,-p;p+1;-\frac{2(d+ex)}{\left(-3+i\sqrt{3}\right)d},\frac{2(d+ex)}{\left(3+i\sqrt{3}\right)d}\right)}{ep}$$

```
[Out] ((d^3 + e^3*x^3)^p*AppellF1[p, -p, -p, 1 + p, (-2*(d + e*x))/((-3
 + I*Sqrt[3])*d), (2*(d + e*x))/((3 + I*Sqrt[3])*d)])/(e*p*(1 + (
2*(d + e*x))/((-3 + I*Sqrt[3])*d))^p*(1 - (2*(d + e*x))/((3 + I*S
qrt[3])*d))^p)
```

Rubi [F] time = 0.121845, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$

$$\text{Int}\left(\frac{(d^3+e^3x^3)^p}{d+ex},x\right)$$

Verification is Not applicable to the result.

```
[In]  Int[(d^3 + e^3*x^3)^p/(d + e*x),x]
```

```
[Out] Defer[Int][(d^3 + e^3*x^3)^p/(d + e*x), x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d^3+e^3x^3)^p}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e**3*x**3+d**3)**p/(e*x+d),x)
```

```
[Out] Integral((d**3 + e**3*x**3)**p/(d + e*x), x)
```

Mathematica [A] time = 0.0431622, size = 0, normalized size = 0.

$$\int \frac{(d^3 + e^3x^3)^p}{d + ex} \, dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(d^3 + e^3*x^3)^p/(d + e*x),x]
```

```
[Out] Integrate[(d^3 + e^3*x^3)^p/(d + e*x), x]
```

Maple [F] time = 0.087, size = 0, normalized size = 0.

$$\int \frac{(e^3x^3 + d^3)^p}{ex + d} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e^3*x^3+d^3)^p/(e*x+d),x)
```

```
[Out] int((e^3*x^3+d^3)^p/(e*x+d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e^3x^3 + d^3)^p}{ex + d} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e^3*x^3 + d^3)^p/(e*x + d),x, algorithm="maxima")
```

```
[Out] integrate((e^3*x^3 + d^3)^p/(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(e^3x^3 + d^3)^p}{ex + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e^3*x^3 + d^3)^p/(e*x + d),x, algorithm="fricas")
```

```
[Out] integral((e^3*x^3 + d^3)^p/(e*x + d), x)
```

Sympy [A] time = 164.096, size = 638, normalized size = 4.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e**3*x**3+d**3)**p/(e*x+d),x)
```

```
[Out] 0**p*log(1 + e**3*x**3/d**3)*gamma(-2/3)*gamma(-1/3)*gamma(4/3)*g
amma(5/3)/(4*pi**2*e) - 0**p*exp(10*I*pi/3)*log(1 - e*x*exp_polar
(I*pi/3)/d)*gamma(-1/3)*gamma(1/3)*gamma(2/3)**2*gamma(4/3)/(6*pi
**2*e*gamma(5/3)) - 0**p*exp(5*I*pi/3)*log(1 - e*x*exp_polar(I*pi
/3)/d)*gamma(1/3)**3*gamma(2/3)**2/(12*pi**2*e*gamma(4/3)) - 0**p
*log(1 - e*x*exp_polar(I*pi)/d)*gamma(-1/3)*gamma(1/3)*gamma(2/3)
**2*gamma(4/3)/(6*pi**2*e*gamma(5/3)) + 0**p*log(1 - e*x*exp_pola
r(I*pi)/d)*gamma(1/3)**3*gamma(2/3)**2/(12*pi**2*e*gamma(4/3)) -
0**p*exp(I*pi/3)*log(1 - e*x*exp_polar(5*I*pi/3)/d)*gamma(1/3)**3
*gamma(2/3)**2/(12*pi**2*e*gamma(4/3)) - 0**p*exp(2*I*pi/3)*log(1
 - e*x*exp_polar(5*I*pi/3)/d)*gamma(-1/3)*gamma(1/3)*gamma(2/3)**
2*gamma(4/3)/(6*pi**2*e*gamma(5/3)) - d**2*e**(3*p)*p*x**(3*p)*ga
mma(-2/3)*gamma(-1/3)*gamma(4/3)*gamma(5/3)*gamma(p)*gamma(-p + 2
/3)*hyper((-p + 1, -p + 2/3), (-p + 5/3,), d**3*exp_polar(I*pi)/(
e**3*x**3))/(4*pi**2*e**3*x**2*gamma(-p + 5/3)*gamma(p + 1)) - d*
e**(3*p)*p*x**(3*p)*gamma(-1/3)*gamma(1/3)*gamma(2/3)*gamma(4/3)*
gamma(p)*gamma(-p + 1/3)*hyper((-p + 1, -p + 1/3), (-p + 4/3,), d
**3*exp_polar(I*pi)/(e**3*x**3))/(4*pi**2*e**2*x*gamma(-p + 4/3)*
gamma(p + 1)) - d**(3*p)*e**2*x**3*gamma(1/3)**2*gamma(2/3)**2*ga
mma(p)*gamma(-p + 1)*hyper((2, 1, -p + 1), (2, 2), e**3*x**3*exp_
polar(I*pi)/d**3)/(4*pi**2*d**3*gamma(-p)*gamma(p + 1))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(e^3x^3+d^3\right)^p}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e^3*x^3 + d^3)^p/(e*x + d),x, algorithm="giac")
```

```
[Out] integrate((e^3*x^3 + d^3)^p/(e*x + d), x)
```

3.175 $\int \frac{2-2x-x^2}{(2+x^2)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=16

$$2\tan^{-1}\left(\frac{x+1}{\sqrt{x^3+1}}\right)$$

```
[Out] 2*ArcTan[(1 + x)/Sqrt[1 + x^3]]
```

Rubi [A] time = 0.108783, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074

$$2\tan^{-1}\left(\frac{x+1}{\sqrt{x^3+1}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(2 - 2*x - x^2)/((2 + x^2)*Sqrt[1 + x^3]),x]

[Out] 2*ArcTan[(1 + x)/Sqrt[1 + x^3]]
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2-2*x+2)/(x**2+2)/(x**3+1)**(1/2),x)

[Out] Timed out
```

Mathematica [C] time = 1.36196, size = 296, normalized size = 18.5

$$\frac{2\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\sqrt{x^2-x+1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{-1}\right)\left(\sqrt[3]{-1}-x\right)F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{(-1)^{2/3}x+1}-\frac{3i\left(\sqrt{2}-i\right)\left(\frac{2\sqrt{3}}{-i-2\sqrt{2}+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{(-1)^{5/6}+\sqrt{2}}+\frac{3\left(5+i\sqrt{2}+i\sqrt{3}\cdots\right.}{\cdots}\right.}{3\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 - 2*x - x^2)/((2 + x^2)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*Sqrt[1 - x + x^2]*((Sqrt[3]*(1
+ (-1)^(1/3))*((-1)^(1/3) - x)*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2
/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(1 + (-1)^(2/3)*x) - ((3*
I)*(-I + Sqrt[2])*EllipticPi[(2*Sqrt[3])/(-I - 2*Sqrt[2] + Sqrt[3
]), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)
])/((-1)^(5/6) + Sqrt[2]) + (3*(5 + I*Sqrt[2] + I*Sqrt[3] + Sqrt[
6])*EllipticPi[(2*Sqrt[3])/(-I + 2*Sqrt[2] + Sqrt[3]), ArcSin[Sqr
t[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(5*I + 2*Sq
rt[2] + Sqrt[3] + (2*I)*Sqrt[6])))/(3*Sqrt[1 + x^3])
```

Maple [C] time = 0.097, size = 1640, normalized size = 102.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2-2*x+2)/(x^2+2)/(x^3+1)^(1/2),x)
```

```
[Out] -2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-
1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))
/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-
1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))
^(1/2))-3*I*2^(1/2)*(1/(3/2-1/2*I*3^(1/2))+1/(3/2-1/2*I*3^(1/2))*
x)^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I
/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/
2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/
(x^3+1)^(1/2)/(-1-I*2^(1/2))*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)
))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1-I*2^(1/2)),((-3/2+1/2*I*3^(1/2)
)/(-3/2-1/2*I*3^(1/2)))^(1/2))-2^(1/2)*(1/(3/2-1/2*I*3^(1/2))+1/(
3/2-1/2*I*3^(1/2))*x)^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1
/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+
1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/
2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1-I*2^(1/2))*EllipticPi(((1+x)
/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1-I*2^(1/2)),(
(-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)-3*(1/(3
/2-1/2*I*3^(1/2))+1/(3/2-1/2*I*3^(1/2))*x)^(1/2)*(1/(-3/2-1/2*I*3
^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(
1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/
2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1-I*2^(1/
2))*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1
/2))/(-1-I*2^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(
1/2))+I*(1/(3/2-1/2*I*3^(1/2))+1/(3/2-1/2*I*3^(1/2))*x)^(1/2)*(1/
(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I
*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*
```

```
I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2
)/(-1-I*2^(1/2))*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3
/2+1/2*I*3^(1/2))/(-1-I*2^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*
I*3^(1/2)))^(1/2))*3^(1/2)+3*I*2^(1/2)*(1/(3/2-1/2*I*3^(1/2))+1/(
3/2-1/2*I*3^(1/2))*x)^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1
/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+
1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/
2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(I*2^(1/2)-1)*EllipticPi(((1+x)/
(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(I*2^(1/2)-1),((-
3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+2^(1/2)*(1/(3/2-1
/2*I*3^(1/2))+1/(3/2-1/2*I*3^(1/2))*x)^(1/2)*(1/(-3/2-1/2*I*3^(1/
2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2)
)^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/
(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(I*2^(1/2)-1)*E
llipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/
(I*2^(1/2)-1),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*
3^(1/2)-3*(1/(3/2-1/2*I*3^(1/2))+1/(3/2-1/2*I*3^(1/2))*x)^(1/2)*(
1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2
*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/
2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1
/2)/(I*2^(1/2)-1)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-
3/2+1/2*I*3^(1/2))/(I*2^(1/2)-1),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*
I*3^(1/2)))^(1/2))+I*(1/(3/2-1/2*I*3^(1/2))+1/(3/2-1/2*I*3^(1/2))
*x)^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*
I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1
/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)
/(x^3+1)^(1/2)/(I*2^(1/2)-1)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)
))^(1/2),(-3/2+1/2*I*3^(1/2))/(I*2^(1/2)-1),((-3/2+1/2*I*3^(1/2))
/(-3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2 + 2x - 2}{\sqrt{x^3 + 1}(x^2 + 2)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x^2 + 2*x - 2)/(sqrt(x^3 + 1)*(x^2 + 2)),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 + 2*x - 2)/(sqrt(x^3 + 1)*(x^2 + 2)), x)
```

Fricas [A] time = 0.297622, size = 26, normalized size = 1.62

$$-\arctan\left(\frac{x^2 - 2x}{2\sqrt{x^3 + 1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 + 2*x - 2)/(sqrt(x^3 + 1)*(x^2 + 2)),x, algorithm="fricas")
```

```
[Out] -arctan(1/2*(x^2 - 2*x)/sqrt(x^3 + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{2x}{x^2\sqrt{x^3+1}+2\sqrt{x^3+1}}\,dx-\int \frac{x^2}{x^2\sqrt{x^3+1}+2\sqrt{x^3+1}}\,dx-\int \left(-\frac{2}{x^2\sqrt{x^3+1}+2\sqrt{x^3+1}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2-2*x+2)/(x**2+2)/(x**3+1)**(1/2),x)
```

```
[Out] -Integral(2*x/(x**2*sqrt(x**3 + 1) + 2*sqrt(x**3 + 1)), x) - Inte
gral(x**2/(x**2*sqrt(x**3 + 1) + 2*sqrt(x**3 + 1)), x) - Integral
(-2/(x**2*sqrt(x**3 + 1) + 2*sqrt(x**3 + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2+2\,x-2}{\sqrt{x^3+1}(x^2+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 + 2*x - 2)/(sqrt(x^3 + 1)*(x^2 + 2)),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 + 2*x - 2)/(sqrt(x^3 + 1)*(x^2 + 2)), x)
```

3.176 $\int \frac{2+2x-x^2}{(2+x^2)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=20

$$-2\tan^{-1}\left(\frac{1-x}{\sqrt{1-x^3}}\right)$$

```
[Out] -2*ArcTan[(1 - x)/Sqrt[1 - x^3]]
```

Rubi [A] time = 0.132165, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069

$$-2\tan^{-1}\left(\frac{1-x}{\sqrt{1-x^3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + 2*x - x^2)/((2 + x^2)*Sqrt[1 - x^3]),x]
```

```
[Out] -2*ArcTan[(1 - x)/Sqrt[1 - x^3]]
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2+2*x+2)/(x**2+2)/(-x**3+1)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 1.04607, size = 280, normalized size = 14.

$$\frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\sqrt{x^2+x+1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{-1}\right)\left(x+\sqrt[3]{-1}\right)F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{(-1)^{2/3}x-1}+\frac{6\left(1+i\sqrt{2}\right)\left(\frac{2\sqrt{3}}{-i-2\sqrt{2}+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{i+2\sqrt{2}-\sqrt{3}}+\frac{3\left(1-i\sqrt{2}\right)\left(\frac{\quad}{-i+\cdots}\right.}{\cdots}\right.}{3\sqrt{1-x^3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + 2*x - x^2)/((2 + x^2)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*Sqrt[1 + x + x^2]*((Sqrt[3]*(1
+ (-1)^(1/3))*((-1)^(1/3) + x)*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2
/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(-1 + (-1)^(2/3)*x) + (6*
(1 + I*Sqrt[2])*EllipticPi[(2*Sqrt[3])/(-I - 2*Sqrt[2] + Sqrt[3])
, ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])
/(I + 2*Sqrt[2] - Sqrt[3]) + (3*(1 - I*Sqrt[2])*EllipticPi[(2*Sqr
t[3])/(-I + 2*Sqrt[2] + Sqrt[3]), ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/
(1 + (-1)^(1/3))]], (-1)^(1/3)])/((-1)^(5/6) - Sqrt[2])))/(3*Sqrt
[1 - x^3])
```

Maple [C] time = 0.097, size = 732, normalized size = 36.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2+2*x+2)/(x^2+2)/(-x^3+1)^(1/2),x)
```

```
[Out] 2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/(-3
/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)
/(-x^3+1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^
(1/2))^(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))-2/3*I*3^(1/2
)*(I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x
-1/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(
1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)-I*2^(1/2))*EllipticPi(1/3
*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+
1/2*I*3^(1/2)-I*2^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))-
2/3*2^(1/2)*3^(1/2)*(I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*(1/(-3/
2+1/2*I*3^(1/2))*x-1/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x-1/
2*I*3^(1/2)+3/2)^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)-I*2^(1/
2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2
),I*3^(1/2)/(-1/2+1/2*I*3^(1/2)-I*2^(1/2)),(I*3^(1/2)/(-3/2+1/2*I
*3^(1/2)))^(1/2))+2/3*2^(1/2)*3^(1/2)*(I*3^(1/2)*x+1/2*I*3^(1/2)+
3/2)^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/(-3/2+1/2*I*3^(1/2)))^(1/2
)*(-I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2
*I*3^(1/2)+I*2^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1
/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2)+I*2^(1/2)),(I*3
^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))-2/3*I*3^(1/2)*(I*3^(1/2)*x+1/
2*I*3^(1/2)+3/2)^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/(-3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3+1)^(1/
2)/(-1/2+1/2*I*3^(1/2)+I*2^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/
2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2)+I*2
^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2 - 2x - 2}{\sqrt{-x^3 + 1}(x^2 + 2)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x^2 - 2*x - 2)/(sqrt(-x^3 + 1)*(x^2 + 2)),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 2*x - 2)/(sqrt(-x^3 + 1)*(x^2 + 2)), x)
```

Fricas [A] time = 0.306091, size = 26, normalized size = 1.3

$$\arctan\left(\frac{x^2 + 2x}{2\sqrt{-x^3 + 1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x^2 - 2*x - 2)/(sqrt(-x^3 + 1)*(x^2 + 2)),x, algorithm="fricas")
```

```
[Out] arctan(1/2*(x^2 + 2*x)/sqrt(-x^3 + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \left(-\frac{2x}{x^2\sqrt{-x^3+1} + 2\sqrt{-x^3+1}}\right) dx - \int \frac{x^2}{x^2\sqrt{-x^3+1} + 2\sqrt{-x^3+1}}\, dx - \int \left(-\frac{2}{x^2\sqrt{-x^3+1} + 2\sqrt{-x^3+1}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+2*x+2)/(x**2+2)/(-x**3+1)**(1/2),x)
```

```
[Out] -Integral(-2*x/(x**2*sqrt(-x**3 + 1) + 2*sqrt(-x**3 + 1)), x) - I
ntegral(x**2/(x**2*sqrt(-x**3 + 1) + 2*sqrt(-x**3 + 1)), x) - Int
egral(-2/(x**2*sqrt(-x**3 + 1) + 2*sqrt(-x**3 + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2 - 2x - 2}{\sqrt{-x^3 + 1}(x^2 + 2)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 2*x - 2)/(sqrt(-x^3 + 1)*(x^2 + 2)),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 2*x - 2)/(sqrt(-x^3 + 1)*(x^2 + 2)), x)
```

3.177 $\int \frac{2+2x-x^2}{(2+x^2)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=18

$$-2\tanh^{-1}\left(\frac{1-x}{\sqrt{x^3-1}}\right)$$

```
[Out] -2*ArcTanh[(1 - x)/Sqrt[-1 + x^3]]
```

Rubi [A] time = 0.111001, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$

$$-2\tanh^{-1}\left(\frac{1-x}{\sqrt{x^3-1}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + 2*x - x^2)/((2 + x^2)*Sqrt[-1 + x^3]),x]
```

```
[Out] -2*ArcTanh[(1 - x)/Sqrt[-1 + x^3]]
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2+2*x+2)/(x**2+2)/(x**3-1)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 1.01832, size = 278, normalized size = 15.44

$$\frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\sqrt{x^2+x+1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{-1}\right)\left(x+\sqrt[3]{-1}\right)F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{(-1)^{2/3}x-1}+\frac{6\left(1+i\sqrt{2}\right)\left(\frac{2\sqrt{3}}{-i-2\sqrt{2}+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{i+2\sqrt{2}-\sqrt{3}}+\frac{3\left(1-i\sqrt{2}\right)\left(\frac{}{-i+}\right.}{}\right.}{3\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + 2*x - x^2)/((2 + x^2)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*Sqrt[1 + x + x^2]*((Sqrt[3]*(1
+ (-1)^(1/3))*((-1)^(1/3) + x)*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2
/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(-1 + (-1)^(2/3)*x) + (6*
(1 + I*Sqrt[2])*EllipticPi[(2*Sqrt[3])/(-I - 2*Sqrt[2] + Sqrt[3])
, ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])
/(I + 2*Sqrt[2] - Sqrt[3]) + (3*(1 - I*Sqrt[2])*EllipticPi[(2*Sqr
t[3])/(-I + 2*Sqrt[2] + Sqrt[3]), ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/
(1 + (-1)^(1/3))]], (-1)^(1/3)])/((-1)^(5/6) - Sqrt[2])))/(3*Sqrt
[-1 + x^3])
```

Maple [C] time = 0.075, size = 1656, normalized size = 92.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2+2*x+2)/(x^2+2)/(x^3-1)^(1/2),x)
```

```
[Out] -2*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1
/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2
))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((-1+x)/(-3
/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2))
)^(1/2))-3*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2
)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2
*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*
I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)
/(-I*2^(1/2)+1)*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3
/2+1/2*I*3^(1/2))/(-I*2^(1/2)+1),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*
3^(1/2)))^(1/2))-I*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2
)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/
(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(
3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-
1)^(1/2)/(-I*2^(1/2)+1)*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^
(1/2),(3/2+1/2*I*3^(1/2))/(-I*2^(1/2)+1),((3/2+1/2*I*3^(1/2))/(3/
2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)+3*I*2^(1/2)*(1/(-3/2-1/2*I*3^(1/
2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/
(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(
3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(
1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(-I*2^(1/2)+1)*EllipticPi(((-1
+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(-I*2^(1/2)+1
),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-2^(1/2)*(1/(-3
/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3
^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/
2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/
```

```
(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(-I*2^(1/2)+1)*E
llipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))
/(-I*2^(1/2)+1),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*
3^(1/2)-3*I*2^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/
2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I
/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/
(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3
-1)^(1/2)/(I*2^(1/2)+1)*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^
(1/2),(3/2+1/2*I*3^(1/2))/(I*2^(1/2)+1),((3/2+1/2*I*3^(1/2))/(3/2
-1/2*I*3^(1/2)))^(1/2))+2^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2
-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^
(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(
1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2)
)^(1/2)/(x^3-1)^(1/2)/(I*2^(1/2)+1)*EllipticPi(((-1+x)/(-3/2-1/2*
I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(I*2^(1/2)+1),((3/2+1/2*I*3
^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)-3*(1/(-3/2-1/2*I*3^(1
/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2
/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/
(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^
(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(I*2^(1/2)+1)*EllipticPi(((-1
+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(I*2^(1/2)+1)
,((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-I*(1/(-3/2-1/2*
I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))
*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/
2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/
2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(I*2^(1/2)+1)*EllipticP
i(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(I*2^(1
/2)+1),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2 - 2x - 2}{\sqrt{x^3 - 1}(x^2 + 2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x^2 - 2*x - 2)/(sqrt(x^3 - 1)*(x^2 + 2)),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 2*x - 2)/(sqrt(x^3 - 1)*(x^2 + 2)), x)
```

Fricas [A] time = 0.271031, size = 34, normalized size = 1.89

$$\log\left(\frac{x^2 + 2x + 2\sqrt{x^3 - 1}}{x^2 + 2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x^2 - 2*x - 2)/(sqrt(x^3 - 1)*(x^2 + 2)),x, algorithm="fricas")
```

```
[Out] log((x^2 + 2*x + 2*sqrt(x^3 - 1))/(x^2 + 2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\left(-\frac{2x}{x^2\sqrt{x^3-1}+2\sqrt{x^3-1}}\right)dx-\int\frac{x^2}{x^2\sqrt{x^3-1}+2\sqrt{x^3-1}}\,dx-\int\left(-\frac{2}{x^2\sqrt{x^3-1}+2\sqrt{x^3-1}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+2*x+2)/(x**2+2)/(x**3-1)**(1/2),x)
```

```
[Out] -Integral(-2*x/(x**2*sqrt(x**3 - 1) + 2*sqrt(x**3 - 1)), x) - Int
egral(x**2/(x**2*sqrt(x**3 - 1) + 2*sqrt(x**3 - 1)), x) - Integra
l(-2/(x**2*sqrt(x**3 - 1) + 2*sqrt(x**3 - 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int-\frac{x^2-2\,x-2}{\sqrt{x^3-1}(x^2+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x^2 - 2*x - 2)/(sqrt(x^3 - 1)*(x^2 + 2)),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 2*x - 2)/(sqrt(x^3 - 1)*(x^2 + 2)), x)
```

3.178 $\int \frac{2-2x-x^2}{(2+x^2)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=18

$$2\tanh^{-1}\left(\frac{x+1}{\sqrt{-x^3-1}}\right)$$

```
[Out] 2*ArcTanh[(1 + x)/Sqrt[-1 - x^3]]
```

Rubi [A] time = 0.123203, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$

$$2\tanh^{-1}\left(\frac{x+1}{\sqrt{-x^3-1}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(2 - 2*x - x^2)/((2 + x^2)*Sqrt[-1 - x^3]),x]
```

```
[Out] 2*ArcTanh[(1 + x)/Sqrt[-1 - x^3]]
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2-2*x+2)/(x**2+2)/(-x**3-1)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 1.18729, size = 298, normalized size = 16.56

$$\frac{2\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\sqrt{x^2-x+1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{-1}\right)\left(\sqrt[3]{-1}-x\right)F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{(-1)^{2/3}x+1}-\frac{3i\left(\sqrt{2}-i\right)\left(\frac{2\sqrt{3}}{-i-2\sqrt{2}+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{(-1)^{5/6}+\sqrt{2}}+\frac{3\left(5+i\sqrt{2}+i\sqrt{3}\cdots\right.}{\cdots}\right.}{3\sqrt{-x^3-1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 - 2*x - x^2)/((2 + x^2)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*Sqrt[1 - x + x^2]*((Sqrt[3]*(1
+ (-1)^(1/3))*((-1)^(1/3) - x)*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2
/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(1 + (-1)^(2/3)*x) - ((3*
I)*(-I + Sqrt[2])*EllipticPi[(2*Sqrt[3])/(-I - 2*Sqrt[2] + Sqrt[3
]), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)
])/((-1)^(5/6) + Sqrt[2]) + (3*(5 + I*Sqrt[2] + I*Sqrt[3] + Sqrt[
6])*EllipticPi[(2*Sqrt[3])/(-I + 2*Sqrt[2] + Sqrt[3]), ArcSin[Sqr
t[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(5*I + 2*Sq
rt[2] + Sqrt[3] + (2*I)*Sqrt[6])))/(3*Sqrt[-1 - x^3])
```

Maple [C] time = 0.092, size = 724, normalized size = 40.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2-2*x+2)/(x^2+2)/(-x^3-1)^(1/2),x)
```

```
[Out] 2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2
+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(
-x^3-1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1
/2))^(1/2),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))-2/3*2^(1/2)*3^(
1/2)*(I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))
+1/(3/2+1/2*I*3^(1/2))*x)^(1/2)*(-I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^
(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2)-I*2^(1/2))*EllipticPi(1/3
*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1
/2*I*3^(1/2)-I*2^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))+2/
3*I*3^(1/2)*(I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3
^(1/2))+1/(3/2+1/2*I*3^(1/2))*x)^(1/2)*(-I*3^(1/2)*x+1/2*I*3^(1/2
)+3/2)^(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2)-I*2^(1/2))*Ellipti
cPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)
/(1/2+1/2*I*3^(1/2)-I*2^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1
/2))+2/3*2^(1/2)*3^(1/2)*(I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)*(1
/(3/2+1/2*I*3^(1/2))+1/(3/2+1/2*I*3^(1/2))*x)^(1/2)*(-I*3^(1/2)*x
+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2)+I*2^(
1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1
/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+I*2^(1/2)),(I*3^(1/2)/(3/2+1/2*I
*3^(1/2)))^(1/2))+2/3*I*3^(1/2)*(I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(
1/2)*(1/(3/2+1/2*I*3^(1/2))+1/(3/2+1/2*I*3^(1/2))*x)^(1/2)*(-I*3^
(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2
)+I*2^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1
/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+I*2^(1/2)),(I*3^(1/2)/(3/
2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2 + 2x - 2}{\sqrt{-x^3 - 1}(x^2 + 2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x^2 + 2*x - 2)/(sqrt(-x^3 - 1)*(x^2 + 2)),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 + 2*x - 2)/(sqrt(-x^3 - 1)*(x^2 + 2)), x)
```

Fricas [A] time = 0.269407, size = 38, normalized size = 2.11

$$\log\left(-\frac{x^2 - 2x - 2\sqrt{-x^3 - 1}}{x^2 + 2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x^2 + 2*x - 2)/(sqrt(-x^3 - 1)*(x^2 + 2)),x, algorithm="fricas")
```

```
[Out] log(-(x^2 - 2*x - 2*sqrt(-x^3 - 1))/(x^2 + 2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{2x}{x^2\sqrt{-x^3 - 1} + 2\sqrt{-x^3 - 1}}\,dx - \int \frac{x^2}{x^2\sqrt{-x^3 - 1} + 2\sqrt{-x^3 - 1}}\,dx - \int \left(-\frac{2}{x^2\sqrt{-x^3 - 1} + 2\sqrt{-x^3 - 1}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2-2*x+2)/(x**2+2)/(-x**3-1)**(1/2),x)
```

```
[Out] -Integral(2*x/(x**2*sqrt(-x**3 - 1) + 2*sqrt(-x**3 - 1)), x) - In
tegral(x**2/(x**2*sqrt(-x**3 - 1) + 2*sqrt(-x**3 - 1)), x) - Inte
gral(-2/(x**2*sqrt(-x**3 - 1) + 2*sqrt(-x**3 - 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2 + 2x - 2}{\sqrt{-x^3 - 1}(x^2 + 2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x^2 + 2*x - 2)/(sqrt(-x^3 - 1)*(x^2 + 2)),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 + 2*x - 2)/(sqrt(-x^3 - 1)*(x^2 + 2)), x)
```

3.179 $\int \frac{2-2x-x^2}{(2+d+dx+x^2)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=30

$$\frac{2\tan^{-1}\left(\frac{\sqrt{d+1}(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{d+1}}$$

```
[Out] (2*ArcTan[(Sqrt[1 + d]*(1 + x))/Sqrt[1 + x^3]])/Sqrt[1 + d]
```

Rubi [A] time = 0.143266, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065

$$\frac{2\tan^{-1}\left(\frac{\sqrt{d+1}(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{d+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(2 - 2*x - x^2)/((2 + d + d*x + x^2)*Sqrt[1 + x^3]),x]

[Out] (2*ArcTan[(Sqrt[1 + d]*(1 + x))/Sqrt[1 + x^3]])/Sqrt[1 + d]
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2-2*x+2)/(d*x+x**2+d+2)/(x**3+1)**(1/2),x)

[Out] Timed out
```

Mathematica [C] time = 2.05576, size = 424, normalized size = 14.13

$$\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\sqrt{x^2-x+1}\left(\frac{2\sqrt{3}\left(1+\sqrt[3]{-1}\right)\left(\sqrt[3]{-1}-x\right)F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{(-1)^{2/3}x+1}-\frac{3i\left(\left(-\left(1+\sqrt[3]{-1}\right)d^2+\left(1+\sqrt[3]{-1}\right)\left(\sqrt{d^2-4d-8}+4\right)d-2\sqrt[3]{-1}\sqrt{d^2-4d-\cdots}\right.\right.}{\cdots}\right.$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(2 - 2*x - x^2)/((2 + d + d*x + x^2)*Sqrt[1 + x^3]),x]
```

```
[Out] (Sqrt[(1 + x)/(1 + (-1)^(1/3))]*Sqrt[1 - x + x^2]*((2*Sqrt[3]*(1
+ (-1)^(1/3))*((-1)^(1/3) - x)*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2
/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(1 + (-1)^(2/3)*x) - ((3*
I)*((8 + 8*(-1)^(1/3) - (1 + (-1)^(1/3))*d^2 + 4*Sqrt[-8 - 4*d +
d^2] - 2*(-1)^(1/3)*Sqrt[-8 - 4*d + d^2] + (1 + (-1)^(1/3))*d*(4
+ Sqrt[-8 - 4*d + d^2]))*EllipticPi[((2*I)*Sqrt[3])/(2*(-1)^(1/3)
 + d - Sqrt[-8 - 4*d + d^2]), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 +
 (-1)^(1/3))]], (-1)^(1/3)] + ((1 + (-1)^(1/3))*d^2 + (1 + (-1)^(
1/3))*d*(-4 + Sqrt[-8 - 4*d + d^2]) - 2*(4 + 4*(-1)^(1/3) - 2*Sqr
t[-8 - 4*d + d^2] + (-1)^(1/3)*Sqrt[-8 - 4*d + d^2]))*EllipticPi[
((2*I)*Sqrt[3])/(2*(-1)^(1/3) + d + Sqrt[-8 - 4*d + d^2]), ArcSin
[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)]))/((2 +
(-1)^(2/3) + d + (-1)^(1/3)*d)*Sqrt[-8 - 4*d + d^2])))/(3*Sqrt[1
+ x^3])
```

Maple [C]   time = 0.057, size = 4397, normalized size = 146.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2-2*x+2)/(d*x+x^2+d+2)/(x^3+1)^(1/2),x)
```

```
[Out] -2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-
1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))
/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-
1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))
^(1/2))-3/2/(d^2-4*d-8)^(1/2)*(1/(3/2-1/2*I*3^(1/2))+1/(3/2-1/2*I
*3^(1/2))*x)^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1
/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(
1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/
2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2))*Elliptic
Pi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1+1/2
*d-1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/
2)))^(1/2))*d^2-4*I/(d^2-4*d-8)^(1/2)*(1/(3/2-1/2*I*3^(1/2))+1/(3
/2-1/2*I*3^(1/2))*x)^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/
2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1
/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2
))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2))*
EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))
/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2
*I*3^(1/2)))^(1/2))*3^(1/2)+3/2*(1/(3/2-1/2*I*3^(1/2))+1/(3/2-1/2
*I*3^(1/2))*x)^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^
(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3
```

```
^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(
1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2))*Ellipt
icPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1+1
/2*d-1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(
1/2)))^(1/2))*d+1/2*I/(d^2-4*d-8)^(1/2)*(1/(3/2-1/2*I*3^(1/2))+1/
(3/2-1/2*I*3^(1/2))*x)^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-
1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2
+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1
/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2)
)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2
))/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1
/2*I*3^(1/2)))^(1/2))*d^2*3^(1/2)+6/(d^2-4*d-8)^(1/2)*(1/(3/2-1/2
*I*3^(1/2))+1/(3/2-1/2*I*3^(1/2))*x)^(1/2)*(1/(-3/2-1/2*I*3^(1/2)
)*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^
(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-
3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d-1/2*(d^
2-4*d-8)^(1/2))*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/
2+1/2*I*3^(1/2))/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^
(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*d-1/2*I*(1/(3/2-1/2*I*3^(1/2)
)+1/(3/2-1/2*I*3^(1/2))*x)^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-
3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(
-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*
3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d-1/2*(d^2-4*d-8)^(
1/2))*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^
(1/2))/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3
/2-1/2*I*3^(1/2)))^(1/2))*d*3^(1/2)-3*(1/(3/2-1/2*I*3^(1/2))+1/(3
/2-1/2*I*3^(1/2))*x)^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/
2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1
/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2
))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2))*
EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))
/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2
*I*3^(1/2)))^(1/2))+2*I/(d^2-4*d-8)^(1/2)*(1/(3/2-1/2*I*3^(1/2))+
1/(3/2-1/2*I*3^(1/2))*x)^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/
2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3
/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^
(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d+1/2*(d^2-4*d-8)^(1/
2))*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1
/2))/(-1+1/2*d+1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2
-1/2*I*3^(1/2)))^(1/2))*d*3^(1/2)+12/(d^2-4*d-8)^(1/2)*(1/(3/2-1/
2*I*3^(1/2))+1/(3/2-1/2*I*3^(1/2))*x)^(1/2)*(1/(-3/2-1/2*I*3^(1/2
))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))
^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(
-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d-1/2*(d
^2-4*d-8)^(1/2))*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3
/2+1/2*I*3^(1/2))/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3
^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-1/2*I/(d^2-4*d-8)^(1/2)*(1/(
3/2-1/2*I*3^(1/2))+1/(3/2-1/2*I*3^(1/2))*x)^(1/2)*(1/(-3/2-1/2*I*
3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^
(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1
/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d+
1/2*(d^2-4*d-8)^(1/2))*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/
2),(-3/2+1/2*I*3^(1/2))/(-1+1/2*d+1/2*(d^2-4*d-8)^(1/2)),((-3/2+1
/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*d^2*3^(1/2)+3/2/(d^2-4
```

```
*d-8)^(1/2)*(1/(3/2-1/2*I*3^(1/2))+1/(3/2-1/2*I*3^(1/2))*x)^(1/2)
*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1
/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+
1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^
(1/2)/(-1+1/2*d+1/2*(d^2-4*d-8)^(1/2))*EllipticPi(((1+x)/(3/2-1/2
*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1+1/2*d+1/2*(d^2-4*d-8)
^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*d^2+I*
(1/(3/2-1/2*I*3^(1/2))+1/(3/2-1/2*I*3^(1/2))*x)^(1/2)*(1/(-3/2-1/
2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2)
)*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2
))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/
2*d+1/2*(d^2-4*d-8)^(1/2))*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))
^(1/2),(-3/2+1/2*I*3^(1/2))/(-1+1/2*d+1/2*(d^2-4*d-8)^(1/2)),((-3
/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)+3/2*(1/(3/
2-1/2*I*3^(1/2))+1/(3/2-1/2*I*3^(1/2))*x)^(1/2)*(1/(-3/2-1/2*I*3^
(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1
/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2
*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d+1/
2*(d^2-4*d-8)^(1/2))*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)
,(-3/2+1/2*I*3^(1/2))/(-1+1/2*d+1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2
*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*d-1/2*I*(1/(3/2-1/2*I*3^
(1/2))+1/(3/2-1/2*I*3^(1/2))*x)^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1
/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)
*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1
/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d+1/2*(d^2-4*d
-8)^(1/2))*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2
*I*3^(1/2))/(-1+1/2*d+1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2)
)/(-3/2-1/2*I*3^(1/2)))^(1/2))*d*3^(1/2)-6/(d^2-4*d-8)^(1/2)*(1/(
3/2-1/2*I*3^(1/2))+1/(3/2-1/2*I*3^(1/2))*x)^(1/2)*(1/(-3/2-1/2*I*
3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^
(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1
/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d+
1/2*(d^2-4*d-8)^(1/2))*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/
2),(-3/2+1/2*I*3^(1/2))/(-1+1/2*d+1/2*(d^2-4*d-8)^(1/2)),((-3/2+1
/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*d+I*(1/(3/2-1/2*I*3^(1
/2))+1/(3/2-1/2*I*3^(1/2))*x)^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2
/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(
1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2
*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d-1/2*(d^2-4*d-8
)^(1/2))*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I
*3^(1/2))/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/
(-3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)-3*(1/(3/2-1/2*I*3^(1/2))+1/(
3/2-1/2*I*3^(1/2))*x)^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1
/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+
1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/
2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d+1/2*(d^2-4*d-8)^(1/2))
*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2)
)/(-1+1/2*d+1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/
2*I*3^(1/2)))^(1/2))+4*I/(d^2-4*d-8)^(1/2)*(1/(3/2-1/2*I*3^(1/2))
+1/(3/2-1/2*I*3^(1/2))*x)^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3
/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-
3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3
^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d+1/2*(d^2-4*d-8)^(1
/2))*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(
```

```
1/2))/(-1+1/2*d+1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/
2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)-12/(d^2-4*d-8)^(1/2)*(1/(3/2-1/2
*I*3^(1/2))+1/(3/2-1/2*I*3^(1/2))*x)^(1/2)*(1/(-3/2-1/2*I*3^(1/2)
)*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^
(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-
3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d+1/2*(d^
2-4*d-8)^(1/2))*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/
2+1/2*I*3^(1/2))/(-1+1/2*d+1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^
(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-2*I/(d^2-4*d-8)^(1/2)*(1/(3/2
-1/2*I*3^(1/2))+1/(3/2-1/2*I*3^(1/2))*x)^(1/2)*(1/(-3/2-1/2*I*3^(
1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/
2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*
I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d-1/2
*(d^2-4*d-8)^(1/2))*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),
(-3/2+1/2*I*3^(1/2))/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*
I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*d*3^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x^2 + 2*x - 2)/(sqrt(x^3 + 1)*(d*x + x^2 + d + 2)),x, algorithm="maxima
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.291163, size = 1, normalized size = 0.03

$$\left[\frac{\log\left(-\frac{4\left((d+1)x^2-d^2-\left(d^2+3d+2\right)x-d\right)\sqrt{x^3+1}+\left(2(3d+4)x^3-x^4-\left(d^2+2d+4\right)x^2-d^2-2\left(d^2+2d\right)x+4d+4\right)\sqrt{-d-1}}{2dx^3+x^4+(d^2+2d+4)x^2+d^2+2(d^2+2d)x+4d+4}\right)}{2\sqrt{-d-1}}, -\frac{\arctan\left(-\frac{(d+2)x-x^2+d}{2\sqrt{x^3+1}\sqrt{d+1}}\right)}{\sqrt{d+1}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x^2 + 2*x - 2)/(sqrt(x^3 + 1)*(d*x + x^2 + d + 2)),x, algorithm="frica
```

```
[Out] [1/2*log(-(4*((d + 1)*x^2 - d^2 - (d^2 + 3*d + 2)*x - d)*sqrt(x^3
 + 1) + (2*(3*d + 4)*x^3 - x^4 - (d^2 + 2*d + 4)*x^2 - d^2 - 2*(d
```

```
^2 + 2*d)*x + 4*d + 4)*sqrt(-d - 1))/(2*d*x^3 + x^4 + (d^2 + 2*d
+ 4)*x^2 + d^2 + 2*(d^2 + 2*d)*x + 4*d + 4))/sqrt(-d - 1), -arcta
n(-1/2*((d + 2)*x - x^2 + d)/(sqrt(x^3 + 1)*sqrt(d + 1)))/sqrt(d
+ 1)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{2x}{dx\sqrt{x^3+1}+d\sqrt{x^3+1}+x^2\sqrt{x^3+1}+2\sqrt{x^3+1}}\,dx - \int \frac{x^2}{dx\sqrt{x^3+1}+d\sqrt{x^3+1}+x^2\sqrt{x^3+1}+2\sqrt{x^3+1}}\,dx - \int \left(-\frac{2}{dx\sqrt{x^3+1}+d\sqrt{x^3+1}+x^2\sqrt{x^3+1}+2\sqrt{x^3+1}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2-2*x+2)/(d*x+x**2+d+2)/(x**3+1)**(1/2),x)
```

```
[Out] -Integral(2*x/(d*x*sqrt(x**3 + 1) + d*sqrt(x**3 + 1) + x**2*sqrt(
x**3 + 1) + 2*sqrt(x**3 + 1)), x) - Integral(x**2/(d*x*sqrt(x**3
+ 1) + d*sqrt(x**3 + 1) + x**2*sqrt(x**3 + 1) + 2*sqrt(x**3 + 1))
, x) - Integral(-2/(d*x*sqrt(x**3 + 1) + d*sqrt(x**3 + 1) + x**2*
sqrt(x**3 + 1) + 2*sqrt(x**3 + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2+2x-2}{\sqrt{x^3+1}(dx+x^2+d+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x^2 + 2*x - 2)/(sqrt(x^3 + 1)*(d*x + x^2 + d + 2)),x, algorithm="giac"
```

```
[Out] integrate(-(x^2 + 2*x - 2)/(sqrt(x^3 + 1)*(d*x + x^2 + d + 2)), x
)
```

3.180 $\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=38

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{1-d}(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{1-d}}$$

```
[Out] (-2*ArcTan[(Sqrt[1 - d]*(1 - x))/Sqrt[1 - x^3]])/Sqrt[1 - d]
```

Rubi [A] time = 0.182616, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{1-d}(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{1-d}}$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + 2*x - x^2)/((2 - d + d*x + x^2)*Sqrt[1 - x^3]),x]

[Out] (-2*ArcTan[(Sqrt[1 - d]*(1 - x))/Sqrt[1 - x^3]])/Sqrt[1 - d]
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2+2*x+2)/(d*x+x**2-d+2)/(-x**3+1)**(1/2),x)

[Out] Timed out
```

Mathematica [C] time = 2.0837, size = 427, normalized size = 11.24

$$\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\sqrt{x^2+x+1}\left(\frac{2\sqrt{3}\left(1+\sqrt[3]{-1}\right)\left(x+\sqrt[3]{-1}\right)F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{(-1)^{2/3}x-1}+\frac{3i\left(\left(-\left(1+\sqrt[3]{-1}\right)d^2+\left(1+\sqrt[3]{-1}\right)\left(\sqrt{d^2+4d-8}-4\right)d+2\sqrt[3]{-1}\sqrt{d^2+4d-8}\right.\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(2 + 2*x - x^2)/((2 - d + d*x + x^2)*Sqrt[1 - x^3]),x]
```

```
[Out] (Sqrt[(1 - x)/(1 + (-1)^(1/3))]*Sqrt[1 + x + x^2]*((2*Sqrt[3]*(1
+ (-1)^(1/3))*((-1)^(1/3) + x)*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2
/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(-1 + (-1)^(2/3)*x) + ((3
*I)*((8 + 8*(-1)^(1/3) - (1 + (-1)^(1/3))*d^2 - 4*Sqrt[-8 + 4*d +
 d^2] + 2*(-1)^(1/3)*Sqrt[-8 + 4*d + d^2] + (1 + (-1)^(1/3))*d*(-
4 + Sqrt[-8 + 4*d + d^2]))*EllipticPi[((2*I)*Sqrt[3])/(2*(-1)^(1/
3) - d + Sqrt[-8 + 4*d + d^2]), ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1
 + (-1)^(1/3))]], (-1)^(1/3)] + (-8 - 8*(-1)^(1/3) + (1 + (-1)^(1
/3))*d^2 - 4*Sqrt[-8 + 4*d + d^2] + 2*(-1)^(1/3)*Sqrt[-8 + 4*d +
d^2] + (1 + (-1)^(1/3))*d*(4 + Sqrt[-8 + 4*d + d^2]))*EllipticPi[
((-2*I)*Sqrt[3])/(-2*(-1)^(1/3) + d + Sqrt[-8 + 4*d + d^2]), ArcS
in[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)]))/((-2
 - (-1)^(2/3) + d + (-1)^(1/3)*d)*Sqrt[-8 + 4*d + d^2])))/(3*Sqrt
[1 - x^3])
```

Maple [C] time = 0.069, size = 1908, normalized size = 50.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2+2*x+2)/(d*x+x^2-d+2)/(-x^3+1)^(1/2),x)
```

```
[Out] 2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/(-3
/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)
/(-x^3+1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^
(1/2))^(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))+1/3*I/(d^2+4
*d-8)^(1/2)*3^(1/2)*(I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*(1/(-3/
2+1/2*I*3^(1/2))*x-1/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x-1/
2*I*3^(1/2)+3/2)^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d-1
/2*(d^2+4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1
/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2+
4*d-8)^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))*d^2-1/3*I*3
^(1/2)*(I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*(1/(-3/2+1/2*I*3^(1/
2))*x-1/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x-1/2*I*3^(1/2)+3
/2)^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2+4*d-8
)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))
^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2+4*d-8)^(1/2))
,(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))*d+4/3*I/(d^2+4*d-8)^(1/2
)*3^(1/2)*(I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*(1/(-3/2+1/2*I*3^
(1/2))*x-1/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x-1/2*I*3^(1/2
)+3/2)^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2+4*
d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/
2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2+4*d-8)^(1/
```

```
2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))*d-2/3*I*3^(1/2)*(I*3^
(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/(-3/
2+1/2*I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)/(-
x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2+4*d-8)^(1/2))*Ell
ipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(
1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2+4*d-8)^(1/2)),(I*3^(1/2)/
(-3/2+1/2*I*3^(1/2)))^(1/2))-8/3*I/(d^2+4*d-8)^(1/2)*3^(1/2)*(I*3
^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/(-3
/2+1/2*I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)/(
-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2+4*d-8)^(1/2))*El
lipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^
(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2+4*d-8)^(1/2)),(I*3^(1/2)
/(-3/2+1/2*I*3^(1/2)))^(1/2))-1/3*I/(d^2+4*d-8)^(1/2)*3^(1/2)*(I*
3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/(-
3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)/
(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2+4*d-8)^(1/2))*E
llipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3
^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2+4*d-8)^(1/2)),(I*3^(1/2
)/(-3/2+1/2*I*3^(1/2)))^(1/2))*d^2-1/3*I*3^(1/2)*(I*3^(1/2)*x+1/2
*I*3^(1/2)+3/2)^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/(-3/2+1/2*I*3^(
1/2)))^(1/2)*(-I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3+1)^(1/2
)/(-1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2+4*d-8)^(1/2))*EllipticPi(1/3
*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+
1/2*I*3^(1/2)+1/2*d+1/2*(d^2+4*d-8)^(1/2)),(I*3^(1/2)/(-3/2+1/2*I
*3^(1/2)))^(1/2))*d-4/3*I/(d^2+4*d-8)^(1/2)*3^(1/2)*(I*3^(1/2)*x+
1/2*I*3^(1/2)+3/2)^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/(-3/2+1/2*I*
3^(1/2)))^(1/2)*(-I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3+1)^(
1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2+4*d-8)^(1/2))*EllipticPi(
1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1
/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2+4*d-8)^(1/2)),(I*3^(1/2)/(-3/2+1/
2*I*3^(1/2)))^(1/2))*d-2/3*I*3^(1/2)*(I*3^(1/2)*x+1/2*I*3^(1/2)+3
/2)^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/(-3/2+1/2*I*3^(1/2)))^(1/2)
*(-I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*
I*3^(1/2)+1/2*d+1/2*(d^2+4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*
(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2
)+1/2*d+1/2*(d^2+4*d-8)^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(
1/2))+8/3*I/(d^2+4*d-8)^(1/2)*3^(1/2)*(I*3^(1/2)*x+1/2*I*3^(1/2)+
3/2)^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/(-3/2+1/2*I*3^(1/2)))^(1/2
)*(-I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2
*I*3^(1/2)+1/2*d+1/2*(d^2+4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I
*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/
2)+1/2*d+1/2*(d^2+4*d-8)^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^
(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x^2 - 2*x - 2)/(sqrt(-x^3 + 1)*(d*x + x^2 - d + 2)),x, algorithm="maxi
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.289378, size = 1, normalized size = 0.03

$$\left[\frac{\log\left(-\frac{4\left((d-1)x^2+d^2-(d^2-3d+2)x-d\right)\sqrt{-x^3+1}+\left(2(3d-4)x^3-x^4-(d^2-2d+4)x^2-d^2+2(d^2-2d)x-4d+4\right)\sqrt{d-1}}{2dx^3+x^4+(d^2-2d+4)x^2+d^2-2(d^2-2d)x-4d+4}\right)}{2\sqrt{d-1}},\right.$$
$$\left.-\frac{\arctan\left(-\frac{\left((d-2)x-x^2-d\right)\sqrt{-d+1}}{2\sqrt{-x^3+1}(d-1)}\right)}{\sqrt{-d+1}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x^2 - 2*x - 2)/(sqrt(-x^3 + 1)*(d*x + x^2 - d + 2)),x, algorithm="fric
```

```
[Out] [1/2*log(-(4*((d - 1)*x^2 + d^2 - (d^2 - 3*d + 2)*x - d)*sqrt(-x^
3 + 1) + (2*(3*d - 4)*x^3 - x^4 - (d^2 - 2*d + 4)*x^2 - d^2 + 2*(
d^2 - 2*d)*x - 4*d + 4)*sqrt(d - 1))/(2*d*x^3 + x^4 + (d^2 - 2*d
+ 4)*x^2 + d^2 - 2*(d^2 - 2*d)*x - 4*d + 4))/sqrt(d - 1), -arctan
(-1/2*((d - 2)*x - x^2 - d)*sqrt(-d + 1)/(sqrt(-x^3 + 1)*(d - 1))
)/sqrt(-d + 1)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\left(-\frac{2x}{dx\sqrt{-x^3+1}-d\sqrt{-x^3+1}+x^2\sqrt{-x^3+1}+2\sqrt{-x^3+1}}\right)dx$$
$$-\int\frac{x^2}{dx\sqrt{-x^3+1}-d\sqrt{-x^3+1}+x^2\sqrt{-x^3+1}+2\sqrt{-x^3+1}}\,dx$$
$$-\int\left(-\frac{2}{dx\sqrt{-x^3+1}-d\sqrt{-x^3+1}+x^2\sqrt{-x^3+1}+2\sqrt{-x^3+1}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+2*x+2)/(d*x+x**2-d+2)/(-x**3+1)**(1/2),x)
```

```
[Out] -Integral(-2*x/(d*x*sqrt(-x**3 + 1) - d*sqrt(-x**3 + 1) + x**2*sq
rt(-x**3 + 1) + 2*sqrt(-x**3 + 1)), x) - Integral(x**2/(d*x*sqrt(
-x**3 + 1) - d*sqrt(-x**3 + 1) + x**2*sqrt(-x**3 + 1) + 2*sqrt(-x
```

```
**3 + 1)), x) - Integral(-2/(d*x*sqrt(-x**3 + 1) - d*sqrt(-x**3 +
 1) + x**2*sqrt(-x**3 + 1) + 2*sqrt(-x**3 + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2 - 2\,x - 2}{\sqrt{-x^3 + 1}(dx + x^2 - d + 2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x^2 - 2*x - 2)/(sqrt(-x^3 + 1)*(d*x + x^2 - d + 2)),x, algorithm="giac
```

```
[Out] integrate(-(x^2 - 2*x - 2)/(sqrt(-x^3 + 1)*(d*x + x^2 - d + 2)),
x)
```

3.181 $\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=36

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{1-d}(1-x)}{\sqrt{x^3-1}}\right)}{\sqrt{1-d}}$$

```
[Out] (-2*ArcTanh[(Sqrt[1 - d]*(1 - x))/Sqrt[-1 + x^3]])/Sqrt[1 - d]
```

Rubi [A] time = 0.143648, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{1-d}(1-x)}{\sqrt{x^3-1}}\right)}{\sqrt{1-d}}$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + 2*x - x^2)/((2 - d + d*x + x^2)*Sqrt[-1 + x^3]),x]

[Out] (-2*ArcTanh[(Sqrt[1 - d]*(1 - x))/Sqrt[-1 + x^3]])/Sqrt[1 - d]
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2+2*x+2)/(d*x+x**2-d+2)/(x**3-1)**(1/2),x)

[Out] Timed out
```

Mathematica [C] time = 0.615835, size = 425, normalized size = 11.81

$$\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\sqrt{x^2+x+1}\left(\frac{2\sqrt{3}\left(1+\sqrt[3]{-1}\right)\left(x+\sqrt[3]{-1}\right)F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{(-1)^{2/3}x-1}+\frac{3i\left(\left(-\left(1+\sqrt[3]{-1}\right)d^2+\left(1+\sqrt[3]{-1}\right)\left(\sqrt{d^2+4d-8}-4\right)d+2\sqrt[3]{-1}\sqrt{d^2+4d-8}\right.\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(2 + 2*x - x^2)/((2 - d + d*x + x^2)*Sqrt[-1 + x^3]),x]
```

```
[Out] (Sqrt[(1 - x)/(1 + (-1)^(1/3))]*Sqrt[1 + x + x^2]*((2*Sqrt[3]*(1
+ (-1)^(1/3))*((-1)^(1/3) + x)*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2
/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(-1 + (-1)^(2/3)*x) + ((3
*I)*((8 + 8*(-1)^(1/3) - (1 + (-1)^(1/3))*d^2 - 4*Sqrt[-8 + 4*d +
 d^2] + 2*(-1)^(1/3)*Sqrt[-8 + 4*d + d^2] + (1 + (-1)^(1/3))*d*(-
4 + Sqrt[-8 + 4*d + d^2]))*EllipticPi[((2*I)*Sqrt[3])/(2*(-1)^(1/
3) - d + Sqrt[-8 + 4*d + d^2]), ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1
 + (-1)^(1/3))]], (-1)^(1/3)] + (-8 - 8*(-1)^(1/3) + (1 + (-1)^(1
/3))*d^2 - 4*Sqrt[-8 + 4*d + d^2] + 2*(-1)^(1/3)*Sqrt[-8 + 4*d +
d^2] + (1 + (-1)^(1/3))*d*(4 + Sqrt[-8 + 4*d + d^2]))*EllipticPi[
((-2*I)*Sqrt[3])/(-2*(-1)^(1/3) + d + Sqrt[-8 + 4*d + d^2]), ArcS
in[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)]))/((-2
 - (-1)^(2/3) + d + (-1)^(1/3)*d)*Sqrt[-8 + 4*d + d^2])))/(3*Sqrt
[-1 + x^3])
```

Maple [C] time = 0.043, size = 4437, normalized size = 123.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2+2*x+2)/(d*x+x^2-d+2)/(x^3-1)^(1/2),x)
```

```
[Out] -2*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1
/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2
))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((-1+x)/(-3
/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2))
)^(1/2))+3/2/(d^2+4*d-8)^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-
1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(
1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1
/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))
^(1/2)/(x^3-1)^(1/2)/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi((
(-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(1+1/2*d-1
/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(
1/2))*d^2-4*I/(d^2+4*d-8)^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2
-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^
(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(
1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2)
)^(1/2)/(x^3-1)^(1/2)/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(
((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(1+1/2*d-
1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^
(1/2))*3^(1/2)-3/2*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2
)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/
(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(
```

```
3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-
1)^(1/2)/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(((-1+x)/(-3/2
-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(1+1/2*d-1/2*(d^2+4*d-
8)^(1/2)),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*d-I*(1
/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2
*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3
^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/
2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(1+1/2*d-1/2
*(d^2+4*d-8)^(1/2))*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2
),(3/2+1/2*I*3^(1/2))/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I
*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)+6/(d^2+4*d-8)^(1/2)
*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-
1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2)
)*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))
+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(1+1/2*d-
1/2*(d^2+4*d-8)^(1/2))*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(
1/2),(3/2+1/2*I*3^(1/2))/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2)),((3/2+1/
2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*d+4*I/(d^2+4*d-8)^(1/2)*
(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1
/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))
*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+
1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(1+1/2*d+1
/2*(d^2+4*d-8)^(1/2))*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1
/2),(3/2+1/2*I*3^(1/2))/(1+1/2*d+1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2
*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)-3*(1/(-3/2-1/2*I*
3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x
+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)
*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*
I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(1+1/2*d-1/2*(d^2+4*d-8)^
(1/2))*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*
3^(1/2))/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*3^(1/2))/(3/
2-1/2*I*3^(1/2)))^(1/2))-I*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*
I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2)
)-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))
*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/
2)/(x^3-1)^(1/2)/(1+1/2*d+1/2*(d^2+4*d-8)^(1/2))*EllipticPi(((-1+
x)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(1+1/2*d+1/2*(
d^2+4*d-8)^(1/2)),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)
)*3^(1/2)-12/(d^2+4*d-8)^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-
1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(
1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1
/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))
^(1/2)/(x^3-1)^(1/2)/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi((
(-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(1+1/2*d-1
/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(
1/2))+1/2*I/(d^2+4*d-8)^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1
/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1
/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/
2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^
(1/2)/(x^3-1)^(1/2)/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(((
-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(1+1/2*d-1/
2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1
/2))*d^2*3^(1/2)-3/2/(d^2+4*d-8)^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-
1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1
```

```
/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/
2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*
3^(1/2))^(1/2)/(x^3-1)^(1/2)/(1+1/2*d+1/2*(d^2+4*d-8)^(1/2))*Elli
pticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(1
+1/2*d+1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(
1/2)))^(1/2))*d^2-1/2*I/(d^2+4*d-8)^(1/2)*(1/(-3/2-1/2*I*3^(1/2))
*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/
2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2
+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2
))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(1+1/2*d+1/2*(d^2+4*d-8)^(1/2))*E
llipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))
/(1+1/2*d+1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*
3^(1/2)))^(1/2))*d^2*3^(1/2)-3/2*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/
2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3
^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^
(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2
))^(1/2)/(x^3-1)^(1/2)/(1+1/2*d+1/2*(d^2+4*d-8)^(1/2))*EllipticPi
(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(1+1/2*d
+1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))
^(1/2))*d-1/2*I*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))
^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/
2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2
+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^
(1/2)/(1+1/2*d+1/2*(d^2+4*d-8)^(1/2))*EllipticPi(((-1+x)/(-3/2-1/
2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(1+1/2*d+1/2*(d^2+4*d-8)^
(1/2)),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*d*3^(1/2)
-6/(d^2+4*d-8)^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1
/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*
I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2
/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^
3-1)^(1/2)/(1+1/2*d+1/2*(d^2+4*d-8)^(1/2))*EllipticPi(((-1+x)/(-3
/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(1+1/2*d+1/2*(d^2+4*
d-8)^(1/2)),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*d+2*
I/(d^2+4*d-8)^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/
2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I
/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/
(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3
-1)^(1/2)/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(((-1+x)/(-3/
2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(1+1/2*d-1/2*(d^2+4*d
-8)^(1/2)),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*d*3^(
1/2)-3*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1
/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3
^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^
(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(1+
1/2*d+1/2*(d^2+4*d-8)^(1/2))*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/
2)))^(1/2),(3/2+1/2*I*3^(1/2))/(1+1/2*d+1/2*(d^2+4*d-8)^(1/2)),((
3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-1/2*I*(1/(-3/2-1/2
*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2)
)*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1
/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1
/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(1+1/2*d-1/2*(d^2+4*d-
8)^(1/2))*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2
*I*3^(1/2))/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*3^(1/2))/
(3/2-1/2*I*3^(1/2)))^(1/2))*d*3^(1/2)+12/(d^2+4*d-8)^(1/2)*(1/(-3
```

```
/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3
^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/
2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/
(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(1+1/2*d+1/2*(d^
2+4*d-8)^(1/2))*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3
/2+1/2*I*3^(1/2))/(1+1/2*d+1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*3^(
1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-2*I/(d^2+4*d-8)^(1/2)*(1/(-3/2-
1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1
/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))
^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/
2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(1+1/2*d+1/2*(d^2+4
*d-8)^(1/2))*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+
1/2*I*3^(1/2))/(1+1/2*d+1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*3^(1/2
))/(3/2-1/2*I*3^(1/2)))^(1/2))*d*3^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x^2 - 2*x - 2)/(sqrt(x^3 - 1)*(d*x + x^2 - d + 2)),x, algorithm="maxim
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.287666, size = 1, normalized size = 0.03

$$\left[\frac{\log\left(-\frac{4\left((d-1)x^2+d^2-\left(d^2-3\,d+2\right)x-d\right)\sqrt{x^3-1}+\left(2\,(3\,d-4)x^3-x^4-\left(d^2-2\,d+4\right)x^2-d^2+2\left(d^2-2\,d\right)x-4\,d+4\right)\sqrt{-d+1}}{2\,dx^3+x^4+(d^2-2\,d+4)x^2+d^2-2\,(d^2-2\,d)x-4\,d+4}\right)}{2\sqrt{-d+1}},\right.$$
$$\left.-\frac{\arctan\left(-\frac{(d-2)x-x^2-d}{2\sqrt{x^3-1}\sqrt{d-1}}\right)}{\sqrt{d-1}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x^2 - 2*x - 2)/(sqrt(x^3 - 1)*(d*x + x^2 - d + 2)),x, algorithm="frica
```

```
[Out] [1/2*log(-(4*((d - 1)*x^2 + d^2 - (d^2 - 3*d + 2)*x - d)*sqrt(x^3
 - 1) + (2*(3*d - 4)*x^3 - x^4 - (d^2 - 2*d + 4)*x^2 - d^2 + 2*(d
^2 - 2*d)*x - 4*d + 4)*sqrt(-d + 1))/(2*d*x^3 + x^4 + (d^2 - 2*d
+ 4)*x^2 + d^2 - 2*(d^2 - 2*d)*x - 4*d + 4))/sqrt(-d + 1), -arcta
```

```
n(-1/2*((d - 2)*x - x^2 - d)/(sqrt(x^3 - 1)*sqrt(d - 1)))/sqrt(d
- 1)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\left(-\frac{2x}{dx\sqrt{x^3-1}-d\sqrt{x^3-1}+x^2\sqrt{x^3-1}+2\sqrt{x^3-1}}\right)dx$$
$$-\int\frac{x^2}{dx\sqrt{x^3-1}-d\sqrt{x^3-1}+x^2\sqrt{x^3-1}+2\sqrt{x^3-1}}\,dx$$
$$-\int\left(-\frac{2}{dx\sqrt{x^3-1}-d\sqrt{x^3-1}+x^2\sqrt{x^3-1}+2\sqrt{x^3-1}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+2*x+2)/(d*x+x**2-d+2)/(x**3-1)**(1/2),x)
```

```
[Out] -Integral(-2*x/(d*x*sqrt(x**3 - 1) - d*sqrt(x**3 - 1) + x**2*sqrt
(x**3 - 1) + 2*sqrt(x**3 - 1)), x) - Integral(x**2/(d*x*sqrt(x**3
 - 1) - d*sqrt(x**3 - 1) + x**2*sqrt(x**3 - 1) + 2*sqrt(x**3 - 1)
), x) - Integral(-2/(d*x*sqrt(x**3 - 1) - d*sqrt(x**3 - 1) + x**2
*sqrt(x**3 - 1) + 2*sqrt(x**3 - 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int-\frac{x^2-2\,x-2}{\sqrt{x^3-1}(dx+x^2-d+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x^2 - 2*x - 2)/(sqrt(x^3 - 1)*(d*x + x^2 - d + 2)),x, algorithm="giac"
```

```
[Out] integrate(-(x^2 - 2*x - 2)/(sqrt(x^3 - 1)*(d*x + x^2 - d + 2)), x
)
```

3.182 $\int \frac{2-2x-x^2}{(2+d+dx+x^2)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=32

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{d+1}(x+1)}{\sqrt{-x^3-1}}\right)}{\sqrt{d+1}}$$

```
[Out] (2*ArcTanh[(Sqrt[1 + d]*(1 + x))/Sqrt[-1 - x^3]])/Sqrt[1 + d]
```

Rubi [A] time = 0.149565, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{d+1}(x+1)}{\sqrt{-x^3-1}}\right)}{\sqrt{d+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(2 - 2*x - x^2)/((2 + d + d*x + x^2)*Sqrt[-1 - x^3]),x]

[Out] (2*ArcTanh[(Sqrt[1 + d]*(1 + x))/Sqrt[-1 - x^3]])/Sqrt[1 + d]
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2-2*x+2)/(d*x+x**2+d+2)/(-x**3-1)**(1/2),x)

[Out] Timed out
```

Mathematica [C] time = 2.06181, size = 426, normalized size = 13.31

$$\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\sqrt{x^2-x+1}\left(\frac{2\sqrt{3}\left(1+\sqrt[3]{-1}\right)\left(\sqrt[3]{-1}-x\right)F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{(-1)^{2/3}x+1}-\frac{3i\left(\left(-\left(1+\sqrt[3]{-1}\right)d^2+\left(1+\sqrt[3]{-1}\right)\left(\sqrt{d^2-4d-8}+4\right)d-2\sqrt[3]{-1}\sqrt{d^2-4d-\right.\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(2 - 2*x - x^2)/((2 + d + d*x + x^2)*Sqrt[-1 - x^3]),x]
```

```
[Out] (Sqrt[(1 + x)/(1 + (-1)^(1/3))]*Sqrt[1 - x + x^2]*((2*Sqrt[3]*(1
+ (-1)^(1/3))*((-1)^(1/3) - x)*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2
/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(1 + (-1)^(2/3)*x) - ((3*
I)*((8 + 8*(-1)^(1/3) - (1 + (-1)^(1/3))*d^2 + 4*Sqrt[-8 - 4*d +
d^2] - 2*(-1)^(1/3)*Sqrt[-8 - 4*d + d^2] + (1 + (-1)^(1/3))*d*(4
+ Sqrt[-8 - 4*d + d^2]))*EllipticPi[((2*I)*Sqrt[3])/(2*(-1)^(1/3)
 + d - Sqrt[-8 - 4*d + d^2]), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 +
 (-1)^(1/3))]], (-1)^(1/3)] + ((1 + (-1)^(1/3))*d^2 + (1 + (-1)^(
1/3))*d*(-4 + Sqrt[-8 - 4*d + d^2]) - 2*(4 + 4*(-1)^(1/3) - 2*Sqr
t[-8 - 4*d + d^2] + (-1)^(1/3)*Sqrt[-8 - 4*d + d^2]))*EllipticPi[
((2*I)*Sqrt[3])/(2*(-1)^(1/3) + d + Sqrt[-8 - 4*d + d^2]), ArcSin
[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)]))/((2 +
(-1)^(2/3) + d + (-1)^(1/3)*d)*Sqrt[-8 - 4*d + d^2])))/(3*Sqrt[-1
 - x^3])
```

Maple [C] time = 0.053, size = 1888, normalized size = 59.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2-2*x+2)/(d*x+x^2+d+2)/(-x^3-1)^(1/2),x)
```

```
[Out] 2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2
+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(
-x^3-1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1
/2))^(1/2),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))+1/3*I/(d^2-4*d-
8)^(1/2)*3^(1/2)*(I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)*(1/(3/2+1/
2*I*3^(1/2))+1/(3/2+1/2*I*3^(1/2))*x)^(1/2)*(-I*3^(1/2)*x+1/2*I*3
^(1/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^
2-4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3
^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2-4*d-8)^
(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))*d^2-1/3*I*3^(1/2)*(
I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))+1/(3/
2+1/2*I*3^(1/2))*x)^(1/2)*(-I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)/
(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2-4*d-8)^(1/2))*El
lipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^
(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2-4*d-8)^(1/2)),(I*3^(1/2)/
(3/2+1/2*I*3^(1/2)))^(1/2))*d-4/3*I/(d^2-4*d-8)^(1/2)*3^(1/2)*(I*
3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))+1/(3/2+
1/2*I*3^(1/2))*x)^(1/2)*(-I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)/(-
x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2-4*d-8)^(1/2))*Elli
pticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1
/2)/(1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2-4*d-8)^(1/2)),(I*3^(1/2)/(3
```

```
/2+1/2*I*3^(1/2)))^(1/2))*d+2/3*I*3^(1/2)*(I*3^(1/2)*x-1/2*I*3^(1
/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))+1/(3/2+1/2*I*3^(1/2))*x)^(1
/2)*(-I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/(1/2+1/
2*I*3^(1/2)+1/2*d-1/2*(d^2-4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(
I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/
2)+1/2*d-1/2*(d^2-4*d-8)^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(
1/2))-8/3*I/(d^2-4*d-8)^(1/2)*3^(1/2)*(I*3^(1/2)*x-1/2*I*3^(1/2)+
3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))+1/(3/2+1/2*I*3^(1/2))*x)^(1/2)*
(-I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*
3^(1/2)+1/2*d-1/2*(d^2-4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x
-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+1
/2*d-1/2*(d^2-4*d-8)^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2)
)-1/3*I/(d^2-4*d-8)^(1/2)*3^(1/2)*(I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)
^(1/2)*(1/(3/2+1/2*I*3^(1/2))+1/(3/2+1/2*I*3^(1/2))*x)^(1/2)*(-I*
3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1
/2)+1/2*d+1/2*(d^2-4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2
-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d
+1/2*(d^2-4*d-8)^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))*d^
2-1/3*I*3^(1/2)*(I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)*(1/(3/2+1/2
*I*3^(1/2))+1/(3/2+1/2*I*3^(1/2))*x)^(1/2)*(-I*3^(1/2)*x+1/2*I*3^
(1/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2
-4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^
(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2-4*d-8)^(
1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))*d+4/3*I/(d^2-4*d-8)^
(1/2)*3^(1/2)*(I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)*(1/(3/2+1/2*I
*3^(1/2))+1/(3/2+1/2*I*3^(1/2))*x)^(1/2)*(-I*3^(1/2)*x+1/2*I*3^(1
/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2-4
*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1
/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2-4*d-8)^(1/
2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))*d+2/3*I*3^(1/2)*(I*3^(
1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))+1/(3/2+1/2
*I*3^(1/2))*x)^(1/2)*(-I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3
-1)^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2-4*d-8)^(1/2))*Ellipti
cPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)
/(1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2-4*d-8)^(1/2)),(I*3^(1/2)/(3/2+
1/2*I*3^(1/2)))^(1/2))+8/3*I/(d^2-4*d-8)^(1/2)*3^(1/2)*(I*3^(1/2)
*x-1/2*I*3^(1/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))+1/(3/2+1/2*I*3
^(1/2))*x)^(1/2)*(-I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3-1)^
(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2-4*d-8)^(1/2))*EllipticPi(
1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/
2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2-4*d-8)^(1/2)),(I*3^(1/2)/(3/2+1/2*
I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x^2 + 2*x - 2)/(sqrt(-x^3 - 1)*(d*x + x^2 + d + 2)),x, algorithm="maxi
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.295897, size = 1, normalized size = 0.03

$$\left[\frac{\log\left(-\frac{4\left((d+1)x^2-d^2-\left(d^2+3\,d+2\right)x-d\right)\sqrt{-x^3-1}+\left(2\,(3\,d+4)x^3-x^4-\left(d^2+2\,d+4\right)x^2-d^2-2\left(d^2+2\,d\right)x+4\,d+4\right)\sqrt{d+1}}{2\,dx^3+x^4+(d^2+2\,d+4)x^2+d^2+2\,(d^2+2\,d)x+4\,d+4}\right)}{2\sqrt{d+1}},\right.$$
$$\left.-\frac{\arctan\left(-\frac{((d+2)x-x^2+d)\sqrt{-d-1}}{2\sqrt{-x^3-1}(d+1)}\right)}{\sqrt{-d-1}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x^2 + 2*x - 2)/(sqrt(-x^3 - 1)*(d*x + x^2 + d + 2)),x, algorithm="fric
```

```
[Out] [1/2*log(-(4*((d + 1)*x^2 - d^2 - (d^2 + 3*d + 2)*x - d)*sqrt(-x^
3 - 1) + (2*(3*d + 4)*x^3 - x^4 - (d^2 + 2*d + 4)*x^2 - d^2 - 2*(
d^2 + 2*d)*x + 4*d + 4)*sqrt(d + 1))/(2*d*x^3 + x^4 + (d^2 + 2*d
+ 4)*x^2 + d^2 + 2*(d^2 + 2*d)*x + 4*d + 4))/sqrt(d + 1), -arctan
(-1/2*((d + 2)*x - x^2 + d)*sqrt(-d - 1)/(sqrt(-x^3 - 1)*(d + 1))
)/sqrt(-d - 1)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{2x}{dx\sqrt{-x^3-1}+d\sqrt{-x^3-1}+x^2\sqrt{-x^3-1}+2\sqrt{-x^3-1}}\,dx$$
$$-\int\frac{x^2}{dx\sqrt{-x^3-1}+d\sqrt{-x^3-1}+x^2\sqrt{-x^3-1}+2\sqrt{-x^3-1}}\,dx$$
$$-\int\left(-\frac{2}{dx\sqrt{-x^3-1}+d\sqrt{-x^3-1}+x^2\sqrt{-x^3-1}+2\sqrt{-x^3-1}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2-2*x+2)/(d*x+x**2+d+2)/(-x**3-1)**(1/2),x)
```

```
[Out] -Integral(2*x/(d*x*sqrt(-x**3 - 1) + d*sqrt(-x**3 - 1) + x**2*sqr
t(-x**3 - 1) + 2*sqrt(-x**3 - 1)), x) - Integral(x**2/(d*x*sqrt(-
x**3 - 1) + d*sqrt(-x**3 - 1) + x**2*sqrt(-x**3 - 1) + 2*sqrt(-x*
*3 - 1)), x) - Integral(-2/(d*x*sqrt(-x**3 - 1) + d*sqrt(-x**3 -
1) + x**2*sqrt(-x**3 - 1) + 2*sqrt(-x**3 - 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2 + 2x - 2}{\sqrt{-x^3 - 1}(dx + x^2 + d + 2)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x^2 + 2*x - 2)/(sqrt(-x^3 - 1)*(d*x + x^2 + d + 2)),x, algorithm="giac
```

```
[Out] integrate(-(x^2 + 2*x - 2)/(sqrt(-x^3 - 1)*(d*x + x^2 + d + 2)),
x)
```

3.183 $\int (d+ex)^3\sqrt{a+cx^4}\,dx$

Optimal. Leaf size=355

$$\frac{a^{3/4}d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(9\sqrt{a}e^2+5\sqrt{c}d^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15c^{3/4}\sqrt{a+cx^4}}-\frac{6a^{5/4}de^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5c^{3/4}\sqrt{a+cx^4}}+\frac{1}{15}dx\sqrt{a+cx^4}\left(5d^2+9e^2x^2\right)+\frac{3}{4}d^2ex^2\sqrt{a+cx^4}+\frac{3ad^2e\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{4\sqrt{c}}+\frac{6ade^2x\sqrt{a+cx^4}}{5\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{e^3\left(a+cx^4\right)^{3/2}}{6c}$$

```
[Out] (3*d^2*e*x^2*Sqrt[a + c*x^4])/4 + (6*a*d*e^2*x*Sqrt[a + c*x^4])/(
5*Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) + (d*x*(5*d^2 + 9*e^2*x^2)*Sqr
t[a + c*x^4])/15 + (e^3*(a + c*x^4)^(3/2))/(6*c) + (3*a*d^2*e*Arc
Tanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]])/(4*Sqrt[c]) - (6*a^(5/4)*d*e
^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^
2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(5*c^(3/4)*S
qrt[a + c*x^4]) + (a^(3/4)*d*(5*Sqrt[c]*d^2 + 9*Sqrt[a]*e^2)*(Sqr
t[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*E
llipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(15*c^(3/4)*Sqrt[a
+ c*x^4])
```

Rubi [A] time = 0.503341, antiderivative size = 355, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$

$$\frac{a^{3/4}d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(9\sqrt{a}e^2+5\sqrt{c}d^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15c^{3/4}\sqrt{a+cx^4}}-\frac{6a^{5/4}de^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5c^{3/4}\sqrt{a+cx^4}}+\frac{1}{15}dx\sqrt{a+cx^4}\left(5d^2+9e^2x^2\right)+\frac{3}{4}d^2ex^2\sqrt{a+cx^4}+\frac{3ad^2e\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{4\sqrt{c}}+\frac{6ade^2x\sqrt{a+cx^4}}{5\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{e^3\left(a+cx^4\right)^{3/2}}{6c}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x)^3*Sqrt[a + c*x^4],x]
```

```
[Out] (3*d^2*e*x^2*Sqrt[a + c*x^4])/4 + (6*a*d*e^2*x*Sqrt[a + c*x^4])/(
5*Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) + (d*x*(5*d^2 + 9*e^2*x^2)*Sqr
```

```
t[a + c*x^4])/15 + (e^3*(a + c*x^4)^(3/2))/(6*c) + (3*a*d^2*e*Arc
Tanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]])/(4*Sqrt[c]) - (6*a^(5/4)*d*e
^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^
2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(5*c^(3/4)*S
qrt[a + c*x^4]) + (a^(3/4)*d*(5*Sqrt[c]*d^2 + 9*Sqrt[a]*e^2)*(Sqr
t[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*E
llipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(15*c^(3/4)*Sqrt[a
+ c*x^4])
```

Rubi in Sympy [A] time = 54.5314, size = 332, normalized size = 0.94

$$-\frac{6a^{\frac{5}{4}}de^2\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{5c^{\frac{3}{4}}\sqrt{a+cx^4}}+\frac{a^{\frac{3}{4}}d\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(9\sqrt{a}e^2+5\sqrt{c}d^2\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{15c^{\frac{3}{4}}\sqrt{a+cx^4}}+\frac{3ad^2e\operatorname{atanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{4\sqrt{c}}+\frac{6ade^2x\sqrt{a+cx^4}}{5\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{3d^2ex^2\sqrt{a+cx^4}}{4}+\frac{dx\sqrt{a+cx^4}\left(5d^2+9e^2x^2\right)}{15}+\frac{e^3\left(a+cx^4\right)^{\frac{3}{2}}}{6c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x+d)**3*(c*x**4+a)**(1/2),x)
```

```
[Out] -6*a**(5/4)*d*e**2*sqrt((a + c*x**4)/(sqrt(a) + sqrt(c)*x**2)**2)
*(sqrt(a) + sqrt(c)*x**2)*elliptic_e(2*atan(c**(1/4)*x/a**(1/4)),
 1/2)/(5*c**(3/4)*sqrt(a + c*x**4)) + a**(3/4)*d*sqrt((a + c*x**4
)/(sqrt(a) + sqrt(c)*x**2)**2)*(sqrt(a) + sqrt(c)*x**2)*(9*sqrt(a
)*e**2 + 5*sqrt(c)*d**2)*elliptic_f(2*atan(c**(1/4)*x/a**(1/4)),
1/2)/(15*c**(3/4)*sqrt(a + c*x**4)) + 3*a*d**2*e*atanh(sqrt(c)*x*
*2/sqrt(a + c*x**4))/(4*sqrt(c)) + 6*a*d*e**2*x*sqrt(a + c*x**4)/
(5*sqrt(c)*(sqrt(a) + sqrt(c)*x**2)) + 3*d**2*e*x**2*sqrt(a + c*x
**4)/4 + d*x*sqrt(a + c*x**4)*(5*d**2 + 9*e**2*x**2)/15 + e**3*(a
 + c*x**4)**(3/2)/(6*c)
```

Mathematica [C] time = 0.724068, size = 310, normalized size = 0.87

$$72a^{3/2}\sqrt{c}de^2\sqrt{\frac{cx^4}{a}+1}E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)+\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\left(10a^2e^3+45a\sqrt{c}d^2e\sqrt{a+cx^4}\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)+acx\left(20d^3+45c\right.\right.$$

60

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x)^3*Sqrt[a + c*x^4],x]
```

```
[Out] (Sqrt[(I*Sqrt[c])/Sqrt[a]]*(10*a^2*e^3 + c^2*x^5*(20*d^3 + 45*d^2
*e*x + 36*d*e^2*x^2 + 10*e^3*x^3) + a*c*x*(20*d^3 + 45*d^2*e*x +
36*d*e^2*x^2 + 20*e^3*x^3) + 45*a*Sqrt[c]*d^2*e*Sqrt[a + c*x^4]*A
rcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]]) + 72*a^(3/2)*Sqrt[c]*d*e^2
*Sqrt[1 + (c*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]
]*x], -1] - 8*a*Sqrt[c]*d*((5*I)*Sqrt[c]*d^2 + 9*Sqrt[a]*e^2)*Sqr
t[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x]
, -1])/(60*Sqrt[(I*Sqrt[c])/Sqrt[a]]*c*Sqrt[a + c*x^4])
```

Maple [C] time = 0.048, size = 334, normalized size = 0.9

$$\frac{d^3x}{3}\sqrt{cx^4+a}+\frac{2\,ad^3}{3}\sqrt{1-ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4+a}}$$
$$+\frac{e^3}{6\,c}\left(cx^4+a\right)^{\frac{3}{2}}+\frac{3\,d^2ex^2}{4}\sqrt{cx^4+a}+\frac{3\,ad^2e}{4}\ln\left(x^2\sqrt{c}+\sqrt{cx^4+a}\right)\frac{1}{\sqrt{c}}+\frac{3\,de^2x^3}{5}\sqrt{cx^4+a}$$
$$+\frac{6\,i}{5}e^2da^{\frac{3}{2}}\sqrt{1-ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4+a}}\frac{1}{\sqrt{c}}$$
$$-\frac{6\,i}{5}e^2da^{\frac{3}{2}}\sqrt{1-ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\mathit{EllipticE}\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4+a}}\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x+d)^3*(c*x^4+a)^(1/2),x)
```

```
[Out] 1/3*d^3*x*(c*x^4+a)^(1/2)+2/3*d^3*a/(I/a^(1/2)*c^(1/2))^(1/2)*(1-
I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x
^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+1/6*e^3*(c*x
^4+a)^(3/2)/c+3/4*d^2*e*x^2*(c*x^4+a)^(1/2)+3/4*d^2*e*a/c^(1/2)*l
n(x^2*c^(1/2)+(c*x^4+a)^(1/2))+3/5*e^2*d*x^3*(c*x^4+a)^(1/2)+6/5*
I*e^2*d*a^(3/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^
2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*
EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-6/5*I*e^2*d*a^(3/2)/(I/a
^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2
)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*EllipticE(x*(I/a^(1/
2)*c^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{cx^4+a}(ex+d)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + a)*(e*x + d)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + a)*(e*x + d)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^3x^3 + 3\,de^2x^2 + 3\,d^2ex + d^3\right)\sqrt{cx^4 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + a)*(e*x + d)^3,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*sqrt(c*x^4 + a
), x)
```

Sympy [A] time = 10.1082, size = 175, normalized size = 0.49

$$\frac{\sqrt{a}d^3x\,\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\,\Gamma\left(\frac{5}{4}\right)} + \frac{3\sqrt{a}d^2ex^2\sqrt{1+\frac{cx^4}{a}}}{4} + \frac{3\sqrt{a}de^2x^3\,\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\,\Gamma\left(\frac{7}{4}\right)} + \frac{3ad^2e\,\operatorname{asinh}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4\sqrt{c}} + e^3\left(\begin{cases}\frac{\sqrt{a}x^4}{4} & \text{for } c=0\\ \frac{\left(a+cx^4\right)^{\frac{3}{2}}}{6c} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x+d)**3*(c*x**4+a)**(1/2),x)
```

```
[Out] sqrt(a)*d**3*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), c*x**4*exp_p
olar(I*pi)/a)/(4*gamma(5/4)) + 3*sqrt(a)*d**2*e*x**2*sqrt(1 + c*x
**4/a)/4 + 3*sqrt(a)*d*e**2*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7
/4,), c*x**4*exp_polar(I*pi)/a)/(4*gamma(7/4)) + 3*a*d**2*e*asinh
(sqrt(c)*x**2/sqrt(a))/(4*sqrt(c)) + e**3*Piecewise((sqrt(a)*x**4
/4, Eq(c, 0)), ((a + c*x**4)**(3/2)/(6*c), True))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^4 + a}(ex + d)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + a)*(e*x + d)^3,x, algorithm="giac")

[Out] integrate(sqrt(c*x^4 + a)*(e*x + d)^3, x)
```

3.184 $\int (d+ex)^2\sqrt{a+cx^4}\,dx$

Optimal. Leaf size=326

$$\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(3\sqrt{a}e^2+5\sqrt{c}d^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15c^{3/4}\sqrt{a+cx^4}} - \frac{2a^{5/4}e^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5c^{3/4}\sqrt{a+cx^4}} + \frac{1}{15}x\sqrt{a+cx^4}\left(5d^2+3e^2x^2\right)+\frac{1}{2}dex^2\sqrt{a+cx^4}+\frac{ade\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}}+\frac{2ae^2x\sqrt{a+cx^4}}{5\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

```
[Out] (d*e*x^2*Sqrt[a + c*x^4])/2 + (2*a*e^2*x*Sqrt[a + c*x^4])/(5*Sqrt
[c]*(Sqrt[a] + Sqrt[c]*x^2)) + (x*(5*d^2 + 3*e^2*x^2)*Sqrt[a + c*
x^4])/15 + (a*d*e*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]])/(2*Sqrt
[c]) - (2*a^(5/4)*e^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(S
qrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)],
 1/2])/(5*c^(3/4)*Sqrt[a + c*x^4]) + (a^(3/4)*(5*Sqrt[c]*d^2 + 3*
Sqrt[a]*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] +
Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(1
5*c^(3/4)*Sqrt[a + c*x^4])
```

Rubi [A] time = 0.403576, antiderivative size = 326, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474

$$\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(3\sqrt{a}e^2+5\sqrt{c}d^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15c^{3/4}\sqrt{a+cx^4}} - \frac{2a^{5/4}e^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5c^{3/4}\sqrt{a+cx^4}} + \frac{1}{15}x\sqrt{a+cx^4}\left(5d^2+3e^2x^2\right)+\frac{1}{2}dex^2\sqrt{a+cx^4}+\frac{ade\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}}+\frac{2ae^2x\sqrt{a+cx^4}}{5\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x)^2*Sqrt[a + c*x^4],x]
```

```
[Out] (d*e*x^2*Sqrt[a + c*x^4])/2 + (2*a*e^2*x*Sqrt[a + c*x^4])/(5*Sqrt
[c]*(Sqrt[a] + Sqrt[c]*x^2)) + (x*(5*d^2 + 3*e^2*x^2)*Sqrt[a + c*
x^4])/15 + (a*d*e*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]])/(2*Sqrt
```

```
[c]) - (2*a^(5/4)*e^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(S
qrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)],
 1/2])/(5*c^(3/4)*Sqrt[a + c*x^4]) + (a^(3/4)*(5*Sqrt[c]*d^2 + 3*
Sqrt[a]*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] +
Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(1
5*c^(3/4)*Sqrt[a + c*x^4])
```

Rubi in Sympy [A] time = 44.4783, size = 301, normalized size = 0.92

$$-\frac{2a^{\frac{5}{4}}e^2\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{5c^{\frac{3}{4}}\sqrt{a+cx^4}}+\frac{a^{\frac{3}{4}}\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(3\sqrt{a}e^2+5\sqrt{c}d^2\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{15c^{\frac{3}{4}}\sqrt{a+cx^4}}+\frac{ade\operatorname{atanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}}+\frac{2ae^2x\sqrt{a+cx^4}}{5\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{dex^2\sqrt{a+cx^4}}{2}+\frac{x\sqrt{a+cx^4}\left(5d^2+3e^2x^2\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x+d)**2*(c*x**4+a)**(1/2),x)
```

```
[Out] -2*a**(5/4)*e**2*sqrt((a + c*x**4)/(sqrt(a) + sqrt(c)*x**2)**2)*(
sqrt(a) + sqrt(c)*x**2)*elliptic_e(2*atan(c**(1/4)*x/a**(1/4)), 1
/2)/(5*c**(3/4)*sqrt(a + c*x**4)) + a**(3/4)*sqrt((a + c*x**4)/(s
qrt(a) + sqrt(c)*x**2)**2)*(sqrt(a) + sqrt(c)*x**2)*(3*sqrt(a)*e*
*2 + 5*sqrt(c)*d**2)*elliptic_f(2*atan(c**(1/4)*x/a**(1/4)), 1/2)
/(15*c**(3/4)*sqrt(a + c*x**4)) + a*d*e*atanh(sqrt(c)*x**2/sqrt(a
 + c*x**4))/(2*sqrt(c)) + 2*a*e**2*x*sqrt(a + c*x**4)/(5*sqrt(c)*
(sqrt(a) + sqrt(c)*x**2)) + d*e*x**2*sqrt(a + c*x**4)/2 + x*sqrt(
a + c*x**4)*(5*d**2 + 3*e**2*x**2)/15
```

Mathematica [C] time = 0.699326, size = 247, normalized size = 0.76

$$\frac{12a^{3/2}e^2\sqrt{\frac{cx^4}{a}+1}E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)-4a\sqrt{\frac{cx^4}{a}+1}\left(3\sqrt{a}e^2+5i\sqrt{c}d^2\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)+\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\left(\sqrt{c}x\left(\right.\right.}{30\sqrt{c}\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x)^2*Sqrt[a + c*x^4],x]
```

```
[Out] (Sqrt[(I*Sqrt[c])/Sqrt[a]]*(Sqrt[c]*x*(10*d^2 + 15*d*e*x + 6*e^2*
x^2)*(a + c*x^4) + 15*a*d*e*Sqrt[a + c*x^4]*ArcTanh[(Sqrt[c]*x^2)
/Sqrt[a + c*x^4]]) + 12*a^(3/2)*e^2*Sqrt[1 + (c*x^4)/a]*EllipticE
[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] - 4*a*((5*I)*Sqrt[c]
*d^2 + 3*Sqrt[a]*e^2)*Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqr
t[(I*Sqrt[c])/Sqrt[a]]*x], -1])/(30*Sqrt[(I*Sqrt[c])/Sqrt[a]]*Sqr
t[c]*Sqrt[a + c*x^4])
```

Maple [C] time = 0.01, size = 310, normalized size = 1.

$$\frac{d^2x}{3}\sqrt{cx^4+a}+\frac{2\,ad^2}{3}\sqrt{1-ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4+a}}$$
$$+\frac{e^2x^3}{5}\sqrt{cx^4+a}$$
$$+\frac{2\,i}{5}e^2a^{\frac{3}{2}}\sqrt{1-ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4+a}}\frac{1}{\sqrt{c}}$$
$$-\frac{2\,i}{5}e^2a^{\frac{3}{2}}\sqrt{1-ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\mathit{EllipticE}\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4+a}}\frac{1}{\sqrt{c}}$$
$$+\frac{dex^2}{2}\sqrt{cx^4+a}+\frac{ade}{2}\ln\left(x^2\sqrt{c}+\sqrt{cx^4+a}\right)\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x+d)^2*(c*x^4+a)^(1/2),x)
```

```
[Out] 1/3*d^2*x*(c*x^4+a)^(1/2)+2/3*d^2*a/(I/a^(1/2)*c^(1/2))^(1/2)*(1-
I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x
^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+1/5*e^2*x^3*
(c*x^4+a)^(1/2)+2/5*I*e^2*a^(3/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/
a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4
+a)^(1/2)/c^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-2/5*I*
e^2*a^(3/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(
1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*Elli
pticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+1/2*d*e*x^2*(c*x^4+a)^(1/2)+
1/2*d*e*a/c^(1/2)*ln(x^2*c^(1/2)+(c*x^4+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{cx^4+a}(ex+d)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + a)*(e*x + d)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + a)*(e*x + d)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{cx^4 + a}\left(e^2x^2 + 2\,dex + d^2\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + a)*(e*x + d)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + a)*(e^2*x^2 + 2*d*e*x + d^2), x)
```

Sympy [A] time = 9.74102, size = 138, normalized size = 0.42

$$\frac{\sqrt{a}d^2x\,\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\,\Gamma\left(\frac{5}{4}\right)} + \frac{\sqrt{a}dex^2\sqrt{1+\frac{cx^4}{a}}}{2} + \frac{\sqrt{a}e^2x^3\,\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\,\Gamma\left(\frac{7}{4}\right)} + \frac{ade\operatorname{asinh}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x+d)**2*(c*x**4+a)**(1/2),x)
```

```
[Out] sqrt(a)*d**2*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), c*x**4*exp_p
olar(I*pi)/a)/(4*gamma(5/4)) + sqrt(a)*d*e*x**2*sqrt(1 + c*x**4/a
)/2 + sqrt(a)*e**2*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), c*x
**4*exp_polar(I*pi)/a)/(4*gamma(7/4)) + a*d*e*asinh(sqrt(c)*x**2/
sqrt(a))/(2*sqrt(c))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^4 + a}(ex + d)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + a)*(e*x + d)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + a)*(e*x + d)^2, x)
```

3.185 $\int (d+ex)\sqrt{a+cx^4}\,dx$

Optimal. Leaf size=158

$$\frac{a^{3/4}d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{1}{3}dx\sqrt{a+cx^4}+\frac{1}{4}ex^2\sqrt{a+cx^4}+\frac{ae\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{4\sqrt{c}}$$

```
[Out] (d*x*Sqrt[a + c*x^4])/3 + (e*x^2*Sqrt[a + c*x^4])/4 + (a*e*ArcTan
h[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]])/(4*Sqrt[c]) + (a^(3/4)*d*(Sqrt[
a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ell
ipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(3*c^(1/4)*Sqrt[a + c
*x^4])
```

Rubi [A] time = 0.181423, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353

$$\frac{a^{3/4}d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{1}{3}dx\sqrt{a+cx^4}+\frac{1}{4}ex^2\sqrt{a+cx^4}+\frac{ae\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{4\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x)*Sqrt[a + c*x^4],x]
```

```
[Out] (d*x*Sqrt[a + c*x^4])/3 + (e*x^2*Sqrt[a + c*x^4])/4 + (a*e*ArcTan
h[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]])/(4*Sqrt[c]) + (a^(3/4)*d*(Sqrt[
a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ell
ipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(3*c^(1/4)*Sqrt[a + c
*x^4])
```

Rubi in Sympy [A] time = 16.5768, size = 143, normalized size = 0.91

$$\frac{a^{\frac{3}{4}}d\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{3\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{ae\operatorname{atanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{4\sqrt{c}}+\frac{dx\sqrt{a+cx^4}}{3}+\frac{ex^2\sqrt{a+cx^4}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x+d)*(c*x**4+a)**(1/2),x)
```

```
[Out] a**(3/4)*d*sqrt((a + c*x**4)/(sqrt(a) + sqrt(c)*x**2)**2)*(sqrt(a
) + sqrt(c)*x**2)*elliptic_f(2*atan(c**(1/4)*x/a**(1/4)), 1/2)/(3
*c**(1/4)*sqrt(a + c*x**4)) + a*e*atanh(sqrt(c)*x**2/sqrt(a + c*x
**4))/(4*sqrt(c)) + d*x*sqrt(a + c*x**4)/3 + e*x**2*sqrt(a + c*x*
*4)/4
```

Mathematica [C] time = 0.553328, size = 132, normalized size = 0.84

$$\frac{1}{12}\left(x\sqrt{a+cx^4}(4d+3ex)-\frac{8iad\sqrt{\frac{cx^4}{a}+1}F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{a+cx^4}}+\frac{3ae\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x)*Sqrt[a + c*x^4],x]
```

```
[Out] (x*(4*d + 3*e*x)*Sqrt[a + c*x^4] + (3*a*e*ArcTanh[(Sqrt[c]*x^2)/S
qrt[a + c*x^4]])/Sqrt[c] - ((8*I)*a*d*Sqrt[1 + (c*x^4)/a]*Ellipti
cF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1])/(Sqrt[(I*Sqrt[c])
/Sqrt[a]]*Sqrt[a + c*x^4]))/12
```

Maple [C] time = 0.006, size = 127, normalized size = 0.8

$$\frac{dx}{3}\sqrt{cx^4+a}+\frac{2\,ad}{3}\sqrt{1-ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{c}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4+a}}$$
$$+\frac{ex^2}{4}\sqrt{cx^4+a}+\frac{ae}{4}\ln\left(x^2\sqrt{c}+\sqrt{cx^4+a}\right)\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x+d)*(c*x^4+a)^(1/2),x)
```

```
[Out] 1/3*d*x*(c*x^4+a)^(1/2)+2/3*d*a/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^
(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a
)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+1/4*e*x^2*(c*x^4
+a)^(1/2)+1/4*e*a/c^(1/2)*ln(x^2*c^(1/2)+(c*x^4+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^4 + a}(ex + d)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + a)*(e*x + d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + a)*(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{cx^4 + a}(ex + d), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + a)*(e*x + d),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + a)*(e*x + d), x)
```

Sympy [A] time = 8.5021, size = 88, normalized size = 0.56

$$\frac{\sqrt{a}dx\,\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{1}{4} \\ \frac{5}{4} \end{matrix}\middle| \frac{cx^4 e^{i\pi}}{a}\right)}{4\,\Gamma\left(\frac{5}{4}\right)} + \frac{\sqrt{a}ex^2\sqrt{1 + \frac{cx^4}{a}}}{4} + \frac{ae \operatorname{asinh}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x+d)*(c*x**4+a)**(1/2),x)
```

```
[Out] sqrt(a)*d*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), c*x**4*exp_pola
r(I*pi)/a)/(4*gamma(5/4)) + sqrt(a)*e*x**2*sqrt(1 + c*x**4/a)/4 +
 a*e*asinh(sqrt(c)*x**2/sqrt(a))/(4*sqrt(c))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^4 + a}(ex + d)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + a)*(e*x + d),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + a)*(e*x + d), x)
```

3.186 $\int \sqrt{a+cx^4}\,dx$

Optimal. Leaf size=105

$$\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{1}{3}x\sqrt{a+cx^4}$$

```
[Out] (x*Sqrt[a + c*x^4])/3 + (a^(3/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a
+ c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x
)/a^(1/4)], 1/2])/(3*c^(1/4)*Sqrt[a + c*x^4])
```

Rubi [A] time = 0.0561471, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{1}{3}x\sqrt{a+cx^4}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + c*x^4],x]
```

```
[Out] (x*Sqrt[a + c*x^4])/3 + (a^(3/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a
+ c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x
)/a^(1/4)], 1/2])/(3*c^(1/4)*Sqrt[a + c*x^4])
```

Rubi in Sympy [A] time = 5.44448, size = 92, normalized size = 0.88

$$\frac{a^{\frac{3}{4}}\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{3\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{x\sqrt{a+cx^4}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+a)**(1/2),x)
```

```
[Out] a**(3/4)*sqrt((a + c*x**4)/(sqrt(a) + sqrt(c)*x**2)**2)*(sqrt(a)
+ sqrt(c)*x**2)*elliptic_f(2*atan(c**(1/4)*x/a**(1/4)), 1/2)/(3*c
**(1/4)*sqrt(a + c*x**4)) + x*sqrt(a + c*x**4)/3
```

Mathematica [C] time = 0.175041, size = 89, normalized size = 0.85

$$\frac{x\left(a+cx^4\right)-\frac{2ia\sqrt{\frac{cx^4}{a}+1}F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}}{3\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + c*x^4],x]
```

```
[Out] (x*(a + c*x^4) - ((2*I)*a*Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh
[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1])/Sqrt[(I*Sqrt[c])/Sqrt[a]])/(3
*Sqrt[a + c*x^4])
```

Maple [C] time = 0.003, size = 85, normalized size = 0.8

$$\frac{x}{3}\sqrt{cx^4+a}+\frac{2a}{3}\sqrt{1-ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{c}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+a)^(1/2),x)
```

```
[Out] 1/3*x*(c*x^4+a)^(1/2)+2/3*a/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2
)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1
/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{cx^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{cx^4 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + a),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + a), x)
```

Sympy [A] time = 2.10837, size = 37, normalized size = 0.35

$$\frac{\sqrt{a}x\,\Gamma\left(\frac{1}{4}\right)\,{}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+a)**(1/2),x)
```

```
[Out] sqrt(a)*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), c*x**4*exp_polar(
I*pi)/a)/(4*gamma(5/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^4 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + a), x)
```

3.187 $\int \frac{\sqrt{a+cx^4}}{d+ex}\,dx$

Optimal. Leaf size=737

$$\frac{\sqrt{c}d^2\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2e^3}-\frac{\sqrt[4]{a}\sqrt[4]{c}d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2e^4\sqrt{a+cx^4}}$$
$$-\frac{d\sqrt{-\frac{ae^4+cd^4}{d^2e^2}}\tan^{-1}\left(\frac{x\sqrt{-\frac{ae^4+cd^4}{d^2e^2}}}{\sqrt{a+cx^4}}\right)}{2e^2}$$
$$+\frac{\sqrt[4]{c}d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(ae^4+cd^4\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}e^4\sqrt{a+cx^4}\left(\sqrt{a}e^2+\sqrt{c}d^2\right)}$$
$$-\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(ae^4+cd^4\right)\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}de^4\sqrt{a+cx^4}\left(\sqrt{a}e^2+\sqrt{c}d^2\right)}$$
$$-\frac{\sqrt{ae^4+cd^4}\tanh^{-1}\left(\frac{ae^2+cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{2e^3}-\frac{\sqrt{c}dx\sqrt{a+cx^4}}{e^2\left(\sqrt{a}+\sqrt{c}x^2\right)}$$
$$+\frac{\sqrt[4]{a}\sqrt[4]{c}d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{e^2\sqrt{a+cx^4}}+\frac{\sqrt{a+cx^4}}{2e}$$

```
[Out] Sqrt[a + c*x^4]/(2*e) - (Sqrt[c]*d*x*Sqrt[a + c*x^4])/(e^2*(Sqrt[
a] + Sqrt[c]*x^2)) - (d*Sqrt[-((c*d^4 + a*e^4)/(d^2*e^2))]*ArcTan
[(Sqrt[-((c*d^4 + a*e^4)/(d^2*e^2))]*x)/Sqrt[a + c*x^4]])/(2*e^2)
 + (Sqrt[c]*d^2*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]])/(2*e^3) -
 (Sqrt[c*d^4 + a*e^4]*ArcTanh[(a*e^2 + c*d^2*x^2)/(Sqrt[c*d^4 + a
*e^4]*Sqrt[a + c*x^4])]])/(2*e^3) + (a^(1/4)*c^(1/4)*d*(Sqrt[a] +
Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Elliptic
E[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(e^2*Sqrt[a + c*x^4]) - (a
^(1/4)*c^(1/4)*d*((Sqrt[c]*d^2)/Sqrt[a] + e^2)*(Sqrt[a] + Sqrt[c]
*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*Arc
Tan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*e^4*Sqrt[a + c*x^4]) + (c^(1/4
)*d*(c*d^4 + a*e^4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqr
t[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1
/2])/(2*a^(1/4)*e^4*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sqrt[a + c*x^4])
- ((Sqrt[c]*d^2 - Sqrt[a]*e^2)*(c*d^4 + a*e^4)*(Sqrt[a] + Sqrt[c]
*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[(Sqr
t[c]*d^2 + Sqrt[a]*e^2)^2/(4*Sqrt[a]*Sqrt[c]*d^2*e^2), 2*ArcTan[(
c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*c^(1/4)*d*e^4*(Sqrt[c]*d^2
+ Sqrt[a]*e^2)*Sqrt[a + c*x^4])
```

Rubi [A] time = 1.39254, antiderivative size = 737, normalized size of antiderivative = 1., number of

steps used = 14, number of rules used = 12, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$

$$\frac{\sqrt{c}d^2\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2e^3}-\frac{\sqrt[4]{a}\sqrt[4]{c}d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2e^4\sqrt{a+cx^4}}$$
$$-\frac{d\sqrt{-\frac{ae^4+cd^4}{d^2e^2}}\tan^{-1}\left(\frac{x\sqrt{-\frac{ae^4+cd^4}{d^2e^2}}}{\sqrt{a+cx^4}}\right)}{2e^2}$$
$$+\frac{\sqrt[4]{c}d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(ae^4+cd^4\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}e^4\sqrt{a+cx^4}\left(\sqrt{a}e^2+\sqrt{c}d^2\right)}$$
$$-\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(ae^4+cd^4\right)\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}de^4\sqrt{a+cx^4}\left(\sqrt{a}e^2+\sqrt{c}d^2\right)}$$
$$-\frac{\sqrt{ae^4+cd^4}\tanh^{-1}\left(\frac{ae^2+cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{2e^3}-\frac{\sqrt{c}dx\sqrt{a+cx^4}}{e^2\left(\sqrt{a}+\sqrt{c}x^2\right)}$$
$$+\frac{\sqrt[4]{a}\sqrt[4]{c}d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{e^2\sqrt{a+cx^4}}+\frac{\sqrt{a+cx^4}}{2e}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[Sqrt[a + c*x^4]/(d + e*x),x]
```

```
[Out] Sqrt[a + c*x^4]/(2*e) - (Sqrt[c]*d*x*Sqrt[a + c*x^4])/(e^2*(Sqrt[
a] + Sqrt[c]*x^2)) - (d*Sqrt[-((c*d^4 + a*e^4)/(d^2*e^2))]*ArcTan
[(Sqrt[-((c*d^4 + a*e^4)/(d^2*e^2))]*x)/Sqrt[a + c*x^4]])/(2*e^2)
 + (Sqrt[c]*d^2*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]])/(2*e^3) -
 (Sqrt[c*d^4 + a*e^4]*ArcTanh[(a*e^2 + c*d^2*x^2)/(Sqrt[c*d^4 + a
*e^4]*Sqrt[a + c*x^4])])/(2*e^3) + (a^(1/4)*c^(1/4)*d*(Sqrt[a] +
Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Elliptic
E[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(e^2*Sqrt[a + c*x^4]) - (a
^(1/4)*c^(1/4)*d*((Sqrt[c]*d^2)/Sqrt[a] + e^2)*(Sqrt[a] + Sqrt[c]
*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*Arc
Tan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*e^4*Sqrt[a + c*x^4]) + (c^(1/4
)*d*(c*d^4 + a*e^4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqr
t[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1
/2])/(2*a^(1/4)*e^4*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sqrt[a + c*x^4])
- ((Sqrt[c]*d^2 - Sqrt[a]*e^2)*(c*d^4 + a*e^4)*(Sqrt[a] + Sqrt[c]
*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[(Sqr
t[c]*d^2 + Sqrt[a]*e^2)^2/(4*Sqrt[a]*Sqrt[c]*d^2*e^2), 2*ArcTan[(
c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*c^(1/4)*d*e^4*(Sqrt[c]*d^2
+ Sqrt[a]*e^2)*Sqrt[a + c*x^4])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + cx^4}}{d + ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+a)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(sqrt(a + c*x**4)/(d + e*x), x)
```

Mathematica [C] time = 1.63642, size = 451, normalized size = 0.61

$$\frac{2c^{3/4}d^2\sqrt{\frac{cx^4}{a}+1}\left(\sqrt{a}e^2+i\sqrt{c}d^2\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)-2\sqrt{a}c^{3/4}d^2e^2\sqrt{\frac{cx^4}{a}+1}E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)+\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + c*x^4]/(d + e*x),x]
```

```
[Out] (-2*Sqrt[a]*c^(3/4)*d^2*e^2*Sqrt[1 + (c*x^4)/a]*EllipticE[I*ArcSi
nh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] + 2*c^(3/4)*d^2*(I*Sqrt[c]*d
^2 + Sqrt[a]*e^2)*Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I
*Sqrt[c])/Sqrt[a]]*x], -1] + Sqrt[(I*Sqrt[c])/Sqrt[a]]*(-2*(-1)^(
1/4)*a^(1/4)*(c*d^4 + a*e^4)*Sqrt[1 + (c*x^4)/a]*EllipticPi[(I*Sq
rt[a]*e^2)/(Sqrt[c]*d^2), ArcSin[((-1)^(3/4)*c^(1/4)*x)/a^(1/4)],
 -1] + c^(1/4)*d*e*(a*e^2 + c*e^2*x^4 + Sqrt[c*d^4 + a*e^4]*Sqrt[
a + c*x^4]*Log[-d^2 + e^2*x^2] + Sqrt[c]*d^2*Sqrt[a + c*x^4]*Log[
c*x^2 + Sqrt[c]*Sqrt[a + c*x^4]] - Sqrt[c*d^4 + a*e^4]*Sqrt[a + c
*x^4]*Log[a*e^2 + c*d^2*x^2 + Sqrt[c*d^4 + a*e^4]*Sqrt[a + c*x^4]
])))/(2*Sqrt[(I*Sqrt[c])/Sqrt[a]]*c^(1/4)*d*e^4*Sqrt[a + c*x^4])
```

Maple [C] time = 0.021, size = 565, normalized size = 0.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+a)^(1/2)/(e*x+d),x)
```

```
[Out] 1/2*(c*x^4+a)^(1/2)/e-c*d^3/e^4/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^
(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a
)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+1/2*d^2/e^3*c^(1
/2)*ln(2*x^2*c^(1/2)+2*(c*x^4+a)^(1/2))-I*c^(1/2)*d/e^2*a^(1/2)/(
I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(
1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*c
^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I))-1/2/e/
(c*d^4/e^4+a)^(1/2)*arctanh(1/2*(2*c*x^2*d^2/e^2+2*a)/(c*d^4/e^4+
a)^(1/2)/(c*x^4+a)^(1/2))*a-1/2/e^5/(c*d^4/e^4+a)^(1/2)*arctanh(1
/2*(2*c*x^2*d^2/e^2+2*a)/(c*d^4/e^4+a)^(1/2)/(c*x^4+a)^(1/2))*c*d
^4+1/(I/a^(1/2)*c^(1/2))^(1/2)/d*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*
(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticPi(x*(I/a
^(1/2)*c^(1/2))^(1/2),-I*a^(1/2)/c^(1/2)/d^2*e^2,(-I/a^(1/2)*c^(1
/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2))*a+1/e^4/(I/a^(1/2)*c^(1/2))
^(1/2)*d^3*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x
^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*c^(1/2))^(1/2),
-I*a^(1/2)/c^(1/2)/d^2*e^2,(-I/a^(1/2)*c^(1/2))^(1/2)/(I/a^(1/2)*
c^(1/2))^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + a}}{ex + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + a)/(e*x + d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + a)/(e*x + d), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + a)/(e*x + d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + cx^4}}{d + ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+a)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(sqrt(a + c*x**4)/(d + e*x), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + a}}{ex + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + a)/(e*x + d),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + a)/(e*x + d), x)
```

3.188 $\int \frac{\sqrt{a+cx^4}}{(d+ex)^2} \, dx$

Optimal. Leaf size=1385

result too large to display

```
[Out] (2*Sqrt[c]*x*Sqrt[a + c*x^4])/(e^2*(Sqrt[a] + Sqrt[c]*x^2)) - (d*
Sqrt[a + c*x^4])/(e*(d^2 - e^2*x^2)) + (x*Sqrt[a + c*x^4])/(d^2 -
 e^2*x^2) - ((c*d^4 - a*e^4)*ArcTan[(Sqrt[-((c*d^4 + a*e^4)/(d^2*
e^2))]*x)/Sqrt[a + c*x^4]])/(2*d^2*e^4*Sqrt[-((c*d^4 + a*e^4)/(d^
2*e^2))]) + (Sqrt[-((c*d^4 + a*e^4)/(d^2*e^2))]*ArcTan[(Sqrt[-((c
*d^4 + a*e^4)/(d^2*e^2))]*x)/Sqrt[a + c*x^4]])/(2*e^2) - (Sqrt[c]
*d*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]])/e^3 + (c*d^3*ArcTanh[(
a*e^2 + c*d^2*x^2)/(Sqrt[c*d^4 + a*e^4]*Sqrt[a + c*x^4])])/(e^3*S
qrt[c*d^4 + a*e^4]) - (2*a^(1/4)*c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*
Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c
^(1/4)*x)/a^(1/4)], 1/2])/(e^2*Sqrt[a + c*x^4]) + (3*a^(1/4)*c^(1
/4)*((Sqrt[c]*d^2)/Sqrt[a] + e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a
 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*
x)/a^(1/4)], 1/2])/(4*e^4*Sqrt[a + c*x^4]) - (c^(1/4)*(Sqrt[c]*d^
2 - Sqrt[a]*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a
] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2]
)/(2*a^(1/4)*e^4*Sqrt[a + c*x^4]) + (c^(1/4)*(Sqrt[c]*d^2 + Sqrt[
a]*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[
c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1
/4)*e^4*Sqrt[a + c*x^4]) - (c^(1/4)*(c*d^4 + a*e^4)*(Sqrt[a] + Sq
rt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[
2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*e^4*(Sqrt[c]*d^2
+ Sqrt[a]*e^2)*Sqrt[a + c*x^4]) + ((Sqrt[c]*d^2 - Sqrt[a]*e^2)^2*
(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^
2]*EllipticPi[(Sqrt[a]*((Sqrt[c]*d^2)/Sqrt[a] + e^2)^2)/(4*Sqrt[c
]*d^2*e^2), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(8*a^(1/4)*c^(1/
4)*d^2*e^4*Sqrt[a + c*x^4]) + ((Sqrt[c]*d^2 - Sqrt[a]*e^2)*(c*d^4
 + a*e^4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqr
t[c]*x^2)^2]*EllipticPi[(Sqrt[a]*((Sqrt[c]*d^2)/Sqrt[a] + e^2)^2)
/(4*Sqrt[c]*d^2*e^2), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(
1/4)*c^(1/4)*d^2*e^4*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sqrt[a + c*x^4])
 + ((Sqrt[c]*d^2 - Sqrt[a]*e^2)^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a
 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[(Sqrt[c]*d^2 + Sq
rt[a]*e^2)^2/(4*Sqrt[a]*Sqrt[c]*d^2*e^2), 2*ArcTan[(c^(1/4)*x)/a^
(1/4)], 1/2])/(8*a^(1/4)*c^(1/4)*d^2*e^4*Sqrt[a + c*x^4])
```

Rubi [A] time = 4.06253, antiderivative size = 1385, normalized size of antiderivative = 1., number of steps used = 29, number of rules used = 14, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.737$

result too large to display

Warning: Unable to verify antiderivative.

```
[In]  Int[Sqrt[a + c*x^4]/(d + e*x)^2,x]
```

```
[Out] (2*Sqrt[c]*x*Sqrt[a + c*x^4])/(e^2*(Sqrt[a] + Sqrt[c]*x^2)) - (d*
Sqrt[a + c*x^4])/(e*(d^2 - e^2*x^2)) + (x*Sqrt[a + c*x^4])/(d^2 -
 e^2*x^2) - ((c*d^4 - a*e^4)*ArcTan[(Sqrt[-((c*d^4 + a*e^4)/(d^2*
e^2))]*x)/Sqrt[a + c*x^4]])/(2*d^2*e^4*Sqrt[-((c*d^4 + a*e^4)/(d^
2*e^2))]) + (Sqrt[-((c*d^4 + a*e^4)/(d^2*e^2))]*ArcTan[(Sqrt[-((c
*d^4 + a*e^4)/(d^2*e^2))]*x)/Sqrt[a + c*x^4]])/(2*e^2) - (Sqrt[c]
*d*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]])/e^3 + (c*d^3*ArcTanh[(
a*e^2 + c*d^2*x^2)/(Sqrt[c*d^4 + a*e^4]*Sqrt[a + c*x^4])])/(e^3*S
qrt[c*d^4 + a*e^4]) - (2*a^(1/4)*c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*
Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c
^(1/4)*x)/a^(1/4)], 1/2])/(e^2*Sqrt[a + c*x^4]) + (3*a^(1/4)*c^(1
/4)*((Sqrt[c]*d^2)/Sqrt[a] + e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a
 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*
x)/a^(1/4)], 1/2])/(4*e^4*Sqrt[a + c*x^4]) - (c^(1/4)*(Sqrt[c]*d^
2 - Sqrt[a]*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a
] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2]
)/(2*a^(1/4)*e^4*Sqrt[a + c*x^4]) + (c^(1/4)*(Sqrt[c]*d^2 + Sqrt[
a]*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[
c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1
/4)*e^4*Sqrt[a + c*x^4]) - (c^(1/4)*(c*d^4 + a*e^4)*(Sqrt[a] + Sq
rt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[
2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*e^4*(Sqrt[c]*d^2
+ Sqrt[a]*e^2)*Sqrt[a + c*x^4]) + ((Sqrt[c]*d^2 - Sqrt[a]*e^2)^2*
(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^
2]*EllipticPi[(Sqrt[a]*((Sqrt[c]*d^2)/Sqrt[a] + e^2)^2)/(4*Sqrt[c
]*d^2*e^2), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(8*a^(1/4)*c^(1/
4)*d^2*e^4*Sqrt[a + c*x^4]) + ((Sqrt[c]*d^2 - Sqrt[a]*e^2)*(c*d^4
 + a*e^4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqr
t[c]*x^2)^2]*EllipticPi[(Sqrt[a]*((Sqrt[c]*d^2)/Sqrt[a] + e^2)^2)
/(4*Sqrt[c]*d^2*e^2), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(
1/4)*c^(1/4)*d^2*e^4*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sqrt[a + c*x^4])
 + ((Sqrt[c]*d^2 - Sqrt[a]*e^2)^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a
 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[(Sqrt[c]*d^2 + Sq
rt[a]*e^2)^2/(4*Sqrt[a]*Sqrt[c]*d^2*e^2), 2*ArcTan[(c^(1/4)*x)/a^
(1/4)], 1/2])/(8*a^(1/4)*c^(1/4)*d^2*e^4*Sqrt[a + c*x^4])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + cx^4}}{(d + ex)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+a)**(1/2)/(e*x+d)**2,x)
```

```
[Out] Integral(sqrt(a + c*x**4)/(d + e*x)**2, x)
```

Mathematica [C] time = 6.29073, size = 924, normalized size = 0.67

$$2c\left(\left(-\frac{e\left(-2\sqrt[4]{-1}\sqrt[4]{a}\sqrt{\frac{cd^4}{ae^4}+1}e\sqrt{1-\frac{i\sqrt{c}x^2}{\sqrt{a}}}\sqrt{\frac{i\sqrt{c}x^2}{\sqrt{a}}+1}\left(\frac{i\sqrt{a}e^2}{\sqrt{c}d^2};\sin^{-1}\left(\frac{(-1)^{3/4}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)-\sqrt[4]{c}d\sqrt{\frac{cx^4}{a}+1}\log\left(\frac{e^2x^2-d^2}{ae^2+a\sqrt{\frac{cd^4}{ae^4}+1}\sqrt{\frac{cx^4}{a}+1}e^2+cd^2x^2}\right)\right)}{4\sqrt[4]{c}d^2\sqrt{\frac{cd^4}{ae^4}+1}\sqrt{cx^4+a}}-\frac{e\left(\sqrt[4]{c}d\sqrt{\frac{cx^4}{a}+1}\log\left(\frac{\cdots}{ae^2+a\sqrt{\cdots}}\right.\right.}{\cdots}\right.\right.$$

$$\frac{\cdots}{e^5}$$

$$-\frac{\sqrt{cx^4+a}}{e(d+ex)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + c*x^4]/(d + e*x)^2,x]
```

```
[Out] -(Sqrt[a + c*x^4]/(e*(d + e*x))) + (2*c*((Sqrt[a]*Sqrt[1 - (I*Sqr
t[c]*x^2)/Sqrt[a]]*Sqrt[1 + (I*Sqrt[c]*x^2)/Sqrt[a]]*(EllipticE[I
*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] - EllipticF[I*ArcSinh[
Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1]))/(Sqrt[(I*Sqrt[c])/Sqrt[a]]*Sq
rt[c]*e*Sqrt[a + c*x^4]) - (I*d^2*Sqrt[1 - (I*Sqrt[c]*x^2)/Sqrt[a
]]*Sqrt[1 + (I*Sqrt[c]*x^2)/Sqrt[a]]*EllipticF[I*ArcSinh[Sqrt[(I*
Sqrt[c])/Sqrt[a]]*x], -1])/(Sqrt[(I*Sqrt[c])/Sqrt[a]]*e^3*Sqrt[a
+ c*x^4]) - (d*((d^2*Log[-d^2 + e^2*x^2])/Sqrt[c*d^4 + a*e^4] + L
og[c*x^2 + Sqrt[c]*Sqrt[a + c*x^4]]/Sqrt[c] - (d^2*Log[a*e^2 + c*
d^2*x^2 + Sqrt[c*d^4 + a*e^4]*Sqrt[a + c*x^4]])/Sqrt[c*d^4 + a*e^
4]))/(2*e^2) + (d^4*(-(e*(-2*(-1)^(1/4)*a^(1/4)*Sqrt[1 + (c*d^4)/
(a*e^4)]*e*Sqrt[1 - (I*Sqrt[c]*x^2)/Sqrt[a]]*Sqrt[1 + (I*Sqrt[c]*
x^2)/Sqrt[a]]*EllipticPi[(I*Sqrt[a]*e^2)/(Sqrt[c]*d^2), ArcSin[((
-1)^(3/4)*c^(1/4)*x)/a^(1/4)], -1] - c^(1/4)*d*Sqrt[1 + (c*x^4)/a
]*Log[(-d^2 + e^2*x^2)/(a*e^2 + c*d^2*x^2 + a*Sqrt[1 + (c*d^4)/(a
*e^4)]*e^2*Sqrt[1 + (c*x^4)/a])]))/(4*c^(1/4)*d^2*Sqrt[1 + (c*d^4
)/(a*e^4)]*Sqrt[a + c*x^4]) - (e*(-2*(-1)^(1/4)*a^(1/4)*Sqrt[1 +
(c*d^4)/(a*e^4)]*e*Sqrt[1 - (I*Sqrt[c]*x^2)/Sqrt[a]]*Sqrt[1 + (I*
Sqrt[c]*x^2)/Sqrt[a]]*EllipticPi[(I*Sqrt[a]*e^2)/(Sqrt[c]*d^2), A
rcSin[((-1)^(3/4)*c^(1/4)*x)/a^(1/4)], -1] + c^(1/4)*d*Sqrt[1 + (
c*x^4)/a]*Log[(-d^2 + e^2*x^2)/(a*e^2 + c*d^2*x^2 + a*Sqrt[1 + (c
*d^4)/(a*e^4)]*e^2*Sqrt[1 + (c*x^4)/a])]))/(4*c^(1/4)*d^2*Sqrt[1
+ (c*d^4)/(a*e^4)]*Sqrt[a + c*x^4])))/e^5))/e
```

Maple [C] time = 0.025, size = 402, normalized size = 0.3

$$-\frac{1}{e\left(ex+d\right)}\sqrt{cx^4+a}+2\,\frac{cd^2}{e^4\sqrt{cx^4+a}}\sqrt{1-\frac{i\sqrt{c}x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}x^2}{\sqrt{a}}}EllipticF\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)\frac{1}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}$$
$$-\frac{d}{e^3}\sqrt{c}\ln\left(2\,x^2\sqrt{c}+2\,\sqrt{cx^4+a}\right)$$
$$+\frac{2\,i}{e^2}\sqrt{a}\sqrt{c}\sqrt{1-ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\left(EllipticF\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)-EllipticE\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\right)\frac{1}{\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4+a}}$$
$$+\frac{cd^3}{e^5}Artanh\left(\frac{1}{2}\left(2\,\frac{cd^2x^2}{e^2}+2\,a\right)\frac{1}{\sqrt{\frac{cd^4}{e^4}+a}}\frac{1}{\sqrt{cx^4+a}}\right)\frac{1}{\sqrt{\frac{cd^4}{e^4}+a}}$$
$$-2\,\frac{cd^2}{e^4\sqrt{cx^4+a}}\sqrt{1-\frac{i\sqrt{c}x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}x^2}{\sqrt{a}}}EllipticPi\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},\frac{-i\sqrt{a}e^2}{d^2\sqrt{c}},1\sqrt{\frac{-i\sqrt{c}}{\sqrt{a}}}\frac{1}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}\right)\frac{1}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+a)^(1/2)/(e*x+d)^2,x)
```

```
[Out] -1/e*(c*x^4+a)^(1/2)/(e*x+d)+2*c*d^2/e^4/(I/a^(1/2)*c^(1/2))^(1/2
)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)
/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-c^(1/2)
*d/e^3*ln(2*x^2*c^(1/2)+2*(c*x^4+a)^(1/2))+2*I*c^(1/2)/e^2*a^(1/2
)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/
a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2
)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I))+c*d
^3/e^5/(c*d^4/e^4+a)^(1/2)*arctanh(1/2*(2*c*x^2*d^2/e^2+2*a)/(c*d
^4/e^4+a)^(1/2)/(c*x^4+a)^(1/2))-2*c*d^2/e^4/(I/a^(1/2)*c^(1/2))^
(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(
1/2)/(c*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*c^(1/2))^(1/2),-I*a^
(1/2)/c^(1/2)/d^2*e^2,(-I/a^(1/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/
2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{cx^4+a}}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + a)/(e*x + d)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + a)/(e*x + d)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4+a}}{e^2x^2+2\,dex+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + a)/(e*x + d)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + a)/(e^2*x^2 + 2*d*e*x + d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{a+cx^4}}{(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+a)**(1/2)/(e*x+d)**2,x)
```

```
[Out] Integral(sqrt(a + c*x**4)/(d + e*x)**2, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{cx^4+a}}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + a)/(e*x + d)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + a)/(e*x + d)^2, x)
```

3.189 $\int \frac{(d+ex)^3}{\sqrt{a+cx^4}}\,dx$

Optimal. Leaf size=295

$$\frac{d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(3\sqrt{a}e^2+\sqrt{c}d^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}c^{3/4}\sqrt{a+cx^4}}-\frac{3\sqrt[4]{a}de^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}}+\frac{3d^2e\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}}+\frac{3de^2x\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{e^3\sqrt{a+cx^4}}{2c}$$

```
[Out] (e^3*Sqrt[a + c*x^4])/(2*c) + (3*d*e^2*x*Sqrt[a + c*x^4])/(Sqrt[c
]*(Sqrt[a] + Sqrt[c]*x^2)) + (3*d^2*e*ArcTanh[(Sqrt[c]*x^2)/Sqrt[
a + c*x^4]])/(2*Sqrt[c]) - (3*a^(1/4)*d*e^2*(Sqrt[a] + Sqrt[c]*x^
2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan
[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(3/4)*Sqrt[a + c*x^4]) + (d*(Sqrt
[c]*d^2 + 3*Sqrt[a]*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)
/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4
)], 1/2])/(2*a^(1/4)*c^(3/4)*Sqrt[a + c*x^4])
```

Rubi [A] time = 0.38285, antiderivative size = 295, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421

$$\frac{d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(3\sqrt{a}e^2+\sqrt{c}d^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}c^{3/4}\sqrt{a+cx^4}}-\frac{3\sqrt[4]{a}de^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}}+\frac{3d^2e\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}}+\frac{3de^2x\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{e^3\sqrt{a+cx^4}}{2c}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x)^3/Sqrt[a + c*x^4],x]
```

```
[Out] (e^3*Sqrt[a + c*x^4])/(2*c) + (3*d*e^2*x*Sqrt[a + c*x^4])/(Sqrt[c
]*(Sqrt[a] + Sqrt[c]*x^2)) + (3*d^2*e*ArcTanh[(Sqrt[c]*x^2)/Sqrt[
a + c*x^4]])/(2*Sqrt[c]) - (3*a^(1/4)*d*e^2*(Sqrt[a] + Sqrt[c]*x^
```

```
2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan
[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(3/4)*Sqrt[a + c*x^4]) + (d*(Sqrt
[c]*d^2 + 3*Sqrt[a]*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)
/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4
)], 1/2])/(2*a^(1/4)*c^(3/4)*Sqrt[a + c*x^4])
```

Rubi in Sympy [A] time = 42.9493, size = 272, normalized size = 0.92

$$-\frac{3\sqrt[4]{a}de^2\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{c^{\frac{3}{4}}\sqrt{a+cx^4}}+\frac{e^3\sqrt{a+cx^4}}{2c}+\frac{3d^2e\operatorname{atanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}}+\frac{3de^2x\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{d\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(3\sqrt{a}e^2+\sqrt{c}d^2\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{2\sqrt[4]{a}c^{\frac{3}{4}}\sqrt{a+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x+d)**3/(c*x**4+a)**(1/2),x)
```

```
[Out] -3*a**(1/4)*d*e**2*sqrt((a + c*x**4)/(sqrt(a) + sqrt(c)*x**2)**2)
*(sqrt(a) + sqrt(c)*x**2)*elliptic_e(2*atan(c**(1/4)*x/a**(1/4)),
 1/2)/(c**(3/4)*sqrt(a + c*x**4)) + e**3*sqrt(a + c*x**4)/(2*c) +
 3*d**2*e*atanh(sqrt(c)*x**2/sqrt(a + c*x**4))/(2*sqrt(c)) + 3*d*
e**2*x*sqrt(a + c*x**4)/(sqrt(c)*(sqrt(a) + sqrt(c)*x**2)) + d*sq
rt((a + c*x**4)/(sqrt(a) + sqrt(c)*x**2)**2)*(sqrt(a) + sqrt(c)*x
**2)*(3*sqrt(a)*e**2 + sqrt(c)*d**2)*elliptic_f(2*atan(c**(1/4)*x
/a**(1/4)), 1/2)/(2*a**(1/4)*c**(3/4)*sqrt(a + c*x**4))
```

Mathematica [C] time = 0.504441, size = 240, normalized size = 0.81

$$\frac{-2\sqrt{c}d\sqrt{\frac{cx^4}{a}+1}\left(3\sqrt{a}e^2+i\sqrt{c}d^2\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)+e\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\left(3\sqrt{c}d^2\sqrt{a+cx^4}\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)+e^2\left(a+cx^4\right)\right)+}{2c\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x)^3/Sqrt[a + c*x^4],x]
```

```
[Out] (Sqrt[(I*Sqrt[c])/Sqrt[a]]*e*(e^2*(a + c*x^4) + 3*Sqrt[c]*d^2*Sqr
t[a + c*x^4]*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]]) + 6*Sqrt[a]*
Sqrt[c]*d*e^2*Sqrt[1 + (c*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[(I*Sqr
```

```
t[c])/Sqrt[a]]*x], -1] - 2*Sqrt[c]*d*(I*Sqrt[c]*d^2 + 3*Sqrt[a]*e
^2)*Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt
[a]]*x], -1])/(2*Sqrt[(I*Sqrt[c])/Sqrt[a]]*c*Sqrt[a + c*x^4])
```

Maple [C] time = 0.012, size = 218, normalized size = 0.7

$$d^3\sqrt{1-ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{c}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4+a}}$$
$$+\frac{e^3}{2c}\sqrt{cx^4+a}+\frac{3d^2e}{2}\ln\left(x^2\sqrt{c}+\sqrt{cx^4+a}\right)\frac{1}{\sqrt{c}}$$
$$+3ie^2d\sqrt{a}\sqrt{1-ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\left(EllipticF\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)-EllipticE\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\right)\frac{1}{\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4+a}}\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x+d)^3/(c*x^4+a)^(1/2),x)
```

```
[Out] d^3/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+
I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/
2)*c^(1/2))^(1/2),I)+1/2*e^3*(c*x^4+a)^(1/2)/c+3/2*d^2*e*ln(x^2*c
^(1/2)+(c*x^4+a)^(1/2))/c^(1/2)+3*I*e^2*d*a^(1/2)/(I/a^(1/2)*c^(1
/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x
^2)^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2)
)^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(ex+d)^3}{\sqrt{cx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)^3/sqrt(c*x^4 + a),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^3/sqrt(c*x^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^3x^3+3\,de^2x^2+3\,d^2ex+d^3}{\sqrt{cx^4+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)^3/sqrt(c*x^4 + a),x, algorithm="fricas")
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)/sqrt(c*x^4 + a
), x)
```

Sympy [A] time = 8.35472, size = 141, normalized size = 0.48

$$e^3\left(\begin{cases}\frac{x^4}{4\sqrt{a}} & \text{for } c = 0\\ \frac{\sqrt{a+cx^4}}{2c} & \text{otherwise}\end{cases}\right) + \frac{3d^2e\,\text{asinh}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{c}} + \frac{d^3x\,\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)} + \frac{3de^2x^3\,\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x+d)**3/(c*x**4+a)**(1/2),x)
```

```
[Out] e**3*Piecewise((x**4/(4*sqrt(a)), Eq(c, 0)), (sqrt(a + c*x**4)/(2
*c), True)) + 3*d**2*e*asinh(sqrt(c)*x**2/sqrt(a))/(2*sqrt(c)) +
d**3*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi
)/a)/(4*sqrt(a)*gamma(5/4)) + 3*d*e**2*x**3*gamma(3/4)*hyper((1/2
, 3/4), (7/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(7/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^3}{\sqrt{cx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)^3/sqrt(c*x^4 + a),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3/sqrt(c*x^4 + a), x)
```

3.190 $\int \frac{(d+ex)^2}{\sqrt{a+cx^4}}\,dx$

Optimal. Leaf size=263

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2c^{3/4}\sqrt{a+cx^4}} - \frac{\sqrt[4]{a}e^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}} + \frac{de\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{\sqrt{c}} + \frac{e^2x\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

```
[Out] (e^2*x*Sqrt[a + c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) + (d*e*
ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]])/Sqrt[c] - (a^(1/4)*e^2*(S
qrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]
*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(3/4)*Sqrt[a +
 c*x^4]) + (a^(1/4)*((Sqrt[c]*d^2)/Sqrt[a] + e^2)*(Sqrt[a] + Sqrt
[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*
ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*c^(3/4)*Sqrt[a + c*x^4])
```

Rubi [A] time = 0.299818, antiderivative size = 263, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2c^{3/4}\sqrt{a+cx^4}} - \frac{\sqrt[4]{a}e^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}} + \frac{de\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{\sqrt{c}} + \frac{e^2x\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x)^2/Sqrt[a + c*x^4],x]
```

```
[Out] (e^2*x*Sqrt[a + c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) + (d*e*
ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]])/Sqrt[c] - (a^(1/4)*e^2*(S
qrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]
*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(3/4)*Sqrt[a +
 c*x^4]) + (a^(1/4)*((Sqrt[c]*d^2)/Sqrt[a] + e^2)*(Sqrt[a] + Sqrt
[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*
ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*c^(3/4)*Sqrt[a + c*x^4])
```

Rubi in Sympy [A] time = 33.2493, size = 240, normalized size = 0.91

$$-\frac{\sqrt[4]{a}e^{2}\sqrt{\frac{a+cx^{4}}{\left(\sqrt{a}+\sqrt{c}x^{2}\right)^{2}}}\left(\sqrt{a}+\sqrt{c}x^{2}\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{c^{\frac{3}{4}}\sqrt{a+cx^{4}}}+\frac{de\operatorname{atanh}\left(\frac{\sqrt{c}x^{2}}{\sqrt{a+cx^{4}}}\right)}{\sqrt{c}}+\frac{e^{2}x\sqrt{a+cx^{4}}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^{2}\right)}+\frac{\sqrt{\frac{a+cx^{4}}{\left(\sqrt{a}+\sqrt{c}x^{2}\right)^{2}}}\left(\sqrt{a}+\sqrt{c}x^{2}\right)\left(\sqrt{a}e^{2}+\sqrt{c}d^{2}\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{2\sqrt[4]{a}c^{\frac{3}{4}}\sqrt{a+cx^{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x+d)**2/(c*x**4+a)**(1/2),x)
```

```
[Out] -a**(1/4)*e**2*sqrt((a + c*x**4)/(sqrt(a) + sqrt(c)*x**2)**2)*(sq
rt(a) + sqrt(c)*x**2)*elliptic_e(2*atan(c**(1/4)*x/a**(1/4)), 1/2
)/(c**(3/4)*sqrt(a + c*x**4)) + d*e*atanh(sqrt(c)*x**2/sqrt(a + c
*x**4))/sqrt(c) + e**2*x*sqrt(a + c*x**4)/(sqrt(c)*(sqrt(a) + sqr
t(c)*x**2)) + sqrt((a + c*x**4)/(sqrt(a) + sqrt(c)*x**2)**2)*(sqr
t(a) + sqrt(c)*x**2)*(sqrt(a)*e**2 + sqrt(c)*d**2)*elliptic_f(2*a
tan(c**(1/4)*x/a**(1/4)), 1/2)/(2*a**(1/4)*c**(3/4)*sqrt(a + c*x*
*4))
```

Mathematica [C] time = 0.333624, size = 204, normalized size = 0.78

$$\frac{-\sqrt{\frac{cx^{4}}{a}+1}\left(\sqrt{a}e^{2}+i\sqrt{c}d^{2}\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)+de\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{a+cx^{4}}\tanh^{-1}\left(\frac{\sqrt{c}x^{2}}{\sqrt{a+cx^{4}}}\right)+\sqrt{a}e^{2}\sqrt{\frac{cx^{4}}{a}+1}E\left(i\sinh^{-1}\left(\right.\right.}{\sqrt{c}\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{a+cx^{4}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x)^2/Sqrt[a + c*x^4],x]
```

```
[Out] (Sqrt[(I*Sqrt[c])/Sqrt[a]]*d*e*Sqrt[a + c*x^4]*ArcTanh[(Sqrt[c]*x
^2)/Sqrt[a + c*x^4]] + Sqrt[a]*e^2*Sqrt[1 + (c*x^4)/a]*EllipticE[
I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] - (I*Sqrt[c]*d^2 + Sq
rt[a]*e^2)*Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c
])/Sqrt[a]]*x], -1])/(Sqrt[(I*Sqrt[c])/Sqrt[a]]*Sqrt[c]*Sqrt[a +
c*x^4])
```

Maple [C] time = 0.01, size = 197, normalized size = 0.8

$$d^2\sqrt{1-ix^2\sqrt{c}\,\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{c}\,\frac{1}{\sqrt{a}}}\,\mathit{EllipticF}\left(x\sqrt{i\sqrt{c}\,\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{c}\,\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4+a}}$$
$$+ie^2\sqrt{a}\sqrt{1-ix^2\sqrt{c}\,\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{c}\,\frac{1}{\sqrt{a}}}\left(\mathit{EllipticF}\left(x\sqrt{i\sqrt{c}\,\frac{1}{\sqrt{a}}},i\right)-\mathit{EllipticE}\left(x\sqrt{i\sqrt{c}\,\frac{1}{\sqrt{a}}},i\right)\right)\frac{1}{\sqrt{i\sqrt{c}\,\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4+a}}\frac{1}{\sqrt{c}}$$
$$+de\ln\left(x^2\sqrt{c}+\sqrt{cx^4+a}\right)\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x+d)^2/(c*x^4+a)^(1/2),x)
```

```
[Out] d^2/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+
I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/
2)*c^(1/2))^(1/2),I)+I*e^2*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I
/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^
4+a)^(1/2)/c^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-Elli
pticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I))+d*e*ln(x^2*c^(1/2)+(c*x^4+a
)^(1/2))/c^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^2}{\sqrt{cx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)^2/sqrt(c*x^4 + a),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^2/sqrt(c*x^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{e^2x^2+2\,dex+d^2}{\sqrt{cx^4+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)^2/sqrt(c*x^4 + a),x, algorithm="fricas")
```

```
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)/sqrt(c*x^4 + a), x)
```

Sympy [A] time = 7.53781, size = 105, normalized size = 0.4

$$\frac{de\operatorname{asinh}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{\sqrt{c}}+\frac{d^2x\,\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)}+\frac{e^2x^3\,\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x+d)**2/(c*x**4+a)**(1/2),x)
```

```
[Out] d*e*asinh(sqrt(c)*x**2/sqrt(a))/sqrt(c) + d**2*x*gamma(1/4)*hyper
((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5
/4)) + e**2*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4*exp_
polar(I*pi)/a)/(4*sqrt(a)*gamma(7/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^2}{\sqrt{cx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)^2/sqrt(c*x^4 + a),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2/sqrt(c*x^4 + a), x)
```

3.191 $\int \frac{d+ex}{\sqrt{a+cx^4}}\,dx$

Optimal. Leaf size=121

$$\frac{d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{e\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}}$$

```
[Out] (e*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]])/(2*Sqrt[c]) + (d*(Sqrt
[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*El
lipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*c^(1/4)*S
qrt[a + c*x^4])
```

Rubi [A] time = 0.129752, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294

$$\frac{d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{e\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x)/Sqrt[a + c*x^4],x]
```

```
[Out] (e*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]])/(2*Sqrt[c]) + (d*(Sqrt
[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*El
lipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*c^(1/4)*S
qrt[a + c*x^4])
```

Rubi in Sympy [A] time = 13.8098, size = 109, normalized size = 0.9

$$\frac{e\,\mathrm{atanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}}+\frac{d\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x+d)/(c*x**4+a)**(1/2),x)
```

```
[Out] e*atanh(sqrt(c)*x**2/sqrt(a + c*x**4))/(2*sqrt(c)) + d*sqrt((a +
c*x**4)/(sqrt(a) + sqrt(c)*x**2)**2)*(sqrt(a) + sqrt(c)*x**2)*ell
```

```
iptic_f(2*atan(c**(1/4)*x/a**(1/4)), 1/2)/(2*a**(1/4)*c**(1/4)*sq
rt(a + c*x**4))
```

Mathematica [C] time = 0.212945, size = 107, normalized size = 0.88

$$\frac{e\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}}-\frac{id\sqrt{\frac{cx^4}{a}+1}F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x)/Sqrt[a + c*x^4],x]
```

```
[Out] (e*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]])/(2*Sqrt[c]) - (I*d*Sqr
t[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x]
, -1])/(Sqrt[(I*Sqrt[c])/Sqrt[a]]*Sqrt[a + c*x^4])
```

Maple [C] time = 0.005, size = 96, normalized size = 0.8

$$d\sqrt{1-ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{c}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4+a}}+\frac{e}{2}\ln\left(x^2\sqrt{c}+\sqrt{cx^4+a}\right)\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x+d)/(c*x^4+a)^(1/2),x)
```

```
[Out] d/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/
a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)
*c^(1/2))^(1/2),I)+1/2*e*ln(x^2*c^(1/2)+(c*x^4+a)^(1/2))/c^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{ex+d}{\sqrt{cx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)/sqrt(c*x^4 + a),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)/sqrt(c*x^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ex+d}{\sqrt{cx^4+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)/sqrt(c*x^4 + a),x, algorithm="fricas")
```

```
[Out] integral((e*x + d)/sqrt(c*x^4 + a), x)
```

Sympy [A] time = 5.81879, size = 61, normalized size = 0.5

$$\frac{e\operatorname{asinh}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{c}}+\frac{dx\,\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x+d)/(c*x**4+a)**(1/2),x)
```

```
[Out] e*asinh(sqrt(c)*x**2/sqrt(a))/(2*sqrt(c)) + d*x*gamma(1/4)*hyper(
(1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/
4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{ex+d}{\sqrt{cx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)/sqrt(c*x^4 + a),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)/sqrt(c*x^4 + a), x)
```

3.192 $\int \frac{1}{\sqrt{a+cx^4}}\,dx$

Optimal. Leaf size=88

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}$$

```
[Out] ((Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)
^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*c^(
1/4)*Sqrt[a + c*x^4])
```

Rubi [A] time = 0.0344219, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[a + c*x^4],x]
```

```
[Out] ((Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)
^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*c^(
1/4)*Sqrt[a + c*x^4])
```

Rubi in Sympy [A] time = 3.9159, size = 78, normalized size = 0.89

$$\frac{\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c*x**4+a)**(1/2),x)
```

```
[Out] sqrt((a + c*x**4)/(sqrt(a) + sqrt(c)*x**2)**2)*(sqrt(a) + sqrt(c)
*x**2)*elliptic_f(2*atan(c**(1/4)*x/a**(1/4)), 1/2)/(2*a**(1/4)*c
**(1/4)*sqrt(a + c*x**4))
```

Mathematica [C] time = 0.0427462, size = 74, normalized size = 0.84

$$-\frac{i\sqrt{\frac{cx^4}{a}+1}F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[a + c*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sq
rt[a]]*x], -1])/(Sqrt[(I*Sqrt[c])/Sqrt[a]]*Sqrt[a + c*x^4])
```

Maple [C] time = 0.003, size = 70, normalized size = 0.8

$$1\sqrt{1-ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{c}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c*x^4+a)^(1/2),x)
```

```
[Out] 1/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/
a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)
*c^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{cx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(c*x^4 + a),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(c*x^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{cx^4+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(c*x^4 + a),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(c*x^4 + a), x)
```

Sympy [A] time = 2.02507, size = 36, normalized size = 0.41

$$\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\right|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x**4+a)**(1/2),x)
```

```
[Out] x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/
(4*sqrt(a)*gamma(5/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(c*x^4 + a),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(c*x^4 + a), x)
```

3.193 $\int \frac{1}{(d+ex)\sqrt{a+cx^4}}\,dx$

Optimal. Leaf size=413

$$\frac{\sqrt[4]{c}d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+cx^4}\left(\sqrt{a}e^2+\sqrt{c}d^2\right)}-\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}d\sqrt{a+cx^4}\left(\sqrt{a}e^2+\sqrt{c}d^2\right)}+\frac{\tan^{-1}\left(\frac{x\sqrt{-\frac{ae^4+cd^4}{d^2e^2}}}{\sqrt{a+cx^4}}\right)}{2d\sqrt{-\frac{ae^4+cd^4}{d^2e^2}}}-\frac{e\tanh^{-1}\left(\frac{ae^2+cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{2\sqrt{ae^4+cd^4}}$$

```
[Out] ArcTan[(Sqrt[-((c*d^4 + a*e^4)/(d^2*e^2))]*x)/Sqrt[a + c*x^4]]/(2
*d*Sqrt[-((c*d^4 + a*e^4)/(d^2*e^2))]) - (e*ArcTanh[(a*e^2 + c*d^
2*x^2)/(Sqrt[c*d^4 + a*e^4]*Sqrt[a + c*x^4])])/(2*Sqrt[c*d^4 + a*
e^4]) + (c^(1/4)*d*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt
[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/
2])/(2*a^(1/4)*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sqrt[a + c*x^4]) - ((S
qrt[c]*d^2 - Sqrt[a]*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4
)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[(Sqrt[c]*d^2 + Sqrt[a]*e^
2)^2/(4*Sqrt[a]*Sqrt[c]*d^2*e^2), 2*ArcTan[(c^(1/4)*x)/a^(1/4)],
1/2])/(4*a^(1/4)*c^(1/4)*d*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sqrt[a + c
*x^4])
```

Rubi [A] time = 0.602332, antiderivative size = 413, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$

$$\frac{\sqrt[4]{c}d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+cx^4}\left(\sqrt{a}e^2+\sqrt{c}d^2\right)}-\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}d\sqrt{a+cx^4}\left(\sqrt{a}e^2+\sqrt{c}d^2\right)}+\frac{\tan^{-1}\left(\frac{x\sqrt{-\frac{ae^4+cd^4}{d^2e^2}}}{\sqrt{a+cx^4}}\right)}{2d\sqrt{-\frac{ae^4+cd^4}{d^2e^2}}}-\frac{e\tanh^{-1}\left(\frac{ae^2+cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{2\sqrt{ae^4+cd^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d + e*x)*Sqrt[a + c*x^4]),x]
```

```
[Out] ArcTan[(Sqrt[-((c*d^4 + a*e^4)/(d^2*e^2))]*x)/Sqrt[a + c*x^4]]/(2
*d*Sqrt[-((c*d^4 + a*e^4)/(d^2*e^2))]) - (e*ArcTanh[(a*e^2 + c*d^
2*x^2)/(Sqrt[c*d^4 + a*e^4]*Sqrt[a + c*x^4])])/(2*Sqrt[c*d^4 + a*
e^4]) + (c^(1/4)*d*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt
[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/
2])/(2*a^(1/4)*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sqrt[a + c*x^4]) - ((S
qrt[c]*d^2 - Sqrt[a]*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4
)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[(Sqrt[c]*d^2 + Sqrt[a]*e^
2)^2/(4*Sqrt[a]*Sqrt[c]*d^2*e^2), 2*ArcTan[(c^(1/4)*x)/a^(1/4)],
1/2])/(4*a^(1/4)*c^(1/4)*d*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sqrt[a + c
*x^4])
```

Rubi in Sympy [A] time = 56.3311, size = 367, normalized size = 0.89

$$\frac{e\operatorname{atanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{2\sqrt{ae^4+cd^4}}+\frac{\operatorname{atan}\left(\frac{x\sqrt{-\frac{ae^2}{d^2}-\frac{cd^2}{e^2}}}{\sqrt{a+cx^4}}\right)}{2d\sqrt{-\frac{ae^2}{d^2}-\frac{cd^2}{e^2}}}+\frac{\sqrt[4]{c}d\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+cx^4}\left(\sqrt{a}e^2+\sqrt{c}d^2\right)}+\frac{\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(\sqrt{a}e^2-\sqrt{c}d^2\right)\Pi\left(\frac{\left(\sqrt{a}e^2+\sqrt{c}d^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2};2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}d\sqrt{a+cx^4}\left(\sqrt{a}e^2+\sqrt{c}d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x+d)/(c*x**4+a)**(1/2),x)
```

```
[Out] e*atanh((-a*e**2 - c*d**2*x**2)/(sqrt(a + c*x**4)*sqrt(a*e**4 + c
*d**4)))/(2*sqrt(a*e**4 + c*d**4)) + atan(x*sqrt(-a*e**2/d**2 - c
*d**2/e**2)/sqrt(a + c*x**4))/(2*d*sqrt(-a*e**2/d**2 - c*d**2/e**
2)) + c**(1/4)*d*sqrt((a + c*x**4)/(sqrt(a) + sqrt(c)*x**2)**2)*(
sqrt(a) + sqrt(c)*x**2)*elliptic_f(2*atan(c**(1/4)*x/a**(1/4)), 1
/2)/(2*a**(1/4)*sqrt(a + c*x**4)*(sqrt(a)*e**2 + sqrt(c)*d**2)) +
 sqrt((a + c*x**4)/(sqrt(a) + sqrt(c)*x**2)**2)*(sqrt(a) + sqrt(c
)*x**2)*(sqrt(a)*e**2 - sqrt(c)*d**2)*elliptic_pi((sqrt(a)*e**2 +
 sqrt(c)*d**2)**2/(4*sqrt(a)*sqrt(c)*d**2*e**2), 2*atan(c**(1/4)*
x/a**(1/4)), 1/2)/(4*a**(1/4)*c**(1/4)*d*sqrt(a + c*x**4)*(sqrt(a
)*e**2 + sqrt(c)*d**2))
```

Mathematica [C] time = 0.433786, size = 200, normalized size = 0.48

$$\frac{\sqrt{\frac{cx^4}{a}+1}\left(\sqrt[4]{c}d\log\left(\frac{e^2x^2-d^2}{ae^2\left(\sqrt{\frac{cx^4}{a}+1}\sqrt{\frac{cd^4}{ae^4}+1}+1\right)+cd^2x^2}\right)-2\sqrt[4]{-1}\sqrt[4]{a}e\sqrt{\frac{cd^4}{ae^4}+1}\,\Pi\left(\frac{i\sqrt{a}e^2}{\sqrt{c}d^2};\sin^{-1}\left(\frac{(-1)^{3/4}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)\right)}{2\sqrt[4]{c}de\sqrt{a+cx^4}\sqrt{\frac{cd^4}{ae^4}+1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d + e*x)*Sqrt[a + c*x^4]),x]
```

```
[Out] (Sqrt[1 + (c*x^4)/a]*(-2*(-1)^(1/4)*a^(1/4)*Sqrt[1 + (c*d^4)/(a*e
^4)]*e*EllipticPi[(I*Sqrt[a]*e^2)/(Sqrt[c]*d^2), ArcSin[((-1)^(3/
4)*c^(1/4)*x)/a^(1/4)], -1] + c^(1/4)*d*Log[(-d^2 + e^2*x^2)/(c*d
^2*x^2 + a*e^2*(1 + Sqrt[1 + (c*d^4)/(a*e^4)]*Sqrt[1 + (c*x^4)/a]
))]))/(2*c^(1/4)*d*Sqrt[1 + (c*d^4)/(a*e^4)]*e*Sqrt[a + c*x^4])
```

Maple [C] time = 0.008, size = 169, normalized size = 0.4

$$\frac{1}{e}\left(-\frac{1}{2}Artanh\left(\frac{1}{2}\left(2\,\frac{cd^2x^2}{e^2}+2\,a\right)\frac{1}{\sqrt{\frac{cd^4}{e^4}+a}}\frac{1}{\sqrt{cx^4+a}}\right)\frac{1}{\sqrt{\frac{cd^4}{e^4}+a}}+\frac{e}{d}\sqrt{1-ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{c}\frac{1}{\sqrt{a}}}EllipticPi\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*x+d)/(c*x^4+a)^(1/2),x)
```

```
[Out] 1/e*(-1/2/(c*d^4/e^4+a)^(1/2)*arctanh(1/2*(2*c*x^2*d^2/e^2+2*a)/(
c*d^4/e^4+a)^(1/2)/(c*x^4+a)^(1/2))+1/(I/a^(1/2)*c^(1/2))^(1/2)/d
*e*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2
)/(c*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*c^(1/2))^(1/2),-I*a^(1/
2)/c^(1/2)/d^2*e^2,(-I/a^(1/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))
^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{cx^4+a}(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + a)*(e*x + d)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + a)*(e*x + d)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + a)*(e*x + d)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + cx^4}\,(d + ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*x+d)/(c*x**4+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + c*x**4)*(d + e*x)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 + a}(ex + d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + a)*(e*x + d)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + a)*(e*x + d)), x)
```

3.194 $\int \frac{1}{(d+ex)^2\sqrt{a+cx^4}}\,dx$

Optimal. Leaf size=749

$$\frac{c^{5/4}d^4\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt{a+cx^4}\left(\sqrt{a}e^2+\sqrt{c}d^2\right)(ae^4+cd^4)}$$
$$-\frac{c^{3/4}d^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+cx^4}\left(\sqrt{a}e^2+\sqrt{c}d^2\right)(ae^4+cd^4)}$$
$$-\frac{e^3\sqrt{a+cx^4}}{(d+ex)(ae^4+cd^4)}+\frac{\sqrt{c}e^2x\sqrt{a+cx^4}}{\left(\sqrt{a}+\sqrt{c}x^2\right)(ae^4+cd^4)}$$
$$-\frac{\sqrt[4]{a}\sqrt[4]{c}e^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt{a+cx^4}(ae^4+cd^4)}-\frac{c\tan^{-1}\left(\frac{x\sqrt{-\frac{ae^4+cd^4}{d^2e^2}}}{\sqrt{a+cx^4}}\right)}{e^2\left(-\frac{ae^4+cd^4}{d^2e^2}\right)^{3/2}}$$
$$-\frac{\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d^2}{\sqrt{a}}-e^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt{a+cx^4}(ae^4+cd^4)}$$
$$-\frac{cd^3e\tanh^{-1}\left(\frac{ae^2+cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{(ae^4+cd^4)^{3/2}}$$

```
[Out] -((e^3*Sqrt[a + c*x^4])/((c*d^4 + a*e^4)*(d + e*x))) + (Sqrt[c]*e
^2*x*Sqrt[a + c*x^4])/((c*d^4 + a*e^4)*(Sqrt[a] + Sqrt[c]*x^2)) -
 (c*ArcTan[(Sqrt[-((c*d^4 + a*e^4)/(d^2*e^2))]*x)/Sqrt[a + c*x^4]
])/(e^2*(-((c*d^4 + a*e^4)/(d^2*e^2)))^(3/2)) - (c*d^3*e*ArcTanh[
(a*e^2 + c*d^2*x^2)/(Sqrt[c*d^4 + a*e^4]*Sqrt[a + c*x^4])])/(c*d^
4 + a*e^4)^(3/2) - (a^(1/4)*c^(1/4)*e^2*(Sqrt[a] + Sqrt[c]*x^2)*S
qrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^
(1/4)*x)/a^(1/4)], 1/2])/((c*d^4 + a*e^4)*Sqrt[a + c*x^4]) - (a^(
1/4)*c^(1/4)*((Sqrt[c]*d^2)/Sqrt[a] - e^2)*(Sqrt[a] + Sqrt[c]*x^2
)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[
(c^(1/4)*x)/a^(1/4)], 1/2])/(2*(c*d^4 + a*e^4)*Sqrt[a + c*x^4]) +
 (c^(5/4)*d^4*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] +
 Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(
a^(1/4)*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*(c*d^4 + a*e^4)*Sqrt[a + c*x^
4]) - (c^(3/4)*d^2*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*(Sqrt[a] + Sqrt[c]
*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[(Sqr
t[c]*d^2 + Sqrt[a]*e^2)^2/(4*Sqrt[a]*Sqrt[c]*d^2*e^2), 2*ArcTan[(
c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*(Sqrt[c]*d^2 + Sqrt[a]*e^2)
*(c*d^4 + a*e^4)*Sqrt[a + c*x^4])
```

Rubi [A] time = 1.16176, antiderivative size = 749, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474

$$\frac{c^{5/4}d^4\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt{a+cx^4}\left(\sqrt{a}e^2+\sqrt{c}d^2\right)(ae^4+cd^4)} - \frac{c^{3/4}d^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+cx^4}\left(\sqrt{a}e^2+\sqrt{c}d^2\right)(ae^4+cd^4)} - \frac{e^3\sqrt{a+cx^4}}{(d+ex)(ae^4+cd^4)} + \frac{\sqrt{c}e^2x\sqrt{a+cx^4}}{\left(\sqrt{a}+\sqrt{c}x^2\right)(ae^4+cd^4)} - \frac{\sqrt[4]{a}\sqrt[4]{c}e^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt{a+cx^4}(ae^4+cd^4)} - \frac{c\tan^{-1}\left(\frac{x\sqrt{-\frac{ae^4+cd^4}{d^2e^2}}}{\sqrt{a+cx^4}}\right)}{e^2\left(-\frac{ae^4+cd^4}{d^2e^2}\right)^{3/2}} - \frac{\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d^2}{\sqrt{a}}-e^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt{a+cx^4}(ae^4+cd^4)} - \frac{cd^3e\tanh^{-1}\left(\frac{ae^2+cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{(ae^4+cd^4)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[1/((d + e*x)^2*Sqrt[a + c*x^4]),x]
```

```
[Out] -((e^3*Sqrt[a + c*x^4])/((c*d^4 + a*e^4)*(d + e*x))) + (Sqrt[c]*e
^2*x*Sqrt[a + c*x^4])/((c*d^4 + a*e^4)*(Sqrt[a] + Sqrt[c]*x^2)) -
 (c*ArcTan[(Sqrt[-((c*d^4 + a*e^4)/(d^2*e^2))]*x)/Sqrt[a + c*x^4]
])/(e^2*(-((c*d^4 + a*e^4)/(d^2*e^2)))^(3/2)) - (c*d^3*e*ArcTanh[
(a*e^2 + c*d^2*x^2)/(Sqrt[c*d^4 + a*e^4]*Sqrt[a + c*x^4])])/(c*d^
4 + a*e^4)^(3/2) - (a^(1/4)*c^(1/4)*e^2*(Sqrt[a] + Sqrt[c]*x^2)*S
qrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^
(1/4)*x)/a^(1/4)], 1/2])/((c*d^4 + a*e^4)*Sqrt[a + c*x^4]) - (a^(
1/4)*c^(1/4)*((Sqrt[c]*d^2)/Sqrt[a] - e^2)*(Sqrt[a] + Sqrt[c]*x^2
)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[
(c^(1/4)*x)/a^(1/4)], 1/2])/(2*(c*d^4 + a*e^4)*Sqrt[a + c*x^4]) +
 (c^(5/4)*d^4*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] +
 Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(
a^(1/4)*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*(c*d^4 + a*e^4)*Sqrt[a + c*x^
4]) - (c^(3/4)*d^2*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*(Sqrt[a] + Sqrt[c]
*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[(Sqr
t[c]*d^2 + Sqrt[a]*e^2)^2/(4*Sqrt[a]*Sqrt[c]*d^2*e^2), 2*ArcTan[(
c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*(Sqrt[c]*d^2 + Sqrt[a]*e^2)
*(c*d^4 + a*e^4)*Sqrt[a + c*x^4])
```

Rubi in Sympy [A] time = 119.817, size = 666, normalized size = 0.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x+d)**2/(c*x**4+a)**(1/2),x)
```

```
[Out] -a**(1/4)*c**(1/4)*e**2*sqrt((a + c*x**4)/(sqrt(a) + sqrt(c)*x**2
)**2)*(sqrt(a) + sqrt(c)*x**2)*elliptic_e(2*atan(c**(1/4)*x/a**(1
/4)), 1/2)/(sqrt(a + c*x**4)*(a*e**4 + c*d**4)) + sqrt(c)*e**2*x*
sqrt(a + c*x**4)/((sqrt(a) + sqrt(c)*x**2)*(a*e**4 + c*d**4)) + c
*d**3*e*atanh((-a*e**2 - c*d**2*x**2)/(sqrt(a + c*x**4)*sqrt(a*e*
*4 + c*d**4)))/(a*e**4 + c*d**4)**(3/2) - c*atan(x*sqrt(-a*e**2/d
**2 - c*d**2/e**2)/sqrt(a + c*x**4))/(e**2*(-a*e**2/d**2 - c*d**2
/e**2)**(3/2)) - e**3*sqrt(a + c*x**4)/((d + e*x)*(a*e**4 + c*d**
4)) + c**(5/4)*d**4*sqrt((a + c*x**4)/(sqrt(a) + sqrt(c)*x**2)**2
)*(sqrt(a) + sqrt(c)*x**2)*elliptic_f(2*atan(c**(1/4)*x/a**(1/4))
, 1/2)/(a**(1/4)*sqrt(a + c*x**4)*(sqrt(a)*e**2 + sqrt(c)*d**2)*(
a*e**4 + c*d**4)) + c**(3/4)*d**2*sqrt((a + c*x**4)/(sqrt(a) + sq
rt(c)*x**2)**2)*(sqrt(a) + sqrt(c)*x**2)*(sqrt(a)*e**2 - sqrt(c)*
d**2)*elliptic_pi((sqrt(a)*e**2 + sqrt(c)*d**2)**2/(4*sqrt(a)*sqr
t(c)*d**2*e**2), 2*atan(c**(1/4)*x/a**(1/4)), 1/2)/(2*a**(1/4)*sq
rt(a + c*x**4)*(sqrt(a)*e**2 + sqrt(c)*d**2)*(a*e**4 + c*d**4)) +
 c**(1/4)*sqrt((a + c*x**4)/(sqrt(a) + sqrt(c)*x**2)**2)*(sqrt(a)
 + sqrt(c)*x**2)*(sqrt(a)*e**2 - sqrt(c)*d**2)*elliptic_f(2*atan(
c**(1/4)*x/a**(1/4)), 1/2)/(2*a**(1/4)*sqrt(a + c*x**4)*(a*e**4 +
 c*d**4))
```

Mathematica [C] time = 2.41905, size = 462, normalized size = 0.62

$$-\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\left(2\sqrt[4]{-1}\sqrt[4]{a}c^{3/4}d^2\sqrt{\frac{cx^4}{a}+1}(d+ex)\sqrt{ae^4+cd^4}\left(\frac{i\sqrt{a}e^2}{\sqrt{c}d^2};\sin^{-1}\left(\frac{(-1)^{3/4}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)+e^3\left(a+cx^4\right)\sqrt{ae^4+cd^4}-cd^3e\sqrt{a+\cdots}\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d + e*x)^2*Sqrt[a + c*x^4]),x]
```

```
[Out] (Sqrt[a]*Sqrt[c]*e^2*Sqrt[c*d^4 + a*e^4]*(d + e*x)*Sqrt[1 + (c*x^
4)/a]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] + I*S
qrt[c]*(Sqrt[c]*d^2 + I*Sqrt[a]*e^2)*Sqrt[c*d^4 + a*e^4]*(d + e*x
)*Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a
]]*x], -1] - Sqrt[(I*Sqrt[c])/Sqrt[a]]*(e^3*Sqrt[c*d^4 + a*e^4]*(
```

```
a + c*x^4) + 2*(-1)^(1/4)*a^(1/4)*c^(3/4)*d^2*Sqrt[c*d^4 + a*e^4]
*(d + e*x)*Sqrt[1 + (c*x^4)/a]*EllipticPi[(I*Sqrt[a]*e^2)/(Sqrt[c
]*d^2), ArcSin[((-1)^(3/4)*c^(1/4)*x)/a^(1/4)], -1] - c*d^3*e*(d
+ e*x)*Sqrt[a + c*x^4]*Log[-d^2 + e^2*x^2] + c*d^3*e*(d + e*x)*Sq
rt[a + c*x^4]*Log[a*e^2 + c*d^2*x^2 + Sqrt[c*d^4 + a*e^4]*Sqrt[a
+ c*x^4]]))/(Sqrt[(I*Sqrt[c])/Sqrt[a]]*(c*d^4 + a*e^4)^(3/2)*(d +
 e*x)*Sqrt[a + c*x^4])
```

Maple [C] time = 0.009, size = 421, normalized size = 0.6

$$-\frac{e^3}{(ae^4+cd^4)(ex+d)}\sqrt{cx^4+a}$$
$$-\frac{cd^2}{ae^4+cd^4}\sqrt{1-ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{c}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4+a}}$$
$$+\frac{ie^2}{ae^4+cd^4}\sqrt{a}\sqrt{c}\sqrt{1-ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\left(EllipticF\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)-EllipticE\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\right)\frac{1}{\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4+a}}$$
$$+2\frac{cd^3}{(ae^4+cd^4)e}\left(-1/2\,1Artanh\left(1/2\frac{1}{\sqrt{cx^4+a}}\left(2\frac{cd^2x^2}{e^2}+2a\right)\frac{1}{\sqrt{\frac{cd^4}{e^4}+a}}\right)\frac{1}{\sqrt{\frac{cd^4}{e^4}+a}}+\frac{e}{d\sqrt{cx^4+a}}\sqrt{1-\frac{i\sqrt{c}x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}x^2}{\sqrt{a}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(1/(e*x+d)^2/(c*x^4+a)^(1/2),x)
```

```
[Out] -e^3*(c*x^4+a)^(1/2)/(a*e^4+c*d^4)/(e*x+d)-d^2*c/(a*e^4+c*d^4)/(I
/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1
/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(
1/2))^(1/2),I)+I*c^(1/2)*e^2/(a*e^4+c*d^4)*a^(1/2)/(I/a^(1/2)*c^(
1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*
x^2)^(1/2)/(c*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2)
,I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I))+2*c*d^3/(a*e^4+c*d^
4)/e*(-1/2/(c*d^4/e^4+a)^(1/2)*arctanh(1/2*(2*c*x^2*d^2/e^2+2*a)/
(c*d^4/e^4+a)^(1/2)/(c*x^4+a)^(1/2))+1/(I/a^(1/2)*c^(1/2))^(1/2)/
d*e*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/
2)/(c*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*c^(1/2))^(1/2),-I*a^(1
/2)/c^(1/2)/d^2*e^2,(-I/a^(1/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2)
)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4+a}(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + a)*(e*x + d)^2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + a)*(e*x + d)^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + a)*(e*x + d)^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + cx^4}\,(d + ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*x+d)**2/(c*x**4+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + c*x**4)*(d + e*x)**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 + a}(ex + d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + a)*(e*x + d)^2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + a)*(e*x + d)^2), x)
```

3.195 $\int \frac{1}{(d+ex)^3\sqrt{a+cx^4}}\,dx$

Optimal. Leaf size=1969

result too large to display

```
[Out] (-3*Sqrt[c]*e^2*x*Sqrt[a + c*x^4])/(2*d*(c*d^4 + a*e^4)*(Sqrt[a]
+ Sqrt[c]*x^2)) + (3*Sqrt[c]*e^2*(3*c*d^4 + a*e^4)*x*Sqrt[a + c*x
^4])/(2*d*(c*d^4 + a*e^4)^2*(Sqrt[a] + Sqrt[c]*x^2)) - (d^2*e^3*S
qrt[a + c*x^4])/((c*d^4 + a*e^4)*(d^2 - e^2*x^2)^2) + (d*e^4*x*Sq
rt[a + c*x^4])/((c*d^4 + a*e^4)*(d^2 - e^2*x^2)^2) - (9*c*d^4*e^3
*Sqrt[a + c*x^4])/(4*(c*d^4 + a*e^4)^2*(d^2 - e^2*x^2)) - (e^3*(c
*d^4 - 2*a*e^4)*Sqrt[a + c*x^4])/(4*(c*d^4 + a*e^4)^2*(d^2 - e^2*
x^2)) - (3*e^4*x*Sqrt[a + c*x^4])/(2*d*(c*d^4 + a*e^4)*(d^2 - e^2
*x^2)) + (3*e^4*(3*c*d^4 + a*e^4)*x*Sqrt[a + c*x^4])/(2*d*(c*d^4
+ a*e^4)^2*(d^2 - e^2*x^2)) + (3*(3*c*d^4 + a*e^4)*ArcTan[(Sqrt[-
((c*d^4 + a*e^4)/(d^2*e^2))]*x)/Sqrt[a + c*x^4]])/(4*d^5*e^2*(-((
c*d^4 + a*e^4)/(d^2*e^2)))^(3/2)) + (3*(5*c^2*d^8 + 2*a*c*d^4*e^4
 + a^2*e^8)*ArcTan[(Sqrt[-((c*d^4 + a*e^4)/(d^2*e^2))]*x)/Sqrt[a
+ c*x^4]])/(4*d^3*(c*d^4 + a*e^4)^2*Sqrt[-((c*d^4 + a*e^4)/(d^2*e
^2))]) + (3*a*c*d^2*e^5*ArcTanh[(a*e^2 + c*d^2*x^2)/(Sqrt[c*d^4 +
 a*e^4]*Sqrt[a + c*x^4])])/(4*(c*d^4 + a*e^4)^(5/2)) - (3*c*d^2*e
*(2*c*d^4 - a*e^4)*ArcTanh[(a*e^2 + c*d^2*x^2)/(Sqrt[c*d^4 + a*e^
4]*Sqrt[a + c*x^4])])/(4*(c*d^4 + a*e^4)^(5/2)) + (3*a^(1/4)*c^(1
/4)*e^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[
c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*d*(c
*d^4 + a*e^4)*Sqrt[a + c*x^4]) - (3*a^(1/4)*c^(1/4)*e^2*(3*c*d^4
+ a*e^4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt
[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*d*(
c*d^4 + a*e^4)^2*Sqrt[a + c*x^4]) + (3*a^(1/4)*c^(1/4)*((Sqrt[c]*
d^2)/Sqrt[a] - e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqr
t[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1
/2])/(4*d*(c*d^4 + a*e^4)*Sqrt[a + c*x^4]) - (3*c^(1/4)*(3*c*d^4
+ a*e^4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt
[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(
1/4)*d*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*(c*d^4 + a*e^4)*Sqrt[a + c*x^4
]) - (c^(1/4)*(7*c^(3/2)*d^6 - 9*Sqrt[a]*c*d^4*e^2 + a*Sqrt[c]*d^
2*e^4 - 3*a^(3/2)*e^6)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(
Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)]
, 1/2])/(4*a^(1/4)*d*(c*d^4 + a*e^4)^2*Sqrt[a + c*x^4]) + (3*c^(1
/4)*(5*c^2*d^8 + 2*a*c*d^4*e^4 + a^2*e^8)*(Sqrt[a] + Sqrt[c]*x^2)
*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(
c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*d*(Sqrt[c]*d^2 + Sqrt[a]*e^
2)*(c*d^4 + a*e^4)^2*Sqrt[a + c*x^4]) + (3*(Sqrt[c]*d^2 - Sqrt[a]
*e^2)*(3*c*d^4 + a*e^4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/
(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[(Sqrt[c]*d^2 + Sqrt[a]*e^2)
^2/(4*Sqrt[a]*Sqrt[c]*d^2*e^2), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/
2])/(8*a^(1/4)*c^(1/4)*d^3*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*(c*d^4 + a
*e^4)*Sqrt[a + c*x^4]) - (3*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*(5*c^2*d^
8 + 2*a*c*d^4*e^4 + a^2*e^8)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*
x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[(Sqrt[c]*d^2 + Sqrt[a]
*e^2)^2/(4*Sqrt[a]*Sqrt[c]*d^2*e^2), 2*ArcTan[(c^(1/4)*x)/a^(1/4)
], 1/2])/(8*a^(1/4)*c^(1/4)*d^3*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*(c*d^
```

```
4 + a*e^4)^2*Sqrt[a + c*x^4])
```

Rubi [A] time = 9.25775, antiderivative size = 1969, normalized size of antiderivative = 1., number of steps used = 36, number of rules used = 17, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.895$

result too large to display

Warning: Unable to verify antiderivative.

```
[In]  Int[1/((d + e*x)^3*Sqrt[a + c*x^4]),x]
```

```
[Out] (-3*Sqrt[c]*e^2*x*Sqrt[a + c*x^4])/(2*d*(c*d^4 + a*e^4)*(Sqrt[a]
+ Sqrt[c]*x^2)) + (3*Sqrt[c]*e^2*(3*c*d^4 + a*e^4)*x*Sqrt[a + c*x
^4])/(2*d*(c*d^4 + a*e^4)^2*(Sqrt[a] + Sqrt[c]*x^2)) - (d^2*e^3*S
qrt[a + c*x^4])/((c*d^4 + a*e^4)*(d^2 - e^2*x^2)^2) + (d*e^4*x*Sq
rt[a + c*x^4])/((c*d^4 + a*e^4)*(d^2 - e^2*x^2)^2) - (9*c*d^4*e^3
*Sqrt[a + c*x^4])/(4*(c*d^4 + a*e^4)^2*(d^2 - e^2*x^2)) - (e^3*(c
*d^4 - 2*a*e^4)*Sqrt[a + c*x^4])/(4*(c*d^4 + a*e^4)^2*(d^2 - e^2*
x^2)) - (3*e^4*x*Sqrt[a + c*x^4])/(2*d*(c*d^4 + a*e^4)*(d^2 - e^2
*x^2)) + (3*e^4*(3*c*d^4 + a*e^4)*x*Sqrt[a + c*x^4])/(2*d*(c*d^4
+ a*e^4)^2*(d^2 - e^2*x^2)) + (3*(3*c*d^4 + a*e^4)*ArcTan[(Sqrt[-
((c*d^4 + a*e^4)/(d^2*e^2))]*x)/Sqrt[a + c*x^4]])/(4*d^5*e^2*(-((
c*d^4 + a*e^4)/(d^2*e^2)))^(3/2)) + (3*(5*c^2*d^8 + 2*a*c*d^4*e^4
 + a^2*e^8)*ArcTan[(Sqrt[-((c*d^4 + a*e^4)/(d^2*e^2))]*x)/Sqrt[a
+ c*x^4]])/(4*d^3*(c*d^4 + a*e^4)^2*Sqrt[-((c*d^4 + a*e^4)/(d^2*e
^2))]) + (3*a*c*d^2*e^5*ArcTanh[(a*e^2 + c*d^2*x^2)/(Sqrt[c*d^4 +
 a*e^4]*Sqrt[a + c*x^4])]])/(4*(c*d^4 + a*e^4)^(5/2)) - (3*c*d^2*e
*(2*c*d^4 - a*e^4)*ArcTanh[(a*e^2 + c*d^2*x^2)/(Sqrt[c*d^4 + a*e^
4]*Sqrt[a + c*x^4])]])/(4*(c*d^4 + a*e^4)^(5/2)) + (3*a^(1/4)*c^(1
/4)*e^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[
c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*d*(c
*d^4 + a*e^4)*Sqrt[a + c*x^4]) - (3*a^(1/4)*c^(1/4)*e^2*(3*c*d^4
+ a*e^4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt
[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*d*(
c*d^4 + a*e^4)^2*Sqrt[a + c*x^4]) + (3*a^(1/4)*c^(1/4)*((Sqrt[c]*
d^2)/Sqrt[a] - e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqr
t[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1
/2])/(4*d*(c*d^4 + a*e^4)*Sqrt[a + c*x^4]) - (3*c^(1/4)*(3*c*d^4
+ a*e^4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt
[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(
1/4)*d*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*(c*d^4 + a*e^4)*Sqrt[a + c*x^4
]) - (c^(1/4)*(7*c^(3/2)*d^6 - 9*Sqrt[a]*c*d^4*e^2 + a*Sqrt[c]*d^
2*e^4 - 3*a^(3/2)*e^6)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(
Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)]
, 1/2])/(4*a^(1/4)*d*(c*d^4 + a*e^4)^2*Sqrt[a + c*x^4]) + (3*c^(1
/4)*(5*c^2*d^8 + 2*a*c*d^4*e^4 + a^2*e^8)*(Sqrt[a] + Sqrt[c]*x^2)
*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(
c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*d*(Sqrt[c]*d^2 + Sqrt[a]*e^
2)*(c*d^4 + a*e^4)^2*Sqrt[a + c*x^4]) + (3*(Sqrt[c]*d^2 - Sqrt[a]
```

```
*e^2)*(3*c*d^4 + a*e^4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/
(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[(Sqrt[c]*d^2 + Sqrt[a]*e^2)
^2/(4*Sqrt[a]*Sqrt[c]*d^2*e^2), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/
2])/(8*a^(1/4)*c^(1/4)*d^3*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*(c*d^4 + a
*e^4)*Sqrt[a + c*x^4]) - (3*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*(5*c^2*d^
8 + 2*a*c*d^4*e^4 + a^2*e^8)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*
x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[(Sqrt[c]*d^2 + Sqrt[a]
*e^2)^2/(4*Sqrt[a]*Sqrt[c]*d^2*e^2), 2*ArcTan[(c^(1/4)*x)/a^(1/4)
], 1/2])/(8*a^(1/4)*c^(1/4)*d^3*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*(c*d^
4 + a*e^4)^2*Sqrt[a + c*x^4])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + cx^4}\,(d + ex)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x+d)**3/(c*x**4+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + c*x**4)*(d + e*x)**3), x)
```

Mathematica [C] time = 2.04995, size = 884, normalized size = 0.45

$$\frac{3c^2e(d + ex)^2\sqrt{cx^4 + a}\log\left(e^2x^2 - d^2\right)d^6 - 3c^2e(d + ex)^2\sqrt{cx^4 + a}\log\left(ae^2 + cd^2x^2 + \sqrt{cd^4 + ae^4}\sqrt{cx^4 + a}\right)d^6 + \frac{4ic^2\sqrt{cd^4+ae\ldots}}{\ldots}}{\ldots}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d + e*x)^3*Sqrt[a + c*x^4]),x]
```

```
[Out] (-(e^3*(c*d^4 + a*e^4)^(3/2)*(a + c*x^4)) - 6*c*d^3*e^3*Sqrt[c*d^
4 + a*e^4]*(d + e*x)*(a + c*x^4) - (6*I)*a*Sqrt[(I*Sqrt[c])/Sqrt[
a]]*c*d^3*e^2*Sqrt[c*d^4 + a*e^4]*(d + e*x)^2*Sqrt[1 + (c*x^4)/a]
*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] + ((4*I)*c
^2*d^5*Sqrt[c*d^4 + a*e^4]*(d + e*x)^2*Sqrt[1 + (c*x^4)/a]*Ellipt
icF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1])/Sqrt[(I*Sqrt[c])
/Sqrt[a]] + (6*I)*a*Sqrt[(I*Sqrt[c])/Sqrt[a]]*c*d^3*e^2*Sqrt[c*d^
4 + a*e^4]*(d + e*x)^2*Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sq
rt[(I*Sqrt[c])/Sqrt[a]]*x], -1] - ((2*I)*a*c*d*e^4*Sqrt[c*d^4 + a
*e^4]*(d + e*x)^2*Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I
*Sqrt[c])/Sqrt[a]]*x], -1])/Sqrt[(I*Sqrt[c])/Sqrt[a]] - 6*(-1)^(1
```

```
/4)*a^(1/4)*c^(7/4)*d^5*Sqrt[c*d^4 + a*e^4]*(d + e*x)^2*Sqrt[1 +
(c*x^4)/a]*EllipticPi[(I*Sqrt[a]*e^2)/(Sqrt[c]*d^2), ArcSin[((-1)
^(3/4)*c^(1/4)*x)/a^(1/4)], -1] + 6*(-1)^(1/4)*a^(5/4)*c^(3/4)*d*
e^4*Sqrt[c*d^4 + a*e^4]*(d + e*x)^2*Sqrt[1 + (c*x^4)/a]*EllipticP
i[(I*Sqrt[a]*e^2)/(Sqrt[c]*d^2), ArcSin[((-1)^(3/4)*c^(1/4)*x)/a^
(1/4)], -1] + 3*c^2*d^6*e*(d + e*x)^2*Sqrt[a + c*x^4]*Log[-d^2 +
e^2*x^2] - 3*a*c*d^2*e^5*(d + e*x)^2*Sqrt[a + c*x^4]*Log[-d^2 + e
^2*x^2] - 3*c^2*d^6*e*(d + e*x)^2*Sqrt[a + c*x^4]*Log[a*e^2 + c*d
^2*x^2 + Sqrt[c*d^4 + a*e^4]*Sqrt[a + c*x^4]] + 3*a*c*d^2*e^5*(d
+ e*x)^2*Sqrt[a + c*x^4]*Log[a*e^2 + c*d^2*x^2 + Sqrt[c*d^4 + a*e
^4]*Sqrt[a + c*x^4]])/(2*(c*d^4 + a*e^4)^(5/2)*(d + e*x)^2*Sqrt[a
 + c*x^4])
```

Maple [C] time = 0.025, size = 483, normalized size = 0.3

$$-\frac{e^3}{(2\,ae^4+2\,cd^4)\,(ex+d)^2}\sqrt{cx^4+a}-3\,\frac{ce^3d^3\sqrt{cx^4+a}}{(ae^4+cd^4)^2\,(ex+d)}$$

$$+\frac{cd\,(ae^4-2\,cd^4)}{(ae^4+cd^4)^2}\sqrt{1-ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{c}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4+a}}$$

$$+\frac{3\,id^3e^2}{(ae^4+cd^4)^2}c^{\frac{3}{2}}\sqrt{a}\sqrt{1-ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\left(EllipticF\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)-EllipticE\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\right)\frac{1}{\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4+}}$$

$$-3\,\frac{cd^2\,(ae^4-cd^4)}{(ae^4+cd^4)^2\,e}\left(-1/2\,1Artanh\left(1/2\,\frac{1}{\sqrt{cx^4+a}}\left(2\,\frac{cd^2x^2}{e^2}+2\,a\right)\frac{1}{\sqrt{\frac{cd^4}{e^4}+a}}\right)\frac{1}{\sqrt{\frac{cd^4}{e^4}+a}}+\frac{e}{d\sqrt{cx^4+a}}\sqrt{1-\frac{i\sqrt{c}x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{}}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*x+d)^3/(c*x^4+a)^(1/2),x)
```

```
[Out] -1/2*e^3/(a*e^4+c*d^4)*(c*x^4+a)^(1/2)/(e*x+d)^2-3*c*e^3*d^3/(a*e
^4+c*d^4)^2*(c*x^4+a)^(1/2)/(e*x+d)+c*d*(a*e^4-2*c*d^4)/(a*e^4+c*
d^4)^2/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*
(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^
(1/2)*c^(1/2))^(1/2),I)+3*I*c^(3/2)*d^3*e^2/(a*e^4+c*d^4)^2*a^(1/
2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I
/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/
2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I))-3*
c*d^2*(a*e^4-c*d^4)/(a*e^4+c*d^4)^2/e*(-1/2/(c*d^4/e^4+a)^(1/2)*a
rctanh(1/2*(2*c*x^2*d^2/e^2+2*a)/(c*d^4/e^4+a)^(1/2)/(c*x^4+a)^(1
/2))+1/(I/a^(1/2)*c^(1/2))^(1/2)/d*e*(1-I/a^(1/2)*c^(1/2)*x^2)^(1
/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticPi(x*
(I/a^(1/2)*c^(1/2))^(1/2),-I*a^(1/2)/c^(1/2)/d^2*e^2,(-I/a^(1/2)*
c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 + a}(ex + d)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + a)*(e*x + d)^3),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + a)*(e*x + d)^3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + a)*(e*x + d)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + cx^4}\,(d + ex)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*x+d)**3/(c*x**4+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + c*x**4)*(d + e*x)**3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 + a}(ex + d)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + a)*(e*x + d)^3),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + a)*(e*x + d)^3), x)
```

3.196 $\int \frac{(d+ex)^3}{(a+cx^4)^{3/2}} \, dx$

Optimal. Leaf size=298

$$\frac{d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{c}d^2-3\sqrt{a}e^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}c^{3/4}\sqrt{a+cx^4}}+\frac{3de^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}c^{3/4}\sqrt{a+cx^4}}-\frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{2ac\sqrt{a+cx^4}}-\frac{3de^2x\sqrt{a+cx^4}}{2a\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

```
[Out] (-3*d*e^2*x*Sqrt[a + c*x^4])/(2*a*Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)
) - (a*e^3 - c*x*(d^3 + 3*d^2*e*x + 3*d*e^2*x^2))/(2*a*c*Sqrt[a +
 c*x^4]) + (3*d*e^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqr
t[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1
/2])/(2*a^(3/4)*c^(3/4)*Sqrt[a + c*x^4]) + (d*(Sqrt[c]*d^2 - 3*Sq
rt[a]*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sq
rt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a
^(5/4)*c^(3/4)*Sqrt[a + c*x^4])
```

Rubi [A] time = 0.342391, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$\frac{d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{c}d^2-3\sqrt{a}e^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}c^{3/4}\sqrt{a+cx^4}}+\frac{3de^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}c^{3/4}\sqrt{a+cx^4}}-\frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{2ac\sqrt{a+cx^4}}-\frac{3de^2x\sqrt{a+cx^4}}{2a\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x)^3/(a + c*x^4)^(3/2),x]
```


```
[Out] (-3*d*e^2*x*Sqrt[a + c*x^4])/(2*a*Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)
) - (a*e^3 - c*x*(d^3 + 3*d^2*e*x + 3*d*e^2*x^2))/(2*a*c*Sqrt[a +
 c*x^4]) + (3*d*e^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqr
t[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1
```

```
/2])/(2*a^(3/4)*c^(3/4)*Sqrt[a + c*x^4]) + (d*(Sqrt[c]*d^2 - 3*Sq
rt[a]*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sq
rt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a
^(5/4)*c^(3/4)*Sqrt[a + c*x^4])
```

Rubi in Sympy [A] time = 37.383, size = 272, normalized size = 0.91

$$-\frac{ae^3 - cx\left(d^3 + 3d^2ex + 3de^2x^2\right)}{2ac\sqrt{a + cx^4}} - \frac{3de^2x\sqrt{a + cx^4}}{2a\sqrt{c}\left(\sqrt{a} + \sqrt{c}x^2\right)} + \frac{3de^2\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a} + \sqrt{c}x^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{2a^{\frac{3}{4}}c^{\frac{3}{4}}\sqrt{a + cx^4}} - \frac{d\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a} + \sqrt{c}x^2\right)\left(3\sqrt{a}e^2 - \sqrt{c}d^2\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{4a^{\frac{5}{4}}c^{\frac{3}{4}}\sqrt{a + cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x+d)**3/(c*x**4+a)**(3/2),x)
```

```
[Out] -(a*e**3 - c*x*(d**3 + 3*d**2*e*x + 3*d*e**2*x**2))/(2*a*c*sqrt(a
 + c*x**4)) - 3*d*e**2*x*sqrt(a + c*x**4)/(2*a*sqrt(c)*(sqrt(a) +
 sqrt(c)*x**2)) + 3*d*e**2*sqrt((a + c*x**4)/(sqrt(a) + sqrt(c)*x
**2)**2)*(sqrt(a) + sqrt(c)*x**2)*elliptic_e(2*atan(c**(1/4)*x/a*
*(1/4)), 1/2)/(2*a**(3/4)*c**(3/4)*sqrt(a + c*x**4)) - d*sqrt((a
+ c*x**4)/(sqrt(a) + sqrt(c)*x**2)**2)*(sqrt(a) + sqrt(c)*x**2)*(
3*sqrt(a)*e**2 - sqrt(c)*d**2)*elliptic_f(2*atan(c**(1/4)*x/a**(1
/4)), 1/2)/(4*a**(5/4)*c**(3/4)*sqrt(a + c*x**4))
```

Mathematica [C] time = 0.48482, size = 215, normalized size = 0.72

$$\frac{\sqrt{c}d\sqrt{\frac{cx^4}{a}+1}\left(3\sqrt{a}e^2 - i\sqrt{c}d^2\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right) + \sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\left(cdx\left(d^2 + 3dex + 3e^2x^2\right) - ae^3\right) - 3\sqrt{a}\sqrt{c}de^2\sqrt{\frac{cx^4}{a}+1}}{2ac\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{a + cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x)^3/(a + c*x^4)^(3/2),x]
```

```
[Out] (Sqrt[(I*Sqrt[c])/Sqrt[a]]*(-(a*e^3) + c*d*x*(d^2 + 3*d*e*x + 3*e
^2*x^2)) - 3*Sqrt[a]*Sqrt[c]*d*e^2*Sqrt[1 + (c*x^4)/a]*EllipticE[
```

```
I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] + Sqrt[c]*d*((-I)*Sqr
t[c]*d^2 + 3*Sqrt[a]*e^2)*Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh
[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1])/(2*a*Sqrt[(I*Sqrt[c])/Sqrt[a]
]*c*Sqrt[a + c*x^4])
```

Maple [C] time = 0.039, size = 261, normalized size = 0.9

$$d^3\left(\frac{x}{2\,a}\frac{1}{\sqrt{\left(x^4+\frac{a}{c}\right)c}} + \frac{1}{2\,a}\sqrt{1-ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{c}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4+a}}\right) - \frac{e^3}{2\,c}\frac{1}{\sqrt{cx^4+a}} + \frac{3\,d^2ex^2}{2\,a}\frac{1}{\sqrt{cx^4+a}} + 3\,e^2d\left(1/2\,\frac{x^3}{a}\frac{1}{\sqrt{\left(x^4+\frac{a}{c}\right)c}} - \frac{i/2}{\sqrt{a}\sqrt{cx^4+a}\sqrt{c}}\sqrt{1-\frac{i\sqrt{c}x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}x^2}{\sqrt{a}}}\left(EllipticF\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-EllipticE\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)\right)\frac{1}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x+d)^3/(c*x^4+a)^(3/2),x)
```

```
[Out] d^3*(1/2/a*x/((x^4+1/c*a)*c)^(1/2)+1/2/a/(I/a^(1/2)*c^(1/2))^(1/2
)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)
/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I))-1/2*e^
3/c/(c*x^4+a)^(1/2)+3/2*d^2*e*x^2/a/(c*x^4+a)^(1/2)+3*e^2*d*(1/2/
a*x^3/((x^4+1/c*a)*c)^(1/2)-1/2*I/a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/
2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2
)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),
I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^3}{(cx^4+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)^3/(c*x^4 + a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^3/(c*x^4 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^3x^3 + 3\,de^2x^2 + 3\,d^2ex + d^3}{(cx^4 + a)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)^3/(c*x^4 + a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)/(c*x^4 + a)^(3
/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex)^3}{(a + cx^4)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x+d)**3/(c*x**4+a)**(3/2),x)
```

```
[Out] Integral((d + e*x)**3/(a + c*x**4)**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^3}{(cx^4 + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)^3/(c*x^4 + a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3/(c*x^4 + a)^(3/2), x)
```

3.197 $\int \frac{(d+ex)^2}{(a+cx^4)^{3/2}} \, dx$

Optimal. Leaf size=270

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{c}d^2-\sqrt{a}e^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}c^{3/4}\sqrt{a+cx^4}}+\frac{e^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}c^{3/4}\sqrt{a+cx^4}}+\frac{x(d+ex)^2}{2a\sqrt{a+cx^4}}-\frac{e^2x\sqrt{a+cx^4}}{2a\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

```
[Out] (x*(d + e*x)^2)/(2*a*Sqrt[a + c*x^4]) - (e^2*x*Sqrt[a + c*x^4])/(
2*a*Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) + (e^2*(Sqrt[a] + Sqrt[c]*x^
2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan
[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*c^(3/4)*Sqrt[a + c*x^4])
+ ((Sqrt[c]*d^2 - Sqrt[a]*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a +
c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/
a^(1/4)], 1/2])/(4*a^(5/4)*c^(3/4)*Sqrt[a + c*x^4])
```

Rubi [A] time = 0.313343, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{c}d^2-\sqrt{a}e^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}c^{3/4}\sqrt{a+cx^4}}+\frac{e^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}c^{3/4}\sqrt{a+cx^4}}+\frac{x(d+ex)^2}{2a\sqrt{a+cx^4}}-\frac{e^2x\sqrt{a+cx^4}}{2a\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x)^2/(a + c*x^4)^(3/2),x]
```


```
[Out] (x*(d + e*x)^2)/(2*a*Sqrt[a + c*x^4]) - (e^2*x*Sqrt[a + c*x^4])/(
2*a*Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) + (e^2*(Sqrt[a] + Sqrt[c]*x^
2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan
[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*c^(3/4)*Sqrt[a + c*x^4])
+ ((Sqrt[c]*d^2 - Sqrt[a]*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a +
c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/
a^(1/4)], 1/2])/(4*a^(5/4)*c^(3/4)*Sqrt[a + c*x^4])
```

Rubi in Sympy [A] time = 34.6986, size = 238, normalized size = 0.88

$$\frac{x(d+ex)^2}{2a\sqrt{a+cx^4}} - \frac{e^2x\sqrt{a+cx^4}}{2a\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{e^2\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{2a^{\frac{3}{4}}c^{\frac{3}{4}}\sqrt{a+cx^4}} - \frac{\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(\sqrt{a}e^2-\sqrt{c}d^2\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{4a^{\frac{5}{4}}c^{\frac{3}{4}}\sqrt{a+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x+d)**2/(c*x**4+a)**(3/2),x)
```

```
[Out] x*(d + e*x)**2/(2*a*sqrt(a + c*x**4)) - e**2*x*sqrt(a + c*x**4)/(
2*a*sqrt(c)*(sqrt(a) + sqrt(c)*x**2)) + e**2*sqrt((a + c*x**4)/(s
qrt(a) + sqrt(c)*x**2)**2)*(sqrt(a) + sqrt(c)*x**2)*elliptic_e(2*
atan(c**(1/4)*x/a**(1/4)), 1/2)/(2*a**(3/4)*c**(3/4)*sqrt(a + c*x
**4)) - sqrt((a + c*x**4)/(sqrt(a) + sqrt(c)*x**2)**2)*(sqrt(a) +
 sqrt(c)*x**2)*(sqrt(a)*e**2 - sqrt(c)*d**2)*elliptic_f(2*atan(c*
*(1/4)*x/a**(1/4)), 1/2)/(4*a**(5/4)*c**(3/4)*sqrt(a + c*x**4))
```

Mathematica [C] time = 0.364497, size = 188, normalized size = 0.7

$$\frac{i\left(\sqrt{\frac{cx^4}{a}+1}\left(\sqrt{a}e^2-i\sqrt{c}d^2\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)+\sqrt{c}x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}(d+ex)^2-\sqrt{a}e^2\sqrt{\frac{cx^4}{a}+1}E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)\right)}{2a^{3/2}\left(\frac{i\sqrt{c}}{\sqrt{a}}\right)^{3/2}\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x)^2/(a + c*x^4)^(3/2),x]
```

```
[Out] ((I/2)*(Sqrt[(I*Sqrt[c])/Sqrt[a]]*Sqrt[c]*x*(d + e*x)^2 - Sqrt[a]
*e^2*Sqrt[1 + (c*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqr
t[a]]*x], -1] + ((-I)*Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sqrt[1 + (c*x^4)
/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1]))/(a^(3
/2)*((I*Sqrt[c])/Sqrt[a])^(3/2)*Sqrt[a + c*x^4])
```

Maple [C] time = 0.011, size = 239, normalized size = 0.9

$$d^{2}\left(\frac{x}{2a}\frac{1}{\sqrt{\left(x^{4}+\frac{a}{c}\right)c}}+\frac{1}{2a}\sqrt{1-ix^{2}\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+ix^{2}\sqrt{c}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^{4}+a}}\right)$$
$$+e^{2}\left(\frac{x^{3}}{2a}\frac{1}{\sqrt{\left(x^{4}+\frac{a}{c}\right)c}}\right.$$
$$\left.-\frac{i}{2}\sqrt{1-ix^{2}\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+ix^{2}\sqrt{c}\frac{1}{\sqrt{a}}}\left(\mathit{EllipticF}\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)-\mathit{EllipticE}\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\right)\frac{1}{\sqrt{a}}\frac{1}{\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^{4}+a}}\frac{1}{\sqrt{c}}\right)$$
$$+\frac{dex^{2}}{a}\frac{1}{\sqrt{cx^{4}+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x+d)^2/(c*x^4+a)^(3/2),x)
```

```
[Out] d^2*(1/2/a*x/((x^4+1/c*a)*c)^(1/2)+1/2/a/(I/a^(1/2)*c^(1/2))^(1/2
)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)
/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I))+e^2*(1
/2/a*x^3/((x^4+1/c*a)*c)^(1/2)-1/2*I/a^(1/2)/(I/a^(1/2)*c^(1/2))^
(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(
1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/
2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I)))+d*e*x^2/a/(c*x^4
+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{2}}{(cx^{4}+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)^2/(c*x^4 + a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^2/(c*x^4 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{e^{2}x^{2}+2\,dex+d^{2}}{(cx^{4}+a)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)^2/(c*x^4 + a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)/(c*x^4 + a)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^2}{(a+cx^4)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x+d)**2/(c*x**4+a)**(3/2),x)
```

```
[Out] Integral((d + e*x)**2/(a + c*x**4)**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^2}{(cx^4+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)^2/(c*x^4 + a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2/(c*x^4 + a)^(3/2), x)
```

3.198 $\int \frac{d+ex}{(a+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=114

$$\frac{d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{x(d+ex)}{2a\sqrt{a+cx^4}}$$

```
[Out] (x*(d + e*x))/(2*a*Sqrt[a + c*x^4]) + (d*(Sqrt[a] + Sqrt[c]*x^2)*
Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c
^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(5/4)*c^(1/4)*Sqrt[a + c*x^4])
```

Rubi [A] time = 0.110956, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$

$$\frac{d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{x(d+ex)}{2a\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x)/(a + c*x^4)^(3/2),x]
```

```
[Out] (x*(d + e*x))/(2*a*Sqrt[a + c*x^4]) + (d*(Sqrt[a] + Sqrt[c]*x^2)*
Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c
^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(5/4)*c^(1/4)*Sqrt[a + c*x^4])
```

Rubi in Sympy [A] time = 12.695, size = 100, normalized size = 0.88

$$\frac{x\left(d+ex\right)}{2a\sqrt{a+cx^4}}+\frac{d\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{4a^{\frac{5}{4}}\sqrt[4]{c}\sqrt{a+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x+d)/(c*x**4+a)**(3/2),x)
```

```
[Out] x*(d + e*x)/(2*a*sqrt(a + c*x**4)) + d*sqrt((a + c*x**4)/(sqrt(a)
 + sqrt(c)*x**2)**2)*(sqrt(a) + sqrt(c)*x**2)*elliptic_f(2*atan(c
```

```
**(1/4)*x/a**(1/4)), 1/2)/(4*a**(5/4)*c**(1/4)*sqrt(a + c*x**4))
```

Mathematica [C] time = 0.182713, size = 90, normalized size = 0.79

$$\frac{x(d+ex)-\frac{id\sqrt{\frac{cx^4}{a}+1}F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}}{2a\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x)/(a + c*x^4)^(3/2),x]
```

```
[Out] (x*(d + e*x) - (I*d*Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[
(I*Sqrt[c])/Sqrt[a]]*x], -1])/Sqrt[(I*Sqrt[c])/Sqrt[a]])/(2*a*Sqr
t[a + c*x^4])
```

Maple [C] time = 0.006, size = 115, normalized size = 1.

$$d\left(\frac{x}{2\,a}\frac{1}{\sqrt{\left(x^4+\frac{a}{c}\right)c}}+\frac{1}{2\,a}\sqrt{1-ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{c}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4+a}}\right)+\frac{ex^2}{2\,a}\frac{1}{\sqrt{cx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x+d)/(c*x^4+a)^(3/2),x)
```

```
[Out] d*(1/2/a*x/((x^4+1/c*a)*c)^(1/2)+1/2/a/(I/a^(1/2)*c^(1/2))^(1/2)*
(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(
c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I))+1/2*e*x^
2/a/(c*x^4+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{ex+d}{(cx^4+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)/(c*x^4 + a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)/(c*x^4 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ex+d}{(cx^4+a)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)/(c*x^4 + a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((e*x + d)/(c*x^4 + a)^(3/2), x)
```

Sympy [A] time = 24.2276, size = 61, normalized size = 0.54

$$\frac{dx\,\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\,\Gamma\left(\frac{5}{4}\right)}+\frac{ex^2}{2a^{\frac{3}{2}}\sqrt{1+\frac{cx^4}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x+d)/(c*x**4+a)**(3/2),x)
```

```
[Out] d*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), c*x**4*exp_polar(I*pi)/a
)/(4*a**(3/2)*gamma(5/4)) + e*x**2/(2*a**(3/2)*sqrt(1 + c*x**4/a)
)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex+d}{(cx^4+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)/(c*x^4 + a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)/(c*x^4 + a)^(3/2), x)
```

3.199 $\int \frac{1}{(a+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=108

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{x}{2a\sqrt{a+cx^4}}$$

```
[Out] x/(2*a*Sqrt[a + c*x^4]) + ((Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^
4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1
/4)], 1/2])/(4*a^(5/4)*c^(1/4)*Sqrt[a + c*x^4])
```

Rubi [A] time = 0.0638053, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{x}{2a\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + c*x^4)^(-3/2),x]
```

```
[Out] x/(2*a*Sqrt[a + c*x^4]) + ((Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^
4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1
/4)], 1/2])/(4*a^(5/4)*c^(1/4)*Sqrt[a + c*x^4])
```

Rubi in Sympy [A] time = 5.65726, size = 94, normalized size = 0.87

$$\frac{x}{2a\sqrt{a+cx^4}}+\frac{\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{4a^{\frac{5}{4}}\sqrt[4]{c}\sqrt{a+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c*x**4+a)**(3/2),x)
```

```
[Out] x/(2*a*sqrt(a + c*x**4)) + sqrt((a + c*x**4)/(sqrt(a) + sqrt(c)*x
**2)**2)*(sqrt(a) + sqrt(c)*x**2)*elliptic_f(2*atan(c**(1/4)*x/a*
```

```
*(1/4)), 1/2)/(4*a**(5/4)*c**(1/4)*sqrt(a + c*x**4))
```

Mathematica [C] time = 0.0734806, size = 102, normalized size = 0.94

$$\frac{x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}-i\sqrt{\frac{cx^4}{a}+1}F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)}{2a\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + c*x^4)^(-3/2),x]
```

```
[Out] (Sqrt[(I*Sqrt[c])/Sqrt[a]]*x - I*Sqrt[1 + (c*x^4)/a]*EllipticF[I*
ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1])/(2*a*Sqrt[(I*Sqrt[c])/
Sqrt[a]]*Sqrt[a + c*x^4])
```

Maple [C] time = 0.004, size = 94, normalized size = 0.9

$$\frac{x}{2\,a}\frac{1}{\sqrt{\left(x^4+\frac{a}{c}\right)c}}+\frac{1}{2\,a}\sqrt{1-ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{c}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c*x^4+a)^(3/2),x)
```

```
[Out] 1/2/a*x/((x^4+1/c*a)*c)^(1/2)+1/2/a/(I/a^(1/2)*c^(1/2))^(1/2)*(1-
I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x
^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^4+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)^(-3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + a)^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(cx^4+a)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)^(-3/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^4 + a)^(-3/2), x)
```

Sympy [A] time = 2.27501, size = 36, normalized size = 0.33

$$\frac{x\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x**4+a)**(3/2),x)
```

```
[Out] x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/
(4*a**(3/2)*gamma(5/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^4+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)^(-3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + a)^(-3/2), x)
```

3.200 $\int \frac{1}{(d+ex)(a+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=827

$$-\frac{\tanh^{-1}\left(\frac{ae^2+cd^2x^2}{\sqrt{cd^4+ae^4}\sqrt{cx^4+a}}\right)e^5}{2\left(cd^4+ae^4\right)^{3/2}}+\frac{\sqrt[4]{c}d\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)e^4}{2\sqrt[4]{a}\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\left(cd^4+ae^4\right)\sqrt{cx^4+a}}$$
$$-\frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\Pi\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)e^4}{4\sqrt[4]{a}\sqrt[4]{c}d\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\left(cd^4+ae^4\right)\sqrt{cx^4+a}}$$
$$-\frac{\tan^{-1}\left(\frac{\sqrt{-\frac{cd^4+ae^4}{d^2e^2}}x}{\sqrt{cx^4+a}}\right)e^2}{2d^3\left(-\frac{cd^4+ae^4}{d^2e^2}\right)^{3/2}}+\frac{\sqrt[4]{c}d\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)e^2}{2a^{3/4}\left(cd^4+ae^4\right)\sqrt{cx^4+a}}$$
$$-\frac{\sqrt{c}dx\sqrt{cx^4+a}e^2}{2a\left(cd^4+ae^4\right)\left(\sqrt{c}x^2+\sqrt{a}\right)}+\frac{\left(ae^2-cd^2x^2\right)e}{2a\left(cd^4+ae^4\right)\sqrt{cx^4+a}}$$
$$+\frac{\sqrt[4]{c}d\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}\left(cd^4+ae^4\right)\sqrt{cx^4+a}}+\frac{cdx\left(d^2+e^2x^2\right)}{2a\left(cd^4+ae^4\right)\sqrt{cx^4+a}}$$

```
[Out] (e*(a*e^2 - c*d^2*x^2))/(2*a*(c*d^4 + a*e^4)*Sqrt[a + c*x^4]) + (
c*d*x*(d^2 + e^2*x^2))/(2*a*(c*d^4 + a*e^4)*Sqrt[a + c*x^4]) - (S
qrt[c]*d*e^2*x*Sqrt[a + c*x^4])/(2*a*(c*d^4 + a*e^4)*(Sqrt[a] + S
qrt[c]*x^2)) - (e^2*ArcTan[(Sqrt[-((c*d^4 + a*e^4)/(d^2*e^2))]*x)
/Sqrt[a + c*x^4]])/(2*d^3*(-((c*d^4 + a*e^4)/(d^2*e^2)))^(3/2)) -
 (e^5*ArcTanh[(a*e^2 + c*d^2*x^2)/(Sqrt[c*d^4 + a*e^4]*Sqrt[a + c
*x^4])])/(2*(c*d^4 + a*e^4)^(3/2)) + (c^(1/4)*d*e^2*(Sqrt[a] + Sq
rt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[
2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*(c*d^4 + a*e^4)*S
qrt[a + c*x^4]) + (c^(1/4)*d*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*(Sqrt[a]
 + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellip
ticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(5/4)*(c*d^4 + a*e
^4)*Sqrt[a + c*x^4]) + (c^(1/4)*d*e^4*(Sqrt[a] + Sqrt[c]*x^2)*Sqr
t[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1
/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*(c*
d^4 + a*e^4)*Sqrt[a + c*x^4]) - (e^4*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*
(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^
2]*EllipticPi[(Sqrt[c]*d^2 + Sqrt[a]*e^2)^2/(4*Sqrt[a]*Sqrt[c]*d^
2*e^2), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*c^(1/4)*d
*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*(c*d^4 + a*e^4)*Sqrt[a + c*x^4])
```

Rubi [A] time = 1.72462, antiderivative size = 827, normalized size of antiderivative = 1., number of

steps used = 13, number of rules used = 12, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.632

$$-\frac{\tanh^{-1}\left(\frac{ae^2+cd^2x^2}{\sqrt{cd^4+ae^4}\sqrt{cx^4+a}}\right)e^5}{2\left(cd^4+ae^4\right)^{3/2}}+\frac{\sqrt[4]{c}d\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)e^4}{2\sqrt[4]{a}\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\left(cd^4+ae^4\right)\sqrt{cx^4+a}}$$
$$-\frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)e^4}{4\sqrt[4]{a}\sqrt[4]{c}d\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\left(cd^4+ae^4\right)\sqrt{cx^4+a}}$$
$$-\frac{\tan^{-1}\left(\frac{\sqrt{-\frac{cd^4+ae^4}{d^2e^2}}x}{\sqrt{cx^4+a}}\right)e^2}{2d^3\left(-\frac{cd^4+ae^4}{d^2e^2}\right)^{3/2}}+\frac{\sqrt[4]{c}d\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)e^2}{2a^{3/4}\left(cd^4+ae^4\right)\sqrt{cx^4+a}}$$
$$-\frac{\sqrt{c}dx\sqrt{cx^4+a}e^2}{2a\left(cd^4+ae^4\right)\left(\sqrt{c}x^2+\sqrt{a}\right)}+\frac{\left(ae^2-cd^2x^2\right)e}{2a\left(cd^4+ae^4\right)\sqrt{cx^4+a}}$$
$$+\frac{\sqrt[4]{c}d\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}\left(cd^4+ae^4\right)\sqrt{cx^4+a}}+\frac{cdx\left(d^2+e^2x^2\right)}{2a\left(cd^4+ae^4\right)\sqrt{cx^4+a}}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[1/((d + e*x)*(a + c*x^4)^(3/2)),x]
```

```
[Out] (e*(a*e^2 - c*d^2*x^2))/(2*a*(c*d^4 + a*e^4)*Sqrt[a + c*x^4]) + (
c*d*x*(d^2 + e^2*x^2))/(2*a*(c*d^4 + a*e^4)*Sqrt[a + c*x^4]) - (S
qrt[c]*d*e^2*x*Sqrt[a + c*x^4])/(2*a*(c*d^4 + a*e^4)*(Sqrt[a] + S
qrt[c]*x^2)) - (e^2*ArcTan[(Sqrt[-((c*d^4 + a*e^4)/(d^2*e^2))]*x)
/Sqrt[a + c*x^4]])/(2*d^3*(-((c*d^4 + a*e^4)/(d^2*e^2)))^(3/2)) -
 (e^5*ArcTanh[(a*e^2 + c*d^2*x^2)/(Sqrt[c*d^4 + a*e^4]*Sqrt[a + c
*x^4])])/(2*(c*d^4 + a*e^4)^(3/2)) + (c^(1/4)*d*e^2*(Sqrt[a] + Sq
rt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[
2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*(c*d^4 + a*e^4)*S
qrt[a + c*x^4]) + (c^(1/4)*d*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*(Sqrt[a]
 + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellip
ticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(5/4)*(c*d^4 + a*e
^4)*Sqrt[a + c*x^4]) + (c^(1/4)*d*e^4*(Sqrt[a] + Sqrt[c]*x^2)*Sqr
t[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1
/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*(c*
d^4 + a*e^4)*Sqrt[a + c*x^4]) - (e^4*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*
(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^
2]*EllipticPi[(Sqrt[c]*d^2 + Sqrt[a]*e^2)^2/(4*Sqrt[a]*Sqrt[c]*d^
2*e^2), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*c^(1/4)*d
*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*(c*d^4 + a*e^4)*Sqrt[a + c*x^4])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+cx^4)^{\frac{3}{2}}(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x+d)/(c*x**4+a)**(3/2),x)
```

```
[Out] Integral(1/((a + c*x**4)**(3/2)*(d + e*x)), x)
```

Mathematica [C] time = 4.59494, size = 464, normalized size = 0.56

$$\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\left(\sqrt[4]{c}d\left(ae^5\sqrt{a+cx^4}\log\left(e^2x^2-d^2\right)-ae^5\sqrt{a+cx^4}\log\left(\sqrt{a+cx^4}\sqrt{ae^4+cd^4}+ae^2+cd^2x^2\right)+\sqrt{ae^4+cd^4}\left(ae^3+cdx\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d + e*x)*(a + c*x^4)^(3/2)),x]
```

```
[Out] (-(Sqrt[a]*c^(3/4)*d^2*e^2*Sqrt[c*d^4 + a*e^4]*Sqrt[1 + (c*x^4)/a
]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1]) + c^(3/4
)*d^2*((-I)*Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sqrt[c*d^4 + a*e^4]*Sqrt[1
 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -
1] + Sqrt[(I*Sqrt[c])/Sqrt[a]]*(-2*(-1)^(1/4)*a^(5/4)*e^4*Sqrt[c*
d^4 + a*e^4]*Sqrt[1 + (c*x^4)/a]*EllipticPi[(I*Sqrt[a]*e^2)/(Sqrt
[c]*d^2), ArcSin[((-1)^(3/4)*c^(1/4)*x)/a^(1/4)], -1] + c^(1/4)*d
*(Sqrt[c*d^4 + a*e^4]*(a*e^3 + c*d*x*(d^2 - d*e*x + e^2*x^2)) + a
*e^5*Sqrt[a + c*x^4]*Log[-d^2 + e^2*x^2] - a*e^5*Sqrt[a + c*x^4]*
Log[a*e^2 + c*d^2*x^2 + Sqrt[c*d^4 + a*e^4]*Sqrt[a + c*x^4]])))/(
2*a*Sqrt[(I*Sqrt[c])/Sqrt[a]]*c^(1/4)*d*(c*d^4 + a*e^4)^(3/2)*Sqr
t[a + c*x^4])
```

Maple [C] time = 0.025, size = 496, normalized size = 0.6

$$-2\,c\left(-1/4\,\frac{de^2x^3}{a\,(ae^4+cd^4)}+1/4\,\frac{d^2ex^2}{a\,(ae^4+cd^4)}-1/4\,\frac{d^3x}{a\,(ae^4+cd^4)}-1/4\,\frac{e^3}{(ae^4+cd^4)\,c}\right)\frac{1}{\sqrt{\left(x^4+\frac{a}{c}\right)c}}$$
$$+\frac{cd^3}{2\,a\,(ae^4+cd^4)}\sqrt{1-ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{c}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4+a}}$$
$$-\frac{\frac{i}{2}e^2d}{ae^4+cd^4}\sqrt{c}\sqrt{1-ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\left(EllipticF\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)-EllipticE\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\right)\frac{1}{\sqrt{a}}\frac{1}{\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4+a}}$$
$$+\frac{e^3}{ae^4+cd^4}\left(-\frac{1}{2}Artanh\left(\frac{1}{2}\left(2\,\frac{cd^2x^2}{e^2}+2\,a\right)\frac{1}{\sqrt{\frac{cd^4}{e^4}+a}}\frac{1}{\sqrt{cx^4+a}}\right)\frac{1}{\sqrt{\frac{cd^4}{e^4}+a}}+\frac{e}{d}\sqrt{1-ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{c}\frac{1}{\sqrt{a}}}EllipticPi\left(x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*x+d)/(c*x^4+a)^(3/2),x)
```

```
[Out] -2*c*(-1/4/a*e^2*d/(a*e^4+c*d^4)*x^3+1/4/a*d^2*e/(a*e^4+c*d^4)*x^
2-1/4/a*d^3/(a*e^4+c*d^4)*x-1/4*e^3/(a*e^4+c*d^4)/c)/((x^4+1/c*a)
*c)^(1/2)+1/2*c/a*d^3/(a*e^4+c*d^4)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-
I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x
^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-1/2*I*c^(1/2
)/a^(1/2)*e^2*d/(a*e^4+c*d^4)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1
/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^
(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^
(1/2)*c^(1/2))^(1/2),I))+e^3/(a*e^4+c*d^4)*(-1/2/(c*d^4/e^4+a)^(1
/2)*arctanh(1/2*(2*c*x^2*d^2/e^2+2*a)/(c*d^4/e^4+a)^(1/2)/(c*x^4+
a)^(1/2))+1/(I/a^(1/2)*c^(1/2))^(1/2)/d*e*(1-I/a^(1/2)*c^(1/2)*x^
2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*Elliptic
Pi(x*(I/a^(1/2)*c^(1/2))^(1/2),-I*a^(1/2)/c^(1/2)/d^2*e^2,(-I/a^(
1/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(cx^4+a)^{\frac{3}{2}}(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)^(3/2)*(e*x + d)),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + a)^(3/2)*(e*x + d)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)^(3/2)*(e*x + d)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a + cx^{4}\right)^{\frac{3}{2}} \left(d + ex\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*x+d)/(c*x**4+a)**(3/2),x)
```

```
[Out] Integral(1/((a + c*x**4)**(3/2)*(d + e*x)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^{4} + a\right)^{\frac{3}{2}} \left(ex + d\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)^(3/2)*(e*x + d)),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + a)^(3/2)*(e*x + d)), x)
```

3.201 $\int \frac{1}{(d+ex)^2(a+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=2413

result too large to display

```
[Out] (d*e*(a*e^2 - c*d^2*x^2))/(a*(c*d^4 + a*e^4)*(d^2 - e^2*x^2)*Sqrt
[a + c*x^4]) - (c*x*(d^2 + e^2*x^2))/(2*a*(c*d^4 + a*e^4)*Sqrt[a
+ c*x^4]) + (c*d^2*x*(c*d^4 - a*e^4 + 2*c*d^2*e^2*x^2))/(a*(c*d^4
 + a*e^4)^2*Sqrt[a + c*x^4]) + (e^7*Sqrt[a + c*x^4])/(2*(c*d^4 +
a*e^4)^2*(d - e*x)) - (e^7*Sqrt[a + c*x^4])/(2*(c*d^4 + a*e^4)^2*
(d + e*x)) - (2*c^(3/2)*d^4*e^2*x*Sqrt[a + c*x^4])/(a*(c*d^4 + a*
e^4)^2*(Sqrt[a] + Sqrt[c]*x^2)) + (Sqrt[c]*e^6*x*Sqrt[a + c*x^4])
/((c*d^4 + a*e^4)^2*(Sqrt[a] + Sqrt[c]*x^2)) + (Sqrt[c]*e^2*x*Sqr
t[a + c*x^4])/(2*a*(c*d^4 + a*e^4)*(Sqrt[a] + Sqrt[c]*x^2)) + (d*
e^3*(c*d^4 - 2*a*e^4)*Sqrt[a + c*x^4])/(a*(c*d^4 + a*e^4)^2*(d^2
- e^2*x^2)) + (c*ArcTan[(Sqrt[-((c*d^4 + a*e^4)/(d^2*e^2))]*x)/Sq
rt[a + c*x^4]])/(d^2*(-((c*d^4 + a*e^4)/(d^2*e^2)))^(5/2)) + (e^2
*ArcTan[(Sqrt[-((c*d^4 + a*e^4)/(d^2*e^2))]*x)/Sqrt[a + c*x^4]])/
(2*d^4*(-((c*d^4 + a*e^4)/(d^2*e^2)))^(3/2)) + (e^4*(5*c*d^4 + a*
e^4)*ArcTan[(Sqrt[-((c*d^4 + a*e^4)/(d^2*e^2))]*x)/Sqrt[a + c*x^4
]])/(2*d^2*(c*d^4 + a*e^4)^2*Sqrt[-((c*d^4 + a*e^4)/(d^2*e^2))])
- (3*c*d^3*e^5*ArcTanh[(a*e^2 + c*d^2*x^2)/(Sqrt[c*d^4 + a*e^4]*S
qrt[a + c*x^4])])/(c*d^4 + a*e^4)^(5/2) + (2*c^(5/4)*d^4*e^2*(Sqr
t[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*E
llipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2]])/(a^(3/4)*(c*d^4 + a
*e^4)^2*Sqrt[a + c*x^4]) - (a^(1/4)*c^(1/4)*e^6*(Sqrt[a] + Sqrt[c
]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*Ar
cTan[(c^(1/4)*x)/a^(1/4)], 1/2])/((c*d^4 + a*e^4)^2*Sqrt[a + c*x^
4]) - (c^(1/4)*e^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt
[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/
2])/(2*a^(3/4)*(c*d^4 + a*e^4)*Sqrt[a + c*x^4]) - (c^(1/4)*e^4*(S
qrt[c]*d^2 - Sqrt[a]*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4
)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/
4)], 1/2])/(2*a^(1/4)*(c*d^4 + a*e^4)^2*Sqrt[a + c*x^4]) + (c^(5/
4)*d^4*e^4*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sq
rt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(a^(
1/4)*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*(c*d^4 + a*e^4)^2*Sqrt[a + c*x^4
]) + (c^(3/4)*d^2*(c*d^4 - 2*Sqrt[a]*Sqrt[c]*d^2*e^2 - a*e^4)*(Sq
rt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*
EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(5/4)*(c*d^4
+ a*e^4)^2*Sqrt[a + c*x^4]) - (c^(1/4)*(Sqrt[c]*d^2 - Sqrt[a]*e^2
)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2
)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(5/4)*(c
*d^4 + a*e^4)*Sqrt[a + c*x^4]) - (c^(1/4)*e^4*(Sqrt[a] + Sqrt[c]*
x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcT
an[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*(Sqrt[c]*d^2 + Sqrt[a]*
e^2)*(c*d^4 + a*e^4)*Sqrt[a + c*x^4]) + (c^(1/4)*e^4*(5*c*d^4 + a
*e^4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]
*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4
)*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*(c*d^4 + a*e^4)^2*Sqrt[a + c*x^4])
+ (e^4*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(
a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[(Sqrt[a]*((Sqrt[
```

```
c]*d^2)/Sqrt[a] + e^2)^2)/(4*Sqrt[c]*d^2*e^2), 2*ArcTan[(c^(1/4)*
x)/a^(1/4)], 1/2])/(4*a^(1/4)*c^(1/4)*d^2*(Sqrt[c]*d^2 + Sqrt[a]*
e^2)*(c*d^4 + a*e^4)*Sqrt[a + c*x^4]) - (c^(3/4)*d^2*e^4*(Sqrt[c]
*d^2 - Sqrt[a]*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqr
t[a] + Sqrt[c]*x^2)^2]*EllipticPi[(Sqrt[c]*d^2 + Sqrt[a]*e^2)^2/(
4*Sqrt[a]*Sqrt[c]*d^2*e^2), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/
(2*a^(1/4)*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*(c*d^4 + a*e^4)^2*Sqrt[a +
 c*x^4]) - (e^4*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*(5*c*d^4 + a*e^4)*(Sq
rt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*
EllipticPi[(Sqrt[c]*d^2 + Sqrt[a]*e^2)^2/(4*Sqrt[a]*Sqrt[c]*d^2*e
^2), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*c^(1/4)*d^2*
(Sqrt[c]*d^2 + Sqrt[a]*e^2)*(c*d^4 + a*e^4)^2*Sqrt[a + c*x^4])
```

Rubi [A] time = 13.5254, antiderivative size = 2413, normalized size of antiderivative = 1., number of steps used = 72, number of rules used = 16, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.842$

result too large to display

Warning: Unable to verify antiderivative.

```
[In]  Int[1/((d + e*x)^2*(a + c*x^4)^(3/2)),x]
```

```
[Out] (d*e*(a*e^2 - c*d^2*x^2))/(a*(c*d^4 + a*e^4)*(d^2 - e^2*x^2)*Sqrt
[a + c*x^4]) - (c*x*(d^2 + e^2*x^2))/(2*a*(c*d^4 + a*e^4)*Sqrt[a
+ c*x^4]) + (c*d^2*x*(c*d^4 - a*e^4 + 2*c*d^2*e^2*x^2))/(a*(c*d^4
 + a*e^4)^2*Sqrt[a + c*x^4]) + (e^7*Sqrt[a + c*x^4])/(2*(c*d^4 +
a*e^4)^2*(d - e*x)) - (e^7*Sqrt[a + c*x^4])/(2*(c*d^4 + a*e^4)^2*
(d + e*x)) - (2*c^(3/2)*d^4*e^2*x*Sqrt[a + c*x^4])/(a*(c*d^4 + a*
e^4)^2*(Sqrt[a] + Sqrt[c]*x^2)) + (Sqrt[c]*e^6*x*Sqrt[a + c*x^4])
/((c*d^4 + a*e^4)^2*(Sqrt[a] + Sqrt[c]*x^2)) + (Sqrt[c]*e^2*x*Sqr
t[a + c*x^4])/(2*a*(c*d^4 + a*e^4)*(Sqrt[a] + Sqrt[c]*x^2)) + (d*
e^3*(c*d^4 - 2*a*e^4)*Sqrt[a + c*x^4])/(a*(c*d^4 + a*e^4)^2*(d^2
- e^2*x^2)) + (c*ArcTan[(Sqrt[-((c*d^4 + a*e^4)/(d^2*e^2))]*x)/Sq
rt[a + c*x^4]])/(d^2*(-((c*d^4 + a*e^4)/(d^2*e^2)))^(5/2)) + (e^2
*ArcTan[(Sqrt[-((c*d^4 + a*e^4)/(d^2*e^2))]*x)/Sqrt[a + c*x^4]])/
(2*d^4*(-((c*d^4 + a*e^4)/(d^2*e^2)))^(3/2)) + (e^4*(5*c*d^4 + a*
e^4)*ArcTan[(Sqrt[-((c*d^4 + a*e^4)/(d^2*e^2))]*x)/Sqrt[a + c*x^4
]])/(2*d^2*(c*d^4 + a*e^4)^2*Sqrt[-((c*d^4 + a*e^4)/(d^2*e^2))])
- (3*c*d^3*e^5*ArcTanh[(a*e^2 + c*d^2*x^2)/(Sqrt[c*d^4 + a*e^4]*S
qrt[a + c*x^4])])/(c*d^4 + a*e^4)^(5/2) + (2*c^(5/4)*d^4*e^2*(Sqr
t[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*E
llipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(a^(3/4)*(c*d^4 + a
*e^4)^2*Sqrt[a + c*x^4]) - (a^(1/4)*c^(1/4)*e^6*(Sqrt[a] + Sqrt[c
]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*Ar
cTan[(c^(1/4)*x)/a^(1/4)], 1/2])/((c*d^4 + a*e^4)^2*Sqrt[a + c*x^
4]) - (c^(1/4)*e^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt
[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/
2])/(2*a^(3/4)*(c*d^4 + a*e^4)*Sqrt[a + c*x^4]) - (c^(1/4)*e^4*(S
qrt[c]*d^2 - Sqrt[a]*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4
```

```
)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/
4)], 1/2])/(2*a^(1/4)*(c*d^4 + a*e^4)^2*Sqrt[a + c*x^4]) + (c^(5/
4)*d^4*e^4*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sq
rt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(a^(
1/4)*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*(c*d^4 + a*e^4)^2*Sqrt[a + c*x^4
]) + (c^(3/4)*d^2*(c*d^4 - 2*Sqrt[a]*Sqrt[c]*d^2*e^2 - a*e^4)*(Sq
rt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*
EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(5/4)*(c*d^4
+ a*e^4)^2*Sqrt[a + c*x^4]) - (c^(1/4)*(Sqrt[c]*d^2 - Sqrt[a]*e^2
)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2
)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(5/4)*(c
*d^4 + a*e^4)*Sqrt[a + c*x^4]) - (c^(1/4)*e^4*(Sqrt[a] + Sqrt[c]*
x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcT
an[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*(Sqrt[c]*d^2 + Sqrt[a]*
e^2)*(c*d^4 + a*e^4)*Sqrt[a + c*x^4]) + (c^(1/4)*e^4*(5*c*d^4 + a
*e^4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]
*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4
)*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*(c*d^4 + a*e^4)^2*Sqrt[a + c*x^4])
+ (e^4*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(
a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[(Sqrt[a]*((Sqrt[
c]*d^2)/Sqrt[a] + e^2)^2)/(4*Sqrt[c]*d^2*e^2), 2*ArcTan[(c^(1/4)*
x)/a^(1/4)], 1/2])/(4*a^(1/4)*c^(1/4)*d^2*(Sqrt[c]*d^2 + Sqrt[a]*
e^2)*(c*d^4 + a*e^4)*Sqrt[a + c*x^4]) - (c^(3/4)*d^2*e^4*(Sqrt[c]
*d^2 - Sqrt[a]*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqr
t[a] + Sqrt[c]*x^2)^2]*EllipticPi[(Sqrt[c]*d^2 + Sqrt[a]*e^2)^2/(
4*Sqrt[a]*Sqrt[c]*d^2*e^2), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/
(2*a^(1/4)*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*(c*d^4 + a*e^4)^2*Sqrt[a +
 c*x^4]) - (e^4*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*(5*c*d^4 + a*e^4)*(Sq
rt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*
EllipticPi[(Sqrt[c]*d^2 + Sqrt[a]*e^2)^2/(4*Sqrt[a]*Sqrt[c]*d^2*e
^2), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*c^(1/4)*d^2*
(Sqrt[c]*d^2 + Sqrt[a]*e^2)*(c*d^4 + a*e^4)^2*Sqrt[a + c*x^4])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+cx^4)^{\frac{3}{2}}(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x+d)**2/(c*x**4+a)**(3/2),x)
```

```
[Out] Integral(1/((a + c*x**4)**(3/2)*(d + e*x)**2), x)
```

Mathematica [C] time = 4.06581, size = 809, normalized size = 0.34

$$\frac{3\sqrt{a}\sqrt{c}\left(ae^4-cd^4\right)\sqrt{cd^4+ae^4}(d+ex)\sqrt{\frac{cx^4}{a}+1}E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)e^2+\sqrt{c}\sqrt{cd^4+ae^4}\left(-ic^{3/2}d^6+3\sqrt{a}ce^2d^4+5ia\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d + e*x)^2*(a + c*x^4)^(3/2)),x]
```

```
[Out] (3*Sqrt[a]*Sqrt[c]*e^2*(-(c*d^4) + a*e^4)*Sqrt[c*d^4 + a*e^4]*(d
+ e*x)*Sqrt[1 + (c*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[c])/S
qrt[a]]*x], -1] + Sqrt[c]*Sqrt[c*d^4 + a*e^4]*((-I)*c^(3/2)*d^6 +
 3*Sqrt[a]*c*d^4*e^2 + (5*I)*a*Sqrt[c]*d^2*e^4 - 3*a^(3/2)*e^6)*(
d + e*x)*Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])
/Sqrt[a]]*x], -1] + Sqrt[(I*Sqrt[c])/Sqrt[a]]*(4*a*c*d^4*e^3*Sqrt
[c*d^4 + a*e^4] - 2*a^2*e^7*Sqrt[c*d^4 + a*e^4] + c^2*d^7*Sqrt[c*
d^4 + a*e^4]*x + a*c*d^3*e^4*Sqrt[c*d^4 + a*e^4]*x - c^2*d^6*e*Sq
rt[c*d^4 + a*e^4]*x^2 - a*c*d^2*e^5*Sqrt[c*d^4 + a*e^4]*x^2 + c^2
*d^5*e^2*Sqrt[c*d^4 + a*e^4]*x^3 + a*c*d*e^6*Sqrt[c*d^4 + a*e^4]*
x^3 + 3*c^2*d^4*e^3*Sqrt[c*d^4 + a*e^4]*x^4 - 3*a*c*e^7*Sqrt[c*d^
4 + a*e^4]*x^4 - 12*(-1)^(1/4)*a^(5/4)*c^(3/4)*d^2*e^4*Sqrt[c*d^4
 + a*e^4]*(d + e*x)*Sqrt[1 + (c*x^4)/a]*EllipticPi[(I*Sqrt[a]*e^2
)/(Sqrt[c]*d^2), ArcSin[((-1)^(3/4)*c^(1/4)*x)/a^(1/4)], -1] + 6*
a*c*d^3*e^5*(d + e*x)*Sqrt[a + c*x^4]*Log[-d^2 + e^2*x^2] - 6*a*c
*d^4*e^5*Sqrt[a + c*x^4]*Log[a*e^2 + c*d^2*x^2 + Sqrt[c*d^4 + a*e
^4]*Sqrt[a + c*x^4]] - 6*a*c*d^3*e^6*x*Sqrt[a + c*x^4]*Log[a*e^2
+ c*d^2*x^2 + Sqrt[c*d^4 + a*e^4]*Sqrt[a + c*x^4]]))/(2*a*Sqrt[(I
*Sqrt[c])/Sqrt[a]]*(c*d^4 + a*e^4)^(5/2)*(d + e*x)*Sqrt[a + c*x^4
])
```

Maple [C] time = 0.038, size = 642, normalized size = 0.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*x+d)^2/(c*x^4+a)^(3/2),x)
```

```
[Out] -e^7*(c*x^4+a)^(1/2)/(a*e^4+c*d^4)^2/(e*x+d)-2*c*(1/4*e^2*(a*e^4-
3*c*d^4)/a/(a*e^4+c*d^4)^2*x^3-1/2*d*e*(a*e^4-c*d^4)/a/(a*e^4+c*d
^4)^2*x^2+1/4*d^2*(3*a*e^4-c*d^4)/a/(a*e^4+c*d^4)^2*x-d^3*e^3/(a*
e^4+c*d^4)^2)/((x^4+1/c*a)*c)^(1/2)+(-d^2*e^4*c/(a*e^4+c*d^4)^2-1
/2*c*d^2*(3*a*e^4-c*d^4)/a/(a*e^4+c*d^4)^2)/(I/a^(1/2)*c^(1/2))^(
1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1
```

```
/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+I*(c
*e^6/(a*e^4+c*d^4)^2+1/2*c*e^2*(a*e^4-3*c*d^4)/a/(a*e^4+c*d^4)^2)
*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2
)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(Ellipt
icF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2)
)^(1/2),I))+6*c*d^3*e^3/(a*e^4+c*d^4)^2*(-1/2/(c*d^4/e^4+a)^(1/2)
*arctanh(1/2*(2*c*x^2*d^2/e^2+2*a)/(c*d^4/e^4+a)^(1/2)/(c*x^4+a)^
(1/2))+1/(I/a^(1/2)*c^(1/2))^(1/2)/d*e*(1-I/a^(1/2)*c^(1/2)*x^2)^
(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticPi(
x*(I/a^(1/2)*c^(1/2))^(1/2),-I*a^(1/2)/c^(1/2)/d^2*e^2,(-I/a^(1/2
)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^4+a)^{\frac{3}{2}}(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)^(3/2)*(e*x + d)^2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + a)^(3/2)*(e*x + d)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{(ce^2x^6+2\,cdex^5+cd^2x^4+ae^2x^2+2\,adex+ad^2)\sqrt{cx^4+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)^(3/2)*(e*x + d)^2),x, algorithm="fricas")
```

```
[Out] integral(1/((c*e^2*x^6 + 2*c*d*e*x^5 + c*d^2*x^4 + a*e^2*x^2 + 2*
a*d*e*x + a*d^2)*sqrt(c*x^4 + a)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+cx^4)^{\frac{3}{2}}\,(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*x+d)**2/(c*x**4+a)**(3/2),x)

[Out] Integral(1/((a + c*x**4)**(3/2)*(d + e*x)**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^4 + a)^{\frac{3}{2}}(ex + d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)^(3/2)*(e*x + d)^2),x, algorithm="giac")

[Out] integrate(1/((c*x^4 + a)^(3/2)*(e*x + d)^2), x)
```

3.202 $\int \frac{x^3(c+dx)^n}{a+bx^4}\,dx$

Optimal. Leaf size=349

$$-\frac{(c+dx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4b^{3/4}(n+1)\left(\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d\right)}-\frac{(c+dx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4b^{3/4}(n+1)\left(\sqrt{-\sqrt{-a}}d+\sqrt[4]{b}c\right)}-\frac{(c+dx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4b^{3/4}(n+1)\left(\sqrt[4]{b}c-\sqrt[4]{-a}d\right)}-\frac{(c+dx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4b^{3/4}(n+1)\left(\sqrt[4]{-a}d+\sqrt[4]{b}c\right)}$$

```
[Out] -((c + d*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/4)*(
c + d*x))/(b^(1/4)*c - Sqrt[-Sqrt[-a]]*d)])/(4*b^(3/4)*(b^(1/4)*c
 - Sqrt[-Sqrt[-a]]*d)*(1 + n)) - ((c + d*x)^(1 + n)*Hypergeometri
c2F1[1, 1 + n, 2 + n, (b^(1/4)*(c + d*x))/(b^(1/4)*c + Sqrt[-Sqrt
[-a]]*d)])/(4*b^(3/4)*(b^(1/4)*c + Sqrt[-Sqrt[-a]]*d)*(1 + n)) -
((c + d*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/4)*(c
 + d*x))/(b^(1/4)*c - (-a)^(1/4)*d)])/(4*b^(3/4)*(b^(1/4)*c - (-a
)^(1/4)*d)*(1 + n)) - ((c + d*x)^(1 + n)*Hypergeometric2F1[1, 1 +
 n, 2 + n, (b^(1/4)*(c + d*x))/(b^(1/4)*c + (-a)^(1/4)*d)])/(4*b^
(3/4)*(b^(1/4)*c + (-a)^(1/4)*d)*(1 + n))
```

Rubi [A] time = 1.46328, antiderivative size = 349, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$

$$-\frac{(c+dx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4b^{3/4}(n+1)\left(\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d\right)}-\frac{(c+dx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4b^{3/4}(n+1)\left(\sqrt{-\sqrt{-a}}d+\sqrt[4]{b}c\right)}-\frac{(c+dx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4b^{3/4}(n+1)\left(\sqrt[4]{b}c-\sqrt[4]{-a}d\right)}-\frac{(c+dx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4b^{3/4}(n+1)\left(\sqrt[4]{-a}d+\sqrt[4]{b}c\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(c + d*x)^n)/(a + b*x^4),x]
```



```
[Out] -((c + d*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/4)*(
c + d*x))/(b^(1/4)*c - Sqrt[-Sqrt[-a]]*d)])/(4*b^(3/4)*(b^(1/4)*c
 - Sqrt[-Sqrt[-a]]*d)*(1 + n)) - ((c + d*x)^(1 + n)*Hypergeometri
c2F1[1, 1 + n, 2 + n, (b^(1/4)*(c + d*x))/(b^(1/4)*c + Sqrt[-Sqrt
[-a]]*d)])/(4*b^(3/4)*(b^(1/4)*c + Sqrt[-Sqrt[-a]]*d)*(1 + n)) -
```

```
((c + d*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/4)*(c
 + d*x))/(b^(1/4)*c - (-a)^(1/4)*d)])/(4*b^(3/4)*(b^(1/4)*c - (-a
)^(1/4)*d)*(1 + n)) - ((c + d*x)^(1 + n)*Hypergeometric2F1[1, 1 +
 n, 2 + n, (b^(1/4)*(c + d*x))/(b^(1/4)*c + (-a)^(1/4)*d)])/(4*b^
(3/4)*(b^(1/4)*c + (-a)^(1/4)*d)*(1 + n))
```

Rubi in Sympy [A] time = 86.3924, size = 265, normalized size = 0.76

$$-\frac{(c+dx)^{n+1}\,{}_2F_1\left(\begin{matrix}1,n+1\\ n+2\end{matrix}\middle|\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+id\sqrt[4]{-a}}\right)}{4b^{\frac{3}{4}}(n+1)\left(\sqrt[4]{b}c+id\sqrt[4]{-a}\right)}-\frac{(c+dx)^{n+1}\,{}_2F_1\left(\begin{matrix}1,n+1\\ n+2\end{matrix}\middle|\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-id\sqrt[4]{-a}}\right)}{4b^{\frac{3}{4}}(n+1)\left(\sqrt[4]{b}c-id\sqrt[4]{-a}\right)}-\frac{(c+dx)^{n+1}\,{}_2F_1\left(\begin{matrix}1,n+1\\ n+2\end{matrix}\middle|\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+d\sqrt[4]{-a}}\right)}{4b^{\frac{3}{4}}(n+1)\left(\sqrt[4]{b}c+d\sqrt[4]{-a}\right)}-\frac{(c+dx)^{n+1}\,{}_2F_1\left(\begin{matrix}1,n+1\\ n+2\end{matrix}\middle|\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-d\sqrt[4]{-a}}\right)}{4b^{\frac{3}{4}}(n+1)\left(\sqrt[4]{b}c-d\sqrt[4]{-a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(d*x+c)**n/(b*x**4+a),x)
```

```
[Out] -(c + d*x)**(n + 1)*hyper((1, n + 1), (n + 2,), b**(1/4)*(c + d*x
)/(b**(1/4)*c + I*d*(-a)**(1/4)))/(4*b**(3/4)*(n + 1)*(b**(1/4)*c
 + I*d*(-a)**(1/4))) - (c + d*x)**(n + 1)*hyper((1, n + 1), (n +
2,), b**(1/4)*(c + d*x)/(b**(1/4)*c - I*d*(-a)**(1/4)))/(4*b**(3/
4)*(n + 1)*(b**(1/4)*c - I*d*(-a)**(1/4))) - (c + d*x)**(n + 1)*h
yper((1, n + 1), (n + 2,), b**(1/4)*(c + d*x)/(b**(1/4)*c + d*(-a
)**(1/4)))/(4*b**(3/4)*(n + 1)*(b**(1/4)*c + d*(-a)**(1/4))) - (c
 + d*x)**(n + 1)*hyper((1, n + 1), (n + 2,), b**(1/4)*(c + d*x)/(
b**(1/4)*c - d*(-a)**(1/4)))/(4*b**(3/4)*(n + 1)*(b**(1/4)*c - d*
(-a)**(1/4)))
```

Mathematica [C] time = 0.253223, size = 526, normalized size = 1.51

$$(c+dx)^n\left(c^3\text{RootSum}\left[\#1^4b-4\#1^3bc+6\#1^2bc^2-4\#1bc^3+ad^4+bc^4\&,\frac{\left(\frac{c+dx}{-\#1+c+dx}\right)^{-n}{}_2F_1\left(-n,-n;1-n;-\frac{\#1}{c+dx-\#1}\right)}{-\#1^3+3\#1^2c-3\#1c^2+c^3}\&\right]-3c^2\text{Root}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(x^3*(c + d*x)^n)/(a + b*x^4),x]
```

```
[Out] ((c + d*x)^n*(c^3*RootSum[b*c^4 + a*d^4 - 4*b*c^3*#1 + 6*b*c^2*#1
^2 - 4*b*c*#1^3 + b*#1^4 & , Hypergeometric2F1[-n, -n, 1 - n, -(#
1/(c + d*x - #1))]/(((c + d*x)/(c + d*x - #1))^n*(c^3 - 3*c^2*#1
+ 3*c*#1^2 - #1^3)) & ] - 3*c^2*RootSum[b*c^4 + a*d^4 - 4*b*c^3*#
1 + 6*b*c^2*#1^2 - 4*b*c*#1^3 + b*#1^4 & , (Hypergeometric2F1[-n,
 -n, 1 - n, -(#1/(c + d*x - #1))]*#1)/(((c + d*x)/(c + d*x - #1))
^n*(c^3 - 3*c^2*#1 + 3*c*#1^2 - #1^3)) & ] + 3*c*RootSum[b*c^4 +
a*d^4 - 4*b*c^3*#1 + 6*b*c^2*#1^2 - 4*b*c*#1^3 + b*#1^4 & , (Hype
rgeometric2F1[-n, -n, 1 - n, -(#1/(c + d*x - #1))]*#1^2)/(((c + d
*x)/(c + d*x - #1))^n*(c^3 - 3*c^2*#1 + 3*c*#1^2 - #1^3)) & ] - R
ootSum[b*c^4 + a*d^4 - 4*b*c^3*#1 + 6*b*c^2*#1^2 - 4*b*c*#1^3 + b
*#1^4 & , (Hypergeometric2F1[-n, -n, 1 - n, -(#1/(c + d*x - #1))]
*#1^3)/(((c + d*x)/(c + d*x - #1))^n*(c^3 - 3*c^2*#1 + 3*c*#1^2 -
 #1^3)) & ]))/(4*b*n)
```

Maple [F] time = 0.078, size = 0, normalized size = 0.

$$\int \frac{x^3 (dx + c)^n}{bx^4 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(d*x+c)^n/(b*x^4+a),x)
```

```
[Out] int(x^3*(d*x+c)^n/(b*x^4+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx + c)^n x^3}{bx^4 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)^n*x^3/(b*x^4 + a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^n*x^3/(b*x^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx + c)^n x^3}{bx^4 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)^n*x^3/(b*x^4 + a),x, algorithm="fricas")

[Out] integral((d*x + c)^n*x^3/(b*x^4 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(d*x+c)**n/(b*x**4+a),x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^n x^3}{bx^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)^n*x^3/(b*x^4 + a),x, algorithm="giac")

[Out] integrate((d*x + c)^n*x^3/(b*x^4 + a), x)
```

3.203 $\int \frac{x^3(c+dx)^{1+n}}{a+bx^4}\,dx$

Optimal. Leaf size=349

$$-\frac{(c+dx)^{n+2}\,{}_2F_1\left(1,n+2;n+3;\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4b^{3/4}(n+2)\left(\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d\right)}-\frac{(c+dx)^{n+2}\,{}_2F_1\left(1,n+2;n+3;\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4b^{3/4}(n+2)\left(\sqrt{-\sqrt{-a}}d+\sqrt[4]{b}c\right)}-\frac{(c+dx)^{n+2}\,{}_2F_1\left(1,n+2;n+3;\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4b^{3/4}(n+2)\left(\sqrt[4]{b}c-\sqrt[4]{-a}d\right)}-\frac{(c+dx)^{n+2}\,{}_2F_1\left(1,n+2;n+3;\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4b^{3/4}(n+2)\left(\sqrt[4]{-a}d+\sqrt[4]{b}c\right)}$$

```
[Out] -((c + d*x)^(2 + n)*Hypergeometric2F1[1, 2 + n, 3 + n, (b^(1/4)*(
c + d*x))/(b^(1/4)*c - Sqrt[-Sqrt[-a]]*d)])/(4*b^(3/4)*(b^(1/4)*c
 - Sqrt[-Sqrt[-a]]*d)*(2 + n)) - ((c + d*x)^(2 + n)*Hypergeometri
c2F1[1, 2 + n, 3 + n, (b^(1/4)*(c + d*x))/(b^(1/4)*c + Sqrt[-Sqrt
[-a]]*d)])/(4*b^(3/4)*(b^(1/4)*c + Sqrt[-Sqrt[-a]]*d)*(2 + n)) -
((c + d*x)^(2 + n)*Hypergeometric2F1[1, 2 + n, 3 + n, (b^(1/4)*(c
 + d*x))/(b^(1/4)*c - (-a)^(1/4)*d)])/(4*b^(3/4)*(b^(1/4)*c - (-a
)^(1/4)*d)*(2 + n)) - ((c + d*x)^(2 + n)*Hypergeometric2F1[1, 2 +
 n, 3 + n, (b^(1/4)*(c + d*x))/(b^(1/4)*c + (-a)^(1/4)*d)])/(4*b^
(3/4)*(b^(1/4)*c + (-a)^(1/4)*d)*(2 + n))
```

Rubi [A] time = 1.2044, antiderivative size = 349, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$

$$-\frac{(c+dx)^{n+2}\,{}_2F_1\left(1,n+2;n+3;\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4b^{3/4}(n+2)\left(\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d\right)}-\frac{(c+dx)^{n+2}\,{}_2F_1\left(1,n+2;n+3;\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4b^{3/4}(n+2)\left(\sqrt{-\sqrt{-a}}d+\sqrt[4]{b}c\right)}-\frac{(c+dx)^{n+2}\,{}_2F_1\left(1,n+2;n+3;\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4b^{3/4}(n+2)\left(\sqrt[4]{b}c-\sqrt[4]{-a}d\right)}-\frac{(c+dx)^{n+2}\,{}_2F_1\left(1,n+2;n+3;\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4b^{3/4}(n+2)\left(\sqrt[4]{-a}d+\sqrt[4]{b}c\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(c + d*x)^(1 + n))/(a + b*x^4),x]
```


```
[Out] -((c + d*x)^(2 + n)*Hypergeometric2F1[1, 2 + n, 3 + n, (b^(1/4)*(
c + d*x))/(b^(1/4)*c - Sqrt[-Sqrt[-a]]*d)])/(4*b^(3/4)*(b^(1/4)*c
 - Sqrt[-Sqrt[-a]]*d)*(2 + n)) - ((c + d*x)^(2 + n)*Hypergeometri
c2F1[1, 2 + n, 3 + n, (b^(1/4)*(c + d*x))/(b^(1/4)*c + Sqrt[-Sqrt
[-a]]*d)])/(4*b^(3/4)*(b^(1/4)*c + Sqrt[-Sqrt[-a]]*d)*(2 + n)) -
```

```
((c + d*x)^(2 + n)*Hypergeometric2F1[1, 2 + n, 3 + n, (b^(1/4)*(c
 + d*x))/(b^(1/4)*c - (-a)^(1/4)*d)])/(4*b^(3/4)*(b^(1/4)*c - (-a
)^(1/4)*d)*(2 + n)) - ((c + d*x)^(2 + n)*Hypergeometric2F1[1, 2 +
 n, 3 + n, (b^(1/4)*(c + d*x))/(b^(1/4)*c + (-a)^(1/4)*d)])/(4*b^
(3/4)*(b^(1/4)*c + (-a)^(1/4)*d)*(2 + n))
```

Rubi in Sympy [A] time = 90.0766, size = 265, normalized size = 0.76

$$-\frac{(c+dx)^{n+2}\,{}_2F_1\left(\begin{matrix}1,n+2\\n+3\end{matrix}\middle|\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+id\sqrt[4]{-a}}\right)}{4b^{\frac{3}{4}}(n+2)\left(\sqrt[4]{b}c+id\sqrt[4]{-a}\right)}-\frac{(c+dx)^{n+2}\,{}_2F_1\left(\begin{matrix}1,n+2\\n+3\end{matrix}\middle|\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-id\sqrt[4]{-a}}\right)}{4b^{\frac{3}{4}}(n+2)\left(\sqrt[4]{b}c-id\sqrt[4]{-a}\right)}-\frac{(c+dx)^{n+2}\,{}_2F_1\left(\begin{matrix}1,n+2\\n+3\end{matrix}\middle|\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+d\sqrt[4]{-a}}\right)}{4b^{\frac{3}{4}}(n+2)\left(\sqrt[4]{b}c+d\sqrt[4]{-a}\right)}-\frac{(c+dx)^{n+2}\,{}_2F_1\left(\begin{matrix}1,n+2\\n+3\end{matrix}\middle|\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-d\sqrt[4]{-a}}\right)}{4b^{\frac{3}{4}}(n+2)\left(\sqrt[4]{b}c-d\sqrt[4]{-a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(d*x+c)**(1+n)/(b*x**4+a),x)
```

```
[Out] -(c + d*x)**(n + 2)*hyper((1, n + 2), (n + 3,), b**(1/4)*(c + d*x
)/(b**(1/4)*c + I*d*(-a)**(1/4)))/(4*b**(3/4)*(n + 2)*(b**(1/4)*c
 + I*d*(-a)**(1/4))) - (c + d*x)**(n + 2)*hyper((1, n + 2), (n +
3,), b**(1/4)*(c + d*x)/(b**(1/4)*c - I*d*(-a)**(1/4)))/(4*b**(3/
4)*(n + 2)*(b**(1/4)*c - I*d*(-a)**(1/4))) - (c + d*x)**(n + 2)*h
yper((1, n + 2), (n + 3,), b**(1/4)*(c + d*x)/(b**(1/4)*c + d*(-a
)**(1/4)))/(4*b**(3/4)*(n + 2)*(b**(1/4)*c + d*(-a)**(1/4))) - (c
 + d*x)**(n + 2)*hyper((1, n + 2), (n + 3,), b**(1/4)*(c + d*x)/(
b**(1/4)*c - d*(-a)**(1/4)))/(4*b**(3/4)*(n + 2)*(b**(1/4)*c - d*
(-a)**(1/4)))
```

Mathematica [C] time = 0.348139, size = 691, normalized size = 1.98

$$\frac{(c+dx)^n\left((n+1)\left(ad^4+bc^4\right)\text{RootSum}\left[\#1^4b-4\#1^3bc+6\#1^2bc^2-4\#1bc^3+ad^4+bc^4\&,\frac{\left(\frac{c+dx}{-\#1+c+dx}\right)^{-n}{}_2F_1\left(-n,-n;1-n;-\frac{\#1}{c+dx-\#}\right.}{-\#1^3+3\#1^2c-3\#1c^2+c^3}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(x^3*(c + d*x)^(1 + n))/(a + b*x^4),x]
```

```
[Out] ((c + d*x)^n*((b*c^4 + a*d^4)*(1 + n)*RootSum[b*c^4 + a*d^4 - 4*b
*c^3*#1 + 6*b*c^2*#1^2 - 4*b*c*#1^3 + b*#1^4 & , Hypergeometric2F
1[-n, -n, 1 - n, -(#1/(c + d*x - #1))]/(((c + d*x)/(c + d*x - #1)
)^n*(c^3 - 3*c^2*#1 + 3*c*#1^2 - #1^3)) & ] - b*(-4*c*n - 4*d*n*x
 + 3*c^3*(1 + n)*RootSum[b*c^4 + a*d^4 - 4*b*c^3*#1 + 6*b*c^2*#1^
2 - 4*b*c*#1^3 + b*#1^4 & , (Hypergeometric2F1[-n, -n, 1 - n, -(#
1/(c + d*x - #1))]*#1)/(((c + d*x)/(c + d*x - #1))^n*(c^3 - 3*c^2
*#1 + 3*c*#1^2 - #1^3)) & ] - 3*c^2*(1 + n)*RootSum[b*c^4 + a*d^4
 - 4*b*c^3*#1 + 6*b*c^2*#1^2 - 4*b*c*#1^3 + b*#1^4 & , (Hypergeom
etric2F1[-n, -n, 1 - n, -(#1/(c + d*x - #1))]*#1^2)/(((c + d*x)/(
c + d*x - #1))^n*(c^3 - 3*c^2*#1 + 3*c*#1^2 - #1^3)) & ] + c*Root
Sum[b*c^4 + a*d^4 - 4*b*c^3*#1 + 6*b*c^2*#1^2 - 4*b*c*#1^3 + b*#1
^4 & , (Hypergeometric2F1[-n, -n, 1 - n, -(#1/(c + d*x - #1))]*#1
^3)/(((c + d*x)/(c + d*x - #1))^n*(c^3 - 3*c^2*#1 + 3*c*#1^2 - #1
^3)) & ] + c*n*RootSum[b*c^4 + a*d^4 - 4*b*c^3*#1 + 6*b*c^2*#1^2
- 4*b*c*#1^3 + b*#1^4 & , (Hypergeometric2F1[-n, -n, 1 - n, -(#1/
(c + d*x - #1))]*#1^3)/(((c + d*x)/(c + d*x - #1))^n*(c^3 - 3*c^2
*#1 + 3*c*#1^2 - #1^3)) & ])))/(4*b^2*n*(1 + n))
```

Maple [F] time = 0.076, size = 0, normalized size = 0.

$$\int \frac{x^3 (dx + c)^{1+n}}{bx^4 + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(d*x+c)^(1+n)/(b*x^4+a),x)
```

```
[Out] int(x^3*(d*x+c)^(1+n)/(b*x^4+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx + c)^{n+1} x^3}{bx^4 + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)^(n + 1)*x^3/(b*x^4 + a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(n + 1)*x^3/(b*x^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx+c)^{n+1}x^3}{bx^4+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)^(n + 1)*x^3/(b*x^4 + a),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^(n + 1)*x^3/(b*x^4 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(d*x+c)**(1+n)/(b*x**4+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{n+1}x^3}{bx^4+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)^(n + 1)*x^3/(b*x^4 + a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(n + 1)*x^3/(b*x^4 + a), x)
```

3.204 $\int \frac{1}{(c+dx+ex^2)\sqrt{a+bx^4}}\,dx$

Optimal. Leaf size=1493

result too large to display

```
[Out] (2*e*ArcTan[(Sqrt[-((16*a*e^4 + b*(d - Sqrt[d^2 - 4*c*e])^4)/(e^2
*(d - Sqrt[d^2 - 4*c*e])^2))]*x)/(2*Sqrt[a + b*x^4])])/(Sqrt[d^2
- 4*c*e]*(d - Sqrt[d^2 - 4*c*e])*Sqrt[-((16*a*e^4 + b*(d - Sqrt[d
^2 - 4*c*e])^4)/(e^2*(d - Sqrt[d^2 - 4*c*e])^2))]) - (2*e*ArcTan[
(Sqrt[-((16*a*e^4 + b*(d + Sqrt[d^2 - 4*c*e])^4)/(e^2*(d + Sqrt[d
^2 - 4*c*e])^2))]*x)/(2*Sqrt[a + b*x^4])])/(Sqrt[d^2 - 4*c*e]*(d
+ Sqrt[d^2 - 4*c*e])*Sqrt[-((16*a*e^4 + b*(d + Sqrt[d^2 - 4*c*e])
^4)/(e^2*(d + Sqrt[d^2 - 4*c*e])^2))]) - (e^2*ArcTanh[(4*a*e^2 +
b*(d - Sqrt[d^2 - 4*c*e])^2*x^2)/(2*Sqrt[2]*Sqrt[b*d^4 - 4*b*c*d^
2*e + 2*b*c^2*e^2 + 2*a*e^4 - b*d*Sqrt[d^2 - 4*c*e]*(d^2 - 2*c*e)
]*Sqrt[a + b*x^4])])/(Sqrt[2]*Sqrt[d^2 - 4*c*e]*Sqrt[b*d^4 - 4*b*
c*d^2*e + 2*b*c^2*e^2 + 2*a*e^4 - b*d*Sqrt[d^2 - 4*c*e]*(d^2 - 2*
c*e)]) + (e^2*ArcTanh[(4*a*e^2 + b*(d + Sqrt[d^2 - 4*c*e])^2*x^2)
/(2*Sqrt[2]*Sqrt[b*d^4 - 4*b*c*d^2*e + 2*b*c^2*e^2 + 2*a*e^4 + b*
d*Sqrt[d^2 - 4*c*e]*(d^2 - 2*c*e)]*Sqrt[a + b*x^4])])/(Sqrt[2]*Sq
rt[d^2 - 4*c*e]*Sqrt[b*d^4 - 4*b*c*d^2*e + 2*b*c^2*e^2 + 2*a*e^4
+ b*d*Sqrt[d^2 - 4*c*e]*(d^2 - 2*c*e)]) + (b^(1/4)*e*(d - Sqrt[d^
2 - 4*c*e])*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + S
qrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(a^
(3/4)*Sqrt[d^2 - 4*c*e]*(4*e^2 + (Sqrt[b]*(d - Sqrt[d^2 - 4*c*e])
^2)/Sqrt[a])*Sqrt[a + b*x^4]) - (b^(1/4)*e*(d + Sqrt[d^2 - 4*c*e]
)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2
)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(a^(3/4)*Sqrt
[d^2 - 4*c*e]*(4*e^2 + (Sqrt[b]*(d + Sqrt[d^2 - 4*c*e])^2)/Sqrt[a
])*Sqrt[a + b*x^4]) + (e*(4*e^2 - (Sqrt[b]*(d - Sqrt[d^2 - 4*c*e]
)^2)/Sqrt[a])*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] +
 Sqrt[b]*x^2)^2]*EllipticPi[(Sqrt[a]*(4*e^2 + (Sqrt[b]*(d - Sqrt[
d^2 - 4*c*e])^2)/Sqrt[a])^2)/(16*Sqrt[b]*e^2*(d - Sqrt[d^2 - 4*c*
e])^2), 2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*b^(1/4)*S
qrt[d^2 - 4*c*e]*(d - Sqrt[d^2 - 4*c*e])*(4*e^2 + (Sqrt[b]*(d - S
qrt[d^2 - 4*c*e])^2)/Sqrt[a])*Sqrt[a + b*x^4]) - (e*(4*e^2 - (Sqr
t[b]*(d + Sqrt[d^2 - 4*c*e])^2)/Sqrt[a])*(Sqrt[a] + Sqrt[b]*x^2)*
Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[(Sqrt[a]*(
4*e^2 + (Sqrt[b]*(d + Sqrt[d^2 - 4*c*e])^2)/Sqrt[a])^2)/(16*Sqrt[
b]*e^2*(d + Sqrt[d^2 - 4*c*e])^2), 2*ArcTan[(b^(1/4)*x)/a^(1/4)],
 1/2])/(2*a^(1/4)*b^(1/4)*Sqrt[d^2 - 4*c*e]*(d + Sqrt[d^2 - 4*c*e
])*(4*e^2 + (Sqrt[b]*(d + Sqrt[d^2 - 4*c*e])^2)/Sqrt[a])*Sqrt[a +
 b*x^4])
```

Rubi [A] time = 15.2873, antiderivative size = 1493, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$

result too large to display

Warning: Unable to verify antiderivative.

```
[In]  Int[1/((c + d*x + e*x^2)*Sqrt[a + b*x^4]),x]
```

```
[Out] (2*e*ArcTan[(Sqrt[-((16*a*e^4 + b*(d - Sqrt[d^2 - 4*c*e])^4)/(e^2
*(d - Sqrt[d^2 - 4*c*e])^2))]*x)/(2*Sqrt[a + b*x^4])])/(Sqrt[d^2
- 4*c*e]*(d - Sqrt[d^2 - 4*c*e])*Sqrt[-((16*a*e^4 + b*(d - Sqrt[d
^2 - 4*c*e])^4)/(e^2*(d - Sqrt[d^2 - 4*c*e])^2))]) - (2*e*ArcTan[
(Sqrt[-((16*a*e^4 + b*(d + Sqrt[d^2 - 4*c*e])^4)/(e^2*(d + Sqrt[d
^2 - 4*c*e])^2))]*x)/(2*Sqrt[a + b*x^4])])/(Sqrt[d^2 - 4*c*e]*(d
+ Sqrt[d^2 - 4*c*e])*Sqrt[-((16*a*e^4 + b*(d + Sqrt[d^2 - 4*c*e])
^4)/(e^2*(d + Sqrt[d^2 - 4*c*e])^2))]) - (e^2*ArcTanh[(4*a*e^2 +
b*(d - Sqrt[d^2 - 4*c*e])^2*x^2)/(2*Sqrt[2]*Sqrt[b*d^4 - 4*b*c*d^
2*e + 2*b*c^2*e^2 + 2*a*e^4 - b*d*Sqrt[d^2 - 4*c*e]*(d^2 - 2*c*e)
]*Sqrt[a + b*x^4])])/(Sqrt[2]*Sqrt[d^2 - 4*c*e]*Sqrt[b*d^4 - 4*b*
c*d^2*e + 2*b*c^2*e^2 + 2*a*e^4 - b*d*Sqrt[d^2 - 4*c*e]*(d^2 - 2*
c*e)]) + (e^2*ArcTanh[(4*a*e^2 + b*(d + Sqrt[d^2 - 4*c*e])^2*x^2)
/(2*Sqrt[2]*Sqrt[b*d^4 - 4*b*c*d^2*e + 2*b*c^2*e^2 + 2*a*e^4 + b*
d*Sqrt[d^2 - 4*c*e]*(d^2 - 2*c*e)]*Sqrt[a + b*x^4])])/(Sqrt[2]*Sq
rt[d^2 - 4*c*e]*Sqrt[b*d^4 - 4*b*c*d^2*e + 2*b*c^2*e^2 + 2*a*e^4
+ b*d*Sqrt[d^2 - 4*c*e]*(d^2 - 2*c*e)]) + (b^(1/4)*e*(d - Sqrt[d^
2 - 4*c*e])*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + S
qrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(a^
(3/4)*Sqrt[d^2 - 4*c*e]*(4*e^2 + (Sqrt[b]*(d - Sqrt[d^2 - 4*c*e])
^2)/Sqrt[a])*Sqrt[a + b*x^4]) - (b^(1/4)*e*(d + Sqrt[d^2 - 4*c*e]
)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2
)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(a^(3/4)*Sqrt
[d^2 - 4*c*e]*(4*e^2 + (Sqrt[b]*(d + Sqrt[d^2 - 4*c*e])^2)/Sqrt[a
])*Sqrt[a + b*x^4]) + (e*(4*e^2 - (Sqrt[b]*(d - Sqrt[d^2 - 4*c*e]
)^2)/Sqrt[a])*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] +
 Sqrt[b]*x^2)^2]*EllipticPi[(Sqrt[a]*(4*e^2 + (Sqrt[b]*(d - Sqrt[
d^2 - 4*c*e])^2)/Sqrt[a])^2)/(16*Sqrt[b]*e^2*(d - Sqrt[d^2 - 4*c*
e])^2), 2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*b^(1/4)*S
qrt[d^2 - 4*c*e]*(d - Sqrt[d^2 - 4*c*e])*(4*e^2 + (Sqrt[b]*(d - S
qrt[d^2 - 4*c*e])^2)/Sqrt[a])*Sqrt[a + b*x^4]) - (e*(4*e^2 - (Sqr
t[b]*(d + Sqrt[d^2 - 4*c*e])^2)/Sqrt[a])*(Sqrt[a] + Sqrt[b]*x^2)*
Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[(Sqrt[a]*(
4*e^2 + (Sqrt[b]*(d + Sqrt[d^2 - 4*c*e])^2)/Sqrt[a])^2)/(16*Sqrt[
b]*e^2*(d + Sqrt[d^2 - 4*c*e])^2), 2*ArcTan[(b^(1/4)*x)/a^(1/4)],
 1/2])/(2*a^(1/4)*b^(1/4)*Sqrt[d^2 - 4*c*e]*(d + Sqrt[d^2 - 4*c*e
])*(4*e^2 + (Sqrt[b]*(d + Sqrt[d^2 - 4*c*e])^2)/Sqrt[a])*Sqrt[a +
 b*x^4])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x**2+d*x+c)/(b*x**4+a)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 8.50006, size = 653, normalized size = 0.44

$$-\frac{\sqrt[4]{-1}\sqrt{2}\sqrt{-\frac{i\left(\sqrt[4]{-1}\sqrt[4]{a}+\sqrt[4]{b}x\right)}{\sqrt[4]{-1}\sqrt[4]{a}-\sqrt[4]{b}x}}\left(\sqrt{b}x^2+i\sqrt{a}\right)\left(\sqrt[4]{-1}\sqrt[4]{a}\left(-\left(\sqrt{b}\left(d\sqrt{d^2-4ce}-2ce+d^2\right)-2i\sqrt{a}e^2\right)\left(\frac{2(-1)^{3/4}\sqrt[4]{a}e-i\sqrt[4]{b}\left(\sqrt{d^2-4c\ldots}\right.}{2\sqrt[4]{-1}\sqrt[4]{a}e+\sqrt[4]{b}\left(\sqrt{d^2-4c\ldots}\right.}\right.\right.\right.}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((c + d*x + e*x^2)*Sqrt[a + b*x^4]),x]
```

```
[Out] -(((-1)^(1/4)*Sqrt[2]*Sqrt[((-I)*((-1)^(1/4)*a^(1/4) + b^(1/4)*x)
)/((-1)^(1/4)*a^(1/4) - b^(1/4)*x)]*(I*Sqrt[a] + Sqrt[b]*x^2)*(b^
(1/4)*(-(Sqrt[b]*c) + (-1)^(1/4)*a^(1/4)*b^(1/4)*d - I*Sqrt[a]*e)
*Sqrt[d^2 - 4*c*e]*EllipticF[ArcSin[Sqrt[((-I)*((-1)^(1/4)*a^(1/4
) + b^(1/4)*x))/((-1)^(1/4)*a^(1/4) - b^(1/4)*x)]], -1] + (-1)^(1
/4)*a^(1/4)*(-(((-2*I)*Sqrt[a]*e^2 + Sqrt[b]*(d^2 - 2*c*e + d*Sqr
t[d^2 - 4*c*e]))*EllipticPi[(2*(-1)^(3/4)*a^(1/4)*e - I*b^(1/4)*(
-d + Sqrt[d^2 - 4*c*e]))/(2*(-1)^(1/4)*a^(1/4)*e + b^(1/4)*(-d +
Sqrt[d^2 - 4*c*e])), ArcSin[Sqrt[((-I)*((-1)^(1/4)*a^(1/4) + b^(1
/4)*x))/((-1)^(1/4)*a^(1/4) - b^(1/4)*x)]], -1]) - ((2*I)*Sqrt[a]
*e^2 + Sqrt[b]*(-d^2 + 2*c*e + d*Sqrt[d^2 - 4*c*e]))*EllipticPi[(
(-I)*(2*(-1)^(1/4)*a^(1/4)*e + b^(1/4)*(d + Sqrt[d^2 - 4*c*e])))/
(-2*(-1)^(1/4)*a^(1/4)*e + b^(1/4)*(d + Sqrt[d^2 - 4*c*e])), ArcS
in[Sqrt[((-I)*((-1)^(1/4)*a^(1/4) + b^(1/4)*x))/((-1)^(1/4)*a^(1/
4) - b^(1/4)*x)]], -1])))/(a^(1/4)*Sqrt[d^2 - 4*c*e]*(b*c^2 - a*e
^2 - I*Sqrt[a]*Sqrt[b]*(d^2 - 2*c*e))*Sqrt[(I*Sqrt[a] + Sqrt[b]*x
^2)/((-1)^(1/4)*a^(1/4) - b^(1/4)*x)^2]*Sqrt[a + b*x^4]))
```

Maple [C] time = 0.102, size = 1153, normalized size = 0.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*x^2+d*x+c)/(b*x^4+a)^(1/2),x)
```

```
[Out] -1/2/(-4*c*e+d^2)^(1/2)/(1/2*b/e^4*d^4-1/2*b/e^4*(-4*c*e+d^2)^(1/
2)*d^3-2*b/e^3*d^2*c+b/e^3*d*(-4*c*e+d^2)^(1/2)*c+b/e^2*c^2+a)^(1
```

```
/2)*arctanh(1/2/(1/2*b/e^4*d^4-1/2*b/e^4*(-4*c*e+d^2)^(1/2)*d^3-2
*b/e^3*d^2*c+b/e^3*d*(-4*c*e+d^2)^(1/2)*c+b/e^2*c^2+a)^(1/2)/(b*x
^4+a)^(1/2)*b*x^2/e^2*d^2-1/2/(1/2*b/e^4*d^4-1/2*b/e^4*(-4*c*e+d^
2)^(1/2)*d^3-2*b/e^3*d^2*c+b/e^3*d*(-4*c*e+d^2)^(1/2)*c+b/e^2*c^2
+a)^(1/2)/(b*x^4+a)^(1/2)*b*x^2/e^2*d*(-4*c*e+d^2)^(1/2)-1/(1/2*b
/e^4*d^4-1/2*b/e^4*(-4*c*e+d^2)^(1/2)*d^3-2*b/e^3*d^2*c+b/e^3*d*(
-4*c*e+d^2)^(1/2)*c+b/e^2*c^2+a)^(1/2)/(b*x^4+a)^(1/2)*b*x^2/e*c+
1/(1/2*b/e^4*d^4-1/2*b/e^4*(-4*c*e+d^2)^(1/2)*d^3-2*b/e^3*d^2*c+b
/e^3*d*(-4*c*e+d^2)^(1/2)*c+b/e^2*c^2+a)^(1/2)/(b*x^4+a)^(1/2)*a)
-2/(-4*c*e+d^2)^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*e/(-d+(-4*c*e+d^2
)^(1/2))*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2
)^(1/2)/(b*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*b^(1/2))^(1/2),-4
*I*a^(1/2)/b^(1/2)*e^2/(-d+(-4*c*e+d^2)^(1/2))^2,(-I/a^(1/2)*b^(1
/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2))+1/2/(-4*c*e+d^2)^(1/2)/(1/2
*b/e^4*d^4+1/2*b/e^4*(-4*c*e+d^2)^(1/2)*d^3-2*b/e^3*d^2*c-b/e^3*d
*(-4*c*e+d^2)^(1/2)*c+b/e^2*c^2+a)^(1/2)*arctanh(1/2/(1/2*b/e^4*d
^4+1/2*b/e^4*(-4*c*e+d^2)^(1/2)*d^3-2*b/e^3*d^2*c-b/e^3*d*(-4*c*e
+d^2)^(1/2)*c+b/e^2*c^2+a)^(1/2)/(b*x^4+a)^(1/2)*b*x^2/e^2*d^2+1/
2/(1/2*b/e^4*d^4+1/2*b/e^4*(-4*c*e+d^2)^(1/2)*d^3-2*b/e^3*d^2*c-b
/e^3*d*(-4*c*e+d^2)^(1/2)*c+b/e^2*c^2+a)^(1/2)/(b*x^4+a)^(1/2)*b*
x^2/e^2*d*(-4*c*e+d^2)^(1/2)-1/(1/2*b/e^4*d^4+1/2*b/e^4*(-4*c*e+d
^2)^(1/2)*d^3-2*b/e^3*d^2*c-b/e^3*d*(-4*c*e+d^2)^(1/2)*c+b/e^2*c^
2+a)^(1/2)/(b*x^4+a)^(1/2)*b*x^2/e*c+1/(1/2*b/e^4*d^4+1/2*b/e^4*(
-4*c*e+d^2)^(1/2)*d^3-2*b/e^3*d^2*c-b/e^3*d*(-4*c*e+d^2)^(1/2)*c+
b/e^2*c^2+a)^(1/2)/(b*x^4+a)^(1/2)*a)-2/(-4*c*e+d^2)^(1/2)/(I/a^(
1/2)*b^(1/2))^(1/2)/(d+(-4*c*e+d^2)^(1/2))*e*(1-I/a^(1/2)*b^(1/2)
*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*Ellip
ticPi(x*(I/a^(1/2)*b^(1/2))^(1/2),-4*I*a^(1/2)/b^(1/2)/(d+(-4*c*e
+d^2)^(1/2))^2*e^2,(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))
^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^4+a}(ex^2+dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(b*x^4 + a)*(e*x^2 + d*x + c)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^4 + a)*(e*x^2 + d*x + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(b*x^4 + a)*(e*x^2 + d*x + c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+bx^4}\,(c+dx+ex^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*x**2+d*x+c)/(b*x**4+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*x**4)*(c + d*x + e*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^4+a}(ex^2+dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(b*x^4 + a)*(e*x^2 + d*x + c)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^4 + a)*(e*x^2 + d*x + c)), x)
```

3.205 $\int \frac{\sqrt{ax^{23}}}{\sqrt{1+x^5}}\,dx$

Optimal. Leaf size=75

$$\frac{3\sqrt{ax^{23}}\sinh^{-1}\left(x^{5/2}\right)}{20x^{23/2}}-\frac{3\sqrt{x^5+1}\sqrt{ax^{23}}}{20x^9}+\frac{\sqrt{x^5+1}\sqrt{ax^{23}}}{10x^4}$$

```
[Out] (-3*Sqrt[a*x^23]*Sqrt[1 + x^5])/(20*x^9) + (Sqrt[a*x^23]*Sqrt[1 +
 x^5])/(10*x^4) + (3*Sqrt[a*x^23]*ArcSinh[x^(5/2)])/(20*x^(23/2))
```

Rubi [A] time = 0.0557023, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263

$$\frac{3\sqrt{ax^{23}}\sinh^{-1}\left(x^{5/2}\right)}{20x^{23/2}}-\frac{3\sqrt{x^5+1}\sqrt{ax^{23}}}{20x^9}+\frac{\sqrt{x^5+1}\sqrt{ax^{23}}}{10x^4}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a*x^23]/Sqrt[1 + x^5],x]
```

```
[Out] (-3*Sqrt[a*x^23]*Sqrt[1 + x^5])/(20*x^9) + (Sqrt[a*x^23]*Sqrt[1 +
 x^5])/(10*x^4) + (3*Sqrt[a*x^23]*ArcSinh[x^(5/2)])/(20*x^(23/2))
```

Rubi in Sympy [A] time = 11.5955, size = 68, normalized size = 0.91

$$\frac{\sqrt{ax^{23}}\sqrt{x^5+1}}{10x^4}-\frac{3\sqrt{ax^{23}}\sqrt{x^5+1}}{20x^9}+\frac{3\sqrt{ax^{23}}\operatorname{asinh}\left(x^{\frac{5}{2}}\right)}{20x^{\frac{23}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a*x**23)**(1/2)/(x**5+1)**(1/2),x)
```

```
[Out] sqrt(a*x**23)*sqrt(x**5 + 1)/(10*x**4) - 3*sqrt(a*x**23)*sqrt(x**
5 + 1)/(20*x**9) + 3*sqrt(a*x**23)*asinh(x**(5/2))/(20*x**(23/2))
```

Mathematica [A] time = 0.109455, size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax^{23}}}{\sqrt{1+x^5}}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[Sqrt[a*x^23]/Sqrt[1 + x^5],x]
```

```
[Out] Integrate[Sqrt[a*x^23]/Sqrt[1 + x^5], x]
```

Maple [A] time = 0.193, size = 64, normalized size = 0.9

$$\frac{2x^5-3}{20x^9}\sqrt{x^5+1}\sqrt{ax^{23}}+\frac{3}{20x^{12}}Arcsinh\left(x^{\frac{5}{2}}\right)\sqrt{ax^{23}}\sqrt{ax\left(x^5+1\right)}\frac{1}{\sqrt{a}}\frac{1}{\sqrt{x^5+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a*x^23)^(1/2)/(x^5+1)^(1/2),x)
```

```
[Out] 1/20/x^9*(2*x^5-3)*(x^5+1)^(1/2)*(a*x^23)^(1/2)+3/20/a^(1/2)*arcs
inh(x^(5/2))*(a*x^23)^(1/2)/x^12*(a*x*(x^5+1))^(1/2)/(x^5+1)^(1/2
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{ax^{23}}}{\sqrt{x^5+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a*x^23)/sqrt(x^5 + 1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x^23)/sqrt(x^5 + 1), x)
```

Fricas [A] time = 0.395604, size = 1, normalized size = 0.01

$$\left[\frac{3\sqrt{a}x^9\log\left(-\frac{8ax^{19}+8ax^{14}+ax^9+4\sqrt{ax^{23}}\left(2x^5+1\right)\sqrt{x^5+1}\sqrt{a}}{x^9}\right)+4\sqrt{ax^{23}}\left(2x^5-3\right)\sqrt{x^5+1}}{80x^9},\right.$$
$$\left.-\frac{3\sqrt{-a}x^9\arctan\left(\frac{\left(2x^{14}+x^9\right)\sqrt{-a}}{2\sqrt{ax^{23}}\sqrt{x^5+1}}\right)-2\sqrt{ax^{23}}\left(2x^5-3\right)\sqrt{x^5+1}}{40x^9}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a*x^23)/sqrt(x^5 + 1),x, algorithm="fricas")
```

```
[Out] [1/80*(3*sqrt(a)*x^9*log(-(8*a*x^19 + 8*a*x^14 + a*x^9 + 4*sqrt(a
*x^23)*(2*x^5 + 1)*sqrt(x^5 + 1)*sqrt(a))/x^9) + 4*sqrt(a*x^23)*(
2*x^5 - 3)*sqrt(x^5 + 1))/x^9, -1/40*(3*sqrt(-a)*x^9*arctan(1/2*(
2*x^14 + x^9)*sqrt(-a)/(sqrt(a*x^23)*sqrt(x^5 + 1))) - 2*sqrt(a*x
^23)*(2*x^5 - 3)*sqrt(x^5 + 1))/x^9]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax^{23}}}{\sqrt{(x+1)(x^4-x^3+x^2-x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a*x**23)**(1/2)/(x**5+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a*x**23)/sqrt((x + 1)*(x**4 - x**3 + x**2 - x + 1))
, x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a*x^23)/sqrt(x^5 + 1),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.206 $\int \frac{\sqrt{ax^{13}}}{\sqrt{1+x^5}}\,dx$

Optimal. Leaf size=50

$$\frac{\sqrt{x^5+1}\sqrt{ax^{13}}}{5x^4}-\frac{\sqrt{ax^{13}}\sinh^{-1}\left(x^{5/2}\right)}{5x^{13/2}}$$

```
[Out] (Sqrt[a*x^13]*Sqrt[1 + x^5])/(5*x^4) - (Sqrt[a*x^13]*ArcSinh[x^(5
/2)])/(5*x^(13/2))
```

Rubi [A] time = 0.0400788, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263

$$\frac{\sqrt{x^5+1}\sqrt{ax^{13}}}{5x^4}-\frac{\sqrt{ax^{13}}\sinh^{-1}\left(x^{5/2}\right)}{5x^{13/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a*x^13]/Sqrt[1 + x^5],x]
```

```
[Out] (Sqrt[a*x^13]*Sqrt[1 + x^5])/(5*x^4) - (Sqrt[a*x^13]*ArcSinh[x^(5
/2)])/(5*x^(13/2))
```

Rubi in Sympy [A] time = 10.2477, size = 42, normalized size = 0.84

$$\frac{\sqrt{ax^{13}}\sqrt{x^5+1}}{5x^4}-\frac{\sqrt{ax^{13}}\operatorname{asinh}\left(x^{\frac{5}{2}}\right)}{5x^{\frac{13}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a*x**13)**(1/2)/(x**5+1)**(1/2),x)
```

```
[Out] sqrt(a*x**13)*sqrt(x**5 + 1)/(5*x**4) - sqrt(a*x**13)*asinh(x**(5
/2))/(5*x**(13/2))
```

Mathematica [A] time = 0.0880008, size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax^{13}}}{\sqrt{1+x^5}}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[Sqrt[a*x^13]/Sqrt[1 + x^5],x]
```

```
[Out] Integrate[Sqrt[a*x^13]/Sqrt[1 + x^5], x]
```

Maple [A] time = 0.066, size = 57, normalized size = 1.1

$$\frac{1}{5\,x^{4}}\sqrt{a x^{13}}\sqrt{x^{5}+1}-\frac{1}{5\,x^{7}}Arcsinh\left(x^{\frac{5}{2}}\right)\sqrt{a x^{13}}\sqrt{a x\,(x^{5}+1)}\frac{1}{\sqrt{a}}\frac{1}{\sqrt{x^{5}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a*x^13)^(1/2)/(x^5+1)^(1/2),x)
```

```
[Out] 1/5*(a*x^13)^(1/2)*(x^5+1)^(1/2)/x^4-1/5/a^(1/2)*arcsinh(x^(5/2))
*(a*x^13)^(1/2)/x^7*(a*x*(x^5+1))^(1/2)/(x^5+1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a x^{13}}}{\sqrt{x^{5}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a*x^13)/sqrt(x^5 + 1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x^13)/sqrt(x^5 + 1), x)
```

Fricas [A] time = 0.392661, size = 1, normalized size = 0.02

$$\left[\frac{\sqrt{a}x^{4}\log\left(-\frac{8\,a x^{14}+8\,a x^{9}+a x^{4}-4\,\sqrt{a x^{13}}(2\,x^{5}+1)\,\sqrt{x^{5}+1}\sqrt{a}}{x^{4}}\right)+4\,\sqrt{a x^{13}}\sqrt{x^{5}+1}}{20\,x^{4}},\ \frac{\sqrt{-a}x^{4}\arctan\left(\frac{(2\,x^{9}+x^{4})\,\sqrt{-a}}{2\,\sqrt{a x^{13}}\sqrt{x^{5}+1}}\right)+2\,\sqrt{a x^{13}}\sqrt{x^{5}+1}}{10\,x^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a*x^13)/sqrt(x^5 + 1),x, algorithm="fricas")
```

```
[Out] [1/20*(sqrt(a)*x^4*log(-(8*a*x^14 + 8*a*x^9 + a*x^4 - 4*sqrt(a*x^
13)*(2*x^5 + 1)*sqrt(x^5 + 1)*sqrt(a))/x^4) + 4*sqrt(a*x^13)*sqrt
(x^5 + 1))/x^4, 1/10*(sqrt(-a)*x^4*arctan(1/2*(2*x^9 + x^4)*sqrt(
-a)/(sqrt(a*x^13)*sqrt(x^5 + 1))) + 2*sqrt(a*x^13)*sqrt(x^5 + 1))
/x^4]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax^{13}}}{\sqrt{(x+1)(x^4-x^3+x^2-x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a*x**13)**(1/2)/(x**5+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a*x**13)/sqrt((x + 1)*(x**4 - x**3 + x**2 - x + 1))
, x)
```

GIAC/XCAS [A] time = 0.282395, size = 92, normalized size = 1.84

$$\frac{a^{\frac{11}{2}}\ln\left(-\sqrt{ax}a^{\frac{5}{2}}x^2+\sqrt{a^6x^5+a^6}\right)}{5\,|a|^5}+\frac{\sqrt{a^6x^5+a^6}\sqrt{ax}x^2}{5\,a^2|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a*x^13)/sqrt(x^5 + 1),x, algorithm="giac")
```

```
[Out] 1/5*a^(11/2)*ln(-sqrt(a*x)*a^(5/2)*x^2 + sqrt(a^6*x^5 + a^6))/abs
(a)^5 + 1/5*sqrt(a^6*x^5 + a^6)*sqrt(a*x)*x^2/(a^2*abs(a))
```

3.207 $\int \frac{\sqrt{ax^3}}{\sqrt{1+x^5}}\,dx$

Optimal. Leaf size=24

$$\frac{2\sqrt{ax^3}\sinh^{-1}\left(x^{5/2}\right)}{5x^{3/2}}$$

```
[Out] (2*Sqrt[a*x^3]*ArcSinh[x^(5/2)])/(5*x^(3/2))
```

Rubi [A] time = 0.0251148, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$\frac{2\sqrt{ax^3}\sinh^{-1}\left(x^{5/2}\right)}{5x^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a*x^3]/Sqrt[1 + x^5],x]
```

```
[Out] (2*Sqrt[a*x^3]*ArcSinh[x^(5/2)])/(5*x^(3/2))
```

Rubi in Sympy [A] time = 8.86404, size = 22, normalized size = 0.92

$$\frac{2\sqrt{ax^3}\operatorname{asinh}\left(x^{\frac{5}{2}}\right)}{5x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a*x**3)**(1/2)/(x**5+1)**(1/2),x)
```

```
[Out] 2*sqrt(a*x**3)*asinh(x**(5/2))/(5*x**(3/2))
```

Mathematica [A] time = 0.0338001, size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^5}}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[Sqrt[a*x^3]/Sqrt[1 + x^5],x]
```

```
[Out] Integrate[Sqrt[a*x^3]/Sqrt[1 + x^5], x]
```

Maple [A] time = 0.058, size = 17, normalized size = 0.7

$$\frac{2}{5}Arcsinh\left(x^{\frac{5}{2}}\right)\sqrt{ax^3}x^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a*x^3)^(1/2)/(x^5+1)^(1/2),x)
```

```
[Out] 2/5*arcsinh(x^(5/2))*(a*x^3)^(1/2)/x^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax^3}}{\sqrt{x^5+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a*x^3)/sqrt(x^5 + 1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x^3)/sqrt(x^5 + 1), x)
```

Fricas [A] time = 0.358236, size = 1, normalized size = 0.04

$$\left[\frac{1}{10}\sqrt{a}\log\left(-8\,ax^{10}-8\,ax^5-4\left(2\,x^6+x\right)\sqrt{x^5+1}\sqrt{ax^3}\sqrt{a}-a\right),-\frac{1}{5}\sqrt{-a}\arctan\left(\frac{\left(2\,x^5+1\right)\sqrt{-a}}{2\sqrt{x^5+1}\sqrt{ax^3}x}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a*x^3)/sqrt(x^5 + 1),x, algorithm="fricas")
```

```
[Out] [1/10*sqrt(a)*log(-8*a*x^10 - 8*a*x^5 - 4*(2*x^6 + x)*sqrt(x^5 +
1)*sqrt(a*x^3)*sqrt(a) - a), -1/5*sqrt(-a)*arctan(1/2*(2*x^5 + 1)
*sqrt(-a)/(sqrt(x^5 + 1)*sqrt(a*x^3)*x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax^3}}{\sqrt{(x+1)\,(x^4-x^3+x^2-x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a*x**3)**(1/2)/(x**5+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a*x**3)/sqrt((x + 1)*(x**4 - x**3 + x**2 - x + 1)),
 x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a*x^3)/sqrt(x^5 + 1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.208 $\int \frac{\sqrt{\frac{a}{x^7}}}{\sqrt{1+x^5}}\,dx$

Optimal. Leaf size=23

$$-\frac{2}{5}x\sqrt{x^5+1}\sqrt{\frac{a}{x^7}}$$

```
[Out] (-2*Sqrt[a/x^7]*x*Sqrt[1 + x^5])/5
```

Rubi [A] time = 0.0156065, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$-\frac{2}{5}x\sqrt{x^5+1}\sqrt{\frac{a}{x^7}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a/x^7]/Sqrt[1 + x^5],x]

[Out] (-2*Sqrt[a/x^7]*x*Sqrt[1 + x^5])/5
```

Rubi in Sympy [A] time = 7.01607, size = 22, normalized size = 0.96

$$-\frac{2x\sqrt{\frac{a}{x^7}}\sqrt{x^5+1}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a/x**7)**(1/2)/(x**5+1)**(1/2),x)

[Out] -2*x*sqrt(a/x**7)*sqrt(x**5 + 1)/5
```

Mathematica [A] time = 0.0160049, size = 23, normalized size = 1.

$$-\frac{2}{5}x\sqrt{x^5+1}\sqrt{\frac{a}{x^7}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a/x^7]/Sqrt[1 + x^5],x]

[Out] (-2*Sqrt[a/x^7]*x*Sqrt[1 + x^5])/5
```

Maple [B] time = 0.008, size = 37, normalized size = 1.6

$$-\frac{2x(1+x)\left(x^4-x^3+x^2-x+1\right)}{5}\sqrt{\frac{a}{x^7}}\frac{1}{\sqrt{x^5+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a/x^7)^(1/2)/(x^5+1)^(1/2),x)

[Out] -2/5*x*(1+x)*(x^4-x^3+x^2-x+1)*(a/x^7)^(1/2)/(x^5+1)^(1/2)
```

Maxima [A] time = 0.779543, size = 55, normalized size = 2.39

$$-\frac{2\left(\sqrt{a}x^6+\sqrt{a}x\right)}{5\sqrt{x^4-x^3+x^2-x+1}\sqrt{x+1}x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a/x^7)/sqrt(x^5 + 1),x, algorithm="maxima")

[Out] -2/5*(sqrt(a)*x^6 + sqrt(a)*x)/(sqrt(x^4 - x^3 + x^2 - x + 1)*sqr
t(x + 1)*x^(7/2))
```

Fricas [A] time = 0.286137, size = 23, normalized size = 1.

$$-\frac{2}{5}\sqrt{x^5+1}x\sqrt{\frac{a}{x^7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a/x^7)/sqrt(x^5 + 1),x, algorithm="fricas")

[Out] -2/5*sqrt(x^5 + 1)*x*sqrt(a/x^7)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{a}{x^7}}}{\sqrt{(x+1)(x^4-x^3+x^2-x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a/x**7)**(1/2)/(x**5+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a/x**7)/sqrt((x + 1)*(x**4 - x**3 + x**2 - x + 1)),
 x)
```

GIAC/XCAS [A] time = 0.274373, size = 38, normalized size = 1.65

$$-\frac{2\,a^3\left(\frac{\sqrt{a+\frac{a}{x^5}}}{a^2}-\frac{1}{a^{\frac{3}{2}}}\right)}{5\,|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a/x^7)/sqrt(x^5 + 1),x, algorithm="giac")
```

```
[Out] -2/5*a^3*(sqrt(a + a/x^5)/a^2 - 1/a^(3/2))/abs(a)
```

3.209 $\int \frac{\sqrt{\frac{a}{x^{17}}}}{\sqrt{1+x^5}}\,dx$

Optimal. Leaf size=49

$$\frac{4}{15}x^6\sqrt{x^5+1}\sqrt{\frac{a}{x^{17}}}-\frac{2}{15}x\sqrt{x^5+1}\sqrt{\frac{a}{x^{17}}}$$

```
[Out] (-2*Sqrt[a/x^17]*x*Sqrt[1 + x^5])/15 + (4*Sqrt[a/x^17]*x^6*Sqrt[1
 + x^5])/15
```

Rubi [A] time = 0.0302922, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$\frac{4}{15}x^6\sqrt{x^5+1}\sqrt{\frac{a}{x^{17}}}-\frac{2}{15}x\sqrt{x^5+1}\sqrt{\frac{a}{x^{17}}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a/x^17]/Sqrt[1 + x^5],x]
```

```
[Out] (-2*Sqrt[a/x^17]*x*Sqrt[1 + x^5])/15 + (4*Sqrt[a/x^17]*x^6*Sqrt[1
 + x^5])/15
```

Rubi in Sympy [A] time = 8.22471, size = 44, normalized size = 0.9

$$\frac{4x^6\sqrt{\frac{a}{x^{17}}}\sqrt{x^5+1}}{15}-\frac{2x\sqrt{\frac{a}{x^{17}}}\sqrt{x^5+1}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a/x**17)**(1/2)/(x**5+1)**(1/2),x)
```

```
[Out] 4*x**6*sqrt(a/x**17)*sqrt(x**5 + 1)/15 - 2*x*sqrt(a/x**17)*sqrt(x
**5 + 1)/15
```

Mathematica [A] time = 0.014932, size = 30, normalized size = 0.61

$$\frac{2}{15}x\sqrt{x^5+1}\left(2x^5-1\right)\sqrt{\frac{a}{x^{17}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a/x^17]/Sqrt[1 + x^5],x]
```

```
[Out] (2*Sqrt[a/x^17]*x*Sqrt[1 + x^5]*(-1 + 2*x^5))/15
```

Maple [A] time = 0.008, size = 44, normalized size = 0.9

$$\frac{2x(1+x)\left(x^4-x^3+x^2-x+1\right)\left(2x^5-1\right)}{15}\sqrt{\frac{a}{x^{17}}}\frac{1}{\sqrt{x^5+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a/x^17)^(1/2)/(x^5+1)^(1/2),x)
```

```
[Out] 2/15*x*(1+x)*(x^4-x^3+x^2-x+1)*(2*x^5-1)*(a/x^17)^(1/2)/(x^5+1)^(
1/2)
```

Maxima [A] time = 0.782597, size = 68, normalized size = 1.39

$$\frac{2\left(2\sqrt{a}x^{11}+\sqrt{a}x^6-\sqrt{a}x\right)}{15\sqrt{x^4-x^3+x^2-x+1}\sqrt{x+1}x^{\frac{17}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a/x^17)/sqrt(x^5 + 1),x, algorithm="maxima")
```

```
[Out] 2/15*(2*sqrt(a)*x^11 + sqrt(a)*x^6 - sqrt(a)*x)/(sqrt(x^4 - x^3 +
 x^2 - x + 1)*sqrt(x + 1)*x^(17/2))
```

Fricas [A] time = 0.276903, size = 34, normalized size = 0.69

$$\frac{2}{15}\left(2x^6-x\right)\sqrt{x^5+1}\sqrt{\frac{a}{x^{17}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a/x^17)/sqrt(x^5 + 1),x, algorithm="fricas")
```

```
[Out] 2/15*(2*x^6 - x)*sqrt(x^5 + 1)*sqrt(a/x^17)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{a}{x^{17}}}}{\sqrt{(x+1)(x^4-x^3+x^2-x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a/x**17)**(1/2)/(x**5+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a/x**17)/sqrt((x + 1)*(x**4 - x**3 + x**2 - x + 1))
, x)
```

GIAC/XCAS [A] time = 0.279049, size = 58, normalized size = 1.18

$$-\frac{2\,a^3\left(\frac{2}{a^{\frac{3}{2}}}+\frac{\left(a+\frac{a}{x^5}\right)^{\frac{3}{2}}-3\,\sqrt{a+\frac{a}{x^5}}\,a}{a^3}\right)\mathrm{sign}\left(x\right)}{15\,|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a/x^17)/sqrt(x^5 + 1),x, algorithm="giac")
```

```
[Out] -2/15*a^3*(2/a^(3/2) + ((a + a/x^5)^(3/2) - 3*sqrt(a + a/x^5)*a)/
a^3)*sign(x)/abs(a)
```

3.210 $\int \frac{\sqrt{ax^6}}{x(1-x^4)} dx$

Optimal. Leaf size=37

$$\frac{\sqrt{ax^6}\tanh^{-1}(x)}{2x^3}-\frac{\sqrt{ax^6}\tan^{-1}(x)}{2x^3}$$

```
[Out] -(Sqrt[a*x^6]*ArcTan[x])/(2*x^3) + (Sqrt[a*x^6]*ArcTanh[x])/(2*x^
3)
```

Rubi [A] time = 0.0224638, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{\sqrt{ax^6}\tanh^{-1}(x)}{2x^3}-\frac{\sqrt{ax^6}\tan^{-1}(x)}{2x^3}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a*x^6]/(x*(1 - x^4)),x]
```

```
[Out] -(Sqrt[a*x^6]*ArcTan[x])/(2*x^3) + (Sqrt[a*x^6]*ArcTanh[x])/(2*x^
3)
```

Rubi in Sympy [A] time = 13.9603, size = 32, normalized size = 0.86

$$-\frac{\sqrt{ax^6}\operatorname{atan}(x)}{2x^3}+\frac{\sqrt{ax^6}\operatorname{atanh}(x)}{2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a*x**6)**(1/2)/x/(-x**4+1),x)
```

```
[Out] -sqrt(a*x**6)*atan(x)/(2*x**3) + sqrt(a*x**6)*atanh(x)/(2*x**3)
```

Mathematica [A] time = 0.0677394, size = 33, normalized size = 0.89

$$-\frac{\sqrt{ax^6}\left(\log(1-x)-\log(x+1)+2\tan^{-1}(x)\right)}{4x^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a*x^6]/(x*(1 - x^4)),x]

[Out] -(Sqrt[a*x^6]*(2*ArcTan[x] + Log[1 - x] - Log[1 + x]))/(4*x^3)
```

Maple [A] time = 0.016, size = 28, normalized size = 0.8

$$-\frac{\ln(-1+x)-\ln(1+x)+2\arctan(x)}{4x^3}\sqrt{ax^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a*x^6)^(1/2)/x/(-x^4+1),x)

[Out] -1/4*(a*x^6)^(1/2)*(ln(-1+x)-ln(1+x)+2*arctan(x))/x^3
```

Maxima [A] time = 0.767813, size = 35, normalized size = 0.95

$$-\frac{1}{2}\sqrt{a}\arctan(x)+\frac{1}{4}\sqrt{a}\log(x+1)-\frac{1}{4}\sqrt{a}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sqrt(a*x^6)/((x^4 - 1)*x),x, algorithm="maxima")

[Out] -1/2*sqrt(a)*arctan(x) + 1/4*sqrt(a)*log(x + 1) - 1/4*sqrt(a)*log
(x - 1)
```

Fricas [A] time = 0.293477, size = 39, normalized size = 1.05

$$-\frac{\sqrt{ax^6}\left(2\arctan(x)-\log\left(\frac{x+1}{x-1}\right)\right)}{4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sqrt(a*x^6)/((x^4 - 1)*x),x, algorithm="fricas")

[Out] -1/4*sqrt(a*x^6)*(2*arctan(x) - log((x + 1)/(x - 1)))/x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{ax^6}}{x^5 - x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a*x**6)**(1/2)/x/(-x**4+1),x)
```

```
[Out] -Integral(sqrt(a*x**6)/(x**5 - x), x)
```

GIAC/XCAS [A] time = 0.261351, size = 39, normalized size = 1.05

$$-\frac{1}{4}\left(2\,\arctan\left(x\right)\operatorname{sign}\left(x\right) - \ln\left(|x+1|\right)\operatorname{sign}\left(x\right) + \ln\left(|x-1|\right)\operatorname{sign}\left(x\right)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sqrt(a*x^6)/((x^4 - 1)*x),x, algorithm="giac")
```

```
[Out] -1/4*(2*arctan(x)*sign(x) - ln(abs(x + 1))*sign(x) + ln(abs(x - 1
))*sign(x))*sqrt(a)
```

3.211 $\int \frac{\sqrt{ax^6}}{x-x^5} \, dx$

Optimal. Leaf size=37

$$\frac{\sqrt{ax^6}\tanh^{-1}(x)}{2x^3} - \frac{\sqrt{ax^6}\tan^{-1}(x)}{2x^3}$$

```
[Out] -(Sqrt[a*x^6]*ArcTan[x])/(2*x^3) + (Sqrt[a*x^6]*ArcTanh[x])/(2*x^
3)
```

Rubi [A] time = 0.0247574, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263

$$\frac{\sqrt{ax^6}\tanh^{-1}(x)}{2x^3} - \frac{\sqrt{ax^6}\tan^{-1}(x)}{2x^3}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a*x^6]/(x - x^5),x]
```

```
[Out] -(Sqrt[a*x^6]*ArcTan[x])/(2*x^3) + (Sqrt[a*x^6]*ArcTanh[x])/(2*x^
3)
```

Rubi in Sympy [A] time = 14.9667, size = 32, normalized size = 0.86

$$-\frac{\sqrt{ax^6}\,\mathrm{atan}\,(x)}{2x^3} + \frac{\sqrt{ax^6}\,\mathrm{atanh}\,(x)}{2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a*x**6)**(1/2)/(-x**5+x),x)
```

```
[Out] -sqrt(a*x**6)*atan(x)/(2*x**3) + sqrt(a*x**6)*atanh(x)/(2*x**3)
```

Mathematica [A] time = 0.00652605, size = 33, normalized size = 0.89

$$-\frac{\sqrt{ax^6}\left(\log(1-x) - \log(x+1) + 2\tan^{-1}(x)\right)}{4x^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a*x^6]/(x - x^5),x]

[Out] -(Sqrt[a*x^6]*(2*ArcTan[x] + Log[1 - x] - Log[1 + x]))/(4*x^3)
```

Maple [A] time = 0.01, size = 28, normalized size = 0.8

$$-\frac{\ln\left(-1+x\right)-\ln\left(1+x\right)+2\arctan\left(x\right)}{4\,x^{3}}\sqrt{a x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a*x^6)^(1/2)/(-x^5+x),x)

[Out] -1/4*(a*x^6)^(1/2)*(ln(-1+x)-ln(1+x)+2*arctan(x))/x^3
```

Maxima [A] time = 0.769744, size = 35, normalized size = 0.95

$$-\frac{1}{2}\sqrt{a}\arctan\left(x\right)+\frac{1}{4}\sqrt{a}\log\left(x+1\right)-\frac{1}{4}\sqrt{a}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sqrt(a*x^6)/(x^5 - x),x, algorithm="maxima")

[Out] -1/2*sqrt(a)*arctan(x) + 1/4*sqrt(a)*log(x + 1) - 1/4*sqrt(a)*log
(x - 1)
```

Fricas [A] time = 0.298899, size = 39, normalized size = 1.05

$$-\frac{\sqrt{a x^{6}}\left(2\arctan\left(x\right)-\log\left(\frac{x+1}{x-1}\right)\right)}{4\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sqrt(a*x^6)/(x^5 - x),x, algorithm="fricas")

[Out] -1/4*sqrt(a*x^6)*(2*arctan(x) - log((x + 1)/(x - 1)))/x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{ax^6}}{x^5 - x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a*x**6)**(1/2)/(-x**5+x),x)
```

```
[Out] -Integral(sqrt(a*x**6)/(x**5 - x), x)
```

GIAC/XCAS [A] time = 0.259908, size = 39, normalized size = 1.05

$$-\frac{1}{4}\left(2\,\arctan\left(x\right)\mathrm{sign}\left(x\right) - \ln\left(\left|x+1\right|\right)\mathrm{sign}\left(x\right) + \ln\left(\left|x-1\right|\right)\mathrm{sign}\left(x\right)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sqrt(a*x^6)/(x^5 - x),x, algorithm="giac")
```

```
[Out] -1/4*(2*arctan(x)*sign(x) - ln(abs(x + 1))*sign(x) + ln(abs(x - 1
))*sign(x))*sqrt(a)
```

3.212 $\int \frac{(ax^6)^{3/2}}{x(1-x^4)} \, dx$

Optimal. Leaf size=71

$$\frac{a\sqrt{ax^6}\tan^{-1}(x)}{2x^3}+\frac{a\sqrt{ax^6}\tanh^{-1}(x)}{2x^3}-\frac{1}{5}ax^2\sqrt{ax^6}-\frac{a\sqrt{ax^6}}{x^2}$$

```
[Out] -((a*Sqrt[a*x^6])/x^2) - (a*x^2*Sqrt[a*x^6])/5 + (a*Sqrt[a*x^6]*A
rcTan[x])/(2*x^3) + (a*Sqrt[a*x^6]*ArcTanh[x])/(2*x^3)
```

Rubi [A] time = 0.0374281, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227

$$\frac{a\sqrt{ax^6}\tan^{-1}(x)}{2x^3}+\frac{a\sqrt{ax^6}\tanh^{-1}(x)}{2x^3}-\frac{1}{5}ax^2\sqrt{ax^6}-\frac{a\sqrt{ax^6}}{x^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a*x^6)^(3/2)/(x*(1 - x^4)),x]
```

```
[Out] -((a*Sqrt[a*x^6])/x^2) - (a*x^2*Sqrt[a*x^6])/5 + (a*Sqrt[a*x^6]*A
rcTan[x])/(2*x^3) + (a*Sqrt[a*x^6]*ArcTanh[x])/(2*x^3)
```

Rubi in Sympy [A] time = 15.0726, size = 65, normalized size = 0.92

$$-\frac{ax^2\sqrt{ax^6}}{5}-\frac{a\sqrt{ax^6}}{x^2}+\frac{a\sqrt{ax^6}\operatorname{atan}(x)}{2x^3}+\frac{a\sqrt{ax^6}\operatorname{atanh}(x)}{2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a*x**6)**(3/2)/x/(-x**4+1),x)
```

```
[Out] -a*x**2*sqrt(a*x**6)/5 - a*sqrt(a*x**6)/x**2 + a*sqrt(a*x**6)*ata
n(x)/(2*x**3) + a*sqrt(a*x**6)*atanh(x)/(2*x**3)
```

Mathematica [A] time = 0.037031, size = 44, normalized size = 0.62

$$-\frac{a\sqrt{ax^6}\left(4x^5+20x+5\log(1-x)-5\log(x+1)-10\tan^{-1}(x)\right)}{20x^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a*x^6)^(3/2)/(x*(1 - x^4)),x]
```

```
[Out] -(a*Sqrt[a*x^6]*(20*x + 4*x^5 - 10*ArcTan[x] + 5*Log[1 - x] - 5*L
og[1 + x]))/(20*x^3)
```

Maple [A] time = 0.014, size = 38, normalized size = 0.5

$$-\frac{4\,x^{5}+5\,\ln\left(-1+x\right)-5\,\ln\left(1+x\right)-10\,\arctan\left(x\right)+20\,x}{20\,x^{9}}\left(ax^{6}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a*x^6)^(3/2)/x/(-x^4+1),x)
```

```
[Out] -1/20*(a*x^6)^(3/2)*(4*x^5+5*ln(-1+x)-5*ln(1+x)-10*arctan(x)+20*x
)/x^9
```

Maxima [A] time = 0.774762, size = 54, normalized size = 0.76

$$-\frac{1}{5}\,a^{\frac{3}{2}}x^{5}-a^{\frac{3}{2}}x+\frac{1}{2}\,a^{\frac{3}{2}}\arctan\left(x\right)+\frac{1}{4}\,a^{\frac{3}{2}}\log\left(x+1\right)-\frac{1}{4}\,a^{\frac{3}{2}}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(a*x^6)^(3/2)/((x^4 - 1)*x),x, algorithm="maxima")
```

```
[Out] -1/5*a^(3/2)*x^5 - a^(3/2)*x + 1/2*a^(3/2)*arctan(x) + 1/4*a^(3/2
)*log(x + 1) - 1/4*a^(3/2)*log(x - 1)
```

Fricas [A] time = 0.293685, size = 55, normalized size = 0.77

$$-\frac{\sqrt{ax^{6}}\left(4\,ax^{5}+20\,ax-10\,a\arctan\left(x\right)-5\,a\log\left(\frac{x+1}{x-1}\right)\right)}{20\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(a*x^6)^(3/2)/((x^4 - 1)*x),x, algorithm="fricas")
```

```
[Out] -1/20*sqrt(a*x^6)*(4*a*x^5 + 20*a*x - 10*a*arctan(x) - 5*a*log((x
 + 1)/(x - 1)))/x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(ax^{6}\right)^{\frac{3}{2}}}{x^{5}-x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a*x**6)**(3/2)/x/(-x**4+1),x)
```

```
[Out] -Integral((a*x**6)**(3/2)/(x**5 - x), x)
```

GIAC/XCAS [A] time = 0.264066, size = 57, normalized size = 0.8

$$-\frac{1}{20}\left(4\,x^{5}\mathrm{sign}\left(x\right)+20\,x\mathrm{sign}\left(x\right)-10\,\arctan\left(x\right)\mathrm{sign}\left(x\right)-5\,\ln\left(|x+1|\right)\mathrm{sign}\left(x\right)+5\,\ln\left(|x-1|\right)\mathrm{sign}\left(x\right)\right)a^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(a*x^6)^(3/2)/((x^4 - 1)*x),x, algorithm="giac")
```

```
[Out] -1/20*(4*x^5*sign(x) + 20*x*sign(x) - 10*arctan(x)*sign(x) - 5*ln
(abs(x + 1))*sign(x) + 5*ln(abs(x - 1))*sign(x))*a^(3/2)
```

3.213 $\int \left(\frac{1}{1-x^4} - \frac{\sqrt{ax^6}}{x(1-x^4)} \right) dx$

Optimal. Leaf size=49

$$\frac{\sqrt{ax^6}\tan^{-1}(x)}{2x^3} - \frac{\sqrt{ax^6}\tanh^{-1}(x)}{2x^3} + \frac{1}{2}\tan^{-1}(x) + \frac{1}{2}\tanh^{-1}(x)$$

```
[Out] ArcTan[x]/2 + (Sqrt[a*x^6]*ArcTan[x])/(2*x^3) + ArcTanh[x]/2 - (S
qrt[a*x^6]*ArcTanh[x])/(2*x^3)
```

Rubi [A] time = 0.0354551, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152

$$\frac{\sqrt{ax^6}\tan^{-1}(x)}{2x^3} - \frac{\sqrt{ax^6}\tanh^{-1}(x)}{2x^3} + \frac{1}{2}\tan^{-1}(x) + \frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - x^4)^(-1) - Sqrt[a*x^6]/(x*(1 - x^4)),x]
```

```
[Out] ArcTan[x]/2 + (Sqrt[a*x^6]*ArcTan[x])/(2*x^3) + ArcTanh[x]/2 - (S
qrt[a*x^6]*ArcTanh[x])/(2*x^3)
```

Rubi in Sympy [A] time = 14.4498, size = 42, normalized size = 0.86

$$\frac{\operatorname{atan}(x)}{2} + \frac{\operatorname{atanh}(x)}{2} + \frac{\sqrt{ax^6}\operatorname{atan}(x)}{2x^3} - \frac{\sqrt{ax^6}\operatorname{atanh}(x)}{2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-x**4+1)-(a*x**6)**(1/2)/x/(-x**4+1),x)
```

```
[Out] atan(x)/2 + atanh(x)/2 + sqrt(a*x**6)*atan(x)/(2*x**3) - sqrt(a*x
**6)*atanh(x)/(2*x**3)
```

Mathematica [A] time = 0.13873, size = 0, normalized size = 0.

$$\int \left(\frac{1}{1-x^4} - \frac{\sqrt{ax^6}}{x(1-x^4)} \right) dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(1 - x^4)^(-1) - Sqrt[a*x^6]/(x*(1 - x^4)),x]
```

```
[Out] Integrate[(1 - x^4)^(-1) - Sqrt[a*x^6]/(x*(1 - x^4)), x]
```

Maple [A] time = 0.005, size = 37, normalized size = 0.8

$$\frac{Artanh(x)}{2}+\frac{\arctan(x)}{2}+\frac{\ln(-1+x)-\ln(1+x)+2\arctan(x)}{4x^{3}}\sqrt{ax^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-x^4+1)-(a*x^6)^(1/2)/x/(-x^4+1),x)
```

```
[Out] 1/2*arctanh(x)+1/2*arctan(x)+1/4*(a*x^6)^(1/2)*(ln(-1+x)-ln(1+x)+
2*arctan(x))/x^3
```

Maxima [A] time = 0.777124, size = 57, normalized size = 1.16

$$\frac{1}{2}\sqrt{a}\arctan(x)-\frac{1}{4}\sqrt{a}\log(x+1)+\frac{1}{4}\sqrt{a}\log(x-1)+\frac{1}{2}\arctan(x)+\frac{1}{4}\log(x+1)-\frac{1}{4}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(x^4 - 1) + sqrt(a*x^6)/((x^4 - 1)*x),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(a)*arctan(x) - 1/4*sqrt(a)*log(x + 1) + 1/4*sqrt(a)*log(
x - 1) + 1/2*arctan(x) + 1/4*log(x + 1) - 1/4*log(x - 1)
```

Fricas [A] time = 0.292584, size = 1, normalized size = 0.02

$$\left[\frac{x^{3}\sqrt{-\frac{(a+1)x^{3}+2\sqrt{ax^{6}}}{x^{3}}}\log\left(\frac{(a-1)x^{4}-(a-1)x^{2}-2\left(x^{3}-\sqrt{ax^{6}}\right)\sqrt{-\frac{(a+1)x^{3}+2\sqrt{ax^{6}}}{x^{3}}}}{x^{4}+x^{2}}\right)+x^{3}\log(x+1)-x^{3}\log(x-1)-\sqrt{ax^{6}}(\log(x+1)-\log}{4x^{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/(x^4 - 1) + sqrt(a*x^6)/((x^4 - 1)*x),x, algorithm="fricas")
```

```
[Out] [1/4*(x^3*sqrt(-((a + 1)*x^3 + 2*sqrt(a*x^6))/x^3)*log(((a - 1)*x
^4 - (a - 1)*x^2 - 2*(x^3 - sqrt(a*x^6))*sqrt(-((a + 1)*x^3 + 2*s
qrt(a*x^6))/x^3))/(x^4 + x^2)) + x^3*log(x + 1) - x^3*log(x - 1)
- sqrt(a*x^6)*(log(x + 1) - log(x - 1)))/x^3, 1/4*(2*x^3*sqrt(((a
 + 1)*x^3 + 2*sqrt(a*x^6))/x^3)*arctan(-(a - 1)*x^4/((x^3 - sqrt(
a*x^6))*sqrt(((a + 1)*x^3 + 2*sqrt(a*x^6))/x^3))) + x^3*log(x + 1
) - x^3*log(x - 1) - sqrt(a*x^6)*(log(x + 1) - log(x - 1)))/x^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{x^5 - x}\,dx - \int \left(-\frac{\sqrt{ax^6}}{x^5 - x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**4+1)-(a*x**6)**(1/2)/x/(-x**4+1),x)
```

```
[Out] -Integral(x/(x**5 - x), x) - Integral(-sqrt(a*x**6)/(x**5 - x), x
)
```

GIAC/XCAS [A] time = 0.26123, size = 65, normalized size = 1.33

$$\frac{1}{4}\left(2\arctan\left(x\right)\mathrm{sign}\left(x\right) - \ln\left(|x+1|\right)\mathrm{sign}\left(x\right) + \ln\left(|x-1|\right)\mathrm{sign}\left(x\right)\right)\sqrt{a} + \frac{1}{2}\arctan\left(x\right) + \frac{1}{4}\ln\left(|x+1|\right) - \frac{1}{4}\ln\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/(x^4 - 1) + sqrt(a*x^6)/((x^4 - 1)*x),x, algorithm="giac")
```

```
[Out] 1/4*(2*arctan(x)*sign(x) - ln(abs(x + 1))*sign(x) + ln(abs(x - 1)
)*sign(x))*sqrt(a) + 1/2*arctan(x) + 1/4*ln(abs(x + 1)) - 1/4*ln(
abs(x - 1))
```

3.214 $\int \left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x-x^5}\right) dx$

Optimal. Leaf size=49

$$\frac{\sqrt{ax^6}\tan^{-1}(x)}{2x^3}-\frac{\sqrt{ax^6}\tanh^{-1}(x)}{2x^3}+\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)$$

```
[Out] ArcTan[x]/2 + (Sqrt[a*x^6]*ArcTan[x])/(2*x^3) + ArcTanh[x]/2 - (S
qrt[a*x^6]*ArcTanh[x])/(2*x^3)
```

Rubi [A] time = 0.0366121, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{\sqrt{ax^6}\tan^{-1}(x)}{2x^3}-\frac{\sqrt{ax^6}\tanh^{-1}(x)}{2x^3}+\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - x^4)^(-1) - Sqrt[a*x^6]/(x - x^5),x]
```

```
[Out] ArcTan[x]/2 + (Sqrt[a*x^6]*ArcTan[x])/(2*x^3) + ArcTanh[x]/2 - (S
qrt[a*x^6]*ArcTanh[x])/(2*x^3)
```

Rubi in Sympy [A] time = 15.2205, size = 42, normalized size = 0.86

$$\frac{\operatorname{atan}(x)}{2}+\frac{\operatorname{atanh}(x)}{2}+\frac{\sqrt{ax^6}\operatorname{atan}(x)}{2x^3}-\frac{\sqrt{ax^6}\operatorname{atanh}(x)}{2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-x**4+1)-(a*x**6)**(1/2)/(-x**5+x),x)
```

```
[Out] atan(x)/2 + atanh(x)/2 + sqrt(a*x**6)*atan(x)/(2*x**3) - sqrt(a*x
**6)*atanh(x)/(2*x**3)
```

Mathematica [A] time = 0.0684988, size = 0, normalized size = 0.

$$\int \left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x-x^5}\right) dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(1 - x^4)^(-1) - Sqrt[a*x^6]/(x - x^5),x]
```

```
[Out] Integrate[(1 - x^4)^(-1) - Sqrt[a*x^6]/(x - x^5), x]
```

Maple [A] time = 0.004, size = 37, normalized size = 0.8

$$\frac{Artanh(x)}{2} + \frac{\arctan(x)}{2} + \frac{\ln(-1+x) - \ln(1+x) + 2\arctan(x)}{4x^3}\sqrt{ax^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-x^4+1)-(a*x^6)^(1/2)/(-x^5+x),x)
```

```
[Out] 1/2*arctanh(x)+1/2*arctan(x)+1/4*(a*x^6)^(1/2)*(ln(-1+x)-ln(1+x)+
2*arctan(x))/x^3
```

Maxima [A] time = 0.775652, size = 57, normalized size = 1.16

$$\frac{1}{2}\sqrt{a}\arctan(x) - \frac{1}{4}\sqrt{a}\log(x+1) + \frac{1}{4}\sqrt{a}\log(x-1) + \frac{1}{2}\arctan(x) + \frac{1}{4}\log(x+1) - \frac{1}{4}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a*x^6)/(x^5 - x) - 1/(x^4 - 1),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(a)*arctan(x) - 1/4*sqrt(a)*log(x + 1) + 1/4*sqrt(a)*log(
x - 1) + 1/2*arctan(x) + 1/4*log(x + 1) - 1/4*log(x - 1)
```

Fricas [A] time = 0.291491, size = 1, normalized size = 0.02

$$\left[\frac{x^3\sqrt{-\frac{(a+1)x^3+2\sqrt{ax^6}}{x^3}}\log\left(\frac{(a-1)x^4-(a-1)x^2-2\left(x^3-\sqrt{ax^6}\right)\sqrt{-\frac{(a+1)x^3+2\sqrt{ax^6}}{x^3}}}{x^4+x^2}\right) + x^3\log(x+1) - x^3\log(x-1) - \sqrt{ax^6}(\log(x+1) - \log}{4x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(a*x^6)/(x^5 - x) - 1/(x^4 - 1),x, algorithm="fricas")
```

```
[Out] [1/4*(x^3*sqrt(-((a + 1)*x^3 + 2*sqrt(a*x^6))/x^3)*log(((a - 1)*x
^4 - (a - 1)*x^2 - 2*(x^3 - sqrt(a*x^6))*sqrt(-((a + 1)*x^3 + 2*s
qrt(a*x^6))/x^3))/(x^4 + x^2)) + x^3*log(x + 1) - x^3*log(x - 1)
- sqrt(a*x^6)*(log(x + 1) - log(x - 1)))/x^3, 1/4*(2*x^3*sqrt(((a
 + 1)*x^3 + 2*sqrt(a*x^6))/x^3)*arctan(-(a - 1)*x^4/((x^3 - sqrt(
a*x^6))*sqrt(((a + 1)*x^3 + 2*sqrt(a*x^6))/x^3))) + x^3*log(x + 1
) - x^3*log(x - 1) - sqrt(a*x^6)*(log(x + 1) - log(x - 1)))/x^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{x^5 - x}\,dx - \int \left(-\frac{\sqrt{ax^6}}{x^5 - x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**4+1)-(a*x**6)**(1/2)/(-x**5+x),x)
```

```
[Out] -Integral(x/(x**5 - x), x) - Integral(-sqrt(a*x**6)/(x**5 - x), x
)
```

GIAC/XCAS [A] time = 0.261921, size = 65, normalized size = 1.33

$$\frac{1}{4}\,(2\,\arctan\left(x\right)\operatorname{sign}\left(x\right) - \ln\left(|x+1|\right)\operatorname{sign}\left(x\right) + \ln\left(|x-1|\right)\operatorname{sign}\left(x\right))\sqrt{a} + \frac{1}{2}\,\arctan\left(x\right) + \frac{1}{4}\,\ln\left(|x+1|\right) - \frac{1}{4}\,\ln\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(a*x^6)/(x^5 - x) - 1/(x^4 - 1),x, algorithm="giac")
```

```
[Out] 1/4*(2*arctan(x)*sign(x) - ln(abs(x + 1))*sign(x) + ln(abs(x - 1)
)*sign(x))*sqrt(a) + 1/2*arctan(x) + 1/4*ln(abs(x + 1)) - 1/4*ln(
abs(x - 1))
```

3.215 $\int \frac{\sqrt{ax^3}}{x-x^3}\,dx$

Optimal. Leaf size=44

$$\frac{\sqrt{ax^3}\tanh^{-1}\left(\sqrt{x}\right)}{x^{3/2}}-\frac{\sqrt{ax^3}\tan^{-1}\left(\sqrt{x}\right)}{x^{3/2}}$$

```
[Out] -((Sqrt[a*x^3]*ArcTan[Sqrt[x]])/x^(3/2)) + (Sqrt[a*x^3]*ArcTanh[S
qrt[x]])/x^(3/2)
```

Rubi [A] time = 0.0327554, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316

$$\frac{\sqrt{ax^3}\tanh^{-1}\left(\sqrt{x}\right)}{x^{3/2}}-\frac{\sqrt{ax^3}\tan^{-1}\left(\sqrt{x}\right)}{x^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a*x^3]/(x - x^3),x]
```

```
[Out] -((Sqrt[a*x^3]*ArcTan[Sqrt[x]])/x^(3/2)) + (Sqrt[a*x^3]*ArcTanh[S
qrt[x]])/x^(3/2)
```

Rubi in Sympy [A] time = 18.3921, size = 39, normalized size = 0.89

$$-\frac{\sqrt{ax^3}\,\mathrm{atan}\left(\sqrt{x}\right)}{x^{\frac{3}{2}}}+\frac{\sqrt{ax^3}\,\mathrm{atanh}\left(\sqrt{x}\right)}{x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a*x**3)**(1/2)/(-x**3+x),x)
```

```
[Out] -sqrt(a*x**3)*atan(sqrt(x))/x**(3/2) + sqrt(a*x**3)*atanh(sqrt(x)
)/x**(3/2)
```

Mathematica [A] time = 0.0327653, size = 47, normalized size = 1.07

$$-\frac{\sqrt{ax^3}\left(\log\left(1-\sqrt{x}\right)-\log\left(\sqrt{x}+1\right)+2\tan^{-1}\left(\sqrt{x}\right)\right)}{2x^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a*x^3]/(x - x^3),x]
```

```
[Out] -(Sqrt[a*x^3]*(2*ArcTan[Sqrt[x]] + Log[1 - Sqrt[x]] - Log[1 + Sqr
t[x]]))/(2*x^(3/2))
```

Maple [A] time = 0.019, size = 43, normalized size = 1.

$$\frac{1}{x}\sqrt{ax^3}\sqrt{a}\left(Artanh\left(1\sqrt{ax}\frac{1}{\sqrt{a}}\right)-\arctan\left(1\sqrt{ax}\frac{1}{\sqrt{a}}\right)\right)\frac{1}{\sqrt{ax}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a*x^3)^(1/2)/(-x^3+x),x)
```

```
[Out] (a*x^3)^(1/2)*a^(1/2)*(arctanh((a*x)^(1/2)/a^(1/2))-arctan((a*x)^
(1/2)/a^(1/2)))/x/(a*x)^(1/2)
```

Maxima [A] time = 0.79082, size = 43, normalized size = 0.98

$$-\sqrt{a}\arctan\left(\sqrt{x}\right)+\frac{1}{2}\sqrt{a}\log\left(\sqrt{x}+1\right)-\frac{1}{2}\sqrt{a}\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sqrt(a*x^3)/(x^3 - x),x, algorithm="maxima")
```

```
[Out] -sqrt(a)*arctan(sqrt(x)) + 1/2*sqrt(a)*log(sqrt(x) + 1) - 1/2*sqr
t(a)*log(sqrt(x) - 1)
```

Fricas [A] time = 0.288693, size = 1, normalized size = 0.02

$$\left[-\sqrt{a}\arctan\left(\frac{\sqrt{ax^3}}{\sqrt{ax}}\right)+\frac{1}{2}\sqrt{a}\log\left(\frac{ax^2+ax+2\sqrt{ax^3}\sqrt{a}}{x^2-x}\right),\sqrt{-a}\arctan\left(\frac{\sqrt{ax^3}}{\sqrt{-ax}}\right)+\frac{1}{2}\sqrt{-a}\log\left(\frac{ax^2-ax-2\sqrt{ax^3}\sqrt{-a}}{x^2+x}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sqrt(a*x^3)/(x^3 - x),x, algorithm="fricas")
```

```
[Out] [-sqrt(a)*arctan(sqrt(a*x^3)/(sqrt(a)*x)) + 1/2*sqrt(a)*log((a*x^
2 + a*x + 2*sqrt(a*x^3)*sqrt(a))/(x^2 - x)), sqrt(-a)*arctan(sqrt
(a*x^3)/(sqrt(-a)*x)) + 1/2*sqrt(-a)*log((a*x^2 - a*x - 2*sqrt(a*
x^3)*sqrt(-a))/(x^2 + x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{ax^3}}{x^3 - x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a*x**3)**(1/2)/(-x**3+x),x)
```

```
[Out] -Integral(sqrt(a*x**3)/(x**3 - x), x)
```

GIAC/XCAS [A] time = 0.265323, size = 51, normalized size = 1.16

$$-\left(\frac{a\arctan\left(\frac{\sqrt{ax}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \sqrt{a}\arctan\left(\frac{\sqrt{ax}}{\sqrt{a}}\right)\right)\mathrm{sign}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sqrt(a*x^3)/(x^3 - x),x, algorithm="giac")
```

```
[Out] -(a*arctan(sqrt(a*x)/sqrt(-a))/sqrt(-a) + sqrt(a)*arctan(sqrt(a*x
)/sqrt(a)))*sign(x)
```

3.216 $\int \frac{\sqrt{ax^4}}{\sqrt{1+x^2}} \, dx$

Optimal. Leaf size=44

$$\frac{\sqrt{x^2+1}\sqrt{ax^4}}{2x} - \frac{\sqrt{ax^4}\sinh^{-1}(x)}{2x^2}$$

```
[Out] (Sqrt[a*x^4]*Sqrt[1 + x^2])/(2*x) - (Sqrt[a*x^4]*ArcSinh[x])/(2*x
^2)
```

Rubi [A] time = 0.0230602, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$

$$\frac{\sqrt{x^2+1}\sqrt{ax^4}}{2x} - \frac{\sqrt{ax^4}\sinh^{-1}(x)}{2x^2}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a*x^4]/Sqrt[1 + x^2],x]
```

```
[Out] (Sqrt[a*x^4]*Sqrt[1 + x^2])/(2*x) - (Sqrt[a*x^4]*ArcSinh[x])/(2*x
^2)
```

Rubi in Sympy [A] time = 8.76722, size = 36, normalized size = 0.82

$$\frac{\sqrt{ax^4}\sqrt{x^2+1}}{2x} - \frac{\sqrt{ax^4}\,\text{asinh}\,(x)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a*x**4)**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] sqrt(a*x**4)*sqrt(x**2 + 1)/(2*x) - sqrt(a*x**4)*asinh(x)/(2*x**2
)
```

Mathematica [A] time = 0.017481, size = 32, normalized size = 0.73

$$\frac{\sqrt{ax^4}\left(x\sqrt{x^2+1} - \sinh^{-1}(x)\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a*x^4]/Sqrt[1 + x^2],x]
```

```
[Out] (Sqrt[a*x^4]*(x*Sqrt[1 + x^2] - ArcSinh[x]))/(2*x^2)
```

Maple [A] time = 0.01, size = 26, normalized size = 0.6

$$-\frac{1}{2\,x^2}\sqrt{ax^4}\left(-x\sqrt{x^2+1}+Arcsinh\,(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a*x^4)^(1/2)/(x^2+1)^(1/2),x)
```

```
[Out] -1/2*(a*x^4)^(1/2)*(-x*(x^2+1)^(1/2)+arcsinh(x))/x^2
```

Maxima [A] time = 0.786085, size = 26, normalized size = 0.59

$$\frac{1}{2}\left(\sqrt{x^2+1}x-\operatorname{arsinh}(x)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a*x^4)/sqrt(x^2 + 1),x, algorithm="maxima")
```

```
[Out] 1/2*(sqrt(x^2 + 1)*x - arcsinh(x))*sqrt(a)
```

Fricas [A] time = 0.299278, size = 177, normalized size = 4.02

$$\frac{\sqrt{ax^4}\left(8\,x^4+8\,x^2-4\left(2\,x^3+x\right)\sqrt{x^2+1}+1\right)\log\left(-x+\sqrt{x^2+1}\right)-\left(8\,x^6+12\,x^4+4\,x^2-\left(8\,x^5+8\,x^3+x\right)\sqrt{x^2+1}\right)\sqrt{ax^4}}{2\left(8\,x^6+8\,x^4+x^2-4\,(2\,x^5+x^3)\sqrt{x^2+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a*x^4)/sqrt(x^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(a*x^4)*(8*x^4 + 8*x^2 - 4*(2*x^3 + x)*sqrt(x^2 + 1) + 1
)*log(-x + sqrt(x^2 + 1)) - (8*x^6 + 12*x^4 + 4*x^2 - (8*x^5 + 8*
```

```
x^3 + x)*sqrt(x^2 + 1))*sqrt(a*x^4))/(8*x^6 + 8*x^4 + x^2 - 4*(2*
x^5 + x^3)*sqrt(x^2 + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax^4}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a*x**4)**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a*x**4)/sqrt(x**2 + 1), x)
```

GIAC/XCAS [A] time = 0.260717, size = 36, normalized size = 0.82

$$\frac{1}{2}\left(\sqrt{x^2+1}x+\ln\left(-x+\sqrt{x^2+1}\right)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a*x^4)/sqrt(x^2 + 1),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(x^2 + 1)*x + ln(-x + sqrt(x^2 + 1)))*sqrt(a)
```

3.217 $\int \frac{\sqrt{ax^3}}{\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=83

$$\frac{2\sqrt{x^2+1}\sqrt{ax^3}}{3x}-\frac{(x+1)\sqrt{\frac{x^2+1}{(x+1)^2}}\sqrt{ax^3}F\left(2\tan^{-1}\left(\sqrt{x}\right)\mid\frac{1}{2}\right)}{3x^{3/2}\sqrt{x^2+1}}$$

```
[Out] (2*Sqrt[a*x^3]*Sqrt[1 + x^2])/(3*x) - (Sqrt[a*x^3]*(1 + x)*Sqrt[(
1 + x^2)/(1 + x)^2]*EllipticF[2*ArcTan[Sqrt[x]], 1/2])/(3*x^(3/2)
*Sqrt[1 + x^2])
```

Rubi [A] time = 0.0605491, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21

$$\frac{2\sqrt{x^2+1}\sqrt{ax^3}}{3x}-\frac{(x+1)\sqrt{\frac{x^2+1}{(x+1)^2}}\sqrt{ax^3}F\left(2\tan^{-1}\left(\sqrt{x}\right)\mid\frac{1}{2}\right)}{3x^{3/2}\sqrt{x^2+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a*x^3]/Sqrt[1 + x^2],x]
```

```
[Out] (2*Sqrt[a*x^3]*Sqrt[1 + x^2])/(3*x) - (Sqrt[a*x^3]*(1 + x)*Sqrt[(
1 + x^2)/(1 + x)^2]*EllipticF[2*ArcTan[Sqrt[x]], 1/2])/(3*x^(3/2)
*Sqrt[1 + x^2])
```

Rubi in Sympy [A] time = 10.7222, size = 73, normalized size = 0.88

$$\frac{2\sqrt{ax^3}\sqrt{x^2+1}}{3x}-\frac{\sqrt{ax^3}\sqrt{\frac{x^2+1}{(x+1)^2}}(x+1)F\left(2\operatorname{atan}\left(\sqrt{x}\right)\middle|\frac{1}{2}\right)}{3x^{\frac{3}{2}}\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a*x**3)**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] 2*sqrt(a*x**3)*sqrt(x**2 + 1)/(3*x) - sqrt(a*x**3)*sqrt((x**2 + 1
)/(x + 1)**2)*(x + 1)*elliptic_f(2*atan(sqrt(x)), 1/2)/(3*x**(3/2
)*sqrt(x**2 + 1))
```

Mathematica [C] time = 0.0690683, size = 77, normalized size = 0.93

$$\frac{2\sqrt{x^2+1}\sqrt{ax^3}\left(\sqrt{\frac{1}{x^2}+1}x^{3/2}-\sqrt[4]{-1}F\left(i\sinh^{-1}\left(\frac{\sqrt[4]{-1}}{\sqrt{x}}\right)\middle|-1\right)\right)}{3\sqrt{\frac{1}{x^2}+1}x^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a*x^3]/Sqrt[1 + x^2],x]
```

```
[Out] (2*Sqrt[a*x^3]*Sqrt[1 + x^2]*(Sqrt[1 + x^(-2)]*x^(3/2) - (-1)^(1/
4)*EllipticF[I*ArcSinh[(-1)^(1/4)/Sqrt[x]], -1]))/(3*Sqrt[1 + x^(
-2)]*x^(5/2))
```

Maple [C] time = 0.031, size = 76, normalized size = 0.9

$$-\frac{1}{3\,x^2}\sqrt{ax^3}\left(i\sqrt{-i\,(x+i)}\sqrt{-i\,(-x+i)}\sqrt{ix}EllipticF\left(\sqrt{-i\,(x+i)},\frac{\sqrt{2}}{2}\right)\sqrt{2}-2\,x^3-2\,x\right)\frac{1}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a*x^3)^(1/2)/(x^2+1)^(1/2),x)
```

```
[Out] -1/3*(a*x^3)^(1/2)/x^2/(x^2+1)^(1/2)*(I*(-I*(x+I))^(1/2)*(-I*(-x+
I))^(1/2)*(I*x)^(1/2)*EllipticF((-I*(x+I))^(1/2),1/2*2^(1/2))*2^(
1/2)-2*x^3-2*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{ax^3}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a*x^3)/sqrt(x^2 + 1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x^3)/sqrt(x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ax^3}}{\sqrt{x^2+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a*x^3)/sqrt(x^2 + 1),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*x^3)/sqrt(x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax^3}}{\sqrt{x^2+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a*x**3)**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a*x**3)/sqrt(x**2 + 1), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax^3}}{\sqrt{x^2+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a*x^3)/sqrt(x^2 + 1),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x^3)/sqrt(x^2 + 1), x)
```

3.218 $\int \frac{\sqrt{ax^2}}{\sqrt{1+x^2}} \, dx$

Optimal. Leaf size=22

$$\frac{\sqrt{x^2+1}\sqrt{ax^2}}{x}$$

```
[Out] (Sqrt[a*x^2]*Sqrt[1 + x^2])/x
```

Rubi [A] time = 0.00918767, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{\sqrt{x^2+1}\sqrt{ax^2}}{x}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a*x^2]/Sqrt[1 + x^2],x]

[Out] (Sqrt[a*x^2]*Sqrt[1 + x^2])/x
```

Rubi in Sympy [A] time = 7.45317, size = 17, normalized size = 0.77

$$\frac{\sqrt{ax^2}\sqrt{x^2+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a*x**2)**(1/2)/(x**2+1)**(1/2),x)

[Out] sqrt(a*x**2)*sqrt(x**2 + 1)/x
```

Mathematica [A] time = 0.0079487, size = 22, normalized size = 1.

$$\frac{\sqrt{x^2+1}\sqrt{ax^2}}{x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a*x^2]/Sqrt[1 + x^2],x]
```

```
[Out] (Sqrt[a*x^2]*Sqrt[1 + x^2])/x
```

Maple [A] time = 0.003, size = 19, normalized size = 0.9

$$\frac{1}{x}\sqrt{a x^2}\sqrt{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a*x^2)^(1/2)/(x^2+1)^(1/2),x)
```

```
[Out] (a*x^2)^(1/2)*(x^2+1)^(1/2)/x
```

Maxima [A] time = 0.768608, size = 26, normalized size = 1.18

$$\frac{\sqrt{a}x^2+\sqrt{a}}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a*x^2)/sqrt(x^2 + 1),x, algorithm="maxima")
```

```
[Out] (sqrt(a)*x^2 + sqrt(a))/sqrt(x^2 + 1)
```

Fricas [A] time = 0.290682, size = 74, normalized size = 3.36

$$-\frac{\sqrt{a x^2}\left(2 x^3-\left(2 x^2+1\right)\sqrt{x^2+1}+2 x\right)}{2 x^3-2 \sqrt{x^2+1}x^2+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a*x^2)/sqrt(x^2 + 1),x, algorithm="fricas")
```

```
[Out] -sqrt(a*x^2)*(2*x^3 - (2*x^2 + 1)*sqrt(x^2 + 1) + 2*x)/(2*x^3 - 2
*sqrt(x^2 + 1)*x^2 + x)
```

Sympy [A] time = 1.11581, size = 20, normalized size = 0.91

$$\frac{\sqrt{a}\sqrt{x^2+1}\sqrt{x^2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a*x**2)**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] sqrt(a)*sqrt(x**2 + 1)*sqrt(x**2)/x
```

GIAC/XCAS [A] time = 0.264582, size = 26, normalized size = 1.18

$$\left(\sqrt{x^2+1}\operatorname{sign}(x) - \operatorname{sign}(x)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a*x^2)/sqrt(x^2 + 1),x, algorithm="giac")
```

```
[Out] (sqrt(x^2 + 1)*sign(x) - sign(x))*sqrt(a)
```

3.219 $\int \frac{\sqrt{ax}}{\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=131

$$\frac{2\sqrt{x^2+1}\sqrt{ax}}{x+1}+\frac{\sqrt{a}(x+1)\sqrt{\frac{x^2+1}{(x+1)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt{ax}}{\sqrt{a}}\right)|\frac{1}{2}\right)}{\sqrt{x^2+1}}-\frac{2\sqrt{a}(x+1)\sqrt{\frac{x^2+1}{(x+1)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt{ax}}{\sqrt{a}}\right)|\frac{1}{2}\right)}{\sqrt{x^2+1}}$$

```
[Out] (2*Sqrt[a*x]*Sqrt[1 + x^2])/(1 + x) - (2*Sqrt[a]*(1 + x)*Sqrt[(1
+ x^2)/(1 + x)^2]*EllipticE[2*ArcTan[Sqrt[a*x]/Sqrt[a]], 1/2])/Sq
rt[1 + x^2] + (Sqrt[a]*(1 + x)*Sqrt[(1 + x^2)/(1 + x)^2]*Elliptic
F[2*ArcTan[Sqrt[a*x]/Sqrt[a]], 1/2])/Sqrt[1 + x^2]
```

Rubi [A] time = 0.181553, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235

$$\frac{2\sqrt{x^2+1}\sqrt{ax}}{x+1}+\frac{\sqrt{a}(x+1)\sqrt{\frac{x^2+1}{(x+1)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt{ax}}{\sqrt{a}}\right)|\frac{1}{2}\right)}{\sqrt{x^2+1}}-\frac{2\sqrt{a}(x+1)\sqrt{\frac{x^2+1}{(x+1)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt{ax}}{\sqrt{a}}\right)|\frac{1}{2}\right)}{\sqrt{x^2+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a*x]/Sqrt[1 + x^2],x]
```

```
[Out] (2*Sqrt[a*x]*Sqrt[1 + x^2])/(1 + x) - (2*Sqrt[a]*(1 + x)*Sqrt[(1
+ x^2)/(1 + x)^2]*EllipticE[2*ArcTan[Sqrt[a*x]/Sqrt[a]], 1/2])/Sq
rt[1 + x^2] + (Sqrt[a]*(1 + x)*Sqrt[(1 + x^2)/(1 + x)^2]*Elliptic
F[2*ArcTan[Sqrt[a*x]/Sqrt[a]], 1/2])/Sqrt[1 + x^2]
```

Rubi in Sympy [A] time = 16.5096, size = 119, normalized size = 0.91

$$-\frac{2\sqrt{a}\sqrt{\frac{x^2+1}{(x+1)^2}}\,(x+1)\,E\left(2\,\mathrm{atan}\left(\frac{\sqrt{ax}}{\sqrt{a}}\right)\middle|\frac{1}{2}\right)}{\sqrt{x^2+1}}+\frac{\sqrt{a}\sqrt{\frac{x^2+1}{(x+1)^2}}\,(x+1)\,F\left(2\,\mathrm{atan}\left(\frac{\sqrt{ax}}{\sqrt{a}}\right)\middle|\frac{1}{2}\right)}{\sqrt{x^2+1}}+\frac{2\sqrt{ax}\sqrt{x^2+1}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a*x)**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] -2*sqrt(a)*sqrt((x**2 + 1)/(x + 1)**2)*(x + 1)*elliptic_e(2*atan(
sqrt(a*x)/sqrt(a)), 1/2)/sqrt(x**2 + 1) + sqrt(a)*sqrt((x**2 + 1)
/(x + 1)**2)*(x + 1)*elliptic_f(2*atan(sqrt(a*x)/sqrt(a)), 1/2)/s
```

```
qrt(x**2 + 1) + 2*sqrt(a*x)*sqrt(x**2 + 1)/(x + 1)
```

Mathematica [C] time = 0.0521742, size = 58, normalized size = 0.44

$$\frac{2(-1)^{3/4}\sqrt{ax}\left(F\left(i\sinh^{-1}\left(\sqrt[4]{-1}\sqrt{x}\right)\middle|-1\right)-E\left(i\sinh^{-1}\left(\sqrt[4]{-1}\sqrt{x}\right)\middle|-1\right)\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a*x]/Sqrt[1 + x^2],x]
```

```
[Out] (2*(-1)^(3/4)*Sqrt[a*x]*(-EllipticE[I*ArcSinh[(-1)^(1/4)*Sqrt[x]]
, -1] + EllipticF[I*ArcSinh[(-1)^(1/4)*Sqrt[x]], -1]))/Sqrt[x]
```

Maple [C] time = 0.024, size = 81, normalized size = 0.6

$$\frac{\sqrt{2}}{x}\sqrt{ax}\sqrt{-i(x+i)}\sqrt{-i(-x+i)}\sqrt{ix}\left(2\,EllipticE\left(\sqrt{-i(x+i)},1/2\sqrt{2}\right)-EllipticF\left(\sqrt{-i(x+i)},\frac{\sqrt{2}}{2}\right)\right)\frac{1}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a*x)^(1/2)/(x^2+1)^(1/2),x)
```

```
[Out] (a*x)^(1/2)/(x^2+1)^(1/2)*(-I*(x+I))^(1/2)*2^(1/2)*(-I*(-x+I))^(1
/2)*(I*x)^(1/2)*(2*EllipticE((-I*(x+I))^(1/2),1/2*2^(1/2))-Ellipt
icF((-I*(x+I))^(1/2),1/2*2^(1/2)))/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a*x)/sqrt(x^2 + 1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x)/sqrt(x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ax}}{\sqrt{x^2+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a*x)/sqrt(x^2 + 1),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*x)/sqrt(x^2 + 1), x)
```

Sympy [A] time = 3.48374, size = 36, normalized size = 0.27

$$\frac{\sqrt{a}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|x^2e^{i\pi}\right)}{2\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a*x)**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] sqrt(a)*x**(3/2)*gamma(3/4)*hyper((1/2, 3/4), (7/4,), x**2*exp_po
lar(I*pi))/(2*gamma(7/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{ax}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a*x)/sqrt(x^2 + 1),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x)/sqrt(x^2 + 1), x)
```

3.220 $\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^2}} \, dx$

Optimal. Leaf size=54

$$\frac{\sqrt{x}(x+1)\sqrt{\frac{x^2+1}{(x+1)^2}}\sqrt{\frac{a}{x}}F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{2}\right)}{\sqrt{x^2+1}}$$

```
[Out] (Sqrt[a/x]*Sqrt[x]*(1 + x)*Sqrt[(1 + x^2)/(1 + x)^2]*EllipticF[2*
ArcTan[Sqrt[x]], 1/2])/Sqrt[1 + x^2]
```

Rubi [A] time = 0.042399, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$\frac{\sqrt{x}(x+1)\sqrt{\frac{x^2+1}{(x+1)^2}}\sqrt{\frac{a}{x}}F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{2}\right)}{\sqrt{x^2+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a/x]/Sqrt[1 + x^2],x]
```

```
[Out] (Sqrt[a/x]*Sqrt[x]*(1 + x)*Sqrt[(1 + x^2)/(1 + x)^2]*EllipticF[2*
ArcTan[Sqrt[x]], 1/2])/Sqrt[1 + x^2]
```

Rubi in Sympy [A] time = 8.72322, size = 48, normalized size = 0.89

$$\frac{\sqrt{x}\sqrt{\frac{a}{x}}\sqrt{\frac{x^2+1}{(x+1)^2}}\,(x+1)\,F\left(2\,\mathrm{atan}\left(\sqrt{x}\right)\middle|\frac{1}{2}\right)}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a/x)**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] sqrt(x)*sqrt(a/x)*sqrt((x**2 + 1)/(x + 1)**2)*(x + 1)*elliptic_f(
2*atan(sqrt(x)), 1/2)/sqrt(x**2 + 1)
```

Mathematica [C] time = 0.02865, size = 57, normalized size = 1.06

$$\frac{2\sqrt[4]{-1}\sqrt{x^2+1}\sqrt{\frac{a}{x}}F\left(i\sinh^{-1}\left(\frac{\sqrt[4]{-1}}{\sqrt{x}}\right)\middle|-1\right)}{\sqrt{\frac{1}{x^2}+1}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a/x]/Sqrt[1 + x^2],x]
```

```
[Out] (2*(-1)^(1/4)*Sqrt[a/x]*Sqrt[1 + x^2]*EllipticF[I*ArcSinh[(-1)^(1
/4)/Sqrt[x]], -1])/(Sqrt[1 + x^(-2)]*Sqrt[x])
```

Maple [C] time = 0.038, size = 62, normalized size = 1.2

$$i\sqrt{2}\sqrt{\frac{a}{x}}\sqrt{-i(x+i)}\sqrt{-i(-x+i)}\sqrt{ix}EllipticF\left(\sqrt{-i(x+i)},\frac{\sqrt{2}}{2}\right)\frac{1}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a/x)^(1/2)/(x^2+1)^(1/2),x)
```

```
[Out] I*(a/x)^(1/2)/(x^2+1)^(1/2)*(-I*(x+I))^(1/2)*2^(1/2)*(-I*(-x+I))^
(1/2)*(I*x)^(1/2)*EllipticF((-I*(x+I))^(1/2),1/2*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a/x)/sqrt(x^2 + 1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a/x)/sqrt(x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\frac{a}{x}}}{\sqrt{x^2+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a/x)/sqrt(x^2 + 1),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a/x)/sqrt(x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a/x)**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a/x)/sqrt(x**2 + 1), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a/x)/sqrt(x^2 + 1),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a/x)/sqrt(x^2 + 1), x)
```

3.221 $\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=22

$$x\left(-\sqrt{\frac{a}{x^2}}\right)\tanh^{-1}\left(\sqrt{x^2+1}\right)$$

```
[Out] -(Sqrt[a/x^2]*x*ArcTanh[Sqrt[1 + x^2]])
```

Rubi [A] time = 0.0249561, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21

$$x\left(-\sqrt{\frac{a}{x^2}}\right)\tanh^{-1}\left(\sqrt{x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a/x^2]/Sqrt[1 + x^2],x]

[Out] -(Sqrt[a/x^2]*x*ArcTanh[Sqrt[1 + x^2]])
```

Rubi in Sympy [A] time = 8.99198, size = 20, normalized size = 0.91

$$-x\sqrt{\frac{a}{x^2}}\,\mathrm{atanh}\left(\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a/x**2)**(1/2)/(x**2+1)**(1/2),x)

[Out] -x*sqrt(a/x**2)*atanh(sqrt(x**2 + 1))
```

Mathematica [A] time = 0.0122486, size = 28, normalized size = 1.27

$$x\sqrt{\frac{a}{x^2}}\left(\log(x)-\log\left(\sqrt{x^2+1}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a/x^2]/Sqrt[1 + x^2],x]
```

```
[Out] Sqrt[a/x^2]*x*(Log[x] - Log[1 + Sqrt[1 + x^2]])
```

Maple [A]   time = 0.008, size = 19, normalized size = 0.9

$$-\sqrt{\frac{a}{x^2}}\,x\,Artanh\left(\frac{1}{\sqrt{x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a/x^2)^(1/2)/(x^2+1)^(1/2),x)
```

```
[Out] -(a/x^2)^(1/2)*x*arctanh(1/(x^2+1)^(1/2))
```

Maxima [A]   time = 0.783211, size = 14, normalized size = 0.64

$$-\sqrt{a}\,\operatorname{arsinh}\left(\frac{1}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a/x^2)/sqrt(x^2 + 1),x, algorithm="maxima")
```

```
[Out] -sqrt(a)*arcsinh(1/abs(x))
```

Fricas [A]   time = 0.280684, size = 1, normalized size = 0.05

$$\left[x\sqrt{\frac{a}{x^2}}\log\left(\frac{x^2-\sqrt{x^2+1}(x+1)+x+1}{x^2-\sqrt{x^2+1}x}\right), -2\sqrt{-a}\arctan\left(-\frac{ax-\sqrt{x^2+1}a}{\sqrt{-a}x\sqrt{\frac{a}{x^2}}}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a/x^2)/sqrt(x^2 + 1),x, algorithm="fricas")
```

```
[Out] [x*sqrt(a/x^2)*log((x^2 - sqrt(x^2 + 1)*(x + 1) + x + 1)/(x^2 - s
qrt(x^2 + 1)*x)), -2*sqrt(-a)*arctan(-(a*x - sqrt(x^2 + 1)*a)/(sq
```

```
rt(-a)*x*sqrt(a/x^2)))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a/x**2)**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a/x**2)/sqrt(x**2 + 1), x)
```

GIAC/XCAS [A] time = 0.266763, size = 41, normalized size = 1.86

$$-\frac{1}{2}\sqrt{a}\left(\ln\left(\sqrt{x^2+1}+1\right)-\ln\left(\sqrt{x^2+1}-1\right)\right)\operatorname{sign}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a/x^2)/sqrt(x^2 + 1),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(a)*(ln(sqrt(x^2 + 1) + 1) - ln(sqrt(x^2 + 1) - 1))*sign
(x)
```

3.222 $\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^2}} \, dx$

Optimal. Leaf size=159

$$\frac{2\sqrt{x^2+1}x^2\sqrt{\frac{a}{x^3}}}{x+1} - 2\sqrt{x^2+1}x\sqrt{\frac{a}{x^3}} + \frac{(x+1)\sqrt{\frac{x^2+1}{(x+1)^2}}x^{3/2}\sqrt{\frac{a}{x^3}}F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{2}\right)}{\sqrt{x^2+1}} - \frac{2(x+1)\sqrt{\frac{x^2+1}{(x+1)^2}}x^{3/2}\sqrt{\frac{a}{x^3}}E\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{2}\right)}{\sqrt{x^2+1}}$$

```
[Out] -2*Sqrt[a/x^3]*x*Sqrt[1 + x^2] + (2*Sqrt[a/x^3]*x^2*Sqrt[1 + x^2]
)/(1 + x) - (2*Sqrt[a/x^3]*x^(3/2)*(1 + x)*Sqrt[(1 + x^2)/(1 + x)
^2]*EllipticE[2*ArcTan[Sqrt[x]], 1/2])/Sqrt[1 + x^2] + (Sqrt[a/x^
3]*x^(3/2)*(1 + x)*Sqrt[(1 + x^2)/(1 + x)^2]*EllipticF[2*ArcTan[S
qrt[x]], 1/2])/Sqrt[1 + x^2]
```

Rubi [A] time = 0.120332, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$

$$\frac{2\sqrt{x^2+1}x^2\sqrt{\frac{a}{x^3}}}{x+1} - 2\sqrt{x^2+1}x\sqrt{\frac{a}{x^3}} + \frac{(x+1)\sqrt{\frac{x^2+1}{(x+1)^2}}x^{3/2}\sqrt{\frac{a}{x^3}}F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{2}\right)}{\sqrt{x^2+1}} - \frac{2(x+1)\sqrt{\frac{x^2+1}{(x+1)^2}}x^{3/2}\sqrt{\frac{a}{x^3}}E\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{2}\right)}{\sqrt{x^2+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a/x^3]/Sqrt[1 + x^2],x]
```

```
[Out] -2*Sqrt[a/x^3]*x*Sqrt[1 + x^2] + (2*Sqrt[a/x^3]*x^2*Sqrt[1 + x^2]
)/(1 + x) - (2*Sqrt[a/x^3]*x^(3/2)*(1 + x)*Sqrt[(1 + x^2)/(1 + x)
^2]*EllipticE[2*ArcTan[Sqrt[x]], 1/2])/Sqrt[1 + x^2] + (Sqrt[a/x^
3]*x^(3/2)*(1 + x)*Sqrt[(1 + x^2)/(1 + x)^2]*EllipticF[2*ArcTan[S
qrt[x]], 1/2])/Sqrt[1 + x^2]
```

Rubi in Sympy [A] time = 15.9624, size = 148, normalized size = 0.93

$$-\frac{2x^{\frac{3}{2}}\sqrt{\frac{a}{x^3}}\sqrt{\frac{x^2+1}{(x+1)^2}}\,(x+1)\,E\left(2\operatorname{atan}\left(\sqrt{x}\right)\middle|\frac{1}{2}\right)}{\sqrt{x^2+1}}+\frac{x^{\frac{3}{2}}\sqrt{\frac{a}{x^3}}\sqrt{\frac{x^2+1}{(x+1)^2}}\,(x+1)\,F\left(2\operatorname{atan}\left(\sqrt{x}\right)\middle|\frac{1}{2}\right)}{\sqrt{x^2+1}}+\frac{2x^2\sqrt{\frac{a}{x^3}}\sqrt{x^2+1}}{x+1}-2x\sqrt{\frac{a}{x^3}}\sqrt{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a/x**3)**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] -2*x**(3/2)*sqrt(a/x**3)*sqrt((x**2 + 1)/(x + 1)**2)*(x + 1)*elli
ptic_e(2*atan(sqrt(x)), 1/2)/sqrt(x**2 + 1) + x**(3/2)*sqrt(a/x**
3)*sqrt((x**2 + 1)/(x + 1)**2)*(x + 1)*elliptic_f(2*atan(sqrt(x))
, 1/2)/sqrt(x**2 + 1) + 2*x**2*sqrt(a/x**3)*sqrt(x**2 + 1)/(x + 1
) - 2*x*sqrt(a/x**3)*sqrt(x**2 + 1)
```

Mathematica [C] time = 0.0337988, size = 74, normalized size = 0.47

$$2x\sqrt{\frac{a}{x^3}}\left(-\sqrt{x^2+1}+(-1)^{3/4}\sqrt{x}\left(F\left(i\sinh^{-1}\left(\sqrt[4]{-1}\sqrt{x}\right)\middle|-1\right)-E\left(i\sinh^{-1}\left(\sqrt[4]{-1}\sqrt{x}\right)\middle|-1\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a/x^3]/Sqrt[1 + x^2],x]
```

```
[Out] 2*Sqrt[a/x^3]*x*(-Sqrt[1 + x^2] + (-1)^(3/4)*Sqrt[x]*(-EllipticE[
I*ArcSinh[(-1)^(1/4)*Sqrt[x]], -1] + EllipticF[I*ArcSinh[(-1)^(1/
4)*Sqrt[x]], -1]))
```

Maple [C] time = 0.044, size = 116, normalized size = 0.7

$$x\sqrt{\frac{a}{x^3}}\left(2\sqrt{-i(x+i)}\sqrt{-i(-x+i)}\sqrt{ix}EllipticE\left(\sqrt{-i(x+i)},1/2\sqrt{2}\right)\sqrt{2}-\sqrt{-i(x+i)}\sqrt{-i(-x+i)}\sqrt{ix}EllipticF\left(\sqrt{-i(x+i)},\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a/x^3)^(1/2)/(x^2+1)^(1/2),x)
```

```
[Out] (a/x^3)^(1/2)*x*(2*(-I*(x+I))^(1/2)*(-I*(-x+I))^(1/2)*(I*x)^(1/2)
*EllipticE((-I*(x+I))^(1/2),1/2*2^(1/2))*2^(1/2)-(-I*(x+I))^(1/2)
*(-I*(-x+I))^(1/2)*(I*x)^(1/2)*EllipticF((-I*(x+I))^(1/2),1/2*2^(
1/2))*2^(1/2)-2*x^2-2)/(x^2+1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(a/x^3)/sqrt(x^2 + 1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a/x^3)/sqrt(x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\frac{a}{x^3}}}{\sqrt{x^2+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(a/x^3)/sqrt(x^2 + 1),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a/x^3)/sqrt(x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x**3)**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a/x**3)/sqrt(x**2 + 1), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{x^2 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a/x^3)/sqrt(x^2 + 1),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a/x^3)/sqrt(x^2 + 1), x)
```

3.223 $\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=21

$$x\sqrt{x^2+1}\left(-\sqrt{\frac{a}{x^4}}\right)$$

```
[Out] -(Sqrt[a/x^4]*x*Sqrt[1 + x^2])
```

Rubi [A] time = 0.01534, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$x\sqrt{x^2+1}\left(-\sqrt{\frac{a}{x^4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a/x^4]/Sqrt[1 + x^2],x]
```

```
[Out] -(Sqrt[a/x^4]*x*Sqrt[1 + x^2])
```

Rubi in Sympy [A] time = 7.87076, size = 19, normalized size = 0.9

$$-x\sqrt{\frac{a}{x^4}}\sqrt{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((a/x**4)**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] -x*sqrt(a/x**4)*sqrt(x**2 + 1)
```

Mathematica [A] time = 0.00980076, size = 21, normalized size = 1.

$$x\sqrt{x^2+1}\left(-\sqrt{\frac{a}{x^4}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a/x^4]/Sqrt[1 + x^2],x]
```

```
[Out] -(Sqrt[a/x^4]*x*Sqrt[1 + x^2])
```

Maple [A] time = 0.006, size = 18, normalized size = 0.9

$$-x\sqrt{\frac{a}{x^4}}\sqrt{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a/x^4)^(1/2)/(x^2+1)^(1/2),x)
```

```
[Out] -x*(a/x^4)^(1/2)*(x^2+1)^(1/2)
```

Maxima [A] time = 0.775118, size = 31, normalized size = 1.48

$$-\frac{\sqrt{a}x^2+\sqrt{a}}{\sqrt{x^2+1}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a/x^4)/sqrt(x^2 + 1),x, algorithm="maxima")
```

```
[Out] -(sqrt(a)*x^2 + sqrt(a))/(sqrt(x^2 + 1)*x)
```

Fricas [A] time = 0.289448, size = 66, normalized size = 3.14

$$\frac{x^2\sqrt{\frac{a}{x^4}}-\sqrt{x^2+1}x\sqrt{\frac{a}{x^4}}}{2x^2-2\sqrt{x^2+1}x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a/x^4)/sqrt(x^2 + 1),x, algorithm="fricas")
```

```
[Out] (x^2*sqrt(a/x^4) - sqrt(x^2 + 1)*x*sqrt(a/x^4))/(2*x^2 - 2*sqrt(x
^2 + 1)*x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a/x**4)**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a/x**4)/sqrt(x**2 + 1), x)
```

GIAC/XCAS [A] time = 0.262143, size = 30, normalized size = 1.43

$$\frac{2\sqrt{a}}{\left(x-\sqrt{x^2+1}\right)^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a/x^4)/sqrt(x^2 + 1),x, algorithm="giac")
```

```
[Out] 2*sqrt(a)/((x - sqrt(x^2 + 1))^2 - 1)
```

3.224 $\int \frac{\sqrt{ax^4}}{\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=25

$$\frac{2\sqrt{x^3+1}\sqrt{ax^4}}{3x^2}$$

```
[Out] (2*Sqrt[a*x^4]*Sqrt[1 + x^3])/(3*x^2)
```

Rubi [A] time = 0.00999083, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$\frac{2\sqrt{x^3+1}\sqrt{ax^4}}{3x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^4]/Sqrt[1 + x^3],x]

[Out] (2*Sqrt[a*x^4]*Sqrt[1 + x^3])/(3*x^2)
```

Rubi in Sympy [A] time = 6.52583, size = 22, normalized size = 0.88

$$\frac{2\sqrt{ax^4}\sqrt{x^3+1}}{3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((a*x**4)**(1/2)/(x**3+1)**(1/2),x)

[Out] 2*sqrt(a*x**4)*sqrt(x**3 + 1)/(3*x**2)
```

Mathematica [A] time = 0.0087445, size = 25, normalized size = 1.

$$\frac{2\sqrt{x^3+1}\sqrt{ax^4}}{3x^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a*x^4]/Sqrt[1 + x^3],x]

[Out] (2*Sqrt[a*x^4]*Sqrt[1 + x^3])/(3*x^2)
```

Maple [A] time = 0.007, size = 31, normalized size = 1.2

$$\frac{(2+2x)(x^2-x+1)}{3x^2}\sqrt{ax^4}\frac{1}{\sqrt{x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a*x^4)^(1/2)/(x^3+1)^(1/2),x)

[Out] 2/3*(1+x)*(x^2-x+1)/x^2*(a*x^4)^(1/2)/(x^3+1)^(1/2)
```

Maxima [A] time = 0.794808, size = 38, normalized size = 1.52

$$\frac{2\left(\sqrt{a}x^3+\sqrt{a}\right)}{3\sqrt{x^2-x+1}\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a*x^4)/sqrt(x^3 + 1),x, algorithm="maxima")

[Out] 2/3*(sqrt(a)*x^3 + sqrt(a))/(sqrt(x^2 - x + 1)*sqrt(x + 1))
```

Fricas [A] time = 0.274466, size = 26, normalized size = 1.04

$$\frac{2\sqrt{ax^4}\sqrt{x^3+1}}{3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a*x^4)/sqrt(x^3 + 1),x, algorithm="fricas")

[Out] 2/3*sqrt(a*x^4)*sqrt(x^3 + 1)/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax^4}}{\sqrt{(x+1)(x^2-x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a*x**4)**(1/2)/(x**3+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a*x**4)/sqrt((x + 1)*(x**2 - x + 1)), x)
```

GIAC/XCAS [A] time = 0.26073, size = 16, normalized size = 0.64

$$\frac{2}{3}\sqrt{x^3+1}\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a*x^4)/sqrt(x^3 + 1),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(x^3 + 1)*sqrt(a)
```

3.225 $\int \frac{\sqrt{ax^3}}{\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=292

$$\frac{\left(1+\sqrt{3}\right)\sqrt{x^3+1}\sqrt{ax^3}}{x\left(\left(1+\sqrt{3}\right)x+1\right)}-\frac{\left(1-\sqrt{3}\right)(x+1)\sqrt{\frac{x^2-x+1}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\sqrt{ax^3}F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)x+1}{\left(1+\sqrt{3}\right)x+1}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2\sqrt[4]{3}x\sqrt{\frac{x(x+1)}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\sqrt{ax^3}E\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)x+1}{\left(1+\sqrt{3}\right)x+1}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{x\sqrt{\frac{x(x+1)}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\sqrt{x^3+1}}$$

```
[Out] ((1 + Sqrt[3])*Sqrt[a*x^3]*Sqrt[1 + x^3])/(x*(1 + (1 + Sqrt[3])*x
)) - (3^(1/4)*Sqrt[a*x^3]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + (1 + Sq
rt[3])*x)^2]*EllipticE[ArcCos[(1 + (1 - Sqrt[3])*x)/(1 + (1 + Sqr
t[3])*x)], (2 + Sqrt[3])/4])/(x*Sqrt[(x*(1 + x))/(1 + (1 + Sqrt[3
])*x)^2]*Sqrt[1 + x^3]) - ((1 - Sqrt[3])*Sqrt[a*x^3]*(1 + x)*Sqrt
[(1 - x + x^2)/(1 + (1 + Sqrt[3])*x)^2]*EllipticF[ArcCos[(1 + (1
- Sqrt[3])*x)/(1 + (1 + Sqrt[3])*x)], (2 + Sqrt[3])/4])/(2*3^(1/4
)*x*Sqrt[(x*(1 + x))/(1 + (1 + Sqrt[3])*x)^2]*Sqrt[1 + x^3])
```

Rubi [A] time = 0.424752, antiderivative size = 292, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263

$$\frac{\left(1+\sqrt{3}\right)\sqrt{x^3+1}\sqrt{ax^3}}{x\left(\left(1+\sqrt{3}\right)x+1\right)}-\frac{\left(1-\sqrt{3}\right)(x+1)\sqrt{\frac{x^2-x+1}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\sqrt{ax^3}F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)x+1}{\left(1+\sqrt{3}\right)x+1}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2\sqrt[4]{3}x\sqrt{\frac{x(x+1)}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\sqrt{ax^3}E\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)x+1}{\left(1+\sqrt{3}\right)x+1}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{x\sqrt{\frac{x(x+1)}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a*x^3]/Sqrt[1 + x^3],x]
```


```
[Out] ((1 + Sqrt[3])*Sqrt[a*x^3]*Sqrt[1 + x^3])/(x*(1 + (1 + Sqrt[3])*x
)) - (3^(1/4)*Sqrt[a*x^3]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + (1 + Sq
```

```
rt[3])*x)^2]*EllipticE[ArcCos[(1 + (1 - Sqrt[3])*x)/(1 + (1 + Sqr
t[3])*x)], (2 + Sqrt[3])/4])/(x*Sqrt[(x*(1 + x))/(1 + (1 + Sqrt[3
])*x)^2]*Sqrt[1 + x^3]) - ((1 - Sqrt[3])*Sqrt[a*x^3]*(1 + x)*Sqrt
[(1 - x + x^2)/(1 + (1 + Sqrt[3])*x)^2]*EllipticF[ArcCos[(1 + (1
- Sqrt[3])*x)/(1 + (1 + Sqrt[3])*x)], (2 + Sqrt[3])/4])/(2*3^(1/4
)*x*Sqrt[(x*(1 + x))/(1 + (1 + Sqrt[3])*x)^2]*Sqrt[1 + x^3])
```

Rubi in Sympy [A] time = 19.2598, size = 253, normalized size = 0.87

$$-\frac{\sqrt[4]{3}\sqrt{ax^3}\sqrt{\frac{x^2-x+1}{\left(x\left(1+\sqrt{3}\right)+1\right)^2}}(x+1)E\left(\operatorname{acos}\left(\frac{x\left(-\sqrt{3}+1\right)+1}{x\left(1+\sqrt{3}\right)+1}\right)\middle|\frac{\sqrt{3}}{4}+\frac{1}{2}\right)}{x\sqrt{\frac{x(x+1)}{\left(x\left(1+\sqrt{3}\right)+1\right)^2}}\sqrt{x^3+1}} - \frac{3^{\frac{3}{4}}\sqrt{ax^3}\sqrt{\frac{x^2-x+1}{\left(x\left(1+\sqrt{3}\right)+1\right)^2}}\left(-\frac{\sqrt{3}}{2}+\frac{1}{2}\right)(x+1)F\left(\operatorname{acos}\left(\frac{x\left(-\sqrt{3}+1\right)+1}{x\left(1+\sqrt{3}\right)+1}\right)\middle|\frac{\sqrt{3}}{4}+\frac{1}{2}\right)}{3x\sqrt{\frac{x(x+1)}{\left(x\left(1+\sqrt{3}\right)+1\right)^2}}\sqrt{x^3+1}} + \frac{\sqrt{ax^3}\left(2+2\sqrt{3}\right)\sqrt{x^3+1}}{x\left(x\left(2+2\sqrt{3}\right)+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a*x**3)**(1/2)/(x**3+1)**(1/2),x)
```

```
[Out] -3**(1/4)*sqrt(a*x**3)*sqrt((x**2 - x + 1)/(x*(1 + sqrt(3)) + 1)*
*2)*(x + 1)*elliptic_e(acos((x*(-sqrt(3) + 1) + 1)/(x*(1 + sqrt(3
)) + 1)), sqrt(3)/4 + 1/2)/(x*sqrt(x*(x + 1)/(x*(1 + sqrt(3)) + 1
)**2)*sqrt(x**3 + 1)) - 3**(3/4)*sqrt(a*x**3)*sqrt((x**2 - x + 1)
/(x*(1 + sqrt(3)) + 1)**2)*(-sqrt(3)/2 + 1/2)*(x + 1)*elliptic_f(
acos((x*(-sqrt(3) + 1) + 1)/(x*(1 + sqrt(3)) + 1)), sqrt(3)/4 + 1
/2)/(3*x*sqrt(x*(x + 1)/(x*(1 + sqrt(3)) + 1)**2)*sqrt(x**3 + 1))
 + sqrt(a*x**3)*(2 + 2*sqrt(3))*sqrt(x**3 + 1)/(x*(x*(2 + 2*sqrt(
3)) + 2))
```

Mathematica [C] time = 0.570049, size = 174, normalized size = 0.6

$$\frac{ax\left(x^3+\frac{\left(1-(-1)^{2/3}\right)\sqrt{\frac{x-\sqrt[3]{-1}}{\left(1+\sqrt[3]{-1}\right)x}}\sqrt{\frac{(x+1)\left(2x+i\sqrt{3}-1\right)}{x^2}}x^2\left(\left(1+\sqrt[3]{-1}\right)E\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}(x+1)}{\left(-1+(-1)^{2/3}\right)x}}\right)\middle|1+(-1)^{2/3}\right)-F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}(x+1)}{\left(-1+(-1)^{2/3}\right)x}}\right)\middle|1+(-1)^{2/3}\right)\right)}{\sqrt{6}}+1\right)}{\sqrt{x^3+1}\sqrt{ax^3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a*x^3]/Sqrt[1 + x^3],x]
```

```
[Out] (a*x*(1 + x^3 + ((1 - (-1)^(2/3))*x^2*Sqrt[(-(-1)^(1/3) + x)/((1
+ (-1)^(1/3))*x)]*Sqrt[((1 + x)*(-1 + I*Sqrt[3] + 2*x))/x^2]*((1
+ (-1)^(1/3))*EllipticE[ArcSin[Sqrt[((-1)^(2/3)*(1 + x))/((-1 + (
-1)^(2/3))*x)]], 1 + (-1)^(2/3)] - EllipticF[ArcSin[Sqrt[((-1)^(2
/3)*(1 + x))/((-1 + (-1)^(2/3))*x)]], 1 + (-1)^(2/3)]))/Sqrt[6]))
/(Sqrt[a*x^3]*Sqrt[1 + x^3])
```

Maple [C] time = 0.376, size = 1521, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a*x^3)^(1/2)/(x^3+1)^(1/2),x)
```

```
[Out] -2*(a*x^3)^(1/2)/x*(x^3+1)^(1/2)*a*(I*3^(1/2)*((I*3^(1/2)+3)*x/(I
*3^(1/2)+1)/(1+x))^(1/2)*((I*3^(1/2)+2*x-1)/(-1+I*3^(1/2))/(1+x))
^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+1)/(1+x))^(1/2)*EllipticE(((
I*3^(1/2)+3)*x/(I*3^(1/2)+1)/(1+x))^(1/2),((-3+I*3^(1/2))*(I*3^(1
/2)+1)/(-1+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*x^2+2*I*3^(1/2)*((I*3
^(1/2)+3)*x/(I*3^(1/2)+1)/(1+x))^(1/2)*((I*3^(1/2)+2*x-1)/(-1+I*3
^(1/2))/(1+x))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+1)/(1+x))^(1/2
)*EllipticE(((I*3^(1/2)+3)*x/(I*3^(1/2)+1)/(1+x))^(1/2),((-3+I*3^
(1/2))*(I*3^(1/2)+1)/(-1+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*x+I*3^(
1/2)*((I*3^(1/2)+3)*x/(I*3^(1/2)+1)/(1+x))^(1/2)*((I*3^(1/2)+2*x-
1)/(-1+I*3^(1/2))/(1+x))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+1)/(
1+x))^(1/2)*EllipticE(((I*3^(1/2)+3)*x/(I*3^(1/2)+1)/(1+x))^(1/2)
,((-3+I*3^(1/2))*(I*3^(1/2)+1)/(-1+I*3^(1/2))/(I*3^(1/2)+3))^(1/2
))-2*((I*3^(1/2)+3)*x/(I*3^(1/2)+1)/(1+x))^(1/2)*((I*3^(1/2)+2*x-
1)/(-1+I*3^(1/2))/(1+x))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+1)/(
1+x))^(1/2)*EllipticF(((I*3^(1/2)+3)*x/(I*3^(1/2)+1)/(1+x))^(1/2)
,((-3+I*3^(1/2))*(I*3^(1/2)+1)/(-1+I*3^(1/2))/(I*3^(1/2)+3))^(1/2
))*x^2+3*((I*3^(1/2)+3)*x/(I*3^(1/2)+1)/(1+x))^(1/2)*((I*3^(1/2)+
2*x-1)/(-1+I*3^(1/2))/(1+x))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+
1)/(1+x))^(1/2)*EllipticE(((I*3^(1/2)+3)*x/(I*3^(1/2)+1)/(1+x))^(
1/2),((-3+I*3^(1/2))*(I*3^(1/2)+1)/(-1+I*3^(1/2))/(I*3^(1/2)+3))^
(1/2))*x^2-I*3^(1/2)*x^3-4*((I*3^(1/2)+3)*x/(I*3^(1/2)+1)/(1+x))^
(1/2)*((I*3^(1/2)+2*x-1)/(-1+I*3^(1/2))/(1+x))^(1/2)*((I*3^(1/2)-
2*x+1)/(I*3^(1/2)+1)/(1+x))^(1/2)*EllipticF(((I*3^(1/2)+3)*x/(I*3
^(1/2)+1)/(1+x))^(1/2),((-3+I*3^(1/2))*(I*3^(1/2)+1)/(-1+I*3^(1/2
))/(I*3^(1/2)+3))^(1/2))*x+6*((I*3^(1/2)+3)*x/(I*3^(1/2)+1)/(1+x)
)^(1/2)*((I*3^(1/2)+2*x-1)/(-1+I*3^(1/2))/(1+x))^(1/2)*((I*3^(1/2
)-2*x+1)/(I*3^(1/2)+1)/(1+x))^(1/2)*EllipticE(((I*3^(1/2)+3)*x/(I
*3^(1/2)+1)/(1+x))^(1/2),((-3+I*3^(1/2))*(I*3^(1/2)+1)/(-1+I*3^(1
```

```
/2))/(I*3^(1/2)+3))^(1/2))*x+I*3^(1/2)*x^2-2*((I*3^(1/2)+3)*x/(I*
3^(1/2)+1)/(1+x))^(1/2)*((I*3^(1/2)+2*x-1)/(-1+I*3^(1/2))/(1+x))^
(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+1)/(1+x))^(1/2)*EllipticF(((I
*3^(1/2)+3)*x/(I*3^(1/2)+1)/(1+x))^(1/2),((-3+I*3^(1/2))*(I*3^(1/
2)+1)/(-1+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))+3*((I*3^(1/2)+3)*x/(I*
3^(1/2)+1)/(1+x))^(1/2)*((I*3^(1/2)+2*x-1)/(-1+I*3^(1/2))/(1+x))^
(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+1)/(1+x))^(1/2)*EllipticE(((I
*3^(1/2)+3)*x/(I*3^(1/2)+1)/(1+x))^(1/2),((-3+I*3^(1/2))*(I*3^(1/
2)+1)/(-1+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))-I*3^(1/2)*x-3*x^3+3*x^
2-3*x)/(x*(x^3+1)*a)^(1/2)/(I*3^(1/2)+3)/(-a*x*(1+x)*(I*3^(1/2)+2
*x-1)*(I*3^(1/2)-2*x+1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax^3}}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a*x^3)/sqrt(x^3 + 1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x^3)/sqrt(x^3 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ax^3}}{\sqrt{x^3+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a*x^3)/sqrt(x^3 + 1),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*x^3)/sqrt(x^3 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax^3}}{\sqrt{(x+1)(x^2-x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a*x**3)**(1/2)/(x**3+1)**(1/2),x)

[Out] Integral(sqrt(a*x**3)/sqrt((x + 1)*(x**2 - x + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax^3}}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a*x^3)/sqrt(x^3 + 1),x, algorithm="giac")

[Out] integrate(sqrt(a*x^3)/sqrt(x^3 + 1), x)
```

3.226 $\int \frac{\sqrt{ax^2}}{\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=260

$$\frac{2\sqrt{x^3+1}\sqrt{ax^2}}{x\left(x+\sqrt{3}+1\right)}+\frac{2\sqrt{2}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{ax^2}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}x\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{ax^2}E\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{x\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

```
[Out] (2*Sqrt[a*x^2]*Sqrt[1 + x^3])/(x*(1 + Sqrt[3] + x)) - (3^(1/4)*Sq
rt[2 - Sqrt[3]]*Sqrt[a*x^2]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[
3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)],
 -7 - 4*Sqrt[3]])/(x*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x
^3]) + (2*Sqrt[2]*Sqrt[a*x^2]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqr
t[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)
], -7 - 4*Sqrt[3]])/(3^(1/4)*x*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*
Sqrt[1 + x^3])
```

Rubi [A] time = 0.144361, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$\frac{2\sqrt{x^3+1}\sqrt{ax^2}}{x\left(x+\sqrt{3}+1\right)}+\frac{2\sqrt{2}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{ax^2}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}x\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{ax^2}E\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{x\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a*x^2]/Sqrt[1 + x^3],x]
```

```
[Out] (2*Sqrt[a*x^2]*Sqrt[1 + x^3])/(x*(1 + Sqrt[3] + x)) - (3^(1/4)*Sq
rt[2 - Sqrt[3]]*Sqrt[a*x^2]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[
3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)],
 -7 - 4*Sqrt[3]])/(x*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x
^3]) + (2*Sqrt[2]*Sqrt[a*x^2]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqr
```

```
t[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)
], -7 - 4*Sqrt[3]])/(3^(1/4)*x*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*
Sqrt[1 + x^3])
```

Rubi in Sympy [A] time = 15.0113, size = 233, normalized size = 0.9

$$\frac{2\sqrt{ax^2}\sqrt{x^3+1}}{x\left(x+1+\sqrt{3}\right)} - \frac{\sqrt[4]{3}\sqrt{ax^2}\sqrt{\frac{x^2-x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{-\sqrt{3}+2}\,(x+1)\,E\left(\operatorname{asin}\left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{x\sqrt{\frac{x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{x^3+1}} + \frac{2\sqrt{2}\cdot 3^{\frac{3}{4}}\sqrt{ax^2}\sqrt{\frac{x^2-x+1}{\left(x+1+\sqrt{3}\right)^2}}\,(x+1)\,F\left(\operatorname{asin}\left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{3x\sqrt{\frac{x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a*x**2)**(1/2)/(x**3+1)**(1/2),x)
```

```
[Out] 2*sqrt(a*x**2)*sqrt(x**3 + 1)/(x*(x + 1 + sqrt(3))) - 3**(1/4)*sq
rt(a*x**2)*sqrt((x**2 - x + 1)/(x + 1 + sqrt(3))**2)*sqrt(-sqrt(3
) + 2)*(x + 1)*elliptic_e(asin((x - sqrt(3) + 1)/(x + 1 + sqrt(3)
)), -7 - 4*sqrt(3))/(x*sqrt((x + 1)/(x + 1 + sqrt(3))**2)*sqrt(x*
*3 + 1)) + 2*sqrt(2)*3**(3/4)*sqrt(a*x**2)*sqrt((x**2 - x + 1)/(x
 + 1 + sqrt(3))**2)*(x + 1)*elliptic_f(asin((x - sqrt(3) + 1)/(x
+ 1 + sqrt(3))), -7 - 4*sqrt(3))/(3*x*sqrt((x + 1)/(x + 1 + sqrt(
3))**2)*sqrt(x**3 + 1))
```

Mathematica [A] time = 0.131403, size = 134, normalized size = 0.52

$$-\frac{2ax\sqrt{-\sqrt[6]{-1}\left(x+(-1)^{2/3}\right)}\sqrt{(-1)^{2/3}x^2+\sqrt[3]{-1}x+1}\left((-1)^{5/6}F\left(\sin^{-1}\left(\frac{\sqrt{-(-1)^{5/6}(x+1)}}{\sqrt[4]{3}}\right)\middle|\sqrt[3]{-1}\right)+\sqrt{3}E\left(\sin^{-1}\left(\frac{\sqrt{-(-1)^{5/6}(x+1)}}{\sqrt[4]{3}}\right)\right|\right.\right.}{\sqrt[4]{3}\sqrt{x^3+1}\sqrt{ax^2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[Sqrt[a*x^2]/Sqrt[1 + x^3],x]
```

```
[Out] (-2*a*x*Sqrt[-((-1)^(1/6)*((-1)^(2/3) + x))]*Sqrt[1 + (-1)^(1/3)*
x + (-1)^(2/3)*x^2]*(Sqrt[3]*EllipticE[ArcSin[Sqrt[-((-1)^(5/6)*(
1 + x))]/3^(1/4)], (-1)^(1/3)] + (-1)^(5/6)*EllipticF[ArcSin[Sqrt
[-((-1)^(5/6)*(1 + x))]/3^(1/4)], (-1)^(1/3)]))/(3^(1/4)*Sqrt[a*x
```

```
^2]*Sqrt[1 + x^3])
```

Maple [A] time = 0.029, size = 270, normalized size = 1.

$$\frac{-3+i\sqrt{3}}{2\,x}\sqrt{a x^2}\sqrt{-2\,\frac{1+x}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2\,x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2\,x-1}{-3+i\sqrt{3}}}\left(iEllipticE\left(\sqrt{-2\,\frac{1+x}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)\sqrt{3}-iEllipticF\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a*x^2)^(1/2)/(x^3+1)^(1/2),x)
```

```
[Out] 1/2*(a*x^2)^(1/2)*(-3+I*3^(1/2))*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2)*
((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*
3^(1/2)))^(1/2)*(I*EllipticE((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),(-(-
3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*3^(1/2)-I*EllipticF((-2*(1+x)/
(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*3^(1
/2)+3*EllipticE((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/
(I*3^(1/2)+3))^(1/2))-EllipticF((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),(
-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2)))/x/(x^3+1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a x^2}}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a*x^2)/sqrt(x^3 + 1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x^2)/sqrt(x^3 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a x^2}}{\sqrt{x^3+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a*x^2)/sqrt(x^3 + 1),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*x^2)/sqrt(x^3 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax^2}}{\sqrt{(x+1)(x^2-x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a*x**2)**(1/2)/(x**3+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a*x**2)/sqrt((x + 1)*(x**2 - x + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax^2}}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a*x^2)/sqrt(x^3 + 1),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x^2)/sqrt(x^3 + 1), x)
```

3.227 $\int \frac{\sqrt{ax}}{\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=23

$$\frac{2}{3}\sqrt{a}\sinh^{-1}\left(\frac{(ax)^{3/2}}{a^{3/2}}\right)$$

```
[Out] (2*Sqrt[a]*ArcSinh[(a*x)^(3/2)/a^(3/2)])/3
```

Rubi [A] time = 0.0470897, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$\frac{2}{3}\sqrt{a}\sinh^{-1}\left(\frac{(ax)^{3/2}}{a^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a*x]/Sqrt[1 + x^3],x]
```

```
[Out] (2*Sqrt[a]*ArcSinh[(a*x)^(3/2)/a^(3/2)])/3
```

Rubi in Sympy [A] time = 6.19283, size = 20, normalized size = 0.87

$$\frac{2\sqrt{a}\operatorname{asinh}\left(\frac{(ax)^{\frac{3}{2}}}{a^{\frac{3}{2}}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a*x)**(1/2)/(x**3+1)**(1/2),x)
```

```
[Out] 2*sqrt(a)*asinh((a*x)**(3/2)/a**(3/2))/3
```

Mathematica [A] time = 0.0250192, size = 22, normalized size = 0.96

$$\frac{2\sqrt{ax}\sinh^{-1}\left(x^{3/2}\right)}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a*x]/Sqrt[1 + x^3],x]
```

```
[Out] (2*Sqrt[a*x]*ArcSinh[x^(3/2)])/(3*Sqrt[x])
```

Maple [C] time = 0.086, size = 321, normalized size = 14.

$$-4\frac{\sqrt{ax}\sqrt{x^3+1}a\left(i\sqrt{3}+1\right)(1+x)^2}{\sqrt{x(x^3+1)a}\left(i\sqrt{3}+3\right)\sqrt{-ax(1+x)\left(i\sqrt{3}+2x-1\right)\left(i\sqrt{3}-2x+1\right)}}\sqrt{\frac{\left(i\sqrt{3}+3\right)x}{\left(i\sqrt{3}+1\right)(1+x)}}\sqrt{\frac{i\sqrt{3}+2x-1}{\left(-1+i\sqrt{3}\right)(1+x)}}\sqrt{\frac{i\sqrt{}}{(i\sqrt{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a*x)^(1/2)/(x^3+1)^(1/2),x)
```

```
[Out] -4*(a*x)^(1/2)*(x^3+1)^(1/2)*a*(I*3^(1/2)+1)*((I*3^(1/2)+3)*x/(I*
3^(1/2)+1)/(1+x))^(1/2)*(1+x)^2*((I*3^(1/2)+2*x-1)/(-1+I*3^(1/2))
/(1+x))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+1)/(1+x))^(1/2)*(Elli
pticF(((I*3^(1/2)+3)*x/(I*3^(1/2)+1)/(1+x))^(1/2),((-3+I*3^(1/2))
*(I*3^(1/2)+1)/(-1+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))-EllipticPi(((
I*3^(1/2)+3)*x/(I*3^(1/2)+1)/(1+x))^(1/2),(I*3^(1/2)+1)/(I*3^(1/2
)+3),((-3+I*3^(1/2))*(I*3^(1/2)+1)/(-1+I*3^(1/2))/(I*3^(1/2)+3))^
(1/2)))/(x*(x^3+1)*a)^(1/2)/(I*3^(1/2)+3)/(-a*x*(1+x)*(I*3^(1/2)+
2*x-1)*(I*3^(1/2)-2*x+1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax}}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a*x)/sqrt(x^3 + 1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x)/sqrt(x^3 + 1), x)
```

Fricas [A] time = 0.325601, size = 1, normalized size = 0.04

$$\left[\frac{1}{6}\sqrt{a}\log\left(-8\,ax^6-8\,ax^3-4\left(2\,x^4+x\right)\sqrt{x^3+1}\sqrt{ax}\sqrt{a}-a\right),\ \frac{1}{3}\sqrt{-a}\arctan\left(\frac{2\sqrt{x^3+1}\sqrt{ax}x}{(2\,x^3+1)\sqrt{-a}}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a*x)/sqrt(x^3 + 1),x, algorithm="fricas")
```

```
[Out] [1/6*sqrt(a)*log(-8*a*x^6 - 8*a*x^3 - 4*(2*x^4 + x)*sqrt(x^3 + 1)
*sqrt(a*x)*sqrt(a) - a), 1/3*sqrt(-a)*arctan(2*sqrt(x^3 + 1)*sqrt
(a*x)*x/((2*x^3 + 1)*sqrt(-a)))]
```

Sympy [A] time = 3.65279, size = 14, normalized size = 0.61

$$\frac{2\sqrt{a}\operatorname{asinh}\left(x^{\frac{3}{2}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a*x)**(1/2)/(x**3+1)**(1/2),x)
```

```
[Out] 2*sqrt(a)*asinh(x**(3/2))/3
```

GIAC/XCAS [A] time = 0.263575, size = 47, normalized size = 2.04

$$-\frac{2\,a^{\frac{5}{2}}\ln\left(-\sqrt{ax}a^{\frac{3}{2}}x + \sqrt{a^4x^3 + a^4}\right)}{3\,|a|^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a*x)/sqrt(x^3 + 1),x, algorithm="giac")
```

```
[Out] -2/3*a^(5/2)*ln(-sqrt(a*x)*a^(3/2)*x + sqrt(a^4*x^3 + a^4))/abs(a
)^2
```

3.228 $\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=116

$$\frac{x(x+1)\sqrt{\frac{x^2-x+1}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\sqrt{\frac{a}{x}}F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)x+1}{\left(1+\sqrt{3}\right)x+1}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt{\frac{x(x+1)}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\sqrt{x^3+1}}$$

```
[Out] (Sqrt[a/x]*x*(1 + x)*Sqrt[(1 - x + x^2)/(1 + (1 + Sqrt[3])*x)^2]*
EllipticF[ArcCos[(1 + (1 - Sqrt[3])*x)/(1 + (1 + Sqrt[3])*x)], (2
 + Sqrt[3])/4])/(3^(1/4)*Sqrt[(x*(1 + x))/(1 + (1 + Sqrt[3])*x)^2
]*Sqrt[1 + x^3])
```

Rubi [A] time = 0.1479, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$\frac{x(x+1)\sqrt{\frac{x^2-x+1}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\sqrt{\frac{a}{x}}F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)x+1}{\left(1+\sqrt{3}\right)x+1}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt{\frac{x(x+1)}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a/x]/Sqrt[1 + x^3],x]
```

```
[Out] (Sqrt[a/x]*x*(1 + x)*Sqrt[(1 - x + x^2)/(1 + (1 + Sqrt[3])*x)^2]*
EllipticF[ArcCos[(1 + (1 - Sqrt[3])*x)/(1 + (1 + Sqrt[3])*x)], (2
 + Sqrt[3])/4])/(3^(1/4)*Sqrt[(x*(1 + x))/(1 + (1 + Sqrt[3])*x)^2
]*Sqrt[1 + x^3])
```

Rubi in Sympy [A] time = 9.03999, size = 100, normalized size = 0.86

$$\frac{3^{\frac{3}{4}}x\sqrt{\frac{a}{x}}\sqrt{\frac{x^2-x+1}{\left(x\left(1+\sqrt{3}\right)+1\right)^2}}(x+1)\,F\left(\operatorname{acos}\left(\frac{x\left(-\sqrt{3}+1\right)+1}{x\left(1+\sqrt{3}\right)+1}\right)\middle|\frac{\sqrt{3}}{4}+\frac{1}{2}\right)}{3\sqrt{\frac{x(x+1)}{\left(x\left(1+\sqrt{3}\right)+1\right)^2}}\sqrt{x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a/x)**(1/2)/(x**3+1)**(1/2),x)
```

```
[Out] 3**(3/4)*x*sqrt(a/x)*sqrt((x**2 - x + 1)/(x*(1 + sqrt(3)) + 1)**2
)*(x + 1)*elliptic_f(acos((x*(-sqrt(3) + 1) + 1)/(x*(1 + sqrt(3))
 + 1)), sqrt(3)/4 + 1/2)/(3*sqrt(x*(x + 1)/(x*(1 + sqrt(3)) + 1)*
*2)*sqrt(x**3 + 1))
```

Mathematica [A] time = 0.16617, size = 106, normalized size = 0.91

$$-\frac{2\sqrt[6]{-1}\sqrt{-\sqrt[6]{-1}\left(\frac{1}{x}+(-1)^{2/3}\right)}\sqrt{\frac{(-1)^{2/3}}{x^2}+\frac{\sqrt[3]{-1}}{x}+1}x^2\sqrt{\frac{a}{x}}F\left(\sin^{-1}\left(\frac{\sqrt{-(-1)^{5/6}\left(1+\frac{1}{x}\right)}}{\sqrt[4]{3}}\right)|\sqrt[3]{-1}\right)}{\sqrt[4]{3}\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[Sqrt[a/x]/Sqrt[1 + x^3],x]
```

```
[Out] (-2*(-1)^(1/6)*Sqrt[-((-1)^(1/6)*((-1)^(2/3) + x^(-1)))]*Sqrt[1 +
 (-1)^(2/3)/x^2 + (-1)^(1/3)/x]*Sqrt[a/x]*x^2*EllipticF[ArcSin[Sq
rt[-((-1)^(5/6)*(1 + x^(-1)))]/3^(1/4)], (-1)^(1/3)])/(3^(1/4)*Sq
rt[1 + x^3])
```

Maple [C] time = 0.146, size = 232, normalized size = 2.

$$4\frac{x\sqrt{x^3+1}\left(i\sqrt{3}+1\right)(1+x)^2}{\sqrt{x(x^3+1)}\left(i\sqrt{3}+3\right)\sqrt{-x(1+x)\left(i\sqrt{3}+2x-1\right)\left(i\sqrt{3}-2x+1\right)}}\sqrt{\frac{a}{x}}\sqrt{\frac{\left(i\sqrt{3}+3\right)x}{\left(i\sqrt{3}+1\right)(1+x)}}\sqrt{\frac{i\sqrt{3}+2x-1}{\left(-1+i\sqrt{3}\right)(1+x)}}\sqrt{\frac{i\sqrt{3}}{(i\sqrt{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a/x)^(1/2)/(x^3+1)^(1/2),x)
```

```
[Out] 4*(a/x)^(1/2)*x*(x^3+1)^(1/2)*(I*3^(1/2)+1)*((I*3^(1/2)+3)*x/(I*3
^(1/2)+1)/(1+x))^(1/2)*(1+x)^2*((I*3^(1/2)+2*x-1)/(-1+I*3^(1/2))/
(1+x))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+1)/(1+x))^(1/2)*Ellipt
icF(((I*3^(1/2)+3)*x/(I*3^(1/2)+1)/(1+x))^(1/2),((-3+I*3^(1/2))*(
I*3^(1/2)+1)/(-1+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))/(x*(x^3+1))^(1/
2)/(I*3^(1/2)+3)/(-x*(1+x)*(I*3^(1/2)+2*x-1)*(I*3^(1/2)-2*x+1))^(
1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a/x)/sqrt(x^3 + 1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a/x)/sqrt(x^3 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\frac{a}{x}}}{\sqrt{x^3+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a/x)/sqrt(x^3 + 1),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a/x)/sqrt(x^3 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{(x+1)(x^2-x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a/x)**(1/2)/(x**3+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a/x)/sqrt((x + 1)*(x**2 - x + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a/x)/sqrt(x^3 + 1),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a/x)/sqrt(x^3 + 1), x)
```

3.229 $\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=24

$$-\frac{2}{3}x\sqrt{\frac{a}{x^2}}\tanh^{-1}\left(\sqrt{x^3+1}\right)$$

```
[Out] (-2*Sqrt[a/x^2]*x*ArcTanh[Sqrt[1 + x^3]])/3
```

Rubi [A] time = 0.0255874, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21

$$-\frac{2}{3}x\sqrt{\frac{a}{x^2}}\tanh^{-1}\left(\sqrt{x^3+1}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a/x^2]/Sqrt[1 + x^3],x]

[Out] (-2*Sqrt[a/x^2]*x*ArcTanh[Sqrt[1 + x^3]])/3
```

Rubi in Sympy [A] time = 8.47861, size = 24, normalized size = 1.

$$-\frac{2x\sqrt{\frac{a}{x^2}}\operatorname{atanh}\left(\sqrt{x^3+1}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a/x**2)**(1/2)/(x**3+1)**(1/2),x)

[Out] -2*x*sqrt(a/x**2)*atanh(sqrt(x**3 + 1))/3
```

Mathematica [A] time = 0.018103, size = 24, normalized size = 1.

$$-\frac{2}{3}x\sqrt{\frac{a}{x^2}}\tanh^{-1}\left(\sqrt{x^3+1}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a/x^2]/Sqrt[1 + x^3],x]
```

```
[Out] (-2*Sqrt[a/x^2]*x*ArcTanh[Sqrt[1 + x^3]])/3
```

Maple [A] time = 0.007, size = 19, normalized size = 0.8

$$-\frac{2x}{3} Artanh\left(\sqrt{x^3+1}\right)\sqrt{\frac{a}{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a/x^2)^(1/2)/(x^3+1)^(1/2),x)
```

```
[Out] -2/3*x*arctanh((x^3+1)^(1/2))*(a/x^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a/x^2)/sqrt(x^3 + 1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a/x^2)/sqrt(x^3 + 1), x)
```

Fricas [A] time = 0.285416, size = 1, normalized size = 0.04

$$\left[\frac{1}{3}\,x\sqrt{\frac{a}{x^2}}\log\left(\frac{x^3-2\sqrt{x^3+1}+2}{x^3}\right),\ \frac{2}{3}\sqrt{-a}\arctan\left(\frac{\sqrt{-a}x\sqrt{\frac{a}{x^2}}}{\sqrt{x^3+1}a}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a/x^2)/sqrt(x^3 + 1),x, algorithm="fricas")
```

```
[Out] [1/3*x*sqrt(a/x^2)*log((x^3 - 2*sqrt(x^3 + 1) + 2)/x^3), 2/3*sqrt
(-a)*arctan(sqrt(-a)*x*sqrt(a/x^2)/(sqrt(x^3 + 1)*a))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{(x+1)(x^2-x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a/x**2)**(1/2)/(x**3+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a/x**2)/sqrt((x + 1)*(x**2 - x + 1)), x)
```

GIAC/XCAS [A] time = 0.263721, size = 42, normalized size = 1.75

$$-\frac{1}{3}\sqrt{a}\left(\ln\left(\sqrt{x^3+1}+1\right)-\ln\left(\left|\sqrt{x^3+1}-1\right|\right)\right)\operatorname{sign}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a/x^2)/sqrt(x^3 + 1),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(a)*(ln(sqrt(x^3 + 1) + 1) - ln(abs(sqrt(x^3 + 1) - 1)))
*sign(x)
```

3.230 $\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=312

$$-2\sqrt{x^3+1}x\sqrt{\frac{a}{x^3}}+\frac{2\left(1+\sqrt{3}\right)\sqrt{x^3+1}x^2\sqrt{\frac{a}{x^3}}}{\left(1+\sqrt{3}\right)x+1}-\frac{\left(1-\sqrt{3}\right)(x+1)\sqrt{\frac{x^2-x+1}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}x^2\sqrt{\frac{a}{x^3}}F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)x+1}{\left(1+\sqrt{3}\right)x+1}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt{\frac{x(x+1)}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\sqrt{x^3+1}}-\frac{2\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}x^2\sqrt{\frac{a}{x^3}}E\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)x+1}{\left(1+\sqrt{3}\right)x+1}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt{\frac{x(x+1)}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\sqrt{x^3+1}}$$

```
[Out] -2*Sqrt[a/x^3]*x*Sqrt[1 + x^3] + (2*(1 + Sqrt[3])*Sqrt[a/x^3]*x^2
*Sqrt[1 + x^3])/(1 + (1 + Sqrt[3])*x) - (2*3^(1/4)*Sqrt[a/x^3]*x^
2*(1 + x)*Sqrt[(1 - x + x^2)/(1 + (1 + Sqrt[3])*x)^2]*EllipticE[A
rcCos[(1 + (1 - Sqrt[3])*x)/(1 + (1 + Sqrt[3])*x)], (2 + Sqrt[3])
/4])/(Sqrt[(x*(1 + x))/(1 + (1 + Sqrt[3])*x)^2]*Sqrt[1 + x^3]) -
((1 - Sqrt[3])*Sqrt[a/x^3]*x^2*(1 + x)*Sqrt[(1 - x + x^2)/(1 + (1
 + Sqrt[3])*x)^2]*EllipticF[ArcCos[(1 + (1 - Sqrt[3])*x)/(1 + (1
+ Sqrt[3])*x)], (2 + Sqrt[3])/4])/(3^(1/4)*Sqrt[(x*(1 + x))/(1 +
(1 + Sqrt[3])*x)^2]*Sqrt[1 + x^3])
```

Rubi [A] time = 0.44452, antiderivative size = 312, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316

$$-2\sqrt{x^3+1}x\sqrt{\frac{a}{x^3}}+\frac{2\left(1+\sqrt{3}\right)\sqrt{x^3+1}x^2\sqrt{\frac{a}{x^3}}}{\left(1+\sqrt{3}\right)x+1}-\frac{\left(1-\sqrt{3}\right)(x+1)\sqrt{\frac{x^2-x+1}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}x^2\sqrt{\frac{a}{x^3}}F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)x+1}{\left(1+\sqrt{3}\right)x+1}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt{\frac{x(x+1)}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\sqrt{x^3+1}}-\frac{2\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}x^2\sqrt{\frac{a}{x^3}}E\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)x+1}{\left(1+\sqrt{3}\right)x+1}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt{\frac{x(x+1)}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a/x^3]/Sqrt[1 + x^3],x]
```

```
[Out] -2*Sqrt[a/x^3]*x*Sqrt[1 + x^3] + (2*(1 + Sqrt[3])*Sqrt[a/x^3]*x^2
*Sqrt[1 + x^3])/(1 + (1 + Sqrt[3])*x) - (2*3^(1/4)*Sqrt[a/x^3]*x^
2*(1 + x)*Sqrt[(1 - x + x^2)/(1 + (1 + Sqrt[3])*x)^2]*EllipticE[A
rcCos[(1 + (1 - Sqrt[3])*x)/(1 + (1 + Sqrt[3])*x)], (2 + Sqrt[3])
/4])/(Sqrt[(x*(1 + x))/(1 + (1 + Sqrt[3])*x)^2]*Sqrt[1 + x^3]) -
((1 - Sqrt[3])*Sqrt[a/x^3]*x^2*(1 + x)*Sqrt[(1 - x + x^2)/(1 + (1
 + Sqrt[3])*x)^2]*EllipticF[ArcCos[(1 + (1 - Sqrt[3])*x)/(1 + (1
+ Sqrt[3])*x)], (2 + Sqrt[3])/4])/(3^(1/4)*Sqrt[(x*(1 + x))/(1 +
(1 + Sqrt[3])*x)^2]*Sqrt[1 + x^3])
```

Rubi in Sympy [A] time = 21.1696, size = 282, normalized size = 0.9

$$-\frac{2\sqrt[4]{3}x^2\sqrt{\frac{a}{x^3}}\sqrt{\frac{x^2-x+1}{\left(x\left(1+\sqrt{3}\right)+1\right)^2}}\,(x+1)\,E\left(\operatorname{acos}\left(\frac{x\left(-\sqrt{3}+1\right)+1}{x\left(1+\sqrt{3}\right)+1}\right)\middle|\frac{\sqrt{3}}{4}+\frac{1}{2}\right)}{\sqrt{\frac{x(x+1)}{\left(x\left(1+\sqrt{3}\right)+1\right)^2}}\sqrt{x^3+1}} - \frac{2\cdot 3^{\frac{3}{4}}x^2\sqrt{\frac{a}{x^3}}\sqrt{\frac{x^2-x+1}{\left(x\left(1+\sqrt{3}\right)+1\right)^2}}\left(-\frac{\sqrt{3}}{2}+\frac{1}{2}\right)(x+1)\,F\left(\operatorname{acos}\left(\frac{x\left(-\sqrt{3}+1\right)+1}{x\left(1+\sqrt{3}\right)+1}\right)\middle|\frac{\sqrt{3}}{4}+\frac{1}{2}\right)}{3\sqrt{\frac{x(x+1)}{\left(x\left(1+\sqrt{3}\right)+1\right)^2}}\sqrt{x^3+1}} + \frac{x^2\sqrt{\frac{a}{x^3}}\left(4+4\sqrt{3}\right)\sqrt{x^3+1}}{x\left(2+2\sqrt{3}\right)+2} - 2x\sqrt{\frac{a}{x^3}}\sqrt{x^3+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a/x**3)**(1/2)/(x**3+1)**(1/2),x)
```

```
[Out] -2*3**(1/4)*x**2*sqrt(a/x**3)*sqrt((x**2 - x + 1)/(x*(1 + sqrt(3)
) + 1)**2)*(x + 1)*elliptic_e(acos((x*(-sqrt(3) + 1) + 1)/(x*(1 +
 sqrt(3)) + 1)), sqrt(3)/4 + 1/2)/(sqrt(x*(x + 1)/(x*(1 + sqrt(3)
) + 1)**2)*sqrt(x**3 + 1)) - 2*3**(3/4)*x**2*sqrt(a/x**3)*sqrt((x
**2 - x + 1)/(x*(1 + sqrt(3)) + 1)**2)*(-sqrt(3)/2 + 1/2)*(x + 1)
*elliptic_f(acos((x*(-sqrt(3) + 1) + 1)/(x*(1 + sqrt(3)) + 1)), s
qrt(3)/4 + 1/2)/(3*sqrt(x*(x + 1)/(x*(1 + sqrt(3)) + 1)**2)*sqrt(
x**3 + 1)) + x**2*sqrt(a/x**3)*(4 + 4*sqrt(3))*sqrt(x**3 + 1)/(x*
(2 + 2*sqrt(3)) + 2) - 2*x*sqrt(a/x**3)*sqrt(x**3 + 1)
```

Mathematica [C] time = 0.461757, size = 165, normalized size = 0.53

$$-\frac{\sqrt{\frac{2}{3}}\left((-1)^{2/3}-1\right)a\sqrt{\frac{x-\sqrt[3]{-1}}{\left(1+\sqrt[3]{-1}\right)x}}\sqrt{\frac{(x+1)\left(2x+i\sqrt{3}-1\right)}{x^2}}\left(\left(1+\sqrt[3]{-1}\right)E\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}(x+1)}{\left(-1+(-1)^{2/3}\right)x}}\right)|1+(-1)^{2/3}\right)-F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2}}{(-1+(}}\right.\right.\right.}{\sqrt{x^3+1}\sqrt{\frac{a}{x^3}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a/x^3]/Sqrt[1 + x^3],x]
```

```
[Out] -((Sqrt[2/3]*(-1 + (-1)^(2/3))*a*Sqrt[(-(-1)^(1/3) + x)/((1 + (-1
)^(1/3))*x)]*Sqrt[((1 + x)*(-1 + I*Sqrt[3] + 2*x))/x^2]*((1 + (-1
)^(1/3))*EllipticE[ArcSin[Sqrt[((-1)^(2/3)*(1 + x))/((-1 + (-1)^(
2/3))*x)]], 1 + (-1)^(2/3)] - EllipticF[ArcSin[Sqrt[((-1)^(2/3)*(
1 + x))/((-1 + (-1)^(2/3))*x)]], 1 + (-1)^(2/3)]))/(Sqrt[a/x^3]*S
qrt[1 + x^3]))
```

Maple [C] time = 0.096, size = 1784, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a/x^3)^(1/2)/(x^3+1)^(1/2),x)
```

```
[Out] -2*(a/x^3)^(1/2)*x/(x^3+1)^(1/2)*(4*I*3^(1/2)*(x*(x^3+1))^(1/2)*(
(I*3^(1/2)+3)*x/(I*3^(1/2)+1)/(1+x))^(1/2)*((I*3^(1/2)+2*x-1)/(-1
+I*3^(1/2))/(1+x))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+1)/(1+x))^
(1/2)*EllipticE(((I*3^(1/2)+3)*x/(I*3^(1/2)+1)/(1+x))^(1/2),((-3+
I*3^(1/2))*(I*3^(1/2)+1)/(-1+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*x+I
*3^(1/2)*(-x*(1+x)*(I*3^(1/2)+2*x-1)*(I*3^(1/2)-2*x+1))^(1/2)*x^3
+2*I*3^(1/2)*(x*(x^3+1))^(1/2)*x^2-4*(x*(x^3+1))^(1/2)*((I*3^(1/2
)+3)*x/(I*3^(1/2)+1)/(1+x))^(1/2)*((I*3^(1/2)+2*x-1)/(-1+I*3^(1/2
))/(1+x))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+1)/(1+x))^(1/2)*Ell
ipticF(((I*3^(1/2)+3)*x/(I*3^(1/2)+1)/(1+x))^(1/2),((-3+I*3^(1/2)
)*(I*3^(1/2)+1)/(-1+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*x^2+6*(x*(x^
3+1))^(1/2)*((I*3^(1/2)+3)*x/(I*3^(1/2)+1)/(1+x))^(1/2)*((I*3^(1/
2)+2*x-1)/(-1+I*3^(1/2))/(1+x))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/
2)+1)/(1+x))^(1/2)*EllipticE(((I*3^(1/2)+3)*x/(I*3^(1/2)+1)/(1+x)
)^(1/2),((-3+I*3^(1/2))*(I*3^(1/2)+1)/(-1+I*3^(1/2))/(I*3^(1/2)+3
))^(1/2))*x^2+2*I*3^(1/2)*(x*(x^3+1))^(1/2)*((I*3^(1/2)+3)*x/(I*3
^(1/2)+1)/(1+x))^(1/2)*((I*3^(1/2)+2*x-1)/(-1+I*3^(1/2))/(1+x))^(
1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+1)/(1+x))^(1/2)*EllipticE(((I*
3^(1/2)+3)*x/(I*3^(1/2)+1)/(1+x))^(1/2),((-3+I*3^(1/2))*(I*3^(1/2
)+1)/(-1+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*x^2-2*I*3^(1/2)*(x*(x^3
```

```
+1))^(1/2)*x^3-8*(x*(x^3+1))^(1/2)*((I*3^(1/2)+3)*x/(I*3^(1/2)+1)
/(1+x))^(1/2)*((I*3^(1/2)+2*x-1)/(-1+I*3^(1/2))/(1+x))^(1/2)*((I*
3^(1/2)-2*x+1)/(I*3^(1/2)+1)/(1+x))^(1/2)*EllipticF(((I*3^(1/2)+3
)*x/(I*3^(1/2)+1)/(1+x))^(1/2),((-3+I*3^(1/2))*(I*3^(1/2)+1)/(-1+
I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*x+12*(x*(x^3+1))^(1/2)*((I*3^(1/
2)+3)*x/(I*3^(1/2)+1)/(1+x))^(1/2)*((I*3^(1/2)+2*x-1)/(-1+I*3^(1/
2))/(1+x))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+1)/(1+x))^(1/2)*El
lipticE(((I*3^(1/2)+3)*x/(I*3^(1/2)+1)/(1+x))^(1/2),((-3+I*3^(1/2
))*(I*3^(1/2)+1)/(-1+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*x+I*3^(1/2)
*(-x*(1+x)*(I*3^(1/2)+2*x-1)*(I*3^(1/2)-2*x+1))^(1/2)-4*(x*(x^3+1
))^(1/2)*((I*3^(1/2)+3)*x/(I*3^(1/2)+1)/(1+x))^(1/2)*((I*3^(1/2)+
2*x-1)/(-1+I*3^(1/2))/(1+x))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+
1)/(1+x))^(1/2)*EllipticF(((I*3^(1/2)+3)*x/(I*3^(1/2)+1)/(1+x))^(
1/2),((-3+I*3^(1/2))*(I*3^(1/2)+1)/(-1+I*3^(1/2))/(I*3^(1/2)+3))^
(1/2))+6*(x*(x^3+1))^(1/2)*((I*3^(1/2)+3)*x/(I*3^(1/2)+1)/(1+x))^
(1/2)*((I*3^(1/2)+2*x-1)/(-1+I*3^(1/2))/(1+x))^(1/2)*((I*3^(1/2)-
2*x+1)/(I*3^(1/2)+1)/(1+x))^(1/2)*EllipticE(((I*3^(1/2)+3)*x/(I*3
^(1/2)+1)/(1+x))^(1/2),((-3+I*3^(1/2))*(I*3^(1/2)+1)/(-1+I*3^(1/2
))/(I*3^(1/2)+3))^(1/2))+2*I*3^(1/2)*(x*(x^3+1))^(1/2)*((I*3^(1/2
)+3)*x/(I*3^(1/2)+1)/(1+x))^(1/2)*((I*3^(1/2)+2*x-1)/(-1+I*3^(1/2
))/(1+x))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+1)/(1+x))^(1/2)*Ell
ipticE(((I*3^(1/2)+3)*x/(I*3^(1/2)+1)/(1+x))^(1/2),((-3+I*3^(1/2)
)*(I*3^(1/2)+1)/(-1+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))-6*(x*(x^3+1)
)^(1/2)*x^3+3*(-x*(1+x)*(I*3^(1/2)+2*x-1)*(I*3^(1/2)-2*x+1))^(1/2
)*x^3-2*I*3^(1/2)*(x*(x^3+1))^(1/2)*x+6*(x*(x^3+1))^(1/2)*x^2-6*(
x*(x^3+1))^(1/2)*x+3*(-x*(1+x)*(I*3^(1/2)+2*x-1)*(I*3^(1/2)-2*x+1
))^(1/2))/(I*3^(1/2)+3)/(-x*(1+x)*(I*3^(1/2)+2*x-1)*(I*3^(1/2)-2*
x+1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a/x^3)/sqrt(x^3 + 1),x, algorithm="maxima")

[Out] integrate(sqrt(a/x^3)/sqrt(x^3 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\frac{a}{x^3}}}{\sqrt{x^3+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a/x^3)/sqrt(x^3 + 1),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a/x^3)/sqrt(x^3 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{(x+1)(x^2-x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a/x**3)**(1/2)/(x**3+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a/x**3)/sqrt((x + 1)*(x**2 - x + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a/x^3)/sqrt(x^3 + 1),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a/x^3)/sqrt(x^3 + 1), x)
```

3.231 $\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=281

$$-\sqrt{x^3+1}x\sqrt{\frac{a}{x^4}}+\frac{\sqrt{x^3+1}x^2\sqrt{\frac{a}{x^4}}}{x+\sqrt{3}+1}+\frac{\sqrt{2}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}x^2\sqrt{\frac{a}{x^4}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}x^2\sqrt{\frac{a}{x^4}}E\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{2\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

```
[Out] -(Sqrt[a/x^4]*x*Sqrt[1 + x^3]) + (Sqrt[a/x^4]*x^2*Sqrt[1 + x^3])/
(1 + Sqrt[3] + x) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*Sqrt[a/x^4]*x^2*(1
 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1
 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(2*Sqrt[(1 +
 x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) + (Sqrt[2]*Sqrt[a/x^4]*x^
2*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSi
n[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)
*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rubi [A] time = 0.17353, antiderivative size = 281, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263

$$-\sqrt{x^3+1}x\sqrt{\frac{a}{x^4}}+\frac{\sqrt{x^3+1}x^2\sqrt{\frac{a}{x^4}}}{x+\sqrt{3}+1}+\frac{\sqrt{2}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}x^2\sqrt{\frac{a}{x^4}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}x^2\sqrt{\frac{a}{x^4}}E\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{2\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a/x^4]/Sqrt[1 + x^3],x]
```


```
[Out] -(Sqrt[a/x^4]*x*Sqrt[1 + x^3]) + (Sqrt[a/x^4]*x^2*Sqrt[1 + x^3])/
(1 + Sqrt[3] + x) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*Sqrt[a/x^4]*x^2*(1
 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1
 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(2*Sqrt[(1 +
```

```
 x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) + (Sqrt[2]*Sqrt[a/x^4]*x^
2*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSi
n[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)
*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rubi in Sympy [A] time = 17.4682, size = 255, normalized size = 0.91

$$\frac{x^2\sqrt{\frac{a}{x^4}}\sqrt{x^3+1}}{x+1+\sqrt{3}}-\frac{\sqrt[4]{3}x^2\sqrt{\frac{a}{x^4}}\sqrt{\frac{x^2-x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{-\sqrt{3}+2}\,(x+1)\,E\left(\operatorname{asin}\left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{2\sqrt{\frac{x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{x^3+1}}+\frac{\sqrt{2}\cdot 3^{\frac{3}{4}}x^2\sqrt{\frac{a}{x^4}}\sqrt{\frac{x^2-x+1}{\left(x+1+\sqrt{3}\right)^2}}\,(x+1)\,F\left(\operatorname{asin}\left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{3\sqrt{\frac{x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{x^3+1}}-x\sqrt{\frac{a}{x^4}}\sqrt{x^3+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a/x**4)**(1/2)/(x**3+1)**(1/2),x)
```

```
[Out] x**2*sqrt(a/x**4)*sqrt(x**3 + 1)/(x + 1 + sqrt(3)) - 3**(1/4)*x**
2*sqrt(a/x**4)*sqrt((x**2 - x + 1)/(x + 1 + sqrt(3))**2)*sqrt(-sq
rt(3) + 2)*(x + 1)*elliptic_e(asin((x - sqrt(3) + 1)/(x + 1 + sqr
t(3))), -7 - 4*sqrt(3))/(2*sqrt((x + 1)/(x + 1 + sqrt(3))**2)*sqr
t(x**3 + 1)) + sqrt(2)*3**(3/4)*x**2*sqrt(a/x**4)*sqrt((x**2 - x
+ 1)/(x + 1 + sqrt(3))**2)*(x + 1)*elliptic_f(asin((x - sqrt(3) +
 1)/(x + 1 + sqrt(3))), -7 - 4*sqrt(3))/(3*sqrt((x + 1)/(x + 1 +
sqrt(3))**2)*sqrt(x**3 + 1)) - x*sqrt(a/x**4)*sqrt(x**3 + 1)
```

Mathematica [A] time = 0.396525, size = 146, normalized size = 0.52

$$\frac{x\sqrt{\frac{a}{x^4}}\left(-3\left(x^3+1\right)-3^{3/4}x\sqrt{-\sqrt[6]{-1}\left(x+(-1)^{2/3}\right)}\sqrt{(-1)^{2/3}x^2+\sqrt[3]{-1}x+1}\left((-1)^{5/6}F\left(\sin^{-1}\left(\frac{\sqrt{-(-1)^{5/6}(x+1)}}{\sqrt[4]{3}}\right)\middle|\sqrt[3]{-1}\right)+\sqrt{3}E\left(s\right.\right.\right.}{3\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[Sqrt[a/x^4]/Sqrt[1 + x^3],x]
```

```
[Out] (Sqrt[a/x^4]*x*(-3*(1 + x^3) - 3^(3/4)*x*Sqrt[-((-1)^(1/6)*((-1)^
(2/3) + x))]*Sqrt[1 + (-1)^(1/3)*x + (-1)^(2/3)*x^2]*(Sqrt[3]*Ell
ipticE[ArcSin[Sqrt[-((-1)^(5/6)*(1 + x))]/3^(1/4)], (-1)^(1/3)] +
```

```
 (-1)^(5/6)*EllipticF[ArcSin[Sqrt[-((-1)^(5/6)*(1 + x))]/3^(1/4)]
, (-1)^(1/3)])))/(3*Sqrt[1 + x^3])
```

Maple [A] time = 0.032, size = 353, normalized size = 1.3

$$\frac{x}{2}\sqrt{\frac{a}{x^4}}\left(i\sqrt{-2\,\frac{1+x}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2\,x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2\,x-1}{-3+i\sqrt{3}}}\mathit{EllipticF}\left(\sqrt{-2\,\frac{1+x}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)\sqrt{3}x+3\sqrt{-2\,\frac{1+x}{-3+i\sqrt{3}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a/x^4)^(1/2)/(x^3+1)^(1/2),x)
```

```
[Out] 1/2*(a/x^4)^(1/2)*x*(I*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2)*((I*3^(1/2
)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/2)))^
(1/2)*EllipticF((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/
(I*3^(1/2)+3))^(1/2))*3^(1/2)*x+3*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2)
*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I
*3^(1/2)))^(1/2)*EllipticF((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),(-(-3+
I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*x-6*(-2*(1+x)/(-3+I*3^(1/2)))^(1
/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-
3+I*3^(1/2)))^(1/2)*EllipticE((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),(-(
-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*x-2*x^3-2)/(x^3+1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a/x^4)/sqrt(x^3 + 1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a/x^4)/sqrt(x^3 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\frac{a}{x^4}}}{\sqrt{x^3+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a/x^4)/sqrt(x^3 + 1),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a/x^4)/sqrt(x^3 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{(x+1)(x^2-x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a/x**4)**(1/2)/(x**3+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a/x**4)/sqrt((x + 1)*(x**2 - x + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a/x^4)/sqrt(x^3 + 1),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a/x^4)/sqrt(x^3 + 1), x)
```

3.232 $\int \frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}} \, dx$

Optimal. Leaf size=37

$$\frac{x\sqrt{ax^{2n}}\, {}_2F_1\left(\frac{1}{2}, 1+\frac{1}{n}; 2+\frac{1}{n}; -x^n\right)}{n+1}$$

```
[Out] (x*Sqrt[a*x^(2*n)]*Hypergeometric2F1[1/2, 1 + n^(-1), 2 + n^(-1),
 -x^n])/(1 + n)
```

Rubi [A] time = 0.0314402, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095

$$\frac{x\sqrt{ax^{2n}}\, {}_2F_1\left(\frac{1}{2}, 1+\frac{1}{n}; 2+\frac{1}{n}; -x^n\right)}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^(2*n)]/Sqrt[1 + x^n],x]
```

```
[Out] (x*Sqrt[a*x^(2*n)]*Hypergeometric2F1[1/2, 1 + n^(-1), 2 + n^(-1),
 -x^n])/(1 + n)
```

Rubi in Sympy [A] time = 8.88256, size = 36, normalized size = 0.97

$$\frac{x^{-n}x^{n+1}\sqrt{ax^{2n}}\, {}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{n+1}{n} \\ 2+\frac{1}{n} \end{matrix}\middle| -x^n\right)}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((a*x**(2*n))**(1/2)/(1+x**n)**(1/2),x)
```

```
[Out] x**(-n)*x**(n + 1)*sqrt(a*x**(2*n))*hyper((1/2, (n + 1)/n), (2 +
1/n,), -x**n)/(n + 1)
```

Mathematica [A] time = 0.0489049, size = 53, normalized size = 1.43

$$\frac{2ax^{n+1}\left(\sqrt{x^n+1} - {}_2F_1\left(\frac{1}{2}, \frac{1}{n}; 1+\frac{1}{n}; -x^n\right)\right)}{(n+2)\sqrt{ax^{2n}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a*x^(2*n)]/Sqrt[1 + x^n],x]
```

```
[Out] (2*a*x^(1 + n)*(Sqrt[1 + x^n] - Hypergeometric2F1[1/2, n^(-1), 1
+ n^(-1), -x^n]))/((2 + n)*Sqrt[a*x^(2*n)])
```

Maple [F] time = 0.102, size = 0, normalized size = 0.

$$\int 1\sqrt{a x^{2 n}}\,\frac{1}{\sqrt{1+x^{n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a*x^(2*n))^(1/2)/(1+x^n)^(1/2),x)
```

```
[Out] int((a*x^(2*n))^(1/2)/(1+x^n)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a x^{2 n}}}{\sqrt{x^{n}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a*x^(2*n))/sqrt(x^n + 1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x^(2*n))/sqrt(x^n + 1), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a*x^(2*n))/sqrt(x^n + 1),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax^{2n}}}{\sqrt{x^n + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a*x**(2*n))**(1/2)/(1+x**n)**(1/2),x)

[Out] Integral(sqrt(a*x**(2*n))/sqrt(x**n + 1), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax^{2\,n}}}{\sqrt{x^n + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a*x^(2*n))/sqrt(x^n + 1),x, algorithm="giac")

[Out] integrate(sqrt(a*x^(2*n))/sqrt(x^n + 1), x)
```

3.233 $\int \frac{\sqrt{ax^n}}{\sqrt{1+x^n}} dx$

Optimal. Leaf size=48

$$\frac{2x\sqrt{ax^n}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(1+\frac{2}{n}\right);\frac{1}{2}\left(3+\frac{2}{n}\right);-x^n\right)}{n+2}$$

```
[Out] (2*x*Sqrt[a*x^n]*Hypergeometric2F1[1/2, (1 + 2/n)/2, (3 + 2/n)/2,
 -x^n])/(2 + n)
```

Rubi [A] time = 0.0331198, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$\frac{2x\sqrt{ax^n}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(1+\frac{2}{n}\right);\frac{1}{2}\left(3+\frac{2}{n}\right);-x^n\right)}{n+2}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a*x^n]/Sqrt[1 + x^n],x]
```

```
[Out] (2*x*Sqrt[a*x^n]*Hypergeometric2F1[1/2, (1 + 2/n)/2, (3 + 2/n)/2,
 -x^n])/(2 + n)
```

Rubi in Sympy [A] time = 9.17569, size = 42, normalized size = 0.88

$$\frac{2x^{-\frac{n}{2}}x^{\frac{n}{2}+1}\sqrt{ax^n}\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{n+2}{2n}\\ \frac{3}{2}+\frac{1}{n}\end{matrix}\middle|-x^n\right)}{n+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a*x**n)**(1/2)/(1+x**n)**(1/2),x)
```

```
[Out] 2*x**(-n/2)*x**(n/2 + 1)*sqrt(a*x**n)*hyper((1/2, (n + 2)/(2*n)),
 (3/2 + 1/n,), -x**n)/(n + 2)
```

Mathematica [A] time = 0.027451, size = 40, normalized size = 0.83

$$\frac{2x\sqrt{ax^n}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}+\frac{1}{n};\frac{3}{2}+\frac{1}{n};-x^n\right)}{n+2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a*x^n]/Sqrt[1 + x^n],x]
```

```
[Out] (2*x*Sqrt[a*x^n]*Hypergeometric2F1[1/2, 1/2 + n^(-1), 3/2 + n^(-1
), -x^n])/(2 + n)
```

Maple [A] time = 0.063, size = 35, normalized size = 0.7

$$2\,\frac{x\,{}_2F_1(1/2,1/2+n^{-1};\,3/2+n^{-1};\,-x^n)\sqrt{ax^n}}{2+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a*x^n)^(1/2)/(1+x^n)^(1/2),x)
```

```
[Out] 2*x*hypergeom([1/2,1/2+1/n],[3/2+1/n],-x^n)*(a*x^n)^(1/2)/(2+n)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax^n}}{\sqrt{x^n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a*x^n)/sqrt(x^n + 1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x^n)/sqrt(x^n + 1), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a*x^n)/sqrt(x^n + 1),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax^n}}{\sqrt{x^n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a*x**n)**(1/2)/(1+x**n)**(1/2),x)

[Out] Integral(sqrt(a*x**n)/sqrt(x**n + 1), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax^n}}{\sqrt{x^n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a*x^n)/sqrt(x^n + 1),x, algorithm="giac")

[Out] integrate(sqrt(a*x^n)/sqrt(x^n + 1), x)
```

3.234 $\int \frac{\sqrt{ax^{n/2}}}{\sqrt{1+x^n}}\,dx$

Optimal. Leaf size=52

$$\frac{4x\sqrt{ax^{n/2}}\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}\left(1+\frac{4}{n}\right);\frac{1}{4}\left(5+\frac{4}{n}\right);-x^n\right)}{n+4}$$

```
[Out] (4*x*Sqrt[a*x^(n/2)]*Hypergeometric2F1[1/2, (1 + 4/n)/4, (5 + 4/n
)/4, -x^n])/(4 + n)
```

Rubi [A] time = 0.0361213, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087

$$\frac{4x\sqrt{ax^{n/2}}\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}\left(1+\frac{4}{n}\right);\frac{1}{4}\left(5+\frac{4}{n}\right);-x^n\right)}{n+4}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a*x^(n/2)]/Sqrt[1 + x^n],x]
```

```
[Out] (4*x*Sqrt[a*x^(n/2)]*Hypergeometric2F1[1/2, (1 + 4/n)/4, (5 + 4/n
)/4, -x^n])/(4 + n)
```

Rubi in Sympy [A] time = 9.07168, size = 44, normalized size = 0.85

$$\frac{4x^{-\frac{n}{4}}x^{\frac{n}{4}+1}\sqrt{ax^{\frac{n}{2}}}\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{n+4}{4n}\\\frac{5}{4}+\frac{1}{n}\end{matrix}\middle|-x^n\right)}{n+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a*x**(1/2*n))**(1/2)/(1+x**n)**(1/2),x)
```

```
[Out] 4*x**(-n/4)*x**(n/4 + 1)*sqrt(a*x**(n/2))*hyper((1/2, (n + 4)/(4*
n)), (5/4 + 1/n,), -x**n)/(n + 4)
```

Mathematica [A] time = 0.0293149, size = 44, normalized size = 0.85

$$\frac{4x\sqrt{ax^{n/2}}\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}+\frac{1}{n};\frac{5}{4}+\frac{1}{n};-x^n\right)}{n+4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a*x^(n/2)]/Sqrt[1 + x^n],x]
```

```
[Out] (4*x*Sqrt[a*x^(n/2)]*Hypergeometric2F1[1/2, 1/4 + n^(-1), 5/4 + n
^(-1), -x^n])/(4 + n)
```

Maple [A] time = 0.089, size = 37, normalized size = 0.7

$$4\,{\frac {x{}_{2}F_{1}(1/2,1/4+{n}^{-1};\,5/4+{n}^{-1};\,-{x}^{n})\sqrt {a{x}^{n/2}}}{4+n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a*x^(1/2*n))^(1/2)/(1+x^n)^(1/2),x)
```

```
[Out] 4*x*hypergeom([1/2,1/4+1/n],[5/4+1/n],-x^n)*(a*x^(1/2*n))^(1/2)/(
4+n)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a x^{\frac{1}{2}\, n}}}{\sqrt{x^{n} + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a*x^(1/2*n))/sqrt(x^n + 1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x^(1/2*n))/sqrt(x^n + 1), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a*x^(1/2*n))/sqrt(x^n + 1),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax^{\frac{n}{2}}}}{\sqrt{x^n + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a*x**(1/2*n))**(1/2)/(1+x**n)**(1/2),x)
```

```
[Out] Integral(sqrt(a*x**(n/2))/sqrt(x**n + 1), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax^{\frac{1}{2}n}}}{\sqrt{x^n + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a*x^(1/2*n))/sqrt(x^n + 1),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x^(1/2*n))/sqrt(x^n + 1), x)
```

3.235 $\int \left(\frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}}+\frac{2x^{-n}\sqrt{ax^{2n}}}{(2+n)\sqrt{1+x^n}}\right)\,dx$

Optimal. Leaf size=34

$$\frac{2x^{1-n}\sqrt{x^n+1}\sqrt{ax^{2n}}}{n+2}$$

```
[Out] (2*x^(1 - n)*Sqrt[a*x^(2*n)]*Sqrt[1 + x^n])/(2 + n)
```

Rubi [C] time = 0.0645534, antiderivative size = 80, normalized size of antiderivative = 2.35, number of steps used = 5, number of rules used = 3, integrand size = 54, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056

$$\frac{2x^{1-n}\sqrt{ax^{2n}}\,{}_2F_1\left(\frac{1}{2},\frac{1}{n};1+\frac{1}{n};-x^n\right)}{n+2}+\frac{x\sqrt{ax^{2n}}\,{}_2F_1\left(\frac{1}{2},1+\frac{1}{n};2+\frac{1}{n};-x^n\right)}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^(2*n)]/Sqrt[1 + x^n] + (2*Sqrt[a*x^(2*n)])/((2 + n)*x^n*Sqrt[1 + x^n
```

```
[Out] (x*Sqrt[a*x^(2*n)]*Hypergeometric2F1[1/2, 1 + n^(-1), 2 + n^(-1),
 -x^n])/(1 + n) + (2*x^(1 - n)*Sqrt[a*x^(2*n)]*Hypergeometric2F1[
1/2, n^(-1), 1 + n^(-1), -x^n])/(2 + n)
```

Rubi in Sympy [A] time = 21.9847, size = 70, normalized size = 2.06

$$\frac{2xx^{-n}\sqrt{ax^{2n}}{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{1}{n}\\1+\frac{1}{n}\end{matrix}\middle|-x^n\right)}{n+2}+\frac{x^{-n}x^{n+1}\sqrt{ax^{2n}}{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{n+1}{n}\\2+\frac{1}{n}\end{matrix}\middle|-x^n\right)}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((a*x**(2*n))**(1/2)/(1+x**n)**(1/2)+2*(a*x**(2*n))**(1/2)/(2+n)
```

```
[Out] 2*x*x**(-n)*sqrt(a*x**(2*n))*hyper((1/2, 1/n), (1 + 1/n,), -x**n)
/(n + 2) + x**(-n)*x**(n + 1)*sqrt(a*x**(2*n))*hyper((1/2, (n + 1
)/n), (2 + 1/n,), -x**n)/(n + 1)
```

Mathematica [A] time = 0.0587505, size = 33, normalized size = 0.97

$$\frac{2ax^{n+1}\sqrt{x^n+1}}{(n+2)\sqrt{ax^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^(2*n)]/Sqrt[1 + x^n] + (2*Sqrt[a*x^(2*n)])/((2 + n)*x^n*Sqrt[
```

```
[Out] (2*a*x^(1 + n)*Sqrt[1 + x^n])/((2 + n)*Sqrt[a*x^(2*n)])
```

Maple [A] time = 0.037, size = 30, normalized size = 0.9

$$2\,{\frac{x\sqrt{1+x^n}\sqrt{a\left(x^n\right)^2}}{\left(2+n\right)x^n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^(2*n))^(1/2)/(1+x^n)^(1/2)+2*(a*x^(2*n))^(1/2)/(2+n)/(x^n)/(1+x^n)^(1
```

```
[Out] 2*x*(1+x^n)^(1/2)/(2+n)*(a*(x^n)^2)^(1/2)/(x^n)
```

Maxima [A] time = 0.843897, size = 24, normalized size = 0.71

$$\frac{2\sqrt{a}\sqrt{x^n+1}x}{n+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(a*x^(2*n))/sqrt(x^n + 1) + 2*sqrt(a*x^(2*n))/((n + 2)*sqrt(x^n + 1
```

```
[Out] 2*sqrt(a)*sqrt(x^n + 1)*x/(n + 2)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(a*x^(2*n))/sqrt(x^n + 1) + 2*sqrt(a*x^(2*n))/((n + 2)*sqrt(x^n + 1
```

```
[Out] Exception raised: TypeError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{2\sqrt{ax^{2n}}}{\sqrt{x^n+1}}\, dx + \int \frac{n\sqrt{ax^{2n}}}{\sqrt{x^n+1}}\, dx + \int \frac{2x^{-n}\sqrt{ax^{2n}}}{\sqrt{x^n+1}}\, dx}{n+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**(2*n))**(1/2)/(1+x**n)**(1/2)+2*(a*x**(2*n))**(1/2)/(2+n)/(x**
```

```
[Out] (Integral(2*sqrt(a*x**(2*n))/sqrt(x**n + 1), x) + Integral(n*sqrt
(a*x**(2*n))/sqrt(x**n + 1), x) + Integral(2*x**(-n)*sqrt(a*x**(2
*n))/sqrt(x**n + 1), x))/(n + 2)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax^{2\,n}}}{\sqrt{x^n + 1}} + \frac{2\,\sqrt{ax^{2\,n}}}{(n+2)\sqrt{x^n + 1}x^n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(a*x^(2*n))/sqrt(x^n + 1) + 2*sqrt(a*x^(2*n))/((n + 2)*sqrt(x^n + 1
```

```
[Out] integrate(sqrt(a*x^(2*n))/sqrt(x^n + 1) + 2*sqrt(a*x^(2*n))/((n +
 2)*sqrt(x^n + 1)*x^n), x)
```

3.236 $\int \frac{\sqrt{ax}}{\sqrt{d+ex}\sqrt{e+fx}}\,dx$

Optimal. Leaf size=114

$$\frac{2\sqrt{ax}\sqrt{df-e^2}\sqrt{\frac{e(e+fx)}{e^2-df}}E\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{d+ex}}{\sqrt{df-e^2}}\right)|1-\frac{e^2}{df}\right)}{e\sqrt{f}\sqrt{-\frac{ex}{d}}\sqrt{e+fx}}$$

```
[Out] (2*Sqrt[-e^2 + d*f]*Sqrt[a*x]*Sqrt[(e*(e + f*x))/(e^2 - d*f)]*Ell
ipticE[ArcSin[(Sqrt[f]*Sqrt[d + e*x])/Sqrt[-e^2 + d*f]], 1 - e^2/
(d*f)])/(e*Sqrt[f]*Sqrt[-((e*x)/d)]*Sqrt[e + f*x])
```

Rubi [A] time = 0.230828, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{2\sqrt{ax}\sqrt{df-e^2}\sqrt{\frac{e(e+fx)}{e^2-df}}E\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{d+ex}}{\sqrt{df-e^2}}\right)|1-\frac{e^2}{df}\right)}{e\sqrt{f}\sqrt{-\frac{ex}{d}}\sqrt{e+fx}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a*x]/(Sqrt[d + e*x]*Sqrt[e + f*x]),x]
```

```
[Out] (2*Sqrt[-e^2 + d*f]*Sqrt[a*x]*Sqrt[(e*(e + f*x))/(e^2 - d*f)]*Ell
ipticE[ArcSin[(Sqrt[f]*Sqrt[d + e*x])/Sqrt[-e^2 + d*f]], 1 - e^2/
(d*f)])/(e*Sqrt[f]*Sqrt[-((e*x)/d)]*Sqrt[e + f*x])
```

Rubi in Sympy [A] time = 27.888, size = 95, normalized size = 0.83

$$\frac{2\sqrt{ax}\sqrt{\frac{e(-e-fx)}{df-e^2}}\sqrt{df-e^2}E\left(\operatorname{asin}\left(\frac{\sqrt{f}\sqrt{d+ex}}{\sqrt{df-e^2}}\right)\middle|1-\frac{e^2}{df}\right)}{e\sqrt{f}\sqrt{-\frac{ex}{d}}\sqrt{e+fx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a*x)**(1/2)/(e*x+d)**(1/2)/(f*x+e)**(1/2),x)
```

```
[Out] 2*sqrt(a*x)*sqrt(e*(-e - f*x)/(d*f - e**2))*sqrt(d*f - e**2)*elli
ptic_e(asin(sqrt(f)*sqrt(d + e*x)/sqrt(d*f - e**2)), 1 - e**2/(d*
```

```
f))/(e*sqrt(f)*sqrt(-e*x/d)*sqrt(e + f*x))
```

Mathematica [C] time = 0.30274, size = 106, normalized size = 0.93

$$-\frac{2ie\sqrt{ax}\sqrt{\frac{fx}{e}+1}\left(E\left(i\sinh^{-1}\left(\sqrt{\frac{ex}{d}}\right)|\frac{df}{e^2}\right)-F\left(i\sinh^{-1}\left(\sqrt{\frac{ex}{d}}\right)|\frac{df}{e^2}\right)\right)}{f\sqrt{\frac{ex}{d+ex}}\sqrt{d+ex}\sqrt{e+fx}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a*x]/(Sqrt[d + e*x]*Sqrt[e + f*x]),x]
```

```
[Out] ((-2*I)*e*Sqrt[a*x]*Sqrt[1 + (f*x)/e]*(EllipticE[I*ArcSinh[Sqrt[(
e*x)/d]], (d*f)/e^2] - EllipticF[I*ArcSinh[Sqrt[(e*x)/d]], (d*f)/
e^2]))/(f*Sqrt[(e*x)/(d + e*x)]*Sqrt[d + e*x]*Sqrt[e + f*x])
```

Maple [A] time = 0.084, size = 191, normalized size = 1.7

$$-2\,\frac{d\sqrt{fx+e}\sqrt{ex+d}\sqrt{ax}}{e^2fx\,(efx^2+dfx+e^2x+de)}\left(e^2EllipticF\left(\sqrt{\frac{ex+d}{d}},\sqrt{\frac{df}{df-e^2}}\right)+EllipticE\left(\sqrt{\frac{ex+d}{d}},\sqrt{\frac{df}{df-e^2}}\right)df-EllipticE\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a*x)^(1/2)/(e*x+d)^(1/2)/(f*x+e)^(1/2),x)
```

```
[Out] -2*(e^2*EllipticF(((e*x+d)/d)^(1/2),(d*f/(d*f-e^2))^(1/2))+Ellipt
icE(((e*x+d)/d)^(1/2),(d*f/(d*f-e^2))^(1/2))*d*f-EllipticE(((e*x+
d)/d)^(1/2),(d*f/(d*f-e^2))^(1/2))*e^2)*(-e*x/d)^(1/2)*(-(f*x+e)*
e/(d*f-e^2))^(1/2)*((e*x+d)/d)^(1/2)*d*(f*x+e)^(1/2)*(e*x+d)^(1/2
)*(a*x)^(1/2)/f/e^2/x/(e*f*x^2+d*f*x+e^2*x+d*e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{ax}}{\sqrt{ex+d}\sqrt{fx+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(a*x)/(sqrt(e*x + d)*sqrt(f*x + e)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x)/(sqrt(e*x + d)*sqrt(f*x + e)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ax}}{\sqrt{ex+d}\sqrt{fx+e}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(a*x)/(sqrt(e*x + d)*sqrt(f*x + e)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*x)/(sqrt(e*x + d)*sqrt(f*x + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax}}{\sqrt{d+ex}\sqrt{e+fx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x)**(1/2)/(e*x+d)**(1/2)/(f*x+e)**(1/2),x)
```

```
[Out] Integral(sqrt(a*x)/(sqrt(d + e*x)*sqrt(e + f*x)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax}}{\sqrt{ex+d}\sqrt{fx+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(a*x)/(sqrt(e*x + d)*sqrt(f*x + e)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x)/(sqrt(e*x + d)*sqrt(f*x + e)), x)
```

3.237 $\int (ax^m)^r \, dx$

Optimal. Leaf size=16

$$\frac{x (ax^m)^r}{mr + 1}$$

```
[Out] (x*(a*x^m)^r)/(1 + m*r)
```

Rubi [A] time = 0.0126771, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286

$$\frac{x (ax^m)^r}{mr + 1}$$

Antiderivative was successfully verified.

```
[In]  Int[(a*x^m)^r,x]
```

```
[Out] (x*(a*x^m)^r)/(1 + m*r)
```

Rubi in Sympy [A] time = 2.02921, size = 22, normalized size = 1.38

$$\frac{x^{-mr} x^{mr+1} (ax^m)^r}{mr + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a*x**m)**r,x)
```

```
[Out] x**(-m*r)*x**(m*r + 1)*(a*x**m)**r/(m*r + 1)
```

Mathematica [A] time = 0.00507077, size = 16, normalized size = 1.

$$\frac{x (ax^m)^r}{mr + 1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a*x^m)^r,x]
```

```
[Out] (x*(a*x^m)^r)/(1 + m*r)
```

Maple [A] time = 0.002, size = 17, normalized size = 1.1

$$\frac{x\,(a x^m)^r}{mr+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a*x^m)^r,x)
```

```
[Out] x*(a*x^m)^r/(m*r+1)
```

Maxima [A] time = 0.705845, size = 23, normalized size = 1.44

$$\frac{a^r x (x^m)^r}{mr+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a*x^m)^r,x, algorithm="maxima")
```

```
[Out] a^r*x*(x^m)^r/(m*r + 1)
```

Fricas [A] time = 0.279681, size = 27, normalized size = 1.69

$$\frac{xe^{(mr\log(x)+r\log(a))}}{mr+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a*x^m)^r,x, algorithm="fricas")
```

```
[Out] x*e^(m*r*log(x) + r*log(a))/(m*r + 1)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a*x**m)**r,x)
```

```
[Out] Exception raised: TypeError
```

GIAC/XCAS [A] time = 0.262485, size = 27, normalized size = 1.69

$$\frac{xe^{(mr\ln(x)+r\ln(a))}}{mr+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a*x^m)^r,x, algorithm="giac")
```

```
[Out] x*e^(m*r*ln(x) + r*ln(a))/(m*r + 1)
```

3.238 $\int (ax^m)^r (bx^n)^s \, dx$

Optimal. Leaf size=26

$$\frac{x (ax^m)^r (bx^n)^s}{mr + ns + 1}$$

```
[Out] (x*(a*x^m)^r*(b*x^n)^s)/(1 + m*r + n*s)
```

Rubi [A] time = 0.0207064, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$\frac{x (ax^m)^r (bx^n)^s}{mr + ns + 1}$$

Antiderivative was successfully verified.

```
[In]  Int[(a*x^m)^r*(b*x^n)^s,x]
```

```
[Out] (x*(a*x^m)^r*(b*x^n)^s)/(1 + m*r + n*s)
```

Rubi in Sympy [A] time = 7.1562, size = 41, normalized size = 1.58

$$\frac{x^{-mr} x^{-ns} x^{mr+ns+1} (ax^m)^r (bx^n)^s}{mr + ns + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a*x**m)**r*(b*x**n)**s,x)
```

```
[Out] x**(-m*r)*x**(-n*s)*x**(m*r + n*s + 1)*(a*x**m)**r*(b*x**n)**s/(m
*r + n*s + 1)
```

Mathematica [A] time = 0.010642, size = 26, normalized size = 1.

$$\frac{x (ax^m)^r (bx^n)^s}{mr + ns + 1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a*x^m)^r*(b*x^n)^s,x]
```

```
[Out] (x*(a*x^m)^r*(b*x^n)^s)/(1 + m*r + n*s)
```

Maple [A] time = 0.003, size = 27, normalized size = 1.

$$\frac{x\,(ax^{m})^{r}\,(bx^{n})^{s}}{mr+ns+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a*x^m)^r*(b*x^n)^s,x)
```

```
[Out] x*(a*x^m)^r*(b*x^n)^s/(m*r+n*s+1)
```

Maxima [A] time = 0.722567, size = 43, normalized size = 1.65

$$\frac{a^{r}b^{s}xe^{\left(r\log\left(x^{m}\right)+s\log\left(x^{n}\right)\right)}}{mr+ns+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a*x^m)^r*(b*x^n)^s,x, algorithm="maxima")
```

```
[Out] a^r*b^s*x*e^(r*log(x^m) + s*log(x^n))/(m*r + n*s + 1)
```

Fricas [A] time = 0.333926, size = 43, normalized size = 1.65

$$\frac{xe^{\left(mr\log\left(x\right)+ns\log\left(x\right)+r\log\left(a\right)+s\log\left(b\right)\right)}}{mr+ns+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a*x^m)^r*(b*x^n)^s,x, algorithm="fricas")
```

```
[Out] x*e^(m*r*log(x) + n*s*log(x) + r*log(a) + s*log(b))/(m*r + n*s +
1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a*x**m)**r*(b*x**n)**s,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.265955, size = 43, normalized size = 1.65

$$\frac{xe^{(mr\ln(x)+ns\ln(x)+r\ln(a)+s\ln(b))}}{mr+ns+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a*x^m)^r*(b*x^n)^s,x, algorithm="giac")
```

```
[Out] x*e^(m*r*ln(x) + n*s*ln(x) + r*ln(a) + s*ln(b))/(m*r + n*s + 1)
```

3.239 $\int (ax^m)^r (bx^n)^s (cx^p)^t \, dx$

Optimal. Leaf size=36

$$\frac{x (ax^m)^r (bx^n)^s (cx^p)^t}{mr + ns + pt + 1}$$

```
[Out] (x*(a*x^m)^r*(b*x^n)^s*(c*x^p)^t)/(1 + m*r + n*s + p*t)
```

Rubi [A] time = 0.0323343, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{x (ax^m)^r (bx^n)^s (cx^p)^t}{mr + ns + pt + 1}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^m)^r*(b*x^n)^s*(c*x^p)^t,x]
```

```
[Out] (x*(a*x^m)^r*(b*x^n)^s*(c*x^p)^t)/(1 + m*r + n*s + p*t)
```

Rubi in Sympy [A] time = 20.9521, size = 60, normalized size = 1.67

$$\frac{x^{-mr} x^{-ns} x^{-pt} x^{mr+ns+pt+1} (ax^m)^r (bx^n)^s (cx^p)^t}{mr + ns + pt + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((a*x**m)**r*(b*x**n)**s*(c*x**p)**t,x)
```

```
[Out] x**(-m*r)*x**(-n*s)*x**(-p*t)*x**(m*r + n*s + p*t + 1)*(a*x**m)**
r*(b*x**n)**s*(c*x**p)**t/(m*r + n*s + p*t + 1)
```

Mathematica [A] time = 0.016587, size = 36, normalized size = 1.

$$\frac{x (ax^m)^r (bx^n)^s (cx^p)^t}{mr + ns + pt + 1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a*x^m)^r*(b*x^n)^s*(c*x^p)^t,x]
```

```
[Out] (x*(a*x^m)^r*(b*x^n)^s*(c*x^p)^t)/(1 + m*r + n*s + p*t)
```

Maple [A] time = 0.003, size = 37, normalized size = 1.

$$\frac{x\left(a x^{m}\right)^{r}\left(b x^{n}\right)^{s}\left(c x^{p}\right)^{t}}{m r+n s+p t+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a*x^m)^r*(b*x^n)^s*(c*x^p)^t,x)
```

```
[Out] x*(a*x^m)^r*(b*x^n)^s*(c*x^p)^t/(m*r+n*s+p*t+1)
```

Maxima [A] time = 0.756761, size = 59, normalized size = 1.64

$$\frac{a^{r} b^{s} c^{t} x e^{\left(r \log\left(x^{m}\right)+s \log\left(x^{n}\right)+t \log\left(x^{p}\right)\right)}}{m r+n s+p t+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a*x^m)^r*(b*x^n)^s*(c*x^p)^t,x, algorithm="maxima")
```

```
[Out] a^r*b^s*c^t*x*e^(r*log(x^m) + s*log(x^n) + t*log(x^p))/(m*r + n*s
 + p*t + 1)
```

Fricas [A] time = 0.299202, size = 59, normalized size = 1.64

$$\frac{x e^{\left(m r \log\left(x\right)+n s \log\left(x\right)+p t \log\left(x\right)+r \log\left(a\right)+s \log\left(b\right)+t \log\left(c\right)\right)}}{m r+n s+p t+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a*x^m)^r*(b*x^n)^s*(c*x^p)^t,x, algorithm="fricas")
```

```
[Out] x*e^(m*r*log(x) + n*s*log(x) + p*t*log(x) + r*log(a) + s*log(b) +
 t*log(c))/(m*r + n*s + p*t + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a*x**m)**r*(b*x**n)**s*(c*x**p)**t,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.265416, size = 59, normalized size = 1.64

$$\frac{xe^{\left(mr\ln\left(x\right)+ns\ln\left(x\right)+pt\ln\left(x\right)+r\ln\left(a\right)+s\ln\left(b\right)+t\ln\left(c\right)\right)}}{mr+ns+pt+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a*x^m)^r*(b*x^n)^s*(c*x^p)^t,x, algorithm="giac")
```

```
[Out] x*e^(m*r*ln(x) + n*s*ln(x) + p*t*ln(x) + r*ln(a) + s*ln(b) + t*ln
(c))/(m*r + n*s + p*t + 1)
```

3.240 $\int \frac{x^2}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx$

Optimal. Leaf size=147

$$\frac{2a^2(a+bx)^{3/2}}{3b^3(a-c)}-\frac{2c^2(bx+c)^{3/2}}{3b^3(a-c)}+\frac{2(a+bx)^{7/2}}{7b^3(a-c)}-\frac{4a(a+bx)^{5/2}}{5b^3(a-c)}-\frac{2(bx+c)^{7/2}}{7b^3(a-c)}+\frac{4c(bx+c)^{5/2}}{5b^3(a-c)}$$

```
[Out] (2*a^2*(a + b*x)^(3/2))/(3*b^3*(a - c)) - (4*a*(a + b*x)^(5/2))/(
5*b^3*(a - c)) + (2*(a + b*x)^(7/2))/(7*b^3*(a - c)) - (2*c^2*(c
+ b*x)^(3/2))/(3*b^3*(a - c)) + (4*c*(c + b*x)^(5/2))/(5*b^3*(a -
 c)) - (2*(c + b*x)^(7/2))/(7*b^3*(a - c))
```

Rubi [A] time = 0.250968, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$

$$\frac{2a^2(a+bx)^{3/2}}{3b^3(a-c)}-\frac{2c^2(bx+c)^{3/2}}{3b^3(a-c)}+\frac{2(a+bx)^{7/2}}{7b^3(a-c)}-\frac{4a(a+bx)^{5/2}}{5b^3(a-c)}-\frac{2(bx+c)^{7/2}}{7b^3(a-c)}+\frac{4c(bx+c)^{5/2}}{5b^3(a-c)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(Sqrt[a + b*x] + Sqrt[c + b*x]),x]
```

```
[Out] (2*a^2*(a + b*x)^(3/2))/(3*b^3*(a - c)) - (4*a*(a + b*x)^(5/2))/(
5*b^3*(a - c)) + (2*(a + b*x)^(7/2))/(7*b^3*(a - c)) - (2*c^2*(c
+ b*x)^(3/2))/(3*b^3*(a - c)) + (4*c*(c + b*x)^(5/2))/(5*b^3*(a -
 c)) - (2*(c + b*x)^(7/2))/(7*b^3*(a - c))
```

Rubi in Sympy [A] time = 27.5271, size = 121, normalized size = 0.82

$$\frac{2a^2\left(a+bx\right)^{\frac{3}{2}}}{3b^3\left(a-c\right)}-\frac{4a\left(a+bx\right)^{\frac{5}{2}}}{5b^3\left(a-c\right)}-\frac{2c^2\left(bx+c\right)^{\frac{3}{2}}}{3b^3\left(a-c\right)}+\frac{4c\left(bx+c\right)^{\frac{5}{2}}}{5b^3\left(a-c\right)}+\frac{2\left(a+bx\right)^{\frac{7}{2}}}{7b^3\left(a-c\right)}-\frac{2\left(bx+c\right)^{\frac{7}{2}}}{7b^3\left(a-c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/((b*x+a)**(1/2)+(b*x+c)**(1/2)),x)
```

```
[Out] 2*a**2*(a + b*x)**(3/2)/(3*b**3*(a - c)) - 4*a*(a + b*x)**(5/2)/(
5*b**3*(a - c)) - 2*c**2*(b*x + c)**(3/2)/(3*b**3*(a - c)) + 4*c*
(b*x + c)**(5/2)/(5*b**3*(a - c)) + 2*(a + b*x)**(7/2)/(7*b**3*(a
 - c)) - 2*(b*x + c)**(7/2)/(7*b**3*(a - c))
```

Mathematica [A] time = 0.154433, size = 140, normalized size = 0.95

$$\frac{2\left(8a^3\sqrt{a+bx}-4a^2bx\sqrt{a+bx}+15b^3x^3\left(\sqrt{a+bx}-\sqrt{bx+c}\right)+3ab^2x^2\sqrt{a+bx}-3b^2cx^2\sqrt{bx+c}-8c^3\sqrt{bx+c}+4bc^2x\sqrt{b\right.}{105b^3(a-c)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(Sqrt[a + b*x] + Sqrt[c + b*x]),x]
```

```
[Out] (2*(8*a^3*Sqrt[a + b*x] - 4*a^2*b*x*Sqrt[a + b*x] + 3*a*b^2*x^2*S
qrt[a + b*x] - 8*c^3*Sqrt[c + b*x] + 4*b*c^2*x*Sqrt[c + b*x] - 3*
b^2*c*x^2*Sqrt[c + b*x] + 15*b^3*x^3*(Sqrt[a + b*x] - Sqrt[c + b*
x])))/(105*b^3*(a - c))
```

Maple [A] time = 0.007, size = 90, normalized size = 0.6

$$2\,\frac{1/7\,\left(bx+a\right)^{7/2}-2/5\,\left(bx+a\right)^{5/2}a+1/3\,a^2\left(bx+a\right)^{3/2}}{\left(a-c\right)b^3}-2\,\frac{1/7\,\left(bx+c\right)^{7/2}-2/5\,\left(bx+c\right)^{5/2}c+1/3\,c^2\left(bx+c\right)^{3/2}}{\left(a-c\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x)
```

```
[Out] 2/(a-c)/b^3*(1/7*(b*x+a)^(7/2)-2/5*(b*x+a)^(5/2)*a+1/3*a^2*(b*x+a
)^(3/2))-2/(a-c)/b^3*(1/7*(b*x+c)^(7/2)-2/5*(b*x+c)^(5/2)*c+1/3*c
^2*(b*x+c)^(3/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2}{\sqrt{bx+a}+\sqrt{bx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(sqrt(b*x + a) + sqrt(b*x + c)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(b*x + a) + sqrt(b*x + c)), x)
```

Fricas [A] time = 0.296346, size = 127, normalized size = 0.86

$$\frac{2\left(\left(15\,b^3x^3+3\,ab^2x^2-4\,a^2bx+8\,a^3\right)\sqrt{bx+a}-\left(15\,b^3x^3+3\,b^2cx^2-4\,bc^2x+8\,c^3\right)\sqrt{bx+c}\right)}{105\,(ab^3-b^3c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(sqrt(b*x + a) + sqrt(b*x + c)),x, algorithm="fricas")
```

```
[Out] 2/105*((15*b^3*x^3 + 3*a*b^2*x^2 - 4*a^2*b*x + 8*a^3)*sqrt(b*x +
a) - (15*b^3*x^3 + 3*b^2*c*x^2 - 4*b*c^2*x + 8*c^3)*sqrt(b*x + c)
)/(a*b^3 - b^3*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{a+bx}+\sqrt{bx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/((b*x+a)**(1/2)+(b*x+c)**(1/2)),x)
```

```
[Out] Integral(x**2/(sqrt(a + b*x) + sqrt(b*x + c)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(sqrt(b*x + a) + sqrt(b*x + c)),x, algorithm="giac")
```

```
[Out] undef
```

3.241 $\int \frac{x}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx$

Optimal. Leaf size=95

$$\frac{2(a+bx)^{5/2}}{5b^2(a-c)}-\frac{2a(a+bx)^{3/2}}{3b^2(a-c)}-\frac{2(bx+c)^{5/2}}{5b^2(a-c)}+\frac{2c(bx+c)^{3/2}}{3b^2(a-c)}$$

```
[Out] (-2*a*(a + b*x)^(3/2))/(3*b^2*(a - c)) + (2*(a + b*x)^(5/2))/(5*b
^2*(a - c)) + (2*c*(c + b*x)^(3/2))/(3*b^2*(a - c)) - (2*(c + b*x
)^(5/2))/(5*b^2*(a - c))
```

Rubi [A] time = 0.150951, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087

$$\frac{2(a+bx)^{5/2}}{5b^2(a-c)}-\frac{2a(a+bx)^{3/2}}{3b^2(a-c)}-\frac{2(bx+c)^{5/2}}{5b^2(a-c)}+\frac{2c(bx+c)^{3/2}}{3b^2(a-c)}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(Sqrt[a + b*x] + Sqrt[c + b*x]),x]
```

```
[Out] (-2*a*(a + b*x)^(3/2))/(3*b^2*(a - c)) + (2*(a + b*x)^(5/2))/(5*b
^2*(a - c)) + (2*c*(c + b*x)^(3/2))/(3*b^2*(a - c)) - (2*(c + b*x
)^(5/2))/(5*b^2*(a - c))
```

Rubi in Sympy [A] time = 17.4956, size = 76, normalized size = 0.8

$$-\frac{2a\,(a+bx)^{\frac{3}{2}}}{3b^2\,(a-c)}+\frac{2c\,(bx+c)^{\frac{3}{2}}}{3b^2\,(a-c)}+\frac{2\,(a+bx)^{\frac{5}{2}}}{5b^2\,(a-c)}-\frac{2\,(bx+c)^{\frac{5}{2}}}{5b^2\,(a-c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/((b*x+a)**(1/2)+(b*x+c)**(1/2)),x)
```

```
[Out] -2*a*(a + b*x)**(3/2)/(3*b**2*(a - c)) + 2*c*(b*x + c)**(3/2)/(3*
b**2*(a - c)) + 2*(a + b*x)**(5/2)/(5*b**2*(a - c)) - 2*(b*x + c)
**(5/2)/(5*b**2*(a - c))
```

Mathematica [A] time = 0.0992523, size = 100, normalized size = 1.05

$$\frac{-4a^2\sqrt{a+bx}+6b^2x^2\left(\sqrt{a+bx}-\sqrt{bx+c}\right)+2abx\sqrt{a+bx}+4c^2\sqrt{bx+c}-2bcx\sqrt{bx+c}}{15b^2(a-c)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(Sqrt[a + b*x] + Sqrt[c + b*x]),x]
```

```
[Out] (-4*a^2*Sqrt[a + b*x] + 2*a*b*x*Sqrt[a + b*x] + 4*c^2*Sqrt[c + b*
x] - 2*b*c*x*Sqrt[c + b*x] + 6*b^2*x^2*(Sqrt[a + b*x] - Sqrt[c +
b*x]))/(15*b^2*(a - c))
```

Maple [A] time = 0.004, size = 66, normalized size = 0.7

$$2\,\frac{1/5\,(bx+a)^{5/2}-1/3\,(bx+a)^{3/2}a}{(a-c)\,b^2}-2\,\frac{1/5\,(bx+c)^{5/2}-1/3\,(bx+c)^{3/2}c}{(a-c)\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x)
```

```
[Out] 2/(a-c)/b^2*(1/5*(b*x+a)^(5/2)-1/3*(b*x+a)^(3/2)*a)-2/(a-c)/b^2*(
1/5*(b*x+c)^(5/2)-1/3*(b*x+c)^(3/2)*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{bx+a}+\sqrt{bx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(b*x + a) + sqrt(b*x + c)),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(b*x + a) + sqrt(b*x + c)), x)
```

Fricas [A] time = 0.273224, size = 95, normalized size = 1.

$$\frac{2\left(\left(3\,b^2x^2+abx-2\,a^2\right)\sqrt{bx+a}-\left(3\,b^2x^2+bcx-2\,c^2\right)\sqrt{bx+c}\right)}{15\,(ab^2-b^2c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(b*x + a) + sqrt(b*x + c)),x, algorithm="fricas")
```

```
[Out] 2/15*((3*b^2*x^2 + a*b*x - 2*a^2)*sqrt(b*x + a) - (3*b^2*x^2 + b*
c*x - 2*c^2)*sqrt(b*x + c))/(a*b^2 - b^2*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a+bx}+\sqrt{bx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((b*x+a)**(1/2)+(b*x+c)**(1/2)),x)
```

```
[Out] Integral(x/(sqrt(a + b*x) + sqrt(b*x + c)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\mathit{undef}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(b*x + a) + sqrt(b*x + c)),x, algorithm="giac")
```

```
[Out] undef
```

3.242 $\int \frac{1}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx$

Optimal. Leaf size=47

$$\frac{2(a+bx)^{3/2}}{3b(a-c)}-\frac{2(bx+c)^{3/2}}{3b(a-c)}$$

```
[Out] (2*(a + b*x)^(3/2))/(3*b*(a - c)) - (2*(c + b*x)^(3/2))/(3*b*(a -
 c))
```

Rubi [A] time = 0.0816449, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048

$$\frac{2(a+bx)^{3/2}}{3b(a-c)}-\frac{2(bx+c)^{3/2}}{3b(a-c)}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[a + b*x] + Sqrt[c + b*x])^(-1),x]
```

```
[Out] (2*(a + b*x)^(3/2))/(3*b*(a - c)) - (2*(c + b*x)^(3/2))/(3*b*(a -
 c))
```

Rubi in Sympy [A] time = 5.57357, size = 32, normalized size = 0.68

$$\frac{2\,(a+bx)^{\frac{3}{2}}}{3b\,(a-c)}-\frac{2\,(bx+c)^{\frac{3}{2}}}{3b\,(a-c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/((b*x+a)**(1/2)+(b*x+c)**(1/2)),x)
```

```
[Out] 2*(a + b*x)**(3/2)/(3*b*(a - c)) - 2*(b*x + c)**(3/2)/(3*b*(a - c
))
```

Mathematica [A] time = 0.0766401, size = 63, normalized size = 1.34

$$\frac{2a\sqrt{a+bx}+2bx\sqrt{a+bx}-2c\sqrt{bx+c}-2bx\sqrt{bx+c}}{3ab-3bc}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[a + b*x] + Sqrt[c + b*x])^(-1),x]
```

```
[Out] (2*a*Sqrt[a + b*x] + 2*b*x*Sqrt[a + b*x] - 2*c*Sqrt[c + b*x] - 2*
b*x*Sqrt[c + b*x])/(3*a*b - 3*b*c)
```

Maple [A] time = 0.003, size = 40, normalized size = 0.9

$$\frac{2}{3\,b\,(a-c)}\,(bx+a)^{\frac{3}{2}}-\frac{2}{3\,b\,(a-c)}\,(bx+c)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x)
```

```
[Out] 2/3*(b*x+a)^(3/2)/b/(a-c)-2/3*(b*x+c)^(3/2)/b/(a-c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx+a}+\sqrt{bx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(b*x + a) + sqrt(b*x + c)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x + a) + sqrt(b*x + c)), x)
```

Fricas [A] time = 0.274852, size = 39, normalized size = 0.83

$$\frac{2\left((bx+a)^{\frac{3}{2}}-(bx+c)^{\frac{3}{2}}\right)}{3\,(ab-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(b*x + a) + sqrt(b*x + c)),x, algorithm="fricas")
```

```
[Out] 2/3*((b*x + a)^(3/2) - (b*x + c)^(3/2))/(a*b - b*c)
```

Sympy [A] time = 2.4676, size = 136, normalized size = 2.89

$$\begin{cases} \frac{2a}{3b\sqrt{a+bx}+3b\sqrt{bx+c}} + \frac{4bx}{3b\sqrt{a+bx}+3b\sqrt{bx+c}} + \frac{2c}{3b\sqrt{a+bx}+3b\sqrt{bx+c}} + \frac{2\sqrt{a+bx}\sqrt{bx+c}}{3b\sqrt{a+bx}+3b\sqrt{bx+c}} & \text{for } b \neq 0 \\ \frac{x}{\sqrt{a}+\sqrt{c}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x+a)**(1/2)+(b*x+c)**(1/2)),x)
```

```
[Out] Piecewise((2*a/(3*b*sqrt(a + b*x) + 3*b*sqrt(b*x + c)) + 4*b*x/(3
*b*sqrt(a + b*x) + 3*b*sqrt(b*x + c)) + 2*c/(3*b*sqrt(a + b*x) +
3*b*sqrt(b*x + c)) + 2*sqrt(a + b*x)*sqrt(b*x + c)/(3*b*sqrt(a +
b*x) + 3*b*sqrt(b*x + c)), Ne(b, 0)), (x/(sqrt(a) + sqrt(c)), Tru
e))
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(b*x + a) + sqrt(b*x + c)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.243 $\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}\,dx$

Optimal. Leaf size=97

$$\frac{2\sqrt{a+bx}}{a-c}-\frac{2\sqrt{bx+c}}{a-c}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a-c}+\frac{2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{a-c}$$

```
[Out] (2*Sqrt[a + b*x])/(a - c) - (2*Sqrt[c + b*x])/(a - c) - (2*Sqrt[a
]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(a - c) + (2*Sqrt[c]*ArcTanh[Sq
rt[c + b*x]/Sqrt[c]])/(a - c)
```

Rubi [A] time = 0.197203, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16

$$\frac{2\sqrt{a+bx}}{a-c}-\frac{2\sqrt{bx+c}}{a-c}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a-c}+\frac{2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{a-c}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(Sqrt[a + b*x] + Sqrt[c + b*x])),x]
```

```
[Out] (2*Sqrt[a + b*x])/(a - c) - (2*Sqrt[c + b*x])/(a - c) - (2*Sqrt[a
]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(a - c) + (2*Sqrt[c]*ArcTanh[Sq
rt[c + b*x]/Sqrt[c]])/(a - c)
```

Rubi in Sympy [A] time = 18.0611, size = 76, normalized size = 0.78

$$-\frac{2\sqrt{a}\operatorname{atanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a-c}+\frac{2\sqrt{c}\operatorname{atanh}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{a-c}+\frac{2\sqrt{a+bx}}{a-c}-\frac{2\sqrt{bx+c}}{a-c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/((b*x+a)**(1/2)+(b*x+c)**(1/2)),x)
```

```
[Out] -2*sqrt(a)*atanh(sqrt(a + b*x)/sqrt(a))/(a - c) + 2*sqrt(c)*atanh
(sqrt(b*x + c)/sqrt(c))/(a - c) + 2*sqrt(a + b*x)/(a - c) - 2*sqr
t(b*x + c)/(a - c)
```

Mathematica [A] time = 0.0769796, size = 75, normalized size = 0.77

$$\frac{2\left(\sqrt{a+bx}-\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)-\sqrt{bx+c}+\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)\right)}{a-c}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(Sqrt[a + b*x] + Sqrt[c + b*x])),x]
```

```
[Out] (2*(Sqrt[a + b*x] - Sqrt[c + b*x] - Sqrt[a]*ArcTanh[Sqrt[a + b*x]
/Sqrt[a]] + Sqrt[c]*ArcTanh[Sqrt[c + b*x]/Sqrt[c]]))/(a - c)
```

Maple [A] time = 0.008, size = 73, normalized size = 0.8

$$\frac{1}{a-c}\left(2\sqrt{bx+a}-2\sqrt{a}Artanh\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)-\frac{1}{a-c}\left(2\sqrt{bx+c}-2\sqrt{c}Artanh\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x)
```

```
[Out] 1/(a-c)*(2*(b*x+a)^(1/2)-2*a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2))
)-1/(a-c)*(2*(b*x+c)^(1/2)-2*c^(1/2)*arctanh((b*x+c)^(1/2)/c^(1/2
)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\sqrt{bx+a}+\sqrt{bx+c}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(sqrt(b*x + a) + sqrt(b*x + c))),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*(sqrt(b*x + a) + sqrt(b*x + c))), x)
```

Fricas [A] time = 0.342296, size = 1, normalized size = 0.01

$$\left[-\frac{\sqrt{a}\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+\sqrt{c}\log\left(\frac{bx-2\sqrt{bx+c}\sqrt{c}+2c}{x}\right)-2\sqrt{bx+a}+2\sqrt{bx+c}}{a-c}, \frac{2\sqrt{-c}\arctan\left(\frac{\sqrt{bx+c}}{\sqrt{-c}}\right)-\sqrt{a}\log\left(\frac{bx+2\sqrt{\ldots}}{\ldots}\right.}{a-\ldots}\right.$$

$$-\frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)+\sqrt{c}\log\left(\frac{bx-2\sqrt{bx+c}\sqrt{c}+2c}{x}\right)-2\sqrt{bx+a}+2\sqrt{bx+c}}{a-c},$$

$$\left.-\frac{2\left(\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)-\sqrt{-c}\arctan\left(\frac{\sqrt{bx+c}}{\sqrt{-c}}\right)-\sqrt{bx+a}+\sqrt{bx+c}\right)}{a-c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(sqrt(b*x + a) + sqrt(b*x + c))),x, algorithm="fricas")
```

```
[Out] [-(sqrt(a)*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + sqrt(c)
*log((b*x - 2*sqrt(b*x + c)*sqrt(c) + 2*c)/x) - 2*sqrt(b*x + a) +
 2*sqrt(b*x + c))/(a - c), (2*sqrt(-c)*arctan(sqrt(b*x + c)/sqrt(
-c)) - sqrt(a)*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*s
qrt(b*x + a) - 2*sqrt(b*x + c))/(a - c), -(2*sqrt(-a)*arctan(sqrt
(b*x + a)/sqrt(-a)) + sqrt(c)*log((b*x - 2*sqrt(b*x + c)*sqrt(c)
+ 2*c)/x) - 2*sqrt(b*x + a) + 2*sqrt(b*x + c))/(a - c), -2*(sqrt(
-a)*arctan(sqrt(b*x + a)/sqrt(-a)) - sqrt(-c)*arctan(sqrt(b*x + c
)/sqrt(-c)) - sqrt(b*x + a) + sqrt(b*x + c))/(a - c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{bx+c}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)**(1/2)+(b*x+c)**(1/2)),x)
```

```
[Out] Integral(1/(x*(sqrt(a + b*x) + sqrt(b*x + c))), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x*(sqrt(b*x + a) + sqrt(b*x + c))),x, algorithm="giac")

[Out] Exception raised: TypeError
```

3.244 $\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}\,dx$

Optimal. Leaf size=103

$$-\frac{\sqrt{a+bx}}{x(a-c)}+\frac{\sqrt{bx+c}}{x(a-c)}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}(a-c)}+\frac{b\tanh^{-1}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{\sqrt{c}(a-c)}$$

```
[Out] -(Sqrt[a + b*x]/((a - c)*x)) + Sqrt[c + b*x]/((a - c)*x) - (b*Arc
Tanh[Sqrt[a + b*x]/Sqrt[a]])/(Sqrt[a]*(a - c)) + (b*ArcTanh[Sqrt[
c + b*x]/Sqrt[c]])/((a - c)*Sqrt[c])
```

Rubi [A] time = 0.199084, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$

$$-\frac{\sqrt{a+bx}}{x(a-c)}+\frac{\sqrt{bx+c}}{x(a-c)}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}(a-c)}+\frac{b\tanh^{-1}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{\sqrt{c}(a-c)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(Sqrt[a + b*x] + Sqrt[c + b*x])),x]
```

```
[Out] -(Sqrt[a + b*x]/((a - c)*x)) + Sqrt[c + b*x]/((a - c)*x) - (b*Arc
Tanh[Sqrt[a + b*x]/Sqrt[a]])/(Sqrt[a]*(a - c)) + (b*ArcTanh[Sqrt[
c + b*x]/Sqrt[c]])/((a - c)*Sqrt[c])
```

Rubi in Sympy [A] time = 18.6214, size = 76, normalized size = 0.74

$$\frac{b\,\text{atanh}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{\sqrt{c}\,(a-c)}-\frac{\sqrt{a+bx}}{x\,(a-c)}+\frac{\sqrt{bx+c}}{x\,(a-c)}-\frac{b\,\text{atanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}\,(a-c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/((b*x+a)**(1/2)+(b*x+c)**(1/2)),x)
```

```
[Out] b*atanh(sqrt(b*x + c)/sqrt(c))/(sqrt(c)*(a - c)) - sqrt(a + b*x)/
(x*(a - c)) + sqrt(b*x + c)/(x*(a - c)) - b*atanh(sqrt(a + b*x)/s
qrt(a))/(sqrt(a)*(a - c))
```

Mathematica [A] time = 0.18027, size = 81, normalized size = 0.79

$$\frac{-\sqrt{a+bx}-\frac{bx\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}+\sqrt{bx+c}+\frac{bx\tanh^{-1}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{\sqrt{c}}}{ax-cx}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(Sqrt[a + b*x] + Sqrt[c + b*x])),x]
```

```
[Out] (-Sqrt[a + b*x] + Sqrt[c + b*x] - (b*x*ArcTanh[Sqrt[a + b*x]/Sqrt
[a]])/Sqrt[a] + (b*x*ArcTanh[Sqrt[c + b*x]/Sqrt[c]])/Sqrt[c])/(a*
x - c*x)
```

Maple [A] time = 0.02, size = 88, normalized size = 0.9

$$2\frac{b}{a-c}\left(-1/2\frac{\sqrt{bx+a}}{bx}-1/2\frac{1}{\sqrt{a}}Artanh\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)-2\frac{b}{a-c}\left(-1/2\frac{\sqrt{bx+c}}{bx}-1/2\frac{1}{\sqrt{c}}Artanh\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x)
```

```
[Out] 2/(a-c)*b*(-1/2*(b*x+a)^(1/2)/x/b-1/2*arctanh((b*x+a)^(1/2)/a^(1/
2))/a^(1/2))-2/(a-c)*b*(-1/2*(b*x+c)^(1/2)/x/b-1/2/c^(1/2)*arctan
h((b*x+c)^(1/2)/c^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x^2\left(\sqrt{bx+a}+\sqrt{bx+c}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^2*(sqrt(b*x + a) + sqrt(b*x + c))),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^2*(sqrt(b*x + a) + sqrt(b*x + c))), x)
```

Fricas [A] time = 0.313997, size = 1, normalized size = 0.01

$$\left[-\frac{b\sqrt{c}x\log\left(\frac{(bx+2\,a)\sqrt{a}+2\,\sqrt{bx+a}a}{x}\right)+\sqrt{a}bx\log\left(\frac{(bx+2\,c)\sqrt{c}-2\,\sqrt{bx+c}c}{x}\right)+2\,\sqrt{bx+a}\sqrt{a}\sqrt{c}-2\,\sqrt{bx+c}\sqrt{a}\sqrt{c}}{2\,(a-c)\sqrt{a}\sqrt{c}x},\right.$$
$$-\frac{2\,\sqrt{a}bx\arctan\left(\frac{c}{\sqrt{bx+c}\sqrt{-c}}\right)+b\sqrt{-c}x\log\left(\frac{(bx+2\,a)\sqrt{a}+2\,\sqrt{bx+a}a}{x}\right)+2\,\sqrt{bx+a}\sqrt{a}\sqrt{-c}-2\,\sqrt{bx+c}\sqrt{a}\sqrt{-c}}{2\,(a-c)\sqrt{a}\sqrt{-c}x},\ \frac{2\,b\sqrt{c}x\arctan\left(\frac{}{\sqrt{b}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(sqrt(b*x + a) + sqrt(b*x + c))),x, algorithm="fricas")
```

```
[Out] [-1/2*(b*sqrt(c)*x*log(((b*x + 2*a)*sqrt(a) + 2*sqrt(b*x + a)*a)/
x) + sqrt(a)*b*x*log(((b*x + 2*c)*sqrt(c) - 2*sqrt(b*x + c)*c)/x)
 + 2*sqrt(b*x + a)*sqrt(a)*sqrt(c) - 2*sqrt(b*x + c)*sqrt(a)*sqrt
(c))/((a - c)*sqrt(a)*sqrt(c)*x), -1/2*(2*sqrt(a)*b*x*arctan(c/(s
qrt(b*x + c)*sqrt(-c))) + b*sqrt(-c)*x*log(((b*x + 2*a)*sqrt(a) +
 2*sqrt(b*x + a)*a)/x) + 2*sqrt(b*x + a)*sqrt(a)*sqrt(-c) - 2*sqr
t(b*x + c)*sqrt(a)*sqrt(-c))/((a - c)*sqrt(a)*sqrt(-c)*x), 1/2*(2
*b*sqrt(c)*x*arctan(a/(sqrt(b*x + a)*sqrt(-a))) - sqrt(-a)*b*x*lo
g(((b*x + 2*c)*sqrt(c) - 2*sqrt(b*x + c)*c)/x) - 2*sqrt(b*x + a)*
sqrt(-a)*sqrt(c) + 2*sqrt(b*x + c)*sqrt(-a)*sqrt(c))/(sqrt(-a)*(a
 - c)*sqrt(c)*x), (b*sqrt(-c)*x*arctan(a/(sqrt(b*x + a)*sqrt(-a))
) - sqrt(-a)*b*x*arctan(c/(sqrt(b*x + c)*sqrt(-c))) - sqrt(b*x +
a)*sqrt(-a)*sqrt(-c) + sqrt(b*x + c)*sqrt(-a)*sqrt(-c))/(sqrt(-a)
*(a - c)*sqrt(-c)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{bx+c}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/((b*x+a)**(1/2)+(b*x+c)**(1/2)),x)
```

```
[Out] Integral(1/(x**2*(sqrt(a + b*x) + sqrt(b*x + c))), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^2*(sqrt(b*x + a) + sqrt(b*x + c))),x, algorithm="giac")
```

```
[Out] Timed out
```

3.245 $\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx$

Optimal. Leaf size=228

$$\frac{5(a+c)(a+bx)^{3/2}(bx+c)^{3/2}}{12b^3(a-c)^2}+\frac{\left(4ac-5(a+c)^2\right)(a+bx)^{3/2}\sqrt{bx+c}}{16b^3(a-c)^2}-\frac{\left(4ac-5(a+c)^2\right)\sqrt{a+bx}\sqrt{bx+c}}{32b^3(a-c)}-\frac{\left(4ac-5(a+c)^2\right)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{bx+c}}\right)}{32b^3}-\frac{x(a+bx)^{3/2}(bx+c)^{3/2}}{2b^2(a-c)^2}+\frac{bx^4}{2(a-c)^2}+\frac{x^3(a+c)}{3(a-c)^2}$$

```
[Out] ((a + c)*x^3)/(3*(a - c)^2) + (b*x^4)/(2*(a - c)^2) - ((4*a*c - 5
*(a + c)^2)*Sqrt[a + b*x]*Sqrt[c + b*x])/(32*b^3*(a - c)) + ((4*a
*c - 5*(a + c)^2)*(a + b*x)^(3/2)*Sqrt[c + b*x])/(16*b^3*(a - c)^
2) + (5*(a + c)*(a + b*x)^(3/2)*(c + b*x)^(3/2))/(12*b^3*(a - c)^
2) - (x*(a + b*x)^(3/2)*(c + b*x)^(3/2))/(2*b^2*(a - c)^2) - ((4*
a*c - 5*(a + c)^2)*ArcTanh[Sqrt[a + b*x]/Sqrt[c + b*x]])/(32*b^3)
```

Rubi [A] time = 0.751523, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28

$$\frac{5(a+c)(a+bx)^{3/2}(bx+c)^{3/2}}{12b^3(a-c)^2}+\frac{\left(4ac-5(a+c)^2\right)(a+bx)^{3/2}\sqrt{bx+c}}{16b^3(a-c)^2}-\frac{\left(4ac-5(a+c)^2\right)\sqrt{a+bx}\sqrt{bx+c}}{32b^3(a-c)}-\frac{\left(4ac-5(a+c)^2\right)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{bx+c}}\right)}{32b^3}-\frac{x(a+bx)^{3/2}(bx+c)^{3/2}}{2b^2(a-c)^2}+\frac{bx^4}{2(a-c)^2}+\frac{x^3(a+c)}{3(a-c)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(Sqrt[a + b*x] + Sqrt[c + b*x])^2,x]
```



```
[Out] ((a + c)*x^3)/(3*(a - c)^2) + (b*x^4)/(2*(a - c)^2) - ((4*a*c - 5
*(a + c)^2)*Sqrt[a + b*x]*Sqrt[c + b*x])/(32*b^3*(a - c)) + ((4*a
*c - 5*(a + c)^2)*(a + b*x)^(3/2)*Sqrt[c + b*x])/(16*b^3*(a - c)^
2) + (5*(a + c)*(a + b*x)^(3/2)*(c + b*x)^(3/2))/(12*b^3*(a - c)^
2) - (x*(a + b*x)^(3/2)*(c + b*x)^(3/2))/(2*b^2*(a - c)^2) - ((4*
a*c - 5*(a + c)^2)*ArcTanh[Sqrt[a + b*x]/Sqrt[c + b*x]])/(32*b^3)
```

Rubi in Sympy [A] time = 61.8664, size = 197, normalized size = 0.86

$$\frac{bx^4}{2(a-c)^2}+\frac{x^3(a+c)}{3(a-c)^2}-\frac{x(a+bx)^{\frac{3}{2}}(bx+c)^{\frac{3}{2}}}{2b^2(a-c)^2}-\frac{\left(ac-\frac{5(a+c)^2}{4}\right)\operatorname{atanh}\left(\frac{\sqrt{a+bx}}{\sqrt{bx+c}}\right)}{8b^3}+\frac{\sqrt{a+bx}\left(4ac-5(a+c)^2\right)\sqrt{bx+c}}{32b^3(a-c)}+\frac{5(a+c)(a+bx)^{\frac{3}{2}}(bx+c)^{\frac{3}{2}}}{12b^3(a-c)^2}+\frac{\sqrt{a+bx}\left(ac-\frac{5(a+c)^2}{4}\right)(bx+c)^{\frac{3}{2}}}{4b^3(a-c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/((b*x+a)**(1/2)+(b*x+c)**(1/2))**2,x)
```

```
[Out] b*x**4/(2*(a - c)**2) + x**3*(a + c)/(3*(a - c)**2) - x*(a + b*x)
**(3/2)*(b*x + c)**(3/2)/(2*b**2*(a - c)**2) - (a*c - 5*(a + c)**
2/4)*atanh(sqrt(a + b*x)/sqrt(b*x + c))/(8*b**3) + sqrt(a + b*x)*
(4*a*c - 5*(a + c)**2)*sqrt(b*x + c)/(32*b**3*(a - c)) + 5*(a + c
)*(a + b*x)**(3/2)*(b*x + c)**(3/2)/(12*b**3*(a - c)**2) + sqrt(a
 + b*x)*(a*c - 5*(a + c)**2/4)*(b*x + c)**(3/2)/(4*b**3*(a - c)**
2)
```

Mathematica [A] time = 0.184766, size = 167, normalized size = 0.73

$$\frac{3(a-c)^2\left(5a^2+6ac+5c^2\right)\log\left(2\sqrt{a+bx}\sqrt{bx+c}+a+2bx+c\right)-2\sqrt{a+bx}\sqrt{bx+c}\left(15a^3-2bx\left(5a^2-2ac+5c^2\right)-7a^2c\right.}{192b^3(a-c)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(Sqrt[a + b*x] + Sqrt[c + b*x])^2,x]
```

```
[Out] (64*b^3*(a + c)*x^3 + 96*b^4*x^4 - 2*Sqrt[a + b*x]*Sqrt[c + b*x]*
(15*a^3 - 7*a^2*c - 7*a*c^2 + 15*c^3 - 2*b*(5*a^2 - 2*a*c + 5*c^2
)*x + 8*b^2*(a + c)*x^2 + 48*b^3*x^3) + 3*(a - c)^2*(5*a^2 + 6*a*
c + 5*c^2)*Log[a + c + 2*b*x + 2*Sqrt[a + b*x]*Sqrt[c + b*x]])/(1
92*b^3*(a - c)^2)
```

Maple [C] time = 0.027, size = 604, normalized size = 2.7

$$\frac{ax^3}{3(a-c)^2}+\frac{cx^3}{3(a-c)^2}+\frac{bx^4}{2(a-c)^2}-\frac{\operatorname{csgn}(b)}{192(a-c)^2b^3}\sqrt{bx+a}\sqrt{bx+c}\left(96\operatorname{csgn}(b)x^3b^3\sqrt{b^2x^2+abx+bcx+ac}+16\operatorname{csgn}(b)x^2ab^2\sqrt{b^2x^2+abx+bcx+ac}+16\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x)
```

```
[Out] 1/3*x^3/(a-c)^2*a+1/3*x^3/(a-c)^2*c+1/2*b*x^4/(a-c)^2-1/192/(a-c)
^2*(b*x+a)^(1/2)*(b*x+c)^(1/2)*(96*csgn(b)*x^3*b^3*(b^2*x^2+a*b*x
+b*c*x+a*c)^(1/2)+16*csgn(b)*x^2*a*b^2*(b^2*x^2+a*b*x+b*c*x+a*c)^
(1/2)+16*csgn(b)*x^2*b^2*c*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)-20*csg
n(b)*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*x*a^2*b+8*csgn(b)*(b^2*x^2+a
*b*x+b*c*x+a*c)^(1/2)*x*a*b*c-20*csgn(b)*(b^2*x^2+a*b*x+b*c*x+a*c
)^(1/2)*x*b*c^2+30*csgn(b)*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*a^3-14
*csgn(b)*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*a^2*c-14*csgn(b)*(b^2*x^
2+a*b*x+b*c*x+a*c)^(1/2)*a*c^2+30*csgn(b)*(b^2*x^2+a*b*x+b*c*x+a*
c)^(1/2)*c^3-15*ln(1/2*(2*csgn(b)*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)
+2*b*x+a+c)*csgn(b))*a^4+12*ln(1/2*(2*csgn(b)*(b^2*x^2+a*b*x+b*c*
x+a*c)^(1/2)+2*b*x+a+c)*csgn(b))*a^3*c+6*ln(1/2*(2*csgn(b)*(b^2*x
^2+a*b*x+b*c*x+a*c)^(1/2)+2*b*x+a+c)*csgn(b))*a^2*c^2+12*ln(1/2*(
2*csgn(b)*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)+2*b*x+a+c)*csgn(b))*a*c
^3-15*ln(1/2*(2*csgn(b)*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)+2*b*x+a+c
)*csgn(b))*c^4)*csgn(b)/b^3/(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(sqrt(b*x + a) + sqrt(b*x + c))^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(b*x + a) + sqrt(b*x + c))^2, x)
```

Fricas [A] time = 0.309121, size = 2361, normalized size = 10.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(sqrt(b*x + a) + sqrt(b*x + c))^2,x, algorithm="fricas")
```

```
[Out] 1/1536*(196608*b^8*x^8 - 59*a^8 + 224*a^7*c + 7212*a^6*c^2 + 2336
*a^5*c^3 - 9186*a^4*c^4 + 2336*a^3*c^5 + 7212*a^2*c^6 + 224*a*c^7
 - 59*c^8 + 524288*(a*b^7 + b^7*c)*x^7 + 8192*(59*a^2*b^6 + 154*a
```

```
*b^6*c + 59*b^6*c^2)*x^6 + 8192*(23*a^3*b^5 + 121*a^2*b^5*c + 121
*a*b^5*c^2 + 23*b^5*c^3)*x^5 + 128*(471*a^4*b^4 + 2188*a^3*b^4*c
+ 4490*a^2*b^4*c^2 + 2188*a*b^4*c^3 + 471*b^4*c^4)*x^4 + 256*(147
*a^5*b^3 + 303*a^4*b^3*c + 142*a^3*b^3*c^2 + 142*a^2*b^3*c^3 + 30
3*a*b^3*c^4 + 147*b^3*c^5)*x^3 + 32*(325*a^6*b^2 + 2114*a^5*b^2*c
 - 37*a^4*b^2*c^2 - 2884*a^3*b^2*c^3 - 37*a^2*b^2*c^4 + 2114*a*b^
2*c^5 + 325*b^2*c^6)*x^2 - 8*(24576*b^7*x^7 - 29*a^7 + 369*a^6*c
+ 1003*a^5*c^2 - 703*a^4*c^3 - 703*a^3*c^4 + 1003*a^2*c^5 + 369*a
*c^6 - 29*c^7 + 53248*(a*b^6 + b^6*c)*x^6 + 12288*(3*a^2*b^5 + 8*
a*b^5*c + 3*b^5*c^2)*x^5 + 10240*(a^3*b^4 + 5*a^2*b^4*c + 5*a*b^4
*c^2 + b^4*c^3)*x^4 + 16*(291*a^4*b^3 + 412*a^3*b^3*c + 722*a^2*b
^3*c^2 + 412*a*b^3*c^3 + 291*b^3*c^4)*x^3 + 24*(115*a^5*b^2 + 207
*a^4*b^2*c - 242*a^3*b^2*c^2 - 242*a^2*b^2*c^3 + 207*a*b^2*c^4 +
115*b^2*c^5)*x^2 + 2*(175*a^6*b + 1814*a^5*b*c + 209*a^4*b*c^2 -
2476*a^3*b*c^3 + 209*a^2*b*c^4 + 1814*a*b*c^5 + 175*b*c^6)*x)*sqr
t(b*x + a)*sqrt(b*x + c) + 32*(a^7*b + 555*a^6*b*c + 1033*a^5*b*c
^2 - 949*a^4*b*c^3 - 949*a^3*b*c^4 + 1033*a^2*b*c^5 + 555*a*b*c^6
 + b*c^7)*x - 24*(5*a^8 + 136*a^7*c + 236*a^6*c^2 - 200*a^5*c^3 -
 354*a^4*c^4 - 200*a^3*c^5 + 236*a^2*c^6 + 136*a*c^7 + 5*c^8 + 12
8*(5*a^4*b^4 - 4*a^3*b^4*c - 2*a^2*b^4*c^2 - 4*a*b^4*c^3 + 5*b^4*
c^4)*x^4 + 256*(5*a^5*b^3 + a^4*b^3*c - 6*a^3*b^3*c^2 - 6*a^2*b^3
*c^3 + a*b^3*c^4 + 5*b^3*c^5)*x^3 + 32*(25*a^6*b^2 + 50*a^5*b^2*c
 - 41*a^4*b^2*c^2 - 68*a^3*b^2*c^3 - 41*a^2*b^2*c^4 + 50*a*b^2*c^
5 + 25*b^2*c^6)*x^2 - 8*(5*a^7 + 31*a^6*c + 5*a^5*c^2 - 41*a^4*c^
3 - 41*a^3*c^4 + 5*a^2*c^5 + 31*a*c^6 + 5*c^7 + 16*(5*a^4*b^3 - 4
*a^3*b^3*c - 2*a^2*b^3*c^2 - 4*a*b^3*c^3 + 5*b^3*c^4)*x^3 + 24*(5
*a^5*b^2 + a^4*b^2*c - 6*a^3*b^2*c^2 - 6*a^2*b^2*c^3 + a*b^2*c^4
+ 5*b^2*c^5)*x^2 + 2*(25*a^6*b + 50*a^5*b*c - 41*a^4*b*c^2 - 68*a
^3*b*c^3 - 41*a^2*b*c^4 + 50*a*b*c^5 + 25*b*c^6)*x)*sqrt(b*x + a)
*sqrt(b*x + c) + 32*(5*a^7*b + 31*a^6*b*c + 5*a^5*b*c^2 - 41*a^4*
b*c^3 - 41*a^3*b*c^4 + 5*a^2*b*c^5 + 31*a*b*c^6 + 5*b*c^7)*x)*log
(-2*b*x + 2*sqrt(b*x + a)*sqrt(b*x + c) - a - c))/(a^6*b^3 + 26*a
^5*b^3*c + 15*a^4*b^3*c^2 - 84*a^3*b^3*c^3 + 15*a^2*b^3*c^4 + 26*
a*b^3*c^5 + b^3*c^6 + 128*(a^2*b^7 - 2*a*b^7*c + b^7*c^2)*x^4 + 2
56*(a^3*b^6 - a^2*b^6*c - a*b^6*c^2 + b^6*c^3)*x^3 + 32*(5*a^4*b^
5 + 4*a^3*b^5*c - 18*a^2*b^5*c^2 + 4*a*b^5*c^3 + 5*b^5*c^4)*x^2 -
 8*(a^5*b^3 + 5*a^4*b^3*c - 6*a^3*b^3*c^2 - 6*a^2*b^3*c^3 + 5*a*b
^3*c^4 + b^3*c^5 + 16*(a^2*b^6 - 2*a*b^6*c + b^6*c^2)*x^3 + 24*(a
^3*b^5 - a^2*b^5*c - a*b^5*c^2 + b^5*c^3)*x^2 + 2*(5*a^4*b^4 + 4*
a^3*b^4*c - 18*a^2*b^4*c^2 + 4*a*b^4*c^3 + 5*b^4*c^4)*x)*sqrt(b*x
 + a)*sqrt(b*x + c) + 32*(a^5*b^4 + 5*a^4*b^4*c - 6*a^3*b^4*c^2 -
 6*a^2*b^4*c^3 + 5*a*b^4*c^4 + b^4*c^5)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/((b*x+a)**(1/2)+(b*x+c)**(1/2))**2,x)
```

```
[Out] Integral(x**2/(sqrt(a + b*x) + sqrt(b*x + c))**2, x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(sqrt(b*x + a) + sqrt(b*x + c))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.246 $\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx$

Optimal. Leaf size=165

$$-\frac{2(a+bx)^{3/2}(bx+c)^{3/2}}{3b^2(a-c)^2}+\frac{(a+c)(a+bx)^{3/2}\sqrt{bx+c}}{2b^2(a-c)^2}-\frac{(a+c)\sqrt{a+bx}\sqrt{bx+c}}{4b^2(a-c)} -\frac{(a+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{bx+c}}\right)}{4b^2}+\frac{2bx^3}{3(a-c)^2}+\frac{x^2(a+c)}{2(a-c)^2}$$

```
[Out] ((a + c)*x^2)/(2*(a - c)^2) + (2*b*x^3)/(3*(a - c)^2) - ((a + c)*
Sqrt[a + b*x]*Sqrt[c + b*x])/(4*b^2*(a - c)) + ((a + c)*(a + b*x)
^(3/2)*Sqrt[c + b*x])/(2*b^2*(a - c)^2) - (2*(a + b*x)^(3/2)*(c +
 b*x)^(3/2))/(3*b^2*(a - c)^2) - ((a + c)*ArcTanh[Sqrt[a + b*x]/S
qrt[c + b*x]])/(4*b^2)
```

Rubi [A] time = 0.44974, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$

$$-\frac{2(a+bx)^{3/2}(bx+c)^{3/2}}{3b^2(a-c)^2}+\frac{(a+c)(a+bx)^{3/2}\sqrt{bx+c}}{2b^2(a-c)^2}-\frac{(a+c)\sqrt{a+bx}\sqrt{bx+c}}{4b^2(a-c)} -\frac{(a+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{bx+c}}\right)}{4b^2}+\frac{2bx^3}{3(a-c)^2}+\frac{x^2(a+c)}{2(a-c)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(Sqrt[a + b*x] + Sqrt[c + b*x])^2,x]
```

```
[Out] ((a + c)*x^2)/(2*(a - c)^2) + (2*b*x^3)/(3*(a - c)^2) - ((a + c)*
Sqrt[a + b*x]*Sqrt[c + b*x])/(4*b^2*(a - c)) + ((a + c)*(a + b*x)
^(3/2)*Sqrt[c + b*x])/(2*b^2*(a - c)^2) - (2*(a + b*x)^(3/2)*(c +
 b*x)^(3/2))/(3*b^2*(a - c)^2) - ((a + c)*ArcTanh[Sqrt[a + b*x]/S
qrt[c + b*x]])/(4*b^2)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{2bx^3}{3\left(a-c\right)^2}+\frac{(a+c)\int x\,dx}{(a-c)^2}-\frac{(a+c)\,\mathrm{atanh}\left(\frac{\sqrt{bx+c}}{\sqrt{a+bx}}\right)}{4b^2}+\frac{(a+c)\sqrt{a+bx}\sqrt{bx+c}}{4b^2\left(a-c\right)} +\frac{(a+c)\sqrt{a+bx}\left(bx+c\right)^{\frac{3}{2}}}{2b^2\left(a-c\right)^2}-\frac{2\left(a+bx\right)^{\frac{3}{2}}\left(bx+c\right)^{\frac{3}{2}}}{3b^2\left(a-c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/((b*x+a)**(1/2)+(b*x+c)**(1/2))**2,x)
```

```
[Out] 2*b*x**3/(3*(a - c)**2) + (a + c)*Integral(x, x)/(a - c)**2 - (a
+ c)*atanh(sqrt(b*x + c)/sqrt(a + b*x))/(4*b**2) + (a + c)*sqrt(a
 + b*x)*sqrt(b*x + c)/(4*b**2*(a - c)) + (a + c)*sqrt(a + b*x)*(b
*x + c)**(3/2)/(2*b**2*(a - c)**2) - 2*(a + b*x)**(3/2)*(b*x + c)
**(3/2)/(3*b**2*(a - c)**2)
```

Mathematica [A] time = 0.28808, size = 124, normalized size = 0.75

$$\frac{2\sqrt{a+bx}\sqrt{bx+c}\left(3a^2-2bx(a+c)-2ac-8b^2x^2+3c^2\right)+12b^2x^2(a+c)-3(a-c)^2(a+c)\log\left(2\sqrt{a+bx}\sqrt{bx+c}+a+2bx\right.}{24b^2(a-c)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(Sqrt[a + b*x] + Sqrt[c + b*x])^2,x]
```

```
[Out] (12*b^2*(a + c)*x^2 + 16*b^3*x^3 + 2*Sqrt[a + b*x]*Sqrt[c + b*x]*
(3*a^2 - 2*a*c + 3*c^2 - 2*b*(a + c)*x - 8*b^2*x^2) - 3*(a - c)^2
*(a + c)*Log[a + c + 2*b*x + 2*Sqrt[a + b*x]*Sqrt[c + b*x]])/(24*
b^2*(a - c)^2)
```

Maple [C] time = 0.015, size = 431, normalized size = 2.6

$$\frac{ax^2}{2\,(a-c)^2}+\frac{cx^2}{2\,(a-c)^2}+\frac{2\,bx^3}{3\,(a-c)^2}$$
$$-\frac{csgn\,(b)}{24\,(a-c)^2\,b^2}\sqrt{bx+a}\sqrt{bx+c}\left(16\,csgn\,(b)\,x^2b^2\sqrt{b^2x^2+abx+bcx+ac}+4\,csgn\,(b)\,\sqrt{b^2x^2+abx+bcx+ac}xab+4\,csgn\,(b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x)
```

```
[Out] 1/2*x^2/(a-c)^2*a+1/2*x^2/(a-c)^2*c+2/3*b*x^3/(a-c)^2-1/24/(a-c)^
2*(b*x+a)^(1/2)*(b*x+c)^(1/2)*(16*csgn(b)*x^2*b^2*(b^2*x^2+a*b*x+
b*c*x+a*c)^(1/2)+4*csgn(b)*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*x*a*b+
4*csgn(b)*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*x*b*c-6*csgn(b)*(b^2*x^
2+a*b*x+b*c*x+a*c)^(1/2)*a^2+4*csgn(b)*(b^2*x^2+a*b*x+b*c*x+a*c)^
(1/2)*a*c-6*csgn(b)*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*c^2+3*ln(1/2*
(2*csgn(b)*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)+2*b*x+a+c)*csgn(b))*a^
```

```
3-3*ln(1/2*(2*csgn(b)*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)+2*b*x+a+c)*
csgn(b))*a^2*c-3*ln(1/2*(2*csgn(b)*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2
)+2*b*x+a+c)*csgn(b))*a*c^2+3*ln(1/2*(2*csgn(b)*(b^2*x^2+a*b*x+b*
c*x+a*c)^(1/2)+2*b*x+a+c)*csgn(b))*c^3)*csgn(b)/b^2/(b^2*x^2+a*b*
x+b*c*x+a*c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(b*x + a) + sqrt(b*x + c))^2,x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(b*x + a) + sqrt(b*x + c))^2, x)
```

Fricas [A] time = 0.312584, size = 1385, normalized size = 8.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(b*x + a) + sqrt(b*x + c))^2,x, algorithm="fricas")
```

```
[Out] 1/96*(4096*b^6*x^6 + 5*a^6 - 66*a^5*c - 261*a^4*c^2 + 132*a^3*c^3
 - 261*a^2*c^4 - 66*a*c^5 + 5*c^6 + 9216*(a*b^5 + b^5*c)*x^5 + 61
44*(a^2*b^4 + 3*a*b^4*c + b^4*c^2)*x^4 + 32*(17*a^3*b^3 + 327*a^2
*b^3*c + 327*a*b^3*c^2 + 17*b^3*c^3)*x^3 - 48*(11*a^4*b^2 - 8*a^3
*b^2*c - 102*a^2*b^2*c^2 - 8*a*b^2*c^3 + 11*b^2*c^4)*x^2 - 2*(204
8*b^5*x^5 + 3*a^5 - 113*a^4*c - 18*a^3*c^2 - 18*a^2*c^3 - 113*a*c
^4 + 3*c^5 + 3584*(a*b^4 + b^4*c)*x^4 + 512*(3*a^2*b^3 + 10*a*b^3
*c + 3*b^3*c^2)*x^3 - 176*(a^3*b^2 - 9*a^2*b^2*c - 9*a*b^2*c^2 +
b^2*c^3)*x^2 - 64*(2*a^4*b + 5*a^3*b*c - 6*a^2*b*c^2 + 5*a*b*c^3
+ 2*b*c^4)*x)*sqrt(b*x + a)*sqrt(b*x + c) - 6*(9*a^5*b + 141*a^4*
b*c - 22*a^3*b*c^2 - 22*a^2*b*c^3 + 141*a*b*c^4 + 9*b*c^5)*x + 12
*(a^6 + 14*a^5*c - a^4*c^2 - 28*a^3*c^3 - a^2*c^4 + 14*a*c^5 + c^
6 + 32*(a^3*b^3 - a^2*b^3*c - a*b^3*c^2 + b^3*c^3)*x^3 + 48*(a^4*
b^2 - 2*a^2*b^2*c^2 + b^2*c^4)*x^2 - 2*(3*a^5 + 7*a^4*c - 10*a^3*
c^2 - 10*a^2*c^3 + 7*a*c^4 + 3*c^5 + 16*(a^3*b^2 - a^2*b^2*c - a*
b^2*c^2 + b^2*c^3)*x^2 + 16*(a^4*b - 2*a^2*b*c^2 + b*c^4)*x)*sqrt
(b*x + a)*sqrt(b*x + c) + 6*(3*a^5*b + 7*a^4*b*c - 10*a^3*b*c^2 -
 10*a^2*b*c^3 + 7*a*b*c^4 + 3*b*c^5)*x)*log(-2*b*x + 2*sqrt(b*x +
 a)*sqrt(b*x + c) - a - c))/(a^5*b^2 + 13*a^4*b^2*c - 14*a^3*b^2*
```

```
c^2 - 14*a^2*b^2*c^3 + 13*a*b^2*c^4 + b^2*c^5 + 32*(a^2*b^5 - 2*a
*b^5*c + b^5*c^2)*x^3 + 48*(a^3*b^4 - a^2*b^4*c - a*b^4*c^2 + b^4
*c^3)*x^2 - 2*(3*a^4*b^2 + 4*a^3*b^2*c - 14*a^2*b^2*c^2 + 4*a*b^2
*c^3 + 3*b^2*c^4 + 16*(a^2*b^4 - 2*a*b^4*c + b^4*c^2)*x^2 + 16*(a
^3*b^3 - a^2*b^3*c - a*b^3*c^2 + b^3*c^3)*x)*sqrt(b*x + a)*sqrt(b
*x + c) + 6*(3*a^4*b^3 + 4*a^3*b^3*c - 14*a^2*b^3*c^2 + 4*a*b^3*c
^3 + 3*b^3*c^4)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((b*x+a)**(1/2)+(b*x+c)**(1/2))**2,x)
```

```
[Out] Integral(x/(sqrt(a + b*x) + sqrt(b*x + c))**2, x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(b*x + a) + sqrt(b*x + c))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.247 $\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx$

Optimal. Leaf size=63

$$\frac{(a-c)^2}{8b\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^4}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{bx+c}}\right)}{2b}$$

```
[Out] (a - c)^2/(8*b*(Sqrt[a + b*x] + Sqrt[c + b*x])^4) + ArcTanh[Sqrt[
a + b*x]/Sqrt[c + b*x]]/(2*b)
```

Rubi [A] time = 0.216258, antiderivative size = 114, normalized size of antiderivative = 1.81, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238

$$\frac{bx^2}{(a-c)^2}-\frac{(a+bx)^{3/2}\sqrt{bx+c}}{b(a-c)^2}+\frac{\sqrt{a+bx}\sqrt{bx+c}}{2b(a-c)}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{bx+c}}\right)}{2b}+\frac{x(a+c)}{(a-c)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[a + b*x] + Sqrt[c + b*x])^(-2),x]
```

```
[Out] ((a + c)*x)/(a - c)^2 + (b*x^2)/(a - c)^2 + (Sqrt[a + b*x]*Sqrt[c
 + b*x])/(2*b*(a - c)) - ((a + b*x)^(3/2)*Sqrt[c + b*x])/(b*(a -
c)^2) + ArcTanh[Sqrt[a + b*x]/Sqrt[c + b*x]]/(2*b)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{2b\int x\,dx}{(a-c)^2}+\frac{\operatorname{atanh}\left(\frac{\sqrt{bx+c}}{\sqrt{a+bx}}\right)}{2b}-\frac{\sqrt{a+bx}\sqrt{bx+c}}{2b\,(a-c)}-\frac{\sqrt{a+bx}\,(bx+c)^{\frac{3}{2}}}{b\,(a-c)^2}+\frac{(a+c)\int a\,dx}{a\,(a-c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/((b*x+a)**(1/2)+(b*x+c)**(1/2))**2,x)
```

```
[Out] 2*b*Integral(x, x)/(a - c)**2 + atanh(sqrt(b*x + c)/sqrt(a + b*x)
)/(2*b) - sqrt(a + b*x)*sqrt(b*x + c)/(2*b*(a - c)) - sqrt(a + b*
x)*(b*x + c)**(3/2)/(b*(a - c)**2) + (a + c)*Integral(a, x)/(a*(a
 - c)**2)
```

Mathematica [A] time = 0.123261, size = 93, normalized size = 1.48

$$\frac{4bx(a+c)-2\sqrt{a+bx}\sqrt{bx+c}(a+2bx+c)+(a-c)^2\log\left(2\sqrt{a+bx}\sqrt{bx+c}+a+2bx+c\right)+4b^2x^2}{4b(a-c)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[a + b*x] + Sqrt[c + b*x])^(-2),x]
```

```
[Out] (4*b*(a + c)*x + 4*b^2*x^2 - 2*Sqrt[a + b*x]*Sqrt[c + b*x]*(a + c
 + 2*b*x) + (a - c)^2*Log[a + c + 2*b*x + 2*Sqrt[a + b*x]*Sqrt[c
+ b*x]])/(4*b*(a - c)^2)
```

Maple [B] time = 0.01, size = 377, normalized size = 6.

$$\frac{ax}{(a-c)^2}+\frac{cx}{(a-c)^2}+\frac{bx^2}{(a-c)^2}-\frac{1}{(a-c)^2 b}\sqrt{bx+a}\,(bx+c)^{\frac{3}{2}}$$
$$-\frac{a}{2\,(a-c)^2 b}\sqrt{bx+c}\sqrt{bx+a}+\frac{c}{2\,(a-c)^2 b}\sqrt{bx+c}\sqrt{bx+a}$$
$$+\frac{a^2}{4\,(a-c)^2}\sqrt{(bx+c)\,(bx+a)}\ln\left(1\left(\frac{ab}{2}+\frac{bc}{2}+b^2x\right)\frac{1}{\sqrt{b^2}}+\sqrt{b^2x^2+(ab+bc)\,x+ac}\right)\frac{1}{\sqrt{bx+c}}\frac{1}{\sqrt{bx+a}}\frac{1}{\sqrt{b^2}}$$
$$-\frac{ac}{2\,(a-c)^2}\sqrt{(bx+c)\,(bx+a)}\ln\left(1\left(\frac{ab}{2}+\frac{bc}{2}+b^2x\right)\frac{1}{\sqrt{b^2}}+\sqrt{b^2x^2+(ab+bc)\,x+ac}\right)\frac{1}{\sqrt{bx+c}}\frac{1}{\sqrt{bx+a}}\frac{1}{\sqrt{b^2}}$$
$$+\frac{c^2}{4\,(a-c)^2}\sqrt{(bx+c)\,(bx+a)}\ln\left(1\left(\frac{ab}{2}+\frac{bc}{2}+b^2x\right)\frac{1}{\sqrt{b^2}}+\sqrt{b^2x^2+(ab+bc)\,x+ac}\right)\frac{1}{\sqrt{bx+c}}\frac{1}{\sqrt{bx+a}}\frac{1}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x)
```

```
[Out] x/(a-c)^2*a+x/(a-c)^2*c+b*x^2/(a-c)^2-1/(a-c)^2/b*(b*x+a)^(1/2)*(
b*x+c)^(3/2)-1/2/(a-c)^2/b*(b*x+c)^(1/2)*(b*x+a)^(1/2)*a+1/2/(a-c
)^2/b*(b*x+c)^(1/2)*(b*x+a)^(1/2)*c+1/4/(a-c)^2*((b*x+c)*(b*x+a))
^(1/2)/(b*x+c)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*b+1/2*b*c+b^2*x)/(b^
2)^(1/2)+(b^2*x^2+(a*b+b*c)*x+a*c)^(1/2))/(b^2)^(1/2)*a^2-1/2/(a-
c)^2*((b*x+c)*(b*x+a))^(1/2)/(b*x+c)^(1/2)/(b*x+a)^(1/2)*ln((1/2*
a*b+1/2*b*c+b^2*x)/(b^2)^(1/2)+(b^2*x^2+(a*b+b*c)*x+a*c)^(1/2))/(
b^2)^(1/2)*a*c+1/4/(a-c)^2*((b*x+c)*(b*x+a))^(1/2)/(b*x+c)^(1/2)/
(b*x+a)^(1/2)*ln((1/2*a*b+1/2*b*c+b^2*x)/(b^2)^(1/2)+(b^2*x^2+(a*
b+b*c)*x+a*c)^(1/2))/(b^2)^(1/2)*c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sqrt(b*x + a) + sqrt(b*x + c))^(-2),x, algorithm="maxima")
```

```
[Out] integrate((sqrt(b*x + a) + sqrt(b*x + c))^(-2), x)
```

Fricas [A] time = 0.275395, size = 711, normalized size = 11.29

$$\frac{256\,b^4x^4 - a^4 + 24\,a^3c + 50\,a^2c^2 + 24\,ac^3 - c^4 + 512\,(ab^3 + b^3c)\,x^3 + 8\,(37\,a^2b^2 + 98\,ab^2c + 37\,b^2c^2)\,x^2 - 4\,(64\,b^3x^3 + a^3 + 1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sqrt(b*x + a) + sqrt(b*x + c))^(-2),x, algorithm="fricas")
```

```
[Out] 1/16*(256*b^4*x^4 - a^4 + 24*a^3*c + 50*a^2*c^2 + 24*a*c^3 - c^4
+ 512*(a*b^3 + b^3*c)*x^3 + 8*(37*a^2*b^2 + 98*a*b^2*c + 37*b^2*c
^2)*x^2 - 4*(64*b^3*x^3 + a^3 + 11*a^2*c + 11*a*c^2 + c^3 + 96*(a
*b^2 + b^2*c)*x^2 + 2*(17*a^2*b + 42*a*b*c + 17*b*c^2)*x)*sqrt(b*
x + a)*sqrt(b*x + c) + 8*(5*a^3*b + 39*a^2*b*c + 39*a*b*c^2 + 5*b
*c^3)*x - 4*(a^4 + 4*a^3*c - 10*a^2*c^2 + 4*a*c^3 + c^4 + 8*(a^2*
b^2 - 2*a*b^2*c + b^2*c^2)*x^2 - 4*(a^3 - a^2*c - a*c^2 + c^3 + 2
*(a^2*b - 2*a*b*c + b*c^2)*x)*sqrt(b*x + a)*sqrt(b*x + c) + 8*(a^
3*b - a^2*b*c - a*b*c^2 + b*c^3)*x)*log(-2*b*x + 2*sqrt(b*x + a)*
sqrt(b*x + c) - a - c))/(a^4*b + 4*a^3*b*c - 10*a^2*b*c^2 + 4*a*b
*c^3 + b*c^4 + 8*(a^2*b^3 - 2*a*b^3*c + b^3*c^2)*x^2 - 4*(a^3*b -
 a^2*b*c - a*b*c^2 + b*c^3 + 2*(a^2*b^2 - 2*a*b^2*c + b^2*c^2)*x)
*sqrt(b*x + a)*sqrt(b*x + c) + 8*(a^3*b^2 - a^2*b^2*c - a*b^2*c^2
 + b^2*c^3)*x)
```

Sympy [A] time = 3.59794, size = 388, normalized size = 6.16

$$\begin{cases} \frac{2a\log\left(\sqrt{a+bx}+\sqrt{bx+c}\right)}{4ab+8b^2x+4bc+8b\sqrt{a+bx}\sqrt{bx+c}} + \frac{a}{4ab+8b^2x+4bc+8b\sqrt{a+bx}\sqrt{bx+c}} + \frac{4bx\log\left(\sqrt{a+bx}+\sqrt{bx+c}\right)}{4ab+8b^2x+4bc+8b\sqrt{a+bx}\sqrt{bx+c}} + \frac{2bx}{4ab+8b^2x+4bc+8b\sqrt{a+bx}\sqrt{bx+c}} + \frac{2}{4ab+8} \\ \frac{x}{\left(\sqrt{a}+\sqrt{c}\right)^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x+a)**(1/2)+(b*x+c)**(1/2))**2,x)
```

```
[Out] Piecewise((2*a*log(sqrt(a + b*x) + sqrt(b*x + c))/(4*a*b + 8*b**2
*x + 4*b*c + 8*b*sqrt(a + b*x)*sqrt(b*x + c)) + a/(4*a*b + 8*b**2
*x + 4*b*c + 8*b*sqrt(a + b*x)*sqrt(b*x + c)) + 4*b*x*log(sqrt(a
+ b*x) + sqrt(b*x + c))/(4*a*b + 8*b**2*x + 4*b*c + 8*b*sqrt(a +
b*x)*sqrt(b*x + c)) + 2*b*x/(4*a*b + 8*b**2*x + 4*b*c + 8*b*sqrt(
a + b*x)*sqrt(b*x + c)) + 2*c*log(sqrt(a + b*x) + sqrt(b*x + c))/
(4*a*b + 8*b**2*x + 4*b*c + 8*b*sqrt(a + b*x)*sqrt(b*x + c)) + c/
(4*a*b + 8*b**2*x + 4*b*c + 8*b*sqrt(a + b*x)*sqrt(b*x + c)) + 4*
sqrt(a + b*x)*sqrt(b*x + c)*log(sqrt(a + b*x) + sqrt(b*x + c))/(4
*a*b + 8*b**2*x + 4*b*c + 8*b*sqrt(a + b*x)*sqrt(b*x + c)), Ne(b,
 0)), (x/(sqrt(a) + sqrt(c))**2, True))
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(b*x + a) + sqrt(b*x + c))^(-2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.248 $\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx$

Optimal. Leaf size=133

$$\frac{2bx}{(a-c)^2}-\frac{2\sqrt{a+bx}\sqrt{bx+c}}{(a-c)^2}-\frac{2(a+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{bx+c}}\right)}{(a-c)^2}+\frac{4\sqrt{a}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{a}\sqrt{bx+c}}\right)}{(a-c)^2}+\frac{(a+c)\log(x)}{(a-c)^2}$$

```
[Out] (2*b*x)/(a - c)^2 - (2*Sqrt[a + b*x]*Sqrt[c + b*x])/(a - c)^2 - (
2*(a + c)*ArcTanh[Sqrt[a + b*x]/Sqrt[c + b*x]])/(a - c)^2 + (4*Sq
rt[a]*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[a + b*x])/(Sqrt[a]*Sqrt[c + b
*x])])/(a - c)^2 + ((a + c)*Log[x])/(a - c)^2
```

Rubi [A] time = 0.542579, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.32$

$$\frac{2bx}{(a-c)^2}-\frac{2\sqrt{a+bx}\sqrt{bx+c}}{(a-c)^2}-\frac{2(a+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{bx+c}}\right)}{(a-c)^2}+\frac{4\sqrt{a}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{a}\sqrt{bx+c}}\right)}{(a-c)^2}+\frac{(a+c)\log(x)}{(a-c)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(Sqrt[a + b*x] + Sqrt[c + b*x])^2),x]
```

```
[Out] (2*b*x)/(a - c)^2 - (2*Sqrt[a + b*x]*Sqrt[c + b*x])/(a - c)^2 - (
2*(a + c)*ArcTanh[Sqrt[a + b*x]/Sqrt[c + b*x]])/(a - c)^2 + (4*Sq
rt[a]*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[a + b*x])/(Sqrt[a]*Sqrt[c + b
*x])])/(a - c)^2 + ((a + c)*Log[x])/(a - c)^2
```

Rubi in Sympy [A] time = 46.3208, size = 119, normalized size = 0.89

$$\frac{4\sqrt{a}\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{a}\sqrt{bx+c}}\right)}{(a-c)^2}+\frac{2bx}{(a-c)^2}+\frac{(a+c)\log(x)}{(a-c)^2}-\frac{2(a+c)\,\mathrm{atanh}\left(\frac{\sqrt{a+bx}}{\sqrt{bx+c}}\right)}{(a-c)^2}-\frac{2\sqrt{a+bx}\sqrt{bx+c}}{(a-c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/((b*x+a)**(1/2)+(b*x+c)**(1/2))**2,x)
```

```
[Out] 4*sqrt(a)*sqrt(c)*atanh(sqrt(c)*sqrt(a + b*x)/(sqrt(a)*sqrt(b*x +
 c)))/(a - c)**2 + 2*b*x/(a - c)**2 + (a + c)*log(x)/(a - c)**2 -
```

```
 2*(a + c)*atanh(sqrt(a + b*x)/sqrt(b*x + c))/(a - c)**2 - 2*sqrt
(a + b*x)*sqrt(b*x + c)/(a - c)**2
```

Mathematica [A] time = 0.137924, size = 140, normalized size = 1.05

$$\frac{-2\sqrt{a+bx}\sqrt{bx+c}-(a+c)\log\left(2\sqrt{a+bx}\sqrt{bx+c}+a+2bx+c\right)+2\sqrt{a}\sqrt{c}\log\left(2\sqrt{a}\sqrt{c}\sqrt{a+bx}\sqrt{bx+c}+abx+2ac+bcx\right.}{(a-c)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(Sqrt[a + b*x] + Sqrt[c + b*x])^2),x]
```

```
[Out] (2*b*x - 2*Sqrt[a + b*x]*Sqrt[c + b*x] + (a - 2*Sqrt[a]*Sqrt[c] +
 c)*Log[x] - (a + c)*Log[a + c + 2*b*x + 2*Sqrt[a + b*x]*Sqrt[c +
 b*x]] + 2*Sqrt[a]*Sqrt[c]*Log[2*a*c + a*b*x + b*c*x + 2*Sqrt[a]*
Sqrt[c]*Sqrt[a + b*x]*Sqrt[c + b*x]])/(a - c)^2
```

Maple [C] time = 0.015, size = 258, normalized size = 1.9

$$\frac{a\ln(x)}{(a-c)^2}+\frac{c\ln(x)}{(a-c)^2}+2\,\frac{bx}{(a-c)^2}$$
$$+\frac{csgn(b)}{(a-c)^2}\sqrt{bx+a}\sqrt{bx+c}\left(2\,csgn(b)\ln\left(\frac{abx+bcx+2\,\sqrt{ac}\sqrt{b^2x^2+abx+bcx+ac}+2\,ac}{x}\right)ac-2\,csgn(b)\,\sqrt{ac}\sqrt{b^2x^2+abx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x)
```

```
[Out] 1/(a-c)^2*a*ln(x)+1/(a-c)^2*c*ln(x)+2*b*x/(a-c)^2+1/(a-c)^2*(b*x+
a)^(1/2)*(b*x+c)^(1/2)*(2*csgn(b)*ln((a*b*x+b*c*x+2*(a*c)^(1/2)*(
b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)+2*a*c)/x)*a*c-2*csgn(b)*(a*c)^(1/2
)*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)-ln(1/2*(2*csgn(b)*(b^2*x^2+a*b*
x+b*c*x+a*c)^(1/2)+2*b*x+a+c)*csgn(b))*(a*c)^(1/2)*a-ln(1/2*(2*cs
gn(b)*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)+2*b*x+a+c)*csgn(b))*(a*c)^(
1/2)*c)*csgn(b)/(a*c)^(1/2)/(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(sqrt(b*x + a) + sqrt(b*x + c))^2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*(sqrt(b*x + a) + sqrt(b*x + c))^2), x)
```

Fricas [A] time = 0.308151, size = 1, normalized size = 0.01

$$\left[\frac{16\,b^2x^2 - 2\,(8\,bx + 2\,(a + c)\log{(x)} + a + c)\sqrt{bx + a}\sqrt{bx + c} - a^2 + 6\,ac - c^2 + 10\,(ab + bc)x - 2\,\left(2\,\sqrt{bx + a}\sqrt{bx + c}(a + c) - \right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(sqrt(b*x + a) + sqrt(b*x + c))^2),x, algorithm="fricas")
```

```
[Out] [1/2*(16*b^2*x^2 - 2*(8*b*x + 2*(a + c)*log(x) + a + c)*sqrt(b*x
+ a)*sqrt(b*x + c) - a^2 + 6*a*c - c^2 + 10*(a*b + b*c)*x - 2*(2*
sqrt(b*x + a)*sqrt(b*x + c)*(a + c) - a^2 - 2*a*c - c^2 - 2*(a*b
+ b*c)*x)*log(-2*b*x + 2*sqrt(b*x + a)*sqrt(b*x + c) - a - c) + 2
*(a^2 + 2*a*c + c^2 + 2*(a*b + b*c)*x)*log(x) + 4*(sqrt(a*c)*(2*b
*x + a + c) - 2*sqrt(a*c)*sqrt(b*x + a)*sqrt(b*x + c))*log((2*b^2
*x^2 - 2*sqrt(a*c)*b*x - 2*sqrt(b*x + a)*sqrt(b*x + c)*(b*x - sqr
t(a*c)) + 2*a*c + (a*b + b*c)*x)/(2*b*x^2 - 2*sqrt(b*x + a)*sqrt(
b*x + c)*x + (a + c)*x)))/(a^3 - a^2*c - a*c^2 + c^3 - 2*(a^2 - 2
*a*c + c^2)*sqrt(b*x + a)*sqrt(b*x + c) + 2*(a^2*b - 2*a*b*c + b*
c^2)*x), 1/2*(16*b^2*x^2 - 2*(8*b*x + 2*(a + c)*log(x) + a + c)*s
qrt(b*x + a)*sqrt(b*x + c) - a^2 + 6*a*c - c^2 + 10*(a*b + b*c)*x
 + 8*(sqrt(-a*c)*(2*b*x + a + c) - 2*sqrt(-a*c)*sqrt(b*x + a)*sqr
t(b*x + c))*arctan(-(b*x - sqrt(b*x + a)*sqrt(b*x + c))/sqrt(-a*c
)) - 2*(2*sqrt(b*x + a)*sqrt(b*x + c)*(a + c) - a^2 - 2*a*c - c^2
 - 2*(a*b + b*c)*x)*log(-2*b*x + 2*sqrt(b*x + a)*sqrt(b*x + c) -
a - c) + 2*(a^2 + 2*a*c + c^2 + 2*(a*b + b*c)*x)*log(x))/(a^3 - a
^2*c - a*c^2 + c^3 - 2*(a^2 - 2*a*c + c^2)*sqrt(b*x + a)*sqrt(b*x
 + c) + 2*(a^2*b - 2*a*b*c + b*c^2)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\sqrt{a + bx} + \sqrt{bx + c}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/((b*x+a)**(1/2)+(b*x+c)**(1/2))**2,x)
```

```
[Out] Integral(1/(x*(sqrt(a + b*x) + sqrt(b*x + c))**2), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x*(sqrt(b*x + a) + sqrt(b*x + c))^2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.249 $\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx$

Optimal. Leaf size=141

$$\frac{2\sqrt{a+bx}\sqrt{bx+c}}{x(a-c)^2}+\frac{2b\log(x)}{(a-c)^2}+\frac{2b(a+c)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{a}\sqrt{bx+c}}\right)}{\sqrt{a}\sqrt{c}(a-c)^2}-\frac{4b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{bx+c}}\right)}{(a-c)^2}-\frac{a+c}{x(a-c)^2}$$

```
[Out] -((a + c)/((a - c)^2*x)) + (2*Sqrt[a + b*x]*Sqrt[c + b*x])/((a -
c)^2*x) - (4*b*ArcTanh[Sqrt[a + b*x]/Sqrt[c + b*x]])/(a - c)^2 +
(2*b*(a + c)*ArcTanh[(Sqrt[c]*Sqrt[a + b*x])/(Sqrt[a]*Sqrt[c + b*
x])])/(Sqrt[a]*(a - c)^2*Sqrt[c]) + (2*b*Log[x])/(a - c)^2
```

Rubi [A] time = 0.533415, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32

$$\frac{2\sqrt{a+bx}\sqrt{bx+c}}{x(a-c)^2}+\frac{2b\log(x)}{(a-c)^2}+\frac{2b(a+c)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{a}\sqrt{bx+c}}\right)}{\sqrt{a}\sqrt{c}(a-c)^2}-\frac{4b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{bx+c}}\right)}{(a-c)^2}-\frac{a+c}{x(a-c)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(Sqrt[a + b*x] + Sqrt[c + b*x])^2),x]
```

```
[Out] -((a + c)/((a - c)^2*x)) + (2*Sqrt[a + b*x]*Sqrt[c + b*x])/((a -
c)^2*x) - (4*b*ArcTanh[Sqrt[a + b*x]/Sqrt[c + b*x]])/(a - c)^2 +
(2*b*(a + c)*ArcTanh[(Sqrt[c]*Sqrt[a + b*x])/(Sqrt[a]*Sqrt[c + b*
x])])/(Sqrt[a]*(a - c)^2*Sqrt[c]) + (2*b*Log[x])/(a - c)^2
```

Rubi in Sympy [A] time = 44.2423, size = 124, normalized size = 0.88

$$\frac{2b\log(x)}{(a-c)^2}-\frac{4b\,\text{atanh}\left(\frac{\sqrt{bx+c}}{\sqrt{a+bx}}\right)}{(a-c)^2}-\frac{a+c}{x(a-c)^2}+\frac{2\sqrt{a+bx}\sqrt{bx+c}}{x(a-c)^2}+\frac{2b(a+c)\,\text{atanh}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{a}\sqrt{bx+c}}\right)}{\sqrt{a}\sqrt{c}(a-c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/((b*x+a)**(1/2)+(b*x+c)**(1/2))**2,x)
```

```
[Out] 2*b*log(x)/(a - c)**2 - 4*b*atanh(sqrt(b*x + c)/sqrt(a + b*x))/(a
 - c)**2 - (a + c)/(x*(a - c)**2) + 2*sqrt(a + b*x)*sqrt(b*x + c)
```

```
/(x*(a - c)**2) + 2*b*(a + c)*atanh(sqrt(c)*sqrt(a + b*x)/(sqrt(a
)*sqrt(b*x + c)))/(sqrt(a)*sqrt(c)*(a - c)**2)
```

Mathematica [A] time = 0.314713, size = 153, normalized size = 1.09

$$\frac{\frac{2\sqrt{a+bx}\sqrt{bx+c}}{x}-\frac{b(a+c)\log(x)}{\sqrt{a}\sqrt{c}}+\frac{b(a+c)\log\left(2\sqrt{a}\sqrt{c}\sqrt{a+bx}\sqrt{bx+c}+abx+2ac+bcx\right)}{\sqrt{a}\sqrt{c}}-2b\log\left(2\sqrt{a+bx}\sqrt{bx+c}+a+2bx+c\right)-\frac{a+c}{x}+2b}{(a-c)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(Sqrt[a + b*x] + Sqrt[c + b*x])^2),x]
```

```
[Out] (-((a + c)/x) + (2*Sqrt[a + b*x]*Sqrt[c + b*x])/x + 2*b*Log[x] -
(b*(a + c)*Log[x])/(Sqrt[a]*Sqrt[c]) - 2*b*Log[a + c + 2*b*x + 2*
Sqrt[a + b*x]*Sqrt[c + b*x]] + (b*(a + c)*Log[2*a*c + a*b*x + b*c
*x + 2*Sqrt[a]*Sqrt[c]*Sqrt[a + b*x]*Sqrt[c + b*x]])/(Sqrt[a]*Sqr
t[c]))/(a - c)^2
```

Maple [C] time = 0.017, size = 274, normalized size = 1.9

$$-\frac{a}{x(a-c)^2}-\frac{c}{x(a-c)^2}+2\frac{b\ln(x)}{(a-c)^2}$$
$$+\frac{csgn(b)}{x(a-c)^2}\sqrt{bx+a}\sqrt{bx+c}\left(csgn(b)\ln\left(\frac{1}{x}\left(abx+bcx+2\sqrt{ac}\sqrt{b^2x^2+abx+bcx+ac}+2ac\right)\right)xab+csgn(b)\ln\left(\frac{1}{x}\left(abx+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x)
```

```
[Out] -1/x/(a-c)^2*a-1/x/(a-c)^2*c+2*b*ln(x)/(a-c)^2+1/(a-c)^2*(b*x+a)^
(1/2)*(b*x+c)^(1/2)*(csgn(b)*ln((a*b*x+b*c*x+2*(a*c)^(1/2)*(b^2*x
^2+a*b*x+b*c*x+a*c)^(1/2)+2*a*c)/x)*x*a*b+csgn(b)*ln((a*b*x+b*c*x
+2*(a*c)^(1/2)*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)+2*a*c)/x)*x*b*c-2*
ln(1/2*(2*csgn(b)*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)+2*b*x+a+c)*csgn
(b))*x*b*(a*c)^(1/2)+2*csgn(b)*(a*c)^(1/2)*(b^2*x^2+a*b*x+b*c*x+a
*c)^(1/2))*csgn(b)/(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)/x/(a*c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^2*(sqrt(b*x + a) + sqrt(b*x + c))^2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^2*(sqrt(b*x + a) + sqrt(b*x + c))^2), x)
```

Fricas [A] time = 0.30657, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^2*(sqrt(b*x + a) + sqrt(b*x + c))^2),x, algorithm="fricas")
```

```
[Out] [(4*(b*x*log(x) - a - c)*sqrt(a*c)*sqrt(b*x + a)*sqrt(b*x + c) +
2*(2*sqrt(a*c)*sqrt(b*x + a)*sqrt(b*x + c)*b*x - (2*b^2*x^2 + (a*
b + b*c)*x)*sqrt(a*c))*log(-2*b*x + 2*sqrt(b*x + a)*sqrt(b*x + c)
 - a - c) + (2*(a*b + b*c)*sqrt(b*x + a)*sqrt(b*x + c)*x - 2*(a*b
^2 + b^2*c)*x^2 - (a^2*b + 2*a*b*c + b*c^2)*x)*log(-(2*a*b*c*x +
2*(sqrt(a*c)*b*x - a*c)*sqrt(b*x + a)*sqrt(b*x + c) - (2*b^2*x^2
+ 2*a*c + (a*b + b*c)*x)*sqrt(a*c))/(2*b*x^2 - 2*sqrt(b*x + a)*sq
rt(b*x + c)*x + (a + c)*x)) + (a^2 + 6*a*c + c^2 + 4*(a*b + b*c)*
x - 2*(2*b^2*x^2 + (a*b + b*c)*x)*log(x))*sqrt(a*c))/(2*(a^2 - 2*
a*c + c^2)*sqrt(a*c)*sqrt(b*x + a)*sqrt(b*x + c)*x - (2*(a^2*b -
2*a*b*c + b*c^2)*x^2 + (a^3 - a^2*c - a*c^2 + c^3)*x)*sqrt(a*c)),
 (4*(b*x*log(x) - a - c)*sqrt(-a*c)*sqrt(b*x + a)*sqrt(b*x + c) +
 2*(2*(a*b + b*c)*sqrt(b*x + a)*sqrt(b*x + c)*x - 2*(a*b^2 + b^2*
c)*x^2 - (a^2*b + 2*a*b*c + b*c^2)*x)*arctan(-(sqrt(-a*c)*b*x - s
qrt(-a*c)*sqrt(b*x + a)*sqrt(b*x + c))/(a*c)) + 2*(2*sqrt(-a*c)*s
qrt(b*x + a)*sqrt(b*x + c)*b*x - (2*b^2*x^2 + (a*b + b*c)*x)*sqrt
(-a*c))*log(-2*b*x + 2*sqrt(b*x + a)*sqrt(b*x + c) - a - c) + (a^
2 + 6*a*c + c^2 + 4*(a*b + b*c)*x - 2*(2*b^2*x^2 + (a*b + b*c)*x)
*log(x))*sqrt(-a*c))/(2*(a^2 - 2*a*c + c^2)*sqrt(-a*c)*sqrt(b*x +
 a)*sqrt(b*x + c)*x - (2*(a^2*b - 2*a*b*c + b*c^2)*x^2 + (a^3 - a
^2*c - a*c^2 + c^3)*x)*sqrt(-a*c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/((b*x+a)**(1/2)+(b*x+c)**(1/2))**2,x)
```

```
[Out] Integral(1/(x**2*(sqrt(a + b*x) + sqrt(b*x + c))**2), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(sqrt(b*x + a) + sqrt(b*x + c))^2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.250 $\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3} dx$

Optimal. Leaf size=375

$$-\frac{8a^3(a+bx)^{3/2}}{3b^3(a-c)^3}+\frac{24a^2(a+bx)^{5/2}}{5b^3(a-c)^3}+\frac{2a^2(a+3c)(a+bx)^{3/2}}{3b^3(a-c)^3}+\frac{8c^3(bx+c)^{3/2}}{3b^3(a-c)^3}-\frac{24c^2(bx+c)^{5/2}}{5b^3(a-c)^3}-\frac{2c^2(3a+c)(bx+c)^{3/2}}{3b^3(a-c)^3}+\frac{8(a+bx)^{9/2}}{9b^3(a-c)^3}+\frac{2(a+3c)(a+bx)^{7/2}}{7b^3(a-c)^3}-\frac{24a(a+bx)^{7/2}}{7b^3(a-c)^3}-\frac{4a(a+3c)(a+bx)^{5/2}}{5b^3(a-c)^3}-\frac{8(bx+c)^{9/2}}{9b^3(a-c)^3}+\frac{24c(bx+c)^{7/2}}{7b^3(a-c)^3}-\frac{2(3a+c)(bx+c)^{7/2}}{7b^3(a-c)^3}+\frac{4c(3a+c)(bx+c)^{5/2}}{5b^3(a-c)^3}$$

```
[Out] (-8*a^3*(a + b*x)^(3/2))/(3*b^3*(a - c)^3) + (2*a^2*(a + 3*c)*(a
+ b*x)^(3/2))/(3*b^3*(a - c)^3) + (24*a^2*(a + b*x)^(5/2))/(5*b^3
*(a - c)^3) - (4*a*(a + 3*c)*(a + b*x)^(5/2))/(5*b^3*(a - c)^3) -
 (24*a*(a + b*x)^(7/2))/(7*b^3*(a - c)^3) + (2*(a + 3*c)*(a + b*x
)^(7/2))/(7*b^3*(a - c)^3) + (8*(a + b*x)^(9/2))/(9*b^3*(a - c)^3
) + (8*c^3*(c + b*x)^(3/2))/(3*b^3*(a - c)^3) - (2*c^2*(3*a + c)*
(c + b*x)^(3/2))/(3*b^3*(a - c)^3) - (24*c^2*(c + b*x)^(5/2))/(5*
b^3*(a - c)^3) + (4*c*(3*a + c)*(c + b*x)^(5/2))/(5*b^3*(a - c)^3
) + (24*c*(c + b*x)^(7/2))/(7*b^3*(a - c)^3) - (2*(3*a + c)*(c +
b*x)^(7/2))/(7*b^3*(a - c)^3) - (8*(c + b*x)^(9/2))/(9*b^3*(a - c
)^3)
```

Rubi [A] time = 0.7333, antiderivative size = 375, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$

$$-\frac{8a^3(a+bx)^{3/2}}{3b^3(a-c)^3}+\frac{24a^2(a+bx)^{5/2}}{5b^3(a-c)^3}+\frac{2a^2(a+3c)(a+bx)^{3/2}}{3b^3(a-c)^3}+\frac{8c^3(bx+c)^{3/2}}{3b^3(a-c)^3}-\frac{24c^2(bx+c)^{5/2}}{5b^3(a-c)^3}-\frac{2c^2(3a+c)(bx+c)^{3/2}}{3b^3(a-c)^3}+\frac{8(a+bx)^{9/2}}{9b^3(a-c)^3}+\frac{2(a+3c)(a+bx)^{7/2}}{7b^3(a-c)^3}-\frac{24a(a+bx)^{7/2}}{7b^3(a-c)^3}-\frac{4a(a+3c)(a+bx)^{5/2}}{5b^3(a-c)^3}-\frac{8(bx+c)^{9/2}}{9b^3(a-c)^3}+\frac{24c(bx+c)^{7/2}}{7b^3(a-c)^3}-\frac{2(3a+c)(bx+c)^{7/2}}{7b^3(a-c)^3}+\frac{4c(3a+c)(bx+c)^{5/2}}{5b^3(a-c)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(Sqrt[a + b*x] + Sqrt[c + b*x])^3,x]
```


```
[Out] (-8*a^3*(a + b*x)^(3/2))/(3*b^3*(a - c)^3) + (2*a^2*(a + 3*c)*(a
+ b*x)^(3/2))/(3*b^3*(a - c)^3) + (24*a^2*(a + b*x)^(5/2))/(5*b^3
*(a - c)^3) - (4*a*(a + 3*c)*(a + b*x)^(5/2))/(5*b^3*(a - c)^3) -
 (24*a*(a + b*x)^(7/2))/(7*b^3*(a - c)^3) + (2*(a + 3*c)*(a + b*x
)^(7/2))/(7*b^3*(a - c)^3) + (8*(a + b*x)^(9/2))/(9*b^3*(a - c)^3
) + (8*c^3*(c + b*x)^(3/2))/(3*b^3*(a - c)^3) - (2*c^2*(3*a + c)*
(c + b*x)^(3/2))/(3*b^3*(a - c)^3) - (24*c^2*(c + b*x)^(5/2))/(5*
```

```
b^3*(a - c)^3) + (4*c*(3*a + c)*(c + b*x)^(5/2))/(5*b^3*(a - c)^3
) + (24*c*(c + b*x)^(7/2))/(7*b^3*(a - c)^3) - (2*(3*a + c)*(c +
b*x)^(7/2))/(7*b^3*(a - c)^3) - (8*(c + b*x)^(9/2))/(9*b^3*(a - c
)^3)
```

Rubi in Sympy [A] time = 78.8331, size = 342, normalized size = 0.91

$$-\frac{8a^3(a+bx)^{\frac{3}{2}}}{3b^3(a-c)^3}+\frac{2a^2(a+3c)(a+bx)^{\frac{3}{2}}}{3b^3(a-c)^3}+\frac{24a^2(a+bx)^{\frac{5}{2}}}{5b^3(a-c)^3}-\frac{4a(a+3c)(a+bx)^{\frac{5}{2}}}{5b^3(a-c)^3}-\frac{24a(a+bx)^{\frac{7}{2}}}{7b^3(a-c)^3}+\frac{8c^3(bx+c)^{\frac{3}{2}}}{3b^3(a-c)^3}-\frac{2c^2(3a+c)(bx+c)^{\frac{3}{2}}}{3b^3(a-c)^3}-\frac{24c^2(bx+c)^{\frac{5}{2}}}{5b^3(a-c)^3}+\frac{4c(3a+c)(bx+c)^{\frac{5}{2}}}{5b^3(a-c)^3}+\frac{24c(bx+c)^{\frac{7}{2}}}{7b^3(a-c)^3}+\frac{2(a+3c)(a+bx)^{\frac{7}{2}}}{7b^3(a-c)^3}+\frac{8(a+bx)^{\frac{9}{2}}}{9b^3(a-c)^3}-\frac{2(3a+c)(bx+c)^{\frac{7}{2}}}{7b^3(a-c)^3}-\frac{8(bx+c)^{\frac{9}{2}}}{9b^3(a-c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/((b*x+a)**(1/2)+(b*x+c)**(1/2))**3,x)
```

```
[Out] -8*a**3*(a + b*x)**(3/2)/(3*b**3*(a - c)**3) + 2*a**2*(a + 3*c)*(
a + b*x)**(3/2)/(3*b**3*(a - c)**3) + 24*a**2*(a + b*x)**(5/2)/(5
*b**3*(a - c)**3) - 4*a*(a + 3*c)*(a + b*x)**(5/2)/(5*b**3*(a - c
)**3) - 24*a*(a + b*x)**(7/2)/(7*b**3*(a - c)**3) + 8*c**3*(b*x +
 c)**(3/2)/(3*b**3*(a - c)**3) - 2*c**2*(3*a + c)*(b*x + c)**(3/2
)/(3*b**3*(a - c)**3) - 24*c**2*(b*x + c)**(5/2)/(5*b**3*(a - c)*
*3) + 4*c*(3*a + c)*(b*x + c)**(5/2)/(5*b**3*(a - c)**3) + 24*c*(
b*x + c)**(7/2)/(7*b**3*(a - c)**3) + 2*(a + 3*c)*(a + b*x)**(7/2
)/(7*b**3*(a - c)**3) + 8*(a + b*x)**(9/2)/(9*b**3*(a - c)**3) -
2*(3*a + c)*(b*x + c)**(7/2)/(7*b**3*(a - c)**3) - 8*(b*x + c)**(
9/2)/(9*b**3*(a - c)**3)
```

Mathematica [A] time = 0.536062, size = 138, normalized size = 0.37

$$-\frac{2\left((a+bx)^{3/2}\left(40a^3-12a^2(5bx+6c)+3abx(25bx+36c)-5b^2x^2(28bx+27c)\right)+(bx+c)^{3/2}\left(9a\left(15b^2x^2-12bcx+8c^2\right)\right.\right.}{315b^3(a-c)^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(Sqrt[a + b*x] + Sqrt[c + b*x])^3,x]
```

```
[Out] (-2*((a + b*x)^(3/2)*(40*a^3 - 12*a^2*(6*c + 5*b*x) + 3*a*b*x*(36
*c + 25*b*x) - 5*b^2*x^2*(27*c + 28*b*x)) + (c + b*x)^(3/2)*(9*a*
(8*c^2 - 12*b*c*x + 15*b^2*x^2) + 5*(-8*c^3 + 12*b*c^2*x - 15*b^2
```

```
*c*x^2 + 28*b^3*x^3)))))/(315*b^3*(a - c)^3)
```

Maple [A] time = 0.007, size = 294, normalized size = 0.8

$$2\,{\frac {a \left(1/7\, \left(bx+a \right) ^{7/2}-2/5\, \left(bx+a \right) ^{5/2}a+1/3\,{a}^{2} \left(bx+a \right) ^{3/2} \right) }{ \left(a-c \right) ^{3}{b}^{3}}}+6\,{\frac {c \left(1/7\, \left(bx+a \right) ^{7/2}-2/5\, \left(bx+a \right) ^{5/2}a+1/3\,{a}^{2} \left(bx+a \right) ^{3/2} \right) }{ \left(a-c \right) ^{3}{b}^{3}}}-6\,{\frac {a \left(1/7\, \left(bx+c \right) ^{7/2}-2/5\, \left(bx+c \right) ^{5/2}c+1/3\,{c}^{2} \left(bx+c \right) ^{3/2} \right) }{ \left(a-c \right) ^{3}{b}^{3}}}-2\,{\frac {c \left(1/7\, \left(bx+c \right) ^{7/2}-2/5\, \left(bx+c \right) ^{5/2}c+1/3\,{c}^{2} \left(bx+c \right) ^{3/2} \right) }{ \left(a-c \right) ^{3}{b}^{3}}}+8\,{\frac {1/9\, \left(bx+a \right) ^{9/2}-3/7\,a \left(bx+a \right) ^{7/2}+3/5\,{a}^{2} \left(bx+a \right) ^{5/2}-1/3\,{a}^{3} \left(bx+a \right) ^{3/2}}{ \left(a-c \right) ^{3}{b}^{3}}}-8\,{\frac {1/9\, \left(bx+c \right) ^{9/2}-3/7\, \left(bx+c \right) ^{7/2}c+3/5\, \left(bx+c \right) ^{5/2}{c}^{2}-1/3\,{c}^{3} \left(bx+c \right) ^{3/2}}{ \left(a-c \right) ^{3}{b}^{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x)
```

```
[Out] 2/(a-c)^3*a/b^3*(1/7*(b*x+a)^(7/2)-2/5*(b*x+a)^(5/2)*a+1/3*a^2*(b
*x+a)^(3/2))+6/(a-c)^3*c/b^3*(1/7*(b*x+a)^(7/2)-2/5*(b*x+a)^(5/2)
*a+1/3*a^2*(b*x+a)^(3/2))-6/(a-c)^3*a/b^3*(1/7*(b*x+c)^(7/2)-2/5*
(b*x+c)^(5/2)*c+1/3*c^2*(b*x+c)^(3/2))-2/(a-c)^3*c/b^3*(1/7*(b*x+
c)^(7/2)-2/5*(b*x+c)^(5/2)*c+1/3*c^2*(b*x+c)^(3/2))+8/(a-c)^3/b^3
*(1/9*(b*x+a)^(9/2)-3/7*a*(b*x+a)^(7/2)+3/5*a^2*(b*x+a)^(5/2)-1/3
*a^3*(b*x+a)^(3/2))-8/(a-c)^3/b^3*(1/9*(b*x+c)^(9/2)-3/7*(b*x+c)^
(7/2)*c+3/5*(b*x+c)^(5/2)*c^2-1/3*c^3*(b*x+c)^(3/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(sqrt(b*x + a) + sqrt(b*x + c))^3,x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(b*x + a) + sqrt(b*x + c))^3, x)
```

Fricas [A] time = 0.27866, size = 281, normalized size = 0.75

$$\frac{2\left(\left(140\,b^4x^4-40\,a^4+72\,a^3c+5\left(13\,ab^3+27\,b^3c\right)x^3-3\left(5\,a^2b^2-9\,ab^2c\right)x^2+4\left(5\,a^3b-9\,a^2bc\right)x\right)\sqrt{bx+a}-\left(140\,b^4x^4+\right.\right.}{315\left(a^3b^3-3\,a^2b^3c+3\,ab^3c^2-b^{\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(sqrt(b*x + a) + sqrt(b*x + c))^3,x, algorithm="fricas")
```

```
[Out] 2/315*((140*b^4*x^4 - 40*a^4 + 72*a^3*c + 5*(13*a*b^3 + 27*b^3*c)
*x^3 - 3*(5*a^2*b^2 - 9*a*b^2*c)*x^2 + 4*(5*a^3*b - 9*a^2*b*c)*x)
*sqrt(b*x + a) - (140*b^4*x^4 + 72*a*c^3 - 40*c^4 + 5*(27*a*b^3 +
 13*b^3*c)*x^3 + 3*(9*a*b^2*c - 5*b^2*c^2)*x^2 - 4*(9*a*b*c^2 - 5
*b*c^3)*x)*sqrt(b*x + c))/(a^3*b^3 - 3*a^2*b^3*c + 3*a*b^3*c^2 -
b^3*c^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/((b*x+a)**(1/2)+(b*x+c)**(1/2))**3,x)
```

```
[Out] Integral(x**2/(sqrt(a + b*x) + sqrt(b*x + c))**3, x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(sqrt(b*x + a) + sqrt(b*x + c))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.251 $\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx$

Optimal. Leaf size=261

$$\frac{8a^2(a+bx)^{3/2}}{3b^2(a-c)^3}-\frac{8c^2(bx+c)^{3/2}}{3b^2(a-c)^3}+\frac{8(a+bx)^{7/2}}{7b^2(a-c)^3}+\frac{2(a+3c)(a+bx)^{5/2}}{5b^2(a-c)^3}-\frac{16a(a+bx)^{5/2}}{5b^2(a-c)^3}-\frac{2a(a+3c)(a+bx)^{3/2}}{3b^2(a-c)^3}-\frac{8(bx+c)^{7/2}}{7b^2(a-c)^3}+\frac{16c(bx+c)^{5/2}}{5b^2(a-c)^3}-\frac{2(3a+c)(bx+c)^{5/2}}{5b^2(a-c)^3}+\frac{2c(3a+c)(bx+c)^{3/2}}{3b^2(a-c)^3}$$

```
[Out] (8*a^2*(a + b*x)^(3/2))/(3*b^2*(a - c)^3) - (2*a*(a + 3*c)*(a + b
*x)^(3/2))/(3*b^2*(a - c)^3) - (16*a*(a + b*x)^(5/2))/(5*b^2*(a -
 c)^3) + (2*(a + 3*c)*(a + b*x)^(5/2))/(5*b^2*(a - c)^3) + (8*(a
+ b*x)^(7/2))/(7*b^2*(a - c)^3) - (8*c^2*(c + b*x)^(3/2))/(3*b^2*
(a - c)^3) + (2*c*(3*a + c)*(c + b*x)^(3/2))/(3*b^2*(a - c)^3) +
(16*c*(c + b*x)^(5/2))/(5*b^2*(a - c)^3) - (2*(3*a + c)*(c + b*x)
^(5/2))/(5*b^2*(a - c)^3) - (8*(c + b*x)^(7/2))/(7*b^2*(a - c)^3)
```

Rubi [A] time = 0.50518, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$

$$\frac{8a^2(a+bx)^{3/2}}{3b^2(a-c)^3}-\frac{8c^2(bx+c)^{3/2}}{3b^2(a-c)^3}+\frac{8(a+bx)^{7/2}}{7b^2(a-c)^3}+\frac{2(a+3c)(a+bx)^{5/2}}{5b^2(a-c)^3}-\frac{16a(a+bx)^{5/2}}{5b^2(a-c)^3}-\frac{2a(a+3c)(a+bx)^{3/2}}{3b^2(a-c)^3}-\frac{8(bx+c)^{7/2}}{7b^2(a-c)^3}+\frac{16c(bx+c)^{5/2}}{5b^2(a-c)^3}-\frac{2(3a+c)(bx+c)^{5/2}}{5b^2(a-c)^3}+\frac{2c(3a+c)(bx+c)^{3/2}}{3b^2(a-c)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(Sqrt[a + b*x] + Sqrt[c + b*x])^3,x]
```

```
[Out] (8*a^2*(a + b*x)^(3/2))/(3*b^2*(a - c)^3) - (2*a*(a + 3*c)*(a + b
*x)^(3/2))/(3*b^2*(a - c)^3) - (16*a*(a + b*x)^(5/2))/(5*b^2*(a -
 c)^3) + (2*(a + 3*c)*(a + b*x)^(5/2))/(5*b^2*(a - c)^3) + (8*(a
+ b*x)^(7/2))/(7*b^2*(a - c)^3) - (8*c^2*(c + b*x)^(3/2))/(3*b^2*
(a - c)^3) + (2*c*(3*a + c)*(c + b*x)^(3/2))/(3*b^2*(a - c)^3) +
(16*c*(c + b*x)^(5/2))/(5*b^2*(a - c)^3) - (2*(3*a + c)*(c + b*x)
^(5/2))/(5*b^2*(a - c)^3) - (8*(c + b*x)^(7/2))/(7*b^2*(a - c)^3)
```

Rubi in Sympy [A] time = 53.0561, size = 236, normalized size = 0.9

$$\frac{8a^2(a+bx)^{\frac{3}{2}}}{3b^2(a-c)^3}-\frac{2a(a+3c)(a+bx)^{\frac{3}{2}}}{3b^2(a-c)^3}-\frac{16a(a+bx)^{\frac{5}{2}}}{5b^2(a-c)^3}-\frac{8c^2(bx+c)^{\frac{3}{2}}}{3b^2(a-c)^3}+\frac{2c(3a+c)(bx+c)^{\frac{3}{2}}}{3b^2(a-c)^3}+\frac{16c(bx+c)^{\frac{5}{2}}}{5b^2(a-c)^3}+\frac{2(a+3c)(a+bx)^{\frac{5}{2}}}{5b^2(a-c)^3}+\frac{8(a+bx)^{\frac{7}{2}}}{7b^2(a-c)^3}-\frac{2(3a+c)(bx+c)^{\frac{5}{2}}}{5b^2(a-c)^3}-\frac{8(bx+c)^{\frac{7}{2}}}{7b^2(a-c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/((b*x+a)**(1/2)+(b*x+c)**(1/2))**3,x)
```

```
[Out] 8*a**2*(a + b*x)**(3/2)/(3*b**2*(a - c)**3) - 2*a*(a + 3*c)*(a +
b*x)**(3/2)/(3*b**2*(a - c)**3) - 16*a*(a + b*x)**(5/2)/(5*b**2*(
a - c)**3) - 8*c**2*(b*x + c)**(3/2)/(3*b**2*(a - c)**3) + 2*c*(3
*a + c)*(b*x + c)**(3/2)/(3*b**2*(a - c)**3) + 16*c*(b*x + c)**(5
/2)/(5*b**2*(a - c)**3) + 2*(a + 3*c)*(a + b*x)**(5/2)/(5*b**2*(a
 - c)**3) + 8*(a + b*x)**(7/2)/(7*b**2*(a - c)**3) - 2*(3*a + c)*
(b*x + c)**(5/2)/(5*b**2*(a - c)**3) - 8*(b*x + c)**(7/2)/(7*b**2
*(a - c)**3)
```

Mathematica [A] time = 0.397595, size = 93, normalized size = 0.36

$$\frac{2\left((a+bx)^{3/2}\left(6a^2-a(9bx+14c)+bx(20bx+21c)\right)+(bx+c)^{3/2}\left(7a(2c-3bx)-20b^2x^2+9bcx-6c^2\right)\right)}{35b^2(a-c)^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(Sqrt[a + b*x] + Sqrt[c + b*x])^3,x]
```

```
[Out] (2*((c + b*x)^(3/2)*(-6*c^2 + 9*b*c*x - 20*b^2*x^2 + 7*a*(2*c - 3
*b*x)) + (a + b*x)^(3/2)*(6*a^2 - a*(14*c + 9*b*x) + b*x*(21*c +
20*b*x))))/(35*b^2*(a - c)^3)
```

Maple [A] time = 0.004, size = 222, normalized size = 0.9

$$2\,\frac{a\left(1/5\,\left(bx+a\right)^{5/2}-1/3\,\left(bx+a\right)^{3/2}a\right)}{\left(a-c\right)^{3}b^{2}}+6\,\frac{c\left(1/5\,\left(bx+a\right)^{5/2}-1/3\,\left(bx+a\right)^{3/2}a\right)}{\left(a-c\right)^{3}b^{2}}-6\,\frac{a\left(1/5\,\left(bx+c\right)^{5/2}-1/3\,\left(bx+c\right)^{3/2}c\right)}{\left(a-c\right)^{3}b^{2}}-2\,\frac{c\left(1/5\,\left(bx+c\right)^{5/2}-1/3\,\left(bx+c\right)^{3/2}c\right)}{\left(a-c\right)^{3}b^{2}}+8\,\frac{1/7\,\left(bx+a\right)^{7/2}-2/5\,\left(bx+a\right)^{5/2}a+1/3\,a^{2}\left(bx+a\right)^{3/2}}{\left(a-c\right)^{3}b^{2}}-8\,\frac{1/7\,\left(bx+c\right)^{7/2}-2/5\,\left(bx+c\right)^{5/2}c+1/3\,c^{2}\left(bx+c\right)^{3/2}}{\left(a-c\right)^{3}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x)
```

```
[Out] 2/(a-c)^3*a/b^2*(1/5*(b*x+a)^(5/2)-1/3*(b*x+a)^(3/2)*a)+6/(a-c)^3
*c/b^2*(1/5*(b*x+a)^(5/2)-1/3*(b*x+a)^(3/2)*a)-6/(a-c)^3*a/b^2*(1
/5*(b*x+c)^(5/2)-1/3*(b*x+c)^(3/2)*c)-2/(a-c)^3*c/b^2*(1/5*(b*x+c
)^(5/2)-1/3*(b*x+c)^(3/2)*c)+8/(a-c)^3/b^2*(1/7*(b*x+a)^(7/2)-2/5
*(b*x+a)^(5/2)*a+1/3*a^2*(b*x+a)^(3/2))-8/(a-c)^3/b^2*(1/7*(b*x+c
)^(7/2)-2/5*(b*x+c)^(5/2)*c+1/3*c^2*(b*x+c)^(3/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(b*x + a) + sqrt(b*x + c))^3,x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(b*x + a) + sqrt(b*x + c))^3, x)
```

Fricas [A] time = 0.275982, size = 215, normalized size = 0.82

$$\frac{2\left(\left(20\,b^{3}x^{3}+6\,a^{3}-14\,a^{2}c+\left(11\,ab^{2}+21\,b^{2}c\right)x^{2}-\left(3\,a^{2}b-7\,abc\right)x\right)\sqrt{bx+a}-\left(20\,b^{3}x^{3}-14\,ac^{2}+6\,c^{3}+\left(21\,ab^{2}+11\,b^{2}c\right)\right.}{35\left(a^{3}b^{2}-3\,a^{2}b^{2}c+3\,ab^{2}c^{2}-b^{2}c^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(b*x + a) + sqrt(b*x + c))^3,x, algorithm="fricas")
```

```
[Out] 2/35*((20*b^3*x^3 + 6*a^3 - 14*a^2*c + (11*a*b^2 + 21*b^2*c)*x^2
- (3*a^2*b - 7*a*b*c)*x)*sqrt(b*x + a) - (20*b^3*x^3 - 14*a*c^2 +
 6*c^3 + (21*a*b^2 + 11*b^2*c)*x^2 + (7*a*b*c - 3*b*c^2)*x)*sqrt(
b*x + c))/(a^3*b^2 - 3*a^2*b^2*c + 3*a*b^2*c^2 - b^2*c^3)
```

Sympy [A] time = 7.27523, size = 942, normalized size = 3.61

$$\begin{cases} \frac{12a^2}{35ab^2\sqrt{a+bx}+105ab^2\sqrt{bx+c}+140b^3x\sqrt{a+bx}+140b^3x\sqrt{bx+c}+105b^2c\sqrt{a+bx}+35b^2c\sqrt{bx+c}} + \frac{54abx}{35ab^2\sqrt{a+bx}+105ab^2\sqrt{bx+c}+140b^3x\sqrt{a+bx}+140b^3x\sqrt{bx+c}+ \\ \frac{x^2}{2\left(\sqrt{a}+\sqrt{c}\right)^3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((b*x+a)**(1/2)+(b*x+c)**(1/2))**3,x)
```

```
[Out] Piecewise((12*a**2/(35*a*b**2*sqrt(a + b*x) + 105*a*b**2*sqrt(b*x
 + c) + 140*b**3*x*sqrt(a + b*x) + 140*b**3*x*sqrt(b*x + c) + 105
*b**2*c*sqrt(a + b*x) + 35*b**2*c*sqrt(b*x + c)) + 54*a*b*x/(35*a
*b**2*sqrt(a + b*x) + 105*a*b**2*sqrt(b*x + c) + 140*b**3*x*sqrt(
a + b*x) + 140*b**3*x*sqrt(b*x + c) + 105*b**2*c*sqrt(a + b*x) +
35*b**2*c*sqrt(b*x + c)) + 44*a*c/(35*a*b**2*sqrt(a + b*x) + 105*
a*b**2*sqrt(b*x + c) + 140*b**3*x*sqrt(a + b*x) + 140*b**3*x*sqrt
(b*x + c) + 105*b**2*c*sqrt(a + b*x) + 35*b**2*c*sqrt(b*x + c)) +
 36*a*sqrt(a + b*x)*sqrt(b*x + c)/(35*a*b**2*sqrt(a + b*x) + 105*
a*b**2*sqrt(b*x + c) + 140*b**3*x*sqrt(a + b*x) + 140*b**3*x*sqrt
(b*x + c) + 105*b**2*c*sqrt(a + b*x) + 35*b**2*c*sqrt(b*x + c)) +
 40*b**2*x**2/(35*a*b**2*sqrt(a + b*x) + 105*a*b**2*sqrt(b*x + c)
 + 140*b**3*x*sqrt(a + b*x) + 140*b**3*x*sqrt(b*x + c) + 105*b**2
*c*sqrt(a + b*x) + 35*b**2*c*sqrt(b*x + c)) + 54*b*c*x/(35*a*b**2
*sqrt(a + b*x) + 105*a*b**2*sqrt(b*x + c) + 140*b**3*x*sqrt(a + b
*x) + 140*b**3*x*sqrt(b*x + c) + 105*b**2*c*sqrt(a + b*x) + 35*b*
*2*c*sqrt(b*x + c)) + 30*b*x*sqrt(a + b*x)*sqrt(b*x + c)/(35*a*b*
*2*sqrt(a + b*x) + 105*a*b**2*sqrt(b*x + c) + 140*b**3*x*sqrt(a +
 b*x) + 140*b**3*x*sqrt(b*x + c) + 105*b**2*c*sqrt(a + b*x) + 35*
b**2*c*sqrt(b*x + c)) + 12*c**2/(35*a*b**2*sqrt(a + b*x) + 105*a*
b**2*sqrt(b*x + c) + 140*b**3*x*sqrt(a + b*x) + 140*b**3*x*sqrt(b
*x + c) + 105*b**2*c*sqrt(a + b*x) + 35*b**2*c*sqrt(b*x + c)) + 3
6*c*sqrt(a + b*x)*sqrt(b*x + c)/(35*a*b**2*sqrt(a + b*x) + 105*a*
b**2*sqrt(b*x + c) + 140*b**3*x*sqrt(a + b*x) + 140*b**3*x*sqrt(b
*x + c) + 105*b**2*c*sqrt(a + b*x) + 35*b**2*c*sqrt(b*x + c)), Ne
(b, 0)), (x**2/(2*(sqrt(a) + sqrt(c))**3), True))
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(b*x + a) + sqrt(b*x + c))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.252 $\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3} \, dx$

Optimal. Leaf size=64

$$\frac{(a-c)^2}{10b\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^5}-\frac{1}{2b\left(\sqrt{a+bx}+\sqrt{bx+c}\right)}$$

```
[Out] (a - c)^2/(10*b*(Sqrt[a + b*x] + Sqrt[c + b*x])^5) - 1/(2*b*(Sqrt
[a + b*x] + Sqrt[c + b*x]))
```

Rubi [B] time = 0.213021, antiderivative size = 151, normalized size of antiderivative = 2.36, number of steps used = 6, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095

$$\frac{8(a+bx)^{5/2}}{5b(a-c)^3}+\frac{2(a+3c)(a+bx)^{3/2}}{3b(a-c)^3}-\frac{8a(a+bx)^{3/2}}{3b(a-c)^3}-\frac{8(bx+c)^{5/2}}{5b(a-c)^3}+\frac{8c(bx+c)^{3/2}}{3b(a-c)^3}-\frac{2(3a+c)(bx+c)^{3/2}}{3b(a-c)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[a + b*x] + Sqrt[c + b*x])^(-3),x]
```

```
[Out] (-8*a*(a + b*x)^(3/2))/(3*b*(a - c)^3) + (2*(a + 3*c)*(a + b*x)^(
3/2))/(3*b*(a - c)^3) + (8*(a + b*x)^(5/2))/(5*b*(a - c)^3) + (8*
c*(c + b*x)^(3/2))/(3*b*(a - c)^3) - (2*(3*a + c)*(c + b*x)^(3/2)
)/(3*b*(a - c)^3) - (8*(c + b*x)^(5/2))/(5*b*(a - c)^3)
```

Rubi in Sympy [A] time = 25.8229, size = 124, normalized size = 1.94

$$-\frac{8a\left(a+bx\right)^{\frac{3}{2}}}{3b\left(a-c\right)^{3}}+\frac{8c\left(bx+c\right)^{\frac{3}{2}}}{3b\left(a-c\right)^{3}}+\frac{2\left(a+3c\right)\left(a+bx\right)^{\frac{3}{2}}}{3b\left(a-c\right)^{3}}+\frac{8\left(a+bx\right)^{\frac{5}{2}}}{5b\left(a-c\right)^{3}}-\frac{2\left(3a+c\right)\left(bx+c\right)^{\frac{3}{2}}}{3b\left(a-c\right)^{3}}-\frac{8\left(bx+c\right)^{\frac{5}{2}}}{5b\left(a-c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/((b*x+a)**(1/2)+(b*x+c)**(1/2))**3,x)
```

```
[Out] -8*a*(a + b*x)**(3/2)/(3*b*(a - c)**3) + 8*c*(b*x + c)**(3/2)/(3*
b*(a - c)**3) + 2*(a + 3*c)*(a + b*x)**(3/2)/(3*b*(a - c)**3) + 8
*(a + b*x)**(5/2)/(5*b*(a - c)**3) - 2*(3*a + c)*(b*x + c)**(3/2)
/(3*b*(a - c)**3) - 8*(b*x + c)**(5/2)/(5*b*(a - c)**3)
```

Mathematica [A] time = 0.252068, size = 55, normalized size = 0.86

$$-\frac{2\left((a+bx)^{3/2}(a-4bx-5c)+(bx+c)^{3/2}(5a+4bx-c)\right)}{5b(a-c)^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[a + b*x] + Sqrt[c + b*x])^(-3),x]
```

```
[Out] (-2*((a - 5*c - 4*b*x)*(a + b*x)^(3/2) + (c + b*x)^(3/2)*(5*a - c
 + 4*b*x)))/(5*b*(a - c)^3)
```

Maple [B] time = 0.004, size = 146, normalized size = 2.3

$$\frac{2\,a}{3\,b\,(a-c)^3}\,(bx+a)^{\frac{3}{2}}+2\,\frac{c\,(bx+a)^{3/2}}{b\,(a-c)^3}-2\,\frac{a\,(bx+c)^{3/2}}{b\,(a-c)^3}-\frac{2\,c}{3\,b\,(a-c)^3}\,(bx+c)^{\frac{3}{2}}+8\,\frac{1/5\,(bx+a)^{5/2}-1/3\,(bx+a)^{3/2}\,a}{b\,(a-c)^3}-8\,\frac{1/5\,(bx+c)^{5/2}-1/3\,(bx+c)^{3/2}\,c}{b\,(a-c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x)
```

```
[Out] 2/3*a*(b*x+a)^(3/2)/b/(a-c)^3+2/(a-c)^3*c*(b*x+a)^(3/2)/b-2/(a-c)
^3*a*(b*x+c)^(3/2)/b-2/3*c*(b*x+c)^(3/2)/b/(a-c)^3+8/(a-c)^3/b*(1
/5*(b*x+a)^(5/2)-1/3*(b*x+a)^(3/2)*a)-8/(a-c)^3/b*(1/5*(b*x+c)^(5
/2)-1/3*(b*x+c)^(3/2)*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(b*x + a) + sqrt(b*x + c))^(-3),x, algorithm="maxima")
```

```
[Out] integrate((sqrt(b*x + a) + sqrt(b*x + c))^(-3), x)
```

Fricas [A] time = 0.262813, size = 143, normalized size = 2.23

$$\frac{2\left(\left(4\,b^2x^2-a^2+5\,ac+(3\,ab+5\,bc)x\right)\sqrt{bx+a}-\left(4\,b^2x^2+5\,ac-c^2+(5\,ab+3\,bc)x\right)\sqrt{bx+c}\right)}{5\,(a^3b-3\,a^2bc+3\,abc^2-bc^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sqrt(b*x + a) + sqrt(b*x + c))^(-3),x, algorithm="fricas")
```

```
[Out] 2/5*((4*b^2*x^2 - a^2 + 5*a*c + (3*a*b + 5*b*c)*x)*sqrt(b*x + a)
- (4*b^2*x^2 + 5*a*c - c^2 + (5*a*b + 3*b*c)*x)*sqrt(b*x + c))/(a
^3*b - 3*a^2*b*c + 3*a*b*c^2 - b*c^3)
```

Sympy [A] time = 6.79849, size = 384, normalized size = 6.

$$\begin{cases} -\frac{2a}{5ab\sqrt{a+bx}+15ab\sqrt{bx+c}+20b^2x\sqrt{a+bx}+20b^2x\sqrt{bx+c}+15bc\sqrt{a+bx}+5bc\sqrt{bx+c}} - \frac{4bx}{5ab\sqrt{a+bx}+15ab\sqrt{bx+c}+20b^2x\sqrt{a+bx}+20b^2x\sqrt{bx+c}+15bc\sqrt{a+bx}+5 \\ \frac{x}{\left(\sqrt{a}+\sqrt{c}\right)^3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x+a)**(1/2)+(b*x+c)**(1/2))**3,x)
```

```
[Out] Piecewise((-2*a/(5*a*b*sqrt(a + b*x) + 15*a*b*sqrt(b*x + c) + 20*
b**2*x*sqrt(a + b*x) + 20*b**2*x*sqrt(b*x + c) + 15*b*c*sqrt(a +
b*x) + 5*b*c*sqrt(b*x + c)) - 4*b*x/(5*a*b*sqrt(a + b*x) + 15*a*b
*sqrt(b*x + c) + 20*b**2*x*sqrt(a + b*x) + 20*b**2*x*sqrt(b*x + c
) + 15*b*c*sqrt(a + b*x) + 5*b*c*sqrt(b*x + c)) - 2*c/(5*a*b*sqrt
(a + b*x) + 15*a*b*sqrt(b*x + c) + 20*b**2*x*sqrt(a + b*x) + 20*b
**2*x*sqrt(b*x + c) + 15*b*c*sqrt(a + b*x) + 5*b*c*sqrt(b*x + c))
 - 6*sqrt(a + b*x)*sqrt(b*x + c)/(5*a*b*sqrt(a + b*x) + 15*a*b*sq
rt(b*x + c) + 20*b**2*x*sqrt(a + b*x) + 20*b**2*x*sqrt(b*x + c) +
 15*b*c*sqrt(a + b*x) + 5*b*c*sqrt(b*x + c)), Ne(b, 0)), (x/(sqrt
(a) + sqrt(c))**3, True))
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(b*x + a) + sqrt(b*x + c))^(-3),x, algorithm="giac")
```

```
[Out] Timed out
```

3.253 $\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx$

Optimal. Leaf size=157

$$\frac{8(a+bx)^{3/2}}{3(a-c)^3}+\frac{2(a+3c)\sqrt{a+bx}}{(a-c)^3}-\frac{8(bx+c)^{3/2}}{3(a-c)^3}-\frac{2(3a+c)\sqrt{bx+c}}{(a-c)^3}-\frac{2\sqrt{a}(a+3c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(a-c)^3}+\frac{2\sqrt{c}(3a+c)\tanh^{-1}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{(a-c)^3}$$

```
[Out] (2*(a + 3*c)*Sqrt[a + b*x])/(a - c)^3 + (8*(a + b*x)^(3/2))/(3*(a
 - c)^3) - (2*(3*a + c)*Sqrt[c + b*x])/(a - c)^3 - (8*(c + b*x)^(
3/2))/(3*(a - c)^3) - (2*Sqrt[a]*(a + 3*c)*ArcTanh[Sqrt[a + b*x]/
Sqrt[a]])/(a - c)^3 + (2*Sqrt[c]*(3*a + c)*ArcTanh[Sqrt[c + b*x]/
Sqrt[c]])/(a - c)^3
```

Rubi [A] time = 0.46371, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16

$$\frac{8(a+bx)^{3/2}}{3(a-c)^3}+\frac{2(a+3c)\sqrt{a+bx}}{(a-c)^3}-\frac{8(bx+c)^{3/2}}{3(a-c)^3}-\frac{2(3a+c)\sqrt{bx+c}}{(a-c)^3}-\frac{2\sqrt{a}(a+3c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(a-c)^3}+\frac{2\sqrt{c}(3a+c)\tanh^{-1}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{(a-c)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(Sqrt[a + b*x] + Sqrt[c + b*x])^3),x]
```

```
[Out] (2*(a + 3*c)*Sqrt[a + b*x])/(a - c)^3 + (8*(a + b*x)^(3/2))/(3*(a
 - c)^3) - (2*(3*a + c)*Sqrt[c + b*x])/(a - c)^3 - (8*(c + b*x)^(
3/2))/(3*(a - c)^3) - (2*Sqrt[a]*(a + 3*c)*ArcTanh[Sqrt[a + b*x]/
Sqrt[a]])/(a - c)^3 + (2*Sqrt[c]*(3*a + c)*ArcTanh[Sqrt[c + b*x]/
Sqrt[c]])/(a - c)^3
```

Rubi in Sympy [A] time = 34.0811, size = 138, normalized size = 0.88

$$-\frac{2\sqrt{a}\,(a+3c)\operatorname{atanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(a-c)^3}+\frac{2\sqrt{c}\,(3a+c)\operatorname{atanh}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{(a-c)^3}+\frac{2\,(a+3c)\,\sqrt{a+bx}}{(a-c)^3}+\frac{8\,(a+bx)^{\frac{3}{2}}}{3\,(a-c)^3}-\frac{2\,(3a+c)\,\sqrt{bx+c}}{(a-c)^3}-\frac{8\,(bx+c)^{\frac{3}{2}}}{3\,(a-c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/((b*x+a)**(1/2)+(b*x+c)**(1/2))**3,x)
```

```
[Out] -2*sqrt(a)*(a + 3*c)*atanh(sqrt(a + b*x)/sqrt(a))/(a - c)**3 + 2*
sqrt(c)*(3*a + c)*atanh(sqrt(b*x + c)/sqrt(c))/(a - c)**3 + 2*(a
+ 3*c)*sqrt(a + b*x)/(a - c)**3 + 8*(a + b*x)**(3/2)/(3*(a - c)**
3) - 2*(3*a + c)*sqrt(b*x + c)/(a - c)**3 - 8*(b*x + c)**(3/2)/(3
*(a - c)**3)
```

Mathematica [A] time = 0.242034, size = 142, normalized size = 0.9

$$\frac{2\left(-9a\sqrt{bx+c}+9c\sqrt{a+bx}-3\sqrt{a}(a+3c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+3\sqrt{c}(3a+c)\tanh^{-1}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)+7a\sqrt{a+bx}+4bx\sqrt{a+bx}-7c\right.}{3(a-c)^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(Sqrt[a + b*x] + Sqrt[c + b*x])^3),x]
```

```
[Out] (2*(7*a*Sqrt[a + b*x] + 9*c*Sqrt[a + b*x] + 4*b*x*Sqrt[a + b*x] -
 9*a*Sqrt[c + b*x] - 7*c*Sqrt[c + b*x] - 4*b*x*Sqrt[c + b*x] - 3*
Sqrt[a]*(a + 3*c)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]] + 3*Sqrt[c]*(3*a
 + c)*ArcTanh[Sqrt[c + b*x]/Sqrt[c]]))/(3*(a - c)^3)
```

Maple [A] time = 0.005, size = 181, normalized size = 1.2

$$\frac{a}{(a-c)^3}\left(2\sqrt{bx+a}-2\sqrt{a}Artanh\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)+\frac{8}{3\,(a-c)^3}(bx+a)^{\frac{3}{2}}-\frac{8}{3\,(a-c)^3}(bx+c)^{\frac{3}{2}}+3\frac{c}{(a-c)^3}\left(2\sqrt{bx+a}-2\sqrt{a}Artanh\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)-3\frac{a}{(a-c)^3}\left(2\sqrt{bx+c}-2\sqrt{c}Artanh\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)\right)-\frac{c}{(a-c)^3}\left(2\sqrt{bx+c}-2\sqrt{c}Artanh\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x)
```

```
[Out] 1/(a-c)^3*a*(2*(b*x+a)^(1/2)-2*a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1
/2)))+8/3*(b*x+a)^(3/2)/(a-c)^3-8/3*(b*x+c)^(3/2)/(a-c)^3+3/(a-c)
^3*c*(2*(b*x+a)^(1/2)-2*a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2)))-3
/(a-c)^3*a*(2*(b*x+c)^(1/2)-2*c^(1/2)*arctanh((b*x+c)^(1/2)/c^(1/
2)))-1/(a-c)^3*c*(2*(b*x+c)^(1/2)-2*c^(1/2)*arctanh((b*x+c)^(1/2)
/c^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(sqrt(b*x + a) + sqrt(b*x + c))^3),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*(sqrt(b*x + a) + sqrt(b*x + c))^3), x)
```

Fricas [A] time = 0.31121, size = 1, normalized size = 0.01

$$\left[-\frac{3\,(a+3\,c)\sqrt{a}\log\left(\frac{bx+2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+3\,(3\,a+c)\sqrt{c}\log\left(\frac{bx-2\,\sqrt{bx+c}\sqrt{c}+2\,c}{x}\right)-2\,(4\,bx+7\,a+9\,c)\sqrt{bx+a}+2\,(4\,bx+9\,a+7}{3\,(a^3-3\,a^2c+3\,ac^2-c^3)}\right.$$
$$-\frac{6\,\sqrt{-a}(a+3\,c)\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)+3\,(3\,a+c)\sqrt{c}\log\left(\frac{bx-2\,\sqrt{bx+c}\sqrt{c}+2\,c}{x}\right)-2\,(4\,bx+7\,a+9\,c)\sqrt{bx+a}+2\,(4\,bx+9\,a+7\,c)\sqrt{bx}}{3\,(a^3-3\,a^2c+3\,ac^2-c^3)}$$
$$-\frac{2\left(3\,\sqrt{-a}(a+3\,c)\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)-3\,(3\,a+c)\sqrt{-c}\arctan\left(\frac{\sqrt{bx+c}}{\sqrt{-c}}\right)-(4\,bx+7\,a+9\,c)\sqrt{bx+a}+(4\,bx+9\,a+7\,c)\sqrt{bx+c}\right)}{3\,(a^3-3\,a^2c+3\,ac^2-c^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(sqrt(b*x + a) + sqrt(b*x + c))^3),x, algorithm="fricas")
```

```
[Out] [-1/3*(3*(a + 3*c)*sqrt(a)*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2
*a)/x) + 3*(3*a + c)*sqrt(c)*log((b*x - 2*sqrt(b*x + c)*sqrt(c) +
 2*c)/x) - 2*(4*b*x + 7*a + 9*c)*sqrt(b*x + a) + 2*(4*b*x + 9*a +
 7*c)*sqrt(b*x + c))/(a^3 - 3*a^2*c + 3*a*c^2 - c^3), 1/3*(6*(3*a
 + c)*sqrt(-c)*arctan(sqrt(b*x + c)/sqrt(-c)) - 3*(a + 3*c)*sqrt(
a)*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*(4*b*x + 7*a
+ 9*c)*sqrt(b*x + a) - 2*(4*b*x + 9*a + 7*c)*sqrt(b*x + c))/(a^3
- 3*a^2*c + 3*a*c^2 - c^3), -1/3*(6*sqrt(-a)*(a + 3*c)*arctan(sqr
```

```
t(b*x + a)/sqrt(-a)) + 3*(3*a + c)*sqrt(c)*log((b*x - 2*sqrt(b*x
+ c)*sqrt(c) + 2*c)/x) - 2*(4*b*x + 7*a + 9*c)*sqrt(b*x + a) + 2*
(4*b*x + 9*a + 7*c)*sqrt(b*x + c))/(a^3 - 3*a^2*c + 3*a*c^2 - c^3
), -2/3*(3*sqrt(-a)*(a + 3*c)*arctan(sqrt(b*x + a)/sqrt(-a)) - 3*
(3*a + c)*sqrt(-c)*arctan(sqrt(b*x + c)/sqrt(-c)) - (4*b*x + 7*a
+ 9*c)*sqrt(b*x + a) + (4*b*x + 9*a + 7*c)*sqrt(b*x + c))/(a^3 -
3*a^2*c + 3*a*c^2 - c^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/((b*x+a)**(1/2)+(b*x+c)**(1/2))**3,x)

[Out] Integral(1/(x*(sqrt(a + b*x) + sqrt(b*x + c))**3), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(sqrt(b*x + a) + sqrt(b*x + c))^3),x, algorithm="giac")

[Out] Timed out
```

3.254 $\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx$

Optimal. Leaf size=162

$$\frac{8b\sqrt{a+bx}}{(a-c)^3}-\frac{8b\sqrt{bx+c}}{(a-c)^3}-\frac{(a+3c)\sqrt{a+bx}}{x(a-c)^3}+\frac{(3a+c)\sqrt{bx+c}}{x(a-c)^3}-\frac{3b(3a+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}(a-c)^3}-\frac{3b(a+3c)\tanh^{-1}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{\sqrt{c}(c-a)^3}$$

```
[Out] (8*b*Sqrt[a + b*x])/(a - c)^3 - ((a + 3*c)*Sqrt[a + b*x])/((a - c
)^3*x) - (8*b*Sqrt[c + b*x])/(a - c)^3 + ((3*a + c)*Sqrt[c + b*x]
)/((a - c)^3*x) - (3*b*(3*a + c)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/
(Sqrt[a]*(a - c)^3) - (3*b*(a + 3*c)*ArcTanh[Sqrt[c + b*x]/Sqrt[c
]])/(Sqrt[c]*(-a + c)^3)
```

Rubi [A] time = 0.564657, antiderivative size = 223, normalized size of antiderivative = 1.38, number of steps used = 14, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{8b\sqrt{a+bx}}{(a-c)^3}-\frac{8b\sqrt{bx+c}}{(a-c)^3}-\frac{(a+3c)\sqrt{a+bx}}{x(a-c)^3}+\frac{(3a+c)\sqrt{bx+c}}{x(a-c)^3}-\frac{b(a+3c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}(a-c)^3}-\frac{8\sqrt{a}b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(a-c)^3}+\frac{b(3a+c)\tanh^{-1}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{\sqrt{c}(a-c)^3}+\frac{8b\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{(a-c)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(Sqrt[a + b*x] + Sqrt[c + b*x])^3),x]
```

```
[Out] (8*b*Sqrt[a + b*x])/(a - c)^3 - ((a + 3*c)*Sqrt[a + b*x])/((a - c
)^3*x) - (8*b*Sqrt[c + b*x])/(a - c)^3 + ((3*a + c)*Sqrt[c + b*x]
)/((a - c)^3*x) - (8*Sqrt[a]*b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(a
 - c)^3 - (b*(a + 3*c)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(Sqrt[a]*(
a - c)^3) + (8*b*Sqrt[c]*ArcTanh[Sqrt[c + b*x]/Sqrt[c]])/(a - c)^
3 + (b*(3*a + c)*ArcTanh[Sqrt[c + b*x]/Sqrt[c]])/((a - c)^3*Sqrt[
c])
```

Rubi in Sympy [A] time = 41.0077, size = 196, normalized size = 1.21

$$-\frac{8\sqrt{a}b\,\mathrm{atanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(a-c)^3}+\frac{8b\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{(a-c)^3}+\frac{8b\sqrt{a+bx}}{(a-c)^3}-\frac{8b\sqrt{bx+c}}{(a-c)^3}+\frac{b(3a+c)\,\mathrm{atanh}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{\sqrt{c}(a-c)^3}-\frac{(a+3c)\sqrt{a+bx}}{x(a-c)^3}+\frac{(3a+c)\sqrt{bx+c}}{x(a-c)^3}-\frac{b(a+3c)\,\mathrm{atanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}(a-c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/((b*x+a)**(1/2)+(b*x+c)**(1/2))**3,x)
```

```
[Out] -8*sqrt(a)*b*atanh(sqrt(a + b*x)/sqrt(a))/(a - c)**3 + 8*b*sqrt(c
)*atanh(sqrt(b*x + c)/sqrt(c))/(a - c)**3 + 8*b*sqrt(a + b*x)/(a
- c)**3 - 8*b*sqrt(b*x + c)/(a - c)**3 + b*(3*a + c)*atanh(sqrt(b
*x + c)/sqrt(c))/(sqrt(c)*(a - c)**3) - (a + 3*c)*sqrt(a + b*x)/(
x*(a - c)**3) + (3*a + c)*sqrt(b*x + c)/(x*(a - c)**3) - b*(a + 3
*c)*atanh(sqrt(a + b*x)/sqrt(a))/(sqrt(a)*(a - c)**3)
```

Mathematica [A] time = 0.511731, size = 112, normalized size = 0.69

$$\frac{\frac{\sqrt{bx+c}(3a-8bx+c)}{x}-\frac{\sqrt{a+bx}(a-8bx+3c)}{x}-\frac{3b(3a+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{3b(a+3c)\tanh^{-1}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{\sqrt{c}}}{(a-c)^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(Sqrt[a + b*x] + Sqrt[c + b*x])^3),x]
```

```
[Out] (-(((a + 3*c - 8*b*x)*Sqrt[a + b*x])/x) + ((3*a + c - 8*b*x)*Sqrt
[c + b*x])/x - (3*b*(3*a + c)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqr
t[a] + (3*b*(a + 3*c)*ArcTanh[Sqrt[c + b*x]/Sqrt[c]])/Sqrt[c])/(a
 - c)^3
```

Maple [A] time = 0.004, size = 252, normalized size = 1.6

$$
\begin{aligned}
&2\frac{ab}{(a-c)^3}\left(-1/2\,\frac{\sqrt{bx+a}}{bx}-1/2\,\frac{1}{\sqrt{a}}Artanh\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)\\
&+6\,\frac{bc}{(a-c)^3}\left(-1/2\,\frac{\sqrt{bx+a}}{bx}-1/2\,\frac{1}{\sqrt{a}}Artanh\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)\\
&-6\,\frac{ab}{(a-c)^3}\left(-1/2\,\frac{\sqrt{bx+c}}{bx}-1/2\,\frac{1}{\sqrt{c}}Artanh\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)\right)\\
&-2\,\frac{bc}{(a-c)^3}\left(-1/2\,\frac{\sqrt{bx+c}}{bx}-1/2\,\frac{1}{\sqrt{c}}Artanh\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)\right)\\
&+4\,\frac{b}{(a-c)^3}\left(2\sqrt{bx+a}-2\sqrt{a}Artanh\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)\\
&-4\,\frac{b}{(a-c)^3}\left(2\sqrt{bx+c}-2\sqrt{c}Artanh\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)\right)
\end{aligned}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x)
```

```
[Out] 2/(a-c)^3*a*b*(-1/2*(b*x+a)^(1/2)/x/b-1/2*arctanh((b*x+a)^(1/2)/a
^(1/2))/a^(1/2))+6/(a-c)^3*c*b*(-1/2*(b*x+a)^(1/2)/x/b-1/2*arctan
h((b*x+a)^(1/2)/a^(1/2))/a^(1/2))-6/(a-c)^3*a*b*(-1/2*(b*x+c)^(1/
2)/x/b-1/2/c^(1/2)*arctanh((b*x+c)^(1/2)/c^(1/2)))-2/(a-c)^3*c*b*
(-1/2*(b*x+c)^(1/2)/x/b-1/2/c^(1/2)*arctanh((b*x+c)^(1/2)/c^(1/2)
))+4/(a-c)^3*b*(2*(b*x+a)^(1/2)-2*a^(1/2)*arctanh((b*x+a)^(1/2)/a
^(1/2)))-4/(a-c)^3*b*(2*(b*x+c)^(1/2)-2*c^(1/2)*arctanh((b*x+c)^(
1/2)/c^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(sqrt(b*x + a) + sqrt(b*x + c))^3),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^2*(sqrt(b*x + a) + sqrt(b*x + c))^3), x)
```

Fricas [A] time = 0.317618, size = 1, normalized size = 0.01

$$\left[-\frac{3\,(3\,ab+bc)\sqrt{c}x\log\left(\frac{(bx+2\,a)\sqrt{a}+2\,\sqrt{bx+a}a}{x}\right)+3\,(ab+3\,bc)\sqrt{a}x\log\left(\frac{(bx+2\,c)\sqrt{c}-2\,\sqrt{bx+c}c}{x}\right)-2\,(8\,bx-a-3\,c)\sqrt{bx+a}\sqrt{a}\sqrt{c}}{2\,(a^3-3\,a^2c+3\,ac^2-c^3)\sqrt{a}\sqrt{c}x}\right.$$

$$-\frac{6\,(ab+3\,bc)\sqrt{a}x\arctan\left(\frac{c}{\sqrt{bx+c}\sqrt{-c}}\right)+3\,(3\,ab+bc)\sqrt{-c}x\log\left(\frac{(bx+2\,a)\sqrt{a}+2\,\sqrt{bx+a}a}{x}\right)-2\,(8\,bx-a-3\,c)\sqrt{bx+a}\sqrt{a}\sqrt{-c}+2}{2\,(a^3-3\,a^2c+3\,ac^2-c^3)\sqrt{a}\sqrt{-c}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(sqrt(b*x + a) + sqrt(b*x + c))^3),x, algorithm="fricas")
```

```
[Out] [-1/2*(3*(3*a*b + b*c)*sqrt(c)*x*log(((b*x + 2*a)*sqrt(a) + 2*sqr
t(b*x + a)*a)/x) + 3*(a*b + 3*b*c)*sqrt(a)*x*log(((b*x + 2*c)*sqr
t(c) - 2*sqrt(b*x + c)*c)/x) - 2*(8*b*x - a - 3*c)*sqrt(b*x + a)*
sqrt(a)*sqrt(c) + 2*(8*b*x - 3*a - c)*sqrt(b*x + c)*sqrt(a)*sqrt(
c))/((a^3 - 3*a^2*c + 3*a*c^2 - c^3)*sqrt(a)*sqrt(c)*x), -1/2*(6*
(a*b + 3*b*c)*sqrt(a)*x*arctan(c/(sqrt(b*x + c)*sqrt(-c))) + 3*(3
*a*b + b*c)*sqrt(-c)*x*log(((b*x + 2*a)*sqrt(a) + 2*sqrt(b*x + a)
*a)/x) - 2*(8*b*x - a - 3*c)*sqrt(b*x + a)*sqrt(a)*sqrt(-c) + 2*(
8*b*x - 3*a - c)*sqrt(b*x + c)*sqrt(a)*sqrt(-c))/((a^3 - 3*a^2*c
+ 3*a*c^2 - c^3)*sqrt(a)*sqrt(-c)*x), 1/2*(6*(3*a*b + b*c)*sqrt(c
)*x*arctan(a/(sqrt(b*x + a)*sqrt(-a))) - 3*(a*b + 3*b*c)*sqrt(-a)
*x*log(((b*x + 2*c)*sqrt(c) - 2*sqrt(b*x + c)*c)/x) + 2*(8*b*x -
a - 3*c)*sqrt(b*x + a)*sqrt(-a)*sqrt(c) - 2*(8*b*x - 3*a - c)*sqr
t(b*x + c)*sqrt(-a)*sqrt(c))/((a^3 - 3*a^2*c + 3*a*c^2 - c^3)*sqr
t(-a)*sqrt(c)*x), (3*(3*a*b + b*c)*sqrt(-c)*x*arctan(a/(sqrt(b*x
+ a)*sqrt(-a))) - 3*(a*b + 3*b*c)*sqrt(-a)*x*arctan(c/(sqrt(b*x +
 c)*sqrt(-c))) + (8*b*x - a - 3*c)*sqrt(b*x + a)*sqrt(-a)*sqrt(-c
) - (8*b*x - 3*a - c)*sqrt(b*x + c)*sqrt(-a)*sqrt(-c))/((a^3 - 3*
a^2*c + 3*a*c^2 - c^3)*sqrt(-a)*sqrt(-c)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/((b*x+a)**(1/2)+(b*x+c)**(1/2))**3,x)
```

```
[Out] Integral(1/(x**2*(sqrt(a + b*x) + sqrt(b*x + c))**3), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(sqrt(b*x + a) + sqrt(b*x + c))^3),x, algorithm="giac")
```

```
[Out] Timed out
```

3.255 $\int \frac{1}{\sqrt{x}+\sqrt{1+x}}\,dx$

Optimal. Leaf size=21

$$\frac{2}{3}(x+1)^{3/2}-\frac{2x^{3/2}}{3}$$

```
[Out] (-2*x^(3/2))/3 + (2*(1 + x)^(3/2))/3
```

Rubi [A] time = 0.0135762, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{2}{3}(x+1)^{3/2}-\frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[x] + Sqrt[1 + x])^(-1),x]
```

```
[Out] (-2*x^(3/2))/3 + (2*(1 + x)^(3/2))/3
```

Rubi in Sympy [A] time = 1.42524, size = 17, normalized size = 0.81

$$-\frac{2x^{\frac{3}{2}}}{3}+\frac{2\,(x+1)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**(1/2)+(1+x)**(1/2)),x)
```

```
[Out] -2*x**(3/2)/3 + 2*(x + 1)**(3/2)/3
```

Mathematica [A] time = 0.0224673, size = 19, normalized size = 0.9

$$\frac{2}{3}\left((x+1)^{3/2}-x^{3/2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[x] + Sqrt[1 + x])^(-1),x]
```

```
[Out] (2*(-x^(3/2) + (1 + x)^(3/2)))/3
```

Maple [A] time = 0.003, size = 14, normalized size = 0.7

$$-\frac{2}{3}x^{\frac{3}{2}}+\frac{2}{3}\left(1+x\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^(1/2)+(1+x)^(1/2)),x)
```

```
[Out] -2/3*x^(3/2)+2/3*(1+x)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x+1}+\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x + 1) + sqrt(x)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x + 1) + sqrt(x)), x)
```

Fricas [A] time = 0.308378, size = 18, normalized size = 0.86

$$\frac{2}{3}\left(x+1\right)^{\frac{3}{2}}-\frac{2}{3}\,x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x + 1) + sqrt(x)),x, algorithm="fricas")
```

```
[Out] 2/3*(x + 1)^(3/2) - 2/3*x^(3/2)
```

Sympy [A] time = 1.41415, size = 63, normalized size = 3.

$$\frac{2\sqrt{x}\sqrt{x+1}}{3\sqrt{x}+3\sqrt{x+1}}+\frac{4x}{3\sqrt{x}+3\sqrt{x+1}}+\frac{2}{3\sqrt{x}+3\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**(1/2)+(1+x)**(1/2)),x)
```

```
[Out] 2*sqrt(x)*sqrt(x + 1)/(3*sqrt(x) + 3*sqrt(x + 1)) + 4*x/(3*sqrt(x
) + 3*sqrt(x + 1)) + 2/(3*sqrt(x) + 3*sqrt(x + 1))
```

GIAC/XCAS [A] time = 0.292285, size = 18, normalized size = 0.86

$$\frac{2}{3}(x+1)^{\frac{3}{2}}-\frac{2}{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x + 1) + sqrt(x)),x, algorithm="giac")
```

```
[Out] 2/3*(x + 1)^(3/2) - 2/3*x^(3/2)
```

3.256 $\int \frac{1}{\sqrt{-1+x}+\sqrt{x}} \, dx$

Optimal. Leaf size=21

$$\frac{2x^{3/2}}{3} - \frac{2}{3}(x-1)^{3/2}$$

```
[Out] (-2*(-1 + x)^(3/2))/3 + (2*x^(3/2))/3
```

Rubi [A] time = 0.0144696, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{2x^{3/2}}{3} - \frac{2}{3}(x-1)^{3/2}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[-1 + x] + Sqrt[x])^(-1),x]
```

```
[Out] (-2*(-1 + x)^(3/2))/3 + (2*x^(3/2))/3
```

Rubi in Sympy [A] time = 1.46981, size = 17, normalized size = 0.81

$$\frac{2x^{\frac{3}{2}}}{3} - \frac{2(x-1)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/((-1+x)**(1/2)+x**(1/2)),x)
```

```
[Out] 2*x**(3/2)/3 - 2*(x - 1)**(3/2)/3
```

Mathematica [A] time = 0.0234333, size = 19, normalized size = 0.9

$$\frac{2}{3}\left(x^{3/2} - (x-1)^{3/2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[-1 + x] + Sqrt[x])^(-1),x]

[Out] (2*(-(-1 + x)^(3/2) + x^(3/2)))/3
```

Maple [A] time = 0.002, size = 14, normalized size = 0.7

$$-\frac{2}{3}(-1+x)^{\frac{3}{2}}+\frac{2}{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/((-1+x)^(1/2)+x^(1/2)),x)

[Out] -2/3*(-1+x)^(3/2)+2/3*x^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x-1}+\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x - 1) + sqrt(x)),x, algorithm="maxima")

[Out] integrate(1/(sqrt(x - 1) + sqrt(x)), x)
```

Fricas [A] time = 0.27261, size = 18, normalized size = 0.86

$$-\frac{2}{3}(x-1)^{\frac{3}{2}}+\frac{2}{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x - 1) + sqrt(x)),x, algorithm="fricas")

[Out] -2/3*(x - 1)^(3/2) + 2/3*x^(3/2)
```

Sympy [A] time = 1.44117, size = 63, normalized size = 3.

$$\frac{2\sqrt{x}\sqrt{x-1}}{3\sqrt{x}+3\sqrt{x-1}}+\frac{4x}{3\sqrt{x}+3\sqrt{x-1}}-\frac{2}{3\sqrt{x}+3\sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((-1+x)**(1/2)+x**(1/2)),x)
```

```
[Out] 2*sqrt(x)*sqrt(x - 1)/(3*sqrt(x) + 3*sqrt(x - 1)) + 4*x/(3*sqrt(x
) + 3*sqrt(x - 1)) - 2/(3*sqrt(x) + 3*sqrt(x - 1))
```

GIAC/XCAS [A] time = 0.278179, size = 18, normalized size = 0.86

$$-\frac{2}{3}\,(x-1)^{\frac{3}{2}}+\frac{2}{3}\,x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x - 1) + sqrt(x)),x, algorithm="giac")
```

```
[Out] -2/3*(x - 1)^(3/2) + 2/3*x^(3/2)
```

3.257 $\int \frac{1}{\sqrt{-1+x}+\sqrt{1+x}}\,dx$

Optimal. Leaf size=23

$$\frac{1}{3}(x+1)^{3/2}-\frac{1}{3}(x-1)^{3/2}$$

```
[Out] -(-1 + x)^(3/2)/3 + (1 + x)^(3/2)/3
```

Rubi [A] time = 0.0397659, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059

$$\frac{1}{3}(x+1)^{3/2}-\frac{1}{3}(x-1)^{3/2}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[-1 + x] + Sqrt[1 + x])^(-1),x]

[Out] -(-1 + x)^(3/2)/3 + (1 + x)^(3/2)/3
```

Rubi in Sympy [A] time = 2.57501, size = 15, normalized size = 0.65

$$-\frac{(x-1)^{\frac{3}{2}}}{3}+\frac{(x+1)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/((-1+x)**(1/2)+(1+x)**(1/2)),x)

[Out] -(x - 1)**(3/2)/3 + (x + 1)**(3/2)/3
```

Mathematica [A] time = 0.0218059, size = 31, normalized size = 1.35

$$\left(\frac{x-1}{3}+\frac{2}{3}\right)\sqrt{x+1}-\frac{1}{3}(x-1)^{3/2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[-1 + x] + Sqrt[1 + x])^(-1),x]

[Out] -(-1 + x)^(3/2)/3 + (2/3 + (-1 + x)/3)*Sqrt[1 + x]
```

Maple [A] time = 0.002, size = 16, normalized size = 0.7

$$-\frac{1}{3}(-1+x)^{\frac{3}{2}}+\frac{1}{3}(1+x)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/((-1+x)^(1/2)+(1+x)^(1/2)),x)

[Out] -1/3*(-1+x)^(3/2)+1/3*(1+x)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x+1}+\sqrt{x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x + 1) + sqrt(x - 1)),x, algorithm="maxima")

[Out] integrate(1/(sqrt(x + 1) + sqrt(x - 1)), x)
```

Fricas [A] time = 0.264395, size = 20, normalized size = 0.87

$$\frac{1}{3}(x+1)^{\frac{3}{2}}-\frac{1}{3}(x-1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x + 1) + sqrt(x - 1)),x, algorithm="fricas")

[Out] 1/3*(x + 1)^(3/2) - 1/3*(x - 1)^(3/2)
```

Sympy [A] time = 1.48457, size = 51, normalized size = 2.22

$$\frac{4x}{3\sqrt{x-1}+3\sqrt{x+1}}+\frac{2\sqrt{x-1}\sqrt{x+1}}{3\sqrt{x-1}+3\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((-1+x)**(1/2)+(1+x)**(1/2)),x)
```

```
[Out] 4*x/(3*sqrt(x - 1) + 3*sqrt(x + 1)) + 2*sqrt(x - 1)*sqrt(x + 1)/(
3*sqrt(x - 1) + 3*sqrt(x + 1))
```

GIAC/XCAS [A] time = 0.279648, size = 20, normalized size = 0.87

$$\frac{1}{3}\,(x+1)^{\frac{3}{2}}-\frac{1}{3}\,(x-1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x + 1) + sqrt(x - 1)),x, algorithm="giac")
```

```
[Out] 1/3*(x + 1)^(3/2) - 1/3*(x - 1)^(3/2)
```

3.258 $\int x^3 \left(\sqrt{1-x}+\sqrt{1+x}\right)^2 dx$

Optimal. Leaf size=38

$$\frac{x^4}{2}+\frac{2}{5}\left(1-x^2\right)^{5/2}-\frac{2}{3}\left(1-x^2\right)^{3/2}$$

```
[Out] x^4/2 - (2*(1 - x^2)^(3/2))/3 + (2*(1 - x^2)^(5/2))/5
```

Rubi [A] time = 0.197017, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13

$$\frac{x^4}{2}+\frac{2}{5}\left(1-x^2\right)^{5/2}-\frac{2}{3}\left(1-x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*(Sqrt[1 - x] + Sqrt[1 + x])^2,x]

[Out] x^4/2 - (2*(1 - x^2)^(3/2))/3 + (2*(1 - x^2)^(5/2))/5
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*((1-x)**(1/2)+(1+x)**(1/2))**2,x)

[Out] Timed out
```

Mathematica [A] time = 0.04812, size = 44, normalized size = 1.16

$$\frac{1}{30}\left(x^2-1\right)\left(3\left(4\sqrt{1-x^2}+5\right)x^2+8\sqrt{1-x^2}+15\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*(Sqrt[1 - x] + Sqrt[1 + x])^2,x]
```

```
[Out] ((-1 + x^2)*(15 + 8*Sqrt[1 - x^2] + 3*x^2*(5 + 4*Sqrt[1 - x^2])))
/30
```

Maple [A] time = 0.006, size = 33, normalized size = 0.9

$$\frac{x^4}{2} + \frac{(2x^2-2)(3x^2+2)}{15}\sqrt{1-x}\sqrt{1+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*((1-x)^(1/2)+(1+x)^(1/2))^2,x)
```

```
[Out] 1/2*x^4+2/15*(1+x)^(1/2)*(1-x)^(1/2)*(x^2-1)*(3*x^2+2)
```

Maxima [A] time = 0.792452, size = 42, normalized size = 1.11

$$\frac{1}{2}x^4 - \frac{2}{5}\left(-x^2+1\right)^{\frac{3}{2}}x^2 - \frac{4}{15}\left(-x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*(sqrt(x + 1) + sqrt(-x + 1))^2,x, algorithm="maxima")
```

```
[Out] 1/2*x^4 - 2/5*(-x^2 + 1)^(3/2)*x^2 - 4/15*(-x^2 + 1)^(3/2)
```

Fricas [A] time = 0.276094, size = 109, normalized size = 2.87

$$\frac{12x^{10} - 85x^8 + 80x^6 + 5\left(9x^8 - 16x^6\right)\sqrt{x+1}\sqrt{-x+1}}{30\left(5x^4 - 20x^2 - (x^4 - 12x^2 + 16)\sqrt{x+1}\sqrt{-x+1} + 16\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*(sqrt(x + 1) + sqrt(-x + 1))^2,x, algorithm="fricas")
```

```
[Out] 1/30*(12*x^10 - 85*x^8 + 80*x^6 + 5*(9*x^8 - 16*x^6)*sqrt(x + 1)*
sqrt(-x + 1))/(5*x^4 - 20*x^2 - (x^4 - 12*x^2 + 16)*sqrt(x + 1)*s
qrt(-x + 1) + 16)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*((1-x)**(1/2)+(1+x)**(1/2))**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.281216, size = 76, normalized size = 2.

$$\frac{1}{2}(x+1)^4 - 2(x+1)^3 + \frac{2}{15}((3(x+1)(x-3)+17)(x+1)-10)(x+1)^{\frac{3}{2}}\sqrt{-x+1} + 3(x+1)^2 - 2x - 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*(sqrt(x + 1) + sqrt(-x + 1))^2,x, algorithm="giac")
```

```
[Out] 1/2*(x + 1)^4 - 2*(x + 1)^3 + 2/15*((3*(x + 1)*(x - 3) + 17)*(x +
 1) - 10)*(x + 1)^(3/2)*sqrt(-x + 1) + 3*(x + 1)^2 - 2*x - 2
```

3.259 $\int x^2\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx$

Optimal. Leaf size=48

$$\frac{2x^3}{3}-\frac{1}{4}\sqrt{1-x^2}x+\frac{1}{2}\sqrt{1-x^2}x^3+\frac{1}{4}\sin^{-1}(x)$$

```
[Out] (2*x^3)/3 - (x*Sqrt[1 - x^2])/4 + (x^3*Sqrt[1 - x^2])/2 + ArcSin[
x]/4
```

Rubi [A] time = 0.168421, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.174$

$$\frac{2x^3}{3}-\frac{1}{4}\sqrt{1-x^2}x+\frac{1}{2}\sqrt{1-x^2}x^3+\frac{1}{4}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(Sqrt[1 - x] + Sqrt[1 + x])^2,x]
```

```
[Out] (2*x^3)/3 - (x*Sqrt[1 - x^2])/4 + (x^3*Sqrt[1 - x^2])/2 + ArcSin[
x]/4
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*((1-x)**(1/2)+(1+x)**(1/2))**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.0669366, size = 55, normalized size = 1.15

$$\frac{1}{12}\left(-3\sqrt{1-x^2}x+\left(6\sqrt{1-x^2}+8\right)x^3+6\sin^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{2}}\right)+8\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(Sqrt[1 - x] + Sqrt[1 + x])^2,x]
```

```
[Out] (8 - 3*x*Sqrt[1 - x^2] + x^3*(8 + 6*Sqrt[1 - x^2]) + 6*ArcSin[Sqr
t[1 + x]/Sqrt[2]])/12
```

Maple [A] time = 0.009, size = 59, normalized size = 1.2

$$\frac{2x^3}{3} + \frac{1}{4}\sqrt{1-x}\sqrt{1+x}\left(2x^3\sqrt{-x^2+1} - x\sqrt{-x^2+1} + \arcsin(x)\right)\frac{1}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*((1-x)^(1/2)+(1+x)^(1/2))^2,x)
```

```
[Out] 2/3*x^3+1/4*(1-x)^(1/2)*(1+x)^(1/2)*(2*x^3*(-x^2+1)^(1/2)-x*(-x^2
+1)^(1/2)+arcsin(x))/(-x^2+1)^(1/2)
```

Maxima [A] time = 0.790349, size = 46, normalized size = 0.96

$$\frac{2}{3}x^3 - \frac{1}{2}\left(-x^2+1\right)^{\frac{3}{2}}x + \frac{1}{4}\sqrt{-x^2+1}x + \frac{1}{4}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*(sqrt(x + 1) + sqrt(-x + 1))^2,x, algorithm="maxima")
```

```
[Out] 2/3*x^3 - 1/2*(-x^2 + 1)^(3/2)*x + 1/4*sqrt(-x^2 + 1)*x + 1/4*arc
sin(x)
```

Fricas [A] time = 0.268288, size = 184, normalized size = 3.83

$$-\frac{16x^7 - 20x^5 + 20x^3 - \left(6x^7 - 19x^5 + 8x^3 - 24x\right)\sqrt{x+1}\sqrt{-x+1} + 6\left(x^4 - 8x^2 + 4\left(x^2-2\right)\sqrt{x+1}\sqrt{-x+1} + 8\right)\arctan}{12\left(x^4 - 8x^2 + 4(x^2-2)\sqrt{x+1}\sqrt{-x+1} + 8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*(sqrt(x + 1) + sqrt(-x + 1))^2,x, algorithm="fricas")
```

```
[Out] -1/12*(16*x^7 - 20*x^5 + 20*x^3 - (6*x^7 - 19*x^5 + 8*x^3 - 24*x)
*sqrt(x + 1)*sqrt(-x + 1) + 6*(x^4 - 8*x^2 + 4*(x^2 - 2)*sqrt(x +
 1)*sqrt(-x + 1) + 8)*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) -
24*x)/(x^4 - 8*x^2 + 4*(x^2 - 2)*sqrt(x + 1)*sqrt(-x + 1) + 8)
```

Sympy [A] time = 107.557, size = 194, normalized size = 4.04

$$\frac{2x^3}{3} + 4\left(\begin{cases} \frac{x\sqrt{-x+1}\sqrt{x+1}}{4} + \frac{\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{2} & \text{for}\ x \geq -1 \wedge x < 1 \end{cases}\right) - 8\left(\begin{cases} \frac{x\sqrt{-x+1}\sqrt{x+1}}{4} - \frac{(-x+1)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}{6} + \frac{\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{2} & \text{for}\ x \geq -1 \wedge x < 1 \end{cases}\right) + 4\left(\begin{cases} \frac{x\sqrt{-x+1}\sqrt{x+1}}{4} - \frac{(-x+1)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}{3} - \frac{\sqrt{-x+1}\sqrt{x+1}\left(-5x-2(x+1)^3+6(x+1)^2-4\right)}{16} + \frac{5\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{8} & \text{for}\ x \geq -1 \wedge x < 1 \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*((1-x)**(1/2)+(1+x)**(1/2))**2,x)
```

```
[Out] 2*x**3/3 + 4*Piecewise((x*sqrt(-x + 1)*sqrt(x + 1)/4 + asin(sqrt(
2)*sqrt(x + 1)/2)/2, (x >= -1) & (x < 1))) - 8*Piecewise((x*sqrt(
-x + 1)*sqrt(x + 1)/4 - (-x + 1)**(3/2)*(x + 1)**(3/2)/6 + asin(s
qrt(2)*sqrt(x + 1)/2)/2, (x >= -1) & (x < 1))) + 4*Piecewise((x*s
qrt(-x + 1)*sqrt(x + 1)/4 - (-x + 1)**(3/2)*(x + 1)**(3/2)/3 - sq
rt(-x + 1)*sqrt(x + 1)*(-5*x - 2*(x + 1)**3 + 6*(x + 1)**2 - 4)/1
6 + 5*asin(sqrt(2)*sqrt(x + 1)/2)/8, (x >= -1) & (x < 1)))
```

GIAC/XCAS [A] time = 0.283223, size = 84, normalized size = 1.75

$$\frac{2}{3}(x+1)^3 - 2(x+1)^2 + \frac{1}{4}\left((2(x+1)(x-2)+5)(x+1)-1\right)\sqrt{x+1}\sqrt{-x+1} + 2x + \frac{1}{2}\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right) + 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*(sqrt(x + 1) + sqrt(-x + 1))^2,x, algorithm="giac")
```

```
[Out] 2/3*(x + 1)^3 - 2*(x + 1)^2 + 1/4*((2*(x + 1)*(x - 2) + 5)*(x + 1
) - 1)*sqrt(x + 1)*sqrt(-x + 1) + 2*x + 1/2*arcsin(1/2*sqrt(2)*sq
rt(x + 1)) + 2
```

3.260 $\int x\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx$

Optimal. Leaf size=19

$$x^2-\frac{2}{3}\left(1-x^2\right)^{3/2}$$

```
[Out] x^2 - (2*(1 - x^2)^(3/2))/3
```

Rubi [A] time = 0.0971286, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095

$$x^2-\frac{2}{3}\left(1-x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(Sqrt[1 - x] + Sqrt[1 + x])^2,x]

[Out] x^2 - (2*(1 - x^2)^(3/2))/3
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$2\int^{\sqrt{x+1}} x\left(x+\sqrt{-x^2+2}\right)^2\left(x^2-1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*((1-x)**(1/2)+(1+x)**(1/2))**2,x)

[Out] 2*Integral(x*(x + sqrt(-x**2 + 2))**2*(x**2 - 1), (x, sqrt(x + 1)
))
```

Mathematica [A] time = 0.022019, size = 24, normalized size = 1.26

$$\frac{1}{3}\left(x^2-1\right)\left(2\sqrt{1-x^2}+3\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(Sqrt[1 - x] + Sqrt[1 + x])^2,x]

[Out] ((-1 + x^2)*(3 + 2*Sqrt[1 - x^2]))/3
```

Maple [A] time = 0.004, size = 24, normalized size = 1.3

$$x^2+\frac{2x^2-2}{3}\sqrt{1-x}\sqrt{1+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*((1-x)^(1/2)+(1+x)^(1/2))^2,x)

[Out] x^2+2/3*(1-x)^(1/2)*(1+x)^(1/2)*(x^2-1)
```

Maxima [A] time = 0.78458, size = 20, normalized size = 1.05

$$x^2-\frac{2}{3}\left(-x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(sqrt(x + 1) + sqrt(-x + 1))^2,x, algorithm="maxima")

[Out] x^2 - 2/3*(-x^2 + 1)^(3/2)
```

Fricas [A] time = 0.268092, size = 78, normalized size = 4.11

$$\frac{2x^6+3\sqrt{x+1}x^4\sqrt{-x+1}-3x^4}{3\left(3x^2-(x^2-4)\sqrt{x+1}\sqrt{-x+1}-4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(sqrt(x + 1) + sqrt(-x + 1))^2,x, algorithm="fricas")

[Out] 1/3*(2*x^6 + 3*sqrt(x + 1)*x^4*sqrt(-x + 1) - 3*x^4)/(3*x^2 - (x^
2 - 4)*sqrt(x + 1)*sqrt(-x + 1) - 4)
```

Sympy [A] time = 53.0237, size = 99, normalized size = 5.21

$$x^2 - 4\left(\begin{cases} \frac{x\sqrt{-x+1}\sqrt{x+1}}{4} + \frac{\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{2} & \text{for } x \geq -1 \wedge x < 1 \end{cases}\right) + 4\left(\begin{cases} \frac{x\sqrt{-x+1}\sqrt{x+1}}{4} - \frac{(-x+1)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}{6} + \frac{\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{2} & \text{for } x \geq -1 \wedge x < 1 \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*((1-x)**(1/2)+(1+x)**(1/2))**2,x)
```

```
[Out] x**2 - 4*Piecewise((x*sqrt(-x + 1)*sqrt(x + 1)/4 + asin(sqrt(2)*s
qrt(x + 1)/2)/2, (x >= -1) & (x < 1))) + 4*Piecewise((x*sqrt(-x +
 1)*sqrt(x + 1)/4 - (-x + 1)**(3/2)*(x + 1)**(3/2)/6 + asin(sqrt(
2)*sqrt(x + 1)/2)/2, (x >= -1) & (x < 1)))
```

GIAC/XCAS [A] time = 0.286396, size = 36, normalized size = 1.89

$$\frac{2}{3}(x+1)^{\frac{3}{2}}(x-1)\sqrt{-x+1} + (x+1)^2 - 2x - 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(sqrt(x + 1) + sqrt(-x + 1))^2,x, algorithm="giac")
```

```
[Out] 2/3*(x + 1)^(3/2)*(x - 1)*sqrt(-x + 1) + (x + 1)^2 - 2*x - 2
```

3.261 $\int \left(\sqrt{1-x}+\sqrt{1+x}\right)^2 \, dx$

Optimal. Leaf size=19

$$\sqrt{1-x^2}x+2x+\sin^{-1}(x)$$

```
[Out] 2*x + x*Sqrt[1 - x^2] + ArcSin[x]
```

Rubi [A] time = 0.0436127, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$\sqrt{1-x^2}x+2x+\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[1 - x] + Sqrt[1 + x])^2,x]

[Out] 2*x + x*Sqrt[1 - x^2] + ArcSin[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$2\int^{\sqrt{x+1}} x\left(x+\sqrt{-x^2+2}\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((1-x)**(1/2)+(1+x)**(1/2))**2,x)

[Out] 2*Integral(x*(x + sqrt(-x**2 + 2))**2, (x, sqrt(x + 1)))
```

Mathematica [A] time = 0.0224705, size = 33, normalized size = 1.74

$$x\left(\sqrt{1-x^2}+2\right)+2\sin^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{2}}\right)+2$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[1 - x] + Sqrt[1 + x])^2,x]
```

```
[Out] 2 + x*(2 + Sqrt[1 - x^2]) + 2*ArcSin[Sqrt[1 + x]/Sqrt[2]]
```

Maple [B] time = 0.007, size = 58, normalized size = 3.1

$$2x + \sqrt{1-x}\,(1+x)^{\frac{3}{2}} - \sqrt{1-x}\sqrt{1+x} + \arcsin(x)\sqrt{(1+x)(1-x)}\frac{1}{\sqrt{1-x}}\frac{1}{\sqrt{1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((1-x)^(1/2)+(1+x)^(1/2))^2,x)
```

```
[Out] 2*x+(1-x)^(1/2)*(1+x)^(3/2)-(1-x)^(1/2)*(1+x)^(1/2)+((1+x)*(1-x))
^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [A] time = 0.783593, size = 23, normalized size = 1.21

$$\sqrt{-x^2+1}x + 2x + \arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(x + 1) + sqrt(-x + 1))^2,x, algorithm="maxima")
```

```
[Out] sqrt(-x^2 + 1)*x + 2*x + arcsin(x)
```

Fricas [A] time = 0.274992, size = 117, normalized size = 6.16

$$\frac{(x^3+2x)\sqrt{x+1}\sqrt{-x+1} - 2\left(x^2+2\sqrt{x+1}\sqrt{-x+1}-2\right)\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right) - 2x}{x^2+2\sqrt{x+1}\sqrt{-x+1}-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(x + 1) + sqrt(-x + 1))^2,x, algorithm="fricas")
```

```
[Out] ((x^3 + 2*x)*sqrt(x + 1)*sqrt(-x + 1) - 2*(x^2 + 2*sqrt(x + 1)*sq
rt(-x + 1) - 2)*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) - 2*x)/(
```

```
x^2 + 2*sqrt(x + 1)*sqrt(-x + 1) - 2)
```

Sympy [A] time = 30.0789, size = 42, normalized size = 2.21

$$2x + 4\left(\begin{cases} \frac{x\sqrt{-x+1}\sqrt{x+1}}{4} + \frac{\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{2} & \text{for}\ x \geq -1 \wedge x < 1 \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((1-x)**(1/2)+(1+x)**(1/2))**2,x)
```

```
[Out] 2*x + 4*Piecewise((x*sqrt(-x + 1)*sqrt(x + 1)/4 + asin(sqrt(2)*sq
rt(x + 1)/2)/2, (x >= -1) & (x < 1)))
```

GIAC/XCAS [A] time = 0.282978, size = 43, normalized size = 2.26

$$\sqrt{x+1}x\sqrt{-x+1} + 2\,x + 2\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\sqrt{x+1}\right) + 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(x + 1) + sqrt(-x + 1))^2,x, algorithm="giac")
```

```
[Out] sqrt(x + 1)*x*sqrt(-x + 1) + 2*x + 2*arcsin(1/2*sqrt(2)*sqrt(x +
1)) + 2
```

3.262 $\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x}\,dx$

Optimal. Leaf size=32

$$2\sqrt{1-x^2}-2\tanh^{-1}\left(\sqrt{1-x^2}\right)+2\log(x)$$

```
[Out] 2*Sqrt[1 - x^2] - 2*ArcTanh[Sqrt[1 - x^2]] + 2*Log[x]
```

Rubi [A] time = 0.166174, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217

$$2\sqrt{1-x^2}-2\tanh^{-1}\left(\sqrt{1-x^2}\right)+2\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[1 - x] + Sqrt[1 + x])^2/x,x]
```

```
[Out] 2*Sqrt[1 - x^2] - 2*ArcTanh[Sqrt[1 - x^2]] + 2*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$2\int^{\sqrt{x+1}}\frac{x\sqrt{-x^2+2}+1}{x-1}\,dx+2\int^{\sqrt{x+1}}\frac{x\sqrt{-x^2+2}+1}{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((1-x)**(1/2)+(1+x)**(1/2))**2/x,x)
```

```
[Out] 2*Integral((x*sqrt(-x**2 + 2) + 1)/(x - 1), (x, sqrt(x + 1))) + 2
*Integral((x*sqrt(-x**2 + 2) + 1)/(x + 1), (x, sqrt(x + 1)))
```

Mathematica [B] time = 0.039875, size = 84, normalized size = 2.62

$$2\left(\sqrt{1-x^2}+\log(-x)+\log\left(1-\sqrt{x+1}\right)-\log\left(\sqrt{1-x}-\sqrt{x+1}+2\right)\right.$$
$$\left.-\log\left(\sqrt{x+1}+1\right)+\log\left(\sqrt{1-x}+\sqrt{x+1}+2\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(Sqrt[1 - x] + Sqrt[1 + x])^2/x,x]
```

```
[Out] 2*(Sqrt[1 - x^2] + Log[-x] + Log[1 - Sqrt[1 + x]] - Log[2 + Sqrt[
1 - x] - Sqrt[1 + x]] - Log[1 + Sqrt[1 + x]] + Log[2 + Sqrt[1 - x
] + Sqrt[1 + x]])
```

Maple [A] time = 0.009, size = 51, normalized size = 1.6

$$2\ \ln(x) + 2\,\frac{\sqrt{1-x}\sqrt{1+x}\left(\sqrt{-x^2+1} - \mathit{Artanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)\right)}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((1-x)^(1/2)+(1+x)^(1/2))^2/x,x)
```

```
[Out] 2*ln(x)+2*(1-x)^(1/2)*(1+x)^(1/2)/(-x^2+1)^(1/2)*((-x^2+1)^(1/2)-
arctanh(1/(-x^2+1)^(1/2)))
```

Maxima [A] time = 0.790988, size = 55, normalized size = 1.72

$$2\sqrt{-x^2+1} + 2\ \log(x) - 2\ \log\left(\frac{2\sqrt{-x^2+1}}{|x|} + \frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(x + 1) + sqrt(-x + 1))^2/x,x, algorithm="maxima")
```

```
[Out] 2*sqrt(-x^2 + 1) + 2*log(x) - 2*log(2*sqrt(-x^2 + 1)/abs(x) + 2/a
bs(x))
```

Fricas [A] time = 0.2789, size = 105, normalized size = 3.28

$$-\frac{2\left(x^2 - \sqrt{x+1}\sqrt{-x+1}\log(x) - \left(\sqrt{x+1}\sqrt{-x+1} - 1\right)\log\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right) + \log(x)\right)}{\sqrt{x+1}\sqrt{-x+1} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(x + 1) + sqrt(-x + 1))^2/x,x, algorithm="fricas")
```

```
[Out] -2*(x^2 - sqrt(x + 1)*sqrt(-x + 1)*log(x) - (sqrt(x + 1)*sqrt(-x
+ 1) - 1)*log((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + log(x))/(sqrt(x
 + 1)*sqrt(-x + 1) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\sqrt{-x+1}+\sqrt{x+1}\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((1-x)**(1/2)+(1+x)**(1/2))**2/x,x)
```

```
[Out] Integral((sqrt(-x + 1) + sqrt(x + 1))**2/x, x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(x + 1) + sqrt(-x + 1))^2/x,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.263 $\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^2} \, dx$

Optimal. Leaf size=26

$$-\frac{2\sqrt{1-x^2}}{x}-\frac{2}{x}-2\sin^{-1}(x)$$

```
[Out] -2/x - (2*Sqrt[1 - x^2])/x - 2*ArcSin[x]
```

Rubi [A] time = 0.139159, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13

$$-\frac{2\sqrt{1-x^2}}{x}-\frac{2}{x}-2\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[1 - x] + Sqrt[1 + x])^2/x^2,x]

[Out] -2/x - (2*Sqrt[1 - x^2])/x - 2*ArcSin[x]
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((1-x)**(1/2)+(1+x)**(1/2))**2/x**2,x)

[Out] Timed out
```

Mathematica [A] time = 0.0469396, size = 35, normalized size = 1.35

$$-\frac{2\left(\sqrt{1-x^2}+2x\sin^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{2}}\right)+1\right)}{x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[1 - x] + Sqrt[1 + x])^2/x^2,x]
```

```
[Out] (-2*(1 + Sqrt[1 - x^2] + 2*x*ArcSin[Sqrt[1 + x]/Sqrt[2]]))/x
```

Maple [B] time = 0.016, size = 50, normalized size = 1.9

$$-2\,x^{-1}+2\,\frac{\left(-\arcsin(x)\,x-\sqrt{-x^2+1}\right)\sqrt{1-x}\sqrt{1+x}}{x\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((1-x)^(1/2)+(1+x)^(1/2))^2/x^2,x)
```

```
[Out] -2/x+2*(-arcsin(x)*x-(-x^2+1)^(1/2))*(1-x)^(1/2)*(1+x)^(1/2)/x/(-
x^2+1)^(1/2)
```

Maxima [A] time = 0.770877, size = 32, normalized size = 1.23

$$-\frac{2\sqrt{-x^2+1}}{x}-\frac{2}{x}-2\,\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(x + 1) + sqrt(-x + 1))^2/x^2,x, algorithm="maxima")
```

```
[Out] -2*sqrt(-x^2 + 1)/x - 2/x - 2*arcsin(x)
```

Fricas [A] time = 0.269924, size = 78, normalized size = 3.

$$\frac{2\left(2\left(\sqrt{x+1}\sqrt{-x+1}-1\right)\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)+x\right)}{\sqrt{x+1}\sqrt{-x+1}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(x + 1) + sqrt(-x + 1))^2/x^2,x, algorithm="fricas")
```

```
[Out] 2*(2*(sqrt(x + 1)*sqrt(-x + 1) - 1)*arctan((sqrt(x + 1)*sqrt(-x +
 1) - 1)/x) + x)/(sqrt(x + 1)*sqrt(-x + 1) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\sqrt{-x+1}+\sqrt{x+1}\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((1-x)**(1/2)+(1+x)**(1/2))**2/x**2,x)
```

```
[Out] Integral((sqrt(-x + 1) + sqrt(x + 1))**2/x**2, x)
```

GIAC/XCAS [A] time = 0.298895, size = 201, normalized size = 7.73

$$-2\pi - \frac{8\left(\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}} - \frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}\right)}{\left(\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}} - \frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}\right)^2 - 4} - \frac{2}{x} - 4\arctan\left(\frac{\sqrt{x+1}\left(\frac{\left(\sqrt{2}-\sqrt{-x+1}\right)^2}{x+1} - 1\right)}{2\left(\sqrt{2}-\sqrt{-x+1}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(x + 1) + sqrt(-x + 1))^2/x^2,x, algorithm="giac")
```

```
[Out] -2*pi - 8*((sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - sqrt(x + 1)/(sq
rt(2) - sqrt(-x + 1)))/(((sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - s
qrt(x + 1)/(sqrt(2) - sqrt(-x + 1)))^2 - 4) - 2/x - 4*arctan(1/2*
sqrt(x + 1)*((sqrt(2) - sqrt(-x + 1))^2/(x + 1) - 1)/(sqrt(2) - s
qrt(-x + 1)))
```

3.264 $\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^3}\,dx$

Optimal. Leaf size=34

$$-\frac{\sqrt{1-x^2}}{x^2}-\frac{1}{x^2}+\tanh^{-1}\left(\sqrt{1-x^2}\right)$$

```
[Out] -x^(-2) - Sqrt[1 - x^2]/x^2 + ArcTanh[Sqrt[1 - x^2]]
```

Rubi [A] time = 0.168146, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217

$$-\frac{\sqrt{1-x^2}}{x^2}-\frac{1}{x^2}+\tanh^{-1}\left(\sqrt{1-x^2}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[1 - x] + Sqrt[1 + x])^2/x^3,x]

[Out] -x^(-2) - Sqrt[1 - x^2]/x^2 + ArcTanh[Sqrt[1 - x^2]]
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((1-x)**(1/2)+(1+x)**(1/2))**2/x**3,x)

[Out] Timed out
```

Mathematica [B] time = 0.0558687, size = 88, normalized size = 2.59

$$-\frac{\sqrt{1-x^2}}{x^2}-\frac{1}{x^2}-\log\left(1-\sqrt{x+1}\right)+\log\left(\sqrt{1-x}-\sqrt{x+1}+2\right)+\log\left(\sqrt{x+1}+1\right)-\log\left(\sqrt{1-x}+\sqrt{x+1}+2\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[1 - x] + Sqrt[1 + x])^2/x^3,x]
```

```
[Out] -x^(-2) - Sqrt[1 - x^2]/x^2 - Log[1 - Sqrt[1 + x]] + Log[2 + Sqrt
[1 - x] - Sqrt[1 + x]] + Log[1 + Sqrt[1 + x]] - Log[2 + Sqrt[1 -
x] + Sqrt[1 + x]]
```

Maple [A] time = 0.017, size = 58, normalized size = 1.7

$$-x^{-2}+\frac{1}{x^{2}}\sqrt{1-x}\sqrt{1+x}\left(Artanh\left(\frac{1}{\sqrt{-x^{2}+1}}\right)x^{2}-\sqrt{-x^{2}+1}\right)\frac{1}{\sqrt{-x^{2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((1-x)^(1/2)+(1+x)^(1/2))^2/x^3,x)
```

```
[Out] -1/x^2+(1-x)^(1/2)*(1+x)^(1/2)*(arctanh(1/(-x^2+1)^(1/2))*x^2-(-x
^2+1)^(1/2))/x^2/(-x^2+1)^(1/2)
```

Maxima [A] time = 0.776452, size = 73, normalized size = 2.15

$$-\sqrt{-x^{2}+1}-\frac{\left(-x^{2}+1\right)^{\frac{3}{2}}}{x^{2}}-\frac{1}{x^{2}}+\log\left(\frac{2\sqrt{-x^{2}+1}}{|x|}+\frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(x + 1) + sqrt(-x + 1))^2/x^3,x, algorithm="maxima")
```

```
[Out] -sqrt(-x^2 + 1) - (-x^2 + 1)^(3/2)/x^2 - 1/x^2 + log(2*sqrt(-x^2
+ 1)/abs(x) + 2/abs(x))
```

Fricas [A] time = 0.294051, size = 105, normalized size = 3.09

$$-\frac{\left(x^{2}+2\sqrt{x+1}\sqrt{-x+1}-2\right)\log\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)+\sqrt{x+1}\sqrt{-x+1}-1}{x^{2}+2\sqrt{x+1}\sqrt{-x+1}-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(x + 1) + sqrt(-x + 1))^2/x^3,x, algorithm="fricas")
```

```
[Out] -((x^2 + 2*sqrt(x + 1)*sqrt(-x + 1) - 2)*log((sqrt(x + 1)*sqrt(-x
 + 1) - 1)/x) + sqrt(x + 1)*sqrt(-x + 1) - 1)/(x^2 + 2*sqrt(x + 1
)*sqrt(-x + 1) - 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\sqrt{-x+1}+\sqrt{x+1}\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((1-x)**(1/2)+(1+x)**(1/2))**2/x**3,x)
```

```
[Out] Integral((sqrt(-x + 1) + sqrt(x + 1))**2/x**3, x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(x + 1) + sqrt(-x + 1))^2/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.265 $\int \frac{x^3}{\sqrt{a+bx}+\sqrt{a+cx}} dx$

Optimal. Leaf size=147

$$\frac{2a^2(a+bx)^{3/2}}{3b^3(b-c)}-\frac{2a^2(a+cx)^{3/2}}{3c^3(b-c)}+\frac{2(a+bx)^{7/2}}{7b^3(b-c)}-\frac{4a(a+bx)^{5/2}}{5b^3(b-c)}-\frac{2(a+cx)^{7/2}}{7c^3(b-c)}+\frac{4a(a+cx)^{5/2}}{5c^3(b-c)}$$

```
[Out] (2*a^2*(a + b*x)^(3/2))/(3*b^3*(b - c)) - (4*a*(a + b*x)^(5/2))/(
5*b^3*(b - c)) + (2*(a + b*x)^(7/2))/(7*b^3*(b - c)) - (2*a^2*(a
+ c*x)^(3/2))/(3*(b - c)*c^3) + (4*a*(a + c*x)^(5/2))/(5*(b - c)*
c^3) - (2*(a + c*x)^(7/2))/(7*(b - c)*c^3)
```

Rubi [A] time = 0.233471, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$

$$\frac{2a^2(a+bx)^{3/2}}{3b^3(b-c)}-\frac{2a^2(a+cx)^{3/2}}{3c^3(b-c)}+\frac{2(a+bx)^{7/2}}{7b^3(b-c)}-\frac{4a(a+bx)^{5/2}}{5b^3(b-c)}-\frac{2(a+cx)^{7/2}}{7c^3(b-c)}+\frac{4a(a+cx)^{5/2}}{5c^3(b-c)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/(Sqrt[a + b*x] + Sqrt[a + c*x]),x]
```

```
[Out] (2*a^2*(a + b*x)^(3/2))/(3*b^3*(b - c)) - (4*a*(a + b*x)^(5/2))/(
5*b^3*(b - c)) + (2*(a + b*x)^(7/2))/(7*b^3*(b - c)) - (2*a^2*(a
+ c*x)^(3/2))/(3*(b - c)*c^3) + (4*a*(a + c*x)^(5/2))/(5*(b - c)*
c^3) - (2*(a + c*x)^(7/2))/(7*(b - c)*c^3)
```

Rubi in Sympy [A] time = 27.4787, size = 121, normalized size = 0.82

$$-\frac{2a^2\left(a+cx\right)^{\frac{3}{2}}}{3c^3\left(b-c\right)}+\frac{2a^2\left(a+bx\right)^{\frac{3}{2}}}{3b^3\left(b-c\right)}+\frac{4a\left(a+cx\right)^{\frac{5}{2}}}{5c^3\left(b-c\right)}-\frac{4a\left(a+bx\right)^{\frac{5}{2}}}{5b^3\left(b-c\right)}-\frac{2\left(a+cx\right)^{\frac{7}{2}}}{7c^3\left(b-c\right)}+\frac{2\left(a+bx\right)^{\frac{7}{2}}}{7b^3\left(b-c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/((b*x+a)**(1/2)+(c*x+a)**(1/2)),x)
```

```
[Out] -2*a**2*(a + c*x)**(3/2)/(3*c**3*(b - c)) + 2*a**2*(a + b*x)**(3/
2)/(3*b**3*(b - c)) + 4*a*(a + c*x)**(5/2)/(5*c**3*(b - c)) - 4*a
*(a + b*x)**(5/2)/(5*b**3*(b - c)) - 2*(a + c*x)**(7/2)/(7*c**3*(
b - c)) + 2*(a + b*x)**(7/2)/(7*b**3*(b - c))
```

Mathematica [A] time = 0.205245, size = 157, normalized size = 1.07

$$\sqrt{a+bx}\left(\frac{16a^3}{105b^3(b-c)}-\frac{8a^2x}{105b^2(b-c)}+\frac{2ax^2}{35b(b-c)}+\frac{2x^3}{7(b-c)}\right)+\sqrt{a+cx}\left(-\frac{16a^3}{105c^3(b-c)}+\frac{8a^2x}{105c^2(b-c)}-\frac{2ax^2}{35c(b-c)}-\frac{2x^3}{7(b-c)}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/(Sqrt[a + b*x] + Sqrt[a + c*x]),x]
```

```
[Out] Sqrt[a + c*x]*((-16*a^3)/(105*(b - c)*c^3) + (8*a^2*x)/(105*(b -
c)*c^2) - (2*a*x^2)/(35*(b - c)*c) - (2*x^3)/(7*(b - c))) + Sqrt[
a + b*x]*((16*a^3)/(105*b^3*(b - c)) - (8*a^2*x)/(105*b^2*(b - c)
) + (2*a*x^2)/(35*b*(b - c)) + (2*x^3)/(7*(b - c)))
```

Maple [A] time = 0.005, size = 90, normalized size = 0.6

$$2\,\frac{1/7\,(bx+a)^{7/2}-2/5\,(bx+a)^{5/2}\,a+1/3\,a^2\,(bx+a)^{3/2}}{(b-c)\,b^3}-2\,\frac{1/7\,(cx+a)^{7/2}-2/5\,(cx+a)^{5/2}\,a+1/3\,a^2\,(cx+a)^{3/2}}{(b-c)\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x)
```

```
[Out] 2/(b-c)/b^3*(1/7*(b*x+a)^(7/2)-2/5*(b*x+a)^(5/2)*a+1/3*a^2*(b*x+a
)^(3/2))-2/(b-c)/c^3*(1/7*(c*x+a)^(7/2)-2/5*(c*x+a)^(5/2)*a+1/3*a
^2*(c*x+a)^(3/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{bx+a}+\sqrt{cx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(sqrt(b*x + a) + sqrt(c*x + a)),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(sqrt(b*x + a) + sqrt(c*x + a)), x)
```

Fricas [A] time = 0.288328, size = 165, normalized size = 1.12

$$\frac{2\left(\left(15\,b^3c^3x^3+3\,ab^2c^3x^2-4\,a^2bc^3x+8\,a^3c^3\right)\sqrt{bx+a}-\left(15\,b^3c^3x^3+3\,ab^3c^2x^2-4\,a^2b^3cx+8\,a^3b^3\right)\sqrt{cx+a}\right)}{105\,(b^4c^3-b^3c^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(sqrt(b*x + a) + sqrt(c*x + a)),x, algorithm="fricas")
```

```
[Out] 2/105*((15*b^3*c^3*x^3 + 3*a*b^2*c^3*x^2 - 4*a^2*b*c^3*x + 8*a^3*
c^3)*sqrt(b*x + a) - (15*b^3*c^3*x^3 + 3*a*b^3*c^2*x^2 - 4*a^2*b^
3*c*x + 8*a^3*b^3)*sqrt(c*x + a))/(b^4*c^3 - b^3*c^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^3}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/((b*x+a)**(1/2)+(c*x+a)**(1/2)),x)
```

```
[Out] Integral(x**3/(sqrt(a + b*x) + sqrt(a + c*x)), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(sqrt(b*x + a) + sqrt(c*x + a)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.266 $\int \frac{x^2}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$

Optimal. Leaf size=95

$$\frac{2(a+bx)^{5/2}}{5b^2(b-c)}-\frac{2a(a+bx)^{3/2}}{3b^2(b-c)}-\frac{2(a+cx)^{5/2}}{5c^2(b-c)}+\frac{2a(a+cx)^{3/2}}{3c^2(b-c)}$$

```
[Out] (-2*a*(a + b*x)^(3/2))/(3*b^2*(b - c)) + (2*(a + b*x)^(5/2))/(5*b
^2*(b - c)) + (2*a*(a + c*x)^(3/2))/(3*(b - c)*c^2) - (2*(a + c*x
)^(5/2))/(5*(b - c)*c^2)
```

Rubi [A] time = 0.181601, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$

$$\frac{2(a+bx)^{5/2}}{5b^2(b-c)}-\frac{2a(a+bx)^{3/2}}{3b^2(b-c)}-\frac{2(a+cx)^{5/2}}{5c^2(b-c)}+\frac{2a(a+cx)^{3/2}}{3c^2(b-c)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(Sqrt[a + b*x] + Sqrt[a + c*x]),x]
```

```
[Out] (-2*a*(a + b*x)^(3/2))/(3*b^2*(b - c)) + (2*(a + b*x)^(5/2))/(5*b
^2*(b - c)) + (2*a*(a + c*x)^(3/2))/(3*(b - c)*c^2) - (2*(a + c*x
)^(5/2))/(5*(b - c)*c^2)
```

Rubi in Sympy [A] time = 18.5712, size = 76, normalized size = 0.8

$$\frac{2a\left(a+cx\right)^{\frac{3}{2}}}{3c^2\left(b-c\right)}-\frac{2a\left(a+bx\right)^{\frac{3}{2}}}{3b^2\left(b-c\right)}-\frac{2\left(a+cx\right)^{\frac{5}{2}}}{5c^2\left(b-c\right)}+\frac{2\left(a+bx\right)^{\frac{5}{2}}}{5b^2\left(b-c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/((b*x+a)**(1/2)+(c*x+a)**(1/2)),x)
```

```
[Out] 2*a*(a + c*x)**(3/2)/(3*c**2*(b - c)) - 2*a*(a + b*x)**(3/2)/(3*b
**2*(b - c)) - 2*(a + c*x)**(5/2)/(5*c**2*(b - c)) + 2*(a + b*x)*
*(5/2)/(5*b**2*(b - c))
```

Mathematica [A] time = 0.106714, size = 113, normalized size = 1.19

$$\frac{a^2\left(4b^2\sqrt{a+cx}-4c^2\sqrt{a+bx}\right)+6b^2c^2x^2\left(\sqrt{a+bx}-\sqrt{a+cx}\right)+2abcx\left(c\sqrt{a+bx}-b\sqrt{a+cx}\right)}{15b^2c^2(b-c)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(Sqrt[a + b*x] + Sqrt[a + c*x]),x]
```

```
[Out] (6*b^2*c^2*x^2*(Sqrt[a + b*x] - Sqrt[a + c*x]) + 2*a*b*c*x*(c*Sqr
t[a + b*x] - b*Sqrt[a + c*x]) + a^2*(-4*c^2*Sqrt[a + b*x] + 4*b^2
*Sqrt[a + c*x]))/(15*b^2*(b - c)*c^2)
```

Maple [A] time = 0.004, size = 66, normalized size = 0.7

$$2\,\frac{1/5\,(bx+a)^{5/2}-1/3\,(bx+a)^{3/2}a}{(b-c)\,b^2}-2\,\frac{1/5\,(cx+a)^{5/2}-1/3\,(cx+a)^{3/2}a}{(b-c)\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x)
```

```
[Out] 2/(b-c)/b^2*(1/5*(b*x+a)^(5/2)-1/3*(b*x+a)^(3/2)*a)-2/(b-c)/c^2*(
1/5*(c*x+a)^(5/2)-1/3*(c*x+a)^(3/2)*a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{bx+a}+\sqrt{cx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(sqrt(b*x + a) + sqrt(c*x + a)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(b*x + a) + sqrt(c*x + a)), x)
```

Fricas [A] time = 0.270817, size = 124, normalized size = 1.31

$$\frac{2\left(\left(3\,b^2c^2x^2+abc^2x-2\,a^2c^2\right)\sqrt{bx+a}-\left(3\,b^2c^2x^2+ab^2cx-2\,a^2b^2\right)\sqrt{cx+a}\right)}{15\left(b^3c^2-b^2c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(sqrt(b*x + a) + sqrt(c*x + a)),x, algorithm="fricas")
```

```
[Out] 2/15*((3*b^2*c^2*x^2 + a*b*c^2*x - 2*a^2*c^2)*sqrt(b*x + a) - (3*
b^2*c^2*x^2 + a*b^2*c*x - 2*a^2*b^2)*sqrt(c*x + a))/(b^3*c^2 - b^
2*c^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/((b*x+a)**(1/2)+(c*x+a)**(1/2)),x)
```

```
[Out] Integral(x**2/(sqrt(a + b*x) + sqrt(a + c*x)), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(sqrt(b*x + a) + sqrt(c*x + a)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.267 $\int \frac{x}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$

Optimal. Leaf size=47

$$\frac{2(a+bx)^{3/2}}{3b(b-c)}-\frac{2(a+cx)^{3/2}}{3c(b-c)}$$

```
[Out] (2*(a + b*x)^(3/2))/(3*b*(b - c)) - (2*(a + c*x)^(3/2))/(3*(b - c
)*c)
```

Rubi [A] time = 0.0987983, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087

$$\frac{2(a+bx)^{3/2}}{3b(b-c)}-\frac{2(a+cx)^{3/2}}{3c(b-c)}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(Sqrt[a + b*x] + Sqrt[a + c*x]),x]
```

```
[Out] (2*(a + b*x)^(3/2))/(3*b*(b - c)) - (2*(a + c*x)^(3/2))/(3*(b - c
)*c)
```

Rubi in Sympy [A] time = 7.23393, size = 32, normalized size = 0.68

$$-\frac{2\,(a+cx)^{\frac{3}{2}}}{3c\,(b-c)}+\frac{2\,(a+bx)^{\frac{3}{2}}}{3b\,(b-c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/((b*x+a)**(1/2)+(c*x+a)**(1/2)),x)
```

```
[Out] -2*(a + c*x)**(3/2)/(3*c*(b - c)) + 2*(a + b*x)**(3/2)/(3*b*(b -
c))
```

Mathematica [A] time = 0.0797935, size = 71, normalized size = 1.51

$$\frac{2bcx\sqrt{a+bx}-2ab\sqrt{a+cx}-2bcx\sqrt{a+cx}+2ac\sqrt{a+bx}}{3b^2c-3bc^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(Sqrt[a + b*x] + Sqrt[a + c*x]),x]
```

```
[Out] (2*a*c*Sqrt[a + b*x] + 2*b*c*x*Sqrt[a + b*x] - 2*a*b*Sqrt[a + c*x
] - 2*b*c*x*Sqrt[a + c*x])/(3*b^2*c - 3*b*c^2)
```

Maple [A] time = 0.004, size = 40, normalized size = 0.9

$$\frac{2}{3\,b\,(b-c)}\,(bx+a)^{\frac{3}{2}}-\frac{2}{(3\,b-3\,c)\,c}\,(cx+a)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x)
```

```
[Out] 2/3*(b*x+a)^(3/2)/b/(b-c)-2/3*(c*x+a)^(3/2)/(b-c)/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{bx+a}+\sqrt{cx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(b*x + a) + sqrt(c*x + a)),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(b*x + a) + sqrt(c*x + a)), x)
```

Fricas [A] time = 0.275037, size = 68, normalized size = 1.45

$$\frac{2\left((bcx+ac)\sqrt{bx+a}-(bcx+ab)\sqrt{cx+a}\right)}{3\,(b^2c-bc^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(b*x + a) + sqrt(c*x + a)),x, algorithm="fricas")
```

```
[Out] 2/3*((b*c*x + a*c)*sqrt(b*x + a) - (b*c*x + a*b)*sqrt(c*x + a))/(
b^2*c - b*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((b*x+a)**(1/2)+(c*x+a)**(1/2)),x)
```

```
[Out] Integral(x/(sqrt(a + b*x) + sqrt(a + c*x)), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(b*x + a) + sqrt(c*x + a)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.268 $\int \frac{1}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$

Optimal. Leaf size=97

$$\frac{2\sqrt{a+bx}}{b-c}-\frac{2\sqrt{a+cx}}{b-c}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{b-c}+\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{b-c}$$

```
[Out] (2*Sqrt[a + b*x])/(b - c) - (2*Sqrt[a + c*x])/(b - c) - (2*Sqrt[a
]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(b - c) + (2*Sqrt[a]*ArcTanh[Sq
rt[a + c*x]/Sqrt[a]])/(b - c)
```

Rubi [A] time = 0.150429, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$

$$\frac{2\sqrt{a+bx}}{b-c}-\frac{2\sqrt{a+cx}}{b-c}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{b-c}+\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{b-c}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[a + b*x] + Sqrt[a + c*x])^(-1),x]
```

```
[Out] (2*Sqrt[a + b*x])/(b - c) - (2*Sqrt[a + c*x])/(b - c) - (2*Sqrt[a
]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(b - c) + (2*Sqrt[a]*ArcTanh[Sq
rt[a + c*x]/Sqrt[a]])/(b - c)
```

Rubi in Sympy [A] time = 13.1754, size = 76, normalized size = 0.78

$$-\frac{2\sqrt{a}\,\text{atanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{b-c}+\frac{2\sqrt{a}\,\text{atanh}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{b-c}+\frac{2\sqrt{a+bx}}{b-c}-\frac{2\sqrt{a+cx}}{b-c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/((b*x+a)**(1/2)+(c*x+a)**(1/2)),x)
```

```
[Out] -2*sqrt(a)*atanh(sqrt(a + b*x)/sqrt(a))/(b - c) + 2*sqrt(a)*atanh
(sqrt(a + c*x)/sqrt(a))/(b - c) + 2*sqrt(a + b*x)/(b - c) - 2*sqr
t(a + c*x)/(b - c)
```

Mathematica [A] time = 0.0620588, size = 75, normalized size = 0.77

$$\frac{2\left(\sqrt{a+bx}-\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)-\sqrt{a+cx}+\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)\right)}{b-c}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[a + b*x] + Sqrt[a + c*x])^(-1),x]
```

```
[Out] (2*(Sqrt[a + b*x] - Sqrt[a + c*x] - Sqrt[a]*ArcTanh[Sqrt[a + b*x]
/Sqrt[a]] + Sqrt[a]*ArcTanh[Sqrt[a + c*x]/Sqrt[a]]))/(b - c)
```

Maple [A] time = 0.005, size = 73, normalized size = 0.8

$$\frac{1}{b-c}\left(2\sqrt{bx+a}-2\sqrt{a}Artanh\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)-\frac{1}{b-c}\left(2\sqrt{cx+a}-2\sqrt{a}Artanh\left(\frac{\sqrt{cx+a}}{\sqrt{a}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x)
```

```
[Out] 1/(b-c)*(2*(b*x+a)^(1/2)-2*a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2))
)-1/(b-c)*(2*(c*x+a)^(1/2)-2*a^(1/2)*arctanh((c*x+a)^(1/2)/a^(1/2
)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{bx+a}+\sqrt{cx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(b*x + a) + sqrt(c*x + a)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x + a) + sqrt(c*x + a)), x)
```

Fricas [A] time = 0.279846, size = 1, normalized size = 0.01

$$\left[-\frac{\sqrt{a}\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+\sqrt{a}\log\left(\frac{cx-2\sqrt{cx+a}\sqrt{a}+2a}{x}\right)-2\sqrt{bx+a}+2\sqrt{cx+a}}{b-c},\right.$$
$$\left.-\frac{2\left(\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)-\sqrt{-a}\arctan\left(\frac{\sqrt{cx+a}}{\sqrt{-a}}\right)-\sqrt{bx+a}+\sqrt{cx+a}\right)}{b-c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(b*x + a) + sqrt(c*x + a)),x, algorithm="fricas")
```

```
[Out] [-(sqrt(a)*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + sqrt(a)
*log((c*x - 2*sqrt(c*x + a)*sqrt(a) + 2*a)/x) - 2*sqrt(b*x + a) +
 2*sqrt(c*x + a))/(b - c), -2*(sqrt(-a)*arctan(sqrt(b*x + a)/sqrt
(-a)) - sqrt(-a)*arctan(sqrt(c*x + a)/sqrt(-a)) - sqrt(b*x + a) +
 sqrt(c*x + a))/(b - c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x+a)**(1/2)+(c*x+a)**(1/2)),x)
```

```
[Out] Integral(1/(sqrt(a + b*x) + sqrt(a + c*x)), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(b*x + a) + sqrt(c*x + a)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.269 $\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx$

Optimal. Leaf size=103

$$-\frac{\sqrt{a+bx}}{x(b-c)}+\frac{\sqrt{a+cx}}{x(b-c)}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}(b-c)}+\frac{c\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{\sqrt{a}(b-c)}$$

```
[Out] -(Sqrt[a + b*x]/((b - c)*x)) + Sqrt[a + c*x]/((b - c)*x) - (b*Arc
Tanh[Sqrt[a + b*x]/Sqrt[a]])/(Sqrt[a]*(b - c)) + (c*ArcTanh[Sqrt[
a + c*x]/Sqrt[a]])/(Sqrt[a]*(b - c))
```

Rubi [A] time = 0.193023, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16

$$-\frac{\sqrt{a+bx}}{x(b-c)}+\frac{\sqrt{a+cx}}{x(b-c)}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}(b-c)}+\frac{c\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{\sqrt{a}(b-c)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(Sqrt[a + b*x] + Sqrt[a + c*x])),x]
```

```
[Out] -(Sqrt[a + b*x]/((b - c)*x)) + Sqrt[a + c*x]/((b - c)*x) - (b*Arc
Tanh[Sqrt[a + b*x]/Sqrt[a]])/(Sqrt[a]*(b - c)) + (c*ArcTanh[Sqrt[
a + c*x]/Sqrt[a]])/(Sqrt[a]*(b - c))
```

Rubi in Sympy [A] time = 17.4099, size = 76, normalized size = 0.74

$$-\frac{\sqrt{a+bx}}{x\,(b-c)}+\frac{\sqrt{a+cx}}{x\,(b-c)}-\frac{b\,\text{atanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}\,(b-c)}+\frac{c\,\text{atanh}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{\sqrt{a}\,(b-c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/((b*x+a)**(1/2)+(c*x+a)**(1/2)),x)
```

```
[Out] -sqrt(a + b*x)/(x*(b - c)) + sqrt(a + c*x)/(x*(b - c)) - b*atanh(
sqrt(a + b*x)/sqrt(a))/(sqrt(a)*(b - c)) + c*atanh(sqrt(a + c*x)/
sqrt(a))/(sqrt(a)*(b - c))
```

Mathematica [A] time = 0.120457, size = 81, normalized size = 0.79

$$\frac{-\sqrt{a+bx}-\frac{bx\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}+\sqrt{a+cx}+\frac{cx\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{\sqrt{a}}}{bx-cx}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(Sqrt[a + b*x] + Sqrt[a + c*x])),x]
```

```
[Out] (-Sqrt[a + b*x] + Sqrt[a + c*x] - (b*x*ArcTanh[Sqrt[a + b*x]/Sqrt
[a]])/Sqrt[a] + (c*x*ArcTanh[Sqrt[a + c*x]/Sqrt[a]])/Sqrt[a])/(b*
x - c*x)
```

Maple [A] time = 0.006, size = 88, normalized size = 0.9

$$2\,\frac{b}{b-c}\left(-1/2\,\frac{\sqrt{bx+a}}{bx}-1/2\,\frac{1}{\sqrt{a}}Artanh\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)-2\,\frac{c}{b-c}\left(-1/2\,\frac{\sqrt{cx+a}}{cx}-1/2\,\frac{1}{\sqrt{a}}Artanh\left(\frac{\sqrt{cx+a}}{\sqrt{a}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x)
```

```
[Out] 2/(b-c)*b*(-1/2*(b*x+a)^(1/2)/x/b-1/2*arctanh((b*x+a)^(1/2)/a^(1/
2))/a^(1/2))-2/(b-c)*c*(-1/2*(c*x+a)^(1/2)/c/x-1/2/a^(1/2)*arctan
h((c*x+a)^(1/2)/a^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\sqrt{bx+a}+\sqrt{cx+a}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x*(sqrt(b*x + a) + sqrt(c*x + a))),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*(sqrt(b*x + a) + sqrt(c*x + a))), x)
```

Fricas [A] time = 0.290659, size = 1, normalized size = 0.01

$$\left[-\frac{bx\log\left(\frac{(bx+2a)\sqrt{a}+2\sqrt{bx+a}a}{x}\right)+cx\log\left(\frac{(cx+2a)\sqrt{a}-2\sqrt{cx+a}a}{x}\right)+2\sqrt{bx+a}\sqrt{a}-2\sqrt{cx+a}\sqrt{a}}{2\sqrt{a}(b-c)x}, \frac{bx\arctan\left(\frac{a}{\sqrt{bx+a}\sqrt{-a}}\right)-cx\,\mathrm{ar}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(sqrt(b*x + a) + sqrt(c*x + a))),x, algorithm="fricas")
```

```
[Out] [-1/2*(b*x*log(((b*x + 2*a)*sqrt(a) + 2*sqrt(b*x + a)*a)/x) + c*x
*log(((c*x + 2*a)*sqrt(a) - 2*sqrt(c*x + a)*a)/x) + 2*sqrt(b*x +
a)*sqrt(a) - 2*sqrt(c*x + a)*sqrt(a))/(sqrt(a)*(b - c)*x), (b*x*a
rctan(a/(sqrt(b*x + a)*sqrt(-a))) - c*x*arctan(a/(sqrt(c*x + a)*s
qrt(-a))) - sqrt(b*x + a)*sqrt(-a) + sqrt(c*x + a)*sqrt(-a))/(sqr
t(-a)*(b - c)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)**(1/2)+(c*x+a)**(1/2)),x)
```

```
[Out] Integral(1/(x*(sqrt(a + b*x) + sqrt(a + c*x))), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(sqrt(b*x + a) + sqrt(c*x + a))),x, algorithm="giac")
```

```
[Out] Timed out
```

3.270 $\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx$

Optimal. Leaf size=171

$$\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{3/2}(b-c)}-\frac{c^2\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{4a^{3/2}(b-c)}-\frac{\sqrt{a+bx}}{2x^2(b-c)}+\frac{\sqrt{a+cx}}{2x^2(b-c)}-\frac{b\sqrt{a+bx}}{4ax(b-c)}+\frac{c\sqrt{a+cx}}{4ax(b-c)}$$

```
[Out] -Sqrt[a + b*x]/(2*(b - c)*x^2) - (b*Sqrt[a + b*x])/(4*a*(b - c)*x
) + Sqrt[a + c*x]/(2*(b - c)*x^2) + (c*Sqrt[a + c*x])/(4*a*(b - c
)*x) + (b^2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(4*a^(3/2)*(b - c)) -
 (c^2*ArcTanh[Sqrt[a + c*x]/Sqrt[a]])/(4*a^(3/2)*(b - c))
```

Rubi [A] time = 0.247181, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{3/2}(b-c)}-\frac{c^2\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{4a^{3/2}(b-c)}-\frac{\sqrt{a+bx}}{2x^2(b-c)}+\frac{\sqrt{a+cx}}{2x^2(b-c)}-\frac{b\sqrt{a+bx}}{4ax(b-c)}+\frac{c\sqrt{a+cx}}{4ax(b-c)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(Sqrt[a + b*x] + Sqrt[a + c*x])),x]
```

```
[Out] -Sqrt[a + b*x]/(2*(b - c)*x^2) - (b*Sqrt[a + b*x])/(4*a*(b - c)*x
) + Sqrt[a + c*x]/(2*(b - c)*x^2) + (c*Sqrt[a + c*x])/(4*a*(b - c
)*x) + (b^2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(4*a^(3/2)*(b - c)) -
 (c^2*ArcTanh[Sqrt[a + c*x]/Sqrt[a]])/(4*a^(3/2)*(b - c))
```

Rubi in Sympy [A] time = 24.6208, size = 128, normalized size = 0.75

$$-\frac{\sqrt{a+bx}}{2x^2\,(b-c)}+\frac{\sqrt{a+cx}}{2x^2\,(b-c)}-\frac{b\sqrt{a+bx}}{4ax\,(b-c)}+\frac{c\sqrt{a+cx}}{4ax\,(b-c)}+\frac{b^2\,\operatorname{atanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{\frac{3}{2}}\,(b-c)}-\frac{c^2\,\operatorname{atanh}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{4a^{\frac{3}{2}}\,(b-c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/((b*x+a)**(1/2)+(c*x+a)**(1/2)),x)
```

```
[Out] -sqrt(a + b*x)/(2*x**2*(b - c)) + sqrt(a + c*x)/(2*x**2*(b - c))
- b*sqrt(a + b*x)/(4*a*x*(b - c)) + c*sqrt(a + c*x)/(4*a*x*(b - c
```

```
)) + b**2*atanh(sqrt(a + b*x)/sqrt(a))/(4*a**(3/2)*(b - c)) - c**
2*atanh(sqrt(a + c*x)/sqrt(a))/(4*a**(3/2)*(b - c))
```

Mathematica [A] time = 0.183354, size = 123, normalized size = 0.72

$$\frac{b^2x^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+\sqrt{a}\left(-2a\sqrt{a+bx}-bx\sqrt{a+bx}+2a\sqrt{a+cx}+cx\sqrt{a+cx}\right)-c^2x^2\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{4a^{3/2}x^2(b-c)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(Sqrt[a + b*x] + Sqrt[a + c*x])),x]
```

```
[Out] (Sqrt[a]*(-2*a*Sqrt[a + b*x] - b*x*Sqrt[a + b*x] + 2*a*Sqrt[a + c
*x] + c*x*Sqrt[a + c*x]) + b^2*x^2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
 - c^2*x^2*ArcTanh[Sqrt[a + c*x]/Sqrt[a]])/(4*a^(3/2)*(b - c)*x^2
)
```

Maple [A] time = 0.015, size = 120, normalized size = 0.7

$$2\frac{b^2}{b-c}\left(\frac{1}{b^2x^2}\left(-1/8\frac{(bx+a)^{3/2}}{a}-1/8\sqrt{bx+a}\right)+1/8\frac{1}{a^{3/2}}Artanh\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)$$
$$-2\frac{c^2}{b-c}\left(\frac{1}{c^2x^2}\left(-1/8\frac{(cx+a)^{3/2}}{a}-1/8\sqrt{cx+a}\right)+1/8\frac{1}{a^{3/2}}Artanh\left(\frac{\sqrt{cx+a}}{\sqrt{a}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x)
```

```
[Out] 2/(b-c)*b^2*((-1/8/a*(b*x+a)^(3/2)-1/8*(b*x+a)^(1/2))/x^2/b^2+1/8
/a^(3/2)*arctanh((b*x+a)^(1/2)/a^(1/2)))-2/(b-c)*c^2*((-1/8/a*(c*
x+a)^(3/2)-1/8*(c*x+a)^(1/2))/c^2/x^2+1/8/a^(3/2)*arctanh((c*x+a)
^(1/2)/a^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x^2\left(\sqrt{bx+a}+\sqrt{cx+a}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(sqrt(b*x + a) + sqrt(c*x + a))),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^2*(sqrt(b*x + a) + sqrt(c*x + a))), x)
```

Fricas [A] time = 0.310516, size = 1, normalized size = 0.01

$$\left[-\frac{b^2x^2\log\left(\frac{(bx+2a)\sqrt{a}-2\sqrt{bx+a}a}{x}\right)+c^2x^2\log\left(\frac{(cx+2a)\sqrt{a}+2\sqrt{cx+a}a}{x}\right)+2(bx+2a)\sqrt{bx+a}\sqrt{a}-2(cx+2a)\sqrt{cx+a}\sqrt{a}}{8(ab-ac)\sqrt{a}x^2},\right.$$
$$\left.-\frac{b^2x^2\arctan\left(\frac{a}{\sqrt{bx+a}\sqrt{-a}}\right)-c^2x^2\arctan\left(\frac{a}{\sqrt{cx+a}\sqrt{-a}}\right)+(bx+2a)\sqrt{bx+a}\sqrt{-a}-(cx+2a)\sqrt{cx+a}\sqrt{-a}}{4(ab-ac)\sqrt{-a}x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(sqrt(b*x + a) + sqrt(c*x + a))),x, algorithm="fricas")
```

```
[Out] [-1/8*(b^2*x^2*log(((b*x + 2*a)*sqrt(a) - 2*sqrt(b*x + a)*a)/x) +
 c^2*x^2*log(((c*x + 2*a)*sqrt(a) + 2*sqrt(c*x + a)*a)/x) + 2*(b*
x + 2*a)*sqrt(b*x + a)*sqrt(a) - 2*(c*x + 2*a)*sqrt(c*x + a)*sqrt
(a))/((a*b - a*c)*sqrt(a)*x^2), -1/4*(b^2*x^2*arctan(a/(sqrt(b*x
+ a)*sqrt(-a))) - c^2*x^2*arctan(a/(sqrt(c*x + a)*sqrt(-a))) + (b
*x + 2*a)*sqrt(b*x + a)*sqrt(-a) - (c*x + 2*a)*sqrt(c*x + a)*sqrt
(-a))/((a*b - a*c)*sqrt(-a)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/((b*x+a)**(1/2)+(c*x+a)**(1/2)),x)
```

```
[Out] Integral(1/(x**2*(sqrt(a + b*x) + sqrt(a + c*x))), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^2*(sqrt(b*x + a) + sqrt(c*x + a))),x, algorithm="giac")
```

```
[Out] Timed out
```

3.271 $\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2} \, dx$

Optimal. Leaf size=195

$$-\frac{a^3(b+c)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}}\right)}{4b^{5/2}c^{5/2}}+\frac{a^2(b+c)\sqrt{a+bx}\sqrt{a+cx}}{4b^2c^2(b-c)}+\frac{a(b+c)(a+bx)^{3/2}\sqrt{a+cx}}{2b^2c(b-c)^2}+\frac{ax^2}{(b-c)^2}-\frac{2(a+bx)^{3/2}(a+cx)^{3/2}}{3bc(b-c)^2}+\frac{x^3(b+c)}{3(b-c)^2}$$

```
[Out] (a*x^2)/(b - c)^2 + ((b + c)*x^3)/(3*(b - c)^2) + (a^2*(b + c)*Sq
rt[a + b*x]*Sqrt[a + c*x])/(4*b^2*(b - c)*c^2) + (a*(b + c)*(a +
b*x)^(3/2)*Sqrt[a + c*x])/(2*b^2*(b - c)^2*c) - (2*(a + b*x)^(3/2
)*(a + c*x)^(3/2))/(3*b*(b - c)^2*c) - (a^3*(b + c)*ArcTanh[(Sqrt
[c]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[a + c*x])])/(4*b^(5/2)*c^(5/2))
```

Rubi [A] time = 0.644019, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\frac{a^3(b+c)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}}\right)}{4b^{5/2}c^{5/2}}+\frac{a^2(b+c)\sqrt{a+bx}\sqrt{a+cx}}{4b^2c^2(b-c)}+\frac{a(b+c)(a+bx)^{3/2}\sqrt{a+cx}}{2b^2c(b-c)^2}+\frac{ax^2}{(b-c)^2}-\frac{2(a+bx)^{3/2}(a+cx)^{3/2}}{3bc(b-c)^2}+\frac{x^3(b+c)}{3(b-c)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/(Sqrt[a + b*x] + Sqrt[a + c*x])^2,x]
```

```
[Out] (a*x^2)/(b - c)^2 + ((b + c)*x^3)/(3*(b - c)^2) + (a^2*(b + c)*Sq
rt[a + b*x]*Sqrt[a + c*x])/(4*b^2*(b - c)*c^2) + (a*(b + c)*(a +
b*x)^(3/2)*Sqrt[a + c*x])/(2*b^2*(b - c)^2*c) - (2*(a + b*x)^(3/2
)*(a + c*x)^(3/2))/(3*b*(b - c)^2*c) - (a^3*(b + c)*ArcTanh[(Sqrt
[c]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[a + c*x])])/(4*b^(5/2)*c^(5/2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^3\,(b+c)\operatorname{atanh}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}}\right)}{4b^{\frac{5}{2}}c^{\frac{5}{2}}}+\frac{a^2\sqrt{a+bx}\sqrt{a+cx}\,(b+c)}{4b^2c^2\,(b-c)}+\frac{2a\int x\,dx}{(b-c)^2}+\frac{a\,(a+bx)^{\frac{3}{2}}\,\sqrt{a+cx}\,(b+c)}{2b^2c\,(b-c)^2}+\frac{x^3\,(b+c)}{3\,(b-c)^2}-\frac{2\,(a+bx)^{\frac{3}{2}}\,(a+cx)^{\frac{3}{2}}}{3bc\,(b-c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/((b*x+a)**(1/2)+(c*x+a)**(1/2))**2,x)
```

```
[Out] -a**3*(b + c)*atanh(sqrt(c)*sqrt(a + b*x)/(sqrt(b)*sqrt(a + c*x))
)/(4*b**(5/2)*c**(5/2)) + a**2*sqrt(a + b*x)*sqrt(a + c*x)*(b + c
)/(4*b**2*c**2*(b - c)) + 2*a*Integral(x, x)/(b - c)**2 + a*(a +
b*x)**(3/2)*sqrt(a + c*x)*(b + c)/(2*b**2*c*(b - c)**2) + x**3*(b
 + c)/(3*(b - c)**2) - 2*(a + b*x)**(3/2)*(a + c*x)**(3/2)/(3*b*c
*(b - c)**2)
```

Mathematica [A] time = 0.161263, size = 168, normalized size = 0.86

$$-\frac{a^3(b+c)\log\left(2\sqrt{b}\sqrt{c}\sqrt{a+bx}\sqrt{a+cx}+ab+ac+2bcx\right)}{8b^{5/2}c^{5/2}}+\frac{\sqrt{a+bx}\sqrt{a+cx}\left(a^2\left(3b^2-2bc+3c^2\right)-2abcx(b+c)-8b^2c^2x^2\right)}{12b^2c^2(b-c)^2}+\frac{ax^2}{(b-c)^2}+\frac{x^3(b+c)}{3(b-c)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/(Sqrt[a + b*x] + Sqrt[a + c*x])^2,x]
```

```
[Out] (a*x^2)/(b - c)^2 + ((b + c)*x^3)/(3*(b - c)^2) + (Sqrt[a + b*x]*
Sqrt[a + c*x]*(a^2*(3*b^2 - 2*b*c + 3*c^2) - 2*a*b*c*(b + c)*x -
8*b^2*c^2*x^2))/(12*b^2*(b - c)^2*c^2) - (a^3*(b + c)*Log[a*b + a
*c + 2*b*c*x + 2*Sqrt[b]*Sqrt[c]*Sqrt[a + b*x]*Sqrt[a + c*x]])/(8
*b^(5/2)*c^(5/2))
```

Maple [B] time = 0.019, size = 517, normalized size = 2.7

$$\frac{bx^3}{3\,(b-c)^2}+\frac{cx^3}{3\,(b-c)^2}+\frac{ax^2}{(b-c)^2}-\frac{1}{24\,(b-c)^2\,b^2c^2}\sqrt{bx+a}\sqrt{cx+a}\left(16\,x^2b^2c^2\sqrt{bcx^2+abx+acx+a^2}\sqrt{bc}+3\,\ln\left(1/2\,\frac{2\,bcx+2\,\sqrt{bcx^2+abx+acx+a^2}\sqrt{bc}}{\sqrt{bc}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x)
```

```
[Out] 1/3*x^3/(b-c)^2*b+1/3*x^3/(b-c)^2*c+a*x^2/(b-c)^2-1/24/(b-c)^2*(b
*x+a)^(1/2)*(c*x+a)^(1/2)*(16*x^2*b^2*c^2*(b*c*x^2+a*b*x+a*c*x+a^
```

```
2)^(1/2)*(b*c)^(1/2)+3*ln(1/2*(2*b*c*x+2*(b*c*x^2+a*b*x+a*c*x+a^2
)^(1/2)*(b*c)^(1/2)+a*b+a*c)/(b*c)^(1/2))*a^3*b^3-3*ln(1/2*(2*b*c
*x+2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*(b*c)^(1/2)+a*b+a*c)/(b*c)^(
1/2))*a^3*b^2*c-3*ln(1/2*(2*b*c*x+2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/
2)*(b*c)^(1/2)+a*b+a*c)/(b*c)^(1/2))*a^3*b*c^2+3*ln(1/2*(2*b*c*x+
2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*(b*c)^(1/2)+a*b+a*c)/(b*c)^(1/2
))*a^3*c^3+4*(b*c)^(1/2)*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*x*a*b^2*
c+4*(b*c)^(1/2)*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*x*a*b*c^2-6*(b*c)
^(1/2)*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*a^2*b^2+4*(b*c)^(1/2)*(b*c
*x^2+a*b*x+a*c*x+a^2)^(1/2)*a^2*b*c-6*(b*c)^(1/2)*(b*c*x^2+a*b*x+
a*c*x+a^2)^(1/2)*a^2*c^2)/(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)/b^2/c^2
/(b*c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(sqrt(b*x + a) + sqrt(c*x + a))^2,x, algorithm="maxima")
```

```
[Out] integrate(x^3/(sqrt(b*x + a) + sqrt(c*x + a))^2, x)
```

Fricas [A] time = 0.324604, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(sqrt(b*x + a) + sqrt(c*x + a))^2,x, algorithm="fricas")
```

```
[Out] [1/24*(2*(32*(b^5*c^2 + 7*b^4*c^3 + 7*b^3*c^4 + b^2*c^5)*x^5 + 2*
(a*b^5*c + 188*a*b^4*c^2 + 518*a*b^3*c^3 + 188*a*b^2*c^4 + a*b*c^
5)*x^4 - (3*a^2*b^5 + 7*a^2*b^4*c - 1034*a^2*b^3*c^2 - 1034*a^2*b
^2*c^3 + 7*a^2*b*c^4 + 3*a^2*c^5)*x^3 - 12*(3*a^3*b^4 - 70*a^3*b^
2*c^2 + 3*a^3*c^4)*x^2 - 48*(a^4*b^3 - a^4*b^2*c - a^4*b*c^2 + a^
4*c^3)*x)*sqrt(b*c)*sqrt(b*x + a)*sqrt(c*x + a) - 3*(32*a^6*b^3 -
 32*a^6*b^2*c - 32*a^6*b*c^2 + 32*a^6*c^3 + (a^3*b^6 + 14*a^3*b^5
*c - a^3*b^4*c^2 - 28*a^3*b^3*c^3 - a^3*b^2*c^4 + 14*a^3*b*c^5 +
a^3*c^6)*x^3 + 6*(3*a^4*b^5 + 7*a^4*b^4*c - 10*a^4*b^3*c^2 - 10*a
^4*b^2*c^3 + 7*a^4*b*c^4 + 3*a^4*c^5)*x^2 - 2*(16*a^5*b^3 - 16*a^
5*b^2*c - 16*a^5*b*c^2 + 16*a^5*c^3 + (3*a^3*b^5 + 7*a^3*b^4*c -
10*a^3*b^3*c^2 - 10*a^3*b^2*c^3 + 7*a^3*b*c^4 + 3*a^3*c^5)*x^2 +
```

```
16*(a^4*b^4 - 2*a^4*b^2*c^2 + a^4*c^4)*x)*sqrt(b*x + a)*sqrt(c*x
+ a) + 48*(a^5*b^4 - 2*a^5*b^2*c^2 + a^5*c^4)*x)*log((2*a*b*c*x -
 2*(b*c*x + sqrt(b*c)*a)*sqrt(b*x + a)*sqrt(c*x + a) + (2*b*c*x^2
 + 2*a^2 + (a*b + a*c)*x)*sqrt(b*c))/((b + c)*x - 2*sqrt(b*x + a)
*sqrt(c*x + a) + 2*a)) - 2*(4*(b^6*c^2 + 28*b^5*c^3 + 70*b^4*c^4
+ 28*b^3*c^5 + b^2*c^6)*x^6 + 144*(a*b^5*c^2 + 7*a*b^4*c^3 + 7*a*
b^3*c^4 + a*b^2*c^5)*x^5 - 6*(a^2*b^5*c - 132*a^2*b^4*c^2 - 378*a
^2*b^3*c^3 - 132*a^2*b^2*c^4 + a^2*b*c^5)*x^4 - (15*a^3*b^5 + 43*
a^3*b^4*c - 1466*a^3*b^3*c^2 - 1466*a^3*b^2*c^3 + 43*a^3*b*c^4 +
15*a^3*c^5)*x^3 - 12*(5*a^4*b^4 - 74*a^4*b^2*c^2 + 5*a^4*c^4)*x^2
 - 48*(a^5*b^3 - a^5*b^2*c - a^5*b*c^2 + a^5*c^3)*x)*sqrt(b*c))/(
2*(16*a^2*b^4*c^2 - 32*a^2*b^3*c^3 + 16*a^2*b^2*c^4 + (3*b^6*c^2
+ 4*b^5*c^3 - 14*b^4*c^4 + 4*b^3*c^5 + 3*b^2*c^6)*x^2 + 16*(a*b^5
*c^2 - a*b^4*c^3 - a*b^3*c^4 + a*b^2*c^5)*x)*sqrt(b*c)*sqrt(b*x +
 a)*sqrt(c*x + a) - (32*a^3*b^4*c^2 - 64*a^3*b^3*c^3 + 32*a^3*b^2
*c^4 + (b^7*c^2 + 13*b^6*c^3 - 14*b^5*c^4 - 14*b^4*c^5 + 13*b^3*c
^6 + b^2*c^7)*x^3 + 6*(3*a*b^6*c^2 + 4*a*b^5*c^3 - 14*a*b^4*c^4 +
 4*a*b^3*c^5 + 3*a*b^2*c^6)*x^2 + 48*(a^2*b^5*c^2 - a^2*b^4*c^3 -
 a^2*b^3*c^4 + a^2*b^2*c^5)*x)*sqrt(b*c)), 1/12*((32*(b^5*c^2 + 7
*b^4*c^3 + 7*b^3*c^4 + b^2*c^5)*x^5 + 2*(a*b^5*c + 188*a*b^4*c^2
+ 518*a*b^3*c^3 + 188*a*b^2*c^4 + a*b*c^5)*x^4 - (3*a^2*b^5 + 7*a
^2*b^4*c - 1034*a^2*b^3*c^2 - 1034*a^2*b^2*c^3 + 7*a^2*b*c^4 + 3*
a^2*c^5)*x^3 - 12*(3*a^3*b^4 - 70*a^3*b^2*c^2 + 3*a^3*c^4)*x^2 -
48*(a^4*b^3 - a^4*b^2*c - a^4*b*c^2 + a^4*c^3)*x)*sqrt(-b*c)*sqrt
(b*x + a)*sqrt(c*x + a) + 3*(32*a^6*b^3 - 32*a^6*b^2*c - 32*a^6*b
*c^2 + 32*a^6*c^3 + (a^3*b^6 + 14*a^3*b^5*c - a^3*b^4*c^2 - 28*a^
3*b^3*c^3 - a^3*b^2*c^4 + 14*a^3*b*c^5 + a^3*c^6)*x^3 + 6*(3*a^4*
b^5 + 7*a^4*b^4*c - 10*a^4*b^3*c^2 - 10*a^4*b^2*c^3 + 7*a^4*b*c^4
 + 3*a^4*c^5)*x^2 - 2*(16*a^5*b^3 - 16*a^5*b^2*c - 16*a^5*b*c^2 +
 16*a^5*c^3 + (3*a^3*b^5 + 7*a^3*b^4*c - 10*a^3*b^3*c^2 - 10*a^3*
b^2*c^3 + 7*a^3*b*c^4 + 3*a^3*c^5)*x^2 + 16*(a^4*b^4 - 2*a^4*b^2*
c^2 + a^4*c^4)*x)*sqrt(b*x + a)*sqrt(c*x + a) + 48*(a^5*b^4 - 2*a
^5*b^2*c^2 + a^5*c^4)*x)*arctan((sqrt(-b*c)*sqrt(b*x + a)*sqrt(c*
x + a) - sqrt(-b*c)*a)/(b*c*x)) - (4*(b^6*c^2 + 28*b^5*c^3 + 70*b
^4*c^4 + 28*b^3*c^5 + b^2*c^6)*x^6 + 144*(a*b^5*c^2 + 7*a*b^4*c^3
 + 7*a*b^3*c^4 + a*b^2*c^5)*x^5 - 6*(a^2*b^5*c - 132*a^2*b^4*c^2
- 378*a^2*b^3*c^3 - 132*a^2*b^2*c^4 + a^2*b*c^5)*x^4 - (15*a^3*b^
5 + 43*a^3*b^4*c - 1466*a^3*b^3*c^2 - 1466*a^3*b^2*c^3 + 43*a^3*b
*c^4 + 15*a^3*c^5)*x^3 - 12*(5*a^4*b^4 - 74*a^4*b^2*c^2 + 5*a^4*c
^4)*x^2 - 48*(a^5*b^3 - a^5*b^2*c - a^5*b*c^2 + a^5*c^3)*x)*sqrt(
-b*c))/(2*(16*a^2*b^4*c^2 - 32*a^2*b^3*c^3 + 16*a^2*b^2*c^4 + (3*
b^6*c^2 + 4*b^5*c^3 - 14*b^4*c^4 + 4*b^3*c^5 + 3*b^2*c^6)*x^2 + 1
6*(a*b^5*c^2 - a*b^4*c^3 - a*b^3*c^4 + a*b^2*c^5)*x)*sqrt(-b*c)*s
qrt(b*x + a)*sqrt(c*x + a) - (32*a^3*b^4*c^2 - 64*a^3*b^3*c^3 + 3
2*a^3*b^2*c^4 + (b^7*c^2 + 13*b^6*c^3 - 14*b^5*c^4 - 14*b^4*c^5 +
 13*b^3*c^6 + b^2*c^7)*x^3 + 6*(3*a*b^6*c^2 + 4*a*b^5*c^3 - 14*a*
b^4*c^4 + 4*a*b^3*c^5 + 3*a*b^2*c^6)*x^2 + 48*(a^2*b^5*c^2 - a^2*
b^4*c^3 - a^2*b^3*c^4 + a^2*b^2*c^5)*x)*sqrt(-b*c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/((b*x+a)**(1/2)+(c*x+a)**(1/2))**2,x)
```

```
[Out] Integral(x**3/(sqrt(a + b*x) + sqrt(a + c*x))**2, x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(sqrt(b*x + a) + sqrt(c*x + a))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.272 $\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$

Optimal. Leaf size=142

$$\frac{a^2\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}}\right)}{2b^{3/2}c^{3/2}}+\frac{2ax}{(b-c)^2}-\frac{a\sqrt{a+bx}\sqrt{a+cx}}{2bc(b-c)}-\frac{(a+bx)^{3/2}\sqrt{a+cx}}{b(b-c)^2}+\frac{x^2(b+c)}{2(b-c)^2}$$

```
[Out] (2*a*x)/(b - c)^2 + ((b + c)*x^2)/(2*(b - c)^2) - (a*Sqrt[a + b*x
]*Sqrt[a + c*x])/(2*b*(b - c)*c) - ((a + b*x)^(3/2)*Sqrt[a + c*x]
)/(b*(b - c)^2) + (a^2*ArcTanh[(Sqrt[c]*Sqrt[a + b*x])/(Sqrt[b]*S
qrt[a + c*x])])/(2*b^(3/2)*c^(3/2))
```

Rubi [A] time = 0.447945, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$

$$\frac{a^2\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}}\right)}{2b^{3/2}c^{3/2}}+\frac{2ax}{(b-c)^2}-\frac{a\sqrt{a+bx}\sqrt{a+cx}}{2bc(b-c)}-\frac{(a+bx)^{3/2}\sqrt{a+cx}}{b(b-c)^2}+\frac{x^2(b+c)}{2(b-c)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(Sqrt[a + b*x] + Sqrt[a + c*x])^2,x]
```

```
[Out] (2*a*x)/(b - c)^2 + ((b + c)*x^2)/(2*(b - c)^2) - (a*Sqrt[a + b*x
]*Sqrt[a + c*x])/(2*b*(b - c)*c) - ((a + b*x)^(3/2)*Sqrt[a + c*x]
)/(b*(b - c)^2) + (a^2*ArcTanh[(Sqrt[c]*Sqrt[a + b*x])/(Sqrt[b]*S
qrt[a + c*x])])/(2*b^(3/2)*c^(3/2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^2\operatorname{atanh}\left(\frac{\sqrt{b}\sqrt{a+cx}}{\sqrt{c}\sqrt{a+bx}}\right)}{2b^{\frac{3}{2}}c^{\frac{3}{2}}}+\frac{2ax}{(b-c)^2}-\frac{a\sqrt{a+bx}\sqrt{a+cx}}{2bc\,(b-c)}+\frac{(b+c)\int x\,dx}{(b-c)^2}-\frac{(a+bx)^{\frac{3}{2}}\sqrt{a+cx}}{b\,(b-c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/((b*x+a)**(1/2)+(c*x+a)**(1/2))**2,x)
```

```
[Out] a**2*atanh(sqrt(b)*sqrt(a + c*x)/(sqrt(c)*sqrt(a + b*x)))/(2*b**(
3/2)*c**(3/2)) + 2*a*x/(b - c)**2 - a*sqrt(a + b*x)*sqrt(a + c*x)
```

```
/(2*b*c*(b - c)) + (b + c)*Integral(x, x)/(b - c)**2 - (a + b*x)*
*(3/2)*sqrt(a + c*x)/(b*(b - c)**2)
```

Mathematica [A] time = 0.275629, size = 132, normalized size = 0.93

$$\frac{1}{4}\left(\frac{a^2\log\left(2\sqrt{b}\sqrt{c}\sqrt{a+bx}\sqrt{a+cx}+ab+ac+2bcx\right)}{b^{3/2}c^{3/2}}+\frac{8ax}{(b-c)^2}-\frac{2\sqrt{a+bx}\sqrt{a+cx}(a(b+c)+2bcx)}{bc(b-c)^2}+\frac{2x^2(b+c)}{(b-c)^2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(Sqrt[a + b*x] + Sqrt[a + c*x])^2,x]
```

```
[Out] ((8*a*x)/(b - c)^2 + (2*(b + c)*x^2)/(b - c)^2 - (2*Sqrt[a + b*x]
*Sqrt[a + c*x]*(a*(b + c) + 2*b*c*x))/(b*(b - c)^2*c) + (a^2*Log[
a*b + a*c + 2*b*c*x + 2*Sqrt[b]*Sqrt[c]*Sqrt[a + b*x]*Sqrt[a + c*
x]])/(b^(3/2)*c^(3/2)))/4
```

Maple [B] time = 0.009, size = 385, normalized size = 2.7

$$\frac{bx^2}{2\,(b-c)^2}+\frac{cx^2}{2\,(b-c)^2}+2\,\frac{ax}{(b-c)^2}-\frac{1}{(b-c)^2\,c}\sqrt{bx+a}\,(cx+a)^{\frac{3}{2}}$$
$$+\frac{a}{2\,(b-c)^2\,c}\sqrt{cx+a}\sqrt{bx+a}-\frac{a}{2\,(b-c)^2\,b}\sqrt{cx+a}\sqrt{bx+a}$$
$$+\frac{a^2b}{4\,(b-c)^2\,c}\sqrt{(cx+a)\,(bx+a)}\ln\left(1\left(\frac{ab}{2}+\frac{ac}{2}+bcx\right)\frac{1}{\sqrt{bc}}+\sqrt{bcx^2+(ab+ac)\,x+a^2}\right)\frac{1}{\sqrt{cx+a}}\frac{1}{\sqrt{bx+a}}\frac{1}{\sqrt{bc}}$$
$$-\frac{a^2}{2\,(b-c)^2}\sqrt{(cx+a)\,(bx+a)}\ln\left(1\left(\frac{ab}{2}+\frac{ac}{2}+bcx\right)\frac{1}{\sqrt{bc}}+\sqrt{bcx^2+(ab+ac)\,x+a^2}\right)\frac{1}{\sqrt{cx+a}}\frac{1}{\sqrt{bx+a}}\frac{1}{\sqrt{bc}}$$
$$+\frac{a^2c}{4\,(b-c)^2\,b}\sqrt{(cx+a)\,(bx+a)}\ln\left(1\left(\frac{ab}{2}+\frac{ac}{2}+bcx\right)\frac{1}{\sqrt{bc}}+\sqrt{bcx^2+(ab+ac)\,x+a^2}\right)\frac{1}{\sqrt{cx+a}}\frac{1}{\sqrt{bx+a}}\frac{1}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x)
```

```
[Out] 1/2*x^2/(b-c)^2*b+1/2*x^2/(b-c)^2*c+2*a*x/(b-c)^2-1/(b-c)^2/c*(b*
x+a)^(1/2)*(c*x+a)^(3/2)+1/2/(b-c)^2/c*(c*x+a)^(1/2)*(b*x+a)^(1/2
)*a-1/2/(b-c)^2/b*(c*x+a)^(1/2)*(b*x+a)^(1/2)*a+1/4/(b-c)^2/c*((c
```

```
*x+a)*(b*x+a))^(1/2)/(c*x+a)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*b+1/2*
a*c+b*c*x)/(b*c)^(1/2)+(b*c*x^2+(a*b+a*c)*x+a^2)^(1/2))/(b*c)^(1/
2)*a^2*b-1/2/(b-c)^2*((c*x+a)*(b*x+a))^(1/2)/(c*x+a)^(1/2)/(b*x+a
)^(1/2)*ln((1/2*a*b+1/2*a*c+b*c*x)/(b*c)^(1/2)+(b*c*x^2+(a*b+a*c)
*x+a^2)^(1/2))/(b*c)^(1/2)*a^2+1/4/(b-c)^2*c/b*((c*x+a)*(b*x+a))^
(1/2)/(c*x+a)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*b+1/2*a*c+b*c*x)/(b*c
)^(1/2)+(b*c*x^2+(a*b+a*c)*x+a^2)^(1/2))/(b*c)^(1/2)*a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(sqrt(b*x + a) + sqrt(c*x + a))^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(b*x + a) + sqrt(c*x + a))^2, x)
```

Fricas [A] time = 0.294556, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(sqrt(b*x + a) + sqrt(c*x + a))^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(2*(3*b^3*c + 10*b^2*c^2 + 3*b*c^3)*x^3 + (a*b^3 + 47*a*b
^2*c + 47*a*b*c^2 + a*c^3)*x^2 + 4*(a^2*b^2 + 14*a^2*b*c + a^2*c^
2)*x)*sqrt(b*c)*sqrt(b*x + a)*sqrt(c*x + a) - (8*a^4*b^2 - 16*a^4
*b*c + 8*a^4*c^2 + (a^2*b^4 + 4*a^2*b^3*c - 10*a^2*b^2*c^2 + 4*a^
2*b*c^3 + a^2*c^4)*x^2 - 4*(2*a^3*b^2 - 4*a^3*b*c + 2*a^3*c^2 + (
a^2*b^3 - a^2*b^2*c - a^2*b*c^2 + a^2*c^3)*x)*sqrt(b*x + a)*sqrt(
c*x + a) + 8*(a^3*b^3 - a^3*b^2*c - a^3*b*c^2 + a^3*c^3)*x)*log(-
(2*a*b*c*x - 2*(b*c*x - sqrt(b*c)*a)*sqrt(b*x + a)*sqrt(c*x + a)
- (2*b*c*x^2 + 2*a^2 + (a*b + a*c)*x)*sqrt(b*c))/((b + c)*x - 2*s
qrt(b*x + a)*sqrt(c*x + a) + 2*a)) - 2*((b^4*c + 15*b^3*c^2 + 15*
b^2*c^3 + b*c^4)*x^4 + 8*(3*a*b^3*c + 10*a*b^2*c^2 + 3*a*b*c^3)*x
^3 + (3*a^2*b^3 + 77*a^2*b^2*c + 77*a^2*b*c^2 + 3*a^2*c^3)*x^2 +
4*(a^3*b^2 + 14*a^3*b*c + a^3*c^2)*x)*sqrt(b*c))/(4*(2*a*b^3*c -
4*a*b^2*c^2 + 2*a*b*c^3 + (b^4*c - b^3*c^2 - b^2*c^3 + b*c^4)*x)*
sqrt(b*c)*sqrt(b*x + a)*sqrt(c*x + a) - (8*a^2*b^3*c - 16*a^2*b^2
*c^2 + 8*a^2*b*c^3 + (b^5*c + 4*b^4*c^2 - 10*b^3*c^3 + 4*b^2*c^4
+ b*c^5)*x^2 + 8*(a*b^4*c - a*b^3*c^2 - a*b^2*c^3 + a*b*c^4)*x)*s
```

```
qrt(b*c)), 1/2*((2*(3*b^3*c + 10*b^2*c^2 + 3*b*c^3)*x^3 + (a*b^3
+ 47*a*b^2*c + 47*a*b*c^2 + a*c^3)*x^2 + 4*(a^2*b^2 + 14*a^2*b*c
+ a^2*c^2)*x)*sqrt(-b*c)*sqrt(b*x + a)*sqrt(c*x + a) - (8*a^4*b^2
 - 16*a^4*b*c + 8*a^4*c^2 + (a^2*b^4 + 4*a^2*b^3*c - 10*a^2*b^2*c
^2 + 4*a^2*b*c^3 + a^2*c^4)*x^2 - 4*(2*a^3*b^2 - 4*a^3*b*c + 2*a^
3*c^2 + (a^2*b^3 - a^2*b^2*c - a^2*b*c^2 + a^2*c^3)*x)*sqrt(b*x +
 a)*sqrt(c*x + a) + 8*(a^3*b^3 - a^3*b^2*c - a^3*b*c^2 + a^3*c^3)
*x)*arctan((sqrt(-b*c)*sqrt(b*x + a)*sqrt(c*x + a) - sqrt(-b*c)*a
)/(b*c*x)) - ((b^4*c + 15*b^3*c^2 + 15*b^2*c^3 + b*c^4)*x^4 + 8*(
3*a*b^3*c + 10*a*b^2*c^2 + 3*a*b*c^3)*x^3 + (3*a^2*b^3 + 77*a^2*b
^2*c + 77*a^2*b*c^2 + 3*a^2*c^3)*x^2 + 4*(a^3*b^2 + 14*a^3*b*c +
a^3*c^2)*x)*sqrt(-b*c))/(4*(2*a*b^3*c - 4*a*b^2*c^2 + 2*a*b*c^3 +
 (b^4*c - b^3*c^2 - b^2*c^3 + b*c^4)*x)*sqrt(-b*c)*sqrt(b*x + a)*
sqrt(c*x + a) - (8*a^2*b^3*c - 16*a^2*b^2*c^2 + 8*a^2*b*c^3 + (b^
5*c + 4*b^4*c^2 - 10*b^3*c^3 + 4*b^2*c^4 + b*c^5)*x^2 + 8*(a*b^4*
c - a*b^3*c^2 - a*b^2*c^3 + a*b*c^4)*x)*sqrt(-b*c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/((b*x+a)**(1/2)+(c*x+a)**(1/2))**2,x)

[Out] Integral(x**2/(sqrt(a + b*x) + sqrt(a + c*x))**2, x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(sqrt(b*x + a) + sqrt(c*x + a))^2,x, algorithm="giac")

[Out] Timed out
```

3.273 $\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$

Optimal. Leaf size=135

$$-\frac{2\sqrt{a+bx}\sqrt{a+cx}}{(b-c)^2}+\frac{2a\log(x)}{(b-c)^2}+\frac{4a\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{(b-c)^2}-\frac{2a(b+c)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}}\right)}{\sqrt{b}\sqrt{c}(b-c)^2}+\frac{x(b+c)}{(b-c)^2}$$

```
[Out] ((b + c)*x)/(b - c)^2 - (2*Sqrt[a + b*x]*Sqrt[a + c*x])/(b - c)^2
 + (4*a*ArcTanh[Sqrt[a + b*x]/Sqrt[a + c*x]])/(b - c)^2 - (2*a*(b
 + c)*ArcTanh[(Sqrt[c]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[a + c*x])])/(
Sqrt[b]*(b - c)^2*Sqrt[c]) + (2*a*Log[x])/(b - c)^2
```

Rubi [A] time = 0.453467, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261

$$-\frac{2\sqrt{a+bx}\sqrt{a+cx}}{(b-c)^2}+\frac{2a\log(x)}{(b-c)^2}+\frac{4a\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{(b-c)^2}-\frac{2a(b+c)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}}\right)}{\sqrt{b}\sqrt{c}(b-c)^2}+\frac{x(b+c)}{(b-c)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(Sqrt[a + b*x] + Sqrt[a + c*x])^2,x]
```

```
[Out] ((b + c)*x)/(b - c)^2 - (2*Sqrt[a + b*x]*Sqrt[a + c*x])/(b - c)^2
 + (4*a*ArcTanh[Sqrt[a + b*x]/Sqrt[a + c*x]])/(b - c)^2 - (2*a*(b
 + c)*ArcTanh[(Sqrt[c]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[a + c*x])])/(
Sqrt[b]*(b - c)^2*Sqrt[c]) + (2*a*Log[x])/(b - c)^2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{2a\log{(x)}}{(b-c)^2}+\frac{4a\,\mathrm{atanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{(b-c)^2}-\frac{2a\,(b+c)\,\mathrm{atanh}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}}\right)}{\sqrt{b}\sqrt{c}\,(b-c)^2}-\frac{2\sqrt{a+bx}\sqrt{a+cx}}{(b-c)^2}+\frac{(b+c)\int b\,dx}{b\,(b-c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/((b*x+a)**(1/2)+(c*x+a)**(1/2))**2,x)
```

```
[Out] 2*a*log(x)/(b - c)**2 + 4*a*atanh(sqrt(a + b*x)/sqrt(a + c*x))/(b
 - c)**2 - 2*a*(b + c)*atanh(sqrt(c)*sqrt(a + b*x)/(sqrt(b)*sqrt(
```

```
a + c*x)))/(sqrt(b)*sqrt(c)*(b - c)**2) - 2*sqrt(a + b*x)*sqrt(a
+ c*x)/(b - c)**2 + (b + c)*Integral(b, x)/(b*(b - c)**2)
```

Mathematica [A] time = 0.17939, size = 128, normalized size = 0.95

$$\frac{-2\sqrt{a+bx}\sqrt{a+cx}+2a\log\left(2\sqrt{a+bx}\sqrt{a+cx}+2a+bx+cx\right)-\frac{a(b+c)\log\left(2\sqrt{b}\sqrt{c}\sqrt{a+bx}\sqrt{a+cx}+ab+ac+2bcx\right)}{\sqrt{b}\sqrt{c}}+bx+cx}{(b-c)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(Sqrt[a + b*x] + Sqrt[a + c*x])^2,x]
```

```
[Out] (b*x + c*x - 2*Sqrt[a + b*x]*Sqrt[a + c*x] + 2*a*Log[2*a + b*x +
c*x + 2*Sqrt[a + b*x]*Sqrt[a + c*x]] - (a*(b + c)*Log[a*b + a*c +
 2*b*c*x + 2*Sqrt[b]*Sqrt[c]*Sqrt[a + b*x]*Sqrt[a + c*x]])/(Sqrt[
b]*Sqrt[c]))/(b - c)^2
```

Maple [C] time = 0.018, size = 266, normalized size = 2.

$$\frac{bx}{(b-c)^2}+\frac{cx}{(b-c)^2}+2\,\frac{a\ln(x)}{(b-c)^2}$$
$$-\frac{csgn(a)}{(b-c)^2}\sqrt{bx+a}\sqrt{cx+a}\left(csgn(a)\ln\left(\frac{1}{2}\left(2\,bcx+2\,\sqrt{bcx^2+abx+acx+a^2}\sqrt{bc}+ab+ac\right)\frac{1}{\sqrt{bc}}\right)ab+csgn(a)\ln\left(\frac{1}{2}\left(2\,bcx\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x)
```

```
[Out] x/(b-c)^2*b+x/(b-c)^2*c+2*a*ln(x)/(b-c)^2-1/(b-c)^2*(b*x+a)^(1/2)
*(c*x+a)^(1/2)*(csgn(a)*ln(1/2*(2*b*c*x+2*(b*c*x^2+a*b*x+a*c*x+a^
2)^(1/2)*(b*c)^(1/2)+a*b+a*c)/(b*c)^(1/2))*a*b+csgn(a)*ln(1/2*(2*
b*c*x+2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*(b*c)^(1/2)+a*b+a*c)/(b*c
)^(1/2))*a*c+2*csgn(a)*(b*c)^(1/2)*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2
)-2*ln(a*(2*csgn(a)*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)+b*x+c*x+2*a)/
x)*(b*c)^(1/2)*a)*csgn(a)/(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)/(b*c)^(
1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(b*x + a) + sqrt(c*x + a))^2,x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(b*x + a) + sqrt(c*x + a))^2, x)
```

Fricas [A] time = 0.318313, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(b*x + a) + sqrt(c*x + a))^2,x, algorithm="fricas")
```

```
[Out] [(4*sqrt(b*c)*((b + c)*x + a*log(x))*sqrt(b*x + a)*sqrt(c*x + a)
- (2*a^2*b + 2*a^2*c - 2*(a*b + a*c)*sqrt(b*x + a)*sqrt(c*x + a)
+ (a*b^2 + 2*a*b*c + a*c^2)*x)*log((2*a*b*c*x - 2*(b*c*x + sqrt(b
*c)*a)*sqrt(b*x + a)*sqrt(c*x + a) + (2*b*c*x^2 + 2*a^2 + (a*b +
a*c)*x)*sqrt(b*c))/((b + c)*x - 2*sqrt(b*x + a)*sqrt(c*x + a) + 2
*a)) - 2*(2*sqrt(b*c)*sqrt(b*x + a)*sqrt(c*x + a)*a - (2*a^2 + (a
*b + a*c)*x)*sqrt(b*c))*log(-((b + c)*x - 2*sqrt(b*x + a)*sqrt(c*
x + a) + 2*a)/x) - ((b^2 + 6*b*c + c^2)*x^2 + 4*(a*b + a*c)*x + 2
*(2*a^2 + (a*b + a*c)*x)*log(x))*sqrt(b*c))/(2*(b^2 - 2*b*c + c^2
)*sqrt(b*c)*sqrt(b*x + a)*sqrt(c*x + a) - (2*a*b^2 - 4*a*b*c + 2*
a*c^2 + (b^3 - b^2*c - b*c^2 + c^3)*x)*sqrt(b*c)), (4*sqrt(-b*c)*
((b + c)*x + a*log(x))*sqrt(b*x + a)*sqrt(c*x + a) + 2*(2*a^2*b +
 2*a^2*c - 2*(a*b + a*c)*sqrt(b*x + a)*sqrt(c*x + a) + (a*b^2 + 2
*a*b*c + a*c^2)*x)*arctan((sqrt(-b*c)*sqrt(b*x + a)*sqrt(c*x + a)
 - sqrt(-b*c)*a)/(b*c*x)) - 2*(2*sqrt(-b*c)*sqrt(b*x + a)*sqrt(c*
x + a)*a - (2*a^2 + (a*b + a*c)*x)*sqrt(-b*c))*log(-((b + c)*x -
2*sqrt(b*x + a)*sqrt(c*x + a) + 2*a)/x) - ((b^2 + 6*b*c + c^2)*x^
2 + 4*(a*b + a*c)*x + 2*(2*a^2 + (a*b + a*c)*x)*log(x))*sqrt(-b*c
))/(2*(b^2 - 2*b*c + c^2)*sqrt(-b*c)*sqrt(b*x + a)*sqrt(c*x + a)
- (2*a*b^2 - 4*a*b*c + 2*a*c^2 + (b^3 - b^2*c - b*c^2 + c^3)*x)*s
qrt(-b*c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((b*x+a)**(1/2)+(c*x+a)**(1/2))**2,x)
```

```
[Out] Integral(x/(sqrt(a + b*x) + sqrt(a + c*x))**2, x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(b*x + a) + sqrt(c*x + a))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.274 $\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$

Optimal. Leaf size=138

$$-\frac{2a}{x(b-c)^2}+\frac{2\sqrt{a+bx}\sqrt{a+cx}}{x(b-c)^2}+\frac{2(b+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{(b-c)^2}-\frac{4\sqrt{b}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}}\right)}{(b-c)^2}+\frac{(b+c)\log(x)}{(b-c)^2}$$

```
[Out] (-2*a)/((b - c)^2*x) + (2*Sqrt[a + b*x]*Sqrt[a + c*x])/((b - c)^2
*x) + (2*(b + c)*ArcTanh[Sqrt[a + b*x]/Sqrt[a + c*x]])/(b - c)^2
- (4*Sqrt[b]*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[a + b*x])/(Sqrt[b]*Sqr
t[a + c*x])])/(b - c)^2 + ((b + c)*Log[x])/(b - c)^2
```

Rubi [A] time = 0.336803, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286

$$-\frac{2a}{x(b-c)^2}+\frac{2\sqrt{a+bx}\sqrt{a+cx}}{x(b-c)^2}+\frac{2(b+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{(b-c)^2}-\frac{4\sqrt{b}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}}\right)}{(b-c)^2}+\frac{(b+c)\log(x)}{(b-c)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[a + b*x] + Sqrt[a + c*x])^(-2),x]
```

```
[Out] (-2*a)/((b - c)^2*x) + (2*Sqrt[a + b*x]*Sqrt[a + c*x])/((b - c)^2
*x) + (2*(b + c)*ArcTanh[Sqrt[a + b*x]/Sqrt[a + c*x]])/(b - c)^2
- (4*Sqrt[b]*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[a + b*x])/(Sqrt[b]*Sqr
t[a + c*x])])/(b - c)^2 + ((b + c)*Log[x])/(b - c)^2
```

Rubi in Sympy [A] time = 33.632, size = 121, normalized size = 0.88

$$-\frac{2a}{x\,(b-c)^2}-\frac{4\sqrt{b}\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{b}\sqrt{a+cx}}{\sqrt{c}\sqrt{a+bx}}\right)}{(b-c)^2}+\frac{(b+c)\log{(x)}}{(b-c)^2}+\frac{2\,(b+c)\,\mathrm{atanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{(b-c)^2}+\frac{2\sqrt{a+bx}\sqrt{a+cx}}{x\,(b-c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/((b*x+a)**(1/2)+(c*x+a)**(1/2))**2,x)
```

```
[Out] -2*a/(x*(b - c)**2) - 4*sqrt(b)*sqrt(c)*atanh(sqrt(b)*sqrt(a + c*
x)/(sqrt(c)*sqrt(a + b*x)))/(b - c)**2 + (b + c)*log(x)/(b - c)**
```

```
2 + 2*(b + c)*atanh(sqrt(a + b*x)/sqrt(a + c*x))/(b - c)**2 + 2*s
qrt(a + b*x)*sqrt(a + c*x)/(x*(b - c)**2)
```

Mathematica [A] time = 0.0839888, size = 127, normalized size = 0.92

$$\frac{2\sqrt{a+bx}\sqrt{a+cx}+x(b+c)\log\left(2\sqrt{a+bx}\sqrt{a+cx}+2a+bx+cx\right)-2\sqrt{b}\sqrt{c}x\log\left(2\sqrt{b}\sqrt{c}\sqrt{a+bx}\sqrt{a+cx}+ab+ac+2bc\right.}{x(b-c)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[a + b*x] + Sqrt[a + c*x])^(-2),x]
```

```
[Out] (-2*a + 2*Sqrt[a + b*x]*Sqrt[a + c*x] + (b + c)*x*Log[2*a + b*x +
 c*x + 2*Sqrt[a + b*x]*Sqrt[a + c*x]] - 2*Sqrt[b]*Sqrt[c]*x*Log[a
*b + a*c + 2*b*c*x + 2*Sqrt[b]*Sqrt[c]*Sqrt[a + b*x]*Sqrt[a + c*x
]])/((b - c)^2*x)
```

Maple [C] time = 0.017, size = 272, normalized size = 2.

$$\frac{b\ln(x)}{(b-c)^2}+\frac{c\ln(x)}{(b-c)^2}-2\frac{a}{(b-c)^2x}$$
$$-\frac{csgn(a)}{(b-c)^2x}\sqrt{bx+a}\sqrt{cx+a}\left(2\,csgn(a)\ln\left(1/2\,\frac{2\,bcx+2\sqrt{bcx^2+abx+acx+a^2}\sqrt{bc}+ab+ac}{\sqrt{bc}}\right)xbc-\ln\left(\frac{a}{x}\left(2\,csgn(a)\sqrt{b}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x)
```

```
[Out] 1/(b-c)^2*b*ln(x)+1/(b-c)^2*c*ln(x)-2*a/(b-c)^2/x-1/(b-c)^2*(b*x+
a)^(1/2)*(c*x+a)^(1/2)*(2*csgn(a)*ln(1/2*(2*b*c*x+2*(b*c*x^2+a*b*
x+a*c*x+a^2)^(1/2)*(b*c)^(1/2)+a*b+a*c)/(b*c)^(1/2))*x*b*c-ln(a*(
2*csgn(a)*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)+b*x+c*x+2*a)/x)*x*b*(b*
c)^(1/2)-ln(a*(2*csgn(a)*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)+b*x+c*x+
2*a)/x)*x*c*(b*c)^(1/2)-2*csgn(a)*(b*c)^(1/2)*(b*c*x^2+a*b*x+a*c*
x+a^2)^(1/2))*csgn(a)/(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)/x/(b*c)^(1/
2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sqrt(b*x + a) + sqrt(c*x + a))^(-2),x, algorithm="maxima")
```

```
[Out] integrate((sqrt(b*x + a) + sqrt(c*x + a))^(-2), x)
```

Fricas [A] time = 0.302274, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sqrt(b*x + a) + sqrt(c*x + a))^(-2),x, algorithm="fricas")
```

```
[Out] [-1/2*((b^2 - 6*b*c + c^2)*x^2 - 2*(2*(b + c)*x*log(x) - (b + c)*
x - 8*a)*sqrt(b*x + a)*sqrt(c*x + a) - 16*a^2 - 10*(a*b + a*c)*x
+ 2*((b^2 + 2*b*c + c^2)*x^2 + 2*(a*b + a*c)*x)*log(x) - 4*(2*sqr
t(b*c)*sqrt(b*x + a)*sqrt(c*x + a)*x - ((b + c)*x^2 + 2*a*x)*sqrt
(b*c))*log((2*b*c*x^2 + 2*sqrt(b*c)*a*x - 2*sqrt(b*x + a)*sqrt(c*
x + a)*(sqrt(b*c)*x + a) + 2*a^2 + (a*b + a*c)*x)/((b + c)*x - 2*
sqrt(b*x + a)*sqrt(c*x + a) + 2*a)) + 2*(2*sqrt(b*x + a)*sqrt(c*x
 + a)*(b + c)*x - (b^2 + 2*b*c + c^2)*x^2 - 2*(a*b + a*c)*x)*log(
-((b + c)*x - 2*sqrt(b*x + a)*sqrt(c*x + a) + 2*a)/x))/(2*(b^2 -
2*b*c + c^2)*sqrt(b*x + a)*sqrt(c*x + a)*x - (b^3 - b^2*c - b*c^2
 + c^3)*x^2 - 2*(a*b^2 - 2*a*b*c + a*c^2)*x), -1/2*((b^2 - 6*b*c
+ c^2)*x^2 - 2*(2*(b + c)*x*log(x) - (b + c)*x - 8*a)*sqrt(b*x +
a)*sqrt(c*x + a) - 16*a^2 - 10*(a*b + a*c)*x + 8*(2*sqrt(-b*c)*sq
rt(b*x + a)*sqrt(c*x + a)*x - ((b + c)*x^2 + 2*a*x)*sqrt(-b*c))*a
rctan((sqrt(b*x + a)*sqrt(c*x + a) - a)/(sqrt(-b*c)*x)) + 2*((b^2
 + 2*b*c + c^2)*x^2 + 2*(a*b + a*c)*x)*log(x) + 2*(2*sqrt(b*x + a
)*sqrt(c*x + a)*(b + c)*x - (b^2 + 2*b*c + c^2)*x^2 - 2*(a*b + a*
c)*x)*log(-((b + c)*x - 2*sqrt(b*x + a)*sqrt(c*x + a) + 2*a)/x))/
(2*(b^2 - 2*b*c + c^2)*sqrt(b*x + a)*sqrt(c*x + a)*x - (b^3 - b^2
*c - b*c^2 + c^3)*x^2 - 2*(a*b^2 - 2*a*b*c + a*c^2)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\sqrt{a + bx} + \sqrt{a + cx}\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)**(1/2)+(c*x+a)**(1/2))**2,x)
```

```
[Out] Integral((sqrt(a + b*x) + sqrt(a + c*x))**(-2), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(b*x + a) + sqrt(c*x + a))^(-2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.275 $\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$

Optimal. Leaf size=123

$$\frac{\sqrt{a+bx}(a+cx)^{3/2}}{ax^2(b-c)^2}-\frac{a}{x^2(b-c)^2}+\frac{\sqrt{a+bx}\sqrt{a+cx}}{2ax(b-c)}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{2a}-\frac{b+c}{x(b-c)^2}$$

```
[Out] -(a/((b - c)^2*x^2)) - (b + c)/((b - c)^2*x) + (Sqrt[a + b*x]*Sqr
t[a + c*x])/(2*a*(b - c)*x) + (Sqrt[a + b*x]*(a + c*x)^(3/2))/(a*
(b - c)^2*x^2) - ArcTanh[Sqrt[a + b*x]/Sqrt[a + c*x]]/(2*a)
```

Rubi [A] time = 0.452643, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$

$$\frac{\sqrt{a+bx}(a+cx)^{3/2}}{ax^2(b-c)^2}-\frac{a}{x^2(b-c)^2}+\frac{\sqrt{a+bx}\sqrt{a+cx}}{2ax(b-c)}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{2a}-\frac{b+c}{x(b-c)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(Sqrt[a + b*x] + Sqrt[a + c*x])^2),x]
```

```
[Out] -(a/((b - c)^2*x^2)) - (b + c)/((b - c)^2*x) + (Sqrt[a + b*x]*Sqr
t[a + c*x])/(2*a*(b - c)*x) + (Sqrt[a + b*x]*(a + c*x)^(3/2))/(a*
(b - c)^2*x^2) - ArcTanh[Sqrt[a + b*x]/Sqrt[a + c*x]]/(2*a)
```

Rubi in Sympy [A] time = 28.1656, size = 94, normalized size = 0.76

$$-\frac{a}{x^2\,(b-c)^2}-\frac{b+c}{x\,(b-c)^2}-\frac{\operatorname{atanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{2a}+\frac{\sqrt{a+bx}\sqrt{a+cx}}{2ax\,(b-c)}+\frac{\sqrt{a+bx}\,(a+cx)^{\frac{3}{2}}}{ax^2\,(b-c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/((b*x+a)**(1/2)+(c*x+a)**(1/2))**2,x)
```

```
[Out] -a/(x**2*(b - c)**2) - (b + c)/(x*(b - c)**2) - atanh(sqrt(a + b*
x)/sqrt(a + c*x))/(2*a) + sqrt(a + b*x)*sqrt(a + c*x)/(2*a*x*(b -
 c)) + sqrt(a + b*x)*(a + c*x)**(3/2)/(a*x**2*(b - c)**2)
```

Mathematica [A] time = 0.212935, size = 130, normalized size = 1.06

$$-\frac{a}{x^2(b-c)^2}+\sqrt{a+bx}\sqrt{a+cx}\left(\frac{b+c}{2ax(b-c)^2}+\frac{1}{x^2(b-c)^2}\right)-\frac{\log\left(2\sqrt{a+bx}\sqrt{a+cx}+2a+bx+cx\right)}{4a}+\frac{\log(x)}{4a}+\frac{-b-c}{x(b-c)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(Sqrt[a + b*x] + Sqrt[a + c*x])^2),x]
```

```
[Out] -(a/((b - c)^2*x^2)) + (-b - c)/((b - c)^2*x) + (1/((b - c)^2*x^2
) + (b + c)/(2*a*(b - c)^2*x))*Sqrt[a + b*x]*Sqrt[a + c*x] + Log[
x]/(4*a) - Log[2*a + b*x + c*x + 2*Sqrt[a + b*x]*Sqrt[a + c*x]]/(
4*a)
```

Maple [C] time = 0.017, size = 313, normalized size = 2.5

$$-\frac{b}{x(b-c)^2}-\frac{c}{x(b-c)^2}-\frac{a}{(b-c)^2x^2}+\frac{csgn(a)}{4(b-c)^2ax^2}\sqrt{bx+a}\sqrt{cx+a}\left(-\ln\left(\frac{a}{x}\left(2\,csgn(a)\sqrt{bcx^2+abx+acx+a^2}+bx+cx+2a\right)\right)x^2b^2+2\ln\left(\frac{a\left(2\,csgn(a)\sqrt{b\right.}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x)
```

```
[Out] -1/x/(b-c)^2*b-1/x/(b-c)^2*c-a/(b-c)^2/x^2+1/4/(b-c)^2*(b*x+a)^(1
/2)*(c*x+a)^(1/2)/a*(-ln(a*(2*csgn(a)*(b*c*x^2+a*b*x+a*c*x+a^2)^(
1/2)+b*x+c*x+2*a)/x)*x^2*b^2+2*ln(a*(2*csgn(a)*(b*c*x^2+a*b*x+a*c
*x+a^2)^(1/2)+b*x+c*x+2*a)/x)*x^2*b*c-ln(a*(2*csgn(a)*(b*c*x^2+a*
b*x+a*c*x+a^2)^(1/2)+b*x+c*x+2*a)/x)*x^2*c^2+2*csgn(a)*(b*c*x^2+a
*b*x+a*c*x+a^2)^(1/2)*x*b+2*csgn(a)*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/
2)*x*c+4*csgn(a)*a*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2))*csgn(a)/(b*c*
x^2+a*b*x+a*c*x+a^2)^(1/2)/x^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(sqrt(b*x + a) + sqrt(c*x + a))^2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*(sqrt(b*x + a) + sqrt(c*x + a))^2), x)
```

Fricas [A] time = 0.28425, size = 753, normalized size = 6.12

$$\frac{\left(b^4-24\,b^3c-50\,b^2c^2-24\,bc^3+c^4\right)x^4-256\,a^4-8\left(5\,ab^3+39\,ab^2c+39\,abc^2+5\,ac^3\right)x^3-8\left(37\,a^2b^2+98\,a^2bc+37\,a^2c^2\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(sqrt(b*x + a) + sqrt(c*x + a))^2),x, algorithm="fricas")
```

```
[Out] 1/16*((b^4 - 24*b^3*c - 50*b^2*c^2 - 24*b*c^3 + c^4)*x^4 - 256*a^
4 - 8*(5*a*b^3 + 39*a*b^2*c + 39*a*b*c^2 + 5*a*c^3)*x^3 - 8*(37*a
^2*b^2 + 98*a^2*b*c + 37*a^2*c^2)*x^2 + 4*((b^3 + 11*b^2*c + 11*b
*c^2 + c^3)*x^3 + 64*a^3 + 2*(17*a*b^2 + 42*a*b*c + 17*a*c^2)*x^2
 + 96*(a^2*b + a^2*c)*x)*sqrt(b*x + a)*sqrt(c*x + a) - 512*(a^3*b
 + a^3*c)*x + 4*((b^4 + 4*b^3*c - 10*b^2*c^2 + 4*b*c^3 + c^4)*x^4
 + 8*(a*b^3 - a*b^2*c - a*b*c^2 + a*c^3)*x^3 + 8*(a^2*b^2 - 2*a^2
*b*c + a^2*c^2)*x^2 - 4*((b^3 - b^2*c - b*c^2 + c^3)*x^3 + 2*(a*b
^2 - 2*a*b*c + a*c^2)*x^2)*sqrt(b*x + a)*sqrt(c*x + a))*log(-((b
+ c)*x - 2*sqrt(b*x + a)*sqrt(c*x + a) + 2*a)/x))/((a*b^4 + 4*a*b
^3*c - 10*a*b^2*c^2 + 4*a*b*c^3 + a*c^4)*x^4 + 8*(a^2*b^3 - a^2*b
^2*c - a^2*b*c^2 + a^2*c^3)*x^3 + 8*(a^3*b^2 - 2*a^3*b*c + a^3*c^
2)*x^2 - 4*((a*b^3 - a*b^2*c - a*b*c^2 + a*c^3)*x^3 + 2*(a^2*b^2
- 2*a^2*b*c + a^2*c^2)*x^2)*sqrt(b*x + a)*sqrt(c*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)**(1/2)+(c*x+a)**(1/2))**2,x)
```

```
[Out] Integral(1/(x*(sqrt(a + b*x) + sqrt(a + c*x))**2), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x*(sqrt(b*x + a) + sqrt(c*x + a))^2),x, algorithm="giac")

[Out] Timed out
```

3.276 $\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2} \, dx$

Optimal. Leaf size=174

$$\frac{2(a+bx)^{3/2}(a+cx)^{3/2}}{3a^2x^3(b-c)^2}-\frac{(b+c)\sqrt{a+bx}(a+cx)^{3/2}}{2a^2x^2(b-c)^2}-\frac{(b+c)\sqrt{a+bx}\sqrt{a+cx}}{4a^2x(b-c)}+\frac{(b+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{4a^2}-\frac{2a}{3x^3(b-c)^2}-\frac{b+c}{2x^2(b-c)^2}$$

```
[Out] (-2*a)/(3*(b - c)^2*x^3) - (b + c)/(2*(b - c)^2*x^2) - ((b + c)*S
qrt[a + b*x]*Sqrt[a + c*x])/(4*a^2*(b - c)*x) - ((b + c)*Sqrt[a +
 b*x]*(a + c*x)^(3/2))/(2*a^2*(b - c)^2*x^2) + (2*(a + b*x)^(3/2)
*(a + c*x)^(3/2))/(3*a^2*(b - c)^2*x^3) + ((b + c)*ArcTanh[Sqrt[a
 + b*x]/Sqrt[a + c*x]])/(4*a^2)
```

Rubi [A] time = 0.537159, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{2(a+bx)^{3/2}(a+cx)^{3/2}}{3a^2x^3(b-c)^2}-\frac{(b+c)\sqrt{a+bx}(a+cx)^{3/2}}{2a^2x^2(b-c)^2}-\frac{(b+c)\sqrt{a+bx}\sqrt{a+cx}}{4a^2x(b-c)}+\frac{(b+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{4a^2}-\frac{2a}{3x^3(b-c)^2}-\frac{b+c}{2x^2(b-c)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(Sqrt[a + b*x] + Sqrt[a + c*x])^2),x]
```

```
[Out] (-2*a)/(3*(b - c)^2*x^3) - (b + c)/(2*(b - c)^2*x^2) - ((b + c)*S
qrt[a + b*x]*Sqrt[a + c*x])/(4*a^2*(b - c)*x) - ((b + c)*Sqrt[a +
 b*x]*(a + c*x)^(3/2))/(2*a^2*(b - c)^2*x^2) + (2*(a + b*x)^(3/2)
*(a + c*x)^(3/2))/(3*a^2*(b - c)^2*x^3) + ((b + c)*ArcTanh[Sqrt[a
 + b*x]/Sqrt[a + c*x]])/(4*a^2)
```

Rubi in Sympy [A] time = 36.1827, size = 150, normalized size = 0.86

$$-\frac{2a}{3x^3\left(b-c\right)^2}-\frac{b+c}{2x^2\left(b-c\right)^2}+\frac{\left(b+c\right)\operatorname{atanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{4a^2}+\frac{\sqrt{a+bx}\sqrt{a+cx}\left(b+c\right)}{4a^2x\left(b-c\right)}-\frac{\left(a+bx\right)^{\frac{3}{2}}\sqrt{a+cx}\left(b+c\right)}{2a^2x^2\left(b-c\right)^2}+\frac{2\left(a+bx\right)^{\frac{3}{2}}\left(a+cx\right)^{\frac{3}{2}}}{3a^2x^3\left(b-c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/((b*x+a)**(1/2)+(c*x+a)**(1/2))**2,x)
```

```
[Out] -2*a/(3*x**3*(b - c)**2) - (b + c)/(2*x**2*(b - c)**2) + (b + c)*
atanh(sqrt(a + b*x)/sqrt(a + c*x))/(4*a**2) + sqrt(a + b*x)*sqrt(
a + c*x)*(b + c)/(4*a**2*x*(b - c)) - (a + b*x)**(3/2)*sqrt(a + c
*x)*(b + c)/(2*a**2*x**2*(b - c)**2) + 2*(a + b*x)**(3/2)*(a + c*
x)**(3/2)/(3*a**2*x**3*(b - c)**2)
```

Mathematica [A] time = 0.43688, size = 164, normalized size = 0.94

$$\frac{\frac{2\left(-8a^3+a^2\left(8\sqrt{a+bx}\sqrt{a+cx}-6bx-6cx\right)+x^2\left(-3b^2+2bc-3c^2\right)\sqrt{a+bx}\sqrt{a+cx}+2ax(b+c)\sqrt{a+bx}\sqrt{a+cx}\right)}{x^3(b-c)^2}+3(b+c)\log\left(2\sqrt{a+bx}\sqrt{a+cx}+2a+bx\right.}{24a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(Sqrt[a + b*x] + Sqrt[a + c*x])^2),x]
```

```
[Out] ((2*(-8*a^3 + 2*a*(b + c)*x*Sqrt[a + b*x]*Sqrt[a + c*x] + (-3*b^2
 + 2*b*c - 3*c^2)*x^2*Sqrt[a + b*x]*Sqrt[a + c*x] + a^2*(-6*b*x -
 6*c*x + 8*Sqrt[a + b*x]*Sqrt[a + c*x])))/((b - c)^2*x^3) - 3*(b
+ c)*Log[x] + 3*(b + c)*Log[2*a + b*x + c*x + 2*Sqrt[a + b*x]*Sqr
t[a + c*x]])/(24*a^2)
```

Maple [C] time = 0.019, size = 457, normalized size = 2.6

$$-\frac{b}{2\,x^2\,(b-c)^2}-\frac{c}{2\,x^2\,(b-c)^2}-\frac{2\,a}{3\,(b-c)^2\,x^3}$$
$$-\frac{csgn\,(a)}{24\,(b-c)^2\,a^2x^3}\sqrt{bx+a}\sqrt{cx+a}\left(-3\,\ln\left(\frac{a\left(2\,csgn\,(a)\,\sqrt{bcx^2+abx+acx+a^2}+bx+cx+2\,a\right)}{x}\right)x^3b^3+3\,\ln\left(\frac{a\left(2\,csgn\,(a\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x)
```

```
[Out] -1/2/x^2/(b-c)^2*b-1/2/x^2/(b-c)^2*c-2/3*a/(b-c)^2/x^3-1/24/(b-c)
^2*(b*x+a)^(1/2)*(c*x+a)^(1/2)/a^2*(-3*ln(a*(2*csgn(a)*(b*c*x^2+a
*b*x+a*c*x+a^2)^(1/2)+b*x+c*x+2*a)/x)*x^3*b^3+3*ln(a*(2*csgn(a)*(
b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)+b*x+c*x+2*a)/x)*x^3*b^2*c+3*ln(a*(
```

```
2*csgn(a)*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)+b*x+c*x+2*a)/x)*x^3*b*c
^2-3*ln(a*(2*csgn(a)*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)+b*x+c*x+2*a)
/x)*x^3*c^3+6*csgn(a)*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*x^2*b^2-4*c
sgn(a)*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*x^2*b*c+6*csgn(a)*(b*c*x^2
+a*b*x+a*c*x+a^2)^(1/2)*x^2*c^2-4*csgn(a)*(b*c*x^2+a*b*x+a*c*x+a^
2)^(1/2)*x*a*b-4*csgn(a)*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*x*a*c-16
*csgn(a)*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*a^2)*csgn(a)/(b*c*x^2+a*
b*x+a*c*x+a^2)^(1/2)/x^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(sqrt(b*x + a) + sqrt(c*x + a))^2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^2*(sqrt(b*x + a) + sqrt(c*x + a))^2), x)
```

Fricas [A] time = 0.285803, size = 1432, normalized size = 8.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(sqrt(b*x + a) + sqrt(c*x + a))^2),x, algorithm="fricas")
```

```
[Out] -1/96*((5*b^6 - 66*b^5*c - 261*b^4*c^2 + 132*b^3*c^3 - 261*b^2*c^
4 - 66*b*c^5 + 5*c^6)*x^6 + 4096*a^6 - 6*(9*a*b^5 + 141*a*b^4*c -
 22*a*b^3*c^2 - 22*a*b^2*c^3 + 141*a*b*c^4 + 9*a*c^5)*x^5 - 48*(1
1*a^2*b^4 - 8*a^2*b^3*c - 102*a^2*b^2*c^2 - 8*a^2*b*c^3 + 11*a^2*
c^4)*x^4 + 32*(17*a^3*b^3 + 327*a^3*b^2*c + 327*a^3*b*c^2 + 17*a^
3*c^3)*x^3 + 6144*(a^4*b^2 + 3*a^4*b*c + a^4*c^2)*x^2 - 2*((3*b^5
 - 113*b^4*c - 18*b^3*c^2 - 18*b^2*c^3 - 113*b*c^4 + 3*c^5)*x^5 +
 2048*a^5 - 64*(2*a*b^4 + 5*a*b^3*c - 6*a*b^2*c^2 + 5*a*b*c^3 + 2
*a*c^4)*x^4 - 176*(a^2*b^3 - 9*a^2*b^2*c - 9*a^2*b*c^2 + a^2*c^3)
*x^3 + 512*(3*a^3*b^2 + 10*a^3*b*c + 3*a^3*c^2)*x^2 + 3584*(a^4*b
 + a^4*c)*x)*sqrt(b*x + a)*sqrt(c*x + a) + 9216*(a^5*b + a^5*c)*x
 + 12*((b^6 + 14*b^5*c - b^4*c^2 - 28*b^3*c^3 - b^2*c^4 + 14*b*c^
5 + c^6)*x^6 + 6*(3*a*b^5 + 7*a*b^4*c - 10*a*b^3*c^2 - 10*a*b^2*c
^3 + 7*a*b*c^4 + 3*a*c^5)*x^5 + 48*(a^2*b^4 - 2*a^2*b^2*c^2 + a^2
*c^4)*x^4 + 32*(a^3*b^3 - a^3*b^2*c - a^3*b*c^2 + a^3*c^3)*x^3 -
2*((3*b^5 + 7*b^4*c - 10*b^3*c^2 - 10*b^2*c^3 + 7*b*c^4 + 3*c^5)*
```

```
x^5 + 16*(a*b^4 - 2*a*b^2*c^2 + a*c^4)*x^4 + 16*(a^2*b^3 - a^2*b^
2*c - a^2*b*c^2 + a^2*c^3)*x^3)*sqrt(b*x + a)*sqrt(c*x + a))*log(
-((b + c)*x - 2*sqrt(b*x + a)*sqrt(c*x + a) + 2*a)/x))/((a^2*b^5
+ 13*a^2*b^4*c - 14*a^2*b^3*c^2 - 14*a^2*b^2*c^3 + 13*a^2*b*c^4 +
 a^2*c^5)*x^6 + 6*(3*a^3*b^4 + 4*a^3*b^3*c - 14*a^3*b^2*c^2 + 4*a
^3*b*c^3 + 3*a^3*c^4)*x^5 + 48*(a^4*b^3 - a^4*b^2*c - a^4*b*c^2 +
 a^4*c^3)*x^4 + 32*(a^5*b^2 - 2*a^5*b*c + a^5*c^2)*x^3 - 2*((3*a^
2*b^4 + 4*a^2*b^3*c - 14*a^2*b^2*c^2 + 4*a^2*b*c^3 + 3*a^2*c^4)*x
^5 + 16*(a^3*b^3 - a^3*b^2*c - a^3*b*c^2 + a^3*c^3)*x^4 + 16*(a^4
*b^2 - 2*a^4*b*c + a^4*c^2)*x^3)*sqrt(b*x + a)*sqrt(c*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/((b*x+a)**(1/2)+(c*x+a)**(1/2))**2,x)
```

```
[Out] Integral(1/(x**2*(sqrt(a + b*x) + sqrt(a + c*x))**2), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(sqrt(b*x + a) + sqrt(c*x + a))^2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.277 $\int \frac{x^4}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$

Optimal. Leaf size=277

$$\frac{2a^2(b+3c)(a+bx)^{3/2}}{3b^3(b-c)^3}-\frac{8a^2(a+bx)^{3/2}}{3b^2(b-c)^3}-\frac{2a^2(3b+c)(a+cx)^{3/2}}{3c^3(b-c)^3}+\frac{8a^2(a+cx)^{3/2}}{3c^2(b-c)^3}+\frac{2(b+3c)(a+bx)^{7/2}}{7b^3(b-c)^3}-\frac{4a(b+3c)(a+bx)^{5/2}}{5b^3(b-c)^3}+\frac{8a(a+bx)^{5/2}}{5b^2(b-c)^3}-\frac{2(3b+c)(a+cx)^{7/2}}{7c^3(b-c)^3}+\frac{4a(3b+c)(a+cx)^{5/2}}{5c^3(b-c)^3}-\frac{8a(a+cx)^{5/2}}{5c^2(b-c)^3}$$

```
[Out] (-8*a^2*(a + b*x)^(3/2))/(3*b^2*(b - c)^3) + (2*a^2*(b + 3*c)*(a
+ b*x)^(3/2))/(3*b^3*(b - c)^3) + (8*a*(a + b*x)^(5/2))/(5*b^2*(b
 - c)^3) - (4*a*(b + 3*c)*(a + b*x)^(5/2))/(5*b^3*(b - c)^3) + (2
*(b + 3*c)*(a + b*x)^(7/2))/(7*b^3*(b - c)^3) + (8*a^2*(a + c*x)^
(3/2))/(3*(b - c)^3*c^2) - (2*a^2*(3*b + c)*(a + c*x)^(3/2))/(3*(
b - c)^3*c^3) - (8*a*(a + c*x)^(5/2))/(5*(b - c)^3*c^2) + (4*a*(3
*b + c)*(a + c*x)^(5/2))/(5*(b - c)^3*c^3) - (2*(3*b + c)*(a + c*
x)^(7/2))/(7*(b - c)^3*c^3)
```

Rubi [A] time = 0.609445, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08

$$\frac{2a^2(b+3c)(a+bx)^{3/2}}{3b^3(b-c)^3}-\frac{8a^2(a+bx)^{3/2}}{3b^2(b-c)^3}-\frac{2a^2(3b+c)(a+cx)^{3/2}}{3c^3(b-c)^3}+\frac{8a^2(a+cx)^{3/2}}{3c^2(b-c)^3}+\frac{2(b+3c)(a+bx)^{7/2}}{7b^3(b-c)^3}-\frac{4a(b+3c)(a+bx)^{5/2}}{5b^3(b-c)^3}+\frac{8a(a+bx)^{5/2}}{5b^2(b-c)^3}-\frac{2(3b+c)(a+cx)^{7/2}}{7c^3(b-c)^3}+\frac{4a(3b+c)(a+cx)^{5/2}}{5c^3(b-c)^3}-\frac{8a(a+cx)^{5/2}}{5c^2(b-c)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

```
[Out] (-8*a^2*(a + b*x)^(3/2))/(3*b^2*(b - c)^3) + (2*a^2*(b + 3*c)*(a
+ b*x)^(3/2))/(3*b^3*(b - c)^3) + (8*a*(a + b*x)^(5/2))/(5*b^2*(b
 - c)^3) - (4*a*(b + 3*c)*(a + b*x)^(5/2))/(5*b^3*(b - c)^3) + (2
*(b + 3*c)*(a + b*x)^(7/2))/(7*b^3*(b - c)^3) + (8*a^2*(a + c*x)^
(3/2))/(3*(b - c)^3*c^2) - (2*a^2*(3*b + c)*(a + c*x)^(3/2))/(3*(
b - c)^3*c^3) - (8*a*(a + c*x)^(5/2))/(5*(b - c)^3*c^2) + (4*a*(3
*b + c)*(a + c*x)^(5/2))/(5*(b - c)^3*c^3) - (2*(3*b + c)*(a + c*
x)^(7/2))/(7*(b - c)^3*c^3)
```

Rubi in Sympy [A] time = 58.3058, size = 253, normalized size = 0.91

$$\frac{8a^2(a+cx)^{\frac{3}{2}}}{3c^2(b-c)^3}-\frac{2a^2(a+cx)^{\frac{3}{2}}(3b+c)}{3c^3(b-c)^3}-\frac{8a^2(a+bx)^{\frac{3}{2}}}{3b^2(b-c)^3}+\frac{2a^2(a+bx)^{\frac{3}{2}}(b+3c)}{3b^3(b-c)^3}-\frac{8a(a+cx)^{\frac{5}{2}}}{5c^2(b-c)^3}+\frac{4a(a+cx)^{\frac{5}{2}}(3b+c)}{5c^3(b-c)^3}+\frac{8a(a+bx)^{\frac{5}{2}}}{5b^2(b-c)^3}-\frac{4a(a+bx)^{\frac{5}{2}}(b+3c)}{5b^3(b-c)^3}-\frac{2(a+cx)^{\frac{7}{2}}(3b+c)}{7c^3(b-c)^3}+\frac{2(a+bx)^{\frac{7}{2}}(b+3c)}{7b^3(b-c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/((b*x+a)**(1/2)+(c*x+a)**(1/2))**3,x)
```

```
[Out] 8*a**2*(a + c*x)**(3/2)/(3*c**2*(b - c)**3) - 2*a**2*(a + c*x)**(
3/2)*(3*b + c)/(3*c**3*(b - c)**3) - 8*a**2*(a + b*x)**(3/2)/(3*b
**2*(b - c)**3) + 2*a**2*(a + b*x)**(3/2)*(b + 3*c)/(3*b**3*(b -
c)**3) - 8*a*(a + c*x)**(5/2)/(5*c**2*(b - c)**3) + 4*a*(a + c*x)
**(5/2)*(3*b + c)/(5*c**3*(b - c)**3) + 8*a*(a + b*x)**(5/2)/(5*b
**2*(b - c)**3) - 4*a*(a + b*x)**(5/2)*(b + 3*c)/(5*b**3*(b - c)*
*3) - 2*(a + c*x)**(7/2)*(3*b + c)/(7*c**3*(b - c)**3) + 2*(a + b
*x)**(7/2)*(b + 3*c)/(7*b**3*(b - c)**3)
```

Mathematica [A] time = 0.51002, size = 114, normalized size = 0.41

$$-\frac{2\left(b^3(a+cx)^{3/2}\left(8a^2(b-2c)-12acx(b-2c)+5c^2x^2(3b+c)\right)+c^3(a+bx)^{3/2}\left(8a^2(2b-c)+12abx(c-2b)-5b^2x^2(b+3c\right.\right.}{35b^3c^3(b-c)^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

```
[Out] (-2*(b^3*(a + c*x)^(3/2)*(8*a^2*(b - 2*c) - 12*a*(b - 2*c)*c*x +
5*c^2*(3*b + c)*x^2) + c^3*(a + b*x)^(3/2)*(8*a^2*(2*b - c) + 12*
a*b*(-2*b + c)*x - 5*b^2*(b + 3*c)*x^2)))/(35*b^3*(b - c)^3*c^3)
```

Maple [A] time = 0.005, size = 246, normalized size = 0.9

$$
\begin{aligned}
&2\,\frac{1/7\,\left(bx+a\right)^{7/2}-2/5\,\left(bx+a\right)^{5/2}a+1/3\,a^{2}\left(bx+a\right)^{3/2}}{\left(b-c\right)^{3}b^{2}}\\
&+8\,\frac{a\left(1/5\,\left(bx+a\right)^{5/2}-1/3\,\left(bx+a\right)^{3/2}a\right)}{\left(b-c\right)^{3}b^{2}}-8\,\frac{a\left(1/5\,\left(cx+a\right)^{5/2}-1/3\,\left(cx+a\right)^{3/2}a\right)}{\left(b-c\right)^{3}c^{2}}\\
&+6\,\frac{c\left(1/7\,\left(bx+a\right)^{7/2}-2/5\,\left(bx+a\right)^{5/2}a+1/3\,a^{2}\left(bx+a\right)^{3/2}\right)}{\left(b-c\right)^{3}b^{3}}\\
&-6\,\frac{b\left(1/7\,\left(cx+a\right)^{7/2}-2/5\,\left(cx+a\right)^{5/2}a+1/3\,a^{2}\left(cx+a\right)^{3/2}\right)}{\left(b-c\right)^{3}c^{3}}\\
&-2\,\frac{1/7\,\left(cx+a\right)^{7/2}-2/5\,\left(cx+a\right)^{5/2}a+1/3\,a^{2}\left(cx+a\right)^{3/2}}{\left(b-c\right)^{3}c^{2}}
\end{aligned}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x)
```

```
[Out] 2/(b-c)^3/b^2*(1/7*(b*x+a)^(7/2)-2/5*(b*x+a)^(5/2)*a+1/3*a^2*(b*x
+a)^(3/2))+8/(b-c)^3*a/b^2*(1/5*(b*x+a)^(5/2)-1/3*(b*x+a)^(3/2)*a
)-8/(b-c)^3*a/c^2*(1/5*(c*x+a)^(5/2)-1/3*(c*x+a)^(3/2)*a)+6/(b-c)
^3*c/b^3*(1/7*(b*x+a)^(7/2)-2/5*(b*x+a)^(5/2)*a+1/3*a^2*(b*x+a)^(
3/2))-6/(b-c)^3*b/c^3*(1/7*(c*x+a)^(7/2)-2/5*(c*x+a)^(5/2)*a+1/3*
a^2*(c*x+a)^(3/2))-2/(b-c)^3/c^2*(1/7*(c*x+a)^(7/2)-2/5*(c*x+a)^(
5/2)*a+1/3*a^2*(c*x+a)^(3/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(sqrt(b*x + a) + sqrt(c*x + a))^3,x, algorithm="maxima")
```

```
[Out] integrate(x^4/(sqrt(b*x + a) + sqrt(c*x + a))^3, x)
```

Fricas [A] time = 0.267037, size = 304, normalized size = 1.1

$$-\frac{2\left(\left(16\,a^{3}bc^{3}-8\,a^{3}c^{4}-5\,\left(b^{4}c^{3}+3\,b^{3}c^{4}\right)x^{3}-\left(29\,ab^{3}c^{3}+3\,ab^{2}c^{4}\right)x^{2}-4\,\left(2\,a^{2}b^{2}c^{3}-a^{2}bc^{4}\right)x\right)\sqrt{bx+a}+\left(8\,a^{3}b^{4}-16\,a^{3}b^{3}\right.}{35\,\left(b^{6}c^{3}-3\,b^{5}c^{4}+3\,b^{4}c^{5}-b^{3}c^{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(sqrt(b*x + a) + sqrt(c*x + a))^3,x, algorithm="fricas")
```

```
[Out] -2/35*((16*a^3*b*c^3 - 8*a^3*c^4 - 5*(b^4*c^3 + 3*b^3*c^4)*x^3 -
(29*a*b^3*c^3 + 3*a*b^2*c^4)*x^2 - 4*(2*a^2*b^2*c^3 - a^2*b*c^4)*
x)*sqrt(b*x + a) + (8*a^3*b^4 - 16*a^3*b^3*c + 5*(3*b^4*c^3 + b^3
*c^4)*x^3 + (3*a*b^4*c^2 + 29*a*b^3*c^3)*x^2 - 4*(a^2*b^4*c - 2*a
^2*b^3*c^2)*x)*sqrt(c*x + a))/(b^6*c^3 - 3*b^5*c^4 + 3*b^4*c^5 -
b^3*c^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{4}}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/((b*x+a)**(1/2)+(c*x+a)**(1/2))**3,x)
```

```
[Out] Integral(x**4/(sqrt(a + b*x) + sqrt(a + c*x))**3, x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(sqrt(b*x + a) + sqrt(c*x + a))^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.278 $\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$

Optimal. Leaf size=163

$$\frac{2(b+3c)(a+bx)^{5/2}}{5b^2(b-c)^3}-\frac{2a(b+3c)(a+bx)^{3/2}}{3b^2(b-c)^3}-\frac{2(3b+c)(a+cx)^{5/2}}{5c^2(b-c)^3}+\frac{2a(3b+c)(a+cx)^{3/2}}{3c^2(b-c)^3}+\frac{8a(a+bx)^{3/2}}{3b(b-c)^3}-\frac{8a(a+cx)^{3/2}}{3c(b-c)^3}$$

```
[Out] (8*a*(a + b*x)^(3/2))/(3*b*(b - c)^3) - (2*a*(b + 3*c)*(a + b*x)^
(3/2))/(3*b^2*(b - c)^3) + (2*(b + 3*c)*(a + b*x)^(5/2))/(5*b^2*(
b - c)^3) - (8*a*(a + c*x)^(3/2))/(3*(b - c)^3*c) + (2*a*(3*b + c
)*(a + c*x)^(3/2))/(3*(b - c)^3*c^2) - (2*(3*b + c)*(a + c*x)^(5/
2))/(5*(b - c)^3*c^2)
```

Rubi [A] time = 0.437136, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08

$$\frac{2(b+3c)(a+bx)^{5/2}}{5b^2(b-c)^3}-\frac{2a(b+3c)(a+bx)^{3/2}}{3b^2(b-c)^3}-\frac{2(3b+c)(a+cx)^{5/2}}{5c^2(b-c)^3}+\frac{2a(3b+c)(a+cx)^{3/2}}{3c^2(b-c)^3}+\frac{8a(a+bx)^{3/2}}{3b(b-c)^3}-\frac{8a(a+cx)^{3/2}}{3c(b-c)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

```
[Out] (8*a*(a + b*x)^(3/2))/(3*b*(b - c)^3) - (2*a*(b + 3*c)*(a + b*x)^
(3/2))/(3*b^2*(b - c)^3) + (2*(b + 3*c)*(a + b*x)^(5/2))/(5*b^2*(
b - c)^3) - (8*a*(a + c*x)^(3/2))/(3*(b - c)^3*c) + (2*a*(3*b + c
)*(a + c*x)^(3/2))/(3*(b - c)^3*c^2) - (2*(3*b + c)*(a + c*x)^(5/
2))/(5*(b - c)^3*c^2)
```

Rubi in Sympy [A] time = 35.1659, size = 144, normalized size = 0.88

$$-\frac{8a\left(a+cx\right)^{\frac{3}{2}}}{3c\left(b-c\right)^{3}}+\frac{2a\left(a+cx\right)^{\frac{3}{2}}\left(3b+c\right)}{3c^{2}\left(b-c\right)^{3}}+\frac{8a\left(a+bx\right)^{\frac{3}{2}}}{3b\left(b-c\right)^{3}}-\frac{2a\left(a+bx\right)^{\frac{3}{2}}\left(b+3c\right)}{3b^{2}\left(b-c\right)^{3}}-\frac{2\left(a+cx\right)^{\frac{5}{2}}\left(3b+c\right)}{5c^{2}\left(b-c\right)^{3}}+\frac{2\left(a+bx\right)^{\frac{5}{2}}\left(b+3c\right)}{5b^{2}\left(b-c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/((b*x+a)**(1/2)+(c*x+a)**(1/2))**3,x)
```

```
[Out] -8*a*(a + c*x)**(3/2)/(3*c*(b - c)**3) + 2*a*(a + c*x)**(3/2)*(3*
b + c)/(3*c**2*(b - c)**3) + 8*a*(a + b*x)**(3/2)/(3*b*(b - c)**3
) - 2*a*(a + b*x)**(3/2)*(b + 3*c)/(3*b**2*(b - c)**3) - 2*(a + c
*x)**(5/2)*(3*b + c)/(5*c**2*(b - c)**3) + 2*(a + b*x)**(5/2)*(b
+ 3*c)/(5*b**2*(b - c)**3)
```

Mathematica [A] time = 0.340223, size = 80, normalized size = 0.49

$$\frac{2\left(c^2(a+bx)^{3/2}(a(6b-2c)+bx(b+3c))-b^2(a+cx)^{3/2}(cx(3b+c)-2a(b-3c))\right)}{5b^2c^2(b-c)^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

```
[Out] (2*(-(b^2*(a + c*x)^(3/2)*(-2*a*(b - 3*c) + c*(3*b + c)*x)) + c^2
*(a + b*x)^(3/2)*(a*(6*b - 2*c) + b*(b + 3*c)*x)))/(5*b^2*(b - c)
^3*c^2)
```

Maple [A] time = 0.005, size = 172, normalized size = 1.1

$$2\,{\frac{1/5\,(bx+a)^{5/2}-1/3\,(bx+a)^{3/2}a}{(b-c)^3b}}+{\frac{8\,a}{3\,(b-c)^3b}(bx+a)^{\frac{3}{2}}}-{\frac{8\,a}{3\,(b-c)^3c}(cx+a)^{\frac{3}{2}}}+6\,{\frac{c\left(1/5\,(bx+a)^{5/2}-1/3\,(bx+a)^{3/2}a\right)}{(b-c)^3b^2}}-6\,{\frac{b\left(1/5\,(cx+a)^{5/2}-1/3\,(cx+a)^{3/2}a\right)}{(b-c)^3c^2}}-2\,{\frac{1/5\,(cx+a)^{5/2}-1/3\,(cx+a)^{3/2}a}{(b-c)^3c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x)
```

```
[Out] 2/(b-c)^3/b*(1/5*(b*x+a)^(5/2)-1/3*(b*x+a)^(3/2)*a)+8/3*a*(b*x+a)
^(3/2)/b/(b-c)^3-8/3*a*(c*x+a)^(3/2)/(b-c)^3/c+6/(b-c)^3*c/b^2*(1
/5*(b*x+a)^(5/2)-1/3*(b*x+a)^(3/2)*a)-6/(b-c)^3*b/c^2*(1/5*(c*x+a
)^(5/2)-1/3*(c*x+a)^(3/2)*a)-2/(b-c)^3/c*(1/5*(c*x+a)^(5/2)-1/3*(
c*x+a)^(3/2)*a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(sqrt(b*x + a) + sqrt(c*x + a))^3,x, algorithm="maxima")
```

```
[Out] integrate(x^3/(sqrt(b*x + a) + sqrt(c*x + a))^3, x)
```

Fricas [A] time = 0.273866, size = 225, normalized size = 1.38

$$\frac{2\left(\left(6\,a^2bc^2-2\,a^2c^3+\left(b^3c^2+3\,b^2c^3\right)x^2+\left(7\,ab^2c^2+abc^3\right)x\right)\sqrt{bx+a}+\left(2\,a^2b^3-6\,a^2b^2c-\left(3\,b^3c^2+b^2c^3\right)x^2-\left(ab^3c+7\,a\right.\right.}{5\left(b^5c^2-3\,b^4c^3+3\,b^3c^4-b^2c^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(sqrt(b*x + a) + sqrt(c*x + a))^3,x, algorithm="fricas")
```

```
[Out] 2/5*((6*a^2*b*c^2 - 2*a^2*c^3 + (b^3*c^2 + 3*b^2*c^3)*x^2 + (7*a*
b^2*c^2 + a*b*c^3)*x)*sqrt(b*x + a) + (2*a^2*b^3 - 6*a^2*b^2*c -
(3*b^3*c^2 + b^2*c^3)*x^2 - (a*b^3*c + 7*a*b^2*c^2)*x)*sqrt(c*x +
 a))/(b^5*c^2 - 3*b^4*c^3 + 3*b^3*c^4 - b^2*c^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/((b*x+a)**(1/2)+(c*x+a)**(1/2))**3,x)
```

```
[Out] Integral(x**3/(sqrt(a + b*x) + sqrt(a + c*x))**3, x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(sqrt(b*x + a) + sqrt(c*x + a))^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.279 $\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3} \, dx$

Optimal. Leaf size=155

$$-\frac{8a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(b-c)^3}+\frac{8a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{(b-c)^3}+\frac{8a\sqrt{a+bx}}{(b-c)^3}-\frac{8a\sqrt{a+cx}}{(b-c)^3}+\frac{2(b+3c)(a+bx)^{3/2}}{3b(b-c)^3}-\frac{2(3b+c)(a+cx)^{3/2}}{3c(b-c)^3}$$

```
[Out] (8*a*Sqrt[a + b*x])/(b - c)^3 + (2*(b + 3*c)*(a + b*x)^(3/2))/(3*
b*(b - c)^3) - (8*a*Sqrt[a + c*x])/(b - c)^3 - (2*(3*b + c)*(a +
c*x)^(3/2))/(3*(b - c)^3*c) - (8*a^(3/2)*ArcTanh[Sqrt[a + b*x]/Sq
rt[a]])/(b - c)^3 + (8*a^(3/2)*ArcTanh[Sqrt[a + c*x]/Sqrt[a]])/(b
 - c)^3
```

Rubi [A] time = 0.396067, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$

$$-\frac{8a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(b-c)^3}+\frac{8a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{(b-c)^3}+\frac{8a\sqrt{a+bx}}{(b-c)^3}-\frac{8a\sqrt{a+cx}}{(b-c)^3}+\frac{2(b+3c)(a+bx)^{3/2}}{3b(b-c)^3}-\frac{2(3b+c)(a+cx)^{3/2}}{3c(b-c)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

```
[Out] (8*a*Sqrt[a + b*x])/(b - c)^3 + (2*(b + 3*c)*(a + b*x)^(3/2))/(3*
b*(b - c)^3) - (8*a*Sqrt[a + c*x])/(b - c)^3 - (2*(3*b + c)*(a +
c*x)^(3/2))/(3*(b - c)^3*c) - (8*a^(3/2)*ArcTanh[Sqrt[a + b*x]/Sq
rt[a]])/(b - c)^3 + (8*a^(3/2)*ArcTanh[Sqrt[a + c*x]/Sqrt[a]])/(b
 - c)^3
```

Rubi in Sympy [A] time = 32.8726, size = 131, normalized size = 0.85

$$-\frac{8a^{\frac{3}{2}}\operatorname{atanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(b-c)^3}+\frac{8a^{\frac{3}{2}}\operatorname{atanh}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{(b-c)^3}+\frac{8a\sqrt{a+bx}}{(b-c)^3}-\frac{8a\sqrt{a+cx}}{(b-c)^3}-\frac{2\left(a+cx\right)^{\frac{3}{2}}\left(b+\frac{c}{3}\right)}{c\left(b-c\right)^3}+\frac{2\left(a+bx\right)^{\frac{3}{2}}\left(\frac{b}{3}+c\right)}{b\left(b-c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/((b*x+a)**(1/2)+(c*x+a)**(1/2))**3,x)
```

```
[Out] -8*a**(3/2)*atanh(sqrt(a + b*x)/sqrt(a))/(b - c)**3 + 8*a**(3/2)*
atanh(sqrt(a + c*x)/sqrt(a))/(b - c)**3 + 8*a*sqrt(a + b*x)/(b -
c)**3 - 8*a*sqrt(a + c*x)/(b - c)**3 - 2*(a + c*x)**(3/2)*(b + c/
3)/(c*(b - c)**3) + 2*(a + b*x)**(3/2)*(b/3 + c)/(b*(b - c)**3)
```

Mathematica [A] time = 0.286157, size = 127, normalized size = 0.82

$$-\frac{2\left(12a^{3/2}bc\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)-12a^{3/2}bc\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)+b\sqrt{a+cx}(a(3b+13c)+cx(3b+c))-c\sqrt{a+bx}(a(13b+3c)+b\right.}{3bc(b-c)^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

```
[Out] (-2*(b*Sqrt[a + c*x]*(a*(3*b + 13*c) + c*(3*b + c)*x) - c*Sqrt[a
+ b*x]*(a*(13*b + 3*c) + b*(b + 3*c)*x) + 12*a^(3/2)*b*c*ArcTanh[
Sqrt[a + b*x]/Sqrt[a]] - 12*a^(3/2)*b*c*ArcTanh[Sqrt[a + c*x]/Sqr
t[a]]))/(3*b*(b - c)^3*c)
```

Maple [A] time = 0.005, size = 148, normalized size = 1.

$$\frac{2}{3\,(b-c)^3}(bx+a)^{\frac{3}{2}}+2\,\frac{c\,(bx+a)^{3/2}}{(b-c)^3\,b}-2\,\frac{b\,(cx+a)^{3/2}}{(b-c)^3\,c}-\frac{2}{3\,(b-c)^3}(cx+a)^{\frac{3}{2}}+4\,\frac{a}{(b-c)^3}\left(2\,\sqrt{bx+a}-2\,\sqrt{a}Artanh\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)-4\,\frac{a}{(b-c)^3}\left(2\,\sqrt{cx+a}-2\,\sqrt{a}Artanh\left(\frac{\sqrt{cx+a}}{\sqrt{a}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x)
```

```
[Out] 2/3/(b-c)^3*(b*x+a)^(3/2)+2/(b-c)^3*c*(b*x+a)^(3/2)/b-2/(b-c)^3*b
*(c*x+a)^(3/2)/c-2/3/(b-c)^3*(c*x+a)^(3/2)+4/(b-c)^3*a*(2*(b*x+a)
^(1/2)-2*a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2)))-4/(b-c)^3*a*(2*(
c*x+a)^(1/2)-2*a^(1/2)*arctanh((c*x+a)^(1/2)/a^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(sqrt(b*x + a) + sqrt(c*x + a))^3,x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(b*x + a) + sqrt(c*x + a))^3, x)
```

Fricas [A] time = 0.280438, size = 1, normalized size = 0.01

$$\left[-\frac{2\left(6\,a^{\frac{3}{2}}bc\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+6\,a^{\frac{3}{2}}bc\log\left(\frac{cx-2\sqrt{cx+a}\sqrt{a}+2\,a}{x}\right)-\left(13\,abc+3\,ac^2+\left(b^2c+3\,bc^2\right)x\right)\sqrt{bx+a}+\left(3\,ab^2+1\right.\right.}{3\left(b^4c-3\,b^3c^2+3\,b^2c^3-bc^4\right)}\right.$$

$$-\frac{2\left(12\sqrt{-a}abc\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)-12\sqrt{-a}abc\arctan\left(\frac{\sqrt{cx+a}}{\sqrt{-a}}\right)-\left(13\,abc+3\,ac^2+\left(b^2c+3\,bc^2\right)x\right)\sqrt{bx+a}+\left(3\,ab^2+13\,abc\right.\right.}{3\left(b^4c-3\,b^3c^2+3\,b^2c^3-bc^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(sqrt(b*x + a) + sqrt(c*x + a))^3,x, algorithm="fricas")
```

```
[Out] [-2/3*(6*a^(3/2)*b*c*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x)
 + 6*a^(3/2)*b*c*log((c*x - 2*sqrt(c*x + a)*sqrt(a) + 2*a)/x) - (
13*a*b*c + 3*a*c^2 + (b^2*c + 3*b*c^2)*x)*sqrt(b*x + a) + (3*a*b^
2 + 13*a*b*c + (3*b^2*c + b*c^2)*x)*sqrt(c*x + a))/(b^4*c - 3*b^3
*c^2 + 3*b^2*c^3 - b*c^4), -2/3*(12*sqrt(-a)*a*b*c*arctan(sqrt(b*
x + a)/sqrt(-a)) - 12*sqrt(-a)*a*b*c*arctan(sqrt(c*x + a)/sqrt(-a
)) - (13*a*b*c + 3*a*c^2 + (b^2*c + 3*b*c^2)*x)*sqrt(b*x + a) + (
3*a*b^2 + 13*a*b*c + (3*b^2*c + b*c^2)*x)*sqrt(c*x + a))/(b^4*c -
 3*b^3*c^2 + 3*b^2*c^3 - b*c^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/((b*x+a)**(1/2)+(c*x+a)**(1/2))**3,x)
```

```
[Out] Integral(x**2/(sqrt(a + b*x) + sqrt(a + c*x))**3, x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(sqrt(b*x + a) + sqrt(c*x + a))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.280 $\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$

Optimal. Leaf size=157

$$-\frac{4a\sqrt{a+bx}}{x(b-c)^3}+\frac{4a\sqrt{a+cx}}{x(b-c)^3}+\frac{2(b+3c)\sqrt{a+bx}}{(b-c)^3}-\frac{2(3b+c)\sqrt{a+cx}}{(b-c)^3}-\frac{6\sqrt{a}(b+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(b-c)^3}+\frac{6\sqrt{a}(b+c)\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{(b-c)^3}$$

```
[Out] (2*(b + 3*c)*Sqrt[a + b*x])/(b - c)^3 - (4*a*Sqrt[a + b*x])/((b -
 c)^3*x) - (2*(3*b + c)*Sqrt[a + c*x])/(b - c)^3 + (4*a*Sqrt[a +
c*x])/((b - c)^3*x) - (6*Sqrt[a]*(b + c)*ArcTanh[Sqrt[a + b*x]/Sq
rt[a]])/(b - c)^3 + (6*Sqrt[a]*(b + c)*ArcTanh[Sqrt[a + c*x]/Sqrt
[a]])/(b - c)^3
```

Rubi [A] time = 0.451189, antiderivative size = 223, normalized size of antiderivative = 1.42, number of steps used = 14, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217

$$-\frac{4a\sqrt{a+bx}}{x(b-c)^3}+\frac{4a\sqrt{a+cx}}{x(b-c)^3}+\frac{2(b+3c)\sqrt{a+bx}}{(b-c)^3}-\frac{2(3b+c)\sqrt{a+cx}}{(b-c)^3}-\frac{2\sqrt{a}(b+3c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(b-c)^3}-\frac{4\sqrt{a}b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(b-c)^3}+\frac{4\sqrt{a}c\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{(b-c)^3}+\frac{2\sqrt{a}(3b+c)\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{(b-c)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

```
[Out] (2*(b + 3*c)*Sqrt[a + b*x])/(b - c)^3 - (4*a*Sqrt[a + b*x])/((b -
 c)^3*x) - (2*(3*b + c)*Sqrt[a + c*x])/(b - c)^3 + (4*a*Sqrt[a +
c*x])/((b - c)^3*x) - (4*Sqrt[a]*b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
)/(b - c)^3 - (2*Sqrt[a]*(b + 3*c)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
)/(b - c)^3 + (4*Sqrt[a]*c*ArcTanh[Sqrt[a + c*x]/Sqrt[a]])/(b - c
)^3 + (2*Sqrt[a]*(3*b + c)*ArcTanh[Sqrt[a + c*x]/Sqrt[a]])/(b - c
)^3
```

Rubi in Sympy [A] time = 38.5623, size = 199, normalized size = 1.27

$$-\frac{4\sqrt{a}b\,\mathrm{atanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(b-c)^3}+\frac{4\sqrt{a}c\,\mathrm{atanh}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{(b-c)^3}-\frac{2\sqrt{a}\,(b+3c)\,\mathrm{atanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(b-c)^3}+\frac{2\sqrt{a}\,(3b+c)\,\mathrm{atanh}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{(b-c)^3}-\frac{4a\sqrt{a+bx}}{x\,(b-c)^3}+\frac{4a\sqrt{a+cx}}{x\,(b-c)^3}+\frac{2\sqrt{a+bx}\,(b+3c)}{(b-c)^3}-\frac{2\sqrt{a+cx}\,(3b+c)}{(b-c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/((b*x+a)**(1/2)+(c*x+a)**(1/2))**3,x)
```

```
[Out] -4*sqrt(a)*b*atanh(sqrt(a + b*x)/sqrt(a))/(b - c)**3 + 4*sqrt(a)*
c*atanh(sqrt(a + c*x)/sqrt(a))/(b - c)**3 - 2*sqrt(a)*(b + 3*c)*a
tanh(sqrt(a + b*x)/sqrt(a))/(b - c)**3 + 2*sqrt(a)*(3*b + c)*atan
h(sqrt(a + c*x)/sqrt(a))/(b - c)**3 - 4*a*sqrt(a + b*x)/(x*(b - c
)**3) + 4*a*sqrt(a + c*x)/(x*(b - c)**3) + 2*sqrt(a + b*x)*(b + 3
*c)/(b - c)**3 - 2*sqrt(a + c*x)*(3*b + c)/(b - c)**3
```

Mathematica [A] time = 0.23465, size = 143, normalized size = 0.91

$$\sqrt{a+bx}\left(\frac{2(b+3c)}{(b-c)^3}-\frac{4a}{x(b-c)^3}\right)+\sqrt{a+cx}\left(\frac{4a}{x(b-c)^3}-\frac{2(3b+c)}{(b-c)^3}\right)-\frac{6\sqrt{a}(b+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(b-c)^3}+\frac{6\sqrt{a}(b+c)\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{(b-c)^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

```
[Out] ((2*(b + 3*c))/(b - c)^3 - (4*a)/((b - c)^3*x))*Sqrt[a + b*x] + (
(-2*(3*b + c))/(b - c)^3 + (4*a)/((b - c)^3*x))*Sqrt[a + c*x] - (
6*Sqrt[a]*(b + c)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(b - c)^3 + (6*
Sqrt[a]*(b + c)*ArcTanh[Sqrt[a + c*x]/Sqrt[a]])/(b - c)^3
```

Maple [A] time = 0.005, size = 237, normalized size = 1.5

$$\frac{b}{(b-c)^3}\left(2\sqrt{bx+a}-2\sqrt{a}Artanh\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)+8\frac{ab}{(b-c)^3}\left(-1/2\frac{\sqrt{bx+a}}{bx}-1/2\frac{1}{\sqrt{a}}Artanh\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)-8\frac{ac}{(b-c)^3}\left(-1/2\frac{\sqrt{cx+a}}{cx}-1/2\frac{1}{\sqrt{a}}Artanh\left(\frac{\sqrt{cx+a}}{\sqrt{a}}\right)\right)+3\frac{c}{(b-c)^3}\left(2\sqrt{bx+a}-2\sqrt{a}Artanh\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)-3\frac{b}{(b-c)^3}\left(2\sqrt{cx+a}-2\sqrt{a}Artanh\left(\frac{\sqrt{cx+a}}{\sqrt{a}}\right)\right)-\frac{c}{(b-c)^3}\left(2\sqrt{cx+a}-2\sqrt{a}Artanh\left(\frac{\sqrt{cx+a}}{\sqrt{a}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x)
```

```
[Out] 1/(b-c)^3*b*(2*(b*x+a)^(1/2)-2*a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1
/2)))+8/(b-c)^3*a*b*(-1/2*(b*x+a)^(1/2)/x/b-1/2*arctanh((b*x+a)^(
1/2)/a^(1/2))/a^(1/2))-8/(b-c)^3*a*c*(-1/2*(c*x+a)^(1/2)/c/x-1/2/
a^(1/2)*arctanh((c*x+a)^(1/2)/a^(1/2)))+3/(b-c)^3*c*(2*(b*x+a)^(1
/2)-2*a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2)))-3/(b-c)^3*b*(2*(c*x
+a)^(1/2)-2*a^(1/2)*arctanh((c*x+a)^(1/2)/a^(1/2)))-1/(b-c)^3*c*(
2*(c*x+a)^(1/2)-2*a^(1/2)*arctanh((c*x+a)^(1/2)/a^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(b*x + a) + sqrt(c*x + a))^3,x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(b*x + a) + sqrt(c*x + a))^3, x)
```

Fricas [A] time = 0.292785, size = 1, normalized size = 0.01

$$\left[-\frac{3\sqrt{a}(b+c)x\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+3\sqrt{a}(b+c)x\log\left(\frac{cx-2\sqrt{cx+a}\sqrt{a}+2a}{x}\right)-2\left((b+3c)x-2a\right)\sqrt{bx+a}+2\left((3b+c)x-2\right.}{(b^3-3b^2c+3bc^2-c^3)x}\right.$$

$$-\frac{2\left(3\sqrt{-a}(b+c)x\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)-3\sqrt{-a}(b+c)x\arctan\left(\frac{\sqrt{cx+a}}{\sqrt{-a}}\right)-((b+3c)x-2a)\sqrt{bx+a}+((3b+c)x-2a)\sqrt{cx+a}\right)}{(b^3-3b^2c+3bc^2-c^3)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(b*x + a) + sqrt(c*x + a))^3,x, algorithm="fricas")
```

```
[Out] [-(3*sqrt(a)*(b + c)*x*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/
x) + 3*sqrt(a)*(b + c)*x*log((c*x - 2*sqrt(c*x + a)*sqrt(a) + 2*a
)/x) - 2*((b + 3*c)*x - 2*a)*sqrt(b*x + a) + 2*((3*b + c)*x - 2*a
)*sqrt(c*x + a))/((b^3 - 3*b^2*c + 3*b*c^2 - c^3)*x), -2*(3*sqrt(
-a)*(b + c)*x*arctan(sqrt(b*x + a)/sqrt(-a)) - 3*sqrt(-a)*(b + c)
*x*arctan(sqrt(c*x + a)/sqrt(-a)) - ((b + 3*c)*x - 2*a)*sqrt(b*x
+ a) + ((3*b + c)*x - 2*a)*sqrt(c*x + a))/((b^3 - 3*b^2*c + 3*b*c
^2 - c^3)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((b*x+a)**(1/2)+(c*x+a)**(1/2))**3,x)
```

```
[Out] Integral(x/(sqrt(a + b*x) + sqrt(a + c*x))**3, x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(b*x + a) + sqrt(c*x + a))^3,x, algorithm="giac")
```

[Out] Timed out

3.281 $\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$

Optimal. Leaf size=164

$$-\frac{2a\sqrt{a+bx}}{x^2(b-c)^3}+\frac{2a\sqrt{a+cx}}{x^2(b-c)^3}-\frac{(2b+3c)\sqrt{a+bx}}{x(b-c)^3}+\frac{(3b+2c)\sqrt{a+cx}}{x(b-c)^3}-\frac{3bc\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}(b-c)^3}+\frac{3bc\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{\sqrt{a}(b-c)^3}$$

```
[Out] (-2*a*Sqrt[a + b*x])/((b - c)^3*x^2) - ((2*b + 3*c)*Sqrt[a + b*x]
)/((b - c)^3*x) + (2*a*Sqrt[a + c*x])/((b - c)^3*x^2) + ((3*b + 2
*c)*Sqrt[a + c*x])/((b - c)^3*x) - (3*b*c*ArcTanh[Sqrt[a + b*x]/S
qrt[a]])/(Sqrt[a]*(b - c)^3) + (3*b*c*ArcTanh[Sqrt[a + c*x]/Sqrt[
a]])/(Sqrt[a]*(b - c)^3)
```

Rubi [A] time = 0.410479, antiderivative size = 275, normalized size of antiderivative = 1.68, number of steps used = 16, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$

$$\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}(b-c)^3}-\frac{c^2\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{\sqrt{a}(b-c)^3}-\frac{2a\sqrt{a+bx}}{x^2(b-c)^3}+\frac{2a\sqrt{a+cx}}{x^2(b-c)^3}-\frac{b\sqrt{a+bx}}{x(b-c)^3}-\frac{(b+3c)\sqrt{a+bx}}{x(b-c)^3}+\frac{c\sqrt{a+cx}}{x(b-c)^3}+\frac{(3b+c)\sqrt{a+cx}}{x(b-c)^3}-\frac{b(b+3c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}(b-c)^3}+\frac{c(3b+c)\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{\sqrt{a}(b-c)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[a + b*x] + Sqrt[a + c*x])^(-3),x]
```

```
[Out] (-2*a*Sqrt[a + b*x])/((b - c)^3*x^2) - (b*Sqrt[a + b*x])/((b - c)
^3*x) - ((b + 3*c)*Sqrt[a + b*x])/((b - c)^3*x) + (2*a*Sqrt[a + c
*x])/((b - c)^3*x^2) + (c*Sqrt[a + c*x])/((b - c)^3*x) + ((3*b +
c)*Sqrt[a + c*x])/((b - c)^3*x) + (b^2*ArcTanh[Sqrt[a + b*x]/Sqrt
[a]])/(Sqrt[a]*(b - c)^3) - (b*(b + 3*c)*ArcTanh[Sqrt[a + b*x]/Sq
rt[a]])/(Sqrt[a]*(b - c)^3) - (c^2*ArcTanh[Sqrt[a + c*x]/Sqrt[a]]
)/(Sqrt[a]*(b - c)^3) + (c*(3*b + c)*ArcTanh[Sqrt[a + c*x]/Sqrt[a
]])/(Sqrt[a]*(b - c)^3)
```

Rubi in Sympy [A] time = 43.3717, size = 236, normalized size = 1.44

$$-\frac{2a\sqrt{a+bx}}{x^2(b-c)^3}+\frac{2a\sqrt{a+cx}}{x^2(b-c)^3}-\frac{b\sqrt{a+bx}}{x(b-c)^3}+\frac{c\sqrt{a+cx}}{x(b-c)^3}-\frac{\sqrt{a+bx}\,(b+3c)}{x(b-c)^3}+\frac{\sqrt{a+cx}\,(3b+c)}{x(b-c)^3}+\frac{b^2\operatorname{atanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}\,(b-c)^3}-\frac{b\,(b+3c)\operatorname{atanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}\,(b-c)^3}-\frac{c^2\operatorname{atanh}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{\sqrt{a}\,(b-c)^3}+\frac{c\,(3b+c)\operatorname{atanh}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{\sqrt{a}\,(b-c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/((b*x+a)**(1/2)+(c*x+a)**(1/2))**3,x)
```

```
[Out] -2*a*sqrt(a + b*x)/(x**2*(b - c)**3) + 2*a*sqrt(a + c*x)/(x**2*(b
 - c)**3) - b*sqrt(a + b*x)/(x*(b - c)**3) + c*sqrt(a + c*x)/(x*(
b - c)**3) - sqrt(a + b*x)*(b + 3*c)/(x*(b - c)**3) + sqrt(a + c*
x)*(3*b + c)/(x*(b - c)**3) + b**2*atanh(sqrt(a + b*x)/sqrt(a))/(
sqrt(a)*(b - c)**3) - b*(b + 3*c)*atanh(sqrt(a + b*x)/sqrt(a))/(s
qrt(a)*(b - c)**3) - c**2*atanh(sqrt(a + c*x)/sqrt(a))/(sqrt(a)*(
b - c)**3) + c*(3*b + c)*atanh(sqrt(a + c*x)/sqrt(a))/(sqrt(a)*(b
 - c)**3)
```

Mathematica [A] time = 0.274927, size = 146, normalized size = 0.89

$$\frac{-3bcx^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+3bcx^2\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)+\sqrt{a}\left(-3cx\sqrt{a+bx}+3bx\sqrt{a+cx}-2a\sqrt{a+bx}-2bx\sqrt{a+bx}+2a\sqrt{a+}\right.}{\sqrt{a}x^2(b-c)^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[a + b*x] + Sqrt[a + c*x])^(-3),x]
```

```
[Out] (Sqrt[a]*(-2*a*Sqrt[a + b*x] - 2*b*x*Sqrt[a + b*x] - 3*c*x*Sqrt[a
 + b*x] + 2*a*Sqrt[a + c*x] + 3*b*x*Sqrt[a + c*x] + 2*c*x*Sqrt[a
+ c*x]) - 3*b*c*x^2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]] + 3*b*c*x^2*Ar
cTanh[Sqrt[a + c*x]/Sqrt[a]])/(Sqrt[a]*(b - c)^3*x^2)
```

Maple [B] time = 0.004, size = 300, normalized size = 1.8

$$2\frac{b^{2}}{(b-c)^{3}}\left(-1/2\,\frac{\sqrt{bx+a}}{bx}-1/2\,\frac{1}{\sqrt{a}}\mathit{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)$$
$$+8\,\frac{ab^{2}}{(b-c)^{3}}\left(\frac{1}{b^{2}x^{2}}\left(-1/8\,\frac{(bx+a)^{3/2}}{a}-1/8\,\sqrt{bx+a}\right)+1/8\,\frac{1}{a^{3/2}}\mathit{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)$$
$$-8\,\frac{ac^{2}}{(b-c)^{3}}\left(\frac{1}{c^{2}x^{2}}\left(-1/8\,\frac{(cx+a)^{3/2}}{a}-1/8\,\sqrt{cx+a}\right)+1/8\,\frac{1}{a^{3/2}}\mathit{Artanh}\left(\frac{\sqrt{cx+a}}{\sqrt{a}}\right)\right)$$
$$+6\,\frac{bc}{(b-c)^{3}}\left(-1/2\,\frac{\sqrt{bx+a}}{bx}-1/2\,\frac{1}{\sqrt{a}}\mathit{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)$$
$$-6\,\frac{bc}{(b-c)^{3}}\left(-1/2\,\frac{\sqrt{cx+a}}{cx}-1/2\,\frac{1}{\sqrt{a}}\mathit{Artanh}\left(\frac{\sqrt{cx+a}}{\sqrt{a}}\right)\right)$$
$$-2\,\frac{c^{2}}{(b-c)^{3}}\left(-1/2\,\frac{\sqrt{cx+a}}{cx}-1/2\,\frac{1}{\sqrt{a}}\mathit{Artanh}\left(\frac{\sqrt{cx+a}}{\sqrt{a}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x)
```

```
[Out] 2/(b-c)^3*b^2*(-1/2*(b*x+a)^(1/2)/x/b-1/2*arctanh((b*x+a)^(1/2)/a
^(1/2))/a^(1/2))+8/(b-c)^3*a*b^2*((-1/8/a*(b*x+a)^(3/2)-1/8*(b*x+
a)^(1/2))/x^2/b^2+1/8/a^(3/2)*arctanh((b*x+a)^(1/2)/a^(1/2)))-8/(
b-c)^3*a*c^2*((-1/8/a*(c*x+a)^(3/2)-1/8*(c*x+a)^(1/2))/c^2/x^2+1/
8/a^(3/2)*arctanh((c*x+a)^(1/2)/a^(1/2)))+6/(b-c)^3*c*b*(-1/2*(b*
x+a)^(1/2)/x/b-1/2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2))-6/(b-c
)^3*b*c*(-1/2*(c*x+a)^(1/2)/c/x-1/2/a^(1/2)*arctanh((c*x+a)^(1/2)
/a^(1/2)))-2/(b-c)^3*c^2*(-1/2*(c*x+a)^(1/2)/c/x-1/2/a^(1/2)*arct
anh((c*x+a)^(1/2)/a^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(b*x + a) + sqrt(c*x + a))^(-3),x, algorithm="maxima")
```

```
[Out] integrate((sqrt(b*x + a) + sqrt(c*x + a))^(-3), x)
```

Fricas [A] time = 0.296504, size = 1, normalized size = 0.01

$$\left[-\frac{3\,bcx^2\log\left(\frac{(bx+2\,a)\sqrt{a}+2\,\sqrt{bx+a}a}{x}\right)+3\,bcx^2\log\left(\frac{(cx+2\,a)\sqrt{a}-2\,\sqrt{cx+a}a}{x}\right)+2\,((2\,b+3\,c)x+2\,a)\sqrt{bx+a}\sqrt{a}-2\,((3\,b+2\,c)x+2\,a}{2\,(b^3-3\,b^2c+3\,bc^2-c^3)\sqrt{a}x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sqrt(b*x + a) + sqrt(c*x + a))^(-3),x, algorithm="fricas")
```

```
[Out] [-1/2*(3*b*c*x^2*log(((b*x + 2*a)*sqrt(a) + 2*sqrt(b*x + a)*a)/x)
 + 3*b*c*x^2*log(((c*x + 2*a)*sqrt(a) - 2*sqrt(c*x + a)*a)/x) + 2
*((2*b + 3*c)*x + 2*a)*sqrt(b*x + a)*sqrt(a) - 2*((3*b + 2*c)*x +
 2*a)*sqrt(c*x + a)*sqrt(a))/((b^3 - 3*b^2*c + 3*b*c^2 - c^3)*sqr
t(a)*x^2), (3*b*c*x^2*arctan(a/(sqrt(b*x + a)*sqrt(-a))) - 3*b*c*
x^2*arctan(a/(sqrt(c*x + a)*sqrt(-a))) - ((2*b + 3*c)*x + 2*a)*sq
rt(b*x + a)*sqrt(-a) + ((3*b + 2*c)*x + 2*a)*sqrt(c*x + a)*sqrt(-
a))/((b^3 - 3*b^2*c + 3*b*c^2 - c^3)*sqrt(-a)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)**(1/2)+(c*x+a)**(1/2))**3,x)
```

```
[Out] Integral((sqrt(a + b*x) + sqrt(a + c*x))**(-3), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sqrt(b*x + a) + sqrt(c*x + a))^(-3),x, algorithm="giac")
```

```
[Out] Timed out
```

3.282 $\int \sqrt{1-x}\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx$

Optimal. Leaf size=31

$$-\frac{x^2}{2}+\frac{1}{2}\sqrt{1-x^2}x+x+\frac{1}{2}\sin^{-1}(x)$$

```
[Out] x - x^2/2 + (x*Sqrt[1 - x^2])/2 + ArcSin[x]/2
```

Rubi [A] time = 0.0824615, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$-\frac{x^2}{2}+\frac{1}{2}\sqrt{1-x^2}x+x+\frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 - x]*(Sqrt[1 - x] + Sqrt[1 + x]),x]
```

```
[Out] x - x^2/2 + (x*Sqrt[1 - x^2])/2 + ArcSin[x]/2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-2\int^{\sqrt{-x+1}} x^2\left(x+\sqrt{-x^2+2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1-x)**(1/2)*((1-x)**(1/2)+(1+x)**(1/2)),x)
```

```
[Out] -2*Integral(x**2*(x + sqrt(-x**2 + 2)), (x, sqrt(-x + 1)))
```

Mathematica [A] time = 0.0220164, size = 31, normalized size = 1.

$$-\frac{x^2}{2}+\frac{1}{2}\sqrt{1-x^2}x+x+\frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 - x]*(Sqrt[1 - x] + Sqrt[1 + x]),x]
```

```
[Out] x - x^2/2 + (x*Sqrt[1 - x^2])/2 + ArcSin[x]/2
```

Maple [B] time = 0.002, size = 63, normalized size = 2.

$$\frac{1}{2}\sqrt{1-x}\,(1+x)^{\frac{3}{2}}-\frac{1}{2}\sqrt{1-x}\sqrt{1+x}+\frac{\arcsin(x)}{2}\sqrt{(1+x)(1-x)}\frac{1}{\sqrt{1-x}}\frac{1}{\sqrt{1+x}}-\frac{x^2}{2}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1-x)^(1/2)*((1-x)^(1/2)+(1+x)^(1/2)),x)
```

```
[Out] 1/2*(1-x)^(1/2)*(1+x)^(3/2)-1/2*(1-x)^(1/2)*(1+x)^(1/2)+1/2*((1+x
)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)-1/2*x^2+x
```

Maxima [A] time = 0.766558, size = 31, normalized size = 1.

$$-\frac{1}{2}\,x^2+\frac{1}{2}\,\sqrt{-x^2+1}x+x+\frac{1}{2}\,\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x + 1)*(sqrt(x + 1) + sqrt(-x + 1)),x, algorithm="maxima")
```

```
[Out] -1/2*x^2 + 1/2*sqrt(-x^2 + 1)*x + x + 1/2*arcsin(x)
```

Fricas [A] time = 0.298677, size = 138, normalized size = 4.45

$$-\frac{x^4-2\,x^2-\left(x^3-2\,x^2+2\,x\right)\sqrt{x+1}\sqrt{-x+1}+2\left(x^2+2\,\sqrt{x+1}\sqrt{-x+1}-2\right)\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)+2\,x}{2\left(x^2+2\,\sqrt{x+1}\sqrt{-x+1}-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x + 1)*(sqrt(x + 1) + sqrt(-x + 1)),x, algorithm="fricas")
```

```
[Out] -1/2*(x^4 - 2*x^2 - (x^3 - 2*x^2 + 2*x)*sqrt(x + 1)*sqrt(-x + 1)
+ 2*(x^2 + 2*sqrt(x + 1)*sqrt(-x + 1) - 2)*arctan((sqrt(x + 1)*sq
```

```
rt(-x + 1) - 1)/x) + 2*x)/(x^2 + 2*sqrt(x + 1)*sqrt(-x + 1) - 2)
```

Sympy [A] time = 6.65975, size = 48, normalized size = 1.55

$$-\frac{(-x+1)^2}{2}-2\left(\begin{cases}-\frac{x\sqrt{-x+1}\sqrt{x+1}}{4}+\frac{\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{-x+1}}{2}\right)}{2} & \text{for } x\leq 1\wedge x>-1\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1-x)**(1/2)*((1-x)**(1/2)+(1+x)**(1/2)),x)
```

```
[Out] -(-x + 1)**2/2 - 2*Piecewise((-x*sqrt(-x + 1)*sqrt(x + 1)/4 + asi
n(sqrt(2)*sqrt(-x + 1)/2)/2, (x <= 1) & (x > -1)))
```

GIAC/XCAS [A] time = 0.302178, size = 51, normalized size = 1.65

$$-\frac{1}{2}(x-1)^2+\frac{1}{2}\sqrt{x+1}x\sqrt{-x+1}-\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{-x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x + 1)*(sqrt(x + 1) + sqrt(-x + 1)),x, algorithm="giac")
```

```
[Out] -1/2*(x - 1)^2 + 1/2*sqrt(x + 1)*x*sqrt(-x + 1) - arcsin(1/2*sqrt
(2)*sqrt(-x + 1))
```

3.283 $\int x^3 \left(-\sqrt{1-x}-\sqrt{1+x}\right) \left(\sqrt{1-x}+\sqrt{1+x}\right) \, dx$

Optimal. Leaf size=38

$$-\frac{x^4}{2}-\frac{2}{5}\left(1-x^2\right)^{5/2}+\frac{2}{3}\left(1-x^2\right)^{3/2}$$

```
[Out] -x^4/2 + (2*(1 - x^2)^(3/2))/3 - (2*(1 - x^2)^(5/2))/5
```

Rubi [A] time = 0.6136, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095

$$-\frac{x^4}{2}-\frac{2}{5}\left(1-x^2\right)^{5/2}+\frac{2}{3}\left(1-x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]
```

```
[Out] -x^4/2 + (2*(1 - x^2)^(3/2))/3 - (2*(1 - x^2)^(5/2))/5
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(-(1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2)),x
```

```
[Out] Timed out
```

Mathematica [A] time = 0.0443343, size = 44, normalized size = 1.16

$$-\frac{1}{30}\left(x^2-1\right)\left(3\left(4\sqrt{1-x^2}+5\right)x^2+8\sqrt{1-x^2}+15\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]
```

```
[Out] -((-1 + x^2)*(15 + 8*Sqrt[1 - x^2] + 3*x^2*(5 + 4*Sqrt[1 - x^2]))
)/30
```

Maple [A] time = 0.003, size = 33, normalized size = 0.9

$$-\frac{x^4}{2}-\frac{(2x^2-2)(3x^2+2)}{15}\sqrt{1-x}\sqrt{1+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x)
```

```
[Out] -1/2*x^4-2/15*(1+x)^(1/2)*(1-x)^(1/2)*(x^2-1)*(3*x^2+2)
```

Maxima [A] time = 0.766081, size = 42, normalized size = 1.11

$$-\frac{1}{2}x^4+\frac{2}{5}\left(-x^2+1\right)^{\frac{3}{2}}x^2+\frac{4}{15}\left(-x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^3*(sqrt(x + 1) + sqrt(-x + 1))^2,x, algorithm="maxima")
```

```
[Out] -1/2*x^4 + 2/5*(-x^2 + 1)^(3/2)*x^2 + 4/15*(-x^2 + 1)^(3/2)
```

Fricas [A] time = 0.271582, size = 109, normalized size = 2.87

$$-\frac{12x^{10}-85x^8+80x^6+5\left(9x^8-16x^6\right)\sqrt{x+1}\sqrt{-x+1}}{30\left(5x^4-20x^2-(x^4-12x^2+16)\sqrt{x+1}\sqrt{-x+1}+16\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^3*(sqrt(x + 1) + sqrt(-x + 1))^2,x, algorithm="fricas")
```

```
[Out] -1/30*(12*x^10 - 85*x^8 + 80*x^6 + 5*(9*x^8 - 16*x^6)*sqrt(x + 1)
*sqrt(-x + 1))/(5*x^4 - 20*x^2 - (x^4 - 12*x^2 + 16)*sqrt(x + 1)*
sqrt(-x + 1) + 16)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-(1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.287635, size = 76, normalized size = 2.

$$-\frac{1}{2}(x+1)^4+2(x+1)^3-\frac{2}{15}((3(x+1)(x-3)+17)(x+1)-10)(x+1)^{\frac{3}{2}}\sqrt{-x+1}-3(x+1)^2+2x+2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^3*(sqrt(x + 1) + sqrt(-x + 1))^2,x, algorithm="giac")
```

```
[Out] -1/2*(x + 1)^4 + 2*(x + 1)^3 - 2/15*((3*(x + 1)*(x - 3) + 17)*(x
+ 1) - 10)*(x + 1)^(3/2)*sqrt(-x + 1) - 3*(x + 1)^2 + 2*x + 2
```

3.284 $\int x^2\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx$

Optimal. Leaf size=48

$$-\frac{2x^3}{3}+\frac{1}{4}\sqrt{1-x^2}x-\frac{1}{2}\sqrt{1-x^2}x^3-\frac{1}{4}\sin^{-1}(x)$$

```
[Out] (-2*x^3)/3 + (x*Sqrt[1 - x^2])/4 - (x^3*Sqrt[1 - x^2])/2 - ArcSin
[x]/4
```

Rubi [A] time = 0.566827, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119

$$-\frac{2x^3}{3}+\frac{1}{4}\sqrt{1-x^2}x-\frac{1}{2}\sqrt{1-x^2}x^3-\frac{1}{4}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]
```

```
[Out] (-2*x^3)/3 + (x*Sqrt[1 - x^2])/4 - (x^3*Sqrt[1 - x^2])/2 - ArcSin
[x]/4
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(-(1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2)),x
```

```
[Out] Timed out
```

Mathematica [A] time = 0.0590548, size = 56, normalized size = 1.17

$$\frac{1}{12}\left(3\sqrt{1-x^2}x-\left(6\sqrt{1-x^2}+8\right)x^3-6\sin^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{2}}\right)-8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]
```

```
[Out] (-8 + 3*x*Sqrt[1 - x^2] - x^3*(8 + 6*Sqrt[1 - x^2]) - 6*ArcSin[Sq
rt[1 + x]/Sqrt[2]])/12
```

Maple [A] time = 0.002, size = 59, normalized size = 1.2

$$-\frac{2x^3}{3}-\frac{1}{4}\sqrt{1-x}\sqrt{1+x}\left(2x^3\sqrt{-x^2+1}-x\sqrt{-x^2+1}+\arcsin(x)\right)\frac{1}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x)
```

```
[Out] -2/3*x^3-1/4*(1-x)^(1/2)*(1+x)^(1/2)*(2*x^3*(-x^2+1)^(1/2)-x*(-x^
2+1)^(1/2)+arcsin(x))/(-x^2+1)^(1/2)
```

Maxima [A] time = 0.763824, size = 46, normalized size = 0.96

$$-\frac{2}{3}x^3+\frac{1}{2}\left(-x^2+1\right)^{\frac{3}{2}}x-\frac{1}{4}\sqrt{-x^2+1}x-\frac{1}{4}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2*(sqrt(x + 1) + sqrt(-x + 1))^2,x, algorithm="maxima")
```

```
[Out] -2/3*x^3 + 1/2*(-x^2 + 1)^(3/2)*x - 1/4*sqrt(-x^2 + 1)*x - 1/4*ar
csin(x)
```

Fricas [A] time = 0.273695, size = 184, normalized size = 3.83

$$\frac{16x^7-20x^5+20x^3-\left(6x^7-19x^5+8x^3-24x\right)\sqrt{x+1}\sqrt{-x+1}+6\left(x^4-8x^2+4\left(x^2-2\right)\sqrt{x+1}\sqrt{-x+1}+8\right)\arctan\left(\right.}{12\left(x^4-8x^2+4(x^2-2)\sqrt{x+1}\sqrt{-x+1}+8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2*(sqrt(x + 1) + sqrt(-x + 1))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(16*x^7 - 20*x^5 + 20*x^3 - (6*x^7 - 19*x^5 + 8*x^3 - 24*x)*
sqrt(x + 1)*sqrt(-x + 1) + 6*(x^4 - 8*x^2 + 4*(x^2 - 2)*sqrt(x +
1)*sqrt(-x + 1) + 8)*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) - 2
4*x)/(x^4 - 8*x^2 + 4*(x^2 - 2)*sqrt(x + 1)*sqrt(-x + 1) + 8)
```

Sympy [A] time = 147.376, size = 219, normalized size = 4.56

$$\frac{x^4}{4} - \frac{x^3}{3} - \frac{(x+1)^4}{4} + \frac{2(x+1)^3}{3} - \frac{(x+1)^2}{2} - 4\left(\begin{cases} \frac{x\sqrt{-x+1}\sqrt{x+1}}{4} + \frac{\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{2} & \text{for } x \geq -1 \wedge x < 1 \end{cases}\right) + 8\left(\begin{cases} \frac{x\sqrt{-x+1}\sqrt{x+1}}{4} - \frac{(-x+1)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}{6} + \frac{\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{2} & \text{for } x \geq -1 \wedge x < 1 \end{cases}\right) - 4\left(\begin{cases} \frac{x\sqrt{-x+1}\sqrt{x+1}}{4} - \frac{(-x+1)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}{3} - \frac{\sqrt{-x+1}\sqrt{x+1}\left(-5x-2(x+1)^3+6(x+1)^2-4\right)}{16} + \frac{5\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{8} & \text{for } x \geq -1 \wedge x < 1 \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-(1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2)),x)
```

```
[Out] x**4/4 - x**3/3 - (x + 1)**4/4 + 2*(x + 1)**3/3 - (x + 1)**2/2 -
4*Piecewise((x*sqrt(-x + 1)*sqrt(x + 1)/4 + asin(sqrt(2)*sqrt(x +
 1)/2)/2, (x >= -1) & (x < 1))) + 8*Piecewise((x*sqrt(-x + 1)*sqr
t(x + 1)/4 - (-x + 1)**(3/2)*(x + 1)**(3/2)/6 + asin(sqrt(2)*sqrt
(x + 1)/2)/2, (x >= -1) & (x < 1))) - 4*Piecewise((x*sqrt(-x + 1)
*sqrt(x + 1)/4 - (-x + 1)**(3/2)*(x + 1)**(3/2)/3 - sqrt(-x + 1)*
sqrt(x + 1)*(-5*x - 2*(x + 1)**3 + 6*(x + 1)**2 - 4)/16 + 5*asin(
sqrt(2)*sqrt(x + 1)/2)/8, (x >= -1) & (x < 1)))
```

GIAC/XCAS [A] time = 0.291801, size = 84, normalized size = 1.75

$$-\frac{2}{3}(x+1)^3 + 2(x+1)^2 - \frac{1}{4}((2(x+1)(x-2)+5)(x+1)-1)\sqrt{x+1}\sqrt{-x+1} - 2x - \frac{1}{2}\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right) - 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2*(sqrt(x + 1) + sqrt(-x + 1))^2,x, algorithm="giac")
```

```
[Out] -2/3*(x + 1)^3 + 2*(x + 1)^2 - 1/4*((2*(x + 1)*(x - 2) + 5)*(x +
1) - 1)*sqrt(x + 1)*sqrt(-x + 1) - 2*x - 1/2*arcsin(1/2*sqrt(2)*s
qrt(x + 1)) - 2
```

3.285 $\int x\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx$

Optimal. Leaf size=21

$$\frac{2}{3}\left(1-x^2\right)^{3/2}-x^2$$

```
[Out] -x^2 + (2*(1 - x^2)^(3/2))/3
```

Rubi [A] time = 0.227074, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075

$$\frac{2}{3}\left(1-x^2\right)^{3/2}-x^2$$

Antiderivative was successfully verified.

```
[In]  Int[x*(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]
```

```
[Out] -x^2 + (2*(1 - x^2)^(3/2))/3
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-2\int^{\sqrt{x+1}} x\left(x+\sqrt{-x^2+2}\right)^2\left(x^2-1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(-(1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2)),x)
```

```
[Out] -2*Integral(x*(x + sqrt(-x**2 + 2))**2*(x**2 - 1), (x, sqrt(x + 1
)))
```

Mathematica [A] time = 0.0234544, size = 24, normalized size = 1.14

$$-\frac{1}{3}\left(x^2-1\right)\left(2\sqrt{1-x^2}+3\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]
```

```
[Out] -((-1 + x^2)*(3 + 2*Sqrt[1 - x^2]))/3
```

Maple [A]   time = 0.002, size = 26, normalized size = 1.2

$$-x^{2}-\frac{2 x^{2}-2}{3}\sqrt{1-x}\sqrt{1+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x)
```

```
[Out] -x^2-2/3*(1-x)^(1/2)*(1+x)^(1/2)*(x^2-1)
```

Maxima [A]   time = 0.760666, size = 23, normalized size = 1.1

$$-x^{2}+\frac{2}{3}\left(-x^{2}+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x*(sqrt(x + 1) + sqrt(-x + 1))^2,x, algorithm="maxima")
```

```
[Out] -x^2 + 2/3*(-x^2 + 1)^(3/2)
```

Fricas [A]   time = 0.268052, size = 78, normalized size = 3.71

$$-\frac{2 x^{6}+3 \sqrt{x+1}x^{4}\sqrt{-x+1}-3 x^{4}}{3\left(3 x^{2}-(x^{2}-4)\sqrt{x+1}\sqrt{-x+1}-4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x*(sqrt(x + 1) + sqrt(-x + 1))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(2*x^6 + 3*sqrt(x + 1)*x^4*sqrt(-x + 1) - 3*x^4)/(3*x^2 - (x
^2 - 4)*sqrt(x + 1)*sqrt(-x + 1) - 4)
```

Sympy [A] time = 72.5871, size = 110, normalized size = 5.24

$$\frac{x^3}{3}+x-\frac{(x+1)^3}{3}+4\left(\begin{cases}\frac{x\sqrt{-x+1}\sqrt{x+1}}{4}+\frac{\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{2} & \text{for } x\geq -1\wedge x<1\end{cases}\right)-4\left(\begin{cases}\frac{x\sqrt{-x+1}\sqrt{x+1}}{4}-\frac{(-x+1)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}{6}+\frac{\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{2} & \text{for } x\geq -1\wedge x<1\end{cases}\right)+1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(-(1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2)),x)
```

```
[Out] x**3/3 + x - (x + 1)**3/3 + 4*Piecewise((x*sqrt(-x + 1)*sqrt(x +
1)/4 + asin(sqrt(2)*sqrt(x + 1)/2)/2, (x >= -1) & (x < 1))) - 4*P
iecewise((x*sqrt(-x + 1)*sqrt(x + 1)/4 - (-x + 1)**(3/2)*(x + 1)*
*(3/2)/6 + asin(sqrt(2)*sqrt(x + 1)/2)/2, (x >= -1) & (x < 1))) +
 1
```

GIAC/XCAS [A] time = 0.299002, size = 39, normalized size = 1.86

$$-\frac{2}{3}(x+1)^{\frac{3}{2}}(x-1)\sqrt{-x+1}-(x+1)^2+2x+2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x*(sqrt(x + 1) + sqrt(-x + 1))^2,x, algorithm="giac")
```

```
[Out] -2/3*(x + 1)^(3/2)*(x - 1)*sqrt(-x + 1) - (x + 1)^2 + 2*x + 2
```

3.286 $\int \left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx$

Optimal. Leaf size=22

$$-\sqrt{1-x^2}x-2x-\sin^{-1}(x)$$

```
[Out] -2*x - x*Sqrt[1 - x^2] - ArcSin[x]
```

Rubi [A] time = 0.0925122, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$

$$-\sqrt{1-x^2}x-2x-\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]

[Out] -2*x - x*Sqrt[1 - x^2] - ArcSin[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-2\int^{\sqrt{x+1}} x\left(x+\sqrt{-x^2+2}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-(1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2)),x)

[Out] -2*Integral(x*(x + sqrt(-x**2 + 2))**2, (x, sqrt(x + 1)))
```

Mathematica [A] time = 0.0226164, size = 34, normalized size = 1.55

$$-x\left(\sqrt{1-x^2}+2\right)-2\sin^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{2}}\right)-2$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]
```

```
[Out] -2 - x*(2 + Sqrt[1 - x^2]) - 2*ArcSin[Sqrt[1 + x]/Sqrt[2]]
```

Maple [B] time = 0.002, size = 59, normalized size = 2.7

$$-2x-\sqrt{1-x}\,(1+x)^{\frac{3}{2}}+\sqrt{1-x}\sqrt{1+x}-\arcsin(x)\sqrt{(1+x)(1-x)}\frac{1}{\sqrt{1-x}}\frac{1}{\sqrt{1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x)
```

```
[Out] -2*x-(1-x)^(1/2)*(1+x)^(3/2)+(1-x)^(1/2)*(1+x)^(1/2)-((1+x)*(1-x)
)^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [A] time = 0.768891, size = 27, normalized size = 1.23

$$-\sqrt{-x^2+1}x-2x-\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(sqrt(x + 1) + sqrt(-x + 1))^2,x, algorithm="maxima")
```

```
[Out] -sqrt(-x^2 + 1)*x - 2*x - arcsin(x)
```

Fricas [A] time = 0.276811, size = 119, normalized size = 5.41

$$-\frac{\left(x^3+2x\right)\sqrt{x+1}\sqrt{-x+1}-2\left(x^2+2\sqrt{x+1}\sqrt{-x+1}-2\right)\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)-2x}{x^2+2\sqrt{x+1}\sqrt{-x+1}-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(sqrt(x + 1) + sqrt(-x + 1))^2,x, algorithm="fricas")
```

```
[Out] -((x^3 + 2*x)*sqrt(x + 1)*sqrt(-x + 1) - 2*(x^2 + 2*sqrt(x + 1)*s
qrt(-x + 1) - 2)*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) - 2*x)/
(x^2 + 2*sqrt(x + 1)*sqrt(-x + 1) - 2)
```

Sympy [A] time = 39.2566, size = 46, normalized size = 2.09

$$-2x-4\left(\begin{cases}\frac{x\sqrt{-x+1}\sqrt{x+1}}{4}+\frac{\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{2} & \text{for}\ x\geq -1\wedge x<1\end{cases}\right)-2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-(1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2)),x)
```

```
[Out] -2*x - 4*Piecewise((x*sqrt(-x + 1)*sqrt(x + 1)/4 + asin(sqrt(2)*s
qrt(x + 1)/2)/2, (x >= -1) & (x < 1))) - 2
```

GIAC/XCAS [A] time = 0.292131, size = 45, normalized size = 2.05

$$-\sqrt{x+1}x\sqrt{-x+1}-2\,x-2\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\sqrt{x+1}\right)-2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(sqrt(x + 1) + sqrt(-x + 1))^2,x, algorithm="giac")
```

```
[Out] -sqrt(x + 1)*x*sqrt(-x + 1) - 2*x - 2*arcsin(1/2*sqrt(2)*sqrt(x +
 1)) - 2
```

3.287 $\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x}\,dx$

Optimal. Leaf size=32

$$-2\sqrt{1-x^2}+2\tanh^{-1}\left(\sqrt{1-x^2}\right)-2\log(x)$$

```
[Out] -2*Sqrt[1 - x^2] + 2*ArcTanh[Sqrt[1 - x^2]] - 2*Log[x]
```

Rubi [A] time = 0.381187, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$-2\sqrt{1-x^2}+2\tanh^{-1}\left(\sqrt{1-x^2}\right)-2\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[((-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]))/x,x]
```

```
[Out] -2*Sqrt[1 - x^2] + 2*ArcTanh[Sqrt[1 - x^2]] - 2*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-2\int^{\sqrt{x+1}}\frac{x\sqrt{-x^2+2}+1}{x-1}\,dx-2\int^{\sqrt{x+1}}\frac{x\sqrt{-x^2+2}+1}{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-(1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2))/x,x)
```

```
[Out] -2*Integral((x*sqrt(-x**2 + 2) + 1)/(x - 1), (x, sqrt(x + 1))) -
2*Integral((x*sqrt(-x**2 + 2) + 1)/(x + 1), (x, sqrt(x + 1)))
```

Mathematica [B] time = 0.04144, size = 84, normalized size = 2.62

$$-2\left(\sqrt{1-x^2}+\log(-x)+\log\left(1-\sqrt{x+1}\right)-\log\left(\sqrt{1-x}-\sqrt{x+1}+2\right)\right.$$
$$\left.-\log\left(\sqrt{x+1}+1\right)+\log\left(\sqrt{1-x}+\sqrt{x+1}+2\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[((-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]))/x,x]
```

```
[Out] -2*(Sqrt[1 - x^2] + Log[-x] + Log[1 - Sqrt[1 + x]] - Log[2 + Sqrt
[1 - x] - Sqrt[1 + x]] - Log[1 + Sqrt[1 + x]] + Log[2 + Sqrt[1 -
x] + Sqrt[1 + x]])
```

Maple [A] time = 0.003, size = 51, normalized size = 1.6

$$-2\ln(x)-2\,\frac{\sqrt{1-x}\sqrt{1+x}\left(\sqrt{-x^2+1}-Artanh\left(\frac{1}{\sqrt{-x^2+1}}\right)\right)}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x,x)
```

```
[Out] -2*ln(x)-2*(1-x)^(1/2)*(1+x)^(1/2)/(-x^2+1)^(1/2)*((-x^2+1)^(1/2)
-arctanh(1/(-x^2+1)^(1/2)))
```

Maxima [A] time = 0.768683, size = 55, normalized size = 1.72

$$-2\sqrt{-x^2+1}-2\log(x)+2\log\left(\frac{2\sqrt{-x^2+1}}{|x|}+\frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(sqrt(x + 1) + sqrt(-x + 1))^2/x,x, algorithm="maxima")
```

```
[Out] -2*sqrt(-x^2 + 1) - 2*log(x) + 2*log(2*sqrt(-x^2 + 1)/abs(x) + 2/
abs(x))
```

Fricas [A] time = 0.265748, size = 105, normalized size = 3.28

$$\frac{2\left(x^2-\sqrt{x+1}\sqrt{-x+1}\log(x)-\left(\sqrt{x+1}\sqrt{-x+1}-1\right)\log\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)+\log(x)\right)}{\sqrt{x+1}\sqrt{-x+1}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(sqrt(x + 1) + sqrt(-x + 1))^2/x,x, algorithm="fricas")
```

```
[Out] 2*(x^2 - sqrt(x + 1)*sqrt(-x + 1)*log(x) - (sqrt(x + 1)*sqrt(-x +
 1) - 1)*log((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + log(x))/(sqrt(x
+ 1)*sqrt(-x + 1) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{2}{x}\,dx - \int \frac{2\sqrt{-x+1}\sqrt{x+1}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-(1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2))/x,x)
```

```
[Out] -Integral(2/x, x) - Integral(2*sqrt(-x + 1)*sqrt(x + 1)/x, x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(sqrt(x + 1) + sqrt(-x + 1))^2/x,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.288 $\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^2}\,dx$

Optimal. Leaf size=26

$$\frac{2\sqrt{1-x^2}}{x}+\frac{2}{x}+2\sin^{-1}(x)$$

```
[Out] 2/x + (2*Sqrt[1 - x^2])/x + 2*ArcSin[x]
```

Rubi [A] time = 0.411712, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095

$$\frac{2\sqrt{1-x^2}}{x}+\frac{2}{x}+2\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[((-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]))/x^2,x]

[Out] 2/x + (2*Sqrt[1 - x^2])/x + 2*ArcSin[x]
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-(1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2))/x**2,x

[Out] Timed out
```

Mathematica [A] time = 0.0407194, size = 35, normalized size = 1.35

$$\frac{2\left(\sqrt{1-x^2}+2x\sin^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{2}}\right)+1\right)}{x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]))/x^2,x]
```

```
[Out] (2*(1 + Sqrt[1 - x^2] + 2*x*ArcSin[Sqrt[1 + x]/Sqrt[2]]))/x
```

Maple [B] time = 0.002, size = 50, normalized size = 1.9

$$2x^{-1} - 2\,\frac{\left(-\arcsin(x)\,x - \sqrt{-x^2+1}\right)\sqrt{1-x}\sqrt{1+x}}{x\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x^2,x)
```

```
[Out] 2/x-2*(-arcsin(x)*x-(-x^2+1)^(1/2))*(1-x)^(1/2)*(1+x)^(1/2)/x/(-x
^2+1)^(1/2)
```

Maxima [A] time = 0.765417, size = 32, normalized size = 1.23

$$\frac{2\sqrt{-x^2+1}}{x} + \frac{2}{x} + 2\,\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(sqrt(x + 1) + sqrt(-x + 1))^2/x^2,x, algorithm="maxima")
```

```
[Out] 2*sqrt(-x^2 + 1)/x + 2/x + 2*arcsin(x)
```

Fricas [A] time = 0.276682, size = 78, normalized size = 3.

$$-\frac{2\left(2\left(\sqrt{x+1}\sqrt{-x+1}-1\right)\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)+x\right)}{\sqrt{x+1}\sqrt{-x+1}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(sqrt(x + 1) + sqrt(-x + 1))^2/x^2,x, algorithm="fricas")
```

```
[Out] -2*(2*(sqrt(x + 1)*sqrt(-x + 1) - 1)*arctan((sqrt(x + 1)*sqrt(-x
+ 1) - 1)/x) + x)/(sqrt(x + 1)*sqrt(-x + 1) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{2}{x^2}\,dx-\int \frac{2\sqrt{-x+1}\,\sqrt{x+1}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-(1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2))/x**2,x)
```

```
[Out] -Integral(2/x**2, x) - Integral(2*sqrt(-x + 1)*sqrt(x + 1)/x**2,
x)
```

GIAC/XCAS [A] time = 0.316073, size = 201, normalized size = 7.73

$$2\pi+\frac{8\left(\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}}-\frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}\right)}{\left(\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}}-\frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}\right)^2-4}+\frac{2}{x}+4\arctan\left(\frac{\sqrt{x+1}\left(\frac{\left(\sqrt{2}-\sqrt{-x+1}\right)^2}{x+1}-1\right)}{2\left(\sqrt{2}-\sqrt{-x+1}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(sqrt(x + 1) + sqrt(-x + 1))^2/x^2,x, algorithm="giac")
```

```
[Out] 2*pi + 8*((sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - sqrt(x + 1)/(sqr
t(2) - sqrt(-x + 1)))/(((sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - sq
rt(x + 1)/(sqrt(2) - sqrt(-x + 1)))^2 - 4) + 2/x + 4*arctan(1/2*s
qrt(x + 1)*((sqrt(2) - sqrt(-x + 1))^2/(x + 1) - 1)/(sqrt(2) - sq
rt(-x + 1)))
```

3.289 $\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^3}\,dx$

Optimal. Leaf size=33

$$\frac{\sqrt{1-x^2}}{x^2}+\frac{1}{x^2}-\tanh^{-1}\left(\sqrt{1-x^2}\right)$$

```
[Out] x^(-2) + Sqrt[1 - x^2]/x^2 - ArcTanh[Sqrt[1 - x^2]]
```

Rubi [A] time = 0.451368, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{\sqrt{1-x^2}}{x^2}+\frac{1}{x^2}-\tanh^{-1}\left(\sqrt{1-x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]))/x^3,x]

[Out] x^(-2) + Sqrt[1 - x^2]/x^2 - ArcTanh[Sqrt[1 - x^2]]
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-(1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2))/x**3,x

[Out] Timed out
```

Mathematica [B] time = 0.0566216, size = 85, normalized size = 2.58

$$\frac{\sqrt{1-x^2}}{x^2}+\frac{1}{x^2}+\log\left(1-\sqrt{x+1}\right)-\log\left(\sqrt{1-x}-\sqrt{x+1}+2\right)$$
$$-\log\left(\sqrt{x+1}+1\right)+\log\left(\sqrt{1-x}+\sqrt{x+1}+2\right)$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[((-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]))/x^3,x]
```

```
[Out] x^(-2) + Sqrt[1 - x^2]/x^2 + Log[1 - Sqrt[1 + x]] - Log[2 + Sqrt[
1 - x] - Sqrt[1 + x]] - Log[1 + Sqrt[1 + x]] + Log[2 + Sqrt[1 - x
] + Sqrt[1 + x]]
```

Maple [A] time = 0.003, size = 57, normalized size = 1.7

$$x^{-2}-\frac{1}{x^{2}}\sqrt{1-x}\sqrt{1+x}\left(Artanh\left(\frac{1}{\sqrt{-x^{2}+1}}\right)x^{2}-\sqrt{-x^{2}+1}\right)\frac{1}{\sqrt{-x^{2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x^3,x)
```

```
[Out] 1/x^2-(1-x)^(1/2)*(1+x)^(1/2)*(arctanh(1/(-x^2+1)^(1/2))*x^2-(-x^
2+1)^(1/2))/x^2/(-x^2+1)^(1/2)
```

Maxima [A] time = 0.764547, size = 69, normalized size = 2.09

$$\sqrt{-x^{2}+1}+\frac{\left(-x^{2}+1\right)^{\frac{3}{2}}}{x^{2}}+\frac{1}{x^{2}}-\log\left(\frac{2\sqrt{-x^{2}+1}}{|x|}+\frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(sqrt(x + 1) + sqrt(-x + 1))^2/x^3,x, algorithm="maxima")
```

```
[Out] sqrt(-x^2 + 1) + (-x^2 + 1)^(3/2)/x^2 + 1/x^2 - log(2*sqrt(-x^2 +
 1)/abs(x) + 2/abs(x))
```

Fricas [A] time = 0.271468, size = 104, normalized size = 3.15

$$\frac{\left(x^{2}+2\sqrt{x+1}\sqrt{-x+1}-2\right)\log\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)+\sqrt{x+1}\sqrt{-x+1}-1}{x^{2}+2\sqrt{x+1}\sqrt{-x+1}-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(sqrt(x + 1) + sqrt(-x + 1))^2/x^3,x, algorithm="fricas")
```

```
[Out] ((x^2 + 2*sqrt(x + 1)*sqrt(-x + 1) - 2)*log((sqrt(x + 1)*sqrt(-x
+ 1) - 1)/x) + sqrt(x + 1)*sqrt(-x + 1) - 1)/(x^2 + 2*sqrt(x + 1)
*sqrt(-x + 1) - 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{2}{x^3}\,dx - \int \frac{2\sqrt{-x+1}\sqrt{x+1}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2))/x**3,x)
```

```
[Out] -Integral(2/x**3, x) - Integral(2*sqrt(-x + 1)*sqrt(x + 1)/x**3,
x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(sqrt(x + 1) + sqrt(-x + 1))^2/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.290 $\int \frac{\sqrt{1-x}+\sqrt{1+x}}{-\sqrt{1-x}+\sqrt{1+x}}\,dx$

Optimal. Leaf size=28

$$\sqrt{1-x^2}-\tanh^{-1}\left(\sqrt{1-x^2}\right)+\log(x)$$

```
[Out] Sqrt[1 - x^2] - ArcTanh[Sqrt[1 - x^2]] + Log[x]
```

Rubi [A] time = 0.586311, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 7, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.18

$$\sqrt{1-x^2}-\tanh^{-1}\left(\sqrt{1-x^2}\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[1 - x] + Sqrt[1 + x])/(-Sqrt[1 - x] + Sqrt[1 + x]),x]

[Out] Sqrt[1 - x^2] - ArcTanh[Sqrt[1 - x^2]] + Log[x]
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((1-x)**(1/2)+(1+x)**(1/2))/(-(1-x)**(1/2)+(1+x)**(1/2)),x)

[Out] Timed out
```

Mathematica [B] time = 0.0432623, size = 82, normalized size = 2.93

$$\sqrt{1-x^2}+\log(-x)+\log\left(1-\sqrt{x+1}\right)-\log\left(\sqrt{1-x}-\sqrt{x+1}+2\right)-\log\left(\sqrt{x+1}+1\right)+\log\left(\sqrt{1-x}+\sqrt{x+1}+2\right)$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(Sqrt[1 - x] + Sqrt[1 + x])/(-Sqrt[1 - x] + Sqrt[1 + x]),x]
```

```
[Out] Sqrt[1 - x^2] + Log[-x] + Log[1 - Sqrt[1 + x]] - Log[2 + Sqrt[1 -
 x] - Sqrt[1 + x]] - Log[1 + Sqrt[1 + x]] + Log[2 + Sqrt[1 - x] +
 Sqrt[1 + x]]
```

Maple [A] time = 0.004, size = 48, normalized size = 1.7

$$\ln(x) + 1\sqrt{1-x}\sqrt{1+x}\left(\sqrt{-x^2+1} - Artanh\left(\frac{1}{\sqrt{-x^2+1}}\right)\right)\frac{1}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((1-x)^(1/2)+(1+x)^(1/2))/(-(1-x)^(1/2)+(1+x)^(1/2)),x)
```

```
[Out] ln(x)+(1-x)^(1/2)*(1+x)^(1/2)/(-x^2+1)^(1/2)*((-x^2+1)^(1/2)-arct
anh(1/(-x^2+1)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x+1} + \sqrt{-x+1}}{\sqrt{x+1} - \sqrt{-x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(x + 1) + sqrt(-x + 1))/(sqrt(x + 1) - sqrt(-
x + 1)),x, algorithm="maxima")
```

```
[Out] integrate((sqrt(x + 1) + sqrt(-x + 1))/(sqrt(x + 1) - sqrt(-x + 1
)), x)
```

Fricas [A] time = 0.266512, size = 105, normalized size = 3.75

$$-\frac{x^2 - \sqrt{x+1}\sqrt{-x+1}\log(x) - \left(\sqrt{x+1}\sqrt{-x+1} - 1\right)\log\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right) + \log(x)}{\sqrt{x+1}\sqrt{-x+1} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(x + 1) + sqrt(-x + 1))/(sqrt(x + 1) - sqrt(-
x + 1)),x, algorithm="fricas")
```

```
[Out] -(x^2 - sqrt(x + 1)*sqrt(-x + 1)*log(x) - (sqrt(x + 1)*sqrt(-x +
1) - 1)*log((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + log(x))/(sqrt(x +
 1)*sqrt(-x + 1) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{-x+1}}{\sqrt{-x+1}-\sqrt{x+1}}\,dx-\int \frac{\sqrt{x+1}}{\sqrt{-x+1}-\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)**(1/2)+(1+x)**(1/2))/(-(1-x)**(1/2)+(1+x)**(1/2)),x)
```

```
[Out] -Integral(sqrt(-x + 1)/(sqrt(-x + 1) - sqrt(x + 1)), x) - Integra
l(sqrt(x + 1)/(sqrt(-x + 1) - sqrt(x + 1)), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(x + 1) + sqrt(-x + 1))/(sqrt(x + 1) - sqrt(-
x + 1)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.291 $\int \frac{-\sqrt{-1+x}+\sqrt{1+x}}{\sqrt{-1+x}+\sqrt{1+x}}\,dx$

Optimal. Leaf size=33

$$\frac{x^2}{2}-\frac{1}{2}\sqrt{x-1}\sqrt{x+1}x+\frac{1}{2}\cosh^{-1}(x)$$

```
[Out] x^2/2 - (Sqrt[-1 + x]*x*Sqrt[1 + x])/2 + ArcCosh[x]/2
```

Rubi [A] time = 0.25071, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114

$$\frac{x^2}{2}-\frac{1}{2}\sqrt{x-1}\sqrt{x+1}x+\frac{1}{2}\cosh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(-Sqrt[-1 + x] + Sqrt[1 + x])/(Sqrt[-1 + x] + Sqrt[1 + x]),x]
```

```
[Out] x^2/2 - (Sqrt[-1 + x]*x*Sqrt[1 + x])/2 + ArcCosh[x]/2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\log\left(\left(-\sqrt{x-1}+\sqrt{x+1}\right)^2\right)}{2}+\frac{\int^{\left(-\sqrt{x-1}+\sqrt{x+1}\right)^2} x\,dx}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-(-1+x)**(1/2)+(1+x)**(1/2))/((-1+x)**(1/2)+(1+x)**(1/2)),x)
```

```
[Out] -log((-sqrt(x - 1) + sqrt(x + 1))**2)/2 + Integral(x, (x, (-sqrt(
x - 1) + sqrt(x + 1))**2))/8
```

Mathematica [A] time = 0.0285646, size = 42, normalized size = 1.27

$$\frac{1}{2}\left(x^2-\sqrt{x-1}\sqrt{x+1}x+2\sinh^{-1}\left(\frac{\sqrt{x-1}}{\sqrt{2}}\right)+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Sqrt[-1 + x] + Sqrt[1 + x])/(Sqrt[-1 + x] + Sqrt[1 + x]),x]
```

```
[Out] (1 + x^2 - Sqrt[-1 + x]*x*Sqrt[1 + x] + 2*ArcSinh[Sqrt[-1 + x]/Sq
rt[2]])/2
```

Maple [B] time = 0.008, size = 62, normalized size = 1.9

$$-\frac{1}{2}\sqrt{-1+x}\,(1+x)^{\frac{3}{2}}+\frac{1}{2}\sqrt{-1+x}\sqrt{1+x}+\frac{1}{2}\sqrt{(-1+x)\,(1+x)}\ln\left(x+\sqrt{x^2-1}\right)\frac{1}{\sqrt{-1+x}}\frac{1}{\sqrt{1+x}}+\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(-1+x)^(1/2)+(1+x)^(1/2))/((-1+x)^(1/2)+(1+x)^(1/2)),x)
```

```
[Out] -1/2*(-1+x)^(1/2)*(1+x)^(3/2)+1/2*(-1+x)^(1/2)*(1+x)^(1/2)+1/2*((
-1+x)*(1+x))^(1/2)/(1+x)^(1/2)/(-1+x)^(1/2)*ln(x+(x^2-1)^(1/2))+1
/2*x^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x+1}-\sqrt{x-1}}{\sqrt{x+1}+\sqrt{x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sqrt(x + 1) - sqrt(x - 1))/(sqrt(x + 1) + sqrt(x - 1)),x, algorithm="ma
```

```
[Out] integrate((sqrt(x + 1) - sqrt(x - 1))/(sqrt(x + 1) + sqrt(x - 1))
, x)
```

Fricas [A] time = 0.308273, size = 126, normalized size = 3.82

$$-\frac{4\,x^4-\left(4\,x^3-x\right)\sqrt{x+1}\sqrt{x-1}-3\,x^2+\left(2\,\sqrt{x+1}\sqrt{x-1}x-2\,x^2+1\right)\log\left(\sqrt{x+1}\sqrt{x-1}-x\right)}{2\left(2\,\sqrt{x+1}\sqrt{x-1}x-2\,x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sqrt(x + 1) - sqrt(x - 1))/(sqrt(x + 1) + sqrt(x - 1)),x, algorithm="fr
```

```
[Out] -1/2*(4*x^4 - (4*x^3 - x)*sqrt(x + 1)*sqrt(x - 1) - 3*x^2 + (2*sq
rt(x + 1)*sqrt(x - 1)*x - 2*x^2 + 1)*log(sqrt(x + 1)*sqrt(x - 1)
- x))/(2*sqrt(x + 1)*sqrt(x - 1)*x - 2*x^2 + 1)
```

Sympy [A] time = 48.7695, size = 226, normalized size = 6.85

$$-\frac{(x-1)^{\frac{5}{2}}}{4\sqrt{x+1}}-\frac{3(x-1)^{\frac{3}{2}}}{4\sqrt{x+1}}-\frac{\sqrt{x-1}}{2\sqrt{x+1}}+\frac{(x-1)^2}{4}+2\left(\begin{cases}\frac{(x+1)^2}{8}+\frac{\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4}-\frac{(x+1)^{\frac{5}{2}}}{8\sqrt{x-1}}+\frac{3(x+1)^{\frac{3}{2}}}{8\sqrt{x-1}}-\frac{\sqrt{x+1}}{4\sqrt{x-1}} & \text{for } \frac{|x+1|}{2}>1\\ \frac{(x+1)^2}{8}-\frac{i\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4}+\frac{i(x+1)^{\frac{5}{2}}}{8\sqrt{-x+1}}-\frac{3i(x+1)^{\frac{3}{2}}}{8\sqrt{-x+1}}+\frac{i\sqrt{x+1}}{4\sqrt{-x+1}} & \text{otherwise}\end{cases}\right)+\frac{\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{x-1}}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(-1+x)**(1/2)+(1+x)**(1/2))/((-1+x)**(1/2)+(1+x)**(1/2)),x)
```

```
[Out] -(x - 1)**(5/2)/(4*sqrt(x + 1)) - 3*(x - 1)**(3/2)/(4*sqrt(x + 1)
) - sqrt(x - 1)/(2*sqrt(x + 1)) + (x - 1)**2/4 + 2*Piecewise(((x
+ 1)**2/8 + acosh(sqrt(2)*sqrt(x + 1)/2)/4 - (x + 1)**(5/2)/(8*sq
rt(x - 1)) + 3*(x + 1)**(3/2)/(8*sqrt(x - 1)) - sqrt(x + 1)/(4*sq
rt(x - 1)), Abs(x + 1)/2 > 1), ((x + 1)**2/8 - I*asin(sqrt(2)*sqr
t(x + 1)/2)/4 + I*(x + 1)**(5/2)/(8*sqrt(-x + 1)) - 3*I*(x + 1)**
(3/2)/(8*sqrt(-x + 1)) + I*sqrt(x + 1)/(4*sqrt(-x + 1)), True)) +
 asinh(sqrt(2)*sqrt(x - 1)/2)/2
```

GIAC/XCAS [A] time = 0.311212, size = 57, normalized size = 1.73

$$\frac{1}{2}(x+1)^2-\frac{1}{2}\sqrt{x+1}\sqrt{x-1}x-x-\ln\left(\left|-\sqrt{x+1}+\sqrt{x-1}\right|\right)-1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sqrt(x + 1) - sqrt(x - 1))/(sqrt(x + 1) + sqrt(x - 1)),x, algorithm="gi
```

```
[Out] 1/2*(x + 1)^2 - 1/2*sqrt(x + 1)*sqrt(x - 1)*x - x - ln(abs(-sqrt(
x + 1) + sqrt(x - 1))) - 1
```

3.292 $\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^n\,dx$

Optimal. Leaf size=121

$$\frac{af^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}{}_2F_1\left(2,n+1;n+2;\frac{d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}}{d}\right)}{2d^2e(n+1)}+\frac{\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}}{2e(n+1)}$$

```
[Out] (d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(1 + n)/(2*e*(1 + n)) + (a*
f^2*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(1 + n)*Hypergeometric2
F1[2, 1 + n, 2 + n, (d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])/d])/(2*
d^2*e*(1 + n))
```

Rubi [A] time = 0.234842, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12

$$\frac{af^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}{}_2F_1\left(2,n+1;n+2;\frac{d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}}{d}\right)}{2d^2e(n+1)}+\frac{\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}}{2e(n+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^n,x]
```

```
[Out] (d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(1 + n)/(2*e*(1 + n)) + (a*
f^2*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(1 + n)*Hypergeometric2
F1[2, 1 + n, 2 + n, (d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])/d])/(2*
d^2*e*(1 + n))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**n,x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + e**2*x**2/f**2))**n, x)
```

Mathematica [A] time = 0.123827, size = 0, normalized size = 0.

$$\int \left(d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)^n dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^n,x]
```

```
[Out] Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^n, x]
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int \left(d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^n,x)
```

```
[Out] int((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex + \sqrt{\frac{e^2x^2}{f^2} + a}f + d\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^n,x, algorithm="maxima")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(ex + f\sqrt{\frac{e^2x^2 + af^2}{f^2}} + d\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^n,x, algorithm="fricas")
```

```
[Out] integral((e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**n,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex + \sqrt{\frac{e^2x^2}{f^2} + a}f + d\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^n,x, algorithm="giac")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^n, x)
```

3.293 $\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^3 dx$

Optimal. Leaf size=175

$$-\frac{ad^3f^2}{2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}+\frac{3ad^2f^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{2e}+\frac{\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^4}{8e}+\frac{af^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^2}{4e}+\frac{adf^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{e}$$

```
[Out] -(a*d^3*f^2)/(2*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])) + (a*d*f^2*(
e*x + f*Sqrt[a + (e^2*x^2)/f^2]))/e + (a*f^2*(d + e*x + f*Sqrt[a
+ (e^2*x^2)/f^2])^2)/(4*e) + (d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]
)^4/(8*e) + (3*a*d^2*f^2*Log[e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2
*e)
```

Rubi [A] time = 0.283324, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$

$$-\frac{ad^3f^2}{2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}+\frac{3ad^2f^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{2e}+\frac{\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^4}{8e}+\frac{af^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^2}{4e}+\frac{adf^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{e}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^3,x]
```

```
[Out] -(a*d^3*f^2)/(2*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])) + (a*d*f^2*(
e*x + f*Sqrt[a + (e^2*x^2)/f^2]))/e + (a*f^2*(d + e*x + f*Sqrt[a
+ (e^2*x^2)/f^2])^2)/(4*e) + (d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]
)^4/(8*e) + (3*a*d^2*f^2*Log[e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2
*e)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**3,x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + e**2*x**2/f**2))**3, x)
```

Mathematica [A] time = 0.598262, size = 143, normalized size = 0.82

$$\frac{1}{2}\left(\frac{3ad^2f^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{e}+\frac{\sqrt{a+\frac{e^2x^2}{f^2}}\left(2af^3(2d+ex)+efx\left(3d^2+4dex+2e^2x^2\right)\right)}{e}+3ex^2\left(af^2+d^2\right)+2dx\left(3af^2+d^2\right)+4de^2x^3+2e^3x^4\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^3,x]
```

```
[Out] (2*d*(d^2 + 3*a*f^2)*x + 3*e*(d^2 + a*f^2)*x^2 + 4*d*e^2*x^3 + 2*
e^3*x^4 + (Sqrt[a + (e^2*x^2)/f^2]*(2*a*f^3*(2*d + e*x) + e*f*x*(
3*d^2 + 4*d*e*x + 2*e^2*x^2)))/e + (3*a*d^2*f^2*Log[e*x + f*Sqrt[
a + (e^2*x^2)/f^2]])/e)/2
```

Maple [A] time = 0.018, size = 175, normalized size = 1.

$$f^3x\left(a+\frac{e^2x^2}{f^2}\right)^{\frac{3}{2}}+e^3x^4+2x^3e^2d+\frac{3f^2aex^2}{2}+3f^2adx+\frac{3fd^2x}{2}\sqrt{a+\frac{e^2x^2}{f^2}}+\frac{3fd^2a}{2}\ln\left(\frac{e^2x}{f^2}\frac{1}{\sqrt{\frac{e^2}{f^2}}}+\sqrt{a+\frac{e^2x^2}{f^2}}\right)\frac{1}{\sqrt{\frac{e^2}{f^2}}}+2\frac{df^3}{e}\left(\frac{e^2x^2+af^2}{f^2}\right)^{3/2}+\frac{3x^2d^2e}{2}+d^3x+\frac{d^4}{4e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^3,x)
```

```
[Out] f^3*x*(a+e^2*x^2/f^2)^(3/2)+e^3*x^4+2*x^3*e^2*d+3/2*f^2*a*e*x^2+3
*f^2*a*d*x+3/2*f*d^2*x*(a+e^2*x^2/f^2)^(1/2)+3/2*f*d^2*a*ln(e^2*x
/f^2/(1/f^2*e^2)^(1/2)+(a+e^2*x^2/f^2)^(1/2))/(1/f^2*e^2)^(1/2)+2
*d/e*f^3*((e^2*x^2+a*f^2)/f^2)^(3/2)+3/2*x^2*d^2*e+d^3*x+1/4*d^4/
```

e

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.291018, size = 217, normalized size = 1.24

$$\frac{2\,e^{4}x^{4}+4\,de^{3}x^{3}-3\,ad^{2}f^{2}\log\left(-ex+f\sqrt{\frac{e^{2}x^{2}+af^{2}}{f^{2}}}\right)+3\,\left(ae^{2}f^{2}+d^{2}e^{2}\right)x^{2}+2\,\left(3\,adef^{2}+d^{3}e\right)x+\left(2\,e^{3}fx^{3}+4\,de^{2}fx^{2}+4\,a\right.}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*e^4*x^4 + 4*d*e^3*x^3 - 3*a*d^2*f^2*log(-e*x + f*sqrt((e^2
*x^2 + a*f^2)/f^2)) + 3*(a*e^2*f^2 + d^2*e^2)*x^2 + 2*(3*a*d*e*f^
2 + d^3*e)*x + (2*e^3*f*x^3 + 4*d*e^2*f*x^2 + 4*a*d*f^3 + (2*a*e*
f^3 + 3*d^2*e*f)*x)*sqrt((e^2*x^2 + a*f^2)/f^2))/e
```

Sympy [A] time = 25.4691, size = 279, normalized size = 1.59

$$\frac{a^{\frac{3}{2}}f^{3}x\sqrt{1+\frac{e^{2}x^{2}}{af^{2}}}}{2}+\frac{a^{\frac{3}{2}}f^{3}x}{2\sqrt{1+\frac{e^{2}x^{2}}{af^{2}}}}+\frac{3\sqrt{a}d^{2}fx\sqrt{1+\frac{e^{2}x^{2}}{af^{2}}}}{2}+\frac{3\sqrt{a}e^{2}fx^{3}}{2\sqrt{1+\frac{e^{2}x^{2}}{af^{2}}}}+\frac{3ad^{2}f^{2}\operatorname{asinh}\left(\frac{ex}{\sqrt{a}f}\right)}{2e}+3adf^{2}x$$
$$+\frac{3aef^{2}x^{2}}{2}+d^{3}x+\frac{3d^{2}ex^{2}}{2}+2de^{2}x^{3}+6def\left(\begin{cases}\frac{\sqrt{a}x^{2}}{2} & \text{for}\ e^{2}=0\\ \frac{f^{2}\left(a+\frac{e^{2}x^{2}}{f^{2}}\right)^{\frac{3}{2}}}{3e^{2}} & \text{otherwise}\end{cases}\right)+e^{3}x^{4}+\frac{e^{4}x^{5}}{\sqrt{a}f\sqrt{1+\frac{e^{2}x^{2}}{af^{2}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**3,x)
```

```
[Out] a**(3/2)*f**3*x*sqrt(1 + e**2*x**2/(a*f**2))/2 + a**(3/2)*f**3*x/
(2*sqrt(1 + e**2*x**2/(a*f**2))) + 3*sqrt(a)*d**2*f*x*sqrt(1 + e*
*2*x**2/(a*f**2))/2 + 3*sqrt(a)*e**2*f*x**3/(2*sqrt(1 + e**2*x**2
/(a*f**2))) + 3*a*d**2*f**2*asinh(e*x/(sqrt(a)*f))/(2*e) + 3*a*d*
f**2*x + 3*a*e*f**2*x**2/2 + d**3*x + 3*d**2*e*x**2/2 + 2*d*e**2*
x**3 + 6*d*e*f*Piecewise((sqrt(a)*x**2/2, Eq(e**2, 0)), (f**2*(a
+ e**2*x**2/f**2)**(3/2)/(3*e**2), True)) + e**3*x**4 + e**4*x**5
/(sqrt(a)*f*sqrt(1 + e**2*x**2/(a*f**2)))
```

GIAC/XCAS [A] time = 0.288999, size = 220, normalized size = 1.26

$$-\frac{3}{2}\,ad^2f|f|e^{(-1)}\ln\left(\left|-xe+\sqrt{af^2+x^2e^2}\right|\right)+\frac{3}{2}\,af^2x^2e+3\,adf^2x+x^4e^3+2\,dx^3e^2+\frac{3}{2}\,d^2x^2e+d^3x+\frac{1}{2}\left(4\,adf|f|e^{(-1)}+\left(2\left(\frac{x|f|e^2}{f}+\frac{2\,d|f|e}{f}\right)x+\frac{\left(2\,af^6|f|e^4+3\,d^2f^4|f|e^4\right)e^{(-4)}}{f^5}\right)x\right)\sqrt{af^2+x^2e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^3,x, algorithm="giac")
```

```
[Out] -3/2*a*d^2*f*abs(f)*e^(-1)*ln(abs(-x*e + sqrt(a*f^2 + x^2*e^2)))
+ 3/2*a*f^2*x^2*e + 3*a*d*f^2*x + x^4*e^3 + 2*d*x^3*e^2 + 3/2*d^2
*x^2*e + d^3*x + 1/2*(4*a*d*f*abs(f)*e^(-1) + (2*(x*abs(f)*e^2/f
+ 2*d*abs(f)*e/f)*x + (2*a*f^6*abs(f)*e^4 + 3*d^2*f^4*abs(f)*e^4)
*e^(-4)/f^5)*x)*sqrt(a*f^2 + x^2*e^2)
```

3.294 $\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2 dx$

Optimal. Leaf size=136

$$-\frac{ad^2f^2}{2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}+\frac{\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^3}{6e}+\frac{adf^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{e}+\frac{af^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{2e}$$

```
[Out] -(a*d^2*f^2)/(2*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])) + (a*f^2*(e*
x + f*Sqrt[a + (e^2*x^2)/f^2]))/(2*e) + (d + e*x + f*Sqrt[a + (e^
2*x^2)/f^2])^3/(6*e) + (a*d*f^2*Log[e*x + f*Sqrt[a + (e^2*x^2)/f^
2]])/e
```

Rubi [A] time = 0.228167, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08

$$-\frac{ad^2f^2}{2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}+\frac{\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^3}{6e}+\frac{adf^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{e}+\frac{af^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{2e}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^2,x]
```

```
[Out] -(a*d^2*f^2)/(2*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])) + (a*f^2*(e*
x + f*Sqrt[a + (e^2*x^2)/f^2]))/(2*e) + (d + e*x + f*Sqrt[a + (e^
2*x^2)/f^2])^3/(6*e) + (a*d*f^2*Log[e*x + f*Sqrt[a + (e^2*x^2)/f^
2]])/e
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**2,x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + e**2*x**2/f**2))**2, x)
```

Mathematica [A] time = 0.393465, size = 102, normalized size = 0.75

$$x\left(af^2+d^2\right)+\frac{adf^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{e}+\frac{\sqrt{a+\frac{e^2x^2}{f^2}}\left(2af^3+efx(3d+2ex)\right)}{3e}+dex^2+\frac{2e^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^2,x]
```

```
[Out] (d^2 + a*f^2)*x + d*e*x^2 + (2*e^2*x^3)/3 + (Sqrt[a + (e^2*x^2)/f
^2]*(2*a*f^3 + e*f*x*(3*d + 2*e*x)))/(3*e) + (a*d*f^2*Log[e*x + f
*Sqrt[a + (e^2*x^2)/f^2]])/e
```

Maple [A] time = 0.007, size = 126, normalized size = 0.9

$$f^2xa+\frac{2x^3e^2}{3}+fdx\sqrt{a+\frac{e^2x^2}{f^2}}+adf\ln\left(\frac{e^2x}{f^2}\frac{1}{\sqrt{\frac{e^2}{f^2}}}+\sqrt{a+\frac{e^2x^2}{f^2}}\right)\frac{1}{\sqrt{\frac{e^2}{f^2}}}+\frac{2f^3}{3e}\left(\frac{e^2x^2+af^2}{f^2}\right)^{\frac{3}{2}}+x^2de+xd^2+\frac{d^3}{3e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^2,x)
```

```
[Out] f^2*x*a+2/3*x^3*e^2+f*d*x*(a+e^2*x^2/f^2)^(1/2)+f*d*a*ln(e^2*x/f^
2/(1/f^2*e^2)^(1/2)+(a+e^2*x^2/f^2)^(1/2))/(1/f^2*e^2)^(1/2)+2/3/
e*f^3*((e^2*x^2+a*f^2)/f^2)^(3/2)+x^2*d*e+x*d^2+1/3*d^3/e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.292464, size = 154, normalized size = 1.13

$$\frac{2 e^3 x^3 + 3 d e^2 x^2 - 3 a d f^2 \log\left(-ex + f\sqrt{\frac{e^2x^2+af^2}{f^2}}\right) + 3\left(aef^2 + d^2 e\right)x + \left(2 e^2 f x^2 + 2 a f^3 + 3 d e f x\right)\sqrt{\frac{e^2x^2+af^2}{f^2}}}{3 e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(2*e^3*x^3 + 3*d*e^2*x^2 - 3*a*d*f^2*log(-e*x + f*sqrt((e^2*x
^2 + a*f^2)/f^2)) + 3*(a*e*f^2 + d^2*e)*x + (2*e^2*f*x^2 + 2*a*f^
3 + 3*d*e*f*x)*sqrt((e^2*x^2 + a*f^2)/f^2))/e
```

Sympy [A] time = 8.60567, size = 116, normalized size = 0.85

$$\sqrt{a} d f x \sqrt{1 + \frac{e^2x^2}{af^2}} + \frac{a d f^2 \operatorname{asinh}\left(\frac{ex}{\sqrt{a}f}\right)}{e} + a f^2 x + d^2 x + d e x^2 + \frac{2e^2x^3}{3} + 2ef\left(\begin{cases} \frac{\sqrt{a}x^2}{2} & \text{for}\ e^2 = 0 \\ \frac{f^2\left(a+\frac{e^2x^2}{f^2}\right)^{\frac{3}{2}}}{3e^2} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**2,x)
```

```
[Out] sqrt(a)*d*f*x*sqrt(1 + e**2*x**2/(a*f**2)) + a*d*f**2*asinh(e*x/(
sqrt(a)*f))/e + a*f**2*x + d**2*x + d*e*x**2 + 2*e**2*x**3/3 + 2*
e*f*Piecewise((sqrt(a)*x**2/2, Eq(e**2, 0)), (f**2*(a + e**2*x**2
/f**2)**(3/2)/(3*e**2), True))
```

GIAC/XCAS [A] time = 0.28552, size = 139, normalized size = 1.02

$$-adf|f|e^{(-1)}\ln\left(\left|-xe + \sqrt{af^2 + x^2e^2}\right|\right) + af^2x + \frac{2}{3}x^3e^2 + dx^2e + d^2x + \frac{1}{3}\left(2\,af|f|e^{(-1)} + \left(\frac{2\,x|f|e}{f} + \frac{3\,d|f|}{f}\right)x\right)\sqrt{af^2 + x^2e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^2,x, algorithm="giac")
```

```
[Out] -a*d*f*abs(f)*e^(-1)*ln(abs(-x*e + sqrt(a*f^2 + x^2*e^2))) + a*f^
2*x + 2/3*x^3*e^2 + d*x^2*e + d^2*x + 1/3*(2*a*f*abs(f)*e^(-1) +
(2*x*abs(f)*e/f + 3*d*abs(f)/f)*x)*sqrt(a*f^2 + x^2*e^2)
```

3.295 $\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right) dx$

Optimal. Leaf size=68

$$\frac{1}{2}fx\sqrt{a+\frac{e^2x^2}{f^2}}+\frac{af^2\tanh^{-1}\left(\frac{ex}{f\sqrt{a+\frac{e^2x^2}{f^2}}}\right)}{2e}+dx+\frac{ex^2}{2}$$

```
[Out] d*x + (e*x^2)/2 + (f*x*Sqrt[a + (e^2*x^2)/f^2])/2 + (a*f^2*ArcTan
h[(e*x)/(f*Sqrt[a + (e^2*x^2)/f^2])])/(2*e)
```

Rubi [A] time = 0.0699131, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13

$$\frac{1}{2}fx\sqrt{a+\frac{e^2x^2}{f^2}}+\frac{af^2\tanh^{-1}\left(\frac{ex}{f\sqrt{a+\frac{e^2x^2}{f^2}}}\right)}{2e}+dx+\frac{ex^2}{2}$$

Antiderivative was successfully verified.

```
[In]  Int[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2],x]
```

```
[Out] d*x + (e*x^2)/2 + (f*x*Sqrt[a + (e^2*x^2)/f^2])/2 + (a*f^2*ArcTan
h[(e*x)/(f*Sqrt[a + (e^2*x^2)/f^2])])/(2*e)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{af^2\operatorname{atanh}\left(\frac{ex}{f\sqrt{a+\frac{e^2x^2}{f^2}}}\right)}{2e}+e\int x\,dx+\frac{fx\sqrt{a+\frac{e^2x^2}{f^2}}}{2}+\int d\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(d+e*x+f*(a+e**2*x**2/f**2)**(1/2),x)
```

```
[Out] a*f**2*atanh(e*x/(f*sqrt(a + e**2*x**2/f**2)))/(2*e) + e*Integral
(x, x) + f*x*sqrt(a + e**2*x**2/f**2)/2 + Integral(d, x)
```

Mathematica [A] time = 0.0794617, size = 81, normalized size = 1.19

$$\frac{1}{2}fx\sqrt{\frac{af^2+e^2x^2}{f^2}}+\frac{af^2\log\left(ef\sqrt{\frac{af^2+e^2x^2}{f^2}}+e^2x\right)}{2e}+dx+\frac{ex^2}{2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2],x]
```

```
[Out] d*x + (e*x^2)/2 + (f*x*Sqrt[(a*f^2 + e^2*x^2)/f^2])/2 + (a*f^2*Lo
g[e^2*x + e*f*Sqrt[(a*f^2 + e^2*x^2)/f^2]])/(2*e)
```

Maple [A] time = 0.006, size = 75, normalized size = 1.1

$$dx+\frac{ex^2}{2}+\frac{fx}{2}\sqrt{a+\frac{e^2x^2}{f^2}}+\frac{fa}{2}\ln\left(\frac{e^2x}{f^2}\frac{1}{\sqrt{\frac{e^2}{f^2}}}+\sqrt{a+\frac{e^2x^2}{f^2}}\right)\frac{1}{\sqrt{\frac{e^2}{f^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(d+e*x+f*(a+e^2*x^2/f^2)^(1/2),x)
```

```
[Out] d*x+1/2*e*x^2+1/2*f*x*(a+e^2*x^2/f^2)^(1/2)+1/2*f*a*ln(e^2*x/f^2/
(1/f^2*e^2)^(1/2)+(a+e^2*x^2/f^2)^(1/2))/(1/f^2*e^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e*x + sqrt(e^2*x^2/f^2 + a)*f + d,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.282706, size = 100, normalized size = 1.47

$$\frac{e^2x^2 - af^2 \log\left(-ex + f\sqrt{\frac{e^2x^2+af^2}{f^2}}\right) + efx\sqrt{\frac{e^2x^2+af^2}{f^2}} + 2\,dex}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e*x + sqrt(e^2*x^2/f^2 + a)*f + d,x, algorithm="fricas")
```

```
[Out] 1/2*(e^2*x^2 - a*f^2*log(-e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2)) +
e*f*x*sqrt((e^2*x^2 + a*f^2)/f^2) + 2*d*e*x)/e
```

Sympy [A] time = 7.18205, size = 54, normalized size = 0.79

$$dx + \frac{ex^2}{2} + f\left(\frac{\sqrt{a}x\sqrt{1+\frac{e^2x^2}{af^2}}}{2} + \frac{af\operatorname{asinh}\left(\frac{ex}{\sqrt{a}f}\right)}{2e}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(d+e*x+f*(a+e**2*x**2/f**2)**(1/2),x)
```

```
[Out] d*x + e*x**2/2 + f*(sqrt(a)*x*sqrt(1 + e**2*x**2/(a*f**2))/2 + a*
f*asinh(e*x/(sqrt(a)*f))/(2*e))
```

GIAC/XCAS [A] time = 0.282809, size = 88, normalized size = 1.29

$$\frac{1}{2}x^2e + dx - \frac{\left(af^2e^{(-1)}\ln\left(\left|-xe + \sqrt{af^2 + x^2e^2}\right|\right) - \sqrt{af^2 + x^2e^2}x\right)|f|}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e*x + sqrt(e^2*x^2/f^2 + a)*f + d,x, algorithm="giac")
```

```
[Out] 1/2*x^2*e + d*x - 1/2*(a*f^2*e^(-1)*ln(abs(-x*e + sqrt(a*f^2 + x^
2*e^2))) - sqrt(a*f^2 + x^2*e^2)*x)*abs(f)/f
```

3.296 $\int \frac{1}{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\,dx$

Optimal. Leaf size=117

$$-\frac{af^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{2d^2e}+\frac{\left(\frac{af^2}{d^2}+1\right)\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)}{2e}-\frac{af^2}{2de\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}$$

```
[Out] -(a*f^2)/(2*d*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])) - (a*f^2*Log[e
*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*d^2*e) + ((1 + (a*f^2)/d^2)*L
og[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*e)
```

Rubi [A] time = 0.203482, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08

$$-\frac{af^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{2d^2e}+\frac{\left(\frac{af^2}{d^2}+1\right)\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)}{2e}-\frac{af^2}{2de\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-1),x]
```

```
[Out] -(a*f^2)/(2*d*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])) - (a*f^2*Log[e
*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*d^2*e) + ((1 + (a*f^2)/d^2)*L
og[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*e)
```

Rubi in Sympy [A] time = 22.418, size = 102, normalized size = 0.87

$$-\frac{af}{2de\left(\frac{ex}{f}+\sqrt{a+\frac{e^2x^2}{f^2}}\right)}-\frac{af^2\log\left(\frac{ex}{f}+\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2d^2e}+\frac{\left(af^2+d^2\right)\log\left(d+f\left(\frac{ex}{f}+\sqrt{a+\frac{e^2x^2}{f^2}}\right)\right)}{2d^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(d+e*x+f*(a+e**2*x**2/f**2)**(1/2)),x)
```

```
[Out] -a*f/(2*d*e*(e*x/f + sqrt(a + e**2*x**2/f**2))) - a*f**2*log(e*x/
f + sqrt(a + e**2*x**2/f**2))/(2*d**2*e) + (a*f**2 + d**2)*log(d
```

```
+ f*(e*x/f + sqrt(a + e**2*x**2/f**2)))/(2*d**2*e)
```

Mathematica [A] time = 0.224731, size = 141, normalized size = 1.21

$$\frac{\left(d^2-af^2\right)\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)+\left(af^2+d^2\right)\log\left(d^2\left(ex-f\sqrt{a+\frac{e^2x^2}{f^2}}\right)-af^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+2d+ex\right)\right)+2d\left(ex-f\sqrt{}\right.}{4d^2e}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-1),x]
```

```
[Out] (2*d*(e*x - f*Sqrt[a + (e^2*x^2)/f^2]) + (d^2 - a*f^2)*Log[e*x +
f*Sqrt[a + (e^2*x^2)/f^2]] + (d^2 + a*f^2)*Log[d^2*(e*x - f*Sqrt[
a + (e^2*x^2)/f^2]) - a*f^2*(2*d + e*x + f*Sqrt[a + (e^2*x^2)/f^2
])])/(4*d^2*e)
```

Maple [B] time = 0.043, size = 1325, normalized size = 11.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2)),x)
```

```
[Out] -1/4*f/d/e*(4*e^2*(x+1/2*(-a*f^2+d^2)/d/e)^2/f^2+4*e*(a*f^2-d^2)/
d/f^2*(x+1/2*(-a*f^2+d^2)/d/e)+(a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)
^(1/2)-1/4*f/d^2*ln((1/2*e*(a*f^2-d^2)/d/f^2+e^2*(x+1/2*(-a*f^2+d
^2)/d/e)/f^2)/(1/f^2*e^2)^(1/2)+(e^2*(x+1/2*(-a*f^2+d^2)/d/e)^2/f
^2+e*(a*f^2-d^2)/d/f^2*(x+1/2*(-a*f^2+d^2)/d/e)+1/4*(a^2*f^4+2*a*
d^2*f^2+d^4)/f^2/d^2)^(1/2))/(1/f^2*e^2)^(1/2)*a+1/4/f*ln((1/2*e*
(a*f^2-d^2)/d/f^2+e^2*(x+1/2*(-a*f^2+d^2)/d/e)/f^2)/(1/f^2*e^2)^(
1/2)+(e^2*(x+1/2*(-a*f^2+d^2)/d/e)^2/f^2+e*(a*f^2-d^2)/d/f^2*(x+1
/2*(-a*f^2+d^2)/d/e)+1/4*(a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/2)
)/(1/f^2*e^2)^(1/2)+1/4*f^3/d^3/e/((a^2*f^4+2*a*d^2*f^2+d^4)/f^2/
d^2)^(1/2)*ln((1/2*(a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2+e*(a*f^2-d^2
)/d/f^2*(x+1/2*(-a*f^2+d^2)/d/e)+1/2*((a^2*f^4+2*a*d^2*f^2+d^4)/f
^2/d^2)^(1/2)*(4*e^2*(x+1/2*(-a*f^2+d^2)/d/e)^2/f^2+4*e*(a*f^2-d^
2)/d/f^2*(x+1/2*(-a*f^2+d^2)/d/e)+(a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d
^2)^(1/2))/(x+1/2*(-a*f^2+d^2)/d/e))*a^2+1/2*f/d/e/((a^2*f^4+2*a*
d^2*f^2+d^4)/f^2/d^2)^(1/2)*ln((1/2*(a^2*f^4+2*a*d^2*f^2+d^4)/f^2
/d^2+e*(a*f^2-d^2)/d/f^2*(x+1/2*(-a*f^2+d^2)/d/e)+1/2*((a^2*f^4+2
*a*d^2*f^2+d^4)/f^2/d^2)^(1/2)*(4*e^2*(x+1/2*(-a*f^2+d^2)/d/e)^2/
f^2+4*e*(a*f^2-d^2)/d/f^2*(x+1/2*(-a*f^2+d^2)/d/e)+(a^2*f^4+2*a*d
^2*f^2+d^4)/f^2/d^2)^(1/2))/(x+1/2*(-a*f^2+d^2)/d/e))*a+1/4/f*d/e
```

```
/((a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/2)*ln((1/2*(a^2*f^4+2*a*d
^2*f^2+d^4)/f^2/d^2+e*(a*f^2-d^2)/d/f^2*(x+1/2*(-a*f^2+d^2)/d/e)+
1/2*((a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/2)*(4*e^2*(x+1/2*(-a*f
^2+d^2)/d/e)^2/f^2+4*e*(a*f^2-d^2)/d/f^2*(x+1/2*(-a*f^2+d^2)/d/e)
+(a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/2))/(x+1/2*(-a*f^2+d^2)/d/
e))+1/2*ln(a*f^2-2*d*e*x-d^2)/e+1/2/d*x+1/4/d^2/e*ln(-a*f^2+2*d*e
*x+d^2)*a*f^2-1/4/e*ln(-a*f^2+2*d*e*x+d^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{ex + \sqrt{\frac{e^2x^2}{f^2} + a}f + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x + sqrt(e^2*x^2/f^2 + a)*f + d),x, algorithm="maxima")
```

```
[Out] integrate(1/(e*x + sqrt(e^2*x^2/f^2 + a)*f + d), x)
```

Fricas [A] time = 0.302596, size = 252, normalized size = 2.15

$$\frac{2\,dex - 2\,df\sqrt{\frac{e^2x^2+af^2}{f^2}} + \left(af^2 + d^2\right)\log\left(af^2 - dex + df\sqrt{\frac{e^2x^2+af^2}{f^2}}\right) + \left(af^2 + d^2\right)\log\left(-af^2 + 2\,dex + d^2\right) - \left(af^2 + d^2\right)\log}{4\,d^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x + sqrt(e^2*x^2/f^2 + a)*f + d),x, algorithm="fricas")
```

```
[Out] 1/4*(2*d*e*x - 2*d*f*sqrt((e^2*x^2 + a*f^2)/f^2) + (a*f^2 + d^2)*
log(a*f^2 - d*e*x + d*f*sqrt((e^2*x^2 + a*f^2)/f^2)) + (a*f^2 + d
^2)*log(-a*f^2 + 2*d*e*x + d^2) - (a*f^2 + d^2)*log(-e*x + f*sqrt
((e^2*x^2 + a*f^2)/f^2) - d) + (a*f^2 - d^2)*log(-e*x + f*sqrt((e
^2*x^2 + a*f^2)/f^2)))/(d^2*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(d+e*x+f*(a+e**2*x**2/f**2)**(1/2)),x)
```

```
[Out] Integral(1/(d + e*x + f*sqrt(a + e**2*x**2/f**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$[undef, +\infty, 1]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*x + sqrt(e^2*x^2/f^2 + a)*f + d),x, algorithm="giac")
```

```
[Out] [undef, +Infinity, 1]
```

3.297 $\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2}\,dx$

Optimal. Leaf size=151

$$-\frac{af^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{d^3e}+\frac{af^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)}{d^3e}-\frac{af^2}{2d^2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}-\frac{\frac{af^2}{d^2}+1}{2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)}$$

```
[Out] -(a*f^2)/(2*d^2*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])) - (1 + (a*f^
2)/d^2)/(2*e*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])) - (a*f^2*Log[
e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(d^3*e) + (a*f^2*Log[d + e*x +
f*Sqrt[a + (e^2*x^2)/f^2]])/(d^3*e)
```

Rubi [A] time = 0.255772, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08

$$-\frac{af^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{d^3e}+\frac{af^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)}{d^3e}-\frac{af^2}{2d^2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}-\frac{\frac{af^2}{d^2}+1}{2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-2),x]
```

```
[Out] -(a*f^2)/(2*d^2*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])) - (1 + (a*f^
2)/d^2)/(2*e*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])) - (a*f^2*Log[
e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(d^3*e) + (a*f^2*Log[d + e*x +
f*Sqrt[a + (e^2*x^2)/f^2]])/(d^3*e)
```

Rubi in Sympy [A] time = 39.648, size = 136, normalized size = 0.9

$$-\frac{af}{2d^2e\left(\frac{ex}{f}+\sqrt{a+\frac{e^2x^2}{f^2}}\right)}+\frac{af^2\log\left(d+f\left(\frac{ex}{f}+\sqrt{a+\frac{e^2x^2}{f^2}}\right)\right)}{d^3e}-\frac{af^2\log\left(\frac{ex}{f}+\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{d^3e}-\frac{af^2+d^2}{2d^2e\left(d+f\left(\frac{ex}{f}+\sqrt{a+\frac{e^2x^2}{f^2}}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**2,x)
```

```
[Out] -a*f/(2*d**2*e*(e*x/f + sqrt(a + e**2*x**2/f**2))) + a*f**2*log(d
 + f*(e*x/f + sqrt(a + e**2*x**2/f**2)))/(d**3*e) - a*f**2*log(e*
x/f + sqrt(a + e**2*x**2/f**2))/(d**3*e) - (a*f**2 + d**2)/(2*d**
2*e*(d + f*(e*x/f + sqrt(a + e**2*x**2/f**2))))
```

Mathematica [A] time = 0.654703, size = 248, normalized size = 1.64

$$\frac{\frac{4df\sqrt{a+\frac{e^2x^2}{f^2}}\left(af^2-dex\right)}{e\left(-af^2+d^2+2dex\right)}+\frac{2af^2\log\left(d^2\left(ex-f\sqrt{a+\frac{e^2x^2}{f^2}}\right)-af^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+2d+ex\right)\right)}{e}-\frac{\left(af^2+d^2\right)^2}{e\left(-af^2+d^2+2dex\right)}-\frac{2af^2\log\left(af^2-d^2-2dex\right)}{e}+\frac{2af^2\log(-a \cdots}{\cdots}}{4d^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-2),x]
```

```
[Out] (2*d*x - (d^2 + a*f^2)^2/(e*(d^2 - a*f^2 + 2*d*e*x)) + (4*d*f*(a*
f^2 - d*e*x)*Sqrt[a + (e^2*x^2)/f^2])/(e*(d^2 - a*f^2 + 2*d*e*x))
 - (2*a*f^2*Log[-d^2 + a*f^2 - 2*d*e*x])/e + (2*a*f^2*Log[d^2 - a
*f^2 + 2*d*e*x])/e - (2*a*f^2*Log[e*x + f*Sqrt[a + (e^2*x^2)/f^2]
])/e + (2*a*f^2*Log[d^2*(e*x - f*Sqrt[a + (e^2*x^2)/f^2]) - a*f^2
*(2*d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])])/e)/(4*d^3)
```

Maple [B] time = 0.042, size = 4136, normalized size = 27.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^2,x)
```

```
[Out] 1/2/d^2*f^5/e/(a^2*f^4+2*a*d^2*f^2+d^4)/((a^2*f^4+2*a*d^2*f^2+d^4
)/f^2/d^2)^(1/2)*ln((1/2*(a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2+e*(a*f
^2-d^2)/d/f^2*(x+1/2*(-a*f^2+d^2)/d/e)+1/2*((a^2*f^4+2*a*d^2*f^2+
d^4)/f^2/d^2)^(1/2)*(4*e^2*(x+1/2*(-a*f^2+d^2)/d/e)^2/f^2+4*e*(a*
f^2-d^2)/d/f^2*(x+1/2*(-a*f^2+d^2)/d/e)+(a^2*f^4+2*a*d^2*f^2+d^4)
/f^2/d^2)^(1/2))/(x+1/2*(-a*f^2+d^2)/d/e))*a^3-1/2*d^2*f/e/(a^2*f
^4+2*a*d^2*f^2+d^4)/((a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/2)*ln(
(1/2*(a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2+e*(a*f^2-d^2)/d/f^2*(x+1/2
*(-a*f^2+d^2)/d/e)+1/2*((a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/2)*
(4*e^2*(x+1/2*(-a*f^2+d^2)/d/e)^2/f^2+4*e*(a*f^2-d^2)/d/f^2*(x+1/
2*(-a*f^2+d^2)/d/e)+(a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/2))/(x+
1/2*(-a*f^2+d^2)/d/e))*a+1/2/d^2*x-1/4/e*f/d^2*(4*e^2*(x+1/2*(-a*
f^2+d^2)/d/e)^2/f^2+4*e*(a*f^2-d^2)/d/f^2*(x+1/2*(-a*f^2+d^2)/d/e
)+(a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/2)+1/4/f/d*ln((1/2*e*(a*f
^2-d^2)/d/f^2+e^2*(x+1/2*(-a*f^2+d^2)/d/e)/f^2)/(1/f^2*e^2)^(1/2)
+(e^2*(x+1/2*(-a*f^2+d^2)/d/e)^2/f^2+e*(a*f^2-d^2)/d/f^2*(x+1/2*(
-a*f^2+d^2)/d/e)+1/4*(a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/2))/(1
/f^2*e^2)^(1/2)+1/4/e/f/((a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/2)
*ln((1/2*(a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2+e*(a*f^2-d^2)/d/f^2*(x
+1/2*(-a*f^2+d^2)/d/e)+1/2*((a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1
/2)*(4*e^2*(x+1/2*(-a*f^2+d^2)/d/e)^2/f^2+4*e*(a*f^2-d^2)/d/f^2*(
x+1/2*(-a*f^2+d^2)/d/e)+(a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/2))
/(x+1/2*(-a*f^2+d^2)/d/e))-1/4*f/d^3*ln((1/2*e*(a*f^2-d^2)/d/f^2+
e^2*(x+1/2*(-a*f^2+d^2)/d/e)/f^2)/(1/f^2*e^2)^(1/2)+(e^2*(x+1/2*(
-a*f^2+d^2)/d/e)^2/f^2+e*(a*f^2-d^2)/d/f^2*(x+1/2*(-a*f^2+d^2)/d/
e)+1/4*(a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/2))/(1/f^2*e^2)^(1/2
)*a-d*f/(a^2*f^4+2*a*d^2*f^2+d^4)*(e^2*(x+1/2*(-a*f^2+d^2)/d/e)^2
/f^2+e*(a*f^2-d^2)/d/f^2*(x+1/2*(-a*f^2+d^2)/d/e)+1/4*(a^2*f^4+2*
a*d^2*f^2+d^4)/f^2/d^2)^(1/2)*x-1/2*a*f^2/(-a*f^2+2*d*e*x+d^2)/d/
e-1/4/e/d^3/(-a*f^2+2*d*e*x+d^2)*a^2*f^4+1/4*d^2*f/e/(a^2*f^4+2*a
*d^2*f^2+d^4)*(4*e^2*(x+1/2*(-a*f^2+d^2)/d/e)^2/f^2+4*e*(a*f^2-d^
2)/d/f^2*(x+1/2*(-a*f^2+d^2)/d/e)+(a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d
^2)^(1/2)+1/2/e/d^3*ln(-a*f^2+2*d*e*x+d^2)*a*f^2-1/4*d^3/f/(a^2*f
^4+2*a*d^2*f^2+d^4)*ln((1/2*e*(a*f^2-d^2)/d/f^2+e^2*(x+1/2*(-a*f^
2+d^2)/d/e)/f^2)/(1/f^2*e^2)^(1/2)+(e^2*(x+1/2*(-a*f^2+d^2)/d/e)^
2/f^2+e*(a*f^2-d^2)/d/f^2*(x+1/2*(-a*f^2+d^2)/d/e)+1/4*(a^2*f^4+2
*a*d^2*f^2+d^4)/f^2/d^2)^(1/2))/(1/f^2*e^2)^(1/2)+1/e^2*f^5/d/(a^
2*f^4+2*a*d^2*f^2+d^4)/(x-1/2/d/e*a*f^2+1/2*d/e)*(e^2*(x+1/2*(-a*
f^2+d^2)/d/e)^2/f^2+e*(a*f^2-d^2)/d/f^2*(x+1/2*(-a*f^2+d^2)/d/e)+
1/4*(a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(3/2)*a+1/4/e*f^7/d^4/(a^2
*f^4+2*a*d^2*f^2+d^4)/((a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/2)*l
n((1/2*(a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2+e*(a*f^2-d^2)/d/f^2*(x+1
/2*(-a*f^2+d^2)/d/e)+1/2*((a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/2
)*(4*e^2*(x+1/2*(-a*f^2+d^2)/d/e)^2/f^2+4*e*(a*f^2-d^2)/d/f^2*(x+
1/2*(-a*f^2+d^2)/d/e)+(a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/2))/(
x+1/2*(-a*f^2+d^2)/d/e))*a^4+d*f^3/e^2/(a^2*f^4+2*a*d^2*f^2+d^4)/
(x-1/2/d/e*a*f^2+1/2*d/e)*(e^2*(x+1/2*(-a*f^2+d^2)/d/e)^2/f^2+e*(
a*f^2-d^2)/d/f^2*(x+1/2*(-a*f^2+d^2)/d/e)+1/4*(a^2*f^4+2*a*d^2*f^
2+d^4)/f^2/d^2)^(3/2)-3/4/d*f^3/(a^2*f^4+2*a*d^2*f^2+d^4)*ln((1/2
*e*(a*f^2-d^2)/d/f^2+e^2*(x+1/2*(-a*f^2+d^2)/d/e)/f^2)/(1/f^2*e^2
)^(1/2)+(e^2*(x+1/2*(-a*f^2+d^2)/d/e)^2/f^2+e*(a*f^2-d^2)/d/f^2*(
```

```
x+1/2*(-a*f^2+d^2)/d/e)+1/4*(a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1
/2))/(1/f^2*e^2)^(1/2)*a^2-3/4*d*f/(a^2*f^4+2*a*d^2*f^2+d^4)*ln((
1/2*e*(a*f^2-d^2)/d/f^2+e^2*(x+1/2*(-a*f^2+d^2)/d/e)/f^2)/(1/f^2*
e^2)^(1/2)+(e^2*(x+1/2*(-a*f^2+d^2)/d/e)^2/f^2+e*(a*f^2-d^2)/d/f^
2*(x+1/2*(-a*f^2+d^2)/d/e)+1/4*(a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)
^(1/2))/(1/f^2*e^2)^(1/2)*a-1/4*d^4/f/e/(a^2*f^4+2*a*d^2*f^2+d^4)
/((a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/2)*ln((1/2*(a^2*f^4+2*a*d
^2*f^2+d^4)/f^2/d^2+e*(a*f^2-d^2)/d/f^2*(x+1/2*(-a*f^2+d^2)/d/e)+
1/2*((a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/2)*(4*e^2*(x+1/2*(-a*f
^2+d^2)/d/e)^2/f^2+4*e*(a*f^2-d^2)/d/f^2*(x+1/2*(-a*f^2+d^2)/d/e)
+(a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/2))/(x+1/2*(-a*f^2+d^2)/d/
e))-f^3/d/(a^2*f^4+2*a*d^2*f^2+d^4)*(e^2*(x+1/2*(-a*f^2+d^2)/d/e)
^2/f^2+e*(a*f^2-d^2)/d/f^2*(x+1/2*(-a*f^2+d^2)/d/e)+1/4*(a^2*f^4+
2*a*d^2*f^2+d^4)/f^2/d^2)^(1/2)*x*a+1/4/e*f^3/d^4/((a^2*f^4+2*a*d
^2*f^2+d^4)/f^2/d^2)^(1/2)*ln((1/2*(a^2*f^4+2*a*d^2*f^2+d^4)/f^2/
d^2+e*(a*f^2-d^2)/d/f^2*(x+1/2*(-a*f^2+d^2)/d/e)+1/2*((a^2*f^4+2*
a*d^2*f^2+d^4)/f^2/d^2)^(1/2)*(4*e^2*(x+1/2*(-a*f^2+d^2)/d/e)^2/f
^2+4*e*(a*f^2-d^2)/d/f^2*(x+1/2*(-a*f^2+d^2)/d/e)+(a^2*f^4+2*a*d^
2*f^2+d^4)/f^2/d^2)^(1/2))/(x+1/2*(-a*f^2+d^2)/d/e))*a^2+1/2/e*f/
d^2/((a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/2)*ln((1/2*(a^2*f^4+2*
a*d^2*f^2+d^4)/f^2/d^2+e*(a*f^2-d^2)/d/f^2*(x+1/2*(-a*f^2+d^2)/d/
e)+1/2*((a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/2)*(4*e^2*(x+1/2*(-
a*f^2+d^2)/d/e)^2/f^2+4*e*(a*f^2-d^2)/d/f^2*(x+1/2*(-a*f^2+d^2)/d
/e)+(a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/2))/(x+1/2*(-a*f^2+d^2)
/d/e))*a-1/4/e*f^5/d^2/(a^2*f^4+2*a*d^2*f^2+d^4)*(4*e^2*(x+1/2*(-
a*f^2+d^2)/d/e)^2/f^2+4*e*(a*f^2-d^2)/d/f^2*(x+1/2*(-a*f^2+d^2)/d
/e)+(a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/2)*a^2-1/4*f^5/d^3/(a^2
*f^4+2*a*d^2*f^2+d^4)*ln((1/2*e*(a*f^2-d^2)/d/f^2+e^2*(x+1/2*(-a*
f^2+d^2)/d/e)/f^2)/(1/f^2*e^2)^(1/2)+(e^2*(x+1/2*(-a*f^2+d^2)/d/e
)^2/f^2+e*(a*f^2-d^2)/d/f^2*(x+1/2*(-a*f^2+d^2)/d/e)+1/4*(a^2*f^4
+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/2))/(1/f^2*e^2)^(1/2)*a^3-1/4*d/(-a
*f^2+2*d*e*x+d^2)/e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ex + \sqrt{\frac{e^2x^2}{f^2} + a}f + d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(-2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(-2), x)
```

Fricas [A] time = 0.30386, size = 383, normalized size = 2.54

$$\frac{a^2f^4 - 2\,d^2e^2x^2 + ad^2f^2 - 2\,d^3ex + \left(a^2f^4 - 2\,adef^2x - ad^2f^2\right)\log\left(-aef^2x + 2\,de^2x^2 + adf^2 + \left(af^3 - 2\,defx\right)\sqrt{\frac{e^2x^2+af}{f^2}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(-2),x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*f^4 - 2*d^2*e^2*x^2 + a*d^2*f^2 - 2*d^3*e*x + (a^2*f^4 -
 2*a*d*e*f^2*x - a*d^2*f^2)*log(-a*e*f^2*x + 2*d*e^2*x^2 + a*d*f^
2 + (a*f^3 - 2*d*e*f*x)*sqrt((e^2*x^2 + a*f^2)/f^2)) + (a^2*f^4 -
 2*a*d*e*f^2*x - a*d^2*f^2)*log(-a*f^2 + 2*d*e*x + d^2) - (a^2*f^
4 - 2*a*d*e*f^2*x - a*d^2*f^2)*log(-e*x + f*sqrt((e^2*x^2 + a*f^2
)/f^2) - d) - 2*(a*d*f^3 - d^2*e*f*x)*sqrt((e^2*x^2 + a*f^2)/f^2)
)/(a*d^3*e*f^2 - 2*d^4*e^2*x - d^5*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**2,x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + e**2*x**2/f**2))**(-2), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(-2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.298 $\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^3}\,dx$

Optimal. Leaf size=193

$$-\frac{3af^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{2d^4e}+\frac{3af^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)}{2d^4e}-\frac{af^2}{2d^3e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}-\frac{af^2}{d^3e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)}-\frac{\frac{af^2}{d^2}+1}{4e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^2}$$

```
[Out] -(a*f^2)/(2*d^3*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])) - (1 + (a*f^
2)/d^2)/(4*e*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^2) - (a*f^2)/(
d^3*e*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])) - (3*a*f^2*Log[e*x +
 f*Sqrt[a + (e^2*x^2)/f^2]])/(2*d^4*e) + (3*a*f^2*Log[d + e*x + f
*Sqrt[a + (e^2*x^2)/f^2]])/(2*d^4*e)
```

Rubi [A] time = 0.287864, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$

$$-\frac{3af^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{2d^4e}+\frac{3af^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)}{2d^4e}-\frac{af^2}{2d^3e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}-\frac{af^2}{d^3e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)}-\frac{\frac{af^2}{d^2}+1}{4e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-3),x]
```

```
[Out] -(a*f^2)/(2*d^3*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])) - (1 + (a*f^
2)/d^2)/(4*e*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^2) - (a*f^2)/(
d^3*e*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])) - (3*a*f^2*Log[e*x +
 f*Sqrt[a + (e^2*x^2)/f^2]])/(2*d^4*e) + (3*a*f^2*Log[d + e*x + f
*Sqrt[a + (e^2*x^2)/f^2]])/(2*d^4*e)
```

Rubi in Sympy [A] time = 46.1686, size = 178, normalized size = 0.92

$$-\frac{af^2}{d^3e\left(d+f\left(\frac{ex}{f}+\sqrt{a+\frac{e^2x^2}{f^2}}\right)\right)}-\frac{af}{2d^3e\left(\frac{ex}{f}+\sqrt{a+\frac{e^2x^2}{f^2}}\right)}+\frac{3af^2\log\left(d+f\left(\frac{ex}{f}+\sqrt{a+\frac{e^2x^2}{f^2}}\right)\right)}{2d^4e}-\frac{3af^2\log\left(\frac{ex}{f}+\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2d^4e}-\frac{af^2+d^2}{4d^2e\left(d+f\left(\frac{ex}{f}+\sqrt{a+\frac{e^2x^2}{f^2}}\right)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**3,x)
```

```
[Out] -a*f**2/(d**3*e*(d + f*(e*x/f + sqrt(a + e**2*x**2/f**2)))) - a*f
/(2*d**3*e*(e*x/f + sqrt(a + e**2*x**2/f**2))) + 3*a*f**2*log(d +
 f*(e*x/f + sqrt(a + e**2*x**2/f**2)))/(2*d**4*e) - 3*a*f**2*log(
e*x/f + sqrt(a + e**2*x**2/f**2))/(2*d**4*e) - (a*f**2 + d**2)/(4
*d**2*e*(d + f*(e*x/f + sqrt(a + e**2*x**2/f**2)))**2)
```

Mathematica [A] time = 1.15352, size = 309, normalized size = 1.6

$$-\frac{\frac{4df\sqrt{a+\frac{e^2x^2}{f^2}}\left(3a^2f^4-adf^2(5d+9ex)+d^2ex(3d+4ex)\right)}{e\left(-af^2+d^2+2dex\right)^2}-\frac{6af^2\log\left(d^2\left(ex-f\sqrt{a+\frac{e^2x^2}{f^2}}\right)-af^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+2d+ex\right)\right)}{e}+\frac{\left(af^2+d^2\right)^3}{e\left(-af^2+d^2+2dex\right)^2}+\cdots}{8d^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-3),x]
```

```
[Out] -(-4*d*x + (d^2 + a*f^2)^3/(e*(d^2 - a*f^2 + 2*d*e*x)^2) + (6*a*f
^2*(d^2 + a*f^2))/(e*(d^2 - a*f^2 + 2*d*e*x)) + (4*d*f*Sqrt[a + (
e^2*x^2)/f^2]*(3*a^2*f^4 + d^2*e*x*(3*d + 4*e*x) - a*d*f^2*(5*d +
 9*e*x)))/(e*(d^2 - a*f^2 + 2*d*e*x)^2) + (6*a*f^2*Log[-d^2 + a*f
^2 - 2*d*e*x])/e - (6*a*f^2*Log[d^2 - a*f^2 + 2*d*e*x])/e + (6*a*
f^2*Log[e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/e - (6*a*f^2*Log[d^2*(e
*x - f*Sqrt[a + (e^2*x^2)/f^2]) - a*f^2*(2*d + e*x + f*Sqrt[a + (
e^2*x^2)/f^2])])/e)/(8*d^4)
```

Maple [B] time = 0.064, size = 9721, normalized size = 50.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^3,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ex + \sqrt{\frac{e^2x^2}{f^2} + a}f + d\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(-3),x, algorithm="maxima")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(-3), x)
```

Fricas [A] time = 0.365868, size = 724, normalized size = 3.75

$$\frac{5\,a^3f^6 + 8\,d^3e^3x^3 - 6\,a^2d^2f^4 - 3\,ad^4f^2 + 2\,\left(ad^2e^2f^2 + 5\,d^4e^2\right)x^2 - 2\,\left(7\,a^2def^4 + ad^3ef^2 - 2\,d^5e\right)x + 3\,\left(a^3f^6 + 4\,ad^2e^2f^2x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(-3),x, algorithm="fricas")
```

```
[Out] 1/4*(5*a^3*f^6 + 8*d^3*e^3*x^3 - 6*a^2*d^2*f^4 - 3*a*d^4*f^2 + 2*
(a*d^2*e^2*f^2 + 5*d^4*e^2)*x^2 - 2*(7*a^2*d*e*f^4 + a*d^3*e*f^2
- 2*d^5*e)*x + 3*(a^3*f^6 + 4*a*d^2*e^2*f^2*x^2 - 2*a^2*d^2*f^4 +
 a*d^4*f^2 - 4*(a^2*d*e*f^4 - a*d^3*e*f^2)*x)*log(-a*e*f^2*x + 2*
d*e^2*x^2 + a*d*f^2 + (a*f^3 - 2*d*e*f*x)*sqrt((e^2*x^2 + a*f^2)/
f^2)) + 3*(a^3*f^6 + 4*a*d^2*e^2*f^2*x^2 - 2*a^2*d^2*f^4 + a*d^4*
f^2 - 4*(a^2*d*e*f^4 - a*d^3*e*f^2)*x)*log(-a*f^2 + 2*d*e*x + d^2
) - 3*(a^3*f^6 + 4*a*d^2*e^2*f^2*x^2 - 2*a^2*d^2*f^4 + a*d^4*f^2
- 4*(a^2*d*e*f^4 - a*d^3*e*f^2)*x)*log(-e*x + f*sqrt((e^2*x^2 + a
*f^2)/f^2) - d) - 2*(3*a^2*d*f^5 + 4*d^3*e^2*f*x^2 - 5*a*d^3*f^3
- 3*(3*a*d^2*e*f^3 - d^4*e*f)*x)*sqrt((e^2*x^2 + a*f^2)/f^2))/(a^
2*d^4*e*f^4 + 4*d^6*e^3*x^2 - 2*a*d^6*e*f^2 + d^8*e - 4*(a*d^5*e^
2*f^2 - d^7*e^2)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**3,x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + e**2*x**2/f**2))**(-3), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(-3),x, algorithm="giac")
```

```
[Out] Timed out
```

3.299 $\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{5/2} dx$

Optimal. Leaf size=268

$$-\frac{5ad^{3/2}f^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2e}+\frac{\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{7/2}}{7e}-\frac{af^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{7/2}}{2de\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}+\frac{af^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{5/2}}{2de}+\frac{5af^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}}{6e}+\frac{5adf^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2e}$$

```
[Out] (5*a*d*f^2*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*e) + (5*
a*f^2*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(3/2))/(6*e) + (a*f^2
*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(5/2))/(2*d*e) + (d + e*x
+ f*Sqrt[a + (e^2*x^2)/f^2])^(7/2)/(7*e) - (a*f^2*(d + e*x + f*Sq
rt[a + (e^2*x^2)/f^2])^(7/2))/(2*d*e*(e*x + f*Sqrt[a + (e^2*x^2)/
f^2])) - (5*a*d^(3/2)*f^2*ArcTanh[Sqrt[d + e*x + f*Sqrt[a + (e^2*
x^2)/f^2]]/Sqrt[d]])/(2*e)
```

Rubi [A] time = 0.477121, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259

$$-\frac{5ad^{3/2}f^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2e}+\frac{\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{7/2}}{7e}-\frac{af^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{7/2}}{2de\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}+\frac{af^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{5/2}}{2de}+\frac{5af^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}}{6e}+\frac{5adf^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2e}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(5/2),x]
```


```
[Out] (5*a*d*f^2*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*e) + (5*
a*f^2*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(3/2))/(6*e) + (a*f^2
*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(5/2))/(2*d*e) + (d + e*x
+ f*Sqrt[a + (e^2*x^2)/f^2])^(7/2)/(7*e) - (a*f^2*(d + e*x + f*Sq
rt[a + (e^2*x^2)/f^2])^(7/2))/(2*d*e*(e*x + f*Sqrt[a + (e^2*x^2)/
```

```
f^2])) - (5*a*d^(3/2)*f^2*ArcTanh[Sqrt[d + e*x + f*Sqrt[a + (e^2*
x^2)/f^2]]/Sqrt[d]])/(2*e)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**(5/2),x)

[Out] Integral((d + e*x + f*sqrt(a + e**2*x**2/f**2))**(5/2), x)
```

Mathematica [A] time = 0.631118, size = 0, normalized size = 0.

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{5/2}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(5/2),x]

[Out] Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(5/2), x]
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(5/2),x)

[Out] int((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex + \sqrt{\frac{e^2x^2}{f^2} + a}f + d\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(5/2), x)
```

Fricas [A] time = 0.348758, size = 1, normalized size = 0.

$$\left[\frac{105\,ad^{\frac{3}{2}}f^2\log\left(\frac{ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}-2\sqrt{ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}+d}\sqrt{d}+2\,d}{ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}}\right)+2\left(24\,e^3x^3+36\,de^2x^2+116\,adf^2+6\,d^3+\left(32\,aef^2+39\,d^2e\right)x\right.}{84\,e}\right.$$

$$-\frac{105\,a\sqrt{-d}df^2\arctan\left(\frac{\sqrt{ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}+d}}{\sqrt{-d}}\right)-\left(24\,e^3x^3+36\,de^2x^2+116\,adf^2+6\,d^3+\left(32\,aef^2+39\,d^2e\right)x+\left(24\,e^2fx^2+20\right.\right.}{42\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/84*(105*a*d^(3/2)*f^2*log((e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2)
 - 2*sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d)*sqrt(d) + 2*d)
/(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2))) + 2*(24*e^3*x^3 + 36*d*e^
2*x^2 + 116*a*d*f^2 + 6*d^3 + (32*a*e*f^2 + 39*d^2*e)*x + (24*e^2
*f*x^2 + 20*a*f^3 + 36*d*e*f*x - 3*d^2*f)*sqrt((e^2*x^2 + a*f^2)/
f^2))*sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d))/e, -1/42*(10
5*a*sqrt(-d)*d*f^2*arctan(sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2
) + d)/sqrt(-d)) - (24*e^3*x^3 + 36*d*e^2*x^2 + 116*a*d*f^2 + 6*d
^3 + (32*a*e*f^2 + 39*d^2*e)*x + (24*e^2*f*x^2 + 20*a*f^3 + 36*d*
```

```
e*f*x - 3*d^2*f)*sqrt((e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((
e^2*x^2 + a*f^2)/f^2) + d))/e]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**(5/2),x)

[Out] Integral((d + e*x + f*sqrt(a + e**2*x**2/f**2))**(5/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex + \sqrt{\frac{e^2x^2}{f^2} + a}f + d\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(5/2),x, algorithm="giac")

[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(5/2), x)
```

3.300 $\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2} dx$

Optimal. Leaf size=229

$$\frac{\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{5/2}}{5e}-\frac{af^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{5/2}}{2de\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}+\frac{af^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}}{2de}+\frac{3af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2e}-\frac{3a\sqrt{d}f^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2e}$$

```
[Out] (3*a*f^2*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*e) + (a*f^
2*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(3/2))/(2*d*e) + (d + e*x
 + f*Sqrt[a + (e^2*x^2)/f^2])^(5/2)/(5*e) - (a*f^2*(d + e*x + f*S
qrt[a + (e^2*x^2)/f^2])^(5/2))/(2*d*e*(e*x + f*Sqrt[a + (e^2*x^2)
/f^2])) - (3*a*Sqrt[d]*f^2*ArcTanh[Sqrt[d + e*x + f*Sqrt[a + (e^2
*x^2)/f^2]]/Sqrt[d]])/(2*e)
```

Rubi [A] time = 0.440032, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259

$$\frac{\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{5/2}}{5e}-\frac{af^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{5/2}}{2de\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}+\frac{af^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}}{2de}+\frac{3af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2e}-\frac{3a\sqrt{d}f^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2e}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(3/2),x]
```


```
[Out] (3*a*f^2*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*e) + (a*f^
2*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(3/2))/(2*d*e) + (d + e*x
 + f*Sqrt[a + (e^2*x^2)/f^2])^(5/2)/(5*e) - (a*f^2*(d + e*x + f*S
qrt[a + (e^2*x^2)/f^2])^(5/2))/(2*d*e*(e*x + f*Sqrt[a + (e^2*x^2)
/f^2])) - (3*a*Sqrt[d]*f^2*ArcTanh[Sqrt[d + e*x + f*Sqrt[a + (e^2
*x^2)/f^2]]/Sqrt[d]])/(2*e)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**(3/2),x)

[Out] Integral((d + e*x + f*sqrt(a + e**2*x**2/f**2))**(3/2), x)
```

Mathematica [A] time = 0.222256, size = 0, normalized size = 0.

$$\int \left(d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)^{3/2} dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(3/2),x]

[Out] Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(3/2), x]
```

Maple [F] time = 0.024, size = 0, normalized size = 0.

$$\int \left(d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2),x)

[Out] int((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex + \sqrt{\frac{e^2x^2}{f^2} + a}f + d\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(3/2), x)
```

Fricas [A] time = 0.341659, size = 1, normalized size = 0.

$$\left[\frac{15\,a\sqrt{d}f^2\log\left(\frac{ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}-2\sqrt{ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}+d}\sqrt{d}+2\,d}{ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}}\right)+2\left(4\,e^2x^2+12\,af^2+9\,dex+2\,d^2+(4\,efx-df)\sqrt{\frac{e^2x^2+af^2}{f^2}}\right)\sqrt{e\ldots}}{20\,e}\right.$$

$$-\frac{15\,a\sqrt{-d}f^2\arctan\left(\frac{\sqrt{ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}+d}}{\sqrt{-d}}\right)-\left(4\,e^2x^2+12\,af^2+9\,dex+2\,d^2+(4\,efx-df)\sqrt{\frac{e^2x^2+af^2}{f^2}}\right)\sqrt{ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}+\ldots}}{10\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/20*(15*a*sqrt(d)*f^2*log((e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2)
- 2*sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d)*sqrt(d) + 2*d)/
(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2))) + 2*(4*e^2*x^2 + 12*a*f^2
+ 9*d*e*x + 2*d^2 + (4*e*f*x - d*f)*sqrt((e^2*x^2 + a*f^2)/f^2))*
sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d))/e, -1/10*(15*a*sqr
t(-d)*f^2*arctan(sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d)/sq
rt(-d)) - (4*e^2*x^2 + 12*a*f^2 + 9*d*e*x + 2*d^2 + (4*e*f*x - d*
f)*sqrt((e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((e^2*x^2 + a*f^
2)/f^2) + d))/e]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**(3/2),x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + e**2*x**2/f**2))**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex + \sqrt{\frac{e^2x^2}{f^2} + a}f + d\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(3/2), x)
```

3.301 $\int \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\,dx$

Optimal. Leaf size=191

$$-\frac{af^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}}{2de\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}+\frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2de}+\frac{\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}}{3e}-\frac{af^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2\sqrt{d}e}$$

```
[Out] (a*f^2*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*d*e) + (d +
e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(3/2)/(3*e) - (a*f^2*(d + e*x +
f*Sqrt[a + (e^2*x^2)/f^2])^(3/2))/(2*d*e*(e*x + f*Sqrt[a + (e^2*x
^2)/f^2])) - (a*f^2*ArcTanh[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f
^2]]/Sqrt[d]])/(2*Sqrt[d]*e)
```

Rubi [A] time = 0.376297, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$

$$-\frac{af^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}}{2de\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}+\frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2de}+\frac{\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}}{3e}-\frac{af^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2\sqrt{d}e}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]],x]
```


```
[Out] (a*f^2*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*d*e) + (d +
e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(3/2)/(3*e) - (a*f^2*(d + e*x +
f*Sqrt[a + (e^2*x^2)/f^2])^(3/2))/(2*d*e*(e*x + f*Sqrt[a + (e^2*x
^2)/f^2])) - (a*f^2*ArcTanh[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f
^2]]/Sqrt[d]])/(2*Sqrt[d]*e)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(d + e*x + f*sqrt(a + e**2*x**2/f**2)), x)
```

Mathematica [A] time = 0.412064, size = 139, normalized size = 0.73

$$\frac{-\frac{3af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{f\sqrt{a+\frac{e^2x^2}{f^2}}+ex}+2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}-\frac{3af^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{\sqrt{d}}}{6e}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]],x]
```

```
[Out] ((-3*a*f^2*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(e*x + f*Sq
rt[a + (e^2*x^2)/f^2]) + 2*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^
(3/2) - (3*a*f^2*ArcTanh[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]
]/Sqrt[d]])/Sqrt[d])/(6*e)
```

Maple [F] time = 0.01, size = 0, normalized size = 0.

$$\int \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2),x)
```

```
[Out] int((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ex + \sqrt{\frac{e^2x^2}{f^2} + a}f + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(e*x + sqrt(e^2*x^2/f^2 + a)*f + d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + sqrt(e^2*x^2/f^2 + a)*f + d), x)
```

Fricas [A] time = 0.337377, size = 1, normalized size = 0.01

$$\left[\frac{3\,a\sqrt{d}f^2\log\left(\frac{\sqrt{d}f\sqrt{\frac{e^2x^2+af^2}{f^2}}+(ex+2\,d)\sqrt{d}-2\,\sqrt{ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}+d}d}{ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}}\right)+2\left(5\,dex-df\sqrt{\frac{e^2x^2+af^2}{f^2}}+2\,d^2\right)\sqrt{ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}+d}}{12\,de},\right.$$
$$\left.-\frac{3\,a\sqrt{-d}f^2\arctan\left(\frac{d}{\sqrt{ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}+d}\sqrt{-d}}\right)-\left(5\,dex-df\sqrt{\frac{e^2x^2+af^2}{f^2}}+2\,d^2\right)\sqrt{ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}+d}}{6\,de}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(e*x + sqrt(e^2*x^2/f^2 + a)*f + d),x, algorithm="fricas")
```

```
[Out] [1/12*(3*a*sqrt(d)*f^2*log((sqrt(d)*f*sqrt((e^2*x^2 + a*f^2)/f^2)
 + (e*x + 2*d)*sqrt(d) - 2*sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^
2) + d)*d)/(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2))) + 2*(5*d*e*x -
d*f*sqrt((e^2*x^2 + a*f^2)/f^2) + 2*d^2)*sqrt(e*x + f*sqrt((e^2*x
^2 + a*f^2)/f^2) + d))/(d*e), -1/6*(3*a*sqrt(-d)*f^2*arctan(d/(sq
rt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d)*sqrt(-d))) - (5*d*e*x
```

```
 - d*f*sqrt((e^2*x^2 + a*f^2)/f^2) + 2*d^2)*sqrt(e*x + f*sqrt((e^
2*x^2 + a*f^2)/f^2) + d))/(d*e)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**(1/2),x)

[Out] Integral(sqrt(d + e*x + f*sqrt(a + e**2*x**2/f**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ex + \sqrt{\frac{e^2x^2}{f^2} + a}f + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(e*x + sqrt(e^2*x^2/f^2 + a)*f + d),x, algorithm="giac")

[Out] integrate(sqrt(e*x + sqrt(e^2*x^2/f^2 + a)*f + d), x)
```

3.302 $\int \frac{1}{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}\,dx$

Optimal. Leaf size=147

$$\frac{af^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2d^{3/2}e}-\frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2de\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}+\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{e}$$

```
[Out] Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/e - (a*f^2*Sqrt[d + e*x
 + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*d*e*(e*x + f*Sqrt[a + (e^2*x^2)
/f^2])) + (a*f^2*ArcTanh[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]
]/Sqrt[d]])/(2*d^(3/2)*e)
```

Rubi [A] time = 0.24196, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185

$$\frac{af^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2d^{3/2}e}-\frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2de\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}+\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{e}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]],x]
```

```
[Out] Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/e - (a*f^2*Sqrt[d + e*x
 + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*d*e*(e*x + f*Sqrt[a + (e^2*x^2)
/f^2])) + (a*f^2*ArcTanh[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]
]/Sqrt[d]])/(2*d^(3/2)*e)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/sqrt(d + e*x + f*sqrt(a + e**2*x**2/f**2)), x)
```

Mathematica [A] time = 0.671794, size = 141, normalized size = 0.96

$$\frac{\frac{af^2\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}\right)}{2d^{3/2}}-\frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2d\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}+\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{e}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]],x]
```

```
[Out] (Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]] - (a*f^2*Sqrt[d + e*x
+ f*Sqrt[a + (e^2*x^2)/f^2]])/(2*d*(e*x + f*Sqrt[a + (e^2*x^2)/f^
2])) + (a*f^2*ArcTanh[Sqrt[d]/Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)
/f^2]]])/(2*d^(3/2)))/e
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2),x)
```

```
[Out] int(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ex+\sqrt{\frac{e^2x^2}{f^2}+a}f+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sqrt(e*x + sqrt(e^2*x^2/f^2 + a)*f + d),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(e*x + sqrt(e^2*x^2/f^2 + a)*f + d), x)
```

Fricas [A] time = 0.33397, size = 1, normalized size = 0.01

$$\left[\frac{a\sqrt{d}f^2\log\left(\frac{\sqrt{d}f\sqrt{\frac{e^2x^2+af^2}{f^2}}+(ex+2d)\sqrt{d}+2\sqrt{ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}+d}d}{ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}}\right)+2\left(dex-df\sqrt{\frac{e^2x^2+af^2}{f^2}}+2d^2\right)\sqrt{ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}+d}}{4d^2e}, \frac{a\sqrt{-d}\ldots}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sqrt(e*x + sqrt(e^2*x^2/f^2 + a)*f + d),x, algorithm="fricas")
```

```
[Out] [1/4*(a*sqrt(d)*f^2*log((sqrt(d)*f*sqrt((e^2*x^2 + a*f^2)/f^2) +
(e*x + 2*d)*sqrt(d) + 2*sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2)
+ d)*d)/(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2))) + 2*(d*e*x - d*f*s
qrt((e^2*x^2 + a*f^2)/f^2) + 2*d^2)*sqrt(e*x + f*sqrt((e^2*x^2 +
a*f^2)/f^2) + d))/(d^2*e), 1/2*(a*sqrt(-d)*f^2*arctan(d/(sqrt(e*x
 + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d)*sqrt(-d))) + (d*e*x - d*f*s
qrt((e^2*x^2 + a*f^2)/f^2) + 2*d^2)*sqrt(e*x + f*sqrt((e^2*x^2 +
a*f^2)/f^2) + d))/(d^2*e)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/sqrt(d + e*x + f*sqrt(a + e**2*x**2/f**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ex + \sqrt{\frac{e^2x^2}{f^2} + a}f + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(e*x + sqrt(e^2*x^2/f^2 + a)*f + d),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(e*x + sqrt(e^2*x^2/f^2 + a)*f + d), x)
```

3.303 $\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2}}\,dx$

Optimal. Leaf size=158

$$\frac{3af^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2d^{5/2}e}-\frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2d^2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}-\frac{\frac{af^2}{d^2}+1}{e\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}$$

```
[Out] -((1 + (a*f^2)/d^2)/(e*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])
) - (a*f^2*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*d^2*e*(e
*x + f*Sqrt[a + (e^2*x^2)/f^2])) + (3*a*f^2*ArcTanh[Sqrt[d + e*x
+ f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/(2*d^(5/2)*e)
```

Rubi [A] time = 0.341548, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$

$$\frac{3af^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2d^{5/2}e}-\frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2d^2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}-\frac{\frac{af^2}{d^2}+1}{e\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-3/2),x]
```

```
[Out] -((1 + (a*f^2)/d^2)/(e*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])
) - (a*f^2*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*d^2*e*(e
*x + f*Sqrt[a + (e^2*x^2)/f^2])) + (3*a*f^2*ArcTanh[Sqrt[d + e*x
+ f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/(2*d^(5/2)*e)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**(3/2),x)

[Out] Integral((d + e*x + f*sqrt(a + e**2*x**2/f**2))**(-3/2), x)
```

Mathematica [A] time = 0.403787, size = 0, normalized size = 0.

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-3/2),x]

[Out] Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-3/2), x]
```

Maple [F] time = 0.013, size = 0, normalized size = 0.

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2),x)

[Out] int(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a}f+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(-3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(-3/2), x)
```

Fricas [A] time = 0.343136, size = 1, normalized size = 0.01

$$\left[\frac{3\left(a\sqrt{d}f^3\sqrt{\frac{e^2x^2+af^2}{f^2}}+\left(aef^2x+adf^2\right)\sqrt{d}\right)\log\left(\frac{\sqrt{d}f\sqrt{\frac{e^2x^2+af^2}{f^2}}+(ex+2\,d)\sqrt{d}+2\sqrt{ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}+d}\,d}{ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}}\right)-2\left(3\,adf^2-d^2ex+d^2f\sqrt{\cdots}\right.}{4\left(d^3e^2x+d^3ef\sqrt{\frac{e^2x^2+af^2}{f^2}}+d^4e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(-3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*(a*sqrt(d)*f^3*sqrt((e^2*x^2 + a*f^2)/f^2) + (a*e*f^2*x +
 a*d*f^2)*sqrt(d))*log((sqrt(d)*f*sqrt((e^2*x^2 + a*f^2)/f^2) + (
e*x + 2*d)*sqrt(d) + 2*sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) +
 d)*d)/(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2))) - 2*(3*a*d*f^2 - d^
2*e*x + d^2*f*sqrt((e^2*x^2 + a*f^2)/f^2) + 2*d^3)*sqrt(e*x + f*s
qrt((e^2*x^2 + a*f^2)/f^2) + d))/(d^3*e^2*x + d^3*e*f*sqrt((e^2*x
^2 + a*f^2)/f^2) + d^4*e), 1/2*(3*(a*sqrt(-d)*f^3*sqrt((e^2*x^2 +
 a*f^2)/f^2) + (a*e*f^2*x + a*d*f^2)*sqrt(-d))*arctan(d/(sqrt(e*x
 + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d)*sqrt(-d))) - (3*a*d*f^2 - d
^2*e*x + d^2*f*sqrt((e^2*x^2 + a*f^2)/f^2) + 2*d^3)*sqrt(e*x + f*
sqrt((e^2*x^2 + a*f^2)/f^2) + d))/(d^3*e^2*x + d^3*e*f*sqrt((e^2*
x^2 + a*f^2)/f^2) + d^4*e)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**(3/2),x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + e**2*x**2/f**2))**(-3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ex + \sqrt{\frac{e^2x^2}{f^2} + a}f + d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(-3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(-3/2), x)
```

3.304 $\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{5/2}}\,dx$

Optimal. Leaf size=199

$$\frac{5af^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2d^{7/2}e}-\frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2d^3e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}-\frac{2af^2}{d^3e\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}-\frac{\frac{af^2}{d^2}+1}{3e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}}$$

```
[Out] -(1 + (a*f^2)/d^2)/(3*e*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(3/
2)) - (2*a*f^2)/(d^3*e*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])
 - (a*f^2*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*d^3*e*(e*
x + f*Sqrt[a + (e^2*x^2)/f^2])) + (5*a*f^2*ArcTanh[Sqrt[d + e*x +
 f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/(2*d^(7/2)*e)
```

Rubi [A] time = 0.42237, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185

$$\frac{5af^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2d^{7/2}e}-\frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2d^3e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}-\frac{2af^2}{d^3e\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}-\frac{\frac{af^2}{d^2}+1}{3e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-5/2),x]
```

```
[Out] -(1 + (a*f^2)/d^2)/(3*e*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(3/
2)) - (2*a*f^2)/(d^3*e*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])
 - (a*f^2*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*d^3*e*(e*
x + f*Sqrt[a + (e^2*x^2)/f^2])) + (5*a*f^2*ArcTanh[Sqrt[d + e*x +
 f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/(2*d^(7/2)*e)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**(5/2),x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + e**2*x**2/f**2))**(-5/2), x)
```

Mathematica [A] time = 0.722339, size = 0, normalized size = 0.

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-5/2),x]
```

```
[Out] Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-5/2), x]
```

Maple [F] time = 0.013, size = 0, normalized size = 0.

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(5/2),x)
```

```
[Out] int(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ex + \sqrt{\frac{e^2x^2}{f^2} + a}f + d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(-5/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(-5/2), x)
```

Fricas [A] time = 0.356918, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(-5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(15*((a^2*f^5 - 2*a*d*e*f^3*x - a*d^2*f^3)*sqrt(d)*sqrt((e
^2*x^2 + a*f^2)/f^2) - (2*a*d*e^2*f^2*x^2 - a^2*d*f^4 + a*d^3*f^2
 - (a^2*e*f^4 - 3*a*d^2*e*f^2)*x)*sqrt(d))*log((sqrt(d)*f*sqrt((e
^2*x^2 + a*f^2)/f^2) + (e*x + 2*d)*sqrt(d) + 2*sqrt(e*x + f*sqrt(
(e^2*x^2 + a*f^2)/f^2) + d)*d)/(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^
2))) - 2*(15*a^2*d*f^4 + 6*d^3*e^2*x^2 - 17*a*d^3*f^2 - 2*d^5 - (
35*a*d^2*e*f^2 - d^4*e)*x + (5*a*d^2*f^3 - 6*d^3*e*f*x - d^4*f)*s
qrt((e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f
^2) + d))/(2*d^5*e^3*x^2 - a*d^5*e*f^2 + d^7*e - (a*d^4*e^2*f^2 -
 3*d^6*e^2)*x - (a*d^4*e*f^3 - 2*d^5*e^2*f*x - d^6*e*f)*sqrt((e^2
*x^2 + a*f^2)/f^2)), -1/6*(15*((a^2*f^5 - 2*a*d*e*f^3*x - a*d^2*f
^3)*sqrt(-d)*sqrt((e^2*x^2 + a*f^2)/f^2) - (2*a*d*e^2*f^2*x^2 - a
^2*d*f^4 + a*d^3*f^2 - (a^2*e*f^4 - 3*a*d^2*e*f^2)*x)*sqrt(-d))*a
rctan(d/(sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d)*sqrt(-d)))
 - (15*a^2*d*f^4 + 6*d^3*e^2*x^2 - 17*a*d^3*f^2 - 2*d^5 - (35*a*d
^2*e*f^2 - d^4*e)*x + (5*a*d^2*f^3 - 6*d^3*e*f*x - d^4*f)*sqrt((e
^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) +
d))/(2*d^5*e^3*x^2 - a*d^5*e*f^2 + d^7*e - (a*d^4*e^2*f^2 - 3*d^6
*e^2)*x - (a*d^4*e*f^3 - 2*d^5*e^2*f*x - d^6*e*f)*sqrt((e^2*x^2 +
 a*f^2)/f^2))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**(5/2),x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + e**2*x**2/f**2))**(-5/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ex + \sqrt{\frac{e^2x^2}{f^2} + a}f + d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(-5/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(-5/2), x)
```

3.305 $\int \sqrt{x-\sqrt{-4+x^2}}\,dx$

Optimal. Leaf size=41

$$\frac{1}{3}\left(x-\sqrt{x^2-4}\right)^{3/2}+\frac{4}{\sqrt{x-\sqrt{x^2-4}}}$$

```
[Out] 4/Sqrt[x - Sqrt[-4 + x^2]] + (x - Sqrt[-4 + x^2])^(3/2)/3
```

Rubi [A] time = 0.0319785, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118

$$\frac{1}{3}\left(x-\sqrt{x^2-4}\right)^{3/2}+\frac{4}{\sqrt{x-\sqrt{x^2-4}}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x - Sqrt[-4 + x^2]],x]

[Out] 4/Sqrt[x - Sqrt[-4 + x^2]] + (x - Sqrt[-4 + x^2])^(3/2)/3
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x-\sqrt{x^2-4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x-(x**2-4)**(1/2))**(1/2),x)

[Out] Integral(sqrt(x - sqrt(x**2 - 4)), x)
```

Mathematica [A] time = 0.0244141, size = 34, normalized size = 0.83

$$\frac{2}{3}\sqrt{x-\sqrt{x^2-4}}\left(\sqrt{x^2-4}+2x\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[x - Sqrt[-4 + x^2]],x]
```

```
[Out] (2*Sqrt[x - Sqrt[-4 + x^2]]*(2*x + Sqrt[-4 + x^2]))/3
```

Maple [F] time = 0.051, size = 0, normalized size = 0.

$$\int \sqrt{x-\sqrt{x^2-4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x-(x^2-4)^(1/2))^(1/2),x)
```

```
[Out] int((x-(x^2-4)^(1/2))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x-\sqrt{x^2-4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x - sqrt(x^2 - 4)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x - sqrt(x^2 - 4)), x)
```

Fricas [A] time = 0.280239, size = 41, normalized size = 1.

$$\frac{2\left(x^2-\sqrt{x^2-4}x+4\right)}{3\sqrt{x-\sqrt{x^2-4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x - sqrt(x^2 - 4)),x, algorithm="fricas")
```

```
[Out] 2/3*(x^2 - sqrt(x^2 - 4)*x + 4)/sqrt(x - sqrt(x^2 - 4))
```

Sympy [A] time = 0.902583, size = 42, normalized size = 1.02

$$\frac{4x\sqrt{x-\sqrt{x^2-4}}}{3}+\frac{2\sqrt{x-\sqrt{x^2-4}}\sqrt{x^2-4}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x-(x**2-4)**(1/2))**(1/2),x)
```

```
[Out] 4*x*sqrt(x - sqrt(x**2 - 4))/3 + 2*sqrt(x - sqrt(x**2 - 4))*sqrt(
x**2 - 4)/3
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x-\sqrt{x^2-4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x - sqrt(x^2 - 4)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x - sqrt(x^2 - 4)), x)
```

3.306 $\int \sqrt{ax + b\sqrt{c + \frac{a^2x^2}{b^2}}}\, dx$

Optimal. Leaf size=69

$$\frac{\left(b\sqrt{\frac{a^2x^2}{b^2}+c}+ax\right)^{3/2}}{3a}-\frac{b^2c}{a\sqrt{b\sqrt{\frac{a^2x^2}{b^2}+c}+ax}}$$

```
[Out] -((b^2*c)/(a*Sqrt[a*x + b*Sqrt[c + (a^2*x^2)/b^2]])) + (a*x + b*S
qrt[c + (a^2*x^2)/b^2])^(3/2)/(3*a)
```

Rubi [A] time = 0.108216, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{\left(b\sqrt{\frac{a^2x^2}{b^2}+c}+ax\right)^{3/2}}{3a}-\frac{b^2c}{a\sqrt{b\sqrt{\frac{a^2x^2}{b^2}+c}+ax}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a*x + b*Sqrt[c + (a^2*x^2)/b^2]],x]
```

```
[Out] -((b^2*c)/(a*Sqrt[a*x + b*Sqrt[c + (a^2*x^2)/b^2]])) + (a*x + b*S
qrt[c + (a^2*x^2)/b^2])^(3/2)/(3*a)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ax + b\sqrt{\frac{a^2x^2}{b^2} + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a*x+b*(c+a**2*x**2/b**2)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(a*x + b*sqrt(a**2*x**2/b**2 + c)), x)
```

Mathematica [A] time = 0.133149, size = 57, normalized size = 0.83

$$\frac{2\left(2ax-b\sqrt{\frac{a^2x^2}{b^2}+c}\right)\sqrt{b\sqrt{\frac{a^2x^2}{b^2}+c}+ax}}{3a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a*x + b*Sqrt[c + (a^2*x^2)/b^2]],x]
```

```
[Out] (2*(2*a*x - b*Sqrt[c + (a^2*x^2)/b^2])*Sqrt[a*x + b*Sqrt[c + (a^2
*x^2)/b^2]])/(3*a)
```

Maple [F] time = 0.028, size = 0, normalized size = 0.

$$\int \sqrt{ax+b\sqrt{c+\frac{a^2x^2}{b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a*x+b*(c+a^2*x^2/b^2)^(1/2))^(1/2),x)
```

```
[Out] int((a*x+b*(c+a^2*x^2/b^2)^(1/2))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ax+\sqrt{\frac{a^2x^2}{b^2}+c}\,b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a*x + sqrt(a^2*x^2/b^2 + c)*b),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x + sqrt(a^2*x^2/b^2 + c)*b), x)
```

Fricas [A] time = 0.331644, size = 130, normalized size = 1.88

$$\frac{2\left(a^2x^2 + abx\sqrt{\frac{a^2x^2+b^2c}{b^2}} - b^2c\right)\sqrt{ax + b\sqrt{\frac{a^2x^2+b^2c}{b^2}}}}{3\left(a^2x + ab\sqrt{\frac{a^2x^2+b^2c}{b^2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a*x + sqrt(a^2*x^2/b^2 + c)*b),x, algorithm="fricas")
```

```
[Out] 2/3*(a^2*x^2 + a*b*x*sqrt((a^2*x^2 + b^2*c)/b^2) - b^2*c)*sqrt(a*
x + b*sqrt((a^2*x^2 + b^2*c)/b^2))/(a^2*x + a*b*sqrt((a^2*x^2 + b
^2*c)/b^2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{ax + b\sqrt{\frac{a^2x^2}{b^2} + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a*x+b*(c+a**2*x**2/b**2)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(a*x + b*sqrt(a**2*x**2/b**2 + c)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{ax + \sqrt{\frac{a^2x^2}{b^2} + c}b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a*x + sqrt(a^2*x^2/b^2 + c)*b),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x + sqrt(a^2*x^2/b^2 + c)*b), x)
```

3.307 $\int \sqrt{1+\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=45

$$\frac{2x}{\sqrt{\sqrt{1-x^2}+1}}-\frac{2x^3}{3\left(\sqrt{1-x^2}+1\right)^{3/2}}$$

```
[Out] (-2*x^3)/(3*(1 + Sqrt[1 - x^2])^(3/2)) + (2*x)/Sqrt[1 + Sqrt[1 -
x^2]]
```

Rubi [A] time = 0.0249283, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059

$$\frac{2x}{\sqrt{\sqrt{1-x^2}+1}}-\frac{2x^3}{3\left(\sqrt{1-x^2}+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 + Sqrt[1 - x^2]],x]
```

```
[Out] (-2*x^3)/(3*(1 + Sqrt[1 - x^2])^(3/2)) + (2*x)/Sqrt[1 + Sqrt[1 -
x^2]]
```

Rubi in Sympy [A] time = 1.17951, size = 36, normalized size = 0.8

$$-\frac{2x^3}{3\left(\sqrt{-x^2+1}+1\right)^{\frac{3}{2}}}+\frac{2x}{\sqrt{\sqrt{-x^2+1}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+(-x**2+1)**(1/2))**(1/2),x)
```

```
[Out] -2*x**3/(3*(sqrt(-x**2 + 1) + 1)**(3/2)) + 2*x/sqrt(sqrt(-x**2 +
1) + 1)
```

Mathematica [A] time = 0.0898973, size = 35, normalized size = 0.78

$$\frac{2x\left(\sqrt{1-x^2}+2\right)}{3\sqrt{\sqrt{1-x^2}+1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 + Sqrt[1 - x^2]],x]
```

```
[Out] (2*x*(2 + Sqrt[1 - x^2]))/(3*Sqrt[1 + Sqrt[1 - x^2]])
```

Maple [C] time = 0.12, size = 60, normalized size = 1.3

$$\frac{\frac{i}{8}}{\sqrt{\pi}}\left(\frac{32\,i}{3}\sqrt{\pi}\sqrt{2}x^3\cos\left(\frac{3\,\arcsin(x)}{2}\right)-8\,i\sqrt{\pi}\sqrt{2}\left(-\frac{4\,x^4}{3}+\frac{2\,x^2}{3}+\frac{2}{3}\right)\sin\left(\frac{3\,\arcsin(x)}{2}\right)\frac{1}{\sqrt{-x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+(-x^2+1)^(1/2))^(1/2),x)
```

```
[Out] 1/8*I/Pi^(1/2)*(32/3*I*Pi^(1/2)*2^(1/2)*x^3*cos(3/2*arcsin(x))-8*
I*Pi^(1/2)*2^(1/2)*(-4/3*x^4+2/3*x^2+2/3)*sin(3/2*arcsin(x))/(-x^
2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\sqrt{-x^2+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(sqrt(-x^2 + 1) + 1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sqrt(-x^2 + 1) + 1), x)
```

Fricas [A] time = 0.294554, size = 46, normalized size = 1.02

$$\frac{2\left(x^2-\sqrt{-x^2+1}+1\right)\sqrt{\sqrt{-x^2+1}+1}}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(sqrt(-x^2 + 1) + 1),x, algorithm="fricas")
```

```
[Out] 2/3*(x^2 - sqrt(-x^2 + 1) + 1)*sqrt(sqrt(-x^2 + 1) + 1)/x
```

Sympy [A] time = 3.91283, size = 413, normalized size = 9.18

$$\begin{cases} \frac{\sqrt{2}x^3\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12i\pi\sqrt{x^2-1}\sqrt{i\sqrt{x^2-1}+1}+12\pi\sqrt{i\sqrt{x^2-1}+1}} - \frac{3\sqrt{2}ix\sqrt{x^2-1}\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12i\pi\sqrt{x^2-1}\sqrt{i\sqrt{x^2-1}+1}+12\pi\sqrt{i\sqrt{x^2-1}+1}} - \frac{3\sqrt{2}x\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12i\pi\sqrt{x^2-1}\sqrt{i\sqrt{x^2-1}+1}+12\pi\sqrt{i\sqrt{x^2-1}+1}} & \text{for } \left|x^2\right| > 1 \\ \frac{\sqrt{2}x^3\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{-x^2+1}\sqrt{\sqrt{-x^2+1}+1}+12\pi\sqrt{\sqrt{-x^2+1}+1}} - \frac{3\sqrt{2}x\sqrt{-x^2+1}\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{-x^2+1}\sqrt{\sqrt{-x^2+1}+1}+12\pi\sqrt{\sqrt{-x^2+1}+1}} - \frac{3\sqrt{2}x\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{-x^2+1}\sqrt{\sqrt{-x^2+1}+1}+12\pi\sqrt{\sqrt{-x^2+1}+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+(-x**2+1)**(1/2))**(1/2),x)
```

```
[Out] Piecewise((sqrt(2)*x**3*gamma(-1/4)*gamma(1/4)/(12*I*pi*sqrt(x**2
 - 1)*sqrt(I*sqrt(x**2 - 1) + 1) + 12*pi*sqrt(I*sqrt(x**2 - 1) +
1)) - 3*sqrt(2)*I*x*sqrt(x**2 - 1)*gamma(-1/4)*gamma(1/4)/(12*I*p
i*sqrt(x**2 - 1)*sqrt(I*sqrt(x**2 - 1) + 1) + 12*pi*sqrt(I*sqrt(x
**2 - 1) + 1)) - 3*sqrt(2)*x*gamma(-1/4)*gamma(1/4)/(12*I*pi*sqrt
(x**2 - 1)*sqrt(I*sqrt(x**2 - 1) + 1) + 12*pi*sqrt(I*sqrt(x**2 -
1) + 1)), Abs(x**2) > 1), (sqrt(2)*x**3*gamma(-1/4)*gamma(1/4)/(1
2*pi*sqrt(-x**2 + 1)*sqrt(sqrt(-x**2 + 1) + 1) + 12*pi*sqrt(sqrt(
-x**2 + 1) + 1)) - 3*sqrt(2)*x*sqrt(-x**2 + 1)*gamma(-1/4)*gamma(
1/4)/(12*pi*sqrt(-x**2 + 1)*sqrt(sqrt(-x**2 + 1) + 1) + 12*pi*sqr
t(sqrt(-x**2 + 1) + 1)) - 3*sqrt(2)*x*gamma(-1/4)*gamma(1/4)/(12*
pi*sqrt(-x**2 + 1)*sqrt(sqrt(-x**2 + 1) + 1) + 12*pi*sqrt(sqrt(-x
**2 + 1) + 1)), True))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sqrt{-x^2+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(sqrt(-x^2 + 1) + 1),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sqrt(-x^2 + 1) + 1), x)
```

3.308 $\int \sqrt{1+\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=41

$$\frac{2x}{\sqrt{\sqrt{x^2+1}+1}}+\frac{2x^3}{3\left(\sqrt{x^2+1}+1\right)^{3/2}}$$

```
[Out] (2*x^3)/(3*(1 + Sqrt[1 + x^2])^(3/2)) + (2*x)/Sqrt[1 + Sqrt[1 + x
^2]]
```

Rubi [A] time = 0.0194902, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$\frac{2x}{\sqrt{\sqrt{x^2+1}+1}}+\frac{2x^3}{3\left(\sqrt{x^2+1}+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 + Sqrt[1 + x^2]],x]
```

```
[Out] (2*x^3)/(3*(1 + Sqrt[1 + x^2])^(3/2)) + (2*x)/Sqrt[1 + Sqrt[1 + x
^2]]
```

Rubi in Sympy [A] time = 1.15661, size = 36, normalized size = 0.88

$$\frac{2x^3}{3\left(\sqrt{x^2+1}+1\right)^{\frac{3}{2}}}+\frac{2x}{\sqrt{\sqrt{x^2+1}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+(x**2+1)**(1/2))**(1/2),x)
```

```
[Out] 2*x**3/(3*(sqrt(x**2 + 1) + 1)**(3/2)) + 2*x/sqrt(sqrt(x**2 + 1)
+ 1)
```

Mathematica [A] time = 0.080765, size = 44, normalized size = 1.07

$$\frac{2\left(\sqrt{x^2+1}-1\right)\sqrt{\sqrt{x^2+1}+1}\left(\sqrt{x^2+1}+2\right)}{3x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 + Sqrt[1 + x^2]],x]
```

```
[Out] (2*(-1 + Sqrt[1 + x^2])*Sqrt[1 + Sqrt[1 + x^2]]*(2 + Sqrt[1 + x^2
]))/(3*x)
```

Maple [C] time = 0.04, size = 55, normalized size = 1.3

$$-\frac{1}{8\sqrt{\pi}}\left(-\frac{32\sqrt{\pi}\sqrt{2}x^3}{3}\cosh\left(\frac{3\,Arcsinh\,(x)}{2}\right)-8\,\frac{\sqrt{\pi}\sqrt{2}\left(-4/3\,x^4-2/3\,x^2+2/3\right)\sinh\left(3/2\,Arcsinh\,(x)\right)}{\sqrt{x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+(x^2+1)^(1/2))^(1/2),x)
```

```
[Out] -1/8/Pi^(1/2)*(-32/3*Pi^(1/2)*2^(1/2)*x^3*cosh(3/2*arcsinh(x))-8*
Pi^(1/2)*2^(1/2)*(-4/3*x^4-2/3*x^2+2/3)*sinh(3/2*arcsinh(x))/(x^2
+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\sqrt{x^2+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(sqrt(x^2 + 1) + 1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sqrt(x^2 + 1) + 1), x)
```

Fricas [A] time = 0.291077, size = 38, normalized size = 0.93

$$\frac{2\left(x^2+\sqrt{x^2+1}-1\right)\sqrt{\sqrt{x^2+1}+1}}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(sqrt(x^2 + 1) + 1),x, algorithm="fricas")
```

```
[Out] 2/3*(x^2 + sqrt(x^2 + 1) - 1)*sqrt(sqrt(x^2 + 1) + 1)/x
```

Sympy [A] time = 3.8663, size = 197, normalized size = 4.8

$$-\frac{\sqrt{2}x^3\,\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{x^2+1}\sqrt{\sqrt{x^2+1}+1}+12\pi\sqrt{\sqrt{x^2+1}+1}}-\frac{3\sqrt{2}x\sqrt{x^2+1}\,\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{x^2+1}\sqrt{\sqrt{x^2+1}+1}+12\pi\sqrt{\sqrt{x^2+1}+1}}-\frac{3\sqrt{2}x\,\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{x^2+1}\sqrt{\sqrt{x^2+1}+1}+12\pi\sqrt{\sqrt{x^2+1}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+(x**2+1)**(1/2))**(1/2),x)
```

```
[Out] -sqrt(2)*x**3*gamma(-1/4)*gamma(1/4)/(12*pi*sqrt(x**2 + 1)*sqrt(s
qrt(x**2 + 1) + 1) + 12*pi*sqrt(sqrt(x**2 + 1) + 1)) - 3*sqrt(2)*
x*sqrt(x**2 + 1)*gamma(-1/4)*gamma(1/4)/(12*pi*sqrt(x**2 + 1)*sqr
t(sqrt(x**2 + 1) + 1) + 12*pi*sqrt(sqrt(x**2 + 1) + 1)) - 3*sqrt(
2)*x*gamma(-1/4)*gamma(1/4)/(12*pi*sqrt(x**2 + 1)*sqrt(sqrt(x**2
+ 1) + 1) + 12*pi*sqrt(sqrt(x**2 + 1) + 1))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\sqrt{x^2+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(sqrt(x^2 + 1) + 1),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sqrt(x^2 + 1) + 1), x)
```

3.309 $\int \sqrt{5+\sqrt{25+x^2}}\,dx$

Optimal. Leaf size=41

$$\frac{10x}{\sqrt{\sqrt{x^2+25}+5}}+\frac{2x^3}{3\left(\sqrt{x^2+25}+5\right)^{3/2}}$$

```
[Out] (2*x^3)/(3*(5 + Sqrt[25 + x^2])^(3/2)) + (10*x)/Sqrt[5 + Sqrt[25
+ x^2]]
```

Rubi [A] time = 0.0199868, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$\frac{10x}{\sqrt{\sqrt{x^2+25}+5}}+\frac{2x^3}{3\left(\sqrt{x^2+25}+5\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[5 + Sqrt[25 + x^2]],x]
```

```
[Out] (2*x^3)/(3*(5 + Sqrt[25 + x^2])^(3/2)) + (10*x)/Sqrt[5 + Sqrt[25
+ x^2]]
```

Rubi in Sympy [A] time = 1.14524, size = 36, normalized size = 0.88

$$\frac{2x^3}{3\left(\sqrt{x^2+25}+5\right)^{\frac{3}{2}}}+\frac{10x}{\sqrt{\sqrt{x^2+25}+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5+(x**2+25)**(1/2))**(1/2),x)
```

```
[Out] 2*x**3/(3*(sqrt(x**2 + 25) + 5)**(3/2)) + 10*x/sqrt(sqrt(x**2 + 2
5) + 5)
```

Mathematica [A] time = 0.0820932, size = 44, normalized size = 1.07

$$\frac{2\left(\sqrt{x^2+25}-5\right)\sqrt{\sqrt{x^2+25}+5}\left(\sqrt{x^2+25}+10\right)}{3x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[5 + Sqrt[25 + x^2]],x]
```

```
[Out] (2*(-5 + Sqrt[25 + x^2])*Sqrt[5 + Sqrt[25 + x^2]]*(10 + Sqrt[25 +
 x^2]))/(3*x)
```

Maple [C] time = 0.022, size = 64, normalized size = 1.6

$$-\frac{5\sqrt{5}}{8\sqrt{\pi}}\left(-\frac{32\sqrt{\pi}\sqrt{2}x^3}{375}\cosh\left(\frac{3}{2}Arcsinh\left(\frac{x}{5}\right)\right)-8\frac{\sqrt{\pi}\sqrt{2}\sinh\left(3/2\,Arcsinh\left(x/5\right)\right)}{\sqrt{1/25\,x^2+1}}\left(-\frac{4\,x^4}{1875}-\frac{2\,x^2}{75}+2/3\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5+(x^2+25)^(1/2))^(1/2),x)
```

```
[Out] -5/8*5^(1/2)/Pi^(1/2)*(-32/375*Pi^(1/2)*2^(1/2)*x^3*cosh(3/2*arcs
inh(1/5*x))-8*Pi^(1/2)*2^(1/2)*(-4/1875*x^4-2/75*x^2+2/3)*sinh(3/
2*arcsinh(1/5*x))/(1/25*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\sqrt{x^2+25}+5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(sqrt(x^2 + 25) + 5),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sqrt(x^2 + 25) + 5), x)
```

Fricas [A] time = 0.287792, size = 41, normalized size = 1.

$$\frac{2\left(x^2+5\sqrt{x^2+25}-25\right)\sqrt{\sqrt{x^2+25}+5}}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(sqrt(x^2 + 25) + 5),x, algorithm="fricas")
```

```
[Out] 2/3*(x^2 + 5*sqrt(x^2 + 25) - 25)*sqrt(sqrt(x^2 + 25) + 5)/x
```

Sympy [A] time = 3.93124, size = 197, normalized size = 4.8

$$-\frac{\sqrt{2}x^3\,(-\tfrac{1}{4})\,(\tfrac{1}{4})}{12\pi\sqrt{x^2+25}\sqrt{\sqrt{x^2+25}+5}+60\pi\sqrt{\sqrt{x^2+25}+5}} - \frac{15\sqrt{2}x\sqrt{x^2+25}\,(-\tfrac{1}{4})\,(\tfrac{1}{4})}{12\pi\sqrt{x^2+25}\sqrt{\sqrt{x^2+25}+5}+60\pi\sqrt{\sqrt{x^2+25}+5}} - \frac{75\sqrt{2}x\,(-\tfrac{1}{4})\,(\tfrac{1}{4})}{12\pi\sqrt{x^2+25}\sqrt{\sqrt{x^2+25}+5}+60\pi\sqrt{\sqrt{x^2+25}+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5+(x**2+25)**(1/2))**(1/2),x)
```

```
[Out] -sqrt(2)*x**3*gamma(-1/4)*gamma(1/4)/(12*pi*sqrt(x**2 + 25)*sqrt(
sqrt(x**2 + 25) + 5) + 60*pi*sqrt(sqrt(x**2 + 25) + 5)) - 15*sqrt
(2)*x*sqrt(x**2 + 25)*gamma(-1/4)*gamma(1/4)/(12*pi*sqrt(x**2 + 2
5)*sqrt(sqrt(x**2 + 25) + 5) + 60*pi*sqrt(sqrt(x**2 + 25) + 5)) -
 75*sqrt(2)*x*gamma(-1/4)*gamma(1/4)/(12*pi*sqrt(x**2 + 25)*sqrt(
sqrt(x**2 + 25) + 5) + 60*pi*sqrt(sqrt(x**2 + 25) + 5))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\sqrt{x^2+25}+5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(sqrt(x^2 + 25) + 5),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sqrt(x^2 + 25) + 5), x)
```

3.310 $\int \sqrt{a+b\sqrt{\frac{a^2}{b^2}+cx^2}}\,dx$

Optimal. Leaf size=66

$$\frac{2ax}{\sqrt{b\sqrt{\frac{a^2}{b^2}+cx^2}+a}}+\frac{2b^2cx^3}{3\left(b\sqrt{\frac{a^2}{b^2}+cx^2}+a\right)^{3/2}}$$

```
[Out] (2*b^2*c*x^3)/(3*(a + b*Sqrt[a^2/b^2 + c*x^2])^(3/2)) + (2*a*x)/S
qrt[a + b*Sqrt[a^2/b^2 + c*x^2]]
```

Rubi [A] time = 0.0755461, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04

$$\frac{2ax}{\sqrt{b\sqrt{\frac{a^2}{b^2}+cx^2}+a}}+\frac{2b^2cx^3}{3\left(b\sqrt{\frac{a^2}{b^2}+cx^2}+a\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*Sqrt[a^2/b^2 + c*x^2]],x]
```

```
[Out] (2*b^2*c*x^3)/(3*(a + b*Sqrt[a^2/b^2 + c*x^2])^(3/2)) + (2*a*x)/S
qrt[a + b*Sqrt[a^2/b^2 + c*x^2]]
```

Rubi in Sympy [A] time = 2.37248, size = 60, normalized size = 0.91

$$\frac{2ax}{\sqrt{a+b\sqrt{\frac{a^2}{b^2}+cx^2}}}+\frac{2b^2cx^3}{3\left(a+b\sqrt{\frac{a^2}{b^2}+cx^2}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b*(a**2/b**2+c*x**2)**(1/2))**(1/2),x)
```

```
[Out] 2*a*x/sqrt(a + b*sqrt(a**2/b**2 + c*x**2)) + 2*b**2*c*x**3/(3*(a
+ b*sqrt(a**2/b**2 + c*x**2))**(3/2))
```

Mathematica [A] time = 0.306316, size = 55, normalized size = 0.83

$$\frac{2bx\sqrt{\frac{a^2}{b^2}+cx^2}+4ax}{3\sqrt{b\sqrt{\frac{a^2}{b^2}+cx^2}+a}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b*Sqrt[a^2/b^2 + c*x^2]],x]
```

```
[Out] (4*a*x + 2*b*x*Sqrt[a^2/b^2 + c*x^2])/(3*Sqrt[a + b*Sqrt[a^2/b^2
+ c*x^2]])
```

Maple [F] time = 0.026, size = 0, normalized size = 0.

$$\int\sqrt{a+b\sqrt{\frac{a^2}{b^2}+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b*(1/b^2*a^2+c*x^2)^(1/2))^(1/2),x)
```

```
[Out] int((a+b*(1/b^2*a^2+c*x^2)^(1/2))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\sqrt{cx^2+\frac{a^2}{b^2}}b+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(sqrt(c*x^2 + a^2/b^2)*b + a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sqrt(c*x^2 + a^2/b^2)*b + a), x)
```

Fricas [A] time = 0.352361, size = 95, normalized size = 1.44

$$\frac{2\left(b^{2}cx^{2}+ab\sqrt{\frac{b^{2}cx^{2}+a^{2}}{b^{2}}}-a^{2}\right)\sqrt{b\sqrt{\frac{b^{2}cx^{2}+a^{2}}{b^{2}}}+a}}{3\,b^{2}cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(sqrt(c*x^2 + a^2/b^2)*b + a),x, algorithm="fricas")
```

```
[Out] 2/3*(b^2*c*x^2 + a*b*sqrt((b^2*c*x^2 + a^2)/b^2) - a^2)*sqrt(b*sq
rt((b^2*c*x^2 + a^2)/b^2) + a)/(b^2*c*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a+b\sqrt{\frac{a^{2}}{b^{2}}+cx^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b*(a**2/b**2+c*x**2)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sqrt(a**2/b**2 + c*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\sqrt{cx^{2}+\frac{a^{2}}{b^{2}}}b+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(sqrt(c*x^2 + a^2/b^2)*b + a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sqrt(c*x^2 + a^2/b^2)*b + a), x)
```

3.311 $\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^n dx$

Optimal. Leaf size=166

$$\frac{f^2(4ae^2-b^2f^2)\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}{}_2F_1\left(2,n+1;n+2;\frac{2e\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)}{2de-bf^2}\right)}{2e(n+1)(2de-bf^2)^2}+\frac{\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}}{2e(n+1)}$$

```
[Out] (d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(1 + n)/(2*e*(1 + n))
 + (f^2*(4*a*e^2 - b^2*f^2)*(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)
/f^2])^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, (2*e*(d + e*x +
 f*Sqrt[a + b*x + (e^2*x^2)/f^2]))/(2*d*e - b*f^2)])/(2*e*(2*d*e
- b*f^2)^2*(1 + n))
```

Rubi [A] time = 0.386172, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$

$$\frac{f^2(4ae^2-b^2f^2)\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}{}_2F_1\left(2,n+1;n+2;\frac{2e\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)}{2de-bf^2}\right)}{2e(n+1)(2de-bf^2)^2}+\frac{\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}}{2e(n+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^n,x]
```

```
[Out] (d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(1 + n)/(2*e*(1 + n))
 + (f^2*(4*a*e^2 - b^2*f^2)*(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)
/f^2])^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, (2*e*(d + e*x +
 f*Sqrt[a + b*x + (e^2*x^2)/f^2]))/(2*d*e - b*f^2)])/(2*e*(2*d*e
- b*f^2)^2*(1 + n))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**n,x)

[Out] Integral((d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2))**n, x)
```

Mathematica [A] time = 0.20409, size = 0, normalized size = 0.

$$\int \left(d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}\right)^n dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^n,x]

[Out] Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^n, x]
```

Maple [F] time = 0.024, size = 0, normalized size = 0.

$$\int \left(d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^n,x)

[Out] int((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex + \sqrt{bx + \frac{e^2x^2}{f^2} + a}f + d\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^n,x, algorithm="maxima")
```

```
[Out] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^n, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^n,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}\right)^n\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**n,x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2))**n, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex + \sqrt{bx + \frac{e^2x^2}{f^2} + a}f + d\right)^n\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^n,x, algorithm="giac")
```

```
[Out] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^n, x)
```

3.312 $\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^3\,dx$

Optimal. Leaf size=303

$$-\frac{f^2\left(4ae^2-b^2f^2\right)\left(2de-bf^2\right)^3}{32e^5\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)}+\frac{3f^2\left(4ae^2-b^2f^2\right)\left(2de-bf^2\right)^2\log\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)}{32e^5}+\frac{f^2\left(4ae^2-b^2f^2\right)\left(2de-bf^2\right)\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+ex\right)}{8e^4}+\frac{f^2\left(4ae^2-b^2f^2\right)\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^2}{16e^3}+\frac{\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^4}{8e}$$

```
[Out] (f^2*(2*d*e - b*f^2)*(4*a*e^2 - b^2*f^2)*(e*x + f*Sqrt[a + b*x +
(e^2*x^2)/f^2]))/(8*e^4) + (f^2*(4*a*e^2 - b^2*f^2)*(d + e*x + f*
Sqrt[a + b*x + (e^2*x^2)/f^2])^2)/(16*e^3) + (d + e*x + f*Sqrt[a
+ b*x + (e^2*x^2)/f^2])^4/(8*e) - (f^2*(2*d*e - b*f^2)^3*(4*a*e^2
 - b^2*f^2))/(32*e^5*(b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e
^2*x))/f^2]))) + (3*f^2*(2*d*e - b*f^2)^2*(4*a*e^2 - b^2*f^2)*Log
[b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2])])/(32*e^
5)
```

Rubi [A] time = 0.768512, antiderivative size = 303, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$

$$-\frac{f^2\left(4ae^2-b^2f^2\right)\left(2de-bf^2\right)^3}{32e^5\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)}+\frac{3f^2\left(4ae^2-b^2f^2\right)\left(2de-bf^2\right)^2\log\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)}{32e^5}+\frac{f^2\left(4ae^2-b^2f^2\right)\left(2de-bf^2\right)\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+ex\right)}{8e^4}+\frac{f^2\left(4ae^2-b^2f^2\right)\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^2}{16e^3}+\frac{\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^4}{8e}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^3,x]
```

```
[Out] (f^2*(2*d*e - b*f^2)*(4*a*e^2 - b^2*f^2)*(e*x + f*Sqrt[a + b*x +
(e^2*x^2)/f^2]))/(8*e^4) + (f^2*(4*a*e^2 - b^2*f^2)*(d + e*x + f*
Sqrt[a + b*x + (e^2*x^2)/f^2])^2)/(16*e^3) + (d + e*x + f*Sqrt[a
+ b*x + (e^2*x^2)/f^2])^4/(8*e) - (f^2*(2*d*e - b*f^2)^3*(4*a*e^2
 - b^2*f^2))/(32*e^5*(b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e
^2*x))/f^2]))) + (3*f^2*(2*d*e - b*f^2)^2*(4*a*e^2 - b^2*f^2)*Log
[b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2])])/(32*e^
5)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.412209, size = 255, normalized size = 0.84

$$-\frac{3\left(b^2f^2-4ae^2\right)\left(bf^3-2def\right)^2\log\left(2e\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+ex\right)+bf^2\right)}{32e^5}$$
$$+\frac{\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\left(4be^2f^3\left(-2af^2+3d^2+2dex+2e^2x^2\right)+8e^3f\left(2af^2(2d+ex)+ex\left(3d^2+4dex+2e^2x^2\right)\right)+3b^3f^7-2b\right.}{16e^4}$$
$$+\frac{3}{2}x^2\left(aef^2+bdf^2+d^2e\right)+dx\left(3af^2+d^2\right)+ex^3\left(bf^2+2de\right)+e^3x^4$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^3,x]
```

```
[Out] d*(d^2 + 3*a*f^2)*x + (3*(d^2*e + b*d*f^2 + a*e*f^2)*x^2)/2 + e*(
2*d*e + b*f^2)*x^3 + e^3*x^4 + (Sqrt[a + x*(b + (e^2*x)/f^2)]*(3*
b^3*f^7 - 2*b^2*e*f^5*(6*d + e*x) + 4*b*e^2*f^3*(3*d^2 - 2*a*f^2
+ 2*d*e*x + 2*e^2*x^2) + 8*e^3*f*(2*a*f^2*(2*d + e*x) + e*x*(3*d^
2 + 4*d*e*x + 2*e^2*x^2))))/(16*e^4) - (3*(-4*a*e^2 + b^2*f^2)*(-
2*d*e*f + b*f^3)^2*Log[b*f^2 + 2*e*(e*x + f*Sqrt[a + x*(b + (e^2*
```

```
x)/f^2)])])/(32*e^5)
```

Maple [B] time = 0.022, size = 685, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^3,x)
```

```
[Out] d^3*x+e^3*x^4-3/8*d^2*f^3/e^2*ln((1/2*b+e^2*x/f^2)/(1/f^2*e^2)^(1
/2)+(a+b*x+e^2*x^2/f^2)^(1/2))/(1/f^2*e^2)^(1/2)*b^2-3/2*d/e*b*f^
3*(a+b*x+e^2*x^2/f^2)^(1/2)*x+3/8*d/e^3*b^3*f^5*ln((1/2*b+e^2*x/f
^2)/(1/f^2*e^2)^(1/2)+(a+b*x+e^2*x^2/f^2)^(1/2))/(1/f^2*e^2)^(1/2
)+3/2*f^2*a*e*x^2+3*f^2*a*d*x+1/4*d^4/e+2*x^3*e^2*d+3/2*x^2*d^2*e
+f^3*(a+b*x+e^2*x^2/f^2)^(3/2)*x+2*d/e*(a+b*x+e^2*x^2/f^2)^(3/2)*
f^3+3/4*d^2*f^3/e^2*(a+b*x+e^2*x^2/f^2)^(1/2)*b+3/2*f*d^2*ln((1/2
*b+e^2*x/f^2)/(1/f^2*e^2)^(1/2)+(a+b*x+e^2*x^2/f^2)^(1/2))/(1/f^2
*e^2)^(1/2)*a+3/8*f^5/e^2*a*ln((1/2*b+e^2*x/f^2)/(1/f^2*e^2)^(1/2
)+(a+b*x+e^2*x^2/f^2)^(1/2))/(1/f^2*e^2)^(1/2)*b^2-3/2*d/e*b*f^3*
ln((1/2*b+e^2*x/f^2)/(1/f^2*e^2)^(1/2)+(a+b*x+e^2*x^2/f^2)^(1/2))
/(1/f^2*e^2)^(1/2)*a-3/4*d/e^3*b^2*f^5*(a+b*x+e^2*x^2/f^2)^(1/2)+
3/2*f*d^2*(a+b*x+e^2*x^2/f^2)^(1/2)*x-1/2*f^5*(a+b*x+e^2*x^2/f^2)
^(3/2)/e^2*b+3/16*f^7/e^4*b^3*(a+b*x+e^2*x^2/f^2)^(1/2)+f^2*x^3*b
*e+3/2*f^2*x^2*b*d+3/8*f^5/e^2*b^2*(a+b*x+e^2*x^2/f^2)^(1/2)*x-3/
32*f^7/e^4*b^4*ln((1/2*b+e^2*x/f^2)/(1/f^2*e^2)^(1/2)+(a+b*x+e^2*
x^2/f^2)^(1/2))/(1/f^2*e^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.320528, size = 466, normalized size = 1.54

$$\frac{32\,e^{8}x^{4} + 32\left(be^{6}f^{2} + 2\,de^{7}\right)x^{3} + 48\left(d^{2}e^{6} + \left(bde^{5} + ae^{6}\right)f^{2}\right)x^{2} + 32\left(3\,ade^{5}f^{2} + d^{3}e^{5}\right)x + 3\left(b^{4}f^{8} - 16\,ad^{2}e^{4}f^{2} - 4\left(b^{3}de - \right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^3,x, algorithm="fricas")
```

```
[Out] 1/32*(32*e^8*x^4 + 32*(b*e^6*f^2 + 2*d*e^7)*x^3 + 48*(d^2*e^6 + (
b*d*e^5 + a*e^6)*f^2)*x^2 + 32*(3*a*d*e^5*f^2 + d^3*e^5)*x + 3*(b
^4*f^8 - 16*a*d^2*e^4*f^2 - 4*(b^3*d*e + a*b^2*e^2)*f^6 + 4*(b^2*
d^2*e^2 + 4*a*b*d*e^3)*f^4)*log(-b*f^2 - 2*e^2*x + 2*e*f*sqrt((b*
f^2*x + e^2*x^2 + a*f^2)/f^2)) + 2*(3*b^3*e*f^7 + 16*e^7*f*x^3 -
4*(3*b^2*d*e^2 + 2*a*b*e^3)*f^5 + 4*(3*b*d^2*e^3 + 8*a*d*e^4)*f^3
 + 8*(b*e^5*f^3 + 4*d*e^6*f)*x^2 - 2*(b^2*e^3*f^5 - 12*d^2*e^5*f
- 4*(b*d*e^4 + 2*a*e^5)*f^3)*x)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/
f^2))/e^5
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**3,x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2))**3, x)
```

GIAC/XCAS [A] time = 0.283567, size = 504, normalized size = 1.66

$$bf^2x^3e + \frac{3}{2}bdf^2x^2 + \frac{3}{2}af^2x^2e + 3adf^2x + x^4e^3 + 2dx^3e^2 + \frac{3}{2}d^2x^2e + d^3x$$
$$+ \frac{3}{32}\left(b^4f^7|f| - 4b^3df^5|f|e - 4ab^2f^5|f|e^2 + 4b^2d^2f^3|f|e^2 + 16abdf^3|f|e^3 - 16ad^2f|f|e^4\right)e^{(-5)}\ln\left(\left|-bf^2 - 2\left(xe - \sqrt{b\right.\right.\right.$$
$$+ \frac{1}{16}\sqrt{bf^2x + af^2 + x^2e^2}\left(2\left(4\left(\frac{2x|f|e^2}{f} + \frac{\left(bf^6|f|e^6 + 4df^4|f|e^7\right)e^{(-6)}}{f^5}\right)x - \frac{\left(b^2f^8|f|e^4 - 4bdf^6|f|e^5 - 8af^6|f|e^6 - 12\right.}{f^5}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^3,x, algorithm="giac")
```

```
[Out] b*f^2*x^3*e + 3/2*b*d*f^2*x^2 + 3/2*a*f^2*x^2*e + 3*a*d*f^2*x + x
^4*e^3 + 2*d*x^3*e^2 + 3/2*d^2*x^2*e + d^3*x + 3/32*(b^4*f^7*abs(
f) - 4*b^3*d*f^5*abs(f)*e - 4*a*b^2*f^5*abs(f)*e^2 + 4*b^2*d^2*f^
3*abs(f)*e^2 + 16*a*b*d*f^3*abs(f)*e^3 - 16*a*d^2*f*abs(f)*e^4)*e
```

```
^(-5)*ln(abs(-b*f^2 - 2*(x*e - sqrt(b*f^2*x + a*f^2 + x^2*e^2))*e
)) + 1/16*sqrt(b*f^2*x + a*f^2 + x^2*e^2)*(2*(4*(2*x*abs(f)*e^2/f
 + (b*f^6*abs(f)*e^6 + 4*d*f^4*abs(f)*e^7)*e^(-6)/f^5)*x - (b^2*f
^8*abs(f)*e^4 - 4*b*d*f^6*abs(f)*e^5 - 8*a*f^6*abs(f)*e^6 - 12*d^
2*f^4*abs(f)*e^6)*e^(-6)/f^5)*x + (3*b^3*f^10*abs(f)*e^2 - 12*b^2
*d*f^8*abs(f)*e^3 - 8*a*b*f^8*abs(f)*e^4 + 12*b*d^2*f^6*abs(f)*e^
4 + 32*a*d*f^6*abs(f)*e^5)*e^(-6)/f^5)
```

3.313 $\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2 dx$

Optimal. Leaf size=237

$$-\frac{f^2(4ae^2-b^2f^2)(2de-bf^2)^2}{16e^4\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)}+\frac{f^2(4ae^2-b^2f^2)(2de-bf^2)\log\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)}{8e^4}+\frac{f^2(4ae^2-b^2f^2)\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+ex\right)}{8e^3}+\frac{\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^3}{6e}$$

```
[Out] (f^2*(4*a*e^2 - b^2*f^2)*(e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]))
/(8*e^3) + (d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^3/(6*e) -
(f^2*(2*d*e - b*f^2)^2*(4*a*e^2 - b^2*f^2))/(16*e^4*(b*f^2 + 2*e*
(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2]))) + (f^2*(2*d*e - b*f
^2)*(4*a*e^2 - b^2*f^2)*Log[b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f
^2 + e^2*x))/f^2])])/(8*e^4)
```

Rubi [A] time = 0.604273, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071

$$-\frac{f^2(4ae^2-b^2f^2)(2de-bf^2)^2}{16e^4\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)}+\frac{f^2(4ae^2-b^2f^2)(2de-bf^2)\log\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)}{8e^4}+\frac{f^2(4ae^2-b^2f^2)\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+ex\right)}{8e^3}+\frac{\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^3}{6e}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^2,x]
```

```
[Out] (f^2*(4*a*e^2 - b^2*f^2)*(e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]))
/(8*e^3) + (d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^3/(6*e) -
(f^2*(2*d*e - b*f^2)^2*(4*a*e^2 - b^2*f^2))/(16*e^4*(b*f^2 + 2*e*
(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2]))) + (f^2*(2*d*e - b*f
^2)*(4*a*e^2 - b^2*f^2)*Log[b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f
```

```
^2 + e^2*x))/f^2])])/(8*e^4)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**2,x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2))**2, x)
```

Mathematica [A] time = 0.293019, size = 179, normalized size = 0.76

$$\frac{f^2\left(b^2f^2-4ae^2\right)\left(bf^2-2de\right)\log\left(2e\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+ex\right)+bf^2\right)}{8e^4}+\frac{\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\left(4e^2f\left(2af^2+ex(3d+2ex)\right)-3b^2f^5+2bef^3(3d+ex)\right)}{12e^3}+x\left(af^2+d^2\right)+\frac{1}{2}x^2\left(bf^2+2de\right)+\frac{2e^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^2,x]
```

```
[Out] (d^2 + a*f^2)*x + ((2*d*e + b*f^2)*x^2)/2 + (2*e^2*x^3)/3 + (Sqrt
[a + x*(b + (e^2*x)/f^2)]*(-3*b^2*f^5 + 2*b*e*f^3*(3*d + e*x) + 4
*e^2*f*(2*a*f^2 + e*x*(3*d + 2*e*x))))/(12*e^3) + (f^2*(-2*d*e +
b*f^2)*(-4*a*e^2 + b^2*f^2)*Log[b*f^2 + 2*e*(e*x + f*Sqrt[a + x*(
b + (e^2*x)/f^2)])])/(8*e^4)
```

Maple [A] time = 0.01, size = 409, normalized size = 1.7

$$f^{2}xa+\frac{x^{2}bf^{2}}{2}+\frac{2\,x^{3}e^{2}}{3}+\frac{df^{3}b}{2\,e^{2}}\sqrt{a+bx+\frac{e^{2}x^{2}}{f^{2}}}+fd\sqrt{a+bx+\frac{e^{2}x^{2}}{f^{2}}}x$$
$$+adf\ln\left(1\left(\frac{b}{2}+\frac{e^{2}x}{f^{2}}\right)\frac{1}{\sqrt{\frac{e^{2}}{f^{2}}}}+\sqrt{a+bx+\frac{e^{2}x^{2}}{f^{2}}}\right)\frac{1}{\sqrt{\frac{e^{2}}{f^{2}}}}$$
$$-\frac{df^{3}b^{2}}{4\,e^{2}}\ln\left(1\left(\frac{b}{2}+\frac{e^{2}x}{f^{2}}\right)\frac{1}{\sqrt{\frac{e^{2}}{f^{2}}}}+\sqrt{a+bx+\frac{e^{2}x^{2}}{f^{2}}}\right)\frac{1}{\sqrt{\frac{e^{2}}{f^{2}}}}$$
$$+\frac{2\,f^{3}}{3\,e}\left(a+bx+\frac{e^{2}x^{2}}{f^{2}}\right)^{\frac{3}{2}}-\frac{b^{2}f^{5}}{4\,e^{3}}\sqrt{a+bx+\frac{e^{2}x^{2}}{f^{2}}}-\frac{bf^{3}x}{2\,e}\sqrt{a+bx+\frac{e^{2}x^{2}}{f^{2}}}$$
$$-\frac{bf^{3}a}{2\,e}\ln\left(1\left(\frac{b}{2}+\frac{e^{2}x}{f^{2}}\right)\frac{1}{\sqrt{\frac{e^{2}}{f^{2}}}}+\sqrt{a+bx+\frac{e^{2}x^{2}}{f^{2}}}\right)\frac{1}{\sqrt{\frac{e^{2}}{f^{2}}}}$$
$$+\frac{b^{3}f^{5}}{8\,e^{3}}\ln\left(1\left(\frac{b}{2}+\frac{e^{2}x}{f^{2}}\right)\frac{1}{\sqrt{\frac{e^{2}}{f^{2}}}}+\sqrt{a+bx+\frac{e^{2}x^{2}}{f^{2}}}\right)\frac{1}{\sqrt{\frac{e^{2}}{f^{2}}}}+x^{2}de+xd^{2}+\frac{d^{3}}{3\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^2,x)
```

```
[Out] f^2*x*a+1/2*x^2*b*f^2+2/3*x^3*e^2+1/2*d*f^3/e^2*(a+b*x+e^2*x^2/f^
2)^(1/2)*b+f*d*(a+b*x+e^2*x^2/f^2)^(1/2)*x+f*d*ln((1/2*b+e^2*x/f^
2)/(1/f^2*e^2)^(1/2)+(a+b*x+e^2*x^2/f^2)^(1/2))/(1/f^2*e^2)^(1/2)
*a-1/4*d*f^3/e^2*ln((1/2*b+e^2*x/f^2)/(1/f^2*e^2)^(1/2)+(a+b*x+e^
2*x^2/f^2)^(1/2))/(1/f^2*e^2)^(1/2)*b^2+2/3*(a+b*x+e^2*x^2/f^2)^(
3/2)*f^3/e-1/4*b^2*f^5/e^3*(a+b*x+e^2*x^2/f^2)^(1/2)-1/2*b*f^3/e*
(a+b*x+e^2*x^2/f^2)^(1/2)*x-1/2*b*f^3/e*ln((1/2*b+e^2*x/f^2)/(1/f
^2*e^2)^(1/2)+(a+b*x+e^2*x^2/f^2)^(1/2))/(1/f^2*e^2)^(1/2)*a+1/8*
b^3*f^5/e^3*ln((1/2*b+e^2*x/f^2)/(1/f^2*e^2)^(1/2)+(a+b*x+e^2*x^2
/f^2)^(1/2))/(1/f^2*e^2)^(1/2)+x^2*d*e+x*d^2+1/3*d^3/e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.301302, size = 296, normalized size = 1.25

$$\frac{16\,e^6x^3+12\left(be^4f^2+2\,de^5\right)x^2+24\left(ae^4f^2+d^2e^4\right)x-3\left(b^3f^6+8\,ade^3f^2-2\left(b^2de+2\,abe^2\right)f^4\right)\log\left(-bf^2-2\,e^2x+2\,ef\right.}{24\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(16*e^6*x^3 + 12*(b*e^4*f^2 + 2*d*e^5)*x^2 + 24*(a*e^4*f^2 +
 d^2*e^4)*x - 3*(b^3*f^6 + 8*a*d*e^3*f^2 - 2*(b^2*d*e + 2*a*b*e^2
)*f^4)*log(-b*f^2 - 2*e^2*x + 2*e*f*sqrt((b*f^2*x + e^2*x^2 + a*f
^2)/f^2)) - 2*(3*b^2*e*f^5 - 8*e^5*f*x^2 - 2*(3*b*d*e^2 + 4*a*e^3
)*f^3 - 2*(b*e^3*f^3 + 6*d*e^4*f)*x)*sqrt((b*f^2*x + e^2*x^2 + a*
f^2)/f^2))/e^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**2,x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2))**2, x)
```

GIAC/XCAS [A] time = 0.27989, size = 302, normalized size = 1.27

$$\frac{1}{2}\,bf^2x^2+af^2x+\frac{2}{3}\,x^3e^2+dx^2e+d^2x$$
$$-\frac{1}{8}\left(b^3f^5|f|-2\,b^2df^3|f|e-4\,abf^3|f|e^2+8\,adf|f|e^3\right)e^{(-4)}\ln\left(\left|-bf^2-2\left(xe-\sqrt{bf^2x+af^2+x^2e^2}\right)e\right|\right)$$
$$+\frac{1}{12}\sqrt{bf^2x+af^2+x^2e^2}\left(2\left(\frac{4\,x|f|e}{f}+\frac{\left(bf^3|f|e^3+6\,df|f|e^4\right)e^{(-4)}}{f^2}\right)x-\frac{\left(3\,b^2f^5|f|e-6\,bdf^3|f|e^2-8\,af^3|f|e^3\right)e^{(-4)}}{f^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^2,x, algorithm="giac")
```

```
[Out] 1/2*b*f^2*x^2 + a*f^2*x + 2/3*x^3*e^2 + d*x^2*e + d^2*x - 1/8*(b^
3*f^5*abs(f) - 2*b^2*d*f^3*abs(f)*e - 4*a*b*f^3*abs(f)*e^2 + 8*a*
d*f*abs(f)*e^3)*e^(-4)*ln(abs(-b*f^2 - 2*(x*e - sqrt(b*f^2*x + a*
f^2 + x^2*e^2))*e)) + 1/12*sqrt(b*f^2*x + a*f^2 + x^2*e^2)*(2*(4*
x*abs(f)*e/f + (b*f^3*abs(f)*e^3 + 6*d*f*abs(f)*e^4)*e^(-4)/f^2)*
x - (3*b^2*f^5*abs(f)*e - 6*b*d*f^3*abs(f)*e^2 - 8*a*f^3*abs(f)*e
^3)*e^(-4)/f^2)
```

3.314 $\int \left(d+e x+f \sqrt{a+b x+\frac{e^2 x^2}{f^2}}\right) \, dx$

Optimal. Leaf size=118

$$\frac{f^2 \left(4 a e^2-b^2 f^2\right) \tanh ^{-1}\left(\frac{b f^2+2 e^2 x}{2 e f \sqrt{a+b x+\frac{e^2 x^2}{f^2}}}\right)}{8 e^3}+\frac{f \left(b f^2+2 e^2 x\right) \sqrt{a+b x+\frac{e^2 x^2}{f^2}}}{4 e^2}+d x+\frac{e x^2}{2}$$

```
[Out] d*x + (e*x^2)/2 + (f*(b*f^2 + 2*e^2*x)*Sqrt[a + b*x + (e^2*x^2)/f
^2])/(4*e^2) + (f^2*(4*a*e^2 - b^2*f^2)*ArcTanh[(b*f^2 + 2*e^2*x)
/(2*e*f*Sqrt[a + b*x + (e^2*x^2)/f^2])])/(8*e^3)
```

Rubi [A] time = 0.129191, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{f^2 \left(4 a e^2-b^2 f^2\right) \tanh ^{-1}\left(\frac{b f^2+2 e^2 x}{2 e f \sqrt{a+b x+\frac{e^2 x^2}{f^2}}}\right)}{8 e^3}+\frac{f \left(b f^2+2 e^2 x\right) \sqrt{a+b x+\frac{e^2 x^2}{f^2}}}{4 e^2}+d x+\frac{e x^2}{2}$$

Antiderivative was successfully verified.

```
[In]  Int[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2],x]
```

```
[Out] d*x + (e*x^2)/2 + (f*(b*f^2 + 2*e^2*x)*Sqrt[a + b*x + (e^2*x^2)/f
^2])/(4*e^2) + (f^2*(4*a*e^2 - b^2*f^2)*ArcTanh[(b*f^2 + 2*e^2*x)
/(2*e*f*Sqrt[a + b*x + (e^2*x^2)/f^2])])/(8*e^3)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$e \int x \, dx+\int d \, dx+\frac{f \left(b f^2+2 e^2 x\right) \sqrt{a+b x+\frac{e^2 x^2}{f^2}}}{4 e^2}+\frac{f^2 \left(4 a e^2-b^2 f^2\right) \operatorname{atanh}\left(\frac{b f^2+2 e^2 x}{2 e f \sqrt{a+b x+\frac{e^2 x^2}{f^2}}}\right)}{8 e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2),x)
```

```
[Out] e*Integral(x, x) + Integral(d, x) + f*(b*f**2 + 2*e**2*x)*sqrt(a
+ b*x + e**2*x**2/f**2)/(4*e**2) + f**2*(4*a*e**2 - b**2*f**2)*at
anh((b*f**2 + 2*e**2*x)/(2*e*f*sqrt(a + b*x + e**2*x**2/f**2)))/(
8*e**3)
```

Mathematica [A] time = 0.400028, size = 120, normalized size = 1.02

$$\frac{1}{8}\left(\frac{\left(4ae^2f^2-b^2f^4\right)\log\left(2e\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+ex\right)+bf^2\right)}{e^3}+4fx\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+\frac{2bf^3\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}}{e^2}+8dx+4ex^2\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2],x]
```

```
[Out] (8*d*x + 4*e*x^2 + (2*b*f^3*Sqrt[a + x*(b + (e^2*x)/f^2)])/e^2 +
4*f*x*Sqrt[a + x*(b + (e^2*x)/f^2)] + ((4*a*e^2*f^2 - b^2*f^4)*Lo
g[b*f^2 + 2*e*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])])/e^3)/8
```

Maple [A] time = 0.007, size = 173, normalized size = 1.5

$$dx+\frac{ex^2}{2}+\frac{f^3b}{4\,e^2}\sqrt{a+bx+\frac{e^2x^2}{f^2}}+\frac{fx}{2}\sqrt{a+bx+\frac{e^2x^2}{f^2}}+\frac{fa}{2}\ln\left(1\left(\frac{b}{2}+\frac{e^2x}{f^2}\right)\frac{1}{\sqrt{\frac{e^2}{f^2}}}+\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)\frac{1}{\sqrt{\frac{e^2}{f^2}}}-\frac{f^3b^2}{8\,e^2}\ln\left(1\left(\frac{b}{2}+\frac{e^2x}{f^2}\right)\frac{1}{\sqrt{\frac{e^2}{f^2}}}+\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)\frac{1}{\sqrt{\frac{e^2}{f^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2),x)
```

```
[Out] d*x+1/2*e*x^2+1/4*f^3/e^2*(a+b*x+e^2*x^2/f^2)^(1/2)*b+1/2*f*(a+b*
x+e^2*x^2/f^2)^(1/2)*x+1/2*f*ln((1/2*b+e^2*x/f^2)/(1/f^2*e^2)^(1/
```

```
2)+(a+b*x+e^2*x^2/f^2)^(1/2))/(1/f^2*e^2)^(1/2)*a-1/8*f^3/e^2*ln(
(1/2*b+e^2*x/f^2)/(1/f^2*e^2)^(1/2)+(a+b*x+e^2*x^2/f^2)^(1/2))/(1
/f^2*e^2)^(1/2)*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.288769, size = 166, normalized size = 1.41

$$\frac{4\,e^4x^2 + 8\,de^3x + \left(b^2f^4 - 4\,ae^2f^2\right)\log\left(-bf^2 - 2\,e^2x + 2\,ef\sqrt{\frac{bf^2x+e^2x^2+af^2}{f^2}}\right) + 2\left(bef^3 + 2\,e^3fx\right)\sqrt{\frac{bf^2x+e^2x^2+af^2}{f^2}}}{8\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d,x, algorithm="fricas")
```

```
[Out] 1/8*(4*e^4*x^2 + 8*d*e^3*x + (b^2*f^4 - 4*a*e^2*f^2)*log(-b*f^2 -
 2*e^2*x + 2*e*f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2)) + 2*(b*e*
f^3 + 2*e^3*f*x)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))/e^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2),x)
```

```
[Out] Integral(d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2), x)
```

GIAC/XCAS [A] time = 0.273574, size = 150, normalized size = 1.27

$$\frac{1}{2}x^2e + dx + \frac{\left(\left(b^2f^4 - 4af^2e^2\right)e^{(-3)}\ln\left(\left|-bf^2 - 2\left(xe - \sqrt{bf^2x + af^2 + x^2e^2}\right)e\right|\right) + 2\sqrt{bf^2x + af^2 + x^2e^2}\left(bf^2e^{(-2)} + 2x\right)\right)|f|}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d,x, algorithm="giac")
```

```
[Out] 1/2*x^2*e + d*x + 1/8*((b^2*f^4 - 4*a*f^2*e^2)*e^(-3)*ln(abs(-b*f
^2 - 2*(x*e - sqrt(b*f^2*x + a*f^2 + x^2*e^2))*e)) + 2*sqrt(b*f^2
*x + a*f^2 + x^2*e^2)*(b*f^2*e^(-2) + 2*x))*abs(f)/f
```

3.315 $\int \frac{1}{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\,dx$

Optimal. Leaf size=215

$$-\frac{f^2\left(4ae^2-b^2f^2\right)}{2e\left(2de-bf^2\right)\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)}-\frac{f^2\left(4ae^2-b^2f^2\right)\log\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)}{2e\left(2de-bf^2\right)^2}+\frac{2\left(aef^2-bdf^2+d^2e\right)\log\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)}{\left(2de-bf^2\right)^2}$$

```
[Out] -(f^2*(4*a*e^2 - b^2*f^2))/(2*e*(2*d*e - b*f^2)*(b*f^2 + 2*e*(e*x
 + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2]))) + (2*(d^2*e - b*d*f^2 +
 a*e*f^2)*Log[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(2*d*e
- b*f^2)^2 - (f^2*(4*a*e^2 - b^2*f^2)*Log[b*f^2 + 2*e*(e*x + f*Sq
rt[a + (x*(b*f^2 + e^2*x))/f^2])])/(2*e*(2*d*e - b*f^2)^2)
```

Rubi [A] time = 0.449108, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$

$$-\frac{f^2\left(4ae^2-b^2f^2\right)}{2e\left(2de-bf^2\right)\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)}-\frac{f^2\left(4ae^2-b^2f^2\right)\log\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)}{2e\left(2de-bf^2\right)^2}+\frac{2\left(aef^2-bdf^2+d^2e\right)\log\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)}{\left(2de-bf^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-1),x]
```

```
[Out] -(f^2*(4*a*e^2 - b^2*f^2))/(2*e*(2*d*e - b*f^2)*(b*f^2 + 2*e*(e*x
 + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2]))) + (2*(d^2*e - b*d*f^2 +
 a*e*f^2)*Log[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(2*d*e
- b*f^2)^2 - (f^2*(4*a*e^2 - b^2*f^2)*Log[b*f^2 + 2*e*(e*x + f*Sq
```

```
rt[a + (x*(b*f^2 + e^2*x))/f^2])])/(2*e*(2*d*e - b*f^2)^2)
```

Rubi in Sympy [A] time = 136.281, size = 187, normalized size = 0.87

$$\frac{2\left(aef^2-bdf^2+d^2e\right)\log\left(d+f\left(\frac{ex}{f}+\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)\right)}{\left(bf^2-2de\right)^2}-\frac{f^2\left(4ae^2-b^2f^2\right)\log\left(bf+e\left(\frac{2ex}{f}+2\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)\right)}{2e\left(bf^2-2de\right)^2}+\frac{f\left(4ae^2-b^2f^2\right)}{2e\left(bf+e\left(\frac{2ex}{f}+2\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)\right)\left(bf^2-2de\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2)),x)
```

```
[Out] 2*(a*e*f**2 - b*d*f**2 + d**2*e)*log(d + f*(e*x/f + sqrt(a + b*x
+ e**2*x**2/f**2)))/(b*f**2 - 2*d*e)**2 - f**2*(4*a*e**2 - b**2*f
**2)*log(b*f + e*(2*e*x/f + 2*sqrt(a + b*x + e**2*x**2/f**2)))/(2
*e*(b*f**2 - 2*d*e)**2) + f*(4*a*e**2 - b**2*f**2)/(2*e*(b*f + e*
(2*e*x/f + 2*sqrt(a + b*x + e**2*x**2/f**2)))*(b*f**2 - 2*d*e))
```

Mathematica [A] time = 0.551367, size = 275, normalized size = 1.28

$$\frac{2e\left(aef^2-bdf^2+d^2e\right)\log\left(bf^2\left(2df\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+af^2+d^2-2dex\right)+2d^2e\left(ex-f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)-2aef^2\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-1),x]
```

```
[Out] (2*e^2*(2*d*e - b*f^2)*x + 2*e*f*(-2*d*e + b*f^2)*Sqrt[a + x*(b +
 (e^2*x)/f^2)] + (2*d^2*e^2 - 2*b*d*e*f^2 - 2*a*e^2*f^2 + b^2*f^4
)*Log[b*f^2 + 2*e*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])] + 2*e*
(d^2*e - b*d*f^2 + a*e*f^2)*Log[b^2*f^4*x + 2*d^2*e*(e*x - f*Sqrt
[a + x*(b + (e^2*x)/f^2)]) - 2*a*e*f^2*(2*d + e*x + f*Sqrt[a + x*
(b + (e^2*x)/f^2)]) + b*f^2*(d^2 + a*f^2 - 2*d*e*x + 2*d*f*Sqrt[a
 + x*(b + (e^2*x)/f^2)])])/(2*e*(-2*d*e + b*f^2)^2)
```

Maple [B] time = 0.075, size = 4918, normalized size = 22.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2)),x)
```

```
[Out] f/(b*f^2-2*d*e)*(e^2*(x+(a*f^2-d^2)/(b*f^2-2*d*e))^2/f^2-(-b^2*f^
4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-
d^2)/(b*f^2-2*d*e))+(a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^
2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2)+1/2*f
^3/(b*f^2-2*d*e)^2*ln((-1/2*(-b^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d
^2*e^2)/f^2/(b*f^2-2*d*e)+e^2*(x+(a*f^2-d^2)/(b*f^2-2*d*e))/f^2)/
(1/f^2*e^2)^(1/2)+(e^2*(x+(a*f^2-d^2)/(b*f^2-2*d*e))^2/f^2-(-b^2*
f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^
2-d^2)/(b*f^2-2*d*e))+(a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*
d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2))/(1
/f^2*e^2)^(1/2)*b^2-f/(b*f^2-2*d*e)^2*ln((-1/2*(-b^2*f^4+2*a*e^2*
f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)+e^2*(x+(a*f^2-d^2)/(
b*f^2-2*d*e))/f^2)/(1/f^2*e^2)^(1/2)+(e^2*(x+(a*f^2-d^2)/(b*f^2-2
*d*e))^2/f^2-(-b^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*
f^2-2*d*e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e))+(a^2*e^2*f^4-2*a*b*d*e*f
^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-
2*d*e)^2)^(1/2))/(1/f^2*e^2)^(1/2)*a*e^2-f/(b*f^2-2*d*e)^2*ln((-1
/2*(-b^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)
+e^2*(x+(a*f^2-d^2)/(b*f^2-2*d*e))/f^2)/(1/f^2*e^2)^(1/2)+(e^2*(x
+(a*f^2-d^2)/(b*f^2-2*d*e))^2/f^2-(-b^2*f^4+2*a*e^2*f^2+2*b*d*e*f
^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e))+(a^
2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2
+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2))/(1/f^2*e^2)^(1/2)*b*d*e+1/f
/(b*f^2-2*d*e)^2*ln((-1/2*(-b^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2
*e^2)/f^2/(b*f^2-2*d*e)+e^2*(x+(a*f^2-d^2)/(b*f^2-2*d*e))/f^2)/(1
/f^2*e^2)^(1/2)+(e^2*(x+(a*f^2-d^2)/(b*f^2-2*d*e))^2/f^2-(-b^2*f^
4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-
d^2)/(b*f^2-2*d*e))+(a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^
2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2))/(1/f
^2*e^2)^(1/2)*d^2*e^2-f^3/(b*f^2-2*d*e)^3/((a^2*e^2*f^4-2*a*b*d*e
*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^
2-2*d*e)^2)^(1/2)*ln((2*(a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*
a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2-(-b^2*f^
4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-
d^2)/(b*f^2-2*d*e))+2*((a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a
*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2)*(e
^2*(x+(a*f^2-d^2)/(b*f^2-2*d*e))^2/f^2-(-b^2*f^4+2*a*e^2*f^2+2*b*
d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e)
)+(a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*
e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2))/(x+(a*f^2-d^2)/(b*f^2-
2*d*e)))*a^2*e^2+2*f^3/(b*f^2-2*d*e)^3/((a^2*e^2*f^4-2*a*b*d*e*f^
4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2
*d*e)^2)^(1/2)*ln((2*(a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d
^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2-(-b^2*f^4+2
```

*a*e^2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^2
)/(b*f^2-2*d*e))+2*((a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^
2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2)*(e^2*
(x+(a*f^2-d^2)/(b*f^2-2*d*e))^2/f^2-(-b^2*f^4+2*a*e^2*f^2+2*b*d*e
*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e))+(
a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f
^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2))/(x+(a*f^2-d^2)/(b*f^2-2*d
*e)))*a*b*d*e-f^3/(b*f^2-2*d*e)^3/((a^2*e^2*f^4-2*a*b*d*e*f^4+b^2
*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)
^2)^(1/2)*ln((2*(a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^
2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2-(-b^2*f^4+2*a*e^
2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^2)/(b*
f^2-2*d*e))+2*((a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2
*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2)*(e^2*(x+(a
*f^2-d^2)/(b*f^2-2*d*e))^2/f^2-(-b^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-
2*d^2*e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e))+(a^2*e
^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^
4*e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2))/(x+(a*f^2-d^2)/(b*f^2-2*d*e)))
*b^2*d^2-2*f/(b*f^2-2*d*e)^3/((a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*
f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^(
1/2)*ln((2*(a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2
-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2-(-b^2*f^4+2*a*e^2*f^2
+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^2)/(b*f^2-2
*d*e))+2*((a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-
2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2)*(e^2*(x+(a*f^2-
d^2)/(b*f^2-2*d*e))^2/f^2-(-b^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2
*e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e))+(a^2*e^2*f^
4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2
)/f^2/(b*f^2-2*d*e)^2)^(1/2))/(x+(a*f^2-d^2)/(b*f^2-2*d*e)))*a*d^
2*e^2+2*f/(b*f^2-2*d*e)^3/((a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4
+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2
)*ln((2*(a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*
b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2-(-b^2*f^4+2*a*e^2*f^2+2*
b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^2)/(b*f^2-2*d*
e))+2*((a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b
*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2)*(e^2*(x+(a*f^2-d^2
)/(b*f^2-2*d*e))^2/f^2-(-b^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*e^
2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e))+(a^2*e^2*f^4-2
*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f
^2/(b*f^2-2*d*e)^2)^(1/2))/(x+(a*f^2-d^2)/(b*f^2-2*d*e)))*b*d^3*e
-1/f/(b*f^2-2*d*e)^3/((a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*
d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2)*ln(
(2*(a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3
*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2-(-b^2*f^4+2*a*e^2*f^2+2*b*d*e
*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e))+2
*((a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*
e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2)*(e^2*(x+(a*f^2-d^2)/(b*
f^2-2*d*e))^2/f^2-(-b^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^
2/(b*f^2-2*d*e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e))+(a^2*e^2*f^4-2*a*b*
d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b
*f^2-2*d*e)^2)^(1/2))/(x+(a*f^2-d^2)/(b*f^2-2*d*e)))*d^4*e^2-d*ln
((b*f^2-2*d*e)*x+a*f^2-d^2)/(b*f^2-2*d*e)-e/(b*f^2-2*d*e)*x+e/(b*
f^2-2*d*e)^2*ln(b*f^2*x+a*f^2-2*d*e*x-d^2)*a*f^2-e/(b*f^2-2*d*e)^
2*ln(b*f^2*x+a*f^2-2*d*e*x-d^2)*d^2

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{ex + \sqrt{bx + \frac{e^2x^2}{f^2} + a}f + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d),x, algorithm="maxima")
```

```
[Out] integrate(1/(e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d), x)
```

Fricas [A] time = 4.61315, size = 501, normalized size = 2.33

$$-\frac{2\left(be^2f^2 - 2\,de^3\right)x - 2\left(d^2e^2 - \left(bde - ae^2\right)f^2\right)\log\left((bd - 2\,ae)f^2 - \left(bef^2 - 2\,de^2\right)x + \left(bf^3 - 2\,def\right)\sqrt{\frac{bf^2x + e^2x^2 + af^2}{f^2}}\right) -}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(b*e^2*f^2 - 2*d*e^3)*x - 2*(d^2*e^2 - (b*d*e - a*e^2)*f^
2)*log((b*d - 2*a*e)*f^2 - (b*e*f^2 - 2*d*e^2)*x + (b*f^3 - 2*d*e
*f)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2)) - 2*(d^2*e^2 - (b*d*e
- a*e^2)*f^2)*log(a*f^2 - d^2 + (b*f^2 - 2*d*e)*x) + (b^2*f^4 + 2
*d^2*e^2 - 2*(b*d*e + a*e^2)*f^2)*log(-b*f^2 - 2*e^2*x + 2*e*f*sq
rt((b*f^2*x + e^2*x^2 + a*f^2)/f^2)) + 2*(d^2*e^2 - (b*d*e - a*e^
2)*f^2)*log(-e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) - d) -
 2*(b*e*f^3 - 2*d*e^2*f)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))/(
b^2*e*f^4 - 4*b*d*e^2*f^2 + 4*d^2*e^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2)),x)

[Out] Integral(1/(d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$[undef, +\infty, 1]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d),x, algorithm="giac")

[Out] [undef, +Infinity, 1]
```

3.316 $\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2}\,dx$

Optimal. Leaf size=266

$$\frac{2f^2\left(4ae^2-b^2f^2\right)\log\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)}{\left(2de-bf^2\right)^3}-\frac{f^2\left(4ae^2-b^2f^2\right)}{\left(2de-bf^2\right)^2\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)}-\frac{2f^2\left(4ae^2-b^2f^2\right)\log\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)}{\left(2de-bf^2\right)^3}-\frac{2\left(aef^2-bdf^2+d^2e\right)}{\left(2de-bf^2\right)^2\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)}$$

```
[Out] (-2*(d^2*e - b*d*f^2 + a*e*f^2))/((2*d*e - b*f^2)^2*(d + e*x + f*
Sqrt[a + b*x + (e^2*x^2)/f^2])) - (f^2*(4*a*e^2 - b^2*f^2))/((2*d
*e - b*f^2)^2*(b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/
f^2]))) + (2*f^2*(4*a*e^2 - b^2*f^2)*Log[d + e*x + f*Sqrt[a + b*x
 + (e^2*x^2)/f^2]])/(2*d*e - b*f^2)^3 - (2*f^2*(4*a*e^2 - b^2*f^2
)*Log[b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2])])/(
2*d*e - b*f^2)^3
```

Rubi [A] time = 0.548343, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$

$$\frac{2f^2\left(4ae^2-b^2f^2\right)\log\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)}{\left(2de-bf^2\right)^3}-\frac{f^2\left(4ae^2-b^2f^2\right)}{\left(2de-bf^2\right)^2\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)}-\frac{2f^2\left(4ae^2-b^2f^2\right)\log\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)}{\left(2de-bf^2\right)^3}-\frac{2\left(aef^2-bdf^2+d^2e\right)}{\left(2de-bf^2\right)^2\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-2),x]
```

```
[Out] (-2*(d^2*e - b*d*f^2 + a*e*f^2))/((2*d*e - b*f^2)^2*(d + e*x + f*
Sqrt[a + b*x + (e^2*x^2)/f^2])) - (f^2*(4*a*e^2 - b^2*f^2))/((2*d
*e - b*f^2)^2*(b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/
f^2]))) + (2*f^2*(4*a*e^2 - b^2*f^2)*Log[d + e*x + f*Sqrt[a + b*x
 + (e^2*x^2)/f^2]])/(2*d*e - b*f^2)^3 - (2*f^2*(4*a*e^2 - b^2*f^2
)*Log[b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2])])/(
2*d*e - b*f^2)^3
```

Rubi in Sympy [A] time = 71.4719, size = 262, normalized size = 0.98

$$\frac{4f^2\left(4ae^2-b^2f^2\right)\operatorname{atanh}\left(\frac{bf^2+2de+ef\left(\frac{4ex}{f}+4\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)}{bf^2-2de}\right)}{\left(bf^2-2de\right)^3}-\frac{f\left(2abef^2+4ade^2-3b^2df^2+2bd^2e\right)-\left(\frac{ex}{f}+\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)\left(-8ae^2f^2+b^2f^4+4bdef^2-4d^2e^2\right)}{\left(bf^2-2de\right)^2\left(bdf+2ef\left(\frac{ex}{f}+\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2+\left(bf^2+2de\right)\left(\frac{ex}{f}+\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**2,x)
```

```
[Out] 4*f**2*(4*a*e**2 - b**2*f**2)*atanh((b*f**2 + 2*d*e + e*f*(4*e*x/
f + 4*sqrt(a + b*x + e**2*x**2/f**2)))/(b*f**2 - 2*d*e))/(b*f**2
- 2*d*e)**3 - (f*(2*a*b*e*f**2 + 4*a*d*e**2 - 3*b**2*d*f**2 + 2*b
*d**2*e) - (e*x/f + sqrt(a + b*x + e**2*x**2/f**2))*(-8*a*e**2*f*
*2 + b**2*f**4 + 4*b*d*e*f**2 - 4*d**2*e**2))/((b*f**2 - 2*d*e)**
2*(b*d*f + 2*e*f*(e*x/f + sqrt(a + b*x + e**2*x**2/f**2))**2 + (b
*f**2 + 2*d*e)*(e*x/f + sqrt(a + b*x + e**2*x**2/f**2))))
```

Mathematica [A] time = 1.02532, size = 421, normalized size = 1.58

$$\frac{f^2\left(4ae^2-b^2f^2\right)\log\left(f^2(a+bx)-d^2-2dex\right)+\left(4ae^2f^2-b^2f^4\right)\log\left(-f^2(a+bx)+d^2+2dex\right)+f^2\left(b^2f^2-4ae^2\right)\log\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-2),x]
```

```
[Out] (2*e^2*(2*d*e - b*f^2)*x - (2*(d^2*e - b*d*f^2 + a*e*f^2)^2)/(d^2
 + 2*d*e*x - f^2*(a + b*x)) + (2*f*(-2*d*e + b*f^2)*(b*f^2*(-d +
e*x) + 2*e*(a*f^2 - d*e*x))*Sqrt[a + x*(b + (e^2*x)/f^2)])/(-d^2
- 2*d*e*x + f^2*(a + b*x)) + (4*a*e^2*f^2 - b^2*f^4)*Log[d^2 + 2*
d*e*x - f^2*(a + b*x)] + f^2*(4*a*e^2 - b^2*f^2)*Log[-d^2 - 2*d*e
*x + f^2*(a + b*x)] + f^2*(-4*a*e^2 + b^2*f^2)*Log[b*f^2 + 2*e*(e
*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])] + f^2*(-4*a*e^2 + b^2*f^2)
*Log[b^2*f^4*x + b*f^2*(d^2 + a*f^2 - 2*d*(e*x + f*Sqrt[a + x*(b
+ (e^2*x)/f^2)])) + 2*e*(a*f^2*(-2*d - e*x + f*Sqrt[a + x*(b + (e
^2*x)/f^2)]) + d^2*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]))])/(2*
d*e - b*f^2)^3
```

Maple [B] time = 0.069, size = 58067, normalized size = 218.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^2,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ex + \sqrt{bx + \frac{e^2x^2}{f^2} + a}f + d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(-2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(-2), x)
```

Fricas [A] time = 2.43574, size = 1115, normalized size = 4.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(-2),x, algorithm="fricas")
```

```
[Out] -1/2*(a*b^2*f^6 + (3*b^2*d^2 - 14*a*b*d*e + 8*a^2*e^2)*f^4 - 2*(b
*d^3*e - 4*a*d^2*e^2)*f^2 - 4*(b^2*e^2*f^4 - 4*b*d*e^3*f^2 + 4*d^
2*e^4)*x^2 + (b^3*f^6 - 8*b^2*d*e*f^4 + 20*b*d^2*e^2*f^2 - 16*d^3
*e^3)*x - 2*(a*b^2*f^6 + 4*a*d^2*e^2*f^2 - (b^2*d^2 + 4*a^2*e^2)*
f^4 + (b^3*f^6 + 8*a*d*e^3*f^2 - 2*(b^2*d*e + 2*a*b*e^2)*f^4)*x)*
log(-4*a*d*e^2*f^2 - (b^2*d - 4*a*b*e)*f^4 + 4*(b*e^3*f^2 - 2*d*e
^4)*x^2 + (3*b^2*e*f^4 - 4*(2*b*d*e^2 - a*e^3)*f^2)*x - (b^2*f^5
- 4*(b*d*e - a*e^2)*f^3 + 4*(b*e^2*f^3 - 2*d*e^3*f)*x)*sqrt((b*f^
2*x + e^2*x^2 + a*f^2)/f^2)) - 2*(a*b^2*f^6 + 4*a*d^2*e^2*f^2 - (
b^2*d^2 + 4*a^2*e^2)*f^4 + (b^3*f^6 + 8*a*d*e^3*f^2 - 2*(b^2*d*e
+ 2*a*b*e^2)*f^4)*x)*log(a*f^2 - d^2 + (b*f^2 - 2*d*e)*x) + 2*(a*
b^2*f^6 + 4*a*d^2*e^2*f^2 - (b^2*d^2 + 4*a^2*e^2)*f^4 + (b^3*f^6
+ 8*a*d*e^3*f^2 - 2*(b^2*d*e + 2*a*b*e^2)*f^4)*x)*log(-e*x + f*sq
rt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) - d) - 4*((b^2*d - 2*a*b*e)*f
^5 - 2*(b*d^2*e - 2*a*d*e^2)*f^3 - (b^2*e*f^5 - 4*b*d*e^2*f^3 + 4
*d^2*e^3*f)*x)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))/(a*b^3*f^8
+ 8*d^5*e^3 - (b^3*d^2 + 6*a*b^2*d*e)*f^6 + 6*(b^2*d^3*e + 2*a*b*
d^2*e^2)*f^4 - 4*(3*b*d^4*e^2 + 2*a*d^3*e^3)*f^2 + (b^4*f^8 - 8*b
^3*d*e*f^6 + 24*b^2*d^2*e^2*f^4 - 32*b*d^3*e^3*f^2 + 16*d^4*e^4)*
x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**2,x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2))**(-2), x)
```

GIAC/XCAS [A] time = 1.0465, size = 4, normalized size = 0.02

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(-2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.317 $\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^3}\,dx$

Optimal. Leaf size=330

$$-\frac{2f^2\left(4ae^2-b^2f^2\right)}{\left(2de-bf^2\right)^3\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)}+\frac{6ef^2\left(4ae^2-b^2f^2\right)\log\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)}{\left(2de-bf^2\right)^4}-\frac{2ef^2\left(4ae^2-b^2f^2\right)}{\left(2de-bf^2\right)^3\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)}-\frac{6ef^2\left(4ae^2-b^2f^2\right)\log\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)}{\left(2de-bf^2\right)^4}-\frac{aef^2-bdf^2+d^2e}{\left(2de-bf^2\right)^2\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^2}$$

```
[Out] -((d^2*e - b*d*f^2 + a*e*f^2)/((2*d*e - b*f^2)^2*(d + e*x + f*Sqr
t[a + b*x + (e^2*x^2)/f^2])^2)) - (2*f^2*(4*a*e^2 - b^2*f^2))/((2
*d*e - b*f^2)^3*(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])) - (2
*e*f^2*(4*a*e^2 - b^2*f^2))/((2*d*e - b*f^2)^3*(b*f^2 + 2*e*(e*x
+ f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2]))) + (6*e*f^2*(4*a*e^2 - b^
2*f^2)*Log[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(2*d*e - b
*f^2)^4 - (6*e*f^2*(4*a*e^2 - b^2*f^2)*Log[b*f^2 + 2*e*(e*x + f*S
qrt[a + (x*(b*f^2 + e^2*x))/f^2])])/(2*d*e - b*f^2)^4
```

Rubi [A] time = 0.70135, antiderivative size = 330, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$

$$-\frac{2f^2\left(4ae^2-b^2f^2\right)}{\left(2de-bf^2\right)^3\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)}+\frac{6ef^2\left(4ae^2-b^2f^2\right)\log\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)}{\left(2de-bf^2\right)^4}-\frac{2ef^2\left(4ae^2-b^2f^2\right)}{\left(2de-bf^2\right)^3\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)}-\frac{6ef^2\left(4ae^2-b^2f^2\right)\log\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)}{\left(2de-bf^2\right)^4}-\frac{aef^2-bdf^2+d^2e}{\left(2de-bf^2\right)^2\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-3),x]
```

```
[Out] -((d^2*e - b*d*f^2 + a*e*f^2)/((2*d*e - b*f^2)^2*(d + e*x + f*Sqr
t[a + b*x + (e^2*x^2)/f^2])^2)) - (2*f^2*(4*a*e^2 - b^2*f^2))/((2
*d*e - b*f^2)^3*(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])) - (2
*e*f^2*(4*a*e^2 - b^2*f^2))/((2*d*e - b*f^2)^3*(b*f^2 + 2*e*(e*x
+ f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2]))) + (6*e*f^2*(4*a*e^2 - b^
2*f^2)*Log[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(2*d*e - b
*f^2)^4 - (6*e*f^2*(4*a*e^2 - b^2*f^2)*Log[b*f^2 + 2*e*(e*x + f*S
qrt[a + (x*(b*f^2 + e^2*x))/f^2])])/(2*d*e - b*f^2)^4
```

Rubi in Sympy [A] time = 117.511, size = 306, normalized size = 0.93

$$\frac{6ef^2\left(4ae^2-b^2f^2\right)\log\left(d+f\left(\frac{ex}{f}+\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)\right)}{(bf^2-2de)^4} - \frac{6ef^2\left(4ae^2-b^2f^2\right)\log\left(bf+e\left(\frac{2ex}{f}+2\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)\right)}{(bf^2-2de)^4} + \frac{2ef\left(4ae^2-b^2f^2\right)}{\left(bf+e\left(\frac{2ex}{f}+2\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)\right)(bf^2-2de)^3} + \frac{2f^2\left(4ae^2-b^2f^2\right)}{\left(d+f\left(\frac{ex}{f}+\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)\right)(bf^2-2de)^3} - \frac{aef^2-bdf^2+d^2e}{\left(d+f\left(\frac{ex}{f}+\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)\right)^2(bf^2-2de)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**3,x)
```

```
[Out] 6*e*f**2*(4*a*e**2 - b**2*f**2)*log(d + f*(e*x/f + sqrt(a + b*x +
 e**2*x**2/f**2)))/(b*f**2 - 2*d*e)**4 - 6*e*f**2*(4*a*e**2 - b**
2*f**2)*log(b*f + e*(2*e*x/f + 2*sqrt(a + b*x + e**2*x**2/f**2)))
/(b*f**2 - 2*d*e)**4 + 2*e*f*(4*a*e**2 - b**2*f**2)/((b*f + e*(2*
e*x/f + 2*sqrt(a + b*x + e**2*x**2/f**2)))*(b*f**2 - 2*d*e)**3) +
 2*f**2*(4*a*e**2 - b**2*f**2)/((d + f*(e*x/f + sqrt(a + b*x + e*
*2*x**2/f**2)))*(b*f**2 - 2*d*e)**3) - (a*e*f**2 - b*d*f**2 + d**
2*e)/((d + f*(e*x/f + sqrt(a + b*x + e**2*x**2/f**2)))**2*(b*f**2
 - 2*d*e)**2)
```

Mathematica [B] time = 1.54139, size = 665, normalized size = 2.02

$$-\frac{3\left(4a^2e^3f^4+aef^2\left(-b^2f^4-4bdef^2+4d^2e^2\right)+b^2df^4\left(bf^2-de\right)\right)}{\left(bf^2-2de\right)^4\left(-f^2(a+bx)+d^2+2dex\right)}$$
$$-\frac{2f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\left(-2e^2\left(3a^2f^4-adf^2(5d+9ex)+d^2ex(3d+4ex)\right)+b^2\left(af^6-ef^4x(d+2ex)\right)+bef^2\left(-adf^2-9ae\right.\right.}{\left(bf^2-2de\right)^3\left(-f^2(a+bx)+d^2+2dex\right)^2}$$
$$+\frac{3\left(4ae^3f^2-b^2ef^4\right)\log\left(-f^2(a+bx)+d^2+2dex\right)}{\left(bf^2-2de\right)^4}-\frac{3ef^2\left(4ae^2-b^2f^2\right)\log\left(-f^2(a+bx)+d^2+2dex\right)}{\left(bf^2-2de\right)^4}$$
$$+\frac{3ef^2\left(4ae^2-b^2f^2\right)\log\left(bf^2\left(2df\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+af^2+d^2-2dex\right)+2d^2e\left(ex-f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)-2aef^2\left(f\sqrt{\right.\right.}{\left(bf^2-2de\right)^4}$$
$$-\frac{3ef^2\left(4ae^2-b^2f^2\right)\log\left(2e\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+ex\right)+bf^2\right)}{\left(bf^2-2de\right)^4}$$
$$-\frac{2\left(aef^2-bdf^2+d^2e\right)^3}{\left(bf^2-2de\right)^4\left(-f^2(a+bx)+d^2+2dex\right)^2}+\frac{4e^3x}{\left(2de-bf^2\right)^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-3),x]
```

```
[Out] (4*e^3*x)/(2*d*e - b*f^2)^3 - (2*(d^2*e - b*d*f^2 + a*e*f^2)^3)/(
(-2*d*e + b*f^2)^4*(d^2 + 2*d*e*x - f^2*(a + b*x))^2) - (3*(4*a^2
*e^3*f^4 + b^2*d*f^4*(-(d*e) + b*f^2) + a*e*f^2*(4*d^2*e^2 - 4*b*
d*e*f^2 - b^2*f^4)))/((-2*d*e + b*f^2)^4*(d^2 + 2*d*e*x - f^2*(a
+ b*x))) - (2*f*Sqrt[a + x*(b + (e^2*x)/f^2)]*(b^3*f^6*x + b*e*f^
2*(-3*d^3 - a*d*f^2 + d^2*e*x - 9*a*e*f^2*x + 8*d*e^2*x^2) + b^2*
(a*f^6 - e*f^4*x*(d + 2*e*x)) - 2*e^2*(3*a^2*f^4 + d^2*e*x*(3*d +
 4*e*x) - a*d*f^2*(5*d + 9*e*x))))/((-2*d*e + b*f^2)^3*(d^2 + 2*d
*e*x - f^2*(a + b*x))^2) - (3*e*f^2*(4*a*e^2 - b^2*f^2)*Log[d^2 +
 2*d*e*x - f^2*(a + b*x)])/(-2*d*e + b*f^2)^4 + (3*(4*a*e^3*f^2 -
 b^2*e*f^4)*Log[d^2 + 2*d*e*x - f^2*(a + b*x)])/(-2*d*e + b*f^2)^
4 - (3*e*f^2*(4*a*e^2 - b^2*f^2)*Log[b*f^2 + 2*e*(e*x + f*Sqrt[a
+ x*(b + (e^2*x)/f^2)])])/(-2*d*e + b*f^2)^4 + (3*e*f^2*(4*a*e^2
- b^2*f^2)*Log[b^2*f^4*x + 2*d^2*e*(e*x - f*Sqrt[a + x*(b + (e^2*
x)/f^2)]) - 2*a*e*f^2*(2*d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)
]) + b*f^2*(d^2 + a*f^2 - 2*d*e*x + 2*d*f*Sqrt[a + x*(b + (e^2*x)
/f^2)])])/(-2*d*e + b*f^2)^4
```

Maple [B] time = 0.205, size = 295147, normalized size = 894.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^3,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ex+\sqrt{bx+\frac{e^2x^2}{f^2}+a}f+d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(-3),x, algorithm="maxima")
```

```
[Out] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(-3), x)
```

Fricas [A] time = 10.5942, size = 2638, normalized size = 7.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(-3),x, algorithm="fricas")
```

```
[Out] ((3*a*b^3*d - 4*a^2*b^2*e)*f^8 - (b^3*d^3 + 4*a*b^2*d^2*e + 10*a^
2*b*d*e^2 - 20*a^3*e^3)*f^6 - 4*(b^2*d^4*e - 8*a*b*d^3*e^2 + 6*a^
2*d^2*e^3)*f^4 - 4*(b^3*e^3*f^6 - 6*b^2*d*e^4*f^4 + 12*b*d^2*e^5*
f^2 - 8*d^3*e^6)*x^3 + 2*(b*d^5*e^2 - 6*a*d^4*e^3)*f^2 - (b^4*e*f
^8 - 2*a*b^2*e^3*f^6 - 40*d^4*e^5 - 2*(11*b^2*d^2*e^3 - 4*a*b*d*e
^4)*f^4 + 8*(7*b*d^3*e^4 - a*d^2*e^5)*f^2)*x^2 + (16*d^5*e^4 + (3
*b^4*d - 5*a*b^3*e)*f^8 - (7*b^3*d^2*e + 10*a*b^2*d*e^2 - 28*a^2*
b*e^3)*f^6 + 2*(5*b^2*d^3*e^2 + 22*a*b*d^2*e^3 - 28*a^2*d*e^4)*f^
4 - 8*(3*b*d^4*e^3 + a*d^3*e^4)*f^2)*x - 3*(a^2*b^2*e*f^8 - 4*a*d
^4*e^3*f^2 - 2*(a*b^2*d^2*e + 2*a^3*e^3)*f^6 + (b^2*d^4*e + 8*a^2
*d^2*e^3)*f^4 + (b^4*e*f^8 - 16*a*d^2*e^5*f^2 - 4*(b^3*d*e^2 + a*
b^2*e^3)*f^6 + 4*(b^2*d^2*e^3 + 4*a*b*d*e^4)*f^4)*x^2 + 2*(a*b^3*
e*f^8 - 8*a*d^3*e^4*f^2 - (b^3*d^2*e + 2*a*b^2*d*e^2 + 4*a^2*b*e^
3)*f^6 + 2*(b^2*d^3*e^2 + 2*a*b*d^2*e^3 + 4*a^2*d*e^4)*f^4)*x)*lo
g(-4*a*d*e^2*f^2 - (b^2*d - 4*a*b*e)*f^4 + 4*(b*e^3*f^2 - 2*d*e^4
)*x^2 + (3*b^2*e*f^4 - 4*(2*b*d*e^2 - a*e^3)*f^2)*x - (b^2*f^5 -
4*(b*d*e - a*e^2)*f^3 + 4*(b*e^2*f^3 - 2*d*e^3*f)*x)*sqrt((b*f^2*
x + e^2*x^2 + a*f^2)/f^2)) - 3*(a^2*b^2*e*f^8 - 4*a*d^4*e^3*f^2 -
```

```
 2*(a*b^2*d^2*e + 2*a^3*e^3)*f^6 + (b^2*d^4*e + 8*a^2*d^2*e^3)*f^
4 + (b^4*e*f^8 - 16*a*d^2*e^5*f^2 - 4*(b^3*d*e^2 + a*b^2*e^3)*f^6
 + 4*(b^2*d^2*e^3 + 4*a*b*d*e^4)*f^4)*x^2 + 2*(a*b^3*e*f^8 - 8*a*
d^3*e^4*f^2 - (b^3*d^2*e + 2*a*b^2*d*e^2 + 4*a^2*b*e^3)*f^6 + 2*(
b^2*d^3*e^2 + 2*a*b*d^2*e^3 + 4*a^2*d*e^4)*f^4)*x)*log(a*f^2 - d^
2 + (b*f^2 - 2*d*e)*x) + 3*(a^2*b^2*e*f^8 - 4*a*d^4*e^3*f^2 - 2*(
a*b^2*d^2*e + 2*a^3*e^3)*f^6 + (b^2*d^4*e + 8*a^2*d^2*e^3)*f^4 +
(b^4*e*f^8 - 16*a*d^2*e^5*f^2 - 4*(b^3*d*e^2 + a*b^2*e^3)*f^6 + 4
*(b^2*d^2*e^3 + 4*a*b*d*e^4)*f^4)*x^2 + 2*(a*b^3*e*f^8 - 8*a*d^3*
e^4*f^2 - (b^3*d^2*e + 2*a*b^2*d*e^2 + 4*a^2*b*e^3)*f^6 + 2*(b^2*
d^3*e^2 + 2*a*b*d^2*e^3 + 4*a^2*d*e^4)*f^4)*x)*log(-e*x + f*sqrt(
(b*f^2*x + e^2*x^2 + a*f^2)/f^2) - d) - 2*(a*b^3*f^9 - 3*(a*b^2*d
*e + 2*a^2*b*e^2)*f^7 - 3*(b^2*d^3*e - 4*a*b*d^2*e^2 - 4*a^2*d*e^
3)*f^5 + 2*(3*b*d^4*e^2 - 10*a*d^3*e^3)*f^3 - 2*(b^3*e^2*f^7 - 6*
b^2*d*e^3*f^5 + 12*b*d^2*e^4*f^3 - 8*d^3*e^5*f)*x^2 + (b^4*f^9 +
12*d^4*e^4*f - 3*(b^3*d*e + 3*a*b^2*e^2)*f^7 + 3*(b^2*d^2*e^2 + 1
2*a*b*d*e^3)*f^5 - 4*(2*b*d^3*e^3 + 9*a*d^2*e^4)*f^3)*x)*sqrt((b*
f^2*x + e^2*x^2 + a*f^2)/f^2))/(a^2*b^4*f^12 + 16*d^8*e^4 - 2*(a*
b^4*d^2 + 4*a^2*b^3*d*e)*f^10 + (b^4*d^4 + 16*a*b^3*d^3*e + 24*a^
2*b^2*d^2*e^2)*f^8 - 8*(b^3*d^5*e + 6*a*b^2*d^4*e^2 + 4*a^2*b*d^3
*e^3)*f^6 + 8*(3*b^2*d^6*e^2 + 8*a*b*d^5*e^3 + 2*a^2*d^4*e^4)*f^4
 - 32*(b*d^7*e^3 + a*d^6*e^4)*f^2 + (b^6*f^12 - 12*b^5*d*e*f^10 +
 60*b^4*d^2*e^2*f^8 - 160*b^3*d^3*e^3*f^6 + 240*b^2*d^4*e^4*f^4 -
 192*b*d^5*e^5*f^2 + 64*d^6*e^6)*x^2 + 2*(a*b^5*f^12 + 32*d^7*e^5
 - (b^5*d^2 + 10*a*b^4*d*e)*f^10 + 10*(b^4*d^3*e + 4*a*b^3*d^2*e^
2)*f^8 - 40*(b^3*d^4*e^2 + 2*a*b^2*d^3*e^3)*f^6 + 80*(b^2*d^5*e^3
 + a*b*d^4*e^4)*f^4 - 16*(5*b*d^6*e^4 + 2*a*d^5*e^5)*f^2)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(-3),x, algorithm="giac")
```

[Out] Timed out

3.318 $\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{5/2} dx$

Optimal. Leaf size=436

$$-\frac{f^2\left(4ae-\frac{b^2f^2}{e}\right)\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^{7/2}}{2\left(2de-bf^2\right)\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)}+\frac{f^2\left(4ae^2-b^2f^2\right)\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^{5/2}}{4e^2\left(2de-bf^2\right)}-\frac{5f^2\left(4ae^2-b^2f^2\right)\left(2de-bf^2\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{2de-bf^2}}\right)}{16\sqrt{2}e^{9/2}}+\frac{5f^2\left(4ae^2-b^2f^2\right)\left(2de-bf^2\right)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{16e^4}+\frac{5f^2\left(4ae^2-b^2f^2\right)\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}}{24e^3}+\frac{\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^{7/2}}{7e}$$

```
[Out] (5*f^2*(2*d*e - b*f^2)*(4*a*e^2 - b^2*f^2)*Sqrt[d + e*x + f*Sqrt[
a + b*x + (e^2*x^2)/f^2]])/(16*e^4) + (5*f^2*(4*a*e^2 - b^2*f^2)*
(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(3/2))/(24*e^3) + (f^
2*(4*a*e^2 - b^2*f^2)*(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])
^(5/2))/(4*e^2*(2*d*e - b*f^2)) + (d + e*x + f*Sqrt[a + b*x + (e^
2*x^2)/f^2])^(7/2)/(7*e) - (f^2*(4*a*e - (b^2*f^2)/e)*(d + e*x +
f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(7/2))/(2*(2*d*e - b*f^2)*(b*f^2
 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2]))) - (5*f^2*(2*
d*e - b*f^2)^(3/2)*(4*a*e^2 - b^2*f^2)*ArcTanh[(Sqrt[2]*Sqrt[e]*S
qrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/Sqrt[2*d*e - b*f^
2]])/(16*Sqrt[2]*e^(9/2))
```

Rubi [A] time = 1.41367, antiderivative size = 436, normalized size of antiderivative = 1., number of

steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$

$$-\frac{f^2\left(4ae-\frac{b^2f^2}{e}\right)\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^{7/2}}{2\left(2de-bf^2\right)\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)}+\frac{f^2\left(4ae^2-b^2f^2\right)\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^{5/2}}{4e^2\left(2de-bf^2\right)}-\frac{5f^2\left(4ae^2-b^2f^2\right)\left(2de-bf^2\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{2de-bf^2}}\right)}{16\sqrt{2}e^{9/2}}+\frac{5f^2\left(4ae^2-b^2f^2\right)\left(2de-bf^2\right)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{16e^4}+\frac{5f^2\left(4ae^2-b^2f^2\right)\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}}{24e^3}+\frac{\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^{7/2}}{7e}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(5/2),x]
```

```
[Out] (5*f^2*(2*d*e - b*f^2)*(4*a*e^2 - b^2*f^2)*Sqrt[d + e*x + f*Sqrt[
a + b*x + (e^2*x^2)/f^2]])/(16*e^4) + (5*f^2*(4*a*e^2 - b^2*f^2)*
(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(3/2))/(24*e^3) + (f^
2*(4*a*e^2 - b^2*f^2)*(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])
^(5/2))/(4*e^2*(2*d*e - b*f^2)) + (d + e*x + f*Sqrt[a + b*x + (e^
2*x^2)/f^2])^(7/2)/(7*e) - (f^2*(4*a*e - (b^2*f^2)/e)*(d + e*x +
f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(7/2))/(2*(2*d*e - b*f^2)*(b*f^2
 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2]))) - (5*f^2*(2*
d*e - b*f^2)^(3/2)*(4*a*e^2 - b^2*f^2)*ArcTanh[(Sqrt[2]*Sqrt[e]*S
qrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/Sqrt[2*d*e - b*f^
2]])/(16*Sqrt[2]*e^(9/2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**(5/2),x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2))**(5/2), x)
```

Mathematica [A] time = 0.797402, size = 0, normalized size = 0.

$$\int \left(d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}\right)^{5/2} dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(5/2),x]

[Out] Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(5/2), x]
```

Maple [F] time = 0.045, size = 0, normalized size = 0.

$$\int \left(d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(5/2),x)

[Out] int((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex + \sqrt{bx + \frac{e^2x^2}{f^2} + a}f + d\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(5/2),x, algorithm="maxima")

[Out] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(5/2), x)
```

Fricas [A] time = 0.527733, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/672*(105*sqrt(1/2)*(b^3*f^6 + 8*a*d*e^3*f^2 - 2*(b^2*d*e + 2*a
*b*e^2)*f^4)*sqrt(-(b*f^2 - 2*d*e)/e)*log(-(b*f^2 - 2*e^2*x + 4*s
qrt(1/2)*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d)*
e*sqrt(-(b*f^2 - 2*d*e)/e) - 2*e*f*sqrt((b*f^2*x + e^2*x^2 + a*f^
2)/f^2) - 4*d*e)/(b*f^2 + 2*e^2*x + 2*e*f*sqrt((b*f^2*x + e^2*x^2
 + a*f^2)/f^2))) + 2*(105*b^3*f^6 + 192*e^6*x^3 + 48*d^3*e^3 - 56
*(5*b^2*d*e + 6*a*b*e^2)*f^4 + 4*(21*b*d^2*e^2 + 232*a*d*e^3)*f^2
 + 144*(b*e^4*f^2 + 2*d*e^5)*x^2 + 2*(7*b^2*e^2*f^4 + 156*d^2*e^4
 - 4*(3*b*d*e^3 - 32*a*e^4)*f^2)*x - 2*(35*b^2*e*f^5 - 96*e^5*f*x
^2 + 12*d^2*e^3*f - 4*(21*b*d*e^2 + 20*a*e^3)*f^3 - 24*(b*e^3*f^3
 + 6*d*e^4*f)*x)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))*sqrt(e*x
+ f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d))/e^4, -1/336*(105*
sqrt(1/2)*(b^3*f^6 + 8*a*d*e^3*f^2 - 2*(b^2*d*e + 2*a*b*e^2)*f^4)
*sqrt((b*f^2 - 2*d*e)/e)*arctan(2*sqrt(1/2)*sqrt(e*x + f*sqrt((b*
f^2*x + e^2*x^2 + a*f^2)/f^2) + d)/sqrt((b*f^2 - 2*d*e)/e)) - (10
5*b^3*f^6 + 192*e^6*x^3 + 48*d^3*e^3 - 56*(5*b^2*d*e + 6*a*b*e^2)
*f^4 + 4*(21*b*d^2*e^2 + 232*a*d*e^3)*f^2 + 144*(b*e^4*f^2 + 2*d*
e^5)*x^2 + 2*(7*b^2*e^2*f^4 + 156*d^2*e^4 - 4*(3*b*d*e^3 - 32*a*e
^4)*f^2)*x - 2*(35*b^2*e*f^5 - 96*e^5*f*x^2 + 12*d^2*e^3*f - 4*(2
1*b*d*e^2 + 20*a*e^3)*f^3 - 24*(b*e^3*f^3 + 6*d*e^4*f)*x)*sqrt((b
*f^2*x + e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((b*f^2*x + e^2*
x^2 + a*f^2)/f^2) + d))/e^4]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex + \sqrt{bx + \frac{e^2x^2}{f^2} + a}f + d\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(5/2), x)
```

3.319 $\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2} dx$

Optimal. Leaf size=370

$$-\frac{f^2\left(4ae-\frac{b^2f^2}{e}\right)\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^{5/2}}{2\left(2de-bf^2\right)\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)}+\frac{f^2\left(4ae^2-b^2f^2\right)\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}}{4e^2\left(2de-bf^2\right)}-\frac{3f^2\left(4ae^2-b^2f^2\right)\sqrt{2de-bf^2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{2de-bf^2}}\right)}{8\sqrt{2}e^{7/2}}+\frac{3f^2\left(4ae^2-b^2f^2\right)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{8e^3}+\frac{\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^{5/2}}{5e}$$

```
[Out] (3*f^2*(4*a*e^2 - b^2*f^2)*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x
^2)/f^2]])/(8*e^3) + (f^2*(4*a*e^2 - b^2*f^2)*(d + e*x + f*Sqrt[a
 + b*x + (e^2*x^2)/f^2])^(3/2))/(4*e^2*(2*d*e - b*f^2)) + (d + e*
x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(5/2)/(5*e) - (f^2*(4*a*e -
(b^2*f^2)/e)*(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(5/2))/(
2*(2*d*e - b*f^2)*(b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*
x))/f^2]))) - (3*f^2*Sqrt[2*d*e - b*f^2]*(4*a*e^2 - b^2*f^2)*ArcT
anh[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^
2]])/Sqrt[2*d*e - b*f^2]])/(8*Sqrt[2]*e^(7/2))
```

Rubi [A] time = 1.01787, antiderivative size = 370, normalized size of antiderivative = 1., number of

steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$

$$-\frac{f^2\left(4ae-\frac{b^2f^2}{e}\right)\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^{5/2}}{2\left(2de-bf^2\right)\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)}+\frac{f^2\left(4ae^2-b^2f^2\right)\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}}{4e^2\left(2de-bf^2\right)}-\frac{3f^2\left(4ae^2-b^2f^2\right)\sqrt{2de-bf^2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{2de-bf^2}}\right)}{8\sqrt{2}e^{7/2}}+\frac{3f^2\left(4ae^2-b^2f^2\right)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{8e^3}+\frac{\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^{5/2}}{5e}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(3/2),x]
```

```
[Out] (3*f^2*(4*a*e^2 - b^2*f^2)*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x
^2)/f^2]])/(8*e^3) + (f^2*(4*a*e^2 - b^2*f^2)*(d + e*x + f*Sqrt[a
 + b*x + (e^2*x^2)/f^2])^(3/2))/(4*e^2*(2*d*e - b*f^2)) + (d + e*
x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(5/2)/(5*e) - (f^2*(4*a*e -
(b^2*f^2)/e)*(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(5/2))/(
2*(2*d*e - b*f^2)*(b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*
x))/f^2]))) - (3*f^2*Sqrt[2*d*e - b*f^2]*(4*a*e^2 - b^2*f^2)*ArcT
anh[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^
2]])/Sqrt[2*d*e - b*f^2]])/(8*Sqrt[2]*e^(7/2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**(3/2),x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2))**(3/2), x)
```

Mathematica [A] time = 0.427944, size = 0, normalized size = 0.

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(3/2),x]
```

```
[Out] Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(3/2), x]
```

Maple [F] time = 0.023, size = 0, normalized size = 0.

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(3/2),x)
```

```
[Out] int((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex+\sqrt{bx+\frac{e^2x^2}{f^2}+a}f+d\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(3/2), x)
```

Fricas [A] time = 0.496602, size = 1, normalized size = 0.

$$\left[-\frac{15\sqrt{\frac{1}{2}}\left(b^2f^4-4\,ae^2f^2\right)\sqrt{-\frac{bf^2-2\,de}{e}}\log\left(-\frac{bf^2-2\,e^2x+4\,\sqrt{\frac{1}{2}}\sqrt{ex+f\sqrt{\frac{bf^2x+e^2x^2+af^2}{f^2}}+de}\sqrt{-\frac{bf^2-2\,de}{e}}-2\,ef\sqrt{\frac{bf^2x+e^2x^2+af^2}{f^2}}-4\,de}{bf^2+2\,e^2x+2\,ef\sqrt{\frac{bf^2x+e^2x^2+af^2}{f^2}}}\right)+2\,\left(15\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/80*(15*sqrt(1/2)*(b^2*f^4 - 4*a*e^2*f^2)*sqrt(-(b*f^2 - 2*d*e
)/e)*log(-(b*f^2 - 2*e^2*x + 4*sqrt(1/2)*sqrt(e*x + f*sqrt((b*f^2
*x + e^2*x^2 + a*f^2)/f^2) + d)*e*sqrt(-(b*f^2 - 2*d*e)/e) - 2*e*
f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) - 4*d*e)/(b*f^2 + 2*e^2*x
 + 2*e*f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))) + 2*(15*b^2*f^4
- 16*e^4*x^2 - 8*d^2*e^2 - 2*(5*b*d*e + 24*a*e^2)*f^2 + 2*(b*e^2*
f^2 - 18*d*e^3)*x - 2*(5*b*e*f^3 + 8*e^3*f*x - 2*d*e^2*f)*sqrt((b
*f^2*x + e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((b*f^2*x + e^2*
x^2 + a*f^2)/f^2) + d))/e^3, 1/40*(15*sqrt(1/2)*(b^2*f^4 - 4*a*e^
2*f^2)*sqrt((b*f^2 - 2*d*e)/e)*arctan(2*sqrt(1/2)*sqrt(e*x + f*sq
rt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d)/sqrt((b*f^2 - 2*d*e)/e))
 - (15*b^2*f^4 - 16*e^4*x^2 - 8*d^2*e^2 - 2*(5*b*d*e + 24*a*e^2)*
f^2 + 2*(b*e^2*f^2 - 18*d*e^3)*x - 2*(5*b*e*f^3 + 8*e^3*f*x - 2*d
*e^2*f)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt(
(b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d))/e^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**(3/2),x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2))**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex + \sqrt{bx + \frac{e^2x^2}{f^2} + a}f + d\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(3/2),x, algorithm="giac")

[Out] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(3/2), x)
```

3.320 $\int \sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\,dx$

Optimal. Leaf size=315

$$\frac{f^2(4ae^2-b^2f^2)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{4e^2(2de-bf^2)}-\frac{f^2\left(4ae-\frac{b^2f^2}{e}\right)\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}}{2(2de-bf^2)\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)}-\frac{f^2(4ae^2-b^2f^2)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{2de-bf^2}}\right)}{4\sqrt{2}e^{5/2}\sqrt{2de-bf^2}}+\frac{\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}}{3e}$$

```
[Out] (f^2*(4*a*e^2 - b^2*f^2)*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2
)/f^2]])/(4*e^2*(2*d*e - b*f^2)) + (d + e*x + f*Sqrt[a + b*x + (e
^2*x^2)/f^2])^(3/2)/(3*e) - (f^2*(4*a*e - (b^2*f^2)/e)*(d + e*x +
 f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(3/2))/(2*(2*d*e - b*f^2)*(b*f^
2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2]))) - (f^2*(4*a
*e^2 - b^2*f^2)*ArcTanh[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a
+ b*x + (e^2*x^2)/f^2]])/Sqrt[2*d*e - b*f^2]])/(4*Sqrt[2]*e^(5/2)
*Sqrt[2*d*e - b*f^2])
```

Rubi [A] time = 0.86417, antiderivative size = 315, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233

$$\frac{f^2(4ae^2-b^2f^2)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{4e^2(2de-bf^2)}-\frac{f^2\left(4ae-\frac{b^2f^2}{e}\right)\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}}{2(2de-bf^2)\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)}-\frac{f^2(4ae^2-b^2f^2)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{2de-bf^2}}\right)}{4\sqrt{2}e^{5/2}\sqrt{2de-bf^2}}+\frac{\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}}{3e}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]],x]
```

```
[Out] (f^2*(4*a*e^2 - b^2*f^2)*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2
)/f^2]])/(4*e^2*(2*d*e - b*f^2)) + (d + e*x + f*Sqrt[a + b*x + (e
```

```
^2*x^2)/f^2])^(3/2)/(3*e) - (f^2*(4*a*e - (b^2*f^2)/e)*(d + e*x +
 f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(3/2))/(2*(2*d*e - b*f^2)*(b*f^
2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2]))) - (f^2*(4*a
*e^2 - b^2*f^2)*ArcTanh[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a
+ b*x + (e^2*x^2)/f^2]])/Sqrt[2*d*e - b*f^2]])/(4*Sqrt[2]*e^(5/2)
*Sqrt[2*d*e - b*f^2])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2)), x)
```

Mathematica [A] time = 0.732814, size = 222, normalized size = 0.7

$$\frac{\left(b^2f^4-4ae^2f^2\right)\sqrt{f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex}}{4e^2\left(2e\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+ex\right)+bf^2\right)}+\frac{f^2\left(4ae^2-b^2f^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex}}{\sqrt{bf^2-2de}}\right)}{4e^{5/2}\sqrt{2bf^2-4de}}+\frac{\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex\right)^{3/2}}{3e}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]],x]
```

```
[Out] (d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])^(3/2)/(3*e) + ((-4*a*
e^2*f^2 + b^2*f^4)*Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]
])/(4*e^2*(b*f^2 + 2*e*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])))
+ (f^2*(4*a*e^2 - b^2*f^2)*ArcTan[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x +
 f*Sqrt[a + x*(b + (e^2*x)/f^2)]])/Sqrt[-2*d*e + b*f^2]])/(4*e^(5
/2)*Sqrt[-4*d*e + 2*b*f^2])
```

Maple [F] time = 0.012, size = 0, normalized size = 0.

$$\int \sqrt{d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2),x)
```

```
[Out] int((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ex + \sqrt{bx + \frac{e^2x^2}{f^2} + a}f + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d), x)
```

Fricas [A] time = 0.502121, size = 1, normalized size = 0.

$$\left[-\frac{3\left(b^2f^4 - 4\,ae^2f^2\right)\sqrt{-2\,bef^2 + 4\,de^2}\log\left(\frac{2\sqrt{-2\,bef^2+4\,de^2}ef\sqrt{\frac{bf^2x+e^2x^2+af^2}{f^2}} - \sqrt{-2\,bef^2+4\,de^2}\left(bf^2-2\,e^2x-4\,de\right) - 4\left(bef^2-2\,de^2\right)\sqrt{ex+f \cdots}}{bf^2+2\,e^2x+2\,ef\sqrt{\frac{bf^2x+e^2x^2+af^2}{f^2}}}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d),x, algorithm="fricas")
```

```
[Out] [-1/48*(3*(b^2*f^4 - 4*a*e^2*f^2)*sqrt(-2*b*e*f^2 + 4*d*e^2)*log(
(2*sqrt(-2*b*e*f^2 + 4*d*e^2)*e*f*sqrt((b*f^2*x + e^2*x^2 + a*f^2
)/f^2) - sqrt(-2*b*e*f^2 + 4*d*e^2)*(b*f^2 - 2*e^2*x - 4*d*e) - 4
*(b*e*f^2 - 2*d*e^2)*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2
)/f^2) + d))/(b*f^2 + 2*e^2*x + 2*e*f*sqrt((b*f^2*x + e^2*x^2 + a
*f^2)/f^2))) - 4*(3*b^2*e*f^4 - 2*b*d*e^2*f^2 - 8*d^2*e^3 + 10*(b
*e^3*f^2 - 2*d*e^4)*x - 2*(b*e^2*f^3 - 2*d*e^3*f)*sqrt((b*f^2*x +
 e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*
f^2)/f^2) + d))/(b*e^3*f^2 - 2*d*e^4), 1/24*(3*(b^2*f^4 - 4*a*e^2
*f^2)*sqrt(2*b*e*f^2 - 4*d*e^2)*arctan((b*f^2 - 2*d*e)/(sqrt(2*b*
e*f^2 - 4*d*e^2)*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^
2) + d))) + 2*(3*b^2*e*f^4 - 2*b*d*e^2*f^2 - 8*d^2*e^3 + 10*(b*e^
3*f^2 - 2*d*e^4)*x - 2*(b*e^2*f^3 - 2*d*e^3*f)*sqrt((b*f^2*x + e^
2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2
)/f^2) + d))/(b*e^3*f^2 - 2*d*e^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ex + \sqrt{bx + \frac{e^2x^2}{f^2} + a}f + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d), x)
```

3.321 $\int \frac{1}{\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}\,dx$

Optimal. Leaf size=244

$$-\frac{f^2\left(4ae-\frac{b^2f^2}{e}\right)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{2\left(2de-bf^2\right)\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)}+\frac{f^2\left(4ae^2-b^2f^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{2de-bf^2}}\right)}{2\sqrt{2}e^{3/2}\left(2de-bf^2\right)^{3/2}}+\frac{\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{e}$$

```
[Out] Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]]/e - (f^2*(4*a*e -
 (b^2*f^2)/e)*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(2
*(2*d*e - b*f^2)*(b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x
))/f^2]))) + (f^2*(4*a*e^2 - b^2*f^2)*ArcTanh[(Sqrt[2]*Sqrt[e]*Sq
rt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/Sqrt[2*d*e - b*f^2
]])/(2*Sqrt[2]*e^(3/2)*(2*d*e - b*f^2)^(3/2))
```

Rubi [A] time = 0.628531, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$-\frac{f^2\left(4ae-\frac{b^2f^2}{e}\right)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{2\left(2de-bf^2\right)\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)}+\frac{f^2\left(4ae^2-b^2f^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{2de-bf^2}}\right)}{2\sqrt{2}e^{3/2}\left(2de-bf^2\right)^{3/2}}+\frac{\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{e}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]],x]
```

```
[Out] Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]]/e - (f^2*(4*a*e -
 (b^2*f^2)/e)*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(2
*(2*d*e - b*f^2)*(b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x
```

```
))/f^2]))) + (f^2*(4*a*e^2 - b^2*f^2)*ArcTanh[(Sqrt[2]*Sqrt[e]*Sq
rt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/Sqrt[2*d*e - b*f^2
]])/(2*Sqrt[2]*e^(3/2)*(2*d*e - b*f^2)^(3/2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/sqrt(d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2)), x)
```

Mathematica [A] time = 1.53682, size = 238, normalized size = 0.98

$$\frac{f^2\left(b^2f^2-4ae^2\right)\sqrt{f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex}}{2e\left(2de-bf^2\right)\left(2e\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+ex\right)+bf^2\right)}+\frac{f^2\left(4ae^2-b^2f^2\right)\tanh^{-1}\left(\frac{\sqrt{2de-bf^2}}{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex}}\right)}{2\sqrt{2}e^{3/2}\left(2de-bf^2\right)^{3/2}}+\frac{\sqrt{f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex}}{e}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]],x]
```

```
[Out] Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]]/e + (f^2*(-4*a*e^
2 + b^2*f^2)*Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]])/(2*
e*(2*d*e - b*f^2)*(b*f^2 + 2*e*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f
^2)]))) + (f^2*(4*a*e^2 - b^2*f^2)*ArcTanh[Sqrt[2*d*e - b*f^2]/(S
qrt[2]*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]])])
/(2*Sqrt[2]*e^(3/2)*(2*d*e - b*f^2)^(3/2))
```

Maple [F] time = 0.022, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2),x)

[Out] int(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ex+\sqrt{bx+\frac{e^2x^2}{f^2}+a}\,f+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d),x, algorithm="maxima")

[Out] integrate(1/sqrt(e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d), x)
```

Fricas [A] time = 0.499333, size = 1, normalized size = 0.

$$\left[\frac{\left(b^2f^4-4\,ae^2f^2\right)\sqrt{-2\,bef^2+4\,de^2}\log\left(\frac{2\sqrt{-2\,bef^2+4\,de^2}ef\sqrt{\frac{bf^2x+e^2x^2+af^2}{f^2}}-\sqrt{-2\,bef^2+4\,de^2}\left(bf^2-2\,e^2x-4\,de\right)+4\left(bef^2-2\,de^2\right)\sqrt{ex+f\sqrt{\frac{bf}{}}}}{bf^2+2\,e^2x+2\,ef\sqrt{\frac{bf^2x+e^2x^2+af^2}{f^2}}}\right.}{8\left(b^2e^2f^4\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d),x, algorithm="fricas")

[Out] [1/8*((b^2*f^4 - 4*a*e^2*f^2)*sqrt(-2*b*e*f^2 + 4*d*e^2)*log((2*s
qrt(-2*b*e*f^2 + 4*d*e^2)*e*f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^
```

```
2) - sqrt(-2*b*e*f^2 + 4*d*e^2)*(b*f^2 - 2*e^2*x - 4*d*e) + 4*(b*
e*f^2 - 2*d*e^2)*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^
2) + d))/(b*f^2 + 2*e^2*x + 2*e*f*sqrt((b*f^2*x + e^2*x^2 + a*f^2
)/f^2))) + 4*(b^2*e*f^4 - 6*b*d*e^2*f^2 + 8*d^2*e^3 - 2*(b*e^3*f^
2 - 2*d*e^4)*x + 2*(b*e^2*f^3 - 2*d*e^3*f)*sqrt((b*f^2*x + e^2*x^
2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^
2) + d))/(b^2*e^2*f^4 - 4*b*d*e^3*f^2 + 4*d^2*e^4), 1/4*((b^2*f^4
 - 4*a*e^2*f^2)*sqrt(2*b*e*f^2 - 4*d*e^2)*arctan((b*f^2 - 2*d*e)/
(sqrt(2*b*e*f^2 - 4*d*e^2)*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2 +
 a*f^2)/f^2) + d))) + 2*(b^2*e*f^4 - 6*b*d*e^2*f^2 + 8*d^2*e^3 -
2*(b*e^3*f^2 - 2*d*e^4)*x + 2*(b*e^2*f^3 - 2*d*e^3*f)*sqrt((b*f^2
*x + e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2
+ a*f^2)/f^2) + d))/(b^2*e^2*f^4 - 4*b*d*e^3*f^2 + 4*d^2*e^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/sqrt(d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ex + \sqrt{bx + \frac{e^2x^2}{f^2} + a}\,f + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sqrt(e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d), x)
```

3.322 $\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2}}\,dx$

Optimal. Leaf size=269

$$-\frac{f^2\left(4ae^2-b^2f^2\right)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\left(2de-bf^2\right)^2\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)}+\frac{3f^2\left(4ae^2-b^2f^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{2de-bf^2}}\right)}{\sqrt{2}\sqrt{e}\left(2de-bf^2\right)^{5/2}}-\frac{4\left(aef^2-bdf^2+d^2e\right)}{\left(2de-bf^2\right)^2\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}$$

```
[Out] (-4*(d^2*e - b*d*f^2 + a*e*f^2))/((2*d*e - b*f^2)^2*Sqrt[d + e*x
+ f*Sqrt[a + b*x + (e^2*x^2)/f^2]]) - (f^2*(4*a*e^2 - b^2*f^2)*Sq
rt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/((2*d*e - b*f^2)^2
*(b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2]))) + (3*
f^2*(4*a*e^2 - b^2*f^2)*ArcTanh[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f
*Sqrt[a + b*x + (e^2*x^2)/f^2]])/Sqrt[2*d*e - b*f^2]])/(Sqrt[2]*S
qrt[e]*(2*d*e - b*f^2)^(5/2))
```

Rubi [A] time = 0.960809, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$-\frac{f^2\left(4ae^2-b^2f^2\right)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\left(2de-bf^2\right)^2\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)}+\frac{3f^2\left(4ae^2-b^2f^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{2de-bf^2}}\right)}{\sqrt{2}\sqrt{e}\left(2de-bf^2\right)^{5/2}}-\frac{4\left(aef^2-bdf^2+d^2e\right)}{\left(2de-bf^2\right)^2\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-3/2),x]
```

```
[Out] (-4*(d^2*e - b*d*f^2 + a*e*f^2))/((2*d*e - b*f^2)^2*Sqrt[d + e*x
+ f*Sqrt[a + b*x + (e^2*x^2)/f^2]]) - (f^2*(4*a*e^2 - b^2*f^2)*Sq
rt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/((2*d*e - b*f^2)^2
*(b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2]))) + (3*
f^2*(4*a*e^2 - b^2*f^2)*ArcTanh[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f
*Sqrt[a + b*x + (e^2*x^2)/f^2]])/Sqrt[2*d*e - b*f^2]])/(Sqrt[2]*S
qrt[e]*(2*d*e - b*f^2)^(5/2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**(3/2),x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2))**(-3/2), x)
```

Mathematica [A] time = 0.805268, size = 0, normalized size = 0.

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-3/2),x]
```

```
[Out] Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-3/2), x]
```

Maple [F] time = 0.015, size = 0, normalized size = 0.

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(3/2),x)
```

```
[Out] int(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ex + \sqrt{bx + \frac{e^2x^2}{f^2} + a}f + d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(-3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(-3/2), x)
```

Fricas [A] time = 0.526124, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(-3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*((b^2*f^5 - 4*a*e^2*f^3)*sqrt(-2*b*e*f^2 + 4*d*e^2)*sqrt(
(b*f^2*x + e^2*x^2 + a*f^2)/f^2) + (b^2*d*f^4 - 4*a*d*e^2*f^2 + (
b^2*e*f^4 - 4*a*e^3*f^2)*x)*sqrt(-2*b*e*f^2 + 4*d*e^2))*log((2*sq
rt(-2*b*e*f^2 + 4*d*e^2)*e*f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2
) - sqrt(-2*b*e*f^2 + 4*d*e^2)*(b*f^2 - 2*e^2*x - 4*d*e) - 4*(b*e
*f^2 - 2*d*e^2)*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2
) + d))/(b*f^2 + 2*e^2*x + 2*e*f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)
/f^2))) + 4*(8*d^3*e^3 + (5*b^2*d*e - 6*a*b*e^2)*f^4 - 2*(7*b*d^2
*e^2 - 6*a*d*e^3)*f^2 - (b^2*e^2*f^4 - 4*b*d*e^3*f^2 + 4*d^2*e^4)
*x + (b^2*e*f^5 - 4*b*d*e^2*f^3 + 4*d^2*e^3*f)*sqrt((b*f^2*x + e^
2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2
)/f^2) + d))/(b^3*d*e*f^6 - 6*b^2*d^2*e^2*f^4 + 12*b*d^3*e^3*f^2
- 8*d^4*e^4 + (b^3*e^2*f^6 - 6*b^2*d*e^3*f^4 + 12*b*d^2*e^4*f^2 -
 8*d^3*e^5)*x + (b^3*e*f^7 - 6*b^2*d*e^2*f^5 + 12*b*d^2*e^3*f^3 -
 8*d^3*e^4*f)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2)), -1/2*(3*((b
^2*f^5 - 4*a*e^2*f^3)*sqrt(2*b*e*f^2 - 4*d*e^2)*sqrt((b*f^2*x + e
^2*x^2 + a*f^2)/f^2) + (b^2*d*f^4 - 4*a*d*e^2*f^2 + (b^2*e*f^4 -
4*a*e^3*f^2)*x)*sqrt(2*b*e*f^2 - 4*d*e^2))*arctan((b*f^2 - 2*d*e)
/(sqrt(2*b*e*f^2 - 4*d*e^2)*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2
+ a*f^2)/f^2) + d))) - 2*(8*d^3*e^3 + (5*b^2*d*e - 6*a*b*e^2)*f^4
 - 2*(7*b*d^2*e^2 - 6*a*d*e^3)*f^2 - (b^2*e^2*f^4 - 4*b*d*e^3*f^2
 + 4*d^2*e^4)*x + (b^2*e*f^5 - 4*b*d*e^2*f^3 + 4*d^2*e^3*f)*sqrt(
(b*f^2*x + e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((b*f^2*x + e^
```

```
2*x^2 + a*f^2)/f^2) + d))/(b^3*d*e*f^6 - 6*b^2*d^2*e^2*f^4 + 12*b
*d^3*e^3*f^2 - 8*d^4*e^4 + (b^3*e^2*f^6 - 6*b^2*d*e^3*f^4 + 12*b*
d^2*e^4*f^2 - 8*d^3*e^5)*x + (b^3*e*f^7 - 6*b^2*d*e^2*f^5 + 12*b*
d^2*e^3*f^3 - 8*d^3*e^4*f)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))
]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**(3/2),x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2))**(-3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ex+\sqrt{bx+\frac{e^2x^2}{f^2}+a}f+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(-3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(-3/2), x)
```

3.323 $\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{5/2}}\,dx$

Optimal. Leaf size=335

$$-\frac{4f^2(4ae^2-b^2f^2)}{(2de-bf^2)^3\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}-\frac{2ef^2(4ae^2-b^2f^2)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{(2de-bf^2)^3\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)}+\frac{5\sqrt{2}\sqrt{e}f^2(4ae^2-b^2f^2)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{2de-bf^2}}\right)}{(2de-bf^2)^{7/2}}-\frac{4(aef^2-bdf^2+d^2e)}{3(2de-bf^2)^2\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}}$$

```
[Out] (-4*(d^2*e - b*d*f^2 + a*e*f^2))/(3*(2*d*e - b*f^2)^2*(d + e*x +
f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(3/2)) - (4*f^2*(4*a*e^2 - b^2*f
^2))/((2*d*e - b*f^2)^3*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)
/f^2]]) - (2*e*f^2*(4*a*e^2 - b^2*f^2)*Sqrt[d + e*x + f*Sqrt[a +
b*x + (e^2*x^2)/f^2]])/((2*d*e - b*f^2)^3*(b*f^2 + 2*e*(e*x + f*S
qrt[a + (x*(b*f^2 + e^2*x))/f^2]))) + (5*Sqrt[2]*Sqrt[e]*f^2*(4*a
*e^2 - b^2*f^2)*ArcTanh[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a
+ b*x + (e^2*x^2)/f^2]])/Sqrt[2*d*e - b*f^2]])/(2*d*e - b*f^2)^(7
/2)
```

Rubi [A] time = 1.37831, antiderivative size = 335, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$-\frac{4f^2(4ae^2-b^2f^2)}{(2de-bf^2)^3\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}-\frac{2ef^2(4ae^2-b^2f^2)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{(2de-bf^2)^3\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)}+\frac{5\sqrt{2}\sqrt{e}f^2(4ae^2-b^2f^2)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{2de-bf^2}}\right)}{(2de-bf^2)^{7/2}}-\frac{4(aef^2-bdf^2+d^2e)}{3(2de-bf^2)^2\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-5/2),x]
```

```
[Out] (-4*(d^2*e - b*d*f^2 + a*e*f^2))/(3*(2*d*e - b*f^2)^2*(d + e*x +
f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(3/2)) - (4*f^2*(4*a*e^2 - b^2*f
^2))/((2*d*e - b*f^2)^3*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)
/f^2]]) - (2*e*f^2*(4*a*e^2 - b^2*f^2)*Sqrt[d + e*x + f*Sqrt[a +
b*x + (e^2*x^2)/f^2]])/((2*d*e - b*f^2)^3*(b*f^2 + 2*e*(e*x + f*S
qrt[a + (x*(b*f^2 + e^2*x))/f^2]))) + (5*Sqrt[2]*Sqrt[e]*f^2*(4*a
*e^2 - b^2*f^2)*ArcTanh[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a
+ b*x + (e^2*x^2)/f^2]])/Sqrt[2*d*e - b*f^2]])/(2*d*e - b*f^2)^(7
/2)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**(5/2),x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2))**(-5/2), x)
```

Mathematica [A] time = 1.13015, size = 0, normalized size = 0.

$$\int \frac{1}{\left(d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-5/2),x]
```

```
[Out] Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-5/2), x]
```

Maple [F] time = 0.015, size = 0, normalized size = 0.

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(5/2),x)

[Out] int(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ex+\sqrt{bx+\frac{e^2x^2}{f^2}+a}f+d\right)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(-5/2),x, algorithm="maxima")

[Out] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(-5/2), x)
```

Fricas [A] time = 0.834223, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(-5/2),x, algorithm="fricas")

[Out] [-1/6*(15*(sqrt(2)*(a*b^2*f^7 + 4*a*d^2*e^2*f^3 - (b^2*d^2 + 4*a^
2*e^2)*f^5 + (b^3*f^7 + 8*a*d*e^3*f^3 - 2*(b^2*d*e + 2*a*b*e^2)*f
^5)*x)*sqrt(-e/(b*f^2 - 2*d*e))*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/
f^2) + sqrt(2)*(a*b^2*d*f^6 + 4*a*d^3*e^2*f^2 - (b^2*d^3 + 4*a^2*
d*e^2)*f^4 + (b^3*e*f^6 + 8*a*d*e^4*f^2 - 2*(b^2*d*e^2 + 2*a*b*e^
3)*f^4)*x^2 + (12*a*d^2*e^3*f^2 + (b^3*d + a*b^2*e)*f^6 - (3*b^2*
d^2*e + 4*a*b*d*e^2 + 4*a^2*e^3)*f^4)*x)*sqrt(-e/(b*f^2 - 2*d*e))
)*log(-(b*f^2 - 2*e^2*x - 2*sqrt(2)*(b*f^2 - 2*d*e)*sqrt(e*x + f*
sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d)*sqrt(-e/(b*f^2 - 2*d*e
)) - 2*e*f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) - 4*d*e)/(b*f^2
```

```
+ 2*e^2*x + 2*e*f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))) + 4*(6*
a*b^2*f^6 + 4*d^4*e^2 - (4*b^2*d^2 - a*b*d*e + 30*a^2*e^2)*f^4 -
(9*b*d^3*e - 34*a*d^2*e^2)*f^2 - 3*(b^2*e^2*f^4 - 4*b*d*e^3*f^2 +
 4*d^2*e^4)*x^2 + (6*b^3*f^6 - 2*d^3*e^3 - 7*(b^2*d*e + 5*a*b*e^2
)*f^4 - (9*b*d^2*e^2 - 70*a*d*e^3)*f^2)*x + (2*d^3*e^2*f - (2*b^2
*d - 5*a*b*e)*f^5 + (3*b*d^2*e - 10*a*d*e^2)*f^3 + 3*(b^2*e*f^5 -
 4*b*d*e^2*f^3 + 4*d^2*e^3*f)*x)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)
/f^2))*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d))/(
a*b^3*d*f^8 + 8*d^6*e^3 - (b^3*d^3 + 6*a*b^2*d^2*e)*f^6 + 6*(b^2*
d^4*e + 2*a*b*d^3*e^2)*f^4 - 4*(3*b*d^5*e^2 + 2*a*d^4*e^3)*f^2 +
(b^4*e*f^8 - 8*b^3*d*e^2*f^6 + 24*b^2*d^2*e^3*f^4 - 32*b*d^3*e^4*
f^2 + 16*d^4*e^5)*x^2 + (24*d^5*e^4 + (b^4*d + a*b^3*e)*f^8 - 3*(
3*b^3*d^2*e + 2*a*b^2*d*e^2)*f^6 + 6*(5*b^2*d^3*e^2 + 2*a*b*d^2*e
^3)*f^4 - 4*(11*b*d^4*e^3 + 2*a*d^3*e^4)*f^2)*x + (a*b^3*f^9 + 8*
d^5*e^3*f - (b^3*d^2 + 6*a*b^2*d*e)*f^7 + 6*(b^2*d^3*e + 2*a*b*d^
2*e^2)*f^5 - 4*(3*b*d^4*e^2 + 2*a*d^3*e^3)*f^3 + (b^4*f^9 - 8*b^3
*d*e*f^7 + 24*b^2*d^2*e^2*f^5 - 32*b*d^3*e^3*f^3 + 16*d^4*e^4*f)*
x)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2)), 1/3*(15*(sqrt(2)*(a*b^
2*f^7 + 4*a*d^2*e^2*f^3 - (b^2*d^2 + 4*a^2*e^2)*f^5 + (b^3*f^7 +
8*a*d*e^3*f^3 - 2*(b^2*d*e + 2*a*b*e^2)*f^5)*x)*sqrt(e/(b*f^2 - 2
*d*e))*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + sqrt(2)*(a*b^2*d*f
^6 + 4*a*d^3*e^2*f^2 - (b^2*d^3 + 4*a^2*d*e^2)*f^4 + (b^3*e*f^6 +
 8*a*d*e^4*f^2 - 2*(b^2*d*e^2 + 2*a*b*e^3)*f^4)*x^2 + (12*a*d^2*e
^3*f^2 + (b^3*d + a*b^2*e)*f^6 - (3*b^2*d^2*e + 4*a*b*d*e^2 + 4*a
^2*e^3)*f^4)*x)*sqrt(e/(b*f^2 - 2*d*e)))*arctan(1/2*sqrt(2)*(b*f^
2 - 2*d*e)*sqrt(e/(b*f^2 - 2*d*e))/(sqrt(e*x + f*sqrt((b*f^2*x +
e^2*x^2 + a*f^2)/f^2) + d)*e)) - 2*(6*a*b^2*f^6 + 4*d^4*e^2 - (4*
b^2*d^2 - a*b*d*e + 30*a^2*e^2)*f^4 - (9*b*d^3*e - 34*a*d^2*e^2)*
f^2 - 3*(b^2*e^2*f^4 - 4*b*d*e^3*f^2 + 4*d^2*e^4)*x^2 + (6*b^3*f^
6 - 2*d^3*e^3 - 7*(b^2*d*e + 5*a*b*e^2)*f^4 - (9*b*d^2*e^2 - 70*a
*d*e^3)*f^2)*x + (2*d^3*e^2*f - (2*b^2*d - 5*a*b*e)*f^5 + (3*b*d^
2*e - 10*a*d*e^2)*f^3 + 3*(b^2*e*f^5 - 4*b*d*e^2*f^3 + 4*d^2*e^3*
f)*x)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((b
*f^2*x + e^2*x^2 + a*f^2)/f^2) + d))/(a*b^3*d*f^8 + 8*d^6*e^3 - (
b^3*d^3 + 6*a*b^2*d^2*e)*f^6 + 6*(b^2*d^4*e + 2*a*b*d^3*e^2)*f^4
- 4*(3*b*d^5*e^2 + 2*a*d^4*e^3)*f^2 + (b^4*e*f^8 - 8*b^3*d*e^2*f^
6 + 24*b^2*d^2*e^3*f^4 - 32*b*d^3*e^4*f^2 + 16*d^4*e^5)*x^2 + (24
*d^5*e^4 + (b^4*d + a*b^3*e)*f^8 - 3*(3*b^3*d^2*e + 2*a*b^2*d*e^2
)*f^6 + 6*(5*b^2*d^3*e^2 + 2*a*b*d^2*e^3)*f^4 - 4*(11*b*d^4*e^3 +
 2*a*d^3*e^4)*f^2)*x + (a*b^3*f^9 + 8*d^5*e^3*f - (b^3*d^2 + 6*a*
b^2*d*e)*f^7 + 6*(b^2*d^3*e + 2*a*b*d^2*e^2)*f^5 - 4*(3*b*d^4*e^2
 + 2*a*d^3*e^3)*f^3 + (b^4*f^9 - 8*b^3*d*e*f^7 + 24*b^2*d^2*e^2*f
^5 - 32*b*d^3*e^3*f^3 + 16*d^4*e^4*f)*x)*sqrt((b*f^2*x + e^2*x^2
+ a*f^2)/f^2))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**(5/2),x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2))**(-5/2), x)
```

GIAC/XCAS [A] time = 1.41446, size = 4, normalized size = 0.01

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(-5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.324 $\int (a+x^2)^2 \left(x+\sqrt{a+x^2}\right)^n \, dx$

Optimal. Leaf size=164

$$-\frac{a^5\left(\sqrt{a+x^2}+x\right)^{n-5}}{32(5-n)}-\frac{5a^4\left(\sqrt{a+x^2}+x\right)^{n-3}}{32(3-n)}-\frac{5a^3\left(\sqrt{a+x^2}+x\right)^{n-1}}{16(1-n)}+\frac{5a^2\left(\sqrt{a+x^2}+x\right)^{n+1}}{16(n+1)}+\frac{5a\left(\sqrt{a+x^2}+x\right)^{n+3}}{32(n+3)}+\frac{\left(\sqrt{a+x^2}+x\right)^{n+5}}{32(n+5)}$$

```
[Out] -(a^5*(x + Sqrt[a + x^2])^(-5 + n))/(32*(5 - n)) - (5*a^4*(x + Sq
rt[a + x^2])^(-3 + n))/(32*(3 - n)) - (5*a^3*(x + Sqrt[a + x^2])^
(-1 + n))/(16*(1 - n)) + (5*a^2*(x + Sqrt[a + x^2])^(1 + n))/(16*
(1 + n)) + (5*a*(x + Sqrt[a + x^2])^(3 + n))/(32*(3 + n)) + (x +
Sqrt[a + x^2])^(5 + n)/(32*(5 + n))
```

Rubi [A] time = 0.205213, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095

$$-\frac{a^5\left(\sqrt{a+x^2}+x\right)^{n-5}}{32(5-n)}-\frac{5a^4\left(\sqrt{a+x^2}+x\right)^{n-3}}{32(3-n)}-\frac{5a^3\left(\sqrt{a+x^2}+x\right)^{n-1}}{16(1-n)}+\frac{5a^2\left(\sqrt{a+x^2}+x\right)^{n+1}}{16(n+1)}+\frac{5a\left(\sqrt{a+x^2}+x\right)^{n+3}}{32(n+3)}+\frac{\left(\sqrt{a+x^2}+x\right)^{n+5}}{32(n+5)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + x^2)^2*(x + Sqrt[a + x^2])^n,x]
```

```
[Out] -(a^5*(x + Sqrt[a + x^2])^(-5 + n))/(32*(5 - n)) - (5*a^4*(x + Sq
rt[a + x^2])^(-3 + n))/(32*(3 - n)) - (5*a^3*(x + Sqrt[a + x^2])^
(-1 + n))/(16*(1 - n)) + (5*a^2*(x + Sqrt[a + x^2])^(1 + n))/(16*
(1 + n)) + (5*a*(x + Sqrt[a + x^2])^(3 + n))/(32*(3 + n)) + (x +
Sqrt[a + x^2])^(5 + n)/(32*(5 + n))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int^{x+\sqrt{a+x^2}} \frac{x^n\left(a+x^2\right)^5}{x^6}\,dx}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+a)**2*(x+(x**2+a)**(1/2))**n,x)
```

```
[Out] Integral(x**n*(a + x**2)**5/x**6, (x, x + sqrt(a + x**2)))/32
```

Mathematica [B] time = 2.75552, size = 338, normalized size = 2.06

$$\frac{1}{2}\left(\sqrt{a+x^2}+x\right)^n\left(-\frac{2a^2\left(x-n\sqrt{a+x^2}\right)}{n^2-1}\right.$$
$$+\frac{4a\sqrt{a+x^2}\left(2a^3n+a^2(n-3)nx\left((n-3)x-2\sqrt{a+x^2}\right)+a\left(n^2-4n+3\right)x^3\left((3n+1)\sqrt{a+x^2}+(5n+3)x\right)+4\left(n^3-3n^2-\right.\right.}{(n-3)(n-1)(n+1)(n+3)\left(\sqrt{a+x^2}+x\right)^2\left(x\left(\sqrt{a+x^2}+x\right)+a\right)}$$
$$+\frac{1}{16}\left(\frac{a^5}{(n-5)\left(\sqrt{a+x^2}+x\right)^5}-\frac{3a^4}{(n-3)\left(\sqrt{a+x^2}+x\right)^3}+\frac{2a^3}{(n-1)\left(\sqrt{a+x^2}+x\right)}\right.$$
$$\left.\left.+\frac{2a^2\left(\sqrt{a+x^2}+x\right)}{n+1}-\frac{3a\left(\sqrt{a+x^2}+x\right)^3}{n+3}+\frac{\left(\sqrt{a+x^2}+x\right)^5}{n+5}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + x^2)^2*(x + Sqrt[a + x^2])^n,x]
```

```
[Out] ((x + Sqrt[a + x^2])^n*((-2*a^2*(x - n*Sqrt[a + x^2]))/(-1 + n^2)
 + (a^5/((-5 + n)*(x + Sqrt[a + x^2])^5) - (3*a^4)/((-3 + n)*(x +
 Sqrt[a + x^2])^3) + (2*a^3)/((-1 + n)*(x + Sqrt[a + x^2])) + (2*
a^2*(x + Sqrt[a + x^2]))/(1 + n) - (3*a*(x + Sqrt[a + x^2])^3)/(3
 + n) + (x + Sqrt[a + x^2])^5/(5 + n))/16 + (4*a*Sqrt[a + x^2]*(2
*a^3*n + a^2*(-3 + n)*n*x*((-3 + n)*x - 2*Sqrt[a + x^2]) + 4*(3 -
 n - 3*n^2 + n^3)*x^5*(x + Sqrt[a + x^2]) + a*(3 - 4*n + n^2)*x^3
*((3 + 5*n)*x + (1 + 3*n)*Sqrt[a + x^2])))/((-3 + n)*(-1 + n)*(1
+ n)*(3 + n)*(x + Sqrt[a + x^2])^2*(a + x*(x + Sqrt[a + x^2])))))
/2
```

Maple [C] time = 0.103, size = 216, normalized size = 1.3

$$\frac{2^n x^{5+n}}{5+n} {}_3\mathrm{F}_2(-\frac{n}{2}, -\frac{5}{2}-\frac{n}{2}, \frac{1}{2}-\frac{n}{2}; 1-n, -\frac{3}{2}-\frac{n}{2}; -\frac{a}{x^2}) + \frac{2^{1+n} a x^{3+n}}{3+n} {}_3\mathrm{F}_2(-\frac{n}{2}, -\frac{3}{2}-\frac{n}{2}, \frac{1}{2}-\frac{n}{2}; 1-n, -\frac{1}{2}-\frac{n}{2}; -\frac{a}{x^2}) + \frac{n}{4\sqrt{\pi}} a^{\frac{5}{2}+\frac{n}{2}} \left(8 \frac{\sqrt{\pi} x^{1+n} a^{-1/2-n/2}}{(1+n)\, n\, (-2+2\,n)} \left(\frac{an}{x^2}+n-1\right) \left(\sqrt{\frac{a}{x^2}+1}+1\right)^{-1+n} + 4 \frac{\sqrt{\pi} x^{1+n} a^{-1/2-n/2}}{(1+n)\, n} \sqrt{\frac{a}{x^2}+1} \left(\sqrt{\frac{a}{x^2}+1}+1\right)^{-1+n}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+a)^2*(x+(x^2+a)^(1/2))^n,x)
```

```
[Out] 2^n/(5+n)*x^(5+n)*hypergeom([-1/2*n,-5/2-1/2*n,1/2-1/2*n],[1-n,-3
/2-1/2*n],-a/x^2)+2^(1+n)*a/(3+n)*x^(3+n)*hypergeom([-1/2*n,-3/2-
1/2*n,1/2-1/2*n],[1-n,-1/2-1/2*n],-a/x^2)+1/4*a^(5/2+1/2*n)/Pi^(1
/2)*n*(8*Pi^(1/2)/(1+n)/n*x^(1+n)*a^(-1/2-1/2*n)*(a/x^2*n+n-1)/(-
2+2*n)*((a/x^2+1)^(1/2)+1)^(-1+n)+4*Pi^(1/2)/(1+n)/n*x^(1+n)*a^(-
1/2-1/2*n)*(a/x^2+1)^(1/2)*((a/x^2+1)^(1/2)+1)^(-1+n))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^2+a\right)^2 \left(x+\sqrt{x^2+a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + a)^2*(x + sqrt(x^2 + a))^n,x, algorithm="maxima")
```

```
[Out] integrate((x^2 + a)^2*(x + sqrt(x^2 + a))^n, x)
```

Fricas [A] time = 0.313923, size = 213, normalized size = 1.3

$$-\frac{\left(5\left(n^4-10\,n^2+9\right)x^5+10\left(an^4-16\,an^2+15\,a\right)x^3+5\left(a^2n^4-22\,a^2n^2+45\,a^2\right)x-\left(a^2n^5-30\,a^2n^3+\left(n^5-10\,n^3+9\,n\right)x^4\right.}{n^6-35\,n^4+259\,n^2-225}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + a)^2*(x + sqrt(x^2 + a))^n,x, algorithm="fricas")
```

```
[Out] -(5*(n^4 - 10*n^2 + 9)*x^5 + 10*(a*n^4 - 16*a*n^2 + 15*a)*x^3 + 5
*(a^2*n^4 - 22*a^2*n^2 + 45*a^2)*x - (a^2*n^5 - 30*a^2*n^3 + (n^5
 - 10*n^3 + 9*n)*x^4 + 149*a^2*n + 2*(a*n^5 - 20*a*n^3 + 19*a*n)*
x^2)*sqrt(x^2 + a))*(x + sqrt(x^2 + a))^n/(n^6 - 35*n^4 + 259*n^2
 - 225)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+a)**2*(x+(x**2+a)**(1/2))**n,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^2 + a\right)^2 \left(x + \sqrt{x^2 + a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + a)^2*(x + sqrt(x^2 + a))^n,x, algorithm="giac")
```

```
[Out] integrate((x^2 + a)^2*(x + sqrt(x^2 + a))^n, x)
```

3.325 $\int (a+x^2) \left(x+\sqrt{a+x^2}\right)^n \, dx$

Optimal. Leaf size=108

$$-\frac{a^3\left(\sqrt{a+x^2}+x\right)^{n-3}}{8(3-n)}-\frac{3a^2\left(\sqrt{a+x^2}+x\right)^{n-1}}{8(1-n)}+\frac{3a\left(\sqrt{a+x^2}+x\right)^{n+1}}{8(n+1)}+\frac{\left(\sqrt{a+x^2}+x\right)^{n+3}}{8(n+3)}$$

```
[Out] -(a^3*(x + Sqrt[a + x^2])^(-3 + n))/(8*(3 - n)) - (3*a^2*(x + Sqr
t[a + x^2])^(-1 + n))/(8*(1 - n)) + (3*a*(x + Sqrt[a + x^2])^(1 +
 n))/(8*(1 + n)) + (x + Sqrt[a + x^2])^(3 + n)/(8*(3 + n))
```

Rubi [A] time = 0.124702, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$-\frac{a^3\left(\sqrt{a+x^2}+x\right)^{n-3}}{8(3-n)}-\frac{3a^2\left(\sqrt{a+x^2}+x\right)^{n-1}}{8(1-n)}+\frac{3a\left(\sqrt{a+x^2}+x\right)^{n+1}}{8(n+1)}+\frac{\left(\sqrt{a+x^2}+x\right)^{n+3}}{8(n+3)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + x^2)*(x + Sqrt[a + x^2])^n,x]
```

```
[Out] -(a^3*(x + Sqrt[a + x^2])^(-3 + n))/(8*(3 - n)) - (3*a^2*(x + Sqr
t[a + x^2])^(-1 + n))/(8*(1 - n)) + (3*a*(x + Sqrt[a + x^2])^(1 +
 n))/(8*(1 + n)) + (x + Sqrt[a + x^2])^(3 + n)/(8*(3 + n))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int^{x+\sqrt{a+x^2}} \frac{x^n\left(a+x^2\right)^3}{x^4} \, dx}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+a)*(x+(x**2+a)**(1/2))**n,x)
```

```
[Out] Integral(x**n*(a + x**2)**3/x**4, (x, x + sqrt(a + x**2)))/8
```

Mathematica [A] time = 0.359624, size = 202, normalized size = 1.87

$$\frac{\left(\sqrt{a+x^2}+x\right)^{n-2}\left(a^3\left(n\left(n^2-7\right)\sqrt{a+x^2}+3\left(n^3-n^2-7n+3\right)x\right)+a^2(n-3)x^2\left(3\left(2n^2+3n-3\right)\sqrt{a+x^2}+2\left(5n^2+6n-\right.\right.}{(n-3)(n-1)(n+1)(n+3)\left(x\left(\sqrt{a}\right.\right.}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + x^2)*(x + Sqrt[a + x^2])^n,x]
```

```
[Out] ((x + Sqrt[a + x^2])^(-2 + n)*(4*(3 - n - 3*n^2 + n^3)*x^6*(x + S
qrt[a + x^2]) + a*(3 - 4*n + n^2)*x^4*((17 + 11*n)*x + 3*(5 + 3*n
)*Sqrt[a + x^2]) + a^3*(3*(3 - 7*n - n^2 + n^3)*x + n*(-7 + n^2)*
Sqrt[a + x^2]) + a^2*(-3 + n)*x^2*(2*(-8 + 6*n + 5*n^2)*x + 3*(-3
 + 3*n + 2*n^2)*Sqrt[a + x^2])))/((-3 + n)*(-1 + n)*(1 + n)*(3 +
n)*(a + x*(x + Sqrt[a + x^2])))
```

Maple [C] time = 0.015, size = 167, normalized size = 1.6

$$\frac{2^n x^{3+n}}{3+n}\,{}_3F_2(-\frac{n}{2},-\frac{3}{2}-\frac{n}{2},\frac{1}{2}-\frac{n}{2};1-n,-\frac{1}{2}-\frac{n}{2};-\frac{a}{x^2})$$
$$+\frac{n}{4\sqrt{\pi}}a^{\frac{3}{2}+\frac{n}{2}}\left(8\frac{\sqrt{\pi}x^{1+n}a^{-1/2-n/2}}{(1+n)\,n\,(-2+2\,n)}\left(\frac{an}{x^2}+n-1\right)\left(\sqrt{\frac{a}{x^2}+1}+1\right)^{-1+n}+4\frac{\sqrt{\pi}x^{1+n}a^{-1/2-n/2}}{(1+n)\,n}\sqrt{\frac{a}{x^2}+1}\left(\sqrt{\frac{a}{x^2}+1}+1\right)^{-1+n}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+a)*(x+(x^2+a)^(1/2))^n,x)
```

```
[Out] 2^n/(3+n)*x^(3+n)*hypergeom([-1/2*n,-3/2-1/2*n,1/2-1/2*n],[1-n,-1
/2-1/2*n],-a/x^2)+1/4*a^(3/2+1/2*n)/Pi^(1/2)*n*(8*Pi^(1/2)/(1+n)/
n*x^(1+n)*a^(-1/2-1/2*n)*(a/x^2*n+n-1)/(-2+2*n)*((a/x^2+1)^(1/2)+
1)^(-1+n)+4*Pi^(1/2)/(1+n)/n*x^(1+n)*a^(-1/2-1/2*n)*(a/x^2+1)^(1/
2)*((a/x^2+1)^(1/2)+1)^(-1+n))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(x^2+a\right)\left(x+\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + a)*(x + sqrt(x^2 + a))^n,x, algorithm="maxima")
```

```
[Out] integrate((x^2 + a)*(x + sqrt(x^2 + a))^n, x)
```

Fricas [A] time = 0.319104, size = 105, normalized size = 0.97

$$-\frac{\left(3\left(n^{2}-1\right)x^{3}+3\left(an^{2}-3\,a\right)x-\left(an^{3}+\left(n^{3}-n\right)x^{2}-7\,an\right)\sqrt{x^{2}+a}\right)\left(x+\sqrt{x^{2}+a}\right)^{n}}{n^{4}-10\,n^{2}+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 + a)*(x + sqrt(x^2 + a))^n,x, algorithm="fricas")
```

```
[Out] -(3*(n^2 - 1)*x^3 + 3*(a*n^2 - 3*a)*x - (a*n^3 + (n^3 - n)*x^2 -
7*a*n)*sqrt(x^2 + a))*(x + sqrt(x^2 + a))^n/(n^4 - 10*n^2 + 9)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+a)*(x+(x**2+a)**(1/2))**n,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^{2}+a\right)\left(x+\sqrt{x^{2}+a}\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 + a)*(x + sqrt(x^2 + a))^n,x, algorithm="giac")
```

```
[Out] integrate((x^2 + a)*(x + sqrt(x^2 + a))^n, x)
```

3.326 $\int \left(x+\sqrt{a+x^2}\right)^n dx$

Optimal. Leaf size=52

$$\frac{\left(\sqrt{a+x^2}+x\right)^{n+1}}{2(n+1)}-\frac{a\left(\sqrt{a+x^2}+x\right)^{n-1}}{2(1-n)}$$

```
[Out] -(a*(x + Sqrt[a + x^2])^(-1 + n))/(2*(1 - n)) + (x + Sqrt[a + x^2
])^(1 + n)/(2*(1 + n))
```

Rubi [A] time = 0.0445983, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$\frac{\left(\sqrt{a+x^2}+x\right)^{n+1}}{2(n+1)}-\frac{a\left(\sqrt{a+x^2}+x\right)^{n-1}}{2(1-n)}$$

Antiderivative was successfully verified.

```
[In]  Int[(x + Sqrt[a + x^2])^n,x]
```

```
[Out] -(a*(x + Sqrt[a + x^2])^(-1 + n))/(2*(1 - n)) + (x + Sqrt[a + x^2
])^(1 + n)/(2*(1 + n))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x+\sqrt{a+x^2}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x+(x**2+a)**(1/2))**n,x)
```

```
[Out] Integral((x + sqrt(a + x**2))**n, x)
```

Mathematica [A] time = 0.0271864, size = 36, normalized size = 0.69

$$\frac{\left(\sqrt{a+x^2}+x\right)^n\left(n\sqrt{a+x^2}-x\right)}{n^2-1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x + Sqrt[a + x^2])^n,x]
```

```
[Out] ((x + Sqrt[a + x^2])^n*(-x + n*Sqrt[a + x^2]))/(-1 + n^2)
```

Maple [B] time = 0.012, size = 120, normalized size = 2.3

$$\frac{n}{4\sqrt{\pi}}a^{\frac{1}{2}+\frac{n}{2}}\left(8\frac{\sqrt{\pi}x^{1+n}a^{-1/2-n/2}}{(1+n)\,n\,(-2+2\,n)}\left(\frac{an}{x^2}+n-1\right)\left(\sqrt{\frac{a}{x^2}+1}+1\right)^{-1+n}+4\frac{\sqrt{\pi}x^{1+n}a^{-1/2-n/2}}{(1+n)\,n}\sqrt{\frac{a}{x^2}+1}\left(\sqrt{\frac{a}{x^2}+1}+1\right)^{-1+n}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x+(x^2+a)^(1/2))^n,x)
```

```
[Out] 1/4*a^(1/2+1/2*n)/Pi^(1/2)*n*(8*Pi^(1/2)/(1+n)/n*x^(1+n)*a^(-1/2-
1/2*n)*(a/x^2*n+n-1)/(-2+2*n)*((a/x^2+1)^(1/2)+1)^(-1+n)+4*Pi^(1/
2)/(1+n)/n*x^(1+n)*a^(-1/2-1/2*n)*(a/x^2+1)^(1/2)*((a/x^2+1)^(1/2
)+1)^(-1+n))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(x+\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sqrt(x^2 + a))^n,x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(x^2 + a))^n, x)
```

Fricas [A] time = 0.324031, size = 43, normalized size = 0.83

$$\frac{\left(\sqrt{x^2+a}n-x\right)\left(x+\sqrt{x^2+a}\right)^n}{n^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sqrt(x^2 + a))^n,x, algorithm="fricas")
```

```
[Out] (sqrt(x^2 + a)*n - x)*(x + sqrt(x^2 + a))^n/(n^2 - 1)
```

Sympy [A] time = 13.1818, size = 2147, normalized size = 41.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x+(x**2+a)**(1/2))**n,x)
```

```
[Out] Piecewise((-a**(9/2)*a**(n/2)*n**2*x*sqrt(a/x**2 + 1)*sinh(n*asin
h(x/sqrt(a)))*gamma(-n/2)/(2*a**(9/2)*n**2*gamma(-n/2 + 1) - 2*a*
*(9/2)*gamma(-n/2 + 1) + 2*a**(7/2)*n**2*x**2*gamma(-n/2 + 1) - 2
*a**(7/2)*x**2*gamma(-n/2 + 1)) + a**(9/2)*a**(n/2)*n*x*cosh(n*as
inh(x/sqrt(a)))*gamma(-n/2)/(2*a**(9/2)*n**2*gamma(-n/2 + 1) - 2*
a**(9/2)*gamma(-n/2 + 1) + 2*a**(7/2)*n**2*x**2*gamma(-n/2 + 1) -
 2*a**(7/2)*x**2*gamma(-n/2 + 1)) - a**(7/2)*a**(n/2)*n**2*x**3*s
qrt(a/x**2 + 1)*sinh(n*asinh(x/sqrt(a)))*gamma(-n/2)/(2*a**(9/2)*
n**2*gamma(-n/2 + 1) - 2*a**(9/2)*gamma(-n/2 + 1) + 2*a**(7/2)*n*
*2*x**2*gamma(-n/2 + 1) - 2*a**(7/2)*x**2*gamma(-n/2 + 1)) + a**(
7/2)*a**(n/2)*n*x**3*cosh(n*asinh(x/sqrt(a)))*gamma(-n/2)/(2*a**(
9/2)*n**2*gamma(-n/2 + 1) - 2*a**(9/2)*gamma(-n/2 + 1) + 2*a**(7/
2)*n**2*x**2*gamma(-n/2 + 1) - 2*a**(7/2)*x**2*gamma(-n/2 + 1)) +
 2*a**5*a**(n/2)*n*cosh(n*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*ga
mma(-n/2 + 1)/(2*a**(9/2)*n**2*gamma(-n/2 + 1) - 2*a**(9/2)*gamma
(-n/2 + 1) + 2*a**(7/2)*n**2*x**2*gamma(-n/2 + 1) - 2*a**(7/2)*x*
*2*gamma(-n/2 + 1)) - 2*a**5*a**(n/2)*n*gamma(-n/2 + 1)/(2*a**(9/
2)*n**2*gamma(-n/2 + 1) - 2*a**(9/2)*gamma(-n/2 + 1) + 2*a**(7/2)
*n**2*x**2*gamma(-n/2 + 1) - 2*a**(7/2)*x**2*gamma(-n/2 + 1)) - 2
*a**4*a**(n/2)*n*x**2*sqrt(a/x**2 + 1)*sinh(n*asinh(x/sqrt(a)) +
asinh(x/sqrt(a)))*gamma(-n/2 + 1)/(2*a**(9/2)*n**2*gamma(-n/2 + 1
) - 2*a**(9/2)*gamma(-n/2 + 1) + 2*a**(7/2)*n**2*x**2*gamma(-n/2
+ 1) - 2*a**(7/2)*x**2*gamma(-n/2 + 1)) + 4*a**4*a**(n/2)*n*x**2*
cosh(n*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(-n/2 + 1)/(2*a*
*(9/2)*n**2*gamma(-n/2 + 1) - 2*a**(9/2)*gamma(-n/2 + 1) + 2*a**(
7/2)*n**2*x**2*gamma(-n/2 + 1) - 2*a**(7/2)*x**2*gamma(-n/2 + 1))
 - 2*a**4*a**(n/2)*n*x**2*gamma(-n/2 + 1)/(2*a**(9/2)*n**2*gamma(
-n/2 + 1) - 2*a**(9/2)*gamma(-n/2 + 1) + 2*a**(7/2)*n**2*x**2*gam
ma(-n/2 + 1) - 2*a**(7/2)*x**2*gamma(-n/2 + 1)) - 2*a**4*a**(n/2)
*x**2*sqrt(a/x**2 + 1)*sinh(n*asinh(x/sqrt(a)) + asinh(x/sqrt(a))
)*gamma(-n/2 + 1)/(2*a**(9/2)*n**2*gamma(-n/2 + 1) - 2*a**(9/2)*g
amma(-n/2 + 1) + 2*a**(7/2)*n**2*x**2*gamma(-n/2 + 1) - 2*a**(7/2
)*x**2*gamma(-n/2 + 1)) + 2*a**4*a**(n/2)*x**2*cosh(n*asinh(x/sqr
t(a)) + asinh(x/sqrt(a)))*gamma(-n/2 + 1)/(2*a**(9/2)*n**2*gamma(
-n/2 + 1) - 2*a**(9/2)*gamma(-n/2 + 1) + 2*a**(7/2)*n**2*x**2*gam
ma(-n/2 + 1) - 2*a**(7/2)*x**2*gamma(-n/2 + 1)) - 2*a**3*a**(n/2)
*n*x**4*sqrt(a/x**2 + 1)*sinh(n*asinh(x/sqrt(a)) + asinh(x/sqrt(a
)))*gamma(-n/2 + 1)/(2*a**(9/2)*n**2*gamma(-n/2 + 1) - 2*a**(9/2)
*gamma(-n/2 + 1) + 2*a**(7/2)*n**2*x**2*gamma(-n/2 + 1) - 2*a**(7
```

```
/2)*x**2*gamma(-n/2 + 1)) + 2*a**3*a**(n/2)*n*x**4*cosh(n*asinh(x
/sqrt(a)) + asinh(x/sqrt(a)))*gamma(-n/2 + 1)/(2*a**(9/2)*n**2*ga
mma(-n/2 + 1) - 2*a**(9/2)*gamma(-n/2 + 1) + 2*a**(7/2)*n**2*x**2
*gamma(-n/2 + 1) - 2*a**(7/2)*x**2*gamma(-n/2 + 1)) - 2*a**3*a**(
n/2)*x**4*sqrt(a/x**2 + 1)*sinh(n*asinh(x/sqrt(a)) + asinh(x/sqrt
(a)))*gamma(-n/2 + 1)/(2*a**(9/2)*n**2*gamma(-n/2 + 1) - 2*a**(9/
2)*gamma(-n/2 + 1) + 2*a**(7/2)*n**2*x**2*gamma(-n/2 + 1) - 2*a**
(7/2)*x**2*gamma(-n/2 + 1)) + 2*a**3*a**(n/2)*x**4*cosh(n*asinh(x
/sqrt(a)) + asinh(x/sqrt(a)))*gamma(-n/2 + 1)/(2*a**(9/2)*n**2*ga
mma(-n/2 + 1) - 2*a**(9/2)*gamma(-n/2 + 1) + 2*a**(7/2)*n**2*x**2
*gamma(-n/2 + 1) - 2*a**(7/2)*x**2*gamma(-n/2 + 1)), Abs(x**2/a)
> 1), (-2*a**(5/2)*a**(n/2)*n*x*sqrt(1 + x**2/a)*sinh(n*asinh(x/s
qrt(a)) + asinh(x/sqrt(a)))*gamma(-n/2 + 1)/(2*a**(5/2)*n**2*gamm
a(-n/2 + 1) - 2*a**(5/2)*gamma(-n/2 + 1)) + a**(5/2)*a**(n/2)*n*x
*cosh(n*asinh(x/sqrt(a)))*gamma(-n/2)/(2*a**(5/2)*n**2*gamma(-n/2
 + 1) - 2*a**(5/2)*gamma(-n/2 + 1)) - 2*a**(5/2)*a**(n/2)*x*sqrt(
1 + x**2/a)*sinh(n*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(-n/
2 + 1)/(2*a**(5/2)*n**2*gamma(-n/2 + 1) - 2*a**(5/2)*gamma(-n/2 +
 1)) - a**3*a**(n/2)*n**2*sqrt(1 + x**2/a)*sinh(n*asinh(x/sqrt(a)
))*gamma(-n/2)/(2*a**(5/2)*n**2*gamma(-n/2 + 1) - 2*a**(5/2)*gamm
a(-n/2 + 1)) + 2*a**3*a**(n/2)*n*cosh(n*asinh(x/sqrt(a)) + asinh(
x/sqrt(a)))*gamma(-n/2 + 1)/(2*a**(5/2)*n**2*gamma(-n/2 + 1) - 2*
a**(5/2)*gamma(-n/2 + 1)) + 2*a**2*a**(n/2)*n*x**2*cosh(n*asinh(x
/sqrt(a)) + asinh(x/sqrt(a)))*gamma(-n/2 + 1)/(2*a**(5/2)*n**2*ga
mma(-n/2 + 1) - 2*a**(5/2)*gamma(-n/2 + 1)) + 2*a**2*a**(n/2)*x**
2*cosh(n*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(-n/2 + 1)/(2*
a**(5/2)*n**2*gamma(-n/2 + 1) - 2*a**(5/2)*gamma(-n/2 + 1)), True
))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x + \sqrt{x^2 + a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sqrt(x^2 + a))^n,x, algorithm="giac")
```

```
[Out] integrate((x + sqrt(x^2 + a))^n, x)
```

3.327 $\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{a+x^2}\,dx$

Optimal. Leaf size=59

$$\frac{2\left(\sqrt{a+x^2}+x\right)^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};-\frac{\left(x+\sqrt{x^2+a}\right)^2}{a}\right)}{a(n+1)}$$

```
[Out] (2*(x + Sqrt[a + x^2])^(1 + n)*Hypergeometric2F1[1, (1 + n)/2, (3
 + n)/2, -((x + Sqrt[a + x^2])^2/a)])/(a*(1 + n))
```

Rubi [A] time = 0.122796, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$

$$\frac{2\left(\sqrt{a+x^2}+x\right)^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};-\frac{\left(x+\sqrt{x^2+a}\right)^2}{a}\right)}{a(n+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(x + Sqrt[a + x^2])^n/(a + x^2),x]
```

```
[Out] (2*(x + Sqrt[a + x^2])^(1 + n)*Hypergeometric2F1[1, (1 + n)/2, (3
 + n)/2, -((x + Sqrt[a + x^2])^2/a)])/(a*(1 + n))
```

Rubi in Sympy [A] time = 12.2769, size = 46, normalized size = 0.78

$$\frac{2\left(x+\sqrt{a+x^2}\right)^{n+1}\,{}_2F_1\left(\begin{matrix}1,\frac{n}{2}+\frac{1}{2}\\ \frac{n}{2}+\frac{3}{2}\end{matrix}\middle|-\frac{\left(x+\sqrt{a+x^2}\right)^2}{a}\right)}{a\,(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x+(x**2+a)**(1/2))**n/(x**2+a),x)
```

```
[Out] 2*(x + sqrt(a + x**2))**(n + 1)*hyper((1, n/2 + 1/2), (n/2 + 3/2,
), -(x + sqrt(a + x**2))**2/a)/(a*(n + 1))
```

Mathematica [A] time = 0.0404769, size = 0, normalized size = 0.

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{a+x^2}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(x + Sqrt[a + x^2])^n/(a + x^2),x]
```

```
[Out] Integrate[(x + Sqrt[a + x^2])^n/(a + x^2), x]
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int \frac{1}{x^2+a}\left(x+\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x+(x^2+a)^(1/2))^n/(x^2+a),x)
```

```
[Out] int((x+(x^2+a)^(1/2))^n/(x^2+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x+\sqrt{x^2+a}\right)^n}{x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sqrt(x^2 + a))^n/(x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(x^2 + a))^n/(x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(x+\sqrt{x^2+a}\right)^n}{x^2+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sqrt(x^2 + a))^n/(x^2 + a),x, algorithm="fricas")
```

```
[Out] integral((x + sqrt(x^2 + a))^n/(x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x + \sqrt{a + x^2}\right)^n}{a + x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x+(x**2+a)**(1/2))**n/(x**2+a),x)
```

```
[Out] Integral((x + sqrt(a + x**2))**n/(a + x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x + \sqrt{x^2 + a}\right)^n}{x^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sqrt(x^2 + a))^n/(x^2 + a),x, algorithm="giac")
```

```
[Out] integrate((x + sqrt(x^2 + a))^n/(x^2 + a), x)
```

3.328 $\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^2}\,dx$

Optimal. Leaf size=59

$$\frac{8\left(\sqrt{a+x^2}+x\right)^{n+3}\,{}_2F_1\left(3,\frac{n+3}{2};\frac{n+5}{2};-\frac{\left(x+\sqrt{x^2+a}\right)^2}{a}\right)}{a^3(n+3)}$$

```
[Out] (8*(x + Sqrt[a + x^2])^(3 + n)*Hypergeometric2F1[3, (3 + n)/2, (5
 + n)/2, -((x + Sqrt[a + x^2])^2/a)])/(a^3*(3 + n))
```

Rubi [A] time = 0.114322, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095

$$\frac{8\left(\sqrt{a+x^2}+x\right)^{n+3}\,{}_2F_1\left(3,\frac{n+3}{2};\frac{n+5}{2};-\frac{\left(x+\sqrt{x^2+a}\right)^2}{a}\right)}{a^3(n+3)}$$

Antiderivative was successfully verified.

```
[In]  Int[(x + Sqrt[a + x^2])^n/(a + x^2)^2,x]
```

```
[Out] (8*(x + Sqrt[a + x^2])^(3 + n)*Hypergeometric2F1[3, (3 + n)/2, (5
 + n)/2, -((x + Sqrt[a + x^2])^2/a)])/(a^3*(3 + n))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$8\int^{x+\sqrt{a+x^2}}\frac{x^2x^n}{\left(a+x^2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x+(x**2+a)**(1/2))**n/(x**2+a)**2,x)
```

```
[Out] 8*Integral(x**2*x**n/(a + x**2)**3, (x, x + sqrt(a + x**2)))
```

Mathematica [A] time = 0.0420448, size = 0, normalized size = 0.

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^2} \, dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(x + Sqrt[a + x^2])^n/(a + x^2)^2,x]

[Out] Integrate[(x + Sqrt[a + x^2])^n/(a + x^2)^2, x]
```

Maple [F] time = 0.025, size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^2+a\right)^2} \left(x+\sqrt{x^2+a}\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x+(x^2+a)^(1/2))^n/(x^2+a)^2,x)

[Out] int((x+(x^2+a)^(1/2))^n/(x^2+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x+\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sqrt(x^2 + a))^n/(x^2 + a)^2,x, algorithm="maxima")

[Out] integrate((x + sqrt(x^2 + a))^n/(x^2 + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(x+\sqrt{x^2+a}\right)^n}{x^4+2\,ax^2+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sqrt(x^2 + a))^n/(x^2 + a)^2,x, algorithm="fricas")

[Out] integral((x + sqrt(x^2 + a))^n/(x^4 + 2*a*x^2 + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x + \sqrt{a + x^2}\right)^n}{\left(a + x^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x+(x**2+a)**(1/2))**n/(x**2+a)**2,x)

[Out] Integral((x + sqrt(a + x**2))**n/(a + x**2)**2, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x + \sqrt{x^2 + a}\right)^n}{\left(x^2 + a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sqrt(x^2 + a))^n/(x^2 + a)^2,x, algorithm="giac")

[Out] integrate((x + sqrt(x^2 + a))^n/(x^2 + a)^2, x)
```

3.329 $\int (a+x^2)^2 \left(x-\sqrt{a+x^2}\right)^n \, dx$

Optimal. Leaf size=176

$$-\frac{a^5 \left(x-\sqrt{a+x^2}\right)^{n-5}}{32(5-n)}-\frac{5a^4 \left(x-\sqrt{a+x^2}\right)^{n-3}}{32(3-n)}-\frac{5a^3 \left(x-\sqrt{a+x^2}\right)^{n-1}}{16(1-n)}+\frac{5a^2 \left(x-\sqrt{a+x^2}\right)^{n+1}}{16(n+1)}+\frac{5a \left(x-\sqrt{a+x^2}\right)^{n+3}}{32(n+3)}+\frac{\left(x-\sqrt{a+x^2}\right)^{n+5}}{32(n+5)}$$

```
[Out] -(a^5*(x - Sqrt[a + x^2])^(-5 + n))/(32*(5 - n)) - (5*a^4*(x - Sq
rt[a + x^2])^(-3 + n))/(32*(3 - n)) - (5*a^3*(x - Sqrt[a + x^2])^
(-1 + n))/(16*(1 - n)) + (5*a^2*(x - Sqrt[a + x^2])^(1 + n))/(16*
(1 + n)) + (5*a*(x - Sqrt[a + x^2])^(3 + n))/(32*(3 + n)) + (x -
Sqrt[a + x^2])^(5 + n)/(32*(5 + n))
```

Rubi [A] time = 0.203304, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$

$$-\frac{a^5 \left(x-\sqrt{a+x^2}\right)^{n-5}}{32(5-n)}-\frac{5a^4 \left(x-\sqrt{a+x^2}\right)^{n-3}}{32(3-n)}-\frac{5a^3 \left(x-\sqrt{a+x^2}\right)^{n-1}}{16(1-n)}+\frac{5a^2 \left(x-\sqrt{a+x^2}\right)^{n+1}}{16(n+1)}+\frac{5a \left(x-\sqrt{a+x^2}\right)^{n+3}}{32(n+3)}+\frac{\left(x-\sqrt{a+x^2}\right)^{n+5}}{32(n+5)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + x^2)^2*(x - Sqrt[a + x^2])^n,x]
```

```
[Out] -(a^5*(x - Sqrt[a + x^2])^(-5 + n))/(32*(5 - n)) - (5*a^4*(x - Sq
rt[a + x^2])^(-3 + n))/(32*(3 - n)) - (5*a^3*(x - Sqrt[a + x^2])^
(-1 + n))/(16*(1 - n)) + (5*a^2*(x - Sqrt[a + x^2])^(1 + n))/(16*
(1 + n)) + (5*a*(x - Sqrt[a + x^2])^(3 + n))/(32*(3 + n)) + (x -
Sqrt[a + x^2])^(5 + n)/(32*(5 + n))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int^{x-\sqrt{a+x^2}} \frac{x^n \left(a+x^2\right)^5}{x^6} \, dx}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+a)**2*(x-(x**2+a)**(1/2))**n,x)
```

```
[Out] Integral(x**n*(a + x**2)**5/x**6, (x, x - sqrt(a + x**2)))/32
```

Mathematica [B] time = 3.11133, size = 361, normalized size = 2.05

$$\frac{1}{2}\left(x-\sqrt{a+x^2}\right)^n\left(-\frac{2a^2\left(n\sqrt{a+x^2}+x\right)}{n^2-1}\right.$$
$$+\frac{4a\sqrt{a+x^2}\left(2a^3n+a^2(n-3)nx\left(2\sqrt{a+x^2}+(n-3)x\right)-a\left(n^2-4n+3\right)x^3\left((3n+1)\sqrt{a+x^2}-(5n+3)x\right)-4\left(n^3-3n^2-\right.\right.}{(n-3)(n-1)(n+1)(n+3)\left(x-\sqrt{a+x^2}\right)^2\left(x\left(\sqrt{a+x^2}-x\right)-a\right)}$$
$$+\frac{1}{16}\left(\frac{a^5}{(n-5)\left(x-\sqrt{a+x^2}\right)^5}+\frac{3a^4}{(n-3)\left(\sqrt{a+x^2}-x\right)^3}+\frac{2a^3}{(n-1)\left(x-\sqrt{a+x^2}\right)}\right.$$
$$\left.\left.+\frac{2a^2\left(x-\sqrt{a+x^2}\right)}{n+1}+\frac{3a\left(\sqrt{a+x^2}-x\right)^3}{n+3}+\frac{\left(x-\sqrt{a+x^2}\right)^5}{n+5}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + x^2)^2*(x - Sqrt[a + x^2])^n,x]
```

```
[Out] ((x - Sqrt[a + x^2])^n*((-2*a^2*(x + n*Sqrt[a + x^2]))/(-1 + n^2)
 + (a^5/((-5 + n)*(x - Sqrt[a + x^2])^5) + (2*a^3)/((-1 + n)*(x -
 Sqrt[a + x^2])) + (2*a^2*(x - Sqrt[a + x^2]))/(1 + n) + (x - Sqr
t[a + x^2])^5/(5 + n) + (3*a^4)/((-3 + n)*(-x + Sqrt[a + x^2])^3)
 + (3*a*(-x + Sqrt[a + x^2])^3)/(3 + n))/16 + (4*a*Sqrt[a + x^2]*
(2*a^3*n - 4*(3 - n - 3*n^2 + n^3)*x^5*(-x + Sqrt[a + x^2]) + a^2
*(-3 + n)*n*x*((-3 + n)*x + 2*Sqrt[a + x^2]) - a*(3 - 4*n + n^2)*
x^3*(-((3 + 5*n)*x) + (1 + 3*n)*Sqrt[a + x^2])))/((-3 + n)*(-1 +
n)*(1 + n)*(3 + n)*(x - Sqrt[a + x^2])^2*(-a + x*(-x + Sqrt[a + x
^2])))))/2
```

Maple [F] time = 0.047, size = 0, normalized size = 0.

$$\int \left(x^2+a\right)^2\left(x-\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+a)^2*(x-(x^2+a)^(1/2))^n,x)
```

```
[Out] int((x^2+a)^2*(x-(x^2+a)^(1/2))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (x^2+a)^2\left(x-\sqrt{x^2+a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + a)^2*(x - sqrt(x^2 + a))^n,x, algorithm="maxima")
```

```
[Out] integrate((x^2 + a)^2*(x - sqrt(x^2 + a))^n, x)
```

Fricas [A] time = 0.31672, size = 215, normalized size = 1.22

$$-\frac{\left(5\left(n^4-10\,n^2+9\right)x^5+10\left(an^4-16\,an^2+15\,a\right)x^3+5\left(a^2n^4-22\,a^2n^2+45\,a^2\right)x+\left(a^2n^5-30\,a^2n^3+\left(n^5-10\,n^3+9\,n\right)x^4\right.}{n^6-35\,n^4+259\,n^2-225}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + a)^2*(x - sqrt(x^2 + a))^n,x, algorithm="fricas")
```

```
[Out] -(5*(n^4 - 10*n^2 + 9)*x^5 + 10*(a*n^4 - 16*a*n^2 + 15*a)*x^3 + 5
*(a^2*n^4 - 22*a^2*n^2 + 45*a^2)*x + (a^2*n^5 - 30*a^2*n^3 + (n^5
 - 10*n^3 + 9*n)*x^4 + 149*a^2*n + 2*(a*n^5 - 20*a*n^3 + 19*a*n)*
x^2)*sqrt(x^2 + a))*(x - sqrt(x^2 + a))^n/(n^6 - 35*n^4 + 259*n^2
 - 225)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+a)**2*(x-(x**2+a)**(1/2))**n,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^2+a\right)^2 \left(x-\sqrt{x^2+a}\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + a)^2*(x - sqrt(x^2 + a))^n,x, algorithm="giac")
```

```
[Out] integrate((x^2 + a)^2*(x - sqrt(x^2 + a))^n, x)
```

3.330 $\int (a+x^2) \left(x-\sqrt{a+x^2}\right)^n \, dx$

Optimal. Leaf size=116

$$-\frac{a^3 \left(x-\sqrt{a+x^2}\right)^{n-3}}{8(3-n)}-\frac{3a^2 \left(x-\sqrt{a+x^2}\right)^{n-1}}{8(1-n)}+\frac{3a \left(x-\sqrt{a+x^2}\right)^{n+1}}{8(n+1)}+\frac{\left(x-\sqrt{a+x^2}\right)^{n+3}}{8(n+3)}$$

```
[Out] -(a^3*(x - Sqrt[a + x^2])^(-3 + n))/(8*(3 - n)) - (3*a^2*(x - Sqr
t[a + x^2])^(-1 + n))/(8*(1 - n)) + (3*a*(x - Sqrt[a + x^2])^(1 +
 n))/(8*(1 + n)) + (x - Sqrt[a + x^2])^(3 + n)/(8*(3 + n))
```

Rubi [A] time = 0.125904, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$

$$-\frac{a^3 \left(x-\sqrt{a+x^2}\right)^{n-3}}{8(3-n)}-\frac{3a^2 \left(x-\sqrt{a+x^2}\right)^{n-1}}{8(1-n)}+\frac{3a \left(x-\sqrt{a+x^2}\right)^{n+1}}{8(n+1)}+\frac{\left(x-\sqrt{a+x^2}\right)^{n+3}}{8(n+3)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + x^2)*(x - Sqrt[a + x^2])^n,x]
```

```
[Out] -(a^3*(x - Sqrt[a + x^2])^(-3 + n))/(8*(3 - n)) - (3*a^2*(x - Sqr
t[a + x^2])^(-1 + n))/(8*(1 - n)) + (3*a*(x - Sqrt[a + x^2])^(1 +
 n))/(8*(1 + n)) + (x - Sqrt[a + x^2])^(3 + n)/(8*(3 + n))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int^{x-\sqrt{a+x^2}} \frac{x^n (a+x^2)^3}{x^4} \, dx}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+a)*(x-(x**2+a)**(1/2))**n,x)
```

```
[Out] Integral(x**n*(a + x**2)**3/x**4, (x, x - sqrt(a + x**2)))/8
```

Mathematica [A] time = 0.370003, size = 211, normalized size = 1.82

$$\frac{\left(x-\sqrt{a+x^2}\right)^{n-2}\left(a^3\left(n\left(n^2-7\right)\sqrt{a+x^2}-3\left(n^3-n^2-7n+3\right)x\right)+a^2(n-3)x^2\left(3\left(2n^2+3n-3\right)\sqrt{a+x^2}-2\left(5n^2+6n\right.\right.\right.}{(n-3)(n-1)(n+1)(n+3)\left(x\left(\sqrt{a\right.\right.}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + x^2)*(x - Sqrt[a + x^2])^n,x]
```

```
[Out] ((x - Sqrt[a + x^2])^(-2 + n)*(4*(3 - n - 3*n^2 + n^3)*x^6*(-x +
Sqrt[a + x^2]) + a*(3 - 4*n + n^2)*x^4*(-((17 + 11*n)*x) + 3*(5 +
 3*n)*Sqrt[a + x^2]) + a^3*(-3*(3 - 7*n - n^2 + n^3)*x + n*(-7 +
n^2)*Sqrt[a + x^2]) + a^2*(-3 + n)*x^2*(-2*(-8 + 6*n + 5*n^2)*x +
 3*(-3 + 3*n + 2*n^2)*Sqrt[a + x^2])))/((-3 + n)*(-1 + n)*(1 + n)
*(3 + n)*(-a + x*(-x + Sqrt[a + x^2])))
```

Maple [F] time = 0.024, size = 0, normalized size = 0.

$$\int \left(x^2+a\right)\left(x-\sqrt{x^2+a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+a)*(x-(x^2+a)^(1/2))^n,x)
```

```
[Out] int((x^2+a)*(x-(x^2+a)^(1/2))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^2+a\right)\left(x-\sqrt{x^2+a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + a)*(x - sqrt(x^2 + a))^n,x, algorithm="maxima")
```

```
[Out] integrate((x^2 + a)*(x - sqrt(x^2 + a))^n, x)
```

Fricas [A] time = 0.334651, size = 107, normalized size = 0.92

$$-\frac{\left(3\left(n^{2}-1\right)x^{3}+3\left(an^{2}-3\,a\right)x+\left(an^{3}+\left(n^{3}-n\right)x^{2}-7\,an\right)\sqrt{x^{2}+a}\right)\left(x-\sqrt{x^{2}+a}\right)^{n}}{n^{4}-10\,n^{2}+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + a)*(x - sqrt(x^2 + a))^n,x, algorithm="fricas")
```

```
[Out] -(3*(n^2 - 1)*x^3 + 3*(a*n^2 - 3*a)*x + (a*n^3 + (n^3 - n)*x^2 -
7*a*n)*sqrt(x^2 + a))*(x - sqrt(x^2 + a))^n/(n^4 - 10*n^2 + 9)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+x^{2}\right)\left(x-\sqrt{a+x^{2}}\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+a)*(x-(x**2+a)**(1/2))**n,x)
```

```
[Out] Integral((a + x**2)*(x - sqrt(a + x**2))**n, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\left(x^{2}+a\right)\left(x-\sqrt{x^{2}+a}\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + a)*(x - sqrt(x^2 + a))^n,x, algorithm="giac")
```

```
[Out] integrate((x^2 + a)*(x - sqrt(x^2 + a))^n, x)
```

3.331 $\int \left(x-\sqrt{a+x^2}\right)^n \, dx$

Optimal. Leaf size=56

$$\frac{\left(x-\sqrt{a+x^2}\right)^{n+1}}{2(n+1)}-\frac{a\left(x-\sqrt{a+x^2}\right)^{n-1}}{2(1-n)}$$

```
[Out] -(a*(x - Sqrt[a + x^2])^(-1 + n))/(2*(1 - n)) + (x - Sqrt[a + x^2
])^(1 + n)/(2*(1 + n))
```

Rubi [A] time = 0.0453643, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$\frac{\left(x-\sqrt{a+x^2}\right)^{n+1}}{2(n+1)}-\frac{a\left(x-\sqrt{a+x^2}\right)^{n-1}}{2(1-n)}$$

Antiderivative was successfully verified.

```
[In]  Int[(x - Sqrt[a + x^2])^n,x]
```

```
[Out] -(a*(x - Sqrt[a + x^2])^(-1 + n))/(2*(1 - n)) + (x - Sqrt[a + x^2
])^(1 + n)/(2*(1 + n))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x-\sqrt{a+x^2}\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x-(x**2+a)**(1/2))**n,x)
```

```
[Out] Integral((x - sqrt(a + x**2))**n, x)
```

Mathematica [A] time = 0.0235853, size = 39, normalized size = 0.7

$$\frac{\left(x-\sqrt{a+x^2}\right)^n \left(n\left(-\sqrt{a+x^2}\right)-x\right)}{n^2-1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x - Sqrt[a + x^2])^n,x]

[Out] ((x - Sqrt[a + x^2])^n*(-x - n*Sqrt[a + x^2]))/(-1 + n^2)
```

Maple [F] time = 0.024, size = 0, normalized size = 0.

$$\int \left(x-\sqrt{x^2+a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x-(x^2+a)^(1/2))^n,x)

[Out] int((x-(x^2+a)^(1/2))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x-\sqrt{x^2+a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - sqrt(x^2 + a))^n,x, algorithm="maxima")

[Out] integrate((x - sqrt(x^2 + a))^n, x)
```

Fricas [A] time = 0.299274, size = 45, normalized size = 0.8

$$-\frac{\left(\sqrt{x^2+a}n+x\right)\left(x-\sqrt{x^2+a}\right)^n}{n^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - sqrt(x^2 + a))^n,x, algorithm="fricas")

[Out] -(sqrt(x^2 + a)*n + x)*(x - sqrt(x^2 + a))^n/(n^2 - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x - \sqrt{a + x^2}\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x-(x**2+a)**(1/2))**n,x)
```

```
[Out] Integral((x - sqrt(a + x**2))**n, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x - \sqrt{x^2 + a}\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - sqrt(x^2 + a))^n,x, algorithm="giac")
```

```
[Out] integrate((x - sqrt(x^2 + a))^n, x)
```

3.332 $\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{a+x^2}\,dx$

Optimal. Leaf size=63

$$\frac{2\left(x-\sqrt{a+x^2}\right)^{n+1} {}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};-\frac{\left(x-\sqrt{x^2+a}\right)^2}{a}\right)}{a(n+1)}$$

```
[Out] (2*(x - Sqrt[a + x^2])^(1 + n)*Hypergeometric2F1[1, (1 + n)/2, (3
 + n)/2, -((x - Sqrt[a + x^2])^2/a)])/(a*(1 + n))
```

Rubi [A] time = 0.123828, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087

$$\frac{2\left(x-\sqrt{a+x^2}\right)^{n+1} {}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};-\frac{\left(x-\sqrt{x^2+a}\right)^2}{a}\right)}{a(n+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(x - Sqrt[a + x^2])^n/(a + x^2),x]
```

```
[Out] (2*(x - Sqrt[a + x^2])^(1 + n)*Hypergeometric2F1[1, (1 + n)/2, (3
 + n)/2, -((x - Sqrt[a + x^2])^2/a)])/(a*(1 + n))
```

Rubi in Sympy [A] time = 12.7166, size = 46, normalized size = 0.73

$$\frac{2\left(x-\sqrt{a+x^2}\right)^{n+1} {}_2F_1\left(\begin{matrix} 1,\frac{n}{2}+\frac{1}{2} \\ \frac{n}{2}+\frac{3}{2} \end{matrix}\middle| -\frac{\left(x-\sqrt{a+x^2}\right)^2}{a}\right)}{a\left(n+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x-(x**2+a)**(1/2))**n/(x**2+a),x)
```

```
[Out] 2*(x - sqrt(a + x**2))**(n + 1)*hyper((1, n/2 + 1/2), (n/2 + 3/2,
), -(x - sqrt(a + x**2))**2/a)/(a*(n + 1))
```

Mathematica [A] time = 0.04136, size = 0, normalized size = 0.

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{a+x^2}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(x - Sqrt[a + x^2])^n/(a + x^2),x]
```

```
[Out] Integrate[(x - Sqrt[a + x^2])^n/(a + x^2), x]
```

Maple [F] time = 0.038, size = 0, normalized size = 0.

$$\int \frac{1}{x^2+a}\left(x-\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x-(x^2+a)^(1/2))^n/(x^2+a),x)
```

```
[Out] int((x-(x^2+a)^(1/2))^n/(x^2+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x-\sqrt{x^2+a}\right)^n}{x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - sqrt(x^2 + a))^n/(x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate((x - sqrt(x^2 + a))^n/(x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(x-\sqrt{x^2+a}\right)^n}{x^2+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - sqrt(x^2 + a))^n/(x^2 + a),x, algorithm="fricas")
```

```
[Out] integral((x - sqrt(x^2 + a))^n/(x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{a+x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x-(x**2+a)**(1/2))**n/(x**2+a),x)
```

```
[Out] Integral((x - sqrt(a + x**2))**n/(a + x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x-\sqrt{x^2+a}\right)^n}{x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - sqrt(x^2 + a))^n/(x^2 + a),x, algorithm="giac")
```

```
[Out] integrate((x - sqrt(x^2 + a))^n/(x^2 + a), x)
```

3.333 $\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{(a+x^2)^2}\,dx$

Optimal. Leaf size=63

$$\frac{8\left(x-\sqrt{a+x^2}\right)^{n+3}\,{}_2F_1\left(3,\frac{n+3}{2};\frac{n+5}{2};-\frac{\left(x-\sqrt{x^2+a}\right)^2}{a}\right)}{a^3(n+3)}$$

```
[Out] (8*(x - Sqrt[a + x^2])^(3 + n)*Hypergeometric2F1[3, (3 + n)/2, (5
 + n)/2, -((x - Sqrt[a + x^2])^2/a)])/(a^3*(3 + n))
```

Rubi [A] time = 0.1164, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087

$$\frac{8\left(x-\sqrt{a+x^2}\right)^{n+3}\,{}_2F_1\left(3,\frac{n+3}{2};\frac{n+5}{2};-\frac{\left(x-\sqrt{x^2+a}\right)^2}{a}\right)}{a^3(n+3)}$$

Antiderivative was successfully verified.

```
[In]  Int[(x - Sqrt[a + x^2])^n/(a + x^2)^2,x]
```

```
[Out] (8*(x - Sqrt[a + x^2])^(3 + n)*Hypergeometric2F1[3, (3 + n)/2, (5
 + n)/2, -((x - Sqrt[a + x^2])^2/a)])/(a^3*(3 + n))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$8\int^{x-\sqrt{a+x^2}} \frac{x^2x^n}{(a+x^2)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x-(x**2+a)**(1/2))**n/(x**2+a)**2,x)
```

```
[Out] 8*Integral(x**2*x**n/(a + x**2)**3, (x, x - sqrt(a + x**2)))
```

Mathematica [A] time = 0.0422912, size = 0, normalized size = 0.

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{(a+x^2)^2}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(x - Sqrt[a + x^2])^n/(a + x^2)^2,x]

[Out] Integrate[(x - Sqrt[a + x^2])^n/(a + x^2)^2, x]
```

Maple [F] time = 0.04, size = 0, normalized size = 0.

$$\int \frac{1}{(x^2+a)^2}\left(x-\sqrt{x^2+a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x-(x^2+a)^(1/2))^n/(x^2+a)^2,x)

[Out] int((x-(x^2+a)^(1/2))^n/(x^2+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x-\sqrt{x^2+a}\right)^n}{(x^2+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - sqrt(x^2 + a))^n/(x^2 + a)^2,x, algorithm="maxima")

[Out] integrate((x - sqrt(x^2 + a))^n/(x^2 + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(x-\sqrt{x^2+a}\right)^n}{x^4+2\,ax^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - sqrt(x^2 + a))^n/(x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] integral((x - sqrt(x^2 + a))^n/(x^4 + 2*a*x^2 + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x-(x**2+a)**(1/2))**n/(x**2+a)**2,x)
```

```
[Out] Integral((x - sqrt(a + x**2))**n/(a + x**2)**2, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x-\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - sqrt(x^2 + a))^n/(x^2 + a)^2,x, algorithm="giac")
```

```
[Out] integrate((x - sqrt(x^2 + a))^n/(x^2 + a)^2, x)
```

3.334 $\int (a+x^2)^{5/2} \left(x+\sqrt{a+x^2}\right)^n \, dx$

Optimal. Leaf size=187

$$-\frac{a^6 \left(\sqrt{a+x^2}+x\right)^{n-6}}{64(6-n)}-\frac{3a^5 \left(\sqrt{a+x^2}+x\right)^{n-4}}{32(4-n)}-\frac{15a^4 \left(\sqrt{a+x^2}+x\right)^{n-2}}{64(2-n)}+\frac{5a^3 \left(\sqrt{a+x^2}+x\right)^{n}}{16n}+\frac{15a^2 \left(\sqrt{a+x^2}+x\right)^{n+2}}{64(n+2)}+\frac{3a \left(\sqrt{a+x^2}+x\right)^{n+4}}{32(n+4)}+\frac{\left(\sqrt{a+x^2}+x\right)^{n+6}}{64(n+6)}$$

```
[Out] -(a^6*(x + Sqrt[a + x^2])^(-6 + n))/(64*(6 - n)) - (3*a^5*(x + Sq
rt[a + x^2])^(-4 + n))/(32*(4 - n)) - (15*a^4*(x + Sqrt[a + x^2])
^(-2 + n))/(64*(2 - n)) + (5*a^3*(x + Sqrt[a + x^2])^n)/(16*n) +
(15*a^2*(x + Sqrt[a + x^2])^(2 + n))/(64*(2 + n)) + (3*a*(x + Sqr
t[a + x^2])^(4 + n))/(32*(4 + n)) + (x + Sqrt[a + x^2])^(6 + n)/(
64*(6 + n))
```

Rubi [A] time = 0.223775, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$

$$-\frac{a^6 \left(\sqrt{a+x^2}+x\right)^{n-6}}{64(6-n)}-\frac{3a^5 \left(\sqrt{a+x^2}+x\right)^{n-4}}{32(4-n)}-\frac{15a^4 \left(\sqrt{a+x^2}+x\right)^{n-2}}{64(2-n)}+\frac{5a^3 \left(\sqrt{a+x^2}+x\right)^{n}}{16n}+\frac{15a^2 \left(\sqrt{a+x^2}+x\right)^{n+2}}{64(n+2)}+\frac{3a \left(\sqrt{a+x^2}+x\right)^{n+4}}{32(n+4)}+\frac{\left(\sqrt{a+x^2}+x\right)^{n+6}}{64(n+6)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + x^2)^(5/2)*(x + Sqrt[a + x^2])^n,x]
```

```
[Out] -(a^6*(x + Sqrt[a + x^2])^(-6 + n))/(64*(6 - n)) - (3*a^5*(x + Sq
rt[a + x^2])^(-4 + n))/(32*(4 - n)) - (15*a^4*(x + Sqrt[a + x^2])
^(-2 + n))/(64*(2 - n)) + (5*a^3*(x + Sqrt[a + x^2])^n)/(16*n) +
(15*a^2*(x + Sqrt[a + x^2])^(2 + n))/(64*(2 + n)) + (3*a*(x + Sqr
t[a + x^2])^(4 + n))/(32*(4 + n)) + (x + Sqrt[a + x^2])^(6 + n)/(
64*(6 + n))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int^{x+\sqrt{a+x^2}} \frac{x^n \left(a+x^2\right)^6}{x^7}\, dx}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+a)**(5/2)*(x+(x**2+a)**(1/2))**n,x)
```

```
[Out] Integral(x**n*(a + x**2)**6/x**7, (x, x + sqrt(a + x**2)))/64
```

Mathematica [B]   time = 15.851, size = 659, normalized size = 3.52

$$\frac{a^2\left(a+x^2\right)\left(a^2\left(n^2-2\right)+a(n-2)x\left(2(n+1)\sqrt{a+x^2}+(3n+2)x\right)+2(n-2)nx^3\left(\sqrt{a+x^2}+x\right)\right)\left(\sqrt{a+x^2}+x\right)^n}{n\left(n^2-4\right)\left(x\left(\sqrt{a+x^2}+x\right)+a\right)^2}$$
$$+\frac{2a\sqrt{a+x^2}\left(2a^4+a^3(n-4)x\left((n-4)x-2\sqrt{a+x^2}\right)+a^2(n-4)x^3\left(4(n-1)\sqrt{a+x^2}+(9n-4)x\right)+8(n-4)nx^7\left(\sqrt{a+x^2}+\cdots\right.\right.}{(n-4)n(n+4)\left(a^4\left(\sqrt{a+x^2}+8x\right)+8a^3x^2\left(4\sqrt{a+x^2}+11x\right)+16a^2x^4\left(10\sqrt{a+x^2}+17x\right)+128x^8\left(\sqrt{a\cdots\right.\right.}$$
$$+\frac{\left(x\left(\sqrt{a+x^2}+x\right)+a\right)\left(\frac{a^6}{(n-6)\left(\sqrt{a+x^2}+x\right)^6}-\frac{2a^5}{(n-4)\left(\sqrt{a+x^2}+x\right)^4}-\frac{a^4}{(n-2)\left(\sqrt{a+x^2}+x\right)^2}+\frac{4a^3}{n}-\frac{a^2\left(\sqrt{a+x^2}+x\right)^2}{n+2}-\frac{2a\left(\sqrt{a+x^2}+x\right)^4}{n+4}\cdots\right.}{64\left(a^5\left(\sqrt{a+x^2}+10x\right)+10a^4x^2\left(5\sqrt{a+x^2}+17x\right)+16a^3x^4\left(25\sqrt{a+x^2}+52x\right)+32a^2x^6\left(35\sqrt{a+x^2}+53x\right)+512x^{10}\left(\cdots\right.\right.}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + x^2)^(5/2)*(x + Sqrt[a + x^2])^n,x]
```

```
[Out] ((x + Sqrt[a + x^2])^(9 + n)*(a + x*(x + Sqrt[a + x^2]))*((4*a^3)
/n + a^6/((-6 + n)*(x + Sqrt[a + x^2])^6) - (2*a^5)/((-4 + n)*(x
+ Sqrt[a + x^2])^4) - a^4/((-2 + n)*(x + Sqrt[a + x^2])^2) - (a^2
*(x + Sqrt[a + x^2])^2)/(2 + n) - (2*a*(x + Sqrt[a + x^2])^4)/(4
+ n) + (x + Sqrt[a + x^2])^6/(6 + n)))/(64*(512*x^10*(x + Sqrt[a
+ x^2]) + a^5*(10*x + Sqrt[a + x^2]) + 256*a*x^8*(6*x + 5*Sqrt[a
+ x^2]) + 10*a^4*x^2*(17*x + 5*Sqrt[a + x^2]) + 16*a^3*x^4*(52*x
+ 25*Sqrt[a + x^2]) + 32*a^2*x^6*(53*x + 35*Sqrt[a + x^2]))) + (2
*a*Sqrt[a + x^2]*(x + Sqrt[a + x^2])^(4 + n)*(2*a^4 + a^3*(-4 + n
)*x*((-4 + n)*x - 2*Sqrt[a + x^2]) + 8*(-4 + n)*n*x^7*(x + Sqrt[a
 + x^2]) + 4*a*(-4 + n)*n*x^5*(4*x + 3*Sqrt[a + x^2]) + a^2*(-4 +
 n)*x^3*((-4 + 9*n)*x + 4*(-1 + n)*Sqrt[a + x^2])))/((-4 + n)*n*(
4 + n)*(128*x^8*(x + Sqrt[a + x^2]) + a^4*(8*x + Sqrt[a + x^2]) +
 64*a*x^6*(5*x + 4*Sqrt[a + x^2]) + 8*a^3*x^2*(11*x + 4*Sqrt[a +
x^2]) + 16*a^2*x^4*(17*x + 10*Sqrt[a + x^2]))) + (a^2*(a + x^2)*(
x + Sqrt[a + x^2])^n*(a^2*(-2 + n^2) + 2*(-2 + n)*n*x^3*(x + Sqrt
[a + x^2]) + a*(-2 + n)*x*((2 + 3*n)*x + 2*(1 + n)*Sqrt[a + x^2])
))/(n*(-4 + n^2)*(a + x*(x + Sqrt[a + x^2]))^2)
```

Maple [F] time = 0.025, size = 0, normalized size = 0.

$$\int \left(x^2+a\right)^{\frac{5}{2}} \left(x+\sqrt{x^2+a}\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+a)^(5/2)*(x+(x^2+a)^(1/2))^n,x)
```

```
[Out] int((x^2+a)^(5/2)*(x+(x^2+a)^(1/2))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^2+a\right)^{\frac{5}{2}} \left(x+\sqrt{x^2+a}\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 + a)^(5/2)*(x + sqrt(x^2 + a))^n,x, algorithm="maxima")
```

```
[Out] integrate((x^2 + a)^(5/2)*(x + sqrt(x^2 + a))^n, x)
```

Fricas [A] time = 0.298639, size = 271, normalized size = 1.45

$$\frac{\left(a^3n^6 - 50\,a^3n^4 + \left(n^6 - 20\,n^4 + 64\,n^2\right)x^6 + 544\,a^3n^2 + 3\,\left(an^6 - 30\,an^4 + 104\,an^2\right)x^4 - 720\,a^3 + 3\,\left(a^2n^6 - 40\,a^2n^4 + 264\,a^2n^2\right.\right.}{n^7 - 56\,n^5 + 784}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 + a)^(5/2)*(x + sqrt(x^2 + a))^n,x, algorithm="fricas")
```

```
[Out] (a^3*n^6 - 50*a^3*n^4 + (n^6 - 20*n^4 + 64*n^2)*x^6 + 544*a^3*n^2
 + 3*(a*n^6 - 30*a*n^4 + 104*a*n^2)*x^4 - 720*a^3 + 3*(a^2*n^6 -
40*a^2*n^4 + 264*a^2*n^2)*x^2 - 6*((n^5 - 20*n^3 + 64*n)*x^5 + 2*
(a*n^5 - 30*a*n^3 + 104*a*n)*x^3 + (a^2*n^5 - 40*a^2*n^3 + 264*a^
2*n)*x)*sqrt(x^2 + a))*(x + sqrt(x^2 + a))^n/(n^7 - 56*n^5 + 784*
n^3 - 2304*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+a)**(5/2)*(x+(x**2+a)**(1/2))**n,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int (x^2+a)^{\frac{5}{2}} \left(x+\sqrt{x^2+a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + a)^(5/2)*(x + sqrt(x^2 + a))^n,x, algorithm="giac")
```

```
[Out] integrate((x^2 + a)^(5/2)*(x + sqrt(x^2 + a))^n, x)
```

3.335 $\int \left(a+x^2\right)^{3/2} \left(x+\sqrt{a+x^2}\right)^n \, dx$

Optimal. Leaf size=131

$$-\frac{a^4\left(\sqrt{a+x^2}+x\right)^{n-4}}{16(4-n)}-\frac{a^3\left(\sqrt{a+x^2}+x\right)^{n-2}}{4(2-n)}+\frac{3a^2\left(\sqrt{a+x^2}+x\right)^{n}}{8n}+\frac{a\left(\sqrt{a+x^2}+x\right)^{n+2}}{4(n+2)}+\frac{\left(\sqrt{a+x^2}+x\right)^{n+4}}{16(n+4)}$$

```
[Out] -(a^4*(x + Sqrt[a + x^2])^(-4 + n))/(16*(4 - n)) - (a^3*(x + Sqrt
[a + x^2])^(-2 + n))/(4*(2 - n)) + (3*a^2*(x + Sqrt[a + x^2])^n)/
(8*n) + (a*(x + Sqrt[a + x^2])^(2 + n))/(4*(2 + n)) + (x + Sqrt[a
 + x^2])^(4 + n)/(16*(4 + n))
```

Rubi [A] time = 0.176048, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087

$$-\frac{a^4\left(\sqrt{a+x^2}+x\right)^{n-4}}{16(4-n)}-\frac{a^3\left(\sqrt{a+x^2}+x\right)^{n-2}}{4(2-n)}+\frac{3a^2\left(\sqrt{a+x^2}+x\right)^{n}}{8n}+\frac{a\left(\sqrt{a+x^2}+x\right)^{n+2}}{4(n+2)}+\frac{\left(\sqrt{a+x^2}+x\right)^{n+4}}{16(n+4)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + x^2)^(3/2)*(x + Sqrt[a + x^2])^n,x]
```

```
[Out] -(a^4*(x + Sqrt[a + x^2])^(-4 + n))/(16*(4 - n)) - (a^3*(x + Sqrt
[a + x^2])^(-2 + n))/(4*(2 - n)) + (3*a^2*(x + Sqrt[a + x^2])^n)/
(8*n) + (a*(x + Sqrt[a + x^2])^(2 + n))/(4*(2 + n)) + (x + Sqrt[a
 + x^2])^(4 + n)/(16*(4 + n))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int^{x+\sqrt{a+x^2}} \frac{x^n\left(a+x^2\right)^4}{x^5}\, dx}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+a)**(3/2)*(x+(x**2+a)**(1/2))**n,x)
```

```
[Out] Integral(x**n*(a + x**2)**4/x**5, (x, x + sqrt(a + x**2)))/16
```

Mathematica [B] time = 4.06056, size = 355, normalized size = 2.71

$$\frac{\sqrt{a+x^2}\left(\sqrt{a+x^2}+x\right)^n\left(\frac{a\sqrt{a+x^2}\left(a^2\left(n^2-2\right)+a(n-2)x\left(2(n+1)\sqrt{a+x^2}+(3n+2)x\right)+2(n-2)nx^3\left(\sqrt{a+x^2}+x\right)\right)}{\left(n^2-4\right)\left(x\left(\sqrt{a+x^2}+x\right)+a\right)^2}+\frac{\left(\sqrt{a+x^2}+x\right)^4\left(2a^4+a^3(n-4)x\left((n-4)x-2\sqrt{}\right.\right.}{(n-4)(n+4)\left(a^4\left(\sqrt{a+x^2}+8x\right)+8a\right.}\right.}{n}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + x^2)^(3/2)*(x + Sqrt[a + x^2])^n,x]
```

```
[Out] (Sqrt[a + x^2]*(x + Sqrt[a + x^2])^n*(((x + Sqrt[a + x^2])^4*(2*a
^4 + a^3*(-4 + n)*x*((-4 + n)*x - 2*Sqrt[a + x^2]) + 8*(-4 + n)*n
*x^7*(x + Sqrt[a + x^2]) + 4*a*(-4 + n)*n*x^5*(4*x + 3*Sqrt[a + x
^2]) + a^2*(-4 + n)*x^3*((-4 + 9*n)*x + 4*(-1 + n)*Sqrt[a + x^2])
))/((-4 + n)*(4 + n)*(128*x^8*(x + Sqrt[a + x^2]) + a^4*(8*x + Sq
rt[a + x^2]) + 64*a*x^6*(5*x + 4*Sqrt[a + x^2]) + 8*a^3*x^2*(11*x
 + 4*Sqrt[a + x^2]) + 16*a^2*x^4*(17*x + 10*Sqrt[a + x^2]))) + (a
*Sqrt[a + x^2]*(a^2*(-2 + n^2) + 2*(-2 + n)*n*x^3*(x + Sqrt[a + x
^2]) + a*(-2 + n)*x*((2 + 3*n)*x + 2*(1 + n)*Sqrt[a + x^2])))/((-
4 + n^2)*(a + x*(x + Sqrt[a + x^2]))^2)))/n
```

Maple [F] time = 0.024, size = 0, normalized size = 0.

$$\int \left(x^2+a\right)^{\frac{3}{2}}\left(x+\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+a)^(3/2)*(x+(x^2+a)^(1/2))^n,x)
```

```
[Out] int((x^2+a)^(3/2)*(x+(x^2+a)^(1/2))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^2+a\right)^{\frac{3}{2}}\left(x+\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 + a)^(3/2)*(x + sqrt(x^2 + a))^n,x, algorithm="maxima")
```

```
[Out] integrate((x^2 + a)^(3/2)*(x + sqrt(x^2 + a))^n, x)
```

Fricas [A] time = 0.295498, size = 149, normalized size = 1.14

$$\frac{\left(a^{2} n^{4} + \left(n^{4} - 4\, n^{2}\right) x^{4} - 16\, a^{2} n^{2} + 2\, \left(a n^{4} - 10\, a n^{2}\right) x^{2} + 24\, a^{2} - 4\, \left(\left(n^{3} - 4\, n\right) x^{3} + \left(a n^{3} - 10\, a n\right) x\right) \sqrt{x^{2} + a}\right) \left(x + \sqrt{x^{2} + a}\right)^{n}}{n^{5} - 20\, n^{3} + 64\, n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 + a)^(3/2)*(x + sqrt(x^2 + a))^n,x, algorithm="fricas")
```

```
[Out] (a^2*n^4 + (n^4 - 4*n^2)*x^4 - 16*a^2*n^2 + 2*(a*n^4 - 10*a*n^2)*
x^2 + 24*a^2 - 4*((n^3 - 4*n)*x^3 + (a*n^3 - 10*a*n)*x)*sqrt(x^2
+ a))*(x + sqrt(x^2 + a))^n/(n^5 - 20*n^3 + 64*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+a)**(3/2)*(x+(x**2+a)**(1/2))**n,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^{2} + a\right)^{\frac{3}{2}} \left(x + \sqrt{x^{2} + a}\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 + a)^(3/2)*(x + sqrt(x^2 + a))^n,x, algorithm="giac")
```

```
[Out] integrate((x^2 + a)^(3/2)*(x + sqrt(x^2 + a))^n, x)
```

3.336 $\int \sqrt{a+x^2}\left(x+\sqrt{a+x^2}\right)^n\,dx$

Optimal. Leaf size=75

$$-\frac{a^2\left(\sqrt{a+x^2}+x\right)^{n-2}}{4(2-n)}+\frac{a\left(\sqrt{a+x^2}+x\right)^n}{2n}+\frac{\left(\sqrt{a+x^2}+x\right)^{n+2}}{4(n+2)}$$

```
[Out] -(a^2*(x + Sqrt[a + x^2])^(-2 + n))/(4*(2 - n)) + (a*(x + Sqrt[a
+ x^2])^n)/(2*n) + (x + Sqrt[a + x^2])^(2 + n)/(4*(2 + n))
```

Rubi [A] time = 0.133209, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087

$$-\frac{a^2\left(\sqrt{a+x^2}+x\right)^{n-2}}{4(2-n)}+\frac{a\left(\sqrt{a+x^2}+x\right)^n}{2n}+\frac{\left(\sqrt{a+x^2}+x\right)^{n+2}}{4(n+2)}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + x^2]*(x + Sqrt[a + x^2])^n,x]
```

```
[Out] -(a^2*(x + Sqrt[a + x^2])^(-2 + n))/(4*(2 - n)) + (a*(x + Sqrt[a
+ x^2])^n)/(2*n) + (x + Sqrt[a + x^2])^(2 + n)/(4*(2 + n))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int^{x+\sqrt{a+x^2}} \frac{x^n\left(a+x^2\right)^2}{x^3}\,dx}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+a)**(1/2)*(x+(x**2+a)**(1/2))**n,x)
```

```
[Out] Integral(x**n*(a + x**2)**2/x**3, (x, x + sqrt(a + x**2)))/4
```

Mathematica [A] time = 0.762825, size = 104, normalized size = 1.39

$$\frac{(a+x^2)\left(\sqrt{a+x^2}+x\right)^n\left(a^2\left(n^2-2\right)+a(n-2)x\left(2(n+1)\sqrt{a+x^2}+(3n+2)x\right)+2(n-2)nx^3\left(\sqrt{a+x^2}+x\right)\right)}{n\left(n^2-4\right)\left(x\left(\sqrt{a+x^2}+x\right)+a\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + x^2]*(x + Sqrt[a + x^2])^n,x]
```

```
[Out] ((a + x^2)*(x + Sqrt[a + x^2])^n*(a^2*(-2 + n^2) + 2*(-2 + n)*n*x
^3*(x + Sqrt[a + x^2]) + a*(-2 + n)*x*((2 + 3*n)*x + 2*(1 + n)*Sq
rt[a + x^2])))/(n*(-4 + n^2)*(a + x*(x + Sqrt[a + x^2]))^2)
```

Maple [F] time = 0.024, size = 0, normalized size = 0.

$$\int \sqrt{x^2+a}\left(x+\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+a)^(1/2)*(x+(x^2+a)^(1/2))^n,x)
```

```
[Out] int((x^2+a)^(1/2)*(x+(x^2+a)^(1/2))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^2+a}\left(x+\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + a)*(x + sqrt(x^2 + a))^n,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 + a)*(x + sqrt(x^2 + a))^n, x)
```

Fricas [A] time = 0.302129, size = 65, normalized size = 0.87

$$\frac{\left(n^2x^2+an^2-2\sqrt{x^2+a}\,anx-2\,a\right)\left(x+\sqrt{x^2+a}\right)^n}{n^3-4\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + a)*(x + sqrt(x^2 + a))^n,x, algorithm="fricas")
```

```
[Out] (n^2*x^2 + a*n^2 - 2*sqrt(x^2 + a)*n*x - 2*a)*(x + sqrt(x^2 + a))
^n/(n^3 - 4*n)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + x^2} \left(x + \sqrt{a + x^2}\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+a)**(1/2)*(x+(x**2+a)**(1/2))**n,x)
```

```
[Out] Integral(sqrt(a + x**2)*(x + sqrt(a + x**2))**n, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^2 + a}\left(x + \sqrt{x^2 + a}\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + a)*(x + sqrt(x^2 + a))^n,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 + a)*(x + sqrt(x^2 + a))^n, x)
```

3.337 $\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\sqrt{a+x^2}}\,dx$

Optimal. Leaf size=17

$$\frac{\left(\sqrt{a+x^2}+x\right)^n}{n}$$

```
[Out] (x + Sqrt[a + x^2])^n/n
```

Rubi [A] time = 0.0895184, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087

$$\frac{\left(\sqrt{a+x^2}+x\right)^n}{n}$$

Antiderivative was successfully verified.

```
[In]  Int[(x + Sqrt[a + x^2])^n/Sqrt[a + x^2],x]

[Out] (x + Sqrt[a + x^2])^n/n
```

Rubi in Sympy [A] time = 9.21112, size = 12, normalized size = 0.71

$$\frac{\left(x+\sqrt{a+x^2}\right)^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x+(x**2+a)**(1/2))**n/(x**2+a)**(1/2),x)

[Out] (x + sqrt(a + x**2))**n/n
```

Mathematica [A] time = 0.0291316, size = 17, normalized size = 1.

$$\frac{\left(\sqrt{a+x^2}+x\right)^n}{n}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x + Sqrt[a + x^2])^n/Sqrt[a + x^2],x]
```

```
[Out] (x + Sqrt[a + x^2])^n/n
```

Maple [F] time = 0.028, size = 0, normalized size = 0.

$$\int 1\left(x+\sqrt{x^{2}+a}\right)^{n}\frac{1}{\sqrt{x^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x+(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x)
```

```
[Out] int((x+(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x)
```

Maxima [A] time = 0.727793, size = 20, normalized size = 1.18

$$\frac{\left(x+\sqrt{x^{2}+a}\right)^{n}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sqrt(x^2 + a))^n/sqrt(x^2 + a),x, algorithm="maxima")
```

```
[Out] (x + sqrt(x^2 + a))^n/n
```

Fricas [A] time = 0.293211, size = 20, normalized size = 1.18

$$\frac{\left(x+\sqrt{x^{2}+a}\right)^{n}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sqrt(x^2 + a))^n/sqrt(x^2 + a),x, algorithm="fricas")
```

```
[Out] (x + sqrt(x^2 + a))^n/n
```

Sympy [A] time = 14.3653, size = 311, normalized size = 18.29

$$\begin{cases} -\frac{\sqrt{a}a^{\frac{n}{2}}\sinh\left(-n\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)+\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{nx\sqrt{\frac{a}{x^2}+1}} - \frac{2a^{\frac{n}{2}}\cosh\left(n\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)\left(-\frac{n}{2}+1\right)}{n^2\left(-\frac{n}{2}\right)} + \frac{a^{\frac{n}{2}}x\cosh\left(-n\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)+\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{\sqrt{a}n} - \frac{a^{\frac{n}{2}}x\sinh\left(-n\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right.}{\sqrt{a}n\sqrt{\frac{a}{x^2}+1}} \\ -\frac{a^{\frac{n}{2}}\sinh\left(-n\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)+\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{n\sqrt{1+\frac{x^2}{a}}} - \frac{2a^{\frac{n}{2}}\cosh\left(n\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)\left(-\frac{n}{2}+1\right)}{n^2\left(-\frac{n}{2}\right)} - \frac{a^{\frac{n}{2}}x^2\sinh\left(-n\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)+\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{an\sqrt{1+\frac{x^2}{a}}} + \frac{a^{\frac{n}{2}}x\cosh\left(-n\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)+\right.}{\sqrt{a}n} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x+(x**2+a)**(1/2))**n/(x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-sqrt(a)*a**(n/2)*sinh(-n*asinh(x/sqrt(a)) + asinh(x/s
qrt(a)))/(n*x*sqrt(a/x**2 + 1)) - 2*a**(n/2)*cosh(n*asinh(x/sqrt(
a)))*gamma(-n/2 + 1)/(n**2*gamma(-n/2)) + a**(n/2)*x*cosh(-n*asin
h(x/sqrt(a)) + asinh(x/sqrt(a)))/(sqrt(a)*n) - a**(n/2)*x*sinh(-n
*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))/(sqrt(a)*n*sqrt(a/x**2 + 1)
), Abs(x**2/a) > 1), (-a**(n/2)*sinh(-n*asinh(x/sqrt(a)) + asinh(
x/sqrt(a)))/(n*sqrt(1 + x**2/a)) - 2*a**(n/2)*cosh(n*asinh(x/sqrt
(a)))*gamma(-n/2 + 1)/(n**2*gamma(-n/2)) - a**(n/2)*x**2*sinh(-n*
asinh(x/sqrt(a)) + asinh(x/sqrt(a)))/(a*n*sqrt(1 + x**2/a)) + a**
(n/2)*x*cosh(-n*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))/(sqrt(a)*n),
 True))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x+\sqrt{x^2+a}\right)^n}{\sqrt{x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sqrt(x^2 + a))^n/sqrt(x^2 + a),x, algorithm="giac")
```

```
[Out] integrate((x + sqrt(x^2 + a))^n/sqrt(x^2 + a), x)
```

3.338 $\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=59

$$\frac{4\left(\sqrt{a+x^2}+x\right)^{n+2} {}_2F_1\left(2,\frac{n+2}{2};\frac{n+4}{2};-\frac{\left(x+\sqrt{x^2+a}\right)^2}{a}\right)}{a^2(n+2)}$$

```
[Out] (4*(x + Sqrt[a + x^2])^(2 + n)*Hypergeometric2F1[2, (2 + n)/2, (4
 + n)/2, -((x + Sqrt[a + x^2])^2/a)])/(a^2*(2 + n))
```

Rubi [A] time = 0.120211, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087

$$\frac{4\left(\sqrt{a+x^2}+x\right)^{n+2} {}_2F_1\left(2,\frac{n+2}{2};\frac{n+4}{2};-\frac{\left(x+\sqrt{x^2+a}\right)^2}{a}\right)}{a^2(n+2)}$$

Antiderivative was successfully verified.

```
[In]  Int[(x + Sqrt[a + x^2])^n/(a + x^2)^(3/2),x]
```

```
[Out] (4*(x + Sqrt[a + x^2])^(2 + n)*Hypergeometric2F1[2, (2 + n)/2, (4
 + n)/2, -((x + Sqrt[a + x^2])^2/a)])/(a^2*(2 + n))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$4\int^{x+\sqrt{a+x^2}} \frac{xx^n}{\left(a+x^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x+(x**2+a)**(1/2))**n/(x**2+a)**(3/2),x)
```

```
[Out] 4*Integral(x*x**n/(a + x**2)**2, (x, x + sqrt(a + x**2)))
```

Mathematica [A] time = 0.0484099, size = 0, normalized size = 0.

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{(a+x^2)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(x + Sqrt[a + x^2])^n/(a + x^2)^(3/2),x]

[Out] Integrate[(x + Sqrt[a + x^2])^n/(a + x^2)^(3/2), x]
```

Maple [F] time = 0.025, size = 0, normalized size = 0.

$$\int 1\left(x+\sqrt{x^2+a}\right)^n\left(x^2+a\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x+(x^2+a)^(1/2))^n/(x^2+a)^(3/2),x)

[Out] int((x+(x^2+a)^(1/2))^n/(x^2+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x+\sqrt{x^2+a}\right)^n}{(x^2+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x + sqrt(x^2 + a))^n/(x^2 + a)^(3/2),x, algorithm="maxima")

[Out] integrate((x + sqrt(x^2 + a))^n/(x^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(x+\sqrt{x^2+a}\right)^n}{(x^2+a)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sqrt(x^2 + a))^n/(x^2 + a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((x + sqrt(x^2 + a))^n/(x^2 + a)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x + \sqrt{a + x^2}\right)^n}{\left(a + x^2\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x+(x**2+a)**(1/2))**n/(x**2+a)**(3/2),x)
```

```
[Out] Integral((x + sqrt(a + x**2))**n/(a + x**2)**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x + \sqrt{x^2 + a}\right)^n}{\left(x^2 + a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sqrt(x^2 + a))^n/(x^2 + a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x + sqrt(x^2 + a))^n/(x^2 + a)^(3/2), x)
```

3.339 $\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{(a+x^2)^{5/2}}\,dx$

Optimal. Leaf size=59

$$\frac{16\left(\sqrt{a+x^2}+x\right)^{n+4}\,{}_2F_1\left(4,\frac{n+4}{2};\frac{n+6}{2};-\frac{\left(x+\sqrt{x^2+a}\right)^2}{a}\right)}{a^4(n+4)}$$

```
[Out] (16*(x + Sqrt[a + x^2])^(4 + n)*Hypergeometric2F1[4, (4 + n)/2, (
6 + n)/2, -((x + Sqrt[a + x^2])^2/a)])/(a^4*(4 + n))
```

Rubi [A] time = 0.122221, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087

$$\frac{16\left(\sqrt{a+x^2}+x\right)^{n+4}\,{}_2F_1\left(4,\frac{n+4}{2};\frac{n+6}{2};-\frac{\left(x+\sqrt{x^2+a}\right)^2}{a}\right)}{a^4(n+4)}$$

Antiderivative was successfully verified.

```
[In]  Int[(x + Sqrt[a + x^2])^n/(a + x^2)^(5/2),x]
```

```
[Out] (16*(x + Sqrt[a + x^2])^(4 + n)*Hypergeometric2F1[4, (4 + n)/2, (
6 + n)/2, -((x + Sqrt[a + x^2])^2/a)])/(a^4*(4 + n))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$16\int^{x+\sqrt{a+x^2}}\frac{x^3x^n}{(a+x^2)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x+(x**2+a)**(1/2))**n/(x**2+a)**(5/2),x)
```

```
[Out] 16*Integral(x**3*x**n/(a + x**2)**4, (x, x + sqrt(a + x**2)))
```

Mathematica [A] time = 0.0491116, size = 0, normalized size = 0.

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{(a+x^2)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(x + Sqrt[a + x^2])^n/(a + x^2)^(5/2),x]

[Out] Integrate[(x + Sqrt[a + x^2])^n/(a + x^2)^(5/2), x]
```

Maple [F] time = 0.028, size = 0, normalized size = 0.

$$\int 1\left(x+\sqrt{x^2+a}\right)^n\left(x^2+a\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x+(x^2+a)^(1/2))^n/(x^2+a)^(5/2),x)

[Out] int((x+(x^2+a)^(1/2))^n/(x^2+a)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x+\sqrt{x^2+a}\right)^n}{(x^2+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sqrt(x^2 + a))^n/(x^2 + a)^(5/2),x, algorithm="maxima")

[Out] integrate((x + sqrt(x^2 + a))^n/(x^2 + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(x+\sqrt{x^2+a}\right)^n}{(x^4+2\,ax^2+a^2)\sqrt{x^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sqrt(x^2 + a))^n/(x^2 + a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((x + sqrt(x^2 + a))^n/((x^4 + 2*a*x^2 + a^2)*sqrt(x^2 +
a)), x)
```

Sympy [F(-1)]   time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x+(x**2+a)**(1/2))**n/(x**2+a)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x + \sqrt{x^2 + a}\right)^n}{(x^2 + a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sqrt(x^2 + a))^n/(x^2 + a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((x + sqrt(x^2 + a))^n/(x^2 + a)^(5/2), x)
```

3.340 $\int (a+x^2)^{5/2} \left(x-\sqrt{a+x^2}\right)^n \, dx$

Optimal. Leaf size=201

$$\frac{a^6\left(x-\sqrt{a+x^2}\right)^{n-6}}{64(6-n)}+\frac{3a^5\left(x-\sqrt{a+x^2}\right)^{n-4}}{32(4-n)}+\frac{15a^4\left(x-\sqrt{a+x^2}\right)^{n-2}}{64(2-n)}-\frac{5a^3\left(x-\sqrt{a+x^2}\right)^{n}}{16n}-\frac{15a^2\left(x-\sqrt{a+x^2}\right)^{n+2}}{64(n+2)}-\frac{3a\left(x-\sqrt{a+x^2}\right)^{n+4}}{32(n+4)}-\frac{\left(x-\sqrt{a+x^2}\right)^{n+6}}{64(n+6)}$$

```
[Out] (a^6*(x - Sqrt[a + x^2])^(-6 + n))/(64*(6 - n)) + (3*a^5*(x - Sqr
t[a + x^2])^(-4 + n))/(32*(4 - n)) + (15*a^4*(x - Sqrt[a + x^2])^
(-2 + n))/(64*(2 - n)) - (5*a^3*(x - Sqrt[a + x^2])^n)/(16*n) - (
15*a^2*(x - Sqrt[a + x^2])^(2 + n))/(64*(2 + n)) - (3*a*(x - Sqrt
[a + x^2])^(4 + n))/(32*(4 + n)) - (x - Sqrt[a + x^2])^(6 + n)/(6
4*(6 + n))
```

Rubi [A] time = 0.21591, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$

$$\frac{a^6\left(x-\sqrt{a+x^2}\right)^{n-6}}{64(6-n)}+\frac{3a^5\left(x-\sqrt{a+x^2}\right)^{n-4}}{32(4-n)}+\frac{15a^4\left(x-\sqrt{a+x^2}\right)^{n-2}}{64(2-n)}-\frac{5a^3\left(x-\sqrt{a+x^2}\right)^{n}}{16n}-\frac{15a^2\left(x-\sqrt{a+x^2}\right)^{n+2}}{64(n+2)}-\frac{3a\left(x-\sqrt{a+x^2}\right)^{n+4}}{32(n+4)}-\frac{\left(x-\sqrt{a+x^2}\right)^{n+6}}{64(n+6)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + x^2)^(5/2)*(x - Sqrt[a + x^2])^n,x]
```


```
[Out] (a^6*(x - Sqrt[a + x^2])^(-6 + n))/(64*(6 - n)) + (3*a^5*(x - Sqr
t[a + x^2])^(-4 + n))/(32*(4 - n)) + (15*a^4*(x - Sqrt[a + x^2])^
(-2 + n))/(64*(2 - n)) - (5*a^3*(x - Sqrt[a + x^2])^n)/(16*n) - (
15*a^2*(x - Sqrt[a + x^2])^(2 + n))/(64*(2 + n)) - (3*a*(x - Sqrt
[a + x^2])^(4 + n))/(32*(4 + n)) - (x - Sqrt[a + x^2])^(6 + n)/(6
4*(6 + n))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int^{x-\sqrt{a+x^2}} \frac{x^n\left(a+x^2\right)^6}{x^7}\, dx}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+a)**(5/2)*(x-(x**2+a)**(1/2))**n,x)
```

```
[Out] -Integral(x**n*(a + x**2)**6/x**7, (x, x - sqrt(a + x**2)))/64
```

Mathematica [B] time = 17.5732, size = 692, normalized size = 3.44

$$\frac{a^2\left(a+x^2\right)\left(-a^2\left(n^2-2\right)+a(n-2)x\left(2(n+1)\sqrt{a+x^2}-(3n+2)x\right)+2(n-2)nx^3\left(\sqrt{a+x^2}-x\right)\right)\left(x-\sqrt{a+x^2}\right)^n}{n\left(n^2-4\right)\left(x\left(x-\sqrt{a+x^2}\right)+a\right)^2}+\frac{2a\sqrt{a+x^2}\left(-2a^4-a^3(n-4)x\left(2\sqrt{a+x^2}+(n-4)x\right)+a^2(n-4)x^3\left(4(n-1)\sqrt{a+x^2}+(4-9n)x\right)+8(n-4)nx^7\left(\sqrt{a+x^2}\right.\right.}{(n-4)n(n+4)\left(a^4\left(\sqrt{a+x^2}-8x\right)+8a^3x^2\left(4\sqrt{a+x^2}-11x\right)+16a^2x^4\left(10\sqrt{a+x^2}-17x\right)+128x^8\left(\sqrt{\right.\right.}+\frac{\left(x\left(x-\sqrt{a+x^2}\right)+a\right)\left(\frac{a^6}{(n-6)\left(x-\sqrt{a+x^2}\right)^6}-\frac{2a^5}{(n-4)\left(x-\sqrt{a+x^2}\right)^4}-\frac{a^4}{(n-2)\left(x-\sqrt{a+x^2}\right)^2}+\frac{4a^3}{n}-\frac{a^2\left(x-\sqrt{a+x^2}\right)^2}{n+2}-\frac{2a\left(x-\sqrt{a+x^2}\right)^4}{n+4}\right.}{64\left(a^5\left(\sqrt{a+x^2}-10x\right)+10a^4x^2\left(5\sqrt{a+x^2}-17x\right)+16a^3x^4\left(25\sqrt{a+x^2}-52x\right)+32a^2x^6\left(35\sqrt{a+x^2}-53x\right)+512x^{10}\left(\right.\right.}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + x^2)^(5/2)*(x - Sqrt[a + x^2])^n,x]
```

```
[Out] ((x - Sqrt[a + x^2])^(9 + n)*(a + x*(x - Sqrt[a + x^2]))*((4*a^3)
/n + a^6/((-6 + n)*(x - Sqrt[a + x^2])^6) - (2*a^5)/((-4 + n)*(x
- Sqrt[a + x^2])^4) - a^4/((-2 + n)*(x - Sqrt[a + x^2])^2) - (a^2
*(x - Sqrt[a + x^2])^2)/(2 + n) - (2*a*(x - Sqrt[a + x^2])^4)/(4
+ n) + (x - Sqrt[a + x^2])^6/(6 + n)))/(64*(a^5*(-10*x + Sqrt[a +
 x^2]) + 512*x^10*(-x + Sqrt[a + x^2]) + 10*a^4*x^2*(-17*x + 5*Sq
rt[a + x^2]) + 256*a*x^8*(-6*x + 5*Sqrt[a + x^2]) + 16*a^3*x^4*(-
52*x + 25*Sqrt[a + x^2]) + 32*a^2*x^6*(-53*x + 35*Sqrt[a + x^2]))
) + (2*a*Sqrt[a + x^2]*(x - Sqrt[a + x^2])^(4 + n)*(-2*a^4 + 8*(-
4 + n)*n*x^7*(-x + Sqrt[a + x^2]) - a^3*(-4 + n)*x*((-4 + n)*x +
2*Sqrt[a + x^2]) + 4*a*(-4 + n)*n*x^5*(-4*x + 3*Sqrt[a + x^2]) +
a^2*(-4 + n)*x^3*((4 - 9*n)*x + 4*(-1 + n)*Sqrt[a + x^2])))/((-4
+ n)*n*(4 + n)*(a^4*(-8*x + Sqrt[a + x^2]) + 128*x^8*(-x + Sqrt[a
 + x^2]) + 8*a^3*x^2*(-11*x + 4*Sqrt[a + x^2]) + 64*a*x^6*(-5*x +
 4*Sqrt[a + x^2]) + 16*a^2*x^4*(-17*x + 10*Sqrt[a + x^2]))) + (a^
2*(a + x^2)*(x - Sqrt[a + x^2])^n*(-(a^2*(-2 + n^2)) + 2*(-2 + n)
*n*x^3*(-x + Sqrt[a + x^2]) + a*(-2 + n)*x*(-((2 + 3*n)*x) + 2*(1
 + n)*Sqrt[a + x^2])))/(n*(-4 + n^2)*(a + x*(x - Sqrt[a + x^2]))^
2)
```

Maple [F] time = 0.039, size = 0, normalized size = 0.

$$\int \left(x^2+a\right)^{\frac{5}{2}} \left(x-\sqrt{x^2+a}\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+a)^(5/2)*(x-(x^2+a)^(1/2))^n,x)
```

```
[Out] int((x^2+a)^(5/2)*(x-(x^2+a)^(1/2))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^2+a\right)^{\frac{5}{2}} \left(x-\sqrt{x^2+a}\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 + a)^(5/2)*(x - sqrt(x^2 + a))^n,x, algorithm="maxima")
```

```
[Out] integrate((x^2 + a)^(5/2)*(x - sqrt(x^2 + a))^n, x)
```

Fricas [A] time = 0.30303, size = 275, normalized size = 1.37

$$-\frac{\left(a^3n^6 - 50\,a^3n^4 + \left(n^6 - 20\,n^4 + 64\,n^2\right)x^6 + 544\,a^3n^2 + 3\left(an^6 - 30\,an^4 + 104\,an^2\right)x^4 - 720\,a^3 + 3\left(a^2n^6 - 40\,a^2n^4 + 264\,a^2n\right.}{n^7 - 56\,n^5 + 78}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 + a)^(5/2)*(x - sqrt(x^2 + a))^n,x, algorithm="fricas")
```

```
[Out] -(a^3*n^6 - 50*a^3*n^4 + (n^6 - 20*n^4 + 64*n^2)*x^6 + 544*a^3*n^
2 + 3*(a*n^6 - 30*a*n^4 + 104*a*n^2)*x^4 - 720*a^3 + 3*(a^2*n^6 -
 40*a^2*n^4 + 264*a^2*n^2)*x^2 + 6*((n^5 - 20*n^3 + 64*n)*x^5 + 2
*(a*n^5 - 30*a*n^3 + 104*a*n)*x^3 + (a^2*n^5 - 40*a^2*n^3 + 264*a
^2*n)*x)*sqrt(x^2 + a))*(x - sqrt(x^2 + a))^n/(n^7 - 56*n^5 + 784
*n^3 - 2304*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+a)**(5/2)*(x-(x**2+a)**(1/2))**n,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^2+a\right)^{\frac{5}{2}}\left(x-\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + a)^(5/2)*(x - sqrt(x^2 + a))^n,x, algorithm="giac")
```

```
[Out] integrate((x^2 + a)^(5/2)*(x - sqrt(x^2 + a))^n, x)
```

3.341 $\int \left(a+x^2\right)^{3/2} \left(x-\sqrt{a+x^2}\right)^n \, dx$

Optimal. Leaf size=141

$$\frac{a^4 \left(x-\sqrt{a+x^2}\right)^{n-4}}{16(4-n)}+\frac{a^3 \left(x-\sqrt{a+x^2}\right)^{n-2}}{4(2-n)}-\frac{3a^2 \left(x-\sqrt{a+x^2}\right)^{n}}{8n}-\frac{a \left(x-\sqrt{a+x^2}\right)^{n+2}}{4(n+2)}-\frac{\left(x-\sqrt{a+x^2}\right)^{n+4}}{16(n+4)}$$

```
[Out] (a^4*(x - Sqrt[a + x^2])^(-4 + n))/(16*(4 - n)) + (a^3*(x - Sqrt[
a + x^2])^(-2 + n))/(4*(2 - n)) - (3*a^2*(x - Sqrt[a + x^2])^n)/(
8*n) - (a*(x - Sqrt[a + x^2])^(2 + n))/(4*(2 + n)) - (x - Sqrt[a
+ x^2])^(4 + n)/(16*(4 + n))
```

Rubi [A] time = 0.180091, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08

$$\frac{a^4 \left(x-\sqrt{a+x^2}\right)^{n-4}}{16(4-n)}+\frac{a^3 \left(x-\sqrt{a+x^2}\right)^{n-2}}{4(2-n)}-\frac{3a^2 \left(x-\sqrt{a+x^2}\right)^{n}}{8n}-\frac{a \left(x-\sqrt{a+x^2}\right)^{n+2}}{4(n+2)}-\frac{\left(x-\sqrt{a+x^2}\right)^{n+4}}{16(n+4)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + x^2)^(3/2)*(x - Sqrt[a + x^2])^n,x]
```

```
[Out] (a^4*(x - Sqrt[a + x^2])^(-4 + n))/(16*(4 - n)) + (a^3*(x - Sqrt[
a + x^2])^(-2 + n))/(4*(2 - n)) - (3*a^2*(x - Sqrt[a + x^2])^n)/(
8*n) - (a*(x - Sqrt[a + x^2])^(2 + n))/(4*(2 + n)) - (x - Sqrt[a
+ x^2])^(4 + n)/(16*(4 + n))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int^{x-\sqrt{a+x^2}} \frac{x^n \left(a+x^2\right)^4}{x^5}\, dx}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+a)**(3/2)*(x-(x**2+a)**(1/2))**n,x)
```

```
[Out] -Integral(x**n*(a + x**2)**4/x**5, (x, x - sqrt(a + x**2)))/16
```

Mathematica [B] time = 3.19027, size = 366, normalized size = 2.6

$$\frac{\left(x-\sqrt{a+x^2}\right)^n\left(\frac{a\left(a+x^2\right)\left(-a^2\left(n^2-2\right)+a(n-2)x\left(2(n+1)\sqrt{a+x^2}-(3n+2)x\right)+2(n-2)nx^3\left(\sqrt{a+x^2}-x\right)\right)}{\left(n^2-4\right)\left(x\left(x-\sqrt{a+x^2}\right)+a\right)^2}+\frac{\sqrt{a+x^2}\left(x-\sqrt{a+x^2}\right)^4\left(-2a^4-a^3(n-4)x\left(2\sqrt{a+x^2}+(\cdots\right.\right.}{(n-4)(n+4)\left(a^4\left(\sqrt{a+x^2}-8x\right)+8a^3x^2\cdots\right.}\right.}{n}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + x^2)^(3/2)*(x - Sqrt[a + x^2])^n,x]
```

```
[Out] ((x - Sqrt[a + x^2])^n*((Sqrt[a + x^2]*(x - Sqrt[a + x^2])^4*(-2*
a^4 + 8*(-4 + n)*n*x^7*(-x + Sqrt[a + x^2]) - a^3*(-4 + n)*x*((-4
 + n)*x + 2*Sqrt[a + x^2]) + 4*a*(-4 + n)*n*x^5*(-4*x + 3*Sqrt[a
+ x^2]) + a^2*(-4 + n)*x^3*((4 - 9*n)*x + 4*(-1 + n)*Sqrt[a + x^2
])))/((-4 + n)*(4 + n)*(a^4*(-8*x + Sqrt[a + x^2]) + 128*x^8*(-x
+ Sqrt[a + x^2]) + 8*a^3*x^2*(-11*x + 4*Sqrt[a + x^2]) + 64*a*x^6
*(-5*x + 4*Sqrt[a + x^2]) + 16*a^2*x^4*(-17*x + 10*Sqrt[a + x^2])
)) + (a*(a + x^2)*(-(a^2*(-2 + n^2)) + 2*(-2 + n)*n*x^3*(-x + Sqr
t[a + x^2]) + a*(-2 + n)*x*(-((2 + 3*n)*x) + 2*(1 + n)*Sqrt[a + x
^2])))/((-4 + n^2)*(a + x*(x - Sqrt[a + x^2]))^2)))/n
```

Maple [F] time = 0.04, size = 0, normalized size = 0.

$$\int \left(x^2+a\right)^{\frac{3}{2}}\left(x-\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+a)^(3/2)*(x-(x^2+a)^(1/2))^n,x)
```

```
[Out] int((x^2+a)^(3/2)*(x-(x^2+a)^(1/2))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^2+a\right)^{\frac{3}{2}}\left(x-\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 + a)^(3/2)*(x - sqrt(x^2 + a))^n,x, algorithm="maxima")
```

```
[Out] integrate((x^2 + a)^(3/2)*(x - sqrt(x^2 + a))^n, x)
```

Fricas [A] time = 0.298217, size = 153, normalized size = 1.09

$$-\frac{\left(a^2n^4 + \left(n^4 - 4\,n^2\right)x^4 - 16\,a^2n^2 + 2\left(an^4 - 10\,an^2\right)x^2 + 24\,a^2 + 4\left(\left(n^3 - 4\,n\right)x^3 + \left(an^3 - 10\,an\right)x\right)\sqrt{x^2 + a}\right)\left(x - \sqrt{x^2 + a}\right)}{n^5 - 20\,n^3 + 64\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 + a)^(3/2)*(x - sqrt(x^2 + a))^n,x, algorithm="fricas")
```

```
[Out] -(a^2*n^4 + (n^4 - 4*n^2)*x^4 - 16*a^2*n^2 + 2*(a*n^4 - 10*a*n^2)
*x^2 + 24*a^2 + 4*((n^3 - 4*n)*x^3 + (a*n^3 - 10*a*n)*x)*sqrt(x^2
 + a))*(x - sqrt(x^2 + a))^n/(n^5 - 20*n^3 + 64*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+a)**(3/2)*(x-(x**2+a)**(1/2))**n,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\left(x^2 + a\right)^{\frac{3}{2}}\left(x - \sqrt{x^2 + a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 + a)^(3/2)*(x - sqrt(x^2 + a))^n,x, algorithm="giac")
```

```
[Out] integrate((x^2 + a)^(3/2)*(x - sqrt(x^2 + a))^n, x)
```

3.342 $\int \sqrt{a+x^2}\left(x-\sqrt{a+x^2}\right)^n\,dx$

Optimal. Leaf size=81

$$\frac{a^2\left(x-\sqrt{a+x^2}\right)^{n-2}}{4(2-n)}-\frac{a\left(x-\sqrt{a+x^2}\right)^n}{2n}-\frac{\left(x-\sqrt{a+x^2}\right)^{n+2}}{4(n+2)}$$

```
[Out] (a^2*(x - Sqrt[a + x^2])^(-2 + n))/(4*(2 - n)) - (a*(x - Sqrt[a +
 x^2])^n)/(2*n) - (x - Sqrt[a + x^2])^(2 + n)/(4*(2 + n))
```

Rubi [A] time = 0.139141, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08

$$\frac{a^2\left(x-\sqrt{a+x^2}\right)^{n-2}}{4(2-n)}-\frac{a\left(x-\sqrt{a+x^2}\right)^n}{2n}-\frac{\left(x-\sqrt{a+x^2}\right)^{n+2}}{4(n+2)}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + x^2]*(x - Sqrt[a + x^2])^n,x]
```

```
[Out] (a^2*(x - Sqrt[a + x^2])^(-2 + n))/(4*(2 - n)) - (a*(x - Sqrt[a +
 x^2])^n)/(2*n) - (x - Sqrt[a + x^2])^(2 + n)/(4*(2 + n))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int^{x-\sqrt{a+x^2}}\frac{x^n\left(a+x^2\right)^2}{x^3}\,dx}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+a)**(1/2)*(x-(x**2+a)**(1/2))**n,x)
```

```
[Out] -Integral(x**n*(a + x**2)**2/x**3, (x, x - sqrt(a + x**2)))/4
```

Mathematica [A] time = 0.849468, size = 112, normalized size = 1.38

$$\frac{\left(a+x^2\right)\left(x-\sqrt{a+x^2}\right)^n\left(-a^2\left(n^2-2\right)+a(n-2)x\left(2(n+1)\sqrt{a+x^2}-(3n+2)x\right)+2(n-2)nx^3\left(\sqrt{a+x^2}-x\right)\right)}{n\left(n^2-4\right)\left(x\left(x-\sqrt{a+x^2}\right)+a\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + x^2]*(x - Sqrt[a + x^2])^n,x]
```

```
[Out] ((a + x^2)*(x - Sqrt[a + x^2])^n*(-(a^2*(-2 + n^2)) + 2*(-2 + n)*
n*x^3*(-x + Sqrt[a + x^2]) + a*(-2 + n)*x*(-((2 + 3*n)*x) + 2*(1
+ n)*Sqrt[a + x^2])))/(n*(-4 + n^2)*(a + x*(x - Sqrt[a + x^2]))^2
)
```

Maple [F] time = 0.039, size = 0, normalized size = 0.

$$\int \sqrt{x^2+a}\left(x-\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+a)^(1/2)*(x-(x^2+a)^(1/2))^n,x)
```

```
[Out] int((x^2+a)^(1/2)*(x-(x^2+a)^(1/2))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^2+a}\left(x-\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + a)*(x - sqrt(x^2 + a))^n,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 + a)*(x - sqrt(x^2 + a))^n, x)
```

Fricas [A] time = 0.323089, size = 69, normalized size = 0.85

$$-\frac{\left(n^2x^2+an^2+2\sqrt{x^2+a}\,anx-2\,a\right)\left(x-\sqrt{x^2+a}\right)^n}{n^3-4\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + a)*(x - sqrt(x^2 + a))^n,x, algorithm="fricas")
```

```
[Out] -(n^2*x^2 + a*n^2 + 2*sqrt(x^2 + a)*n*x - 2*a)*(x - sqrt(x^2 + a)
)^n/(n^3 - 4*n)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + x^2} \left(x - \sqrt{a + x^2}\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+a)**(1/2)*(x-(x**2+a)**(1/2))**n,x)
```

```
[Out] Integral(sqrt(a + x**2)*(x - sqrt(a + x**2))**n, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^2 + a}\left(x - \sqrt{x^2 + a}\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + a)*(x - sqrt(x^2 + a))^n,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 + a)*(x - sqrt(x^2 + a))^n, x)
```

3.343 $\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\sqrt{a+x^2}}\,dx$

Optimal. Leaf size=20

$$-\frac{\left(x-\sqrt{a+x^2}\right)^n}{n}$$

```
[Out] -((x - Sqrt[a + x^2])^n/n)
```

Rubi [A] time = 0.094739, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08

$$-\frac{\left(x-\sqrt{a+x^2}\right)^n}{n}$$

Antiderivative was successfully verified.

```
[In]  Int[(x - Sqrt[a + x^2])^n/Sqrt[a + x^2],x]
```

```
[Out] -((x - Sqrt[a + x^2])^n/n)
```

Rubi in Sympy [A] time = 9.91696, size = 14, normalized size = 0.7

$$-\frac{\left(x-\sqrt{a+x^2}\right)^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x-(x**2+a)**(1/2))**n/(x**2+a)**(1/2),x)
```

```
[Out] -(x - sqrt(a + x**2))**n/n
```

Mathematica [A] time = 0.0308736, size = 20, normalized size = 1.

$$-\frac{\left(x-\sqrt{a+x^2}\right)^n}{n}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x - Sqrt[a + x^2])^n/Sqrt[a + x^2],x]

[Out] -((x - Sqrt[a + x^2])^n/n)
```

Maple [F] time = 0.042, size = 0, normalized size = 0.

$$\int 1\left(x-\sqrt{x^2+a}\right)^n \frac{1}{\sqrt{x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x-(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x)

[Out] int((x-(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x)
```

Maxima [A] time = 0.738753, size = 24, normalized size = 1.2

$$-\frac{\left(x-\sqrt{x^2+a}\right)^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - sqrt(x^2 + a))^n/sqrt(x^2 + a),x, algorithm="maxima")

[Out] -(x - sqrt(x^2 + a))^n/n
```

Fricas [A] time = 0.312866, size = 24, normalized size = 1.2

$$-\frac{\left(x-\sqrt{x^2+a}\right)^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - sqrt(x^2 + a))^n/sqrt(x^2 + a),x, algorithm="fricas")
```

```
[Out] -(x - sqrt(x^2 + a))^n/n
```

Sympy [A] time = 10.3882, size = 36, normalized size = 1.8

$$\begin{cases} -\frac{\left(x-\sqrt{a+x^2}\right)^n}{n} & \text{for}\ n \neq 0 \\ \begin{cases} \operatorname{asinh}\left(x\sqrt{\frac{1}{a}}\right) & \text{for}\ a > 0 \\ \operatorname{acosh}\left(x\sqrt{-\frac{1}{a}}\right) & \text{for}\ a < 0 \end{cases} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x-(x**2+a)**(1/2))**n/(x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-(x - sqrt(a + x**2))**n/n, Ne(n, 0)), (Piecewise((asi
nh(x*sqrt(1/a)), a > 0), (acosh(x*sqrt(-1/a)), a < 0)), True))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x-\sqrt{x^2+a}\right)^n}{\sqrt{x^2+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - sqrt(x^2 + a))^n/sqrt(x^2 + a),x, algorithm="giac")
```

```
[Out] integrate((x - sqrt(x^2 + a))^n/sqrt(x^2 + a), x)
```

3.344 $\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{(a+x^2)^{3/2}}\,dx$

Optimal. Leaf size=63

$$-\frac{4\left(x-\sqrt{a+x^2}\right)^{n+2}\,{}_2F_1\left(2,\frac{n+2}{2};\frac{n+4}{2};-\frac{\left(x-\sqrt{x^2+a}\right)^2}{a}\right)}{a^2(n+2)}$$

```
[Out] (-4*(x - Sqrt[a + x^2])^(2 + n)*Hypergeometric2F1[2, (2 + n)/2, (
4 + n)/2, -((x - Sqrt[a + x^2])^2/a)])/(a^2*(2 + n))
```

Rubi [A] time = 0.125568, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$

$$-\frac{4\left(x-\sqrt{a+x^2}\right)^{n+2}\,{}_2F_1\left(2,\frac{n+2}{2};\frac{n+4}{2};-\frac{\left(x-\sqrt{x^2+a}\right)^2}{a}\right)}{a^2(n+2)}$$

Antiderivative was successfully verified.

```
[In]  Int[(x - Sqrt[a + x^2])^n/(a + x^2)^(3/2),x]
```

```
[Out] (-4*(x - Sqrt[a + x^2])^(2 + n)*Hypergeometric2F1[2, (2 + n)/2, (
4 + n)/2, -((x - Sqrt[a + x^2])^2/a)])/(a^2*(2 + n))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-4\int^{x-\sqrt{a+x^2}} \frac{xx^n}{(a+x^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x-(x**2+a)**(1/2))**n/(x**2+a)**(3/2),x)
```

```
[Out] -4*Integral(x*x**n/(a + x**2)**2, (x, x - sqrt(a + x**2)))
```

Mathematica [A] time = 0.0507669, size = 0, normalized size = 0.

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{(a+x^2)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(x - Sqrt[a + x^2])^n/(a + x^2)^(3/2),x]
```

```
[Out] Integrate[(x - Sqrt[a + x^2])^n/(a + x^2)^(3/2), x]
```

Maple [F] time = 0.039, size = 0, normalized size = 0.

$$\int 1\left(x-\sqrt{x^2+a}\right)^n\left(x^2+a\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x-(x^2+a)^(1/2))^n/(x^2+a)^(3/2),x)
```

```
[Out] int((x-(x^2+a)^(1/2))^n/(x^2+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x-\sqrt{x^2+a}\right)^n}{(x^2+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x - sqrt(x^2 + a))^n/(x^2 + a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x - sqrt(x^2 + a))^n/(x^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(x-\sqrt{x^2+a}\right)^n}{(x^2+a)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - sqrt(x^2 + a))^n/(x^2 + a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((x - sqrt(x^2 + a))^n/(x^2 + a)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x - \sqrt{a + x^2}\right)^n}{(a + x^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x-(x**2+a)**(1/2))**n/(x**2+a)**(3/2),x)
```

```
[Out] Integral((x - sqrt(a + x**2))**n/(a + x**2)**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x - \sqrt{x^2 + a}\right)^n}{(x^2 + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - sqrt(x^2 + a))^n/(x^2 + a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x - sqrt(x^2 + a))^n/(x^2 + a)^(3/2), x)
```

3.345 $\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{(a+x^2)^{5/2}}\,dx$

Optimal. Leaf size=63

$$-\frac{16\left(x-\sqrt{a+x^2}\right)^{n+4}\,{}_2F_1\left(4,\frac{n+4}{2};\frac{n+6}{2};-\frac{\left(x-\sqrt{x^2+a}\right)^2}{a}\right)}{a^4(n+4)}$$

```
[Out] (-16*(x - Sqrt[a + x^2])^(4 + n)*Hypergeometric2F1[4, (4 + n)/2,
(6 + n)/2, -((x - Sqrt[a + x^2])^2/a)])/(a^4*(4 + n))
```

Rubi [A] time = 0.12499, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$

$$-\frac{16\left(x-\sqrt{a+x^2}\right)^{n+4}\,{}_2F_1\left(4,\frac{n+4}{2};\frac{n+6}{2};-\frac{\left(x-\sqrt{x^2+a}\right)^2}{a}\right)}{a^4(n+4)}$$

Antiderivative was successfully verified.

```
[In]  Int[(x - Sqrt[a + x^2])^n/(a + x^2)^(5/2),x]
```

```
[Out] (-16*(x - Sqrt[a + x^2])^(4 + n)*Hypergeometric2F1[4, (4 + n)/2,
(6 + n)/2, -((x - Sqrt[a + x^2])^2/a)])/(a^4*(4 + n))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-16\int^{x-\sqrt{a+x^2}}\frac{x^3x^n}{(a+x^2)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x-(x**2+a)**(1/2))**n/(x**2+a)**(5/2),x)
```

```
[Out] -16*Integral(x**3*x**n/(a + x**2)**4, (x, x - sqrt(a + x**2)))
```

Mathematica [A] time = 0.0502313, size = 0, normalized size = 0.

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{(a+x^2)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(x - Sqrt[a + x^2])^n/(a + x^2)^(5/2),x]

[Out] Integrate[(x - Sqrt[a + x^2])^n/(a + x^2)^(5/2), x]
```

Maple [F] time = 0.039, size = 0, normalized size = 0.

$$\int 1\left(x-\sqrt{x^2+a}\right)^n\left(x^2+a\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x-(x^2+a)^(1/2))^n/(x^2+a)^(5/2),x)

[Out] int((x-(x^2+a)^(1/2))^n/(x^2+a)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x-\sqrt{x^2+a}\right)^n}{(x^2+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - sqrt(x^2 + a))^n/(x^2 + a)^(5/2),x, algorithm="maxima")

[Out] integrate((x - sqrt(x^2 + a))^n/(x^2 + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(x-\sqrt{x^2+a}\right)^n}{(x^4+2\,ax^2+a^2)\sqrt{x^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - sqrt(x^2 + a))^n/(x^2 + a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((x - sqrt(x^2 + a))^n/((x^4 + 2*a*x^2 + a^2)*sqrt(x^2 +
a)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x-(x**2+a)**(1/2))**n/(x**2+a)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x - \sqrt{x^2 + a}\right)^n}{(x^2 + a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - sqrt(x^2 + a))^n/(x^2 + a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((x - sqrt(x^2 + a))^n/(x^2 + a)^(5/2), x)
```

3.346 $\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^2 \left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$

Optimal. Leaf size=365

$$\frac{(d^2-af^2)^5\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-5}}{32ef^4(5-n)}-\frac{5(d^2-af^2)^4\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-3}}{32ef^4(3-n)}+\frac{5(d^2-af^2)^3\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-1}}{16ef^4(1-n)}+\frac{5(d^2-af^2)^2\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}}{16ef^4(n+1)}-\frac{5(d^2-af^2)\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+3}}{32ef^4(n+3)}+\frac{\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+5}}{32ef^4(n+5)}$$

```
[Out] ((d^2 - a*f^2)^5*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/
f^2])^(-5 + n))/(32*e*f^4*(5 - n)) - (5*(d^2 - a*f^2)^4*(d + e*x
+ f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(-3 + n))/(32*e*f^4*
(3 - n)) + (5*(d^2 - a*f^2)^3*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2
 + (e^2*x^2)/f^2])^(-1 + n))/(16*e*f^4*(1 - n)) + (5*(d^2 - a*f^2
)^2*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(1 + n)
)/(16*e*f^4*(1 + n)) - (5*(d^2 - a*f^2)*(d + e*x + f*Sqrt[a + (2*
d*e*x)/f^2 + (e^2*x^2)/f^2])^(3 + n))/(32*e*f^4*(3 + n)) + (d + e
*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(5 + n)/(32*e*f^4
*(5 + n))
```

Rubi [A] time = 0.863627, antiderivative size = 365, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 56, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$

$$\frac{(d^2-af^2)^5\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-5}}{32ef^4(5-n)}-\frac{5(d^2-af^2)^4\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-3}}{32ef^4(3-n)}+\frac{5(d^2-af^2)^3\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-1}}{16ef^4(1-n)}+\frac{5(d^2-af^2)^2\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}}{16ef^4(n+1)}-\frac{5(d^2-af^2)\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+3}}{32ef^4(n+3)}+\frac{\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+5}}{32ef^4(n+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2)^2*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e
```

```
[Out] ((d^2 - a*f^2)^5*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/
f^2])^(-5 + n))/(32*e*f^4*(5 - n)) - (5*(d^2 - a*f^2)^4*(d + e*x
+ f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(-3 + n))/(32*e*f^4*
(3 - n)) + (5*(d^2 - a*f^2)^3*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2
 + (e^2*x^2)/f^2])^(-1 + n))/(16*e*f^4*(1 - n)) + (5*(d^2 - a*f^2
)^2*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(1 + n)
)/(16*e*f^4*(1 + n)) - (5*(d^2 - a*f^2)*(d + e*x + f*Sqrt[a + (2*
d*e*x)/f^2 + (e^2*x^2)/f^2])^(3 + n))/(32*e*f^4*(3 + n)) + (d + e
*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(5 + n)/(32*e*f^4
*(5 + n))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((a+2*d*e*x/f**2+e**2*x**2/f**2)**2*(d+e*x+f*(a+2*d*e*x/f**2+e**
```

```
[Out] Timed out
```

Mathematica [A] time = 0.646068, size = 0, normalized size = 0.

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^2\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2)^2*(d + e*x + f*Sqrt[a + (2*d*e*x)/f
```

```
[Out] Integrate[(a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2)^2*(d + e*x + f*Sqrt
[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n, x]
```

Maple [F] time = 0.124, size = 0, normalized size = 0.

$$\int \left(a+2\,\frac{dex}{f^2}+\frac{e^2x^2}{f^2}\right)^2\left(d+ex+f\sqrt{a+2\,\frac{dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+2*d*e*x/f^2+e^2*x^2/f^2)^2*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^
```

```
[Out] int((a+2*d*e*x/f^2+e^2*x^2/f^2)^2*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2
/f^2)^(1/2))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}\right)^2 \left(ex + \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}}f + d\right)^n \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e^2*x^2/f^2 + a + 2*d*e*x/f^2)^2*(e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x
```

```
[Out] integrate((e^2*x^2/f^2 + a + 2*d*e*x/f^2)^2*(e*x + sqrt(e^2*x^2/f
^2 + a + 2*d*e*x/f^2)*f + d)^n, x)
```

Fricas [A] time = 0.332434, size = 883, normalized size = 2.42

$$-\frac{\left(5\,a^2df^4n^4 + 225\,a^2df^4 - 300\,ad^3f^2 + 5\,\left(e^5n^4 - 10\,e^5n^2 + 9\,e^5\right)x^5 + 120\,d^5 + 25\,\left(de^4n^4 - 10\,de^4n^2 + 9\,de^4\right)x^4 + 10\,(15\,ae\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e^2*x^2/f^2 + a + 2*d*e*x/f^2)^2*(e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x
```

```
[Out] -(5*a^2*d*f^4*n^4 + 225*a^2*d*f^4 - 300*a*d^3*f^2 + 5*(e^5*n^4 -
10*e^5*n^2 + 9*e^5)*x^5 + 120*d^5 + 25*(d*e^4*n^4 - 10*d*e^4*n^2
+ 9*d*e^4)*x^4 + 10*(15*a*e^3*f^2 + 30*d^2*e^3 + (a*e^3*f^2 + 4*d
^2*e^3)*n^4 - 2*(8*a*e^3*f^2 + 17*d^2*e^3)*n^2)*x^3 - 10*(11*a^2*
d*f^4 - 6*a*d^3*f^2)*n^2 + 10*(45*a*d*e^2*f^2 + (3*a*d*e^2*f^2 +
2*d^3*e^2)*n^4 - 2*(24*a*d*e^2*f^2 + d^3*e^2)*n^2)*x^2 + 5*(45*a^
2*e*f^4 + (a^2*e*f^4 + 4*a*d^2*e*f^2)*n^4 - 2*(11*a^2*e*f^4 + 26*
a*d^2*e*f^2 - 12*d^4*e)*n^2)*x - (a^2*f^5*n^5 + (e^4*f*n^5 - 10*e
^4*f*n^3 + 9*e^4*f*n)*x^4 - 10*(3*a^2*f^5 - 2*a*d^2*f^3)*n^3 + 4*
(d*e^3*f*n^5 - 10*d*e^3*f*n^3 + 9*d*e^3*f*n)*x^3 + 2*((a*e^2*f^3
+ 2*d^2*e^2*f)*n^5 - 10*(2*a*e^2*f^3 + d^2*e^2*f)*n^3 + (19*a*e^2
*f^3 + 8*d^2*e^2*f)*n)*x^2 + (149*a^2*f^5 - 260*a*d^2*f^3 + 120*d
^4*f)*n + 4*(a*d*e*f^3*n^5 - 10*(2*a*d*e*f^3 - d^3*e*f)*n^3 + (19
```

```
*a*d*e*f^3 - 10*d^3*e*f)*n)*x)*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f
^2))*(e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) + d)^n/(e*f^4
*n^6 - 35*e*f^4*n^4 + 259*e*f^4*n^2 - 225*e*f^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f**2+e**2*x**2/f**2)**2*(d+e*x+f*(a+2*d*e*x/f**2+e**2*x**
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}\right)^2\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}\,f+d\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e^2*x^2/f^2 + a + 2*d*e*x/f^2)^2*(e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x
```

```
[Out] integrate((e^2*x^2/f^2 + a + 2*d*e*x/f^2)^2*(e*x + sqrt(e^2*x^2/f
^2 + a + 2*d*e*x/f^2)*f + d)^n, x)
```

3.347 $\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$

Optimal. Leaf size=239

$$\frac{(d^2-af^2)^3\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-3}}{8ef^2(3-n)}-\frac{3(d^2-af^2)^2\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-1}}{8ef^2(1-n)}-\frac{3(d^2-af^2)\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}}{8ef^2(n+1)}+\frac{\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+3}}{8ef^2(n+3)}$$

```
[Out] ((d^2 - a*f^2)^3*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/
f^2])^(-3 + n))/(8*e*f^2*(3 - n)) - (3*(d^2 - a*f^2)^2*(d + e*x +
 f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(-1 + n))/(8*e*f^2*(1
 - n)) - (3*(d^2 - a*f^2)*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (
e^2*x^2)/f^2])^(1 + n))/(8*e*f^2*(1 + n)) + (d + e*x + f*Sqrt[a +
 (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(3 + n)/(8*e*f^2*(3 + n))
```

Rubi [A] time = 0.482507, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 54, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$

$$\frac{(d^2-af^2)^3\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-3}}{8ef^2(3-n)}-\frac{3(d^2-af^2)^2\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-1}}{8ef^2(1-n)}-\frac{3(d^2-af^2)\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}}{8ef^2(n+1)}+\frac{\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+3}}{8ef^2(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2)*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2
```

```
[Out] ((d^2 - a*f^2)^3*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/
f^2])^(-3 + n))/(8*e*f^2*(3 - n)) - (3*(d^2 - a*f^2)^2*(d + e*x +
 f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(-1 + n))/(8*e*f^2*(1
 - n)) - (3*(d^2 - a*f^2)*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (
e^2*x^2)/f^2])^(1 + n))/(8*e*f^2*(1 + n)) + (d + e*x + f*Sqrt[a +
 (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(3 + n)/(8*e*f^2*(3 + n))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int^{d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}\frac{e^3x^n\left(af^2-d^2+x^2\right)^3}{x^4}\,dx}{8e^4f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((a+2*d*e*x/f**2+e**2*x**2/f**2)*(d+e*x+f*(a+2*d*e*x/f**2+e**2*x
```

```
[Out] Integral(e**3*x**n*(a*f**2 - d**2 + x**2)**3/x**4, (x, d + e*x +
f*sqrt(a + 2*d*e*x/f**2 + e**2*x**2/f**2)))/(8*e**4*f**2)
```

Mathematica [A] time = 0.227906, size = 0, normalized size = 0.

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2)*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2
```

```
[Out] Integrate[(a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2)*(d + e*x + f*Sqrt[a
 + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n, x]
```

Maple [F] time = 0.016, size = 0, normalized size = 0.

$$\int \left(a+2\,\frac{dex}{f^2}+\frac{e^2x^2}{f^2}\right)\left(d+ex+f\sqrt{a+2\,\frac{dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+2*d*e*x/f^2+e^2*x^2/f^2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n,
```

```
[Out] int((a+2*d*e*x/f^2+e^2*x^2/f^2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f
^2)^(1/2))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}\right)\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}f+d\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e^2*x^2/f^2 + a + 2*d*e*x/f^2)*(e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f
```

```
[Out] integrate((e^2*x^2/f^2 + a + 2*d*e*x/f^2)*(e*x + sqrt(e^2*x^2/f^2
 + a + 2*d*e*x/f^2)*f + d)^n, x)
```

Fricas [A] time = 0.327124, size = 323, normalized size = 1.35

$$-\frac{\left(3\,adf^2n^2 - 9\,adf^2 + 3\left(e^3n^2 - e^3\right)x^3 + 6\,d^3 + 9\left(de^2n^2 - de^2\right)x^2 - 3\left(3\,aef^2 - \left(aef^2 + 2\,d^2e\right)n^2\right)x - \left(af^3n^3 + \left(e^2fn^3 - \right.\right.}{ef^2n^4 - 10\,ef^2n^2 + 9\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e^2*x^2/f^2 + a + 2*d*e*x/f^2)*(e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f
```

```
[Out] -(3*a*d*f^2*n^2 - 9*a*d*f^2 + 3*(e^3*n^2 - e^3)*x^3 + 6*d^3 + 9*(
d*e^2*n^2 - d*e^2)*x^2 - 3*(3*a*e*f^2 - (a*e*f^2 + 2*d^2*e)*n^2)*
x - (a*f^3*n^3 + (e^2*f*n^3 - e^2*f*n)*x^2 - (7*a*f^3 - 6*d^2*f)*
n + 2*(d*e*f*n^3 - d*e*f*n)*x)*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f
^2))*(e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) + d)^n/(e*f^2
*n^4 - 10*e*f^2*n^2 + 9*e*f^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f**2+e**2*x**2/f**2)*(d+e*x+f*(a+2*d*e*x/f**2+e**2*x**2/f
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\left(\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}\right)\left(ex + \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}}f + d\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e^2*x^2/f^2 + a + 2*d*e*x/f^2)*(e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f
```

```
[Out] integrate((e^2*x^2/f^2 + a + 2*d*e*x/f^2)*(e*x + sqrt(e^2*x^2/f^2
 + a + 2*d*e*x/f^2)*f + d)^n, x)
```

3.348 $\int \left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$

Optimal. Leaf size=107

$$\frac{(d^2-af^2)\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-1}}{2e(1-n)}+\frac{\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}}{2e(n+1)}$$

```
[Out] ((d^2 - a*f^2)*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^
2])^(-1 + n))/(2*e*(1 - n)) + (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2
 + (e^2*x^2)/f^2])^(1 + n)/(2*e*(1 + n))
```

Rubi [A] time = 0.172607, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{(d^2-af^2)\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-1}}{2e(1-n)}+\frac{\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}}{2e(n+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n,x]
```

```
[Out] ((d^2 - a*f^2)*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^
2])^(-1 + n))/(2*e*(1 - n)) + (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2
 + (e^2*x^2)/f^2])^(1 + n)/(2*e*(1 + n))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((d+e*x+f*(a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2))**n,x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + 2*d*e*x/f**2 + e**2*x**2/f**2))**n
, x)
```

Mathematica [A] time = 0.0595444, size = 0, normalized size = 0.

$$\int \left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n,x]
```

```
[Out] Integrate[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n
, x]
```

Maple [F] time = 0.015, size = 0, normalized size = 0.

$$\int \left(d+ex+f\sqrt{a+2\,\frac{dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n,x)
```

```
[Out] int((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}f+d\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n,x, algorithm="maxim
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n, x)
```

Fricas [A] time = 0.345711, size = 108, normalized size = 1.01

$$\frac{\left(fn\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}-ex-d\right)\left(ex+f\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}+d\right)^n}{en^2-e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n,x, algorithm="frica
```

```
[Out] (f*n*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) - e*x - d)*(e*x + f*sq
rt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) + d)^n/(e*n^2 - e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d+e*x+f*(a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2))**n,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex + \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}}f + d\right)^n \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n,x, algorithm="giac"
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n, x)
```

3.349 $\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\,dx$

Optimal. Leaf size=122

$$-\frac{2f^2\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^2}{d^2-af^2}\right)}{e(n+1)\left(d^2-af^2\right)}$$

```
[Out] (-2*f^2*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(1
+ n)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, (d + e*x + f*Sqrt
[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^2/(d^2 - a*f^2)])/(e*(d^2 -
a*f^2)*(1 + n))
```

Rubi [A] time = 0.485895, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 56, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036

$$-\frac{2f^2\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^2}{d^2-af^2}\right)}{e(n+1)\left(d^2-af^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/(a + (2*d*e*x)/f^2 + (
```

```
[Out] (-2*f^2*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(1
+ n)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, (d + e*x + f*Sqrt
[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^2/(d^2 - a*f^2)])/(e*(d^2 -
a*f^2)*(1 + n))
```

Rubi in Sympy [A] time = 111.889, size = 110, normalized size = 0.9

$$-\frac{2f^2\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{n+1}{}_2F_1\left(\begin{matrix}1,\frac{n}{2}+\frac{1}{2}\\\frac{n}{2}+\frac{3}{2}\end{matrix}\middle|-\frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^2}{af^2-d^2}\right)}{e(n+1)\left(-af^2+d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((d+e*x+f*(a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2))**n/(a+2*d*e*x/
```

```
[Out] -2*f**2*(d + e*x + f*sqrt(a + 2*d*e*x/f**2 + e**2*x**2/f**2))**(n
 + 1)*hyper((1, n/2 + 1/2), (n/2 + 3/2,), -(d + e*x + f*sqrt(a +
2*d*e*x/f**2 + e**2*x**2/f**2))**2/(a*f**2 - d**2))/(e*(n + 1)*(-
a*f**2 + d**2))
```

Mathematica [A] time = 0.152238, size = 0, normalized size = 0.

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/(a + (2*d*e*x)/
```

```
[Out] Integrate[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n
/(a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2), x]
```

Maple [F] time = 0.133, size = 0, normalized size = 0.

$$\int 1\left(d+ex+f\sqrt{a+2\,\frac{dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n\left(a+2\,\frac{dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*x^2/f^2),
```

```
[Out] int((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+
e^2*x^2/f^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}f+d\right)^n}{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/(e^2*x^2/f^2 + a + 2
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/(e^
2*x^2/f^2 + a + 2*d*e*x/f^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(ex+f\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}+d\right)^n f^2}{e^2x^2+af^2+2\,dex},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/(e^2*x^2/f^2 + a + 2
```

```
[Out] integral((e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) + d)^n*f^
2/(e^2*x^2 + a*f^2 + 2*d*e*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2))**n/(a+2*d*e*x/f**2+
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}f+d\right)^n}{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/(e^2*x^2/f^2 + a + 2
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/(e^
2*x^2/f^2 + a + 2*d*e*x/f^2), x)
```

3.350 $\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^2}\,dx$

Optimal. Leaf size=122

$$-\frac{8f^4\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+3}{}_2F_1\left(3,\frac{n+3}{2};\frac{n+5}{2};\frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^2}{d^2-af^2}\right)}{e(n+3)\left(d^2-af^2\right)^3}$$

```
[Out] (-8*f^4*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(3
+ n)*Hypergeometric2F1[3, (3 + n)/2, (5 + n)/2, (d + e*x + f*Sqrt
[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^2/(d^2 - a*f^2)])/(e*(d^2 -
a*f^2)^3*(3 + n))
```

Rubi [A] time = 0.439299, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 56, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$

$$-\frac{8f^4\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+3}{}_2F_1\left(3,\frac{n+3}{2};\frac{n+5}{2};\frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^2}{d^2-af^2}\right)}{e(n+3)\left(d^2-af^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/(a + (2*d*e*x)/f^2 + (
```

```
[Out] (-8*f^4*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(3
+ n)*Hypergeometric2F1[3, (3 + n)/2, (5 + n)/2, (d + e*x + f*Sqrt
[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^2/(d^2 - a*f^2)])/(e*(d^2 -
a*f^2)^3*(3 + n))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((d+e*x+f*(a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2))**n/(a+2*d*e*x/
```

```
[Out] Timed out
```

Mathematica [A] time = 0.157778, size = 0, normalized size = 0.

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/(a + (2*d*e*x)/
```

```
[Out] Integrate[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n
/(a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2)^2, x]
```

Maple [F] time = 0.107, size = 0, normalized size = 0.

$$\int 1\left(d+ex+f\sqrt{a+2\,\frac{dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n\left(a+2\,\frac{dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*x^2/f^2)^
```

```
[Out] int((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+
e^2*x^2/f^2)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}f+d\right)^n}{\left(\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/(e^2*x^2/f^2 + a + 2
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/(e^
2*x^2/f^2 + a + 2*d*e*x/f^2)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(ex + f\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}} + d\right)^n f^4}{e^4x^4 + 4\,de^3x^3 + a^2f^4 + 4\,adef^2x + 2\,(ae^2f^2 + 2\,d^2e^2)x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/(e^2*x^2/f^2 + a + 2
```

```
[Out] integral((e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) + d)^n*f^
4/(e^4*x^4 + 4*d*e^3*x^3 + a^2*f^4 + 4*a*d*e*f^2*x + 2*(a*e^2*f^2
 + 2*d^2*e^2)*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2))**n/(a+2*d*e*x/f**2+
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex + \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}}f + d\right)^n}{\left(\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/(e^2*x^2/f^2 + a + 2
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/(e^
2*x^2/f^2 + a + 2*d*e*x/f^2)^2, x)
```

3.351 $\int \left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n dx$

Optimal. Leaf size=107

$$\frac{(d^2-af^2)\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-1}}{2e(1-n)}+\frac{\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}}{2e(n+1)}$$

```
[Out] ((d^2 - a*f^2)*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^
2])^(-1 + n))/(2*e*(1 - n)) + (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2
 + (e^2*x^2)/f^2])^(1 + n)/(2*e*(1 + n))
```

Rubi [A] time = 0.234673, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121

$$\frac{(d^2-af^2)\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-1}}{2e(1-n)}+\frac{\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}}{2e(n+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n,x]
```

```
[Out] ((d^2 - a*f^2)*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^
2])^(-1 + n))/(2*e*(1 - n)) + (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2
 + (e^2*x^2)/f^2])^(1 + n)/(2*e*(1 + n))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d+ex+f\sqrt{\frac{af^2+ex\,(2d+ex)}{f^2}}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((d+e*x+f*((a*f**2+e*x*(e*x+2*d))/f**2)**(1/2))**n,x)
```

```
[Out] Integral((d + e*x + f*sqrt((a*f**2 + e*x*(2*d + e*x))/f**2))**n,
x)
```

Mathematica [A] time = 0.0833649, size = 0, normalized size = 0.

$$\int \left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n,x]
```

```
[Out] Integrate[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n, x]
```

Maple [F] time = 0.014, size = 0, normalized size = 0.

$$\int \left(d+ex+f\sqrt{\frac{af^2+ex\,(ex+2\,d)}{f^2}}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n,x)
```

```
[Out] int((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex+f\sqrt{\frac{af^2+(ex+2\,d)ex}{f^2}}+d\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + f*sqrt((a*f^2 + (e*x + 2*d)*e*x)/f^2) + d)^n,x, algorithm="maxim
```

```
[Out] integrate((e*x + f*sqrt((a*f^2 + (e*x + 2*d)*e*x)/f^2) + d)^n, x)
```

Fricas [A] time = 0.302633, size = 108, normalized size = 1.01

$$\frac{\left(fn\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}-ex-d\right)\left(ex+f\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}+d\right)^n}{en^2-e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + f*sqrt((a*f^2 + (e*x + 2*d)*e*x)/f^2) + d)^n,x, algorithm="frica
```

```
[Out] (f*n*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) - e*x - d)*(e*x + f*sq
rt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) + d)^n/(e*n^2 - e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*((a*f**2+e*x*(e*x+2*d))/f**2)**(1/2))**n,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex + f\sqrt{\frac{af^2 + (ex + 2\,d)ex}{f^2}} + d\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + f*sqrt((a*f^2 + (e*x + 2*d)*e*x)/f^2) + d)^n,x, algorithm="giac"
```

```
[Out] integrate((e*x + f*sqrt((a*f^2 + (e*x + 2*d)*e*x)/f^2) + d)^n, x)
```

3.352 $\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\,dx$

Optimal. Leaf size=122

$$-\frac{2f^2\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^2}{d^2-af^2}\right)}{e(n+1)\left(d^2-af^2\right)}$$

```
[Out] (-2*f^2*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(1
+ n)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, (d + e*x + f*Sqrt
[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^2/(d^2 - a*f^2)])/(e*(d^2 -
a*f^2)*(1 + n))
```

Rubi [A] time = 0.766204, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 56, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054

$$-\frac{2f^2\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^2}{d^2-af^2}\right)}{e(n+1)\left(d^2-af^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n/(a + (2*d*e*x)/f^2 + (e^2*
```


```
[Out] (-2*f^2*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(1
+ n)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, (d + e*x + f*Sqrt
[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^2/(d^2 - a*f^2)])/(e*(d^2 -
a*f^2)*(1 + n))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((d+e*x+f*((a*f**2+e*x*(e*x+2*d))/f**2)**(1/2))**n/(a+2*d*e*x/f*
```

```
[Out] Timed out
```

Mathematica [A] time = 0.106441, size = 0, normalized size = 0.

$$\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n/(a + (2*d*e*x)/f^2 +
```

```
[Out] Integrate[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n/(a
+ (2*d*e*x)/f^2 + (e^2*x^2)/f^2), x]
```

Maple [F] time = 0.105, size = 0, normalized size = 0.

$$\int 1\left(d+ex+f\sqrt{\frac{af^2+ex\,(ex+2\,d)}{f^2}}\right)^n\left(a+2\,\frac{dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{-1}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*x^2/f^2),
```

```
[Out] int((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/(a+2*d*e*x/f^2+
e^2*x^2/f^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex+f\sqrt{\frac{af^2+(ex+2\,d)ex}{f^2}}+d\right)^n}{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + f*sqrt((a*f^2 + (e*x + 2*d)*e*x)/f^2) + d)^n/(e^2*x^2/f^2 + a + 2
```

```
[Out] integrate((e*x + f*sqrt((a*f^2 + (e*x + 2*d)*e*x)/f^2) + d)^n/(e^
2*x^2/f^2 + a + 2*d*e*x/f^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(ex+f\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}+d\right)^n f^2}{e^2x^2+af^2+2\,dex},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + f*sqrt((a*f^2 + (e*x + 2*d)*e*x)/f^2) + d)^n/(e^2*x^2/f^2 + a + 2
```

```
[Out] integral((e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) + d)^n*f^
2/(e^2*x^2 + a*f^2 + 2*d*e*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*((a*f**2+e*x*(e*x+2*d))/f**2)**(1/2))**n/(a+2*d*e*x/f**2+e*
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(ex+f\sqrt{\frac{af^2+(ex+2\,d)ex}{f^2}}+d\right)^n}{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + f*sqrt((a*f^2 + (e*x + 2*d)*e*x)/f^2) + d)^n/(e^2*x^2/f^2 + a + 2
```

```
[Out] integrate((e*x + f*sqrt((a*f^2 + (e*x + 2*d)*e*x)/f^2) + d)^n/(e^
2*x^2/f^2 + a + 2*d*e*x/f^2), x)
```

3.353 $\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{3/2}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$

Optimal. Leaf size=297

$$-\frac{(d^2-af^2)^4\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-4}}{16ef^3(4-n)}+\frac{(d^2-af^2)^3\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-2}}{4ef^3(2-n)}+\frac{3(d^2-af^2)^2\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n}}{8ef^3n}-\frac{(d^2-af^2)\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+2}}{4ef^3(n+2)}+\frac{\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+4}}{16ef^3(n+4)}$$

```
[Out] -((d^2 - a*f^2)^4*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)
/f^2])^(-4 + n))/(16*e*f^3*(4 - n)) + ((d^2 - a*f^2)^3*(d + e*x +
 f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(-2 + n))/(4*e*f^3*(2
 - n)) + (3*(d^2 - a*f^2)^2*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 +
 (e^2*x^2)/f^2])^n)/(8*e*f^3*n) - ((d^2 - a*f^2)*(d + e*x + f*Sqr
t[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(2 + n))/(4*e*f^3*(2 + n))
+ (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(4 + n)/(
16*e*f^3*(4 + n))
```

Rubi [A] time = 0.729178, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 58, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.052

$$-\frac{(d^2-af^2)^4\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-4}}{16ef^3(4-n)}+\frac{(d^2-af^2)^3\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-2}}{4ef^3(2-n)}+\frac{3(d^2-af^2)^2\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n}}{8ef^3n}-\frac{(d^2-af^2)\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+2}}{4ef^3(n+2)}+\frac{\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+4}}{16ef^3(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2)^(3/2)*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2
```

```
[Out] -((d^2 - a*f^2)^4*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)
/f^2])^(-4 + n))/(16*e*f^3*(4 - n)) + ((d^2 - a*f^2)^3*(d + e*x +
 f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(-2 + n))/(4*e*f^3*(2
 - n)) + (3*(d^2 - a*f^2)^2*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 +
 (e^2*x^2)/f^2])^n)/(8*e*f^3*n) - ((d^2 - a*f^2)*(d + e*x + f*Sqr
```

```
t[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(2 + n))/(4*e*f^3*(2 + n))
+ (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(4 + n)/(
16*e*f^3*(4 + n))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((a+2*d*e*x/f**2+e**2*x**2/f**2)**(3/2)*(d+e*x+f*(a+2*d*e*x/f**2
```

```
[Out] Timed out
```

Mathematica [A] time = 0.433868, size = 0, normalized size = 0.

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{3/2}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2)^(3/2)*(d + e*x + f*Sqrt[a + (2*d*e*
```

```
[Out] Integrate[(a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2)^(3/2)*(d + e*x + f*
Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n, x]
```

Maple [F] time = 0.108, size = 0, normalized size = 0.

$$\int \left(a+2\,\frac{dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{\frac{3}{2}}\left(d+ex+f\sqrt{a+2\,\frac{dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+2*d*e*x/f^2+e^2*x^2/f^2)^(3/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/
```

```
[Out] int((a+2*d*e*x/f^2+e^2*x^2/f^2)^(3/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2
*x^2/f^2)^(1/2))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}\right)^{\frac{3}{2}} \left(ex + \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}}\,f + d\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e^2*x^2/f^2 + a + 2*d*e*x/f^2)^(3/2)*(e*x + sqrt(e^2*x^2/f^2 + a + 2*d
```

```
[Out] integrate((e^2*x^2/f^2 + a + 2*d*e*x/f^2)^(3/2)*(e*x + sqrt(e^2*x
^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n, x)
```

Fricas [A] time = 0.30795, size = 509, normalized size = 1.71

$$\frac{\left(a^2f^4n^4 + 24\,a^2f^4 - 48\,ad^2f^2 + \left(e^4n^4 - 4\,e^4n^2\right)x^4 + 24\,d^4 + 4\left(de^3n^4 - 4\,de^3n^2\right)x^3 - 4\left(4\,a^2f^4 - 3\,ad^2f^2\right)n^2 + 2\left(\left(ae^2f^2 + \right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e^2*x^2/f^2 + a + 2*d*e*x/f^2)^(3/2)*(e*x + sqrt(e^2*x^2/f^2 + a + 2*d
```

```
[Out] (a^2*f^4*n^4 + 24*a^2*f^4 - 48*a*d^2*f^2 + (e^4*n^4 - 4*e^4*n^2)*
x^4 + 24*d^4 + 4*(d*e^3*n^4 - 4*d*e^3*n^2)*x^3 - 4*(4*a^2*f^4 - 3
*a*d^2*f^2)*n^2 + 2*((a*e^2*f^2 + 2*d^2*e^2)*n^4 - 2*(5*a*e^2*f^2
 + d^2*e^2)*n^2)*x^2 + 4*(a*d*e*f^2*n^4 - 2*(5*a*d*e*f^2 - 3*d^3*
e)*n^2)*x - 4*(a*d*f^3*n^3 + (e^3*f*n^3 - 4*e^3*f*n)*x^3 + 3*(d*e
^2*f*n^3 - 4*d*e^2*f*n)*x^2 - 2*(5*a*d*f^3 - 3*d^3*f)*n + ((a*e*f
^3 + 2*d^2*e*f)*n^3 - 2*(5*a*e*f^3 + d^2*e*f)*n)*x)*sqrt((e^2*x^2
 + a*f^2 + 2*d*e*x)/f^2))*(e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*
x)/f^2) + d)^n/(e*f^3*n^5 - 20*e*f^3*n^3 + 64*e*f^3*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f**2+e**2*x**2/f**2)**(3/2)*(d+e*x+f*(a+2*d*e*x/f**2+e**2
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}\right)^{\frac{3}{2}} \left(ex + \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}}f + d\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e^2*x^2/f^2 + a + 2*d*e*x/f^2)^(3/2)*(e*x + sqrt(e^2*x^2/f^2 + a + 2*d
```

```
[Out] integrate((e^2*x^2/f^2 + a + 2*d*e*x/f^2)^(3/2)*(e*x + sqrt(e^2*x
^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n, x)
```

3.354 $\int \sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$

Optimal. Leaf size=171

$$-\frac{(d^2-af^2)^2\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-2}}{4ef(2-n)}-\frac{(d^2-af^2)\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n}{2efn}+\frac{\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+2}}{4ef(n+2)}$$

```
[Out] -((d^2 - a*f^2)^2*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)
/f^2])^(-2 + n))/(4*e*f*(2 - n)) - ((d^2 - a*f^2)*(d + e*x + f*Sq
rt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n)/(2*e*f*n) + (d + e*x +
f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(2 + n)/(4*e*f*(2 + n)
)
```

Rubi [A] time = 0.561311, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 58, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.052

$$-\frac{(d^2-af^2)^2\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-2}}{4ef(2-n)}-\frac{(d^2-af^2)\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n}{2efn}+\frac{\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+2}}{4ef(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 +
```

```
[Out] -((d^2 - a*f^2)^2*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)
/f^2])^(-2 + n))/(4*e*f*(2 - n)) - ((d^2 - a*f^2)*(d + e*x + f*Sq
rt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n)/(2*e*f*n) + (d + e*x +
f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(2 + n)/(4*e*f*(2 + n)
)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2)*(d+e*x+f*(a+2*d*e*x/f**2
```

```
[Out] Timed out
```

Mathematica [A] time = 0.113423, size = 0, normalized size = 0.

$$\int \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} \left(d + ex + f\sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}}\right)^n \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2*d*e*x)
```

```
[Out] Integrate[Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]*(d + e*x + f*Sq
rt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n, x]
```

Maple [F] time = 0.104, size = 0, normalized size = 0.

$$\int \sqrt{a + 2\,\frac{dex}{f^2} + \frac{e^2x^2}{f^2}} \left(d + ex + f\sqrt{a + 2\,\frac{dex}{f^2} + \frac{e^2x^2}{f^2}}\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/
```

```
[Out] int((a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2
*x^2/f^2)^(1/2))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}} \left(ex + \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}}f + d\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*(e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e
```

```
[Out] integrate(sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*(e*x + sqrt(e^2*x^2
/f^2 + a + 2*d*e*x/f^2)*f + d)^n, x)
```

Fricas [A] time = 0.306425, size = 165, normalized size = 0.96

$$\frac{\left(e^2n^2x^2 + af^2n^2 + 2\,den^2x - 2\,af^2 + 2\,d^2 - 2\,(efnx + dfn)\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}\right)\left(ex + f\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}} + d\right)^n}{efn^3 - 4\,efn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*(e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e
```

```
[Out] (e^2*n^2*x^2 + a*f^2*n^2 + 2*d*e*n^2*x - 2*a*f^2 + 2*d^2 - 2*(e*f
*n*x + d*f*n)*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2))*(e*x + f*sqr
t((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) + d)^n/(e*f*n^3 - 4*e*f*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2)*(d+e*x+f*(a+2*d*e*x/f**2+e**2
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}}\left(ex + \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}}\,f + d\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*(e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e
```

```
[Out] integrate(sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*(e*x + sqrt(e^2*x^2
/f^2 + a + 2*d*e*x/f^2)*f + d)^n, x)
```

3.355 $\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}\,dx$

Optimal. Leaf size=41

$$\frac{f\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n}{en}$$

```
[Out] (f*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n)/(e*n)
```

Rubi [A] time = 0.38793, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 58, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.052

$$\frac{f\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n}{en}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/Sqrt[a + (2*d*e*x)/f^
```

```
[Out] (f*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n)/(e*n)
```

Rubi in Sympy [A] time = 96.1647, size = 37, normalized size = 0.9

$$\frac{f\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{en}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((d+e*x+f*(a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2))**n/(a+2*d*e*x/
```

```
[Out] f*(d + e*x + f*sqrt(a + 2*d*e*x/f**2 + e**2*x**2/f**2))**n/(e*n)
```

Mathematica [A] time = 0.198977, size = 0, normalized size = 0.

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/Sqrt[a + (2*d*e
```

```
[Out] Integrate[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n
/Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2], x]
```

Maple [F] time = 0.109, size = 0, normalized size = 0.

$$\int 1\left(d+ex+f\sqrt{a+2\,\frac{dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n\frac{1}{\sqrt{a+2\,\frac{dex}{f^2}+\frac{e^2x^2}{f^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*x^2/f^2)^
```

```
[Out] int((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+
e^2*x^2/f^2)^(1/2),x)
```

Maxima [A] time = 0.850073, size = 47, normalized size = 1.15

$$\frac{\left(ex+d+\sqrt{e^2x^2+af^2+2\,dex}\right)^n f}{en}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/sqrt(e^2*x^2/f^2 + a
```

```
[Out] (e*x + d + sqrt(e^2*x^2 + a*f^2 + 2*d*e*x))^n*f/(e*n)
```

Fricas [A] time = 0.324129, size = 55, normalized size = 1.34

$$\frac{\left(ex + f\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}} + d\right)^n f}{en}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/sqrt(e^2*x^2/f^2 + a
```

```
[Out] (e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) + d)^n*f/(e*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2))**n/(a+2*d*e*x/f**2+
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex + \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}}f + d\right)^n}{\sqrt{\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/sqrt(e^2*x^2/f^2 + a
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/sqr
t(e^2*x^2/f^2 + a + 2*d*e*x/f^2), x)
```

3.356 $\displaystyle \int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=122

$$\frac{4f^3\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+2}{}_2F_1\left(2,\frac{n+2}{2};\frac{n+4}{2};\frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^2}{d^2-af^2}\right)}{e(n+2)\left(d^2-af^2\right)^2}$$

```
[Out] (4*f^3*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(2 +
 n)*Hypergeometric2F1[2, (2 + n)/2, (4 + n)/2, (d + e*x + f*Sqrt[
a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^2/(d^2 - a*f^2)])/(e*(d^2 - a
*f^2)^2*(2 + n))
```

Rubi [A] time = 0.497419, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 58, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.052

$$\frac{4f^3\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+2}{}_2F_1\left(2,\frac{n+2}{2};\frac{n+4}{2};\frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^2}{d^2-af^2}\right)}{e(n+2)\left(d^2-af^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/(a + (2*d*e*x)/f^2 + (
```

```
[Out] (4*f^3*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(2 +
 n)*Hypergeometric2F1[2, (2 + n)/2, (4 + n)/2, (d + e*x + f*Sqrt[
a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^2/(d^2 - a*f^2)])/(e*(d^2 - a
*f^2)^2*(2 + n))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$4ef^3\int^{d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}\frac{xx^n}{e^2\left(af^2-d^2+x^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((d+e*x+f*(a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2))**n/(a+2*d*e*x/
```

```
[Out] 4*e*f**3*Integral(x*x**n/(e**2*(a*f**2 - d**2 + x**2)**2), (x, d
+ e*x + f*sqrt(a + 2*d*e*x/f**2 + e**2*x**2/f**2)))
```

Mathematica [A] time = 0.297942, size = 0, normalized size = 0.

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/(a + (2*d*e*x)/
```

```
[Out] Integrate[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n
/(a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2)^(3/2), x]
```

Maple [F] time = 0.111, size = 0, normalized size = 0.

$$\int 1\left(d+ex+f\sqrt{a+2\,\frac{dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n\left(a+2\,\frac{dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*x^2/f^2)^
```

```
[Out] int((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+
e^2*x^2/f^2)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}f+d\right)^n}{\left(\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/(e^2*x^2/f^2 + a + 2
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/(e^
2*x^2/f^2 + a + 2*d*e*x/f^2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(ex+f\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}+d\right)^n f^2}{(e^2x^2+af^2+2\,dex)\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/(e^2*x^2/f^2 + a + 2
```

```
[Out] integral((e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) + d)^n*f^
2/((e^2*x^2 + a*f^2 + 2*d*e*x)*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f
^2)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2))**n/(a+2*d*e*x/f**2+
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}f+d\right)^n}{\left(\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/(e^2*x^2/f^2 + a + 2
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/(e^
2*x^2/f^2 + a + 2*d*e*x/f^2)^(3/2), x)
```

3.357 $\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}}\,dx$

Optimal. Leaf size=41

$$\frac{f\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n}{en}$$

```
[Out] (f*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n)/(e*n)
```

Rubi [A] time = 0.668488, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 58, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069

$$\frac{f\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n}{en}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n/Sqrt[(a*f^2 + e*x*(2*d + e

[Out] (f*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n)/(e*n)
```

Rubi in Sympy [A] time = 109.276, size = 37, normalized size = 0.9

$$\frac{f\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{en}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((d+e*x+f*((a*f**2+e*x*(e*x+2*d))/f**2)**(1/2))**n/((a*f**2+e*x*

[Out] f*(d + e*x + f*sqrt(a + 2*d*e*x/f**2 + e**2*x**2/f**2))**n/(e*n)
```

Mathematica [A] time = 0.123001, size = 0, normalized size = 0.

$$\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n/Sqrt[(a*f^2 + e*x*(
```

```
[Out] Integrate[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n/Sqr
t[(a*f^2 + e*x*(2*d + e*x))/f^2], x]
```

Maple [F] time = 0.111, size = 0, normalized size = 0.

$$\int 1\left(d+ex+f\sqrt{\frac{af^2+ex\,(ex+2\,d)}{f^2}}\right)^n\frac{1}{\sqrt{\frac{af^2+ex(ex+2\,d)}{f^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/((a*f^2+e*x*(e*x+2*d))/f^2)^
```

```
[Out] int((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/((a*f^2+e*x*(e*
x+2*d))/f^2)^(1/2),x)
```

Maxima [A] time = 0.851402, size = 47, normalized size = 1.15

$$\frac{\left(ex+d+\sqrt{e^2x^2+af^2+2\,dex}\right)^n f}{en}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + f*sqrt((a*f^2 + (e*x + 2*d)*e*x)/f^2) + d)^n/sqrt((a*f^2 + (e*x +
```

```
[Out] (e*x + d + sqrt(e^2*x^2 + a*f^2 + 2*d*e*x))^n*f/(e*n)
```

Fricas [A] time = 0.312753, size = 55, normalized size = 1.34

$$\frac{\left(ex + f\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}} + d\right)^n f}{en}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + f*sqrt((a*f^2 + (e*x + 2*d)*e*x)/f^2) + d)^n/sqrt((a*f^2 + (e*x +
```

```
[Out] (e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) + d)^n*f/(e*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*((a*f**2+e*x*(e*x+2*d))/f**2)**(1/2))**n/((a*f**2+e*x*(e*x+
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex + f\sqrt{\frac{af^2+(ex+2\,d)ex}{f^2}} + d\right)^n}{\sqrt{\frac{af^2+(ex+2\,d)ex}{f^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + f*sqrt((a*f^2 + (e*x + 2*d)*e*x)/f^2) + d)^n/sqrt((a*f^2 + (e*x +
```

```
[Out] integrate((e*x + f*sqrt((a*f^2 + (e*x + 2*d)*e*x)/f^2) + d)^n/sqr
t((a*f^2 + (e*x + 2*d)*e*x)/f^2), x)
```

3.358 $\int \sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}} \left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$

Optimal. Leaf size=327

$$-\frac{(d^2-af^2)^2\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-2}}{4ef(2-n)\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}-\frac{(d^2-af^2)\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n}}{2efn\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}+\frac{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+2}}{4ef(n+2)\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}$$

```
[Out] -((d^2 - a*f^2)^2*Sqrt[a*g + (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2]*(
d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(-2 + n))/(4
*e*f*(2 - n)*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]) - ((d^2 - a
*f^2)*Sqrt[a*g + (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2]*(d + e*x + f*
Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n)/(2*e*f*n*Sqrt[a + (2*
d*e*x)/f^2 + (e^2*x^2)/f^2]) + (Sqrt[a*g + (2*d*e*g*x)/f^2 + (e^2
*g*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]
)^(2 + n))/(4*e*f*(2 + n)*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]
)
```

Rubi [A] time = 1.05213, antiderivative size = 327, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 62, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$

$$-\frac{(d^2-af^2)^2\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-2}}{4ef(2-n)\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}-\frac{(d^2-af^2)\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n}}{2efn\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}+\frac{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+2}}{4ef(n+2)\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*g + (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2*d*e*x)
```

```
[Out] -((d^2 - a*f^2)^2*Sqrt[a*g + (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2]*(
d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(-2 + n))/(4
*e*f*(2 - n)*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]) - ((d^2 - a
*f^2)*Sqrt[a*g + (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2]*(d + e*x + f*
Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n)/(2*e*f*n*Sqrt[a + (2*
d*e*x)/f^2 + (e^2*x^2)/f^2]) + (Sqrt[a*g + (2*d*e*g*x)/f^2 + (e^2
*g*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]
)^(2 + n))/(4*e*f*(2 + n)*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]
)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((a*g+2*d*e*g*x/f**2+e**2*g*x**2/f**2)**(1/2)*(d+e*x+f*(a+2*d*e*
```

```
[Out] Timed out
```

Mathematica [A] time = 0.123497, size = 0, normalized size = 0.

$$\int \sqrt{ag + \frac{2degx}{f^2} + \frac{e^2gx^2}{f^2}} \left(d + ex + f\sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}}\right)^n \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a*g + (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2*
```

```
[Out] Integrate[Sqrt[a*g + (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2]*(d + e*x
+ f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n, x]
```

Maple [F] time = 0.114, size = 0, normalized size = 0.

$$\int \sqrt{ag + 2\,\frac{degx}{f^2} + \frac{e^2gx^2}{f^2}} \left(d + ex + f\sqrt{a + 2\,\frac{dex}{f^2} + \frac{e^2x^2}{f^2}}\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g+2*d*e*g*x/f^2+e^2*g*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^
```

```
[Out] int((a*g+2*d*e*g*x/f^2+e^2*g*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e*x/f
^2+e^2*x^2/f^2)^(1/2))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{e^2gx^2}{f^2} + ag + \frac{2\,degx}{f^2}}\left(ex + \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}}f + d\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(e^2*g*x^2/f^2 + a*g + 2*d*e*g*x/f^2)*(e*x + sqrt(e^2*x^2/f^2 + a +
```

```
[Out] integrate(sqrt(e^2*g*x^2/f^2 + a*g + 2*d*e*g*x/f^2)*(e*x + sqrt(e
^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n, x)
```

Fricas [A] time = 0.323563, size = 312, normalized size = 0.95

$$-\frac{\left(2\,e^3nx^3 + 6\,de^2nx^2 + 2\,adf^2n + 2\left(aef^2 + 2\,d^2e\right)nx - \left(e^2fn^2x^2 + af^3n^2 + 2\,defn^2x - 2\,af^3 + 2\,d^2f\right)\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}\right)\left(e\right.}{aef^2n^3 - 4\,aef^2n + (e^3n^3 - 4\,e^3n)x^2 + 2\,(de^2n^3 - 4\,de^2n)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(e^2*g*x^2/f^2 + a*g + 2*d*e*g*x/f^2)*(e*x + sqrt(e^2*x^2/f^2 + a +
```

```
[Out] -(2*e^3*n*x^3 + 6*d*e^2*n*x^2 + 2*a*d*f^2*n + 2*(a*e*f^2 + 2*d^2*
e)*n*x - (e^2*f*n^2*x^2 + a*f^3*n^2 + 2*d*e*f*n^2*x - 2*a*f^3 + 2
*d^2*f)*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2))*(e*x + f*sqrt((e^2
*x^2 + a*f^2 + 2*d*e*x)/f^2) + d)^n*sqrt((e^2*g*x^2 + a*f^2*g + 2
*d*e*g*x)/f^2)/(a*e*f^2*n^3 - 4*a*e*f^2*n + (e^3*n^3 - 4*e^3*n)*x
^2 + 2*(d*e^2*n^3 - 4*d*e^2*n)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{g\left(a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}\right)}\left(d + ex + f\sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*g+2*d*e*g*x/f**2+e**2*g*x**2/f**2)**(1/2)*(d+e*x+f*(a+2*d*e*x/f**
```

```
[Out] Integral(sqrt(g*(a + 2*d*e*x/f**2 + e**2*x**2/f**2))*(d + e*x + f
*sqrt(a + 2*d*e*x/f**2 + e**2*x**2/f**2))**n, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{e^2 g x^2}{f^2} + ag + \frac{2\,degx}{f^2}}\left(ex + \sqrt{\frac{e^2 x^2}{f^2} + a + \frac{2\,dex}{f^2}}f + d\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(e^2*g*x^2/f^2 + a*g + 2*d*e*g*x/f^2)*(e*x + sqrt(e^2*x^2/f^2 + a +
```

```
[Out] integrate(sqrt(e^2*g*x^2/f^2 + a*g + 2*d*e*g*x/f^2)*(e*x + sqrt(e
^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n, x)
```

3.359 $\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}\,dx$

Optimal. Leaf size=93

$$\frac{f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n}{en\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

```
[Out] (f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]*(d + e*x + f*Sqrt[a +
(2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n)/(e*n*Sqrt[a*g + (2*d*e*g*x)/f^
2 + (e^2*g*x^2)/f^2])
```

Rubi [A] time = 0.839321, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 62, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065

$$\frac{f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n}{en\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/Sqrt[a*g + (2*d*e*g*x
```

```
[Out] (f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]*(d + e*x + f*Sqrt[a +
(2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n)/(e*n*Sqrt[a*g + (2*d*e*g*x)/f^
2 + (e^2*g*x^2)/f^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((d+e*x+f*(a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2))**n/(a*g+2*d*e*
```

```
[Out] Timed out
```

Mathematica [A] time = 0.133682, size = 0, normalized size = 0.

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/Sqrt[a*g + (2*d
```

```
[Out] Integrate[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n
/Sqrt[a*g + (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2], x]
```

Maple [F] time = 0.11, size = 0, normalized size = 0.

$$\int 1\left(d+ex+f\sqrt{a+2\,\frac{dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n \frac{1}{\sqrt{ag+2\,\frac{degx}{f^2}+\frac{e^2gx^2}{f^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a*g+2*d*e*g*x/f^2+e^2*g*x^2
```

```
[Out] int((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a*g+2*d*e*g*x/
f^2+e^2*g*x^2/f^2)^(1/2),x)
```

Maxima [A] time = 0.811837, size = 51, normalized size = 0.55

$$\frac{\left(ex+d+\sqrt{e^2x^2+af^2+2\,dex}\right)^n f}{e\sqrt{g}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/sqrt(e^2*g*x^2/f^2
```

```
[Out] (e*x + d + sqrt(e^2*x^2 + a*f^2 + 2*d*e*x))^n*f/(e*sqrt(g)*n)
```

Fricas [A] time = 0.311372, size = 158, normalized size = 1.7

$$\frac{\left(ex+f\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}+d\right)^n f^3\sqrt{\frac{e^2gx^2+af^2g+2\,degx}{f^2}}\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}}{e^3gnx^2+aef^2gn+2\,de^2gnx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/sqrt(e^2*g*x^2/f^2
```

```
[Out] (e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) + d)^n*f^3*sqrt((e
^2*g*x^2 + a*f^2*g + 2*d*e*g*x)/f^2)*sqrt((e^2*x^2 + a*f^2 + 2*d*
e*x)/f^2)/(e^3*g*n*x^2 + a*e*f^2*g*n + 2*d*e^2*g*n*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{g\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2))**n/(a*g+2*d*e*g*x/f
```

```
[Out] Integral((d + e*x + f*sqrt(a + 2*d*e*x/f**2 + e**2*x**2/f**2))**n
/sqrt(g*(a + 2*d*e*x/f**2 + e**2*x**2/f**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}f+d\right)^n}{\sqrt{\frac{e^2gx^2}{f^2}+ag+\frac{2\,degx}{f^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/sqrt(e^2*g*x^2/f^2
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/sqr
t(e^2*g*x^2/f^2 + a*g + 2*d*e*g*x/f^2), x)
```

3.360 $\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=177

$$\frac{4f^3\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+2}{}_2F_1\left(2,\frac{n+2}{2};\frac{n+4}{2};\frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^2}{d^2-af^2}\right)}{eg(n+2)\left(d^2-af^2\right)^2\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

```
[Out] (4*f^3*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]*(d + e*x + f*Sqrt[
a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(2 + n)*Hypergeometric2F1[2,
(2 + n)/2, (4 + n)/2, (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*
x^2)/f^2])^2/(d^2 - a*f^2)])/(e*(d^2 - a*f^2)^2*g*(2 + n)*Sqrt[a*
g + (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2])
```

Rubi [A] time = 0.945467, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 62, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065

$$\frac{4f^3\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+2}{}_2F_1\left(2,\frac{n+2}{2};\frac{n+4}{2};\frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^2}{d^2-af^2}\right)}{eg(n+2)\left(d^2-af^2\right)^2\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/(a*g + (2*d*e*g*x)/f^
```

```
[Out] (4*f^3*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]*(d + e*x + f*Sqrt[
a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(2 + n)*Hypergeometric2F1[2,
(2 + n)/2, (4 + n)/2, (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*
x^2)/f^2])^2/(d^2 - a*f^2)])/(e*(d^2 - a*f^2)^2*g*(2 + n)*Sqrt[a*
g + (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((d+e*x+f*(a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2))**n/(a*g+2*d*e*
```

```
[Out] Timed out
```

Mathematica [A] time = 0.225036, size = 0, normalized size = 0.

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/(a*g + (2*d*e*g
```

```
[Out] Integrate[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n
/(a*g + (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2)^(3/2), x]
```

Maple [F] time = 0.115, size = 0, normalized size = 0.

$$\int 1\left(d+ex+f\sqrt{a+2\,\frac{dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n\left(ag+2\,\frac{degx}{f^2}+\frac{e^2gx^2}{f^2}\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a*g+2*d*e*g*x/f^2+e^2*g*x^2
```

```
[Out] int((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a*g+2*d*e*g*x/
f^2+e^2*g*x^2/f^2)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}f+d\right)^n}{\left(\frac{e^2gx^2}{f^2}+ag+\frac{2\,degx}{f^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/(e^2*g*x^2/f^2 + a*
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/(e^
2*g*x^2/f^2 + a*g + 2*d*e*g*x/f^2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(ex+f\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}+d\right)^n f^2}{(e^2gx^2+af^2g+2\,degx)\sqrt{\frac{e^2gx^2+af^2g+2\,degx}{f^2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/(e^2*g*x^2/f^2 + a*g
```

```
[Out] integral((e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) + d)^n*f^
2/((e^2*g*x^2 + a*f^2*g + 2*d*e*g*x)*sqrt((e^2*g*x^2 + a*f^2*g +
2*d*e*g*x)/f^2)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2))**n/(a*g+2*d*e*g*x/f
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}f+d\right)^n}{\left(\frac{e^2gx^2}{f^2}+ag+\frac{2\,degx}{f^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/(e^2*g*x^2/f^2 + a*g
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/(e^
2*g*x^2/f^2 + a*g + 2*d*e*g*x/f^2)^(3/2), x)
```

3.361 $\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{\sqrt{\frac{af^2g+egx(2d+ex)}{f^2}}}\,dx$

Optimal. Leaf size=93

$$\frac{f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n}{en\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

```
[Out] (f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]*(d + e*x + f*Sqrt[a +
(2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n)/(e*n*Sqrt[a*g + (2*d*e*g*x)/f^
2 + (e^2*g*x^2)/f^2])
```

Rubi [A] time = 1.12844, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 60, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$\frac{f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n}{en\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n/Sqrt[(a*f^2*g + e*g*x*(2*
```

```
[Out] (f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]*(d + e*x + f*Sqrt[a +
(2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n)/(e*n*Sqrt[a*g + (2*d*e*g*x)/f^
2 + (e^2*g*x^2)/f^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((d+e*x+f*((a*f**2+e*x*(e*x+2*d))/f**2)**(1/2))**n/((a*f**2*g+e*
```

```
[Out] Timed out
```

Mathematica [A] time = 0.123786, size = 0, normalized size = 0.

$$\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{\sqrt{\frac{af^2g+egx(2d+ex)}{f^2}}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n/Sqrt[(a*f^2*g + e*g
```

```
[Out] Integrate[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n/Sqr
t[(a*f^2*g + e*g*x*(2*d + e*x))/f^2], x]
```

Maple [F] time = 0.118, size = 0, normalized size = 0.

$$\int 1\left(d+ex+f\sqrt{\frac{af^2+ex(ex+2d)}{f^2}}\right)^n\frac{1}{\sqrt{\frac{af^2g+egx(ex+2d)}{f^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/((a*f^2*g+e*g*x*(e*x+2*d))/f
```

```
[Out] int((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/((a*f^2*g+e*g*x
*(e*x+2*d))/f^2)^(1/2),x)
```

Maxima [A] time = 0.818051, size = 51, normalized size = 0.55

$$\frac{\left(ex+d+\sqrt{e^2x^2+af^2+2\,dex}\right)^n f}{e\sqrt{g}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + f*sqrt((a*f^2 + (e*x + 2*d)*e*x)/f^2) + d)^n/sqrt((a*f^2*g + (e*
```

```
[Out] (e*x + d + sqrt(e^2*x^2 + a*f^2 + 2*d*e*x))^n*f/(e*sqrt(g)*n)
```

Fricas [A] time = 0.312657, size = 158, normalized size = 1.7

$$\frac{\left(ex+f\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}+d\right)^n f^3\sqrt{\frac{e^2gx^2+af^2g+2\,degx}{f^2}}\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}}{e^3gnx^2+aef^2gn+2\,de^2gnx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + f*sqrt((a*f^2 + (e*x + 2*d)*e*x)/f^2) + d)^n/sqrt((a*f^2*g + (e*
```

```
[Out] (e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) + d)^n*f^3*sqrt((e
^2*g*x^2 + a*f^2*g + 2*d*e*g*x)/f^2)*sqrt((e^2*x^2 + a*f^2 + 2*d*
e*x)/f^2)/(e^3*g*n*x^2 + a*e*f^2*g*n + 2*d*e^2*g*n*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*((a*f**2+e*x*(e*x+2*d))/f**2)**(1/2))**n/((a*f**2*g+e*g*x*(
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex+f\sqrt{\frac{af^2+(ex+2\,d)ex}{f^2}}+d\right)^n}{\sqrt{\frac{af^2g+(ex+2\,d)egx}{f^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + f*sqrt((a*f^2 + (e*x + 2*d)*e*x)/f^2) + d)^n/sqrt((a*f^2*g + (e*
```

```
[Out] integrate((e*x + f*sqrt((a*f^2 + (e*x + 2*d)*e*x)/f^2) + d)^n/sqr
t((a*f^2*g + (e*x + 2*d)*e*g*x)/f^2), x)
```

3.362 $\int \frac{1}{(a+bx)\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=191

$$\frac{\sqrt{-c}\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\,\Pi\left(-\frac{b^2c}{a^2d};\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{-c}}\right)|\frac{cf}{de}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{e+fx^2}}-\frac{b\tanh^{-1}\left(\frac{\sqrt{c+dx^2}\sqrt{a^2f+b^2e}}{\sqrt{e+fx^2}\sqrt{a^2d+b^2c}}\right)}{\sqrt{a^2d+b^2c}\sqrt{a^2f+b^2e}}$$

```
[Out] -((b*ArcTanh[(Sqrt[b^2*e + a^2*f]*Sqrt[c + d*x^2])/(Sqrt[b^2*c +
a^2*d]*Sqrt[e + f*x^2])]])/(Sqrt[b^2*c + a^2*d]*Sqrt[b^2*e + a^2*f
])) + (Sqrt[-c]*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticP
i[-((b^2*c)/(a^2*d)), ArcSin[(Sqrt[d]*x)/Sqrt[-c]], (c*f)/(d*e)])
/(a*Sqrt[d]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Rubi [A] time = 1.22223, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{\sqrt{-c}\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\,\Pi\left(-\frac{b^2c}{a^2d};\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{-c}}\right)|\frac{cf}{de}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{e+fx^2}}-\frac{b\tanh^{-1}\left(\frac{\sqrt{c+dx^2}\sqrt{a^2f+b^2e}}{\sqrt{e+fx^2}\sqrt{a^2d+b^2c}}\right)}{\sqrt{a^2d+b^2c}\sqrt{a^2f+b^2e}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

```
[Out] -((b*ArcTanh[(Sqrt[b^2*e + a^2*f]*Sqrt[c + d*x^2])/(Sqrt[b^2*c +
a^2*d]*Sqrt[e + f*x^2])]])/(Sqrt[b^2*c + a^2*d]*Sqrt[b^2*e + a^2*f
])) + (Sqrt[-c]*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticP
i[-((b^2*c)/(a^2*d)), ArcSin[(Sqrt[d]*x)/Sqrt[-c]], (c*f)/(d*e)])
/(a*Sqrt[d]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Rubi in Sympy [A] time = 115.082, size = 264, normalized size = 1.38

$$\frac{a\sqrt{c}\sqrt{d}\sqrt{e+fx^2}F\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\frac{cf}{de}+1\right)}{e\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\sqrt{c+dx^2}\,(a^2d+b^2c)}-\frac{b\operatorname{atanh}\left(\frac{\sqrt{c+dx^2}\sqrt{a^2f+b^2e}}{\sqrt{e+fx^2}\sqrt{a^2d+b^2c}}\right)}{\sqrt{a^2d+b^2c}\sqrt{a^2f+b^2e}}+\frac{b^2c^{\frac{3}{2}}\sqrt{e+fx^2}\left(1+\frac{b^2c}{a^2d};\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\frac{cf}{de}+1\right)}{a\sqrt{d}e\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\sqrt{c+dx^2}\,(a^2d+b^2c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x+a)/(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] a*sqrt(c)*sqrt(d)*sqrt(e + f*x**2)*elliptic_f(atan(sqrt(d)*x/sqrt
(c)), -c*f/(d*e) + 1)/(e*sqrt(c*(e + f*x**2)/(e*(c + d*x**2)))*sq
rt(c + d*x**2)*(a**2*d + b**2*c)) - b*atanh(sqrt(c + d*x**2)*sqrt
(a**2*f + b**2*e)/(sqrt(e + f*x**2)*sqrt(a**2*d + b**2*c)))/(sqrt
(a**2*d + b**2*c)*sqrt(a**2*f + b**2*e)) + b**2*c**(3/2)*sqrt(e +
 f*x**2)*elliptic_pi(1 + b**2*c/(a**2*d), atan(sqrt(d)*x/sqrt(c))
, -c*f/(d*e) + 1)/(a*sqrt(d)*e*sqrt(c*(e + f*x**2)/(e*(c + d*x**2
)))*sqrt(c + d*x**2)*(a**2*d + b**2*c))
```

Mathematica [A] time = 0.800918, size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx)\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[1/((a + b*x)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

```
[Out] Integrate[1/((a + b*x)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x]
```

Maple [B] time = 0.094, size = 352, normalized size = 1.8

$$-\frac{1}{2\,ab\,(dfx^4+cx^2f+x^2de+ce)}\left(-2\sqrt{\frac{dx^2+c}{c}}\sqrt{\frac{fx^2+e}{e}}\sqrt{\frac{a^4df+a^2b^2cf+a^2b^2de+b^4ce}{b^4}}\mathit{EllipticPi}\left(x\sqrt{-\frac{d}{c}},-\frac{b^2c}{a^2d},1\sqrt{-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

```
[Out] -1/2*(-2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*((a^4*d*f+a^2*b^
2*c*f+a^2*b^2*d*e+b^4*c*e)/b^4)^(1/2)*EllipticPi(x*(-1/c*d)^(1/2)
,-b^2*c/a^2/d,(-f/e)^(1/2)/(-1/c*d)^(1/2))*b+(d*f*x^4+c*f*x^2+d*e
*x^2+c*e)^(1/2)*arctanh(1/2*(2*a^2*d*f*x^2+b^2*c*f*x^2+b^2*d*e*x^
2+a^2*c*f+a^2*d*e+2*b^2*c*e)/b^2/((a^4*d*f+a^2*b^2*c*f+a^2*b^2*d*
e+b^4*c*e)/b^4)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2))*(-1/c*
d)^(1/2)*a)*(f*x^2+e)^(1/2)*(d*x^2+c)^(1/2)/b/((a^4*d*f+a^2*b^2*c
*f+a^2*b^2*d*e+b^4*c*e)/b^4)^(1/2)/a/(-1/c*d)^(1/2)/(d*f*x^4+c*f*
```

```
x^2+d*e*x^2+c*e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)*(b*x + a)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)*(b*x + a)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)*(b*x + a)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx)\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x)*sqrt(c + d*x**2)*sqrt(e + f*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)*(b*x + a)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)*(b*x + a)), x)
```

3.363 $\int \frac{e-2fx^2}{e^2+4dfx^2+4efx^2+4f^2x^4}\,dx$

Optimal. Leaf size=81

$$\frac{\log\left(2\sqrt{-d}\sqrt{f}x+e+2fx^2\right)}{4\sqrt{-d}\sqrt{f}}-\frac{\log\left(-2\sqrt{-d}\sqrt{f}x+e+2fx^2\right)}{4\sqrt{-d}\sqrt{f}}$$

```
[Out] -Log[e - 2*Sqrt[-d]*Sqrt[f]*x + 2*f*x^2]/(4*Sqrt[-d]*Sqrt[f]) + L
og[e + 2*Sqrt[-d]*Sqrt[f]*x + 2*f*x^2]/(4*Sqrt[-d]*Sqrt[f])
```

Rubi [A] time = 0.113775, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.081

$$\frac{\log\left(2\sqrt{-d}\sqrt{f}x+e+2fx^2\right)}{4\sqrt{-d}\sqrt{f}}-\frac{\log\left(-2\sqrt{-d}\sqrt{f}x+e+2fx^2\right)}{4\sqrt{-d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In]  Int[(e - 2*f*x^2)/(e^2 + 4*d*f*x^2 + 4*e*f*x^2 + 4*f^2*x^4),x]
```

```
[Out] -Log[e - 2*Sqrt[-d]*Sqrt[f]*x + 2*f*x^2]/(4*Sqrt[-d]*Sqrt[f]) + L
og[e + 2*Sqrt[-d]*Sqrt[f]*x + 2*f*x^2]/(4*Sqrt[-d]*Sqrt[f])
```

Rubi in Sympy [A] time = 70.6325, size = 73, normalized size = 0.9

$$-\frac{\log\left(\frac{e}{2f}+x^2-\frac{x\sqrt{-d}}{\sqrt{f}}\right)}{4\sqrt{f}\sqrt{-d}}+\frac{\log\left(\frac{e}{2f}+x^2+\frac{x\sqrt{-d}}{\sqrt{f}}\right)}{4\sqrt{f}\sqrt{-d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-2*f*x**2+e)/(4*f**2*x**4+4*d*f*x**2+4*e*f*x**2+e**2),x)
```

```
[Out] -log(e/(2*f) + x**2 - x*sqrt(-d)/sqrt(f))/(4*sqrt(f)*sqrt(-d)) +
log(e/(2*f) + x**2 + x*sqrt(-d)/sqrt(f))/(4*sqrt(f)*sqrt(-d))
```

Mathematica [B] time = 0.187654, size = 191, normalized size = 2.36

$$\frac{-\frac{\left(\sqrt{d}\sqrt{d+2e}-d-2e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{-\sqrt{d}\sqrt{d+2e}+d+e}}\right)}{\sqrt{-\sqrt{d}\sqrt{d+2e}+d+e}}-\frac{\left(\sqrt{d}\sqrt{d+2e}+d+2e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{\sqrt{d}\sqrt{d+2e}+d+e}}\right)}{\sqrt{\sqrt{d}\sqrt{d+2e}+d+e}}}{2\sqrt{2}\sqrt{d}\sqrt{f}\sqrt{d+2e}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(e - 2*f*x^2)/(e^2 + 4*d*f*x^2 + 4*e*f*x^2 + 4*f^2*x^4),x]
```

```
[Out] (-(((-d - 2*e + Sqrt[d]*Sqrt[d + 2*e])*ArcTan[(Sqrt[2]*Sqrt[f]*x)
/Sqrt[d + e - Sqrt[d]*Sqrt[d + 2*e]]])/Sqrt[d + e - Sqrt[d]*Sqrt[
d + 2*e]]) - ((d + 2*e + Sqrt[d]*Sqrt[d + 2*e])*ArcTan[(Sqrt[2]*S
qrt[f]*x)/Sqrt[d + e + Sqrt[d]*Sqrt[d + 2*e]]])/Sqrt[d + e + Sqrt
[d]*Sqrt[d + 2*e]])/(2*Sqrt[2]*Sqrt[d]*Sqrt[d + 2*e]*Sqrt[f])
```

Maple [B] time = 0.07, size = 394, normalized size = 4.9

$$-\frac{f\sqrt{2}d}{4}Artanh\left(fx\sqrt{2}\frac{1}{\sqrt{-df-ef+\sqrt{df^2(d+2e)}}}\right)\frac{1}{\sqrt{df^2(d+2e)}}\frac{1}{\sqrt{-df-ef+\sqrt{df^2(d+2e)}}}$$
$$-\frac{f\sqrt{2}e}{2}Artanh\left(fx\sqrt{2}\frac{1}{\sqrt{-df-ef+\sqrt{df^2(d+2e)}}}\right)\frac{1}{\sqrt{df^2(d+2e)}}\frac{1}{\sqrt{-df-ef+\sqrt{df^2(d+2e)}}}$$
$$+\frac{\sqrt{2}}{4}Artanh\left(fx\sqrt{2}\frac{1}{\sqrt{-df-ef+\sqrt{df^2(d+2e)}}}\right)\frac{1}{\sqrt{-df-ef+\sqrt{df^2(d+2e)}}}$$
$$-\frac{f\sqrt{2}d}{4}\arctan\left(fx\sqrt{2}\frac{1}{\sqrt{df+ef+\sqrt{df^2(d+2e)}}}\right)\frac{1}{\sqrt{df^2(d+2e)}}\frac{1}{\sqrt{df+ef+\sqrt{df^2(d+2e)}}}$$
$$-\frac{f\sqrt{2}e}{2}\arctan\left(fx\sqrt{2}\frac{1}{\sqrt{df+ef+\sqrt{df^2(d+2e)}}}\right)\frac{1}{\sqrt{df^2(d+2e)}}\frac{1}{\sqrt{df+ef+\sqrt{df^2(d+2e)}}}$$
$$-\frac{\sqrt{2}}{4}\arctan\left(fx\sqrt{2}\frac{1}{\sqrt{df+ef+\sqrt{df^2(d+2e)}}}\right)\frac{1}{\sqrt{df+ef+\sqrt{df^2(d+2e)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-2*f*x^2+e)/(4*f^2*x^4+4*d*f*x^2+4*e*f*x^2+e^2),x)
```

```
[Out] -1/4*f/(d*f^2*(d+2*e))^(1/2)*2^(1/2)/(-d*f-e*f+(d*f^2*(d+2*e))^(1
/2))^(1/2)*arctanh(f*x*2^(1/2)/(-d*f-e*f+(d*f^2*(d+2*e))^(1/2))^(
1/2))*d-1/2*f/(d*f^2*(d+2*e))^(1/2)*2^(1/2)/(-d*f-e*f+(d*f^2*(d+2
*e))^(1/2))^(1/2)*arctanh(f*x*2^(1/2)/(-d*f-e*f+(d*f^2*(d+2*e))^(
1/2))^(1/2))*e+1/4*2^(1/2)/(-d*f-e*f+(d*f^2*(d+2*e))^(1/2))^(1/2)
*arctanh(f*x*2^(1/2)/(-d*f-e*f+(d*f^2*(d+2*e))^(1/2))^(1/2))-1/4*
f/(d*f^2*(d+2*e))^(1/2)*2^(1/2)/(d*f+e*f+(d*f^2*(d+2*e))^(1/2))^(
1/2)*arctan(f*x*2^(1/2)/(d*f+e*f+(d*f^2*(d+2*e))^(1/2))^(1/2))*d-
1/2*f/(d*f^2*(d+2*e))^(1/2)*2^(1/2)/(d*f+e*f+(d*f^2*(d+2*e))^(1/2
))^(1/2)*arctan(f*x*2^(1/2)/(d*f+e*f+(d*f^2*(d+2*e))^(1/2))^(1/2)
)*e-1/4*2^(1/2)/(d*f+e*f+(d*f^2*(d+2*e))^(1/2))^(1/2)*arctan(f*x*
2^(1/2)/(d*f+e*f+(d*f^2*(d+2*e))^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{2\,fx^2 - e}{4\,f^2x^4 + 4\,dfx^2 + 4\,efx^2 + e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(2*f*x^2 - e)/(4*f^2*x^4 + 4*d*f*x^2 + 4*e*f*x^2 + e^2),x, algorithm="
```

```
[Out] -integrate((2*f*x^2 - e)/(4*f^2*x^4 + 4*d*f*x^2 + 4*e*f*x^2 + e^2
), x)
```

Fricas [A] time = 0.300178, size = 1, normalized size = 0.01

$$\left[\frac{\log\left(-\frac{8\,df^2x^3+4\,defx-\left(4\,f^2x^4-4\,(d-e)fx^2+e^2\right)\sqrt{-df}}{4\,f^2x^4+4\,(d+e)fx^2+e^2}\right)}{4\,\sqrt{-df}}, -\frac{\arctan\left(\frac{\sqrt{df}\,x}{d}\right)-\arctan\left(\frac{2\,f^2x^3+(2\,d+e)fx}{\sqrt{df}\,e}\right)}{2\,\sqrt{df}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(2*f*x^2 - e)/(4*f^2*x^4 + 4*d*f*x^2 + 4*e*f*x^2 + e^2),x, algorithm="
```

```
[Out] [1/4*log(-(8*d*f^2*x^3 + 4*d*e*f*x - (4*f^2*x^4 - 4*(d - e)*f*x^2
 + e^2)*sqrt(-d*f))/(4*f^2*x^4 + 4*(d + e)*f*x^2 + e^2))/sqrt(-d*
f), -1/2*(arctan(sqrt(d*f)*x/d) - arctan((2*f^2*x^3 + (2*d + e)*f
*x)/(sqrt(d*f)*e)))/sqrt(d*f)]
```

Sympy [A] time = 2.45002, size = 70, normalized size = 0.86

$$\frac{\sqrt{-\frac{1}{df}}\log\left(-dx\sqrt{-\frac{1}{df}}+\frac{e}{2f}+x^2\right)}{4}-\frac{\sqrt{-\frac{1}{df}}\log\left(dx\sqrt{-\frac{1}{df}}+\frac{e}{2f}+x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*f*x**2+e)/(4*f**2*x**4+4*d*f*x**2+4*e*f*x**2+e**2),x)
```

```
[Out] sqrt(-1/(d*f))*log(-d*x*sqrt(-1/(d*f)) + e/(2*f) + x**2)/4 - sqrt
(-1/(d*f))*log(d*x*sqrt(-1/(d*f)) + e/(2*f) + x**2)/4
```

GIAC/XCAS [A] time = 0.761543, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(2*f*x^2 - e)/(4*f^2*x^4 + 4*d*f*x^2 + 4*e*f*x^2 + e^2),x, algorithm="
```

```
[Out] Done
```

3.364 $\int \frac{e-2fx^2}{e^2-4dfx^2+4efx^2+4f^2x^4}\,dx$

Optimal. Leaf size=73

$$\frac{\log\left(2\sqrt{d}\sqrt{f}x+e+2fx^2\right)}{4\sqrt{d}\sqrt{f}}-\frac{\log\left(-2\sqrt{d}\sqrt{f}x+e+2fx^2\right)}{4\sqrt{d}\sqrt{f}}$$

```
[Out] -Log[e - 2*Sqrt[d]*Sqrt[f]*x + 2*f*x^2]/(4*Sqrt[d]*Sqrt[f]) + Log
[e + 2*Sqrt[d]*Sqrt[f]*x + 2*f*x^2]/(4*Sqrt[d]*Sqrt[f])
```

Rubi [A] time = 0.0875291, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.081

$$\frac{\log\left(2\sqrt{d}\sqrt{f}x+e+2fx^2\right)}{4\sqrt{d}\sqrt{f}}-\frac{\log\left(-2\sqrt{d}\sqrt{f}x+e+2fx^2\right)}{4\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In]  Int[(e - 2*f*x^2)/(e^2 - 4*d*f*x^2 + 4*e*f*x^2 + 4*f^2*x^4),x]
```

```
[Out] -Log[e - 2*Sqrt[d]*Sqrt[f]*x + 2*f*x^2]/(4*Sqrt[d]*Sqrt[f]) + Log
[e + 2*Sqrt[d]*Sqrt[f]*x + 2*f*x^2]/(4*Sqrt[d]*Sqrt[f])
```

Rubi in Sympy [A] time = 68.9749, size = 66, normalized size = 0.9

$$-\frac{\log\left(-\frac{\sqrt{d}x}{\sqrt{f}}+\frac{e}{2f}+x^2\right)}{4\sqrt{d}\sqrt{f}}+\frac{\log\left(\frac{\sqrt{d}x}{\sqrt{f}}+\frac{e}{2f}+x^2\right)}{4\sqrt{d}\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-2*f*x**2+e)/(4*f**2*x**4-4*d*f*x**2+4*e*f*x**2+e**2),x)
```

```
[Out] -log(-sqrt(d)*x/sqrt(f) + e/(2*f) + x**2)/(4*sqrt(d)*sqrt(f)) + l
og(sqrt(d)*x/sqrt(f) + e/(2*f) + x**2)/(4*sqrt(d)*sqrt(f))
```

Mathematica [C] time = 0.218115, size = 233, normalized size = 3.19

$$\frac{-\frac{\left(\sqrt{d}\sqrt{2e-d}-id+2ie\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{-i\sqrt{d}\sqrt{2e-d}-d+e}}\right)}{\sqrt{-i\sqrt{d}\sqrt{2e-d}-d+e}}-\frac{\left(\sqrt{d}\sqrt{2e-d}+id-2ie\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{i\sqrt{d}\sqrt{2e-d}-d+e}}\right)}{\sqrt{i\sqrt{d}\sqrt{2e-d}-d+e}}}{2\sqrt{2}\sqrt{d}\sqrt{f}\sqrt{2e-d}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(e - 2*f*x^2)/(e^2 - 4*d*f*x^2 + 4*e*f*x^2 + 4*f^2*x^4),x]
```

```
[Out] (-(((((-I)*d + (2*I)*e + Sqrt[d]*Sqrt[-d + 2*e])*ArcTan[(Sqrt[2]*S
qrt[f]*x)/Sqrt[-d + e - I*Sqrt[d]*Sqrt[-d + 2*e]]])/Sqrt[-d + e -
 I*Sqrt[d]*Sqrt[-d + 2*e]]) - ((I*d - (2*I)*e + Sqrt[d]*Sqrt[-d +
 2*e])*ArcTan[(Sqrt[2]*Sqrt[f]*x)/Sqrt[-d + e + I*Sqrt[d]*Sqrt[-d
 + 2*e]]])/Sqrt[-d + e + I*Sqrt[d]*Sqrt[-d + 2*e]])/(2*Sqrt[2]*Sq
rt[d]*Sqrt[-d + 2*e]*Sqrt[f])
```

Maple [B] time = 0.069, size = 394, normalized size = 5.4

$$\frac{f\sqrt{2}d}{4}\arctan\left(fx\sqrt{2}\frac{1}{\sqrt{-df+ef+\sqrt{df^2(d-2e)}}}\right)\frac{1}{\sqrt{df^2(d-2e)}}\frac{1}{\sqrt{-df+ef+\sqrt{df^2(d-2e)}}}$$
$$-\frac{f\sqrt{2}e}{2}\arctan\left(fx\sqrt{2}\frac{1}{\sqrt{-df+ef+\sqrt{df^2(d-2e)}}}\right)\frac{1}{\sqrt{df^2(d-2e)}}\frac{1}{\sqrt{-df+ef+\sqrt{df^2(d-2e)}}}$$
$$-\frac{\sqrt{2}}{4}\arctan\left(fx\sqrt{2}\frac{1}{\sqrt{-df+ef+\sqrt{df^2(d-2e)}}}\right)\frac{1}{\sqrt{-df+ef+\sqrt{df^2(d-2e)}}}$$
$$+\frac{f\sqrt{2}d}{4}Artanh\left(fx\sqrt{2}\frac{1}{\sqrt{df-ef+\sqrt{df^2(d-2e)}}}\right)\frac{1}{\sqrt{df^2(d-2e)}}\frac{1}{\sqrt{df-ef+\sqrt{df^2(d-2e)}}}$$
$$-\frac{f\sqrt{2}e}{2}Artanh\left(fx\sqrt{2}\frac{1}{\sqrt{df-ef+\sqrt{df^2(d-2e)}}}\right)\frac{1}{\sqrt{df^2(d-2e)}}\frac{1}{\sqrt{df-ef+\sqrt{df^2(d-2e)}}}$$
$$+\frac{\sqrt{2}}{4}Artanh\left(fx\sqrt{2}\frac{1}{\sqrt{df-ef+\sqrt{df^2(d-2e)}}}\right)\frac{1}{\sqrt{df-ef+\sqrt{df^2(d-2e)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-2*f*x^2+e)/(4*f^2*x^4-4*d*f*x^2+4*e*f*x^2+e^2),x)
```

```
[Out] 1/4*f/(d*f^2*(d-2*e))^(1/2)*2^(1/2)/(-d*f+e*f+(d*f^2*(d-2*e))^(1/
2))^(1/2)*arctan(f*x*2^(1/2)/(-d*f+e*f+(d*f^2*(d-2*e))^(1/2))^(1/
2))*d-1/2*f/(d*f^2*(d-2*e))^(1/2)*2^(1/2)/(-d*f+e*f+(d*f^2*(d-2*e
))^(1/2))^(1/2)*arctan(f*x*2^(1/2)/(-d*f+e*f+(d*f^2*(d-2*e))^(1/2
))^(1/2))*e-1/4*2^(1/2)/(-d*f+e*f+(d*f^2*(d-2*e))^(1/2))^(1/2)*ar
ctan(f*x*2^(1/2)/(-d*f+e*f+(d*f^2*(d-2*e))^(1/2))^(1/2))+1/4*f/(d
*f^2*(d-2*e))^(1/2)*2^(1/2)/(d*f-e*f+(d*f^2*(d-2*e))^(1/2))^(1/2)
*arctanh(f*x*2^(1/2)/(d*f-e*f+(d*f^2*(d-2*e))^(1/2))^(1/2))*d-1/2
*f/(d*f^2*(d-2*e))^(1/2)*2^(1/2)/(d*f-e*f+(d*f^2*(d-2*e))^(1/2))^
(1/2)*arctanh(f*x*2^(1/2)/(d*f-e*f+(d*f^2*(d-2*e))^(1/2))^(1/2))*
e+1/4*2^(1/2)/(d*f-e*f+(d*f^2*(d-2*e))^(1/2))^(1/2)*arctanh(f*x*2
^(1/2)/(d*f-e*f+(d*f^2*(d-2*e))^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{2\,fx^2-e}{4\,f^2x^4-4\,dfx^2+4\,efx^2+e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(2*f*x^2 - e)/(4*f^2*x^4 - 4*d*f*x^2 + 4*e*f*x^2 + e^2),x, algorithm="
```

```
[Out] -integrate((2*f*x^2 - e)/(4*f^2*x^4 - 4*d*f*x^2 + 4*e*f*x^2 + e^2
), x)
```

Fricas [A] time = 0.285423, size = 1, normalized size = 0.01

$$\left[\frac{\log\left(\frac{8\,df^2x^3+4\,defx+\left(4\,f^2x^4+4\,(d+e)fx^2+e^2\right)\sqrt{df}}{4\,f^2x^4-4\,(d-e)fx^2+e^2}\right)}{4\,\sqrt{df}},\ \frac{\arctan\left(\frac{\sqrt{-df}\,x}{d}\right)+\arctan\left(\frac{2\,f^2x^3-(2\,d-e)fx}{\sqrt{-df}\,e}\right)}{2\,\sqrt{-df}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(2*f*x^2 - e)/(4*f^2*x^4 - 4*d*f*x^2 + 4*e*f*x^2 + e^2),x, algorithm="
```

```
[Out] [1/4*log((8*d*f^2*x^3 + 4*d*e*f*x + (4*f^2*x^4 + 4*(d + e)*f*x^2
+ e^2)*sqrt(d*f))/(4*f^2*x^4 - 4*(d - e)*f*x^2 + e^2))/sqrt(d*f),
 1/2*(arctan(sqrt(-d*f)*x/d) + arctan((2*f^2*x^3 - (2*d - e)*f*x)
/(sqrt(-d*f)*e)))/sqrt(-d*f)]
```

Sympy [A] time = 2.51022, size = 63, normalized size = 0.86

$$-\frac{\sqrt{\frac{1}{df}}\log\left(-dx\sqrt{\frac{1}{df}}+\frac{e}{2f}+x^2\right)}{4}+\frac{\sqrt{\frac{1}{df}}\log\left(dx\sqrt{\frac{1}{df}}+\frac{e}{2f}+x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*f*x**2+e)/(4*f**2*x**4-4*d*f*x**2+4*e*f*x**2+e**2),x)
```

```
[Out] -sqrt(1/(d*f))*log(-d*x*sqrt(1/(d*f)) + e/(2*f) + x**2)/4 + sqrt(
1/(d*f))*log(d*x*sqrt(1/(d*f)) + e/(2*f) + x**2)/4
```

GIAC/XCAS [A] time = 0.748517, size = 1, normalized size = 0.01

$$\textit{Done}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(2*f*x^2 - e)/(4*f^2*x^4 - 4*d*f*x^2 + 4*e*f*x^2 + e^2),x, algorithm="
```

```
[Out] Done
```

3.365 $\int \frac{e-4fx^3}{e^2+4dfx^2+4efx^3+4f^2x^6}\,dx$

Optimal. Leaf size=38

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x)/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f])
```

Rubi [A] time = 0.10521, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In]  Int[(e - 4*f*x^3)/(e^2 + 4*d*f*x^2 + 4*e*f*x^3 + 4*f^2*x^6),x]

[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x)/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f])
```

Rubi in Sympy [A] time = 46.3091, size = 36, normalized size = 0.95

$$\frac{\operatorname{atan}\left(\frac{4\sqrt{d}\sqrt{f}x}{2e+4fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-4*f*x**3+e)/(4*f**2*x**6+4*e*f*x**3+4*d*f*x**2+e**2),x)

[Out] atan(4*sqrt(d)*sqrt(f)*x/(2*e + 4*f*x**3))/(2*sqrt(d)*sqrt(f))
```

Mathematica [C] time = 0.0929538, size = 87, normalized size = 2.29

$$-\frac{\text{RootSum}\left[4\#1^6f^2+4\#1^3ef+4\#1^2df+e^2\&,\frac{4\#1^3f\log(x-\#1)-e\log(x-\#1)}{6\#1^5f+3\#1^2e+2\#1d}\&\right]}{4f}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(e - 4*f*x^3)/(e^2 + 4*d*f*x^2 + 4*e*f*x^3 + 4*f^2*x^6),x]
```

```
[Out] -RootSum[e^2 + 4*d*f*#1^2 + 4*e*f*#1^3 + 4*f^2*#1^6 & , (-(e*Log[
x - #1]) + 4*f*Log[x - #1]*#1^3)/(2*d*#1 + 3*e*#1^2 + 6*f*#1^5) &
 ]/(4*f)
```

Maple [C] time = 0.012, size = 70, normalized size = 1.8

$$\frac{1}{4f}\sum_{\_R=RootOf(4f^2\_Z^6+4ef\_Z^3+4df\_Z^2+e^2)}\frac{(-4\_R^3f+e)\ln(x-\_R)}{6f\_R^5+3e\_R^2+2d\_R}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-4*f*x^3+e)/(4*f^2*x^6+4*e*f*x^3+4*d*f*x^2+e^2),x)
```

```
[Out] 1/4/f*sum((-4*_R^3*f+e)/(6*_R^5*f+3*_R^2*e+2*_R*d)*ln(x-_R),_R=Ro
otOf(4*_Z^6*f^2+4*_Z^3*e*f+4*_Z^2*d*f+e^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{4fx^3-e}{4f^2x^6+4efx^3+4dfx^2+e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(4*f*x^3 - e)/(4*f^2*x^6 + 4*e*f*x^3 + 4*d*f*x^2 + e^2),x, algorithm="
```

```
[Out] -integrate((4*f*x^3 - e)/(4*f^2*x^6 + 4*e*f*x^3 + 4*d*f*x^2 + e^2
), x)
```

Fricas [A] time = 0.286313, size = 1, normalized size = 0.03

$$\left[\frac{\log\left(-\frac{8\,df^2x^4+4\,defx-(4f^2x^6+4\,efx^3-4\,dfx^2+e^2)\sqrt{-df}}{4f^2x^6+4\,efx^3+4\,dfx^2+e^2}\right)}{4\sqrt{-df}},-\frac{\arctan\left(\frac{\sqrt{df}x^2}{d}\right)-\arctan\left(\frac{2f^2x^5+efx^2+2\,dfx}{\sqrt{df}e}\right)}{2\sqrt{df}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(4*f*x^3 - e)/(4*f^2*x^6 + 4*e*f*x^3 + 4*d*f*x^2 + e^2),x, algorithm="
```

```
[Out] [1/4*log(-(8*d*f^2*x^4 + 4*d*e*f*x - (4*f^2*x^6 + 4*e*f*x^3 - 4*d
*f*x^2 + e^2)*sqrt(-d*f))/(4*f^2*x^6 + 4*e*f*x^3 + 4*d*f*x^2 + e^
2))/sqrt(-d*f), -1/2*(arctan(sqrt(d*f)*x^2/d) - arctan((2*f^2*x^5
 + e*f*x^2 + 2*d*f*x)/(sqrt(d*f)*e)))/sqrt(d*f)]
```

Sympy [A] time = 3.05482, size = 70, normalized size = 1.84

$$\frac{\sqrt{-\frac{1}{df}}\log\left(-dx\sqrt{-\frac{1}{df}}+\frac{e}{2f}+x^3\right)}{4}-\frac{\sqrt{-\frac{1}{df}}\log\left(dx\sqrt{-\frac{1}{df}}+\frac{e}{2f}+x^3\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*f*x**3+e)/(4*f**2*x**6+4*e*f*x**3+4*d*f*x**2+e**2),x)
```

```
[Out] sqrt(-1/(d*f))*log(-d*x*sqrt(-1/(d*f)) + e/(2*f) + x**3)/4 - sqrt
(-1/(d*f))*log(d*x*sqrt(-1/(d*f)) + e/(2*f) + x**3)/4
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{4\,fx^3-e}{4\,f^2x^6+4\,efx^3+4\,dfx^2+e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(4*f*x^3 - e)/(4*f^2*x^6 + 4*e*f*x^3 + 4*d*f*x^2 + e^2),x, algorithm="
```

```
[Out] integrate(-(4*f*x^3 - e)/(4*f^2*x^6 + 4*e*f*x^3 + 4*d*f*x^2 + e^2
), x)
```

3.366 $\int \frac{e-4fx^3}{e^2-4dfx^2+4efx^3+4f^2x^6}\,dx$

Optimal. Leaf size=38

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x)/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f])
```

Rubi [A] time = 0.104376, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In]  Int[(e - 4*f*x^3)/(e^2 - 4*d*f*x^2 + 4*e*f*x^3 + 4*f^2*x^6),x]

[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x)/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f])
```

Rubi in Sympy [A] time = 47.9102, size = 36, normalized size = 0.95

$$\frac{\operatorname{atanh}\left(\frac{4\sqrt{d}\sqrt{f}x}{2e+4fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-4*f*x**3+e)/(4*f**2*x**6+4*e*f*x**3-4*d*f*x**2+e**2),x)

[Out] atanh(4*sqrt(d)*sqrt(f)*x/(2*e + 4*f*x**3))/(2*sqrt(d)*sqrt(f))
```

Mathematica [C] time = 0.0928706, size = 87, normalized size = 2.29

$$-\frac{\text{RootSum}\left[4\#1^6f^2+4\#1^3ef-4\#1^2df+e^2\&,\frac{4\#1^3f\log(x-\#1)-e\log(x-\#1)}{6\#1^5f+3\#1^2e-2\#1d}\&\right]}{4f}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(e - 4*f*x^3)/(e^2 - 4*d*f*x^2 + 4*e*f*x^3 + 4*f^2*x^6),x]
```

```
[Out] -RootSum[e^2 - 4*d*f*#1^2 + 4*e*f*#1^3 + 4*f^2*#1^6 & , (-(e*Log[
x - #1]) + 4*f*Log[x - #1]*#1^3)/(-2*d*#1 + 3*e*#1^2 + 6*f*#1^5)
& ]/(4*f)
```

Maple [C] time = 0.013, size = 70, normalized size = 1.8

$$\frac{1}{4f}\sum_{\_R=RootOf(4f^2\_Z^6+4ef\_Z^3-4df\_Z^2+e^2)}\frac{(-4\_R^3f+e)\ln(x-\_R)}{6f\_R^5+3e\_R^2-2d\_R}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-4*f*x^3+e)/(4*f^2*x^6+4*e*f*x^3-4*d*f*x^2+e^2),x)
```

```
[Out] 1/4/f*sum((-4*_R^3*f+e)/(6*_R^5*f+3*_R^2*e-2*_R*d)*ln(x-_R),_R=Ro
otOf(4*_Z^6*f^2+4*_Z^3*e*f-4*_Z^2*d*f+e^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{4fx^3-e}{4f^2x^6+4efx^3-4dfx^2+e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(4*f*x^3 - e)/(4*f^2*x^6 + 4*e*f*x^3 - 4*d*f*x^2 + e^2),x, algorithm="
```

```
[Out] -integrate((4*f*x^3 - e)/(4*f^2*x^6 + 4*e*f*x^3 - 4*d*f*x^2 + e^2
), x)
```

Fricas [A] time = 0.283552, size = 1, normalized size = 0.03

$$\left[\frac{\log\left(\frac{8df^2x^4+4defx+(4f^2x^6+4efx^3+4dfx^2+e^2)\sqrt{df}}{4f^2x^6+4efx^3-4dfx^2+e^2}\right)}{4\sqrt{df}},\ \frac{\arctan\left(\frac{\sqrt{-df}x^2}{d}\right)+\arctan\left(\frac{2f^2x^5+efx^2-2dfx}{\sqrt{-df}e}\right)}{2\sqrt{-df}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(4*f*x^3 - e)/(4*f^2*x^6 + 4*e*f*x^3 - 4*d*f*x^2 + e^2),x, algorithm="
```

```
[Out] [1/4*log((8*d*f^2*x^4 + 4*d*e*f*x + (4*f^2*x^6 + 4*e*f*x^3 + 4*d*
f*x^2 + e^2)*sqrt(d*f))/(4*f^2*x^6 + 4*e*f*x^3 - 4*d*f*x^2 + e^2)
)/sqrt(d*f), 1/2*(arctan(sqrt(-d*f)*x^2/d) + arctan((2*f^2*x^5 +
e*f*x^2 - 2*d*f*x)/(sqrt(-d*f)*e)))/sqrt(-d*f)]
```

Sympy [A] time = 3.08941, size = 63, normalized size = 1.66

$$-\frac{\sqrt{\frac{1}{df}}\log\left(-dx\sqrt{\frac{1}{df}}+\frac{e}{2f}+x^3\right)}{4}+\frac{\sqrt{\frac{1}{df}}\log\left(dx\sqrt{\frac{1}{df}}+\frac{e}{2f}+x^3\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*f*x**3+e)/(4*f**2*x**6+4*e*f*x**3-4*d*f*x**2+e**2),x)
```

```
[Out] -sqrt(1/(d*f))*log(-d*x*sqrt(1/(d*f)) + e/(2*f) + x**3)/4 + sqrt(
1/(d*f))*log(d*x*sqrt(1/(d*f)) + e/(2*f) + x**3)/4
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{4\,fx^3-e}{4\,f^2x^6+4\,efx^3-4\,dfx^2+e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(4*f*x^3 - e)/(4*f^2*x^6 + 4*e*f*x^3 - 4*d*f*x^2 + e^2),x, algorithm="
```

```
[Out] integrate(-(4*f*x^3 - e)/(4*f^2*x^6 + 4*e*f*x^3 - 4*d*f*x^2 + e^2
), x)
```

3.367 $\int \frac{e-2f(-1+n)x^n}{e^2+4dfx^2+4efx^n+4f^2x^{2n}}\,dx$

Optimal. Leaf size=38

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x)/(e + 2*f*x^n)]/(2*Sqrt[d]*Sqrt[f])
```

Rubi [A] time = 0.155725, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(e - 2*f*(-1 + n)*x^n)/(e^2 + 4*d*f*x^2 + 4*e*f*x^n + 4*f^2*x^(2*n)),x]
```

```
[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x)/(e + 2*f*x^n)]/(2*Sqrt[d]*Sqrt[f])
```

Rubi in Sympy [A] time = 69.6738, size = 44, normalized size = 1.16

$$\frac{\operatorname{atan}\left(\frac{2\sqrt{d}\sqrt{f}x(n-1)}{e(n-1)+2fx^n(n-1)}\right)}{2\sqrt{d}\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((e-2*f*(-1+n)*x**n)/(e**2+4*d*f*x**2+4*e*f*x**n+4*f**2*x**(2*n)
```

```
[Out] atan(2*sqrt(d)*sqrt(f)*x*(n - 1)/(e*(n - 1) + 2*f*x**n*(n - 1)))/
(2*sqrt(d)*sqrt(f))
```

Mathematica [A] time = 0.150596, size = 0, normalized size = 0.

$$\int \frac{e-2f(-1+n)x^n}{e^2+4dfx^2+4efx^n+4f^2x^{2n}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e - 2*f*(-1 + n)*x^n)/(e^2 + 4*d*f*x^2 + 4*e*f*x^n + 4*f^2*x^(2*n)),x]
```

```
[Out] Integrate[(e - 2*f*(-1 + n)*x^n)/(e^2 + 4*d*f*x^2 + 4*e*f*x^n + 4
*f^2*x^(2*n)), x]
```

Maple [B] time = 0.073, size = 78, normalized size = 2.1

$$-\frac{1}{4}\ln\left(x^{n}+\frac{1}{2f}\left(2dfx+e\sqrt{-df}\right)\frac{1}{\sqrt{-df}}\right)\frac{1}{\sqrt{-df}}+\frac{1}{4}\ln\left(x^{n}+\frac{1}{2f}\left(-2dfx+e\sqrt{-df}\right)\frac{1}{\sqrt{-df}}\right)\frac{1}{\sqrt{-df}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e-2*f*(-1+n)*x^n)/(e^2+4*d*f*x^2+4*e*f*x^n+4*f^2*x^(2*n)),x)
```

```
[Out] -1/4/(-d*f)^(1/2)*ln(x^n+1/2*(2*d*f*x+e*(-d*f)^(1/2))/(-d*f)^(1/2
)/f)+1/4/(-d*f)^(1/2)*ln(x^n+1/2*(-2*d*f*x+e*(-d*f)^(1/2))/(-d*f)
^(1/2)/f)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{2f(n-1)x^{n}-e}{4dfx^{2}+4f^{2}x^{2n}+4efx^{n}+e^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(2*f*(n - 1)*x^n - e)/(4*d*f*x^2 + 4*f^2*x^(2*n) + 4*e*f*x^n + e^2),x,
```

```
[Out] -integrate((2*f*(n - 1)*x^n - e)/(4*d*f*x^2 + 4*f^2*x^(2*n) + 4*e
*f*x^n + e^2), x)
```

Fricas [A] time = 0.323055, size = 1, normalized size = 0.03

$$\left[\frac{\log\left(-\frac{4defx-4\sqrt{-df}f^{2}x^{2n}+4\left(2df^{2}x-\sqrt{-df}ef\right)x^{n}+\left(4dfx^{2}-e^{2}\right)\sqrt{-df}}{4dfx^{2}+4f^{2}x^{2n}+4efx^{n}+e^{2}}\right)}{4\sqrt{-df}},-\frac{\arctan\left(\frac{2\sqrt{df}fx^{n}+\sqrt{df}e}{2dfx}\right)}{2\sqrt{df}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(2*f*(n - 1)*x^n - e)/(4*d*f*x^2 + 4*f^2*x^(2*n) + 4*e*f*x^n + e^2),x,
```

```
[Out] [1/4*log(-(4*d*e*f*x - 4*sqrt(-d*f)*f^2*x^(2*n) + 4*(2*d*f^2*x -
sqrt(-d*f)*e*f)*x^n + (4*d*f*x^2 - e^2)*sqrt(-d*f))/(4*d*f*x^2 +
4*f^2*x^(2*n) + 4*e*f*x^n + e^2))/sqrt(-d*f), -1/2*arctan(1/2*(2*
sqrt(d*f)*f*x^n + sqrt(d*f)*e)/(d*f*x))/sqrt(d*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e-2*f*(-1+n)*x**n)/(e**2+4*d*f*x**2+4*e*f*x**n+4*f**2*x**(2*n)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{2\,f(n-1)x^{n} - e}{4\,dfx^{2} + 4\,f^{2}x^{2\,n} + 4\,efx^{n} + e^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(2*f*(n - 1)*x^n - e)/(4*d*f*x^2 + 4*f^2*x^(2*n) + 4*e*f*x^n + e^2),x,
```

```
[Out] integrate(-(2*f*(n - 1)*x^n - e)/(4*d*f*x^2 + 4*f^2*x^(2*n) + 4*e
*f*x^n + e^2), x)
```

3.368 $\int \frac{e-2f(-1+n)x^n}{e^2-4dfx^2+4efx^n+4f^2x^{2n}}\,dx$

Optimal. Leaf size=38

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x)/(e + 2*f*x^n)]/(2*Sqrt[d]*Sqrt[f])
```

Rubi [A] time = 0.153447, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(e - 2*f*(-1 + n)*x^n)/(e^2 - 4*d*f*x^2 + 4*e*f*x^n + 4*f^2*x^(2*n)),x]
```

```
[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x)/(e + 2*f*x^n)]/(2*Sqrt[d]*Sqrt[f])
```

Rubi in Sympy [A] time = 74.1072, size = 44, normalized size = 1.16

$$\frac{\operatorname{atanh}\left(\frac{2\sqrt{d}\sqrt{f}x(n-1)}{e(n-1)+2fx^n(n-1)}\right)}{2\sqrt{d}\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((e-2*f*(-1+n)*x**n)/(e**2-4*d*f*x**2+4*e*f*x**n+4*f**2*x**(2*n)
```

```
[Out] atanh(2*sqrt(d)*sqrt(f)*x*(n - 1)/(e*(n - 1) + 2*f*x**n*(n - 1)))
/(2*sqrt(d)*sqrt(f))
```

Mathematica [A] time = 0.141701, size = 0, normalized size = 0.

$$\int \frac{e-2f(-1+n)x^n}{e^2-4dfx^2+4efx^n+4f^2x^{2n}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e - 2*f*(-1 + n)*x^n)/(e^2 - 4*d*f*x^2 + 4*e*f*x^n + 4*f^2*x^(2*n)),x]
```

```
[Out] Integrate[(e - 2*f*(-1 + n)*x^n)/(e^2 - 4*d*f*x^2 + 4*e*f*x^n + 4
*f^2*x^(2*n)), x]
```

Maple [B] time = 0.074, size = 72, normalized size = 1.9

$$\frac{1}{4}\ln\left(x^{n}+\frac{1}{2f}\left(2dfx+e\sqrt{df}\right)\frac{1}{\sqrt{df}}\right)\frac{1}{\sqrt{df}}-\frac{1}{4}\ln\left(x^{n}+\frac{1}{2f}\left(-2dfx+e\sqrt{df}\right)\frac{1}{\sqrt{df}}\right)\frac{1}{\sqrt{df}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e-2*f*(-1+n)*x^n)/(e^2-4*d*f*x^2+4*e*f*x^n+4*f^2*x^(2*n)),x)
```

```
[Out] 1/4/(d*f)^(1/2)*ln(x^n+1/2*(2*d*f*x+e*(d*f)^(1/2))/(d*f)^(1/2)/f)
-1/4/(d*f)^(1/2)*ln(x^n+1/2*(-2*d*f*x+e*(d*f)^(1/2))/(d*f)^(1/2)/
f)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2\,f(n-1)x^{n}-e}{4\,dfx^{2}-4\,f^{2}x^{2\,n}-4\,efx^{n}-e^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*f*(n - 1)*x^n - e)/(4*d*f*x^2 - 4*f^2*x^(2*n) - 4*e*f*x^n - e^2),x, a
```

```
[Out] integrate((2*f*(n - 1)*x^n - e)/(4*d*f*x^2 - 4*f^2*x^(2*n) - 4*e*
f*x^n - e^2), x)
```

Fricas [A] time = 0.307275, size = 1, normalized size = 0.03

$$\left[\frac{\log\left(-\frac{4\,defx+4\,\sqrt{df}f^{2}x^{2\,n}+4\left(2\,df^{2}x+\sqrt{df}ef\right)x^{n}+\left(4\,dfx^{2}+e^{2}\right)\sqrt{df}}{4\,dfx^{2}-4\,f^{2}x^{2\,n}-4\,efx^{n}-e^{2}}\right)}{4\,\sqrt{df}},\ \frac{\arctan\left(\frac{2\,\sqrt{-df}fx^{n}+\sqrt{-df}e}{2\,dfx}\right)}{2\,\sqrt{-df}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*f*(n - 1)*x^n - e)/(4*d*f*x^2 - 4*f^2*x^(2*n) - 4*e*f*x^n - e^2),x, a
```

```
[Out] [1/4*log(-(4*d*e*f*x + 4*sqrt(d*f)*f^2*x^(2*n) + 4*(2*d*f^2*x + s
qrt(d*f)*e*f)*x^n + (4*d*f*x^2 + e^2)*sqrt(d*f))/(4*d*f*x^2 - 4*f
^2*x^(2*n) - 4*e*f*x^n - e^2))/sqrt(d*f), 1/2*arctan(1/2*(2*sqrt(
-d*f)*f*x^n + sqrt(-d*f)*e)/(d*f*x))/sqrt(-d*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e-2*f*(-1+n)*x**n)/(e**2-4*d*f*x**2+4*e*f*x**n+4*f**2*x**(2*n)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2\,f(n-1)x^n - e}{4\,dfx^2 - 4\,f^2x^{2\,n} - 4\,efx^n - e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*f*(n - 1)*x^n - e)/(4*d*f*x^2 - 4*f^2*x^(2*n) - 4*e*f*x^n - e^2),x, a
```

```
[Out] integrate((2*f*(n - 1)*x^n - e)/(4*d*f*x^2 - 4*f^2*x^(2*n) - 4*e*
f*x^n - e^2), x)
```

3.369 $\int \frac{x}{e^2+4efx^2+4dfx^4+4f^2x^4}\,dx$

Optimal. Leaf size=42

$$\frac{\tan^{-1}\left(\frac{\sqrt{f}\left(2x^2(d+f)+e\right)}{\sqrt{d}e}\right)}{4\sqrt{d}e\sqrt{f}}$$

```
[Out] ArcTan[(Sqrt[f]*(e + 2*(d + f)*x^2))/(Sqrt[d]*e)]/(4*Sqrt[d]*e*Sq
rt[f])
```

Rubi [A] time = 0.128848, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$\frac{\tan^{-1}\left(\frac{\sqrt{f}\left(2x^2(d+f)+e\right)}{\sqrt{d}e}\right)}{4\sqrt{d}e\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(e^2 + 4*e*f*x^2 + 4*d*f*x^4 + 4*f^2*x^4),x]
```

```
[Out] ArcTan[(Sqrt[f]*(e + 2*(d + f)*x^2))/(Sqrt[d]*e)]/(4*Sqrt[d]*e*Sq
rt[f])
```

Rubi in Sympy [A] time = 43.4651, size = 37, normalized size = 0.88

$$\frac{\operatorname{atan}\left(\frac{\sqrt{f}\left(e+x^2(2d+2f)\right)}{\sqrt{d}e}\right)}{4\sqrt{d}e\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(4*d*f*x**4+4*f**2*x**4+4*e*f*x**2+e**2),x)
```

```
[Out] atan(sqrt(f)*(e + x**2*(2*d + 2*f))/(sqrt(d)*e))/(4*sqrt(d)*e*sqr
t(f))
```

Mathematica [A] time = 0.0320959, size = 42, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{f}(2x^2(d+f)+e)}{\sqrt{d}e}\right)}{4\sqrt{d}e\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(e^2 + 4*e*f*x^2 + 4*d*f*x^4 + 4*f^2*x^4),x]
```

```
[Out] ArcTan[(Sqrt[f]*(e + 2*(d + f)*x^2))/(Sqrt[d]*e)]/(4*Sqrt[d]*e*Sq
rt[f])
```

Maple [A] time = 0.008, size = 42, normalized size = 1.

$$\frac{1}{4e}\arctan\left(\frac{2\left(4df+4f^2\right)x^2+4ef}{4e}\frac{1}{\sqrt{df}}\right)\frac{1}{\sqrt{df}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(4*d*f*x^4+4*f^2*x^4+4*e*f*x^2+e^2),x)
```

```
[Out] 1/4/e/(d*f)^(1/2)*arctan(1/4*(2*(4*d*f+4*f^2)*x^2+4*e*f)/e/(d*f)^
(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4*d*f*x^4 + 4*f^2*x^4 + 4*e*f*x^2 + e^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.300433, size = 1, normalized size = 0.02

$$\left[\frac{\log\left(\frac{2\,de^2f+4\,(d^2ef+def^2)\,x^2+\left(4\,(d^2f+2\,df^2+f^3)\,x^4-de^2+e^2f+4\,(def+ef^2)\,x^2\right)\sqrt{-df}}{4\,(df+f^2)x^4+4\,efx^2+e^2}\right)}{8\,\sqrt{-df}e},\ \frac{\arctan\left(\frac{(2\,(d+f)x^2+e)\,\sqrt{df}}{de}\right)}{4\,\sqrt{df}e}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4*d*f*x^4 + 4*f^2*x^4 + 4*e*f*x^2 + e^2),x, algorithm="fricas")
```

```
[Out] [1/8*log((2*d*e^2*f + 4*(d^2*e*f + d*e*f^2)*x^2 + (4*(d^2*f + 2*d
*f^2 + f^3)*x^4 - d*e^2 + e^2*f + 4*(d*e*f + e*f^2)*x^2)*sqrt(-d*
f))/(4*(d*f + f^2)*x^4 + 4*e*f*x^2 + e^2))/(sqrt(-d*f)*e), 1/4*ar
ctan((2*(d + f)*x^2 + e)*sqrt(d*f)/(d*e))/(sqrt(d*f)*e)]
```

Sympy [A] time = 1.87182, size = 78, normalized size = 1.86

$$\frac{-\frac{\sqrt{-\frac{1}{df}}\log\left(x^2+\frac{-de\sqrt{-\frac{1}{df}}+e}{2d+2f}\right)}{8}+\frac{\sqrt{-\frac{1}{df}}\log\left(x^2+\frac{de\sqrt{-\frac{1}{df}}+e}{2d+2f}\right)}{8}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4*d*f*x**4+4*f**2*x**4+4*e*f*x**2+e**2),x)
```

```
[Out] (-sqrt(-1/(d*f))*log(x**2 + (-d*e*sqrt(-1/(d*f)) + e)/(2*d + 2*f)
)/8 + sqrt(-1/(d*f))*log(x**2 + (d*e*sqrt(-1/(d*f)) + e)/(2*d + 2
*f))/8)/e
```

GIAC/XCAS [A] time = 0.342735, size = 51, normalized size = 1.21

$$\frac{\arctan\left(\frac{(2\,dfx^2+2\,f^2x^2+fe)\,e^{(-1)}}{\sqrt{df}}\right)e^{(-1)}}{4\,\sqrt{df}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4*d*f*x^4 + 4*f^2*x^4 + 4*e*f*x^2 + e^2),x, algorithm="giac")
```

```
[Out] 1/4*arctan((2*d*f*x^2 + 2*f^2*x^2 + f*e)*e^(-1)/sqrt(d*f))*e^(-1)
/sqrt(d*f)
```

3.370 $\int \frac{x}{e^2+4efx^2-4dfx^4+4f^2x^4}\,dx$

Optimal. Leaf size=44

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{f}\,(e-2x^2(d-f))}{\sqrt{d}e}\right)}{4\sqrt{d}e\sqrt{f}}$$

```
[Out] -ArcTanh[(Sqrt[f]*(e - 2*(d - f)*x^2))/(Sqrt[d]*e)]/(4*Sqrt[d]*e*
Sqrt[f])
```

Rubi [A] time = 0.134713, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{f}\,(e-2x^2(d-f))}{\sqrt{d}e}\right)}{4\sqrt{d}e\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(e^2 + 4*e*f*x^2 - 4*d*f*x^4 + 4*f^2*x^4),x]
```

```
[Out] -ArcTanh[(Sqrt[f]*(e - 2*(d - f)*x^2))/(Sqrt[d]*e)]/(4*Sqrt[d]*e*
Sqrt[f])
```

Rubi in Sympy [A] time = 44.4472, size = 39, normalized size = 0.89

$$-\frac{\operatorname{atanh}\left(\frac{\sqrt{f}\,(e+x^2(-2d+2f))}{\sqrt{d}e}\right)}{4\sqrt{d}e\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(-4*d*f*x**4+4*f**2*x**4+4*e*f*x**2+e**2),x)
```

```
[Out] -atanh(sqrt(f)*(e + x**2*(-2*d + 2*f))/(sqrt(d)*e))/(4*sqrt(d)*e*
sqrt(f))
```

Mathematica [A] time = 0.0356423, size = 46, normalized size = 1.05

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{f}(-2dx^2+e+2fx^2)}{\sqrt{d}e}\right)}{4\sqrt{d}e\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(e^2 + 4*e*f*x^2 - 4*d*f*x^4 + 4*f^2*x^4),x]
```

```
[Out] -ArcTanh[(Sqrt[f]*(e - 2*d*x^2 + 2*f*x^2))/(Sqrt[d]*e)]/(4*Sqrt[d
]*e*Sqrt[f])
```

Maple [A] time = 0.004, size = 42, normalized size = 1.

$$\frac{1}{4\,e}\mathit{Artanh}\left(\frac{2\,\left(4\,df-4\,f^2\right)x^2-4\,ef}{4\,e}\frac{1}{\sqrt{df}}\right)\frac{1}{\sqrt{df}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(-4*d*f*x^4+4*f^2*x^4+4*e*f*x^2+e^2),x)
```

```
[Out] 1/4/e/(d*f)^(1/2)*arctanh(1/4*(2*(4*d*f-4*f^2)*x^2-4*e*f)/e/(d*f)
^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x/(4*d*f*x^4 - 4*f^2*x^4 - 4*e*f*x^2 - e^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.284293, size = 1, normalized size = 0.02

$$\left[\frac{\log\left(\frac{2\,de^2f-4\,(d^2ef-def^2)\,x^2-\left(4\,(d^2f-2\,df^2+f^3)\,x^4+de^2+e^2f-4\,(def-ef^2)\,x^2\right)\sqrt{df}}{4\,(df-f^2)x^4-4\,efx^2-e^2}\right)}{8\,\sqrt{df}e}, -\frac{\arctan\left(-\frac{(2\,(d-f)x^2-e)\,\sqrt{-df}}{de}\right)}{4\,\sqrt{-df}e}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x/(4*d*f*x^4 - 4*f^2*x^4 - 4*e*f*x^2 - e^2),x, algorithm="fricas")
```

```
[Out] [1/8*log((2*d*e^2*f - 4*(d^2*e*f - d*e*f^2)*x^2 - (4*(d^2*f - 2*d
*f^2 + f^3)*x^4 + d*e^2 + e^2*f - 4*(d*e*f - e*f^2)*x^2)*sqrt(d*f
))/(4*(d*f - f^2)*x^4 - 4*e*f*x^2 - e^2))/(sqrt(d*f)*e), -1/4*arc
tan(-(2*(d - f)*x^2 - e)*sqrt(-d*f)/(d*e))/(sqrt(-d*f)*e)]
```

Sympy [A] time = 2.00816, size = 75, normalized size = 1.7

$$-\frac{\frac{\sqrt{\frac{1}{df}}\log\left(x^2+\frac{-de\sqrt{\frac{1}{df}}-e}{2d-2f}\right)}{8}-\frac{\sqrt{\frac{1}{df}}\log\left(x^2+\frac{de\sqrt{\frac{1}{df}}-e}{2d-2f}\right)}{8}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-4*d*f*x**4+4*f**2*x**4+4*e*f*x**2+e**2),x)
```

```
[Out] -(sqrt(1/(d*f))*log(x**2 + (-d*e*sqrt(1/(d*f)) - e)/(2*d - 2*f))/
8 - sqrt(1/(d*f))*log(x**2 + (d*e*sqrt(1/(d*f)) - e)/(2*d - 2*f))
/8)/e
```

GIAC/XCAS [A] time = 0.346464, size = 55, normalized size = 1.25

$$-\frac{\arctan\left(\frac{2\,dfx^2-2\,f^2x^2-fe}{\sqrt{-dfe^2}}\right)}{4\,\sqrt{-dfe^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x/(4*d*f*x^4 - 4*f^2*x^4 - 4*e*f*x^2 - e^2),x, algorithm="giac")
```

```
[Out] -1/4*arctan((2*d*f*x^2 - 2*f^2*x^2 - f*e)/sqrt(-d*f*e^2))/sqrt(-d
*f*e^2)
```

3.371 $\int \frac{x^2(3e+2fx^2)}{e^2+4efx^2+4f^2x^4+4dfx^6}\,dx$

Optimal. Leaf size=40

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^3}{e+2fx^2}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x^3)/(e + 2*f*x^2)]/(2*Sqrt[d]*Sqrt[f])
```

Rubi [A] time = 0.204955, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^3}{e+2fx^2}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(3*e + 2*f*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4 + 4*d*f*x^6),x]
```

```
[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x^3)/(e + 2*f*x^2)]/(2*Sqrt[d]*Sqrt[f])
```

Rubi in Sympy [A] time = 64.986, size = 37, normalized size = 0.92

$$\frac{\operatorname{atan}\left(\frac{6\sqrt{d}\sqrt{f}x^3}{3e+6fx^2}\right)}{2\sqrt{d}\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**2*(2*f*x**2+3*e)/(4*d*f*x**6+4*f**2*x**4+4*e*f*x**2+e**2),x)
```

```
[Out] atan(6*sqrt(d)*sqrt(f)*x**3/(3*e + 6*f*x**2))/(2*sqrt(d)*sqrt(f))
```

Mathematica [C] time = 0.0755973, size = 85, normalized size = 2.12

$$\frac{\text{RootSum}\left[4\#1^6 df + 4\#1^4 f^2 + 4\#1^2 ef + e^2\&, \frac{2\#1^3 f\log(x-\#1)+3\#1 e\log(x-\#1)}{3\#1^4 d+2\#1^2 f+e}\&\right]}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(3*e + 2*f*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4 + 4*d*f*x^6),x]
```

```
[Out] RootSum[e^2 + 4*e*f*#1^2 + 4*f^2*#1^4 + 4*d*f*#1^6 & , (3*e*Log[x
 - #1]*#1 + 2*f*Log[x - #1]*#1^3)/(e + 2*f*#1^2 + 3*d*#1^4) & ]/(
8*f)
```

Maple [C] time = 0.419, size = 74, normalized size = 1.9

$$\frac{1}{8f}\sum_{\_R=RootOf\left(4\,df\_Z^6+4f^2\_Z^4+4\,ef\_Z^2+e^2\right)}\frac{\left(2\,\_R^4f+3\,\_R^2e\right)\ln\left(x-\_R\right)}{3\,d\_R^5+2\,f\_R^3+e\_R}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*f*x^2+3*e)/(4*d*f*x^6+4*f^2*x^4+4*e*f*x^2+e^2),x)
```

```
[Out] 1/8/f*sum((2*_R^4*f+3*_R^2*e)/(3*_R^5*d+2*_R^3*f+_R*e)*ln(x-_R),_
R=RootOf(4*_Z^6*d*f+4*_Z^4*f^2+4*_Z^2*e*f+e^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(2\,fx^2+3\,e\right)x^2}{4\,dfx^6+4\,f^2x^4+4\,efx^2+e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*f*x^2 + 3*e)*x^2/(4*d*f*x^6 + 4*f^2*x^4 + 4*e*f*x^2 + e^2),x, algori
```

```
[Out] integrate((2*f*x^2 + 3*e)*x^2/(4*d*f*x^6 + 4*f^2*x^4 + 4*e*f*x^2
+ e^2), x)
```

Fricas [A] time = 0.286408, size = 1, normalized size = 0.02

$$\left[\frac{\log\left(\frac{8\,df^2x^5+4\,defx^3+\left(4\,dfx^6-4\,f^2x^4-4\,efx^2-e^2\right)\sqrt{-df}}{4\,dfx^6+4\,f^2x^4+4\,efx^2+e^2}\right)}{4\,\sqrt{-df}},\ \frac{\arctan\left(\frac{\sqrt{df}\,x}{f}\right)+\arctan\left(\frac{2\,dfx^3-\left(de-2\,f^2\right)x}{\sqrt{df}\,e}\right)-\arctan\left(\frac{2\left(2\,df^2x^5+ef^2x-(d\right.}{\sqrt{df}\,e^2}\right.}{2\,\sqrt{df}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*f*x^2 + 3*e)*x^2/(4*d*f*x^6 + 4*f^2*x^4 + 4*e*f*x^2 + e^2),x, algori
```

```
[Out] [1/4*log((8*d*f^2*x^5 + 4*d*e*f*x^3 + (4*d*f*x^6 - 4*f^2*x^4 - 4*
e*f*x^2 - e^2)*sqrt(-d*f))/(4*d*f*x^6 + 4*f^2*x^4 + 4*e*f*x^2 + e
^2))/sqrt(-d*f), 1/2*(arctan(sqrt(d*f)*x/f) + arctan((2*d*f*x^3 -
 (d*e - 2*f^2)*x)/(sqrt(d*f)*e)) - arctan(2*(2*d*f^2*x^5 + e*f^2*
x - (d*e*f - 2*f^3)*x^3)/(sqrt(d*f)*e^2)))/sqrt(d*f)]
```

Sympy [A] time = 4.23546, size = 90, normalized size = 2.25

$$-\frac{\sqrt{-\frac{1}{df}}\log\left(-\frac{e\sqrt{-\frac{1}{df}}}{2}-fx^2\sqrt{-\frac{1}{df}}+x^3\right)}{4}+\frac{\sqrt{-\frac{1}{df}}\log\left(\frac{e\sqrt{-\frac{1}{df}}}{2}+fx^2\sqrt{-\frac{1}{df}}+x^3\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*f*x**2+3*e)/(4*d*f*x**6+4*f**2*x**4+4*e*f*x**2+e**2),x)
```

```
[Out] -sqrt(-1/(d*f))*log(-e*sqrt(-1/(d*f))/2 - f*x**2*sqrt(-1/(d*f)) +
 x**3)/4 + sqrt(-1/(d*f))*log(e*sqrt(-1/(d*f))/2 + f*x**2*sqrt(-1
/(d*f)) + x**3)/4
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(2fx^2+3e)x^2}{4dfx^6+4f^2x^4+4efx^2+e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*f*x^2 + 3*e)*x^2/(4*d*f*x^6 + 4*f^2*x^4 + 4*e*f*x^2 + e^2),x, algori
```

```
[Out] integrate((2*f*x^2 + 3*e)*x^2/(4*d*f*x^6 + 4*f^2*x^4 + 4*e*f*x^2
+ e^2), x)
```

3.372 $\int \frac{x^2(3e+2fx^2)}{e^2+4efx^2+4f^2x^4-4dfx^6}\,dx$

Optimal. Leaf size=40

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^3}{e+2fx^2}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x^3)/(e + 2*f*x^2)]/(2*Sqrt[d]*Sqrt[f]
)
```

Rubi [A] time = 0.205808, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^3}{e+2fx^2}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(3*e + 2*f*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4 - 4*d*f*x^6),x]
```

```
[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x^3)/(e + 2*f*x^2)]/(2*Sqrt[d]*Sqrt[f]
)
```

Rubi in Sympy [A] time = 68.4054, size = 37, normalized size = 0.92

$$\frac{\operatorname{atanh}\left(\frac{6\sqrt{d}\sqrt{f}x^3}{3e+6fx^2}\right)}{2\sqrt{d}\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**2*(2*f*x**2+3*e)/(-4*d*f*x**6+4*f**2*x**4+4*e*f*x**2+e**2),x
```

```
[Out] atanh(6*sqrt(d)*sqrt(f)*x**3/(3*e + 6*f*x**2))/(2*sqrt(d)*sqrt(f)
)
```

Mathematica [C] time = 0.0829876, size = 85, normalized size = 2.12

$$\frac{\text{RootSum}\left[-4\#1^6df+4\#1^4f^2+4\#1^2ef+e^2\&,\frac{2\#1^3f\log(x-\#1)+3\#1e\log(x-\#1)}{-3\#1^4d+2\#1^2f+e}\&\right]}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(3*e + 2*f*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4 - 4*d*f*x^6),x]
```

```
[Out] RootSum[e^2 + 4*e*f*#1^2 + 4*f^2*#1^4 - 4*d*f*#1^6 & , (3*e*Log[x
 - #1]*#1 + 2*f*Log[x - #1]*#1^3)/(e + 2*f*#1^2 - 3*d*#1^4) & ]/(
8*f)
```

Maple [C] time = 0.377, size = 77, normalized size = 1.9

$$-\frac{1}{8f}\sum_{\_R=RootOf\left(4df\_Z^6-4f^2\_Z^4-4ef\_Z^2-e^2\right)}\frac{\left(2\_R^4f+3\_R^2e\right)\ln\left(x-\_R\right)}{3d\_R^5-2f\_R^3-e\_R}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*f*x^2+3*e)/(-4*d*f*x^6+4*f^2*x^4+4*e*f*x^2+e^2),x)
```

```
[Out] -1/8/f*sum((2*_R^4*f+3*_R^2*e)/(3*_R^5*d-2*_R^3*f-_R*e)*ln(x-_R),
_R=RootOf(4*_Z^6*d*f-4*_Z^4*f^2-4*_Z^2*e*f-e^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\left(2fx^2+3e\right)x^2}{4dfx^6-4f^2x^4-4efx^2-e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(2*f*x^2 + 3*e)*x^2/(4*d*f*x^6 - 4*f^2*x^4 - 4*e*f*x^2 - e^2),x, algor
```

```
[Out] -integrate((2*f*x^2 + 3*e)*x^2/(4*d*f*x^6 - 4*f^2*x^4 - 4*e*f*x^2
 - e^2), x)
```

Fricas [A] time = 0.288159, size = 1, normalized size = 0.02

$$\left[\frac{\log\left(\frac{8\,df^{2}x^{5}+4\,defx^{3}+\left(4\,dfx^{6}+4\,f^{2}x^{4}+4\,efx^{2}+e^{2}\right)\sqrt{df}}{4\,dfx^{6}-4\,f^{2}x^{4}-4\,efx^{2}-e^{2}}\right)}{4\,\sqrt{df}}, \frac{\arctan\left(\frac{\sqrt{-df}x}{f}\right)-\arctan\left(\frac{2\,dfx^{3}-\left(de+2\,f^{2}\right)x}{\sqrt{-df}e}\right)+\arctan\left(\frac{2\,\left(2\,df^{2}x^{5}-ef^{2}x-\left(de\right.\right.}{\sqrt{-df}e^{2}}\right.}{2\,\sqrt{-df}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(2*f*x^2 + 3*e)*x^2/(4*d*f*x^6 - 4*f^2*x^4 - 4*e*f*x^2 - e^2),x, algor
```

```
[Out] [1/4*log((8*d*f^2*x^5 + 4*d*e*f*x^3 + (4*d*f*x^6 + 4*f^2*x^4 + 4*
e*f*x^2 + e^2)*sqrt(d*f))/(4*d*f*x^6 - 4*f^2*x^4 - 4*e*f*x^2 - e^
2))/sqrt(d*f), 1/2*(arctan(sqrt(-d*f)*x/f) - arctan((2*d*f*x^3 -
(d*e + 2*f^2)*x)/(sqrt(-d*f)*e)) + arctan(2*(2*d*f^2*x^5 - e*f^2*
x - (d*e*f + 2*f^3)*x^3)/(sqrt(-d*f)*e^2)))/sqrt(-d*f)]
```

Sympy [A] time = 4.30452, size = 80, normalized size = 2.

$$-\frac{\sqrt{\frac{1}{df}}\log\left(-\frac{e\sqrt{\frac{1}{df}}}{2}-fx^{2}\sqrt{\frac{1}{df}}+x^{3}\right)}{4}+\frac{\sqrt{\frac{1}{df}}\log\left(\frac{e\sqrt{\frac{1}{df}}}{2}+fx^{2}\sqrt{\frac{1}{df}}+x^{3}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*f*x**2+3*e)/(-4*d*f*x**6+4*f**2*x**4+4*e*f*x**2+e**2),x)
```

```
[Out] -sqrt(1/(d*f))*log(-e*sqrt(1/(d*f))/2 - f*x**2*sqrt(1/(d*f)) + x*
*3)/4 + sqrt(1/(d*f))*log(e*sqrt(1/(d*f))/2 + f*x**2*sqrt(1/(d*f)
) + x**3)/4
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(2\,fx^{2}+3\,e\right)x^{2}}{4\,dfx^{6}-4\,f^{2}x^{4}-4\,efx^{2}-e^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(2*f*x^2 + 3*e)*x^2/(4*d*f*x^6 - 4*f^2*x^4 - 4*e*f*x^2 - e^2),x, algor
```

```
[Out] integrate(-(2*f*x^2 + 3*e)*x^2/(4*d*f*x^6 - 4*f^2*x^4 - 4*e*f*x^2
 - e^2), x)
```

3.373 $\int \frac{x^m \left(e(1+m)+2 f(-1+m) x^2\right)}{e^2+4 e f x^2+4 f^2 x^4+4 d f x^{2+2 m}} \, dx$

Optimal. Leaf size=42

$$\frac{\tan^{-1}\left(\frac{2 \sqrt{d} \sqrt{f} x^{m+1}}{e+2 f x^2}\right)}{2 \sqrt{d} \sqrt{f}}$$

```
[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^2)]/(2*Sqrt[d]*Sq
rt[f])
```

Rubi [A] time = 0.340527, antiderivative size = 61, normalized size of antiderivative = 1.45, number of steps used = 2, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.039

$$\frac{\tan^{-1}\left(\frac{2 \sqrt{d} \sqrt{f} x^{m+1}}{e+2 f x^2}\right)}{2 \sqrt{d} \sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(e*(1 + m) + 2*f*(-1 + m)*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4 + 4*d*f*x^(2
```

```
[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^2)]/(2*Sqrt[d]*Sq
rt[f])
```

Rubi in Sympy [A] time = 106.903, size = 37, normalized size = 0.88

$$\frac{\operatorname{atan}\left(\frac{2 \sqrt{d} \sqrt{f} x^{m+1}}{e+2 f x^2}\right)}{2 \sqrt{d} \sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**m*(e*(1+m)+2*f*(-1+m)*x**2)/(e**2+4*e*f*x**2+4*f**2*x**4+4*d
```

```
[Out] atan(2*sqrt(d)*sqrt(f)*x**(m + 1)/(e + 2*f*x**2))/(2*sqrt(d)*sqrt
(f))
```

Mathematica [A] time = 0.0835232, size = 42, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{m+1}}{e+2fx^2}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(e*(1 + m) + 2*f*(-1 + m)*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4 + 4*d*
```

```
[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^2)]/(2*Sqrt[d]*Sq
rt[f])
```

Maple [B] time = 0.09, size = 78, normalized size = 1.9

$$-\frac{1}{4}\ln\left(x^m+\frac{2fx^2+e}{2dfx}\sqrt{-df}\right)\frac{1}{\sqrt{-df}}+\frac{1}{4}\ln\left(x^m-\frac{2fx^2+e}{2dfx}\sqrt{-df}\right)\frac{1}{\sqrt{-df}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(e*(1+m)+2*f*(-1+m)*x^2)/(e^2+4*e*f*x^2+4*f^2*x^4+4*d*f*x^(2+2*m)),x)
```

```
[Out] -1/4/(-d*f)^(1/2)*ln(x^m+1/2*(2*f*x^2+e)*(-d*f)^(1/2)/d/f/x)+1/4/
(-d*f)^(1/2)*ln(x^m-1/2*(2*f*x^2+e)*(-d*f)^(1/2)/d/f/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(2\,f(m-1)x^2+e(m+1)\right)x^m}{4\,f^2x^4+4\,efx^2+4\,dfx^{2\,m+2}+e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*f*(m - 1)*x^2 + e*(m + 1))*x^m/(4*f^2*x^4 + 4*e*f*x^2 + 4*d*f*x^(2*m
```

```
[Out] integrate((2*f*(m - 1)*x^2 + e*(m + 1))*x^m/(4*f^2*x^4 + 4*e*f*x^
2 + 4*d*f*x^(2*m + 2) + e^2), x)
```

Fricas [A] time = 0.30025, size = 1, normalized size = 0.02

$$\left[\frac{\log\left(\frac{4\sqrt{-df}dfx^2x^{2m}+4\left(2df^2x^3+defx\right)x^m-\left(4f^2x^4+4efx^2+e^2\right)\sqrt{-df}}{4f^2x^4+4dfx^2x^{2m}+4efx^2+e^2}\right)}{4\sqrt{-df}}, -\frac{\arctan\left(\frac{2fx^2+e}{2\sqrt{df}xx^m}\right)}{2\sqrt{df}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*f*(m - 1)*x^2 + e*(m + 1))*x^m/(4*f^2*x^4 + 4*e*f*x^2 + 4*d*f*x^(2*m
```

```
[Out] [1/4*log((4*sqrt(-d*f)*d*f*x^2*x^(2*m) + 4*(2*d*f^2*x^3 + d*e*f*x
)*x^m - (4*f^2*x^4 + 4*e*f*x^2 + e^2)*sqrt(-d*f))/(4*f^2*x^4 + 4*
d*f*x^2*x^(2*m) + 4*e*f*x^2 + e^2))/sqrt(-d*f), -1/2*arctan(1/2*(
2*f*x^2 + e)/(sqrt(d*f)*x*x^m))/sqrt(d*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(e*(1+m)+2*f*(-1+m)*x**2)/(e**2+4*e*f*x**2+4*f**2*x**4+4*d*f*x*
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(2f(m-1)x^2 + e(m+1)\right)x^m}{4f^2x^4 + 4efx^2 + 4dfx^{2m+2} + e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*f*(m - 1)*x^2 + e*(m + 1))*x^m/(4*f^2*x^4 + 4*e*f*x^2 + 4*d*f*x^(2*m
```

```
[Out] integrate((2*f*(m - 1)*x^2 + e*(m + 1))*x^m/(4*f^2*x^4 + 4*e*f*x^
2 + 4*d*f*x^(2*m + 2) + e^2), x)
```

3.374 $\int \frac{x^m\left(e(1+m)+2f(-1+m)x^2\right)}{e^2+4efx^2+4f^2x^4-4dfx^{2+2m}}\,dx$

Optimal. Leaf size=42

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{m+1}}{e+2fx^2}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^2)]/(2*Sqrt[d]*S
qrt[f])
```

Rubi [A] time = 0.332903, antiderivative size = 61, normalized size of antiderivative = 1.45, number of steps used = 2, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.039

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{m+1}}{e+2fx^2}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(e*(1 + m) + 2*f*(-1 + m)*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4 - 4*d*f*x^(2
```

```
[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^2)]/(2*Sqrt[d]*S
qrt[f])
```

Rubi in Sympy [A] time = 116.428, size = 37, normalized size = 0.88

$$\frac{\operatorname{atanh}\left(\frac{2\sqrt{d}\sqrt{f}x^{m+1}}{e+2fx^2}\right)}{2\sqrt{d}\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**m*(e*(1+m)+2*f*(-1+m)*x**2)/(e**2+4*e*f*x**2+4*f**2*x**4-
4*d*f*x**(2+2*m)),x)
```

```
[Out] atanh(2*sqrt(d)*sqrt(f)*x**(m + 1)/(e + 2*f*x**2))/(2*sqrt(d)*sqr
t(f))
```

Mathematica [A] time = 0.0750872, size = 42, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{m+1}}{e+2fx^2}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(e*(1 + m) + 2*f*(-1 + m)*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4 - 4*d*
```

```
[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^2)]/(2*Sqrt[d]*S
qrt[f])
```

Maple [B] time = 0.09, size = 74, normalized size = 1.8

$$\frac{1}{4}\ln\left(x^m+\frac{2fx^2+e}{2dfx}\sqrt{df}\right)\frac{1}{\sqrt{df}}-\frac{1}{4}\ln\left(x^m-\frac{2fx^2+e}{2dfx}\sqrt{df}\right)\frac{1}{\sqrt{df}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(e*(1+m)+2*f*(-1+m)*x^2)/(e^2+4*e*f*x^2+4*f^2*x^4-4*d*f*x^(2+2*m)),x)
```

```
[Out] 1/4/(d*f)^(1/2)*ln(x^m+1/2*(2*f*x^2+e)*(d*f)^(1/2)/d/f/x)-1/4/(d*
f)^(1/2)*ln(x^m-1/2*(2*f*x^2+e)*(d*f)^(1/2)/d/f/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(2f(m-1)x^2+e(m+1)\right)x^m}{4f^2x^4+4efx^2-4dfx^{2m+2}+e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*f*(m - 1)*x^2 + e*(m + 1))*x^m/(4*f^2*x^4 + 4*e*f*x^2 - 4*d*f*x^(2*m
```

```
[Out] integrate((2*f*(m - 1)*x^2 + e*(m + 1))*x^m/(4*f^2*x^4 + 4*e*f*x^
2 - 4*d*f*x^(2*m + 2) + e^2), x)
```

Fricas [A] time = 0.29847, size = 1, normalized size = 0.02

$$\left[\frac{\log\left(-\frac{4\sqrt{df}dfx^2x^{2m}+4\left(2df^2x^3+defx\right)x^m+\left(4f^2x^4+4efx^2+e^2\right)\sqrt{df}}{4f^2x^4-4dfx^2x^{2m}+4efx^2+e^2}\right)}{4\sqrt{df}}, -\frac{\arctan\left(\frac{2fx^2+e}{2\sqrt{-df}xx^m}\right)}{2\sqrt{-df}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*f*(m - 1)*x^2 + e*(m + 1))*x^m/(4*f^2*x^4 + 4*e*f*x^2 - 4*d*f*x^(2*m
```

```
[Out] [1/4*log(-(4*sqrt(d*f)*d*f*x^2*x^(2*m) + 4*(2*d*f^2*x^3 + d*e*f*x
)*x^m + (4*f^2*x^4 + 4*e*f*x^2 + e^2)*sqrt(d*f))/(4*f^2*x^4 - 4*d
*f*x^2*x^(2*m) + 4*e*f*x^2 + e^2))/sqrt(d*f), -1/2*arctan(1/2*(2*
f*x^2 + e)/(sqrt(-d*f)*x*x^m))/sqrt(-d*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(e*(1+m)+2*f*(-1+m)*x**2)/(e**2+4*e*f*x**2+4*f**2*x**4-
4*d*f*x**(2+2*m)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(2f(m-1)x^2 + e(m+1)\right)x^m}{4f^2x^4 + 4efx^2 - 4dfx^{2m+2} + e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*f*(m - 1)*x^2 + e*(m + 1))*x^m/(4*f^2*x^4 + 4*e*f*x^2 - 4*d*f*x^(2*m
```

```
[Out] integrate((2*f*(m - 1)*x^2 + e*(m + 1))*x^m/(4*f^2*x^4 + 4*e*f*x^
2 - 4*d*f*x^(2*m + 2) + e^2), x)
```

3.375 $\int \frac{x(2e-2fx^3)}{e^2+4efx^3+4dfx^4+4f^2x^6}\,dx$

Optimal. Leaf size=40

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^2}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x^2)/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f])
```

Rubi [A] time = 0.142368, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^2}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(2*e - 2*f*x^3))/(e^2 + 4*e*f*x^3 + 4*d*f*x^4 + 4*f^2*x^6),x]
```

```
[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x^2)/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f])
```

Rubi in Sympy [A] time = 48.5976, size = 41, normalized size = 1.02

$$-\frac{\operatorname{atan}\left(\frac{4\sqrt{d}\sqrt{f}x^2}{-2e-4fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x*(-2*f*x**3+2*e)/(4*f**2*x**6+4*d*f*x**4+4*e*f*x**3+e**2),x)
```

```
[Out] -atan(4*sqrt(d)*sqrt(f)*x**2/(-2*e - 4*f*x**3))/(2*sqrt(d)*sqrt(f
))
```

Mathematica [C] time = 0.0768785, size = 86, normalized size = 2.15

$$-\frac{\text{RootSum}\left[4\#1^6f^2+4\#1^4df+4\#1^3ef+e^2\&,\frac{\#1^3f\log(x-\#1)-e\log(x-\#1)}{6\#1^4f+4\#1^2d+3\#1e}\&\right]}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(2*e - 2*f*x^3))/(e^2 + 4*e*f*x^3 + 4*d*f*x^4 + 4*f^2*x^6),x]
```

```
[Out] -RootSum[e^2 + 4*e*f*#1^3 + 4*d*f*#1^4 + 4*f^2*#1^6 & , (-(e*Log[
x - #1]) + f*Log[x - #1]*#1^3)/(3*e*#1 + 4*d*#1^2 + 6*f*#1^4) & ]
/(2*f)
```

Maple [C] time = 0.013, size = 74, normalized size = 1.9

$$-\frac{1}{2f}\sum_{\_R=RootOf\left(4f^2\_Z^6+4df\_Z^4+4ef\_Z^3+e^2\right)}\frac{\left(\_R^4f-\_Re\right)\ln\left(x-\_R\right)}{6f\_R^5+4d\_R^3+3e\_R^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-2*f*x^3+2*e)/(4*f^2*x^6+4*d*f*x^4+4*e*f*x^3+e^2),x)
```

```
[Out] -1/2/f*sum((_R^4*f-_R*e)/(6*_R^5*f+4*_R^3*d+3*_R^2*e)*ln(x-_R),_R
=RootOf(4*_Z^6*f^2+4*_Z^4*d*f+4*_Z^3*e*f+e^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-2\int\frac{\left(fx^3-e\right)x}{4f^2x^6+4dfx^4+4efx^3+e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2*(f*x^3 - e)*x/(4*f^2*x^6 + 4*d*f*x^4 + 4*e*f*x^3 + e^2),x, algorithm
```

```
[Out] -2*integrate((f*x^3 - e)*x/(4*f^2*x^6 + 4*d*f*x^4 + 4*e*f*x^3 + e
^2), x)
```

Fricas [A] time = 0.282742, size = 1, normalized size = 0.02

$$\left[\frac{\log\left(-\frac{8\,df^{2}x^{5}+4\,defx^{2}-\left(4\,f^{2}x^{6}-4\,dfx^{4}+4\,efx^{3}+e^{2}\right)\sqrt{-df}}{4\,f^{2}x^{6}+4\,dfx^{4}+4\,efx^{3}+e^{2}}\right)}{4\,\sqrt{-df}}, -\frac{\arctan\left(\frac{\sqrt{df}x}{d}\right)-\arctan\left(\frac{2\,f^{2}x^{4}+2\,dfx^{2}+efx}{\sqrt{df}e}\right)}{2\,\sqrt{df}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2*(f*x^3 - e)*x/(4*f^2*x^6 + 4*d*f*x^4 + 4*e*f*x^3 + e^2),x, algorithm
```

```
[Out] [1/4*log(-(8*d*f^2*x^5 + 4*d*e*f*x^2 - (4*f^2*x^6 - 4*d*f*x^4 + 4
*e*f*x^3 + e^2)*sqrt(-d*f))/(4*f^2*x^6 + 4*d*f*x^4 + 4*e*f*x^3 +
e^2))/sqrt(-d*f), -1/2*(arctan(sqrt(d*f)*x/d) - arctan((2*f^2*x^4
 + 2*d*f*x^2 + e*f*x)/(sqrt(d*f)*e)))/sqrt(d*f)]
```

Sympy [A] time = 4.45304, size = 73, normalized size = 1.82

$$\frac{\sqrt{-\frac{1}{df}}\log\left(-dx^{2}\sqrt{-\frac{1}{df}}+\frac{e}{2f}+x^{3}\right)}{4}-\frac{\sqrt{-\frac{1}{df}}\log\left(dx^{2}\sqrt{-\frac{1}{df}}+\frac{e}{2f}+x^{3}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*f*x**3+2*e)/(4*f**2*x**6+4*d*f*x**4+4*e*f*x**3+e**2),x)
```

```
[Out] sqrt(-1/(d*f))*log(-d*x**2*sqrt(-1/(d*f)) + e/(2*f) + x**3)/4 - s
qrt(-1/(d*f))*log(d*x**2*sqrt(-1/(d*f)) + e/(2*f) + x**3)/4
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{2\left(fx^{3}-e\right)x}{4\,f^{2}x^{6}+4\,dfx^{4}+4\,efx^{3}+e^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2*(f*x^3 - e)*x/(4*f^2*x^6 + 4*d*f*x^4 + 4*e*f*x^3 + e^2),x, algorithm
```

```
[Out] integrate(-2*(f*x^3 - e)*x/(4*f^2*x^6 + 4*d*f*x^4 + 4*e*f*x^3 + e
^2), x)
```

3.376 $\int \frac{x(2e-2fx^3)}{e^2+4efx^3-4dfx^4+4f^2x^6}\,dx$

Optimal. Leaf size=40

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^2}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x^2)/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f]
)
```

Rubi [A] time = 0.145437, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^2}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(2*e - 2*f*x^3))/(e^2 + 4*e*f*x^3 - 4*d*f*x^4 + 4*f^2*x^6),x]
```

```
[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x^2)/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f]
)
```

Rubi in Sympy [A] time = 50.0624, size = 41, normalized size = 1.02

$$-\frac{\operatorname{atanh}\left(\frac{4\sqrt{d}\sqrt{f}x^2}{-2e-4fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x*(-2*f*x**3+2*e)/(4*f**2*x**6-4*d*f*x**4+4*e*f*x**3+e**2),x)
```

```
[Out] -atanh(4*sqrt(d)*sqrt(f)*x**2/(-2*e - 4*f*x**3))/(2*sqrt(d)*sqrt(
f))
```

Mathematica [C] time = 0.0740578, size = 86, normalized size = 2.15

$$-\frac{\text{RootSum}\left[4\#1^6f^2-4\#1^4df+4\#1^3ef+e^2\&,\frac{\#1^3f\log(x-\#1)-e\log(x-\#1)}{6\#1^4f-4\#1^2d+3\#1e}\&\right]}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(2*e - 2*f*x^3))/(e^2 + 4*e*f*x^3 - 4*d*f*x^4 + 4*f^2*x^6),x]
```

```
[Out] -RootSum[e^2 + 4*e*f*#1^3 - 4*d*f*#1^4 + 4*f^2*#1^6 & , (-(e*Log[
x - #1]) + f*Log[x - #1]*#1^3)/(3*e*#1 - 4*d*#1^2 + 6*f*#1^4) & ]
/(2*f)
```

Maple [C] time = 0.013, size = 74, normalized size = 1.9

$$-\frac{1}{2f}\sum_{\_R=RootOf\left(4f^2\_Z^6-4df\_Z^4+4ef\_Z^3+e^2\right)}\frac{\left(\_R^4f-\_Re\right)\ln\left(x-\_R\right)}{6f\_R^5-4d\_R^3+3e\_R^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-2*f*x^3+2*e)/(4*f^2*x^6-4*d*f*x^4+4*e*f*x^3+e^2),x)
```

```
[Out] -1/2/f*sum((_R^4*f-_R*e)/(6*_R^5*f-4*_R^3*d+3*_R^2*e)*ln(x-_R),_R
=RootOf(4*_Z^6*f^2-4*_Z^4*d*f+4*_Z^3*e*f+e^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-2\int\frac{\left(fx^3-e\right)x}{4f^2x^6-4dfx^4+4efx^3+e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2*(f*x^3 - e)*x/(4*f^2*x^6 - 4*d*f*x^4 + 4*e*f*x^3 + e^2),x, algorithm
```

```
[Out] -2*integrate((f*x^3 - e)*x/(4*f^2*x^6 - 4*d*f*x^4 + 4*e*f*x^3 + e
^2), x)
```

Fricas [A] time = 0.302629, size = 1, normalized size = 0.02

$$\left[\frac{\log\left(\frac{8\,df^{2}x^{5}+4\,defx^{2}+\left(4\,f^{2}x^{6}+4\,dfx^{4}+4\,efx^{3}+e^{2}\right)\sqrt{df}}{4\,f^{2}x^{6}-4\,dfx^{4}+4\,efx^{3}+e^{2}}\right)}{4\,\sqrt{df}},\ \frac{\arctan\left(\frac{\sqrt{-df}x}{d}\right)+\arctan\left(\frac{2\,f^{2}x^{4}-2\,dfx^{2}+efx}{\sqrt{-df}e}\right)}{2\,\sqrt{-df}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2*(f*x^3 - e)*x/(4*f^2*x^6 - 4*d*f*x^4 + 4*e*f*x^3 + e^2),x, algorithm
```

```
[Out] [1/4*log((8*d*f^2*x^5 + 4*d*e*f*x^2 + (4*f^2*x^6 + 4*d*f*x^4 + 4*
e*f*x^3 + e^2)*sqrt(d*f))/(4*f^2*x^6 - 4*d*f*x^4 + 4*e*f*x^3 + e^
2))/sqrt(d*f), 1/2*(arctan(sqrt(-d*f)*x/d) + arctan((2*f^2*x^4 -
2*d*f*x^2 + e*f*x)/(sqrt(-d*f)*e)))/sqrt(-d*f)]
```

Sympy [A] time = 4.44465, size = 66, normalized size = 1.65

$$-\frac{\sqrt{\frac{1}{df}}\log\left(-dx^{2}\sqrt{\frac{1}{df}}+\frac{e}{2f}+x^{3}\right)}{4}+\frac{\sqrt{\frac{1}{df}}\log\left(dx^{2}\sqrt{\frac{1}{df}}+\frac{e}{2f}+x^{3}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*f*x**3+2*e)/(4*f**2*x**6-4*d*f*x**4+4*e*f*x**3+e**2),x)
```

```
[Out] -sqrt(1/(d*f))*log(-d*x**2*sqrt(1/(d*f)) + e/(2*f) + x**3)/4 + sq
rt(1/(d*f))*log(d*x**2*sqrt(1/(d*f)) + e/(2*f) + x**3)/4
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{2\,\left(fx^{3}-e\right)x}{4\,f^{2}x^{6}-4\,dfx^{4}+4\,efx^{3}+e^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2*(f*x^3 - e)*x/(4*f^2*x^6 - 4*d*f*x^4 + 4*e*f*x^3 + e^2),x, algorithm
```

```
[Out] integrate(-2*(f*x^3 - e)*x/(4*f^2*x^6 - 4*d*f*x^4 + 4*e*f*x^3 + e
^2), x)
```

3.377 $\int \frac{x^2}{e^2+4efx^3+4dfx^6+4f^2x^6}\,dx$

Optimal. Leaf size=42

$$\frac{\tan^{-1}\left(\frac{\sqrt{f}\left(2x^3(d+f)+e\right)}{\sqrt{d}e}\right)}{6\sqrt{d}e\sqrt{f}}$$

```
[Out] ArcTan[(Sqrt[f]*(e + 2*(d + f)*x^3))/(Sqrt[d]*e)]/(6*Sqrt[d]*e*Sq
rt[f])
```

Rubi [A] time = 0.124634, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{\tan^{-1}\left(\frac{\sqrt{f}\left(2x^3(d+f)+e\right)}{\sqrt{d}e}\right)}{6\sqrt{d}e\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(e^2 + 4*e*f*x^3 + 4*d*f*x^6 + 4*f^2*x^6),x]
```

```
[Out] ArcTan[(Sqrt[f]*(e + 2*(d + f)*x^3))/(Sqrt[d]*e)]/(6*Sqrt[d]*e*Sq
rt[f])
```

Rubi in Sympy [A] time = 47.9681, size = 37, normalized size = 0.88

$$\frac{\operatorname{atan}\left(\frac{\sqrt{f}\left(e+x^3(2d+2f)\right)}{\sqrt{d}e}\right)}{6\sqrt{d}e\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(4*d*f*x**6+4*f**2*x**6+4*e*f*x**3+e**2),x)
```

```
[Out] atan(sqrt(f)*(e + x**3*(2*d + 2*f))/(sqrt(d)*e))/(6*sqrt(d)*e*sqr
t(f))
```

Mathematica [A] time = 0.0315529, size = 42, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{f}(2x^3(d+f)+e)}{\sqrt{d}e}\right)}{6\sqrt{d}e\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(e^2 + 4*e*f*x^3 + 4*d*f*x^6 + 4*f^2*x^6),x]
```

```
[Out] ArcTan[(Sqrt[f]*(e + 2*(d + f)*x^3))/(Sqrt[d]*e)]/(6*Sqrt[d]*e*Sq
rt[f])
```

Maple [A] time = 0.003, size = 42, normalized size = 1.

$$\frac{1}{6e}\arctan\left(\frac{2\left(4df+4f^2\right)x^3+4ef}{4e}\frac{1}{\sqrt{df}}\right)\frac{1}{\sqrt{df}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(4*d*f*x^6+4*f^2*x^6+4*e*f*x^3+e^2),x)
```

```
[Out] 1/6/e/(d*f)^(1/2)*arctan(1/4*(2*(4*d*f+4*f^2)*x^3+4*e*f)/e/(d*f)^
(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(4*d*f*x^6 + 4*f^2*x^6 + 4*e*f*x^3 + e^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.312027, size = 1, normalized size = 0.02

$$\left[\frac{\log\left(\frac{2\,de^2f+4\,(d^2ef+def^2)\,x^3+(4\,(d^2f+2\,df^2+f^3)\,x^6+4\,(def+ef^2)\,x^3-de^2+e^2f)\,\sqrt{-df}}{4\,(df+f^2)x^6+4\,efx^3+e^2}\right)}{12\,\sqrt{-df}e},\ \frac{\arctan\left(\frac{(2\,(d+f)x^3+e)\,\sqrt{df}}{de}\right)}{6\,\sqrt{df}e}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(4*d*f*x^6 + 4*f^2*x^6 + 4*e*f*x^3 + e^2),x, algorithm="fricas")
```

```
[Out] [1/12*log((2*d*e^2*f + 4*(d^2*e*f + d*e*f^2)*x^3 + (4*(d^2*f + 2*
d*f^2 + f^3)*x^6 + 4*(d*e*f + e*f^2)*x^3 - d*e^2 + e^2*f)*sqrt(-d
*f))/(4*(d*f + f^2)*x^6 + 4*e*f*x^3 + e^2))/(sqrt(-d*f)*e), 1/6*a
rctan((2*(d + f)*x^3 + e)*sqrt(d*f)/(d*e))/(sqrt(d*f)*e)]
```

Sympy [A] time = 2.46519, size = 78, normalized size = 1.86

$$\frac{-\frac{\sqrt{-\frac{1}{df}}\log\left(x^3+\frac{-de\sqrt{-\frac{1}{df}}+e}{2d+2f}\right)}{12}+\frac{\sqrt{-\frac{1}{df}}\log\left(x^3+\frac{de\sqrt{-\frac{1}{df}}+e}{2d+2f}\right)}{12}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(4*d*f*x**6+4*f**2*x**6+4*e*f*x**3+e**2),x)
```

```
[Out] (-sqrt(-1/(d*f))*log(x**3 + (-d*e*sqrt(-1/(d*f)) + e)/(2*d + 2*f)
)/12 + sqrt(-1/(d*f))*log(x**3 + (d*e*sqrt(-1/(d*f)) + e)/(2*d +
2*f))/12)/e
```

GIAC/XCAS [A] time = 0.27136, size = 51, normalized size = 1.21

$$\frac{\arctan\left(\frac{(2\,dfx^3+2\,f^2x^3+fe)\,e^{(-1)}}{\sqrt{df}}\right)e^{(-1)}}{6\,\sqrt{df}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(4*d*f*x^6 + 4*f^2*x^6 + 4*e*f*x^3 + e^2),x, algorithm="giac")
```

```
[Out] 1/6*arctan((2*d*f*x^3 + 2*f^2*x^3 + f*e)*e^(-1)/sqrt(d*f))*e^(-1)
/sqrt(d*f)
```

3.378 $\int \frac{x^2}{e^2+4efx^3-4dfx^6+4f^2x^6}\,dx$

Optimal. Leaf size=44

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{f}(e-2x^3(d-f))}{\sqrt{d}e}\right)}{6\sqrt{d}e\sqrt{f}}$$

```
[Out] -ArcTanh[(Sqrt[f]*(e - 2*(d - f)*x^3))/(Sqrt[d]*e)]/(6*Sqrt[d]*e*
Sqrt[f])
```

Rubi [A] time = 0.135682, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{f}(e-2x^3(d-f))}{\sqrt{d}e}\right)}{6\sqrt{d}e\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(e^2 + 4*e*f*x^3 - 4*d*f*x^6 + 4*f^2*x^6),x]
```

```
[Out] -ArcTanh[(Sqrt[f]*(e - 2*(d - f)*x^3))/(Sqrt[d]*e)]/(6*Sqrt[d]*e*
Sqrt[f])
```

Rubi in Sympy [A] time = 49.0763, size = 39, normalized size = 0.89

$$-\frac{\operatorname{atanh}\left(\frac{\sqrt{f}(e+x^3(-2d+2f))}{\sqrt{d}e}\right)}{6\sqrt{d}e\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(-4*d*f*x**6+4*f**2*x**6+4*e*f*x**3+e**2),x)
```

```
[Out] -atanh(sqrt(f)*(e + x**3*(-2*d + 2*f))/(sqrt(d)*e))/(6*sqrt(d)*e*
sqrt(f))
```

Mathematica [A] time = 0.035074, size = 46, normalized size = 1.05

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{f}(-2dx^3+e+2fx^3)}{\sqrt{d}e}\right)}{6\sqrt{d}e\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(e^2 + 4*e*f*x^3 - 4*d*f*x^6 + 4*f^2*x^6),x]
```

```
[Out] -ArcTanh[(Sqrt[f]*(e - 2*d*x^3 + 2*f*x^3))/(Sqrt[d]*e)]/(6*Sqrt[d
]*e*Sqrt[f])
```

Maple [A] time = 0.002, size = 42, normalized size = 1.

$$\frac{1}{6e}Artanh\left(\frac{2\left(4df-4f^2\right)x^3-4ef}{4e}\frac{1}{\sqrt{df}}\right)\frac{1}{\sqrt{df}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(-4*d*f*x^6+4*f^2*x^6+4*e*f*x^3+e^2),x)
```

```
[Out] 1/6/e/(d*f)^(1/2)*arctanh(1/4*(2*(4*d*f-4*f^2)*x^3-4*e*f)/e/(d*f)
^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2/(4*d*f*x^6 - 4*f^2*x^6 - 4*e*f*x^3 - e^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.318452, size = 1, normalized size = 0.02

$$\left[\frac{\log\left(\frac{2\,de^{2}f-4\,\left(d^{2}ef-def^{2}\right)x^{3}-\left(4\,\left(d^{2}f-2\,df^{2}+f^{3}\right)x^{6}-4\,\left(def-ef^{2}\right)x^{3}+de^{2}+e^{2}f\right)\sqrt{df}}{4\,(df-f^{2})x^{6}-4\,efx^{3}-e^{2}}\right)}{12\,\sqrt{df}e}, -\frac{\arctan\left(-\frac{\left(2\,(d-f)x^{3}-e\right)\sqrt{-df}}{de}\right)}{6\,\sqrt{-df}e}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2/(4*d*f*x^6 - 4*f^2*x^6 - 4*e*f*x^3 - e^2),x, algorithm="fricas")
```

```
[Out] [1/12*log((2*d*e^2*f - 4*(d^2*e*f - d*e*f^2)*x^3 - (4*(d^2*f - 2*
d*f^2 + f^3)*x^6 - 4*(d*e*f - e*f^2)*x^3 + d*e^2 + e^2*f)*sqrt(d*
f))/(4*(d*f - f^2)*x^6 - 4*e*f*x^3 - e^2))/(sqrt(d*f)*e), -1/6*ar
ctan(-(2*(d - f)*x^3 - e)*sqrt(-d*f)/(d*e))/(sqrt(-d*f)*e)]
```

Sympy [A] time = 2.64861, size = 75, normalized size = 1.7

$$-\frac{\frac{\sqrt{\frac{1}{df}}\log\left(x^{3}+\frac{-de\sqrt{\frac{1}{df}}-e}{2d-2f}\right)}{12}-\frac{\sqrt{\frac{1}{df}}\log\left(x^{3}+\frac{de\sqrt{\frac{1}{df}}-e}{2d-2f}\right)}{12}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-4*d*f*x**6+4*f**2*x**6+4*e*f*x**3+e**2),x)
```

```
[Out] -(sqrt(1/(d*f))*log(x**3 + (-d*e*sqrt(1/(d*f)) - e)/(2*d - 2*f))/
12 - sqrt(1/(d*f))*log(x**3 + (d*e*sqrt(1/(d*f)) - e)/(2*d - 2*f)
)/12)/e
```

GIAC/XCAS [A] time = 0.271215, size = 55, normalized size = 1.25

$$-\frac{\arctan\left(\frac{2\,dfx^{3}-2\,f^{2}x^{3}-fe}{\sqrt{-dfe^{2}}}\right)}{6\,\sqrt{-dfe^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2/(4*d*f*x^6 - 4*f^2*x^6 - 4*e*f*x^3 - e^2),x, algorithm="giac")
```

```
[Out] -1/6*arctan((2*d*f*x^3 - 2*f^2*x^3 - f*e)/sqrt(-d*f*e^2))/sqrt(-d
*f*e^2)
```

3.379 $\int \frac{x^m\left(e(1+m)+2f(-2+m)x^3\right)}{e^2+4efx^3+4f^2x^6+4dfx^{2+2m}}\,dx$

Optimal. Leaf size=42

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{m+1}}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^3)]/(2*Sqrt[d]*Sq
rt[f])
```

Rubi [A] time = 0.348507, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.039

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{m+1}}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(e*(1 + m) + 2*f*(-2 + m)*x^3))/(e^2 + 4*e*f*x^3 + 4*f^2*x^6 + 4*d*f*x^(2
```

```
[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^3)]/(2*Sqrt[d]*Sq
rt[f])
```

Rubi in Sympy [A] time = 81.4092, size = 37, normalized size = 0.88

$$\frac{\operatorname{atan}\left(\frac{2\sqrt{d}\sqrt{f}x^{m+1}}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**m*(e*(1+m)+2*f*(-2+m)*x**3)/(e**2+4*e*f*x**3+4*f**2*x**6+4*d
```

```
[Out] atan(2*sqrt(d)*sqrt(f)*x**(m + 1)/(e + 2*f*x**3))/(2*sqrt(d)*sqrt
(f))
```

Mathematica [A] time = 0.0875569, size = 42, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{m+1}}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(e*(1 + m) + 2*f*(-2 + m)*x^3))/(e^2 + 4*e*f*x^3 + 4*f^2*x^6 + 4*d*
```

```
[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^3)]/(2*Sqrt[d]*Sq
rt[f])
```

Maple [B] time = 0.055, size = 78, normalized size = 1.9

$$-\frac{1}{4}\ln\left(x^m+\frac{2fx^3+e}{2dfx}\sqrt{-df}\right)\frac{1}{\sqrt{-df}}+\frac{1}{4}\ln\left(x^m-\frac{2fx^3+e}{2dfx}\sqrt{-df}\right)\frac{1}{\sqrt{-df}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(e*(1+m)+2*f*(-2+m)*x^3)/(e^2+4*e*f*x^3+4*f^2*x^6+4*d*f*x^(2+2*m)),x)
```

```
[Out] -1/4/(-d*f)^(1/2)*ln(x^m+1/2*(2*f*x^3+e)*(-d*f)^(1/2)/d/f/x)+1/4/
(-d*f)^(1/2)*ln(x^m-1/2*(2*f*x^3+e)*(-d*f)^(1/2)/d/f/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(2f(m-2)x^3+e(m+1)\right)x^m}{4f^2x^6+4efx^3+4dfx^{2m+2}+e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*f*(m - 2)*x^3 + e*(m + 1))*x^m/(4*f^2*x^6 + 4*e*f*x^3 + 4*d*f*x^(2*m
```

```
[Out] integrate((2*f*(m - 2)*x^3 + e*(m + 1))*x^m/(4*f^2*x^6 + 4*e*f*x^
3 + 4*d*f*x^(2*m + 2) + e^2), x)
```

Fricas [A] time = 0.320855, size = 1, normalized size = 0.02

$$\left[\frac{\log\left(\frac{4\sqrt{-df}dfx^2x^{2m}+4\left(2df^2x^4+defx\right)x^m-\left(4f^2x^6+4efx^3+e^2\right)\sqrt{-df}}{4f^2x^6+4dfx^2x^{2m}+4efx^3+e^2}\right)}{4\sqrt{-df}}, -\frac{\arctan\left(\frac{2fx^3+e}{2\sqrt{df}xx^m}\right)}{2\sqrt{df}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*f*(m - 2)*x^3 + e*(m + 1))*x^m/(4*f^2*x^6 + 4*e*f*x^3 + 4*d*f*x^(2*m
```

```
[Out] [1/4*log((4*sqrt(-d*f)*d*f*x^2*x^(2*m) + 4*(2*d*f^2*x^4 + d*e*f*x
)*x^m - (4*f^2*x^6 + 4*e*f*x^3 + e^2)*sqrt(-d*f))/(4*f^2*x^6 + 4*
d*f*x^2*x^(2*m) + 4*e*f*x^3 + e^2))/sqrt(-d*f), -1/2*arctan(1/2*(
2*f*x^3 + e)/(sqrt(d*f)*x*x^m))/sqrt(d*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(e*(1+m)+2*f*(-2+m)*x**3)/(e**2+4*e*f*x**3+4*f**2*x**6+4*d*f*x*
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(2f(m-2)x^3+e(m+1)\right)x^m}{4f^2x^6+4efx^3+4dfx^{2m+2}+e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*f*(m - 2)*x^3 + e*(m + 1))*x^m/(4*f^2*x^6 + 4*e*f*x^3 + 4*d*f*x^(2*m
```

```
[Out] integrate((2*f*(m - 2)*x^3 + e*(m + 1))*x^m/(4*f^2*x^6 + 4*e*f*x^
3 + 4*d*f*x^(2*m + 2) + e^2), x)
```

3.380 $\int \frac{x^m\left(e(1+m)+2f(-2+m)x^3\right)}{e^2+4efx^3+4f^2x^6-4dfx^{2+2m}}\,dx$

Optimal. Leaf size=42

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{m+1}}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^3)]/(2*Sqrt[d]*S
qrt[f])
```

Rubi [A] time = 0.340291, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.039

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{m+1}}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(e*(1 + m) + 2*f*(-2 + m)*x^3))/(e^2 + 4*e*f*x^3 + 4*f^2*x^6 - 4*d*f*x^(2
```

```
[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^3)]/(2*Sqrt[d]*S
qrt[f])
```

Rubi in Sympy [A] time = 57.8138, size = 37, normalized size = 0.88

$$\frac{\operatorname{atanh}\left(\frac{2\sqrt{d}\sqrt{f}x^{m+1}}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**m*(e*(1+m)+2*f*(-2+m)*x**3)/(e**2+4*e*f*x**3+4*f**2*x**6-
4*d*f*x**(2+2*m)),x)
```

```
[Out] atanh(2*sqrt(d)*sqrt(f)*x**(m + 1)/(e + 2*f*x**3))/(2*sqrt(d)*sqr
t(f))
```

Mathematica [A] time = 0.0709946, size = 42, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{m+1}}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(e*(1 + m) + 2*f*(-2 + m)*x^3))/(e^2 + 4*e*f*x^3 + 4*f^2*x^6 - 4*d*
```

```
[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^3)]/(2*Sqrt[d]*S
qrt[f])
```

Maple [B] time = 0.055, size = 74, normalized size = 1.8

$$\frac{1}{4}\ln\left(x^m+\frac{2fx^3+e}{2dfx}\sqrt{df}\right)\frac{1}{\sqrt{df}}-\frac{1}{4}\ln\left(x^m-\frac{2fx^3+e}{2dfx}\sqrt{df}\right)\frac{1}{\sqrt{df}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(e*(1+m)+2*f*(-2+m)*x^3)/(e^2+4*e*f*x^3+4*f^2*x^6-4*d*f*x^(2+2*m)),x)
```

```
[Out] 1/4/(d*f)^(1/2)*ln(x^m+1/2*(2*f*x^3+e)*(d*f)^(1/2)/d/f/x)-1/4/(d*
f)^(1/2)*ln(x^m-1/2*(2*f*x^3+e)*(d*f)^(1/2)/d/f/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(2\,f(m-2)x^3+e(m+1)\right)x^m}{4\,f^2x^6+4\,efx^3-4\,dfx^{2\,m+2}+e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*f*(m - 2)*x^3 + e*(m + 1))*x^m/(4*f^2*x^6 + 4*e*f*x^3 - 4*d*f*x^(2*m
```

```
[Out] integrate((2*f*(m - 2)*x^3 + e*(m + 1))*x^m/(4*f^2*x^6 + 4*e*f*x^
3 - 4*d*f*x^(2*m + 2) + e^2), x)
```

Fricas [A] time = 0.330728, size = 1, normalized size = 0.02

$$\left[\frac{\log\left(-\frac{4\sqrt{df}dfx^2x^{2m}+4\left(2df^2x^4+defx\right)x^m+\left(4f^2x^6+4efx^3+e^2\right)\sqrt{df}}{4f^2x^6-4dfx^2x^{2m}+4efx^3+e^2}\right)}{4\sqrt{df}}, -\frac{\arctan\left(\frac{2fx^3+e}{2\sqrt{-df}xx^m}\right)}{2\sqrt{-df}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*f*(m - 2)*x^3 + e*(m + 1))*x^m/(4*f^2*x^6 + 4*e*f*x^3 - 4*d*f*x^(2*m
```

```
[Out] [1/4*log(-(4*sqrt(d*f)*d*f*x^2*x^(2*m) + 4*(2*d*f^2*x^4 + d*e*f*x
)*x^m + (4*f^2*x^6 + 4*e*f*x^3 + e^2)*sqrt(d*f))/(4*f^2*x^6 - 4*d
*f*x^2*x^(2*m) + 4*e*f*x^3 + e^2))/sqrt(d*f), -1/2*arctan(1/2*(2*
f*x^3 + e)/(sqrt(-d*f)*x*x^m))/sqrt(-d*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(e*(1+m)+2*f*(-2+m)*x**3)/(e**2+4*e*f*x**3+4*f**2*x**6-
4*d*f*x**(2+2*m)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(2f(m-2)x^3+e(m+1)\right)x^m}{4f^2x^6+4efx^3-4dfx^{2m+2}+e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*f*(m - 2)*x^3 + e*(m + 1))*x^m/(4*f^2*x^6 + 4*e*f*x^3 - 4*d*f*x^(2*m
```

```
[Out] integrate((2*f*(m - 2)*x^3 + e*(m + 1))*x^m/(4*f^2*x^6 + 4*e*f*x^
3 - 4*d*f*x^(2*m + 2) + e^2), x)
```

3.381 $\int \frac{x^m(e(1+m)+2f(1+m-n)x^n)}{e^2+4dfx^{2+2m}+4efx^n+4f^2x^{2n}}\,dx$

Optimal. Leaf size=42

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{m+1}}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^n)]/(2*Sqrt[d]*Sq
rt[f])
```

Rubi [A] time = 0.382072, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 56, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{m+1}}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(e*(1 + m) + 2*f*(1 + m - n)*x^n))/(e^2 + 4*d*f*x^(2 + 2*m) + 4*e*f*x^n +
```

```
[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^n)]/(2*Sqrt[d]*Sq
rt[f])
```

Rubi in Sympy [A] time = 74.6395, size = 63, normalized size = 1.5

$$\frac{\operatorname{atan}\left(\frac{2\sqrt{d}\sqrt{f}x^{m+1}(m+1)(m-n+1)}{e(m+1)(m-n+1)+2fx^n(m+1)(m-n+1)}\right)}{2\sqrt{d}\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**m*(e*(1+m)+2*f*(1+m-n)*x**n)/(e**2+4*d*f*x**(2+2*m)+4*e*f*x*
```

```
[Out] atan(2*sqrt(d)*sqrt(f)*x**(m + 1)*(m + 1)*(m - n + 1)/(e*(m + 1)*
(m - n + 1) + 2*f*x**n*(m + 1)*(m - n + 1)))/(2*sqrt(d)*sqrt(f))
```

Mathematica [A] time = 0.267333, size = 0, normalized size = 0.

$$\int \frac{x^m\left(e(1+m)+2f(1+m-n)x^n\right)}{e^2+4dfx^{2+2m}+4efx^n+4f^2x^{2n}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*(e*(1 + m) + 2*f*(1 + m - n)*x^n))/(e^2 + 4*d*f*x^(2 + 2*m) + 4*e*f*
```

```
[Out] Integrate[(x^m*(e*(1 + m) + 2*f*(1 + m - n)*x^n))/(e^2 + 4*d*f*x^
(2 + 2*m) + 4*e*f*x^n + 4*f^2*x^(2*n)), x]
```

Maple [B] time = 0.111, size = 84, normalized size = 2.

$$-\frac{1}{4}\ln\left(x^n+\frac{1}{2f}\left(2\,dfxx^m+e\sqrt{-df}\right)\frac{1}{\sqrt{-df}}\right)\frac{1}{\sqrt{-df}}+\frac{1}{4}\ln\left(x^n+\frac{1}{2f}\left(-2\,dfxx^m+e\sqrt{-df}\right)\frac{1}{\sqrt{-df}}\right)\frac{1}{\sqrt{-df}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(e*(1+m)+2*f*(1+m-n)*x^n)/(e^2+4*d*f*x^(2+2*m)+4*e*f*x^n+4*f^2*x^(2*n)
```

```
[Out] -1/4/(-d*f)^(1/2)*ln(x^n+1/2*(2*d*f*x*x^m+e*(-d*f)^(1/2))/(-d*f)^
(1/2)/f)+1/4/(-d*f)^(1/2)*ln(x^n+1/2*(-2*d*f*x*x^m+e*(-d*f)^(1/2)
)/(-d*f)^(1/2)/f)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(2\,f(m-n+1)x^n+e(m+1)\right)x^m}{4\,dfx^{2\,m+2}+4\,f^2x^{2\,n}+4\,efx^n+e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*f*(m - n + 1)*x^n + e*(m + 1))*x^m/(4*d*f*x^(2*m + 2) + 4*f^2*x^(2*n)
```

```
[Out] integrate((2*f*(m - n + 1)*x^n + e*(m + 1))*x^m/(4*d*f*x^(2*m + 2
) + 4*f^2*x^(2*n) + 4*e*f*x^n + e^2), x)
```

Fricas [A] time = 0.343854, size = 1, normalized size = 0.02

$$\left[\frac{\log\left(-\frac{4\sqrt{-df}\,dfx^2x^{2m}+4\,defxx^m-4\sqrt{-df}\,f^2x^{2n}-\sqrt{-df}\,e^2+4\left(2\,df^2xx^m-\sqrt{-df}\,ef\right)x^n}{4\,dfx^2x^{2m}+4\,f^2x^{2n}+4\,efx^n+e^2}\right)}{4\sqrt{-df}},\,-\frac{\arctan\left(\frac{2\sqrt{df}\,fx^n+\sqrt{df}\,e}{2\,dfxx^m}\right)}{2\sqrt{df}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*f*(m - n + 1)*x^n + e*(m + 1))*x^m/(4*d*f*x^(2*m + 2) + 4*f^2*x^(2*n)
```

```
[Out] [1/4*log(-(4*sqrt(-d*f)*d*f*x^2*x^(2*m) + 4*d*e*f*x*x^m - 4*sqrt(
-d*f)*f^2*x^(2*n) - sqrt(-d*f)*e^2 + 4*(2*d*f^2*x*x^m - sqrt(-d*f
)*e*f)*x^n)/(4*d*f*x^2*x^(2*m) + 4*f^2*x^(2*n) + 4*e*f*x^n + e^2)
)/sqrt(-d*f), -1/2*arctan(1/2*(2*sqrt(d*f)*f*x^n + sqrt(d*f)*e)/(
d*f*x*x^m))/sqrt(d*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(e*(1+m)+2*f*(1+m-n)*x**n)/(e**2+4*d*f*x**(2+2*m)+4*e*f*x**n+4*
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2\,f(m-n+1)x^n + e(m+1))x^m}{4\,dfx^{2\,m+2} + 4\,f^2x^{2\,n} + 4\,efx^n + e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*f*(m - n + 1)*x^n + e*(m + 1))*x^m/(4*d*f*x^(2*m + 2) + 4*f^2*x^(2*n)
```

```
[Out] integrate((2*f*(m - n + 1)*x^n + e*(m + 1))*x^m/(4*d*f*x^(2*m + 2
) + 4*f^2*x^(2*n) + 4*e*f*x^n + e^2), x)
```

3.382 $\int \frac{x^m(e(1+m)+2f(1+m-n)x^n)}{e^2-4dfx^{2+2m}+4efx^n+4f^2x^{2n}}\,dx$

Optimal. Leaf size=42

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{m+1}}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^n)]/(2*Sqrt[d]*S
qrt[f])
```

Rubi [A] time = 0.373787, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 56, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{m+1}}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(e*(1 + m) + 2*f*(1 + m - n)*x^n))/(e^2 - 4*d*f*x^(2 + 2*m) + 4*e*f*x^n + 
```

```
[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^n)]/(2*Sqrt[d]*S
qrt[f])
```

Rubi in Sympy [A] time = 84.3159, size = 63, normalized size = 1.5

$$\frac{\operatorname{atanh}\left(\frac{2\sqrt{d}\sqrt{f}x^{m+1}(m+1)(m-n+1)}{e(m+1)(m-n+1)+2fx^n(m+1)(m-n+1)}\right)}{2\sqrt{d}\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**m*(e*(1+m)+2*f*(1+m-n)*x**n)/(e**2-4*d*f*x**(2+2*m)+4*e*f*x*
```

```
[Out] atanh(2*sqrt(d)*sqrt(f)*x**(m + 1)*(m + 1)*(m - n + 1)/(e*(m + 1)
*(m - n + 1) + 2*f*x**n*(m + 1)*(m - n + 1)))/(2*sqrt(d)*sqrt(f))
```

Mathematica [A] time = 0.280598, size = 0, normalized size = 0.

$$\int \frac{x^m\left(e(1+m)+2f(1+m-n)x^n\right)}{e^2-4dfx^{2+2m}+4efx^n+4f^2x^{2n}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*(e*(1 + m) + 2*f*(1 + m - n)*x^n))/(e^2 - 4*d*f*x^(2 + 2*m) + 4*e*f*
```

```
[Out] Integrate[(x^m*(e*(1 + m) + 2*f*(1 + m - n)*x^n))/(e^2 - 4*d*f*x^
(2 + 2*m) + 4*e*f*x^n + 4*f^2*x^(2*n)), x]
```

Maple [B] time = 0.108, size = 78, normalized size = 1.9

$$\frac{1}{4}\ln\left(x^n+\frac{1}{2f}\left(2dfxx^m+e\sqrt{df}\right)\frac{1}{\sqrt{df}}\right)\frac{1}{\sqrt{df}}-\frac{1}{4}\ln\left(x^n+\frac{1}{2f}\left(-2dfxx^m+e\sqrt{df}\right)\frac{1}{\sqrt{df}}\right)\frac{1}{\sqrt{df}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(e*(1+m)+2*f*(1+m-n)*x^n)/(e^2-4*d*f*x^(2+2*m)+4*e*f*x^n+4*f^2*x^(2*n)
```

```
[Out] 1/4/(d*f)^(1/2)*ln(x^n+1/2*(2*d*f*x*x^m+e*(d*f)^(1/2))/(d*f)^(1/2
)/f)-1/4/(d*f)^(1/2)*ln(x^n+1/2*(-2*d*f*x*x^m+e*(d*f)^(1/2))/(d*f
)^(1/2)/f)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(2f(m-n+1)x^n+e(m+1))x^m}{4dfx^{2m+2}-4f^2x^{2n}-4efx^n-e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(2*f*(m - n + 1)*x^n + e*(m + 1))*x^m/(4*d*f*x^(2*m + 2) - 4*f^2*x^(2*n
```

```
[Out] -integrate((2*f*(m - n + 1)*x^n + e*(m + 1))*x^m/(4*d*f*x^(2*m +
2) - 4*f^2*x^(2*n) - 4*e*f*x^n - e^2), x)
```

Fricas [A] time = 0.335337, size = 1, normalized size = 0.02

$$\left[\frac{\log\left(-\frac{4\sqrt{df}dfx^2x^{2m}+4defxx^m+4\sqrt{df}f^2x^{2n}+\sqrt{df}e^2+4\left(2df^2xx^m+\sqrt{df}ef\right)x^n}{4dfx^2x^{2m}-4f^2x^{2n}-4efx^n-e^2}\right)}{4\sqrt{df}}, \frac{\arctan\left(\frac{2\sqrt{-df}fx^n+\sqrt{-df}e}{2dfxx^m}\right)}{2\sqrt{-df}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(2*f*(m - n + 1)*x^n + e*(m + 1))*x^m/(4*d*f*x^(2*m + 2) - 4*f^2*x^(2*n
```

```
[Out] [1/4*log(-(4*sqrt(d*f)*d*f*x^2*x^(2*m) + 4*d*e*f*x*x^m + 4*sqrt(d
*f)*f^2*x^(2*n) + sqrt(d*f)*e^2 + 4*(2*d*f^2*x*x^m + sqrt(d*f)*e*
f)*x^n)/(4*d*f*x^2*x^(2*m) - 4*f^2*x^(2*n) - 4*e*f*x^n - e^2))/sq
rt(d*f), 1/2*arctan(1/2*(2*sqrt(-d*f)*f*x^n + sqrt(-d*f)*e)/(d*f*
x*x^m))/sqrt(-d*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(e*(1+m)+2*f*(1+m-n)*x**n)/(e**2-4*d*f*x**(2+2*m)+4*e*f*x**n+4*
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(2\,f(m-n+1)x^n + e(m+1))x^m}{4\,dfx^{2\,m+2} - 4\,f^2x^{2\,n} - 4\,efx^n - e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(2*f*(m - n + 1)*x^n + e*(m + 1))*x^m/(4*d*f*x^(2*m + 2) - 4*f^2*x^(2*n
```

```
[Out] integrate(-(2*f*(m - n + 1)*x^n + e*(m + 1))*x^m/(4*d*f*x^(2*m +
2) - 4*f^2*x^(2*n) - 4*e*f*x^n - e^2), x)
```

3.383 $\int \frac{x^5}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=134

$$-\frac{d\left(a+bx^2\right)^{3/2}}{3b^3c^2}+\frac{\left(ac^2-d^2\right)^2\log\left(c\sqrt{a+bx^2}+d\right)}{b^3c^5}+\frac{d\sqrt{a+bx^2}\left(2ac^2-d^2\right)}{b^3c^4}+\frac{\left(a+bx^2\right)^2}{4b^3c}-\frac{x^2\left(2ac^2-d^2\right)}{2b^2c^3}$$

```
[Out] -((2*a*c^2 - d^2)*x^2)/(2*b^2*c^3) + (d*(2*a*c^2 - d^2)*Sqrt[a +
b*x^2])/(b^3*c^4) - (d*(a + b*x^2)^(3/2))/(3*b^3*c^2) + (a + b*x^
2)^2/(4*b^3*c) + ((a*c^2 - d^2)^2*Log[d + c*Sqrt[a + b*x^2]])/(b^
3*c^5)
```

Rubi [A] time = 0.619294, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069

$$-\frac{d\left(a+bx^2\right)^{3/2}}{3b^3c^2}+\frac{\left(ac^2-d^2\right)^2\log\left(c\sqrt{a+bx^2}+d\right)}{b^3c^5}+\frac{d\sqrt{a+bx^2}\left(2ac^2-d^2\right)}{b^3c^4}+\frac{\left(a+bx^2\right)^2}{4b^3c}-\frac{x^2\left(2ac^2-d^2\right)}{2b^2c^3}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]),x]
```

```
[Out] -((2*a*c^2 - d^2)*x^2)/(2*b^2*c^3) + (d*(2*a*c^2 - d^2)*Sqrt[a +
b*x^2])/(b^3*c^4) - (d*(a + b*x^2)^(3/2))/(3*b^3*c^2) + (a + b*x^
2)^2/(4*b^3*c) + ((a*c^2 - d^2)^2*Log[d + c*Sqrt[a + b*x^2]])/(b^
3*c^5)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(a+bx^2\right)^2}{4b^3c}-\frac{d\left(a+bx^2\right)^{\frac{3}{2}}}{3b^3c^2}+\frac{\left(-2ac^2+d^2\right)\int^{\sqrt{a+bx^2}}x\,dx}{b^3c^3}-\frac{\left(-2ac^2+d^2\right)\int^{\sqrt{a+bx^2}}d\,dx}{b^3c^4}+\frac{\left(-ac^2+d^2\right)^2\log\left(c\sqrt{a+bx^2}+d\right)}{b^3c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)
```

```
[Out] (a + b*x**2)**2/(4*b**3*c) - d*(a + b*x**2)**(3/2)/(3*b**3*c**2)
+ (-2*a*c**2 + d**2)*Integral(x, (x, sqrt(a + b*x**2)))/(b**3*c**
```

```
3) - (-2*a*c**2 + d**2)*Integral(d, (x, sqrt(a + b*x**2)))/(b**3*
c**4) + (-a*c**2 + d**2)**2*log(c*sqrt(a + b*x**2) + d)/(b**3*c**
5)
```

Mathematica [A] time = 0.335107, size = 161, normalized size = 1.2

$$\frac{c\left(a\left(20c^2d\sqrt{a+bx^2}-6bc^3x^2\right)+2bcdx^2\left(3d-2c\sqrt{a+bx^2}\right)-12d^3\sqrt{a+bx^2}+3b^2c^3x^4\right)+6\left(d^2-ac^2\right)^2\log\left(ac^2+bc^2x^2-\right.}{12b^3c^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]),x]
```

```
[Out] (c*(3*b^2*c^3*x^4 - 12*d^3*Sqrt[a + b*x^2] + 2*b*c*d*x^2*(3*d - 2
*c*Sqrt[a + b*x^2]) + a*(-6*b*c^3*x^2 + 20*c^2*d*Sqrt[a + b*x^2])
) + 12*(-(a*c^2) + d^2)^2*ArcTanh[(c*Sqrt[a + b*x^2])/d] + 6*(-(a
*c^2) + d^2)^2*Log[a*c^2 - d^2 + b*c^2*x^2])/(12*b^3*c^5)
```

Maple [B] time = 0.086, size = 4947, normalized size = 36.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x)
```

```
[Out] 1/b^2/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)
*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))/c^2*d^5/(d^2/c^2)^(1/2)*ln((2*d^
2/c^2+2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x-(-c^2*b*(a*c^2-d^2))^(1
/2)/c^2/b)+2*(d^2/c^2)^(1/2)*((x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b
)^2*b+2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x-(-c^2*b*(a*c^2-d^2))^(1
/2)/c^2/b)+d^2/c^2)^(1/2))/(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b))*
a+1/4/b/c*x^4+1/2*a^2/c/b^3*ln(b*c^2*x^2+a*c^2-d^2)+1/2/b^2/c^3*x
^2*d^2+1/2/b^3/c^5*d^4*ln(b*c^2*x^2+a*c^2-d^2)+1/b^(5/2)/((-a*b)^
(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(
a*c^2-d^2))^(1/2))/c^2*(-c^2*b*(a*c^2-d^2))^(1/2)*ln((-(-c^2*b*(a
*c^2-d^2))^(1/2)/c^2+(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)*b)/b^(1
/2)+((x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b-2*(-c^2*b*(a*c^2-d^
2))^(1/2)/c^2*(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+d^2/c^2)^(1/2)
)*a*d^3+1/b^2/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*
b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))/c^2*d^5/(d^2/c^2)^(1/2)*
ln((2*d^2/c^2-2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x+(-c^2*b*(a*c^2-
d^2))^(1/2)/c^2/b)+2*(d^2/c^2)^(1/2)*((x+(-c^2*b*(a*c^2-d^2))^(1/
2)/c^2/b)^2*b-2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x+(-c^2*b*(a*c^2-
```

d^2))^(1/2)/c^2/b)+d^2/c^2)^(1/2))/(x+(-c^2*b*(a*c^2-d^2))^(1/2)/
c^2/b))*a-1/2*d/b^(5/2)*c^2*a^2/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-
d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))*(-a*b)
^(1/2)*ln(((x-1/b*(-a*b)^(1/2))*b+(-a*b)^(1/2))/b^(1/2)+((x-1/b*(
-a*b)^(1/2))^2*b+2*(-a*b)^(1/2)*(x-1/b*(-a*b)^(1/2)))^(1/2))+1/2*
d/b^(5/2)*c^2*a^2/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/(
(-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))*(-a*b)^(1/2)*ln(((x+
1/b*(-a*b)^(1/2))*b-(-a*b)^(1/2))/b^(1/2)+((x+1/b*(-a*b)^(1/2))^2
b-2(-a*b)^(1/2)*(x+1/b*(-a*b)^(1/2)))^(1/2))-1/b^(5/2)/((-a*b)^
(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(
a*c^2-d^2))^(1/2))/c^2*(-c^2*b*(a*c^2-d^2))^(1/2)*ln(((-c^2*b*(a*
c^2-d^2))^(1/2)/c^2+(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)*b)/b^(1/
2)+((x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b+2*(-c^2*b*(a*c^2-d^2
))^(1/2)/c^2*(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+d^2/c^2)^(1/2))
*a*d^3+1/2/b^2/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a
*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))/c^2*((x+(-c^2*b*(a*c^2-
d^2))^(1/2)/c^2/b)^2*b-2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x+(-c^2*
b*(a*c^2-d^2))^(1/2)/c^2/b)+d^2/c^2)^(1/2)*d^5+1/2*d/b^2/((-a*b)^
(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(
a*c^2-d^2))^(1/2))*c^2*((x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b+
2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^
2/b)+d^2/c^2)^(1/2)*a^2+1/2*d/b^(5/2)/((-a*b)^(1/2)*c^2+(-c^2*b*(
a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))*
(-c^2*b*(a*c^2-d^2))^(1/2)*ln(((-c^2*b*(a*c^2-d^2))^(1/2)/c^2+(x-
(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)*b)/b^(1/2)+((x-(-c^2*b*(a*c^2-d
^2))^(1/2)/c^2/b)^2*b+2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x-(-c^2*b
*(a*c^2-d^2))^(1/2)/c^2/b)+d^2/c^2)^(1/2))*a^2+1/2/b^(5/2)/((-a*b
)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b
*(a*c^2-d^2))^(1/2))/c^4*(-c^2*b*(a*c^2-d^2))^(1/2)*ln(((-c^2*b*(
a*c^2-d^2))^(1/2)/c^2+(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)*b)/b^(
1/2)+((x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b+2*(-c^2*b*(a*c^2-d
^2))^(1/2)/c^2*(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+d^2/c^2)^(1/2
))*d^5-1/2/b^2/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a
*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))*d^3/(d^2/c^2)^(1/2)*ln(
(2*d^2/c^2+2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x-(-c^2*b*(a*c^2-d^2
))^(1/2)/c^2/b)+2*(d^2/c^2)^(1/2)*((x-(-c^2*b*(a*c^2-d^2))^(1/2)/
c^2/b)^2*b+2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x-(-c^2*b*(a*c^2-d^2
))^(1/2)/c^2/b)+d^2/c^2)^(1/2))/(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2
/b))*a^2-1/2/b^2/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((
-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))/c^4*d^7/(d^2/c^2)^(1/
2)*ln((2*d^2/c^2+2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x-(-c^2*b*(a*c
^2-d^2))^(1/2)/c^2/b)+2*(d^2/c^2)^(1/2)*((x-(-c^2*b*(a*c^2-d^2))^
(1/2)/c^2/b)^2*b+2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x-(-c^2*b*(a*c
^2-d^2))^(1/2)/c^2/b)+d^2/c^2)^(1/2))/(x-(-c^2*b*(a*c^2-d^2))^(1/
2)/c^2/b))+1/2*d/b^2/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2)
)/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))*c^2*((x+(-c^2*b*(
a*c^2-d^2))^(1/2)/c^2/b)^2*b-2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x+
(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+d^2/c^2)^(1/2)*a^2-1/2*d/b^(5/2
)/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2
-(-c^2*b*(a*c^2-d^2))^(1/2))*(-c^2*b*(a*c^2-d^2))^(1/2)*ln((-(-c^
2*b*(a*c^2-d^2))^(1/2)/c^2+(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)*b
)/b^(1/2)+((x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b-2*(-c^2*b*(a*
c^2-d^2))^(1/2)/c^2*(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+d^2/c^2)
^(1/2))*a^2-1/2/b^(5/2)/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1

```
/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))/c^4*(-c^2*b*(a
*c^2-d^2))^(1/2)*ln((-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2+(x+(-c^2*b*(
a*c^2-d^2))^(1/2)/c^2/b)*b)/b^(1/2)+((x+(-c^2*b*(a*c^2-d^2))^(1/2
)/c^2/b)^2*b-2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x+(-c^2*b*(a*c^2-d
^2))^(1/2)/c^2/b)+d^2/c^2)^(1/2))*d^5-1/2/b^2/((-a*b)^(1/2)*c^2+(
-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))
^(1/2))*d^3/(d^2/c^2)^(1/2)*ln((2*d^2/c^2-2*(-c^2*b*(a*c^2-d^2))^
(1/2)/c^2*(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+2*(d^2/c^2)^(1/2)*
((x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b-2*(-c^2*b*(a*c^2-d^2))^
(1/2)/c^2*(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+d^2/c^2)^(1/2))/(x
+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b))*a^2-1/2/b^2/((-a*b)^(1/2)*c^2
+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2
))^(1/2))/c^4*d^7/(d^2/c^2)^(1/2)*ln((2*d^2/c^2-2*(-c^2*b*(a*c^2-
d^2))^(1/2)/c^2*(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+2*(d^2/c^2)^
(1/2)*((x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b-2*(-c^2*b*(a*c^2-
d^2))^(1/2)/c^2*(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+d^2/c^2)^(1/
2))/(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b))-1/2*d/b^2*c^2*a^2/((-a*
b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*
b*(a*c^2-d^2))^(1/2))*((x-1/b*(-a*b)^(1/2))^2*b+2*(-a*b)^(1/2)*(x
-1/b*(-a*b)^(1/2)))^(1/2)-1/2*d/b^2*c^2*a^2/((-a*b)^(1/2)*c^2+(-c
^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(
1/2))*((x+1/b*(-a*b)^(1/2))^2*b-2*(-a*b)^(1/2)*(x+1/b*(-a*b)^(1/2
)))^(1/2)-1/2*a/c/b^2*x^2-1/b^2/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-
d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))*((x-(-
c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b+2*(-c^2*b*(a*c^2-d^2))^(1/2)/
c^2*(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+d^2/c^2)^(1/2)*a*d^3+1/2
/b^2/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*
c^2-(-c^2*b*(a*c^2-d^2))^(1/2))/c^2*((x-(-c^2*b*(a*c^2-d^2))^(1/2
)/c^2/b)^2*b+2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x-(-c^2*b*(a*c^2-d
^2))^(1/2)/c^2/b)+d^2/c^2)^(1/2)*d^5-1/b^2/((-a*b)^(1/2)*c^2+(-c^
2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1
/2))*((x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b-2*(-c^2*b*(a*c^2-d
^2))^(1/2)/c^2*(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+d^2/c^2)^(1/2
)*a*d^3-a/c^3/b^3*d^2*ln(b*c^2*x^2+a*c^2-d^2)-1/3*d*(b*x^2+a)^(3/
2)/b^3/c^2
```

Maxima [A] time = 0.707384, size = 169, normalized size = 1.26

$$\frac{\frac{3\left(bx^2+a\right)^2c^3-4\left(bx^2+a\right)^{\frac{3}{2}}c^2d-6\left(2\,ac^3-cd^2\right)\left(bx^2+a\right)+12\left(2\,ac^2d-d^3\right)\sqrt{bx^2+a}}{c^4}+\frac{12\left(a^2c^4-2\,ac^2d^2+d^4\right)\log\left(\sqrt{bx^2+a}c+d\right)}{c^5}}{12\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*c*x^2 + a*c + sqrt(b*x^2 + a)*d),x, algorithm="maxima")
```

```
[Out] 1/12*((3*(b*x^2 + a)^2*c^3 - 4*(b*x^2 + a)^(3/2)*c^2*d - 6*(2*a*c
^3 - c*d^2)*(b*x^2 + a) + 12*(2*a*c^2*d - d^3)*sqrt(b*x^2 + a))/c
^4 + 12*(a^2*c^4 - 2*a*c^2*d^2 + d^4)*log(sqrt(b*x^2 + a)*c + d)/
c^5)/b^3
```

Fricas [A] time = 0.353589, size = 315, normalized size = 2.35

$$\frac{3\,b^{2}c^{4}x^{4}-6\,\left(abc^{4}-bc^{2}d^{2}\right)x^{2}+6\,\left(a^{2}c^{4}-2\,ac^{2}d^{2}+d^{4}\right)\log\left(bc^{2}x^{2}+ac^{2}-d^{2}\right)+3\,\left(a^{2}c^{4}-2\,ac^{2}d^{2}+d^{4}\right)\log\left(-\frac{bc^{2}x^{2}+ac^{2}+2\,\sqrt{\cdots}}{x^{2}}\cdots\right.}{12\,b^{3}c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*c*x^2 + a*c + sqrt(b*x^2 + a)*d),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^2*c^4*x^4 - 6*(a*b*c^4 - b*c^2*d^2)*x^2 + 6*(a^2*c^4 -
2*a*c^2*d^2 + d^4)*log(b*c^2*x^2 + a*c^2 - d^2) + 3*(a^2*c^4 - 2*
a*c^2*d^2 + d^4)*log(-(b*c^2*x^2 + a*c^2 + 2*sqrt(b*x^2 + a)*c*d
+ d^2)/x^2) - 3*(a^2*c^4 - 2*a*c^2*d^2 + d^4)*log(-(b*c^2*x^2 + a
*c^2 - 2*sqrt(b*x^2 + a)*c*d + d^2)/x^2) - 4*(b*c^3*d*x^2 - 5*a*c
^3*d + 3*c*d^3)*sqrt(b*x^2 + a))/(b^3*c^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{5}}{ac+bcx^{2}+d\sqrt{a+bx^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)
```

```
[Out] Integral(x**5/(a*c + b*c*x**2 + d*sqrt(a + b*x**2)), x)
```

GIAC/XCAS [A] time = 0.277159, size = 209, normalized size = 1.56

$$\frac{\left(a^{2}c^{4}-2\,ac^{2}d^{2}+d^{4}\right)\ln\left(\left|\sqrt{bx^{2}+a}c+d\right|\right)}{b^{3}c^{5}}+\frac{3\,\left(bx^{2}+a\right)^{2}b^{9}c^{3}-12\,\left(bx^{2}+a\right)ab^{9}c^{3}-4\,\left(bx^{2}+a\right)^{\frac{3}{2}}b^{9}c^{2}d+24\,\sqrt{bx^{2}+a}ab^{9}c^{2}d+6\,\left(bx^{2}+a\right)b^{9}cd^{2}-12\,\sqrt{bx^{2}+a}b^{9}d^{3}}{12\,b^{12}c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*c*x^2 + a*c + sqrt(b*x^2 + a)*d),x, algorithm="giac")
```

```
[Out] (a^2*c^4 - 2*a*c^2*d^2 + d^4)*ln(abs(sqrt(b*x^2 + a)*c + d))/(b^3
*c^5) + 1/12*(3*(b*x^2 + a)^2*b^9*c^3 - 12*(b*x^2 + a)*a*b^9*c^3
- 4*(b*x^2 + a)^(3/2)*b^9*c^2*d + 24*sqrt(b*x^2 + a)*a*b^9*c^2*d
+ 6*(b*x^2 + a)*b^9*c*d^2 - 12*sqrt(b*x^2 + a)*b^9*d^3)/(b^12*c^4
)
```

3.384 $\int \frac{x^3}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=69

$$-\frac{d\sqrt{a+bx^2}}{b^2c^2}-\frac{(ac^2-d^2)\log\left(c\sqrt{a+bx^2}+d\right)}{b^2c^3}+\frac{x^2}{2bc}$$

```
[Out] x^2/(2*b*c) - (d*Sqrt[a + b*x^2])/(b^2*c^2) - ((a*c^2 - d^2)*Log[
d + c*Sqrt[a + b*x^2]])/(b^2*c^3)
```

Rubi [A] time = 0.372471, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069

$$-\frac{d\sqrt{a+bx^2}}{b^2c^2}-\frac{(ac^2-d^2)\log\left(c\sqrt{a+bx^2}+d\right)}{b^2c^3}+\frac{x^2}{2bc}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]),x]
```

```
[Out] x^2/(2*b*c) - (d*Sqrt[a + b*x^2])/(b^2*c^2) - ((a*c^2 - d^2)*Log[
d + c*Sqrt[a + b*x^2]])/(b^2*c^3)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int^{\sqrt{a+bx^2}} x\,dx}{b^2c}-\frac{\int^{\sqrt{a+bx^2}} d\,dx}{b^2c^2}+\frac{(-ac^2+d^2)\log\left(c\sqrt{a+bx^2}+d\right)}{b^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)
```

```
[Out] Integral(x, (x, sqrt(a + b*x**2)))/(b**2*c) - Integral(d, (x, sqr
t(a + b*x**2)))/(b**2*c**2) + (-a*c**2 + d**2)*log(c*sqrt(a + b*x
**2) + d)/(b**2*c**3)
```

Mathematica [A] time = 0.10938, size = 95, normalized size = 1.38

$$\frac{(d^2 - ac^2)\log(ac^2 + bc^2x^2 - d^2) + (2d^2 - 2ac^2)\tanh^{-1}\left(\frac{c\sqrt{a+bx^2}}{d}\right) + c\left(bcx^2 - 2d\sqrt{a + bx^2}\right)}{2b^2c^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]),x]
```

```
[Out] (c*(b*c*x^2 - 2*d*Sqrt[a + b*x^2]) + (-2*a*c^2 + 2*d^2)*ArcTanh[(
c*Sqrt[a + b*x^2])/d] + (-(a*c^2) + d^2)*Log[a*c^2 - d^2 + b*c^2*
x^2])/(2*b^2*c^3)
```

Maple [B] time = 0.03, size = 3426, normalized size = 49.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x)
```

```
[Out] 1/2*d*c^2*a/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)
^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))/b*((x-1/b*(-a*b)^(1/2))^2*
b+2*(-a*b)^(1/2)*(x-1/b*(-a*b)^(1/2)))^(1/2)+1/2*d*c^2*a/((-a*b)^
(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(
a*c^2-d^2))^(1/2))/b^(3/2)*(-a*b)^(1/2)*ln(((x-1/b*(-a*b)^(1/2))*
b+(-a*b)^(1/2))/b^(1/2)+((x-1/b*(-a*b)^(1/2))^2*b+2*(-a*b)^(1/2)*
(x-1/b*(-a*b)^(1/2)))^(1/2))+1/2*d*c^2*a/((-a*b)^(1/2)*c^2+(-c^2*
b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2
))/b*((x+1/b*(-a*b)^(1/2))^2*b-2*(-a*b)^(1/2)*(x+1/b*(-a*b)^(1/2)
))^(1/2)-1/2*d*c^2*a/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2)
)/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))/b^(3/2)*(-a*b)^(1
/2)*ln(((x+1/b*(-a*b)^(1/2))*b-(-a*b)^(1/2))/b^(1/2)+((x+1/b*(-a*
b)^(1/2))^2*b-2*(-a*b)^(1/2)*(x+1/b*(-a*b)^(1/2)))^(1/2))-1/2*d/(
(-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-
c^2*b*(a*c^2-d^2))^(1/2))/b*((x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)
^2*b+2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x-(-c^2*b*(a*c^2-d^2))^(1/
2)/c^2/b)+d^2/c^2)^(1/2)*a*c^2+1/2/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c
^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))/b*(
(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b+2*(-c^2*b*(a*c^2-d^2))^(
1/2)/c^2*(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+d^2/c^2)^(1/2)*d^3-
1/2*d/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)
*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))/b^(3/2)*(-c^2*b*(a*c^2-d^2))^(1/
2)*ln(((-c^2*b*(a*c^2-d^2))^(1/2)/c^2+(x-(-c^2*b*(a*c^2-d^2))^(1/
2)/c^2/b)*b)/b^(1/2)+((x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b+2*
(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/
b)+d^2/c^2)^(1/2))*a+1/2/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(
```

```
1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))/b^(3/2)*(-c^2
*b*(a*c^2-d^2))^(1/2)/c^2*ln(((-c^2*b*(a*c^2-d^2))^(1/2)/c^2+(x-(
-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)*b)/b^(1/2)+((x-(-c^2*b*(a*c^2-d^
2))^(1/2)/c^2/b)^2*b+2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x-(-c^2*b*
(a*c^2-d^2))^(1/2)/c^2/b)+d^2/c^2)^(1/2))*d^3+1/2/((-a*b)^(1/2)*c
^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d
^2))^(1/2))/b*d^3/(d^2/c^2)^(1/2)*ln((2*d^2/c^2+2*(-c^2*b*(a*c^2-
d^2))^(1/2)/c^2*(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+2*(d^2/c^2)^
(1/2)*((x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b+2*(-c^2*b*(a*c^2-
d^2))^(1/2)/c^2*(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+d^2/c^2)^(1/
2))/(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b))*a-1/2/((-a*b)^(1/2)*c^2
+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2
))^(1/2))/b*d^5/c^2/(d^2/c^2)^(1/2)*ln((2*d^2/c^2+2*(-c^2*b*(a*c^
2-d^2))^(1/2)/c^2*(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+2*(d^2/c^2
)^(1/2)*((x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b+2*(-c^2*b*(a*c^
2-d^2))^(1/2)/c^2*(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+d^2/c^2)^(
1/2))/(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b))-1/2*d/((-a*b)^(1/2)*c
^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d
^2))^(1/2))/b*((x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b-2*(-c^2*b
*(a*c^2-d^2))^(1/2)/c^2*(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+d^2/
c^2)^(1/2)*a*c^2+1/2/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2)
)/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))/b*((x+(-c^2*b*(a*
c^2-d^2))^(1/2)/c^2/b)^2*b-2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x+(-
c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+d^2/c^2)^(1/2)*d^3+1/2*d/((-a*b)^
(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(
a*c^2-d^2))^(1/2))/b^(3/2)*(-c^2*b*(a*c^2-d^2))^(1/2)*ln((-(-c^2*
b*(a*c^2-d^2))^(1/2)/c^2+(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)*b)/
b^(1/2)+((x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b-2*(-c^2*b*(a*c^
2-d^2))^(1/2)/c^2*(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+d^2/c^2)^(
1/2))*a-1/2/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)
^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))/b^(3/2)*(-c^2*b*(a*c^2-d^2
))^(1/2)/c^2*ln((-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2+(x+(-c^2*b*(a*c^
2-d^2))^(1/2)/c^2/b)*b)/b^(1/2)+((x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^
2/b)^2*b-2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x+(-c^2*b*(a*c^2-d^2))
^(1/2)/c^2/b)+d^2/c^2)^(1/2))*d^3+1/2/((-a*b)^(1/2)*c^2+(-c^2*b*(
a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))/
b*d^3/(d^2/c^2)^(1/2)*ln((2*d^2/c^2-2*(-c^2*b*(a*c^2-d^2))^(1/2)/
c^2*(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+2*(d^2/c^2)^(1/2)*((x+(-
c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b-2*(-c^2*b*(a*c^2-d^2))^(1/2)/
c^2*(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+d^2/c^2)^(1/2))/(x+(-c^2
*b*(a*c^2-d^2))^(1/2)/c^2/b))*a-1/2/((-a*b)^(1/2)*c^2+(-c^2*b*(a*
c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))/b*
d^5/c^2/(d^2/c^2)^(1/2)*ln((2*d^2/c^2-2*(-c^2*b*(a*c^2-d^2))^(1/2
)/c^2*(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+2*(d^2/c^2)^(1/2)*((x+
(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b-2*(-c^2*b*(a*c^2-d^2))^(1/2
)/c^2*(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+d^2/c^2)^(1/2))/(x+(-c
^2*b*(a*c^2-d^2))^(1/2)/c^2/b))-1/2*a/c/b^2*ln(b*c^2*x^2+a*c^2-d^
2)+1/2*x^2/b/c+1/2/b^2/c^3*d^2*ln(b*c^2*x^2+a*c^2-d^2)
```

Maxima [A] time = 0.701929, size = 84, normalized size = 1.22

$$\frac{\frac{(bx^2+a)c-2\sqrt{bx^2+a}d}{c^2} - \frac{2(ac^2-d^2)\log\left(\sqrt{bx^2+a}c+d\right)}{c^3}}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*c*x^2 + a*c + sqrt(b*x^2 + a)*d),x, algorithm="maxima")
```

```
[Out] 1/2*(((b*x^2 + a)*c - 2*sqrt(b*x^2 + a)*d)/c^2 - 2*(a*c^2 - d^2)*
log(sqrt(b*x^2 + a)*c + d)/c^3)/b^2
```

Fricas [A] time = 0.346862, size = 217, normalized size = 3.14

$$\frac{2bc^2x^2 - 4\sqrt{bx^2+a}cd - 2(ac^2-d^2)\log(bc^2x^2+ac^2-d^2) - (ac^2-d^2)\log\left(-\frac{bc^2x^2+ac^2+2\sqrt{bx^2+a}cd+d^2}{x^2}\right) + (ac^2-d^2)\log}{4b^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*c*x^2 + a*c + sqrt(b*x^2 + a)*d),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*c^2*x^2 - 4*sqrt(b*x^2 + a)*c*d - 2*(a*c^2 - d^2)*log(b*
c^2*x^2 + a*c^2 - d^2) - (a*c^2 - d^2)*log(-(b*c^2*x^2 + a*c^2 +
2*sqrt(b*x^2 + a)*c*d + d^2)/x^2) + (a*c^2 - d^2)*log(-(b*c^2*x^2
 + a*c^2 - 2*sqrt(b*x^2 + a)*c*d + d^2)/x^2))/(b^2*c^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)
```

```
[Out] Integral(x**3/(a*c + b*c*x**2 + d*sqrt(a + b*x**2)), x)
```

GIAC/XCAS [A] time = 0.27769, size = 97, normalized size = 1.41

$$-\frac{\frac{2\,(ac^2-d^2)\ln\left(\left|\sqrt{bx^2+a}c+d\right|\right)}{bc^3}-\frac{(bx^2+a)\,bc-2\,\sqrt{bx^2+a}bd}{b^2c^2}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*c*x^2 + a*c + sqrt(b*x^2 + a)*d),x, algorithm="giac")
```

```
[Out] -1/2*(2*(a*c^2 - d^2)*ln(abs(sqrt(b*x^2 + a)*c + d))/(b*c^3) - ((
b*x^2 + a)*b*c - 2*sqrt(b*x^2 + a)*b*d)/(b^2*c^2))/b
```

3.385 $\int \frac{x}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=23

$$\frac{\log\left(c\sqrt{a+bx^2}+d\right)}{bc}$$

```
[Out] Log[d + c*Sqrt[a + b*x^2]]/(b*c)
```

Rubi [A] time = 0.136047, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074

$$\frac{\log\left(c\sqrt{a+bx^2}+d\right)}{bc}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]),x]
```

```
[Out] Log[d + c*Sqrt[a + b*x^2]]/(b*c)
```

Rubi in Sympy [A] time = 7.2676, size = 17, normalized size = 0.74

$$\frac{\log\left(c\sqrt{a+bx^2}+d\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)
```

```
[Out] log(c*sqrt(a + b*x**2) + d)/(b*c)
```

Mathematica [A] time = 0.0165623, size = 23, normalized size = 1.

$$\frac{\log\left(c\sqrt{a+bx^2}+d\right)}{bc}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]),x]
```

```
[Out] Log[d + c*Sqrt[a + b*x^2]]/(b*c)
```

Maple [B] time = 0.026, size = 1941, normalized size = 84.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x)
```

```
[Out] 1/2*d*c^2/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(
1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))*((x-(-c^2*b*(a*c^2-d^2))^(1/
2)/c^2/b)^2*b+2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x-(-c^2*b*(a*c^2-
d^2))^(1/2)/c^2/b)+d^2/c^2)^(1/2)+1/2*d/((-a*b)^(1/2)*c^2+(-c^2*b
*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2)
)*(-c^2*b*(a*c^2-d^2))^(1/2)*ln(((-c^2*b*(a*c^2-d^2))^(1/2)/c^2+(
x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)*b)/b^(1/2)+((x-(-c^2*b*(a*c^2
-d^2))^(1/2)/c^2/b)^2*b+2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x-(-c^2
*b*(a*c^2-d^2))^(1/2)/c^2/b)+d^2/c^2)^(1/2))/b^(1/2)-1/2/((-a*b)^
(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(
a*c^2-d^2))^(1/2))*d^3/(d^2/c^2)^(1/2)*ln((2*d^2/c^2+2*(-c^2*b*(a
*c^2-d^2))^(1/2)/c^2*(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+2*(d^2/
c^2)^(1/2)*((x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b+2*(-c^2*b*(a
*c^2-d^2))^(1/2)/c^2*(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+d^2/c^2
)^(1/2))/(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b))+1/2*d*c^2/((-a*b)^
(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(
a*c^2-d^2))^(1/2))*((x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b-2*(-
c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)
+d^2/c^2)^(1/2)-1/2*d/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2
))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))*(-c^2*b*(a*c^2-d
^2))^(1/2)*ln((-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2+(x+(-c^2*b*(a*c^2-
d^2))^(1/2)/c^2/b)*b)/b^(1/2)+((x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/
b)^2*b-2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x+(-c^2*b*(a*c^2-d^2))^(
1/2)/c^2/b)+d^2/c^2)^(1/2))/b^(1/2)-1/2/((-a*b)^(1/2)*c^2+(-c^2*b
*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2)
)*d^3/(d^2/c^2)^(1/2)*ln((2*d^2/c^2-2*(-c^2*b*(a*c^2-d^2))^(1/2)/
c^2*(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+2*(d^2/c^2)^(1/2)*((x+(-
c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b-2*(-c^2*b*(a*c^2-d^2))^(1/2)/
c^2*(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+d^2/c^2)^(1/2))/(x+(-c^2
*b*(a*c^2-d^2))^(1/2)/c^2/b))-1/2*d*c^2/((-a*b)^(1/2)*c^2+(-c^2*b
*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2)
)*((x-1/b*(-a*b)^(1/2))^2*b+2*(-a*b)^(1/2)*(x-1/b*(-a*b)^(1/2)))^
(1/2)-1/2*d*c^2/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-
a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))*(-a*b)^(1/2)*ln(((x-1/
b*(-a*b)^(1/2))*b+(-a*b)^(1/2))/b^(1/2)+((x-1/b*(-a*b)^(1/2))^2*b
+2*(-a*b)^(1/2)*(x-1/b*(-a*b)^(1/2)))^(1/2))/b^(1/2)-1/2*d*c^2/((
-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c
```

```
^2*b*(a*c^2-d^2))^(1/2))*((x+1/b*(-a*b)^(1/2))^2*b-2*(-a*b)^(1/2)
*(x+1/b*(-a*b)^(1/2)))^(1/2)+1/2*d*c^2/((-a*b)^(1/2)*c^2+(-c^2*b*
(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))
*(-a*b)^(1/2)*ln(((x+1/b*(-a*b)^(1/2))*b-(-a*b)^(1/2))/b^(1/2)+((
x+1/b*(-a*b)^(1/2))^2*b-2*(-a*b)^(1/2)*(x+1/b*(-a*b)^(1/2)))^(1/2
))/b^(1/2)+1/2/b/c*ln(b*c^2*x^2+a*c^2-d^2)
```

Maxima [A] time = 0.697275, size = 28, normalized size = 1.22

$$\frac{\log\left(\sqrt{bx^2 + a}c + d\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*c*x^2 + a*c + sqrt(b*x^2 + a)*d),x, algorithm="maxima")
```

```
[Out] log(sqrt(b*x^2 + a)*c + d)/(b*c)
```

Fricas [A] time = 0.306467, size = 142, normalized size = 6.17

$$\frac{2\log\left(bc^2x^2 + ac^2 - d^2\right) + \log\left(-\frac{bc^2x^2 + ac^2 + 2\sqrt{bx^2 + a}cd + d^2}{x^2}\right) - \log\left(-\frac{bc^2x^2 + ac^2 - 2\sqrt{bx^2 + a}cd + d^2}{x^2}\right)}{4\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*c*x^2 + a*c + sqrt(b*x^2 + a)*d),x, algorithm="fricas")
```

```
[Out] 1/4*(2*log(b*c^2*x^2 + a*c^2 - d^2) + log(-(b*c^2*x^2 + a*c^2 + 2
*sqrt(b*x^2 + a)*c*d + d^2)/x^2) - log(-(b*c^2*x^2 + a*c^2 - 2*sq
rt(b*x^2 + a)*c*d + d^2)/x^2))/(b*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{ac + bcx^2 + d\sqrt{a + bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)
```

```
[Out] Integral(x/(a*c + b*c*x**2 + d*sqrt(a + b*x**2)), x)
```

GIAC/XCAS [A] time = 0.273608, size = 30, normalized size = 1.3

$$\frac{\ln\left(\left|\sqrt{bx^2 + a}c + d\right|\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*c*x^2 + a*c + sqrt(b*x^2 + a)*d),x, algorithm="giac")
```

```
[Out] ln(abs(sqrt(b*x^2 + a)*c + d))/(b*c)
```

3.386 $\int \frac{1}{x\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx$

Optimal. Leaf size=88

$$-\frac{c\log\left(c\sqrt{a+bx^2}+d\right)}{ac^2-d^2}+\frac{d\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}\,(ac^2-d^2)}+\frac{c\log(x)}{ac^2-d^2}$$

```
[Out] (d*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(Sqrt[a]*(a*c^2 - d^2)) + (c
*Log[x])/(a*c^2 - d^2) - (c*Log[d + c*Sqrt[a + b*x^2]])/(a*c^2 -
d^2)
```

Rubi [A] time = 0.415814, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$

$$-\frac{c\log\left(c\sqrt{a+bx^2}+d\right)}{ac^2-d^2}+\frac{d\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}\,(ac^2-d^2)}+\frac{c\log(x)}{ac^2-d^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])),x]
```

```
[Out] (d*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(Sqrt[a]*(a*c^2 - d^2)) + (c
*Log[x])/(a*c^2 - d^2) - (c*Log[d + c*Sqrt[a + b*x^2]])/(a*c^2 -
d^2)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.91718, size = 282, normalized size = 3.2

$$-\frac{c\log\left(ac^2+bc^2x^2-d^2\right)+c\log\left(\frac{\left(ac^2-d^2\right)\left(-i\sqrt{b}x\sqrt{ac^2-d^2}+d\sqrt{a+bx^2}+ac\right)}{\sqrt{b}cd^2\left(\sqrt{b}cx+i\sqrt{ac^2-d^2}\right)}\right)+c\log\left(\frac{\left(ac^2-d^2\right)\left(i\sqrt{b}x\sqrt{ac^2-d^2}+d\sqrt{a+bx^2}+ac\right)}{\sqrt{b}cd^2\left(\sqrt{b}cx-i\sqrt{ac^2-d^2}\right)}\right)-\frac{2d\log(}{}}{2ac^2-2d^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])),x]
```

```
[Out] -((c*Log[4] + (-2*c + (2*d)/Sqrt[a])*Log[x] + c*Log[a*c^2 - d^2 +
 b*c^2*x^2] - (2*d*Log[a + Sqrt[a]*Sqrt[a + b*x^2]])/Sqrt[a] + c*
Log[((a*c^2 - d^2)*(a*c - I*Sqrt[b]*Sqrt[a*c^2 - d^2]*x + d*Sqrt[
a + b*x^2]))/(Sqrt[b]*c*d^2*(I*Sqrt[a*c^2 - d^2] + Sqrt[b]*c*x))]
 + c*Log[((a*c^2 - d^2)*(a*c + I*Sqrt[b]*Sqrt[a*c^2 - d^2]*x + d*
Sqrt[a + b*x^2]))/(Sqrt[b]*c*d^2*((-I)*Sqrt[a*c^2 - d^2] + Sqrt[b
]*c*x))])/(2*a*c^2 - 2*d^2))
```

Maple [B] time = 0.042, size = 2175, normalized size = 24.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x)
```

```
[Out] c*ln(x)/(a*c^2-d^2)-1/2*a*c^3/(a*c^2-d^2)/d^2*ln(b*c^2*x^2+a*c^2-
d^2)+1/2*c/d^2*ln(b*c^2*x^2+a*c^2-d^2)+d/a^(1/2)/(a*c^2-d^2)*ln((
2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)-d/a/(a*c^2-d^2)*(b*x^2+a)^(1/2)
+1/2*d*b*c^2/a/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a
*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))*((x-1/b*(-a*b)^(1/2))^2
*b+2*(-a*b)^(1/2)*(x-1/b*(-a*b)^(1/2)))^(1/2)+1/2*d*b^(1/2)*c^2/a
/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-
(-c^2*b*(a*c^2-d^2))^(1/2))*(-a*b)^(1/2)*ln(((x-1/b*(-a*b)^(1/2))
*b+(-a*b)^(1/2))/b^(1/2)+((x-1/b*(-a*b)^(1/2))^2*b+2*(-a*b)^(1/2)
*(x-1/b*(-a*b)^(1/2)))^(1/2))+1/2*d*b*c^2/a/((-a*b)^(1/2)*c^2+(-c
^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(
1/2))*((x+1/b*(-a*b)^(1/2))^2*b-2*(-a*b)^(1/2)*(x+1/b*(-a*b)^(1/2
)))^(1/2)-1/2*d*b^(1/2)*c^2/a/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^
2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))*(-a*b)^(
1/2)*ln(((x+1/b*(-a*b)^(1/2))*b-(-a*b)^(1/2))/b^(1/2)+((x+1/b*(-a
*b)^(1/2))^2*b-2*(-a*b)^(1/2)*(x+1/b*(-a*b)^(1/2)))^(1/2))-1/2*d*
b*c^4/(a*c^2-d^2)/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/(
(-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))*((x-(-c^2*b*(a*c^2-d
^2))^(1/2)/c^2/b)^2*b+2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x-(-c^2*b
*(a*c^2-d^2))^(1/2)/c^2/b)+d^2/c^2)^(1/2)-1/2*d*b^(1/2)*c^2/(a*c^
```

```
2-d^2)/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2
)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))*(-c^2*b*(a*c^2-d^2))^(1/2)*ln((
(-c^2*b*(a*c^2-d^2))^(1/2)/c^2+(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/
b)*b)/b^(1/2)+((x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b+2*(-c^2*b
*(a*c^2-d^2))^(1/2)/c^2*(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+d^2/
c^2)^(1/2))+1/2*b*c^2/(a*c^2-d^2)/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^
2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))*d^3/
(d^2/c^2)^(1/2)*ln((2*d^2/c^2+2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x
-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+2*(d^2/c^2)^(1/2)*((x-(-c^2*b*
(a*c^2-d^2))^(1/2)/c^2/b)^2*b+2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x
-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+d^2/c^2)^(1/2))/(x-(-c^2*b*(a*
c^2-d^2))^(1/2)/c^2/b))-1/2*d*b*c^4/(a*c^2-d^2)/((-a*b)^(1/2)*c^2
+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2
))^(1/2))*((x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b-2*(-c^2*b*(a*
c^2-d^2))^(1/2)/c^2*(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+d^2/c^2)
^(1/2)+1/2*d*b^(1/2)*c^2/(a*c^2-d^2)/((-a*b)^(1/2)*c^2+(-c^2*b*(a
*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))*(
-c^2*b*(a*c^2-d^2))^(1/2)*ln((-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2+(x+
(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)*b)/b^(1/2)+((x+(-c^2*b*(a*c^2-d
^2))^(1/2)/c^2/b)^2*b-2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x+(-c^2*b
*(a*c^2-d^2))^(1/2)/c^2/b)+d^2/c^2)^(1/2))+1/2*b*c^2/(a*c^2-d^2)/
((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(
-c^2*b*(a*c^2-d^2))^(1/2))*d^3/(d^2/c^2)^(1/2)*ln((2*d^2/c^2-2*(-
c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)
+2*(d^2/c^2)^(1/2)*((x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b-2*(-
c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)
+d^2/c^2)^(1/2))/(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bcx^2 + ac + \sqrt{bx^2 + ad}\right)x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*c*x^2 + a*c + sqrt(b*x^2 + a)*d)*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*c*x^2 + a*c + sqrt(b*x^2 + a)*d)*x), x)
```

Fricas [A] time = 0.374207, size = 1, normalized size = 0.01

$$\left[-\frac{\sqrt{a}c\log\left(-\frac{bc^2x^2+ac^2+2\sqrt{bx^2+a}cd+d^2}{x^2}\right)-\sqrt{a}c\log\left(-\frac{bc^2x^2+ac^2-2\sqrt{bx^2+a}cd+d^2}{x^2}\right)+2\,d\log\left(-\frac{(bx^2+2\,a)\sqrt{a}-2\sqrt{bx^2+a}a}{x^2}\right)+2\left(c\log\left(b\right.\right.}{4\,(ac^2-d^2)\sqrt{a}}\right.$$

$$-\frac{\sqrt{-a}c\log\left(-\frac{bc^2x^2+ac^2+2\sqrt{bx^2+a}cd+d^2}{x^2}\right)-\sqrt{-a}c\log\left(-\frac{bc^2x^2+ac^2-2\sqrt{bx^2+a}cd+d^2}{x^2}\right)-4\,d\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+2\left(c\log\left(bc^2x^2+ac\right.\right.}{4\,(ac^2-d^2)\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*c*x^2 + a*c + sqrt(b*x^2 + a)*d)*x),x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(a)*c*log(-(b*c^2*x^2 + a*c^2 + 2*sqrt(b*x^2 + a)*c*d
+ d^2)/x^2) - sqrt(a)*c*log(-(b*c^2*x^2 + a*c^2 - 2*sqrt(b*x^2 +
a)*c*d + d^2)/x^2) + 2*d*log(-((b*x^2 + 2*a)*sqrt(a) - 2*sqrt(b*x
^2 + a)*a)/x^2) + 2*(c*log(b*c^2*x^2 + a*c^2 - d^2) - 2*c*log(x))
*sqrt(a))/((a*c^2 - d^2)*sqrt(a)), -1/4*(sqrt(-a)*c*log(-(b*c^2*x
^2 + a*c^2 + 2*sqrt(b*x^2 + a)*c*d + d^2)/x^2) - sqrt(-a)*c*log(-
(b*c^2*x^2 + a*c^2 - 2*sqrt(b*x^2 + a)*c*d + d^2)/x^2) - 4*d*arct
an(sqrt(-a)/sqrt(b*x^2 + a)) + 2*(c*log(b*c^2*x^2 + a*c^2 - d^2)
- 2*c*log(x))*sqrt(-a))/((a*c^2 - d^2)*sqrt(-a))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)
```

```
[Out] Integral(1/(x*(a*c + b*c*x**2 + d*sqrt(a + b*x**2))), x)
```

GIAC/XCAS [A] time = 0.281115, size = 127, normalized size = 1.44

$$-\frac{c^2\ln\left(\left|\sqrt{bx^2+a}c+d\right|\right)}{ac^3-cd^2}+\frac{c\ln\left(bx^2\right)}{2\,(ac^2-d^2)}-\frac{d\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{(ac^2-d^2)\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*c*x^2 + a*c + sqrt(b*x^2 + a)*d)*x),x, algorithm="giac")
```

```
[Out] -c^2*ln(abs(sqrt(b*x^2 + a)*c + d))/(a*c^3 - c*d^2) + 1/2*c*ln(b*
x^2)/(a*c^2 - d^2) - d*arctan(sqrt(b*x^2 + a)/sqrt(-a))/((a*c^2 -
 d^2)*sqrt(-a))
```

3.387 $\int \frac{1}{x^3\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx$

Optimal. Leaf size=151

$$-\frac{bd\left(3ac^2-d^2\right)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}\left(ac^2-d^2\right)^2}-\frac{ac-d\sqrt{a+bx^2}}{2ax^2\left(ac^2-d^2\right)}+\frac{bc^3\log\left(c\sqrt{a+bx^2}+d\right)}{\left(ac^2-d^2\right)^2}-\frac{bc^3\log(x)}{\left(ac^2-d^2\right)^2}$$

```
[Out] -(a*c - d*Sqrt[a + b*x^2])/(2*a*(a*c^2 - d^2)*x^2) - (b*d*(3*a*c^
2 - d^2)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(3/2)*(a*c^2 - d^
2)^2) - (b*c^3*Log[x])/(a*c^2 - d^2)^2 + (b*c^3*Log[d + c*Sqrt[a
+ b*x^2]])/(a*c^2 - d^2)^2
```

Rubi [A] time = 0.637981, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207

$$-\frac{bd\left(3ac^2-d^2\right)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}\left(ac^2-d^2\right)^2}-\frac{ac-d\sqrt{a+bx^2}}{2ax^2\left(ac^2-d^2\right)}+\frac{bc^3\log\left(c\sqrt{a+bx^2}+d\right)}{\left(ac^2-d^2\right)^2}-\frac{bc^3\log(x)}{\left(ac^2-d^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])),x]
```

```
[Out] -(a*c - d*Sqrt[a + b*x^2])/(2*a*(a*c^2 - d^2)*x^2) - (b*d*(3*a*c^
2 - d^2)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(3/2)*(a*c^2 - d^
2)^2) - (b*c^3*Log[x])/(a*c^2 - d^2)^2 + (b*c^3*Log[d + c*Sqrt[a
+ b*x^2]])/(a*c^2 - d^2)^2
```

Rubi in Sympy [A] time = 42.6553, size = 133, normalized size = 0.88

$$-\frac{bc^3\log\left(-bx^2\right)}{2\left(-ac^2+d^2\right)^2}+\frac{bc^3\log\left(c\sqrt{a+bx^2}+d\right)}{\left(-ac^2+d^2\right)^2}+\frac{ac-d\sqrt{a+bx^2}}{2ax^2\left(-ac^2+d^2\right)}+\frac{bd\left(-3ac^2+d^2\right)\operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}\left(-ac^2+d^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)
```

```
[Out] -b*c**3*log(-b*x**2)/(2*(-a*c**2 + d**2)**2) + b*c**3*log(c*sqrt(
a + b*x**2) + d)/(-a*c**2 + d**2)**2 + (a*c - d*sqrt(a + b*x**2))
```

```
/(2*a*x**2*(-a*c**2 + d**2)) + b*d*(-3*a*c**2 + d**2)*atanh(sqrt(
a + b*x**2)/sqrt(a))/(2*a**(3/2)*(-a*c**2 + d**2)**2)
```

Mathematica [C] time = 0.94616, size = 430, normalized size = 2.85

$$\frac{a^{3/2}bc^3x^2\log\left(-\frac{2(d^2-ac^2)^2\left(-i\sqrt{b}x\sqrt{ac^2-d^2}+d\sqrt{a+bx^2}+ac\right)}{b^{3/2}c^3d^2\left(\sqrt{b}cx+i\sqrt{ac^2-d^2}\right)}\right)+a^{3/2}bc^3x^2\log\left(-\frac{2(d^2-ac^2)^2\left(i\sqrt{b}x\sqrt{ac^2-d^2}+d\sqrt{a+bx^2}+ac\right)}{b^{3/2}c^3d^2\left(\sqrt{b}cx-i\sqrt{ac^2-d^2}\right)}\right)+a^{3/2}c^2d\sqrt{a}}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])),x]
```

```
[Out] (-(a^(5/2)*c^3) + a^(3/2)*c*d^2 + a^(3/2)*c^2*d*Sqrt[a + b*x^2] -
 Sqrt[a]*d^3*Sqrt[a + b*x^2] - b*(2*a^(3/2)*c^3 - 3*a*c^2*d + d^3
)*x^2*Log[x] + a^(3/2)*b*c^3*x^2*Log[a*c^2 - d^2 + b*c^2*x^2] - 3
*a*b*c^2*d*x^2*Log[a + Sqrt[a]*Sqrt[a + b*x^2]] + b*d^3*x^2*Log[a
 + Sqrt[a]*Sqrt[a + b*x^2]] + a^(3/2)*b*c^3*x^2*Log[(-2*(-(a*c^2)
 + d^2)^2*(a*c - I*Sqrt[b]*Sqrt[a*c^2 - d^2]*x + d*Sqrt[a + b*x^2
]))/(b^(3/2)*c^3*d^2*(I*Sqrt[a*c^2 - d^2] + Sqrt[b]*c*x))] + a^(3
/2)*b*c^3*x^2*Log[(-2*(-(a*c^2) + d^2)^2*(a*c + I*Sqrt[b]*Sqrt[a*
c^2 - d^2]*x + d*Sqrt[a + b*x^2]))/(b^(3/2)*c^3*d^2*((-I)*Sqrt[a*
c^2 - d^2] + Sqrt[b]*c*x))])/(2*a^(3/2)*(-(a*c^2) + d^2)^2*x^2)
```

Maple [B] time = 0.067, size = 2459, normalized size = 16.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x)
```

```
[Out] -1/2*c/(a*c^2-d^2)/x^2-2*b*c^3*ln(x)/(a*c^2-d^2)^2+1/a*c*b/(a*c^2
-d^2)^2*ln(x)*d^2+1/2*a*c^5*b/(a*c^2-d^2)^2/d^2*ln(b*c^2*x^2+a*c^
2-d^2)+b*c/a/(a*c^2-d^2)*ln(x)-1/2*b*c^3/(a*c^2-d^2)/d^2*ln(b*c^2
*x^2+a*c^2-d^2)+1/2*d/a^2/(a*c^2-d^2)/x^2*(b*x^2+a)^(3/2)+1/2*d/a
^(3/2)/(a*c^2-d^2)*b*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)-1/2*d/
a^2/(a*c^2-d^2)*b*(b*x^2+a)^(1/2)-2*d*b/a^(1/2)/(a*c^2-d^2)^2*ln(
(2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)*c^2+2*d*b/a/(a*c^2-d^2)^2*(b*x
^2+a)^(1/2)*c^2+b/a^(3/2)/(a*c^2-d^2)^2*ln((2*a+2*a^(1/2)*(b*x^2+
a)^(1/2))/x)*d^3-b/a^2/(a*c^2-d^2)^2*(b*x^2+a)^(1/2)*d^3-1/2*d*b^
2*c^2/a^2/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(
1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))*((x-1/b*(-a*b)^(1/2))^2*b+2*
(-a*b)^(1/2)*(x-1/b*(-a*b)^(1/2)))^(1/2)-1/2*d*b^(3/2)*c^2/a^2/((
```

```
-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c
^2*b*(a*c^2-d^2))^(1/2))*(-a*b)^(1/2)*ln(((x-1/b*(-a*b)^(1/2))*b+
(-a*b)^(1/2))/b^(1/2)+((x-1/b*(-a*b)^(1/2))^2*b+2*(-a*b)^(1/2)*(x
-1/b*(-a*b)^(1/2)))^(1/2))-1/2*d*b^2*c^2/a^2/((-a*b)^(1/2)*c^2+(-
c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^
(1/2))*((x+1/b*(-a*b)^(1/2))^2*b-2*(-a*b)^(1/2)*(x+1/b*(-a*b)^(1/
2)))^(1/2)+1/2*d*b^(3/2)*c^2/a^2/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2
-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))*(-a*b
)^(1/2)*ln(((x+1/b*(-a*b)^(1/2))*b-(-a*b)^(1/2))/b^(1/2)+((x+1/b*
(-a*b)^(1/2))^2*b-2*(-a*b)^(1/2)*(x+1/b*(-a*b)^(1/2)))^(1/2))+1/2
*d*b^2*c^6/(a*c^2-d^2)^2/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(
1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))*((x-(-c^2*b*(
a*c^2-d^2))^(1/2)/c^2/b)^2*b+2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x-
(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+d^2/c^2)^(1/2)+1/2*d*b^(3/2)*c^
4/(a*c^2-d^2)^2/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-
a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))*(-c^2*b*(a*c^2-d^2))^(
1/2)*ln(((-c^2*b*(a*c^2-d^2))^(1/2)/c^2+(x-(-c^2*b*(a*c^2-d^2))^(
1/2)/c^2/b)*b)/b^(1/2)+((x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b+
2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^
2/b)+d^2/c^2)^(1/2))-1/2*b^2*c^4/(a*c^2-d^2)^2/((-a*b)^(1/2)*c^2+
(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2)
)^(1/2))*d^3/(d^2/c^2)^(1/2)*ln((2*d^2/c^2+2*(-c^2*b*(a*c^2-d^2))
^(1/2)/c^2*(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+2*(d^2/c^2)^(1/2)
*((x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b+2*(-c^2*b*(a*c^2-d^2))
^(1/2)/c^2*(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+d^2/c^2)^(1/2))/(
x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b))+1/2*d*b^2*c^6/(a*c^2-d^2)^2/
((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(
-c^2*b*(a*c^2-d^2))^(1/2))*((x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^
2*b-2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x+(-c^2*b*(a*c^2-d^2))^(1/2
)/c^2/b)+d^2/c^2)^(1/2)-1/2*d*b^(3/2)*c^4/(a*c^2-d^2)^2/((-a*b)^(
1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a
*c^2-d^2))^(1/2))*(-c^2*b*(a*c^2-d^2))^(1/2)*ln((-(-c^2*b*(a*c^2-
d^2))^(1/2)/c^2+(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)*b)/b^(1/2)+(
(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b-2*(-c^2*b*(a*c^2-d^2))^(
1/2)/c^2*(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+d^2/c^2)^(1/2))-1/2
*b^2*c^4/(a*c^2-d^2)^2/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/
2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))*d^3/(d^2/c^2)^(
1/2)*ln((2*d^2/c^2-2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x+(-c^2*b*(a
*c^2-d^2))^(1/2)/c^2/b)+2*(d^2/c^2)^(1/2)*((x+(-c^2*b*(a*c^2-d^2)
)^(1/2)/c^2/b)^2*b-2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x+(-c^2*b*(a
*c^2-d^2))^(1/2)/c^2/b)+d^2/c^2)^(1/2))/(x+(-c^2*b*(a*c^2-d^2))^(
1/2)/c^2/b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bcx^2 + ac + \sqrt{bx^2 + ad}\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*c*x^2 + a*c + sqrt(b*x^2 + a)*d)*x^3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*c*x^2 + a*c + sqrt(b*x^2 + a)*d)*x^3), x)
```

Fricas [A] time = 0.9211, size = 1, normalized size = 0.01

$$\left[\frac{a^{\frac{3}{2}}bc^3x^2\log\left(-\frac{bc^2x^2+ac^2+2\sqrt{bx^2+a}cd+d^2}{x^2}\right)-a^{\frac{3}{2}}bc^3x^2\log\left(-\frac{bc^2x^2+ac^2-2\sqrt{bx^2+a}cd+d^2}{x^2}\right)-\left(3\,abc^2d-bd^3\right)x^2\log\left(-\frac{(bx^2+2\,a)\sqrt{a}+}{x^2}\right.}{4\left(a^3c^4-2\,a^2c^2d^2+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*c*x^2 + a*c + sqrt(b*x^2 + a)*d)*x^3),x, algorithm="fricas")
```

```
[Out] [1/4*(a^(3/2)*b*c^3*x^2*log(-(b*c^2*x^2 + a*c^2 + 2*sqrt(b*x^2 +
a)*c*d + d^2)/x^2) - a^(3/2)*b*c^3*x^2*log(-(b*c^2*x^2 + a*c^2 -
2*sqrt(b*x^2 + a)*c*d + d^2)/x^2) - (3*a*b*c^2*d - b*d^3)*x^2*log
(-((b*x^2 + 2*a)*sqrt(a) + 2*sqrt(b*x^2 + a)*a)/x^2) + 2*(a*c^2*d
 - d^3)*sqrt(b*x^2 + a)*sqrt(a) + 2*(a*b*c^3*x^2*log(b*c^2*x^2 +
a*c^2 - d^2) - 2*a*b*c^3*x^2*log(x) - a^2*c^3 + a*c*d^2)*sqrt(a))
/((a^3*c^4 - 2*a^2*c^2*d^2 + a*d^4)*sqrt(a)*x^2), 1/4*(sqrt(-a)*a
*b*c^3*x^2*log(-(b*c^2*x^2 + a*c^2 + 2*sqrt(b*x^2 + a)*c*d + d^2)
/x^2) - sqrt(-a)*a*b*c^3*x^2*log(-(b*c^2*x^2 + a*c^2 - 2*sqrt(b*x
^2 + a)*c*d + d^2)/x^2) - 2*(3*a*b*c^2*d - b*d^3)*x^2*arctan(sqrt
(-a)/sqrt(b*x^2 + a)) + 2*(a*c^2*d - d^3)*sqrt(b*x^2 + a)*sqrt(-a
) + 2*(a*b*c^3*x^2*log(b*c^2*x^2 + a*c^2 - d^2) - 2*a*b*c^3*x^2*l
og(x) - a^2*c^3 + a*c*d^2)*sqrt(-a))/((a^3*c^4 - 2*a^2*c^2*d^2 +
a*d^4)*sqrt(-a)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)
```

```
[Out] Integral(1/(x**3*(a*c + b*c*x**2 + d*sqrt(a + b*x**2))), x)
```

GIAC/XCAS [A] time = 0.278815, size = 275, normalized size = 1.82

$$\frac{1}{2}\left(\frac{2\,c^4\ln\left(\left|\sqrt{bx^2+a}c+d\right|\right)}{a^2c^5-2\,ac^3d^2+cd^4}-\frac{c^3\ln\left(bx^2\right)}{a^2c^4-2\,ac^2d^2+d^4}+\frac{\left(3\,ac^2d-d^3\right)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\left(a^3c^4-2\,a^2c^2d^2+ad^4\right)\sqrt{-a}}-\frac{a^2c^3-acd^2-\left(ac^2d-d^3\right)\sqrt{bx^2+a}}{\left(ac^2-d^2\right)^2abx^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*c*x^2 + a*c + sqrt(b*x^2 + a)*d)*x^3),x, algorithm="giac")
```

```
[Out] 1/2*(2*c^4*ln(abs(sqrt(b*x^2 + a)*c + d))/(a^2*c^5 - 2*a*c^3*d^2
+ c*d^4) - c^3*ln(b*x^2)/(a^2*c^4 - 2*a*c^2*d^2 + d^4) + (3*a*c^2
*d - d^3)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/((a^3*c^4 - 2*a^2*c^2*
d^2 + a*d^4)*sqrt(-a)) - (a^2*c^3 - a*c*d^2 - (a*c^2*d - d^3)*sqr
t(b*x^2 + a))/((a*c^2 - d^2)^2*a*b*x^2))*b
```

3.388 $\int \frac{x^2}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=147

$$\frac{\sqrt{ac^2-d^2}\tan^{-1}\left(\frac{\sqrt{b}dx}{\sqrt{a+bx^2}\sqrt{ac^2-d^2}}\right)}{b^{3/2}c^2}-\frac{\sqrt{ac^2-d^2}\tan^{-1}\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right)}{b^{3/2}c^2}-\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}c^2}+\frac{x}{bc}$$

```
[Out] x/(b*c) - (Sqrt[a*c^2 - d^2]*ArcTan[(Sqrt[b]*c*x)/Sqrt[a*c^2 - d^
2]])/(b^(3/2)*c^2) + (Sqrt[a*c^2 - d^2]*ArcTan[(Sqrt[b]*d*x)/(Sqr
t[a*c^2 - d^2]*Sqrt[a + b*x^2])])/(b^(3/2)*c^2) - (d*ArcTanh[(Sqr
t[b]*x)/Sqrt[a + b*x^2]])/(b^(3/2)*c^2)
```

Rubi [A] time = 0.478813, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241

$$\frac{\sqrt{ac^2-d^2}\tan^{-1}\left(\frac{\sqrt{b}dx}{\sqrt{a+bx^2}\sqrt{ac^2-d^2}}\right)}{b^{3/2}c^2}-\frac{\sqrt{ac^2-d^2}\tan^{-1}\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right)}{b^{3/2}c^2}-\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}c^2}+\frac{x}{bc}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]),x]
```

```
[Out] x/(b*c) - (Sqrt[a*c^2 - d^2]*ArcTan[(Sqrt[b]*c*x)/Sqrt[a*c^2 - d^
2]])/(b^(3/2)*c^2) + (Sqrt[a*c^2 - d^2]*ArcTan[(Sqrt[b]*d*x)/(Sqr
t[a*c^2 - d^2]*Sqrt[a + b*x^2])])/(b^(3/2)*c^2) - (d*ArcTanh[(Sqr
t[b]*x)/Sqrt[a + b*x^2]])/(b^(3/2)*c^2)
```

Rubi in Sympy [A] time = 45.5677, size = 128, normalized size = 0.87

$$\frac{x}{bc}-\frac{d\,\text{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{\frac{3}{2}}c^2}-\frac{\sqrt{-ac^2+d^2}\,\text{atanh}\left(\frac{\sqrt{b}cx}{\sqrt{-ac^2+d^2}}\right)}{b^{\frac{3}{2}}c^2}+\frac{\sqrt{-ac^2+d^2}\,\text{atanh}\left(\frac{\sqrt{b}dx}{\sqrt{a+bx^2}\sqrt{-ac^2+d^2}}\right)}{b^{\frac{3}{2}}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)
```

```
[Out] x/(b*c) - d*atanh(sqrt(b)*x/sqrt(a + b*x**2))/(b**(3/2)*c**2) - s
qrt(-a*c**2 + d**2)*atanh(sqrt(b)*c*x/sqrt(-a*c**2 + d**2))/(b**(
3/2)*c**2) + sqrt(-a*c**2 + d**2)*atanh(sqrt(b)*d*x/(sqrt(a + b*x
```

```
**2)*sqrt(-a*c**2 + d**2)))/(b**(3/2)*c**2)
```

Mathematica [A] time = 0.190331, size = 157, normalized size = 1.07

$$\frac{\sqrt{ac^2-d^2}\left(\sqrt{b}cx-d\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)\right)+\left(ac^2-d^2\right)\tan^{-1}\left(\frac{\sqrt{b}dx}{\sqrt{a+bx^2}\sqrt{ac^2-d^2}}\right)+\left(d^2-ac^2\right)\tan^{-1}\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right)}{b^{3/2}c^2\sqrt{ac^2-d^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]),x]
```

```
[Out] ((-(a*c^2) + d^2)*ArcTan[(Sqrt[b]*c*x)/Sqrt[a*c^2 - d^2]] + (a*c^
2 - d^2)*ArcTan[(Sqrt[b]*d*x)/(Sqrt[a*c^2 - d^2]*Sqrt[a + b*x^2])
] + Sqrt[a*c^2 - d^2]*(Sqrt[b]*c*x - d*Log[b*x + Sqrt[b]*Sqrt[a +
 b*x^2]]))/(b^(3/2)*c^2*Sqrt[a*c^2 - d^2])
```

Maple [B] time = 0.043, size = 3501, normalized size = 23.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x)
```

```
[Out] 1/2*d*c^2*a/(-a*b)^(1/2)/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(
1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))*((x-1/b*(-a*b
)^(1/2))^2*b+2*(-a*b)^(1/2)*(x-1/b*(-a*b)^(1/2)))^(1/2)+1/2*d*c^2
*a/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^
2-(-c^2*b*(a*c^2-d^2))^(1/2))*ln(((x-1/b*(-a*b)^(1/2))*b+(-a*b)^(
1/2))/b^(1/2)+((x-1/b*(-a*b)^(1/2))^2*b+2*(-a*b)^(1/2)*(x-1/b*(-a
*b)^(1/2)))^(1/2))/b^(1/2)-1/2*d*c^2*a/(-a*b)^(1/2)/((-a*b)^(1/2)
*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2
-d^2))^(1/2))*((x+1/b*(-a*b)^(1/2))^2*b-2*(-a*b)^(1/2)*(x+1/b*(-a
*b)^(1/2)))^(1/2)+1/2*d*c^2*a/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^
2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))*ln(((x+1
/b*(-a*b)^(1/2))*b-(-a*b)^(1/2))/b^(1/2)+((x+1/b*(-a*b)^(1/2))^2*
b-2*(-a*b)^(1/2)*(x+1/b*(-a*b)^(1/2)))^(1/2))/b^(1/2)-1/2*d*c^4/(
(-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-
c^2*b*(a*c^2-d^2))^(1/2))/(-c^2*b*(a*c^2-d^2))^(1/2)*((x-(-c^2*b*
(a*c^2-d^2))^(1/2)/c^2/b)^2*b+2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x
-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+d^2/c^2)^(1/2)*a+1/2*c^2/((-a*
b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*
b*(a*c^2-d^2))^(1/2))/(-c^2*b*(a*c^2-d^2))^(1/2)*((x-(-c^2*b*(a*c
^2-d^2))^(1/2)/c^2/b)^2*b+2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x-(-c
```

^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+d^2/c^2)^(1/2)*d^3-1/2*d*c^2/((-a*
b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*
b*(a*c^2-d^2))^(1/2))*ln(((-c^2*b*(a*c^2-d^2))^(1/2)/c^2+(x-(-c^2
b(a*c^2-d^2))^(1/2)/c^2/b)*b)/b^(1/2)+((x-(-c^2*b*(a*c^2-d^2))^
(1/2)/c^2/b)^2*b+2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x-(-c^2*b*(a*c
^2-d^2))^(1/2)/c^2/b)+d^2/c^2)^(1/2))/b^(1/2)*a+1/2/((-a*b)^(1/2)
*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2
-d^2))^(1/2))*ln(((-c^2*b*(a*c^2-d^2))^(1/2)/c^2+(x-(-c^2*b*(a*c^
2-d^2))^(1/2)/c^2/b)*b)/b^(1/2)+((x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^
2/b)^2*b+2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x-(-c^2*b*(a*c^2-d^2))
^(1/2)/c^2/b)+d^2/c^2)^(1/2))/b^(1/2)*d^3+1/2*c^2/((-a*b)^(1/2)*c
^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d
^2))^(1/2))/(-c^2*b*(a*c^2-d^2))^(1/2)*d^3/(d^2/c^2)^(1/2)*ln((2*
d^2/c^2+2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x-(-c^2*b*(a*c^2-d^2))^
(1/2)/c^2/b)+2*(d^2/c^2)^(1/2)*((x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2
/b)^2*b+2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x-(-c^2*b*(a*c^2-d^2))^
(1/2)/c^2/b)+d^2/c^2)^(1/2))/(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)
)*a-1/2/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/
2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))/(-c^2*b*(a*c^2-d^2))^(1/2)*d^5
/(d^2/c^2)^(1/2)*ln((2*d^2/c^2+2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(
x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+2*(d^2/c^2)^(1/2)*((x-(-c^2*b
*(a*c^2-d^2))^(1/2)/c^2/b)^2*b+2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(
x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+d^2/c^2)^(1/2))/(x-(-c^2*b*(a
*c^2-d^2))^(1/2)/c^2/b))+1/2*d*c^4/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c
^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))/(-c
^2*b*(a*c^2-d^2))^(1/2)*((x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b
-2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c
^2/b)+d^2/c^2)^(1/2)*a-1/2*c^2/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d
^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))/(-c^2*b
*(a*c^2-d^2))^(1/2)*((x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b-2*(
-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b
)+d^2/c^2)^(1/2)*d^3-1/2*d*c^2/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d
^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))*ln((-(-
c^2*b*(a*c^2-d^2))^(1/2)/c^2+(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)
*b)/b^(1/2)+((x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b-2*(-c^2*b*(
a*c^2-d^2))^(1/2)/c^2*(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+d^2/c^
2)^(1/2))/b^(1/2)*a+1/2/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1
/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))*ln((-(-c^2*b*(
a*c^2-d^2))^(1/2)/c^2+(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)*b)/b^(
1/2)+((x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b-2*(-c^2*b*(a*c^2-d
^2))^(1/2)/c^2*(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+d^2/c^2)^(1/2
))/b^(1/2)*d^3-1/2*c^2/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/
2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))/(-c^2*b*(a*c^2-
d^2))^(1/2)*d^3/(d^2/c^2)^(1/2)*ln((2*d^2/c^2-2*(-c^2*b*(a*c^2-d^
2))^(1/2)/c^2*(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+2*(d^2/c^2)^(1
/2)*((x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b-2*(-c^2*b*(a*c^2-d^
2))^(1/2)/c^2*(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+d^2/c^2)^(1/2)
)/(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b))*a+1/2/((-a*b)^(1/2)*c^2+(
-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))
^(1/2))/(-c^2*b*(a*c^2-d^2))^(1/2)*d^5/(d^2/c^2)^(1/2)*ln((2*d^2/
c^2-2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x+(-c^2*b*(a*c^2-d^2))^(1/2
)/c^2/b)+2*(d^2/c^2)^(1/2)*((x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^
2*b-2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x+(-c^2*b*(a*c^2-d^2))^(1/2
)/c^2/b)+d^2/c^2)^(1/2))/(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b))-a/

```
b/(b*(a*c^2-d^2))^(1/2)*arctan(b*c*x/(b*(a*c^2-d^2))^(1/2))+x/b/c
+1/b/c^2*d^2/(b*(a*c^2-d^2))^(1/2)*arctan(b*c*x/(b*(a*c^2-d^2))^(
1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{bcx^2 + ac + \sqrt{bx^2 + a}\,d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*c*x^2 + a*c + sqrt(b*x^2 + a)*d),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b*c*x^2 + a*c + sqrt(b*x^2 + a)*d), x)
```

Fricas [A] time = 0.422737, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*c*x^2 + a*c + sqrt(b*x^2 + a)*d),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(b)*c*x + 2*d*log(2*sqrt(b*x^2 + a)*b*x - (2*b*x^2 +
a)*sqrt(b)) + sqrt(b)*sqrt(-(a*c^2 - d^2)/b)*log((a^4*c^4 - 2*a^3
*c^2*d^2 + a^2*d^4 + (a^2*b^2*c^4 - 8*a*b^2*c^2*d^2 + 8*b^2*d^4)*
x^4 + 2*(a^3*b*c^4 - 5*a^2*b*c^2*d^2 + 4*a*b*d^4)*x^2 - 4*((a*b^2
*c^2*d - 2*b^2*d^3)*x^3 + (a^2*b*c^2*d - a*b*d^3)*x)*sqrt(b*x^2 +
 a)*sqrt(-(a*c^2 - d^2)/b))/(b^2*c^4*x^4 + a^2*c^4 - 2*a*c^2*d^2
+ d^4 + 2*(a*b*c^4 - b*c^2*d^2)*x^2)) + 2*sqrt(b)*sqrt(-(a*c^2 -
d^2)/b)*log((b*c^2*x^2 - 2*b*c*x*sqrt(-(a*c^2 - d^2)/b) - a*c^2 +
 d^2)/(b*c^2*x^2 + a*c^2 - d^2)))/(b^(3/2)*c^2), 1/4*(4*sqrt(-b)*
c*x - 4*d*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + sqrt(-b)*sqrt(-(a*
c^2 - d^2)/b)*log((a^4*c^4 - 2*a^3*c^2*d^2 + a^2*d^4 + (a^2*b^2*c
^4 - 8*a*b^2*c^2*d^2 + 8*b^2*d^4)*x^4 + 2*(a^3*b*c^4 - 5*a^2*b*c^
2*d^2 + 4*a*b*d^4)*x^2 - 4*((a*b^2*c^2*d - 2*b^2*d^3)*x^3 + (a^2*
b*c^2*d - a*b*d^3)*x)*sqrt(b*x^2 + a)*sqrt(-(a*c^2 - d^2)/b))/(b^
2*c^4*x^4 + a^2*c^4 - 2*a*c^2*d^2 + d^4 + 2*(a*b*c^4 - b*c^2*d^2)
*x^2)) + 2*sqrt(-b)*sqrt(-(a*c^2 - d^2)/b)*log((b*c^2*x^2 - 2*b*c
*x*sqrt(-(a*c^2 - d^2)/b) - a*c^2 + d^2)/(b*c^2*x^2 + a*c^2 - d^2
)))/(sqrt(-b)*b*c^2), 1/2*(2*sqrt(b)*c*x - 2*sqrt(b)*sqrt((a*c^2
- d^2)/b)*arctan(c*x/sqrt((a*c^2 - d^2)/b)) + sqrt(b)*sqrt((a*c^2
 - d^2)/b)*arctan(-1/2*(a^2*c^2 - a*d^2 + (a*b*c^2 - 2*b*d^2)*x^2
)/(sqrt(b*x^2 + a)*b*d*x*sqrt((a*c^2 - d^2)/b))) + d*log(2*sqrt(b
*x^2 + a)*b*x - (2*b*x^2 + a)*sqrt(b)))/(b^(3/2)*c^2), 1/2*(2*sqr
```

```
t(-b)*c*x - 2*sqrt(-b)*sqrt((a*c^2 - d^2)/b)*arctan(c*x/sqrt((a*c
^2 - d^2)/b)) - 2*d*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + sqrt(-b)
*sqrt((a*c^2 - d^2)/b)*arctan(-1/2*(a^2*c^2 - a*d^2 + (a*b*c^2 -
2*b*d^2)*x^2)/(sqrt(b*x^2 + a)*b*d*x*sqrt((a*c^2 - d^2)/b))))/(sq
rt(-b)*b*c^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{ac + bcx^2 + d\sqrt{a + bx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)
```

```
[Out] Integral(x**2/(a*c + b*c*x**2 + d*sqrt(a + b*x**2)), x)
```

GIAC/XCAS [A] time = 0.286272, size = 244, normalized size = 1.66

$$\frac{x}{bc} - \frac{\left(ac^2 - d^2\right)\arctan\left(\frac{bcx}{\sqrt{abc^2 - bd^2}}\right)}{\sqrt{abc^2 - bd^2}\,bc^2} + \frac{d\ln\left(\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^2\right)}{2\,b^{\frac{3}{2}}c^2} - \frac{\left(ac^2d - d^3\right)\arctan\left(\frac{\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^2 c^2 + ac^2 - 2\,d^2}{2\,\sqrt{ac^2 - d^2}\,d}\right)}{\sqrt{ac^2 - d^2}\,b^{\frac{3}{2}}c^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(b*c*x^2 + a*c + sqrt(b*x^2 + a)*d),x, algorithm="giac")
```

```
[Out] x/(b*c) - (a*c^2 - d^2)*arctan(b*c*x/sqrt(a*b*c^2 - b*d^2))/(sqrt
(a*b*c^2 - b*d^2)*b*c^2) + 1/2*d*ln((sqrt(b)*x - sqrt(b*x^2 + a))
^2)/(b^(3/2)*c^2) - (a*c^2*d - d^3)*arctan(1/2*((sqrt(b)*x - sqrt
(b*x^2 + a))^2*c^2 + a*c^2 - 2*d^2)/(sqrt(a*c^2 - d^2)*d))/(sqrt(
a*c^2 - d^2)*b^(3/2)*c^2*d)
```

3.389 $\int \frac{1}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=103

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right)}{\sqrt{b}\sqrt{ac^2-d^2}}-\frac{\tan^{-1}\left(\frac{\sqrt{b}dx}{\sqrt{a+bx^2}\sqrt{ac^2-d^2}}\right)}{\sqrt{b}\sqrt{ac^2-d^2}}$$

```
[Out] ArcTan[(Sqrt[b]*c*x)/Sqrt[a*c^2 - d^2]]/(Sqrt[b]*Sqrt[a*c^2 - d^2
]) - ArcTan[(Sqrt[b]*d*x)/(Sqrt[a*c^2 - d^2]*Sqrt[a + b*x^2])]/(S
qrt[b]*Sqrt[a*c^2 - d^2])
```

Rubi [A] time = 0.147746, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right)}{\sqrt{b}\sqrt{ac^2-d^2}}-\frac{\tan^{-1}\left(\frac{\sqrt{b}dx}{\sqrt{a+bx^2}\sqrt{ac^2-d^2}}\right)}{\sqrt{b}\sqrt{ac^2-d^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])^(-1),x]
```

```
[Out] ArcTan[(Sqrt[b]*c*x)/Sqrt[a*c^2 - d^2]]/(Sqrt[b]*Sqrt[a*c^2 - d^2
]) - ArcTan[(Sqrt[b]*d*x)/(Sqrt[a*c^2 - d^2]*Sqrt[a + b*x^2])]/(S
qrt[b]*Sqrt[a*c^2 - d^2])
```

Rubi in Sympy [A] time = 23.8457, size = 87, normalized size = 0.84

$$-\frac{\operatorname{atanh}\left(\frac{\sqrt{b}cx}{\sqrt{-ac^2+d^2}}\right)}{\sqrt{b}\sqrt{-ac^2+d^2}}+\frac{\operatorname{atanh}\left(\frac{\sqrt{b}dx}{\sqrt{a+bx^2}\sqrt{-ac^2+d^2}}\right)}{\sqrt{b}\sqrt{-ac^2+d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)
```

```
[Out] -atanh(sqrt(b)*c*x/sqrt(-a*c**2 + d**2))/(sqrt(b)*sqrt(-a*c**2 +
d**2)) + atanh(sqrt(b)*d*x/(sqrt(a + b*x**2)*sqrt(-a*c**2 + d**2)
))/(sqrt(b)*sqrt(-a*c**2 + d**2))
```

Mathematica [A] time = 0.0584958, size = 83, normalized size = 0.81

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right)-\tan^{-1}\left(\frac{\sqrt{b}dx}{\sqrt{a+bx^2}\sqrt{ac^2-d^2}}\right)}{\sqrt{b}\sqrt{ac^2-d^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])^(-1),x]
```

```
[Out] (ArcTan[(Sqrt[b]*c*x)/Sqrt[a*c^2 - d^2]] - ArcTan[(Sqrt[b]*d*x)/(
Sqrt[a*c^2 - d^2]*Sqrt[a + b*x^2])])/(Sqrt[b]*Sqrt[a*c^2 - d^2])
```

Maple [B] time = 0.031, size = 2005, normalized size = 19.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x)
```

```
[Out] -1/2*d*b*c^2/(-a*b)^(1/2)/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^
(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))*((x-1/b*(-a*
b)^(1/2))^2*b+2*(-a*b)^(1/2)*(x-1/b*(-a*b)^(1/2)))^(1/2)-1/2*d*b^
(1/2)*c^2/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(
1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))*ln(((x-1/b*(-a*b)^(1/2))*b+(
-a*b)^(1/2))/b^(1/2)+((x-1/b*(-a*b)^(1/2))^2*b+2*(-a*b)^(1/2)*(x-
1/b*(-a*b)^(1/2)))^(1/2))+1/2*d*b*c^2/(-a*b)^(1/2)/((-a*b)^(1/2)*
c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-
d^2))^(1/2))*((x+1/b*(-a*b)^(1/2))^2*b-2*(-a*b)^(1/2)*(x+1/b*(-a*
b)^(1/2)))^(1/2)-1/2*d*b^(1/2)*c^2/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c
^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))*ln(
((x+1/b*(-a*b)^(1/2))*b-(-a*b)^(1/2))/b^(1/2)+((x+1/b*(-a*b)^(1/2
))^2*b-2*(-a*b)^(1/2)*(x+1/b*(-a*b)^(1/2)))^(1/2))+1/2*d*b*c^4/((
-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c
^2*b*(a*c^2-d^2))^(1/2))/(-c^2*b*(a*c^2-d^2))^(1/2)*((x-(-c^2*b*(
a*c^2-d^2))^(1/2)/c^2/b)^2*b+2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x-
(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+d^2/c^2)^(1/2)+1/2*d*b^(1/2)*c^
2/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2
-(-c^2*b*(a*c^2-d^2))^(1/2))*ln(((-c^2*b*(a*c^2-d^2))^(1/2)/c^2+(
x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)*b)/b^(1/2)+((x-(-c^2*b*(a*c^2
-d^2))^(1/2)/c^2/b)^2*b+2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x-(-c^2
*b*(a*c^2-d^2))^(1/2)/c^2/b)+d^2/c^2)^(1/2))-1/2*b*c^2/((-a*b)^(1
/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*
c^2-d^2))^(1/2))/(-c^2*b*(a*c^2-d^2))^(1/2)*d^3/(d^2/c^2)^(1/2)*l
n((2*d^2/c^2+2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x-(-c^2*b*(a*c^2-d
^2))^(1/2)/c^2/b)+2*(d^2/c^2)^(1/2)*((x-(-c^2*b*(a*c^2-d^2))^(1/2
```

```
)/c^2/b)^2*b+2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x-(-c^2*b*(a*c^2-d
^2))^(1/2)/c^2/b)+d^2/c^2)^(1/2))/(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c
^2/b))-1/2*d*b*c^4/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/
((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))/(-c^2*b*(a*c^2-d^2)
)^(1/2)*((x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b-2*(-c^2*b*(a*c^
2-d^2))^(1/2)/c^2*(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+d^2/c^2)^(
1/2)+1/2*d*b^(1/2)*c^2/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/
2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))*ln((-(-c^2*b*(a
*c^2-d^2))^(1/2)/c^2+(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)*b)/b^(1
/2)+((x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b-2*(-c^2*b*(a*c^2-d^
2))^(1/2)/c^2*(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+d^2/c^2)^(1/2)
)+1/2*b*c^2/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)
^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))/(-c^2*b*(a*c^2-d^2))^(1/2)
*d^3/(d^2/c^2)^(1/2)*ln((2*d^2/c^2-2*(-c^2*b*(a*c^2-d^2))^(1/2)/c
^2*(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+2*(d^2/c^2)^(1/2)*((x+(-c
^2*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b-2*(-c^2*b*(a*c^2-d^2))^(1/2)/c
^2*(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+d^2/c^2)^(1/2))/(x+(-c^2*
b*(a*c^2-d^2))^(1/2)/c^2/b))+1/(b*(a*c^2-d^2))^(1/2)*arctan(b*c*x
/(b*(a*c^2-d^2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{bcx^2 + ac + \sqrt{bx^2 + a}d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c*x^2 + a*c + sqrt(b*x^2 + a)*d),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*c*x^2 + a*c + sqrt(b*x^2 + a)*d), x)
```

Fricas [A] time = 0.305486, size = 1, normalized size = 0.01

$$\left[\frac{2\,\log\left(\frac{2\,(abc^3-bcd^2)x+(bc^2x^2-ac^2+d^2)\sqrt{-abc^2+bd^2}}{bc^2x^2+ac^2-d^2}\right)+\log\left(\frac{(a^4c^4-2\,a^3c^2d^2+a^2d^4+(a^2b^2c^4-8\,ab^2c^2d^2+8\,b^2d^4)x^4+2\,(a^3bc^4-5\,a^2bc^2d^2+4\,abd^4)x}{b^2c^4x^4+a^2c^4-2\,ac^2}\right.}{4\sqrt{-abc^2+bd^2}}\right.$$

$$\left.-\frac{2\,\arctan\left(-\frac{\sqrt{abc^2-bd^2}cx}{ac^2-d^2}\right)-\arctan\left(\frac{(a^2c^2-ad^2+(abc^2-2\,bd^2)x^2)\sqrt{abc^2-bd^2}}{2\,(abc^2d-bd^3)\sqrt{bx^2+a}x}\right)}{2\sqrt{abc^2-bd^2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*c*x^2 + a*c + sqrt(b*x^2 + a)*d),x, algorithm="fricas")
```

```
[Out] [1/4*(2*log((2*(a*b*c^3 - b*c*d^2)*x + (b*c^2*x^2 - a*c^2 + d^2)*
sqrt(-a*b*c^2 + b*d^2))/(b*c^2*x^2 + a*c^2 - d^2)) + log(((a^4*c^
4 - 2*a^3*c^2*d^2 + a^2*d^4 + (a^2*b^2*c^4 - 8*a*b^2*c^2*d^2 + 8*
b^2*d^4)*x^4 + 2*(a^3*b*c^4 - 5*a^2*b*c^2*d^2 + 4*a*b*d^4)*x^2)*s
qrt(-a*b*c^2 + b*d^2) + 4*((a^2*b^2*c^4*d - 3*a*b^2*c^2*d^3 + 2*b
^2*d^5)*x^3 + (a^3*b*c^4*d - 2*a^2*b*c^2*d^3 + a*b*d^5)*x)*sqrt(b
*x^2 + a))/(b^2*c^4*x^4 + a^2*c^4 - 2*a*c^2*d^2 + d^4 + 2*(a*b*c^
4 - b*c^2*d^2)*x^2)))/sqrt(-a*b*c^2 + b*d^2), -1/2*(2*arctan(-sqr
t(a*b*c^2 - b*d^2)*c*x/(a*c^2 - d^2)) - arctan(1/2*(a^2*c^2 - a*d
^2 + (a*b*c^2 - 2*b*d^2)*x^2)*sqrt(a*b*c^2 - b*d^2)/((a*b*c^2*d -
 b*d^3)*sqrt(b*x^2 + a)*x)))/sqrt(a*b*c^2 - b*d^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{ac + bcx^2 + d\sqrt{a + bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)
```

```
[Out] Integral(1/(a*c + b*c*x**2 + d*sqrt(a + b*x**2)), x)
```

GIAC/XCAS [A] time = 0.279895, size = 144, normalized size = 1.4

$$\frac{\arctan\left(\frac{bcx}{\sqrt{abc^2 - bd^2}}\right)}{\sqrt{abc^2 - bd^2}} + \frac{\arctan\left(\frac{\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^2 c^2 + ac^2 - 2\,d^2}{2\,\sqrt{ac^2 - d^2}d}\right)}{\sqrt{ac^2 - d^2}\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*c*x^2 + a*c + sqrt(b*x^2 + a)*d),x, algorithm="giac")
```

```
[Out] arctan(b*c*x/sqrt(a*b*c^2 - b*d^2))/sqrt(a*b*c^2 - b*d^2) + arcta
n(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*c^2 + a*c^2 - 2*d^2)/(sqrt
(a*c^2 - d^2)*d))/(sqrt(a*c^2 - d^2)*sqrt(b))
```

3.390 $\int \frac{1}{x^2\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx$

Optimal. Leaf size=160

$$\frac{d\sqrt{a+bx^2}}{ax\left(ac^2-d^2\right)}+\frac{\sqrt{b}c^2\tan^{-1}\left(\frac{\sqrt{b}dx}{\sqrt{a+bx^2}\sqrt{ac^2-d^2}}\right)}{\left(ac^2-d^2\right)^{3/2}}-\frac{\sqrt{b}c^2\tan^{-1}\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right)}{\left(ac^2-d^2\right)^{3/2}}-\frac{c}{x\left(ac^2-d^2\right)}$$

```
[Out] -(c/((a*c^2 - d^2)*x)) + (d*Sqrt[a + b*x^2])/(a*(a*c^2 - d^2)*x)
- (Sqrt[b]*c^2*ArcTan[(Sqrt[b]*c*x)/Sqrt[a*c^2 - d^2]])/(a*c^2 -
d^2)^(3/2) + (Sqrt[b]*c^2*ArcTan[(Sqrt[b]*d*x)/(Sqrt[a*c^2 - d^2]
*Sqrt[a + b*x^2])])/(a*c^2 - d^2)^(3/2)
```

Rubi [A] time = 0.505991, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207

$$\frac{d\sqrt{a+bx^2}}{ax\left(ac^2-d^2\right)}+\frac{\sqrt{b}c^2\tan^{-1}\left(\frac{\sqrt{b}dx}{\sqrt{a+bx^2}\sqrt{ac^2-d^2}}\right)}{\left(ac^2-d^2\right)^{3/2}}-\frac{\sqrt{b}c^2\tan^{-1}\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right)}{\left(ac^2-d^2\right)^{3/2}}-\frac{c}{x\left(ac^2-d^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])),x]
```

```
[Out] -(c/((a*c^2 - d^2)*x)) + (d*Sqrt[a + b*x^2])/(a*(a*c^2 - d^2)*x)
- (Sqrt[b]*c^2*ArcTan[(Sqrt[b]*c*x)/Sqrt[a*c^2 - d^2]])/(a*c^2 -
d^2)^(3/2) + (Sqrt[b]*c^2*ArcTan[(Sqrt[b]*d*x)/(Sqrt[a*c^2 - d^2]
*Sqrt[a + b*x^2])])/(a*c^2 - d^2)^(3/2)
```

Rubi in Sympy [A] time = 50.8058, size = 129, normalized size = 0.81

$$-\frac{\sqrt{b}c^2\operatorname{atanh}\left(\frac{\sqrt{b}cx}{\sqrt{-ac^2+d^2}}\right)}{\left(-ac^2+d^2\right)^{\frac{3}{2}}}+\frac{\sqrt{b}c^2\operatorname{atanh}\left(\frac{\sqrt{b}dx}{\sqrt{a+bx^2}\sqrt{-ac^2+d^2}}\right)}{\left(-ac^2+d^2\right)^{\frac{3}{2}}}+\frac{c}{x\left(-ac^2+d^2\right)}-\frac{d\sqrt{a+bx^2}}{ax\left(-ac^2+d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)
```

```
[Out] -sqrt(b)*c**2*atanh(sqrt(b)*c*x/sqrt(-a*c**2 + d**2))/(-a*c**2 +
d**2)**(3/2) + sqrt(b)*c**2*atanh(sqrt(b)*d*x/(sqrt(a + b*x**2)*s
```

```
qrt(-a*c**2 + d**2)))/(-a*c**2 + d**2)**(3/2) + c/(x*(-a*c**2 + d
**2)) - d*sqrt(a + b*x**2)/(a*x*(-a*c**2 + d**2))
```

Mathematica [A] time = 0.17119, size = 139, normalized size = 0.87

$$\frac{\sqrt{ac^2-d^2}\left(d\sqrt{a+bx^2}-ac\right)+a\sqrt{b}c^2x\tan^{-1}\left(\frac{\sqrt{b}dx}{\sqrt{a+bx^2}\sqrt{ac^2-d^2}}\right)-a\sqrt{b}c^2x\tan^{-1}\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right)}{ax\left(ac^2-d^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])),x]
```

```
[Out] (Sqrt[a*c^2 - d^2]*(-(a*c) + d*Sqrt[a + b*x^2]) - a*Sqrt[b]*c^2*x
*ArcTan[(Sqrt[b]*c*x)/Sqrt[a*c^2 - d^2]] + a*Sqrt[b]*c^2*x*ArcTan
[(Sqrt[b]*d*x)/(Sqrt[a*c^2 - d^2]*Sqrt[a + b*x^2])])/(a*(a*c^2 -
d^2)^(3/2)*x)
```

Maple [B] time = 0.044, size = 2289, normalized size = 14.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x)
```

```
[Out] b*c^2/d^2/(b*(a*c^2-d^2))^(1/2)*arctan(b*c*x/(b*(a*c^2-d^2))^(1/2
))-a*c^4/(a*c^2-d^2)*b/d^2/(b*(a*c^2-d^2))^(1/2)*arctan(b*c*x/(b*
(a*c^2-d^2))^(1/2))-c/(a*c^2-d^2)/x+d/a^2/(a*c^2-d^2)/x*(b*x^2+a)
^(3/2)-d/a^2/(a*c^2-d^2)*b*x*(b*x^2+a)^(1/2)-d/a/(a*c^2-d^2)*b^(1
/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+1/2*d*b^2*c^2/a/(-a*b)^(1/2)/((
-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c
^2*b*(a*c^2-d^2))^(1/2))*((x-1/b*(-a*b)^(1/2))^2*b+2*(-a*b)^(1/2)
*(x-1/b*(-a*b)^(1/2)))^(1/2)+1/2*d*b^(3/2)*c^2/a/((-a*b)^(1/2)*c^
2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^
2))^(1/2))*ln(((x-1/b*(-a*b)^(1/2))*b+(-a*b)^(1/2))/b^(1/2)+((x-1
/b*(-a*b)^(1/2))^2*b+2*(-a*b)^(1/2)*(x-1/b*(-a*b)^(1/2)))^(1/2))-
1/2*d*b^2*c^2/a/(-a*b)^(1/2)/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2
))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))*((x+1/b*(
-a*b)^(1/2))^2*b-2*(-a*b)^(1/2)*(x+1/b*(-a*b)^(1/2)))^(1/2)+1/2*d
*b^(3/2)*c^2/a/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a
*b)^(1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))*ln(((x+1/b*(-a*b)^(1/2)
)*b-(-a*b)^(1/2))/b^(1/2)+((x+1/b*(-a*b)^(1/2))^2*b-2*(-a*b)^(1/2
)*(x+1/b*(-a*b)^(1/2)))^(1/2))-1/2*d*b^2*c^6/(a*c^2-d^2)/((-a*b)^
(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(
```

```
a*c^2-d^2))^(1/2))/(-c^2*b*(a*c^2-d^2))^(1/2)*((x-(-c^2*b*(a*c^2-
d^2))^(1/2)/c^2/b)^2*b+2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x-(-c^2*
b*(a*c^2-d^2))^(1/2)/c^2/b)+d^2/c^2)^(1/2)-1/2*d*b^(3/2)*c^4/(a*c
^2-d^2)/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/
2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))*ln(((-c^2*b*(a*c^2-d^2))^(1/2)
/c^2+(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)*b)/b^(1/2)+((x-(-c^2*b*
(a*c^2-d^2))^(1/2)/c^2/b)^2*b+2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x
-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+d^2/c^2)^(1/2))+1/2*b^2*c^4/(a
*c^2-d^2)/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(
1/2)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))/(-c^2*b*(a*c^2-d^2))^(1/2)*d
^3/(d^2/c^2)^(1/2)*ln((2*d^2/c^2+2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2
*(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+2*(d^2/c^2)^(1/2)*((x-(-c^2
*b*(a*c^2-d^2))^(1/2)/c^2/b)^2*b+2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2
*(x-(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+d^2/c^2)^(1/2))/(x-(-c^2*b*
(a*c^2-d^2))^(1/2)/c^2/b))+1/2*d*b^2*c^6/(a*c^2-d^2)/((-a*b)^(1/2
)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c^2-(-c^2*b*(a*c^
2-d^2))^(1/2))/(-c^2*b*(a*c^2-d^2))^(1/2)*((x+(-c^2*b*(a*c^2-d^2)
)^(1/2)/c^2/b)^2*b-2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x+(-c^2*b*(a
*c^2-d^2))^(1/2)/c^2/b)+d^2/c^2)^(1/2)-1/2*d*b^(3/2)*c^4/(a*c^2-d
^2)/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2)*c
^2-(-c^2*b*(a*c^2-d^2))^(1/2))*ln((-(-c^2*b*(a*c^2-d^2))^(1/2)/c^
2+(x+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)*b)/b^(1/2)+((x+(-c^2*b*(a*
c^2-d^2))^(1/2)/c^2/b)^2*b-2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x+(-
c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+d^2/c^2)^(1/2))-1/2*b^2*c^4/(a*c^
2-d^2)/((-a*b)^(1/2)*c^2+(-c^2*b*(a*c^2-d^2))^(1/2))/((-a*b)^(1/2
)*c^2-(-c^2*b*(a*c^2-d^2))^(1/2))/(-c^2*b*(a*c^2-d^2))^(1/2)*d^3/
(d^2/c^2)^(1/2)*ln((2*d^2/c^2-2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x
+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+2*(d^2/c^2)^(1/2)*((x+(-c^2*b*
(a*c^2-d^2))^(1/2)/c^2/b)^2*b-2*(-c^2*b*(a*c^2-d^2))^(1/2)/c^2*(x
+(-c^2*b*(a*c^2-d^2))^(1/2)/c^2/b)+d^2/c^2)^(1/2))/(x+(-c^2*b*(a*
c^2-d^2))^(1/2)/c^2/b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bcx^2 + ac + \sqrt{bx^2 + a}d\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*c*x^2 + a*c + sqrt(b*x^2 + a)*d)*x^2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*c*x^2 + a*c + sqrt(b*x^2 + a)*d)*x^2), x)
```

Fricas [A] time = 0.39092, size = 1, normalized size = 0.01

$$\left[-\frac{ac^2x\sqrt{-\frac{b}{ac^2-d^2}}\log\left(\frac{a^4c^4-2\,a^3c^2d^2+a^2d^4+\left(a^2b^2c^4-8\,ab^2c^2d^2+8\,b^2d^4\right)x^4+2\left(a^3bc^4-5\,a^2bc^2d^2+4\,abd^4\right)x^2+4\left(\left(a^2bc^4d-3\,abc^2d^3+2\,bd^5\right)x^3+\left(a^3c^4d\right.\right.}{b^2c^4x^4+a^2c^4-2\,ac^2d^2+d^4+2\left(abc^4-bc^2d^2\right)x^2}\right.}{4\left(a^2c^2-ad\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*c*x^2 + a*c + sqrt(b*x^2 + a)*d)*x^2),x, algorithm="fricas")
```

```
[Out] [-1/4*(a*c^2*x*sqrt(-b/(a*c^2 - d^2))*log((a^4*c^4 - 2*a^3*c^2*d^
2 + a^2*d^4 + (a^2*b^2*c^4 - 8*a*b^2*c^2*d^2 + 8*b^2*d^4)*x^4 + 2
*(a^3*b*c^4 - 5*a^2*b*c^2*d^2 + 4*a*b*d^4)*x^2 + 4*((a^2*b*c^4*d
- 3*a*b*c^2*d^3 + 2*b*d^5)*x^3 + (a^3*c^4*d - 2*a^2*c^2*d^3 + a*d
^5)*x)*sqrt(b*x^2 + a)*sqrt(-b/(a*c^2 - d^2)))/(b^2*c^4*x^4 + a^2
*c^4 - 2*a*c^2*d^2 + d^4 + 2*(a*b*c^4 - b*c^2*d^2)*x^2)) + 2*a*c^
2*x*sqrt(-b/(a*c^2 - d^2))*log((b*c^2*x^2 - a*c^2 + 2*(a*c^3 - c*
d^2)*x*sqrt(-b/(a*c^2 - d^2)) + d^2)/(b*c^2*x^2 + a*c^2 - d^2)) +
 4*a*c - 4*sqrt(b*x^2 + a)*d)/((a^2*c^2 - a*d^2)*x), 1/2*(2*a*c^2
*x*sqrt(b/(a*c^2 - d^2))*arctan(-b*c*x/((a*c^2 - d^2)*sqrt(b/(a*c
^2 - d^2)))) - a*c^2*x*sqrt(b/(a*c^2 - d^2))*arctan(1/2*(a^2*c^2
- a*d^2 + (a*b*c^2 - 2*b*d^2)*x^2)/((a*c^2*d - d^3)*sqrt(b*x^2 +
a)*x*sqrt(b/(a*c^2 - d^2)))) - 2*a*c + 2*sqrt(b*x^2 + a)*d)/((a^2
*c^2 - a*d^2)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)
```

```
[Out] Integral(1/(x**2*(a*c + b*c*x**2 + d*sqrt(a + b*x**2))), x)
```

GIAC/XCAS [A] time = 0.284054, size = 285, normalized size = 1.78

$$-b^{\frac{3}{2}}d\left(\frac{c^2\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2c^2+ac^2-2\,d^2}{2\,\sqrt{ac^2-d^2}d}\right)}{(abc^2-bd^2)\sqrt{ac^2-d^2}d}+\frac{2}{(abc^2-bd^2)\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)}\right)-\frac{bc^2\arctan\left(\frac{bcx}{\sqrt{abc^2-bd^2}}\right)}{\sqrt{abc^2-bd^2}(ac^2-d^2)}-\frac{c}{(ac^2-d^2)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*c*x^2 + a*c + sqrt(b*x^2 + a)*d)*x^2),x, algorithm="giac")
```

```
[Out] -b^(3/2)*d*(c^2*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*c^2 +
 a*c^2 - 2*d^2)/(sqrt(a*c^2 - d^2)*d))/((a*b*c^2 - b*d^2)*sqrt(a*
c^2 - d^2)*d) + 2/((a*b*c^2 - b*d^2)*((sqrt(b)*x - sqrt(b*x^2 + a
))^2 - a))) - b*c^2*arctan(b*c*x/sqrt(a*b*c^2 - b*d^2))/(sqrt(a*b
*c^2 - b*d^2)*(a*c^2 - d^2)) - c/((a*c^2 - d^2)*x)
```

3.391 $\int \frac{x^8}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=140

$$-\frac{2d\left(a+bx^3\right)^{3/2}}{9b^3c^2}+\frac{2\left(ac^2-d^2\right)^2\log\left(c\sqrt{a+bx^3}+d\right)}{3b^3c^5}+\frac{2d\sqrt{a+bx^3}\left(2ac^2-d^2\right)}{3b^3c^4}+\frac{\left(a+bx^3\right)^2}{6b^3c}-\frac{x^3\left(2ac^2-d^2\right)}{3b^2c^3}$$

```
[Out] -((2*a*c^2 - d^2)*x^3)/(3*b^2*c^3) + (2*d*(2*a*c^2 - d^2)*Sqrt[a
+ b*x^3])/(3*b^3*c^4) - (2*d*(a + b*x^3)^(3/2))/(9*b^3*c^2) + (a
+ b*x^3)^2/(6*b^3*c) + (2*(a*c^2 - d^2)^2*Log[d + c*Sqrt[a + b*x^
3]])/(3*b^3*c^5)
```

Rubi [A] time = 0.610609, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069

$$-\frac{2d\left(a+bx^3\right)^{3/2}}{9b^3c^2}+\frac{2\left(ac^2-d^2\right)^2\log\left(c\sqrt{a+bx^3}+d\right)}{3b^3c^5}+\frac{2d\sqrt{a+bx^3}\left(2ac^2-d^2\right)}{3b^3c^4}+\frac{\left(a+bx^3\right)^2}{6b^3c}-\frac{x^3\left(2ac^2-d^2\right)}{3b^2c^3}$$

Antiderivative was successfully verified.

```
[In]  Int[x^8/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]
```

```
[Out] -((2*a*c^2 - d^2)*x^3)/(3*b^2*c^3) + (2*d*(2*a*c^2 - d^2)*Sqrt[a
+ b*x^3])/(3*b^3*c^4) - (2*d*(a + b*x^3)^(3/2))/(9*b^3*c^2) + (a
+ b*x^3)^2/(6*b^3*c) + (2*(a*c^2 - d^2)^2*Log[d + c*Sqrt[a + b*x^
3]])/(3*b^3*c^5)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(a+bx^3\right)^2}{6b^3c}-\frac{2d\left(a+bx^3\right)^{\frac{3}{2}}}{9b^3c^2}+\frac{2\left(-2ac^2+d^2\right)\int^{\sqrt{a+bx^3}}x\,dx}{3b^3c^3}-\frac{2\left(-2ac^2+d^2\right)\int^{\sqrt{a+bx^3}}d\,dx}{3b^3c^4}+\frac{2\left(-ac^2+d^2\right)^2\log\left(c\sqrt{a+bx^3}+d\right)}{3b^3c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**8/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
```

```
[Out] (a + b*x**3)**2/(6*b**3*c) - 2*d*(a + b*x**3)**(3/2)/(9*b**3*c**2
) + 2*(-2*a*c**2 + d**2)*Integral(x, (x, sqrt(a + b*x**3)))/(3*b*
*3*c**3) - 2*(-2*a*c**2 + d**2)*Integral(d, (x, sqrt(a + b*x**3))
)/(3*b**3*c**4) + 2*(-a*c**2 + d**2)**2*log(c*sqrt(a + b*x**3) +
d)/(3*b**3*c**5)
```

Mathematica [A] time = 0.266094, size = 161, normalized size = 1.15

$$\frac{c\left(a\left(20c^2d\sqrt{a+bx^3}-6bc^3x^3\right)+2bcdx^3\left(3d-2c\sqrt{a+bx^3}\right)-12d^3\sqrt{a+bx^3}+3b^2c^3x^6\right)+6\left(d^2-ac^2\right)^2\log\left(ac^2+bc^2x^3\right.}{18b^3c^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^8/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]
```

```
[Out] (c*(3*b^2*c^3*x^6 - 12*d^3*Sqrt[a + b*x^3] + 2*b*c*d*x^3*(3*d - 2
*c*Sqrt[a + b*x^3]) + a*(-6*b*c^3*x^3 + 20*c^2*d*Sqrt[a + b*x^3])
) + 12*(-(a*c^2) + d^2)^2*ArcTanh[(c*Sqrt[a + b*x^3])/d] + 6*(-(a
*c^2) + d^2)^2*Log[a*c^2 - d^2 + b*c^2*x^3])/(18*b^3*c^5)
```

Maple [C] time = 0.183, size = 1473, normalized size = 10.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^8/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x)
```

```
[Out] -2/9*d*(b*x^3+a)^(3/2)/b^3/c^2+4/3*d/b^3/c^2*(b*x^3+a)^(1/2)*a-2/
3/b^3/c^4*d^3*(b*x^3+a)^(1/2)-1/3*I/b^5/d*2^(1/2)*sum((-a*b^2)^(1
/3)*(1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)-I*3^(1/2)*(-a*b^2)^(1/3)))/
(-a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(-a*b^2)^(1/3))/(-3*(-a*b^2)^(1/3
)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(2*x+1/b*((-a*b^2)^(
1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(
1/2)*(I*(-a*b^2)^(1/3)*3^(1/2)*_alpha*b-I*(-a*b^2)^(2/3)*3^(1/2)+
2*_alpha^2*b^2-(-a*b^2)^(1/3)*_alpha*b-(-a*b^2)^(2/3))*EllipticPi
(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),-1/2*c^2/b*(2*I*3^(1/2)*(-
a*b^2)^(1/3)*_alpha^2*b-I*3^(1/2)*(-a*b^2)^(2/3)*_alpha+I*3^(1/2)
*a*b-3*(-a*b^2)^(2/3)*_alpha-3*a*b)/d^2,(I*3^(1/2)/b*(-a*b^2)^(1/
3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

```
,_alpha=RootOf(_Z^3*b*c^2+a*c^2-d^2))*a^2+2/3*I*d/b^5/c^2*2^(1/2)
*sum((-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)-I*3^(1/2)*(
-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(-a*b^2)^(1/3))/(
-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(2*x
+1/b*((-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(
1/2)/(b*x^3+a)^(1/2)*(I*(-a*b^2)^(1/3)*3^(1/2)*_alpha*b-I*(-a*b^2
)^(2/3)*3^(1/2)+2*_alpha^2*b^2-(-a*b^2)^(1/3)*_alpha*b-(-a*b^2)^(
2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),-1/2*c^2/b
*(2*I*3^(1/2)*(-a*b^2)^(1/3)*_alpha^2*b-I*3^(1/2)*(-a*b^2)^(2/3)*
_alpha+I*3^(1/2)*a*b-3*(-a*b^2)^(2/3)*_alpha-3*a*b)/d^2,(I*3^(1/2
)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b*c^2+a*c^2-d^2))*a-1/3*I/b^5
/c^4*d^3*2^(1/2)*sum((-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/b*((-a*b^2)^(
1/3)-I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(
-a*b^2)^(1/3))/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2
)*(-1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))/(
-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*(I*(-a*b^2)^(1/3)*3^(1/2)*_a
lpha*b-I*(-a*b^2)^(2/3)*3^(1/2)+2*_alpha^2*b^2-(-a*b^2)^(1/3)*_al
pha*b-(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)
^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^
(1/2),-1/2*c^2/b*(2*I*3^(1/2)*(-a*b^2)^(1/3)*_alpha^2*b-I*3^(1/2)
*(-a*b^2)^(2/3)*_alpha+I*3^(1/2)*a*b-3*(-a*b^2)^(2/3)*_alpha-3*a*
b)/d^2,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b*c^2+a*c^2-d
^2))-1/3*a/c/b^2*x^3+1/3*a^2/c/b^3*ln(b*c^2*x^3+a*c^2-d^2)-2/3*a/
c^3/b^3*d^2*ln(b*c^2*x^3+a*c^2-d^2)+1/6/b/c*x^6+1/3/b^2/c^3*x^3*d
^2+1/3/b^3/c^5*d^4*ln(b*c^2*x^3+a*c^2-d^2)
```

Maxima [A] time = 0.695355, size = 169, normalized size = 1.21

$$\frac{\frac{3\,(bx^3+a)^2c^3-4\,(bx^3+a)^{\frac{3}{2}}c^2d-6\,(2\,ac^3-cd^2)\,(bx^3+a)+12\,(2\,ac^2d-d^3)\sqrt{bx^3+a}}{c^4}+\frac{12\,(a^2c^4-2\,ac^2d^2+d^4)\log\left(\sqrt{bx^3+a}c+d\right)}{c^5}}{18\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*c*x^3 + a*c + sqrt(b*x^3 + a)*d),x, algorithm="maxima")
```

```
[Out] 1/18*((3*(b*x^3 + a)^2*c^3 - 4*(b*x^3 + a)^(3/2)*c^2*d - 6*(2*a*c
^3 - c*d^2)*(b*x^3 + a) + 12*(2*a*c^2*d - d^3)*sqrt(b*x^3 + a))/c
^4 + 12*(a^2*c^4 - 2*a*c^2*d^2 + d^4)*log(sqrt(b*x^3 + a)*c + d)/
c^5)/b^3
```

Fricas [A] time = 0.276609, size = 258, normalized size = 1.84

$$\frac{3\,b^2c^4x^6-6\left(abc^4-bc^2d^2\right)x^3+6\left(a^2c^4-2\,ac^2d^2+d^4\right)\log\left(bc^2x^3+ac^2-d^2\right)+6\left(a^2c^4-2\,ac^2d^2+d^4\right)\log\left(\sqrt{bx^3+a}c+d\right)}{18\,b^3c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*c*x^3 + a*c + sqrt(b*x^3 + a)*d),x, algorithm="fricas")
```

```
[Out] 1/18*(3*b^2*c^4*x^6 - 6*(a*b*c^4 - b*c^2*d^2)*x^3 + 6*(a^2*c^4 -
2*a*c^2*d^2 + d^4)*log(b*c^2*x^3 + a*c^2 - d^2) + 6*(a^2*c^4 - 2*
a*c^2*d^2 + d^4)*log(sqrt(b*x^3 + a)*c + d) - 6*(a^2*c^4 - 2*a*c^
2*d^2 + d^4)*log(sqrt(b*x^3 + a)*c - d) - 4*(b*c^3*d*x^3 - 5*a*c^
3*d + 3*c*d^3)*sqrt(b*x^3 + a))/(b^3*c^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.276099, size = 211, normalized size = 1.51

$$\frac{2\left(a^2c^4-2\,ac^2d^2+d^4\right)\ln\left(\left|\sqrt{bx^3+a}c+d\right|\right)}{3\,b^3c^5}+\frac{3\left(bx^3+a\right)^2b^9c^3-12\left(bx^3+a\right)ab^9c^3-4\left(bx^3+a\right)^{\frac{3}{2}}b^9c^2d+24\sqrt{bx^3+a}ab^9c^2d+6\left(bx^3+a\right)b^9cd^2-12\sqrt{bx^3+a}b^9d^3}{18\,b^{12}c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*c*x^3 + a*c + sqrt(b*x^3 + a)*d),x, algorithm="giac")
```

```
[Out] 2/3*(a^2*c^4 - 2*a*c^2*d^2 + d^4)*ln(abs(sqrt(b*x^3 + a)*c + d))/
(b^3*c^5) + 1/18*(3*(b*x^3 + a)^2*b^9*c^3 - 12*(b*x^3 + a)*a*b^9*
c^3 - 4*(b*x^3 + a)^(3/2)*b^9*c^2*d + 24*sqrt(b*x^3 + a)*a*b^9*c^
2*d + 6*(b*x^3 + a)*b^9*c*d^2 - 12*sqrt(b*x^3 + a)*b^9*d^3)/(b^12
*c^4)
```

3.392 $\int \frac{x^5}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=73

$$-\frac{2d\sqrt{a+bx^3}}{3b^2c^2}-\frac{2\left(ac^2-d^2\right)\log\left(c\sqrt{a+bx^3}+d\right)}{3b^2c^3}+\frac{x^3}{3bc}$$

```
[Out] x^3/(3*b*c) - (2*d*Sqrt[a + b*x^3])/(3*b^2*c^2) - (2*(a*c^2 - d^2
)*Log[d + c*Sqrt[a + b*x^3]])/(3*b^2*c^3)
```

Rubi [A] time = 0.367028, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069

$$-\frac{2d\sqrt{a+bx^3}}{3b^2c^2}-\frac{2\left(ac^2-d^2\right)\log\left(c\sqrt{a+bx^3}+d\right)}{3b^2c^3}+\frac{x^3}{3bc}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]
```

```
[Out] x^3/(3*b*c) - (2*d*Sqrt[a + b*x^3])/(3*b^2*c^2) - (2*(a*c^2 - d^2
)*Log[d + c*Sqrt[a + b*x^3]])/(3*b^2*c^3)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\int^{\sqrt{a+bx^3}}x\,dx}{3b^2c}-\frac{2\int^{\sqrt{a+bx^3}}d\,dx}{3b^2c^2}+\frac{2\left(-ac^2+d^2\right)\log\left(c\sqrt{a+bx^3}+d\right)}{3b^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
```

```
[Out] 2*Integral(x, (x, sqrt(a + b*x**3)))/(3*b**2*c) - 2*Integral(d, (
x, sqrt(a + b*x**3)))/(3*b**2*c**2) + 2*(-a*c**2 + d**2)*log(c*sq
rt(a + b*x**3) + d)/(3*b**2*c**3)
```

Mathematica [A] time = 0.102777, size = 95, normalized size = 1.3

$$\frac{(d^2-ac^2)\log(ac^2+bc^2x^3-d^2)+(2d^2-2ac^2)\tanh^{-1}\left(\frac{c\sqrt{a+bx^3}}{d}\right)+c\left(bcx^3-2d\sqrt{a+bx^3}\right)}{3b^2c^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]
```

```
[Out] (c*(b*c*x^3 - 2*d*Sqrt[a + b*x^3]) + (-2*a*c^2 + 2*d^2)*ArcTanh[(
c*Sqrt[a + b*x^3])/d] + (-(a*c^2) + d^2)*Log[a*c^2 - d^2 + b*c^2*
x^3])/(3*b^2*c^3)
```

Maple [C] time = 0.021, size = 943, normalized size = 12.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x)
```

```
[Out] -2/3*d*(b*x^3+a)^(1/2)/b^2/c^2+1/3*I/b^4/d*2^(1/2)*sum((-a*b^2)^(
1/3)*(1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)-I*3^(1/2)*(-a*b^2)^(1/3)))
/(-a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(-a*b^2)^(1/3))/(-3*(-a*b^2)^(1/
3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(2*x+1/b*((-a*b^2)^
(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^
(1/2)*(I*(-a*b^2)^(1/3)*3^(1/2)*_alpha*b-I*(-a*b^2)^(2/3)*3^(1/2)
+2*_alpha^2*b^2-(-a*b^2)^(1/3)*_alpha*b-(-a*b^2)^(2/3))*EllipticP
i(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),-1/2*c^2/b*(2*I*3^(1/2)*(
-a*b^2)^(1/3)*_alpha^2*b-I*3^(1/2)*(-a*b^2)^(2/3)*_alpha+I*3^(1/2
)*a*b-3*(-a*b^2)^(2/3)*_alpha-3*a*b)/d^2,(I*3^(1/2)/b*(-a*b^2)^(1
/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)
),_alpha=RootOf(_Z^3*b*c^2+a*c^2-d^2))*a-1/3*I*d/b^4/c^2*2^(1/2)*
sum((-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)-I*3^(1/2)*(-
a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(-a*b^2)^(1/3))/(-
3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(2*x+
1/b*((-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1
/2)/(b*x^3+a)^(1/2)*(I*(-a*b^2)^(1/3)*3^(1/2)*_alpha*b-I*(-a*b^2)
^(2/3)*3^(1/2)+2*_alpha^2*b^2-(-a*b^2)^(1/3)*_alpha*b-(-a*b^2)^(2
/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),-1/2*c^2/b*
(2*I*3^(1/2)*(-a*b^2)^(1/3)*_alpha^2*b-I*3^(1/2)*(-a*b^2)^(2/3)*_
alpha+I*3^(1/2)*a*b-3*(-a*b^2)^(2/3)*_alpha-3*a*b)/d^2,(I*3^(1/2)
/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b*c^2+a*c^2-d^2))-1/3*a/c/b^2*
ln(b*c^2*x^3+a*c^2-d^2)+1/3*x^3/b/c+1/3/b^2/c^3*d^2*ln(b*c^2*x^3+
```

```
a*c^2-d^2)
```

Maxima [A] time = 0.701437, size = 84, normalized size = 1.15

$$\frac{\frac{(bx^3+a)c-2\sqrt{bx^3+a}d}{c^2}-\frac{2(ac^2-d^2)\log\left(\sqrt{bx^3+a}c+d\right)}{c^3}}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*c*x^3 + a*c + sqrt(b*x^3 + a)*d),x, algorithm="maxima")
```

```
[Out] 1/3*(((b*x^3 + a)*c - 2*sqrt(b*x^3 + a)*d)/c^2 - 2*(a*c^2 - d^2)*
log(sqrt(b*x^3 + a)*c + d)/c^3)/b^2
```

Fricas [A] time = 0.279972, size = 159, normalized size = 2.18

$$\frac{bc^2x^3-2\sqrt{bx^3+a}cd-(ac^2-d^2)\log\left(bc^2x^3+ac^2-d^2\right)-(ac^2-d^2)\log\left(\sqrt{bx^3+a}c+d\right)+(ac^2-d^2)\log\left(\sqrt{bx^3+a}c-\right.}{3b^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*c*x^3 + a*c + sqrt(b*x^3 + a)*d),x, algorithm="fricas")
```

```
[Out] 1/3*(b*c^2*x^3 - 2*sqrt(b*x^3 + a)*c*d - (a*c^2 - d^2)*log(b*c^2*
x^3 + a*c^2 - d^2) - (a*c^2 - d^2)*log(sqrt(b*x^3 + a)*c + d) + (
a*c^2 - d^2)*log(sqrt(b*x^3 + a)*c - d))/(b^2*c^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.271632, size = 97, normalized size = 1.33

$$-\frac{\frac{2\,(ac^2-d^2)\ln\left(\left|\sqrt{bx^3+a}c+d\right|\right)}{bc^3}-\frac{(bx^3+a)\,bc-2\,\sqrt{bx^3+a}bd}{b^2c^2}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*c*x^3 + a*c + sqrt(b*x^3 + a)*d),x, algorithm="giac")
```

```
[Out] -1/3*(2*(a*c^2 - d^2)*ln(abs(sqrt(b*x^3 + a)*c + d))/(b*c^3) - ((
b*x^3 + a)*b*c - 2*sqrt(b*x^3 + a)*b*d)/(b^2*c^2))/b
```

3.393 $\int \frac{x^2}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=26

$$\frac{2\log\left(c\sqrt{a+bx^3}+d\right)}{3bc}$$

```
[Out] (2*Log[d + c*Sqrt[a + b*x^3]])/(3*b*c)
```

Rubi [A] time = 0.176762, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069

$$\frac{2\log\left(c\sqrt{a+bx^3}+d\right)}{3bc}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]

[Out] (2*Log[d + c*Sqrt[a + b*x^3]])/(3*b*c)
```

Rubi in Sympy [A] time = 8.79665, size = 20, normalized size = 0.77

$$\frac{2\log\left(c\sqrt{a+bx^3}+d\right)}{3bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)

[Out] 2*log(c*sqrt(a + b*x**3) + d)/(3*b*c)
```

Mathematica [A] time = 0.0160763, size = 26, normalized size = 1.

$$\frac{2\log\left(c\sqrt{a+bx^3}+d\right)}{3bc}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*Log[d + c*Sqrt[a + b*x^3]])/(3*b*c)
```

Maple [C] time = 0.02, size = 455, normalized size = 17.5

$$\frac{-\frac{i}{3}\sqrt{2}}{b^3 d} \sum_{\_alpha = RootOf\left(\_Z^3 b c^2 + a c^2 - d^2\right)} 1\sqrt[3]{-ab^2}\sqrt{\frac{i}{2} b\left(2x + \frac{1}{b}\left(\sqrt[3]{-ab^2} - i\sqrt{3}\sqrt[3]{-ab^2}\right)\right)\frac{1}{\sqrt[3]{-ab^2}}}\sqrt{b\left(x - \frac{1}{b}\sqrt[3]{-ab^2}\right)\left(-3\sqrt[3]{-ab^2} + i\right.}$$
$$+ \frac{\ln\left(bc^2x^3 + ac^2 - d^2\right)}{3bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x)
```

```
[Out] -1/3*I/d/b^3*2^(1/2)*sum((-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/b*((-a*b^
2)^(1/3)-I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-1
/b*(-a*b^2)^(1/3))/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^
(1/2)*(-1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)
))/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*(I*(-a*b^2)^(1/3)*3^(1/2
)*_alpha*b-I*(-a*b^2)^(2/3)*3^(1/2)+2*_alpha^2*b^2-(-a*b^2)^(1/3)
*_alpha*b-(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/b*(-a*
b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/
3))^(1/2),-1/2*c^2/b*(2*I*3^(1/2)*(-a*b^2)^(1/3)*_alpha^2*b-I*3^(
1/2)*(-a*b^2)^(2/3)*_alpha+I*3^(1/2)*a*b-3*(-a*b^2)^(2/3)*_alpha-
3*a*b)/d^2,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b*c^2+a*c
^2-d^2))+1/3/b/c*ln(b*c^2*x^3+a*c^2-d^2)
```

Maxima [A] time = 0.690906, size = 30, normalized size = 1.15

$$\frac{2\log\left(\sqrt{bx^3 + a}c + d\right)}{3bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(b*c*x^3 + a*c + sqrt(b*x^3 + a)*d),x, algorithm="maxima")
```

```
[Out] 2/3*log(sqrt(b*x^3 + a)*c + d)/(b*c)
```

Fricas [A] time = 0.273522, size = 82, normalized size = 3.15

$$\frac{\log\left(bc^2x^3 + ac^2 - d^2\right) + \log\left(\sqrt{bx^3 + a}c + d\right) - \log\left(\sqrt{bx^3 + a}c - d\right)}{3\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*c*x^3 + a*c + sqrt(b*x^3 + a)*d),x, algorithm="fricas")
```

```
[Out] 1/3*(log(b*c^2*x^3 + a*c^2 - d^2) + log(sqrt(b*x^3 + a)*c + d) -
log(sqrt(b*x^3 + a)*c - d))/(b*c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.273732, size = 31, normalized size = 1.19

$$\frac{2\ln\left(\left|\sqrt{bx^3 + a}c + d\right|\right)}{3\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*c*x^3 + a*c + sqrt(b*x^3 + a)*d),x, algorithm="giac")
```

```
[Out] 2/3*ln(abs(sqrt(b*x^3 + a)*c + d))/(b*c)
```

3.394 $\int \frac{1}{x\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx$

Optimal. Leaf size=93

$$-\frac{2c\log\left(c\sqrt{a+bx^3}+d\right)}{3\left(ac^2-d^2\right)}+\frac{2d\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}\left(ac^2-d^2\right)}+\frac{c\log(x)}{ac^2-d^2}$$

```
[Out] (2*d*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]*(a*c^2 - d^2))
+ (c*Log[x])/(a*c^2 - d^2) - (2*c*Log[d + c*Sqrt[a + b*x^3]])/(3*
(a*c^2 - d^2))
```

Rubi [A] time = 0.40595, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207

$$-\frac{2c\log\left(c\sqrt{a+bx^3}+d\right)}{3\left(ac^2-d^2\right)}+\frac{2d\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}\left(ac^2-d^2\right)}+\frac{c\log(x)}{ac^2-d^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])),x]
```

```
[Out] (2*d*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]*(a*c^2 - d^2))
+ (c*Log[x])/(a*c^2 - d^2) - (2*c*Log[d + c*Sqrt[a + b*x^3]])/(3*
(a*c^2 - d^2))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.122586, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[1/(x*(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])),x]
```

```
[Out] Integrate[1/(x*(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])), x]
```

Maple [C] time = 0.048, size = 636, normalized size = 6.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x)
```

```
[Out] c*ln(x)/(a*c^2-d^2)-1/3*a*c^3/(a*c^2-d^2)/d^2*ln(b*c^2*x^3+a*c^2-
d^2)+1/3*c/d^2*ln(b*c^2*x^3+a*c^2-d^2)-2/3*d/a/(a*c^2-d^2)*(b*x^3
+a)^(1/2)+2/3*d*arctanh((b*x^3+a)^(1/2)/a^(1/2))/(a*c^2-d^2)/a^(1
/2)-2/3/a/d*(b*x^3+a)^(1/2)+2/3*c^2/(a*c^2-d^2)/d*(b*x^3+a)^(1/2)
+1/3*I/b^2*c^2/(a*c^2-d^2)/d*2^(1/2)*sum((-a*b^2)^(1/3)*(1/2*I*b*
(2*x+1/b*((-a*b^2)^(1/3)-I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3
))^(1/2)*(b*(x-1/b*(-a*b^2)^(1/3))/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(
-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)+I*3^(1/2
)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*(I*(-a*b
^2)^(1/3)*3^(1/2)*_alpha*b-I*(-a*b^2)^(2/3)*3^(1/2)+2*_alpha^2*b^
2-(-a*b^2)^(1/3)*_alpha*b-(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*
(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2
)*b/(-a*b^2)^(1/3))^(1/2),-1/2*c^2/b*(2*I*3^(1/2)*(-a*b^2)^(1/3)*
_alpha^2*b-I*3^(1/2)*(-a*b^2)^(2/3)*_alpha+I*3^(1/2)*a*b-3*(-a*b^
2)^(2/3)*_alpha-3*a*b)/d^2,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)),_alpha=RootO
f(_Z^3*b*c^2+a*c^2-d^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bcx^3+ac+\sqrt{bx^3+a}\,d\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*c*x^3 + a*c + sqrt(b*x^3 + a)*d)*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*c*x^3 + a*c + sqrt(b*x^3 + a)*d)*x), x)
```

Fricas [A] time = 0.338577, size = 1, normalized size = 0.01

$$\left[-\frac{\sqrt{a}c\log\left(\sqrt{bx^3+a}c+d\right)-\sqrt{a}c\log\left(\sqrt{bx^3+a}c-d\right)-d\log\left(\frac{(bx^3+2a)\sqrt{a}+2\sqrt{bx^3+a}a}{x^3}\right)+\left(c\log\left(bc^2x^3+ac^2-d^2\right)-3c\log\right.}{3(ac^2-d^2)\sqrt{a}}\right.$$

$$-\frac{\sqrt{-a}c\log\left(\sqrt{bx^3+a}c+d\right)-\sqrt{-a}c\log\left(\sqrt{bx^3+a}c-d\right)+2d\arctan\left(\frac{a}{\sqrt{bx^3+a}\sqrt{-a}}\right)+\left(c\log\left(bc^2x^3+ac^2-d^2\right)-3c\log(x\right.}{3(ac^2-d^2)\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*c*x^3 + a*c + sqrt(b*x^3 + a)*d)*x),x, algorithm="fricas")
```

```
[Out] [-1/3*(sqrt(a)*c*log(sqrt(b*x^3 + a)*c + d) - sqrt(a)*c*log(sqrt(
b*x^3 + a)*c - d) - d*log(((b*x^3 + 2*a)*sqrt(a) + 2*sqrt(b*x^3 +
 a)*a)/x^3) + (c*log(b*c^2*x^3 + a*c^2 - d^2) - 3*c*log(x))*sqrt(
a))/((a*c^2 - d^2)*sqrt(a)), -1/3*(sqrt(-a)*c*log(sqrt(b*x^3 + a)
*c + d) - sqrt(-a)*c*log(sqrt(b*x^3 + a)*c - d) + 2*d*arctan(a/(s
qrt(b*x^3 + a)*sqrt(-a))) + (c*log(b*c^2*x^3 + a*c^2 - d^2) - 3*c
*log(x))*sqrt(-a))/((a*c^2 - d^2)*sqrt(-a))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.27708, size = 127, normalized size = 1.37

$$-\frac{2c^2\ln\left(\left|\sqrt{bx^3+a}c+d\right|\right)}{3(ac^3-cd^2)}+\frac{c\ln\left(bx^3\right)}{3(ac^2-d^2)}-\frac{2d\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3(ac^2-d^2)\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*c*x^3 + a*c + sqrt(b*x^3 + a)*d)*x),x, algorithm="giac")
```

```
[Out] -2/3*c^2*ln(abs(sqrt(b*x^3 + a)*c + d))/(a*c^3 - c*d^2) + 1/3*c*l
n(b*x^3)/(a*c^2 - d^2) - 2/3*d*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(
(a*c^2 - d^2)*sqrt(-a))
```

3.395 $\int \frac{1}{x^4\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx$

Optimal. Leaf size=154

$$-\frac{bd\left(3ac^2-d^2\right)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}\left(ac^2-d^2\right)^2}-\frac{ac-d\sqrt{a+bx^3}}{3ax^3\left(ac^2-d^2\right)}+\frac{2bc^3\log\left(c\sqrt{a+bx^3}+d\right)}{3\left(ac^2-d^2\right)^2}-\frac{bc^3\log(x)}{\left(ac^2-d^2\right)^2}$$

```
[Out] -(a*c - d*Sqrt[a + b*x^3])/(3*a*(a*c^2 - d^2)*x^3) - (b*d*(3*a*c^
2 - d^2)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*a^(3/2)*(a*c^2 - d^
2)^2) - (b*c^3*Log[x])/(a*c^2 - d^2)^2 + (2*b*c^3*Log[d + c*Sqrt[
a + b*x^3]])/(3*(a*c^2 - d^2)^2)
```

Rubi [A] time = 0.617852, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207

$$-\frac{bd\left(3ac^2-d^2\right)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}\left(ac^2-d^2\right)^2}-\frac{ac-d\sqrt{a+bx^3}}{3ax^3\left(ac^2-d^2\right)}+\frac{2bc^3\log\left(c\sqrt{a+bx^3}+d\right)}{3\left(ac^2-d^2\right)^2}-\frac{bc^3\log(x)}{\left(ac^2-d^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^4*(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])),x]
```

```
[Out] -(a*c - d*Sqrt[a + b*x^3])/(3*a*(a*c^2 - d^2)*x^3) - (b*d*(3*a*c^
2 - d^2)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*a^(3/2)*(a*c^2 - d^
2)^2) - (b*c^3*Log[x])/(a*c^2 - d^2)^2 + (2*b*c^3*Log[d + c*Sqrt[
a + b*x^3]])/(3*(a*c^2 - d^2)^2)
```

Rubi in Sympy [A] time = 42.5427, size = 136, normalized size = 0.88

$$-\frac{bc^3\log\left(-bx^3\right)}{3\left(-ac^2+d^2\right)^2}+\frac{2bc^3\log\left(c\sqrt{a+bx^3}+d\right)}{3\left(-ac^2+d^2\right)^2}+\frac{ac-d\sqrt{a+bx^3}}{3ax^3\left(-ac^2+d^2\right)}+\frac{bd\left(-3ac^2+d^2\right)\operatorname{atanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{\frac{3}{2}}\left(-ac^2+d^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**4/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
```

```
[Out] -b*c**3*log(-b*x**3)/(3*(-a*c**2 + d**2)**2) + 2*b*c**3*log(c*sqr
t(a + b*x**3) + d)/(3*(-a*c**2 + d**2)**2) + (a*c - d*sqrt(a + b*
```

```
x**3))/(3*a*x**3*(-a*c**2 + d**2)) + b*d*(-3*a*c**2 + d**2)*atanh
(sqrt(a + b*x**3)/sqrt(a))/(3*a**(3/2)*(-a*c**2 + d**2)**2)
```

Mathematica [C] time = 6.76996, size = 860, normalized size = 5.58

$$\frac{5b^2dx^3F_1\left(\frac{3}{2};\frac{1}{2},1;\frac{5}{2};-\frac{a}{bx^3},\frac{d^2-ac^2}{bc^2x^3}\right)c^4}{3\left(ac^2-d^2\right)\sqrt{bx^3+a}\left(bc^2x^3+ac^2-d^2\right)\left(-5bc^2F_1\left(\frac{3}{2};\frac{1}{2},1;\frac{5}{2};-\frac{a}{bx^3},\frac{d^2-ac^2}{bc^2x^3}\right)x^3+2\left(ac^2-d^2\right)F_1\left(\frac{5}{2};\frac{1}{2},2;\frac{7}{2};-\frac{a}{bx^3},\frac{d^2-ac^2}{bc^2x^3}\right)+a\right.}$$

$$-\frac{b\log(x)c^3}{\left(ac^2-d^2\right)^2}+\frac{b\log\left(bc^2x^3+ac^2-d^2\right)c^3}{3\left(ac^2-d^2\right)^2}$$

$$+\frac{2b^2dx^3F_1\left(1;\frac{1}{2},1;2;-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)c^2}{3\sqrt{bx^3+a}\left(bc^2x^3+ac^2-d^2\right)\left(b\left(\left(d^2-ac^2\right)F_1\left(2;\frac{3}{2},1;3;-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)-2ac^2F_1\left(2;\frac{1}{2},2;3;-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)\right)x^3+4a\left(ac^2\right.\right.}$$

$$-\frac{5b^2d^3x^3F_1\left(\frac{3}{2};\frac{1}{2},1;\frac{5}{2};-\frac{a}{bx^3},\frac{d^2-ac^2}{bc^2x^3}\right)c^2}{9a\left(ac^2-d^2\right)\sqrt{bx^3+a}\left(bc^2x^3+ac^2-d^2\right)\left(-5bc^2F_1\left(\frac{3}{2};\frac{1}{2},1;\frac{5}{2};-\frac{a}{bx^3},\frac{d^2-ac^2}{bc^2x^3}\right)x^3+2\left(ac^2-d^2\right)F_1\left(\frac{5}{2};\frac{1}{2},2;\frac{7}{2};-\frac{a}{bx^3},\frac{d^2-ac^2}{bc^2x^3}\right)\right.}$$

$$-\frac{c}{3\left(ac^2-d^2\right)x^3}+\frac{d\sqrt{bx^3+a}}{3a\left(ac^2-d^2\right)x^3}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/(x^4*(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])),x]
```

```
[Out] -c/(3*(a*c^2 - d^2)*x^3) + (d*Sqrt[a + b*x^3])/(3*a*(a*c^2 - d^2)
*x^3) + (2*b^2*c^2*d*x^3*AppellF1[1, 1/2, 1, 2, -((b*x^3)/a), -((
b*c^2*x^3)/(a*c^2 - d^2))])/(3*Sqrt[a + b*x^3]*(a*c^2 - d^2 + b*c
^2*x^3)*(4*a*(a*c^2 - d^2)*AppellF1[1, 1/2, 1, 2, -((b*x^3)/a), -
((b*c^2*x^3)/(a*c^2 - d^2))] + b*x^3*(-2*a*c^2*AppellF1[2, 1/2, 2
, 3, -((b*x^3)/a), -((b*c^2*x^3)/(a*c^2 - d^2))] + (-(a*c^2) + d^
2)*AppellF1[2, 3/2, 1, 3, -((b*x^3)/a), -((b*c^2*x^3)/(a*c^2 - d^
2))]))) + (5*b^2*c^4*d*x^3*AppellF1[3/2, 1/2, 1, 5/2, -(a/(b*x^3)
), (-(a*c^2) + d^2)/(b*c^2*x^3)])/(3*(a*c^2 - d^2)*Sqrt[a + b*x^3
]*(a*c^2 - d^2 + b*c^2*x^3)*(-5*b*c^2*x^3*AppellF1[3/2, 1/2, 1, 5
/2, -(a/(b*x^3)), (-(a*c^2) + d^2)/(b*c^2*x^3)] + 2*(a*c^2 - d^2)
*AppellF1[5/2, 1/2, 2, 7/2, -(a/(b*x^3)), (-(a*c^2) + d^2)/(b*c^2
*x^3)] + a*c^2*AppellF1[5/2, 3/2, 1, 7/2, -(a/(b*x^3)), (-(a*c^2)
 + d^2)/(b*c^2*x^3)])) - (5*b^2*c^2*d^3*x^3*AppellF1[3/2, 1/2, 1,
 5/2, -(a/(b*x^3)), (-(a*c^2) + d^2)/(b*c^2*x^3)])/(9*a*(a*c^2 -
d^2)*Sqrt[a + b*x^3]*(a*c^2 - d^2 + b*c^2*x^3)*(-5*b*c^2*x^3*Appe
llF1[3/2, 1/2, 1, 5/2, -(a/(b*x^3)), (-(a*c^2) + d^2)/(b*c^2*x^3)
] + 2*(a*c^2 - d^2)*AppellF1[5/2, 1/2, 2, 7/2, -(a/(b*x^3)), (-(a
*c^2) + d^2)/(b*c^2*x^3)] + a*c^2*AppellF1[5/2, 3/2, 1, 7/2, -(a/
(b*x^3)), (-(a*c^2) + d^2)/(b*c^2*x^3)])) - (b*c^3*Log[x])/(a*c^2
 - d^2)^2 + (b*c^3*Log[a*c^2 - d^2 + b*c^2*x^3])/(3*(a*c^2 - d^2)
^2)
```

Maple [C] time = 0.053, size = 863, normalized size = 5.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^4/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x)
```

```
[Out] -1/3*c/(a*c^2-d^2)/x^3-2*b*c^3*ln(x)/(a*c^2-d^2)^2+1/a*c*b/(a*c^2
-d^2)^2*ln(x)*d^2+1/3*a*c^5*b/(a*c^2-d^2)^2/d^2*ln(b*c^2*x^3+a*c^
2-d^2)+b*c/a/(a*c^2-d^2)*ln(x)-1/3*b*c^3/(a*c^2-d^2)/d^2*ln(b*c^2
*x^3+a*c^2-d^2)+1/3*d/a/(a*c^2-d^2)*(b*x^3+a)^(1/2)/x^3+1/3*d/a^(
3/2)/(a*c^2-d^2)*b*arctanh((b*x^3+a)^(1/2)/a^(1/2))-2/3*b*c^4/(a*
c^2-d^2)^2/d*(b*x^3+a)^(1/2)-1/3*I/b*c^4/(a*c^2-d^2)^2/d*2^(1/2)*
sum((-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)-I*3^(1/2)*(-
a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(-a*b^2)^(1/3))/(-
3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(2*x+
1/b*((-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1
/2)/(b*x^3+a)^(1/2)*(I*(-a*b^2)^(1/3)*3^(1/2)*_alpha*b-I*(-a*b^2)
^(2/3)*3^(1/2)+2*_alpha^2*b^2-(-a*b^2)^(1/3)*_alpha*b-(-a*b^2)^(2
/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),-1/2*c^2/b*
(2*I*3^(1/2)*(-a*b^2)^(1/3)*_alpha^2*b-I*3^(1/2)*(-a*b^2)^(2/3)*_
alpha+I*3^(1/2)*a*b-3*(-a*b^2)^(2/3)*_alpha-3*a*b)/d^2,(I*3^(1/2)
/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b*c^2+a*c^2-d^2))+2/3*b/a^2/d*
(b*x^3+a)^(1/2)+4/3*d*b/a/(a*c^2-d^2)^2*(b*x^3+a)^(1/2)*c^2-2/3*b
/a^2/(a*c^2-d^2)^2*(b*x^3+a)^(1/2)*d^3-4/3*d*b/a^(1/2)/(a*c^2-d^2
)^2*arctanh((b*x^3+a)^(1/2)/a^(1/2))*c^2+2/3*b/a^(3/2)/(a*c^2-d^2
)^2*arctanh((b*x^3+a)^(1/2)/a^(1/2))*d^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bcx^3 + ac + \sqrt{bx^3 + a}d\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*c*x^3 + a*c + sqrt(b*x^3 + a)*d)*x^4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*c*x^3 + a*c + sqrt(b*x^3 + a)*d)*x^4), x)
```

Fricas [A] time = 0.394049, size = 1, normalized size = 0.01

$$\left[\frac{2\,a^{\frac{3}{2}}bc^3x^3\log\left(\sqrt{bx^3+ac}+d\right)-2\,a^{\frac{3}{2}}bc^3x^3\log\left(\sqrt{bx^3+ac}-d\right)-\left(3\,abc^2d-bd^3\right)x^3\log\left(\frac{(bx^3+2\,a)\sqrt{a}+2\,\sqrt{bx^3+a}a}{x^3}\right)+2\,(ac^2\cdots}{6\,(a^3c^4-2\,a^2c^2d^2+ad^4)\sqrt{a}x^3\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*c*x^3 + a*c + sqrt(b*x^3 + a)*d)*x^4),x, algorithm="fricas")
```

```
[Out] [1/6*(2*a^(3/2)*b*c^3*x^3*log(sqrt(b*x^3 + a)*c + d) - 2*a^(3/2)*
b*c^3*x^3*log(sqrt(b*x^3 + a)*c - d) - (3*a*b*c^2*d - b*d^3)*x^3*
log(((b*x^3 + 2*a)*sqrt(a) + 2*sqrt(b*x^3 + a)*a)/x^3) + 2*(a*c^2
*d - d^3)*sqrt(b*x^3 + a)*sqrt(a) + 2*(a*b*c^3*x^3*log(b*c^2*x^3
+ a*c^2 - d^2) - 3*a*b*c^3*x^3*log(x) - a^2*c^3 + a*c*d^2)*sqrt(a
))/((a^3*c^4 - 2*a^2*c^2*d^2 + a*d^4)*sqrt(a)*x^3), 1/3*(sqrt(-a)
*a*b*c^3*x^3*log(sqrt(b*x^3 + a)*c + d) - sqrt(-a)*a*b*c^3*x^3*lo
g(sqrt(b*x^3 + a)*c - d) + (3*a*b*c^2*d - b*d^3)*x^3*arctan(a/(sq
rt(b*x^3 + a)*sqrt(-a))) + (a*c^2*d - d^3)*sqrt(b*x^3 + a)*sqrt(-
a) + (a*b*c^3*x^3*log(b*c^2*x^3 + a*c^2 - d^2) - 3*a*b*c^3*x^3*lo
g(x) - a^2*c^3 + a*c*d^2)*sqrt(-a))/((a^3*c^4 - 2*a^2*c^2*d^2 + a
*d^4)*sqrt(-a)*x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.289417, size = 275, normalized size = 1.79

$$\frac{1}{3}\left(\frac{2\,c^4\ln\left(\left|\sqrt{bx^3+ac}+d\right|\right)}{a^2c^5-2\,ac^3d^2+cd^4}-\frac{c^3\ln\left(bx^3\right)}{a^2c^4-2\,ac^2d^2+d^4}+\frac{\left(3\,ac^2d-d^3\right)\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{(a^3c^4-2\,a^2c^2d^2+ad^4)\sqrt{-a}}-\frac{a^2c^3-acd^2-\left(ac^2d-d^3\right)\sqrt{bx^3+a}}{(ac^2-d^2)^2abx^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*c*x^3 + a*c + sqrt(b*x^3 + a)*d)*x^4),x, algorithm="giac")
```

```
[Out] 1/3*(2*c^4*ln(abs(sqrt(b*x^3 + a)*c + d))/(a^2*c^5 - 2*a*c^3*d^2
+ c*d^4) - c^3*ln(b*x^3)/(a^2*c^4 - 2*a*c^2*d^2 + d^4) + (3*a*c^2
*d - d^3)*arctan(sqrt(b*x^3 + a)/sqrt(-a))/((a^3*c^4 - 2*a^2*c^2*
d^2 + a*d^4)*sqrt(-a)) - (a^2*c^3 - a*c*d^2 - (a*c^2*d - d^3)*sqr
t(b*x^3 + a))/((a*c^2 - d^2)^2*a*b*x^3))*b
```

3.396 $\int \frac{x^3}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=311

$$-\frac{dx^4\sqrt{\frac{bx^3}{a}+1}F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{4\sqrt{a+bx^3}\,(ac^2-d^2)}+\frac{\sqrt[3]{ac^2-d^2}\log\left(-\sqrt[3]{b}c^{2/3}x\sqrt[3]{ac^2-d^2}+\left(ac^2-d^2\right)^{2/3}+b^{2/3}c^{4/3}x^2\right)}{6b^{4/3}c^{5/3}}-\frac{\sqrt[3]{ac^2-d^2}\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3b^{4/3}c^{5/3}}+\frac{\sqrt[3]{ac^2-d^2}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right)}{\sqrt{3}b^{4/3}c^{5/3}}+\frac{x}{bc}$$

```
[Out] x/(b*c) - (d*x^4*Sqrt[1 + (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, -
((b*x^3)/a), -((b*c^2*x^3)/(a*c^2 - d^2))])/(4*(a*c^2 - d^2)*Sqrt
[a + b*x^3]) + ((a*c^2 - d^2)^(1/3)*ArcTan[(1 - (2*b^(1/3)*c^(2/3
)*x)/(a*c^2 - d^2)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(4/3)*c^(5/3)) - (
(a*c^2 - d^2)^(1/3)*Log[(a*c^2 - d^2)^(1/3) + b^(1/3)*c^(2/3)*x])
/(3*b^(4/3)*c^(5/3)) + ((a*c^2 - d^2)^(1/3)*Log[(a*c^2 - d^2)^(2/
3) - b^(1/3)*c^(2/3)*(a*c^2 - d^2)^(1/3)*x + b^(2/3)*c^(4/3)*x^2]
)/(6*b^(4/3)*c^(5/3))
```

Rubi [A] time = 1.02733, antiderivative size = 311, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$

$$-\frac{dx^4\sqrt{\frac{bx^3}{a}+1}F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{4\sqrt{a+bx^3}\,(ac^2-d^2)}+\frac{\sqrt[3]{ac^2-d^2}\log\left(-\sqrt[3]{b}c^{2/3}x\sqrt[3]{ac^2-d^2}+\left(ac^2-d^2\right)^{2/3}+b^{2/3}c^{4/3}x^2\right)}{6b^{4/3}c^{5/3}}-\frac{\sqrt[3]{ac^2-d^2}\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3b^{4/3}c^{5/3}}+\frac{\sqrt[3]{ac^2-d^2}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right)}{\sqrt{3}b^{4/3}c^{5/3}}+\frac{x}{bc}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]
```

```
[Out] x/(b*c) - (d*x^4*Sqrt[1 + (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, -
((b*x^3)/a), -((b*c^2*x^3)/(a*c^2 - d^2))])/(4*(a*c^2 - d^2)*Sqrt
```

```
[a + b*x^3]) + ((a*c^2 - d^2)^(1/3)*ArcTan[(1 - (2*b^(1/3)*c^(2/3
)*x)/(a*c^2 - d^2)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(4/3)*c^(5/3)) - (
(a*c^2 - d^2)^(1/3)*Log[(a*c^2 - d^2)^(1/3) + b^(1/3)*c^(2/3)*x])
/(3*b^(4/3)*c^(5/3)) + ((a*c^2 - d^2)^(1/3)*Log[(a*c^2 - d^2)^(2/
3) - b^(1/3)*c^(2/3)*(a*c^2 - d^2)^(1/3)*x + b^(2/3)*c^(4/3)*x^2]
)/(6*b^(4/3)*c^(5/3))
```

Rubi in Sympy [A] time = 91.7342, size = 280, normalized size = 0.9

$$\frac{x}{bc}+\frac{\sqrt[3]{-ac^2+d^2}\log\left(\sqrt[3]{b}c^{\frac{2}{3}}x-\sqrt[3]{-ac^2+d^2}\right)}{3b^{\frac{4}{3}}c^{\frac{5}{3}}}-\frac{\sqrt[3]{-ac^2+d^2}\log\left(a^{\frac{2}{3}}b^{\frac{2}{3}}c^{\frac{4}{3}}x^2+a^{\frac{2}{3}}\sqrt[3]{b}c^{\frac{2}{3}}x\sqrt[3]{-ac^2+d^2}+a^{\frac{2}{3}}\left(-ac^2+d^2\right)^{\frac{2}{3}}\right)}{6b^{\frac{4}{3}}c^{\frac{5}{3}}}-\frac{\sqrt{3}\sqrt[3]{-ac^2+d^2}\operatorname{atan}\left(\sqrt{3}\left(\frac{2\sqrt[3]{b}c^{\frac{2}{3}}x}{3\sqrt[3]{-ac^2+d^2}}+\frac{1}{3}\right)\right)}{3b^{\frac{4}{3}}c^{\frac{5}{3}}}+\frac{dx^4\sqrt{a+bx^3}\operatorname{appellf}_1\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{4a\sqrt{1+\frac{bx^3}{a}}\left(-ac^2+d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
```

```
[Out] x/(b*c) + (-a*c**2 + d**2)**(1/3)*log(b**(1/3)*c**(2/3)*x - (-a*c
**2 + d**2)**(1/3))/(3*b**(4/3)*c**(5/3)) - (-a*c**2 + d**2)**(1/
3)*log(a**(2/3)*b**(2/3)*c**(4/3)*x**2 + a**(2/3)*b**(1/3)*c**(2/
3)*x*(-a*c**2 + d**2)**(1/3) + a**(2/3)*(-a*c**2 + d**2)**(2/3))/
(6*b**(4/3)*c**(5/3)) - sqrt(3)*(-a*c**2 + d**2)**(1/3)*atan(sqrt
(3)*(2*b**(1/3)*c**(2/3)*x/(3*(-a*c**2 + d**2)**(1/3)) + 1/3))/(3
*b**(4/3)*c**(5/3)) + d*x**4*sqrt(a + b*x**3)*appellf1(4/3, 1/2,
1, 7/3, -b*x**3/a, -b*c**2*x**3/(a*c**2 - d**2))/(4*a*sqrt(1 + b*
x**3/a)*(-a*c**2 + d**2))
```

Mathematica [A] time = 1.4971, size = 470, normalized size = 1.51

$$\frac{1}{6}\left(\frac{\sqrt[3]{ac^2-d^2}\log\left(-\sqrt[3]{b}c^{2/3}x\sqrt[3]{ac^2-d^2}+\left(ac^2-d^2\right)^{2/3}+b^{2/3}c^{4/3}x^2\right)-2\sqrt[3]{ac^2-d^2}\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)-2\sqrt{3}\sqrt[3]{ac^2}}{b^{4/3}c^{5/3}}\right.$$

$$-\frac{21adx^4\left(ac^2-d^2\right)F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{\sqrt{a+bx^3}\left(ac^2+bc^2x^3-d^2\right)\left(14a\left(ac^2-d^2\right)F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)-3bx^3\left(2ac^2F_1\left(\frac{7}{3};\frac{1}{2},2;\frac{10}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)+(a\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^3/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]
```

```
[Out] ((-21*a*d*(a*c^2 - d^2)*x^4*AppellF1[4/3, 1/2, 1, 7/3, -((b*x^3)/
a), -((b*c^2*x^3)/(a*c^2 - d^2))])/(Sqrt[a + b*x^3]*(a*c^2 - d^2
+ b*c^2*x^3)*(14*a*(a*c^2 - d^2)*AppellF1[4/3, 1/2, 1, 7/3, -((b*
x^3)/a), -((b*c^2*x^3)/(a*c^2 - d^2))] - 3*b*x^3*(2*a*c^2*AppellF
1[7/3, 1/2, 2, 10/3, -((b*x^3)/a), -((b*c^2*x^3)/(a*c^2 - d^2))]
+ (a*c^2 - d^2)*AppellF1[7/3, 3/2, 1, 10/3, -((b*x^3)/a), -((b*c^
2*x^3)/(a*c^2 - d^2))]))) + (6*b^(1/3)*c^(2/3)*x - 2*Sqrt[3]*(a*c
^2 - d^2)^(1/3)*ArcTan[(-1 + (2*b^(1/3)*c^(2/3)*x)/(a*c^2 - d^2)^
(1/3))/Sqrt[3]] - 2*(a*c^2 - d^2)^(1/3)*Log[(a*c^2 - d^2)^(1/3) +
 b^(1/3)*c^(2/3)*x] + (a*c^2 - d^2)^(1/3)*Log[(a*c^2 - d^2)^(2/3)
 - b^(1/3)*c^(2/3)*(a*c^2 - d^2)^(1/3)*x + b^(2/3)*c^(4/3)*x^2])/
(b^(4/3)*c^(5/3)))/6
```

Maple [C] time = 0.067, size = 1544, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x)
```

```
[Out] 2/3*I*d/b^2/c^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)
-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*
```

```
((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*E
llipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b
^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))
^(1/2))+1/3*I/b^4/d*2^(1/2)*sum(1/_alpha^2*(-a*b^2)^(1/3)*(1/2*I*
b*(2*x+1/b*((-a*b^2)^(1/3)-I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1
/3))^(1/2)*(b*(x-1/b*(-a*b^2)^(1/3))/(-3*(-a*b^2)^(1/3)+I*3^(1/2)
*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)+I*3^(1
/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*(I*(-a
*b^2)^(1/3)*3^(1/2)*_alpha*b-I*(-a*b^2)^(2/3)*3^(1/2)+2*_alpha^2*
b^2-(-a*b^2)^(1/3)*_alpha*b-(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2
)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1
/2)*b/(-a*b^2)^(1/3))^(1/2),-1/2*c^2/b*(2*I*3^(1/2)*(-a*b^2)^(1/3
)*_alpha^2*b-I*3^(1/2)*(-a*b^2)^(2/3)*_alpha+I*3^(1/2)*a*b-3*(-a*
b^2)^(2/3)*_alpha-3*a*b)/d^2,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)),_alpha=Roo
tOf(_Z^3*b*c^2+a*c^2-d^2))*a-1/3*I*d/b^4/c^2*2^(1/2)*sum(1/_alpha
^2*(-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)-I*3^(1/2)*(-a
*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(-a*b^2)^(1/3))/(-3
*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(2*x+1
/b*((-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/
2)/(b*x^3+a)^(1/2)*(I*(-a*b^2)^(1/3)*3^(1/2)*_alpha*b-I*(-a*b^2)^
(2/3)*3^(1/2)+2*_alpha^2*b^2-(-a*b^2)^(1/3)*_alpha*b-(-a*b^2)^(2/
3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),-1/2*c^2/b*(
2*I*3^(1/2)*(-a*b^2)^(1/3)*_alpha^2*b-I*3^(1/2)*(-a*b^2)^(2/3)*_a
lpha+I*3^(1/2)*a*b-3*(-a*b^2)^(2/3)*_alpha-3*a*b)/d^2,(I*3^(1/2)/
b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b*c^2+a*c^2-d^2))-1/3*a/c/b^2/(
1/c^2/b*(a*c^2-d^2))^(2/3)*ln(x+(1/c^2/b*(a*c^2-d^2))^(1/3))+1/6*
a/c/b^2/(1/c^2/b*(a*c^2-d^2))^(2/3)*ln(x^2-x*(1/c^2/b*(a*c^2-d^2)
)^(1/3)+(1/c^2/b*(a*c^2-d^2))^(2/3))-1/3*a/c/b^2/(1/c^2/b*(a*c^2-
d^2))^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/c^2/b*(a*c^2-d^2))^(
1/3)*x-1))+x/b/c+1/3/b^2/c^3*d^2/(1/c^2/b*(a*c^2-d^2))^(2/3)*ln(x
+(1/c^2/b*(a*c^2-d^2))^(1/3))-1/6/b^2/c^3*d^2/(1/c^2/b*(a*c^2-d^2
))^(2/3)*ln(x^2-x*(1/c^2/b*(a*c^2-d^2))^(1/3)+(1/c^2/b*(a*c^2-d^2
))^(2/3))+1/3/b^2/c^3*d^2/(1/c^2/b*(a*c^2-d^2))^(2/3)*3^(1/2)*arc
tan(1/3*3^(1/2)*(2/(1/c^2/b*(a*c^2-d^2))^(1/3)*x-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{bcx^3 + ac + \sqrt{bx^3 + a}d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*c*x^3 + a*c + sqrt(b*x^3 + a)*d),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(b*c*x^3 + a*c + sqrt(b*x^3 + a)*d), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*c*x^3 + a*c + sqrt(b*x^3 + a)*d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{bcx^3 + ac + \sqrt{bx^3 + a}\,d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*c*x^3 + a*c + sqrt(b*x^3 + a)*d),x, algorithm="giac")
```

```
[Out] integrate(x^3/(b*c*x^3 + a*c + sqrt(b*x^3 + a)*d), x)
```

3.397 $\int \frac{x}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=304

$$-\frac{dx^2\sqrt{\frac{bx^3}{a}+1}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{2\sqrt{a+bx^3}\,(ac^2-d^2)}+\frac{\log\left(-\sqrt[3]{b}c^{2/3}x\sqrt[3]{ac^2-d^2}+\left(ac^2-d^2\right)^{2/3}+b^{2/3}c^{4/3}x^2\right)}{6b^{2/3}\sqrt[3]{c}\sqrt[3]{ac^2-d^2}}-\frac{\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3b^{2/3}\sqrt[3]{c}\sqrt[3]{ac^2-d^2}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}\sqrt[3]{c}\sqrt[3]{ac^2-d^2}}$$

```
[Out] -(d*x^2*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, -((b*x^3)/
a), -((b*c^2*x^3)/(a*c^2 - d^2))])/(2*(a*c^2 - d^2)*Sqrt[a + b*x^
3]) - ArcTan[(1 - (2*b^(1/3)*c^(2/3)*x)/(a*c^2 - d^2)^(1/3))/Sqrt
[3]]/(Sqrt[3]*b^(2/3)*c^(1/3)*(a*c^2 - d^2)^(1/3)) - Log[(a*c^2 -
 d^2)^(1/3) + b^(1/3)*c^(2/3)*x]/(3*b^(2/3)*c^(1/3)*(a*c^2 - d^2)
^(1/3)) + Log[(a*c^2 - d^2)^(2/3) - b^(1/3)*c^(2/3)*(a*c^2 - d^2)
^(1/3)*x + b^(2/3)*c^(4/3)*x^2]/(6*b^(2/3)*c^(1/3)*(a*c^2 - d^2)^
(1/3))
```

Rubi [A] time = 0.730165, antiderivative size = 304, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$-\frac{dx^2\sqrt{\frac{bx^3}{a}+1}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{2\sqrt{a+bx^3}\,(ac^2-d^2)}+\frac{\log\left(-\sqrt[3]{b}c^{2/3}x\sqrt[3]{ac^2-d^2}+\left(ac^2-d^2\right)^{2/3}+b^{2/3}c^{4/3}x^2\right)}{6b^{2/3}\sqrt[3]{c}\sqrt[3]{ac^2-d^2}}-\frac{\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3b^{2/3}\sqrt[3]{c}\sqrt[3]{ac^2-d^2}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}\sqrt[3]{c}\sqrt[3]{ac^2-d^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]
```

```
[Out] -(d*x^2*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, -((b*x^3)/
a), -((b*c^2*x^3)/(a*c^2 - d^2))])/(2*(a*c^2 - d^2)*Sqrt[a + b*x^
3]) - ArcTan[(1 - (2*b^(1/3)*c^(2/3)*x)/(a*c^2 - d^2)^(1/3))/Sqrt
[3]]/(Sqrt[3]*b^(2/3)*c^(1/3)*(a*c^2 - d^2)^(1/3)) - Log[(a*c^2 -
 d^2)^(1/3) + b^(1/3)*c^(2/3)*x]/(3*b^(2/3)*c^(1/3)*(a*c^2 - d^2)
^(1/3)) + Log[(a*c^2 - d^2)^(2/3) - b^(1/3)*c^(2/3)*(a*c^2 - d^2)
^(1/3)*x + b^(2/3)*c^(4/3)*x^2]/(6*b^(2/3)*c^(1/3)*(a*c^2 - d^2)^
(1/3))
```

Rubi in Sympy [A] time = 77.517, size = 275, normalized size = 0.9

$$\frac{\log\left(\sqrt[3]{b}c^{\frac{2}{3}}x-\sqrt[3]{-ac^{2}+d^{2}}\right)}{3b^{\frac{2}{3}}\sqrt[3]{c}\sqrt[3]{-ac^{2}+d^{2}}}-\frac{\log\left(a^{\frac{2}{3}}b^{\frac{2}{3}}c^{\frac{4}{3}}x^{2}+a^{\frac{2}{3}}\sqrt[3]{b}c^{\frac{2}{3}}x\sqrt[3]{-ac^{2}+d^{2}}+a^{\frac{2}{3}}\left(-ac^{2}+d^{2}\right)^{\frac{2}{3}}\right)}{6b^{\frac{2}{3}}\sqrt[3]{c}\sqrt[3]{-ac^{2}+d^{2}}}+\frac{\sqrt{3}\operatorname{atan}\left(\sqrt{3}\left(\frac{2\sqrt[3]{b}c^{\frac{2}{3}}x}{3\sqrt[3]{-ac^{2}+d^{2}}}+\frac{1}{3}\right)\right)}{3b^{\frac{2}{3}}\sqrt[3]{c}\sqrt[3]{-ac^{2}+d^{2}}}+\frac{dx^{2}\sqrt{a+bx^{3}}\operatorname{appellf}_{1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{bx^{3}}{a},-\frac{bc^{2}x^{3}}{ac^{2}-d^{2}}\right)}{2a\sqrt{1+\frac{bx^{3}}{a}}\left(-ac^{2}+d^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
```

```
[Out] log(b**(1/3)*c**(2/3)*x - (-a*c**2 + d**2)**(1/3))/(3*b**(2/3)*c*
*(1/3)*(-a*c**2 + d**2)**(1/3)) - log(a**(2/3)*b**(2/3)*c**(4/3)*
x**2 + a**(2/3)*b**(1/3)*c**(2/3)*x*(-a*c**2 + d**2)**(1/3) + a**
(2/3)*(-a*c**2 + d**2)**(2/3))/(6*b**(2/3)*c**(1/3)*(-a*c**2 + d*
*2)**(1/3)) + sqrt(3)*atan(sqrt(3)*(2*b**(1/3)*c**(2/3)*x/(3*(-a*
c**2 + d**2)**(1/3)) + 1/3))/(3*b**(2/3)*c**(1/3)*(-a*c**2 + d**2
)**(1/3)) + d*x**2*sqrt(a + b*x**3)*appellf1(2/3, 1/2, 1, 5/3, -b
*x**3/a, -b*c**2*x**3/(a*c**2 - d**2))/(2*a*sqrt(1 + b*x**3/a)*(-
a*c**2 + d**2))
```

Mathematica [A] time = 0.147551, size = 0, normalized size = 0.

$$\int\frac{x}{ac+bcx^{3}+d\sqrt{a+bx^{3}}}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[x/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]
```

```
[Out] Integrate[x/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]), x]
```

Maple [C] time = 0.071, size = 619, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x)
```

```
[Out] -1/3*I/d/b^3*2^(1/2)*sum(1/_alpha*(-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/
b*((-a*b^2)^(1/3)-I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2
)*(b*(x-1/b*(-a*b^2)^(1/3))/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)
^(1/3)))^(1/2)*(-1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)+I*3^(1/2)*(-a*b
^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*(I*(-a*b^2)^(1/
3)*3^(1/2)*_alpha*b-I*(-a*b^2)^(2/3)*3^(1/2)+2*_alpha^2*b^2-(-a*b
^2)^(1/3)*_alpha*b-(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1
/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a
*b^2)^(1/3))^(1/2),-1/2*c^2/b*(2*I*3^(1/2)*(-a*b^2)^(1/3)*_alpha^
2*b-I*3^(1/2)*(-a*b^2)^(2/3)*_alpha+I*3^(1/2)*a*b-3*(-a*b^2)^(2/3
)*_alpha-3*a*b)/d^2,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*
b*c^2+a*c^2-d^2))-1/3/b/c/(1/c^2/b*(a*c^2-d^2))^(1/3)*ln(x+(1/c^2
/b*(a*c^2-d^2))^(1/3))+1/6/b/c/(1/c^2/b*(a*c^2-d^2))^(1/3)*ln(x^2
-x*(1/c^2/b*(a*c^2-d^2))^(1/3)+(1/c^2/b*(a*c^2-d^2))^(2/3))+1/3/b
/c*3^(1/2)/(1/c^2/b*(a*c^2-d^2))^(1/3)*arctan(1/3*3^(1/2)*(2/(1/c
^2/b*(a*c^2-d^2))^(1/3)*x-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{bcx^3 + ac + \sqrt{bx^3 + a}\,d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(b*c*x^3 + a*c + sqrt(b*x^3 + a)*d),x, algorithm="maxima")
```

```
[Out] integrate(x/(b*c*x^3 + a*c + sqrt(b*x^3 + a)*d), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*c*x^3 + a*c + sqrt(b*x^3 + a)*d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{bcx^3 + ac + \sqrt{bx^3 + a}\,d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*c*x^3 + a*c + sqrt(b*x^3 + a)*d),x, algorithm="giac")
```

```
[Out] integrate(x/(b*c*x^3 + a*c + sqrt(b*x^3 + a)*d), x)
```

3.398 $\int \frac{1}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=300

$$-\frac{dx\sqrt{\frac{bx^3}{a}+1}F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{\sqrt{a+bx^3}\,(ac^2-d^2)}-\frac{\sqrt[3]{c}\log\left(-\sqrt[3]{b}c^{2/3}x\sqrt[3]{ac^2-d^2}+\left(ac^2-d^2\right)^{2/3}+b^{2/3}c^{4/3}x^2\right)}{6\sqrt[3]{b}\,(ac^2-d^2)^{2/3}}+\frac{\sqrt[3]{c}\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3\sqrt[3]{b}\,(ac^2-d^2)^{2/3}}-\frac{\sqrt[3]{c}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}\,(ac^2-d^2)^{2/3}}$$

```
[Out] -((d*x*Sqrt[1 + (b*x^3)/a]*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a
), -((b*c^2*x^3)/(a*c^2 - d^2))])/((a*c^2 - d^2)*Sqrt[a + b*x^3])
) - (c^(1/3)*ArcTan[(1 - (2*b^(1/3)*c^(2/3)*x)/(a*c^2 - d^2)^(1/3
))/Sqrt[3]])/(Sqrt[3]*b^(1/3)*(a*c^2 - d^2)^(2/3)) + (c^(1/3)*Log
[(a*c^2 - d^2)^(1/3) + b^(1/3)*c^(2/3)*x])/(3*b^(1/3)*(a*c^2 - d^
2)^(2/3)) - (c^(1/3)*Log[(a*c^2 - d^2)^(2/3) - b^(1/3)*c^(2/3)*(a
*c^2 - d^2)^(1/3)*x + b^(2/3)*c^(4/3)*x^2])/(6*b^(1/3)*(a*c^2 - d
^2)^(2/3))
```

Rubi [A] time = 0.608751, antiderivative size = 300, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.36$

$$-\frac{dx\sqrt{\frac{bx^3}{a}+1}F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{\sqrt{a+bx^3}\,(ac^2-d^2)}-\frac{\sqrt[3]{c}\log\left(-\sqrt[3]{b}c^{2/3}x\sqrt[3]{ac^2-d^2}+\left(ac^2-d^2\right)^{2/3}+b^{2/3}c^{4/3}x^2\right)}{6\sqrt[3]{b}\,(ac^2-d^2)^{2/3}}+\frac{\sqrt[3]{c}\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3\sqrt[3]{b}\,(ac^2-d^2)^{2/3}}-\frac{\sqrt[3]{c}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}\,(ac^2-d^2)^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])^(-1),x]
```

```
[Out] -((d*x*Sqrt[1 + (b*x^3)/a]*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a
), -((b*c^2*x^3)/(a*c^2 - d^2))])/((a*c^2 - d^2)*Sqrt[a + b*x^3])
) - (c^(1/3)*ArcTan[(1 - (2*b^(1/3)*c^(2/3)*x)/(a*c^2 - d^2)^(1/3
))/Sqrt[3]])/(Sqrt[3]*b^(1/3)*(a*c^2 - d^2)^(2/3)) + (c^(1/3)*Log
[(a*c^2 - d^2)^(1/3) + b^(1/3)*c^(2/3)*x])/(3*b^(1/3)*(a*c^2 - d^
2)^(2/3)) - (c^(1/3)*Log[(a*c^2 - d^2)^(2/3) - b^(1/3)*c^(2/3)*(a
*c^2 - d^2)^(1/3)*x + b^(2/3)*c^(4/3)*x^2])/(6*b^(1/3)*(a*c^2 - d
^2)^(2/3))
```

Rubi in Sympy [A] time = 83.1627, size = 272, normalized size = 0.91

$$\frac{\sqrt[3]{c}\log\left(\sqrt[3]{b}c^{\frac{2}{3}}x-\sqrt[3]{-ac^2+d^2}\right)}{3\sqrt[3]{b}\left(-ac^2+d^2\right)^{\frac{2}{3}}}-\frac{\sqrt[3]{c}\log\left(a^{\frac{2}{3}}b^{\frac{2}{3}}c^{\frac{4}{3}}x^2+a^{\frac{2}{3}}\sqrt[3]{b}c^{\frac{2}{3}}x\sqrt[3]{-ac^2+d^2}+a^{\frac{2}{3}}\left(-ac^2+d^2\right)^{\frac{2}{3}}\right)}{6\sqrt[3]{b}\left(-ac^2+d^2\right)^{\frac{2}{3}}}-\frac{\sqrt{3}\sqrt[3]{c}\operatorname{atan}\left(\sqrt{3}\left(\frac{2\sqrt[3]{b}c^{\frac{2}{3}}x}{3\sqrt[3]{-ac^2+d^2}}+\frac{1}{3}\right)\right)}{3\sqrt[3]{b}\left(-ac^2+d^2\right)^{\frac{2}{3}}}+\frac{dx\sqrt{a+bx^3}\operatorname{appellf_1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{a\sqrt{1+\frac{bx^3}{a}}\left(-ac^2+d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
```

```
[Out] c**(1/3)*log(b**(1/3)*c**(2/3)*x - (-a*c**2 + d**2)**(1/3))/(3*b*
*(1/3)*(-a*c**2 + d**2)**(2/3)) - c**(1/3)*log(a**(2/3)*b**(2/3)*
c**(4/3)*x**2 + a**(2/3)*b**(1/3)*c**(2/3)*x*(-a*c**2 + d**2)**(1
/3) + a**(2/3)*(-a*c**2 + d**2)**(2/3))/(6*b**(1/3)*(-a*c**2 + d*
*2)**(2/3)) - sqrt(3)*c**(1/3)*atan(sqrt(3)*(2*b**(1/3)*c**(2/3)*
x/(3*(-a*c**2 + d**2)**(1/3)) + 1/3))/(3*b**(1/3)*(-a*c**2 + d**2
)**(2/3)) + d*x*sqrt(a + b*x**3)*appellf1(1/3, 1/2, 1, 4/3, -b*x*
*3/a, -b*c**2*x**3/(a*c**2 - d**2))/(a*sqrt(1 + b*x**3/a)*(-a*c**
2 + d**2))
```

Mathematica [A] time = 0.0502271, size = 0, normalized size = 0.

$$\int \frac{1}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])^(-1),x]
```

```
[Out] Integrate[(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])^(-1), x]
```

Maple [C] time = 0.021, size = 619, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x)
```

```
[Out] -1/3*I/d/b^3*2^(1/2)*sum(1/_alpha^2*(-a*b^2)^(1/3)*(1/2*I*b*(2*x+
1/b*((-a*b^2)^(1/3)-I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1
/2)*(b*(x-1/b*(-a*b^2)^(1/3))/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^
2)^(1/3)))^(1/2)*(-1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)+I*3^(1/2)*(-a
*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*(I*(-a*b^2)^(
1/3)*3^(1/2)*_alpha*b-I*(-a*b^2)^(2/3)*3^(1/2)+2*_alpha^2*b^2-(-a
*b^2)^(1/3)*_alpha*b-(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x
+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(
-a*b^2)^(1/3))^(1/2),-1/2*c^2/b*(2*I*3^(1/2)*(-a*b^2)^(1/3)*_alph
a^2*b-I*3^(1/2)*(-a*b^2)^(2/3)*_alpha+I*3^(1/2)*a*b-3*(-a*b^2)^(2
/3)*_alpha-3*a*b)/d^2,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^
3*b*c^2+a*c^2-d^2))+1/3/b/c/(1/c^2/b*(a*c^2-d^2))^(2/3)*ln(x+(1/c
^2/b*(a*c^2-d^2))^(1/3))-1/6/b/c/(1/c^2/b*(a*c^2-d^2))^(2/3)*ln(x
^2-x*(1/c^2/b*(a*c^2-d^2))^(1/3)+(1/c^2/b*(a*c^2-d^2))^(2/3))+1/3
/b/c/(1/c^2/b*(a*c^2-d^2))^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1
/c^2/b*(a*c^2-d^2))^(1/3)*x-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{bcx^3 + ac + \sqrt{bx^3 + a}\,d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*c*x^3 + a*c + sqrt(b*x^3 + a)*d),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*c*x^3 + a*c + sqrt(b*x^3 + a)*d), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*c*x^3 + a*c + sqrt(b*x^3 + a)*d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{bcx^3 + ac + \sqrt{bx^3 + a}d} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*c*x^3 + a*c + sqrt(b*x^3 + a)*d),x, algorithm="giac")
```

```
[Out] integrate(1/(b*c*x^3 + a*c + sqrt(b*x^3 + a)*d), x)
```

3.399 $\int \frac{1}{x^2\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx$

Optimal. Leaf size=319

$$\frac{d\sqrt{\frac{bx^3}{a}+1}F_1\left(-\frac{1}{3};\frac{1}{2},1;\frac{2}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{x\sqrt{a+bx^3}\,(ac^2-d^2)}-\frac{\sqrt[3]{b}c^{5/3}\log\left(-\sqrt[3]{b}c^{2/3}x\sqrt[3]{ac^2-d^2}+\left(ac^2-d^2\right)^{2/3}+b^{2/3}c^{4/3}x^2\right)}{6\,(ac^2-d^2)^{4/3}}+\frac{\sqrt[3]{b}c^{5/3}\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3\,(ac^2-d^2)^{4/3}}+\frac{\sqrt[3]{b}c^{5/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right)}{\sqrt{3}\,(ac^2-d^2)^{4/3}}-\frac{c}{x\,(ac^2-d^2)}$$

```
[Out] -(c/((a*c^2 - d^2)*x)) + (d*Sqrt[1 + (b*x^3)/a]*AppellF1[-1/3, 1/
2, 1, 2/3, -((b*x^3)/a), -((b*c^2*x^3)/(a*c^2 - d^2))])/((a*c^2 -
 d^2)*x*Sqrt[a + b*x^3]) + (b^(1/3)*c^(5/3)*ArcTan[(1 - (2*b^(1/3
)*c^(2/3)*x)/(a*c^2 - d^2)^(1/3))/Sqrt[3]])/(Sqrt[3]*(a*c^2 - d^2
)^(4/3)) + (b^(1/3)*c^(5/3)*Log[(a*c^2 - d^2)^(1/3) + b^(1/3)*c^(
2/3)*x])/(3*(a*c^2 - d^2)^(4/3)) - (b^(1/3)*c^(5/3)*Log[(a*c^2 -
d^2)^(2/3) - b^(1/3)*c^(2/3)*(a*c^2 - d^2)^(1/3)*x + b^(2/3)*c^(4
/3)*x^2])/(6*(a*c^2 - d^2)^(4/3))
```

Rubi [A] time = 0.949746, antiderivative size = 319, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$

$$\frac{d\sqrt{\frac{bx^3}{a}+1}F_1\left(-\frac{1}{3};\frac{1}{2},1;\frac{2}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{x\sqrt{a+bx^3}\,(ac^2-d^2)}-\frac{\sqrt[3]{b}c^{5/3}\log\left(-\sqrt[3]{b}c^{2/3}x\sqrt[3]{ac^2-d^2}+\left(ac^2-d^2\right)^{2/3}+b^{2/3}c^{4/3}x^2\right)}{6\,(ac^2-d^2)^{4/3}}+\frac{\sqrt[3]{b}c^{5/3}\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3\,(ac^2-d^2)^{4/3}}+\frac{\sqrt[3]{b}c^{5/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right)}{\sqrt{3}\,(ac^2-d^2)^{4/3}}-\frac{c}{x\,(ac^2-d^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])),x]
```

```
[Out] -(c/((a*c^2 - d^2)*x)) + (d*Sqrt[1 + (b*x^3)/a]*AppellF1[-1/3, 1/
2, 1, 2/3, -((b*x^3)/a), -((b*c^2*x^3)/(a*c^2 - d^2))])/((a*c^2 -
 d^2)*x*Sqrt[a + b*x^3]) + (b^(1/3)*c^(5/3)*ArcTan[(1 - (2*b^(1/3
)*c^(2/3)*x)/(a*c^2 - d^2)^(1/3))/Sqrt[3]])/(Sqrt[3]*(a*c^2 - d^2
)^(4/3)) + (b^(1/3)*c^(5/3)*Log[(a*c^2 - d^2)^(1/3) + b^(1/3)*c^(
2/3)*x])/(3*(a*c^2 - d^2)^(4/3)) - (b^(1/3)*c^(5/3)*Log[(a*c^2 -
d^2)^(2/3) - b^(1/3)*c^(2/3)*(a*c^2 - d^2)^(1/3)*x + b^(2/3)*c^(4
/3)*x^2])/(6*(a*c^2 - d^2)^(4/3))
```

Rubi in Sympy [A] time = 97.038, size = 286, normalized size = 0.9

$$\frac{\sqrt[3]{b}c^{\frac{5}{3}}\log\left(\sqrt[3]{b}c^{\frac{2}{3}}x-\sqrt[3]{-ac^2+d^2}\right)}{3\left(-ac^2+d^2\right)^{\frac{4}{3}}} - \frac{\sqrt[3]{b}c^{\frac{5}{3}}\log\left(a^{\frac{2}{3}}b^{\frac{2}{3}}c^{\frac{4}{3}}x^2+a^{\frac{2}{3}}\sqrt[3]{b}c^{\frac{2}{3}}x\sqrt[3]{-ac^2+d^2}+a^{\frac{2}{3}}\left(-ac^2+d^2\right)^{\frac{2}{3}}\right)}{6\left(-ac^2+d^2\right)^{\frac{4}{3}}} + \frac{\sqrt{3}\sqrt[3]{b}c^{\frac{5}{3}}\operatorname{atan}\left(\sqrt{3}\left(\frac{2\sqrt[3]{b}c^{\frac{2}{3}}x}{3\sqrt[3]{-ac^2+d^2}}+\frac{1}{3}\right)\right)}{3\left(-ac^2+d^2\right)^{\frac{4}{3}}} + \frac{c}{x\left(-ac^2+d^2\right)} - \frac{d\sqrt{a+bx^3}\operatorname{appellf_1}\left(-\frac{1}{3},\frac{1}{2},1,\frac{2}{3},-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{ax\sqrt{1+\frac{bx^3}{a}}\left(-ac^2+d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
```

```
[Out] b**(1/3)*c**(5/3)*log(b**(1/3)*c**(2/3)*x - (-a*c**2 + d**2)**(1/
3))/(3*(-a*c**2 + d**2)**(4/3)) - b**(1/3)*c**(5/3)*log(a**(2/3)*
b**(2/3)*c**(4/3)*x**2 + a**(2/3)*b**(1/3)*c**(2/3)*x*(-a*c**2 +
d**2)**(1/3) + a**(2/3)*(-a*c**2 + d**2)**(2/3))/(6*(-a*c**2 + d*
*2)**(4/3)) + sqrt(3)*b**(1/3)*c**(5/3)*atan(sqrt(3)*(2*b**(1/3)*
c**(2/3)*x/(3*(-a*c**2 + d**2)**(1/3)) + 1/3))/(3*(-a*c**2 + d**2
)**(4/3)) + c/(x*(-a*c**2 + d**2)) - d*sqrt(a + b*x**3)*appellf1(
-1/3, 1/2, 1, 2/3, -b*x**3/a, -b*c**2*x**3/(a*c**2 - d**2))/(a*x*
sqrt(1 + b*x**3/a)*(-a*c**2 + d**2))
```

Mathematica [B] time = 6.71123, size = 1047, normalized size = 3.28

$$-\frac{8b^2c^2dF_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)x^5}{5\sqrt{bx^3+a}\left(bc^2x^3+ac^2-d^2\right)\left(16a\left(ac^2-d^2\right)F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)-3bx^3\left(2aF_1\left(\frac{8}{3};\frac{1}{2},2;\frac{11}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)c^2+\left(\cdots\right.\right.\right.}$$
$$+\frac{5bd^3F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)x^2}{2\sqrt{bx^3+a}\left(bc^2x^3+ac^2-d^2\right)\left(10a\left(ac^2-d^2\right)F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)-3bx^3\left(2aF_1\left(\frac{5}{3};\frac{1}{2},2;\frac{8}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)c^2+\left(a\cdots\right.\right.\right.}$$
$$+\frac{5abc^2dF_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)x^2}{2\sqrt{bx^3+a}\left(bc^2x^3+ac^2-d^2\right)\left(10a\left(ac^2-d^2\right)F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)-3bx^3\left(2aF_1\left(\frac{5}{3};\frac{1}{2},2;\frac{8}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)c^2+\left(a\cdots\right.\right.\right.}$$
$$-\frac{\sqrt[3]{b}c^{5/3}\tan^{-1}\left(\frac{2\sqrt[3]{b}c^{2/3}x-\sqrt[3]{ac^2-d^2}}{\sqrt{3}\sqrt[3]{ac^2-d^2}}\right)}{\sqrt{3}\left(ac^2-d^2\right)^{4/3}}+\frac{\sqrt[3]{b}c^{5/3}\log\left(\sqrt[3]{b}c^{2/3}x+\sqrt[3]{ac^2-d^2}\right)}{3\left(ac^2-d^2\right)^{4/3}}$$
$$-\frac{\sqrt[3]{b}c^{5/3}\log\left(b^{2/3}c^{4/3}x^2-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+\left(ac^2-d^2\right)^{2/3}\right)}{6\left(ac^2-d^2\right)^{4/3}}+\frac{d\sqrt{bx^3+a}}{a\left(ac^2-d^2\right)x}-\frac{c}{\left(ac^2-d^2\right)x}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/(x^2*(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])),x]
```

```
[Out] -(c/((a*c^2 - d^2)*x)) + (d*Sqrt[a + b*x^3])/(a*(a*c^2 - d^2)*x)
+ (5*a*b*c^2*d*x^2*AppellF1[2/3, 1/2, 1, 5/3, -((b*x^3)/a), -((b*
c^2*x^3)/(a*c^2 - d^2))])/(2*Sqrt[a + b*x^3]*(a*c^2 - d^2 + b*c^2
*x^3)*(10*a*(a*c^2 - d^2)*AppellF1[2/3, 1/2, 1, 5/3, -((b*x^3)/a)
, -((b*c^2*x^3)/(a*c^2 - d^2))] - 3*b*x^3*(2*a*c^2*AppellF1[5/3,
1/2, 2, 8/3, -((b*x^3)/a), -((b*c^2*x^3)/(a*c^2 - d^2))] + (a*c^2
 - d^2)*AppellF1[5/3, 3/2, 1, 8/3, -((b*x^3)/a), -((b*c^2*x^3)/(a
*c^2 - d^2))]))) + (5*b*d^3*x^2*AppellF1[2/3, 1/2, 1, 5/3, -((b*x
^3)/a), -((b*c^2*x^3)/(a*c^2 - d^2))])/(2*Sqrt[a + b*x^3]*(a*c^2
- d^2 + b*c^2*x^3)*(10*a*(a*c^2 - d^2)*AppellF1[2/3, 1/2, 1, 5/3,
 -((b*x^3)/a), -((b*c^2*x^3)/(a*c^2 - d^2))] - 3*b*x^3*(2*a*c^2*A
ppellF1[5/3, 1/2, 2, 8/3, -((b*x^3)/a), -((b*c^2*x^3)/(a*c^2 - d^
2))] + (a*c^2 - d^2)*AppellF1[5/3, 3/2, 1, 8/3, -((b*x^3)/a), -((
b*c^2*x^3)/(a*c^2 - d^2))]))) - (8*b^2*c^2*d*x^5*AppellF1[5/3, 1/
2, 1, 8/3, -((b*x^3)/a), -((b*c^2*x^3)/(a*c^2 - d^2))])/(5*Sqrt[a
 + b*x^3]*(a*c^2 - d^2 + b*c^2*x^3)*(16*a*(a*c^2 - d^2)*AppellF1[
5/3, 1/2, 1, 8/3, -((b*x^3)/a), -((b*c^2*x^3)/(a*c^2 - d^2))] - 3
*b*x^3*(2*a*c^2*AppellF1[8/3, 1/2, 2, 11/3, -((b*x^3)/a), -((b*c^
2*x^3)/(a*c^2 - d^2))] + (a*c^2 - d^2)*AppellF1[8/3, 3/2, 1, 11/3
, -((b*x^3)/a), -((b*c^2*x^3)/(a*c^2 - d^2))]))) - (b^(1/3)*c^(5/
3)*ArcTan[(-(a*c^2 - d^2)^(1/3) + 2*b^(1/3)*c^(2/3)*x)/(Sqrt[3]*(
a*c^2 - d^2)^(1/3))])/(Sqrt[3]*(a*c^2 - d^2)^(4/3)) + (b^(1/3)*c^
(5/3)*Log[(a*c^2 - d^2)^(1/3) + b^(1/3)*c^(2/3)*x])/(3*(a*c^2 - d
^2)^(4/3)) - (b^(1/3)*c^(5/3)*Log[(a*c^2 - d^2)^(2/3) - b^(1/3)*c
^(2/3)*(a*c^2 - d^2)^(1/3)*x + b^(2/3)*c^(4/3)*x^2])/(6*(a*c^2 -
d^2)^(4/3))
```

Maple [C] time = 0.052, size = 3560, normalized size = 11.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x)

[Out] -c/(a*c^2-d^2)/x+1/3*a*c^3/(a*c^2-d^2)/d^2/(1/c^2/b*(a*c^2-d^2))^(1/3)*ln(x+(1/c^2/b*(a*c^2-d^2))^(1/3))-1/6*a*c^3/(a*c^2-d^2)/d^2/(1/c^2/b*(a*c^2-d^2))^(1/3)*ln(x^2-x*(1/c^2/b*(a*c^2-d^2))^(1/3)+(1/c^2/b*(a*c^2-d^2))^(2/3))-1/3*a*c^3/(a*c^2-d^2)/d^2*3^(1/2)/(1/c^2/b*(a*c^2-d^2))^(1/3)*arctan(1/3*3^(1/2)*(2/(1/c^2/b*(a*c^2-d^2))^(1/3)*x-1))-1/3*c/d^2/(1/c^2/b*(a*c^2-d^2))^(1/3)*ln(x+(1/c^2/b*(a*c^2-d^2))^(1/3))+1/6*c/d^2/(1/c^2/b*(a*c^2-d^2))^(1/3)*ln(x^2-x*(1/c^2/b*(a*c^2-d^2))^(1/3)+(1/c^2/b*(a*c^2-d^2))^(2/3))+1/3*c/d^2*3^(1/2)/(1/c^2/b*(a*c^2-d^2))^(1/3)*arctan(1/3*3^(1/2)*(2/(1/c^2/b*(a*c^2-d^2))^(1/3)*x-1))+d/a/(a*c^2-d^2)/x*(b*x^3+a)^(1/2)-3/2*I*d/a/(a*c^2-d^2)*3^(1/2)*(-a*b^2)^(2/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))/b-3/2*d/a/(a*c^2-d^2)*(-a*b^2)^(2/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))/b-I/a/d*3^(1/2)*(-a*b^2)^(2/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))/b-2/3*I/b*c^2/(a*c^2-d^2)/d*3^(1/2)*(-a*b^2)^(2/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(

```
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-1/a/d*(-a*b^2)^(2/3)
*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/
2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/
3))^(1/2)/(b*x^3+a)^(1/2)*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b
^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3
))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))/b+2/3*I/a/d*3^(1/2)*(-a*b^2)^(
2/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3
^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)
^(1/3))^(1/2)/(b*x^3+a)^(1/2)/b*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b
*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2
)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+I*d/a/(a*c^2-d^2)*3^(1/2
)*(-a*b^2)^(2/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3)
)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(
-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2
)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)/b*EllipticF(1/3*3^(1/2)
*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/
2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*c^2/(a*c
^2-d^2)/d*(-a*b^2)^(2/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^
2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))
^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticE(1/3*3
^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/3*I/
b^2*c^2/(a*c^2-d^2)/d*2^(1/2)*sum(1/_alpha*(-a*b^2)^(1/3)*(1/2*I*
b*(2*x+1/b*((-a*b^2)^(1/3)-I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1
/3))^(1/2)*(b*(x-1/b*(-a*b^2)^(1/3))/(-3*(-a*b^2)^(1/3)+I*3^(1/2)
*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)+I*3^(1
/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*(I*(-a
*b^2)^(1/3)*3^(1/2)*_alpha*b-I*(-a*b^2)^(2/3)*3^(1/2)+2*_alpha^2*
b^2-(-a*b^2)^(1/3)*_alpha*b-(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2
)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1
/2)*b/(-a*b^2)^(1/3))^(1/2),-1/2*c^2/b*(2*I*3^(1/2)*(-a*b^2)^(1/3
)*_alpha^2*b-I*3^(1/2)*(-a*b^2)^(2/3)*_alpha+I*3^(1/2)*a*b-3*(-a*
b^2)^(2/3)*_alpha-3*a*b)/d^2,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)),_alpha=Roo
tOf(_Z^3*b*c^2+a*c^2-d^2))+I/b*c^2/(a*c^2-d^2)/d*3^(1/2)*(-a*b^2)
^(2/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^
2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b
*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2
)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bcx^3 + ac + \sqrt{bx^3 + a}d\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*c*x^3 + a*c + sqrt(b*x^3 + a)*d)*x^2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*c*x^3 + a*c + sqrt(b*x^3 + a)*d)*x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*c*x^3 + a*c + sqrt(b*x^3 + a)*d)*x^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bcx^3 + ac + \sqrt{bx^3 + a}d\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*c*x^3 + a*c + sqrt(b*x^3 + a)*d)*x^2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*c*x^3 + a*c + sqrt(b*x^3 + a)*d)*x^2), x)
```

3.400 $\int \frac{1}{x^3\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx$

Optimal. Leaf size=324

$$\frac{d\sqrt{\frac{bx^3}{a}+1}F_1\left(-\frac{2}{3};\frac{1}{2},1;\frac{1}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{2x^2\sqrt{a+bx^3}\,(ac^2-d^2)}+\frac{b^{2/3}c^{7/3}\log\left(-\sqrt[3]{b}c^{2/3}x\sqrt[3]{ac^2-d^2}+\left(ac^2-d^2\right)^{2/3}+b^{2/3}c^{4/3}x^2\right)}{6\,(ac^2-d^2)^{5/3}}-\frac{b^{2/3}c^{7/3}\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3\,(ac^2-d^2)^{5/3}}+\frac{b^{2/3}c^{7/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right)}{\sqrt{3}\,(ac^2-d^2)^{5/3}}-\frac{c}{2x^2\,(ac^2-d^2)}$$

```
[Out] -c/(2*(a*c^2 - d^2)*x^2) + (d*Sqrt[1 + (b*x^3)/a]*AppellF1[-2/3,
1/2, 1, 1/3, -((b*x^3)/a), -((b*c^2*x^3)/(a*c^2 - d^2))])/(2*(a*c
^2 - d^2)*x^2*Sqrt[a + b*x^3]) + (b^(2/3)*c^(7/3)*ArcTan[(1 - (2*
b^(1/3)*c^(2/3)*x)/(a*c^2 - d^2)^(1/3))/Sqrt[3]])/(Sqrt[3]*(a*c^2
 - d^2)^(5/3)) - (b^(2/3)*c^(7/3)*Log[(a*c^2 - d^2)^(1/3) + b^(1/
3)*c^(2/3)*x])/(3*(a*c^2 - d^2)^(5/3)) + (b^(2/3)*c^(7/3)*Log[(a*
c^2 - d^2)^(2/3) - b^(1/3)*c^(2/3)*(a*c^2 - d^2)^(1/3)*x + b^(2/3
)*c^(4/3)*x^2])/(6*(a*c^2 - d^2)^(5/3))
```

Rubi [A] time = 0.955046, antiderivative size = 324, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345

$$\frac{d\sqrt{\frac{bx^3}{a}+1}F_1\left(-\frac{2}{3};\frac{1}{2},1;\frac{1}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{2x^2\sqrt{a+bx^3}\,(ac^2-d^2)}+\frac{b^{2/3}c^{7/3}\log\left(-\sqrt[3]{b}c^{2/3}x\sqrt[3]{ac^2-d^2}+\left(ac^2-d^2\right)^{2/3}+b^{2/3}c^{4/3}x^2\right)}{6\,(ac^2-d^2)^{5/3}}-\frac{b^{2/3}c^{7/3}\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3\,(ac^2-d^2)^{5/3}}+\frac{b^{2/3}c^{7/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right)}{\sqrt{3}\,(ac^2-d^2)^{5/3}}-\frac{c}{2x^2\,(ac^2-d^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])),x]
```

```
[Out] -c/(2*(a*c^2 - d^2)*x^2) + (d*Sqrt[1 + (b*x^3)/a]*AppellF1[-2/3,
1/2, 1, 1/3, -((b*x^3)/a), -((b*c^2*x^3)/(a*c^2 - d^2))])/(2*(a*c
^2 - d^2)*x^2*Sqrt[a + b*x^3]) + (b^(2/3)*c^(7/3)*ArcTan[(1 - (2*
b^(1/3)*c^(2/3)*x)/(a*c^2 - d^2)^(1/3))/Sqrt[3]])/(Sqrt[3]*(a*c^2
 - d^2)^(5/3)) - (b^(2/3)*c^(7/3)*Log[(a*c^2 - d^2)^(1/3) + b^(1/
3)*c^(2/3)*x])/(3*(a*c^2 - d^2)^(5/3)) + (b^(2/3)*c^(7/3)*Log[(a*
c^2 - d^2)^(2/3) - b^(1/3)*c^(2/3)*(a*c^2 - d^2)^(1/3)*x + b^(2/3
)*c^(4/3)*x^2])/(6*(a*c^2 - d^2)^(5/3))
```

Rubi in Sympy [A] time = 92.3923, size = 292, normalized size = 0.9

$$\frac{b^{\frac{2}{3}}c^{\frac{7}{3}}\log\left(\sqrt[3]{b}c^{\frac{2}{3}}x-\sqrt[3]{-ac^2+d^2}\right)}{3\left(-ac^2+d^2\right)^{\frac{5}{3}}} - \frac{b^{\frac{2}{3}}c^{\frac{7}{3}}\log\left(a^{\frac{2}{3}}b^{\frac{2}{3}}c^{\frac{4}{3}}x^2+a^{\frac{2}{3}}\sqrt[3]{b}c^{\frac{2}{3}}x\sqrt[3]{-ac^2+d^2}+a^{\frac{2}{3}}\left(-ac^2+d^2\right)^{\frac{2}{3}}\right)}{6\left(-ac^2+d^2\right)^{\frac{5}{3}}} - \frac{\sqrt{3}b^{\frac{2}{3}}c^{\frac{7}{3}}\operatorname{atan}\left(\sqrt{3}\left(\frac{2\sqrt[3]{b}c^{\frac{2}{3}}x}{3\sqrt[3]{-ac^2+d^2}}+\frac{1}{3}\right)\right)}{3\left(-ac^2+d^2\right)^{\frac{5}{3}}} + \frac{c}{2x^2\left(-ac^2+d^2\right)} - \frac{d\sqrt{a+bx^3}\operatorname{appellf_1}\left(-\frac{2}{3},\frac{1}{2},1,\frac{1}{3},-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{2ax^2\sqrt{1+\frac{bx^3}{a}}\left(-ac^2+d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
```

```
[Out] b**(2/3)*c**(7/3)*log(b**(1/3)*c**(2/3)*x - (-a*c**2 + d**2)**(1/
3))/(3*(-a*c**2 + d**2)**(5/3)) - b**(2/3)*c**(7/3)*log(a**(2/3)*
b**(2/3)*c**(4/3)*x**2 + a**(2/3)*b**(1/3)*c**(2/3)*x*(-a*c**2 +
d**2)**(1/3) + a**(2/3)*(-a*c**2 + d**2)**(2/3))/(6*(-a*c**2 + d*
*2)**(5/3)) - sqrt(3)*b**(2/3)*c**(7/3)*atan(sqrt(3)*(2*b**(1/3)*
c**(2/3)*x/(3*(-a*c**2 + d**2)**(1/3)) + 1/3))/(3*(-a*c**2 + d**2
)**(5/3)) + c/(2*x**2*(-a*c**2 + d**2)) - d*sqrt(a + b*x**3)*appe
llf1(-2/3, 1/2, 1, 1/3, -b*x**3/a, -b*c**2*x**3/(a*c**2 - d**2))/
(2*a*x**2*sqrt(1 + b*x**3/a)*(-a*c**2 + d**2))
```

Mathematica [B] time = 6.67882, size = 1044, normalized size = 3.22

$$\frac{7b^2c^2dF_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)x^4}{8\sqrt{bx^3+a}\,(bc^2x^3+ac^2-d^2)\left(14a\,(ac^2-d^2)\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)-3bx^3\left(2aF_1\left(\frac{7}{3};\frac{1}{2},2;\frac{10}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)c^2+(a\right.\right.}$$
$$-\frac{2bd^3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)x}{\sqrt{bx^3+a}\,(bc^2x^3+ac^2-d^2)\left(8a\,(ac^2-d^2)\,F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)-3bx^3\left(2aF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)c^2+(ac^2\right.\right.}$$
$$+\frac{10abc^2dF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)x}{\sqrt{bx^3+a}\,(bc^2x^3+ac^2-d^2)\left(8a\,(ac^2-d^2)\,F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)-3bx^3\left(2aF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)c^2+(ac^2\right.\right.}$$
$$-\frac{b^{2/3}c^{7/3}\tan^{-1}\left(\frac{2\sqrt[3]{b}c^{2/3}x-\sqrt[3]{ac^2-d^2}}{\sqrt{3}\sqrt[3]{ac^2-d^2}}\right)}{\sqrt{3}\,(ac^2-d^2)^{5/3}}-\frac{b^{2/3}c^{7/3}\log\left(\sqrt[3]{b}c^{2/3}x+\sqrt[3]{ac^2-d^2}\right)}{3\,(ac^2-d^2)^{5/3}}$$
$$+\frac{b^{2/3}c^{7/3}\log\left(b^{2/3}c^{4/3}x^2-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+\left(ac^2-d^2\right)^{2/3}\right)}{6\,(ac^2-d^2)^{5/3}}+\frac{d\sqrt{bx^3+a}}{2a\,(ac^2-d^2)\,x^2}-\frac{c}{2\,(ac^2-d^2)\,x^2}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/(x^3*(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])),x]
```

```
[Out] -c/(2*(a*c^2 - d^2)*x^2) + (d*Sqrt[a + b*x^3])/(2*a*(a*c^2 - d^2)
*x^2) + (10*a*b*c^2*d*x*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a),
-((b*c^2*x^3)/(a*c^2 - d^2))])/(Sqrt[a + b*x^3]*(a*c^2 - d^2 + b*
c^2*x^3)*(8*a*(a*c^2 - d^2)*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/
a), -((b*c^2*x^3)/(a*c^2 - d^2))] - 3*b*x^3*(2*a*c^2*AppellF1[4/3
, 1/2, 2, 7/3, -((b*x^3)/a), -((b*c^2*x^3)/(a*c^2 - d^2))] + (a*c
^2 - d^2)*AppellF1[4/3, 3/2, 1, 7/3, -((b*x^3)/a), -((b*c^2*x^3)/
(a*c^2 - d^2))]))) - (2*b*d^3*x*AppellF1[1/3, 1/2, 1, 4/3, -((b*x
^3)/a), -((b*c^2*x^3)/(a*c^2 - d^2))])/(Sqrt[a + b*x^3]*(a*c^2 -
d^2 + b*c^2*x^3)*(8*a*(a*c^2 - d^2)*AppellF1[1/3, 1/2, 1, 4/3, -(
(b*x^3)/a), -((b*c^2*x^3)/(a*c^2 - d^2))] - 3*b*x^3*(2*a*c^2*Appe
llF1[4/3, 1/2, 2, 7/3, -((b*x^3)/a), -((b*c^2*x^3)/(a*c^2 - d^2))
] + (a*c^2 - d^2)*AppellF1[4/3, 3/2, 1, 7/3, -((b*x^3)/a), -((b*c
^2*x^3)/(a*c^2 - d^2))]))) + (7*b^2*c^2*d*x^4*AppellF1[4/3, 1/2,
1, 7/3, -((b*x^3)/a), -((b*c^2*x^3)/(a*c^2 - d^2))])/(8*Sqrt[a +
b*x^3]*(a*c^2 - d^2 + b*c^2*x^3)*(14*a*(a*c^2 - d^2)*AppellF1[4/3
, 1/2, 1, 7/3, -((b*x^3)/a), -((b*c^2*x^3)/(a*c^2 - d^2))] - 3*b*
x^3*(2*a*c^2*AppellF1[7/3, 1/2, 2, 10/3, -((b*x^3)/a), -((b*c^2*x
^3)/(a*c^2 - d^2))] + (a*c^2 - d^2)*AppellF1[7/3, 3/2, 1, 10/3, -
((b*x^3)/a), -((b*c^2*x^3)/(a*c^2 - d^2))]))) - (b^(2/3)*c^(7/3)*
ArcTan[(-(a*c^2 - d^2)^(1/3) + 2*b^(1/3)*c^(2/3)*x)/(Sqrt[3]*(a*c
^2 - d^2)^(1/3))])/(Sqrt[3]*(a*c^2 - d^2)^(5/3)) - (b^(2/3)*c^(7/
3)*Log[(a*c^2 - d^2)^(1/3) + b^(1/3)*c^(2/3)*x])/(3*(a*c^2 - d^2)
^(5/3)) + (b^(2/3)*c^(7/3)*Log[(a*c^2 - d^2)^(2/3) - b^(1/3)*c^(2
/3)*(a*c^2 - d^2)^(1/3)*x + b^(2/3)*c^(4/3)*x^2])/(6*(a*c^2 - d^2
)^(5/3))
```

Maple [C] time = 0.052, size = 1789, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x)
```

```
[Out] 1/3*c/d^2/(1/c^2/b*(a*c^2-d^2))^(2/3)*ln(x+(1/c^2/b*(a*c^2-d^2))^
(1/3))-1/6*c/d^2/(1/c^2/b*(a*c^2-d^2))^(2/3)*ln(x^2-x*(1/c^2/b*(a
*c^2-d^2))^(1/3)+(1/c^2/b*(a*c^2-d^2))^(2/3))+1/3*c/d^2/(1/c^2/b*
(a*c^2-d^2))^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/c^2/b*(a*c^2-
d^2))^(1/3)*x-1))-1/3*a*c^3/(a*c^2-d^2)/d^2/(1/c^2/b*(a*c^2-d^2))
^(2/3)*ln(x+(1/c^2/b*(a*c^2-d^2))^(1/3))+1/6*a*c^3/(a*c^2-d^2)/d^
2/(1/c^2/b*(a*c^2-d^2))^(2/3)*ln(x^2-x*(1/c^2/b*(a*c^2-d^2))^(1/3
)+(1/c^2/b*(a*c^2-d^2))^(2/3))-1/3*a*c^3/(a*c^2-d^2)/d^2/(1/c^2/b
*(a*c^2-d^2))^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/c^2/b*(a*c^2
-d^2))^(1/3)*x-1))-1/2*c/(a*c^2-d^2)/x^2+1/2*d/a/(a*c^2-d^2)/x^2*
(b*x^3+a)^(1/2)+1/2*I*d/a/(a*c^2-d^2)*3^(1/2)*(-a*b^2)^(1/3)*(I*(
x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/
(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(
1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1
/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3)))^(1/2))+2/3*I/a/d*3^(1/2)*(-a*b^2)^(1/3)*(I
*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*
b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))
^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)
^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^
(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-2/3*I/(a*c^2-d^2)*c^2/d*3^(1/2)*(
-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*
(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b
/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(
x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/
(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/3*I/(a*c^2-d^2)
/b^2*c^2/d*2^(1/2)*sum(1/_alpha^2*(-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/
b*((-a*b^2)^(1/3)-I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2
)*(b*(x-1/b*(-a*b^2)^(1/3))/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)
^(1/3)))^(1/2)*(-1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)+I*3^(1/2)*(-a*b
^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*(I*(-a*b^2)^(1/
3)*3^(1/2)*_alpha*b-I*(-a*b^2)^(2/3)*3^(1/2)+2*_alpha^2*b^2-(-a*b
^2)^(1/3)*_alpha*b-(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1
```

```
/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a
*b^2)^(1/3))^(1/2),-1/2*c^2/b*(2*I*3^(1/2)*(-a*b^2)^(1/3)*_alpha^
2*b-I*3^(1/2)*(-a*b^2)^(2/3)*_alpha+I*3^(1/2)*a*b-3*(-a*b^2)^(2/3
)*_alpha-3*a*b)/d^2,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*
b*c^2+a*c^2-d^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bcx^3 + ac + \sqrt{bx^3 + ad}\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*c*x^3 + a*c + sqrt(b*x^3 + a)*d)*x^3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*c*x^3 + a*c + sqrt(b*x^3 + a)*d)*x^3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*c*x^3 + a*c + sqrt(b*x^3 + a)*d)*x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bcx^3 + ac + \sqrt{bx^3 + a}\,d\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*c*x^3 + a*c + sqrt(b*x^3 + a)*d)*x^3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*c*x^3 + a*c + sqrt(b*x^3 + a)*d)*x^3), x)
```

3.401 $\int \frac{1}{ac+bcx^n+d\sqrt{a+bx^n}}\,dx$

Optimal. Leaf size=135

$$\frac{cx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bc^2x^n}{ac^2-d^2}\right)}{ac^2-d^2}-\frac{dx\sqrt{\frac{bx^n}{a}+1}F_1\left(\frac{1}{n};\frac{1}{2},1;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{bc^2x^n}{ac^2-d^2}\right)}{(ac^2-d^2)\sqrt{a+bx^n}}$$

```
[Out] -((d*x*Sqrt[1 + (b*x^n)/a]*AppellF1[n^(-1), 1/2, 1, 1 + n^(-1), -
((b*x^n)/a), -((b*c^2*x^n)/(a*c^2 - d^2))])/((a*c^2 - d^2)*Sqrt[a
 + b*x^n])) + (c*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*
c^2*x^n)/(a*c^2 - d^2))])/(a*c^2 - d^2)
```

Rubi [A] time = 0.216469, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16

$$\frac{cx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bc^2x^n}{ac^2-d^2}\right)}{ac^2-d^2}-\frac{dx\sqrt{\frac{bx^n}{a}+1}F_1\left(\frac{1}{n};\frac{1}{2},1;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{bc^2x^n}{ac^2-d^2}\right)}{(ac^2-d^2)\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a*c + b*c*x^n + d*Sqrt[a + b*x^n])^(-1),x]
```

```
[Out] -((d*x*Sqrt[1 + (b*x^n)/a]*AppellF1[n^(-1), 1/2, 1, 1 + n^(-1), -
((b*x^n)/a), -((b*c^2*x^n)/(a*c^2 - d^2))])/((a*c^2 - d^2)*Sqrt[a
 + b*x^n])) + (c*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*
c^2*x^n)/(a*c^2 - d^2))])/(a*c^2 - d^2)
```

Rubi in Sympy [A] time = 42.4942, size = 105, normalized size = 0.78

$$-\frac{cx{}_2F_1\left(\begin{matrix}1,\frac{1}{n}\\1+\frac{1}{n}\end{matrix}\middle|-\frac{bc^2x^n}{ac^2-d^2}\right)}{-ac^2+d^2}+\frac{dx\sqrt{a+bx^n}\,\mathrm{appellf}_1\left(\frac{1}{n},\frac{1}{2},1,1+\frac{1}{n},-\frac{bx^n}{a},-\frac{bc^2x^n}{ac^2-d^2}\right)}{a\sqrt{1+\frac{bx^n}{a}}\left(-ac^2+d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a*c+b*c*x**n+d*(a+b*x**n)**(1/2)),x)
```

```
[Out] -c*x*hyper((1, 1/n), (1 + 1/n,), -b*c**2*x**n/(a*c**2 - d**2))/(-
a*c**2 + d**2) + d*x*sqrt(a + b*x**n)*appellf1(1/n, 1/2, 1, 1 + 1
```

```
/n, -b*x**n/a, -b*c**2*x**n/(a*c**2 - d**2))/(a*sqrt(1 + b*x**n/a
)*(-a*c**2 + d**2))
```

Mathematica [B] time = 0.895518, size = 320, normalized size = 2.37

$$\frac{cx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bc^2x^n}{ac^2-d^2}\right)}{ac^2-d^2}-\frac{2ad(n+1)x\left(ac^2-d^2\right)F_1\left(\frac{1}{n};\frac{1}{2},1;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{bc^2x^n}{ac^2-d^2}\right)}{\sqrt{a+bx^n}\left(ac^2+bc^2x^n-d^2\right)\left(\left(ac^2-d^2\right)\left(2a(n+1)F_1\left(\frac{1}{n};\frac{1}{2},1;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{bc^2x^n}{ac^2-d^2}\right)-bnx^nF_1\left(1+\frac{1}{n};\frac{3}{2},1;2+\frac{1}{n};-\frac{bx^n}{a},-\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a*c + b*c*x^n + d*Sqrt[a + b*x^n])^(-1),x]
```

```
[Out] (-2*a*d*(a*c^2 - d^2)*(1 + n)*x*AppellF1[n^(-1), 1/2, 1, 1 + n^(-
1), -((b*x^n)/a), -((b*c^2*x^n)/(a*c^2 - d^2))])/(Sqrt[a + b*x^n]
*(a*c^2 - d^2 + b*c^2*x^n)*(-2*a*b*c^2*n*x^n*AppellF1[1 + n^(-1),
 1/2, 2, 2 + n^(-1), -((b*x^n)/a), -((b*c^2*x^n)/(a*c^2 - d^2))]
+ (a*c^2 - d^2)*(-(b*n*x^n*AppellF1[1 + n^(-1), 3/2, 1, 2 + n^(-1
), -((b*x^n)/a), -((b*c^2*x^n)/(a*c^2 - d^2))]) + 2*a*(1 + n)*App
ellF1[n^(-1), 1/2, 1, 1 + n^(-1), -((b*x^n)/a), -((b*c^2*x^n)/(a*
c^2 - d^2))]))) + (c*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -
((b*c^2*x^n)/(a*c^2 - d^2))])/(a*c^2 - d^2)
```

Maple [F] time = 0.014, size = 0, normalized size = 0.

$$\int \left(ac+bc\,x^n+d\sqrt{a+b\,x^n}\right)^{-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x)
```

```
[Out] int(1/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{bcx^n+ac+\sqrt{bx^n+a}d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c*x^n + a*c + sqrt(b*x^n + a)*d),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*c*x^n + a*c + sqrt(b*x^n + a)*d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{bcx^n + ac + \sqrt{bx^n + a}d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c*x^n + a*c + sqrt(b*x^n + a)*d),x, algorithm="fricas")
```

```
[Out] integral(1/(b*c*x^n + a*c + sqrt(b*x^n + a)*d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{ac + bcx^n + d\sqrt{a + bx^n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c+b*c*x**n+d*(a+b*x**n)**(1/2)),x)
```

```
[Out] Integral(1/(a*c + b*c*x**n + d*sqrt(a + b*x**n)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{bcx^n + ac + \sqrt{bx^n + a}d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c*x^n + a*c + sqrt(b*x^n + a)*d),x, algorithm="giac")
```

```
[Out] integrate(1/(b*c*x^n + a*c + sqrt(b*x^n + a)*d), x)
```

3.402 $\int \frac{x^m}{ac+bcx^n+d\sqrt{a+bx^n}}\,dx$

Optimal. Leaf size=167

$$\frac{cx^{m+1}\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{bc^2x^n}{ac^2-d^2}\right)}{(m+1)\,(ac^2-d^2)}-\frac{dx^{m+1}\sqrt{\frac{bx^n}{a}+1}F_1\left(\frac{m+1}{n};\frac{1}{2},1;\frac{m+n+1}{n};-\frac{bx^n}{a},-\frac{bc^2x^n}{ac^2-d^2}\right)}{(m+1)\,(ac^2-d^2)\,\sqrt{a+bx^n}}$$

```
[Out] -((d*x^(1 + m)*Sqrt[1 + (b*x^n)/a]*AppellF1[(1 + m)/n, 1/2, 1, (1
 + m + n)/n, -((b*x^n)/a), -((b*c^2*x^n)/(a*c^2 - d^2))])/((a*c^2
 - d^2)*(1 + m)*Sqrt[a + b*x^n])) + (c*x^(1 + m)*Hypergeometric2F
1[1, (1 + m)/n, (1 + m + n)/n, -((b*c^2*x^n)/(a*c^2 - d^2))])/((a
*c^2 - d^2)*(1 + m))
```

Rubi [A] time = 0.455837, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$

$$\frac{cx^{m+1}\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{bc^2x^n}{ac^2-d^2}\right)}{(m+1)\,(ac^2-d^2)}-\frac{dx^{m+1}\sqrt{\frac{bx^n}{a}+1}F_1\left(\frac{m+1}{n};\frac{1}{2},1;\frac{m+n+1}{n};-\frac{bx^n}{a},-\frac{bc^2x^n}{ac^2-d^2}\right)}{(m+1)\,(ac^2-d^2)\,\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^m/(a*c + b*c*x^n + d*Sqrt[a + b*x^n]),x]
```

```
[Out] -((d*x^(1 + m)*Sqrt[1 + (b*x^n)/a]*AppellF1[(1 + m)/n, 1/2, 1, (1
 + m + n)/n, -((b*x^n)/a), -((b*c^2*x^n)/(a*c^2 - d^2))])/((a*c^2
 - d^2)*(1 + m)*Sqrt[a + b*x^n])) + (c*x^(1 + m)*Hypergeometric2F
1[1, (1 + m)/n, (1 + m + n)/n, -((b*c^2*x^n)/(a*c^2 - d^2))])/((a
*c^2 - d^2)*(1 + m))
```

Rubi in Sympy [A] time = 43.1399, size = 126, normalized size = 0.75

$$-\frac{cx^{m+1}{}_2F_1\left(\begin{matrix}1,\frac{m+1}{n}\\\frac{m+n+1}{n}\end{matrix}\middle|-\frac{bc^2x^n}{ac^2-d^2}\right)}{(m+1)\,(-ac^2+d^2)}+\frac{dx^{m+1}\sqrt{a+bx^n}\,\operatorname{appellf}_1\left(\frac{m+1}{n},\frac{1}{2},1,\frac{m+n+1}{n},-\frac{bx^n}{a},-\frac{bc^2x^n}{ac^2-d^2}\right)}{a\sqrt{1+\frac{bx^n}{a}}\,(m+1)\,(-ac^2+d^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**m/(a*c+b*c*x**n+d*(a+b*x**n)**(1/2)),x)
```

```
[Out] -c*x**(m + 1)*hyper((1, (m + 1)/n), ((m + n + 1)/n,), -b*c**2*x**
n/(a*c**2 - d**2))/((m + 1)*(-a*c**2 + d**2)) + d*x**(m + 1)*sqrt
(a + b*x**n)*appellf1((m + 1)/n, 1/2, 1, (m + n + 1)/n, -b*x**n/a
, -b*c**2*x**n/(a*c**2 - d**2))/(a*sqrt(1 + b*x**n/a)*(m + 1)*(-a
*c**2 + d**2))
```

Mathematica [B] time = 1.27272, size = 373, normalized size = 2.23

$$\frac{x^{m+1}\left(c\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{bc^2x^n}{ac^2-d^2}\right)-\frac{2ad(m+n+1)\left(d^2-ac^2\right)^2F_1\left(\frac{m+1}{n};\frac{1}{2},1;\frac{m+n+}{n}\right.}{\sqrt{a+bx^n}(ac^2+bc^2x^n-d^2)\left(2a(m+n+1)(ac^2-d^2)F_1\left(\frac{m+1}{n};\frac{1}{2},1;\frac{m+n+1}{n};-\frac{bx^n}{a},-\frac{bc^2x^n}{ac^2-d^2}\right)-bnx^n\left(2ac^2F_1\left(\frac{m+n}{n}\right.\right.\right.}\right.}{(m+1)\left(ac^2-d^2\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^m/(a*c + b*c*x^n + d*Sqrt[a + b*x^n]),x]
```

```
[Out] (x^(1 + m)*((-2*a*d*(-(a*c^2) + d^2)^2*(1 + m + n)*AppellF1[(1 +
m)/n, 1/2, 1, (1 + m + n)/n, -((b*x^n)/a), -((b*c^2*x^n)/(a*c^2 -
 d^2))])/(Sqrt[a + b*x^n]*(a*c^2 - d^2 + b*c^2*x^n)*(2*a*(a*c^2 -
 d^2)*(1 + m + n)*AppellF1[(1 + m)/n, 1/2, 1, (1 + m + n)/n, -((b
*x^n)/a), -((b*c^2*x^n)/(a*c^2 - d^2))] - b*n*x^n*(2*a*c^2*Appell
F1[(1 + m + n)/n, 1/2, 2, 2 + (1 + m)/n, -((b*x^n)/a), -((b*c^2*x
^n)/(a*c^2 - d^2))] + (a*c^2 - d^2)*AppellF1[(1 + m + n)/n, 3/2,
1, 2 + (1 + m)/n, -((b*x^n)/a), -((b*c^2*x^n)/(a*c^2 - d^2))])))
+ c*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((b*c^2*x^n)/
(a*c^2 - d^2))]))/((a*c^2 - d^2)*(1 + m))
```

Maple [F] time = 0.019, size = 0, normalized size = 0.

$$\int x^m\left(ac+bcx^n+d\sqrt{a+bx^n}\right)^{-1}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^m/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x)
```

```
[Out] int(x^m/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{bcx^n+ac+\sqrt{bx^n+ad}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*c*x^n + a*c + sqrt(b*x^n + a)*d),x, algorithm="maxima")
```

```
[Out] integrate(x^m/(b*c*x^n + a*c + sqrt(b*x^n + a)*d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^m}{bcx^n + ac + \sqrt{bx^n + a}d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*c*x^n + a*c + sqrt(b*x^n + a)*d),x, algorithm="fricas")
```

```
[Out] integral(x^m/(b*c*x^n + a*c + sqrt(b*x^n + a)*d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{ac + bcx^n + d\sqrt{a + bx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a*c+b*c*x**n+d*(a+b*x**n)**(1/2)),x)
```

```
[Out] Integral(x**m/(a*c + b*c*x**n + d*sqrt(a + b*x**n)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{bcx^n + ac + \sqrt{bx^n + a}d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*c*x^n + a*c + sqrt(b*x^n + a)*d),x, algorithm="giac")
```

```
[Out] integrate(x^m/(b*c*x^n + a*c + sqrt(b*x^n + a)*d), x)
```

3.403 $\int \frac{x^{-1+n}}{ac+bcx^n+d\sqrt{a+bx^n}}\,dx$

Optimal. Leaf size=27

$$\frac{2\log\left(c\sqrt{a+bx^n}+d\right)}{bcn}$$

```
[Out] (2*Log[d + c*Sqrt[a + b*x^n]])/(b*c*n)
```

Rubi [A] time = 0.188998, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065

$$\frac{2\log\left(c\sqrt{a+bx^n}+d\right)}{bcn}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(-1 + n)/(a*c + b*c*x^n + d*Sqrt[a + b*x^n]),x]

[Out] (2*Log[d + c*Sqrt[a + b*x^n]])/(b*c*n)
```

Rubi in Sympy [A] time = 10.5515, size = 20, normalized size = 0.74

$$\frac{2\log\left(c\sqrt{a+bx^n}+d\right)}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(-1+n)/(a*c+b*c*x**n+d*(a+b*x**n)**(1/2)),x)

[Out] 2*log(c*sqrt(a + b*x**n) + d)/(b*c*n)
```

Mathematica [A] time = 0.0308307, size = 27, normalized size = 1.

$$\frac{2\log\left(c\sqrt{a+bx^n}+d\right)}{bcn}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(-1 + n)/(a*c + b*c*x^n + d*Sqrt[a + b*x^n]),x]
```

```
[Out] (2*Log[d + c*Sqrt[a + b*x^n]])/(b*c*n)
```

Maple [F] time = 0.021, size = 0, normalized size = 0.

$$\int x^{-1+n} \left(ac + bcx^n + d\sqrt{a + bx^n}\right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(-1+n)/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x)
```

```
[Out] int(x^(-1+n)/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x)
```

Maxima [A] time = 0.770031, size = 82, normalized size = 3.04

$$-\frac{\log\left(\frac{bx^n+a}{b}\right)}{bcn} + \frac{2\log\left(\frac{bcx^n+ac+\sqrt{bx^n+a}d}{d}\right)}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(n - 1)/(b*c*x^n + a*c + sqrt(b*x^n + a)*d),x, algorithm="maxima")
```

```
[Out] -log((b*x^n + a)/b)/(b*c*n) + 2*log((b*c*x^n + a*c + sqrt(b*x^n +
 a)*d)/d)/(b*c*n)
```

Fricas [A] time = 0.285102, size = 34, normalized size = 1.26

$$\frac{2\log\left(\sqrt{bx^n + a}c + d\right)}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(n - 1)/(b*c*x^n + a*c + sqrt(b*x^n + a)*d),x, algorithm="fricas")
```

```
[Out] 2*log(sqrt(b*x^n + a)*c + d)/(b*c*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(-1+n)/(a*c+b*c*x**n+d*(a+b*x**n)**(1/2)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.384767, size = 55, normalized size = 2.04

$$\frac{2\ln\left(\left|\sqrt{bx^n+a}c+d\right|\right)}{bcn}-\frac{2\ln\left(|d|\right)}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(n - 1)/(b*c*x^n + a*c + sqrt(b*x^n + a)*d),x, algorithm="giac")
```

```
[Out] 2*ln(abs(sqrt(b*x^n + a)*c + d))/(b*c*n) - 2*ln(abs(d))/(b*c*n)
```

3.404 $\int \frac{1}{\sqrt{x}+4x^{3/2}}\,dx$

Optimal. Leaf size=8

$$\tan^{-1}\left(2\sqrt{x}\right)$$

```
[Out] ArcTan[2*Sqrt[x]]
```

Rubi [A] time = 0.0102897, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\tan^{-1}\left(2\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x] + 4*x^(3/2))^(-1),x]
```

```
[Out] ArcTan[2*Sqrt[x]]
```

Rubi in Sympy [A] time = 1.32601, size = 7, normalized size = 0.88

$$\operatorname{atan}\left(2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(4*x**(3/2)+x**(1/2)),x)
```

```
[Out] atan(2*sqrt(x))
```

Mathematica [A] time = 0.00550019, size = 8, normalized size = 1.

$$\tan^{-1}\left(2\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x] + 4*x^(3/2))^(-1),x]
```

```
[Out] ArcTan[2*Sqrt[x]]
```

Maple [A] time = 0.006, size = 7, normalized size = 0.9

$$\arctan\left(2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^(3/2)+x^(1/2)),x)
```

```
[Out] arctan(2*x^(1/2))
```

Maxima [A] time = 0.796948, size = 8, normalized size = 1.

$$\arctan\left(2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^(3/2) + sqrt(x)),x, algorithm="maxima")
```

```
[Out] arctan(2*sqrt(x))
```

Fricas [A] time = 0.271161, size = 8, normalized size = 1.

$$\arctan\left(2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^(3/2) + sqrt(x)),x, algorithm="fricas")
```

```
[Out] arctan(2*sqrt(x))
```

Sympy [A] time = 0.555798, size = 7, normalized size = 0.88

$$\operatorname{atan}\left(2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(4*x**(3/2)+x**(1/2)),x)
```

```
[Out] atan(2*sqrt(x))
```

GIAC/XCAS [A] time = 0.277453, size = 8, normalized size = 1.

$$\arctan\left(2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(4*x^(3/2) + sqrt(x)),x, algorithm="giac")
```

```
[Out] arctan(2*sqrt(x))
```

3.405 $\int \frac{1}{\sqrt{x}-x^{5/2}}\,dx$

Optimal. Leaf size=13

$$\tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)$$

```
[Out] ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]
```

Rubi [A] time = 0.0169297, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[x] - x^(5/2))^(-1),x]

[Out] ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]
```

Rubi in Sympy [A] time = 2.09033, size = 12, normalized size = 0.92

$$\operatorname{atan}\left(\sqrt{x}\right)+\operatorname{atanh}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-x**(5/2)+x**(1/2)),x)

[Out] atan(sqrt(x)) + atanh(sqrt(x))
```

Mathematica [B] time = 0.00963981, size = 33, normalized size = 2.54

$$-\frac{1}{2}\log\left(1-\sqrt{x}\right)+\frac{1}{2}\log\left(\sqrt{x}+1\right)+\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[x] - x^(5/2))^(-1),x]
```

```
[Out] ArcTan[Sqrt[x]] - Log[1 - Sqrt[x]]/2 + Log[1 + Sqrt[x]]/2
```

Maple [A] time = 0.008, size = 10, normalized size = 0.8

$$\arctan\left(\sqrt{x}\right)+\mathit{Artanh}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-x^(5/2)+x^(1/2)),x)

[Out] arctan(x^(1/2))+arctanh(x^(1/2))
```

Maxima [A] time = 0.839129, size = 28, normalized size = 2.15

$$\arctan\left(\sqrt{x}\right)+\frac{1}{2}\log\left(\sqrt{x}+1\right)-\frac{1}{2}\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(x^(5/2) - sqrt(x)),x, algorithm="maxima")

[Out] arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(sqrt(x) - 1)
```

Fricas [A] time = 0.277136, size = 28, normalized size = 2.15

$$\arctan\left(\sqrt{x}\right)+\frac{1}{2}\log\left(\sqrt{x}+1\right)-\frac{1}{2}\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(x^(5/2) - sqrt(x)),x, algorithm="fricas")

[Out] arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(sqrt(x) - 1)
```

Sympy [A] time = 1.23389, size = 26, normalized size = 2.

$$-\frac{\log\left(\sqrt{x}-1\right)}{2}+\frac{\log\left(\sqrt{x}+1\right)}{2}+\operatorname{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-x**(5/2)+x**(1/2)),x)
```

```
[Out] -log(sqrt(x) - 1)/2 + log(sqrt(x) + 1)/2 + atan(sqrt(x))
```

GIAC/XCAS [A] time = 0.280916, size = 30, normalized size = 2.31

$$\arctan\left(\sqrt{x}\right) + \frac{1}{2}\ln\left(\sqrt{x} + 1\right) - \frac{1}{2}\ln\left(\left|\sqrt{x} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(x^(5/2) - sqrt(x)),x, algorithm="giac")
```

```
[Out] arctan(sqrt(x)) + 1/2*ln(sqrt(x) + 1) - 1/2*ln(abs(sqrt(x) - 1))
```

3.406 $\int \frac{1}{-\sqrt[4]{x}+\sqrt{x}}\,dx$

Optimal. Leaf size=27

$$2\sqrt{x}+4\sqrt[4]{x}+4\log\left(1-\sqrt[4]{x}\right)$$

```
[Out] 4*x^(1/4) + 2*Sqrt[x] + 4*Log[1 - x^(1/4)]
```

Rubi [A] time = 0.0248985, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$2\sqrt{x}+4\sqrt[4]{x}+4\log\left(1-\sqrt[4]{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(-x^(1/4) + Sqrt[x])^(-1),x]

[Out] 4*x^(1/4) + 2*Sqrt[x] + 4*Log[1 - x^(1/4)]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$4\sqrt[4]{x}+4\log\left(-\sqrt[4]{x}+1\right)+4\int^{\sqrt[4]{x}} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-x**(1/4)+x**(1/2)),x)

[Out] 4*x**(1/4) + 4*log(-x**(1/4) + 1) + 4*Integral(x, (x, x**(1/4)))
```

Mathematica [A] time = 0.0128025, size = 27, normalized size = 1.

$$2\sqrt{x}+4\sqrt[4]{x}+4\log\left(1-\sqrt[4]{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-x^(1/4) + Sqrt[x])^(-1),x]
```

```
[Out] 4*x^(1/4) + 2*Sqrt[x] + 4*Log[1 - x^(1/4)]
```

Maple [A] time = 0.011, size = 20, normalized size = 0.7

$$4\sqrt[4]{x}+2\sqrt{x}+4\ \ln\left(\sqrt[4]{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-x^(1/4)+x^(1/2)),x)
```

```
[Out] 4*x^(1/4)+2*x^(1/2)+4*ln(x^(1/4)-1)
```

Maxima [A] time = 0.733607, size = 26, normalized size = 0.96

$$2\sqrt{x}+4\,x^{\frac{1}{4}}+4\ \log\left(x^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x) - x^(1/4)),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x) + 4*x^(1/4) + 4*log(x^(1/4) - 1)
```

Fricas [A] time = 0.270019, size = 26, normalized size = 0.96

$$2\sqrt{x}+4\,x^{\frac{1}{4}}+4\ \log\left(x^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x) - x^(1/4)),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x) + 4*x^(1/4) + 4*log(x^(1/4) - 1)
```

Sympy [A] time = 0.580411, size = 22, normalized size = 0.81

$$4\sqrt[4]{x}+2\sqrt{x}+4\log\left(\sqrt[4]{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-x**(1/4)+x**(1/2)),x)
```

```
[Out] 4*x**(1/4) + 2*sqrt(x) + 4*log(x**(1/4) - 1)
```

GIAC/XCAS [A] time = 0.282616, size = 27, normalized size = 1.

$$2\sqrt{x} + 4x^{\frac{1}{4}} + 4\ln\left(\left|x^{\frac{1}{4}} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x) - x^(1/4)),x, algorithm="giac")
```

```
[Out] 2*sqrt(x) + 4*x^(1/4) + 4*ln(abs(x^(1/4) - 1))
```

3.407 $\int \frac{1}{\sqrt[3]{x}+\sqrt{x}} \, dx$

Optimal. Leaf size=32

$$2\sqrt{x} - 3\sqrt[3]{x} + 6\sqrt[6]{x} - 6\log\left(\sqrt[6]{x} + 1\right)$$

```
[Out] 6*x^(1/6) - 3*x^(1/3) + 2*Sqrt[x] - 6*Log[1 + x^(1/6)]
```

Rubi [A] time = 0.0298855, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$2\sqrt{x} - 3\sqrt[3]{x} + 6\sqrt[6]{x} - 6\log\left(\sqrt[6]{x} + 1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(x^(1/3) + Sqrt[x])^(-1),x]
```

```
[Out] 6*x^(1/6) - 3*x^(1/3) + 2*Sqrt[x] - 6*Log[1 + x^(1/6)]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$6\sqrt[6]{x} + 2\sqrt{x} - 6\log\left(\sqrt[6]{x} + 1\right) - 6\int^{\sqrt[6]{x}} x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**(1/3)+x**(1/2)),x)
```

```
[Out] 6*x**(1/6) + 2*sqrt(x) - 6*log(x**(1/6) + 1) - 6*Integral(x, (x,
x**(1/6)))
```

Mathematica [A] time = 0.0134489, size = 32, normalized size = 1.

$$2\sqrt{x} - 3\sqrt[3]{x} + 6\sqrt[6]{x} - 6\log\left(\sqrt[6]{x} + 1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^(1/3) + Sqrt[x])^(-1),x]
```

```
[Out] 6*x^(1/6) - 3*x^(1/3) + 2*Sqrt[x] - 6*Log[1 + x^(1/6)]
```

Maple [B] time = 0.037, size = 92, normalized size = 2.9

$$-\ln\left(\sqrt[3]{x}+\sqrt[6]{x}+1\right)+2\ln\left(\sqrt[6]{x}-1\right)-2\ln\left(1+\sqrt[6]{x}\right)+\ln\left(1-\sqrt[6]{x}+\sqrt[3]{x}\right)+2\sqrt{x}+\ln\left(-1+\sqrt{x}\right)-\ln\left(1+\sqrt{x}\right)+6\sqrt[6]{x}-\ln\left(-1+x\right)+\ln\left(x^{\frac{2}{3}}+\sqrt[3]{x}+1\right)-2\ln\left(\sqrt[3]{x}-1\right)-3\sqrt[3]{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^(1/3)+x^(1/2)),x)
```

```
[Out] -ln(x^(1/3)+x^(1/6)+1)+2*ln(x^(1/6)-1)-2*ln(1+x^(1/6))+ln(1-x^(1/
6)+x^(1/3))+2*x^(1/2)+ln(-1+x^(1/2))-ln(1+x^(1/2))+6*x^(1/6)-ln(-
1+x)+ln(x^(2/3)+x^(1/3)+1)-2*ln(x^(1/3)-1)-3*x^(1/3)
```

Maxima [A] time = 0.694591, size = 32, normalized size = 1.

$$2\sqrt{x}-3x^{\frac{1}{3}}+6x^{\frac{1}{6}}-6\log\left(x^{\frac{1}{6}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x) + x^(1/3)),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x) - 3*x^(1/3) + 6*x^(1/6) - 6*log(x^(1/6) + 1)
```

Fricas [A] time = 0.268479, size = 32, normalized size = 1.

$$2\sqrt{x}-3x^{\frac{1}{3}}+6x^{\frac{1}{6}}-6\log\left(x^{\frac{1}{6}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x) + x^(1/3)),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x) - 3*x^(1/3) + 6*x^(1/6) - 6*log(x^(1/6) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{x} + \sqrt{x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**(1/3)+x**(1/2)),x)
```

```
[Out] Integral(1/(x**(1/3) + sqrt(x)), x)
```

GIAC/XCAS [A] time = 0.277637, size = 32, normalized size = 1.

$$2\sqrt{x} - 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}} - 6\ln\left(x^{\frac{1}{6}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x) + x^(1/3)),x, algorithm="giac")
```

```
[Out] 2*sqrt(x) - 3*x^(1/3) + 6*x^(1/6) - 6*ln(x^(1/6) + 1)
```

3.408 $\int \frac{1}{\sqrt[4]{x}+\sqrt{x}}\,dx$

Optimal. Leaf size=25

$$2\sqrt{x}-4\sqrt[4]{x}+4\log\left(\sqrt[4]{x}+1\right)$$

```
[Out] -4*x^(1/4) + 2*Sqrt[x] + 4*Log[1 + x^(1/4)]
```

Rubi [A] time = 0.0240742, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$2\sqrt{x}-4\sqrt[4]{x}+4\log\left(\sqrt[4]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(x^(1/4) + Sqrt[x])^(-1),x]

[Out] -4*x^(1/4) + 2*Sqrt[x] + 4*Log[1 + x^(1/4)]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-4\sqrt[4]{x}+4\log\left(\sqrt[4]{x}+1\right)+4\int^{\sqrt[4]{x}} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**(1/4)+x**(1/2)),x)

[Out] -4*x**(1/4) + 4*log(x**(1/4) + 1) + 4*Integral(x, (x, x**(1/4)))
```

Mathematica [A] time = 0.00990667, size = 25, normalized size = 1.

$$2\sqrt{x}-4\sqrt[4]{x}+4\log\left(\sqrt[4]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^(1/4) + Sqrt[x])^(-1),x]
```

```
[Out] -4*x^(1/4) + 2*Sqrt[x] + 4*Log[1 + x^(1/4)]
```

Maple [A] time = 0.008, size = 20, normalized size = 0.8

$$-4\sqrt[4]{x} + 4\ln\left(1 + \sqrt[4]{x}\right) + 2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/4)+x^(1/2)),x)
```

```
[Out] -4*x^(1/4)+4*ln(1+x^(1/4))+2*x^(1/2)
```

Maxima [A] time = 0.745472, size = 26, normalized size = 1.04

$$2\sqrt{x} - 4x^{\frac{1}{4}} + 4\log\left(x^{\frac{1}{4}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(x) + x^(1/4)),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x) - 4*x^(1/4) + 4*log(x^(1/4) + 1)
```

Fricas [A] time = 0.271106, size = 26, normalized size = 1.04

$$2\sqrt{x} - 4x^{\frac{1}{4}} + 4\log\left(x^{\frac{1}{4}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(x) + x^(1/4)),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x) - 4*x^(1/4) + 4*log(x^(1/4) + 1)
```

Sympy [A] time = 0.555739, size = 22, normalized size = 0.88

$$-4\sqrt[4]{x} + 2\sqrt{x} + 4\log\left(\sqrt[4]{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**(1/4)+x**(1/2)),x)
```

```
[Out] -4*x**(1/4) + 2*sqrt(x) + 4*log(x**(1/4) + 1)
```

GIAC/XCAS [A] time = 0.280659, size = 26, normalized size = 1.04

$$2\sqrt{x} - 4x^{\frac{1}{4}} + 4\ln\left(x^{\frac{1}{4}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x) + x^(1/4)),x, algorithm="giac")
```

```
[Out] 2*sqrt(x) - 4*x^(1/4) + 4*ln(x^(1/4) + 1)
```

3.409 $\int \frac{1}{-\sqrt[3]{x}+x^{2/3}}\,dx$

Optimal. Leaf size=20

$$3\sqrt[3]{x}+3\log\left(1-\sqrt[3]{x}\right)$$

```
[Out] 3*x^(1/3) + 3*Log[1 - x^(1/3)]
```

Rubi [A] time = 0.0202348, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$3\sqrt[3]{x}+3\log\left(1-\sqrt[3]{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(-x^(1/3) + x^(2/3))^(-1),x]

[Out] 3*x^(1/3) + 3*Log[1 - x^(1/3)]
```

Rubi in Sympy [A] time = 1.90405, size = 15, normalized size = 0.75

$$3\sqrt[3]{x}+3\log\left(-\sqrt[3]{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-x**(1/3)+x**(2/3)),x)

[Out] 3*x**(1/3) + 3*log(-x**(1/3) + 1)
```

Mathematica [A] time = 0.0093595, size = 20, normalized size = 1.

$$3\sqrt[3]{x}+3\log\left(1-\sqrt[3]{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-x^(1/3) + x^(2/3))^(-1),x]
```

```
[Out] 3*x^(1/3) + 3*Log[1 - x^(1/3)]
```

Maple [A] time = 0.002, size = 15, normalized size = 0.8

$$3\sqrt[3]{x}+3\ln\left(\sqrt[3]{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-x^(1/3)+x^(2/3)),x)
```

```
[Out] 3*x^(1/3)+3*ln(x^(1/3)-1)
```

Maxima [A] time = 0.716103, size = 19, normalized size = 0.95

$$3\,x^{\frac{1}{3}}+3\,\log\left(x^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^(2/3) - x^(1/3)),x, algorithm="maxima")
```

```
[Out] 3*x^(1/3) + 3*log(x^(1/3) - 1)
```

Fricas [A] time = 0.274374, size = 19, normalized size = 0.95

$$3\,x^{\frac{1}{3}}+3\,\log\left(x^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^(2/3) - x^(1/3)),x, algorithm="fricas")
```

```
[Out] 3*x^(1/3) + 3*log(x^(1/3) - 1)
```

Sympy [A] time = 0.340885, size = 15, normalized size = 0.75

$$3\sqrt[3]{x}+3\log\left(\sqrt[3]{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-x**(1/3)+x**(2/3)),x)

[Out] 3*x**(1/3) + 3*log(x**(1/3) - 1)
```

GIAC/XCAS [A] time = 0.277788, size = 20, normalized size = 1.

$$3\,x^{\frac{1}{3}} + 3\ln\left(\left|x^{\frac{1}{3}} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^(2/3) - x^(1/3)),x, algorithm="giac")

[Out] 3*x^(1/3) + 3*ln(abs(x^(1/3) - 1))
```

3.410 $\int \frac{1}{\frac{1}{\sqrt[4]{x}}+\sqrt{x}}\,dx$

Optimal. Leaf size=62

$$2\sqrt{x}+\frac{4}{3}\log\left(\sqrt[4]{x}+1\right)-\frac{2}{3}\log\left(\sqrt{x}-\sqrt[4]{x}+1\right)+\frac{4\tan^{-1}\left(\frac{1-2\sqrt[4]{x}}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] 2*Sqrt[x] + (4*ArcTan[(1 - 2*x^(1/4))/Sqrt[3]])/Sqrt[3] + (4*Log[
1 + x^(1/4)])/3 - (2*Log[1 - x^(1/4) + Sqrt[x]])/3
```

Rubi [A] time = 0.0786717, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692

$$2\sqrt{x}+\frac{4}{3}\log\left(\sqrt[4]{x}+1\right)-\frac{2}{3}\log\left(\sqrt{x}-\sqrt[4]{x}+1\right)+\frac{4\tan^{-1}\left(\frac{1-2\sqrt[4]{x}}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^(-1/4) + Sqrt[x])^(-1),x]
```

```
[Out] 2*Sqrt[x] + (4*ArcTan[(1 - 2*x^(1/4))/Sqrt[3]])/Sqrt[3] + (4*Log[
1 + x^(1/4)])/3 - (2*Log[1 - x^(1/4) + Sqrt[x]])/3
```

Rubi in Sympy [A] time = 5.24184, size = 61, normalized size = 0.98

$$2\sqrt{x}+\frac{4\log\left(\sqrt[4]{x}+1\right)}{3}-\frac{2\log\left(-\sqrt[4]{x}+\sqrt{x}+1\right)}{3}-\frac{4\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2\sqrt[4]{x}}{3}-\frac{1}{3}\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1/x**(1/4)+x**(1/2)),x)
```

```
[Out] 2*sqrt(x) + 4*log(x**(1/4) + 1)/3 - 2*log(-x**(1/4) + sqrt(x) + 1
)/3 - 4*sqrt(3)*atan(sqrt(3)*(2*x**(1/4)/3 - 1/3))/3
```

Mathematica [A] time = 0.0238147, size = 62, normalized size = 1.

$$2\sqrt{x}+\frac{4}{3}\log\left(\sqrt[4]{x}+1\right)-\frac{2}{3}\log\left(\sqrt{x}-\sqrt[4]{x}+1\right)-\frac{4\tan^{-1}\left(\frac{2\sqrt[4]{x}-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^(-1/4) + Sqrt[x])^(-1),x]
```

```
[Out] 2*Sqrt[x] - (4*ArcTan[(-1 + 2*x^(1/4))/Sqrt[3]])/Sqrt[3] + (4*Log
[1 + x^(1/4)])/3 - (2*Log[1 - x^(1/4) + Sqrt[x]])/3
```

Maple [A] time = 0.006, size = 46, normalized size = 0.7

$$2\sqrt{x}+\frac{4}{3}\ln\left(1+\sqrt[4]{x}\right)-\frac{2}{3}\ln\left(1-\sqrt[4]{x}+\sqrt{x}\right)-\frac{4\sqrt{3}}{3}\arctan\left(\frac{\sqrt{3}}{3}\left(2\sqrt[4]{x}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1/x^(1/4)+x^(1/2)),x)
```

```
[Out] 2*x^(1/2)+4/3*ln(1+x^(1/4))-2/3*ln(1-x^(1/4)+x^(1/2))-4/3*3^(1/2)
*arctan(1/3*(2*x^(1/4)-1)*3^(1/2))
```

Maxima [A] time = 0.838069, size = 61, normalized size = 0.98

$$-\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{4}}-1\right)\right)+2\sqrt{x}-\frac{2}{3}\log\left(\sqrt{x}-x^{\frac{1}{4}}+1\right)+\frac{4}{3}\log\left(x^{\frac{1}{4}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x) + 1/x^(1/4)),x, algorithm="maxima")
```

```
[Out] -4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/4) - 1)) + 2*sqrt(x) - 2/
3*log(sqrt(x) - x^(1/4) + 1) + 4/3*log(x^(1/4) + 1)
```

Fricas [A] time = 0.28245, size = 77, normalized size = 1.24

$$-\frac{2}{9}\sqrt{3}\left(\sqrt{3}\log\left(\sqrt{x}-x^{\frac{1}{4}}+1\right)-2\sqrt{3}\log\left(x^{\frac{1}{4}}+1\right)-3\sqrt{3}\sqrt{x}+6\arctan\left(\frac{2}{3}\sqrt{3}x^{\frac{1}{4}}-\frac{1}{3}\sqrt{3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x) + 1/x^(1/4)),x, algorithm="fricas")
```

```
[Out] -2/9*sqrt(3)*(sqrt(3)*log(sqrt(x) - x^(1/4) + 1) - 2*sqrt(3)*log(
x^(1/4) + 1) - 3*sqrt(3)*sqrt(x) + 6*arctan(2/3*sqrt(3)*x^(1/4) -
 1/3*sqrt(3)))
```

Sympy [A] time = 2.47825, size = 68, normalized size = 1.1

$$2\sqrt{x} + \frac{4\log\left(\sqrt[4]{x} + 1\right)}{3} - \frac{2\log\left(-4\sqrt[4]{x} + 4\sqrt{x} + 4\right)}{3} - \frac{4\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}\sqrt[4]{x}}{3} - \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1/x**(1/4)+x**(1/2)),x)
```

```
[Out] 2*sqrt(x) + 4*log(x**(1/4) + 1)/3 - 2*log(-4*x**(1/4) + 4*sqrt(x)
 + 4)/3 - 4*sqrt(3)*atan(2*sqrt(3)*x**(1/4)/3 - sqrt(3)/3)/3
```

GIAC/XCAS [A] time = 0.28011, size = 61, normalized size = 0.98

$$-\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{4}} - 1\right)\right) + 2\sqrt{x} - \frac{2}{3}\ln\left(\sqrt{x} - x^{\frac{1}{4}} + 1\right) + \frac{4}{3}\ln\left(x^{\frac{1}{4}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x) + 1/x^(1/4)),x, algorithm="giac")
```

```
[Out] -4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/4) - 1)) + 2*sqrt(x) - 2/
3*ln(sqrt(x) - x^(1/4) + 1) + 4/3*ln(x^(1/4) + 1)
```

3.411 $\int \frac{1}{\sqrt[4]{x}+\sqrt[3]{x}}\,dx$

Optimal. Leaf size=73

$$\frac{3x^{2/3}}{2}-\frac{12x^{7/12}}{7}-\frac{12x^{5/12}}{5}+2\sqrt{x}+3\sqrt[3]{x}-4\sqrt[4]{x}+6\sqrt[6]{x}-12\sqrt[12]{x}+12\log\left(\sqrt[12]{x}+1\right)$$

```
[Out] -12*x^(1/12) + 6*x^(1/6) - 4*x^(1/4) + 3*x^(1/3) - (12*x^(5/12))/
5 + 2*Sqrt[x] - (12*x^(7/12))/7 + (3*x^(2/3))/2 + 12*Log[1 + x^(1
/12)]
```

Rubi [A] time = 0.0555871, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$\frac{3x^{2/3}}{2}-\frac{12x^{7/12}}{7}-\frac{12x^{5/12}}{5}+2\sqrt{x}+3\sqrt[3]{x}-4\sqrt[4]{x}+6\sqrt[6]{x}-12\sqrt[12]{x}+12\log\left(\sqrt[12]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(x^(1/4) + x^(1/3))^(-1),x]
```

```
[Out] -12*x^(1/12) + 6*x^(1/6) - 4*x^(1/4) + 3*x^(1/3) - (12*x^(5/12))/
5 + 2*Sqrt[x] - (12*x^(7/12))/7 + (3*x^(2/3))/2 + 12*Log[1 + x^(1
/12)]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{12x^{\frac{7}{12}}}{7}-\frac{12x^{\frac{5}{12}}}{5}-12\sqrt[12]{x}-4\sqrt[4]{x}+\frac{3x^{\frac{2}{3}}}{2}+3\sqrt[3]{x}+2\sqrt{x}+12\log\left(\sqrt[12]{x}+1\right)+12\int^{\sqrt[12]{x}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**(1/4)+x**(1/3)),x)
```

```
[Out] -12*x**(7/12)/7 - 12*x**(5/12)/5 - 12*x**(1/12) - 4*x**(1/4) + 3*
x**(2/3)/2 + 3*x**(1/3) + 2*sqrt(x) + 12*log(x**(1/12) + 1) + 12*
Integral(x, (x, x**(1/12)))
```

Mathematica [A] time = 0.0186307, size = 73, normalized size = 1.

$$\frac{3x^{2/3}}{2}-\frac{12x^{7/12}}{7}-\frac{12x^{5/12}}{5}+2\sqrt{x}+3\sqrt[3]{x}-4\sqrt[4]{x}+6\sqrt[6]{x}-12\sqrt[12]{x}+12\log\left(\sqrt[12]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^(1/4) + x^(1/3))^(-1),x]
```

```
[Out] -12*x^(1/12) + 6*x^(1/6) - 4*x^(1/4) + 3*x^(1/3) - (12*x^(5/12))/
5 + 2*Sqrt[x] - (12*x^(7/12))/7 + (3*x^(2/3))/2 + 12*Log[1 + x^(1
/12)]
```

Maple [B] time = 0.153, size = 173, normalized size = 2.4

$$\frac{3}{2}x^{\frac{2}{3}}+6\sqrt[6]{x}-4\sqrt[4]{x}+3\sqrt[3]{x}-12x^{1/12}-2\ln\left(\sqrt[4]{x}-1\right)+2\ln\left(1+\sqrt[4]{x}\right)+2\sqrt{x}+\ln\left(-1+x\right)-4\ln\left(x^{1/12}-1\right)$$
$$+4\ln\left(1+x^{1/12}\right)-2\ln\left(1-x^{1/12}+\sqrt[6]{x}\right)+2\ln\left(\sqrt[6]{x}+x^{1/12}+1\right)+2\ln\left(\sqrt[3]{x}-1\right)-\ln\left(x^{\frac{2}{3}}+\sqrt[3]{x}+1\right)-\ln\left(1+\sqrt{x}\right)+\ln\left(-1+\sqrt{x}\right)-2\,\mathrm{l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^(1/4)+x^(1/3)),x)
```

```
[Out] 3/2*x^(2/3)+6*x^(1/6)-4*x^(1/4)+3*x^(1/3)-12*x^(1/12)-2*ln(x^(1/4
)-1)+2*ln(1+x^(1/4))+2*x^(1/2)+ln(-1+x)-4*ln(x^(1/12)-1)+4*ln(1+x
^(1/12))-2*ln(1-x^(1/12)+x^(1/6))+2*ln(x^(1/6)+x^(1/12)+1)+2*ln(x
^(1/3)-1)-ln(x^(2/3)+x^(1/3)+1)-ln(1+x^(1/2))+ln(-1+x^(1/2))-2*ln
(1+x^(1/6))+ln(1-x^(1/6)+x^(1/3))+2*ln(x^(1/6)-1)-ln(x^(1/3)+x^(1
/6)+1)-12/5*x^(5/12)-12/7*x^(7/12)
```

Maxima [A] time = 0.731938, size = 66, normalized size = 0.9

$$\frac{3}{2}\,x^{\frac{2}{3}}-\frac{12}{7}\,x^{\frac{7}{12}}+2\,\sqrt{x}-\frac{12}{5}\,x^{\frac{5}{12}}+3\,x^{\frac{1}{3}}-4\,x^{\frac{1}{4}}+6\,x^{\frac{1}{6}}-12\,x^{\frac{1}{12}}+12\,\log\left(x^{\frac{1}{12}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^(1/3) + x^(1/4)),x, algorithm="maxima")
```

```
[Out] 3/2*x^(2/3) - 12/7*x^(7/12) + 2*sqrt(x) - 12/5*x^(5/12) + 3*x^(1/
3) - 4*x^(1/4) + 6*x^(1/6) - 12*x^(1/12) + 12*log(x^(1/12) + 1)
```

Fricas [A] time = 0.27571, size = 66, normalized size = 0.9

$$\frac{3}{2}\,x^{\frac{2}{3}} - \frac{12}{7}\,x^{\frac{7}{12}} + 2\,\sqrt{x} - \frac{12}{5}\,x^{\frac{5}{12}} + 3\,x^{\frac{1}{3}} - 4\,x^{\frac{1}{4}} + 6\,x^{\frac{1}{6}} - 12\,x^{\frac{1}{12}} + 12\,\log\left(x^{\frac{1}{12}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^(1/3) + x^(1/4)),x, algorithm="fricas")
```

```
[Out] 3/2*x^(2/3) - 12/7*x^(7/12) + 2*sqrt(x) - 12/5*x^(5/12) + 3*x^(1/
3) - 4*x^(1/4) + 6*x^(1/6) - 12*x^(1/12) + 12*log(x^(1/12) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[4]{x} + \sqrt[3]{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**(1/4)+x**(1/3)),x)
```

```
[Out] Integral(1/(x**(1/4) + x**(1/3)), x)
```

GIAC/XCAS [A] time = 0.280707, size = 66, normalized size = 0.9

$$\frac{3}{2}\,x^{\frac{2}{3}} - \frac{12}{7}\,x^{\frac{7}{12}} + 2\,\sqrt{x} - \frac{12}{5}\,x^{\frac{5}{12}} + 3\,x^{\frac{1}{3}} - 4\,x^{\frac{1}{4}} + 6\,x^{\frac{1}{6}} - 12\,x^{\frac{1}{12}} + 12\ln\left(x^{\frac{1}{12}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^(1/3) + x^(1/4)),x, algorithm="giac")
```

```
[Out] 3/2*x^(2/3) - 12/7*x^(7/12) + 2*sqrt(x) - 12/5*x^(5/12) + 3*x^(1/
3) - 4*x^(1/4) + 6*x^(1/6) - 12*x^(1/12) + 12*ln(x^(1/12) + 1)
```

3.412 $\int \frac{1}{\frac{1}{\sqrt[3]{x}}+\frac{1}{\sqrt[4]{x}}}\,dx$

Optimal. Leaf size=130

$$\frac{4x^{5/4}}{5}-\frac{6x^{7/6}}{7}+\frac{12x^{13/12}}{13}+\frac{12x^{11/12}}{11}-\frac{6x^{5/6}}{5}+\frac{4x^{3/4}}{3}-\frac{3x^{2/3}}{2}+\frac{12x^{7/12}}{7}+\frac{12x^{5/12}}{5}-x-2\sqrt{x}-3\sqrt[3]{x}+4\sqrt[4]{x}-6\sqrt[6]{x}+12\sqrt[12]{x}-12\log\left(\sqrt[12]{x}+1\right)$$

```
[Out] 12*x^(1/12) - 6*x^(1/6) + 4*x^(1/4) - 3*x^(1/3) + (12*x^(5/12))/5
 - 2*Sqrt[x] + (12*x^(7/12))/7 - (3*x^(2/3))/2 + (4*x^(3/4))/3 -
(6*x^(5/6))/5 + (12*x^(11/12))/11 - x + (12*x^(13/12))/13 - (6*x^
(7/6))/7 + (4*x^(5/4))/5 - 12*Log[1 + x^(1/12)]
```

Rubi [A] time = 0.0879713, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$\frac{4x^{5/4}}{5}-\frac{6x^{7/6}}{7}+\frac{12x^{13/12}}{13}+\frac{12x^{11/12}}{11}-\frac{6x^{5/6}}{5}+\frac{4x^{3/4}}{3}-\frac{3x^{2/3}}{2}+\frac{12x^{7/12}}{7}+\frac{12x^{5/12}}{5}-x-2\sqrt{x}-3\sqrt[3]{x}+4\sqrt[4]{x}-6\sqrt[6]{x}+12\sqrt[12]{x}-12\log\left(\sqrt[12]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(x^(-1/3) + x^(-1/4))^(-1),x]
```

```
[Out] 12*x^(1/12) - 6*x^(1/6) + 4*x^(1/4) - 3*x^(1/3) + (12*x^(5/12))/5
 - 2*Sqrt[x] + (12*x^(7/12))/7 - (3*x^(2/3))/2 + (4*x^(3/4))/3 -
(6*x^(5/6))/5 + (12*x^(11/12))/11 - x + (12*x^(13/12))/13 - (6*x^
(7/6))/7 + (4*x^(5/4))/5 - 12*Log[1 + x^(1/12)]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{12x^{\frac{13}{12}}}{13}+\frac{12x^{\frac{11}{12}}}{11}+\frac{12x^{\frac{7}{12}}}{7}+\frac{12x^{\frac{5}{12}}}{5}+12\sqrt[12]{x}-\frac{6x^{\frac{7}{6}}}{7}-\frac{6x^{\frac{5}{6}}}{5}+\frac{4x^{\frac{5}{4}}}{5}+\frac{4x^{\frac{3}{4}}}{3}+4\sqrt[4]{x}-\frac{3x^{\frac{2}{3}}}{2}-3\sqrt[3]{x}-2\sqrt{x}-x-12\log\left(\sqrt[12]{x}+1\right)-12\int^{\sqrt[12]{x}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1/x**(1/3)+1/x**(1/4)),x)
```

```
[Out] 12*x**(13/12)/13 + 12*x**(11/12)/11 + 12*x**(7/12)/7 + 12*x**(5/1
2)/5 + 12*x**(1/12) - 6*x**(7/6)/7 - 6*x**(5/6)/5 + 4*x**(5/4)/5
+ 4*x**(3/4)/3 + 4*x**(1/4) - 3*x**(2/3)/2 - 3*x**(1/3) - 2*sqrt(
x) - x - 12*log(x**(1/12) + 1) - 12*Integral(x, (x, x**(1/12)))
```

Mathematica [A] time = 0.029298, size = 130, normalized size = 1.

$$\frac{4x^{5/4}}{5}-\frac{6x^{7/6}}{7}+\frac{12x^{13/12}}{13}+\frac{12x^{11/12}}{11}-\frac{6x^{5/6}}{5}+\frac{4x^{3/4}}{3}-\frac{3x^{2/3}}{2}+\frac{12x^{7/12}}{7}+\frac{12x^{5/12}}{5}-x-2\sqrt{x}-3\sqrt[3]{x}+4\sqrt[4]{x}-6\sqrt[6]{x}+12\sqrt[12]{x}-12\log\left(\sqrt[12]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^(-1/3) + x^(-1/4))^(-1),x]
```

```
[Out] 12*x^(1/12) - 6*x^(1/6) + 4*x^(1/4) - 3*x^(1/3) + (12*x^(5/12))/5
 - 2*Sqrt[x] + (12*x^(7/12))/7 - (3*x^(2/3))/2 + (4*x^(3/4))/3 -
(6*x^(5/6))/5 + (12*x^(11/12))/11 - x + (12*x^(13/12))/13 - (6*x^
(7/6))/7 + (4*x^(5/4))/5 - 12*Log[1 + x^(1/12)]
```

Maple [A] time = 0.005, size = 83, normalized size = 0.6

$$12\,x^{1/12}-6\,\sqrt[6]{x}+4\,\sqrt[4]{x}-3\,\sqrt[3]{x}+\frac{12}{5}x^{\frac{5}{12}}+\frac{12}{7}x^{\frac{7}{12}}-\frac{3}{2}x^{\frac{2}{3}}+\frac{4}{3}x^{\frac{3}{4}}-\frac{6}{5}x^{\frac{5}{6}}+\frac{12}{11}x^{\frac{11}{12}}-x+\frac{12}{13}x^{\frac{13}{12}}-\frac{6}{7}x^{\frac{7}{6}}+\frac{4}{5}x^{\frac{5}{4}}-12\,\ln\left(1+x^{1/12}\right)-2\,\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1/x^(1/3)+1/x^(1/4)),x)
```

```
[Out] 12*x^(1/12)-6*x^(1/6)+4*x^(1/4)-3*x^(1/3)+12/5*x^(5/12)+12/7*x^(7
/12)-3/2*x^(2/3)+4/3*x^(3/4)-6/5*x^(5/6)+12/11*x^(11/12)-x+12/13*
x^(13/12)-6/7*x^(7/6)+4/5*x^(5/4)-12*ln(1+x^(1/12))-2*x^(1/2)
```

Maxima [A] time = 0.695713, size = 111, normalized size = 0.85

$$\frac{4}{5}\,x^{\frac{5}{4}}-\frac{6}{7}\,x^{\frac{7}{6}}+\frac{12}{13}\,x^{\frac{13}{12}}-x+\frac{12}{11}\,x^{\frac{11}{12}}-\frac{6}{5}\,x^{\frac{5}{6}}+\frac{4}{3}\,x^{\frac{3}{4}}-\frac{3}{2}\,x^{\frac{2}{3}}+\frac{12}{7}\,x^{\frac{7}{12}}-2\,\sqrt{x}+\frac{12}{5}\,x^{\frac{5}{12}}-3\,x^{\frac{1}{3}}+4\,x^{\frac{1}{4}}-6\,x^{\frac{1}{6}}+12\,x^{\frac{1}{12}}-12\,\log\left(x^{\frac{1}{12}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1/x^(1/4) + 1/x^(1/3)),x, algorithm="maxima")
```

```
[Out] 4/5*x^(5/4) - 6/7*x^(7/6) + 12/13*x^(13/12) - x + 12/11*x^(11/12)
 - 6/5*x^(5/6) + 4/3*x^(3/4) - 3/2*x^(2/3) + 12/7*x^(7/12) - 2*sq
rt(x) + 12/5*x^(5/12) - 3*x^(1/3) + 4*x^(1/4) - 6*x^(1/6) + 12*x^
(1/12) - 12*log(x^(1/12) + 1)
```

Fricas [A] time = 0.272816, size = 103, normalized size = 0.79

$$\frac{4}{5}(x+5)x^{\frac{1}{4}} - \frac{6}{7}(x+7)x^{\frac{1}{6}} + \frac{12}{13}(x+13)x^{\frac{1}{12}} - x + \frac{12}{11}x^{\frac{11}{12}} - \frac{6}{5}x^{\frac{5}{6}} + \frac{4}{3}x^{\frac{3}{4}} - \frac{3}{2}x^{\frac{2}{3}} + \frac{12}{7}x^{\frac{7}{12}} - 2\sqrt{x} + \frac{12}{5}x^{\frac{5}{12}} - 3x^{\frac{1}{3}} - 12\log\left(x^{\frac{1}{12}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1/x^(1/4) + 1/x^(1/3)),x, algorithm="fricas")
```

```
[Out] 4/5*(x + 5)*x^(1/4) - 6/7*(x + 7)*x^(1/6) + 12/13*(x + 13)*x^(1/1
2) - x + 12/11*x^(11/12) - 6/5*x^(5/6) + 4/3*x^(3/4) - 3/2*x^(2/3
) + 12/7*x^(7/12) - 2*sqrt(x) + 12/5*x^(5/12) - 3*x^(1/3) - 12*lo
g(x^(1/12) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{7}{12}}}{\sqrt[4]{x}+\sqrt[3]{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1/x**(1/3)+1/x**(1/4)),x)
```

```
[Out] Integral(x**(7/12)/(x**(1/4) + x**(1/3)), x)
```

GIAC/XCAS [A] time = 0.279229, size = 111, normalized size = 0.85

$$\frac{4}{5}x^{\frac{5}{4}} - \frac{6}{7}x^{\frac{7}{6}} + \frac{12}{13}x^{\frac{13}{12}} - x + \frac{12}{11}x^{\frac{11}{12}} - \frac{6}{5}x^{\frac{5}{6}} + \frac{4}{3}x^{\frac{3}{4}} - \frac{3}{2}x^{\frac{2}{3}} + \frac{12}{7}x^{\frac{7}{12}} - 2\sqrt{x} + \frac{12}{5}x^{\frac{5}{12}} - 3x^{\frac{1}{3}} + 4x^{\frac{1}{4}} - 6x^{\frac{1}{6}} + 12x^{\frac{1}{12}} - 12\ln\left(x^{\frac{1}{12}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1/x^(1/4) + 1/x^(1/3)),x, algorithm="giac")
```

```
[Out] 4/5*x^(5/4) - 6/7*x^(7/6) + 12/13*x^(13/12) - x + 12/11*x^(11/12)
 - 6/5*x^(5/6) + 4/3*x^(3/4) - 3/2*x^(2/3) + 12/7*x^(7/12) - 2*sq
rt(x) + 12/5*x^(5/12) - 3*x^(1/3) + 4*x^(1/4) - 6*x^(1/6) + 12*x^
(1/12) - 12*ln(x^(1/12) + 1)
```

3.413 $\int \frac{1}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}} \, dx$

Optimal. Leaf size=200

$$2\sqrt{x}+\frac{6}{5}\log\left(1-\sqrt[6]{x}\right)-\frac{3}{10}\left(1+\sqrt{5}\right)\log\left(2\sqrt[3]{x}-\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}+2\right)-\frac{3}{10}\left(1-\sqrt{5}\right)\log\left(2\sqrt[3]{x}+\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}+2\right)+\frac{3}{5}\sqrt{2\left(5-\sqrt{5}\right)}\tan^{-1}\left(\frac{4\sqrt[6]{x}-\sqrt{5}+1}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)-\frac{3}{5}\sqrt{2\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{1}{2}\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(4\sqrt[6]{x}+\sqrt{5}+1\right)\right)$$

```
[Out] 2*Sqrt[x] + (3*Sqrt[2*(5 - Sqrt[5])]*ArcTan[(1 - Sqrt[5] + 4*x^(1
/6))/Sqrt[2*(5 + Sqrt[5])]])/5 - (3*Sqrt[2*(5 + Sqrt[5])]*ArcTan[
(Sqrt[(5 + Sqrt[5])/10]*(1 + Sqrt[5] + 4*x^(1/6)))/2])/5 + (6*Log
[1 - x^(1/6)])/5 - (3*(1 + Sqrt[5])*Log[2 + x^(1/6) - Sqrt[5]*x^(
1/6) + 2*x^(1/3)])/10 - (3*(1 - Sqrt[5])*Log[2 + x^(1/6) + Sqrt[5
]*x^(1/6) + 2*x^(1/3)])/10
```

Rubi [A] time = 0.684933, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6

$$2\sqrt{x}+\frac{6}{5}\log\left(1-\sqrt[6]{x}\right)-\frac{3}{10}\left(1+\sqrt{5}\right)\log\left(2\sqrt[3]{x}-\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}+2\right)-\frac{3}{10}\left(1-\sqrt{5}\right)\log\left(2\sqrt[3]{x}+\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}+2\right)+\frac{3}{5}\sqrt{2\left(5-\sqrt{5}\right)}\tan^{-1}\left(\frac{4\sqrt[6]{x}-\sqrt{5}+1}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)-\frac{3}{5}\sqrt{2\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{1}{2}\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(4\sqrt[6]{x}+\sqrt{5}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(-x^(-1/3) + Sqrt[x])^(-1),x]
```


```
[Out] 2*Sqrt[x] + (3*Sqrt[2*(5 - Sqrt[5])]*ArcTan[(1 - Sqrt[5] + 4*x^(1
/6))/Sqrt[2*(5 + Sqrt[5])]])/5 - (3*Sqrt[2*(5 + Sqrt[5])]*ArcTan[
(Sqrt[(5 + Sqrt[5])/10]*(1 + Sqrt[5] + 4*x^(1/6)))/2])/5 + (6*Log
[1 - x^(1/6)])/5 - (3*(1 + Sqrt[5])*Log[2 + x^(1/6) - Sqrt[5]*x^(
1/6) + 2*x^(1/3)])/10 - (3*(1 - Sqrt[5])*Log[2 + x^(1/6) + Sqrt[5
]*x^(1/6) + 2*x^(1/3)])/10
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-1/x**(1/3)+x**(1/2)),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.237335, size = 180, normalized size = 0.9

$$\frac{1}{10}\left(20\sqrt{x}+12\log\left(1-\sqrt[6]{x}\right)-3\left(1+\sqrt{5}\right)\log\left(\sqrt[3]{x}-\frac{1}{2}\left(\sqrt{5}-1\right)\sqrt[6]{x}+1\right)+3\left(\sqrt{5}-1\right)\log\left(\sqrt[3]{x}+\frac{1}{2}\left(1+\sqrt{5}\right)\sqrt[6]{x}+1\right)+6\sqrt{10-2\sqrt{5}}\tan^{-1}\left(\frac{4\sqrt[6]{x}-\sqrt{5}+1}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)-6\sqrt{2\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{4\sqrt[6]{x}+\sqrt{5}+1}{\sqrt{10-2\sqrt{5}}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-x^(-1/3) + Sqrt[x])^(-1),x]
```

```
[Out] (20*Sqrt[x] + 6*Sqrt[10 - 2*Sqrt[5]]*ArcTan[(1 - Sqrt[5] + 4*x^(1
/6))/Sqrt[2*(5 + Sqrt[5])]] - 6*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(1 +
 Sqrt[5] + 4*x^(1/6))/Sqrt[10 - 2*Sqrt[5]]] + 12*Log[1 - x^(1/6)]
 - 3*(1 + Sqrt[5])*Log[1 - ((-1 + Sqrt[5])*x^(1/6))/2 + x^(1/3)]
+ 3*(-1 + Sqrt[5])*Log[1 + ((1 + Sqrt[5])*x^(1/6))/2 + x^(1/3)])/
10
```

Maple [A] time = 0.052, size = 175, normalized size = 0.9

$$2\sqrt{x}+\frac{6}{5}\ln\left(\sqrt[6]{x}-1\right)+\frac{3\sqrt{5}}{10}\ln\left(2+\sqrt[6]{x}+2\sqrt[3]{x}+\sqrt[6]{x}\sqrt{5}\right)-\frac{3}{10}\ln\left(2+\sqrt[6]{x}+2\sqrt[3]{x}+\sqrt[6]{x}\sqrt{5}\right)-\frac{12\sqrt{5}}{5\sqrt{10-2\sqrt{5}}}\arctan\left(\frac{1}{\sqrt{10-2\sqrt{5}}}\left(1+4\sqrt[6]{x}+\sqrt{5}\right)\right)-\frac{3\sqrt{5}}{10}\ln\left(2+\sqrt[6]{x}+2\sqrt[3]{x}-\sqrt[6]{x}\sqrt{5}\right)-\frac{3}{10}\ln\left(2+\sqrt[6]{x}+2\sqrt[3]{x}-\sqrt[6]{x}\sqrt{5}\right)+\frac{12\sqrt{5}}{5\sqrt{10+2\sqrt{5}}}\arctan\left(\frac{1}{\sqrt{10+2\sqrt{5}}}\left(1+4\sqrt[6]{x}-\sqrt{5}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-1/x^(1/3)+x^(1/2)),x)
```

```
[Out] 2*x^(1/2)+6/5*ln(x^(1/6)-1)+3/10*ln(2+x^(1/6)+2*x^(1/3)+x^(1/6)*5
^(1/2))*5^(1/2)-3/10*ln(2+x^(1/6)+2*x^(1/3)+x^(1/6)*5^(1/2))-12/5
/(10-2*5^(1/2))^(1/2)*arctan((1+4*x^(1/6)+5^(1/2))/(10-2*5^(1/2))
^(1/2))*5^(1/2)-3/10*ln(2+x^(1/6)+2*x^(1/3)-x^(1/6)*5^(1/2))*5^(1
/2)-3/10*ln(2+x^(1/6)+2*x^(1/3)-x^(1/6)*5^(1/2))+12/5/(10+2*5^(1/
2))^(1/2)*arctan((1+4*x^(1/6)-5^(1/2))/(10+2*5^(1/2))^(1/2))*5^(1
/2)
```

Maxima [A] time = 0.837875, size = 367, normalized size = 1.84

$$-\frac{6}{5}\left(-1\right)^{\frac{3}{5}}\log\left(\left(-1\right)^{\frac{1}{5}}+x^{\frac{1}{6}}\right)-\frac{6\sqrt{5}\left(-1\right)^{\frac{3}{5}}\log\left(\frac{\sqrt{5}(-1)^{\frac{1}{5}}+(-1)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}+(-1)^{\frac{1}{5}}-4x^{\frac{1}{6}}}{\sqrt{5}(-1)^{\frac{1}{5}}-(-1)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}+(-1)^{\frac{1}{5}}-4x^{\frac{1}{6}}}\right)}{5\sqrt{2\sqrt{5}-10}}+\frac{6\sqrt{5}\left(-1\right)^{\frac{3}{5}}\log\left(\frac{\sqrt{5}(-1)^{\frac{1}{5}}-(-1)^{\frac{1}{5}}\sqrt{-2\sqrt{5}-10}-(-1)^{\frac{1}{5}}+4x^{\frac{1}{6}}}{\sqrt{5}(-1)^{\frac{1}{5}}+(-1)^{\frac{1}{5}}\sqrt{-2\sqrt{5}-10}-(-1)^{\frac{1}{5}}+4x^{\frac{1}{6}}}\right)}{5\sqrt{-2\sqrt{5}-10}}+2\sqrt{x}+\frac{6\log\left(-x^{\frac{1}{6}}\left(\sqrt{5}\left(-1\right)^{\frac{1}{5}}+\left(-1\right)^{\frac{1}{5}}\right)+2\left(-1\right)^{\frac{2}{5}}+2x^{\frac{1}{3}}\right)}{5\left(\sqrt{5}\left(-1\right)^{\frac{2}{5}}+\left(-1\right)^{\frac{2}{5}}\right)}-\frac{6\log\left(x^{\frac{1}{6}}\left(\sqrt{5}\left(-1\right)^{\frac{1}{5}}-\left(-1\right)^{\frac{1}{5}}\right)+2\left(-1\right)^{\frac{2}{5}}+2x^{\frac{1}{3}}\right)}{5\left(\sqrt{5}\left(-1\right)^{\frac{2}{5}}-\left(-1\right)^{\frac{2}{5}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x) - 1/x^(1/3)),x, algorithm="maxima")
```

```
[Out] -6/5*(-1)^(3/5)*log((-1)^(1/5) + x^(1/6)) - 6/5*sqrt(5)*(-1)^(3/5
)*log((sqrt(5)*(-1)^(1/5) + (-1)^(1/5)*sqrt(2*sqrt(5) - 10) + (-1
```

```
)^(1/5) - 4*x^(1/6))/(sqrt(5)*(-1)^(1/5) - (-1)^(1/5)*sqrt(2*sqrt
(5) - 10) + (-1)^(1/5) - 4*x^(1/6)))/sqrt(2*sqrt(5) - 10) + 6/5*s
qrt(5)*(-1)^(3/5)*log((sqrt(5)*(-1)^(1/5) - (-1)^(1/5)*sqrt(-2*sq
rt(5) - 10) - (-1)^(1/5) + 4*x^(1/6))/(sqrt(5)*(-1)^(1/5) + (-1)^
(1/5)*sqrt(-2*sqrt(5) - 10) - (-1)^(1/5) + 4*x^(1/6)))/sqrt(-2*sq
rt(5) - 10) + 2*sqrt(x) + 6/5*log(-x^(1/6)*(sqrt(5)*(-1)^(1/5) +
(-1)^(1/5)) + 2*(-1)^(2/5) + 2*x^(1/3))/(sqrt(5)*(-1)^(2/5) + (-1
)^(2/5)) - 6/5*log(x^(1/6)*(sqrt(5)*(-1)^(1/5) - (-1)^(1/5)) + 2*
(-1)^(2/5) + 2*x^(1/3))/(sqrt(5)*(-1)^(2/5) - (-1)^(2/5))
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x) - 1/x^(1/3)),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{x}}{\left(\sqrt[6]{x}-1\right)\left(\sqrt[6]{x}+x^{\frac{2}{3}}+\sqrt[3]{x}+\sqrt{x}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-1/x**(1/3)+x**(1/2)),x)
```

```
[Out] Integral(x**(1/3)/((x**(1/6) - 1)*(x**(1/6) + x**(2/3) + x**(1/3)
 + sqrt(x) + 1)), x)
```

GIAC/XCAS [A] time = 0.379049, size = 188, normalized size = 0.94

$$\frac{3}{5}\sqrt{-2\sqrt{5}+10}\arctan\left(-\frac{\sqrt{5}-4x^{\frac{1}{6}}-1}{\sqrt{2\sqrt{5}+10}}\right)-\frac{3}{5}\sqrt{2\sqrt{5}+10}\arctan\left(\frac{\sqrt{5}+4x^{\frac{1}{6}}+1}{\sqrt{-2\sqrt{5}+10}}\right)+\frac{3}{10}\sqrt{5}\ln\left(\frac{1}{2}x^{\frac{1}{6}}\left(\sqrt{5}+1\right)+x^{\frac{1}{3}}+1\right)-\frac{3}{10}\sqrt{5}\ln\left(-\frac{1}{2}x^{\frac{1}{6}}\left(\sqrt{5}-1\right)+x^{\frac{1}{3}}+1\right)+2\sqrt{x}-\frac{3}{10}\ln\left(x^{\frac{2}{3}}+\sqrt{x}+x^{\frac{1}{3}}+x^{\frac{1}{6}}+1\right)+\frac{6}{5}\ln\left(\left|x^{\frac{1}{6}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x) - 1/x^(1/3)),x, algorithm="giac")
```

```
[Out] 3/5*sqrt(-2*sqrt(5) + 10)*arctan(-(sqrt(5) - 4*x^(1/6) - 1)/sqrt(
2*sqrt(5) + 10)) - 3/5*sqrt(2*sqrt(5) + 10)*arctan((sqrt(5) + 4*x
^(1/6) + 1)/sqrt(-2*sqrt(5) + 10)) + 3/10*sqrt(5)*ln(1/2*x^(1/6)*
(sqrt(5) + 1) + x^(1/3) + 1) - 3/10*sqrt(5)*ln(-1/2*x^(1/6)*(sqrt
(5) - 1) + x^(1/3) + 1) + 2*sqrt(x) - 3/10*ln(x^(2/3) + sqrt(x) +
 x^(1/3) + x^(1/6) + 1) + 6/5*ln(abs(x^(1/6) - 1))
```

3.414 $\int \frac{\sqrt{x}}{x+x^2} dx$

Optimal. Leaf size=8

$$2\tan^{-1}\left(\sqrt{x}\right)$$

```
[Out] 2*ArcTan[Sqrt[x]]
```

Rubi [A] time = 0.0102523, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$2\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x]/(x + x^2),x]

[Out] 2*ArcTan[Sqrt[x]]
```

Rubi in Sympy [A] time = 1.3472, size = 7, normalized size = 0.88

$$2\,\text{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(1/2)/(x**2+x),x)

[Out] 2*atan(sqrt(x))
```

Mathematica [A] time = 0.00505989, size = 8, normalized size = 1.

$$2\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[x]/(x + x^2),x]
```

```
[Out] 2*ArcTan[Sqrt[x]]
```

Maple [A] time = 0.004, size = 7, normalized size = 0.9

$$2 \arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(1/2)/(x^2+x),x)
```

```
[Out] 2*arctan(x^(1/2))
```

Maxima [A] time = 0.791403, size = 8, normalized size = 1.

$$2 \arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/(x^2 + x),x, algorithm="maxima")
```

```
[Out] 2*arctan(sqrt(x))
```

Fricas [A] time = 0.278572, size = 8, normalized size = 1.

$$2 \arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/(x^2 + x),x, algorithm="fricas")
```

```
[Out] 2*arctan(sqrt(x))
```

Sympy [A] time = 1.84693, size = 7, normalized size = 0.88

$$2 \operatorname{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(1/2)/(x**2+x),x)
```

```
[Out] 2*atan(sqrt(x))
```

GIAC/XCAS [A] time = 0.276827, size = 8, normalized size = 1.

$$2 \arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/(x^2 + x),x, algorithm="giac")
```

```
[Out] 2*arctan(sqrt(x))
```

3.415 $\int \frac{x}{4\sqrt{x}+x} \, dx$

Optimal. Leaf size=19

$$x - 8\sqrt{x} + 32\log\left(\sqrt{x} + 4\right)$$

```
[Out] -8*Sqrt[x] + x + 32*Log[4 + Sqrt[x]]
```

Rubi [A] time = 0.0287146, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$x - 8\sqrt{x} + 32\log\left(\sqrt{x} + 4\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x/(4*Sqrt[x] + x),x]

[Out] -8*Sqrt[x] + x + 32*Log[4 + Sqrt[x]]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-8\sqrt{x} + 32\log\left(\sqrt{x} + 4\right) + 2\int^{\sqrt{x}} x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(x+4*x**(1/2)),x)

[Out] -8*sqrt(x) + 32*log(sqrt(x) + 4) + 2*Integral(x, (x, sqrt(x)))
```

Mathematica [A] time = 0.0089928, size = 19, normalized size = 1.

$$x - 8\sqrt{x} + 32\log\left(\sqrt{x} + 4\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(4*Sqrt[x] + x),x]
```

```
[Out] -8*Sqrt[x] + x + 32*Log[4 + Sqrt[x]]
```

Maple [A] time = 0.004, size = 16, normalized size = 0.8

$$x + 32 \ln \left(4 + \sqrt{x}\right) - 8 \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(x+4*x^(1/2)),x)
```

```
[Out] x+32*ln(4+x^(1/2))-8*x^(1/2)
```

Maxima [A] time = 0.690106, size = 20, normalized size = 1.05

$$x - 8 \sqrt{x} + 32 \log \left(\sqrt{x} + 4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x + 4*sqrt(x)),x, algorithm="maxima")
```

```
[Out] x - 8*sqrt(x) + 32*log(sqrt(x) + 4)
```

Fricas [A] time = 0.26943, size = 20, normalized size = 1.05

$$x - 8 \sqrt{x} + 32 \log \left(\sqrt{x} + 4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x + 4*sqrt(x)),x, algorithm="fricas")
```

```
[Out] x - 8*sqrt(x) + 32*log(sqrt(x) + 4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{4\sqrt{x} + x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x+4*x**(1/2)),x)
```

```
[Out] Integral(x/(4*sqrt(x) + x), x)
```

GIAC/XCAS [A] time = 0.278019, size = 20, normalized size = 1.05

$$x - 8\sqrt{x} + 32\ln\left(\sqrt{x} + 4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x + 4*sqrt(x)),x, algorithm="giac")
```

```
[Out] x - 8*sqrt(x) + 32*ln(sqrt(x) + 4)
```

3.416 $\int \frac{\sqrt{x}}{\sqrt[3]{x}+x} \, dx$

Optimal. Leaf size=108

$$2\sqrt{x} - \frac{3\log\left(\sqrt[3]{x} - \sqrt{2}\sqrt[6]{x} + 1\right)}{2\sqrt{2}} + \frac{3\log\left(\sqrt[3]{x} + \sqrt{2}\sqrt[6]{x} + 1\right)}{2\sqrt{2}} + \frac{3\tan^{-1}\left(1 - \sqrt{2}\sqrt[6]{x}\right)}{\sqrt{2}} - \frac{3\tan^{-1}\left(\sqrt{2}\sqrt[6]{x} + 1\right)}{\sqrt{2}}$$

```
[Out] 2*Sqrt[x] + (3*ArcTan[1 - Sqrt[2]*x^(1/6)])/Sqrt[2] - (3*ArcTan[1
 + Sqrt[2]*x^(1/6)])/Sqrt[2] - (3*Log[1 - Sqrt[2]*x^(1/6) + x^(1/
3)])/(2*Sqrt[2]) + (3*Log[1 + Sqrt[2]*x^(1/6) + x^(1/3)])/(2*Sqrt
[2])
```

Rubi [A] time = 0.149362, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667

$$2\sqrt{x} - \frac{3\log\left(\sqrt[3]{x} - \sqrt{2}\sqrt[6]{x} + 1\right)}{2\sqrt{2}} + \frac{3\log\left(\sqrt[3]{x} + \sqrt{2}\sqrt[6]{x} + 1\right)}{2\sqrt{2}} + \frac{3\tan^{-1}\left(1 - \sqrt{2}\sqrt[6]{x}\right)}{\sqrt{2}} - \frac{3\tan^{-1}\left(\sqrt{2}\sqrt[6]{x} + 1\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x]/(x^(1/3) + x),x]
```

```
[Out] 2*Sqrt[x] + (3*ArcTan[1 - Sqrt[2]*x^(1/6)])/Sqrt[2] - (3*ArcTan[1
 + Sqrt[2]*x^(1/6)])/Sqrt[2] - (3*Log[1 - Sqrt[2]*x^(1/6) + x^(1/
3)])/(2*Sqrt[2]) + (3*Log[1 + Sqrt[2]*x^(1/6) + x^(1/3)])/(2*Sqrt
[2])
```

Rubi in Sympy [A] time = 10.477, size = 104, normalized size = 0.96

$$2\sqrt{x} - \frac{3\sqrt{2}\log\left(-\sqrt{2}\sqrt[6]{x} + \sqrt[3]{x} + 1\right)}{4} + \frac{3\sqrt{2}\log\left(\sqrt{2}\sqrt[6]{x} + \sqrt[3]{x} + 1\right)}{4} - \frac{3\sqrt{2}\,\text{atan}\left(\sqrt{2}\sqrt[6]{x} - 1\right)}{2} - \frac{3\sqrt{2}\,\text{atan}\left(\sqrt{2}\sqrt[6]{x} + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(1/2)/(x**(1/3)+x),x)
```

```
[Out] 2*sqrt(x) - 3*sqrt(2)*log(-sqrt(2)*x**(1/6) + x**(1/3) + 1)/4 + 3
*sqrt(2)*log(sqrt(2)*x**(1/6) + x**(1/3) + 1)/4 - 3*sqrt(2)*atan(
sqrt(2)*x**(1/6) - 1)/2 - 3*sqrt(2)*atan(sqrt(2)*x**(1/6) + 1)/2
```

Mathematica [A] time = 0.0460104, size = 108, normalized size = 1.

$$\frac{1}{4}\left(8\sqrt{x}-3\sqrt{2}\log\left(\sqrt[3]{x}-\sqrt{2}\sqrt[6]{x}+1\right)+3\sqrt{2}\log\left(\sqrt[3]{x}+\sqrt{2}\sqrt[6]{x}+1\right)\right.$$
$$\left.+6\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt[6]{x}\right)-6\sqrt{2}\tan^{-1}\left(\sqrt{2}\sqrt[6]{x}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[x]/(x^(1/3) + x),x]
```

```
[Out] (8*Sqrt[x] + 6*Sqrt[2]*ArcTan[1 - Sqrt[2]*x^(1/6)] - 6*Sqrt[2]*Ar
cTan[1 + Sqrt[2]*x^(1/6)] - 3*Sqrt[2]*Log[1 - Sqrt[2]*x^(1/6) + x
^(1/3)] + 3*Sqrt[2]*Log[1 + Sqrt[2]*x^(1/6) + x^(1/3)])/4
```

Maple [A] time = 0.007, size = 71, normalized size = 0.7

$$2\sqrt{x}-\frac{3\sqrt{2}}{2}\arctan\left(\sqrt[6]{x}\sqrt{2}-1\right)$$
$$-\frac{3\sqrt{2}}{4}\ln\left(1\left(1+\sqrt[3]{x}-\sqrt[6]{x}\sqrt{2}\right)\left(1+\sqrt[3]{x}+\sqrt[6]{x}\sqrt{2}\right)^{-1}\right)-\frac{3\sqrt{2}}{2}\arctan\left(1+\sqrt[6]{x}\sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(1/2)/(x^(1/3)+x),x)
```

```
[Out] 2*x^(1/2)-3/2*arctan(x^(1/6)*2^(1/2)-1)*2^(1/2)-3/4*2^(1/2)*ln((1
+x^(1/3)-x^(1/6)*2^(1/2))/(1+x^(1/3)+x^(1/6)*2^(1/2)))-3/2*arctan
(1+x^(1/6)*2^(1/2))*2^(1/2)
```

Maxima [A] time = 0.808029, size = 112, normalized size = 1.04

$$-\frac{3}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2x^{\frac{1}{6}}\right)\right)-\frac{3}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2x^{\frac{1}{6}}\right)\right)$$
$$+\frac{3}{4}\sqrt{2}\log\left(\sqrt{2}x^{\frac{1}{6}}+x^{\frac{1}{3}}+1\right)-\frac{3}{4}\sqrt{2}\log\left(-\sqrt{2}x^{\frac{1}{6}}+x^{\frac{1}{3}}+1\right)+2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/(x + x^(1/3)),x, algorithm="maxima")
```

```
[Out] -3/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*x^(1/6))) - 3/2*sqrt
(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*x^(1/6))) + 3/4*sqrt(2)*log(
sqrt(2)*x^(1/6) + x^(1/3) + 1) - 3/4*sqrt(2)*log(-sqrt(2)*x^(1/6)
 + x^(1/3) + 1) + 2*sqrt(x)
```

Fricas [A] time = 0.28404, size = 157, normalized size = 1.45

$$3\sqrt{2}\arctan\left(\frac{1}{\sqrt{2}x^{\frac{1}{6}} + \sqrt{2\sqrt{2}x^{\frac{1}{6}} + 2x^{\frac{1}{3}} + 2} + 1}\right) + 3\sqrt{2}\arctan\left(\frac{1}{\sqrt{2}x^{\frac{1}{6}} + \sqrt{-2\sqrt{2}x^{\frac{1}{6}} + 2x^{\frac{1}{3}} + 2} - 1}\right) + \frac{3}{4}\sqrt{2}\log\left(2\sqrt{2}x^{\frac{1}{6}} + 2x^{\frac{1}{3}} + 2\right) - \frac{3}{4}\sqrt{2}\log\left(-2\sqrt{2}x^{\frac{1}{6}} + 2x^{\frac{1}{3}} + 2\right) + 2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/(x + x^(1/3)),x, algorithm="fricas")
```

```
[Out] 3*sqrt(2)*arctan(1/(sqrt(2)*x^(1/6) + sqrt(2*sqrt(2)*x^(1/6) + 2*
x^(1/3) + 2) + 1)) + 3*sqrt(2)*arctan(1/(sqrt(2)*x^(1/6) + sqrt(-
2*sqrt(2)*x^(1/6) + 2*x^(1/3) + 2) - 1)) + 3/4*sqrt(2)*log(2*sqrt
(2)*x^(1/6) + 2*x^(1/3) + 2) - 3/4*sqrt(2)*log(-2*sqrt(2)*x^(1/6)
 + 2*x^(1/3) + 2) + 2*sqrt(x)
```

Sympy [A] time = 4.88837, size = 110, normalized size = 1.02

$$2\sqrt{x} - \frac{3\sqrt{2}\log\left(-4\sqrt{2}\sqrt[6]{x} + 4\sqrt[3]{x} + 4\right)}{4} + \frac{3\sqrt{2}\log\left(4\sqrt{2}\sqrt[6]{x} + 4\sqrt[3]{x} + 4\right)}{4} - \frac{3\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt[6]{x} - 1\right)}{2} - \frac{3\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt[6]{x} + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(1/2)/(x**(1/3)+x),x)
```

```
[Out] 2*sqrt(x) - 3*sqrt(2)*log(-4*sqrt(2)*x**(1/6) + 4*x**(1/3) + 4)/4
 + 3*sqrt(2)*log(4*sqrt(2)*x**(1/6) + 4*x**(1/3) + 4)/4 - 3*sqrt(
2)*atan(sqrt(2)*x**(1/6) - 1)/2 - 3*sqrt(2)*atan(sqrt(2)*x**(1/6)
```

```
 + 1)/2
```

GIAC/XCAS [A] time = 0.280433, size = 112, normalized size = 1.04

$$-\frac{3}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\,x^{\frac{1}{6}}\right)\right)-\frac{3}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\,x^{\frac{1}{6}}\right)\right)+\frac{3}{4}\sqrt{2}\ln\left(\sqrt{2}x^{\frac{1}{6}}+x^{\frac{1}{3}}+1\right)-\frac{3}{4}\sqrt{2}\ln\left(-\sqrt{2}x^{\frac{1}{6}}+x^{\frac{1}{3}}+1\right)+2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/(x + x^(1/3)),x, algorithm="giac")
```

```
[Out] -3/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*x^(1/6))) - 3/2*sqrt
(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*x^(1/6))) + 3/4*sqrt(2)*ln(s
qrt(2)*x^(1/6) + x^(1/3) + 1) - 3/4*sqrt(2)*ln(-sqrt(2)*x^(1/6) +
 x^(1/3) + 1) + 2*sqrt(x)
```

3.417 $\int \frac{\sqrt[3]{x}}{\sqrt[4]{x}+\sqrt{x}}\,dx$

Optimal. Leaf size=76

$$\frac{6x^{5/6}}{5}-\frac{12x^{7/12}}{7}+3\sqrt[3]{x}-12\sqrt[12]{x}+6\log\left(\sqrt[12]{x}+1\right)-2\log\left(\sqrt[4]{x}+1\right)-4\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[12]{x}}{\sqrt{3}}\right)$$

```
[Out] -12*x^(1/12) + 3*x^(1/3) - (12*x^(7/12))/7 + (6*x^(5/6))/5 - 4*Sq
rt[3]*ArcTan[(1 - 2*x^(1/12))/Sqrt[3]] + 6*Log[1 + x^(1/12)] - 2*
Log[1 + x^(1/4)]
```

Rubi [A] time = 0.0758517, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368

$$\frac{6x^{5/6}}{5}-\frac{12x^{7/12}}{7}+3\sqrt[3]{x}-12\sqrt[12]{x}+6\log\left(\sqrt[12]{x}+1\right)-2\log\left(\sqrt[4]{x}+1\right)-4\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[12]{x}}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^(1/3)/(x^(1/4) + Sqrt[x]),x]
```

```
[Out] -12*x^(1/12) + 3*x^(1/3) - (12*x^(7/12))/7 + (6*x^(5/6))/5 - 4*Sq
rt[3]*ArcTan[(1 - 2*x^(1/12))/Sqrt[3]] + 6*Log[1 + x^(1/12)] - 2*
Log[1 + x^(1/4)]
```

Rubi in Sympy [A] time = 3.87906, size = 75, normalized size = 0.99

$$-\frac{12x^{\frac{7}{12}}}{7}-12\sqrt[12]{x}+\frac{6x^{\frac{5}{6}}}{5}+3\sqrt[3]{x}+6\log\left(\sqrt[12]{x}+1\right)-2\log\left(\sqrt[4]{x}+1\right)+4\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2\sqrt[12]{x}}{3}-\frac{1}{3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(1/3)/(x**(1/4)+x**(1/2)),x)
```

```
[Out] -12*x**(7/12)/7 - 12*x**(1/12) + 6*x**(5/6)/5 + 3*x**(1/3) + 6*lo
g(x**(1/12) + 1) - 2*log(x**(1/4) + 1) + 4*sqrt(3)*atan(sqrt(3)*(
2*x**(1/12)/3 - 1/3))
```

Mathematica [A] time = 0.0260146, size = 83, normalized size = 1.09

$$\frac{6x^{5/6}}{5}-\frac{12x^{7/12}}{7}+3\sqrt[3]{x}-12\sqrt[12]{x}+4\log\left(\sqrt[12]{x}+1\right)-2\log\left(\sqrt[6]{x}-\sqrt[12]{x}+1\right)+4\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[12]{x}-1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(1/3)/(x^(1/4) + Sqrt[x]),x]
```

```
[Out] -12*x^(1/12) + 3*x^(1/3) - (12*x^(7/12))/7 + (6*x^(5/6))/5 + 4*Sq
rt[3]*ArcTan[(-1 + 2*x^(1/12))/Sqrt[3]] + 4*Log[1 + x^(1/12)] - 2
*Log[1 - x^(1/12) + x^(1/6)]
```

Maple [A] time = 0.006, size = 61, normalized size = 0.8

$$\frac{6}{5}x^{\frac{5}{6}}-\frac{12}{7}x^{\frac{7}{12}}+3\sqrt[3]{x}-12x^{1/12}+4\ln\left(1+x^{1/12}\right)-2\ln\left(1-x^{1/12}+\sqrt[6]{x}\right)+4\sqrt{3}\arctan\left(1/3\left(2x^{1/12}-1\right)\sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(1/3)/(x^(1/4)+x^(1/2)),x)
```

```
[Out] 6/5*x^(5/6)-12/7*x^(7/12)+3*x^(1/3)-12*x^(1/12)+4*ln(1+x^(1/12))-
2*ln(1-x^(1/12)+x^(1/6))+4*3^(1/2)*arctan(1/3*(2*x^(1/12)-1)*3^(1
/2))
```

Maxima [A] time = 0.81326, size = 81, normalized size = 1.07

$$4\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{12}}-1\right)\right)+\frac{6}{5}x^{\frac{5}{6}}-\frac{12}{7}x^{\frac{7}{12}}+3x^{\frac{1}{3}}-12x^{\frac{1}{12}}-2\log\left(x^{\frac{1}{6}}-x^{\frac{1}{12}}+1\right)+4\log\left(x^{\frac{1}{12}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(1/3)/(sqrt(x) + x^(1/4)),x, algorithm="maxima")
```

```
[Out] 4*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/12) - 1)) + 6/5*x^(5/6) - 12
/7*x^(7/12) + 3*x^(1/3) - 12*x^(1/12) - 2*log(x^(1/6) - x^(1/12)
+ 1) + 4*log(x^(1/12) + 1)
```

Fricas [A] time = 0.281759, size = 81, normalized size = 1.07

$$4\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{12}}-1\right)\right)+\frac{6}{5}x^{\frac{5}{6}}-\frac{12}{7}x^{\frac{7}{12}}+3x^{\frac{1}{3}}-12x^{\frac{1}{12}}-2\log\left(x^{\frac{1}{6}}-x^{\frac{1}{12}}+1\right)+4\log\left(x^{\frac{1}{12}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(1/3)/(sqrt(x) + x^(1/4)),x, algorithm="fricas")
```

```
[Out] 4*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/12) - 1)) + 6/5*x^(5/6) - 12
/7*x^(7/12) + 3*x^(1/3) - 12*x^(1/12) - 2*log(x^(1/6) - x^(1/12)
+ 1) + 4*log(x^(1/12) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{x}}{\sqrt[4]{x}+\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(1/3)/(x**(1/4)+x**(1/2)),x)
```

```
[Out] Integral(x**(1/3)/(x**(1/4) + sqrt(x)), x)
```

GIAC/XCAS [A] time = 0.285673, size = 81, normalized size = 1.07

$$4\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{12}}-1\right)\right)+\frac{6}{5}x^{\frac{5}{6}}-\frac{12}{7}x^{\frac{7}{12}}+3x^{\frac{1}{3}}-12x^{\frac{1}{12}}-2\ln\left(x^{\frac{1}{6}}-x^{\frac{1}{12}}+1\right)+4\ln\left(x^{\frac{1}{12}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(1/3)/(sqrt(x) + x^(1/4)),x, algorithm="giac")
```

```
[Out] 4*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/12) - 1)) + 6/5*x^(5/6) - 12
/7*x^(7/12) + 3*x^(1/3) - 12*x^(1/12) - 2*ln(x^(1/6) - x^(1/12) +
 1) + 4*ln(x^(1/12) + 1)
```

3.418 $\int \frac{\sqrt{x}}{\sqrt[4]{x}+\sqrt[3]{x}}\,dx$

Optimal. Leaf size=119

$$\frac{6x^{7/6}}{7}-\frac{12x^{13/12}}{13}-\frac{12x^{11/12}}{11}+\frac{6x^{5/6}}{5}-\frac{4x^{3/4}}{3}+\frac{3x^{2/3}}{2}-\frac{12x^{7/12}}{7}-\frac{12x^{5/12}}{5}+x+2\sqrt{x}+3\sqrt[3]{x}-4\sqrt[4]{x}+6\sqrt[6]{x}-12\sqrt[12]{x}+12\log\left(\sqrt[12]{x}+1\right)$$

```
[Out] -12*x^(1/12) + 6*x^(1/6) - 4*x^(1/4) + 3*x^(1/3) - (12*x^(5/12))/
5 + 2*Sqrt[x] - (12*x^(7/12))/7 + (3*x^(2/3))/2 - (4*x^(3/4))/3 +
 (6*x^(5/6))/5 - (12*x^(11/12))/11 + x - (12*x^(13/12))/13 + (6*x
^(7/6))/7 + 12*Log[1 + x^(1/12)]
```

Rubi [A] time = 0.0950388, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$\frac{6x^{7/6}}{7}-\frac{12x^{13/12}}{13}-\frac{12x^{11/12}}{11}+\frac{6x^{5/6}}{5}-\frac{4x^{3/4}}{3}+\frac{3x^{2/3}}{2}-\frac{12x^{7/12}}{7}-\frac{12x^{5/12}}{5}+x+2\sqrt{x}+3\sqrt[3]{x}-4\sqrt[4]{x}+6\sqrt[6]{x}-12\sqrt[12]{x}+12\log\left(\sqrt[12]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x]/(x^(1/4) + x^(1/3)),x]
```

```
[Out] -12*x^(1/12) + 6*x^(1/6) - 4*x^(1/4) + 3*x^(1/3) - (12*x^(5/12))/
5 + 2*Sqrt[x] - (12*x^(7/12))/7 + (3*x^(2/3))/2 - (4*x^(3/4))/3 +
 (6*x^(5/6))/5 - (12*x^(11/12))/11 + x - (12*x^(13/12))/13 + (6*x
^(7/6))/7 + 12*Log[1 + x^(1/12)]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{12x^{\frac{13}{12}}}{13}-\frac{12x^{\frac{11}{12}}}{11}-\frac{12x^{\frac{7}{12}}}{7}-\frac{12x^{\frac{5}{12}}}{5}-12\sqrt[12]{x}+\frac{6x^{\frac{7}{6}}}{7}+\frac{6x^{\frac{5}{6}}}{5}-\frac{4x^{\frac{3}{4}}}{3}-4\sqrt[4]{x}+\frac{3x^{\frac{2}{3}}}{2}+3\sqrt[3]{x}+2\sqrt{x}+x+12\log\left(\sqrt[12]{x}+1\right)+12\int^{\sqrt[12]{x}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(1/2)/(x**(1/4)+x**(1/3)),x)
```

```
[Out] -12*x**(13/12)/13 - 12*x**(11/12)/11 - 12*x**(7/12)/7 - 12*x**(5/
12)/5 - 12*x**(1/12) + 6*x**(7/6)/7 + 6*x**(5/6)/5 - 4*x**(3/4)/3
 - 4*x**(1/4) + 3*x**(2/3)/2 + 3*x**(1/3) + 2*sqrt(x) + x + 12*lo
g(x**(1/12) + 1) + 12*Integral(x, (x, x**(1/12)))
```

Mathematica [A] time = 0.0267483, size = 119, normalized size = 1.

$$\frac{6x^{7/6}}{7}-\frac{12x^{13/12}}{13}-\frac{12x^{11/12}}{11}+\frac{6x^{5/6}}{5}-\frac{4x^{3/4}}{3}+\frac{3x^{2/3}}{2}-\frac{12x^{7/12}}{7}-\frac{12x^{5/12}}{5}+x+2\sqrt{x}+3\sqrt[3]{x}-4\sqrt[4]{x}+6\sqrt[6]{x}-12\sqrt[12]{x}+12\log\left(\sqrt[12]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[x]/(x^(1/4) + x^(1/3)),x]
```

```
[Out] -12*x^(1/12) + 6*x^(1/6) - 4*x^(1/4) + 3*x^(1/3) - (12*x^(5/12))/
5 + 2*Sqrt[x] - (12*x^(7/12))/7 + (3*x^(2/3))/2 - (4*x^(3/4))/3 +
 (6*x^(5/6))/5 - (12*x^(11/12))/11 + x - (12*x^(13/12))/13 + (6*x
^(7/6))/7 + 12*Log[1 + x^(1/12)]
```

Maple [A] time = 0.005, size = 76, normalized size = 0.6

$$-12\,x^{1/12}+6\,\sqrt[6]{x}-4\,\sqrt[4]{x}+3\,\sqrt[3]{x}-\frac{12}{5}x^{\frac{5}{12}}-\frac{12}{7}x^{\frac{7}{12}}+\frac{3}{2}x^{\frac{2}{3}}-\frac{4}{3}x^{\frac{3}{4}}+\frac{6}{5}x^{\frac{5}{6}}-\frac{12}{11}x^{\frac{11}{12}}+x-\frac{12}{13}x^{\frac{13}{12}}+\frac{6}{7}x^{\frac{7}{6}}+12\,\ln\left(1+x^{1/12}\right)+2\,\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(1/2)/(x^(1/4)+x^(1/3)),x)
```

```
[Out] -12*x^(1/12)+6*x^(1/6)-4*x^(1/4)+3*x^(1/3)-12/5*x^(5/12)-12/7*x^(
7/12)+3/2*x^(2/3)-4/3*x^(3/4)+6/5*x^(5/6)-12/11*x^(11/12)+x-12/13
*x^(13/12)+6/7*x^(7/6)+12*ln(1+x^(1/12))+2*x^(1/2)
```

Maxima [A] time = 0.699702, size = 101, normalized size = 0.85

$$\frac{6}{7}\,x^{\frac{7}{6}}-\frac{12}{13}\,x^{\frac{13}{12}}+x-\frac{12}{11}\,x^{\frac{11}{12}}+\frac{6}{5}\,x^{\frac{5}{6}}-\frac{4}{3}\,x^{\frac{3}{4}}+\frac{3}{2}\,x^{\frac{2}{3}}-\frac{12}{7}\,x^{\frac{7}{12}}+2\,\sqrt{x}-\frac{12}{5}\,x^{\frac{5}{12}}+3\,x^{\frac{1}{3}}-4\,x^{\frac{1}{4}}+6\,x^{\frac{1}{6}}-12\,x^{\frac{1}{12}}+12\,\log\left(x^{\frac{1}{12}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/(x^(1/3) + x^(1/4)),x, algorithm="maxima")
```

```
[Out] 6/7*x^(7/6) - 12/13*x^(13/12) + x - 12/11*x^(11/12) + 6/5*x^(5/6)
 - 4/3*x^(3/4) + 3/2*x^(2/3) - 12/7*x^(7/12) + 2*sqrt(x) - 12/5*x
^(5/12) + 3*x^(1/3) - 4*x^(1/4) + 6*x^(1/6) - 12*x^(1/12) + 12*lo
g(x^(1/12) + 1)
```

Fricas [A] time = 0.283121, size = 96, normalized size = 0.81

$$\frac{6}{7}(x+7)x^{\frac{1}{6}} - \frac{12}{13}(x+13)x^{\frac{1}{12}} + x - \frac{12}{11}x^{\frac{11}{12}} + \frac{6}{5}x^{\frac{5}{6}} - \frac{4}{3}x^{\frac{3}{4}} + \frac{3}{2}x^{\frac{2}{3}} - \frac{12}{7}x^{\frac{7}{12}} + 2\sqrt{x} - \frac{12}{5}x^{\frac{5}{12}} + 3x^{\frac{1}{3}} - 4x^{\frac{1}{4}} + 12\log\left(x^{\frac{1}{12}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/(x^(1/3) + x^(1/4)),x, algorithm="fricas")
```

```
[Out] 6/7*(x + 7)*x^(1/6) - 12/13*(x + 13)*x^(1/12) + x - 12/11*x^(11/1
2) + 6/5*x^(5/6) - 4/3*x^(3/4) + 3/2*x^(2/3) - 12/7*x^(7/12) + 2*
sqrt(x) - 12/5*x^(5/12) + 3*x^(1/3) - 4*x^(1/4) + 12*log(x^(1/12)
 + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{\sqrt[4]{x} + \sqrt[3]{x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(1/2)/(x**(1/4)+x**(1/3)),x)
```

```
[Out] Integral(sqrt(x)/(x**(1/4) + x**(1/3)), x)
```

GIAC/XCAS [A] time = 0.290448, size = 101, normalized size = 0.85

$$\frac{6}{7}x^{\frac{7}{6}} - \frac{12}{13}x^{\frac{13}{12}} + x - \frac{12}{11}x^{\frac{11}{12}} + \frac{6}{5}x^{\frac{5}{6}} - \frac{4}{3}x^{\frac{3}{4}} + \frac{3}{2}x^{\frac{2}{3}} - \frac{12}{7}x^{\frac{7}{12}} + 2\sqrt{x} - \frac{12}{5}x^{\frac{5}{12}} + 3x^{\frac{1}{3}} - 4x^{\frac{1}{4}} + 6x^{\frac{1}{6}} - 12x^{\frac{1}{12}} + 12\ln\left(x^{\frac{1}{12}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/(x^(1/3) + x^(1/4)),x, algorithm="giac")
```

```
[Out] 6/7*x^(7/6) - 12/13*x^(13/12) + x - 12/11*x^(11/12) + 6/5*x^(5/6)
 - 4/3*x^(3/4) + 3/2*x^(2/3) - 12/7*x^(7/12) + 2*sqrt(x) - 12/5*x
^(5/12) + 3*x^(1/3) - 4*x^(1/4) + 6*x^(1/6) - 12*x^(1/12) + 12*ln
(x^(1/12) + 1)
```

3.419 $\int \frac{\sqrt{x}}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx$

Optimal. Leaf size=201

$$x+6\sqrt[6]{x}+\frac{6}{5}\log\left(1-\sqrt[6]{x}\right)-\frac{3}{10}\left(1-\sqrt{5}\right)\log\left(2\sqrt[3]{x}-\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}+2\right)-\frac{3}{10}\left(1+\sqrt{5}\right)\log\left(2\sqrt[3]{x}+\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}+2\right)-\frac{3}{5}\sqrt{2\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{4\sqrt[6]{x}-\sqrt{5}+1}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)-\frac{3}{5}\sqrt{2\left(5-\sqrt{5}\right)}\tan^{-1}\left(\frac{1}{2}\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(4\sqrt[6]{x}+\sqrt{5}+1\right)\right)$$

```
[Out] 6*x^(1/6) + x - (3*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(1 - Sqrt[5] + 4*
x^(1/6))/Sqrt[2*(5 + Sqrt[5])]])/5 - (3*Sqrt[2*(5 - Sqrt[5])]*Arc
Tan[(Sqrt[(5 + Sqrt[5])/10]*(1 + Sqrt[5] + 4*x^(1/6)))/2])/5 + (6
*Log[1 - x^(1/6)])/5 - (3*(1 - Sqrt[5])*Log[2 + x^(1/6) - Sqrt[5]
*x^(1/6) + 2*x^(1/3)])/10 - (3*(1 + Sqrt[5])*Log[2 + x^(1/6) + Sq
rt[5]*x^(1/6) + 2*x^(1/3)])/10
```

Rubi [A] time = 0.517531, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429

$$x+6\sqrt[6]{x}+\frac{6}{5}\log\left(1-\sqrt[6]{x}\right)-\frac{3}{10}\left(1-\sqrt{5}\right)\log\left(2\sqrt[3]{x}-\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}+2\right)-\frac{3}{10}\left(1+\sqrt{5}\right)\log\left(2\sqrt[3]{x}+\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}+2\right)-\frac{3}{5}\sqrt{2\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{4\sqrt[6]{x}-\sqrt{5}+1}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)-\frac{3}{5}\sqrt{2\left(5-\sqrt{5}\right)}\tan^{-1}\left(\frac{1}{2}\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(4\sqrt[6]{x}+\sqrt{5}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x]/(-x^(-1/3) + Sqrt[x]),x]
```


```
[Out] 6*x^(1/6) + x - (3*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(1 - Sqrt[5] + 4*
x^(1/6))/Sqrt[2*(5 + Sqrt[5])]])/5 - (3*Sqrt[2*(5 - Sqrt[5])]*Arc
Tan[(Sqrt[(5 + Sqrt[5])/10]*(1 + Sqrt[5] + 4*x^(1/6)))/2])/5 + (6
*Log[1 - x^(1/6)])/5 - (3*(1 - Sqrt[5])*Log[2 + x^(1/6) - Sqrt[5]
*x^(1/6) + 2*x^(1/3)])/10 - (3*(1 + Sqrt[5])*Log[2 + x^(1/6) + Sq
```

```
rt[5]*x^(1/6) + 2*x^(1/3)])/10
```

Rubi in Sympy [A] time = 122.632, size = 267, normalized size = 1.33

$$6\sqrt[6]{x}+x+\frac{6\log\left(-\sqrt[6]{x}+1\right)}{5}-\left(\frac{3}{10}+\frac{3\sqrt{5}}{10}\right)\log\left(\sqrt[6]{x}\left(\frac{1}{2}+\frac{\sqrt{5}}{2}\right)+\sqrt[3]{x}+1\right)-\left(-\frac{3\sqrt{5}}{10}+\frac{3}{10}\right)\log\left(\sqrt[6]{x}\left(-\frac{\sqrt{5}}{2}+\frac{1}{2}\right)+\sqrt[3]{x}+1\right)-\frac{12\left(-\left(\frac{1}{4}+\frac{\sqrt{5}}{4}\right)^2+1\right)\operatorname{atan}\left(\frac{\sqrt[6]{x}+\frac{1}{4}+\frac{\sqrt{5}}{4}}{\sqrt{-\frac{\sqrt{5}}{4}+\frac{3}{4}}\sqrt{\frac{\sqrt{5}}{4}+\frac{5}{4}}}\right)}{5\sqrt{-\frac{\sqrt{5}}{4}+\frac{3}{4}}\sqrt{\frac{\sqrt{5}}{4}+\frac{5}{4}}}-\frac{12\left(-\left(-\frac{\sqrt{5}}{4}+\frac{1}{4}\right)^2+1\right)\operatorname{atan}\left(\frac{\sqrt[6]{x}-\frac{\sqrt{5}}{4}+\frac{1}{4}}{\sqrt{-\frac{\sqrt{5}}{4}+\frac{5}{4}}\sqrt{\frac{\sqrt{5}}{4}+\frac{3}{4}}}\right)}{5\sqrt{-\frac{\sqrt{5}}{4}+\frac{5}{4}}\sqrt{\frac{\sqrt{5}}{4}+\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(1/2)/(-1/x**(1/3)+x**(1/2)),x)
```

```
[Out] 6*x**(1/6) + x + 6*log(-x**(1/6) + 1)/5 - (3/10 + 3*sqrt(5)/10)*l
og(x**(1/6)*(1/2 + sqrt(5)/2) + x**(1/3) + 1) - (-3*sqrt(5)/10 +
3/10)*log(x**(1/6)*(-sqrt(5)/2 + 1/2) + x**(1/3) + 1) - 12*(-(1/4
 + sqrt(5)/4)**2 + 1)*atan((x**(1/6) + 1/4 + sqrt(5)/4)/(sqrt(-sq
rt(5)/4 + 3/4)*sqrt(sqrt(5)/4 + 5/4)))/(5*sqrt(-sqrt(5)/4 + 3/4)*
sqrt(sqrt(5)/4 + 5/4)) - 12*(-(-sqrt(5)/4 + 1/4)**2 + 1)*atan((x*
*(1/6) - sqrt(5)/4 + 1/4)/(sqrt(-sqrt(5)/4 + 5/4)*sqrt(sqrt(5)/4
+ 3/4)))/(5*sqrt(-sqrt(5)/4 + 5/4)*sqrt(sqrt(5)/4 + 3/4))
```

Mathematica [A] time = 0.241995, size = 183, normalized size = 0.91

$$\frac{1}{10}\left(10x+60\sqrt[6]{x}+12\log\left(1-\sqrt[6]{x}\right)+3\left(\sqrt{5}-1\right)\log\left(\sqrt[3]{x}-\frac{1}{2}\left(\sqrt{5}-1\right)\sqrt[6]{x}+1\right)-3\left(1+\sqrt{5}\right)\log\left(\sqrt[3]{x}+\frac{1}{2}\left(1+\sqrt{5}\right)\sqrt[6]{x}+1\right)-6\sqrt{2\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{4\sqrt[6]{x}-\sqrt{5}+1}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)-6\sqrt{10-2\sqrt{5}}\tan^{-1}\left(\frac{4\sqrt[6]{x}+\sqrt{5}+1}{\sqrt{10-2\sqrt{5}}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[x]/(-x^(-1/3) + Sqrt[x]),x]
```

```
[Out] (60*x^(1/6) + 10*x - 6*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(1 - Sqrt[5]
+ 4*x^(1/6))/Sqrt[2*(5 + Sqrt[5])]] - 6*Sqrt[10 - 2*Sqrt[5]]*ArcT
an[(1 + Sqrt[5] + 4*x^(1/6))/Sqrt[10 - 2*Sqrt[5]]] + 12*Log[1 - x
^(1/6)] + 3*(-1 + Sqrt[5])*Log[1 - ((-1 + Sqrt[5])*x^(1/6))/2 + x
^(1/3)] - 3*(1 + Sqrt[5])*Log[1 + ((1 + Sqrt[5])*x^(1/6))/2 + x^(
1/3)])/10
```

Maple [A] time = 0.021, size = 242, normalized size = 1.2

$$x+6\sqrt[6]{x}+\frac{6}{5}\ln\left(\sqrt[6]{x}-1\right)-\frac{3\sqrt{5}}{10}\ln\left(2+\sqrt[6]{x}+2\sqrt[3]{x}+\sqrt[6]{x}\sqrt{5}\right)-\frac{3}{10}\ln\left(2+\sqrt[6]{x}+2\sqrt[3]{x}+\sqrt[6]{x}\sqrt{5}\right)$$
$$-6\frac{1}{\sqrt{10-2\sqrt{5}}}\arctan\left(\frac{1+4\sqrt[6]{x}+\sqrt{5}}{\sqrt{10-2\sqrt{5}}}\right)+\frac{6\sqrt{5}}{5\sqrt{10-2\sqrt{5}}}\arctan\left(\frac{1}{\sqrt{10-2\sqrt{5}}}\left(1+4\sqrt[6]{x}+\sqrt{5}\right)\right)$$
$$+\frac{3\sqrt{5}}{10}\ln\left(2+\sqrt[6]{x}+2\sqrt[3]{x}-\sqrt[6]{x}\sqrt{5}\right)-\frac{3}{10}\ln\left(2+\sqrt[6]{x}+2\sqrt[3]{x}-\sqrt[6]{x}\sqrt{5}\right)$$
$$-6\frac{1}{\sqrt{10+2\sqrt{5}}}\arctan\left(\frac{1+4\sqrt[6]{x}-\sqrt{5}}{\sqrt{10+2\sqrt{5}}}\right)-\frac{6\sqrt{5}}{5\sqrt{10+2\sqrt{5}}}\arctan\left(\frac{1}{\sqrt{10+2\sqrt{5}}}\left(1+4\sqrt[6]{x}-\sqrt{5}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(1/2)/(-1/x^(1/3)+x^(1/2)),x)
```

```
[Out] x+6*x^(1/6)+6/5*ln(x^(1/6)-1)-3/10*ln(2+x^(1/6)+2*x^(1/3)+x^(1/6)
*5^(1/2))*5^(1/2)-3/10*ln(2+x^(1/6)+2*x^(1/3)+x^(1/6)*5^(1/2))-6/
(10-2*5^(1/2))^(1/2)*arctan((1+4*x^(1/6)+5^(1/2))/(10-2*5^(1/2))^
(1/2))+6/5/(10-2*5^(1/2))^(1/2)*arctan((1+4*x^(1/6)+5^(1/2))/(10-
2*5^(1/2))^(1/2))*5^(1/2)+3/10*ln(2+x^(1/6)+2*x^(1/3)-x^(1/6)*5^(
1/2))*5^(1/2)-3/10*ln(2+x^(1/6)+2*x^(1/3)-x^(1/6)*5^(1/2))-6/(10+
2*5^(1/2))^(1/2)*arctan((1+4*x^(1/6)-5^(1/2))/(10+2*5^(1/2))^(1/2
))-6/5/(10+2*5^(1/2))^(1/2)*arctan((1+4*x^(1/6)-5^(1/2))/(10+2*5^
(1/2))^(1/2))*5^(1/2)
```

Maxima [A] time = 0.831169, size = 396, normalized size = 1.97

$$-\frac{3\,\sqrt{5}\,(-1)^{\frac{1}{5}}\left(\sqrt{5}-1\right)\log\left(\frac{\sqrt{5}(-1)^{\frac{1}{5}}+(-1)^{\frac{1}{5}}\sqrt{2\,\sqrt{5}-10}+(-1)^{\frac{1}{5}}-4\,x^{\frac{1}{6}}}{\sqrt{5}(-1)^{\frac{1}{5}}-(-1)^{\frac{1}{5}}\sqrt{2\,\sqrt{5}-10}+(-1)^{\frac{1}{5}}-4\,x^{\frac{1}{6}}}\right)}{5\,\sqrt{2\,\sqrt{5}-10}} - \frac{3\,\sqrt{5}\,(-1)^{\frac{1}{5}}\left(\sqrt{5}+1\right)\log\left(\frac{\sqrt{5}(-1)^{\frac{1}{5}}-(-1)^{\frac{1}{5}}\sqrt{-2\,\sqrt{5}-10}-(-1)^{\frac{1}{5}}+4\,x^{\frac{1}{6}}}{\sqrt{5}(-1)^{\frac{1}{5}}+(-1)^{\frac{1}{5}}\sqrt{-2\,\sqrt{5}-10}-(-1)^{\frac{1}{5}}+4\,x^{\frac{1}{6}}}\right)}{5\,\sqrt{-2\,\sqrt{5}-10}} - \frac{6}{5}\,(-1)^{\frac{1}{5}}\log\left((-1)^{\frac{1}{5}}+x^{\frac{1}{6}}\right) + x - \frac{3\left(\sqrt{5}+3\right)\log\left(-x^{\frac{1}{6}}\left(\sqrt{5}\,(-1)^{\frac{1}{5}}+(-1)^{\frac{1}{5}}\right)+2\,(-1)^{\frac{2}{5}}+2\,x^{\frac{1}{3}}\right)}{5\left(\sqrt{5}\,(-1)^{\frac{4}{5}}+(-1)^{\frac{4}{5}}\right)} - \frac{3\left(\sqrt{5}-3\right)\log\left(x^{\frac{1}{6}}\left(\sqrt{5}\,(-1)^{\frac{1}{5}}-(-1)^{\frac{1}{5}}\right)+2\,(-1)^{\frac{2}{5}}+2\,x^{\frac{1}{3}}\right)}{5\left(\sqrt{5}\,(-1)^{\frac{4}{5}}-(-1)^{\frac{4}{5}}\right)} + 6\,x^{\frac{1}{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/(sqrt(x) - 1/x^(1/3)),x, algorithm="maxima")
```

```
[Out] -3/5*sqrt(5)*(-1)^(1/5)*(sqrt(5) - 1)*log((sqrt(5)*(-1)^(1/5) + (
-1)^(1/5)*sqrt(2*sqrt(5) - 10) + (-1)^(1/5) - 4*x^(1/6))/(sqrt(5)
*(-1)^(1/5) - (-1)^(1/5)*sqrt(2*sqrt(5) - 10) + (-1)^(1/5) - 4*x^
(1/6)))/sqrt(2*sqrt(5) - 10) - 3/5*sqrt(5)*(-1)^(1/5)*(sqrt(5) +
1)*log((sqrt(5)*(-1)^(1/5) - (-1)^(1/5)*sqrt(-2*sqrt(5) - 10) - (
-1)^(1/5) + 4*x^(1/6))/(sqrt(5)*(-1)^(1/5) + (-1)^(1/5)*sqrt(-2*s
qrt(5) - 10) - (-1)^(1/5) + 4*x^(1/6)))/sqrt(-2*sqrt(5) - 10) - 6
/5*(-1)^(1/5)*log((-1)^(1/5) + x^(1/6)) + x - 3/5*(sqrt(5) + 3)*l
og(-x^(1/6)*(sqrt(5)*(-1)^(1/5) + (-1)^(1/5)) + 2*(-1)^(2/5) + 2*
x^(1/3))/(sqrt(5)*(-1)^(4/5) + (-1)^(4/5)) - 3/5*(sqrt(5) - 3)*lo
g(x^(1/6)*(sqrt(5)*(-1)^(1/5) - (-1)^(1/5)) + 2*(-1)^(2/5) + 2*x^
(1/3))/(sqrt(5)*(-1)^(4/5) - (-1)^(4/5)) + 6*x^(1/6)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/(sqrt(x) - 1/x^(1/3)),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{5}{6}}}{\left(\sqrt[6]{x}-1\right)\left(\sqrt[6]{x}+x^{\frac{2}{3}}+\sqrt[3]{x}+\sqrt{x}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(1/2)/(-1/x**(1/3)+x**(1/2)),x)
```

```
[Out] Integral(x**(5/6)/((x**(1/6) - 1)*(x**(1/6) + x**(2/3) + x**(1/3)
 + sqrt(x) + 1)), x)
```

GIAC/XCAS [A] time = 0.386902, size = 189, normalized size = 0.94

$$-\frac{3}{5}\sqrt{2\sqrt{5}+10}\arctan\left(-\frac{\sqrt{5}-4x^{\frac{1}{6}}-1}{\sqrt{2\sqrt{5}+10}}\right)-\frac{3}{5}\sqrt{-2\sqrt{5}+10}\arctan\left(\frac{\sqrt{5}+4x^{\frac{1}{6}}+1}{\sqrt{-2\sqrt{5}+10}}\right)-\frac{3}{10}\sqrt{5}\ln\left(\frac{1}{2}x^{\frac{1}{6}}\left(\sqrt{5}+1\right)+x^{\frac{1}{3}}+1\right)+\frac{3}{10}\sqrt{5}\ln\left(-\frac{1}{2}x^{\frac{1}{6}}\left(\sqrt{5}-1\right)+x^{\frac{1}{3}}+1\right)+x+6x^{\frac{1}{6}}-\frac{3}{10}\ln\left(x^{\frac{2}{3}}+\sqrt{x}+x^{\frac{1}{3}}+x^{\frac{1}{6}}+1\right)+\frac{6}{5}\ln\left(\left|x^{\frac{1}{6}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/(sqrt(x) - 1/x^(1/3)),x, algorithm="giac")
```

```
[Out] -3/5*sqrt(2*sqrt(5) + 10)*arctan(-(sqrt(5) - 4*x^(1/6) - 1)/sqrt(
2*sqrt(5) + 10)) - 3/5*sqrt(-2*sqrt(5) + 10)*arctan((sqrt(5) + 4*
x^(1/6) + 1)/sqrt(-2*sqrt(5) + 10)) - 3/10*sqrt(5)*ln(1/2*x^(1/6)
*(sqrt(5) + 1) + x^(1/3) + 1) + 3/10*sqrt(5)*ln(-1/2*x^(1/6)*(sqr
t(5) - 1) + x^(1/3) + 1) + x + 6*x^(1/6) - 3/10*ln(x^(2/3) + sqrt
(x) + x^(1/3) + x^(1/6) + 1) + 6/5*ln(abs(x^(1/6) - 1))
```

3.420 $\int \frac{\sqrt{b-\frac{a}{x}}x^m}{\sqrt{a-bx}}\,dx$

Optimal. Leaf size=36

$$\frac{2x^{m+1}\sqrt{b-\frac{a}{x}}}{(2m+1)\sqrt{a-bx}}$$

```
[Out] (2*Sqrt[b - a/x]*x^(1 + m))/((1 + 2*m)*Sqrt[a - b*x])
```

Rubi [A] time = 0.130518, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{2x^{m+1}\sqrt{b-\frac{a}{x}}}{(2m+1)\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[b - a/x]*x^m)/Sqrt[a - b*x],x]
```

```
[Out] (2*Sqrt[b - a/x]*x^(1 + m))/((1 + 2*m)*Sqrt[a - b*x])
```

Rubi in Sympy [A] time = 5.74633, size = 36, normalized size = 1.

$$-\frac{2x^{m+\frac{1}{2}}\sqrt{a-bx}}{\sqrt{x}\,(2m+1)\,\sqrt{-\frac{a}{x}+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**m*(b-a/x)**(1/2)/(-b*x+a)**(1/2),x)
```

```
[Out] -2*x**(m + 1/2)*sqrt(a - b*x)/(sqrt(x)*(2*m + 1)*sqrt(-a/x + b))
```

Mathematica [A] time = 0.133393, size = 34, normalized size = 0.94

$$-\frac{2x^m\sqrt{a-bx}}{(2m+1)\sqrt{b-\frac{a}{x}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[b - a/x]*x^m)/Sqrt[a - b*x],x]
```

```
[Out] (-2*x^m*Sqrt[a - b*x])/((1 + 2*m)*Sqrt[b - a/x])
```

Maple [A] time = 0.005, size = 36, normalized size = 1.

$$2\,\frac{x^{1+m}}{(1+2\,m)\,\sqrt{-bx+a}}\sqrt{-\frac{-bx+a}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^m*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x)
```

```
[Out] 2*x^(1+m)/(1+2*m)*(-(-b*x+a)/x)^(1/2)/(-b*x+a)^(1/2)
```

Maxima [A] time = 0.740365, size = 20, normalized size = 0.56

$$\frac{2\,\sqrt{x}x^{m}}{2i\,m+i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b - a/x)*x^m/sqrt(-b*x + a),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x)*x^m/(2*I*m + I)
```

Fricas [A] time = 0.288123, size = 59, normalized size = 1.64

$$\frac{2\,\sqrt{-bx+a}xx^{m}\sqrt{\frac{bx-a}{x}}}{2\,am-(2\,bm+b)x+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b - a/x)*x^m/sqrt(-b*x + a),x, algorithm="fricas")
```

```
[Out] 2*sqrt(-b*x + a)*x*x^m*sqrt((b*x - a)/x)/(2*a*m - (2*b*m + b)*x +
 a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m\sqrt{-\frac{a}{x}+b}}{\sqrt{a-bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**m*(b-a/x)**(1/2)/(-b*x+a)**(1/2),x)
```

```
[Out] Integral(x**m*sqrt(-a/x + b)/sqrt(a - b*x), x)
```

GIAC/XCAS [A] time = 0.29791, size = 212, normalized size = 5.89

$$\frac{2\sqrt{-ab}a|b|e^{\left(m\ln\left(\frac{a}{b}\right)-\ln\left(\frac{a}{b}\right)\right)}\operatorname{sign}(x)}{2b^3m+b^3}-\frac{2\left(\frac{\sqrt{-ab}ae^{\left(m\ln\left(\frac{a}{b}\right)-\ln\left(\frac{a}{b}\right)\right)}}{2m+1}+\frac{(-(bx-a)b-ab)^{\frac{3}{2}}e^{\left(m\ln\left(\frac{(bx-a)b+ab}{b^2}\right)-\ln\left(\frac{(bx-a)b+ab}{b^2}\right)\right)}}{b(2m+1)}\right)|b|\operatorname{sign}(x)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b - a/x)*x^m/sqrt(-b*x + a),x, algorithm="giac")
```

```
[Out] 2*sqrt(-a*b)*a*abs(b)*e^(m*ln(a/b) - ln(a/b))*sign(x)/(2*b^3*m +
b^3) - 2*(sqrt(-a*b)*a*e^(m*ln(a/b) - ln(a/b))/(2*m + 1) + (-(b*x
 - a)*b - a*b)^(3/2)*e^(m*ln(((b*x - a)*b + a*b)/b^2) - ln(((b*x
- a)*b + a*b)/b^2))/(b*(2*m + 1)))*abs(b)*sign(x)/b^3
```

3.421 $\int \frac{\sqrt{b-\frac{a}{x}}x^2}{\sqrt{a-bx}} dx$

Optimal. Leaf size=29

$$\frac{2x^3\sqrt{b-\frac{a}{x}}}{5\sqrt{a-bx}}$$

```
[Out] (2*Sqrt[b - a/x]*x^3)/(5*Sqrt[a - b*x])
```

Rubi [A] time = 0.131094, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{2x^3\sqrt{b-\frac{a}{x}}}{5\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b - a/x]*x^2)/Sqrt[a - b*x],x]
```

```
[Out] (2*Sqrt[b - a/x]*x^3)/(5*Sqrt[a - b*x])
```

Rubi in Sympy [A] time = 5.26386, size = 24, normalized size = 0.83

$$-\frac{2x^2\sqrt{a-bx}}{5\sqrt{-\frac{a}{x}+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**2*(b-a/x)**(1/2)/(-b*x+a)**(1/2),x)
```

```
[Out] -2*x**2*sqrt(a - b*x)/(5*sqrt(-a/x + b))
```

Mathematica [A] time = 0.0341236, size = 29, normalized size = 1.

$$\frac{2x^3\sqrt{b-\frac{a}{x}}}{5\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[b - a/x]*x^2)/Sqrt[a - b*x],x]
```

```
[Out] (2*Sqrt[b - a/x]*x^3)/(5*Sqrt[a - b*x])
```

Maple [A] time = 0.003, size = 27, normalized size = 0.9

$$\frac{2x^3}{5}\sqrt{-\frac{-bx+a}{x}}\frac{1}{\sqrt{-bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x)
```

```
[Out] 2/5*x^3*(-(-b*x+a)/x)^(1/2)/(-b*x+a)^(1/2)
```

Maxima [A] time = 0.782081, size = 7, normalized size = 0.24

$$-\frac{2}{5}i\,x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b - a/x)*x^2/sqrt(-b*x + a),x, algorithm="maxima")
```

```
[Out] -2/5*I*x^(5/2)
```

Fricas [A] time = 0.271508, size = 47, normalized size = 1.62

$$\frac{2\left(bx^3-ax^2\right)}{5\sqrt{-bx+a}\sqrt{\frac{bx-a}{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b - a/x)*x^2/sqrt(-b*x + a),x, algorithm="fricas")
```

```
[Out] 2/5*(b*x^3 - a*x^2)/(sqrt(-b*x + a)*sqrt((b*x - a)/x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{-\frac{a}{x}+b}}{\sqrt{a-bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(b-a/x)**(1/2)/(-b*x+a)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-a/x + b)/sqrt(a - b*x), x)
```

GIAC/XCAS [A] time = 0.285674, size = 170, normalized size = 5.86

$$\frac{2\sqrt{-ab}a^2|b|\operatorname{sign}(x)}{5\,b^4}-\frac{2\left(3\sqrt{-ab}a^2+\frac{5\,(-(bx-a)b-ab)^{\frac{3}{2}}a-\frac{5\,(-(bx-a)b-ab)^{\frac{3}{2}}ab+3\,((bx-a)b+ab)^2\sqrt{-(bx-a)b-ab}}{b}}{b}\right)|b|\operatorname{sign}(x)}{15\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b - a/x)*x^2/sqrt(-b*x + a),x, algorithm="giac")
```

```
[Out] 2/5*sqrt(-a*b)*a^2*abs(b)*sign(x)/b^4 - 2/15*(3*sqrt(-a*b)*a^2 +
(5*(-(b*x - a)*b - a*b)^(3/2)*a - (5*(-(b*x - a)*b - a*b)^(3/2)*a
*b + 3*((b*x - a)*b + a*b)^2*sqrt(-(b*x - a)*b - a*b))/b)/b)*abs(
b)*sign(x)/b^4
```

3.422 $\int \frac{\sqrt{b-\frac{a}{x}}\,x}{\sqrt{a-bx}}\,dx$

Optimal. Leaf size=29

$$\frac{2x^2\sqrt{b-\frac{a}{x}}}{3\sqrt{a-bx}}$$

```
[Out] (2*Sqrt[b - a/x]*x^2)/(3*Sqrt[a - b*x])
```

Rubi [A] time = 0.0919785, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{2x^2\sqrt{b-\frac{a}{x}}}{3\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[b - a/x]*x)/Sqrt[a - b*x],x]

[Out] (2*Sqrt[b - a/x]*x^2)/(3*Sqrt[a - b*x])
```

Rubi in Sympy [A] time = 4.68248, size = 22, normalized size = 0.76

$$-\frac{2x\sqrt{a-bx}}{3\sqrt{-\frac{a}{x}+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(b-a/x)**(1/2)/(-b*x+a)**(1/2),x)

[Out] -2*x*sqrt(a - b*x)/(3*sqrt(-a/x + b))
```

Mathematica [A] time = 0.0291623, size = 29, normalized size = 1.

$$\frac{2x^2\sqrt{b-\frac{a}{x}}}{3\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[b - a/x]*x)/Sqrt[a - b*x],x]
```

```
[Out] (2*Sqrt[b - a/x]*x^2)/(3*Sqrt[a - b*x])
```

Maple [A] time = 0.003, size = 27, normalized size = 0.9

$$\frac{2x^2}{3}\sqrt{-\frac{-bx+a}{x}}\,\frac{1}{\sqrt{-bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x)
```

```
[Out] 2/3*x^2*(-(-b*x+a)/x)^(1/2)/(-b*x+a)^(1/2)
```

Maxima [A] time = 0.759298, size = 7, normalized size = 0.24

$$-\frac{2}{3}i\,x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b - a/x)*x/sqrt(-b*x + a),x, algorithm="maxima")
```

```
[Out] -2/3*I*x^(3/2)
```

Fricas [A] time = 0.267406, size = 45, normalized size = 1.55

$$\frac{2\left(bx^2-ax\right)}{3\sqrt{-bx+a}\sqrt{\frac{bx-a}{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b - a/x)*x/sqrt(-b*x + a),x, algorithm="fricas")
```

```
[Out] 2/3*(b*x^2 - a*x)/(sqrt(-b*x + a)*sqrt((b*x - a)/x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{-\frac{a}{x}+b}}{\sqrt{a-bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(b-a/x)**(1/2)/(-b*x+a)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-a/x + b)/sqrt(a - b*x), x)
```

GIAC/XCAS [A] time = 0.282147, size = 76, normalized size = 2.62

$$\frac{2\sqrt{-ab}a|b|\operatorname{sign}(x)}{3\,b^3}-\frac{2\left(\sqrt{-ab}a+\frac{(-(bx-a)b-ab)^{\frac{3}{2}}}{b}\right)|b|\operatorname{sign}(x)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b - a/x)*x/sqrt(-b*x + a),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(-a*b)*a*abs(b)*sign(x)/b^3 - 2/3*(sqrt(-a*b)*a + (-(b*x
- a)*b - a*b)^(3/2)/b)*abs(b)*sign(x)/b^3
```

3.423 $\int \frac{\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}} dx$

Optimal. Leaf size=25

$$\frac{2x\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}$$

```
[Out] (2*Sqrt[b - a/x]*x)/Sqrt[a - b*x]
```

Rubi [A] time = 0.042153, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13

$$\frac{2x\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[b - a/x]/Sqrt[a - b*x],x]
```

```
[Out] (2*Sqrt[b - a/x]*x)/Sqrt[a - b*x]
```

Rubi in Sympy [A] time = 4.35024, size = 19, normalized size = 0.76

$$-\frac{2\sqrt{a-bx}}{\sqrt{-\frac{a}{x}+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b-a/x)**(1/2)/(-b*x+a)**(1/2),x)
```

```
[Out] -2*sqrt(a - b*x)/sqrt(-a/x + b)
```

Mathematica [A] time = 0.0265951, size = 25, normalized size = 1.

$$\frac{2x\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[b - a/x]/Sqrt[a - b*x],x]
```

```
[Out] (2*Sqrt[b - a/x]*x)/Sqrt[a - b*x]
```

Maple [A] time = 0.003, size = 25, normalized size = 1.

$$2\,{\frac{x}{\sqrt{-bx+a}}\sqrt{-{\frac{-bx+a}{x}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b-a/x)^(1/2)/(-b*x+a)^(1/2),x)
```

```
[Out] 2*x*(-(-b*x+a)/x)^(1/2)/(-b*x+a)^(1/2)
```

Maxima [A] time = 0.783961, size = 7, normalized size = 0.28

$$-2i\,\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b - a/x)/sqrt(-b*x + a),x, algorithm="maxima")
```

```
[Out] -2*I*sqrt(x)
```

Fricas [A] time = 0.260108, size = 41, normalized size = 1.64

$$\frac{2\,(bx-a)}{\sqrt{-bx+a}\sqrt{\frac{bx-a}{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b - a/x)/sqrt(-b*x + a),x, algorithm="fricas")
```

```
[Out] 2*(b*x - a)/(sqrt(-b*x + a)*sqrt((b*x - a)/x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-\frac{a}{x}+b}}{\sqrt{a-bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b-a/x)**(1/2)/(-b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(-a/x + b)/sqrt(a - b*x), x)
```

GIAC/XCAS [A] time = 0.282074, size = 69, normalized size = 2.76

$$\frac{2\left(\sqrt{-(bx-a)b-ab}-\sqrt{-ab}\right)|b|\operatorname{sign}(x)}{b^2}+\frac{2\sqrt{-ab}|b|\operatorname{sign}(x)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b - a/x)/sqrt(-b*x + a),x, algorithm="giac")
```

```
[Out] 2*(sqrt(-(b*x - a)*b - a*b) - sqrt(-a*b))*abs(b)*sign(x)/b^2 + 2*
sqrt(-a*b)*abs(b)*sign(x)/b^2
```

3.424 $\int \frac{\sqrt{b-\frac{a}{x}}}{x\sqrt{a-bx}}\,dx$

Optimal. Leaf size=24

$$-\frac{2\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}$$

```
[Out] (-2*Sqrt[b - a/x])/Sqrt[a - b*x]
```

Rubi [A] time = 0.129611, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$-\frac{2\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[b - a/x]/(x*Sqrt[a - b*x]),x]

[Out] (-2*Sqrt[b - a/x])/Sqrt[a - b*x]
```

Rubi in Sympy [A] time = 5.55558, size = 19, normalized size = 0.79

$$\frac{2\sqrt{a-bx}}{x\sqrt{-\frac{a}{x}+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b-a/x)**(1/2)/x/(-b*x+a)**(1/2),x)

[Out] 2*sqrt(a - b*x)/(x*sqrt(-a/x + b))
```

Mathematica [A] time = 0.0265813, size = 24, normalized size = 1.

$$-\frac{2\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[b - a/x]/(x*Sqrt[a - b*x]),x]
```

```
[Out] (-2*Sqrt[b - a/x])/Sqrt[a - b*x]
```

Maple [A] time = 0.003, size = 24, normalized size = 1.

$$-2\,\frac{1}{\sqrt{-bx+a}}\sqrt{-\frac{-bx+a}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b-a/x)^(1/2)/x/(-b*x+a)^(1/2),x)
```

```
[Out] -2*(-(-b*x+a)/x)^(1/2)/(-b*x+a)^(1/2)
```

Maxima [A] time = 0.757043, size = 7, normalized size = 0.29

$$\frac{2i}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b - a/x)/(sqrt(-b*x + a)*x),x, algorithm="maxima")
```

```
[Out] 2*I/sqrt(x)
```

Fricas [A] time = 0.262101, size = 43, normalized size = 1.79

$$\frac{2\,\sqrt{-bx+a}\sqrt{\frac{bx-a}{x}}}{bx-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b - a/x)/(sqrt(-b*x + a)*x),x, algorithm="fricas")
```

```
[Out] 2*sqrt(-b*x + a)*sqrt((b*x - a)/x)/(b*x - a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-\frac{a}{x}+b}}{x\sqrt{a-bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b-a/x)**(1/2)/x/(-b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(-a/x + b)/(x*sqrt(a - b*x)), x)
```

GIAC/XCAS [A] time = 0.287364, size = 57, normalized size = 2.38

$$\frac{2\left(\frac{b^3}{\sqrt{-(bx-a)b-ab}}-\frac{b^3}{\sqrt{-ab}}\right)|b|\mathrm{sign}\left(x\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b - a/x)/(sqrt(-b*x + a)*x),x, algorithm="giac")
```

```
[Out] 2*(b^3/sqrt(-(b*x - a)*b - a*b) - b^3/sqrt(-a*b))*abs(b)*sign(x)/
b^3
```

3.425 $\int \frac{\sqrt{b-\frac{a}{x}}}{x^2\sqrt{a-bx}}\,dx$

Optimal. Leaf size=29

$$-\frac{2\sqrt{b-\frac{a}{x}}}{3x\sqrt{a-bx}}$$

```
[Out] (-2*Sqrt[b - a/x])/(3*x*Sqrt[a - b*x])
```

Rubi [A] time = 0.123482, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$-\frac{2\sqrt{b-\frac{a}{x}}}{3x\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[b - a/x]/(x^2*Sqrt[a - b*x]),x]

[Out] (-2*Sqrt[b - a/x])/(3*x*Sqrt[a - b*x])
```

Rubi in Sympy [A] time = 5.40434, size = 22, normalized size = 0.76

$$\frac{2\sqrt{a-bx}}{3x^2\sqrt{-\frac{a}{x}+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b-a/x)**(1/2)/x**2/(-b*x+a)**(1/2),x)

[Out] 2*sqrt(a - b*x)/(3*x**2*sqrt(-a/x + b))
```

Mathematica [A] time = 0.0339624, size = 29, normalized size = 1.

$$-\frac{2\sqrt{b-\frac{a}{x}}}{3x\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[b - a/x]/(x^2*Sqrt[a - b*x]),x]

[Out] (-2*Sqrt[b - a/x])/(3*x*Sqrt[a - b*x])
```

Maple [A] time = 0.003, size = 27, normalized size = 0.9

$$-\frac{2}{3\,x}\sqrt{-\frac{-bx+a}{x}}\frac{1}{\sqrt{-bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b-a/x)^(1/2)/x^2/(-b*x+a)^(1/2),x)

[Out] -2/3*(-(-b*x+a)/x)^(1/2)/x/(-b*x+a)^(1/2)
```

Maxima [A] time = 0.802035, size = 7, normalized size = 0.24

$$\frac{2i}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b - a/x)/(sqrt(-b*x + a)*x^2),x, algorithm="maxima")

[Out] 2/3*I/x^(3/2)
```

Fricas [A] time = 0.265394, size = 47, normalized size = 1.62

$$\frac{2\,\sqrt{-bx+a}\sqrt{\frac{bx-a}{x}}}{3\,(bx^2-ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b - a/x)/(sqrt(-b*x + a)*x^2),x, algorithm="fricas")

[Out] 2/3*sqrt(-b*x + a)*sqrt((b*x - a)/x)/(b*x^2 - a*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-\frac{a}{x}+b}}{x^2\sqrt{a-bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b-a/x)**(1/2)/x**2/(-b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(-a/x + b)/(x**2*sqrt(a - b*x)), x)
```

GIAC/XCAS [A] time = 0.282479, size = 81, normalized size = 2.79

$$\frac{2\left(\frac{b^5}{((bx-a)b+ab)\sqrt{-(bx-a)b-ab}}-\frac{b^4}{\sqrt{-ab}a}\right)|b|\operatorname{sign}(x)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b - a/x)/(sqrt(-b*x + a)*x^2),x, algorithm="giac")
```

```
[Out] 2/3*(b^5/(((b*x - a)*b + a*b)*sqrt(-(b*x - a)*b - a*b)) - b^4/(sq
rt(-a*b)*a))*abs(b)*sign(x)/b^3
```

3.426 $\int \left(a+\frac{b}{x}\right)^m (c+dx)^n\,dx$

Optimal. Leaf size=80

$$\frac{x\left(a+\frac{b}{x}\right)^m\left(\frac{ax}{b}+1\right)^{-m}(c+dx)^n\left(\frac{dx}{c}+1\right)^{-n}F_1\left(1-m;-m,-n;2-m;-\frac{ax}{b},-\frac{dx}{c}\right)}{1-m}$$

```
[Out] ((a + b/x)^m*x*(c + d*x)^n*AppellF1[1 - m, -m, -n, 2 - m, -((a*x)
/b), -((d*x)/c)])/((1 - m)*(1 + (a*x)/b)^m*(1 + (d*x)/c)^n)
```

Rubi [A] time = 0.148823, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$

$$\frac{x\left(a+\frac{b}{x}\right)^m\left(\frac{ax}{b}+1\right)^{-m}(c+dx)^n\left(\frac{dx}{c}+1\right)^{-n}F_1\left(1-m;-m,-n;2-m;-\frac{ax}{b},-\frac{dx}{c}\right)}{1-m}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b/x)^m*(c + d*x)^n,x]
```

```
[Out] ((a + b/x)^m*x*(c + d*x)^n*AppellF1[1 - m, -m, -n, 2 - m, -((a*x)
/b), -((d*x)/c)])/((1 - m)*(1 + (a*x)/b)^m*(1 + (d*x)/c)^n)
```

Rubi in Sympy [A] time = 11.3662, size = 61, normalized size = 0.76

$$\frac{x^m x^{-m+1}\left(1+\frac{dx}{c}\right)^{-n}\left(a+\frac{b}{x}\right)^m(c+dx)^n\left(\frac{ax}{b}+1\right)^{-m}\operatorname{appellf}_1\left(-m+1,-m,-n,-m+2,-\frac{ax}{b},-\frac{dx}{c}\right)}{-m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b/x)**m*(d*x+c)**n,x)
```

```
[Out] x**m*x**(-m + 1)*(1 + d*x/c)**(-n)*(a + b/x)**m*(c + d*x)**n*(a*x
/b + 1)**(-m)*appellf1(-m + 1, -m, -n, -m + 2, -a*x/b, -d*x/c)/(-
m + 1)
```

Mathematica [A] time = 0.0849088, size = 0, normalized size = 0.

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)^n\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(a + b/x)^m*(c + d*x)^n,x]

[Out] Integrate[(a + b/x)^m*(c + d*x)^n, x]
```

Maple [F] time = 0.132, size = 0, normalized size = 0.

$$\int \left(a+\frac{b}{x}\right)^m (dx+c)^n\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b/x)^m*(d*x+c)^n,x)

[Out] int((a+b/x)^m*(d*x+c)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^n \left(a+\frac{b}{x}\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)^n*(a + b/x)^m,x, algorithm="maxima")

[Out] integrate((d*x + c)^n*(a + b/x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((dx+c)^n \left(\frac{ax+b}{x}\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)^n*(a + b/x)^m,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^n*((a*x + b)/x)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + \frac{b}{x}\right)^m (c + dx)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b/x)**m*(d*x+c)**n,x)
```

```
[Out] Integral((a + b/x)**m*(c + d*x)**n, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^n \left(a + \frac{b}{x}\right)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)^n*(a + b/x)^m,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^n*(a + b/x)^m, x)
```

3.427 $\int \left(a+\frac{b}{x}\right)^m (c+dx)^2\,dx$

Optimal. Leaf size=138

$$\frac{dx^2\left(a+\frac{b}{x}\right)^{m+1}(6ac-bd(2-m))}{6a^2} - \frac{b\left(a+\frac{b}{x}\right)^{m+1}\left(6a^2c^2-6abcd(1-m)+b^2d^2\left(m^2-3m+2\right)\right)\,{}_2F_1\left(2,m+1;m+2;\frac{b}{ax}+1\right)}{6a^4(m+1)} + \frac{d^2x^3\left(a+\frac{b}{x}\right)^{m+1}}{3a}$$

```
[Out] (d*(6*a*c - b*d*(2 - m))*(a + b/x)^(1 + m)*x^2)/(6*a^2) + (d^2*(a
 + b/x)^(1 + m)*x^3)/(3*a) - (b*(6*a^2*c^2 - 6*a*b*c*d*(1 - m) +
b^2*d^2*(2 - 3*m + m^2))*(a + b/x)^(1 + m)*Hypergeometric2F1[2, 1
 + m, 2 + m, 1 + b/(a*x)])/(6*a^4*(1 + m))
```

Rubi [A] time = 0.28323, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$

$$\frac{dx^2\left(a+\frac{b}{x}\right)^{m+1}(6ac-bd(2-m))}{6a^2} - \frac{b\left(a+\frac{b}{x}\right)^{m+1}\left(6a^2c^2-6abcd(1-m)+b^2d^2\left(m^2-3m+2\right)\right)\,{}_2F_1\left(2,m+1;m+2;\frac{b}{ax}+1\right)}{6a^4(m+1)} + \frac{d^2x^3\left(a+\frac{b}{x}\right)^{m+1}}{3a}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b/x)^m*(c + d*x)^2,x]
```

```
[Out] (d*(6*a*c - b*d*(2 - m))*(a + b/x)^(1 + m)*x^2)/(6*a^2) + (d^2*(a
 + b/x)^(1 + m)*x^3)/(3*a) - (b*(6*a^2*c^2 - 6*a*b*c*d*(1 - m) +
b^2*d^2*(2 - 3*m + m^2))*(a + b/x)^(1 + m)*Hypergeometric2F1[2, 1
 + m, 2 + m, 1 + b/(a*x)])/(6*a^4*(1 + m))
```

Rubi in Sympy [A] time = 12.7217, size = 105, normalized size = 0.76

$$\frac{d^2x^3\left(a+\frac{b}{x}\right)^{m+1}}{3a}+\frac{dx^2\left(a+\frac{b}{x}\right)^{m+1}(6ac-bd(-m+2))}{6a^2}-\frac{b\left(a+\frac{b}{x}\right)^{m+1}\left(6a^2c^2-bd(-m+1)(6ac-bd(-m+2))\right){}_2F_1\left(\begin{matrix}2,m+1\\m+2\end{matrix}\middle|1+\frac{b}{ax}\right)}{6a^4(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b/x)**m*(d*x+c)**2,x)
```

```
[Out] d**2*x**3*(a + b/x)**(m + 1)/(3*a) + d*x**2*(a + b/x)**(m + 1)*(6
*a*c - b*d*(-m + 2))/(6*a**2) - b*(a + b/x)**(m + 1)*(6*a**2*c**2
 - b*d*(-m + 1)*(6*a*c - b*d*(-m + 2)))*hyper((2, m + 1), (m + 2,
), 1 + b/(a*x))/(6*a**4*(m + 1))
```

Mathematica [A] time = 0.131228, size = 134, normalized size = 0.97

$$-\frac{x\left(a+\frac{b}{x}\right)^m\left(\frac{ax}{b}+1\right)^{-m}\left(c^2\left(m^2-5m+6\right){}_2F_1\left(1-m,-m;2-m;-\frac{ax}{b}\right)+d(m-1)x\left(2c(m-3){}_2F_1\left(2-m,-m;3-m;-\right.\right.\right.}{(m-3)(m-2)(m-1)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b/x)^m*(c + d*x)^2,x]
```

```
[Out] -(((a + b/x)^m*x*(c^2*(6 - 5*m + m^2)*Hypergeometric2F1[1 - m, -m
, 2 - m, -((a*x)/b)] + d*(-1 + m)*x*(2*c*(-3 + m)*Hypergeometric2
F1[2 - m, -m, 3 - m, -((a*x)/b)] + d*(-2 + m)*x*Hypergeometric2F1
[3 - m, -m, 4 - m, -((a*x)/b)])))/((-3 + m)*(-2 + m)*(-1 + m)*(1
+ (a*x)/b)^m))
```

Maple [F] time = 0.048, size = 0, normalized size = 0.

$$\int\left(a+\frac{b}{x}\right)^m(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b/x)^m*(d*x+c)^2,x)
```

```
[Out] int((a+b/x)^m*(d*x+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^2\left(a+\frac{b}{x}\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)^2*(a + b/x)^m,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^2*(a + b/x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(d^2x^2+2\,cdx+c^2\right)\left(\frac{ax+b}{x}\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)^2*(a + b/x)^m,x, algorithm="fricas")
```

```
[Out] integral((d^2*x^2 + 2*c*d*x + c^2)*((a*x + b)/x)^m, x)
```

Sympy [A] time = 16.921, size = 121, normalized size = 0.88

$$\frac{b^mc^2xx^{-m}(-m+1)\,{}_2F_1\left(\begin{matrix}-m,-m+1\\-m+2\end{matrix}\middle|\frac{axe^{i\pi}}{b}\right)}{(-m+2)}+\frac{2b^mcdx^2x^{-m}(-m+2)\,{}_2F_1\left(\begin{matrix}-m,-m+2\\-m+3\end{matrix}\middle|\frac{axe^{i\pi}}{b}\right)}{(-m+3)}+\frac{b^md^2x^3x^{-m}(-m+3)\,{}_2F_1\left(\begin{matrix}-m,-m+3\\-m+4\end{matrix}\middle|\frac{axe^{i\pi}}{b}\right)}{(-m+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b/x)**m*(d*x+c)**2,x)
```

```
[Out] b**m*c**2*x*x**(-m)*gamma(-m + 1)*hyper((-m, -m + 1), (-m + 2,),
a*x*exp_polar(I*pi)/b)/gamma(-m + 2) + 2*b**m*c*d*x**2*x**(-m)*ga
mma(-m + 2)*hyper((-m, -m + 2), (-m + 3,), a*x*exp_polar(I*pi)/b)
/gamma(-m + 3) + b**m*d**2*x**3*x**(-m)*gamma(-m + 3)*hyper((-m,
-m + 3), (-m + 4,), a*x*exp_polar(I*pi)/b)/gamma(-m + 4)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^2\left(a+\frac{b}{x}\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)^2*(a + b/x)^m,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*(a + b/x)^m, x)
```

3.428 $\int \left(a+\frac{b}{x}\right)^m (c+dx)\,dx$

Optimal. Leaf size=79

$$\frac{dx^2\left(a+\frac{b}{x}\right)^{m+1}}{2a}-\frac{b\left(a+\frac{b}{x}\right)^{m+1}(2ac-bd(1-m))\,{}_2F_1\left(2,m+1;m+2;\frac{b}{ax}+1\right)}{2a^3(m+1)}$$

```
[Out] (d*(a + b/x)^(1 + m)*x^2)/(2*a) - (b*(2*a*c - b*d*(1 - m))*(a + b
/x)^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, 1 + b/(a*x)])/(2*a
^3*(1 + m))
```

Rubi [A] time = 0.105425, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\frac{dx^2\left(a+\frac{b}{x}\right)^{m+1}}{2a}-\frac{b\left(a+\frac{b}{x}\right)^{m+1}(2ac-bd(1-m))\,{}_2F_1\left(2,m+1;m+2;\frac{b}{ax}+1\right)}{2a^3(m+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b/x)^m*(c + d*x),x]
```

```
[Out] (d*(a + b/x)^(1 + m)*x^2)/(2*a) - (b*(2*a*c - b*d*(1 - m))*(a + b
/x)^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, 1 + b/(a*x)])/(2*a
^3*(1 + m))
```

Rubi in Sympy [A] time = 6.47905, size = 58, normalized size = 0.73

$$\frac{dx^2\left(a+\frac{b}{x}\right)^{m+1}}{2a}-\frac{b\left(a+\frac{b}{x}\right)^{m+1}(2ac-bd\,(-m+1))\,{}_2F_1\left(\begin{matrix}2,m+1\\m+2\end{matrix}\middle|1+\frac{b}{ax}\right)}{2a^3\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b/x)**m*(d*x+c),x)
```

```
[Out] d*x**2*(a + b/x)**(m + 1)/(2*a) - b*(a + b/x)**(m + 1)*(2*a*c - b
*d*(-m + 1))*hyper((2, m + 1), (m + 2,), 1 + b/(a*x))/(2*a**3*(m
+ 1))
```

Mathematica [A] time = 0.048839, size = 88, normalized size = 1.11

$$-\frac{x\left(a+\frac{b}{x}\right)^m\left(\frac{ax}{b}+1\right)^{-m}\left(c(m-2)\,{}_2F_1\left(1-m,-m;2-m;-\frac{ax}{b}\right)+d(m-1)x\,{}_2F_1\left(2-m,-m;3-m;-\frac{ax}{b}\right)\right)}{(m-2)(m-1)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b/x)^m*(c + d*x),x]
```

```
[Out] -(((a + b/x)^m*x*(c*(-2 + m)*Hypergeometric2F1[1 - m, -m, 2 - m,
-((a*x)/b)] + d*(-1 + m)*x*Hypergeometric2F1[2 - m, -m, 3 - m, -(
(a*x)/b)]))/((-2 + m)*(-1 + m)*(1 + (a*x)/b)^m))
```

Maple [F] time = 0.028, size = 0, normalized size = 0.

$$\int\left(a+\frac{b}{x}\right)^m(dx+c)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b/x)^m*(d*x+c),x)
```

```
[Out] int((a+b/x)^m*(d*x+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int(dx+c)\left(a+\frac{b}{x}\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)*(a + b/x)^m,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)*(a + b/x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((dx+c)\left(\frac{ax+b}{x}\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)*(a + b/x)^m,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)*((a*x + b)/x)^m, x)
```

Sympy [A] time = 10.7185, size = 75, normalized size = 0.95

$$\frac{b^m cxx^{-m}\,(-m+1)\,{}_2F_1\left(\begin{matrix}-m,-m+1\\-m+2\end{matrix}\middle|\frac{axe^{i\pi}}{b}\right)}{(-m+2)}+\frac{b^m dx^2x^{-m}\,(-m+2)\,{}_2F_1\left(\begin{matrix}-m,-m+2\\-m+3\end{matrix}\middle|\frac{axe^{i\pi}}{b}\right)}{(-m+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b/x)**m*(d*x+c),x)
```

```
[Out] b**m*c*x*x**(-m)*gamma(-m + 1)*hyper((-m, -m + 1), (-m + 2,), a*x
*exp_polar(I*pi)/b)/gamma(-m + 2) + b**m*d*x**2*x**(-m)*gamma(-m
+ 2)*hyper((-m, -m + 2), (-m + 3,), a*x*exp_polar(I*pi)/b)/gamma(
-m + 3)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)\left(a+\frac{b}{x}\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)*(a + b/x)^m,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*(a + b/x)^m, x)
```

3.429 $\int \left(a+\frac{b}{x}\right)^m dx$

Optimal. Leaf size=40

$$-\frac{b\left(a+\frac{b}{x}\right)^{m+1} {}_2F_1\left(2,m+1;m+2;\frac{b}{ax}+1\right)}{a^2(m+1)}$$

```
[Out] -((b*(a + b/x)^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, 1 + b/(
a*x)])/(a^2*(1 + m)))
```

Rubi [A] time = 0.0292528, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$-\frac{b\left(a+\frac{b}{x}\right)^{m+1} {}_2F_1\left(2,m+1;m+2;\frac{b}{ax}+1\right)}{a^2(m+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b/x)^m,x]
```

```
[Out] -((b*(a + b/x)^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, 1 + b/(
a*x)])/(a^2*(1 + m)))
```

Rubi in Sympy [A] time = 2.04972, size = 29, normalized size = 0.72

$$-\frac{b\left(a+\frac{b}{x}\right)^{m+1} {}_2F_1\left(\begin{matrix}2,m+1\\ m+2\end{matrix}\middle|1+\frac{b}{ax}\right)}{a^2\left(m+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b/x)**m,x)
```

```
[Out] -b*(a + b/x)**(m + 1)*hyper((2, m + 1), (m + 2,), 1 + b/(a*x))/(a
**2*(m + 1))
```

Mathematica [A] time = 0.0129052, size = 50, normalized size = 1.25

$$-\frac{x\left(a+\frac{b}{x}\right)^m\left(\frac{ax}{b}+1\right)^{-m}\,{}_2F_1\left(1-m,-m;2-m;-\frac{ax}{b}\right)}{m-1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b/x)^m,x]
```

```
[Out] -(((a + b/x)^m*x*Hypergeometric2F1[1 - m, -m, 2 - m, -((a*x)/b)])
/((-1 + m)*(1 + (a*x)/b)^m))
```

Maple [F] time = 0.018, size = 0, normalized size = 0.

$$\int\left(a+\frac{b}{x}\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b/x)^m,x)
```

```
[Out] int((a+b/x)^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+\frac{b}{x}\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x)^m,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\frac{ax+b}{x}\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x)^m,x, algorithm="fricas")
```

```
[Out] integral(((a*x + b)/x)^m, x)
```

Sympy [A] time = 4.79641, size = 34, normalized size = 0.85

$$\frac{b^m x x^{-m} (-m+1)\, {}_2F_1\left(\begin{matrix} -m, -m+1 \\ -m+2 \end{matrix}\middle| \frac{axe^{i\pi}}{b}\right)}{(-m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b/x)**m,x)
```

```
[Out] b**m*x*x**(-m)*gamma(-m + 1)*hyper((-m, -m + 1), (-m + 2,), a*x*e
xp_polar(I*pi)/b)/gamma(-m + 2)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + \frac{b}{x}\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x)^m,x, algorithm="giac")
```

```
[Out] integrate((a + b/x)^m, x)
```

3.430 $\int \frac{\left(a+\frac{b}{x}\right)^m}{c+dx}\,dx$

Optimal. Leaf size=101

$$\frac{\left(a+\frac{b}{x}\right)^{m+1}{}_2F_1\left(1,m+1;m+2;\frac{b}{ax}+1\right)}{ad(m+1)}-\frac{c\left(a+\frac{b}{x}\right)^{m+1}{}_2F_1\left(1,m+1;m+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d(m+1)(ac-bd)}$$

```
[Out] -((c*(a + b/x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (c*(a +
 b/x))/(a*c - b*d)])/(d*(a*c - b*d)*(1 + m))) + ((a + b/x)^(1 + m
)*Hypergeometric2F1[1, 1 + m, 2 + m, 1 + b/(a*x)])/(a*d*(1 + m))
```

Rubi [A] time = 0.161163, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294

$$\frac{\left(a+\frac{b}{x}\right)^{m+1}{}_2F_1\left(1,m+1;m+2;\frac{b}{ax}+1\right)}{ad(m+1)}-\frac{c\left(a+\frac{b}{x}\right)^{m+1}{}_2F_1\left(1,m+1;m+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d(m+1)(ac-bd)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b/x)^m/(c + d*x),x]
```

```
[Out] -((c*(a + b/x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (c*(a +
 b/x))/(a*c - b*d)])/(d*(a*c - b*d)*(1 + m))) + ((a + b/x)^(1 + m
)*Hypergeometric2F1[1, 1 + m, 2 + m, 1 + b/(a*x)])/(a*d*(1 + m))
```

Rubi in Sympy [A] time = 10.9484, size = 66, normalized size = 0.65

$$-\frac{c\left(a+\frac{b}{x}\right)^{m+1}{}_2F_1\left(\begin{matrix}1,m+1\\m+2\end{matrix}\middle|\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d\,(m+1)\,(ac-bd)}+\frac{\left(a+\frac{b}{x}\right)^{m+1}{}_2F_1\left(\begin{matrix}1,m+1\\m+2\end{matrix}\middle|1+\frac{b}{ax}\right)}{ad\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b/x)**m/(d*x+c),x)
```

```
[Out] -c*(a + b/x)**(m + 1)*hyper((1, m + 1), (m + 2,), c*(a + b/x)/(a*
c - b*d))/(d*(m + 1)*(a*c - b*d)) + (a + b/x)**(m + 1)*hyper((1,
```

```
m + 1), (m + 2,), 1 + b/(a*x))/(a*d*(m + 1))
```

Mathematica [A] time = 0.0406183, size = 0, normalized size = 0.

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(a + b/x)^m/(c + d*x),x]

[Out] Integrate[(a + b/x)^m/(c + d*x), x]
```

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int \frac{1}{dx+c}\left(a+\frac{b}{x}\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b/x)^m/(d*x+c),x)

[Out] int((a+b/x)^m/(d*x+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x)^m/(d*x + c),x, algorithm="maxima")

[Out] integrate((a + b/x)^m/(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(\frac{ax+b}{x}\right)^m}{dx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x)^m/(d*x + c),x, algorithm="fricas")

[Out] integral(((a*x + b)/x)^m/(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b/x)**m/(d*x+c),x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{dx+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x)^m/(d*x + c),x, algorithm="giac")

[Out] integrate((a + b/x)^m/(d*x + c), x)
```

3.431 $\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^2}\,dx$

Optimal. Leaf size=56

$$-\frac{b\left(a+\frac{b}{x}\right)^{m+1}{}_2F_1\left(2,m+1;m+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{(m+1)(ac-bd)^2}$$

```
[Out] -((b*(a + b/x)^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, (c*(a +
 b/x))/(a*c - b*d)])/((a*c - b*d)^2*(1 + m)))
```

Rubi [A] time = 0.0948967, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$-\frac{b\left(a+\frac{b}{x}\right)^{m+1}{}_2F_1\left(2,m+1;m+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{(m+1)(ac-bd)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b/x)^m/(c + d*x)^2,x]
```

```
[Out] -((b*(a + b/x)^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, (c*(a +
 b/x))/(a*c - b*d)])/((a*c - b*d)^2*(1 + m)))
```

Rubi in Sympy [A] time = 6.58278, size = 41, normalized size = 0.73

$$-\frac{b\left(a+\frac{b}{x}\right)^{m+1}{}_2F_1\left(\begin{matrix}2,m+1\\ m+2\end{matrix}\middle|\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{(m+1)\,(ac-bd)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b/x)**m/(d*x+c)**2,x)
```

```
[Out] -b*(a + b/x)**(m + 1)*hyper((2, m + 1), (m + 2,), c*(a + b/x)/(a*
c - b*d))/((m + 1)*(a*c - b*d)**2)
```

Mathematica [A] time = 0.0440716, size = 0, normalized size = 0.

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(a + b/x)^m/(c + d*x)^2,x]
```

```
[Out] Integrate[(a + b/x)^m/(c + d*x)^2, x]
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)^2}\left(a+\frac{b}{x}\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b/x)^m/(d*x+c)^2,x)
```

```
[Out] int((a+b/x)^m/(d*x+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x)^m/(d*x + c)^2,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^m/(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(\frac{ax+b}{x}\right)^m}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x)^m/(d*x + c)^2,x, algorithm="fricas")
```

```
[Out] integral(((a*x + b)/x)^m/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b/x)**m/(d*x+c)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + \frac{b}{x}\right)^m}{(dx + c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x)^m/(d*x + c)^2,x, algorithm="giac")
```

```
[Out] integrate((a + b/x)^m/(d*x + c)^2, x)
```

3.432 $\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^3}\,dx$

Optimal. Leaf size=112

$$-\frac{b\left(a+\frac{b}{x}\right)^{m+1}(2ac-bd(m+1))\,{}_2F_1\left(2,m+1;m+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{2c(m+1)(ac-bd)^3}-\frac{d\left(a+\frac{b}{x}\right)^{m+1}}{2c\left(\frac{c}{x}+d\right)^2(ac-bd)}$$

```
[Out] -(d*(a + b/x)^(1 + m))/(2*c*(a*c - b*d)*(d + c/x)^2) - (b*(2*a*c
- b*d*(1 + m))*(a + b/x)^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 +
m, (c*(a + b/x))/(a*c - b*d)])/(2*c*(a*c - b*d)^3*(1 + m))
```

Rubi [A] time = 0.165579, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235

$$-\frac{b\left(a+\frac{b}{x}\right)^{m+1}(2ac-bd(m+1))\,{}_2F_1\left(2,m+1;m+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{2c(m+1)(ac-bd)^3}-\frac{d\left(a+\frac{b}{x}\right)^{m+1}}{2c\left(\frac{c}{x}+d\right)^2(ac-bd)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b/x)^m/(c + d*x)^3,x]
```

```
[Out] -(d*(a + b/x)^(1 + m))/(2*c*(a*c - b*d)*(d + c/x)^2) - (b*(2*a*c
- b*d*(1 + m))*(a + b/x)^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 +
m, (c*(a + b/x))/(a*c - b*d)])/(2*c*(a*c - b*d)^3*(1 + m))
```

Rubi in Sympy [A] time = 11.2758, size = 83, normalized size = 0.74

$$-\frac{b\left(a+\frac{b}{x}\right)^{m+1}(2ac-bd\,(m+1))\,{}_2F_1\left(\begin{matrix}2,m+1\\m+2\end{matrix}\middle|\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{2c\,(m+1)\,(ac-bd)^3}-\frac{d\left(a+\frac{b}{x}\right)^{m+1}}{2c\,(ac-bd)\left(\frac{c}{x}+d\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b/x)**m/(d*x+c)**3,x)
```

```
[Out] -b*(a + b/x)**(m + 1)*(2*a*c - b*d*(m + 1))*hyper((2, m + 1), (m
+ 2,), c*(a + b/x)/(a*c - b*d))/(2*c*(m + 1)*(a*c - b*d)**3) - d*
(a + b/x)**(m + 1)/(2*c*(a*c - b*d)*(c/x + d)**2)
```

Mathematica [A] time = 0.0937313, size = 0, normalized size = 0.

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^3}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(a + b/x)^m/(c + d*x)^3,x]
```

```
[Out] Integrate[(a + b/x)^m/(c + d*x)^3, x]
```

Maple [F] time = 0.087, size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)^3}\left(a+\frac{b}{x}\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b/x)^m/(d*x+c)^3,x)
```

```
[Out] int((a+b/x)^m/(d*x+c)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(dx+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x)^m/(d*x + c)^3,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^m/(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(\frac{ax+b}{x}\right)^m}{d^3x^3 + 3\,cd^2x^2 + 3\,c^2dx + c^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x)^m/(d*x + c)^3,x, algorithm="fricas")
```

```
[Out] integral(((a*x + b)/x)^m/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3
), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b/x)**m/(d*x+c)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + \frac{b}{x}\right)^m}{(dx + c)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x)^m/(d*x + c)^3,x, algorithm="giac")
```

```
[Out] integrate((a + b/x)^m/(d*x + c)^3, x)
```

3.433 $\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^4}\,dx$

Optimal. Leaf size=185

$$-\frac{b\left(a+\frac{b}{x}\right)^{m+1}\left(6a^2c^2-6abcd(m+1)+b^2d^2\left(m^2+3m+2\right)\right)\,{}_2F_1\left(2,m+1;m+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{6c^2(m+1)(ac-bd)^4}+\frac{d^2\left(a+\frac{b}{x}\right)^{m+1}}{3c^2\left(\frac{c}{x}+d\right)^3(ac-bd)}-\frac{d\left(a+\frac{b}{x}\right)^{m+1}(6ac-bd(m+4))}{6c^2\left(\frac{c}{x}+d\right)^2(ac-bd)^2}$$

```
[Out] (d^2*(a + b/x)^(1 + m))/(3*c^2*(a*c - b*d)*(d + c/x)^3) - (d*(6*a
*c - b*d*(4 + m))*(a + b/x)^(1 + m))/(6*c^2*(a*c - b*d)^2*(d + c/
x)^2) - (b*(6*a^2*c^2 - 6*a*b*c*d*(1 + m) + b^2*d^2*(2 + 3*m + m^
2))*(a + b/x)^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, (c*(a +
b/x))/(a*c - b*d)])/(6*c^2*(a*c - b*d)^4*(1 + m))
```

Rubi [A] time = 0.40771, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$

$$-\frac{b\left(a+\frac{b}{x}\right)^{m+1}\left(6a^2c^2-6abcd(m+1)+b^2d^2\left(m^2+3m+2\right)\right)\,{}_2F_1\left(2,m+1;m+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{6c^2(m+1)(ac-bd)^4}+\frac{d^2\left(a+\frac{b}{x}\right)^{m+1}}{3c^2\left(\frac{c}{x}+d\right)^3(ac-bd)}-\frac{d\left(a+\frac{b}{x}\right)^{m+1}(6ac-bd(m+4))}{6c^2\left(\frac{c}{x}+d\right)^2(ac-bd)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b/x)^m/(c + d*x)^4,x]
```

```
[Out] (d^2*(a + b/x)^(1 + m))/(3*c^2*(a*c - b*d)*(d + c/x)^3) - (d*(6*a
*c - b*d*(4 + m))*(a + b/x)^(1 + m))/(6*c^2*(a*c - b*d)^2*(d + c/
x)^2) - (b*(6*a^2*c^2 - 6*a*b*c*d*(1 + m) + b^2*d^2*(2 + 3*m + m^
2))*(a + b/x)^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, (c*(a +
b/x))/(a*c - b*d)])/(6*c^2*(a*c - b*d)^4*(1 + m))
```

Rubi in Sympy [A] time = 21.976, size = 162, normalized size = 0.88

$$-\frac{b\left(a+\frac{b}{x}\right)^{m+1}\left(bd\left(-m+1\right)\left(3ac-bd\left(m+1\right)\right)+\left(2ac-bd\left(m+1\right)\right)\left(3ac-3bd\right)\right){}_2F_1\left(\begin{matrix}2,m+1\\m+2\end{matrix}\middle|\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{6c^2\left(m+1\right)\left(ac-bd\right)^4}+\frac{d^2\left(a+\frac{b}{x}\right)^{m+1}}{3c^2\left(ac-bd\right)\left(\frac{c}{x}+d\right)^3}-\frac{d\left(a+\frac{b}{x}\right)^{m+1}\left(6ac-bdm-4bd\right)}{6c^2\left(ac-bd\right)^2\left(\frac{c}{x}+d\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b/x)**m/(d*x+c)**4,x)
```

```
[Out] -b*(a + b/x)**(m + 1)*(b*d*(-m + 1)*(3*a*c - b*d*(m + 1)) + (2*a*
c - b*d*(m + 1))*(3*a*c - 3*b*d))*hyper((2, m + 1), (m + 2,), c*(
a + b/x)/(a*c - b*d))/(6*c**2*(m + 1)*(a*c - b*d)**4) + d**2*(a +
 b/x)**(m + 1)/(3*c**2*(a*c - b*d)*(c/x + d)**3) - d*(a + b/x)**(
m + 1)*(6*a*c - b*d*m - 4*b*d)/(6*c**2*(a*c - b*d)**2*(c/x + d)**
2)
```

Mathematica [A] time = 0.223834, size = 0, normalized size = 0.

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^4}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(a + b/x)^m/(c + d*x)^4,x]
```

```
[Out] Integrate[(a + b/x)^m/(c + d*x)^4, x]
```

Maple [F] time = 0.136, size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)^4}\left(a+\frac{b}{x}\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b/x)^m/(d*x+c)^4,x)
```

```
[Out] int((a+b/x)^m/(d*x+c)^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+\frac{b}{x}\right)^{m}}{(dx+c)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x)^m/(d*x + c)^4,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^m/(d*x + c)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(\frac{ax+b}{x}\right)^{m}}{d^{4}x^{4}+4\,cd^{3}x^{3}+6\,c^{2}d^{2}x^{2}+4\,c^{3}dx+c^{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x)^m/(d*x + c)^4,x, algorithm="fricas")
```

```
[Out] integral(((a*x + b)/x)^m/(d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 +
 4*c^3*d*x + c^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b/x)**m/(d*x+c)**4,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(dx+c)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x)^m/(d*x + c)^4,x, algorithm="giac")

[Out] integrate((a + b/x)^m/(d*x + c)^4, x)
```

3.434 $\int \frac{\sqrt{b-\frac{a}{x^2}}x^m}{\sqrt{a-bx^2}}\,dx$

Optimal. Leaf size=33

$$\frac{x^{m+1}\sqrt{b-\frac{a}{x^2}}}{m\sqrt{a-bx^2}}$$

```
[Out] (Sqrt[b - a/x^2]*x^(1 + m))/(m*Sqrt[a - b*x^2])
```

Rubi [A] time = 0.111759, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107

$$\frac{x^{m+1}\sqrt{b-\frac{a}{x^2}}}{m\sqrt{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[b - a/x^2]*x^m)/Sqrt[a - b*x^2],x]

[Out] (Sqrt[b - a/x^2]*x^(1 + m))/(m*Sqrt[a - b*x^2])
```

Rubi in Sympy [A] time = 6.00444, size = 27, normalized size = 0.82

$$-\frac{x^m\sqrt{a-bx^2}}{mx\sqrt{-\frac{a}{x^2}+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**m*(b-a/x**2)**(1/2)/(-b*x**2+a)**(1/2),x)

[Out] -x**m*sqrt(a - b*x**2)/(m*x*sqrt(-a/x**2 + b))
```

Mathematica [A] time = 0.0927068, size = 33, normalized size = 1.

$$\frac{x^{m+1}\sqrt{b-\frac{a}{x^2}}}{m\sqrt{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[b - a/x^2]*x^m)/Sqrt[a - b*x^2],x]
```

```
[Out] (Sqrt[b - a/x^2]*x^(1 + m))/(m*Sqrt[a - b*x^2])
```

Maple [A] time = 0.004, size = 35, normalized size = 1.1

$$\frac{x^{1+m}}{m}\sqrt{-\frac{-bx^2+a}{x^2}}\frac{1}{\sqrt{-bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^m*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x)
```

```
[Out] x^(1+m)/m*(-(-b*x^2+a)/x^2)^(1/2)/(-b*x^2+a)^(1/2)
```

Maxima [A] time = 0.71332, size = 11, normalized size = 0.33

$$-\frac{i\,x^m}{m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b - a/x^2)*x^m/sqrt(-b*x^2 + a),x, algorithm="maxima")
```

```
[Out] -I*x^m/m
```

Fricas [A] time = 0.279225, size = 59, normalized size = 1.79

$$-\frac{\sqrt{-bx^2+a}\,xx^m\sqrt{\frac{bx^2-a}{x^2}}}{bmx^2-am}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b - a/x^2)*x^m/sqrt(-b*x^2 + a),x, algorithm="fricas")
```

```
[Out] -sqrt(-b*x^2 + a)*x*x^m*sqrt((b*x^2 - a)/x^2)/(b*m*x^2 - a*m)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m\sqrt{-\frac{a}{x^2}+b}}{\sqrt{a-bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**m*(b-a/x**2)**(1/2)/(-b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**m*sqrt(-a/x**2 + b)/sqrt(a - b*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b-\frac{a}{x^2}}x^m}{\sqrt{-bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b - a/x^2)*x^m/sqrt(-b*x^2 + a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b - a/x^2)*x^m/sqrt(-b*x^2 + a), x)
```

3.435 $\int \frac{\sqrt{b-\frac{a}{x^2}}x^2}{\sqrt{a-bx^2}} dx$

Optimal. Leaf size=31

$$\frac{x^3\sqrt{b-\frac{a}{x^2}}}{2\sqrt{a-bx^2}}$$

```
[Out] (Sqrt[b - a/x^2]*x^3)/(2*Sqrt[a - b*x^2])
```

Rubi [A] time = 0.120606, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107

$$\frac{x^3\sqrt{b-\frac{a}{x^2}}}{2\sqrt{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[b - a/x^2]*x^2)/Sqrt[a - b*x^2],x]

[Out] (Sqrt[b - a/x^2]*x^3)/(2*Sqrt[a - b*x^2])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\sqrt{a-bx^2}\int x\,dx}{x\sqrt{-\frac{a}{x^2}+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(b-a/x**2)**(1/2)/(-b*x**2+a)**(1/2),x)

[Out] -sqrt(a - b*x**2)*Integral(x, x)/(x*sqrt(-a/x**2 + b))
```

Mathematica [A] time = 0.0156379, size = 31, normalized size = 1.

$$\frac{x^3\sqrt{b-\frac{a}{x^2}}}{2\sqrt{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[b - a/x^2]*x^2)/Sqrt[a - b*x^2],x]

[Out] (Sqrt[b - a/x^2]*x^3)/(2*Sqrt[a - b*x^2])
```

Maple [A] time = 0.003, size = 31, normalized size = 1.

$$\frac{x^3}{2}\sqrt{-\frac{-bx^2+a}{x^2}}\frac{1}{\sqrt{-bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x)

[Out] 1/2*x^3*(-(-b*x^2+a)/x^2)^(1/2)/(-b*x^2+a)^(1/2)
```

Maxima [A] time = 0.726032, size = 7, normalized size = 0.23

$$-\frac{1}{2}i\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b - a/x^2)*x^2/sqrt(-b*x^2 + a),x, algorithm="maxima")

[Out] -1/2*I*x^2
```

Fricas [A] time = 0.274983, size = 55, normalized size = 1.77

$$-\frac{\sqrt{-bx^2+a}x^3\sqrt{\frac{bx^2-a}{x^2}}}{2\,(bx^2-a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b - a/x^2)*x^2/sqrt(-b*x^2 + a),x, algorithm="fricas")

[Out] -1/2*sqrt(-b*x^2 + a)*x^3*sqrt((b*x^2 - a)/x^2)/(b*x^2 - a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{-\frac{a}{x^2}+b}}{\sqrt{a-bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(b-a/x**2)**(1/2)/(-b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-a/x**2 + b)/sqrt(a - b*x**2), x)
```

GIAC/XCAS [A] time = 0.273877, size = 53, normalized size = 1.71

$$-\frac{\left(bx^2-a\right)i\mathrm{sign}\left(bx^2-a\right)\,\mathrm{sign}\left(x\right)}{2\,b}+\frac{ai\mathrm{sign}\left(a\right)\mathrm{sign}\left(x\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b - a/x^2)*x^2/sqrt(-b*x^2 + a),x, algorithm="giac")
```

```
[Out] -1/2*(b*x^2 - a)*i*sign(b*x^2 - a)*sign(x)/b + 1/2*a*i*sign(a)*si
gn(x)/b
```

3.436 $\int \frac{\sqrt{b-\frac{a}{x^2}}x}{\sqrt{a-bx^2}} \, dx$

Optimal. Leaf size=28

$$\frac{x^2\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}$$

```
[Out] (Sqrt[b - a/x^2]*x^2)/Sqrt[a - b*x^2]
```

Rubi [A] time = 0.089545, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{x^2\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[b - a/x^2]*x)/Sqrt[a - b*x^2],x]
```

```
[Out] (Sqrt[b - a/x^2]*x^2)/Sqrt[a - b*x^2]
```

Rubi in Sympy [A] time = 4.93753, size = 20, normalized size = 0.71

$$-\frac{\sqrt{a-bx^2}}{\sqrt{-\frac{a}{x^2}+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(b-a/x**2)**(1/2)/(-b*x**2+a)**(1/2),x)
```

```
[Out] -sqrt(a - b*x**2)/sqrt(-a/x**2 + b)
```

Mathematica [A] time = 0.0283499, size = 28, normalized size = 1.

$$\frac{x^2\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[b - a/x^2]*x)/Sqrt[a - b*x^2],x]
```

```
[Out] (Sqrt[b - a/x^2]*x^2)/Sqrt[a - b*x^2]
```

Maple [A] time = 0.015, size = 42, normalized size = 1.5

$$-\frac{x^2}{bx^2-a}\sqrt{\frac{bx^2-a}{x^2}}\sqrt{-bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x)
```

```
[Out] -((b*x^2-a)/x^2)^(1/2)*x^2/(b*x^2-a)*(-b*x^2+a)^(1/2)
```

Maxima [A] time = 0.722078, size = 9, normalized size = 0.32

$$-i\sqrt{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b - a/x^2)*x/sqrt(-b*x^2 + a),x, algorithm="maxima")
```

```
[Out] -I*sqrt(x^2)
```

Fricas [A] time = 0.267732, size = 55, normalized size = 1.96

$$-\frac{\sqrt{-bx^2+a}x^2\sqrt{\frac{bx^2-a}{x^2}}}{bx^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b - a/x^2)*x/sqrt(-b*x^2 + a),x, algorithm="fricas")
```

```
[Out] -sqrt(-b*x^2 + a)*x^2*sqrt((b*x^2 - a)/x^2)/(b*x^2 - a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{-\frac{a}{x^2}+b}}{\sqrt{a-bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(b-a/x**2)**(1/2)/(-b*x**2+a)**(1/2),x)

[Out] Integral(x*sqrt(-a/x**2 + b)/sqrt(a - b*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b-\frac{a}{x^2}}\,x}{\sqrt{-bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b - a/x^2)*x/sqrt(-b*x^2 + a),x, algorithm="giac")

[Out] integrate(sqrt(b - a/x^2)*x/sqrt(-b*x^2 + a), x)
```

3.437 $\int \frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}\,dx$

Optimal. Leaf size=28

$$\frac{x\log(x)\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}$$

```
[Out] (Sqrt[b - a/x^2]*x*Log[x])/Sqrt[a - b*x^2]
```

Rubi [A] time = 0.0530788, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12

$$\frac{x\log(x)\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[b - a/x^2]/Sqrt[a - b*x^2],x]

[Out] (Sqrt[b - a/x^2]*x*Log[x])/Sqrt[a - b*x^2]
```

Rubi in Sympy [A] time = 5.15388, size = 26, normalized size = 0.93

$$-\frac{\sqrt{a-bx^2}\log(x)}{x\sqrt{-\frac{a}{x^2}+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b-a/x**2)**(1/2)/(-b*x**2+a)**(1/2),x)

[Out] -sqrt(a - b*x**2)*log(x)/(x*sqrt(-a/x**2 + b))
```

Mathematica [A] time = 0.11024, size = 0, normalized size = 0.

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[Sqrt[b - a/x^2]/Sqrt[a - b*x^2],x]
```

```
[Out] Integrate[Sqrt[b - a/x^2]/Sqrt[a - b*x^2], x]
```

Maple [A] time = 0.011, size = 42, normalized size = 1.5

$$-\frac{x\ln(x)}{bx^2-a}\sqrt{\frac{bx^2-a}{x^2}}\sqrt{-bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x)
```

```
[Out] -((b*x^2-a)/x^2)^(1/2)*x/(b*x^2-a)*(-b*x^2+a)^(1/2)*ln(x)
```

Maxima [A] time = 0.719973, size = 5, normalized size = 0.18

$$-i\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b - a/x^2)/sqrt(-b*x^2 + a),x, algorithm="maxima")
```

```
[Out] -I*log(x)
```

Fricas [A] time = 0.291698, size = 69, normalized size = 2.46

$$-\arctan\left(\frac{\sqrt{-bx^2+a}\left(x^3+x\right)\sqrt{\frac{bx^2-a}{x^2}}}{bx^4-(a+b)x^2+a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b - a/x^2)/sqrt(-b*x^2 + a),x, algorithm="fricas")
```

```
[Out] -arctan(sqrt(-b*x^2 + a)*(x^3 + x)*sqrt((b*x^2 - a)/x^2)/(b*x^4 -
 (a + b)*x^2 + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-\frac{a}{x^2} + b}}{\sqrt{a - bx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b-a/x**2)**(1/2)/(-b*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(-a/x**2 + b)/sqrt(a - b*x**2), x)
```

GIAC/XCAS [A] time = 0.275199, size = 42, normalized size = 1.5

$$-\frac{1}{2}\, i\ln\left(\left(bx^2 - a\right)i + ai\right)\,\mathrm{sign}\left(bx^2 - a\right)\,\mathrm{sign}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b - a/x^2)/sqrt(-b*x^2 + a),x, algorithm="giac")
```

```
[Out] -1/2*i*ln((b*x^2 - a)*i + a*i)*sign(b*x^2 - a)*sign(x)
```

3.438 $\int \frac{\sqrt{b-\frac{a}{x^2}}}{x\sqrt{a-bx^2}} \, dx$

Optimal. Leaf size=26

$$-\frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}$$

```
[Out] -(Sqrt[b - a/x^2]/Sqrt[a - b*x^2])
```

Rubi [A] time = 0.112253, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107

$$-\frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[b - a/x^2]/(x*Sqrt[a - b*x^2]),x]

[Out] -(Sqrt[b - a/x^2]/Sqrt[a - b*x^2])
```

Rubi in Sympy [A] time = 5.9837, size = 22, normalized size = 0.85

$$\frac{\sqrt{a-bx^2}}{x^2\sqrt{-\frac{a}{x^2}+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b-a/x**2)**(1/2)/x/(-b*x**2+a)**(1/2),x)

[Out] sqrt(a - b*x**2)/(x**2*sqrt(-a/x**2 + b))
```

Mathematica [A] time = 0.0334872, size = 26, normalized size = 1.

$$-\frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[b - a/x^2]/(x*Sqrt[a - b*x^2]),x]
```

```
[Out] -(Sqrt[b - a/x^2]/Sqrt[a - b*x^2])
```

Maple [A] time = 0.004, size = 28, normalized size = 1.1

$$-1\sqrt{-\frac{-bx^2+a}{x^2}}\frac{1}{\sqrt{-bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b-a/x^2)^(1/2)/x/(-b*x^2+a)^(1/2),x)
```

```
[Out] -(-(-b*x^2+a)/x^2)^(1/2)/(-b*x^2+a)^(1/2)
```

Maxima [A] time = 0.731808, size = 9, normalized size = 0.35

$$\frac{i}{\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b - a/x^2)/(sqrt(-b*x^2 + a)*x),x, algorithm="maxima")
```

```
[Out] I/sqrt(x^2)
```

Fricas [A] time = 0.266722, size = 55, normalized size = 2.12

$$-\frac{\sqrt{-bx^2+a}(x-1)\sqrt{\frac{bx^2-a}{x^2}}}{bx^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b - a/x^2)/(sqrt(-b*x^2 + a)*x),x, algorithm="fricas")
```

```
[Out] -sqrt(-b*x^2 + a)*(x - 1)*sqrt((b*x^2 - a)/x^2)/(b*x^2 - a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-\frac{a}{x^2} + b}}{x\sqrt{a - bx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b-a/x**2)**(1/2)/x/(-b*x**2+a)**(1/2),x)

[Out] Integral(sqrt(-a/x**2 + b)/(x*sqrt(a - b*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b - \frac{a}{x^2}}}{\sqrt{-bx^2 + a}x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b - a/x^2)/(sqrt(-b*x^2 + a)*x),x, algorithm="giac")

[Out] integrate(sqrt(b - a/x^2)/(sqrt(-b*x^2 + a)*x), x)
```

3.439 $\int \frac{\sqrt{b-\frac{a}{x^2}}}{x^2\sqrt{a-bx^2}}\,dx$

Optimal. Leaf size=31

$$-\frac{\sqrt{b-\frac{a}{x^2}}}{2x\sqrt{a-bx^2}}$$

```
[Out] -Sqrt[b - a/x^2]/(2*x*Sqrt[a - b*x^2])
```

Rubi [A] time = 0.114928, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107

$$-\frac{\sqrt{b-\frac{a}{x^2}}}{2x\sqrt{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[b - a/x^2]/(x^2*Sqrt[a - b*x^2]),x]

[Out] -Sqrt[b - a/x^2]/(2*x*Sqrt[a - b*x^2])
```

Rubi in Sympy [A] time = 5.78084, size = 24, normalized size = 0.77

$$\frac{\sqrt{a-bx^2}}{2x^3\sqrt{-\frac{a}{x^2}+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b-a/x**2)**(1/2)/x**2/(-b*x**2+a)**(1/2),x)

[Out] sqrt(a - b*x**2)/(2*x**3*sqrt(-a/x**2 + b))
```

Mathematica [A] time = 0.0250569, size = 38, normalized size = 1.23

$$\frac{\sqrt{b-\frac{a}{x^2}}\sqrt{a-bx^2}}{2bx^3-2ax}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[b - a/x^2]/(x^2*Sqrt[a - b*x^2]),x]

[Out] (Sqrt[b - a/x^2]*Sqrt[a - b*x^2])/(-2*a*x + 2*b*x^3)
```

Maple [A] time = 0.003, size = 31, normalized size = 1.

$$-\frac{1}{2x}\sqrt{-\frac{-bx^2+a}{x^2}}\frac{1}{\sqrt{-bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b-a/x^2)^(1/2)/x^2/(-b*x^2+a)^(1/2),x)

[Out] -1/2*(-(-b*x^2+a)/x^2)^(1/2)/x/(-b*x^2+a)^(1/2)
```

Maxima [A] time = 0.774443, size = 7, normalized size = 0.23

$$\frac{i}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b - a/x^2)/(sqrt(-b*x^2 + a)*x^2),x, algorithm="maxima")

[Out] 1/2*I/x^2
```

Fricas [A] time = 0.270704, size = 59, normalized size = 1.9

$$-\frac{\sqrt{-bx^2+a}\left(x^2-1\right)\sqrt{\frac{bx^2-a}{x^2}}}{2\left(bx^3-ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b - a/x^2)/(sqrt(-b*x^2 + a)*x^2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(-b*x^2 + a)*(x^2 - 1)*sqrt((b*x^2 - a)/x^2)/(b*x^3 - a*
x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-\frac{a}{x^2}+b}}{x^2\sqrt{a-bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b-a/x**2)**(1/2)/x**2/(-b*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(-a/x**2 + b)/(x**2*sqrt(a - b*x**2)), x)
```

GIAC/XCAS [A] time = 0.275152, size = 27, normalized size = 0.87

$$-\frac{\mathrm{sign}\left(bx^2-a\right)\,\mathrm{sign}\left(x\right)}{2\,ix^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b - a/x^2)/(sqrt(-b*x^2 + a)*x^2),x, algorithm="giac")
```

```
[Out] -1/2*sign(b*x^2 - a)*sign(x)/(i*x^2)
```

3.440 $\int \frac{(c+dx)^{3/2}}{\sqrt{a+\frac{b}{x^2}}}\,dx$

Optimal. Leaf size=406

$$-\frac{2\sqrt{b}c\sqrt{\frac{ax^2}{b}+1}\left(ac^2+bd^2\right)\sqrt{\frac{a(c+dx)}{ac-\sqrt{-a}\sqrt{b}d}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{-a}x}{\sqrt{b}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{-a}\sqrt{b}d}{ac-\sqrt{-a}\sqrt{b}d}\right)}{5(-a)^{3/2}dx\sqrt{a+\frac{b}{x^2}}\sqrt{c+dx}}+\frac{2\sqrt{b}\sqrt{\frac{ax^2}{b}+1}\sqrt{c+dx}\left(ac^2-3bd^2\right)E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{-a}x}{\sqrt{b}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{-a}\sqrt{b}d}{ac-\sqrt{-a}\sqrt{b}d}\right)}{5(-a)^{3/2}dx\sqrt{a+\frac{b}{x^2}}\sqrt{\frac{a(c+dx)}{ac-\sqrt{-a}\sqrt{b}d}}}+\frac{2\left(ax^2+b\right)(c+dx)^{3/2}}{5ax\sqrt{a+\frac{b}{x^2}}}+\frac{2c\left(ax^2+b\right)\sqrt{c+dx}}{5ax\sqrt{a+\frac{b}{x^2}}}$$

```
[Out] (2*c*Sqrt[c + d*x]*(b + a*x^2))/(5*a*Sqrt[a + b/x^2]*x) + (2*(c +
 d*x)^(3/2)*(b + a*x^2))/(5*a*Sqrt[a + b/x^2]*x) + (2*Sqrt[b]*(a*
c^2 - 3*b*d^2)*Sqrt[c + d*x]*Sqrt[1 + (a*x^2)/b]*EllipticE[ArcSin
[Sqrt[1 - (Sqrt[-a]*x)/Sqrt[b]]/Sqrt[2]], (-2*Sqrt[-a]*Sqrt[b]*d)
/(a*c - Sqrt[-a]*Sqrt[b]*d)])/(5*(-a)^(3/2)*d*Sqrt[a + b/x^2]*x*S
qrt[(a*(c + d*x))/(a*c - Sqrt[-a]*Sqrt[b]*d)]) - (2*Sqrt[b]*c*(a*
c^2 + b*d^2)*Sqrt[(a*(c + d*x))/(a*c - Sqrt[-a]*Sqrt[b]*d)]*Sqrt[
1 + (a*x^2)/b]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[-a]*x)/Sqrt[b]]/Sq
rt[2]], (-2*Sqrt[-a]*Sqrt[b]*d)/(a*c - Sqrt[-a]*Sqrt[b]*d)])/(5*(
-a)^(3/2)*d*Sqrt[a + b/x^2]*x*Sqrt[c + d*x])
```

Rubi [A] time = 1.13368, antiderivative size = 406, normalized size of antiderivative = 1., number of

steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$-\frac{2\sqrt{b}c\sqrt{\frac{ax^2}{b}+1}\left(ac^2+bd^2\right)\sqrt{\frac{a(c+dx)}{ac-\sqrt{-a}\sqrt{b}d}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{-a}x}{\sqrt{b}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{-a}\sqrt{b}d}{ac-\sqrt{-a}\sqrt{b}d}\right)}{5(-a)^{3/2}dx\sqrt{a+\frac{b}{x^2}}\sqrt{c+dx}}+\frac{2\sqrt{b}\sqrt{\frac{ax^2}{b}+1}\sqrt{c+dx}\left(ac^2-3bd^2\right)E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{-a}x}{\sqrt{b}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{-a}\sqrt{b}d}{ac-\sqrt{-a}\sqrt{b}d}\right)}{5(-a)^{3/2}dx\sqrt{a+\frac{b}{x^2}}\sqrt{\frac{a(c+dx)}{ac-\sqrt{-a}\sqrt{b}d}}}+\frac{2\left(ax^2+b\right)(c+dx)^{3/2}}{5ax\sqrt{a+\frac{b}{x^2}}}+\frac{2c\left(ax^2+b\right)\sqrt{c+dx}}{5ax\sqrt{a+\frac{b}{x^2}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x)^(3/2)/Sqrt[a + b/x^2],x]
```

```
[Out] (2*c*Sqrt[c + d*x]*(b + a*x^2))/(5*a*Sqrt[a + b/x^2]*x) + (2*(c +
 d*x)^(3/2)*(b + a*x^2))/(5*a*Sqrt[a + b/x^2]*x) + (2*Sqrt[b]*(a*
c^2 - 3*b*d^2)*Sqrt[c + d*x]*Sqrt[1 + (a*x^2)/b]*EllipticE[ArcSin
[Sqrt[1 - (Sqrt[-a]*x)/Sqrt[b]]/Sqrt[2]], (-2*Sqrt[-a]*Sqrt[b]*d)
/(a*c - Sqrt[-a]*Sqrt[b]*d)])/(5*(-a)^(3/2)*d*Sqrt[a + b/x^2]*x*S
qrt[(a*(c + d*x))/(a*c - Sqrt[-a]*Sqrt[b]*d)]) - (2*Sqrt[b]*c*(a*
c^2 + b*d^2)*Sqrt[(a*(c + d*x))/(a*c - Sqrt[-a]*Sqrt[b]*d)]*Sqrt[
1 + (a*x^2)/b]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[-a]*x)/Sqrt[b]]/Sq
rt[2]], (-2*Sqrt[-a]*Sqrt[b]*d)/(a*c - Sqrt[-a]*Sqrt[b]*d)])/(5*(
-a)^(3/2)*d*Sqrt[a + b/x^2]*x*Sqrt[c + d*x])
```

Rubi in Sympy [A] time = 75.39, size = 357, normalized size = 0.88

$$-\frac{2\sqrt{b}cx\sqrt{\frac{a(c+dx)}{ac-\sqrt{b}d\sqrt{-a}}}\sqrt{a+\frac{b}{x^2}}\left(ac^2+bd^2\right)\sqrt{\frac{ax^2}{b}+1}F\left(\operatorname{asin}\left(\sqrt{\frac{1}{2}-\frac{x\sqrt{-a}}{2\sqrt{b}}}\right)\middle|-\frac{2\sqrt{b}d\sqrt{-a}}{ac-\sqrt{b}d\sqrt{-a}}\right)}{5d\left(-a\right)^{\frac{3}{2}}\sqrt{c+dx}\left(ax^2+b\right)}+\frac{2\sqrt{b}x\sqrt{a+\frac{b}{x^2}}\sqrt{c+dx}\left(ac^2-3bd^2\right)\sqrt{\frac{ax^2}{b}+1}E\left(\operatorname{asin}\left(\sqrt{\frac{1}{2}-\frac{x\sqrt{-a}}{2\sqrt{b}}}\right)\middle|-\frac{2\sqrt{b}d\sqrt{-a}}{ac-\sqrt{b}d\sqrt{-a}}\right)}{5d\left(-a\right)^{\frac{3}{2}}\sqrt{\frac{a(c+dx)}{ac-\sqrt{b}d\sqrt{-a}}}\left(ax^2+b\right)}+\frac{2cx\sqrt{a+\frac{b}{x^2}}\sqrt{c+dx}}{5a}+\frac{2x\sqrt{a+\frac{b}{x^2}}\left(c+dx\right)^{\frac{3}{2}}}{5a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x+c)**(3/2)/(a+b/x**2)**(1/2),x)
```

```
[Out] -2*sqrt(b)*c*x*sqrt(a*(c + d*x)/(a*c - sqrt(b)*d*sqrt(-a)))*sqrt(
a + b/x**2)*(a*c**2 + b*d**2)*sqrt(a*x**2/b + 1)*elliptic_f(asin(
sqrt(1/2 - x*sqrt(-a)/(2*sqrt(b)))), -2*sqrt(b)*d*sqrt(-a)/(a*c -
 sqrt(b)*d*sqrt(-a)))/(5*d*(-a)**(3/2)*sqrt(c + d*x)*(a*x**2 + b)
) + 2*sqrt(b)*x*sqrt(a + b/x**2)*sqrt(c + d*x)*(a*c**2 - 3*b*d**2
)*sqrt(a*x**2/b + 1)*elliptic_e(asin(sqrt(1/2 - x*sqrt(-a)/(2*sqr
t(b)))), -2*sqrt(b)*d*sqrt(-a)/(a*c - sqrt(b)*d*sqrt(-a)))/(5*d*(
-a)**(3/2)*sqrt(a*(c + d*x)/(a*c - sqrt(b)*d*sqrt(-a)))*(a*x**2 +
 b)) + 2*c*x*sqrt(a + b/x**2)*sqrt(c + d*x)/(5*a) + 2*x*sqrt(a +
b/x**2)*(c + d*x)**(3/2)/(5*a)
```

Mathematica [C] time = 4.59329, size = 540, normalized size = 1.33

$$\frac{\sqrt{c+dx}\left(\frac{2\left(ax^2+b\right)(2c+dx)}{a}+\frac{2\left(\sqrt{a}(c+dx)^{3/2}\left(-ia^{3/2}c^3+a\sqrt{b}c^2d+3i\sqrt{a}bcd^2-3b^{3/2}d^3\right)\sqrt{\frac{d\left(x+\frac{i\sqrt{b}}{\sqrt{a}}\right)}{c+dx}}\sqrt{-\frac{-dx+\frac{i\sqrt{b}d}{\sqrt{a}}}{c+dx}}E\left(i\sinh^{-1}\left(\frac{\sqrt{-c-\frac{i\sqrt{b}d}{\sqrt{a}}}}{\sqrt{c+dx}}\right)|\frac{\sqrt{a}c-i\sqrt{b}d}{\sqrt{a}c+i\sqrt{b}d}\right)+d^2 \cdots}{\cdots}\right)}{5x\sqrt{a \cdots}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x)^(3/2)/Sqrt[a + b/x^2],x]
```

```
[Out] (Sqrt[c + d*x]*((2*(2*c + d*x)*(b + a*x^2))/a + (2*(d^2*Sqrt[-c -
 (I*Sqrt[b]*d)/Sqrt[a]]*(-3*b^2*d^2 + a^2*c^2*x^2 + a*b*(c^2 - 3*
d^2*x^2)) + Sqrt[a]*((-I)*a^(3/2)*c^3 + a*Sqrt[b]*c^2*d + (3*I)*S
qrt[a]*b*c*d^2 - 3*b^(3/2)*d^3)*Sqrt[(d*((I*Sqrt[b])/Sqrt[a] + x)
)/(c + d*x)]*Sqrt[-(((I*Sqrt[b]*d)/Sqrt[a] - d*x)/(c + d*x))]*(c
+ d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c - (I*Sqrt[b]*d)/Sqrt[a]]
/Sqrt[c + d*x]], (Sqrt[a]*c - I*Sqrt[b]*d)/(Sqrt[a]*c + I*Sqrt[b]
*d)] - Sqrt[a]*Sqrt[b]*d*(a*c^2 + (4*I)*Sqrt[a]*Sqrt[b]*c*d - 3*b
*d^2)*Sqrt[(d*((I*Sqrt[b])/Sqrt[a] + x))/(c + d*x)]*Sqrt[-(((I*Sq
rt[b]*d)/Sqrt[a] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticF[I*A
rcSinh[Sqrt[-c - (I*Sqrt[b]*d)/Sqrt[a]]/Sqrt[c + d*x]], (Sqrt[a]*
c - I*Sqrt[b]*d)/(Sqrt[a]*c + I*Sqrt[b]*d)]))/(a^2*d^2*Sqrt[-c -
(I*Sqrt[b]*d)/Sqrt[a]]*(c + d*x))))/(5*Sqrt[a + b/x^2]*x)
```

Maple [B] time = 0.15, size = 1145, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x+c)^(3/2)/(a+b/x^2)^(1/2),x)
```

```
[Out] 2/5*((-a*b)^(1/2)*(-(d*x+c)*a/((-a*b)^(1/2)*d-a*c))^(1/2)*((-a*x+
(-a*b)^(1/2))*d/((-a*b)^(1/2)*d+a*c))^(1/2)*((a*x+(-a*b)^(1/2))*d
/((-a*b)^(1/2)*d-a*c))^(1/2)*EllipticF((-(d*x+c)*a/((-a*b)^(1/2)*
d-a*c))^(1/2),(-((-a*b)^(1/2)*d-a*c)/((-a*b)^(1/2)*d+a*c))^(1/2))
*a*c^3*d+(-a*b)^(1/2)*(-(d*x+c)*a/((-a*b)^(1/2)*d-a*c))^(1/2)*((-
a*x+(-a*b)^(1/2))*d/((-a*b)^(1/2)*d+a*c))^(1/2)*((a*x+(-a*b)^(1/2
))*d/((-a*b)^(1/2)*d-a*c))^(1/2)*EllipticF((-(d*x+c)*a/((-a*b)^(1
/2)*d-a*c))^(1/2),(-((-a*b)^(1/2)*d-a*c)/((-a*b)^(1/2)*d+a*c))^(1
/2))*b*c*d^3-3*(-(d*x+c)*a/((-a*b)^(1/2)*d-a*c))^(1/2)*((-a*x+(-a
*b)^(1/2))*d/((-a*b)^(1/2)*d+a*c))^(1/2)*((a*x+(-a*b)^(1/2))*d/((
-a*b)^(1/2)*d-a*c))^(1/2)*EllipticF((-(d*x+c)*a/((-a*b)^(1/2)*d-a
*c))^(1/2),(-((-a*b)^(1/2)*d-a*c)/((-a*b)^(1/2)*d+a*c))^(1/2))*a*
b*c^2*d^2-3*b^2*(-(d*x+c)*a/((-a*b)^(1/2)*d-a*c))^(1/2)*((-a*x+(-
a*b)^(1/2))*d/((-a*b)^(1/2)*d+a*c))^(1/2)*((a*x+(-a*b)^(1/2))*d/(
(-a*b)^(1/2)*d-a*c))^(1/2)*EllipticF((-(d*x+c)*a/((-a*b)^(1/2)*d-
a*c))^(1/2),(-((-a*b)^(1/2)*d-a*c)/((-a*b)^(1/2)*d+a*c))^(1/2))*d
^4-(-(d*x+c)*a/((-a*b)^(1/2)*d-a*c))^(1/2)*((-a*x+(-a*b)^(1/2))*d
/((-a*b)^(1/2)*d+a*c))^(1/2)*((a*x+(-a*b)^(1/2))*d/((-a*b)^(1/2)*
d-a*c))^(1/2)*EllipticE((-(d*x+c)*a/((-a*b)^(1/2)*d-a*c))^(1/2),(
-((-a*b)^(1/2)*d-a*c)/((-a*b)^(1/2)*d+a*c))^(1/2))*a^2*c^4+2*(-(d
*x+c)*a/((-a*b)^(1/2)*d-a*c))^(1/2)*((-a*x+(-a*b)^(1/2))*d/((-a*b
)^(1/2)*d+a*c))^(1/2)*((a*x+(-a*b)^(1/2))*d/((-a*b)^(1/2)*d-a*c))
^(1/2)*EllipticE((-(d*x+c)*a/((-a*b)^(1/2)*d-a*c))^(1/2),(-((-a*b
)^(1/2)*d-a*c)/((-a*b)^(1/2)*d+a*c))^(1/2))*a*b*c^2*d^2+3*b^2*(-(
d*x+c)*a/((-a*b)^(1/2)*d-a*c))^(1/2)*((-a*x+(-a*b)^(1/2))*d/((-a*
b)^(1/2)*d+a*c))^(1/2)*((a*x+(-a*b)^(1/2))*d/((-a*b)^(1/2)*d-a*c)
)^(1/2)*EllipticE((-(d*x+c)*a/((-a*b)^(1/2)*d-a*c))^(1/2),(-((-a*
b)^(1/2)*d-a*c)/((-a*b)^(1/2)*d+a*c))^(1/2))*d^4+x^4*a^2*d^4+3*x^
3*a^2*c*d^3+2*x^2*a^2*c^2*d^2+x^2*a*b*d^4+3*x*a*b*c*d^3+2*a*b*c^2
*d^2)/(d*x+c)^(1/2)/d^2/a^2/x/((a*x^2+b)/x^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^{\frac{3}{2}}}{\sqrt{a+\frac{b}{x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)^(3/2)/sqrt(a + b/x^2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(3/2)/sqrt(a + b/x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx+c)^{\frac{3}{2}}}{\sqrt{\frac{ax^2+b}{x^2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)^(3/2)/sqrt(a + b/x^2),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^(3/2)/sqrt((a*x^2 + b)/x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x+c)**(3/2)/(a+b/x**2)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)^(3/2)/sqrt(a + b/x^2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.441 $\int \frac{-1+x^3}{(-4x+x^4)^{2/3}}\,dx$

Optimal. Leaf size=15

$$\frac{3}{4}\sqrt[3]{x^4-4x}$$

```
[Out] (3*(-4*x + x^4)^(1/3))/4
```

Rubi [A] time = 0.00760728, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$

$$\frac{3}{4}\sqrt[3]{x^4-4x}$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + x^3)/(-4*x + x^4)^(2/3),x]

[Out] (3*(-4*x + x^4)^(1/3))/4
```

Rubi in Sympy [A] time = 1.29084, size = 12, normalized size = 0.8

$$\frac{3\sqrt[3]{x^4-4x}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**3-1)/(x**4-4*x)**(2/3),x)

[Out] 3*(x**4 - 4*x)**(1/3)/4
```

Mathematica [A] time = 0.01844, size = 15, normalized size = 1.

$$\frac{3}{4}\sqrt[3]{x\left(x^3-4\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + x^3)/(-4*x + x^4)^(2/3),x]
```

```
[Out] (3*(x*(-4 + x^3))^(1/3))/4
```

Maple [A] time = 0.01, size = 18, normalized size = 1.2

$$\frac{3\,x\left(x^{3}-4\right)}{4}\left(x^{4}-4\,x\right)^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^3-1)/(x^4-4*x)^(2/3),x)
```

```
[Out] 3/4*x*(x^3-4)/(x^4-4*x)^(2/3)
```

Maxima [A] time = 0.724625, size = 15, normalized size = 1.

$$\frac{3}{4}\left(x^{4}-4\,x\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - 1)/(x^4 - 4*x)^(2/3),x, algorithm="maxima")
```

```
[Out] 3/4*(x^4 - 4*x)^(1/3)
```

Fricas [A] time = 0.259559, size = 15, normalized size = 1.

$$\frac{3}{4}\left(x^{4}-4\,x\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - 1)/(x^4 - 4*x)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/4*(x^4 - 4*x)^(1/3)
```

Sympy [A] time = 0.518973, size = 12, normalized size = 0.8

$$\frac{3\sqrt[3]{x^{4}-4x}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3-1)/(x**4-4*x)**(2/3),x)
```

```
[Out] 3*(x**4 - 4*x)**(1/3)/4
```

GIAC/XCAS [A] time = 0.284484, size = 15, normalized size = 1.

$$\frac{3}{4}\left(x^{4}-4\,x\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - 1)/(x^4 - 4*x)^(2/3),x, algorithm="giac")
```

```
[Out] 3/4*(x^4 - 4*x)^(1/3)
```

3.442 $\int \left(2-x^2\right) \sqrt[4]{6x-x^3}\,dx$

Optimal. Leaf size=17

$$\frac{4}{15}\left(6x-x^3\right)^{5/4}$$

```
[Out] (4*(6*x - x^3)^(5/4))/15
```

Rubi [A] time = 0.00850163, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$

$$\frac{4}{15}\left(6x-x^3\right)^{5/4}$$

Antiderivative was successfully verified.

```
[In]  Int[(2 - x^2)*(6*x - x^3)^(1/4),x]

[Out] (4*(6*x - x^3)^(5/4))/15
```

Rubi in Sympy [A] time = 1.63635, size = 12, normalized size = 0.71

$$\frac{4\left(-x^3+6x\right)^{\frac{5}{4}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2+2)*(-x**3+6*x)**(1/4),x)

[Out] 4*(-x**3 + 6*x)**(5/4)/15
```

Mathematica [A] time = 0.0167495, size = 16, normalized size = 0.94

$$\frac{4}{15}\left(-x\left(x^2-6\right)\right)^{5/4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 - x^2)*(6*x - x^3)^(1/4),x]
```

```
[Out] (4*(-(x*(-6 + x^2)))^(5/4))/15
```

Maple [A] time = 0.007, size = 20, normalized size = 1.2

$$-\frac{4\,x\left(x^{2}-6\right)}{15}\sqrt[4]{-x^{3}+6\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2+2)*(-x^3+6*x)^(1/4),x)
```

```
[Out] -4/15*(-x^3+6*x)^(1/4)*x*(x^2-6)
```

Maxima [A] time = 0.733525, size = 18, normalized size = 1.06

$$\frac{4}{15}\left(-x^{3}+6\,x\right)^{\frac{5}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(-x^3 + 6*x)^(1/4)*(x^2 - 2),x, algorithm="maxima")
```

```
[Out] 4/15*(-x^3 + 6*x)^(5/4)
```

Fricas [A] time = 0.259894, size = 27, normalized size = 1.59

$$-\frac{4}{15}\left(x^{3}-6\,x\right)\left(-x^{3}+6\,x\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(-x^3 + 6*x)^(1/4)*(x^2 - 2),x, algorithm="fricas")
```

```
[Out] -4/15*(x^3 - 6*x)*(-x^3 + 6*x)^(1/4)
```

Sympy [A] time = 0.689727, size = 31, normalized size = 1.82

$$-\frac{4x^{3}\sqrt[4]{-x^{3}+6x}}{15}+\frac{8x\sqrt[4]{-x^{3}+6x}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+2)*(-x**3+6*x)**(1/4),x)
```

```
[Out] -4*x**3*(-x**3 + 6*x)**(1/4)/15 + 8*x*(-x**3 + 6*x)**(1/4)/5
```

GIAC/XCAS [A] time = 0.274269, size = 18, normalized size = 1.06

$$\frac{4}{15}\left(-x^3+6\,x\right)^{\frac{5}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(-x^3 + 6*x)^(1/4)*(x^2 - 2),x, algorithm="giac")
```

```
[Out] 4/15*(-x^3 + 6*x)^(5/4)
```

3.443 $\int (1+x^4)\sqrt{5x+x^5}\,dx$

Optimal. Leaf size=15

$$\frac{2}{15}\left(x^5+5x\right)^{3/2}$$

```
[Out] (2*(5*x + x^5)^(3/2))/15
```

Rubi [A] time = 0.00727577, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059

$$\frac{2}{15}\left(x^5+5x\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)*Sqrt[5*x + x^5],x]
```

```
[Out] (2*(5*x + x^5)^(3/2))/15
```

Rubi in Sympy [A] time = 1.27445, size = 12, normalized size = 0.8

$$\frac{2\left(x^5+5x\right)^{\frac{3}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((x**4+1)*(x**5+5*x)**(1/2),x)
```

```
[Out] 2*(x**5 + 5*x)**(3/2)/15
```

Mathematica [A] time = 0.0149563, size = 15, normalized size = 1.

$$\frac{2}{15}\left(x\left(x^4+5\right)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)*Sqrt[5*x + x^5],x]
```

```
[Out] (2*(x*(5 + x^4))^(3/2))/15
```

Maple [A] time = 0.007, size = 18, normalized size = 1.2

$$\frac{2x(x^4+5)}{15}\sqrt{x^5+5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^4+1)*(x^5+5*x)^(1/2),x)
```

```
[Out] 2/15*x*(x^4+5)*(x^5+5*x)^(1/2)
```

Maxima [A] time = 0.691572, size = 15, normalized size = 1.

$$\frac{2}{15}\left(x^5+5x\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^5 + 5*x)*(x^4 + 1),x, algorithm="maxima")
```

```
[Out] 2/15*(x^5 + 5*x)^(3/2)
```

Fricas [A] time = 0.261134, size = 15, normalized size = 1.

$$\frac{2}{15}\left(x^5+5x\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^5 + 5*x)*(x^4 + 1),x, algorithm="fricas")
```

```
[Out] 2/15*(x^5 + 5*x)^(3/2)
```

Sympy [A] time = 0.684449, size = 31, normalized size = 2.07

$$\frac{2x^5\sqrt{x^5+5x}}{15}+\frac{2x\sqrt{x^5+5x}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**4+1)*(x**5+5*x)**(1/2),x)
```

```
[Out] 2*x**5*sqrt(x**5 + 5*x)/15 + 2*x*sqrt(x**5 + 5*x)/3
```

GIAC/XCAS [A] time = 0.271071, size = 15, normalized size = 1.

$$\frac{2}{15}\left(x^{5}+5\,x\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^5 + 5*x)*(x^4 + 1),x, algorithm="giac")
```

```
[Out] 2/15*(x^5 + 5*x)^(3/2)
```

3.444 $\int \left(2+5x^4\right) \sqrt{2x+x^5}\,dx$

Optimal. Leaf size=15

$$\frac{2}{3}\left(x^5+2x\right)^{3/2}$$

```
[Out] (2*(2*x + x^5)^(3/2))/3
```

Rubi [A] time = 0.00718842, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$

$$\frac{2}{3}\left(x^5+2x\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + 5*x^4)*Sqrt[2*x + x^5],x]

[Out] (2*(2*x + x^5)^(3/2))/3
```

Rubi in Sympy [A] time = 1.43104, size = 12, normalized size = 0.8

$$\frac{2\left(x^5+2x\right)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**4+2)*(x**5+2*x)**(1/2),x)

[Out] 2*(x**5 + 2*x)**(3/2)/3
```

Mathematica [A] time = 0.0158078, size = 15, normalized size = 1.

$$\frac{2}{3}\left(x\left(x^4+2\right)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + 5*x^4)*Sqrt[2*x + x^5],x]
```

```
[Out] (2*(x*(2 + x^4))^(3/2))/3
```

Maple [A] time = 0.006, size = 18, normalized size = 1.2

$$\frac{2\,x\,(x^4+2)}{3}\sqrt{x^5+2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4+2)*(x^5+2*x)^(1/2),x)
```

```
[Out] 2/3*x*(x^4+2)*(x^5+2*x)^(1/2)
```

Maxima [A] time = 0.690947, size = 15, normalized size = 1.

$$\frac{2}{3}\,(x^5+2\,x)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x^5 + 2*x)*(5*x^4 + 2),x, algorithm="maxima")
```

```
[Out] 2/3*(x^5 + 2*x)^(3/2)
```

Fricas [A] time = 0.258986, size = 15, normalized size = 1.

$$\frac{2}{3}\,(x^5+2\,x)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x^5 + 2*x)*(5*x^4 + 2),x, algorithm="fricas")
```

```
[Out] 2/3*(x^5 + 2*x)^(3/2)
```

Sympy [A] time = 0.678819, size = 31, normalized size = 2.07

$$\frac{2x^5\sqrt{x^5+2x}}{3}+\frac{4x\sqrt{x^5+2x}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x**4+2)*(x**5+2*x)**(1/2),x)
```

```
[Out] 2*x**5*sqrt(x**5 + 2*x)/3 + 4*x*sqrt(x**5 + 2*x)/3
```

GIAC/XCAS [A] time = 0.272422, size = 15, normalized size = 1.

$$\frac{2}{3}\left(x^5 + 2\,x\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^5 + 2*x)*(5*x^4 + 2),x, algorithm="giac")
```

```
[Out] 2/3*(x^5 + 2*x)^(3/2)
```

3.445 $\int \frac{x+3x^2}{\sqrt{x^2+2x^3}} \, dx$

Optimal. Leaf size=13

$$\sqrt{2x^3 + x^2}$$

```
[Out] Sqrt[x^2 + 2*x^3]
```

Rubi [A] time = 0.0072265, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048

$$\sqrt{2x^3 + x^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x + 3*x^2)/Sqrt[x^2 + 2*x^3],x]

[Out] Sqrt[x^2 + 2*x^3]
```

Rubi in Sympy [A] time = 1.91949, size = 10, normalized size = 0.77

$$\sqrt{2x^3 + x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+x)/(2*x**3+x**2)**(1/2),x)

[Out] sqrt(2*x**3 + x**2)
```

Mathematica [A] time = 0.0132076, size = 13, normalized size = 1.

$$\sqrt{x^2(2x + 1)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x + 3*x^2)/Sqrt[x^2 + 2*x^3],x]
```

```
[Out] Sqrt[x^2*(1 + 2*x)]
```

Maple [A] time = 0.004, size = 21, normalized size = 1.6

$$x^{2}\left(1+2\,x\right)\frac{1}{\sqrt{2\,x^{3}+x^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2+x)/(2*x^3+x^2)^(1/2),x)
```

```
[Out] x^2*(1+2*x)/(2*x^3+x^2)^(1/2)
```

Maxima [A] time = 0.692179, size = 15, normalized size = 1.15

$$\sqrt{2\,x^{3}+x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + x)/sqrt(2*x^3 + x^2),x, algorithm="maxima")
```

```
[Out] sqrt(2*x^3 + x^2)
```

Fricas [A] time = 0.272712, size = 15, normalized size = 1.15

$$\sqrt{2\,x^{3}+x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + x)/sqrt(2*x^3 + x^2),x, algorithm="fricas")
```

```
[Out] sqrt(2*x^3 + x^2)
```

Sympy [A] time = 0.340961, size = 10, normalized size = 0.77

$$\sqrt{2x^{3}+x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**2+x)/(2*x**3+x**2)**(1/2),x)
```

```
[Out] sqrt(2*x**3 + x**2)
```

GIAC/XCAS [A] time = 0.27028, size = 15, normalized size = 1.15

$$\sqrt{2x^3 + x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + x)/sqrt(2*x^3 + x^2),x, algorithm="giac")
```

```
[Out] sqrt(2*x^3 + x^2)
```

3.446 $\int \frac{2+\sqrt[3]{1-5x}}{3+\sqrt[3]{1-5x}}\,dx$

Optimal. Leaf size=44

$$x+\frac{3}{10}(1-5x)^{2/3}-\frac{9}{5}\sqrt[3]{1-5x}+\frac{27}{5}\log\left(\sqrt[3]{1-5x}+3\right)$$

```
[Out] (-9*(1 - 5*x)^(1/3))/5 + (3*(1 - 5*x)^(2/3))/10 + x + (27*Log[3 +
 (1 - 5*x)^(1/3)])/5
```

Rubi [A] time = 0.0588394, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12

$$x+\frac{3}{10}(1-5x)^{2/3}-\frac{9}{5}\sqrt[3]{1-5x}+\frac{27}{5}\log\left(\sqrt[3]{1-5x}+3\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + (1 - 5*x)^(1/3))/(3 + (1 - 5*x)^(1/3)),x]
```

```
[Out] (-9*(1 - 5*x)^(1/3))/5 + (3*(1 - 5*x)^(2/3))/10 + x + (27*Log[3 +
 (1 - 5*x)^(1/3)])/5
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$x-\frac{9\sqrt[3]{-5x+1}}{5}+\frac{27\log\left(\sqrt[3]{-5x+1}+3\right)}{5}+\frac{3\int^{\sqrt[3]{-5x+1}} x\,dx}{5}-\frac{1}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2+(1-5*x)**(1/3))/(3+(1-5*x)**(1/3)),x)
```

```
[Out] x - 9*(-5*x + 1)**(1/3)/5 + 27*log((-5*x + 1)**(1/3) + 3)/5 + 3*I
ntegral(x, (x, (-5*x + 1)**(1/3)))/5 - 1/5
```

Mathematica [A] time = 0.0197138, size = 45, normalized size = 1.02

$$\frac{1}{10}\left(10x+3(1-5x)^{2/3}-18\sqrt[3]{1-5x}+54\log\left(\sqrt[3]{1-5x}+3\right)-2\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + (1 - 5*x)^(1/3))/(3 + (1 - 5*x)^(1/3)),x]
```

```
[Out] (-2 - 18*(1 - 5*x)^(1/3) + 3*(1 - 5*x)^(2/3) + 10*x + 54*Log[3 +
(1 - 5*x)^(1/3)])/10
```

Maple [A] time = 0.004, size = 34, normalized size = 0.8

$$-\frac{1}{5}+x+\frac{3}{10}(1-5x)^{\frac{2}{3}}-\frac{9}{5}\sqrt[3]{1-5x}+\frac{27}{5}\ln\left(3+\sqrt[3]{1-5x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2+(1-5*x)^(1/3))/(3+(1-5*x)^(1/3)),x)
```

```
[Out] -1/5+x+3/10*(1-5*x)^(2/3)-9/5*(1-5*x)^(1/3)+27/5*ln(3+(1-5*x)^(1/
3))
```

Maxima [A] time = 0.698118, size = 45, normalized size = 1.02

$$x+\frac{3}{10}(-5x+1)^{\frac{2}{3}}-\frac{9}{5}(-5x+1)^{\frac{1}{3}}+\frac{27}{5}\log\left((-5x+1)^{\frac{1}{3}}+3\right)-\frac{1}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((-5*x + 1)^(1/3) + 2)/((-5*x + 1)^(1/3) + 3),x, algorithm="maxima")
```

```
[Out] x + 3/10*(-5*x + 1)^(2/3) - 9/5*(-5*x + 1)^(1/3) + 27/5*log((-5*x
 + 1)^(1/3) + 3) - 1/5
```

Fricas [A] time = 0.262665, size = 43, normalized size = 0.98

$$x+\frac{3}{10}(-5x+1)^{\frac{2}{3}}-\frac{9}{5}(-5x+1)^{\frac{1}{3}}+\frac{27}{5}\log\left((-5x+1)^{\frac{1}{3}}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((-5*x + 1)^(1/3) + 2)/((-5*x + 1)^(1/3) + 3),x, algorithm="fricas")
```

```
[Out] x + 3/10*(-5*x + 1)^(2/3) - 9/5*(-5*x + 1)^(1/3) + 27/5*log((-5*x
 + 1)^(1/3) + 3)
```

Sympy [A] time = 0.468949, size = 39, normalized size = 0.89

$$x + \frac{3(-5x+1)^{\frac{2}{3}}}{10} - \frac{9\sqrt[3]{-5x+1}}{5} + \frac{27\log\left(\sqrt[3]{-5x+1}+3\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2+(1-5*x)**(1/3))/(3+(1-5*x)**(1/3)),x)
```

```
[Out] x + 3*(-5*x + 1)**(2/3)/10 - 9*(-5*x + 1)**(1/3)/5 + 27*log((-5*x
 + 1)**(1/3) + 3)/5
```

GIAC/XCAS [A] time = 0.278195, size = 45, normalized size = 1.02

$$x + \frac{3}{10}(-5x+1)^{\frac{2}{3}} - \frac{9}{5}(-5x+1)^{\frac{1}{3}} + \frac{27}{5}\ln\left((-5x+1)^{\frac{1}{3}}+3\right) - \frac{1}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((-5*x + 1)^(1/3) + 2)/((-5*x + 1)^(1/3) + 3),x, algorithm="giac")
```

```
[Out] x + 3/10*(-5*x + 1)^(2/3) - 9/5*(-5*x + 1)^(1/3) + 27/5*ln((-5*x
+ 1)^(1/3) + 3) - 1/5
```

3.447 $\int \frac{1+\sqrt{x}}{-1+\sqrt{x}}\,dx$

Optimal. Leaf size=21

$$x + 4\sqrt{x} + 4\log\left(1 - \sqrt{x}\right)$$

```
[Out] 4*Sqrt[x] + x + 4*Log[1 - Sqrt[x]]
```

Rubi [A] time = 0.0307865, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$

$$x + 4\sqrt{x} + 4\log\left(1 - \sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + Sqrt[x])/(-1 + Sqrt[x]),x]

[Out] 4*Sqrt[x] + x + 4*Log[1 - Sqrt[x]]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$4\sqrt{x} + 4\log\left(-\sqrt{x} + 1\right) + 2\int^{\sqrt{x}} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+x**(1/2))/(-1+x**(1/2)),x)

[Out] 4*sqrt(x) + 4*log(-sqrt(x) + 1) + 2*Integral(x, (x, sqrt(x)))
```

Mathematica [A] time = 0.00834228, size = 20, normalized size = 0.95

$$x + 4\sqrt{x} + 4\log\left(\sqrt{x} - 1\right) - 5$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + Sqrt[x])/(-1 + Sqrt[x]),x]
```

```
[Out] -5 + 4*Sqrt[x] + x + 4*Log[-1 + Sqrt[x]]
```

Maple [A] time = 0.003, size = 16, normalized size = 0.8

$$x + 4\sqrt{x} + 4\ln\left(-1 + \sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x^(1/2))/(-1+x^(1/2)),x)
```

```
[Out] x+4*x^(1/2)+4*ln(-1+x^(1/2))
```

Maxima [A] time = 0.693775, size = 20, normalized size = 0.95

$$x + 4\sqrt{x} + 4\log\left(\sqrt{x} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sqrt(x) + 1)/(sqrt(x) - 1),x, algorithm="maxima")
```

```
[Out] x + 4*sqrt(x) + 4*log(sqrt(x) - 1)
```

Fricas [A] time = 0.261775, size = 20, normalized size = 0.95

$$x + 4\sqrt{x} + 4\log\left(\sqrt{x} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sqrt(x) + 1)/(sqrt(x) - 1),x, algorithm="fricas")
```

```
[Out] x + 4*sqrt(x) + 4*log(sqrt(x) - 1)
```

Sympy [A] time = 0.312255, size = 17, normalized size = 0.81

$$4\sqrt{x} + x + 4\log\left(\sqrt{x} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+x**(1/2))/(-1+x**(1/2)),x)
```

```
[Out] 4*sqrt(x) + x + 4*log(sqrt(x) - 1)
```

GIAC/XCAS [A] time = 0.272663, size = 22, normalized size = 1.05

$$x + 4\sqrt{x} + 4\ln\left(\left|\sqrt{x} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(x) + 1)/(sqrt(x) - 1),x, algorithm="giac")
```

```
[Out] x + 4*sqrt(x) + 4*ln(abs(sqrt(x) - 1))
```

3.448 $\int \frac{1-\sqrt{2+3x}}{1+\sqrt{2+3x}}\,dx$

Optimal. Leaf size=33

$$-x+\frac{4}{3}\sqrt{3x+2}-\frac{4}{3}\log\left(\sqrt{3x+2}+1\right)$$

```
[Out] -x + (4*Sqrt[2 + 3*x])/3 - (4*Log[1 + Sqrt[2 + 3*x]])/3
```

Rubi [A] time = 0.0520846, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$-x+\frac{4}{3}\sqrt{3x+2}-\frac{4}{3}\log\left(\sqrt{3x+2}+1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - Sqrt[2 + 3*x])/(1 + Sqrt[2 + 3*x]),x]
```

```
[Out] -x + (4*Sqrt[2 + 3*x])/3 - (4*Log[1 + Sqrt[2 + 3*x]])/3
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{4\sqrt{3x+2}}{3}-\frac{4\log\left(\sqrt{3x+2}+1\right)}{3}-\frac{2\int^{\sqrt{3x+2}}x\,dx}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1-(2+3*x)**(1/2))/(1+(2+3*x)**(1/2)),x)
```

```
[Out] 4*sqrt(3*x + 2)/3 - 4*log(sqrt(3*x + 2) + 1)/3 - 2*Integral(x, (x
, sqrt(3*x + 2)))/3
```

Mathematica [A] time = 0.0167514, size = 34, normalized size = 1.03

$$\frac{1}{3}\left(-3x+4\sqrt{3x+2}-4\log\left(\sqrt{3x+2}+1\right)+3\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - Sqrt[2 + 3*x])/(1 + Sqrt[2 + 3*x]),x]

[Out] (3 - 3*x + 4*Sqrt[2 + 3*x] - 4*Log[1 + Sqrt[2 + 3*x]])/3
```

Maple [A] time = 0.005, size = 27, normalized size = 0.8

$$-\frac{2}{3}-x+\frac{4}{3}\sqrt{2+3x}-\frac{4}{3}\ln\left(1+\sqrt{2+3x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1-(2+3*x)^(1/2))/(1+(2+3*x)^(1/2)),x)

[Out] -2/3-x+4/3*(2+3*x)^(1/2)-4/3*ln(1+(2+3*x)^(1/2))
```

Maxima [A] time = 0.69277, size = 35, normalized size = 1.06

$$-x+\frac{4}{3}\sqrt{3x+2}-\frac{4}{3}\log\left(\sqrt{3x+2}+1\right)-\frac{2}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(sqrt(3*x + 2) - 1)/(sqrt(3*x + 2) + 1),x, algorithm="maxima")

[Out] -x + 4/3*sqrt(3*x + 2) - 4/3*log(sqrt(3*x + 2) + 1) - 2/3
```

Fricas [A] time = 0.261773, size = 34, normalized size = 1.03

$$-x+\frac{4}{3}\sqrt{3x+2}-\frac{4}{3}\log\left(\sqrt{3x+2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(sqrt(3*x + 2) - 1)/(sqrt(3*x + 2) + 1),x, algorithm="fricas")

[Out] -x + 4/3*sqrt(3*x + 2) - 4/3*log(sqrt(3*x + 2) + 1)
```

Sympy [A] time = 0.371301, size = 27, normalized size = 0.82

$$-x+\frac{4\sqrt{3x+2}}{3}-\frac{4\log\left(\sqrt{3x+2}+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1-(2+3*x)**(1/2))/(1+(2+3*x)**(1/2)),x)
```

```
[Out] -x + 4*sqrt(3*x + 2)/3 - 4*log(sqrt(3*x + 2) + 1)/3
```

GIAC/XCAS [A] time = 0.273814, size = 35, normalized size = 1.06

$$-x+\frac{4}{3}\sqrt{3x+2}-\frac{4}{3}\ln\left(\sqrt{3x+2}+1\right)-\frac{2}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(sqrt(3*x + 2) - 1)/(sqrt(3*x + 2) + 1),x, algorithm="giac")
```

```
[Out] -x + 4/3*sqrt(3*x + 2) - 4/3*ln(sqrt(3*x + 2) + 1) - 2/3
```

3.449 $\int \frac{-1+\sqrt{a+bx}}{1+\sqrt{a+bx}}\,dx$

Optimal. Leaf size=33

$$-\frac{4\sqrt{a+bx}}{b}+\frac{4\log\left(\sqrt{a+bx}+1\right)}{b}+x$$

```
[Out] x - (4*Sqrt[a + b*x])/b + (4*Log[1 + Sqrt[a + b*x]])/b
```

Rubi [A] time = 0.0575947, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$

$$-\frac{4\sqrt{a+bx}}{b}+\frac{4\log\left(\sqrt{a+bx}+1\right)}{b}+x$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + Sqrt[a + b*x])/(1 + Sqrt[a + b*x]),x]
```

```
[Out] x - (4*Sqrt[a + b*x])/b + (4*Log[1 + Sqrt[a + b*x]])/b
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{4\sqrt{a+bx}}{b}+\frac{4\log\left(\sqrt{a+bx}+1\right)}{b}+\frac{2\int^{\sqrt{a+bx}} x\,dx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-1+(b*x+a)**(1/2))/(1+(b*x+a)**(1/2)),x)
```

```
[Out] -4*sqrt(a + b*x)/b + 4*log(sqrt(a + b*x) + 1)/b + 2*Integral(x, (
x, sqrt(a + b*x)))/b
```

Mathematica [A] time = 0.0221819, size = 35, normalized size = 1.06

$$\frac{-4\sqrt{a+bx}+4\log\left(\sqrt{a+bx}+1\right)+a+bx-5}{b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + Sqrt[a + b*x])/(1 + Sqrt[a + b*x]),x]
```

```
[Out] (-5 + a + b*x - 4*Sqrt[a + b*x] + 4*Log[1 + Sqrt[a + b*x]])/b
```

Maple [A] time = 0.003, size = 35, normalized size = 1.1

$$x+\frac{a}{b}-4\,\frac{\sqrt{bx+a}}{b}+4\,\frac{\ln\left(1+\sqrt{bx+a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-1+(b*x+a)^(1/2))/(1+(b*x+a)^(1/2)),x)
```

```
[Out] x+a/b-4*(b*x+a)^(1/2)/b+4*ln(1+(b*x+a)^(1/2))/b
```

Maxima [A] time = 0.694523, size = 41, normalized size = 1.24

$$\frac{bx+a-4\,\sqrt{bx+a}+4\,\log\left(\sqrt{bx+a}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(b*x + a) - 1)/(sqrt(b*x + a) + 1),x, algorithm="maxima")
```

```
[Out] (b*x + a - 4*sqrt(b*x + a) + 4*log(sqrt(b*x + a) + 1))/b
```

Fricas [A] time = 0.265543, size = 39, normalized size = 1.18

$$\frac{bx-4\,\sqrt{bx+a}+4\,\log\left(\sqrt{bx+a}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(b*x + a) - 1)/(sqrt(b*x + a) + 1),x, algorithm="fricas")
```

```
[Out] (b*x - 4*sqrt(b*x + a) + 4*log(sqrt(b*x + a) + 1))/b
```

Sympy [A] time = 1.67776, size = 42, normalized size = 1.27

$$\begin{cases} x - \frac{4\sqrt{a+bx}}{b} + \frac{4\log\left(\sqrt{a+bx}+1\right)}{b} & \text{for } b \neq 0 \\ \frac{x\left(\sqrt{a}-1\right)}{\sqrt{a}+1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-1+(b*x+a)**(1/2))/(1+(b*x+a)**(1/2)),x)
```

```
[Out] Piecewise((x - 4*sqrt(a + b*x)/b + 4*log(sqrt(a + b*x) + 1)/b, Ne
(b, 0)), (x*(sqrt(a) - 1)/(sqrt(a) + 1), True))
```

GIAC/XCAS [A] time = 0.275703, size = 51, normalized size = 1.55

$$\frac{4\ln\left(\sqrt{bx+a}+1\right)}{b} + \frac{(bx+a)b - 4\sqrt{bx+a}b}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(b*x + a) - 1)/(sqrt(b*x + a) + 1),x, algorithm="giac")
```

```
[Out] 4*ln(sqrt(b*x + a) + 1)/b + ((b*x + a)*b - 4*sqrt(b*x + a)*b)/b^2
```

3.450 $\int \frac{a+bnx^{-1+n}}{ax+bx^n}\,dx$

Optimal. Leaf size=10

$$\log\left(ax+bx^n\right)$$

```
[Out] Log[a*x + b*x^n]
```

Rubi [A] time = 0.140964, antiderivative size = 17, normalized size of antiderivative = 1.7, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\log\left(ax^{1-n}+b\right)+n\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*n*x^(-1 + n))/(a*x + b*x^n),x]

[Out] n*Log[x] + Log[b + a*x^(1 - n)]
```

Rubi in Sympy [A] time = 8.96809, size = 20, normalized size = 2.

$$\frac{n\log\left(x^{-n+1}\right)}{-n+1}+\log\left(ax^{-n+1}+b\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b*n*x**(-1+n))/(a*x+b*x**n),x)

[Out] n*log(x**(-n + 1))/(-n + 1) + log(a*x**(-n + 1) + b)
```

Mathematica [A] time = 0.018744, size = 10, normalized size = 1.

$$\log\left(ax+bx^n\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*n*x^(-1 + n))/(a*x + b*x^n),x]
```

```
[Out] Log[a*x + b*x^n]
```

Maple [A] time = 0.024, size = 13, normalized size = 1.3

$$\ln\left(ax + b\,e^{n\ln(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b*n*x^(-1+n))/(a*x+b*x^n),x)

[Out] ln(a*x+b*exp(n*ln(x)))
```

Maxima [A] time = 0.686981, size = 14, normalized size = 1.4

$$\log\left(ax + bx^{n}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*n*x^(n - 1) + a)/(a*x + b*x^n),x, algorithm="maxima")

[Out] log(a*x + b*x^n)
```

Fricas [A] time = 0.308787, size = 14, normalized size = 1.4

$$\log\left(ax + bx^{n}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*n*x^(n - 1) + a)/(a*x + b*x^n),x, algorithm="fricas")

[Out] log(a*x + b*x^n)
```

Sympy [A] time = 30.6255, size = 32, normalized size = 3.2

$$\begin{cases} \log\left(x + \frac{bx^{n}}{a}\right) & \text{for}\: a \neq 0 \\ n\left(\frac{n^{2}\log(x)}{n^{2}-n} - \frac{n\log(x)}{n^{2}-n}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b*n*x**(-1+n))/(a*x+b*x**n),x)
```

```
[Out] Piecewise((log(x + b*x**n/a), Ne(a, 0)), (n*(n**2*log(x)/(n**2 -
n) - n*log(x)/(n**2 - n)), True))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bnx^{n-1} + a}{ax + bx^n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*n*x^(n - 1) + a)/(a*x + b*x^n),x, algorithm="giac")
```

```
[Out] integrate((b*n*x^(n - 1) + a)/(a*x + b*x^n), x)
```

3.451 $\int \frac{x^{-n}\left(a+bnx^{-1+n}\right)}{b+ax^{1-n}}\,dx$

Optimal. Leaf size=17

$$\log\left(ax^{1-n}+b\right)+n\log(x)$$

```
[Out] n*Log[x] + Log[b + a*x^(1 - n)]
```

Rubi [A] time = 0.124116, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103

$$\log\left(ax^{1-n}+b\right)+n\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*n*x^(-1 + n))/(x^n*(b + a*x^(1 - n))),x]

[Out] n*Log[x] + Log[b + a*x^(1 - n)]
```

Rubi in Sympy [A] time = 8.12144, size = 20, normalized size = 1.18

$$\frac{n\log\left(x^{-n+1}\right)}{-n+1}+\log\left(ax^{-n+1}+b\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b*n*x**(-1+n))/(x**n)/(b+a*x**(1-n)),x)

[Out] n*log(x**(-n + 1))/(-n + 1) + log(a*x**(-n + 1) + b)
```

Mathematica [A] time = 0.0125254, size = 10, normalized size = 0.59

$$\log\left(ax+bx^n\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*n*x^(-1 + n))/(x^n*(b + a*x^(1 - n))),x]
```

```
[Out] Log[a*x + b*x^n]
```

Maple [A] time = 0.035, size = 13, normalized size = 0.8

$$\ln\left(ax+b\,\mathrm{e}^{n\ln(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b*n*x^(-1+n))/(x^n)/(b+a*x^(1-n)),x)
```

```
[Out] ln(a*x+b*exp(n*ln(x)))
```

Maxima [A] time = 0.701296, size = 116, normalized size = 6.82

$$bn\left(\frac{\log(x)}{b}-\frac{n\log(x)}{b(n-1)}+\frac{\log\left(\frac{ax+bx^n}{b}\right)}{b(n-1)}\right)+a\left(\frac{n\log(x)}{a(n-1)}-\frac{\log\left(\frac{ax+bx^n}{b}\right)}{a(n-1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*n*x^(n - 1) + a)/((a*x^(-n + 1) + b)*x^n),x, algorithm="maxima")
```

```
[Out] b*n*(log(x)/b - n*log(x)/(b*(n - 1)) + log((a*x + b*x^n)/b)/(b*(n
 - 1))) + a*(n*log(x)/(a*(n - 1)) - log((a*x + b*x^n)/b)/(a*(n -
1)))
```

Fricas [A] time = 0.275187, size = 14, normalized size = 0.82

$$\log\left(ax+bx^n\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*n*x^(n - 1) + a)/((a*x^(-n + 1) + b)*x^n),x, algorithm="fricas")
```

```
[Out] log(a*x + b*x^n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b*n*x**(-1+n))/(x**n)/(b+a*x**(1-n)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bnx^{n-1} + a}{(ax^{-n+1} + b)x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*n*x^(n - 1) + a)/((a*x^(-n + 1) + b)*x^n),x, algorithm="giac")
```

```
[Out] integrate((b*n*x^(n - 1) + a)/((a*x^(-n + 1) + b)*x^n), x)
```

3.452 $\int x\left(a+bx+cx^2\right)^m\left(d+ex+fx^2+gx^3\right)^n\left(2ad+(3bd+3ae+bdm\right.$

Optimal. Leaf size=37

$$x^2\left(a+bx+cx^2\right)^{m+1}\left(d+ex+fx^2+gx^3\right)^{n+1}$$

```
[Out] x^2*(a + b*x + c*x^2)^(1 + m)*(d + e*x + f*x^2 + g*x^3)^(1 + n)
```

Rubi [A] time = 0.0604921, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 176, $\frac{\text{number of rules}}{\text{integrand size}} = 0.006$

$$x^2\left(a+bx+cx^2\right)^{m+1}\left(d+ex+fx^2+gx^3\right)^{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(2*a*d + (3*b*d + 3*a*e + b*d
```

```
[Out] x^2*(a + b*x + c*x^2)^(1 + m)*(d + e*x + f*x^2 + g*x^3)^(1 + n)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x*(c*x**2+b*x+a)**m*(g*x**3+f*x**2+e*x+d)**n*(2*a*d+(a*e*n+b*d*
```

```
[Out] Timed out
```

Mathematica [A] time = 0.566681, size = 34, normalized size = 0.92

$$x^2(a+x(b+cx))^{m+1}(d+x(e+x(f+gx)))^{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(2*a*d + (3*b*d + 3*a*e
```

```
[Out] x^2*(a + x*(b + c*x))^(1 + m)*(d + x*(e + x*(f + g*x)))^(1 + n)
```

Maple [A] time = 0.034, size = 38, normalized size = 1.

$$x^{2} \left(c x^{2}+b x +a \right)^{1+m} \left(g \,x^{3}+f \,x^{2}+e x +d \right)^{1+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(2*a*d+(a*e*n+b*d*m+3*a*e+3*b*d)*x
```

```
[Out] x^2*(c*x^2+b*x+a)^(1+m)*(g*x^3+f*x^2+e*x+d)^(1+n)
```

Maxima [A] time = 0.904313, size = 131, normalized size = 3.54

$$\left(cgx^{7} + (cf + bg)x^{6} + (ce + bf + ag)x^{5} + (cd + be + af)x^{4} + adx^{2} + (bd + ae)x^{3}\right)e^{\left(n\log\left(gx^{3}+fx^{2}+ex+d\right)+m\log\left(cx^{2}+bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*g*(2*m + 3*n + 7)*x^5 + (2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n + 6*c*f +
```

```
[Out] (c*g*x^7 + (c*f + b*g)*x^6 + (c*e + b*f + a*g)*x^5 + (c*d + b*e +
 a*f)*x^4 + a*d*x^2 + (b*d + a*e)*x^3)*e^(n*log(g*x^3 + f*x^2 + e
*x + d) + m*log(c*x^2 + b*x + a))
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*g*(2*m + 3*n + 7)*x^5 + (2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n + 6*c*f +
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2+b*x+a)**m*(g*x**3+f*x**2+e*x+d)**n*(2*a*d+(a*e*n+b*d*m+3*a
```

```
[Out] Timed out
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*g*(2*m + 3*n + 7)*x^5 + (2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n + 6*c*f +
```

```
[Out] Timed out
```

3.453 $\int \left(a+bx+cx^2\right)^m \left(d+ex+fx^2+gx^3\right)^n \left(ad+(2bd+2ae+bdm+\right.$

Optimal. Leaf size=35

$$x\left(a+bx+cx^2\right)^{m+1}\left(d+ex+fx^2+gx^3\right)^{n+1}$$

```
[Out] x*(a + b*x + c*x^2)^(1 + m)*(d + e*x + f*x^2 + g*x^3)^(1 + n)
```

Rubi [A] time = 0.0367923, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 174, $\frac{\text{number of rules}}{\text{integrand size}} = 0.006$

$$x\left(a+bx+cx^2\right)^{m+1}\left(d+ex+fx^2+gx^3\right)^{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(a*d + (2*b*d + 2*a*e + b*d*m +
```

```
[Out] x*(a + b*x + c*x^2)^(1 + m)*(d + e*x + f*x^2 + g*x^3)^(1 + n)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((c*x**2+b*x+a)**m*(g*x**3+f*x**2+e*x+d)**n*(a*d+(a*e*n+b*d*m+2*
```

```
[Out] Timed out
```

Mathematica [A] time = 0.578451, size = 32, normalized size = 0.91

$$x(a+x(b+cx))^{m+1}(d+x(e+x(f+gx)))^{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(a*d + (2*b*d + 2*a*e + b
```

```
[Out] x*(a + x*(b + c*x))^(1 + m)*(d + x*(e + x*(f + g*x)))^(1 + n)
```

Maple [A] time = 0.034, size = 36, normalized size = 1.

$$x\left(cx^{2}+bx+a\right)^{1+m}\left(gx^{3}+fx^{2}+ex+d\right)^{1+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(a*d+(a*e*n+b*d*m+2*a*e+2*b*d)*x+(2*
```

```
[Out] x*(c*x^2+b*x+a)^(1+m)*(g*x^3+f*x^2+e*x+d)^(1+n)
```

Maxima [A] time = 0.95988, size = 128, normalized size = 3.66

$$\left(cgx^{6}+(cf+bg)x^{5}+(ce+bf+ag)x^{4}+(cd+be+af)x^{3}+adx+(bd+ae)x^{2}\right)e^{\left(n\log\left(gx^{3}+fx^{2}+ex+d\right)+m\log\left(cx^{2}+bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*g*(2*m + 3*n + 6)*x^5 + (2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n + 5*c*f +
```

```
[Out] (c*g*x^6 + (c*f + b*g)*x^5 + (c*e + b*f + a*g)*x^4 + (c*d + b*e +
 a*f)*x^3 + a*d*x + (b*d + a*e)*x^2)*e^(n*log(g*x^3 + f*x^2 + e*x
 + d) + m*log(c*x^2 + b*x + a))
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*g*(2*m + 3*n + 6)*x^5 + (2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n + 5*c*f +
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**m*(g*x**3+f*x**2+e*x+d)**n*(a*d+(a*e*n+b*d*m+2*a*e+2
```

```
[Out] Timed out
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*g*(2*m + 3*n + 6)*x^5 + (2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n + 5*c*f +
```

```
[Out] Timed out
```

3.454 $\int (a + bx + cx^2)^m (d + ex + fx^2 + gx^3)^n (bd + ae + bdm + aen + (2$

Optimal. Leaf size=34

$$(a + bx + cx^2)^{m+1} (d + ex + fx^2 + gx^3)^{n+1}$$

```
[Out] (a + b*x + c*x^2)^(1 + m)*(d + e*x + f*x^2 + g*x^3)^(1 + n)
```

Rubi [A] time = 0.0411668, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 164, $\frac{\text{number of rules}}{\text{integrand size}} = 0.006$

$$(a + bx + cx^2)^{m+1} (d + ex + fx^2 + gx^3)^{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(b*d + a*e + b*d*m + a*e*n + (2*
```

```
[Out] (a + b*x + c*x^2)^(1 + m)*(d + e*x + f*x^2 + g*x^3)^(1 + n)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((c*x**2+b*x+a)**m*(g*x**3+f*x**2+e*x+d)**n*(b*d+a*e+b*d*m+a*e*n
```

```
[Out] Timed out
```

Mathematica [A] time = 0.536428, size = 31, normalized size = 0.91

$$(a + x(b + cx))^{m+1}(d + x(e + x(f + gx)))^{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(b*d + a*e + b*d*m + a*e*
```

```
[Out] (a + x*(b + c*x))^(1 + m)*(d + x*(e + x*(f + g*x)))^(1 + n)
```

Maple [A] time = 0.031, size = 35, normalized size = 1.

$$\left(c x^{2}+b x+a\right)^{1+m} \left(g x^{3}+f x^{2}+e x+d\right)^{1+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(b*d+a*e+b*d*m+a*e*n+(2*a*f*n+b*e*m+
```

```
[Out] (c*x^2+b*x+a)^(1+m)*(g*x^3+f*x^2+e*x+d)^(1+n)
```

Maxima [A] time = 0.919864, size = 124, normalized size = 3.65

$$\left(cgx^{5}+(cf+bg)x^{4}+(ce+bf+ag)x^{3}+(cd+be+af)x^{2}+ad+(bd+ae)x\right)e^{\left(n\log\left(gx^{3}+fx^{2}+ex+d\right)+m\log\left(cx^{2}+bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*g*(2*m + 3*n + 5)*x^4 + (2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n + 4*c*f +
```

```
[Out] (c*g*x^5 + (c*f + b*g)*x^4 + (c*e + b*f + a*g)*x^3 + (c*d + b*e +
 a*f)*x^2 + a*d + (b*d + a*e)*x)*e^(n*log(g*x^3 + f*x^2 + e*x + d
) + m*log(c*x^2 + b*x + a))
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*g*(2*m + 3*n + 5)*x^4 + (2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n + 4*c*f +
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**m*(g*x**3+f*x**2+e*x+d)**n*(b*d+a*e+b*d*m+a*e*n+(2*a
```

```
[Out] Timed out
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*g*(2*m + 3*n + 5)*x^4 + (2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n + 4*c*f +
```

```
[Out] Timed out
```

3.455 $\int \frac{(a+bx+cx^2)^m(d+ex+fx^2+gx^3)^n(-ad+(bdm+aen)x+(cd+be+af+2cdm+bem+ben+2afn)x$

Optimal. Leaf size=37

$$\frac{(a+bx+cx^2)^{m+1}(d+ex+fx^2+gx^3)^{n+1}}{x}$$

```
[Out] ((a + b*x + c*x^2)^(1 + m)*(d + e*x + f*x^2 + g*x^3)^(1 + n))/x
```

Rubi [F] time = 8.29014, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$

$$\text{Int}\left(\frac{(a+bx+cx^2)^m(d+ex+fx^2+gx^3)^n(-ad+(bdm+aen)x+(cd+be+af+2cdm+bem+ben+2afn)x^2+(2ce+}{x}\right.$$

Verification is Not applicable to the result.

```
[In] Int[((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(-(a*d) + (b*d*m + a*e*n)*x +
```

```
[Out] (c*(d + 2*d*m) + b*e*(1 + m + n) + a*f*(1 + 2*n))*Defer[Int][(a +
 b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n, x] - a*d*Defer[Int][
((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n)/x^2, x] + (b*d*
m + a*e*n)*Defer[Int][((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x
^3)^n)/x, x] + (c*e*(2 + 2*m + n) + b*f*(2 + m + 2*n) + a*g*(2 +
3*n))*Defer[Int][x*(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^
n, x] + (c*f*(3 + 2*m + 2*n) + b*g*(3 + m + 3*n))*Defer[Int][x^2*
(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n, x] + c*g*(4 + 2*
m + 3*n)*Defer[Int][x^3*(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*
x^3)^n, x]
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((c*x**2+b*x+a)**m*(g*x**3+f*x**2+e*x+d)**n*(-a*d+(a*e*n+b*d*m)*
```

```
[Out] Timed out
```

Mathematica [A] time = 1.52376, size = 34, normalized size = 0.92

$$\frac{(a+x(b+cx))^{m+1}(d+x(e+x(f+gx)))^{n+1}}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(-
(a*d) + (b*d*m + a*e*n)*x + (c*d + b*e + a*f + 2*c*d*m + b*e*m + b*e*n + 2*a*f*n)*x^2 + (2
```

```
[Out] ((a + x*(b + c*x))^(1 + m)*(d + x*(e + x*(f + g*x)))^(1 + n))/x
```

Maple [A] time = 0.035, size = 38, normalized size = 1.

$$\frac{\left(cx^2+bx+a\right)^{1+m}\left(gx^3+fx^2+ex+d\right)^{1+n}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(-a*d+(a*e*n+b*d*m)*x+(2*a*f*n+b*e*m
```

```
[Out] (c*x^2+b*x+a)^(1+m)*(g*x^3+f*x^2+e*x+d)^(1+n)/x
```

Maxima [A] time = 0.964169, size = 128, normalized size = 3.46

$$\frac{\left(cgx^5+(cf+bg)x^4+(ce+bf+ag)x^3+(cd+be+af)x^2+ad+(bd+ae)x\right)e^{\left(n\log\left(gx^3+fx^2+ex+d\right)+m\log\left(cx^2+bx+a\right)\right)}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*g*(2*m + 3*n + 4)*x^5 + (2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n + 3*c*f +
```

```
[Out] (c*g*x^5 + (c*f + b*g)*x^4 + (c*e + b*f + a*g)*x^3 + (c*d + b*e +
 a*f)*x^2 + a*d + (b*d + a*e)*x)*e^(n*log(g*x^3 + f*x^2 + e*x + d
) + m*log(c*x^2 + b*x + a))/x
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*g*(2*m + 3*n + 4)*x^5 + (2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n + 3*c*f +
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**m*(g*x**3+f*x**2+e*x+d)**n*(-a*d+(a*e*n+b*d*m)*x+(2*
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cg(2\,m+3\,n+4)x^5+(2\,cfm+bgm+2\,cfn+3\,bgn+3\,cf+3\,bg)x^4+(2\,cem+bfm+cen+2\,bfn+3\,agn+2\,ce+2\,bf}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*g*(2*m + 3*n + 4)*x^5 + (2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n + 3*c*f +
```

```
[Out] integrate((c*g*(2*m + 3*n + 4)*x^5 + (2*c*f*m + b*g*m + 2*c*f*n +
 3*b*g*n + 3*c*f + 3*b*g)*x^4 + (2*c*e*m + b*f*m + c*e*n + 2*b*f*
n + 3*a*g*n + 2*c*e + 2*b*f + 2*a*g)*x^3 + (2*c*d*m + b*e*m + b*e
*n + 2*a*f*n + c*d + b*e + a*f)*x^2 - a*d + (b*d*m + a*e*n)*x)*(g
*x^3 + f*x^2 + e*x + d)^n*(c*x^2 + b*x + a)^m/x^2, x)
```

3.456 $\int \frac{(a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (-2ad+(-bd-ae+bdm+aen)x+(2cdm+bem+ben+2afn)x^2}{}$

Optimal. Leaf size=37

$$\frac{(a+bx+cx^2)^{m+1}(d+ex+fx^2+gx^3)^{n+1}}{x^2}$$

```
[Out] ((a + b*x + c*x^2)^(1 + m)*(d + e*x + f*x^2 + g*x^3)^(1 + n))/x^2
```

Rubi [F] time = 9.37199, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$

$$\text{Int}\left(\frac{(a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (-2ad+(-bd-ae+bdm+aen)x+(2cdm+bem+ben+2afn)x^2+(ce+bf}{x^3}\right.$$

Verification is Not applicable to the result.

```
[In]  Int[((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(-2*a*d + (-
(b*d) - a*e + b*d*m + a*e*n)*x + (2*c*d*m + b*e*m + b*e*n + 2*a*f*n)*x^2 + (c*e + b*f + a*
```

```
[Out] (c*e*(1 + 2*m + n) + b*f*(1 + m + 2*n) + a*g*(1 + 3*n))*Defer[Int
][(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n, x] - 2*a*d*Def
er[Int][((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n)/x^3, x]
 - (b*d*(1 - m) + a*e*(1 - n))*Defer[Int][((a + b*x + c*x^2)^m*(d
 + e*x + f*x^2 + g*x^3)^n)/x^2, x] + (2*c*d*m + 2*a*f*n + b*e*(m
+ n))*Defer[Int][((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n
)/x, x] + (2*c*f*(1 + m + n) + b*g*(2 + m + 3*n))*Defer[Int][x*(a
 + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n, x] + c*g*(3 + 2*m
+ 3*n)*Defer[Int][x^2*(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^
3)^n, x]
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((c*x**2+b*x+a)**m*(g*x**3+f*x**2+e*x+d)**n*(-2*a*d+(a*e*n+b*d*m
a*e-b*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m)*x**2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c*e*n+a
```

```
[Out] Timed out
```

Mathematica [A] time = 2.20482, size = 34, normalized size = 0.92

$$\frac{(a+x(b+cx))^{m+1}(d+x(e+x(f+gx)))^{n+1}}{x^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(-
2*a*d + (-(b*d) - a*e + b*d*m + a*e*n)*x + (2*c*d*m + b*e*m + b*e*n + 2*a*f*n)*x^2 + (c*e
```

```
[Out] ((a + x*(b + c*x))^(1 + m)*(d + x*(e + x*(f + g*x)))^(1 + n))/x^2
```

Maple [A] time = 0.036, size = 38, normalized size = 1.

$$\frac{\left(cx^2+bx+a\right)^{1+m}\left(gx^3+fx^2+ex+d\right)^{1+n}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(-2*a*d+(a*e*n+b*d*m-a*e-
b*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m)*x^2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c*e*n+a*g+b
```

```
[Out] (c*x^2+b*x+a)^(1+m)*(g*x^3+f*x^2+e*x+d)^(1+n)/x^2
```

Maxima [A] time = 0.90355, size = 128, normalized size = 3.46

$$\frac{\left(cgx^5+(cf+bg)x^4+(ce+bf+ag)x^3+(cd+be+af)x^2+ad+(bd+ae)x\right)e^{\left(n\log\left(gx^3+fx^2+ex+d\right)+m\log\left(cx^2+bx+a\right)\right)}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*g*(2*m + 3*n + 3)*x^5 + (2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n + 2*c*f +
```

```
[Out] (c*g*x^5 + (c*f + b*g)*x^4 + (c*e + b*f + a*g)*x^3 + (c*d + b*e +
 a*f)*x^2 + a*d + (b*d + a*e)*x)*e^(n*log(g*x^3 + f*x^2 + e*x + d
) + m*log(c*x^2 + b*x + a))/x^2
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*g*(2*m + 3*n + 3)*x^5 + (2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n + 2*c*f +
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**m*(g*x**3+f*x**2+e*x+d)**n*(-2*a*d+(a*e*n+b*d*m-
a*e-b*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m)*x**2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c*e*n+a
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cg(2m+3n+3)x^5 + (2cfm + bgm + 2cfn + 3bgn + 2cf + 2bg)x^4 + (2cem + bfm + cen + 2bfn + 3agn + ce + bf + a}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*g*(2*m + 3*n + 3)*x^5 + (2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n + 2*c*f +
```

```
[Out] integrate((c*g*(2*m + 3*n + 3)*x^5 + (2*c*f*m + b*g*m + 2*c*f*n +
 3*b*g*n + 2*c*f + 2*b*g)*x^4 + (2*c*e*m + b*f*m + c*e*n + 2*b*f*
n + 3*a*g*n + c*e + b*f + a*g)*x^3 + (2*c*d*m + b*e*m + b*e*n + 2
*a*f*n)*x^2 - 2*a*d + (b*d*m + a*e*n - b*d - a*e)*x)*(g*x^3 + f*x
^2 + e*x + d)^n*(c*x^2 + b*x + a)^m/x^3, x)
```

3.457 $\int x^3 \left(a+b\sqrt{c+dx}\right)^2 \, dx$

Optimal. Leaf size=185

$$\frac{c^2 \left(3a^2-b^2c\right) (c+dx)^2}{2d^4}+\frac{\left(a^2-3b^2c\right) (c+dx)^4}{4d^4}-\frac{c \left(a^2-b^2c\right) (c+dx)^3}{d^4}-\frac{a^2c^3x}{d^3}-\frac{4abc^3(c+dx)^{3/2}}{3d^4}+\frac{12abc^2(c+dx)^{5/2}}{5d^4}+\frac{4ab(c+dx)^{9/2}}{9d^4}-\frac{12abc(c+dx)^{7/2}}{7d^4}+\frac{b^2(c+dx)^5}{5d^4}$$

```
[Out] -((a^2*c^3*x)/d^3) - (4*a*b*c^3*(c + d*x)^(3/2))/(3*d^4) + (c^2*(
3*a^2 - b^2*c)*(c + d*x)^2)/(2*d^4) + (12*a*b*c^2*(c + d*x)^(5/2)
)/(5*d^4) - (c*(a^2 - b^2*c)*(c + d*x)^3)/d^4 - (12*a*b*c*(c + d*
x)^(7/2))/(7*d^4) + ((a^2 - 3*b^2*c)*(c + d*x)^4)/(4*d^4) + (4*a*
b*(c + d*x)^(9/2))/(9*d^4) + (b^2*(c + d*x)^5)/(5*d^4)
```

Rubi [A] time = 0.511152, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$\frac{c^2 \left(3a^2-b^2c\right) (c+dx)^2}{2d^4}+\frac{\left(a^2-3b^2c\right) (c+dx)^4}{4d^4}-\frac{c \left(a^2-b^2c\right) (c+dx)^3}{d^4}-\frac{a^2c^3x}{d^3}-\frac{4abc^3(c+dx)^{3/2}}{3d^4}+\frac{12abc^2(c+dx)^{5/2}}{5d^4}+\frac{4ab(c+dx)^{9/2}}{9d^4}-\frac{12abc(c+dx)^{7/2}}{7d^4}+\frac{b^2(c+dx)^5}{5d^4}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] -((a^2*c^3*x)/d^3) - (4*a*b*c^3*(c + d*x)^(3/2))/(3*d^4) + (c^2*(
3*a^2 - b^2*c)*(c + d*x)^2)/(2*d^4) + (12*a*b*c^2*(c + d*x)^(5/2)
)/(5*d^4) - (c*(a^2 - b^2*c)*(c + d*x)^3)/d^4 - (12*a*b*c*(c + d*
x)^(7/2))/(7*d^4) + ((a^2 - 3*b^2*c)*(c + d*x)^4)/(4*d^4) + (4*a*
b*(c + d*x)^(9/2))/(9*d^4) + (b^2*(c + d*x)^5)/(5*d^4)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2a^2c^3 \int^{\sqrt{c+dx}} x \, dx}{d^4}-\frac{4abc^3 \left(c+dx\right)^{\frac{3}{2}}}{3d^4}+\frac{12abc^2 \left(c+dx\right)^{\frac{5}{2}}}{5d^4}-\frac{12abc \left(c+dx\right)^{\frac{7}{2}}}{7d^4}+\frac{4ab \left(c+dx\right)^{\frac{9}{2}}}{9d^4}+\frac{b^2 \left(c+dx\right)^5}{5d^4}+\frac{c^2 \left(3a^2-b^2c\right) \left(c+dx\right)^2}{2d^4}-\frac{c \left(a^2-b^2c\right) \left(c+dx\right)^3}{d^4}+\frac{\left(a^2-3b^2c\right) \left(c+dx\right)^4}{4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] -2*a**2*c**3*Integral(x, (x, sqrt(c + d*x)))/d**4 - 4*a*b*c**3*(c
 + d*x)**(3/2)/(3*d**4) + 12*a*b*c**2*(c + d*x)**(5/2)/(5*d**4) -
 12*a*b*c*(c + d*x)**(7/2)/(7*d**4) + 4*a*b*(c + d*x)**(9/2)/(9*d
**4) + b**2*(c + d*x)**5/(5*d**4) + c**2*(3*a**2 - b**2*c)*(c + d
*x)**2/(2*d**4) - c*(a**2 - b**2*c)*(c + d*x)**3/d**4 + (a**2 - 3
*b**2*c)*(c + d*x)**4/(4*d**4)
```

Mathematica [A] time = 0.114188, size = 88, normalized size = 0.48

$$\frac{a^2x^4}{4}+\frac{4ab\sqrt{c+dx}\left(-16c^4+8c^3dx-6c^2d^2x^2+5cd^3x^3+35d^4x^4\right)}{315d^4}+\frac{1}{20}b^2x^4(5c+4dx)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] (a^2*x^4)/4 + (b^2*x^4*(5*c + 4*d*x))/20 + (4*a*b*Sqrt[c + d*x]*(
-16*c^4 + 8*c^3*d*x - 6*c^2*d^2*x^2 + 5*c*d^3*x^3 + 35*d^4*x^4))/
(315*d^4)
```

Maple [A] time = 0.004, size = 78, normalized size = 0.4

$$b^2\left(\frac{x^5d}{5}+\frac{cx^4}{4}\right)+4\,\frac{ab\left(1/9\,\left(dx+c\right)^{9/2}-3/7\,\left(dx+c\right)^{7/2}c+3/5\,\left(dx+c\right)^{5/2}c^2-1/3\,c^3\left(dx+c\right)^{3/2}\right)}{d^4}+\frac{a^2x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(a+b*(d*x+c)^(1/2))^2,x)
```

```
[Out] b^2*(1/5*x^5*d+1/4*c*x^4)+4*a*b/d^4*(1/9*(d*x+c)^(9/2)-3/7*(d*x+c
)^(7/2)*c+3/5*(d*x+c)^(5/2)*c^2-1/3*c^3*(d*x+c)^(3/2))+1/4*a^2*x^
4
```

Maxima [A] time = 0.69538, size = 204, normalized size = 1.1

$$\frac{252\,(dx+c)^5b^2+560\,(dx+c)^{\frac{9}{2}}ab-2160\,(dx+c)^{\frac{7}{2}}abc+3024\,(dx+c)^{\frac{5}{2}}abc^2-1680\,(dx+c)^{\frac{3}{2}}abc^3-1260\,(dx+c)a^2c^3-315}{1260\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(d*x + c)*b + a)^2*x^3,x, algorithm="maxima")
```

```
[Out] 1/1260*(252*(d*x + c)^5*b^2 + 560*(d*x + c)^(9/2)*a*b - 2160*(d*x
 + c)^(7/2)*a*b*c + 3024*(d*x + c)^(5/2)*a*b*c^2 - 1680*(d*x + c)
^(3/2)*a*b*c^3 - 1260*(d*x + c)*a^2*c^3 - 315*(3*b^2*c - a^2)*(d*
x + c)^4 + 1260*(b^2*c^2 - a^2*c)*(d*x + c)^3 - 630*(b^2*c^3 - 3*
a^2*c^2)*(d*x + c)^2)/d^4
```

Fricas [A] time = 0.36167, size = 127, normalized size = 0.69

$$\frac{252\,b^2d^5x^5 + 315\,\left(b^2c + a^2\right)d^4x^4 + 16\,\left(35\,abd^4x^4 + 5\,abcd^3x^3 - 6\,abc^2d^2x^2 + 8\,abc^3dx - 16\,abc^4\right)\sqrt{dx + c}}{1260\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(d*x + c)*b + a)^2*x^3,x, algorithm="fricas")
```

```
[Out] 1/1260*(252*b^2*d^5*x^5 + 315*(b^2*c + a^2)*d^4*x^4 + 16*(35*a*b*
d^4*x^4 + 5*a*b*c*d^3*x^3 - 6*a*b*c^2*d^2*x^2 + 8*a*b*c^3*d*x - 1
6*a*b*c^4)*sqrt(d*x + c))/d^4
```

Sympy [A] time = 2.58479, size = 88, normalized size = 0.48

$$\frac{a^2x^4}{4} + \frac{4ab\left(-\frac{c^3(c+dx)^{\frac{3}{2}}}{3} + \frac{3c^2(c+dx)^{\frac{5}{2}}}{5} - \frac{3c(c+dx)^{\frac{7}{2}}}{7} + \frac{(c+dx)^{\frac{9}{2}}}{9}\right)}{d^4} + \frac{b^2cx^4}{4} + \frac{b^2dx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] a**2*x**4/4 + 4*a*b*(-c**3*(c + d*x)**(3/2)/3 + 3*c**2*(c + d*x)*
*(5/2)/5 - 3*c*(c + d*x)**(7/2)/7 + (c + d*x)**(9/2)/9)/d**4 + b*
*2*c*x**4/4 + b**2*d*x**5/5
```

GIAC/XCAS [A] time = 0.276203, size = 205, normalized size = 1.11

$$\frac{315\left(dx^4 - \frac{c^4}{d^3}\right)a^2 + \frac{63\left(4\,(dx+c)^5d^{12} - 15\,(dx+c)^4cd^{12} + 20\,(dx+c)^3c^2d^{12} - 10\,(dx+c)^2c^3d^{12}\right)b^2}{d^{15}} + \frac{16\left(35\,(dx+c)^{\frac{9}{2}}d^{24} - 135\,(dx+c)^{\frac{7}{2}}cd^{24} + 189\,(dx+c)^{\frac{5}{2}}c^2d^{24}}{d^{27}}}{1260\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(d*x + c)*b + a)^2*x^3,x, algorithm="giac")
```

```
[Out] 1/1260*(315*(d*x^4 - c^4/d^3)*a^2 + 63*(4*(d*x + c)^5*d^12 - 15*(
d*x + c)^4*c*d^12 + 20*(d*x + c)^3*c^2*d^12 - 10*(d*x + c)^2*c^3*
d^12)*b^2/d^15 + 16*(35*(d*x + c)^(9/2)*d^24 - 135*(d*x + c)^(7/2
)*c*d^24 + 189*(d*x + c)^(5/2)*c^2*d^24 - 105*(d*x + c)^(3/2)*c^3
*d^24)*a*b/d^27)/d
```

3.458 $\int x^2 \left(a+b\sqrt{c+dx}\right)^2 \, dx$

Optimal. Leaf size=138

$$\frac{\left(a^2-2b^2c\right)(c+dx)^3}{3d^3}-\frac{c\left(2a^2-b^2c\right)(c+dx)^2}{2d^3}+\frac{a^2c^2x}{d^2}+\frac{4abc^2(c+dx)^{3/2}}{3d^3}+\frac{4ab(c+dx)^{7/2}}{7d^3}-\frac{8abc(c+dx)^{5/2}}{5d^3}+\frac{b^2(c+dx)^4}{4d^3}$$

```
[Out] (a^2*c^2*x)/d^2 + (4*a*b*c^2*(c + d*x)^(3/2))/(3*d^3) - (c*(2*a^2
 - b^2*c)*(c + d*x)^2)/(2*d^3) - (8*a*b*c*(c + d*x)^(5/2))/(5*d^3
) + ((a^2 - 2*b^2*c)*(c + d*x)^3)/(3*d^3) + (4*a*b*(c + d*x)^(7/2
))/(7*d^3) + (b^2*(c + d*x)^4)/(4*d^3)
```

Rubi [A] time = 0.358372, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$\frac{\left(a^2-2b^2c\right)(c+dx)^3}{3d^3}-\frac{c\left(2a^2-b^2c\right)(c+dx)^2}{2d^3}+\frac{a^2c^2x}{d^2}+\frac{4abc^2(c+dx)^{3/2}}{3d^3}+\frac{4ab(c+dx)^{7/2}}{7d^3}-\frac{8abc(c+dx)^{5/2}}{5d^3}+\frac{b^2(c+dx)^4}{4d^3}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] (a^2*c^2*x)/d^2 + (4*a*b*c^2*(c + d*x)^(3/2))/(3*d^3) - (c*(2*a^2
 - b^2*c)*(c + d*x)^2)/(2*d^3) - (8*a*b*c*(c + d*x)^(5/2))/(5*d^3
) + ((a^2 - 2*b^2*c)*(c + d*x)^3)/(3*d^3) + (4*a*b*(c + d*x)^(7/2
))/(7*d^3) + (b^2*(c + d*x)^4)/(4*d^3)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{2a^2c^2\int^{\sqrt{c+dx}} x\,dx}{d^3}+\frac{4abc^2\left(c+dx\right)^{\frac{3}{2}}}{3d^3}-\frac{8abc\left(c+dx\right)^{\frac{5}{2}}}{5d^3}+\frac{4ab\left(c+dx\right)^{\frac{7}{2}}}{7d^3}+\frac{b^2\left(c+dx\right)^4}{4d^3}-\frac{c\left(2a^2-b^2c\right)\left(c+dx\right)^2}{2d^3}+\frac{\left(a^2-2b^2c\right)\left(c+dx\right)^3}{3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] 2*a**2*c**2*Integral(x, (x, sqrt(c + d*x)))/d**3 + 4*a*b*c**2*(c
+ d*x)**(3/2)/(3*d**3) - 8*a*b*c*(c + d*x)**(5/2)/(5*d**3) + 4*a*
b*(c + d*x)**(7/2)/(7*d**3) + b**2*(c + d*x)**4/(4*d**3) - c*(2*a
**2 - b**2*c)*(c + d*x)**2/(2*d**3) + (a**2 - 2*b**2*c)*(c + d*x)
**3/(3*d**3)
```

Mathematica [A] time = 0.0967523, size = 77, normalized size = 0.56

$$\frac{a^2x^3}{3}+\frac{4ab\sqrt{c+dx}\left(8c^3-4c^2dx+3cd^2x^2+15d^3x^3\right)}{105d^3}+\frac{1}{12}b^2x^3(4c+3dx)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] (a^2*x^3)/3 + (b^2*x^3*(4*c + 3*d*x))/12 + (4*a*b*Sqrt[c + d*x]*(
8*c^3 - 4*c^2*d*x + 3*c*d^2*x^2 + 15*d^3*x^3))/(105*d^3)
```

Maple [A] time = 0.003, size = 66, normalized size = 0.5

$$b^2\left(\frac{dx^4}{4}+\frac{cx^3}{3}\right)+4\,\frac{ab\left(1/7\,(dx+c)^{7/2}-2/5\,(dx+c)^{5/2}\,c+1/3\,c^2\,(dx+c)^{3/2}\right)}{d^3}+\frac{a^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(a+b*(d*x+c)^(1/2))^2,x)
```

```
[Out] b^2*(1/4*d*x^4+1/3*c*x^3)+4*a*b/d^3*(1/7*(d*x+c)^(7/2)-2/5*(d*x+c
)^(5/2)*c+1/3*c^2*(d*x+c)^(3/2))+1/3*a^2*x^3
```

Maxima [A] time = 0.70696, size = 151, normalized size = 1.09

$$\frac{105\,(dx+c)^4b^2+240\,(dx+c)^{\frac{7}{2}}ab-672\,(dx+c)^{\frac{5}{2}}abc+560\,(dx+c)^{\frac{3}{2}}abc^2+420\,(dx+c)a^2c^2-140\left(2\,b^2c-a^2\right)(dx+c)^3+2}{420\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(d*x + c)*b + a)^2*x^2,x, algorithm="maxima")
```

```
[Out] 1/420*(105*(d*x + c)^4*b^2 + 240*(d*x + c)^(7/2)*a*b - 672*(d*x +
 c)^(5/2)*a*b*c + 560*(d*x + c)^(3/2)*a*b*c^2 + 420*(d*x + c)*a^2
*c^2 - 140*(2*b^2*c - a^2)*(d*x + c)^3 + 210*(b^2*c^2 - 2*a^2*c)*
(d*x + c)^2)/d^3
```

Fricas [A] time = 0.288538, size = 109, normalized size = 0.79

$$\frac{105\,b^2d^4x^4 + 140\,\left(b^2c + a^2\right)d^3x^3 + 16\,\left(15\,abd^3x^3 + 3\,abcd^2x^2 - 4\,abc^2dx + 8\,abc^3\right)\sqrt{dx + c}}{420\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(d*x + c)*b + a)^2*x^2,x, algorithm="fricas")
```

```
[Out] 1/420*(105*b^2*d^4*x^4 + 140*(b^2*c + a^2)*d^3*x^3 + 16*(15*a*b*d
^3*x^3 + 3*a*b*c*d^2*x^2 - 4*a*b*c^2*d*x + 8*a*b*c^3)*sqrt(d*x +
c))/d^3
```

Sympy [A] time = 2.43874, size = 73, normalized size = 0.53

$$\frac{a^2x^3}{3} + \frac{4ab\left(\frac{c^2(c+dx)^{\frac{3}{2}}}{3} - \frac{2c(c+dx)^{\frac{5}{2}}}{5} + \frac{(c+dx)^{\frac{7}{2}}}{7}\right)}{d^3} + \frac{b^2cx^3}{3} + \frac{b^2dx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] a**2*x**3/3 + 4*a*b*(c**2*(c + d*x)**(3/2)/3 - 2*c*(c + d*x)**(5/
2)/5 + (c + d*x)**(7/2)/7)/d**3 + b**2*c*x**3/3 + b**2*d*x**4/4
```

GIAC/XCAS [A] time = 0.27438, size = 163, normalized size = 1.18

$$\frac{140\left(dx^3 + \frac{c^3}{d^2}\right)a^2 + \frac{35\left(3\,(dx+c)^4d^6 - 8\,(dx+c)^3cd^6 + 6\,(dx+c)^2c^2d^6\right)b^2}{d^8} + \frac{16\left(15\,(dx+c)^{\frac{7}{2}}d^{12} - 42\,(dx+c)^{\frac{5}{2}}cd^{12} + 35\,(dx+c)^{\frac{3}{2}}c^2d^{12}\right)ab}{d^{14}}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(d*x + c)*b + a)^2*x^2,x, algorithm="giac")
```

```
[Out] 1/420*(140*(d*x^3 + c^3/d^2)*a^2 + 35*(3*(d*x + c)^4*d^6 - 8*(d*x
 + c)^3*c*d^6 + 6*(d*x + c)^2*c^2*d^6)*b^2/d^8 + 16*(15*(d*x + c)
^(7/2)*d^12 - 42*(d*x + c)^(5/2)*c*d^12 + 35*(d*x + c)^(3/2)*c^2*
d^12)*a*b/d^14)/d
```

3.459 $\int x\left(a+b\sqrt{c+dx}\right)^2\,dx$

Optimal. Leaf size=89

$$\frac{\left(a^2-b^2c\right)(c+dx)^2}{2d^2}-\frac{a^2cx}{d}+\frac{4ab(c+dx)^{5/2}}{5d^2}-\frac{4abc(c+dx)^{3/2}}{3d^2}+\frac{b^2(c+dx)^3}{3d^2}$$

```
[Out] -((a^2*c*x)/d) - (4*a*b*c*(c + d*x)^(3/2))/(3*d^2) + ((a^2 - b^2*
c)*(c + d*x)^2)/(2*d^2) + (4*a*b*(c + d*x)^(5/2))/(5*d^2) + (b^2*
(c + d*x)^3)/(3*d^2)
```

Rubi [A] time = 0.20363, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$\frac{\left(a^2-b^2c\right)(c+dx)^2}{2d^2}-\frac{a^2cx}{d}+\frac{4ab(c+dx)^{5/2}}{5d^2}-\frac{4abc(c+dx)^{3/2}}{3d^2}+\frac{b^2(c+dx)^3}{3d^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] -((a^2*c*x)/d) - (4*a*b*c*(c + d*x)^(3/2))/(3*d^2) + ((a^2 - b^2*
c)*(c + d*x)^2)/(2*d^2) + (4*a*b*(c + d*x)^(5/2))/(5*d^2) + (b^2*
(c + d*x)^3)/(3*d^2)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2a^2c\int^{\sqrt{c+dx}}x\,dx}{d^2}-\frac{4abc\,(c+dx)^{\frac{3}{2}}}{3d^2}+\frac{4ab\,(c+dx)^{\frac{5}{2}}}{5d^2}+\frac{b^2\,(c+dx)^3}{3d^2}+\frac{\left(a^2-b^2c\right)(c+dx)^2}{2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] -2*a**2*c*Integral(x, (x, sqrt(c + d*x)))/d**2 - 4*a*b*c*(c + d*x
)**(3/2)/(3*d**2) + 4*a*b*(c + d*x)**(5/2)/(5*d**2) + b**2*(c + d
*x)**3/(3*d**2) + (a**2 - b**2*c)*(c + d*x)**2/(2*d**2)
```

Mathematica [A] time = 0.0746338, size = 67, normalized size = 0.75

$$-\frac{(c+dx)\left(15a^2(c-dx)+8ab(2c-3dx)\sqrt{c+dx}+5b^2\left(c^2-cdx-2d^2x^2\right)\right)}{30d^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] -((c + d*x)*(15*a^2*(c - d*x) + 8*a*b*(2*c - 3*d*x)*Sqrt[c + d*x]
 + 5*b^2*(c^2 - c*d*x - 2*d^2*x^2)))/(30*d^2)
```

Maple [A] time = 0.004, size = 54, normalized size = 0.6

$$b^2\left(\frac{dx^3}{3}+\frac{cx^2}{2}\right)+4\,\frac{ab\left(1/5\,(dx+c)^{5/2}-1/3\,(dx+c)^{3/2}c\right)}{d^2}+\frac{a^2x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(a+b*(d*x+c)^(1/2))^2,x)
```

```
[Out] b^2*(1/3*d*x^3+1/2*c*x^2)+4*a*b/d^2*(1/5*(d*x+c)^(5/2)-1/3*(d*x+c
)^(3/2)*c)+1/2*a^2*x^2
```

Maxima [A] time = 0.716909, size = 97, normalized size = 1.09

$$\frac{10\,(dx+c)^3b^2+24\,(dx+c)^{\frac{5}{2}}ab-40\,(dx+c)^{\frac{3}{2}}abc-30\,(dx+c)a^2c-15\left(b^2c-a^2\right)(dx+c)^2}{30\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(d*x + c)*b + a)^2*x,x, algorithm="maxima")
```

```
[Out] 1/30*(10*(d*x + c)^3*b^2 + 24*(d*x + c)^(5/2)*a*b - 40*(d*x + c)^
(3/2)*a*b*c - 30*(d*x + c)*a^2*c - 15*(b^2*c - a^2)*(d*x + c)^2)/
d^2
```

Fricas [A] time = 0.294338, size = 90, normalized size = 1.01

$$\frac{10\,b^2d^3x^3 + 15\,\left(b^2c + a^2\right)d^2x^2 + 8\,\left(3\,abd^2x^2 + abcdx - 2\,abc^2\right)\sqrt{dx + c}}{30\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(d*x + c)*b + a)^2*x,x, algorithm="fricas")
```

```
[Out] 1/30*(10*b^2*d^3*x^3 + 15*(b^2*c + a^2)*d^2*x^2 + 8*(3*a*b*d^2*x^
2 + a*b*c*d*x - 2*a*b*c^2)*sqrt(d*x + c))/d^2
```

Sympy [A] time = 2.44111, size = 58, normalized size = 0.65

$$\frac{a^2x^2}{2} + \frac{4ab\left(-\frac{c(c+dx)^{\frac{3}{2}}}{3} + \frac{(c+dx)^{\frac{5}{2}}}{5}\right)}{d^2} + \frac{b^2cx^2}{2} + \frac{b^2dx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] a**2*x**2/2 + 4*a*b*(-c*(c + d*x)**(3/2)/3 + (c + d*x)**(5/2)/5)/
d**2 + b**2*c*x**2/2 + b**2*d*x**3/3
```

GIAC/XCAS [A] time = 0.27682, size = 115, normalized size = 1.29

$$\frac{\frac{15\left((dx+c)^2-2\,(dx+c)c\right)a^2}{d} + \frac{8\left(3\,(dx+c)^{\frac{5}{2}}-5\,(dx+c)^{\frac{3}{2}}c\right)ab}{d} + \frac{5\left(2\,(dx+c)^3-3\,(dx+c)^2c\right)b^2}{d}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(d*x + c)*b + a)^2*x,x, algorithm="giac")
```

```
[Out] 1/30*(15*((d*x + c)^2 - 2*(d*x + c)*c)*a^2/d + 8*(3*(d*x + c)^(5/
2) - 5*(d*x + c)^(3/2)*c)*a*b/d + 5*(2*(d*x + c)^3 - 3*(d*x + c)^
2*c)*b^2/d)/d
```

3.460 $\int \left(a+b\sqrt{c+dx}\right)^2\,dx$

Optimal. Leaf size=41

$$a^2x+\frac{4ab(c+dx)^{3/2}}{3d}+\frac{b^2(c+dx)^2}{2d}$$

```
[Out] a^2*x + (4*a*b*(c + d*x)^(3/2))/(3*d) + (b^2*(c + d*x)^2)/(2*d)
```

Rubi [A] time = 0.0629925, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$a^2x+\frac{4ab(c+dx)^{3/2}}{3d}+\frac{b^2(c+dx)^2}{2d}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] a^2*x + (4*a*b*(c + d*x)^(3/2))/(3*d) + (b^2*(c + d*x)^2)/(2*d)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{4ab\left(c+dx\right)^{\frac{3}{2}}}{3d}+\frac{b^2\int^{c+dx}x\,dx}{d}+\frac{\int^{c+dx}a^2\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] 4*a*b*(c + d*x)**(3/2)/(3*d) + b**2*Integral(x, (x, c + d*x))/d +
 Integral(a**2, (x, c + d*x))/d
```

Mathematica [A] time = 0.0246211, size = 41, normalized size = 1.

$$\frac{(c+dx)\left(6a^2+8ab\sqrt{c+dx}+3b^2(c+dx)\right)}{6d}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] ((c + d*x)*(6*a^2 + 8*a*b*Sqrt[c + d*x] + 3*b^2*(c + d*x)))/(6*d)
```

Maple [A] time = 0.002, size = 35, normalized size = 0.9

$$b^2\left(\frac{dx^2}{2}+cx\right)+\frac{4\,ab}{3\,d}\,(dx+c)^{\frac{3}{2}}+a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b*(d*x+c)^(1/2))^2,x)
```

```
[Out] b^2*(1/2*d*x^2+c*x)+4/3*a*b*(d*x+c)^(3/2)/d+a^2*x
```

Maxima [A] time = 0.726655, size = 47, normalized size = 1.15

$$\frac{1}{2}\left(dx^2+2\,cx\right)b^2+a^2x+\frac{4\,(dx+c)^{\frac{3}{2}}ab}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(d*x + c)*b + a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(d*x^2 + 2*c*x)*b^2 + a^2*x + 4/3*(d*x + c)^(3/2)*a*b/d
```

Fricas [A] time = 0.295526, size = 66, normalized size = 1.61

$$\frac{3\,b^2d^2x^2+6\left(b^2c+a^2\right)dx+8\,(abdx+abc)\sqrt{dx+c}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(d*x + c)*b + a)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(3*b^2*d^2*x^2 + 6*(b^2*c + a^2)*d*x + 8*(a*b*d*x + a*b*c)*sq
rt(d*x + c))/d
```

Sympy [A] time = 0.538076, size = 68, normalized size = 1.66

$$\begin{cases} a^2x + \frac{4abc\sqrt{c+dx}}{3d} + \frac{4abx\sqrt{c+dx}}{3} + b^2cx + \frac{b^2dx^2}{2} & \text{for } d \neq 0 \\ x\left(a + b\sqrt{c}\right)^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] Piecewise((a**2*x + 4*a*b*c*sqrt(c + d*x)/(3*d) + 4*a*b*x*sqrt(c
+ d*x)/3 + b**2*c*x + b**2*d*x**2/2, Ne(d, 0)), (x*(a + b*sqrt(c)
)**2, True))
```

GIAC/XCAS [A] time = 0.273487, size = 53, normalized size = 1.29

$$\frac{3\,(dx+c)^2b^2 + 8\,(dx+c)^{\frac{3}{2}}ab + 6\,(dx+c)a^2}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(d*x + c)*b + a)^2,x, algorithm="giac")
```

```
[Out] 1/6*(3*(d*x + c)^2*b^2 + 8*(d*x + c)^(3/2)*a*b + 6*(d*x + c)*a^2)
/d
```

3.461 $\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x}\,dx$

Optimal. Leaf size=57

$$\log(x)\left(a^2+b^2c\right)+4ab\sqrt{c+dx}-4ab\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)+b^2dx$$

```
[Out] b^2*d*x + 4*a*b*Sqrt[c + d*x] - 4*a*b*Sqrt[c]*ArcTanh[Sqrt[c + d*
x]/Sqrt[c]] + (a^2 + b^2*c)*Log[x]
```

Rubi [A] time = 0.152317, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316

$$\log(x)\left(a^2+b^2c\right)+4ab\sqrt{c+dx}-4ab\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)+b^2dx$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*Sqrt[c + d*x])^2/x,x]
```

```
[Out] b^2*d*x + 4*a*b*Sqrt[c + d*x] - 4*a*b*Sqrt[c]*ArcTanh[Sqrt[c + d*
x]/Sqrt[c]] + (a^2 + b^2*c)*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-4ab\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)+4ab\sqrt{c+dx}+2b^2\int^{\sqrt{c+dx}}x\,dx+\left(a^2+b^2c\right)\log\left(-dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b*(d*x+c)**(1/2))**2/x,x)
```

```
[Out] -4*a*b*sqrt(c)*atanh(sqrt(c + d*x)/sqrt(c)) + 4*a*b*sqrt(c + d*x)
 + 2*b**2*Integral(x, (x, sqrt(c + d*x))) + (a**2 + b**2*c)*log(-
d*x)
```

Mathematica [A] time = 0.0592721, size = 62, normalized size = 1.09

$$\left(a^2+b^2c\right)\log(dx)+b\left(4a\sqrt{c+dx}+bc+bdx\right)-4ab\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*Sqrt[c + d*x])^2/x,x]
```

```
[Out] b*(b*c + b*d*x + 4*a*Sqrt[c + d*x]) - 4*a*b*Sqrt[c]*ArcTanh[Sqrt[
c + d*x]/Sqrt[c]] + (a^2 + b^2*c)*Log[d*x]
```

Maple [A] time = 0.006, size = 51, normalized size = 0.9

$$b^2c\ln(x)+b^2dx-4\,abArtanh\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)\sqrt{c}+4\,ab\sqrt{dx+c}+a^2\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b*(d*x+c)^(1/2))^2/x,x)
```

```
[Out] b^2*c*ln(x)+b^2*d*x-4*a*b*arctanh((d*x+c)^(1/2)/c^(1/2))*c^(1/2)+
4*a*b*(d*x+c)^(1/2)+a^2*ln(x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(d*x + c)*b + a)^2/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.31156, size = 1, normalized size = 0.02

$$\left[b^2dx + 2\,ab\sqrt{c}\log\left(\frac{dx - 2\,\sqrt{dx + c}\sqrt{c} + 2\,c}{x}\right) + 4\,\sqrt{dx + c}ab + \left(b^2c + a^2\right)\log\left(x\right), b^2dx - 4\,ab\sqrt{-c}\arctan\left(\frac{\sqrt{dx + c}}{\sqrt{-c}}\right) + 4\,\sqrt{dx + c}ab + \left(b^2c + a^2\right)\log\left(x\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(d*x + c)*b + a)^2/x,x, algorithm="fricas")
```

```
[Out] [b^2*d*x + 2*a*b*sqrt(c)*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c
)/x) + 4*sqrt(d*x + c)*a*b + (b^2*c + a^2)*log(x), b^2*d*x - 4*a*
b*sqrt(-c)*arctan(sqrt(d*x + c)/sqrt(-c)) + 4*sqrt(d*x + c)*a*b +
 (b^2*c + a^2)*log(x)]
```

Sympy [A] time = 9.05637, size = 129, normalized size = 2.26

$$a^2\log\left(-dx\right) - 4abc\left(\begin{cases} -\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{-c}}\right)}{\sqrt{-c}} & \text{for}\ -c > 0 \\ \frac{\operatorname{acoth}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}} & \text{for}\ -c < 0 \wedge c < c + dx \\ \frac{\operatorname{atanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}} & \text{for}\ c > c + dx \wedge -c < 0 \end{cases}\right) + 4ab\sqrt{c + dx} + b^2c\log\left(-dx\right) + b^2\left(c + dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b*(d*x+c)**(1/2))**2/x,x)
```

```
[Out] a**2*log(-d*x) - 4*a*b*c*Piecewise((-atan(sqrt(c + d*x)/sqrt(-c))
/sqrt(-c), -c > 0), (acoth(sqrt(c + d*x)/sqrt(c))/sqrt(c), (-c <
0) & (c < c + d*x)), (atanh(sqrt(c + d*x)/sqrt(c))/sqrt(c), (-c <
 0) & (c > c + d*x))) + 4*a*b*sqrt(c + d*x) + b**2*c*log(-d*x) +
b**2*(c + d*x)
```

GIAC/XCAS [A] time = 0.275968, size = 105, normalized size = 1.84

$$-b^2c\ln\left(-c\right) + \frac{4\,abc\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} + (dx + c)b^2 - a^2\ln\left(-c\right) + 4\,\sqrt{dx + c}ab + \left(b^2c + a^2\right)\ln\left(dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(d*x + c)*b + a)^2/x,x, algorithm="giac")
```

```
[Out] -b^2*c*ln(-c) + 4*a*b*c*arctan(sqrt(d*x + c)/sqrt(-c))/sqrt(-c) +
 (d*x + c)*b^2 - a^2*ln(-c) + 4*sqrt(d*x + c)*a*b + (b^2*c + a^2)
*ln(d*x)
```

3.462 $\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^2}\,dx$

Optimal. Leaf size=54

$$-\frac{\left(a+b\sqrt{c+dx}\right)^2}{x}-\frac{2abd\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}}+b^2d\log(x)$$

```
[Out] -((a + b*Sqrt[c + d*x])^2/x) - (2*a*b*d*ArcTanh[Sqrt[c + d*x]/Sqr
t[c]])/Sqrt[c] + b^2*d*Log[x]
```

Rubi [A] time = 0.153833, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316

$$-\frac{\left(a+b\sqrt{c+dx}\right)^2}{x}-\frac{2abd\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}}+b^2d\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*Sqrt[c + d*x])^2/x^2,x]
```

```
[Out] -((a + b*Sqrt[c + d*x])^2/x) - (2*a*b*d*ArcTanh[Sqrt[c + d*x]/Sqr
t[c]])/Sqrt[c] + b^2*d*Log[x]
```

Rubi in Sympy [A] time = 10.8642, size = 68, normalized size = 1.26

$$-\frac{2abd\,\mathrm{atanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}}+b^2d\log(-dx)-\frac{\left(a+b\sqrt{c+dx}\right)\left(2a+2b\sqrt{c+dx}\right)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b*(d*x+c)**(1/2))**2/x**2,x)
```

```
[Out] -2*a*b*d*atanh(sqrt(c + d*x)/sqrt(c))/sqrt(c) + b**2*d*log(-d*x)
- (a + b*sqrt(c + d*x))*(2*a + 2*b*sqrt(c + d*x))/(2*x)
```

Mathematica [A] time = 0.121485, size = 63, normalized size = 1.17

$$-\frac{a^2+2ab\sqrt{c+dx}+\frac{2abdx\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}}+b^2c-b^2dx\log(x)}{x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*Sqrt[c + d*x])^2/x^2,x]
```

```
[Out] -((a^2 + b^2*c + 2*a*b*Sqrt[c + d*x] + (2*a*b*d*x*ArcTanh[Sqrt[c
+ d*x]/Sqrt[c]])/Sqrt[c] - b^2*d*x*Log[x])/x)
```

Maple [A] time = 0.01, size = 60, normalized size = 1.1

$$b^2d\ln(x)-\frac{b^2c}{x}-2\frac{ab\sqrt{dx+c}}{x}-2\frac{abd}{\sqrt{c}}Artanh\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)-\frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b*(d*x+c)^(1/2))^2/x^2,x)
```

```
[Out] b^2*d*ln(x)-b^2*c/x-2*a*b/x*(d*x+c)^(1/2)-2*a*b*d*arctanh((d*x+c)
^(1/2)/c^(1/2))/c^(1/2)-a^2/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(d*x + c)*b + a)^2/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.28203, size = 1, normalized size = 0.02

$$\left[\frac{abdx\log\left(\frac{(dx+2c)\sqrt{c}-2\sqrt{dx+c}c}{x}\right)-2\sqrt{dx+c}ab\sqrt{c}+\left(b^2dx\log(x)-b^2c-a^2\right)\sqrt{c}}{\sqrt{c}x},\ \frac{2abdx\arctan\left(\frac{c}{\sqrt{dx+c}\sqrt{-c}}\right)-2\sqrt{dx+c}ab}{\sqrt{-c}x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(d*x + c)*b + a)^2/x^2,x, algorithm="fricas")
```

```
[Out] [(a*b*d*x*log(((d*x + 2*c)*sqrt(c) - 2*sqrt(d*x + c)*c)/x) - 2*sq
rt(d*x + c)*a*b*sqrt(c) + (b^2*d*x*log(x) - b^2*c - a^2)*sqrt(c))
/(sqrt(c)*x), (2*a*b*d*x*arctan(c/(sqrt(d*x + c)*sqrt(-c))) - 2*s
qrt(d*x + c)*a*b*sqrt(-c) + (b^2*d*x*log(x) - b^2*c - a^2)*sqrt(-
c))/(sqrt(-c)*x)]
```

Sympy [A] time = 19.2259, size = 196, normalized size = 3.63

$$-\frac{a^2}{x} - abcd\sqrt{\frac{1}{c^3}}\log\left(-c^2\sqrt{\frac{1}{c^3}} + \sqrt{c+dx}\right) + abcd\sqrt{\frac{1}{c^3}}\log\left(c^2\sqrt{\frac{1}{c^3}} + \sqrt{c+dx}\right) - 4abd\left(\begin{cases} -\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{-c}}\right)}{\sqrt{-c}} & \text{for } -c > 0 \\ \frac{\operatorname{acoth}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}} & \text{for } -c < 0 \wedge c < c + dx \\ \frac{\operatorname{atanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}} & \text{for } c > c + dx \wedge -c < 0 \end{cases}\right) - \frac{2ab\sqrt{c+dx}}{x} - \frac{b^2c}{x} + b^2d\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b*(d*x+c)**(1/2))**2/x**2,x)
```

```
[Out] -a**2/x - a*b*c*d*sqrt(c**(-3))*log(-c**2*sqrt(c**(-3)) + sqrt(c
+ d*x)) + a*b*c*d*sqrt(c**(-3))*log(c**2*sqrt(c**(-3)) + sqrt(c +
 d*x)) - 4*a*b*d*Piecewise((-atan(sqrt(c + d*x)/sqrt(-c))/sqrt(-c
), -c > 0), (acoth(sqrt(c + d*x)/sqrt(c))/sqrt(c), (-c < 0) & (c
< c + d*x)), (atanh(sqrt(c + d*x)/sqrt(c))/sqrt(c), (-c < 0) & (c
 > c + d*x))) - 2*a*b*sqrt(c + d*x)/x - b**2*c/x + b**2*d*log(x)
```

GIAC/XCAS [A] time = 0.2943, size = 153, normalized size = 2.83

$$\frac{b^2d^2\ln(dx) + \frac{2\,abd^2\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} - \frac{b^2cd^2\ln(-c)+b^2cd^2+a^2d^2}{c} - \frac{b^2cd^2+2\sqrt{dx+c}\,abd^2+a^2d^2}{dx}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(d*x + c)*b + a)^2/x^2,x, algorithm="giac")
```

```
[Out] (b^2*d^2*ln(d*x) + 2*a*b*d^2*arctan(sqrt(d*x + c)/sqrt(-c))/sqrt(
-c) - (b^2*c*d^2*ln(-c) + b^2*c*d^2 + a^2*d^2)/c - (b^2*c*d^2 + 2
*sqrt(d*x + c)*a*b*d^2 + a^2*d^2)/(d*x))/d
```

3.463 $\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^3}\,dx$

Optimal. Leaf size=80

$$\frac{abd^2\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{2c^{3/2}}-\frac{\left(a+b\sqrt{c+dx}\right)^2}{2x^2}-\frac{bd\left(a\sqrt{c+dx}+bc\right)}{2cx}$$

```
[Out] -(b*d*(b*c + a*Sqrt[c + d*x]))/(2*c*x) - (a + b*Sqrt[c + d*x])^2/
(2*x^2) + (a*b*d^2*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/(2*c^(3/2))
```

Rubi [A] time = 0.179639, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316

$$\frac{abd^2\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{2c^{3/2}}-\frac{\left(a+b\sqrt{c+dx}\right)^2}{2x^2}-\frac{bd\left(a\sqrt{c+dx}+bc\right)}{2cx}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*Sqrt[c + d*x])^2/x^3,x]
```

```
[Out] -(b*d*(b*c + a*Sqrt[c + d*x]))/(2*c*x) - (a + b*Sqrt[c + d*x])^2/
(2*x^2) + (a*b*d^2*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/(2*c^(3/2))
```

Rubi in Sympy [A] time = 10.6154, size = 68, normalized size = 0.85

$$\frac{abd^2\,\mathrm{atanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{2c^{\frac{3}{2}}}-\frac{bd\left(a\sqrt{c+dx}+bc\right)}{2cx}-\frac{\left(a+b\sqrt{c+dx}\right)^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b*(d*x+c)**(1/2))**2/x**3,x)
```

```
[Out] a*b*d**2*atanh(sqrt(c + d*x)/sqrt(c))/(2*c**(3/2)) - b*d*(a*sqrt(
c + d*x) + b*c)/(2*c*x) - (a + b*sqrt(c + d*x))**2/(2*x**2)
```

Mathematica [A] time = 0.129455, size = 77, normalized size = 0.96

$$\frac{abd^2\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{2c^{3/2}}-\frac{a^2c+ab\sqrt{c+dx}(2c+dx)+b^2c(c+2dx)}{2cx^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*Sqrt[c + d*x])^2/x^3,x]
```

```
[Out] -(a^2*c + a*b*Sqrt[c + d*x]*(2*c + d*x) + b^2*c*(c + 2*d*x))/(2*c
*x^2) + (a*b*d^2*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/(2*c^(3/2))
```

Maple [A] time = 0.017, size = 81, normalized size = 1.

$$b^2\left(-\frac{d}{x}-\frac{c}{2x^2}\right)+4abd^2\left(\frac{1}{d^2x^2}\left(-1/8\frac{(dx+c)^{3/2}}{c}-1/8\sqrt{dx+c}\right)+1/8\frac{1}{c^{3/2}}Artanh\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)\right)-\frac{a^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b*(d*x+c)^(1/2))^2/x^3,x)
```

```
[Out] b^2*(-d/x-1/2*c/x^2)+4*a*b*d^2*((-1/8/c*(d*x+c)^(3/2)-1/8*(d*x+c)
^(1/2))/x^2/d^2+1/8/c^(3/2)*arctanh((d*x+c)^(1/2)/c^(1/2)))-1/2*a
^2/x^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(d*x + c)*b + a)^2/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.278916, size = 1, normalized size = 0.01

$$\left[\frac{abd^2x^2\log\left(\frac{(dx+2\,c)\sqrt{c}+2\,\sqrt{dx+c}c}{x}\right)-2\,(abdx+2\,abc)\sqrt{dx+c}\sqrt{c}-2\,\left(2\,b^2cdx+b^2c^2+a^2c\right)\sqrt{c}}{4\,c^{\frac{3}{2}}x^2},\ -\frac{abd^2x^2\arctan\left(\frac{c}{\sqrt{dx+c}\sqrt{-c}}\right)+(abdx+2\,abc)\sqrt{dx+c}\sqrt{-c}+\left(2\,b^2cdx+b^2c^2+a^2c\right)\sqrt{-c}}{2\,\sqrt{-c}cx^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(d*x + c)*b + a)^2/x^3,x, algorithm="fricas")
```

```
[Out] [1/4*(a*b*d^2*x^2*log(((d*x + 2*c)*sqrt(c) + 2*sqrt(d*x + c)*c)/x
) - 2*(a*b*d*x + 2*a*b*c)*sqrt(d*x + c)*sqrt(c) - 2*(2*b^2*c*d*x
+ b^2*c^2 + a^2*c)*sqrt(c))/(c^(3/2)*x^2), -1/2*(a*b*d^2*x^2*arct
an(c/(sqrt(d*x + c)*sqrt(-c))) + (a*b*d*x + 2*a*b*c)*sqrt(d*x + c
)*sqrt(-c) + (2*b^2*c*d*x + b^2*c^2 + a^2*c)*sqrt(-c))/(sqrt(-c)*
c*x^2)]
```

Sympy [A] time = 38.4755, size = 292, normalized size = 3.65

$$-\frac{a^2}{2x^2}-\frac{20abc^2d^2\sqrt{c+dx}}{-8c^4-16c^3dx+8c^2(c+dx)^2}+\frac{12abcd^2(c+dx)^{\frac{3}{2}}}{-8c^4-16c^3dx+8c^2(c+dx)^2}+\frac{3abcd^2\sqrt{\frac{1}{c^5}}\log\left(-c^3\sqrt{\frac{1}{c^5}}+\sqrt{c+dx}\right)}{4}-\frac{3abcd^2\sqrt{\frac{1}{c^5}}\log\left(c^3\sqrt{\frac{1}{c^5}}+\sqrt{c+dx}\right)}{4}-abd^2\sqrt{\frac{1}{c^3}}\log\left(-c^2\sqrt{\frac{1}{c^3}}+\sqrt{c+dx}\right)+abd^2\sqrt{\frac{1}{c^3}}\log\left(c^2\sqrt{\frac{1}{c^3}}+\sqrt{c+dx}\right)-\frac{2abd\sqrt{c+dx}}{cx}-\frac{b^2c}{2x^2}-\frac{b^2d}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b*(d*x+c)**(1/2))**2/x**3,x)
```

```
[Out] -a**2/(2*x**2) - 20*a*b*c**2*d**2*sqrt(c + d*x)/(-8*c**4 - 16*c**
3*d*x + 8*c**2*(c + d*x)**2) + 12*a*b*c*d**2*(c + d*x)**(3/2)/(-8
*c**4 - 16*c**3*d*x + 8*c**2*(c + d*x)**2) + 3*a*b*c*d**2*sqrt(c*
*(-5))*log(-c**3*sqrt(c**(-5)) + sqrt(c + d*x))/4 - 3*a*b*c*d**2*
sqrt(c**(-5))*log(c**3*sqrt(c**(-5)) + sqrt(c + d*x))/4 - a*b*d**
2*sqrt(c**(-3))*log(-c**2*sqrt(c**(-3)) + sqrt(c + d*x)) + a*b*d*
```

```
*2*sqrt(c**(-3))*log(c**2*sqrt(c**(-3)) + sqrt(c + d*x)) - 2*a*b*
d*sqrt(c + d*x)/(c*x) - b**2*c/(2*x**2) - b**2*d/x
```

GIAC/XCAS [A] time = 0.2944, size = 170, normalized size = 2.12

$$-\frac{\frac{abd^3\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}c}+\frac{b^2cd^3-a^2d^3}{c^2}+\frac{2(dx+c)b^2cd^3-b^2c^2d^3+(dx+c)^{\frac{3}{2}}abd^3+\sqrt{dx+c}abcd^3+a^2cd^3}{cd^2x^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(d*x + c)*b + a)^2/x^3,x, algorithm="giac")
```

```
[Out] -1/2*(a*b*d^3*arctan(sqrt(d*x + c)/sqrt(-c))/(sqrt(-c)*c) + (b^2*
c*d^3 - a^2*d^3)/c^2 + (2*(d*x + c)*b^2*c*d^3 - b^2*c^2*d^3 + (d*
x + c)^(3/2)*a*b*d^3 + sqrt(d*x + c)*a*b*c*d^3 + a^2*c*d^3)/(c*d^
2*x^2))/d
```

3.464 $\int x^3\sqrt{a+b\sqrt{c+dx}}\,dx$

Optimal. Leaf size=326

$$\frac{12\left(7a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{13/2}}{13b^8d^4}-\frac{20a\left(7a^2-3b^2c\right)\left(a+b\sqrt{c+dx}\right)^{11/2}}{11b^8d^4}-\frac{12a\left(7a^2-3b^2c\right)\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^8d^4}+\frac{4\left(a^2-b^2c\right)^2\left(7a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^8d^4}-\frac{4a\left(a^2-b^2c\right)^3\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^8d^4}+\frac{4\left(35a^4-30a^2b^2c+3b^4c^2\right)\left(a+b\sqrt{c+dx}\right)^{9/2}}{9b^8d^4}+\frac{4\left(a+b\sqrt{c+dx}\right)^{17/2}}{17b^8d^4}-\frac{28a\left(a+b\sqrt{c+dx}\right)^{15/2}}{15b^8d^4}$$

```
[Out] (-4*a*(a^2 - b^2*c)^3*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^8*d^4) +
(4*(a^2 - b^2*c)^2*(7*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(5/2))/(
5*b^8*d^4) - (12*a*(7*a^2 - 3*b^2*c)*(a^2 - b^2*c)*(a + b*Sqrt[c
+ d*x])^(7/2))/(7*b^8*d^4) + (4*(35*a^4 - 30*a^2*b^2*c + 3*b^4*c^
2)*(a + b*Sqrt[c + d*x])^(9/2))/(9*b^8*d^4) - (20*a*(7*a^2 - 3*b^
2*c)*(a + b*Sqrt[c + d*x])^(11/2))/(11*b^8*d^4) + (12*(7*a^2 - b^
2*c)*(a + b*Sqrt[c + d*x])^(13/2))/(13*b^8*d^4) - (28*a*(a + b*Sq
rt[c + d*x])^(15/2))/(15*b^8*d^4) + (4*(a + b*Sqrt[c + d*x])^(17/
2))/(17*b^8*d^4)
```

Rubi [A] time = 0.530122, antiderivative size = 326, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{12\left(7a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{13/2}}{13b^8d^4}-\frac{20a\left(7a^2-3b^2c\right)\left(a+b\sqrt{c+dx}\right)^{11/2}}{11b^8d^4}-\frac{12a\left(7a^2-3b^2c\right)\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^8d^4}+\frac{4\left(a^2-b^2c\right)^2\left(7a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^8d^4}-\frac{4a\left(a^2-b^2c\right)^3\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^8d^4}+\frac{4\left(35a^4-30a^2b^2c+3b^4c^2\right)\left(a+b\sqrt{c+dx}\right)^{9/2}}{9b^8d^4}+\frac{4\left(a+b\sqrt{c+dx}\right)^{17/2}}{17b^8d^4}-\frac{28a\left(a+b\sqrt{c+dx}\right)^{15/2}}{15b^8d^4}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-4*a*(a^2 - b^2*c)^3*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^8*d^4) +
(4*(a^2 - b^2*c)^2*(7*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(5/2))/(
5*b^8*d^4) - (12*a*(7*a^2 - 3*b^2*c)*(a^2 - b^2*c)*(a + b*Sqrt[c
+ d*x])^(7/2))/(7*b^8*d^4) + (4*(35*a^4 - 30*a^2*b^2*c + 3*b^4*c^
2)*(a + b*Sqrt[c + d*x])^(9/2))/(9*b^8*d^4) - (20*a*(7*a^2 - 3*b^
2*c)*(a + b*Sqrt[c + d*x])^(11/2))/(11*b^8*d^4) + (12*(7*a^2 - b^
2*c)*(a + b*Sqrt[c + d*x])^(13/2))/(13*b^8*d^4) - (28*a*(a + b*Sq
rt[c + d*x])^(15/2))/(15*b^8*d^4) + (4*(a + b*Sqrt[c + d*x])^(17/
2))/(17*b^8*d^4)
```

Rubi in Sympy [A] time = 32.3162, size = 306, normalized size = 0.94

$$-\frac{28a\left(a+b\sqrt{c+dx}\right)^{\frac{15}{2}}}{15b^8d^4}-\frac{20a\left(a+b\sqrt{c+dx}\right)^{\frac{11}{2}}\left(7a^2-3b^2c\right)}{11b^8d^4}-\frac{12a\left(a+b\sqrt{c+dx}\right)^{\frac{7}{2}}\left(a^2-b^2c\right)\left(7a^2-3b^2c\right)}{7b^8d^4}-\frac{4a\left(a+b\sqrt{c+dx}\right)^{\frac{3}{2}}\left(a^2-b^2c\right)^3}{3b^8d^4}+\frac{4\left(a+b\sqrt{c+dx}\right)^{\frac{17}{2}}}{17b^8d^4}+\frac{12\left(a+b\sqrt{c+dx}\right)^{\frac{13}{2}}\left(7a^2-b^2c\right)}{13b^8d^4}+\frac{4\left(a+b\sqrt{c+dx}\right)^{\frac{9}{2}}\left(35a^4-30a^2b^2c+3b^4c^2\right)}{9b^8d^4}+\frac{4\left(a+b\sqrt{c+dx}\right)^{\frac{5}{2}}\left(a^2-b^2c\right)^2\left(7a^2-b^2c\right)}{5b^8d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] -28*a*(a + b*sqrt(c + d*x))**(15/2)/(15*b**8*d**4) - 20*a*(a + b*
sqrt(c + d*x))**(11/2)*(7*a**2 - 3*b**2*c)/(11*b**8*d**4) - 12*a*
(a + b*sqrt(c + d*x))**(7/2)*(a**2 - b**2*c)*(7*a**2 - 3*b**2*c)/
(7*b**8*d**4) - 4*a*(a + b*sqrt(c + d*x))**(3/2)*(a**2 - b**2*c)*
*3/(3*b**8*d**4) + 4*(a + b*sqrt(c + d*x))**(17/2)/(17*b**8*d**4)
 + 12*(a + b*sqrt(c + d*x))**(13/2)*(7*a**2 - b**2*c)/(13*b**8*d*
*4) + 4*(a + b*sqrt(c + d*x))**(9/2)*(35*a**4 - 30*a**2*b**2*c +
3*b**4*c**2)/(9*b**8*d**4) + 4*(a + b*sqrt(c + d*x))**(5/2)*(a**2
 - b**2*c)**2*(7*a**2 - b**2*c)/(5*b**8*d**4)
```

Mathematica [A] time = 0.290161, size = 232, normalized size = 0.71

$$\frac{4\left(a+b\sqrt{c+dx}\right)^{3/2}\left(-14336a^7+21504a^6b\sqrt{c+dx}+3840a^5b^2(10c-7dx)-640a^4b^3(104c-49dx)\sqrt{c+dx}-48a^3b^4\left(616c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (4*(a + b*Sqrt[c + d*x])^(3/2)*(-14336*a^7 + 3840*a^5*b^2*(10*c -
 7*d*x) + 21504*a^6*b*Sqrt[c + d*x] - 640*a^4*b^3*(104*c - 49*d*x
)*Sqrt[c + d*x] - 48*a^3*b^4*(616*c^2 - 1080*c*d*x + 735*d^2*x^2)
 + 24*a^2*b^5*Sqrt[c + d*x]*(2960*c^2 - 2716*c*d*x + 1617*d^2*x^2
) + 6*a*b^6*(320*c^3 - 3936*c^2*d*x + 5754*c*d^2*x^2 - 7007*d^3*x
^3) - 231*b^7*Sqrt[c + d*x]*(128*c^3 - 160*c^2*d*x + 180*c*d^2*x^
2 - 195*d^3*x^3)))/(765765*b^8*d^4)
```

Maple [A] time = 0.006, size = 383, normalized size = 1.2

$$4\,\frac{1}{d^4b^8}\left(1/17\,\left(a+b\sqrt{dx+c}\right)^{17/2}-\frac{7\,a\,\left(a+b\sqrt{dx+c}\right)^{15/2}}{15}+1/13\,\left(-3\,b^2c+21\,a^2\right)\left(a+b\sqrt{dx+c}\right)^{13/2}+1/11\,\left(-8\,\left(-b^2c\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(a+b*(d*x+c)^(1/2))^(1/2),x)
```

```
[Out] 4/d^4/b^8*(1/17*(a+b*(d*x+c)^(1/2))^(17/2)-7/15*a*(a+b*(d*x+c)^(1
/2))^(15/2)+1/13*(-3*b^2*c+21*a^2)*(a+b*(d*x+c)^(1/2))^(13/2)+1/1
1*(-8*(-b^2*c+a^2)*a-2*a*(-2*b^2*c+6*a^2)-(-3*b^2*c+15*a^2)*a)*(a
+b*(d*x+c)^(1/2))^(11/2)+1/9*((-b^2*c+a^2)*(-2*b^2*c+6*a^2)+8*a^2
*(-b^2*c+a^2)+(-b^2*c+a^2)^2-(-8*(-b^2*c+a^2)*a-2*a*(-2*b^2*c+6*a
^2))*a)*(a+b*(d*x+c)^(1/2))^(9/2)+1/7*(-6*(-b^2*c+a^2)^2*a-((-b^2
*c+a^2)*(-2*b^2*c+6*a^2)+8*a^2*(-b^2*c+a^2)+(-b^2*c+a^2)^2)*a)*(a
+b*(d*x+c)^(1/2))^(7/2)+1/5*((-b^2*c+a^2)^3+6*(-b^2*c+a^2)^2*a^2)
*(a+b*(d*x+c)^(1/2))^(5/2)-1/3*(-b^2*c+a^2)^3*a*(a+b*(d*x+c)^(1/2
))^(3/2))
```

Maxima [A] time = 0.725595, size = 362, normalized size = 1.11

$$\frac{4\left(45045\left(\sqrt{dx+c}b+a\right)^{\frac{17}{2}}-357357\left(\sqrt{dx+c}b+a\right)^{\frac{15}{2}}a-176715\left(b^2c-7\,a^2\right)\left(\sqrt{dx+c}b+a\right)^{\frac{13}{2}}+348075\left(3\,ab^2c-7\,a^3\right)\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(sqrt(d*x + c)*b + a)*x^3,x, algorithm="maxima")
```

```
[Out] 4/765765*(45045*(sqrt(d*x + c)*b + a)^(17/2) - 357357*(sqrt(d*x +
 c)*b + a)^(15/2)*a - 176715*(b^2*c - 7*a^2)*(sqrt(d*x + c)*b + a
```

```
)^(13/2) + 348075*(3*a*b^2*c - 7*a^3)*(sqrt(d*x + c)*b + a)^(11/2
) + 85085*(3*b^4*c^2 - 30*a^2*b^2*c + 35*a^4)*(sqrt(d*x + c)*b +
a)^(9/2) - 328185*(3*a*b^4*c^2 - 10*a^3*b^2*c + 7*a^5)*(sqrt(d*x
+ c)*b + a)^(7/2) - 153153*(b^6*c^3 - 9*a^2*b^4*c^2 + 15*a^4*b^2*
c - 7*a^6)*(sqrt(d*x + c)*b + a)^(5/2) + 255255*(a*b^6*c^3 - 3*a^
3*b^4*c^2 + 3*a^5*b^2*c - a^7)*(sqrt(d*x + c)*b + a)^(3/2))/(b^8*
d^4)
```

Fricas [A] time = 0.346331, size = 386, normalized size = 1.18

$$\frac{4\left(45045\,b^{8}d^{4}x^{4} - 29568\,b^{8}c^{4} + 72960\,a^{2}b^{6}c^{3} - 96128\,a^{4}b^{4}c^{2} + 59904\,a^{6}b^{2}c - 14336\,a^{8} + 231\left(15\,b^{8}c - 14\,a^{2}b^{6}\right)d^{3}x^{3} - 28\,(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(sqrt(d*x + c)*b + a)*x^3,x, algorithm="fricas")
```

```
[Out] 4/765765*(45045*b^8*d^4*x^4 - 29568*b^8*c^4 + 72960*a^2*b^6*c^3 -
 96128*a^4*b^4*c^2 + 59904*a^6*b^2*c - 14336*a^8 + 231*(15*b^8*c
- 14*a^2*b^6)*d^3*x^3 - 28*(165*b^8*c^2 - 291*a^2*b^6*c + 140*a^4
*b^4)*d^2*x^2 + 32*(231*b^8*c^3 - 555*a^2*b^6*c^2 + 520*a^4*b^4*c
 - 168*a^6*b^2)*d*x + (3003*a*b^7*d^3*x^3 - 27648*a*b^7*c^3 + 414
72*a^3*b^5*c^2 - 28160*a^5*b^3*c + 7168*a^7*b - 3528*(2*a*b^7*c -
 a^3*b^5)*d^2*x^2 + 32*(417*a*b^7*c^2 - 417*a^3*b^5*c + 140*a^5*b
^3)*d*x)*sqrt(d*x + c))*sqrt(sqrt(d*x + c)*b + a)/(b^8*d^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{3}\sqrt{a + b\sqrt{c + dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(a + b*sqrt(c + d*x)), x)
```

GIAC/XCAS [A] time = 0.375332, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(sqrt(d*x + c)*b + a)*x^3,x, algorithm="giac")
```

```
[Out] Done
```

3.465 $\int x^2\sqrt{a+b\sqrt{c+dx}}\,dx$

Optimal. Leaf size=224

$$\frac{8(5a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{9/2}}{9b^6d^3}-\frac{8a(5a^2-3b^2c)\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^6d^3}-\frac{4a(a^2-b^2c)^2\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^6d^3}+\frac{4(5a^4-6a^2b^2c+b^4c^2)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^6d^3}+\frac{4\left(a+b\sqrt{c+dx}\right)^{13/2}}{13b^6d^3}-\frac{20a\left(a+b\sqrt{c+dx}\right)^{11/2}}{11b^6d^3}$$

```
[Out] (-4*a*(a^2 - b^2*c)^2*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^6*d^3) +
(4*(5*a^4 - 6*a^2*b^2*c + b^4*c^2)*(a + b*Sqrt[c + d*x])^(5/2))/(
5*b^6*d^3) - (8*a*(5*a^2 - 3*b^2*c)*(a + b*Sqrt[c + d*x])^(7/2))/
(7*b^6*d^3) + (8*(5*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(9/2))/(9*
b^6*d^3) - (20*a*(a + b*Sqrt[c + d*x])^(11/2))/(11*b^6*d^3) + (4*
(a + b*Sqrt[c + d*x])^(13/2))/(13*b^6*d^3)
```

Rubi [A] time = 0.371229, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$\frac{8(5a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{9/2}}{9b^6d^3}-\frac{8a(5a^2-3b^2c)\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^6d^3}-\frac{4a(a^2-b^2c)^2\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^6d^3}+\frac{4(5a^4-6a^2b^2c+b^4c^2)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^6d^3}+\frac{4\left(a+b\sqrt{c+dx}\right)^{13/2}}{13b^6d^3}-\frac{20a\left(a+b\sqrt{c+dx}\right)^{11/2}}{11b^6d^3}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-4*a*(a^2 - b^2*c)^2*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^6*d^3) +
(4*(5*a^4 - 6*a^2*b^2*c + b^4*c^2)*(a + b*Sqrt[c + d*x])^(5/2))/(
5*b^6*d^3) - (8*a*(5*a^2 - 3*b^2*c)*(a + b*Sqrt[c + d*x])^(7/2))/
(7*b^6*d^3) + (8*(5*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(9/2))/(9*
b^6*d^3) - (20*a*(a + b*Sqrt[c + d*x])^(11/2))/(11*b^6*d^3) + (4*
(a + b*Sqrt[c + d*x])^(13/2))/(13*b^6*d^3)
```

Rubi in Sympy [A] time = 22.5591, size = 211, normalized size = 0.94

$$-\frac{20a\left(a+b\sqrt{c+dx}\right)^{\frac{11}{2}}}{11b^6d^3}-\frac{8a\left(a+b\sqrt{c+dx}\right)^{\frac{7}{2}}\left(5a^2-3b^2c\right)}{7b^6d^3}-\frac{4a\left(a+b\sqrt{c+dx}\right)^{\frac{3}{2}}\left(a^2-b^2c\right)^2}{3b^6d^3}+\frac{4\left(a+b\sqrt{c+dx}\right)^{\frac{13}{2}}}{13b^6d^3}+\frac{8\left(a+b\sqrt{c+dx}\right)^{\frac{9}{2}}\left(5a^2-b^2c\right)}{9b^6d^3}+\frac{4\left(a+b\sqrt{c+dx}\right)^{\frac{5}{2}}\left(5a^4-6a^2b^2c+b^4c^2\right)}{5b^6d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] -20*a*(a + b*sqrt(c + d*x))**(11/2)/(11*b**6*d**3) - 8*a*(a + b*s
qrt(c + d*x))**(7/2)*(5*a**2 - 3*b**2*c)/(7*b**6*d**3) - 4*a*(a +
 b*sqrt(c + d*x))**(3/2)*(a**2 - b**2*c)**2/(3*b**6*d**3) + 4*(a
+ b*sqrt(c + d*x))**(13/2)/(13*b**6*d**3) + 8*(a + b*sqrt(c + d*x
))**(9/2)*(5*a**2 - b**2*c)/(9*b**6*d**3) + 4*(a + b*sqrt(c + d*x
))**(5/2)*(5*a**4 - 6*a**2*b**2*c + b**4*c**2)/(5*b**6*d**3)
```

Mathematica [A] time = 0.183388, size = 147, normalized size = 0.66

$$\frac{4\left(a+b\sqrt{c+dx}\right)^{3/2}\left(-1280a^5+1920a^4b\sqrt{c+dx}+32a^3b^2(68c-75dx)+16a^2b^3\sqrt{c+dx}(175dx-254c)-6ab^4\left(96c^2-380\right.\right.}{45045b^6d^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (4*(a + b*Sqrt[c + d*x])^(3/2)*(-1280*a^5 + 32*a^3*b^2*(68*c - 75
*d*x) + 1920*a^4*b*Sqrt[c + d*x] + 16*a^2*b^3*Sqrt[c + d*x]*(-254
*c + 175*d*x) + 77*b^5*Sqrt[c + d*x]*(32*c^2 - 40*c*d*x + 45*d^2*
x^2) - 6*a*b^4*(96*c^2 - 380*c*d*x + 525*d^2*x^2)))/(45045*b^6*d^
3)
```

Maple [A] time = 0.003, size = 183, normalized size = 0.8

$$4\,\frac{1}{d^3b^6}\left(1/13\,\left(a+b\sqrt{dx+c}\right)^{13/2}-\frac{5\,a\left(a+b\sqrt{dx+c}\right)^{11/2}}{11}+1/9\,\left(-2\,b^2c+10\,a^2\right)\left(a+b\sqrt{dx+c}\right)^{9/2}+1/7\,\left(-4\,\left(-b^2c+a^2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(a+b*(d*x+c)^(1/2))^(1/2),x)
```

```
[Out] 4/d^3/b^6*(1/13*(a+b*(d*x+c)^(1/2))^(13/2)-5/11*a*(a+b*(d*x+c)^(1
/2))^(11/2)+1/9*(-2*b^2*c+10*a^2)*(a+b*(d*x+c)^(1/2))^(9/2)+1/7*(
-4*(-b^2*c+a^2)*a-a*(-2*b^2*c+6*a^2))*(a+b*(d*x+c)^(1/2))^(7/2)+1
/5*((-b^2*c+a^2)^2+4*a^2*(-b^2*c+a^2))*(a+b*(d*x+c)^(1/2))^(5/2)-
1/3*(-b^2*c+a^2)^2*a*(a+b*(d*x+c)^(1/2))^(3/2))
```

Maxima [A] time = 0.725133, size = 225, normalized size = 1.

$$\frac{4\left(3465\left(\sqrt{dx+c}b+a\right)^{\frac{13}{2}}-20475\left(\sqrt{dx+c}b+a\right)^{\frac{11}{2}}a-10010\left(b^2c-5a^2\right)\left(\sqrt{dx+c}b+a\right)^{\frac{9}{2}}+12870\left(3ab^2c-5a^3\right)\left(\sqrt{dx+\right.\right.}{45045b^6d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(sqrt(d*x + c)*b + a)*x^2,x, algorithm="maxima")
```

```
[Out] 4/45045*(3465*(sqrt(d*x + c)*b + a)^(13/2) - 20475*(sqrt(d*x + c)
*b + a)^(11/2)*a - 10010*(b^2*c - 5*a^2)*(sqrt(d*x + c)*b + a)^(9
/2) + 12870*(3*a*b^2*c - 5*a^3)*(sqrt(d*x + c)*b + a)^(7/2) + 900
9*(b^4*c^2 - 6*a^2*b^2*c + 5*a^4)*(sqrt(d*x + c)*b + a)^(5/2) - 1
5015*(a*b^4*c^2 - 2*a^3*b^2*c + a^5)*(sqrt(d*x + c)*b + a)^(3/2))
/(b^6*d^3)
```

Fricas [A] time = 0.358349, size = 248, normalized size = 1.11

$$\frac{4\left(3465b^6d^3x^3+2464b^6c^3-4640a^2b^4c^2+4096a^4b^2c-1280a^6+35\left(11b^6c-10a^2b^4\right)d^2x^2-8\left(77b^6c^2-127a^2b^4c+60\right.\right.}{45045b^6d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(sqrt(d*x + c)*b + a)*x^2,x, algorithm="fricas")
```

```
[Out] 4/45045*(3465*b^6*d^3*x^3 + 2464*b^6*c^3 - 4640*a^2*b^4*c^2 + 409
6*a^4*b^2*c - 1280*a^6 + 35*(11*b^6*c - 10*a^2*b^4)*d^2*x^2 - 8*(
77*b^6*c^2 - 127*a^2*b^4*c + 60*a^4*b^2)*d*x + (315*a*b^5*d^2*x^2
 + 1888*a*b^5*c^2 - 1888*a^3*b^3*c + 640*a^5*b - 400*(2*a*b^5*c -
 a^3*b^3)*d*x)*sqrt(d*x + c))*sqrt(sqrt(d*x + c)*b + a)/(b^6*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sqrt{a+b\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(a + b*sqrt(c + d*x)), x)
```

GIAC/XCAS [A] time = 0.335948, size = 923, normalized size = 4.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(sqrt(d*x + c)*b + a)*x^2,x, algorithm="giac")
```

```
[Out] 4/45045*(9009*sqrt((sqrt(d*x + c)*b + a)*b^2)*(sqrt(d*x + c)*b +
a)^2*b^6*c^2*sign((sqrt(d*x + c)*b + a)*b - a*b) - 15015*sqrt((sq
rt(d*x + c)*b + a)*b^2)*(sqrt(d*x + c)*b + a)*a*b^6*c^2*sign((sqr
t(d*x + c)*b + a)*b - a*b) - 10010*sqrt((sqrt(d*x + c)*b + a)*b^2
)*(sqrt(d*x + c)*b + a)^4*b^4*c*sign((sqrt(d*x + c)*b + a)*b - a*
b) + 38610*sqrt((sqrt(d*x + c)*b + a)*b^2)*(sqrt(d*x + c)*b + a)^
3*a*b^4*c*sign((sqrt(d*x + c)*b + a)*b - a*b) - 54054*sqrt((sqrt(
d*x + c)*b + a)*b^2)*(sqrt(d*x + c)*b + a)^2*a^2*b^4*c*sign((sqrt
(d*x + c)*b + a)*b - a*b) + 30030*sqrt((sqrt(d*x + c)*b + a)*b^2)
*(sqrt(d*x + c)*b + a)*a^3*b^4*c*sign((sqrt(d*x + c)*b + a)*b - a
*b) + 3465*sqrt((sqrt(d*x + c)*b + a)*b^2)*(sqrt(d*x + c)*b + a)^
6*b^2*sign((sqrt(d*x + c)*b + a)*b - a*b) - 20475*sqrt((sqrt(d*x
+ c)*b + a)*b^2)*(sqrt(d*x + c)*b + a)^5*a*b^2*sign((sqrt(d*x + c
)*b + a)*b - a*b) + 50050*sqrt((sqrt(d*x + c)*b + a)*b^2)*(sqrt(d
*x + c)*b + a)^4*a^2*b^2*sign((sqrt(d*x + c)*b + a)*b - a*b) - 64
350*sqrt((sqrt(d*x + c)*b + a)*b^2)*(sqrt(d*x + c)*b + a)^3*a^3*b
^2*sign((sqrt(d*x + c)*b + a)*b - a*b) + 45045*sqrt((sqrt(d*x + c
)*b + a)*b^2)*(sqrt(d*x + c)*b + a)^2*a^4*b^2*sign((sqrt(d*x + c)
*b + a)*b - a*b) - 15015*sqrt((sqrt(d*x + c)*b + a)*b^2)*(sqrt(d*
x + c)*b + a)*a^5*b^2*sign((sqrt(d*x + c)*b + a)*b - a*b))*abs(b)
/(b^10*d^3)
```

3.466 $\int x\sqrt{a+b\sqrt{c+dx}}\,dx$

Optimal. Leaf size=133

$$\frac{4(3a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^4d^2}-\frac{4a(a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^4d^2}+\frac{4\left(a+b\sqrt{c+dx}\right)^{9/2}}{9b^4d^2}-\frac{12a\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^4d^2}$$

```
[Out] (-4*a*(a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^4*d^2) + (4
*(3*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(5/2))/(5*b^4*d^2) - (12*a
*(a + b*Sqrt[c + d*x])^(7/2))/(7*b^4*d^2) + (4*(a + b*Sqrt[c + d*
x])^(9/2))/(9*b^4*d^2)
```

Rubi [A] time = 0.227356, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$\frac{4(3a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^4d^2}-\frac{4a(a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^4d^2}+\frac{4\left(a+b\sqrt{c+dx}\right)^{9/2}}{9b^4d^2}-\frac{12a\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^4d^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x*Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-4*a*(a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^4*d^2) + (4
*(3*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(5/2))/(5*b^4*d^2) - (12*a
*(a + b*Sqrt[c + d*x])^(7/2))/(7*b^4*d^2) + (4*(a + b*Sqrt[c + d*
x])^(9/2))/(9*b^4*d^2)
```

Rubi in Sympy [A] time = 11.8205, size = 122, normalized size = 0.92

$$-\frac{12a\left(a+b\sqrt{c+dx}\right)^{\frac{7}{2}}}{7b^4d^2}-\frac{4a\left(a+b\sqrt{c+dx}\right)^{\frac{3}{2}}(a^2-b^2c)}{3b^4d^2}+\frac{4\left(a+b\sqrt{c+dx}\right)^{\frac{9}{2}}}{9b^4d^2}+\frac{4\left(a+b\sqrt{c+dx}\right)^{\frac{5}{2}}(3a^2-b^2c)}{5b^4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] -12*a*(a + b*sqrt(c + d*x))**(7/2)/(7*b**4*d**2) - 4*a*(a + b*sqr
t(c + d*x))**(3/2)*(a**2 - b**2*c)/(3*b**4*d**2) + 4*(a + b*sqrt(
c + d*x))**(9/2)/(9*b**4*d**2) + 4*(a + b*sqrt(c + d*x))**(5/2)*(
3*a**2 - b**2*c)/(5*b**4*d**2)
```

Mathematica [A] time = 0.0965162, size = 109, normalized size = 0.82

$$\frac{4\sqrt{a+b\sqrt{c+dx}}\left(-16a^4+8a^3b\sqrt{c+dx}+6a^2b^2(6c-dx)+ab^3\sqrt{c+dx}(5dx-16c)+7b^4\left(-4c^2+cdx+5d^2x^2\right)\right)}{315b^4d^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (4*Sqrt[a + b*Sqrt[c + d*x]]*(-16*a^4 + 6*a^2*b^2*(6*c - d*x) + 8
*a^3*b*Sqrt[c + d*x] + a*b^3*Sqrt[c + d*x]*(-16*c + 5*d*x) + 7*b^
4*(-4*c^2 + c*d*x + 5*d^2*x^2)))/(315*b^4*d^2)
```

Maple [A] time = 0.003, size = 94, normalized size = 0.7

$$4\,\frac{1/9\,\left(a+b\sqrt{dx+c}\right)^{9/2}-3/7\,a\left(a+b\sqrt{dx+c}\right)^{7/2}+1/5\,\left(-b^2c+3\,a^2\right)\left(a+b\sqrt{dx+c}\right)^{5/2}-1/3\,\left(-b^2c+a^2\right)a\left(a+b\sqrt{dx}\right.}{b^4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(a+b*(d*x+c)^(1/2))^(1/2),x)
```

```
[Out] 4/d^2/b^4*(1/9*(a+b*(d*x+c)^(1/2))^(9/2)-3/7*a*(a+b*(d*x+c)^(1/2)
)^(7/2)+1/5*(-b^2*c+3*a^2)*(a+b*(d*x+c)^(1/2))^(5/2)-1/3*(-b^2*c+
a^2)*a*(a+b*(d*x+c)^(1/2))^(3/2))
```

Maxima [A] time = 0.71097, size = 126, normalized size = 0.95

$$\frac{4\left(35\left(\sqrt{dx+c}b+a\right)^{\frac{9}{2}}-135\left(\sqrt{dx+c}b+a\right)^{\frac{7}{2}}a-63\left(b^2c-3\,a^2\right)\left(\sqrt{dx+c}b+a\right)^{\frac{5}{2}}+105\left(ab^2c-a^3\right)\left(\sqrt{dx+c}b+a\right)^{\frac{3}{2}}\right)}{315\,b^4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(sqrt(d*x + c)*b + a)*x,x, algorithm="maxima")
```

```
[Out] 4/315*(35*(sqrt(d*x + c)*b + a)^(9/2) - 135*(sqrt(d*x + c)*b + a)
^(7/2)*a - 63*(b^2*c - 3*a^2)*(sqrt(d*x + c)*b + a)^(5/2) + 105*(
a*b^2*c - a^3)*(sqrt(d*x + c)*b + a)^(3/2))/(b^4*d^2)
```

Fricas [A] time = 0.352166, size = 139, normalized size = 1.05

$$\frac{4\left(35\,b^4d^2x^2-28\,b^4c^2+36\,a^2b^2c-16\,a^4+\left(7\,b^4c-6\,a^2b^2\right)dx+\left(5\,ab^3dx-16\,ab^3c+8\,a^3b\right)\sqrt{dx+c}\right)\sqrt{\sqrt{dx+c}b+a}}{315\,b^4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(sqrt(d*x + c)*b + a)*x,x, algorithm="fricas")
```

```
[Out] 4/315*(35*b^4*d^2*x^2 - 28*b^4*c^2 + 36*a^2*b^2*c - 16*a^4 + (7*b
^4*c - 6*a^2*b^2)*d*x + (5*a*b^3*d*x - 16*a*b^3*c + 8*a^3*b)*sqrt
(d*x + c))*sqrt(sqrt(d*x + c)*b + a)/(b^4*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{a+b\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] Integral(x*sqrt(a + b*sqrt(c + d*x)), x)
```

GIAC/XCAS [A] time = 0.302967, size = 460, normalized size = 3.46

$$-\frac{4\left(63\sqrt{\left(\sqrt{dx+c}b+a\right)b^2}\left(\sqrt{dx+c}b+a\right)^2b^4\mathrm{csign}\left(\left(\sqrt{dx+c}b+a\right)b-ab\right)-105\sqrt{\left(\sqrt{dx+c}b+a\right)b^2}\left(\sqrt{dx+c}b+a\right)ab^4\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(sqrt(d*x + c)*b + a)*x,x, algorithm="giac")
```

```
[Out] -4/315*(63*sqrt((sqrt(d*x + c)*b + a)*b^2)*(sqrt(d*x + c)*b + a)^
2*b^4*c*sign((sqrt(d*x + c)*b + a)*b - a*b) - 105*sqrt((sqrt(d*x
+ c)*b + a)*b^2)*(sqrt(d*x + c)*b + a)*a*b^4*c*sign((sqrt(d*x + c
)*b + a)*b - a*b) - 35*sqrt((sqrt(d*x + c)*b + a)*b^2)*(sqrt(d*x
+ c)*b + a)^4*b^2*sign((sqrt(d*x + c)*b + a)*b - a*b) + 135*sqrt(
(sqrt(d*x + c)*b + a)*b^2)*(sqrt(d*x + c)*b + a)^3*a*b^2*sign((sq
rt(d*x + c)*b + a)*b - a*b) - 189*sqrt((sqrt(d*x + c)*b + a)*b^2)
*(sqrt(d*x + c)*b + a)^2*a^2*b^2*sign((sqrt(d*x + c)*b + a)*b - a
*b) + 105*sqrt((sqrt(d*x + c)*b + a)*b^2)*(sqrt(d*x + c)*b + a)*a
^3*b^2*sign((sqrt(d*x + c)*b + a)*b - a*b))*abs(b)/(b^8*d^2)
```

3.467 $\int \sqrt{a+b\sqrt{c+dx}}\,dx$

Optimal. Leaf size=56

$$\frac{4\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^2d}-\frac{4a\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^2d}$$

```
[Out] (-4*a*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^2*d) + (4*(a + b*Sqrt[c +
 d*x])^(5/2))/(5*b^2*d)
```

Rubi [A] time = 0.0679516, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$\frac{4\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^2d}-\frac{4a\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^2d}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-4*a*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^2*d) + (4*(a + b*Sqrt[c +
 d*x])^(5/2))/(5*b^2*d)
```

Rubi in Sympy [A] time = 4.01873, size = 48, normalized size = 0.86

$$-\frac{4a\left(a+b\sqrt{c+dx}\right)^{\frac{3}{2}}}{3b^2d}+\frac{4\left(a+b\sqrt{c+dx}\right)^{\frac{5}{2}}}{5b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] -4*a*(a + b*sqrt(c + d*x))**(3/2)/(3*b**2*d) + 4*(a + b*sqrt(c +
d*x))**(5/2)/(5*b**2*d)
```

Mathematica [A] time = 0.0493423, size = 55, normalized size = 0.98

$$\frac{4\sqrt{a+b\sqrt{c+dx}}\left(-2a^2+ab\sqrt{c+dx}+3b^2(c+dx)\right)}{15b^2d}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (4*Sqrt[a + b*Sqrt[c + d*x]]*(-2*a^2 + a*b*Sqrt[c + d*x] + 3*b^2*
(c + d*x)))/(15*b^2*d)
```

Maple [A] time = 0.003, size = 41, normalized size = 0.7

$$4\,\frac{1/5\,\left(a+b\sqrt{dx+c}\right)^{5/2}-1/3\,\left(a+b\sqrt{dx+c}\right)^{3/2}a}{b^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b*(d*x+c)^(1/2))^(1/2),x)
```

```
[Out] 4/d/b^2*(1/5*(a+b*(d*x+c)^(1/2))^(5/2)-1/3*(a+b*(d*x+c)^(1/2))^(3
/2)*a)
```

Maxima [A] time = 0.693783, size = 58, normalized size = 1.04

$$\frac{4\left(\frac{3\left(\sqrt{dx+c}b+a\right)^{\frac{5}{2}}}{b^{2}}-\frac{5\left(\sqrt{dx+c}b+a\right)^{\frac{3}{2}}a}{b^{2}}\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(sqrt(d*x + c)*b + a),x, algorithm="maxima")
```

```
[Out] 4/15*(3*(sqrt(d*x + c)*b + a)^(5/2)/b^2 - 5*(sqrt(d*x + c)*b + a)
^(3/2)*a/b^2)/d
```

Fricas [A] time = 0.341986, size = 68, normalized size = 1.21

$$\frac{4\left(3\,b^{2}dx+3\,b^{2}c+\sqrt{dx+c}ab-2\,a^{2}\right)\sqrt{\sqrt{dx+c}b+a}}{15\,b^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(sqrt(d*x + c)*b + a),x, algorithm="fricas")
```

```
[Out] 4/15*(3*b^2*d*x + 3*b^2*c + sqrt(d*x + c)*a*b - 2*a^2)*sqrt(sqrt(
d*x + c)*b + a)/(b^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b\sqrt{c + dx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sqrt(c + d*x)), x)
```

GIAC/XCAS [A] time = 0.28698, size = 159, normalized size = 2.84

$$\frac{4\left(3\sqrt{\left(\sqrt{dx+c}b+a\right)b^2}\left(\sqrt{dx+c}b+a\right)^2b^2\mathrm{sign}\left(\left(\sqrt{dx+c}b+a\right)b-ab\right)-5\sqrt{\left(\sqrt{dx+c}b+a\right)b^2}\left(\sqrt{dx+c}b+a\right)ab^2\mathrm{sign}\left(\right.\right.}{15\,b^6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(sqrt(d*x + c)*b + a),x, algorithm="giac")
```

```
[Out] 4/15*(3*sqrt((sqrt(d*x + c)*b + a)*b^2)*(sqrt(d*x + c)*b + a)^2*b
^2*sign((sqrt(d*x + c)*b + a)*b - a*b) - 5*sqrt((sqrt(d*x + c)*b
+ a)*b^2)*(sqrt(d*x + c)*b + a)*a*b^2*sign((sqrt(d*x + c)*b + a)*
b - a*b))*abs(b)/(b^6*d)
```

3.468 $\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x}\,dx$

Optimal. Leaf size=116

$$4\sqrt{a+b\sqrt{c+dx}}-2\sqrt{a-b\sqrt{c}}\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)-2\sqrt{a+b\sqrt{c}}\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)$$

```
[Out] 4*Sqrt[a + b*Sqrt[c + d*x]] - 2*Sqrt[a - b*Sqrt[c]]*ArcTanh[Sqrt[
a + b*Sqrt[c + d*x]]/Sqrt[a - b*Sqrt[c]]] - 2*Sqrt[a + b*Sqrt[c]]
*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a + b*Sqrt[c]]]
```

Rubi [A] time = 0.349722, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$4\sqrt{a+b\sqrt{c+dx}}-2\sqrt{a-b\sqrt{c}}\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)-2\sqrt{a+b\sqrt{c}}\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*Sqrt[c + d*x]]/x,x]
```

```
[Out] 4*Sqrt[a + b*Sqrt[c + d*x]] - 2*Sqrt[a - b*Sqrt[c]]*ArcTanh[Sqrt[
a + b*Sqrt[c + d*x]]/Sqrt[a - b*Sqrt[c]]] - 2*Sqrt[a + b*Sqrt[c]]
*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a + b*Sqrt[c]]]
```

Rubi in Sympy [A] time = 27.8602, size = 100, normalized size = 0.86

$$-2\sqrt{a-b\sqrt{c}}\,\text{atanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)-2\sqrt{a+b\sqrt{c}}\,\text{atanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)+4\sqrt{a+b\sqrt{c+dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b*(d*x+c)**(1/2))**(1/2)/x,x)
```

```
[Out] -2*sqrt(a - b*sqrt(c))*atanh(sqrt(a + b*sqrt(c + d*x))/sqrt(a - b
*sqrt(c))) - 2*sqrt(a + b*sqrt(c))*atanh(sqrt(a + b*sqrt(c + d*x)
)/sqrt(a + b*sqrt(c))) + 4*sqrt(a + b*sqrt(c + d*x))
```

Mathematica [A] time = 0.235681, size = 116, normalized size = 1.

$$4\sqrt{a+b\sqrt{c+dx}}-2\sqrt{a-b\sqrt{c}}\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)-2\sqrt{a+b\sqrt{c}}\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b*Sqrt[c + d*x]]/x,x]
```

```
[Out] 4*Sqrt[a + b*Sqrt[c + d*x]] - 2*Sqrt[a - b*Sqrt[c]]*ArcTanh[Sqrt[
a + b*Sqrt[c + d*x]]/Sqrt[a - b*Sqrt[c]]] - 2*Sqrt[a + b*Sqrt[c]]
*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a + b*Sqrt[c]]]
```

Maple [B] time = 0.051, size = 221, normalized size = 1.9

$$4\sqrt{a+b\sqrt{dx+c}}-2\frac{b^2c}{\sqrt{b^2c}\sqrt{\sqrt{b^2c}-a}}\arctan\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{\sqrt{b^2c}-a}}\right)$$
$$+2\frac{a}{\sqrt{\sqrt{b^2c}-a}}\arctan\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{\sqrt{b^2c}-a}}\right)+2\frac{b^2c}{\sqrt{b^2c}\sqrt{-\sqrt{b^2c}-a}}\arctan\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{-\sqrt{b^2c}-a}}\right)$$
$$+2\frac{a}{\sqrt{-\sqrt{b^2c}-a}}\arctan\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{-\sqrt{b^2c}-a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b*(d*x+c)^(1/2))^(1/2)/x,x)
```

```
[Out] 4*(a+b*(d*x+c)^(1/2))^(1/2)-2/(b^2*c)^(1/2)/((b^2*c)^(1/2)-a)^(1/
2)*arctan((a+b*(d*x+c)^(1/2))^(1/2)/((b^2*c)^(1/2)-a)^(1/2))*b^2*
c+2/((b^2*c)^(1/2)-a)^(1/2)*arctan((a+b*(d*x+c)^(1/2))^(1/2)/((b^
2*c)^(1/2)-a)^(1/2))*a+2/(b^2*c)^(1/2)/(-(b^2*c)^(1/2)-a)^(1/2)*a
rctan((a+b*(d*x+c)^(1/2))^(1/2)/(-(b^2*c)^(1/2)-a)^(1/2))*b^2*c+2
/(-(b^2*c)^(1/2)-a)^(1/2)*arctan((a+b*(d*x+c)^(1/2))^(1/2)/(-(b^2
*c)^(1/2)-a)^(1/2))*a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{\sqrt{dx+c}b+a}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(sqrt(d*x + c)*b + a)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sqrt(d*x + c)*b + a)/x, x)
```

Fricas [A] time = 0.349505, size = 262, normalized size = 2.26

$$-\sqrt{a+\sqrt{b^2c}}\log\left(2\sqrt{\sqrt{dx+c}b+a}+2\sqrt{a+\sqrt{b^2c}}\right)+\sqrt{a+\sqrt{b^2c}}\log\left(2\sqrt{\sqrt{dx+c}b+a}-2\sqrt{a+\sqrt{b^2c}}\right)-\sqrt{a-\sqrt{b^2c}}\log\left(2\sqrt{\sqrt{dx+c}b+a}+2\sqrt{a-\sqrt{b^2c}}\right)+\sqrt{a-\sqrt{b^2c}}\log\left(2\sqrt{\sqrt{dx+c}b+a}-2\sqrt{a-\sqrt{b^2c}}\right)+4\sqrt{\sqrt{dx+c}b+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(sqrt(d*x + c)*b + a)/x,x, algorithm="fricas")
```

```
[Out] -sqrt(a + sqrt(b^2*c))*log(2*sqrt(sqrt(d*x + c)*b + a) + 2*sqrt(a
 + sqrt(b^2*c))) + sqrt(a + sqrt(b^2*c))*log(2*sqrt(sqrt(d*x + c)
*b + a) - 2*sqrt(a + sqrt(b^2*c))) - sqrt(a - sqrt(b^2*c))*log(2*
sqrt(sqrt(d*x + c)*b + a) + 2*sqrt(a - sqrt(b^2*c))) + sqrt(a - s
qrt(b^2*c))*log(2*sqrt(sqrt(d*x + c)*b + a) - 2*sqrt(a - sqrt(b^2
*c))) + 4*sqrt(sqrt(d*x + c)*b + a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b*(d*x+c)**(1/2))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(a + b*sqrt(c + d*x))/x, x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(sqrt(d*x + c)*b + a)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.469 $\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^2}\,dx$

Optimal. Leaf size=137

$$-\frac{\sqrt{a+b\sqrt{c+dx}}}{x}+\frac{bd\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{2\sqrt{c}\sqrt{a-b\sqrt{c}}}-\frac{bd\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{2\sqrt{c}\sqrt{a+b\sqrt{c}}}$$

```
[Out] -(Sqrt[a + b*Sqrt[c + d*x]]/x) + (b*d*ArcTanh[Sqrt[a + b*Sqrt[c +
 d*x]]/Sqrt[a - b*Sqrt[c]]])/(2*Sqrt[a - b*Sqrt[c]]*Sqrt[c]) - (b
*d*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a + b*Sqrt[c]]])/(2*Sqr
t[a + b*Sqrt[c]]*Sqrt[c])
```

Rubi [A] time = 0.349087, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$-\frac{\sqrt{a+b\sqrt{c+dx}}}{x}+\frac{bd\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{2\sqrt{c}\sqrt{a-b\sqrt{c}}}-\frac{bd\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{2\sqrt{c}\sqrt{a+b\sqrt{c}}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*Sqrt[c + d*x]]/x^2,x]
```

```
[Out] -(Sqrt[a + b*Sqrt[c + d*x]]/x) + (b*d*ArcTanh[Sqrt[a + b*Sqrt[c +
 d*x]]/Sqrt[a - b*Sqrt[c]]])/(2*Sqrt[a - b*Sqrt[c]]*Sqrt[c]) - (b
*d*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a + b*Sqrt[c]]])/(2*Sqr
t[a + b*Sqrt[c]]*Sqrt[c])
```

Rubi in Sympy [A] time = 23.0225, size = 117, normalized size = 0.85

$$-\frac{bd\,\operatorname{atanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{2\sqrt{c}\sqrt{a+b\sqrt{c}}}+\frac{bd\,\operatorname{atanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{2\sqrt{c}\sqrt{a-b\sqrt{c}}}-\frac{\sqrt{a+b\sqrt{c+dx}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b*(d*x+c)**(1/2))**(1/2)/x**2,x)
```

```
[Out] -b*d*atanh(sqrt(a + b*sqrt(c + d*x))/sqrt(a + b*sqrt(c)))/(2*sqrt
(c)*sqrt(a + b*sqrt(c))) + b*d*atanh(sqrt(a + b*sqrt(c + d*x))/sq
```

```
rt(a - b*sqrt(c)))/(2*sqrt(c)*sqrt(a - b*sqrt(c))) - sqrt(a + b*s
qrt(c + d*x))/x
```

Mathematica [A] time = 0.555435, size = 144, normalized size = 1.05

$$\frac{1}{2}\left(-\frac{2\sqrt{a+b\sqrt{c+dx}}}{x}+\frac{bd\tan^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{-a-b\sqrt{c}}}\right)}{\sqrt{c}\sqrt{-a-b\sqrt{c}}}-\frac{bd\tan^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{b\sqrt{c}-a}}\right)}{\sqrt{c}\sqrt{b\sqrt{c}-a}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b*Sqrt[c + d*x]]/x^2,x]
```

```
[Out] ((-2*Sqrt[a + b*Sqrt[c + d*x]])/x + (b*d*ArcTan[Sqrt[a + b*Sqrt[c
 + d*x]]/Sqrt[-a - b*Sqrt[c]]])/(Sqrt[-a - b*Sqrt[c]]*Sqrt[c]) -
(b*d*ArcTan[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[-a + b*Sqrt[c]]])/(Sqr
t[-a + b*Sqrt[c]]*Sqrt[c]))/2
```

Maple [A] time = 0.03, size = 151, normalized size = 1.1

$$-\frac{b^2d}{b^2(dx+c)-b^2c}\sqrt{a+b\sqrt{dx+c}}-\frac{b^2d}{2}\arctan\left(1\sqrt{a+b\sqrt{dx+c}}\frac{1}{\sqrt{\sqrt{b^2c}-a}}\right)\frac{1}{\sqrt{b^2c}}\frac{1}{\sqrt{\sqrt{b^2c}-a}}+\frac{b^2d}{2}\arctan\left(1\sqrt{a+b\sqrt{dx+c}}\frac{1}{\sqrt{-\sqrt{b^2c}-a}}\right)\frac{1}{\sqrt{b^2c}}\frac{1}{\sqrt{-\sqrt{b^2c}-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b*(d*x+c)^(1/2))^(1/2)/x^2,x)
```

```
[Out] -b^2*d*(a+b*(d*x+c)^(1/2))^(1/2)/(b^2*(d*x+c)-b^2*c)-1/2*b^2*d/(b
^2*c)^(1/2)/((b^2*c)^(1/2)-a)^(1/2)*arctan((a+b*(d*x+c)^(1/2))^(1
/2)/((b^2*c)^(1/2)-a)^(1/2))+1/2*b^2*d/(b^2*c)^(1/2)/(-(b^2*c)^(1
/2)-a)^(1/2)*arctan((a+b*(d*x+c)^(1/2))^(1/2)/(-(b^2*c)^(1/2)-a)^
(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{\sqrt{dx+c}b+a}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(sqrt(d*x + c)*b + a)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sqrt(d*x + c)*b + a)/x^2, x)
```

Fricas [A] time = 0.366363, size = 1354, normalized size = 9.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(sqrt(d*x + c)*b + a)/x^2,x, algorithm="fricas")
```

```
[Out] -1/4*(x*sqrt(-(a*b^2*d^2 + sqrt(b^6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2
+ a^4*c))*(b^2*c^2 - a^2*c))/(b^2*c^2 - a^2*c))*log(sqrt(sqrt(d*x
 + c)*b + a)*b^4*d^3 + (b^4*c*d^2 - sqrt(b^6*d^4/(b^4*c^3 - 2*a^2
*b^2*c^2 + a^4*c))*(a*b^2*c^2 - a^3*c))*sqrt(-(a*b^2*d^2 + sqrt(b
^6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))*(b^2*c^2 - a^2*c))/(b^2
*c^2 - a^2*c))) - x*sqrt(-(a*b^2*d^2 + sqrt(b^6*d^4/(b^4*c^3 - 2*
a^2*b^2*c^2 + a^4*c))*(b^2*c^2 - a^2*c))/(b^2*c^2 - a^2*c))*log(s
qrt(sqrt(d*x + c)*b + a)*b^4*d^3 - (b^4*c*d^2 - sqrt(b^6*d^4/(b^4
*c^3 - 2*a^2*b^2*c^2 + a^4*c))*(a*b^2*c^2 - a^3*c))*sqrt(-(a*b^2*
d^2 + sqrt(b^6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))*(b^2*c^2 -
a^2*c))/(b^2*c^2 - a^2*c))) + x*sqrt(-(a*b^2*d^2 - sqrt(b^6*d^4/(
b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))*(b^2*c^2 - a^2*c))/(b^2*c^2 - a
^2*c))*log(sqrt(sqrt(d*x + c)*b + a)*b^4*d^3 + (b^4*c*d^2 + sqrt(
b^6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))*(a*b^2*c^2 - a^3*c))*s
qrt(-(a*b^2*d^2 - sqrt(b^6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))
*(b^2*c^2 - a^2*c))/(b^2*c^2 - a^2*c))) - x*sqrt(-(a*b^2*d^2 - sq
rt(b^6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))*(b^2*c^2 - a^2*c))/
(b^2*c^2 - a^2*c))*log(sqrt(sqrt(d*x + c)*b + a)*b^4*d^3 - (b^4*c
*d^2 + sqrt(b^6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))*(a*b^2*c^2
 - a^3*c))*sqrt(-(a*b^2*d^2 - sqrt(b^6*d^4/(b^4*c^3 - 2*a^2*b^2*c
^2 + a^4*c))*(b^2*c^2 - a^2*c))/(b^2*c^2 - a^2*c))) + 4*sqrt(sqrt
(d*x + c)*b + a))/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + b\sqrt{c + dx}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b*(d*x+c)**(1/2))**(1/2)/x**2,x)

[Out] Integral(sqrt(a + b*sqrt(c + d*x))/x**2, x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(sqrt(d*x + c)*b + a)/x^2,x, algorithm="giac")

[Out] Exception raised: TypeError
```

3.470 $\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^3}\,dx$

Optimal. Leaf size=224

$$\frac{bd\left(bc-a\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{8cx\left(a^2-b^2c\right)}-\frac{bd^2\left(2a-3b\sqrt{c}\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{16c^{3/2}\left(a-b\sqrt{c}\right)^{3/2}}+\frac{bd^2\left(2a+3b\sqrt{c}\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{16c^{3/2}\left(a+b\sqrt{c}\right)^{3/2}}-\frac{\sqrt{a+b\sqrt{c+dx}}}{2x^2}$$

```
[Out] -Sqrt[a + b*Sqrt[c + d*x]]/(2*x^2) + (b*d*(b*c - a*Sqrt[c + d*x])
*Sqrt[a + b*Sqrt[c + d*x]])/(8*c*(a^2 - b^2*c)*x) - (b*(2*a - 3*b
*Sqrt[c])*d^2*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a - b*Sqrt[c
]]])/(16*(a - b*Sqrt[c])^(3/2)*c^(3/2)) + (b*(2*a + 3*b*Sqrt[c])*
d^2*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a + b*Sqrt[c]]])/(16*(
a + b*Sqrt[c])^(3/2)*c^(3/2))
```

Rubi [A] time = 0.848323, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$

$$\frac{bd\left(bc-a\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{8cx\left(a^2-b^2c\right)}-\frac{bd^2\left(2a-3b\sqrt{c}\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{16c^{3/2}\left(a-b\sqrt{c}\right)^{3/2}}+\frac{bd^2\left(2a+3b\sqrt{c}\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{16c^{3/2}\left(a+b\sqrt{c}\right)^{3/2}}-\frac{\sqrt{a+b\sqrt{c+dx}}}{2x^2}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*Sqrt[c + d*x]]/x^3,x]
```


```
[Out] -Sqrt[a + b*Sqrt[c + d*x]]/(2*x^2) + (b*d*(b*c - a*Sqrt[c + d*x])
*Sqrt[a + b*Sqrt[c + d*x]])/(8*c*(a^2 - b^2*c)*x) - (b*(2*a - 3*b
*Sqrt[c])*d^2*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a - b*Sqrt[c
]]])/(16*(a - b*Sqrt[c])^(3/2)*c^(3/2)) + (b*(2*a + 3*b*Sqrt[c])*
d^2*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a + b*Sqrt[c]]])/(16*(
a + b*Sqrt[c])^(3/2)*c^(3/2))
```

Rubi in Sympy [A] time = 81.5484, size = 228, normalized size = 1.02

$$-\frac{bd\sqrt{a+b\sqrt{c+dx}}\left(a\sqrt{c+dx}-bc\right)}{8cx\left(a^2-b^2c\right)}+\frac{bd^2\left(2a^2+ab\sqrt{c}-3b^2c\right)\operatorname{atanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{16c^{\frac{3}{2}}\sqrt{a+b\sqrt{c}}\left(a^2-b^2c\right)}-\frac{bd^2\left(2a^2-ab\sqrt{c}-3b^2c\right)\operatorname{atanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{16c^{\frac{3}{2}}\sqrt{a-b\sqrt{c}}\left(a^2-b^2c\right)}-\frac{\sqrt{a+b\sqrt{c+dx}}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b*(d*x+c)**(1/2))**(1/2)/x**3,x)
```

```
[Out] -b*d*sqrt(a + b*sqrt(c + d*x))*(a*sqrt(c + d*x) - b*c)/(8*c*x*(a*
*2 - b**2*c)) + b*d**2*(2*a**2 + a*b*sqrt(c) - 3*b**2*c)*atanh(sq
rt(a + b*sqrt(c + d*x))/sqrt(a + b*sqrt(c)))/(16*c**(3/2)*sqrt(a
+ b*sqrt(c))*(a**2 - b**2*c)) - b*d**2*(2*a**2 - a*b*sqrt(c) - 3*
b**2*c)*atanh(sqrt(a + b*sqrt(c + d*x))/sqrt(a - b*sqrt(c)))/(16*
c**(3/2)*sqrt(a - b*sqrt(c))*(a**2 - b**2*c)) - sqrt(a + b*sqrt(c
 + d*x))/(2*x**2)
```

Mathematica [A] time = 1.96749, size = 230, normalized size = 1.03

$$\frac{1}{16}d^2\left(\frac{2b\sqrt{a+b\sqrt{c+dx}}\left(a\sqrt{c+dx}-bc\right)}{cdx\left(b^2c-a^2\right)}+\frac{b\left(2a+3b\sqrt{c}\right)\tan^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{-a-b\sqrt{c}}}\right)}{c^{3/2}\left(-a-b\sqrt{c}\right)^{3/2}}-\frac{b\left(2a-3b\sqrt{c}\right)\tan^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{b\sqrt{c}-a}}\right)}{c^{3/2}\left(b\sqrt{c}-a\right)^{3/2}}-\frac{8\sqrt{a+b\sqrt{c+dx}}}{d^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b*Sqrt[c + d*x]]/x^3,x]
```

```
[Out] (d^2*((-8*Sqrt[a + b*Sqrt[c + d*x]])/(d^2*x^2) + (2*b*(-(b*c) + a
*Sqrt[c + d*x])*Sqrt[a + b*Sqrt[c + d*x]])/(c*(-a^2 + b^2*c)*d*x)
 + (b*(2*a + 3*b*Sqrt[c])*ArcTan[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[-
a - b*Sqrt[c]]])/((-a - b*Sqrt[c])^(3/2)*c^(3/2)) - (b*(2*a - 3*b
*Sqrt[c])*ArcTan[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[-a + b*Sqrt[c]]])
/((-a + b*Sqrt[c])^(3/2)*c^(3/2))))/16
```

Maple [B] time = 0.14, size = 2530, normalized size = 11.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b*(d*x+c)^(1/2))^(1/2)/x^3,x)
```

```
[Out] -1/8*b^2*d^2/(b^2*(d*x+c)-b^2*c)^2*a/c/(-b^2*c+a^2)*(a+b*(d*x+c)^
(1/2))^(7/2)+1/8*b^4*d^2/(b^2*(d*x+c)-b^2*c)^2/(-b^2*c+a^2)*(a+b*
(d*x+c)^(1/2))^(5/2)+3/8*b^2*d^2/(b^2*(d*x+c)-b^2*c)^2/c/(-b^2*c+
a^2)*(a+b*(d*x+c)^(1/2))^(5/2)*a^2-1/8*b^4*d^2/(b^2*(d*x+c)-b^2*c
)^2*a/(-b^2*c+a^2)*(a+b*(d*x+c)^(1/2))^(3/2)-3/8*b^2*d^2/(b^2*(d*
x+c)-b^2*c)^2*a^3/c/(-b^2*c+a^2)*(a+b*(d*x+c)^(1/2))^(3/2)-3/8*b^
4*d^2/(b^2*(d*x+c)-b^2*c)^2*(a+b*(d*x+c)^(1/2))^(1/2)+1/8*b^2*d^2
/(b^2*(d*x+c)-b^2*c)^2/c*(a+b*(d*x+c)^(1/2))^(1/2)*a^2+3/16*b^11*
d^2*c^4/(b^6*c^3*(-b^2*c+a^2)^2)^(1/2)/(-b^2*c+a^2)/(-c*(-b^2*c+a
^2)*(a*b^4*c^2-a^3*b^2*c+(b^6*c^3*(-b^2*c+a^2)^2)^(1/2)))^(1/2)*a
rctanh((-b^4*c^2+a^2*b^2*c)*(a+b*(d*x+c)^(1/2))^(1/2)/b/(-c*(-b^2
*c+a^2)*(a*b^4*c^2-a^3*b^2*c+(b^6*c^3*(-b^2*c+a^2)^2)^(1/2)))^(1/
2))-1/2*b^9*d^2*c^3/(b^6*c^3*(-b^2*c+a^2)^2)^(1/2)/(-b^2*c+a^2)/(
-c*(-b^2*c+a^2)*(a*b^4*c^2-a^3*b^2*c+(b^6*c^3*(-b^2*c+a^2)^2)^(1/
2)))^(1/2)*arctanh((-b^4*c^2+a^2*b^2*c)*(a+b*(d*x+c)^(1/2))^(1/2)
/b/(-c*(-b^2*c+a^2)*(a*b^4*c^2-a^3*b^2*c+(b^6*c^3*(-b^2*c+a^2)^2)
^(1/2)))^(1/2))*a^2+7/16*b^7*d^2*c^2/(b^6*c^3*(-b^2*c+a^2)^2)^(1/
2)/(-b^2*c+a^2)/(-c*(-b^2*c+a^2)*(a*b^4*c^2-a^3*b^2*c+(b^6*c^3*(-
b^2*c+a^2)^2)^(1/2)))^(1/2)*arctanh((-b^4*c^2+a^2*b^2*c)*(a+b*(d*
x+c)^(1/2))^(1/2)/b/(-c*(-b^2*c+a^2)*(a*b^4*c^2-a^3*b^2*c+(b^6*c^
3*(-b^2*c+a^2)^2)^(1/2)))^(1/2))*a^4-1/8*b^5*d^2*c/(b^6*c^3*(-b^2
*c+a^2)^2)^(1/2)/(-b^2*c+a^2)/(-c*(-b^2*c+a^2)*(a*b^4*c^2-a^3*b^2
*c+(b^6*c^3*(-b^2*c+a^2)^2)^(1/2)))^(1/2)*arctanh((-b^4*c^2+a^2*b
^2*c)*(a+b*(d*x+c)^(1/2))^(1/2)/b/(-c*(-b^2*c+a^2)*(a*b^4*c^2-a^3
*b^2*c+(b^6*c^3*(-b^2*c+a^2)^2)^(1/2)))^(1/2))*a^6-1/16*b^5*d^2*c
/(-b^2*c+a^2)/(-c*(-b^2*c+a^2)*(a*b^4*c^2-a^3*b^2*c+(b^6*c^3*(-b^
2*c+a^2)^2)^(1/2)))^(1/2)*arctanh((-b^4*c^2+a^2*b^2*c)*(a+b*(d*x+
c)^(1/2))^(1/2)/b/(-c*(-b^2*c+a^2)*(a*b^4*c^2-a^3*b^2*c+(b^6*c^3*
(-b^2*c+a^2)^2)^(1/2)))^(1/2))*a+1/16*b^3*d^2/(-b^2*c+a^2)/(-c*(-
b^2*c+a^2)*(a*b^4*c^2-a^3*b^2*c+(b^6*c^3*(-b^2*c+a^2)^2)^(1/2)))^
(1/2)*arctanh((-b^4*c^2+a^2*b^2*c)*(a+b*(d*x+c)^(1/2))^(1/2)/b/(-
c*(-b^2*c+a^2)*(a*b^4*c^2-a^3*b^2*c+(b^6*c^3*(-b^2*c+a^2)^2)^(1/2
)))^(1/2))*a^3-3/16*b^11*d^2*c^4/(b^6*c^3*(-b^2*c+a^2)^2)^(1/2)/(
-b^2*c+a^2)/(-(-a*b^4*c^2+a^3*b^2*c+(b^6*c^3*(-b^2*c+a^2)^2)^(1/2
))*c*(-b^2*c+a^2))^(1/2)*arctan((b^4*c^2-a^2*b^2*c)*(a+b*(d*x+c)^
(1/2))^(1/2)/b/(-(-a*b^4*c^2+a^3*b^2*c+(b^6*c^3*(-b^2*c+a^2)^2)^(
1/2))*c*(-b^2*c+a^2))^(1/2))+1/2*b^9*d^2*c^3/(b^6*c^3*(-b^2*c+a^2
)^2)^(1/2)/(-b^2*c+a^2)/(-(-a*b^4*c^2+a^3*b^2*c+(b^6*c^3*(-b^2*c+
a^2)^2)^(1/2))*c*(-b^2*c+a^2))^(1/2)*arctan((b^4*c^2-a^2*b^2*c)*(
a+b*(d*x+c)^(1/2))^(1/2)/b/(-(-a*b^4*c^2+a^3*b^2*c+(b^6*c^3*(-b^2
*c+a^2)^2)^(1/2))*c*(-b^2*c+a^2))^(1/2))*a^2-7/16*b^7*d^2*c^2/(b^
6*c^3*(-b^2*c+a^2)^2)^(1/2)/(-b^2*c+a^2)/(-(-a*b^4*c^2+a^3*b^2*c+
```

```
(b^6*c^3*(-b^2*c+a^2)^2)^(1/2))*c*(-b^2*c+a^2))^(1/2)*arctan((b^4
*c^2-a^2*b^2*c)*(a+b*(d*x+c)^(1/2))^(1/2)/b/(-(-a*b^4*c^2+a^3*b^2
*c+(b^6*c^3*(-b^2*c+a^2)^2)^(1/2))*c*(-b^2*c+a^2))^(1/2))*a^4+1/8
*b^5*d^2*c/(b^6*c^3*(-b^2*c+a^2)^2)^(1/2)/(-b^2*c+a^2)/(-(-a*b^4*
c^2+a^3*b^2*c+(b^6*c^3*(-b^2*c+a^2)^2)^(1/2))*c*(-b^2*c+a^2))^(1/
2)*arctan((b^4*c^2-a^2*b^2*c)*(a+b*(d*x+c)^(1/2))^(1/2)/b/(-(-a*b
^4*c^2+a^3*b^2*c+(b^6*c^3*(-b^2*c+a^2)^2)^(1/2))*c*(-b^2*c+a^2))^
(1/2))*a^6-1/16*b^5*d^2*c/(-b^2*c+a^2)/(-(-a*b^4*c^2+a^3*b^2*c+(b
^6*c^3*(-b^2*c+a^2)^2)^(1/2))*c*(-b^2*c+a^2))^(1/2)*arctan((b^4*c
^2-a^2*b^2*c)*(a+b*(d*x+c)^(1/2))^(1/2)/b/(-(-a*b^4*c^2+a^3*b^2*c
+(b^6*c^3*(-b^2*c+a^2)^2)^(1/2))*c*(-b^2*c+a^2))^(1/2))*a+1/16*b^
3*d^2/(-b^2*c+a^2)/(-(-a*b^4*c^2+a^3*b^2*c+(b^6*c^3*(-b^2*c+a^2)^
2)^(1/2))*c*(-b^2*c+a^2))^(1/2)*arctan((b^4*c^2-a^2*b^2*c)*(a+b*(
d*x+c)^(1/2))^(1/2)/b/(-(-a*b^4*c^2+a^3*b^2*c+(b^6*c^3*(-b^2*c+a^
2)^2)^(1/2))*c*(-b^2*c+a^2))^(1/2))*a^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sqrt{dx+c}b+a}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(sqrt(d*x + c)*b + a)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sqrt(d*x + c)*b + a)/x^3, x)
```

Fricas [A] time = 0.474253, size = 3856, normalized size = 17.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(sqrt(d*x + c)*b + a)/x^3,x, algorithm="fricas")
```

```
[Out] 1/32*((b^2*c^2 - a^2*c)*x^2*sqrt(-((15*a*b^6*c^2 - 15*a^3*b^4*c +
 4*a^5*b^2)*d^4 + (b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*
c^3)*sqrt((81*b^14*c^2 - 90*a^2*b^12*c + 25*a^4*b^10)*d^8/(b^12*c
^9 - 6*a^2*b^10*c^8 + 15*a^4*b^8*c^7 - 20*a^6*b^6*c^6 + 15*a^8*b^
4*c^5 - 6*a^10*b^2*c^4 + a^12*c^3)))/(b^6*c^6 - 3*a^2*b^4*c^5 + 3
*a^4*b^2*c^4 - a^6*c^3))*log((81*b^10*c^2 - 81*a^2*b^8*c + 20*a^4
*b^6)*sqrt(sqrt(d*x + c)*b + a)*d^6 + ((27*b^10*c^4 - 24*a^2*b^8*
c^3 + 5*a^4*b^6*c^2)*d^4 - 2*(2*a*b^8*c^7 - 7*a^3*b^6*c^6 + 9*a^5
*b^4*c^5 - 5*a^7*b^2*c^4 + a^9*c^3)*sqrt((81*b^14*c^2 - 90*a^2*b^
```

12*c + 25*a^4*b^10)*d^8/(b^12*c^9 - 6*a^2*b^10*c^8 + 15*a^4*b^8*c
^7 - 20*a^6*b^6*c^6 + 15*a^8*b^4*c^5 - 6*a^10*b^2*c^4 + a^12*c^3)
))*sqrt(-((15*a*b^6*c^2 - 15*a^3*b^4*c + 4*a^5*b^2)*d^4 + (b^6*c^
6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3)*sqrt((81*b^14*c^2 -
90*a^2*b^12*c + 25*a^4*b^10)*d^8/(b^12*c^9 - 6*a^2*b^10*c^8 + 15*
a^4*b^8*c^7 - 20*a^6*b^6*c^6 + 15*a^8*b^4*c^5 - 6*a^10*b^2*c^4 +
a^12*c^3)))/(b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3)))
 - (b^2*c^2 - a^2*c)*x^2*sqrt(-((15*a*b^6*c^2 - 15*a^3*b^4*c + 4*
a^5*b^2)*d^4 + (b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3
)*sqrt((81*b^14*c^2 - 90*a^2*b^12*c + 25*a^4*b^10)*d^8/(b^12*c^9
- 6*a^2*b^10*c^8 + 15*a^4*b^8*c^7 - 20*a^6*b^6*c^6 + 15*a^8*b^4*c
^5 - 6*a^10*b^2*c^4 + a^12*c^3)))/(b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^
4*b^2*c^4 - a^6*c^3))*log((81*b^10*c^2 - 81*a^2*b^8*c + 20*a^4*b^
6)*sqrt(sqrt(d*x + c)*b + a)*d^6 - ((27*b^10*c^4 - 24*a^2*b^8*c^3
 + 5*a^4*b^6*c^2)*d^4 - 2*(2*a*b^8*c^7 - 7*a^3*b^6*c^6 + 9*a^5*b^
4*c^5 - 5*a^7*b^2*c^4 + a^9*c^3)*sqrt((81*b^14*c^2 - 90*a^2*b^12*
c + 25*a^4*b^10)*d^8/(b^12*c^9 - 6*a^2*b^10*c^8 + 15*a^4*b^8*c^7
- 20*a^6*b^6*c^6 + 15*a^8*b^4*c^5 - 6*a^10*b^2*c^4 + a^12*c^3)))*
sqrt(-((15*a*b^6*c^2 - 15*a^3*b^4*c + 4*a^5*b^2)*d^4 + (b^6*c^6 -
 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3)*sqrt((81*b^14*c^2 - 90*
a^2*b^12*c + 25*a^4*b^10)*d^8/(b^12*c^9 - 6*a^2*b^10*c^8 + 15*a^4
*b^8*c^7 - 20*a^6*b^6*c^6 + 15*a^8*b^4*c^5 - 6*a^10*b^2*c^4 + a^1
2*c^3)))/(b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3))) +
(b^2*c^2 - a^2*c)*x^2*sqrt(-((15*a*b^6*c^2 - 15*a^3*b^4*c + 4*a^5
*b^2)*d^4 - (b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3)*s
qrt((81*b^14*c^2 - 90*a^2*b^12*c + 25*a^4*b^10)*d^8/(b^12*c^9 - 6
*a^2*b^10*c^8 + 15*a^4*b^8*c^7 - 20*a^6*b^6*c^6 + 15*a^8*b^4*c^5
- 6*a^10*b^2*c^4 + a^12*c^3)))/(b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b
^2*c^4 - a^6*c^3))*log((81*b^10*c^2 - 81*a^2*b^8*c + 20*a^4*b^6)*
sqrt(sqrt(d*x + c)*b + a)*d^6 + ((27*b^10*c^4 - 24*a^2*b^8*c^3 +
5*a^4*b^6*c^2)*d^4 + 2*(2*a*b^8*c^7 - 7*a^3*b^6*c^6 + 9*a^5*b^4*c
^5 - 5*a^7*b^2*c^4 + a^9*c^3)*sqrt((81*b^14*c^2 - 90*a^2*b^12*c +
 25*a^4*b^10)*d^8/(b^12*c^9 - 6*a^2*b^10*c^8 + 15*a^4*b^8*c^7 - 2
0*a^6*b^6*c^6 + 15*a^8*b^4*c^5 - 6*a^10*b^2*c^4 + a^12*c^3)))*sqr
t(-((15*a*b^6*c^2 - 15*a^3*b^4*c + 4*a^5*b^2)*d^4 - (b^6*c^6 - 3*
a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3)*sqrt((81*b^14*c^2 - 90*a^2
*b^12*c + 25*a^4*b^10)*d^8/(b^12*c^9 - 6*a^2*b^10*c^8 + 15*a^4*b^
8*c^7 - 20*a^6*b^6*c^6 + 15*a^8*b^4*c^5 - 6*a^10*b^2*c^4 + a^12*c
^3)))/(b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3))) - (b^
2*c^2 - a^2*c)*x^2*sqrt(-((15*a*b^6*c^2 - 15*a^3*b^4*c + 4*a^5*b^
2)*d^4 - (b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3)*sqrt
((81*b^14*c^2 - 90*a^2*b^12*c + 25*a^4*b^10)*d^8/(b^12*c^9 - 6*a^
2*b^10*c^8 + 15*a^4*b^8*c^7 - 20*a^6*b^6*c^6 + 15*a^8*b^4*c^5 - 6
*a^10*b^2*c^4 + a^12*c^3)))/(b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*
c^4 - a^6*c^3))*log((81*b^10*c^2 - 81*a^2*b^8*c + 20*a^4*b^6)*sqr
t(sqrt(d*x + c)*b + a)*d^6 - ((27*b^10*c^4 - 24*a^2*b^8*c^3 + 5*a
^4*b^6*c^2)*d^4 + 2*(2*a*b^8*c^7 - 7*a^3*b^6*c^6 + 9*a^5*b^4*c^5
- 5*a^7*b^2*c^4 + a^9*c^3)*sqrt((81*b^14*c^2 - 90*a^2*b^12*c + 25
*a^4*b^10)*d^8/(b^12*c^9 - 6*a^2*b^10*c^8 + 15*a^4*b^8*c^7 - 20*a
^6*b^6*c^6 + 15*a^8*b^4*c^5 - 6*a^10*b^2*c^4 + a^12*c^3)))*sqrt(-
((15*a*b^6*c^2 - 15*a^3*b^4*c + 4*a^5*b^2)*d^4 - (b^6*c^6 - 3*a^2
*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3)*sqrt((81*b^14*c^2 - 90*a^2*b^
12*c + 25*a^4*b^10)*d^8/(b^12*c^9 - 6*a^2*b^10*c^8 + 15*a^4*b^8*c
^7 - 20*a^6*b^6*c^6 + 15*a^8*b^4*c^5 - 6*a^10*b^2*c^4 + a^12*c^3)

```
))/(b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3))) - 4*(b^2
*c*d*x - sqrt(d*x + c)*a*b*d*x + 4*b^2*c^2 - 4*a^2*c)*sqrt(sqrt(d
*x + c)*b + a))/((b^2*c^2 - a^2*c)*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + b\sqrt{c + dx}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b*(d*x+c)**(1/2))**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(a + b*sqrt(c + d*x))/x**3, x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(sqrt(d*x + c)*b + a)/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.471 $\int \frac{x^3}{a+b\sqrt{c+dx}}\,dx$

Optimal. Leaf size=230

$$-\frac{2a\left(a^2-b^2c\right)^3\log\left(a+b\sqrt{c+dx}\right)}{b^8d^4}+\frac{2\left(a^2-b^2c\right)^3\sqrt{c+dx}}{b^7d^4}-\frac{a\left(a^2-3b^2c\right)(c+dx)^2}{2b^4d^4}+\frac{2\left(a^2-3b^2c\right)(c+dx)^{5/2}}{5b^3d^4}-\frac{ax\left(a^4-3a^2b^2c+3b^4c^2\right)}{b^6d^3}+\frac{2\left(a^4-3a^2b^2c+3b^4c^2\right)(c+dx)^{3/2}}{3b^5d^4}-\frac{a(c+dx)^3}{3b^2d^4}+\frac{2(c+dx)^{7/2}}{7bd^4}$$

```
[Out] -((a*(a^4 - 3*a^2*b^2*c + 3*b^4*c^2)*x)/(b^6*d^3)) + (2*(a^2 - b^
2*c)^3*Sqrt[c + d*x])/(b^7*d^4) + (2*(a^4 - 3*a^2*b^2*c + 3*b^4*c
^2)*(c + d*x)^(3/2))/(3*b^5*d^4) - (a*(a^2 - 3*b^2*c)*(c + d*x)^2
)/(2*b^4*d^4) + (2*(a^2 - 3*b^2*c)*(c + d*x)^(5/2))/(5*b^3*d^4) -
 (a*(c + d*x)^3)/(3*b^2*d^4) + (2*(c + d*x)^(7/2))/(7*b*d^4) - (2
*a*(a^2 - b^2*c)^3*Log[a + b*Sqrt[c + d*x]])/(b^8*d^4)
```

Rubi [A] time = 0.52168, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$

$$-\frac{2a\left(a^2-b^2c\right)^3\log\left(a+b\sqrt{c+dx}\right)}{b^8d^4}+\frac{2\left(a^2-b^2c\right)^3\sqrt{c+dx}}{b^7d^4}-\frac{a\left(a^2-3b^2c\right)(c+dx)^2}{2b^4d^4}+\frac{2\left(a^2-3b^2c\right)(c+dx)^{5/2}}{5b^3d^4}-\frac{ax\left(a^4-3a^2b^2c+3b^4c^2\right)}{b^6d^3}+\frac{2\left(a^4-3a^2b^2c+3b^4c^2\right)(c+dx)^{3/2}}{3b^5d^4}-\frac{a(c+dx)^3}{3b^2d^4}+\frac{2(c+dx)^{7/2}}{7bd^4}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/(a + b*Sqrt[c + d*x]),x]
```


```
[Out] -((a*(a^4 - 3*a^2*b^2*c + 3*b^4*c^2)*x)/(b^6*d^3)) + (2*(a^2 - b^
2*c)^3*Sqrt[c + d*x])/(b^7*d^4) + (2*(a^4 - 3*a^2*b^2*c + 3*b^4*c
^2)*(c + d*x)^(3/2))/(3*b^5*d^4) - (a*(a^2 - 3*b^2*c)*(c + d*x)^2
)/(2*b^4*d^4) + (2*(a^2 - 3*b^2*c)*(c + d*x)^(5/2))/(5*b^3*d^4) -
 (a*(c + d*x)^3)/(3*b^2*d^4) + (2*(c + d*x)^(7/2))/(7*b*d^4) - (2
*a*(a^2 - b^2*c)^3*Log[a + b*Sqrt[c + d*x]])/(b^8*d^4)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a(c+dx)^3}{3b^2d^4}-\frac{a(a^2-3b^2c)(c+dx)^2}{2b^4d^4}-\frac{2a(a^4-3a^2b^2c+3b^4c^2)\int^{\sqrt{c+dx}}x\,dx}{b^6d^4}-\frac{2a(a^2-b^2c)^3\log\left(a+b\sqrt{c+dx}\right)}{b^8d^4}+\frac{2(a^2-b^2c)^3\int^{\sqrt{c+dx}}\frac{1}{b^7}\,dx}{d^4}+\frac{2(c+dx)^{\frac{7}{2}}}{7bd^4}+\frac{2(a^2-3b^2c)(c+dx)^{\frac{5}{2}}}{5b^3d^4}+\frac{2(c+dx)^{\frac{3}{2}}(a^4-3a^2b^2c+3b^4c^2)}{3b^5d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] -a*(c + d*x)**3/(3*b**2*d**4) - a*(a**2 - 3*b**2*c)*(c + d*x)**2/
(2*b**4*d**4) - 2*a*(a**4 - 3*a**2*b**2*c + 3*b**4*c**2)*Integral
(x, (x, sqrt(c + d*x)))/(b**6*d**4) - 2*a*(a**2 - b**2*c)**3*log(
a + b*sqrt(c + d*x))/(b**8*d**4) + 2*(a**2 - b**2*c)**3*Integral(
b**(-7), (x, sqrt(c + d*x)))/d**4 + 2*(c + d*x)**(7/2)/(7*b*d**4)
 + 2*(a**2 - 3*b**2*c)*(c + d*x)**(5/2)/(5*b**3*d**4) + 2*(c + d*
x)**(3/2)*(a**4 - 3*a**2*b**2*c + 3*b**4*c**2)/(3*b**5*d**4)
```

Mathematica [A] time = 0.69608, size = 244, normalized size = 1.06

$$\frac{-210a\left(a^2-b^2c\right)^3\log\left(a^2-b^2(c+dx)\right)-420a\left(a^2-b^2c\right)^3\tanh^{-1}\left(\frac{b\sqrt{c+dx}}{a}\right)+b\left(420a^6\sqrt{c+dx}-210a^5bdx-140a^4b^2(8c\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/(a + b*Sqrt[c + d*x]),x]
```

```
[Out] (b*(-210*a^5*b*d*x - 105*a^3*b^3*d*x*(-4*c + d*x) + 420*a^6*Sqrt[
c + d*x] - 140*a^4*b^2*(8*c - d*x)*Sqrt[c + d*x] + 84*a^2*b^4*Sqr
t[c + d*x]*(11*c^2 - 3*c*d*x + d^2*x^2) - 35*a*b^5*d*x*(6*c^2 - 3
*c*d*x + 2*d^2*x^2) + 12*b^6*Sqrt[c + d*x]*(-16*c^3 + 8*c^2*d*x -
 6*c*d^2*x^2 + 5*d^3*x^3)) - 420*a*(a^2 - b^2*c)^3*ArcTanh[(b*Sqr
t[c + d*x])/a] - 210*a*(a^2 - b^2*c)^3*Log[a^2 - b^2*(c + d*x)])/
(210*b^8*d^4)
```

Maple [A] time = 0.008, size = 394, normalized size = 1.7

$$-\frac{ax^3}{3\,b^2d}+\frac{2}{7\,bd^4}(dx+c)^{\frac{7}{2}}+2\,\frac{a^6\sqrt{dx+c}}{d^4b^7}+\frac{5\,c^2a^3}{2\,d^4b^4}-2\,\frac{(dx+c)^{3/2}\,a^2c}{d^4b^3}+6\,\frac{a^2c^2\sqrt{dx+c}}{d^4b^3}-6\,\frac{a^4c\sqrt{dx+c}}{d^4b^5}+2\,\frac{a^3xc}{b^4d^3}-\frac{11\,ac^3}{6\,d^4b^2}-\frac{a^5c}{d^4b^6}-\frac{axc^2}{b^2d^3}-\frac{6\,c}{5\,bd^4}(dx+c)^{\frac{5}{2}}+\frac{2\,a^2}{5\,d^4b^3}(dx+c)^{\frac{5}{2}}+2\,\frac{c^2\,(dx+c)^{3/2}}{bd^4}-2\,\frac{c^3\sqrt{dx+c}}{bd^4}+\frac{2\,a^4}{3\,d^4b^5}(dx+c)^{\frac{3}{2}}-\frac{xa^5}{d^3b^6}+\frac{ax^2c}{2\,b^2d^2}-\frac{x^2a^3}{2\,b^4d^2}+2\,\frac{a\ln\left(a+b\sqrt{dx+c}\right)c^3}{d^4b^2}-6\,\frac{a^3\ln\left(a+b\sqrt{dx+c}\right)c^2}{d^4b^4}+6\,\frac{a^5\ln\left(a+b\sqrt{dx+c}\right)c}{d^4b^6}-2\,\frac{a^7\ln\left(a+b\sqrt{dx+c}\right)}{d^4b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(a+b*(d*x+c)^(1/2)),x)
```

```
[Out] -1/3/d/b^2*x^3*a+2/7*(d*x+c)^(7/2)/b/d^4+2/d^4/b^7*a^6*(d*x+c)^(1
/2)+5/2/d^4/b^4*a^3*c^2-2/d^4/b^3*(d*x+c)^(3/2)*a^2*c+6/d^4/b^3*a
^2*c^2*(d*x+c)^(1/2)-6/d^4/b^5*a^4*c*(d*x+c)^(1/2)+2/d^3/b^4*x*a^
3*c-11/6/d^4/b^2*a*c^3-1/d^4/b^6*c*a^5-1/d^3/b^2*x*a*c^2-6/5/d^4/
b*(d*x+c)^(5/2)*c+2/5/d^4/b^3*a^2*(d*x+c)^(5/2)+2/d^4/b*(d*x+c)^(
3/2)*c^2-2/d^4/b*c^3*(d*x+c)^(1/2)+2/3/d^4/b^5*a^4*(d*x+c)^(3/2)-
1/d^3/b^6*x*a^5+1/2/d^2/b^2*x^2*a*c-1/2/d^2/b^4*x^2*a^3+2/d^4*a/b
^2*ln(a+b*(d*x+c)^(1/2))*c^3-6/d^4*a^3/b^4*ln(a+b*(d*x+c)^(1/2))*
c^2+6/d^4*a^5/b^6*ln(a+b*(d*x+c)^(1/2))*c-2/d^4*a^7/b^8*ln(a+b*(d
*x+c)^(1/2))
```

Maxima [A] time = 0.706678, size = 328, normalized size = 1.43

$$\frac{\frac{60\,(dx+c)^{\frac{7}{2}}b^6-70\,(dx+c)^3ab^5-84\,\left(3\,b^6c-a^2b^4\right)(dx+c)^{\frac{5}{2}}+105\,\left(3\,ab^5c-a^3b^3\right)(dx+c)^2+140\,\left(3\,b^6c^2-3\,a^2b^4c+a^4b^2\right)(dx+c)^{\frac{3}{2}}-210\,\left(3\,ab^5c^2-3\,a^3b^3c+a^5b\right)(dx+c)}{b^7}}{210\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(sqrt(d*x + c)*b + a),x, algorithm="maxima")
```

```
[Out] 1/210*((60*(d*x + c)^(7/2)*b^6 - 70*(d*x + c)^3*a*b^5 - 84*(3*b^6
*c - a^2*b^4)*(d*x + c)^(5/2) + 105*(3*a*b^5*c - a^3*b^3)*(d*x +
c)^2 + 140*(3*b^6*c^2 - 3*a^2*b^4*c + a^4*b^2)*(d*x + c)^(3/2) -
210*(3*a*b^5*c^2 - 3*a^3*b^3*c + a^5*b)*(d*x + c) - 420*(b^6*c^3
- 3*a^2*b^4*c^2 + 3*a^4*b^2*c - a^6)*sqrt(d*x + c))/b^7 + 420*(a*
b^6*c^3 - 3*a^3*b^4*c^2 + 3*a^5*b^2*c - a^7)*log(sqrt(d*x + c)*b
```

```
+ a)/b^8)/d^4
```

Fricas [A] time = 0.2837, size = 308, normalized size = 1.34

$$-\frac{70\,ab^6d^3x^3 - 105\,\left(ab^6c - a^3b^4\right)d^2x^2 + 210\,\left(ab^6c^2 - 2\,a^3b^4c + a^5b^2\right)dx - 420\,\left(ab^6c^3 - 3\,a^3b^4c^2 + 3\,a^5b^2c - a^7\right)\log\left(\sqrt{dx\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(sqrt(d*x + c)*b + a),x, algorithm="fricas")
```

```
[Out] -1/210*(70*a*b^6*d^3*x^3 - 105*(a*b^6*c - a^3*b^4)*d^2*x^2 + 210*
(a*b^6*c^2 - 2*a^3*b^4*c + a^5*b^2)*d*x - 420*(a*b^6*c^3 - 3*a^3*
b^4*c^2 + 3*a^5*b^2*c - a^7)*log(sqrt(d*x + c)*b + a) - 4*(15*b^7
*d^3*x^3 - 48*b^7*c^3 + 231*a^2*b^5*c^2 - 280*a^4*b^3*c + 105*a^6
*b - 3*(6*b^7*c - 7*a^2*b^5)*d^2*x^2 + (24*b^7*c^2 - 63*a^2*b^5*c
 + 35*a^4*b^3)*d*x)*sqrt(d*x + c))/(b^8*d^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{a + b\sqrt{c + dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] Integral(x**3/(a + b*sqrt(c + d*x)), x)
```

GIAC/XCAS [A] time = 0.306543, size = 533, normalized size = 2.32

$$\frac{2\left(ab^6c^3 - 3\,a^3b^4c^2 + 3\,a^5b^2c - a^7\right)\ln\left(\left|\sqrt{dx + c}b + a\right|\right)}{b^8d^4} - \frac{2\left(ab^6c^3\ln\left(|a|\right) - 3\,a^3b^4c^2\ln\left(|a|\right) + 3\,a^5b^2c\ln\left(|a|\right) - a^7\ln\left(|a|\right)\right)}{b^8d^4} + \frac{60\,(dx + c)^{\frac{7}{2}}b^6d^{24} - 252\,(dx + c)^{\frac{5}{2}}b^6cd^{24} + 420\,(dx + c)^{\frac{3}{2}}b^6c^2d^{24} - 420\,\sqrt{dx + c}b^6c^3d^{24} - 70\,(dx + c)^3ab^5d^{24} + 315\,(dx + c)^2\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(sqrt(d*x + c)*b + a),x, algorithm="giac")
```

```
[Out] 2*(a*b^6*c^3 - 3*a^3*b^4*c^2 + 3*a^5*b^2*c - a^7)*ln(abs(sqrt(d*x
 + c)*b + a))/(b^8*d^4) - 2*(a*b^6*c^3*ln(abs(a)) - 3*a^3*b^4*c^2
*ln(abs(a)) + 3*a^5*b^2*c*ln(abs(a)) - a^7*ln(abs(a)))/(b^8*d^4)
+ 1/210*(60*(d*x + c)^(7/2)*b^6*d^24 - 252*(d*x + c)^(5/2)*b^6*c*
d^24 + 420*(d*x + c)^(3/2)*b^6*c^2*d^24 - 420*sqrt(d*x + c)*b^6*c
^3*d^24 - 70*(d*x + c)^3*a*b^5*d^24 + 315*(d*x + c)^2*a*b^5*c*d^2
4 - 630*(d*x + c)*a*b^5*c^2*d^24 + 84*(d*x + c)^(5/2)*a^2*b^4*d^2
4 - 420*(d*x + c)^(3/2)*a^2*b^4*c*d^24 + 1260*sqrt(d*x + c)*a^2*b
^4*c^2*d^24 - 105*(d*x + c)^2*a^3*b^3*d^24 + 630*(d*x + c)*a^3*b^
3*c*d^24 + 140*(d*x + c)^(3/2)*a^4*b^2*d^24 - 1260*sqrt(d*x + c)*
a^4*b^2*c*d^24 - 210*(d*x + c)*a^5*b*d^24 + 420*sqrt(d*x + c)*a^6
*d^24)/(b^7*d^28)
```

3.472 $\int \frac{x^2}{a+b\sqrt{c+dx}}\,dx$

Optimal. Leaf size=151

$$-\frac{2a\left(a^2-b^2c\right)^2\log\left(a+b\sqrt{c+dx}\right)}{b^6d^3}+\frac{2\left(a^2-b^2c\right)^2\sqrt{c+dx}}{b^5d^3}-\frac{ax\left(a^2-2b^2c\right)}{b^4d^2}+\frac{2\left(a^2-2b^2c\right)(c+dx)^{3/2}}{3b^3d^3}-\frac{a(c+dx)^2}{2b^2d^3}+\frac{2(c+dx)^{5/2}}{5bd^3}$$

```
[Out] -((a*(a^2 - 2*b^2*c)*x)/(b^4*d^2)) + (2*(a^2 - b^2*c)^2*Sqrt[c +
d*x])/(b^5*d^3) + (2*(a^2 - 2*b^2*c)*(c + d*x)^(3/2))/(3*b^3*d^3)
 - (a*(c + d*x)^2)/(2*b^2*d^3) + (2*(c + d*x)^(5/2))/(5*b*d^3) -
(2*a*(a^2 - b^2*c)^2*Log[a + b*Sqrt[c + d*x]])/(b^6*d^3)
```

Rubi [A] time = 0.327907, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$-\frac{2a\left(a^2-b^2c\right)^2\log\left(a+b\sqrt{c+dx}\right)}{b^6d^3}+\frac{2\left(a^2-b^2c\right)^2\sqrt{c+dx}}{b^5d^3}-\frac{ax\left(a^2-2b^2c\right)}{b^4d^2}+\frac{2\left(a^2-2b^2c\right)(c+dx)^{3/2}}{3b^3d^3}-\frac{a(c+dx)^2}{2b^2d^3}+\frac{2(c+dx)^{5/2}}{5bd^3}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(a + b*Sqrt[c + d*x]),x]
```

```
[Out] -((a*(a^2 - 2*b^2*c)*x)/(b^4*d^2)) + (2*(a^2 - b^2*c)^2*Sqrt[c +
d*x])/(b^5*d^3) + (2*(a^2 - 2*b^2*c)*(c + d*x)^(3/2))/(3*b^3*d^3)
 - (a*(c + d*x)^2)/(2*b^2*d^3) + (2*(c + d*x)^(5/2))/(5*b*d^3) -
(2*a*(a^2 - b^2*c)^2*Log[a + b*Sqrt[c + d*x]])/(b^6*d^3)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a\,(c+dx)^2}{2b^2d^3}-\frac{2a\left(a^2-2b^2c\right)\int^{\sqrt{c+dx}}x\,dx}{b^4d^3}-\frac{2a\left(a^2-b^2c\right)^2\log\left(a+b\sqrt{c+dx}\right)}{b^6d^3}+\frac{2\left(a^2-b^2c\right)^2\int^{\sqrt{c+dx}}\frac{1}{b^5}\,dx}{d^3}+\frac{2\,(c+dx)^{\frac{5}{2}}}{5bd^3}+\frac{2\left(a^2-2b^2c\right)(c+dx)^{\frac{3}{2}}}{3b^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] -a*(c + d*x)**2/(2*b**2*d**3) - 2*a*(a**2 - 2*b**2*c)*Integral(x,
 (x, sqrt(c + d*x)))/(b**4*d**3) - 2*a*(a**2 - b**2*c)**2*log(a +
 b*sqrt(c + d*x))/(b**6*d**3) + 2*(a**2 - b**2*c)**2*Integral(b**
(-5), (x, sqrt(c + d*x)))/d**3 + 2*(c + d*x)**(5/2)/(5*b*d**3) +
2*(a**2 - 2*b**2*c)*(c + d*x)**(3/2)/(3*b**3*d**3)
```

Mathematica [A] time = 0.351033, size = 169, normalized size = 1.12

$$\frac{-30a\left(a^2-b^2c\right)^2\log\left(a^2-b^2(c+dx)\right)-60a\left(a^2-b^2c\right)^2\tanh^{-1}\left(\frac{b\sqrt{c+dx}}{a}\right)+b\left(60a^4\sqrt{c+dx}-30a^3bdx-20a^2b^2(5c-dx)\right.}{30b^6d^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(a + b*Sqrt[c + d*x]),x]
```

```
[Out] (b*(-30*a^3*b*d*x - 15*a*b^3*d*x*(-2*c + d*x) + 60*a^4*Sqrt[c + d
*x] - 20*a^2*b^2*(5*c - d*x)*Sqrt[c + d*x] + 4*b^4*Sqrt[c + d*x]*
(8*c^2 - 4*c*d*x + 3*d^2*x^2)) - 60*a*(a^2 - b^2*c)^2*ArcTanh[(b*
Sqrt[c + d*x])/a] - 30*a*(a^2 - b^2*c)^2*Log[a^2 - b^2*(c + d*x)]
)/(30*b^6*d^3)
```

Maple [A] time = 0.007, size = 235, normalized size = 1.6

$$\frac{2}{5\,bd^3}\left(dx+c\right)^{\frac{5}{2}}-\frac{ax^2}{2\,b^2d}+\frac{acx}{b^2d^2}+\frac{3\,ac^2}{2\,b^2d^3}-\frac{4\,c}{3\,bd^3}\left(dx+c\right)^{\frac{3}{2}}+\frac{2\,a^2}{3\,b^3d^3}\left(dx+c\right)^{\frac{3}{2}}$$
$$+2\,\frac{c^2\sqrt{dx+c}}{bd^3}-\frac{a^3x}{b^4d^2}-\frac{a^3c}{b^4d^3}-4\,\frac{a^2c\sqrt{dx+c}}{b^3d^3}+2\,\frac{\sqrt{dx+c}a^4}{d^3b^5}$$
$$-2\,\frac{a\ln\left(a+b\sqrt{dx+c}\right)c^2}{b^2d^3}+4\,\frac{a^3\ln\left(a+b\sqrt{dx+c}\right)c}{b^4d^3}-2\,\frac{a^5\ln\left(a+b\sqrt{dx+c}\right)}{d^3b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(a+b*(d*x+c)^(1/2)),x)
```

```
[Out] 2/5*(d*x+c)^(5/2)/b/d^3-1/2/d/b^2*x^2*a+1/d^2/b^2*x*a*c+3/2/d^3/b
^2*a*c^2-4/3/d^3/b*(d*x+c)^(3/2)*c+2/3/d^3/b^3*a^2*(d*x+c)^(3/2)+
2/d^3/b*c^2*(d*x+c)^(1/2)-1/d^2/b^4*x*a^3-1/d^3/b^4*a^3*c-4/d^3/b
^3*c*a^2*(d*x+c)^(1/2)+2/d^3/b^5*(d*x+c)^(1/2)*a^4-2/d^3*a/b^2*ln
(a+b*(d*x+c)^(1/2))*c^2+4/d^3*a^3/b^4*ln(a+b*(d*x+c)^(1/2))*c-2/d
```

```
^3*a^5/b^6*ln(a+b*(d*x+c)^(1/2))
```

Maxima [A] time = 0.704924, size = 200, normalized size = 1.32

$$\frac{\frac{12\,(dx+c)^{\frac{5}{2}}b^4-15\,(dx+c)^2ab^3-20\,\left(2\,b^4c-a^2b^2\right)(dx+c)^{\frac{3}{2}}+30\,\left(2\,ab^3c-a^3b\right)(dx+c)+60\,\left(b^4c^2-2\,a^2b^2c+a^4\right)\sqrt{dx+c}}{b^5}-\frac{60\,\left(ab^4c^2-2\,a^3b^2c+a^5\right)\log\left(\sqrt{dx+c}b+a\right)}{b^6}}{30\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(sqrt(d*x + c)*b + a),x, algorithm="maxima")
```

```
[Out] 1/30*((12*(d*x + c)^(5/2)*b^4 - 15*(d*x + c)^2*a*b^3 - 20*(2*b^4*
c - a^2*b^2)*(d*x + c)^(3/2) + 30*(2*a*b^3*c - a^3*b)*(d*x + c) +
 60*(b^4*c^2 - 2*a^2*b^2*c + a^4)*sqrt(d*x + c))/b^5 - 60*(a*b^4*
c^2 - 2*a^3*b^2*c + a^5)*log(sqrt(d*x + c)*b + a)/b^6)/d^3
```

Fricas [A] time = 0.293567, size = 186, normalized size = 1.23

$$-\frac{15\,ab^4d^2x^2-30\,\left(ab^4c-a^3b^2\right)dx+60\,\left(ab^4c^2-2\,a^3b^2c+a^5\right)\log\left(\sqrt{dx+c}b+a\right)-4\,\left(3\,b^5d^2x^2+8\,b^5c^2-25\,a^2b^3c+15\,a^4}{30\,b^6d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(sqrt(d*x + c)*b + a),x, algorithm="fricas")
```

```
[Out] -1/30*(15*a*b^4*d^2*x^2 - 30*(a*b^4*c - a^3*b^2)*d*x + 60*(a*b^4*
c^2 - 2*a^3*b^2*c + a^5)*log(sqrt(d*x + c)*b + a) - 4*(3*b^5*d^2*
x^2 + 8*b^5*c^2 - 25*a^2*b^3*c + 15*a^4*b - (4*b^5*c - 5*a^2*b^3)
*d*x)*sqrt(d*x + c))/(b^6*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{a+b\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] Integral(x**2/(a + b*sqrt(c + d*x)), x)
```

GIAC/XCAS [A] time = 0.283787, size = 320, normalized size = 2.12

$$-\frac{2\left(ab^4c^2 - 2a^3b^2c + a^5\right)\ln\left(\left|\sqrt{dx+c}b + a\right|\right)}{b^6d^3} + \frac{2\left(ab^4c^2\ln\left(|a|\right) - 2a^3b^2c\ln\left(|a|\right) + a^5\ln\left(|a|\right)\right)}{b^6d^3} + \frac{12\left(dx+c\right)^{\frac{5}{2}}b^4d^{12} - 40\left(dx+c\right)^{\frac{3}{2}}b^4cd^{12} + 60\sqrt{dx+c}b^4c^2d^{12} - 15\left(dx+c\right)^2ab^3d^{12} + 60\left(dx+c\right)ab^3cd^{12} + 20\left(dx+c\right)^{\frac{3}{2}}a^2b^2d}{30b^5d^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(sqrt(d*x + c)*b + a),x, algorithm="giac")
```

```
[Out] -2*(a*b^4*c^2 - 2*a^3*b^2*c + a^5)*ln(abs(sqrt(d*x + c)*b + a))/(
b^6*d^3) + 2*(a*b^4*c^2*ln(abs(a)) - 2*a^3*b^2*c*ln(abs(a)) + a^5
*ln(abs(a)))/(b^6*d^3) + 1/30*(12*(d*x + c)^(5/2)*b^4*d^12 - 40*(
d*x + c)^(3/2)*b^4*c*d^12 + 60*sqrt(d*x + c)*b^4*c^2*d^12 - 15*(d
*x + c)^2*a*b^3*d^12 + 60*(d*x + c)*a*b^3*c*d^12 + 20*(d*x + c)^(
3/2)*a^2*b^2*d^12 - 120*sqrt(d*x + c)*a^2*b^2*c*d^12 - 30*(d*x +
c)*a^3*b*d^12 + 60*sqrt(d*x + c)*a^4*d^12)/(b^5*d^15)
```

3.473 $\int \frac{x}{a+b\sqrt{c+dx}}\,dx$

Optimal. Leaf size=90

$$-\frac{2a\left(a^2-b^2c\right)\log\left(a+b\sqrt{c+dx}\right)}{b^4d^2}+\frac{2\left(a^2-b^2c\right)\sqrt{c+dx}}{b^3d^2}-\frac{ax}{b^2d}+\frac{2(c+dx)^{3/2}}{3bd^2}$$

```
[Out] -((a*x)/(b^2*d)) + (2*(a^2 - b^2*c)*Sqrt[c + d*x])/(b^3*d^2) + (2
*(c + d*x)^(3/2))/(3*b*d^2) - (2*a*(a^2 - b^2*c)*Log[a + b*Sqrt[c
 + d*x]])/(b^4*d^2)
```

Rubi [A] time = 0.178688, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$-\frac{2a\left(a^2-b^2c\right)\log\left(a+b\sqrt{c+dx}\right)}{b^4d^2}+\frac{2\left(a^2-b^2c\right)\sqrt{c+dx}}{b^3d^2}-\frac{ax}{b^2d}+\frac{2(c+dx)^{3/2}}{3bd^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(a + b*Sqrt[c + d*x]),x]
```

```
[Out] -((a*x)/(b^2*d)) + (2*(a^2 - b^2*c)*Sqrt[c + d*x])/(b^3*d^2) + (2
*(c + d*x)^(3/2))/(3*b*d^2) - (2*a*(a^2 - b^2*c)*Log[a + b*Sqrt[c
 + d*x]])/(b^4*d^2)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2a\int^{\sqrt{c+dx}} x\,dx}{b^2d^2}-\frac{2a\left(a^2-b^2c\right)\log\left(a+b\sqrt{c+dx}\right)}{b^4d^2}+\frac{2\left(a^2-b^2c\right)\int^{\sqrt{c+dx}}\frac{1}{b^3}\,dx}{d^2}+\frac{2\left(c+dx\right)^{\frac{3}{2}}}{3bd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] -2*a*Integral(x, (x, sqrt(c + d*x)))/(b**2*d**2) - 2*a*(a**2 - b*
*2*c)*log(a + b*sqrt(c + d*x))/(b**4*d**2) + 2*(a**2 - b**2*c)*In
tegral(b**(-3), (x, sqrt(c + d*x)))/d**2 + 2*(c + d*x)**(3/2)/(3*
b*d**2)
```

Mathematica [A] time = 0.083584, size = 85, normalized size = 0.94

$$\frac{b\left(6a^2\sqrt{c+dx}-3ab(c+dx)+2b^2(dx-2c)\sqrt{c+dx}\right)-6\left(a^3-ab^2c\right)\log\left(a+b\sqrt{c+dx}\right)}{3b^4d^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(a + b*Sqrt[c + d*x]),x]
```

```
[Out] (b*(6*a^2*Sqrt[c + d*x] + 2*b^2*(-2*c + d*x)*Sqrt[c + d*x] - 3*a*
b*(c + d*x)) - 6*(a^3 - a*b^2*c)*Log[a + b*Sqrt[c + d*x]])/(3*b^4
*d^2)
```

Maple [A] time = 0.006, size = 116, normalized size = 1.3

$$\frac{2}{3\,bd^2}(dx+c)^{\frac{3}{2}}-\frac{ax}{b^2d}-\frac{ac}{b^2d^2}-2\,\frac{c\sqrt{dx+c}}{bd^2}+2\,\frac{\sqrt{dx+c}a^2}{b^3d^2}+2\,\frac{a\ln\left(a+b\sqrt{dx+c}\right)c}{b^2d^2}-2\,\frac{a^3\ln\left(a+b\sqrt{dx+c}\right)}{b^4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(a+b*(d*x+c)^(1/2)),x)
```

```
[Out] 2/3*(d*x+c)^(3/2)/b/d^2-a*x/b^2/d-1/d^2/b^2*a*c-2/d^2/b*c*(d*x+c)
^(1/2)+2/d^2/b^3*(d*x+c)^(1/2)*a^2+2/d^2*a/b^2*ln(a+b*(d*x+c)^(1/
2))*c-2/d^2*a^3/b^4*ln(a+b*(d*x+c)^(1/2))
```

Maxima [A] time = 0.699251, size = 109, normalized size = 1.21

$$\frac{\frac{2\,(dx+c)^{\frac{3}{2}}\,b^2-3\,(dx+c)ab-6\,\left(b^2c-a^2\right)\sqrt{dx+c}}{b^3}+\frac{6\,\left(ab^2c-a^3\right)\log\left(\sqrt{dx+c}b+a\right)}{b^4}}{3\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(d*x + c)*b + a),x, algorithm="maxima")
```

```
[Out] 1/3*((2*(d*x + c)^(3/2)*b^2 - 3*(d*x + c)*a*b - 6*(b^2*c - a^2)*s
qrt(d*x + c))/b^3 + 6*(a*b^2*c - a^3)*log(sqrt(d*x + c)*b + a)/b^
4)/d^2
```

Fricas [A] time = 0.28963, size = 96, normalized size = 1.07

$$-\frac{3\,ab^2dx-6\,\left(ab^2c-a^3\right)\,\log\left(\sqrt{dx+c}b+a\right)-2\,\left(b^3dx-2\,b^3c+3\,a^2b\right)\sqrt{dx+c}}{3\,b^4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(d*x + c)*b + a),x, algorithm="fricas")
```

```
[Out] -1/3*(3*a*b^2*d*x - 6*(a*b^2*c - a^3)*log(sqrt(d*x + c)*b + a) -
2*(b^3*d*x - 2*b^3*c + 3*a^2*b)*sqrt(d*x + c))/(b^4*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{a+b\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] Integral(x/(a + b*sqrt(c + d*x)), x)
```

GIAC/XCAS [A] time = 0.276789, size = 177, normalized size = 1.97

$$\frac{\frac{6\,\left(ab^2c-a^3\right)\ln\left(\left|\sqrt{dx+c}b+a\right|\right)}{b^4d}-\frac{6\,\left(ab^2c\ln\left(|a|\right)-a^3\ln\left(|a|\right)\right)}{b^4d}+\frac{2\,(dx+c)^{\frac{3}{2}}b^2d^2-6\,\sqrt{dx+c}b^2cd^2-3\,(dx+c)abd^2+6\,\sqrt{dx+c}a^2d^2}{b^3d^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(d*x + c)*b + a),x, algorithm="giac")
```

```
[Out] 1/3*(6*(a*b^2*c - a^3)*ln(abs(sqrt(d*x + c)*b + a))/(b^4*d) - 6*(
a*b^2*c*ln(abs(a)) - a^3*ln(abs(a)))/(b^4*d) + (2*(d*x + c)^(3/2)
*b^2*d^2 - 6*sqrt(d*x + c)*b^2*c*d^2 - 3*(d*x + c)*a*b*d^2 + 6*sq
rt(d*x + c)*a^2*d^2)/(b^3*d^3))/d
```

3.474 $\int \frac{1}{a+b\sqrt{c+dx}} \, dx$

Optimal. Leaf size=41

$$\frac{2\sqrt{c+dx}}{bd} - \frac{2a\log\left(a+b\sqrt{c+dx}\right)}{b^2d}$$

```
[Out] (2*Sqrt[c + d*x])/(b*d) - (2*a*Log[a + b*Sqrt[c + d*x]])/(b^2*d)
```

Rubi [A] time = 0.0511365, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{2\sqrt{c+dx}}{bd} - \frac{2a\log\left(a+b\sqrt{c+dx}\right)}{b^2d}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*Sqrt[c + d*x])^(-1),x]

[Out] (2*Sqrt[c + d*x])/(b*d) - (2*a*Log[a + b*Sqrt[c + d*x]])/(b^2*d)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2a\log\left(a+b\sqrt{c+dx}\right)}{b^2d} + \frac{2\int^{\sqrt{c+dx}} \frac{1}{b}\, dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a+b*(d*x+c)**(1/2)),x)

[Out] -2*a*log(a + b*sqrt(c + d*x))/(b**2*d) + 2*Integral(1/b, (x, sqrt
(c + d*x)))/d
```

Mathematica [A] time = 0.0176275, size = 37, normalized size = 0.9

$$\frac{2b\sqrt{c+dx} - 2a\log\left(a+b\sqrt{c+dx}\right)}{b^2d}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*Sqrt[c + d*x])^(-1),x]
```

```
[Out] (2*b*Sqrt[c + d*x] - 2*a*Log[a + b*Sqrt[c + d*x]])/(b^2*d)
```

Maple [B] time = 0.012, size = 87, normalized size = 2.1

$$2\,\frac{\sqrt{dx+c}}{bd}-\frac{a}{b^2d}\ln\left(a+b\sqrt{dx+c}\right)+\frac{a}{b^2d}\ln\left(-a+b\sqrt{dx+c}\right)-\frac{a\ln\left(b^2dx+b^2c-a^2\right)}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a+b*(d*x+c)^(1/2)),x)
```

```
[Out] 2*(d*x+c)^(1/2)/b/d-a*ln(a+b*(d*x+c)^(1/2))/b^2/d+1/b^2/d*a*ln(-a
+b*(d*x+c)^(1/2))-a*ln(b^2*d*x+b^2*c-a^2)/b^2/d
```

Maxima [A] time = 0.696424, size = 47, normalized size = 1.15

$$-\frac{2\left(\frac{a\log\left(\sqrt{dx+c}b+a\right)}{b^2}-\frac{\sqrt{dx+c}}{b}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(d*x + c)*b + a),x, algorithm="maxima")
```

```
[Out] -2*(a*log(sqrt(d*x + c)*b + a)/b^2 - sqrt(d*x + c)/b)/d
```

Fricas [A] time = 0.285814, size = 45, normalized size = 1.1

$$-\frac{2\left(a\log\left(\sqrt{dx+c}b+a\right)-\sqrt{dx+c}b\right)}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(d*x + c)*b + a),x, algorithm="fricas")
```

```
[Out] -2*(a*log(sqrt(d*x + c)*b + a) - sqrt(d*x + c)*b)/(b^2*d)
```

Sympy [A] time = 1.92524, size = 49, normalized size = 1.2

$$\begin{cases} \frac{x}{a} & \text{for } b = 0 \wedge (b = 0 \vee d = 0) \\ \frac{x}{a+b\sqrt{c}} & \text{for } d = 0 \\ -\frac{2a\log\left(\frac{a}{b}+\sqrt{c+dx}\right)}{b^2d} + \frac{2\sqrt{c+dx}}{bd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] Piecewise((x/a, Eq(b, 0) & (Eq(b, 0) | Eq(d, 0))), (x/(a + b*sqrt
(c)), Eq(d, 0)), (-2*a*log(a/b + sqrt(c + d*x))/(b**2*d) + 2*sqrt
(c + d*x)/(b*d), True))
```

GIAC/XCAS [A] time = 0.277209, size = 68, normalized size = 1.66

$$-\frac{2\,a\ln\left(\left|\sqrt{dx+c}b+a\right|\right)}{b^2d} + \frac{2\,a\ln\left(|a|\right)}{b^2d} + \frac{2\,\sqrt{dx+c}}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(d*x + c)*b + a),x, algorithm="giac")
```

```
[Out] -2*a*ln(abs(sqrt(d*x + c)*b + a))/(b^2*d) + 2*a*ln(abs(a))/(b^2*d
) + 2*sqrt(d*x + c)/(b*d)
```

3.475 $\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)}\,dx$

Optimal. Leaf size=82

$$-\frac{2a\log\left(a+b\sqrt{c+dx}\right)}{a^2-b^2c}+\frac{2b\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a^2-b^2c}+\frac{a\log(x)}{a^2-b^2c}$$

```
[Out] (2*b*Sqrt[c]*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/(a^2 - b^2*c) + (a*L
og[x])/(a^2 - b^2*c) - (2*a*Log[a + b*Sqrt[c + d*x]])/(a^2 - b^2*
c)
```

Rubi [A] time = 0.175363, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316

$$-\frac{2a\log\left(a+b\sqrt{c+dx}\right)}{a^2-b^2c}+\frac{2b\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a^2-b^2c}+\frac{a\log(x)}{a^2-b^2c}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(a + b*Sqrt[c + d*x])),x]
```

```
[Out] (2*b*Sqrt[c]*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/(a^2 - b^2*c) + (a*L
og[x])/(a^2 - b^2*c) - (2*a*Log[a + b*Sqrt[c + d*x]])/(a^2 - b^2*
c)
```

Rubi in Sympy [A] time = 11.1031, size = 73, normalized size = 0.89

$$\frac{a\log\left(-dx\right)}{a^2-b^2c}-\frac{2a\log\left(a+b\sqrt{c+dx}\right)}{a^2-b^2c}+\frac{2b\sqrt{c}\operatorname{atanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a^2-b^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] a*log(-d*x)/(a**2 - b**2*c) - 2*a*log(a + b*sqrt(c + d*x))/(a**2
- b**2*c) + 2*b*sqrt(c)*atanh(sqrt(c + d*x)/sqrt(c))/(a**2 - b**2
*c)
```

Mathematica [A] time = 0.0583137, size = 61, normalized size = 0.74

$$\frac{-2a\log\left(a+b\sqrt{c+dx}\right)+a\log(dx)+2b\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a^2-b^2c}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(a + b*Sqrt[c + d*x])),x]
```

```
[Out] (2*b*Sqrt[c]*ArcTanh[Sqrt[c + d*x]/Sqrt[c]] + a*Log[d*x] - 2*a*Lo
g[a + b*Sqrt[c + d*x]])/(a^2 - b^2*c)
```

Maple [A] time = 0.008, size = 77, normalized size = 0.9

$$\frac{a\ln(dx)}{-b^2c+a^2}+2\,\frac{b\sqrt{c}}{-b^2c+a^2}Artanh\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)-2\,\frac{a\ln\left(a+b\sqrt{dx+c}\right)}{-b^2c+a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(a+b*(d*x+c)^(1/2)),x)
```

```
[Out] 1/(-b^2*c+a^2)*a*ln(d*x)+2*b*arctanh((d*x+c)^(1/2)/c^(1/2))*c^(1/
2)/(-b^2*c+a^2)-2*a*ln(a+b*(d*x+c)^(1/2))/(-b^2*c+a^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((sqrt(d*x + c)*b + a)*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.287991, size = 1, normalized size = 0.01

$$\left[\frac{b\sqrt{c}\log\left(\frac{dx-2\sqrt{dx+c}\sqrt{c}+2c}{x}\right)+2a\log\left(\sqrt{dx+c}b+a\right)-a\log\left(x\right)}{b^2c-a^2},\ -\frac{2b\sqrt{-c}\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)-2a\log\left(\sqrt{dx+c}b+a\right)+a\log\left(x\right)}{b^2c-a^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((sqrt(d*x + c)*b + a)*x),x, algorithm="fricas")
```

```
[Out] [(b*sqrt(c)*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) + 2*a*lo
g(sqrt(d*x + c)*b + a) - a*log(x))/(b^2*c - a^2), -(2*b*sqrt(-c)*
arctan(sqrt(d*x + c)/sqrt(-c)) - 2*a*log(sqrt(d*x + c)*b + a) + a
*log(x))/(b^2*c - a^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x\left(a+b\sqrt{c+dx}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] Integral(1/(x*(a + b*sqrt(c + d*x))), x)
```

GIAC/XCAS [A] time = 0.283629, size = 155, normalized size = 1.89

$$\frac{2ab\ln\left(\left|\sqrt{dx+c}b+a\right|\right)}{b^3c-a^2b}+\frac{2bc\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{(b^2c-a^2)\sqrt{-c}}-\frac{a\ln\left(dx\right)}{b^2c-a^2}+\frac{a\ln\left(-c\right)-2a\ln\left(|a|\right)}{b^2c-a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((sqrt(d*x + c)*b + a)*x),x, algorithm="giac")
```

```
[Out] 2*a*b*ln(abs(sqrt(d*x + c)*b + a))/(b^3*c - a^2*b) + 2*b*c*arctan
(sqrt(d*x + c)/sqrt(-c))/((b^2*c - a^2)*sqrt(-c)) - a*ln(d*x)/(b^
```

```
2*c - a^2) + (a*ln(-c) - 2*a*ln(abs(a)))/(b^2*c - a^2)
```

3.476 $\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)}\,dx$

Optimal. Leaf size=130

$$-\frac{a-b\sqrt{c+dx}}{x\left(a^2-b^2c\right)}+\frac{ab^2d\log(x)}{\left(a^2-b^2c\right)^2}-\frac{2ab^2d\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^2}+\frac{bd\left(a^2+b^2c\right)\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}\left(a^2-b^2c\right)^2}$$

```
[Out] -((a - b*Sqrt[c + d*x])/((a^2 - b^2*c)*x)) + (b*(a^2 + b^2*c)*d*A
rcTanh[Sqrt[c + d*x]/Sqrt[c]])/(Sqrt[c]*(a^2 - b^2*c)^2) + (a*b^2
*d*Log[x])/(a^2 - b^2*c)^2 - (2*a*b^2*d*Log[a + b*Sqrt[c + d*x]])
/(a^2 - b^2*c)^2
```

Rubi [A] time = 0.363361, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368

$$-\frac{a-b\sqrt{c+dx}}{x\left(a^2-b^2c\right)}+\frac{ab^2d\log(x)}{\left(a^2-b^2c\right)^2}-\frac{2ab^2d\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^2}+\frac{bd\left(a^2+b^2c\right)\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}\left(a^2-b^2c\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(a + b*Sqrt[c + d*x])),x]
```

```
[Out] -((a - b*Sqrt[c + d*x])/((a^2 - b^2*c)*x)) + (b*(a^2 + b^2*c)*d*A
rcTanh[Sqrt[c + d*x]/Sqrt[c]])/(Sqrt[c]*(a^2 - b^2*c)^2) + (a*b^2
*d*Log[x])/(a^2 - b^2*c)^2 - (2*a*b^2*d*Log[a + b*Sqrt[c + d*x]])
/(a^2 - b^2*c)^2
```

Rubi in Sympy [A] time = 26.3431, size = 119, normalized size = 0.92

$$\frac{ab^2d\log(-dx)}{\left(a^2-b^2c\right)^2}-\frac{2ab^2d\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^2}+\frac{bd\left(a^2+b^2c\right)\operatorname{atanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}\left(a^2-b^2c\right)^2}-\frac{a-b\sqrt{c+dx}}{x\left(a^2-b^2c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] a*b**2*d*log(-d*x)/(a**2 - b**2*c)**2 - 2*a*b**2*d*log(a + b*sqrt
(c + d*x))/(a**2 - b**2*c)**2 + b*d*(a**2 + b**2*c)*atanh(sqrt(c
```

```
+ d*x)/sqrt(c))/(sqrt(c)*(a**2 - b**2*c)**2) - (a - b*sqrt(c + d*
x))/(x*(a**2 - b**2*c))
```

Mathematica [A] time = 0.325795, size = 144, normalized size = 1.11

$$\frac{\sqrt{c}\left(-\left(a^2-b^2c\right)\left(a-b\sqrt{c+dx}\right)-ab^2dx\log\left(a^2-b^2(c+dx)\right)+ab^2dx\log(x)\right)+bdx\left(a^2+b^2c\right)\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)-2ab^2\sqrt{\ldots}}{\sqrt{c}x\left(a^2-b^2c\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(a + b*Sqrt[c + d*x])),x]
```

```
[Out] (-2*a*b^2*Sqrt[c]*d*x*ArcTanh[(b*Sqrt[c + d*x])/a] + b*(a^2 + b^2
*c)*d*x*ArcTanh[Sqrt[c + d*x]/Sqrt[c]] + Sqrt[c]*(-((a^2 - b^2*c)
*(a - b*Sqrt[c + d*x])) + a*b^2*d*x*Log[x] - a*b^2*d*x*Log[a^2 -
b^2*(c + d*x)]))/(Sqrt[c]*(a^2 - b^2*c)^2*x)
```

Maple [A] time = 0.027, size = 216, normalized size = 1.7

$$-\frac{b^3c}{\left(-b^2c+a^2\right)^2x}\sqrt{dx+c}+\frac{a^2b}{\left(-b^2c+a^2\right)^2x}\sqrt{dx+c}+\frac{ab^2c}{\left(-b^2c+a^2\right)^2x}-\frac{a^3}{\left(-b^2c+a^2\right)^2x}+\frac{ab^2d\ln(dx)}{\left(-b^2c+a^2\right)^2}+\frac{b^3d}{\left(-b^2c+a^2\right)^2}\sqrt{c}Artanh\left(1\sqrt{dx+c}\frac{1}{\sqrt{c}}\right)+\frac{a^2bd}{\left(-b^2c+a^2\right)^2}Artanh\left(1\sqrt{dx+c}\frac{1}{\sqrt{c}}\right)\frac{1}{\sqrt{c}}-2\frac{ab^2d\ln\left(a+b\sqrt{dx+c}\right)}{\left(-b^2c+a^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(a+b*(d*x+c)^(1/2)),x)
```

```
[Out] -1/(-b^2*c+a^2)^2/x*(d*x+c)^(1/2)*b^3*c+1/(-b^2*c+a^2)^2/x*(d*x+c
)^(1/2)*a^2*b+1/(-b^2*c+a^2)^2/x*a*b^2*c-1/(-b^2*c+a^2)^2/x*a^3+d
/(-b^2*c+a^2)^2*a*b^2*ln(d*x)+d/(-b^2*c+a^2)^2*b^3*c^(1/2)*arctan
h((d*x+c)^(1/2)/c^(1/2))+d/(-b^2*c+a^2)^2*b/c^(1/2)*arctanh((d*x+
c)^(1/2)/c^(1/2))*a^2-2*a*b^2*d*ln(a+b*(d*x+c)^(1/2))/(-b^2*c+a^2
)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((sqrt(d*x + c)*b + a)*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.352942, size = 1, normalized size = 0.01

$$\left[-\frac{4\,ab^2\sqrt{c}dx\log\left(\sqrt{dx+c}b+a\right)-\left(b^3c+a^2b\right)dx\log\left(\frac{(dx+2\,c)\sqrt{c}+2\,\sqrt{dx+c}c}{x}\right)+2\left(b^3c-a^2b\right)\sqrt{dx+c}\sqrt{c}-2\left(ab^2dx\log\left(x\right)+\right.}{2\left(b^4c^2-2\,a^2b^2c+a^4\right)\sqrt{c}x}\right.$$
$$-\frac{2\,ab^2\sqrt{-c}dx\log\left(\sqrt{dx+c}b+a\right)+\left(b^3c+a^2b\right)dx\arctan\left(\frac{c}{\sqrt{dx+c}\sqrt{-c}}\right)+\left(b^3c-a^2b\right)\sqrt{dx+c}\sqrt{-c}-\left(ab^2dx\log\left(x\right)+ab^2c-\right.}{\left(b^4c^2-2\,a^2b^2c+a^4\right)\sqrt{-c}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((sqrt(d*x + c)*b + a)*x^2),x, algorithm="fricas")
```

```
[Out] [-1/2*(4*a*b^2*sqrt(c)*d*x*log(sqrt(d*x + c)*b + a) - (b^3*c + a^
2*b)*d*x*log(((d*x + 2*c)*sqrt(c) + 2*sqrt(d*x + c)*c)/x) + 2*(b^
3*c - a^2*b)*sqrt(d*x + c)*sqrt(c) - 2*(a*b^2*d*x*log(x) + a*b^2*
c - a^3)*sqrt(c))/((b^4*c^2 - 2*a^2*b^2*c + a^4)*sqrt(c)*x), -(2*
a*b^2*sqrt(-c)*d*x*log(sqrt(d*x + c)*b + a) + (b^3*c + a^2*b)*d*x
*arctan(c/(sqrt(d*x + c)*sqrt(-c))) + (b^3*c - a^2*b)*sqrt(d*x +
c)*sqrt(-c) - (a*b^2*d*x*log(x) + a*b^2*c - a^3)*sqrt(-c))/((b^4*
c^2 - 2*a^2*b^2*c + a^4)*sqrt(-c)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] Integral(1/(x**2*(a + b*sqrt(c + d*x))), x)
```

GIAC/XCAS [A] time = 0.286877, size = 342, normalized size = 2.63

$$-\frac{2\,ab^{3}d\ln\left(\left|\sqrt{dx+c}b+a\right|\right)}{b^{5}c^{2}-2\,a^{2}b^{3}c+a^{4}b}+\frac{ab^{2}d\ln\left(-dx\right)}{b^{4}c^{2}-2\,a^{2}b^{2}c+a^{4}}-\frac{\left(b^{3}cd+a^{2}bd\right)\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{\left(b^{4}c^{2}-2\,a^{2}b^{2}c+a^{4}\right)\sqrt{-c}}-\frac{ab^{2}cd\ln\left(c\right)-2\,ab^{2}cd\ln\left(|a|\right)-ab^{2}cd+a^{3}d}{b^{4}c^{3}-2\,a^{2}b^{2}c^{2}+a^{4}c}+\frac{ab^{2}cd-a^{3}d-\left(b^{3}cd-a^{2}bd\right)\sqrt{dx+c}}{\left(b^{2}c-a^{2}\right)^{2}dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((sqrt(d*x + c)*b + a)*x^2),x, algorithm="giac")
```

```
[Out] -2*a*b^3*d*ln(abs(sqrt(d*x + c)*b + a))/(b^5*c^2 - 2*a^2*b^3*c +
a^4*b) + a*b^2*d*ln(-d*x)/(b^4*c^2 - 2*a^2*b^2*c + a^4) - (b^3*c*
d + a^2*b*d)*arctan(sqrt(d*x + c)/sqrt(-c))/((b^4*c^2 - 2*a^2*b^2
*c + a^4)*sqrt(-c)) - (a*b^2*c*d*ln(c) - 2*a*b^2*c*d*ln(abs(a)) -
 a*b^2*c*d + a^3*d)/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c) + (a*b^2*c*
d - a^3*d - (b^3*c*d - a^2*b*d)*sqrt(d*x + c))/((b^2*c - a^2)^2*d
*x)
```

3.477 $\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)}\,dx$

Optimal. Leaf size=204

$$-\frac{a-b\sqrt{c+dx}}{2x^2\,(a^2-b^2c)}-\frac{bd\left(4abc-\left(a^2+3b^2c\right)\sqrt{c+dx}\right)}{4cx\,(a^2-b^2c)^2}+\frac{ab^4d^2\log(x)}{(a^2-b^2c)^3}-\frac{2ab^4d^2\log\left(a+b\sqrt{c+dx}\right)}{(a^2-b^2c)^3}-\frac{bd^2\left(a^4-6a^2b^2c-3b^4c^2\right)\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{4c^{3/2}\,(a^2-b^2c)^3}$$

```
[Out] -(a - b*Sqrt[c + d*x])/(2*(a^2 - b^2*c)*x^2) - (b*d*(4*a*b*c - (a
^2 + 3*b^2*c)*Sqrt[c + d*x]))/(4*c*(a^2 - b^2*c)^2*x) - (b*(a^4 -
 6*a^2*b^2*c - 3*b^4*c^2)*d^2*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/(4*
c^(3/2)*(a^2 - b^2*c)^3) + (a*b^4*d^2*Log[x])/(a^2 - b^2*c)^3 - (
2*a*b^4*d^2*Log[a + b*Sqrt[c + d*x]])/(a^2 - b^2*c)^3
```

Rubi [A] time = 0.59139, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368

$$-\frac{a-b\sqrt{c+dx}}{2x^2\,(a^2-b^2c)}-\frac{bd\left(4abc-\left(a^2+3b^2c\right)\sqrt{c+dx}\right)}{4cx\,(a^2-b^2c)^2}+\frac{ab^4d^2\log(x)}{(a^2-b^2c)^3}-\frac{2ab^4d^2\log\left(a+b\sqrt{c+dx}\right)}{(a^2-b^2c)^3}-\frac{bd^2\left(a^4-6a^2b^2c-3b^4c^2\right)\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{4c^{3/2}\,(a^2-b^2c)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*(a + b*Sqrt[c + d*x])),x]
```


```
[Out] -(a - b*Sqrt[c + d*x])/(2*(a^2 - b^2*c)*x^2) - (b*d*(4*a*b*c - (a
^2 + 3*b^2*c)*Sqrt[c + d*x]))/(4*c*(a^2 - b^2*c)^2*x) - (b*(a^4 -
 6*a^2*b^2*c - 3*b^4*c^2)*d^2*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/(4*
c^(3/2)*(a^2 - b^2*c)^3) + (a*b^4*d^2*Log[x])/(a^2 - b^2*c)^3 - (
2*a*b^4*d^2*Log[a + b*Sqrt[c + d*x]])/(a^2 - b^2*c)^3
```

Rubi in Sympy [A] time = 47.5834, size = 187, normalized size = 0.92

$$\frac{ab^4d^2\log(-dx)}{(a^2-b^2c)^3}-\frac{2ab^4d^2\log\left(a+b\sqrt{c+dx}\right)}{(a^2-b^2c)^3}+\frac{bd\left(-4abc+\left(a^2+3b^2c\right)\sqrt{c+dx}\right)}{4cx\left(a^2-b^2c\right)^2}-\frac{bd^2\left(a^4-6a^2b^2c-3b^4c^2\right)\operatorname{atanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{4c^{\frac{3}{2}}\left(a^2-b^2c\right)^3}-\frac{a-b\sqrt{c+dx}}{2x^2\left(a^2-b^2c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] a*b**4*d**2*log(-d*x)/(a**2 - b**2*c)**3 - 2*a*b**4*d**2*log(a +
b*sqrt(c + d*x))/(a**2 - b**2*c)**3 + b*d*(-4*a*b*c + (a**2 + 3*b
**2*c)*sqrt(c + d*x))/(4*c*x*(a**2 - b**2*c)**2) - b*d**2*(a**4 -
 6*a**2*b**2*c - 3*b**4*c**2)*atanh(sqrt(c + d*x)/sqrt(c))/(4*c**
(3/2)*(a**2 - b**2*c)**3) - (a - b*sqrt(c + d*x))/(2*x**2*(a**2 -
 b**2*c))
```

Mathematica [A] time = 0.732582, size = 228, normalized size = 1.12

$$\frac{bd^2x^2\left(a^4-6a^2b^2c-3b^4c^2\right)\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)+\sqrt{c}\left(4ab^4cd^2x^2\log\left(a^2-b^2(c+dx)\right)+\left(a^2-b^2c\right)\left(2a^3c-a^2b\sqrt{c+dx}(2c+\right.\right.}{4c^{3/2}x^2\left(b^2c-a^2\right)^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*(a + b*Sqrt[c + d*x])),x]
```

```
[Out] (8*a*b^4*c^(3/2)*d^2*x^2*ArcTanh[(b*Sqrt[c + d*x])/a] + b*(a^4 -
6*a^2*b^2*c - 3*b^4*c^2)*d^2*x^2*ArcTanh[Sqrt[c + d*x]/Sqrt[c]] +
 Sqrt[c]*((a^2 - b^2*c)*(2*a^3*c - 2*a*b^2*c*(c - 2*d*x) + b^3*c*
(2*c - 3*d*x)*Sqrt[c + d*x] - a^2*b*Sqrt[c + d*x]*(2*c + d*x)) -
4*a*b^4*c*d^2*x^2*Log[x] + 4*a*b^4*c*d^2*x^2*Log[a^2 - b^2*(c + d
*x)]))/(4*c^(3/2)*(-a^2 + b^2*c)^3*x^2)
```

Maple [B] time = 0.021, size = 460, normalized size = 2.3

$$
\begin{aligned}
&-\frac{3\,b^5c}{4\,(-b^2c+a^2)^3\,x^2}(dx+c)^{\frac{3}{2}}+\frac{b^3a^2}{2\,(-b^2c+a^2)^3\,x^2}(dx+c)^{\frac{3}{2}}\\
&+\frac{ba^4}{4\,(-b^2c+a^2)^3\,x^2c}(dx+c)^{\frac{3}{2}}+\frac{ab^4cd}{(-b^2c+a^2)^3\,x}-\frac{ab^4c^2}{2\,(-b^2c+a^2)^3\,x^2}-\frac{a^3db^2}{(-b^2c+a^2)^3\,x}\\
&+\frac{a^3b^2c}{(-b^2c+a^2)^3\,x^2}-\frac{3\,a^2b^3c}{2\,(-b^2c+a^2)^3\,x^2}\sqrt{dx+c}+\frac{ba^4}{4\,(-b^2c+a^2)^3\,x^2}\sqrt{dx+c}\\
&+\frac{5\,b^5c^2}{4\,(-b^2c+a^2)^3\,x^2}\sqrt{dx+c}-\frac{a^5}{2\,(-b^2c+a^2)^3\,x^2}+\frac{ab^4d^2\ln(cdx)}{(-b^2c+a^2)^3}\\
&+\frac{3\,d^2b^5}{4\,(-b^2c+a^2)^3}\sqrt{c}Artanh\left(1\sqrt{dx+c}\frac{1}{\sqrt{c}}\right)+\frac{3\,a^2b^3d^2}{2\,(-b^2c+a^2)^3}Artanh\left(1\sqrt{dx+c}\frac{1}{\sqrt{c}}\right)\frac{1}{\sqrt{c}}\\
&-\frac{bd^2a^4}{4\,(-b^2c+a^2)^3}Artanh\left(1\sqrt{dx+c}\frac{1}{\sqrt{c}}\right)c^{-\frac{3}{2}}-2\,\frac{ab^4d^2\ln\left(a+b\sqrt{dx+c}\right)}{(-b^2c+a^2)^3}
\end{aligned}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(a+b*(d*x+c)^(1/2)),x)
```

```
[Out] -3/4/(-b^2*c+a^2)^3/x^2*b^5*c*(d*x+c)^(3/2)+1/2/(-b^2*c+a^2)^3/x^
2*b^3*(d*x+c)^(3/2)*a^2+1/4/(-b^2*c+a^2)^3/x^2*b/c*(d*x+c)^(3/2)*
a^4+d/(-b^2*c+a^2)^3/x*a*b^4*c-1/2/(-b^2*c+a^2)^3/x^2*a*b^4*c^2-d
/(-b^2*c+a^2)^3/x*a^3*b^2+1/(-b^2*c+a^2)^3/x^2*a^3*b^2*c-3/2/(-b^
2*c+a^2)^3/x^2*(d*x+c)^(1/2)*a^2*b^3*c+1/4/(-b^2*c+a^2)^3/x^2*(d*
x+c)^(1/2)*b*a^4+5/4/(-b^2*c+a^2)^3/x^2*(d*x+c)^(1/2)*b^5*c^2-1/2
/(-b^2*c+a^2)^3/x^2*a^5+d^2/(-b^2*c+a^2)^3*a*b^4*ln(c*d*x)+3/4*d^
2/(-b^2*c+a^2)^3*b^5*c^(1/2)*arctanh((d*x+c)^(1/2)/c^(1/2))+3/2*d
^2/(-b^2*c+a^2)^3*b^3/c^(1/2)*arctanh((d*x+c)^(1/2)/c^(1/2))*a^2-
1/4*d^2/(-b^2*c+a^2)^3*b/c^(3/2)*arctanh((d*x+c)^(1/2)/c^(1/2))*a
^4-2*a*b^4*d^2*ln(a+b*(d*x+c)^(1/2))/(-b^2*c+a^2)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((sqrt(d*x + c)*b + a)*x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.873207, size = 1, normalized size = 0.

$$\left[\frac{16\,ab^4c^{\frac{3}{2}}d^2x^2\log\left(\sqrt{dx+c}b+a\right)+\left(3\,b^5c^2+6\,a^2b^3c-a^4b\right)d^2x^2\log\left(\frac{(dx+2\,c)\sqrt{c}-2\,\sqrt{dx+c}c}{x}\right)-2\left(2\,b^5c^3-4\,a^2b^3c^2+2\,a^4bc-\right.}{8\left(b^6c^4-3\,a^2b^4c^3+3\,a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((sqrt(d*x + c)*b + a)*x^3),x, algorithm="fricas")
```

```
[Out] [1/8*(16*a*b^4*c^(3/2)*d^2*x^2*log(sqrt(d*x + c)*b + a) + (3*b^5*
c^2 + 6*a^2*b^3*c - a^4*b)*d^2*x^2*log(((d*x + 2*c)*sqrt(c) - 2*s
qrt(d*x + c)*c)/x) - 2*(2*b^5*c^3 - 4*a^2*b^3*c^2 + 2*a^4*b*c - (
3*b^5*c^2 - 2*a^2*b^3*c - a^4*b)*d*x)*sqrt(d*x + c)*sqrt(c) - 4*(
2*a*b^4*c*d^2*x^2*log(x) - a*b^4*c^3 + 2*a^3*b^2*c^2 - a^5*c + 2*
(a*b^4*c^2 - a^3*b^2*c)*d*x)*sqrt(c))/((b^6*c^4 - 3*a^2*b^4*c^3 +
 3*a^4*b^2*c^2 - a^6*c)*sqrt(c)*x^2), 1/4*(8*a*b^4*sqrt(-c)*c*d^2
*x^2*log(sqrt(d*x + c)*b + a) + (3*b^5*c^2 + 6*a^2*b^3*c - a^4*b)
*d^2*x^2*arctan(c/(sqrt(d*x + c)*sqrt(-c))) - (2*b^5*c^3 - 4*a^2*
b^3*c^2 + 2*a^4*b*c - (3*b^5*c^2 - 2*a^2*b^3*c - a^4*b)*d*x)*sqrt
(d*x + c)*sqrt(-c) - 2*(2*a*b^4*c*d^2*x^2*log(x) - a*b^4*c^3 + 2*
a^3*b^2*c^2 - a^5*c + 2*(a*b^4*c^2 - a^3*b^2*c)*d*x)*sqrt(-c))/((
b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c)*sqrt(-c)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x^3\left(a+b\sqrt{c+dx}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] Integral(1/(x**3*(a + b*sqrt(c + d*x))), x)
```

GIAC/XCAS [A] time = 0.295693, size = 648, normalized size = 3.18

$$\frac{2\,ab^5d^2\ln\left(\left|\sqrt{dx+c}b+a\right|\right)}{b^7c^3-3\,a^2b^5c^2+3\,a^4b^3c-a^6b}-\frac{ab^4d^2\ln\left(-dx\right)}{b^6c^3-3\,a^2b^4c^2+3\,a^4b^2c-a^6}+\frac{\left(3\,b^5c^2d^2+6\,a^2b^3cd^2-a^4bd^2\right)\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{4\,(b^6c^4-3\,a^2b^4c^3+3\,a^4b^2c^2-a^6c)\sqrt{-c}}+\frac{2\,ab^4c^2d^2\ln\left(c\right)-4\,ab^4c^2d^2\ln\left(|a|\right)-3\,ab^4c^2d^2+4\,a^3b^2cd^2-a^5d^2}{2\,(b^6c^5-3\,a^2b^4c^4+3\,a^4b^2c^3-a^6c^2)}+\frac{6\,ab^4c^3d^2-8\,a^3b^2c^2d^2+2\,a^5cd^2+\left(3\,b^5c^2d^2-2\,a^2b^3cd^2-a^4bd^2\right)(dx+c)^{\frac{3}{2}}-4\left(ab^4c^2d^2-a^3b^2cd^2\right)(dx+c)-\left(5\,b^5c^3d^2-\right.}{4\,(b^2c-a^2)^3cd^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((sqrt(d*x + c)*b + a)*x^3),x, algorithm="giac")
```

```
[Out] 2*a*b^5*d^2*ln(abs(sqrt(d*x + c)*b + a))/(b^7*c^3 - 3*a^2*b^5*c^2
 + 3*a^4*b^3*c - a^6*b) - a*b^4*d^2*ln(-d*x)/(b^6*c^3 - 3*a^2*b^4
*c^2 + 3*a^4*b^2*c - a^6) + 1/4*(3*b^5*c^2*d^2 + 6*a^2*b^3*c*d^2
- a^4*b*d^2)*arctan(sqrt(d*x + c)/sqrt(-c))/((b^6*c^4 - 3*a^2*b^4
*c^3 + 3*a^4*b^2*c^2 - a^6*c)*sqrt(-c)) + 1/2*(2*a*b^4*c^2*d^2*ln
(c) - 4*a*b^4*c^2*d^2*ln(abs(a)) - 3*a*b^4*c^2*d^2 + 4*a^3*b^2*c*
d^2 - a^5*d^2)/(b^6*c^5 - 3*a^2*b^4*c^4 + 3*a^4*b^2*c^3 - a^6*c^2
) + 1/4*(6*a*b^4*c^3*d^2 - 8*a^3*b^2*c^2*d^2 + 2*a^5*c*d^2 + (3*b
^5*c^2*d^2 - 2*a^2*b^3*c*d^2 - a^4*b*d^2)*(d*x + c)^(3/2) - 4*(a*
b^4*c^2*d^2 - a^3*b^2*c*d^2)*(d*x + c) - (5*b^5*c^3*d^2 - 6*a^2*b
^3*c^2*d^2 + a^4*b*c*d^2)*sqrt(d*x + c))/((b^2*c - a^2)^3*c*d^2*x
^2)
```

3.478 $\int \frac{x^3}{\left(a+b\sqrt{c+dx}\right)^2}\,dx$

Optimal. Leaf size=240

$$\frac{2a\left(a^2-b^2c\right)^3}{b^8d^4\left(a+b\sqrt{c+dx}\right)}+\frac{2\left(7a^2-b^2c\right)\left(a^2-b^2c\right)^2\log\left(a+b\sqrt{c+dx}\right)}{b^8d^4}-\frac{12a\left(a^2-b^2c\right)^2\sqrt{c+dx}}{b^7d^4}-\frac{4a\left(2a^2-3b^2c\right)(c+dx)^{3/2}}{3b^5d^4}+\frac{3\left(a^2-b^2c\right)(c+dx)^2}{2b^4d^4}+\frac{x\left(5a^4-9a^2b^2c+3b^4c^2\right)}{b^6d^3}-\frac{4a(c+dx)^{5/2}}{5b^3d^4}+\frac{(c+dx)^3}{3b^2d^4}$$

```
[Out] ((5*a^4 - 9*a^2*b^2*c + 3*b^4*c^2)*x)/(b^6*d^3) - (12*a*(a^2 - b^
2*c)^2*Sqrt[c + d*x])/(b^7*d^4) - (4*a*(2*a^2 - 3*b^2*c)*(c + d*x
)^(3/2))/(3*b^5*d^4) + (3*(a^2 - b^2*c)*(c + d*x)^2)/(2*b^4*d^4)
- (4*a*(c + d*x)^(5/2))/(5*b^3*d^4) + (c + d*x)^3/(3*b^2*d^4) + (
2*a*(a^2 - b^2*c)^3)/(b^8*d^4*(a + b*Sqrt[c + d*x])) + (2*(a^2 -
b^2*c)^2*(7*a^2 - b^2*c)*Log[a + b*Sqrt[c + d*x]])/(b^8*d^4)
```

Rubi [A] time = 0.575377, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$

$$\frac{2a\left(a^2-b^2c\right)^3}{b^8d^4\left(a+b\sqrt{c+dx}\right)}+\frac{2\left(7a^2-b^2c\right)\left(a^2-b^2c\right)^2\log\left(a+b\sqrt{c+dx}\right)}{b^8d^4}-\frac{12a\left(a^2-b^2c\right)^2\sqrt{c+dx}}{b^7d^4}-\frac{4a\left(2a^2-3b^2c\right)(c+dx)^{3/2}}{3b^5d^4}+\frac{3\left(a^2-b^2c\right)(c+dx)^2}{2b^4d^4}+\frac{x\left(5a^4-9a^2b^2c+3b^4c^2\right)}{b^6d^3}-\frac{4a(c+dx)^{5/2}}{5b^3d^4}+\frac{(c+dx)^3}{3b^2d^4}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/(a + b*Sqrt[c + d*x])^2,x]
```



```
[Out] ((5*a^4 - 9*a^2*b^2*c + 3*b^4*c^2)*x)/(b^6*d^3) - (12*a*(a^2 - b^
2*c)^2*Sqrt[c + d*x])/(b^7*d^4) - (4*a*(2*a^2 - 3*b^2*c)*(c + d*x
)^(3/2))/(3*b^5*d^4) + (3*(a^2 - b^2*c)*(c + d*x)^2)/(2*b^4*d^4)
- (4*a*(c + d*x)^(5/2))/(5*b^3*d^4) + (c + d*x)^3/(3*b^2*d^4) + (
2*a*(a^2 - b^2*c)^3)/(b^8*d^4*(a + b*Sqrt[c + d*x])) + (2*(a^2 -
b^2*c)^2*(7*a^2 - b^2*c)*Log[a + b*Sqrt[c + d*x]])/(b^8*d^4)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{4a(c+dx)^{\frac{5}{2}}}{5b^3d^4}-\frac{4a(2a^2-3b^2c)(c+dx)^{\frac{3}{2}}}{3b^5d^4}-\frac{12a(a^2-b^2c)^2\sqrt{c+dx}}{b^7d^4}+\frac{2a(a^2-b^2c)^3}{b^8d^4\left(a+b\sqrt{c+dx}\right)}+\frac{(c+dx)^3}{3b^2d^4}+\frac{3(a^2-b^2c)(c+dx)^2}{2b^4d^4}+\frac{2(5a^4-9a^2b^2c+3b^4c^2)\int^{\sqrt{c+dx}}x\,dx}{b^6d^4}+\frac{2(a^2-b^2c)^2(7a^2-b^2c)\log\left(a+b\sqrt{c+dx}\right)}{b^8d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] -4*a*(c + d*x)**(5/2)/(5*b**3*d**4) - 4*a*(2*a**2 - 3*b**2*c)*(c
+ d*x)**(3/2)/(3*b**5*d**4) - 12*a*(a**2 - b**2*c)**2*sqrt(c + d*
x)/(b**7*d**4) + 2*a*(a**2 - b**2*c)**3/(b**8*d**4*(a + b*sqrt(c
+ d*x))) + (c + d*x)**3/(3*b**2*d**4) + 3*(a**2 - b**2*c)*(c + d*
x)**2/(2*b**4*d**4) + 2*(5*a**4 - 9*a**2*b**2*c + 3*b**4*c**2)*In
tegral(x, (x, sqrt(c + d*x)))/(b**6*d**4) + 2*(a**2 - b**2*c)**2*
(7*a**2 - b**2*c)*log(a + b*sqrt(c + d*x))/(b**8*d**4)
```

Mathematica [A] time = 0.451785, size = 301, normalized size = 1.25

$$\frac{\frac{60a^2(a^2-b^2c)^3}{a^2-b^2(c+dx)}+30(a^2-b^2c)^2(7a^2-b^2c)\log\left(a^2-b^2(c+dx)\right)+60(a^2-b^2c)^2(7a^2-b^2c)\tanh^{-1}\left(\frac{b\sqrt{c+dx}}{a}\right)-15b^4d^2x^2\,(}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] (30*b^2*(5*a^4 - 6*a^2*b^2*c + b^4*c^2)*d*x - 15*b^4*(-3*a^2 + b^
2*c)*d^2*x^2 + 10*b^6*d^3*x^3 + (60*a^2*(a^2 - b^2*c)^3)/(a^2 - b
^2*(c + d*x)) - (4*a*b*Sqrt[c + d*x]*(-105*a^6 + 5*a^4*b^2*(59*c
+ 14*d*x) + a^2*b^4*(-271*c^2 - 122*c*d*x + 14*d^2*x^2) + 3*b^6*(
27*c^3 + 16*c^2*d*x - 4*c*d^2*x^2 + 2*d^3*x^3)))/(-a^2 + b^2*(c +
 d*x)) + 60*(a^2 - b^2*c)^2*(7*a^2 - b^2*c)*ArcTanh[(b*Sqrt[c + d
*x])/a] + 30*(a^2 - b^2*c)^2*(7*a^2 - b^2*c)*Log[a^2 - b^2*(c + d
*x)])/(30*b^8*d^4)
```

Maple [A] time = 0.014, size = 416, normalized size = 1.7

$$\frac{x^3}{3\,b^2 d} - \frac{c x^2}{2\,b^2 d^2} + \frac{c^2 x}{b^2 d^3} + \frac{11\,c^3}{6\,d^4 b^2} - \frac{4\,a}{5\,b^3 d^4}(dx+c)^{\frac{5}{2}} + \frac{3\,a^2 x^2}{2\,b^4 d^2} - 6\,\frac{a^2 x c}{b^4 d^3} - \frac{15\,a^2 c^2}{2\,d^4 b^4} + 4\,\frac{(dx+c)^{3/2}\,ac}{b^3 d^4} - \frac{8\,a^3}{3\,d^4 b^5}(dx+c)^{\frac{3}{2}} - 12\,\frac{ac^2\sqrt{dx+c}}{b^3 d^4} + 5\,\frac{x a^4}{d^3 b^6} + 5\,\frac{a^4 c}{d^4 b^6} + 24\,\frac{a^3 c\sqrt{dx+c}}{d^4 b^5} - 12\,\frac{a^5\sqrt{dx+c}}{d^4 b^7} - 2\,\frac{\ln\left(a+b\sqrt{dx+c}\right)c^3}{d^4 b^2} + 18\,\frac{\ln\left(a+b\sqrt{dx+c}\right)a^2 c^2}{d^4 b^4} - 30\,\frac{\ln\left(a+b\sqrt{dx+c}\right)a^4 c}{d^4 b^6} + 14\,\frac{\ln\left(a+b\sqrt{dx+c}\right)a^6}{d^4 b^8} - 2\,\frac{ac^3}{d^4 b^2\left(a+b\sqrt{dx+c}\right)} + 6\,\frac{c^2 a^3}{d^4 b^4\left(a+b\sqrt{dx+c}\right)} - 6\,\frac{a^5 c}{d^4 b^6\left(a+b\sqrt{dx+c}\right)} + 2\,\frac{a^7}{d^4 b^8\left(a+b\sqrt{dx+c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(a+b*(d*x+c)^(1/2))^2,x)
```

```
[Out] 1/3/d/b^2*x^3-1/2/d^2/b^2*x^2*c+1/d^3/b^2*x*c^2+11/6/d^4*c^3/b^2-
4/5*a*(d*x+c)^(5/2)/b^3/d^4+3/2/d^2/b^4*x^2*a^2-6/d^3/b^4*x*a^2*c
-15/2/d^4/b^4*a^2*c^2+4/d^4/b^3*(d*x+c)^(3/2)*a*c-8/3/d^4/b^5*a^3
*(d*x+c)^(3/2)-12/d^4/b^3*a*c^2*(d*x+c)^(1/2)+5/d^3/b^6*x*a^4+5/d
^4/b^6*a^4*c+24/d^4/b^5*a^3*c*(d*x+c)^(1/2)-12/d^4/b^7*a^5*(d*x+c
)^(1/2)-2/d^4/b^2*ln(a+b*(d*x+c)^(1/2))*c^3+18/d^4/b^4*ln(a+b*(d*
x+c)^(1/2))*a^2*c^2-30/d^4/b^6*ln(a+b*(d*x+c)^(1/2))*a^4*c+14/d^4
/b^8*ln(a+b*(d*x+c)^(1/2))*a^6-2/d^4*a/b^2/(a+b*(d*x+c)^(1/2))*c^
3+6/d^4*a^3/b^4/(a+b*(d*x+c)^(1/2))*c^2-6/d^4*a^5/b^6/(a+b*(d*x+c
)^(1/2))*c+2/d^4*a^7/b^8/(a+b*(d*x+c)^(1/2))
```

Maxima [A] time = 0.698218, size = 339, normalized size = 1.41

$$-\frac{\frac{60\left(ab^6c^3-3\,a^3b^4c^2+3\,a^5b^2c-a^7\right)}{\sqrt{dx+c}\,b^9+ab^8} - \frac{10\,(dx+c)^3b^5-24\,(dx+c)^{\frac{5}{2}}ab^4-45\left(b^5c-a^2b^3\right)(dx+c)^2+40\left(3\,ab^4c-2\,a^3b^2\right)(dx+c)^{\frac{3}{2}}+30\left(3\,b^5c^2-9\,a^2b^3c+5\,a^4b\right)(dx+c)}{b^7}}{30\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(sqrt(d*x + c)*b + a)^2,x, algorithm="maxima")
```

```
[Out] -1/30*(60*(a*b^6*c^3 - 3*a^3*b^4*c^2 + 3*a^5*b^2*c - a^7)/(sqrt(d
*x + c)*b^9 + a*b^8) - (10*(d*x + c)^3*b^5 - 24*(d*x + c)^(5/2)*a
*b^4 - 45*(b^5*c - a^2*b^3)*(d*x + c)^2 + 40*(3*a*b^4*c - 2*a^3*b
^2)*(d*x + c)^(3/2) + 30*(3*b^5*c^2 - 9*a^2*b^3*c + 5*a^4*b)*(d*x
 + c) - 360*(a*b^4*c^2 - 2*a^3*b^2*c + a^5)*sqrt(d*x + c))/b^7 +
```

```
60*(b^6*c^3 - 9*a^2*b^4*c^2 + 15*a^4*b^2*c - 7*a^6)*log(sqrt(d*x
+ c)*b + a)/b^8)/d^4
```

Fricas [A] time = 0.278942, size = 441, normalized size = 1.84

$$-\frac{14\,ab^6d^3x^3 + 269\,ab^6c^3 - 595\,a^3b^4c^2 + 390\,a^5b^2c - 60\,a^7 - \left(33\,ab^6c - 35\,a^3b^4\right)d^2x^2 + 2\left(81\,ab^6c^2 - 190\,a^3b^4c + 105\,a^5b^2\right)\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(sqrt(d*x + c)*b + a)^2,x, algorithm="fricas")
```

```
[Out] -1/30*(14*a*b^6*d^3*x^3 + 269*a*b^6*c^3 - 595*a^3*b^4*c^2 + 390*a
^5*b^2*c - 60*a^7 - (33*a*b^6*c - 35*a^3*b^4)*d^2*x^2 + 2*(81*a*b
^6*c^2 - 190*a^3*b^4*c + 105*a^5*b^2)*d*x + 60*(a*b^6*c^3 - 9*a^3
*b^4*c^2 + 15*a^5*b^2*c - 7*a^7 + (b^7*c^3 - 9*a^2*b^5*c^2 + 15*a
^4*b^3*c - 7*a^6*b)*sqrt(d*x + c))*log(sqrt(d*x + c)*b + a) - (10
*b^7*d^3*x^3 + 55*b^7*c^3 - 489*a^2*b^5*c^2 + 790*a^4*b^3*c - 360
*a^6*b - 3*(5*b^7*c - 7*a^2*b^5)*d^2*x^2 + 2*(15*b^7*c^2 - 54*a^2
*b^5*c + 35*a^4*b^3)*d*x)*sqrt(d*x + c))/(sqrt(d*x + c)*b^9*d^4 +
 a*b^8*d^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(a + b\sqrt{c + dx}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] Integral(x**3/(a + b*sqrt(c + d*x))**2, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(sqrt(d*x + c)*b + a)^2,x, algorithm="giac")
```

```
[Out] undef
```

3.479 $\int \frac{x^2}{\left(a+b\sqrt{c+dx}\right)^2}\,dx$

Optimal. Leaf size=166

$$\frac{2a\left(a^2-b^2c\right)^2}{b^6d^3\left(a+b\sqrt{c+dx}\right)}-\frac{8a\left(a^2-b^2c\right)\sqrt{c+dx}}{b^5d^3}+\frac{x\left(3a^2-2b^2c\right)}{b^4d^2}+\frac{2\left(5a^4-6a^2b^2c+b^4c^2\right)\log\left(a+b\sqrt{c+dx}\right)}{b^6d^3}-\frac{4a(c+dx)^{3/2}}{3b^3d^3}+\frac{(c+dx)^2}{2b^2d^3}$$

```
[Out] ((3*a^2 - 2*b^2*c)*x)/(b^4*d^2) - (8*a*(a^2 - b^2*c)*Sqrt[c + d*x
])/(b^5*d^3) - (4*a*(c + d*x)^(3/2))/(3*b^3*d^3) + (c + d*x)^2/(2
*b^2*d^3) + (2*a*(a^2 - b^2*c)^2)/(b^6*d^3*(a + b*Sqrt[c + d*x]))
 + (2*(5*a^4 - 6*a^2*b^2*c + b^4*c^2)*Log[a + b*Sqrt[c + d*x]])/(
b^6*d^3)
```

Rubi [A] time = 0.371511, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$

$$\frac{2a\left(a^2-b^2c\right)^2}{b^6d^3\left(a+b\sqrt{c+dx}\right)}-\frac{8a\left(a^2-b^2c\right)\sqrt{c+dx}}{b^5d^3}+\frac{x\left(3a^2-2b^2c\right)}{b^4d^2}+\frac{2\left(5a^4-6a^2b^2c+b^4c^2\right)\log\left(a+b\sqrt{c+dx}\right)}{b^6d^3}-\frac{4a(c+dx)^{3/2}}{3b^3d^3}+\frac{(c+dx)^2}{2b^2d^3}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] ((3*a^2 - 2*b^2*c)*x)/(b^4*d^2) - (8*a*(a^2 - b^2*c)*Sqrt[c + d*x
])/(b^5*d^3) - (4*a*(c + d*x)^(3/2))/(3*b^3*d^3) + (c + d*x)^2/(2
*b^2*d^3) + (2*a*(a^2 - b^2*c)^2)/(b^6*d^3*(a + b*Sqrt[c + d*x]))
 + (2*(5*a^4 - 6*a^2*b^2*c + b^4*c^2)*Log[a + b*Sqrt[c + d*x]])/(
b^6*d^3)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{4a(c+dx)^{\frac{3}{2}}}{3b^3d^3}-\frac{8a(a^2-b^2c)\sqrt{c+dx}}{b^5d^3}+\frac{2a(a^2-b^2c)^2}{b^6d^3\left(a+b\sqrt{c+dx}\right)}+\frac{(c+dx)^2}{2b^2d^3}+\frac{2(3a^2-2b^2c)\int^{\sqrt{c+dx}}x\,dx}{b^4d^3}+\frac{2(a^2-b^2c)(5a^2-b^2c)\log\left(a+b\sqrt{c+dx}\right)}{b^6d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] -4*a*(c + d*x)**(3/2)/(3*b**3*d**3) - 8*a*(a**2 - b**2*c)*sqrt(c
+ d*x)/(b**5*d**3) + 2*a*(a**2 - b**2*c)**2/(b**6*d**3*(a + b*sqr
t(c + d*x))) + (c + d*x)**2/(2*b**2*d**3) + 2*(3*a**2 - 2*b**2*c)
*Integral(x, (x, sqrt(c + d*x)))/(b**4*d**3) + 2*(a**2 - b**2*c)*
(5*a**2 - b**2*c)*log(a + b*sqrt(c + d*x))/(b**6*d**3)
```

Mathematica [A] time = 0.357992, size = 224, normalized size = 1.35

$$\frac{-6b^2dx(b^2c-3a^2)+12(a^2-b^2c)(5a^2-b^2c)\tanh^{-1}\left(\frac{b\sqrt{c+dx}}{a}\right)+\frac{4ab\sqrt{c+dx}(15a^4-2a^2b^2(14c+5dx)+b^4(13c^2+8cdx-2d^2x^2))}{b^2(c+dx)-a^2}+6(5a}{6b^6d^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] (-6*b^2*(-3*a^2 + b^2*c)*d*x + 3*b^4*d^2*x^2 + (12*(a^3 - a*b^2*c
)^2)/(a^2 - b^2*(c + d*x)) + (4*a*b*Sqrt[c + d*x]*(15*a^4 - 2*a^2
*b^2*(14*c + 5*d*x) + b^4*(13*c^2 + 8*c*d*x - 2*d^2*x^2)))/(-a^2
+ b^2*(c + d*x)) + 12*(a^2 - b^2*c)*(5*a^2 - b^2*c)*ArcTanh[(b*Sq
rt[c + d*x])/a] + 6*(5*a^4 - 6*a^2*b^2*c + b^4*c^2)*Log[a^2 - b^2
*(c + d*x)])/(6*b^6*d^3)
```

Maple [A] time = 0.013, size = 253, normalized size = 1.5

$$\frac{x^2}{2b^2d}-\frac{cx}{b^2d^2}-\frac{3c^2}{2b^2d^3}-\frac{4a}{3b^3d^3}(dx+c)^{\frac{3}{2}}+3\frac{a^2x}{b^4d^2}+3\frac{a^2c}{b^4d^3}+8\frac{ac\sqrt{dx+c}}{b^3d^3}-8\frac{a^3\sqrt{dx+c}}{d^3b^5}+2\frac{\ln\left(a+b\sqrt{dx+c}\right)c^2}{b^2d^3}-12\frac{\ln\left(a+b\sqrt{dx+c}\right)ca^2}{b^4d^3}+10\frac{\ln\left(a+b\sqrt{dx+c}\right)a^4}{d^3b^6}+2\frac{ac^2}{b^2d^3\left(a+b\sqrt{dx+c}\right)}-4\frac{a^3c}{b^4d^3\left(a+b\sqrt{dx+c}\right)}+2\frac{a^5}{d^3b^6\left(a+b\sqrt{dx+c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(a+b*(d*x+c)^(1/2))^2,x)
```

```
[Out] 1/2/d/b^2*x^2-1/d^2/b^2*x*c-3/2/d^3/b^2*c^2-4/3*a*(d*x+c)^(3/2)/b
^3/d^3+3/d^2/b^4*x*a^2+3/d^3/b^4*a^2*c+8/d^3/b^3*a*c*(d*x+c)^(1/2
)-8/d^3/b^5*a^3*(d*x+c)^(1/2)+2/d^3/b^2*ln(a+b*(d*x+c)^(1/2))*c^2
-12/d^3/b^4*ln(a+b*(d*x+c)^(1/2))*c*a^2+10/d^3/b^6*ln(a+b*(d*x+c)
^(1/2))*a^4+2/d^3*a/b^2/(a+b*(d*x+c)^(1/2))*c^2-4/d^3*a^3/b^4/(a+
b*(d*x+c)^(1/2))*c+2/d^3*a^5/b^6/(a+b*(d*x+c)^(1/2))
```

Maxima [A] time = 0.703476, size = 213, normalized size = 1.28

$$\frac{\frac{12\left(ab^4c^2-2\,a^3b^2c+a^5\right)}{\sqrt{dx+c}b^7+ab^6}+\frac{3\,(dx+c)^2b^3-8\,(dx+c)^{\frac{3}{2}}ab^2-6\left(2\,b^3c-3\,a^2b\right)(dx+c)+48\left(ab^2c-a^3\right)\sqrt{dx+c}}{b^5}+\frac{12\left(b^4c^2-6\,a^2b^2c+5\,a^4\right)\log\left(\sqrt{dx+c}b+a\right)}{b^6}}{6\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(sqrt(d*x + c)*b + a)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(12*(a*b^4*c^2 - 2*a^3*b^2*c + a^5)/(sqrt(d*x + c)*b^7 + a*b^
6) + (3*(d*x + c)^2*b^3 - 8*(d*x + c)^(3/2)*a*b^2 - 6*(2*b^3*c -
3*a^2*b)*(d*x + c) + 48*(a*b^2*c - a^3)*sqrt(d*x + c))/b^5 + 12*(
b^4*c^2 - 6*a^2*b^2*c + 5*a^4)*log(sqrt(d*x + c)*b + a)/b^6)/d^3
```

Fricas [A] time = 0.268676, size = 289, normalized size = 1.74

$$-\frac{5\,ab^4d^2x^2-43\,ab^4c^2+54\,a^3b^2c-12\,a^5-2\left(13\,ab^4c-15\,a^3b^2\right)dx-12\left(ab^4c^2-6\,a^3b^2c+5\,a^5+\left(b^5c^2-6\,a^2b^3c+5\,a^4b\right)\right.}{6\left(\sqrt{dx+c}b^7d^3+ab^6\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(sqrt(d*x + c)*b + a)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(5*a*b^4*d^2*x^2 - 43*a*b^4*c^2 + 54*a^3*b^2*c - 12*a^5 - 2*
(13*a*b^4*c - 15*a^3*b^2)*d*x - 12*(a*b^4*c^2 - 6*a^3*b^2*c + 5*a
^5 + (b^5*c^2 - 6*a^2*b^3*c + 5*a^4*b)*sqrt(d*x + c))*log(sqrt(d*
x + c)*b + a) - (3*b^5*d^2*x^2 - 9*b^5*c^2 + 58*a^2*b^3*c - 48*a^
4*b - 2*(3*b^5*c - 5*a^2*b^3)*d*x)*sqrt(d*x + c))/(sqrt(d*x + c)*
b^7*d^3 + a*b^6*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(a + b\sqrt{c + dx}\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(a+b*(d*x+c)**(1/2))**2,x)

[Out] Integral(x**2/(a + b*sqrt(c + d*x))**2, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(sqrt(d*x + c)*b + a)^2,x, algorithm="giac")

[Out] undef
```

3.480 $\int \frac{x}{\left(a+b\sqrt{c+dx}\right)^2}\,dx$

Optimal. Leaf size=95

$$\frac{2a\left(a^2-b^2c\right)}{b^4d^2\left(a+b\sqrt{c+dx}\right)}+\frac{2\left(3a^2-b^2c\right)\log\left(a+b\sqrt{c+dx}\right)}{b^4d^2}-\frac{4a\sqrt{c+dx}}{b^3d^2}+\frac{x}{b^2d}$$

```
[Out] x/(b^2*d) - (4*a*Sqrt[c + d*x])/(b^3*d^2) + (2*a*(a^2 - b^2*c))/(
b^4*d^2*(a + b*Sqrt[c + d*x])) + (2*(3*a^2 - b^2*c)*Log[a + b*Sqr
t[c + d*x]])/(b^4*d^2)
```

Rubi [A] time = 0.199244, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$

$$\frac{2a\left(a^2-b^2c\right)}{b^4d^2\left(a+b\sqrt{c+dx}\right)}+\frac{2\left(3a^2-b^2c\right)\log\left(a+b\sqrt{c+dx}\right)}{b^4d^2}-\frac{4a\sqrt{c+dx}}{b^3d^2}+\frac{x}{b^2d}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] x/(b^2*d) - (4*a*Sqrt[c + d*x])/(b^3*d^2) + (2*a*(a^2 - b^2*c))/(
b^4*d^2*(a + b*Sqrt[c + d*x])) + (2*(3*a^2 - b^2*c)*Log[a + b*Sqr
t[c + d*x]])/(b^4*d^2)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{4a\sqrt{c+dx}}{b^3d^2}+\frac{2a\left(a^2-b^2c\right)}{b^4d^2\left(a+b\sqrt{c+dx}\right)}+\frac{2\int^{\sqrt{c+dx}} x\,dx}{b^2d^2}+\frac{2\left(3a^2-b^2c\right)\log\left(a+b\sqrt{c+dx}\right)}{b^4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] -4*a*sqrt(c + d*x)/(b**3*d**2) + 2*a*(a**2 - b**2*c)/(b**4*d**2*(
a + b*sqrt(c + d*x))) + 2*Integral(x, (x, sqrt(c + d*x)))/(b**2*d
```

```
**2) + 2*(3*a**2 - b**2*c)*log(a + b*sqrt(c + d*x))/(b**4*d**2)
```

Mathematica [A] time = 0.123418, size = 86, normalized size = 0.91

$$\frac{\frac{2(a^3-ab^2c)}{a+b\sqrt{c+dx}}+2\left(3a^2-b^2c\right)\log\left(a+b\sqrt{c+dx}\right)-4ab\sqrt{c+dx}+b^2(c+dx)}{b^4d^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] (-4*a*b*Sqrt[c + d*x] + b^2*(c + d*x) + (2*(a^3 - a*b^2*c))/(a +
b*Sqrt[c + d*x]) + 2*(3*a^2 - b^2*c)*Log[a + b*Sqrt[c + d*x]])/(b
^4*d^2)
```

Maple [A] time = 0.012, size = 125, normalized size = 1.3

$$\frac{x}{b^2d}+\frac{c}{b^2d^2}-4\,\frac{a\sqrt{dx+c}}{b^3d^2}-2\,\frac{\ln\left(a+b\sqrt{dx+c}\right)c}{b^2d^2}+6\,\frac{\ln\left(a+b\sqrt{dx+c}\right)a^2}{b^4d^2}-2\,\frac{ac}{b^2d^2\left(a+b\sqrt{dx+c}\right)}+2\,\frac{a^3}{b^4d^2\left(a+b\sqrt{dx+c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(a+b*(d*x+c)^(1/2))^2,x)
```

```
[Out] x/b^2/d+1/d^2/b^2*c-4*a*(d*x+c)^(1/2)/b^3/d^2-2/d^2/b^2*ln(a+b*(d
*x+c)^(1/2))*c+6/d^2/b^4*ln(a+b*(d*x+c)^(1/2))*a^2-2/d^2*a/b^2/(a
+b*(d*x+c)^(1/2))*c+2/d^2*a^3/b^4/(a+b*(d*x+c)^(1/2))
```

Maxima [A] time = 0.693321, size = 122, normalized size = 1.28

$$-\frac{\frac{2\left(ab^2c-a^3\right)}{\sqrt{dx+c}b^5+ab^4}-\frac{(dx+c)b-4\sqrt{dx+c}a}{b^3}+\frac{2\left(b^2c-3a^2\right)\log\left(\sqrt{dx+c}b+a\right)}{b^4}}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(d*x + c)*b + a)^2,x, algorithm="maxima")
```

```
[Out] -(2*(a*b^2*c - a^3)/(sqrt(d*x + c)*b^5 + a*b^4) - ((d*x + c)*b -
4*sqrt(d*x + c)*a)/b^3 + 2*(b^2*c - 3*a^2)*log(sqrt(d*x + c)*b +
a)/b^4)/d^2
```

Fricas [A] time = 0.266919, size = 163, normalized size = 1.72

$$-\frac{3\,ab^2dx+5\,ab^2c-2\,a^3+2\left(ab^2c-3\,a^3+\left(b^3c-3\,a^2b\right)\sqrt{dx+c}\right)\log\left(\sqrt{dx+c}b+a\right)-\left(b^3dx+b^3c-4\,a^2b\right)\sqrt{dx+c}}{\sqrt{dx+c}b^5d^2+ab^4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(d*x + c)*b + a)^2,x, algorithm="fricas")
```

```
[Out] -(3*a*b^2*d*x + 5*a*b^2*c - 2*a^3 + 2*(a*b^2*c - 3*a^3 + (b^3*c -
 3*a^2*b)*sqrt(d*x + c))*log(sqrt(d*x + c)*b + a) - (b^3*d*x + b^
3*c - 4*a^2*b)*sqrt(d*x + c))/(sqrt(d*x + c)*b^5*d^2 + a*b^4*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{\left(a+b\sqrt{c+dx}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] Integral(x/(a + b*sqrt(c + d*x))**2, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(d*x + c)*b + a)^2,x, algorithm="giac")
```

```
[Out] undef
```

3.481 $\int \frac{1}{\left(a+b\sqrt{c+dx}\right)^2} dx$

Optimal. Leaf size=47

$$\frac{2a}{b^2d\left(a+b\sqrt{c+dx}\right)}+\frac{2\log\left(a+b\sqrt{c+dx}\right)}{b^2d}$$

```
[Out] (2*a)/(b^2*d*(a + b*Sqrt[c + d*x])) + (2*Log[a + b*Sqrt[c + d*x]]
)/(b^2*d)
```

Rubi [A] time = 0.0643198, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{2a}{b^2d\left(a+b\sqrt{c+dx}\right)}+\frac{2\log\left(a+b\sqrt{c+dx}\right)}{b^2d}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*Sqrt[c + d*x])^(-2),x]
```

```
[Out] (2*a)/(b^2*d*(a + b*Sqrt[c + d*x])) + (2*Log[a + b*Sqrt[c + d*x]]
)/(b^2*d)
```

Rubi in Sympy [A] time = 4.28379, size = 39, normalized size = 0.83

$$\frac{2a}{b^2d\left(a+b\sqrt{c+dx}\right)}+\frac{2\log\left(a+b\sqrt{c+dx}\right)}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] 2*a/(b**2*d*(a + b*sqrt(c + d*x))) + 2*log(a + b*sqrt(c + d*x))/(
b**2*d)
```

Mathematica [A] time = 0.0294848, size = 40, normalized size = 0.85

$$\frac{2\left(\frac{a}{a+b\sqrt{c+dx}}+\log\left(a+b\sqrt{c+dx}\right)\right)}{b^2d}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*Sqrt[c + d*x])^(-2),x]
```

```
[Out] (2*(a/(a + b*Sqrt[c + d*x]) + Log[a + b*Sqrt[c + d*x]]))/(b^2*d)
```

Maple [B] time = 0.027, size = 142, normalized size = 3.

$$-2\,\frac{a^2}{(b^2dx+b^2c-a^2)\,b^2d}+\frac{\ln\left(b^2dx+b^2c-a^2\right)}{b^2d}+\frac{a}{b^2d}\left(a+b\sqrt{dx+c}\right)^{-1}+\frac{1}{b^2d}\ln\left(a+b\sqrt{dx+c}\right)+\frac{a}{b^2d}\left(-a+b\sqrt{dx+c}\right)^{-1}-\frac{1}{b^2d}\ln\left(-a+b\sqrt{dx+c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a+b*(d*x+c)^(1/2))^2,x)
```

```
[Out] -2*a^2/(b^2*d*x+b^2*c-a^2)/b^2/d+ln(b^2*d*x+b^2*c-a^2)/b^2/d+a/b^
2/d/(a+b*(d*x+c)^(1/2))+ln(a+b*(d*x+c)^(1/2))/b^2/d+a/b^2/d/(-a+b
*(d*x+c)^(1/2))-1/b^2/d*ln(-a+b*(d*x+c)^(1/2))
```

Maxima [A] time = 0.701641, size = 58, normalized size = 1.23

$$\frac{2\left(\frac{a}{\sqrt{dx+c}\,b^3+ab^2}+\frac{\log\left(\sqrt{dx+c}\,b+a\right)}{b^2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(d*x + c)*b + a)^(-2),x, algorithm="maxima")
```

```
[Out] 2*(a/(sqrt(d*x + c)*b^3 + a*b^2) + log(sqrt(d*x + c)*b + a)/b^2)/
d
```

Fricas [A] time = 0.276779, size = 66, normalized size = 1.4

$$\frac{2\left(\left(\sqrt{dx+c}b+a\right)\log\left(\sqrt{dx+c}b+a\right)+a\right)}{\sqrt{dx+c}b^3d+ab^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(d*x + c)*b + a)^(-2),x, algorithm="fricas")
```

```
[Out] 2*((sqrt(d*x + c)*b + a)*log(sqrt(d*x + c)*b + a) + a)/(sqrt(d*x
+ c)*b^3*d + a*b^2*d)
```

Sympy [A] time = 3.43724, size = 124, normalized size = 2.64

$$\begin{cases}\frac{x}{a^2} & \text{for } b=0 \wedge (b=0 \vee d=0)\\ \frac{x}{\left(a+b\sqrt{c}\right)^2} & \text{for } d=0\\ \frac{2a\log\left(\frac{a}{b}+\sqrt{c+dx}\right)}{ab^2d+b^3d\sqrt{c+dx}}+\frac{2a}{ab^2d+b^3d\sqrt{c+dx}}+\frac{2b\sqrt{c+dx}\log\left(\frac{a}{b}+\sqrt{c+dx}\right)}{ab^2d+b^3d\sqrt{c+dx}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] Piecewise((x/a**2, Eq(b, 0) & (Eq(b, 0) | Eq(d, 0))), (x/(a + b*s
qrt(c))**2, Eq(d, 0)), (2*a*log(a/b + sqrt(c + d*x))/(a*b**2*d +
b**3*d*sqrt(c + d*x)) + 2*a/(a*b**2*d + b**3*d*sqrt(c + d*x)) + 2
*b*sqrt(c + d*x)*log(a/b + sqrt(c + d*x))/(a*b**2*d + b**3*d*sqrt
(c + d*x)), True))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

undef

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(d*x + c)*b + a)^(-2),x, algorithm="giac")
```

```
[Out] undef
```

3.482 $\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)^2}\,dx$

Optimal. Leaf size=129

$$\frac{2a}{(a^2-b^2c)\left(a+b\sqrt{c+dx}\right)}-\frac{2\left(a^2+b^2c\right)\log\left(a+b\sqrt{c+dx}\right)}{(a^2-b^2c)^2}+\frac{4ab\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{(a^2-b^2c)^2}+\frac{\log(x)\left(a^2+b^2c\right)}{(a^2-b^2c)^2}$$

```
[Out] (2*a)/((a^2 - b^2*c)*(a + b*Sqrt[c + d*x])) + (4*a*b*Sqrt[c]*ArcT
anh[Sqrt[c + d*x]/Sqrt[c]])/(a^2 - b^2*c)^2 + ((a^2 + b^2*c)*Log[
x])/(a^2 - b^2*c)^2 - (2*(a^2 + b^2*c)*Log[a + b*Sqrt[c + d*x]])/
(a^2 - b^2*c)^2
```

Rubi [A] time = 0.261699, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316

$$\frac{2a}{(a^2-b^2c)\left(a+b\sqrt{c+dx}\right)}-\frac{2\left(a^2+b^2c\right)\log\left(a+b\sqrt{c+dx}\right)}{(a^2-b^2c)^2}+\frac{4ab\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{(a^2-b^2c)^2}+\frac{\log(x)\left(a^2+b^2c\right)}{(a^2-b^2c)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(a + b*Sqrt[c + d*x])^2),x]
```

```
[Out] (2*a)/((a^2 - b^2*c)*(a + b*Sqrt[c + d*x])) + (4*a*b*Sqrt[c]*ArcT
anh[Sqrt[c + d*x]/Sqrt[c]])/(a^2 - b^2*c)^2 + ((a^2 + b^2*c)*Log[
x])/(a^2 - b^2*c)^2 - (2*(a^2 + b^2*c)*Log[a + b*Sqrt[c + d*x]])/
(a^2 - b^2*c)^2
```

Rubi in Sympy [A] time = 17.4796, size = 117, normalized size = 0.91

$$\frac{4ab\sqrt{c}\operatorname{atanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{(a^2-b^2c)^2}+\frac{2a}{\left(a+b\sqrt{c+dx}\right)(a^2-b^2c)}+\frac{\left(a^2+b^2c\right)\log(-dx)}{(a^2-b^2c)^2}-\frac{2\left(a^2+b^2c\right)\log\left(a+b\sqrt{c+dx}\right)}{(a^2-b^2c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] 4*a*b*sqrt(c)*atanh(sqrt(c + d*x)/sqrt(c))/(a**2 - b**2*c)**2 + 2
*a/((a + b*sqrt(c + d*x))*(a**2 - b**2*c)) + (a**2 + b**2*c)*log(
-d*x)/(a**2 - b**2*c)**2 - 2*(a**2 + b**2*c)*log(a + b*sqrt(c + d
*x))/(a**2 - b**2*c)**2
```

Mathematica [A] time = 0.281962, size = 106, normalized size = 0.82

$$\frac{\frac{2a(a^2-b^2c)}{a+b\sqrt{c+dx}} + \left(a^2+b^2c\right)\log(dx) - 2\left(a^2+b^2c\right)\log\left(a+b\sqrt{c+dx}\right) + 4ab\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\left(a^2-b^2c\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(a + b*Sqrt[c + d*x])^2),x]
```

```
[Out] ((2*a*(a^2 - b^2*c))/(a + b*Sqrt[c + d*x]) + 4*a*b*Sqrt[c]*ArcTan
h[Sqrt[c + d*x]/Sqrt[c]] + (a^2 + b^2*c)*Log[d*x] - 2*(a^2 + b^2*
c)*Log[a + b*Sqrt[c + d*x]])/(a^2 - b^2*c)^2
```

Maple [A] time = 0.013, size = 161, normalized size = 1.3

$$\frac{\ln(dx)\,b^2c}{\left(-b^2c+a^2\right)^2} + \frac{\ln(dx)\,a^2}{\left(-b^2c+a^2\right)^2} + 4\,\frac{\sqrt{c}ab}{\left(-b^2c+a^2\right)^2}Artanh\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right) + 2\,\frac{a}{\left(-b^2c+a^2\right)\left(a+b\sqrt{dx+c}\right)} - 2\,\frac{\ln\left(a+b\sqrt{dx+c}\right)b^2c}{\left(-b^2c+a^2\right)^2} - 2\,\frac{\ln\left(a+b\sqrt{dx+c}\right)a^2}{\left(-b^2c+a^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(a+b*(d*x+c)^(1/2))^2,x)
```

```
[Out] 1/(-b^2*c+a^2)^2*ln(d*x)*b^2*c+1/(-b^2*c+a^2)^2*ln(d*x)*a^2+4*a*b
*arctanh((d*x+c)^(1/2)/c^(1/2))*c^(1/2)/(-b^2*c+a^2)^2+2*a/(-b^2*
c+a^2)/(a+b*(d*x+c)^(1/2))-2/(-b^2*c+a^2)^2*ln(a+b*(d*x+c)^(1/2))
*b^2*c-2/(-b^2*c+a^2)^2*ln(a+b*(d*x+c)^(1/2))*a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((sqrt(d*x + c)*b + a)^2*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.304268, size = 1, normalized size = 0.01

$$\left[-\frac{2\,ab^2c-2\,a^3-\left(b^3c+a^2b\right)\sqrt{dx+c}\log\left(x\right)+2\left(ab^2c+a^3+\left(b^3c+a^2b\right)\sqrt{dx+c}\right)\log\left(\sqrt{dx+c}b+a\right)-\left(ab^2c+a^3\right)\log\left(x\right.}{ab^4c^2-2\,a^3b^2c+a^5+(b^5c^2-2\,a^2b^3c+a^4b)\sqrt{dx+c}}\right.$$

$$-\frac{2\,ab^2c-2\,a^3-\left(b^3c+a^2b\right)\sqrt{dx+c}\log\left(x\right)-4\left(\sqrt{dx+c}ab^2\sqrt{-c}+a^2b\sqrt{-c}\right)\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)+2\left(ab^2c+a^3+\left(b^3c+a^2b\right)\right.}{ab^4c^2-2\,a^3b^2c+a^5+(b^5c^2-2\,a^2b^3c+a^4b)\sqrt{dx+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((sqrt(d*x + c)*b + a)^2*x),x, algorithm="fricas")
```

```
[Out] [-(2*a*b^2*c - 2*a^3 - (b^3*c + a^2*b)*sqrt(d*x + c)*log(x) + 2*(
a*b^2*c + a^3 + (b^3*c + a^2*b)*sqrt(d*x + c))*log(sqrt(d*x + c)*
b + a) - (a*b^2*c + a^3)*log(x) - 2*(sqrt(d*x + c)*a*b^2*sqrt(c)
+ a^2*b*sqrt(c))*log((d*x + 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x))/(a
*b^4*c^2 - 2*a^3*b^2*c + a^5 + (b^5*c^2 - 2*a^2*b^3*c + a^4*b)*sq
rt(d*x + c)), -(2*a*b^2*c - 2*a^3 - (b^3*c + a^2*b)*sqrt(d*x + c)
*log(x) - 4*(sqrt(d*x + c)*a*b^2*sqrt(-c) + a^2*b*sqrt(-c))*arcta
n(sqrt(d*x + c)/sqrt(-c)) + 2*(a*b^2*c + a^3 + (b^3*c + a^2*b)*sq
rt(d*x + c))*log(sqrt(d*x + c)*b + a) - (a*b^2*c + a^3)*log(x))/(
a*b^4*c^2 - 2*a^3*b^2*c + a^5 + (b^5*c^2 - 2*a^2*b^3*c + a^4*b)*s
qrt(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] Integral(1/(x*(a + b*sqrt(c + d*x))**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((sqrt(d*x + c)*b + a)^2*x),x, algorithm="giac")
```

```
[Out] undef
```

3.483 $\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)^2}\,dx$

Optimal. Leaf size=202

$$\frac{4ab^2d}{\left(a^2-b^2c\right)^2\left(a+b\sqrt{c+dx}\right)}-\frac{a-b\sqrt{c+dx}}{x\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)}+\frac{b^2d\log(x)\left(3a^2+b^2c\right)}{\left(a^2-b^2c\right)^3}-\frac{2b^2d\left(3a^2+b^2c\right)\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^3}+\frac{2abd\left(a^2+3b^2c\right)\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}\left(a^2-b^2c\right)^3}$$

```
[Out] (4*a*b^2*d)/((a^2 - b^2*c)^2*(a + b*Sqrt[c + d*x])) - (a - b*Sqrt
[c + d*x])/((a^2 - b^2*c)*x*(a + b*Sqrt[c + d*x])) + (2*a*b*(a^2
+ 3*b^2*c)*d*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/(Sqrt[c]*(a^2 - b^2*
c)^3) + (b^2*(3*a^2 + b^2*c)*d*Log[x])/(a^2 - b^2*c)^3 - (2*b^2*(
3*a^2 + b^2*c)*d*Log[a + b*Sqrt[c + d*x]])/(a^2 - b^2*c)^3
```

Rubi [A] time = 0.518407, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368

$$\frac{4ab^2d}{\left(a^2-b^2c\right)^2\left(a+b\sqrt{c+dx}\right)}-\frac{a-b\sqrt{c+dx}}{x\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)}+\frac{b^2d\log(x)\left(3a^2+b^2c\right)}{\left(a^2-b^2c\right)^3}-\frac{2b^2d\left(3a^2+b^2c\right)\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^3}+\frac{2abd\left(a^2+3b^2c\right)\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}\left(a^2-b^2c\right)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(a + b*Sqrt[c + d*x])^2),x]
```


```
[Out] (4*a*b^2*d)/((a^2 - b^2*c)^2*(a + b*Sqrt[c + d*x])) - (a - b*Sqrt
[c + d*x])/((a^2 - b^2*c)*x*(a + b*Sqrt[c + d*x])) + (2*a*b*(a^2
+ 3*b^2*c)*d*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/(Sqrt[c]*(a^2 - b^2*
c)^3) + (b^2*(3*a^2 + b^2*c)*d*Log[x])/(a^2 - b^2*c)^3 - (2*b^2*(
3*a^2 + b^2*c)*d*Log[a + b*Sqrt[c + d*x]])/(a^2 - b^2*c)^3
```

Rubi in Sympy [A] time = 40.3978, size = 184, normalized size = 0.91

$$\frac{4ab^2d}{\left(a+b\sqrt{c+dx}\right)(a^2-b^2c)^2}+\frac{2abd\left(a^2+3b^2c\right)\operatorname{atanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}\,(a^2-b^2c)^3}+\frac{b^2d\left(3a^2+b^2c\right)\log\left(-dx\right)}{(a^2-b^2c)^3}-\frac{2b^2d\left(3a^2+b^2c\right)\log\left(a+b\sqrt{c+dx}\right)}{(a^2-b^2c)^3}-\frac{a-b\sqrt{c+dx}}{x\left(a+b\sqrt{c+dx}\right)(a^2-b^2c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] 4*a*b**2*d/((a + b*sqrt(c + d*x))*(a**2 - b**2*c)**2) + 2*a*b*d*(
a**2 + 3*b**2*c)*atanh(sqrt(c + d*x)/sqrt(c))/(sqrt(c)*(a**2 - b*
*2*c)**3) + b**2*d*(3*a**2 + b**2*c)*log(-d*x)/(a**2 - b**2*c)**3
 - 2*b**2*d*(3*a**2 + b**2*c)*log(a + b*sqrt(c + d*x))/(a**2 - b*
*2*c)**3 - (a - b*sqrt(c + d*x))/(x*(a + b*sqrt(c + d*x))*(a**2 -
 b**2*c))
```

Mathematica [A] time = 1.72961, size = 292, normalized size = 1.45

$$\frac{2a^2b^2d}{(a^2-b^2c)^2\,(a^2-b^2(c+dx))}+\frac{b^2d\log(x)\left(3a^2+b^2c\right)}{(a^2-b^2c)^3}-\frac{b^2d\left(3a^2+b^2c\right)\log\left(a^2-b^2(c+dx)\right)}{(a^2-b^2c)^3}+\frac{2b^2d\left(3a^2+b^2c\right)\tanh^{-1}\left(\frac{b\sqrt{c+dx}}{a}\right)}{(b^2c-a^2)^3}+\frac{2abd\left(a^2+3b^2c\right)\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}\,(a^2-b^2c)^3}-\frac{a^2+b^2c}{x\,(a^2-b^2c)^2}+\frac{2a\sqrt{c+dx}\left(b^3(c+2dx)-a^2b\right)}{x\,(a^2-b^2c)^2\,(b^2(c+dx)-a^2)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(a + b*Sqrt[c + d*x])^2),x]
```

```
[Out] -((a^2 + b^2*c)/((a^2 - b^2*c)^2*x)) + (2*a^2*b^2*d)/((a^2 - b^2*
c)^2*(a^2 - b^2*(c + d*x))) + (2*a*Sqrt[c + d*x]*(-(a^2*b) + b^3*
(c + 2*d*x)))/((a^2 - b^2*c)^2*x*(-a^2 + b^2*(c + d*x))) + (2*b^2
*(3*a^2 + b^2*c)*d*ArcTanh[(b*Sqrt[c + d*x])/a])/(-a^2 + b^2*c)^3
 + (2*a*b*(a^2 + 3*b^2*c)*d*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/(Sqrt
[c]*(a^2 - b^2*c)^3) + (b^2*(3*a^2 + b^2*c)*d*Log[x])/(a^2 - b^2*
c)^3 - (b^2*(3*a^2 + b^2*c)*d*Log[a^2 - b^2*(c + d*x)])/(a^2 - b^
2*c)^3
```

Maple [A] time = 0.022, size = 312, normalized size = 1.5

$$-2\frac{a\sqrt{dx+c}b^{3}c}{\left(-b^{2}c+a^{2}\right)^{3}x}+2\frac{\sqrt{dx+c}a^{3}b}{\left(-b^{2}c+a^{2}\right)^{3}x}+\frac{b^{4}c^{2}}{\left(-b^{2}c+a^{2}\right)^{3}x}-\frac{a^{4}}{\left(-b^{2}c+a^{2}\right)^{3}x}+\frac{d\ln\left(dx\right)b^{4}c}{\left(-b^{2}c+a^{2}\right)^{3}}+3\frac{d\ln\left(dx\right)a^{2}b^{2}}{\left(-b^{2}c+a^{2}\right)^{3}}+6\frac{b^{3}d\sqrt{c}a}{\left(-b^{2}c+a^{2}\right)^{3}}Artanh\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)+2\frac{bda^{3}}{\left(-b^{2}c+a^{2}\right)^{3}\sqrt{c}}Artanh\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)+2\frac{ab^{2}d}{\left(-b^{2}c+a^{2}\right)^{2}\left(a+b\sqrt{dx+c}\right)}-2\frac{b^{4}d\ln\left(a+b\sqrt{dx+c}\right)c}{\left(-b^{2}c+a^{2}\right)^{3}}-6\frac{b^{2}d\ln\left(a+b\sqrt{dx+c}\right)a^{2}}{\left(-b^{2}c+a^{2}\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(a+b*(d*x+c)^(1/2))^2,x)
```

```
[Out] -2/(-b^2*c+a^2)^3/x*(d*x+c)^(1/2)*a*b^3*c+2/(-b^2*c+a^2)^3/x*(d*x
+c)^(1/2)*a^3*b+1/(-b^2*c+a^2)^3/x*b^4*c^2-1/(-b^2*c+a^2)^3/x*a^4
+d/(-b^2*c+a^2)^3*ln(d*x)*b^4*c+3*d/(-b^2*c+a^2)^3*ln(d*x)*a^2*b^
2+6*d/(-b^2*c+a^2)^3*b^3*c^(1/2)*arctanh((d*x+c)^(1/2)/c^(1/2))*a
+2*d/(-b^2*c+a^2)^3*b/c^(1/2)*arctanh((d*x+c)^(1/2)/c^(1/2))*a^3+
2*a*b^2*d/(-b^2*c+a^2)^2/(a+b*(d*x+c)^(1/2))-2*d*b^4/(-b^2*c+a^2)
^3*ln(a+b*(d*x+c)^(1/2))*c-6*d*b^2/(-b^2*c+a^2)^3*ln(a+b*(d*x+c)^
(1/2))*a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((sqrt(d*x + c)*b + a)^2*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.605176, size = 1, normalized size = 0.

$$\left[-\frac{\left(b^{5}c^{2}-2\,a^{2}b^{3}c+a^{4}b+\left(b^{5}c+3\,a^{2}b^{3}\right)dx\log\left(x\right)\right)\sqrt{dx+c}\sqrt{c}-2\left(\left(b^{5}c+3\,a^{2}b^{3}\right)\sqrt{dx+c}\sqrt{c}dx+\left(ab^{4}c+3\,a^{3}b^{2}\right)\sqrt{c}dx\right)\text{lo}}{\left(b^{7}c^{3}-3\,a^{2}b^{5}c^{2}\right.}\right.$$

$$-\frac{\left(b^{5}c^{2}-2\,a^{2}b^{3}c+a^{4}b+\left(b^{5}c+3\,a^{2}b^{3}\right)dx\log\left(x\right)\right)\sqrt{dx+c}\sqrt{-c}-2\left(\left(3\,ab^{4}c+a^{3}b^{2}\right)\sqrt{dx+c}dx+\left(3\,a^{2}b^{3}c+a^{4}b\right)dx\right)\text{arctar}}{\left(b^{7}c^{3}-3\,a^{2}b^{5}c^{2}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((sqrt(d*x + c)*b + a)^2*x^2),x, algorithm="fricas")
```

```
[Out] [-((b^5*c^2 - 2*a^2*b^3*c + a^4*b + (b^5*c + 3*a^2*b^3)*d*x*log(x
))*sqrt(d*x + c)*sqrt(c) - 2*((b^5*c + 3*a^2*b^3)*sqrt(d*x + c)*s
qrt(c)*d*x + (a*b^4*c + 3*a^3*b^2)*sqrt(c)*d*x)*log(sqrt(d*x + c)
*b + a) - ((3*a*b^4*c + a^3*b^2)*sqrt(d*x + c)*d*x + (3*a^2*b^3*c
 + a^4*b)*d*x)*log(((d*x + 2*c)*sqrt(c) - 2*sqrt(d*x + c)*c)/x) -
 (a*b^4*c^2 - 2*a^3*b^2*c + a^5 - (a*b^4*c + 3*a^3*b^2)*d*x*log(x
) + 4*(a*b^4*c - a^3*b^2)*d*x)*sqrt(c))/((b^7*c^3 - 3*a^2*b^5*c^2
 + 3*a^4*b^3*c - a^6*b)*sqrt(d*x + c)*sqrt(c)*x + (a*b^6*c^3 - 3*
a^3*b^4*c^2 + 3*a^5*b^2*c - a^7)*sqrt(c)*x), -((b^5*c^2 - 2*a^2*b
^3*c + a^4*b + (b^5*c + 3*a^2*b^3)*d*x*log(x))*sqrt(d*x + c)*sqrt
(-c) - 2*((3*a*b^4*c + a^3*b^2)*sqrt(d*x + c)*d*x + (3*a^2*b^3*c
+ a^4*b)*d*x)*arctan(c/(sqrt(d*x + c)*sqrt(-c))) - 2*((b^5*c + 3*
a^2*b^3)*sqrt(d*x + c)*sqrt(-c)*d*x + (a*b^4*c + 3*a^3*b^2)*sqrt(
-c)*d*x)*log(sqrt(d*x + c)*b + a) - (a*b^4*c^2 - 2*a^3*b^2*c + a^
5 - (a*b^4*c + 3*a^3*b^2)*d*x*log(x) + 4*(a*b^4*c - a^3*b^2)*d*x)
*sqrt(-c))/((b^7*c^3 - 3*a^2*b^5*c^2 + 3*a^4*b^3*c - a^6*b)*sqrt(
d*x + c)*sqrt(-c)*x + (a*b^6*c^3 - 3*a^3*b^4*c^2 + 3*a^5*b^2*c -
a^7)*sqrt(-c)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \left(a + b\sqrt{c + dx}\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] Integral(1/(x**2*(a + b*sqrt(c + d*x))**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((sqrt(d*x + c)*b + a)^2*x^2),x, algorithm="giac")
```

```
[Out] undef
```

3.484 $\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)^2}\,dx$

Optimal. Leaf size=306

$$\frac{ab^2d^2\left(a^2+11b^2c\right)}{2c\left(a^2-b^2c\right)^3\left(a+b\sqrt{c+dx}\right)}-\frac{a-b\sqrt{c+dx}}{2x^2\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)}-\frac{bd\left(3abc-\left(a^2+2b^2c\right)\sqrt{c+dx}\right)}{2cx\left(a^2-b^2c\right)^2\left(a+b\sqrt{c+dx}\right)}+\frac{b^4d^2\log(x)\left(5a^2+b^2c\right)}{\left(a^2-b^2c\right)^4}-\frac{2b^4d^2\left(5a^2+b^2c\right)\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^4}-\frac{abd^2\left(a^4-10a^2b^2c-15b^4c^2\right)\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{2c^{3/2}\left(a^2-b^2c\right)^4}$$

```
[Out] (a*b^2*(a^2 + 11*b^2*c)*d^2)/(2*c*(a^2 - b^2*c)^3*(a + b*Sqrt[c +
 d*x])) - (a - b*Sqrt[c + d*x])/(2*(a^2 - b^2*c)*x^2*(a + b*Sqrt[
c + d*x])) - (b*d*(3*a*b*c - (a^2 + 2*b^2*c)*Sqrt[c + d*x]))/(2*c
*(a^2 - b^2*c)^2*x*(a + b*Sqrt[c + d*x])) - (a*b*(a^4 - 10*a^2*b^
2*c - 15*b^4*c^2)*d^2*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/(2*c^(3/2)*
(a^2 - b^2*c)^4) + (b^4*(5*a^2 + b^2*c)*d^2*Log[x])/(a^2 - b^2*c)
^4 - (2*b^4*(5*a^2 + b^2*c)*d^2*Log[a + b*Sqrt[c + d*x]])/(a^2 -
b^2*c)^4
```

Rubi [A] time = 0.837213, antiderivative size = 306, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$

$$\frac{ab^2d^2\left(a^2+11b^2c\right)}{2c\left(a^2-b^2c\right)^3\left(a+b\sqrt{c+dx}\right)}-\frac{a-b\sqrt{c+dx}}{2x^2\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)}-\frac{bd\left(3abc-\left(a^2+2b^2c\right)\sqrt{c+dx}\right)}{2cx\left(a^2-b^2c\right)^2\left(a+b\sqrt{c+dx}\right)}+\frac{b^4d^2\log(x)\left(5a^2+b^2c\right)}{\left(a^2-b^2c\right)^4}-\frac{2b^4d^2\left(5a^2+b^2c\right)\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^4}-\frac{abd^2\left(a^4-10a^2b^2c-15b^4c^2\right)\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{2c^{3/2}\left(a^2-b^2c\right)^4}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*(a + b*Sqrt[c + d*x])^2),x]
```

```
[Out] (a*b^2*(a^2 + 11*b^2*c)*d^2)/(2*c*(a^2 - b^2*c)^3*(a + b*Sqrt[c +
 d*x])) - (a - b*Sqrt[c + d*x])/(2*(a^2 - b^2*c)*x^2*(a + b*Sqrt[
c + d*x])) - (b*d*(3*a*b*c - (a^2 + 2*b^2*c)*Sqrt[c + d*x]))/(2*c
```

```
*(a^2 - b^2*c)^2*x*(a + b*Sqrt[c + d*x])) - (a*b*(a^4 - 10*a^2*b^
2*c - 15*b^4*c^2)*d^2*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/(2*c^(3/2)*
(a^2 - b^2*c)^4) + (b^4*(5*a^2 + b^2*c)*d^2*Log[x])/(a^2 - b^2*c)
^4 - (2*b^4*(5*a^2 + b^2*c)*d^2*Log[a + b*Sqrt[c + d*x]])/(a^2 -
b^2*c)^4
```

Rubi in Sympy [A] time = 71.6826, size = 275, normalized size = 0.9

$$\frac{ab^2d^2\left(a^2+11b^2c\right)}{2c\left(a+b\sqrt{c+dx}\right)\left(a^2-b^2c\right)^3}-\frac{abd^2\left(a^4-10a^2b^2c-15b^4c^2\right)\operatorname{atanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{2c^{\frac{3}{2}}\left(a^2-b^2c\right)^4}+\frac{b^4d^2\left(5a^2+b^2c\right)\log\left(-dx\right)}{\left(a^2-b^2c\right)^4}-\frac{2b^4d^2\left(5a^2+b^2c\right)\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^4}+\frac{bd\left(-6abc+\left(2a^2+4b^2c\right)\sqrt{c+dx}\right)}{4cx\left(a+b\sqrt{c+dx}\right)\left(a^2-b^2c\right)^2}-\frac{a-b\sqrt{c+dx}}{2x^2\left(a+b\sqrt{c+dx}\right)\left(a^2-b^2c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] a*b**2*d**2*(a**2 + 11*b**2*c)/(2*c*(a + b*sqrt(c + d*x))*(a**2 -
 b**2*c)**3) - a*b*d**2*(a**4 - 10*a**2*b**2*c - 15*b**4*c**2)*at
anh(sqrt(c + d*x)/sqrt(c))/(2*c**(3/2)*(a**2 - b**2*c)**4) + b**4
*d**2*(5*a**2 + b**2*c)*log(-d*x)/(a**2 - b**2*c)**4 - 2*b**4*d**
2*(5*a**2 + b**2*c)*log(a + b*sqrt(c + d*x))/(a**2 - b**2*c)**4 +
 b*d*(-6*a*b*c + (2*a**2 + 4*b**2*c)*sqrt(c + d*x))/(4*c*x*(a + b
*sqrt(c + d*x))*(a**2 - b**2*c)**2) - (a - b*sqrt(c + d*x))/(2*x*
*2*(a + b*sqrt(c + d*x))*(a**2 - b**2*c))
```

Mathematica [A] time = 0.616692, size = 390, normalized size = 1.27

$$\frac{1}{2}\left(-\frac{2b^2d\left(3a^2+b^2c\right)}{x\left(a^2-b^2c\right)^3}-\frac{a^2+b^2c}{x^2\left(a^2-b^2c\right)^2}+\frac{4a^2b^4d^2}{\left(a^2-b^2c\right)^3\left(a^2-b^2(c+dx)\right)}+\frac{2b^4d^2\log(x)\left(5a^2+b^2c\right)}{\left(a^2-b^2c\right)^4}\right.$$
$$-\frac{2b^4d^2\left(5a^2+b^2c\right)\log\left(a^2-b^2(c+dx)\right)}{\left(a^2-b^2c\right)^4}-\frac{4b^4d^2\left(5a^2+b^2c\right)\tanh^{-1}\left(\frac{b\sqrt{c+dx}}{a}\right)}{\left(a^2-b^2c\right)^4}$$
$$+\frac{a\sqrt{c+dx}\left(a^4b(2c+dx)-a^2b^3(dx-2c)^2+b^5c\left(2c^2-5cdx-11d^2x^2\right)\right)}{cx^2\left(b^2c-a^2\right)^3\left(b^2(c+dx)-a^2\right)}$$
$$\left.+\frac{d^2\left(a^5(-b)+10a^3b^3c+15ab^5c^2\right)\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{c^{3/2}\left(a^2-b^2c\right)^4}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*(a + b*Sqrt[c + d*x])^2),x]
```

```
[Out] (-((a^2 + b^2*c)/((a^2 - b^2*c)^2*x^2)) - (2*b^2*(3*a^2 + b^2*c)*
d)/((a^2 - b^2*c)^3*x) + (4*a^2*b^4*d^2)/((a^2 - b^2*c)^3*(a^2 -
b^2*(c + d*x))) + (a*Sqrt[c + d*x]*(-(a^2*b^3*(-2*c + d*x)^2) + a
^4*b*(2*c + d*x) + b^5*c*(2*c^2 - 5*c*d*x - 11*d^2*x^2)))/(c*(-a^
2 + b^2*c)^3*x^2*(-a^2 + b^2*(c + d*x))) - (4*b^4*(5*a^2 + b^2*c)
*d^2*ArcTanh[(b*Sqrt[c + d*x])/a])/(a^2 - b^2*c)^4 + ((-(a^5*b) +
 10*a^3*b^3*c + 15*a*b^5*c^2)*d^2*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])
/(c^(3/2)*(a^2 - b^2*c)^4) + (2*b^4*(5*a^2 + b^2*c)*d^2*Log[x])/(
a^2 - b^2*c)^4 - (2*b^4*(5*a^2 + b^2*c)*d^2*Log[a^2 - b^2*(c + d*
x)])/(a^2 - b^2*c)^4)/2
```

Maple [B] time = 0.027, size = 612, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(a+b*(d*x+c)^(1/2))^2,x)
```

```
[Out] -7/2/(-b^2*c+a^2)^4/x^2*a*b^5*c*(d*x+c)^(3/2)+3/(-b^2*c+a^2)^4/x^
2*a^3*b^3*(d*x+c)^(3/2)+1/2/(-b^2*c+a^2)^4/x^2*a^5*b/c*(d*x+c)^(3
/2)+d/(-b^2*c+a^2)^4/x*b^6*c^2-1/2/(-b^2*c+a^2)^4/x^2*b^6*c^3+2*d
/(-b^2*c+a^2)^4/x*a^2*b^4*c+1/2/(-b^2*c+a^2)^4/x^2*a^2*b^4*c^2-3*
d/(-b^2*c+a^2)^4/x*a^4*b^2+1/2/(-b^2*c+a^2)^4/x^2*a^4*b^2*c+9/2/(
-b^2*c+a^2)^4/x^2*(d*x+c)^(1/2)*a*b^5*c^2-5/(-b^2*c+a^2)^4/x^2*(d
*x+c)^(1/2)*a^3*b^3*c+1/2/(-b^2*c+a^2)^4/x^2*(d*x+c)^(1/2)*b*a^5-
```

```
1/2/(-b^2*c+a^2)^4/x^2*a^6+d^2/(-b^2*c+a^2)^4*b^6*c*ln(c*d*x)+5*d
^2/(-b^2*c+a^2)^4*b^4*ln(c*d*x)*a^2+15/2*d^2/(-b^2*c+a^2)^4*b^5*c
^(1/2)*arctanh((d*x+c)^(1/2)/c^(1/2))*a+5*d^2/(-b^2*c+a^2)^4*b^3/
c^(1/2)*arctanh((d*x+c)^(1/2)/c^(1/2))*a^3-1/2*d^2/(-b^2*c+a^2)^4
*b/c^(3/2)*arctanh((d*x+c)^(1/2)/c^(1/2))*a^5+2*d^2*b^4/(-b^2*c+a
^2)^3*a/(a+b*(d*x+c)^(1/2))-2*d^2*b^6/(-b^2*c+a^2)^4*ln(a+b*(d*x+
c)^(1/2))*c-10*d^2*b^4/(-b^2*c+a^2)^4*ln(a+b*(d*x+c)^(1/2))*a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((sqrt(d*x + c)*b + a)^2*x^3),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 1.60741, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((sqrt(d*x + c)*b + a)^2*x^3),x, algorithm="fricas")

[Out] [-1/4*(2*(b^7*c^4 - 3*a^2*b^5*c^3 + 3*a^4*b^3*c^2 - a^6*b*c - 2*(
b^7*c^2 + 5*a^2*b^5*c)*d^2*x^2*log(x) - (2*b^7*c^3 - 3*a^2*b^5*c^
2 + a^6*b)*d*x)*sqrt(d*x + c)*sqrt(c) + 8*((b^7*c^2 + 5*a^2*b^5*c
)*sqrt(d*x + c)*sqrt(c)*d^2*x^2 + (a*b^6*c^2 + 5*a^3*b^4*c)*sqrt(
c)*d^2*x^2)*log(sqrt(d*x + c)*b + a) - ((15*a*b^6*c^2 + 10*a^3*b^
4*c - a^5*b^2)*sqrt(d*x + c)*d^2*x^2 + (15*a^2*b^5*c^2 + 10*a^4*b
^3*c - a^6*b)*d^2*x^2)*log(((d*x + 2*c)*sqrt(c) + 2*sqrt(d*x + c)
*c)/x) - 2*(a*b^6*c^4 - 3*a^3*b^4*c^3 + 3*a^5*b^2*c^2 - a^7*c + 2
*(a*b^6*c^2 + 5*a^3*b^4*c)*d^2*x^2*log(x) - (11*a*b^6*c^2 - 10*a^
3*b^4*c - a^5*b^2)*d^2*x^2 - 3*(a*b^6*c^3 - 2*a^3*b^4*c^2 + a^5*b
^2*c)*d*x)*sqrt(c))/((b^9*c^5 - 4*a^2*b^7*c^4 + 6*a^4*b^5*c^3 - 4
*a^6*b^3*c^2 + a^8*b*c)*sqrt(d*x + c)*sqrt(c)*x^2 + (a*b^8*c^5 -
4*a^3*b^6*c^4 + 6*a^5*b^4*c^3 - 4*a^7*b^2*c^2 + a^9*c)*sqrt(c)*x^
2), -1/2*((b^7*c^4 - 3*a^2*b^5*c^3 + 3*a^4*b^3*c^2 - a^6*b*c - 2*
(b^7*c^2 + 5*a^2*b^5*c)*d^2*x^2*log(x) - (2*b^7*c^3 - 3*a^2*b^5*c
^2 + a^6*b)*d*x)*sqrt(d*x + c)*sqrt(-c) + ((15*a*b^6*c^2 + 10*a^3
*b^4*c - a^5*b^2)*sqrt(d*x + c)*d^2*x^2 + (15*a^2*b^5*c^2 + 10*a^
4*b^3*c - a^6*b)*d^2*x^2)*arctan(c/(sqrt(d*x + c)*sqrt(-c))) + 4*
((b^7*c^2 + 5*a^2*b^5*c)*sqrt(d*x + c)*sqrt(-c)*d^2*x^2 + (a*b^6*
```

```
c^2 + 5*a^3*b^4*c)*sqrt(-c)*d^2*x^2)*log(sqrt(d*x + c)*b + a) - (
a*b^6*c^4 - 3*a^3*b^4*c^3 + 3*a^5*b^2*c^2 - a^7*c + 2*(a*b^6*c^2
+ 5*a^3*b^4*c)*d^2*x^2*log(x) - (11*a*b^6*c^2 - 10*a^3*b^4*c - a^
5*b^2)*d^2*x^2 - 3*(a*b^6*c^3 - 2*a^3*b^4*c^2 + a^5*b^2*c)*d*x)*s
qrt(-c))/((b^9*c^5 - 4*a^2*b^7*c^4 + 6*a^4*b^5*c^3 - 4*a^6*b^3*c^
2 + a^8*b*c)*sqrt(d*x + c)*sqrt(-c)*x^2 + (a*b^8*c^5 - 4*a^3*b^6*
c^4 + 6*a^5*b^4*c^3 - 4*a^7*b^2*c^2 + a^9*c)*sqrt(-c)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \left(a + b\sqrt{c + dx}\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] Integral(1/(x**3*(a + b*sqrt(c + d*x))**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((sqrt(d*x + c)*b + a)^2*x^3),x, algorithm="giac")
```

```
[Out] undef
```

3.485 $\int \frac{x^3}{\sqrt{a+b\sqrt{c+dx}}}\,dx$

Optimal. Leaf size=324

$$\frac{12\left(7a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{11/2}}{11b^8d^4}-\frac{20a\left(7a^2-3b^2c\right)\left(a+b\sqrt{c+dx}\right)^{9/2}}{9b^8d^4}-\frac{12a\left(7a^2-3b^2c\right)\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^8d^4}+\frac{4\left(a^2-b^2c\right)^2\left(7a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^8d^4}-\frac{4a\left(a^2-b^2c\right)^3\sqrt{a+b\sqrt{c+dx}}}{b^8d^4}+\frac{4\left(35a^4-30a^2b^2c+3b^4c^2\right)\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^8d^4}+\frac{4\left(a+b\sqrt{c+dx}\right)^{15/2}}{15b^8d^4}-\frac{28a\left(a+b\sqrt{c+dx}\right)^{13/2}}{13b^8d^4}$$

```
[Out] (-4*a*(a^2 - b^2*c)^3*Sqrt[a + b*Sqrt[c + d*x]])/(b^8*d^4) + (4*(
a^2 - b^2*c)^2*(7*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^
8*d^4) - (12*a*(7*a^2 - 3*b^2*c)*(a^2 - b^2*c)*(a + b*Sqrt[c + d*
x])^(5/2))/(5*b^8*d^4) + (4*(35*a^4 - 30*a^2*b^2*c + 3*b^4*c^2)*(
a + b*Sqrt[c + d*x])^(7/2))/(7*b^8*d^4) - (20*a*(7*a^2 - 3*b^2*c)
*(a + b*Sqrt[c + d*x])^(9/2))/(9*b^8*d^4) + (12*(7*a^2 - b^2*c)*(
a + b*Sqrt[c + d*x])^(11/2))/(11*b^8*d^4) - (28*a*(a + b*Sqrt[c +
 d*x])^(13/2))/(13*b^8*d^4) + (4*(a + b*Sqrt[c + d*x])^(15/2))/(1
5*b^8*d^4)
```

Rubi [A] time = 0.532948, antiderivative size = 324, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$\frac{12\left(7a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{11/2}}{11b^8d^4}-\frac{20a\left(7a^2-3b^2c\right)\left(a+b\sqrt{c+dx}\right)^{9/2}}{9b^8d^4}-\frac{12a\left(7a^2-3b^2c\right)\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^8d^4}+\frac{4\left(a^2-b^2c\right)^2\left(7a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^8d^4}-\frac{4a\left(a^2-b^2c\right)^3\sqrt{a+b\sqrt{c+dx}}}{b^8d^4}+\frac{4\left(35a^4-30a^2b^2c+3b^4c^2\right)\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^8d^4}+\frac{4\left(a+b\sqrt{c+dx}\right)^{15/2}}{15b^8d^4}-\frac{28a\left(a+b\sqrt{c+dx}\right)^{13/2}}{13b^8d^4}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-4*a*(a^2 - b^2*c)^3*Sqrt[a + b*Sqrt[c + d*x]])/(b^8*d^4) + (4*(
a^2 - b^2*c)^2*(7*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^
8*d^4) - (12*a*(7*a^2 - 3*b^2*c)*(a^2 - b^2*c)*(a + b*Sqrt[c + d*
x])^(5/2))/(5*b^8*d^4) + (4*(35*a^4 - 30*a^2*b^2*c + 3*b^4*c^2)*(
a + b*Sqrt[c + d*x])^(7/2))/(7*b^8*d^4) - (20*a*(7*a^2 - 3*b^2*c)
*(a + b*Sqrt[c + d*x])^(9/2))/(9*b^8*d^4) + (12*(7*a^2 - b^2*c)*(
a + b*Sqrt[c + d*x])^(11/2))/(11*b^8*d^4) - (28*a*(a + b*Sqrt[c +
 d*x])^(13/2))/(13*b^8*d^4) + (4*(a + b*Sqrt[c + d*x])^(15/2))/(1
5*b^8*d^4)
```

Rubi in Sympy [A] time = 32.0165, size = 304, normalized size = 0.94

$$-\frac{28a\left(a+b\sqrt{c+dx}\right)^{\frac{13}{2}}}{13b^8d^4}-\frac{20a\left(a+b\sqrt{c+dx}\right)^{\frac{9}{2}}\left(7a^2-3b^2c\right)}{9b^8d^4}-\frac{12a\left(a+b\sqrt{c+dx}\right)^{\frac{5}{2}}\left(a^2-b^2c\right)\left(7a^2-3b^2c\right)}{5b^8d^4}-\frac{4a\sqrt{a+b\sqrt{c+dx}}\left(a^2-b^2c\right)^3}{b^8d^4}+\frac{4\left(a+b\sqrt{c+dx}\right)^{\frac{15}{2}}}{15b^8d^4}+\frac{12\left(a+b\sqrt{c+dx}\right)^{\frac{11}{2}}\left(7a^2-b^2c\right)}{11b^8d^4}+\frac{4\left(a+b\sqrt{c+dx}\right)^{\frac{7}{2}}\left(35a^4-30a^2b^2c+3b^4c^2\right)}{7b^8d^4}+\frac{4\left(a+b\sqrt{c+dx}\right)^{\frac{3}{2}}\left(a^2-b^2c\right)^2\left(7a^2-b^2c\right)}{3b^8d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] -28*a*(a + b*sqrt(c + d*x))**(13/2)/(13*b**8*d**4) - 20*a*(a + b*
sqrt(c + d*x))**(9/2)*(7*a**2 - 3*b**2*c)/(9*b**8*d**4) - 12*a*(a
 + b*sqrt(c + d*x))**(5/2)*(a**2 - b**2*c)*(7*a**2 - 3*b**2*c)/(5
*b**8*d**4) - 4*a*sqrt(a + b*sqrt(c + d*x))*(a**2 - b**2*c)**3/(b
**8*d**4) + 4*(a + b*sqrt(c + d*x))**(15/2)/(15*b**8*d**4) + 12*(
a + b*sqrt(c + d*x))**(11/2)*(7*a**2 - b**2*c)/(11*b**8*d**4) + 4
*(a + b*sqrt(c + d*x))**(7/2)*(35*a**4 - 30*a**2*b**2*c + 3*b**4*
c**2)/(7*b**8*d**4) + 4*(a + b*sqrt(c + d*x))**(3/2)*(a**2 - b**2
*c)**2*(7*a**2 - b**2*c)/(3*b**8*d**4)
```

Mathematica [A] time = 0.486199, size = 285, normalized size = 0.88

$$\frac{4\left(-\frac{5}{9}\left(7a^3-3ab^2c\right)\left(a+b\sqrt{c+dx}\right)^{9/2}+\frac{3}{11}\left(7a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{11/2}+\frac{1}{3}\left(a^2-b^2c\right)^2\left(7a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{3/2}\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (4*(-(a*(a^2 - b^2*c)^3*Sqrt[a + b*Sqrt[c + d*x]]) + ((a^2 - b^2*
c)^2*(7*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(3/2))/3 - (3*(7*a^5 -
 10*a^3*b^2*c + 3*a*b^4*c^2)*(a + b*Sqrt[c + d*x])^(5/2))/5 + ((3
5*a^4 - 30*a^2*b^2*c + 3*b^4*c^2)*(a + b*Sqrt[c + d*x])^(7/2))/7
- (5*(7*a^3 - 3*a*b^2*c)*(a + b*Sqrt[c + d*x])^(9/2))/9 + (3*(7*a
^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(11/2))/11 - (7*a*(a + b*Sqrt[c
 + d*x])^(13/2))/13 + (a + b*Sqrt[c + d*x])^(15/2)/15))/(b^8*d^4)
```

Maple [A] time = 0.004, size = 383, normalized size = 1.2

$$4\,{\frac {1}{{d}^{4}{b}^{8}}\left(1/15\, \left(a+b\sqrt {dx+c} \right) ^{15/2}-{\frac {7\,a}{13} \left(a+b\sqrt {dx+c} \right) ^{13/2}}+1/11\, \left(-3\,{b}^{2}c+21\,{a}^{2} \right) \left(a+b\sqrt {dx+c} \right) ^{11/2}+1/9\, \left(-8\, \left(-{b}^{2}c+ \right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(a+b*(d*x+c)^(1/2))^(1/2),x)
```

```
[Out] 4/d^4/b^8*(1/15*(a+b*(d*x+c)^(1/2))^(15/2)-7/13*a*(a+b*(d*x+c)^(1
/2))^(13/2)+1/11*(-3*b^2*c+21*a^2)*(a+b*(d*x+c)^(1/2))^(11/2)+1/9
*(-8*(-b^2*c+a^2)*a-2*a*(-2*b^2*c+6*a^2)-(-3*b^2*c+15*a^2)*a)*(a+
b*(d*x+c)^(1/2))^(9/2)+1/7*((-b^2*c+a^2)*(-2*b^2*c+6*a^2)+8*a^2*(
-b^2*c+a^2)+(-b^2*c+a^2)^2-(-8*(-b^2*c+a^2)*a-2*a*(-2*b^2*c+6*a^2
))*a)*(a+b*(d*x+c)^(1/2))^(7/2)+1/5*(-6*(-b^2*c+a^2)^2*a-((-b^2*c
+a^2)*(-2*b^2*c+6*a^2)+8*a^2*(-b^2*c+a^2)+(-b^2*c+a^2)^2)*a)*(a+b
*(d*x+c)^(1/2))^(5/2)+1/3*((-b^2*c+a^2)^3+6*(-b^2*c+a^2)^2*a^2)*(
a+b*(d*x+c)^(1/2))^(3/2)-(-b^2*c+a^2)^3*a*(a+b*(d*x+c)^(1/2))^(1/
2))
```

Maple [A] time = 0.69431, size = 362, normalized size = 1.12

$$\frac{4\left(3003\left(\sqrt{dx+c}b+a\right)^{\frac{15}{2}}-24255\left(\sqrt{dx+c}b+a\right)^{\frac{13}{2}}a-12285\left(b^2c-7a^2\right)\left(\sqrt{dx+c}b+a\right)^{\frac{11}{2}}+25025\left(3ab^2c-7a^3\right)\left(\sqrt{dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/sqrt(sqrt(d*x + c)*b + a),x, algorithm="maxima")
```

```
[Out] 4/45045*(3003*(sqrt(d*x + c)*b + a)^(15/2) - 24255*(sqrt(d*x + c)
*b + a)^(13/2)*a - 12285*(b^2*c - 7*a^2)*(sqrt(d*x + c)*b + a)^(1
```

```
1/2) + 25025*(3*a*b^2*c - 7*a^3)*(sqrt(d*x + c)*b + a)^(9/2) + 64
35*(3*b^4*c^2 - 30*a^2*b^2*c + 35*a^4)*(sqrt(d*x + c)*b + a)^(7/2
) - 27027*(3*a*b^4*c^2 - 10*a^3*b^2*c + 7*a^5)*(sqrt(d*x + c)*b +
 a)^(5/2) - 15015*(b^6*c^3 - 9*a^2*b^4*c^2 + 15*a^4*b^2*c - 7*a^6
)*(sqrt(d*x + c)*b + a)^(3/2) + 45045*(a*b^6*c^3 - 3*a^3*b^4*c^2
+ 3*a^5*b^2*c - a^7)*sqrt(sqrt(d*x + c)*b + a))/(b^8*d^4)
```

Fricas [A] time = 0.33755, size = 312, normalized size = 0.96

$$-\frac{4\left(3234\,ab^6d^3x^3-17280\,ab^6c^3+46976\,a^3b^4c^2-44544\,a^5b^2c+14336\,a^7-28\left(141\,ab^6c-140\,a^3b^4\right)d^2x^2+64\left(87\,ab^6c^2-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/sqrt(sqrt(d*x + c)*b + a),x, algorithm="fricas")
```

```
[Out] -4/45045*(3234*a*b^6*d^3*x^3 - 17280*a*b^6*c^3 + 46976*a^3*b^4*c^
2 - 44544*a^5*b^2*c + 14336*a^7 - 28*(141*a*b^6*c - 140*a^3*b^4)*
d^2*x^2 + 64*(87*a*b^6*c^2 - 170*a^3*b^4*c + 84*a^5*b^2)*d*x - (3
003*b^7*d^3*x^3 - 4992*b^7*c^3 + 18816*a^2*b^5*c^2 - 20480*a^4*b^
3*c + 7168*a^6*b - 252*(13*b^7*c - 14*a^2*b^5)*d^2*x^2 + 32*(117*
b^7*c^2 - 267*a^2*b^5*c + 140*a^4*b^3)*d*x)*sqrt(d*x + c))*sqrt(s
qrt(d*x + c)*b + a)/(b^8*d^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{a+b\sqrt{c+dx}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(a + b*sqrt(c + d*x)), x)
```

GIAC/XCAS [A] time = 0.338504, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/sqrt(sqrt(d*x + c)*b + a),x, algorithm="giac")
```

```
[Out] Done
```

3.486 $\int \frac{x^2}{\sqrt{a+b\sqrt{c+dx}}}\,dx$

Optimal. Leaf size=222

$$\frac{8(5a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^6d^3}-\frac{8a(5a^2-3b^2c)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^6d^3}-\frac{4a(a^2-b^2c)^2\sqrt{a+b\sqrt{c+dx}}}{b^6d^3}+\frac{4(5a^4-6a^2b^2c+b^4c^2)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^6d^3}+\frac{4\left(a+b\sqrt{c+dx}\right)^{11/2}}{11b^6d^3}-\frac{20a\left(a+b\sqrt{c+dx}\right)^{9/2}}{9b^6d^3}$$

```
[Out] (-4*a*(a^2 - b^2*c)^2*Sqrt[a + b*Sqrt[c + d*x]])/(b^6*d^3) + (4*(
5*a^4 - 6*a^2*b^2*c + b^4*c^2)*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^
6*d^3) - (8*a*(5*a^2 - 3*b^2*c)*(a + b*Sqrt[c + d*x])^(5/2))/(5*b
^6*d^3) + (8*(5*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(7/2))/(7*b^6*
d^3) - (20*a*(a + b*Sqrt[c + d*x])^(9/2))/(9*b^6*d^3) + (4*(a + b
*Sqrt[c + d*x])^(11/2))/(11*b^6*d^3)
```

Rubi [A] time = 0.3718, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{8(5a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^6d^3}-\frac{8a(5a^2-3b^2c)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^6d^3}-\frac{4a(a^2-b^2c)^2\sqrt{a+b\sqrt{c+dx}}}{b^6d^3}+\frac{4(5a^4-6a^2b^2c+b^4c^2)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^6d^3}+\frac{4\left(a+b\sqrt{c+dx}\right)^{11/2}}{11b^6d^3}-\frac{20a\left(a+b\sqrt{c+dx}\right)^{9/2}}{9b^6d^3}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/Sqrt[a + b*Sqrt[c + d*x]],x]
```


```
[Out] (-4*a*(a^2 - b^2*c)^2*Sqrt[a + b*Sqrt[c + d*x]])/(b^6*d^3) + (4*(
5*a^4 - 6*a^2*b^2*c + b^4*c^2)*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^
6*d^3) - (8*a*(5*a^2 - 3*b^2*c)*(a + b*Sqrt[c + d*x])^(5/2))/(5*b
^6*d^3) + (8*(5*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(7/2))/(7*b^6*
d^3) - (20*a*(a + b*Sqrt[c + d*x])^(9/2))/(9*b^6*d^3) + (4*(a + b
*Sqrt[c + d*x])^(11/2))/(11*b^6*d^3)
```

Rubi in Sympy [A] time = 22.3999, size = 209, normalized size = 0.94

$$-\frac{20a\left(a+b\sqrt{c+dx}\right)^{\frac{9}{2}}}{9b^6d^3}-\frac{8a\left(a+b\sqrt{c+dx}\right)^{\frac{5}{2}}\left(5a^2-3b^2c\right)}{5b^6d^3}-\frac{4a\sqrt{a+b\sqrt{c+dx}}\left(a^2-b^2c\right)^2}{b^6d^3}+\frac{4\left(a+b\sqrt{c+dx}\right)^{\frac{11}{2}}}{11b^6d^3}+\frac{8\left(a+b\sqrt{c+dx}\right)^{\frac{7}{2}}\left(5a^2-b^2c\right)}{7b^6d^3}+\frac{4\left(a+b\sqrt{c+dx}\right)^{\frac{3}{2}}\left(5a^4-6a^2b^2c+b^4c^2\right)}{3b^6d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] -20*a*(a + b*sqrt(c + d*x))**(9/2)/(9*b**6*d**3) - 8*a*(a + b*sqr
t(c + d*x))**(5/2)*(5*a**2 - 3*b**2*c)/(5*b**6*d**3) - 4*a*sqrt(a
 + b*sqrt(c + d*x))*(a**2 - b**2*c)**2/(b**6*d**3) + 4*(a + b*sqr
t(c + d*x))**(11/2)/(11*b**6*d**3) + 8*(a + b*sqrt(c + d*x))**(7/
2)*(5*a**2 - b**2*c)/(7*b**6*d**3) + 4*(a + b*sqrt(c + d*x))**(3/
2)*(5*a**4 - 6*a**2*b**2*c + b**4*c**2)/(3*b**6*d**3)
```

Mathematica [A] time = 0.347656, size = 147, normalized size = 0.66

$$\frac{4\sqrt{a+b\sqrt{c+dx}}\left(-1280a^5+640a^4b\sqrt{c+dx}+96a^3b^2(28c-5dx)-16a^2b^3(74c-25dx)\sqrt{c+dx}-2ab^4\left(736c^2-244cdx+\right.\right.}{3465b^6d^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (4*Sqrt[a + b*Sqrt[c + d*x]]*(-1280*a^5 + 96*a^3*b^2*(28*c - 5*d*
x) + 640*a^4*b*Sqrt[c + d*x] - 16*a^2*b^3*(74*c - 25*d*x)*Sqrt[c
+ d*x] + 15*b^5*Sqrt[c + d*x]*(32*c^2 - 24*c*d*x + 21*d^2*x^2) -
2*a*b^4*(736*c^2 - 244*c*d*x + 175*d^2*x^2)))/(3465*b^6*d^3)
```

Maple [A] time = 0.003, size = 183, normalized size = 0.8

$$4\,\frac{1/11\,\left(a+b\sqrt{dx+c}\right)^{11/2}-5/9\,a\left(a+b\sqrt{dx+c}\right)^{9/2}+1/7\,\left(-2\,b^2c+10\,a^2\right)\left(a+b\sqrt{dx+c}\right)^{7/2}+1/5\,\left(-4\,\left(-b^2c+a^2\right)a-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(a+b*(d*x+c)^(1/2))^(1/2),x)
```

```
[Out] 4/d^3/b^6*(1/11*(a+b*(d*x+c)^(1/2))^(11/2)-5/9*a*(a+b*(d*x+c)^(1/
2))^(9/2)+1/7*(-2*b^2*c+10*a^2)*(a+b*(d*x+c)^(1/2))^(7/2)+1/5*(-4
*(-b^2*c+a^2)*a-a*(-2*b^2*c+6*a^2))*(a+b*(d*x+c)^(1/2))^(5/2)+1/3
*((-b^2*c+a^2)^2+4*a^2*(-b^2*c+a^2))*(a+b*(d*x+c)^(1/2))^(3/2)-(-
b^2*c+a^2)^2*a*(a+b*(d*x+c)^(1/2))^(1/2))
```

Maxima [A] time = 0.703136, size = 225, normalized size = 1.01

$$\frac{4\left(315\left(\sqrt{dx+c}b+a\right)^{\frac{11}{2}}-1925\left(\sqrt{dx+c}b+a\right)^{\frac{9}{2}}a-990\left(b^{2}c-5a^{2}\right)\left(\sqrt{dx+c}b+a\right)^{\frac{7}{2}}+1386\left(3ab^{2}c-5a^{3}\right)\left(\sqrt{dx+c}b+\cdots\right.\right.}{3465\,b^{6}d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt(sqrt(d*x + c)*b + a),x, algorithm="maxima")
```

```
[Out] 4/3465*(315*(sqrt(d*x + c)*b + a)^(11/2) - 1925*(sqrt(d*x + c)*b
+ a)^(9/2)*a - 990*(b^2*c - 5*a^2)*(sqrt(d*x + c)*b + a)^(7/2) +
1386*(3*a*b^2*c - 5*a^3)*(sqrt(d*x + c)*b + a)^(5/2) + 1155*(b^4*
c^2 - 6*a^2*b^2*c + 5*a^4)*(sqrt(d*x + c)*b + a)^(3/2) - 3465*(a*
b^4*c^2 - 2*a^3*b^2*c + a^5)*sqrt(sqrt(d*x + c)*b + a))/(b^6*d^3)
```

Fricas [A] time = 0.335953, size = 189, normalized size = 0.85

$$-\frac{4\left(350\,ab^{4}d^{2}x^{2}+1472\,ab^{4}c^{2}-2688\,a^{3}b^{2}c+1280\,a^{5}-8\left(61\,ab^{4}c-60\,a^{3}b^{2}\right)dx-\left(315\,b^{5}d^{2}x^{2}+480\,b^{5}c^{2}-1184\,a^{2}b^{3}c+6\cdots\right.\right.}{3465\,b^{6}d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt(sqrt(d*x + c)*b + a),x, algorithm="fricas")
```

```
[Out] -4/3465*(350*a*b^4*d^2*x^2 + 1472*a*b^4*c^2 - 2688*a^3*b^2*c + 12
80*a^5 - 8*(61*a*b^4*c - 60*a^3*b^2)*d*x - (315*b^5*d^2*x^2 + 480
*b^5*c^2 - 1184*a^2*b^3*c + 640*a^4*b - 40*(9*b^5*c - 10*a^2*b^3)
*d*x)*sqrt(d*x + c))*sqrt(sqrt(d*x + c)*b + a)/(b^6*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2}}{\sqrt{a+b\sqrt{c+dx}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(a + b*sqrt(c + d*x)), x)
```

GIAC/XCAS [A] time = 0.31854, size = 849, normalized size = 3.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt(sqrt(d*x + c)*b + a),x, algorithm="giac")
```

```
[Out] 4/3465*(1155*sqrt((sqrt(d*x + c)*b + a)*b^2)*(sqrt(d*x + c)*b + a
)*b^4*c^2*sign((sqrt(d*x + c)*b + a)*b - a*b) - 3465*sqrt((sqrt(d
*x + c)*b + a)*b^2)*a*b^4*c^2*sign((sqrt(d*x + c)*b + a)*b - a*b)
 - 990*sqrt((sqrt(d*x + c)*b + a)*b^2)*(sqrt(d*x + c)*b + a)^3*b^
2*c*sign((sqrt(d*x + c)*b + a)*b - a*b) + 4158*sqrt((sqrt(d*x + c
)*b + a)*b^2)*(sqrt(d*x + c)*b + a)^2*a*b^2*c*sign((sqrt(d*x + c)
*b + a)*b - a*b) - 6930*sqrt((sqrt(d*x + c)*b + a)*b^2)*(sqrt(d*x
 + c)*b + a)*a^2*b^2*c*sign((sqrt(d*x + c)*b + a)*b - a*b) + 6930
*sqrt((sqrt(d*x + c)*b + a)*b^2)*a^3*b^2*c*sign((sqrt(d*x + c)*b
+ a)*b - a*b) + 315*sqrt((sqrt(d*x + c)*b + a)*b^2)*(sqrt(d*x + c
)*b + a)^5*sign((sqrt(d*x + c)*b + a)*b - a*b) - 1925*sqrt((sqrt(
d*x + c)*b + a)*b^2)*(sqrt(d*x + c)*b + a)^4*a*sign((sqrt(d*x + c
)*b + a)*b - a*b) + 4950*sqrt((sqrt(d*x + c)*b + a)*b^2)*(sqrt(d*
x + c)*b + a)^3*a^2*sign((sqrt(d*x + c)*b + a)*b - a*b) - 6930*sq
rt((sqrt(d*x + c)*b + a)*b^2)*(sqrt(d*x + c)*b + a)^2*a^3*sign((s
qrt(d*x + c)*b + a)*b - a*b) + 5775*sqrt((sqrt(d*x + c)*b + a)*b^
2)*(sqrt(d*x + c)*b + a)*a^4*sign((sqrt(d*x + c)*b + a)*b - a*b)
- 3465*sqrt((sqrt(d*x + c)*b + a)*b^2)*a^5*sign((sqrt(d*x + c)*b
+ a)*b - a*b))/(b^6*d^3*abs(b))
```

3.487 $\int \frac{x}{\sqrt{a+b\sqrt{c+dx}}}\,dx$

Optimal. Leaf size=131

$$\frac{4(3a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^4d^2}-\frac{4a(a^2-b^2c)\sqrt{a+b\sqrt{c+dx}}}{b^4d^2}+\frac{4\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^4d^2}-\frac{12a\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^4d^2}$$

```
[Out] (-4*a*(a^2 - b^2*c)*Sqrt[a + b*Sqrt[c + d*x]])/(b^4*d^2) + (4*(3*
a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^4*d^2) - (12*a*(a
+ b*Sqrt[c + d*x])^(5/2))/(5*b^4*d^2) + (4*(a + b*Sqrt[c + d*x])^
(7/2))/(7*b^4*d^2)
```

Rubi [A] time = 0.227495, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$\frac{4(3a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^4d^2}-\frac{4a(a^2-b^2c)\sqrt{a+b\sqrt{c+dx}}}{b^4d^2}+\frac{4\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^4d^2}-\frac{12a\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^4d^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x/Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-4*a*(a^2 - b^2*c)*Sqrt[a + b*Sqrt[c + d*x]])/(b^4*d^2) + (4*(3*
a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^4*d^2) - (12*a*(a
+ b*Sqrt[c + d*x])^(5/2))/(5*b^4*d^2) + (4*(a + b*Sqrt[c + d*x])^
(7/2))/(7*b^4*d^2)
```

Rubi in Sympy [A] time = 12.0638, size = 121, normalized size = 0.92

$$-\frac{12a\left(a+b\sqrt{c+dx}\right)^{\frac{5}{2}}}{5b^4d^2}-\frac{4a\sqrt{a+b\sqrt{c+dx}}\,(a^2-b^2c)}{b^4d^2}+\frac{4\left(a+b\sqrt{c+dx}\right)^{\frac{7}{2}}}{7b^4d^2}+\frac{4\left(a+b\sqrt{c+dx}\right)^{\frac{3}{2}}(3a^2-b^2c)}{3b^4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] -12*a*(a + b*sqrt(c + d*x))**(5/2)/(5*b**4*d**2) - 4*a*sqrt(a + b
*sqrt(c + d*x))*(a**2 - b**2*c)/(b**4*d**2) + 4*(a + b*sqrt(c + d
*x))**(7/2)/(7*b**4*d**2) + 4*(a + b*sqrt(c + d*x))**(3/2)*(3*a**
2 - b**2*c)/(3*b**4*d**2)
```

Mathematica [A] time = 0.0896397, size = 84, normalized size = 0.64

$$\frac{4\sqrt{a+b\sqrt{c+dx}}\left(-48a^3+24a^2b\sqrt{c+dx}+2ab^2(26c-9dx)+5b^3\sqrt{c+dx}(3dx-4c)\right)}{105b^4d^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (4*Sqrt[a + b*Sqrt[c + d*x]]*(-48*a^3 + 2*a*b^2*(26*c - 9*d*x) +
24*a^2*b*Sqrt[c + d*x] + 5*b^3*Sqrt[c + d*x]*(-4*c + 3*d*x)))/(10
5*b^4*d^2)
```

Maple [A] time = 0.003, size = 94, normalized size = 0.7

$$4\,\frac{1/7\,\left(a+b\sqrt{dx+c}\right)^{7/2}-3/5\,\left(a+b\sqrt{dx+c}\right)^{5/2}a+1/3\,\left(-b^2c+3\,a^2\right)\left(a+b\sqrt{dx+c}\right)^{3/2}-\left(-b^2c+a^2\right)a\sqrt{a+b\sqrt{dx+c}}}{b^4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(a+b*(d*x+c)^(1/2))^(1/2),x)
```

```
[Out] 4/d^2/b^4*(1/7*(a+b*(d*x+c)^(1/2))^(7/2)-3/5*(a+b*(d*x+c)^(1/2))^
(5/2)*a+1/3*(-b^2*c+3*a^2)*(a+b*(d*x+c)^(1/2))^(3/2)-(-b^2*c+a^2)
*a*(a+b*(d*x+c)^(1/2))^(1/2))
```

Maxima [A] time = 0.699476, size = 126, normalized size = 0.96

$$\frac{4\left(15\left(\sqrt{dx+c}b+a\right)^{\frac{7}{2}}-63\left(\sqrt{dx+c}b+a\right)^{\frac{5}{2}}a-35\left(b^2c-3\,a^2\right)\left(\sqrt{dx+c}b+a\right)^{\frac{3}{2}}+105\left(ab^2c-a^3\right)\sqrt{\sqrt{dx+c}b+a}\right)}{105\,b^4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(sqrt(d*x + c)*b + a),x, algorithm="maxima")
```

```
[Out] 4/105*(15*(sqrt(d*x + c)*b + a)^(7/2) - 63*(sqrt(d*x + c)*b + a)^
(5/2)*a - 35*(b^2*c - 3*a^2)*(sqrt(d*x + c)*b + a)^(3/2) + 105*(a
*b^2*c - a^3)*sqrt(sqrt(d*x + c)*b + a))/(b^4*d^2)
```

Fricas [A] time = 0.330886, size = 96, normalized size = 0.73

$$-\frac{4\left(18\,ab^2dx - 52\,ab^2c + 48\,a^3 - \left(15\,b^3dx - 20\,b^3c + 24\,a^2b\right)\sqrt{dx+c}\right)\sqrt{\sqrt{dx+c}b + a}}{105\,b^4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(sqrt(d*x + c)*b + a),x, algorithm="fricas")
```

```
[Out] -4/105*(18*a*b^2*d*x - 52*a*b^2*c + 48*a^3 - (15*b^3*d*x - 20*b^3
*c + 24*a^2*b)*sqrt(d*x + c))*sqrt(sqrt(d*x + c)*b + a)/(b^4*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a + b\sqrt{c + dx}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] Integral(x/sqrt(a + b*sqrt(c + d*x)), x)
```

GIAC/XCAS [A] time = 0.292773, size = 412, normalized size = 3.15

$$-\frac{4\left(35\sqrt{\left(\sqrt{dx+c}b + a\right)b^2}\left(\sqrt{dx+c}b + a\right)b^2\mathrm{csign}\left(\left(\sqrt{dx+c}b + a\right)b - ab\right) - 105\sqrt{\left(\sqrt{dx+c}b + a\right)b^2}ab^2\mathrm{csign}\left(\left(\sqrt{dx+c}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(sqrt(d*x + c)*b + a),x, algorithm="giac")
```

```
[Out] -4/105*(35*sqrt((sqrt(d*x + c)*b + a)*b^2)*(sqrt(d*x + c)*b + a)*
b^2*c*sign((sqrt(d*x + c)*b + a)*b - a*b) - 105*sqrt((sqrt(d*x +
c)*b + a)*b^2)*a*b^2*c*sign((sqrt(d*x + c)*b + a)*b - a*b) - 15*s
qrt((sqrt(d*x + c)*b + a)*b^2)*(sqrt(d*x + c)*b + a)^3*sign((sqrt
(d*x + c)*b + a)*b - a*b) + 63*sqrt((sqrt(d*x + c)*b + a)*b^2)*(s
qrt(d*x + c)*b + a)^2*a*sign((sqrt(d*x + c)*b + a)*b - a*b) - 105
*sqrt((sqrt(d*x + c)*b + a)*b^2)*(sqrt(d*x + c)*b + a)*a^2*sign((
sqrt(d*x + c)*b + a)*b - a*b) + 105*sqrt((sqrt(d*x + c)*b + a)*b^
2)*a^3*sign((sqrt(d*x + c)*b + a)*b - a*b))/(b^4*d^2*abs(b))
```

3.488 $\int \frac{1}{\sqrt{a+b\sqrt{c+dx}}}\,dx$

Optimal. Leaf size=54

$$\frac{4\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^2d}-\frac{4a\sqrt{a+b\sqrt{c+dx}}}{b^2d}$$

```
[Out] (-4*a*Sqrt[a + b*Sqrt[c + d*x]])/(b^2*d) + (4*(a + b*Sqrt[c + d*x
])^(3/2))/(3*b^2*d)
```

Rubi [A] time = 0.0688968, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$\frac{4\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^2d}-\frac{4a\sqrt{a+b\sqrt{c+dx}}}{b^2d}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-4*a*Sqrt[a + b*Sqrt[c + d*x]])/(b^2*d) + (4*(a + b*Sqrt[c + d*x
])^(3/2))/(3*b^2*d)
```

Rubi in Sympy [A] time = 4.02939, size = 46, normalized size = 0.85

$$-\frac{4a\sqrt{a+b\sqrt{c+dx}}}{b^2d}+\frac{4\left(a+b\sqrt{c+dx}\right)^{\frac{3}{2}}}{3b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] -4*a*sqrt(a + b*sqrt(c + d*x))/(b**2*d) + 4*(a + b*sqrt(c + d*x))
**(3/2)/(3*b**2*d)
```

Mathematica [A] time = 0.0247305, size = 42, normalized size = 0.78

$$\frac{4\left(b\sqrt{c+dx}-2a\right)\sqrt{a+b\sqrt{c+dx}}}{3b^2d}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (4*(-2*a + b*Sqrt[c + d*x])*Sqrt[a + b*Sqrt[c + d*x]])/(3*b^2*d)
```

Maple [A] time = 0.008, size = 41, normalized size = 0.8

$$4\,{\frac{1/3\,\left(a+b\sqrt{dx+c}\right)^{3/2}-\sqrt{a+b\sqrt{dx+c}}a}{b^{2}d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a+b*(d*x+c)^(1/2))^(1/2),x)
```

```
[Out] 4/d/b^2*(1/3*(a+b*(d*x+c)^(1/2))^(3/2)-(a+b*(d*x+c)^(1/2))^(1/2)*
a)
```

Maxima [A] time = 0.691078, size = 57, normalized size = 1.06

$$\frac{4\left(\frac{\left(\sqrt{dx+c}b+a\right)^{\frac{3}{2}}}{b^{2}}-\frac{3\sqrt{\sqrt{dx+c}b+a}a}{b^{2}}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(sqrt(d*x + c)*b + a),x, algorithm="maxima")
```

```
[Out] 4/3*((sqrt(d*x + c)*b + a)^(3/2)/b^2 - 3*sqrt(sqrt(d*x + c)*b + a
)*a/b^2)/d
```

Fricas [A] time = 0.333463, size = 46, normalized size = 0.85

$$\frac{4\,\sqrt{\sqrt{dx+c}b+a}\left(\sqrt{dx+c}b-2\,a\right)}{3\,b^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(sqrt(d*x + c)*b + a),x, algorithm="fricas")
```

```
[Out] 4/3*sqrt(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b - 2*a)/(b^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+b\sqrt{c+dx}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*sqrt(c + d*x)), x)
```

GIAC/XCAS [A] time = 0.276814, size = 135, normalized size = 2.5

$$\frac{4\left(\sqrt{\left(\sqrt{dx+c}b+a\right)b^2}\left(\sqrt{dx+c}b+a\right)\mathrm{sign}\left(\left(\sqrt{dx+c}b+a\right)b-ab\right)-3\sqrt{\left(\sqrt{dx+c}b+a\right)b^2}a\mathrm{sign}\left(\left(\sqrt{dx+c}b+a\right)b-ab\right.}{3\,b^2d|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(sqrt(d*x + c)*b + a),x, algorithm="giac")
```

```
[Out] 4/3*(sqrt((sqrt(d*x + c)*b + a)*b^2)*(sqrt(d*x + c)*b + a)*sign((
sqrt(d*x + c)*b + a)*b - a*b) - 3*sqrt((sqrt(d*x + c)*b + a)*b^2)
*a*sign((sqrt(d*x + c)*b + a)*b - a*b))/(b^2*d*abs(b))
```

3.489 $\int \frac{1}{x\sqrt{a+b\sqrt{c+dx}}}\,dx$

Optimal. Leaf size=97

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{\sqrt{a-b\sqrt{c}}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{\sqrt{a+b\sqrt{c}}}$$

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a - b*Sqrt[c]]])/Sqrt[
a - b*Sqrt[c]] - (2*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a + b*
Sqrt[c]]])/Sqrt[a + b*Sqrt[c]]
```

Rubi [A] time = 0.217547, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{\sqrt{a-b\sqrt{c}}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{\sqrt{a+b\sqrt{c}}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*Sqrt[a + b*Sqrt[c + d*x]]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a - b*Sqrt[c]]])/Sqrt[
a - b*Sqrt[c]] - (2*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a + b*
Sqrt[c]]])/Sqrt[a + b*Sqrt[c]]
```

Rubi in Sympy [A] time = 16.371, size = 85, normalized size = 0.88

$$-\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{\sqrt{a+b\sqrt{c}}}-\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{\sqrt{a-b\sqrt{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] -2*atanh(sqrt(a + b*sqrt(c + d*x))/sqrt(a + b*sqrt(c)))/sqrt(a +
b*sqrt(c)) - 2*atanh(sqrt(a + b*sqrt(c + d*x))/sqrt(a - b*sqrt(c)
```

```
))/sqrt(a - b*sqrt(c))
```

Mathematica [A] time = 0.0959898, size = 97, normalized size = 1.

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{\sqrt{a-b\sqrt{c}}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{\sqrt{a+b\sqrt{c}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*Sqrt[a + b*Sqrt[c + d*x]]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a - b*Sqrt[c]]])/Sqrt[
a - b*Sqrt[c]] - (2*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a + b*
Sqrt[c]]])/Sqrt[a + b*Sqrt[c]]
```

Maple [A] time = 0.02, size = 92, normalized size = 1.

$$2\frac{1}{\sqrt{\sqrt{b^2c}-a}}\arctan\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{\sqrt{b^2c}-a}}\right)+2\frac{1}{\sqrt{-\sqrt{b^2c}-a}}\arctan\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{-\sqrt{b^2c}-a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(a+b*(d*x+c)^(1/2))^(1/2),x)
```

```
[Out] 2/((b^2*c)^(1/2)-a)^(1/2)*arctan((a+b*(d*x+c)^(1/2))^(1/2)/((b^2*
c)^(1/2)-a)^(1/2))+2/(-(b^2*c)^(1/2)-a)^(1/2)*arctan((a+b*(d*x+c)
^(1/2))^(1/2)/(-(b^2*c)^(1/2)-a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{\sqrt{dx+c}b+a}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(sqrt(d*x + c)*b + a)*x),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(sqrt(d*x + c)*b + a)*x), x)
```

Fricas [A] time = 0.348471, size = 1003, normalized size = 10.34

$$\sqrt{-\frac{(b^2c-a^2)\sqrt{\frac{b^2c}{b^4c^2-2\,a^2b^2c+a^4}}+a}{b^2c-a^2}}\log\left(4\left((b^2c-a^2)\sqrt{\frac{b^2c}{b^4c^2-2\,a^2b^2c+a^4}}-a\right)\sqrt{-\frac{(b^2c-a^2)\sqrt{\frac{b^2c}{b^4c^2-2\,a^2b^2c+a^4}}+a}{b^2c-a^2}}+4\sqrt{\sqrt{dx+c}b+a}\right)$$
$$-\sqrt{-\frac{(b^2c-a^2)\sqrt{\frac{b^2c}{b^4c^2-2\,a^2b^2c+a^4}}+a}{b^2c-a^2}}\log\left(-4\left((b^2c-a^2)\sqrt{\frac{b^2c}{b^4c^2-2\,a^2b^2c+a^4}}-a\right)\sqrt{-\frac{(b^2c-a^2)\sqrt{\frac{b^2c}{b^4c^2-2\,a^2b^2c+a^4}}+a}{b^2c-a^2}}+4\sqrt{\sqrt{dx+c}b+a}\right)$$
$$-\sqrt{\frac{(b^2c-a^2)\sqrt{\frac{b^2c}{b^4c^2-2\,a^2b^2c+a^4}}-a}{b^2c-a^2}}\log\left(4\left((b^2c-a^2)\sqrt{\frac{b^2c}{b^4c^2-2\,a^2b^2c+a^4}}+a\right)\sqrt{\frac{(b^2c-a^2)\sqrt{\frac{b^2c}{b^4c^2-2\,a^2b^2c+a^4}}-a}{b^2c-a^2}}+4\sqrt{\sqrt{dx+c}b+a}\right)$$
$$+\sqrt{\frac{(b^2c-a^2)\sqrt{\frac{b^2c}{b^4c^2-2\,a^2b^2c+a^4}}-a}{b^2c-a^2}}\log\left(-4\left((b^2c-a^2)\sqrt{\frac{b^2c}{b^4c^2-2\,a^2b^2c+a^4}}+a\right)\sqrt{\frac{(b^2c-a^2)\sqrt{\frac{b^2c}{b^4c^2-2\,a^2b^2c+a^4}}-a}{b^2c-a^2}}+4\sqrt{\sqrt{dx+c}b+a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(sqrt(d*x + c)*b + a)*x),x, algorithm="fricas")
```

```
[Out] sqrt(-((b^2*c - a^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) +
a)/(b^2*c - a^2))*log(4*((b^2*c - a^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^
2*b^2*c + a^4)) - a)*sqrt(-((b^2*c - a^2)*sqrt(b^2*c/(b^4*c^2 - 2
*a^2*b^2*c + a^4)) + a)/(b^2*c - a^2)) + 4*sqrt(sqrt(d*x + c)*b +
```

```
 a)) - sqrt(-((b^2*c - a^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a
^4)) + a)/(b^2*c - a^2))*log(-4*((b^2*c - a^2)*sqrt(b^2*c/(b^4*c^
2 - 2*a^2*b^2*c + a^4)) - a)*sqrt(-((b^2*c - a^2)*sqrt(b^2*c/(b^4
*c^2 - 2*a^2*b^2*c + a^4)) + a)/(b^2*c - a^2)) + 4*sqrt(sqrt(d*x
+ c)*b + a)) - sqrt(((b^2*c - a^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*b^
2*c + a^4)) - a)/(b^2*c - a^2))*log(4*((b^2*c - a^2)*sqrt(b^2*c/(
b^4*c^2 - 2*a^2*b^2*c + a^4)) + a)*sqrt(((b^2*c - a^2)*sqrt(b^2*c
/(b^4*c^2 - 2*a^2*b^2*c + a^4)) - a)/(b^2*c - a^2)) + 4*sqrt(sqrt
(d*x + c)*b + a)) + sqrt(((b^2*c - a^2)*sqrt(b^2*c/(b^4*c^2 - 2*a
^2*b^2*c + a^4)) - a)/(b^2*c - a^2))*log(-4*((b^2*c - a^2)*sqrt(b
^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) + a)*sqrt(((b^2*c - a^2)*sqrt
(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) - a)/(b^2*c - a^2)) + 4*sqr
t(sqrt(d*x + c)*b + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{a + b\sqrt{c + dx}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + b*sqrt(c + d*x))), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(sqrt(d*x + c)*b + a)*x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.490 $\int \frac{1}{x^2\sqrt{a+b\sqrt{c+dx}}}\,dx$

Optimal. Leaf size=163

$$-\frac{\sqrt{a+b\sqrt{c+dx}}\left(a-b\sqrt{c+dx}\right)}{x\left(a^2-b^2c\right)}-\frac{bd\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{2\sqrt{c}\left(a-b\sqrt{c}\right)^{3/2}}+\frac{bd\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{2\sqrt{c}\left(a+b\sqrt{c}\right)^{3/2}}$$

```
[Out] -(((a - b*Sqrt[c + d*x])*Sqrt[a + b*Sqrt[c + d*x]])/((a^2 - b^2*c
)*x)) - (b*d*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a - b*Sqrt[c]
]])/(2*(a - b*Sqrt[c])^(3/2)*Sqrt[c]) + (b*d*ArcTanh[Sqrt[a + b*S
qrt[c + d*x]]/Sqrt[a + b*Sqrt[c]]])/(2*(a + b*Sqrt[c])^(3/2)*Sqrt
[c])
```

Rubi [A] time = 0.471403, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286

$$-\frac{\sqrt{a+b\sqrt{c+dx}}\left(a-b\sqrt{c+dx}\right)}{x\left(a^2-b^2c\right)}-\frac{bd\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{2\sqrt{c}\left(a-b\sqrt{c}\right)^{3/2}}+\frac{bd\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{2\sqrt{c}\left(a+b\sqrt{c}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*Sqrt[a + b*Sqrt[c + d*x]]),x]
```

```
[Out] -(((a - b*Sqrt[c + d*x])*Sqrt[a + b*Sqrt[c + d*x]])/((a^2 - b^2*c
)*x)) - (b*d*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a - b*Sqrt[c]
]])/(2*(a - b*Sqrt[c])^(3/2)*Sqrt[c]) + (b*d*ArcTanh[Sqrt[a + b*S
qrt[c + d*x]]/Sqrt[a + b*Sqrt[c]]])/(2*(a + b*Sqrt[c])^(3/2)*Sqrt
[c])
```

Rubi in Sympy [A] time = 41.9321, size = 138, normalized size = 0.85

$$\frac{bd\,\mathrm{atanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{2\sqrt{c}\left(a+b\sqrt{c}\right)^{\frac{3}{2}}}-\frac{bd\,\mathrm{atanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{2\sqrt{c}\left(a-b\sqrt{c}\right)^{\frac{3}{2}}}-\frac{\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{x\left(a^2-b^2c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] b*d*atanh(sqrt(a + b*sqrt(c + d*x))/sqrt(a + b*sqrt(c)))/(2*sqrt(
c)*(a + b*sqrt(c))**(3/2)) - b*d*atanh(sqrt(a + b*sqrt(c + d*x))/
sqrt(a - b*sqrt(c)))/(2*sqrt(c)*(a - b*sqrt(c))**(3/2)) - (a - b*
sqrt(c + d*x))*sqrt(a + b*sqrt(c + d*x))/(x*(a**2 - b**2*c))
```

Mathematica [A] time = 2.40075, size = 250, normalized size = 1.53

$$\frac{\sqrt{a+b\sqrt{c+dx}}\left(a-b\sqrt{c+dx}\right)}{x\left(b^2c-a^2\right)}+\frac{bd\sqrt{a^2-b^2c}\tan^{-1}\left(\frac{\sqrt{a^2-b^2c}}{\sqrt{-a-b\sqrt{c}}\sqrt{a+b\sqrt{c+dx}}}\right)}{2\sqrt{c}\sqrt{-a-b\sqrt{c}}\left(a-b\sqrt{c}\right)^2}-\frac{bd\sqrt{a^2-b^2c}\tan^{-1}\left(\frac{\sqrt{a^2-b^2c}}{\sqrt{b\sqrt{c}-a}\sqrt{a+b\sqrt{c+dx}}}\right)}{2\sqrt{c}\sqrt{b\sqrt{c}-a}\left(a+b\sqrt{c}\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*Sqrt[a + b*Sqrt[c + d*x]]),x]
```

```
[Out] ((a - b*Sqrt[c + d*x])*Sqrt[a + b*Sqrt[c + d*x]])/((-a^2 + b^2*c)
*x) + (b*Sqrt[a^2 - b^2*c]*d*ArcTan[Sqrt[a^2 - b^2*c]/(Sqrt[-a -
b*Sqrt[c]]*Sqrt[a + b*Sqrt[c + d*x]])])/(2*Sqrt[-a - b*Sqrt[c]]*(
a - b*Sqrt[c])^2*Sqrt[c]) - (b*Sqrt[a^2 - b^2*c]*d*ArcTan[Sqrt[a^
2 - b^2*c]/(Sqrt[-a + b*Sqrt[c]]*Sqrt[a + b*Sqrt[c + d*x]])])/(2*
Sqrt[-a + b*Sqrt[c]]*(a + b*Sqrt[c])^2*Sqrt[c])
```

Maple [B] time = 0.033, size = 265, normalized size = 1.6

$$-2\,\frac{d\sqrt{b^2c}\sqrt{a+b\sqrt{dx+c}}}{c\left(4\,\sqrt{b^2c}-4\,a\right)\left(b\sqrt{dx+c}+\sqrt{b^2c}\right)}-2\,\frac{d\sqrt{b^2c}}{c\left(4\,\sqrt{b^2c}-4\,a\right)\sqrt{\sqrt{b^2c}-a}}\arctan\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{\sqrt{b^2c}-a}}\right)-2\,\frac{d\sqrt{b^2c}\sqrt{a+b\sqrt{dx+c}}}{c\left(-4\,\sqrt{b^2c}-4\,a\right)\left(-b\sqrt{dx+c}+\sqrt{b^2c}\right)}+2\,\frac{d\sqrt{b^2c}}{c\left(-4\,\sqrt{b^2c}-4\,a\right)\sqrt{-\sqrt{b^2c}-a}}\arctan\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{-\sqrt{b^2c}-a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(a+b*(d*x+c)^(1/2))^(1/2),x)
```

```
[Out] -2*d*(b^2*c)^(1/2)/c*(a+b*(d*x+c)^(1/2))^(1/2)/(4*(b^2*c)^(1/2)-4
*a)/(b*(d*x+c)^(1/2)+(b^2*c)^(1/2))-2*d*(b^2*c)^(1/2)/c/(4*(b^2*c
)^(1/2)-4*a)/((b^2*c)^(1/2)-a)^(1/2)*arctan((a+b*(d*x+c)^(1/2))^(
1/2)/((b^2*c)^(1/2)-a)^(1/2))-2*d*(b^2*c)^(1/2)/c*(a+b*(d*x+c)^(1
/2))^(1/2)/(-4*(b^2*c)^(1/2)-4*a)/(-b*(d*x+c)^(1/2)+(b^2*c)^(1/2)
)+2*d*(b^2*c)^(1/2)/c/(-4*(b^2*c)^(1/2)-4*a)/(-(b^2*c)^(1/2)-a)^(
1/2)*arctan((a+b*(d*x+c)^(1/2))^(1/2)/(-(b^2*c)^(1/2)-a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\sqrt{dx+c}b+a}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(sqrt(d*x + c)*b + a)*x^2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(sqrt(d*x + c)*b + a)*x^2), x)
```

Fricas [A] time = 0.387123, size = 3366, normalized size = 20.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(sqrt(d*x + c)*b + a)*x^2),x, algorithm="fricas")
```

```
[Out] 1/4*((b^2*c - a^2)*x*sqrt(-((3*a*b^4*c + a^3*b^2)*d^2 + (b^6*c^4
- 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c)*sqrt((b^10*c^2 + 6*a^2*b
^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^2*b^10*c^6 + 15*a^4*b^8*c^5
 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2*c^2 + a^12*c)))/(
b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c))*log((b^6*c + 3*
a^2*b^4)*sqrt(sqrt(d*x + c)*b + a)*d^3 + (2*(a*b^6*c^2 + 3*a^3*b^
4*c)*d^2 - (b^8*c^5 - 2*a^2*b^6*c^4 + 2*a^6*b^2*c^2 - a^8*c)*sqrt
((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^2*b^10*
c^6 + 15*a^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b
^2*c^2 + a^12*c)))*sqrt(-((3*a*b^4*c + a^3*b^2)*d^2 + (b^6*c^4 -
3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c)*sqrt((b^10*c^2 + 6*a^2*b^8
*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^2*b^10*c^6 + 15*a^4*b^8*c^5 -
 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2*c^2 + a^12*c)))/(b^
6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c))) - (b^2*c - a^2)*
x*sqrt(-((3*a*b^4*c + a^3*b^2)*d^2 + (b^6*c^4 - 3*a^2*b^4*c^3 + 3
*a^4*b^2*c^2 - a^6*c)*sqrt((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d
^4/(b^12*c^7 - 6*a^2*b^10*c^6 + 15*a^4*b^8*c^5 - 20*a^6*b^6*c^4 +
 15*a^8*b^4*c^3 - 6*a^10*b^2*c^2 + a^12*c)))/(b^6*c^4 - 3*a^2*b^4
```

```
*c^3 + 3*a^4*b^2*c^2 - a^6*c))*log((b^6*c + 3*a^2*b^4)*sqrt(sqrt(
d*x + c)*b + a)*d^3 - (2*(a*b^6*c^2 + 3*a^3*b^4*c)*d^2 - (b^8*c^5
 - 2*a^2*b^6*c^4 + 2*a^6*b^2*c^2 - a^8*c)*sqrt((b^10*c^2 + 6*a^2*
b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^2*b^10*c^6 + 15*a^4*b^8*c^
5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2*c^2 + a^12*c)))*
sqrt(-((3*a*b^4*c + a^3*b^2)*d^2 + (b^6*c^4 - 3*a^2*b^4*c^3 + 3*a
^4*b^2*c^2 - a^6*c)*sqrt((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4
/(b^12*c^7 - 6*a^2*b^10*c^6 + 15*a^4*b^8*c^5 - 20*a^6*b^6*c^4 + 1
5*a^8*b^4*c^3 - 6*a^10*b^2*c^2 + a^12*c)))/(b^6*c^4 - 3*a^2*b^4*c
^3 + 3*a^4*b^2*c^2 - a^6*c))) + (b^2*c - a^2)*x*sqrt(-((3*a*b^4*c
 + a^3*b^2)*d^2 - (b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*
c)*sqrt((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^
2*b^10*c^6 + 15*a^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6
*a^10*b^2*c^2 + a^12*c)))/(b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^
2 - a^6*c))*log((b^6*c + 3*a^2*b^4)*sqrt(sqrt(d*x + c)*b + a)*d^3
 + (2*(a*b^6*c^2 + 3*a^3*b^4*c)*d^2 + (b^8*c^5 - 2*a^2*b^6*c^4 +
2*a^6*b^2*c^2 - a^8*c)*sqrt((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*
d^4/(b^12*c^7 - 6*a^2*b^10*c^6 + 15*a^4*b^8*c^5 - 20*a^6*b^6*c^4
+ 15*a^8*b^4*c^3 - 6*a^10*b^2*c^2 + a^12*c)))*sqrt(-((3*a*b^4*c +
 a^3*b^2)*d^2 - (b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c)
*sqrt((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^2*
b^10*c^6 + 15*a^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a
^10*b^2*c^2 + a^12*c)))/(b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2
- a^6*c))) - (b^2*c - a^2)*x*sqrt(-((3*a*b^4*c + a^3*b^2)*d^2 - (
b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c)*sqrt((b^10*c^2 +
 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^2*b^10*c^6 + 15*a^4
*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2*c^2 + a^1
2*c)))/(b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c))*log((b^
6*c + 3*a^2*b^4)*sqrt(sqrt(d*x + c)*b + a)*d^3 - (2*(a*b^6*c^2 +
3*a^3*b^4*c)*d^2 + (b^8*c^5 - 2*a^2*b^6*c^4 + 2*a^6*b^2*c^2 - a^8
*c)*sqrt((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a
^2*b^10*c^6 + 15*a^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 -
6*a^10*b^2*c^2 + a^12*c)))*sqrt(-((3*a*b^4*c + a^3*b^2)*d^2 - (b^
6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c)*sqrt((b^10*c^2 + 6
*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^2*b^10*c^6 + 15*a^4*b
^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2*c^2 + a^12*
c)))/(b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c))) - 4*sqrt
(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b - a))/((b^2*c - a^2)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{a+b\sqrt{c+dx}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(a + b*sqrt(c + d*x))), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(sqrt(d*x + c)*b + a)*x^2),x, algorithm="giac")

[Out] Exception raised: TypeError
```

3.491 $\int \frac{1}{x^3\sqrt{a+b\sqrt{c+dx}}}\,dx$

Optimal. Leaf size=261

$$-\frac{\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{2x^2\left(a^2-b^2c\right)}-\frac{bd\sqrt{a+b\sqrt{c+dx}}\left(6abc-\left(a^2+5b^2c\right)\sqrt{c+dx}\right)}{8cx\left(a^2-b^2c\right)^2}+\frac{bd^2\left(2a-5b\sqrt{c}\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{16c^{3/2}\left(a-b\sqrt{c}\right)^{5/2}}-\frac{bd^2\left(2a+5b\sqrt{c}\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{16c^{3/2}\left(a+b\sqrt{c}\right)^{5/2}}$$

```
[Out] -((a - b*Sqrt[c + d*x])*Sqrt[a + b*Sqrt[c + d*x]])/(2*(a^2 - b^2*
c)*x^2) - (b*d*Sqrt[a + b*Sqrt[c + d*x]]*(6*a*b*c - (a^2 + 5*b^2*
c)*Sqrt[c + d*x]))/(8*c*(a^2 - b^2*c)^2*x) + (b*(2*a - 5*b*Sqrt[c
])*d^2*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a - b*Sqrt[c]]])/(1
6*(a - b*Sqrt[c])^(5/2)*c^(3/2)) - (b*(2*a + 5*b*Sqrt[c])*d^2*Arc
Tanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a + b*Sqrt[c]]])/(16*(a + b*S
qrt[c])^(5/2)*c^(3/2))
```

Rubi [A] time = 1.01265, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286

$$-\frac{\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{2x^2\left(a^2-b^2c\right)}-\frac{bd\sqrt{a+b\sqrt{c+dx}}\left(6abc-\left(a^2+5b^2c\right)\sqrt{c+dx}\right)}{8cx\left(a^2-b^2c\right)^2}+\frac{bd^2\left(2a-5b\sqrt{c}\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{16c^{3/2}\left(a-b\sqrt{c}\right)^{5/2}}-\frac{bd^2\left(2a+5b\sqrt{c}\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{16c^{3/2}\left(a+b\sqrt{c}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*Sqrt[a + b*Sqrt[c + d*x]]),x]
```

```
[Out] -((a - b*Sqrt[c + d*x])*Sqrt[a + b*Sqrt[c + d*x]])/(2*(a^2 - b^2*
c)*x^2) - (b*d*Sqrt[a + b*Sqrt[c + d*x]]*(6*a*b*c - (a^2 + 5*b^2*
c)*Sqrt[c + d*x]))/(8*c*(a^2 - b^2*c)^2*x) + (b*(2*a - 5*b*Sqrt[c
])*d^2*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a - b*Sqrt[c]]])/(1
6*(a - b*Sqrt[c])^(5/2)*c^(3/2)) - (b*(2*a + 5*b*Sqrt[c])*d^2*Arc
Tanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a + b*Sqrt[c]]])/(16*(a + b*S
qrt[c])^(5/2)*c^(3/2))
```

Rubi in Sympy [A] time = 91.3238, size = 289, normalized size = 1.11

$$\frac{bd\sqrt{a+b\sqrt{c+dx}}\left(-3abc+\left(\frac{a^2}{2}+\frac{5b^2c}{2}\right)\sqrt{c+dx}\right)}{4cx\left(a^2-b^2c\right)^2}-\frac{bd^2\left(2a\left(a^2-4b^2c\right)+b\sqrt{c}\left(a^2+5b^2c\right)\right)\operatorname{atanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{16c^{\frac{3}{2}}\sqrt{a+b\sqrt{c}}\left(a^2-b^2c\right)^2}+\frac{bd^2\left(2a\left(a^2-4b^2c\right)-b\sqrt{c}\left(a^2+5b^2c\right)\right)\operatorname{atanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{16c^{\frac{3}{2}}\sqrt{a-b\sqrt{c}}\left(a^2-b^2c\right)^2}-\frac{\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{2x^2\left(a^2-b^2c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] b*d*sqrt(a + b*sqrt(c + d*x))*(-3*a*b*c + (a**2/2 + 5*b**2*c/2)*s
qrt(c + d*x))/(4*c*x*(a**2 - b**2*c)**2) - b*d**2*(2*a*(a**2 - 4*
b**2*c) + b*sqrt(c)*(a**2 + 5*b**2*c))*atanh(sqrt(a + b*sqrt(c +
d*x))/sqrt(a + b*sqrt(c)))/(16*c**(3/2)*sqrt(a + b*sqrt(c))*(a**2
 - b**2*c)**2) + b*d**2*(2*a*(a**2 - 4*b**2*c) - b*sqrt(c)*(a**2
+ 5*b**2*c))*atanh(sqrt(a + b*sqrt(c + d*x))/sqrt(a - b*sqrt(c)))
/(16*c**(3/2)*sqrt(a - b*sqrt(c))*(a**2 - b**2*c)**2) - (a - b*sq
rt(c + d*x))*sqrt(a + b*sqrt(c + d*x))/(2*x**2*(a**2 - b**2*c))
```

Mathematica [A] time = 2.83799, size = 410, normalized size = 1.57

$$\frac{1}{16}\left(-\frac{bd^2\left(2a-5b\sqrt{c}\right)\sqrt{a^2-b^2c}\tan^{-1}\left(\frac{\sqrt{a^2-b^2c}}{\sqrt{-a-b\sqrt{c}}\sqrt{a+b\sqrt{c+dx}}}\right)}{c^{3/2}\sqrt{-a-b\sqrt{c}}\left(a-b\sqrt{c}\right)^3}+\frac{bd^2\left(2a+5b\sqrt{c}\right)\sqrt{a^2-b^2c}\tan^{-1}\left(\frac{\sqrt{a^2-b^2c}}{\sqrt{b\sqrt{c}-a}\sqrt{a+b\sqrt{c+dx}}}\right)}{c^{3/2}\sqrt{b\sqrt{c}-a}\left(a+b\sqrt{c}\right)^3}-\frac{8\left(a^3-3a^2b\sqrt{c+dx}+3ab^2c-b^3c\sqrt{c+dx}\right)\left(a+b\sqrt{c+dx}\right)^{5/2}}{x^2\left(a^2-b^2c\right)^3}+\frac{2bd\left(a^4\left(-\sqrt{c+dx}\right)+26a^3bc+8a^2b^2c\sqrt{c+dx}-10ab^3c^2+9b^4c^2\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{cx\left(b^2c-a^2\right)^3}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*Sqrt[a + b*Sqrt[c + d*x]]),x]
```

```
[Out] ((-8*(a + b*Sqrt[c + d*x])^(5/2)*(a^3 + 3*a*b^2*c - 3*a^2*b*Sqrt[
c + d*x] - b^3*c*Sqrt[c + d*x]))/((a^2 - b^2*c)^3*x^2) + (2*b*d*S
qrt[a + b*Sqrt[c + d*x]]*(26*a^3*b*c - 10*a*b^3*c^2 - a^4*Sqrt[c
+ d*x] + 8*a^2*b^2*c*Sqrt[c + d*x] + 9*b^4*c^2*Sqrt[c + d*x]))/(c
*(-a^2 + b^2*c)^3*x) - (b*(2*a - 5*b*Sqrt[c])*Sqrt[a^2 - b^2*c]*d
^2*ArcTan[Sqrt[a^2 - b^2*c]/(Sqrt[-a - b*Sqrt[c]]*Sqrt[a + b*Sqrt
[c + d*x]])])/(Sqrt[-a - b*Sqrt[c]]*(a - b*Sqrt[c])^3*c^(3/2)) +
(b*(2*a + 5*b*Sqrt[c])*Sqrt[a^2 - b^2*c]*d^2*ArcTan[Sqrt[a^2 - b^
2*c]/(Sqrt[-a + b*Sqrt[c]]*Sqrt[a + b*Sqrt[c + d*x]])])/(Sqrt[-a
+ b*Sqrt[c]]*(a + b*Sqrt[c])^3*c^(3/2)))/16
```

Maple [B] time = 0.117, size = 834, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(a+b*(d*x+c)^(1/2))^(1/2),x)
```

```
[Out] 5/16*b^2*d^2/c/(b*(d*x+c)^(1/2)+(b^2*c)^(1/2))^2/(b^2*c-2*a*(b^2*
c)^(1/2)+a^2)*(a+b*(d*x+c)^(1/2))^(3/2)-1/8*b^2*d^2/c/(b^2*c)^(1/
2)/(b*(d*x+c)^(1/2)+(b^2*c)^(1/2))^2/(b^2*c-2*a*(b^2*c)^(1/2)+a^2
)*(a+b*(d*x+c)^(1/2))^(3/2)*a-7/16*b^2*d^2/c/(b*(d*x+c)^(1/2)+(b^
2*c)^(1/2))^2/(-(b^2*c)^(1/2)+a)*(a+b*(d*x+c)^(1/2))^(1/2)+1/8*b^
2*d^2/c/(b^2*c)^(1/2)/(b*(d*x+c)^(1/2)+(b^2*c)^(1/2))^2/(-(b^2*c)
^(1/2)+a)*(a+b*(d*x+c)^(1/2))^(1/2)*a+5/16*b^2*d^2/c/(b^2*c-2*a*(
b^2*c)^(1/2)+a^2)/((b^2*c)^(1/2)-a)^(1/2)*arctan((a+b*(d*x+c)^(1/
2))^(1/2)/((b^2*c)^(1/2)-a)^(1/2))-1/8*b^2*d^2/c/(b^2*c)^(1/2)/(b
^2*c-2*a*(b^2*c)^(1/2)+a^2)/((b^2*c)^(1/2)-a)^(1/2)*arctan((a+b*(
d*x+c)^(1/2))^(1/2)/((b^2*c)^(1/2)-a)^(1/2))*a+5/16*b^2*d^2/c/(b*
(d*x+c)^(1/2)-(b^2*c)^(1/2))^2/(b^2*c+2*a*(b^2*c)^(1/2)+a^2)*(a+b
*(d*x+c)^(1/2))^(3/2)+1/8*b^2*d^2/c/(b^2*c)^(1/2)/(b*(d*x+c)^(1/2
)-(b^2*c)^(1/2))^2/(b^2*c+2*a*(b^2*c)^(1/2)+a^2)*(a+b*(d*x+c)^(1/
2))^(3/2)*a-7/16*b^2*d^2/c/(b*(d*x+c)^(1/2)-(b^2*c)^(1/2))^2/((b^
2*c)^(1/2)+a)*(a+b*(d*x+c)^(1/2))^(1/2)-1/8*b^2*d^2/c/(b^2*c)^(1/
2)/(b*(d*x+c)^(1/2)-(b^2*c)^(1/2))^2/((b^2*c)^(1/2)+a)*(a+b*(d*x+
c)^(1/2))^(1/2)*a+5/16*b^2*d^2/c/(b^2*c+2*a*(b^2*c)^(1/2)+a^2)/(-
(b^2*c)^(1/2)-a)^(1/2)*arctan((a+b*(d*x+c)^(1/2))^(1/2)/(-(b^2*c)
^(1/2)-a)^(1/2))+1/8*b^2*d^2/c/(b^2*c)^(1/2)/(b^2*c+2*a*(b^2*c)^(
1/2)+a^2)/(-(b^2*c)^(1/2)-a)^(1/2)*arctan((a+b*(d*x+c)^(1/2))^(1/
2)/(-(b^2*c)^(1/2)-a)^(1/2))*a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\sqrt{dx+c}b+a}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(sqrt(d*x + c)*b + a)*x^3),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(sqrt(d*x + c)*b + a)*x^3), x)
```

Fricas [A] time = 0.956999, size = 5927, normalized size = 22.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(sqrt(d*x + c)*b + a)*x^3),x, algorithm="fricas")
```

```
[Out] -1/32*((b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)*x^2*sqrt(-((105*a*b^8*c^
3 + 70*a^3*b^6*c^2 - 35*a^5*b^4*c + 4*a^7*b^2)*d^4 + (b^10*c^8 -
5*a^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 -
 a^10*c^3)*sqrt((625*b^18*c^4 + 7700*a^2*b^16*c^3 + 21966*a^4*b^1
4*c^2 - 10780*a^6*b^12*c + 1225*a^8*b^10)*d^8/(b^20*c^13 - 10*a^2
*b^18*c^12 + 45*a^4*b^16*c^11 - 120*a^6*b^14*c^10 + 210*a^8*b^12*
c^9 - 252*a^10*b^10*c^8 + 210*a^12*b^8*c^7 - 120*a^14*b^6*c^6 + 4
5*a^16*b^4*c^5 - 10*a^18*b^2*c^4 + a^20*c^3)))/(b^10*c^8 - 5*a^2*
b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*
c^3))*log((625*b^12*c^3 + 3750*a^2*b^10*c^2 - 1491*a^4*b^8*c + 14
0*a^6*b^6)*sqrt(sqrt(d*x + c)*b + a)*d^6 + ((325*a*b^12*c^5 + 197
7*a^3*b^10*c^4 - 609*a^5*b^8*c^3 + 35*a^7*b^6*c^2)*d^4 - (5*b^14*
c^10 - 16*a^2*b^12*c^9 + 3*a^4*b^10*c^8 + 50*a^6*b^8*c^7 - 85*a^8
*b^6*c^6 + 60*a^10*b^4*c^5 - 19*a^12*b^2*c^4 + 2*a^14*c^3)*sqrt((
625*b^18*c^4 + 7700*a^2*b^16*c^3 + 21966*a^4*b^14*c^2 - 10780*a^6
*b^12*c + 1225*a^8*b^10)*d^8/(b^20*c^13 - 10*a^2*b^18*c^12 + 45*a
^4*b^16*c^11 - 120*a^6*b^14*c^10 + 210*a^8*b^12*c^9 - 252*a^10*b^
10*c^8 + 210*a^12*b^8*c^7 - 120*a^14*b^6*c^6 + 45*a^16*b^4*c^5 -
10*a^18*b^2*c^4 + a^20*c^3)))*sqrt(-((105*a*b^8*c^3 + 70*a^3*b^6*
c^2 - 35*a^5*b^4*c + 4*a^7*b^2)*d^4 + (b^10*c^8 - 5*a^2*b^8*c^7 +
 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3)*sqrt
((625*b^18*c^4 + 7700*a^2*b^16*c^3 + 21966*a^4*b^14*c^2 - 10780*a
^6*b^12*c + 1225*a^8*b^10)*d^8/(b^20*c^13 - 10*a^2*b^18*c^12 + 45
*a^4*b^16*c^11 - 120*a^6*b^14*c^10 + 210*a^8*b^12*c^9 - 252*a^10*
b^10*c^8 + 210*a^12*b^8*c^7 - 120*a^14*b^6*c^6 + 45*a^16*b^4*c^5
- 10*a^18*b^2*c^4 + a^20*c^3)))/(b^10*c^8 - 5*a^2*b^8*c^7 + 10*a^
4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3))) - (b^4*c
^3 - 2*a^2*b^2*c^2 + a^4*c)*x^2*sqrt(-((105*a*b^8*c^3 + 70*a^3*b^
```

6*c^2 - 35*a^5*b^4*c + 4*a^7*b^2)*d^4 + (b^10*c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3)*sqrt((625*b^18*c^4 + 7700*a^2*b^16*c^3 + 21966*a^4*b^14*c^2 - 10780*a^6*b^12*c + 1225*a^8*b^10)*d^8/(b^20*c^13 - 10*a^2*b^18*c^12 + 45*a^4*b^16*c^11 - 120*a^6*b^14*c^10 + 210*a^8*b^12*c^9 - 252*a^10*b^10*c^8 + 210*a^12*b^8*c^7 - 120*a^14*b^6*c^6 + 45*a^16*b^4*c^5 - 10*a^18*b^2*c^4 + a^20*c^3)))/(b^10*c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3))*log((625*b^12*c^3 + 3750*a^2*b^10*c^2 - 1491*a^4*b^8*c + 140*a^6*b^6)*sqrt(sqrt(d*x + c)*b + a)*d^6 - ((325*a*b^12*c^5 + 1977*a^3*b^10*c^4 - 609*a^5*b^8*c^3 + 35*a^7*b^6*c^2)*d^4 - (5*b^14*c^10 - 16*a^2*b^12*c^9 + 3*a^4*b^10*c^8 + 50*a^6*b^8*c^7 - 85*a^8*b^6*c^6 + 60*a^10*b^4*c^5 - 19*a^12*b^2*c^4 + 2*a^14*c^3)*sqrt((625*b^18*c^4 + 7700*a^2*b^16*c^3 + 21966*a^4*b^14*c^2 - 10780*a^6*b^12*c + 1225*a^8*b^10)*d^8/(b^20*c^13 - 10*a^2*b^18*c^12 + 45*a^4*b^16*c^11 - 120*a^6*b^14*c^10 + 210*a^8*b^12*c^9 - 252*a^10*b^10*c^8 + 210*a^12*b^8*c^7 - 120*a^14*b^6*c^6 + 45*a^16*b^4*c^5 - 10*a^18*b^2*c^4 + a^20*c^3)))*sqrt(-((105*a*b^8*c^3 + 70*a^3*b^6*c^2 - 35*a^5*b^4*c + 4*a^7*b^2)*d^4 + (b^10*c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3)*sqrt((625*b^18*c^4 + 7700*a^2*b^16*c^3 + 21966*a^4*b^14*c^2 - 10780*a^6*b^12*c + 1225*a^8*b^10)*d^8/(b^20*c^13 - 10*a^2*b^18*c^12 + 45*a^4*b^16*c^11 - 120*a^6*b^14*c^10 + 210*a^8*b^12*c^9 - 252*a^10*b^10*c^8 + 210*a^12*b^8*c^7 - 120*a^14*b^6*c^6 + 45*a^16*b^4*c^5 - 10*a^18*b^2*c^4 + a^20*c^3)))/(b^10*c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3))) + (b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)*x^2*sqrt(-((105*a*b^8*c^3 + 70*a^3*b^6*c^2 - 35*a^5*b^4*c + 4*a^7*b^2)*d^4 - (b^10*c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3)*sqrt((625*b^18*c^4 + 7700*a^2*b^16*c^3 + 21966*a^4*b^14*c^2 - 10780*a^6*b^12*c + 1225*a^8*b^10)*d^8/(b^20*c^13 - 10*a^2*b^18*c^12 + 45*a^4*b^16*c^11 - 120*a^6*b^14*c^10 + 210*a^8*b^12*c^9 - 252*a^10*b^10*c^8 + 210*a^12*b^8*c^7 - 120*a^14*b^6*c^6 + 45*a^16*b^4*c^5 - 10*a^18*b^2*c^4 + a^20*c^3)))/(b^10*c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3))*log((625*b^12*c^3 + 3750*a^2*b^10*c^2 - 1491*a^4*b^8*c + 140*a^6*b^6)*sqrt(sqrt(d*x + c)*b + a)*d^6 + ((325*a*b^12*c^5 + 1977*a^3*b^10*c^4 - 609*a^5*b^8*c^3 + 35*a^7*b^6*c^2)*d^4 + (5*b^14*c^10 - 16*a^2*b^12*c^9 + 3*a^4*b^10*c^8 + 50*a^6*b^8*c^7 - 85*a^8*b^6*c^6 + 60*a^10*b^4*c^5 - 19*a^12*b^2*c^4 + 2*a^14*c^3)*sqrt((625*b^18*c^4 + 7700*a^2*b^16*c^3 + 21966*a^4*b^14*c^2 - 10780*a^6*b^12*c + 1225*a^8*b^10)*d^8/(b^20*c^13 - 10*a^2*b^18*c^12 + 45*a^4*b^16*c^11 - 120*a^6*b^14*c^10 + 210*a^8*b^12*c^9 - 252*a^10*b^10*c^8 + 210*a^12*b^8*c^7 - 120*a^14*b^6*c^6 + 45*a^16*b^4*c^5 - 10*a^18*b^2*c^4 + a^20*c^3)))*sqrt(-((105*a*b^8*c^3 + 70*a^3*b^6*c^2 - 35*a^5*b^4*c + 4*a^7*b^2)*d^4 - (b^10*c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3)*sqrt((625*b^18*c^4 + 7700*a^2*b^16*c^3 + 21966*a^4*b^14*c^2 - 10780*a^6*b^12*c + 1225*a^8*b^10)*d^8/(b^20*c^13 - 10*a^2*b^18*c^12 + 45*a^4*b^16*c^11 - 120*a^6*b^14*c^10 + 210*a^8*b^12*c^9 - 252*a^10*b^10*c^8 + 210*a^12*b^8*c^7 - 120*a^14*b^6*c^6 + 45*a^16*b^4*c^5 - 10*a^18*b^2*c^4 + a^20*c^3)))/(b^10*c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3))) - (b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)*x^2*sqrt(-((105*a*b^8*c^3 + 70*a^3*b^6*c^2 - 35*a^5*b^4*c + 4*a

```
^7*b^2)*d^4 - (b^10*c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6
*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3)*sqrt((625*b^18*c^4 + 7700*a^
2*b^16*c^3 + 21966*a^4*b^14*c^2 - 10780*a^6*b^12*c + 1225*a^8*b^1
0)*d^8/(b^20*c^13 - 10*a^2*b^18*c^12 + 45*a^4*b^16*c^11 - 120*a^6
*b^14*c^10 + 210*a^8*b^12*c^9 - 252*a^10*b^10*c^8 + 210*a^12*b^8*
c^7 - 120*a^14*b^6*c^6 + 45*a^16*b^4*c^5 - 10*a^18*b^2*c^4 + a^20
*c^3)))/(b^10*c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c
^5 + 5*a^8*b^2*c^4 - a^10*c^3))*log((625*b^12*c^3 + 3750*a^2*b^10
*c^2 - 1491*a^4*b^8*c + 140*a^6*b^6)*sqrt(sqrt(d*x + c)*b + a)*d^
6 - ((325*a*b^12*c^5 + 1977*a^3*b^10*c^4 - 609*a^5*b^8*c^3 + 35*a
^7*b^6*c^2)*d^4 + (5*b^14*c^10 - 16*a^2*b^12*c^9 + 3*a^4*b^10*c^8
 + 50*a^6*b^8*c^7 - 85*a^8*b^6*c^6 + 60*a^10*b^4*c^5 - 19*a^12*b^
2*c^4 + 2*a^14*c^3)*sqrt((625*b^18*c^4 + 7700*a^2*b^16*c^3 + 2196
6*a^4*b^14*c^2 - 10780*a^6*b^12*c + 1225*a^8*b^10)*d^8/(b^20*c^13
 - 10*a^2*b^18*c^12 + 45*a^4*b^16*c^11 - 120*a^6*b^14*c^10 + 210*
a^8*b^12*c^9 - 252*a^10*b^10*c^8 + 210*a^12*b^8*c^7 - 120*a^14*b^
6*c^6 + 45*a^16*b^4*c^5 - 10*a^18*b^2*c^4 + a^20*c^3)))*sqrt(-((1
05*a*b^8*c^3 + 70*a^3*b^6*c^2 - 35*a^5*b^4*c + 4*a^7*b^2)*d^4 - (
b^10*c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^
8*b^2*c^4 - a^10*c^3)*sqrt((625*b^18*c^4 + 7700*a^2*b^16*c^3 + 21
966*a^4*b^14*c^2 - 10780*a^6*b^12*c + 1225*a^8*b^10)*d^8/(b^20*c^
13 - 10*a^2*b^18*c^12 + 45*a^4*b^16*c^11 - 120*a^6*b^14*c^10 + 21
0*a^8*b^12*c^9 - 252*a^10*b^10*c^8 + 210*a^12*b^8*c^7 - 120*a^14*
b^6*c^6 + 45*a^16*b^4*c^5 - 10*a^18*b^2*c^4 + a^20*c^3)))/(b^10*c
^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*
c^4 - a^10*c^3))) + 4*(6*a*b^2*c*d*x - 4*a*b^2*c^2 + 4*a^3*c + (4
*b^3*c^2 - 4*a^2*b*c - (5*b^3*c + a^2*b)*d*x)*sqrt(d*x + c))*sqrt
(sqrt(d*x + c)*b + a))/((b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\sqrt{a+b\sqrt{c+dx}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(a + b*sqrt(c + d*x))), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(sqrt(d*x + c)*b + a)*x^3),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.492 $\int x^3 \left(a+b\sqrt{c+dx}\right)^p dx$

Optimal. Leaf size=350

$$-\frac{2a(a^2-b^2c)^3\left(a+b\sqrt{c+dx}\right)^{p+1}}{b^8d^4(p+1)}+\frac{2(a^2-b^2c)^2(7a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{p+2}}{b^8d^4(p+2)}-\frac{6a(7a^2-3b^2c)(a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{p+3}}{b^8d^4(p+3)}-\frac{10a(7a^2-3b^2c)\left(a+b\sqrt{c+dx}\right)^{p+5}}{b^8d^4(p+5)}+\frac{6(7a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{p+6}}{b^8d^4(p+6)}+\frac{2(35a^4-30a^2b^2c+3b^4c^2)\left(a+b\sqrt{c+dx}\right)^{p+4}}{b^8d^4(p+4)}-\frac{14a\left(a+b\sqrt{c+dx}\right)^{p+7}}{b^8d^4(p+7)}+\frac{2\left(a+b\sqrt{c+dx}\right)^{p+8}}{b^8d^4(p+8)}$$

```
[Out] (-2*a*(a^2 - b^2*c)^3*(a + b*Sqrt[c + d*x])^(1 + p))/(b^8*d^4*(1
+ p)) + (2*(a^2 - b^2*c)^2*(7*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^
(2 + p))/(b^8*d^4*(2 + p)) - (6*a*(7*a^2 - 3*b^2*c)*(a^2 - b^2*c)
*(a + b*Sqrt[c + d*x])^(3 + p))/(b^8*d^4*(3 + p)) + (2*(35*a^4 -
30*a^2*b^2*c + 3*b^4*c^2)*(a + b*Sqrt[c + d*x])^(4 + p))/(b^8*d^4
*(4 + p)) - (10*a*(7*a^2 - 3*b^2*c)*(a + b*Sqrt[c + d*x])^(5 + p)
)/(b^8*d^4*(5 + p)) + (6*(7*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(6
 + p))/(b^8*d^4*(6 + p)) - (14*a*(a + b*Sqrt[c + d*x])^(7 + p))/(
b^8*d^4*(7 + p)) + (2*(a + b*Sqrt[c + d*x])^(8 + p))/(b^8*d^4*(8
+ p))
```

Rubi [A] time = 0.616953, antiderivative size = 350, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$

$$-\frac{2a(a^2-b^2c)^3\left(a+b\sqrt{c+dx}\right)^{p+1}}{b^8d^4(p+1)}+\frac{2(a^2-b^2c)^2(7a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{p+2}}{b^8d^4(p+2)}-\frac{6a(7a^2-3b^2c)(a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{p+3}}{b^8d^4(p+3)}-\frac{10a(7a^2-3b^2c)\left(a+b\sqrt{c+dx}\right)^{p+5}}{b^8d^4(p+5)}+\frac{6(7a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{p+6}}{b^8d^4(p+6)}+\frac{2(35a^4-30a^2b^2c+3b^4c^2)\left(a+b\sqrt{c+dx}\right)^{p+4}}{b^8d^4(p+4)}-\frac{14a\left(a+b\sqrt{c+dx}\right)^{p+7}}{b^8d^4(p+7)}+\frac{2\left(a+b\sqrt{c+dx}\right)^{p+8}}{b^8d^4(p+8)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*(a + b*Sqrt[c + d*x])^p,x]
```

```
[Out] (-2*a*(a^2 - b^2*c)^3*(a + b*Sqrt[c + d*x])^(1 + p))/(b^8*d^4*(1
+ p)) + (2*(a^2 - b^2*c)^2*(7*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^
(2 + p))/(b^8*d^4*(2 + p)) - (6*a*(7*a^2 - 3*b^2*c)*(a^2 - b^2*c)
*(a + b*Sqrt[c + d*x])^(3 + p))/(b^8*d^4*(3 + p)) + (2*(35*a^4 -
30*a^2*b^2*c + 3*b^4*c^2)*(a + b*Sqrt[c + d*x])^(4 + p))/(b^8*d^4
*(4 + p)) - (10*a*(7*a^2 - 3*b^2*c)*(a + b*Sqrt[c + d*x])^(5 + p)
)/(b^8*d^4*(5 + p)) + (6*(7*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(6
 + p))/(b^8*d^4*(6 + p)) - (14*a*(a + b*Sqrt[c + d*x])^(7 + p))/(
b^8*d^4*(7 + p)) + (2*(a + b*Sqrt[c + d*x])^(8 + p))/(b^8*d^4*(8
+ p))
```

Rubi in Sympy [A] time = 49.9932, size = 320, normalized size = 0.91

$$-\frac{2a\left(a+b\sqrt{c+dx}\right)^{p+1}\left(a^2-b^2c\right)^3}{b^8d^4\left(p+1\right)}-\frac{6a\left(a+b\sqrt{c+dx}\right)^{p+3}\left(a^2-b^2c\right)\left(7a^2-3b^2c\right)}{b^8d^4\left(p+3\right)}-\frac{10a\left(a+b\sqrt{c+dx}\right)^{p+5}\left(7a^2-3b^2c\right)}{b^8d^4\left(p+5\right)}-\frac{14a\left(a+b\sqrt{c+dx}\right)^{p+7}}{b^8d^4\left(p+7\right)}+\frac{2\left(a+b\sqrt{c+dx}\right)^{p+2}\left(a^2-b^2c\right)^2\left(7a^2-b^2c\right)}{b^8d^4\left(p+2\right)}+\frac{2\left(a+b\sqrt{c+dx}\right)^{p+4}\left(35a^4-30a^2b^2c+3b^4c^2\right)}{b^8d^4\left(p+4\right)}+\frac{6\left(a+b\sqrt{c+dx}\right)^{p+6}\left(7a^2-b^2c\right)}{b^8d^4\left(p+6\right)}+\frac{2\left(a+b\sqrt{c+dx}\right)^{p+8}}{b^8d^4\left(p+8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(a+b*(d*x+c)**(1/2))**p,x)
```

```
[Out] -2*a*(a + b*sqrt(c + d*x))**(p + 1)*(a**2 - b**2*c)**3/(b**8*d**4
*(p + 1)) - 6*a*(a + b*sqrt(c + d*x))**(p + 3)*(a**2 - b**2*c)*(7
*a**2 - 3*b**2*c)/(b**8*d**4*(p + 3)) - 10*a*(a + b*sqrt(c + d*x)
)**(p + 5)*(7*a**2 - 3*b**2*c)/(b**8*d**4*(p + 5)) - 14*a*(a + b*
sqrt(c + d*x))**(p + 7)/(b**8*d**4*(p + 7)) + 2*(a + b*sqrt(c + d
*x))**(p + 2)*(a**2 - b**2*c)**2*(7*a**2 - b**2*c)/(b**8*d**4*(p
+ 2)) + 2*(a + b*sqrt(c + d*x))**(p + 4)*(35*a**4 - 30*a**2*b**2*
c + 3*b**4*c**2)/(b**8*d**4*(p + 4)) + 6*(a + b*sqrt(c + d*x))**(
p + 6)*(7*a**2 - b**2*c)/(b**8*d**4*(p + 6)) + 2*(a + b*sqrt(c +
d*x))**(p + 8)/(b**8*d**4*(p + 8))
```

Mathematica [A] time = 1.48302, size = 554, normalized size = 1.58

$$-\frac{2\left(a+b\sqrt{c+dx}\right)^{p+1}\left(5040a^7-5040a^6b(p+1)\sqrt{c+dx}-360a^5b^2\left(-6c\left(p^2+p-7\right)-7d\left(p^2+3p+2\right)x\right)+120a^4b^3(p+1\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*(a + b*Sqrt[c + d*x])^p,x]
```

```
[Out] (-2*(a + b*Sqrt[c + d*x])^(1 + p)*(5040*a^7 - 5040*a^6*b*(1 + p)*
Sqrt[c + d*x] - 360*a^5*b^2*(-6*c*(-7 + p + p^2) - 7*d*(2 + 3*p +
 p^2)*x) + 120*a^4*b^3*(1 + p)*Sqrt[c + d*x]*(c*(126 + 10*p - 4*p
^2) - 7*d*(6 + 5*p + p^2)*x) + 6*a^3*b^4*(8*c^2*(315 - 124*p - 13
9*p^2 - 14*p^3 + p^4) + 40*c*d*(-42 - 61*p - 16*p^2 + 4*p^3 + p^4
)*x + 35*d^2*(24 + 50*p + 35*p^2 + 10*p^3 + p^4)*x^2) - 6*a^2*b^5
*(1 + p)*Sqrt[c + d*x]*(-24*c^2*(-105 - 24*p + 5*p^2 + p^3) + 4*c
*d*(-420 - 386*p - 94*p^2 - p^3 + p^4)*x + 7*d^2*(120 + 154*p + 7
1*p^2 + 14*p^3 + p^4)*x^2) + b^7*(105 + 176*p + 86*p^2 + 16*p^3 +
 p^4)*Sqrt[c + d*x]*(48*c^3 - 24*c^2*d*(2 + p)*x + 6*c*d^2*(8 + 6
*p + p^2)*x^2 - d^3*(48 + 44*p + 12*p^2 + p^3)*x^3) + a*b^6*(48*c
^3*(-105 + 103*p + 138*p^2 + 38*p^3 + 3*p^4) - 24*c^2*d*(-210 - 2
83*p - 21*p^2 + 74*p^3 + 24*p^4 + 2*p^5)*x + 6*c*d^2*(-840 - 1726
*p - 1151*p^2 - 265*p^3 + 10*p^4 + 11*p^5 + p^6)*x^2 + 7*d^3*(720
 + 1764*p + 1624*p^2 + 735*p^3 + 175*p^4 + 21*p^5 + p^6)*x^3)))/(
b^8*d^4*(1 + p)*(2 + p)*(3 + p)*(4 + p)*(5 + p)*(6 + p)*(7 + p)*(
8 + p))
```

Maple [F] time = 0.008, size = 0, normalized size = 0.

$$\int x^3\left(a+b\sqrt{dx+c}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(a+b*(d*x+c)^(1/2))^p,x)
```

```
[Out] int(x^3*(a+b*(d*x+c)^(1/2))^p,x)
```

Maxima [A] time = 0.723975, size = 983, normalized size = 2.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(d*x + c)*b + a)^p*x^3,x, algorithm="maxima")
```

```
[Out] -2*(((d*x + c)*b^2*(p + 1) + sqrt(d*x + c)*a*b*p - a^2)*(sqrt(d*x
 + c)*b + a)^p*c^3/((p^2 + 3*p + 2)*b^2) - 3*((p^3 + 6*p^2 + 11*p
 + 6)*(d*x + c)^2*b^4 + (p^3 + 3*p^2 + 2*p)*(d*x + c)^(3/2)*a*b^3
 - 3*(p^2 + p)*(d*x + c)*a^2*b^2 + 6*sqrt(d*x + c)*a^3*b*p - 6*a^
4)*(sqrt(d*x + c)*b + a)^p*c^2/((p^4 + 10*p^3 + 35*p^2 + 50*p + 2
4)*b^4) + 3*((p^5 + 15*p^4 + 85*p^3 + 225*p^2 + 274*p + 120)*(d*x
 + c)^3*b^6 + (p^5 + 10*p^4 + 35*p^3 + 50*p^2 + 24*p)*(d*x + c)^(
5/2)*a*b^5 - 5*(p^4 + 6*p^3 + 11*p^2 + 6*p)*(d*x + c)^2*a^2*b^4 +
 20*(p^3 + 3*p^2 + 2*p)*(d*x + c)^(3/2)*a^3*b^3 - 60*(p^2 + p)*(d
*x + c)*a^4*b^2 + 120*sqrt(d*x + c)*a^5*b*p - 120*a^6)*(sqrt(d*x
+ c)*b + a)^p*c/((p^6 + 21*p^5 + 175*p^4 + 735*p^3 + 1624*p^2 + 1
764*p + 720)*b^6) - ((p^7 + 28*p^6 + 322*p^5 + 1960*p^4 + 6769*p^
3 + 13132*p^2 + 13068*p + 5040)*(d*x + c)^4*b^8 + (p^7 + 21*p^6 +
 175*p^5 + 735*p^4 + 1624*p^3 + 1764*p^2 + 720*p)*(d*x + c)^(7/2)
*a*b^7 - 7*(p^6 + 15*p^5 + 85*p^4 + 225*p^3 + 274*p^2 + 120*p)*(d
*x + c)^3*a^2*b^6 + 42*(p^5 + 10*p^4 + 35*p^3 + 50*p^2 + 24*p)*(d
*x + c)^(5/2)*a^3*b^5 - 210*(p^4 + 6*p^3 + 11*p^2 + 6*p)*(d*x + c
)^2*a^4*b^4 + 840*(p^3 + 3*p^2 + 2*p)*(d*x + c)^(3/2)*a^5*b^3 - 2
520*(p^2 + p)*(d*x + c)*a^6*b^2 + 5040*sqrt(d*x + c)*a^7*b*p - 50
40*a^8)*(sqrt(d*x + c)*b + a)^p/((p^8 + 36*p^7 + 546*p^6 + 4536*p
^5 + 22449*p^4 + 67284*p^3 + 118124*p^2 + 109584*p + 40320)*b^8))
/d^4
```

Fricas [A] time = 0.439962, size = 1912, normalized size = 5.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(d*x + c)*b + a)^p*x^3,x, algorithm="fricas")
```

```
[Out] -2*(5040*b^8*c^4 - 20160*a^2*b^6*c^3 + 30240*a^4*b^4*c^2 - 20160*
a^6*b^2*c + 5040*a^8 + 48*(b^8*c^4 + 6*a^2*b^6*c^3 + a^4*b^4*c^2)
*p^4 - (b^8*d^4*p^7 + 28*b^8*d^4*p^6 + 322*b^8*d^4*p^5 + 1960*b^8
*d^4*p^4 + 6769*b^8*d^4*p^3 + 13132*b^8*d^4*p^2 + 13068*b^8*d^4*p
 + 5040*b^8*d^4)*x^4 + 384*(2*b^8*c^4 + 7*a^2*b^6*c^3 - 3*a^4*b^4
*c^2)*p^3 - (b^8*c*d^3*p^7 + (22*b^8*c - 7*a^2*b^6)*d^3*p^6 + 5*(
38*b^8*c - 21*a^2*b^6)*d^3*p^5 + 5*(164*b^8*c - 119*a^2*b^6)*d^3*
p^4 + (1849*b^8*c - 1575*a^2*b^6)*d^3*p^3 + 2*(1019*b^8*c - 959*a
^2*b^6)*d^3*p^2 + 840*(b^8*c - a^2*b^6)*d^3*p)*x^3 + 48*(86*b^8*c
^4 + 81*a^2*b^6*c^3 - 124*a^4*b^4*c^2 + 45*a^6*b^2*c)*p^2 + 6*(18
*b^8*c^2*d^2*p^5 + (b^8*c^2 + a^2*b^6*c)*d^2*p^6 + (118*b^8*c^2 -
 95*a^2*b^6*c + 35*a^4*b^4)*d^2*p^4 + 6*(58*b^8*c^2 - 80*a^2*b^6*
c + 35*a^4*b^4)*d^2*p^3 + (457*b^8*c^2 - 806*a^2*b^6*c + 385*a^4*
b^4)*d^2*p^2 + 210*(b^8*c^2 - 2*a^2*b^6*c + a^4*b^4)*d^2*p)*x^2 +
 192*(44*b^8*c^4 - 71*a^2*b^6*c^3 + 54*a^4*b^4*c^2 - 15*a^6*b^2*c
)*p - 24*((b^8*c^3 + 3*a^2*b^6*c^2)*d*p^5 + 2*(8*b^8*c^3 + 9*a^2*
```

```
b^6*c^2 - 5*a^4*b^4*c)*d*p^4 + (86*b^8*c^3 - 57*a^2*b^6*c^2 + 15*
a^4*b^4*c)*d*p^3 + (176*b^8*c^3 - 387*a^2*b^6*c^2 + 340*a^4*b^4*c
 - 105*a^6*b^2)*d*p^2 + 105*(b^8*c^3 - 3*a^2*b^6*c^2 + 3*a^4*b^4*
c - a^6*b^2)*d*p)*x + (192*(a*b^7*c^3 + a^3*b^5*c^2)*p^4 + 96*(27
*a*b^7*c^3 + 2*a^3*b^5*c^2 - 5*a^5*b^3*c)*p^3 - (a*b^7*d^3*p^7 +
21*a*b^7*d^3*p^6 + 175*a*b^7*d^3*p^5 + 735*a*b^7*d^3*p^4 + 1624*a
*b^7*d^3*p^3 + 1764*a*b^7*d^3*p^2 + 720*a*b^7*d^3*p)*x^3 + 192*(5
6*a*b^7*c^3 - 49*a^3*b^5*c^2 + 15*a^5*b^3*c)*p^2 + 6*(2*a*b^7*c*d
^2*p^6 + (33*a*b^7*c - 7*a^3*b^5)*d^2*p^5 + 10*(20*a*b^7*c - 7*a^
3*b^5)*d^2*p^4 + 5*(111*a*b^7*c - 49*a^3*b^5)*d^2*p^3 + 2*(349*a*
b^7*c - 175*a^3*b^5)*d^2*p^2 + 24*(13*a*b^7*c - 7*a^3*b^5)*d^2*p)
*x^2 + 48*(279*a*b^7*c^3 - 511*a^3*b^5*c^2 + 385*a^5*b^3*c - 105*
a^7*b)*p - 24*((3*a*b^7*c^2 + a^3*b^5*c)*d*p^5 + 2*(21*a*b^7*c^2
- 5*a^3*b^5*c)*d*p^4 + (192*a*b^7*c^2 - 135*a^3*b^5*c + 35*a^5*b^
3)*d*p^3 + (327*a*b^7*c^2 - 320*a^3*b^5*c + 105*a^5*b^3)*d*p^2 +
2*(87*a*b^7*c^2 - 98*a^3*b^5*c + 35*a^5*b^3)*d*p)*x)*sqrt(d*x + c
))*(sqrt(d*x + c)*b + a)^p/(b^8*d^4*p^8 + 36*b^8*d^4*p^7 + 546*b^
8*d^4*p^6 + 4536*b^8*d^4*p^5 + 22449*b^8*d^4*p^4 + 67284*b^8*d^4*
p^3 + 118124*b^8*d^4*p^2 + 109584*b^8*d^4*p + 40320*b^8*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(a+b*(d*x+c)**(1/2))**p,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(d*x + c)*b + a)^p*x^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.493 $\int x^2 \left(a+b\sqrt{c+dx}\right)^p \, dx$

Optimal. Leaf size=242

$$-\frac{2a\left(a^2-b^2c\right)^2\left(a+b\sqrt{c+dx}\right)^{p+1}}{b^6d^3(p+1)}-\frac{4a\left(5a^2-3b^2c\right)\left(a+b\sqrt{c+dx}\right)^{p+3}}{b^6d^3(p+3)}+\frac{4\left(5a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{p+4}}{b^6d^3(p+4)}+\frac{2\left(5a^4-6a^2b^2c+b^4c^2\right)\left(a+b\sqrt{c+dx}\right)^{p+2}}{b^6d^3(p+2)}-\frac{10a\left(a+b\sqrt{c+dx}\right)^{p+5}}{b^6d^3(p+5)}+\frac{2\left(a+b\sqrt{c+dx}\right)^{p+6}}{b^6d^3(p+6)}$$

```
[Out] (-2*a*(a^2 - b^2*c)^2*(a + b*Sqrt[c + d*x])^(1 + p))/(b^6*d^3*(1
+ p)) + (2*(5*a^4 - 6*a^2*b^2*c + b^4*c^2)*(a + b*Sqrt[c + d*x])^
(2 + p))/(b^6*d^3*(2 + p)) - (4*a*(5*a^2 - 3*b^2*c)*(a + b*Sqrt[c
 + d*x])^(3 + p))/(b^6*d^3*(3 + p)) + (4*(5*a^2 - b^2*c)*(a + b*S
qrt[c + d*x])^(4 + p))/(b^6*d^3*(4 + p)) - (10*a*(a + b*Sqrt[c +
d*x])^(5 + p))/(b^6*d^3*(5 + p)) + (2*(a + b*Sqrt[c + d*x])^(6 +
p))/(b^6*d^3*(6 + p))
```

Rubi [A] time = 0.410663, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$

$$-\frac{2a\left(a^2-b^2c\right)^2\left(a+b\sqrt{c+dx}\right)^{p+1}}{b^6d^3(p+1)}-\frac{4a\left(5a^2-3b^2c\right)\left(a+b\sqrt{c+dx}\right)^{p+3}}{b^6d^3(p+3)}+\frac{4\left(5a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{p+4}}{b^6d^3(p+4)}+\frac{2\left(5a^4-6a^2b^2c+b^4c^2\right)\left(a+b\sqrt{c+dx}\right)^{p+2}}{b^6d^3(p+2)}-\frac{10a\left(a+b\sqrt{c+dx}\right)^{p+5}}{b^6d^3(p+5)}+\frac{2\left(a+b\sqrt{c+dx}\right)^{p+6}}{b^6d^3(p+6)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(a + b*Sqrt[c + d*x])^p,x]
```


```
[Out] (-2*a*(a^2 - b^2*c)^2*(a + b*Sqrt[c + d*x])^(1 + p))/(b^6*d^3*(1
+ p)) + (2*(5*a^4 - 6*a^2*b^2*c + b^4*c^2)*(a + b*Sqrt[c + d*x])^
(2 + p))/(b^6*d^3*(2 + p)) - (4*a*(5*a^2 - 3*b^2*c)*(a + b*Sqrt[c
 + d*x])^(3 + p))/(b^6*d^3*(3 + p)) + (4*(5*a^2 - b^2*c)*(a + b*S
qrt[c + d*x])^(4 + p))/(b^6*d^3*(4 + p)) - (10*a*(a + b*Sqrt[c +
d*x])^(5 + p))/(b^6*d^3*(5 + p)) + (2*(a + b*Sqrt[c + d*x])^(6 +
```

```
p))/(b^6*d^3*(6 + p))
```

Rubi in Sympy [A] time = 33.7381, size = 221, normalized size = 0.91

$$-\frac{2a\left(a+b\sqrt{c+dx}\right)^{p+1}\left(a^2-b^2c\right)^2}{b^6d^3\left(p+1\right)}-\frac{4a\left(a+b\sqrt{c+dx}\right)^{p+3}\left(5a^2-3b^2c\right)}{b^6d^3\left(p+3\right)}-\frac{10a\left(a+b\sqrt{c+dx}\right)^{p+5}}{b^6d^3\left(p+5\right)}+\frac{2\left(a+b\sqrt{c+dx}\right)^{p+2}\left(5a^4-6a^2b^2c+b^4c^2\right)}{b^6d^3\left(p+2\right)}+\frac{4\left(a+b\sqrt{c+dx}\right)^{p+4}\left(5a^2-b^2c\right)}{b^6d^3\left(p+4\right)}+\frac{2\left(a+b\sqrt{c+dx}\right)^{p+6}}{b^6d^3\left(p+6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(a+b*(d*x+c)**(1/2))**p,x)
```

```
[Out] -2*a*(a + b*sqrt(c + d*x))**(p + 1)*(a**2 - b**2*c)**2/(b**6*d**3
*(p + 1)) - 4*a*(a + b*sqrt(c + d*x))**(p + 3)*(5*a**2 - 3*b**2*c
)/(b**6*d**3*(p + 3)) - 10*a*(a + b*sqrt(c + d*x))**(p + 5)/(b**6
*d**3*(p + 5)) + 2*(a + b*sqrt(c + d*x))**(p + 2)*(5*a**4 - 6*a**
2*b**2*c + b**4*c**2)/(b**6*d**3*(p + 2)) + 4*(a + b*sqrt(c + d*x
))**(p + 4)*(5*a**2 - b**2*c)/(b**6*d**3*(p + 4)) + 2*(a + b*sqrt
(c + d*x))**(p + 6)/(b**6*d**3*(p + 6))
```

Mathematica [A] time = 0.57644, size = 285, normalized size = 1.18

$$\frac{2\left(a+b\sqrt{c+dx}\right)^{p+1}\left(-120a^5+120a^4b(p+1)\sqrt{c+dx}+12a^3b^2\left(-4c\left(p^2+p-5\right)-5d\left(p^2+3p+2\right)x\right)-4a^2b^3(p+1)\sqrt{c+\cdots}\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(a + b*Sqrt[c + d*x])^p,x]
```

```
[Out] (2*(a + b*Sqrt[c + d*x])^(1 + p)*(-120*a^5 + 120*a^4*b*(1 + p)*Sq
rt[c + d*x] + 12*a^3*b^2*(-4*c*(-5 + p + p^2) - 5*d*(2 + 3*p + p^
2)*x) - 4*a^2*b^3*(1 + p)*Sqrt[c + d*x]*(c*(60 + 8*p - 2*p^2) - 5
*d*(6 + 5*p + p^2)*x) + b^5*(15 + 23*p + 9*p^2 + p^3)*Sqrt[c + d*
x]*(8*c^2 - 4*c*d*(2 + p)*x + d^2*(8 + 6*p + p^2)*x^2) - a*b^4*(-
8*c^2*(-15 + 10*p + 12*p^2 + 2*p^3) + 4*c*d*(-30 - 43*p - 10*p^2
+ 4*p^3 + p^4)*x + 5*d^2*(24 + 50*p + 35*p^2 + 10*p^3 + p^4)*x^2)
))/(b^6*d^3*(1 + p)*(2 + p)*(3 + p)*(4 + p)*(5 + p)*(6 + p))
```

Maple [F] time = 0.006, size = 0, normalized size = 0.

$$\int x^2 \left(a + b\sqrt{dx + c}\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(a+b*(d*x+c)^(1/2))^p,x)
```

```
[Out] int(x^2*(a+b*(d*x+c)^(1/2))^p,x)
```

Maxima [A] time = 0.729036, size = 543, normalized size = 2.24

$$2\left(\frac{\left((dx+c)b^2(p+1)+\sqrt{dx+c}\,abp-a^2\right)\left(\sqrt{dx+c}\,b+a\right)^p c^2}{(p^2+3\,p+2)b^2} - \frac{2\left((p^3+6\,p^2+11\,p+6)\,(dx+c)^2b^4+\left(p^3+3\,p^2+2\,p\right)(dx+c)^{\frac{3}{2}}\,ab^3-3\,\left(p^2+p\right)(dx+c)a^2b^2+6\,\sqrt{dx+c}\,a^3bp-6\right.}{(p^4+10\,p^3+35\,p^2+50\,p+24)b^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(d*x + c)*b + a)^p*x^2,x, algorithm="maxima")
```

```
[Out] 2*((((d*x + c)*b^2*(p + 1) + sqrt(d*x + c)*a*b*p - a^2)*(sqrt(d*x
+ c)*b + a)^p*c^2/((p^2 + 3*p + 2)*b^2) - 2*((p^3 + 6*p^2 + 11*p
+ 6)*(d*x + c)^2*b^4 + (p^3 + 3*p^2 + 2*p)*(d*x + c)^(3/2)*a*b^3
- 3*(p^2 + p)*(d*x + c)*a^2*b^2 + 6*sqrt(d*x + c)*a^3*b*p - 6*a^4
)*(sqrt(d*x + c)*b + a)^p*c/((p^4 + 10*p^3 + 35*p^2 + 50*p + 24)*
b^4) + ((p^5 + 15*p^4 + 85*p^3 + 225*p^2 + 274*p + 120)*(d*x + c)
^3*b^6 + (p^5 + 10*p^4 + 35*p^3 + 50*p^2 + 24*p)*(d*x + c)^(5/2)*
a*b^5 - 5*(p^4 + 6*p^3 + 11*p^2 + 6*p)*(d*x + c)^2*a^2*b^4 + 20*(
p^3 + 3*p^2 + 2*p)*(d*x + c)^(3/2)*a^3*b^3 - 60*(p^2 + p)*(d*x +
c)*a^4*b^2 + 120*sqrt(d*x + c)*a^5*b*p - 120*a^6)*(sqrt(d*x + c)*
b + a)^p/((p^6 + 21*p^5 + 175*p^4 + 735*p^3 + 1624*p^2 + 1764*p +
 720)*b^6))/d^3
```

Fricas [A] time = 0.359207, size = 961, normalized size = 3.97

$$\frac{2\left(120\,b^6c^3 - 360\,a^2b^4c^2 + 360\,a^4b^2c - 120\,a^6 + 8\,\left(b^6c^3 + 3\,a^2b^4c^2\right)p^3 + \left(b^6d^3p^5 + 15\,b^6d^3p^4 + 85\,b^6d^3p^3 + 225\,b^6d^3p^2 + 274\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(d*x + c)*b + a)^p*x^2,x, algorithm="fricas")
```

```
[Out] 2*(120*b^6*c^3 - 360*a^2*b^4*c^2 + 360*a^4*b^2*c - 120*a^6 + 8*(b
^6*c^3 + 3*a^2*b^4*c^2)*p^3 + (b^6*d^3*p^5 + 15*b^6*d^3*p^4 + 85*
b^6*d^3*p^3 + 225*b^6*d^3*p^2 + 274*b^6*d^3*p + 120*b^6*d^3)*x^3
+ 24*(3*b^6*c^3 + 3*a^2*b^4*c^2 - 2*a^4*b^2*c)*p^2 + (b^6*c*d^2*p
^5 + (11*b^6*c - 5*a^2*b^4)*d^2*p^4 + (41*b^6*c - 30*a^2*b^4)*d^2
*p^3 + (61*b^6*c - 55*a^2*b^4)*d^2*p^2 + 30*(b^6*c - a^2*b^4)*d^2
*p)*x^2 + 8*(23*b^6*c^3 - 24*a^2*b^4*c^2 + 9*a^4*b^2*c)*p - 4*((b
^6*c^2 + a^2*b^4*c)*d*p^4 + 3*(3*b^6*c^2 - a^2*b^4*c)*d*p^3 + (23
*b^6*c^2 - 34*a^2*b^4*c + 15*a^4*b^2)*d*p^2 + 15*(b^6*c^2 - 2*a^2
*b^4*c + a^4*b^2)*d*p)*x + (8*(3*a*b^5*c^2 + a^3*b^3*c)*p^3 + 24*
(7*a*b^5*c^2 - 3*a^3*b^3*c)*p^2 + (a*b^5*d^2*p^5 + 10*a*b^5*d^2*p
^4 + 35*a*b^5*d^2*p^3 + 50*a*b^5*d^2*p^2 + 24*a*b^5*d^2*p)*x^2 +
8*(33*a*b^5*c^2 - 40*a^3*b^3*c + 15*a^5*b)*p - 4*(2*a*b^5*c*d*p^4
 + 5*(3*a*b^5*c - a^3*b^3)*d*p^3 + (31*a*b^5*c - 15*a^3*b^3)*d*p^
2 + 2*(9*a*b^5*c - 5*a^3*b^3)*d*p)*x)*sqrt(d*x + c))*(sqrt(d*x +
c)*b + a)^p/(b^6*d^3*p^6 + 21*b^6*d^3*p^5 + 175*b^6*d^3*p^4 + 735
*b^6*d^3*p^3 + 1624*b^6*d^3*p^2 + 1764*b^6*d^3*p + 720*b^6*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(a+b*(d*x+c)**(1/2))**p,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 2.27939, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(d*x + c)*b + a)^p*x^2,x, algorithm="giac")
```

```
[Out] Done
```

3.494 $\int x\left(a+b\sqrt{c+dx}\right)^p\,dx$

Optimal. Leaf size=145

$$-\frac{2a\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{p+1}}{b^4d^2(p+1)}+\frac{2\left(3a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{p+2}}{b^4d^2(p+2)}-\frac{6a\left(a+b\sqrt{c+dx}\right)^{p+3}}{b^4d^2(p+3)}+\frac{2\left(a+b\sqrt{c+dx}\right)^{p+4}}{b^4d^2(p+4)}$$

```
[Out] (-2*a*(a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(1 + p))/(b^4*d^2*(1 +
p)) + (2*(3*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(2 + p))/(b^4*d^2*
(2 + p)) - (6*a*(a + b*Sqrt[c + d*x])^(3 + p))/(b^4*d^2*(3 + p))
+ (2*(a + b*Sqrt[c + d*x])^(4 + p))/(b^4*d^2*(4 + p))
```

Rubi [A] time = 0.241775, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$-\frac{2a\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{p+1}}{b^4d^2(p+1)}+\frac{2\left(3a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{p+2}}{b^4d^2(p+2)}-\frac{6a\left(a+b\sqrt{c+dx}\right)^{p+3}}{b^4d^2(p+3)}+\frac{2\left(a+b\sqrt{c+dx}\right)^{p+4}}{b^4d^2(p+4)}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(a + b*Sqrt[c + d*x])^p,x]
```

```
[Out] (-2*a*(a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(1 + p))/(b^4*d^2*(1 +
p)) + (2*(3*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(2 + p))/(b^4*d^2*
(2 + p)) - (6*a*(a + b*Sqrt[c + d*x])^(3 + p))/(b^4*d^2*(3 + p))
+ (2*(a + b*Sqrt[c + d*x])^(4 + p))/(b^4*d^2*(4 + p))
```

Rubi in Sympy [A] time = 17.4348, size = 129, normalized size = 0.89

$$-\frac{2a\left(a+b\sqrt{c+dx}\right)^{p+1}\left(a^2-b^2c\right)}{b^4d^2\left(p+1\right)}-\frac{6a\left(a+b\sqrt{c+dx}\right)^{p+3}}{b^4d^2\left(p+3\right)}+\frac{2\left(a+b\sqrt{c+dx}\right)^{p+2}\left(3a^2-b^2c\right)}{b^4d^2\left(p+2\right)}+\frac{2\left(a+b\sqrt{c+dx}\right)^{p+4}}{b^4d^2\left(p+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(a+b*(d*x+c)**(1/2))**p,x)
```

```
[Out] -2*a*(a + b*sqrt(c + d*x))**(p + 1)*(a**2 - b**2*c)/(b**4*d**2*(p
 + 1)) - 6*a*(a + b*sqrt(c + d*x))**(p + 3)/(b**4*d**2*(p + 3)) +
 2*(a + b*sqrt(c + d*x))**(p + 2)*(3*a**2 - b**2*c)/(b**4*d**2*(p
 + 2)) + 2*(a + b*sqrt(c + d*x))**(p + 4)/(b**4*d**2*(p + 4))
```

Mathematica [A] time = 0.292388, size = 128, normalized size = 0.88

$$-\frac{2\left(a+b\sqrt{c+dx}\right)^{p+1}\left(6a^3-6a^2b(p+1)\sqrt{c+dx}+ab^2\left(2c\left(p^2+p-3\right)+3d\left(p^2+3p+2\right)x\right)-b^3\left(p^2+4p+3\right)\sqrt{c+dx}(d(\ldots}{b^4d^2(p+1)(p+2)(p+3)(p+4)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(a + b*Sqrt[c + d*x])^p,x]
```

```
[Out] (-2*(a + b*Sqrt[c + d*x])^(1 + p)*(6*a^3 - 6*a^2*b*(1 + p)*Sqrt[c
 + d*x] - b^3*(3 + 4*p + p^2)*Sqrt[c + d*x]*(-2*c + d*(2 + p)*x)
+ a*b^2*(2*c*(-3 + p + p^2) + 3*d*(2 + 3*p + p^2)*x)))/(b^4*d^2*(
1 + p)*(2 + p)*(3 + p)*(4 + p))
```

Maple [F] time = 0.006, size = 0, normalized size = 0.

$$\int x\left(a+b\sqrt{dx+c}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(a+b*(d*x+c)^(1/2))^p,x)
```

```
[Out] int(x*(a+b*(d*x+c)^(1/2))^p,x)
```

Maxima [A] time = 0.71969, size = 252, normalized size = 1.74

$$-\frac{2\left(\frac{\left((dx+c)b^2(p+1)+\sqrt{dx+c}\,abp-a^2\right)\left(\sqrt{dx+c}\,b+a\right)^p c}{(p^2+3p+2)b^2}-\frac{\left((p^3+6p^2+11p+6)(dx+c)^2b^4+(p^3+3p^2+2p)(dx+c)^{\frac{3}{2}}ab^3-3\left(p^2+p\right)(dx+c)a^2b^2+6\sqrt{dx+c}\,a^3bp-6\ldots}{(p^4+10p^3+35p^2+50p+24)b^4}\right.}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(d*x + c)*b + a)^p*x,x, algorithm="maxima")
```

```
[Out] -2*(((d*x + c)*b^2*(p + 1) + sqrt(d*x + c)*a*b*p - a^2)*(sqrt(d*x
 + c)*b + a)^p*c/((p^2 + 3*p + 2)*b^2) - ((p^3 + 6*p^2 + 11*p + 6
)*(d*x + c)^2*b^4 + (p^3 + 3*p^2 + 2*p)*(d*x + c)^(3/2)*a*b^3 - 3
*(p^2 + p)*(d*x + c)*a^2*b^2 + 6*sqrt(d*x + c)*a^3*b*p - 6*a^4)*(
sqrt(d*x + c)*b + a)^p/((p^4 + 10*p^3 + 35*p^2 + 50*p + 24)*b^4))
/d^2
```

Fricas [A] time = 0.318172, size = 397, normalized size = 2.74

$$-\frac{2\left(6\,b^4c^2-12\,a^2b^2c+6\,a^4+2\left(b^4c^2+a^2b^2c\right)p^2-\left(b^4d^2p^3+6\,b^4d^2p^2+11\,b^4d^2p+6\,b^4d^2\right)x^2+4\left(2\,b^4c^2-a^2b^2c\right)p-\left(b^4c\cdots\right.\right.}{b^4d^2p^4+10\,b^4d\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(d*x + c)*b + a)^p*x,x, algorithm="fricas")
```

```
[Out] -2*(6*b^4*c^2 - 12*a^2*b^2*c + 6*a^4 + 2*(b^4*c^2 + a^2*b^2*c)*p^
2 - (b^4*d^2*p^3 + 6*b^4*d^2*p^2 + 11*b^4*d^2*p + 6*b^4*d^2)*x^2
+ 4*(2*b^4*c^2 - a^2*b^2*c)*p - (b^4*c*d*p^3 + (4*b^4*c - 3*a^2*b
^2)*d*p^2 + 3*(b^4*c - a^2*b^2)*d*p)*x + (4*a*b^3*c*p^2 + 2*(5*a*
b^3*c - 3*a^3*b)*p - (a*b^3*d*p^3 + 3*a*b^3*d*p^2 + 2*a*b^3*d*p)*
x)*sqrt(d*x + c))*(sqrt(d*x + c)*b + a)^p/(b^4*d^2*p^4 + 10*b^4*d
^2*p^3 + 35*b^4*d^2*p^2 + 50*b^4*d^2*p + 24*b^4*d^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(a+b*(d*x+c)**(1/2))**p,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.858036, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(d*x + c)*b + a)^p*x,x, algorithm="giac")
```

```
[Out] Done
```

3.495 $\int \left(a+b\sqrt{c+dx}\right)^p\,dx$

Optimal. Leaf size=62

$$\frac{2\left(a+b\sqrt{c+dx}\right)^{p+2}}{b^2d(p+2)}-\frac{2a\left(a+b\sqrt{c+dx}\right)^{p+1}}{b^2d(p+1)}$$

```
[Out] (-2*a*(a + b*Sqrt[c + d*x])^(1 + p))/(b^2*d*(1 + p)) + (2*(a + b*
Sqrt[c + d*x])^(2 + p))/(b^2*d*(2 + p))
```

Rubi [A] time = 0.0832109, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{2\left(a+b\sqrt{c+dx}\right)^{p+2}}{b^2d(p+2)}-\frac{2a\left(a+b\sqrt{c+dx}\right)^{p+1}}{b^2d(p+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*Sqrt[c + d*x])^p,x]
```

```
[Out] (-2*a*(a + b*Sqrt[c + d*x])^(1 + p))/(b^2*d*(1 + p)) + (2*(a + b*
Sqrt[c + d*x])^(2 + p))/(b^2*d*(2 + p))
```

Rubi in Sympy [A] time = 6.21274, size = 51, normalized size = 0.82

$$-\frac{2a\left(a+b\sqrt{c+dx}\right)^{p+1}}{b^2d\,(p+1)}+\frac{2\left(a+b\sqrt{c+dx}\right)^{p+2}}{b^2d\,(p+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b*(d*x+c)**(1/2))**p,x)
```

```
[Out] -2*a*(a + b*sqrt(c + d*x))**(p + 1)/(b**2*d*(p + 1)) + 2*(a + b*s
qrt(c + d*x))**(p + 2)/(b**2*d*(p + 2))
```

Mathematica [A] time = 0.0529063, size = 64, normalized size = 1.03

$$\frac{2\left(a+b\sqrt{c+dx}\right)^{p}\left(-a^{2}+abp\sqrt{c+dx}+b^{2}(p+1)(c+dx)\right)}{b^{2}d(p+1)(p+2)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*Sqrt[c + d*x])^p,x]
```

```
[Out] (2*(a + b*Sqrt[c + d*x])^p*(-a^2 + a*b*p*Sqrt[c + d*x] + b^2*(1 +
 p)*(c + d*x)))/(b^2*d*(1 + p)*(2 + p))
```

Maple [F] time = 0.007, size = 0, normalized size = 0.

$$\int\left(a+b\sqrt{dx+c}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b*(d*x+c)^(1/2))^p,x)
```

```
[Out] int((a+b*(d*x+c)^(1/2))^p,x)
```

Maxima [A] time = 0.706817, size = 81, normalized size = 1.31

$$\frac{2\left((dx+c)b^{2}(p+1)+\sqrt{dx+c}abp-a^{2}\right)\left(\sqrt{dx+c}b+a\right)^{p}}{(p^{2}+3\,p+2)b^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(d*x + c)*b + a)^p,x, algorithm="maxima")
```

```
[Out] 2*((d*x + c)*b^2*(p + 1) + sqrt(d*x + c)*a*b*p - a^2)*(sqrt(d*x +
 c)*b + a)^p/((p^2 + 3*p + 2)*b^2*d)
```

Fricas [A] time = 0.298043, size = 109, normalized size = 1.76

$$\frac{2\left(b^{2}cp+\sqrt{dx+c}abp+b^{2}c-a^{2}+\left(b^{2}dp+b^{2}d\right)x\right)\left(\sqrt{dx+c}b+a\right)^{p}}{b^{2}dp^{2}+3\,b^{2}dp+2\,b^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(d*x + c)*b + a)^p,x, algorithm="fricas")
```

```
[Out] 2*(b^2*c*p + sqrt(d*x + c)*a*b*p + b^2*c - a^2 + (b^2*d*p + b^2*d
)*x)*(sqrt(d*x + c)*b + a)^p/(b^2*d*p^2 + 3*b^2*d*p + 2*b^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b\sqrt{c+dx}\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b*(d*x+c)**(1/2))**p,x)
```

```
[Out] Integral((a + b*sqrt(c + d*x))**p, x)
```

GIAC/XCAS [A] time = 0.356475, size = 836, normalized size = 13.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(d*x + c)*b + a)^p,x, algorithm="giac")
```

```
[Out] 2*((sqrt(d*x + c)*b + a)*a*b*p*e^(p*ln((sqrt(d*x + c)*b + a)*sign
((sqrt(d*x + c)*b + a)*b - a*b) - a*sign((sqrt(d*x + c)*b + a)*b
- a*b) + a))*sign((sqrt(d*x + c)*b + a)*b - a*b) - a^2*b*p*e^(p*l
n((sqrt(d*x + c)*b + a)*sign((sqrt(d*x + c)*b + a)*b - a*b) - a*s
ign((sqrt(d*x + c)*b + a)*b - a*b) + a))*sign((sqrt(d*x + c)*b +
a)*b - a*b) + (sqrt(d*x + c)*b + a)^2*b*p*e^(p*ln((sqrt(d*x + c)*
b + a)*sign((sqrt(d*x + c)*b + a)*b - a*b) - a*sign((sqrt(d*x + c
)*b + a)*b - a*b) + a)) - 2*(sqrt(d*x + c)*b + a)*a*b*p*e^(p*ln((
sqrt(d*x + c)*b + a)*sign((sqrt(d*x + c)*b + a)*b - a*b) - a*sign
((sqrt(d*x + c)*b + a)*b - a*b) + a)) + a^2*b*p*e^(p*ln((sqrt(d*x
 + c)*b + a)*sign((sqrt(d*x + c)*b + a)*b - a*b) - a*sign((sqrt(d
*x + c)*b + a)*b - a*b) + a)) + (sqrt(d*x + c)*b + a)^2*b*e^(p*ln
((sqrt(d*x + c)*b + a)*sign((sqrt(d*x + c)*b + a)*b - a*b) - a*si
gn((sqrt(d*x + c)*b + a)*b - a*b) + a)) - 2*(sqrt(d*x + c)*b + a)
*a*b*e^(p*ln((sqrt(d*x + c)*b + a)*sign((sqrt(d*x + c)*b + a)*b -
 a*b) - a*sign((sqrt(d*x + c)*b + a)*b - a*b) + a)))/((p^2*sign((
sqrt(d*x + c)*b + a)*b - a*b) + 3*p*sign((sqrt(d*x + c)*b + a)*b
- a*b) + 2*sign((sqrt(d*x + c)*b + a)*b - a*b))*b^3*d)
```

3.496 $\int \frac{\left(a+b\sqrt{c+dx}\right)^p}{x}\,dx$

Optimal. Leaf size=139

$$-\frac{\left(a+b\sqrt{c+dx}\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{a+b\sqrt{c+dx}}{a-b\sqrt{c}}\right)}{(p+1)\left(a-b\sqrt{c}\right)}-\frac{\left(a+b\sqrt{c+dx}\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{a+b\sqrt{c+dx}}{a+b\sqrt{c}}\right)}{(p+1)\left(a+b\sqrt{c}\right)}$$

```
[Out] -(((a + b*Sqrt[c + d*x])^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 +
p, (a + b*Sqrt[c + d*x])/(a - b*Sqrt[c])])/((a - b*Sqrt[c])*(1 +
p))) - ((a + b*Sqrt[c + d*x])^(1 + p)*Hypergeometric2F1[1, 1 + p,
 2 + p, (a + b*Sqrt[c + d*x])/(a + b*Sqrt[c])])/((a + b*Sqrt[c])*
(1 + p))
```

Rubi [A] time = 0.256577, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21

$$-\frac{\left(a+b\sqrt{c+dx}\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{a+b\sqrt{c+dx}}{a-b\sqrt{c}}\right)}{(p+1)\left(a-b\sqrt{c}\right)}-\frac{\left(a+b\sqrt{c+dx}\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{a+b\sqrt{c+dx}}{a+b\sqrt{c}}\right)}{(p+1)\left(a+b\sqrt{c}\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*Sqrt[c + d*x])^p/x,x]
```

```
[Out] -(((a + b*Sqrt[c + d*x])^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 +
p, (a + b*Sqrt[c + d*x])/(a - b*Sqrt[c])])/((a - b*Sqrt[c])*(1 +
p))) - ((a + b*Sqrt[c + d*x])^(1 + p)*Hypergeometric2F1[1, 1 + p,
 2 + p, (a + b*Sqrt[c + d*x])/(a + b*Sqrt[c])])/((a + b*Sqrt[c])*
(1 + p))
```

Rubi in Sympy [A] time = 11.6762, size = 105, normalized size = 0.76

$$-\frac{\left(a+b\sqrt{c+dx}\right)^{p+1}{}_2F_1\left(\begin{matrix}1,p+1\\p+2\end{matrix}\middle|\frac{a+b\sqrt{c+dx}}{a+b\sqrt{c}}\right)}{\left(a+b\sqrt{c}\right)(p+1)}-\frac{\left(a+b\sqrt{c+dx}\right)^{p+1}{}_2F_1\left(\begin{matrix}1,p+1\\p+2\end{matrix}\middle|\frac{a+b\sqrt{c+dx}}{a-b\sqrt{c}}\right)}{\left(a-b\sqrt{c}\right)(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b*(d*x+c)**(1/2))**p/x,x)
```

```
[Out] -(a + b*sqrt(c + d*x))**(p + 1)*hyper((1, p + 1), (p + 2,), (a +
b*sqrt(c + d*x))/(a + b*sqrt(c)))/((a + b*sqrt(c))*(p + 1)) - (a
+ b*sqrt(c + d*x))**(p + 1)*hyper((1, p + 1), (p + 2,), (a + b*sq
rt(c + d*x))/(a - b*sqrt(c)))/((a - b*sqrt(c))*(p + 1))
```

Mathematica [A] time = 0.474265, size = 189, normalized size = 1.36

$$\frac{\left(a+b\sqrt{c+dx}\right)^p\left(\left(\frac{a+b\sqrt{c+dx}}{b\sqrt{c+dx}-b\sqrt{c}}\right)^{-p}{}_2F_1\left(-p,-p;1-p;\frac{a+b\sqrt{c}}{b\sqrt{c}-b\sqrt{c+dx}}\right)+\left(\frac{a+b\sqrt{c+dx}}{b\sqrt{c+dx}+b\sqrt{c}}\right)^{-p}{}_2F_1\left(-p,-p;1-p;\frac{b\sqrt{c}-a}{b\left(\sqrt{c}+\sqrt{c+dx}\right)}\right)\right)}{p}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*Sqrt[c + d*x])^p/x,x]
```

```
[Out] ((a + b*Sqrt[c + d*x])^p*(Hypergeometric2F1[-p, -p, 1 - p, (-a +
b*Sqrt[c])/(b*(Sqrt[c] + Sqrt[c + d*x]))]/((a + b*Sqrt[c + d*x])/
(b*Sqrt[c] + b*Sqrt[c + d*x]))^p + Hypergeometric2F1[-p, -p, 1 -
p, (a + b*Sqrt[c])/(b*Sqrt[c] - b*Sqrt[c + d*x])]/((a + b*Sqrt[c
+ d*x])/(-(b*Sqrt[c]) + b*Sqrt[c + d*x]))^p))/p
```

Maple [F] time = 0.006, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left(a+b\sqrt{dx+c}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b*(d*x+c)^(1/2))^p/x,x)
```

```
[Out] int((a+b*(d*x+c)^(1/2))^p/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\sqrt{dx+c}b+a\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(d*x + c)*b + a)^p/x,x, algorithm="maxima")

[Out] integrate((sqrt(d*x + c)*b + a)^p/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(\sqrt{dx+c}b+a\right)^{p}}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(d*x + c)*b + a)^p/x,x, algorithm="fricas")

[Out] integral((sqrt(d*x + c)*b + a)^p/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+b\sqrt{c+dx}\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b*(d*x+c)**(1/2))**p/x,x)

[Out] Integral((a + b*sqrt(c + d*x))**p/x, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(\sqrt{dx+c}b+a\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(d*x + c)*b + a)^p/x,x, algorithm="giac")

[Out] integrate((sqrt(d*x + c)*b + a)^p/x, x)
```

3.497 $\int \frac{(a+b(cx)^n)^{5/2}}{x} dx$

Optimal. Leaf size=93

$$-\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}+\frac{2a^2\sqrt{a+b(cx)^n}}{n}+\frac{2a\left(a+b(cx)^n\right)^{3/2}}{3n}+\frac{2\left(a+b(cx)^n\right)^{5/2}}{5n}$$

```
[Out] (2*a^2*Sqrt[a + b*(c*x)^n])/n + (2*a*(a + b*(c*x)^n)^(3/2))/(3*n)
 + (2*(a + b*(c*x)^n)^(5/2))/(5*n) - (2*a^(5/2)*ArcTanh[Sqrt[a +
b*(c*x)^n]/Sqrt[a]])/n
```

Rubi [A] time = 0.170548, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353

$$-\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}+\frac{2a^2\sqrt{a+b(cx)^n}}{n}+\frac{2a\left(a+b(cx)^n\right)^{3/2}}{3n}+\frac{2\left(a+b(cx)^n\right)^{5/2}}{5n}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*(c*x)^n)^(5/2)/x,x]
```

```
[Out] (2*a^2*Sqrt[a + b*(c*x)^n])/n + (2*a*(a + b*(c*x)^n)^(3/2))/(3*n)
 + (2*(a + b*(c*x)^n)^(5/2))/(5*n) - (2*a^(5/2)*ArcTanh[Sqrt[a +
b*(c*x)^n]/Sqrt[a]])/n
```

Rubi in Sympy [A] time = 7.18671, size = 80, normalized size = 0.86

$$-\frac{2a^{\frac{5}{2}}\operatorname{atanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}+\frac{2a^2\sqrt{a+b\left(cx\right)^n}}{n}+\frac{2a\left(a+b\left(cx\right)^n\right)^{\frac{3}{2}}}{3n}+\frac{2\left(a+b\left(cx\right)^n\right)^{\frac{5}{2}}}{5n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b*(c*x)**n)**(5/2)/x,x)
```

```
[Out] -2*a**(5/2)*atanh(sqrt(a + b*(c*x)**n)/sqrt(a))/n + 2*a**2*sqrt(a
 + b*(c*x)**n)/n + 2*a*(a + b*(c*x)**n)**(3/2)/(3*n) + 2*(a + b*(
c*x)**n)**(5/2)/(5*n)
```

Mathematica [A] time = 0.0969101, size = 77, normalized size = 0.83

$$\frac{2\sqrt{a+b(cx)^n}\left(23a^2+11ab(cx)^n+3b^2(cx)^{2n}\right)-30a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{15n}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*(c*x)^n)^(5/2)/x,x]
```

```
[Out] (2*Sqrt[a + b*(c*x)^n]*(23*a^2 + 11*a*b*(c*x)^n + 3*b^2*(c*x)^(2*
n)) - 30*a^(5/2)*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/(15*n)
```

Maple [A] time = 0.009, size = 70, normalized size = 0.8

$$\frac{1}{n}\left(\frac{2}{5}(a+b(cx)^n)^{\frac{5}{2}}+\frac{2a}{3}(a+b(cx)^n)^{\frac{3}{2}}+2\sqrt{a+b(cx)^n}a^2-2a^{5/2}Artanh\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b*(c*x)^n)^(5/2)/x,x)
```

```
[Out] 1/n*(2/5*(a+b*(c*x)^n)^(5/2)+2/3*(a+b*(c*x)^n)^(3/2)*a+2*(a+b*(c*
x)^n)^(1/2)*a^2-2*a^(5/2)*arctanh((a+b*(c*x)^n)^(1/2)/a^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((c*x)^n*b + a)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.284281, size = 1, normalized size = 0.01

$$\left[\frac{15\,a^{\frac{5}{2}}\log\left(\frac{(cx)^{n}b-2\,\sqrt{(cx)^{n}b+a}\sqrt{a}+2\,a}{(cx)^{n}}\right)+2\left(11\,(cx)^{n}ab+3\,(cx)^{2\,n}b^{2}+23\,a^{2}\right)\sqrt{(cx)^{n}b+a}}{15\,n},\ -\frac{2\left(15\,\sqrt{-a}a^{2}\arctan\left(\frac{\sqrt{(cx)^{n}b+a}}{\sqrt{-a}}\right)-\left(11\,(cx)^{n}ab+3\,(cx)^{2\,n}b^{2}+23\,a^{2}\right)\sqrt{(cx)^{n}b+a}\right)}{15\,n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((c*x)^n*b + a)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] [1/15*(15*a^(5/2)*log(((c*x)^n*b - 2*sqrt((c*x)^n*b + a)*sqrt(a)
+ 2*a)/(c*x)^n) + 2*(11*(c*x)^n*a*b + 3*(c*x)^(2*n)*b^2 + 23*a^2)
*sqrt((c*x)^n*b + a))/n, -2/15*(15*sqrt(-a)*a^2*arctan(sqrt((c*x)
^n*b + a)/sqrt(-a)) - (11*(c*x)^n*a*b + 3*(c*x)^(2*n)*b^2 + 23*a^
2)*sqrt((c*x)^n*b + a))/n]
```

Sympy [A] time = 108.802, size = 189, normalized size = 2.03

$$\begin{cases}\frac{-2a^{3}\left(\begin{cases}-\frac{\operatorname{atan}\left(\frac{\sqrt{a+b(cx)^{n}}}{\sqrt{-a}}\right)}{\sqrt{-a}} & \text{for}\ -a>0\\ \frac{\operatorname{acoth}\left(\frac{\sqrt{a+b(cx)^{n}}}{\sqrt{a}}\right)}{\sqrt{a}} & \text{for}\ -a<0\wedge a<a+b\,(cx)^{n}\\ \frac{\operatorname{atanh}\left(\frac{\sqrt{a+b(cx)^{n}}}{\sqrt{a}}\right)}{\sqrt{a}} & \text{for}\ a>a+b\,(cx)^{n}\wedge -a<0\end{cases}\right)+2a^{2}\sqrt{a+b(cx)^{n}}+\frac{2a\left(a+b(cx)^{n}\right)^{\frac{3}{2}}}{3}+\frac{2\left(a+b(cx)^{n}\right)^{\frac{5}{2}}}{5}}{n} & \text{for}\ n\neq 0\\ \left(a^{2}\sqrt{a+b}+2ab\sqrt{a+b}+b^{2}\sqrt{a+b}\right)\log(cx) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b*(c*x)**n)**(5/2)/x,x)
```

```
[Out] Piecewise(((-2*a**3*Piecewise((-atan(sqrt(a + b*(c*x)**n)/sqrt(-a
))/sqrt(-a), -a > 0), (acoth(sqrt(a + b*(c*x)**n)/sqrt(a))/sqrt(a
), (-a < 0) & (a < a + b*(c*x)**n)), (atanh(sqrt(a + b*(c*x)**n)/
sqrt(a))/sqrt(a), (-a < 0) & (a > a + b*(c*x)**n))) + 2*a**2*sqrt
(a + b*(c*x)**n) + 2*a*(a + b*(c*x)**n)**(3/2)/3 + 2*(a + b*(c*x)
**n)**(5/2)/5)/n, Ne(n, 0)), ((a**2*sqrt(a + b) + 2*a*b*sqrt(a +
b) + b**2*sqrt(a + b))*log(c*x), True))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{((cx)^n b + a)^{\frac{5}{2}}}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((c*x)^n*b + a)^(5/2)/x,x, algorithm="giac")

[Out] integrate(((c*x)^n*b + a)^(5/2)/x, x)
```

3.498 $\int \frac{(a+b(cx)^n)^{3/2}}{x} \, dx$

Optimal. Leaf size=70

$$-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}+\frac{2a\sqrt{a+b(cx)^n}}{n}+\frac{2\left(a+b(cx)^n\right)^{3/2}}{3n}$$

```
[Out] (2*a*Sqrt[a + b*(c*x)^n])/n + (2*(a + b*(c*x)^n)^(3/2))/(3*n) - (
2*a^(3/2)*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/n
```

Rubi [A] time = 0.132647, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353

$$-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}+\frac{2a\sqrt{a+b(cx)^n}}{n}+\frac{2\left(a+b(cx)^n\right)^{3/2}}{3n}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*(c*x)^n)^(3/2)/x,x]
```

```
[Out] (2*a*Sqrt[a + b*(c*x)^n])/n + (2*(a + b*(c*x)^n)^(3/2))/(3*n) - (
2*a^(3/2)*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/n
```

Rubi in Sympy [A] time = 5.74466, size = 60, normalized size = 0.86

$$-\frac{2a^{\frac{3}{2}}\operatorname{atanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}+\frac{2a\sqrt{a+b\,(cx)^n}}{n}+\frac{2\left(a+b\,(cx)^n\right)^{\frac{3}{2}}}{3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b*(c*x)**n)**(3/2)/x,x)
```

```
[Out] -2*a**(3/2)*atanh(sqrt(a + b*(c*x)**n)/sqrt(a))/n + 2*a*sqrt(a +
b*(c*x)**n)/n + 2*(a + b*(c*x)**n)**(3/2)/(3*n)
```

Mathematica [A] time = 0.0596122, size = 61, normalized size = 0.87

$$\frac{2\sqrt{a+b(cx)^n}\,(4a+b(cx)^n)-6a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{3n}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*(c*x)^n)^(3/2)/x,x]
```

```
[Out] (2*Sqrt[a + b*(c*x)^n]*(4*a + b*(c*x)^n) - 6*a^(3/2)*ArcTanh[Sqrt
[a + b*(c*x)^n]/Sqrt[a]])/(3*n)
```

Maple [A] time = 0.005, size = 54, normalized size = 0.8

$$\frac{1}{n}\left(\frac{2}{3}\left(a+b\left(cx\right)^{n}\right)^{\frac{3}{2}}+2\sqrt{a+b\left(cx\right)^{n}}a-2a^{3/2}Artanh\left(\frac{\sqrt{a+b\left(cx\right)^{n}}}{\sqrt{a}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b*(c*x)^n)^(3/2)/x,x)
```

```
[Out] 1/n*(2/3*(a+b*(c*x)^n)^(3/2)+2*(a+b*(c*x)^n)^(1/2)*a-2*a^(3/2)*ar
ctanh((a+b*(c*x)^n)^(1/2)/a^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((c*x)^n*b + a)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.289642, size = 1, normalized size = 0.01

$$\left[\frac{3\,a^{\frac{3}{2}}\log\left(\frac{(cx)^{n}b-2\,\sqrt{(cx)^{n}b+a}\sqrt{a}+2\,a}{(cx)^{n}}\right)+2\,((cx)^{n}\,b+4\,a)\sqrt{(cx)^{n}\,b+a}}{3\,n},\right.$$
$$\left.-\frac{2\left(3\,\sqrt{-a}a\arctan\left(\frac{\sqrt{(cx)^{n}b+a}}{\sqrt{-a}}\right)-((cx)^{n}\,b+4\,a)\sqrt{(cx)^{n}\,b+a}\right)}{3\,n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((c*x)^n*b + a)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/3*(3*a^(3/2)*log(((c*x)^n*b - 2*sqrt((c*x)^n*b + a)*sqrt(a) +
2*a)/(c*x)^n) + 2*((c*x)^n*b + 4*a)*sqrt((c*x)^n*b + a))/n, -2/3*
(3*sqrt(-a)*a*arctan(sqrt((c*x)^n*b + a)/sqrt(-a)) - ((c*x)^n*b +
 4*a)*sqrt((c*x)^n*b + a))/n]
```

Sympy [A] time = 14.2866, size = 153, normalized size = 2.19

$$\begin{cases}\frac{-2a^{2}\left(\begin{cases}-\frac{\operatorname{atan}\left(\frac{\sqrt{a+b(cx)^{n}}}{\sqrt{-a}}\right)}{\sqrt{-a}} & \text{for}\ -a>0\\ \frac{\operatorname{acoth}\left(\frac{\sqrt{a+b(cx)^{n}}}{\sqrt{a}}\right)}{\sqrt{a}} & \text{for}\ -a<0\wedge a<a+b\,(cx)^{n}\\ \frac{\operatorname{atanh}\left(\frac{\sqrt{a+b(cx)^{n}}}{\sqrt{a}}\right)}{\sqrt{a}} & \text{for}\ a>a+b\,(cx)^{n}\wedge -a<0\end{cases}\right)+2a\sqrt{a+b(cx)^{n}}+\frac{2\left(a+b(cx)^{n}\right)^{\frac{3}{2}}}{3}}{n} & \text{for}\ n\neq 0\\ \left(a\sqrt{a+b}+b\sqrt{a+b}\right)\log(x) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b*(c*x)**n)**(3/2)/x,x)
```

```
[Out] Piecewise(((-2*a**2*Piecewise((-atan(sqrt(a + b*(c*x)**n)/sqrt(-a
))/sqrt(-a), -a > 0), (acoth(sqrt(a + b*(c*x)**n)/sqrt(a))/sqrt(a
), (-a < 0) & (a < a + b*(c*x)**n)), (atanh(sqrt(a + b*(c*x)**n)/
sqrt(a))/sqrt(a), (-a < 0) & (a > a + b*(c*x)**n))) + 2*a*sqrt(a
+ b*(c*x)**n) + 2*(a + b*(c*x)**n)**(3/2)/3)/n, Ne(n, 0)), ((a*sq
rt(a + b) + b*sqrt(a + b))*log(x), True))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{((cx)^n b + a)^{\frac{3}{2}}}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((c*x)^n*b + a)^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate(((c*x)^n*b + a)^(3/2)/x, x)
```

3.499 $\int \frac{\sqrt{a+b(cx)^n}}{x}\,dx$

Optimal. Leaf size=49

$$\frac{2\sqrt{a+b(cx)^n}}{n}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}$$

```
[Out] (2*Sqrt[a + b*(c*x)^n])/n - (2*Sqrt[a]*ArcTanh[Sqrt[a + b*(c*x)^n
]/Sqrt[a]])/n
```

Rubi [A] time = 0.101153, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353

$$\frac{2\sqrt{a+b(cx)^n}}{n}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*(c*x)^n]/x,x]
```

```
[Out] (2*Sqrt[a + b*(c*x)^n])/n - (2*Sqrt[a]*ArcTanh[Sqrt[a + b*(c*x)^n
]/Sqrt[a]])/n
```

Rubi in Sympy [A] time = 4.67254, size = 41, normalized size = 0.84

$$-\frac{2\sqrt{a}\,\operatorname{atanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}+\frac{2\sqrt{a+b\,(cx)^n}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b*(c*x)**n)**(1/2)/x,x)
```

```
[Out] -2*sqrt(a)*atanh(sqrt(a + b*(c*x)**n)/sqrt(a))/n + 2*sqrt(a + b*(
c*x)**n)/n
```

Mathematica [A] time = 0.0294167, size = 46, normalized size = 0.94

$$\frac{2\left(\sqrt{a+b(cx)^n}-\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)\right)}{n}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b*(c*x)^n]/x,x]
```

```
[Out] (2*(Sqrt[a + b*(c*x)^n] - Sqrt[a]*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqr
t[a]]))/n
```

Maple [A] time = 0.002, size = 40, normalized size = 0.8

$$\frac{1}{n}\left(2\sqrt{a+b\,(cx)^{n}}-2\,\sqrt{a}Artanh\left(\frac{\sqrt{a+b\,(cx)^{n}}}{\sqrt{a}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b*(c*x)^n)^(1/2)/x,x)
```

```
[Out] 1/n*(2*(a+b*(c*x)^n)^(1/2)-2*a^(1/2)*arctanh((a+b*(c*x)^n)^(1/2)/
a^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((c*x)^n*b + a)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.284205, size = 1, normalized size = 0.02

$$\left[\frac{\sqrt{a}\log\left(\frac{(cx)^{n}b-2\,\sqrt{(cx)^{n}b+a}\sqrt{a}+2\,a}{(cx)^{n}}\right)+2\,\sqrt{(cx)^{n}\,b+a}}{n},\,-\frac{2\left(\sqrt{-a}\arctan\left(\frac{\sqrt{(cx)^{n}b+a}}{\sqrt{-a}}\right)-\sqrt{(cx)^{n}\,b+a}\right)}{n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((c*x)^n*b + a)/x,x, algorithm="fricas")
```

```
[Out] [(sqrt(a)*log(((c*x)^n*b - 2*sqrt((c*x)^n*b + a)*sqrt(a) + 2*a)/(
c*x)^n) + 2*sqrt((c*x)^n*b + a))/n, -2*(sqrt(-a)*arctan(sqrt((c*x
)^n*b + a)/sqrt(-a)) - sqrt((c*x)^n*b + a))/n]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + b\left(cx\right)^{n}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b*(c*x)**n)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(a + b*(c*x)**n)/x, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\left(cx\right)^{n} b + a}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((c*x)^n*b + a)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt((c*x)^n*b + a)/x, x)
```

3.500 $\int \frac{1}{x\sqrt{a+b(cx)^n}}\,dx$

Optimal. Leaf size=30

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{\sqrt{a}n}$$

```
[Out] (-2*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/(Sqrt[a]*n)
```

Rubi [A] time = 0.0772551, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{\sqrt{a}n}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*Sqrt[a + b*(c*x)^n]),x]

[Out] (-2*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/(Sqrt[a]*n)
```

Rubi in Sympy [A] time = 3.6891, size = 27, normalized size = 0.9

$$-\frac{2\,\text{atanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{\sqrt{a}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(a+b*(c*x)**n)**(1/2),x)

[Out] -2*atanh(sqrt(a + b*(c*x)**n)/sqrt(a))/(sqrt(a)*n)
```

Mathematica [A] time = 0.0234624, size = 30, normalized size = 1.

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{\sqrt{a}n}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*Sqrt[a + b*(c*x)^n]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/(Sqrt[a]*n)
```

Maple [A] time = 0.007, size = 25, normalized size = 0.8

$$-2\,\frac{1}{n\sqrt{a}}\mathit{Artanh}\left(\frac{\sqrt{a+b\,(cx)^n}}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(a+b*(c*x)^n)^(1/2),x)
```

```
[Out] -2*arctanh((a+b*(c*x)^n)^(1/2)/a^(1/2))/n/a^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt((c*x)^n*b + a)*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.281912, size = 1, normalized size = 0.03

$$\left[\frac{\log\left(\frac{(cx)^n\sqrt{a}b-2\sqrt{(cx)^n b+a}a+2\,a^{\frac{3}{2}}}{(cx)^n}\right)}{\sqrt{a}n},\ \frac{2\,\arctan\left(\frac{a}{\sqrt{(cx)^n b+a}\sqrt{-a}}\right)}{\sqrt{-a}n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt((c*x)^n*b + a)*x),x, algorithm="fricas")
```

```
[Out] [log(((c*x)^n*sqrt(a)*b - 2*sqrt((c*x)^n*b + a)*a + 2*a^(3/2))/(c
*x)^n)/(sqrt(a)*n), 2*arctan(a/(sqrt((c*x)^n*b + a)*sqrt(-a)))/(s
qrt(-a)*n)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{a + b\,(cx)^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(a+b*(c*x)**n)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + b*(c*x)**n)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(cx)^n\,b + a}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt((c*x)^n*b + a)*x),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt((c*x)^n*b + a)*x), x)
```

3.501 $\int \frac{1}{x(a+b(cx)^n)^{3/2}}\,dx$

Optimal. Leaf size=52

$$\frac{2}{an\sqrt{a+b(cx)^n}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{3/2}n}$$

```
[Out] 2/(a*n*Sqrt[a + b*(c*x)^n]) - (2*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt
[a]])/(a^(3/2)*n)
```

Rubi [A] time = 0.112795, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353

$$\frac{2}{an\sqrt{a+b(cx)^n}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{3/2}n}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(a + b*(c*x)^n)^(3/2)),x]
```

```
[Out] 2/(a*n*Sqrt[a + b*(c*x)^n]) - (2*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt
[a]])/(a^(3/2)*n)
```

Rubi in Sympy [A] time = 4.9284, size = 42, normalized size = 0.81

$$\frac{2}{an\sqrt{a+b\,(cx)^n}}-\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(a+b*(c*x)**n)**(3/2),x)
```

```
[Out] 2/(a*n*sqrt(a + b*(c*x)**n)) - 2*atanh(sqrt(a + b*(c*x)**n)/sqrt(
a))/(a**(3/2)*n)
```

Mathematica [A] time = 0.0604115, size = 52, normalized size = 1.

$$\frac{2}{an\sqrt{a+b(cx)^n}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{3/2}n}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(a + b*(c*x)^n)^(3/2)),x]
```

```
[Out] 2/(a*n*Sqrt[a + b*(c*x)^n]) - (2*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt
[a]])/(a^(3/2)*n)
```

Maple [A] time = 0.007, size = 43, normalized size = 0.8

$$\frac{1}{n}\left(-2\,\frac{1}{a^{3/2}}Artanh\left(\frac{\sqrt{a+b\,(cx)^n}}{\sqrt{a}}\right)+2\,\frac{1}{\sqrt{a+b\,(cx)^n}\,a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(a+b*(c*x)^n)^(3/2),x)
```

```
[Out] 1/n*(-2/a^(3/2)*arctanh((a+b*(c*x)^n)^(1/2)/a^(1/2))+2/a/(a+b*(c*
x)^n)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{((cx)^n\,b+a)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(((c*x)^n*b + a)^(3/2)*x),x, algorithm="maxima")
```

```
[Out] integrate(1/(((c*x)^n*b + a)^(3/2)*x), x)
```

Fricas [A] time = 0.285816, size = 1, normalized size = 0.02

$$\left[\frac{\sqrt{(cx)^n b + a}\log\left(\frac{(cx)^n\sqrt{a}b - 2\sqrt{(cx)^n b + a}a + 2a^{\frac{3}{2}}}{(cx)^n}\right) + 2\sqrt{a}}{\sqrt{(cx)^n b + a}a^{\frac{3}{2}}n}, \frac{2\left(\sqrt{(cx)^n b + a}\arctan\left(\frac{a}{\sqrt{(cx)^n b + a}\sqrt{-a}}\right) + \sqrt{-a}\right)}{\sqrt{(cx)^n b + a}\sqrt{-a}an}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((((c*x)^n*b + a)^(3/2)*x),x, algorithm="fricas")
```

```
[Out] [(sqrt((c*x)^n*b + a)*log(((c*x)^n*sqrt(a)*b - 2*sqrt((c*x)^n*b +
 a)*a + 2*a^(3/2))/(c*x)^n) + 2*sqrt(a))/(sqrt((c*x)^n*b + a)*a^(
3/2)*n), 2*(sqrt((c*x)^n*b + a)*arctan(a/(sqrt((c*x)^n*b + a)*sqr
t(-a))) + sqrt(-a))/(sqrt((c*x)^n*b + a)*sqrt(-a)*a*n)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a + b\left(cx\right)^{n}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(a+b*(c*x)**n)**(3/2),x)
```

```
[Out] Integral(1/(x*(a + b*(c*x)**n)**(3/2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left((cx)^n b + a\right)^{\frac{3}{2}}x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((((c*x)^n*b + a)^(3/2)*x),x, algorithm="giac")
```

```
[Out] integrate(1/((((c*x)^n*b + a)^(3/2)*x), x)
```

3.502 $\int \frac{1}{x(a+b(cx)^n)^{5/2}}\,dx$

Optimal. Leaf size=75

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{5/2}n}+\frac{2}{a^2n\sqrt{a+b(cx)^n}}+\frac{2}{3an\left(a+b(cx)^n\right)^{3/2}}$$

```
[Out] 2/(3*a*n*(a + b*(c*x)^n)^(3/2)) + 2/(a^2*n*Sqrt[a + b*(c*x)^n]) -
 (2*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/(a^(5/2)*n)
```

Rubi [A] time = 0.146465, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{5/2}n}+\frac{2}{a^2n\sqrt{a+b(cx)^n}}+\frac{2}{3an\left(a+b(cx)^n\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(a + b*(c*x)^n)^(5/2)),x]
```

```
[Out] 2/(3*a*n*(a + b*(c*x)^n)^(3/2)) + 2/(a^2*n*Sqrt[a + b*(c*x)^n]) -
 (2*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/(a^(5/2)*n)
```

Rubi in Sympy [A] time = 6.5052, size = 63, normalized size = 0.84

$$\frac{2}{3an\left(a+b\left(cx\right)^{n}\right)^{\frac{3}{2}}}+\frac{2}{a^{2}n\sqrt{a+b\left(cx\right)^{n}}}-\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(a+b*(c*x)**n)**(5/2),x)
```

```
[Out] 2/(3*a*n*(a + b*(c*x)**n)**(3/2)) + 2/(a**2*n*sqrt(a + b*(c*x)**n
)) - 2*atanh(sqrt(a + b*(c*x)**n)/sqrt(a))/(a**(5/2)*n)
```

Mathematica [A] time = 0.153711, size = 66, normalized size = 0.88

$$\frac{2\left(\frac{\sqrt{a}(4a+3b(cx)^n)}{(a+b(cx)^n)^{3/2}}-3\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)\right)}{3a^{5/2}n}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(a + b*(c*x)^n)^(5/2)),x]
```

```
[Out] (2*((Sqrt[a]*(4*a + 3*b*(c*x)^n))/(a + b*(c*x)^n)^(3/2) - 3*ArcTa
nh[Sqrt[a + b*(c*x)^n]/Sqrt[a]]))/(3*a^(5/2)*n)
```

Maple [A] time = 0.011, size = 59, normalized size = 0.8

$$\frac{1}{n}\left(-2\frac{1}{a^{5/2}}Artanh\left(\frac{\sqrt{a+b\,(cx)^n}}{\sqrt{a}}\right)+2\frac{1}{\sqrt{a+b\,(cx)^n}\,a^2}+\frac{2}{3\,a}\left(a+b\,(cx)^n\right)^{-\frac{3}{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(a+b*(c*x)^n)^(5/2),x)
```

```
[Out] 1/n*(-2/a^(5/2)*arctanh((a+b*(c*x)^n)^(1/2)/a^(1/2))+2/a^2/(a+b*(
c*x)^n)^(1/2)+2/3/a/(a+b*(c*x)^n)^(3/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{((cx)^n\,b+a)^{\frac{5}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(((c*x)^n*b + a)^(5/2)*x),x, algorithm="maxima")
```

```
[Out] integrate(1/(((c*x)^n*b + a)^(5/2)*x), x)
```

Fricas [A] time = 0.288377, size = 1, normalized size = 0.01

$$\left[\frac{6\,(cx)^n\sqrt{a}b+3\,((cx)^n\,b+a)^{\frac{3}{2}}\log\left(\frac{(cx)^n\sqrt{a}b-2\,\sqrt{(cx)^n\,b+a}a+2\,a^{\frac{3}{2}}}{(cx)^n}\right)+8\,a^{\frac{3}{2}}}{3\,\left((cx)^n\,a^{\frac{5}{2}}bn+a^{\frac{7}{2}}n\right)\sqrt{(cx)^n\,b+a}},\ \frac{2\left(3\,(cx)^n\,\sqrt{-a}b+3\,((cx)^n\,b+a)^{\frac{3}{2}}\arctan\left(\frac{a}{\sqrt{(cx)^n\,b+a}\cdots}\right.\right.}{3\,\left((cx)^n\,\sqrt{-a}a^2bn+\sqrt{-a}a^3n\right)\sqrt{(cx)^n\,b\cdots}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((((c*x)^n*b + a)^(5/2)*x),x, algorithm="fricas")
```

```
[Out] [1/3*(6*(c*x)^n*sqrt(a)*b + 3*((c*x)^n*b + a)^(3/2)*log(((c*x)^n*
sqrt(a)*b - 2*sqrt((c*x)^n*b + a)*a + 2*a^(3/2))/(c*x)^n) + 8*a^(
3/2))/(((c*x)^n*a^(5/2)*b*n + a^(7/2)*n)*sqrt((c*x)^n*b + a)), 2/
3*(3*(c*x)^n*sqrt(-a)*b + 3*((c*x)^n*b + a)^(3/2)*arctan(a/(sqrt(
(c*x)^n*b + a)*sqrt(-a))) + 4*sqrt(-a)*a)/(((c*x)^n*sqrt(-a)*a^2*
b*n + sqrt(-a)*a^3*n)*sqrt((c*x)^n*b + a))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x\left(a+b\left(cx\right)^{n}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(a+b*(c*x)**n)**(5/2),x)
```

```
[Out] Integral(1/(x*(a + b*(c*x)**n)**(5/2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{((cx)^n\,b+a)^{\frac{5}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((((c*x)^n*b + a)^(5/2)*x),x, algorithm="giac")
```

```
[Out] integrate(1/((((c*x)^n*b + a)^(5/2)*x), x)
```

3.503 $\int \frac{(-a+b(cx)^n)^{5/2}}{x} dx$

Optimal. Leaf size=101

$$-\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{n}+\frac{2a^2\sqrt{b(cx)^n-a}}{n}-\frac{2a\,(b(cx)^n-a)^{3/2}}{3n}+\frac{2\,(b(cx)^n-a)^{5/2}}{5n}$$

```
[Out] (2*a^2*Sqrt[-a + b*(c*x)^n])/n - (2*a*(-a + b*(c*x)^n)^(3/2))/(3*
n) + (2*(-a + b*(c*x)^n)^(5/2))/(5*n) - (2*a^(5/2)*ArcTan[Sqrt[-a
 + b*(c*x)^n]/Sqrt[a]])/n
```

Rubi [A] time = 0.178208, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316

$$-\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{n}+\frac{2a^2\sqrt{b(cx)^n-a}}{n}-\frac{2a\,(b(cx)^n-a)^{3/2}}{3n}+\frac{2\,(b(cx)^n-a)^{5/2}}{5n}$$

Antiderivative was successfully verified.

```
[In]  Int[(-a + b*(c*x)^n)^(5/2)/x,x]
```

```
[Out] (2*a^2*Sqrt[-a + b*(c*x)^n])/n - (2*a*(-a + b*(c*x)^n)^(3/2))/(3*
n) + (2*(-a + b*(c*x)^n)^(5/2))/(5*n) - (2*a^(5/2)*ArcTan[Sqrt[-a
 + b*(c*x)^n]/Sqrt[a]])/n
```

Rubi in Sympy [A] time = 8.07787, size = 80, normalized size = 0.79

$$-\frac{2a^{\frac{5}{2}}\operatorname{atan}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{n}+\frac{2a^2\sqrt{-a+b\,(cx)^n}}{n}-\frac{2a\,(-a+b\,(cx)^n)^{\frac{3}{2}}}{3n}+\frac{2\,(-a+b\,(cx)^n)^{\frac{5}{2}}}{5n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-a+b*(c*x)**n)**(5/2)/x,x)
```

```
[Out] -2*a**(5/2)*atan(sqrt(-a + b*(c*x)**n)/sqrt(a))/n + 2*a**2*sqrt(-
a + b*(c*x)**n)/n - 2*a*(-a + b*(c*x)**n)**(3/2)/(3*n) + 2*(-a +
b*(c*x)**n)**(5/2)/(5*n)
```

Mathematica [A] time = 0.102974, size = 81, normalized size = 0.8

$$\frac{2\sqrt{b(cx)^n-a}\left(23a^2-11ab(cx)^n+3b^2(cx)^{2n}\right)-30a^{5/2}\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{15n}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-a + b*(c*x)^n)^(5/2)/x,x]
```

```
[Out] (2*Sqrt[-a + b*(c*x)^n]*(23*a^2 - 11*a*b*(c*x)^n + 3*b^2*(c*x)^(2
*n)) - 30*a^(5/2)*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/(15*n)
```

Maple [A] time = 0.011, size = 86, normalized size = 0.9

$$-\frac{2\,a}{3\,n}\left(-a+b\,(cx)^n\right)^{\frac{3}{2}}+\frac{2}{5\,n}\left(-a+b\,(cx)^n\right)^{\frac{5}{2}}-2\,\frac{a^{5/2}}{n}\arctan\left(\frac{\sqrt{-a+b\,(cx)^n}}{\sqrt{a}}\right)+2\,\frac{a^2\sqrt{-a+b\,(cx)^n}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-a+b*(c*x)^n)^(5/2)/x,x)
```

```
[Out] -2/3*a*(-a+b*(c*x)^n)^(3/2)/n+2/5*(-a+b*(c*x)^n)^(5/2)/n-2*a^(5/2
)*arctan((-a+b*(c*x)^n)^(1/2)/a^(1/2))/n+2*a^2*(-a+b*(c*x)^n)^(1/
2)/n
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((c*x)^n*b - a)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.285436, size = 1, normalized size = 0.01

$$\left[\frac{15\sqrt{-a}a^2\log\left(\frac{(cx)^n b-2\sqrt{(cx)^n b-a}\sqrt{-a}-2a}{(cx)^n}\right)-2\left(11\,(cx)^n ab-3\,(cx)^{2n}b^2-23\,a^2\right)\sqrt{(cx)^n b-a}}{15\,n},\right.$$
$$\left.-\frac{2\left(15\,a^{\frac{5}{2}}\arctan\left(\frac{\sqrt{(cx)^n b-a}}{\sqrt{a}}\right)+\left(11\,(cx)^n ab-3\,(cx)^{2n}b^2-23\,a^2\right)\sqrt{(cx)^n b-a}\right)}{15\,n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((c*x)^n*b - a)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] [1/15*(15*sqrt(-a)*a^2*log(((c*x)^n*b - 2*sqrt((c*x)^n*b - a)*sqr
t(-a) - 2*a)/(c*x)^n) - 2*(11*(c*x)^n*a*b - 3*(c*x)^(2*n)*b^2 - 2
3*a^2)*sqrt((c*x)^n*b - a))/n, -2/15*(15*a^(5/2)*arctan(sqrt((c*x
)^n*b - a)/sqrt(a)) + (11*(c*x)^n*a*b - 3*(c*x)^(2*n)*b^2 - 23*a^
2)*sqrt((c*x)^n*b - a))/n]
```

Sympy [A] time = 106.39, size = 192, normalized size = 1.9

$$\begin{cases} \frac{-2a^3\left(\begin{cases}\frac{\operatorname{atan}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{\sqrt{a}} & \text{for}\ a>0\\ -\frac{\operatorname{acoth}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{-a}}\right)}{\sqrt{-a}} & \text{for}\ -a<-a+b\,(cx)^n\wedge a<0\\ -\frac{\operatorname{atanh}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{-a}}\right)}{\sqrt{-a}} & \text{for}\ -a>-a+b\,(cx)^n\wedge a<0\end{cases}\right)+2a^2\sqrt{-a+b(cx)^n}-\frac{2a\left(-a+b(cx)^n\right)^{\frac{3}{2}}}{3}+\frac{2\left(-a+b(cx)^n\right)^{\frac{5}{2}}}{5}}{n} & \text{for}\ n\neq 0\\ \left(a^2\sqrt{-a+b}-2ab\sqrt{-a+b}+b^2\sqrt{-a+b}\right)\log(cx) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-a+b*(c*x)**n)**(5/2)/x,x)
```

```
[Out] Piecewise(((-2*a**3*Piecewise((atan(sqrt(-a + b*(c*x)**n)/sqrt(a)
)/sqrt(a), a > 0), (-acoth(sqrt(-a + b*(c*x)**n)/sqrt(-a))/sqrt(-
a), (a < 0) & (-a < -a + b*(c*x)**n)), (-atanh(sqrt(-a + b*(c*x)*
*n)/sqrt(-a))/sqrt(-a), (a < 0) & (-a > -a + b*(c*x)**n))) + 2*a*
*2*sqrt(-a + b*(c*x)**n) - 2*a*(-a + b*(c*x)**n)**(3/2)/3 + 2*(-a
 + b*(c*x)**n)**(5/2)/5)/n, Ne(n, 0)), ((a**2*sqrt(-a + b) - 2*a*
b*sqrt(-a + b) + b**2*sqrt(-a + b))*log(c*x), True))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{((cx)^n b - a)^{\frac{5}{2}}}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((c*x)^n*b - a)^(5/2)/x,x, algorithm="giac")
```

```
[Out] integrate(((c*x)^n*b - a)^(5/2)/x, x)
```

3.504 $\int \frac{(-a+b(cx)^n)^{3/2}}{x} \, dx$

Optimal. Leaf size=76

$$\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{n}-\frac{2a\sqrt{b(cx)^n-a}}{n}+\frac{2\left(b(cx)^n-a\right)^{3/2}}{3n}$$

```
[Out] (-2*a*Sqrt[-a + b*(c*x)^n])/n + (2*(-a + b*(c*x)^n)^(3/2))/(3*n)
+ (2*a^(3/2)*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/n
```

Rubi [A] time = 0.139962, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316

$$\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{n}-\frac{2a\sqrt{b(cx)^n-a}}{n}+\frac{2\left(b(cx)^n-a\right)^{3/2}}{3n}$$

Antiderivative was successfully verified.

```
[In]  Int[(-a + b*(c*x)^n)^(3/2)/x,x]
```

```
[Out] (-2*a*Sqrt[-a + b*(c*x)^n])/n + (2*(-a + b*(c*x)^n)^(3/2))/(3*n)
+ (2*a^(3/2)*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/n
```

Rubi in Sympy [A] time = 6.48878, size = 60, normalized size = 0.79

$$\frac{2a^{\frac{3}{2}}\operatorname{atan}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{n}-\frac{2a\sqrt{-a+b\,(cx)^n}}{n}+\frac{2\left(-a+b\,(cx)^n\right)^{\frac{3}{2}}}{3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-a+b*(c*x)**n)**(3/2)/x,x)
```

```
[Out] 2*a**(3/2)*atan(sqrt(-a + b*(c*x)**n)/sqrt(a))/n - 2*a*sqrt(-a +
b*(c*x)**n)/n + 2*(-a + b*(c*x)**n)**(3/2)/(3*n)
```

Mathematica [A] time = 0.0666771, size = 66, normalized size = 0.87

$$\frac{6a^{3/2}\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)-2\left(4a-b(cx)^n\right)\sqrt{b(cx)^n-a}}{3n}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-a + b*(c*x)^n)^(3/2)/x,x]
```

```
[Out] (-2*(4*a - b*(c*x)^n)*Sqrt[-a + b*(c*x)^n] + 6*a^(3/2)*ArcTan[Sqr
t[-a + b*(c*x)^n]/Sqrt[a]])/(3*n)
```

Maple [A] time = 0.005, size = 65, normalized size = 0.9

$$\frac{2}{3\,n}\left(-a+b\,(cx)^n\right)^{\frac{3}{2}}+2\,\frac{a^{3/2}}{n}\arctan\left(\frac{\sqrt{-a+b\,(cx)^n}}{\sqrt{a}}\right)-2\,\frac{a\sqrt{-a+b\,(cx)^n}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-a+b*(c*x)^n)^(3/2)/x,x)
```

```
[Out] 2/3*(-a+b*(c*x)^n)^(3/2)/n+2*a^(3/2)*arctan((-a+b*(c*x)^n)^(1/2)/
a^(1/2))/n-2*a*(-a+b*(c*x)^n)^(1/2)/n
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((c*x)^n*b - a)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.28255, size = 1, normalized size = 0.01

$$\left[\frac{3\sqrt{-a}a\log\left(\frac{(cx)^n b+2\sqrt{(cx)^n b-a}\sqrt{-a}-2\,a}{(cx)^n}\right)+2\sqrt{(cx)^n\,b-a}((cx)^n\,b-4\,a)}{3\,n},\frac{2\left(3\,a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{(cx)^n b-a}}{\sqrt{a}}\right)+\sqrt{(cx)^n\,b-a}((cx)^n\,b\right.}{3\,n}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((c*x)^n*b - a)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/3*(3*sqrt(-a)*a*log(((c*x)^n*b + 2*sqrt((c*x)^n*b - a)*sqrt(-a
) - 2*a)/(c*x)^n) + 2*sqrt((c*x)^n*b - a)*((c*x)^n*b - 4*a))/n, 2
/3*(3*a^(3/2)*arctan(sqrt((c*x)^n*b - a)/sqrt(a)) + sqrt((c*x)^n*
b - a)*((c*x)^n*b - 4*a))/n]
```

Sympy [A] time = 14.1454, size = 158, normalized size = 2.08

$$-\begin{cases} \left(a\sqrt{-a+b} - b\sqrt{-a+b}\right)\log\left(x\right) & \text{for}\ n = 0 \\ \frac{-2a^2\left(\begin{cases} \frac{\operatorname{atan}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{\sqrt{a}} & \text{for}\ a > 0 \\ -\frac{\operatorname{acoth}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{-a}}\right)}{\sqrt{-a}} & \text{for}\ -a < -a + b\left(cx\right)^n \wedge a < 0 \\ -\frac{\operatorname{atanh}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{-a}}\right)}{\sqrt{-a}} & \text{for}\ -a > -a + b\left(cx\right)^n \wedge a < 0 \end{cases}\right) + 2a\sqrt{-a+b(cx)^n} - \frac{2(-a+b(cx)^n)^{\frac{3}{2}}}{3}}{n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-a+b*(c*x)**n)**(3/2)/x,x)
```

```
[Out] -Piecewise(((a*sqrt(-a + b) - b*sqrt(-a + b))*log(x), Eq(n, 0)),
((-2*a**2*Piecewise((atan(sqrt(-a + b*(c*x)**n)/sqrt(a))/sqrt(a),
 a > 0), (-acoth(sqrt(-a + b*(c*x)**n)/sqrt(-a))/sqrt(-a), (a < 0
) & (-a < -a + b*(c*x)**n)), (-atanh(sqrt(-a + b*(c*x)**n)/sqrt(-
a))/sqrt(-a), (a < 0) & (-a > -a + b*(c*x)**n))) + 2*a*sqrt(-a +
b*(c*x)**n) - 2*(-a + b*(c*x)**n)**(3/2)/3)/n, True))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left((cx)^n b - a\right)^{\frac{3}{2}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((c*x)^n*b - a)^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate(((c*x)^n*b - a)^(3/2)/x, x)
```

3.505 $\int \frac{\sqrt{-a+b(cx)^n}}{x}\,dx$

Optimal. Leaf size=53

$$\frac{2\sqrt{b(cx)^n-a}}{n}-\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{n}$$

```
[Out] (2*Sqrt[-a + b*(c*x)^n])/n - (2*Sqrt[a]*ArcTan[Sqrt[-a + b*(c*x)^
n]/Sqrt[a]])/n
```

Rubi [A] time = 0.106826, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316

$$\frac{2\sqrt{b(cx)^n-a}}{n}-\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{n}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[-a + b*(c*x)^n]/x,x]
```

```
[Out] (2*Sqrt[-a + b*(c*x)^n])/n - (2*Sqrt[a]*ArcTan[Sqrt[-a + b*(c*x)^
n]/Sqrt[a]])/n
```

Rubi in Sympy [A] time = 5.22331, size = 41, normalized size = 0.77

$$-\frac{2\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{n}+\frac{2\sqrt{-a+b\,(cx)^n}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-a+b*(c*x)**n)**(1/2)/x,x)
```

```
[Out] -2*sqrt(a)*atan(sqrt(-a + b*(c*x)**n)/sqrt(a))/n + 2*sqrt(-a + b*
(c*x)**n)/n
```

Mathematica [A] time = 0.0321183, size = 50, normalized size = 0.94

$$\frac{2\left(\sqrt{b(cx)^n-a}-\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)\right)}{n}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[-a + b*(c*x)^n]/x,x]
```

```
[Out] (2*(Sqrt[-a + b*(c*x)^n] - Sqrt[a]*ArcTan[Sqrt[-a + b*(c*x)^n]/Sq
rt[a]]))/n
```

Maple [A] time = 0.005, size = 46, normalized size = 0.9

$$-2\,\frac{\sqrt{a}}{n}\arctan\left(\frac{\sqrt{-a+b\,(cx)^n}}{\sqrt{a}}\right)+2\,\frac{\sqrt{-a+b\,(cx)^n}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-a+b*(c*x)^n)^(1/2)/x,x)
```

```
[Out] -2*arctan((-a+b*(c*x)^n)^(1/2)/a^(1/2))*a^(1/2)/n+2*(-a+b*(c*x)^n
)^(1/2)/n
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((c*x)^n*b - a)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.284697, size = 1, normalized size = 0.02

$$\left[\frac{\sqrt{-a}\log\left(\frac{(cx)^n b-2\sqrt{(cx)^n b-a}\sqrt{-a}-2\,a}{(cx)^n}\right)+2\sqrt{(cx)^n\,b-a}}{n}, -\frac{2\left(\sqrt{a}\arctan\left(\frac{\sqrt{(cx)^n b-a}}{\sqrt{a}}\right)-\sqrt{(cx)^n\,b-a}\right)}{n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((c*x)^n*b - a)/x,x, algorithm="fricas")
```

```
[Out] [(sqrt(-a)*log(((c*x)^n*b - 2*sqrt((c*x)^n*b - a)*sqrt(-a) - 2*a)
/(c*x)^n) + 2*sqrt((c*x)^n*b - a))/n, -2*(sqrt(a)*arctan(sqrt((c*
x)^n*b - a)/sqrt(a)) - sqrt((c*x)^n*b - a))/n]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a + b(cx)^n}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-a+b*(c*x)**n)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-a + b*(c*x)**n)/x, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(cx)^n b - a}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((c*x)^n*b - a)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt((c*x)^n*b - a)/x, x)
```

3.506 $\int \frac{1}{x\sqrt{-a+b(cx)^n}}\,dx$

Optimal. Leaf size=32

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{\sqrt{a}n}$$

```
[Out] (2*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/(Sqrt[a]*n)
```

Rubi [A] time = 0.0811627, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{\sqrt{a}n}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*Sqrt[-a + b*(c*x)^n]),x]
```

```
[Out] (2*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/(Sqrt[a]*n)
```

Rubi in Sympy [A] time = 4.07476, size = 26, normalized size = 0.81

$$\frac{2\,\mathrm{atan}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{\sqrt{a}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(-a+b*(c*x)**n)**(1/2),x)
```

```
[Out] 2*atan(sqrt(-a + b*(c*x)**n)/sqrt(a))/(sqrt(a)*n)
```

Mathematica [A] time = 0.0256412, size = 32, normalized size = 1.

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{\sqrt{a}n}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*Sqrt[-a + b*(c*x)^n]),x]
```

```
[Out] (2*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/(Sqrt[a]*n)
```

Maple [A] time = 0.007, size = 27, normalized size = 0.8

$$2\,{\frac {1}{n\sqrt {a}}\arctan \left({\frac {\sqrt {-a+b \left(cx \right) ^{n}}}{\sqrt {a}}} \right) }$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(-a+b*(c*x)^n)^(1/2),x)
```

```
[Out] 2*arctan((-a+b*(c*x)^n)^(1/2)/a^(1/2))/n/a^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt((c*x)^n*b - a)*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.289236, size = 1, normalized size = 0.03

$$\left[\frac{\log\left(\frac{(cx)^n\sqrt{-a}b+2\sqrt{(cx)^n b-a}a-2\sqrt{-a}a}{(cx)^n}\right)}{\sqrt{-a}n}, -\frac{2\arctan\left(\frac{\sqrt{a}}{\sqrt{(cx)^n b-a}}\right)}{\sqrt{a}n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt((c*x)^n*b - a)*x),x, algorithm="fricas")
```

```
[Out] [log(((c*x)^n*sqrt(-a)*b + 2*sqrt((c*x)^n*b - a)*a - 2*sqrt(-a)*a
)/(c*x)^n)/(sqrt(-a)*n), -2*arctan(sqrt(a)/sqrt((c*x)^n*b - a))/(
sqrt(a)*n)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{-a+b\,(cx)^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(-a+b*(c*x)**n)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(-a + b*(c*x)**n)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(cx)^n\,b-a}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt((c*x)^n*b - a)*x),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt((c*x)^n*b - a)*x), x)
```

3.507 $\int \frac{1}{x(-a+b(cx)^n)^{3/2}}\,dx$

Optimal. Leaf size=56

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{a^{3/2}n}-\frac{2}{an\sqrt{b(cx)^n-a}}$$

```
[Out] -2/(a*n*Sqrt[-a + b*(c*x)^n]) - (2*ArcTan[Sqrt[-a + b*(c*x)^n]/Sq
rt[a]])/(a^(3/2)*n)
```

Rubi [A] time = 0.115704, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{a^{3/2}n}-\frac{2}{an\sqrt{b(cx)^n-a}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(-a + b*(c*x)^n)^(3/2)),x]
```

```
[Out] -2/(a*n*Sqrt[-a + b*(c*x)^n]) - (2*ArcTan[Sqrt[-a + b*(c*x)^n]/Sq
rt[a]])/(a^(3/2)*n)
```

Rubi in Sympy [A] time = 5.53749, size = 44, normalized size = 0.79

$$-\frac{2}{an\sqrt{-a+b\left(cx\right)^{n}}}-\frac{2\,\mathrm{atan}\left(\frac{\sqrt{-a+b(cx)^{n}}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(-a+b*(c*x)**n)**(3/2),x)
```

```
[Out] -2/(a*n*sqrt(-a + b*(c*x)**n)) - 2*atan(sqrt(-a + b*(c*x)**n)/sqr
t(a))/(a**(3/2)*n)
```

Mathematica [A] time = 0.0864088, size = 56, normalized size = 1.

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{a^{3/2}n}-\frac{2}{an\sqrt{b(cx)^n-a}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(-a + b*(c*x)^n)^(3/2)),x]
```

```
[Out] -2/(a*n*Sqrt[-a + b*(c*x)^n]) - (2*ArcTan[Sqrt[-a + b*(c*x)^n]/Sq
rt[a]])/(a^(3/2)*n)
```

Maple [A] time = 0.008, size = 49, normalized size = 0.9

$$-2\,\frac{1}{a^{3/2}n}\arctan\left(\frac{\sqrt{-a+b\,(cx)^n}}{\sqrt{a}}\right)-2\,\frac{1}{an\sqrt{-a+b\,(cx)^n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(-a+b*(c*x)^n)^(3/2),x)
```

```
[Out] -2*arctan((-a+b*(c*x)^n)^(1/2)/a^(1/2))/a^(3/2)/n-2/a/n/(-a+b*(c*
x)^n)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{((cx)^n\,b-a)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(((c*x)^n*b - a)^(3/2)*x),x, algorithm="maxima")
```

```
[Out] integrate(1/(((c*x)^n*b - a)^(3/2)*x), x)
```

Fricas [A] time = 0.282227, size = 1, normalized size = 0.02

$$\left[\frac{\sqrt{(cx)^n b-a}\log\left(\frac{(cx)^n\sqrt{-a}b-2\sqrt{(cx)^n b-a}a-2\sqrt{-a}a}{(cx)^n}\right)-2\sqrt{-a}}{\sqrt{(cx)^n b-a}\sqrt{-a}an}, \frac{2\left(\sqrt{(cx)^n b-a}\arctan\left(\frac{\sqrt{a}}{\sqrt{(cx)^n b-a}}\right)-\sqrt{a}\right)}{\sqrt{(cx)^n b-a}a^{\frac{3}{2}}n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((((c*x)^n*b - a)^(3/2)*x),x, algorithm="fricas")
```

```
[Out] [(sqrt((c*x)^n*b - a)*log(((c*x)^n*sqrt(-a)*b - 2*sqrt((c*x)^n*b
- a)*a - 2*sqrt(-a)*a)/(c*x)^n) - 2*sqrt(-a))/(sqrt((c*x)^n*b - a
)*sqrt(-a)*a*n), 2*(sqrt((c*x)^n*b - a)*arctan(sqrt(a)/sqrt((c*x)
^n*b - a)) - sqrt(a))/(sqrt((c*x)^n*b - a)*a^(3/2)*n)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(-a+b\left(cx\right)^{n}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(-a+b*(c*x)**n)**(3/2),x)
```

```
[Out] Integral(1/(x*(-a + b*(c*x)**n)**(3/2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\left(cx\right)^{n}b-a\right)^{\frac{3}{2}}x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((((c*x)^n*b - a)^(3/2)*x),x, algorithm="giac")
```

```
[Out] integrate(1/((((c*x)^n*b - a)^(3/2)*x), x)
```

3.508 $\int \frac{1}{x(-a+b(cx)^n)^{5/2}} dx$

Optimal. Leaf size=81

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{a^{5/2}n}+\frac{2}{a^2n\sqrt{b(cx)^n-a}}-\frac{2}{3an\,(b(cx)^n-a)^{3/2}}$$

```
[Out] -2/(3*a*n*(-a + b*(c*x)^n)^(3/2)) + 2/(a^2*n*Sqrt[-a + b*(c*x)^n]
) + (2*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/(a^(5/2)*n)
```

Rubi [A] time = 0.1529, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{a^{5/2}n}+\frac{2}{a^2n\sqrt{b(cx)^n-a}}-\frac{2}{3an\,(b(cx)^n-a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(-a + b*(c*x)^n)^(5/2)),x]
```


```
[Out] -2/(3*a*n*(-a + b*(c*x)^n)^(3/2)) + 2/(a^2*n*Sqrt[-a + b*(c*x)^n]
) + (2*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/(a^(5/2)*n)
```

Rubi in Sympy [A] time = 7.26356, size = 63, normalized size = 0.78

$$-\frac{2}{3an\left(-a+b\left(cx\right)^{n}\right)^{\frac{3}{2}}}+\frac{2}{a^{2}n\sqrt{-a+b\left(cx\right)^{n}}}+\frac{2\,\mathrm{atan}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(-a+b*(c*x)**n)**(5/2),x)
```

```
[Out] -2/(3*a*n*(-a + b*(c*x)**n)**(3/2)) + 2/(a**2*n*sqrt(-a + b*(c*x)
**n)) + 2*atan(sqrt(-a + b*(c*x)**n)/sqrt(a))/(a**(5/2)*n)
```

Mathematica [A] time = 0.191232, size = 70, normalized size = 0.86

$$\frac{2\left(\frac{\sqrt{a}(3b(cx)^n-4a)}{(b(cx)^n-a)^{3/2}}+3\tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)\right)}{3a^{5/2}n}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(-a + b*(c*x)^n)^(5/2)),x]
```

```
[Out] (2*((Sqrt[a]*(-4*a + 3*b*(c*x)^n))/(-a + b*(c*x)^n)^(3/2) + 3*Arc
Tan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]]))/(3*a^(5/2)*n)
```

Maple [A] time = 0.01, size = 70, normalized size = 0.9

$$-\frac{2}{3\,an}\left(-a+b\left(cx\right)^{n}\right)^{-\frac{3}{2}}+2\,\frac{1}{a^{5/2}n}\arctan\left(\frac{\sqrt{-a+b\left(cx\right)^{n}}}{\sqrt{a}}\right)+2\,\frac{1}{a^{2}n\sqrt{-a+b\left(cx\right)^{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(-a+b*(c*x)^n)^(5/2),x)
```

```
[Out] -2/3/a/n/(-a+b*(c*x)^n)^(3/2)+2*arctan((-a+b*(c*x)^n)^(1/2)/a^(1/
2))/a^(5/2)/n+2/a^2/n/(-a+b*(c*x)^n)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\left(cx\right)^{n}b-a\right)^{\frac{5}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((((c*x)^n*b - a)^(5/2)*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((((c*x)^n*b - a)^(5/2)*x), x)
```

Fricas [A] time = 0.284092, size = 1, normalized size = 0.01

$$\left[\frac{6\,(cx)^n\sqrt{-a}b+3\,((cx)^n\,b-a)^{\frac{3}{2}}\log\left(\frac{(cx)^n\sqrt{-a}b+2\,\sqrt{(cx)^n\,b-a}a-2\,\sqrt{-a}a}{(cx)^n}\right)-8\,\sqrt{-a}a}{3\,\left((cx)^n\,\sqrt{-a}a^2bn-\sqrt{-a}a^3n\right)\sqrt{(cx)^n\,b-a}},\frac{2\left(3\,(cx)^n\,\sqrt{a}b-3\,((cx)^n\,b-a)^{\frac{3}{2}}\arctan\left(\right.\right.}{3\left((cx)^n\,a^{\frac{5}{2}}bn-a^{\frac{7}{2}}n\right)\sqrt{(cx)^n}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((((c*x)^n*b - a)^(5/2)*x),x, algorithm="fricas")
```

```
[Out] [1/3*(6*(c*x)^n*sqrt(-a)*b + 3*((c*x)^n*b - a)^(3/2)*log(((c*x)^n
*sqrt(-a)*b + 2*sqrt((c*x)^n*b - a)*a - 2*sqrt(-a)*a)/(c*x)^n) -
8*sqrt(-a)*a)/(((c*x)^n*sqrt(-a)*a^2*b*n - sqrt(-a)*a^3*n)*sqrt((
c*x)^n*b - a)), 2/3*(3*(c*x)^n*sqrt(a)*b - 3*((c*x)^n*b - a)^(3/2
)*arctan(sqrt(a)/sqrt((c*x)^n*b - a)) - 4*a^(3/2))/(((c*x)^n*a^(5
/2)*b*n - a^(7/2)*n)*sqrt((c*x)^n*b - a))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\,(-a+b\,(cx)^n)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(-a+b*(c*x)**n)**(5/2),x)
```

```
[Out] Integral(1/(x*(-a + b*(c*x)**n)**(5/2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{((cx)^n\,b-a)^{\frac{5}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((((c*x)^n*b - a)^(5/2)*x),x, algorithm="giac")
```

```
[Out] integrate(1/((((c*x)^n*b - a)^(5/2)*x), x)
```

3.509 $\int \frac{1}{x\sqrt{a+bx}}\,dx$

Optimal. Leaf size=23

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] (-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Rubi [A] time = 0.0218145, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*Sqrt[a + b*x]),x]

[Out] (-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Rubi in Sympy [A] time = 1.64892, size = 22, normalized size = 0.96

$$-\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(b*x+a)**(1/2),x)

[Out] -2*atanh(sqrt(a + b*x)/sqrt(a))/sqrt(a)
```

Mathematica [A] time = 0.0131839, size = 23, normalized size = 1.

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*Sqrt[a + b*x]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Maple [A] time = 0.006, size = 18, normalized size = 0.8

$$-2\frac{1}{\sqrt{a}}Artanh\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(b*x+a)^(1/2),x)
```

```
[Out] -2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(b*x + a)*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.269517, size = 1, normalized size = 0.04

$$\left[\frac{\log\left(\frac{(bx+2a)\sqrt{a}-2\sqrt{bx+a}a}{x}\right)}{\sqrt{a}}, \frac{2\arctan\left(\frac{a}{\sqrt{bx+a}\sqrt{-a}}\right)}{\sqrt{-a}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(b*x + a)*x),x, algorithm="fricas")
```

```
[Out] [log(((b*x + 2*a)*sqrt(a) - 2*sqrt(b*x + a)*a)/x)/sqrt(a), 2*arct
an(a/(sqrt(b*x + a)*sqrt(-a)))/sqrt(-a)]
```

Sympy [A] time = 3.62915, size = 24, normalized size = 1.04

$$-\frac{2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(b*x+a)**(1/2),x)
```

```
[Out] -2*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/sqrt(a)
```

GIAC/XCAS [A] time = 0.259667, size = 28, normalized size = 1.22

$$\frac{2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(b*x + a)*x),x, algorithm="giac")
```

```
[Out] 2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a)
```

3.510 $\int \frac{1}{x\sqrt{a+b(cx)^m}}\,dx$

Optimal. Leaf size=30

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^m}}{\sqrt{a}}\right)}{\sqrt{a}m}$$

```
[Out] (-2*ArcTanh[Sqrt[a + b*(c*x)^m]/Sqrt[a]])/(Sqrt[a]*m)
```

Rubi [A] time = 0.0777392, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^m}}{\sqrt{a}}\right)}{\sqrt{a}m}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*Sqrt[a + b*(c*x)^m]),x]

[Out] (-2*ArcTanh[Sqrt[a + b*(c*x)^m]/Sqrt[a]])/(Sqrt[a]*m)
```

Rubi in Sympy [A] time = 3.68997, size = 27, normalized size = 0.9

$$-\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a+b(cx)^m}}{\sqrt{a}}\right)}{\sqrt{a}m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(a+b*(c*x)**m)**(1/2),x)

[Out] -2*atanh(sqrt(a + b*(c*x)**m)/sqrt(a))/(sqrt(a)*m)
```

Mathematica [A] time = 0.0335413, size = 30, normalized size = 1.

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^m}}{\sqrt{a}}\right)}{\sqrt{a}m}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*Sqrt[a + b*(c*x)^m]),x]

[Out] (-2*ArcTanh[Sqrt[a + b*(c*x)^m]/Sqrt[a]])/(Sqrt[a]*m)
```

Maple [A] time = 0.009, size = 25, normalized size = 0.8

$$-2\,\frac{1}{m\sqrt{a}}Artanh\left(\frac{\sqrt{a+b\,(cx)^m}}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(a+b*(c*x)^m)^(1/2),x)

[Out] -2*arctanh((a+b*(c*x)^m)^(1/2)/a^(1/2))/m/a^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt((c*x)^m*b + a)*x),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 0.282901, size = 1, normalized size = 0.03

$$\left[\frac{\log\left(\frac{(cx)^m\sqrt{a}b-2\sqrt{(cx)^m b+a}a+2\,a^{\frac{3}{2}}}{(cx)^m}\right)}{\sqrt{a}m},\ \frac{2\,\arctan\left(\frac{a}{\sqrt{(cx)^m b+a}\sqrt{-a}}\right)}{\sqrt{-a}m}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt((c*x)^m*b + a)*x),x, algorithm="fricas")
```

```
[Out] [log(((c*x)^m*sqrt(a)*b - 2*sqrt((c*x)^m*b + a)*a + 2*a^(3/2))/(c
*x)^m)/(sqrt(a)*m), 2*arctan(a/(sqrt((c*x)^m*b + a)*sqrt(-a)))/(s
qrt(-a)*m)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{a+b\left(cx\right)^{m}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(a+b*(c*x)**m)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + b*(c*x)**m)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\left(cx\right)^{m}b+a}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt((c*x)^m*b + a)*x),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt((c*x)^m*b + a)*x), x)
```

3.511 $\int \frac{1}{x\sqrt{a+b(c(dx)^m)^n}}\,dx$

Optimal. Leaf size=37

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(c(dx)^m)^n}}{\sqrt{a}}\right)}{\sqrt{a}mn}$$

```
[Out] (-2*ArcTanh[Sqrt[a + b*(c*(d*x)^m)^n]/Sqrt[a]])/(Sqrt[a]*m*n)
```

Rubi [A] time = 0.301416, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(c(dx)^m)^n}}{\sqrt{a}}\right)}{\sqrt{a}mn}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*Sqrt[a + b*(c*(d*x)^m)^n]),x]

[Out] (-2*ArcTanh[Sqrt[a + b*(c*(d*x)^m)^n]/Sqrt[a]])/(Sqrt[a]*m*n)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{a+b\left(c\left(dx\right)^m\right)^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(a+b*(c*(d*x)**m)**n)**(1/2),x)

[Out] Integral(1/(x*sqrt(a + b*(c*(d*x)**m)**n)), x)
```

Mathematica [A] time = 0.112403, size = 37, normalized size = 1.

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(c(dx)^m)^n}}{\sqrt{a}}\right)}{\sqrt{a}mn}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*Sqrt[a + b*(c*(d*x)^m)^n]),x]

[Out] (-2*ArcTanh[Sqrt[a + b*(c*(d*x)^m)^n]/Sqrt[a]])/(Sqrt[a]*m*n)
```

Maple [A] time = 0.01, size = 32, normalized size = 0.9

$$-2\,\frac{1}{mn\sqrt{a}}Artanh\left(\frac{\sqrt{a+b\left(c\left(dx\right)^{m}\right)^{n}}}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(a+b*(c*(d*x)^m)^n)^(1/2),x)

[Out] -2*arctanh((a+b*(c*(d*x)^m)^n)^(1/2)/a^(1/2))/m/n/a^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(((d*x)^m*c)^n*b + a)*x),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 0.282608, size = 1, normalized size = 0.03

$$\left[\frac{\log\left(\left(\sqrt{a}be^{\left(mn\log\left(dx\right)+n\log\left(c\right)\right)}-2\sqrt{be^{\left(mn\log\left(dx\right)+n\log\left(c\right)\right)}+a}a+2\,a^{\frac{3}{2}}\right)e^{\left(-mn\log\left(dx\right)-n\log\left(c\right)\right)}\right)}{\sqrt{a}mn},\ \frac{2\,\arctan\left(\frac{a}{\sqrt{be^{\left(mn\log\left(dx\right)+n\log\left(c\right)\right)}+a}\sqrt{-}}\right.}{\sqrt{-a}mn}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(((d*x)^m*c)^n*b + a)*x),x, algorithm="fricas")
```

```
[Out] [log((sqrt(a)*b*e^(m*n*log(d*x) + n*log(c)) - 2*sqrt(b*e^(m*n*log
(d*x) + n*log(c)) + a)*a + 2*a^(3/2))*e^(-m*n*log(d*x) - n*log(c)
))/(sqrt(a)*m*n), 2*arctan(a/(sqrt(b*e^(m*n*log(d*x) + n*log(c))
+ a)*sqrt(-a)))/(sqrt(-a)*m*n)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{a+b\left(c\left(dx\right)^{m}\right)^{n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(a+b*(c*(d*x)**m)**n)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + b*(c*(d*x)**m)**n)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\left(\left(dx\right)^{m}c\right)^{n}b+a}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(((d*x)^m*c)^n*b + a)*x),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(((d*x)^m*c)^n*b + a)*x), x)
```

3.512 $\int \frac{1}{x\sqrt{a+b(c(d(ex)^m)^n)^p}}\,dx$

Optimal. Leaf size=44

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(c(d(ex)^m)^n)^p}}{\sqrt{a}}\right)}{\sqrt{a}mnp}$$

```
[Out] (-2*ArcTanh[Sqrt[a + b*(c*(d*(e*x)^m)^n)^p]/Sqrt[a]])/(Sqrt[a]*m*
n*p)
```

Rubi [A] time = 0.622821, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(c(d(ex)^m)^n)^p}}{\sqrt{a}}\right)}{\sqrt{a}mnp}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*Sqrt[a + b*(c*(d*(e*x)^m)^n)^p]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*(c*(d*(e*x)^m)^n)^p]/Sqrt[a]])/(Sqrt[a]*m*
n*p)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{a+b\left(c\left(d\left(ex\right)^m\right)^n\right)^p}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(a+b*(c*(d*(e*x)**m)**n)**p)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + b*(c*(d*(e*x)**m)**n)**p)), x)
```

Mathematica [A] time = 0.326151, size = 44, normalized size = 1.

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(c(d(ex)^m)^n)^p}}{\sqrt{a}}\right)}{\sqrt{a}mnp}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*Sqrt[a + b*(c*(d*(e*x)^m)^n)^p]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*(c*(d*(e*x)^m)^n)^p]/Sqrt[a]])/(Sqrt[a]*m*
n*p)
```

Maple [A] time = 0.024, size = 39, normalized size = 0.9

$$-2\,\frac{1}{mnp\sqrt{a}}Artanh\left(\frac{\sqrt{a+b\left(c\left(d\left(ex\right)^{m}\right)^{n}\right)^{p}}}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(a+b*(c*(d*(e*x)^m)^n)^p)^(1/2),x)
```

```
[Out] -2*arctanh((a+b*(c*(d*(e*x)^m)^n)^p)^(1/2)/a^(1/2))/m/n/p/a^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt((((e*x)^m*d)^n*c)^p*b + a)*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.283674, size = 1, normalized size = 0.02

$$\left[\frac{\log\left(\left(\sqrt{a}be^{(mnp\log(ex)+np\log(d)+p\log(c))} - 2\sqrt{be^{(mnp\log(ex)+np\log(d)+p\log(c))}+a}a + 2\,a^{\frac{3}{2}}\right)e^{(-mnp\log(ex)-np\log(d)-p\log(c))}\right)}{\sqrt{a}mnp}, \frac{2\ \mathrm{arc}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt((((e*x)^m*d)^n*c)^p*b + a)*x),x, algorithm="fricas")
```

```
[Out] [log((sqrt(a)*b*e^(m*n*p*log(e*x) + n*p*log(d) + p*log(c)) - 2*sq
rt(b*e^(m*n*p*log(e*x) + n*p*log(d) + p*log(c)) + a)*a + 2*a^(3/2
))*e^(-m*n*p*log(e*x) - n*p*log(d) - p*log(c)))/(sqrt(a)*m*n*p),
2*arctan(a/(sqrt(b*e^(m*n*p*log(e*x) + n*p*log(d) + p*log(c)) + a
)*sqrt(-a)))/(sqrt(-a)*m*n*p)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{a+b\left(c\left(d\left(ex\right)^{m}\right)^{n}\right)^{p}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*(d*(e*x)**m)**n)**p)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + b*(c*(d*(e*x)**m)**n)**p)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\left(\left(\left(ex\right)^{m}d\right)^{n}c\right)^{p}b+a}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt((((e*x)^m*d)^n*c)^p*b + a)*x),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt((((e*x)^m*d)^n*c)^p*b + a)*x), x)
```

3.513 $\int \frac{1}{x\sqrt{a+b\left(c\left(d\left(e(fx)^m\right)^n\right)^p\right)^q}}\,dx$

Optimal. Leaf size=51

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\left(c\left(d\left(e(fx)^m\right)^n\right)^p\right)^q}}{\sqrt{a}}\right)}{\sqrt{a}mnpq}$$

```
[Out] (-2*ArcTanh[Sqrt[a + b*(c*(d*(e*(f*x)^m)^n)^p)^q]/Sqrt[a]])/(Sqrt
[a]*m*n*p*q)
```

Rubi [A] time = 1.10166, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\left(c\left(d\left(e(fx)^m\right)^n\right)^p\right)^q}}{\sqrt{a}}\right)}{\sqrt{a}mnpq}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*Sqrt[a + b*(c*(d*(e*(f*x)^m)^n)^p)^q]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*(c*(d*(e*(f*x)^m)^n)^p)^q]/Sqrt[a]])/(Sqrt
[a]*m*n*p*q)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{a+b\left(c\left(d\left(e\left(fx\right)^m\right)^n\right)^p\right)^q}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(a+b*(c*(d*(e*(f*x)**m)**n)**p)**q)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + b*(c*(d*(e*(f*x)**m)**n)**p)**q)), x)
```

Mathematica [A] time = 2.22159, size = 51, normalized size = 1.

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\left(c\left(d\left(e(fx)^m\right)^n\right)^p\right)^q}}{\sqrt{a}}\right)}{\sqrt{a}mnpq}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*Sqrt[a + b*(c*(d*(e*(f*x)^m)^n)^p)^q]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*(c*(d*(e*(f*x)^m)^n)^p)^q]/Sqrt[a]])/(Sqrt
[a]*m*n*p*q)
```

Maple [A] time = 0.032, size = 46, normalized size = 0.9

$$-2\,\frac{1}{mnpq\sqrt{a}}Artanh\left(\frac{\sqrt{a+b\left(c\left(d\left(e\left(fx\right)^m\right)^n\right)^p\right)^q}}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(a+b*(c*(d*(e*(f*x)^m)^n)^p)^q)^(1/2),x)
```

```
[Out] -2*arctanh((a+b*(c*(d*(e*(f*x)^m)^n)^p)^q)^(1/2)/a^(1/2))/m/n/p/q
/a^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(((((f*x)^m*e)^n*d)^p*c)^q*b + a)*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.288374, size = 1, normalized size = 0.02

$$\left[\frac{\log\left(\left(\sqrt{a}be^{(mnpq\log(fx)+npq\log(e)+pq\log(d)+q\log(c))} - 2\sqrt{be^{(mnpq\log(fx)+npq\log(e)+pq\log(d)+q\log(c))} + a}a + 2a^{\frac{3}{2}}\right)e^{(-mnpq\log(fx)-r}\right.}{\sqrt{a}mnpq}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(((((f*x)^m*e)^n*d)^p*c)^q*b + a)*x),x, algorithm="fricas")
```

```
[Out] [log((sqrt(a)*b*e^(m*n*p*q*log(f*x) + n*p*q*log(e) + p*q*log(d) +
 q*log(c)) - 2*sqrt(b*e^(m*n*p*q*log(f*x) + n*p*q*log(e) + p*q*lo
g(d) + q*log(c)) + a)*a + 2*a^(3/2))*e^(-m*n*p*q*log(f*x) - n*p*q
*log(e) - p*q*log(d) - q*log(c)))/(sqrt(a)*m*n*p*q), 2*arctan(a/(
sqrt(b*e^(m*n*p*q*log(f*x) + n*p*q*log(e) + p*q*log(d) + q*log(c)
) + a)*sqrt(-a)))/(sqrt(-a)*m*n*p*q)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{a + b\left(c\left(d\left(e\left(fx\right)^{m}\right)^{n}\right)^{p}\right)^{q}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(a+b*(c*(d*(e*(f*x)**m)**n)**p)**q)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + b*(c*(d*(e*(f*x)**m)**n)**p)**q)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\left(\left(\left((fx)^{m}e\right)^{n}d\right)^{p}c\right)^{q}b + a}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(((((f*x)^m*e)^n*d)^p*c)^q*b + a)*x),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(((((f*x)^m*e)^n*d)^p*c)^q*b + a)*x), x)
```

3.514 $\int \frac{\sqrt{-1+\frac{1}{x^2}}\left(-1+x^2\right)^3}{x}\,dx$

Optimal. Leaf size=76

$$-\frac{35}{48}\left(\frac{1}{x^2}-1\right)^{3/2}x^2+\frac{35}{16}\sqrt{\frac{1}{x^2}-1}-\frac{35}{16}\tan^{-1}\left(\sqrt{\frac{1}{x^2}-1}\right)-\frac{1}{6}\left(\frac{1}{x^2}-1\right)^{7/2}x^6-\frac{7}{24}\left(\frac{1}{x^2}-1\right)^{5/2}x^4$$

```
[Out] (35*Sqrt[-1 + x^(-2)])/16 - (35*(-1 + x^(-2))^(3/2)*x^2)/48 - (7*
(-1 + x^(-2))^(5/2)*x^4)/24 - ((-1 + x^(-2))^(7/2)*x^6)/6 - (35*A
rcTan[Sqrt[-1 + x^(-2)]])/16
```

Rubi [A] time = 0.0693928, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$-\frac{35}{48}\left(\frac{1}{x^2}-1\right)^{3/2}x^2+\frac{35}{16}\sqrt{\frac{1}{x^2}-1}-\frac{35}{16}\tan^{-1}\left(\sqrt{\frac{1}{x^2}-1}\right)-\frac{1}{6}\left(\frac{1}{x^2}-1\right)^{7/2}x^6-\frac{7}{24}\left(\frac{1}{x^2}-1\right)^{5/2}x^4$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[-1 + x^(-2)]*(-1 + x^2)^3)/x,x]
```

```
[Out] (35*Sqrt[-1 + x^(-2)])/16 - (35*(-1 + x^(-2))^(3/2)*x^2)/48 - (7*
(-1 + x^(-2))^(5/2)*x^4)/24 - ((-1 + x^(-2))^(7/2)*x^6)/6 - (35*A
rcTan[Sqrt[-1 + x^(-2)]])/16
```

Rubi in Sympy [A] time = 4.47909, size = 76, normalized size = 1.

$$-\frac{x^6\left(-1+\frac{1}{x^2}\right)^{\frac{7}{2}}}{6}-\frac{7x^4\left(-1+\frac{1}{x^2}\right)^{\frac{5}{2}}}{24}-\frac{35x^2\left(-1+\frac{1}{x^2}\right)^{\frac{3}{2}}}{48}+\frac{35\sqrt{-1+\frac{1}{x^2}}}{16}-\frac{35\,\mathrm{atan}\left(\sqrt{-1+\frac{1}{x^2}}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2-1)**3*(-1+1/x**2)**(1/2)/x,x)
```

```
[Out] -x**6*(-1 + x**(-2))**(7/2)/6 - 7*x**4*(-1 + x**(-2))**(5/2)/24 -
 35*x**2*(-1 + x**(-2))**(3/2)/48 + 35*sqrt(-1 + x**(-2))/16 - 35
*atan(sqrt(-1 + x**(-2)))/16
```

Mathematica [A] time = 0.0451784, size = 65, normalized size = 0.86

$$\frac{\sqrt{\frac{1}{x^2}-1}\left(\sqrt{x^2-1}\left(8x^6-38x^4+87x^2+48\right)-105x\log\left(\sqrt{x^2-1}+x\right)\right)}{48\sqrt{x^2-1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[-1 + x^(-2)]*(-1 + x^2)^3)/x,x]
```

```
[Out] (Sqrt[-1 + x^(-2)]*(Sqrt[-1 + x^2]*(48 + 87*x^2 - 38*x^4 + 8*x^6)
 - 105*x*Log[x + Sqrt[-1 + x^2]]))/(48*Sqrt[-1 + x^2])
```

Maple [A] time = 0.017, size = 83, normalized size = 1.1

$$\frac{1}{48}\sqrt{-\frac{x^2-1}{x^2}}\left(-8\,x^4\left(-x^2+1\right)^{3/2}+30\,x^2\left(-x^2+1\right)^{3/2}+48\,\left(-x^2+1\right)^{3/2}+105\,x^2\sqrt{-x^2+1}+105\,\arcsin\left(x\right)x\right)\frac{1}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2-1)^3*(-1+1/x^2)^(1/2)/x,x)
```

```
[Out] 1/48*(-(x^2-1)/x^2)^(1/2)*(-8*x^4*(-x^2+1)^(3/2)+30*x^2*(-x^2+1)^
(3/2)+48*(-x^2+1)^(3/2)+105*x^2*(-x^2+1)^(1/2)+105*arcsin(x)*x)/(
-x^2+1)^(1/2)
```

Maxima [A] time = 0.804317, size = 162, normalized size = 2.13

$$\frac{3}{2}\,x^2\sqrt{\frac{1}{x^2}-1}+\sqrt{\frac{1}{x^2}-1}-\frac{3\left(\frac{1}{x^2}-1\right)^{\frac{5}{2}}+8\left(\frac{1}{x^2}-1\right)^{\frac{3}{2}}-3\sqrt{\frac{1}{x^2}-1}}{48\left(\left(\frac{1}{x^2}-1\right)^3+3\left(\frac{1}{x^2}-1\right)^2+\frac{3}{x^2}-2\right)}+\frac{3\left(\left(\frac{1}{x^2}-1\right)^{\frac{3}{2}}-\sqrt{\frac{1}{x^2}-1}\right)}{8\left(\left(\frac{1}{x^2}-1\right)^2+\frac{2}{x^2}-1\right)}-\frac{35}{16}\,\arctan\left(\sqrt{\frac{1}{x^2}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 1)^3*sqrt(1/x^2 - 1)/x,x, algorithm="maxima")
```

```
[Out] 3/2*x^2*sqrt(1/x^2 - 1) + sqrt(1/x^2 - 1) - 1/48*(3*(1/x^2 - 1)^(
5/2) + 8*(1/x^2 - 1)^(3/2) - 3*sqrt(1/x^2 - 1))/((1/x^2 - 1)^3 +
3*(1/x^2 - 1)^2 + 3/x^2 - 2) + 3/8*((1/x^2 - 1)^(3/2) - sqrt(1/x^
2 - 1))/((1/x^2 - 1)^2 + 2/x^2 - 1) - 35/16*arctan(sqrt(1/x^2 - 1
))
```

Fricas [A] time = 0.268988, size = 74, normalized size = 0.97

$$\frac{1}{48}\left(8x^6 - 38x^4 + 87x^2 + 48\right)\sqrt{-\frac{x^2-1}{x^2}} - \frac{35}{8}\arctan\left(\frac{x\sqrt{-\frac{x^2-1}{x^2}} - 1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 1)^3*sqrt(1/x^2 - 1)/x,x, algorithm="fricas")
```

```
[Out] 1/48*(8*x^6 - 38*x^4 + 87*x^2 + 48)*sqrt(-(x^2 - 1)/x^2) - 35/8*a
rctan((x*sqrt(-(x^2 - 1)/x^2) - 1)/x)
```

Sympy [A] time = 32.4403, size = 348, normalized size = 4.58

$$-\begin{cases} -\frac{ix}{\sqrt{x^2-1}} + i\operatorname{acosh}(x) + \frac{i}{x\sqrt{x^2-1}} & \text{for}\ \left|x^2\right| > 1 \\ \frac{x}{\sqrt{-x^2+1}} - \operatorname{asin}(x) - \frac{1}{x\sqrt{-x^2+1}} & \text{otherwise} \end{cases} + 3\left(\begin{cases} \frac{ix^3}{2\sqrt{x^2-1}} - \frac{ix}{2\sqrt{x^2-1}} - \frac{i\operatorname{acosh}(x)}{2} & \text{for}\ \left|x^2\right| > 1 \\ \frac{x\sqrt{-x^2+1}}{2} + \frac{\operatorname{asin}(x)}{2} & \text{otherwise} \end{cases}\right) - 3\left(\begin{cases} \frac{ix^5}{4\sqrt{x^2-1}} - \frac{3ix^3}{8\sqrt{x^2-1}} + \frac{ix}{8\sqrt{x^2-1}} - \frac{i\operatorname{acosh}(x)}{8} & \text{for}\ \left|x^2\right| > 1 \\ -\frac{x^5}{4\sqrt{-x^2+1}} + \frac{3x^3}{8\sqrt{-x^2+1}} - \frac{x}{8\sqrt{-x^2+1}} + \frac{\operatorname{asin}(x)}{8} & \text{otherwise} \end{cases}\right) + \begin{cases} \frac{ix^7}{6\sqrt{x^2-1}} - \frac{5ix^5}{24\sqrt{x^2-1}} - \frac{ix^3}{48\sqrt{x^2-1}} + \frac{ix}{16\sqrt{x^2-1}} - \frac{i\operatorname{acosh}(x)}{16} & \text{for}\ \left|x^2\right| > 1 \\ -\frac{x^7}{6\sqrt{-x^2+1}} + \frac{5x^5}{24\sqrt{-x^2+1}} + \frac{x^3}{48\sqrt{-x^2+1}} - \frac{x}{16\sqrt{-x^2+1}} + \frac{\operatorname{asin}(x)}{16} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2-1)**3*(-1+1/x**2)**(1/2)/x,x)
```

```
[Out] -Piecewise((-I*x/sqrt(x**2 - 1) + I*acosh(x) + I/(x*sqrt(x**2 - 1
)), Abs(x**2) > 1), (x/sqrt(-x**2 + 1) - asin(x) - 1/(x*sqrt(-x**
2 + 1)), True)) + 3*Piecewise((I*x**3/(2*sqrt(x**2 - 1)) - I*x/(2
*sqrt(x**2 - 1)) - I*acosh(x)/2, Abs(x**2) > 1), (x*sqrt(-x**2 +
1)/2 + asin(x)/2, True)) - 3*Piecewise((I*x**5/(4*sqrt(x**2 - 1))
 - 3*I*x**3/(8*sqrt(x**2 - 1)) + I*x/(8*sqrt(x**2 - 1)) - I*acosh
(x)/8, Abs(x**2) > 1), (-x**5/(4*sqrt(-x**2 + 1)) + 3*x**3/(8*sqr
t(-x**2 + 1)) - x/(8*sqrt(-x**2 + 1)) + asin(x)/8, True)) + Piece
```

```
wise((I*x**7/(6*sqrt(x**2 - 1)) - 5*I*x**5/(24*sqrt(x**2 - 1)) -
I*x**3/(48*sqrt(x**2 - 1)) + I*x/(16*sqrt(x**2 - 1)) - I*acosh(x)
/16, Abs(x**2) > 1), (-x**7/(6*sqrt(-x**2 + 1)) + 5*x**5/(24*sqrt
(-x**2 + 1)) + x**3/(48*sqrt(-x**2 + 1)) - x/(16*sqrt(-x**2 + 1))
 + asin(x)/16, True))
```

GIAC/XCAS [A] time = 0.285528, size = 104, normalized size = 1.37

$$\frac{1}{48}\left(2\left(4x^2\operatorname{sign}(x) - 19\operatorname{sign}(x)\right)x^2 + 87\operatorname{sign}(x)\right)\sqrt{-x^2+1}x + \frac{35}{16}\arcsin(x)\operatorname{sign}(x) - \frac{x\operatorname{sign}(x)}{2\left(\sqrt{-x^2+1}-1\right)} + \frac{\left(\sqrt{-x^2+1}-1\right)\operatorname{sign}(x)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 1)^3*sqrt(1/x^2 - 1)/x,x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*x^2*sign(x) - 19*sign(x))*x^2 + 87*sign(x))*sqrt(-x^2
+ 1)*x + 35/16*arcsin(x)*sign(x) - 1/2*x*sign(x)/(sqrt(-x^2 + 1)
- 1) + 1/2*(sqrt(-x^2 + 1) - 1)*sign(x)/x
```

3.515 $\int \frac{\sqrt{-1+\frac{1}{x^2}}\,(-1+x^2)^2}{x}\,dx$

Optimal. Leaf size=60

$$\frac{5}{8}\left(\frac{1}{x^2}-1\right)^{3/2}x^2-\frac{15}{8}\sqrt{\frac{1}{x^2}-1}+\frac{15}{8}\tan^{-1}\left(\sqrt{\frac{1}{x^2}-1}\right)+\frac{1}{4}\left(\frac{1}{x^2}-1\right)^{5/2}x^4$$

```
[Out] (-15*Sqrt[-1 + x^(-2)])/8 + (5*(-1 + x^(-2))^(3/2)*x^2)/8 + ((-1
+ x^(-2))^(5/2)*x^4)/4 + (15*ArcTan[Sqrt[-1 + x^(-2)]])/8
```

Rubi [A] time = 0.0559564, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$\frac{5}{8}\left(\frac{1}{x^2}-1\right)^{3/2}x^2-\frac{15}{8}\sqrt{\frac{1}{x^2}-1}+\frac{15}{8}\tan^{-1}\left(\sqrt{\frac{1}{x^2}-1}\right)+\frac{1}{4}\left(\frac{1}{x^2}-1\right)^{5/2}x^4$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[-1 + x^(-2)]*(-1 + x^2)^2)/x,x]
```

```
[Out] (-15*Sqrt[-1 + x^(-2)])/8 + (5*(-1 + x^(-2))^(3/2)*x^2)/8 + ((-1
+ x^(-2))^(5/2)*x^4)/4 + (15*ArcTan[Sqrt[-1 + x^(-2)]])/8
```

Rubi in Sympy [A] time = 3.97222, size = 60, normalized size = 1.

$$\frac{x^4\left(-1+\frac{1}{x^2}\right)^{\frac{5}{2}}}{4}+\frac{5x^2\left(-1+\frac{1}{x^2}\right)^{\frac{3}{2}}}{8}-\frac{15\sqrt{-1+\frac{1}{x^2}}}{8}+\frac{15\,\mathrm{atan}\left(\sqrt{-1+\frac{1}{x^2}}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2-1)**2*(-1+1/x**2)**(1/2)/x,x)
```

```
[Out] x**4*(-1 + x**(-2))**(5/2)/4 + 5*x**2*(-1 + x**(-2))**(3/2)/8 - 1
5*sqrt(-1 + x**(-2))/8 + 15*atan(sqrt(-1 + x**(-2)))/8
```

Mathematica [A] time = 0.034665, size = 60, normalized size = 1.

$$\frac{\sqrt{\frac{1}{x^2}-1}\left(15x\log\left(\sqrt{x^2-1}+x\right)+\sqrt{x^2-1}\left(2x^4-9x^2-8\right)\right)}{8\sqrt{x^2-1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[-1 + x^(-2)]*(-1 + x^2)^2)/x,x]
```

```
[Out] (Sqrt[-1 + x^(-2)]*(Sqrt[-1 + x^2]*(-8 - 9*x^2 + 2*x^4) + 15*x*Lo
g[x + Sqrt[-1 + x^2]]))/(8*Sqrt[-1 + x^2])
```

Maple [A] time = 0.011, size = 69, normalized size = 1.2

$$-\frac{1}{8}\sqrt{-\frac{x^2-1}{x^2}}\left(2x^2\left(-x^2+1\right)^{3/2}+8\left(-x^2+1\right)^{3/2}+15x^2\sqrt{-x^2+1}+15\arcsin(x)x\right)\frac{1}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2-1)^2*(-1+1/x^2)^(1/2)/x,x)
```

```
[Out] -1/8*(-(x^2-1)/x^2)^(1/2)*(2*x^2*(-x^2+1)^(3/2)+8*(-x^2+1)^(3/2)+
15*x^2*(-x^2+1)^(1/2)+15*arcsin(x)*x)/(-x^2+1)^(1/2)
```

Maxima [A] time = 0.798958, size = 90, normalized size = 1.5

$$-x^2\sqrt{\frac{1}{x^2}-1}-\sqrt{\frac{1}{x^2}-1}-\frac{\left(\frac{1}{x^2}-1\right)^{\frac{3}{2}}-\sqrt{\frac{1}{x^2}-1}}{8\left(\left(\frac{1}{x^2}-1\right)^2+\frac{2}{x^2}-1\right)}+\frac{15}{8}\arctan\left(\sqrt{\frac{1}{x^2}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 1)^2*sqrt(1/x^2 - 1)/x,x, algorithm="maxima")
```

```
[Out] -x^2*sqrt(1/x^2 - 1) - sqrt(1/x^2 - 1) - 1/8*((1/x^2 - 1)^(3/2) -
 sqrt(1/x^2 - 1))/((1/x^2 - 1)^2 + 2/x^2 - 1) + 15/8*arctan(sqrt(
1/x^2 - 1))
```

Fricas [A] time = 0.265638, size = 68, normalized size = 1.13

$$\frac{1}{8}\left(2x^4-9x^2-8\right)\sqrt{-\frac{x^2-1}{x^2}}+\frac{15}{4}\arctan\left(\frac{x\sqrt{-\frac{x^2-1}{x^2}}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 1)^2*sqrt(1/x^2 - 1)/x,x, algorithm="fricas")
```

```
[Out] 1/8*(2*x^4 - 9*x^2 - 8)*sqrt(-(x^2 - 1)/x^2) + 15/4*arctan((x*sqr
t(-(x^2 - 1)/x^2) - 1)/x)
```

Sympy [A] time = 18.6116, size = 216, normalized size = 3.6

$$\begin{cases} -\frac{ix}{\sqrt{x^2-1}}+i\,\mathrm{acosh}\,(x)+\frac{i}{x\sqrt{x^2-1}} & \text{for } \left|x^2\right|>1 \\ \frac{x}{\sqrt{-x^2+1}}-\mathrm{asin}\,(x)-\frac{1}{x\sqrt{-x^2+1}} & \text{otherwise} \end{cases} - 2\left(\begin{cases} \frac{ix^3}{2\sqrt{x^2-1}}-\frac{ix}{2\sqrt{x^2-1}}-\frac{i\,\mathrm{acosh}\,(x)}{2} & \text{for } \left|x^2\right|>1 \\ \frac{x\sqrt{-x^2+1}}{2}+\frac{\mathrm{asin}\,(x)}{2} & \text{otherwise} \end{cases}\right) + \begin{cases} \frac{ix^5}{4\sqrt{x^2-1}}-\frac{3ix^3}{8\sqrt{x^2-1}}+\frac{ix}{8\sqrt{x^2-1}}-\frac{i\,\mathrm{acosh}\,(x)}{8} & \text{for } \left|x^2\right|>1 \\ -\frac{x^5}{4\sqrt{-x^2+1}}+\frac{3x^3}{8\sqrt{-x^2+1}}-\frac{x}{8\sqrt{-x^2+1}}+\frac{\mathrm{asin}\,(x)}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2-1)**2*(-1+1/x**2)**(1/2)/x,x)
```

```
[Out] Piecewise((-I*x/sqrt(x**2 - 1) + I*acosh(x) + I/(x*sqrt(x**2 - 1)
), Abs(x**2) > 1), (x/sqrt(-x**2 + 1) - asin(x) - 1/(x*sqrt(-x**2
 + 1)), True)) - 2*Piecewise((I*x**3/(2*sqrt(x**2 - 1)) - I*x/(2*
sqrt(x**2 - 1)) - I*acosh(x)/2, Abs(x**2) > 1), (x*sqrt(-x**2 + 1
)/2 + asin(x)/2, True)) + Piecewise((I*x**5/(4*sqrt(x**2 - 1)) -
3*I*x**3/(8*sqrt(x**2 - 1)) + I*x/(8*sqrt(x**2 - 1)) - I*acosh(x)
/8, Abs(x**2) > 1), (-x**5/(4*sqrt(-x**2 + 1)) + 3*x**3/(8*sqrt(-
x**2 + 1)) - x/(8*sqrt(-x**2 + 1)) + asin(x)/8, True))
```

GIAC/XCAS [A] time = 0.271025, size = 90, normalized size = 1.5

$$\frac{1}{8}\left(2x^2\mathrm{sign}\,(x)-9\,\mathrm{sign}\,(x)\right)\sqrt{-x^2+1}x-\frac{15}{8}\arcsin\,(x)\,\mathrm{sign}\,(x)+\frac{x\mathrm{sign}\,(x)}{2\left(\sqrt{-x^2+1}-1\right)}-\frac{\left(\sqrt{-x^2+1}-1\right)\mathrm{sign}\,(x)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 1)^2*sqrt(1/x^2 - 1)/x,x, algorithm="giac")
```

```
[Out] 1/8*(2*x^2*sign(x) - 9*sign(x))*sqrt(-x^2 + 1)*x - 15/8*arcsin(x)
*sign(x) + 1/2*x*sign(x)/(sqrt(-x^2 + 1) - 1) - 1/2*(sqrt(-x^2 +
1) - 1)*sign(x)/x
```

3.516 $\int \frac{\sqrt{-1+\frac{1}{x^2}}\,(-1+x^2)}{x}\,dx$

Optimal. Leaf size=44

$$-\frac{1}{2}\left(\frac{1}{x^2}-1\right)^{3/2}x^2+\frac{3}{2}\sqrt{\frac{1}{x^2}-1}-\frac{3}{2}\tan^{-1}\left(\sqrt{\frac{1}{x^2}-1}\right)$$

```
[Out] (3*Sqrt[-1 + x^(-2)])/2 - ((-1 + x^(-2))^(3/2)*x^2)/2 - (3*ArcTan
[Sqrt[-1 + x^(-2)]])/2
```

Rubi [A] time = 0.0409153, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$-\frac{1}{2}\left(\frac{1}{x^2}-1\right)^{3/2}x^2+\frac{3}{2}\sqrt{\frac{1}{x^2}-1}-\frac{3}{2}\tan^{-1}\left(\sqrt{\frac{1}{x^2}-1}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[-1 + x^(-2)]*(-1 + x^2))/x,x]
```

```
[Out] (3*Sqrt[-1 + x^(-2)])/2 - ((-1 + x^(-2))^(3/2)*x^2)/2 - (3*ArcTan
[Sqrt[-1 + x^(-2)]])/2
```

Rubi in Sympy [A] time = 2.97041, size = 42, normalized size = 0.95

$$-\frac{x^2\left(-1+\frac{1}{x^2}\right)^{\frac{3}{2}}}{2}+\frac{3\sqrt{-1+\frac{1}{x^2}}}{2}-\frac{3\,\mathrm{atan}\left(\sqrt{-1+\frac{1}{x^2}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2-1)*(-1+1/x**2)**(1/2)/x,x)
```

```
[Out] -x**2*(-1 + x**(-2))**(3/2)/2 + 3*sqrt(-1 + x**(-2))/2 - 3*atan(s
qrt(-1 + x**(-2)))/2
```

Mathematica [A] time = 0.0261215, size = 53, normalized size = 1.2

$$\frac{\sqrt{\frac{1}{x^2}-1}\left(\sqrt{x^2-1}\left(x^2+2\right)-3x\log\left(\sqrt{x^2-1}+x\right)\right)}{2\sqrt{x^2-1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[-1 + x^(-2)]*(-1 + x^2))/x,x]
```

```
[Out] (Sqrt[-1 + x^(-2)]*(Sqrt[-1 + x^2]*(2 + x^2) - 3*x*Log[x + Sqrt[-
1 + x^2]]))/(2*Sqrt[-1 + x^2])
```

Maple [A] time = 0.011, size = 55, normalized size = 1.3

$$\frac{1}{2}\sqrt{-\frac{x^2-1}{x^2}}\left(2\,\left(-x^2+1\right)^{3/2}+3\,x^2\sqrt{-x^2+1}+3\,\arcsin\left(x\right)x\right)\frac{1}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2-1)*(-1+1/x^2)^(1/2)/x,x)
```

```
[Out] 1/2*(-(x^2-1)/x^2)^(1/2)*(2*(-x^2+1)^(3/2)+3*x^2*(-x^2+1)^(1/2)+3
*arcsin(x)*x)/(-x^2+1)^(1/2)
```

Maxima [A] time = 0.800166, size = 41, normalized size = 0.93

$$\frac{1}{2}\,x^2\sqrt{\frac{1}{x^2}-1}+\sqrt{\frac{1}{x^2}-1}-\frac{3}{2}\,\arctan\left(\sqrt{\frac{1}{x^2}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 1)*sqrt(1/x^2 - 1)/x,x, algorithm="maxima")
```

```
[Out] 1/2*x^2*sqrt(1/x^2 - 1) + sqrt(1/x^2 - 1) - 3/2*arctan(sqrt(1/x^2
 - 1))
```

Fricas [A] time = 0.26889, size = 58, normalized size = 1.32

$$\frac{1}{2}\left(x^{2}+2\right)\sqrt{-\frac{x^{2}-1}{x^{2}}}-3\arctan\left(\frac{x\sqrt{-\frac{x^{2}-1}{x^{2}}}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 1)*sqrt(1/x^2 - 1)/x,x, algorithm="fricas")
```

```
[Out] 1/2*(x^2 + 2)*sqrt(-(x^2 - 1)/x^2) - 3*arctan((x*sqrt(-(x^2 - 1)/
x^2) - 1)/x)
```

Sympy [A] time = 10.4937, size = 112, normalized size = 2.55

$$-\begin{cases}-\frac{ix}{\sqrt{x^{2}-1}}+i\operatorname{acosh}(x)+\frac{i}{x\sqrt{x^{2}-1}} & \text{for}\ \left|x^{2}\right|>1\\ \frac{x}{\sqrt{-x^{2}+1}}-\operatorname{asin}(x)-\frac{1}{x\sqrt{-x^{2}+1}} & \text{otherwise}\end{cases}+\begin{cases}\frac{ix^{3}}{2\sqrt{x^{2}-1}}-\frac{ix}{2\sqrt{x^{2}-1}}-\frac{i\operatorname{acosh}(x)}{2} & \text{for}\ \left|x^{2}\right|>1\\ \frac{x\sqrt{-x^{2}+1}}{2}+\frac{\operatorname{asin}(x)}{2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2-1)*(-1+1/x**2)**(1/2)/x,x)
```

```
[Out] -Piecewise((-I*x/sqrt(x**2 - 1) + I*acosh(x) + I/(x*sqrt(x**2 - 1
)), Abs(x**2) > 1), (x/sqrt(-x**2 + 1) - asin(x) - 1/(x*sqrt(-x**
2 + 1)), True)) + Piecewise((I*x**3/(2*sqrt(x**2 - 1)) - I*x/(2*s
qrt(x**2 - 1)) - I*acosh(x)/2, Abs(x**2) > 1), (x*sqrt(-x**2 + 1)
/2 + asin(x)/2, True))
```

GIAC/XCAS [A] time = 0.269938, size = 77, normalized size = 1.75

$$\frac{1}{2}\sqrt{-x^{2}+1}x\operatorname{sign}(x)+\frac{3}{2}\arcsin(x)\operatorname{sign}(x)-\frac{x\operatorname{sign}(x)}{2\left(\sqrt{-x^{2}+1}-1\right)}+\frac{\left(\sqrt{-x^{2}+1}-1\right)\operatorname{sign}(x)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 1)*sqrt(1/x^2 - 1)/x,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-x^2 + 1)*x*sign(x) + 3/2*arcsin(x)*sign(x) - 1/2*x*sign
(x)/(sqrt(-x^2 + 1) - 1) + 1/2*(sqrt(-x^2 + 1) - 1)*sign(x)/x
```

3.517 $\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x(-1+x^2)}\,dx$

Optimal. Leaf size=9

$$\sqrt{\frac{1}{x^2}-1}$$

```
[Out] Sqrt[-1 + x^(-2)]
```

Rubi [A] time = 0.0154961, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$\sqrt{\frac{1}{x^2}-1}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[-1 + x^(-2)]/(x*(-1 + x^2)),x]

[Out] Sqrt[-1 + x^(-2)]
```

Rubi in Sympy [A] time = 1.47169, size = 8, normalized size = 0.89

$$\sqrt{-1+\frac{1}{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-1+1/x**2)**(1/2)/x/(x**2-1),x)

[Out] sqrt(-1 + x**(-2))
```

Mathematica [A] time = 0.00928303, size = 9, normalized size = 1.

$$\sqrt{\frac{1}{x^2}-1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[-1 + x^(-2)]/(x*(-1 + x^2)),x]

[Out] Sqrt[-1 + x^(-2)]
```

Maple [A] time = 0.005, size = 13, normalized size = 1.4

$$\sqrt{-\frac{x^2-1}{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-1+1/x^2)^(1/2)/x/(x^2-1),x)

[Out] (-(x^2-1)/x^2)^(1/2)
```

Maxima [A] time = 0.736675, size = 22, normalized size = 2.44

$$\frac{\sqrt{x+1}\sqrt{-x+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(1/x^2 - 1)/((x^2 - 1)*x),x, algorithm="maxima")

[Out] sqrt(x + 1)*sqrt(-x + 1)/x
```

Fricas [A] time = 0.263645, size = 16, normalized size = 1.78

$$\sqrt{-\frac{x^2-1}{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(1/x^2 - 1)/((x^2 - 1)*x),x, algorithm="fricas")

[Out] sqrt(-(x^2 - 1)/x^2)
```

Sympy [A] time = 4.06615, size = 8, normalized size = 0.89

$$\sqrt{-1+\frac{1}{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-1+1/x**2)**(1/2)/x/(x**2-1),x)
```

```
[Out] sqrt(-1 + x**(-2))
```

GIAC/XCAS [A] time = 0.266895, size = 50, normalized size = 5.56

$$-\frac{x\text{sign}(x)}{2\left(\sqrt{-x^2+1}-1\right)}+\frac{\left(\sqrt{-x^2+1}-1\right)\text{sign}(x)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(1/x^2 - 1)/((x^2 - 1)*x),x, algorithm="giac")
```

```
[Out] -1/2*x*sign(x)/(sqrt(-x^2 + 1) - 1) + 1/2*(sqrt(-x^2 + 1) - 1)*si
gn(x)/x
```

3.518 $\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x(-1+x^2)^2}\,dx$

Optimal. Leaf size=21

$$\frac{1}{\sqrt{\frac{1}{x^2}-1}}-\sqrt{\frac{1}{x^2}-1}$$

```
[Out] 1/Sqrt[-1 + x^(-2)] - Sqrt[-1 + x^(-2)]
```

Rubi [A] time = 0.0303385, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15

$$\frac{1}{\sqrt{\frac{1}{x^2}-1}}-\sqrt{\frac{1}{x^2}-1}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[-1 + x^(-2)]/(x*(-1 + x^2)^2),x]

[Out] 1/Sqrt[-1 + x^(-2)] - Sqrt[-1 + x^(-2)]
```

Rubi in Sympy [A] time = 2.24651, size = 20, normalized size = 0.95

$$-\sqrt{-1+\frac{1}{x^2}}+\frac{1}{\sqrt{-1+\frac{1}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-1+1/x**2)**(1/2)/x/(x**2-1)**2,x)

[Out] -sqrt(-1 + x**(-2)) + 1/sqrt(-1 + x**(-2))
```

Mathematica [A] time = 0.0142098, size = 24, normalized size = 1.14

$$\frac{\sqrt{\frac{1}{x^2}-1}\,(1-2x^2)}{x^2-1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[-1 + x^(-2)]/(x*(-1 + x^2)^2),x]
```

```
[Out] (Sqrt[-1 + x^(-2)]*(1 - 2*x^2))/(-1 + x^2)
```

Maple [A] time = 0.006, size = 29, normalized size = 1.4

$$-\frac{2x^2-1}{x^2-1}\sqrt{-\frac{x^2-1}{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-1+1/x^2)^(1/2)/x/(x^2-1)^2,x)
```

```
[Out] -(2*x^2-1)*(-(x^2-1)/x^2)^(1/2)/(x^2-1)
```

Maxima [A] time = 0.73913, size = 41, normalized size = 1.95

$$-\frac{(2x^2-1)\sqrt{x+1}\sqrt{-x+1}}{x^3-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(1/x^2 - 1)/((x^2 - 1)^2*x),x, algorithm="maxima")
```

```
[Out] -(2*x^2 - 1)*sqrt(x + 1)*sqrt(-x + 1)/(x^3 - x)
```

Fricas [A] time = 0.263873, size = 38, normalized size = 1.81

$$-\frac{(2x^2-1)\sqrt{-\frac{x^2-1}{x^2}}}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(1/x^2 - 1)/((x^2 - 1)^2*x),x, algorithm="fricas")
```

```
[Out] -(2*x^2 - 1)*sqrt(-(x^2 - 1)/x^2)/(x^2 - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\left(-1+\frac{1}{x}\right)\left(1+\frac{1}{x}\right)}}{x\left(x-1\right)^{2}\left(x+1\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-1+1/x**2)**(1/2)/x/(x**2-1)**2,x)
```

```
[Out] Integral(sqrt((-1 + 1/x)*(1 + 1/x))/(x*(x - 1)**2*(x + 1)**2), x)
```

GIAC/XCAS [A] time = 0.267366, size = 78, normalized size = 3.71

$$-\frac{\sqrt{-x^{2}+1}x\mathrm{sign}\left(x\right)}{x^{2}-1}+\frac{x\mathrm{sign}\left(x\right)}{2\left(\sqrt{-x^{2}+1}-1\right)}-\frac{\left(\sqrt{-x^{2}+1}-1\right)\mathrm{sign}\left(x\right)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(1/x^2 - 1)/((x^2 - 1)^2*x),x, algorithm="giac")
```

```
[Out] -sqrt(-x^2 + 1)*x*sign(x)/(x^2 - 1) + 1/2*x*sign(x)/(sqrt(-x^2 +
1) - 1) - 1/2*(sqrt(-x^2 + 1) - 1)*sign(x)/x
```

3.519 $\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x(-1+x^2)^3}\,dx$

Optimal. Leaf size=34

$$\sqrt{\frac{1}{x^2}-1}-\frac{2}{\sqrt{\frac{1}{x^2}-1}}-\frac{1}{3\left(\frac{1}{x^2}-1\right)^{3/2}}$$

```
[Out] -1/(3*(-1 + x^(-2))^(3/2)) - 2/Sqrt[-1 + x^(-2)] + Sqrt[-1 + x^(-
2)]
```

Rubi [A] time = 0.038637, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15

$$\sqrt{\frac{1}{x^2}-1}-\frac{2}{\sqrt{\frac{1}{x^2}-1}}-\frac{1}{3\left(\frac{1}{x^2}-1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[-1 + x^(-2)]/(x*(-1 + x^2)^3),x]
```

```
[Out] -1/(3*(-1 + x^(-2))^(3/2)) - 2/Sqrt[-1 + x^(-2)] + Sqrt[-1 + x^(-
2)]
```

Rubi in Sympy [A] time = 2.71833, size = 34, normalized size = 1.

$$\sqrt{-1+\frac{1}{x^2}}-\frac{2}{\sqrt{-1+\frac{1}{x^2}}}-\frac{1}{3\left(-1+\frac{1}{x^2}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-1+1/x**2)**(1/2)/x/(x**2-1)**3,x)
```

```
[Out] sqrt(-1 + x**(-2)) - 2/sqrt(-1 + x**(-2)) - 1/(3*(-1 + x**(-2))**
(3/2))
```

Mathematica [A] time = 0.0173236, size = 32, normalized size = 0.94

$$\frac{\sqrt{\frac{1}{x^2}-1}\,(8x^4-12x^2+3)}{3\,(x^2-1)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[-1 + x^(-2)]/(x*(-1 + x^2)^3),x]
```

```
[Out] (Sqrt[-1 + x^(-2)]*(3 - 12*x^2 + 8*x^4))/(3*(-1 + x^2)^2)
```

Maple [A] time = 0.007, size = 34, normalized size = 1.

$$\frac{8\,x^4-12\,x^2+3}{3\,(x^2-1)^2}\sqrt{-\frac{x^2-1}{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-1+1/x^2)^(1/2)/x/(x^2-1)^3,x)
```

```
[Out] 1/3*(8*x^4-12*x^2+3)*(-(x^2-1)/x^2)^(1/2)/(x^2-1)^2
```

Maxima [A] time = 0.741181, size = 51, normalized size = 1.5

$$\frac{(8\,x^4-12\,x^2+3)\sqrt{x+1}\sqrt{-x+1}}{3\,(x^5-2\,x^3+x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(1/x^2 - 1)/((x^2 - 1)^3*x),x, algorithm="maxima")
```

```
[Out] 1/3*(8*x^4 - 12*x^2 + 3)*sqrt(x + 1)*sqrt(-x + 1)/(x^5 - 2*x^3 +
x)
```

Fricas [A] time = 0.267539, size = 51, normalized size = 1.5

$$\frac{(8\,x^4-12\,x^2+3)\sqrt{-\frac{x^2-1}{x^2}}}{3\,(x^4-2\,x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(1/x^2 - 1)/((x^2 - 1)^3*x),x, algorithm="fricas")
```

```
[Out] 1/3*(8*x^4 - 12*x^2 + 3)*sqrt(-(x^2 - 1)/x^2)/(x^4 - 2*x^2 + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\left(-1+\frac{1}{x}\right)\left(1+\frac{1}{x}\right)}}{x\left(x-1\right)^{3}\left(x+1\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-1+1/x**2)**(1/2)/x/(x**2-1)**3,x)
```

```
[Out] Integral(sqrt((-1 + 1/x)*(1 + 1/x))/(x*(x - 1)**3*(x + 1)**3), x)
```

GIAC/XCAS [A] time = 0.26911, size = 92, normalized size = 2.71

$$-\frac{x\mathrm{sign}\left(x\right)}{2\left(\sqrt{-x^{2}+1}-1\right)}+\frac{\left(\sqrt{-x^{2}+1}-1\right)\mathrm{sign}\left(x\right)}{2\,x}-\frac{\left(5\,x^{2}\mathrm{sign}\left(x\right)-6\,\mathrm{sign}\left(x\right)\right)x}{3\left(x^{2}-1\right)\sqrt{-x^{2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(1/x^2 - 1)/((x^2 - 1)^3*x),x, algorithm="giac")
```

```
[Out] -1/2*x*sign(x)/(sqrt(-x^2 + 1) - 1) + 1/2*(sqrt(-x^2 + 1) - 1)*si
gn(x)/x - 1/3*(5*x^2*sign(x) - 6*sign(x))*x/((x^2 - 1)*sqrt(-x^2
+ 1))
```

3.520 $\int \frac{\sqrt{1+\frac{1}{x^2}}x}{(1+x^2)^2} \, dx$

Optimal. Leaf size=9

$$\frac{1}{\sqrt{\frac{1}{x^2}+1}}$$

```
[Out] 1/Sqrt[1 + x^(-2)]
```

Rubi [A] time = 0.0140857, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\frac{1}{\sqrt{\frac{1}{x^2}+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[1 + x^(-2)]*x)/(1 + x^2)^2,x]
```

```
[Out] 1/Sqrt[1 + x^(-2)]
```

Rubi in Sympy [A] time = 1.41876, size = 10, normalized size = 1.11

$$\frac{1}{\sqrt{1+\frac{1}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(1+1/x**2)**(1/2)/(x**2+1)**2,x)
```

```
[Out] 1/sqrt(1 + x**(-2))
```

Mathematica [B] time = 0.0128918, size = 20, normalized size = 2.22

$$\frac{\sqrt{\frac{1}{x^2}+1}x^2}{x^2+1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[1 + x^(-2)]*x)/(1 + x^2)^2,x]
```

```
[Out] (Sqrt[1 + x^(-2)]*x^2)/(1 + x^2)
```

Maple [B] time = 0.005, size = 23, normalized size = 2.6

$$\frac{x^2}{x^2+1}\sqrt{\frac{x^2+1}{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(1+1/x^2)^(1/2)/(x^2+1)^2,x)
```

```
[Out] 1/(x^2+1)*x^2*((x^2+1)/x^2)^(1/2)
```

Maxima [A] time = 0.804031, size = 15, normalized size = 1.67

$$\frac{1}{\sqrt{\frac{x^2+1}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sqrt(1/x^2 + 1)/(x^2 + 1)^2,x, algorithm="maxima")
```

```
[Out] 1/sqrt((x^2 + 1)/x^2)
```

Fricas [A] time = 0.261212, size = 38, normalized size = 4.22

$$\frac{x^2\sqrt{\frac{x^2+1}{x^2}}+x^2+1}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sqrt(1/x^2 + 1)/(x^2 + 1)^2,x, algorithm="fricas")
```

```
[Out] (x^2*sqrt((x^2 + 1)/x^2) + x^2 + 1)/(x^2 + 1)
```

Sympy [A] time = 7.11112, size = 8, normalized size = 0.89

$$\frac{x}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(1+1/x**2)**(1/2)/(x**2+1)**2,x)
```

```
[Out] x/sqrt(x**2 + 1)
```

GIAC/XCAS [A] time = 0.264072, size = 15, normalized size = 1.67

$$\frac{x\text{sign}(x)}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sqrt(1/x^2 + 1)/(x^2 + 1)^2,x, algorithm="giac")
```

```
[Out] x*sign(x)/sqrt(x^2 + 1)
```

3.521 $\int \frac{1}{\sqrt{1+\frac{1}{x^2}}x(1+x^2)}\,dx$

Optimal. Leaf size=9

$$\frac{1}{\sqrt{\frac{1}{x^2}+1}}$$

```
[Out] 1/Sqrt[1 + x^(-2)]
```

Rubi [A] time = 0.0139321, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$\frac{1}{\sqrt{\frac{1}{x^2}+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[1 + x^(-2)]*x*(1 + x^2)),x]

[Out] 1/Sqrt[1 + x^(-2)]
```

Rubi in Sympy [A] time = 1.44591, size = 10, normalized size = 1.11

$$\frac{1}{\sqrt{1+\frac{1}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(x**2+1)/(1+1/x**2)**(1/2),x)

[Out] 1/sqrt(1 + x**(-2))
```

Mathematica [B] time = 0.00927791, size = 20, normalized size = 2.22

$$\frac{\sqrt{\frac{1}{x^2}+1}x^2}{x^2+1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[1 + x^(-2)]*x*(1 + x^2)),x]

[Out] (Sqrt[1 + x^(-2)]*x^2)/(1 + x^2)
```

Maple [A] time = 0.005, size = 12, normalized size = 1.3

$$\frac{1}{\sqrt{\frac{x^2+1}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(x^2+1)/(1+1/x^2)^(1/2),x)

[Out] 1/((x^2+1)/x^2)^(1/2)
```

Maxima [A] time = 0.813992, size = 12, normalized size = 1.33

$$\frac{x}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 1)*x*sqrt(1/x^2 + 1)),x, algorithm="maxima")

[Out] x/sqrt(x^2 + 1)
```

Fricas [A] time = 0.260374, size = 38, normalized size = 4.22

$$\frac{x^2\sqrt{\frac{x^2+1}{x^2}} + x^2 + 1}{x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 1)*x*sqrt(1/x^2 + 1)),x, algorithm="fricas")
```

```
[Out] (x^2*sqrt((x^2 + 1)/x^2) + x^2 + 1)/(x^2 + 1)
```

Sympy [A] time = 5.91692, size = 10, normalized size = 1.11

$$\frac{1}{\sqrt{1+\frac{1}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(x**2+1)/(1+1/x**2)**(1/2),x)

[Out] 1/sqrt(1 + x**(-2))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2+1)x\sqrt{\frac{1}{x^2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 1)*x*sqrt(1/x^2 + 1)),x, algorithm="giac")

[Out] integrate(1/((x^2 + 1)*x*sqrt(1/x^2 + 1)), x)
```

3.522 $\int \frac{x}{a+bx^2+\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=18

$$\frac{\log\left(\sqrt{a+bx^2}+1\right)}{b}$$

```
[Out] Log[1 + Sqrt[a + b*x^2]]/b
```

Rubi [A] time = 0.115315, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{\log\left(\sqrt{a+bx^2}+1\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^2 + Sqrt[a + b*x^2]),x]

[Out] Log[1 + Sqrt[a + b*x^2]]/b
```

Rubi in Sympy [A] time = 4.47737, size = 14, normalized size = 0.78

$$\frac{\log\left(\sqrt{a+bx^2}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x/(a+b*x**2+(b*x**2+a)**(1/2)),x)

[Out] log(sqrt(a + b*x**2) + 1)/b
```

Mathematica [A] time = 0.0170817, size = 18, normalized size = 1.

$$\frac{\log\left(\sqrt{a+bx^2}+1\right)}{b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(a + b*x^2 + Sqrt[a + b*x^2]),x]

[Out] Log[1 + Sqrt[a + b*x^2]]/b
```

Maple [B] time = 0.057, size = 1059, normalized size = 58.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(a+b*x^2+(b*x^2+a)^(1/2)),x)

[Out] 1/2/((-b*(a-1))^(1/2)+(-a*b)^(1/2))/((-b*(a-1))^(1/2)-(-a*b)^(1/2
))*((x-1/b*(-a*b)^(1/2))^2*b+2*(-a*b)^(1/2)*(x-1/b*(-a*b)^(1/2)))
^(1/2)+1/2/((-b*(a-1))^(1/2)+(-a*b)^(1/2))/((-b*(a-1))^(1/2)-(-a*
b)^(1/2))*(-a*b)^(1/2)*ln(((x-1/b*(-a*b)^(1/2))*b+(-a*b)^(1/2))/b
^(1/2)+((x-1/b*(-a*b)^(1/2))^2*b+2*(-a*b)^(1/2)*(x-1/b*(-a*b)^(1/
2)))^(1/2))/b^(1/2)+1/2/((-b*(a-1))^(1/2)+(-a*b)^(1/2))/((-b*(a-1
))^(1/2)-(-a*b)^(1/2))*((x+1/b*(-a*b)^(1/2))^2*b-2*(-a*b)^(1/2)*(
x+1/b*(-a*b)^(1/2)))^(1/2)-1/2/((-b*(a-1))^(1/2)+(-a*b)^(1/2))/((
-b*(a-1))^(1/2)-(-a*b)^(1/2))*(-a*b)^(1/2)*ln(((x+1/b*(-a*b)^(1/2
))*b-(-a*b)^(1/2))/b^(1/2)+((x+1/b*(-a*b)^(1/2))^2*b-2*(-a*b)^(1/
2)*(x+1/b*(-a*b)^(1/2)))^(1/2))/b^(1/2)-1/2/((-b*(a-1))^(1/2)+(-a
*b)^(1/2))/((-b*(a-1))^(1/2)-(-a*b)^(1/2))*((x-(-b*(a-1))^(1/2)/b
)^2*b+2*(-b*(a-1))^(1/2)*(x-(-b*(a-1))^(1/2)/b)+1)^(1/2)-1/2/((-b
*(a-1))^(1/2)+(-a*b)^(1/2))/((-b*(a-1))^(1/2)-(-a*b)^(1/2))*(-b*(
a-1))^(1/2)*ln(((x-(-b*(a-1))^(1/2)/b)*b+(-b*(a-1))^(1/2))/b^(1/2
)+((x-(-b*(a-1))^(1/2)/b)^2*b+2*(-b*(a-1))^(1/2)*(x-(-b*(a-1))^(1
/2)/b)+1)^(1/2))/b^(1/2)+1/2/((-b*(a-1))^(1/2)+(-a*b)^(1/2))/((-b
*(a-1))^(1/2)-(-a*b)^(1/2))*arctanh(1/2*(2+2*(-b*(a-1))^(1/2)*(x-
(-b*(a-1))^(1/2)/b))/((x-(-b*(a-1))^(1/2)/b)^2*b+2*(-b*(a-1))^(1/
2)*(x-(-b*(a-1))^(1/2)/b)+1)^(1/2))-1/2/((-b*(a-1))^(1/2)+(-a*b)^
(1/2))/((-b*(a-1))^(1/2)-(-a*b)^(1/2))*((x+(-b*(a-1))^(1/2)/b)^2*
b-2*(-b*(a-1))^(1/2)*(x+(-b*(a-1))^(1/2)/b)+1)^(1/2)+1/2/((-b*(a-
1))^(1/2)+(-a*b)^(1/2))/((-b*(a-1))^(1/2)-(-a*b)^(1/2))*(-b*(a-1)
)^(1/2)*ln(((x+(-b*(a-1))^(1/2)/b)*b-(-b*(a-1))^(1/2))/b^(1/2)+((
x+(-b*(a-1))^(1/2)/b)^2*b-2*(-b*(a-1))^(1/2)*(x+(-b*(a-1))^(1/2)/
b)+1)^(1/2))/b^(1/2)+1/2/((-b*(a-1))^(1/2)+(-a*b)^(1/2))/((-b*(a-
1))^(1/2)-(-a*b)^(1/2))*arctanh(1/2*(2-2*(-b*(a-1))^(1/2)*(x+(-b*
(a-1))^(1/2)/b))/((x+(-b*(a-1))^(1/2)/b)^2*b-2*(-b*(a-1))^(1/2)*(
x+(-b*(a-1))^(1/2)/b)+1)^(1/2))+1/2/b*ln(b*x^2+a-1)
```

Maxima [A] time = 0.728139, size = 22, normalized size = 1.22

$$\frac{\log\left(\sqrt{bx^2 + a} + 1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(b*x^2 + a + sqrt(b*x^2 + a)),x, algorithm="maxima")
```

```
[Out] log(sqrt(b*x^2 + a) + 1)/b
```

Fricas [A] time = 0.276914, size = 90, normalized size = 5.

$$\frac{2 \log\left(bx^2 + a - 1\right) + \log\left(\frac{bx^2+a+2\sqrt{bx^2+a}+1}{x^2}\right) - \log\left(\frac{bx^2+a-2\sqrt{bx^2+a}+1}{x^2}\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(b*x^2 + a + sqrt(b*x^2 + a)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*log(b*x^2 + a - 1) + log((b*x^2 + a + 2*sqrt(b*x^2 + a) +
1)/x^2) - log((b*x^2 + a - 2*sqrt(b*x^2 + a) + 1)/x^2))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{a + bx^2 + \sqrt{a + bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(a+b*x**2+(b*x**2+a)**(1/2)),x)
```

```
[Out] Integral(x/(a + b*x**2 + sqrt(a + b*x**2)), x)
```

GIAC/XCAS [A] time = 0.261379, size = 22, normalized size = 1.22

$$\frac{\ln\left(\sqrt{bx^2 + a} + 1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(b*x^2 + a + sqrt(b*x^2 + a)),x, algorithm="giac")
```

```
[Out] ln(sqrt(b*x^2 + a) + 1)/b
```

3.523 $\int \frac{x}{x^2-\sqrt[3]{x^2}} \, dx$

Optimal. Leaf size=16

$$\frac{3}{4}\log\left(1-\left(x^2\right)^{2/3}\right)$$

```
[Out] (3*Log[1 - (x^2)^(2/3)])/4
```

Rubi [A] time = 0.0953562, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$

$$\frac{3}{4}\log\left(1-\left(x^2\right)^{2/3}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x/(x^2 - (x^2)^(1/3)),x]

[Out] (3*Log[1 - (x^2)^(2/3)])/4
```

Rubi in Sympy [A] time = 4.7208, size = 12, normalized size = 0.75

$$\frac{3\log\left(-\left(x^2\right)^{\frac{2}{3}}+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(x**2-(x**2)**(1/3)),x)

[Out] 3*log(-(x**2)**(2/3) + 1)/4
```

Mathematica [A] time = 0.0129344, size = 14, normalized size = 0.88

$$\frac{3}{4}\log\left(\left(x^2\right)^{2/3}-1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(x^2 - (x^2)^(1/3)),x]
```

```
[Out] (3*Log[-1 + (x^2)^(2/3)])/4
```

Maple [B] time = 0.056, size = 70, normalized size = 4.4

$$\frac{\ln\left(x^2-1\right)}{4}+\frac{\ln\left(x^2+1\right)}{4}-\frac{1}{4}\ln\left(\left(x^2\right)^{\frac{2}{3}}+\sqrt[3]{x^2}+1\right)+\frac{1}{2}\ln\left(\sqrt[3]{x^2}-1\right)+\frac{1}{2}\ln\left(1+\sqrt[3]{x^2}\right)-\frac{1}{4}\ln\left(\left(x^2\right)^{\frac{2}{3}}-\sqrt[3]{x^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(x^2-(x^2)^(1/3)),x)
```

```
[Out] 1/4*ln(x^2-1)+1/4*ln(x^2+1)-1/4*ln((x^2)^(2/3)+(x^2)^(1/3)+1)+1/2
*ln((x^2)^(1/3)-1)+1/2*ln(1+(x^2)^(1/3))-1/4*ln((x^2)^(2/3)-(x^2)
^(1/3)+1)
```

Maxima [A] time = 0.719974, size = 28, normalized size = 1.75

$$\frac{3}{4}\log\left(\left(x^2\right)^{\frac{1}{3}}+1\right)+\frac{3}{4}\log\left(\left(x^2\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^2 - (x^2)^(1/3)),x, algorithm="maxima")
```

```
[Out] 3/4*log((x^2)^(1/3) + 1) + 3/4*log((x^2)^(1/3) - 1)
```

Fricas [A] time = 0.266958, size = 43, normalized size = 2.69

$$-3\log\left(\frac{\left(x^2\right)^{\frac{1}{3}}}{x}\right)+\frac{3}{4}\log\left(-\frac{x^2-\left(x^2\right)^{\frac{1}{3}}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^2 - (x^2)^(1/3)),x, algorithm="fricas")
```

```
[Out] -3*log((x^2)^(1/3)/x) + 3/4*log(-(x^2 - (x^2)^(1/3))/x^2)
```

Sympy [A] time = 0.584835, size = 19, normalized size = 1.19

$$-\frac{\log(x)}{2}+\frac{3\log\left(x^2-\sqrt[3]{x^2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x**2-(x**2)**(1/3)),x)
```

```
[Out] -log(x)/2 + 3*log(x**2 - (x**2)**(1/3))/4
```

GIAC/XCAS [A] time = 0.266682, size = 22, normalized size = 1.38

$$\frac{3}{4}\ln\left(\left|(x\mathrm{sign}(x))^{\frac{1}{3}}\,x\mathrm{sign}(x)-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^2 - (x^2)^(1/3)),x, algorithm="giac")
```

```
[Out] 3/4*ln(abs((x*sign(x))^(1/3)*x*sign(x) - 1))
```

3.524 $\int x \left(1+x^2\right)^3 \sqrt{2+2x^2+x^4} \, dx$

Optimal. Leaf size=44

$$\frac{1}{10}\left(x^2+1\right)^2\left(x^4+2x^2+2\right)^{3/2}-\frac{1}{15}\left(x^4+2x^2+2\right)^{3/2}$$

```
[Out] -(2 + 2*x^2 + x^4)^(3/2)/15 + ((1 + x^2)^2*(2 + 2*x^2 + x^4)^(3/2
))/10
```

Rubi [A] time = 0.0948337, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$

$$\frac{1}{10}\left(x^2+1\right)^2\left(x^4+2x^2+2\right)^{3/2}-\frac{1}{15}\left(x^4+2x^2+2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(1 + x^2)^3*Sqrt[2 + 2*x^2 + x^4],x]
```

```
[Out] -(2 + 2*x^2 + x^4)^(3/2)/15 + ((1 + x^2)^2*(2 + 2*x^2 + x^4)^(3/2
))/10
```

Rubi in Sympy [A] time = 6.28886, size = 36, normalized size = 0.82

$$\frac{\left(x^2+1\right)^2\left(x^4+2x^2+2\right)^{\frac{3}{2}}}{10}-\frac{\left(x^4+2x^2+2\right)^{\frac{3}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(x**2+1)**3*(x**4+2*x**2+2)**(1/2),x)
```

```
[Out] (x**2 + 1)**2*(x**4 + 2*x**2 + 2)**(3/2)/10 - (x**4 + 2*x**2 + 2)
**(3/2)/15
```

Mathematica [A] time = 0.0211666, size = 30, normalized size = 0.68

$$\frac{1}{30}\left(x^4+2x^2+2\right)^{3/2}\left(3x^4+6x^2+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(1 + x^2)^3*Sqrt[2 + 2*x^2 + x^4],x]
```

```
[Out] ((2 + 2*x^2 + x^4)^(3/2)*(1 + 6*x^2 + 3*x^4))/30
```

Maple [A] time = 0.009, size = 27, normalized size = 0.6

$$\frac{3x^4+6x^2+1}{30}\left(x^4+2x^2+2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(x^2+1)^3*(x^4+2*x^2+2)^(1/2),x)
```

```
[Out] 1/30*(x^4+2*x^2+2)^(3/2)*(3*x^4+6*x^2+1)
```

Maxima [A] time = 0.806657, size = 66, normalized size = 1.5

$$\frac{1}{10}\left(x^4+2x^2+2\right)^{\frac{3}{2}}x^4+\frac{1}{5}\left(x^4+2x^2+2\right)^{\frac{3}{2}}x^2+\frac{1}{30}\left(x^4+2x^2+2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 2*x^2 + 2)*(x^2 + 1)^3*x,x, algorithm="maxima")
```

```
[Out] 1/10*(x^4 + 2*x^2 + 2)^(3/2)*x^4 + 1/5*(x^4 + 2*x^2 + 2)^(3/2)*x^
2 + 1/30*(x^4 + 2*x^2 + 2)^(3/2)
```

Fricas [A] time = 0.26269, size = 49, normalized size = 1.11

$$\frac{1}{30}\left(3x^8+12x^6+19x^4+14x^2+2\right)\sqrt{x^4+2x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 2*x^2 + 2)*(x^2 + 1)^3*x,x, algorithm="fricas")
```

```
[Out] 1/30*(3*x^8 + 12*x^6 + 19*x^4 + 14*x^2 + 2)*sqrt(x^4 + 2*x^2 + 2)
```

Sympy [A] time = 2.06952, size = 94, normalized size = 2.14

$$\frac{x^8\sqrt{x^4+2x^2+2}}{10}+\frac{2x^6\sqrt{x^4+2x^2+2}}{5}+\frac{19x^4\sqrt{x^4+2x^2+2}}{30}+\frac{7x^2\sqrt{x^4+2x^2+2}}{15}+\frac{\sqrt{x^4+2x^2+2}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(x**2+1)**3*(x**4+2*x**2+2)**(1/2),x)
```

```
[Out] x**8*sqrt(x**4 + 2*x**2 + 2)/10 + 2*x**6*sqrt(x**4 + 2*x**2 + 2)/
5 + 19*x**4*sqrt(x**4 + 2*x**2 + 2)/30 + 7*x**2*sqrt(x**4 + 2*x**
2 + 2)/15 + sqrt(x**4 + 2*x**2 + 2)/15
```

GIAC/XCAS [A] time = 0.262465, size = 51, normalized size = 1.16

$$\frac{1}{30}\sqrt{x^4+2\,x^2+2}\left(\left(\left(3\left(x^2+4\right)x^2+19\right)x^2+14\right)x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 2*x^2 + 2)*(x^2 + 1)^3*x,x, algorithm="giac")
```

```
[Out] 1/30*sqrt(x^4 + 2*x^2 + 2)*(((3*(x^2 + 4)*x^2 + 19)*x^2 + 14)*x^2
 + 2)
```

3.525 $\int x\sqrt{\frac{1-x^2}{1+x^2}}\,dx$

Optimal. Leaf size=51

$$\frac{1}{2}\sqrt{\frac{1-x^2}{x^2+1}}\left(x^2+1\right)-\tan^{-1}\left(\sqrt{\frac{1-x^2}{x^2+1}}\right)$$

```
[Out] (Sqrt[(1 - x^2)/(1 + x^2)]*(1 + x^2))/2 - ArcTan[Sqrt[(1 - x^2)/(
1 + x^2)]]
```

Rubi [A] time = 0.0451144, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{1}{2}\sqrt{\frac{1-x^2}{x^2+1}}\left(x^2+1\right)-\tan^{-1}\left(\sqrt{\frac{1-x^2}{x^2+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[(1 - x^2)/(1 + x^2)],x]
```

```
[Out] (Sqrt[(1 - x^2)/(1 + x^2)]*(1 + x^2))/2 - ArcTan[Sqrt[(1 - x^2)/(
1 + x^2)]]
```

Rubi in Sympy [A] time = 2.30847, size = 39, normalized size = 0.76

$$\frac{\sqrt{\frac{-x^2+1}{x^2+1}}}{\frac{-x^2+1}{x^2+1}+1}-\operatorname{atan}\left(\sqrt{\frac{-x^2+1}{x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x*((-x**2+1)/(x**2+1))**(1/2),x)
```

```
[Out] sqrt((-x**2 + 1)/(x**2 + 1))/((-x**2 + 1)/(x**2 + 1) + 1) - atan(
sqrt((-x**2 + 1)/(x**2 + 1)))
```

Mathematica [A] time = 0.0465447, size = 79, normalized size = 1.55

$$\frac{\sqrt{\frac{1-x^2}{x^2+1}}\left(\sqrt{1-x^2}\left(x^2+1\right)+2\sqrt{x^2+1}\sin^{-1}\left(\frac{\sqrt{x^2+1}}{\sqrt{2}}\right)\right)}{2\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*Sqrt[(1 - x^2)/(1 + x^2)],x]
```

```
[Out] (Sqrt[(1 - x^2)/(1 + x^2)]*(Sqrt[1 - x^2]*(1 + x^2) + 2*Sqrt[1 +
x^2]*ArcSin[Sqrt[1 + x^2]/Sqrt[2]]))/(2*Sqrt[1 - x^2])
```

Maple [A] time = 0.024, size = 52, normalized size = 1.

$$\frac{x^2+1}{2}\sqrt{-\frac{x^2-1}{x^2+1}}\left(\sqrt{-x^4+1}+\arcsin\left(x^2\right)\right)\frac{1}{\sqrt{-\left(x^2-1\right)\left(x^2+1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*((-x^2+1)/(x^2+1))^(1/2),x)
```

```
[Out] 1/2*(-(x^2-1)/(x^2+1))^(1/2)*(x^2+1)*((-x^4+1)^(1/2)+arcsin(x^2))
/(-(x^2-1)*(x^2+1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{-\frac{x^2-1}{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sqrt(-(x^2 - 1)/(x^2 + 1)),x, algorithm="maxima")
```

```
[Out] integrate(x*sqrt(-(x^2 - 1)/(x^2 + 1)), x)
```

Fricas [A] time = 0.272159, size = 117, normalized size = 2.29

$$-\frac{x^4+2\left(\left(x^2+1\right)\sqrt{-\frac{x^2-1}{x^2+1}}-1\right)\arctan\left(\frac{\left(x^2+1\right)\sqrt{-\frac{x^2-1}{x^2+1}}-1}{x^2}\right)}{2\left(\left(x^2+1\right)\sqrt{-\frac{x^2-1}{x^2+1}}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sqrt(-(x^2 - 1)/(x^2 + 1)),x, algorithm="fricas")
```

```
[Out] -1/2*(x^4 + 2*((x^2 + 1)*sqrt(-(x^2 - 1)/(x^2 + 1)) - 1)*arctan((
(x^2 + 1)*sqrt(-(x^2 - 1)/(x^2 + 1)) - 1)/x^2))/((x^2 + 1)*sqrt(-
(x^2 - 1)/(x^2 + 1)) - 1)
```

Sympy [A] time = 178.468, size = 39, normalized size = 0.76

$$\begin{cases} \frac{\sqrt{-x^2+1}\sqrt{x^2+1}}{2} - \operatorname{asin}\left(\frac{\sqrt{2}\sqrt{-x^2+1}}{2}\right) & \text{for } x > -1 \wedge x < 1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*((-x**2+1)/(x**2+1))**(1/2),x)
```

```
[Out] Piecewise((sqrt(-x**2 + 1)*sqrt(x**2 + 1)/2 - asin(sqrt(2)*sqrt(-
x**2 + 1)/2), (x > -1) & (x < 1)))
```

GIAC/XCAS [A] time = 0.266698, size = 24, normalized size = 0.47

$$\frac{1}{2}\sqrt{-x^4+1}+\frac{1}{2}\arcsin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sqrt(-(x^2 - 1)/(x^2 + 1)),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-x^4 + 1) + 1/2*arcsin(x^2)
```

3.526 $\int x\sqrt{\frac{5-7x^2}{7+5x^2}}\,dx$

Optimal. Leaf size=72

$$\frac{1}{10}\sqrt{\frac{5-7x^2}{5x^2+7}}\left(5x^2+7\right)-\frac{37\tan^{-1}\left(\sqrt{\frac{5}{7}}\sqrt{\frac{5-7x^2}{5x^2+7}}\right)}{5\sqrt{35}}$$

```
[Out] (Sqrt[(5 - 7*x^2)/(7 + 5*x^2)]*(7 + 5*x^2))/10 - (37*ArcTan[Sqrt[
5/7]*Sqrt[(5 - 7*x^2)/(7 + 5*x^2)]])/(5*Sqrt[35])
```

Rubi [A] time = 0.0653706, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13

$$\frac{1}{10}\sqrt{\frac{5-7x^2}{5x^2+7}}\left(5x^2+7\right)-\frac{37\tan^{-1}\left(\sqrt{\frac{5}{7}}\sqrt{\frac{5-7x^2}{5x^2+7}}\right)}{5\sqrt{35}}$$

Antiderivative was successfully verified.

```
[In]  Int[x*Sqrt[(5 - 7*x^2)/(7 + 5*x^2)],x]
```

```
[Out] (Sqrt[(5 - 7*x^2)/(7 + 5*x^2)]*(7 + 5*x^2))/10 - (37*ArcTan[Sqrt[
5/7]*Sqrt[(5 - 7*x^2)/(7 + 5*x^2)]])/(5*Sqrt[35])
```

Rubi in Sympy [A] time = 2.42523, size = 66, normalized size = 0.92

$$\frac{37\sqrt{\frac{-7x^2+5}{5x^2+7}}}{5\left(\frac{5(-7x^2+5)}{5x^2+7}+7\right)}-\frac{37\sqrt{35}\,\text{atan}\left(\frac{\sqrt{35}\sqrt{\frac{-7x^2+5}{5x^2+7}}}{7}\right)}{175}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*((-7*x**2+5)/(5*x**2+7))**(1/2),x)
```

```
[Out] 37*sqrt((-7*x**2 + 5)/(5*x**2 + 7))/(5*(5*(-7*x**2 + 5)/(5*x**2 +
 7) + 7)) - 37*sqrt(35)*atan(sqrt(35)*sqrt((-7*x**2 + 5)/(5*x**2
+ 7))/7)/175
```

Mathematica [A] time = 0.0989119, size = 95, normalized size = 1.32

$$\frac{\sqrt{\frac{5-7x^2}{5x^2+7}}\left(35\sqrt{5-7x^2}\left(5x^2+7\right)+74\sqrt{35}\sqrt{5x^2+7}\sin^{-1}\left(\sqrt{\frac{7}{74}}\sqrt{5x^2+7}\right)\right)}{350\sqrt{5-7x^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*Sqrt[(5 - 7*x^2)/(7 + 5*x^2)],x]
```

```
[Out] (Sqrt[(5 - 7*x^2)/(7 + 5*x^2)]*(35*Sqrt[5 - 7*x^2]*(7 + 5*x^2) +
74*Sqrt[35]*Sqrt[7 + 5*x^2]*ArcSin[Sqrt[7/74]*Sqrt[7 + 5*x^2]]))/
(350*Sqrt[5 - 7*x^2])
```

Maple [A] time = 0.03, size = 78, normalized size = 1.1

$$\frac{5\,x^2+7}{350}\sqrt{-\frac{7\,x^2-5}{5\,x^2+7}}\left(37\,\sqrt{35}\arcsin\left(\frac{35\,x^2}{37}+\frac{12}{37}\right)+35\,\sqrt{-35\,x^4-24\,x^2+35}\right)\frac{1}{\sqrt{-\left(7\,x^2-5\right)\left(5\,x^2+7\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*((-7*x^2+5)/(5*x^2+7))^(1/2),x)
```

```
[Out] 1/350*(-(7*x^2-5)/(5*x^2+7))^(1/2)*(5*x^2+7)*(37*35^(1/2)*arcsin(
35/37*x^2+12/37)+35*(-35*x^4-24*x^2+35)^(1/2))/(-(7*x^2-5)*(5*x^2
+7))^(1/2)
```

Maxima [A] time = 0.812918, size = 103, normalized size = 1.43

$$-\frac{37}{175}\,\sqrt{35}\arctan\left(\frac{1}{7}\,\sqrt{35}\sqrt{-\frac{7\,x^2-5}{5\,x^2+7}}\right)-\frac{37\,\sqrt{-\frac{7\,x^2-5}{5\,x^2+7}}}{5\left(\frac{5\,(7\,x^2-5)}{5\,x^2+7}-7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sqrt(-(7*x^2 - 5)/(5*x^2 + 7)),x, algorithm="maxima")
```

```
[Out] -37/175*sqrt(35)*arctan(1/7*sqrt(35)*sqrt(-(7*x^2 - 5)/(5*x^2 + 7
))) - 37/5*sqrt(-(7*x^2 - 5)/(5*x^2 + 7))/(5*(7*x^2 - 5)/(5*x^2 +
```

```
7) - 7)
```

Fricas [A] time = 0.272152, size = 109, normalized size = 1.51

$$\frac{1}{350}\sqrt{35}\left(\sqrt{35}\left(5x^2+7\right)\sqrt{-\frac{7x^2-5}{5x^2+7}}+37\arctan\left(\frac{\sqrt{35}\left(35x^2+12\right)}{35\left(5x^2+7\right)\sqrt{-\frac{7x^2-5}{5x^2+7}}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sqrt(-(7*x^2 - 5)/(5*x^2 + 7)),x, algorithm="fricas")
```

```
[Out] 1/350*sqrt(35)*(sqrt(35)*(5*x^2 + 7)*sqrt(-(7*x^2 - 5)/(5*x^2 + 7
)) + 37*arctan(1/35*sqrt(35)*(35*x^2 + 12)/((5*x^2 + 7)*sqrt(-(7*
x^2 - 5)/(5*x^2 + 7)))))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*((-7*x**2+5)/(5*x**2+7))**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.268742, size = 41, normalized size = 0.57

$$\frac{37}{350}\sqrt{35}\arcsin\left(\frac{35}{37}x^2+\frac{12}{37}\right)+\frac{1}{10}\sqrt{-35x^4-24x^2+35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sqrt(-(7*x^2 - 5)/(5*x^2 + 7)),x, algorithm="giac")
```

```
[Out] 37/350*sqrt(35)*arcsin(35/37*x^2 + 12/37) + 1/10*sqrt(-35*x^4 - 2
4*x^2 + 35)
```

3.527 $\int x^2\sqrt{\frac{1-x^3}{1+x^3}}\,dx$

Optimal. Leaf size=53

$$\frac{1}{3}\sqrt{\frac{1-x^3}{x^3+1}}\left(x^3+1\right)-\frac{2}{3}\tan^{-1}\left(\sqrt{\frac{1-x^3}{x^3+1}}\right)$$

```
[Out] (Sqrt[(1 - x^3)/(1 + x^3)]*(1 + x^3))/3 - (2*ArcTan[Sqrt[(1 - x^3
)/(1 + x^3)]])/3
```

Rubi [A] time = 0.0550118, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13

$$\frac{1}{3}\sqrt{\frac{1-x^3}{x^3+1}}\left(x^3+1\right)-\frac{2}{3}\tan^{-1}\left(\sqrt{\frac{1-x^3}{x^3+1}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*Sqrt[(1 - x^3)/(1 + x^3)],x]
```

```
[Out] (Sqrt[(1 - x^3)/(1 + x^3)]*(1 + x^3))/3 - (2*ArcTan[Sqrt[(1 - x^3
)/(1 + x^3)]])/3
```

Rubi in Sympy [A] time = 2.41022, size = 46, normalized size = 0.87

$$\frac{2\sqrt{\frac{-x^3+1}{x^3+1}}}{3\left(\frac{-x^3+1}{x^3+1}+1\right)}-\frac{2\,\mathrm{atan}\left(\sqrt{\frac{-x^3+1}{x^3+1}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*((-x**3+1)/(x**3+1))**(1/2),x)
```

```
[Out] 2*sqrt((-x**3 + 1)/(x**3 + 1))/(3*((-x**3 + 1)/(x**3 + 1) + 1)) -
 2*atan(sqrt((-x**3 + 1)/(x**3 + 1)))/3
```

Mathematica [A] time = 0.0491164, size = 79, normalized size = 1.49

$$\frac{\sqrt{\frac{1-x^3}{x^3+1}}\left(\sqrt{1-x^3}\left(x^3+1\right)+2\sqrt{x^3+1}\sin^{-1}\left(\frac{\sqrt{x^3+1}}{\sqrt{2}}\right)\right)}{3\sqrt{1-x^3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*Sqrt[(1 - x^3)/(1 + x^3)],x]
```

```
[Out] (Sqrt[(1 - x^3)/(1 + x^3)]*(Sqrt[1 - x^3]*(1 + x^3) + 2*Sqrt[1 +
x^3]*ArcSin[Sqrt[1 + x^3]/Sqrt[2]]))/(3*Sqrt[1 - x^3])
```

Maple [A] time = 0.094, size = 68, normalized size = 1.3

$$\frac{x^3+1}{3}\sqrt{-\frac{x^3-1}{x^3+1}}-\frac{\arcsin\left(x^3\right)}{3x^3-3}\sqrt{-\frac{x^3-1}{x^3+1}}\sqrt{-\left(x^3-1\right)\left(x^3+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*((-x^3+1)/(x^3+1))^(1/2),x)
```

```
[Out] 1/3*(x^3+1)*(-(x^3-1)/(x^3+1))^(1/2)-1/3*arcsin(x^3)*(-(x^3-1)/(x
^3+1))^(1/2)*(-(x^3-1)*(x^3+1))^(1/2)/(x^3-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sqrt{-\frac{x^3-1}{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*sqrt(-(x^3 - 1)/(x^3 + 1)),x, algorithm="maxima")
```

```
[Out] integrate(x^2*sqrt(-(x^3 - 1)/(x^3 + 1)), x)
```

Fricas [A] time = 0.267438, size = 117, normalized size = 2.21

$$-\frac{x^{6}+2\left(\left(x^{3}+1\right)\sqrt{-\frac{x^{3}-1}{x^{3}+1}}-1\right)\arctan\left(\frac{\left(x^{3}+1\right)\sqrt{-\frac{x^{3}-1}{x^{3}+1}}-1}{x^{3}}\right)}{3\left(\left(x^{3}+1\right)\sqrt{-\frac{x^{3}-1}{x^{3}+1}}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*sqrt(-(x^3 - 1)/(x^3 + 1)),x, algorithm="fricas")
```

```
[Out] -1/3*(x^6 + 2*((x^3 + 1)*sqrt(-(x^3 - 1)/(x^3 + 1)) - 1)*arctan((
(x^3 + 1)*sqrt(-(x^3 - 1)/(x^3 + 1)) - 1)/x^3))/((x^3 + 1)*sqrt(-
(x^3 - 1)/(x^3 + 1)) - 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*((-x**3+1)/(x**3+1))**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.264491, size = 30, normalized size = 0.57

$$\frac{1}{3}\left(\sqrt{-x^{6}+1}+\arcsin\left(x^{3}\right)\right)\operatorname{sign}\left(x^{3}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*sqrt(-(x^3 - 1)/(x^3 + 1)),x, algorithm="giac")
```

```
[Out] 1/3*(sqrt(-x^6 + 1) + arcsin(x^3))*sign(x^3 + 1)
```

3.528 $\int x^5\sqrt{1-x^3}\left(1+x^9\right)^2\,dx$

Optimal. Leaf size=121

$$\frac{2}{51}\left(1-x^3\right)^{17/2}-\frac{14}{45}\left(1-x^3\right)^{15/2}+\frac{14}{13}\left(1-x^3\right)^{13/2}-\frac{74}{33}\left(1-x^3\right)^{11/2}+\frac{86}{27}\left(1-x^3\right)^{9/2}-\frac{22}{7}\left(1-x^3\right)^{7/2}+\frac{32}{15}\left(1-x^3\right)^{5/2}-\frac{8}{9}\left(1-x^3\right)^{3/2}$$

```
[Out] (-8*(1 - x^3)^(3/2))/9 + (32*(1 - x^3)^(5/2))/15 - (22*(1 - x^3)^
(7/2))/7 + (86*(1 - x^3)^(9/2))/27 - (74*(1 - x^3)^(11/2))/33 + (
14*(1 - x^3)^(13/2))/13 - (14*(1 - x^3)^(15/2))/45 + (2*(1 - x^3)
^(17/2))/51
```

Rubi [A] time = 0.17454, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{2}{51}\left(1-x^3\right)^{17/2}-\frac{14}{45}\left(1-x^3\right)^{15/2}+\frac{14}{13}\left(1-x^3\right)^{13/2}-\frac{74}{33}\left(1-x^3\right)^{11/2}+\frac{86}{27}\left(1-x^3\right)^{9/2}-\frac{22}{7}\left(1-x^3\right)^{7/2}+\frac{32}{15}\left(1-x^3\right)^{5/2}-\frac{8}{9}\left(1-x^3\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5*Sqrt[1 - x^3]*(1 + x^9)^2,x]
```

```
[Out] (-8*(1 - x^3)^(3/2))/9 + (32*(1 - x^3)^(5/2))/15 - (22*(1 - x^3)^
(7/2))/7 + (86*(1 - x^3)^(9/2))/27 - (74*(1 - x^3)^(11/2))/33 + (
14*(1 - x^3)^(13/2))/13 - (14*(1 - x^3)^(15/2))/45 + (2*(1 - x^3)
^(17/2))/51
```

Rubi in Sympy [A] time = 12.639, size = 94, normalized size = 0.78

$$\frac{2\left(-x^3+1\right)^{\frac{17}{2}}}{51}-\frac{14\left(-x^3+1\right)^{\frac{15}{2}}}{45}+\frac{14\left(-x^3+1\right)^{\frac{13}{2}}}{13}-\frac{74\left(-x^3+1\right)^{\frac{11}{2}}}{33}+\frac{86\left(-x^3+1\right)^{\frac{9}{2}}}{27}-\frac{22\left(-x^3+1\right)^{\frac{7}{2}}}{7}+\frac{32\left(-x^3+1\right)^{\frac{5}{2}}}{15}-\frac{8\left(-x^3+1\right)^{\frac{3}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*(x**9+1)**2*(-x**3+1)**(1/2),x)
```

```
[Out] 2*(-x**3 + 1)**(17/2)/51 - 14*(-x**3 + 1)**(15/2)/45 + 14*(-x**3
+ 1)**(13/2)/13 - 74*(-x**3 + 1)**(11/2)/33 + 86*(-x**3 + 1)**(9/
2)/27 - 22*(-x**3 + 1)**(7/2)/7 + 32*(-x**3 + 1)**(5/2)/15 - 8*(-
x**3 + 1)**(3/2)/9
```

Mathematica [A] time = 0.0361146, size = 52, normalized size = 0.43

$$-\frac{2\left(1-x^3\right)^{3/2}\left(45045x^{21}+42042x^{18}+38808x^{15}+174510x^{12}+155120x^9+132960x^6+259521x^3+173014\right)}{2297295}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5*Sqrt[1 - x^3]*(1 + x^9)^2,x]
```

```
[Out] (-2*(1 - x^3)^(3/2)*(173014 + 259521*x^3 + 132960*x^6 + 155120*x^
9 + 174510*x^12 + 38808*x^15 + 42042*x^18 + 45045*x^21))/2297295
```

Maple [A] time = 0.019, size = 58, normalized size = 0.5

$$\frac{\left(90090\,x^{21}+84084\,x^{18}+77616\,x^{15}+349020\,x^{12}+310240\,x^{9}+265920\,x^{6}+519042\,x^{3}+346028\right)\left(-1+x\right)\left(x^{2}+x+1\right)}{2297295}\sqrt{-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(x^9+1)^2*(-x^3+1)^(1/2),x)
```

```
[Out] 2/2297295*(-x^3+1)^(1/2)*(45045*x^21+42042*x^18+38808*x^15+174510
*x^12+155120*x^9+132960*x^6+259521*x^3+173014)*(-1+x)*(x^2+x+1)
```

Maxima [A] time = 0.808153, size = 120, normalized size = 0.99

$$\frac{2}{51}\left(-x^3+1\right)^{\frac{17}{2}}-\frac{14}{45}\left(-x^3+1\right)^{\frac{15}{2}}+\frac{14}{13}\left(-x^3+1\right)^{\frac{13}{2}}-\frac{74}{33}\left(-x^3+1\right)^{\frac{11}{2}}+\frac{86}{27}\left(-x^3+1\right)^{\frac{9}{2}}-\frac{22}{7}\left(-x^3+1\right)^{\frac{7}{2}}+\frac{32}{15}\left(-x^3+1\right)^{\frac{5}{2}}-\frac{8}{9}\left(-x^3+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^9 + 1)^2*sqrt(-x^3 + 1)*x^5,x, algorithm="maxima")
```

```
[Out] 2/51*(-x^3 + 1)^(17/2) - 14/45*(-x^3 + 1)^(15/2) + 14/13*(-x^3 +
1)^(13/2) - 74/33*(-x^3 + 1)^(11/2) + 86/27*(-x^3 + 1)^(9/2) - 22
/7*(-x^3 + 1)^(7/2) + 32/15*(-x^3 + 1)^(5/2) - 8/9*(-x^3 + 1)^(3/
2)
```

Fricas [A] time = 0.294157, size = 72, normalized size = 0.6

$$\frac{2}{2297295}\left(45045\,x^{24}-3003\,x^{21}-3234\,x^{18}+135702\,x^{15}-19390\,x^{12}-22160\,x^{9}+126561\,x^{6}-86507\,x^{3}-173014\right)\sqrt{-x^{3}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^9 + 1)^2*sqrt(-x^3 + 1)*x^5,x, algorithm="fricas")
```

```
[Out] 2/2297295*(45045*x^24 - 3003*x^21 - 3234*x^18 + 135702*x^15 - 193
90*x^12 - 22160*x^9 + 126561*x^6 - 86507*x^3 - 173014)*sqrt(-x^3
+ 1)
```

Sympy [A] time = 54.8838, size = 133, normalized size = 1.1

$$\frac{2x^{24}\sqrt{-x^{3}+1}}{51}-\frac{2x^{21}\sqrt{-x^{3}+1}}{765}-\frac{28x^{18}\sqrt{-x^{3}+1}}{9945}+\frac{1436x^{15}\sqrt{-x^{3}+1}}{12155}-\frac{1108x^{12}\sqrt{-x^{3}+1}}{65637}-\frac{8864x^{9}\sqrt{-x^{3}+1}}{459459}+\frac{84374x^{6}\sqrt{-x^{3}+1}}{765765}-\frac{173014x^{3}\sqrt{-x^{3}+1}}{2297295}-\frac{346028\sqrt{-x^{3}+1}}{2297295}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*(x**9+1)**2*(-x**3+1)**(1/2),x)
```

```
[Out] 2*x**24*sqrt(-x**3 + 1)/51 - 2*x**21*sqrt(-x**3 + 1)/765 - 28*x**
18*sqrt(-x**3 + 1)/9945 + 1436*x**15*sqrt(-x**3 + 1)/12155 - 1108
*x**12*sqrt(-x**3 + 1)/65637 - 8864*x**9*sqrt(-x**3 + 1)/459459 +
 84374*x**6*sqrt(-x**3 + 1)/765765 - 173014*x**3*sqrt(-x**3 + 1)/
2297295 - 346028*sqrt(-x**3 + 1)/2297295
```

GIAC/XCAS [A] time = 0.267418, size = 186, normalized size = 1.54

$$\frac{2}{51}\left(x^{3}-1\right)^{8}\sqrt{-x^{3}+1}+\frac{14}{45}\left(x^{3}-1\right)^{7}\sqrt{-x^{3}+1}+\frac{14}{13}\left(x^{3}-1\right)^{6}\sqrt{-x^{3}+1}+\frac{74}{33}\left(x^{3}-1\right)^{5}\sqrt{-x^{3}+1}+\frac{86}{27}\left(x^{3}-1\right)^{4}\sqrt{-x^{3}+1}+\frac{22}{7}\left(x^{3}-1\right)^{3}\sqrt{-x^{3}+1}+\frac{32}{15}\left(x^{3}-1\right)^{2}\sqrt{-x^{3}+1}-\frac{8}{9}\left(-x^{3}+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^9 + 1)^2*sqrt(-x^3 + 1)*x^5,x, algorithm="giac")
```

```
[Out] 2/51*(x^3 - 1)^8*sqrt(-x^3 + 1) + 14/45*(x^3 - 1)^7*sqrt(-x^3 + 1
) + 14/13*(x^3 - 1)^6*sqrt(-x^3 + 1) + 74/33*(x^3 - 1)^5*sqrt(-x^
3 + 1) + 86/27*(x^3 - 1)^4*sqrt(-x^3 + 1) + 22/7*(x^3 - 1)^3*sqrt
(-x^3 + 1) + 32/15*(x^3 - 1)^2*sqrt(-x^3 + 1) - 8/9*(-x^3 + 1)^(3
/2)
```

3.529 $\int x^8\sqrt{\frac{1-x^3}{1+x^3}}\,dx$

Optimal. Leaf size=113

$$-\frac{1}{9}\left(\frac{1-x^3}{x^3+1}\right)^{3/2}\left(x^3+1\right)^3-\frac{1}{6}\sqrt{\frac{1-x^3}{x^3+1}}\left(x^3+1\right)^2+\frac{1}{2}\sqrt{\frac{1-x^3}{x^3+1}}\left(x^3+1\right)-\frac{1}{3}\tan^{-1}\left(\sqrt{\frac{1-x^3}{x^3+1}}\right)$$

```
[Out] (Sqrt[(1 - x^3)/(1 + x^3)]*(1 + x^3))/2 - (Sqrt[(1 - x^3)/(1 + x^
3)]*(1 + x^3)^2)/6 - (((1 - x^3)/(1 + x^3))^(3/2)*(1 + x^3)^3)/9
- ArcTan[Sqrt[(1 - x^3)/(1 + x^3)]]/3
```

Rubi [A] time = 0.151415, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217

$$-\frac{1}{9}\left(\frac{1-x^3}{x^3+1}\right)^{3/2}\left(x^3+1\right)^3-\frac{1}{6}\sqrt{\frac{1-x^3}{x^3+1}}\left(x^3+1\right)^2+\frac{1}{2}\sqrt{\frac{1-x^3}{x^3+1}}\left(x^3+1\right)-\frac{1}{3}\tan^{-1}\left(\sqrt{\frac{1-x^3}{x^3+1}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^8*Sqrt[(1 - x^3)/(1 + x^3)],x]
```

```
[Out] (Sqrt[(1 - x^3)/(1 + x^3)]*(1 + x^3))/2 - (Sqrt[(1 - x^3)/(1 + x^
3)]*(1 + x^3)^2)/6 - (((1 - x^3)/(1 + x^3))^(3/2)*(1 + x^3)^3)/9
- ArcTan[Sqrt[(1 - x^3)/(1 + x^3)]]/3
```

Rubi in Sympy [A] time = 9.90826, size = 102, normalized size = 0.9

$$-\frac{8\left(\frac{-x^3+1}{x^3+1}\right)^{\frac{3}{2}}}{9\left(\frac{-x^3+1}{x^3+1}+1\right)^3}+\frac{\sqrt{\frac{-x^3+1}{x^3+1}}}{\frac{-x^3+1}{x^3+1}+1}-\frac{2\sqrt{\frac{-x^3+1}{x^3+1}}}{3\left(\frac{-x^3+1}{x^3+1}+1\right)^2}-\frac{\operatorname{atan}\left(\sqrt{\frac{-x^3+1}{x^3+1}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**8*((-x**3+1)/(x**3+1))**(1/2),x)
```

```
[Out] -8*((-x**3 + 1)/(x**3 + 1))**(3/2)/(9*((-x**3 + 1)/(x**3 + 1) + 1
)**3) + sqrt((-x**3 + 1)/(x**3 + 1))/((-x**3 + 1)/(x**3 + 1) + 1)
 - 2*sqrt((-x**3 + 1)/(x**3 + 1))/(3*((-x**3 + 1)/(x**3 + 1) + 1)
```

```
**2) - atan(sqrt((-x**3 + 1)/(x**3 + 1)))/3
```

Mathematica [A] time = 0.0844269, size = 86, normalized size = 0.76

$$\frac{\sqrt{\frac{1-x^3}{x^3+1}}\sqrt{x^3+1}\left(6\sin^{-1}\left(\frac{\sqrt{x^3+1}}{\sqrt{2}}\right)+\sqrt{1-x^6}\left(2x^6-3x^3+4\right)\right)}{18\sqrt{1-x^3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^8*Sqrt[(1 - x^3)/(1 + x^3)],x]
```

```
[Out] (Sqrt[(1 - x^3)/(1 + x^3)]*Sqrt[1 + x^3]*(Sqrt[1 - x^6]*(4 - 3*x^
3 + 2*x^6) + 6*ArcSin[Sqrt[1 + x^3]/Sqrt[2]]))/(18*Sqrt[1 - x^3])
```

Maple [A] time = 0.085, size = 80, normalized size = 0.7

$$\frac{\left(2x^6-3x^3+4\right)\left(x^3+1\right)}{18}\sqrt{-\frac{x^3-1}{x^3+1}}-\frac{\arcsin\left(x^3\right)}{6x^3-6}\sqrt{-\frac{x^3-1}{x^3+1}}\sqrt{-\left(x^3-1\right)\left(x^3+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^8*((-x^3+1)/(x^3+1))^(1/2),x)
```

```
[Out] 1/18*(2*x^6-3*x^3+4)*(x^3+1)*(-(x^3-1)/(x^3+1))^(1/2)-1/6*arcsin(
x^3)*(-(x^3-1)/(x^3+1))^(1/2)*(-(x^3-1)*(x^3+1))^(1/2)/(x^3-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^8\sqrt{-\frac{x^3-1}{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8*sqrt(-(x^3 - 1)/(x^3 + 1)),x, algorithm="maxima")
```

```
[Out] integrate(x^8*sqrt(-(x^3 - 1)/(x^3 + 1)), x)
```

Fricas [A] time = 0.269401, size = 244, normalized size = 2.16

$$\frac{2\,x^{18}-3\,x^{15}-6\,x^{12}+15\,x^{9}-12\,x^{3}-6\left(3\,x^{6}-\left(x^{9}+x^{6}-4\,x^{3}-4\right)\sqrt{-\frac{x^{3}-1}{x^{3}+1}}-4\right)\arctan\left(\frac{\left(x^{3}+1\right)\sqrt{-\frac{x^{3}-1}{x^{3}+1}}-1}{x^{3}}\right)+3\left(2\,x^{15}-x^{12}\right.}{18\left(3\,x^{6}-\left(x^{9}+x^{6}-4\,x^{3}-4\right)\sqrt{-\frac{x^{3}-1}{x^{3}+1}}-4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8*sqrt(-(x^3 - 1)/(x^3 + 1)),x, algorithm="fricas")
```

```
[Out] 1/18*(2*x^18 - 3*x^15 - 6*x^12 + 15*x^9 - 12*x^3 - 6*(3*x^6 - (x^
9 + x^6 - 4*x^3 - 4)*sqrt(-(x^3 - 1)/(x^3 + 1)) - 4)*arctan(((x^3
 + 1)*sqrt(-(x^3 - 1)/(x^3 + 1)) - 1)/x^3) + 3*(2*x^15 - x^12 - 3
*x^9 + 4*x^6 + 4*x^3)*sqrt(-(x^3 - 1)/(x^3 + 1)))/(3*x^6 - (x^9 +
 x^6 - 4*x^3 - 4)*sqrt(-(x^3 - 1)/(x^3 + 1)) - 4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**8*((-x**3+1)/(x**3+1))**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int x^{8}\sqrt{-\frac{x^{3}-1}{x^{3}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8*sqrt(-(x^3 - 1)/(x^3 + 1)),x, algorithm="giac")
```

```
[Out] integrate(x^8*sqrt(-(x^3 - 1)/(x^3 + 1)), x)
```

3.530 $\int x^9\sqrt{\frac{5-7x^5}{7+5x^5}}\,dx$

Optimal. Leaf size=106

$$\frac{1}{250}\sqrt{\frac{5-7x^5}{5x^5+7}}\left(5x^5+7\right)^2-\frac{27}{350}\sqrt{\frac{5-7x^5}{5x^5+7}}\left(5x^5+7\right)+\frac{2257\tan^{-1}\left(\sqrt{\frac{5}{7}}\sqrt{\frac{5-7x^5}{5x^5+7}}\right)}{875\sqrt{35}}$$

```
[Out] (-27*Sqrt[(5 - 7*x^5)/(7 + 5*x^5)]*(7 + 5*x^5))/350 + (Sqrt[(5 -
7*x^5)/(7 + 5*x^5)]*(7 + 5*x^5)^2)/250 + (2257*ArcTan[Sqrt[5/7]*S
qrt[(5 - 7*x^5)/(7 + 5*x^5)]])/(875*Sqrt[35])
```

Rubi [A] time = 0.118541, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16

$$\frac{1}{250}\sqrt{\frac{5-7x^5}{5x^5+7}}\left(5x^5+7\right)^2-\frac{27}{350}\sqrt{\frac{5-7x^5}{5x^5+7}}\left(5x^5+7\right)+\frac{2257\tan^{-1}\left(\sqrt{\frac{5}{7}}\sqrt{\frac{5-7x^5}{5x^5+7}}\right)}{875\sqrt{35}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^9*Sqrt[(5 - 7*x^5)/(7 + 5*x^5)],x]
```

```
[Out] (-27*Sqrt[(5 - 7*x^5)/(7 + 5*x^5)]*(7 + 5*x^5))/350 + (Sqrt[(5 -
7*x^5)/(7 + 5*x^5)]*(7 + 5*x^5)^2)/250 + (2257*ArcTan[Sqrt[5/7]*S
qrt[(5 - 7*x^5)/(7 + 5*x^5)]])/(875*Sqrt[35])
```

Rubi in Sympy [A] time = 6.16264, size = 104, normalized size = 0.98

$$-\frac{999\sqrt{\frac{-7x^5+5}{5x^5+7}}}{175\left(\frac{5(-7x^5+5)}{5x^5+7}+7\right)}+\frac{2738\sqrt{\frac{-7x^5+5}{5x^5+7}}}{125\left(\frac{5(-7x^5+5)}{5x^5+7}+7\right)^2}+\frac{2257\sqrt{35}\,\mathrm{atan}\left(\frac{\sqrt{35}\sqrt{\frac{-7x^5+5}{5x^5+7}}}{7}\right)}{30625}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**9*((-7*x**5+5)/(5*x**5+7))**(1/2),x)
```

```
[Out] -999*sqrt((-7*x**5 + 5)/(5*x**5 + 7))/(175*(5*(-7*x**5 + 5)/(5*x*
*5 + 7) + 7)) + 2738*sqrt((-7*x**5 + 5)/(5*x**5 + 7))/(125*(5*(-7
```

```
*x**5 + 5)/(5*x**5 + 7) + 7)**2) + 2257*sqrt(35)*atan(sqrt(35)*sq
rt((-7*x**5 + 5)/(5*x**5 + 7))/7)/30625
```

Mathematica [A] time = 0.125431, size = 124, normalized size = 1.17

$$\frac{\sqrt{\frac{5-7x^5}{5x^5+7}}\left(2257\sqrt{35}\sqrt{5x^5+7}\tan^{-1}\left(\frac{\sqrt{\frac{1}{7}-\frac{x^5}{5}}(35x^5+12)}{\sqrt{5x^5+7}(7x^5-5)}\right)+35\sqrt{5-7x^5}\left(175x^{10}-185x^5-602\right)\right)}{61250\sqrt{5-7x^5}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^9*Sqrt[(5 - 7*x^5)/(7 + 5*x^5)],x]
```

```
[Out] (Sqrt[(5 - 7*x^5)/(7 + 5*x^5)]*(35*Sqrt[5 - 7*x^5]*(-602 - 185*x^
5 + 175*x^10) + 2257*Sqrt[35]*Sqrt[7 + 5*x^5]*ArcTan[(Sqrt[1/7 -
x^5/5]*(12 + 35*x^5))/(Sqrt[7 + 5*x^5]*(-5 + 7*x^5))]))/(61250*Sq
rt[5 - 7*x^5])
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int x^9\sqrt{\frac{-7\,x^5+5}{5\,x^5+7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^9*((-7*x^5+5)/(5*x^5+7))^(1/2),x)
```

```
[Out] int(x^9*((-7*x^5+5)/(5*x^5+7))^(1/2),x)
```

Maxima [A] time = 0.807534, size = 163, normalized size = 1.54

$$\frac{2257}{30625}\sqrt{35}\arctan\left(\frac{1}{7}\sqrt{35}\sqrt{-\frac{7\,x^5-5}{5\,x^5+7}}\right)-\frac{37\left(675\left(-\frac{7\,x^5-5}{5\,x^5+7}\right)^{\frac{3}{2}}+427\sqrt{-\frac{7\,x^5-5}{5\,x^5+7}}\right)}{875\left(\frac{25\,(7\,x^5-5)^2}{(5\,x^5+7)^2}-\frac{70\,(7\,x^5-5)}{5\,x^5+7}+49\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9*sqrt(-(7*x^5 - 5)/(5*x^5 + 7)),x, algorithm="maxima")
```

```
[Out] 2257/30625*sqrt(35)*arctan(1/7*sqrt(35)*sqrt(-(7*x^5 - 5)/(5*x^5
+ 7))) - 37/875*(675*(-(7*x^5 - 5)/(5*x^5 + 7))^(3/2) + 427*sqrt(
-(7*x^5 - 5)/(5*x^5 + 7)))/(25*(7*x^5 - 5)^2/(5*x^5 + 7)^2 - 70*(
7*x^5 - 5)/(5*x^5 + 7) + 49)
```

Fricas [A] time = 0.27461, size = 116, normalized size = 1.09

$$\frac{1}{61250}\sqrt{35}\left(\sqrt{35}\left(175\,x^{10}-185\,x^{5}-602\right)\sqrt{-\frac{7\,x^{5}-5}{5\,x^{5}+7}}-2257\,\arctan\left(\frac{\sqrt{35}\left(35\,x^{5}+12\right)}{35\,(5\,x^{5}+7)\sqrt{-\frac{7\,x^{5}-5}{5\,x^{5}+7}}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9*sqrt(-(7*x^5 - 5)/(5*x^5 + 7)),x, algorithm="fricas")
```

```
[Out] 1/61250*sqrt(35)*(sqrt(35)*(175*x^10 - 185*x^5 - 602)*sqrt(-(7*x^
5 - 5)/(5*x^5 + 7)) - 2257*arctan(1/35*sqrt(35)*(35*x^5 + 12)/((5
*x^5 + 7)*sqrt(-(7*x^5 - 5)/(5*x^5 + 7)))))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**9*((-7*x**5+5)/(5*x**5+7))**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.268315, size = 63, normalized size = 0.59

$$\frac{1}{61250}\left(35\sqrt{-35\,x^{10}-24\,x^{5}+35}\left(35\,x^{5}-86\right)-2257\sqrt{35}\arcsin\left(\frac{35}{37}\,x^{5}+\frac{12}{37}\right)\right)\mathrm{sign}\left(5\,x^{5}+7\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9*sqrt(-(7*x^5 - 5)/(5*x^5 + 7)),x, algorithm="giac")
```

```
[Out] 1/61250*(35*sqrt(-35*x^10 - 24*x^5 + 35)*(35*x^5 - 86) - 2257*sqr
t(35)*arcsin(35/37*x^5 + 12/37))*sign(5*x^5 + 7)
```

3.531 $\int \left(\frac{x}{(a+bx^2)^{3/2}}+\frac{x}{(1+x^2)\sqrt{a+bx^2}}\right)\,dx$

Optimal. Leaf size=50

$$-\frac{1}{b\sqrt{a+bx^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

```
[Out] -(1/(b*Sqrt[a + b*x^2])) - ArcTanh[Sqrt[a + b*x^2]/Sqrt[a - b]]/S
qrt[a - b]
```

Rubi [A] time = 0.105528, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118

$$-\frac{1}{b\sqrt{a+bx^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(a + b*x^2)^(3/2) + x/((1 + x^2)*Sqrt[a + b*x^2]),x]
```

```
[Out] -(1/(b*Sqrt[a + b*x^2])) - ArcTanh[Sqrt[a + b*x^2]/Sqrt[a - b]]/S
qrt[a - b]
```

Rubi in Sympy [A] time = 5.56334, size = 39, normalized size = 0.78

$$-\frac{\operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}-\frac{1}{b\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(b*x**2+a)**(3/2)+x/(x**2+1)/(b*x**2+a)**(1/2),x)
```

```
[Out] -atanh(sqrt(a + b*x**2)/sqrt(a - b))/sqrt(a - b) - 1/(b*sqrt(a +
b*x**2))
```

Mathematica [A] time = 0.0877979, size = 49, normalized size = 0.98

$$\frac{\tan^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{b-a}}\right)}{\sqrt{b-a}}-\frac{1}{b\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(a + b*x^2)^(3/2) + x/((1 + x^2)*Sqrt[a + b*x^2]),x]
```

```
[Out] -(1/(b*Sqrt[a + b*x^2])) + ArcTan[Sqrt[a + b*x^2]/Sqrt[-a + b]]/S
qrt[-a + b]
```

Maple [A] time = 0.027, size = 42, normalized size = 0.8

$$-\frac{1}{b}\frac{1}{\sqrt{bx^2+a}}+1\arctan\left(1\sqrt{bx^2+a}\frac{1}{\sqrt{-a+b}}\right)\frac{1}{\sqrt{-a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(b*x^2+a)^(3/2)+x/(x^2+1)/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/b/(b*x^2+a)^(1/2)+1/(-a+b)^(1/2)*arctan((b*x^2+a)^(1/2)/(-a+b)
^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2 + a)^(3/2) + x/(sqrt(b*x^2 + a)*(x^2 + 1)),x, algorithm="maxim
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.283675, size = 1, normalized size = 0.02

$$\left[\frac{\left(b^2x^2+ab\right)\log\left(-\frac{4\left(\left(ab-b^2\right)x^2+2a^2-3ab+b^2\right)\sqrt{bx^2+a}-\left(b^2x^4+2\left(4ab-3b^2\right)x^2+8a^2-8ab+b^2\right)\sqrt{a-b}}{x^4+2x^2+1}\right)-4\sqrt{bx^2+a}\sqrt{a-b}}{4\left(b^2x^2+ab\right)\sqrt{a-b}},\ \frac{\left(b^2x^2+ab\right)\ldots}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2 + a)^(3/2) + x/(sqrt(b*x^2 + a)*(x^2 + 1)),x, algorithm="frica
```

```
[Out] [1/4*((b^2*x^2 + a*b)*log(-(4*((a*b - b^2)*x^2 + 2*a^2 - 3*a*b +
b^2)*sqrt(b*x^2 + a) - (b^2*x^4 + 2*(4*a*b - 3*b^2)*x^2 + 8*a^2 -
 8*a*b + b^2)*sqrt(a - b))/(x^4 + 2*x^2 + 1)) - 4*sqrt(b*x^2 + a)
*sqrt(a - b))/((b^2*x^2 + a*b)*sqrt(a - b)), 1/2*((b^2*x^2 + a*b)
*arctan(-1/2*(b*x^2 + 2*a - b)*sqrt(-a + b)/(sqrt(b*x^2 + a)*(a -
 b))) - 2*sqrt(b*x^2 + a)*sqrt(-a + b))/((b^2*x^2 + a*b)*sqrt(-a
+ b))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(a+bx^2+x^2+1\right)}{(a+bx^2)^{\frac{3}{2}}\,(x^2+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(b*x**2+a)**(3/2)+x/(x**2+1)/(b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x*(a + b*x**2 + x**2 + 1)/((a + b*x**2)**(3/2)*(x**2 + 1
)), x)
```

GIAC/XCAS [A] time = 0.266804, size = 55, normalized size = 1.1

$$\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}-\frac{1}{\sqrt{bx^2+a}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2 + a)^(3/2) + x/(sqrt(b*x^2 + a)*(x^2 + 1)),x, algorithm="giac"
```

```
[Out] arctan(sqrt(b*x^2 + a)/sqrt(-a + b))/sqrt(-a + b) - 1/(sqrt(b*x^2
 + a)*b)
```

3.532 $\int \frac{x(1+a+x^2+bx^2)}{(1+x^2)(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=50

$$-\frac{1}{b\sqrt{a+bx^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

```
[Out] -(1/(b*Sqrt[a + b*x^2])) - ArcTanh[Sqrt[a + b*x^2]/Sqrt[a - b]]/S
qrt[a - b]
```

Rubi [A] time = 0.191742, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161

$$-\frac{1}{b\sqrt{a+bx^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(1 + a + x^2 + b*x^2))/((1 + x^2)*(a + b*x^2)^(3/2)),x]
```

```
[Out] -(1/(b*Sqrt[a + b*x^2])) - ArcTanh[Sqrt[a + b*x^2]/Sqrt[a - b]]/S
qrt[a - b]
```

Rubi in Sympy [A] time = 14.9886, size = 39, normalized size = 0.78

$$-\frac{\operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}-\frac{1}{b\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(b*x**2+x**2+a+1)/(x**2+1)/(b*x**2+a)**(3/2),x)
```

```
[Out] -atanh(sqrt(a + b*x**2)/sqrt(a - b))/sqrt(a - b) - 1/(b*sqrt(a +
b*x**2))
```

Mathematica [A] time = 0.0336971, size = 49, normalized size = 0.98

$$\frac{\tan^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{b-a}}\right)}{\sqrt{b-a}}-\frac{1}{b\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(1 + a + x^2 + b*x^2))/((1 + x^2)*(a + b*x^2)^(3/2)),x]
```

```
[Out] -(1/(b*Sqrt[a + b*x^2])) + ArcTan[Sqrt[a + b*x^2]/Sqrt[-a + b]]/S
qrt[-a + b]
```

Maple [B] time = 0.021, size = 133, normalized size = 2.7

$$-\frac{1}{\sqrt{bx^2+a}}-\frac{1}{b}\frac{1}{\sqrt{bx^2+a}}+\frac{a}{a-b}\frac{1}{\sqrt{bx^2+a}}+\frac{a}{a-b}\arctan\left(1\sqrt{bx^2+a}\frac{1}{\sqrt{-a+b}}\right)\frac{1}{\sqrt{-a+b}}$$
$$-\frac{b}{a-b}\frac{1}{\sqrt{bx^2+a}}-\frac{b}{a-b}\arctan\left(1\sqrt{bx^2+a}\frac{1}{\sqrt{-a+b}}\right)\frac{1}{\sqrt{-a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(b*x^2+x^2+a+1)/(x^2+1)/(b*x^2+a)^(3/2),x)
```

```
[Out] -1/(b*x^2+a)^(1/2)-1/b/(b*x^2+a)^(1/2)+a/(a-b)/(b*x^2+a)^(1/2)+a/
(a-b)/(-a+b)^(1/2)*arctan((b*x^2+a)^(1/2)/(-a+b)^(1/2))-b/(a-b)/(
b*x^2+a)^(1/2)-b/(a-b)/(-a+b)^(1/2)*arctan((b*x^2+a)^(1/2)/(-a+b)
^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + x^2 + a + 1)*x/((b*x^2 + a)^(3/2)*(x^2 + 1)),x, algorithm="maxi
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.285327, size = 1, normalized size = 0.02

$$\left[\frac{\left(b^2x^2+ab\right)\log\left(-\frac{4\left(\left(ab-b^2\right)x^2+2a^2-3ab+b^2\right)\sqrt{bx^2+a}-\left(b^2x^4+2\left(4ab-3b^2\right)x^2+8a^2-8ab+b^2\right)\sqrt{a-b}}{x^4+2x^2+1}\right)-4\sqrt{bx^2+a}\sqrt{a-b}}{4\left(b^2x^2+ab\right)\sqrt{a-b}}, \frac{\left(b^2x^2+ab\right)a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + x^2 + a + 1)*x/((b*x^2 + a)^(3/2)*(x^2 + 1)),x, algorithm="fric
```

```
[Out] [1/4*((b^2*x^2 + a*b)*log(-(4*((a*b - b^2)*x^2 + 2*a^2 - 3*a*b +
b^2)*sqrt(b*x^2 + a) - (b^2*x^4 + 2*(4*a*b - 3*b^2)*x^2 + 8*a^2 -
 8*a*b + b^2)*sqrt(a - b))/(x^4 + 2*x^2 + 1)) - 4*sqrt(b*x^2 + a)
*sqrt(a - b))/((b^2*x^2 + a*b)*sqrt(a - b)), 1/2*((b^2*x^2 + a*b)
*arctan(-1/2*(b*x^2 + 2*a - b)*sqrt(-a + b)/(sqrt(b*x^2 + a)*(a -
 b))) - 2*sqrt(b*x^2 + a)*sqrt(-a + b))/((b^2*x^2 + a*b)*sqrt(-a
+ b))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(a+bx^2+x^2+1\right)}{\left(a+bx^2\right)^{\frac{3}{2}}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(b*x**2+x**2+a+1)/(x**2+1)/(b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x*(a + b*x**2 + x**2 + 1)/((a + b*x**2)**(3/2)*(x**2 + 1
)), x)
```

GIAC/XCAS [A] time = 0.265921, size = 55, normalized size = 1.1

$$\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}-\frac{1}{\sqrt{bx^2+a}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + x^2 + a + 1)*x/((b*x^2 + a)^(3/2)*(x^2 + 1)),x, algorithm="giac
```

```
[Out] arctan(sqrt(b*x^2 + a)/sqrt(-a + b))/sqrt(-a + b) - 1/(sqrt(b*x^2
 + a)*b)
```

3.533 $\int \left(\frac{x}{(a+bx^2)^{5/2}}+\frac{x}{(a+bx^2)^{3/2}}+\frac{x}{(1+x^2)\sqrt{a+bx^2}}\right)\,dx$

Optimal. Leaf size=68

$$-\frac{1}{b\sqrt{a+bx^2}}-\frac{1}{3b\,(a+bx^2)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

```
[Out] -1/(3*b*(a + b*x^2)^(3/2)) - 1/(b*Sqrt[a + b*x^2]) - ArcTanh[Sqrt
[a + b*x^2]/Sqrt[a - b]]/Sqrt[a - b]
```

Rubi [A] time = 0.102488, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.085

$$-\frac{1}{b\sqrt{a+bx^2}}-\frac{1}{3b\,(a+bx^2)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^2)^(5/2) + x/(a + b*x^2)^(3/2) + x/((1 + x^2)*Sqrt[a + b*x^2]),x]
```

```
[Out] -1/(3*b*(a + b*x^2)^(3/2)) - 1/(b*Sqrt[a + b*x^2]) - ArcTanh[Sqrt
[a + b*x^2]/Sqrt[a - b]]/Sqrt[a - b]
```

Rubi in Sympy [A] time = 6.05442, size = 54, normalized size = 0.79

$$-\frac{\operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}-\frac{1}{b\sqrt{a+bx^2}}-\frac{1}{3b\,(a+bx^2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x/(b*x**2+a)**(5/2)+x/(b*x**2+a)**(3/2)+x/(x**2+1)/(b*x**2+a)**
```

```
[Out] -atanh(sqrt(a + b*x**2)/sqrt(a - b))/sqrt(a - b) - 1/(b*sqrt(a +
b*x**2)) - 1/(3*b*(a + b*x**2)**(3/2))
```

Mathematica [A] time = 0.180312, size = 63, normalized size = 0.93

$$-\frac{3a+3bx^2+1}{3b\left(a+bx^2\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^2)^(5/2) + x/(a + b*x^2)^(3/2) + x/((1 + x^2)*Sqrt[a + b*x^2]
```

```
[Out] -(1 + 3*a + 3*b*x^2)/(3*b*(a + b*x^2)^(3/2)) - ArcTanh[Sqrt[a + b
*x^2]/Sqrt[a - b]]/Sqrt[a - b]
```

Maple [A] time = 0.016, size = 56, normalized size = 0.8

$$-\frac{1}{3\,b}\left(bx^2+a\right)^{-\frac{3}{2}}-\frac{1}{b}\frac{1}{\sqrt{bx^2+a}}+1\arctan\left(1\sqrt{bx^2+a}\frac{1}{\sqrt{-a+b}}\right)\frac{1}{\sqrt{-a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^(5/2)+x/(b*x^2+a)^(3/2)+x/(x^2+1)/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/3/b/(b*x^2+a)^(3/2)-1/b/(b*x^2+a)^(1/2)+1/(-a+b)^(1/2)*arctan(
(b*x^2+a)^(1/2)/(-a+b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2 + a)^(3/2) + x/(sqrt(b*x^2 + a)*(x^2 + 1)) + x/(b*x^2 + a)^(5/2
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.291352, size = 1, normalized size = 0.01

$$\left[-\frac{4\left(3\,bx^2+3\,a+1\right)\sqrt{bx^2+a}\sqrt{a-b}-3\left(b^3x^4+2\,ab^2x^2+a^2b\right)\log\left(-\frac{4\left(\left(ab-b^2\right)x^2+2\,a^2-3\,ab+b^2\right)\sqrt{bx^2+a}-\left(b^2x^4+2\left(4\,ab-3\,b^2\right)x^2+8\right.}{x^4+2\,x^2+1}\right.}{12\left(b^3x^4+2\,ab^2x^2+a^2b\right)\sqrt{a-b}}\right.$$

$$\left.-\frac{2\left(3\,bx^2+3\,a+1\right)\sqrt{bx^2+a}\sqrt{-a+b}-3\left(b^3x^4+2\,ab^2x^2+a^2b\right)\arctan\left(-\frac{\left(bx^2+2\,a-b\right)\sqrt{-a+b}}{2\sqrt{bx^2+a}(a-b)}\right)}{6\left(b^3x^4+2\,ab^2x^2+a^2b\right)\sqrt{-a+b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2 + a)^(3/2) + x/(sqrt(b*x^2 + a)*(x^2 + 1)) + x/(b*x^2 + a)^(5/2
```

```
[Out] [-1/12*(4*(3*b*x^2 + 3*a + 1)*sqrt(b*x^2 + a)*sqrt(a - b) - 3*(b^
3*x^4 + 2*a*b^2*x^2 + a^2*b)*log(-(4*((a*b - b^2)*x^2 + 2*a^2 - 3
*a*b + b^2)*sqrt(b*x^2 + a) - (b^2*x^4 + 2*(4*a*b - 3*b^2)*x^2 +
8*a^2 - 8*a*b + b^2)*sqrt(a - b))/(x^4 + 2*x^2 + 1)))/((b^3*x^4 +
 2*a*b^2*x^2 + a^2*b)*sqrt(a - b)), -1/6*(2*(3*b*x^2 + 3*a + 1)*s
qrt(b*x^2 + a)*sqrt(-a + b) - 3*(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*a
rctan(-1/2*(b*x^2 + 2*a - b)*sqrt(-a + b)/(sqrt(b*x^2 + a)*(a - b
))))/((b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sqrt(-a + b))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x\left(a^2+2abx^2+ax^2+a+b^2x^4+bx^4+bx^2+x^2+1\right)}{\left(a+bx^2\right)^{\frac{5}{2}}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**(5/2)+x/(b*x**2+a)**(3/2)+x/(x**2+1)/(b*x**2+a)**(1/2)
```

```
[Out] Integral(x*(a**2 + 2*a*b*x**2 + a*x**2 + a + b**2*x**4 + b*x**4 +
 b*x**2 + x**2 + 1)/((a + b*x**2)**(5/2)*(x**2 + 1)), x)
```

GIAC/XCAS [A] time = 0.263541, size = 74, normalized size = 1.09

$$\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}-\frac{1}{\sqrt{bx^2+a}b}-\frac{1}{3\left(bx^2+a\right)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2 + a)^(3/2) + x/(sqrt(b*x^2 + a)*(x^2 + 1)) + x/(b*x^2 + a)^(5/2

[Out] arctan(sqrt(b*x^2 + a)/sqrt(-a + b))/sqrt(-a + b) - 1/(sqrt(b*x^2
 + a)*b) - 1/3/((b*x^2 + a)^(3/2)*b)
```

3.534 $\int \frac{x(1+a+a^2+x^2+ax^2+bx^2+2abx^2+bx^4+b^2x^4)}{(1+x^2)(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=68

$$-\frac{1}{b\sqrt{a+bx^2}}-\frac{1}{3b\,(a+bx^2)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

```
[Out] -1/(3*b*(a + b*x^2)^(3/2)) - 1/(b*Sqrt[a + b*x^2]) - ArcTanh[Sqrt
[a + b*x^2]/Sqrt[a - b]]/Sqrt[a - b]
```

Rubi [A] time = 0.970305, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 58, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086

$$-\frac{1}{b\sqrt{a+bx^2}}-\frac{1}{3b\,(a+bx^2)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 + a + a^2 + x^2 + a*x^2 + b*x^2 + 2*a*b*x^2 + b*x^4 + b^2*x^4))/((1 + x^2)
```

```
[Out] -1/(3*b*(a + b*x^2)^(3/2)) - 1/(b*Sqrt[a + b*x^2]) - ArcTanh[Sqrt
[a + b*x^2]/Sqrt[a - b]]/Sqrt[a - b]
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x*(b**2*x**4+b*x**4+2*a*b*x**2+a*x**2+b*x**2+a**2+x**2+a+1)/(x*
```

```
[Out] Timed out
```

Mathematica [A] time = 0.0893021, size = 63, normalized size = 0.93

$$-\frac{3a+3bx^2+1}{3b\,(a+bx^2)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(1 + a + a^2 + x^2 + a*x^2 + b*x^2 + 2*a*b*x^2 + b*x^4 + b^2*x^4))/((1
```

```
[Out] -(1 + 3*a + 3*b*x^2)/(3*b*(a + b*x^2)^(3/2)) - ArcTanh[Sqrt[a + b
*x^2]/Sqrt[a - b]]/Sqrt[a - b]
```

Maple [B] time = 0.029, size = 314, normalized size = 4.6

$$-bx^2\left(bx^2+a\right)^{-\frac{3}{2}}-x^2\left(bx^2+a\right)^{-\frac{3}{2}}-\frac{4a}{3}\left(bx^2+a\right)^{-\frac{3}{2}}-\frac{a}{b}\left(bx^2+a\right)^{-\frac{3}{2}}+\frac{b}{3}\left(bx^2+a\right)^{-\frac{3}{2}}-\frac{1}{3b}\left(bx^2+a\right)^{-\frac{3}{2}}+\frac{a^2}{(a-b)^2}\frac{1}{\sqrt{bx^2+a}}-2\frac{ab}{(a-b)^2\sqrt{bx^2+a}}+\frac{b^2}{(a-b)^2}\frac{1}{\sqrt{bx^2+a}}+\frac{a^2}{(a-b)^2}\arctan\left(1\sqrt{bx^2+a}\frac{1}{\sqrt{-a+b}}\right)\frac{1}{\sqrt{-a+b}}-2\frac{ab}{(a-b)^2\sqrt{-a+b}}\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a+b}}\right)+\frac{b^2}{(a-b)^2}\arctan\left(1\sqrt{bx^2+a}\frac{1}{\sqrt{-a+b}}\right)\frac{1}{\sqrt{-a+b}}+\frac{a^2}{3a-3b}\left(bx^2+a\right)^{-\frac{3}{2}}-\frac{2ab}{3a-3b}\left(bx^2+a\right)^{-\frac{3}{2}}+\frac{b^2}{3a-3b}\left(bx^2+a\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b^2*x^4+b*x^4+2*a*b*x^2+a*x^2+b*x^2+a^2+x^2+a+1)/(x^2+1)/(b*x^2+a)^(5/2
```

```
[Out] -x^2*b/(b*x^2+a)^(3/2)-x^2/(b*x^2+a)^(3/2)-4/3*a/(b*x^2+a)^(3/2)-
a/b/(b*x^2+a)^(3/2)+1/3*b/(b*x^2+a)^(3/2)-1/3/b/(b*x^2+a)^(3/2)+1
/(a-b)^2/(b*x^2+a)^(1/2)*a^2-2/(a-b)^2/(b*x^2+a)^(1/2)*a*b+1/(a-b
)^2/(b*x^2+a)^(1/2)*b^2+1/(a-b)^2/(-a+b)^(1/2)*arctan((b*x^2+a)^(
1/2)/(-a+b)^(1/2))*a^2-2/(a-b)^2/(-a+b)^(1/2)*arctan((b*x^2+a)^(1
/2)/(-a+b)^(1/2))*a*b+1/(a-b)^2/(-a+b)^(1/2)*arctan((b*x^2+a)^(1/
2)/(-a+b)^(1/2))*b^2+1/3/(a-b)/(b*x^2+a)^(3/2)*a^2-2/3/(a-b)/(b*x
^2+a)^(3/2)*a*b+1/3/(a-b)/(b*x^2+a)^(3/2)*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + b*x^4 + 2*a*b*x^2 + a*x^2 + b*x^2 + a^2 + x^2 + a + 1)*x/((b*x
```

[Out] Exception raised: ValueError

Fricas [A] time = 0.292535, size = 1, normalized size = 0.01

$$\left[-\frac{4\left(3\,bx^2+3\,a+1\right)\sqrt{bx^2+a}\sqrt{a-b}-3\left(b^3x^4+2\,ab^2x^2+a^2b\right)\log\left(-\frac{4\left(\left(ab-b^2\right)x^2+2\,a^2-3\,ab+b^2\right)\sqrt{bx^2+a}-\left(b^2x^4+2\left(4\,ab-3\,b^2\right)x^2+8\right.}{x^4+2\,x^2+1}\right.}{12\left(b^3x^4+2\,ab^2x^2+a^2b\right)\sqrt{a-b}}\right.$$
$$\left.-\frac{2\left(3\,bx^2+3\,a+1\right)\sqrt{bx^2+a}\sqrt{-a+b}-3\left(b^3x^4+2\,ab^2x^2+a^2b\right)\arctan\left(-\frac{\left(bx^2+2\,a-b\right)\sqrt{-a+b}}{2\sqrt{bx^2+a}(a-b)}\right)}{6\left(b^3x^4+2\,ab^2x^2+a^2b\right)\sqrt{-a+b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + b*x^4 + 2*a*b*x^2 + a*x^2 + b*x^2 + a^2 + x^2 + a + 1)*x/((b*x
```

```
[Out] [-1/12*(4*(3*b*x^2 + 3*a + 1)*sqrt(b*x^2 + a)*sqrt(a - b) - 3*(b^
3*x^4 + 2*a*b^2*x^2 + a^2*b)*log(-(4*((a*b - b^2)*x^2 + 2*a^2 - 3
*a*b + b^2)*sqrt(b*x^2 + a) - (b^2*x^4 + 2*(4*a*b - 3*b^2)*x^2 +
8*a^2 - 8*a*b + b^2)*sqrt(a - b))/(x^4 + 2*x^2 + 1)))/((b^3*x^4 +
 2*a*b^2*x^2 + a^2*b)*sqrt(a - b)), -1/6*(2*(3*b*x^2 + 3*a + 1)*s
qrt(b*x^2 + a)*sqrt(-a + b) - 3*(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*a
rctan(-1/2*(b*x^2 + 2*a - b)*sqrt(-a + b)/(sqrt(b*x^2 + a)*(a - b
))))/((b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sqrt(-a + b))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b**2*x**4+b*x**4+2*a*b*x**2+a*x**2+b*x**2+a**2+x**2+a+1)/(x**2+1)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.2764, size = 70, normalized size = 1.03

$$\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}-\frac{3\,bx^2+3\,a+1}{3\left(bx^2+a\right)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + b*x^4 + 2*a*b*x^2 + a*x^2 + b*x^2 + a^2 + x^2 + a + 1)*x/((b*x
```

```
[Out] arctan(sqrt(b*x^2 + a)/sqrt(-a + b))/sqrt(-a + b) - 1/3*(3*b*x^2
+ 3*a + 1)/((b*x^2 + a)^(3/2)*b)
```

3.535 $\int \frac{1}{\sqrt{\sqrt{x}+x}}\,dx$

Optimal. Leaf size=34

$$2\sqrt{x+\sqrt{x}}-2\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{x+\sqrt{x}}}\right)$$

```
[Out] 2*Sqrt[Sqrt[x] + x] - 2*ArcTanh[Sqrt[x]/Sqrt[Sqrt[x] + x]]
```

Rubi [A] time = 0.053362, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364

$$2\sqrt{x+\sqrt{x}}-2\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{x+\sqrt{x}}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[Sqrt[x] + x],x]

[Out] 2*Sqrt[Sqrt[x] + x] - 2*ArcTanh[Sqrt[x]/Sqrt[Sqrt[x] + x]]
```

Rubi in Sympy [A] time = 3.04009, size = 29, normalized size = 0.85

$$2\sqrt{\sqrt{x}+x}-2\,\text{atanh}\left(\frac{\sqrt{x}}{\sqrt{\sqrt{x}+x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x+x**(1/2))**(1/2),x)

[Out] 2*sqrt(sqrt(x) + x) - 2*atanh(sqrt(x)/sqrt(sqrt(x) + x))
```

Mathematica [A] time = 0.0245814, size = 39, normalized size = 1.15

$$2\sqrt{x+\sqrt{x}}-\log\left(2\sqrt{x}+2\sqrt{x+\sqrt{x}}+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[Sqrt[x] + x],x]
```

```
[Out] 2*Sqrt[Sqrt[x] + x] - Log[1 + 2*Sqrt[x] + 2*Sqrt[Sqrt[x] + x]]
```

Maple [A] time = 0.015, size = 44, normalized size = 1.3

$$-1\sqrt{x+\sqrt{x}}\left(-2\sqrt{x+\sqrt{x}}+\ln\left(\frac{1}{2}+\sqrt{x}+\sqrt{x+\sqrt{x}}\right)\right)\frac{1}{\sqrt{\sqrt{x}\left(1+\sqrt{x}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x+x^(1/2))^(1/2),x)
```

```
[Out] -(x+x^(1/2))^(1/2)*(-2*(x+x^(1/2))^(1/2)+ln(1/2+x^(1/2)+(x+x^(1/2
))^(1/2)))/(x^(1/2)*(1+x^(1/2)))^(1/2)
```

Maxima [A] time = 0.768714, size = 76, normalized size = 2.24

$$\frac{2\sqrt{\sqrt{x}+1}}{x^{\frac{1}{4}}\left(\frac{\sqrt{x}+1}{\sqrt{x}}-1\right)}-\log\left(\frac{\sqrt{\sqrt{x}+1}}{x^{\frac{1}{4}}}+1\right)+\log\left(\frac{\sqrt{\sqrt{x}+1}}{x^{\frac{1}{4}}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(x + sqrt(x)),x, algorithm="maxima")
```

```
[Out] 2*sqrt(sqrt(x) + 1)/(x^(1/4)*((sqrt(x) + 1)/sqrt(x) - 1)) - log(s
qrt(sqrt(x) + 1)/x^(1/4) + 1) + log(sqrt(sqrt(x) + 1)/x^(1/4) - 1
)
```

Fricas [A] time = 0.503679, size = 53, normalized size = 1.56

$$2\sqrt{x+\sqrt{x}}+\frac{1}{2}\log\left(4\sqrt{x+\sqrt{x}}\left(2\sqrt{x}+1\right)-8x-8\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(x + sqrt(x)),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x + sqrt(x)) + 1/2*log(4*sqrt(x + sqrt(x))*(2*sqrt(x) + 1)
 - 8*x - 8*sqrt(x) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\sqrt{x}+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x+x**(1/2))**(1/2),x)
```

```
[Out] Integral(1/sqrt(sqrt(x) + x), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(x + sqrt(x)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.536 $\int \sqrt{\sqrt{x}+x}\,dx$

Optimal. Leaf size=74

$$\frac{2}{3}\sqrt{x+\sqrt{x}}x+\frac{1}{6}\sqrt{x+\sqrt{x}}\sqrt{x}-\frac{\sqrt{x+\sqrt{x}}}{4}+\frac{1}{4}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{x+\sqrt{x}}}\right)$$

```
[Out] -Sqrt[Sqrt[x] + x]/4 + (Sqrt[x]*Sqrt[Sqrt[x] + x])/6 + (2*x*Sqrt[
Sqrt[x] + x])/3 + ArcTanh[Sqrt[x]/Sqrt[Sqrt[x] + x]]/4
```

Rubi [A] time = 0.0846573, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546

$$\frac{2}{3}\sqrt{x+\sqrt{x}}x+\frac{1}{6}\sqrt{x+\sqrt{x}}\sqrt{x}-\frac{\sqrt{x+\sqrt{x}}}{4}+\frac{1}{4}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{x+\sqrt{x}}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[Sqrt[x] + x],x]
```

```
[Out] -Sqrt[Sqrt[x] + x]/4 + (Sqrt[x]*Sqrt[Sqrt[x] + x])/6 + (2*x*Sqrt[
Sqrt[x] + x])/3 + ArcTanh[Sqrt[x]/Sqrt[Sqrt[x] + x]]/4
```

Rubi in Sympy [A] time = 4.9406, size = 61, normalized size = 0.82

$$\frac{\sqrt{x}\sqrt{\sqrt{x}+x}}{6}+\frac{2x\sqrt{\sqrt{x}+x}}{3}-\frac{\sqrt{\sqrt{x}+x}}{4}+\frac{\operatorname{atanh}\left(\frac{\sqrt{x}}{\sqrt{\sqrt{x}+x}}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x+x**(1/2))**(1/2),x)
```

```
[Out] sqrt(x)*sqrt(sqrt(x) + x)/6 + 2*x*sqrt(sqrt(x) + x)/3 - sqrt(sqrt
(x) + x)/4 + atanh(sqrt(x)/sqrt(sqrt(x) + x))/4
```

Mathematica [A] time = 0.0247638, size = 55, normalized size = 0.74

$$\frac{1}{12}\sqrt{x+\sqrt{x}}\left(8x+2\sqrt{x}-3\right)+\frac{1}{8}\log\left(2\sqrt{x}+2\sqrt{x+\sqrt{x}}+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[Sqrt[x] + x],x]
```

```
[Out] (Sqrt[Sqrt[x] + x]*(-3 + 2*Sqrt[x] + 8*x))/12 + Log[1 + 2*Sqrt[x]
 + 2*Sqrt[Sqrt[x] + x]]/8
```

Maple [A] time = 0.006, size = 42, normalized size = 0.6

$$\frac{2}{3}\left(x+\sqrt{x}\right)^{\frac{3}{2}}-\frac{1}{4}\left(1+2\sqrt{x}\right)\sqrt{x+\sqrt{x}}+\frac{1}{8}\ln\left(\frac{1}{2}+\sqrt{x}+\sqrt{x+\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x+x^(1/2))^(1/2),x)
```

```
[Out] 2/3*(x+x^(1/2))^(3/2)-1/4*(1+2*x^(1/2))*(x+x^(1/2))^(1/2)+1/8*ln(
1/2+x^(1/2)+(x+x^(1/2))^(1/2))
```

Maxima [A] time = 0.761675, size = 147, normalized size = 1.99

$$-\frac{\frac{3\left(\sqrt{x}+1\right)^{\frac{5}{2}}}{x^{\frac{5}{4}}}-\frac{8\left(\sqrt{x}+1\right)^{\frac{3}{2}}}{x^{\frac{3}{4}}}-\frac{3\sqrt{\sqrt{x}+1}}{x^{\frac{1}{4}}}}{12\left(\frac{\left(\sqrt{x}+1\right)^{3}}{x^{\frac{3}{2}}}-\frac{3\left(\sqrt{x}+1\right)^{2}}{x}+\frac{3\left(\sqrt{x}+1\right)}{\sqrt{x}}-1\right)}+\frac{1}{8}\log\left(\frac{\sqrt{\sqrt{x}+1}}{x^{\frac{1}{4}}}+1\right)-\frac{1}{8}\log\left(\frac{\sqrt{\sqrt{x}+1}}{x^{\frac{1}{4}}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x + sqrt(x)),x, algorithm="maxima")
```

```
[Out] -1/12*(3*(sqrt(x) + 1)^(5/2)/x^(5/4) - 8*(sqrt(x) + 1)^(3/2)/x^(3
/4) - 3*sqrt(sqrt(x) + 1)/x^(1/4))/((sqrt(x) + 1)^3/x^(3/2) - 3*(
sqrt(x) + 1)^2/x + 3*(sqrt(x) + 1)/sqrt(x) - 1) + 1/8*log(sqrt(sq
rt(x) + 1)/x^(1/4) + 1) - 1/8*log(sqrt(sqrt(x) + 1)/x^(1/4) - 1)
```

Fricas [A] time = 0.541969, size = 66, normalized size = 0.89

$$\frac{1}{12}\left(8x+2\sqrt{x}-3\right)\sqrt{x+\sqrt{x}}+\frac{1}{16}\log\left(4\sqrt{x+\sqrt{x}}\left(2\sqrt{x}+1\right)+8x+8\sqrt{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x + sqrt(x)),x, algorithm="fricas")
```

```
[Out] 1/12*(8*x + 2*sqrt(x) - 3)*sqrt(x + sqrt(x)) + 1/16*log(4*sqrt(x
+ sqrt(x))*(2*sqrt(x) + 1) + 8*x + 8*sqrt(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sqrt{x} + x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x+x**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(sqrt(x) + x), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x + sqrt(x)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.537 $\int \sqrt{-x} \left(\sqrt{-x}+x\right) \, dx$

Optimal. Leaf size=19

$$\frac{2}{5}(-x)^{5/2}-\frac{x^2}{2}$$

```
[Out] (2*(-x)^(5/2))/5 - x^2/2
```

Rubi [A] time = 0.0111821, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059

$$\frac{2}{5}(-x)^{5/2}-\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[-x]*(Sqrt[-x] + x),x]
```

```
[Out] (2*(-x)^(5/2))/5 - x^2/2
```

Rubi in Sympy [A] time = 3.49258, size = 14, normalized size = 0.74

$$-\frac{x^2}{2}+\frac{2\,(-x)^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x)**(1/2)*(x+(-x)**(1/2)),x)
```

```
[Out] -x**2/2 + 2*(-x)**(5/2)/5
```

Mathematica [A] time = 0.00753848, size = 19, normalized size = 1.

$$\frac{2}{5}(-x)^{5/2}-\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[-x]*(Sqrt[-x] + x),x]

[Out] (2*(-x)^(5/2))/5 - x^2/2
```

Maple [A] time = 0.003, size = 14, normalized size = 0.7

$$\frac{2}{5}(-x)^{\frac{5}{2}} - \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x)^(1/2)*(x+(-x)^(1/2)),x)

[Out] 2/5*(-x)^(5/2)-1/2*x^2
```

Maxima [A] time = 0.717931, size = 18, normalized size = 0.95

$$\frac{2}{5}(-x)^{\frac{5}{2}} - \frac{1}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x)*(x + sqrt(-x)),x, algorithm="maxima")

[Out] 2/5*(-x)^(5/2) - 1/2*x^2
```

Fricas [A] time = 0.260425, size = 22, normalized size = 1.16

$$\frac{2}{5}\sqrt{-x}x^2 - \frac{1}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x)*(x + sqrt(-x)),x, algorithm="fricas")

[Out] 2/5*sqrt(-x)*x^2 - 1/2*x^2
```

Sympy [A] time = 0.478577, size = 14, normalized size = 0.74

$$\frac{2ix^{\frac{5}{2}}}{5} - \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x)**(1/2)*(x+(-x)**(1/2)),x)
```

```
[Out] 2*I*x**(5/2)/5 - x**2/2
```

GIAC/XCAS [A] time = 0.284895, size = 22, normalized size = 1.16

$$\frac{2}{5}\sqrt{-x}x^2 - \frac{1}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x)*(x + sqrt(-x)),x, algorithm="giac")
```

```
[Out] 2/5*sqrt(-x)*x^2 - 1/2*x^2
```

3.538 $\int \frac{5+\sqrt[4]{x}}{-6+x}\,dx$

Optimal. Leaf size=54

$$4\sqrt[4]{x}+5\log(6-x)-2\sqrt[4]{6}\tan^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{6}}\right)-2\sqrt[4]{6}\tanh^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{6}}\right)$$

```
[Out] 4*x^(1/4) - 2*6^(1/4)*ArcTan[x^(1/4)/6^(1/4)] - 2*6^(1/4)*ArcTanh
[x^(1/4)/6^(1/4)] + 5*Log[6 - x]
```

Rubi [A] time = 0.135973, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462

$$4\sqrt[4]{x}+5\log(6-x)-2\sqrt[4]{6}\tan^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{6}}\right)-2\sqrt[4]{6}\tanh^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{6}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(5 + x^(1/4))/(-6 + x),x]
```

```
[Out] 4*x^(1/4) - 2*6^(1/4)*ArcTan[x^(1/4)/6^(1/4)] - 2*6^(1/4)*ArcTanh
[x^(1/4)/6^(1/4)] + 5*Log[6 - x]
```

Rubi in Sympy [A] time = 4.28562, size = 53, normalized size = 0.98

$$4\sqrt[4]{x}+5\log\left(-x+6\right)-2\sqrt[4]{6}\,\mathrm{atan}\left(\frac{6^{\frac{3}{4}}\sqrt[4]{x}}{6}\right)-2\sqrt[4]{6}\,\mathrm{atanh}\left(\frac{6^{\frac{3}{4}}\sqrt[4]{x}}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5+x**(1/4))/(-6+x),x)
```

```
[Out] 4*x**(1/4) + 5*log(-x + 6) - 2*6**(1/4)*atan(6**(3/4)*x**(1/4)/6)
 - 2*6**(1/4)*atanh(6**(3/4)*x**(1/4)/6)
```

Mathematica [A] time = 0.0461681, size = 77, normalized size = 1.43

$$4\sqrt[4]{x}+\sqrt[4]{6}\log\left(6-6^{3/4}\sqrt[4]{x}\right)-\sqrt[4]{6}\log\left(6^{3/4}\sqrt[4]{x}+6\right)+5\log(6-x)-2\sqrt[4]{6}\tan^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{6}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(5 + x^(1/4))/(-6 + x),x]
```

```
[Out] 4*x^(1/4) - 2*6^(1/4)*ArcTan[x^(1/4)/6^(1/4)] + 6^(1/4)*Log[6 - 6
^(3/4)*x^(1/4)] - 6^(1/4)*Log[6 + 6^(3/4)*x^(1/4)] + 5*Log[6 - x]
```

Maple [A] time = 0.008, size = 52, normalized size = 1.

$$4\sqrt[4]{x}-2\sqrt[4]{6}\arctan\left(1/6\sqrt[4]{x}6^{3/4}\right)-\sqrt[4]{6}\ln\left(1\left(\sqrt[4]{x}+\sqrt[4]{6}\right)\left(\sqrt[4]{x}-\sqrt[4]{6}\right)^{-1}\right)+5\ln(-6+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5+x^(1/4))/(-6+x),x)
```

```
[Out] 4*x^(1/4)-2*6^(1/4)*arctan(1/6*x^(1/4)*6^(3/4))-6^(1/4)*ln((x^(1/
4)+6^(1/4))/(x^(1/4)-6^(1/4)))+5*ln(-6+x)
```

Maxima [A] time = 0.81135, size = 90, normalized size = 1.67

$$-2\cdot 6^{\frac{1}{4}}\arctan\left(\frac{1}{6}\cdot 6^{\frac{3}{4}}x^{\frac{1}{4}}\right)+6^{\frac{1}{4}}\log\left(-\frac{6^{\frac{1}{4}}-x^{\frac{1}{4}}}{6^{\frac{1}{4}}+x^{\frac{1}{4}}}\right)+4x^{\frac{1}{4}}+5\log\left(\sqrt{6}+\sqrt{x}\right)+5\log\left(-\sqrt{6}+\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^(1/4) + 5)/(x - 6),x, algorithm="maxima")
```

```
[Out] -2*6^(1/4)*arctan(1/6*6^(3/4)*x^(1/4)) + 6^(1/4)*log(-(6^(1/4) -
x^(1/4))/(6^(1/4) + x^(1/4))) + 4*x^(1/4) + 5*log(sqrt(6) + sqrt(
x)) + 5*log(-sqrt(6) + sqrt(x))
```

Fricas [A] time = 0.281884, size = 111, normalized size = 2.06

$$-\left(6^{\frac{1}{4}}-5\right)\log\left(2\cdot 6^{\frac{1}{4}}+2x^{\frac{1}{4}}\right)+\left(6^{\frac{1}{4}}+5\right)\log\left(-2\cdot 6^{\frac{1}{4}}+2x^{\frac{1}{4}}\right)+4\cdot 6^{\frac{1}{4}}\arctan\left(\frac{6^{\frac{1}{4}}}{\sqrt{\sqrt{6}+\sqrt{x}}+x^{\frac{1}{4}}}\right)+4x^{\frac{1}{4}}+5\log\left(4\sqrt{6}+4\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^(1/4) + 5)/(x - 6),x, algorithm="fricas")
```

```
[Out] -(6^(1/4) - 5)*log(2*6^(1/4) + 2*x^(1/4)) + (6^(1/4) + 5)*log(-2*
6^(1/4) + 2*x^(1/4)) + 4*6^(1/4)*arctan(6^(1/4)/(sqrt(sqrt(6) + s
qrt(x)) + x^(1/4))) + 4*x^(1/4) + 5*log(4*sqrt(6) + 4*sqrt(x))
```

Sympy [A] time = 4.12421, size = 182, normalized size = 3.37

$$\frac{5\sqrt[4]{x}\,\Gamma\left(\frac{5}{4}\right)}{\Gamma\left(\frac{9}{4}\right)}+5\log\left(x-6\right)+\frac{5\sqrt[4]{6}\log\left(-\frac{6^{\frac{3}{4}}\sqrt[4]{x}}{6}+1\right)\Gamma\left(\frac{5}{4}\right)}{4\Gamma\left(\frac{9}{4}\right)}-\frac{5\sqrt[4]{6}i\log\left(-\frac{6^{\frac{3}{4}}\sqrt[4]{x}e^{\frac{i\pi}{2}}}{6}+1\right)\Gamma\left(\frac{5}{4}\right)}{4\Gamma\left(\frac{9}{4}\right)}-\frac{5\sqrt[4]{6}\log\left(-\frac{6^{\frac{3}{4}}\sqrt[4]{x}e^{i\pi}}{6}+1\right)\Gamma\left(\frac{5}{4}\right)}{4\Gamma\left(\frac{9}{4}\right)}+\frac{5\sqrt[4]{6}i\log\left(-\frac{6^{\frac{3}{4}}\sqrt[4]{x}e^{\frac{3i\pi}{2}}}{6}+1\right)\Gamma\left(\frac{5}{4}\right)}{4\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5+x**(1/4))/(-6+x),x)
```

```
[Out] 5*x**(1/4)*gamma(5/4)/gamma(9/4) + 5*log(x - 6) + 5*6**(1/4)*log(
-6**(3/4)*x**(1/4)/6 + 1)*gamma(5/4)/(4*gamma(9/4)) - 5*6**(1/4)*
I*log(-6**(3/4)*x**(1/4)*exp_polar(I*pi/2)/6 + 1)*gamma(5/4)/(4*g
amma(9/4)) - 5*6**(1/4)*log(-6**(3/4)*x**(1/4)*exp_polar(I*pi)/6
+ 1)*gamma(5/4)/(4*gamma(9/4)) + 5*6**(1/4)*I*log(-6**(3/4)*x**(1
/4)*exp_polar(3*I*pi/2)/6 + 1)*gamma(5/4)/(4*gamma(9/4))
```

GIAC/XCAS [A] time = 0.304351, size = 74, normalized size = 1.37

$$-2\cdot 6^{\frac{1}{4}}\arctan\left(\frac{1}{6}\cdot 6^{\frac{3}{4}}x^{\frac{1}{4}}\right)-6^{\frac{1}{4}}\ln\left(6^{\frac{1}{4}}+x^{\frac{1}{4}}\right)+6^{\frac{1}{4}}\ln\left(\left|-6^{\frac{1}{4}}+x^{\frac{1}{4}}\right|\right)+4\,x^{\frac{1}{4}}+5\,\ln\left(|x-6|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^(1/4) + 5)/(x - 6),x, algorithm="giac")
```

```
[Out] -2*6^(1/4)*arctan(1/6*6^(3/4)*x^(1/4)) - 6^(1/4)*ln(6^(1/4) + x^(
1/4)) + 6^(1/4)*ln(abs(-6^(1/4) + x^(1/4))) + 4*x^(1/4) + 5*ln(ab
s(x - 6))
```

3.539 $\int \frac{1}{4+\sqrt{4-x}-x}\,dx$

Optimal. Leaf size=14

$$-2\log\left(\sqrt{4-x}+1\right)$$

```
[Out] -2*Log[1 + Sqrt[4 - x]]
```

Rubi [A] time = 0.0342119, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062

$$-2\log\left(\sqrt{4-x}+1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(4 + Sqrt[4 - x] - x)^(-1),x]

[Out] -2*Log[1 + Sqrt[4 - x]]
```

Rubi in Sympy [A] time = 1.25729, size = 12, normalized size = 0.86

$$-2\log\left(\sqrt{-x+4}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(4-x+(4-x)**(1/2)),x)

[Out] -2*log(sqrt(-x + 4) + 1)
```

Mathematica [A] time = 0.00647614, size = 14, normalized size = 1.

$$-2\log\left(\sqrt{4-x}+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(4 + Sqrt[4 - x] - x)^(-1),x]
```

```
[Out] -2*Log[1 + Sqrt[4 - x]]
```

Maple [A] time = 0.017, size = 18, normalized size = 1.3

$$-\ln\left(-3+x\right)-2\,\mathit{Artanh}\left(\sqrt{4-x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(4-x+(4-x)^(1/2)),x)

[Out] -ln(-3+x)-2*arctanh((4-x)^(1/2))
```

Maxima [A] time = 0.719177, size = 16, normalized size = 1.14

$$-2\,\log\left(\sqrt{-x+4}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(x - sqrt(-x + 4) - 4),x, algorithm="maxima")

[Out] -2*log(sqrt(-x + 4) + 1)
```

Fricas [A] time = 0.262294, size = 16, normalized size = 1.14

$$-2\,\log\left(\sqrt{-x+4}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(x - sqrt(-x + 4) - 4),x, algorithm="fricas")

[Out] -2*log(sqrt(-x + 4) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{1}{x-\sqrt{-x+4}-4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(4-x+(4-x)**(1/2)),x)
```

```
[Out] -Integral(1/(x - sqrt(-x + 4) - 4), x)
```

GIAC/XCAS [A] time = 0.285926, size = 16, normalized size = 1.14

$$-2\ln\left(\sqrt{-x+4}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(x - sqrt(-x + 4) - 4),x, algorithm="giac")
```

```
[Out] -2*ln(sqrt(-x + 4) + 1)
```

3.540 $\int \frac{1}{1+x-\sqrt{2+x}}\,dx$

Optimal. Leaf size=61

$$\frac{1}{5}\left(5-\sqrt{5}\right)\log\left(-2\sqrt{x+2}-\sqrt{5}+1\right)+\frac{1}{5}\left(5+\sqrt{5}\right)\log\left(-2\sqrt{x+2}+\sqrt{5}+1\right)$$

```
[Out] ((5 - Sqrt[5])*Log[1 - Sqrt[5] - 2*Sqrt[2 + x]])/5 + ((5 + Sqrt[5
])*Log[1 + Sqrt[5] - 2*Sqrt[2 + x]])/5
```

Rubi [A] time = 0.0802047, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$\frac{1}{5}\left(5-\sqrt{5}\right)\log\left(-2\sqrt{x+2}-\sqrt{5}+1\right)+\frac{1}{5}\left(5+\sqrt{5}\right)\log\left(-2\sqrt{x+2}+\sqrt{5}+1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x - Sqrt[2 + x])^(-1),x]
```

```
[Out] ((5 - Sqrt[5])*Log[1 - Sqrt[5] - 2*Sqrt[2 + x]])/5 + ((5 + Sqrt[5
])*Log[1 + Sqrt[5] - 2*Sqrt[2 + x]])/5
```

Rubi in Sympy [A] time = 2.90252, size = 70, normalized size = 1.15

$$\frac{2\sqrt{5}\left(\frac{1}{2}+\frac{\sqrt{5}}{2}\right)\log\left(-2\sqrt{x+2}+1+\sqrt{5}\right)}{5}-\frac{2\sqrt{5}\left(-\frac{\sqrt{5}}{2}+\frac{1}{2}\right)\log\left(-2\sqrt{x+2}-\sqrt{5}+1\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1+x-(2+x)**(1/2)),x)
```

```
[Out] 2*sqrt(5)*(1/2 + sqrt(5)/2)*log(-2*sqrt(x + 2) + 1 + sqrt(5))/5 -
 2*sqrt(5)*(-sqrt(5)/2 + 1/2)*log(-2*sqrt(x + 2) - sqrt(5) + 1)/5
```

Mathematica [A] time = 0.0258799, size = 39, normalized size = 0.64

$$\log\left(-x+\sqrt{x+2}-1\right)-\frac{2\tanh^{-1}\left(\frac{2\sqrt{x+2}-1}{\sqrt{5}}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x - Sqrt[2 + x])^(-1),x]
```

```
[Out] (-2*ArcTanh[(-1 + 2*Sqrt[2 + x])/Sqrt[5]])/Sqrt[5] + Log[-1 - x +
 Sqrt[2 + x]]
```

Maple [A]   time = 0.011, size = 91, normalized size = 1.5

$$\frac{\ln\left(x^2+x-1\right)}{2}-\frac{\sqrt{5}}{5}Artanh\left(\frac{(1+2x)\sqrt{5}}{5}\right)+\frac{1}{2}\ln\left(1+x-\sqrt{2+x}\right)-\frac{\sqrt{5}}{5}Artanh\left(\frac{\sqrt{5}}{5}\left(2\sqrt{2+x}-1\right)\right)-\frac{1}{2}\ln\left(1+x+\sqrt{2+x}\right)-\frac{\sqrt{5}}{5}Artanh\left(\frac{\sqrt{5}}{5}\left(2\sqrt{2+x}+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1+x-(2+x)^(1/2)),x)
```

```
[Out] 1/2*ln(x^2+x-1)-1/5*5^(1/2)*arctanh(1/5*(1+2*x)*5^(1/2))+1/2*ln(1
+x-(2+x)^(1/2))-1/5*5^(1/2)*arctanh(1/5*(2*(2+x)^(1/2)-1)*5^(1/2)
)-1/2*ln(1+x+(2+x)^(1/2))-1/5*5^(1/2)*arctanh(1/5*(2*(2+x)^(1/2)+
1)*5^(1/2))
```

Maxima [A]   time = 0.808924, size = 62, normalized size = 1.02

$$\frac{1}{5}\sqrt{5}\log\left(-\frac{\sqrt{5}-2\sqrt{x+2}+1}{\sqrt{5}+2\sqrt{x+2}-1}\right)+\log\left(x-\sqrt{x+2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x - sqrt(x + 2) + 1),x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(5)*log(-(sqrt(5) - 2*sqrt(x + 2) + 1)/(sqrt(5) + 2*sqrt(
x + 2) - 1)) + log(x - sqrt(x + 2) + 1)
```

Fricas [A]   time = 0.270092, size = 78, normalized size = 1.28

$$\frac{1}{5}\sqrt{5}\left(\sqrt{5}\log\left(x-\sqrt{x+2}+1\right)+\log\left(\frac{\sqrt{5}(2x+7)-2\sqrt{x+2}\left(\sqrt{5}+5\right)+5}{x-\sqrt{x+2}+1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x - sqrt(x + 2) + 1),x, algorithm="fricas")
```

```
[Out] 1/5*sqrt(5)*(sqrt(5)*log(x - sqrt(x + 2) + 1) + log((sqrt(5)*(2*x
 + 7) - 2*sqrt(x + 2)*(sqrt(5) + 5) + 5)/(x - sqrt(x + 2) + 1)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x - \sqrt{x + 2} + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1+x-(2+x)**(1/2)),x)
```

```
[Out] Integral(1/(x - sqrt(x + 2) + 1), x)
```

GIAC/XCAS [A] time = 0.301543, size = 68, normalized size = 1.11

$$\frac{1}{5}\sqrt{5}\ln\left(\frac{\left|-\sqrt{5} + 2\sqrt{x + 2} - 1\right|}{\left|\sqrt{5} + 2\sqrt{x + 2} - 1\right|}\right) + \ln\left(\left|x - \sqrt{x + 2} + 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x - sqrt(x + 2) + 1),x, algorithm="giac")
```

```
[Out] 1/5*sqrt(5)*ln(abs(-sqrt(5) + 2*sqrt(x + 2) - 1)/abs(sqrt(5) + 2*
sqrt(x + 2) - 1)) + ln(abs(x - sqrt(x + 2) + 1))
```

3.541 $\int \frac{1}{4+x+\sqrt{1+x}}\,dx$

Optimal. Leaf size=37

$$\log\left(x+\sqrt{x+1}+4\right)-\frac{2\tan^{-1}\left(\frac{2\sqrt{x+1}+1}{\sqrt{11}}\right)}{\sqrt{11}}$$

```
[Out] (-2*ArcTan[(1 + 2*Sqrt[1 + x])/Sqrt[11]])/Sqrt[11] + Log[4 + x +
Sqrt[1 + x]]
```

Rubi [A] time = 0.0662781, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\log\left(x+\sqrt{x+1}+4\right)-\frac{2\tan^{-1}\left(\frac{2\sqrt{x+1}+1}{\sqrt{11}}\right)}{\sqrt{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x + Sqrt[1 + x])^(-1),x]
```

```
[Out] (-2*ArcTan[(1 + 2*Sqrt[1 + x])/Sqrt[11]])/Sqrt[11] + Log[4 + x +
Sqrt[1 + x]]
```

Rubi in Sympy [A] time = 2.86405, size = 39, normalized size = 1.05

$$\log\left(x+\sqrt{x+1}+4\right)-\frac{2\sqrt{11}\,\mathrm{atan}\left(\sqrt{11}\left(\frac{2\sqrt{x+1}}{11}+\frac{1}{11}\right)\right)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(4+x+(1+x)**(1/2)),x)
```

```
[Out] log(x + sqrt(x + 1) + 4) - 2*sqrt(11)*atan(sqrt(11)*(2*sqrt(x + 1
)/11 + 1/11))/11
```

Mathematica [A] time = 0.0211995, size = 37, normalized size = 1.

$$\log\left(x+\sqrt{x+1}+4\right)-\frac{2\tan^{-1}\left(\frac{2\sqrt{x+1}+1}{\sqrt{11}}\right)}{\sqrt{11}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(4 + x + Sqrt[1 + x])^(-1),x]
```

```
[Out] (-2*ArcTan[(1 + 2*Sqrt[1 + x])/Sqrt[11]])/Sqrt[11] + Log[4 + x +
Sqrt[1 + x]]
```

Maple [B] time = 0.018, size = 93, normalized size = 2.5

$$\frac{1}{2}\ln\left(4+x+\sqrt{1+x}\right)-\frac{\sqrt{11}}{11}\arctan\left(\frac{\sqrt{11}}{11}\left(1+2\sqrt{1+x}\right)\right)-\frac{1}{2}\ln\left(4+x-\sqrt{1+x}\right)-\frac{\sqrt{11}}{11}\arctan\left(\frac{\sqrt{11}}{11}\left(2\sqrt{1+x}-1\right)\right)+\frac{\sqrt{11}}{11}\arctan\left(\frac{(2x+7)\sqrt{11}}{11}\right)+\frac{\ln\left(x^2+7x+15\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(4+x+(1+x)^(1/2)),x)
```

```
[Out] 1/2*ln(4+x+(1+x)^(1/2))-1/11*arctan(1/11*(1+2*(1+x)^(1/2))*11^(1/
2))*11^(1/2)-1/2*ln(4+x-(1+x)^(1/2))-1/11*11^(1/2)*arctan(1/11*(2
*(1+x)^(1/2)-1)*11^(1/2))+1/11*11^(1/2)*arctan(1/11*(2*x+7)*11^(1
/2))+1/2*ln(x^2+7*x+15)
```

Maxima [A] time = 0.802132, size = 41, normalized size = 1.11

$$-\frac{2}{11}\sqrt{11}\arctan\left(\frac{1}{11}\sqrt{11}\left(2\sqrt{x+1}+1\right)\right)+\log\left(x+\sqrt{x+1}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x + sqrt(x + 1) + 4),x, algorithm="maxima")
```

```
[Out] -2/11*sqrt(11)*arctan(1/11*sqrt(11)*(2*sqrt(x + 1) + 1)) + log(x
+ sqrt(x + 1) + 4)
```

Fricas [A] time = 0.264478, size = 51, normalized size = 1.38

$$\frac{1}{11}\sqrt{11}\left(\sqrt{11}\log\left(x+\sqrt{x+1}+4\right)-2\arctan\left(\frac{2}{11}\sqrt{11}\sqrt{x+1}+\frac{1}{11}\sqrt{11}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x + sqrt(x + 1) + 4),x, algorithm="fricas")
```

```
[Out] 1/11*sqrt(11)*(sqrt(11)*log(x + sqrt(x + 1) + 4) - 2*arctan(2/11*
sqrt(11)*sqrt(x + 1) + 1/11*sqrt(11)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x+\sqrt{x+1}+4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(4+x+(1+x)**(1/2)),x)
```

```
[Out] Integral(1/(x + sqrt(x + 1) + 4), x)
```

GIAC/XCAS [A] time = 0.284902, size = 41, normalized size = 1.11

$$-\frac{2}{11}\sqrt{11}\arctan\left(\frac{1}{11}\sqrt{11}\left(2\sqrt{x+1}+1\right)\right)+\ln\left(x+\sqrt{x+1}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x + sqrt(x + 1) + 4),x, algorithm="giac")
```

```
[Out] -2/11*sqrt(11)*arctan(1/11*sqrt(11)*(2*sqrt(x + 1) + 1)) + ln(x +
 sqrt(x + 1) + 4)
```

3.542 $\int \frac{1}{x-\sqrt{1+x}} dx$

Optimal. Leaf size=61

$$\frac{1}{5}\left(5-\sqrt{5}\right)\log\left(-2\sqrt{x+1}-\sqrt{5}+1\right)+\frac{1}{5}\left(5+\sqrt{5}\right)\log\left(-2\sqrt{x+1}+\sqrt{5}+1\right)$$

```
[Out] ((5 - Sqrt[5])*Log[1 - Sqrt[5] - 2*Sqrt[1 + x]])/5 + ((5 + Sqrt[5
])*Log[1 + Sqrt[5] - 2*Sqrt[1 + x]])/5
```

Rubi [A] time = 0.0662003, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$\frac{1}{5}\left(5-\sqrt{5}\right)\log\left(-2\sqrt{x+1}-\sqrt{5}+1\right)+\frac{1}{5}\left(5+\sqrt{5}\right)\log\left(-2\sqrt{x+1}+\sqrt{5}+1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(x - Sqrt[1 + x])^(-1),x]
```

```
[Out] ((5 - Sqrt[5])*Log[1 - Sqrt[5] - 2*Sqrt[1 + x]])/5 + ((5 + Sqrt[5
])*Log[1 + Sqrt[5] - 2*Sqrt[1 + x]])/5
```

Rubi in Sympy [A] time = 2.70687, size = 70, normalized size = 1.15

$$\frac{2\sqrt{5}\left(\frac{1}{2}+\frac{\sqrt{5}}{2}\right)\log\left(-2\sqrt{x+1}+1+\sqrt{5}\right)}{5}-\frac{2\sqrt{5}\left(-\frac{\sqrt{5}}{2}+\frac{1}{2}\right)\log\left(-2\sqrt{x+1}-\sqrt{5}+1\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x-(1+x)**(1/2)),x)
```

```
[Out] 2*sqrt(5)*(1/2 + sqrt(5)/2)*log(-2*sqrt(x + 1) + 1 + sqrt(5))/5 -
 2*sqrt(5)*(-sqrt(5)/2 + 1/2)*log(-2*sqrt(x + 1) - sqrt(5) + 1)/5
```

Mathematica [A] time = 0.0190064, size = 38, normalized size = 0.62

$$\log\left(\sqrt{x+1}-x\right)-\frac{2\tanh^{-1}\left(\frac{2\sqrt{x+1}-1}{\sqrt{5}}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x - Sqrt[1 + x])^(-1),x]
```

```
[Out] (-2*ArcTanh[(-1 + 2*Sqrt[1 + x])/Sqrt[5]])/Sqrt[5] + Log[-x + Sqr
t[1 + x]]
```

Maple [A] time = 0.007, size = 91, normalized size = 1.5

$$\frac{\ln\left(x^2-x-1\right)}{2}-\frac{\sqrt{5}}{5}\mathit{Artanh}\left(\frac{(2x-1)\sqrt{5}}{5}\right)+\frac{1}{2}\ln\left(x-\sqrt{1+x}\right)-\frac{\sqrt{5}}{5}\mathit{Artanh}\left(\frac{\sqrt{5}}{5}\left(2\sqrt{1+x}-1\right)\right)-\frac{1}{2}\ln\left(x+\sqrt{1+x}\right)-\frac{\sqrt{5}}{5}\mathit{Artanh}\left(\frac{\sqrt{5}}{5}\left(1+2\sqrt{1+x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x-(1+x)^(1/2)),x)
```

```
[Out] 1/2*ln(x^2-x-1)-1/5*5^(1/2)*arctanh(1/5*(2*x-1)*5^(1/2))+1/2*ln(x
-(1+x)^(1/2))-1/5*5^(1/2)*arctanh(1/5*(2*(1+x)^(1/2)-1)*5^(1/2))-
1/2*ln(x+(1+x)^(1/2))-1/5*5^(1/2)*arctanh(1/5*(1+2*(1+x)^(1/2))*5
^(1/2))
```

Maxima [A] time = 0.794425, size = 61, normalized size = 1.

$$\frac{1}{5}\sqrt{5}\log\left(-\frac{\sqrt{5}-2\sqrt{x+1}+1}{\sqrt{5}+2\sqrt{x+1}-1}\right)+\log\left(x-\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x - sqrt(x + 1)),x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(5)*log(-(sqrt(5) - 2*sqrt(x + 1) + 1)/(sqrt(5) + 2*sqrt(
x + 1) - 1)) + log(x - sqrt(x + 1))
```

Fricas [A] time = 0.267254, size = 76, normalized size = 1.25

$$\frac{1}{5}\sqrt{5}\left(\sqrt{5}\log\left(x-\sqrt{x+1}\right)+\log\left(\frac{\sqrt{5}(2x+5)-2\sqrt{x+1}\left(\sqrt{5}+5\right)+5}{x-\sqrt{x+1}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x - sqrt(x + 1)),x, algorithm="fricas")
```

```
[Out] 1/5*sqrt(5)*(sqrt(5)*log(x - sqrt(x + 1)) + log((sqrt(5)*(2*x + 5
) - 2*sqrt(x + 1)*(sqrt(5) + 5) + 5)/(x - sqrt(x + 1))))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x - \sqrt{x + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x-(1+x)**(1/2)),x)
```

```
[Out] Integral(1/(x - sqrt(x + 1)), x)
```

GIAC/XCAS [A] time = 0.296464, size = 66, normalized size = 1.08

$$\frac{1}{5}\,\sqrt{5}\ln\left(\frac{\left|-\sqrt{5} + 2\,\sqrt{x + 1} - 1\right|}{\left|\sqrt{5} + 2\,\sqrt{x + 1} - 1\right|}\right) + \ln\left(\left|x - \sqrt{x + 1}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x - sqrt(x + 1)),x, algorithm="giac")
```

```
[Out] 1/5*sqrt(5)*ln(abs(-sqrt(5) + 2*sqrt(x + 1) - 1)/abs(sqrt(5) + 2*
sqrt(x + 1) - 1)) + ln(abs(x - sqrt(x + 1)))
```

3.543 $\int \frac{1}{x-\sqrt{2+x}}\,dx$

Optimal. Leaf size=31

$$\frac{4}{3}\log\left(2-\sqrt{x+2}\right)+\frac{2}{3}\log\left(\sqrt{x+2}+1\right)$$

```
[Out] (4*Log[2 - Sqrt[2 + x]])/3 + (2*Log[1 + Sqrt[2 + x]])/3
```

Rubi [A] time = 0.043598, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$\frac{4}{3}\log\left(2-\sqrt{x+2}\right)+\frac{2}{3}\log\left(\sqrt{x+2}+1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(x - Sqrt[2 + x])^(-1),x]

[Out] (4*Log[2 - Sqrt[2 + x]])/3 + (2*Log[1 + Sqrt[2 + x]])/3
```

Rubi in Sympy [A] time = 2.3903, size = 26, normalized size = 0.84

$$\frac{4\log\left(-\sqrt{x+2}+2\right)}{3}+\frac{2\log\left(\sqrt{x+2}+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x-(2+x)**(1/2)),x)

[Out] 4*log(-sqrt(x + 2) + 2)/3 + 2*log(sqrt(x + 2) + 1)/3
```

Mathematica [A] time = 0.00707898, size = 31, normalized size = 1.

$$\frac{4}{3}\log\left(2-\sqrt{x+2}\right)+\frac{2}{3}\log\left(\sqrt{x+2}+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x - Sqrt[2 + x])^(-1),x]
```

```
[Out] (4*Log[2 - Sqrt[2 + x]])/3 + (2*Log[1 + Sqrt[2 + x]])/3
```

Maple [B] time = 0.02, size = 54, normalized size = 1.7

$$\frac{\ln(1+x)}{3}+\frac{2\ln(x-2)}{3}-\frac{2}{3}\ln\left(\sqrt{2+x}+2\right)-\frac{1}{3}\ln\left(\sqrt{2+x}-1\right)+\frac{1}{3}\ln\left(1+\sqrt{2+x}\right)+\frac{2}{3}\ln\left(\sqrt{2+x}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x-(2+x)^(1/2)),x)
```

```
[Out] 1/3*ln(1+x)+2/3*ln(x-2)-2/3*ln((2+x)^(1/2)+2)-1/3*ln((2+x)^(1/2)-
1)+1/3*ln(1+(2+x)^(1/2))+2/3*ln((2+x)^(1/2)-2)
```

Maxima [A] time = 0.718476, size = 28, normalized size = 0.9

$$\frac{2}{3}\log\left(\sqrt{x+2}+1\right)+\frac{4}{3}\log\left(\sqrt{x+2}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x - sqrt(x + 2)),x, algorithm="maxima")
```

```
[Out] 2/3*log(sqrt(x + 2) + 1) + 4/3*log(sqrt(x + 2) - 2)
```

Fricas [A] time = 0.262678, size = 28, normalized size = 0.9

$$\frac{2}{3}\log\left(\sqrt{x+2}+1\right)+\frac{4}{3}\log\left(\sqrt{x+2}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x - sqrt(x + 2)),x, algorithm="fricas")
```

```
[Out] 2/3*log(sqrt(x + 2) + 1) + 4/3*log(sqrt(x + 2) - 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x - \sqrt{x + 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x-(2+x)**(1/2)),x)
```

```
[Out] Integral(1/(x - sqrt(x + 2)), x)
```

GIAC/XCAS [A] time = 0.278952, size = 30, normalized size = 0.97

$$\frac{2}{3}\ln\left(\sqrt{x + 2} + 1\right) + \frac{4}{3}\ln\left(\left|\sqrt{x + 2} - 2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x - sqrt(x + 2)),x, algorithm="giac")
```

```
[Out] 2/3*ln(sqrt(x + 2) + 1) + 4/3*ln(abs(sqrt(x + 2) - 2))
```

3.544 $\int \frac{1}{-\sqrt{1-x}+x} \, dx$

Optimal. Leaf size=65

$$\frac{1}{5}\left(5-\sqrt{5}\right)\log\left(2\sqrt{1-x}-\sqrt{5}+1\right)+\frac{1}{5}\left(5+\sqrt{5}\right)\log\left(2\sqrt{1-x}+\sqrt{5}+1\right)$$

```
[Out] ((5 - Sqrt[5])*Log[1 - Sqrt[5] + 2*Sqrt[1 - x]])/5 + ((5 + Sqrt[5
])*Log[1 + Sqrt[5] + 2*Sqrt[1 - x]])/5
```

Rubi [A] time = 0.073487, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{1}{5}\left(5-\sqrt{5}\right)\log\left(2\sqrt{1-x}-\sqrt{5}+1\right)+\frac{1}{5}\left(5+\sqrt{5}\right)\log\left(2\sqrt{1-x}+\sqrt{5}+1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(-Sqrt[1 - x] + x)^(-1),x]
```

```
[Out] ((5 - Sqrt[5])*Log[1 - Sqrt[5] + 2*Sqrt[1 - x]])/5 + ((5 + Sqrt[5
])*Log[1 + Sqrt[5] + 2*Sqrt[1 - x]])/5
```

Rubi in Sympy [A] time = 2.5539, size = 70, normalized size = 1.08

$$\frac{2\sqrt{5}\left(\frac{1}{2}+\frac{\sqrt{5}}{2}\right)\log\left(2\sqrt{-x+1}+1+\sqrt{5}\right)}{5}-\frac{2\sqrt{5}\left(-\frac{\sqrt{5}}{2}+\frac{1}{2}\right)\log\left(2\sqrt{-x+1}-\sqrt{5}+1\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x-(1-x)**(1/2)),x)
```

```
[Out] 2*sqrt(5)*(1/2 + sqrt(5)/2)*log(2*sqrt(-x + 1) + 1 + sqrt(5))/5 -
 2*sqrt(5)*(-sqrt(5)/2 + 1/2)*log(2*sqrt(-x + 1) - sqrt(5) + 1)/5
```

Mathematica [A] time = 0.0234023, size = 42, normalized size = 0.65

$$\log\left(x-\sqrt{1-x}\right)+\frac{2\tanh^{-1}\left(\frac{2\sqrt{1-x}+1}{\sqrt{5}}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-Sqrt[1 - x] + x)^(-1),x]
```

```
[Out] (2*ArcTanh[(1 + 2*Sqrt[1 - x])/Sqrt[5]])/Sqrt[5] + Log[-Sqrt[1 -
x] + x]
```

Maple [B] time = 0.006, size = 101, normalized size = 1.6

$$\frac{\ln\left(x^2+x-1\right)}{2}+\frac{\sqrt{5}}{5}Artanh\left(\frac{(1+2x)\sqrt{5}}{5}\right)-\frac{1}{2}\ln\left(-x-\sqrt{1-x}\right)+\frac{\sqrt{5}}{5}Artanh\left(\frac{\sqrt{5}}{5}\left(2\sqrt{1-x}-1\right)\right)+\frac{1}{2}\ln\left(-x+\sqrt{1-x}\right)+\frac{\sqrt{5}}{5}Artanh\left(\frac{\sqrt{5}}{5}\left(2\sqrt{1-x}+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x-(1-x)^(1/2)),x)
```

```
[Out] 1/2*ln(x^2+x-1)+1/5*5^(1/2)*arctanh(1/5*(1+2*x)*5^(1/2))-1/2*ln(-
x-(1-x)^(1/2))+1/5*5^(1/2)*arctanh(1/5*(2*(1-x)^(1/2)-1)*5^(1/2))
+1/2*ln(-x+(1-x)^(1/2))+1/5*5^(1/2)*arctanh(1/5*(2*(1-x)^(1/2)+1)
*5^(1/2))
```

Maxima [A] time = 0.799206, size = 69, normalized size = 1.06

$$-\frac{1}{5}\sqrt{5}\log\left(-\frac{\sqrt{5}-2\sqrt{-x+1}-1}{\sqrt{5}+2\sqrt{-x+1}+1}\right)+\log\left(-x+\sqrt{-x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x - sqrt(-x + 1)),x, algorithm="maxima")
```

```
[Out] -1/5*sqrt(5)*log(-(sqrt(5) - 2*sqrt(-x + 1) - 1)/(sqrt(5) + 2*sqr
t(-x + 1) + 1)) + log(-x + sqrt(-x + 1))
```

Fricas [A] time = 0.270345, size = 84, normalized size = 1.29

$$\frac{1}{5}\sqrt{5}\left(\sqrt{5}\log\left(-x+\sqrt{-x+1}\right)+\log\left(\frac{\sqrt{5}(2x-5)-2\sqrt{-x+1}\left(\sqrt{5}+5\right)-5}{x-\sqrt{-x+1}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x - sqrt(-x + 1)),x, algorithm="fricas")
```

```
[Out] 1/5*sqrt(5)*(sqrt(5)*log(-x + sqrt(-x + 1)) + log((sqrt(5)*(2*x -
 5) - 2*sqrt(-x + 1)*(sqrt(5) + 5) - 5)/(x - sqrt(-x + 1))))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x - \sqrt{-x + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x-(1-x)**(1/2)),x)
```

```
[Out] Integral(1/(x - sqrt(-x + 1)), x)
```

GIAC/XCAS [A] time = 0.293559, size = 73, normalized size = 1.12

$$-\frac{1}{5}\sqrt{5}\ln\left(\frac{\left|-\sqrt{5} + 2\sqrt{-x + 1} + 1\right|}{\sqrt{5} + 2\sqrt{-x + 1} + 1}\right) + \ln\left(\left|-x + \sqrt{-x + 1}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x - sqrt(-x + 1)),x, algorithm="giac")
```

```
[Out] -1/5*sqrt(5)*ln(abs(-sqrt(5) + 2*sqrt(-x + 1) + 1)/(sqrt(5) + 2*s
qrt(-x + 1) + 1)) + ln(abs(-x + sqrt(-x + 1)))
```

3.545 $\int \sqrt{1+\sqrt{x}+x}\,dx$

Optimal. Leaf size=62

$$\frac{2}{3}\left(x+\sqrt{x}+1\right)^{3/2}-\frac{1}{4}\left(2\sqrt{x}+1\right)\sqrt{x+\sqrt{x}+1}-\frac{3}{8}\sinh^{-1}\left(\frac{2\sqrt{x}+1}{\sqrt{3}}\right)$$

```
[Out] -((1 + 2*Sqrt[x])*Sqrt[1 + Sqrt[x] + x])/4 + (2*(1 + Sqrt[x] + x)
^(3/2))/3 - (3*ArcSinh[(1 + 2*Sqrt[x])/Sqrt[3]])/8
```

Rubi [A] time = 0.0524468, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417

$$\frac{2}{3}\left(x+\sqrt{x}+1\right)^{3/2}-\frac{1}{4}\left(2\sqrt{x}+1\right)\sqrt{x+\sqrt{x}+1}-\frac{3}{8}\sinh^{-1}\left(\frac{2\sqrt{x}+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 + Sqrt[x] + x],x]
```

```
[Out] -((1 + 2*Sqrt[x])*Sqrt[1 + Sqrt[x] + x])/4 + (2*(1 + Sqrt[x] + x)
^(3/2))/3 - (3*ArcSinh[(1 + 2*Sqrt[x])/Sqrt[3]])/8
```

Rubi in Sympy [A] time = 2.05082, size = 63, normalized size = 1.02

$$-\frac{\left(2\sqrt{x}+1\right)\sqrt{\sqrt{x}+x+1}}{4}+\frac{2\left(\sqrt{x}+x+1\right)^{\frac{3}{2}}}{3}-\frac{3\,\mathrm{atanh}\left(\frac{2\sqrt{x}+1}{2\sqrt{\sqrt{x}+x+1}}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+x+x**(1/2))**(1/2),x)
```

```
[Out] -(2*sqrt(x) + 1)*sqrt(sqrt(x) + x + 1)/4 + 2*(sqrt(x) + x + 1)**(
3/2)/3 - 3*atanh((2*sqrt(x) + 1)/(2*sqrt(sqrt(x) + x + 1)))/8
```

Mathematica [A] time = 0.0303024, size = 49, normalized size = 0.79

$$\frac{1}{12}\sqrt{x+\sqrt{x}+1}\left(8x+2\sqrt{x}+5\right)-\frac{3}{8}\sinh^{-1}\left(\frac{2\sqrt{x}+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 + Sqrt[x] + x],x]
```

```
[Out] (Sqrt[1 + Sqrt[x] + x]*(5 + 2*Sqrt[x] + 8*x))/12 - (3*ArcSinh[(1
+ 2*Sqrt[x])/Sqrt[3]])/8
```

Maple [A] time = 0.008, size = 42, normalized size = 0.7

$$\frac{2}{3}\left(1+x+\sqrt{x}\right)^{\frac{3}{2}}-\frac{1}{4}\left(1+2\sqrt{x}\right)\sqrt{1+x+\sqrt{x}}-\frac{3}{8}Arcsinh\left(\frac{2\sqrt{3}}{3}\left(\sqrt{x}+\frac{1}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+x+x^(1/2))^(1/2),x)
```

```
[Out] 2/3*(1+x+x^(1/2))^(3/2)-1/4*(1+2*x^(1/2))*(1+x+x^(1/2))^(1/2)-3/8
*arcsinh(2/3*3^(1/2)*(x^(1/2)+1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{x+\sqrt{x}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x + sqrt(x) + 1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x + sqrt(x) + 1), x)
```

Fricas [A] time = 0.555843, size = 69, normalized size = 1.11

$$\frac{1}{12}\left(8x+2\sqrt{x}+5\right)\sqrt{x+\sqrt{x}+1}+\frac{3}{16}\log\left(4\sqrt{x+\sqrt{x}+1}\left(2\sqrt{x}+1\right)-8x-8\sqrt{x}-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x + sqrt(x) + 1),x, algorithm="fricas")
```

```
[Out] 1/12*(8*x + 2*sqrt(x) + 5)*sqrt(x + sqrt(x) + 1) + 3/16*log(4*sqr
t(x + sqrt(x) + 1)*(2*sqrt(x) + 1) - 8*x - 8*sqrt(x) - 5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sqrt{x} + x + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+x+x**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(sqrt(x) + x + 1), x)
```

GIAC/XCAS [A] time = 0.287548, size = 61, normalized size = 0.98

$$\frac{1}{12}\left(2\sqrt{x}\left(4\sqrt{x}+1\right)+5\right)\sqrt{x+\sqrt{x}+1}+\frac{3}{8}\ln\left(2\sqrt{x+\sqrt{x}+1}-2\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x + sqrt(x) + 1),x, algorithm="giac")
```

```
[Out] 1/12*(2*sqrt(x)*(4*sqrt(x) + 1) + 5)*sqrt(x + sqrt(x) + 1) + 3/8*
ln(2*sqrt(x + sqrt(x) + 1) - 2*sqrt(x) - 1)
```

3.546 $\int \sqrt{1+x+\sqrt{1+x}}\,dx$

Optimal. Leaf size=75

$$\frac{2}{3}\left(x+\sqrt{x+1}+1\right)^{3/2}-\frac{1}{4}\left(2\sqrt{x+1}+1\right)\sqrt{x+\sqrt{x+1}+1}+\frac{1}{4}\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{x+\sqrt{x+1}+1}}\right)$$

```
[Out] (2*(1 + x + Sqrt[1 + x])^(3/2))/3 - (Sqrt[1 + x + Sqrt[1 + x]]*(1
 + 2*Sqrt[1 + x]))/4 + ArcTanh[Sqrt[1 + x]/Sqrt[1 + x + Sqrt[1 +
x]]]/4
```

Rubi [A] time = 0.0822884, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357

$$\frac{2}{3}\left(x+\sqrt{x+1}+1\right)^{3/2}-\frac{1}{4}\left(2\sqrt{x+1}+1\right)\sqrt{x+\sqrt{x+1}+1}+\frac{1}{4}\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{x+\sqrt{x+1}+1}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 + x + Sqrt[1 + x]],x]
```

```
[Out] (2*(1 + x + Sqrt[1 + x])^(3/2))/3 - (Sqrt[1 + x + Sqrt[1 + x]]*(1
 + 2*Sqrt[1 + x]))/4 + ArcTanh[Sqrt[1 + x]/Sqrt[1 + x + Sqrt[1 +
x]]]/4
```

Rubi in Sympy [A] time = 2.68395, size = 65, normalized size = 0.87

$$-\frac{\left(2\sqrt{x+1}+1\right)\sqrt{x+\sqrt{x+1}+1}}{4}+\frac{2\left(x+\sqrt{x+1}+1\right)^{\frac{3}{2}}}{3}+\frac{\operatorname{atanh}\left(\frac{\sqrt{x+1}}{\sqrt{x+\sqrt{x+1}+1}}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+x+(1+x)**(1/2))**(1/2),x)
```

```
[Out] -(2*sqrt(x + 1) + 1)*sqrt(x + sqrt(x + 1) + 1)/4 + 2*(x + sqrt(x
+ 1) + 1)**(3/2)/3 + atanh(sqrt(x + 1)/sqrt(x + sqrt(x + 1) + 1))
/4
```

Mathematica [A] time = 0.0514878, size = 65, normalized size = 0.87

$$\frac{1}{24}\left(2\sqrt{x+\sqrt{x+1}+1}\left(8x+2\sqrt{x+1}+5\right)+3\log\left(2\sqrt{x+1}+2\sqrt{x+\sqrt{x+1}+1}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 + x + Sqrt[1 + x]],x]
```

```
[Out] (2*Sqrt[1 + x + Sqrt[1 + x]]*(5 + 8*x + 2*Sqrt[1 + x]) + 3*Log[1
+ 2*Sqrt[1 + x] + 2*Sqrt[1 + x + Sqrt[1 + x]]])/24
```

Maple [A] time = 0.007, size = 55, normalized size = 0.7

$$\frac{2}{3}\left(1+x+\sqrt{1+x}\right)^{\frac{3}{2}}-\frac{1}{4}\left(1+2\sqrt{1+x}\right)\sqrt{1+x+\sqrt{1+x}}+\frac{1}{8}\ln\left(\frac{1}{2}+\sqrt{1+x}+\sqrt{1+x+\sqrt{1+x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+x+(1+x)^(1/2))^(1/2),x)
```

```
[Out] 2/3*(1+x+(1+x)^(1/2))^(3/2)-1/4*(1+2*(1+x)^(1/2))*(1+x+(1+x)^(1/2
))^(1/2)+1/8*ln(1/2+(1+x)^(1/2)+(1+x+(1+x)^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{x+\sqrt{x+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x + sqrt(x + 1) + 1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x + sqrt(x + 1) + 1), x)
```

Fricas [A] time = 0.554688, size = 82, normalized size = 1.09

$$\frac{1}{12}\left(8x+2\sqrt{x+1}+5\right)\sqrt{x+\sqrt{x+1}+1}+\frac{1}{16}\log\left(-4\sqrt{x+\sqrt{x+1}+1}\left(2\sqrt{x+1}+1\right)-8x-8\sqrt{x+1}-9\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x + sqrt(x + 1) + 1),x, algorithm="fricas")
```

```
[Out] 1/12*(8*x + 2*sqrt(x + 1) + 5)*sqrt(x + sqrt(x + 1) + 1) + 1/16*l
og(-4*sqrt(x + sqrt(x + 1) + 1)*(2*sqrt(x + 1) + 1) - 8*x - 8*sqr
t(x + 1) - 9)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x + \sqrt{x + 1} + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+x+(1+x)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(x + sqrt(x + 1) + 1), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x + sqrt(x + 1) + 1),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.547 $\int \sqrt{\sqrt{-1+x}+x}\,dx$

Optimal. Leaf size=68

$$\frac{2}{3}\left(x+\sqrt{x-1}\right)^{3/2}-\frac{1}{4}\left(2\sqrt{x-1}+1\right)\sqrt{x+\sqrt{x-1}}-\frac{3}{8}\sinh^{-1}\left(\frac{2\sqrt{x-1}+1}{\sqrt{3}}\right)$$

```
[Out] -((1 + 2*Sqrt[-1 + x])*Sqrt[Sqrt[-1 + x] + x])/4 + (2*(Sqrt[-1 +
x] + x)^(3/2))/3 - (3*ArcSinh[(1 + 2*Sqrt[-1 + x])/Sqrt[3]])/8
```

Rubi [A] time = 0.0772413, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$

$$\frac{2}{3}\left(x+\sqrt{x-1}\right)^{3/2}-\frac{1}{4}\left(2\sqrt{x-1}+1\right)\sqrt{x+\sqrt{x-1}}-\frac{3}{8}\sinh^{-1}\left(\frac{2\sqrt{x-1}+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[Sqrt[-1 + x] + x],x]
```

```
[Out] -((1 + 2*Sqrt[-1 + x])*Sqrt[Sqrt[-1 + x] + x])/4 + (2*(Sqrt[-1 +
x] + x)^(3/2))/3 - (3*ArcSinh[(1 + 2*Sqrt[-1 + x])/Sqrt[3]])/8
```

Rubi in Sympy [A] time = 2.52094, size = 66, normalized size = 0.97

$$\frac{2\left(x+\sqrt{x-1}\right)^{\frac{3}{2}}}{3}-\frac{\sqrt{x+\sqrt{x-1}}\left(2\sqrt{x-1}+1\right)}{4}-\frac{3\,\mathrm{atanh}\left(\frac{2\sqrt{x-1}+1}{2\sqrt{x+\sqrt{x-1}}}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x+(-1+x)**(1/2))**(1/2),x)
```

```
[Out] 2*(x + sqrt(x - 1))**(3/2)/3 - sqrt(x + sqrt(x - 1))*(2*sqrt(x -
1) + 1)/4 - 3*atanh((2*sqrt(x - 1) + 1)/(2*sqrt(x + sqrt(x - 1)))
)/8
```

Mathematica [A] time = 0.0365456, size = 54, normalized size = 0.79

$$\frac{1}{12}\sqrt{x+\sqrt{x-1}}\left(8x+2\sqrt{x-1}-3\right)-\frac{3}{8}\sinh^{-1}\left(\frac{2\sqrt{x-1}+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[Sqrt[-1 + x] + x],x]
```

```
[Out] (Sqrt[Sqrt[-1 + x] + x]*(-3 + 2*Sqrt[-1 + x] + 8*x))/12 - (3*ArcS
inh[(1 + 2*Sqrt[-1 + x])/Sqrt[3]])/8
```

Maple [A] time = 0.008, size = 48, normalized size = 0.7

$$\frac{2}{3}\left(x+\sqrt{-1+x}\right)^{\frac{3}{2}}-\frac{1}{4}\left(1+2\sqrt{-1+x}\right)\sqrt{x+\sqrt{-1+x}}-\frac{3}{8}Arcsinh\left(\frac{2\sqrt{3}}{3}\left(\sqrt{-1+x}+\frac{1}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x+(-1+x)^(1/2))^(1/2),x)
```

```
[Out] 2/3*(x+(-1+x)^(1/2))^(3/2)-1/4*(1+2*(-1+x)^(1/2))*(x+(-1+x)^(1/2)
)^(1/2)-3/8*arcsinh(2/3*3^(1/2)*((-1+x)^(1/2)+1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{x+\sqrt{x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x + sqrt(x - 1)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x + sqrt(x - 1)), x)
```

Fricas [A] time = 0.606246, size = 80, normalized size = 1.18

$$\frac{1}{12}\left(8\,x+2\,\sqrt{x-1}-3\right)\sqrt{x+\sqrt{x-1}}+\frac{3}{16}\,\log\left(-4\,\sqrt{x+\sqrt{x-1}}\left(2\,\sqrt{x-1}+1\right)+8\,x+8\,\sqrt{x-1}-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x + sqrt(x - 1)),x, algorithm="fricas")
```

```
[Out] 1/12*(8*x + 2*sqrt(x - 1) - 3)*sqrt(x + sqrt(x - 1)) + 3/16*log(-
4*sqrt(x + sqrt(x - 1))*(2*sqrt(x - 1) + 1) + 8*x + 8*sqrt(x - 1)
 - 3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x+\sqrt{x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x+(-1+x)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(x + sqrt(x - 1)), x)
```

GIAC/XCAS [A] time = 0.3034, size = 72, normalized size = 1.06

$$\frac{1}{12}\left(2\sqrt{x-1}\left(4\sqrt{x-1}+1\right)+5\right)\sqrt{x+\sqrt{x-1}}+\frac{3}{8}\ln\left(2\sqrt{x+\sqrt{x-1}}-2\sqrt{x-1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x + sqrt(x - 1)),x, algorithm="giac")
```

```
[Out] 1/12*(2*sqrt(x - 1)*(4*sqrt(x - 1) + 1) + 5)*sqrt(x + sqrt(x - 1)
) + 3/8*ln(2*sqrt(x + sqrt(x - 1)) - 2*sqrt(x - 1) - 1)
```

3.548 $\int \sqrt{2x + \sqrt{-1 + 2x}}\, dx$

Optimal. Leaf size=80

$$\frac{1}{3}\left(2x+\sqrt{2x-1}\right)^{3/2}-\frac{1}{8}\left(2\sqrt{2x-1}+1\right)\sqrt{2x+\sqrt{2x-1}}-\frac{3}{16}\sinh^{-1}\left(\frac{2\sqrt{2x-1}+1}{\sqrt{3}}\right)$$

```
[Out] (2*x + Sqrt[-1 + 2*x])^(3/2)/3 - (Sqrt[2*x + Sqrt[-1 + 2*x]]*(1 +
 2*Sqrt[-1 + 2*x]))/8 - (3*ArcSinh[(1 + 2*Sqrt[-1 + 2*x])/Sqrt[3]
])/16
```

Rubi [A] time = 0.0786265, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$

$$\frac{1}{3}\left(2x+\sqrt{2x-1}\right)^{3/2}-\frac{1}{8}\left(2\sqrt{2x-1}+1\right)\sqrt{2x+\sqrt{2x-1}}-\frac{3}{16}\sinh^{-1}\left(\frac{2\sqrt{2x-1}+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[2*x + Sqrt[-1 + 2*x]],x]
```

```
[Out] (2*x + Sqrt[-1 + 2*x])^(3/2)/3 - (Sqrt[2*x + Sqrt[-1 + 2*x]]*(1 +
 2*Sqrt[-1 + 2*x]))/8 - (3*ArcSinh[(1 + 2*Sqrt[-1 + 2*x])/Sqrt[3]
])/16
```

Rubi in Sympy [A] time = 2.52634, size = 78, normalized size = 0.98

$$\frac{\left(2x+\sqrt{2x-1}\right)^{\frac{3}{2}}}{3}-\frac{\sqrt{2x+\sqrt{2x-1}}\left(2\sqrt{2x-1}+1\right)}{8}-\frac{3\,\text{atanh}\left(\frac{2\sqrt{2x-1}+1}{2\sqrt{2x+\sqrt{2x-1}}}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*x+(-1+2*x)**(1/2))**(1/2),x)
```

```
[Out] (2*x + sqrt(2*x - 1))**(3/2)/3 - sqrt(2*x + sqrt(2*x - 1))*(2*sqr
t(2*x - 1) + 1)/8 - 3*atanh((2*sqrt(2*x - 1) + 1)/(2*sqrt(2*x + s
qrt(2*x - 1))))/16
```

Mathematica [A] time = 0.055038, size = 62, normalized size = 0.78

$$\frac{1}{48}\left(2\sqrt{2x+\sqrt{2x-1}}\left(16x+2\sqrt{2x-1}-3\right)-9\sinh^{-1}\left(\frac{2\sqrt{2x-1}+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[2*x + Sqrt[-1 + 2*x]],x]
```

```
[Out] (2*Sqrt[2*x + Sqrt[-1 + 2*x]]*(-3 + 16*x + 2*Sqrt[-1 + 2*x]) - 9*
ArcSinh[(1 + 2*Sqrt[-1 + 2*x])/Sqrt[3]])/48
```

Maple [A] time = 0.009, size = 60, normalized size = 0.8

$$\frac{1}{3}\left(2x+\sqrt{2x-1}\right)^{\frac{3}{2}}-\frac{1}{8}\left(1+2\sqrt{2x-1}\right)\sqrt{2x+\sqrt{2x-1}}-\frac{3}{16}Arcsinh\left(\frac{2\sqrt{3}}{3}\left(\sqrt{2x-1}+\frac{1}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*x+(2*x-1)^(1/2))^(1/2),x)
```

```
[Out] 1/3*(2*x+(2*x-1)^(1/2))^(3/2)-1/8*(1+2*(2*x-1)^(1/2))*(2*x+(2*x-1
)^(1/2))^(1/2)-3/16*arcsinh(2/3*3^(1/2)*((2*x-1)^(1/2)+1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{2x+\sqrt{2x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(2*x + sqrt(2*x - 1)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(2*x + sqrt(2*x - 1)), x)
```

Fricas [A] time = 0.574076, size = 99, normalized size = 1.24

$$\frac{1}{24}\left(16x+2\sqrt{2x-1}-3\right)\sqrt{2x+\sqrt{2x-1}}$$
$$+\frac{3}{32}\log\left(-4\sqrt{2x+\sqrt{2x-1}}\left(2\sqrt{2x-1}+1\right)+16x+8\sqrt{2x-1}-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(2*x + sqrt(2*x - 1)),x, algorithm="fricas")
```

```
[Out] 1/24*(16*x + 2*sqrt(2*x - 1) - 3)*sqrt(2*x + sqrt(2*x - 1)) + 3/3
2*log(-4*sqrt(2*x + sqrt(2*x - 1))*(2*sqrt(2*x - 1) + 1) + 16*x +
 8*sqrt(2*x - 1) - 3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{2x + \sqrt{2x - 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x+(-1+2*x)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(2*x + sqrt(2*x - 1)), x)
```

GIAC/XCAS [A] time = 0.287032, size = 92, normalized size = 1.15

$$\frac{1}{24}\left(2\sqrt{2x-1}\left(4\sqrt{2x-1}+1\right)+5\right)\sqrt{2x+\sqrt{2x-1}}+\frac{3}{16}\ln\left(2\sqrt{2x+\sqrt{2x-1}}-2\sqrt{2x-1}-1\right)-\frac{5}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(2*x + sqrt(2*x - 1)),x, algorithm="giac")
```

```
[Out] 1/24*(2*sqrt(2*x - 1)*(4*sqrt(2*x - 1) + 1) + 5)*sqrt(2*x + sqrt(
2*x - 1)) + 3/16*ln(2*sqrt(2*x + sqrt(2*x - 1)) - 2*sqrt(2*x - 1)
 - 1) - 5/24
```

3.549 $\int \sqrt{3x+\sqrt{-7+8x}}\,dx$

Optimal. Leaf size=109

$$\frac{\left(-3(7-8x)+8\sqrt{8x-7}+21\right)^{3/2}}{72\sqrt{2}}-\frac{\left(3\sqrt{8x-7}+4\right)\sqrt{-3(7-8x)+8\sqrt{8x-7}+21}}{36\sqrt{2}}-\frac{47\sinh^{-1}\left(\frac{3\sqrt{8x-7}+4}{\sqrt{47}}\right)}{36\sqrt{6}}$$

```
[Out] -((4 + 3*Sqrt[-7 + 8*x])*Sqrt[21 - 3*(7 - 8*x) + 8*Sqrt[-7 + 8*x]
])/(36*Sqrt[2]) + (21 - 3*(7 - 8*x) + 8*Sqrt[-7 + 8*x])^(3/2)/(72
*Sqrt[2]) - (47*ArcSinh[(4 + 3*Sqrt[-7 + 8*x])/Sqrt[47]])/(36*Sqr
t[6])
```

Rubi [A] time = 0.135887, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235

$$\frac{\left(-3(7-8x)+8\sqrt{8x-7}+21\right)^{3/2}}{72\sqrt{2}}-\frac{\left(3\sqrt{8x-7}+4\right)\sqrt{-3(7-8x)+8\sqrt{8x-7}+21}}{36\sqrt{2}}-\frac{47\sinh^{-1}\left(\frac{3\sqrt{8x-7}+4}{\sqrt{47}}\right)}{36\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[3*x + Sqrt[-7 + 8*x]],x]
```

```
[Out] -((4 + 3*Sqrt[-7 + 8*x])*Sqrt[21 - 3*(7 - 8*x) + 8*Sqrt[-7 + 8*x]
])/(36*Sqrt[2]) + (21 - 3*(7 - 8*x) + 8*Sqrt[-7 + 8*x])^(3/2)/(72
*Sqrt[2]) - (47*ArcSinh[(4 + 3*Sqrt[-7 + 8*x])/Sqrt[47]])/(36*Sqr
t[6])
```

Rubi in Sympy [A] time = 3.49236, size = 94, normalized size = 0.86

$$\frac{\left(48x+16\sqrt{8x-7}\right)^{\frac{3}{2}}}{288}-\frac{\sqrt{48x+16\sqrt{8x-7}}\left(12\sqrt{8x-7}+16\right)}{288}-\frac{47\sqrt{6}\,\mathrm{atanh}\left(\frac{\sqrt{6}\left(12\sqrt{8x-7}+16\right)}{12\sqrt{48x+16\sqrt{8x-7}}}\right)}{216}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x+(-7+8*x)**(1/2))**(1/2),x)
```

```
[Out] (48*x + 16*sqrt(8*x - 7))**(3/2)/288 - sqrt(48*x + 16*sqrt(8*x -
7))*(12*sqrt(8*x - 7) + 16)/288 - 47*sqrt(6)*atanh(sqrt(6)*(12*sq
```

```
rt(8*x - 7) + 16)/(12*sqrt(48*x + 16*sqrt(8*x - 7))))/216
```

Mathematica [A] time = 0.0899018, size = 65, normalized size = 0.6

$$\frac{1}{18}\sqrt{3x+\sqrt{8x-7}}\left(12x+\sqrt{8x-7}-4\right)-\frac{47\sinh^{-1}\left(\frac{3\sqrt{8x-7}+4}{\sqrt{47}}\right)}{36\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[3*x + Sqrt[-7 + 8*x]],x]
```

```
[Out] (Sqrt[3*x + Sqrt[-7 + 8*x]]*(-4 + 12*x + Sqrt[-7 + 8*x]))/18 - (4
7*ArcSinh[(4 + 3*Sqrt[-7 + 8*x])/Sqrt[47]])/(36*Sqrt[6])
```

Maple [A] time = 0.011, size = 67, normalized size = 0.6

$$\frac{1}{288}\left(48\,x+16\,\sqrt{-7+8\,x}\right)^{\frac{3}{2}}-\frac{1}{288}\left(12\,\sqrt{-7+8\,x}+16\right)\sqrt{48\,x+16\,\sqrt{-7+8\,x}}-\frac{47\,\sqrt{6}}{216}\mathit{Arcsinh}\left(\frac{3\,\sqrt{47}}{47}\left(\sqrt{-7+8\,x}+\frac{4}{3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x+(-7+8*x)^(1/2))^(1/2),x)
```

```
[Out] 1/288*(48*x+16*(-7+8*x)^(1/2))^(3/2)-1/288*(12*(-7+8*x)^(1/2)+16)
*(48*x+16*(-7+8*x)^(1/2))^(1/2)-47/216*6^(1/2)*arcsinh(3/47*47^(1
/2)*((-7+8*x)^(1/2)+4/3))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{3\,x+\sqrt{8\,x-7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(3*x + sqrt(8*x - 7)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(3*x + sqrt(8*x - 7)), x)
```

Fricas [A] time = 1.32728, size = 153, normalized size = 1.4

$$\frac{1}{864}\sqrt{6}\left(8\left(4\sqrt{6}(3x-1)+\sqrt{6}\sqrt{8x-7}\right)\sqrt{3x+\sqrt{8x-7}}+47\log\left(-192\sqrt{6}(144x-47)\sqrt{8x-7}-\sqrt{6}\left(41472x^2+9792x-\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(3*x + sqrt(8*x - 7)),x, algorithm="fricas")
```

```
[Out] 1/864*sqrt(6)*(8*(4*sqrt(6)*(3*x - 1) + sqrt(6)*sqrt(8*x - 7))*sq
rt(3*x + sqrt(8*x - 7)) + 47*log(-192*sqrt(6)*(144*x - 47)*sqrt(8
*x - 7) - sqrt(6)*(41472*x^2 + 9792*x - 30047) + 48*(3*(144*x + 1
7)*sqrt(8*x - 7) + 1728*x - 1196)*sqrt(3*x + sqrt(8*x - 7))))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{3x+\sqrt{8x-7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x+(-7+8*x)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(3*x + sqrt(8*x - 7)), x)
```

GIAC/XCAS [A] time = 0.298221, size = 174, normalized size = 1.6

$$\frac{1}{72}\sqrt{2}\left(\left(3\sqrt{2}\sqrt{8x-7}+2\sqrt{2}\right)\sqrt{8x-7}+13\sqrt{2}\right)\sqrt{3x+\sqrt{8x-7}} + \frac{47}{216}\sqrt{3}\sqrt{2}\ln\left(-\sqrt{3}\left(\sqrt{3}\sqrt{8x-7}-2\sqrt{2}\sqrt{3x+\sqrt{8x-7}}\right)-4\right) - \frac{1}{432}\sqrt{3}\left(13\sqrt{21}\sqrt{3}\sqrt{2}+94\sqrt{2}\ln\left(\sqrt{21}\sqrt{3}-4\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(3*x + sqrt(8*x - 7)),x, algorithm="giac")
```

```
[Out] 1/72*sqrt(2)*((3*sqrt(2)*sqrt(8*x - 7) + 2*sqrt(2))*sqrt(8*x - 7)
 + 13*sqrt(2))*sqrt(3*x + sqrt(8*x - 7)) + 47/216*sqrt(3)*sqrt(2)
*ln(-sqrt(3)*(sqrt(3)*sqrt(8*x - 7) - 2*sqrt(2)*sqrt(3*x + sqrt(8
*x - 7))) - 4) - 1/432*sqrt(3)*(13*sqrt(21)*sqrt(3)*sqrt(2) + 94*
sqrt(2)*ln(sqrt(21)*sqrt(3) - 4))
```

3.550 $\int \frac{1}{\sqrt{x+\sqrt{1+x}}}\,dx$

Optimal. Leaf size=47

$$2\sqrt{x+\sqrt{x+1}}-\tanh^{-1}\left(\frac{2\sqrt{x+1}+1}{2\sqrt{x+\sqrt{x+1}}}\right)$$

```
[Out] 2*Sqrt[x + Sqrt[1 + x]] - ArcTanh[(1 + 2*Sqrt[1 + x])/(2*Sqrt[x +
 Sqrt[1 + x]])]
```

Rubi [A] time = 0.0576661, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$2\sqrt{x+\sqrt{x+1}}-\tanh^{-1}\left(\frac{2\sqrt{x+1}+1}{2\sqrt{x+\sqrt{x+1}}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[x + Sqrt[1 + x]],x]
```

```
[Out] 2*Sqrt[x + Sqrt[1 + x]] - ArcTanh[(1 + 2*Sqrt[1 + x])/(2*Sqrt[x +
 Sqrt[1 + x]])]
```

Rubi in Sympy [A] time = 2.2801, size = 37, normalized size = 0.79

$$2\sqrt{x+\sqrt{x+1}}-\operatorname{atanh}\left(\frac{2\sqrt{x+1}+1}{2\sqrt{x+\sqrt{x+1}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x+(1+x)**(1/2))**(1/2),x)
```

```
[Out] 2*sqrt(x + sqrt(x + 1)) - atanh((2*sqrt(x + 1) + 1)/(2*sqrt(x + s
qrt(x + 1))))
```

Mathematica [A] time = 0.0217176, size = 45, normalized size = 0.96

$$2\sqrt{x+\sqrt{x+1}}-\log\left(2\sqrt{x+1}+2\sqrt{x+\sqrt{x+1}}+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[x + Sqrt[1 + x]],x]
```

```
[Out] 2*Sqrt[x + Sqrt[1 + x]] - Log[1 + 2*Sqrt[1 + x] + 2*Sqrt[x + Sqrt
[1 + x]]]
```

Maple [A]   time = 0.012, size = 32, normalized size = 0.7

$$2\sqrt{x+\sqrt{1+x}}-\ln\left(\sqrt{1+x}+\frac{1}{2}+\sqrt{x+\sqrt{1+x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x+(1+x)^(1/2))^(1/2),x)
```

```
[Out] 2*(x+(1+x)^(1/2))^(1/2)-ln((1+x)^(1/2)+1/2+(x+(1+x)^(1/2))^(1/2))
```

Maxima [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x+\sqrt{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(x + sqrt(x + 1)),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(x + sqrt(x + 1)), x)
```

Fricas [A]   time = 0.476033, size = 63, normalized size = 1.34

$$2\sqrt{x+\sqrt{x+1}}+\frac{1}{2}\log\left(4\sqrt{x+\sqrt{x+1}}\left(2\sqrt{x+1}+1\right)-8\,x-8\sqrt{x+1}-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(x + sqrt(x + 1)),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x + sqrt(x + 1)) + 1/2*log(4*sqrt(x + sqrt(x + 1))*(2*sqrt
(x + 1) + 1) - 8*x - 8*sqrt(x + 1) - 5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x+\sqrt{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x+(1+x)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/sqrt(x + sqrt(x + 1)), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(x + sqrt(x + 1)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.551 $\int \frac{1+x}{4+x+\sqrt{-9+6x}}\,dx$

Optimal. Leaf size=67

$$x-2\sqrt{3}\sqrt{2x-3}+3\log\left(x+\sqrt{3}\sqrt{2x-3}+4\right)+4\sqrt{6}\tan^{-1}\left(\frac{\sqrt{6x-9}+3}{2\sqrt{6}}\right)$$

```
[Out] x - 2*Sqrt[3]*Sqrt[-3 + 2*x] + 4*Sqrt[6]*ArcTan[(3 + Sqrt[-9 + 6*
x])/(2*Sqrt[6])] + 3*Log[4 + x + Sqrt[3]*Sqrt[-3 + 2*x]]
```

Rubi [A] time = 0.220887, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278

$$x-2\sqrt{3}\sqrt{2x-3}+3\log\left(x+\sqrt{3}\sqrt{2x-3}+4\right)+4\sqrt{6}\tan^{-1}\left(\frac{\sqrt{6x-9}+3}{2\sqrt{6}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x)/(4 + x + Sqrt[-9 + 6*x]),x]
```

```
[Out] x - 2*Sqrt[3]*Sqrt[-3 + 2*x] + 4*Sqrt[6]*ArcTan[(3 + Sqrt[-9 + 6*
x])/(2*Sqrt[6])] + 3*Log[4 + x + Sqrt[3]*Sqrt[-3 + 2*x]]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-2\sqrt{3}\sqrt{2x-3}+3\log\left(2x+2\sqrt{3}\sqrt{2x-3}+8\right)+4\sqrt{6}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{\sqrt{2x-3}}{4}+\frac{\sqrt{3}}{4}\right)\right)+\int^{\sqrt{2x-3}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+x)/(4+x+(-9+6*x)**(1/2)),x)
```

```
[Out] -2*sqrt(3)*sqrt(2*x - 3) + 3*log(2*x + 2*sqrt(3)*sqrt(2*x - 3) +
8) + 4*sqrt(6)*atan(sqrt(2)*(sqrt(2*x - 3)/4 + sqrt(3)/4)) + Inte
gral(x, (x, sqrt(2*x - 3)))
```

Mathematica [A] time = 0.0721075, size = 64, normalized size = 0.96

$$x-2\sqrt{6x-9}+3\log\left(6x+6\left(\sqrt{6x-9}+4\right)\right)+4\sqrt{6}\tan^{-1}\left(\frac{\sqrt{6x-9}+3}{2\sqrt{6}}\right)-\frac{3}{2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x)/(4 + x + Sqrt[-9 + 6*x]),x]
```

```
[Out] -3/2 + x - 2*Sqrt[-9 + 6*x] + 4*Sqrt[6]*ArcTan[(3 + Sqrt[-9 + 6*x
])/(2*Sqrt[6])] + 3*Log[6*x + 6*(4 + Sqrt[-9 + 6*x])]
```

Maple [A]   time = 0.009, size = 52, normalized size = 0.8

$$-2\sqrt{-9+6x}-\frac{3}{2}+x+3\ln\left(24+6x+6\sqrt{-9+6x}\right)+4\sqrt{6}\arctan\left(1/24\left(6+2\sqrt{-9+6x}\right)\sqrt{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+x)/(4+x+(-9+6*x)^(1/2)),x)
```

```
[Out] -2*(-9+6*x)^(1/2)-3/2+x+3*ln(24+6*x+6*(-9+6*x)^(1/2))+4*6^(1/2)*a
rctan(1/24*(6+2*(-9+6*x)^(1/2))*6^(1/2))
```

Maxima [A]   time = 0.806309, size = 66, normalized size = 0.99

$$4\sqrt{6}\arctan\left(\frac{1}{12}\sqrt{6}\left(\sqrt{6x-9}+3\right)\right)+x-2\sqrt{6x-9}+3\log\left(6x+6\sqrt{6x-9}+24\right)-\frac{3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 1)/(x + sqrt(6*x - 9) + 4),x, algorithm="maxima")
```

```
[Out] 4*sqrt(6)*arctan(1/12*sqrt(6)*(sqrt(6*x - 9) + 3)) + x - 2*sqrt(6
*x - 9) + 3*log(6*x + 6*sqrt(6*x - 9) + 24) - 3/2
```

Fricas [A]   time = 0.274862, size = 59, normalized size = 0.88

$$4\sqrt{6}\arctan\left(\frac{1}{12}\sqrt{6}\left(\sqrt{6x-9}+3\right)\right)+x-2\sqrt{6x-9}+3\log\left(x+\sqrt{6x-9}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 1)/(x + sqrt(6*x - 9) + 4),x, algorithm="fricas")
```

```
[Out] 4*sqrt(6)*arctan(1/12*sqrt(6)*(sqrt(6*x - 9) + 3)) + x - 2*sqrt(6
*x - 9) + 3*log(x + sqrt(6*x - 9) + 4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1}{x+\sqrt{3}\sqrt{2x-3}+4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+x)/(4+x+(-9+6*x)**(1/2)),x)
```

```
[Out] Integral((x + 1)/(x + sqrt(3)*sqrt(2*x - 3) + 4), x)
```

GIAC/XCAS [A] time = 0.283529, size = 113, normalized size = 1.69

$$-\frac{1}{2}\sqrt{3}\sqrt{2}\left(\sqrt{3}\sqrt{2}\ln\left(33\right)+8\arctan\left(\frac{1}{4}\sqrt{3}\sqrt{2}\right)\right)+4\sqrt{3}\sqrt{2}\arctan\left(\frac{1}{12}\sqrt{3}\sqrt{2}\left(\sqrt{6x-9}+3\right)\right)+x-2\sqrt{6x-9}+3\ln\left(6x+6\sqrt{6x-9}+24\right)-\frac{3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 1)/(x + sqrt(6*x - 9) + 4),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(3)*sqrt(2)*(sqrt(3)*sqrt(2)*ln(33) + 8*arctan(1/4*sqrt(
3)*sqrt(2))) + 4*sqrt(3)*sqrt(2)*arctan(1/12*sqrt(3)*sqrt(2)*(sqr
t(6*x - 9) + 3)) + x - 2*sqrt(6*x - 9) + 3*ln(6*x + 6*sqrt(6*x -
9) + 24) - 3/2
```

3.552 $\int \frac{12-x}{4+x+\sqrt{-9+6x}}\,dx$

Optimal. Leaf size=71

$$-x+2\sqrt{3}\sqrt{2x-3}+10\log\left(x+\sqrt{3}\sqrt{2x-3}+4\right)-21\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{\sqrt{6x-9}+3}{2\sqrt{6}}\right)$$

```
[Out] -x + 2*Sqrt[3]*Sqrt[-3 + 2*x] - 21*Sqrt[3/2]*ArcTan[(3 + Sqrt[-9
+ 6*x])/(2*Sqrt[6])] + 10*Log[4 + x + Sqrt[3]*Sqrt[-3 + 2*x]]
```

Rubi [A] time = 0.188021, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$-x+2\sqrt{3}\sqrt{2x-3}+10\log\left(x+\sqrt{3}\sqrt{2x-3}+4\right)-21\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{\sqrt{6x-9}+3}{2\sqrt{6}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(12 - x)/(4 + x + Sqrt[-9 + 6*x]),x]
```

```
[Out] -x + 2*Sqrt[3]*Sqrt[-3 + 2*x] - 21*Sqrt[3/2]*ArcTan[(3 + Sqrt[-9
+ 6*x])/(2*Sqrt[6])] + 10*Log[4 + x + Sqrt[3]*Sqrt[-3 + 2*x]]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$2\sqrt{3}\sqrt{2x-3}+10\log\left(2x+2\sqrt{3}\sqrt{2x-3}+8\right)-\frac{21\sqrt{6}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{\sqrt{2x-3}}{4}+\frac{\sqrt{3}}{4}\right)\right)}{2}-\int^{\sqrt{2x-3}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((12-x)/(4+x+(-9+6*x)**(1/2)),x)
```

```
[Out] 2*sqrt(3)*sqrt(2*x - 3) + 10*log(2*x + 2*sqrt(3)*sqrt(2*x - 3) +
8) - 21*sqrt(6)*atan(sqrt(2)*(sqrt(2*x - 3)/4 + sqrt(3)/4))/2 - I
ntegral(x, (x, sqrt(2*x - 3)))
```

Mathematica [A] time = 0.0452376, size = 70, normalized size = 0.99

$$\frac{1}{6}(9-6x)+2\sqrt{6x-9}+10\log\left(6x+6\sqrt{6x-9}+24\right)-21\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{\sqrt{6x-9}+3}{2\sqrt{6}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(12 - x)/(4 + x + Sqrt[-9 + 6*x]),x]
```

```
[Out] (9 - 6*x)/6 + 2*Sqrt[-9 + 6*x] - 21*Sqrt[3/2]*ArcTan[(3 + Sqrt[-9
 + 6*x])/(2*Sqrt[6])] + 10*Log[24 + 6*x + 6*Sqrt[-9 + 6*x]]
```

Maple [A] time = 0.006, size = 54, normalized size = 0.8

$$2\sqrt{-9+6x}+\frac{3}{2}-x+10\ln\left(24+6x+6\sqrt{-9+6x}\right)-\frac{21\sqrt{6}}{2}\arctan\left(\frac{\sqrt{6}}{24}\left(6+2\sqrt{-9+6x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((12-x)/(4+x+(-9+6*x)^(1/2)),x)
```

```
[Out] 2*(-9+6*x)^(1/2)+3/2-x+10*ln(24+6*x+6*(-9+6*x)^(1/2))-21/2*6^(1/2
)*arctan(1/24*(6+2*(-9+6*x)^(1/2))*6^(1/2))
```

Maxima [A] time = 0.801669, size = 69, normalized size = 0.97

$$-\frac{21}{2}\sqrt{6}\arctan\left(\frac{1}{12}\sqrt{6}\left(\sqrt{6x-9}+3\right)\right)-x+2\sqrt{6x-9}+10\log\left(6x+6\sqrt{6x-9}+24\right)+\frac{3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x - 12)/(x + sqrt(6*x - 9) + 4),x, algorithm="maxima")
```

```
[Out] -21/2*sqrt(6)*arctan(1/12*sqrt(6)*(sqrt(6*x - 9) + 3)) - x + 2*sq
rt(6*x - 9) + 10*log(6*x + 6*sqrt(6*x - 9) + 24) + 3/2
```

Fricas [A] time = 0.266952, size = 90, normalized size = 1.27

$$-\frac{1}{2}\sqrt{2}\left(\sqrt{2}x+21\sqrt{3}\arctan\left(\frac{1}{12}\sqrt{3}\left(\sqrt{2}\sqrt{6x-9}+3\sqrt{2}\right)\right)-10\sqrt{2}\log\left(x+\sqrt{6x-9}+4\right)-2\sqrt{2}\sqrt{6x-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x - 12)/(x + sqrt(6*x - 9) + 4),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*(sqrt(2)*x + 21*sqrt(3)*arctan(1/12*sqrt(3)*(sqrt(2)
*sqrt(6*x - 9) + 3*sqrt(2))) - 10*sqrt(2)*log(x + sqrt(6*x - 9) +
 4) - 2*sqrt(2)*sqrt(6*x - 9))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{x+\sqrt{3}\sqrt{2x-3}+4}\,dx-\int \left(-\frac{12}{x+\sqrt{3}\sqrt{2x-3}+4}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((12-x)/(4+x+(-9+6*x)**(1/2)),x)
```

```
[Out] -Integral(x/(x + sqrt(3)*sqrt(2*x - 3) + 4), x) - Integral(-12/(x
 + sqrt(3)*sqrt(2*x - 3) + 4), x)
```

GIAC/XCAS [A] time = 0.269275, size = 117, normalized size = 1.65

$$-\frac{1}{6}\sqrt{3}\sqrt{2}\left(10\sqrt{3}\sqrt{2}\ln(33)-63\arctan\left(\frac{1}{4}\sqrt{3}\sqrt{2}\right)\right)-\frac{21}{2}\sqrt{3}\sqrt{2}\arctan\left(\frac{1}{12}\sqrt{3}\sqrt{2}\left(\sqrt{6x-9}+3\right)\right)-x+2\sqrt{6x-9}+10\ln\left(6x+6\sqrt{6x-9}+24\right)+\frac{3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x - 12)/(x + sqrt(6*x - 9) + 4),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*sqrt(2)*(10*sqrt(3)*sqrt(2)*ln(33) - 63*arctan(1/4*s
qrt(3)*sqrt(2))) - 21/2*sqrt(3)*sqrt(2)*arctan(1/12*sqrt(3)*sqrt(
2)*(sqrt(6*x - 9) + 3)) - x + 2*sqrt(6*x - 9) + 10*ln(6*x + 6*sqr
t(6*x - 9) + 24) + 3/2
```

3.553 $\int \frac{-1+x^3}{\sqrt{x}(1+x^2)}\,dx$

Optimal. Leaf size=52

$$\frac{2x^{3/2}}{3}+\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)-\sqrt{2}\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)$$

```
[Out] (2*x^(3/2))/3 + Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[x]] - Sqrt[2]*Arc
Tan[1 + Sqrt[2]*Sqrt[x]]
```

Rubi [A] time = 0.0959879, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278

$$\frac{2x^{3/2}}{3}+\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)-\sqrt{2}\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + x^3)/(Sqrt[x]*(1 + x^2)),x]
```

```
[Out] (2*x^(3/2))/3 + Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[x]] - Sqrt[2]*Arc
Tan[1 + Sqrt[2]*Sqrt[x]]
```

Rubi in Sympy [A] time = 7.70096, size = 44, normalized size = 0.85

$$\frac{2x^{\frac{3}{2}}}{3}-\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}-1\right)-\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**3-1)/(x**2+1)/x**(1/2),x)
```

```
[Out] 2*x**(3/2)/3 - sqrt(2)*atan(sqrt(2)*sqrt(x) - 1) - sqrt(2)*atan(s
qrt(2)*sqrt(x) + 1)
```

Mathematica [A] time = 0.0323538, size = 52, normalized size = 1.

$$\frac{2x^{3/2}}{3}+\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)-\sqrt{2}\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + x^3)/(Sqrt[x]*(1 + x^2)),x]
```

```
[Out] (2*x^(3/2))/3 + Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[x]] - Sqrt[2]*Arc
Tan[1 + Sqrt[2]*Sqrt[x]]
```

Maple [B] time = 0.013, size = 97, normalized size = 1.9

$$\frac{2}{3}x^{\frac{3}{2}}-\arctan\left(1+\sqrt{2}\sqrt{x}\right)\sqrt{2}-\arctan\left(\sqrt{2}\sqrt{x}-1\right)\sqrt{2}$$
$$-\frac{\sqrt{2}}{4}\ln\left(1\left(x+\sqrt{2}\sqrt{x}+1\right)\left(x-\sqrt{2}\sqrt{x}+1\right)^{-1}\right)-\frac{\sqrt{2}}{4}\ln\left(1\left(x-\sqrt{2}\sqrt{x}+1\right)\left(x+\sqrt{2}\sqrt{x}+1\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^3-1)/(x^2+1)/x^(1/2),x)
```

```
[Out] 2/3*x^(3/2)-arctan(1+2^(1/2)*x^(1/2))*2^(1/2)-arctan(2^(1/2)*x^(1
/2)-1)*2^(1/2)-1/4*2^(1/2)*ln((x+2^(1/2)*x^(1/2)+1)/(x-2^(1/2)*x^
(1/2)+1))-1/4*2^(1/2)*ln((x-2^(1/2)*x^(1/2)+1)/(x+2^(1/2)*x^(1/2)
+1))
```

Maxima [A] time = 0.801757, size = 62, normalized size = 1.19

$$\frac{2}{3}x^{\frac{3}{2}}-\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - 1)/((x^2 + 1)*sqrt(x)),x, algorithm="maxima")
```

```
[Out] 2/3*x^(3/2) - sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) -
 sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x)))
```

Fricas [A] time = 0.266681, size = 31, normalized size = 0.6

$$\frac{2}{3}x^{\frac{3}{2}}-\sqrt{2}\arctan\left(\frac{\sqrt{2}(x-1)}{2\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - 1)/((x^2 + 1)*sqrt(x)),x, algorithm="fricas")
```

```
[Out] 2/3*x^(3/2) - sqrt(2)*arctan(1/2*sqrt(2)*(x - 1)/sqrt(x))
```

Sympy [A] time = 3.3509, size = 44, normalized size = 0.85

$$\frac{2x^{\frac{3}{2}}}{3} - \sqrt{2}\,\operatorname{atan}\left(\sqrt{2}\sqrt{x} - 1\right) - \sqrt{2}\,\operatorname{atan}\left(\sqrt{2}\sqrt{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3-1)/(x**2+1)/x**(1/2),x)
```

```
[Out] 2*x**(3/2)/3 - sqrt(2)*atan(sqrt(2)*sqrt(x) - 1) - sqrt(2)*atan(s
qrt(2)*sqrt(x) + 1)
```

GIAC/XCAS [A] time = 0.261633, size = 62, normalized size = 1.19

$$\frac{2}{3}\,x^{\frac{3}{2}} - \sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2} + 2\,\sqrt{x}\right)\right) - \sqrt{2}\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2} - 2\,\sqrt{x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - 1)/((x^2 + 1)*sqrt(x)),x, algorithm="giac")
```

```
[Out] 2/3*x^(3/2) - sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) -
 sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x)))
```

3.554 $\int \frac{1}{2\sqrt{-1+x}\sqrt{-\sqrt{-1+x}+x}}\,dx$

Optimal. Leaf size=20

$$-\sinh^{-1}\left(\frac{1-2\sqrt{x-1}}{\sqrt{3}}\right)$$

```
[Out] -ArcSinh[(1 - 2*Sqrt[-1 + x])/Sqrt[3]]
```

Rubi [A] time = 0.179992, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$-\sinh^{-1}\left(\frac{1-2\sqrt{x-1}}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(2*Sqrt[-1 + x]*Sqrt[-Sqrt[-1 + x] + x]),x]

[Out] -ArcSinh[(1 - 2*Sqrt[-1 + x])/Sqrt[3]]
```

Rubi in Sympy [A] time = 7.18844, size = 24, normalized size = 1.2

$$\operatorname{atanh}\left(\frac{2\sqrt{x-1}-1}{2\sqrt{x-\sqrt{x-1}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/2/(-1+x)**(1/2)/(x-(-1+x)**(1/2))**(1/2),x)

[Out] atanh((2*sqrt(x - 1) - 1)/(2*sqrt(x - sqrt(x - 1))))
```

Mathematica [A] time = 0.0218932, size = 18, normalized size = 0.9

$$\sinh^{-1}\left(\frac{2\sqrt{x-1}-1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(2*Sqrt[-1 + x]*Sqrt[-Sqrt[-1 + x] + x]),x]
```

```
[Out] ArcSinh[(-1 + 2*Sqrt[-1 + x])/Sqrt[3]]
```

Maple [A] time = 0.009, size = 14, normalized size = 0.7

$$\mathit{Arcsinh}\left(\frac{2\sqrt{3}}{3}\left(\sqrt{-1+x}-\frac{1}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/2/(-1+x)^(1/2)/(x-(-1+x)^(1/2))^(1/2),x)
```

```
[Out] arcsinh(2/3*3^(1/2)*((-1+x)^(1/2)-1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\int \frac{1}{\sqrt{x-\sqrt{x-1}}\sqrt{x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/2/(sqrt(x - sqrt(x - 1))*sqrt(x - 1)),x, algorithm="maxima")
```

```
[Out] 1/2*integrate(1/(sqrt(x - sqrt(x - 1))*sqrt(x - 1)), x)
```

Fricas [A] time = 0.483548, size = 50, normalized size = 2.5

$$\frac{1}{2}\log\left(4\sqrt{x-\sqrt{x-1}}\left(2\sqrt{x-1}-1\right)+8x-8\sqrt{x-1}-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/2/(sqrt(x - sqrt(x - 1))*sqrt(x - 1)),x, algorithm="fricas")
```

```
[Out] 1/2*log(4*sqrt(x - sqrt(x - 1))*(2*sqrt(x - 1) - 1) + 8*x - 8*sqr
t(x - 1) - 3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{\sqrt{x-1}\sqrt{x-\sqrt{x-1}}}\, dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/2/(-1+x)**(1/2)/(x-(-1+x)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x - 1)*sqrt(x - sqrt(x - 1))), x)/2
```

GIAC/XCAS [A] time = 0.262849, size = 34, normalized size = 1.7

$$-\ln\left(2\sqrt{x-\sqrt{x-1}}-2\sqrt{x-1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/2/(sqrt(x - sqrt(x - 1))*sqrt(x - 1)),x, algorithm="giac")
```

```
[Out] -ln(2*sqrt(x - sqrt(x - 1)) - 2*sqrt(x - 1) + 1)
```

3.555 $\int \frac{1+x^{7/2}}{1-x^2}\,dx$

Optimal. Leaf size=43

$$-\frac{2x^{5/2}}{5}-2\sqrt{x}-\log\left(1-\sqrt{x}\right)+\frac{1}{2}\log(x+1)+\tan^{-1}\left(\sqrt{x}\right)$$

```
[Out] -2*Sqrt[x] - (2*x^(5/2))/5 + ArcTan[Sqrt[x]] - Log[1 - Sqrt[x]] +
 Log[1 + x]/2
```

Rubi [A] time = 0.193995, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294

$$-\frac{2x^{5/2}}{5}-2\sqrt{x}-\log\left(1-\sqrt{x}\right)+\frac{1}{2}\log(x+1)+\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^(7/2))/(1 - x^2),x]
```

```
[Out] -2*Sqrt[x] - (2*x^(5/2))/5 + ArcTan[Sqrt[x]] - Log[1 - Sqrt[x]] +
 Log[1 + x]/2
```

Rubi in Sympy [A] time = 13.96, size = 41, normalized size = 0.95

$$-\frac{2x^{\frac{5}{2}}}{5}-2\sqrt{x}-\frac{\log\left(-x+1\right)}{2}+\frac{\log\left(x+1\right)}{2}+\operatorname{atan}\left(\sqrt{x}\right)+\operatorname{atanh}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+x**(7/2))/(-x**2+1),x)
```

```
[Out] -2*x**(5/2)/5 - 2*sqrt(x) - log(-x + 1)/2 + log(x + 1)/2 + atan(s
qrt(x)) + atanh(sqrt(x))
```

Mathematica [A] time = 0.0187024, size = 43, normalized size = 1.

$$-\frac{2x^{5/2}}{5}-2\sqrt{x}-\log\left(1-\sqrt{x}\right)+\frac{1}{2}\log(x+1)+\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^(7/2))/(1 - x^2),x]
```

```
[Out] -2*Sqrt[x] - (2*x^(5/2))/5 + ArcTan[Sqrt[x]] - Log[1 - Sqrt[x]] +
 Log[1 + x]/2
```

Maple [A] time = 0.006, size = 34, normalized size = 0.8

$$-\frac{2}{5}x^{\frac{5}{2}}-2\sqrt{x}-\frac{1}{2}\ln\left(-1+\sqrt{x}\right)+\frac{1}{2}\ln\left(1+\sqrt{x}\right)+\arctan\left(\sqrt{x}\right)+\mathit{Artanh}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+x^(7/2))/(-x^2+1),x)
```

```
[Out] -2/5*x^(5/2)-2*x^(1/2)-1/2*ln(-1+x^(1/2))+1/2*ln(1+x^(1/2))+arcta
n(x^(1/2))+arctanh(x)
```

Maxima [A] time = 0.810667, size = 39, normalized size = 0.91

$$-\frac{2}{5}x^{\frac{5}{2}}-2\sqrt{x}+\arctan\left(\sqrt{x}\right)+\frac{1}{2}\log\left(x+1\right)-\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^(7/2) + 1)/(x^2 - 1),x, algorithm="maxima")
```

```
[Out] -2/5*x^(5/2) - 2*sqrt(x) + arctan(sqrt(x)) + 1/2*log(x + 1) - log
(sqrt(x) - 1)
```

Fricas [A] time = 0.271261, size = 39, normalized size = 0.91

$$-\frac{2}{5}\left(x^{2}+5\right)\sqrt{x}+\arctan\left(\sqrt{x}\right)+\frac{1}{2}\log\left(x+1\right)-\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^(7/2) + 1)/(x^2 - 1),x, algorithm="fricas")
```

```
[Out] -2/5*(x^2 + 5)*sqrt(x) + arctan(sqrt(x)) + 1/2*log(x + 1) - log(s
qrt(x) - 1)
```

Sympy [A] time = 10.6502, size = 36, normalized size = 0.84

$$-\frac{2x^{\frac{5}{2}}}{5} - 2\sqrt{x} - \log\left(\sqrt{x} - 1\right) + \frac{\log\left(x + 1\right)}{2} + \operatorname{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+x**(7/2))/(-x**2+1),x)
```

```
[Out] -2*x**(5/2)/5 - 2*sqrt(x) - log(sqrt(x) - 1) + log(x + 1)/2 + ata
n(sqrt(x))
```

GIAC/XCAS [A] time = 0.260682, size = 41, normalized size = 0.95

$$-\frac{2}{5}\,x^{\frac{5}{2}} - 2\,\sqrt{x} + \arctan\left(\sqrt{x}\right) + \frac{1}{2}\ln\left(x + 1\right) - \ln\left(\left|\sqrt{x} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^(7/2) + 1)/(x^2 - 1),x, algorithm="giac")
```

```
[Out] -2/5*x^(5/2) - 2*sqrt(x) + arctan(sqrt(x)) + 1/2*ln(x + 1) - ln(a
bs(sqrt(x) - 1))
```

3.556 $\int \frac{4+2x}{\sqrt[3]{-1+2x}+\sqrt{-1+2x}}\,dx$

Optimal. Leaf size=116

$$\frac{1}{3}(2x-1)^{3/2}-\frac{3}{8}(2x-1)^{4/3}+\frac{3}{7}(2x-1)^{7/6}+\frac{3}{5}(2x-1)^{5/6}-\frac{3}{4}(2x-1)^{2/3}+6\sqrt{2x-1}-9\sqrt[3]{2x-1}+18\sqrt[6]{2x-1}-x-18\log\left(\sqrt[6]{2x-1}+1\right)$$

```
[Out] -x + 18*(-1 + 2*x)^(1/6) - 9*(-1 + 2*x)^(1/3) + 6*Sqrt[-1 + 2*x]
- (3*(-1 + 2*x)^(2/3))/4 + (3*(-1 + 2*x)^(5/6))/5 + (3*(-1 + 2*x)
^(7/6))/7 - (3*(-1 + 2*x)^(4/3))/8 + (-1 + 2*x)^(3/2)/3 - 18*Log[
1 + (-1 + 2*x)^(1/6)]
```

Rubi [A] time = 0.220101, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037

$$\frac{1}{3}(2x-1)^{3/2}-\frac{3}{8}(2x-1)^{4/3}+\frac{3}{7}(2x-1)^{7/6}+\frac{3}{5}(2x-1)^{5/6}-\frac{3}{4}(2x-1)^{2/3}+6\sqrt{2x-1}-9\sqrt[3]{2x-1}+18\sqrt[6]{2x-1}-x-18\log\left(\sqrt[6]{2x-1}+1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(4 + 2*x)/((-1 + 2*x)^(1/3) + Sqrt[-1 + 2*x]),x]
```

```
[Out] -x + 18*(-1 + 2*x)^(1/6) - 9*(-1 + 2*x)^(1/3) + 6*Sqrt[-1 + 2*x]
- (3*(-1 + 2*x)^(2/3))/4 + (3*(-1 + 2*x)^(5/6))/5 + (3*(-1 + 2*x)
^(7/6))/7 - (3*(-1 + 2*x)^(4/3))/8 + (-1 + 2*x)^(3/2)/3 - 18*Log[
1 + (-1 + 2*x)^(1/6)]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-x+\frac{3\left(2x-1\right)^{\frac{7}{6}}}{7}+\frac{3\left(2x-1\right)^{\frac{5}{6}}}{5}+18\sqrt[6]{2x-1}-\frac{3\left(2x-1\right)^{\frac{4}{3}}}{8}-\frac{3\left(2x-1\right)^{\frac{2}{3}}}{4}+\frac{\left(2x-1\right)^{\frac{3}{2}}}{3}+6\sqrt{2x-1}-18\log\left(\sqrt[6]{2x-1}+1\right)-18\int^{\sqrt[6]{2x-1}}x\,dx+\frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((4+2*x)/((-1+2*x)**(1/3)+(-1+2*x)**(1/2)),x)
```

```
[Out] -x + 3*(2*x - 1)**(7/6)/7 + 3*(2*x - 1)**(5/6)/5 + 18*(2*x - 1)**
(1/6) - 3*(2*x - 1)**(4/3)/8 - 3*(2*x - 1)**(2/3)/4 + (2*x - 1)**
(3/2)/3 + 6*sqrt(2*x - 1) - 18*log((2*x - 1)**(1/6) + 1) - 18*Int
egral(x, (x, (2*x - 1)**(1/6))) + 1/2
```

Mathematica [A] time = 0.200892, size = 156, normalized size = 1.34

$$2\left(x\left(\frac{1}{3}\sqrt{2x-1}-\frac{3}{8}\sqrt[3]{2x-1}+\frac{3}{7}\sqrt[6]{2x-1}+\frac{3}{5\sqrt[6]{2x-1}}-\frac{3}{4\sqrt[3]{2x-1}}-\frac{1}{2}\right)+\frac{17}{6}\sqrt{2x-1}\right.$$
$$\left.-\frac{69}{16}\sqrt[3]{2x-1}+\frac{123}{14}\sqrt[6]{2x-1}-\frac{3}{10\sqrt[6]{2x-1}}+\frac{3}{8\sqrt[3]{2x-1}}-9\log\left(\sqrt[6]{2x-1}+1\right)+\frac{1}{4}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(4 + 2*x)/((-1 + 2*x)^(1/3) + Sqrt[-1 + 2*x]),x]
```

```
[Out] 2*(1/4 + 3/(8*(-1 + 2*x)^(1/3)) - 3/(10*(-1 + 2*x)^(1/6)) + (123*
(-1 + 2*x)^(1/6))/14 - (69*(-1 + 2*x)^(1/3))/16 + (17*Sqrt[-1 + 2
*x])/6 + x*(-1/2 - 3/(4*(-1 + 2*x)^(1/3)) + 3/(5*(-1 + 2*x)^(1/6)
) + (3*(-1 + 2*x)^(1/6))/7 - (3*(-1 + 2*x)^(1/3))/8 + Sqrt[-1 + 2
*x]/3) - 9*Log[1 + (-1 + 2*x)^(1/6)])
```

Maple [A] time = 0.007, size = 90, normalized size = 0.8

$$\frac{1}{3}(2x-1)^{\frac{3}{2}}-\frac{3}{8}(2x-1)^{\frac{4}{3}}+\frac{3}{7}(2x-1)^{\frac{7}{6}}-x+\frac{1}{2}+\frac{3}{5}(2x-1)^{\frac{5}{6}}-\frac{3}{4}(2x-1)^{\frac{2}{3}}$$
$$+6\sqrt{2x-1}-9\sqrt[3]{2x-1}+18\sqrt[6]{2x-1}-18\ln\left(1+\sqrt[6]{2x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((4+2*x)/((2*x-1)^(1/3)+(2*x-1)^(1/2)),x)
```

```
[Out] 1/3*(2*x-1)^(3/2)-3/8*(2*x-1)^(4/3)+3/7*(2*x-1)^(7/6)-x+1/2+3/5*(
2*x-1)^(5/6)-3/4*(2*x-1)^(2/3)+6*(2*x-1)^(1/2)-9*(2*x-1)^(1/3)+18
*(2*x-1)^(1/6)-18*ln(1+(2*x-1)^(1/6))
```

Maxima [A] time = 0.718258, size = 120, normalized size = 1.03

$$\frac{1}{3}(2x-1)^{\frac{3}{2}}-\frac{3}{8}(2x-1)^{\frac{4}{3}}+\frac{3}{7}(2x-1)^{\frac{7}{6}}-x+\frac{3}{5}(2x-1)^{\frac{5}{6}}-\frac{3}{4}(2x-1)^{\frac{2}{3}}$$
$$+6\sqrt{2x-1}-9(2x-1)^{\frac{1}{3}}+18(2x-1)^{\frac{1}{6}}-18\log\left((2x-1)^{\frac{1}{6}}+1\right)+\frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*(x + 2)/(sqrt(2*x - 1) + (2*x - 1)^(1/3)),x, algorithm="maxima")
```

```
[Out] 1/3*(2*x - 1)^(3/2) - 3/8*(2*x - 1)^(4/3) + 3/7*(2*x - 1)^(7/6) -
 x + 3/5*(2*x - 1)^(5/6) - 3/4*(2*x - 1)^(2/3) + 6*sqrt(2*x - 1)
- 9*(2*x - 1)^(1/3) + 18*(2*x - 1)^(1/6) - 18*log((2*x - 1)^(1/6)
 + 1) + 1/2
```

Fricas [A] time = 0.262925, size = 103, normalized size = 0.89

$$\frac{1}{3}(2x+17)\sqrt{2x-1}-\frac{3}{8}(2x+23)(2x-1)^{\frac{1}{3}}+\frac{3}{7}(2x+41)(2x-1)^{\frac{1}{6}}-x+\frac{3}{5}(2x-1)^{\frac{5}{6}}-\frac{3}{4}(2x-1)^{\frac{2}{3}}-18\log\left((2x-1)^{\frac{1}{6}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*(x + 2)/(sqrt(2*x - 1) + (2*x - 1)^(1/3)),x, algorithm="fricas")
```

```
[Out] 1/3*(2*x + 17)*sqrt(2*x - 1) - 3/8*(2*x + 23)*(2*x - 1)^(1/3) + 3
/7*(2*x + 41)*(2*x - 1)^(1/6) - x + 3/5*(2*x - 1)^(5/6) - 3/4*(2*
x - 1)^(2/3) - 18*log((2*x - 1)^(1/6) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$2\left(\int\frac{x}{\sqrt[3]{2x-1}+\sqrt{2x-1}}\,dx+\int\frac{2}{\sqrt[3]{2x-1}+\sqrt{2x-1}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+2*x)/((-1+2*x)**(1/3)+(-1+2*x)**(1/2)),x)
```

```
[Out] 2*(Integral(x/((2*x - 1)**(1/3) + sqrt(2*x - 1)), x) + Integral(2
/((2*x - 1)**(1/3) + sqrt(2*x - 1)), x))
```

GIAC/XCAS [A] time = 0.297345, size = 120, normalized size = 1.03

$$\frac{1}{3}(2x-1)^{\frac{3}{2}}-\frac{3}{8}(2x-1)^{\frac{4}{3}}+\frac{3}{7}(2x-1)^{\frac{7}{6}}-x+\frac{3}{5}(2x-1)^{\frac{5}{6}}-\frac{3}{4}(2x-1)^{\frac{2}{3}}+6\sqrt{2x-1}-9(2x-1)^{\frac{1}{3}}+18(2x-1)^{\frac{1}{6}}-18\ln\left((2x-1)^{\frac{1}{6}}+1\right)+\frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*(x + 2)/(sqrt(2*x - 1) + (2*x - 1)^(1/3)),x, algorithm="giac")
```

```
[Out] 1/3*(2*x - 1)^(3/2) - 3/8*(2*x - 1)^(4/3) + 3/7*(2*x - 1)^(7/6) -
 x + 3/5*(2*x - 1)^(5/6) - 3/4*(2*x - 1)^(2/3) + 6*sqrt(2*x - 1)
- 9*(2*x - 1)^(1/3) + 18*(2*x - 1)^(1/6) - 18*ln((2*x - 1)^(1/6)
+ 1) + 1/2
```

3.557 $\int \frac{1}{\sqrt{2+\sqrt{1+\sqrt{x}}}} \, dx$

Optimal. Leaf size=83

$$\frac{8}{7}\left(\sqrt{\sqrt{x}+1}+2\right)^{7/2}-\frac{48}{5}\left(\sqrt{\sqrt{x}+1}+2\right)^{5/2}+\frac{88}{3}\left(\sqrt{\sqrt{x}+1}+2\right)^{3/2}-48\sqrt{\sqrt{\sqrt{x}+1}+2}$$

```
[Out] -48*Sqrt[2 + Sqrt[1 + Sqrt[x]]] + (88*(2 + Sqrt[1 + Sqrt[x]])^(3/
2))/3 - (48*(2 + Sqrt[1 + Sqrt[x]])^(5/2))/5 + (8*(2 + Sqrt[1 + S
qrt[x]])^(7/2))/7
```

Rubi [A] time = 0.114372, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$\frac{8}{7}\left(\sqrt{\sqrt{x}+1}+2\right)^{7/2}-\frac{48}{5}\left(\sqrt{\sqrt{x}+1}+2\right)^{5/2}+\frac{88}{3}\left(\sqrt{\sqrt{x}+1}+2\right)^{3/2}-48\sqrt{\sqrt{\sqrt{x}+1}+2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[2 + Sqrt[1 + Sqrt[x]]],x]
```

```
[Out] -48*Sqrt[2 + Sqrt[1 + Sqrt[x]]] + (88*(2 + Sqrt[1 + Sqrt[x]])^(3/
2))/3 - (48*(2 + Sqrt[1 + Sqrt[x]])^(5/2))/5 + (8*(2 + Sqrt[1 + S
qrt[x]])^(7/2))/7
```

Rubi in Sympy [A] time = 4.68887, size = 71, normalized size = 0.86

$$\frac{8\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{7}{2}}}{7}-\frac{48\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{5}{2}}}{5}+\frac{88\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{3}{2}}}{3}-48\sqrt{\sqrt{\sqrt{x}+1}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(2+(1+x**(1/2))**(1/2))**(1/2),x)
```

```
[Out] 8*(sqrt(sqrt(x) + 1) + 2)**(7/2)/7 - 48*(sqrt(sqrt(x) + 1) + 2)**
(5/2)/5 + 88*(sqrt(sqrt(x) + 1) + 2)**(3/2)/3 - 48*sqrt(sqrt(sqrt
(x) + 1) + 2)
```

Mathematica [A] time = 0.0425017, size = 58, normalized size = 0.7

$$\frac{8}{105}\sqrt{\sqrt{\sqrt{x}+1}+2}\left(3\sqrt{x}\left(5\sqrt{\sqrt{x}+1}-12\right)+76\sqrt{\sqrt{x}+1}-280\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[2 + Sqrt[1 + Sqrt[x]]],x]
```

```
[Out] (8*Sqrt[2 + Sqrt[1 + Sqrt[x]]]*(-280 + 76*Sqrt[1 + Sqrt[x]] + 3*(
-12 + 5*Sqrt[1 + Sqrt[x]])*Sqrt[x]))/105
```

Maple [A] time = 0.013, size = 54, normalized size = 0.7

$$\frac{88}{3}\left(2+\sqrt{1+\sqrt{x}}\right)^{\frac{3}{2}}-\frac{48}{5}\left(2+\sqrt{1+\sqrt{x}}\right)^{\frac{5}{2}}+\frac{8}{7}\left(2+\sqrt{1+\sqrt{x}}\right)^{\frac{7}{2}}-48\sqrt{2+\sqrt{1+\sqrt{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(2+(1+x^(1/2))^(1/2))^(1/2),x)
```

```
[Out] 88/3*(2+(1+x^(1/2))^(1/2))^(3/2)-48/5*(2+(1+x^(1/2))^(1/2))^(5/2)
+8/7*(2+(1+x^(1/2))^(1/2))^(7/2)-48*(2+(1+x^(1/2))^(1/2))^(1/2)
```

Maxima [A] time = 0.741382, size = 72, normalized size = 0.87

$$\frac{8}{7}\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{7}{2}}-\frac{48}{5}\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{5}{2}}+\frac{88}{3}\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{3}{2}}-48\sqrt{\sqrt{\sqrt{x}+1}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(sqrt(sqrt(x) + 1) + 2),x, algorithm="maxima")
```

```
[Out] 8/7*(sqrt(sqrt(x) + 1) + 2)^(7/2) - 48/5*(sqrt(sqrt(x) + 1) + 2)^
(5/2) + 88/3*(sqrt(sqrt(x) + 1) + 2)^(3/2) - 48*sqrt(sqrt(sqrt(x)
 + 1) + 2)
```

Fricas [A] time = 0.269128, size = 47, normalized size = 0.57

$$\frac{8}{105}\left(\left(15\sqrt{x}+76\right)\sqrt{\sqrt{x}+1}-36\sqrt{x}-280\right)\sqrt{\sqrt{\sqrt{x}+1}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(sqrt(sqrt(x) + 1) + 2),x, algorithm="fricas")
```

```
[Out] 8/105*((15*sqrt(x) + 76)*sqrt(sqrt(x) + 1) - 36*sqrt(x) - 280)*sq
rt(sqrt(sqrt(x) + 1) + 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{\sqrt{\sqrt{x}+1}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(2+(1+x**(1/2))**(1/2))**(1/2),x)
```

```
[Out] Integral(1/sqrt(sqrt(sqrt(x) + 1) + 2), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(sqrt(sqrt(x) + 1) + 2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.558 $\int \sqrt{2+\sqrt{4+\sqrt{x}}}\,dx$

Optimal. Leaf size=64

$$\frac{8}{9}\left(\sqrt{\sqrt{x}+4}+2\right)^{9/2}-\frac{48}{7}\left(\sqrt{\sqrt{x}+4}+2\right)^{7/2}+\frac{64}{5}\left(\sqrt{\sqrt{x}+4}+2\right)^{5/2}$$

```
[Out] (64*(2 + Sqrt[4 + Sqrt[x]])^(5/2))/5 - (48*(2 + Sqrt[4 + Sqrt[x]]
)^(7/2))/7 + (8*(2 + Sqrt[4 + Sqrt[x]])^(9/2))/9
```

Rubi [A] time = 0.10365, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235

$$\frac{8}{9}\left(\sqrt{\sqrt{x}+4}+2\right)^{9/2}-\frac{48}{7}\left(\sqrt{\sqrt{x}+4}+2\right)^{7/2}+\frac{64}{5}\left(\sqrt{\sqrt{x}+4}+2\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[2 + Sqrt[4 + Sqrt[x]]],x]
```

```
[Out] (64*(2 + Sqrt[4 + Sqrt[x]])^(5/2))/5 - (48*(2 + Sqrt[4 + Sqrt[x]]
)^(7/2))/7 + (8*(2 + Sqrt[4 + Sqrt[x]])^(9/2))/9
```

Rubi in Sympy [A] time = 4.79705, size = 54, normalized size = 0.84

$$\frac{8\left(\sqrt{\sqrt{x}+4}+2\right)^{\frac{9}{2}}}{9}-\frac{48\left(\sqrt{\sqrt{x}+4}+2\right)^{\frac{7}{2}}}{7}+\frac{64\left(\sqrt{\sqrt{x}+4}+2\right)^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2+(4+x**(1/2))**(1/2))**(1/2),x)
```

```
[Out] 8*(sqrt(sqrt(x) + 4) + 2)**(9/2)/9 - 48*(sqrt(sqrt(x) + 4) + 2)**
(7/2)/7 + 64*(sqrt(sqrt(x) + 4) + 2)**(5/2)/5
```

Mathematica [A] time = 0.0395384, size = 62, normalized size = 0.97

$$\frac{8}{315}\sqrt{\sqrt{\sqrt{x}+4}+2}\left(-64\left(\sqrt{\sqrt{x}+4}+2\right)+35x+2\left(5\sqrt{\sqrt{x}+4}+2\right)\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[2 + Sqrt[4 + Sqrt[x]]],x]
```

```
[Out] (8*Sqrt[2 + Sqrt[4 + Sqrt[x]]]*(-64*(2 + Sqrt[4 + Sqrt[x]]) + 2*(
2 + 5*Sqrt[4 + Sqrt[x]])*Sqrt[x] + 35*x))/315
```

Maple [A] time = 0.014, size = 41, normalized size = 0.6

$$\frac{64}{5}\left(2+\sqrt{4+\sqrt{x}}\right)^{\frac{5}{2}}-\frac{48}{7}\left(2+\sqrt{4+\sqrt{x}}\right)^{\frac{7}{2}}+\frac{8}{9}\left(2+\sqrt{4+\sqrt{x}}\right)^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2+(4+x^(1/2))^(1/2))^(1/2),x)
```

```
[Out] 64/5*(2+(4+x^(1/2))^(1/2))^(5/2)-48/7*(2+(4+x^(1/2))^(1/2))^(7/2)
+8/9*(2+(4+x^(1/2))^(1/2))^(9/2)
```

Maxima [A] time = 0.733047, size = 54, normalized size = 0.84

$$\frac{8}{9}\left(\sqrt{\sqrt{x}+4}+2\right)^{\frac{9}{2}}-\frac{48}{7}\left(\sqrt{\sqrt{x}+4}+2\right)^{\frac{7}{2}}+\frac{64}{5}\left(\sqrt{\sqrt{x}+4}+2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(sqrt(sqrt(x) + 4) + 2),x, algorithm="maxima")
```

```
[Out] 8/9*(sqrt(sqrt(x) + 4) + 2)^(9/2) - 48/7*(sqrt(sqrt(x) + 4) + 2)^
(7/2) + 64/5*(sqrt(sqrt(x) + 4) + 2)^(5/2)
```

Fricas [A] time = 0.265884, size = 53, normalized size = 0.83

$$\frac{8}{315}\left(2\left(5\sqrt{x}-32\right)\sqrt{\sqrt{x}+4}+35\,x+4\sqrt{x}-128\right)\sqrt{\sqrt{\sqrt{x}+4}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(sqrt(sqrt(x) + 4) + 2),x, algorithm="fricas")
```

```
[Out] 8/315*(2*(5*sqrt(x) - 32)*sqrt(sqrt(x) + 4) + 35*x + 4*sqrt(x) -
128)*sqrt(sqrt(sqrt(x) + 4) + 2)
```

Sympy [A] time = 8.12604, size = 216, normalized size = 3.38

$$-\frac{2\sqrt{2}\sqrt{x}\sqrt{\sqrt{x}+4}\sqrt{\sqrt{\sqrt{x}+4}+2}\,\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{63\pi} - \frac{4\sqrt{2}\sqrt{x}\sqrt{\sqrt{\sqrt{x}+4}+2}\,\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{315\pi} - \frac{\sqrt{2}x\sqrt{\sqrt{\sqrt{x}+4}+2}\,\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{9\pi} + \frac{64\sqrt{2}\sqrt{\sqrt{x}+4}\sqrt{\sqrt{\sqrt{x}+4}+2}\,\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{315\pi} + \frac{128\sqrt{2}\sqrt{\sqrt{\sqrt{x}+4}+2}\,\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{315\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2+(4+x**(1/2))**(1/2))**(1/2),x)
```

```
[Out] -2*sqrt(2)*sqrt(x)*sqrt(sqrt(x) + 4)*sqrt(sqrt(sqrt(x) + 4) + 2)*
gamma(-1/4)*gamma(1/4)/(63*pi) - 4*sqrt(2)*sqrt(x)*sqrt(sqrt(sqrt
(x) + 4) + 2)*gamma(-1/4)*gamma(1/4)/(315*pi) - sqrt(2)*x*sqrt(sq
rt(sqrt(x) + 4) + 2)*gamma(-1/4)*gamma(1/4)/(9*pi) + 64*sqrt(2)*s
qrt(sqrt(x) + 4)*sqrt(sqrt(sqrt(x) + 4) + 2)*gamma(-1/4)*gamma(1/
4)/(315*pi) + 128*sqrt(2)*sqrt(sqrt(sqrt(x) + 4) + 2)*gamma(-1/4)
*gamma(1/4)/(315*pi)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(sqrt(sqrt(x) + 4) + 2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.559 $\int \sqrt{2-\sqrt{4+\sqrt{-9+5x}}}\,dx$

Optimal. Leaf size=82

$$\frac{8}{45}\left(2-\sqrt{\sqrt{5x-9}+4}\right)^{9/2}-\frac{48}{35}\left(2-\sqrt{\sqrt{5x-9}+4}\right)^{7/2}+\frac{64}{25}\left(2-\sqrt{\sqrt{5x-9}+4}\right)^{5/2}$$

```
[Out] (64*(2 - Sqrt[4 + Sqrt[-9 + 5*x]])^(5/2))/25 - (48*(2 - Sqrt[4 +
Sqrt[-9 + 5*x]])^(7/2))/35 + (8*(2 - Sqrt[4 + Sqrt[-9 + 5*x]])^(9
/2))/45
```

Rubi [A] time = 0.167852, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174

$$\frac{8}{45}\left(2-\sqrt{\sqrt{5x-9}+4}\right)^{9/2}-\frac{48}{35}\left(2-\sqrt{\sqrt{5x-9}+4}\right)^{7/2}+\frac{64}{25}\left(2-\sqrt{\sqrt{5x-9}+4}\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[2 - Sqrt[4 + Sqrt[-9 + 5*x]]],x]
```

```
[Out] (64*(2 - Sqrt[4 + Sqrt[-9 + 5*x]])^(5/2))/25 - (48*(2 - Sqrt[4 +
Sqrt[-9 + 5*x]])^(7/2))/35 + (8*(2 - Sqrt[4 + Sqrt[-9 + 5*x]])^(9
/2))/45
```

Rubi in Sympy [A] time = 6.56881, size = 65, normalized size = 0.79

$$\frac{8\left(-\sqrt{\sqrt{5x-9}+4}+2\right)^{\frac{9}{2}}}{45}-\frac{48\left(-\sqrt{\sqrt{5x-9}+4}+2\right)^{\frac{7}{2}}}{35}+\frac{64\left(-\sqrt{\sqrt{5x-9}+4}+2\right)^{\frac{5}{2}}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2-(4+(-9+5*x)**(1/2))**(1/2))**(1/2),x)
```

```
[Out] 8*(-sqrt(sqrt(5*x - 9) + 4) + 2)**(9/2)/45 - 48*(-sqrt(sqrt(5*x -
 9) + 4) + 2)**(7/2)/35 + 64*(-sqrt(sqrt(5*x - 9) + 4) + 2)**(5/2
)/25
```

Mathematica [A] time = 0.0452686, size = 86, normalized size = 1.05

$$-\frac{8\sqrt{2-\sqrt{\sqrt{5x-9}+4}}\left(-175x-4\sqrt{5x-9}+10\sqrt{5x-9}\sqrt{\sqrt{5x-9}+4}-64\sqrt{\sqrt{5x-9}+4}+443\right)}{1575}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[2 - Sqrt[4 + Sqrt[-9 + 5*x]]],x]
```

```
[Out] (-8*Sqrt[2 - Sqrt[4 + Sqrt[-9 + 5*x]]]*(443 - 175*x - 4*Sqrt[-9 +
 5*x] - 64*Sqrt[4 + Sqrt[-9 + 5*x]] + 10*Sqrt[-9 + 5*x]*Sqrt[4 +
Sqrt[-9 + 5*x]]))/1575
```

Maple [A] time = 0.015, size = 59, normalized size = 0.7

$$\frac{64}{25}\left(2-\sqrt{4+\sqrt{-9+5\,x}}\right)^{\frac{5}{2}}-\frac{48}{35}\left(2-\sqrt{4+\sqrt{-9+5\,x}}\right)^{\frac{7}{2}}+\frac{8}{45}\left(2-\sqrt{4+\sqrt{-9+5\,x}}\right)^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2-(4+(-9+5*x)^(1/2))^(1/2))^(1/2),x)
```

```
[Out] 64/25*(2-(4+(-9+5*x)^(1/2))^(1/2))^(5/2)-48/35*(2-(4+(-9+5*x)^(1/
2))^(1/2))^(7/2)+8/45*(2-(4+(-9+5*x)^(1/2))^(1/2))^(9/2)
```

Maxima [A] time = 0.723031, size = 78, normalized size = 0.95

$$\frac{8}{45}\left(-\sqrt{\sqrt{5\,x-9}+4}+2\right)^{\frac{9}{2}}-\frac{48}{35}\left(-\sqrt{\sqrt{5\,x-9}+4}+2\right)^{\frac{7}{2}}+\frac{64}{25}\left(-\sqrt{\sqrt{5\,x-9}+4}+2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-sqrt(sqrt(5*x - 9) + 4) + 2),x, algorithm="maxima")
```

```
[Out] 8/45*(-sqrt(sqrt(5*x - 9) + 4) + 2)^(9/2) - 48/35*(-sqrt(sqrt(5*x
 - 9) + 4) + 2)^(7/2) + 64/25*(-sqrt(sqrt(5*x - 9) + 4) + 2)^(5/2
)
```

Fricas [A] time = 0.267317, size = 77, normalized size = 0.94

$$-\frac{8}{1575}\left(2\left(5\sqrt{5x-9}-32\right)\sqrt{\sqrt{5x-9}+4}-175x-4\sqrt{5x-9}+443\right)\sqrt{-\sqrt{\sqrt{5x-9}+4}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-sqrt(sqrt(5*x - 9) + 4) + 2),x, algorithm="fricas")
```

```
[Out] -8/1575*(2*(5*sqrt(5*x - 9) - 32)*sqrt(sqrt(5*x - 9) + 4) - 175*x
 - 4*sqrt(5*x - 9) + 443)*sqrt(-sqrt(sqrt(5*x - 9) + 4) + 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-\sqrt{\sqrt{5x-9}+4}+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2-(4+(-9+5*x)**(1/2))**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(-sqrt(sqrt(5*x - 9) + 4) + 2), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-sqrt(sqrt(5*x - 9) + 4) + 2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.560 $\int \frac{1}{\sqrt{2+\sqrt{1+\sqrt{x}}}}\,dx$

Optimal. Leaf size=83

$$\frac{8}{7}\left(\sqrt{\sqrt{x}+1}+2\right)^{7/2}-\frac{48}{5}\left(\sqrt{\sqrt{x}+1}+2\right)^{5/2}+\frac{88}{3}\left(\sqrt{\sqrt{x}+1}+2\right)^{3/2}-48\sqrt{\sqrt{\sqrt{x}+1}+2}$$

```
[Out] -48*Sqrt[2 + Sqrt[1 + Sqrt[x]]] + (88*(2 + Sqrt[1 + Sqrt[x]])^(3/
2))/3 - (48*(2 + Sqrt[1 + Sqrt[x]])^(5/2))/5 + (8*(2 + Sqrt[1 + S
qrt[x]])^(7/2))/7
```

Rubi [A] time = 0.101176, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$\frac{8}{7}\left(\sqrt{\sqrt{x}+1}+2\right)^{7/2}-\frac{48}{5}\left(\sqrt{\sqrt{x}+1}+2\right)^{5/2}+\frac{88}{3}\left(\sqrt{\sqrt{x}+1}+2\right)^{3/2}-48\sqrt{\sqrt{\sqrt{x}+1}+2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[2 + Sqrt[1 + Sqrt[x]]],x]
```

```
[Out] -48*Sqrt[2 + Sqrt[1 + Sqrt[x]]] + (88*(2 + Sqrt[1 + Sqrt[x]])^(3/
2))/3 - (48*(2 + Sqrt[1 + Sqrt[x]])^(5/2))/5 + (8*(2 + Sqrt[1 + S
qrt[x]])^(7/2))/7
```

Rubi in Sympy [A] time = 4.69178, size = 71, normalized size = 0.86

$$\frac{8\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{7}{2}}}{7}-\frac{48\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{5}{2}}}{5}+\frac{88\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{3}{2}}}{3}-48\sqrt{\sqrt{\sqrt{x}+1}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(2+(1+x**(1/2))**(1/2))**(1/2),x)
```

```
[Out] 8*(sqrt(sqrt(x) + 1) + 2)**(7/2)/7 - 48*(sqrt(sqrt(x) + 1) + 2)**
(5/2)/5 + 88*(sqrt(sqrt(x) + 1) + 2)**(3/2)/3 - 48*sqrt(sqrt(sqrt
(x) + 1) + 2)
```

Mathematica [A] time = 0.0162142, size = 58, normalized size = 0.7

$$\frac{8}{105}\sqrt{\sqrt{\sqrt{x}+1}+2}\left(3\sqrt{x}\left(5\sqrt{\sqrt{x}+1}-12\right)+76\sqrt{\sqrt{x}+1}-280\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[2 + Sqrt[1 + Sqrt[x]]],x]
```

```
[Out] (8*Sqrt[2 + Sqrt[1 + Sqrt[x]]]*(-280 + 76*Sqrt[1 + Sqrt[x]] + 3*(
-12 + 5*Sqrt[1 + Sqrt[x]])*Sqrt[x]))/105
```

Maple [A] time = 0., size = 54, normalized size = 0.7

$$\frac{88}{3}\left(2+\sqrt{1+\sqrt{x}}\right)^{\frac{3}{2}}-\frac{48}{5}\left(2+\sqrt{1+\sqrt{x}}\right)^{\frac{5}{2}}+\frac{8}{7}\left(2+\sqrt{1+\sqrt{x}}\right)^{\frac{7}{2}}-48\sqrt{2+\sqrt{1+\sqrt{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(2+(1+x^(1/2))^(1/2))^(1/2),x)
```

```
[Out] 88/3*(2+(1+x^(1/2))^(1/2))^(3/2)-48/5*(2+(1+x^(1/2))^(1/2))^(5/2)
+8/7*(2+(1+x^(1/2))^(1/2))^(7/2)-48*(2+(1+x^(1/2))^(1/2))^(1/2)
```

Maxima [A] time = 0.735367, size = 72, normalized size = 0.87

$$\frac{8}{7}\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{7}{2}}-\frac{48}{5}\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{5}{2}}+\frac{88}{3}\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{3}{2}}-48\sqrt{\sqrt{\sqrt{x}+1}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(sqrt(sqrt(x) + 1) + 2),x, algorithm="maxima")
```

```
[Out] 8/7*(sqrt(sqrt(x) + 1) + 2)^(7/2) - 48/5*(sqrt(sqrt(x) + 1) + 2)^
(5/2) + 88/3*(sqrt(sqrt(x) + 1) + 2)^(3/2) - 48*sqrt(sqrt(sqrt(x)
 + 1) + 2)
```

Fricas [A] time = 0.268483, size = 47, normalized size = 0.57

$$\frac{8}{105}\left(\left(15\sqrt{x}+76\right)\sqrt{\sqrt{x}+1}-36\sqrt{x}-280\right)\sqrt{\sqrt{\sqrt{x}+1}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(sqrt(sqrt(x) + 1) + 2),x, algorithm="fricas")
```

```
[Out] 8/105*((15*sqrt(x) + 76)*sqrt(sqrt(x) + 1) - 36*sqrt(x) - 280)*sq
rt(sqrt(sqrt(x) + 1) + 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\sqrt{\sqrt{x}+1}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(2+(1+x**(1/2))**(1/2))**(1/2),x)
```

```
[Out] Integral(1/sqrt(sqrt(sqrt(x) + 1) + 2), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(sqrt(sqrt(x) + 1) + 2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.561 $\int \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{x}}}}\,dx$

Optimal. Leaf size=190

$$\frac{16}{17}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{17/2}-\frac{112}{15}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{15/2}+\frac{288}{13}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{13/2}-\frac{320}{11}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{11/2}+\frac{112}{9}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{9/2}+\frac{48}{7}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{7/2}-\frac{32}{5}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{5/2}$$

```
[Out] (-32*(1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]])^(5/2))/5 + (48*(1 + Sqrt[1
 + Sqrt[1 + Sqrt[x]]])^(7/2))/7 + (112*(1 + Sqrt[1 + Sqrt[1 + Sqr
t[x]]])^(9/2))/9 - (320*(1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]])^(11/2))
/11 + (288*(1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]])^(13/2))/13 - (112*(1
 + Sqrt[1 + Sqrt[1 + Sqrt[x]]])^(15/2))/15 + (16*(1 + Sqrt[1 + Sq
rt[1 + Sqrt[x]]])^(17/2))/17
```

Rubi [A] time = 0.557395, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$

$$\frac{16}{17}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{17/2}-\frac{112}{15}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{15/2}+\frac{288}{13}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{13/2}-\frac{320}{11}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{11/2}+\frac{112}{9}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{9/2}+\frac{48}{7}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{7/2}-\frac{32}{5}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]]],x]
```


```
[Out] (-32*(1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]])^(5/2))/5 + (48*(1 + Sqrt[1
 + Sqrt[1 + Sqrt[x]]])^(7/2))/7 + (112*(1 + Sqrt[1 + Sqrt[1 + Sqr
t[x]]])^(9/2))/9 - (320*(1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]])^(11/2))
/11 + (288*(1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]])^(13/2))/13 - (112*(1
 + Sqrt[1 + Sqrt[1 + Sqrt[x]]])^(15/2))/15 + (16*(1 + Sqrt[1 + Sq
rt[1 + Sqrt[x]]])^(17/2))/17
```

Rubi in Sympy [A] time = 18.1077, size = 165, normalized size = 0.87

$$\frac{16\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{17}{2}}}{17}-\frac{112\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{15}{2}}}{15}+\frac{288\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{13}{2}}}{13}-\frac{320\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{11}{2}}}{11}+\frac{112\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{9}{2}}}{9}+\frac{48\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{7}{2}}}{7}-\frac{32\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+(1+(1+x**(1/2))**(1/2))**(1/2))**(1/2),x)
```

```
[Out] 16*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)**(17/2)/17 - 112*(sqrt(sqrt(
sqrt(x) + 1) + 1) + 1)**(15/2)/15 + 288*(sqrt(sqrt(sqrt(x) + 1) +
 1) + 1)**(13/2)/13 - 320*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)**(11/
2)/11 + 112*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)**(9/2)/9 + 48*(sqrt
(sqrt(sqrt(x) + 1) + 1) + 1)**(7/2)/7 - 32*(sqrt(sqrt(sqrt(x) + 1
) + 1) + 1)**(5/2)/5
```

Mathematica [A] time = 0.128768, size = 135, normalized size = 0.71

$$\frac{16\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{5/2}\left(231\sqrt{x}\left(-377\sqrt{\sqrt{\sqrt{x}+1}+1}+195\sqrt{\sqrt{x}+1}+365\right)+8\left(252\sqrt{\sqrt{x}+1}\sqrt{\sqrt{\sqrt{x}+1}+1}+8642\sqrt{\sqrt{\ldots}}\right.\right.}{765765}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]]],x]
```

```
[Out] (16*(1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]])^(5/2)*(8*(-8221 + 8642*Sqrt
[1 + Sqrt[1 + Sqrt[x]]] - 4865*Sqrt[1 + Sqrt[x]] + 252*Sqrt[1 + S
qrt[1 + Sqrt[x]]]*Sqrt[1 + Sqrt[x]]) + 231*(365 - 377*Sqrt[1 + Sq
rt[1 + Sqrt[x]]] + 195*Sqrt[1 + Sqrt[x]])*Sqrt[x]))/765765
```

Maple [A] time = 0.019, size = 121, normalized size = 0.6

$$-\frac{32}{5}\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{\frac{5}{2}}+\frac{48}{7}\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{\frac{7}{2}}+\frac{112}{9}\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{\frac{9}{2}}-\frac{320}{11}\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{\frac{11}{2}}+\frac{288}{13}\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{\frac{13}{2}}-\frac{112}{15}\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{\frac{15}{2}}+\frac{16}{17}\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{\frac{17}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+(1+(1+x^(1/2))^(1/2))^(1/2))^(1/2),x)
```

```
[Out] -32/5*(1+(1+(1+x^(1/2))^(1/2))^(1/2))^(5/2)+48/7*(1+(1+(1+x^(1/2)
)^(1/2))^(1/2))^(7/2)+112/9*(1+(1+(1+x^(1/2))^(1/2))^(1/2))^(9/2)
-320/11*(1+(1+(1+x^(1/2))^(1/2))^(1/2))^(11/2)+288/13*(1+(1+(1+x^
(1/2))^(1/2))^(1/2))^(13/2)-112/15*(1+(1+(1+x^(1/2))^(1/2))^(1/2)
)^(15/2)+16/17*(1+(1+(1+x^(1/2))^(1/2))^(1/2))^(17/2)
```

Maxima [A] time = 0.739546, size = 162, normalized size = 0.85

$$\frac{16}{17}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{17}{2}}-\frac{112}{15}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{15}{2}}+\frac{288}{13}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{13}{2}}-\frac{320}{11}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{11}{2}}+\frac{112}{9}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{9}{2}}+\frac{48}{7}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{7}{2}}-\frac{32}{5}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(sqrt(sqrt(sqrt(x) + 1) + 1) + 1),x, algorithm="maxima")
```

```
[Out] 16/17*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)^(17/2) - 112/15*(sqrt(sqr
t(sqrt(x) + 1) + 1) + 1)^(15/2) + 288/13*(sqrt(sqrt(sqrt(x) + 1)
+ 1) + 1)^(13/2) - 320/11*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)^(11/2
) + 112/9*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)^(9/2) + 48/7*(sqrt(sq
rt(sqrt(x) + 1) + 1) + 1)^(7/2) - 32/5*(sqrt(sqrt(sqrt(x) + 1) +
1) + 1)^(5/2)
```

Fricas [A] time = 0.270573, size = 103, normalized size = 0.54

$$\frac{16}{765765}\left(\left(231\sqrt{x}-1304\right)\sqrt{\sqrt{x}+1}+\left(\left(3003\sqrt{x}-4672\right)\sqrt{\sqrt{x}+1}-3528\sqrt{x}+8752\right)\sqrt{\sqrt{\sqrt{x}+1}+1}+45045\,x+4613\sqrt{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(sqrt(sqrt(sqrt(x) + 1) + 1) + 1),x, algorithm="fricas")
```

```
[Out] 16/765765*((231*sqrt(x) - 1304)*sqrt(sqrt(x) + 1) + ((3003*sqrt(x
) - 4672)*sqrt(sqrt(x) + 1) - 3528*sqrt(x) + 8752)*sqrt(sqrt(sqrt
(x) + 1) + 1) + 45045*x + 4613*sqrt(x) - 28152)*sqrt(sqrt(sqrt(sq
rt(x) + 1) + 1) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\sqrt{\sqrt{\sqrt{x}+1}+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+(1+(1+x**(1/2))**(1/2))**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(sqrt(sqrt(sqrt(x) + 1) + 1) + 1), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(sqrt(sqrt(sqrt(x) + 1) + 1) + 1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.562 $\int \sqrt{2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}}\,dx$

Optimal. Leaf size=233

$$\frac{4}{17}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{17/2}-\frac{56}{15}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{15/2}+\frac{300}{13}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{13/2}-\frac{760}{11}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{11/2}+\frac{304}{3}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{9/2}-\frac{480}{7}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{7/2}+\frac{136}{5}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)$$

```
[Out] (-16*(2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]])^(3/2))/3 + (136*(2 + Sq
rt[3 + Sqrt[-1 + 2*Sqrt[x]]])^(5/2))/5 - (480*(2 + Sqrt[3 + Sqrt[
-1 + 2*Sqrt[x]]])^(7/2))/7 + (304*(2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[
x]]])^(9/2))/3 - (760*(2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]])^(11/2)
)/11 + (300*(2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]])^(13/2))/13 - (56
*(2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]])^(15/2))/15 + (4*(2 + Sqrt[3
 + Sqrt[-1 + 2*Sqrt[x]]])^(17/2))/17
```

Rubi [A] time = 0.604226, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.04$

$$\frac{4}{17}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{17/2}-\frac{56}{15}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{15/2}+\frac{300}{13}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{13/2}-\frac{760}{11}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{11/2}+\frac{304}{3}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{9/2}-\frac{480}{7}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{7/2}+\frac{136}{5}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]]],x]
```

```
[Out] (-16*(2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]])^(3/2))/3 + (136*(2 + Sq
rt[3 + Sqrt[-1 + 2*Sqrt[x]]])^(5/2))/5 - (480*(2 + Sqrt[3 + Sqrt[
-1 + 2*Sqrt[x]]])^(7/2))/7 + (304*(2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[
x]]])^(9/2))/3 - (760*(2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]])^(11/2)
)/11 + (300*(2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]])^(13/2))/13 - (56
*(2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]])^(15/2))/15 + (4*(2 + Sqrt[3
 + Sqrt[-1 + 2*Sqrt[x]]])^(17/2))/17
```

Rubi in Sympy [A] time = 19.3475, size = 202, normalized size = 0.87

$$\frac{4\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{17}{2}}}{17}-\frac{56\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{15}{2}}}{15}+\frac{300\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{13}{2}}}{13}-\frac{760\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{11}{2}}}{11}+\frac{304\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{9}{2}}}{3}-\frac{480\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{7}{2}}}{7}+\frac{136\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{5}{2}}}{5}-\frac{16\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2+(3+(-1+2*x**(1/2))**(1/2))**(1/2))**(1/2),x)
```

```
[Out] 4*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)**(17/2)/17 - 56*(sqrt(sqrt(
2*sqrt(x) - 1) + 3) + 2)**(15/2)/15 + 300*(sqrt(sqrt(2*sqrt(x) -
1) + 3) + 2)**(13/2)/13 - 760*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)
**(11/2)/11 + 304*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)**(9/2)/3 -
480*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)**(7/2)/7 + 136*(sqrt(sqrt
(2*sqrt(x) - 1) + 3) + 2)**(5/2)/5 - 16*(sqrt(sqrt(2*sqrt(x) - 1)
 + 3) + 2)**(3/2)/3
```

Mathematica [A] time = 0.163324, size = 183, normalized size = 0.79

$$\frac{8\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{3/2}\left(7\sqrt{x}\left(2145\sqrt{2\sqrt{x}-1}\sqrt{\sqrt{2\sqrt{x}-1}+3}+1452\sqrt{\sqrt{2\sqrt{x}-1}+3}-4004\sqrt{2\sqrt{x}-1}-3576\right)+4\left(38\ldots\right.\right.}{255255}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]]],x]
```

```
[Out] (8*(2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]])^(3/2)*(4*(-9786 - 2535*Sq
rt[3 + Sqrt[-1 + 2*Sqrt[x]]] - 4286*Sqrt[-1 + 2*Sqrt[x]] + 3843*S
qrt[3 + Sqrt[-1 + 2*Sqrt[x]]]*Sqrt[-1 + 2*Sqrt[x]]) + 7*(-3576 +
1452*Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]] - 4004*Sqrt[-1 + 2*Sqrt[x]] +
 2145*Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]]*Sqrt[-1 + 2*Sqrt[x]])*Sqrt[x
]))/255255
```

Maple [A] time = 0.025, size = 154, normalized size = 0.7

$$-\frac{16}{3}\left(2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}\right)^{\frac{3}{2}}+\frac{136}{5}\left(2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}\right)^{\frac{5}{2}}-\frac{480}{7}\left(2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}\right)^{\frac{7}{2}}+\frac{304}{3}\left(2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}\right)^{\frac{9}{2}}-\frac{760}{11}\left(2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}\right)^{\frac{11}{2}}+\frac{300}{13}\left(2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}\right)^{\frac{13}{2}}-\frac{56}{15}\left(2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}\right)^{\frac{15}{2}}+\frac{4}{17}\left(2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}\right)^{\frac{17}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(1/2),x)
```

```
[Out] -16/3*(2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(3/2)+136/5*(2+(3+(-1+2*
x^(1/2))^(1/2))^(1/2))^(5/2)-480/7*(2+(3+(-1+2*x^(1/2))^(1/2))^(1
/2))^(7/2)+304/3*(2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(9/2)-760/11*
(2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(11/2)+300/13*(2+(3+(-1+2*x^(1
/2))^(1/2))^(1/2))^(13/2)-56/15*(2+(3+(-1+2*x^(1/2))^(1/2))^(1/2)
)^(15/2)+4/17*(2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(17/2)
```

Maxima [A] time = 0.727288, size = 207, normalized size = 0.89

$$\frac{4}{17}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{17}{2}}-\frac{56}{15}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{15}{2}}+\frac{300}{13}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{13}{2}}-\frac{760}{11}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{11}{2}}+\frac{304}{3}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{9}{2}}-\frac{480}{7}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{7}{2}}+\frac{136}{5}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{5}{2}}-\frac{16}{3}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2),x, algorithm="maxima")
```

```
[Out] 4/17*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)^(17/2) - 56/15*(sqrt(sqr
t(2*sqrt(x) - 1) + 3) + 2)^(15/2) + 300/13*(sqrt(sqrt(2*sqrt(x) -
 1) + 3) + 2)^(13/2) - 760/11*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)
^(11/2) + 304/3*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)^(9/2) - 480/7
*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)^(7/2) + 136/5*(sqrt(sqrt(2*s
qrt(x) - 1) + 3) + 2)^(5/2) - 16/3*(sqrt(sqrt(2*sqrt(x) - 1) + 3)
```

```
+ 2)^(3/2)
```

Fricas [A] time = 0.272282, size = 115, normalized size = 0.49

$$-\frac{8}{255255}\left(\left(847\sqrt{x}-1688\right)\sqrt{2\sqrt{x}-1}-2\left(\left(1001\sqrt{x}+6800\right)\sqrt{2\sqrt{x}-1}-2352\sqrt{x}-29712\right)\sqrt{\sqrt{2\sqrt{x}-1}+3}-30030\,x+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2),x, algorithm="fricas")
```

```
[Out] -8/255255*((847*sqrt(x) - 1688)*sqrt(2*sqrt(x) - 1) - 2*((1001*sq
rt(x) + 6800)*sqrt(2*sqrt(x) - 1) - 2352*sqrt(x) - 29712)*sqrt(sq
rt(2*sqrt(x) - 1) + 3) - 30030*x + 3843*sqrt(x) + 124080)*sqrt(sq
rt(sqrt(2*sqrt(x) - 1) + 3) + 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\sqrt{\sqrt{2\sqrt{x}-1}+3}+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+(3+(-1+2*x**(1/2))**(1/2))**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.563 $\int \sqrt{1+\sqrt{1+\sqrt{-1+x}}}x\,dx$

Optimal. Leaf size=160

$$\frac{8}{17}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{17/2}-\frac{56}{15}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{15/2}+\frac{144}{13}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{13/2}-\frac{160}{11}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{11/2}+8\left(\sqrt{\sqrt{x-1}+1}+1\right)^{9/2}-\frac{24}{7}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{7/2}+\frac{16}{5}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{5/2}$$

```
[Out] (16*(1 + Sqrt[1 + Sqrt[-1 + x]])^(5/2))/5 - (24*(1 + Sqrt[1 + Sqr
t[-1 + x]])^(7/2))/7 + 8*(1 + Sqrt[1 + Sqrt[-1 + x]])^(9/2) - (16
0*(1 + Sqrt[1 + Sqrt[-1 + x]])^(11/2))/11 + (144*(1 + Sqrt[1 + Sq
rt[-1 + x]])^(13/2))/13 - (56*(1 + Sqrt[1 + Sqrt[-1 + x]])^(15/2)
)/15 + (8*(1 + Sqrt[1 + Sqrt[-1 + x]])^(17/2))/17
```

Rubi [A] time = 0.427808, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$

$$\frac{8}{17}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{17/2}-\frac{56}{15}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{15/2}+\frac{144}{13}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{13/2}-\frac{160}{11}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{11/2}+8\left(\sqrt{\sqrt{x-1}+1}+1\right)^{9/2}-\frac{24}{7}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{7/2}+\frac{16}{5}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 + Sqrt[1 + Sqrt[-1 + x]]]*x,x]
```

```
[Out] (16*(1 + Sqrt[1 + Sqrt[-1 + x]])^(5/2))/5 - (24*(1 + Sqrt[1 + Sqr
t[-1 + x]])^(7/2))/7 + 8*(1 + Sqrt[1 + Sqrt[-1 + x]])^(9/2) - (16
0*(1 + Sqrt[1 + Sqrt[-1 + x]])^(11/2))/11 + (144*(1 + Sqrt[1 + Sq
rt[-1 + x]])^(13/2))/13 - (56*(1 + Sqrt[1 + Sqrt[-1 + x]])^(15/2)
)/15 + (8*(1 + Sqrt[1 + Sqrt[-1 + x]])^(17/2))/17
```

Rubi in Sympy [A] time = 16.6857, size = 139, normalized size = 0.87

$$\frac{8\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{17}{2}}}{17}-\frac{56\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{15}{2}}}{15}+\frac{144\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{13}{2}}}{13}-\frac{160\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{11}{2}}}{11}+8\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{9}{2}}-\frac{24\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{7}{2}}}{7}+\frac{16\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(1+(1+(-1+x)**(1/2))**(1/2))**(1/2),x)
```

```
[Out] 8*(sqrt(sqrt(x - 1) + 1) + 1)**(17/2)/17 - 56*(sqrt(sqrt(x - 1) +
 1) + 1)**(15/2)/15 + 144*(sqrt(sqrt(x - 1) + 1) + 1)**(13/2)/13
- 160*(sqrt(sqrt(x - 1) + 1) + 1)**(11/2)/11 + 8*(sqrt(sqrt(x - 1
) + 1) + 1)**(9/2) - 24*(sqrt(sqrt(x - 1) + 1) + 1)**(7/2)/7 + 16
*(sqrt(sqrt(x - 1) + 1) + 1)**(5/2)/5
```

Mathematica [A] time = 0.105612, size = 103, normalized size = 0.64

$$\frac{8\left(\sqrt{\sqrt{x-1}+1}+1\right)^{5/2}\left(8\left(84\sqrt{x-1}\sqrt{\sqrt{x-1}+1}-3030\sqrt{\sqrt{x-1}+1}+1715\sqrt{x-1}+2591\right)+77\left(-377\sqrt{\sqrt{x-1}+1}+19\right.}{255255}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 + Sqrt[1 + Sqrt[-1 + x]]]*x,x]
```

```
[Out] (8*(1 + Sqrt[1 + Sqrt[-1 + x]])^(5/2)*(8*(2591 - 3030*Sqrt[1 + Sq
rt[-1 + x]] + 1715*Sqrt[-1 + x] + 84*Sqrt[1 + Sqrt[-1 + x]]*Sqrt[
-1 + x]) + 77*(365 - 377*Sqrt[1 + Sqrt[-1 + x]] + 195*Sqrt[-1 + x
])*x))/255255
```

Maple [A] time = 0.012, size = 107, normalized size = 0.7

$$\frac{16\left(1+\sqrt{1+\sqrt{-1+x}}\right)^{\frac{5}{2}}}{5}-\frac{24\left(1+\sqrt{1+\sqrt{-1+x}}\right)^{\frac{7}{2}}}{7}+8\left(1+\sqrt{1+\sqrt{-1+x}}\right)^{9/2}-\frac{160\left(1+\sqrt{1+\sqrt{-1+x}}\right)^{\frac{11}{2}}}{11}+\frac{144\left(1+\sqrt{1+\sqrt{-1+x}}\right)^{\frac{13}{2}}}{13}-\frac{56\left(1+\sqrt{1+\sqrt{-1+x}}\right)^{\frac{15}{2}}}{15}+\frac{8\left(1+\sqrt{1+\sqrt{-1+x}}\right)^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(1+(1+(-1+x)^(1/2))^(1/2))^(1/2),x)
```

```
[Out] 16/5*(1+(1+(-1+x)^(1/2))^(1/2))^(5/2)-24/7*(1+(1+(-1+x)^(1/2))^(1
/2))^(7/2)+8*(1+(1+(-1+x)^(1/2))^(1/2))^(9/2)-160/11*(1+(1+(-1+x)
^(1/2))^(1/2))^(11/2)+144/13*(1+(1+(-1+x)^(1/2))^(1/2))^(13/2)-56
/15*(1+(1+(-1+x)^(1/2))^(1/2))^(15/2)+8/17*(1+(1+(-1+x)^(1/2))^(1
/2))^(17/2)
```

Maxima [A] time = 0.727086, size = 143, normalized size = 0.89

$$\frac{8}{17}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{17}{2}}-\frac{56}{15}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{15}{2}}+\frac{144}{13}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{13}{2}}-\frac{160}{11}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{11}{2}}+8\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{9}{2}}-\frac{24}{7}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{7}{2}}+\frac{16}{5}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sqrt(sqrt(sqrt(x - 1) + 1) + 1),x, algorithm="maxima")
```

```
[Out] 8/17*(sqrt(sqrt(x - 1) + 1) + 1)^(17/2) - 56/15*(sqrt(sqrt(x - 1)
 + 1) + 1)^(15/2) + 144/13*(sqrt(sqrt(x - 1) + 1) + 1)^(13/2) - 1
60/11*(sqrt(sqrt(x - 1) + 1) + 1)^(11/2) + 8*(sqrt(sqrt(x - 1) +
1) + 1)^(9/2) - 24/7*(sqrt(sqrt(x - 1) + 1) + 1)^(7/2) + 16/5*(sq
rt(sqrt(x - 1) + 1) + 1)^(5/2)
```

Fricas [A] time = 0.275021, size = 84, normalized size = 0.52

$$\frac{8}{255255}\left(15015\,x^2+(77\,x+1032)\sqrt{x-1}+\left((1001\,x+4544)\sqrt{x-1}-1176\,x-7696\right)\sqrt{\sqrt{x-1}+1}-1799\,x-22088\right)\sqrt{\sqrt{\sqrt{x-1}+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sqrt(sqrt(sqrt(x - 1) + 1) + 1),x, algorithm="fricas")
```

```
[Out] 8/255255*(15015*x^2 + (77*x + 1032)*sqrt(x - 1) + ((1001*x + 4544
)*sqrt(x - 1) - 1176*x - 7696)*sqrt(sqrt(x - 1) + 1) - 1799*x - 2
2088)*sqrt(sqrt(sqrt(x - 1) + 1) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{\sqrt{\sqrt{x-1}+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(1+(1+(-1+x)**(1/2))**(1/2))**(1/2),x)
```

```
[Out] Integral(x*sqrt(sqrt(sqrt(x - 1) + 1) + 1), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sqrt(sqrt(sqrt(x - 1) + 1) + 1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.564 $\int \frac{1}{\sqrt{-1+x}\sqrt{-\sqrt{-1+x}+x}}\,dx$

Optimal. Leaf size=20

$$-2\sinh^{-1}\left(\frac{1-2\sqrt{x-1}}{\sqrt{3}}\right)$$

```
[Out] -2*ArcSinh[(1 - 2*Sqrt[-1 + x])/Sqrt[3]]
```

Rubi [A] time = 0.156734, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087

$$-2\sinh^{-1}\left(\frac{1-2\sqrt{x-1}}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[-1 + x]*Sqrt[-Sqrt[-1 + x] + x]),x]

[Out] -2*ArcSinh[(1 - 2*Sqrt[-1 + x])/Sqrt[3]]
```

Rubi in Sympy [A] time = 7.05316, size = 26, normalized size = 1.3

$$2\,\text{atanh}\left(\frac{2\sqrt{x-1}-1}{2\sqrt{x-\sqrt{x-1}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-1+x)**(1/2)/(x-(-1+x)**(1/2))**(1/2),x)

[Out] 2*atanh((2*sqrt(x - 1) - 1)/(2*sqrt(x - sqrt(x - 1))))
```

Mathematica [A] time = 0.0160907, size = 20, normalized size = 1.

$$2\sinh^{-1}\left(\frac{2\sqrt{x-1}-1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[-1 + x]*Sqrt[-Sqrt[-1 + x] + x]),x]
```

```
[Out] 2*ArcSinh[(-1 + 2*Sqrt[-1 + x])/Sqrt[3]]
```

Maple [A] time = 0.001, size = 16, normalized size = 0.8

$$2\,\mathit{Arcsinh}\left(2/3\,\sqrt{3}\left(\sqrt{-1+x}-1/2\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-1+x)^(1/2)/(x-(-1+x)^(1/2))^(1/2),x)
```

```
[Out] 2*arcsinh(2/3*3^(1/2)*((-1+x)^(1/2)-1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x-\sqrt{x-1}}\sqrt{x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(x - sqrt(x - 1))*sqrt(x - 1)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x - sqrt(x - 1))*sqrt(x - 1)), x)
```

Fricas [A] time = 0.476913, size = 47, normalized size = 2.35

$$\log\left(4\,\sqrt{x-\sqrt{x-1}}\left(2\,\sqrt{x-1}-1\right)+8\,x-8\,\sqrt{x-1}-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(x - sqrt(x - 1))*sqrt(x - 1)),x, algorithm="fricas")
```

```
[Out] log(4*sqrt(x - sqrt(x - 1))*(2*sqrt(x - 1) - 1) + 8*x - 8*sqrt(x
- 1) - 3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x-1}\sqrt{x-\sqrt{x-1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-1+x)**(1/2)/(x-(-1+x)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x - 1)*sqrt(x - sqrt(x - 1))), x)
```

GIAC/XCAS [A] time = 0.286016, size = 34, normalized size = 1.7

$$-2\ln\left(2\sqrt{x-\sqrt{x-1}}-2\sqrt{x-1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x - sqrt(x - 1))*sqrt(x - 1)),x, algorithm="giac")
```

```
[Out] -2*ln(2*sqrt(x - sqrt(x - 1)) - 2*sqrt(x - 1) + 1)
```

3.565 $\int \frac{1}{\sqrt{1+x+\sqrt{-1+2x}}}\,dx$

Optimal. Leaf size=44

$$2\sqrt{x+\sqrt{2x-1}+1}-\sqrt{2}\sinh^{-1}\left(\frac{\sqrt{2x-1}+1}{\sqrt{2}}\right)$$

```
[Out] 2*Sqrt[1 + x + Sqrt[-1 + 2*x]] - Sqrt[2]*ArcSinh[(1 + Sqrt[-1 + 2
*x])/Sqrt[2]]
```

Rubi [A] time = 0.070403, antiderivative size = 52, normalized size of antiderivative = 1.18, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188

$$2\sqrt{x+\sqrt{2x-1}+1}-\sqrt{2}\sinh^{-1}\left(\frac{\sqrt{2x-1}+1}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[1 + x + Sqrt[-1 + 2*x]],x]
```

```
[Out] 2*Sqrt[1 + x + Sqrt[-1 + 2*x]] - Sqrt[2]*ArcSinh[(1 + Sqrt[-1 + 2
*x])/Sqrt[2]]
```

Rubi in Sympy [A] time = 2.82141, size = 61, normalized size = 1.39

$$\sqrt{2}\sqrt{2x+2\sqrt{2x-1}+2}-\sqrt{2}\operatorname{atanh}\left(\frac{2\sqrt{2x-1}+2}{2\sqrt{2x+2\sqrt{2x-1}+2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1+x+(-1+2*x)**(1/2))**(1/2),x)
```

```
[Out] sqrt(2)*sqrt(2*x + 2*sqrt(2*x - 1) + 2) - sqrt(2)*atanh((2*sqrt(2
*x - 1) + 2)/(2*sqrt(2*x + 2*sqrt(2*x - 1) + 2)))
```

Mathematica [A] time = 0.036288, size = 44, normalized size = 1.

$$2\sqrt{x+\sqrt{2x-1}+1}-\sqrt{2}\sinh^{-1}\left(\frac{\sqrt{2x-1}+1}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[1 + x + Sqrt[-1 + 2*x]],x]
```

```
[Out] 2*Sqrt[1 + x + Sqrt[-1 + 2*x]] - Sqrt[2]*ArcSinh[(1 + Sqrt[-1 + 2
*x])/Sqrt[2]]
```

Maple [A] time = 0.01, size = 38, normalized size = 0.9

$$\sqrt{4x+4+4\sqrt{2x-1}}-Arcsinh\left(\frac{\sqrt{2}}{2}\left(1+\sqrt{2x-1}\right)\right)\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1+x+(2*x-1)^(1/2))^(1/2),x)
```

```
[Out] (4*x+4+4*(2*x-1)^(1/2))^(1/2)-arcsinh(1/2*(1+(2*x-1)^(1/2))*2^(1/
2))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x+\sqrt{2x-1}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(x + sqrt(2*x - 1) + 1),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(x + sqrt(2*x - 1) + 1), x)
```

Fricas [A] time = 0.728963, size = 115, normalized size = 2.61

$$\frac{1}{4}\sqrt{2}\log\left(-8x^2-8(2x+1)\sqrt{2x-1}\right.$$
$$\left.+2\left(\sqrt{2}(2x+3)\sqrt{2x-1}+\sqrt{2}(6x-1)\right)\sqrt{x+\sqrt{2x-1}+1}-24x+7\right)+2\sqrt{x+\sqrt{2x-1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(x + sqrt(2*x - 1) + 1),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log(-8*x^2 - 8*(2*x + 1)*sqrt(2*x - 1) + 2*(sqrt(2)*(
2*x + 3)*sqrt(2*x - 1) + sqrt(2)*(6*x - 1))*sqrt(x + sqrt(2*x - 1
) + 1) - 24*x + 7) + 2*sqrt(x + sqrt(2*x - 1) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x + \sqrt{2x - 1} + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1+x+(-1+2*x)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/sqrt(x + sqrt(2*x - 1) + 1), x)
```

GIAC/XCAS [A] time = 0.273461, size = 92, normalized size = 2.09

$$-\sqrt{2}\left(\sqrt{3} + \ln\left(\sqrt{3} - 1\right)\right) + \sqrt{2}\ln\left(\sqrt{2\,x + 2\,\sqrt{2\,x - 1} + 2} - \sqrt{2\,x - 1} - 1\right) + \sqrt{2}\sqrt{2\,x + 2\,\sqrt{2\,x - 1} + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(x + sqrt(2*x - 1) + 1),x, algorithm="giac")
```

```
[Out] -sqrt(2)*(sqrt(3) + ln(sqrt(3) - 1)) + sqrt(2)*ln(sqrt(2*x + 2*sq
rt(2*x - 1) + 2) - sqrt(2*x - 1) - 1) + sqrt(2)*sqrt(2*x + 2*sqrt
(2*x - 1) + 2)
```

3.566 $\int \frac{q+px}{\sqrt{b+ax}\left(f+\sqrt{b+ax}\right)}\,dx$

Optimal. Leaf size=54

$$-\frac{2\left(-aq+bp+f^2(-p)\right)\log\left(\sqrt{ax+b}+f\right)}{a^2}-\frac{2fp\sqrt{ax+b}}{a^2}+\frac{px}{a}$$

```
[Out] (p*x)/a - (2*f*p*Sqrt[b + a*x])/a^2 - (2*(b*p - f^2*p - a*q)*Log[
f + Sqrt[b + a*x]])/a^2
```

Rubi [A] time = 0.602673, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036

$$-\frac{2\left(-aq+bp+f^2(-p)\right)\log\left(\sqrt{ax+b}+f\right)}{a^2}-\frac{2fp\sqrt{ax+b}}{a^2}+\frac{px}{a}$$

Antiderivative was successfully verified.

```
[In]  Int[(q + p*x)/(Sqrt[b + a*x]*(f + Sqrt[b + a*x])),x]
```

```
[Out] (p*x)/a - (2*f*p*Sqrt[b + a*x])/a^2 - (2*(b*p - f^2*p - a*q)*Log[
f + Sqrt[b + a*x]])/a^2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{2q\log\left(f+\sqrt{ax+b}\right)}{a}-\frac{2p\left(b-f^2\right)\log\left(f+\sqrt{ax+b}\right)}{a^2}-\frac{2p\int^{\sqrt{ax+b}}f\,dx}{a^2}+\frac{2p\int^{\sqrt{ax+b}}x\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((p*x+q)/(a*x+b)**(1/2)/(f+(a*x+b)**(1/2)),x)
```

```
[Out] 2*q*log(f + sqrt(a*x + b))/a - 2*p*(b - f**2)*log(f + sqrt(a*x +
b))/a**2 - 2*p*Integral(f, (x, sqrt(a*x + b)))/a**2 + 2*p*Integra
l(x, (x, sqrt(a*x + b)))/a**2
```

Mathematica [A] time = 0.129915, size = 77, normalized size = 1.43

$$\frac{(aq-bp+f^2p)\log(ax+b-f^2)+2(aq-bp+f^2p)\tanh^{-1}\left(\frac{\sqrt{ax+b}}{f}\right)+p\left(ax-2f\sqrt{ax+b}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(q + p*x)/(Sqrt[b + a*x]*(f + Sqrt[b + a*x])),x]
```

```
[Out] (p*(a*x - 2*f*Sqrt[b + a*x]) + 2*(-(b*p) + f^2*p + a*q)*ArcTanh[S
qrt[b + a*x]/f] + (-(b*p) + f^2*p + a*q)*Log[b - f^2 + a*x])/a^2
```

Maple [A] time = 0.007, size = 80, normalized size = 1.5

$$\frac{px}{a}+\frac{pb}{a^2}-2\frac{fp\sqrt{ax+b}}{a^2}+2\frac{\ln\left(f+\sqrt{ax+b}\right)f^2p}{a^2}+2\frac{\ln\left(f+\sqrt{ax+b}\right)q}{a}-2\frac{\ln\left(f+\sqrt{ax+b}\right)bp}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((p*x+q)/(a*x+b)^(1/2)/(f+(a*x+b)^(1/2)),x)
```

```
[Out] p*x/a+1/a^2*p*b-2*f*p*(a*x+b)^(1/2)/a^2+2/a^2*ln(f+(a*x+b)^(1/2))
*f^2*p+2/a*ln(f+(a*x+b)^(1/2))*q-2/a^2*ln(f+(a*x+b)^(1/2))*b*p
```

Maxima [A] time = 0.715266, size = 78, normalized size = 1.44

$$\frac{\frac{2\left((f^2-b)p+aq\right)\log\left(f+\sqrt{ax+b}\right)}{a}-\frac{2\sqrt{ax+b}fp-(ax+b)p}{a}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((p*x + q)/(sqrt(a*x + b)*(f + sqrt(a*x + b))),x, algorithm="maxima")
```

```
[Out] (2*((f^2 - b)*p + a*q)*log(f + sqrt(a*x + b))/a - (2*sqrt(a*x + b
)*f*p - (a*x + b)*p)/a)/a
```

Fricas [A] time = 0.264479, size = 61, normalized size = 1.13

$$\frac{apx - 2\sqrt{ax+b}fp + 2\left(\left(f^2-b\right)p + aq\right)\log\left(f + \sqrt{ax+b}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x + q)/(sqrt(a*x + b)*(f + sqrt(a*x + b))),x, algorithm="fricas")
```

```
[Out] (a*p*x - 2*sqrt(a*x + b)*f*p + 2*((f^2 - b)*p + a*q)*log(f + sqrt
(a*x + b)))/a^2
```

Sympy [A] time = 11.7899, size = 99, normalized size = 1.83

$$-\frac{2fp\sqrt{ax+b}}{a^2} - \frac{2f\left(-aq+bp-f^2p\right)\left(\begin{cases}\frac{1}{\sqrt{ax+b}} & \text{for } f=0\\ \frac{\log\left(\frac{f}{\sqrt{ax+b}}+1\right)}{f} & \text{otherwise}\end{cases}\right)}{a^2} + \frac{p\,(ax+b)}{a^2} + \frac{2\left(-aq+bp-f^2p\right)\log\left(\frac{1}{\sqrt{ax+b}}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((p*x+q)/(a*x+b)**(1/2)/(f+(a*x+b)**(1/2)),x)
```

```
[Out] -2*f*p*sqrt(a*x + b)/a**2 - 2*f*(-a*q + b*p - f**2*p)*Piecewise((
1/sqrt(a*x + b), Eq(f, 0)), (log(f/sqrt(a*x + b) + 1)/f, True))/a
**2 + p*(a*x + b)/a**2 + 2*(-a*q + b*p - f**2*p)*log(1/sqrt(a*x +
 b))/a**2
```

GIAC/XCAS [A] time = 0.275324, size = 119, normalized size = 2.2

$$\frac{2\left(f^2p - bp + aq\right)\ln\left(\left|f + \sqrt{ax+b}\right|\right)}{a^2} - \frac{2\left(f^2p\ln\left(|f|\right) - bp\ln\left(|f|\right) + aq\ln\left(|f|\right)\right)}{a^2} - \frac{2\sqrt{ax+b}a^2fp - (ax+b)a^2p}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x + q)/(sqrt(a*x + b)*(f + sqrt(a*x + b))),x, algorithm="giac")
```

```
[Out] 2*(f^2*p - b*p + a*q)*ln(abs(f + sqrt(a*x + b)))/a^2 - 2*(f^2*p*l
n(abs(f)) - b*p*ln(abs(f)) + a*q*ln(abs(f)))/a^2 - (2*sqrt(a*x +
b)*a^2*f*p - (a*x + b)*a^2*p)/a^4
```

3.567 $\int \sqrt{1-\sqrt{x}-x}\,dx$

Optimal. Leaf size=70

$$-\frac{2}{3}\left(-x-\sqrt{x}+1\right)^{3/2}-\frac{1}{4}\left(2\sqrt{x}+1\right)\sqrt{-x-\sqrt{x}+1}-\frac{5}{8}\sin^{-1}\left(\frac{2\sqrt{x}+1}{\sqrt{5}}\right)$$

```
[Out] -((1 + 2*Sqrt[x])*Sqrt[1 - Sqrt[x] - x])/4 - (2*(1 - Sqrt[x] - x)
^(3/2))/3 - (5*ArcSin[(1 + 2*Sqrt[x])/Sqrt[5]])/8
```

Rubi [A] time = 0.0702571, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312

$$-\frac{2}{3}\left(-x-\sqrt{x}+1\right)^{3/2}-\frac{1}{4}\left(2\sqrt{x}+1\right)\sqrt{-x-\sqrt{x}+1}-\frac{5}{8}\sin^{-1}\left(\frac{2\sqrt{x}+1}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 - Sqrt[x] - x],x]
```

```
[Out] -((1 + 2*Sqrt[x])*Sqrt[1 - Sqrt[x] - x])/4 - (2*(1 - Sqrt[x] - x)
^(3/2))/3 - (5*ArcSin[(1 + 2*Sqrt[x])/Sqrt[5]])/8
```

Rubi in Sympy [A] time = 2.67464, size = 68, normalized size = 0.97

$$-\frac{\left(2\sqrt{x}+1\right)\sqrt{-\sqrt{x}-x+1}}{4}-\frac{2\left(-\sqrt{x}-x+1\right)^{\frac{3}{2}}}{3}-\frac{5\,\mathrm{atan}\left(-\frac{-2\sqrt{x}-1}{2\sqrt{-\sqrt{x}-x+1}}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1-x-x**(1/2))**(1/2),x)
```

```
[Out] -(2*sqrt(x) + 1)*sqrt(-sqrt(x) - x + 1)/4 - 2*(-sqrt(x) - x + 1)*
*(3/2)/3 - 5*atan(-(-2*sqrt(x) - 1)/(2*sqrt(-sqrt(x) - x + 1)))/8
```

Mathematica [A] time = 0.0414724, size = 53, normalized size = 0.76

$$\frac{1}{12}\sqrt{-x-\sqrt{x}+1}\left(8x+2\sqrt{x}-11\right)+\frac{5}{8}\sin^{-1}\left(\frac{-2\sqrt{x}-1}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 - Sqrt[x] - x],x]
```

```
[Out] (Sqrt[1 - Sqrt[x] - x]*(-11 + 2*Sqrt[x] + 8*x))/12 + (5*ArcSin[(-
1 - 2*Sqrt[x])/Sqrt[5]])/8
```

Maple [A] time = 0.009, size = 50, normalized size = 0.7

$$-\frac{2}{3}\left(1-x-\sqrt{x}\right)^{\frac{3}{2}}+\frac{1}{4}\left(-2\sqrt{x}-1\right)\sqrt{1-x-\sqrt{x}}-\frac{5}{8}\arcsin\left(\frac{2\sqrt{5}}{5}\left(\sqrt{x}+\frac{1}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1-x-x^(1/2))^(1/2),x)
```

```
[Out] -2/3*(1-x-x^(1/2))^(3/2)+1/4*(-2*x^(1/2)-1)*(1-x-x^(1/2))^(1/2)-5
/8*arcsin(2/5*5^(1/2)*(x^(1/2)+1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-x-\sqrt{x}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x - sqrt(x) + 1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x - sqrt(x) + 1), x)
```

Fricas [A] time = 0.977254, size = 82, normalized size = 1.17

$$\frac{1}{12}\left(8x+2\sqrt{x}-11\right)\sqrt{-x-\sqrt{x}+1}-\frac{5}{16}\arctan\left(\frac{8x+8\sqrt{x}-3}{4\sqrt{-x-\sqrt{x}+1}\left(2\sqrt{x}+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x - sqrt(x) + 1),x, algorithm="fricas")
```

```
[Out] 1/12*(8*x + 2*sqrt(x) - 11)*sqrt(-x - sqrt(x) + 1) - 5/16*arctan(
1/4*(8*x + 8*sqrt(x) - 3)/(sqrt(-x - sqrt(x) + 1)*(2*sqrt(x) + 1)
))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-\sqrt{x} - x + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1-x-x**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(-sqrt(x) - x + 1), x)
```

GIAC/XCAS [A] time = 0.274965, size = 59, normalized size = 0.84

$$\frac{1}{12}\left(2\sqrt{x}\left(4\sqrt{x}+1\right)-11\right)\sqrt{-x-\sqrt{x}+1}-\frac{5}{8}\arcsin\left(\frac{1}{5}\sqrt{5}\left(2\sqrt{x}+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x - sqrt(x) + 1),x, algorithm="giac")
```

```
[Out] 1/12*(2*sqrt(x)*(4*sqrt(x) + 1) - 11)*sqrt(-x - sqrt(x) + 1) - 5/
8*arcsin(1/5*sqrt(5)*(2*sqrt(x) + 1))
```

3.568 $\int \frac{9+6\sqrt{x}+x}{4\sqrt{x}+x} \, dx$

Optimal. Leaf size=19

$$x + 4\sqrt{x} + 2\log\left(\sqrt{x} + 4\right)$$

```
[Out] 4*Sqrt[x] + x + 2*Log[4 + Sqrt[x]]
```

Rubi [A] time = 0.0410605, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$x + 4\sqrt{x} + 2\log\left(\sqrt{x} + 4\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(9 + 6*Sqrt[x] + x)/(4*Sqrt[x] + x),x]
```

```
[Out] 4*Sqrt[x] + x + 2*Log[4 + Sqrt[x]]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$4\sqrt{x} + 2\log\left(\sqrt{x} + 4\right) + 2\int^{\sqrt{x}} x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((9+x+6*x**(1/2))/(x+4*x**(1/2)),x)
```

```
[Out] 4*sqrt(x) + 2*log(sqrt(x) + 4) + 2*Integral(x, (x, sqrt(x)))
```

Mathematica [A] time = 0.0116253, size = 19, normalized size = 1.

$$x + 4\sqrt{x} + 2\log\left(\sqrt{x} + 4\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(9 + 6*Sqrt[x] + x)/(4*Sqrt[x] + x),x]
```

```
[Out] 4*Sqrt[x] + x + 2*Log[4 + Sqrt[x]]
```

Maple [A] time = 0.004, size = 16, normalized size = 0.8

$$x + 2 \ln \left(4 + \sqrt{x}\right) + 4 \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((9+x+6*x^(1/2))/(x+4*x^(1/2)),x)
```

```
[Out] x+2*ln(4+x^(1/2))+4*x^(1/2)
```

Maxima [A] time = 0.717994, size = 20, normalized size = 1.05

$$x + 4 \sqrt{x} + 2 \log \left(\sqrt{x} + 4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x + 6*sqrt(x) + 9)/(x + 4*sqrt(x)),x, algorithm="maxima")
```

```
[Out] x + 4*sqrt(x) + 2*log(sqrt(x) + 4)
```

Fricas [A] time = 0.266666, size = 20, normalized size = 1.05

$$x + 4 \sqrt{x} + 2 \log \left(\sqrt{x} + 4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x + 6*sqrt(x) + 9)/(x + 4*sqrt(x)),x, algorithm="fricas")
```

```
[Out] x + 4*sqrt(x) + 2*log(sqrt(x) + 4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{6 \sqrt{x} + x + 9}{4 \sqrt{x} + x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((9+x+6*x**(1/2))/(x+4*x**(1/2)),x)

[Out] Integral((6*sqrt(x) + x + 9)/(4*sqrt(x) + x), x)
```

GIAC/XCAS [A] time = 0.267112, size = 20, normalized size = 1.05

$$x + 4\sqrt{x} + 2\ln\left(\sqrt{x} + 4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 6*sqrt(x) + 9)/(x + 4*sqrt(x)),x, algorithm="giac")

[Out] x + 4*sqrt(x) + 2*ln(sqrt(x) + 4)
```

3.569 $\int \frac{6-8x^{7/2}}{5-9\sqrt{x}}\,dx$

Optimal. Leaf size=77

$$\frac{80x^{7/2}}{567}+\frac{400x^{5/2}}{6561}+\frac{50000x^{3/2}}{1594323}+\frac{2x^4}{9}+\frac{200x^3}{2187}+\frac{2500x^2}{59049}+\frac{125000x}{4782969}-\frac{56145628\sqrt{x}}{43046721}-\frac{280728140\log\left(5-9\sqrt{x}\right)}{387420489}$$

```
[Out] (-56145628*Sqrt[x])/43046721 + (125000*x)/4782969 + (50000*x^(3/2
))/1594323 + (2500*x^2)/59049 + (400*x^(5/2))/6561 + (200*x^3)/21
87 + (80*x^(7/2))/567 + (2*x^4)/9 - (280728140*Log[5 - 9*Sqrt[x]]
)/387420489
```

Rubi [A] time = 0.115842, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19

$$\frac{80x^{7/2}}{567}+\frac{400x^{5/2}}{6561}+\frac{50000x^{3/2}}{1594323}+\frac{2x^4}{9}+\frac{200x^3}{2187}+\frac{2500x^2}{59049}+\frac{125000x}{4782969}-\frac{56145628\sqrt{x}}{43046721}-\frac{280728140\log\left(5-9\sqrt{x}\right)}{387420489}$$

Antiderivative was successfully verified.

```
[In]  Int[(6 - 8*x^(7/2))/(5 - 9*Sqrt[x]),x]
```

```
[Out] (-56145628*Sqrt[x])/43046721 + (125000*x)/4782969 + (50000*x^(3/2
))/1594323 + (2500*x^2)/59049 + (400*x^(5/2))/6561 + (200*x^3)/21
87 + (80*x^(7/2))/567 + (2*x^4)/9 - (280728140*Log[5 - 9*Sqrt[x]]
)/387420489
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{80x^{\frac{7}{2}}}{567}+\frac{400x^{\frac{5}{2}}}{6561}+\frac{50000x^{\frac{3}{2}}}{1594323}+\frac{2x^4}{9}+\frac{200x^3}{2187}+\frac{2500x^2}{59049}-\frac{280728140\log\left(-9\sqrt{x}+5\right)}{387420489}+16\int^{\sqrt{x}}\frac{78125}{43046721}\,dx-12\int^{\sqrt{x}}\frac{1}{9}\,dx+\frac{250000\int^{\sqrt{x}}x\,dx}{4782969}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((6-8*x**(7/2))/(5-9*x**(1/2)),x)
```

```
[Out] 80*x**(7/2)/567 + 400*x**(5/2)/6561 + 50000*x**(3/2)/1594323 + 2*
x**4/9 + 200*x**3/2187 + 2500*x**2/59049 - 280728140*log(-9*sqrt(
x) + 5)/387420489 + 16*Integral(78125/43046721, (x, sqrt(x))) - 1
2*Integral(1/9, (x, sqrt(x))) + 250000*Integral(x, (x, sqrt(x)))/
4782969
```

Mathematica [A] time = 0.0322572, size = 66, normalized size = 0.86

$$\frac{2\left(9\left(21257640x^{7/2}+9185400x^{5/2}+4725000x^{3/2}+33480783x^4+13778100x^3+6378750x^2+3937500x-196509698\sqrt{x}\right)-\right.}{2711943423}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(6 - 8*x^(7/2))/(5 - 9*Sqrt[x]),x]
```

```
[Out] (2*(9*(-196509698*Sqrt[x] + 3937500*x + 4725000*x^(3/2) + 6378750
*x^2 + 9185400*x^(5/2) + 13778100*x^3 + 21257640*x^(7/2) + 334807
83*x^4) - 982548490*Log[5 - 9*Sqrt[x]]))/2711943423
```

Maple [A] time = 0.006, size = 50, normalized size = 0.7

$$\frac{2\,x^4}{9}+\frac{80}{567}x^{\frac{7}{2}}+\frac{200\,x^3}{2187}+\frac{400}{6561}x^{\frac{5}{2}}+\frac{2500\,x^2}{59049}+\frac{50000}{1594323}x^{\frac{3}{2}}+\frac{125000\,x}{4782969}-\frac{56145628}{43046721}\sqrt{x}-\frac{280728140}{387420489}\ln\left(-5+9\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((6-8*x^(7/2))/(5-9*x^(1/2)),x)
```

```
[Out] 2/9*x^4+80/567*x^(7/2)+200/2187*x^3+400/6561*x^(5/2)+2500/59049*x
^2+50000/1594323*x^(3/2)+125000/4782969*x-56145628/43046721*x^(1/
2)-280728140/387420489*ln(-5+9*x^(1/2))
```

Maxima [A] time = 0.722884, size = 66, normalized size = 0.86

$$\frac{2}{9}\,x^4+\frac{80}{567}\,x^{\frac{7}{2}}+\frac{200}{2187}\,x^3+\frac{400}{6561}\,x^{\frac{5}{2}}+\frac{2500}{59049}\,x^2+\frac{50000}{1594323}\,x^{\frac{3}{2}}+\frac{125000}{4782969}\,x-\frac{56145628}{43046721}\,\sqrt{x}-\frac{280728140}{387420489}\,\log\left(9\sqrt{x}-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(2*(4*x^(7/2) - 3)/(9*sqrt(x) - 5),x, algorithm="maxima")
```

```
[Out] 2/9*x^4 + 80/567*x^(7/2) + 200/2187*x^3 + 400/6561*x^(5/2) + 2500
/59049*x^2 + 50000/1594323*x^(3/2) + 125000/4782969*x - 56145628/
43046721*sqrt(x) - 280728140/387420489*log(9*sqrt(x) - 5)
```

Fricas [A] time = 0.265334, size = 66, normalized size = 0.86

$$\frac{2}{9}x^4 + \frac{200}{2187}x^3 + \frac{2500}{59049}x^2 + \frac{4}{301327047}\left(10628820\,x^3 + 4592700\,x^2 + 2362500\,x - 98254849\right)\sqrt{x} + \frac{125000}{4782969}x - \frac{280728140}{387420489}\log\left(9\sqrt{x} - 5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(2*(4*x^(7/2) - 3)/(9*sqrt(x) - 5),x, algorithm="fricas")
```

```
[Out] 2/9*x^4 + 200/2187*x^3 + 2500/59049*x^2 + 4/301327047*(10628820*x
^3 + 4592700*x^2 + 2362500*x - 98254849)*sqrt(x) + 125000/4782969
*x - 280728140/387420489*log(9*sqrt(x) - 5)
```

Sympy [A] time = 10.3873, size = 71, normalized size = 0.92

$$\frac{80x^{\frac{7}{2}}}{567} + \frac{400x^{\frac{5}{2}}}{6561} + \frac{50000x^{\frac{3}{2}}}{1594323} - \frac{56145628\sqrt{x}}{43046721} + \frac{2x^4}{9} + \frac{200x^3}{2187} + \frac{2500x^2}{59049} + \frac{125000x}{4782969} - \frac{280728140\log\left(\sqrt{x} - \frac{5}{9}\right)}{387420489}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((6-8*x**(7/2))/(5-9*x**(1/2)),x)
```

```
[Out] 80*x**(7/2)/567 + 400*x**(5/2)/6561 + 50000*x**(3/2)/1594323 - 56
145628*sqrt(x)/43046721 + 2*x**4/9 + 200*x**3/2187 + 2500*x**2/59
049 + 125000*x/4782969 - 280728140*log(sqrt(x) - 5/9)/387420489
```

GIAC/XCAS [A] time = 0.266366, size = 68, normalized size = 0.88

$$\frac{2}{9}x^4 + \frac{80}{567}x^{\frac{7}{2}} + \frac{200}{2187}x^3 + \frac{400}{6561}x^{\frac{5}{2}} + \frac{2500}{59049}x^2 + \frac{50000}{1594323}x^{\frac{3}{2}} + \frac{125000}{4782969}x - \frac{56145628}{43046721}\sqrt{x} - \frac{280728140}{387420489}\ln\left(\left|9\sqrt{x} - 5\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(2*(4*x^(7/2) - 3)/(9*sqrt(x) - 5),x, algorithm="giac")
```

```
[Out] 2/9*x^4 + 80/567*x^(7/2) + 200/2187*x^3 + 400/6561*x^(5/2) + 2500
/59049*x^2 + 50000/1594323*x^(3/2) + 125000/4782969*x - 56145628/
43046721*sqrt(x) - 280728140/387420489*ln(abs(9*sqrt(x) - 5))
```

3.570 $\int \frac{\sqrt{1+x}\left(1+x^3\right)}{1+x^2} \, dx$

Optimal. Leaf size=80

$$\frac{2}{5}(x+1)^{5/2}-\frac{2}{3}(x+1)^{3/2}-2\sqrt{x+1}+(1-i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{1-i}}\right)+(1+i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{1+i}}\right)$$

```
[Out] -2*Sqrt[1 + x] - (2*(1 + x)^(3/2))/3 + (2*(1 + x)^(5/2))/5 + (1 -
 I)^(3/2)*ArcTanh[Sqrt[1 + x]/Sqrt[1 - I]] + (1 + I)^(3/2)*ArcTan
h[Sqrt[1 + x]/Sqrt[1 + I]]
```

Rubi [B] time = 0.566475, antiderivative size = 224, normalized size of antiderivative = 2.8, number of steps used = 16, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$\frac{2}{5}(x+1)^{5/2}-\frac{2}{3}(x+1)^{3/2}-2\sqrt{x+1}-\frac{\log\left(x-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}+1\right)}{2\sqrt{1+\sqrt{2}}}+\frac{\log\left(x+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}+1\right)}{2\sqrt{1+\sqrt{2}}}-\sqrt{1+\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{x+1}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)+\sqrt{1+\sqrt{2}}\tan^{-1}\left(\frac{2\sqrt{x+1}+\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)$$

Warning: Unable to verify antiderivative.

```
[In]  Int[(Sqrt[1 + x]*(1 + x^3))/(1 + x^2),x]
```

```
[Out] -2*Sqrt[1 + x] - (2*(1 + x)^(3/2))/3 + (2*(1 + x)^(5/2))/5 - Sqrt
[1 + Sqrt[2]]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] - 2*Sqrt[1 + x])/Sqrt
[2*(-1 + Sqrt[2])]] + Sqrt[1 + Sqrt[2]]*ArcTan[(Sqrt[2*(1 + Sqrt[
2])] + 2*Sqrt[1 + x])/Sqrt[2*(-1 + Sqrt[2])]] - Log[1 + Sqrt[2] +
 x - Sqrt[2*(1 + Sqrt[2])]*Sqrt[1 + x]]/(2*Sqrt[1 + Sqrt[2]]) + L
og[1 + Sqrt[2] + x + Sqrt[2*(1 + Sqrt[2])]*Sqrt[1 + x]]/(2*Sqrt[1
 + Sqrt[2]])
```

Rubi in Sympy [A] time = 27.6651, size = 211, normalized size = 2.64

$$\frac{2(x+1)^{\frac{5}{2}}}{5}-\frac{2(x+1)^{\frac{3}{2}}}{3}-2\sqrt{x+1}-\frac{\log\left(x-\sqrt{2}\sqrt{1+\sqrt{2}}\sqrt{x+1}+1+\sqrt{2}\right)}{2\sqrt{1+\sqrt{2}}}+\frac{\log\left(x+\sqrt{2}\sqrt{1+\sqrt{2}}\sqrt{x+1}+1+\sqrt{2}\right)}{2\sqrt{1+\sqrt{2}}}+\frac{\operatorname{atan}\left(\frac{\sqrt{2}\left(\sqrt{x+1}-\frac{\sqrt{2+2\sqrt{2}}}{2}\right)}{\sqrt{-1+\sqrt{2}}}\right)}{\sqrt{-1+\sqrt{2}}}+\frac{\operatorname{atan}\left(\frac{\sqrt{2}\left(\sqrt{x+1}+\frac{\sqrt{2+2\sqrt{2}}}{2}\right)}{\sqrt{-1+\sqrt{2}}}\right)}{\sqrt{-1+\sqrt{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**3+1)*(1+x)**(1/2)/(x**2+1),x)
```

```
[Out] 2*(x + 1)**(5/2)/5 - 2*(x + 1)**(3/2)/3 - 2*sqrt(x + 1) - log(x -
 sqrt(2)*sqrt(1 + sqrt(2))*sqrt(x + 1) + 1 + sqrt(2))/(2*sqrt(1 +
 sqrt(2))) + log(x + sqrt(2)*sqrt(1 + sqrt(2))*sqrt(x + 1) + 1 +
sqrt(2))/(2*sqrt(1 + sqrt(2))) + atan(sqrt(2)*(sqrt(x + 1) - sqrt
(2 + 2*sqrt(2))/2)/sqrt(-1 + sqrt(2)))/sqrt(-1 + sqrt(2)) + atan(
sqrt(2)*(sqrt(x + 1) + sqrt(2 + 2*sqrt(2))/2)/sqrt(-1 + sqrt(2)))
/sqrt(-1 + sqrt(2))
```

Mathematica [A] time = 0.097183, size = 70, normalized size = 0.88

$$\frac{2}{15}\sqrt{x+1}\left(3x^2+x-17\right)-(-1-i)^{3/2}\tan^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{-1-i}}\right)-(-1+i)^{3/2}\tan^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{-1+i}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[1 + x]*(1 + x^3))/(1 + x^2),x]
```

```
[Out] (2*Sqrt[1 + x]*(-17 + x + 3*x^2))/15 - (-1 - I)^(3/2)*ArcTan[Sqrt
[1 + x]/Sqrt[-1 - I]] - (-1 + I)^(3/2)*ArcTan[Sqrt[1 + x]/Sqrt[-1
 + I]]
```

Maple [B] time = 0.046, size = 443, normalized size = 5.5

$$\frac{2}{5}(1+x)^{\frac{5}{2}}-\frac{2}{3}(1+x)^{\frac{3}{2}}-2\sqrt{1+x}+\frac{\sqrt{2+2\sqrt{2}}\sqrt{2}}{4}\ln\left(1+x+\sqrt{2}-\sqrt{1+x}\sqrt{2+2\sqrt{2}}\right)$$
$$-\frac{\sqrt{2+2\sqrt{2}}}{2}\ln\left(1+x+\sqrt{2}-\sqrt{1+x}\sqrt{2+2\sqrt{2}}\right)$$
$$+\frac{\left(2+2\sqrt{2}\right)\sqrt{2}}{2\sqrt{-2+2\sqrt{2}}}\arctan\left(\frac{1}{\sqrt{-2+2\sqrt{2}}}\left(2\sqrt{1+x}-\sqrt{2+2\sqrt{2}}\right)\right)$$
$$-\frac{2+2\sqrt{2}}{\sqrt{-2+2\sqrt{2}}}\arctan\left(\frac{1}{\sqrt{-2+2\sqrt{2}}}\left(2\sqrt{1+x}-\sqrt{2+2\sqrt{2}}\right)\right)$$
$$+2\frac{\sqrt{2}}{\sqrt{-2+2\sqrt{2}}}\arctan\left(\frac{2\sqrt{1+x}-\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)$$
$$-\frac{\sqrt{2+2\sqrt{2}}\sqrt{2}}{4}\ln\left(1+x+\sqrt{2}+\sqrt{1+x}\sqrt{2+2\sqrt{2}}\right)$$
$$+\frac{\sqrt{2+2\sqrt{2}}}{2}\ln\left(1+x+\sqrt{2}+\sqrt{1+x}\sqrt{2+2\sqrt{2}}\right)$$
$$+\frac{\left(2+2\sqrt{2}\right)\sqrt{2}}{2\sqrt{-2+2\sqrt{2}}}\arctan\left(\frac{1}{\sqrt{-2+2\sqrt{2}}}\left(2\sqrt{1+x}+\sqrt{2+2\sqrt{2}}\right)\right)$$
$$-\frac{2+2\sqrt{2}}{\sqrt{-2+2\sqrt{2}}}\arctan\left(\frac{1}{\sqrt{-2+2\sqrt{2}}}\left(2\sqrt{1+x}+\sqrt{2+2\sqrt{2}}\right)\right)$$
$$+2\frac{\sqrt{2}}{\sqrt{-2+2\sqrt{2}}}\arctan\left(\frac{2\sqrt{1+x}+\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((x^3+1)*(1+x)^(1/2)/(x^2+1),x)
```

```
[Out] 2/5*(1+x)^(5/2)-2/3*(1+x)^(3/2)-2*(1+x)^(1/2)+1/4*ln(1+x+2^(1/2)-
(1+x)^(1/2)*(2+2*2^(1/2))^(1/2))*(2+2*2^(1/2))^(1/2)*2^(1/2)-1/2*
(2+2*2^(1/2))^(1/2)*ln(1+x+2^(1/2)-(1+x)^(1/2)*(2+2*2^(1/2))^(1/2
))+1/2/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+x)^(1/2)-(2+2*2^(1/2))^(
1/2))/(-2+2*2^(1/2))^(1/2))*(2+2*2^(1/2))*2^(1/2)-1/(-2+2*2^(1/2)
)^(1/2)*arctan((2*(1+x)^(1/2)-(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))
^(1/2))*(2+2*2^(1/2))+2/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+x)^(1/2
)-(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*2^(1/2)-1/4*ln(1+x+2
^(1/2)+(1+x)^(1/2)*(2+2*2^(1/2))^(1/2))*(2+2*2^(1/2))^(1/2)*2^(1/
2)+1/2*(2+2*2^(1/2))^(1/2)*ln(1+x+2^(1/2)+(1+x)^(1/2)*(2+2*2^(1/2
))^(1/2))+1/2/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+x)^(1/2)+(2+2*2^(
1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*(2+2*2^(1/2))*2^(1/2)-1/(-2+2*
2^(1/2))^(1/2)*arctan((2*(1+x)^(1/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2
^(1/2))^(1/2))*(2+2*2^(1/2))+2/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+
```

```
x)^(1/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^3+1)\sqrt{x+1}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + 1)*sqrt(x + 1)/(x^2 + 1),x, algorithm="maxima")
```

```
[Out] integrate((x^3 + 1)*sqrt(x + 1)/(x^2 + 1), x)
```

Fricas [A] time = 0.285619, size = 582, normalized size = 7.28

$$-\frac{\sqrt{2}\left(4\sqrt{2}\left(3x^2-\sqrt{2}\left(3x^2+x-17\right)+x-17\right)\sqrt{x+1}\sqrt{\frac{\sqrt{2}-2}{2\sqrt{2}-3}}-15\cdot 8^{\frac{1}{4}}\left(\sqrt{2}-1\right)\log\left(2\cdot 8^{\frac{1}{4}}\sqrt{2}\sqrt{x+1}\sqrt{\frac{\sqrt{2}-2}{2\sqrt{2}-3}}+4x+4\sqrt{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + 1)*sqrt(x + 1)/(x^2 + 1),x, algorithm="fricas")
```

```
[Out] -1/60*sqrt(2)*(4*sqrt(2)*(3*x^2 - sqrt(2)*(3*x^2 + x - 17) + x -
17)*sqrt(x + 1)*sqrt((sqrt(2) - 2)/(2*sqrt(2) - 3)) - 15*8^(1/4)*
(sqrt(2) - 1)*log(2*8^(1/4)*sqrt(2)*sqrt(x + 1)*sqrt((sqrt(2) - 2
)/(2*sqrt(2) - 3)) + 4*x + 4*sqrt(2) + 4) + 15*8^(1/4)*(sqrt(2) -
 1)*log(-2*8^(1/4)*sqrt(2)*sqrt(x + 1)*sqrt((sqrt(2) - 2)/(2*sqrt
(2) - 3)) + 4*x + 4*sqrt(2) + 4) - 60*8^(1/4)*arctan(8^(1/4)*(sqr
t(2) - 2)/(sqrt(2)*sqrt(2*8^(1/4)*sqrt(2)*sqrt(x + 1)*sqrt((sqrt(
2) - 2)/(2*sqrt(2) - 3)) + 4*x + 4*sqrt(2) + 4)*(sqrt(2) - 1)*sqr
t((sqrt(2) - 2)/(2*sqrt(2) - 3)) + 2*sqrt(2)*sqrt(x + 1)*(sqrt(2)
 - 1)*sqrt((sqrt(2) - 2)/(2*sqrt(2) - 3)) + 8^(1/4)*sqrt(2))) - 6
0*8^(1/4)*arctan(8^(1/4)*(sqrt(2) - 2)/(sqrt(2)*sqrt(-2*8^(1/4)*s
qrt(2)*sqrt(x + 1)*sqrt((sqrt(2) - 2)/(2*sqrt(2) - 3)) + 4*x + 4*
sqrt(2) + 4)*(sqrt(2) - 1)*sqrt((sqrt(2) - 2)/(2*sqrt(2) - 3)) +
2*sqrt(2)*sqrt(x + 1)*(sqrt(2) - 1)*sqrt((sqrt(2) - 2)/(2*sqrt(2)
 - 3)) - 8^(1/4)*sqrt(2))))/((sqrt(2) - 1)*sqrt((sqrt(2) - 2)/(2*
sqrt(2) - 3)))
```

Sympy [A] time = 14.3451, size = 56, normalized size = 0.7

$$\frac{2(x+1)^{\frac{5}{2}}}{5} - \frac{2(x+1)^{\frac{3}{2}}}{3} - 2\sqrt{x+1} + 4\operatorname{RootSum}\left(512t^4 + 32t^2 + 1, \left(t \mapsto t\log\left(-128t^3 + \sqrt{x+1}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3+1)*(1+x)**(1/2)/(x**2+1),x)
```

```
[Out] 2*(x + 1)**(5/2)/5 - 2*(x + 1)**(3/2)/3 - 2*sqrt(x + 1) + 4*RootS
um(512*_t**4 + 32*_t**2 + 1, Lambda(_t, _t*log(-128*_t**3 + sqrt(
x + 1))))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^3+1)\sqrt{x+1}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + 1)*sqrt(x + 1)/(x^2 + 1),x, algorithm="giac")
```

```
[Out] integrate((x^3 + 1)*sqrt(x + 1)/(x^2 + 1), x)
```

3.571 $\int \frac{\sqrt{-1-\sqrt{x}+x}}{(-1+x)\sqrt{x}}\,dx$

Optimal. Leaf size=89

$$\tan^{-1}\left(\frac{3-\sqrt{x}}{2\sqrt{x-\sqrt{x}-1}}\right)-2\tanh^{-1}\left(\frac{1-2\sqrt{x}}{2\sqrt{x-\sqrt{x}-1}}\right)-\tanh^{-1}\left(\frac{3\sqrt{x}+1}{2\sqrt{x-\sqrt{x}-1}}\right)$$

```
[Out] ArcTan[(3 - Sqrt[x])/(2*Sqrt[-1 - Sqrt[x] + x])] - 2*ArcTanh[(1 -
 2*Sqrt[x])/(2*Sqrt[-1 - Sqrt[x] + x])] - ArcTanh[(1 + 3*Sqrt[x])
/(2*Sqrt[-1 - Sqrt[x] + x])]
```

Rubi [A] time = 0.470015, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24

$$\tan^{-1}\left(\frac{3-\sqrt{x}}{2\sqrt{x-\sqrt{x}-1}}\right)-2\tanh^{-1}\left(\frac{1-2\sqrt{x}}{2\sqrt{x-\sqrt{x}-1}}\right)-\tanh^{-1}\left(\frac{3\sqrt{x}+1}{2\sqrt{x-\sqrt{x}-1}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[-1 - Sqrt[x] + x]/((-1 + x)*Sqrt[x]),x]
```

```
[Out] ArcTan[(3 - Sqrt[x])/(2*Sqrt[-1 - Sqrt[x] + x])] - 2*ArcTanh[(1 -
 2*Sqrt[x])/(2*Sqrt[-1 - Sqrt[x] + x])] - ArcTanh[(1 + 3*Sqrt[x])
/(2*Sqrt[-1 - Sqrt[x] + x])]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$2\int^{\sqrt{x}}\left(-\frac{\sqrt{x^2-\sqrt{x^2}-1}}{2\,(x+1)}\right)dx+2\int^{\sqrt{x}}\frac{\sqrt{x^2-\sqrt{x^2}-1}}{2x-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-1+x-x**(1/2))**(1/2)/(-1+x)/x**(1/2),x)
```

```
[Out] 2*Integral(-sqrt(x**2 - sqrt(x**2) - 1)/(2*(x + 1)), (x, sqrt(x))
) + 2*Integral(sqrt(x**2 - sqrt(x**2) - 1)/(2*x - 2), (x, sqrt(x)
))
```

Mathematica [A] time = 0.0186521, size = 93, normalized size = 1.04

$$-\log \left(\sqrt{x}+1\right)+2 \log \left(-2 \sqrt{x}-2 \sqrt{x-\sqrt{x}-1}+1\right)+\log \left(3 \sqrt{x}-2 \sqrt{x-\sqrt{x}-1}+1\right)-\tan ^{-1}\left(\frac{\sqrt{x}-3}{2 \sqrt{x-\sqrt{x}-1}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[-1 - Sqrt[x] + x]/((-1 + x)*Sqrt[x]),x]
```

```
[Out] -ArcTan[(-3 + Sqrt[x])/(2*Sqrt[-1 - Sqrt[x] + x])] - Log[1 + Sqrt
[x]] + 2*Log[1 - 2*Sqrt[x] - 2*Sqrt[-1 - Sqrt[x] + x]] + Log[1 +
3*Sqrt[x] - 2*Sqrt[-1 - Sqrt[x] + x]]
```

Maple [A] time = 0.017, size = 130, normalized size = 1.5

$$-\sqrt{\left(1+\sqrt{x}\right)^{2}-2-3 \sqrt{x}}+\frac{3}{2} \ln \left(-\frac{1}{2}+\sqrt{x}+\sqrt{\left(1+\sqrt{x}\right)^{2}-2-3 \sqrt{x}}\right)+\mathit{Artanh}\left(\frac{1}{2}\left(-1-3 \sqrt{x}\right) \frac{1}{\sqrt{\left(1+\sqrt{x}\right)^{2}-2-3 \sqrt{x}}}\right)+\sqrt{\left(-1+\sqrt{x}\right)^{2}+\sqrt{x}-2}+\frac{1}{2} \ln \left(-\frac{1}{2}+\sqrt{x}+\sqrt{\left(-1+\sqrt{x}\right)^{2}+\sqrt{x}-2}\right)-\arctan \left(\frac{1}{2}\left(\sqrt{x}-3\right) \frac{1}{\sqrt{\left(-1+\sqrt{x}\right)^{2}+\sqrt{x}-2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-1+x-x^(1/2))^(1/2)/(-1+x)/x^(1/2),x)
```

```
[Out] -((1+x^(1/2))^2-2-3*x^(1/2))^(1/2)+3/2*ln(-1/2+x^(1/2)+((1+x^(1/2
))^2-2-3*x^(1/2))^(1/2))+arctanh(1/2*(-1-3*x^(1/2))/((1+x^(1/2))^
2-2-3*x^(1/2))^(1/2))+((-1+x^(1/2))^2+x^(1/2)-2)^(1/2)+1/2*ln(-1/
2+x^(1/2)+((-1+x^(1/2))^2+x^(1/2)-2)^(1/2))-arctan(1/2*(x^(1/2)-3
)/((-1+x^(1/2))^2+x^(1/2)-2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x-\sqrt{x}-1}}{(x-1)\sqrt{x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x - sqrt(x) - 1)/((x - 1)*sqrt(x)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x - sqrt(x) - 1)/((x - 1)*sqrt(x)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x - sqrt(x) - 1)/((x - 1)*sqrt(x)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-\sqrt{x} + x - 1}}{\sqrt{x}\,(x - 1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x-x**(1/2))**(1/2)/(-1+x)/x**(1/2),x)
```

```
[Out] Integral(sqrt(-sqrt(x) + x - 1)/(sqrt(x)*(x - 1)), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x - sqrt(x) - 1)/((x - 1)*sqrt(x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.572 $\int \frac{1+2\sqrt{1+x}}{x\sqrt{1+x}\sqrt{x+\sqrt{1+x}}} \, dx$

Optimal. Leaf size=61

$$3\tanh^{-1}\left(\frac{1-3\sqrt{x+1}}{2\sqrt{x+\sqrt{x+1}}}\right)-\tan^{-1}\left(\frac{\sqrt{x+1}+3}{2\sqrt{x+\sqrt{x+1}}}\right)$$

```
[Out] -ArcTan[(3 + Sqrt[1 + x])/(2*Sqrt[x + Sqrt[1 + x]])] + 3*ArcTanh[
(1 - 3*Sqrt[1 + x])/(2*Sqrt[x + Sqrt[1 + x]])]
```

Rubi [A] time = 0.889224, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114

$$3\tanh^{-1}\left(\frac{1-3\sqrt{x+1}}{2\sqrt{x+\sqrt{x+1}}}\right)-\tan^{-1}\left(\frac{\sqrt{x+1}+3}{2\sqrt{x+\sqrt{x+1}}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + 2*Sqrt[1 + x])/(x*Sqrt[1 + x]*Sqrt[x + Sqrt[1 + x]]),x]
```

```
[Out] -ArcTan[(3 + Sqrt[1 + x])/(2*Sqrt[x + Sqrt[1 + x]])] + 3*ArcTanh[
(1 - 3*Sqrt[1 + x])/(2*Sqrt[x + Sqrt[1 + x]])]
```

Rubi in Sympy [A] time = 71.7827, size = 78, normalized size = 1.28

$$-2\,\mathrm{atan}\left(-\frac{-\sqrt{x+1}-3}{2\sqrt{x+\sqrt{x+1}}}\right)+\mathrm{atan}\left(\frac{\sqrt{x+1}+3}{2\sqrt{x+\sqrt{x+1}}}\right)-3\,\mathrm{atanh}\left(\frac{3\sqrt{x+1}-1}{2\sqrt{x+\sqrt{x+1}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((1+2*(1+x)**(1/2))/x/(1+x)**(1/2)/(x+(1+x)**(1/2))**(1/2),x)
```

```
[Out] -2*atan(-(-sqrt(x + 1) - 3)/(2*sqrt(x + sqrt(x + 1)))) + atan((sq
rt(x + 1) + 3)/(2*sqrt(x + sqrt(x + 1)))) - 3*atanh((3*sqrt(x + 1
) - 1)/(2*sqrt(x + sqrt(x + 1))))
```

Mathematica [A] time = 0.029641, size = 73, normalized size = 1.2

$$3\log\left(1-\sqrt{x+1}\right)-3\log\left(-3\sqrt{x+1}-2\sqrt{x+\sqrt{x+1}}+1\right)-\tan^{-1}\left(\frac{\sqrt{x+1}+3}{2\sqrt{x+\sqrt{x+1}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*Sqrt[1 + x])/(x*Sqrt[1 + x]*Sqrt[x + Sqrt[1 + x]]),x]
```

```
[Out] -ArcTan[(3 + Sqrt[1 + x])/(2*Sqrt[x + Sqrt[1 + x]])] + 3*Log[1 -
Sqrt[1 + x]] - 3*Log[1 - 3*Sqrt[1 + x] - 2*Sqrt[x + Sqrt[1 + x]]]
```

Maple [A] time = 0.021, size = 68, normalized size = 1.1

$$\arctan\left(\frac{1}{2}\left(-3-\sqrt{1+x}\right)\frac{1}{\sqrt{\left(1+\sqrt{1+x}\right)^{2}-2-\sqrt{1+x}}}\right)-3\,Artanh\left(1/2\,\frac{-1+3\,\sqrt{1+x}}{\sqrt{\left(\sqrt{1+x}-1\right)^{2}+3\,\sqrt{1+x}-2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*(1+x)^(1/2))/x/(1+x)^(1/2)/(x+(1+x)^(1/2))^(1/2),x)
```

```
[Out] arctan(1/2*(-3-(1+x)^(1/2))/((1+(1+x)^(1/2))^2-2-(1+x)^(1/2))^(1/
2))-3*arctanh(1/2*(-1+3*(1+x)^(1/2))/(((1+x)^(1/2)-1)^2+3*(1+x)^(
1/2)-2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{2\sqrt{x+1}+1}{\sqrt{x+\sqrt{x+1}}\sqrt{x+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*sqrt(x + 1) + 1)/(sqrt(x + sqrt(x + 1))*sqrt(x + 1)*x),x, algorithm='
```

```
[Out] integrate((2*sqrt(x + 1) + 1)/(sqrt(x + sqrt(x + 1))*sqrt(x + 1)*
x), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*sqrt(x + 1) + 1)/(sqrt(x + sqrt(x + 1))*sqrt(x + 1)*x),x, algorithm='
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2\sqrt{x+1}+1}{x\sqrt{x+1}\sqrt{x+\sqrt{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*(1+x)**(1/2))/x/(1+x)**(1/2)/(x+(1+x)**(1/2))**(1/2),x)
```

```
[Out] Integral((2*sqrt(x + 1) + 1)/(x*sqrt(x + 1)*sqrt(x + sqrt(x + 1))
), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*sqrt(x + 1) + 1)/(sqrt(x + sqrt(x + 1))*sqrt(x + 1)*x),x, algorithm='
```

```
[Out] Timed out
```

3.573 $\int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx$

Optimal. Leaf size=8

$$2\sinh^{-1}\left(\sqrt{x}\right)$$

```
[Out] 2*ArcSinh[Sqrt[x]]
```

Rubi [A] time = 0.00914415, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$2\sinh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[x]*Sqrt[1 + x]),x]

[Out] 2*ArcSinh[Sqrt[x]]
```

Rubi in Sympy [A] time = 1.04692, size = 7, normalized size = 0.88

$$2\,\mathrm{asinh}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**(1/2)/(1+x)**(1/2),x)

[Out] 2*asinh(sqrt(x))
```

Mathematica [A] time = 0.00561474, size = 8, normalized size = 1.

$$2\sinh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[x]*Sqrt[1 + x]),x]
```

```
[Out] 2*ArcSinh[Sqrt[x]]
```

Maple [B] time = 0.005, size = 28, normalized size = 3.5

$$1\sqrt{x(1+x)}\ln\left(\frac{1}{2}+x+\sqrt{x^2+x}\right)\frac{1}{\sqrt{x}}\frac{1}{\sqrt{1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^(1/2)/(1+x)^(1/2),x)
```

```
[Out] (x*(1+x))^(1/2)/x^(1/2)/(1+x)^(1/2)*ln(1/2+x+(x^2+x)^(1/2))
```

Maxima [A] time = 0.722956, size = 36, normalized size = 4.5

$$\log\left(\frac{\sqrt{x+1}}{\sqrt{x}}+1\right)-\log\left(\frac{\sqrt{x+1}}{\sqrt{x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x + 1)*sqrt(x)),x, algorithm="maxima")
```

```
[Out] log(sqrt(x + 1)/sqrt(x) + 1) - log(sqrt(x + 1)/sqrt(x) - 1)
```

Fricas [A] time = 0.300116, size = 24, normalized size = 3.

$$-\log\left(2\sqrt{x+1}\sqrt{x}-2x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x + 1)*sqrt(x)),x, algorithm="fricas")
```

```
[Out] -log(2*sqrt(x + 1)*sqrt(x) - 2*x - 1)
```

Sympy [A] time = 3.57079, size = 26, normalized size = 3.25

$$\begin{cases} 2\operatorname{acosh}\left(\sqrt{x+1}\right) & \text{for } |x+1| > 1 \\ -2i\operatorname{asin}\left(\sqrt{x+1}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**(1/2)/(1+x)**(1/2),x)
```

```
[Out] Piecewise((2*acosh(sqrt(x + 1)), Abs(x + 1) > 1), (-2*I*asin(sqrt
(x + 1)), True))
```

GIAC/XCAS [A] time = 0.308758, size = 20, normalized size = 2.5

$$-2\ln\left(\left|-\sqrt{x+1}+\sqrt{x}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x + 1)*sqrt(x)),x, algorithm="giac")
```

```
[Out] -2*ln(abs(-sqrt(x + 1) + sqrt(x)))
```

3.574 $\int \frac{\sqrt{\frac{x}{1+x}}}{x} \, dx$

Optimal. Leaf size=8

$$2 \sinh^{-1}\left(\sqrt{x}\right)$$

```
[Out] 2*ArcSinh[Sqrt[x]]
```

Rubi [A] time = 0.0217844, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$2 \sinh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x/(1 + x)]/x,x]

[Out] 2*ArcSinh[Sqrt[x]]
```

Rubi in Sympy [A] time = 1.65361, size = 7, normalized size = 0.88

$$2 \operatorname{asinh}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x/(1+x))**(1/2)/x,x)

[Out] 2*asinh(sqrt(x))
```

Mathematica [A] time = 0.0097038, size = 8, normalized size = 1.

$$2 \sinh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[x/(1 + x)]/x,x]
```

```
[Out] 2*ArcSinh[Sqrt[x]]
```

Maple [B] time = 0.015, size = 32, normalized size = 4.

$$(1+x)\sqrt{\frac{x}{1+x}}\ln\left(\frac{1}{2}+x+\sqrt{x^2+x}\right)\frac{1}{\sqrt{x(1+x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x/(1+x))^(1/2)/x,x)

[Out] (x/(1+x))^(1/2)/(x*(1+x))^(1/2)*(1+x)*ln(1/2+x+(x^2+x)^(1/2))
```

Maxima [A] time = 0.722257, size = 36, normalized size = 4.5

$$\log\left(\sqrt{\frac{x}{x+1}}+1\right)-\log\left(\sqrt{\frac{x}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x/(x + 1))/x,x, algorithm="maxima")

[Out] log(sqrt(x/(x + 1)) + 1) - log(sqrt(x/(x + 1)) - 1)
```

Fricas [A] time = 0.273894, size = 36, normalized size = 4.5

$$\log\left(\sqrt{\frac{x}{x+1}}+1\right)-\log\left(\sqrt{\frac{x}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x/(x + 1))/x,x, algorithm="fricas")

[Out] log(sqrt(x/(x + 1)) + 1) - log(sqrt(x/(x + 1)) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{x}{x+1}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x/(1+x))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(x/(x + 1))/x, x)
```

GIAC/XCAS [A] time = 0.26987, size = 30, normalized size = 3.75

$$-\ln\left(\left|-2\,x + 2\,\sqrt{x^2 + x} - 1\right|\right)\operatorname{sign}\left(x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x/(x + 1))/x,x, algorithm="giac")
```

```
[Out] -ln(abs(-2*x + 2*sqrt(x^2 + x) - 1))*sign(x + 1)
```

3.575 $\int \frac{\sqrt{x}}{\sqrt{1+x}} dx$

Optimal. Leaf size=22

$$\sqrt{x}\sqrt{x+1} - \sinh^{-1}\left(\sqrt{x}\right)$$

```
[Out] Sqrt[x]*Sqrt[1 + x] - ArcSinh[Sqrt[x]]
```

Rubi [A] time = 0.0143864, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$\sqrt{x}\sqrt{x+1} - \sinh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x]/Sqrt[1 + x],x]

[Out] Sqrt[x]*Sqrt[1 + x] - ArcSinh[Sqrt[x]]
```

Rubi in Sympy [A] time = 1.45867, size = 17, normalized size = 0.77

$$\sqrt{x}\sqrt{x+1} - \operatorname{asinh}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(1/2)/(1+x)**(1/2),x)

[Out] sqrt(x)*sqrt(x + 1) - asinh(sqrt(x))
```

Mathematica [A] time = 0.0275221, size = 42, normalized size = 1.91

$$\frac{\sqrt{\frac{x}{x+1}}\left(\sqrt{x}(x+1) - \sqrt{x+1}\sinh^{-1}\left(\sqrt{x}\right)\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[x]/Sqrt[1 + x],x]
```

```
[Out] (Sqrt[x/(1 + x)]*(Sqrt[x]*(1 + x) - Sqrt[1 + x]*ArcSinh[Sqrt[x]])
)/Sqrt[x]
```

Maple [B] time = 0.005, size = 39, normalized size = 1.8

$$\sqrt{x}\sqrt{1+x} - \frac{1}{2}\sqrt{x(1+x)}\ln\left(\frac{1}{2}+x+\sqrt{x^2+x}\right)\frac{1}{\sqrt{x}}\frac{1}{\sqrt{1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(1/2)/(1+x)^(1/2),x)
```

```
[Out] x^(1/2)*(1+x)^(1/2)-1/2*(x*(1+x))^(1/2)/x^(1/2)/(1+x)^(1/2)*ln(1/
2+x+(x^2+x)^(1/2))
```

Maxima [A] time = 0.720052, size = 66, normalized size = 3.

$$\frac{\sqrt{x+1}}{\sqrt{x}\left(\frac{x+1}{x}-1\right)} - \frac{1}{2}\log\left(\frac{\sqrt{x+1}}{\sqrt{x}}+1\right) + \frac{1}{2}\log\left(\frac{\sqrt{x+1}}{\sqrt{x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/sqrt(x + 1),x, algorithm="maxima")
```

```
[Out] sqrt(x + 1)/(sqrt(x)*((x + 1)/x - 1)) - 1/2*log(sqrt(x + 1)/sqrt(
x) + 1) + 1/2*log(sqrt(x + 1)/sqrt(x) - 1)
```

Fricas [A] time = 0.273559, size = 104, normalized size = 4.73

$$-\frac{2(4x+1)\sqrt{x+1}\sqrt{x} - 8x^2 - 2\left(2\sqrt{x+1}\sqrt{x}-2x-1\right)\log\left(2\sqrt{x+1}\sqrt{x}-2x-1\right) - 6x + 1}{4\left(2\sqrt{x+1}\sqrt{x}-2x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/sqrt(x + 1),x, algorithm="fricas")
```

```
[Out] -1/4*(2*(4*x + 1)*sqrt(x + 1)*sqrt(x) - 8*x^2 - 2*(2*sqrt(x + 1)*
sqrt(x) - 2*x - 1)*log(2*sqrt(x + 1)*sqrt(x) - 2*x - 1) - 6*x + 1
```

```
)/(2*sqrt(x + 1)*sqrt(x) - 2*x - 1)
```

Sympy [A] time = 5.70949, size = 60, normalized size = 2.73

$$\begin{cases} -\operatorname{acosh}\left(\sqrt{x+1}\right) + \frac{(x+1)^{\frac{3}{2}}}{\sqrt{x}} - \frac{\sqrt{x+1}}{\sqrt{x}} & \text{for } |x+1| > 1 \\ i\sqrt{-x}\sqrt{x+1} + i\operatorname{asin}\left(\sqrt{x+1}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(1/2)/(1+x)**(1/2),x)
```

```
[Out] Piecewise((-acosh(sqrt(x + 1)) + (x + 1)**(3/2)/sqrt(x) - sqrt(x
+ 1)/sqrt(x), Abs(x + 1) > 1), (I*sqrt(-x)*sqrt(x + 1) + I*asin(s
qrt(x + 1)), True))
```

GIAC/XCAS [A] time = 0.293066, size = 31, normalized size = 1.41

$$\sqrt{x+1}\sqrt{x} + \ln\left(\left|-\sqrt{x+1} + \sqrt{x}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/sqrt(x + 1),x, algorithm="giac")
```

```
[Out] sqrt(x + 1)*sqrt(x) + ln(abs(-sqrt(x + 1) + sqrt(x)))
```

3.576 $\int \sqrt{\frac{x}{1+x}}\,dx$

Optimal. Leaf size=22

$$\sqrt{x}\sqrt{x+1}-\sinh^{-1}\left(\sqrt{x}\right)$$

```
[Out] Sqrt[x]*Sqrt[1 + x] - ArcSinh[Sqrt[x]]
```

Rubi [A] time = 0.0167364, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364

$$\sqrt{x}\sqrt{x+1}-\sinh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x/(1 + x)],x]
```

```
[Out] Sqrt[x]*Sqrt[1 + x] - ArcSinh[Sqrt[x]]
```

Rubi in Sympy [A] time = 1.97574, size = 24, normalized size = 1.09

$$\frac{\sqrt{\frac{x}{x+1}}}{-\frac{x}{x+1}+1}-\operatorname{atanh}\left(\sqrt{\frac{x}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x/(1+x))**(1/2),x)
```

```
[Out] sqrt(x/(x + 1))/(-x/(x + 1) + 1) - atanh(sqrt(x/(x + 1)))
```

Mathematica [A] time = 0.00576897, size = 42, normalized size = 1.91

$$\frac{\sqrt{\frac{x}{x+1}}\left(\sqrt{x}(x+1)-\sqrt{x+1}\sinh^{-1}\left(\sqrt{x}\right)\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[x/(1 + x)],x]
```

```
[Out] (Sqrt[x/(1 + x)]*(Sqrt[x]*(1 + x) - Sqrt[1 + x]*ArcSinh[Sqrt[x]])
)/Sqrt[x]
```

Maple [B] time = 0.005, size = 45, normalized size = 2.1

$$\frac{1+x}{2}\sqrt{\frac{x}{1+x}}\left(2\sqrt{x^2+x}-\ln\left(\frac{1}{2}+x+\sqrt{x^2+x}\right)\right)\frac{1}{\sqrt{x(1+x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x/(1+x))^(1/2),x)
```

```
[Out] 1/2*(x/(1+x))^(1/2)*(1+x)*(2*(x^2+x)^(1/2)-ln(1/2+x+(x^2+x)^(1/2)
))/(x*(1+x))^(1/2)
```

Maxima [A] time = 0.713359, size = 69, normalized size = 3.14

$$-\frac{\sqrt{\frac{x}{x+1}}}{\frac{x}{x+1}-1}-\frac{1}{2}\log\left(\sqrt{\frac{x}{x+1}}+1\right)+\frac{1}{2}\log\left(\sqrt{\frac{x}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x/(x + 1)),x, algorithm="maxima")
```

```
[Out] -sqrt(x/(x + 1))/(x/(x + 1) - 1) - 1/2*log(sqrt(x/(x + 1)) + 1) +
 1/2*log(sqrt(x/(x + 1)) - 1)
```

Fricas [A] time = 0.272926, size = 57, normalized size = 2.59

$$(x+1)\sqrt{\frac{x}{x+1}}-\frac{1}{2}\log\left(\sqrt{\frac{x}{x+1}}+1\right)+\frac{1}{2}\log\left(\sqrt{\frac{x}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x/(x + 1)),x, algorithm="fricas")
```

```
[Out] (x + 1)*sqrt(x/(x + 1)) - 1/2*log(sqrt(x/(x + 1)) + 1) + 1/2*log(
sqrt(x/(x + 1)) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{x}{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x/(1+x))**(1/2),x)
```

```
[Out] Integral(sqrt(x/(x + 1)), x)
```

GIAC/XCAS [A] time = 0.272301, size = 47, normalized size = 2.14

$$\frac{1}{2}\ln\left(\left|-2\,x+2\,\sqrt{x^2+x}-1\right|\right)\,\mathrm{sign}\left(x+1\right)+\sqrt{x^2+x}\mathrm{sign}\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x/(x + 1)),x, algorithm="giac")
```

```
[Out] 1/2*ln(abs(-2*x + 2*sqrt(x^2 + x) - 1))*sign(x + 1) + sqrt(x^2 +
x)*sign(x + 1)
```

3.577 $\int \frac{\sqrt{-1+x}}{x^2\sqrt{1+x}}\,dx$

Optimal. Leaf size=36

$$\tan^{-1}\left(\sqrt{x-1}\sqrt{x+1}\right)-\frac{\sqrt{x-1}\sqrt{x+1}}{x}$$

```
[Out] -((Sqrt[-1 + x]*Sqrt[1 + x])/x) + ArcTan[Sqrt[-1 + x]*Sqrt[1 + x]
]
```

Rubi [A] time = 0.0438559, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\tan^{-1}\left(\sqrt{x-1}\sqrt{x+1}\right)-\frac{\sqrt{x-1}\sqrt{x+1}}{x}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[-1 + x]/(x^2*Sqrt[1 + x]),x]

[Out] -((Sqrt[-1 + x]*Sqrt[1 + x])/x) + ArcTan[Sqrt[-1 + x]*Sqrt[1 + x]
]
```

Rubi in Sympy [A] time = 2.1202, size = 29, normalized size = 0.81

$$\operatorname{atan}\left(\sqrt{x-1}\sqrt{x+1}\right)-\frac{\sqrt{x-1}\sqrt{x+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-1+x)**(1/2)/x**2/(1+x)**(1/2),x)

[Out] atan(sqrt(x - 1)*sqrt(x + 1)) - sqrt(x - 1)*sqrt(x + 1)/x
```

Mathematica [A] time = 0.0458833, size = 62, normalized size = 1.72

$$-\frac{\sqrt{\frac{x-1}{x+1}}\left(\sqrt{x-1}(x+1)+x\sqrt{x+1}\tan^{-1}\left(\frac{1}{\sqrt{x-1}\sqrt{x+1}}\right)\right)}{\sqrt{x-1}x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[-1 + x]/(x^2*Sqrt[1 + x]),x]
```

```
[Out] -((Sqrt[(-1 + x)/(1 + x)]*(Sqrt[-1 + x]*(1 + x) + x*Sqrt[1 + x]*A
rcTan[1/(Sqrt[-1 + x]*Sqrt[1 + x])]))/(Sqrt[-1 + x]*x))
```

Maple [A] time = 0.022, size = 43, normalized size = 1.2

$$\frac{1}{x}\left(-\arctan\left(\frac{1}{\sqrt{x^2-1}}\right)x-\sqrt{x^2-1}\right)\sqrt{-1+x}\sqrt{1+x}\frac{1}{\sqrt{x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-1+x)^(1/2)/x^2/(1+x)^(1/2),x)
```

```
[Out] (-arctan(1/(x^2-1)^(1/2))*x-(x^2-1)^(1/2))*(-1+x)^(1/2)*(1+x)^(1/
2)/x/(x^2-1)^(1/2)
```

Maxima [A] time = 0.79807, size = 27, normalized size = 0.75

$$-\frac{\sqrt{x^2-1}}{x}-\arcsin\left(\frac{1}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x - 1)/(sqrt(x + 1)*x^2),x, algorithm="maxima")
```

```
[Out] -sqrt(x^2 - 1)/x - arcsin(1/abs(x))
```

Fricas [A] time = 0.295791, size = 80, normalized size = 2.22

$$\frac{2\left(\sqrt{x+1}\sqrt{x-1}x-x^2\right)\arctan\left(\sqrt{x+1}\sqrt{x-1}-x\right)+1}{\sqrt{x+1}\sqrt{x-1}x-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x - 1)/(sqrt(x + 1)*x^2),x, algorithm="fricas")
```

```
[Out] (2*(sqrt(x + 1)*sqrt(x - 1)*x - x^2)*arctan(sqrt(x + 1)*sqrt(x -
1) - x) + 1)/(sqrt(x + 1)*sqrt(x - 1)*x - x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x-1}}{x^2\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-1+x)**(1/2)/x**2/(1+x)**(1/2),x)
```

```
[Out] Integral(sqrt(x - 1)/(x**2*sqrt(x + 1)), x)
```

GIAC/XCAS [A] time = 0.271331, size = 57, normalized size = 1.58

$$-\frac{8}{\left(\sqrt{x+1}-\sqrt{x-1}\right)^4+4}-2\arctan\left(\frac{1}{2}\left(\sqrt{x+1}-\sqrt{x-1}\right)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x - 1)/(sqrt(x + 1)*x^2),x, algorithm="giac")
```

```
[Out] -8/((sqrt(x + 1) - sqrt(x - 1))^4 + 4) - 2*arctan(1/2*(sqrt(x + 1
) - sqrt(x - 1))^2)
```

3.578 $\int \frac{\sqrt{\frac{-1+x}{1+x}}}{x^2}\,dx$

Optimal. Leaf size=36

$$\tan^{-1}\left(\sqrt{x-1}\sqrt{x+1}\right)-\frac{\sqrt{x-1}\sqrt{x+1}}{x}$$

```
[Out] -((Sqrt[-1 + x]*Sqrt[1 + x])/x) + ArcTan[Sqrt[-1 + x]*Sqrt[1 + x]
]
```

Rubi [A] time = 0.0563378, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235

$$\tan^{-1}\left(\sqrt{x-1}\sqrt{x+1}\right)-\frac{\sqrt{x-1}\sqrt{x+1}}{x}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[(-1 + x)/(1 + x)]/x^2,x]
```

```
[Out] -((Sqrt[-1 + x]*Sqrt[1 + x])/x) + ArcTan[Sqrt[-1 + x]*Sqrt[1 + x]
]
```

Rubi in Sympy [A] time = 2.54127, size = 29, normalized size = 0.81

$$\operatorname{atan}\left(\sqrt{x-1}\sqrt{x+1}\right)-\frac{\sqrt{x-1}\sqrt{x+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((-1+x)/(1+x))**(1/2)/x**2,x)
```

```
[Out] atan(sqrt(x - 1)*sqrt(x + 1)) - sqrt(x - 1)*sqrt(x + 1)/x
```

Mathematica [A] time = 0.0138223, size = 62, normalized size = 1.72

$$-\frac{\sqrt{\frac{x-1}{x+1}}\left(\sqrt{x-1}(x+1)+x\sqrt{x+1}\tan^{-1}\left(\frac{1}{\sqrt{x-1}\sqrt{x+1}}\right)\right)}{\sqrt{x-1}x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[(-1 + x)/(1 + x)]/x^2,x]
```

```
[Out] -((Sqrt[(-1 + x)/(1 + x)]*(Sqrt[-1 + x]*(1 + x) + x*Sqrt[1 + x]*A
rcTan[1/(Sqrt[-1 + x]*Sqrt[1 + x])]))/(Sqrt[-1 + x]*x))
```

Maple [B] time = 0.02, size = 59, normalized size = 1.6

$$\frac{1+x}{x}\sqrt{\frac{-1+x}{1+x}}\left(\left(x^2-1\right)^{\frac{3}{2}}-x^2\sqrt{x^2-1}-\arctan\left(\frac{1}{\sqrt{x^2-1}}\right)x\right)\frac{1}{\sqrt{(-1+x)(1+x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((-1+x)/(1+x))^(1/2)/x^2,x)
```

```
[Out] ((-1+x)/(1+x))^(1/2)*(1+x)*((x^2-1)^(3/2)-x^2*(x^2-1)^(1/2)-arcta
n(1/(x^2-1)^(1/2))*x)/((-1+x)*(1+x))^(1/2)/x
```

Maxima [A] time = 0.797029, size = 55, normalized size = 1.53

$$-\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{x-1}{x+1}+1}+2\arctan\left(\sqrt{\frac{x-1}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((x - 1)/(x + 1))/x^2,x, algorithm="maxima")
```

```
[Out] -2*sqrt((x - 1)/(x + 1))/((x - 1)/(x + 1) + 1) + 2*arctan(sqrt((x
 - 1)/(x + 1)))
```

Fricas [A] time = 0.284026, size = 49, normalized size = 1.36

$$\frac{2\,x\arctan\left(\sqrt{\frac{x-1}{x+1}}\right)-(x+1)\sqrt{\frac{x-1}{x+1}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((x - 1)/(x + 1))/x^2,x, algorithm="fricas")
```

```
[Out] (2*x*arctan(sqrt((x - 1)/(x + 1))) - (x + 1)*sqrt((x - 1)/(x + 1)
))/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{x-1}{x+1}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((-1+x)/(1+x))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt((x - 1)/(x + 1))/x**2, x)
```

GIAC/XCAS [A] time = 0.269447, size = 69, normalized size = 1.92

$$-\frac{1}{2}(\pi-2)\mathrm{sign}(x+1)+2\arctan\left(-x+\sqrt{x^2-1}\right)\mathrm{sign}(x+1)-\frac{2\,\mathrm{sign}(x+1)}{\left(x-\sqrt{x^2-1}\right)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((x - 1)/(x + 1))/x^2,x, algorithm="giac")
```

```
[Out] -1/2*(pi - 2)*sign(x + 1) + 2*arctan(-x + sqrt(x^2 - 1))*sign(x +
 1) - 2*sign(x + 1)/((x - sqrt(x^2 - 1))^2 + 1)
```

3.579 $\int \frac{\sqrt{-1+x}x^3}{\sqrt{1+x}}\,dx$

Optimal. Leaf size=69

$$\frac{1}{4}(x-1)^{3/2}\sqrt{x+1}x^2+\frac{1}{24}(7-2x)(x-1)^{3/2}\sqrt{x+1}-\frac{3}{8}\sqrt{x-1}\sqrt{x+1}+\frac{3}{8}\cosh^{-1}(x)$$

```
[Out] (-3*Sqrt[-1 + x]*Sqrt[1 + x])/8 + ((7 - 2*x)*(-1 + x)^(3/2)*Sqrt[
1 + x])/24 + ((-1 + x)^(3/2)*x^2*Sqrt[1 + x])/4 + (3*ArcCosh[x])/
8
```

Rubi [A] time = 0.0708545, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$\frac{1}{4}(x-1)^{3/2}\sqrt{x+1}x^2+\frac{1}{24}(7-2x)(x-1)^{3/2}\sqrt{x+1}-\frac{3}{8}\sqrt{x-1}\sqrt{x+1}+\frac{3}{8}\cosh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + x]*x^3)/Sqrt[1 + x],x]
```


```
[Out] (-3*Sqrt[-1 + x]*Sqrt[1 + x])/8 + ((7 - 2*x)*(-1 + x)^(3/2)*Sqrt[
1 + x])/24 + ((-1 + x)^(3/2)*x^2*Sqrt[1 + x])/4 + (3*ArcCosh[x])/
8
```

Rubi in Sympy [A] time = 3.89663, size = 61, normalized size = 0.88

$$\frac{x^2(x-1)^{\frac{3}{2}}\sqrt{x+1}}{4}+\frac{(-2x+7)(x-1)^{\frac{3}{2}}\sqrt{x+1}}{24}-\frac{3\sqrt{x-1}\sqrt{x+1}}{8}+\frac{3\,\mathrm{acosh}\,(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**3*(-1+x)**(1/2)/(1+x)**(1/2),x)
```

```
[Out] x**2*(x - 1)**(3/2)*sqrt(x + 1)/4 + (-2*x + 7)*(x - 1)**(3/2)*sqr
t(x + 1)/24 - 3*sqrt(x - 1)*sqrt(x + 1)/8 + 3*acosh(x)/8
```

Mathematica [A] time = 0.0658282, size = 74, normalized size = 1.07

$$\frac{\sqrt{\frac{x-1}{x+1}}\left(\sqrt{x-1}\left(6x^4-2x^3+x^2-7x-16\right)+18\sqrt{x+1}\sinh^{-1}\left(\frac{\sqrt{x-1}}{\sqrt{2}}\right)\right)}{24\sqrt{x-1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[-1 + x]*x^3)/Sqrt[1 + x],x]
```

```
[Out] (Sqrt[(-1 + x)/(1 + x)]*(Sqrt[-1 + x]*(-16 - 7*x + x^2 - 2*x^3 +
6*x^4) + 18*Sqrt[1 + x]*ArcSinh[Sqrt[-1 + x]/Sqrt[2]]))/(24*Sqrt[
-1 + x])
```

Maple [A] time = 0.014, size = 76, normalized size = 1.1

$$\frac{1}{24}\sqrt{-1+x}\sqrt{1+x}\left(6x^3\sqrt{x^2-1}-8x^2\sqrt{x^2-1}+9x\sqrt{x^2-1}+9\ln\left(x+\sqrt{x^2-1}\right)-16\sqrt{x^2-1}\right)\frac{1}{\sqrt{x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(-1+x)^(1/2)/(1+x)^(1/2),x)
```

```
[Out] 1/24*(-1+x)^(1/2)*(1+x)^(1/2)*(6*x^3*(x^2-1)^(1/2)-8*x^2*(x^2-1)^
(1/2)+9*x*(x^2-1)^(1/2)+9*ln(x+(x^2-1)^(1/2))-16*(x^2-1)^(1/2))/(
x^2-1)^(1/2)
```

Maxima [A] time = 0.719345, size = 74, normalized size = 1.07

$$\frac{1}{4}\left(x^2-1\right)^{\frac{3}{2}}x-\frac{1}{3}\left(x^2-1\right)^{\frac{3}{2}}+\frac{5}{8}\sqrt{x^2-1}x-\sqrt{x^2-1}+\frac{3}{8}\log\left(2x+2\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x - 1)*x^3/sqrt(x + 1),x, algorithm="maxima")
```

```
[Out] 1/4*(x^2 - 1)^(3/2)*x - 1/3*(x^2 - 1)^(3/2) + 5/8*sqrt(x^2 - 1)*x
 - sqrt(x^2 - 1) + 3/8*log(2*x + 2*sqrt(x^2 - 1))
```

Fricas [A] time = 0.276165, size = 228, normalized size = 3.3

$$-\frac{48x^8-64x^7-32x^5-84x^4+160x^3-\left(48x^7-64x^6+24x^5-64x^4-66x^3+120x^2+9x-16\right)\sqrt{x+1}\sqrt{x-1}+36x^2}{24\left(8x^4-4\left(2x^3-x\right)\sqrt{x+1}\sqrt{x-1}-8x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x - 1)*x^3/sqrt(x + 1),x, algorithm="fricas")
```

```
[Out] -1/24*(48*x^8 - 64*x^7 - 32*x^5 - 84*x^4 + 160*x^3 - (48*x^7 - 64
*x^6 + 24*x^5 - 64*x^4 - 66*x^3 + 120*x^2 + 9*x - 16)*sqrt(x + 1)
*sqrt(x - 1) + 36*x^2 + 9*(8*x^4 - 4*(2*x^3 - x)*sqrt(x + 1)*sqrt
(x - 1) - 8*x^2 + 1)*log(sqrt(x + 1)*sqrt(x - 1) - x) - 64*x)/(8*
x^4 - 4*(2*x^3 - x)*sqrt(x + 1)*sqrt(x - 1) - 8*x^2 + 1)
```

Sympy [A]   time = 42.1345, size = 83, normalized size = 1.2

$$\frac{(x-1)^{\frac{7}{2}}\sqrt{x+1}}{4}+\frac{5(x-1)^{\frac{5}{2}}\sqrt{x+1}}{12}+\frac{11(x-1)^{\frac{3}{2}}\sqrt{x+1}}{24}-\frac{3\sqrt{x-1}\sqrt{x+1}}{8}+\frac{3\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{x-1}}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(-1+x)**(1/2)/(1+x)**(1/2),x)
```

```
[Out] (x - 1)**(7/2)*sqrt(x + 1)/4 + 5*(x - 1)**(5/2)*sqrt(x + 1)/12 +
11*(x - 1)**(3/2)*sqrt(x + 1)/24 - 3*sqrt(x - 1)*sqrt(x + 1)/8 +
3*asinh(sqrt(2)*sqrt(x - 1)/2)/4
```

GIAC/XCAS [A]   time = 0.292087, size = 65, normalized size = 0.94

$$\frac{1}{24}\left((2(3x-10)(x+1)+43)(x+1)-39\right)\sqrt{x+1}\sqrt{x-1}-\frac{3}{4}\ln\left(\left|-\sqrt{x+1}+\sqrt{x-1}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x - 1)*x^3/sqrt(x + 1),x, algorithm="giac")
```

```
[Out] 1/24*((2*(3*x - 10)*(x + 1) + 43)*(x + 1) - 39)*sqrt(x + 1)*sqrt(
x - 1) - 3/4*ln(abs(-sqrt(x + 1) + sqrt(x - 1)))
```

3.580 $\int x^3\sqrt{\frac{-1+x}{1+x}}\,dx$

Optimal. Leaf size=69

$$\frac{1}{4}(x-1)^{3/2}\sqrt{x+1}x^2+\frac{1}{24}(7-2x)(x-1)^{3/2}\sqrt{x+1}-\frac{3}{8}\sqrt{x-1}\sqrt{x+1}+\frac{3}{8}\cosh^{-1}(x)$$

```
[Out] (-3*Sqrt[-1 + x]*Sqrt[1 + x])/8 + ((7 - 2*x)*(-1 + x)^(3/2)*Sqrt[
1 + x])/24 + ((-1 + x)^(3/2)*x^2*Sqrt[1 + x])/4 + (3*ArcCosh[x])/
8
```

Rubi [A] time = 0.0860815, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$

$$\frac{1}{4}(x-1)^{3/2}\sqrt{x+1}x^2+\frac{1}{24}(7-2x)(x-1)^{3/2}\sqrt{x+1}-\frac{3}{8}\sqrt{x-1}\sqrt{x+1}+\frac{3}{8}\cosh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*Sqrt[(-1 + x)/(1 + x)],x]
```

```
[Out] (-3*Sqrt[-1 + x]*Sqrt[1 + x])/8 + ((7 - 2*x)*(-1 + x)^(3/2)*Sqrt[
1 + x])/24 + ((-1 + x)^(3/2)*x^2*Sqrt[1 + x])/4 + (3*ArcCosh[x])/
8
```

Rubi in Sympy [A] time = 4.4054, size = 61, normalized size = 0.88

$$\frac{x^2(x-1)^{\frac{3}{2}}\sqrt{x+1}}{4}+\frac{(-2x+7)(x-1)^{\frac{3}{2}}\sqrt{x+1}}{24}-\frac{3\sqrt{x-1}\sqrt{x+1}}{8}+\frac{3\,\mathrm{acosh}\,(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*((-1+x)/(1+x))**(1/2),x)
```

```
[Out] x**2*(x - 1)**(3/2)*sqrt(x + 1)/4 + (-2*x + 7)*(x - 1)**(3/2)*sqr
t(x + 1)/24 - 3*sqrt(x - 1)*sqrt(x + 1)/8 + 3*acosh(x)/8
```

Mathematica [A] time = 0.0214597, size = 74, normalized size = 1.07

$$\frac{\sqrt{\frac{x-1}{x+1}}\left(\sqrt{x-1}\left(6x^4-2x^3+x^2-7x-16\right)+18\sqrt{x+1}\sinh^{-1}\left(\frac{\sqrt{x-1}}{\sqrt{2}}\right)\right)}{24\sqrt{x-1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*Sqrt[(-1 + x)/(1 + x)],x]
```

```
[Out] (Sqrt[(-1 + x)/(1 + x)]*(Sqrt[-1 + x]*(-16 - 7*x + x^2 - 2*x^3 +
6*x^4) + 18*Sqrt[1 + x]*ArcSinh[Sqrt[-1 + x]/Sqrt[2]]))/(24*Sqrt[
-1 + x])
```

Maple [A] time = 0.014, size = 79, normalized size = 1.1

$$\frac{1+x}{24}\sqrt{\frac{-1+x}{1+x}}\left(6\,x\left(x^2-1\right)^{3/2}-8\,\left((-1+x)(1+x)\right)^{3/2}+15\,x\sqrt{x^2-1}-24\,\sqrt{x^2-1}+9\,\ln\left(x+\sqrt{x^2-1}\right)\right)\frac{1}{\sqrt{(-1+x)(1+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*((-1+x)/(1+x))^(1/2),x)
```

```
[Out] 1/24*((-1+x)/(1+x))^(1/2)*(1+x)*(6*x*(x^2-1)^(3/2)-8*((-1+x)*(1+x
))^(3/2)+15*x*(x^2-1)^(1/2)-24*(x^2-1)^(1/2)+9*ln(x+(x^2-1)^(1/2)
))/((-1+x)*(1+x))^(1/2)
```

Maxima [A] time = 0.716802, size = 186, normalized size = 2.7

$$-\frac{39\,\left(\frac{x-1}{x+1}\right)^{\frac{7}{2}}-31\,\left(\frac{x-1}{x+1}\right)^{\frac{5}{2}}+49\,\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}}-9\,\sqrt{\frac{x-1}{x+1}}}{12\left(\frac{4(x-1)}{x+1}-\frac{6(x-1)^2}{(x+1)^2}+\frac{4(x-1)^3}{(x+1)^3}-\frac{(x-1)^4}{(x+1)^4}-1\right)}+\frac{3}{8}\,\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\frac{3}{8}\,\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*sqrt((x - 1)/(x + 1)),x, algorithm="maxima")
```

```
[Out] -1/12*(39*((x - 1)/(x + 1))^(7/2) - 31*((x - 1)/(x + 1))^(5/2) +
49*((x - 1)/(x + 1))^(3/2) - 9*sqrt((x - 1)/(x + 1)))/(4*(x - 1)/
(x + 1) - 6*(x - 1)^2/(x + 1)^2 + 4*(x - 1)^3/(x + 1)^3 - (x - 1)
^4/(x + 1)^4 - 1) + 3/8*log(sqrt((x - 1)/(x + 1)) + 1) - 3/8*log(
sqrt((x - 1)/(x + 1)) - 1)
```

Fricas [A] time = 0.276881, size = 86, normalized size = 1.25

$$\frac{1}{24}\left(6x^4 - 2x^3 + x^2 - 7x - 16\right)\sqrt{\frac{x-1}{x+1}} + \frac{3}{8}\log\left(\sqrt{\frac{x-1}{x+1}} + 1\right) - \frac{3}{8}\log\left(\sqrt{\frac{x-1}{x+1}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*sqrt((x - 1)/(x + 1)),x, algorithm="fricas")
```

```
[Out] 1/24*(6*x^4 - 2*x^3 + x^2 - 7*x - 16)*sqrt((x - 1)/(x + 1)) + 3/8
*log(sqrt((x - 1)/(x + 1)) + 1) - 3/8*log(sqrt((x - 1)/(x + 1)) -
 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\sqrt{\frac{x-1}{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*((-1+x)/(1+x))**(1/2),x)
```

```
[Out] Integral(x**3*sqrt((x - 1)/(x + 1)), x)
```

GIAC/XCAS [A] time = 0.270971, size = 84, normalized size = 1.22

$$-\frac{3}{8}\ln\left(\left|-x + \sqrt{x^2 - 1}\right|\right)\operatorname{sign}(x+1) + \frac{1}{24}\left(\left(2\left(3x\operatorname{sign}(x+1) - 4\operatorname{sign}(x+1)\right)x + 9\operatorname{sign}(x+1)\right)x - 16\operatorname{sign}(x+1)\right)\sqrt{x^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*sqrt((x - 1)/(x + 1)),x, algorithm="giac")
```

```
[Out] -3/8*ln(abs(-x + sqrt(x^2 - 1)))*sign(x + 1) + 1/24*((2*(3*x*sign
(x + 1) - 4*sign(x + 1))*x + 9*sign(x + 1))*x - 16*sign(x + 1))*s
qrt(x^2 - 1)
```

3.581 $\int \frac{\sqrt{-\frac{x}{1+x}}}{x} \, dx$

Optimal. Leaf size=15

$$2 \tan^{-1}\left(\sqrt{-\frac{x}{x+1}}\right)$$

```
[Out] 2*ArcTan[Sqrt[-(x/(1 + x))]]
```

Rubi [A] time = 0.0222439, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$2 \tan^{-1}\left(\sqrt{-\frac{x}{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[-(x/(1 + x))]/x,x]

[Out] 2*ArcTan[Sqrt[-(x/(1 + x))]]
```

Rubi in Sympy [A] time = 1.32897, size = 12, normalized size = 0.8

$$2 \operatorname{atan}\left(\sqrt{-\frac{x}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x/(1+x))**(1/2)/x,x)

[Out] 2*atan(sqrt(-x/(x + 1)))
```

Mathematica [B] time = 0.0233332, size = 32, normalized size = 2.13

$$\frac{2\sqrt{-\frac{x}{x+1}}\sqrt{x+1}\sinh^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[-(x/(1 + x))]/x,x]

[Out] (2*Sqrt[-(x/(1 + x))]*Sqrt[1 + x]*ArcSinh[Sqrt[x]])/Sqrt[x]
```

Maple [B] time = 0.006, size = 33, normalized size = 2.2

$$(1+x)\sqrt{-\frac{x}{1+x}}\ln\left(\frac{1}{2}+x+\sqrt{x^2+x}\right)\frac{1}{\sqrt{x(1+x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x/(1+x))^(1/2)/x,x)

[Out] (-x/(1+x))^(1/2)*(1+x)/(x*(1+x))^(1/2)*ln(1/2+x+(x^2+x)^(1/2))
```

Maxima [A] time = 0.797968, size = 18, normalized size = 1.2

$$2\arctan\left(\sqrt{-\frac{x}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x/(x + 1))/x,x, algorithm="maxima")

[Out] 2*arctan(sqrt(-x/(x + 1)))
```

Fricas [A] time = 0.277425, size = 18, normalized size = 1.2

$$2\arctan\left(\sqrt{-\frac{x}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x/(x + 1))/x,x, algorithm="fricas")

[Out] 2*arctan(sqrt(-x/(x + 1)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-\frac{x}{x+1}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x/(1+x))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-x/(x + 1))/x, x)
```

GIAC/XCAS [A] time = 0.269677, size = 27, normalized size = 1.8

$$-\frac{1}{2}\,\pi \operatorname{sign}\left(x + 1\right) - \arcsin\left(2\,x + 1\right) \operatorname{sign}\left(x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x/(x + 1))/x,x, algorithm="giac")
```

```
[Out] -1/2*pi*sign(x + 1) - arcsin(2*x + 1)*sign(x + 1)
```

3.582 $\int \frac{\sqrt{\frac{1-x}{1+x}}}{-1+x} \, dx$

Optimal. Leaf size=18

$$2\tan^{-1}\left(\sqrt{\frac{1-x}{x+1}}\right)$$

```
[Out] 2*ArcTan[Sqrt[(1 - x)/(1 + x)]]
```

Rubi [A] time = 0.0381849, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095

$$2\tan^{-1}\left(\sqrt{\frac{1-x}{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[(1 - x)/(1 + x)]/(-1 + x),x]

[Out] 2*ArcTan[Sqrt[(1 - x)/(1 + x)]]
```

Rubi in Sympy [A] time = 2.11338, size = 12, normalized size = 0.67

$$2\,\mathrm{atan}\left(\sqrt{\frac{-x+1}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((1-x)/(1+x))**(1/2)/(-1+x),x)

[Out] 2*atan(sqrt((-x + 1)/(x + 1)))
```

Mathematica [B] time = 0.026253, size = 47, normalized size = 2.61

$$\frac{2\sqrt{\frac{1-x}{x+1}}\sqrt{1-x^2}\sin^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{2}}\right)}{x-1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[(1 - x)/(1 + x)]/(-1 + x),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + x)]*Sqrt[1 - x^2]*ArcSin[Sqrt[1 + x]/Sqrt[2]
])/(-1 + x)
```

Maple [A] time = 0.016, size = 30, normalized size = 1.7

$$-(1+x)\arcsin(x)\sqrt{-\frac{-1+x}{1+x}}\frac{1}{\sqrt{-(-1+x)(1+x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((1-x)/(1+x))^(1/2)/(-1+x),x)
```

```
[Out] -(-(-1+x)/(1+x))^(1/2)*(1+x)/(-(-1+x)*(1+x))^(1/2)*arcsin(x)
```

Maxima [A] time = 0.803896, size = 20, normalized size = 1.11

$$2 \arctan\left(\sqrt{-\frac{x-1}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-(x - 1)/(x + 1))/(x - 1),x, algorithm="maxima")
```

```
[Out] 2*arctan(sqrt(-(x - 1)/(x + 1)))
```

Fricas [A] time = 0.274667, size = 20, normalized size = 1.11

$$2 \arctan\left(\sqrt{-\frac{x-1}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-(x - 1)/(x + 1))/(x - 1),x, algorithm="fricas")
```

```
[Out] 2*arctan(sqrt(-(x - 1)/(x + 1)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-\frac{x-1}{x+1}}}{x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((1-x)/(1+x))**(1/2)/(-1+x),x)
```

```
[Out] Integral(sqrt(-(x - 1)/(x + 1))/(x - 1), x)
```

GIAC/XCAS [A] time = 0.272292, size = 22, normalized size = 1.22

$$-\frac{1}{2}\,\pi \mathrm{sign}\left(x+1\right)-\arcsin\left(x\right)\mathrm{sign}\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-(x - 1)/(x + 1))/(x - 1),x, algorithm="giac")
```

```
[Out] -1/2*pi*sign(x + 1) - arcsin(x)*sign(x + 1)
```

3.583 $\int \frac{\sqrt{\frac{a+bx}{c-bx}}}{a+bx} \, dx$

Optimal. Leaf size=24

$$\frac{2\tan^{-1}\left(\sqrt{\frac{a+bx}{c-bx}}\right)}{b}$$

```
[Out] (2*ArcTan[Sqrt[(a + b*x)/(c - b*x)]])/b
```

Rubi [A] time = 0.0988306, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{2\tan^{-1}\left(\sqrt{\frac{a+bx}{c-bx}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[(a + b*x)/(c - b*x)]/(a + b*x),x]

[Out] (2*ArcTan[Sqrt[(a + b*x)/(c - b*x)]])/b
```

Rubi in Sympy [A] time = 3.32937, size = 17, normalized size = 0.71

$$\frac{2\,\text{atan}\left(\sqrt{\frac{a+bx}{-bx+c}}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((b*x+a)/(-b*x+c))**(1/2)/(b*x+a),x)

[Out] 2*atan(sqrt((a + b*x)/(-b*x + c)))/b
```

Mathematica [C] time = 0.094124, size = 80, normalized size = 3.33

$$\frac{i\sqrt{c-bx}\sqrt{\frac{a+bx}{c-bx}}\log\left(2\sqrt{a+bx}\sqrt{c-bx}-i(a+2bx-c)\right)}{b\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[(a + b*x)/(c - b*x)]/(a + b*x),x]
```

```
[Out] (I*Sqrt[c - b*x]*Sqrt[(a + b*x)/(c - b*x)]*Log[2*Sqrt[c - b*x]*Sq
rt[a + b*x] - I*(a - c + 2*b*x)])/(b*Sqrt[a + b*x])
```

Maple [B] time = 0.04, size = 85, normalized size = 3.5

$$-(bx-c)\arctan\left(\frac{2\,bx+a-c}{2\,b}\sqrt{b^2}\frac{1}{\sqrt{-(bx+a)(bx-c)}}\right)\sqrt{-\frac{bx+a}{bx-c}}\frac{1}{\sqrt{b^2}}\frac{1}{\sqrt{-(bx+a)(bx-c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((b*x+a)/(-b*x+c))^(1/2)/(b*x+a),x)
```

```
[Out] -arctan(1/2*(b^2)^(1/2)/b*(2*b*x+a-c)/(-(b*x+a)*(b*x-c))^(1/2))*(
b*x-c)*(-(b*x+a)/(b*x-c))^(1/2)/(b^2)^(1/2)/(-(b*x+a)*(b*x-c))^(1
/2)
```

Maxima [A] time = 0.804992, size = 32, normalized size = 1.33

$$\frac{2\,\arctan\left(\sqrt{-\frac{bx+a}{bx-c}}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-(b*x + a)/(b*x - c))/(b*x + a),x, algorithm="maxima")
```

```
[Out] 2*arctan(sqrt(-(b*x + a)/(b*x - c)))/b
```

Fricas [A] time = 0.277303, size = 32, normalized size = 1.33

$$\frac{2\,\arctan\left(\sqrt{-\frac{bx+a}{bx-c}}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-(b*x + a)/(b*x - c))/(b*x + a),x, algorithm="fricas")
```

```
[Out] 2*arctan(sqrt(-(b*x + a)/(b*x - c)))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{a+bx}{-bx+c}}}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((b*x+a)/(-b*x+c))**(1/2)/(b*x+a),x)
```

```
[Out] Integral(sqrt((a + b*x)/(-b*x + c))/(a + b*x), x)
```

GIAC/XCAS [A] time = 0.296876, size = 51, normalized size = 2.12

$$-\frac{\arcsin\left(\frac{2\,bx+a-c}{a+c}\right)\mathrm{sign}\left(ab+bc\right)\mathrm{sign}\left(bx-c\right)}{|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-(b*x + a)/(b*x - c))/(b*x + a),x, algorithm="giac")
```

```
[Out] -arcsin((2*b*x + a - c)/(a + c))*sign(a*b + b*c)*sign(b*x - c)/ab
s(b)
```

3.584 $\int \frac{\sqrt{\frac{a+bx}{c+dx}}}{a+bx}\,dx$

Optimal. Leaf size=41

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{a+bx}{c+dx}}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{d}}$$

```
[Out] (2*ArcTanh[(Sqrt[d]*Sqrt[(a + b*x)/(c + d*x)])/Sqrt[b]])/(Sqrt[b]
*Sqrt[d])
```

Rubi [A] time = 0.11072, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{a+bx}{c+dx}}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[(a + b*x)/(c + d*x)]/(a + b*x),x]
```

```
[Out] (2*ArcTanh[(Sqrt[d]*Sqrt[(a + b*x)/(c + d*x)])/Sqrt[b]])/(Sqrt[b]
*Sqrt[d])
```

Rubi in Sympy [A] time = 5.63461, size = 36, normalized size = 0.88

$$\frac{2\,\text{atanh}\left(\frac{\sqrt{d}\sqrt{\frac{a+bx}{c+dx}}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((b*x+a)/(d*x+c))**(1/2)/(b*x+a),x)
```

```
[Out] 2*atanh(sqrt(d)*sqrt((a + b*x)/(c + d*x))/sqrt(b))/(sqrt(b)*sqrt(
d))
```

Mathematica [B] time = 0.049, size = 89, normalized size = 2.17

$$\frac{\sqrt{c+dx}\sqrt{\frac{a+bx}{c+dx}}\log\left(2\sqrt{b}\sqrt{d}\sqrt{a+bx}\sqrt{c+dx}+ad+bc+2bdx\right)}{\sqrt{b}\sqrt{d}\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[(a + b*x)/(c + d*x)]/(a + b*x),x]
```

```
[Out] (Sqrt[(a + b*x)/(c + d*x)]*Sqrt[c + d*x]*Log[b*c + a*d + 2*b*d*x
+ 2*Sqrt[b]*Sqrt[d]*Sqrt[a + b*x]*Sqrt[c + d*x]])/(Sqrt[b]*Sqrt[d
]*Sqrt[a + b*x])
```

Maple [B] time = 0.034, size = 80, normalized size = 2.

$$(dx+c)\ln\left(\frac{1}{2}\left(2\,bdx+2\,\sqrt{(bx+a)(dx+c)}\sqrt{bd}+ad+bc\right)\frac{1}{\sqrt{bd}}\right)\sqrt{\frac{bx+a}{dx+c}}\frac{1}{\sqrt{(bx+a)(dx+c)}}\frac{1}{\sqrt{bd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((b*x+a)/(d*x+c))^(1/2)/(b*x+a),x)
```

```
[Out] ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b
*d)^(1/2))*(d*x+c)*((b*x+a)/(d*x+c))^(1/2)/((b*x+a)*(d*x+c))^(1/2
)/(b*d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x + a)/(d*x + c))/(b*x + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.279138, size = 1, normalized size = 0.02

$$\left[\frac{\log\left((2\,bdx+bc+ad)\sqrt{bd}+2\left(bd^2x+bcd\right)\sqrt{\frac{bx+a}{dx+c}}\right)}{\sqrt{bd}},-\frac{2\,\arctan\left(\frac{b}{\sqrt{-bd}\sqrt{\frac{bx+a}{dx+c}}}\right)}{\sqrt{-bd}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x + a)/(d*x + c))/(b*x + a),x, algorithm="fricas")
```

```
[Out] [log((2*b*d*x + b*c + a*d)*sqrt(b*d) + 2*(b*d^2*x + b*c*d)*sqrt((
b*x + a)/(d*x + c)))/sqrt(b*d), -2*arctan(b/(sqrt(-b*d)*sqrt((b*x
 + a)/(d*x + c))))/sqrt(-b*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((b*x+a)/(d*x+c))**(1/2)/(b*x+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.296455, size = 100, normalized size = 2.44

$$-\frac{\sqrt{bd}\ln\left(\left|-2\left(\sqrt{bd}x-\sqrt{bdx^2+bcx+adx+ac}\right)bd-\sqrt{bd}bc-\sqrt{bd}ad\right|\right)\mathrm{sign}\left(dx+c\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x + a)/(d*x + c))/(b*x + a),x, algorithm="giac")
```

```
[Out] -sqrt(b*d)*ln(abs(-2*(sqrt(b*d)*x - sqrt(b*d*x^2 + b*c*x + a*d*x
+ a*c))*b*d - sqrt(b*d)*b*c - sqrt(b*d)*a*d))*sign(d*x + c)/(b*d)
```

3.585 $\int \sqrt{-\frac{x}{1+x}}\, dx$

Optimal. Leaf size=32

$$\sqrt{-\frac{x}{x+1}}(x+1)-\tan^{-1}\left(\sqrt{-\frac{x}{x+1}}\right)$$

```
[Out] Sqrt[-(x/(1 + x))]*(1 + x) - ArcTan[Sqrt[-(x/(1 + x))]]
```

Rubi [A] time = 0.0308614, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\sqrt{-\frac{x}{x+1}}(x+1)-\tan^{-1}\left(\sqrt{-\frac{x}{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[-(x/(1 + x))],x]

[Out] Sqrt[-(x/(1 + x))]*(1 + x) - ArcTan[Sqrt[-(x/(1 + x))]]
```

Rubi in Sympy [A] time = 2.0556, size = 27, normalized size = 0.84

$$\frac{\sqrt{-\frac{x}{x+1}}}{-\frac{x}{x+1}+1}-\operatorname{atan}\left(\sqrt{-\frac{x}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x/(1+x))**(1/2),x)

[Out] sqrt(-x/(x + 1))/(-x/(x + 1) + 1) - atan(sqrt(-x/(x + 1)))
```

Mathematica [A] time = 0.0283316, size = 43, normalized size = 1.34

$$\frac{\sqrt{-\frac{x}{x+1}}\left(\sqrt{x}(x+1)-\sqrt{x+1}\sinh^{-1}\left(\sqrt{x}\right)\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[-(x/(1 + x))],x]
```

```
[Out] (Sqrt[-(x/(1 + x))]*(Sqrt[x]*(1 + x) - Sqrt[1 + x]*ArcSinh[Sqrt[x
]]))/Sqrt[x]
```

Maple [A] time = 0.005, size = 46, normalized size = 1.4

$$\frac{1+x}{2}\sqrt{-\frac{x}{1+x}}\left(2\sqrt{x^2+x}-\ln\left(\frac{1}{2}+x+\sqrt{x^2+x}\right)\right)\frac{1}{\sqrt{x(1+x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x/(1+x))^(1/2),x)
```

```
[Out] 1/2*(-x/(1+x))^(1/2)*(1+x)*(2*(x^2+x)^(1/2)-ln(1/2+x+(x^2+x)^(1/2
)))/(x*(1+x))^(1/2)
```

Maxima [A] time = 0.798798, size = 50, normalized size = 1.56

$$-\frac{\sqrt{-\frac{x}{x+1}}}{\frac{x}{x+1}-1}-\arctan\left(\sqrt{-\frac{x}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x/(x + 1)),x, algorithm="maxima")
```

```
[Out] -sqrt(-x/(x + 1))/(x/(x + 1) - 1) - arctan(sqrt(-x/(x + 1)))
```

Fricas [A] time = 0.274071, size = 38, normalized size = 1.19

$$(x+1)\sqrt{-\frac{x}{x+1}}-\arctan\left(\sqrt{-\frac{x}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x/(x + 1)),x, algorithm="fricas")
```

```
[Out] (x + 1)*sqrt(-x/(x + 1)) - arctan(sqrt(-x/(x + 1)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-\frac{x}{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x/(1+x))**(1/2),x)
```

```
[Out] Integral(sqrt(-x/(x + 1)), x)
```

GIAC/XCAS [A] time = 0.274055, size = 49, normalized size = 1.53

$$\frac{1}{4}\,\pi\operatorname{sign}\left(x+1\right)+\frac{1}{2}\,\arcsin\left(2\,x+1\right)\operatorname{sign}\left(x+1\right)+\sqrt{-x^2-x}\operatorname{sign}\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x/(x + 1)),x, algorithm="giac")
```

```
[Out] 1/4*pi*sign(x + 1) + 1/2*arcsin(2*x + 1)*sign(x + 1) + sqrt(-x^2
- x)*sign(x + 1)
```

3.586 $\int \sqrt{\frac{1-x}{1+x}}\,dx$

Optimal. Leaf size=38

$$\sqrt{\frac{1-x}{x+1}}(x+1)-2\tan^{-1}\left(\sqrt{\frac{1-x}{x+1}}\right)$$

```
[Out] Sqrt[(1 - x)/(1 + x)]*(1 + x) - 2*ArcTan[Sqrt[(1 - x)/(1 + x)]]
```

Rubi [A] time = 0.0368016, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\sqrt{\frac{1-x}{x+1}}(x+1)-2\tan^{-1}\left(\sqrt{\frac{1-x}{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[(1 - x)/(1 + x)],x]
```

```
[Out] Sqrt[(1 - x)/(1 + x)]*(1 + x) - 2*ArcTan[Sqrt[(1 - x)/(1 + x)]]
```

Rubi in Sympy [A] time = 2.13615, size = 32, normalized size = 0.84

$$\frac{2\sqrt{\frac{-x+1}{x+1}}}{\frac{-x+1}{x+1}+1}-2\,\mathrm{atan}\left(\sqrt{\frac{-x+1}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((1-x)/(1+x))**(1/2),x)
```

```
[Out] 2*sqrt((-x + 1)/(x + 1))/((-x + 1)/(x + 1) + 1) - 2*atan(sqrt((-x
 + 1)/(x + 1)))
```

Mathematica [A] time = 0.0350465, size = 62, normalized size = 1.63

$$\frac{\sqrt{\frac{1-x}{x+1}}\left(\sqrt{1-x}(x+1)+2\sqrt{x+1}\sin^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{2}}\right)\right)}{\sqrt{1-x}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[(1 - x)/(1 + x)],x]
```

```
[Out] (Sqrt[(1 - x)/(1 + x)]*(Sqrt[1 - x]*(1 + x) + 2*Sqrt[1 + x]*ArcSi
n[Sqrt[1 + x]/Sqrt[2]]))/Sqrt[1 - x]
```

Maple [A] time = 0.006, size = 39, normalized size = 1.

$$(1+x)\sqrt{-\frac{-1+x}{1+x}}\left(\sqrt{-x^2+1}+\arcsin(x)\right)\frac{1}{\sqrt{-(-1+x)(1+x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((1-x)/(1+x))^(1/2),x)
```

```
[Out] (-(-1+x)/(1+x))^(1/2)*(1+x)/(-(-1+x)*(1+x))^(1/2)*((-x^2+1)^(1/2)
+arcsin(x))
```

Maxima [A] time = 0.79345, size = 58, normalized size = 1.53

$$-\frac{2\sqrt{-\frac{x-1}{x+1}}}{\frac{x-1}{x+1}-1}-2\arctan\left(\sqrt{-\frac{x-1}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-(x - 1)/(x + 1)),x, algorithm="maxima")
```

```
[Out] -2*sqrt(-(x - 1)/(x + 1))/((x - 1)/(x + 1) - 1) - 2*arctan(sqrt(-
(x - 1)/(x + 1)))
```

Fricas [A] time = 0.271832, size = 43, normalized size = 1.13

$$(x+1)\sqrt{-\frac{x-1}{x+1}}-2\arctan\left(\sqrt{-\frac{x-1}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-(x - 1)/(x + 1)),x, algorithm="fricas")

[Out] (x + 1)*sqrt(-(x - 1)/(x + 1)) - 2*arctan(sqrt(-(x - 1)/(x + 1)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{-x+1}{x+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((1-x)/(1+x))**(1/2),x)

[Out] Integral(sqrt((-x + 1)/(x + 1)), x)
```

GIAC/XCAS [A] time = 0.272988, size = 39, normalized size = 1.03

$$\frac{1}{2}\,\pi \operatorname{sign}\left(x+1\right)+\arcsin\left(x\right)\operatorname{sign}\left(x+1\right)+\sqrt{-x^2+1}\operatorname{sign}\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-(x - 1)/(x + 1)),x, algorithm="giac")

[Out] 1/2*pi*sign(x + 1) + arcsin(x)*sign(x + 1) + sqrt(-x^2 + 1)*sign(
x + 1)
```

3.587 $\int \sqrt{\frac{a+x}{a-x}}\,dx$

Optimal. Leaf size=42

$$2a\tan^{-1}\left(\sqrt{\frac{a+x}{a-x}}\right)-(a-x)\sqrt{\frac{a+x}{a-x}}$$

```
[Out] -((a - x)*Sqrt[(a + x)/(a - x)]) + 2*a*ArcTan[Sqrt[(a + x)/(a - x
)]]
```

Rubi [A] time = 0.0361946, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$2a\tan^{-1}\left(\sqrt{\frac{a+x}{a-x}}\right)-(a-x)\sqrt{\frac{a+x}{a-x}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[(a + x)/(a - x)],x]
```

```
[Out] -((a - x)*Sqrt[(a + x)/(a - x)]) + 2*a*ArcTan[Sqrt[(a + x)/(a - x
)]]
```

Rubi in Sympy [A] time = 2.01774, size = 36, normalized size = 0.86

$$-\frac{2a\sqrt{\frac{a+x}{a-x}}}{1+\frac{a+x}{a-x}}+2a\operatorname{atan}\left(\sqrt{\frac{a+x}{a-x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((a+x)/(a-x))**(1/2),x)
```

```
[Out] -2*a*sqrt((a + x)/(a - x))/(1 + (a + x)/(a - x)) + 2*a*atan(sqrt(
(a + x)/(a - x)))
```

Mathematica [A] time = 0.0845865, size = 67, normalized size = 1.6

$$\frac{\sqrt{\frac{a+x}{a-x}}\left(\sqrt{a+x}(x-a)+a\sqrt{a-x}\tan^{-1}\left(\frac{x}{\sqrt{a-x}\sqrt{a+x}}\right)\right)}{\sqrt{a+x}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[(a + x)/(a - x)],x]
```

```
[Out] (Sqrt[(a + x)/(a - x)]*((-a + x)*Sqrt[a + x] + a*Sqrt[a - x]*ArcT
an[x/(Sqrt[a - x]*Sqrt[a + x])]))/Sqrt[a + x]
```

Maple [A]   time = 0.023, size = 64, normalized size = 1.5

$$-(-a+x)\sqrt{-\frac{a+x}{-a+x}}\left(a\arctan\left(x\frac{1}{\sqrt{a^2-x^2}}\right)-\sqrt{a^2-x^2}\right)\frac{1}{\sqrt{-(a+x)(-a+x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((a+x)/(a-x))^(1/2),x)
```

```
[Out] -(-(a+x)/(-a+x))^(1/2)*(-a+x)*(a*arctan(x/(a^2-x^2)^(1/2))-(a^2-x
^2)^(1/2))/(-(a+x)*(-a+x))^(1/2)
```

Maxima [A]   time = 0.804052, size = 66, normalized size = 1.57

$$-2\,a\left(\frac{\sqrt{\frac{a+x}{a-x}}}{\frac{a+x}{a-x}+1}-\arctan\left(\sqrt{\frac{a+x}{a-x}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((a + x)/(a - x)),x, algorithm="maxima")
```

```
[Out] -2*a*(sqrt((a + x)/(a - x))/((a + x)/(a - x) + 1) - arctan(sqrt((
a + x)/(a - x))))
```

Fricas [A]   time = 0.278703, size = 51, normalized size = 1.21

$$2\,a\arctan\left(\sqrt{\frac{a+x}{a-x}}\right)-(a-x)\sqrt{\frac{a+x}{a-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((a + x)/(a - x)),x, algorithm="fricas")

[Out] 2*a*arctan(sqrt((a + x)/(a - x))) - (a - x)*sqrt((a + x)/(a - x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{a+x}{a-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((a+x)/(a-x))**(1/2),x)

[Out] Integral(sqrt((a + x)/(a - x)), x)
```

GIAC/XCAS [A] time = 0.27606, size = 49, normalized size = 1.17

$$a \arcsin\left(\frac{x}{a}\right) \operatorname{sign}(a-x) \operatorname{sign}(a) - \sqrt{a^2-x^2} \operatorname{sign}(a-x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((a + x)/(a - x)),x, algorithm="giac")

[Out] a*arcsin(x/a)*sign(a - x)*sign(a) - sqrt(a^2 - x^2)*sign(a - x)
```

3.588 $\int \sqrt{\frac{-a+x}{a+x}}\,dx$

Optimal. Leaf size=41

$$\sqrt{-\frac{a-x}{a+x}}(a+x)-2a\tanh^{-1}\left(\sqrt{-\frac{a-x}{a+x}}\right)$$

```
[Out] Sqrt[-((a - x)/(a + x))]*(a + x) - 2*a*ArcTanh[Sqrt[-((a - x)/(a
+ x))]]
```

Rubi [A] time = 0.0478819, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\sqrt{-\frac{a-x}{a+x}}(a+x)-2a\tanh^{-1}\left(\sqrt{-\frac{a-x}{a+x}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[(-a + x)/(a + x)],x]
```

```
[Out] Sqrt[-((a - x)/(a + x))]*(a + x) - 2*a*ArcTanh[Sqrt[-((a - x)/(a
+ x))]]
```

Rubi in Sympy [A] time = 2.17006, size = 36, normalized size = 0.88

$$\frac{2a\sqrt{\frac{-a+x}{a+x}}}{-\frac{-a+x}{a+x}+1}-2a\operatorname{atanh}\left(\sqrt{\frac{-a+x}{a+x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((-a+x)/(a+x))**(1/2),x)
```

```
[Out] 2*a*sqrt((-a + x)/(a + x))/(-(-a + x)/(a + x) + 1) - 2*a*atanh(sq
rt((-a + x)/(a + x)))
```

Mathematica [A] time = 0.0487526, size = 69, normalized size = 1.68

$$\frac{\sqrt{\frac{x-a}{a+x}}\left(\sqrt{x-a}(a+x)-a\sqrt{a+x}\log\left(\sqrt{x-a}\sqrt{a+x}+x\right)\right)}{\sqrt{x-a}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[(-a + x)/(a + x)],x]
```

```
[Out] (Sqrt[(-a + x)/(a + x)]*(Sqrt[-a + x]*(a + x) - a*Sqrt[a + x]*Log
[x + Sqrt[-a + x]*Sqrt[a + x]]))/Sqrt[-a + x]
```

Maple [A] time = 0.017, size = 60, normalized size = 1.5

$$-(a+x)\sqrt{\frac{-a+x}{a+x}}\left(a\ln\left(x+\sqrt{-a^2+x^2}\right)-\sqrt{-a^2+x^2}\right)\frac{1}{\sqrt{(a+x)(-a+x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((-a+x)/(a+x))^(1/2),x)
```

```
[Out] -((-a+x)/(a+x))^(1/2)*(a+x)*(a*ln(x+(-a^2+x^2)^(1/2))-(-a^2+x^2)^
(1/2))/((a+x)*(-a+x))^(1/2)
```

Maxima [A] time = 0.71399, size = 95, normalized size = 2.32

$$a\left(\frac{2\sqrt{-\frac{a-x}{a+x}}}{\frac{a-x}{a+x}+1}-\log\left(\sqrt{-\frac{a-x}{a+x}}+1\right)+\log\left(\sqrt{-\frac{a-x}{a+x}}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-(a - x)/(a + x)),x, algorithm="maxima")
```

```
[Out] a*(2*sqrt(-(a - x)/(a + x))/((a - x)/(a + x) + 1) - log(sqrt(-(a
- x)/(a + x)) + 1) + log(sqrt(-(a - x)/(a + x)) - 1))
```

Fricas [A] time = 0.275215, size = 78, normalized size = 1.9

$$-a\log\left(\sqrt{-\frac{a-x}{a+x}}+1\right)+a\log\left(\sqrt{-\frac{a-x}{a+x}}-1\right)+(a+x)\sqrt{-\frac{a-x}{a+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-(a - x)/(a + x)),x, algorithm="fricas")
```

```
[Out] -a*log(sqrt(-(a - x)/(a + x)) + 1) + a*log(sqrt(-(a - x)/(a + x))
 - 1) + (a + x)*sqrt(-(a - x)/(a + x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{-a+x}{a+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((-a+x)/(a+x))**(1/2),x)
```

```
[Out] Integral(sqrt((-a + x)/(a + x)), x)
```

GIAC/XCAS [A] time = 0.270772, size = 54, normalized size = 1.32

$$a\ln\left(\left|-x+\sqrt{-a^2+x^2}\right|\right)\operatorname{sign}(a+x)+\sqrt{-a^2+x^2}\operatorname{sign}(a+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-(a - x)/(a + x)),x, algorithm="giac")
```

```
[Out] a*ln(abs(-x + sqrt(-a^2 + x^2)))*sign(a + x) + sqrt(-a^2 + x^2)*s
ign(a + x)
```

3.589 $\int \sqrt{\frac{a+bx}{c+dx}}\,dx$

Optimal. Leaf size=76

$$\frac{(c+dx)\sqrt{\frac{a+bx}{c+dx}}}{d}-\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{a+bx}{c+dx}}}{\sqrt{b}}\right)}{\sqrt{b}d^{3/2}}$$

```
[Out] (Sqrt[(a + b*x)/(c + d*x)]*(c + d*x))/d - ((b*c - a*d)*ArcTanh[(S
qrt[d]*Sqrt[(a + b*x)/(c + d*x)])/Sqrt[b]])/(Sqrt[b]*d^(3/2))
```

Rubi [A] time = 0.0879393, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$\frac{(c+dx)\sqrt{\frac{a+bx}{c+dx}}}{d}-\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{a+bx}{c+dx}}}{\sqrt{b}}\right)}{\sqrt{b}d^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[(a + b*x)/(c + d*x)],x]
```

```
[Out] (Sqrt[(a + b*x)/(c + d*x)]*(c + d*x))/d - ((b*c - a*d)*ArcTanh[(S
qrt[d]*Sqrt[(a + b*x)/(c + d*x)])/Sqrt[b]])/(Sqrt[b]*d^(3/2))
```

Rubi in Sympy [A] time = 4.92557, size = 76, normalized size = 1.

$$-\frac{\sqrt{\frac{a+bx}{c+dx}}\,(ad-bc)}{d\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{(ad-bc)\,\mathrm{atanh}\left(\frac{\sqrt{d}\sqrt{\frac{a+bx}{c+dx}}}{\sqrt{b}}\right)}{\sqrt{b}d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((b*x+a)/(d*x+c))**(1/2),x)
```

```
[Out] -sqrt((a + b*x)/(c + d*x))*(a*d - b*c)/(d*(b - d*(a + b*x)/(c + d
*x))) + (a*d - b*c)*atanh(sqrt(d)*sqrt((a + b*x)/(c + d*x))/sqrt(
b))/(sqrt(b)*d**(3/2))
```

Mathematica [A] time = 0.112044, size = 127, normalized size = 1.67

$$\frac{\sqrt{c+dx}(ad-bc)\sqrt{\frac{a+bx}{c+dx}}\log\left(2\sqrt{b}\sqrt{d}\sqrt{a+bx}\sqrt{c+dx}+ad+bc+2bdx\right)}{2\sqrt{b}d^{3/2}\sqrt{a+bx}}+\frac{(c+dx)\sqrt{\frac{a+bx}{c+dx}}}{d}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[(a + b*x)/(c + d*x)],x]
```

```
[Out] (Sqrt[(a + b*x)/(c + d*x)]*(c + d*x))/d + ((-(b*c) + a*d)*Sqrt[(a
 + b*x)/(c + d*x)]*Sqrt[c + d*x]*Log[b*c + a*d + 2*b*d*x + 2*Sqrt
[b]*Sqrt[d]*Sqrt[a + b*x]*Sqrt[c + d*x]])/(2*Sqrt[b]*d^(3/2)*Sqrt
[a + b*x])
```

Maple [B] time = 0.01, size = 152, normalized size = 2.

$$\frac{dx+c}{2\,d}\sqrt{\frac{bx+a}{dx+c}}\left(\ln\left(\frac{1}{2}\left(2\,bdx+2\,\sqrt{(bx+a)(dx+c)}\sqrt{bd}+ad+bc\right)\frac{1}{\sqrt{bd}}\right)ad-\ln\left(\frac{1}{2}\left(2\,bdx+2\,\sqrt{(bx+a)(dx+c)}\sqrt{bd}+c\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((b*x+a)/(d*x+c))^(1/2),x)
```

```
[Out] 1/2*((b*x+a)/(d*x+c))^(1/2)*(d*x+c)*(ln(1/2*(2*b*d*x+2*((b*x+a)*(
d*x+c))^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a*d-ln(1/2*(2*b*d
*x+2*((b*x+a)*(d*x+c))^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*b*
c+2*((b*x+a)*(d*x+c))^(1/2)*(b*d)^(1/2))/((b*x+a)*(d*x+c))^(1/2)/
d/(b*d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x + a)/(d*x + c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.291685, size = 1, normalized size = 0.01

$$\left[\frac{2\sqrt{bd}(dx+c)\sqrt{\frac{bx+a}{dx+c}}-(bc-ad)\log\left((2\,bdx+bc+ad)\sqrt{bd}+2\left(bd^2x+bcd\right)\sqrt{\frac{bx+a}{dx+c}}\right)}{2\sqrt{bd}d},\ \frac{\sqrt{-bd}(dx+c)\sqrt{\frac{bx+a}{dx+c}}+(bc-ad)\,\mathrm{ar}}{\sqrt{-bd}d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x + a)/(d*x + c)),x, algorithm="fricas")
```

```
[Out] [1/2*(2*sqrt(b*d)*(d*x + c)*sqrt((b*x + a)/(d*x + c)) - (b*c - a*
d)*log((2*b*d*x + b*c + a*d)*sqrt(b*d) + 2*(b*d^2*x + b*c*d)*sqrt
((b*x + a)/(d*x + c))))/(sqrt(b*d)*d), (sqrt(-b*d)*(d*x + c)*sqrt
((b*x + a)/(d*x + c)) + (b*c - a*d)*arctan(b/(sqrt(-b*d)*sqrt((b*
x + a)/(d*x + c)))))/(sqrt(-b*d)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((b*x+a)/(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.296043, size = 161, normalized size = 2.12

$$\frac{\sqrt{bdx^2+bcx+adx+ac}\,\mathrm{sign}\,(dx+c)}{d}+\frac{(bc\,\mathrm{sign}\,(dx+c)-ad\,\mathrm{sign}\,(dx+c))\sqrt{bd}\ln\left(\left|-2\left(\sqrt{bd}x-\sqrt{bdx^2+bcx+adx+ac}\right)bd-\sqrt{bd}bc-\sqrt{bd}ad\right|\right)}{2\,bd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x + a)/(d*x + c)),x, algorithm="giac")
```

```
[Out] sqrt(b*d*x^2 + b*c*x + a*d*x + a*c)*sign(d*x + c)/d + 1/2*(b*c*si
gn(d*x + c) - a*d*sign(d*x + c))*sqrt(b*d)*ln(abs(-2*(sqrt(b*d)*x
 - sqrt(b*d*x^2 + b*c*x + a*d*x + a*c))*b*d - sqrt(b*d)*b*c - sqr
t(b*d)*a*d))/(b*d^2)
```

3.590 $\int \sqrt{\frac{-1+x}{5+3x}}\,dx$

Optimal. Leaf size=49

$$\frac{1}{3}\sqrt{x-1}\sqrt{3x+5}-\frac{8\sinh^{-1}\left(\frac{1}{2}\sqrt{\frac{3}{2}}\sqrt{x-1}\right)}{3\sqrt{3}}$$

```
[Out] (Sqrt[-1 + x]*Sqrt[5 + 3*x])/3 - (8*ArcSinh[(Sqrt[3/2]*Sqrt[-1 +
x])/2])/(3*Sqrt[3])
```

Rubi [A] time = 0.0366841, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267

$$\frac{1}{3}\sqrt{x-1}\sqrt{3x+5}-\frac{8\sinh^{-1}\left(\frac{1}{2}\sqrt{\frac{3}{2}}\sqrt{x-1}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[(-1 + x)/(5 + 3*x)],x]
```

```
[Out] (Sqrt[-1 + x]*Sqrt[5 + 3*x])/3 - (8*ArcSinh[(Sqrt[3/2]*Sqrt[-1 +
x])/2])/(3*Sqrt[3])
```

Rubi in Sympy [A] time = 2.16425, size = 51, normalized size = 1.04

$$\frac{8\sqrt{\frac{x-1}{3x+5}}}{3\left(-\frac{3(x-1)}{3x+5}+1\right)}-\frac{8\sqrt{3}\,\mathrm{atanh}\left(\sqrt{3}\sqrt{\frac{x-1}{3x+5}}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((-1+x)/(5+3*x))**(1/2),x)
```

```
[Out] 8*sqrt((x - 1)/(3*x + 5))/(3*(-3*(x - 1)/(3*x + 5) + 1)) - 8*sqrt
(3)*atanh(sqrt(3)*sqrt((x - 1)/(3*x + 5)))/9
```

Mathematica [A] time = 0.0675081, size = 71, normalized size = 1.45

$$\frac{\sqrt{\frac{x-1}{3x+5}}\left(3\sqrt{x-1}(3x+5)-8\sqrt{9x+15}\sinh^{-1}\left(\frac{1}{2}\sqrt{\frac{3}{2}}\sqrt{x-1}\right)\right)}{9\sqrt{x-1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[(-1 + x)/(5 + 3*x)],x]
```

```
[Out] (Sqrt[(-1 + x)/(5 + 3*x)]*(3*Sqrt[-1 + x]*(5 + 3*x) - 8*Sqrt[15 +
 9*x]*ArcSinh[(Sqrt[3/2]*Sqrt[-1 + x])/2]))/(9*Sqrt[-1 + x])
```

Maple [B] time = 0.015, size = 76, normalized size = 1.6

$$-\frac{5+3\,x}{9}\sqrt{\frac{-1+x}{5+3\,x}}\left(4\,\ln\left(x\sqrt{3}+1/3\,\sqrt{3}+\sqrt{3\,x^{2}+2\,x-5}\right)\sqrt{3}-3\,\sqrt{3\,x^{2}+2\,x-5}\right)\frac{1}{\sqrt{(5+3\,x)(-1+x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((-1+x)/(5+3*x))^(1/2),x)
```

```
[Out] -1/9*((-1+x)/(5+3*x))^(1/2)*(5+3*x)*(4*ln(x*3^(1/2)+1/3*3^(1/2)+(
3*x^2+2*x-5)^(1/2))*3^(1/2)-3*(3*x^2+2*x-5)^(1/2))/((5+3*x)*(-1+x
))^(1/2)
```

Maxima [A] time = 0.803245, size = 108, normalized size = 2.2

$$\frac{4}{9}\sqrt{3}\log\left(-\frac{\sqrt{3}-3\sqrt{\frac{x-1}{3\,x+5}}}{\sqrt{3}+3\sqrt{\frac{x-1}{3\,x+5}}}\right)-\frac{8\sqrt{\frac{x-1}{3\,x+5}}}{3\left(\frac{3\,(x-1)}{3\,x+5}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((x - 1)/(3*x + 5)),x, algorithm="maxima")
```

```
[Out] 4/9*sqrt(3)*log(-(sqrt(3) - 3*sqrt((x - 1)/(3*x + 5)))/(sqrt(3) +
 3*sqrt((x - 1)/(3*x + 5)))) - 8/3*sqrt((x - 1)/(3*x + 5))/(3*(x
- 1)/(3*x + 5) - 1)
```

Fricas [A] time = 0.285896, size = 84, normalized size = 1.71

$$\frac{1}{9}\sqrt{3}\left(\sqrt{3}(3x+5)\sqrt{\frac{x-1}{3x+5}}+4\log\left(-\sqrt{3}(3x+1)+3(3x+5)\sqrt{\frac{x-1}{3x+5}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((x - 1)/(3*x + 5)),x, algorithm="fricas")
```

```
[Out] 1/9*sqrt(3)*(sqrt(3)*(3*x + 5)*sqrt((x - 1)/(3*x + 5)) + 4*log(-s
qrt(3)*(3*x + 1) + 3*(3*x + 5)*sqrt((x - 1)/(3*x + 5))))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\frac{x-1}{3x+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((-1+x)/(5+3*x))**(1/2),x)
```

```
[Out] Integral(sqrt((x - 1)/(3*x + 5)), x)
```

GIAC/XCAS [A] time = 0.275127, size = 100, normalized size = 2.04

$$-\frac{4}{9}\sqrt{3}\ln(4)\,\mathrm{sign}(3x+5)+\frac{4}{9}\sqrt{3}\ln\left(\left|-\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2+2x-5}\right)-1\right|\right)\mathrm{sign}(3x+5)+\frac{1}{3}\sqrt{3x^2+2x-5}\,\mathrm{sign}(3x+5)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((x - 1)/(3*x + 5)),x, algorithm="giac")
```

```
[Out] -4/9*sqrt(3)*ln(4)*sign(3*x + 5) + 4/9*sqrt(3)*ln(abs(-sqrt(3)*(s
qrt(3)*x - sqrt(3*x^2 + 2*x - 5)) - 1))*sign(3*x + 5) + 1/3*sqrt(
3*x^2 + 2*x - 5)*sign(3*x + 5)
```

3.591 $\int \frac{\sqrt{\frac{-1+5x}{1+7x}}}{x^2}\,dx$

Optimal. Leaf size=46

$$-\frac{\sqrt{5x-1}\sqrt{7x+1}}{x}-12\tan^{-1}\left(\frac{\sqrt{7x+1}}{\sqrt{5x-1}}\right)$$

```
[Out] -((Sqrt[-1 + 5*x]*Sqrt[1 + 7*x])/x) - 12*ArcTan[Sqrt[1 + 7*x]/Sqr
t[-1 + 5*x]]
```

Rubi [A] time = 0.0789203, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$

$$-\frac{\sqrt{5x-1}\sqrt{7x+1}}{x}-12\tan^{-1}\left(\frac{\sqrt{7x+1}}{\sqrt{5x-1}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[(-1 + 5*x)/(1 + 7*x)]/x^2,x]
```

```
[Out] -((Sqrt[-1 + 5*x]*Sqrt[1 + 7*x])/x) - 12*ArcTan[Sqrt[1 + 7*x]/Sqr
t[-1 + 5*x]]
```

Rubi in Sympy [A] time = 3.25569, size = 39, normalized size = 0.85

$$-12\,\mathrm{atan}\left(\frac{\sqrt{7x+1}}{\sqrt{5x-1}}\right)-\frac{\sqrt{5x-1}\sqrt{7x+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((-1+5*x)/(1+7*x))**(1/2)/x**2,x)
```

```
[Out] -12*atan(sqrt(7*x + 1)/sqrt(5*x - 1)) - sqrt(5*x - 1)*sqrt(7*x +
1)/x
```

Mathematica [A] time = 0.0635688, size = 82, normalized size = 1.78

$$-\frac{\sqrt{\frac{5x-1}{7x+1}}\left(\sqrt{5x-1}(7x+1)+6x\sqrt{7x+1}\tan^{-1}\left(\frac{x+1}{\sqrt{5x-1}\sqrt{7x+1}}\right)\right)}{x\sqrt{5x-1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[(-1 + 5*x)/(1 + 7*x)]/x^2,x]
```

```
[Out] -((Sqrt[(-1 + 5*x)/(1 + 7*x)]*(Sqrt[-1 + 5*x]*(1 + 7*x) + 6*x*Sqr
t[1 + 7*x]*ArcTan[(1 + x)/(Sqrt[-1 + 5*x]*Sqrt[1 + 7*x])]))/(x*Sq
rt[-1 + 5*x]))
```

Maple [B] time = 0.031, size = 103, normalized size = 2.2

$$\frac{1+7x}{x}\sqrt{\frac{-1+5x}{1+7x}}\left(\left(35x^2-2x-1\right)^{\frac{3}{2}}-35\sqrt{35x^2-2x-1}x^2+2\sqrt{35x^2-2x-1}x-6\arctan\left(\frac{1+x}{\sqrt{35x^2-2x-1}}\right)x\right)\frac{1}{\sqrt{(\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((-1+5*x)/(1+7*x))^(1/2)/x^2,x)
```

```
[Out] ((-1+5*x)/(1+7*x))^(1/2)*(1+7*x)*((35*x^2-2*x-1)^(3/2)-35*(35*x^2
-2*x-1)^(1/2)*x^2+2*(35*x^2-2*x-1)^(1/2)*x-6*arctan((1+x)/(35*x^2
-2*x-1)^(1/2))*x)/((-1+5*x)*(1+7*x))^(1/2)/x
```

Maxima [A] time = 0.801891, size = 72, normalized size = 1.57

$$-\frac{12\sqrt{\frac{5x-1}{7x+1}}}{\frac{5x-1}{7x+1}+1}+12\arctan\left(\sqrt{\frac{5x-1}{7x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((5*x - 1)/(7*x + 1))/x^2,x, algorithm="maxima")
```

```
[Out] -12*sqrt((5*x - 1)/(7*x + 1))/((5*x - 1)/(7*x + 1) + 1) + 12*arct
an(sqrt((5*x - 1)/(7*x + 1)))
```

Fricas [A] time = 0.278061, size = 62, normalized size = 1.35

$$\frac{12x\arctan\left(\sqrt{\frac{5x-1}{7x+1}}\right)-(7x+1)\sqrt{\frac{5x-1}{7x+1}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((5*x - 1)/(7*x + 1))/x^2,x, algorithm="fricas")
```

```
[Out] (12*x*arctan(sqrt((5*x - 1)/(7*x + 1))) - (7*x + 1)*sqrt((5*x - 1
)/(7*x + 1)))/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{5x-1}{7x+1}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((-1+5*x)/(1+7*x))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt((5*x - 1)/(7*x + 1))/x**2, x)
```

GIAC/XCAS [A] time = 0.277704, size = 154, normalized size = 3.35

$$\left(\sqrt{35} - 12\,\arctan\left(\frac{1}{7}\,\sqrt{35}\right)\right)\operatorname{sign}\left(7\,x + 1\right) + 12\,\arctan\left(-\sqrt{35}x + \sqrt{35\,x^2 - 2\,x - 1}\right)\,\operatorname{sign}\left(7\,x + 1\right) - \frac{2\left(\left(\sqrt{35}x - \sqrt{35\,x^2 - 2\,x - 1}\right)\operatorname{sign}\left(7\,x + 1\right) + \sqrt{35}\operatorname{sign}\left(7\,x + 1\right)\right)}{\left(\sqrt{35}x - \sqrt{35\,x^2 - 2\,x - 1}\right)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((5*x - 1)/(7*x + 1))/x^2,x, algorithm="giac")
```

```
[Out] (sqrt(35) - 12*arctan(1/7*sqrt(35)))*sign(7*x + 1) + 12*arctan(-s
qrt(35)*x + sqrt(35*x^2 - 2*x - 1))*sign(7*x + 1) - 2*((sqrt(35)*
x - sqrt(35*x^2 - 2*x - 1))*sign(7*x + 1) + sqrt(35)*sign(7*x + 1
))/((sqrt(35)*x - sqrt(35*x^2 - 2*x - 1))^2 + 1)
```

3.592 $\int \frac{x}{\sqrt{\frac{1-x}{1+x}}(1+x)} \, dx$

Optimal. Leaf size=20

$$-\sqrt{\frac{1-x}{x+1}}(x+1)$$

```
[Out] -(Sqrt[(1 - x)/(1 + x)]*(1 + x))
```

Rubi [A] time = 0.0885777, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$-\sqrt{\frac{1-x}{x+1}}(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[(1 - x)/(1 + x)]*(1 + x)),x]

[Out] -(Sqrt[(1 - x)/(1 + x)]*(1 + x))
```

Rubi in Sympy [A] time = 3.94377, size = 20, normalized size = 1.

$$-\frac{2\sqrt{\frac{-x+1}{x+1}}}{\frac{-x+1}{x+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x/(1+x)/((1-x)/(1+x))**(1/2),x)

[Out] -2*sqrt((-x + 1)/(x + 1))/((-x + 1)/(x + 1) + 1)
```

Mathematica [A] time = 0.0197909, size = 19, normalized size = 0.95

$$\frac{x-1}{\sqrt{\frac{1-x}{x+1}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(Sqrt[(1 - x)/(1 + x)]*(1 + x)),x]
```

```
[Out] (-1 + x)/Sqrt[(1 - x)/(1 + x)]
```

Maple [A] time = 0.005, size = 17, normalized size = 0.9

$$(-1+x)\frac{1}{\sqrt{-\frac{-1+x}{1+x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(1+x)/((1-x)/(1+x))^(1/2),x)
```

```
[Out] (-1+x)/(-(-1+x)/(1+x))^(1/2)
```

Maxima [A] time = 0.716299, size = 36, normalized size = 1.8

$$\frac{2\sqrt{-\frac{x-1}{x+1}}}{\frac{x-1}{x+1}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((x + 1)*sqrt(-(x - 1)/(x + 1))),x, algorithm="maxima")
```

```
[Out] 2*sqrt(-(x - 1)/(x + 1))/((x - 1)/(x + 1) - 1)
```

Fricas [A] time = 0.272183, size = 23, normalized size = 1.15

$$-(x+1)\sqrt{-\frac{x-1}{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((x + 1)*sqrt(-(x - 1)/(x + 1))),x, algorithm="fricas")
```

```
[Out] -(x + 1)*sqrt(-(x - 1)/(x + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-\frac{x-1}{x+1}}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(1+x)/((1-x)/(1+x))**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-(x - 1)/(x + 1))*(x + 1)), x)
```

GIAC/XCAS [A] time = 0.280352, size = 39, normalized size = 1.95

$$-\frac{2}{\sqrt{-\frac{x-1}{x+1}}+\frac{1}{\sqrt{-\frac{x-1}{x+1}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((x + 1)*sqrt(-(x - 1)/(x + 1))),x, algorithm="giac")
```

```
[Out] -2/(sqrt(-(x - 1)/(x + 1)) + 1/sqrt(-(x - 1)/(x + 1)))
```

3.593 $\int \frac{x}{(1+x)\sqrt{-1+\frac{2}{1+x}}}\,dx$

Optimal. Leaf size=18

$$-(x+1)\sqrt{\frac{2}{x+1}-1}$$

```
[Out] -((1 + x)*Sqrt[-1 + 2/(1 + x)])
```

Rubi [A] time = 0.115905, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$-(x+1)\sqrt{\frac{2}{x+1}-1}$$

Antiderivative was successfully verified.

```
[In]  Int[x/((1 + x)*Sqrt[-1 + 2/(1 + x)]),x]

[Out] -((1 + x)*Sqrt[-1 + 2/(1 + x)])
```

Rubi in Sympy [A] time = 6.16638, size = 14, normalized size = 0.78

$$-\sqrt{-1+\frac{2}{x+1}}\,(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(1+x)/(-1+2/(1+x))**(1/2),x)

[Out] -sqrt(-1 + 2/(x + 1))*(x + 1)
```

Mathematica [A] time = 0.0154088, size = 17, normalized size = 0.94

$$\frac{x-1}{\sqrt{\frac{2}{x+1}-1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/((1 + x)*Sqrt[-1 + 2/(1 + x)]),x]

[Out] (-1 + x)/Sqrt[-1 + 2/(1 + x)]
```

Maple [A] time = 0.004, size = 17, normalized size = 0.9

$$(-1+x)\frac{1}{\sqrt{-\frac{-1+x}{1+x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(1+x)/(-1+2/(1+x))^(1/2),x)

[Out] (-1+x)/(-(-1+x)/(1+x))^(1/2)
```

Maxima [A] time = 0.774482, size = 22, normalized size = 1.22

$$\frac{\sqrt{x+1}(x-1)}{\sqrt{-x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((x + 1)*sqrt(2/(x + 1) - 1)),x, algorithm="maxima")

[Out] sqrt(x + 1)*(x - 1)/sqrt(-x + 1)
```

Fricas [A] time = 0.268601, size = 23, normalized size = 1.28

$$-(x+1)\sqrt{-\frac{x-1}{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((x + 1)*sqrt(2/(x + 1) - 1)),x, algorithm="fricas")

[Out] -(x + 1)*sqrt(-(x - 1)/(x + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-\frac{x-1}{x+1}}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(1+x)/(-1+2/(1+x))**(1/2),x)

[Out] Integral(x/(sqrt(-(x - 1)/(x + 1))*(x + 1)), x)
```

GIAC/XCAS [A] time = 0.281223, size = 39, normalized size = 2.17

$$-\frac{2}{\sqrt{-\frac{x-1}{x+1}}+\frac{1}{\sqrt{-\frac{x-1}{x+1}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((x + 1)*sqrt(2/(x + 1) - 1)),x, algorithm="giac")

[Out] -2/(sqrt(-(x - 1)/(x + 1)) + 1/sqrt(-(x - 1)/(x + 1)))
```

3.594 $\int \frac{x}{(1+x)\sqrt{\frac{2+x}{3+x}}}\,dx$

Optimal. Leaf size=54

$$\sqrt{x+2}\sqrt{x+3}-\sinh^{-1}\left(\sqrt{x+2}\right)+2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x+2}}{\sqrt{x+3}}\right)$$

```
[Out] Sqrt[2 + x]*Sqrt[3 + x] - ArcSinh[Sqrt[2 + x]] + 2*Sqrt[2]*ArcTan
h[(Sqrt[2]*Sqrt[2 + x])/Sqrt[3 + x]]
```

Rubi [A] time = 0.176261, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35

$$\sqrt{x+2}\sqrt{x+3}-\sinh^{-1}\left(\sqrt{x+2}\right)+2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x+2}}{\sqrt{x+3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x/((1 + x)*Sqrt[(2 + x)/(3 + x)]),x]
```

```
[Out] Sqrt[2 + x]*Sqrt[3 + x] - ArcSinh[Sqrt[2 + x]] + 2*Sqrt[2]*ArcTan
h[(Sqrt[2]*Sqrt[2 + x])/Sqrt[3 + x]]
```

Rubi in Sympy [A] time = 6.85097, size = 48, normalized size = 0.89

$$\sqrt{x+2}\sqrt{x+3}-\operatorname{asinh}\left(\sqrt{x+2}\right)+2\sqrt{2}\operatorname{atanh}\left(\frac{\sqrt{2}\sqrt{x+2}}{\sqrt{x+3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(1+x)/((2+x)/(3+x))**(1/2),x)
```

```
[Out] sqrt(x + 2)*sqrt(x + 3) - asinh(sqrt(x + 2)) + 2*sqrt(2)*atanh(sq
rt(2)*sqrt(x + 2)/sqrt(x + 3))
```

Mathematica [A] time = 0.102448, size = 101, normalized size = 1.87

$$\sqrt{\frac{x+2}{x+3}}x+3\sqrt{\frac{x+2}{x+3}}-\sqrt{2}\log(x+1)-\frac{1}{2}\log\left(2x+2\sqrt{x+2}\sqrt{x+3}+5\right)+\sqrt{2}\log\left(3x+2\sqrt{2}\sqrt{x+2}\sqrt{x+3}+7\right)$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x/((1 + x)*Sqrt[(2 + x)/(3 + x)]),x]
```

```
[Out] 3*Sqrt[(2 + x)/(3 + x)] + x*Sqrt[(2 + x)/(3 + x)] - Sqrt[2]*Log[1
 + x] - Log[5 + 2*x + 2*Sqrt[2 + x]*Sqrt[3 + x]]/2 + Sqrt[2]*Log[
7 + 3*x + 2*Sqrt[2]*Sqrt[2 + x]*Sqrt[3 + x]]
```

Maple [A] time = 0.026, size = 79, normalized size = 1.5

$$-\frac{2+x}{2}\left(-2\sqrt{2}Artanh\left(1/4\,\frac{(3\,x+7)\sqrt{2}}{\sqrt{x^2+5\,x+6}}\right)+\ln\left(\frac{5}{2}+x+\sqrt{x^2+5\,x+6}\right)-2\sqrt{x^2+5\,x+6}\right)\frac{1}{\sqrt{\frac{2+x}{3+x}}}\frac{1}{\sqrt{(3+x)(2+x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(1+x)/((2+x)/(3+x))^(1/2),x)
```

```
[Out] -1/2*(2+x)*(-2*2^(1/2)*arctanh(1/4*(3*x+7)*2^(1/2)/(x^2+5*x+6)^(1
/2))+ln(5/2+x+(x^2+5*x+6)^(1/2))-2*(x^2+5*x+6)^(1/2))/((2+x)/(3+x
))^(1/2)/((3+x)*(2+x))^(1/2)
```

Maxima [A] time = 0.832791, size = 139, normalized size = 2.57

$$-\sqrt{2}\log\left(-\frac{2\left(\sqrt{2}-2\sqrt{\frac{x+2}{x+3}}\right)}{2\sqrt{2}+4\sqrt{\frac{x+2}{x+3}}}\right)-\frac{\sqrt{\frac{x+2}{x+3}}}{\frac{x+2}{x+3}-1}-\frac{1}{2}\log\left(\sqrt{\frac{x+2}{x+3}}+1\right)+\frac{1}{2}\log\left(\sqrt{\frac{x+2}{x+3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((x + 1)*sqrt((x + 2)/(x + 3))),x, algorithm="maxima")
```

```
[Out] -sqrt(2)*log(-2*(sqrt(2) - 2*sqrt((x + 2)/(x + 3)))/((2*sqrt(2))
+ 4*sqrt((x + 2)/(x + 3)))) - sqrt((x + 2)/(x + 3))/((x + 2)/(x +
 3) - 1) - 1/2*log(sqrt((x + 2)/(x + 3)) + 1) + 1/2*log(sqrt((x +
 2)/(x + 3)) - 1)
```

Fricas [A] time = 0.295916, size = 112, normalized size = 2.07

$$(x+3)\sqrt{\frac{x+2}{x+3}}+\sqrt{2}\log\left(\frac{2\sqrt{2}(x+3)\sqrt{\frac{x+2}{x+3}}+3x+7}{x+1}\right)-\frac{1}{2}\log\left(\sqrt{\frac{x+2}{x+3}}+1\right)+\frac{1}{2}\log\left(\sqrt{\frac{x+2}{x+3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((x + 1)*sqrt((x + 2)/(x + 3))),x, algorithm="fricas")
```

```
[Out] (x + 3)*sqrt((x + 2)/(x + 3)) + sqrt(2)*log((2*sqrt(2)*(x + 3)*sq
rt((x + 2)/(x + 3)) + 3*x + 7)/(x + 1)) - 1/2*log(sqrt((x + 2)/(x
 + 3)) + 1) + 1/2*log(sqrt((x + 2)/(x + 3)) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{\sqrt{\frac{x+2}{x+3}}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(1+x)/((2+x)/(3+x))**(1/2),x)
```

```
[Out] Integral(x/(sqrt((x + 2)/(x + 3))*(x + 1)), x)
```

GIAC/XCAS [A] time = 0.2846, size = 144, normalized size = 2.67

$$-\sqrt{2}\ln\left(\frac{\left|-2\sqrt{2}+4\sqrt{\frac{x+2}{x+3}}\right|}{2\left(\sqrt{2}+2\sqrt{\frac{x+2}{x+3}}\right)}\right)-\frac{\sqrt{\frac{x+2}{x+3}}}{\frac{x+2}{x+3}-1}-\frac{1}{2}\ln\left(\sqrt{\frac{x+2}{x+3}}+1\right)+\frac{1}{2}\ln\left(\left|\sqrt{\frac{x+2}{x+3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((x + 1)*sqrt((x + 2)/(x + 3))),x, algorithm="giac")
```

```
[Out] -sqrt(2)*ln(1/2*abs(-2*sqrt(2) + 4*sqrt((x + 2)/(x + 3)))/(sqrt(2
) + 2*sqrt((x + 2)/(x + 3)))) - sqrt((x + 2)/(x + 3))/((x + 2)/(x
 + 3) - 1) - 1/2*ln(sqrt((x + 2)/(x + 3)) + 1) + 1/2*ln(abs(sqrt(
(x + 2)/(x + 3)) - 1))
```

3.595 $\int \frac{\sqrt{1+\frac{1}{x}}}{(1+x)^2} \, dx$

Optimal. Leaf size=11

$$\frac{2}{\sqrt{\frac{1}{x}+1}}$$

```
[Out] 2/Sqrt[1 + x^(-1)]
```

Rubi [A] time = 0.0135279, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$\frac{2}{\sqrt{\frac{1}{x}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^(-1)]/(1 + x)^2,x]

[Out] 2/Sqrt[1 + x^(-1)]
```

Rubi in Sympy [A] time = 1.34967, size = 8, normalized size = 0.73

$$\frac{2}{\sqrt{1+\frac{1}{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((1+1/x)**(1/2)/(1+x)**2,x)

[Out] 2/sqrt(1 + 1/x)
```

Mathematica [A] time = 0.0147826, size = 17, normalized size = 1.55

$$\frac{2\sqrt{\frac{1}{x}+1}x}{x+1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 + x^(-1)]/(1 + x)^2,x]
```

```
[Out] (2*Sqrt[1 + x^(-1)]*x)/(1 + x)
```

Maple [A] time = 0.005, size = 18, normalized size = 1.6

$$2\,{\frac {x}{1+x}}\sqrt {{\frac {1+x}{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+1/x)^(1/2)/(1+x)^2,x)
```

```
[Out] 2*x/(1+x)*((1+x)/x)^(1/2)
```

Maxima [A] time = 0.88832, size = 15, normalized size = 1.36

$$\frac{2}{\sqrt{\frac{x+1}{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(1/x + 1)/(x + 1)^2,x, algorithm="maxima")
```

```
[Out] 2/sqrt((x + 1)/x)
```

Fricas [A] time = 0.28111, size = 15, normalized size = 1.36

$$\frac{2}{\sqrt{\frac{x+1}{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(1/x + 1)/(x + 1)^2,x, algorithm="fricas")
```

```
[Out] 2/sqrt((x + 1)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{1 + \frac{1}{x}}}{(x + 1)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+1/x)**(1/2)/(1+x)**2,x)
```

```
[Out] Integral(sqrt(1 + 1/x)/(x + 1)**2, x)
```

GIAC/XCAS [A] time = 0.269729, size = 31, normalized size = 2.82

$$\frac{2\,\mathrm{sign}\,(x)}{x - \sqrt{x^2 + x} + 1} - 2\,\mathrm{sign}\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(1/x + 1)/(x + 1)^2,x, algorithm="giac")
```

```
[Out] 2*sign(x)/(x - sqrt(x^2 + x) + 1) - 2*sign(x)
```

3.596 $\int \frac{\sqrt{1+\frac{1}{x}}}{\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=29

$$-\frac{\sqrt{\frac{1}{x}+1}\sqrt{x}\sin^{-1}(1-2x)}{\sqrt{x+1}}$$

```
[Out] -((Sqrt[1 + x^(-1)]*Sqrt[x]*ArcSin[1 - 2*x])/Sqrt[1 + x])
```

Rubi [A] time = 0.0421779, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$

$$-\frac{\sqrt{\frac{1}{x}+1}\sqrt{x}\sin^{-1}(1-2x)}{\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 + x^(-1)]/Sqrt[1 - x^2],x]

[Out] -((Sqrt[1 + x^(-1)]*Sqrt[x]*ArcSin[1 - 2*x])/Sqrt[1 + x])
```

Rubi in Sympy [A] time = 3.71113, size = 26, normalized size = 0.9

$$\frac{\sqrt{x}\sqrt{1+\frac{1}{x}}\,\operatorname{asin}(2x-1)}{\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+1/x)**(1/2)/(-x**2+1)**(1/2),x)

[Out] sqrt(x)*sqrt(1 + 1/x)*asin(2*x - 1)/sqrt(x + 1)
```

Mathematica [A] time = 0.0193164, size = 41, normalized size = 1.41

$$-\tan^{-1}\left(\frac{\sqrt{\frac{x+1}{x}}(2x-1)\sqrt{1-x^2}}{2\left(x^2-1\right)}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 + x^(-1)]/Sqrt[1 - x^2],x]
```

```
[Out] -ArcTan[(Sqrt[(1 + x)/x]*(-1 + 2*x)*Sqrt[1 - x^2])/(2*(-1 + x^2))
]
```

Maple [A] time = 0.026, size = 40, normalized size = 1.4

$$\frac{x\arcsin(2x-1)}{1+x}\sqrt{\frac{1+x}{x}}\sqrt{-x^2+1}\frac{1}{\sqrt{-x(-1+x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+1/x)^(1/2)/(-x^2+1)^(1/2),x)
```

```
[Out] ((1+x)/x)^(1/2)*x*(-x^2+1)^(1/2)/(1+x)/(-x*(-1+x))^(1/2)*arcsin(2
*x-1)
```

Maxima [A] time = 0.862538, size = 19, normalized size = 0.66

$$-2\arctan\left(\frac{\sqrt{-x+1}}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(1/x + 1)/sqrt(-x^2 + 1),x, algorithm="maxima")
```

```
[Out] -2*arctan(sqrt(-x + 1)/sqrt(x))
```

Fricas [A] time = 0.313473, size = 46, normalized size = 1.59

$$-\arctan\left(\frac{2\sqrt{-x^2+1}x\sqrt{\frac{x+1}{x}}}{2x^2+x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(1/x + 1)/sqrt(-x^2 + 1),x, algorithm="fricas")

[Out] -arctan(2*sqrt(-x^2 + 1)*x*sqrt((x + 1)/x)/(2*x^2 + x - 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{1+\frac{1}{x}}}{\sqrt{-(x-1)(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+1/x)**(1/2)/(-x**2+1)**(1/2),x)

[Out] Integral(sqrt(1 + 1/x)/sqrt(-(x - 1)*(x + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{1}{x}+1}}{\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(1/x + 1)/sqrt(-x^2 + 1),x, algorithm="giac")

[Out] integrate(sqrt(1/x + 1)/sqrt(-x^2 + 1), x)
```

3.597 $\int \frac{1}{x+\sqrt{3-2x-x^2}}\, dx$

Optimal. Leaf size=180

$$-\frac{1}{2}\log\left(-\frac{-\sqrt{3}\sqrt{-x^2-2x+3}-x+3}{x^2}\right)+\frac{1}{14}\left(7+\sqrt{7}\right)\log\left(-\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-\sqrt{7}+\sqrt{3}+1\right)+\frac{1}{14}\left(7-\sqrt{7}\right)\log\left(-\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}+\sqrt{7}+\sqrt{3}+1\right)+\tan^{-1}\left(\frac{\sqrt{3}-\sqrt{-x^2-2x+3}}{x}\right)$$

```
[Out] ArcTan[(Sqrt[3] - Sqrt[3 - 2*x - x^2])/x] - Log[-((3 - x - Sqrt[3
]*Sqrt[3 - 2*x - x^2])/x^2)]/2 + ((7 + Sqrt[7])*Log[1 + Sqrt[3] -
 Sqrt[7] - (Sqrt[3]*(Sqrt[3] - Sqrt[3 - 2*x - x^2]))/x])/14 + ((7
 - Sqrt[7])*Log[1 + Sqrt[3] + Sqrt[7] - (Sqrt[3]*(Sqrt[3] - Sqrt[
3 - 2*x - x^2]))/x])/14
```

Rubi [A] time = 0.41385, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$-\frac{1}{2}\log\left(-\frac{-\sqrt{3}\sqrt{-x^2-2x+3}-x+3}{x^2}\right)+\frac{1}{14}\left(7+\sqrt{7}\right)\log\left(-\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-\sqrt{7}+\sqrt{3}+1\right)+\frac{1}{14}\left(7-\sqrt{7}\right)\log\left(-\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}+\sqrt{7}+\sqrt{3}+1\right)+\tan^{-1}\left(\frac{\sqrt{3}-\sqrt{-x^2-2x+3}}{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(x + Sqrt[3 - 2*x - x^2])^(-1),x]
```

```
[Out] ArcTan[(Sqrt[3] - Sqrt[3 - 2*x - x^2])/x] - Log[-((3 - x - Sqrt[3
]*Sqrt[3 - 2*x - x^2])/x^2)]/2 + ((7 + Sqrt[7])*Log[1 + Sqrt[3] -
 Sqrt[7] - (Sqrt[3]*(Sqrt[3] - Sqrt[3 - 2*x - x^2]))/x])/14 + ((7
 - Sqrt[7])*Log[1 + Sqrt[3] + Sqrt[7] - (Sqrt[3]*(Sqrt[3] - Sqrt[
3 - 2*x - x^2]))/x])/14
```

Rubi in Sympy [A] time = 84.3252, size = 180, normalized size = 1.

$$-\frac{\log\left(1+\frac{\left(\sqrt{-x^2-2x+3}-\sqrt{3}\right)^2}{x^2}\right)}{2}+\frac{\log\left(-6+4\sqrt{3}+\frac{\left(4\sqrt{3}+12\right)\left(\sqrt{-x^2-2x+3}-\sqrt{3}\right)}{x}+\frac{\left(\sqrt{-x^2-2x+3}-\sqrt{3}\right)^2}{x^2}\right)}{2}$$
$$-\operatorname{atan}\left(\frac{\sqrt{-x^2-2x+3}-\sqrt{3}}{x}\right)+\frac{\sqrt{2}\left(-2\sqrt{3}+5\right)\operatorname{atanh}\left(\frac{\sqrt{2}\left(\sqrt{3}+3+\frac{\sqrt{-x^2-2x+3}-\sqrt{3}}{2x}\right)}{\sqrt{10\sqrt{3}+27}}\right)}{2\sqrt{10\sqrt{3}+27}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x+(-x**2-2*x+3)**(1/2)),x)
```

```
[Out] -log(1 + (sqrt(-x**2 - 2*x + 3) - sqrt(3))**2/x**2)/2 + log(-6 +
4*sqrt(3) + (4*sqrt(3) + 12)*(sqrt(-x**2 - 2*x + 3) - sqrt(3))/x
+ (sqrt(-x**2 - 2*x + 3) - sqrt(3))**2/x**2)/2 - atan((sqrt(-x**2
 - 2*x + 3) - sqrt(3))/x) + sqrt(2)*(-2*sqrt(3) + 5)*atanh(sqrt(2
)*(sqrt(3) + 3 + (sqrt(-x**2 - 2*x + 3) - sqrt(3))/(2*x))/sqrt(10
*sqrt(3) + 27))/(2*sqrt(10*sqrt(3) + 27))
```

Mathematica [A] time = 0.89127, size = 250, normalized size = 1.39

$$\frac{1}{28}\left(-\sqrt{14\left(4+\sqrt{7}\right)}\log\left(\sqrt{14\left(4+\sqrt{7}\right)}\sqrt{-x^2-2x+3}-\sqrt{7}x+7x+7\sqrt{7}+7\right)\right.$$
$$-\frac{1}{3}\left(\sqrt{7}-4\right)\sqrt{14\left(4+\sqrt{7}\right)}\log\left(-\sqrt{14}\sqrt{\left(\sqrt{7}-4\right)\left(x^2+2x-3\right)}+\left(7+\sqrt{7}\right)x-7\sqrt{7}+7\right)$$
$$-\left(\sqrt{7}-7\right)\log\left(-2x+\sqrt{7}-1\right)+\frac{1}{3}\left(\sqrt{7}-4\right)\sqrt{14\left(4+\sqrt{7}\right)}\log\left(2x-\sqrt{7}+1\right)$$
$$\left.+\left(7+\sqrt{7}\right)\log\left(2x+\sqrt{7}+1\right)+\sqrt{14\left(4+\sqrt{7}\right)}\log\left(2x+\sqrt{7}+1\right)+14\sin^{-1}\left(\frac{x+1}{2}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(x + Sqrt[3 - 2*x - x^2])^(-1),x]
```

```
[Out] (14*ArcSin[(1 + x)/2] - (-7 + Sqrt[7])*Log[-1 + Sqrt[7] - 2*x] +
((-4 + Sqrt[7])*Sqrt[14*(4 + Sqrt[7])]*Log[1 - Sqrt[7] + 2*x])/3
+ Sqrt[14*(4 + Sqrt[7])]*Log[1 + Sqrt[7] + 2*x] + (7 + Sqrt[7])*L
og[1 + Sqrt[7] + 2*x] - Sqrt[14*(4 + Sqrt[7])]*Log[7 + 7*Sqrt[7]
+ 7*x - Sqrt[7]*x + Sqrt[14*(4 + Sqrt[7])]*Sqrt[3 - 2*x - x^2]] -
 ((-4 + Sqrt[7])*Sqrt[14*(4 + Sqrt[7])]*Log[7 - 7*Sqrt[7] + (7 +
```

```
Sqrt[7])*x - Sqrt[14]*Sqrt[(-4 + Sqrt[7])*(-3 + 2*x + x^2)]])/3)/
28
```

Maple [B] time = 0.084, size = 551, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x+(-x^2-2*x+3)^(1/2)),x)
```

```
[Out] -1/28*7^(1/2)*(-4*(x+1/2-1/2*7^(1/2))^2+4*(-1-7^(1/2))*(x+1/2-1/2
*7^(1/2))+8-2*7^(1/2))^(1/2)+1/28*arcsin(1/(2-1/2*7^(1/2)+1/4*(-1
-7^(1/2))^2)^(1/2)*(1+x))*7^(1/2)+1/4*arcsin(1/(2-1/2*7^(1/2)+1/4
*(-1-7^(1/2))^2)^(1/2)*(1+x))+1/7/(-1/2+1/2*7^(1/2))*arctanh((4-7
^(1/2)+(-1-7^(1/2))*(x+1/2-1/2*7^(1/2)))/(-1/2+1/2*7^(1/2))/(-4*(
x+1/2-1/2*7^(1/2))^2+4*(-1-7^(1/2))*(x+1/2-1/2*7^(1/2))+8-2*7^(1/
2))^(1/2))*7^(1/2)-1/4/(-1/2+1/2*7^(1/2))*arctanh((4-7^(1/2)+(-1-
7^(1/2))*(x+1/2-1/2*7^(1/2)))/(-1/2+1/2*7^(1/2))/(-4*(x+1/2-1/2*7
^(1/2))^2+4*(-1-7^(1/2))*(x+1/2-1/2*7^(1/2))+8-2*7^(1/2))^(1/2))+
1/28*7^(1/2)*(-4*(x+1/2+1/2*7^(1/2))^2+4*(-1+7^(1/2))*(x+1/2+1/2*
7^(1/2))+8+2*7^(1/2))^(1/2)-1/28*arcsin(1/(2+1/2*7^(1/2)+1/4*(-1+
7^(1/2))^2)^(1/2)*(1+x))*7^(1/2)+1/4*arcsin(1/(2+1/2*7^(1/2)+1/4*
(-1+7^(1/2))^2)^(1/2)*(1+x))-1/7/(1/2*7^(1/2)+1/2)*arctanh((4+7^(
1/2)+(-1+7^(1/2))*(x+1/2+1/2*7^(1/2)))/(1/2*7^(1/2)+1/2)/(-4*(x+1
/2+1/2*7^(1/2))^2+4*(-1+7^(1/2))*(x+1/2+1/2*7^(1/2))+8+2*7^(1/2))
^(1/2))*7^(1/2)-1/4/(1/2*7^(1/2)+1/2)*arctanh((4+7^(1/2)+(-1+7^(1
/2))*(x+1/2+1/2*7^(1/2)))/(1/2*7^(1/2)+1/2)/(-4*(x+1/2+1/2*7^(1/2
))^2+4*(-1+7^(1/2))*(x+1/2+1/2*7^(1/2))+8+2*7^(1/2))^(1/2))+1/4*l
n(2*x^2+2*x-3)+1/14*7^(1/2)*arctanh(1/14*(4*x+2)*7^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x + \sqrt{-x^2 - 2x + 3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x + sqrt(-x^2 - 2*x + 3)),x, algorithm="maxima")
```

```
[Out] integrate(1/(x + sqrt(-x^2 - 2*x + 3)), x)
```

Fricas [A] time = 0.291535, size = 379, normalized size = 2.11

$$\frac{1}{56}\sqrt{7}\left(4\sqrt{7}\arctan\left(\frac{x+1}{\sqrt{-x^2-2x+3}}\right)+2\sqrt{7}\log\left(2x^2+2x-3\right)-\sqrt{7}\log\left(\frac{2\sqrt{-x^2-2x+3}x+2x-3}{x^2}\right)+\sqrt{7}\log\left(-\frac{2\sqrt{-}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x + sqrt(-x^2 - 2*x + 3)),x, algorithm="fricas")
```

```
[Out] 1/56*sqrt(7)*(4*sqrt(7)*arctan((x + 1)/sqrt(-x^2 - 2*x + 3)) + 2*
sqrt(7)*log(2*x^2 + 2*x - 3) - sqrt(7)*log((2*sqrt(-x^2 - 2*x + 3
)*x + 2*x - 3)/x^2) + sqrt(7)*log(-(2*sqrt(-x^2 - 2*x + 3)*x - 2*
x + 3)/x^2) + 2*log((2*sqrt(7)*(x^2 + x + 2) + 14*x + 7)/(2*x^2 +
 2*x - 3)) + log((28*x^2 + sqrt(7)*(7*x^2 - 30*x + 45) + 3*sqrt(-
x^2 - 2*x + 3)*(4*sqrt(7)*x + 7*x - 21) - 84*x)/(2*sqrt(-x^2 - 2*
x + 3)*x + 2*x - 3)) + log((28*x^2 - sqrt(7)*(7*x^2 - 30*x + 45)
+ 3*sqrt(-x^2 - 2*x + 3)*(4*sqrt(7)*x - 7*x + 21) - 84*x)/(2*sqrt
(-x^2 - 2*x + 3)*x - 2*x + 3)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x+\sqrt{-x^2-2x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x+(-x**2-2*x+3)**(1/2)),x)
```

```
[Out] Integral(1/(x + sqrt(-x**2 - 2*x + 3)), x)
```

GIAC/XCAS [A] time = 0.323625, size = 387, normalized size = 2.15

$$-\frac{1}{28}\sqrt{7}\ln\left(\frac{\left|4x-2\sqrt{7}+2\right|}{\left|4x+2\sqrt{7}+2\right|}\right)+\frac{1}{28}\sqrt{7}\ln\left(\frac{\left|-2\sqrt{7}+\frac{6\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}+4\right|}{\left|2\sqrt{7}+\frac{6\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}+4\right|}\right)-\frac{1}{28}\sqrt{7}\ln\left(\frac{\left|-2\sqrt{7}+\frac{2\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}-4\right|}{\left|2\sqrt{7}+\frac{2\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}-4\right|}\right)+\frac{1}{2}\arcsin\left(\frac{1}{2}x+\frac{1}{2}\right)+\frac{1}{4}\ln\left(\left|2x^2+2x-3\right|\right)+\frac{1}{4}\ln\left(\left|\frac{4\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}+\frac{3\left(\sqrt{-x^2-2x+3}-2\right)^2}{(x+1)^2}-1\right|\right)-\frac{1}{4}\ln\left(\left|-\frac{4\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}+\frac{\left(\sqrt{-x^2-2x+3}-2\right)^2}{(x+1)^2}-3\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x + sqrt(-x^2 - 2*x + 3)),x, algorithm="giac")
```

```
[Out] -1/28*sqrt(7)*ln(abs(4*x - 2*sqrt(7) + 2)/abs(4*x + 2*sqrt(7) + 2
)) + 1/28*sqrt(7)*ln(abs(-2*sqrt(7) + 6*(sqrt(-x^2 - 2*x + 3) - 2
)/(x + 1) + 4)/abs(2*sqrt(7) + 6*(sqrt(-x^2 - 2*x + 3) - 2)/(x +
1) + 4)) - 1/28*sqrt(7)*ln(abs(-2*sqrt(7) + 2*(sqrt(-x^2 - 2*x +
3) - 2)/(x + 1) - 4)/abs(2*sqrt(7) + 2*(sqrt(-x^2 - 2*x + 3) - 2)
/(x + 1) - 4)) + 1/2*arcsin(1/2*x + 1/2) + 1/4*ln(abs(2*x^2 + 2*x
 - 3)) + 1/4*ln(abs(4*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) + 3*(sqr
t(-x^2 - 2*x + 3) - 2)^2/(x + 1)^2 - 1)) - 1/4*ln(abs(-4*(sqrt(-x
^2 - 2*x + 3) - 2)/(x + 1) + (sqrt(-x^2 - 2*x + 3) - 2)^2/(x + 1)
^2 - 3))
```

3.598 $\int \frac{1}{\left(x+\sqrt{3-2x-x^2}\right)^2}\,dx$

Optimal. Leaf size=172

$$\frac{2\left(\frac{3\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-\sqrt{3}+4\right)}{7\left(\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^2}{x^2}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-\sqrt{3}+2\right)}+\frac{8\tanh^{-1}\left(\frac{-\sqrt{3}\sqrt{-x^2-2x+3}-\sqrt{3}x-x+3}{\sqrt{7}x}\right)}{7\sqrt{7}}$$

```
[Out] (2*(4 - Sqrt[3] + (3*(Sqrt[3] - Sqrt[3 - 2*x - x^2]))/x))/(7*(2 -
 Sqrt[3] - (2*(1 + Sqrt[3])*(Sqrt[3] - Sqrt[3 - 2*x - x^2]))/x +
(Sqrt[3]*(Sqrt[3] - Sqrt[3 - 2*x - x^2])^2)/x^2)) + (8*ArcTanh[(3
 - x - Sqrt[3]*x - Sqrt[3]*Sqrt[3 - 2*x - x^2])/(Sqrt[7]*x)])/(7*
Sqrt[7])
```

Rubi [A] time = 0.253153, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$\frac{2\left(\frac{3\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-\sqrt{3}+4\right)}{7\left(\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^2}{x^2}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-\sqrt{3}+2\right)}+\frac{8\tanh^{-1}\left(\frac{-\sqrt{3}\sqrt{-x^2-2x+3}-\sqrt{3}x-x+3}{\sqrt{7}x}\right)}{7\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x + Sqrt[3 - 2*x - x^2])^(-2),x]
```

```
[Out] (2*(4 - Sqrt[3] + (3*(Sqrt[3] - Sqrt[3 - 2*x - x^2]))/x))/(7*(2 -
 Sqrt[3] - (2*(1 + Sqrt[3])*(Sqrt[3] - Sqrt[3 - 2*x - x^2]))/x +
(Sqrt[3]*(Sqrt[3] - Sqrt[3 - 2*x - x^2])^2)/x^2)) + (8*ArcTanh[(3
 - x - Sqrt[3]*x - Sqrt[3]*Sqrt[3 - 2*x - x^2])/(Sqrt[7]*x)])/(7*
Sqrt[7])
```

Rubi in Sympy [A] time = 30.4101, size = 264, normalized size = 1.53

$$\frac{2\sqrt{3}\left(-16\sqrt{3}+12+\frac{\left(\sqrt{3}\left(-\left(2+2\sqrt{3}\right)^2+\sqrt{3}\left(-2\sqrt{3}+4\right)\right)+\sqrt{3}\left(4\sqrt{3}+10\right)\right)\left(-\sqrt{-x^2-2x+3}+\sqrt{3}\right)}{x}\right)}{3\left(-\left(2+2\sqrt{3}\right)^2-\sqrt{3}\left(-8+4\sqrt{3}\right)\right)\left(-\sqrt{3}+2+\frac{\left(2+2\sqrt{3}\right)\left(\sqrt{-x^2-2x+3}-\sqrt{3}\right)}{x}+\frac{\sqrt{3}\left(\sqrt{-x^2-2x+3}-\sqrt{3}\right)^2}{x^2}\right)} - \frac{2\sqrt{21}\left(-\sqrt{3}\left(4\sqrt{3}+10\right)-6\sqrt{3}+12\right)\operatorname{atanh}\left(\sqrt{7}\left(\frac{1}{7}+\frac{\sqrt{3}}{7}+\frac{\sqrt{3}\left(\sqrt{-x^2-2x+3}-\sqrt{3}\right)}{7x}\right)\right)}{21\left(-\left(2+2\sqrt{3}\right)^2-\sqrt{3}\left(-8+4\sqrt{3}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x+(-x**2-2*x+3)**(1/2))**2,x)
```

```
[Out] 2*sqrt(3)*(-16*sqrt(3) + 12 + (sqrt(3)*(-(2 + 2*sqrt(3))**2 + sqr
t(3)*(-2*sqrt(3) + 4)) + sqrt(3)*(4*sqrt(3) + 10))*(-sqrt(-x**2 -
 2*x + 3) + sqrt(3))/x)/(3*(-(2 + 2*sqrt(3))**2 - sqrt(3)*(-8 + 4
*sqrt(3)))*(-sqrt(3) + 2 + (2 + 2*sqrt(3))*(sqrt(-x**2 - 2*x + 3)
 - sqrt(3))/x + sqrt(3)*(sqrt(-x**2 - 2*x + 3) - sqrt(3))**2/x**2
)) - 2*sqrt(21)*(-sqrt(3)*(4*sqrt(3) + 10) - 6*sqrt(3) + 12)*atan
h(sqrt(7)*(1/7 + sqrt(3)/7 + sqrt(3)*(sqrt(-x**2 - 2*x + 3) - sqr
t(3))/(7*x)))/(21*(-(2 + 2*sqrt(3))**2 - sqrt(3)*(-8 + 4*sqrt(3))
))
```

Mathematica [A] time = 1.02062, size = 306, normalized size = 1.78

$$\frac{1}{98}\left(\frac{7(3-8x)}{2x^2+2x-3}-\frac{14(x-3)\sqrt{-x^2-2x+3}}{2x^2+2x-3} - 2\left(1+\sqrt{7}\right)\sqrt{\frac{14}{4+\sqrt{7}}}\log\left(\sqrt{14\left(4+\sqrt{7}\right)}\sqrt{-x^2-2x+3}-\sqrt{7}x+7x+7\sqrt{7}+7\right) - \frac{2}{3}\left(\sqrt{7}-1\right)\sqrt{14\left(4+\sqrt{7}\right)}\log\left(-\sqrt{14}\sqrt{\left(\sqrt{7}-4\right)\left(x^2+2x-3\right)}+\left(7+\sqrt{7}\right)x-7\sqrt{7}+7\right) - 4\sqrt{7}\log\left(-2x+\sqrt{7}-1\right)+\frac{2}{3}\left(\sqrt{7}-1\right)\sqrt{14\left(4+\sqrt{7}\right)}\log\left(2x-\sqrt{7}+1\right) + 2\left(1+\sqrt{7}\right)\sqrt{\frac{14}{4+\sqrt{7}}}\log\left(2x+\sqrt{7}+1\right)+4\sqrt{7}\log\left(2x+\sqrt{7}+1\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(x + Sqrt[3 - 2*x - x^2])^(-2),x]
```

```
[Out] ((7*(3 - 8*x))/(-3 + 2*x + 2*x^2) - (14*(-3 + x)*Sqrt[3 - 2*x - x
^2])/(-3 + 2*x + 2*x^2) - 4*Sqrt[7]*Log[-1 + Sqrt[7] - 2*x] + (2*
(-1 + Sqrt[7])*Sqrt[14*(4 + Sqrt[7])]*Log[1 - Sqrt[7] + 2*x])/3 +
 4*Sqrt[7]*Log[1 + Sqrt[7] + 2*x] + 2*(1 + Sqrt[7])*Sqrt[14/(4 +
Sqrt[7])]*Log[1 + Sqrt[7] + 2*x] - 2*(1 + Sqrt[7])*Sqrt[14/(4 + S
qrt[7])]*Log[7 + 7*Sqrt[7] + 7*x - Sqrt[7]*x + Sqrt[14*(4 + Sqrt[
7])]*Sqrt[3 - 2*x - x^2]] - (2*(-1 + Sqrt[7])*Sqrt[14*(4 + Sqrt[7
])]*Log[7 - 7*Sqrt[7] + (7 + Sqrt[7])*x - Sqrt[14]*Sqrt[(-4 + Sqr
t[7])*(-3 + 2*x + x^2)]])/3)/98
```

Maple [B] time = 0.042, size = 1066, normalized size = 6.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x+(-x^2-2*x+3)^(1/2))^2,x)
```

```
[Out] -3/28*(4*x+2)/(2*x^2+2*x-3)+4/49*7^(1/2)*arctanh(1/14*(4*x+2)*7^(
1/2))+1/14*(-2*x+6)/(2*x^2+2*x-3)-1/49*7^(1/2)*(1/4*(-4*(x+1/2-1/
2*7^(1/2))^2+4*(-1-7^(1/2))*(x+1/2-1/2*7^(1/2))+8-2*7^(1/2))^(1/2
)+1/4*(-1-7^(1/2))*arcsin(1/(2-1/2*7^(1/2)+1/4*(-1-7^(1/2))^2)^(1
/2)*(1+x))-1/2*(2-1/2*7^(1/2))/(-1/2+1/2*7^(1/2))*arctanh((4-7^(1
/2)+(-1-7^(1/2))*(x+1/2-1/2*7^(1/2)))/(-1/2+1/2*7^(1/2))/(-4*(x+1
/2-1/2*7^(1/2))^2+4*(-1-7^(1/2))*(x+1/2-1/2*7^(1/2))+8-2*7^(1/2))
^(1/2)))+1/49*7^(1/2)*(1/4*(-4*(x+1/2+1/2*7^(1/2))^2+4*(-1+7^(1/2
))*(x+1/2+1/2*7^(1/2))+8+2*7^(1/2))^(1/2)+1/4*(-1+7^(1/2))*arcsin
(1/(2+1/2*7^(1/2)+1/4*(-1+7^(1/2))^2)^(1/2)*(1+x))-1/2*(2+1/2*7^(
1/2))/(1/2*7^(1/2)+1/2)*arctanh((4+7^(1/2)+(-1+7^(1/2))*(x+1/2+1/
2*7^(1/2)))/(1/2*7^(1/2)+1/2)/(-4*(x+1/2+1/2*7^(1/2))^2+4*(-1+7^(
1/2))*(x+1/2+1/2*7^(1/2))+8+2*7^(1/2))^(1/2)))-2*(-1/14-1/14*7^(1
/2))*(-1/4/(2+1/2*7^(1/2))/(x+1/2+1/2*7^(1/2))*(-(x+1/2+1/2*7^(1/
2))^2+(-1+7^(1/2))*(x+1/2+1/2*7^(1/2))+2+1/2*7^(1/2))^(3/2)+1/8*(
-1+7^(1/2))/(2+1/2*7^(1/2))*(1/2*(-4*(x+1/2+1/2*7^(1/2))^2+4*(-1+
7^(1/2))*(x+1/2+1/2*7^(1/2))+8+2*7^(1/2))^(1/2)+1/2*(-1+7^(1/2))*
arcsin(1/(2+1/2*7^(1/2)+1/4*(-1+7^(1/2))^2)^(1/2)*(1+x))-(2+1/2*7
^(1/2))/(1/2*7^(1/2)+1/2)*arctanh((4+7^(1/2)+(-1+7^(1/2))*(x+1/2+
1/2*7^(1/2)))/(1/2*7^(1/2)+1/2)/(-4*(x+1/2+1/2*7^(1/2))^2+4*(-1+7
^(1/2))*(x+1/2+1/2*7^(1/2))+8+2*7^(1/2))^(1/2)))-1/2/(2+1/2*7^(1/
2))*(-1/4*(-2*x-2)*(-(x+1/2+1/2*7^(1/2))^2+(-1+7^(1/2))*(x+1/2+1/
2*7^(1/2))+2+1/2*7^(1/2))^(1/2)-1/8*(-8-2*7^(1/2)-(-1+7^(1/2))^2)
*arcsin(1/(2+1/2*7^(1/2)+1/4*(-1+7^(1/2))^2)^(1/2)*(1+x))))-2*(-1
/14+1/14*7^(1/2))*(-1/4/(2-1/2*7^(1/2))/(x+1/2-1/2*7^(1/2))*(-(x+
1/2-1/2*7^(1/2))^2+(-1-7^(1/2))*(x+1/2-1/2*7^(1/2))+2-1/2*7^(1/2)
)^(3/2)+1/8*(-1-7^(1/2))/(2-1/2*7^(1/2))*(1/2*(-4*(x+1/2-1/2*7^(1
/2))^2+4*(-1-7^(1/2))*(x+1/2-1/2*7^(1/2))+8-2*7^(1/2))^(1/2)+1/2*
(-1-7^(1/2))*arcsin(1/(2-1/2*7^(1/2)+1/4*(-1-7^(1/2))^2)^(1/2)*(1
```

```
+x))-(2-1/2*7^(1/2))/(-1/2+1/2*7^(1/2))*arctanh((4-7^(1/2)+(-1-7^
(1/2))*(x+1/2-1/2*7^(1/2)))/(-1/2+1/2*7^(1/2))/(-4*(x+1/2-1/2*7^(
1/2))^2+4*(-1-7^(1/2))*(x+1/2-1/2*7^(1/2))+8-2*7^(1/2))^(1/2)))-1
/2/(2-1/2*7^(1/2))*(-1/4*(-2*x-2)*(-(x+1/2-1/2*7^(1/2))^2+(-1-7^(
1/2))*(x+1/2-1/2*7^(1/2))+2-1/2*7^(1/2))^(1/2)-1/8*(-8+2*7^(1/2)-
(-1-7^(1/2))^2)*arcsin(1/(2-1/2*7^(1/2)+1/4*(-1-7^(1/2))^2)^(1/2)
*(1+x))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x + \sqrt{-x^2 - 2x + 3}\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sqrt(-x^2 - 2*x + 3))^(-2),x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(-x^2 - 2*x + 3))^(-2), x)
```

Fricas [A] time = 0.277265, size = 236, normalized size = 1.37

$$-\frac{\sqrt{7}\left(2\sqrt{7}\sqrt{-x^2-2x+3}(x-3) - 2\left(2x^2+2x-3\right)\log\left(\frac{\sqrt{7}\left(x^4+44x^3+26x^2-276x+207\right)-7\left(3x^3+x^2-45x+45\right)\sqrt{-x^2-2x+3}}{4x^4+8x^3-8x^2-12x+9}\right) - 4\left(2x^2+\right.}{98\left(2x^2+2x-3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sqrt(-x^2 - 2*x + 3))^(-2),x, algorithm="fricas")
```

```
[Out] -1/98*sqrt(7)*(2*sqrt(7)*sqrt(-x^2 - 2*x + 3)*(x - 3) - 2*(2*x^2
+ 2*x - 3)*log((sqrt(7)*(x^4 + 44*x^3 + 26*x^2 - 276*x + 207) - 7
*(3*x^3 + x^2 - 45*x + 45)*sqrt(-x^2 - 2*x + 3))/(4*x^4 + 8*x^3 -
 8*x^2 - 12*x + 9)) - 4*(2*x^2 + 2*x - 3)*log((2*sqrt(7)*(x^2 + x
 + 2) + 14*x + 7)/(2*x^2 + 2*x - 3)) + sqrt(7)*(8*x - 3))/(2*x^2
+ 2*x - 3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x + \sqrt{-x^2 - 2x + 3}\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x+(-x**2-2*x+3)**(1/2))**2,x)
```

```
[Out] Integral((x + sqrt(-x**2 - 2*x + 3))**(-2), x)
```

GIAC/XCAS [A] time = 0.309997, size = 473, normalized size = 2.75

$$-\frac{2}{49}\sqrt{7}\ln\left(\frac{\left|4x-2\sqrt{7}+2\right|}{\left|4x+2\sqrt{7}+2\right|}\right)+\frac{2}{49}\sqrt{7}\ln\left(\frac{\left|-2\sqrt{7}+\frac{6\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}+4\right|}{\left|2\sqrt{7}+\frac{6\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}+4\right|}\right)-\frac{2}{49}\sqrt{7}\ln\left(\frac{\left|-2\sqrt{7}+\frac{2\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}-4\right|}{\left|2\sqrt{7}+\frac{2\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}-4\right|}\right)-\frac{8x-3}{14\left(2x^2+2x-3\right)}-\frac{8\left(\frac{5\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}+\frac{26\left(\sqrt{-x^2-2x+3}-2\right)^2}{(x+1)^2}+\frac{11\left(\sqrt{-x^2-2x+3}-2\right)^3}{(x+1)^3}-6\right)}{21\left(\frac{8\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}+\frac{26\left(\sqrt{-x^2-2x+3}-2\right)^2}{(x+1)^2}+\frac{8\left(\sqrt{-x^2-2x+3}-2\right)^3}{(x+1)^3}-\frac{3\left(\sqrt{-x^2-2x+3}-2\right)^4}{(x+1)^4}-3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sqrt(-x^2 - 2*x + 3))^(-2),x, algorithm="giac")
```

```
[Out] -2/49*sqrt(7)*ln(abs(4*x - 2*sqrt(7) + 2)/abs(4*x + 2*sqrt(7) + 2
)) + 2/49*sqrt(7)*ln(abs(-2*sqrt(7) + 6*(sqrt(-x^2 - 2*x + 3) - 2
)/(x + 1) + 4)/abs(2*sqrt(7) + 6*(sqrt(-x^2 - 2*x + 3) - 2)/(x +
1) + 4)) - 2/49*sqrt(7)*ln(abs(-2*sqrt(7) + 2*(sqrt(-x^2 - 2*x +
3) - 2)/(x + 1) - 4)/abs(2*sqrt(7) + 2*(sqrt(-x^2 - 2*x + 3) - 2)
/(x + 1) - 4)) - 1/14*(8*x - 3)/(2*x^2 + 2*x - 3) - 8/21*(5*(sqrt
(-x^2 - 2*x + 3) - 2)/(x + 1) + 26*(sqrt(-x^2 - 2*x + 3) - 2)^2/(
x + 1)^2 + 11*(sqrt(-x^2 - 2*x + 3) - 2)^3/(x + 1)^3 - 6)/(8*(sqr
t(-x^2 - 2*x + 3) - 2)/(x + 1) + 26*(sqrt(-x^2 - 2*x + 3) - 2)^2/
(x + 1)^2 + 8*(sqrt(-x^2 - 2*x + 3) - 2)^3/(x + 1)^3 - 3*(sqrt(-x
^2 - 2*x + 3) - 2)^4/(x + 1)^4 - 3)
```

3.599 $\int \frac{1}{\left(x+\sqrt{3-2x-x^2}\right)^3}\,dx$

Optimal. Leaf size=307

$$-\frac{4\left(\frac{\left(21+5\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-5\sqrt{3}+9\right)}{21\left(\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^2}{x^2}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-\sqrt{3}+2\right)^2}+\frac{2\left(-\frac{\left(18+49\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-43\sqrt{3}+18\right)}{147\left(\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^2}{x^2}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-\sqrt{3}+2\right)}+\frac{12\tanh^{-1}\left(\frac{-\sqrt{3}\sqrt{-x^2-2x+3}-\sqrt{3}x-x+3}{\sqrt{7}x}\right)}{49\sqrt{7}}$$

```
[Out] (-4*(9 - 5*Sqrt[3] + ((21 + 5*Sqrt[3])*(Sqrt[3] - Sqrt[3 - 2*x -
x^2]))/x))/(21*(2 - Sqrt[3] - (2*(1 + Sqrt[3])*(Sqrt[3] - Sqrt[3
- 2*x - x^2]))/x + (Sqrt[3]*(Sqrt[3] - Sqrt[3 - 2*x - x^2])^2)/x^
2)^2) + (2*(18 - 43*Sqrt[3] - ((18 + 49*Sqrt[3])*(Sqrt[3] - Sqrt[
3 - 2*x - x^2]))/x))/(147*(2 - Sqrt[3] - (2*(1 + Sqrt[3])*(Sqrt[3
] - Sqrt[3 - 2*x - x^2]))/x + (Sqrt[3]*(Sqrt[3] - Sqrt[3 - 2*x -
x^2])^2)/x^2)) + (12*ArcTanh[(3 - x - Sqrt[3]*x - Sqrt[3]*Sqrt[3
- 2*x - x^2])/(Sqrt[7]*x)])/(49*Sqrt[7])
```

Rubi [A] time = 0.445297, antiderivative size = 307, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$-\frac{4\left(\frac{\left(21+5\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-5\sqrt{3}+9\right)}{21\left(\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^2}{x^2}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-\sqrt{3}+2\right)^2}+\frac{2\left(-\frac{\left(18+49\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-43\sqrt{3}+18\right)}{147\left(\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^2}{x^2}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-\sqrt{3}+2\right)}+\frac{12\tanh^{-1}\left(\frac{-\sqrt{3}\sqrt{-x^2-2x+3}-\sqrt{3}x-x+3}{\sqrt{7}x}\right)}{49\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x + Sqrt[3 - 2*x - x^2])^(-3),x]
```

```
[Out] (-4*(9 - 5*Sqrt[3] + ((21 + 5*Sqrt[3])*(Sqrt[3] - Sqrt[3 - 2*x -
x^2]))/x))/(21*(2 - Sqrt[3] - (2*(1 + Sqrt[3])*(Sqrt[3] - Sqrt[3
- 2*x - x^2]))/x + (Sqrt[3]*(Sqrt[3] - Sqrt[3 - 2*x - x^2])^2)/x^
2)^2) + (2*(18 - 43*Sqrt[3] - ((18 + 49*Sqrt[3])*(Sqrt[3] - Sqrt[
3 - 2*x - x^2]))/x))/(147*(2 - Sqrt[3] - (2*(1 + Sqrt[3])*(Sqrt[3
] - Sqrt[3 - 2*x - x^2]))/x + (Sqrt[3]*(Sqrt[3] - Sqrt[3 - 2*x -
x^2])^2)/x^2)) + (12*ArcTanh[(3 - x - Sqrt[3]*x - Sqrt[3]*Sqrt[3
- 2*x - x^2])/(Sqrt[7]*x)])/(49*Sqrt[7])
```

Rubi in Sympy [A] time = 92.8832, size = 733, normalized size = 2.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x+(-x**2-2*x+3)**(1/2))**3,x)
```

```
[Out] sqrt(3)*(-(-3*sqrt(3) + 6)*(sqrt(3)*(-2*sqrt(3) + 4) + (2 + 2*sqr
t(3))**2) + 3*sqrt(3)*(6 + (2 + 2*sqrt(3))*(2*sqrt(3)/3 + 2)))*(2
 + 2*sqrt(3) - 2*sqrt(3)*(-sqrt(-x**2 - 2*x + 3) + sqrt(3))/x)/(3
*(-(2 + 2*sqrt(3))**2 - sqrt(3)*(-8 + 4*sqrt(3)))**2*(-sqrt(3) +
2 + (2 + 2*sqrt(3))*(sqrt(-x**2 - 2*x + 3) - sqrt(3))/x + sqrt(3)
*(sqrt(-x**2 - 2*x + 3) - sqrt(3))**2/x**2)) + sqrt(3)*(-8*sqrt(3
) + 32 - ((-3*sqrt(3) + 6)*(-(2 + 2*sqrt(3))**2 + sqrt(3)*(-2*sqr
t(3) + 4)) + sqrt(3)*(6 + (2 + 2*sqrt(3))*(2*sqrt(3)/3 + 2)))*(-s
qrt(-x**2 - 2*x + 3) + sqrt(3))/x)/(3*(-(2 + 2*sqrt(3))**2 - sqrt
(3)*(-8 + 4*sqrt(3)))*(-sqrt(3) + 2 + (2 + 2*sqrt(3))*(sqrt(-x**2
 - 2*x + 3) - sqrt(3))/x + sqrt(3)*(sqrt(-x**2 - 2*x + 3) - sqrt(
3))**2/x**2)**2) - 2*sqrt(7)*(-(-3*sqrt(3) + 6)*(sqrt(3)*(-2*sqrt
(3) + 4) + (2 + 2*sqrt(3))**2) + 3*sqrt(3)*(6 + (2 + 2*sqrt(3))*(
2*sqrt(3)/3 + 2)))*atanh(sqrt(7)*(1/7 + sqrt(3)/7 + sqrt(3)*(sqrt
(-x**2 - 2*x + 3) - sqrt(3))/(7*x)))/(7*(-(2 + 2*sqrt(3))**2 - sq
rt(3)*(-8 + 4*sqrt(3)))**2) + sqrt(3)*(2*sqrt(3) + 4)*(-sqrt(-x**
2 - 2*x + 3) + sqrt(3))**2/(3*x**2*(-sqrt(3) + 2 + 2*(sqrt(-x**2
- 2*x + 3) - sqrt(3))/x + 2*sqrt(3)*(sqrt(-x**2 - 2*x + 3) - sqrt
(3))/x + sqrt(3)*(sqrt(-x**2 - 2*x + 3) - sqrt(3))**2/x**2)**2) -
 2*(-sqrt(-x**2 - 2*x + 3) + sqrt(3))**3/(x**3*(-sqrt(3) + 2 + 2*
(sqrt(-x**2 - 2*x + 3) - sqrt(3))/x + 2*sqrt(3)*(sqrt(-x**2 - 2*x
 + 3) - sqrt(3))/x + sqrt(3)*(sqrt(-x**2 - 2*x + 3) - sqrt(3))**2
/x**2)**2)
```

Mathematica [A] time = 1.09902, size = 333, normalized size = 1.08

$$\frac{7(37-24x)}{2x^2+2x-3}+\frac{98(11x-12)}{(2x^2+2x-3)^2}-6\left(1+\sqrt{7}\right)\sqrt{\frac{14}{4+\sqrt{7}}}\log\left(\sqrt{14\left(4+\sqrt{7}\right)}\sqrt{-x^2-2x+3}-\sqrt{7}x+7x+7\sqrt{7}+7\right)-2\left(\sqrt{7}-1\right)\sqrt{14\left(4 \cdots\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(x + Sqrt[3 - 2*x - x^2])^(-3),x]
```

```
[Out] ((98*(-12 + 11*x))/(-3 + 2*x + 2*x^2)^2 + (7*(37 - 24*x))/(-3 + 2
*x + 2*x^2) - (14*Sqrt[3 - 2*x - x^2]*(-15 - 83*x + 58*x^2 + 34*x
^3))/(-3 + 2*x + 2*x^2)^2 - 12*Sqrt[7]*Log[-1 + Sqrt[7] - 2*x] +
2*(-1 + Sqrt[7])*Sqrt[14*(4 + Sqrt[7])]*Log[1 - Sqrt[7] + 2*x] +
12*Sqrt[7]*Log[1 + Sqrt[7] + 2*x] + 6*(1 + Sqrt[7])*Sqrt[14/(4 +
Sqrt[7])]*Log[1 + Sqrt[7] + 2*x] - 6*(1 + Sqrt[7])*Sqrt[14/(4 + S
qrt[7])]*Log[7 + 7*Sqrt[7] + 7*x - Sqrt[7]*x + Sqrt[14*(4 + Sqrt[
7])]*Sqrt[3 - 2*x - x^2]] - 2*(-1 + Sqrt[7])*Sqrt[14*(4 + Sqrt[7]
)]*Log[7 - 7*Sqrt[7] + (7 + Sqrt[7])*x - Sqrt[14]*Sqrt[(-4 + Sqrt
[7])*(-3 + 2*x + x^2)]])/1372
```

Maple [B] time = 0.062, size = 6000, normalized size = 19.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x+(-x^2-2*x+3)^(1/2))^3,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x + \sqrt{-x^2 - 2\,x + 3}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sqrt(-x^2 - 2*x + 3))^(-3),x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(-x^2 - 2*x + 3))^(-3), x)
```

Fricas [A] time = 0.27693, size = 306, normalized size = 1.

$$-\frac{\sqrt{7}\left(2\,\sqrt{7}\left(34\,x^3 + 58\,x^2 - 83\,x - 15\right)\sqrt{-x^2 - 2\,x + 3} - 6\,\left(4\,x^4 + 8\,x^3 - 8\,x^2 - 12\,x + 9\right)\log\left(\frac{\sqrt{7}\left(x^4 + 44\,x^3 + 26\,x^2 - 276\,x + 207\right) - 7\,(3\,x}{4\,x^4 + 8\,x^3 - 8\,x^2 -}\right.\right.}{1372\,(4\,x^4 + 8\,x^3 -}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sqrt(-x^2 - 2*x + 3))^(-3),x, algorithm="fricas")
```

```
[Out] -1/1372*sqrt(7)*(2*sqrt(7)*(34*x^3 + 58*x^2 - 83*x - 15)*sqrt(-x^
2 - 2*x + 3) - 6*(4*x^4 + 8*x^3 - 8*x^2 - 12*x + 9)*log((sqrt(7)*
(x^4 + 44*x^3 + 26*x^2 - 276*x + 207) - 7*(3*x^3 + x^2 - 45*x + 4
5)*sqrt(-x^2 - 2*x + 3))/(4*x^4 + 8*x^3 - 8*x^2 - 12*x + 9)) - 12
*(4*x^4 + 8*x^3 - 8*x^2 - 12*x + 9)*log((2*sqrt(7)*(x^2 + x + 2)
+ 14*x + 7)/(2*x^2 + 2*x - 3)) + sqrt(7)*(48*x^3 - 26*x^2 - 300*x
 + 279))/(4*x^4 + 8*x^3 - 8*x^2 - 12*x + 9)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x + \sqrt{-x^2 - 2x + 3}\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x+(-x**2-2*x+3)**(1/2))**3,x)
```

```
[Out] Integral((x + sqrt(-x**2 - 2*x + 3))**(-3), x)
```

GIAC/XCAS [A] time = 0.311365, size = 610, normalized size = 1.99

$$-\frac{3}{343}\sqrt{7}\ln\left(\frac{\left|4\,x - 2\sqrt{7} + 2\right|}{\left|4\,x + 2\sqrt{7} + 2\right|}\right) + \frac{3}{343}\sqrt{7}\ln\left(\frac{\left|-2\sqrt{7} + \frac{6\left(\sqrt{-x^2-2\,x+3}-2\right)}{x+1} + 4\right|}{\left|2\sqrt{7} + \frac{6\left(\sqrt{-x^2-2\,x+3}-2\right)}{x+1} + 4\right|}\right) - \frac{3}{343}\sqrt{7}\ln\left(\frac{\left|-2\sqrt{7} + \frac{2\left(\sqrt{-x^2-2\,x+3}-2\right)}{x+1} - 4\right|}{\left|2\sqrt{7} + \frac{2\left(\sqrt{-x^2-2\,x+3}-2\right)}{x+1} - 4\right|}\right) - \frac{48\,x^3 - 26\,x^2 - 300\,x + 279}{196\left(2\,x^2 + 2\,x - 3\right)^2} + \frac{4\left(\frac{231\left(\sqrt{-x^2-2\,x+3}-2\right)}{x+1} + \frac{3286\left(\sqrt{-x^2-2\,x+3}-2\right)^2}{(x+1)^2} - \frac{4441\left(\sqrt{-x^2-2\,x+3}-2\right)^3}{(x+1)^3} - \frac{18906\left(\sqrt{-x^2-2\,x+3}-2\right)^4}{(x+1)^4} - \frac{12487\left(\sqrt{-x^2-2\,x+3}-2\right)^5}{(x+1)^5} + \frac{946\left(\sqrt{-x^2-2 \cdots}\right.}{(x+1)\cdots}\right.}{441\left(\frac{8\left(\sqrt{-x^2-2\,x+3}-2\right)}{x+1} + \frac{26\left(\sqrt{-x^2-2\,x+3}-2\right)^2}{(x+1)^2} + \frac{8\left(\sqrt{-x^2-2\,x+3}-2\right)^3}{(x+1)^3} - \frac{3\left(\sqrt{-x^2-2\,x+3}-2\right)^4}{(x+1)^4} - 3\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sqrt(-x^2 - 2*x + 3))^(-3),x, algorithm="giac")
```

```
[Out] -3/343*sqrt(7)*ln(abs(4*x - 2*sqrt(7) + 2)/abs(4*x + 2*sqrt(7) +
2)) + 3/343*sqrt(7)*ln(abs(-2*sqrt(7) + 6*(sqrt(-x^2 - 2*x + 3) -
 2)/(x + 1) + 4)/abs(2*sqrt(7) + 6*(sqrt(-x^2 - 2*x + 3) - 2)/(x
+ 1) + 4)) - 3/343*sqrt(7)*ln(abs(-2*sqrt(7) + 2*(sqrt(-x^2 - 2*x
 + 3) - 2)/(x + 1) - 4)/abs(2*sqrt(7) + 2*(sqrt(-x^2 - 2*x + 3) -
 2)/(x + 1) - 4)) - 1/196*(48*x^3 - 26*x^2 - 300*x + 279)/(2*x^2
+ 2*x - 3)^2 + 4/441*(231*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) + 32
86*(sqrt(-x^2 - 2*x + 3) - 2)^2/(x + 1)^2 - 4441*(sqrt(-x^2 - 2*x
 + 3) - 2)^3/(x + 1)^3 - 18906*(sqrt(-x^2 - 2*x + 3) - 2)^4/(x +
1)^4 - 12487*(sqrt(-x^2 - 2*x + 3) - 2)^5/(x + 1)^5 + 946*(sqrt(-
x^2 - 2*x + 3) - 2)^6/(x + 1)^6 + 1977*(sqrt(-x^2 - 2*x + 3) - 2)
^7/(x + 1)^7 - 414)/(8*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) + 26*(s
qrt(-x^2 - 2*x + 3) - 2)^2/(x + 1)^2 + 8*(sqrt(-x^2 - 2*x + 3) -
2)^3/(x + 1)^3 - 3*(sqrt(-x^2 - 2*x + 3) - 2)^4/(x + 1)^4 - 3)^2
```

3.600 $\int \frac{1}{x+\sqrt{-3-2x+x^2}}\,dx$

Optimal. Leaf size=65

$$-\frac{2}{-\sqrt{x^2-2x-3}-x+1}+2\log\left(-\sqrt{x^2-2x-3}-x+1\right)-\frac{3}{2}\log\left(\sqrt{x^2-2x-3}+x\right)$$

```
[Out] -2/(1 - x - Sqrt[-3 - 2*x + x^2]) + 2*Log[1 - x - Sqrt[-3 - 2*x +
 x^2]] - (3*Log[x + Sqrt[-3 - 2*x + x^2]])/2
```

Rubi [A] time = 0.0678783, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$-\frac{2}{-\sqrt{x^2-2x-3}-x+1}+2\log\left(-\sqrt{x^2-2x-3}-x+1\right)-\frac{3}{2}\log\left(\sqrt{x^2-2x-3}+x\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(x + Sqrt[-3 - 2*x + x^2])^(-1),x]
```

```
[Out] -2/(1 - x - Sqrt[-3 - 2*x + x^2]) + 2*Log[1 - x - Sqrt[-3 - 2*x +
 x^2]] - (3*Log[x + Sqrt[-3 - 2*x + x^2]])/2
```

Rubi in Sympy [A] time = 3.97313, size = 53, normalized size = 0.82

$$-\frac{3\log\left(x+\sqrt{x^2-2x-3}\right)}{2}+2\log\left(-x-\sqrt{x^2-2x-3}+1\right)-\frac{2}{-x-\sqrt{x^2-2x-3}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x+(x**2-2*x-3)**(1/2)),x)
```

```
[Out] -3*log(x + sqrt(x**2 - 2*x - 3))/2 + 2*log(-x - sqrt(x**2 - 2*x -
 3) + 1) - 2/(-x - sqrt(x**2 - 2*x - 3) + 1)
```

Mathematica [A] time = 0.0447807, size = 74, normalized size = 1.14

$$\frac{1}{4}\left(-2\sqrt{x^2-2x-3}+3\log\left(-3\sqrt{x^2-2x-3}+5x+3\right)+5\log\left(-\sqrt{x^2-2x-3}-x+1\right)+2x-6\log(2x+3)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x + Sqrt[-3 - 2*x + x^2])^(-1),x]
```

```
[Out] (2*x - 2*Sqrt[-3 - 2*x + x^2] - 6*Log[3 + 2*x] + 3*Log[3 + 5*x -
3*Sqrt[-3 - 2*x + x^2]] + 5*Log[1 - x - Sqrt[-3 - 2*x + x^2]])/4
```

Maple [A]   time = 0.008, size = 71, normalized size = 1.1

$$-\frac{1}{4}\sqrt{4\left(x+3/2\right)^{2}-20x-21}+\frac{5}{4}\ln\left(-1+x+\sqrt{\left(x+\frac{3}{2}\right)^{2}-5x-\frac{21}{4}}\right)+\frac{3}{4}Artanh\left(\frac{-6-10x}{3}\frac{1}{\sqrt{4\left(x+3/2\right)^{2}-20x-21}}\right)+\frac{x}{2}-\frac{3\ln\left(3+2x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x+(x^2-2*x-3)^(1/2)),x)
```

```
[Out] -1/4*(4*(x+3/2)^2-20*x-21)^(1/2)+5/4*ln(-1+x+((x+3/2)^2-5*x-21/4)
^(1/2))+3/4*arctanh(2/3*(-3-5*x)/(4*(x+3/2)^2-20*x-21)^(1/2))+1/2
*x-3/4*ln(3+2*x)
```

Maxima [F]   time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x+\sqrt{x^{2}-2x-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x + sqrt(x^2 - 2*x - 3)),x, algorithm="maxima")
```

```
[Out] integrate(1/(x + sqrt(x^2 - 2*x - 3)), x)
```

Fricas [A]   time = 0.272226, size = 220, normalized size = 3.38

$$\frac{4x^{2}-3(x-1)\log(2x+3)-5\left(x-\sqrt{x^{2}-2x-3}-1\right)\log\left(-x+\sqrt{x^{2}-2x-3}+1\right)+3\left(x-\sqrt{x^{2}-2x-3}-1\right)\log\left(-x\right.}{4\left(x-\sqrt{x\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x + sqrt(x^2 - 2*x - 3)),x, algorithm="fricas")
```

```
[Out] 1/4*(4*x^2 - 3*(x - 1)*log(2*x + 3) - 5*(x - sqrt(x^2 - 2*x - 3)
- 1)*log(-x + sqrt(x^2 - 2*x - 3) + 1) + 3*(x - sqrt(x^2 - 2*x -
3) - 1)*log(-x + sqrt(x^2 - 2*x - 3)) - 3*(x - sqrt(x^2 - 2*x - 3
) - 1)*log(-x + sqrt(x^2 - 2*x - 3) - 3) - sqrt(x^2 - 2*x - 3)*(4
*x - 3*log(2*x + 3) - 1) - 5*x - 7)/(x - sqrt(x^2 - 2*x - 3) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x + \sqrt{x^2 - 2x - 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x+(x**2-2*x-3)**(1/2)),x)
```

```
[Out] Integral(1/(x + sqrt(x**2 - 2*x - 3)), x)
```

GIAC/XCAS [A] time = 0.276838, size = 109, normalized size = 1.68

$$\frac{1}{2}\,x - \frac{1}{2}\,\sqrt{x^2 - 2\,x - 3} - \frac{3}{4}\,\ln\left(|2\,x + 3|\right) - \frac{5}{4}\,\ln\left(\left|-x + \sqrt{x^2 - 2\,x - 3} + 1\right|\right) + \frac{3}{4}\,\ln\left(\left|-x + \sqrt{x^2 - 2\,x - 3}\right|\right) - \frac{3}{4}\,\ln\left(\left|-x + \sqrt{x^2 - 2\,x - 3} - 3\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x + sqrt(x^2 - 2*x - 3)),x, algorithm="giac")
```

```
[Out] 1/2*x - 1/2*sqrt(x^2 - 2*x - 3) - 3/4*ln(abs(2*x + 3)) - 5/4*ln(a
bs(-x + sqrt(x^2 - 2*x - 3) + 1)) + 3/4*ln(abs(-x + sqrt(x^2 - 2*
x - 3))) - 3/4*ln(abs(-x + sqrt(x^2 - 2*x - 3) - 3))
```

3.601 $\int \frac{1}{\left(x+\sqrt{-3-2x+x^2}\right)^2}\,dx$

Optimal. Leaf size=83

$$-\frac{2}{-\sqrt{x^2-2x-3}-x+1}+\frac{3}{2\left(\sqrt{x^2-2x-3}+x\right)}+4\log\left(-\sqrt{x^2-2x-3}-x+1\right)-4\log\left(\sqrt{x^2-2x-3}+x\right)$$

```
[Out] -2/(1 - x - Sqrt[-3 - 2*x + x^2]) + 3/(2*(x + Sqrt[-3 - 2*x + x^2
])) + 4*Log[1 - x - Sqrt[-3 - 2*x + x^2]] - 4*Log[x + Sqrt[-3 - 2
*x + x^2]]
```

Rubi [A] time = 0.0764641, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$-\frac{2}{-\sqrt{x^2-2x-3}-x+1}+\frac{3}{2\left(\sqrt{x^2-2x-3}+x\right)}+4\log\left(-\sqrt{x^2-2x-3}-x+1\right)-4\log\left(\sqrt{x^2-2x-3}+x\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(x + Sqrt[-3 - 2*x + x^2])^(-2),x]
```

```
[Out] -2/(1 - x - Sqrt[-3 - 2*x + x^2]) + 3/(2*(x + Sqrt[-3 - 2*x + x^2
])) + 4*Log[1 - x - Sqrt[-3 - 2*x + x^2]] - 4*Log[x + Sqrt[-3 - 2
*x + x^2]]
```

Rubi in Sympy [A] time = 4.63109, size = 70, normalized size = 0.84

$$-4\log\left(x+\sqrt{x^2-2x-3}\right)+4\log\left(-x-\sqrt{x^2-2x-3}+1\right)-\frac{2}{-x-\sqrt{x^2-2x-3}+1}+\frac{3}{2\left(x+\sqrt{x^2-2x-3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x+(x**2-2*x-3)**(1/2))**2,x)
```

```
[Out] -4*log(x + sqrt(x**2 - 2*x - 3)) + 4*log(-x - sqrt(x**2 - 2*x - 3
) + 1) - 2/(-x - sqrt(x**2 - 2*x - 3) + 1) + 3/(2*(x + sqrt(x**2
- 2*x - 3)))
```

Mathematica [A] time = 0.078783, size = 91, normalized size = 1.1

$$-\frac{(x+3)\sqrt{x^2-2x-3}}{2x+3}+2\log\left(-3\sqrt{x^2-2x-3}+5x+3\right)+2\log\left(-\sqrt{x^2-2x-3}-x+1\right)+\frac{x}{2}-\frac{9}{8x+12}-4\log(2x+3)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x + Sqrt[-3 - 2*x + x^2])^(-2),x]
```

```
[Out] x/2 - 9/(12 + 8*x) - ((3 + x)*Sqrt[-3 - 2*x + x^2])/(3 + 2*x) - 4
*Log[3 + 2*x] + 2*Log[3 + 5*x - 3*Sqrt[-3 - 2*x + x^2]] + 2*Log[1
 - x - Sqrt[-3 - 2*x + x^2]]
```

Maple [A] time = 0.026, size = 118, normalized size = 1.4

$$-2\ln(3+2x)+\frac{x}{2}-\frac{9}{12+8x}-\frac{1}{3}\left(\left(x+\frac{3}{2}\right)^2-5x-\frac{21}{4}\right)^{\frac{3}{2}}\left(x+\frac{3}{2}\right)^{-1}-\frac{2}{3}\sqrt{4(x+3/2)^2-20x-21}+2\ln\left(-1+x+\sqrt{(x+3/2)^2-5x-\frac{21}{4}}\right)+2\,\mathrm{Artanh}\left(2/3\,\frac{-3-5x}{\sqrt{4(x+3/2)^2-20x-21}}\right)+\frac{2x-2}{6}\sqrt{\left(x+\frac{3}{2}\right)^2-5x-\frac{21}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x+(x^2-2*x-3)^(1/2))^2,x)
```

```
[Out] -2*ln(3+2*x)+1/2*x-9/4/(3+2*x)-1/3/(x+3/2)*((x+3/2)^2-5*x-21/4)^(
3/2)-2/3*(4*(x+3/2)^2-20*x-21)^(1/2)+2*ln(-1+x+((x+3/2)^2-5*x-21/
4)^(1/2))+2*arctanh(2/3*(-3-5*x)/(4*(x+3/2)^2-20*x-21)^(1/2))+1/6
*(2*x-2)*((x+3/2)^2-5*x-21/4)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(x+\sqrt{x^2-2x-3}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sqrt(x^2 - 2*x - 3))^(-2),x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(x^2 - 2*x - 3))^(-2), x)
```

Fricas [A] time = 0.267667, size = 317, normalized size = 3.82

$$\frac{8x^4 - 6x^3 - 63x^2 - 8\left(2x^3 - x^2 - \left(2x^2 + x - 3\right)\sqrt{x^2 - 2x - 3} - 8x - 3\right)\log\left(x^2 - \sqrt{x^2 - 2x - 3}(x+1) - 3\right) - 8\left(2x^3 - \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sqrt(x^2 - 2*x - 3))^(-2),x, algorithm="fricas")
```

```
[Out] 1/4*(8*x^4 - 6*x^3 - 63*x^2 - 8*(2*x^3 - x^2 - (2*x^2 + x - 3)*sq
rt(x^2 - 2*x - 3) - 8*x - 3)*log(x^2 - sqrt(x^2 - 2*x - 3)*(x + 1
) - 3) - 8*(2*x^3 - x^2 - 8*x - 3)*log(2*x + 3) + 8*(2*x^3 - x^2
- (2*x^2 + x - 3)*sqrt(x^2 - 2*x - 3) - 8*x - 3)*log(-x + sqrt(x^
2 - 2*x - 3)) - (8*x^3 + 2*x^2 - 8*(2*x^2 + x - 3)*log(2*x + 3) -
 45*x + 3)*sqrt(x^2 - 2*x - 3) + 28*x + 51)/(2*x^3 - x^2 - (2*x^2
 + x - 3)*sqrt(x^2 - 2*x - 3) - 8*x - 3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x + \sqrt{x^2 - 2x - 3}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x+(x**2-2*x-3)**(1/2))**2,x)
```

```
[Out] Integral((x + sqrt(x**2 - 2*x - 3))**(-2), x)
```

GIAC/XCAS [A] time = 0.276419, size = 193, normalized size = 2.33

$$\frac{1}{2}\,x-\frac{1}{2}\,\sqrt{x^2-2\,x-3}-\frac{3\left(5\,x-5\,\sqrt{x^2-2\,x-3}+3\right)}{4\left(\left(x-\sqrt{x^2-2\,x-3}\right)^2+3\,x-3\,\sqrt{x^2-2\,x-3}\right)}-\frac{9}{4\,(2\,x+3)}-2\ln\left(|2\,x+3|\right)-2\ln\left(\left|-x+\sqrt{x^2-2\,x-3}+1\right|\right)+2\ln\left(\left|-x+\sqrt{x^2-2\,x-3}\right|\right)-2\ln\left(\left|-x+\sqrt{x^2-2\,x-3}-3\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sqrt(x^2 - 2*x - 3))^(-2),x, algorithm="giac")
```

```
[Out] 1/2*x - 1/2*sqrt(x^2 - 2*x - 3) - 3/4*(5*x - 5*sqrt(x^2 - 2*x - 3
) + 3)/((x - sqrt(x^2 - 2*x - 3))^2 + 3*x - 3*sqrt(x^2 - 2*x - 3)
) - 9/4/(2*x + 3) - 2*ln(abs(2*x + 3)) - 2*ln(abs(-x + sqrt(x^2 -
 2*x - 3) + 1)) + 2*ln(abs(-x + sqrt(x^2 - 2*x - 3))) - 2*ln(abs(
-x + sqrt(x^2 - 2*x - 3) - 3))
```

3.602 $\int \frac{1}{\left(x+\sqrt{-3-2x+x^2}\right)^3}\,dx$

Optimal. Leaf size=101

$$-\frac{2}{-\sqrt{x^2-2x-3}-x+1}+\frac{4}{\sqrt{x^2-2x-3}+x}+\frac{3}{4\left(\sqrt{x^2-2x-3}+x\right)^2}+6\log\left(-\sqrt{x^2-2x-3}-x+1\right)-6\log\left(\sqrt{x^2-2x-3}+x\right)$$

```
[Out] -2/(1 - x - Sqrt[-3 - 2*x + x^2]) + 3/(4*(x + Sqrt[-3 - 2*x + x^2
])^2) + 4/(x + Sqrt[-3 - 2*x + x^2]) + 6*Log[1 - x - Sqrt[-3 - 2*
x + x^2]] - 6*Log[x + Sqrt[-3 - 2*x + x^2]]
```

Rubi [A] time = 0.0822887, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$-\frac{2}{-\sqrt{x^2-2x-3}-x+1}+\frac{4}{\sqrt{x^2-2x-3}+x}+\frac{3}{4\left(\sqrt{x^2-2x-3}+x\right)^2}+6\log\left(-\sqrt{x^2-2x-3}-x+1\right)-6\log\left(\sqrt{x^2-2x-3}+x\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(x + Sqrt[-3 - 2*x + x^2])^(-3),x]
```

```
[Out] -2/(1 - x - Sqrt[-3 - 2*x + x^2]) + 3/(4*(x + Sqrt[-3 - 2*x + x^2
])^2) + 4/(x + Sqrt[-3 - 2*x + x^2]) + 6*Log[1 - x - Sqrt[-3 - 2*
x + x^2]] - 6*Log[x + Sqrt[-3 - 2*x + x^2]]
```

Rubi in Sympy [A] time = 5.1796, size = 85, normalized size = 0.84

$$-6\log\left(x+\sqrt{x^2-2x-3}\right)+6\log\left(-x-\sqrt{x^2-2x-3}+1\right)-\frac{2}{-x-\sqrt{x^2-2x-3}+1}+\frac{4}{x+\sqrt{x^2-2x-3}}+\frac{3}{4\left(x+\sqrt{x^2-2x-3}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x+(x**2-2*x-3)**(1/2))**3,x)
```

```
[Out] -6*log(x + sqrt(x**2 - 2*x - 3)) + 6*log(-x - sqrt(x**2 - 2*x - 3
) + 1) - 2/(-x - sqrt(x**2 - 2*x - 3) + 1) + 4/(x + sqrt(x**2 - 2
*x - 3)) + 3/(4*(x + sqrt(x**2 - 2*x - 3))**2)
```

Mathematica [A] time = 0.0820744, size = 111, normalized size = 1.1

$$-\frac{\sqrt{x^2-2x-3}\left(4x^2+31x+33\right)}{2(2x+3)^2}+3\log\left(-3\sqrt{x^2-2x-3}+5x+3\right)+3\log\left(-\sqrt{x^2-2x-3}-x+1\right)+\frac{x}{2}-\frac{9}{2x+3}+\frac{27}{8(2x+3)^2}-6\log(2x+3)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x + Sqrt[-3 - 2*x + x^2])^(-3),x]
```

```
[Out] x/2 + 27/(8*(3 + 2*x)^2) - 9/(3 + 2*x) - (Sqrt[-3 - 2*x + x^2]*(3
3 + 31*x + 4*x^2))/(2*(3 + 2*x)^2) - 6*Log[3 + 2*x] + 3*Log[3 + 5
*x - 3*Sqrt[-3 - 2*x + x^2]] + 3*Log[1 - x - Sqrt[-3 - 2*x + x^2]
]
```

Maple [A] time = 0.033, size = 146, normalized size = 1.5

$$-9\,(3+2\,x)^{-1}-3\,\ln(3+2\,x)+\frac{x}{2}+\frac{27}{8\,(3+2\,x)^2}-\frac{1}{2}\left(\left(x+\frac{3}{2}\right)^2-5\,x-\frac{21}{4}\right)^{\frac{3}{2}}\left(x+\frac{3}{2}\right)^{-1}-\sqrt{4\,(x+3/2)^2-20\,x-21}+3\,\mathrm{Artanh}\left(2/3\,\frac{-3-5\,x}{\sqrt{4\,(x+3/2)^2-20\,x-21}}\right)+\frac{2\,x-2}{4}\sqrt{\left(x+\frac{3}{2}\right)^2-5\,x-\frac{21}{4}}+3\,\ln\left(-1+x+\sqrt{(x+3/2)^2-5\,x-\frac{21}{4}}\right)+\frac{1}{4}((x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x+(x^2-2*x-3)^(1/2))^3,x)
```

```
[Out] -9/(3+2*x)-3*ln(3+2*x)+1/2*x+27/8/(3+2*x)^2-1/2/(x+3/2)*((x+3/2)^
2-5*x-21/4)^(3/2)-(4*(x+3/2)^2-20*x-21)^(1/2)+3*arctanh(2/3*(-3-5
*x)/(4*(x+3/2)^2-20*x-21)^(1/2))+1/4*(2*x-2)*((x+3/2)^2-5*x-21/4)
^(1/2)+3*ln(-1+x+((x+3/2)^2-5*x-21/4)^(1/2))+1/4/(x+3/2)^2*((x+3/
2)^2-5*x-21/4)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x+\sqrt{x^2-2x-3}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sqrt(x^2 - 2*x - 3))^(-3),x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(x^2 - 2*x - 3))^(-3), x)
```

Fricas [A] time = 0.26654, size = 431, normalized size = 4.27

$$\frac{16x^6-4x^5-300x^4+159x^3+931x^2-12\left(4x^5-27x^3-19x^2-\left(4x^4+4x^3-15x^2-18x\right)\sqrt{x^2-2x-3}+24x+18\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sqrt(x^2 - 2*x - 3))^(-3),x, algorithm="fricas")
```

```
[Out] 1/4*(16*x^6 - 4*x^5 - 300*x^4 + 159*x^3 + 931*x^2 - 12*(4*x^5 - 2
7*x^3 - 19*x^2 - (4*x^4 + 4*x^3 - 15*x^2 - 18*x)*sqrt(x^2 - 2*x -
 3) + 24*x + 18)*log(x^2 - sqrt(x^2 - 2*x - 3)*(x + 1) - 3) - 12*
(4*x^5 - 27*x^3 - 19*x^2 + 24*x + 18)*log(2*x + 3) + 12*(4*x^5 -
27*x^3 - 19*x^2 - (4*x^4 + 4*x^3 - 15*x^2 - 18*x)*sqrt(x^2 - 2*x
- 3) + 24*x + 18)*log(-x + sqrt(x^2 - 2*x - 3)) - (16*x^5 + 12*x^
4 - 256*x^3 - 41*x^2 - 12*(4*x^4 + 4*x^3 - 15*x^2 - 18*x)*log(2*x
 + 3) + 466*x + 132)*sqrt(x^2 - 2*x - 3) + 84*x - 342)/(4*x^5 - 2
7*x^3 - 19*x^2 - (4*x^4 + 4*x^3 - 15*x^2 - 18*x)*sqrt(x^2 - 2*x -
 3) + 24*x + 18)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x+\sqrt{x^2-2x-3}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x+(x**2-2*x-3)**(1/2))**3,x)
```

```
[Out] Integral((x + sqrt(x**2 - 2*x - 3))**(-3), x)
```

GIAC/XCAS [A] time = 0.274239, size = 248, normalized size = 2.46

$$\frac{1}{2}x - \frac{1}{2}\sqrt{x^2-2x-3} - \frac{104\left(x-\sqrt{x^2-2x-3}\right)^3 + 315\left(x-\sqrt{x^2-2x-3}\right)^2 + 162x - 162\sqrt{x^2-2x-3} + 27}{8\left(\left(x-\sqrt{x^2-2x-3}\right)^2 + 3x - 3\sqrt{x^2-2x-3}\right)^2} - \frac{9(16x+21)}{8(2x+3)^2} - 3\ln(|2x+3|) - 3\ln\left(\left|-x+\sqrt{x^2-2x-3}+1\right|\right) + 3\ln\left(\left|-x+\sqrt{x^2-2x-3}\right|\right) - 3\ln\left(\left|-x+\sqrt{x^2-2x-3}-3\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sqrt(x^2 - 2*x - 3))^(-3),x, algorithm="giac")
```

```
[Out] 1/2*x - 1/2*sqrt(x^2 - 2*x - 3) - 1/8*(104*(x - sqrt(x^2 - 2*x -
3))^3 + 315*(x - sqrt(x^2 - 2*x - 3))^2 + 162*x - 162*sqrt(x^2 -
2*x - 3) + 27)/((x - sqrt(x^2 - 2*x - 3))^2 + 3*x - 3*sqrt(x^2 -
2*x - 3))^2 - 9/8*(16*x + 21)/(2*x + 3)^2 - 3*ln(abs(2*x + 3)) -
3*ln(abs(-x + sqrt(x^2 - 2*x - 3) + 1)) + 3*ln(abs(-x + sqrt(x^2
- 2*x - 3))) - 3*ln(abs(-x + sqrt(x^2 - 2*x - 3) - 3))
```

3.603 $\int \frac{1}{x+\sqrt{-3-4x-x^2}}\,dx$

Optimal. Leaf size=108

$$\frac{1}{2}\log(x+3)+\frac{1}{2}\log\left(\frac{\sqrt{-x-1}x+\sqrt{x+3}x+3\sqrt{-x-1}}{(x+3)^{3/2}}\right)-\tan^{-1}\left(\frac{\sqrt{-x-1}}{\sqrt{x+3}}\right)-\sqrt{2}\tan^{-1}\left(\frac{1-\frac{3\sqrt{-x-1}}{\sqrt{x+3}}}{\sqrt{2}}\right)$$

```
[Out] -ArcTan[Sqrt[-1 - x]/Sqrt[3 + x]] - Sqrt[2]*ArcTan[(1 - (3*Sqrt[-
1 - x])/Sqrt[3 + x])/Sqrt[2]] + Log[3 + x]/2 + Log[(3*Sqrt[-1 - x
] + Sqrt[-1 - x]*x + x*Sqrt[3 + x])/(3 + x)^(3/2)]/2
```

Rubi [A] time = 0.183887, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444

$$\frac{1}{2}\log(x+3)+\frac{1}{2}\log\left(\frac{\sqrt{-x-1}x+\sqrt{x+3}x+3\sqrt{-x-1}}{(x+3)^{3/2}}\right)-\tan^{-1}\left(\frac{\sqrt{-x-1}}{\sqrt{x+3}}\right)-\sqrt{2}\tan^{-1}\left(\frac{1-\frac{3\sqrt{-x-1}}{\sqrt{x+3}}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(x + Sqrt[-3 - 4*x - x^2])^(-1),x]
```

```
[Out] -ArcTan[Sqrt[-1 - x]/Sqrt[3 + x]] - Sqrt[2]*ArcTan[(1 - (3*Sqrt[-
1 - x])/Sqrt[3 + x])/Sqrt[2]] + Log[3 + x]/2 + Log[(3*Sqrt[-1 - x
] + Sqrt[-1 - x]*x + x*Sqrt[3 + x])/(3 + x)^(3/2)]/2
```

Rubi in Sympy [A] time = 7.88414, size = 114, normalized size = 1.06

$$-\frac{\log\left(1+\frac{-x^2-4x-3}{(x+3)^2}\right)}{2}+\frac{\log\left(1-\frac{2\sqrt{-x^2-4x-3}}{x+3}+\frac{3\left(-x^2-4x-3\right)}{(x+3)^2}\right)}{2}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(-\frac{1}{2}+\frac{3\sqrt{-x^2-4x-3}}{2\,(x+3)}\right)\right)-\mathrm{atan}\left(\frac{\sqrt{-x^2-4x-3}}{x+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x+(-x**2-4*x-3)**(1/2)),x)
```

```
[Out] -log(1 + (-x**2 - 4*x - 3)/(x + 3)**2)/2 + log(1 - 2*sqrt(-x**2 -
 4*x - 3)/(x + 3) + 3*(-x**2 - 4*x - 3)/(x + 3)**2)/2 + sqrt(2)*a
```

```
tan(sqrt(2)*(-1/2 + 3*sqrt(-x**2 - 4*x - 3)/(2*(x + 3)))) - atan(
sqrt(-x**2 - 4*x - 3)/(x + 3))
```

Mathematica [C] time = 6.25724, size = 1119, normalized size = 10.36

$$\frac{1}{2}\sin^{-1}(x+2)-\frac{\tan^{-1}\left(\sqrt{2}(x+1)\right)}{\sqrt{2}}$$

$$-\frac{i\left(i+2\sqrt{2}\right)\tan^{-1}\left(\frac{6i\sqrt{2}x^4-16x^4+18i\sqrt{1-2i\sqrt{2}}\sqrt{-x^2-4x-3}x^3+68i\sqrt{2}x^3-68x^3+72i\sqrt{1-2i\sqrt{2}}\sqrt{-x^2-4x-3}x^2+185i\sqrt{2}x^2-44x^2+99i\sqrt{1-2i\sqrt{2}}\sqrt{-x^2-4x-3}}{32\sqrt{2}x^4+66ix^4+208\sqrt{2}x^3+304ix^3+466\sqrt{2}x^2+493ix^2+440\sqrt{2}x+340ix+150\sqrt{2}+93i}\right.}{4\sqrt{1-2i\sqrt{2}}}$$

$$-\frac{i\left(-i+2\sqrt{2}\right)\tan^{-1}\left(\frac{6i\sqrt{2}x^4+16x^4+18i\sqrt{1+2i\sqrt{2}}\sqrt{-x^2-4x-3}x^3+68i\sqrt{2}x^3+68x^3+72i\sqrt{1+2i\sqrt{2}}\sqrt{-x^2-4x-3}x^2+185i\sqrt{2}x^2+44x^2+99i\sqrt{1+2i\sqrt{2}}\sqrt{-x^2-4x-3}}{32\sqrt{2}x^4-66ix^4+208\sqrt{2}x^3-304ix^3+466\sqrt{2}x^2-493ix^2+440\sqrt{2}x-340ix+150\sqrt{2}-9}\right.}{4\sqrt{1+2i\sqrt{2}}}$$

$$+\frac{\left(i+2\sqrt{2}\right)\log\left(\left(-2ix+\sqrt{2}-2i\right)^2\left(2ix+\sqrt{2}+2i\right)^2\right)}{8\sqrt{1-2i\sqrt{2}}}$$

$$+\frac{\left(-i+2\sqrt{2}\right)\log\left(\left(-2ix+\sqrt{2}-2i\right)^2\left(2ix+\sqrt{2}+2i\right)^2\right)}{8\sqrt{1+2i\sqrt{2}}}+\frac{1}{4}\log\left(2x^2+4x+3\right)$$

$$-\frac{\left(i+2\sqrt{2}\right)\log\left(\left(2x^2+4x+3\right)\left(2i\sqrt{2}x^2+2x^2-2\sqrt{2\left(1-2i\sqrt{2}\right)}\sqrt{-x^2-4x-3}x+8i\sqrt{2}x+4x-2\sqrt{2\left(1-2i\sqrt{2}\right)}\sqrt{-x^2}\right.\right.}{8\sqrt{1-2i\sqrt{2}}}$$

$$-\frac{\left(-i+2\sqrt{2}\right)\log\left(\left(2x^2+4x+3\right)\left(-2i\sqrt{2}x^2+2x^2-2\sqrt{2\left(1+2i\sqrt{2}\right)}\sqrt{-x^2-4x-3}x-8i\sqrt{2}x+4x-2\sqrt{2\left(1+2i\sqrt{2}\right)}\sqrt{-}\right.\right.}{8\sqrt{1+2i\sqrt{2}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x + Sqrt[-3 - 4*x - x^2])^(-1),x]
```

```
[Out] ArcSin[2 + x]/2 - ArcTan[Sqrt[2]*(1 + x)]/Sqrt[2] - ((I/4)*(I + 2
*Sqrt[2])*ArcTan[(60 + (51*I)*Sqrt[2] + 68*x + (176*I)*Sqrt[2]*x
- 44*x^2 + (185*I)*Sqrt[2]*x^2 - 68*x^3 + (68*I)*Sqrt[2]*x^3 - 16
*x^4 + (6*I)*Sqrt[2]*x^4 + (54*I)*Sqrt[1 - (2*I)*Sqrt[2]]*Sqrt[-3
 - 4*x - x^2] + (99*I)*Sqrt[1 - (2*I)*Sqrt[2]]*x*Sqrt[-3 - 4*x -
x^2] + (72*I)*Sqrt[1 - (2*I)*Sqrt[2]]*x^2*Sqrt[-3 - 4*x - x^2] +
(18*I)*Sqrt[1 - (2*I)*Sqrt[2]]*x^3*Sqrt[-3 - 4*x - x^2])/(93*I +
150*Sqrt[2] + (340*I)*x + 440*Sqrt[2]*x + (493*I)*x^2 + 466*Sqrt[
2]*x^2 + (304*I)*x^3 + 208*Sqrt[2]*x^3 + (66*I)*x^4 + 32*Sqrt[2]*
x^4)])/Sqrt[1 - (2*I)*Sqrt[2]] - ((I/4)*(-I + 2*Sqrt[2])*ArcTan[(
-60 + (51*I)*Sqrt[2] - 68*x + (176*I)*Sqrt[2]*x + 44*x^2 + (185*I
)*Sqrt[2]*x^2 + 68*x^3 + (68*I)*Sqrt[2]*x^3 + 16*x^4 + (6*I)*Sqrt
```

```
[2]*x^4 + (54*I)*Sqrt[1 + (2*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2] + (
99*I)*Sqrt[1 + (2*I)*Sqrt[2]]*x*Sqrt[-3 - 4*x - x^2] + (72*I)*Sqr
t[1 + (2*I)*Sqrt[2]]*x^2*Sqrt[-3 - 4*x - x^2] + (18*I)*Sqrt[1 + (
2*I)*Sqrt[2]]*x^3*Sqrt[-3 - 4*x - x^2])/(-93*I + 150*Sqrt[2] - (3
40*I)*x + 440*Sqrt[2]*x - (493*I)*x^2 + 466*Sqrt[2]*x^2 - (304*I)
*x^3 + 208*Sqrt[2]*x^3 - (66*I)*x^4 + 32*Sqrt[2]*x^4)])/Sqrt[1 +
(2*I)*Sqrt[2]] + ((-I + 2*Sqrt[2])*Log[(-2*I + Sqrt[2] - (2*I)*x)
^2*(2*I + Sqrt[2] + (2*I)*x)^2])/(8*Sqrt[1 + (2*I)*Sqrt[2]]) + ((
I + 2*Sqrt[2])*Log[(-2*I + Sqrt[2] - (2*I)*x)^2*(2*I + Sqrt[2] +
(2*I)*x)^2])/(8*Sqrt[1 - (2*I)*Sqrt[2]]) + Log[3 + 4*x + 2*x^2]/4
 - ((I + 2*Sqrt[2])*Log[(3 + 4*x + 2*x^2)*(3 + (6*I)*Sqrt[2] + 4*
x + (8*I)*Sqrt[2]*x + 2*x^2 + (2*I)*Sqrt[2]*x^2 - 2*Sqrt[2*(1 - (
2*I)*Sqrt[2])]*Sqrt[-3 - 4*x - x^2] - 2*Sqrt[2*(1 - (2*I)*Sqrt[2]
)]*x*Sqrt[-3 - 4*x - x^2])])/(8*Sqrt[1 - (2*I)*Sqrt[2]]) - ((-I +
 2*Sqrt[2])*Log[(3 + 4*x + 2*x^2)*(3 - (6*I)*Sqrt[2] + 4*x - (8*I
)*Sqrt[2]*x + 2*x^2 - (2*I)*Sqrt[2]*x^2 - 2*Sqrt[2*(1 + (2*I)*Sqr
t[2])]*Sqrt[-3 - 4*x - x^2] - 2*Sqrt[2*(1 + (2*I)*Sqrt[2])]*x*Sqr
t[-3 - 4*x - x^2])])/(8*Sqrt[1 + (2*I)*Sqrt[2]])
```

Maple [B] time = 0.013, size = 370, normalized size = 3.4

$$\frac{\arcsin(2+x)}{2}$$
$$-\frac{\sqrt{3}\sqrt{4}}{12}\sqrt{3\frac{x^2}{(-3/2-x)^2}-12}\left(\sqrt{2}\arctan\left(\frac{\sqrt{2}}{6}\sqrt{3\frac{x^2}{(-3/2-x)^2}-12}\right)-Artanh\left(3\frac{x}{-3/2-x}\frac{1}{\sqrt{3\frac{x^2}{(-3/2-x)^2}-12}}\right)\right)\frac{1}{\sqrt{1\left(x^2\left(-\right.\right.}}$$
$$+\frac{\sqrt{3}\sqrt{4}\sqrt{2}}{3}\sqrt{3\frac{x^2}{(-3/2-x)^2}-12}\arctan\left(\frac{\sqrt{2}}{6}\sqrt{3\frac{x^2}{(-3/2-x)^2}-12}\right)\frac{1}{\sqrt{1\left(x^2\left(-\frac{3}{2}-x\right)^{-2}-4\right)\left(1+x\left(-\frac{3}{2}-x\right)^{-1}\right)^{-2}}}\left(1+x\left(\right.\right.$$
$$-\frac{\sqrt{3}\sqrt{4}}{6}\sqrt{3\frac{x^2}{(-3/2-x)^2}-12}\left(\sqrt{2}\arctan\left(\frac{\sqrt{2}}{6}\sqrt{3\frac{x^2}{(-3/2-x)^2}-12}\right)+Artanh\left(3\frac{x}{-3/2-x}\frac{1}{\sqrt{3\frac{x^2}{(-3/2-x)^2}-12}}\right)\right)\frac{1}{\sqrt{1\left(x^2\left(-\right.\right.}}$$
$$+\frac{\ln(2x^2+4x+3)}{4}-\frac{\sqrt{2}}{2}\arctan\left(\frac{(4+4x)\sqrt{2}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(1/(x+(-x^2-4*x-3)^(1/2)),x)
```

```
[Out] 1/2*arcsin(2+x)-1/12*3^(1/2)*4^(1/2)*(3*x^2/(-3/2-x)^2-12)^(1/2)*
(2^(1/2)*arctan(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))-arctanh(
3*x/(-3/2-x)/(3*x^2/(-3/2-x)^2-12)^(1/2)))/((x^2/(-3/2-x)^2-4)/(1
+x/(-3/2-x))^2)^(1/2)/(1+x/(-3/2-x))+1/3*3^(1/2)*4^(1/2)/((x^2/(-
3/2-x)^2-4)/(1+x/(-3/2-x))^2)^(1/2)/(1+x/(-3/2-x))*(3*x^2/(-3/2-x
```

```
)^2-12)^(1/2)*2^(1/2)*arctan(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1
/2))-1/6*3^(1/2)*4^(1/2)*(3*x^2/(-3/2-x)^2-12)^(1/2)*(2^(1/2)*arc
tan(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))+arctanh(3*x/(-3/2-x)
/(3*x^2/(-3/2-x)^2-12)^(1/2)))/((x^2/(-3/2-x)^2-4)/(1+x/(-3/2-x))
^2)^(1/2)/(1+x/(-3/2-x))+1/4*ln(2*x^2+4*x+3)-1/2*2^(1/2)*arctan(1
/4*(4+4*x)*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x+\sqrt{-x^2-4x-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x + sqrt(-x^2 - 4*x - 3)),x, algorithm="maxima")
```

```
[Out] integrate(1/(x + sqrt(-x^2 - 4*x - 3)), x)
```

Fricas [A] time = 0.281618, size = 248, normalized size = 2.3

$$\frac{1}{16}\sqrt{2}\left(4\sqrt{2}\arctan\left(\frac{x+2}{\sqrt{-x^2-4x-3}}\right)+2\sqrt{2}\log\left(2x^2+4x+3\right)-\sqrt{2}\log\left(-\frac{2\sqrt{-x^2-4x-3}x+4x+3}{x^2}\right)+\sqrt{2}\log\left(\frac{2\sqrt{-}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x + sqrt(-x^2 - 4*x - 3)),x, algorithm="fricas")
```

```
[Out] 1/16*sqrt(2)*(4*sqrt(2)*arctan((x + 2)/sqrt(-x^2 - 4*x - 3)) + 2*
sqrt(2)*log(2*x^2 + 4*x + 3) - sqrt(2)*log(-(2*sqrt(-x^2 - 4*x -
3)*x + 4*x + 3)/x^2) + sqrt(2)*log((2*sqrt(-x^2 - 4*x - 3)*x - 4*
x - 3)/x^2) - 8*arctan(sqrt(2)*(x + 1)) + 4*arctan(1/2*(sqrt(2)*x
 + 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x + 3)) + 4*arctan(-1/2*(sq
rt(2)*x - 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x + 3)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x+\sqrt{-x^2-4x-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x+(-x**2-4*x-3)**(1/2)),x)

[Out] Integral(1/(x + sqrt(-x**2 - 4*x - 3)), x)
```

GIAC/XCAS [A] time = 0.274925, size = 266, normalized size = 2.46

$$-\frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}(x+1)\right)+\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+1\right)\right)+\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4x-3}-1}{x+2}+1\right)\right)+\frac{1}{2}\arcsin(x+2)+\frac{1}{4}\ln\left(2x^2+4x+3\right)+\frac{1}{4}\ln\left(\frac{2\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+\frac{3\left(\sqrt{-x^2-4x-3}-1\right)^2}{(x+2)^2}+1\right)-\frac{1}{4}\ln\left(\frac{2\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+\frac{\left(\sqrt{-x^2-4x-3}-1\right)^2}{(x+2)^2}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x + sqrt(-x^2 - 4*x - 3)),x, algorithm="giac")

[Out] -1/2*sqrt(2)*arctan(sqrt(2)*(x + 1)) + 1/2*sqrt(2)*arctan(1/2*sqr
t(2)*(3*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + 1/2*sqrt(2)*ar
ctan(1/2*sqrt(2)*((sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + 1/2*
arcsin(x + 2) + 1/4*ln(2*x^2 + 4*x + 3) + 1/4*ln(2*(sqrt(-x^2 - 4
*x - 3) - 1)/(x + 2) + 3*(sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 +
 1) - 1/4*ln(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + (sqrt(-x^2 -
4*x - 3) - 1)^2/(x + 2)^2 + 3)
```

3.604 $\int \frac{1}{\left(x+\sqrt{-3-4x-x^2}\right)^2} dx$

Optimal. Leaf size=87

$$\frac{1-\frac{\sqrt{-x-1}}{\sqrt{x+3}}}{-\frac{3(x+1)}{x+3}-\frac{2\sqrt{-x-1}}{\sqrt{x+3}}+1}+\frac{\tan^{-1}\left(\frac{1-\frac{3\sqrt{-x-1}}{\sqrt{x+3}}}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] (1 - Sqrt[-1 - x]/Sqrt[3 + x])/(1 - (3*(1 + x))/(3 + x) - (2*Sqrt
[-1 - x])/Sqrt[3 + x]) + ArcTan[(1 - (3*Sqrt[-1 - x])/Sqrt[3 + x]
)/Sqrt[2]]/Sqrt[2]
```

Rubi [A] time = 0.156246, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$\frac{1-\frac{\sqrt{-x-1}}{\sqrt{x+3}}}{-\frac{3(x+1)}{x+3}-\frac{2\sqrt{-x-1}}{\sqrt{x+3}}+1}+\frac{\tan^{-1}\left(\frac{1-\frac{3\sqrt{-x-1}}{\sqrt{x+3}}}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x + Sqrt[-3 - 4*x - x^2])^(-2),x]
```

```
[Out] (1 - Sqrt[-1 - x]/Sqrt[3 + x])/(1 - (3*(1 + x))/(3 + x) - (2*Sqrt
[-1 - x])/Sqrt[3 + x]) + ArcTan[(1 - (3*Sqrt[-1 - x])/Sqrt[3 + x]
)/Sqrt[2]]/Sqrt[2]
```

Rubi in Sympy [A] time = 3.867, size = 94, normalized size = 1.08

$$\frac{2-\frac{2\sqrt{-x^2-4x-3}}{x+3}}{2\left(1-\frac{2\sqrt{-x^2-4x-3}}{x+3}+\frac{3(-x^2-4x-3)}{(x+3)^2}\right)}-\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(-\frac{1}{2}+\frac{3\sqrt{-x^2-4x-3}}{2(x+3)}\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x+(-x**2-4*x-3)**(1/2))**2,x)
```

```
[Out] (2 - 2*sqrt(-x**2 - 4*x - 3)/(x + 3))/(2*(1 - 2*sqrt(-x**2 - 4*x
- 3)/(x + 3) + 3*(-x**2 - 4*x - 3)/(x + 3)**2)) - sqrt(2)*atan(sq
```

```
rt(2)*(-1/2 + 3*sqrt(-x**2 - 4*x - 3)/(2*(x + 3))))/2
```

Mathematica [C] time = 5.19934, size = 881, normalized size = 10.13

$$\frac{1}{16}\left(\frac{8(x+3)}{2x^2+4x+3}+4\sqrt{2}\tan^{-1}\left(\sqrt{2}(x+1)\right)\right.$$

$$-\frac{2i\left(-2i+\sqrt{2}\right)\tan^{-1}\left(\frac{(x+2)\left(2\left(9+2i\sqrt{2}\right)x^2+16\left(2+i\sqrt{2}\right)x+3\left(5+4i\sqrt{2}\right)\right)}{\left(8i+6\sqrt{2}\right)x^3+\left(-6\sqrt{1+2i\sqrt{2}}\sqrt{-x^2-4x-3}+8\sqrt{2}+36i\right)x^2+\left(-12\sqrt{1+2i\sqrt{2}}\sqrt{-x^2-4x-3}-5\sqrt{2}+40i\right)x-9\sqrt{1+2i\sqrt{2}}\sqrt{-x^2-4x-3}-6\sqrt{2}+12i}\right.}{\sqrt{1+2i\sqrt{2}}}$$

$$+\frac{2\left(2i+\sqrt{2}\right)\tanh^{-1}\left(\frac{(x+2)\left(2\left(9i+2\sqrt{2}\right)x^2+16\left(2i+\sqrt{2}\right)x+3\left(5i+4\sqrt{2}\right)\right)}{\left(-8i+6\sqrt{2}\right)x^3+\left(-6\sqrt{1-2i\sqrt{2}}\sqrt{-x^2-4x-3}+8\sqrt{2}-36i\right)x^2-12\sqrt{1-2i\sqrt{2}}\sqrt{-x^2-4x-3}x-5\left(8i+\sqrt{2}\right)x-3\left(3\sqrt{1-2i\sqrt{2}}\sqrt{-x^2-4x-3}+2\sqrt{2}+4i\right.}\right.}{\sqrt{1-2i\sqrt{2}}}$$

$$-\frac{\left(2i+\sqrt{2}\right)\log\left(4\left(2x^2+4x+3\right)^2\right)}{\sqrt{1-2i\sqrt{2}}}-\frac{\left(-2i+\sqrt{2}\right)\log\left(4\left(2x^2+4x+3\right)^2\right)}{\sqrt{1+2i\sqrt{2}}}$$

$$+\frac{\left(2i+\sqrt{2}\right)\log\left(\left(2x^2+4x+3\right)\left(\left(2+2i\sqrt{2}\right)x^2+\left(-2\sqrt{2-4i\sqrt{2}}\sqrt{-x^2-4x-3}+8i\sqrt{2}+4\right)x-2\sqrt{2-4i\sqrt{2}}\sqrt{-x^2-4x-3\right.\right.}{\sqrt{1-2i\sqrt{2}}}$$

$$+\frac{\left(-2i+\sqrt{2}\right)\log\left(\left(2x^2+4x+3\right)\left(\left(2-2i\sqrt{2}\right)x^2-2\left(\sqrt{2+4i\sqrt{2}}\sqrt{-x^2-4x-3}+4i\sqrt{2}-2\right)x-2\sqrt{2+4i\sqrt{2}}\sqrt{-x^2-4x-\right.\right.}{\sqrt{1+2i\sqrt{2}}}$$

$$\left.+\frac{8(2x+3)\sqrt{-x^2-4x-3}}{2x^2+4x+3}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x + Sqrt[-3 - 4*x - x^2])^(-2),x]
```

```
[Out] ((8*(3 + x))/(3 + 4*x + 2*x^2) + (8*(3 + 2*x)*Sqrt[-3 - 4*x - x^2
])/(3 + 4*x + 2*x^2) + 4*Sqrt[2]*ArcTan[Sqrt[2]*(1 + x)] - ((2*I)
*(-2*I + Sqrt[2])*ArcTan[((2 + x)*(3*(5 + (4*I)*Sqrt[2]) + 16*(2
+ I*Sqrt[2])*x + 2*(9 + (2*I)*Sqrt[2])*x^2))/(12*I - 6*Sqrt[2] +
(8*I + 6*Sqrt[2])*x^3 - 9*Sqrt[1 + (2*I)*Sqrt[2]]*Sqrt[-3 - 4*x -
 x^2] + x*(40*I - 5*Sqrt[2] - 12*Sqrt[1 + (2*I)*Sqrt[2]]*Sqrt[-3
- 4*x - x^2]) + x^2*(36*I + 8*Sqrt[2] - 6*Sqrt[1 + (2*I)*Sqrt[2]]
*Sqrt[-3 - 4*x - x^2]))])/Sqrt[1 + (2*I)*Sqrt[2]] + (2*(2*I + Sqr
t[2])*ArcTanh[((2 + x)*(3*(5*I + 4*Sqrt[2]) + 16*(2*I + Sqrt[2])*
```

```
x + 2*(9*I + 2*Sqrt[2])*x^2))/(-5*(8*I + Sqrt[2])*x + (-8*I + 6*S
qrt[2])*x^3 - 12*Sqrt[1 - (2*I)*Sqrt[2]]*x*Sqrt[-3 - 4*x - x^2] +
 x^2*(-36*I + 8*Sqrt[2] - 6*Sqrt[1 - (2*I)*Sqrt[2]]*Sqrt[-3 - 4*x
 - x^2]) - 3*(4*I + 2*Sqrt[2] + 3*Sqrt[1 - (2*I)*Sqrt[2]]*Sqrt[-3
 - 4*x - x^2]))])/Sqrt[1 - (2*I)*Sqrt[2]] - ((-2*I + Sqrt[2])*Log
[4*(3 + 4*x + 2*x^2)^2])/Sqrt[1 + (2*I)*Sqrt[2]] - ((2*I + Sqrt[2
])*Log[4*(3 + 4*x + 2*x^2)^2])/Sqrt[1 - (2*I)*Sqrt[2]] + ((2*I +
Sqrt[2])*Log[(3 + 4*x + 2*x^2)*(3 + (6*I)*Sqrt[2] + (2 + (2*I)*Sq
rt[2])*x^2 - 2*Sqrt[2 - (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2] + x*(
4 + (8*I)*Sqrt[2] - 2*Sqrt[2 - (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2
]))])/Sqrt[1 - (2*I)*Sqrt[2]] + ((-2*I + Sqrt[2])*Log[(3 + 4*x +
2*x^2)*(3 - (6*I)*Sqrt[2] + (2 - (2*I)*Sqrt[2])*x^2 - 2*Sqrt[2 +
(4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2] - 2*x*(-2 + (4*I)*Sqrt[2] + S
qrt[2 + (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2]))])/Sqrt[1 + (2*I)*Sq
rt[2]])/16
```

Maple [B] time = 0.108, size = 2407, normalized size = 27.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x+(-x^2-4*x-3)^(1/2))^2,x)
```

```
[Out] -3/8*(4+4*x)/(2*x^2+4*x+3)+1/4*2^(1/2)*arctan(1/4*(4+4*x)*2^(1/2)
)-1/2*(-6-4*x)/(2*x^2+4*x+3)+1/36*3^(1/2)*4^(1/2)*(3*x^2/(-3/2-x)
^2-12)^(1/2)*(7*2^(1/2)*arctan(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^
(1/2))+4*arctanh(3*x/(-3/2-x)/(3*x^2/(-3/2-x)^2-12)^(1/2)))/((x^2
/(-3/2-x)^2-4)/(1+x/(-3/2-x))^2)^(1/2)/(1+x/(-3/2-x))+1/72*3^(1/2
)*4^(1/2)*(3*x^2/(-3/2-x)^2-12)^(1/2)*(arctan(1/6*(3*x^2/(-3/2-x)
^2-12)^(1/2)*2^(1/2))*2^(1/2)*x^2/(-3/2-x)^2-8*arctanh(3*x/(-3/2-
x)/(3*x^2/(-3/2-x)^2-12)^(1/2))*x^2/(-3/2-x)^2+2*2^(1/2)*arctan(1
/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))-16*arctanh(3*x/(-3/2-x)/(
3*x^2/(-3/2-x)^2-12)^(1/2))-6*(3*x^2/(-3/2-x)^2-12)^(1/2))/((x^2/
(-3/2-x)^2-4)/(1+x/(-3/2-x))^2)^(1/2)/(1+x/(-3/2-x))/(x^2/(-3/2-x
)^2+2)-2/9*3^(1/2)*4^(1/2)*(3*x^2/(-3/2-x)^2-12)^(1/2)*(2^(1/2)*a
rctan(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))+arctanh(3*x/(-3/2-
x)/(3*x^2/(-3/2-x)^2-12)^(1/2)))/((x^2/(-3/2-x)^2-4)/(1+x/(-3/2-x
))^2)^(1/2)/(1+x/(-3/2-x))-2/9*3^(1/2)*4^(1/2)*(3*x^2/(-3/2-x)^2-
12)^(1/2)*(3*arctan(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))*2^(1
/2)*x^6/(-3/2-x)^6+4*arctanh(3*x/(-3/2-x)/(3*x^2/(-3/2-x)^2-12)^(
1/2))*x^6/(-3/2-x)^6+2*ln(((3*x^2/(-3/2-x)^2-12)^(1/2)*x/(-3/2-x)
-x^2/(-3/2-x)^2+4)/(x^2/(-3/2-x)^2-4))*x^6/(-3/2-x)^6-2*ln(((3*x^
2/(-3/2-x)^2-12)^(1/2)*x/(-3/2-x)+x^2/(-3/2-x)^2-4)/(x^2/(-3/2-x)
^2-4))*x^6/(-3/2-x)^6+(3*x^2/(-3/2-x)^2-12)^(1/2)*x^5/(-3/2-x)^5-
(3*x^2/(-3/2-x)^2-12)^(3/2)*x^2/(-3/2-x)^2+(3*x^2/(-3/2-x)^2-12)^
(1/2)*x^4/(-3/2-x)^4-36*arctan(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^
(1/2))*2^(1/2)*x^2/(-3/2-x)^2-2*(3*x^2/(-3/2-x)^2-12)^(1/2)*x^3/(
-3/2-x)^3-48*arctanh(3*x/(-3/2-x)/(3*x^2/(-3/2-x)^2-12)^(1/2))*x^
```

```
2/(-3/2-x)^2-8*(3*x^2/(-3/2-x)^2-12)^(1/2)*x^2/(-3/2-x)^2-24*ln((
(3*x^2/(-3/2-x)^2-12)^(1/2)*x/(-3/2-x)-x^2/(-3/2-x)^2+4)/(x^2/(-3
/2-x)^2-4))*x^2/(-3/2-x)^2+24*ln(((3*x^2/(-3/2-x)^2-12)^(1/2)*x/(
-3/2-x)+x^2/(-3/2-x)^2-4)/(x^2/(-3/2-x)^2-4))*x^2/(-3/2-x)^2-48*2
^(1/2)*arctan(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))-8*(3*x^2/(
-3/2-x)^2-12)^(1/2)*x/(-3/2-x)-64*arctanh(3*x/(-3/2-x)/(3*x^2/(-3
/2-x)^2-12)^(1/2))+16*(3*x^2/(-3/2-x)^2-12)^(1/2)-32*ln(((3*x^2/(
-3/2-x)^2-12)^(1/2)*x/(-3/2-x)-x^2/(-3/2-x)^2+4)/(x^2/(-3/2-x)^2-
4))+32*ln(((3*x^2/(-3/2-x)^2-12)^(1/2)*x/(-3/2-x)+x^2/(-3/2-x)^2-
4)/(x^2/(-3/2-x)^2-4)))/((x^2/(-3/2-x)^2-4)/(1+x/(-3/2-x))^2)^(1/
2)/(1+x/(-3/2-x))/(x^2/(-3/2-x)^2+2)/((3*x^2/(-3/2-x)^2-12)^(1/2)
*x/(-3/2-x)-x^2/(-3/2-x)^2+4)/((3*x^2/(-3/2-x)^2-12)^(1/2)*x/(-3/
2-x)+x^2/(-3/2-x)^2-4)+1/18*3^(1/2)*4^(1/2)*(3*x^2/(-3/2-x)^2-12)
^(1/2)*(11*arctan(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))*2^(1/2
)*x^6/(-3/2-x)^6+24*arctanh(3*x/(-3/2-x)/(3*x^2/(-3/2-x)^2-12)^(1
/2))*x^6/(-3/2-x)^6+8*ln(((3*x^2/(-3/2-x)^2-12)^(1/2)*x/(-3/2-x)-
x^2/(-3/2-x)^2+4)/(x^2/(-3/2-x)^2-4))*x^6/(-3/2-x)^6-8*ln(((3*x^2
/(-3/2-x)^2-12)^(1/2)*x/(-3/2-x)+x^2/(-3/2-x)^2-4)/(x^2/(-3/2-x)^
2-4))*x^6/(-3/2-x)^6+4*(3*x^2/(-3/2-x)^2-12)^(1/2)*x^5/(-3/2-x)^5
-(3*x^2/(-3/2-x)^2-12)^(3/2)*x^2/(-3/2-x)^2+(3*x^2/(-3/2-x)^2-12)
^(1/2)*x^4/(-3/2-x)^4-132*arctan(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*
2^(1/2))*2^(1/2)*x^2/(-3/2-x)^2-8*(3*x^2/(-3/2-x)^2-12)^(1/2)*x^3
/(-3/2-x)^3-288*arctanh(3*x/(-3/2-x)/(3*x^2/(-3/2-x)^2-12)^(1/2))
*x^2/(-3/2-x)^2-8*(3*x^2/(-3/2-x)^2-12)^(1/2)*x^2/(-3/2-x)^2-96*l
n(((3*x^2/(-3/2-x)^2-12)^(1/2)*x/(-3/2-x)-x^2/(-3/2-x)^2+4)/(x^2/
(-3/2-x)^2-4))*x^2/(-3/2-x)^2+96*ln(((3*x^2/(-3/2-x)^2-12)^(1/2)*
x/(-3/2-x)+x^2/(-3/2-x)^2-4)/(x^2/(-3/2-x)^2-4))*x^2/(-3/2-x)^2-1
76*2^(1/2)*arctan(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))-32*(3*
x^2/(-3/2-x)^2-12)^(1/2)*x/(-3/2-x)-384*arctanh(3*x/(-3/2-x)/(3*x
^2/(-3/2-x)^2-12)^(1/2))+16*(3*x^2/(-3/2-x)^2-12)^(1/2)-128*ln(((
3*x^2/(-3/2-x)^2-12)^(1/2)*x/(-3/2-x)-x^2/(-3/2-x)^2+4)/(x^2/(-3/
2-x)^2-4))+128*ln(((3*x^2/(-3/2-x)^2-12)^(1/2)*x/(-3/2-x)+x^2/(-3
/2-x)^2-4)/(x^2/(-3/2-x)^2-4)))/((x^2/(-3/2-x)^2-4)/(1+x/(-3/2-x)
)^2)^(1/2)/(1+x/(-3/2-x))/(x^2/(-3/2-x)^2+2)/((3*x^2/(-3/2-x)^2-1
2)^(1/2)*x/(-3/2-x)-x^2/(-3/2-x)^2+4)/((3*x^2/(-3/2-x)^2-12)^(1/2
)*x/(-3/2-x)+x^2/(-3/2-x)^2-4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x+\sqrt{-x^2-4x-3}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sqrt(-x^2 - 4*x - 3))^(-2),x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(-x^2 - 4*x - 3))^(-2), x)
```

Fricas [A] time = 0.271421, size = 154, normalized size = 1.77

$$\frac{\sqrt{2}\left(2\,\sqrt{2}\sqrt{-x^2-4\,x-3}(2\,x+3)+2\,\left(2\,x^2+4\,x+3\right)\,\arctan\left(\sqrt{2}(x+1)\right)+\left(2\,x^2+4\,x+3\right)\,\arctan\left(\frac{\sqrt{2}(6\,x^2+20\,x+15)}{4\,\sqrt{-x^2-4\,x-3}(2\,x+3)}\right)+2\,\sqrt{}\right.}{8\,(2\,x^2+4\,x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sqrt(-x^2 - 4*x - 3))^(-2),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2)*(2*sqrt(2)*sqrt(-x^2 - 4*x - 3)*(2*x + 3) + 2*(2*x^2
+ 4*x + 3)*arctan(sqrt(2)*(x + 1)) + (2*x^2 + 4*x + 3)*arctan(1/4
*sqrt(2)*(6*x^2 + 20*x + 15)/(sqrt(-x^2 - 4*x - 3)*(2*x + 3))) +
2*sqrt(2)*(x + 3))/(2*x^2 + 4*x + 3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x+\sqrt{-x^2-4x-3}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x+(-x**2-4*x-3)**(1/2))**2,x)
```

```
[Out] Integral((x + sqrt(-x**2 - 4*x - 3))**(-2), x)
```

GIAC/XCAS [A] time = 0.272446, size = 355, normalized size = 4.08

$$\frac{1}{4}\,\sqrt{2}\arctan\left(\sqrt{2}(x+1)\right)-\frac{1}{4}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4\,x-3}-1\right)}{x+2}+1\right)\right)-\frac{1}{4}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\frac{\sqrt{-x^2-4\,x-3}-1}{x+2}+1\right)\right)+\frac{x+3}{2\,(2\,x^2+4\,x+3)}-\frac{\frac{10\left(\sqrt{-x^2-4\,x-3}-1\right)}{x+2}+\frac{7\left(\sqrt{-x^2-4\,x-3}-1\right)^2}{(x+2)^2}-\frac{2\left(\sqrt{-x^2-4\,x-3}-1\right)^3}{(x+2)^3}+3}{3\left(\frac{8\left(\sqrt{-x^2-4\,x-3}-1\right)}{x+2}+\frac{14\left(\sqrt{-x^2-4\,x-3}-1\right)^2}{(x+2)^2}+\frac{8\left(\sqrt{-x^2-4\,x-3}-1\right)^3}{(x+2)^3}+\frac{3\left(\sqrt{-x^2-4\,x-3}-1\right)^4}{(x+2)^4}+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sqrt(-x^2 - 4*x - 3))^(-2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*arctan(sqrt(2)*(x + 1)) - 1/4*sqrt(2)*arctan(1/2*sqrt
(2)*(3*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) - 1/4*sqrt(2)*arc
tan(1/2*sqrt(2)*((sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + 1/2*(
x + 3)/(2*x^2 + 4*x + 3) - 1/3*(10*(sqrt(-x^2 - 4*x - 3) - 1)/(x
+ 2) + 7*(sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 - 2*(sqrt(-x^2 -
4*x - 3) - 1)^3/(x + 2)^3 + 3)/(8*(sqrt(-x^2 - 4*x - 3) - 1)/(x +
 2) + 14*(sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 8*(sqrt(-x^2 -
4*x - 3) - 1)^3/(x + 2)^3 + 3*(sqrt(-x^2 - 4*x - 3) - 1)^4/(x + 2
)^4 + 3)
```

3.605 $\int \frac{1}{\left(x+\sqrt{-3-4x-x^2}\right)^3}\,dx$

Optimal. Leaf size=149

$$-\frac{13-\frac{27\sqrt{-x-1}}{\sqrt{x+3}}}{18\left(-\frac{3(x+1)}{x+3}-\frac{2\sqrt{-x-1}}{\sqrt{x+3}}+1\right)}-\frac{2\left(2-\frac{\sqrt{-x-1}}{\sqrt{x+3}}\right)}{9\left(-\frac{3(x+1)}{x+3}-\frac{2\sqrt{-x-1}}{\sqrt{x+3}}+1\right)^2}-\frac{3\tan^{-1}\left(\frac{1-\frac{3\sqrt{-x-1}}{\sqrt{x+3}}}{\sqrt{2}}\right)}{2\sqrt{2}}$$

```
[Out] -(13 - (27*Sqrt[-1 - x])/Sqrt[3 + x])/(18*(1 - (3*(1 + x))/(3 + x
) - (2*Sqrt[-1 - x])/Sqrt[3 + x])) - (2*(2 - Sqrt[-1 - x]/Sqrt[3
+ x]))/(9*(1 - (3*(1 + x))/(3 + x) - (2*Sqrt[-1 - x])/Sqrt[3 + x]
)^2) - (3*ArcTan[(1 - (3*Sqrt[-1 - x])/Sqrt[3 + x])/Sqrt[2]])/(2*
Sqrt[2])
```

Rubi [A] time = 0.186621, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278

$$-\frac{13-\frac{27\sqrt{-x-1}}{\sqrt{x+3}}}{18\left(-\frac{3(x+1)}{x+3}-\frac{2\sqrt{-x-1}}{\sqrt{x+3}}+1\right)}-\frac{2\left(2-\frac{\sqrt{-x-1}}{\sqrt{x+3}}\right)}{9\left(-\frac{3(x+1)}{x+3}-\frac{2\sqrt{-x-1}}{\sqrt{x+3}}+1\right)^2}-\frac{3\tan^{-1}\left(\frac{1-\frac{3\sqrt{-x-1}}{\sqrt{x+3}}}{\sqrt{2}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x + Sqrt[-3 - 4*x - x^2])^(-3),x]
```

```
[Out] -(13 - (27*Sqrt[-1 - x])/Sqrt[3 + x])/(18*(1 - (3*(1 + x))/(3 + x
) - (2*Sqrt[-1 - x])/Sqrt[3 + x])) - (2*(2 - Sqrt[-1 - x]/Sqrt[3
+ x]))/(9*(1 - (3*(1 + x))/(3 + x) - (2*Sqrt[-1 - x])/Sqrt[3 + x]
)^2) - (3*ArcTan[(1 - (3*Sqrt[-1 - x])/Sqrt[3 + x])/Sqrt[2]])/(2*
Sqrt[2])
```

Rubi in Sympy [A] time = 10.823, size = 214, normalized size = 1.44

$$-\frac{2-\frac{6\sqrt{-x^2-4x-3}}{x+3}}{12\left(1-\frac{2\sqrt{-x^2-4x-3}}{x+3}+\frac{3(-x^2-4x-3)}{(x+3)^2}\right)}-\frac{8-\frac{4\sqrt{-x^2-4x-3}}{x+3}}{18\left(1-\frac{2\sqrt{-x^2-4x-3}}{x+3}+\frac{3(-x^2-4x-3)}{(x+3)^2}\right)^2}$$
$$-\frac{10-\frac{18\sqrt{-x^2-4x-3}}{x+3}}{18\left(1-\frac{2\sqrt{-x^2-4x-3}}{x+3}+\frac{3(-x^2-4x-3)}{(x+3)^2}\right)}+\frac{3\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(-\frac{1}{2}+\frac{3\sqrt{-x^2-4x-3}}{2(x+3)}\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x+(-x**2-4*x-3)**(1/2))**3,x)
```

```
[Out] -(2 - 6*sqrt(-x**2 - 4*x - 3)/(x + 3))/(12*(1 - 2*sqrt(-x**2 - 4*
x - 3)/(x + 3) + 3*(-x**2 - 4*x - 3)/(x + 3)**2)) - (8 - 4*sqrt(-
x**2 - 4*x - 3)/(x + 3))/(18*(1 - 2*sqrt(-x**2 - 4*x - 3)/(x + 3)
 + 3*(-x**2 - 4*x - 3)/(x + 3)**2)**2) - (10 - 18*sqrt(-x**2 - 4*
x - 3)/(x + 3))/(18*(1 - 2*sqrt(-x**2 - 4*x - 3)/(x + 3) + 3*(-x*
*2 - 4*x - 3)/(x + 3)**2)) + 3*sqrt(2)*atan(sqrt(2)*(-1/2 + 3*sqr
t(-x**2 - 4*x - 3)/(2*(x + 3))))/4
```

Mathematica [C] time = 6.11845, size = 914, normalized size = 6.13

$$\frac{1}{32}\Bigg(\frac{8(2x-3)}{(2x^2+4x+3)^2}-\frac{8\sqrt{-x^2-4x-3}\,(8x^3+22x^2+26x+15)}{(2x^2+4x+3)^2}-12\sqrt{2}\tan^{-1}\left(\sqrt{2}(x+1)\right)$$

$$+\frac{6\left(2+i\sqrt{2}\right)\tan^{-1}\left(\frac{(x+2)\left(2\left(9+2i\sqrt{2}\right)x^2+16\left(2+i\sqrt{2}\right)x+3\left(5+4i\sqrt{2}\right)\right)}{\left(8i+6\sqrt{2}\right)x^3+\left(-6\sqrt{1+2i\sqrt{2}}\sqrt{-x^2-4x-3}+8\sqrt{2}+36i\right)x^2+\left(-12\sqrt{1+2i\sqrt{2}}\sqrt{-x^2-4x-3}-5\sqrt{2}+40i\right)x-9\sqrt{1+2i\sqrt{2}}\sqrt{-x^2-4x-3}-6\sqrt{2}+12i}\right)}{\sqrt{1+2i\sqrt{2}}}$$

$$-\frac{6\left(2i+\sqrt{2}\right)\tanh^{-1}\left(\frac{(x+2)\left(2\left(9i+2\sqrt{2}\right)x^2+16\left(2i+\sqrt{2}\right)x+3\left(5i+4\sqrt{2}\right)\right)}{\left(-8i+6\sqrt{2}\right)x^3+\left(-6\sqrt{1-2i\sqrt{2}}\sqrt{-x^2-4x-3}+8\sqrt{2}-36i\right)x^2-12\sqrt{1-2i\sqrt{2}}\sqrt{-x^2-4x-3}x-5\left(8i+\sqrt{2}\right)x-3\left(3\sqrt{1-2i\sqrt{2}}\sqrt{-x^2-4x-3}+2\sqrt{2}+4 \right.}}{\sqrt{1-2i\sqrt{2}}}$$

$$+\frac{3\left(2i+\sqrt{2}\right)\log\left(4\left(2x^2+4x+3\right)^2\right)}{\sqrt{1-2i\sqrt{2}}}+\frac{3\left(-2i+\sqrt{2}\right)\log\left(4\left(2x^2+4x+3\right)^2\right)}{\sqrt{1+2i\sqrt{2}}}$$

$$-\frac{3\left(2i+\sqrt{2}\right)\log\left(\left(2x^2+4x+3\right)\left(\left(2+2i\sqrt{2}\right)x^2+\left(-2\sqrt{2-4i\sqrt{2}}\sqrt{-x^2-4x-3}+8i\sqrt{2}+4\right)x-2\sqrt{2-4i\sqrt{2}}\sqrt{-x^2-4x \right.\right.\right.}{\sqrt{1-2i\sqrt{2}}}$$

$$-\frac{3\left(-2i+\sqrt{2}\right)\log\left(\left(2x^2+4x+3\right)\left(\left(2-2i\sqrt{2}\right)x^2-2\left(\sqrt{2+4i\sqrt{2}}\sqrt{-x^2-4x-3}+4i\sqrt{2}-2\right)x-2\sqrt{2+4i\sqrt{2}}\sqrt{-x^2-4x \right.\right.\right.}{\sqrt{1+2i\sqrt{2}}}$$

$$-\frac{8(3x+2)}{2x^2+4x+3}\Bigg)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x + Sqrt[-3 - 4*x - x^2])^(-3),x]
```

```
[Out] ((8*(-3 + 2*x))/(3 + 4*x + 2*x^2)^2 - (8*(2 + 3*x))/(3 + 4*x + 2*
x^2) - (8*Sqrt[-3 - 4*x - x^2]*(15 + 26*x + 22*x^2 + 8*x^3))/(3 +
 4*x + 2*x^2)^2 - 12*Sqrt[2]*ArcTan[Sqrt[2]*(1 + x)] + (6*(2 + I*
Sqrt[2])*ArcTan[((2 + x)*(3*(5 + (4*I)*Sqrt[2]) + 16*(2 + I*Sqrt[
2])*x + 2*(9 + (2*I)*Sqrt[2])*x^2))/(12*I - 6*Sqrt[2] + (8*I + 6*
Sqrt[2])*x^3 - 9*Sqrt[1 + (2*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2] + x
*(40*I - 5*Sqrt[2] - 12*Sqrt[1 + (2*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x
^2]) + x^2*(36*I + 8*Sqrt[2] - 6*Sqrt[1 + (2*I)*Sqrt[2]]*Sqrt[-3
- 4*x - x^2]))])/Sqrt[1 + (2*I)*Sqrt[2]] - (6*(2*I + Sqrt[2])*Arc
Tanh[((2 + x)*(3*(5*I + 4*Sqrt[2]) + 16*(2*I + Sqrt[2])*x + 2*(9*
I + 2*Sqrt[2])*x^2))/(-5*(8*I + Sqrt[2])*x + (-8*I + 6*Sqrt[2])*x
^3 - 12*Sqrt[1 - (2*I)*Sqrt[2]]*x*Sqrt[-3 - 4*x - x^2] + x^2*(-36
*I + 8*Sqrt[2] - 6*Sqrt[1 - (2*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])
- 3*(4*I + 2*Sqrt[2] + 3*Sqrt[1 - (2*I)*Sqrt[2]]*Sqrt[-3 - 4*x -
x^2]))])/Sqrt[1 - (2*I)*Sqrt[2]] + (3*(-2*I + Sqrt[2])*Log[4*(3 +
 4*x + 2*x^2)^2])/Sqrt[1 + (2*I)*Sqrt[2]] + (3*(2*I + Sqrt[2])*Lo
g[4*(3 + 4*x + 2*x^2)^2])/Sqrt[1 - (2*I)*Sqrt[2]] - (3*(2*I + Sqr
t[2])*Log[(3 + 4*x + 2*x^2)*(3 + (6*I)*Sqrt[2] + (2 + (2*I)*Sqrt[
2])*x^2 - 2*Sqrt[2 - (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2] + x*(4 +
 (8*I)*Sqrt[2] - 2*Sqrt[2 - (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2]))
])/Sqrt[1 - (2*I)*Sqrt[2]] - (3*(-2*I + Sqrt[2])*Log[(3 + 4*x + 2
*x^2)*(3 - (6*I)*Sqrt[2] + (2 - (2*I)*Sqrt[2])*x^2 - 2*Sqrt[2 + (
4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2] - 2*x*(-2 + (4*I)*Sqrt[2] + Sq
rt[2 + (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2]))])/Sqrt[1 + (2*I)*Sqr
t[2]])/32
```

Maple [B] time = 0.301, size = 14545, normalized size = 97.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x+(-x^2-4*x-3)^(1/2))^3,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x + \sqrt{-x^2 - 4x - 3}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sqrt(-x^2 - 4*x - 3))^(-3),x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(-x^2 - 4*x - 3))^(-3), x)
```

Fricas [A] time = 0.277454, size = 225, normalized size = 1.51

$$-\frac{\sqrt{2}\left(2\sqrt{2}\left(8x^3+22x^2+26x+15\right)\sqrt{-x^2-4x-3}+6\left(4x^4+16x^3+28x^2+24x+9\right)\arctan\left(\sqrt{2}(x+1)\right)+3\left(4x^4+16x\right.\right.}{16\left(4x^4+16x^3+28x^2+24x+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sqrt(-x^2 - 4*x - 3))^(-3),x, algorithm="fricas")
```

```
[Out] -1/16*sqrt(2)*(2*sqrt(2)*(8*x^3 + 22*x^2 + 26*x + 15)*sqrt(-x^2 -
 4*x - 3) + 6*(4*x^4 + 16*x^3 + 28*x^2 + 24*x + 9)*arctan(sqrt(2)
*(x + 1)) + 3*(4*x^4 + 16*x^3 + 28*x^2 + 24*x + 9)*arctan(1/4*sqr
t(2)*(6*x^2 + 20*x + 15)/(sqrt(-x^2 - 4*x - 3)*(2*x + 3))) + 2*sq
rt(2)*(6*x^3 + 16*x^2 + 15*x + 9))/(4*x^4 + 16*x^3 + 28*x^2 + 24*
x + 9)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x+\sqrt{-x^2-4x-3}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x+(-x**2-4*x-3)**(1/2))**3,x)
```

```
[Out] Integral((x + sqrt(-x**2 - 4*x - 3))**(-3), x)
```

GIAC/XCAS [A] time = 0.282357, size = 495, normalized size = 3.32

$$-\frac{3}{8}\sqrt{2}\arctan\left(\sqrt{2}(x+1)\right)+\frac{3}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+1\right)\right)$$
$$+\frac{3}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4x-3}-1}{x+2}+1\right)\right)-\frac{6x^3+16x^2+15x+9}{4(2x^2+4x+3)^2}$$
$$+\frac{\frac{618\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+\frac{1547\left(\sqrt{-x^2-4x-3}-1\right)^2}{(x+2)^2}+\frac{2362\left(\sqrt{-x^2-4x-3}-1\right)^3}{(x+2)^3}+\frac{2223\left(\sqrt{-x^2-4x-3}-1\right)^4}{(x+2)^4}+\frac{1174\left(\sqrt{-x^2-4x-3}-1\right)^5}{(x+2)^5}+\frac{377\left(\sqrt{-x^2-4x-3-\right.}{(x+2)^6}}{18\left(\frac{8\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+\frac{14\left(\sqrt{-x^2-4x-3}-1\right)^2}{(x+2)^2}+\frac{8\left(\sqrt{-x^2-4x-3}-1\right)^3}{(x+2)^3}+\frac{3\left(\sqrt{-x^2-4x-3}-1\right)^4}{(x+2)^4}+3\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sqrt(-x^2 - 4*x - 3))^(-3),x, algorithm="giac")
```

```
[Out] -3/8*sqrt(2)*arctan(sqrt(2)*(x + 1)) + 3/8*sqrt(2)*arctan(1/2*sqr
t(2)*(3*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + 3/8*sqrt(2)*ar
ctan(1/2*sqrt(2)*((sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) - 1/4*
(6*x^3 + 16*x^2 + 15*x + 9)/(2*x^2 + 4*x + 3)^2 + 1/18*(618*(sqrt
(-x^2 - 4*x - 3) - 1)/(x + 2) + 1547*(sqrt(-x^2 - 4*x - 3) - 1)^2
/(x + 2)^2 + 2362*(sqrt(-x^2 - 4*x - 3) - 1)^3/(x + 2)^3 + 2223*(
sqrt(-x^2 - 4*x - 3) - 1)^4/(x + 2)^4 + 1174*(sqrt(-x^2 - 4*x - 3
) - 1)^5/(x + 2)^5 + 377*(sqrt(-x^2 - 4*x - 3) - 1)^6/(x + 2)^6 +
 6*(sqrt(-x^2 - 4*x - 3) - 1)^7/(x + 2)^7 + 117)/(8*(sqrt(-x^2 -
4*x - 3) - 1)/(x + 2) + 14*(sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2
 + 8*(sqrt(-x^2 - 4*x - 3) - 1)^3/(x + 2)^3 + 3*(sqrt(-x^2 - 4*x
- 3) - 1)^4/(x + 2)^4 + 3)^2
```

3.606 $\int x^3(1+x)^3(1+2x)\sqrt{1-x^2-2x^3-x^4}\,dx$

Optimal. Leaf size=42

$$-\frac{1}{15}\left(-x^4-2x^3-x^2+1\right)^{3/2}\left(3x^4+6x^3+3x^2+2\right)$$

```
[Out] -((1 - x^2 - 2*x^3 - x^4)^(3/2)*(2 + 3*x^2 + 6*x^3 + 3*x^4))/15
```

Rubi [A] time = 0.393367, antiderivative size = 59, normalized size of antiderivative = 1.4, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$-\frac{1}{5}x^2\left(-x^4-2x^3-x^2+1\right)^{3/2}(x+1)^2-\frac{2}{15}\left(-x^4-2x^3-x^2+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*(1 + x)^3*(1 + 2*x)*Sqrt[1 - x^2 - 2*x^3 - x^4],x]
```

```
[Out] (-2*(1 - x^2 - 2*x^3 - x^4)^(3/2))/15 - (x^2*(1 + x)^2*(1 - x^2 -
 2*x^3 - x^4)^(3/2))/5
```

Rubi in Sympy [A] time = 23.4367, size = 60, normalized size = 1.43

$$-\frac{\left(-4\left(x+\frac{1}{2}\right)^2+1\right)^2\left(-16\left(x+\frac{1}{2}\right)^4+8\left(x+\frac{1}{2}\right)^2+15\right)^{\frac{3}{2}}}{5120}-\frac{\left(-16\left(x+\frac{1}{2}\right)^4+8\left(x+\frac{1}{2}\right)^2+15\right)^{\frac{3}{2}}}{480}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**3*(1+x)**3*(1+2*x)*(-x**4-2*x**3-x**2+1)**(1/2),x)
```

```
[Out] -(-4*(x + 1/2)**2 + 1)**2*(-16*(x + 1/2)**4 + 8*(x + 1/2)**2 + 15
)**(3/2)/5120 - (-16*(x + 1/2)**4 + 8*(x + 1/2)**2 + 15)**(3/2)/4
80
```

Mathematica [A] time = 0.0827473, size = 62, normalized size = 1.48

$$\frac{1}{15}\sqrt{-x^4-2x^3-x^2+1}\left(3x^8+12x^7+18x^6+12x^5+2x^4-2x^3-x^2-2\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*(1 + x)^3*(1 + 2*x)*Sqrt[1 - x^2 - 2*x^3 - x^4],x]
```

```
[Out] (Sqrt[1 - x^2 - 2*x^3 - x^4]*(-2 - x^2 - 2*x^3 + 2*x^4 + 12*x^5 +
 18*x^6 + 12*x^7 + 3*x^8))/15
```

Maple [A] time = 0.011, size = 51, normalized size = 1.2

$$\frac{\left(x^2+x+1\right)\left(x^2+x-1\right)\left(3x^4+6x^3+3x^2+2\right)}{15}\sqrt{-x^4-2x^3-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(1+x)^3*(1+2*x)*(-x^4-2*x^3-x^2+1)^(1/2),x)
```

```
[Out] 1/15*(x^2+x+1)*(x^2+x-1)*(3*x^4+6*x^3+3*x^2+2)*(-x^4-2*x^3-x^2+1)
^(1/2)
```

Maxima [A] time = 0.884715, size = 80, normalized size = 1.9

$$\frac{1}{15}\left(3x^8+12x^7+18x^6+12x^5+2x^4-2x^3-x^2-2\right)\sqrt{x^2+x+1}\sqrt{-x^2-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-x^4 - 2*x^3 - x^2 + 1)*(2*x + 1)*(x + 1)^3*x^3,x, algorithm="maxim
```

```
[Out] 1/15*(3*x^8 + 12*x^7 + 18*x^6 + 12*x^5 + 2*x^4 - 2*x^3 - x^2 - 2)
*sqrt(x^2 + x + 1)*sqrt(-x^2 - x + 1)
```

Fricas [A] time = 0.260599, size = 78, normalized size = 1.86

$$\frac{1}{15}\left(3x^8+12x^7+18x^6+12x^5+2x^4-2x^3-x^2-2\right)\sqrt{-x^4-2x^3-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-x^4 - 2*x^3 - x^2 + 1)*(2*x + 1)*(x + 1)^3*x^3,x, algorithm="frica
```

```
[Out] 1/15*(3*x^8 + 12*x^7 + 18*x^6 + 12*x^5 + 2*x^4 - 2*x^3 - x^2 - 2)
*sqrt(-x^4 - 2*x^3 - x^2 + 1)
```

Sympy [A] time = 2.15936, size = 182, normalized size = 4.33

$$\frac{x^8\sqrt{-x^4-2x^3-x^2+1}}{5}+\frac{4x^7\sqrt{-x^4-2x^3-x^2+1}}{5}+\frac{6x^6\sqrt{-x^4-2x^3-x^2+1}}{5}+\frac{4x^5\sqrt{-x^4-2x^3-x^2+1}}{5}+\frac{2x^4\sqrt{-x^4-2x^3-x^2+1}}{15}-\frac{2x^3\sqrt{-x^4-2x^3-x^2+1}}{15}-\frac{x^2\sqrt{-x^4-2x^3-x^2+1}}{15}-\frac{2\sqrt{-x^4-2x^3-x^2+1}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(1+x)**3*(1+2*x)*(-x**4-2*x**3-x**2+1)**(1/2),x)
```

```
[Out] x**8*sqrt(-x**4 - 2*x**3 - x**2 + 1)/5 + 4*x**7*sqrt(-x**4 - 2*x*
*3 - x**2 + 1)/5 + 6*x**6*sqrt(-x**4 - 2*x**3 - x**2 + 1)/5 + 4*x
**5*sqrt(-x**4 - 2*x**3 - x**2 + 1)/5 + 2*x**4*sqrt(-x**4 - 2*x**
3 - x**2 + 1)/15 - 2*x**3*sqrt(-x**4 - 2*x**3 - x**2 + 1)/15 - x*
*2*sqrt(-x**4 - 2*x**3 - x**2 + 1)/15 - 2*sqrt(-x**4 - 2*x**3 - x
**2 + 1)/15
```

GIAC/XCAS [A] time = 0.26467, size = 69, normalized size = 1.64

$$\frac{1}{15}\sqrt{-x^4-2\,x^3-x^2+1}\left(((((3\,(((x+4)x+6)x+4)x+2)x-2)x-1)x^2-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^4 - 2*x^3 - x^2 + 1)*(2*x + 1)*(x + 1)^3*x^3,x, algorithm="giac"
```

```
[Out] 1/15*sqrt(-x^4 - 2*x^3 - x^2 + 1)*((((3*(((x + 4)*x + 6)*x + 4)*x
 + 2)*x - 2)*x - 1)*x^2 - 2)
```

3.607 $\int (1+2x)\left(x+x^2\right)^3\sqrt{1-\left(x+x^2\right)^2}\,dx$

Optimal. Leaf size=42

$$-\frac{1}{15}\left(-x^4-2x^3-x^2+1\right)^{3/2}\left(3x^4+6x^3+3x^2+2\right)$$

```
[Out] -((1 - x^2 - 2*x^3 - x^4)^(3/2)*(2 + 3*x^2 + 6*x^3 + 3*x^4))/15
```

Rubi [A] time = 0.395042, antiderivative size = 59, normalized size of antiderivative = 1.4, number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$

$$-\frac{1}{5}x^2\left(-x^4-2x^3-x^2+1\right)^{3/2}(x+1)^2-\frac{2}{15}\left(-x^4-2x^3-x^2+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + 2*x)*(x + x^2)^3*Sqrt[1 - (x + x^2)^2],x]
```

```
[Out] (-2*(1 - x^2 - 2*x^3 - x^4)^(3/2))/15 - (x^2*(1 + x)^2*(1 - x^2 -
 2*x^3 - x^4)^(3/2))/5
```

Rubi in Sympy [A] time = 21.1478, size = 60, normalized size = 1.43

$$-\frac{\left(-4\left(x+\frac{1}{2}\right)^2+1\right)^2\left(-16\left(x+\frac{1}{2}\right)^4+8\left(x+\frac{1}{2}\right)^2+15\right)^{\frac{3}{2}}}{5120}-\frac{\left(-16\left(x+\frac{1}{2}\right)^4+8\left(x+\frac{1}{2}\right)^2+15\right)^{\frac{3}{2}}}{480}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+2*x)*(x**2+x)**3*(1-(x**2+x)**2)**(1/2),x)
```

```
[Out] -(-4*(x + 1/2)**2 + 1)**2*(-16*(x + 1/2)**4 + 8*(x + 1/2)**2 + 15
)**(3/2)/5120 - (-16*(x + 1/2)**4 + 8*(x + 1/2)**2 + 15)**(3/2)/4
80
```

Mathematica [A] time = 0.0714596, size = 62, normalized size = 1.48

$$\frac{1}{15}\sqrt{-x^4-2x^3-x^2+1}\left(3x^8+12x^7+18x^6+12x^5+2x^4-2x^3-x^2-2\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + 2*x)*(x + x^2)^3*Sqrt[1 - (x + x^2)^2],x]
```

```
[Out] (Sqrt[1 - x^2 - 2*x^3 - x^4]*(-2 - x^2 - 2*x^3 + 2*x^4 + 12*x^5 +
 18*x^6 + 12*x^7 + 3*x^8))/15
```

Maple [A] time = 0.008, size = 51, normalized size = 1.2

$$\frac{(3x^4+6x^3+3x^2+2)(x^2+x+1)(x^2+x-1)}{15}\sqrt{-x^4-2x^3-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+2*x)*(x^2+x)^3*(1-(x^2+x)^2)^(1/2),x)
```

```
[Out] 1/15*(x^2+x+1)*(x^2+x-1)*(3*x^4+6*x^3+3*x^2+2)*(-x^4-2*x^3-x^2+1)
^(1/2)
```

Maxima [A] time = 0.876713, size = 80, normalized size = 1.9

$$\frac{1}{15}\left(3x^8+12x^7+18x^6+12x^5+2x^4-2x^3-x^2-2\right)\sqrt{x^2+x+1}\sqrt{-x^2-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-(x^2 + x)^2 + 1)*(x^2 + x)^3*(2*x + 1),x, algorithm="maxima")
```

```
[Out] 1/15*(3*x^8 + 12*x^7 + 18*x^6 + 12*x^5 + 2*x^4 - 2*x^3 - x^2 - 2)
*sqrt(x^2 + x + 1)*sqrt(-x^2 - x + 1)
```

Fricas [A] time = 0.260162, size = 78, normalized size = 1.86

$$\frac{1}{15}\left(3x^8+12x^7+18x^6+12x^5+2x^4-2x^3-x^2-2\right)\sqrt{-x^4-2x^3-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-(x^2 + x)^2 + 1)*(x^2 + x)^3*(2*x + 1),x, algorithm="fricas")
```

```
[Out] 1/15*(3*x^8 + 12*x^7 + 18*x^6 + 12*x^5 + 2*x^4 - 2*x^3 - x^2 - 2)
*sqrt(-x^4 - 2*x^3 - x^2 + 1)
```

Sympy [A] time = 40.747, size = 182, normalized size = 4.33

$$\frac{x^8\sqrt{-x^4-2x^3-x^2+1}}{5}+\frac{4x^7\sqrt{-x^4-2x^3-x^2+1}}{5}+\frac{6x^6\sqrt{-x^4-2x^3-x^2+1}}{5}+\frac{4x^5\sqrt{-x^4-2x^3-x^2+1}}{5}+\frac{2x^4\sqrt{-x^4-2x^3-x^2+1}}{15}-\frac{2x^3\sqrt{-x^4-2x^3-x^2+1}}{15}-\frac{x^2\sqrt{-x^4-2x^3-x^2+1}}{15}-\frac{2\sqrt{-x^4-2x^3-x^2+1}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+2*x)*(x**2+x)**3*(1-(x**2+x)**2)**(1/2),x)
```

```
[Out] x**8*sqrt(-x**4 - 2*x**3 - x**2 + 1)/5 + 4*x**7*sqrt(-x**4 - 2*x*
*3 - x**2 + 1)/5 + 6*x**6*sqrt(-x**4 - 2*x**3 - x**2 + 1)/5 + 4*x
**5*sqrt(-x**4 - 2*x**3 - x**2 + 1)/5 + 2*x**4*sqrt(-x**4 - 2*x**
3 - x**2 + 1)/15 - 2*x**3*sqrt(-x**4 - 2*x**3 - x**2 + 1)/15 - x*
*2*sqrt(-x**4 - 2*x**3 - x**2 + 1)/15 - 2*sqrt(-x**4 - 2*x**3 - x
**2 + 1)/15
```

GIAC/XCAS [A] time = 0.264102, size = 69, normalized size = 1.64

$$\frac{1}{15}\sqrt{-x^4-2\,x^3-x^2+1}\left((((3\,(((x+4)x+6)x+4)x+2)x-2)x-1)x^2-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-(x^2 + x)^2 + 1)*(x^2 + x)^3*(2*x + 1),x, algorithm="giac")
```

```
[Out] 1/15*sqrt(-x^4 - 2*x^3 - x^2 + 1)*((((3*(((x + 4)*x + 6)*x + 4)*x
 + 2)*x - 2)*x - 1)*x^2 - 2)
```

3.608 $\int \left(8x-8x^2+4x^3-x^4\right)^{3/2}\,dx$

Optimal. Leaf size=102

$$\frac{1}{7}(x-1)\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}+\frac{2}{35}\left(13-3(x-1)^2\right)(x-1)\sqrt{-(x-1)^4-2(x-1)^2+3}-\frac{176}{35}\sqrt{3}F\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)+\frac{16}{5}\sqrt{3}E\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)$$

```
[Out] (2*(13 - 3*(-1 + x)^2)*Sqrt[3 - 2*(-1 + x)^2 - (-1 + x)^4]*(-1 +
x))/35 + ((3 - 2*(-1 + x)^2 - (-1 + x)^4)^(3/2)*(-1 + x))/7 + (16
*Sqrt[3]*EllipticE[ArcSin[1 - x], -1/3])/5 - (176*Sqrt[3]*Ellipti
cF[ArcSin[1 - x], -1/3])/35
```

Rubi [A] time = 0.202189, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304

$$-\frac{1}{7}(1-x)\left(-(1-x)^4-2(1-x)^2+3\right)^{3/2}-\frac{2}{35}\left(13-3(1-x)^2\right)(1-x)\sqrt{-(1-x)^4-2(1-x)^2+3}-\frac{176}{35}\sqrt{3}F\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)+\frac{16}{5}\sqrt{3}E\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

```
[Out] (-2*(13 - 3*(1 - x)^2)*Sqrt[3 - 2*(1 - x)^2 - (1 - x)^4]*(1 - x))
/35 - ((3 - 2*(1 - x)^2 - (1 - x)^4)^(3/2)*(1 - x))/7 + (16*Sqrt[
3]*EllipticE[ArcSin[1 - x], -1/3])/5 - (176*Sqrt[3]*EllipticF[Arc
Sin[1 - x], -1/3])/35
```

Rubi in Sympy [A] time = 22.9351, size = 88, normalized size = 0.86

$$\frac{(x-1)\left(-6\,(x-1)^2+26\right)\sqrt{-(x-1)^4-2\,(x-1)^2+3}}{35}+\frac{(x-1)\left(-(x-1)^4-2\,(x-1)^2+3\right)^{\frac{3}{2}}}{7}-\frac{16\sqrt{3}E\left(\operatorname{asin}(x-1)\middle|-\frac{1}{3}\right)}{5}+\frac{176\sqrt{3}F\left(\operatorname{asin}(x-1)\middle|-\frac{1}{3}\right)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**4+4*x**3-8*x**2+8*x)**(3/2),x)
```

```
[Out] (x - 1)*(-6*(x - 1)**2 + 26)*sqrt(-(x - 1)**4 - 2*(x - 1)**2 + 3)
/35 + (x - 1)*(-(x - 1)**4 - 2*(x - 1)**2 + 3)**(3/2)/7 - 16*sqrt
(3)*elliptic_e(asin(x - 1), -1/3)/5 + 176*sqrt(3)*elliptic_f(asin
(x - 1), -1/3)/35
```

Mathematica [C] time = 0.899038, size = 278, normalized size = 2.73

$$\frac{5x^9 - 45x^8 + 206x^7 - 602x^6 + 1152x^5 - 1420x^4 + 848x^3 + 352x^2 - 304i\sqrt{2}\sqrt{-\frac{i(x-2)}{\left(\sqrt{3}-i\right)x}}\sqrt{\frac{x^2-2x+4}{x^2}}x^2F\left(\sin^{-1}\left(\frac{\sqrt{\sqrt{3}+i-\frac{4i}{x}}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\cdots}{-i\cdots}\right.}{35\sqrt{-x\left(x^3-4x^2+8x-8\right)}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

```
[Out] (896 - 1056*x + 352*x^2 + 848*x^3 - 1420*x^4 + 1152*x^5 - 602*x^6
 + 206*x^7 - 45*x^8 + 5*x^9 + ((112*I)*Sqrt[2]*(-2 + x)*x*Sqrt[(4
 - 2*x + x^2)/x^2]*EllipticE[ArcSin[Sqrt[I + Sqrt[3] - (4*I)/x]/(
Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-I + Sqrt[3])])/Sqrt[((-I)*(-2 +
x))/((-I + Sqrt[3])*x)] - (304*I)*Sqrt[2]*Sqrt[((-I)*(-2 + x))/((
-I + Sqrt[3])*x)]*x^2*Sqrt[(4 - 2*x + x^2)/x^2]*EllipticF[ArcSin[
Sqrt[I + Sqrt[3] - (4*I)/x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-I +
 Sqrt[3])])/(35*Sqrt[-(x*(-8 + 8*x - 4*x^2 + x^3))])
```

Maple [B] time = 0.184, size = 1050, normalized size = 10.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^4+4*x^3-8*x^2+8*x)^(3/2),x)
```

```
[Out] -1/7*x^5*(-x^4+4*x^3-8*x^2+8*x)^(1/2)+5/7*x^4*(-x^4+4*x^3-8*x^2+8
*x)^(1/2)-66/35*x^3*(-x^4+4*x^3-8*x^2+8*x)^(1/2)+14/5*x^2*(-x^4+4
*x^3-8*x^2+8*x)^(1/2)-32/35*x*(-x^4+4*x^3-8*x^2+8*x)^(1/2)-4/7*(-
x^4+4*x^3-8*x^2+8*x)^(1/2)+32/7*(-1+I*3^(1/2))*((-I*3^(1/2)-1)*x/
(1-I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-I*3^(1/2)-1)/(I*3^(1/2)+1)
/(x-2))^(1/2)*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)/(-I*3^(
1/2)-1)/(-x*(x-2)*(x-I*3^(1/2)-1)*(x-1+I*3^(1/2)))^(1/2)*Elliptic
F(((-I*3^(1/2)-1)*x/(1-I*3^(1/2))/(x-2))^(1/2),((1-I*3^(1/2))*(-1
+I*3^(1/2))/(-I*3^(1/2)-1)/(I*3^(1/2)+1))^(1/2))+64/5*(-1+I*3^(1/
2))*((-I*3^(1/2)-1)*x/(1-I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-I*3^
```

```
(1/2)-1)/(I*3^(1/2)+1)/(x-2))^(1/2)*((x-1+I*3^(1/2))/(1-I*3^(1/2)
)/(x-2))^(1/2)/(-I*3^(1/2)-1)/(-x*(x-2)*(x-I*3^(1/2)-1)*(x-1+I*3^
(1/2)))^(1/2)*(2*EllipticF(((-I*3^(1/2)-1)*x/(1-I*3^(1/2))/(x-2))
^(1/2),((1-I*3^(1/2))*(-1+I*3^(1/2))/(-I*3^(1/2)-1)/(I*3^(1/2)+1)
)^(1/2))-2*EllipticPi(((-I*3^(1/2)-1)*x/(1-I*3^(1/2))/(x-2))^(1/2
),(1-I*3^(1/2))/(-I*3^(1/2)-1),((1-I*3^(1/2))*(-1+I*3^(1/2))/(-I*
3^(1/2)-1)/(I*3^(1/2)+1))^(1/2)))-16/5*(x*(x-I*3^(1/2)-1)*(x-1+I*
3^(1/2))+2*(-1+I*3^(1/2))*((-I*3^(1/2)-1)*x/(1-I*3^(1/2))/(x-2))^
(1/2)*(x-2)^2*((x-I*3^(1/2)-1)/(I*3^(1/2)+1)/(x-2))^(1/2)*((x-1+I
*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*(1/2*(6-2*I*3^(1/2))/(-I*3^(
1/2)-1)*EllipticF(((-I*3^(1/2)-1)*x/(1-I*3^(1/2))/(x-2))^(1/2),((
1-I*3^(1/2))*(-1+I*3^(1/2))/(-I*3^(1/2)-1)/(I*3^(1/2)+1))^(1/2))+
1/2*(-I*3^(1/2)-1)*EllipticE(((-I*3^(1/2)-1)*x/(1-I*3^(1/2))/(x-2
))^(1/2),((1-I*3^(1/2))*(-1+I*3^(1/2))/(-I*3^(1/2)-1)/(I*3^(1/2)+
1))^(1/2))-4/(-I*3^(1/2)-1)*EllipticPi(((-I*3^(1/2)-1)*x/(1-I*3^(
1/2))/(x-2))^(1/2),(-1+I*3^(1/2))/(I*3^(1/2)+1),((1-I*3^(1/2))*(-
1+I*3^(1/2))/(-I*3^(1/2)-1)/(I*3^(1/2)+1))^(1/2))))/(-x*(x-2)*(x-
I*3^(1/2)-1)*(x-1+I*3^(1/2)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-x^4+4x^3-8x^2+8x\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^4 + 4*x^3 - 8*x^2 + 8*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + 8*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-x^4+4x^3-8x^2+8x\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^4 + 4*x^3 - 8*x^2 + 8*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((-x^4 + 4*x^3 - 8*x^2 + 8*x)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-x^4 + 4x^3 - 8x^2 + 8x\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**4+4*x**3-8*x**2+8*x)**(3/2),x)
```

```
[Out] Integral((-x**4 + 4*x**3 - 8*x**2 + 8*x)**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-x^4 + 4\,x^3 - 8\,x^2 + 8\,x\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^4 + 4*x^3 - 8*x^2 + 8*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + 8*x)^(3/2), x)
```

3.609 $\int \sqrt{8x - 8x^2 + 4x^3 - x^4}\,dx$

Optimal. Leaf size=62

$$\frac{1}{3}\sqrt{-(x-1)^4 - 2(x-1)^2 + 3}(x-1) - \frac{4F\left(\sin^{-1}(1-x)\middle|-\frac{1}{3}\right)}{\sqrt{3}} + \frac{2E\left(\sin^{-1}(1-x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}$$

```
[Out] (Sqrt[3 - 2*(-1 + x)^2 - (-1 + x)^4]*(-1 + x))/3 + (2*EllipticE[A
rcSin[1 - x], -1/3])/Sqrt[3] - (4*EllipticF[ArcSin[1 - x], -1/3])
/Sqrt[3]
```

Rubi [A] time = 0.154578, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$

$$-\frac{1}{3}\sqrt{-(1-x)^4 - 2(1-x)^2 + 3}(1-x) - \frac{4F\left(\sin^{-1}(1-x)\middle|-\frac{1}{3}\right)}{\sqrt{3}} + \frac{2E\left(\sin^{-1}(1-x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] -(Sqrt[3 - 2*(1 - x)^2 - (1 - x)^4]*(1 - x))/3 + (2*EllipticE[Arc
Sin[1 - x], -1/3])/Sqrt[3] - (4*EllipticF[ArcSin[1 - x], -1/3])/S
qrt[3]
```

Rubi in Sympy [A] time = 19.9395, size = 58, normalized size = 0.94

$$\frac{(x-1)\sqrt{-(x-1)^4 - 2(x-1)^2 + 3}}{3} - \frac{2\sqrt{3}E\left(\operatorname{asin}(x-1)\middle|-\frac{1}{3}\right)}{3} + \frac{4\sqrt{3}F\left(\operatorname{asin}(x-1)\middle|-\frac{1}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**4+4*x**3-8*x**2+8*x)**(1/2),x)
```

```
[Out] (x - 1)*sqrt(-(x - 1)**4 - 2*(x - 1)**2 + 3)/3 - 2*sqrt(3)*ellipt
ic_e(asin(x - 1), -1/3)/3 + 4*sqrt(3)*elliptic_f(asin(x - 1), -1/
3)/3
```

Mathematica [C] time = 0.886113, size = 256, normalized size = 4.13

$$-\frac{x^5-5x^4+14x^3-24x^2+8i\sqrt{2}\sqrt{-\frac{i(x-2)}{\left(\sqrt{3}-i\right)x}}\sqrt{\frac{x^2-2x+4}{x^2}}x^2F\left(\sin^{-1}\left(\frac{\sqrt{\sqrt{3}+i-\frac{4i}{x}}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-i+\sqrt{3}}\right)-\frac{2i\sqrt{2}(x-2)\sqrt{\frac{x^2-2x+4}{x^2}}xE\left(\sin^{-1}\left(\frac{\sqrt{\sqrt{3}+i-\frac{4i}{x}}}{\sqrt{2}\sqrt[4]{3}}\right)|\right.}{\sqrt{-\frac{i(x-2)}{\left(\sqrt{3}-i\right)x}}}}{3\sqrt{-x\left(x^3-4x^2+8x-8\right)}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[Sqrt[8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] -(-16 + 24*x - 24*x^2 + 14*x^3 - 5*x^4 + x^5 - ((2*I)*Sqrt[2]*(-2
 + x)*x*Sqrt[(4 - 2*x + x^2)/x^2]*EllipticE[ArcSin[Sqrt[I + Sqrt[
3] - (4*I)/x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-I + Sqrt[3])])/Sq
rt[((-I)*(-2 + x))/((-I + Sqrt[3])*x)] + (8*I)*Sqrt[2]*Sqrt[((-I)
*(-2 + x))/((-I + Sqrt[3])*x)]*x^2*Sqrt[(4 - 2*x + x^2)/x^2]*Elli
pticF[ArcSin[Sqrt[I + Sqrt[3] - (4*I)/x]/(Sqrt[2]*3^(1/4))], (2*S
qrt[3])/(-I + Sqrt[3])])/(3*Sqrt[-(x*(-8 + 8*x - 4*x^2 + x^3))])
```

Maple [B] time = 0.039, size = 946, normalized size = 15.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^4+4*x^3-8*x^2+8*x)^(1/2),x)
```

```
[Out] 1/3*x*(-x^4+4*x^3-8*x^2+8*x)^(1/2)-1/3*(-x^4+4*x^3-8*x^2+8*x)^(1/
2)+8/3*(-1+I*3^(1/2))*((-I*3^(1/2)-1)*x/(1-I*3^(1/2))/(x-2))^(1/2
)*(x-2)^2*((x-I*3^(1/2)-1)/(I*3^(1/2)+1)/(x-2))^(1/2)*((x-1+I*3^(
1/2))/(1-I*3^(1/2))/(x-2))^(1/2)/(-I*3^(1/2)-1)/(-x*(x-2)*(x-I*3^
(1/2)-1)*(x-1+I*3^(1/2)))^(1/2)*EllipticF(((-I*3^(1/2)-1)*x/(1-I*
3^(1/2))/(x-2))^(1/2),((1-I*3^(1/2))*(-1+I*3^(1/2))/(-I*3^(1/2)-1
)/(I*3^(1/2)+1))^(1/2))+8/3*(-1+I*3^(1/2))*((-I*3^(1/2)-1)*x/(1-I
*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-I*3^(1/2)-1)/(I*3^(1/2)+1)/(x-
2))^(1/2)*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)/(-I*3^(1/2)
-1)/(-x*(x-2)*(x-I*3^(1/2)-1)*(x-1+I*3^(1/2)))^(1/2)*(2*EllipticF
(((-I*3^(1/2)-1)*x/(1-I*3^(1/2))/(x-2))^(1/2),((1-I*3^(1/2))*(-1+
I*3^(1/2))/(-I*3^(1/2)-1)/(I*3^(1/2)+1))^(1/2))-2*EllipticPi(((-I
*3^(1/2)-1)*x/(1-I*3^(1/2))/(x-2))^(1/2),(1-I*3^(1/2))/(-I*3^(1/2
)-1),((1-I*3^(1/2))*(-1+I*3^(1/2))/(-I*3^(1/2)-1)/(I*3^(1/2)+1))^
(1/2)))-2/3*(x*(x-I*3^(1/2)-1)*(x-1+I*3^(1/2))+2*(-1+I*3^(1/2))*(
(-I*3^(1/2)-1)*x/(1-I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-I*3^(1/2)
-1)/(I*3^(1/2)+1)/(x-2))^(1/2)*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-
2))^(1/2)*(1/2*(6-2*I*3^(1/2))/(-I*3^(1/2)-1)*EllipticF(((-I*3^(1
/2)-1)*x/(1-I*3^(1/2))/(x-2))^(1/2),((1-I*3^(1/2))*(-1+I*3^(1/2))
```

```
/(-I*3^(1/2)-1)/(I*3^(1/2)+1))^(1/2))+1/2*(-I*3^(1/2)-1)*Elliptic
E(((-I*3^(1/2)-1)*x/(1-I*3^(1/2))/(x-2))^(1/2),((1-I*3^(1/2))*(-1
+I*3^(1/2))/(-I*3^(1/2)-1)/(I*3^(1/2)+1))^(1/2))-4/(-I*3^(1/2)-1)
*EllipticPi(((-I*3^(1/2)-1)*x/(1-I*3^(1/2))/(x-2))^(1/2),(-1+I*3^
(1/2))/(I*3^(1/2)+1),((1-I*3^(1/2))*(-1+I*3^(1/2))/(-I*3^(1/2)-1)
/(I*3^(1/2)+1))^(1/2))))/(-x*(x-2)*(x-I*3^(1/2)-1)*(x-1+I*3^(1/2)
))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x^4+4\,x^3-8\,x^2+8\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x),x, algorithm="maxima")

[Out] integrate(sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{-x^4+4\,x^3-8\,x^2+8\,x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x),x, algorithm="fricas")

[Out] integral(sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x^4+4x^3-8x^2+8x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**4+4*x**3-8*x**2+8*x)**(1/2),x)

[Out] Integral(sqrt(-x**4 + 4*x**3 - 8*x**2 + 8*x), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x^4 + 4\,x^3 - 8\,x^2 + 8\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x),x, algorithm="giac")

[Out] integrate(sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x), x)
```

3.610 $\int \frac{1}{\sqrt{8x-8x^2+4x^3-x^4}}\,dx$

Optimal. Leaf size=17

$$-\frac{F\left(\sin^{-1}(1-x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}$$

```
[Out] -(EllipticF[ArcSin[1 - x], -1/3]/Sqrt[3])
```

Rubi [A] time = 0.0397019, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13

$$-\frac{F\left(\sin^{-1}(1-x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[8*x - 8*x^2 + 4*x^3 - x^4],x]

[Out] -(EllipticF[ArcSin[1 - x], -1/3]/Sqrt[3])
```

Rubi in Sympy [A] time = 10.8188, size = 15, normalized size = 0.88

$$\frac{\sqrt{3}F\left(\operatorname{asin}(x-1)\middle|-\frac{1}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-x**4+4*x**3-8*x**2+8*x)**(1/2),x)

[Out] sqrt(3)*elliptic_f(asin(x - 1), -1/3)/3
```

Mathematica [C] time = 0.215032, size = 156, normalized size = 9.18

$$\frac{\sqrt{\frac{4i}{x}+\sqrt{3}-i}\sqrt{-\frac{i(x-2)}{\left(\sqrt{3}-i\right)x}}x\left(-i\sqrt{3}x+x-4\right)F\left(\sin^{-1}\left(\frac{\sqrt{\sqrt{3}+i-\frac{4i}{x}}}{\sqrt{2}\sqrt[4]{3}}\right)\middle|\frac{2\sqrt{3}}{-i+\sqrt{3}}\right)}{\sqrt{2}\sqrt{-\frac{4i}{x}+\sqrt{3}+i}\sqrt{-x\left(x^3-4x^2+8x-8\right)}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/Sqrt[8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] (Sqrt[-I + Sqrt[3] + (4*I)/x]*Sqrt[((-I)*(-2 + x))/((-I + Sqrt[3]
)*x)]*x*(-4 + x - I*Sqrt[3]*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3]
- (4*I)/x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-I + Sqrt[3])])/(Sqrt
[2]*Sqrt[I + Sqrt[3] - (4*I)/x]*Sqrt[-(x*(-8 + 8*x - 4*x^2 + x^3)
)])
```

Maple [B] time = 0.038, size = 200, normalized size = 11.8

$$2\,\frac{\left(-1+i\sqrt{3}\right)(x-2)^2}{\left(-i\sqrt{3}-1\right)\sqrt{-x\,(x-2)\left(x-i\sqrt{3}-1\right)\left(x-1+i\sqrt{3}\right)}}\sqrt{\frac{\left(-i\sqrt{3}-1\right)x}{\left(1-i\sqrt{3}\right)(x-2)}}\sqrt{\frac{x-i\sqrt{3}-1}{\left(i\sqrt{3}+1\right)(x-2)}}\sqrt{\frac{x-1+i\sqrt{3}}{\left(1-i\sqrt{3}\right)(x-2)}}\,\mathit{Ellipi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-x^4+4*x^3-8*x^2+8*x)^(1/2),x)
```

```
[Out] 2*(-1+I*3^(1/2))*((-I*3^(1/2)-1)*x/(1-I*3^(1/2))/(x-2))^(1/2)*(x-
2)^2*((x-I*3^(1/2)-1)/(I*3^(1/2)+1)/(x-2))^(1/2)*((x-1+I*3^(1/2))
/(1-I*3^(1/2))/(x-2))^(1/2)/(-I*3^(1/2)-1)/(-x*(x-2)*(x-I*3^(1/2)
-1)*(x-1+I*3^(1/2)))^(1/2)*EllipticF(((-I*3^(1/2)-1)*x/(1-I*3^(1/
2))/(x-2))^(1/2),((1-I*3^(1/2))*(-1+I*3^(1/2))/(-I*3^(1/2)-1)/(I*
3^(1/2)+1))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^4+4\,x^3-8\,x^2+8\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{-x^4+4\,x^3-8\,x^2+8\,x}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-x^4+4x^3-8x^2+8x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-x**4+4*x**3-8*x**2+8*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-x**4 + 4*x**3 - 8*x**2 + 8*x), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-x^4+4\,x^3-8\,x^2+8\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x), x)
```

3.611 $\int \frac{1}{(8x-8x^2+4x^3-x^4)^{3/2}}\,dx$

Optimal. Leaf size=73

$$\frac{((x-1)^2+5)\,(x-1)}{24\sqrt{-(x-1)^4-2(x-1)^2+3}}-\frac{F\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)}{4\sqrt{3}}+\frac{E\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)}{8\sqrt{3}}$$

```
[Out] ((5 + (-1 + x)^2)*(-1 + x))/(24*Sqrt[3 - 2*(-1 + x)^2 - (-1 + x)^
4]) + EllipticE[ArcSin[1 - x], -1/3]/(8*Sqrt[3]) - EllipticF[ArcS
in[1 - x], -1/3]/(4*Sqrt[3])
```

Rubi [A] time = 0.159718, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261

$$-\frac{((x-1)^2+5)\,(1-x)}{24\sqrt{-(1-x)^4-2(1-x)^2+3}}-\frac{F\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)}{4\sqrt{3}}+\frac{E\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)}{8\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(8*x - 8*x^2 + 4*x^3 - x^4)^(-3/2),x]
```

```
[Out] -((5 + (-1 + x)^2)*(1 - x))/(24*Sqrt[3 - 2*(1 - x)^2 - (1 - x)^4]
) + EllipticE[ArcSin[1 - x], -1/3]/(8*Sqrt[3]) - EllipticF[ArcSin
[1 - x], -1/3]/(4*Sqrt[3])
```

Rubi in Sympy [A] time = 19.9616, size = 63, normalized size = 0.86

$$\frac{(x-1)\left(2\,(x-1)^2+10\right)}{48\sqrt{-\,(x-1)^4-2\,(x-1)^2+3}}-\frac{\sqrt{3}E\left(\operatorname{asin}(x-1)\middle|-\frac{1}{3}\right)}{24}+\frac{\sqrt{3}F\left(\operatorname{asin}(x-1)\middle|-\frac{1}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-x**4+4*x**3-8*x**2+8*x)**(3/2),x)
```

```
[Out] (x - 1)*(2*(x - 1)**2 + 10)/(48*sqrt(-(x - 1)**4 - 2*(x - 1)**2 +
 3)) - sqrt(3)*elliptic_e(asin(x - 1), -1/3)/24 + sqrt(3)*ellipti
c_f(asin(x - 1), -1/3)/12
```

Mathematica [C] time = 1.35137, size = 261, normalized size = 3.58

$$\frac{\sqrt{-x\left(x^3-4x^2+8x-8\right)}\left(\frac{\sqrt{2}\left(\sqrt{3}-i\right)\sqrt{-\frac{i(x-2)}{\left(\sqrt{3}-i\right)x}}E\left(\sin^{-1}\left(\frac{\sqrt{\sqrt{3}+i-\frac{4i}{x}}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-i+\sqrt{3}}\right)}{\sqrt{\frac{x^2-2x+4}{x^2}}}-\frac{x^2-4i\sqrt{2}\sqrt{-\frac{i(x-2)}{\left(\sqrt{3}-i\right)x}}\sqrt{\frac{x^2-2x+4}{x^2}}x^2F\left(\sin^{-1}\left(\frac{\sqrt{\sqrt{3}+i-\frac{4i}{x}}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-i+\sqrt{3}}\right)+2}{x^2-2x+4}\right)}{24(x-2)x}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(8*x - 8*x^2 + 4*x^3 - x^4)^(-3/2),x]
```

```
[Out] (Sqrt[-(x*(-8 + 8*x - 4*x^2 + x^3))]*((Sqrt[2]*(-I + Sqrt[3])*Sqr
t[((-I)*(-2 + x))/((-I + Sqrt[3])*x)]*EllipticE[ArcSin[Sqrt[I + S
qrt[3] - (4*I)/x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-I + Sqrt[3])]
)/Sqrt[(4 - 2*x + x^2)/x^2] - (2 + x^2 - (4*I)*Sqrt[2]*Sqrt[((-I)
*(-2 + x))/((-I + Sqrt[3])*x)]*x^2*Sqrt[(4 - 2*x + x^2)/x^2]*Elli
pticF[ArcSin[Sqrt[I + Sqrt[3] - (4*I)/x]/(Sqrt[2]*3^(1/4))], (2*S
qrt[3])/(-I + Sqrt[3])])/(4 - 2*x + x^2)))/(24*(-2 + x)*x)
```

Maple [B] time = 0.051, size = 963, normalized size = 13.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-x^4+4*x^3-8*x^2+8*x)^(3/2),x)
```

```
[Out] -1/32*(-x^3+4*x^2-8*x+8)/(x*(-x^3+4*x^2-8*x+8))^(1/2)+2*x*(1/24+1
/192*x^2)/(-x*(x^3-4*x^2+8*x-8))^(1/2)+1/6*(-1+I*3^(1/2))*((-I*3^
(1/2)-1)*x/(1-I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-I*3^(1/2)-1)/(I
*3^(1/2)+1)/(x-2))^(1/2)*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1
/2)/(-I*3^(1/2)-1)/(-x*(x-2)*(x-I*3^(1/2)-1)*(x-1+I*3^(1/2)))^(1/
2)*EllipticF(((-I*3^(1/2)-1)*x/(1-I*3^(1/2))/(x-2))^(1/2),((1-I*3
^(1/2))*(-1+I*3^(1/2))/(-I*3^(1/2)-1)/(I*3^(1/2)+1))^(1/2))+1/6*(
-1+I*3^(1/2))*((-I*3^(1/2)-1)*x/(1-I*3^(1/2))/(x-2))^(1/2)*(x-2)^
2*((x-I*3^(1/2)-1)/(I*3^(1/2)+1)/(x-2))^(1/2)*((x-1+I*3^(1/2))/(1
-I*3^(1/2))/(x-2))^(1/2)/(-I*3^(1/2)-1)/(-x*(x-2)*(x-I*3^(1/2)-1)
*(x-1+I*3^(1/2)))^(1/2)*(2*EllipticF(((-I*3^(1/2)-1)*x/(1-I*3^(1/
2))/(x-2))^(1/2),((1-I*3^(1/2))*(-1+I*3^(1/2))/(-I*3^(1/2)-1)/(I*
3^(1/2)+1))^(1/2))-2*EllipticPi(((-I*3^(1/2)-1)*x/(1-I*3^(1/2))/(
x-2))^(1/2),(1-I*3^(1/2))/(-I*3^(1/2)-1),((1-I*3^(1/2))*(-1+I*3^(
1/2))/(-I*3^(1/2)-1)/(I*3^(1/2)+1))^(1/2)))-1/24*(x*(x-I*3^(1/2)-
1)*(x-1+I*3^(1/2))+2*(-1+I*3^(1/2))*((-I*3^(1/2)-1)*x/(1-I*3^(1/2
))/(x-2))^(1/2)*(x-2)^2*((x-I*3^(1/2)-1)/(I*3^(1/2)+1)/(x-2))^(1/
2)*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*(1/2*(6-2*I*3^(1/2
))/(-I*3^(1/2)-1)*EllipticF(((-I*3^(1/2)-1)*x/(1-I*3^(1/2))/(x-2)
```

```
)^(1/2),((1-I*3^(1/2))*(-1+I*3^(1/2))/(-I*3^(1/2)-1)/(I*3^(1/2)+1
))^(1/2))+1/2*(-I*3^(1/2)-1)*EllipticE(((-I*3^(1/2)-1)*x/(1-I*3^(
1/2))/(x-2))^(1/2),((1-I*3^(1/2))*(-1+I*3^(1/2))/(-I*3^(1/2)-1)/(
I*3^(1/2)+1))^(1/2))-4/(-I*3^(1/2)-1)*EllipticPi(((-I*3^(1/2)-1)*
x/(1-I*3^(1/2))/(x-2))^(1/2),(-1+I*3^(1/2))/(I*3^(1/2)+1),((1-I*3
^(1/2))*(-1+I*3^(1/2))/(-I*3^(1/2)-1)/(I*3^(1/2)+1))^(1/2))))/(-x
*(x-2)*(x-I*3^(1/2)-1)*(x-1+I*3^(1/2)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-x^4+4\,x^3-8\,x^2+8\,x)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^4 + 4*x^3 - 8*x^2 + 8*x)^(-3/2),x, algorithm="maxima")
```

```
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + 8*x)^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{1}{(x^4-4\,x^3+8\,x^2-8\,x)\sqrt{-x^4+4\,x^3-8\,x^2+8\,x}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^4 + 4*x^3 - 8*x^2 + 8*x)^(-3/2),x, algorithm="fricas")
```

```
[Out] integral(-1/((x^4 - 4*x^3 + 8*x^2 - 8*x)*sqrt(-x^4 + 4*x^3 - 8*x^
2 + 8*x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-x^4+4x^3-8x^2+8x)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-x**4+4*x**3-8*x**2+8*x)**(3/2),x)
```

```
[Out] Integral((-x**4 + 4*x**3 - 8*x**2 + 8*x)**(-3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-x^4 + 4\,x^3 - 8\,x^2 + 8\,x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^4 + 4*x^3 - 8*x^2 + 8*x)^(-3/2),x, algorithm="giac")
```

```
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + 8*x)^(-3/2), x)
```

3.612 $\int \frac{1}{(8x-8x^2+4x^3-x^4)^{5/2}}\,dx$

Optimal. Leaf size=109

$$\frac{(7(x-1)^2+26)\,(x-1)}{432\sqrt{-(x-1)^4-2(x-1)^2+3}}+\frac{((x-1)^2+5)\,(x-1)}{72\,(-(x-1)^4-2(x-1)^2+3)^{3/2}}-\frac{11F\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)}{144\sqrt{3}}+\frac{7E\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)}{144\sqrt{3}}$$

```
[Out] ((5 + (-1 + x)^2)*(-1 + x))/(72*(3 - 2*(-1 + x)^2 - (-1 + x)^4)^(
3/2)) + ((26 + 7*(-1 + x)^2)*(-1 + x))/(432*Sqrt[3 - 2*(-1 + x)^2
 - (-1 + x)^4]) + (7*EllipticE[ArcSin[1 - x], -1/3])/(144*Sqrt[3]
) - (11*EllipticF[ArcSin[1 - x], -1/3])/(144*Sqrt[3])
```

Rubi [A] time = 0.20915, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304

$$-\frac{((x-1)^2+5)\,(1-x)}{72\,(-(1-x)^4-2(1-x)^2+3)^{3/2}}-\frac{(7(1-x)^2+26)\,(1-x)}{432\sqrt{-(1-x)^4-2(1-x)^2+3}}-\frac{11F\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)}{144\sqrt{3}}+\frac{7E\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)}{144\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(8*x - 8*x^2 + 4*x^3 - x^4)^(-5/2),x]
```

```
[Out] -((26 + 7*(1 - x)^2)*(1 - x))/(432*Sqrt[3 - 2*(1 - x)^2 - (1 - x)
^4]) - ((5 + (-1 + x)^2)*(1 - x))/(72*(3 - 2*(1 - x)^2 - (1 - x)^
4)^(3/2)) + (7*EllipticE[ArcSin[1 - x], -1/3])/(144*Sqrt[3]) - (1
1*EllipticF[ArcSin[1 - x], -1/3])/(144*Sqrt[3])
```

Rubi in Sympy [A] time = 22.7036, size = 97, normalized size = 0.89

$$\frac{(x-1)\left(2\,(x-1)^2+10\right)}{144\left(-(x-1)^4-2\,(x-1)^2+3\right)^{\frac{3}{2}}}+\frac{(x-1)\left(112\,(x-1)^2+416\right)}{6912\sqrt{-(x-1)^4-2\,(x-1)^2+3}}-\frac{7\sqrt{3}E\left(\operatorname{asin}(x-1)\middle|-\frac{1}{3}\right)}{432}+\frac{11\sqrt{3}F\left(\operatorname{asin}(x-1)\middle|-\frac{1}{3}\right)}{432}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-x**4+4*x**3-8*x**2+8*x)**(5/2),x)
```

```
[Out] (x - 1)*(2*(x - 1)**2 + 10)/(144*(-(x - 1)**4 - 2*(x - 1)**2 + 3)
**(3/2)) + (x - 1)*(112*(x - 1)**2 + 416)/(6912*sqrt(-(x - 1)**4
- 2*(x - 1)**2 + 3)) - 7*sqrt(3)*elliptic_e(asin(x - 1), -1/3)/43
2 + 11*sqrt(3)*elliptic_f(asin(x - 1), -1/3)/432
```

Mathematica [C]   time = 1.7223, size = 298, normalized size = 2.73

$$\frac{\frac{7i\sqrt{2}(x-2)\sqrt{\frac{x^2-2x+4}{x^2}}x^2E\left(\sin^{-1}\left(\frac{\sqrt{\sqrt{3}+i-\frac{4i}{x}}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-i+\sqrt{3}}\right)}{\sqrt{-\frac{i(x-2)}{\left(\sqrt{3}-i\right)x}}}+\frac{7x^6-37x^5+115x^4-226x^3+274x^2-19i\sqrt{2}\sqrt{-\frac{i(x-2)}{\left(\sqrt{3}-i\right)x}}\sqrt{\frac{x^2-2x+4}{x^2}}\left(x^3-4x^2+8x-8\right)x^3F\left(\sin^{-1}\left(\frac{\sqrt{\sqrt{3}+i-\frac{4i}{x}}}{\sqrt{2}\sqrt[4]{3}}\right)| \right.}{x^3-4x^2+8x-8}}{432x\sqrt{-x\left(x^3-4x^2+8x-8\right)}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(8*x - 8*x^2 + 4*x^3 - x^4)^(-5/2),x]
```

```
[Out] (((7*I)*Sqrt[2]*(-2 + x)*x^2*Sqrt[(4 - 2*x + x^2)/x^2]*EllipticE[
ArcSin[Sqrt[I + Sqrt[3] - (4*I)/x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3]
)/(-I + Sqrt[3])])/Sqrt[((-I)*(-2 + x))/((-I + Sqrt[3])*x)] + (36
 - 232*x + 274*x^2 - 226*x^3 + 115*x^4 - 37*x^5 + 7*x^6 - (19*I)*
Sqrt[2]*Sqrt[((-I)*(-2 + x))/((-I + Sqrt[3])*x)]*x^3*Sqrt[(4 - 2*
x + x^2)/x^2]*(-8 + 8*x - 4*x^2 + x^3)*EllipticF[ArcSin[Sqrt[I +
Sqrt[3] - (4*I)/x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-I + Sqrt[3])
])/(-8 + 8*x - 4*x^2 + x^3))/(432*x*Sqrt[-(x*(-8 + 8*x - 4*x^2 +
x^3))])
```

Maple [B]   time = 0.052, size = 1039, normalized size = 9.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-x^4+4*x^3-8*x^2+8*x)^(5/2),x)
```

```
[Out] -1/768*(-x^4+4*x^3-8*x^2+8*x)^(1/2)/x^2-1/96*(-x^3+4*x^2-8*x+8)/(
x*(-x^3+4*x^2-8*x+8))^(1/2)+(1/36+1/288*x^2-1/96*x)*(-x^4+4*x^3-8
*x^2+8*x)^(1/2)/(x^3-4*x^2+8*x-8)^2+2*x*(53/3456+5/1728*x^2-19/46
08*x)/(-x*(x^3-4*x^2+8*x-8))^(1/2)+5/216*(-1+I*3^(1/2))*((-I*3^(1
/2)-1)*x/(1-I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-I*3^(1/2)-1)/(I*3
^(1/2)+1)/(x-2))^(1/2)*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2
)/(-I*3^(1/2)-1)/(-x*(x-2)*(x-I*3^(1/2)-1)*(x-1+I*3^(1/2)))^(1/2)
```

```
*EllipticF(((-I*3^(1/2)-1)*x/(1-I*3^(1/2))/(x-2))^(1/2),((1-I*3^(
1/2))*(-1+I*3^(1/2))/(-I*3^(1/2)-1)/(I*3^(1/2)+1))^(1/2))+7/108*(
-1+I*3^(1/2))*((-I*3^(1/2)-1)*x/(1-I*3^(1/2))/(x-2))^(1/2)*(x-2)^
2*((x-I*3^(1/2)-1)/(I*3^(1/2)+1)/(x-2))^(1/2)*((x-1+I*3^(1/2))/(1
-I*3^(1/2))/(x-2))^(1/2)/(-I*3^(1/2)-1)/(-x*(x-2)*(x-I*3^(1/2)-1)
*(x-1+I*3^(1/2)))^(1/2)*(2*EllipticF(((-I*3^(1/2)-1)*x/(1-I*3^(1/
2))/(x-2))^(1/2),((1-I*3^(1/2))*(-1+I*3^(1/2))/(-I*3^(1/2)-1)/(I*
3^(1/2)+1))^(1/2))-2*EllipticPi(((-I*3^(1/2)-1)*x/(1-I*3^(1/2))/(
x-2))^(1/2),(1-I*3^(1/2))/(-I*3^(1/2)-1),((1-I*3^(1/2))*(-1+I*3^(
1/2))/(-I*3^(1/2)-1)/(I*3^(1/2)+1))^(1/2)))-7/432*(x*(x-I*3^(1/2)
-1)*(x-1+I*3^(1/2))+2*(-1+I*3^(1/2))*((-I*3^(1/2)-1)*x/(1-I*3^(1/
2))/(x-2))^(1/2)*(x-2)^2*((x-I*3^(1/2)-1)/(I*3^(1/2)+1)/(x-2))^(1
/2)*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*(1/2*(6-2*I*3^(1/
2))/(-I*3^(1/2)-1)*EllipticF(((-I*3^(1/2)-1)*x/(1-I*3^(1/2))/(x-2
))^(1/2),((1-I*3^(1/2))*(-1+I*3^(1/2))/(-I*3^(1/2)-1)/(I*3^(1/2)+
1))^(1/2))+1/2*(-I*3^(1/2)-1)*EllipticE(((-I*3^(1/2)-1)*x/(1-I*3^
(1/2))/(x-2))^(1/2),((1-I*3^(1/2))*(-1+I*3^(1/2))/(-I*3^(1/2)-1)/
(I*3^(1/2)+1))^(1/2))-4/(-I*3^(1/2)-1)*EllipticPi(((-I*3^(1/2)-1)
*x/(1-I*3^(1/2))/(x-2))^(1/2),(-1+I*3^(1/2))/(I*3^(1/2)+1),((1-I*
3^(1/2))*(-1+I*3^(1/2))/(-I*3^(1/2)-1)/(I*3^(1/2)+1))^(1/2))))/(-
x*(x-2)*(x-I*3^(1/2)-1)*(x-1+I*3^(1/2)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-x^4+4\,x^3-8\,x^2+8\,x\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^4 + 4*x^3 - 8*x^2 + 8*x)^(-5/2),x, algorithm="maxima")
```

```
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + 8*x)^(-5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\left(x^8-8\,x^7+32\,x^6-80\,x^5+128\,x^4-128\,x^3+64\,x^2\right)\sqrt{-x^4+4\,x^3-8\,x^2+8\,x}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^4 + 4*x^3 - 8*x^2 + 8*x)^(-5/2),x, algorithm="fricas")
```

```
[Out] integral(1/((x^8 - 8*x^7 + 32*x^6 - 80*x^5 + 128*x^4 - 128*x^3 +
64*x^2)*sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-x^4+4x^3-8x^2+8x)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-x**4+4*x**3-8*x**2+8*x)**(5/2),x)
```

```
[Out] Integral((-x**4 + 4*x**3 - 8*x**2 + 8*x)**(-5/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-x^4+4\,x^3-8\,x^2+8\,x)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^4 + 4*x^3 - 8*x^2 + 8*x)^(-5/2),x, algorithm="giac")
```

```
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + 8*x)^(-5/2), x)
```

3.613 $\int \left((2-x)x\left(4-2x+x^2\right)\right)^{3/2}\,dx$

Optimal. Leaf size=102

$$\frac{1}{7}(x-1)\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}+\frac{2}{35}\left(13-3(x-1)^2\right)(x-1)\sqrt{-(x-1)^4-2(x-1)^2+3}-\frac{176}{35}\sqrt{3}F\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)+\frac{16}{5}\sqrt{3}E\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)$$

```
[Out] (2*(13 - 3*(-1 + x)^2)*Sqrt[3 - 2*(-1 + x)^2 - (-1 + x)^4]*(-1 +
x))/35 + ((3 - 2*(-1 + x)^2 - (-1 + x)^4)^(3/2)*(-1 + x))/7 + (16
*Sqrt[3]*EllipticE[ArcSin[1 - x], -1/3])/5 - (176*Sqrt[3]*Ellipti
cF[ArcSin[1 - x], -1/3])/35
```

Rubi [A] time = 0.197234, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368

$$-\frac{1}{7}(1-x)\left(-(1-x)^4-2(1-x)^2+3\right)^{3/2}-\frac{2}{35}\left(13-3(1-x)^2\right)(1-x)\sqrt{-(1-x)^4-2(1-x)^2+3}-\frac{176}{35}\sqrt{3}F\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)+\frac{16}{5}\sqrt{3}E\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[((2 - x)*x*(4 - 2*x + x^2))^(3/2),x]
```

```
[Out] (-2*(13 - 3*(1 - x)^2)*Sqrt[3 - 2*(1 - x)^2 - (1 - x)^4]*(1 - x))
/35 - ((3 - 2*(1 - x)^2 - (1 - x)^4)^(3/2)*(1 - x))/7 + (16*Sqrt[
3]*EllipticE[ArcSin[1 - x], -1/3])/5 - (176*Sqrt[3]*EllipticF[Arc
Sin[1 - x], -1/3])/35
```

Rubi in Sympy [A] time = 15.8143, size = 88, normalized size = 0.86

$$\frac{(x-1)\left(-6(x-1)^2+26\right)\sqrt{-(x-1)^4-2(x-1)^2+3}}{35}+\frac{(x-1)\left(-(x-1)^4-2(x-1)^2+3\right)^{\frac{3}{2}}}{7}-\frac{16\sqrt{3}E\left(\operatorname{asin}(x-1)\middle|-\frac{1}{3}\right)}{5}+\frac{176\sqrt{3}F\left(\operatorname{asin}(x-1)\middle|-\frac{1}{3}\right)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((2-x)*x*(x**2-2*x+4))**(3/2),x)
```

```
[Out] (x - 1)*(-6*(x - 1)**2 + 26)*sqrt(-(x - 1)**4 - 2*(x - 1)**2 + 3)
/35 + (x - 1)*(-(x - 1)**4 - 2*(x - 1)**2 + 3)**(3/2)/7 - 16*sqrt
(3)*elliptic_e(asin(x - 1), -1/3)/5 + 176*sqrt(3)*elliptic_f(asin
(x - 1), -1/3)/35
```

Mathematica [C] time = 1.31495, size = 278, normalized size = 2.73

$$\frac{\sqrt{-x\left(x^3-4x^2+8x-8\right)}\left(\sqrt{\frac{x^2-2x+4}{x^2}}\left(-5x^7+35x^6-116x^5+230x^4-228x^3+44x^2+152x-224\right)+304i\sqrt{2}\sqrt{-\frac{i(x-2)}{\left(\sqrt{3}-i\right)x}}F\left(\right.\right.}{35(x-2)x\sqrt{\frac{x^2-2x+4}{x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[((2 - x)*x*(4 - 2*x + x^2))^(3/2),x]
```

```
[Out] (Sqrt[-(x*(-8 + 8*x - 4*x^2 + x^3))]*(Sqrt[(4 - 2*x + x^2)/x^2]*(
-224 + 152*x + 44*x^2 - 228*x^3 + 230*x^4 - 116*x^5 + 35*x^6 - 5*
x^7) + 112*Sqrt[2]*(-I + Sqrt[3])*Sqrt[((-I)*(-2 + x))/((-I + Sqr
t[3])*x)]*EllipticE[ArcSin[Sqrt[I + Sqrt[3] - (4*I)/x]/(Sqrt[2]*3
^(1/4))], (2*Sqrt[3])/(-I + Sqrt[3])] + (304*I)*Sqrt[2]*Sqrt[((-I
)*(-2 + x))/((-I + Sqrt[3])*x)]*EllipticF[ArcSin[Sqrt[I + Sqrt[3]
 - (4*I)/x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-I + Sqrt[3])]))/(35
*(-2 + x)*x*Sqrt[(4 - 2*x + x^2)/x^2])
```

Maple [B] time = 0.047, size = 1050, normalized size = 10.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((2-x)*x*(x^2-2*x+4))^(3/2),x)
```

```
[Out] -1/7*x^5*(-x^4+4*x^3-8*x^2+8*x)^(1/2)+5/7*x^4*(-x^4+4*x^3-8*x^2+8
*x)^(1/2)-66/35*x^3*(-x^4+4*x^3-8*x^2+8*x)^(1/2)+14/5*x^2*(-x^4+4
*x^3-8*x^2+8*x)^(1/2)-32/35*x*(-x^4+4*x^3-8*x^2+8*x)^(1/2)-4/7*(-
x^4+4*x^3-8*x^2+8*x)^(1/2)+32/7*(-1+I*3^(1/2))*((-I*3^(1/2)-1)*x/
(1-I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-I*3^(1/2)-1)/(I*3^(1/2)+1)
/(x-2))^(1/2)*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)/(-I*3^(
1/2)-1)/(-x*(x-2)*(x-I*3^(1/2)-1)*(x-1+I*3^(1/2)))^(1/2)*Elliptic
F(((-I*3^(1/2)-1)*x/(1-I*3^(1/2))/(x-2))^(1/2),((1-I*3^(1/2))*(-1
+I*3^(1/2))/(-I*3^(1/2)-1)/(I*3^(1/2)+1))^(1/2))+64/5*(-1+I*3^(1/
2))*((-I*3^(1/2)-1)*x/(1-I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-I*3^
(1/2)-1)/(I*3^(1/2)+1)/(x-2))^(1/2)*((x-1+I*3^(1/2))/(1-I*3^(1/2)
```

```
)/(x-2))^(1/2)/(-I*3^(1/2)-1)/(-x*(x-2)*(x-I*3^(1/2)-1)*(x-1+I*3^
(1/2)))^(1/2)*(2*EllipticF(((-I*3^(1/2)-1)*x/(1-I*3^(1/2))/(x-2))
^(1/2),((1-I*3^(1/2))*(-1+I*3^(1/2))/(-I*3^(1/2)-1)/(I*3^(1/2)+1)
)^(1/2))-2*EllipticPi(((-I*3^(1/2)-1)*x/(1-I*3^(1/2))/(x-2))^(1/2
),(1-I*3^(1/2))/(-I*3^(1/2)-1),((1-I*3^(1/2))*(-1+I*3^(1/2))/(-I*
3^(1/2)-1)/(I*3^(1/2)+1))^(1/2)))-16/5*(x*(x-I*3^(1/2)-1)*(x-1+I*
3^(1/2))+2*(-1+I*3^(1/2))*((-I*3^(1/2)-1)*x/(1-I*3^(1/2))/(x-2))^
(1/2)*(x-2)^2*((x-I*3^(1/2)-1)/(I*3^(1/2)+1)/(x-2))^(1/2)*((x-1+I
*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*(1/2*(6-2*I*3^(1/2))/(-I*3^(
1/2)-1)*EllipticF(((-I*3^(1/2)-1)*x/(1-I*3^(1/2))/(x-2))^(1/2),((
1-I*3^(1/2))*(-1+I*3^(1/2))/(-I*3^(1/2)-1)/(I*3^(1/2)+1))^(1/2))+
1/2*(-I*3^(1/2)-1)*EllipticE(((-I*3^(1/2)-1)*x/(1-I*3^(1/2))/(x-2
))^(1/2),((1-I*3^(1/2))*(-1+I*3^(1/2))/(-I*3^(1/2)-1)/(I*3^(1/2)+
1))^(1/2))-4/(-I*3^(1/2)-1)*EllipticPi(((-I*3^(1/2)-1)*x/(1-I*3^(
1/2))/(x-2))^(1/2),(-1+I*3^(1/2))/(I*3^(1/2)+1),((1-I*3^(1/2))*(-
1+I*3^(1/2))/(-I*3^(1/2)-1)/(I*3^(1/2)+1))^(1/2))))/(-x*(x-2)*(x-
I*3^(1/2)-1)*(x-1+I*3^(1/2)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-\left(x^2-2\,x+4\right)(x-2)x\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-(x^2 - 2*x + 4)*(x - 2)*x)^(3/2),x, algorithm="maxima")

[Out] integrate((-(x^2 - 2*x + 4)*(x - 2)*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-x^4+4\,x^3-8\,x^2+8\,x\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-(x^2 - 2*x + 4)*(x - 2)*x)^(3/2),x, algorithm="fricas")

[Out] integral((-x^4 + 4*x^3 - 8*x^2 + 8*x)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((2-x)*x*(x**2-2*x+4))**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-\left(x^2-2x+4\right)(x-2)x\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-(x^2 - 2*x + 4)*(x - 2)*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-(x^2 - 2*x + 4)*(x - 2)*x)^(3/2), x)
```

3.614 $\int \sqrt{(2-x)x\left(4-2x+x^2\right)}\,dx$

Optimal. Leaf size=62

$$\frac{1}{3}\sqrt{-(x-1)^4-2(x-1)^2+3}(x-1)-\frac{4F\left(\sin^{-1}(1-x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}+\frac{2E\left(\sin^{-1}(1-x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}$$

```
[Out] (Sqrt[3 - 2*(-1 + x)^2 - (-1 + x)^4]*(-1 + x))/3 + (2*EllipticE[A
rcSin[1 - x], -1/3]])/Sqrt[3] - (4*EllipticF[ArcSin[1 - x], -1/3]])
/Sqrt[3]
```

Rubi [A] time = 0.150525, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316

$$-\frac{1}{3}\sqrt{-(1-x)^4-2(1-x)^2+3}(1-x)-\frac{4F\left(\sin^{-1}(1-x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}+\frac{2E\left(\sin^{-1}(1-x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[(2 - x)*x*(4 - 2*x + x^2)],x]
```

```
[Out] -(Sqrt[3 - 2*(1 - x)^2 - (1 - x)^4]*(1 - x))/3 + (2*EllipticE[Arc
Sin[1 - x], -1/3]])/Sqrt[3] - (4*EllipticF[ArcSin[1 - x], -1/3]])/S
qrt[3]
```

Rubi in Sympy [A] time = 12.434, size = 58, normalized size = 0.94

$$\frac{(x-1)\sqrt{-(x-1)^4-2(x-1)^2+3}}{3}-\frac{2\sqrt{3}E\left(\operatorname{asin}(x-1)\middle|-\frac{1}{3}\right)}{3}+\frac{4\sqrt{3}F\left(\operatorname{asin}(x-1)\middle|-\frac{1}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((2-x)*x*(x**2-2*x+4))**(1/2),x)
```

```
[Out] (x - 1)*sqrt(-(x - 1)**4 - 2*(x - 1)**2 + 3)/3 - 2*sqrt(3)*ellipt
ic_e(asin(x - 1), -1/3)/3 + 4*sqrt(3)*elliptic_f(asin(x - 1), -1/
3)/3
```

Mathematica [C] time = 0.939238, size = 256, normalized size = 4.13

$$\frac{\sqrt{-x\left(x^3-4x^2+8x-8\right)}\left(\sqrt{\frac{x^2-2x+4}{x^2}}\left(x^3-3x^2+4x-4\right)+8i\sqrt{2}\sqrt{-\frac{i(x-2)}{\left(\sqrt{3}-i\right)x}}F\left(\sin^{-1}\left(\frac{\sqrt{\sqrt{3}+i-\frac{4i}{x}}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-i+\sqrt{3}}\right)+2\sqrt{2}\left(\sqrt{3}-i\right)\right.}{3(x-2)x\sqrt{\frac{x^2-2x+4}{x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[Sqrt[(2 - x)*x*(4 - 2*x + x^2)],x]
```

```
[Out] (Sqrt[-(x*(-8 + 8*x - 4*x^2 + x^3))]*(Sqrt[(4 - 2*x + x^2)/x^2]*(
-4 + 4*x - 3*x^2 + x^3) + 2*Sqrt[2]*(-I + Sqrt[3])*Sqrt[((-I)*(-2
 + x))/((-I + Sqrt[3])*x)]*EllipticE[ArcSin[Sqrt[I + Sqrt[3] - (4
*I)/x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-I + Sqrt[3])] + (8*I)*Sq
rt[2]*Sqrt[((-I)*(-2 + x))/((-I + Sqrt[3])*x)]*EllipticF[ArcSin[S
qrt[I + Sqrt[3] - (4*I)/x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-I +
Sqrt[3])]))/(3*(-2 + x)*x*Sqrt[(4 - 2*x + x^2)/x^2])
```

Maple [B] time = 0.039, size = 946, normalized size = 15.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((2-x)*x*(x^2-2*x+4))^(1/2),x)
```

```
[Out] 1/3*x*(-x^4+4*x^3-8*x^2+8*x)^(1/2)-1/3*(-x^4+4*x^3-8*x^2+8*x)^(1/
2)+8/3*(-1+I*3^(1/2))*((-I*3^(1/2)-1)*x/(1-I*3^(1/2))/(x-2))^(1/2
)*(x-2)^2*((x-I*3^(1/2)-1)/(I*3^(1/2)+1)/(x-2))^(1/2)*((x-1+I*3^(
1/2))/(1-I*3^(1/2))/(x-2))^(1/2)/(-I*3^(1/2)-1)/(-x*(x-2)*(x-I*3^
(1/2)-1)*(x-1+I*3^(1/2)))^(1/2)*EllipticF(((-I*3^(1/2)-1)*x/(1-I*
3^(1/2))/(x-2))^(1/2),((1-I*3^(1/2))*(-1+I*3^(1/2))/(-I*3^(1/2)-1
)/(I*3^(1/2)+1))^(1/2))+8/3*(-1+I*3^(1/2))*((-I*3^(1/2)-1)*x/(1-I
*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-I*3^(1/2)-1)/(I*3^(1/2)+1)/(x-
2))^(1/2)*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)/(-I*3^(1/2)
-1)/(-x*(x-2)*(x-I*3^(1/2)-1)*(x-1+I*3^(1/2)))^(1/2)*(2*EllipticF
(((-I*3^(1/2)-1)*x/(1-I*3^(1/2))/(x-2))^(1/2),((1-I*3^(1/2))*(-1+
I*3^(1/2))/(-I*3^(1/2)-1)/(I*3^(1/2)+1))^(1/2))-2*EllipticPi(((-I
*3^(1/2)-1)*x/(1-I*3^(1/2))/(x-2))^(1/2),(1-I*3^(1/2))/(-I*3^(1/2
)-1),((1-I*3^(1/2))*(-1+I*3^(1/2))/(-I*3^(1/2)-1)/(I*3^(1/2)+1))^
(1/2)))-2/3*(x*(x-I*3^(1/2)-1)*(x-1+I*3^(1/2))+2*(-1+I*3^(1/2))*(
(-I*3^(1/2)-1)*x/(1-I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-I*3^(1/2)
-1)/(I*3^(1/2)+1)/(x-2))^(1/2)*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-
2))^(1/2)*(1/2*(6-2*I*3^(1/2))/(-I*3^(1/2)-1)*EllipticF(((-I*3^(1
/2)-1)*x/(1-I*3^(1/2))/(x-2))^(1/2),((1-I*3^(1/2))*(-1+I*3^(1/2))
```

```
/(-I*3^(1/2)-1)/(I*3^(1/2)+1))^(1/2))+1/2*(-I*3^(1/2)-1)*Elliptic
E(((-I*3^(1/2)-1)*x/(1-I*3^(1/2))/(x-2))^(1/2),((1-I*3^(1/2))*(-1
+I*3^(1/2))/(-I*3^(1/2)-1)/(I*3^(1/2)+1))^(1/2))-4/(-I*3^(1/2)-1)
*EllipticPi(((-I*3^(1/2)-1)*x/(1-I*3^(1/2))/(x-2))^(1/2),(-1+I*3^
(1/2))/(I*3^(1/2)+1),((1-I*3^(1/2))*(-1+I*3^(1/2))/(-I*3^(1/2)-1)
/(I*3^(1/2)+1))^(1/2))))/(-x*(x-2)*(x-I*3^(1/2)-1)*(x-1+I*3^(1/2)
))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-(x^2-2\,x+4)(x-2)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-(x^2 - 2*x + 4)*(x - 2)*x),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-(x^2 - 2*x + 4)*(x - 2)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{-x^4+4\,x^3-8\,x^2+8\,x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-(x^2 - 2*x + 4)*(x - 2)*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x\,(-x+2)\,(x^2-2x+4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((2-x)*x*(x**2-2*x+4))**(1/2),x)
```

```
[Out] Integral(sqrt(x*(-x + 2)*(x**2 - 2*x + 4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-(x^2 - 2x + 4)(x - 2)x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-(x^2 - 2*x + 4)*(x - 2)*x),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-(x^2 - 2*x + 4)*(x - 2)*x), x)
```

3.615 $\int \frac{1}{\sqrt{(2-x)x(4-2x+x^2)}} \, dx$

Optimal. Leaf size=17

$$-\frac{F\left(\sin^{-1}(1-x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}$$

```
[Out] -(EllipticF[ArcSin[1 - x], -1/3]/Sqrt[3])
```

Rubi [A] time = 0.0360589, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$-\frac{F\left(\sin^{-1}(1-x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[(2 - x)*x*(4 - 2*x + x^2)],x]

[Out] -(EllipticF[ArcSin[1 - x], -1/3]/Sqrt[3])
```

Rubi in Sympy [A] time = 3.48265, size = 15, normalized size = 0.88

$$\frac{\sqrt{3}F\left(\operatorname{asin}(x-1)\middle|-\frac{1}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/((2-x)*x*(x**2-2*x+4))**(1/2),x)

[Out] sqrt(3)*elliptic_f(asin(x - 1), -1/3)/3
```

Mathematica [C] time = 0.314301, size = 100, normalized size = 5.88

$$-\frac{\sqrt[3]{-1}(x-2)^2\sqrt{\frac{x\left(x+i\sqrt{3}-1\right)}{(x-2)^2}}\sqrt{\frac{-\sqrt[3]{-1}x+x-2}{x-2}}F\left(\sin^{-1}\left(\sqrt{-\frac{(-1)^{2/3}x}{x-2}}\right)\middle|(-1)^{2/3}\right)}{\sqrt{-x\left(x^3-4x^2+8x-8\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[(2 - x)*x*(4 - 2*x + x^2)],x]
```

```
[Out] -(((-1)^(1/3)*(-2 + x)^2*Sqrt[(x*(-1 + I*Sqrt[3] + x))/(-2 + x)^2
]*Sqrt[(-2 + x - (-1)^(1/3)*x)/(-2 + x)]*EllipticF[ArcSin[Sqrt[-(
((-1)^(2/3)*x)/(-2 + x))]], (-1)^(2/3)])/Sqrt[-(x*(-8 + 8*x - 4*x
^2 + x^3))])
```

Maple [B] time = 0.039, size = 200, normalized size = 11.8

$$2\frac{\left(-1+i\sqrt{3}\right)(x-2)^{2}}{\left(-i\sqrt{3}-1\right)\sqrt{-x(x-2)\left(x-i\sqrt{3}-1\right)\left(x-1+i\sqrt{3}\right)}}\sqrt{\frac{\left(-i\sqrt{3}-1\right)x}{\left(1-i\sqrt{3}\right)(x-2)}}\sqrt{\frac{x-i\sqrt{3}-1}{\left(i\sqrt{3}+1\right)(x-2)}}\sqrt{\frac{x-1+i\sqrt{3}}{\left(1-i\sqrt{3}\right)(x-2)}}\,\mathrm{Ellipt}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/((2-x)*x*(x^2-2*x+4))^(1/2),x)
```

```
[Out] 2*(-1+I*3^(1/2))*((-I*3^(1/2)-1)*x/(1-I*3^(1/2))/(x-2))^(1/2)*(x-
2)^2*((x-I*3^(1/2)-1)/(I*3^(1/2)+1)/(x-2))^(1/2)*((x-1+I*3^(1/2))
/(1-I*3^(1/2))/(x-2))^(1/2)/(-I*3^(1/2)-1)/(-x*(x-2)*(x-I*3^(1/2)
-1)*(x-1+I*3^(1/2)))^(1/2)*EllipticF(((-I*3^(1/2)-1)*x/(1-I*3^(1/
2))/(x-2))^(1/2),((1-I*3^(1/2))*(-1+I*3^(1/2))/(-I*3^(1/2)-1)/(I*
3^(1/2)+1))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-(x^2-2\,x+4)(x-2)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-(x^2 - 2*x + 4)*(x - 2)*x),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-(x^2 - 2*x + 4)*(x - 2)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{-x^4+4\,x^3-8\,x^2+8\,x}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-(x^2 - 2*x + 4)*(x - 2)*x),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{x\,(-x+2)\,(x^2-2x+4)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((2-x)*x*(x**2-2*x+4))**(1/2),x)
```

```
[Out] Integral(1/sqrt(x*(-x + 2)*(x**2 - 2*x + 4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-(x^2-2\,x+4)(x-2)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-(x^2 - 2*x + 4)*(x - 2)*x),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-(x^2 - 2*x + 4)*(x - 2)*x), x)
```

3.616 $\int \frac{1}{((2-x)x(4-2x+x^2))^{3/2}}\,dx$

Optimal. Leaf size=73

$$\frac{((x-1)^2+5)(x-1)}{24\sqrt{-(x-1)^4-2(x-1)^2+3}}-\frac{F\left(\sin^{-1}(1-x)\middle|-\frac{1}{3}\right)}{4\sqrt{3}}+\frac{E\left(\sin^{-1}(1-x)\middle|-\frac{1}{3}\right)}{8\sqrt{3}}$$

```
[Out] ((5 + (-1 + x)^2)*(-1 + x))/(24*Sqrt[3 - 2*(-1 + x)^2 - (-1 + x)^
4]) + EllipticE[ArcSin[1 - x], -1/3]/(8*Sqrt[3]) - EllipticF[ArcS
in[1 - x], -1/3]/(4*Sqrt[3])
```

Rubi [A] time = 0.154776, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316

$$-\frac{((x-1)^2+5)(1-x)}{24\sqrt{-(1-x)^4-2(1-x)^2+3}}-\frac{F\left(\sin^{-1}(1-x)\middle|-\frac{1}{3}\right)}{4\sqrt{3}}+\frac{E\left(\sin^{-1}(1-x)\middle|-\frac{1}{3}\right)}{8\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[((2 - x)*x*(4 - 2*x + x^2))^(-3/2),x]
```

```
[Out] -((5 + (-1 + x)^2)*(1 - x))/(24*Sqrt[3 - 2*(1 - x)^2 - (1 - x)^4]
) + EllipticE[ArcSin[1 - x], -1/3]/(8*Sqrt[3]) - EllipticF[ArcSin
[1 - x], -1/3]/(4*Sqrt[3])
```

Rubi in Sympy [A] time = 11.9846, size = 63, normalized size = 0.86

$$\frac{(x-1)\left(2(x-1)^2+10\right)}{48\sqrt{-(x-1)^4-2(x-1)^2+3}}-\frac{\sqrt{3}E\left(\operatorname{asin}(x-1)\middle|-\frac{1}{3}\right)}{24}+\frac{\sqrt{3}F\left(\operatorname{asin}(x-1)\middle|-\frac{1}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/((2-x)*x*(x**2-2*x+4))**(3/2),x)
```

```
[Out] (x - 1)*(2*(x - 1)**2 + 10)/(48*sqrt(-(x - 1)**4 - 2*(x - 1)**2 +
 3)) - sqrt(3)*elliptic_e(asin(x - 1), -1/3)/24 + sqrt(3)*ellipti
c_f(asin(x - 1), -1/3)/12
```

Mathematica [C] time = 1.27464, size = 298, normalized size = 4.08

$$\frac{(x-2)^2 x\left(x^2-2x+4\right)\left(-\frac{3\left(x^2-2x+4\right)x}{x-2}-3\left(x^2-2x+4\right)-4(2-x)\sqrt{\frac{x^2-2x+4}{(x-2)^2}}\left(\sqrt{\frac{x^2-2x+4}{(x-2)^2}}x+4i\sqrt{2}\sqrt{\frac{ix}{\left(\sqrt{3}+i\right)(x-2)}}F\left(\sin^{-1}\left(\sqrt{\cdots}\right.\right.\right.\right.}{96\left(-x\left(x^3-4x^2+8x-8\right)\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[((2 - x)*x*(4 - 2*x + x^2))^(-3/2),x]
```

```
[Out] ((-2 + x)^2*x*(4 - 2*x + x^2)*(2*(-1 + x)*x - 3*(4 - 2*x + x^2) -
 (3*x*(4 - 2*x + x^2))/(-2 + x) - 4*(2 - x)*Sqrt[(4 - 2*x + x^2)/
(-2 + x)^2]*(x*Sqrt[(4 - 2*x + x^2)/(-2 + x)^2] - Sqrt[2]*(I + Sq
rt[3])*Sqrt[(I*x)/((I + Sqrt[3])*(-2 + x))]*EllipticE[ArcSin[Sqrt
[-I + Sqrt[3] - (4*I)/(-2 + x)]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(
I + Sqrt[3])] + (4*I)*Sqrt[2]*Sqrt[(I*x)/((I + Sqrt[3])*(-2 + x))
]*EllipticF[ArcSin[Sqrt[-I + Sqrt[3] - (4*I)/(-2 + x)]/(Sqrt[2]*3
^(1/4))], (2*Sqrt[3])/(I + Sqrt[3])])))/(96*(-(x*(-8 + 8*x - 4*x^
2 + x^3)))^(3/2))
```

Maple [B] time = 0.045, size = 963, normalized size = 13.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/((2-x)*x*(x^2-2*x+4))^(3/2),x)
```

```
[Out] -1/32*(-x^3+4*x^2-8*x+8)/(x*(-x^3+4*x^2-8*x+8))^(1/2)+2*x*(1/24+1
/192*x^2)/(-x*(x^3-4*x^2+8*x-8))^(1/2)+1/6*(-1+I*3^(1/2))*((-I*3^
(1/2)-1)*x/(1-I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-I*3^(1/2)-1)/(I
*3^(1/2)+1)/(x-2))^(1/2)*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1
/2)/(-I*3^(1/2)-1)/(-x*(x-2)*(x-I*3^(1/2)-1)*(x-1+I*3^(1/2)))^(1/
2)*EllipticF(((-I*3^(1/2)-1)*x/(1-I*3^(1/2))/(x-2))^(1/2),((1-I*3
^(1/2))*(-1+I*3^(1/2))/(-I*3^(1/2)-1)/(I*3^(1/2)+1))^(1/2))+1/6*(
-1+I*3^(1/2))*((-I*3^(1/2)-1)*x/(1-I*3^(1/2))/(x-2))^(1/2)*(x-2)^
2*((x-I*3^(1/2)-1)/(I*3^(1/2)+1)/(x-2))^(1/2)*((x-1+I*3^(1/2))/(1
-I*3^(1/2))/(x-2))^(1/2)/(-I*3^(1/2)-1)/(-x*(x-2)*(x-I*3^(1/2)-1)
*(x-1+I*3^(1/2)))^(1/2)*(2*EllipticF(((-I*3^(1/2)-1)*x/(1-I*3^(1/
2))/(x-2))^(1/2),((1-I*3^(1/2))*(-1+I*3^(1/2))/(-I*3^(1/2)-1)/(I*
3^(1/2)+1))^(1/2))-2*EllipticPi(((-I*3^(1/2)-1)*x/(1-I*3^(1/2))/(
x-2))^(1/2),(1-I*3^(1/2))/(-I*3^(1/2)-1),((1-I*3^(1/2))*(-1+I*3^(
1/2))/(-I*3^(1/2)-1)/(I*3^(1/2)+1))^(1/2)))-1/24*(x*(x-I*3^(1/2)-
1)*(x-1+I*3^(1/2))+2*(-1+I*3^(1/2))*((-I*3^(1/2)-1)*x/(1-I*3^(1/2
))/(x-2))^(1/2)*(x-2)^2*((x-I*3^(1/2)-1)/(I*3^(1/2)+1)/(x-2))^(1/
2)*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*(1/2*(6-2*I*3^(1/2
```

```
))/(-I*3^(1/2)-1)*EllipticF(((-I*3^(1/2)-1)*x/(1-I*3^(1/2))/(x-2)
)^(1/2),((1-I*3^(1/2))*(-1+I*3^(1/2))/(-I*3^(1/2)-1)/(I*3^(1/2)+1
))^(1/2))+1/2*(-I*3^(1/2)-1)*EllipticE(((-I*3^(1/2)-1)*x/(1-I*3^(
1/2))/(x-2))^(1/2),((1-I*3^(1/2))*(-1+I*3^(1/2))/(-I*3^(1/2)-1)/(
I*3^(1/2)+1))^(1/2))-4/(-I*3^(1/2)-1)*EllipticPi(((-I*3^(1/2)-1)*
x/(1-I*3^(1/2))/(x-2))^(1/2),(-1+I*3^(1/2))/(I*3^(1/2)+1),((1-I*3
^(1/2))*(-1+I*3^(1/2))/(-I*3^(1/2)-1)/(I*3^(1/2)+1))^(1/2))))/(-x
*(x-2)*(x-I*3^(1/2)-1)*(x-1+I*3^(1/2)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-(x^2-2x+4)(x-2)x)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-(x^2 - 2*x + 4)*(x - 2)*x)^(-3/2),x, algorithm="maxima")
```

```
[Out] integrate((-(x^2 - 2*x + 4)*(x - 2)*x)^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{1}{(x^4-4x^3+8x^2-8x)\sqrt{-x^4+4x^3-8x^2+8x}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-(x^2 - 2*x + 4)*(x - 2)*x)^(-3/2),x, algorithm="fricas")
```

```
[Out] integral(-1/((x^4 - 4*x^3 + 8*x^2 - 8*x)*sqrt(-x^4 + 4*x^3 - 8*x^
2 + 8*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((2-x)*x*(x**2-2*x+4))**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-(x^2 - 2x + 4)(x - 2)x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-(x^2 - 2*x + 4)*(x - 2)*x)^(-3/2),x, algorithm="giac")
```

```
[Out] integrate((-(x^2 - 2*x + 4)*(x - 2)*x)^(-3/2), x)
```

3.617 $\int \frac{1}{((2-x)x(4-2x+x^2))^{5/2}}\,dx$

Optimal. Leaf size=109

$$\frac{(7(x-1)^2+26)\,(x-1)}{432\sqrt{-(x-1)^4-2(x-1)^2+3}}+\frac{((x-1)^2+5)\,(x-1)}{72\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}}-\frac{11F\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)}{144\sqrt{3}}+\frac{7E\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)}{144\sqrt{3}}$$

```
[Out] ((5 + (-1 + x)^2)*(-1 + x))/(72*(3 - 2*(-1 + x)^2 - (-1 + x)^4)^(
3/2)) + ((26 + 7*(-1 + x)^2)*(-1 + x))/(432*Sqrt[3 - 2*(-1 + x)^2
 - (-1 + x)^4]) + (7*EllipticE[ArcSin[1 - x], -1/3])/(144*Sqrt[3]
) - (11*EllipticF[ArcSin[1 - x], -1/3])/(144*Sqrt[3])
```

Rubi [A] time = 0.201044, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368

$$-\frac{((x-1)^2+5)\,(1-x)}{72\left(-(1-x)^4-2(1-x)^2+3\right)^{3/2}}-\frac{(7(1-x)^2+26)\,(1-x)}{432\sqrt{-(1-x)^4-2(1-x)^2+3}}-\frac{11F\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)}{144\sqrt{3}}+\frac{7E\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)}{144\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[((2 - x)*x*(4 - 2*x + x^2))^(-5/2),x]
```

```
[Out] -((26 + 7*(1 - x)^2)*(1 - x))/(432*Sqrt[3 - 2*(1 - x)^2 - (1 - x)
^4]) - ((5 + (-1 + x)^2)*(1 - x))/(72*(3 - 2*(1 - x)^2 - (1 - x)^
4)^(3/2)) + (7*EllipticE[ArcSin[1 - x], -1/3])/(144*Sqrt[3]) - (1
1*EllipticF[ArcSin[1 - x], -1/3])/(144*Sqrt[3])
```

Rubi in Sympy [A] time = 14.6772, size = 97, normalized size = 0.89

$$\frac{(x-1)\left(2\,(x-1)^2+10\right)}{144\left(-\,(x-1)^4-2\,(x-1)^2+3\right)^{\frac{3}{2}}}+\frac{(x-1)\left(112\,(x-1)^2+416\right)}{6912\sqrt{-\,(x-1)^4-2\,(x-1)^2+3}}-\frac{7\sqrt{3}E\left(\operatorname{asin}(x-1)\middle|-\frac{1}{3}\right)}{432}+\frac{11\sqrt{3}F\left(\operatorname{asin}(x-1)\middle|-\frac{1}{3}\right)}{432}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/((2-x)*x*(x**2-2*x+4))**(5/2),x)
```

```
[Out] (x - 1)*(2*(x - 1)**2 + 10)/(144*(-(x - 1)**4 - 2*(x - 1)**2 + 3)
**(3/2)) + (x - 1)*(112*(x - 1)**2 + 416)/(6912*sqrt(-(x - 1)**4
- 2*(x - 1)**2 + 3)) - 7*sqrt(3)*elliptic_e(asin(x - 1), -1/3)/43
2 + 11*sqrt(3)*elliptic_f(asin(x - 1), -1/3)/432
```

Mathematica [C] time = 1.4368, size = 327, normalized size = 3.

$$\frac{(x-2)^3 x^2 \left(x^2-2x+4\right)^2 \left(-\frac{7x\left(x^2-2x+4\right)}{x-2}-19i\sqrt{2}(x-2)\sqrt{\frac{ix}{\left(\sqrt{3}+i\right)(x-2)}}\sqrt{\frac{x^2-2x+4}{(x-2)^2}}F\left(\sin^{-1}\left(\frac{\sqrt{\sqrt{3}-i-\frac{4i}{x-2}}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{i+\sqrt{3}}\right)+\frac{7i\sqrt{2}x\sqrt{\frac{x^2-2x+4}{(x-2)^2}}}{}\right)}{432\left(-x\left(x^3-4x^2+8x-8\right)\right)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[((2 - x)*x*(4 - 2*x + x^2))^(-5/2),x]
```

```
[Out] ((-2 + x)^3*x^2*(4 - 2*x + x^2)^2*((-7*x*(4 - 2*x + x^2))/(-2 + x
) + (36 + 216*x - 622*x^2 + 670*x^3 - 445*x^4 + 187*x^5 - 49*x^6
+ 7*x^7)/((-2 + x)^2*x*(4 - 2*x + x^2)) + ((7*I)*Sqrt[2]*x*Sqrt[(
4 - 2*x + x^2)/(-2 + x)^2]*EllipticE[ArcSin[Sqrt[-I + Sqrt[3] - (
4*I)/(-2 + x)]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(I + Sqrt[3])])/Sq
rt[(I*x)/((I + Sqrt[3])*(-2 + x))] - (19*I)*Sqrt[2]*(-2 + x)*Sqrt
[(I*x)/((I + Sqrt[3])*(-2 + x))]*Sqrt[(4 - 2*x + x^2)/(-2 + x)^2]
*EllipticF[ArcSin[Sqrt[-I + Sqrt[3] - (4*I)/(-2 + x)]/(Sqrt[2]*3^
(1/4))], (2*Sqrt[3])/(I + Sqrt[3])]))/(432*(-(x*(-8 + 8*x - 4*x^2
 + x^3)))^(5/2))
```

Maple [B] time = 0.051, size = 1039, normalized size = 9.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/((2-x)*x*(x^2-2*x+4))^(5/2),x)
```

```
[Out] -1/768*(-x^4+4*x^3-8*x^2+8*x)^(1/2)/x^2-1/96*(-x^3+4*x^2-8*x+8)/(
x*(-x^3+4*x^2-8*x+8))^(1/2)+(1/36+1/288*x^2-1/96*x)*(-x^4+4*x^3-8
*x^2+8*x)^(1/2)/(x^3-4*x^2+8*x-8)^2+2*x*(53/3456+5/1728*x^2-19/46
08*x)/(-x*(x^3-4*x^2+8*x-8))^(1/2)+5/216*(-1+I*3^(1/2))*((-I*3^(1
/2)-1)*x/(1-I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-I*3^(1/2)-1)/(I*3
```

```
^(1/2)+1)/(x-2))^(1/2)*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2
)/(-I*3^(1/2)-1)/(-x*(x-2)*(x-I*3^(1/2)-1)*(x-1+I*3^(1/2)))^(1/2)
*EllipticF(((-I*3^(1/2)-1)*x/(1-I*3^(1/2))/(x-2))^(1/2),((1-I*3^(
1/2))*(-1+I*3^(1/2))/(-I*3^(1/2)-1)/(I*3^(1/2)+1))^(1/2))+7/108*(
-1+I*3^(1/2))*((-I*3^(1/2)-1)*x/(1-I*3^(1/2))/(x-2))^(1/2)*(x-2)^
2*((x-I*3^(1/2)-1)/(I*3^(1/2)+1)/(x-2))^(1/2)*((x-1+I*3^(1/2))/(1
-I*3^(1/2))/(x-2))^(1/2)/(-I*3^(1/2)-1)/(-x*(x-2)*(x-I*3^(1/2)-1)
*(x-1+I*3^(1/2)))^(1/2)*(2*EllipticF(((-I*3^(1/2)-1)*x/(1-I*3^(1/
2))/(x-2))^(1/2),((1-I*3^(1/2))*(-1+I*3^(1/2))/(-I*3^(1/2)-1)/(I*
3^(1/2)+1))^(1/2))-2*EllipticPi(((-I*3^(1/2)-1)*x/(1-I*3^(1/2))/(
x-2))^(1/2),(1-I*3^(1/2))/(-I*3^(1/2)-1),((1-I*3^(1/2))*(-1+I*3^(
1/2))/(-I*3^(1/2)-1)/(I*3^(1/2)+1))^(1/2)))-7/432*(x*(x-I*3^(1/2)
-1)*(x-1+I*3^(1/2))+2*(-1+I*3^(1/2))*((-I*3^(1/2)-1)*x/(1-I*3^(1/
2))/(x-2))^(1/2)*(x-2)^2*((x-I*3^(1/2)-1)/(I*3^(1/2)+1)/(x-2))^(1
/2)*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*(1/2*(6-2*I*3^(1/
2))/(-I*3^(1/2)-1)*EllipticF(((-I*3^(1/2)-1)*x/(1-I*3^(1/2))/(x-2
))^(1/2),((1-I*3^(1/2))*(-1+I*3^(1/2))/(-I*3^(1/2)-1)/(I*3^(1/2)+
1))^(1/2))+1/2*(-I*3^(1/2)-1)*EllipticE(((-I*3^(1/2)-1)*x/(1-I*3^
(1/2))/(x-2))^(1/2),((1-I*3^(1/2))*(-1+I*3^(1/2))/(-I*3^(1/2)-1)/
(I*3^(1/2)+1))^(1/2))-4/(-I*3^(1/2)-1)*EllipticPi(((-I*3^(1/2)-1)
*x/(1-I*3^(1/2))/(x-2))^(1/2),(-1+I*3^(1/2))/(I*3^(1/2)+1),((1-I*
3^(1/2))*(-1+I*3^(1/2))/(-I*3^(1/2)-1)/(I*3^(1/2)+1))^(1/2))))/(-
x*(x-2)*(x-I*3^(1/2)-1)*(x-1+I*3^(1/2)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-\left(x^2-2\,x+4\right)(x-2)x\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-(x^2 - 2*x + 4)*(x - 2)*x)^(-5/2),x, algorithm="maxima")
```

```
[Out] integrate((-(x^2 - 2*x + 4)*(x - 2)*x)^(-5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\left(x^8-8\,x^7+32\,x^6-80\,x^5+128\,x^4-128\,x^3+64\,x^2\right)\sqrt{-x^4+4\,x^3-8\,x^2+8\,x}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-(x^2 - 2*x + 4)*(x - 2)*x)^(-5/2),x, algorithm="fricas")
```

```
[Out] integral(1/((x^8 - 8*x^7 + 32*x^6 - 80*x^5 + 128*x^4 - 128*x^3 +
64*x^2)*sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((2-x)*x*(x**2-2*x+4))**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-(x^2 - 2x + 4)(x - 2)x\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-(x^2 - 2*x + 4)*(x - 2)*x)^(-5/2),x, algorithm="giac")
```

```
[Out] integrate((-(x^2 - 2*x + 4)*(x - 2)*x)^(-5/2), x)
```

3.618 $\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2}\,dx$

Optimal. Leaf size=730

$$\frac{1}{7}\left(\frac{c}{d}+x\right)\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2}-\frac{16c^3\left(8ad^2+c^3\right)\left(\frac{c}{d}+x\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}{35d^2\sqrt{4ad^2+c^3}\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)}$$
$$+\frac{2c\left(\frac{c}{d}+x\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}\left(20ad^2+7c^3-3cd^2\left(\frac{c}{d}+x\right)^2\right)}{35d^2}$$
$$+\frac{8c^{7/4}\left(4ad^2+c^3\right)^{3/4}\left(\sqrt{4ad^2+c^3}\left(5ad^2+c^3\right)-c^{3/2}\left(8ad^2+c^3\right)\right)\sqrt{\frac{d^2(4ac+4c^2x^2+4cdx^3+d^2x^4)}{(4ad^2+c^3)\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)^2}}\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)F\left(2\tan^{-1}\left(\frac{}{\sqrt[4]{}}\right.\right.}{35d^5\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}$$
$$+\frac{16c^{13/4}\left(4ad^2+c^3\right)^{3/4}\left(8ad^2+c^3\right)\sqrt{\frac{d^2(4ac+4c^2x^2+4cdx^3+d^2x^4)}{(4ad^2+c^3)\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)^2}}\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)E\left(2\tan^{-1}\left(\frac{c+dx}{\sqrt[4]{c}\sqrt[4]{c^3+4ad^2}}\right)|\frac{1}{2}\left(\frac{c^{3/2}}{\sqrt{c^3+4ad^2}}+1\right.\right.}{35d^5\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}$$

```
[Out] ((c/d + x)*(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(3/2))/7 + (
2*c*(c/d + x)*Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4]*(7*c^
3 + 20*a*d^2 - 3*c*d^2*(c/d + x)^2))/(35*d^2) - (16*c^3*(c^3 + 8*
a*d^2)*(c/d + x)*Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4])/(
35*d^2*Sqrt[c^3 + 4*a*d^2]*(Sqrt[c] + (d^2*(c/d + x)^2)/Sqrt[c^3
+ 4*a*d^2])) + (16*c^(13/4)*(c^3 + 4*a*d^2)^(3/4)*(c^3 + 8*a*d^2)
*Sqrt[(d^2*(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4))/((c^3 + 4*a
*d^2)*(Sqrt[c] + (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])^2)]*(Sqrt
[c] + (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])*EllipticE[2*ArcTan[(
c + d*x)/(c^(1/4)*(c^3 + 4*a*d^2)^(1/4))], (1 + c^(3/2)/Sqrt[c^3
+ 4*a*d^2])/2])/(35*d^5*Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*
x^4]) + (8*c^(7/4)*(c^3 + 4*a*d^2)^(3/4)*(Sqrt[c^3 + 4*a*d^2]*(c^
3 + 5*a*d^2) - c^(3/2)*(c^3 + 8*a*d^2))*Sqrt[(d^2*(4*a*c + 4*c^2*
x^2 + 4*c*d*x^3 + d^2*x^4))/((c^3 + 4*a*d^2)*(Sqrt[c] + (d^2*(c/d
 + x)^2)/Sqrt[c^3 + 4*a*d^2])^2)]*(Sqrt[c] + (d^2*(c/d + x)^2)/Sq
rt[c^3 + 4*a*d^2])*EllipticF[2*ArcTan[(c + d*x)/(c^(1/4)*(c^3 + 4
*a*d^2)^(1/4))], (1 + c^(3/2)/Sqrt[c^3 + 4*a*d^2])/2])/(35*d^5*Sq
rt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4])
```

Rubi [A] time = 1.9181, antiderivative size = 730, normalized size of antiderivative = 1., number of

steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$

$$\frac{(c+dx)\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2}}{7d}-\frac{16c^3\left(8ad^2+c^3\right)(c+dx)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}{35d^3\sqrt{4ad^2+c^3}\left(\frac{(c+dx)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)}$$
$$+\frac{2c(c+dx)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}\left(20ad^2+7c^3-3c(c+dx)^2\right)}{35d^3}$$
$$+\frac{8c^{7/4}\left(4ad^2+c^3\right)^{3/4}\left(\sqrt{4ad^2+c^3}\left(5ad^2+c^3\right)-c^{3/2}\left(8ad^2+c^3\right)\right)\sqrt{\frac{d^2(4ac+4c^2x^2+4cdx^3+d^2x^4)}{(4ad^2+c^3)\left(\frac{(c+dx)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)^2}}\left(\frac{(c+dx)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)F\left(2\tan^{-1}\left(\frac{\ }{\sqrt[4]{\ }}\right.\right.}{35d^5\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}$$
$$+\frac{16c^{13/4}\left(4ad^2+c^3\right)^{3/4}\left(8ad^2+c^3\right)\sqrt{\frac{d^2(4ac+4c^2x^2+4cdx^3+d^2x^4)}{(4ad^2+c^3)\left(\frac{(c+dx)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)^2}}\left(\frac{(c+dx)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)E\left(2\tan^{-1}\left(\frac{c+dx}{\sqrt[4]{c}\sqrt[4]{c^3+4ad^2}}\right)|\frac{1}{2}\left(\frac{c^{3/2}}{\sqrt{c^3+4ad^2}}+1\right.\right.}{35d^5\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(3/2),x]
```

```
[Out] ((c + d*x)*(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(3/2))/(7*d)
 + (2*c*(c + d*x)*Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4]*(
7*c^3 + 20*a*d^2 - 3*c*(c + d*x)^2))/(35*d^3) - (16*c^3*(c^3 + 8*
a*d^2)*(c + d*x)*Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4])/(
35*d^3*Sqrt[c^3 + 4*a*d^2]*(Sqrt[c] + (c + d*x)^2/Sqrt[c^3 + 4*a*
d^2])) + (16*c^(13/4)*(c^3 + 4*a*d^2)^(3/4)*(c^3 + 8*a*d^2)*Sqrt[
(d^2*(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4))/((c^3 + 4*a*d^2)*
(Sqrt[c] + (c + d*x)^2/Sqrt[c^3 + 4*a*d^2])^2)]*(Sqrt[c] + (c + d
*x)^2/Sqrt[c^3 + 4*a*d^2])*EllipticE[2*ArcTan[(c + d*x)/(c^(1/4)*
(c^3 + 4*a*d^2)^(1/4))], (1 + c^(3/2)/Sqrt[c^3 + 4*a*d^2])/2])/(3
5*d^5*Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4]) + (8*c^(7/4)
*(c^3 + 4*a*d^2)^(3/4)*(Sqrt[c^3 + 4*a*d^2]*(c^3 + 5*a*d^2) - c^(
3/2)*(c^3 + 8*a*d^2))*Sqrt[(d^2*(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 +
d^2*x^4))/((c^3 + 4*a*d^2)*(Sqrt[c] + (c + d*x)^2/Sqrt[c^3 + 4*a*
d^2])^2)]*(Sqrt[c] + (c + d*x)^2/Sqrt[c^3 + 4*a*d^2])*EllipticF[2
*ArcTan[(c + d*x)/(c^(1/4)*(c^3 + 4*a*d^2)^(1/4))], (1 + c^(3/2)/
Sqrt[c^3 + 4*a*d^2])/2])/(35*d^5*Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x
^3 + d^2*x^4])
```

Rubi in Sympy [A] time = 157.33, size = 734, normalized size = 1.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**(3/2),x)
```

```
[Out] 16*c**(13/4)*sqrt(d**2*(-2*c**2*(c/d + x)**2 + c*(4*a + c**3/d**2
) + d**2*(c/d + x)**4)/((sqrt(c) + d**2*(c/d + x)**2/sqrt(4*a*d**
2 + c**3))**2*(4*a*d**2 + c**3)))*(sqrt(c) + d**2*(c/d + x)**2/sq
rt(4*a*d**2 + c**3))*(4*a*d**2 + c**3)**(3/4)*(8*a*d**2 + c**3)*e
lliptic_e(2*atan(d*(c/d + x)/(c**(1/4)*(4*a*d**2 + c**3)**(1/4)))
, c**(3/2)/(2*sqrt(4*a*d**2 + c**3)) + 1/2)/(35*d**5*sqrt(-2*c**2
*(c/d + x)**2 + c*(4*a + c**3/d**2) + d**2*(c/d + x)**4)) + 8*c**
(7/4)*sqrt(d**2*(-2*c**2*(c/d + x)**2 + c*(4*a + c**3/d**2) + d**
2*(c/d + x)**4)/((sqrt(c) + d**2*(c/d + x)**2/sqrt(4*a*d**2 + c**
3))**2*(4*a*d**2 + c**3)))*(sqrt(c) + d**2*(c/d + x)**2/sqrt(4*a*
d**2 + c**3))*(4*a*d**2 + c**3)**(1/4)*(a*d**2*(20*a*d**2 + 9*c**
3) - c**(3/2)*sqrt(4*a*d**2 + c**3)*(8*a*d**2 + c**3) + c**6)*ell
iptic_f(2*atan(d*(c/d + x)/(c**(1/4)*(4*a*d**2 + c**3)**(1/4))),
c**(3/2)/(2*sqrt(4*a*d**2 + c**3)) + 1/2)/(35*d**5*sqrt(-2*c**2*(
c/d + x)**2 + c*(4*a + c**3/d**2) + d**2*(c/d + x)**4)) - 16*c**3
*(8*a*d**2 + c**3)*(c/d + x)*sqrt(-2*c**2*(c/d + x)**2 + c*(4*a +
 c**3/d**2) + d**2*(c/d + x)**4)/(35*d**2*(sqrt(c) + d**2*(c/d +
x)**2/sqrt(4*a*d**2 + c**3))*sqrt(4*a*d**2 + c**3)) + c*(c/d + x)
*(40*a*d**2 + 14*c**3 - 6*c*d**2*(c/d + x)**2)*sqrt(-2*c**2*(c/d
+ x)**2 + c*(4*a + c**3/d**2) + d**2*(c/d + x)**4)/(35*d**2) + (c
/d + x)*(-2*c**2*(c/d + x)**2 + c*(4*a + c**3/d**2) + d**2*(c/d +
 x)**4)**(3/2)/7
```

Mathematica [C] time = 6.29824, size = 10468, normalized size = 14.34

Result too large to show

Antiderivative was successfully verified.

```
[In]  Integrate[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.268, size = 5229, normalized size = 7.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d^2x^4 + 4\,cdx^3 + 4\,c^2x^2 + 4\,ac\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(d^2x^4 + 4\,cdx^3 + 4\,c^2x^2 + 4\,ac\right)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(4ac + 4c^2x^2 + 4cdx^3 + d^2x^4\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**(3/2),x)
```

```
[Out] Integral((4*a*c + 4*c**2*x**2 + 4*c*d*x**3 + d**2*x**4)**(3/2), x
)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d^2x^4 + 4\,cdx^3 + 4\,c^2x^2 + 4\,ac\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c)^(3/2),x, algorithm="giac")

[Out] integrate((d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c)^(3/2), x)
```

3.619 $\int \sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx$

Optimal. Leaf size=622

$$\frac{1}{3}\left(\frac{c}{d}+x\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}-\frac{2c^2\left(\frac{c}{d}+x\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}{3\sqrt{4ad^2+c^3}\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)}$$
$$+\frac{c^{3/4}\sqrt[4]{4ad^2+c^3}\left(-c^{3/2}\sqrt{4ad^2+c^3}+4ad^2+c^3\right)\sqrt{\frac{d^2(4ac+4c^2x^2+4cdx^3+d^2x^4)}{(4ad^2+c^3)\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)^2}}\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)F\left(2\tan^{-1}\left(\frac{c+dx}{\sqrt[4]{c}\sqrt[4]{c^3+4ad^2}}\right)|\frac{1}{2}\left(\right.\right.}{3d^3\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}$$
$$+\frac{2c^{9/4}\left(4ad^2+c^3\right)^{3/4}\sqrt{\frac{d^2(4ac+4c^2x^2+4cdx^3+d^2x^4)}{(4ad^2+c^3)\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)^2}}\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)E\left(2\tan^{-1}\left(\frac{c+dx}{\sqrt[4]{c}\sqrt[4]{c^3+4ad^2}}\right)|\frac{1}{2}\left(\frac{c^{3/2}}{\sqrt{c^3+4ad^2}}+1\right)\right)}{3d^3\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}$$

```
[Out] ((c/d + x)*Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4])/3 - (2*
c^2*(c/d + x)*Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4])/(3*S
qrt[c^3 + 4*a*d^2]*(Sqrt[c] + (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^
2])) + (2*c^(9/4)*(c^3 + 4*a*d^2)^(3/4)*Sqrt[(d^2*(4*a*c + 4*c^2*
x^2 + 4*c*d*x^3 + d^2*x^4))/((c^3 + 4*a*d^2)*(Sqrt[c] + (d^2*(c/d
 + x)^2)/Sqrt[c^3 + 4*a*d^2])^2)]*(Sqrt[c] + (d^2*(c/d + x)^2)/Sq
rt[c^3 + 4*a*d^2])*EllipticE[2*ArcTan[(c + d*x)/(c^(1/4)*(c^3 + 4
*a*d^2)^(1/4))], (1 + c^(3/2)/Sqrt[c^3 + 4*a*d^2])/2])/(3*d^3*Sqr
t[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4]) + (c^(3/4)*(c^3 + 4*a
*d^2)^(1/4)*(c^3 + 4*a*d^2 - c^(3/2)*Sqrt[c^3 + 4*a*d^2])*Sqrt[(d
^2*(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4))/((c^3 + 4*a*d^2)*(S
qrt[c] + (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])^2)]*(Sqrt[c] + (d
^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])*EllipticF[2*ArcTan[(c + d*x)
/(c^(1/4)*(c^3 + 4*a*d^2)^(1/4))], (1 + c^(3/2)/Sqrt[c^3 + 4*a*d^
2])/2])/(3*d^3*Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4])
```

Rubi [A] time = 1.55441, antiderivative size = 622, normalized size of antiderivative = 1., number of

steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$

$$\frac{(c+dx)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}{3d}-\frac{2c^2(c+dx)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}{3d\sqrt{4ad^2+c^3}\left(\frac{(c+dx)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)}$$
$$+\frac{c^{3/4}\sqrt[4]{4ad^2+c^3}\left(-c^{3/2}\sqrt{4ad^2+c^3}+4ad^2+c^3\right)\sqrt{\frac{d^2(4ac+4c^2x^2+4cdx^3+d^2x^4)}{(4ad^2+c^3)\left(\frac{(c+dx)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)^2}}\left(\frac{(c+dx)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)F\left(2\tan^{-1}\left(\frac{c+dx}{\sqrt[4]{c}\sqrt[4]{c^3+4ad^2}}\right)|\frac{1}{2}\left(\right.\right.}{3d^3\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}$$
$$+\frac{2c^{9/4}\left(4ad^2+c^3\right)^{3/4}\sqrt{\frac{d^2(4ac+4c^2x^2+4cdx^3+d^2x^4)}{(4ad^2+c^3)\left(\frac{(c+dx)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)^2}}\left(\frac{(c+dx)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)E\left(2\tan^{-1}\left(\frac{c+dx}{\sqrt[4]{c}\sqrt[4]{c^3+4ad^2}}\right)|\frac{1}{2}\left(\frac{c^{3/2}}{\sqrt{c^3+4ad^2}}+1\right)\right)}{3d^3\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4],x]
```

```
[Out] ((c + d*x)*Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4])/(3*d) -
 (2*c^2*(c + d*x)*Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4])/
(3*d*Sqrt[c^3 + 4*a*d^2]*(Sqrt[c] + (c + d*x)^2/Sqrt[c^3 + 4*a*d^
2])) + (2*c^(9/4)*(c^3 + 4*a*d^2)^(3/4)*Sqrt[(d^2*(4*a*c + 4*c^2*
x^2 + 4*c*d*x^3 + d^2*x^4))/((c^3 + 4*a*d^2)*(Sqrt[c] + (c + d*x)
^2/Sqrt[c^3 + 4*a*d^2])^2)]*(Sqrt[c] + (c + d*x)^2/Sqrt[c^3 + 4*a
*d^2])*EllipticE[2*ArcTan[(c + d*x)/(c^(1/4)*(c^3 + 4*a*d^2)^(1/4
))], (1 + c^(3/2)/Sqrt[c^3 + 4*a*d^2])/2])/(3*d^3*Sqrt[4*a*c + 4*
c^2*x^2 + 4*c*d*x^3 + d^2*x^4]) + (c^(3/4)*(c^3 + 4*a*d^2)^(1/4)*
(c^3 + 4*a*d^2 - c^(3/2)*Sqrt[c^3 + 4*a*d^2])*Sqrt[(d^2*(4*a*c +
4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4))/((c^3 + 4*a*d^2)*(Sqrt[c] + (c
+ d*x)^2/Sqrt[c^3 + 4*a*d^2])^2)]*(Sqrt[c] + (c + d*x)^2/Sqrt[c^3
 + 4*a*d^2])*EllipticF[2*ArcTan[(c + d*x)/(c^(1/4)*(c^3 + 4*a*d^2
)^(1/4))], (1 + c^(3/2)/Sqrt[c^3 + 4*a*d^2])/2])/(3*d^3*Sqrt[4*a*
c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4])
```

Rubi in Sympy [A] time = 127.364, size = 604, normalized size = 0.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**(1/2),x)
```

```
[Out] 2*c**(9/4)*sqrt(d**2*(-2*c**2*(c/d + x)**2 + c*(4*a + c**3/d**2)
+ d**2*(c/d + x)**4)/((sqrt(c) + d**2*(c/d + x)**2/sqrt(4*a*d**2
+ c**3))**2*(4*a*d**2 + c**3)))*(sqrt(c) + d**2*(c/d + x)**2/sqrt
(4*a*d**2 + c**3))*(4*a*d**2 + c**3)**(3/4)*elliptic_e(2*atan(d*(
```

```
c/d + x)/(c**(1/4)*(4*a*d**2 + c**3)**(1/4))), c**(3/2)/(2*sqrt(4
*a*d**2 + c**3)) + 1/2)/(3*d**3*sqrt(-2*c**2*(c/d + x)**2 + c*(4*
a + c**3/d**2) + d**2*(c/d + x)**4)) - c**(3/4)*sqrt(d**2*(-2*c**
2*(c/d + x)**2 + c*(4*a + c**3/d**2) + d**2*(c/d + x)**4)/((sqrt(
c) + d**2*(c/d + x)**2/sqrt(4*a*d**2 + c**3))**2*(4*a*d**2 + c**3
)))*(sqrt(c) + d**2*(c/d + x)**2/sqrt(4*a*d**2 + c**3))*(c**(3/2)
 - sqrt(4*a*d**2 + c**3))*(4*a*d**2 + c**3)**(3/4)*elliptic_f(2*a
tan(d*(c/d + x)/(c**(1/4)*(4*a*d**2 + c**3)**(1/4))), c**(3/2)/(2
*sqrt(4*a*d**2 + c**3)) + 1/2)/(3*d**3*sqrt(-2*c**2*(c/d + x)**2
+ c*(4*a + c**3/d**2) + d**2*(c/d + x)**4)) - 2*c**2*(c/d + x)*sq
rt(-2*c**2*(c/d + x)**2 + c*(4*a + c**3/d**2) + d**2*(c/d + x)**4
)/(3*(sqrt(c) + d**2*(c/d + x)**2/sqrt(4*a*d**2 + c**3))*sqrt(4*a
*d**2 + c**3)) + (c/d + x)*sqrt(-2*c**2*(c/d + x)**2 + c*(4*a + c
**3/d**2) + d**2*(c/d + x)**4)/3
```

Mathematica [C] time = 6.14725, size = 5218, normalized size = 8.39

Result too large to show

Warning: Unable to verify antiderivative.

```
[In]  Integrate[Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4],x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.05, size = 4890, normalized size = 7.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2),x)
```

```
[Out] 1/3*x*(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2)+1/3*c/d*(d^2*x^4+
4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2)+16/3*a*c*((-c+(2*d*(-a*c)^(1/2)+
c^2)^(1/2))/d+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*((-(c+(-2*d*(-
a*c)^(1/2)+c^2)^(1/2))/d+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x-(
-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^
(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(
1/2)+c^2)^(1/2))/d))^(1/2)*(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)
^2*((-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^
2)^(1/2))/d)*(x-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/((-c+(-2*d*
(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(
x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2)*((-(c+(2*d*(-a*c)^(1
/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x+(c+(-2*
```

d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))
/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2)+c
^2)^(1/2))/d))^(1/2)/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(2*
d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/
d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(d^2*(x-(-c+(2*d*(-a*c)^(1
/2)+c^2)^(1/2))/d)*(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x-(-c+
(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x+(c+(-2*d*(-a*c)^(1/2)+c^2)^(
1/2))/d))^(1/2)*EllipticF(((-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+
(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x-(-c+(2*d*(-a*c)^(1/2)+c^2)
^(1/2))/d)/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^
(1/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2
),((-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(-2*d*(-a*c)^(1/2)+c^
2)^(1/2))/d)*((-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(-2*d*(-a*c)
^(1/2)+c^2)^(1/2))/d)/((-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(-
2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2)
)/d+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2))-8/3*c^3/d*((-c+(
2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/
d)*((-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(2*d*(-a*c)^(1/2)+c^
2)^(1/2))/d)*(x-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(-2*d*(
-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x
+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2)*(x+(c+(2*d*(-a*c)^(1/
2)+c^2)^(1/2))/d)^2*((-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*
d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2
))/d)/((-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)
+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2)*((-
(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/
2))/d)*(x+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(-2*d*(-a*c)^
(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(c+(2
d(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2)/(-(c+(-2*d*(-a*c)^(1/2)+c^2
)^(1/2))/d+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(2*d*(-a*c)^(
1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(d^2*(x-(
-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x+(c+(2*d*(-a*c)^(1/2)+c^2)^
(1/2))/d)*(x-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x+(c+(-2*d*(-
a*c)^(1/2)+c^2)^(1/2))/d))^(1/2)*(-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2
))/d*EllipticF(((-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(2*d*(-a
c)^(1/2)+c^2)^(1/2))/d)(x-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/
(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^
(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2),((-(c+(2*
d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d
)*((-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(-2*d*(-a*c)^(1/2)+c^2)
^(1/2))/d)/((-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(-2*d*(-a*c)^
(1/2)+c^2)^(1/2))/d)/(-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(-2*
d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2))+((-c+(2*d*(-a*c)^(1/2)+c^2)
^(1/2))/d+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*EllipticPi(((-(c+(-
2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d
)*(x-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(-2*d*(-a*c)^(1/2)
+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-
a*c)^(1/2)+c^2)^(1/2))/d))^(1/2),(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/
2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(-2*d*(-a*c)^(1/2
)+c^2)^(1/2))/d+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d),((-(c+(2*d*(-
a*c)^(1/2)+c^2)^(1/2))/d-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*((
-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/
2))/d)/((-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(-2*d*(-a*c)^(1/2
)+c^2)^(1/2))/d)/(-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(-2*d*(-

a*c)^(1/2)+c^2)^(1/2))/d))^(1/2)))-2/3*c^2*((x-(-c+(2*d*(-a*c)^(1
/2)+c^2)^(1/2))/d)*(x-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x+(c
+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)+((-c+(2*d*(-a*c)^(1/2)+c^2)^(1
/2))/d+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*((-(c+(-2*d*(-a*c)^(1
/2)+c^2)^(1/2))/d+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x-(-c+(2*d
*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/
d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2)+c^
2)^(1/2))/d))^(1/2)*(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)^2*((-(
c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2
))/d)*(x-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/((-c+(-2*d*(-a*c)^
(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(c+(2
d(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2)*((-(c+(2*d*(-a*c)^(1/2)+c^2
)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x+(c+(-2*d*(-a*c
)^(1/2)+c^2)^(1/2))/d)/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+
(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/
2))/d))^(1/2)*((-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d^2*(-c+(2*d*(-
a*c)^(1/2)+c^2)^(1/2))+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d^2*(-c+
(2*d*(-a*c)^(1/2)+c^2)^(1/2))+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d
^2*(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/
2))^2/d^2)/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(2*d*(-a*c)^(
1/2)+c^2)^(1/2))/d)/(-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d
*(-a*c)^(1/2)+c^2)^(1/2))/d)*EllipticF(((-(c+(-2*d*(-a*c)^(1/2)+c
^2)^(1/2))/d+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x-(-c+(2*d*(-a*
c)^(1/2)+c^2)^(1/2))/d)/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c
+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1
/2))/d))^(1/2),((-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(-2*d*(-
a*c)^(1/2)+c^2)^(1/2))/d)*((-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c
+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/((-c+(2*d*(-a*c)^(1/2)+c^2)^(1
/2))/d-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(2*d*(-a*c)^(1/
2)+c^2)^(1/2))/d+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2))+((-
c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/
2))/d)*EllipticE(((-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(2*d*(
-a*c)^(1/2)+c^2)^(1/2))/d)*(x-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d
)/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2
)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2),((-(c+(
2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))
/d)*((-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(-2*d*(-a*c)^(1/2)+c^
2)^(1/2))/d)/((-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(-2*d*(-a*c
)^(1/2)+c^2)^(1/2))/d)/(-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(-
2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2))/(-(c+(2*d*(-a*c)^(1/2)+c^
2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)+4*c/d/(-(c+(-2*d
*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*E
llipticPi(((-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(2*d*(-a*c)^(
1/2)+c^2)^(1/2))/d)*(x-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+
(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2)
)/d)/(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2),((-c+(2*d*(-a*
c)^(1/2)+c^2)^(1/2))/d+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+
(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))
/d),((-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(-2*d*(-a*c)^(1/2)+
c^2)^(1/2))/d)*((-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(-2*d*(-a*
c)^(1/2)+c^2)^(1/2))/d)/((-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+
(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/
2))/d+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2))))/(d^2*(x-(-c+
(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/

```
2))/d)*(x-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x+(c+(-2*d*(-a*c
)^(1/2)+c^2)^(1/2))/d))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d^2x^4 + 4\,cdx^3 + 4\,c^2x^2 + 4\,ac}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{d^2x^4 + 4\,cdx^3 + 4\,c^2x^2 + 4\,ac}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{4ac + 4c^2x^2 + 4cdx^3 + d^2x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**(1/2),x)
```

```
[Out] Integral(sqrt(4*a*c + 4*c**2*x**2 + 4*c*d*x**3 + d**2*x**4), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d^2x^4 + 4\,cdx^3 + 4\,c^2x^2 + 4\,ac}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c), x)
```

3.620 $\int \frac{1}{\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}\,dx$

Optimal. Leaf size=227

$$\frac{\sqrt[4]{4ad^2+c^3}\sqrt{\frac{d^2(4ac+4c^2x^2+4cdx^3+d^2x^4)}{(4ad^2+c^3)\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)^2}}\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)F\left(2\tan^{-1}\left(\frac{c+dx}{\sqrt[4]{c}\sqrt[4]{c^3+4ad^2}}\right)|\frac{1}{2}\left(\frac{c^{3/2}}{\sqrt{c^3+4ad^2}}+1\right)\right)}{2\sqrt[4]{c}d\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}$$

```
[Out] ((c^3 + 4*a*d^2)^(1/4)*Sqrt[(d^2*(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 +
 d^2*x^4))/((c^3 + 4*a*d^2)*(Sqrt[c] + (d^2*(c/d + x)^2)/Sqrt[c^3
 + 4*a*d^2])^2)]*(Sqrt[c] + (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2]
)*EllipticF[2*ArcTan[(c + d*x)/(c^(1/4)*(c^3 + 4*a*d^2)^(1/4))],
(1 + c^(3/2)/Sqrt[c^3 + 4*a*d^2])/2])/(2*c^(1/4)*d*Sqrt[4*a*c + 4
*c^2*x^2 + 4*c*d*x^3 + d^2*x^4])
```

Rubi [A] time = 0.468725, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$

$$\frac{\sqrt[4]{4ad^2+c^3}\sqrt{\frac{d^2(4ac+4c^2x^2+4cdx^3+d^2x^4)}{(4ad^2+c^3)\left(\frac{(c+dx)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)^2}}\left(\frac{(c+dx)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)F\left(2\tan^{-1}\left(\frac{c+dx}{\sqrt[4]{c}\sqrt[4]{c^3+4ad^2}}\right)|\frac{1}{2}\left(\frac{c^{3/2}}{\sqrt{c^3+4ad^2}}+1\right)\right)}{2\sqrt[4]{c}d\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4],x]
```

```
[Out] ((c^3 + 4*a*d^2)^(1/4)*Sqrt[(d^2*(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 +
 d^2*x^4))/((c^3 + 4*a*d^2)*(Sqrt[c] + (c + d*x)^2/Sqrt[c^3 + 4*a
*d^2])^2)]*(Sqrt[c] + (c + d*x)^2/Sqrt[c^3 + 4*a*d^2])*EllipticF[
2*ArcTan[(c + d*x)/(c^(1/4)*(c^3 + 4*a*d^2)^(1/4))], (1 + c^(3/2)
/Sqrt[c^3 + 4*a*d^2])/2])/(2*c^(1/4)*d*Sqrt[4*a*c + 4*c^2*x^2 + 4
*c*d*x^3 + d^2*x^4])
```

Rubi in Sympy [A] time = 56.1069, size = 221, normalized size = 0.97

$$\frac{\sqrt{\frac{d^2\left(-2c^2\left(\frac{c}{d}+x\right)^2+c\left(4a+\frac{c^3}{d^2}\right)+d^2\left(\frac{c}{d}+x\right)^4\right)}{\left(\sqrt{c}+\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}\right)^2(4ad^2+c^3)}}\left(\sqrt{c}+\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}\right)\sqrt[4]{4ad^2+c^3}F\left(2\,\mathrm{atan}\left(\frac{d\left(\frac{c}{d}+x\right)}{\sqrt[4]{c}\sqrt[4]{4ad^2+c^3}}\right)\middle|\frac{c^{\frac{3}{2}}}{2\sqrt{4ad^2+c^3}}+\frac{1}{2}\right)}{2\sqrt[4]{c}d\sqrt{-2c^2\left(\frac{c}{d}+x\right)^2+c\left(4a+\frac{c^3}{d^2}\right)+d^2\left(\frac{c}{d}+x\right)^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**(1/2),x)
```

```
[Out] sqrt(d**2*(-2*c**2*(c/d + x)**2 + c*(4*a + c**3/d**2) + d**2*(c/d
 + x)**4)/((sqrt(c) + d**2*(c/d + x)**2/sqrt(4*a*d**2 + c**3))**2
*(4*a*d**2 + c**3)))*(sqrt(c) + d**2*(c/d + x)**2/sqrt(4*a*d**2 +
 c**3))*(4*a*d**2 + c**3)**(1/4)*elliptic_f(2*atan(d*(c/d + x)/(c
**(1/4)*(4*a*d**2 + c**3)**(1/4))), c**(3/2)/(2*sqrt(4*a*d**2 + c
**3)) + 1/2)/(2*c**(1/4)*d*sqrt(-2*c**2*(c/d + x)**2 + c*(4*a + c
**3/d**2) + d**2*(c/d + x)**4))
```

Mathematica [C] time = 3.75966, size = 822, normalized size = 3.62

$$\frac{2\left(-c-dx+\sqrt{c^2-2i\sqrt{a}\sqrt{c}d}\right)\left(c+dx+\sqrt{c^2-2i\sqrt{a}\sqrt{c}d}\right)\sqrt{-\frac{\sqrt{c^2-2i\sqrt{a}\sqrt{c}d}\left(c+dx-\sqrt{c^2+2i\sqrt{a}d\sqrt{c}}\right)}{\left(\sqrt{c^2-2i\sqrt{a}\sqrt{c}d}+\sqrt{c^2+2i\sqrt{a}d\sqrt{c}}\right)\left(-c-dx+\sqrt{c^2-2i\sqrt{a}\sqrt{c}d}\right)}}\sqrt{-\frac{\cdots}{\left(\sqrt{c^2-2i\sqrt{a}\cdots\right.}}}\cdots}{d\sqrt{c^2-2i\sqrt{a}\sqrt{c}d}\sqrt{\frac{\left(\sqrt{c^2-2i\sqrt{a}\sqrt{c}d}-\sqrt{c^2+2i\cdots\right.}{\left(\sqrt{c^2-2i\sqrt{a}\sqrt{c}d}+\sqrt{c^2+2i\cdots\right.}}\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4],x]
```

```
[Out] (2*(-c + Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*d] - d*x)*(c + Sqrt[c^2
 - (2*I)*Sqrt[a]*Sqrt[c]*d] + d*x)*Sqrt[-((Sqrt[c^2 - (2*I)*Sqrt[
a]*Sqrt[c]*d]*(c - Sqrt[c^2 + (2*I)*Sqrt[a]*Sqrt[c]*d] + d*x))/((
Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*d] + Sqrt[c^2 + (2*I)*Sqrt[a]*Sq
rt[c]*d])*(-c + Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*d] - d*x)))]*Sqr
t[-((Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*d]*(c + Sqrt[c^2 + (2*I)*Sq
rt[a]*Sqrt[c]*d] + d*x))/((Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*d] -
Sqrt[c^2 + (2*I)*Sqrt[a]*Sqrt[c]*d])*(-c + Sqrt[c^2 - (2*I)*Sqrt[
a]*Sqrt[c]*d] - d*x)))]*EllipticF[ArcSin[Sqrt[((Sqrt[c^2 - (2*I)*
Sqrt[a]*Sqrt[c]*d] - Sqrt[c^2 + (2*I)*Sqrt[a]*Sqrt[c]*d])*(c + Sq
rt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*d] + d*x))/((Sqrt[c^2 - (2*I)*Sqrt
[a]*Sqrt[c]*d] + Sqrt[c^2 + (2*I)*Sqrt[a]*Sqrt[c]*d])*(-c + Sqrt[
c^2 - (2*I)*Sqrt[a]*Sqrt[c]*d] - d*x))]], (Sqrt[c^2 - (2*I)*Sqrt[
a]*Sqrt[c]*d] + Sqrt[c^2 + (2*I)*Sqrt[a]*Sqrt[c]*d])^2/(Sqrt[c^2
- (2*I)*Sqrt[a]*Sqrt[c]*d] - Sqrt[c^2 + (2*I)*Sqrt[a]*Sqrt[c]*d])
^2])/(d*Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*d]*Sqrt[((Sqrt[c^2 - (2*
I)*Sqrt[a]*Sqrt[c]*d] - Sqrt[c^2 + (2*I)*Sqrt[a]*Sqrt[c]*d])*(c +
 Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*d] + d*x))/((Sqrt[c^2 - (2*I)*S
qrt[a]*Sqrt[c]*d] + Sqrt[c^2 + (2*I)*Sqrt[a]*Sqrt[c]*d])*(-c + Sq
rt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*d] - d*x))]*Sqrt[4*a*c + x^2*(2*c
+ d*x)^2])
```

Maple [B] time = 0.046, size = 1056, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2),x)
```

```
[Out] 2*((-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(-2*d*(-a*c)^(1/2)+c^2)
^(1/2))/d)*((-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(2*d*(-a*c)^
(1/2)+c^2)^(1/2))/d)*(x-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c
+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2
))/d)/(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2)*(x+(c+(2*d*(-
a*c)^(1/2)+c^2)^(1/2))/d)^2*((-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d
-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x-(-c+(-2*d*(-a*c)^(1/2)+c
^2)^(1/2))/d)/((-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*
c)^(1/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(
1/2)*((-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+
c^2)^(1/2))/d)*(x+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(-2*d
*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/
(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2)/(-(c+(-2*d*(-a*c)^(
1/2)+c^2)^(1/2))/d+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(2*d*
(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(
d^2*(x-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x+(c+(2*d*(-a*c)^(1/
2)+c^2)^(1/2))/d)*(x-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x+(c+
(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2)*EllipticF(((-(c+(-2*d*(-
a*c)^(1/2)+c^2)^(1/2))/d+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x-(
-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^
(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(
1/2)+c^2)^(1/2))/d))^(1/2),((-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-
(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*((-c+(2*d*(-a*c)^(1/2)+c^2)
^(1/2))/d+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/((-c+(2*d*(-a*c)^(
1/2)+c^2)^(1/2))/d-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(2*
d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)
)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d^2x^4 + 4\,cdx^3 + 4\,c^2x^2 + 4\,ac}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{d^2x^4 + 4\,cdx^3 + 4\,c^2x^2 + 4\,ac}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sqrt(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c),x, algorithm="fricas")

[Out] integral(1/sqrt(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{4ac + 4c^2x^2 + 4cdx^3 + d^2x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**(1/2),x)

[Out] Integral(1/sqrt(4*a*c + 4*c**2*x**2 + 4*c*d*x**3 + d**2*x**4), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d^2x^4 + 4\,cdx^3 + 4\,c^2x^2 + 4\,ac}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sqrt(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c),x, algorithm="giac")

[Out] integrate(1/sqrt(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c), x)
```

3.621 $\int \frac{1}{(4ac+4c^2x^2+4cdx^3+d^2x^4)^{3/2}}\,dx$

Optimal. Leaf size=674

$$-\frac{d^2\left(\frac{c}{d}+x\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}{8a\left(4ad^2+c^3\right)^{3/2}\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)}-\frac{\left(\frac{c}{d}+x\right)\left(-4ad^2+c^3-cd^2\left(\frac{c}{d}+x\right)^2\right)}{8ac\left(4ad^2+c^3\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}$$
$$+\frac{\left(-c^{3/2}\sqrt{4ad^2+c^3}+4ad^2+c^3\right)\sqrt{\frac{d^2(4ac+4c^2x^2+4cdx^3+d^2x^4)}{(4ad^2+c^3)\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)^2}}\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)F\left(2\tan^{-1}\left(\frac{c+dx}{\sqrt[4]{c}\sqrt[4]{c^3+4ad^2}}\right)|\frac{1}{2}\left(\frac{c^{3/2}}{\sqrt{c^3+4ad^2}}+1\right)\right)}{16ac^{5/4}d\left(4ad^2+c^3\right)^{3/4}\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}$$
$$+\frac{\sqrt[4]{c}\sqrt{\frac{d^2(4ac+4c^2x^2+4cdx^3+d^2x^4)}{(4ad^2+c^3)\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)^2}}\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)E\left(2\tan^{-1}\left(\frac{c+dx}{\sqrt[4]{c}\sqrt[4]{c^3+4ad^2}}\right)|\frac{1}{2}\left(\frac{c^{3/2}}{\sqrt{c^3+4ad^2}}+1\right)\right)}{8ad\sqrt[4]{4ad^2+c^3}\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}$$

```
[Out] -((c/d + x)*(c^3 - 4*a*d^2 - c*d^2*(c/d + x)^2))/(8*a*c*(c^3 + 4*
a*d^2)*Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4]) - (d^2*(c/d
 + x)*Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4])/(8*a*(c^3 +
4*a*d^2)^(3/2)*(Sqrt[c] + (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2]))
 + (c^(1/4)*Sqrt[(d^2*(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4))/
((c^3 + 4*a*d^2)*(Sqrt[c] + (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2]
)^2)]*(Sqrt[c] + (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])*EllipticE
[2*ArcTan[(c + d*x)/(c^(1/4)*(c^3 + 4*a*d^2)^(1/4))], (1 + c^(3/2
)/Sqrt[c^3 + 4*a*d^2])/2])/(8*a*d*(c^3 + 4*a*d^2)^(1/4)*Sqrt[4*a*
c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4]) + ((c^3 + 4*a*d^2 - c^(3/2)
*Sqrt[c^3 + 4*a*d^2])*Sqrt[(d^2*(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 +
d^2*x^4))/((c^3 + 4*a*d^2)*(Sqrt[c] + (d^2*(c/d + x)^2)/Sqrt[c^3
+ 4*a*d^2])^2)]*(Sqrt[c] + (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])
*EllipticF[2*ArcTan[(c + d*x)/(c^(1/4)*(c^3 + 4*a*d^2)^(1/4))], (
1 + c^(3/2)/Sqrt[c^3 + 4*a*d^2])/2])/(16*a*c^(5/4)*d*(c^3 + 4*a*d
^2)^(3/4)*Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4])
```

Rubi [A] time = 1.70507, antiderivative size = 674, normalized size of antiderivative = 1., number of

steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$

$$-\frac{(c+dx)\left(-4ad^2+c^3-c(c+dx)^2\right)}{8acd\left(4ad^2+c^3\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}-\frac{d(c+dx)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}{8a\left(4ad^2+c^3\right)^{3/2}\left(\frac{(c+dx)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)}$$
$$+\frac{\left(-c^{3/2}\sqrt{4ad^2+c^3}+4ad^2+c^3\right)\sqrt{\frac{d^2(4ac+4c^2x^2+4cdx^3+d^2x^4)}{(4ad^2+c^3)\left(\frac{(c+dx)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)^2}}\left(\frac{(c+dx)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)F\left(2\tan^{-1}\left(\frac{c+dx}{\sqrt[4]{c}\sqrt[4]{c^3+4ad^2}}\right)|\frac{1}{2}\left(\frac{c^{3/2}}{\sqrt{c^3+4ad^2}}+1\right)\right)}{16ac^{5/4}d\left(4ad^2+c^3\right)^{3/4}\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}$$
$$+\frac{\sqrt[4]{c}\sqrt{\frac{d^2(4ac+4c^2x^2+4cdx^3+d^2x^4)}{(4ad^2+c^3)\left(\frac{(c+dx)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)^2}}\left(\frac{(c+dx)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)E\left(2\tan^{-1}\left(\frac{c+dx}{\sqrt[4]{c}\sqrt[4]{c^3+4ad^2}}\right)|\frac{1}{2}\left(\frac{c^{3/2}}{\sqrt{c^3+4ad^2}}+1\right)\right)}{8ad\sqrt[4]{4ad^2+c^3}\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(-3/2),x]
```

```
[Out] -((c + d*x)*(c^3 - 4*a*d^2 - c*(c + d*x)^2))/(8*a*c*d*(c^3 + 4*a*
d^2)*Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4]) - (d*(c + d*x
)*Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4])/(8*a*(c^3 + 4*a*
d^2)^(3/2)*(Sqrt[c] + (c + d*x)^2/Sqrt[c^3 + 4*a*d^2])) + (c^(1/4
)*Sqrt[(d^2*(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4))/((c^3 + 4*
a*d^2)*(Sqrt[c] + (c + d*x)^2/Sqrt[c^3 + 4*a*d^2])^2)]*(Sqrt[c] +
 (c + d*x)^2/Sqrt[c^3 + 4*a*d^2])*EllipticE[2*ArcTan[(c + d*x)/(c
^(1/4)*(c^3 + 4*a*d^2)^(1/4))], (1 + c^(3/2)/Sqrt[c^3 + 4*a*d^2])
/2])/(8*a*d*(c^3 + 4*a*d^2)^(1/4)*Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*
x^3 + d^2*x^4]) + ((c^3 + 4*a*d^2 - c^(3/2)*Sqrt[c^3 + 4*a*d^2])*
Sqrt[(d^2*(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4))/((c^3 + 4*a*
d^2)*(Sqrt[c] + (c + d*x)^2/Sqrt[c^3 + 4*a*d^2])^2)]*(Sqrt[c] + (
c + d*x)^2/Sqrt[c^3 + 4*a*d^2])*EllipticF[2*ArcTan[(c + d*x)/(c^(
1/4)*(c^3 + 4*a*d^2)^(1/4))], (1 + c^(3/2)/Sqrt[c^3 + 4*a*d^2])/2
])/(16*a*c^(5/4)*d*(c^3 + 4*a*d^2)^(3/4)*Sqrt[4*a*c + 4*c^2*x^2 +
 4*c*d*x^3 + d^2*x^4])
```

Rubi in Sympy [A] time = 131.157, size = 651, normalized size = 0.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**(3/2),x)
```

```
[Out] c**(1/4)*sqrt(d**2*(-2*c**2*(c/d + x)**2 + c*(4*a + c**3/d**2) +
d**2*(c/d + x)**4)/((sqrt(c) + d**2*(c/d + x)**2/sqrt(4*a*d**2 +
c**3))**2*(4*a*d**2 + c**3)))*(sqrt(c) + d**2*(c/d + x)**2/sqrt(4
```

```
*a*d**2 + c**3))*elliptic_e(2*atan(d*(c/d + x)/(c**(1/4)*(4*a*d**
2 + c**3)**(1/4))), c**(3/2)/(2*sqrt(4*a*d**2 + c**3)) + 1/2)/(8*
a*d*(4*a*d**2 + c**3)**(1/4)*sqrt(-2*c**2*(c/d + x)**2 + c*(4*a +
 c**3/d**2) + d**2*(c/d + x)**4)) - d**2*(c/d + x)*sqrt(-2*c**2*(
c/d + x)**2 + c*(4*a + c**3/d**2) + d**2*(c/d + x)**4)/(8*a*(sqrt
(c) + d**2*(c/d + x)**2/sqrt(4*a*d**2 + c**3))*(4*a*d**2 + c**3)*
*(3/2)) - (c/d + x)*(-8*a*d**2 + 2*c**3 - 2*c*d**2*(c/d + x)**2)/
(16*a*c*(4*a*d**2 + c**3)*sqrt(-2*c**2*(c/d + x)**2 + c*(4*a + c*
*3/d**2) + d**2*(c/d + x)**4)) + sqrt(d**2*(-2*c**2*(c/d + x)**2
+ c*(4*a + c**3/d**2) + d**2*(c/d + x)**4)/((sqrt(c) + d**2*(c/d
+ x)**2/sqrt(4*a*d**2 + c**3))**2*(4*a*d**2 + c**3)))*(sqrt(c) +
d**2*(c/d + x)**2/sqrt(4*a*d**2 + c**3))*(4*a*d**2 - c**(3/2)*sqr
t(4*a*d**2 + c**3) + c**3)*elliptic_f(2*atan(d*(c/d + x)/(c**(1/4
)*(4*a*d**2 + c**3)**(1/4))), c**(3/2)/(2*sqrt(4*a*d**2 + c**3))
+ 1/2)/(16*a*c**(5/4)*d*(4*a*d**2 + c**3)**(3/4)*sqrt(-2*c**2*(c/
d + x)**2 + c*(4*a + c**3/d**2) + d**2*(c/d + x)**4))
```

Mathematica [C] time = 6.20092, size = 5276, normalized size = 7.83

Result too large to show

Antiderivative was successfully verified.

```
[In]  Integrate[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(-3/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.067, size = 5024, normalized size = 7.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d^2x^4 + 4\,cdx^3 + 4\,c^2x^2 + 4\,ac\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c)^(-3/2),x, algorithm="maxima"
```

```
[Out] integrate((d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c)^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(d^2x^4 + 4\,cdx^3 + 4\,c^2x^2 + 4\,ac)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c)^(-3/2),x, algorithm="fricas"
```

```
[Out] integral((d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c)^(-3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(4ac + 4c^2x^2 + 4cdx^3 + d^2x^4)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**(3/2),x)
```

```
[Out] Integral((4*a*c + 4*c**2*x**2 + 4*c*d*x**3 + d**2*x**4)**(-3/2),
x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d^2x^4 + 4\,cdx^3 + 4\,c^2x^2 + 4\,ac)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c)^(-3/2),x, algorithm="giac")
```

```
[Out] integrate((d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c)^(-3/2), x)
```

3.622 $\int \sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}\,dx$

Optimal. Leaf size=663

$$\frac{1}{3}\left(\frac{d}{4e}+x\right)\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}-\frac{2d^2\left(\frac{d}{4e}+x\right)\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}{\sqrt{256ae^3+5d^4}\left(\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{256ae^3+5d^4}}+1\right)}$$

$$+\frac{\sqrt[4]{256ae^3+5d^4}\left(-3d^2\sqrt{256ae^3+5d^4}+256ae^3+5d^4\right)\sqrt{\frac{e(8ae^2-d^3x+8de^2x^3+8e^3x^4)}{(256ae^3+5d^4)\left(\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{256ae^3+5d^4}}+1\right)^2}}\left(\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{256ae^3+5d^4}}+1\right)F\left(2\tan^{-1}\left(\frac{d+4ex}{\sqrt[4]{5d^4+25\cdots}}\right.\right.}{48\sqrt{2}e^2\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}$$

$$+\frac{d^2\left(256ae^3+5d^4\right)^{3/4}\sqrt{\frac{e(8ae^2-d^3x+8de^2x^3+8e^3x^4)}{(256ae^3+5d^4)\left(\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{256ae^3+5d^4}}+1\right)^2}}\left(\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{256ae^3+5d^4}}+1\right)E\left(2\tan^{-1}\left(\frac{d+4ex}{\sqrt[4]{5d^4+256ae^3}}\right)|\frac{1}{2}\left(\frac{3d^2}{\sqrt{5d^4+256ae^3}}+1\right)\right)}{8\sqrt{2}e^2\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}$$

```
[Out] ((d/(4*e) + x)*Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4])/3
 - (2*d^2*(d/(4*e) + x)*Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^
3*x^4])/(Sqrt[5*d^4 + 256*a*e^3]*(1 + (16*e^2*(d/(4*e) + x)^2)/Sq
rt[5*d^4 + 256*a*e^3])) + (d^2*(5*d^4 + 256*a*e^3)^(3/4)*Sqrt[(e*
(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4))/((5*d^4 + 256*a*e^3)
*(1 + (16*e^2*(d/(4*e) + x)^2)/Sqrt[5*d^4 + 256*a*e^3])^2)]*(1 +
(16*e^2*(d/(4*e) + x)^2)/Sqrt[5*d^4 + 256*a*e^3])*EllipticE[2*Arc
Tan[(d + 4*e*x)/(5*d^4 + 256*a*e^3)^(1/4)], (1 + (3*d^2)/Sqrt[5*d
^4 + 256*a*e^3])/2])/(8*Sqrt[2]*e^2*Sqrt[8*a*e^2 - d^3*x + 8*d*e^
2*x^3 + 8*e^3*x^4]) + ((5*d^4 + 256*a*e^3)^(1/4)*(5*d^4 + 256*a*e
^3 - 3*d^2*Sqrt[5*d^4 + 256*a*e^3])*Sqrt[(e*(8*a*e^2 - d^3*x + 8*
d*e^2*x^3 + 8*e^3*x^4))/((5*d^4 + 256*a*e^3)*(1 + (16*e^2*(d/(4*e
) + x)^2)/Sqrt[5*d^4 + 256*a*e^3])^2)]*(1 + (16*e^2*(d/(4*e) + x)
^2)/Sqrt[5*d^4 + 256*a*e^3])*EllipticF[2*ArcTan[(d + 4*e*x)/(5*d^
4 + 256*a*e^3)^(1/4)], (1 + (3*d^2)/Sqrt[5*d^4 + 256*a*e^3])/2])/
(48*Sqrt[2]*e^2*Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4])
```

Rubi [A] time = 1.57782, antiderivative size = 663, normalized size of antiderivative = 1., number of

steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147

$$\frac{(d+4ex)\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}{12e}-\frac{d^2(d+4ex)\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}{2e\sqrt{256ae^3+5d^4}\left(\frac{(d+4ex)^2}{\sqrt{256ae^3+5d^4}}+1\right)}$$
$$+\frac{\sqrt[4]{256ae^3+5d^4}\left(-3d^2\sqrt{256ae^3+5d^4}+256ae^3+5d^4\right)\sqrt{\frac{e(8ae^2-d^3x+8de^2x^3+8e^3x^4)}{(256ae^3+5d^4)\left(\frac{(d+4ex)^2}{\sqrt{256ae^3+5d^4}}+1\right)^2}}\left(\frac{(d+4ex)^2}{\sqrt{256ae^3+5d^4}}+1\right)F\left(2\tan^{-1}\left(\frac{d+4ex}{\sqrt[4]{5d^4+25}}\right.\right.}{48\sqrt{2}e^2\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}$$
$$+\frac{d^2\left(256ae^3+5d^4\right)^{3/4}\sqrt{\frac{e(8ae^2-d^3x+8de^2x^3+8e^3x^4)}{(256ae^3+5d^4)\left(\frac{(d+4ex)^2}{\sqrt{256ae^3+5d^4}}+1\right)^2}}\left(\frac{(d+4ex)^2}{\sqrt{256ae^3+5d^4}}+1\right)E\left(2\tan^{-1}\left(\frac{d+4ex}{\sqrt[4]{5d^4+256ae^3}}\right)|\frac{1}{2}\left(\frac{3d^2}{\sqrt{5d^4+256ae^3}}+1\right)\right)}{8\sqrt{2}e^2\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4],x]
```

```
[Out] ((d + 4*e*x)*Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4])/(12
*e) - (d^2*(d + 4*e*x)*Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3
*x^4])/(2*e*Sqrt[5*d^4 + 256*a*e^3]*(1 + (d + 4*e*x)^2/Sqrt[5*d^4
 + 256*a*e^3])) + (d^2*(5*d^4 + 256*a*e^3)^(3/4)*Sqrt[(e*(8*a*e^2
 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4))/((5*d^4 + 256*a*e^3)*(1 + (d
 + 4*e*x)^2/Sqrt[5*d^4 + 256*a*e^3])^2)]*(1 + (d + 4*e*x)^2/Sqrt[
5*d^4 + 256*a*e^3])*EllipticE[2*ArcTan[(d + 4*e*x)/(5*d^4 + 256*a
*e^3)^(1/4)], (1 + (3*d^2)/Sqrt[5*d^4 + 256*a*e^3])/2])/(8*Sqrt[2
]*e^2*Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4]) + ((5*d^4
+ 256*a*e^3)^(1/4)*(5*d^4 + 256*a*e^3 - 3*d^2*Sqrt[5*d^4 + 256*a*
e^3])*Sqrt[(e*(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4))/((5*d^
4 + 256*a*e^3)*(1 + (d + 4*e*x)^2/Sqrt[5*d^4 + 256*a*e^3])^2)]*(1
 + (d + 4*e*x)^2/Sqrt[5*d^4 + 256*a*e^3])*EllipticF[2*ArcTan[(d +
 4*e*x)/(5*d^4 + 256*a*e^3)^(1/4)], (1 + (3*d^2)/Sqrt[5*d^4 + 256
*a*e^3])/2])/(48*Sqrt[2]*e^2*Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 +
 8*e^3*x^4])
```

Rubi in Sympy [A] time = 139.962, size = 673, normalized size = 1.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2)**(1/2),x)
```

```
[Out] -d**2*(d/(4*e) + x)*sqrt(-192*d**2*e*(d/(4*e) + x)**2 + 512*e**3*
(d/(4*e) + x)**4 + 2*(256*a*e**3 + 5*d**4)/e)/(4*sqrt(256*a*e**3
+ 5*d**4)*(16*e**2*(d/(4*e) + x)**2/sqrt(256*a*e**3 + 5*d**4) + 1
```

```
)) + sqrt(2)*d**2*sqrt((512*a*e**3 + 10*d**4 - 192*d**2*e**2*(d/(
4*e) + x)**2 + 512*e**4*(d/(4*e) + x)**4)/((256*a*e**3 + 5*d**4)*
(16*e**2*(d/(4*e) + x)**2/sqrt(256*a*e**3 + 5*d**4) + 1)**2))*(25
6*a*e**3 + 5*d**4)**(3/4)*(16*e**2*(d/(4*e) + x)**2/sqrt(256*a*e*
*3 + 5*d**4) + 1)*elliptic_e(2*atan(4*e*(d/(4*e) + x)/(256*a*e**3
 + 5*d**4)**(1/4)), 3*d**2/(2*sqrt(256*a*e**3 + 5*d**4)) + 1/2)/(
16*e**2*sqrt(-192*d**2*e*(d/(4*e) + x)**2 + 512*e**3*(d/(4*e) + x
)**4 + 2*(256*a*e**3 + 5*d**4)/e)) + (d/(4*e) + x)*sqrt(-192*d**2
*e*(d/(4*e) + x)**2 + 512*e**3*(d/(4*e) + x)**4 + 2*(256*a*e**3 +
 5*d**4)/e)/24 - sqrt(2)*sqrt((512*a*e**3 + 10*d**4 - 192*d**2*e*
*2*(d/(4*e) + x)**2 + 512*e**4*(d/(4*e) + x)**4)/((256*a*e**3 + 5
*d**4)*(16*e**2*(d/(4*e) + x)**2/sqrt(256*a*e**3 + 5*d**4) + 1)**
2))*(3*d**2 - sqrt(256*a*e**3 + 5*d**4))*(256*a*e**3 + 5*d**4)**(
3/4)*(16*e**2*(d/(4*e) + x)**2/sqrt(256*a*e**3 + 5*d**4) + 1)*ell
iptic_f(2*atan(4*e*(d/(4*e) + x)/(256*a*e**3 + 5*d**4)**(1/4)), 3
*d**2/(2*sqrt(256*a*e**3 + 5*d**4)) + 1/2)/(96*e**2*sqrt(-192*d**
2*e*(d/(4*e) + x)**2 + 512*e**3*(d/(4*e) + x)**4 + 2*(256*a*e**3
+ 5*d**4)/e))
```

Mathematica [B] time = 6.19812, size = 7543, normalized size = 11.38

Result too large to show

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4],x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.283, size = 7887, normalized size = 11.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{8\,e^3x^4 + 8\,de^2x^3 - d^3x + 8\,ae^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{8\,e^{3}x^{4} + 8\,de^{2}x^{3} - d^{3}x + 8\,ae^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{8ae^{2} - d^{3}x + 8de^{2}x^{3} + 8e^{3}x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2)**(1/2),x)
```

```
[Out] Integral(sqrt(8*a*e**2 - d**3*x + 8*d*e**2*x**3 + 8*e**3*x**4), x
)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{8\,e^{3}x^{4} + 8\,de^{2}x^{3} - d^{3}x + 8\,ae^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2), x)
```

3.623 $\int \frac{1}{\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}\,dx$

Optimal. Leaf size=235

$$\frac{\sqrt[4]{256ae^3+5d^4}\sqrt{\frac{e(8ae^2-d^3x+8de^2x^3+8e^3x^4)}{(256ae^3+5d^4)\left(\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{256ae^3+5d^4}}+1\right)^2}}\left(\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{256ae^3+5d^4}}+1\right)F\left(2\tan^{-1}\left(\frac{d+4ex}{\sqrt[4]{5d^4+256ae^3}}\right)|\frac{1}{2}\left(\frac{3d^2}{\sqrt{5d^4+256ae^3}}+1\right)\right)}{\sqrt{2}e\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}$$

```
[Out] ((5*d^4 + 256*a*e^3)^(1/4)*Sqrt[(e*(8*a*e^2 - d^3*x + 8*d*e^2*x^3
 + 8*e^3*x^4))/((5*d^4 + 256*a*e^3)*(1 + (16*e^2*(d/(4*e) + x)^2)
/Sqrt[5*d^4 + 256*a*e^3])^2)]*(1 + (16*e^2*(d/(4*e) + x)^2)/Sqrt[
5*d^4 + 256*a*e^3])*EllipticF[2*ArcTan[(d + 4*e*x)/(5*d^4 + 256*a
*e^3)^(1/4)], (1 + (3*d^2)/Sqrt[5*d^4 + 256*a*e^3])/2])/(Sqrt[2]*
e*Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4])
```

Rubi [A] time = 0.402613, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059

$$\frac{\sqrt[4]{256ae^3+5d^4}\sqrt{\frac{e(8ae^2-d^3x+8de^2x^3+8e^3x^4)}{(256ae^3+5d^4)\left(\frac{(d+4ex)^2}{\sqrt{256ae^3+5d^4}}+1\right)^2}}\left(\frac{(d+4ex)^2}{\sqrt{256ae^3+5d^4}}+1\right)F\left(2\tan^{-1}\left(\frac{d+4ex}{\sqrt[4]{5d^4+256ae^3}}\right)|\frac{1}{2}\left(\frac{3d^2}{\sqrt{5d^4+256ae^3}}+1\right)\right)}{\sqrt{2}e\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4],x]
```

```
[Out] ((5*d^4 + 256*a*e^3)^(1/4)*Sqrt[(e*(8*a*e^2 - d^3*x + 8*d*e^2*x^3
 + 8*e^3*x^4))/((5*d^4 + 256*a*e^3)*(1 + (d + 4*e*x)^2/Sqrt[5*d^4
 + 256*a*e^3])^2)]*(1 + (d + 4*e*x)^2/Sqrt[5*d^4 + 256*a*e^3])*El
lipticF[2*ArcTan[(d + 4*e*x)/(5*d^4 + 256*a*e^3)^(1/4)], (1 + (3*
d^2)/Sqrt[5*d^4 + 256*a*e^3])/2])/(Sqrt[2]*e*Sqrt[8*a*e^2 - d^3*x
 + 8*d*e^2*x^3 + 8*e^3*x^4])
```

Rubi in Sympy [A] time = 60.406, size = 243, normalized size = 1.03

$$\frac{\sqrt{2}\sqrt{\frac{256ae^3+5d^4-96d^2e^2\left(\frac{d}{4e}+x\right)^2+256e^4\left(\frac{d}{4e}+x\right)^4}{(256ae^3+5d^4)\left(\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{256ae^3+5d^4}}+1\right)^2}}\sqrt[4]{256ae^3+5d^4}\left(\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{256ae^3+5d^4}}+1\right)F\left(2\operatorname{atan}\left(\frac{4e\left(\frac{d}{4e}+x\right)}{\sqrt[4]{256ae^3+5d^4}}\right)\middle|\frac{3d^2}{2\sqrt{256ae^3+5d^4}}+\frac{1}{2}\right)}{2e\sqrt{-96d^2e\left(\frac{d}{4e}+x\right)^2+256e^3\left(\frac{d}{4e}+x\right)^4+\frac{256ae^3+5d^4}{e}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2)**(1/2),x)
```

```
[Out] sqrt(2)*sqrt((256*a*e**3 + 5*d**4 - 96*d**2*e**2*(d/(4*e) + x)**2
 + 256*e**4*(d/(4*e) + x)**4)/((256*a*e**3 + 5*d**4)*(16*e**2*(d/
(4*e) + x)**2/sqrt(256*a*e**3 + 5*d**4) + 1)**2))*(256*a*e**3 + 5
*d**4)**(1/4)*(16*e**2*(d/(4*e) + x)**2/sqrt(256*a*e**3 + 5*d**4)
 + 1)*elliptic_f(2*atan(4*e*(d/(4*e) + x)/(256*a*e**3 + 5*d**4)**
(1/4)), 3*d**2/(2*sqrt(256*a*e**3 + 5*d**4)) + 1/2)/(2*e*sqrt(-96
*d**2*e*(d/(4*e) + x)**2 + 256*e**3*(d/(4*e) + x)**4 + (256*a*e**
3 + 5*d**4)/e))
```

Mathematica [B] time = 4.27447, size = 1065, normalized size = 4.53

$$-\frac{\left(-d-4ex+\sqrt{3d^2-2\sqrt{d^4-64ae^3}}\right)\left(d+4ex-\sqrt{3d^2+2\sqrt{d^4-64ae^3}}\right)\sqrt{-\frac{\sqrt{3d^2-2\sqrt{d^4-64ae^3}}\left(d+4ex+\sqrt{3d^2+2\sqrt{d^4-64a}}\cdots\right.}{\left(\sqrt{3d^2-2\sqrt{d^4-64ae^3}}-\sqrt{3d^2+2\sqrt{d^4-64ae^3}}\right)\left(-d-4ex+\sqrt{3d}\cdots\right.}}\cdots}{2e\left(\sqrt{3d^2}\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4],x]
```

```
[Out] -((-d + Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] - 4*e*x)*(d - Sqrt[3
*d^2 + 2*Sqrt[d^4 - 64*a*e^3]] + 4*e*x)*Sqrt[-((Sqrt[3*d^2 - 2*Sq
rt[d^4 - 64*a*e^3]]*(d + Sqrt[3*d^2 + 2*Sqrt[d^4 - 64*a*e^3]] + 4
*e*x))/((Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] - Sqrt[3*d^2 + 2*Sq
rt[d^4 - 64*a*e^3]])*(-d + Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] -
 4*e*x)))]*Sqrt[(3*d^2 - 2*Sqrt[d^4 - 64*a*e^3] - Sqrt[3*d^2 - 2*
Sqrt[d^4 - 64*a*e^3]]*Sqrt[3*d^2 + 2*Sqrt[d^4 - 64*a*e^3]] + d*(S
qrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] - Sqrt[3*d^2 + 2*Sqrt[d^4 - 6
4*a*e^3]]) + 4*e*(Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] - Sqrt[3*d
^2 + 2*Sqrt[d^4 - 64*a*e^3]])*x)/((Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a
*e^3]] + Sqrt[3*d^2 + 2*Sqrt[d^4 - 64*a*e^3]])*(-d + Sqrt[3*d^2 -
 2*Sqrt[d^4 - 64*a*e^3]] - 4*e*x))]*EllipticF[ArcSin[Sqrt[((Sqrt[
3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] - Sqrt[3*d^2 + 2*Sqrt[d^4 - 64*a*
e^3]])*(d + Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] + 4*e*x))/((Sqrt
[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] + Sqrt[3*d^2 + 2*Sqrt[d^4 - 64*a
*e^3]])*(-d + Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] - 4*e*x))]], (
Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] + Sqrt[3*d^2 + 2*Sqrt[d^4 -
64*a*e^3]])^2/(Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] - Sqrt[3*d^2
+ 2*Sqrt[d^4 - 64*a*e^3]])^2])/(2*e*(Sqrt[3*d^2 - 2*Sqrt[d^4 - 64
*a*e^3]] - Sqrt[3*d^2 + 2*Sqrt[d^4 - 64*a*e^3]])*Sqrt[(Sqrt[3*d^2
 - 2*Sqrt[d^4 - 64*a*e^3]]*(-d + Sqrt[3*d^2 + 2*Sqrt[d^4 - 64*a*e
^3]] - 4*e*x))/((Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] + Sqrt[3*d^
2 + 2*Sqrt[d^4 - 64*a*e^3]])*(-d + Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a
```

```
*e^3]] - 4*e*x))]*Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4]
)
```

Maple [B] time = 0.045, size = 1704, normalized size = 7.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2),x)
```

```
[Out] 1/2*(1/4*(-d*e+(3*d^2*e^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2
+1/4*(d*e+(3*d^2*e^2-2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)*((-
1/4*(d*e+(3*d^2*e^2-2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2+1/4*(
d*e+(3*d^2*e^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)*(x-1/4*(-
d*e+(3*d^2*e^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)/(-1/4*(d*
e+(3*d^2*e^2-2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2-1/4*(-d*e+(3
*d^2*e^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)/(x+1/4*(d*e+(3*
d^2*e^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2))^(1/2)*(x+1/4*(d
*e+(3*d^2*e^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)^2*((-1/4*(
d*e+(3*d^2*e^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2-1/4*(-d*e+
(3*d^2*e^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)*(x-1/4*(-d*e+
(3*d^2*e^2-2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)/(1/4*(-d*e+(3
*d^2*e^2-2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2-1/4*(-d*e+(3*d^2
*e^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)/(x+1/4*(d*e+(3*d^2*
e^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2))^(1/2)*((-1/4*(d*e+(
3*d^2*e^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2-1/4*(-d*e+(3*d^
2*e^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)*(x+1/4*(d*e+(3*d^2
*e^2-2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)/(-1/4*(d*e+(3*d^2*e
^2-2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2-1/4*(-d*e+(3*d^2*e^2+2
*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)/(x+1/4*(d*e+(3*d^2*e^2+2*
(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2))^(1/2)/(-1/4*(d*e+(3*d^2*e
^2-2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2+1/4*(d*e+(3*d^2*e^2+2*
(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)/(-1/4*(d*e+(3*d^2*e^2+2*(-
64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2-1/4*(-d*e+(3*d^2*e^2+2*(-64*a
*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)*2^(1/2)/(e^3*(x-1/4*(-d*e+(3*d^2
*e^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)*(x+1/4*(d*e+(3*d^2*
e^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)*(x-1/4*(-d*e+(3*d^2*
e^2-2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)*(x+1/4*(d*e+(3*d^2*e
^2-2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2))^(1/2)*EllipticF(((-1
/4*(d*e+(3*d^2*e^2-2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2+1/4*(d
*e+(3*d^2*e^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)*(x-1/4*(-d
*e+(3*d^2*e^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)/(-1/4*(d*e
+(3*d^2*e^2-2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2-1/4*(-d*e+(3*
d^2*e^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)/(x+1/4*(d*e+(3*d
^2*e^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2))^(1/2),((-1/4*(d*
e+(3*d^2*e^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2-1/4*(-d*e+(3
*d^2*e^2-2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)*(1/4*(-d*e+(3*d
^2*e^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2+1/4*(d*e+(3*d^2*e^
```

```
2-2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)/(1/4*(-d*e+(3*d^2*e^2+
2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2-1/4*(-d*e+(3*d^2*e^2-2*(-
64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)/(-1/4*(d*e+(3*d^2*e^2+2*(-64
*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2+1/4*(d*e+(3*d^2*e^2-2*(-64*a*e^
3+d^4)^(1/2)*e^2)^(1/2))/e^2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{8\,e^3x^4 + 8\,de^2x^3 - d^3x + 8\,ae^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sqrt(8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2),x, algorithm="maxima
```

```
[Out] integrate(1/sqrt(8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{8\,e^3x^4 + 8\,de^2x^3 - d^3x + 8\,ae^2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sqrt(8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2),x, algorithm="fricas
```

```
[Out] integral(1/sqrt(8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(8*a*e**2 - d**3*x + 8*d*e**2*x**3 + 8*e**3*x**4),
 x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{8\,e^3x^4 + 8\,de^2x^3 - d^3x + 8\,ae^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sqrt(8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2), x)
```

3.624 $\int \frac{1}{(8ae^2-d^3x+8de^2x^3+8e^3x^4)^{3/2}}\,dx$

Optimal. Leaf size=748

$$\frac{4e\left(\frac{d}{4e}+x\right)\left(-256ae^3+13d^4-48d^2e^2\left(\frac{d}{4e}+x\right)^2\right)}{(-16384a^2e^6-64ad^4e^3+5d^8)\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}+\frac{384d^2e^2\left(\frac{d}{4e}+x\right)\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}{(d^4-64ae^3)(256ae^3+5d^4)^{3/2}\left(\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{256ae^3+5d^4}}+1\right)}-\frac{2\sqrt{2}\left(-3d^2\sqrt{256ae^3+5d^4}+256ae^3+5d^4\right)\sqrt{\frac{e(8ae^2-d^3x+8de^2x^3+8e^3x^4)}{(256ae^3+5d^4)\left(\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{256ae^3+5d^4}}+1\right)^2}}\left(\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{256ae^3+5d^4}}+1\right)F\left(2\tan^{-1}\left(\frac{d+4ex}{\sqrt[4]{5d^4+256ae^3}}\right)|\frac{1}{2}\right.}{(d^4-64ae^3)(256ae^3+5d^4)^{3/4}\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}-\frac{12\sqrt{2}d^2\sqrt{\frac{e(8ae^2-d^3x+8de^2x^3+8e^3x^4)}{(256ae^3+5d^4)\left(\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{256ae^3+5d^4}}+1\right)^2}}\left(\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{256ae^3+5d^4}}+1\right)E\left(2\tan^{-1}\left(\frac{d+4ex}{\sqrt[4]{5d^4+256ae^3}}\right)|\frac{1}{2}\left(\frac{3d^2}{\sqrt{5d^4+256ae^3}}+1\right)\right)}{(d^4-64ae^3)\sqrt[4]{256ae^3+5d^4}\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}$$

```
[Out] (4*e*(d/(4*e) + x)*(13*d^4 - 256*a*e^3 - 48*d^2*e^2*(d/(4*e) + x)
^2))/((5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)*Sqrt[8*a*e^2 - d^3*x
 + 8*d*e^2*x^3 + 8*e^3*x^4]) + (384*d^2*e^2*(d/(4*e) + x)*Sqrt[8*
a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4])/((d^4 - 64*a*e^3)*(5*d^
4 + 256*a*e^3)^(3/2)*(1 + (16*e^2*(d/(4*e) + x)^2)/Sqrt[5*d^4 + 2
56*a*e^3])) - (12*Sqrt[2]*d^2*Sqrt[(e*(8*a*e^2 - d^3*x + 8*d*e^2*
x^3 + 8*e^3*x^4))/((5*d^4 + 256*a*e^3)*(1 + (16*e^2*(d/(4*e) + x)
^2)/Sqrt[5*d^4 + 256*a*e^3])^2)]*(1 + (16*e^2*(d/(4*e) + x)^2)/Sq
rt[5*d^4 + 256*a*e^3])*EllipticE[2*ArcTan[(d + 4*e*x)/(5*d^4 + 25
6*a*e^3)^(1/4)], (1 + (3*d^2)/Sqrt[5*d^4 + 256*a*e^3])/2])/((d^4
- 64*a*e^3)*(5*d^4 + 256*a*e^3)^(1/4)*Sqrt[8*a*e^2 - d^3*x + 8*d*
e^2*x^3 + 8*e^3*x^4]) - (2*Sqrt[2]*(5*d^4 + 256*a*e^3 - 3*d^2*Sqr
t[5*d^4 + 256*a*e^3])*Sqrt[(e*(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*
e^3*x^4))/((5*d^4 + 256*a*e^3)*(1 + (16*e^2*(d/(4*e) + x)^2)/Sqrt
[5*d^4 + 256*a*e^3])^2)]*(1 + (16*e^2*(d/(4*e) + x)^2)/Sqrt[5*d^4
 + 256*a*e^3])*EllipticF[2*ArcTan[(d + 4*e*x)/(5*d^4 + 256*a*e^3)
^(1/4)], (1 + (3*d^2)/Sqrt[5*d^4 + 256*a*e^3])/2])/((d^4 - 64*a*e
^3)*(5*d^4 + 256*a*e^3)^(3/4)*Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3
+ 8*e^3*x^4])
```

Rubi [A] time = 1.77893, antiderivative size = 748, normalized size of antiderivative = 1., number of

steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$

$$\frac{(d+4ex)\left(-256ae^3+13d^4-3d^2(d+4ex)^2\right)}{(-16384a^2e^6-64ad^4e^3+5d^8)\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}+\frac{96d^2e(d+4ex)\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}{(d^4-64ae^3)\left(256ae^3+5d^4\right)^{3/2}\left(\frac{(d+4ex)^2}{\sqrt{256ae^3+5d^4}}+1\right)}-\frac{2\sqrt{2}\left(-3d^2\sqrt{256ae^3+5d^4}+256ae^3+5d^4\right)\sqrt{\frac{e(8ae^2-d^3x+8de^2x^3+8e^3x^4)}{(256ae^3+5d^4)\left(\frac{(d+4ex)^2}{\sqrt{256ae^3+5d^4}}+1\right)^2}}\left(\frac{(d+4ex)^2}{\sqrt{256ae^3+5d^4}}+1\right)F\left(2\tan^{-1}\left(\frac{d+4ex}{\sqrt[4]{5d^4+256ae^3}}\right)\middle|\frac{1}{2}\left(\right.\right.}{(d^4-64ae^3)\left(256ae^3+5d^4\right)^{3/4}\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}-\frac{12\sqrt{2}d^2\sqrt{\frac{e(8ae^2-d^3x+8de^2x^3+8e^3x^4)}{(256ae^3+5d^4)\left(\frac{(d+4ex)^2}{\sqrt{256ae^3+5d^4}}+1\right)^2}}\left(\frac{(d+4ex)^2}{\sqrt{256ae^3+5d^4}}+1\right)E\left(2\tan^{-1}\left(\frac{d+4ex}{\sqrt[4]{5d^4+256ae^3}}\right)\middle|\frac{1}{2}\left(\frac{3d^2}{\sqrt{5d^4+256ae^3}}+1\right)\right)}{(d^4-64ae^3)\sqrt[4]{256ae^3+5d^4}\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^(-3/2),x]
```

```
[Out] ((d + 4*e*x)*(13*d^4 - 256*a*e^3 - 3*d^2*(d + 4*e*x)^2))/((5*d^8
- 64*a*d^4*e^3 - 16384*a^2*e^6)*Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^
3 + 8*e^3*x^4]) + (96*d^2*e*(d + 4*e*x)*Sqrt[8*a*e^2 - d^3*x + 8*
d*e^2*x^3 + 8*e^3*x^4])/((d^4 - 64*a*e^3)*(5*d^4 + 256*a*e^3)^(3/
2)*(1 + (d + 4*e*x)^2/Sqrt[5*d^4 + 256*a*e^3])) - (12*Sqrt[2]*d^2
*Sqrt[(e*(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4))/((5*d^4 + 2
56*a*e^3)*(1 + (d + 4*e*x)^2/Sqrt[5*d^4 + 256*a*e^3])^2)]*(1 + (d
 + 4*e*x)^2/Sqrt[5*d^4 + 256*a*e^3])*EllipticE[2*ArcTan[(d + 4*e*
x)/(5*d^4 + 256*a*e^3)^(1/4)], (1 + (3*d^2)/Sqrt[5*d^4 + 256*a*e^
3])/2])/((d^4 - 64*a*e^3)*(5*d^4 + 256*a*e^3)^(1/4)*Sqrt[8*a*e^2
- d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4]) - (2*Sqrt[2]*(5*d^4 + 256*a*e
^3 - 3*d^2*Sqrt[5*d^4 + 256*a*e^3])*Sqrt[(e*(8*a*e^2 - d^3*x + 8*
d*e^2*x^3 + 8*e^3*x^4))/((5*d^4 + 256*a*e^3)*(1 + (d + 4*e*x)^2/S
qrt[5*d^4 + 256*a*e^3])^2)]*(1 + (d + 4*e*x)^2/Sqrt[5*d^4 + 256*a
*e^3])*EllipticF[2*ArcTan[(d + 4*e*x)/(5*d^4 + 256*a*e^3)^(1/4)],
 (1 + (3*d^2)/Sqrt[5*d^4 + 256*a*e^3])/2])/((d^4 - 64*a*e^3)*(5*d
^4 + 256*a*e^3)^(3/4)*Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*
x^4])
```

Rubi in Sympy [A] time = 157.726, size = 777, normalized size = 1.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2)**(3/2),x)
```

```
[Out] 48*sqrt(2)*d**2*e**2*(d/(4*e) + x)*sqrt(-96*d**2*e*(d/(4*e) + x)*
*2 + 256*e**3*(d/(4*e) + x)**4 + (256*a*e**3 + 5*d**4)/e)/((-64*a
*e**3 + d**4)*(256*a*e**3 + 5*d**4)**(3/2)*(16*e**2*(d/(4*e) + x)
**2/sqrt(256*a*e**3 + 5*d**4) + 1)) - 12*sqrt(2)*d**2*sqrt((256*a
*e**3 + 5*d**4 - 96*d**2*e**2*(d/(4*e) + x)**2 + 256*e**4*(d/(4*e
) + x)**4)/((256*a*e**3 + 5*d**4)*(16*e**2*(d/(4*e) + x)**2/sqrt(
256*a*e**3 + 5*d**4) + 1)**2))*(16*e**2*(d/(4*e) + x)**2/sqrt(256
*a*e**3 + 5*d**4) + 1)*elliptic_e(2*atan(4*e*(d/(4*e) + x)/(256*a
*e**3 + 5*d**4)**(1/4)), 3*d**2/(2*sqrt(256*a*e**3 + 5*d**4)) + 1
/2)/((-64*a*e**3 + d**4)*(256*a*e**3 + 5*d**4)**(1/4)*sqrt(-96*d*
*2*e*(d/(4*e) + x)**2 + 256*e**3*(d/(4*e) + x)**4 + (256*a*e**3 +
 5*d**4)/e)) + 128*sqrt(2)*e*(d/(4*e) + x)*(-131072*a*e**3 + 6656
*d**4 - 24576*d**2*e**2*(d/(4*e) + x)**2)/((-67108864*a**2*e**6 -
 262144*a*d**4*e**3 + 20480*d**8)*sqrt(-96*d**2*e*(d/(4*e) + x)**
2 + 256*e**3*(d/(4*e) + x)**4 + (256*a*e**3 + 5*d**4)/e)) - 2*sqr
t(2)*sqrt((256*a*e**3 + 5*d**4 - 96*d**2*e**2*(d/(4*e) + x)**2 +
256*e**4*(d/(4*e) + x)**4)/((256*a*e**3 + 5*d**4)*(16*e**2*(d/(4*
e) + x)**2/sqrt(256*a*e**3 + 5*d**4) + 1)**2))*(16*e**2*(d/(4*e)
+ x)**2/sqrt(256*a*e**3 + 5*d**4) + 1)*(256*a*e**3 + 5*d**4 - 3*d
**2*sqrt(256*a*e**3 + 5*d**4))*elliptic_f(2*atan(4*e*(d/(4*e) + x
)/(256*a*e**3 + 5*d**4)**(1/4)), 3*d**2/(2*sqrt(256*a*e**3 + 5*d*
*4)) + 1/2)/((-64*a*e**3 + d**4)*(256*a*e**3 + 5*d**4)**(3/4)*sqr
t(-96*d**2*e*(d/(4*e) + x)**2 + 256*e**3*(d/(4*e) + x)**4 + (256*
a*e**3 + 5*d**4)/e))
```

Mathematica [B] time = 6.23499, size = 7629, normalized size = 10.2

Result too large to show

Antiderivative was successfully verified.

```
[In]  Integrate[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^(-3/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.067, size = 8103, normalized size = 10.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(8\,e^3x^4+8\,de^2x^3-d^3x+8\,ae^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2)^(-3/2),x, algorithm="maxima
```

```
[Out] integrate((8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2)^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\left(8\,e^3x^4+8\,de^2x^3-d^3x+8\,ae^2\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2)^(-3/2),x, algorithm="fricas
```

```
[Out] integral((8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2)^(-3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2)**(3/2),x)
```

```
[Out] Integral((8*a*e**2 - d**3*x + 8*d*e**2*x**3 + 8*e**3*x**4)**(-3/2
), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(8\,e^3x^4+8\,de^2x^3-d^3x+8\,ae^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2)^(-3/2),x, algorithm="giac"

[Out] integrate((8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2)^(-3/2), x)
```

3.625 $\int \left(a+8x-8x^2+4x^3-x^4\right)^{3/2}\,dx$

Optimal. Leaf size=452

$$\frac{1}{7}(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}+\frac{2}{35}(x-1)\left(5a-3(x-1)^2+13\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}-\frac{16(2a+7)\left(1-\sqrt{a+4}\right)(x-1)\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)}{35\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{4(a+3)(5a+16)\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)F\left(\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{35\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{16(2a+7)\left(1-\sqrt{a+4}\right)\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)E\left(\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{35\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

```
[Out] (-16*(7 + 2*a)*(1 - Sqrt[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]
))*(-1 + x))/(35*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (2*(1
3 + 5*a - 3*(-1 + x)^2)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]*(
-1 + x))/35 + ((3 + a - 2*(-1 + x)^2 - (-1 + x)^4)^(3/2)*(-1 + x)
)/7 + (16*(7 + 2*a)*(1 - Sqrt[4 + a])*Sqrt[1 + Sqrt[4 + a]]*(1 +
(-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticE[ArcTan[(-1 + x)/Sqrt[1 +
Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(35*Sqrt[(1 +
 (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))
]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (4*(3 + a)*(16 + 5*a
)*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*Ellipt
icF[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 -
 Sqrt[4 + a])])/(35*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 +
(-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 +
x)^4])
```

Rubi [A] time = 1.40408, antiderivative size = 452, normalized size of antiderivative = 1., number of

steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$-\frac{1}{7}(1-x)\left(a-(1-x)^4-2(1-x)^2+3\right)^{3/2} - \frac{2}{35}(1-x)\left(5a-3(1-x)^2+13\right)\sqrt{a-(1-x)^4-2(1-x)^2+3} + \frac{16(2a+7)\left(1-\sqrt{a+4}\right)(1-x)\left(\frac{(1-x)^2}{1-\sqrt{a+4}}+1\right)}{35\sqrt{a-(1-x)^4-2(1-x)^2+3}} - \frac{4(a+3)(5a+16)\sqrt{\sqrt{a+4}+1}\left(\frac{(1-x)^2}{1-\sqrt{a+4}}+1\right)F\left(\tan^{-1}\left(\frac{1-x}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{35\sqrt{\frac{\frac{(1-x)^2}{1-\sqrt{a+4}}+1}{\frac{(1-x)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(1-x)^4-2(1-x)^2+3}} - \frac{16(2a+7)\left(1-\sqrt{a+4}\right)\sqrt{\sqrt{a+4}+1}\left(\frac{(1-x)^2}{1-\sqrt{a+4}}+1\right)E\left(\tan^{-1}\left(\frac{1-x}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{35\sqrt{\frac{\frac{(1-x)^2}{1-\sqrt{a+4}}+1}{\frac{(1-x)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(1-x)^4-2(1-x)^2+3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

```
[Out] (16*(7 + 2*a)*(1 - Sqrt[4 + a])*(1 + (1 - x)^2/(1 - Sqrt[4 + a]))
*(1 - x))/(35*Sqrt[3 + a - 2*(1 - x)^2 - (1 - x)^4]) - (2*(13 + 5
*a - 3*(1 - x)^2)*Sqrt[3 + a - 2*(1 - x)^2 - (1 - x)^4]*(1 - x))/
35 - ((3 + a - 2*(1 - x)^2 - (1 - x)^4)^(3/2)*(1 - x))/7 - (16*(7
 + 2*a)*(1 - Sqrt[4 + a])*Sqrt[1 + Sqrt[4 + a]]*(1 + (1 - x)^2/(1
 - Sqrt[4 + a]))*EllipticE[ArcTan[(1 - x)/Sqrt[1 + Sqrt[4 + a]]],
 (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(35*Sqrt[(1 + (1 - x)^2/(1
- Sqrt[4 + a]))/(1 + (1 - x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2
*(1 - x)^2 - (1 - x)^4]) - (4*(3 + a)*(16 + 5*a)*Sqrt[1 + Sqrt[4
+ a]]*(1 + (1 - x)^2/(1 - Sqrt[4 + a]))*EllipticF[ArcTan[(1 - x)/
Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(35*
Sqrt[(1 + (1 - x)^2/(1 - Sqrt[4 + a]))/(1 + (1 - x)^2/(1 + Sqrt[4
 + a]))]*Sqrt[3 + a - 2*(1 - x)^2 - (1 - x)^4])
```

Rubi in Sympy [A] time = 78.0523, size = 369, normalized size = 0.82

$$-\frac{16(2a+7)(x-1)\left(\frac{(x-1)^2}{-\sqrt{a+4}+1}+1\right)\left(-\sqrt{a+4}+1\right)}{35\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(x-1)\left(10a-6(x-1)^2+26\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}{35}+\frac{(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{\frac{3}{2}}}{7}+\frac{4(a+3)(5a+16)\left(\frac{(x-1)^2}{-\sqrt{a+4}+1}+1\right)\sqrt{\sqrt{a+4}+1}F\left(\operatorname{atan}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)\middle|\frac{2\sqrt{a+4}}{\sqrt{a+4}-1}\right)}{35\sqrt{\frac{-\frac{(x-1)^2}{\sqrt{a+4}-1}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{16(2a+7)\left(\frac{(x-1)^2}{-\sqrt{a+4}+1}+1\right)\left(-\sqrt{a+4}+1\right)\sqrt{\sqrt{a+4}+1}E\left(\operatorname{atan}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)\middle|\frac{2\sqrt{a+4}}{\sqrt{a+4}-1}\right)}{35\sqrt{\frac{-\frac{(x-1)^2}{\sqrt{a+4}-1}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**4+4*x**3-8*x**2+a+8*x)**(3/2),x)
```

```
[Out] -16*(2*a + 7)*(x - 1)*((x - 1)**2/(-sqrt(a + 4) + 1) + 1)*(-sqrt(
a + 4) + 1)/(35*sqrt(a - (x - 1)**4 - 2*(x - 1)**2 + 3)) + (x - 1
)*(10*a - 6*(x - 1)**2 + 26)*sqrt(a - (x - 1)**4 - 2*(x - 1)**2 +
 3)/35 + (x - 1)*(a - (x - 1)**4 - 2*(x - 1)**2 + 3)**(3/2)/7 + 4
*(a + 3)*(5*a + 16)*((x - 1)**2/(-sqrt(a + 4) + 1) + 1)*sqrt(sqrt
(a + 4) + 1)*elliptic_f(atan((x - 1)/sqrt(sqrt(a + 4) + 1)), 2*sq
rt(a + 4)/(sqrt(a + 4) - 1))/(35*sqrt((-(x - 1)**2/(sqrt(a + 4) -
 1) + 1)/((x - 1)**2/(sqrt(a + 4) + 1) + 1))*sqrt(a - (x - 1)**4
- 2*(x - 1)**2 + 3)) + 16*(2*a + 7)*((x - 1)**2/(-sqrt(a + 4) + 1
) + 1)*(-sqrt(a + 4) + 1)*sqrt(sqrt(a + 4) + 1)*elliptic_e(atan((
x - 1)/sqrt(sqrt(a + 4) + 1)), 2*sqrt(a + 4)/(sqrt(a + 4) - 1))/(
35*sqrt((-(x - 1)**2/(sqrt(a + 4) - 1) + 1)/((x - 1)**2/(sqrt(a +
 4) + 1) + 1))*sqrt(a - (x - 1)**4 - 2*(x - 1)**2 + 3))
```

Mathematica [B] time = 6.18464, size = 6287, normalized size = 13.91

Result too large to show

Antiderivative was successfully verified.

```
[In]  Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.083, size = 2655, normalized size = 5.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x)
```

```
[Out] -1/7*x^5*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+5/7*x^4*(-x^4+4*x^3-8*x^2
+a+8*x)^(1/2)-66/35*x^3*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+14/5*x^2*(
-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+(3/7*a-32/35)*x*(-x^4+4*x^3-8*x^2+a
+8*x)^(1/2)+(-3/7*a-4/7)*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)-(a^2-(3/7
*a-32/35)*a+12/7*a+16/7)*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))
^(1/2))*((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-
1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(
1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(4+a)^(1/2))^(
1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)^(1/2))^(1/2))/(
(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/
2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2)
)^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-
1+(4+a)^(1/2))^(1/2)))^(1/2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(
1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/2)/(-(x-1-(-1+(4+a)^(1/2))^(1/2)
)*(x-1+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+
(-1-(4+a)^(1/2))^(1/2)))^(1/2)*EllipticF(((-(-1-(4+a)^(1/2))^(1/2
)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a
)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2
)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-
(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/
2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2
))^(1/2)))^(1/2))-(64/35*a+32/5)*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a
)^(1/2))^(1/2))*((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))
*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^
(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(4+a)^
(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)^(1/2))^
(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(
4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+
a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))
/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1
+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/2)/(-(x-1-(-1+(4+a)^(1/2
))^(1/2))*(x-1+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2
))*(x-1+(-1-(4+a)^(1/2))^(1/2)))^(1/2)*((1-(-1+(4+a)^(1/2))^(1/2)
)*EllipticF(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-
1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2
))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))
^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^
(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((
-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))+2*(-1+(4+a)
```

```
^(1/2))^(1/2)*EllipticPi(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2
))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(
-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),(-(-1-
(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/
2)+(-1+(4+a)^(1/2))^(1/2)),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1
/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1
-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/
2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2)))+(-32/35*a-16/5)*((x-1-(-1+(4+
a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2
))^(1/2))+((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-1-
(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^
(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+
(4+a)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1
+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2)
)^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/
2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2))^(1/2))/(-(-1-
(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^
(1/2)))^(1/2)*(-1/2*((1-(-1+(4+a)^(1/2))^(1/2))*(1+(-1+(4+a)^(1/2
))^(1/2))-(1-(-1-(4+a)^(1/2))^(1/2))*(1+(-1+(4+a)^(1/2))^(1/2))+(
1-(-1-(4+a)^(1/2))^(1/2))*(1-(-1+(4+a)^(1/2))^(1/2))+(1-(-1+(4+a)
^(1/2))^(1/2))^2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2)
)/(-1+(4+a)^(1/2))^(1/2)*EllipticF(((-(-1-(4+a)^(1/2))^(1/2)+(-1+
(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2
))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1
/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^
(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1
+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/
2)))^(1/2))-1/2*(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*
EllipticE(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-
(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))
^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(
1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1
/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1
-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))/(-1+(4+a)^(1/
2))^(1/2)-4/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*Elli
pticPi(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1
+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1
/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-1-(4+a)^(1/2))^(1/2)+
(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^
(1/2)),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+
a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+
(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^
(1/2)))^(1/2))))/(-(x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1+(-1+(4+a)^(1
/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1
/2)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-x^4 + 4\,x^3 - 8\,x^2 + a + 8\,x\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-x^4+4\,x^3-8\,x^2+a+8\,x\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a-x^4+4x^3-8x^2+8x\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+4*x**3-8*x**2+a+8*x)**(3/2),x)
```

```
[Out] Integral((a - x**4 + 4*x**3 - 8*x**2 + 8*x)**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-x^4+4\,x^3-8\,x^2+a+8\,x\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2), x)
```

3.626 $\int \sqrt{a+8x-8x^2+4x^3-x^4}\,dx$

Optimal. Leaf size=397

$$\frac{1}{3}(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}-\frac{2\left(1-\sqrt{a+4}\right)(x-1)\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)}{3\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{2(a+3)\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)F\left(\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{3\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{2\left(1-\sqrt{a+4}\right)\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)E\left(\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{3\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

```
[Out] (-2*(1 - Sqrt[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(-1 + x)
)/(3*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (Sqrt[3 + a - 2*(
-1 + x)^2 - (-1 + x)^4]*(-1 + x))/3 + (2*(1 - Sqrt[4 + a])*Sqrt[1
 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticE[ArcT
an[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4
+ a])])/(3*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^
2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) +
(2*(3 + a)*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]
))*EllipticF[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 +
 a])/(1 - Sqrt[4 + a])])/(3*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]
))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2
- (-1 + x)^4])
```

Rubi [A] time = 1.07841, antiderivative size = 397, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$

$$-\frac{1}{3}(1-x)\sqrt{a-(1-x)^4-2(1-x)^2+3}+\frac{2\left(1-\sqrt{a+4}\right)(1-x)\left(\frac{(1-x)^2}{1-\sqrt{a+4}}+1\right)}{3\sqrt{a-(1-x)^4-2(1-x)^2+3}}-\frac{2(a+3)\sqrt{\sqrt{a+4}+1}\left(\frac{(1-x)^2}{1-\sqrt{a+4}}+1\right)F\left(\tan^{-1}\left(\frac{1-x}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{3\sqrt{\frac{\frac{(1-x)^2}{1-\sqrt{a+4}}+1}{\frac{(1-x)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(1-x)^4-2(1-x)^2+3}}-\frac{2\left(1-\sqrt{a+4}\right)\sqrt{\sqrt{a+4}+1}\left(\frac{(1-x)^2}{1-\sqrt{a+4}}+1\right)E\left(\tan^{-1}\left(\frac{1-x}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{3\sqrt{\frac{\frac{(1-x)^2}{1-\sqrt{a+4}}+1}{\frac{(1-x)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(1-x)^4-2(1-x)^2+3}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] (2*(1 - Sqrt[4 + a])*(1 + (1 - x)^2/(1 - Sqrt[4 + a]))*(1 - x))/(
3*Sqrt[3 + a - 2*(1 - x)^2 - (1 - x)^4]) - (Sqrt[3 + a - 2*(1 - x
)^2 - (1 - x)^4]*(1 - x))/3 - (2*(1 - Sqrt[4 + a])*Sqrt[1 + Sqrt[
4 + a]]*(1 + (1 - x)^2/(1 - Sqrt[4 + a]))*EllipticE[ArcTan[(1 - x
)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(3
*Sqrt[(1 + (1 - x)^2/(1 - Sqrt[4 + a]))/(1 + (1 - x)^2/(1 + Sqrt[
4 + a]))]*Sqrt[3 + a - 2*(1 - x)^2 - (1 - x)^4]) - (2*(3 + a)*Sqr
t[1 + Sqrt[4 + a]]*(1 + (1 - x)^2/(1 - Sqrt[4 + a]))*EllipticF[Ar
cTan[(1 - x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4
 + a])])/(3*Sqrt[(1 + (1 - x)^2/(1 - Sqrt[4 + a]))/(1 + (1 - x)^2
/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(1 - x)^2 - (1 - x)^4])
```

Rubi in Sympy [A] time = 68.8061, size = 320, normalized size = 0.81

$$-\frac{(x-1)\left(\frac{(x-1)^2}{-\sqrt{a+4}+1}+1\right)\left(-\frac{2\sqrt{a+4}}{3}+\frac{2}{3}\right)}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}}{3}+\frac{2(a+3)\left(\frac{(x-1)^2}{-\sqrt{a+4}+1}+1\right)\sqrt{\sqrt{a+4}+1}F\left(\operatorname{atan}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)\middle|\frac{2\sqrt{a+4}}{\sqrt{a+4}-1}\right)}{3\sqrt{\frac{-\frac{(x-1)^2}{\sqrt{a+4}-1}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{2\left(\frac{(x-1)^2}{-\sqrt{a+4}+1}+1\right)\left(-\sqrt{a+4}+1\right)\sqrt{\sqrt{a+4}+1}E\left(\operatorname{atan}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)\middle|\frac{2\sqrt{a+4}}{\sqrt{a+4}-1}\right)}{3\sqrt{\frac{-\frac{(x-1)^2}{\sqrt{a+4}-1}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**4+4*x**3-8*x**2+a+8*x)**(1/2),x)
```

```
[Out] -(x - 1)*((x - 1)**2/(-sqrt(a + 4) + 1) + 1)*(-2*sqrt(a + 4)/3 +
2/3)/sqrt(a - (x - 1)**4 - 2*(x - 1)**2 + 3) + (x - 1)*sqrt(a - (
x - 1)**4 - 2*(x - 1)**2 + 3)/3 + 2*(a + 3)*((x - 1)**2/(-sqrt(a
+ 4) + 1) + 1)*sqrt(sqrt(a + 4) + 1)*elliptic_f(atan((x - 1)/sqrt
(sqrt(a + 4) + 1)), 2*sqrt(a + 4)/(sqrt(a + 4) - 1))/(3*sqrt((-(x
 - 1)**2/(sqrt(a + 4) - 1) + 1)/((x - 1)**2/(sqrt(a + 4) + 1) + 1
))*sqrt(a - (x - 1)**4 - 2*(x - 1)**2 + 3)) + 2*((x - 1)**2/(-sqr
t(a + 4) + 1) + 1)*(-sqrt(a + 4) + 1)*sqrt(sqrt(a + 4) + 1)*ellip
tic_e(atan((x - 1)/sqrt(sqrt(a + 4) + 1)), 2*sqrt(a + 4)/(sqrt(a
+ 4) - 1))/(3*sqrt((-(x - 1)**2/(sqrt(a + 4) - 1) + 1)/((x - 1)**
2/(sqrt(a + 4) + 1) + 1))*sqrt(a - (x - 1)**4 - 2*(x - 1)**2 + 3)
```

)

Mathematica [B] time = 6.09448, size = 3470, normalized size = 8.74

Result too large to show

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] (-1/3 + x/3)*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4] + (2*((4*(-Sqrt[
-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[
4 + a]] + x)^2*Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4
+ a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]
] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*S
qrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x))/((
Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 -
Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 +
Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 +
a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*EllipticF[ArcSin[Sqrt[((
-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 -
 Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 +
 a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]], ((-Sqrt[-1 - Sqrt[4 +
 a]] - Sqrt[-1 + Sqrt[4 + a]])*(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1
+ Sqrt[4 + a]]))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]
])*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))])/(Sqrt[-1
 - Sqrt[4 + a]]*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]
)*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4]) + (2*a*(-Sqrt[-1 - Sqrt[4
+ a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)
^2*Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 +
 Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1
+ Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1
 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sq
rt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]]
 + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]]
 + x))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - S
qrt[-1 - Sqrt[4 + a]] + x))]*EllipticF[ArcSin[Sqrt[((-Sqrt[-1 - S
qrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]
] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 -
Sqrt[-1 - Sqrt[4 + a]] + x))]], ((-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[
-1 + Sqrt[4 + a]])*(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a
]]))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-Sqrt[-1
 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))])/(Sqrt[-1 - Sqrt[4 +
a]]*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*Sqrt[a + 8
*x - 8*x^2 + 4*x^3 - x^4]) + (4*(-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-
1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((Sqrt
[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt
[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])
*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*
(-1 - Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqr
```

```
t[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sq
rt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1
 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4
+ a]] + x))]*((-1 - Sqrt[-1 - Sqrt[4 + a]])*EllipticF[ArcSin[Sqrt
[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1
 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4
 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 +
a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1
 + Sqrt[4 + a]])^2] + 2*Sqrt[-1 - Sqrt[4 + a]]*EllipticPi[(Sqrt[-
1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])/(-Sqrt[-1 - Sqrt[4 + a
]] + Sqrt[-1 + Sqrt[4 + a]]), ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]
] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((
Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - S
qrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 +
 a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2]))/(S
qrt[-1 - Sqrt[4 + a]]*(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4
+ a]])*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4]) - ((-1 + Sqrt[-1 - Sq
rt[4 + a]] + x)*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x)*(-1 + Sqrt[-1 +
 Sqrt[4 + a]] + x) + 2*(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4
 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((Sqrt[-1 - Sqrt
[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] +
 x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt
[-1 - Sqrt[4 + a]] - x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt
[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqr
t[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sq
rt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4
+ a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)
)]*(((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*EllipticE[
ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-
1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[
-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1
- Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]
] - Sqrt[-1 + Sqrt[4 + a]])^2])/(2*Sqrt[-1 - Sqrt[4 + a]]) + ((-(
(-1 - Sqrt[-1 - Sqrt[4 + a]])*(-2 - Sqrt[-1 - Sqrt[4 + a]] - Sqrt
[-1 + Sqrt[4 + a]])) + (-1 + Sqrt[-1 - Sqrt[4 + a]])*(Sqrt[-1 - S
qrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]]))*EllipticF[ArcSin[Sqrt[((Sq
rt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sq
rt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]
])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] +
 Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sq
rt[4 + a]])^2])/(2*Sqrt[-1 - Sqrt[4 + a]]*(-Sqrt[-1 - Sqrt[4 + a]
] + Sqrt[-1 + Sqrt[4 + a]])) + (4*EllipticPi[(Sqrt[-1 - Sqrt[4 +
a]] + Sqrt[-1 + Sqrt[4 + a]])/(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1
+ Sqrt[4 + a]]), ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 +
 Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqr
t[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] -
 x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt
[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2])/(-Sqrt[-1 - Sqrt
[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])))/Sqrt[a + 8*x - 8*x^2 + 4*x^3
 - x^4]))/3
```

Maple [B] time = 0.027, size = 2519, normalized size = 6.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x)
```

```
[Out] 1/3*x*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)-1/3*(-x^4+4*x^3-8*x^2+a+8*x)
^(1/2)-(2/3*a+4/3)*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2)
)*((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a
)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/
(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(4+a)^(1/2))^(1/2))^
2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)^(1/2))^(1/2))/((-1-(4
+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1
/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2))^(1/2
))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a
)^(1/2))^(1/2)))^(1/2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^
(1/2))/(-1+(4+a)^(1/2))^(1/2)/(-(x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1
+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4
+a)^(1/2))^(1/2)))^(1/2)*EllipticF(((-(-1-(4+a)^(1/2))^(1/2)+(-1+
(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2
))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1
/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^
(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1
+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/
2)))^(1/2))-4/3*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(
(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(
1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-
1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(4+a)^(1/2))^(1/2))^2*(
-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)^(1/2))^(1/2))/((-1-(4+a)
^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)
))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2))^(1/2))/
(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(
1/2))^(1/2)))^(1/2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/
2))/(-1+(4+a)^(1/2))^(1/2)/(-(x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1+(-
1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)
^(1/2))^(1/2)))^(1/2)*((1-(-1+(4+a)^(1/2))^(1/2))*EllipticF(((-(-
1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2)
)^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-
1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^
(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(
-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(
1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))+2*(-1+(4+a)^(1/2))^(1/2)*Ell
ipticPi(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-
1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(
1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),(-(-1-(4+a)^(1/2))^(1/2
)-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2
))^(1/2)),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-
(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/
2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2
))^(1/2)))^(1/2)))-2/3*((x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+
```

```
a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2))+((-1-(4+a)^(1/2))^(
1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^
(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/
2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(x
-1+(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-
(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2
))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)
*(x-1+(-1-(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^
(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-1/2*((1-(-1+(
4+a)^(1/2))^(1/2))*(1+(-1+(4+a)^(1/2))^(1/2))-(1-(-1-(4+a)^(1/2))
^(1/2))*(1+(-1+(4+a)^(1/2))^(1/2))+(1-(-1-(4+a)^(1/2))^(1/2))*(1-
(-1+(4+a)^(1/2))^(1/2))+(1-(-1+(4+a)^(1/2))^(1/2))^2)/(-(-1-(4+a)
^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/2)*Elli
pticF(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+
(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/
2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)
-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))
^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+
a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))-1/2*(-(-1-(4+a)^(
1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*EllipticE(((-(-1-(4+a)^(1/2))
^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1
-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))
^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*
((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2)
)^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)
^(1/2))^(1/2)))^(1/2))/(-1+(4+a)^(1/2))^(1/2)-4/(-(-1-(4+a)^(1/2)
)^(1/2)+(-1+(4+a)^(1/2))^(1/2))*EllipticPi(((-(-1-(4+a)^(1/2))^(1
/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4
+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1
/2)))^(1/2),((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-
(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)),((-(-1-(4+a)^(1/2))^(1
/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/
2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-
(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))))/(-(x-1-(-1+(
4+a)^(1/2))^(1/2))*(x-1+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1
/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x^4+4x^3-8x^2+a+8x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{-x^4 + 4\,x^3 - 8\,x^2 + a + 8\,x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a - x^4 + 4x^3 - 8x^2 + 8x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**4+4*x**3-8*x**2+a+8*x)**(1/2),x)
```

```
[Out] Integral(sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x^4 + 4\,x^3 - 8\,x^2 + a + 8\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x), x)
```

3.627 $\int \frac{1}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx$

Optimal. Leaf size=144

$$\frac{\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)F\left(\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

```
[Out] (Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*Ellipti
cF[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 -
Sqrt[4 + a])])/(Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1
+ x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4
])
```

Rubi [A] time = 0.237258, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$-\frac{\sqrt{\sqrt{a+4}+1}\left(\frac{(1-x)^2}{1-\sqrt{a+4}}+1\right)F\left(\tan^{-1}\left(\frac{1-x}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(1-x)^2}{1-\sqrt{a+4}}+1}{\frac{(1-x)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(1-x)^4-2(1-x)^2+3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] -((Sqrt[1 + Sqrt[4 + a]]*(1 + (1 - x)^2/(1 - Sqrt[4 + a]))*Ellipt
icF[ArcTan[(1 - x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 -
Sqrt[4 + a])])/(Sqrt[(1 + (1 - x)^2/(1 - Sqrt[4 + a]))/(1 + (1 -
x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(1 - x)^2 - (1 - x)^4]))
```

Rubi in Sympy [A] time = 27.1649, size = 112, normalized size = 0.78

$$\frac{\left(\frac{(x-1)^2}{-\sqrt{a+4}+1}+1\right)\sqrt{\sqrt{a+4}+1}F\left(\operatorname{atan}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)\middle|\frac{2\sqrt{a+4}}{\sqrt{a+4}-1}\right)}{\sqrt{\frac{-\frac{(x-1)^2}{\sqrt{a+4}-1}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2\,(x-1)^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-x**4+4*x**3-8*x**2+a+8*x)**(1/2),x)
```

```
[Out] ((x - 1)**2/(-sqrt(a + 4) + 1) + 1)*sqrt(sqrt(a + 4) + 1)*ellipti
c_f(atan((x - 1)/sqrt(sqrt(a + 4) + 1)), 2*sqrt(a + 4)/(sqrt(a +
4) - 1))/(sqrt((-(x - 1)**2/(sqrt(a + 4) - 1) + 1)/((x - 1)**2/(s
qrt(a + 4) + 1) + 1))*sqrt(a - (x - 1)**4 - 2*(x - 1)**2 + 3))
```

Mathematica [B] time = 2.51937, size = 540, normalized size = 3.75

$$\frac{2\left(\sqrt{-\sqrt{a+4}-1}-x+1\right)\sqrt{\frac{\sqrt{-\sqrt{a+4}-1}\left(\sqrt{\sqrt{a+4}-1}-x+1\right)}{\left(\sqrt{-\sqrt{a+4}-1}+\sqrt{\sqrt{a+4}-1}\right)\left(\sqrt{-\sqrt{a+4}-1}-x+1\right)}}\left(\sqrt{-\sqrt{a+4}-1}+x-1\right)\sqrt{\frac{\sqrt{-\sqrt{a+4}-1}\left(\sqrt{\sqrt{a+4}-1}+x-1\right)}{\left(\sqrt{\sqrt{a+4}-1}-\sqrt{-\sqrt{a+4}-1}\right)\left(\sqrt{-\sqrt{a+4}-1}-\right.}}}{\sqrt{-\sqrt{a+4}-1}\sqrt{\frac{\left(\sqrt{-\sqrt{a+4}-1}-\sqrt{\sqrt{a+4}-1}\right)\left(\sqrt{-\sqrt{a+4}-1}+x-1\right)}{\left(\sqrt{-\sqrt{a+4}-1}+\sqrt{\sqrt{a+4}-1}\right)\left(\sqrt{-\sqrt{a+4}-1}-x+1\right)}}\sqrt{a-x(x}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] (2*(1 + Sqrt[-1 - Sqrt[4 + a]] - x)*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*
(1 + Sqrt[-1 + Sqrt[4 + a]] - x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt
[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]*(-1 + Sqrt
[-1 - Sqrt[4 + a]] + x)*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-
1 + Sqrt[4 + a]] + x))/((-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt
[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]*EllipticF[ArcSin[Sqr
t[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-
1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[
4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 +
 a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-
1 + Sqrt[4 + a]])^2])/(Sqrt[-1 - Sqrt[4 + a]]*Sqrt[((Sqrt[-1 - Sq
rt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]]
 + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sq
rt[-1 - Sqrt[4 + a]] - x))]*Sqrt[a - x*(-8 + 8*x - 4*x^2 + x^3)])
```

Maple [B] time = 0.025, size = 530, normalized size = 3.7

$$-1\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\sqrt{1\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(x-1-\sqrt{-1+\sqrt{4+a}}\right)\left(-\sqrt{-1-\sqrt{4+a}}-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x)
```

```
[Out] -((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-1-(4+a)^(1/
2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-
(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/
2))^(1/2)))^(1/2)*(x-1+(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1
/2))^(1/2)*(x-1-(-1-(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(
-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-
1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/
2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(
1/2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(
1/2))^(1/2)/(-(x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1+(-1+(4+a)^(1/2))^
(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2)))
^(1/2)*EllipticF(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2)
)*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)
^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^(
1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(
4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2
))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^4 + 4x^3 - 8x^2 + a + 8x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{-x^4 + 4x^3 - 8x^2 + a + 8x}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a - x^4 + 4x^3 - 8x^2 + 8x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-x**4+4*x**3-8*x**2+a+8*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^4 + 4\,x^3 - 8\,x^2 + a + 8\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x), x)
```

3.628 $\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx$

Optimal. Leaf size=437

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}-\frac{\left(1-\sqrt{a+4}\right)(x-1)\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)}{2(a+3)(a+4)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)F\left(\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{2(a+4)\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{\left(1-\sqrt{a+4}\right)\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)E\left(\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{2(a+3)(a+4)\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

```
[Out] ((5 + a + (-1 + x)^2)*(-1 + x))/(2*(12 + 7*a + a^2)*Sqrt[3 + a -
2*(-1 + x)^2 - (-1 + x)^4]) - ((1 - Sqrt[4 + a])*(1 + (-1 + x)^2/
(1 - Sqrt[4 + a]))*(-1 + x))/(2*(3 + a)*(4 + a)*Sqrt[3 + a - 2*(-
1 + x)^2 - (-1 + x)^4]) + ((1 - Sqrt[4 + a])*Sqrt[1 + Sqrt[4 + a]
]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticE[ArcTan[(-1 + x)/Sq
rt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(2*(3
+ a)*(4 + a)*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x
)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])
+ (Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*Ellip
ticF[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1
- Sqrt[4 + a])])/(2*(4 + a)*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]
))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2
- (-1 + x)^4])
```

Rubi [A] time = 1.13469, antiderivative size = 437, normalized size of antiderivative = 1., number of

steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$

$$-\frac{(1-x)\left(a+(x-1)^2+5\right)}{2\left(a^2+7a+12\right)\sqrt{a-(1-x)^4-2(1-x)^2+3}}+\frac{\left(1-\sqrt{a+4}\right)(1-x)\left(\frac{(1-x)^2}{1-\sqrt{a+4}}+1\right)}{2(a+3)(a+4)\sqrt{a-(1-x)^4-2(1-x)^2+3}}$$
$$-\frac{\sqrt{\sqrt{a+4}+1}\left(\frac{(1-x)^2}{1-\sqrt{a+4}}+1\right)F\left(\tan^{-1}\left(\frac{1-x}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{2(a+4)\sqrt{\frac{\frac{(1-x)^2}{1-\sqrt{a+4}}+1}{\frac{(1-x)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(1-x)^4-2(1-x)^2+3}}$$
$$-\frac{\left(1-\sqrt{a+4}\right)\sqrt{\sqrt{a+4}+1}\left(\frac{(1-x)^2}{1-\sqrt{a+4}}+1\right)E\left(\tan^{-1}\left(\frac{1-x}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{2(a+3)(a+4)\sqrt{\frac{\frac{(1-x)^2}{1-\sqrt{a+4}}+1}{\frac{(1-x)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(1-x)^4-2(1-x)^2+3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-3/2),x]
```

```
[Out] ((1 - Sqrt[4 + a])*(1 + (1 - x)^2/(1 - Sqrt[4 + a]))*(1 - x))/(2*
(3 + a)*(4 + a)*Sqrt[3 + a - 2*(1 - x)^2 - (1 - x)^4]) - ((5 + a
+ (-1 + x)^2)*(1 - x))/(2*(12 + 7*a + a^2)*Sqrt[3 + a - 2*(1 - x)
^2 - (1 - x)^4]) - ((1 - Sqrt[4 + a])*Sqrt[1 + Sqrt[4 + a]]*(1 +
(1 - x)^2/(1 - Sqrt[4 + a]))*EllipticE[ArcTan[(1 - x)/Sqrt[1 + Sq
rt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(2*(3 + a)*(4 +
 a)*Sqrt[(1 + (1 - x)^2/(1 - Sqrt[4 + a]))/(1 + (1 - x)^2/(1 + Sq
rt[4 + a]))]*Sqrt[3 + a - 2*(1 - x)^2 - (1 - x)^4]) - (Sqrt[1 + S
qrt[4 + a]]*(1 + (1 - x)^2/(1 - Sqrt[4 + a]))*EllipticF[ArcTan[(1
 - x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])]
)/(2*(4 + a)*Sqrt[(1 + (1 - x)^2/(1 - Sqrt[4 + a]))/(1 + (1 - x)^
2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(1 - x)^2 - (1 - x)^4])
```

Rubi in Sympy [A] time = 69.56, size = 345, normalized size = 0.79

$$-\frac{(x-1)\left(\frac{(x-1)^2}{-\sqrt{a+4}+1}+1\right)\left(-\sqrt{a+4}+1\right)}{2(a+3)(a+4)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(x-1)\left(2a+2(x-1)^2+10\right)}{4(a+3)(a+4)\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$
$$+\frac{\left(\frac{(x-1)^2}{-\sqrt{a+4}+1}+1\right)\sqrt{\sqrt{a+4}+1}F\left(\operatorname{atan}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)\middle|\frac{2\sqrt{a+4}}{\sqrt{a+4}-1}\right)}{2\sqrt{\frac{-\frac{(x-1)^2}{\sqrt{a+4}-1}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}(a+4)\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$
$$+\frac{\left(\frac{(x-1)^2}{-\sqrt{a+4}+1}+1\right)\left(-\sqrt{a+4}+1\right)\sqrt{\sqrt{a+4}+1}E\left(\operatorname{atan}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)\middle|\frac{2\sqrt{a+4}}{\sqrt{a+4}-1}\right)}{2\sqrt{\frac{-\frac{(x-1)^2}{\sqrt{a+4}-1}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}(a+3)(a+4)\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-x**4+4*x**3-8*x**2+a+8*x)**(3/2),x)
```

```
[Out] -(x - 1)*((x - 1)**2/(-sqrt(a + 4) + 1) + 1)*(-sqrt(a + 4) + 1)/(
2*(a + 3)*(a + 4)*sqrt(a - (x - 1)**4 - 2*(x - 1)**2 + 3)) + (x -
 1)*(2*a + 2*(x - 1)**2 + 10)/(4*(a + 3)*(a + 4)*sqrt(a - (x - 1)
**4 - 2*(x - 1)**2 + 3)) + ((x - 1)**2/(-sqrt(a + 4) + 1) + 1)*sq
rt(sqrt(a + 4) + 1)*elliptic_f(atan((x - 1)/sqrt(sqrt(a + 4) + 1)
), 2*sqrt(a + 4)/(sqrt(a + 4) - 1))/(2*sqrt((-(x - 1)**2/(sqrt(a
+ 4) - 1) + 1)/((x - 1)**2/(sqrt(a + 4) + 1) + 1))*(a + 4)*sqrt(a
 - (x - 1)**4 - 2*(x - 1)**2 + 3)) + ((x - 1)**2/(-sqrt(a + 4) +
1) + 1)*(-sqrt(a + 4) + 1)*sqrt(sqrt(a + 4) + 1)*elliptic_e(atan(
(x - 1)/sqrt(sqrt(a + 4) + 1)), 2*sqrt(a + 4)/(sqrt(a + 4) - 1))/
(2*sqrt((-(x - 1)**2/(sqrt(a + 4) - 1) + 1)/((x - 1)**2/(sqrt(a +
 4) + 1) + 1))*(a + 3)*(a + 4)*sqrt(a - (x - 1)**4 - 2*(x - 1)**2
 + 3))
```

Mathematica [B] time = 6.12438, size = 3526, normalized size = 8.07

Result too large to show

Antiderivative was successfully verified.

```
[In]  Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-3/2),x]
```

```
[Out] ((6 + a - 8*x - a*x + 3*x^2 - x^3)*Sqrt[a + 8*x - 8*x^2 + 4*x^3 -
 x^4])/(2*(3 + a)*(4 + a)*(-a - 8*x + 8*x^2 - 4*x^3 + x^4)) + ((4
*(-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1
 - Sqrt[4 + a]] + x)^2*Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 +
```

Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqr
t[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]]
+ x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[4 + a]]
+ x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sq
rt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sq
rt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + S
qrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*EllipticF[ArcSin
[Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + S
qrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 +
Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]], ((-Sqrt[-1 -
Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(Sqrt[-1 - Sqrt[4 + a]] +
Sqrt[-1 + Sqrt[4 + a]]))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqr
t[4 + a]])*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))])/
(Sqrt[-1 - Sqrt[4 + a]]*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt
[4 + a]])*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4]) + (2*a*(-Sqrt[-1 -
 Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 +
a]] + x)^2*Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]
])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] +
Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[
(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt
[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt
[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt
[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])
*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*EllipticF[ArcSin[Sqrt[((-Sqr
t[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqr
t[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]
)*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]], ((-Sqrt[-1 - Sqrt[4 + a]]
 - Sqrt[-1 + Sqrt[4 + a]])*(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sq
rt[4 + a]]))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(
-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))])/(Sqrt[-1 - S
qrt[4 + a]]*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*Sq
rt[a + 8*x - 8*x^2 + 4*x^3 - x^4]) + (4*(-Sqrt[-1 - Sqrt[4 + a]]
- Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqr
t[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-
1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[
4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]*Sqrt[(Sqrt[-1 - Sqrt[
4 + a]]*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a
]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*
Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/(
(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 -
 Sqrt[4 + a]] + x))]*((-1 - Sqrt[-1 - Sqrt[4 + a]])*EllipticF[Arc
Sin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 +
 Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1
+ Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - S
qrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] -
 Sqrt[-1 + Sqrt[4 + a]])^2] + 2*Sqrt[-1 - Sqrt[4 + a]]*EllipticPi
[(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])/(-Sqrt[-1 - Sq
rt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]), ArcSin[Sqrt[((Sqrt[-1 - Sqr
t[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]]
+ x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqr
t[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 +
Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])
^2]))/(Sqrt[-1 - Sqrt[4 + a]]*(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 +
 Sqrt[4 + a]])*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4]) - ((-1 + Sqrt

```
[-1 - Sqrt[4 + a]] + x)*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x)*(-1 + S
qrt[-1 + Sqrt[4 + a]] + x) + 2*(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1
+ Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((Sqrt[-
1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4
 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(
1 + Sqrt[-1 - Sqrt[4 + a]] - x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-
1 - Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[
-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt
[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 -
 Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 +
a]] + x))]*(((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*El
lipticE[ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 +
 a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]]
 + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (
Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqr
t[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2])/(2*Sqrt[-1 - Sqrt[4 + a]]
) + ((-((-1 - Sqrt[-1 - Sqrt[4 + a]])*(-2 - Sqrt[-1 - Sqrt[4 + a]
] - Sqrt[-1 + Sqrt[4 + a]])) + (-1 + Sqrt[-1 - Sqrt[4 + a]])*(Sqr
t[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]]))*EllipticF[ArcSin[S
qrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt
[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqr
t[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4
 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt
[-1 + Sqrt[4 + a]])^2])/(2*Sqrt[-1 - Sqrt[4 + a]]*(-Sqrt[-1 - Sqr
t[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])) + (4*EllipticPi[(Sqrt[-1 - S
qrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])/(-Sqrt[-1 - Sqrt[4 + a]] +
Sqrt[-1 + Sqrt[4 + a]]), ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - S
qrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[
-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4
 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])
^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2])/(-Sqrt[-
1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])))/Sqrt[a + 8*x - 8*x^2
 + 4*x^3 - x^4])/(2*(3 + a)*(4 + a))
```

Maple [B] time = 0.03, size = 2601, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x)
```

```
[Out] 2*(1/4/(a^2+7*a+12)*x^3-3/4/(a^2+7*a+12)*x^2+1/4*(8+a)/(a^2+7*a+1
2)*x-1/4*(6+a)/(a^2+7*a+12))/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)-((a+5
)/(a^2+7*a+12)-1/2*(8+a)/(a^2+7*a+12))*((-1-(4+a)^(1/2))^(1/2)+(-
1+(4+a)^(1/2))^(1/2))*((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^
(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+
(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+
(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)^(
```

```
1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1
+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(
-1-(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^
(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)/(-(-1-(4+a)^(1/2))^(1/
2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/2)/(-(x-1-(-1+(4+a
)^(1/2))^(1/2))*(x-1+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2)
)^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2)))^(1/2)*EllipticF(((-(-1-(4+
a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/
2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+
a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2)
)^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4
+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-
(-1+(4+a)^(1/2))^(1/2)))^(1/2))-1/(a^2+7*a+12)*((-1-(4+a)^(1/2))^
(1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)
^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1
/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(
x-1+(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1
-(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/
2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2
)*(x-1+(-1-(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)
^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)/(-(-1-(4+a)^(1
/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/2)/(-(x-1-
(-1+(4+a)^(1/2))^(1/2))*(x-1+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+
a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2)))^(1/2)*((1-(-1+(4+a
)^(1/2))^(1/2))*EllipticF(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/
2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-
(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-
1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1
/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1
/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2
))+2*(-1+(4+a)^(1/2))^(1/2)*EllipticPi(((-(-1-(4+a)^(1/2))^(1/2)+
(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^
(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2))
)^(1/2),(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4
+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2)),((-(-1-(4+a)^(1/2))^(1/2
)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2)
)^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4
+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2)))-1/2/(a^2+7*a+12
)*((x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1
+(-1-(4+a)^(1/2))^(1/2))+((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))
^(1/2))*((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-
1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(
1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(4+a)^(1/2))^(
1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)^(1/2))^(1/2))/(
(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/
2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2)
)^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-
1+(4+a)^(1/2))^(1/2)))^(1/2)*(-1/2*((1-(-1+(4+a)^(1/2))^(1/2))*(1
+(-1+(4+a)^(1/2))^(1/2))-(1-(-1-(4+a)^(1/2))^(1/2))*(1+(-1+(4+a)^
(1/2))^(1/2))+(1-(-1-(4+a)^(1/2))^(1/2))*(1-(-1+(4+a)^(1/2))^(1/2
))+(1-(-1+(4+a)^(1/2))^(1/2))^2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+
a)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/2)*EllipticF(((-(-1-(4+a)^(1
/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(
-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1
```

```
/2))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/
2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(
1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(
4+a)^(1/2))^(1/2)))^(1/2))-1/2*(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)
^(1/2))^(1/2))*EllipticE(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2
))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(
-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1
-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/
2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/
2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2)
)/(-1+(4+a)^(1/2))^(1/2)-4/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/
2))^(1/2))*EllipticPi(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^
(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+
(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-1-(4+a
)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-
1+(4+a)^(1/2))^(1/2)),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^
(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a
)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-
1+(4+a)^(1/2))^(1/2)))^(1/2))))/(-(x-1-(-1+(4+a)^(1/2))^(1/2))*(x
-1+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-
(4+a)^(1/2))^(1/2)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-x^4+4\,x^3-8\,x^2+a+8\,x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(-3/2),x, algorithm="maxima")
```

```
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{1}{\left(x^4-4\,x^3+8\,x^2-a-8\,x\right)\sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(-3/2),x, algorithm="fricas")
```

```
[Out] integral(-1/((x^4 - 4*x^3 + 8*x^2 - a - 8*x)*sqrt(-x^4 + 4*x^3 -
8*x^2 + a + 8*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-x**4+4*x**3-8*x**2+a+8*x)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-x^4 + 4\,x^3 - 8\,x^2 + a + 8\,x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(-3/2),x, algorithm="giac")
```

```
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(-3/2), x)
```

3.629 $\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx$

Optimal. Leaf size=517

$$\frac{(x-1)\left(5a^2+4(2a+7)(x-1)^2+47a+104\right)}{12(a+3)^2(a+4)^2\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}-\frac{(2a+7)\left(1-\sqrt{a+4}\right)(x-1)\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)}{3(a+3)^2(a+4)^2\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(5a+16)\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)F\left(\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{12(a+3)(a+4)^2\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(2a+7)\left(1-\sqrt{a+4}\right)\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)E\left(\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{3(a+3)^2(a+4)^2\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

```
[Out] ((5 + a + (-1 + x)^2)*(-1 + x))/(6*(12 + 7*a + a^2)*(3 + a - 2*(-
1 + x)^2 - (-1 + x)^4)^(3/2)) + ((104 + 47*a + 5*a^2 + 4*(7 + 2*a
)*(-1 + x)^2)*(-1 + x))/(12*(3 + a)^2*(4 + a)^2*Sqrt[3 + a - 2*(-
1 + x)^2 - (-1 + x)^4]) - ((7 + 2*a)*(1 - Sqrt[4 + a])*(1 + (-1 +
 x)^2/(1 - Sqrt[4 + a]))*(-1 + x))/(3*(3 + a)^2*(4 + a)^2*Sqrt[3
+ a - 2*(-1 + x)^2 - (-1 + x)^4]) + ((7 + 2*a)*(1 - Sqrt[4 + a])*
Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*Elliptic
E[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - S
qrt[4 + a])])/(3*(3 + a)^2*(4 + a)^2*Sqrt[(1 + (-1 + x)^2/(1 - Sq
rt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-
1 + x)^2 - (-1 + x)^4]) + ((16 + 5*a)*Sqrt[1 + Sqrt[4 + a]]*(1 +
(-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticF[ArcTan[(-1 + x)/Sqrt[1 +
Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(12*(3 + a)*(
4 + a)^2*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/
(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])
```

Rubi [A] time = 1.45568, antiderivative size = 517, normalized size of antiderivative = 1., number of

steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$-\frac{(1-x)\left(a+(x-1)^2+5\right)}{6\left(a^2+7a+12\right)\left(a-(1-x)^4-2(1-x)^2+3\right)^{3/2}} - \frac{(1-x)\left(5a^2+4(2a+7)(1-x)^2+47a+104\right)}{12(a+3)^2(a+4)^2\sqrt{a-(1-x)^4-2(1-x)^2+3}} + \frac{(2a+7)\left(1-\sqrt{a+4}\right)(1-x)\left(\frac{(1-x)^2}{1-\sqrt{a+4}}+1\right)}{3(a+3)^2(a+4)^2\sqrt{a-(1-x)^4-2(1-x)^2+3}} - \frac{(5a+16)\sqrt{\sqrt{a+4}+1}\left(\frac{(1-x)^2}{1-\sqrt{a+4}}+1\right)F\left(\tan^{-1}\left(\frac{1-x}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{12(a+3)(a+4)^2\sqrt{\frac{\frac{(1-x)^2}{1-\sqrt{a+4}}+1}{\frac{(1-x)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(1-x)^4-2(1-x)^2+3}} - \frac{(2a+7)\left(1-\sqrt{a+4}\right)\sqrt{\sqrt{a+4}+1}\left(\frac{(1-x)^2}{1-\sqrt{a+4}}+1\right)E\left(\tan^{-1}\left(\frac{1-x}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{3(a+3)^2(a+4)^2\sqrt{\frac{\frac{(1-x)^2}{1-\sqrt{a+4}}+1}{\frac{(1-x)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(1-x)^4-2(1-x)^2+3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-5/2),x]
```

```
[Out] -((104 + 47*a + 5*a^2 + 4*(7 + 2*a)*(1 - x)^2)*(1 - x))/(12*(3 +
a)^2*(4 + a)^2*Sqrt[3 + a - 2*(1 - x)^2 - (1 - x)^4]) + ((7 + 2*a
)*(1 - Sqrt[4 + a])*(1 + (1 - x)^2/(1 - Sqrt[4 + a]))*(1 - x))/(3
*(3 + a)^2*(4 + a)^2*Sqrt[3 + a - 2*(1 - x)^2 - (1 - x)^4]) - ((5
 + a + (-1 + x)^2)*(1 - x))/(6*(12 + 7*a + a^2)*(3 + a - 2*(1 - x
)^2 - (1 - x)^4)^(3/2)) - ((7 + 2*a)*(1 - Sqrt[4 + a])*Sqrt[1 + S
qrt[4 + a]]*(1 + (1 - x)^2/(1 - Sqrt[4 + a]))*EllipticE[ArcTan[(1
 - x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])]
)/(3*(3 + a)^2*(4 + a)^2*Sqrt[(1 + (1 - x)^2/(1 - Sqrt[4 + a]))/(
1 + (1 - x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(1 - x)^2 - (1 -
 x)^4]) - ((16 + 5*a)*Sqrt[1 + Sqrt[4 + a]]*(1 + (1 - x)^2/(1 - S
qrt[4 + a]))*EllipticF[ArcTan[(1 - x)/Sqrt[1 + Sqrt[4 + a]]], (-2
*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(12*(3 + a)*(4 + a)^2*Sqrt[(1 +
 (1 - x)^2/(1 - Sqrt[4 + a]))/(1 + (1 - x)^2/(1 + Sqrt[4 + a]))]*
Sqrt[3 + a - 2*(1 - x)^2 - (1 - x)^4])
```

Rubi in Sympy [A] time = 87.3058, size = 427, normalized size = 0.83

$$\frac{(x-1)\left(2a+2(x-1)^2+10\right)}{12(a+3)(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)^{\frac{3}{2}}}-\frac{(2a+7)(x-1)\left(\frac{(x-1)^2}{-\sqrt{a+4}+1}+1\right)\left(-\sqrt{a+4}+1\right)}{3(a+3)^2(a+4)^2\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(x-1)\left(20a^2+188a+(32a+112)(x-1)^2+416\right)}{48(a+3)^2(a+4)^2\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(5a+16)\left(\frac{(x-1)^2}{-\sqrt{a+4}+1}+1\right)\sqrt{\sqrt{a+4}+1}F\left(\operatorname{atan}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)\middle|\frac{2\sqrt{a+4}}{\sqrt{a+4}-1}\right)}{12\sqrt{\frac{-\frac{(x-1)^2}{\sqrt{a+4}-1}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}(a+3)(a+4)^2\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(2a+7)\left(\frac{(x-1)^2}{-\sqrt{a+4}+1}+1\right)\left(-\sqrt{a+4}+1\right)\sqrt{\sqrt{a+4}+1}E\left(\operatorname{atan}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)\middle|\frac{2\sqrt{a+4}}{\sqrt{a+4}-1}\right)}{3\sqrt{\frac{-\frac{(x-1)^2}{\sqrt{a+4}-1}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}(a+3)^2(a+4)^2\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-x**4+4*x**3-8*x**2+a+8*x)**(5/2),x)
```

```
[Out] (x - 1)*(2*a + 2*(x - 1)**2 + 10)/(12*(a + 3)*(a + 4)*(a - (x - 1
)**4 - 2*(x - 1)**2 + 3)**(3/2)) - (2*a + 7)*(x - 1)*((x - 1)**2/
(-sqrt(a + 4) + 1) + 1)*(-sqrt(a + 4) + 1)/(3*(a + 3)**2*(a + 4)*
*2*sqrt(a - (x - 1)**4 - 2*(x - 1)**2 + 3)) + (x - 1)*(20*a**2 +
188*a + (32*a + 112)*(x - 1)**2 + 416)/(48*(a + 3)**2*(a + 4)**2*
sqrt(a - (x - 1)**4 - 2*(x - 1)**2 + 3)) + (5*a + 16)*((x - 1)**2
/(-sqrt(a + 4) + 1) + 1)*sqrt(sqrt(a + 4) + 1)*elliptic_f(atan((x
 - 1)/sqrt(sqrt(a + 4) + 1)), 2*sqrt(a + 4)/(sqrt(a + 4) - 1))/(1
2*sqrt((-(x - 1)**2/(sqrt(a + 4) - 1) + 1)/((x - 1)**2/(sqrt(a +
4) + 1) + 1))*(a + 3)*(a + 4)**2*sqrt(a - (x - 1)**4 - 2*(x - 1)*
*2 + 3)) + (2*a + 7)*((x - 1)**2/(-sqrt(a + 4) + 1) + 1)*(-sqrt(a
 + 4) + 1)*sqrt(sqrt(a + 4) + 1)*elliptic_e(atan((x - 1)/sqrt(sqr
t(a + 4) + 1)), 2*sqrt(a + 4)/(sqrt(a + 4) - 1))/(3*sqrt((-(x - 1
)**2/(sqrt(a + 4) - 1) + 1)/((x - 1)**2/(sqrt(a + 4) + 1) + 1))*(
a + 3)**2*(a + 4)**2*sqrt(a - (x - 1)**4 - 2*(x - 1)**2 + 3))
```

Mathematica [B] time = 6.31729, size = 6386, normalized size = 12.35

Result too large to show

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-5/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.039, size = 2757, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x)
```

```
[Out] (1/6/(a^2+7*a+12)*x^3-1/2/(a^2+7*a+12)*x^2+1/6*(8+a)/(a^2+7*a+12)
*x-1/6*(6+a)/(a^2+7*a+12))*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)/(x^4-4*
x^3+8*x^2-a-8*x)^2+2*(1/6*(7+2*a)/(a^2+7*a+12)^2*x^3-1/2*(7+2*a)/
(a^2+7*a+12)^2*x^2+1/24*(5*a^2+71*a+188)/(a^2+7*a+12)^2*x-1/24*(5
*a^2+55*a+132)/(a^2+7*a+12)^2)/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)-(1/
6*(5*a^2+47*a+104)/(a^2+7*a+12)^2-1/12*(5*a^2+71*a+188)/(a^2+7*a+
12)^2)*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-1-(4+
a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/
2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+
a)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4
+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(
1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*
(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2))^(1/2))/(-(-1-(4+
a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/
2)))^(1/2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(
4+a)^(1/2))^(1/2)/(-(x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1+(-1+(4+a)^(
1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(
1/2)))^(1/2)*EllipticF(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))
^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1
+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(
4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)
+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2)
)^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))-
2/3*(7+2*a)/(a^2+7*a+12)^2*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2
))^(1/2))*((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-
(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))
^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(4+a)^(1/2))
^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)^(1/2))^(1/2))
/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(
1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/
2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+
(-1+(4+a)^(1/2))^(1/2)))^(1/2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)
^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/2)/(-(x-1-(-1+(4+a)^(1/2))^(1/
2))*(x-1+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-
1+(-1-(4+a)^(1/2))^(1/2)))^(1/2)*((1-(-1+(4+a)^(1/2))^(1/2))*Elli
pticF(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+
(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/
2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)
-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))
```

```
^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+
a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))+2*(-1+(4+a)^(1/2)
)^(1/2)*EllipticPi(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/
2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+
a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),(-(-1-(4+a)^
(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1
+(4+a)^(1/2))^(1/2)),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(
1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)
^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1
+(4+a)^(1/2))^(1/2)))^(1/2)))-1/3*(7+2*a)/(a^2+7*a+12)^2*((x-1-(-
1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)
^(1/2))^(1/2))+((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*((
-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1
/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1
+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(4+a)^(1/2))^(1/2))^2*(-
2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)^(1/2))^(1/2))/((-1-(4+a)^
(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2))
)^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2))^(1/2))/(
-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1
/2))^(1/2)))^(1/2)*(-1/2*((1-(-1+(4+a)^(1/2))^(1/2))*(1+(-1+(4+a)
^(1/2))^(1/2))-(1-(-1-(4+a)^(1/2))^(1/2))*(1+(-1+(4+a)^(1/2))^(1/
2))+(1-(-1-(4+a)^(1/2))^(1/2))*(1-(-1+(4+a)^(1/2))^(1/2))+(1-(-1+
(4+a)^(1/2))^(1/2))^2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^
(1/2))/(-1+(4+a)^(1/2))^(1/2)*EllipticF(((-(-1-(4+a)^(1/2))^(1/2)
+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)
^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)
))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(
4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2
)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2)
)^(1/2)))^(1/2))-1/2*(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1
/2))*EllipticE(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*
(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(
1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^(1/
2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+
a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))
/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))/(-1+(4+a
)^(1/2))^(1/2)-4/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))
*EllipticPi(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-
1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2
))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-1-(4+a)^(1/2))^(
1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1
/2))^(1/2)),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-
1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(
1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1
/2))^(1/2)))^(1/2))))/(-(x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1+(-1+(4+
a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2
))^(1/2)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-x^4 + 4x^3 - 8x^2 + a + 8x\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(-5/2),x, algorithm="maxima")
```

```
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(-5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(x^8 - 8x^7 + 32x^6 - 2(a-64)x^4 - 80x^5 + 8(a-16)x^3 - 16(a-4)x^2 + a^2 + 16ax)\sqrt{-x^4 + 4x^3 - 8x^2 + a + 8x}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(-5/2),x, algorithm="fricas")
```

```
[Out] integral(1/((x^8 - 8*x^7 + 32*x^6 - 2*(a - 64)*x^4 - 80*x^5 + 8*(
a - 16)*x^3 - 16*(a - 4)*x^2 + a^2 + 16*a*x)*sqrt(-x^4 + 4*x^3 -
8*x^2 + a + 8*x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a - x^4 + 4x^3 - 8x^2 + 8x\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-x**4+4*x**3-8*x**2+a+8*x)**(5/2),x)
```

```
[Out] Integral((a - x**4 + 4*x**3 - 8*x**2 + 8*x)**(-5/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-x^4 + 4x^3 - 8x^2 + a + 8x\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(-5/2),x, algorithm="giac")
```

```
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(-5/2), x)
```

3.630 $\int x \left(a+8x-8x^2+4x^3-x^4\right)^{3/2} dx$

Optimal. Leaf size=558

$$\frac{3}{16}(a+4)\left((x-1)^2+1\right)\sqrt{a-(x-1)^4-2(x-1)^2+3} + \frac{1}{8}\left((x-1)^2+1\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2} + \frac{1}{7}(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2} + \frac{2}{35}(x-1)\left(5a-3(x-1)^2+13\right)\sqrt{a-(x-1)^4-2(x-1)^2+3} - \frac{16(2a+7)\left(1-\sqrt{a+4}\right)(x-1)\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)}{35\sqrt{a-(x-1)^4-2(x-1)^2+3}} + \frac{3}{16}(a+4)^2\tan^{-1}\left(\frac{(x-1)^2+1}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right) + \frac{4(a+3)(5a+16)\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)F\left(\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{35\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}} + \frac{16(2a+7)\left(1-\sqrt{a+4}\right)\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)E\left(\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{35\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

```
[Out] (3*(4 + a)*(1 + (-1 + x)^2)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^
4])/16 + ((1 + (-1 + x)^2)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)^(3
/2))/8 - (16*(7 + 2*a)*(1 - Sqrt[4 + a])*(1 + (-1 + x)^2/(1 - Sqr
t[4 + a]))*(-1 + x))/(35*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])
 + (2*(13 + 5*a - 3*(-1 + x)^2)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 +
 x)^4]*(-1 + x))/35 + ((3 + a - 2*(-1 + x)^2 - (-1 + x)^4)^(3/2)*
(-1 + x))/7 + (3*(4 + a)^2*ArcTan[(1 + (-1 + x)^2)/Sqrt[3 + a - 2
*(-1 + x)^2 - (-1 + x)^4]])/16 + (16*(7 + 2*a)*(1 - Sqrt[4 + a])*
Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*Elliptic
E[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - S
qrt[4 + a])])/(35*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-
1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)
^4]) + (4*(3 + a)*(16 + 5*a)*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^
2/(1 - Sqrt[4 + a]))*EllipticF[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 +
a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(35*Sqrt[(1 + (-1 + x)
^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3
+ a - 2*(-1 + x)^2 - (-1 + x)^4])
```

Rubi [A] time = 1.43727, antiderivative size = 558, normalized size of antiderivative = 1., number of

steps used = 14, number of rules used = 13, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$\frac{3}{16}(a+4)\left((x-1)^2+1\right)\sqrt{a-(1-x)^4-2(1-x)^2+3}$$
$$+\frac{1}{8}\left((x-1)^2+1\right)\left(a-(1-x)^4-2(1-x)^2+3\right)^{3/2}-\frac{1}{7}(1-x)\left(a-(1-x)^4-2(1-x)^2+3\right)^{3/2}$$
$$-\frac{2}{35}(1-x)\left(5a-3(1-x)^2+13\right)\sqrt{a-(1-x)^4-2(1-x)^2+3}$$
$$+\frac{16(2a+7)\left(1-\sqrt{a+4}\right)(1-x)\left(\frac{(1-x)^2}{1-\sqrt{a+4}}+1\right)}{35\sqrt{a-(1-x)^4-2(1-x)^2+3}}$$
$$+\frac{3}{16}(a+4)^2\tan^{-1}\left(\frac{(x-1)^2+1}{\sqrt{a-(1-x)^4-2(1-x)^2+3}}\right)$$
$$-\frac{4(a+3)(5a+16)\sqrt{\sqrt{a+4}+1}\left(\frac{(1-x)^2}{1-\sqrt{a+4}}+1\right)F\left(\tan^{-1}\left(\frac{1-x}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{35\sqrt{\frac{\frac{(1-x)^2}{1-\sqrt{a+4}}+1}{\frac{(1-x)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(1-x)^4-2(1-x)^2+3}}$$
$$-\frac{16(2a+7)\left(1-\sqrt{a+4}\right)\sqrt{\sqrt{a+4}+1}\left(\frac{(1-x)^2}{1-\sqrt{a+4}}+1\right)E\left(\tan^{-1}\left(\frac{1-x}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{35\sqrt{\frac{\frac{(1-x)^2}{1-\sqrt{a+4}}+1}{\frac{(1-x)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(1-x)^4-2(1-x)^2+3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

```
[Out] (3*(4 + a)*Sqrt[3 + a - 2*(1 - x)^2 - (1 - x)^4]*(1 + (-1 + x)^2)
)/16 + ((3 + a - 2*(1 - x)^2 - (1 - x)^4)^(3/2)*(1 + (-1 + x)^2))
/8 + (16*(7 + 2*a)*(1 - Sqrt[4 + a])*(1 + (1 - x)^2/(1 - Sqrt[4 +
 a]))*(1 - x))/(35*Sqrt[3 + a - 2*(1 - x)^2 - (1 - x)^4]) - (2*(1
3 + 5*a - 3*(1 - x)^2)*Sqrt[3 + a - 2*(1 - x)^2 - (1 - x)^4]*(1 -
 x))/35 - ((3 + a - 2*(1 - x)^2 - (1 - x)^4)^(3/2)*(1 - x))/7 + (
3*(4 + a)^2*ArcTan[(1 + (-1 + x)^2)/Sqrt[3 + a - 2*(1 - x)^2 - (1
 - x)^4]])/16 - (16*(7 + 2*a)*(1 - Sqrt[4 + a])*Sqrt[1 + Sqrt[4 +
 a]]*(1 + (1 - x)^2/(1 - Sqrt[4 + a]))*EllipticE[ArcTan[(1 - x)/S
qrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(35*S
qrt[(1 + (1 - x)^2/(1 - Sqrt[4 + a]))/(1 + (1 - x)^2/(1 + Sqrt[4
+ a]))]*Sqrt[3 + a - 2*(1 - x)^2 - (1 - x)^4]) - (4*(3 + a)*(16 +
 5*a)*Sqrt[1 + Sqrt[4 + a]]*(1 + (1 - x)^2/(1 - Sqrt[4 + a]))*Ell
ipticF[ArcTan[(1 - x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1
 - Sqrt[4 + a])])/(35*Sqrt[(1 + (1 - x)^2/(1 - Sqrt[4 + a]))/(1 +
 (1 - x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(1 - x)^2 - (1 - x)
^4])
```

Rubi in Sympy [A] time = 79.6801, size = 476, normalized size = 0.85

$$\left(\frac{3a}{32}+\frac{3}{8}\right)\left(2(x-1)^2+2\right)\sqrt{a-(x-1)^4-2(x-1)^2+3} + \frac{3(a+4)^2\operatorname{atan}\left(-\frac{-2(x-1)^2-2}{2\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)}{16} - \frac{16(2a+7)(x-1)\left(\frac{(x-1)^2}{-\sqrt{a+4}+1}+1\right)\left(-\sqrt{a+4}+1\right)}{35\sqrt{a-(x-1)^4-2(x-1)^2+3}} + \frac{(x-1)\left(10a-6(x-1)^2+26\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}{35} + \frac{(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{\frac{3}{2}}}{7} + \frac{\left(2(x-1)^2+2\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{\frac{3}{2}}}{16} + \frac{4(a+3)(5a+16)\left(\frac{(x-1)^2}{-\sqrt{a+4}+1}+1\right)\sqrt{\sqrt{a+4}+1}F\left(\operatorname{atan}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)\middle|\frac{2\sqrt{a+4}}{\sqrt{a+4}-1}\right)}{35\sqrt{\frac{-\frac{(x-1)^2}{\sqrt{a+4}-1}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}} + \frac{16(2a+7)\left(\frac{(x-1)^2}{-\sqrt{a+4}+1}+1\right)\left(-\sqrt{a+4}+1\right)\sqrt{\sqrt{a+4}+1}E\left(\operatorname{atan}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)\middle|\frac{2\sqrt{a+4}}{\sqrt{a+4}-1}\right)}{35\sqrt{\frac{-\frac{(x-1)^2}{\sqrt{a+4}-1}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(-x**4+4*x**3-8*x**2+a+8*x)**(3/2),x)
```

```
[Out] (3*a/32 + 3/8)*(2*(x - 1)**2 + 2)*sqrt(a - (x - 1)**4 - 2*(x - 1)
**2 + 3) + 3*(a + 4)**2*atan(-(-2*(x - 1)**2 - 2)/(2*sqrt(a - (x
- 1)**4 - 2*(x - 1)**2 + 3)))/16 - 16*(2*a + 7)*(x - 1)*((x - 1)*
*2/(-sqrt(a + 4) + 1) + 1)*(-sqrt(a + 4) + 1)/(35*sqrt(a - (x - 1
)**4 - 2*(x - 1)**2 + 3)) + (x - 1)*(10*a - 6*(x - 1)**2 + 26)*sq
rt(a - (x - 1)**4 - 2*(x - 1)**2 + 3)/35 + (x - 1)*(a - (x - 1)**
4 - 2*(x - 1)**2 + 3)**(3/2)/7 + (2*(x - 1)**2 + 2)*(a - (x - 1)*
*4 - 2*(x - 1)**2 + 3)**(3/2)/16 + 4*(a + 3)*(5*a + 16)*((x - 1)*
*2/(-sqrt(a + 4) + 1) + 1)*sqrt(sqrt(a + 4) + 1)*elliptic_f(atan(
(x - 1)/sqrt(sqrt(a + 4) + 1)), 2*sqrt(a + 4)/(sqrt(a + 4) - 1))/
(35*sqrt((-(x - 1)**2/(sqrt(a + 4) - 1) + 1)/((x - 1)**2/(sqrt(a
+ 4) + 1) + 1))*sqrt(a - (x - 1)**4 - 2*(x - 1)**2 + 3)) + 16*(2*
a + 7)*((x - 1)**2/(-sqrt(a + 4) + 1) + 1)*(-sqrt(a + 4) + 1)*sqr
t(sqrt(a + 4) + 1)*elliptic_e(atan((x - 1)/sqrt(sqrt(a + 4) + 1))
, 2*sqrt(a + 4)/(sqrt(a + 4) - 1))/(35*sqrt((-(x - 1)**2/(sqrt(a
+ 4) - 1) + 1)/((x - 1)**2/(sqrt(a + 4) + 1) + 1))*sqrt(a - (x -
1)**4 - 2*(x - 1)**2 + 3))
```

Mathematica [B] time = 6.28499, size = 7235, normalized size = 12.97

Result too large to show

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.028, size = 2694, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x)
```

```
[Out] -1/8*x^6*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+17/28*x^5*(-x^4+4*x^3-8*x
^2+a+8*x)^(1/2)-43/28*x^4*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+74/35*x^
3*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+(5/16*a-9/20)*x^2*(-x^4+4*x^3-8*
x^2+a+8*x)^(1/2)+(-11/56*a-29/70)*x*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2
)+(11/56*a+13/14)*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)-(-(-11/56*a-29/7
0)*a-11/14*a-26/7)*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2)
)*((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a
)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/
(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(4+a)^(1/2))^(1/2))^
2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)^(1/2))^(1/2))/((-1-(4
+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1
/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2))^(1/2
))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a
)^(1/2))^(1/2)))^(1/2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^
(1/2))/(-1+(4+a)^(1/2))^(1/2)/(-(x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1
+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4
+a)^(1/2))^(1/2)))^(1/2)*EllipticF(((-(-1-(4+a)^(1/2))^(1/2)+(-1+
(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2
))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1
/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^
(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1
+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/
2)))^(1/2))-(a^2-2*(5/16*a-9/20)*a+55/14*a+62/5)*((-1-(4+a)^(1/2)
)^(1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+
a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^
(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)
*(x-1+(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(
-1-(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(
1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1
```

```
/2)*(x-1+(-1-(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+
a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)/(-(-1-(4+a)^
(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/2)/(-(x-
1-(-1+(4+a)^(1/2))^(1/2))*(x-1+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(
4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2)))^(1/2)*((1-(-1+(4
+a)^(1/2))^(1/2))*EllipticF(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(
1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2
)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-
(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^
(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^
(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1
/2))+2*(-1+(4+a)^(1/2))^(1/2)*EllipticPi(((-(-1-(4+a)^(1/2))^(1/2
)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a
)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2
)))^(1/2),(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(-(-1-
(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2)),((-(-1-(4+a)^(1/2))^(1
/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/
2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-
(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2)))+(-32/35*a-16/
5)*((x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-
1+(-1-(4+a)^(1/2))^(1/2))+((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2)
)^(1/2))*((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(
-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^
(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(4+a)^(1/2))^
(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)^(1/2))^(1/2))/
((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1
/2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2
))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(
-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-1/2*((1-(-1+(4+a)^(1/2))^(1/2))*(
1+(-1+(4+a)^(1/2))^(1/2))-(1-(-1-(4+a)^(1/2))^(1/2))*(1+(-1+(4+a)
^(1/2))^(1/2))+(1-(-1-(4+a)^(1/2))^(1/2))*(1-(-1+(4+a)^(1/2))^(1/
2))+(1-(-1+(4+a)^(1/2))^(1/2))^2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4
+a)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/2)*EllipticF(((-(-1-(4+a)^(
1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/
(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(
1/2))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1
/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^
(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+
(4+a)^(1/2))^(1/2)))^(1/2))-1/2*(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a
)^(1/2))^(1/2))*EllipticE(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/
2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-
(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-
1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1
/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1
/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2
))/(-1+(4+a)^(1/2))^(1/2)-4/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1
/2))^(1/2))*EllipticPi(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))
^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1
+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-1-(4+
a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(
-1+(4+a)^(1/2))^(1/2)),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))
^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+
a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(
-1+(4+a)^(1/2))^(1/2)))^(1/2))))/(-(x-1-(-1+(4+a)^(1/2))^(1/2))*(
```

```
x-1+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1
-(4+a)^(1/2))^(1/2)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-x^4+4\,x^3-8\,x^2+a+8\,x\right)^{\frac{3}{2}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2)*x,x, algorithm="maxima")
```

```
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2)*x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(x^5-4\,x^4+8\,x^3-ax-8\,x^2\right)\sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2)*x,x, algorithm="fricas")
```

```
[Out] integral(-(x^5 - 4*x^4 + 8*x^3 - a*x - 8*x^2)*sqrt(-x^4 + 4*x^3 -
 8*x^2 + a + 8*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(a-x^4+4x^3-8x^2+8x\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(-x**4+4*x**3-8*x**2+a+8*x)**(3/2),x)
```

```
[Out] Integral(x*(a - x**4 + 4*x**3 - 8*x**2 + 8*x)**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-x^4 + 4x^3 - 8x^2 + a + 8x\right)^{\frac{3}{2}} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2)*x,x, algorithm="giac")
```

```
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2)*x, x)
```

3.631 $\int x\sqrt{a+8x-8x^2+4x^3-x^4}\,dx$

Optimal. Leaf size=466

$$\frac{1}{4}\left((x-1)^2+1\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}+\frac{1}{3}(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}$$
$$-\frac{2\left(1-\sqrt{a+4}\right)(x-1)\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)}{3\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{1}{4}(a+4)\tan^{-1}\left(\frac{(x-1)^2+1}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)$$
$$+\frac{2(a+3)\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)F\left(\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)\middle|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{3\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$
$$+\frac{2\left(1-\sqrt{a+4}\right)\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)E\left(\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)\middle|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{3\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

```
[Out] ((1 + (-1 + x)^2)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])/4 - (2
*(1 - Sqrt[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(-1 + x))/(
3*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (Sqrt[3 + a - 2*(-1
+ x)^2 - (-1 + x)^4]*(-1 + x))/3 + ((4 + a)*ArcTan[(1 + (-1 + x)^
2)/Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]])/4 + (2*(1 - Sqrt[4 +
 a])*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*Ell
ipticE[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(
1 - Sqrt[4 + a])])/(3*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1
+ (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1
+ x)^4]) + (2*(3 + a)*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 -
Sqrt[4 + a]))*EllipticF[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (
-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(3*Sqrt[(1 + (-1 + x)^2/(1 -
Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*
(-1 + x)^2 - (-1 + x)^4])
```

Rubi [A] time = 1.20298, antiderivative size = 466, normalized size of antiderivative = 1., number of

steps used = 12, number of rules used = 12, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$

$$\frac{1}{4}\left((x-1)^2+1\right)\sqrt{a-(1-x)^4-2(1-x)^2+3}-\frac{1}{3}(1-x)\sqrt{a-(1-x)^4-2(1-x)^2+3}+\frac{2\left(1-\sqrt{a+4}\right)(1-x)\left(\frac{(1-x)^2}{1-\sqrt{a+4}}+1\right)}{3\sqrt{a-(1-x)^4-2(1-x)^2+3}}+\frac{1}{4}(a+4)\tan^{-1}\left(\frac{(x-1)^2+1}{\sqrt{a-(1-x)^4-2(1-x)^2+3}}\right)-\frac{2(a+3)\sqrt{\sqrt{a+4}+1}\left(\frac{(1-x)^2}{1-\sqrt{a+4}}+1\right)F\left(\tan^{-1}\left(\frac{1-x}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{3\sqrt{\frac{\frac{(1-x)^2}{1-\sqrt{a+4}}+1}{\frac{(1-x)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(1-x)^4-2(1-x)^2+3}}-\frac{2\left(1-\sqrt{a+4}\right)\sqrt{\sqrt{a+4}+1}\left(\frac{(1-x)^2}{1-\sqrt{a+4}}+1\right)E\left(\tan^{-1}\left(\frac{1-x}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{3\sqrt{\frac{\frac{(1-x)^2}{1-\sqrt{a+4}}+1}{\frac{(1-x)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(1-x)^4-2(1-x)^2+3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] (Sqrt[3 + a - 2*(1 - x)^2 - (1 - x)^4]*(1 + (-1 + x)^2))/4 + (2*(
1 - Sqrt[4 + a])*(1 + (1 - x)^2/(1 - Sqrt[4 + a]))*(1 - x))/(3*Sq
rt[3 + a - 2*(1 - x)^2 - (1 - x)^4]) - (Sqrt[3 + a - 2*(1 - x)^2
- (1 - x)^4]*(1 - x))/3 + ((4 + a)*ArcTan[(1 + (-1 + x)^2)/Sqrt[3
 + a - 2*(1 - x)^2 - (1 - x)^4]])/4 - (2*(1 - Sqrt[4 + a])*Sqrt[1
 + Sqrt[4 + a]]*(1 + (1 - x)^2/(1 - Sqrt[4 + a]))*EllipticE[ArcTa
n[(1 - x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 +
a])])/(3*Sqrt[(1 + (1 - x)^2/(1 - Sqrt[4 + a]))/(1 + (1 - x)^2/(1
 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(1 - x)^2 - (1 - x)^4]) - (2*(3
+ a)*Sqrt[1 + Sqrt[4 + a]]*(1 + (1 - x)^2/(1 - Sqrt[4 + a]))*Elli
pticF[ArcTan[(1 - x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1
- Sqrt[4 + a])])/(3*Sqrt[(1 + (1 - x)^2/(1 - Sqrt[4 + a]))/(1 + (
1 - x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(1 - x)^2 - (1 - x)^4
])
```

Rubi in Sympy [A] time = 69.8447, size = 388, normalized size = 0.83

$$\left(\frac{a}{4}+1\right)\operatorname{atan}\left(-\frac{-2(x-1)^2-2}{2\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)-\frac{(x-1)\left(\frac{(x-1)^2}{-\sqrt{a+4}+1}+1\right)\left(-\frac{2\sqrt{a+4}}{3}+\frac{2}{3}\right)}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}}{3}+\frac{\left(2(x-1)^2+2\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}{8}+\frac{2(a+3)\left(\frac{(x-1)^2}{-\sqrt{a+4}+1}+1\right)\sqrt{\sqrt{a+4}+1}F\left(\operatorname{atan}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)\middle|\frac{2\sqrt{a+4}}{\sqrt{a+4}-1}\right)}{3\sqrt{\frac{-\frac{(x-1)^2}{\sqrt{a+4}-1}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{2\left(\frac{(x-1)^2}{-\sqrt{a+4}+1}+1\right)\left(-\sqrt{a+4}+1\right)\sqrt{\sqrt{a+4}+1}E\left(\operatorname{atan}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)\middle|\frac{2\sqrt{a+4}}{\sqrt{a+4}-1}\right)}{3\sqrt{\frac{-\frac{(x-1)^2}{\sqrt{a+4}-1}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(-x**4+4*x**3-8*x**2+a+8*x)**(1/2),x)
```

```
[Out] (a/4 + 1)*atan(-(-2*(x - 1)**2 - 2)/(2*sqrt(a - (x - 1)**4 - 2*(x
 - 1)**2 + 3))) - (x - 1)*((x - 1)**2/(-sqrt(a + 4) + 1) + 1)*(-2
*sqrt(a + 4)/3 + 2/3)/sqrt(a - (x - 1)**4 - 2*(x - 1)**2 + 3) + (
x - 1)*sqrt(a - (x - 1)**4 - 2*(x - 1)**2 + 3)/3 + (2*(x - 1)**2
+ 2)*sqrt(a - (x - 1)**4 - 2*(x - 1)**2 + 3)/8 + 2*(a + 3)*((x -
1)**2/(-sqrt(a + 4) + 1) + 1)*sqrt(sqrt(a + 4) + 1)*elliptic_f(at
an((x - 1)/sqrt(sqrt(a + 4) + 1)), 2*sqrt(a + 4)/(sqrt(a + 4) - 1
))/(3*sqrt((-(x - 1)**2/(sqrt(a + 4) - 1) + 1)/((x - 1)**2/(sqrt(
a + 4) + 1) + 1))*sqrt(a - (x - 1)**4 - 2*(x - 1)**2 + 3)) + 2*((
x - 1)**2/(-sqrt(a + 4) + 1) + 1)*(-sqrt(a + 4) + 1)*sqrt(sqrt(a
+ 4) + 1)*elliptic_e(atan((x - 1)/sqrt(sqrt(a + 4) + 1)), 2*sqrt(
a + 4)/(sqrt(a + 4) - 1))/(3*sqrt((-(x - 1)**2/(sqrt(a + 4) - 1)
+ 1)/((x - 1)**2/(sqrt(a + 4) + 1) + 1))*sqrt(a - (x - 1)**4 - 2*
(x - 1)**2 + 3))
```

Mathematica [B] time = 6.1211, size = 4389, normalized size = 9.42

Result too large to show

Antiderivative was successfully verified.

```
[In]  Integrate[x*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] (1/6 - x/6 + x^2/4)*Sqrt[a - x*(-8 + 8*x - 4*x^2 + x^3)] + (Sqrt[
a - x*(-8 + 8*x - 4*x^2 + x^3)]*((-8*(-Sqrt[-1 - Sqrt[4 + a]] - S
qrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[(
(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1
- Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4
+ a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4
 + a]]*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]
] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*S
qrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((
Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 -
Sqrt[4 + a]] + x))]*EllipticF[ArcSin[Sqrt[((-Sqrt[-1 - Sqrt[4 + a
]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/(
(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 -
 Sqrt[4 + a]] + x))]], ((-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt
[4 + a]])*(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))/((Sq
rt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-Sqrt[-1 - Sqrt[4
 + a]] + Sqrt[-1 + Sqrt[4 + a]]))])/(Sqrt[-1 - Sqrt[4 + a]]*(-Sqr
t[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*Sqrt[a + 8*x - 8*x^
2 + 4*x^3 - x^4]) + (2*a*(-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqr
t[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((-Sqrt[-1 -
Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a
]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 -
 Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 -
 Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1
+ Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1
 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sq
rt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]]
 + x))]*EllipticF[ArcSin[Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1
 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - S
qrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]
] + x))]], ((-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(S
qrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))/((Sqrt[-1 - Sqrt
[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-Sqrt[-1 - Sqrt[4 + a]] + Sqr
t[-1 + Sqrt[4 + a]]))])/(Sqrt[-1 - Sqrt[4 + a]]*(-Sqrt[-1 - Sqrt[
4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*Sqrt[a + 8*x - 8*x^2 + 4*x^3 -
x^4]) + (40*(-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-
1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((Sqrt[-1 - Sqrt[4 + a]] -
 Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqr
t[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt
[4 + a]] - x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt
[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])
*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]
*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] - Sq
rt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*((-1 -
Sqrt[-1 - Sqrt[4 + a]])*EllipticF[ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4
+ a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x)
)/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1
 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt
[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2]
+ 2*Sqrt[-1 - Sqrt[4 + a]]*EllipticPi[(Sqrt[-1 - Sqrt[4 + a]] + S
qrt[-1 + Sqrt[4 + a]])/(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[
4 + a]]), ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4
 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a
]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]],
```

```
 (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - S
qrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2]))/(Sqrt[-1 - Sqrt[4 + a]
]*(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*Sqrt[a + 8*x
- 8*x^2 + 4*x^3 - x^4]) + (6*a*(-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1
 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((Sqrt[
-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[
4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*
(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(
-1 - Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt
[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqr
t[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1
- Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 +
 a]] + x))]*((-1 - Sqrt[-1 - Sqrt[4 + a]])*EllipticF[ArcSin[Sqrt[
((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1
- Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4
+ a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a
]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1
+ Sqrt[4 + a]])^2] + 2*Sqrt[-1 - Sqrt[4 + a]]*EllipticPi[(Sqrt[-1
 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])/(-Sqrt[-1 - Sqrt[4 + a]
] + Sqrt[-1 + Sqrt[4 + a]]), ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]]
 - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((S
qrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sq
rt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 +
a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2]))/(Sq
rt[-1 - Sqrt[4 + a]]*(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 +
 a]])*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4]) - (4*((-1 + Sqrt[-1 -
Sqrt[4 + a]] + x)*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x)*(-1 + Sqrt[-1
 + Sqrt[4 + a]] + x) + 2*(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt
[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((Sqrt[-1 - Sq
rt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]]
 + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sq
rt[-1 - Sqrt[4 + a]] - x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sq
rt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + S
qrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 -
Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[
4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] +
x))]*(((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*Elliptic
E[ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*
(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqr
t[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-
1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 +
a]] - Sqrt[-1 + Sqrt[4 + a]])^2])/(2*Sqrt[-1 - Sqrt[4 + a]]) + ((
-((-1 - Sqrt[-1 - Sqrt[4 + a]])*(-2 - Sqrt[-1 - Sqrt[4 + a]] - Sq
rt[-1 + Sqrt[4 + a]])) + (-1 + Sqrt[-1 - Sqrt[4 + a]])*(Sqrt[-1 -
 Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]]))*EllipticF[ArcSin[Sqrt[((
Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 -
Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 +
a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]]
 + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 +
Sqrt[4 + a]])^2])/(2*Sqrt[-1 - Sqrt[4 + a]]*(-Sqrt[-1 - Sqrt[4 +
a]] + Sqrt[-1 + Sqrt[4 + a]])) + (4*EllipticPi[(Sqrt[-1 - Sqrt[4
+ a]] + Sqrt[-1 + Sqrt[4 + a]])/(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-
1 + Sqrt[4 + a]]), ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1
 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - S
```

```
qrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]]
 - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sq
rt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2])/(-Sqrt[-1 - Sq
rt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))))/Sqrt[a + 8*x - 8*x^2 + 4*
x^3 - x^4]))/(6*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4])
```

Maple [B] time = 0.027, size = 2551, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x)
```

```
[Out] 1/4*x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)-1/6*x*(-x^4+4*x^3-8*x^2+a+
8*x)^(1/2)+1/6*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)-(1/6*a-2/3)*((-1-(4
+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-1-(4+a)^(1/2))^(1/2
)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a
)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2
)))^(1/2)*(x-1+(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/
2)*(x-1-(-1-(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)
^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^
(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2
)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)/(-(
-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1
/2)/(-(x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1+(-1+(4+a)^(1/2))^(1/2))*(
x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2)))^(1/2)*E
llipticF(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(
-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^
(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1
/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/
2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-
(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))-(1/2*a+10/3)*(
(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-1-(4+a)^(1/2)
)^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-
1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2)
)^(1/2)))^(1/2)*(x-1+(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2
))^(1/2)*(x-1-(-1-(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1
+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-1+
(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2)
)^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/
2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(1/
2))^(1/2)/(-(x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1+(-1+(4+a)^(1/2))^(1
/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2)))^(
1/2)*((1-(-1+(4+a)^(1/2))^(1/2))*EllipticF(((-(-1-(4+a)^(1/2))^(1
/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4
+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1
/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-
1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(
```

```
1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1
/2))^(1/2)))^(1/2))+2*(-1+(4+a)^(1/2))^(1/2)*EllipticPi(((-(-1-(4
+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1
/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4
+a)^(1/2))^(1/2)))^(1/2),(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2)
)^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2)),((-(-1-
(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2
)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2
))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))
)-2/3*((x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*
(x-1+(-1-(4+a)^(1/2))^(1/2))+((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1
/2))^(1/2))*((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-
1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2
))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(4+a)^(1/2
))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)^(1/2))^(1/2
))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)
^(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(
1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-
1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-1/2*((1-(-1+(4+a)^(1/2))^(1/2)
)*(1+(-1+(4+a)^(1/2))^(1/2))-(1-(-1-(4+a)^(1/2))^(1/2))*(1+(-1+(4
+a)^(1/2))^(1/2))+(1-(-1-(4+a)^(1/2))^(1/2))*(1-(-1+(4+a)^(1/2))^
(1/2))+(1-(-1+(4+a)^(1/2))^(1/2))^2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1
+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/2)*EllipticF(((-(-1-(4+a
)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2
))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a
)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))
^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+
a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(
-1+(4+a)^(1/2))^(1/2)))^(1/2))-1/2*(-(-1-(4+a)^(1/2))^(1/2)+(-1+(
4+a)^(1/2))^(1/2))*EllipticE(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^
(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/
2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((
-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))
^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)
^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(
1/2))/(-1+(4+a)^(1/2))^(1/2)-4/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)
^(1/2))^(1/2))*EllipticPi(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/
2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-
(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-1-
(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2
)-(-1+(4+a)^(1/2))^(1/2)),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/
2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-
(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2
)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))))/(-(x-1-(-1+(4+a)^(1/2))^(1/2)
)*(x-1+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+
(-1-(4+a)^(1/2))^(1/2)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x^4 + 4\,x^3 - 8\,x^2 + a + 8\,x}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}x,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{a-x^4+4x^3-8x^2+8x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(-x**4+4*x**3-8*x**2+a+8*x)**(1/2),x)
```

```
[Out] Integral(x*sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x, x)
```

3.632 $\int \frac{x}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx$

Optimal. Leaf size=179

$$\frac{1}{2}\tan^{-1}\left(\frac{(x-1)^2+1}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)F\left(\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)\middle|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

```
[Out] ArcTan[(1 + (-1 + x)^2)/Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]]/
2 + (Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*Ell
ipticF[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(
1 - Sqrt[4 + a])])/(Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 +
(-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 +
x)^4])
```

Rubi [A] time = 0.334357, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269

$$\frac{1}{2}\tan^{-1}\left(\frac{(x-1)^2+1}{\sqrt{a-(1-x)^4-2(1-x)^2+3}}\right)-\frac{\sqrt{\sqrt{a+4}+1}\left(\frac{(1-x)^2}{1-\sqrt{a+4}}+1\right)F\left(\tan^{-1}\left(\frac{1-x}{\sqrt{\sqrt{a+4}+1}}\right)\middle|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(1-x)^2}{1-\sqrt{a+4}}+1}{\frac{(1-x)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(1-x)^4-2(1-x)^2+3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] ArcTan[(1 + (-1 + x)^2)/Sqrt[3 + a - 2*(1 - x)^2 - (1 - x)^4]]/2
- (Sqrt[1 + Sqrt[4 + a]]*(1 + (1 - x)^2/(1 - Sqrt[4 + a]))*Ellipt
icF[ArcTan[(1 - x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 -
Sqrt[4 + a])])/(Sqrt[(1 + (1 - x)^2/(1 - Sqrt[4 + a]))/(1 + (1 -
x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(1 - x)^2 - (1 - x)^4])
```

Rubi in Sympy [A] time = 26.6278, size = 148, normalized size = 0.83

$$\frac{\operatorname{atan}\left(-\frac{-2(x-1)^2-2}{2\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)}{2}+\frac{\left(\frac{(x-1)^2}{-\sqrt{a+4}+1}+1\right)\sqrt{\sqrt{a+4}+1}F\left(\operatorname{atan}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)\middle|\frac{2\sqrt{a+4}}{\sqrt{a+4}-1}\right)}{\sqrt{\frac{-\frac{(x-1)^2}{\sqrt{a+4}-1}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2\left(x-1\right)^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(-x**4+4*x**3-8*x**2+a+8*x)**(1/2),x)
```

```
[Out] atan(-(-2*(x - 1)**2 - 2)/(2*sqrt(a - (x - 1)**4 - 2*(x - 1)**2 +
 3)))/2 + ((x - 1)**2/(-sqrt(a + 4) + 1) + 1)*sqrt(sqrt(a + 4) +
1)*elliptic_f(atan((x - 1)/sqrt(sqrt(a + 4) + 1)), 2*sqrt(a + 4)/
(sqrt(a + 4) - 1))/(sqrt((-(x - 1)**2/(sqrt(a + 4) - 1) + 1)/((x
- 1)**2/(sqrt(a + 4) + 1) + 1))*sqrt(a - (x - 1)**4 - 2*(x - 1)**
2 + 3))
```

Mathematica [B] time = 5.16382, size = 813, normalized size = 4.54

$$\frac{2\left(-x+\sqrt{-\sqrt{a+4}-1}+1\right)\sqrt{\frac{\sqrt{-\sqrt{a+4}-1}\left(-x+\sqrt{\sqrt{a+4}-1}+1\right)}{\left(\sqrt{-\sqrt{a+4}-1}+\sqrt{\sqrt{a+4}-1}\right)\left(-x+\sqrt{-\sqrt{a+4}-1}+1\right)}}\left(x+\sqrt{-\sqrt{a+4}-1}-1\right)\sqrt{\frac{\sqrt{-\sqrt{a+4}-1}\left(x+\sqrt{\sqrt{a+4}-1}-1\ldots\right.}{\left(\sqrt{\sqrt{a+4}-1}-\sqrt{-\sqrt{a+4}-1}\right)\left(-x+\sqrt{-\sqrt{\ldots}}\right.}}}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] (2*(1 + Sqrt[-1 - Sqrt[4 + a]] - x)*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*
(1 + Sqrt[-1 + Sqrt[4 + a]] - x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt
[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]*(-1 + Sqrt
[-1 - Sqrt[4 + a]] + x)*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-
1 + Sqrt[4 + a]] + x))/((-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt
[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]*((1 + Sqrt[-1 - Sqrt
[4 + a]])*EllipticF[ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-
1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 -
Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]
] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(S
qrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2] - 2*Sqrt[-1 -
Sqrt[4 + a]]*EllipticPi[(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[
4 + a]])/(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]), ArcS
in[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 +
Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 +
 Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sq
rt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] -
Sqrt[-1 + Sqrt[4 + a]])^2]))/(Sqrt[-1 - Sqrt[4 + a]]*Sqrt[((Sqrt[
-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[
4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*
(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]*Sqrt[a - x*(-8 + 8*x - 4*x^2 +
 x^3)])
```

Maple [B] time = 0.026, size = 788, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x)
```

```
[Out] -((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-1-(4+a)^(1/
2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-
(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/
2))^(1/2)))^(1/2)*(x-1+(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1
/2))^(1/2)*(x-1-(-1-(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(
-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-
1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/
2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(
1/2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(
1/2))^(1/2)/(-(x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1+(-1+(4+a)^(1/2))^
(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2)))
^(1/2)*((1-(-1+(4+a)^(1/2))^(1/2))*EllipticF(((-(-1-(4+a)^(1/2))^
(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-
(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^
(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*(
(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))
^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^
(1/2))^(1/2)))^(1/2))+2*(-1+(4+a)^(1/2))^(1/2)*EllipticPi(((-(-1-
(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^
(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+
(4+a)^(1/2))^(1/2)))^(1/2),(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/
2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2)),((-(-
1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1
/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1
/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2
)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-x^4 + 4x^3 - 8x^2 + a + 8x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x}{\sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x),x, algorithm="fricas")
```

```
[Out] integral(x/sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a-x^4+4x^3-8x^2+8x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(-x**4+4*x**3-8*x**2+a+8*x)**(1/2),x)
```

```
[Out] Integral(x/sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x), x)
```

3.633 $\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx$

Optimal. Leaf size=474

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(x-1)^2+1}{2(a+4)\sqrt{a-(x-1)^4-2(x-1)^2+3}}-\frac{\left(1-\sqrt{a+4}\right)(x-1)\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)}{2(a+3)(a+4)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)F\left(\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{2(a+4)\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{\left(1-\sqrt{a+4}\right)\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)E\left(\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{2(a+3)(a+4)\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

```
[Out] (1 + (-1 + x)^2)/(2*(4 + a)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^
4]) + ((5 + a + (-1 + x)^2)*(-1 + x))/(2*(12 + 7*a + a^2)*Sqrt[3
+ a - 2*(-1 + x)^2 - (-1 + x)^4]) - ((1 - Sqrt[4 + a])*(1 + (-1 +
 x)^2/(1 - Sqrt[4 + a]))*(-1 + x))/(2*(3 + a)*(4 + a)*Sqrt[3 + a
- 2*(-1 + x)^2 - (-1 + x)^4]) + ((1 - Sqrt[4 + a])*Sqrt[1 + Sqrt[
4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticE[ArcTan[(-1 +
 x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/
(2*(3 + a)*(4 + a)*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (
-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x
)^4]) + (Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))
*EllipticF[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a
])/(1 - Sqrt[4 + a])])/(2*(4 + a)*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[
4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 +
 x)^2 - (-1 + x)^4])
```

Rubi [A] time = 1.19521, antiderivative size = 474, normalized size of antiderivative = 1., number of

steps used = 10, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385

$$-\frac{(1-x)\left(a+(x-1)^2+5\right)}{2\left(a^2+7a+12\right)\sqrt{a-(1-x)^4-2(1-x)^2+3}}+\frac{(x-1)^2+1}{2(a+4)\sqrt{a-(1-x)^4-2(1-x)^2+3}}+\frac{\left(1-\sqrt{a+4}\right)(1-x)\left(\frac{(1-x)^2}{1-\sqrt{a+4}}+1\right)}{2(a+3)(a+4)\sqrt{a-(1-x)^4-2(1-x)^2+3}}-\frac{\sqrt{\sqrt{a+4}+1}\left(\frac{(1-x)^2}{1-\sqrt{a+4}}+1\right)F\left(\tan^{-1}\left(\frac{1-x}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{2(a+4)\sqrt{\frac{\frac{(1-x)^2}{1-\sqrt{a+4}}+1}{\frac{(1-x)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(1-x)^4-2(1-x)^2+3}}-\frac{\left(1-\sqrt{a+4}\right)\sqrt{\sqrt{a+4}+1}\left(\frac{(1-x)^2}{1-\sqrt{a+4}}+1\right)E\left(\tan^{-1}\left(\frac{1-x}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{2(a+3)(a+4)\sqrt{\frac{\frac{(1-x)^2}{1-\sqrt{a+4}}+1}{\frac{(1-x)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(1-x)^4-2(1-x)^2+3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

```
[Out] (1 + (-1 + x)^2)/(2*(4 + a)*Sqrt[3 + a - 2*(1 - x)^2 - (1 - x)^4]
) + ((1 - Sqrt[4 + a])*(1 + (1 - x)^2/(1 - Sqrt[4 + a]))*(1 - x))
/(2*(3 + a)*(4 + a)*Sqrt[3 + a - 2*(1 - x)^2 - (1 - x)^4]) - ((5
+ a + (-1 + x)^2)*(1 - x))/(2*(12 + 7*a + a^2)*Sqrt[3 + a - 2*(1
- x)^2 - (1 - x)^4]) - ((1 - Sqrt[4 + a])*Sqrt[1 + Sqrt[4 + a]]*(
1 + (1 - x)^2/(1 - Sqrt[4 + a]))*EllipticE[ArcTan[(1 - x)/Sqrt[1
+ Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(2*(3 + a)*
(4 + a)*Sqrt[(1 + (1 - x)^2/(1 - Sqrt[4 + a]))/(1 + (1 - x)^2/(1
+ Sqrt[4 + a]))]*Sqrt[3 + a - 2*(1 - x)^2 - (1 - x)^4]) - (Sqrt[1
 + Sqrt[4 + a]]*(1 + (1 - x)^2/(1 - Sqrt[4 + a]))*EllipticF[ArcTa
n[(1 - x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 +
a])])/(2*(4 + a)*Sqrt[(1 + (1 - x)^2/(1 - Sqrt[4 + a]))/(1 + (1 -
 x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(1 - x)^2 - (1 - x)^4])
```

Rubi in Sympy [A] time = 77.0758, size = 388, normalized size = 0.82

$$-\frac{(x-1)\left(\frac{(x-1)^2}{-\sqrt{a+4}+1}+1\right)\left(-\sqrt{a+4}+1\right)}{2(a+3)(a+4)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(x-1)\left(2a+(2a+10)(x-1)+2(x-1)^3+2(x-1)^2+10\right)}{4(a+3)(a+4)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{\sqrt{a-(x-1)^4-2(x-1)^2+3}}{2(a+3)(a+4)}+\frac{\left(\frac{(x-1)^2}{-\sqrt{a+4}+1}+1\right)\sqrt{\sqrt{a+4}+1}F\left(\operatorname{atan}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)\middle|\frac{2\sqrt{a+4}}{\sqrt{a+4}-1}\right)}{2\sqrt{\frac{-\frac{(x-1)^2}{\sqrt{a+4}-1}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}(a+4)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{\left(\frac{(x-1)^2}{-\sqrt{a+4}+1}+1\right)\left(-\sqrt{a+4}+1\right)\sqrt{\sqrt{a+4}+1}E\left(\operatorname{atan}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)\middle|\frac{2\sqrt{a+4}}{\sqrt{a+4}-1}\right)}{2\sqrt{\frac{-\frac{(x-1)^2}{\sqrt{a+4}-1}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}(a+3)(a+4)\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(-x**4+4*x**3-8*x**2+a+8*x)**(3/2),x)
```

```
[Out] -(x - 1)*((x - 1)**2/(-sqrt(a + 4) + 1) + 1)*(-sqrt(a + 4) + 1)/(
2*(a + 3)*(a + 4)*sqrt(a - (x - 1)**4 - 2*(x - 1)**2 + 3)) + (x -
 1)*(2*a + (2*a + 10)*(x - 1) + 2*(x - 1)**3 + 2*(x - 1)**2 + 10)
/(4*(a + 3)*(a + 4)*sqrt(a - (x - 1)**4 - 2*(x - 1)**2 + 3)) + sq
rt(a - (x - 1)**4 - 2*(x - 1)**2 + 3)/(2*(a + 3)*(a + 4)) + ((x -
 1)**2/(-sqrt(a + 4) + 1) + 1)*sqrt(sqrt(a + 4) + 1)*elliptic_f(a
tan((x - 1)/sqrt(sqrt(a + 4) + 1)), 2*sqrt(a + 4)/(sqrt(a + 4) -
1))/(2*sqrt((-(x - 1)**2/(sqrt(a + 4) - 1) + 1)/((x - 1)**2/(sqrt
(a + 4) + 1) + 1))*(a + 4)*sqrt(a - (x - 1)**4 - 2*(x - 1)**2 + 3
)) + ((x - 1)**2/(-sqrt(a + 4) + 1) + 1)*(-sqrt(a + 4) + 1)*sqrt(
sqrt(a + 4) + 1)*elliptic_e(atan((x - 1)/sqrt(sqrt(a + 4) + 1)),
2*sqrt(a + 4)/(sqrt(a + 4) - 1))/(2*sqrt((-(x - 1)**2/(sqrt(a + 4
) - 1) + 1)/((x - 1)**2/(sqrt(a + 4) + 1) + 1))*(a + 3)*(a + 4)*s
qrt(a - (x - 1)**4 - 2*(x - 1)**2 + 3))
```

Mathematica [B] time = 6.0954, size = 3593, normalized size = 7.58

Result too large to show

Antiderivative was successfully verified.

```
[In]  Integrate[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

```
[Out] ((-a - 2*x + a*x - a*x^2 - x^3)*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2
)/(2*(3 + a)*(4 + a)*(-a - 8*x + 8*x^2 - 4*x^3 + x^4)*(a - x*(-8
+ 8*x - 4*x^2 + x^3))^(3/2)) + ((a + 8*x - 8*x^2 + 4*x^3 - x^4)^(
3/2)*((4*(-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 -
 Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] + S
qrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[
-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[
4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[
4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*
(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*
(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] - Sqr
t[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Elliptic
F[ArcSin[Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])
*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sq
rt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]], ((-Sq
rt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(Sqrt[-1 - Sqrt[4
+ a]] + Sqrt[-1 + Sqrt[4 + a]]))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[
-1 + Sqrt[4 + a]])*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 +
a]]))])/(Sqrt[-1 - Sqrt[4 + a]]*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-
1 + Sqrt[4 + a]])*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4]) + (2*a*(-S
qrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - S
qrt[4 + a]] + x)^2*Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqr
t[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4
+ a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)
)]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x)
)/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-
1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-
1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[
4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*EllipticF[ArcSin[Sqr
t[((-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[
-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt
[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]], ((-Sqrt[-1 - Sqrt
[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(Sqrt[-1 - Sqrt[4 + a]] + Sqrt
[-1 + Sqrt[4 + a]]))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4
+ a]])*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))])/(Sqr
t[-1 - Sqrt[4 + a]]*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 +
 a]])*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4]) + (4*(-Sqrt[-1 - Sqrt[
4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] +
x)^2*Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1
+ Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1
 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]*Sqrt[(Sqrt[-1
 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sq
rt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]]
 + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]]
 + x))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - S
qrt[-1 - Sqrt[4 + a]] + x))]*((-1 - Sqrt[-1 - Sqrt[4 + a]])*Ellip
ticF[ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]
])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] +
Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqr
t[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4
 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2] + 2*Sqrt[-1 - Sqrt[4 + a]]*El
lipticPi[(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])/(-Sqrt
[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]), ArcSin[Sqrt[((Sqrt[
-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[
```

```
4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*
(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sq
rt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[
4 + a]])^2]))/(Sqrt[-1 - Sqrt[4 + a]]*(Sqrt[-1 - Sqrt[4 + a]] - S
qrt[-1 + Sqrt[4 + a]])*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4]) - ((-
1 + Sqrt[-1 - Sqrt[4 + a]] + x)*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x)
*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x) + 2*(Sqrt[-1 - Sqrt[4 + a]] +
Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[
((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1
- Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4
+ a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]*Sqrt[(Sqrt[-1 - Sqrt[4
+ a]]*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]]
 + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sq
rt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((S
qrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - S
qrt[4 + a]] + x))]*(((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 +
 a]])*EllipticE[ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 +
Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt
[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] -
x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[
-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2])/(2*Sqrt[-1 - Sqrt
[4 + a]]) + ((-((-1 - Sqrt[-1 - Sqrt[4 + a]])*(-2 - Sqrt[-1 - Sqr
t[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])) + (-1 + Sqrt[-1 - Sqrt[4 + a
]])*(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]]))*EllipticF[
ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-
1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[
-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1
- Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]
] - Sqrt[-1 + Sqrt[4 + a]])^2])/(2*Sqrt[-1 - Sqrt[4 + a]]*(-Sqrt[
-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])) + (4*EllipticPi[(Sqr
t[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])/(-Sqrt[-1 - Sqrt[4
+ a]] + Sqrt[-1 + Sqrt[4 + a]]), ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 +
 a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))
/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1
- Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[
4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2])/
(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])))/Sqrt[a + 8*x
 - 8*x^2 + 4*x^3 - x^4]))/(2*(3 + a)*(4 + a)*(a - x*(-8 + 8*x - 4
*x^2 + x^3))^(3/2))
```

Maple [B] time = 0.031, size = 2616, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x)
```

```
[Out] 2*(1/4/(a^2+7*a+12)*x^3+1/4*a/(a^2+7*a+12)*x^2-1/4*(a-2)/(a^2+7*a
+12)*x+1/4*a/(a^2+7*a+12))/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)-(2/(a^2
```

+7*a+12)+1/2*(a-2)/(a^2+7*a+12))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a
)^(1/2))^(1/2))*((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))
*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^
(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(4+a)^
(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)^(1/2))^
(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(
4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+
a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))
/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1
+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/2)/(-(x-1-(-1+(4+a)^(1/2
))^(1/2))*(x-1+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2
))*(x-1+(-1-(4+a)^(1/2))^(1/2)))^(1/2)*EllipticF(((-(-1-(4+a)^(1/
2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-
(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/
2))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2
))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1
/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4
+a)^(1/2))^(1/2)))^(1/2))-((1+a)/(a^2+7*a+12)-a/(a^2+7*a+12))*((-
1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-1-(4+a)^(1/2))^
(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-
(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^
(1/2)))^(1/2)*(x-1+(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))
^(1/2)*(x-1-(-1-(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(
4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(4
+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^
(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)
/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(1/2)
)^(1/2)/(-(x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1+(-1+(4+a)^(1/2))^(1/2
))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2)))^(1/
2)*((1-(-1+(4+a)^(1/2))^(1/2))*EllipticF(((-(-1-(4+a)^(1/2))^(1/2
)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a
)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2
)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-
(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/
2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2
))^(1/2)))^(1/2))+2*(-1+(4+a)^(1/2))^(1/2)*EllipticPi(((-(-1-(4+a
)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2
))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a
)^(1/2))^(1/2)))^(1/2),(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^
(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2)),((-(-1-(4
+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+
(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))
^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2)))-
1/2/(a^2+7*a+12)*((x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/
2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2))+((-1-(4+a)^(1/2))^(1/2)+(
-1+(4+a)^(1/2))^(1/2))*((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))
^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1
+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(x-1+(-1
+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)^
(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-
1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+
(-1-(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))
^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-1/2*((1-(-1+(4+a)^(
1/2))^(1/2))*(1+(-1+(4+a)^(1/2))^(1/2))-(1-(-1-(4+a)^(1/2))^(1/2)

```
)*(1+(-1+(4+a)^(1/2))^(1/2))+(1-(-1-(4+a)^(1/2))^(1/2))*(1-(-1+(4
+a)^(1/2))^(1/2))+(1-(-1+(4+a)^(1/2))^(1/2))^2)/(-(-1-(4+a)^(1/2)
)^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/2)*EllipticF(
((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^
(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x
-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(
4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2)
)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/
2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))-1/2*(-(-1-(4+a)^(1/2))^
(1/2)+(-1+(4+a)^(1/2))^(1/2))*EllipticE(((-(-1-(4+a)^(1/2))^(1/2)
+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)
^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)
))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(
4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2
)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2)
)^(1/2)))^(1/2))/(-1+(4+a)^(1/2))^(1/2)-4/(-(-1-(4+a)^(1/2))^(1/2
)+(-1+(4+a)^(1/2))^(1/2))*EllipticPi(((-(-1-(4+a)^(1/2))^(1/2)+(-
1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1
/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^
(1/2),((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^
(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)),((-(-1-(4+a)^(1/2))^(1/2)-(-
1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1
/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^
(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))))/(-(x-1-(-1+(4+a)^(
1/2))^(1/2))*(x-1+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(
1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(-x^4+4\,x^3-8\,x^2+a+8\,x)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{x}{(x^4-4\,x^3+8\,x^2-a-8\,x)\sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-x/((x^4 - 4*x^3 + 8*x^2 - a - 8*x)*sqrt(-x^4 + 4*x^3 -
8*x^2 + a + 8*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**4+4*x**3-8*x**2+a+8*x)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(-x^4 + 4\,x^3 - 8\,x^2 + a + 8\,x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x/(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2), x)
```

3.634 $\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx$

Optimal. Leaf size=591

$$\frac{(x-1)\left(5a^2+4(2a+7)(x-1)^2+47a+104\right)}{12(a+3)^2(a+4)^2\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}+\frac{(x-1)^2+1}{3(a+4)^2\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(x-1)^2+1}{6(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}-\frac{(2a+7)\left(1-\sqrt{a+4}\right)(x-1)\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)}{3(a+3)^2(a+4)^2\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(5a+16)\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)F\left(\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)\middle|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{12(a+3)(a+4)^2\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(2a+7)\left(1-\sqrt{a+4}\right)\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)E\left(\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)\middle|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{3(a+3)^2(a+4)^2\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

```
[Out] (1 + (-1 + x)^2)/(6*(4 + a)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)^(
3/2)) + (1 + (-1 + x)^2)/(3*(4 + a)^2*Sqrt[3 + a - 2*(-1 + x)^2 -
 (-1 + x)^4]) + ((5 + a + (-1 + x)^2)*(-1 + x))/(6*(12 + 7*a + a^
2)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)^(3/2)) + ((104 + 47*a + 5*
a^2 + 4*(7 + 2*a)*(-1 + x)^2)*(-1 + x))/(12*(3 + a)^2*(4 + a)^2*S
qrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) - ((7 + 2*a)*(1 - Sqrt[4
+ a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(-1 + x))/(3*(3 + a)^2*(
4 + a)^2*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + ((7 + 2*a)*(1
 - Sqrt[4 + a])*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4
 + a]))*EllipticE[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqr
t[4 + a])/(1 - Sqrt[4 + a])])/(3*(3 + a)^2*(4 + a)^2*Sqrt[(1 + (-
1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*S
qrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + ((16 + 5*a)*Sqrt[1 + Sq
rt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticF[ArcTan[(-
1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])
])/(12*(3 + a)*(4 + a)^2*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/
(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (
-1 + x)^4])
```

Rubi [A] time = 1.52586, antiderivative size = 591, normalized size of antiderivative = 1., number of

steps used = 12, number of rules used = 12, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$

$$-\frac{(1-x)\left(a+(x-1)^2+5\right)}{6\left(a^2+7a+12\right)\left(a-(1-x)^4-2(1-x)^2+3\right)^{3/2}}-\frac{(1-x)\left(5a^2+4(2a+7)(1-x)^2+47a+104\right)}{12(a+3)^2(a+4)^2\sqrt{a-(1-x)^4-2(1-x)^2+3}}+\frac{(x-1)^2+1}{3(a+4)^2\sqrt{a-(1-x)^4-2(1-x)^2+3}}+\frac{(x-1)^2+1}{6(a+4)\left(a-(1-x)^4-2(1-x)^2+3\right)^{3/2}}+\frac{(2a+7)\left(1-\sqrt{a+4}\right)(1-x)\left(\frac{(1-x)^2}{1-\sqrt{a+4}}+1\right)}{3(a+3)^2(a+4)^2\sqrt{a-(1-x)^4-2(1-x)^2+3}}-\frac{(5a+16)\sqrt{\sqrt{a+4}+1}\left(\frac{(1-x)^2}{1-\sqrt{a+4}}+1\right)F\left(\tan^{-1}\left(\frac{1-x}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{12(a+3)(a+4)^2\sqrt{\frac{\frac{(1-x)^2}{1-\sqrt{a+4}}+1}{\frac{(1-x)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(1-x)^4-2(1-x)^2+3}}-\frac{(2a+7)\left(1-\sqrt{a+4}\right)\sqrt{\sqrt{a+4}+1}\left(\frac{(1-x)^2}{1-\sqrt{a+4}}+1\right)E\left(\tan^{-1}\left(\frac{1-x}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{3(a+3)^2(a+4)^2\sqrt{\frac{\frac{(1-x)^2}{1-\sqrt{a+4}}+1}{\frac{(1-x)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(1-x)^4-2(1-x)^2+3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(5/2),x]
```

```
[Out] (1 + (-1 + x)^2)/(6*(4 + a)*(3 + a - 2*(1 - x)^2 - (1 - x)^4)^(3/
2)) + (1 + (-1 + x)^2)/(3*(4 + a)^2*Sqrt[3 + a - 2*(1 - x)^2 - (1
 - x)^4]) - ((104 + 47*a + 5*a^2 + 4*(7 + 2*a)*(1 - x)^2)*(1 - x)
)/(12*(3 + a)^2*(4 + a)^2*Sqrt[3 + a - 2*(1 - x)^2 - (1 - x)^4])
+ ((7 + 2*a)*(1 - Sqrt[4 + a])*(1 + (1 - x)^2/(1 - Sqrt[4 + a]))*
(1 - x))/(3*(3 + a)^2*(4 + a)^2*Sqrt[3 + a - 2*(1 - x)^2 - (1 - x
)^4]) - ((5 + a + (-1 + x)^2)*(1 - x))/(6*(12 + 7*a + a^2)*(3 + a
 - 2*(1 - x)^2 - (1 - x)^4)^(3/2)) - ((7 + 2*a)*(1 - Sqrt[4 + a])
*Sqrt[1 + Sqrt[4 + a]]*(1 + (1 - x)^2/(1 - Sqrt[4 + a]))*Elliptic
E[ArcTan[(1 - x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sq
rt[4 + a])])/(3*(3 + a)^2*(4 + a)^2*Sqrt[(1 + (1 - x)^2/(1 - Sqrt
[4 + a]))/(1 + (1 - x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(1 -
x)^2 - (1 - x)^4]) - ((16 + 5*a)*Sqrt[1 + Sqrt[4 + a]]*(1 + (1 -
x)^2/(1 - Sqrt[4 + a]))*EllipticF[ArcTan[(1 - x)/Sqrt[1 + Sqrt[4
+ a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(12*(3 + a)*(4 + a)^
2*Sqrt[(1 + (1 - x)^2/(1 - Sqrt[4 + a]))/(1 + (1 - x)^2/(1 + Sqrt
[4 + a]))]*Sqrt[3 + a - 2*(1 - x)^2 - (1 - x)^4])
```

Rubi in Sympy [A] time = 108.201, size = 502, normalized size = 0.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(-x**4+4*x**3-8*x**2+a+8*x)**(5/2),x)
```

```
[Out] (x - 1)*(2*a + (2*a + 10)*(x - 1) + 2*(x - 1)**3 + 2*(x - 1)**2 +
 10)/(12*(a + 3)*(a + 4)*(a - (x - 1)**4 - 2*(x - 1)**2 + 3)**(3/
2)) - (2*a + 7)*(x - 1)*((x - 1)**2/(-sqrt(a + 4) + 1) + 1)*(-sqr
t(a + 4) + 1)/(3*(a + 3)**2*(a + 4)**2*sqrt(a - (x - 1)**4 - 2*(x
 - 1)**2 + 3)) + (3*a + 10)*sqrt(a - (x - 1)**4 - 2*(x - 1)**2 +
3)/(6*(a + 3)**2*(a + 4)**2) + (x - 1)*(20*a**2 + 188*a + (24*a +
 80)*(x - 1)**3 + (32*a + 112)*(x - 1)**2 + (x - 1)*(16*a**2 + 14
4*a + 304) + 416)/(48*(a + 3)**2*(a + 4)**2*sqrt(a - (x - 1)**4 -
 2*(x - 1)**2 + 3)) + (5*a + 16)*((x - 1)**2/(-sqrt(a + 4) + 1) +
 1)*sqrt(sqrt(a + 4) + 1)*elliptic_f(atan((x - 1)/sqrt(sqrt(a + 4
) + 1)), 2*sqrt(a + 4)/(sqrt(a + 4) - 1))/(12*sqrt((-(x - 1)**2/(
sqrt(a + 4) - 1) + 1)/((x - 1)**2/(sqrt(a + 4) + 1) + 1))*(a + 3)
*(a + 4)**2*sqrt(a - (x - 1)**4 - 2*(x - 1)**2 + 3)) + (2*a + 7)*
((x - 1)**2/(-sqrt(a + 4) + 1) + 1)*(-sqrt(a + 4) + 1)*sqrt(sqrt(
a + 4) + 1)*elliptic_e(atan((x - 1)/sqrt(sqrt(a + 4) + 1)), 2*sqr
t(a + 4)/(sqrt(a + 4) - 1))/(3*sqrt((-(x - 1)**2/(sqrt(a + 4) - 1
) + 1)/((x - 1)**2/(sqrt(a + 4) + 1) + 1))*(a + 3)**2*(a + 4)**2*
sqrt(a - (x - 1)**4 - 2*(x - 1)**2 + 3))
```

Mathematica [B] time = 6.16041, size = 6452, normalized size = 10.92

Result too large to show

Antiderivative was successfully verified.

```
[In]  Integrate[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(5/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.042, size = 2777, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x)
```

```
[Out] (1/6/(a^2+7*a+12)*x^3+1/6*a/(a^2+7*a+12)*x^2-1/6*(a-2)/(a^2+7*a+1
2)*x+1/6*a/(a^2+7*a+12))*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)/(x^4-4*x^
```

```
3+8*x^2-a-8*x)^2+2*(1/6*(7+2*a)/(a^2+7*a+12)^2*x^3+1/6*(a^2-12)/(
a^2+7*a+12)^2*x^2-1/24*(3*a^2-23*a-116)/(a^2+7*a+12)^2*x+1/24*(3*
a^2-7*a-60)/(a^2+7*a+12)^2)/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)-(1/6*(
a^2+23*a+68)/(a^2+7*a+12)^2+1/12*(3*a^2-23*a-116)/(a^2+7*a+12)^2)
*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-1-(4+a)^(1/
2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-
(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/
2))^(1/2)))^(1/2)*(x-1+(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1
/2))^(1/2)*(x-1-(-1-(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(
-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-
1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/
2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(
1/2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(
1/2))^(1/2)/(-(x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1+(-1+(4+a)^(1/2))^
(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2)))
^(1/2)*EllipticF(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2)
)*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)
^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^(
1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(
4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2
))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))-(2/3*(
a^2+2*a-5)/(a^2+7*a+12)^2-2/3*(a^2-12)/(a^2+7*a+12)^2)*((-1-(4+a)
^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-1-(4+a)^(1/2))^(1/2)+(
-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(
1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))
^(1/2)*(x-1+(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*
(x-1-(-1-(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1
/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/
2))^(1/2)*(x-1+(-1-(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(
-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)/(-(-1-
(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/2)
/(-(x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1+(-1+(4+a)^(1/2))^(1/2))*(x-1
-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2)))^(1/2)*((1-
(-1+(4+a)^(1/2))^(1/2))*EllipticF(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(
4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2)
)^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/
2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(
1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+
(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2
)))^(1/2))+2*(-1+(4+a)^(1/2))^(1/2)*EllipticPi(((-(-1-(4+a)^(1/2)
)^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-
1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2)
)^(1/2)))^(1/2),(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/
(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2)),((-(-1-(4+a)^(1/
2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+
a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))
/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2)))-1/3*(7+
2*a)/(a^2+7*a+12)^2*((x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^
(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2))+((-1-(4+a)^(1/2))^(1/2
)+(-1+(4+a)^(1/2))^(1/2))*((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/
2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-
(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(x-1+
(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4+
a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/
```

```
(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x
-1+(-1-(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/
2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-1/2*((1-(-1+(4+a
)^(1/2))^(1/2))*(1+(-1+(4+a)^(1/2))^(1/2))-(1-(-1-(4+a)^(1/2))^(1
/2))*(1+(-1+(4+a)^(1/2))^(1/2))+(1-(-1-(4+a)^(1/2))^(1/2))*(1-(-1
+(4+a)^(1/2))^(1/2))+(1-(-1+(4+a)^(1/2))^(1/2))^2)/(-(-1-(4+a)^(1
/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/2)*Ellipti
cF(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+
a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))
/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-
1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1
/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^
(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))-1/2*(-(-1-(4+a)^(1/2
))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*EllipticE(((-(-1-(4+a)^(1/2))^(1
/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4
+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1
/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-
1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(
1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1
/2))^(1/2)))^(1/2))/(-1+(4+a)^(1/2))^(1/2)-4/(-(-1-(4+a)^(1/2))^(
1/2)+(-1+(4+a)^(1/2))^(1/2))*EllipticPi(((-(-1-(4+a)^(1/2))^(1/2)
+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)
^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)
))^(1/2),((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+
a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)),((-(-1-(4+a)^(1/2))^(1/2)
-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))
^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+
a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))))/(-(x-1-(-1+(4+a
)^(1/2))^(1/2))*(x-1+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2)
)^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(-x^4 + 4\,x^3 - 8\,x^2 + a + 8\,x)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x}{(x^8 - 8\,x^7 + 32\,x^6 - 2\,(a-64)x^4 - 80\,x^5 + 8\,(a-16)x^3 - 16\,(a-4)x^2 + a^2 + 16\,ax)\sqrt{-x^4 + 4\,x^3 - 8\,x^2 + a + 8\,x}}, x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(x/((x^8 - 8*x^7 + 32*x^6 - 2*(a - 64)*x^4 - 80*x^5 + 8*(
a - 16)*x^3 - 16*(a - 4)*x^2 + a^2 + 16*a*x)*sqrt(-x^4 + 4*x^3 -
8*x^2 + a + 8*x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(a - x^4 + 4x^3 - 8x^2 + 8x)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(-x**4+4*x**3-8*x**2+a+8*x)**(5/2),x)
```

```
[Out] Integral(x/(a - x**4 + 4*x**3 - 8*x**2 + 8*x)**(5/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(-x^4 + 4\,x^3 - 8\,x^2 + a + 8\,x)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x/(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(5/2), x)
```

3.635 $\int x^2 \left(a+8x-8x^2+4x^3-x^4\right)^{3/2} \, dx$

Optimal. Leaf size=585

$$\frac{4\left(21a^2+111a+140\right)\left(1-\sqrt{a+4}\right)(x-1)\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)}{315\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$
$$-\frac{4\left(21a^2+111a+140\right)\left(1-\sqrt{a+4}\right)\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)E\left(\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{315\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$
$$+\frac{3}{8}(a+4)\left((x-1)^2+1\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}+\frac{1}{4}\left((x-1)^2+1\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$
$$+\frac{1}{63}\left(7(x-1)^2+15\right)(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$+\frac{2}{315}(x-1)\left(3(7a+20)(x-1)^2+2(27a+80)\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}+\frac{3}{8}(a+4)^2\tan^{-1}\left(\frac{(x-1)^2+1}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+$$

```
[Out] (3*(4 + a)*(1 + (-1 + x)^2)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^
4])/8 + ((1 + (-1 + x)^2)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)^(3/
2))/4 + (4*(140 + 111*a + 21*a^2)*(1 - Sqrt[4 + a])*(1 + (-1 + x)
^2/(1 - Sqrt[4 + a]))*(-1 + x))/(315*Sqrt[3 + a - 2*(-1 + x)^2 -
(-1 + x)^4]) + (2*(2*(80 + 27*a) + 3*(20 + 7*a)*(-1 + x)^2)*Sqrt[
3 + a - 2*(-1 + x)^2 - (-1 + x)^4]*(-1 + x))/315 + ((15 + 7*(-1 +
 x)^2)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)^(3/2)*(-1 + x))/63 + (
3*(4 + a)^2*ArcTan[(1 + (-1 + x)^2)/Sqrt[3 + a - 2*(-1 + x)^2 - (
-1 + x)^4]])/8 - (4*(140 + 111*a + 21*a^2)*(1 - Sqrt[4 + a])*Sqrt
[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticE[Ar
cTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[
4 + a])])/(315*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 +
 x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]
) + (4*(3 + a)*(100 + 33*a)*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2
/(1 - Sqrt[4 + a]))*EllipticF[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a
]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(315*Sqrt[(1 + (-1 + x)
^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3
+ a - 2*(-1 + x)^2 - (-1 + x)^4])
```

Rubi [A] time = 1.63525, antiderivative size = 585, normalized size of antiderivative = 1., number of

steps used = 15, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.464$

$$-\frac{4(21a^2+111a+140)\left(1-\sqrt{a+4}\right)(1-x)\left(\frac{(1-x)^2}{1-\sqrt{a+4}}+1\right)}{315\sqrt{a-(1-x)^4-2(1-x)^2+3}}$$
$$+\frac{4(21a^2+111a+140)\left(1-\sqrt{a+4}\right)\sqrt{\sqrt{a+4}+1}\left(\frac{(1-x)^2}{1-\sqrt{a+4}}+1\right)E\left(\tan^{-1}\left(\frac{1-x}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{315\sqrt{\frac{\frac{(1-x)^2}{1-\sqrt{a+4}}+1}{\frac{(1-x)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(1-x)^4-2(1-x)^2+3}}$$
$$+\frac{3}{8}(a+4)\left((x-1)^2+1\right)\sqrt{a-(1-x)^4-2(1-x)^2+3}+\frac{1}{4}\left((x-1)^2+1\right)\left(a-(1-x)^4-2(1-x)^2+3\right)^{3/2}$$
$$-\frac{1}{63}\left(7(1-x)^2+15\right)(1-x)\left(a-(1-x)^4-2(1-x)^2+3\right)^{3/2}$$

$$-\frac{2}{315}(1-x)\left(3(7a+20)(1-x)^2+2(27a+80)\right)\sqrt{a-(1-x)^4-2(1-x)^2+3}+\frac{3}{8}(a+4)^2\tan^{-1}\left(\frac{(x-1)^2+1}{\sqrt{a-(1-x)^4-2(1-x)^2+3}}\right)-$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

```
[Out] (3*(4 + a)*Sqrt[3 + a - 2*(1 - x)^2 - (1 - x)^4]*(1 + (-1 + x)^2)
)/8 + ((3 + a - 2*(1 - x)^2 - (1 - x)^4)^(3/2)*(1 + (-1 + x)^2))/
4 - (4*(140 + 111*a + 21*a^2)*(1 - Sqrt[4 + a])*(1 + (1 - x)^2/(1
 - Sqrt[4 + a]))*(1 - x))/(315*Sqrt[3 + a - 2*(1 - x)^2 - (1 - x)
^4]) - (2*(2*(80 + 27*a) + 3*(20 + 7*a)*(1 - x)^2)*Sqrt[3 + a - 2
*(1 - x)^2 - (1 - x)^4]*(1 - x))/315 - ((15 + 7*(1 - x)^2)*(3 + a
 - 2*(1 - x)^2 - (1 - x)^4)^(3/2)*(1 - x))/63 + (3*(4 + a)^2*ArcT
an[(1 + (-1 + x)^2)/Sqrt[3 + a - 2*(1 - x)^2 - (1 - x)^4]])/8 + (
4*(140 + 111*a + 21*a^2)*(1 - Sqrt[4 + a])*Sqrt[1 + Sqrt[4 + a]]*
(1 + (1 - x)^2/(1 - Sqrt[4 + a]))*EllipticE[ArcTan[(1 - x)/Sqrt[1
 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(315*Sqrt[
(1 + (1 - x)^2/(1 - Sqrt[4 + a]))/(1 + (1 - x)^2/(1 + Sqrt[4 + a]
))]*Sqrt[3 + a - 2*(1 - x)^2 - (1 - x)^4]) - (4*(3 + a)*(100 + 33
*a)*Sqrt[1 + Sqrt[4 + a]]*(1 + (1 - x)^2/(1 - Sqrt[4 + a]))*Ellip
ticF[ArcTan[(1 - x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 -
 Sqrt[4 + a])])/(315*Sqrt[(1 + (1 - x)^2/(1 - Sqrt[4 + a]))/(1 +
(1 - x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(1 - x)^2 - (1 - x)^
4])
```

Rubi in Sympy [A] time = 84.6116, size = 498, normalized size = 0.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(-x**4+4*x**3-8*x**2+a+8*x)**(3/2),x)
```

```
[Out] (3*a/16 + 3/4)*(2*(x - 1)**2 + 2)*sqrt(a - (x - 1)**4 - 2*(x - 1)
**2 + 3) + 3*(a + 4)**2*atan(-(-2*(x - 1)**2 - 2)/(2*sqrt(a - (x
- 1)**4 - 2*(x - 1)**2 + 3)))/8 + 4*(x - 1)*((x - 1)**2/(-sqrt(a
+ 4) + 1) + 1)*(-sqrt(a + 4) + 1)*(21*a**2 + 111*a + 140)/(315*sq
rt(a - (x - 1)**4 - 2*(x - 1)**2 + 3)) + (x - 1)*(7*(x - 1)**2 +
15)*(a - (x - 1)**4 - 2*(x - 1)**2 + 3)**(3/2)/63 + (x - 1)*(108*
a + (42*a + 120)*(x - 1)**2 + 320)*sqrt(a - (x - 1)**4 - 2*(x - 1
)**2 + 3)/315 + (2*(x - 1)**2 + 2)*(a - (x - 1)**4 - 2*(x - 1)**2
 + 3)**(3/2)/8 + 4*(a + 3)*(33*a + 100)*((x - 1)**2/(-sqrt(a + 4)
 + 1) + 1)*sqrt(sqrt(a + 4) + 1)*elliptic_f(atan((x - 1)/sqrt(sqr
t(a + 4) + 1)), 2*sqrt(a + 4)/(sqrt(a + 4) - 1))/(315*sqrt((-(x -
 1)**2/(sqrt(a + 4) - 1) + 1)/((x - 1)**2/(sqrt(a + 4) + 1) + 1))
*sqrt(a - (x - 1)**4 - 2*(x - 1)**2 + 3)) - 4*((x - 1)**2/(-sqrt(
a + 4) + 1) + 1)*(-sqrt(a + 4) + 1)*sqrt(sqrt(a + 4) + 1)*(21*a**
2 + 111*a + 140)*elliptic_e(atan((x - 1)/sqrt(sqrt(a + 4) + 1)),
2*sqrt(a + 4)/(sqrt(a + 4) - 1))/(315*sqrt((-(x - 1)**2/(sqrt(a +
 4) - 1) + 1)/((x - 1)**2/(sqrt(a + 4) + 1) + 1))*sqrt(a - (x - 1
)**4 - 2*(x - 1)**2 + 3))
```

Mathematica [B] time = 6.24083, size = 8500, normalized size = 14.53

Result too large to show

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.033, size = 2733, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x)
```

```
[Out] -1/9*x^7*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+19/36*x^6*(-x^4+4*x^3-8*x
^2+a+8*x)^(1/2)-163/126*x^5*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+71/42*
x^4*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+(11/45*a-16/63)*x^3*(-x^4+4*x^
3-8*x^2+a+8*x)^(1/2)+(-13/120*a-5/18)*x^2*(-x^4+4*x^3-8*x^2+a+8*x
)^(1/2)+(9/140*a+23/63)*x*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+(107/252
```

```
*a+101/63)*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)-(-(9/140*a+23/63)*a-107
/63*a-404/63)*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*((-
(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/
2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+
(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(4+a)^(1/2))^(1/2))^2*(-2
*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)^(1/2))^(1/2))/((-1-(4+a)^(
1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))
^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2))^(1/2))/(-
(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/
2))^(1/2)))^(1/2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2)
)/(-1+(4+a)^(1/2))^(1/2)/(-(x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1+(-1+
(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(
1/2))^(1/2)))^(1/2)*EllipticF(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)
^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1
/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),(
(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2)
)^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a
)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^
(1/2))-(-2*(-13/120*a-5/18)*a+827/315*a+76/9)*((-1-(4+a)^(1/2))^(
1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^
(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/
2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(x
-1+(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-
(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2
))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)
*(x-1+(-1-(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^
(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)/(-(-1-(4+a)^(1/
2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/2)/(-(x-1-(
-1+(4+a)^(1/2))^(1/2))*(x-1+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a
)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2)))^(1/2)*((1-(-1+(4+a)
^(1/2))^(1/2))*EllipticF(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2
))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(
-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1
-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/
2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/
2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2)
)+2*(-1+(4+a)^(1/2))^(1/2)*EllipticPi(((-(-1-(4+a)^(1/2))^(1/2)+(
-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(
1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))
^(1/2),(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+
a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2)),((-(-1-(4+a)^(1/2))^(1/2)
-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))
^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+
a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2)))+(a^2-3*(11/45*a-
16/63)*a+68/105*a+16/9)*((x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4
+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2))+((-1-(4+a)^(1/2))^
(1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)
^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1
/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(
x-1+(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1
-(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/
2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2
)*(x-1+(-1-(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)
^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-1/2*((1-(-1+
```

```
(4+a)^(1/2))^(1/2))*(1+(-1+(4+a)^(1/2))^(1/2))-(1-(-1-(4+a)^(1/2)
)^(1/2))*(1+(-1+(4+a)^(1/2))^(1/2))+(1-(-1-(4+a)^(1/2))^(1/2))*(1
-(-1+(4+a)^(1/2))^(1/2))+(1-(-1+(4+a)^(1/2))^(1/2))^2)/(-(-1-(4+a
)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/2)*Ell
ipticF(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1
+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1
/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2
)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2)
)^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4
+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))-1/2*(-(-1-(4+a)^
(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*EllipticE(((-(-1-(4+a)^(1/2)
)^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-
1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2)
)^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))
*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2
))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a
)^(1/2))^(1/2)))^(1/2))/(-1+(4+a)^(1/2))^(1/2)-4/(-(-1-(4+a)^(1/2
))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*EllipticPi(((-(-1-(4+a)^(1/2))^(
1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(
4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(
1/2)))^(1/2),((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1
-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)),((-(-1-(4+a)^(1/2))^(
1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1
/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1
-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))))/(-(x-1-(-1+
(4+a)^(1/2))^(1/2))*(x-1+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(
1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-x^4 + 4\,x^3 - 8\,x^2 + a + 8\,x\right)^{\frac{3}{2}} x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2)*x^2,x, algorithm="maxima")
```

```
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2)*x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(x^6 - 4\,x^5 + 8\,x^4 - ax^2 - 8\,x^3\right)\sqrt{-x^4 + 4\,x^3 - 8\,x^2 + a + 8\,x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2)*x^2,x, algorithm="fricas")
```

```
[Out] integral(-(x^6 - 4*x^5 + 8*x^4 - a*x^2 - 8*x^3)*sqrt(-x^4 + 4*x^3
 - 8*x^2 + a + 8*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \left(a - x^4 + 4x^3 - 8x^2 + 8x\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-x**4+4*x**3-8*x**2+a+8*x)**(3/2),x)
```

```
[Out] Integral(x**2*(a - x**4 + 4*x**3 - 8*x**2 + 8*x)**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-x^4 + 4\,x^3 - 8\,x^2 + a + 8\,x\right)^{\frac{3}{2}} x^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2)*x^2,x, algorithm="giac")
```

```
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2)*x^2, x)
```

3.636 $\int x^2\sqrt{a+8x-8x^2+4x^3-x^4}\,dx$

Optimal. Leaf size=485

$$\frac{1}{2}\left((x-1)^2+1\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}+\frac{1}{15}\left(3(x-1)^2+7\right)(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}+\frac{2(3a+8)\left(1-\sqrt{a+4}\right)(x-1)\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)}{15\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{1}{2}(a+4)\tan^{-1}\left(\frac{(x-1)^2+1}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{8(a+3)\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)F\left(\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{15\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}-\frac{2(3a+8)\left(1-\sqrt{a+4}\right)\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)E\left(\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{15\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

```
[Out] ((1 + (-1 + x)^2)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])/2 + (2
*(8 + 3*a)*(1 - Sqrt[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(
-1 + x))/(15*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + ((7 + 3*(
-1 + x)^2)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]*(-1 + x))/15 +
 ((4 + a)*ArcTan[(1 + (-1 + x)^2)/Sqrt[3 + a - 2*(-1 + x)^2 - (-1
 + x)^4]])/2 - (2*(8 + 3*a)*(1 - Sqrt[4 + a])*Sqrt[1 + Sqrt[4 + a
]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticE[ArcTan[(-1 + x)/S
qrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(15*S
qrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[
4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (8*(3 + a)*S
qrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticF
[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sq
rt[4 + a])])/(15*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1
 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^
4])
```

Rubi [A] time = 1.35193, antiderivative size = 485, normalized size of antiderivative = 1., number of

steps used = 13, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.464

$$\frac{1}{2}\left((x-1)^2+1\right)\sqrt{a-(1-x)^4-2(1-x)^2+3} - \frac{1}{15}\left(3(1-x)^2+7\right)(1-x)\sqrt{a-(1-x)^4-2(1-x)^2+3} - \frac{2(3a+8)\left(1-\sqrt{a+4}\right)(1-x)\left(\frac{(1-x)^2}{1-\sqrt{a+4}}+1\right)}{15\sqrt{a-(1-x)^4-2(1-x)^2+3}} + \frac{1}{2}(a+4)\tan^{-1}\left(\frac{(x-1)^2+1}{\sqrt{a-(1-x)^4-2(1-x)^2+3}}\right) - \frac{8(a+3)\sqrt{\sqrt{a+4}+1}\left(\frac{(1-x)^2}{1-\sqrt{a+4}}+1\right)F\left(\tan^{-1}\left(\frac{1-x}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{15\sqrt{\frac{\frac{(1-x)^2}{1-\sqrt{a+4}}+1}{\frac{(1-x)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(1-x)^4-2(1-x)^2+3}} + \frac{2(3a+8)\left(1-\sqrt{a+4}\right)\sqrt{\sqrt{a+4}+1}\left(\frac{(1-x)^2}{1-\sqrt{a+4}}+1\right)E\left(\tan^{-1}\left(\frac{1-x}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{15\sqrt{\frac{\frac{(1-x)^2}{1-\sqrt{a+4}}+1}{\frac{(1-x)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(1-x)^4-2(1-x)^2+3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] (Sqrt[3 + a - 2*(1 - x)^2 - (1 - x)^4]*(1 + (-1 + x)^2))/2 - (2*(
8 + 3*a)*(1 - Sqrt[4 + a])*(1 + (1 - x)^2/(1 - Sqrt[4 + a]))*(1 -
 x))/(15*Sqrt[3 + a - 2*(1 - x)^2 - (1 - x)^4]) - ((7 + 3*(1 - x)
^2)*Sqrt[3 + a - 2*(1 - x)^2 - (1 - x)^4]*(1 - x))/15 + ((4 + a)*
ArcTan[(1 + (-1 + x)^2)/Sqrt[3 + a - 2*(1 - x)^2 - (1 - x)^4]])/2
 + (2*(8 + 3*a)*(1 - Sqrt[4 + a])*Sqrt[1 + Sqrt[4 + a]]*(1 + (1 -
 x)^2/(1 - Sqrt[4 + a]))*EllipticE[ArcTan[(1 - x)/Sqrt[1 + Sqrt[4
 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(15*Sqrt[(1 + (1 -
x)^2/(1 - Sqrt[4 + a]))/(1 + (1 - x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3
 + a - 2*(1 - x)^2 - (1 - x)^4]) - (8*(3 + a)*Sqrt[1 + Sqrt[4 + a
]]*(1 + (1 - x)^2/(1 - Sqrt[4 + a]))*EllipticF[ArcTan[(1 - x)/Sqr
t[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(15*Sqr
t[(1 + (1 - x)^2/(1 - Sqrt[4 + a]))/(1 + (1 - x)^2/(1 + Sqrt[4 +
a]))]*Sqrt[3 + a - 2*(1 - x)^2 - (1 - x)^4])
```

Rubi in Sympy [A] time = 72.4568, size = 405, normalized size = 0.84

$$\left(\frac{a}{2}+2\right)\operatorname{atan}\left(-\frac{-2(x-1)^2-2}{2\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{2(3a+8)(x-1)\left(\frac{(x-1)^2}{-\sqrt{a+4}+1}+1\right)\left(-\sqrt{a+4}+1\right)}{15\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(x-1)\left(3(x-1)^2+7\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}{15}+\frac{\left(2(x-1)^2+2\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}{4}+\frac{8(a+3)\left(\frac{(x-1)^2}{-\sqrt{a+4}+1}+1\right)\sqrt{\sqrt{a+4}+1}F\left(\operatorname{atan}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)\middle|\frac{2\sqrt{a+4}}{\sqrt{a+4}-1}\right)}{15\sqrt{\frac{-\frac{(x-1)^2}{\sqrt{a+4}-1}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}-\frac{2(3a+8)\left(\frac{(x-1)^2}{-\sqrt{a+4}+1}+1\right)\left(-\sqrt{a+4}+1\right)\sqrt{\sqrt{a+4}+1}E\left(\operatorname{atan}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)\middle|\frac{2\sqrt{a+4}}{\sqrt{a+4}-1}\right)}{15\sqrt{\frac{-\frac{(x-1)^2}{\sqrt{a+4}-1}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(-x**4+4*x**3-8*x**2+a+8*x)**(1/2),x)
```

```
[Out] (a/2 + 2)*atan(-(-2*(x - 1)**2 - 2)/(2*sqrt(a - (x - 1)**4 - 2*(x
 - 1)**2 + 3))) + 2*(3*a + 8)*(x - 1)*((x - 1)**2/(-sqrt(a + 4) +
 1) + 1)*(-sqrt(a + 4) + 1)/(15*sqrt(a - (x - 1)**4 - 2*(x - 1)**
2 + 3)) + (x - 1)*(3*(x - 1)**2 + 7)*sqrt(a - (x - 1)**4 - 2*(x -
 1)**2 + 3)/15 + (2*(x - 1)**2 + 2)*sqrt(a - (x - 1)**4 - 2*(x -
1)**2 + 3)/4 + 8*(a + 3)*((x - 1)**2/(-sqrt(a + 4) + 1) + 1)*sqrt
(sqrt(a + 4) + 1)*elliptic_f(atan((x - 1)/sqrt(sqrt(a + 4) + 1)),
 2*sqrt(a + 4)/(sqrt(a + 4) - 1))/(15*sqrt((-(x - 1)**2/(sqrt(a +
 4) - 1) + 1)/((x - 1)**2/(sqrt(a + 4) + 1) + 1))*sqrt(a - (x - 1
)**4 - 2*(x - 1)**2 + 3)) - 2*(3*a + 8)*((x - 1)**2/(-sqrt(a + 4)
 + 1) + 1)*(-sqrt(a + 4) + 1)*sqrt(sqrt(a + 4) + 1)*elliptic_e(at
an((x - 1)/sqrt(sqrt(a + 4) + 1)), 2*sqrt(a + 4)/(sqrt(a + 4) - 1
))/(15*sqrt((-(x - 1)**2/(sqrt(a + 4) - 1) + 1)/((x - 1)**2/(sqrt
(a + 4) + 1) + 1))*sqrt(a - (x - 1)**4 - 2*(x - 1)**2 + 3))
```

Mathematica [B] time = 6.16373, size = 5647, normalized size = 11.64

Result too large to show

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] Result too large to show
```

Maple [B]   time = 0.03, size = 2582, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x)
```

```
[Out] 1/5*x^3*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)-1/10*x^2*(-x^4+4*x^3-8*x^2
+a+8*x)^(1/2)+1/15*x*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+1/3*(-x^4+4*x
^3-8*x^2+a+8*x)^(1/2)-(-1/15*a-4/3)*((-1-(4+a)^(1/2))^(1/2)+(-1+(
4+a)^(1/2))^(1/2))*((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/
2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+
a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(4+
a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)^(1/2
))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-
1+(4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-
(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/
2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)/(-(-1-(4+a)^(1/2))^(1/2)+
(-1+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/2)/(-(x-1-(-1+(4+a)^(
1/2))^(1/2))*(x-1+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(
1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2)))^(1/2)*EllipticF(((-(-1-(4+a)^
(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))
/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^
(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(
1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)
^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1
+(4+a)^(1/2))^(1/2)))^(1/2))-(1/5*a+28/15)*((-1-(4+a)^(1/2))^(1/2
)+(-1+(4+a)^(1/2))^(1/2))*((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/
2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-
(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(x-1+
(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4+
a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/
(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x
-1+(-1-(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/
2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)/(-(-1-(4+a)^(1/2))
^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/2)/(-(x-1-(-1+
(4+a)^(1/2))^(1/2))*(x-1+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(
1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2)))^(1/2)*((1-(-1+(4+a)^(1
/2))^(1/2))*EllipticF(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^
(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+
(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4
+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+
(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))
^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))+2
```

```
*(-1+(4+a)^(1/2))^(1/2)*EllipticPi(((-(-1-(4+a)^(1/2))^(1/2)+(-1+
(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2
))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1
/2),(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^
(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2)),((-(-1-(4+a)^(1/2))^(1/2)-(-
1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1
/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^
(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))+(2/5*a+16/15)*((x-1
-(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4
+a)^(1/2))^(1/2))+((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))
*((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)
^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(
x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(4+a)^(1/2))^(1/2))^2
*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)^(1/2))^(1/2))/((-1-(4+
a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/
2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2))^(1/2)
)/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)
^(1/2))^(1/2)))^(1/2)*(-1/2*((1-(-1+(4+a)^(1/2))^(1/2))*(1+(-1+(4
+a)^(1/2))^(1/2))-(1-(-1-(4+a)^(1/2))^(1/2))*(1+(-1+(4+a)^(1/2))^
(1/2))+(1-(-1-(4+a)^(1/2))^(1/2))*(1-(-1+(4+a)^(1/2))^(1/2))+(1-(
-1+(4+a)^(1/2))^(1/2))^2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2
))^(1/2))/(-1+(4+a)^(1/2))^(1/2)*EllipticF(((-(-1-(4+a)^(1/2))^(1
/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4
+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1
/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-
1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(
1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1
/2))^(1/2)))^(1/2))-1/2*(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))
^(1/2))*EllipticE(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2
))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a
)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^
(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+
(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/
2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))/(-1+(
4+a)^(1/2))^(1/2)-4/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/
2))*EllipticPi(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*
(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(
1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-1-(4+a)^(1/2)
)^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)
^(1/2))^(1/2)),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*
((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2)
)^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)
^(1/2))^(1/2)))^(1/2))))/(-(x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1+(-1+
(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(
1/2))^(1/2)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x^4 + 4\,x^3 - 8\,x^2 + a + 8\,x}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{-x^4 + 4\,x^3 - 8\,x^2 + a + 8\,x}x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sqrt{a - x^4 + 4x^3 - 8x^2 + 8x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-x**4+4*x**3-8*x**2+a+8*x)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x^4 + 4\,x^3 - 8\,x^2 + a + 8\,x}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x^2, x)
```

3.637 $\int \frac{x^2}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx$

Optimal. Leaf size=388

$$\frac{\left(1-\sqrt{a+4}\right)(x-1)\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\tan^{-1}\left(\frac{(x-1)^2+1}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)F\left(\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}-\frac{\left(1-\sqrt{a+4}\right)\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)E\left(\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

```
[Out] ((1 - Sqrt[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(-1 + x))/S
qrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4] + ArcTan[(1 + (-1 + x)^2)/
Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]] - ((1 - Sqrt[4 + a])*Sqr
t[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticE[A
rcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt
[4 + a])])/(Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)
^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) +
 (Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*Ellipt
icF[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 -
 Sqrt[4 + a])])/(Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1
 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^
4])
```

Rubi [A] time = 1.03756, antiderivative size = 388, normalized size of antiderivative = 1., number of

steps used = 11, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$

$$-\frac{\left(1-\sqrt{a+4}\right)(1-x)\left(\frac{(1-x)^2}{1-\sqrt{a+4}}+1\right)}{\sqrt{a-(1-x)^4-2(1-x)^2+3}}+\tan^{-1}\left(\frac{(x-1)^2+1}{\sqrt{a-(1-x)^4-2(1-x)^2+3}}\right)-\frac{\sqrt{\sqrt{a+4}+1}\left(\frac{(1-x)^2}{1-\sqrt{a+4}}+1\right)F\left(\tan^{-1}\left(\frac{1-x}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(1-x)^2}{1-\sqrt{a+4}}+1}{\frac{(1-x)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(1-x)^4-2(1-x)^2+3}}+\frac{\left(1-\sqrt{a+4}\right)\sqrt{\sqrt{a+4}+1}\left(\frac{(1-x)^2}{1-\sqrt{a+4}}+1\right)E\left(\tan^{-1}\left(\frac{1-x}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(1-x)^2}{1-\sqrt{a+4}}+1}{\frac{(1-x)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(1-x)^4-2(1-x)^2+3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] -(((1 - Sqrt[4 + a])*(1 + (1 - x)^2/(1 - Sqrt[4 + a]))*(1 - x))/S
qrt[3 + a - 2*(1 - x)^2 - (1 - x)^4]) + ArcTan[(1 + (-1 + x)^2)/S
qrt[3 + a - 2*(1 - x)^2 - (1 - x)^4]] + ((1 - Sqrt[4 + a])*Sqrt[1
 + Sqrt[4 + a]]*(1 + (1 - x)^2/(1 - Sqrt[4 + a]))*EllipticE[ArcTa
n[(1 - x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 +
a])])/(Sqrt[(1 + (1 - x)^2/(1 - Sqrt[4 + a]))/(1 + (1 - x)^2/(1 +
 Sqrt[4 + a]))]*Sqrt[3 + a - 2*(1 - x)^2 - (1 - x)^4]) - (Sqrt[1
+ Sqrt[4 + a]]*(1 + (1 - x)^2/(1 - Sqrt[4 + a]))*EllipticF[ArcTan
[(1 - x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a
])])/(Sqrt[(1 + (1 - x)^2/(1 - Sqrt[4 + a]))/(1 + (1 - x)^2/(1 +
Sqrt[4 + a]))]*Sqrt[3 + a - 2*(1 - x)^2 - (1 - x)^4])
```

Rubi in Sympy [A] time = 58.5489, size = 314, normalized size = 0.81

$$\frac{(x-1)\left(\frac{(x-1)^2}{-\sqrt{a+4}+1}+1\right)\left(-\sqrt{a+4}+1\right)}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\operatorname{atan}\left(-\frac{-2(x-1)^2-2}{2\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)-\frac{\left(\frac{(x-1)^2}{-\sqrt{a+4}+1}+1\right)\left(-\sqrt{a+4}+1\right)\sqrt{\sqrt{a+4}+1}E\left(\operatorname{atan}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)\middle|\frac{2\sqrt{a+4}}{\sqrt{a+4}-1}\right)}{\sqrt{\frac{-\frac{(x-1)^2}{\sqrt{a+4}-1}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{\left(\frac{(x-1)^2}{-\sqrt{a+4}+1}+1\right)\sqrt{\sqrt{a+4}+1}F\left(\operatorname{atan}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)\middle|\frac{2\sqrt{a+4}}{\sqrt{a+4}-1}\right)}{\sqrt{\frac{-\frac{(x-1)^2}{\sqrt{a+4}-1}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(-x**4+4*x**3-8*x**2+a+8*x)**(1/2),x)
```

```
[Out] (x - 1)*((x - 1)**2/(-sqrt(a + 4) + 1) + 1)*(-sqrt(a + 4) + 1)/sq
rt(a - (x - 1)**4 - 2*(x - 1)**2 + 3) + atan(-(-2*(x - 1)**2 - 2)
/(2*sqrt(a - (x - 1)**4 - 2*(x - 1)**2 + 3))) - ((x - 1)**2/(-sqr
t(a + 4) + 1) + 1)*(-sqrt(a + 4) + 1)*sqrt(sqrt(a + 4) + 1)*ellip
tic_e(atan((x - 1)/sqrt(sqrt(a + 4) + 1)), 2*sqrt(a + 4)/(sqrt(a
+ 4) - 1))/(sqrt((-(x - 1)**2/(sqrt(a + 4) - 1) + 1)/((x - 1)**2/
(sqrt(a + 4) + 1) + 1))*sqrt(a - (x - 1)**4 - 2*(x - 1)**2 + 3))
+ ((x - 1)**2/(-sqrt(a + 4) + 1) + 1)*sqrt(sqrt(a + 4) + 1)*ellip
tic_f(atan((x - 1)/sqrt(sqrt(a + 4) + 1)), 2*sqrt(a + 4)/(sqrt(a
+ 4) - 1))/(sqrt((-(x - 1)**2/(sqrt(a + 4) - 1) + 1)/((x - 1)**2/
(sqrt(a + 4) + 1) + 1))*sqrt(a - (x - 1)**4 - 2*(x - 1)**2 + 3))
```

Mathematica [B] time = 6.05408, size = 1247, normalized size = 3.21

$$2\left(\sqrt{-\sqrt{a+4}-1}+\sqrt{\sqrt{a+4}-1}\right)\sqrt{\frac{\left(\sqrt{-\sqrt{a+4}-1}-\sqrt{\sqrt{a+4}-1}\right)\left(x+\sqrt{-\sqrt{a+4}-1}-1\right)}{\left(\sqrt{-\sqrt{a+4}-1}+\sqrt{\sqrt{a+4}-1}\right)\left(-x+\sqrt{-\sqrt{a+4}-1}+1\right)}}\sqrt{\frac{\sqrt{-\sqrt{a+4}-1}\left(x-\sqrt{\sqrt{a+4}-1}-1\right)}{\left(\sqrt{-\sqrt{a+4}-1}+\sqrt{\sqrt{a+4}-1}\right)\left(x-\sqrt{-\sqrt{a+4}-1}-1\right)}}\sqrt{\frac{\sqrt{-\sqrt{\ldots}}}{\left(\sqrt{-\sqrt{a+4}-1\ldots}\right.}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^2/Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] ((-1 + Sqrt[-1 - Sqrt[4 + a]] + x)*(-1 - Sqrt[-1 + Sqrt[4 + a]] +
 x)*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x) + 2*(Sqrt[-1 - Sqrt[4 + a]]
 + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sq
rt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[
-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt
[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]*Sqrt[(Sqrt[-1 - Sqrt
[4 + a]]*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 +
a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]
*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/
((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1
- Sqrt[4 + a]] + x))]*(((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[
4 + a]])*EllipticE[ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1
 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - S
qrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]]
 - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sq
rt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2])/(2*Sqrt[-1 - S
qrt[4 + a]]) + ((-((-1 - Sqrt[-1 - Sqrt[4 + a]])*(-2 - Sqrt[-1 -
Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])) + (-1 + Sqrt[-1 - Sqrt[4
+ a]])*(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]]))*Ellipti
```

```
cF[ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])
*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sq
rt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[
-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 +
 a]] - Sqrt[-1 + Sqrt[4 + a]])^2])/(2*Sqrt[-1 - Sqrt[4 + a]]*(-Sq
rt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])) + (4*EllipticPi[(
Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])/(-Sqrt[-1 - Sqrt
[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]), ArcSin[Sqrt[((Sqrt[-1 - Sqrt[
4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] +
x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[
-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sq
rt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2
])/(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])))/Sqrt[a -
x*(-8 + 8*x - 4*x^2 + x^3)]
```

Maple [B] time = 0.029, size = 1147, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x)
```

```
[Out] ((x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(
-1-(4+a)^(1/2))^(1/2))+((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(
1/2))*((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+
(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/
2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(4+a)^(1/2))^(1/
2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)^(1/2))^(1/2))/((-
1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2)
)^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2))^
(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+
(4+a)^(1/2))^(1/2)))^(1/2)*(-1/2*((1-(-1+(4+a)^(1/2))^(1/2))*(1+(
-1+(4+a)^(1/2))^(1/2))-(1-(-1-(4+a)^(1/2))^(1/2))*(1+(-1+(4+a)^(1
/2))^(1/2))+(1-(-1-(4+a)^(1/2))^(1/2))*(1-(-1+(4+a)^(1/2))^(1/2))
+(1-(-1+(4+a)^(1/2))^(1/2))^2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)
^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/2)*EllipticF(((-(-1-(4+a)^(1/2
))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(
-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2
))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)
)*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/
2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+
a)^(1/2))^(1/2)))^(1/2))-1/2*(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(
1/2))^(1/2))*EllipticE(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))
^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1
+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(
4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)
+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2)
)^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))/
```

```
(-1+(4+a)^(1/2))^(1/2)-4/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2)
)^(1/2))*EllipticPi(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1
/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4
+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-1-(4+a)^
(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+
(4+a)^(1/2))^(1/2)),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1
/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^
(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+
(4+a)^(1/2))^(1/2)))^(1/2))))/(-(x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1
+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4
+a)^(1/2))^(1/2)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{-x^4 + 4x^3 - 8x^2 + a + 8x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{\sqrt{-x^4 + 4x^3 - 8x^2 + a + 8x}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x),x, algorithm="fricas")
```

```
[Out] integral(x^2/sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{a - x^4 + 4x^3 - 8x^2 + 8x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(-x**4+4*x**3-8*x**2+a+8*x)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{-x^4 + 4\,x^3 - 8\,x^2 + a + 8\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x), x)
```

3.638 $\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx$

Optimal. Leaf size=311

$$\frac{(a+4)\left((x-1)^2+2\right)(x-1)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(x-1)^2+1}{(a+4)\sqrt{a-(x-1)^4-2(x-1)^2+3}}-\frac{\left(1-\sqrt{a+4}\right)(x-1)\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)}{2(a+3)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{\left(1-\sqrt{a+4}\right)\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)E\left(\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{2(a+3)\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

```
[Out] (1 + (-1 + x)^2)/((4 + a)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]
) + ((4 + a)*(2 + (-1 + x)^2)*(-1 + x))/(2*(12 + 7*a + a^2)*Sqrt[
3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) - ((1 - Sqrt[4 + a])*(1 + (-1
 + x)^2/(1 - Sqrt[4 + a]))*(-1 + x))/(2*(3 + a)*Sqrt[3 + a - 2*(-
1 + x)^2 - (-1 + x)^4]) + ((1 - Sqrt[4 + a])*Sqrt[1 + Sqrt[4 + a]
]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticE[ArcTan[(-1 + x)/Sq
rt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(2*(3
+ a)*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 +
 Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])
```

Rubi [A] time = 0.803199, antiderivative size = 311, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$

$$\frac{(x-1)^2+1}{(a+4)\sqrt{a-(1-x)^4-2(1-x)^2+3}}-\frac{\left((x-1)^2+2\right)(1-x)}{2(a+3)\sqrt{a-(1-x)^4-2(1-x)^2+3}}+\frac{\left(1-\sqrt{a+4}\right)(1-x)\left(\frac{(1-x)^2}{1-\sqrt{a+4}}+1\right)}{2(a+3)\sqrt{a-(1-x)^4-2(1-x)^2+3}}-\frac{\left(1-\sqrt{a+4}\right)\sqrt{\sqrt{a+4}+1}\left(\frac{(1-x)^2}{1-\sqrt{a+4}}+1\right)E\left(\tan^{-1}\left(\frac{1-x}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{2(a+3)\sqrt{\frac{\frac{(1-x)^2}{1-\sqrt{a+4}}+1}{\frac{(1-x)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(1-x)^4-2(1-x)^2+3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

```
[Out] (1 + (-1 + x)^2)/((4 + a)*Sqrt[3 + a - 2*(1 - x)^2 - (1 - x)^4])
+ ((1 - Sqrt[4 + a])*(1 + (1 - x)^2/(1 - Sqrt[4 + a]))*(1 - x))/(
2*(3 + a)*Sqrt[3 + a - 2*(1 - x)^2 - (1 - x)^4]) - ((2 + (-1 + x)
^2)*(1 - x))/(2*(3 + a)*Sqrt[3 + a - 2*(1 - x)^2 - (1 - x)^4]) -
((1 - Sqrt[4 + a])*Sqrt[1 + Sqrt[4 + a]]*(1 + (1 - x)^2/(1 - Sqrt
[4 + a]))*EllipticE[ArcTan[(1 - x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sq
rt[4 + a])/(1 - Sqrt[4 + a])])/(2*(3 + a)*Sqrt[(1 + (1 - x)^2/(1
- Sqrt[4 + a]))/(1 + (1 - x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2
*(1 - x)^2 - (1 - x)^4])
```

Rubi in Sympy [A] time = 58.0064, size = 264, normalized size = 0.85

$$-\frac{(x-1)\left(\frac{(x-1)^2}{-\sqrt{a+4}+1}+1\right)\left(-\sqrt{a+4}+1\right)}{2\,(a+3)\sqrt{a-(x-1)^4-2\,(x-1)^2+3}}+\frac{(x-1)\left(4a+(2a+8)\,(x-1)^2+(4a+20)\,(x-1)+4\,(x-1)^3+16\right)}{4\,(a+3)\,(a+4)\sqrt{a-(x-1)^4-2\,(x-1)^2+3}}+\frac{\sqrt{a-(x-1)^4-2\,(x-1)^2+3}}{(a+3)\,(a+4)}+\frac{\left(\frac{(x-1)^2}{-\sqrt{a+4}+1}+1\right)\left(-\sqrt{a+4}+1\right)\sqrt{\sqrt{a+4}+1}\,E\left(\operatorname{atan}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)\middle|\frac{2\sqrt{a+4}}{\sqrt{a+4}-1}\right)}{2\sqrt{\frac{-\frac{(x-1)^2}{\sqrt{a+4}-1}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\,(a+3)\sqrt{a-(x-1)^4-2\,(x-1)^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(-x**4+4*x**3-8*x**2+a+8*x)**(3/2),x)
```

```
[Out] -(x - 1)*((x - 1)**2/(-sqrt(a + 4) + 1) + 1)*(-sqrt(a + 4) + 1)/(
2*(a + 3)*sqrt(a - (x - 1)**4 - 2*(x - 1)**2 + 3)) + (x - 1)*(4*a
 + (2*a + 8)*(x - 1)**2 + (4*a + 20)*(x - 1) + 4*(x - 1)**3 + 16)
/(4*(a + 3)*(a + 4)*sqrt(a - (x - 1)**4 - 2*(x - 1)**2 + 3)) + sq
rt(a - (x - 1)**4 - 2*(x - 1)**2 + 3)/((a + 3)*(a + 4)) + ((x - 1
)**2/(-sqrt(a + 4) + 1) + 1)*(-sqrt(a + 4) + 1)*sqrt(sqrt(a + 4)
+ 1)*elliptic_e(atan((x - 1)/sqrt(sqrt(a + 4) + 1)), 2*sqrt(a + 4
)/(sqrt(a + 4) - 1))/(2*sqrt((-(x - 1)**2/(sqrt(a + 4) - 1) + 1)/
((x - 1)**2/(sqrt(a + 4) + 1) + 1))*(a + 3)*sqrt(a - (x - 1)**4 -
 2*(x - 1)**2 + 3))
```

Mathematica [B] time = 6.14818, size = 2941, normalized size = 9.46

Result too large to show

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]


[Out] ((-a - 8*x - a*x + 6*x^2 + a*x^2 - 4*x^3 - a*x^3)*(a + 8*x - 8*x^
2 + 4*x^3 - x^4)^2)/(2*(3 + a)*(4 + a)*(-a - 8*x + 8*x^2 - 4*x^3
+ x^4)*(a - x*(-8 + 8*x - 4*x^2 + x^3))^(3/2)) - ((a + 8*x - 8*x^
2 + 4*x^3 - x^4)^(3/2)*((2*(-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + S
qrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((-Sqrt[-1
- Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 +
 a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1
 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1
 - Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-
1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[
-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 -
Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a
]] + x))]*EllipticF[ArcSin[Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[
-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 -
 Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 +
a]] + x))]], ((-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*
(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))/((Sqrt[-1 - Sq
rt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-Sqrt[-1 - Sqrt[4 + a]] + S
qrt[-1 + Sqrt[4 + a]]))])/(Sqrt[-1 - Sqrt[4 + a]]*(-Sqrt[-1 - Sqr
t[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*Sqrt[a + 8*x - 8*x^2 + 4*x^3
- x^4]) - (4*(-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(
-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((Sqrt[-1 - Sqrt[4 + a]]
- Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sq
rt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqr
t[4 + a]] - x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqr
t[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]
)*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]
]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] - S
qrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*((-1 -
 Sqrt[-1 - Sqrt[4 + a]])*EllipticF[ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4
 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x
))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-
1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqr
t[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2]
 + 2*Sqrt[-1 - Sqrt[4 + a]]*EllipticPi[(Sqrt[-1 - Sqrt[4 + a]] +
Sqrt[-1 + Sqrt[4 + a]])/(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt
[4 + a]]), ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[
4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 +
a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]]
, (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 -
Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2]))/(Sqrt[-1 - Sqrt[4 + a
]]*(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*Sqrt[a + 8*x
 - 8*x^2 + 4*x^3 - x^4]) + ((-1 + Sqrt[-1 - Sqrt[4 + a]] + x)*(-1
 - Sqrt[-1 + Sqrt[4 + a]] + x)*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x)
+ 2*(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[
-1 - Sqrt[4 + a]] + x)^2*Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1
+ Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sq
rt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]]
- x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[4 + a]]
+ x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sq
rt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sq
```

```
rt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + S
qrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*(((Sqrt[-1 - Sqr
t[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*EllipticE[ArcSin[Sqrt[((Sqrt[
-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[
4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*
(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sq
rt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[
4 + a]])^2])/(2*Sqrt[-1 - Sqrt[4 + a]]) + ((-((-1 - Sqrt[-1 - Sqr
t[4 + a]])*(-2 - Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])
) + (-1 + Sqrt[-1 - Sqrt[4 + a]])*(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[
-1 + Sqrt[4 + a]]))*EllipticF[ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]
] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((
Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - S
qrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 +
 a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2])/(2*
Sqrt[-1 - Sqrt[4 + a]]*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[
4 + a]])) + (4*EllipticPi[(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqr
t[4 + a]])/(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]), Ar
cSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1
+ Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1
 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 -
Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]]
- Sqrt[-1 + Sqrt[4 + a]])^2])/(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1
+ Sqrt[4 + a]])))/Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4]))/(2*(3 + a
)*(a - x*(-8 + 8*x - 4*x^2 + x^3))^(3/2))
```

Maple [B] time = 0.035, size = 2607, normalized size = 8.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x)
```

```
[Out] 2*(1/4/(3+a)*x^3-1/4*(6+a)/(a^2+7*a+12)*x^2+1/4*(8+a)/(a^2+7*a+12
)*x+1/4*a/(a^2+7*a+12))/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)-(2/(a^2+7*
a+12)-1/2*(8+a)/(a^2+7*a+12))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(
1/2))^(1/2))*((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x
-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/
2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(4+a)^(1/
2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)^(1/2))^(1/
2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a
)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^
(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x
-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4
+a)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/2)/(-(x-1-(-1+(4+a)^(1/2))^
(1/2))*(x-1+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*
(x-1+(-1-(4+a)^(1/2))^(1/2)))^(1/2)*EllipticF(((-(-1-(4+a)^(1/2))
^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1
```

```
-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))
^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*
((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2)
)^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)
^(1/2))^(1/2)))^(1/2))-(-2/(a^2+7*a+12)+(6+a)/(a^2+7*a+12))*((-1-
(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-1-(4+a)^(1/2))^(1
/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4
+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1
/2)))^(1/2)*(x-1+(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(
1/2)*(x-1-(-1-(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+
a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a
)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1
/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)/(
-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^
(1/2)/(-(x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1+(-1+(4+a)^(1/2))^(1/2))
*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2)))^(1/2)
*((1-(-1+(4+a)^(1/2))^(1/2))*EllipticF(((-(-1-(4+a)^(1/2))^(1/2)+
(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^
(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2))
)^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4
+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)
+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))
^(1/2)))^(1/2))+2*(-1+(4+a)^(1/2))^(1/2)*EllipticPi(((-(-1-(4+a)^
(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))
/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^
(1/2))^(1/2)))^(1/2),(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1
/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2)),((-(-1-(4+a
)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-
1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(
1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2)))-1/
2/(3+a)*((x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2)
)*(x-1+(-1-(4+a)^(1/2))^(1/2))+((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^
(1/2))^(1/2))*((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(
x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1
/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(4+a)^(1
/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)^(1/2))^(1
/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+
a)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)
^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(
x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-1/2*((1-(-1+(4+a)^(1/2))^(1/
2))*(1+(-1+(4+a)^(1/2))^(1/2))-(1-(-1-(4+a)^(1/2))^(1/2))*(1+(-1+
(4+a)^(1/2))^(1/2))+(1-(-1-(4+a)^(1/2))^(1/2))*(1-(-1+(4+a)^(1/2)
)^(1/2))+(1-(-1+(4+a)^(1/2))^(1/2))^2)/(-(-1-(4+a)^(1/2))^(1/2)+(
-1+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/2)*EllipticF(((-(-1-(4
+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1
/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4
+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2
))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(
4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)
-(-1+(4+a)^(1/2))^(1/2)))^(1/2))-1/2*(-(-1-(4+a)^(1/2))^(1/2)+(-1
+(4+a)^(1/2))^(1/2))*EllipticE(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a
)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(
1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),
((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2
```

```
))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+
a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))
^(1/2))/(-1+(4+a)^(1/2))^(1/2)-4/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+
a)^(1/2))^(1/2))*EllipticPi(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(
1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2
)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-
1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1
/2)-(-1+(4+a)^(1/2))^(1/2)),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(
1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-
1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1
/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))))/(-(x-1-(-1+(4+a)^(1/2))^(1/
2))*(x-1+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-
1+(-1-(4+a)^(1/2))^(1/2)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(-x^4+4x^3-8x^2+a+8x)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{x^2}{(x^4-4x^3+8x^2-a-8x)\sqrt{-x^4+4x^3-8x^2+a+8x}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-x^2/((x^4 - 4*x^3 + 8*x^2 - a - 8*x)*sqrt(-x^4 + 4*x^3
- 8*x^2 + a + 8*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(-x**4+4*x**3-8*x**2+a+8*x)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(-x^4+4\,x^3-8\,x^2+a+8\,x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2), x)
```

3.639 $\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx$

Optimal. Leaf size=582

$$\frac{(a+4)\left((x-1)^2+2\right)(x-1)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}+\frac{\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)F\left(\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{12\left(a^2+7a+12\right)\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{2\left((x-1)^2+1\right)}{3(a+4)^2\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(x-1)^2+1}{3(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}+\frac{(x-1)\left((3a+13)(x-1)^2+7a+29\right)}{12(a+3)^2(a+4)\sqrt{a-(x-1)^4-2(x-1)^2+3}}-\frac{(3a+13)\left(1-\sqrt{a+4}\right)(x-1)\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)}{12(a+3)^2(a+4)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(3a+13)\left(1-\sqrt{a+4}\right)\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)E\left(\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{12(a+3)^2(a+4)\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

```
[Out] (1 + (-1 + x)^2)/(3*(4 + a)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)^(
3/2)) + (2*(1 + (-1 + x)^2))/(3*(4 + a)^2*Sqrt[3 + a - 2*(-1 + x)
^2 - (-1 + x)^4]) + ((4 + a)*(2 + (-1 + x)^2)*(-1 + x))/(6*(12 +
7*a + a^2)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)^(3/2)) + ((29 + 7*
a + (13 + 3*a)*(-1 + x)^2)*(-1 + x))/(12*(3 + a)^2*(4 + a)*Sqrt[3
 + a - 2*(-1 + x)^2 - (-1 + x)^4]) - ((13 + 3*a)*(1 - Sqrt[4 + a]
)*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(-1 + x))/(12*(3 + a)^2*(4 +
 a)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + ((13 + 3*a)*(1 - S
qrt[4 + a])*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a
]))*EllipticE[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4
+ a])/(1 - Sqrt[4 + a])])/(12*(3 + a)^2*(4 + a)*Sqrt[(1 + (-1 + x
)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3
 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (Sqrt[1 + Sqrt[4 + a]]*(1 +
(-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticF[ArcTan[(-1 + x)/Sqrt[1 +
Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(12*(12 + 7*a
 + a^2)*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(
1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])
```

Rubi [A] time = 1.64818, antiderivative size = 582, normalized size of antiderivative = 1., number of

steps used = 13, number of rules used = 12, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$

$$-\frac{\sqrt{\sqrt{a+4}+1}\left(\frac{(1-x)^2}{1-\sqrt{a+4}}+1\right)F\left(\tan^{-1}\left(\frac{1-x}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{12\left(a^2+7a+12\right)\sqrt{\frac{\frac{(1-x)^2}{1-\sqrt{a+4}}+1}{\frac{(1-x)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(1-x)^4-2(1-x)^2+3}}+\frac{2\left((x-1)^2+1\right)}{3(a+4)^2\sqrt{a-(1-x)^4-2(1-x)^2+3}}+\frac{(x-1)^2+1}{3(a+4)\left(a-(1-x)^4-2(1-x)^2+3\right)^{3/2}}-\frac{\left((x-1)^2+2\right)(1-x)}{6(a+3)\left(a-(1-x)^4-2(1-x)^2+3\right)^{3/2}}-\frac{(1-x)\left((3a+13)(1-x)^2+7a+29\right)}{12(a+3)^2(a+4)\sqrt{a-(1-x)^4-2(1-x)^2+3}}+\frac{(3a+13)\left(1-\sqrt{a+4}\right)(1-x)\left(\frac{(1-x)^2}{1-\sqrt{a+4}}+1\right)}{12(a+3)^2(a+4)\sqrt{a-(1-x)^4-2(1-x)^2+3}}-\frac{(3a+13)\left(1-\sqrt{a+4}\right)\sqrt{\sqrt{a+4}+1}\left(\frac{(1-x)^2}{1-\sqrt{a+4}}+1\right)E\left(\tan^{-1}\left(\frac{1-x}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{12(a+3)^2(a+4)\sqrt{\frac{\frac{(1-x)^2}{1-\sqrt{a+4}}+1}{\frac{(1-x)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(1-x)^4-2(1-x)^2+3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(5/2),x]
```

```
[Out] (1 + (-1 + x)^2)/(3*(4 + a)*(3 + a - 2*(1 - x)^2 - (1 - x)^4)^(3/
2)) + (2*(1 + (-1 + x)^2))/(3*(4 + a)^2*Sqrt[3 + a - 2*(1 - x)^2
- (1 - x)^4]) - ((29 + 7*a + (13 + 3*a)*(1 - x)^2)*(1 - x))/(12*(
3 + a)^2*(4 + a)*Sqrt[3 + a - 2*(1 - x)^2 - (1 - x)^4]) + ((13 +
3*a)*(1 - Sqrt[4 + a])*(1 + (1 - x)^2/(1 - Sqrt[4 + a]))*(1 - x))
/(12*(3 + a)^2*(4 + a)*Sqrt[3 + a - 2*(1 - x)^2 - (1 - x)^4]) - (
(2 + (-1 + x)^2)*(1 - x))/(6*(3 + a)*(3 + a - 2*(1 - x)^2 - (1 -
x)^4)^(3/2)) - ((13 + 3*a)*(1 - Sqrt[4 + a])*Sqrt[1 + Sqrt[4 + a]
]*(1 + (1 - x)^2/(1 - Sqrt[4 + a]))*EllipticE[ArcTan[(1 - x)/Sqrt
[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(12*(3 +
 a)^2*(4 + a)*Sqrt[(1 + (1 - x)^2/(1 - Sqrt[4 + a]))/(1 + (1 - x)
^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(1 - x)^2 - (1 - x)^4]) - (
Sqrt[1 + Sqrt[4 + a]]*(1 + (1 - x)^2/(1 - Sqrt[4 + a]))*EllipticF
[ArcTan[(1 - x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqr
t[4 + a])])/(12*(12 + 7*a + a^2)*Sqrt[(1 + (1 - x)^2/(1 - Sqrt[4
+ a]))/(1 + (1 - x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(1 - x)^
2 - (1 - x)^4])
```

Rubi in Sympy [A] time = 111.191, size = 500, normalized size = 0.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(-x**4+4*x**3-8*x**2+a+8*x)**(5/2),x)
```

```
[Out] (x - 1)*(4*a + (2*a + 8)*(x - 1)**2 + (4*a + 20)*(x - 1) + 4*(x -
 1)**3 + 16)/(12*(a + 3)*(a + 4)*(a - (x - 1)**4 - 2*(x - 1)**2 +
 3)**(3/2)) - (3*a + 13)*(x - 1)*((x - 1)**2/(-sqrt(a + 4) + 1) +
 1)*(-sqrt(a + 4) + 1)/(12*(a + 3)**2*(a + 4)*sqrt(a - (x - 1)**4
 - 2*(x - 1)**2 + 3)) + (3*a + 10)*sqrt(a - (x - 1)**4 - 2*(x - 1
)**2 + 3)/(3*(a + 3)**2*(a + 4)**2) + (x - 1)*(4*(a + 4)*(3*a + 1
3)*(x - 1)**2 + (4*a + 16)*(7*a + 29) + (48*a + 160)*(x - 1)**3 +
 (x - 1)*(32*a**2 + 288*a + 608))/(48*(a + 3)**2*(a + 4)**2*sqrt(
a - (x - 1)**4 - 2*(x - 1)**2 + 3)) + ((x - 1)**2/(-sqrt(a + 4) +
 1) + 1)*sqrt(sqrt(a + 4) + 1)*elliptic_f(atan((x - 1)/sqrt(sqrt(
a + 4) + 1)), 2*sqrt(a + 4)/(sqrt(a + 4) - 1))/(12*sqrt((-(x - 1)
**2/(sqrt(a + 4) - 1) + 1)/((x - 1)**2/(sqrt(a + 4) + 1) + 1))*(a
 + 3)*(a + 4)*sqrt(a - (x - 1)**4 - 2*(x - 1)**2 + 3)) + (3*a + 1
3)*((x - 1)**2/(-sqrt(a + 4) + 1) + 1)*(-sqrt(a + 4) + 1)*sqrt(sq
rt(a + 4) + 1)*elliptic_e(atan((x - 1)/sqrt(sqrt(a + 4) + 1)), 2*
sqrt(a + 4)/(sqrt(a + 4) - 1))/(12*sqrt((-(x - 1)**2/(sqrt(a + 4)
 - 1) + 1)/((x - 1)**2/(sqrt(a + 4) + 1) + 1))*(a + 3)**2*(a + 4)
*sqrt(a - (x - 1)**4 - 2*(x - 1)**2 + 3))
```

Mathematica [B] time = 6.22607, size = 5812, normalized size = 9.99

Result too large to show

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(5/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.046, size = 2780, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x)
```

```
[Out] (1/6/(3+a)*x^3-1/6*(6+a)/(a^2+7*a+12)*x^2+1/6*(8+a)/(a^2+7*a+12)*
x+1/6*a/(a^2+7*a+12))*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)/(x^4-4*x^3+8
```

```
*x^2-a-8*x)^2+2*(1/24*(13+3*a)/(3+a)/(a^2+7*a+12)*x^3-1/24*(a^2+2
7*a+84)/(a^2+7*a+12)^2*x^2+1/6*(9*a+32)/(a^2+7*a+12)^2*x+1/12*(3*
a^2+7*a-12)/(a^2+7*a+12)^2)/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)-(-1/6*
(a^2-9*a-44)/(a^2+7*a+12)^2-1/3*(9*a+32)/(a^2+7*a+12)^2)*((-1-(4+
a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-1-(4+a)^(1/2))^(1/2)
+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)
^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)
))^(1/2)*(x-1+(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2
)*(x-1-(-1-(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^
(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(
1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)
-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)/(-(-
1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/
2)/(-(x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1+(-1+(4+a)^(1/2))^(1/2))*(x
-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2)))^(1/2)*El
lipticF(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-
1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(
1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/
2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2
))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(
4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))-(1/3*(a^2-a-16)
/(a^2+7*a+12)^2+1/6*(a^2+27*a+84)/(a^2+7*a+12)^2)*((-1-(4+a)^(1/2
))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4
+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))
^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2
)*(x-1+(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-
(-1-(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^
(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(
1/2)*(x-1+(-1-(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4
+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)/(-(-1-(4+a)
^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/2)/(-(x
-1-(-1+(4+a)^(1/2))^(1/2))*(x-1+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-
(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2)))^(1/2)*((1-(-1+(
4+a)^(1/2))^(1/2))*EllipticF(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^
(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/
2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((
-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))
^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)
^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(
1/2))+2*(-1+(4+a)^(1/2))^(1/2)*EllipticPi(((-(-1-(4+a)^(1/2))^(1/
2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+
a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/
2)))^(1/2),(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(-(-1
-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2)),((-(-1-(4+a)^(1/2))^(
1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1
/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1
-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))-1/12*(13+3*a
)/(3+a)/(a^2+7*a+12)*((x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)
^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2))+((-1-(4+a)^(1/2))^(1/
2)+(-1+(4+a)^(1/2))^(1/2))*((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1
/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)
-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(x-1
+(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4
+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))
```

```
/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(
x-1+(-1-(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1
/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-1/2*((1-(-1+(4+
a)^(1/2))^(1/2))*(1+(-1+(4+a)^(1/2))^(1/2))-(1-(-1-(4+a)^(1/2))^(
1/2))*(1+(-1+(4+a)^(1/2))^(1/2))+(1-(-1-(4+a)^(1/2))^(1/2))*(1-(-
1+(4+a)^(1/2))^(1/2))+(1-(-1+(4+a)^(1/2))^(1/2))^2)/(-(-1-(4+a)^(
1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/2)*Ellipt
icF(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4
+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)
)/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(
-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(
1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)
^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))-1/2*(-(-1-(4+a)^(1/
2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*EllipticE(((-(-1-(4+a)^(1/2))^(
1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(
4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(
1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((
-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^
(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(
1/2))^(1/2)))^(1/2))/(-1+(4+a)^(1/2))^(1/2)-4/(-(-1-(4+a)^(1/2))^
(1/2)+(-1+(4+a)^(1/2))^(1/2))*EllipticPi(((-(-1-(4+a)^(1/2))^(1/2
)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a
)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2
)))^(1/2),((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4
+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)),((-(-1-(4+a)^(1/2))^(1/2
)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2)
)^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4
+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))))/(-(x-1-(-1+(4+
a)^(1/2))^(1/2))*(x-1+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2
))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(-x^4 + 4x^3 - 8x^2 + a + 8x)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{(x^8 - 8x^7 + 32x^6 - 2(a-64)x^4 - 80x^5 + 8(a-16)x^3 - 16(a-4)x^2 + a^2 + 16ax)\sqrt{-x^4 + 4x^3 - 8x^2 + a + 8x}}, x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(x^2/((x^8 - 8*x^7 + 32*x^6 - 2*(a - 64)*x^4 - 80*x^5 + 8
*(a - 16)*x^3 - 16*(a - 4)*x^2 + a^2 + 16*a*x)*sqrt(-x^4 + 4*x^3
- 8*x^2 + a + 8*x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(a - x^4 + 4x^3 - 8x^2 + 8x)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(-x**4+4*x**3-8*x**2+a+8*x)**(5/2),x)
```

```
[Out] Integral(x**2/(a - x**4 + 4*x**3 - 8*x**2 + 8*x)**(5/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(-x^4 + 4\,x^3 - 8\,x^2 + a + 8\,x)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(5/2), x)
```

3.640 $\int \frac{1}{\sqrt{8+8x-x^3+8x^4}}\,dx$

Optimal. Leaf size=129

$$-\frac{x^2\sqrt{\frac{\left(\frac{4}{x}+1\right)^4-6\left(\frac{4}{x}+1\right)^2+261}{\left(\frac{\sqrt{29}(x+4)^2}{x^2}+87\right)^2}}\left(\frac{\sqrt{29}(x+4)^2}{x^2}+87\right)F\left(2\tan^{-1}\left(\frac{x+4}{\sqrt{3}\sqrt[4]{29}x}\right)|\frac{1}{58}\left(29+\sqrt{29}\right)\right)}{8\sqrt{3}\sqrt[4]{29}\sqrt{8x^4-x^3+8x+8}}$$

```
[Out] -(x^2*Sqrt[(261 - 6*(1 + 4/x)^2 + (1 + 4/x)^4)/(87 + (Sqrt[29]*(4
 + x)^2)/x^2)^2]*(87 + (Sqrt[29]*(4 + x)^2)/x^2)*EllipticF[2*ArcT
an[(4 + x)/(Sqrt[3]*29^(1/4)*x)], (29 + Sqrt[29])/58])/(8*Sqrt[3]
*29^(1/4)*Sqrt[8 + 8*x - x^3 + 8*x^4])
```

Rubi [A] time = 0.533162, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$-\frac{x^2\sqrt{\frac{\left(\frac{4}{x}+1\right)^4-6\left(\frac{4}{x}+1\right)^2+261}{\left(\frac{\sqrt{29}(x+4)^2}{x^2}+87\right)^2}}\left(\frac{\sqrt{29}(x+4)^2}{x^2}+87\right)F\left(2\tan^{-1}\left(\frac{x+4}{\sqrt{3}\sqrt[4]{29}x}\right)|\frac{1}{58}\left(29+\sqrt{29}\right)\right)}{8\sqrt{3}\sqrt[4]{29}\sqrt{8x^4-x^3+8x+8}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[8 + 8*x - x^3 + 8*x^4],x]
```

```
[Out] -(x^2*Sqrt[(261 - 6*(1 + 4/x)^2 + (1 + 4/x)^4)/(87 + (Sqrt[29]*(4
 + x)^2)/x^2)^2]*(87 + (Sqrt[29]*(4 + x)^2)/x^2)*EllipticF[2*ArcT
an[(4 + x)/(Sqrt[3]*29^(1/4)*x)], (29 + Sqrt[29])/58])/(8*Sqrt[3]
*29^(1/4)*Sqrt[8 + 8*x - x^3 + 8*x^4])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-1024\int^{\frac{1}{4}+\frac{1}{x}}\frac{1}{\sqrt{\frac{8388608x^4-3145728x^2+8552448}{(-32x+8)^4}}\,(-32x+8)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(8*x**4-x**3+8*x+8)**(1/2),x)
```

```
[Out] -1024*Integral(1/(sqrt((8388608*x**4 - 3145728*x**2 + 8552448)/(-
32*x + 8)**4)*(-32*x + 8)**2), (x, 1/4 + 1/x))
```

Mathematica [C] time = 0.554979, size = 927, normalized size = 7.19

result too large to display

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[8 + 8*x - x^3 + 8*x^4],x]
```

```
[Out] (-2*EllipticF[ArcSin[Sqrt[((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & ,
 1, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1
- #1^3 + 8*#1^4 & , 4, 0]))/((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 &
 , 2, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#
1 - #1^3 + 8*#1^4 & , 4, 0]))]], ((Root[8 + 8*#1 - #1^3 + 8*#1^4
& , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0])*(Root[8 + 8*
#1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & ,
4, 0]))/((Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1
 - #1^3 + 8*#1^4 & , 3, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2,
 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))]*(x - Root[8 + 8*
#1 - #1^3 + 8*#1^4 & , 2, 0])^2*Sqrt[((Root[8 + 8*#1 - #1^3 + 8*#
1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(x - Roo
t[8 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0]))/((x - Root[8 + 8*#1 - #1^3
 + 8*#1^4 & , 2, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] - R
oot[8 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0]))]*(Root[8 + 8*#1 - #1^3 +
 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0])*Sqrt
[((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0])*(Root[8 + 8*#1 -
#1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0]
)*(x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0])*(Root[8 + 8*#1 -
#1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]
))/((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])^2*(Root[8 + 8*#
1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4
, 0])^2)])/(Sqrt[8 + 8*x - x^3 + 8*x^4]*(-Root[8 + 8*#1 - #1^3 +
8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(Root
[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1
^4 & , 4, 0]))
```

Maple [C] time = 1.776, size = 965, normalized size = 7.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(8*x^4-x^3+8*x+8)^(1/2),x)
```

```
[Out] 1/2*(-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)+RootOf(8*_Z^4-_Z^3+8*_Z+
8,index=1))*((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z
^3+8*_Z+8,index=2))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(RootO
f(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))
/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2)*(x-RootOf(8*_Z^4-_
Z^3+8*_Z+8,index=2))^2*((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootO
f(8*_Z^4-_Z^3+8*_Z+8,index=1))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index
=3))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3)-RootOf(8*_Z^4-_Z^3+8*_Z+
8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2)*((RootO
f(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))
*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4))/(RootOf(8*_Z^4-_Z^3+8*_Z+
8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_
Z^3+8*_Z+8,index=2)))^(1/2)/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-R
ootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,ind
ex=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*2^(1/2)/((x-RootOf(8*_Z
^4-_Z^3+8*_Z+8,index=1))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))*(
x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+
8,index=4)))^(1/2)*EllipticF(((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)
-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8
,index=1))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3
+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2),(
(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,ind
ex=3))*(-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)+RootOf(8*_Z^4-_Z^3+8*
_Z+8,index=1))/(-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3)+RootOf(8*_Z^4
-_Z^3+8*_Z+8,index=1))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf
(8*_Z^4-_Z^3+8*_Z+8,index=4)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{8x^4 - x^3 + 8x + 8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(8*x^4 - x^3 + 8*x + 8),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(8*x^4 - x^3 + 8*x + 8), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{8x^4 - x^3 + 8x + 8}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(8*x^4 - x^3 + 8*x + 8),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(8*x^4 - x^3 + 8*x + 8), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{8x^4 - x^3 + 8x + 8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(8*x**4-x**3+8*x+8)**(1/2),x)
```

```
[Out] Integral(1/sqrt(8*x**4 - x**3 + 8*x + 8), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{8\,x^4 - x^3 + 8\,x + 8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(8*x^4 - x^3 + 8*x + 8),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(8*x^4 - x^3 + 8*x + 8), x)
```

3.641 $\int \frac{1}{(8+8x-x^3+8x^4)^{3/2}}\,dx$

Optimal. Leaf size=431

$$-\frac{\left(66-\left(\frac{4}{x}+1\right)^2\right)x^2}{1008\sqrt{8x^4-x^3+8x+8}}+\frac{\left(216-7\left(\frac{4}{x}+1\right)^2\right)\left(\frac{4}{x}+1\right)x^2}{12528\sqrt{8x^4-x^3+8x+8}}+\frac{7\left(\left(\frac{4}{x}+1\right)^4-6\left(\frac{4}{x}+1\right)^2+261\right)\left(\frac{4}{x}+1\right)x^2}{432\sqrt{29}\sqrt{8x^4-x^3+8x+8}\left(\frac{\sqrt{29}(x+4)^2}{x^2}+87\right)}+\frac{\left(14-5\sqrt{29}\right)\sqrt{\frac{\left(\frac{4}{x}+1\right)^4-6\left(\frac{4}{x}+1\right)^2+261}{\left(\frac{\sqrt{29}(x+4)^2}{x^2}+87\right)^2}}\left(\frac{\sqrt{29}(x+4)^2}{x^2}+87\right)x^2F\left(2\tan^{-1}\left(\frac{x+4}{\sqrt{3}\sqrt[4]{29}x}\right)|\frac{1}{58}\left(29+\sqrt{29}\right)\right)}{576\sqrt{3}29^{3/4}\sqrt{8x^4-x^3+8x+8}}-\frac{7\sqrt{\frac{\left(\frac{4}{x}+1\right)^4-6\left(\frac{4}{x}+1\right)^2+261}{\left(\frac{\sqrt{29}(x+4)^2}{x^2}+87\right)^2}}\left(\frac{\sqrt{29}(x+4)^2}{x^2}+87\right)x^2E\left(2\tan^{-1}\left(\frac{x+4}{\sqrt{3}\sqrt[4]{29}x}\right)|\frac{1}{58}\left(29+\sqrt{29}\right)\right)}{144\sqrt{3}29^{3/4}\sqrt{8x^4-x^3+8x+8}}$$

```
[Out] -((66 - (1 + 4/x)^2)*x^2)/(1008*Sqrt[8 + 8*x - x^3 + 8*x^4]) + ((
216 - 7*(1 + 4/x)^2)*(1 + 4/x)*x^2)/(12528*Sqrt[8 + 8*x - x^3 + 8
*x^4]) + (7*(261 - 6*(1 + 4/x)^2 + (1 + 4/x)^4)*(1 + 4/x)*x^2)/(4
32*Sqrt[29]*Sqrt[8 + 8*x - x^3 + 8*x^4]*(87 + (Sqrt[29]*(4 + x)^2
)/x^2)) - (7*x^2*Sqrt[(261 - 6*(1 + 4/x)^2 + (1 + 4/x)^4)/(87 + (
Sqrt[29]*(4 + x)^2)/x^2)^2]*(87 + (Sqrt[29]*(4 + x)^2)/x^2)*Ellip
ticE[2*ArcTan[(4 + x)/(Sqrt[3]*29^(1/4)*x)], (29 + Sqrt[29])/58])
/(144*Sqrt[3]*29^(3/4)*Sqrt[8 + 8*x - x^3 + 8*x^4]) + ((14 - 5*Sq
rt[29])*x^2*Sqrt[(261 - 6*(1 + 4/x)^2 + (1 + 4/x)^4)/(87 + (Sqrt[
29]*(4 + x)^2)/x^2)^2]*(87 + (Sqrt[29]*(4 + x)^2)/x^2)*EllipticF[
2*ArcTan[(4 + x)/(Sqrt[3]*29^(1/4)*x)], (29 + Sqrt[29])/58])/(576
*Sqrt[3]*29^(3/4)*Sqrt[8 + 8*x - x^3 + 8*x^4])
```

Rubi [A] time = 0.996573, antiderivative size = 431, normalized size of antiderivative = 1., number

of steps used = 10, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$

$$-\frac{\left(66-\left(\frac{4}{x}+1\right)^2\right)x^2}{1008\sqrt{8x^4-x^3+8x+8}}+\frac{\left(216-7\left(\frac{4}{x}+1\right)^2\right)\left(\frac{4}{x}+1\right)x^2}{12528\sqrt{8x^4-x^3+8x+8}}+\frac{7\left(\left(\frac{4}{x}+1\right)^4-6\left(\frac{4}{x}+1\right)^2+261\right)\left(\frac{4}{x}+1\right)x^2}{432\sqrt{29}\sqrt{8x^4-x^3+8x+8}\left(\frac{\sqrt{29}(x+4)^2}{x^2}+87\right)}+\frac{\left(14-5\sqrt{29}\right)\sqrt{\frac{\left(\frac{4}{x}+1\right)^4-6\left(\frac{4}{x}+1\right)^2+261}{\left(\frac{\sqrt{29}(x+4)^2}{x^2}+87\right)^2}}\left(\frac{\sqrt{29}(x+4)^2}{x^2}+87\right)x^2F\left(2\tan^{-1}\left(\frac{x+4}{\sqrt{3}\sqrt[4]{29}x}\right)|\frac{1}{58}\left(29+\sqrt{29}\right)\right)}{576\sqrt{3}29^{3/4}\sqrt{8x^4-x^3+8x+8}}-\frac{7\sqrt{\frac{\left(\frac{4}{x}+1\right)^4-6\left(\frac{4}{x}+1\right)^2+261}{\left(\frac{\sqrt{29}(x+4)^2}{x^2}+87\right)^2}}\left(\frac{\sqrt{29}(x+4)^2}{x^2}+87\right)x^2E\left(2\tan^{-1}\left(\frac{x+4}{\sqrt{3}\sqrt[4]{29}x}\right)|\frac{1}{58}\left(29+\sqrt{29}\right)\right)}{144\sqrt{3}29^{3/4}\sqrt{8x^4-x^3+8x+8}}$$

Antiderivative was successfully verified.

```
[In]  Int[(8 + 8*x - x^3 + 8*x^4)^(-3/2),x]
```

```
[Out] -((66 - (1 + 4/x)^2)*x^2)/(1008*Sqrt[8 + 8*x - x^3 + 8*x^4]) + ((
216 - 7*(1 + 4/x)^2)*(1 + 4/x)*x^2)/(12528*Sqrt[8 + 8*x - x^3 + 8
*x^4]) + (7*(261 - 6*(1 + 4/x)^2 + (1 + 4/x)^4)*(1 + 4/x)*x^2)/(4
32*Sqrt[29]*Sqrt[8 + 8*x - x^3 + 8*x^4]*(87 + (Sqrt[29]*(4 + x)^2
)/x^2)) - (7*x^2*Sqrt[(261 - 6*(1 + 4/x)^2 + (1 + 4/x)^4)/(87 + (
Sqrt[29]*(4 + x)^2)/x^2)^2]*(87 + (Sqrt[29]*(4 + x)^2)/x^2)*Ellip
ticE[2*ArcTan[(4 + x)/(Sqrt[3]*29^(1/4)*x)], (29 + Sqrt[29])/58])
/(144*Sqrt[3]*29^(3/4)*Sqrt[8 + 8*x - x^3 + 8*x^4]) + ((14 - 5*Sq
rt[29])*x^2*Sqrt[(261 - 6*(1 + 4/x)^2 + (1 + 4/x)^4)/(87 + (Sqrt[
29]*(4 + x)^2)/x^2)^2]*(87 + (Sqrt[29]*(4 + x)^2)/x^2)*EllipticF[
2*ArcTan[(4 + x)/(Sqrt[3]*29^(1/4)*x)], (29 + Sqrt[29])/58])/(576
*Sqrt[3]*29^(3/4)*Sqrt[8 + 8*x - x^3 + 8*x^4])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-1024\int^{\frac{1}{4}+\frac{1}{x}}\frac{1}{\left(\frac{8388608x^4-3145728x^2+8552448}{(-32x+8)^4}\right)^{\frac{3}{2}}(-32x+8)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(8*x**4-x**3+8*x+8)**(3/2),x)
```

```
[Out] -1024*Integral(1/(((8388608*x**4 - 3145728*x**2 + 8552448)/(-32*x
 + 8)**4)**(3/2)*(-32*x + 8)**2), (x, 1/4 + 1/x))
```

Mathematica [C] time = 6.06486, size = 4865, normalized size = 11.29

Result too large to show

Antiderivative was successfully verified.

```
[In]  Integrate[(8 + 8*x - x^3 + 8*x^4)^(-3/2),x]
```

```
[Out] (544 + 1539*x - 1146*x^2 + 784*x^3)/(21924*Sqrt[8 + 8*x - x^3 + 8
*x^4]) + ((28*(x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])^2*(-(
EllipticF[ArcSin[Sqrt[((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1,
0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1
^3 + 8*#1^4 & , 4, 0]))/((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2
, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 -
#1^3 + 8*#1^4 & , 4, 0]))]], -(((Root[8 + 8*#1 - #1^3 + 8*#1^4 &
, 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0])*(Root[8 + 8*#1
 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4,
 0]))/((-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1
- #1^3 + 8*#1^4 & , 3, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2,
0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0])))]*Root[8 + 8*#1 -
#1^3 + 8*#1^4 & , 2, 0]) + EllipticPi[(-Root[8 + 8*#1 - #1^3 + 8*
#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0])/(-Root[
8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^
4 & , 4, 0]), ArcSin[Sqrt[((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & ,
 1, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1
- #1^3 + 8*#1^4 & , 4, 0]))/((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 &
 , 2, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#
1 - #1^3 + 8*#1^4 & , 4, 0]))]], -(((Root[8 + 8*#1 - #1^3 + 8*#1^
4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0])*(Root[8 +
8*#1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 &
, 4, 0]))/((-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8
*#1 - #1^3 + 8*#1^4 & , 3, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & ,
 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0])))]*(-Root[8 + 8
*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & ,
 2, 0]))*Sqrt[((-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8
 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(x - Root[8 + 8*#1 - #1^3 + 8*
#1^4 & , 3, 0]))/((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(
-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 +
 8*#1^4 & , 3, 0]))]*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] - R
oot[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0])*Sqrt[((x - Root[8 + 8*#1
- #1^3 + 8*#1^4 & , 1, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2,
0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))/((x - Root[8 + 8*#
1 - #1^3 + 8*#1^4 & , 2, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1
, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))]*Sqrt[((-Root[8
+ 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4
& , 2, 0])*(x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))/((x - R
oot[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(-Root[8 + 8*#1 - #1^3 +
```

```
8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))])/(S
qrt[8 + 8*x - x^3 + 8*x^4]*(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1,
 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(Root[8 + 8*#1 - #
1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0])
) + (842*EllipticF[ArcSin[Sqrt[((x - Root[8 + 8*#1 - #1^3 + 8*#1^
4 & , 1, 0])*(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] + Root[8 +
 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))/((x - Root[8 + 8*#1 - #1^3 + 8*
#1^4 & , 2, 0])*(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[
8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))]], ((Root[8 + 8*#1 - #1^3 +
8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0])*(Root
[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1
^4 & , 4, 0]))/((Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8
 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4
 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))]*(x - Root
[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])^2*Sqrt[((-Root[8 + 8*#1 - #1
^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*
(x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0]))/((x - Root[8 + 8*#
1 - #1^3 + 8*#1^4 & , 2, 0])*(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & ,
1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0]))]*(Root[8 + 8*#1
 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4,
 0])*Sqrt[((-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8
*#1 - #1^3 + 8*#1^4 & , 2, 0])*(x - Root[8 + 8*#1 - #1^3 + 8*#1^4
 & , 4, 0]))/((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(-Roo
t[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#
1^4 & , 4, 0]))]*Sqrt[((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1,
0])*(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] + Root[8 + 8*#1 - #
1^3 + 8*#1^4 & , 4, 0]))/((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & ,
2, 0])*(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1
- #1^3 + 8*#1^4 & , 4, 0]))])/(Sqrt[8 + 8*x - x^3 + 8*x^4]*(-Root
[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1
^4 & , 2, 0])*(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] + Root[8
+ 8*#1 - #1^3 + 8*#1^4 & , 4, 0])) - (224*((x - Root[8 + 8*#1 - #
1^3 + 8*#1^4 & , 1, 0])*(x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3,
 0])*(x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]) + (x - Root[8
+ 8*#1 - #1^3 + 8*#1^4 & , 2, 0])^2*Sqrt[((-Root[8 + 8*#1 - #1^3
+ 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(x
- Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0]))/((x - Root[8 + 8*#1 -
 #1^3 + 8*#1^4 & , 2, 0])*(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1,
0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0]))]*Sqrt[((x - Root[8
 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4
 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))/((x - Root
[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1
^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))]*Sqrt[((
-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 +
 8*#1^4 & , 2, 0])*(x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))
/((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(-Root[8 + 8*#1 -
 #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0
]))]*(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 -
#1^3 + 8*#1^4 & , 4, 0])*((EllipticE[ArcSin[Sqrt[((x - Root[8 + 8
*#1 - #1^3 + 8*#1^4 & , 1, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & ,
 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))/((x - Root[8 +
 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 &
 , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))]], -(((Root[
8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^
```

```
4 & , 3, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 +
8*#1 - #1^3 + 8*#1^4 & , 4, 0]))/((-Root[8 + 8*#1 - #1^3 + 8*#1^4
 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0])*(Root[8 + 8
*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & ,
 4, 0]))))]*(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8
*#1 - #1^3 + 8*#1^4 & , 3, 0]))/(-Root[8 + 8*#1 - #1^3 + 8*#1^4 &
 , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0]) + (EllipticF[
ArcSin[Sqrt[((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0])*(Root[
8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^
4 & , 4, 0]))/((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(Roo
t[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#
1^4 & , 4, 0]))]], -(((Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] -
Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0])*(Root[8 + 8*#1 - #1^3 +
8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))/((-R
oot[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8
*#1^4 & , 3, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[
8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))))]*(Root[8 + 8*#1 - #1^3 + 8*
#1^4 & , 2, 0]*(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8
 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]) - Root[8 + 8*#1 - #1^3 + 8*#1^
4 & , 1, 0]*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8
*#1 - #1^3 + 8*#1^4 & , 4, 0])))/((-Root[8 + 8*#1 - #1^3 + 8*#1^4
 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(-Root[8 +
8*#1 - #1^3 + 8*#1^4 & , 2, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 &
, 4, 0])) - (EllipticPi[(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0]
 + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0])/(-Root[8 + 8*#1 - #1^
3 + 8*#1^4 & , 2, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]),
ArcSin[Sqrt[((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0])*(Root[
8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^
4 & , 4, 0]))/((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(Roo
t[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#
1^4 & , 4, 0]))]], -(((Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] -
Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0])*(Root[8 + 8*#1 - #1^3 +
8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))/((-R
oot[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8
*#1^4 & , 3, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[
8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))))]*(-Root[8 + 8*#1 - #1^3 + 8
*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[
8 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^
4 & , 4, 0]))/(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] + Root[8
+ 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))))/Sqrt[8 + 8*x - x^3 + 8*x^4])
/6264
```

Maple [C] time = 0.03, size = 4426, normalized size = 10.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(8*x^4-x^3+8*x+8)^(3/2),x)
```

[Out] -16*(-17/10962-57/12992*x+191/58464*x^2-7/3132*x^3)/(8*x^4-x^3+8*x+8)^(1/2)+421/12528*(-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)+RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2)*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))^2*((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2)*((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2)/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*2^(1/2)/((x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)))^(1/2)*EllipticF(((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2),((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3))*(-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)+RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3)+RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)))^(1/2))+7/6264*(-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)+RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2)*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))^2*((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2)*((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2)/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*2^(1/2)/((x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)))^(1/2)*(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)*EllipticF(((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2),((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3))*(-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)+RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3)+RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)))^(1/2))+(-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)+RootOf(8*_Z^4-_Z^3+8*_Z+8,

```
index=1))*EllipticPi(((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(
8*_Z^4-_Z^3+8*_Z+8,index=2))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1
))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,
index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2),(RootOf(8
*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(R
ootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index
=2)),((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z
+8,index=3))*(-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)+RootOf(8*_Z^4-_
Z^3+8*_Z+8,index=1))/(-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3)+RootOf(
8*_Z^4-_Z^3+8*_Z+8,index=1))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-
RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)))^(1/2)))-7/783*((x-RootOf(8*_
Z^4-_Z^3+8*_Z+8,index=1))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3))*
(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4))+(-RootOf(8*_Z^4-_Z^3+8*_Z+
8,index=4)+RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*((RootOf(8*_Z^4-_Z
^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))*(x-RootOf(
8*_Z^4-_Z^3+8*_Z+8,index=1))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-
RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,
index=2)))^(1/2)*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))^2*((RootO
f(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))
*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3))/(RootOf(8*_Z^4-_Z^3+8*_Z+
8,index=3)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_
Z^3+8*_Z+8,index=2)))^(1/2)*((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-
RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,
index=4))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+
8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2)*((
RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)*RootOf(8*_Z^4-_Z^3+8*_Z+8,inde
x=1)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1)*RootOf(8*_Z^4-_Z^3+8*_Z+8
,index=4)+RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)*RootOf(8*_Z^4-_Z^3+8
*_Z+8,index=4)+RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)^2)/(RootOf(8*_Z
^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))/(Root
Of(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1)
)*EllipticF(((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z
^3+8*_Z+8,index=2))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(RootO
f(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))
/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2),((RootOf(8*_Z^4-_Z
^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3))*(-RootOf(8
*_Z^4-_Z^3+8*_Z+8,index=4)+RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(-
RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3)+RootOf(8*_Z^4-_Z^3+8*_Z+8,inde
x=1))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z
+8,index=4)))^(1/2))+(-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3)+RootOf(
8*_Z^4-_Z^3+8*_Z+8,index=1))*EllipticE(((RootOf(8*_Z^4-_Z^3+8*_Z+
8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))*(x-RootOf(8*_Z^4-_
Z^3+8*_Z+8,index=1))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8
*_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)
))^(1/2),((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+
8*_Z+8,index=3))*(-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)+RootOf(8*_Z
^4-_Z^3+8*_Z+8,index=1))/(-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3)+Roo
tOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index
=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)))^(1/2))/(RootOf(8*_Z^4-_Z
^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))-1/8/(RootO
f(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))
*EllipticPi(((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z
^3+8*_Z+8,index=2))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(RootO
f(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))
```

```
/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2),(-RootOf(8*_Z^4-_Z
^3+8*_Z+8,index=4)+RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(RootOf(8*
_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)),((R
ootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index
=3))*(-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)+RootOf(8*_Z^4-_Z^3+8*_Z
+8,index=1))/(-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3)+RootOf(8*_Z^4-_
Z^3+8*_Z+8,index=1))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8
*_Z^4-_Z^3+8*_Z+8,index=4)))^(1/2))))*2^(1/2)/((x-RootOf(8*_Z^4-_
Z^3+8*_Z+8,index=1))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))*(x-Ro
otOf(8*_Z^4-_Z^3+8*_Z+8,index=3))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,in
dex=4)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(8\,x^4 - x^3 + 8\,x + 8)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((8*x^4 - x^3 + 8*x + 8)^(-3/2),x, algorithm="maxima")
```

```
[Out] integrate((8*x^4 - x^3 + 8*x + 8)^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(8\,x^4 - x^3 + 8\,x + 8)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((8*x^4 - x^3 + 8*x + 8)^(-3/2),x, algorithm="fricas")
```

```
[Out] integral((8*x^4 - x^3 + 8*x + 8)^(-3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(8x^4 - x^3 + 8x + 8)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(8*x**4-x**3+8*x+8)**(3/2),x)

[Out] Integral((8*x**4 - x**3 + 8*x + 8)**(-3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(8\,x^{4}-x^{3}+8\,x+8\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((8*x^4 - x^3 + 8*x + 8)^(-3/2),x, algorithm="giac")

[Out] integrate((8*x^4 - x^3 + 8*x + 8)^(-3/2), x)
```

3.642 $\int \frac{1}{\sqrt{1+4x+4x^2+4x^4}}\,dx$

Optimal. Leaf size=108

$$-\frac{\left(\left(\frac{1}{x}+1\right)^2+\sqrt{5}\right)\sqrt{\frac{\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5}{\left(\left(\frac{1}{x}+1\right)^2+\sqrt{5}\right)^2}}x^2F\left(2\tan^{-1}\left(\frac{1+\frac{1}{x}}{\sqrt[4]{5}}\right)|\frac{1}{10}\left(5+\sqrt{5}\right)\right)}{2\sqrt[4]{5}\sqrt{4x^4+4x^2+4x+1}}$$

```
[Out] -((Sqrt[5] + (1 + x^(-1))^2)*Sqrt[(5 - 2*(1 + x^(-1))^2 + (1 + x^
(-1))^4)/(Sqrt[5] + (1 + x^(-1))^2)^2]*x^2*EllipticF[2*ArcTan[(1
+ x^(-1))/5^(1/4)], (5 + Sqrt[5])/10])/(2*5^(1/4)*Sqrt[1 + 4*x +
4*x^2 + 4*x^4])
```

Rubi [A] time = 0.363689, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$

$$-\frac{\left(\left(\frac{1}{x}+1\right)^2+\sqrt{5}\right)\sqrt{\frac{\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5}{\left(\left(\frac{1}{x}+1\right)^2+\sqrt{5}\right)^2}}x^2F\left(2\tan^{-1}\left(\frac{1+\frac{1}{x}}{\sqrt[4]{5}}\right)|\frac{1}{10}\left(5+\sqrt{5}\right)\right)}{2\sqrt[4]{5}\sqrt{4x^4+4x^2+4x+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[1 + 4*x + 4*x^2 + 4*x^4],x]
```

```
[Out] -((Sqrt[5] + (1 + x^(-1))^2)*Sqrt[(5 - 2*(1 + x^(-1))^2 + (1 + x^
(-1))^4)/(Sqrt[5] + (1 + x^(-1))^2)^2]*x^2*EllipticF[2*ArcTan[(1
+ x^(-1))/5^(1/4)], (5 + Sqrt[5])/10])/(2*5^(1/4)*Sqrt[1 + 4*x +
4*x^2 + 4*x^4])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-16\int^{1+\frac{1}{x}}\frac{1}{\sqrt{\frac{256x^4-512x^2+1280}{(-4x+4)^4}}\,(-4x+4)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(4*x**4+4*x**2+4*x+1)**(1/2),x)
```

```
[Out] -16*Integral(1/(sqrt((256*x**4 - 512*x**2 + 1280)/(-4*x + 4)**4)*
(-4*x + 4)**2), (x, 1 + 1/x))
```

Mathematica [C] time = 0.970966, size = 249, normalized size = 2.31

$$\frac{(2-i)\sqrt{-\frac{1}{10}+\frac{i}{5}}\sqrt{\frac{\left(2i+\sqrt{-1-2i}-\sqrt{-1+2i}\right)\left(-2x+\sqrt{-1-2i}-i\right)}{\left(-2i+\sqrt{-1-2i}+\sqrt{-1+2i}\right)\left(2x+\sqrt{-1-2i}+i\right)}}\left(2ix^2+2x+1\right)F\left(\sin^{-1}\left(\frac{\sqrt{\frac{\left(2i+\sqrt{-1-2i}+\sqrt{-1+2i}\right)\left(2x+\sqrt{-1+2i}-i\right)}{\sqrt{-1+2i}\left(2x+\sqrt{-1-2i}+i\right)}}}{\sqrt{2}}\right)|\frac{1}{2}\left(5-\sqrt{5}\right)\right)}{\sqrt{\frac{(1+2i)\left((-1+i)+\sqrt{-1-2i}\right)(2ix^2+2x+1)}{\left(2x+\sqrt{-1-2i}+i\right)^2}}\sqrt{4x^4+4x^2+4x+1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/Sqrt[1 + 4*x + 4*x^2 + 4*x^4],x]
```

```
[Out] ((2 - I)*Sqrt[-1/10 + I/5]*Sqrt[((2*I + Sqrt[-1 - 2*I] - Sqrt[-1
+ 2*I])*(-I + Sqrt[-1 - 2*I] - 2*x))/((-2*I + Sqrt[-1 - 2*I] + Sq
rt[-1 + 2*I])*(I + Sqrt[-1 - 2*I] + 2*x))]*(1 + 2*x + (2*I)*x^2)*
EllipticF[ArcSin[Sqrt[((2*I + Sqrt[-1 - 2*I] + Sqrt[-1 + 2*I])*(-
I + Sqrt[-1 + 2*I] + 2*x))/(Sqrt[-1 + 2*I]*(I + Sqrt[-1 - 2*I] +
2*x))]/Sqrt[2]], (5 - Sqrt[5])/2])/(Sqrt[((1 + 2*I)*((-1 + I) + S
qrt[-1 - 2*I])*(1 + 2*x + (2*I)*x^2))/(I + Sqrt[-1 - 2*I] + 2*x)^
2]*Sqrt[1 + 4*x + 4*x^2 + 4*x^4])
```

Maple [C] time = 1.227, size = 961, normalized size = 8.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(4*x^4+4*x^2+4*x+1)^(1/2),x)
```

```
[Out] (RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1
,index=4))*((RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^4+4
*_Z^2+4*_Z+1,index=2))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(
RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,
index=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)))^(1/2)*(x-Root
Of(4*_Z^4+4*_Z^2+4*_Z+1,index=2))^2*((RootOf(4*_Z^4+4*_Z^2+4*_Z+1
,index=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))*(x-RootOf(4*_Z^4+
4*_Z^2+4*_Z+1,index=3))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=3)-Roo
tOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1
,index=2)))^(1/2)*((RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)-RootOf(4
```

```
*_Z^4+4*_Z^2+4*_Z+1,index=1))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,inde
x=4))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^4+4*_Z^2+
4*_Z+1,index=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)))^(1/2)/
(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1
,index=2))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)-RootOf(4*_Z^4+4*
_Z^2+4*_Z+1,index=1))/((x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))*(
x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2))*(x-RootOf(4*_Z^4+4*_Z^2+4
*_Z+1,index=3))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)))^(1/2)*E
llipticF(((RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^4+4*_
Z^2+4*_Z+1,index=2))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(Ro
otOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,in
dex=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)))^(1/2),((RootOf(
4*_Z^4+4*_Z^2+4*_Z+1,index=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=3
))*(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1)-RootOf(4*_Z^4+4*_Z^2+4*_
Z+1,index=4))/(-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=3)+RootOf(4*_Z^
4+4*_Z^2+4*_Z+1,index=1))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)-R
ootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{4\,x^4+4\,x^2+4\,x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(4*x^4 + 4*x^2 + 4*x + 1),x, algorithm="maxima")

[Out] integrate(1/sqrt(4*x^4 + 4*x^2 + 4*x + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{4\,x^4+4\,x^2+4\,x+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(4*x^4 + 4*x^2 + 4*x + 1),x, algorithm="fricas")

[Out] integral(1/sqrt(4*x^4 + 4*x^2 + 4*x + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{4x^4 + 4x^2 + 4x + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(4*x**4+4*x**2+4*x+1)**(1/2),x)

[Out] Integral(1/sqrt(4*x**4 + 4*x**2 + 4*x + 1), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{4\,x^4 + 4\,x^2 + 4\,x + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(4*x^4 + 4*x^2 + 4*x + 1),x, algorithm="giac")

[Out] integrate(1/sqrt(4*x^4 + 4*x^2 + 4*x + 1), x)
```

3.643 $\int \frac{1}{(1+4x+4x^2+4x^4)^{3/2}}\,dx$

Optimal. Leaf size=367

$$-\frac{\left(3-\left(\frac{1}{x}+1\right)^2\right)x^2}{\sqrt{4x^4+4x^2+4x+1}}+\frac{\left(13-9\left(\frac{1}{x}+1\right)^2\right)\left(\frac{1}{x}+1\right)x^2}{10\sqrt{4x^4+4x^2+4x+1}}+\frac{9\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)\left(\frac{1}{x}+1\right)x^2}{10\left(\left(\frac{1}{x}+1\right)^2+\sqrt{5}\right)\sqrt{4x^4+4x^2+4x+1}}+\frac{3\left(3-\sqrt{5}\right)\left(\left(\frac{1}{x}+1\right)^2+\sqrt{5}\right)\sqrt{\frac{\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5}{\left(\left(\frac{1}{x}+1\right)^2+\sqrt{5}\right)^2}}x^2F\left(2\tan^{-1}\left(\frac{1+\frac{1}{x}}{\sqrt[4]{5}}\right)|\frac{1}{10}\left(5+\sqrt{5}\right)\right)}{4\,5^{3/4}\sqrt{4x^4+4x^2+4x+1}}-\frac{9\left(\left(\frac{1}{x}+1\right)^2+\sqrt{5}\right)\sqrt{\frac{\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5}{\left(\left(\frac{1}{x}+1\right)^2+\sqrt{5}\right)^2}}x^2E\left(2\tan^{-1}\left(\frac{1+\frac{1}{x}}{\sqrt[4]{5}}\right)|\frac{1}{10}\left(5+\sqrt{5}\right)\right)}{2\,5^{3/4}\sqrt{4x^4+4x^2+4x+1}}$$

```
[Out] -(((3 - (1 + x^(-1))^2)*x^2)/Sqrt[1 + 4*x + 4*x^2 + 4*x^4]) + ((1
3 - 9*(1 + x^(-1))^2)*(1 + x^(-1))*x^2)/(10*Sqrt[1 + 4*x + 4*x^2
+ 4*x^4]) + (9*(5 - 2*(1 + x^(-1))^2 + (1 + x^(-1))^4)*(1 + x^(-1
))*x^2)/(10*(Sqrt[5] + (1 + x^(-1))^2)*Sqrt[1 + 4*x + 4*x^2 + 4*x
^4]) - (9*(Sqrt[5] + (1 + x^(-1))^2)*Sqrt[(5 - 2*(1 + x^(-1))^2 +
 (1 + x^(-1))^4)/(Sqrt[5] + (1 + x^(-1))^2)^2]*x^2*EllipticE[2*Ar
cTan[(1 + x^(-1))/5^(1/4)], (5 + Sqrt[5])/10])/(2*5^(3/4)*Sqrt[1
+ 4*x + 4*x^2 + 4*x^4]) + (3*(3 - Sqrt[5])*(Sqrt[5] + (1 + x^(-1)
)^2)*Sqrt[(5 - 2*(1 + x^(-1))^2 + (1 + x^(-1))^4)/(Sqrt[5] + (1 +
 x^(-1))^2)^2]*x^2*EllipticF[2*ArcTan[(1 + x^(-1))/5^(1/4)], (5 +
 Sqrt[5])/10])/(4*5^(3/4)*Sqrt[1 + 4*x + 4*x^2 + 4*x^4])
```

Rubi [A] time = 0.71022, antiderivative size = 367, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$

$$-\frac{\left(3-\left(\frac{1}{x}+1\right)^2\right)x^2}{\sqrt{4x^4+4x^2+4x+1}}+\frac{\left(13-9\left(\frac{1}{x}+1\right)^2\right)\left(\frac{1}{x}+1\right)x^2}{10\sqrt{4x^4+4x^2+4x+1}}+\frac{9\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)\left(\frac{1}{x}+1\right)x^2}{10\left(\left(\frac{1}{x}+1\right)^2+\sqrt{5}\right)\sqrt{4x^4+4x^2+4x+1}}+\frac{3\left(3-\sqrt{5}\right)\left(\left(\frac{1}{x}+1\right)^2+\sqrt{5}\right)\sqrt{\frac{\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5}{\left(\left(\frac{1}{x}+1\right)^2+\sqrt{5}\right)^2}}x^2F\left(2\tan^{-1}\left(\frac{1+\frac{1}{x}}{\sqrt[4]{5}}\right)|\frac{1}{10}\left(5+\sqrt{5}\right)\right)}{4\,5^{3/4}\sqrt{4x^4+4x^2+4x+1}}-\frac{9\left(\left(\frac{1}{x}+1\right)^2+\sqrt{5}\right)\sqrt{\frac{\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5}{\left(\left(\frac{1}{x}+1\right)^2+\sqrt{5}\right)^2}}x^2E\left(2\tan^{-1}\left(\frac{1+\frac{1}{x}}{\sqrt[4]{5}}\right)|\frac{1}{10}\left(5+\sqrt{5}\right)\right)}{2\,5^{3/4}\sqrt{4x^4+4x^2+4x+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + 4*x + 4*x^2 + 4*x^4)^(-3/2),x]
```

```
[Out] -(((3 - (1 + x^(-1))^2)*x^2)/Sqrt[1 + 4*x + 4*x^2 + 4*x^4]) + ((1
3 - 9*(1 + x^(-1))^2)*(1 + x^(-1))*x^2)/(10*Sqrt[1 + 4*x + 4*x^2
+ 4*x^4]) + (9*(5 - 2*(1 + x^(-1))^2 + (1 + x^(-1))^4)*(1 + x^(-1
))*x^2)/(10*(Sqrt[5] + (1 + x^(-1))^2)*Sqrt[1 + 4*x + 4*x^2 + 4*x
^4]) - (9*(Sqrt[5] + (1 + x^(-1))^2)*Sqrt[(5 - 2*(1 + x^(-1))^2 +
 (1 + x^(-1))^4)/(Sqrt[5] + (1 + x^(-1))^2)^2]*x^2*EllipticE[2*Ar
cTan[(1 + x^(-1))/5^(1/4)], (5 + Sqrt[5])/10])/(2*5^(3/4)*Sqrt[1
+ 4*x + 4*x^2 + 4*x^4]) + (3*(3 - Sqrt[5])*(Sqrt[5] + (1 + x^(-1)
)^2)*Sqrt[(5 - 2*(1 + x^(-1))^2 + (1 + x^(-1))^4)/(Sqrt[5] + (1 +
 x^(-1))^2)^2]*x^2*EllipticF[2*ArcTan[(1 + x^(-1))/5^(1/4)], (5 +
 Sqrt[5])/10])/(4*5^(3/4)*Sqrt[1 + 4*x + 4*x^2 + 4*x^4])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-16\int^{1+\frac{1}{x}} \frac{1}{\left(\frac{256x^4-512x^2+1280}{(-4x+4)^4}\right)^{\frac{3}{2}}(-4x+4)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(4*x**4+4*x**2+4*x+1)**(3/2),x)
```

```
[Out] -16*Integral(1/(((256*x**4 - 512*x**2 + 1280)/(-4*x + 4)**4)**(3/
2)*(-4*x + 4)**2), (x, 1 + 1/x))
```

Mathematica [C] time = 6.05141, size = 3334, normalized size = 9.08

Result too large to show

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(1 + 4*x + 4*x^2 + 4*x^4)^(-3/2),x]
```

```
[Out] (19 + 42*x - 16*x^2 + 36*x^3)/(10*Sqrt[1 + 4*x + 4*x^2 + 4*x^4])
- (3*((-2*EllipticF[ArcSin[Sqrt[((x - Root[1 + 4*#1 + 4*#1^2 + 4*
#1^4 & , 1, 0])*(-Root[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , 2, 0] + Roo
t[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , 4, 0]))/((x - Root[1 + 4*#1 + 4*
#1^2 + 4*#1^4 & , 2, 0])*(-Root[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , 1,
 0] + Root[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , 4, 0]))]], ((Root[1 + 4
*#1 + 4*#1^2 + 4*#1^4 & , 2, 0] - Root[1 + 4*#1 + 4*#1^2 + 4*#1^4
 & , 3, 0])*(Root[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , 1, 0] - Root[1 +
```

```
 4*#1 + 4*#1^2 + 4*#1^4 & , 4, 0]))/((Root[1 + 4*#1 + 4*#1^2 + 4*
#1^4 & , 1, 0] - Root[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , 3, 0])*(Root
[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , 2, 0] - Root[1 + 4*#1 + 4*#1^2 +
4*#1^4 & , 4, 0]))]*(x - Root[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , 2, 0
])^2*Sqrt[((-Root[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , 1, 0] + Root[1 +
 4*#1 + 4*#1^2 + 4*#1^4 & , 2, 0])*(x - Root[1 + 4*#1 + 4*#1^2 +
4*#1^4 & , 3, 0]))/((x - Root[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , 2, 0
])*(-Root[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , 1, 0] + Root[1 + 4*#1 +
4*#1^2 + 4*#1^4 & , 3, 0]))]*(Root[1 + 4*#1 + 4*#1^2 + 4*#1^4 & ,
 1, 0] - Root[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , 4, 0])*Sqrt[((-Root[
1 + 4*#1 + 4*#1^2 + 4*#1^4 & , 1, 0] + Root[1 + 4*#1 + 4*#1^2 + 4
*#1^4 & , 2, 0])*(x - Root[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , 4, 0]))
/((x - Root[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , 2, 0])*(-Root[1 + 4*#1
 + 4*#1^2 + 4*#1^4 & , 1, 0] + Root[1 + 4*#1 + 4*#1^2 + 4*#1^4 &
, 4, 0]))]*Sqrt[((x - Root[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , 1, 0])*
(-Root[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , 2, 0] + Root[1 + 4*#1 + 4*#
1^2 + 4*#1^4 & , 4, 0]))/((x - Root[1 + 4*#1 + 4*#1^2 + 4*#1^4 &
, 2, 0])*(-Root[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , 1, 0] + Root[1 + 4
*#1 + 4*#1^2 + 4*#1^4 & , 4, 0]))]])/(Sqrt[1 + 4*x + 4*x^2 + 4*x^4
]*(-Root[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , 1, 0] + Root[1 + 4*#1 + 4
*#1^2 + 4*#1^4 & , 2, 0])*(-Root[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , 2
, 0] + Root[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , 4, 0])) + (6*((x - Roo
t[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , 1, 0])*(x - Root[1 + 4*#1 + 4*#1
^2 + 4*#1^4 & , 3, 0])*(x - Root[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , 4
, 0]) + (x - Root[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , 2, 0])^2*Sqrt[((
-Root[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , 1, 0] + Root[1 + 4*#1 + 4*#1
^2 + 4*#1^4 & , 2, 0])*(x - Root[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , 3
, 0]))/((x - Root[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , 2, 0])*(-Root[1
+ 4*#1 + 4*#1^2 + 4*#1^4 & , 1, 0] + Root[1 + 4*#1 + 4*#1^2 + 4*#
1^4 & , 3, 0]))]*Sqrt[((x - Root[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , 1
, 0])*(Root[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , 2, 0] - Root[1 + 4*#1
+ 4*#1^2 + 4*#1^4 & , 4, 0]))/((x - Root[1 + 4*#1 + 4*#1^2 + 4*#1
^4 & , 2, 0])*(Root[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , 1, 0] - Root[1
 + 4*#1 + 4*#1^2 + 4*#1^4 & , 4, 0]))]*Sqrt[((-Root[1 + 4*#1 + 4*
#1^2 + 4*#1^4 & , 1, 0] + Root[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , 2,
0])*(x - Root[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , 4, 0]))/((x - Root[1
 + 4*#1 + 4*#1^2 + 4*#1^4 & , 2, 0])*(-Root[1 + 4*#1 + 4*#1^2 + 4
*#1^4 & , 1, 0] + Root[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , 4, 0]))]*(-
Root[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , 1, 0] + Root[1 + 4*#1 + 4*#1^
2 + 4*#1^4 & , 4, 0])*((EllipticE[ArcSin[Sqrt[((x - Root[1 + 4*#1
 + 4*#1^2 + 4*#1^4 & , 1, 0])*(Root[1 + 4*#1 + 4*#1^2 + 4*#1^4 &
, 2, 0] - Root[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , 4, 0]))/((x - Root[
1 + 4*#1 + 4*#1^2 + 4*#1^4 & , 2, 0])*(Root[1 + 4*#1 + 4*#1^2 + 4
*#1^4 & , 1, 0] - Root[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , 4, 0]))]],
-(((Root[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , 2, 0] - Root[1 + 4*#1 + 4
*#1^2 + 4*#1^4 & , 3, 0])*(Root[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , 1,
 0] - Root[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , 4, 0]))/((-Root[1 + 4*#
1 + 4*#1^2 + 4*#1^4 & , 1, 0] + Root[1 + 4*#1 + 4*#1^2 + 4*#1^4 &
 , 3, 0])*(Root[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , 2, 0] - Root[1 + 4
*#1 + 4*#1^2 + 4*#1^4 & , 4, 0])))]*(-Root[1 + 4*#1 + 4*#1^2 + 4*
#1^4 & , 1, 0] + Root[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , 3, 0]))/(-Ro
ot[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , 1, 0] + Root[1 + 4*#1 + 4*#1^2
+ 4*#1^4 & , 2, 0]) + (EllipticF[ArcSin[Sqrt[((x - Root[1 + 4*#1
+ 4*#1^2 + 4*#1^4 & , 1, 0])*(Root[1 + 4*#1 + 4*#1^2 + 4*#1^4 & ,
```

```
 2, 0] - Root[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , 4, 0]))/((x - Root[1
 + 4*#1 + 4*#1^2 + 4*#1^4 & , 2, 0])*(Root[1 + 4*#1 + 4*#1^2 + 4*
#1^4 & , 1, 0] - Root[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , 4, 0]))]], -
(((Root[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , 2, 0] - Root[1 + 4*#1 + 4*
#1^2 + 4*#1^4 & , 3, 0])*(Root[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , 1,
0] - Root[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , 4, 0]))/((-Root[1 + 4*#1
 + 4*#1^2 + 4*#1^4 & , 1, 0] + Root[1 + 4*#1 + 4*#1^2 + 4*#1^4 &
, 3, 0])*(Root[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , 2, 0] - Root[1 + 4*
#1 + 4*#1^2 + 4*#1^4 & , 4, 0])))]*(Root[1 + 4*#1 + 4*#1^2 + 4*#1
^4 & , 2, 0]*(-Root[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , 2, 0] - Root[1
 + 4*#1 + 4*#1^2 + 4*#1^4 & , 4, 0]) - Root[1 + 4*#1 + 4*#1^2 + 4
*#1^4 & , 1, 0]*(Root[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , 2, 0] - Root
[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , 4, 0])))/((-Root[1 + 4*#1 + 4*#1^
2 + 4*#1^4 & , 1, 0] + Root[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , 2, 0])
*(-Root[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , 2, 0] + Root[1 + 4*#1 + 4*
#1^2 + 4*#1^4 & , 4, 0])) - (EllipticPi[(-Root[1 + 4*#1 + 4*#1^2
+ 4*#1^4 & , 1, 0] + Root[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , 4, 0])/(
-Root[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , 2, 0] + Root[1 + 4*#1 + 4*#1
^2 + 4*#1^4 & , 4, 0]), ArcSin[Sqrt[((x - Root[1 + 4*#1 + 4*#1^2
+ 4*#1^4 & , 1, 0])*(Root[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , 2, 0] -
Root[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , 4, 0]))/((x - Root[1 + 4*#1 +
 4*#1^2 + 4*#1^4 & , 2, 0])*(Root[1 + 4*#1 + 4*#1^2 + 4*#1^4 & ,
1, 0] - Root[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , 4, 0]))]], -(((Root[1
 + 4*#1 + 4*#1^2 + 4*#1^4 & , 2, 0] - Root[1 + 4*#1 + 4*#1^2 + 4*
#1^4 & , 3, 0])*(Root[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , 1, 0] - Root
[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , 4, 0]))/((-Root[1 + 4*#1 + 4*#1^2
 + 4*#1^4 & , 1, 0] + Root[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , 3, 0])*
(Root[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , 2, 0] - Root[1 + 4*#1 + 4*#1
^2 + 4*#1^4 & , 4, 0])))]*(-Root[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , 1
, 0] - Root[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , 2, 0] - Root[1 + 4*#1
+ 4*#1^2 + 4*#1^4 & , 3, 0] - Root[1 + 4*#1 + 4*#1^2 + 4*#1^4 & ,
 4, 0]))/(-Root[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , 2, 0] + Root[1 + 4
*#1 + 4*#1^2 + 4*#1^4 & , 4, 0]))))/Sqrt[1 + 4*x + 4*x^2 + 4*x^4]
))/5
```

Maple [C] time = 0.028, size = 2564, normalized size = 7.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(4*x^4+4*x^2+4*x+1)^(3/2),x)
```

```
[Out] -8*(-9/20*x^3+1/5*x^2-21/40*x-19/80)/(4*x^4+4*x^2+4*x+1)^(1/2)+3/
5*(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1)-RootOf(4*_Z^4+4*_Z^2+4*_Z
+1,index=4))*((RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^4
+4*_Z^2+4*_Z+1,index=2))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))
/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^4+4*_Z^2+4*_Z+
1,index=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)))^(1/2)*(x-Ro
```

otOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2))^2*((RootOf(4*_Z^4+4*_Z^2+4*_Z
+1,index=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))*(x-RootOf(4*_Z^
4+4*_Z^2+4*_Z+1,index=3))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=3)-R
ootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_Z
+1,index=2)))^(1/2)*((RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)-RootOf
(4*_Z^4+4*_Z^2+4*_Z+1,index=1))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,in
dex=4))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^4+4*_Z^
2+4*_Z+1,index=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)))^(1/2
)/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^4+4*_Z^2+4*_Z
+1,index=2))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)-RootOf(4*_Z^4+
4*_Z^2+4*_Z+1,index=1))/((x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))
*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2))*(x-RootOf(4*_Z^4+4*_Z^2
+4*_Z+1,index=3))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)))^(1/2)
*EllipticF(((RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^4+4
*_Z^2+4*_Z+1,index=2))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(
RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,
index=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)))^(1/2),((RootO
f(4*_Z^4+4*_Z^2+4*_Z+1,index=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index
=3))*(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1)-RootOf(4*_Z^4+4*_Z^2+4
*_Z+1,index=4))/(-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=3)+RootOf(4*_
Z^4+4*_Z^2+4*_Z+1,index=1))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)
-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)))^(1/2))-9/5*((x-RootOf(4*_
Z^4+4*_Z^2+4*_Z+1,index=1))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=
3))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4))+(RootOf(4*_Z^4+4*_Z^
2+4*_Z+1,index=1)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4))*((RootOf(
4*_Z^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2
))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(RootOf(4*_Z^4+4*_Z^2
+4*_Z+1,index=4)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(x-RootOf(
4*_Z^4+4*_Z^2+4*_Z+1,index=2)))^(1/2)*(x-RootOf(4*_Z^4+4*_Z^2+4*_
Z+1,index=2))^2*((RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)-RootOf(4*_
Z^4+4*_Z^2+4*_Z+1,index=1))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=
3))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=3)-RootOf(4*_Z^4+4*_Z^2+4*
_Z+1,index=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)))^(1/2)*((
RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,
index=1))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4))/(RootOf(4*_Z^4
+4*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(x-
RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)))^(1/2)*((RootOf(4*_Z^4+4*_Z
^2+4*_Z+1,index=2)*RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1)-RootOf(4*
_Z^4+4*_Z^2+4*_Z+1,index=1)*RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)+
RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)*RootOf(4*_Z^4+4*_Z^2+4*_Z+1,
index=4)+RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)^2)/(RootOf(4*_Z^4+4
*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2))/(Root
Of(4*_Z^4+4*_Z^2+4*_Z+1,index=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,inde
x=1))*EllipticF(((RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4*_
Z^4+4*_Z^2+4*_Z+1,index=2))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=
1))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^4+4*_Z^2+4*
_Z+1,index=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)))^(1/2),((
RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,
index=3))*(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1)-RootOf(4*_Z^4+4*_
Z^2+4*_Z+1,index=4))/(-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=3)+RootO
f(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,ind
ex=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)))^(1/2))+(-RootOf(4*_Z
^4+4*_Z^2+4*_Z+1,index=3)+RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))*E
llipticE(((RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^4+4*_

```
Z^2+4*_Z+1,index=2))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(Ro
otOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,in
dex=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)))^(1/2),((RootOf(
4*_Z^4+4*_Z^2+4*_Z+1,index=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=3
))*(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1)-RootOf(4*_Z^4+4*_Z^2+4*_
Z+1,index=4))/(-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=3)+RootOf(4*_Z^
4+4*_Z^2+4*_Z+1,index=1))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)-R
ootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)))^(1/2))/(RootOf(4*_Z^4+4*_Z^
2+4*_Z+1,index=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))))/((x-Roo
tOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1
,index=2))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=3))*(x-RootOf(4*_
Z^4+4*_Z^2+4*_Z+1,index=4)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(4\,x^4+4\,x^2+4\,x+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^4 + 4*x^2 + 4*x + 1)^(-3/2),x, algorithm="maxima")
```

```
[Out] integrate((4*x^4 + 4*x^2 + 4*x + 1)^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\left(4\,x^4+4\,x^2+4\,x+1\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^4 + 4*x^2 + 4*x + 1)^(-3/2),x, algorithm="fricas")
```

```
[Out] integral((4*x^4 + 4*x^2 + 4*x + 1)^(-3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(4x^4+4x^2+4x+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(4*x**4+4*x**2+4*x+1)**(3/2),x)
```

```
[Out] Integral((4*x**4 + 4*x**2 + 4*x + 1)**(-3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(4x^4+4x^2+4x+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^4 + 4*x^2 + 4*x + 1)^(-3/2),x, algorithm="giac")
```

```
[Out] integrate((4*x^4 + 4*x^2 + 4*x + 1)^(-3/2), x)
```

3.644 $\int \frac{1}{\sqrt{8+24x+8x^2-15x^3+8x^4}}\,dx$

Optimal. Leaf size=126

$$-\frac{\left(\left(\frac{4}{x}+3\right)^2+\sqrt{517}\right)\sqrt{\frac{\left(\frac{4}{x}+3\right)^4-38\left(\frac{4}{x}+3\right)^2+517}{\left(\left(\frac{4}{x}+3\right)^2+\sqrt{517}\right)^2}}x^2F\left(2\tan^{-1}\left(\frac{3x+4}{\sqrt[4]{517}x}\right)|\frac{517+19\sqrt{517}}{1034}\right)}{8\sqrt[4]{517}\sqrt{8x^4-15x^3+8x^2+24x+8}}$$

```
[Out] -((Sqrt[517] + (3 + 4/x)^2)*Sqrt[(517 - 38*(3 + 4/x)^2 + (3 + 4/x
)^4)/(Sqrt[517] + (3 + 4/x)^2)^2]*x^2*EllipticF[2*ArcTan[(4 + 3*x
)/(517^(1/4)*x)], (517 + 19*Sqrt[517])/1034])/(8*517^(1/4)*Sqrt[8
 + 24*x + 8*x^2 - 15*x^3 + 8*x^4])
```

Rubi [A] time = 0.559767, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$-\frac{\left(\left(\frac{4}{x}+3\right)^2+\sqrt{517}\right)\sqrt{\frac{\left(\frac{4}{x}+3\right)^4-38\left(\frac{4}{x}+3\right)^2+517}{\left(\left(\frac{4}{x}+3\right)^2+\sqrt{517}\right)^2}}x^2F\left(2\tan^{-1}\left(\frac{3x+4}{\sqrt[4]{517}x}\right)|\frac{517+19\sqrt{517}}{1034}\right)}{8\sqrt[4]{517}\sqrt{8x^4-15x^3+8x^2+24x+8}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4],x]
```

```
[Out] -((Sqrt[517] + (3 + 4/x)^2)*Sqrt[(517 - 38*(3 + 4/x)^2 + (3 + 4/x
)^4)/(Sqrt[517] + (3 + 4/x)^2)^2]*x^2*EllipticF[2*ArcTan[(4 + 3*x
)/(517^(1/4)*x)], (517 + 19*Sqrt[517])/1034])/(8*517^(1/4)*Sqrt[8
 + 24*x + 8*x^2 - 15*x^3 + 8*x^4])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-1024\int^{\frac{3}{4}+\frac{1}{x}}\frac{1}{\sqrt{\frac{8388608x^4-19922944x^2+16941056}{(-32x+24)^4}}\,(-32x+24)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(8*x**4-15*x**3+8*x**2+24*x+8)**(1/2),x)
```

```
[Out] -1024*Integral(1/(sqrt((8388608*x**4 - 19922944*x**2 + 16941056)/
(-32*x + 24)**4)*(-32*x + 24)**2), (x, 3/4 + 1/x))
```

Mathematica [C] time = 0.59733, size = 1148, normalized size = 9.11

result too large to display

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4],x]
```

```
[Out] (-2*EllipticF[ArcSin[Sqrt[((x - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3
 + 8*#1^4 & , 1, 0])*(Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4
& , 2, 0] - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 4, 0])
)/((x - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 2, 0])*(Ro
ot[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 1, 0] - Root[8 + 24*
#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 4, 0]))]], ((Root[8 + 24*#1 +
8*#1^2 - 15*#1^3 + 8*#1^4 & , 2, 0] - Root[8 + 24*#1 + 8*#1^2 - 1
5*#1^3 + 8*#1^4 & , 3, 0])*(Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8
*#1^4 & , 1, 0] - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & ,
4, 0]))/((Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 1, 0] -
Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 3, 0])*(Root[8 + 2
4*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 2, 0] - Root[8 + 24*#1 + 8*#
1^2 - 15*#1^3 + 8*#1^4 & , 4, 0]))]*(x - Root[8 + 24*#1 + 8*#1^2
- 15*#1^3 + 8*#1^4 & , 2, 0])^2*Sqrt[((Root[8 + 24*#1 + 8*#1^2 -
15*#1^3 + 8*#1^4 & , 1, 0] - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 +
8*#1^4 & , 2, 0])*(x - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4
 & , 3, 0]))/((x - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & ,
 2, 0])*(Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 1, 0] - R
oot[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 3, 0]))]*(Root[8 +
24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 1, 0] - Root[8 + 24*#1 + 8*
#1^2 - 15*#1^3 + 8*#1^4 & , 4, 0])*Sqrt[((x - Root[8 + 24*#1 + 8*
#1^2 - 15*#1^3 + 8*#1^4 & , 1, 0])*(Root[8 + 24*#1 + 8*#1^2 - 15*
#1^3 + 8*#1^4 & , 1, 0] - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#
1^4 & , 2, 0])*(x - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 &
, 4, 0])*(Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 2, 0] -
Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 4, 0]))/((x - Root
[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 2, 0])^2*(Root[8 + 24*
#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 1, 0] - Root[8 + 24*#1 + 8*#1^
2 - 15*#1^3 + 8*#1^4 & , 4, 0])^2)])/(Sqrt[8 + 24*x + 8*x^2 - 15*
x^3 + 8*x^4]*(-Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 1,
0] + Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 2, 0])*(Root[
8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 2, 0] - Root[8 + 24*#1
+ 8*#1^2 - 15*#1^3 + 8*#1^4 & , 4, 0]))
```

Maple [C] time = 1.678, size = 1180, normalized size = 9.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/2),x)
```

```
[Out] 1/2*(RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=1)-RootOf(8*_Z^4-
15*_Z^3+8*_Z^2+24*_Z+8,index=4))*((RootOf(8*_Z^4-15*_Z^3+8*_Z^2+2
4*_Z+8,index=4)-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2))*(x
-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=1))/(RootOf(8*_Z^4-15
*_Z^3+8*_Z^2+24*_Z+8,index=4)-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+
8,index=1))/(x-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2)))^(1
/2)*(x-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2))^2*((RootOf(
8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2)-RootOf(8*_Z^4-15*_Z^3+8*_Z
^2+24*_Z+8,index=1))*(x-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,inde
x=3))/(RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=3)-RootOf(8*_Z^
4-15*_Z^3+8*_Z^2+24*_Z+8,index=1))/(x-RootOf(8*_Z^4-15*_Z^3+8*_Z^
2+24*_Z+8,index=2)))^(1/2)*((RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8
,index=2)-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=1))*(x-RootO
f(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=4))/(RootOf(8*_Z^4-15*_Z^3+
8*_Z^2+24*_Z+8,index=4)-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,inde
x=1))/(x-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2)))^(1/2)/(R
ootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=4)-RootOf(8*_Z^4-15*_Z^
3+8*_Z^2+24*_Z+8,index=2))/(RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,
index=2)-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=1))*2^(1/2)/(
(x-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=1))*(x-RootOf(8*_Z^
4-15*_Z^3+8*_Z^2+24*_Z+8,index=2))*(x-RootOf(8*_Z^4-15*_Z^3+8*_Z^
2+24*_Z+8,index=3))*(x-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index
=4)))^(1/2)*EllipticF(((RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,inde
x=4)-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2))*(x-RootOf(8*_
Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=1))/(RootOf(8*_Z^4-15*_Z^3+8*_Z^
2+24*_Z+8,index=4)-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=1))
/(x-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2)))^(1/2),((RootO
f(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2)-RootOf(8*_Z^4-15*_Z^3+8*
_Z^2+24*_Z+8,index=3))*(RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,inde
x=1)-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=4))/(-RootOf(8*_Z
^4-15*_Z^3+8*_Z^2+24*_Z+8,index=3)+RootOf(8*_Z^4-15*_Z^3+8*_Z^2+2
4*_Z+8,index=1))/(RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2)-R
ootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=4)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{8\,x^4-15\,x^3+8\,x^2+24\,x+8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sqrt(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{8\,x^4-15\,x^3+8\,x^2+24\,x+8}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sqrt(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{8x^4-15x^3+8x^2+24x+8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x**4-15*x**3+8*x**2+24*x+8)**(1/2),x)
```

```
[Out] Integral(1/sqrt(8*x**4 - 15*x**3 + 8*x**2 + 24*x + 8), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{8\,x^4-15\,x^3+8\,x^2+24\,x+8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sqrt(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8), x)
```

3.645 $\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^{3/2}}\,dx$

Optimal. Leaf size=434

$$-\frac{\left(172-7\left(\frac{4}{x}+3\right)^2\right)x^2}{208\sqrt{8x^4-15x^3+8x^2+24x+8}}+\frac{\left(50896-2455\left(\frac{4}{x}+3\right)^2\right)\left(\frac{4}{x}+3\right)x^2}{322608\sqrt{8x^4-15x^3+8x^2+24x+8}}$$
$$+\frac{2455\left(\left(\frac{4}{x}+3\right)^4-38\left(\frac{4}{x}+3\right)^2+517\right)\left(\frac{4}{x}+3\right)x^2}{322608\left(\left(\frac{4}{x}+3\right)^2+\sqrt{517}\right)\sqrt{8x^4-15x^3+8x^2+24x+8}}$$
$$+\frac{\left(4910-203\sqrt{517}\right)\left(\left(\frac{4}{x}+3\right)^2+\sqrt{517}\right)\sqrt{\frac{\left(\frac{4}{x}+3\right)^4-38\left(\frac{4}{x}+3\right)^2+517}{\left(\left(\frac{4}{x}+3\right)^2+\sqrt{517}\right)^2}}x^2F\left(2\tan^{-1}\left(\frac{3x+4}{\sqrt[4]{517}x}\right)|\frac{517+19\sqrt{517}}{1034}\right)}{2496\ 517^{3/4}\sqrt{8x^4-15x^3+8x^2+24x+8}}$$
$$-\frac{2455\left(\left(\frac{4}{x}+3\right)^2+\sqrt{517}\right)\sqrt{\frac{\left(\frac{4}{x}+3\right)^4-38\left(\frac{4}{x}+3\right)^2+517}{\left(\left(\frac{4}{x}+3\right)^2+\sqrt{517}\right)^2}}x^2E\left(2\tan^{-1}\left(\frac{3x+4}{\sqrt[4]{517}x}\right)|\frac{517+19\sqrt{517}}{1034}\right)}{624\ 517^{3/4}\sqrt{8x^4-15x^3+8x^2+24x+8}}$$

```
[Out] -((172 - 7*(3 + 4/x)^2)*x^2)/(208*Sqrt[8 + 24*x + 8*x^2 - 15*x^3
+ 8*x^4]) + ((50896 - 2455*(3 + 4/x)^2)*(3 + 4/x)*x^2)/(322608*Sq
rt[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4]) + (2455*(517 - 38*(3 + 4/x
)^2 + (3 + 4/x)^4)*(3 + 4/x)*x^2)/(322608*(Sqrt[517] + (3 + 4/x)^
2)*Sqrt[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4]) - (2455*(Sqrt[517] +
(3 + 4/x)^2)*Sqrt[(517 - 38*(3 + 4/x)^2 + (3 + 4/x)^4)/(Sqrt[517]
 + (3 + 4/x)^2)^2]*x^2*EllipticE[2*ArcTan[(4 + 3*x)/(517^(1/4)*x)
], (517 + 19*Sqrt[517])/1034])/(624*517^(3/4)*Sqrt[8 + 24*x + 8*x
^2 - 15*x^3 + 8*x^4]) + ((4910 - 203*Sqrt[517])*(Sqrt[517] + (3 +
 4/x)^2)*Sqrt[(517 - 38*(3 + 4/x)^2 + (3 + 4/x)^4)/(Sqrt[517] + (
3 + 4/x)^2)^2]*x^2*EllipticF[2*ArcTan[(4 + 3*x)/(517^(1/4)*x)], (
517 + 19*Sqrt[517])/1034])/(2496*517^(3/4)*Sqrt[8 + 24*x + 8*x^2
- 15*x^3 + 8*x^4])
```

Rubi [A] time = 1.01155, antiderivative size = 434, normalized size of antiderivative = 1., number of

steps used = 10, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$

$$-\frac{\left(172-7\left(\frac{4}{x}+3\right)^2\right)x^2}{208\sqrt{8x^4-15x^3+8x^2+24x+8}}+\frac{\left(50896-2455\left(\frac{4}{x}+3\right)^2\right)\left(\frac{4}{x}+3\right)x^2}{322608\sqrt{8x^4-15x^3+8x^2+24x+8}}$$
$$+\frac{2455\left(\left(\frac{4}{x}+3\right)^4-38\left(\frac{4}{x}+3\right)^2+517\right)\left(\frac{4}{x}+3\right)x^2}{322608\left(\left(\frac{4}{x}+3\right)^2+\sqrt{517}\right)\sqrt{8x^4-15x^3+8x^2+24x+8}}$$
$$+\frac{\left(4910-203\sqrt{517}\right)\left(\left(\frac{4}{x}+3\right)^2+\sqrt{517}\right)\sqrt{\frac{\left(\frac{4}{x}+3\right)^4-38\left(\frac{4}{x}+3\right)^2+517}{\left(\left(\frac{4}{x}+3\right)^2+\sqrt{517}\right)^2}}x^2F\left(2\tan^{-1}\left(\frac{3x+4}{\sqrt[4]{517}x}\right)|\frac{517+19\sqrt{517}}{1034}\right)}{2496\ 517^{3/4}\sqrt{8x^4-15x^3+8x^2+24x+8}}$$
$$-\frac{2455\left(\left(\frac{4}{x}+3\right)^2+\sqrt{517}\right)\sqrt{\frac{\left(\frac{4}{x}+3\right)^4-38\left(\frac{4}{x}+3\right)^2+517}{\left(\left(\frac{4}{x}+3\right)^2+\sqrt{517}\right)^2}}x^2E\left(2\tan^{-1}\left(\frac{3x+4}{\sqrt[4]{517}x}\right)|\frac{517+19\sqrt{517}}{1034}\right)}{624\ 517^{3/4}\sqrt{8x^4-15x^3+8x^2+24x+8}}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-3/2),x]
```

```
[Out] -((172 - 7*(3 + 4/x)^2)*x^2)/(208*Sqrt[8 + 24*x + 8*x^2 - 15*x^3
+ 8*x^4]) + ((50896 - 2455*(3 + 4/x)^2)*(3 + 4/x)*x^2)/(322608*Sq
rt[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4]) + (2455*(517 - 38*(3 + 4/x
)^2 + (3 + 4/x)^4)*(3 + 4/x)*x^2)/(322608*(Sqrt[517] + (3 + 4/x)^
2)*Sqrt[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4]) - (2455*(Sqrt[517] +
(3 + 4/x)^2)*Sqrt[(517 - 38*(3 + 4/x)^2 + (3 + 4/x)^4)/(Sqrt[517]
 + (3 + 4/x)^2)^2]*x^2*EllipticE[2*ArcTan[(4 + 3*x)/(517^(1/4)*x)
], (517 + 19*Sqrt[517])/1034])/(624*517^(3/4)*Sqrt[8 + 24*x + 8*x
^2 - 15*x^3 + 8*x^4]) + ((4910 - 203*Sqrt[517])*(Sqrt[517] + (3 +
 4/x)^2)*Sqrt[(517 - 38*(3 + 4/x)^2 + (3 + 4/x)^4)/(Sqrt[517] + (
3 + 4/x)^2)^2]*x^2*EllipticF[2*ArcTan[(4 + 3*x)/(517^(1/4)*x)], (
517 + 19*Sqrt[517])/1034])/(2496*517^(3/4)*Sqrt[8 + 24*x + 8*x^2
- 15*x^3 + 8*x^4])
```

Rubi in Sympy [F]   time = 0., size = 0, normalized size = 0.

$$-1024\int^{\frac{3}{4}+\frac{1}{x}}\frac{1}{\left(\frac{8388608x^4-19922944x^2+16941056}{(-32x+24)^4}\right)^{\frac{3}{2}}(-32x+24)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(8*x**4-15*x**3+8*x**2+24*x+8)**(3/2),x)
```

```
[Out] -1024*Integral(1/(((8388608*x**4 - 19922944*x**2 + 16941056)/(-32
*x + 24)**4)**(3/2)*(-32*x + 24)**2), (x, 3/4 + 1/x))
```

Mathematica [C] time = 6.07067, size = 6019, normalized size = 13.87

Result too large to show

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-3/2),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.03, size = 5421, normalized size = 12.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(8\,x^4 - 15\,x^3 + 8\,x^2 + 24\,x + 8\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)^(-3/2),x, algorithm="maxima")
```

```
[Out] integrate((8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\left(8\,x^4 - 15\,x^3 + 8\,x^2 + 24\,x + 8\right)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)^(-3/2),x, algorithm="fricas")
```

```
[Out] integral((8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)^(-3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(8x^4 - 15x^3 + 8x^2 + 24x + 8)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(8*x**4-15*x**3+8*x**2+24*x+8)**(3/2),x)
```

```
[Out] Integral((8*x**4 - 15*x**3 + 8*x**2 + 24*x + 8)**(-3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(8\,x^4 - 15\,x^3 + 8\,x^2 + 24\,x + 8)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)^(-3/2),x, algorithm="giac")
```

```
[Out] integrate((8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)^(-3/2), x)
```

3.646 $\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^{5/2}}\,dx$

Optimal. Leaf size=577

$$-\frac{\left(124415-6308\left(\frac{4}{x}+3\right)^2\right)x^2}{97344\sqrt{8x^4-15x^3+8x^2+24x+8}}+\frac{\left(18932921731-1086525994\left(\frac{4}{x}+3\right)^2\right)\left(\frac{4}{x}+3\right)x^2}{78056941248\sqrt{8x^4-15x^3+8x^2+24x+8}}$$
$$+\frac{543262997\left(\left(\frac{4}{x}+3\right)^4-38\left(\frac{4}{x}+3\right)^2+517\right)\left(\frac{4}{x}+3\right)x^2}{39028470624\left(\left(\frac{4}{x}+3\right)^2+\sqrt{517}\right)\sqrt{8x^4-15x^3+8x^2+24x+8}}$$
$$+\frac{\left(11921698-359497\left(\frac{4}{x}+3\right)^2\right)\left(\frac{4}{x}+3\right)x^2}{483912\left(\left(\frac{4}{x}+3\right)^4-38\left(\frac{4}{x}+3\right)^2+517\right)\sqrt{8x^4-15x^3+8x^2+24x+8}}$$
$$-\frac{\left(64489-1399\left(\frac{4}{x}+3\right)^2\right)x^2}{624\left(\left(\frac{4}{x}+3\right)^4-38\left(\frac{4}{x}+3\right)^2+517\right)\sqrt{8x^4-15x^3+8x^2+24x+8}}$$
$$+\frac{\left(4346103976-175318963\sqrt{517}\right)\left(\left(\frac{4}{x}+3\right)^2+\sqrt{517}\right)\sqrt{\frac{\left(\frac{4}{x}+3\right)^4-38\left(\frac{4}{x}+3\right)^2+517}{\left(\left(\frac{4}{x}+3\right)^2+\sqrt{517}\right)^2}}x^2F\left(2\tan^{-1}\left(\frac{3x+4}{\sqrt[4]{517}x}\right)|\frac{517+19\sqrt{517}}{1034}\right)}{1207844352\ 517^{3/4}\sqrt{8x^4-15x^3+8x^2+24x+8}}$$
$$-\frac{543262997\left(\left(\frac{4}{x}+3\right)^2+\sqrt{517}\right)\sqrt{\frac{\left(\frac{4}{x}+3\right)^4-38\left(\frac{4}{x}+3\right)^2+517}{\left(\left(\frac{4}{x}+3\right)^2+\sqrt{517}\right)^2}}x^2E\left(2\tan^{-1}\left(\frac{3x+4}{\sqrt[4]{517}x}\right)|\frac{517+19\sqrt{517}}{1034}\right)}{75490272\ 517^{3/4}\sqrt{8x^4-15x^3+8x^2+24x+8}}$$

```
[Out] -((124415 - 6308*(3 + 4/x)^2)*x^2)/(97344*Sqrt[8 + 24*x + 8*x^2 -
 15*x^3 + 8*x^4]) - ((64489 - 1399*(3 + 4/x)^2)*x^2)/(624*(517 -
38*(3 + 4/x)^2 + (3 + 4/x)^4)*Sqrt[8 + 24*x + 8*x^2 - 15*x^3 + 8*
x^4]) + ((18932921731 - 1086525994*(3 + 4/x)^2)*(3 + 4/x)*x^2)/(7
8056941248*Sqrt[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4]) + ((11921698
- 359497*(3 + 4/x)^2)*(3 + 4/x)*x^2)/(483912*(517 - 38*(3 + 4/x)^
2 + (3 + 4/x)^4)*Sqrt[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4]) + (5432
62997*(517 - 38*(3 + 4/x)^2 + (3 + 4/x)^4)*(3 + 4/x)*x^2)/(390284
70624*(Sqrt[517] + (3 + 4/x)^2)*Sqrt[8 + 24*x + 8*x^2 - 15*x^3 +
8*x^4]) - (543262997*(Sqrt[517] + (3 + 4/x)^2)*Sqrt[(517 - 38*(3
+ 4/x)^2 + (3 + 4/x)^4)/(Sqrt[517] + (3 + 4/x)^2)^2]*x^2*Elliptic
E[2*ArcTan[(4 + 3*x)/(517^(1/4)*x)], (517 + 19*Sqrt[517])/1034])/
(75490272*517^(3/4)*Sqrt[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4]) + ((
4346103976 - 175318963*Sqrt[517])*(Sqrt[517] + (3 + 4/x)^2)*Sqrt[
(517 - 38*(3 + 4/x)^2 + (3 + 4/x)^4)/(Sqrt[517] + (3 + 4/x)^2)^2]
*x^2*EllipticF[2*ArcTan[(4 + 3*x)/(517^(1/4)*x)], (517 + 19*Sqrt[
517])/1034])/(1207844352*517^(3/4)*Sqrt[8 + 24*x + 8*x^2 - 15*x^3
 + 8*x^4])
```

Rubi [A] time = 1.27776, antiderivative size = 577, normalized size of antiderivative = 1., number of

steps used = 12, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458

$$-\frac{\left(124415-6308\left(\frac{4}{x}+3\right)^2\right)x^2}{97344\sqrt{8x^4-15x^3+8x^2+24x+8}}+\frac{\left(18932921731-1086525994\left(\frac{4}{x}+3\right)^2\right)\left(\frac{4}{x}+3\right)x^2}{78056941248\sqrt{8x^4-15x^3+8x^2+24x+8}}$$
$$+\frac{543262997\left(\left(\frac{4}{x}+3\right)^4-38\left(\frac{4}{x}+3\right)^2+517\right)\left(\frac{4}{x}+3\right)x^2}{39028470624\left(\left(\frac{4}{x}+3\right)^2+\sqrt{517}\right)\sqrt{8x^4-15x^3+8x^2+24x+8}}$$
$$+\frac{\left(11921698-359497\left(\frac{4}{x}+3\right)^2\right)\left(\frac{4}{x}+3\right)x^2}{483912\left(\left(\frac{4}{x}+3\right)^4-38\left(\frac{4}{x}+3\right)^2+517\right)\sqrt{8x^4-15x^3+8x^2+24x+8}}$$
$$-\frac{\left(64489-1399\left(\frac{4}{x}+3\right)^2\right)x^2}{624\left(\left(\frac{4}{x}+3\right)^4-38\left(\frac{4}{x}+3\right)^2+517\right)\sqrt{8x^4-15x^3+8x^2+24x+8}}$$
$$+\frac{\left(4346103976-175318963\sqrt{517}\right)\left(\left(\frac{4}{x}+3\right)^2+\sqrt{517}\right)\sqrt{\frac{\left(\frac{4}{x}+3\right)^4-38\left(\frac{4}{x}+3\right)^2+517}{\left(\left(\frac{4}{x}+3\right)^2+\sqrt{517}\right)^2}}x^2F\left(2\tan^{-1}\left(\frac{3x+4}{\sqrt[4]{517}x}\right)|\frac{517+19\sqrt{517}}{1034}\right)}{1207844352\ 517^{3/4}\sqrt{8x^4-15x^3+8x^2+24x+8}}$$
$$-\frac{543262997\left(\left(\frac{4}{x}+3\right)^2+\sqrt{517}\right)\sqrt{\frac{\left(\frac{4}{x}+3\right)^4-38\left(\frac{4}{x}+3\right)^2+517}{\left(\left(\frac{4}{x}+3\right)^2+\sqrt{517}\right)^2}}x^2E\left(2\tan^{-1}\left(\frac{3x+4}{\sqrt[4]{517}x}\right)|\frac{517+19\sqrt{517}}{1034}\right)}{75490272\ 517^{3/4}\sqrt{8x^4-15x^3+8x^2+24x+8}}$$

Antiderivative was successfully verified.

```
[In]  Int[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-5/2),x]
```

```
[Out] -((124415 - 6308*(3 + 4/x)^2)*x^2)/(97344*Sqrt[8 + 24*x + 8*x^2 -
 15*x^3 + 8*x^4]) - ((64489 - 1399*(3 + 4/x)^2)*x^2)/(624*(517 -
38*(3 + 4/x)^2 + (3 + 4/x)^4)*Sqrt[8 + 24*x + 8*x^2 - 15*x^3 + 8*
x^4]) + ((18932921731 - 1086525994*(3 + 4/x)^2)*(3 + 4/x)*x^2)/(7
8056941248*Sqrt[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4]) + ((11921698
- 359497*(3 + 4/x)^2)*(3 + 4/x)*x^2)/(483912*(517 - 38*(3 + 4/x)^
2 + (3 + 4/x)^4)*Sqrt[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4]) + (5432
62997*(517 - 38*(3 + 4/x)^2 + (3 + 4/x)^4)*(3 + 4/x)*x^2)/(390284
70624*(Sqrt[517] + (3 + 4/x)^2)*Sqrt[8 + 24*x + 8*x^2 - 15*x^3 +
8*x^4]) - (543262997*(Sqrt[517] + (3 + 4/x)^2)*Sqrt[(517 - 38*(3
+ 4/x)^2 + (3 + 4/x)^4)/(Sqrt[517] + (3 + 4/x)^2)^2]*x^2*Elliptic
E[2*ArcTan[(4 + 3*x)/(517^(1/4)*x)], (517 + 19*Sqrt[517])/1034])/
(75490272*517^(3/4)*Sqrt[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4]) + ((
4346103976 - 175318963*Sqrt[517])*(Sqrt[517] + (3 + 4/x)^2)*Sqrt[
(517 - 38*(3 + 4/x)^2 + (3 + 4/x)^4)/(Sqrt[517] + (3 + 4/x)^2)^2]
*x^2*EllipticF[2*ArcTan[(4 + 3*x)/(517^(1/4)*x)], (517 + 19*Sqrt[
517])/1034])/(1207844352*517^(3/4)*Sqrt[8 + 24*x + 8*x^2 - 15*x^3
 + 8*x^4])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-1024\int^{\frac{3}{4}+\frac{1}{x}}\frac{1}{\left(\frac{8388608x^4-19922944x^2+16941056}{(-32x+24)^4}\right)^{\frac{5}{2}}(-32x+24)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(8*x**4-15*x**3+8*x**2+24*x+8)**(5/2),x)
```

```
[Out] -1024*Integral(1/(((8388608*x**4 - 19922944*x**2 + 16941056)/(-32
*x + 24)**4)**(5/2)*(-32*x + 24)**2), (x, 3/4 + 1/x))
```

Mathematica [C] time = 6.0784, size = 6084, normalized size = 10.54

Result too large to show

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-5/2),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.033, size = 5477, normalized size = 9.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(8\,x^4-15\,x^3+8\,x^2+24\,x+8)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)^(-5/2),x, algorithm="maxima")
```

```
[Out] integrate((8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)^(-5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(64\,x^8 - 240\,x^7 + 353\,x^6 + 144\,x^5 - 528\,x^4 + 144\,x^3 + 704\,x^2 + 384\,x + 64)\sqrt{8\,x^4 - 15\,x^3 + 8\,x^2 + 24\,x + 8}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)^(-5/2),x, algorithm="fricas")
```

```
[Out] integral(1/((64*x^8 - 240*x^7 + 353*x^6 + 144*x^5 - 528*x^4 + 144
*x^3 + 704*x^2 + 384*x + 64)*sqrt(8*x^4 - 15*x^3 + 8*x^2 + 24*x +
 8)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(8x^4 - 15x^3 + 8x^2 + 24x + 8)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x**4-15*x**3+8*x**2+24*x+8)**(5/2),x)
```

```
[Out] Integral((8*x**4 - 15*x**3 + 8*x**2 + 24*x + 8)**(-5/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(8\,x^4 - 15\,x^3 + 8\,x^2 + 24\,x + 8)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)^(-5/2),x, algorithm="giac")
```

```
[Out] integrate((8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)^(-5/2), x)
```

3.647 $\int \frac{1}{\sqrt{9-6x-44x^2+15x^3+3x^4}}\,dx$

Optimal. Leaf size=130

$$-\frac{\sqrt{\frac{\left(\frac{6}{x}-1\right)^4-182\left(1-\frac{6}{x}\right)^2+613}{\left(\frac{(6-x)^2}{x^2}+\sqrt{613}\right)^2}}\left(\frac{(6-x)^2}{x^2}+\sqrt{613}\right)x^2F\left(2\tan^{-1}\left(\frac{6-x}{\sqrt[4]{613}x}\right)|\frac{613+91\sqrt{613}}{1226}\right)}{12\sqrt[4]{613}\sqrt{3x^4+15x^3-44x^2-6x+9}}$$

```
[Out] -(Sqrt[(613 - 182*(1 - 6/x)^2 + (-1 + 6/x)^4)/(Sqrt[613] + (6 - x
)^2/x^2)^2]*(Sqrt[613] + (6 - x)^2/x^2)*x^2*EllipticF[2*ArcTan[(6
 - x)/(613^(1/4)*x)], (613 + 91*Sqrt[613])/1226])/(12*613^(1/4)*S
qrt[9 - 6*x - 44*x^2 + 15*x^3 + 3*x^4])
```

Rubi [A] time = 0.428497, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$-\frac{\sqrt{\frac{\left(\frac{6}{x}-1\right)^4-182\left(1-\frac{6}{x}\right)^2+613}{\left(\frac{(6-x)^2}{x^2}+\sqrt{613}\right)^2}}\left(\frac{(6-x)^2}{x^2}+\sqrt{613}\right)x^2F\left(2\tan^{-1}\left(\frac{6-x}{\sqrt[4]{613}x}\right)|\frac{613+91\sqrt{613}}{1226}\right)}{12\sqrt[4]{613}\sqrt{3x^4+15x^3-44x^2-6x+9}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[9 - 6*x - 44*x^2 + 15*x^3 + 3*x^4],x]
```

```
[Out] -(Sqrt[(613 - 182*(1 - 6/x)^2 + (-1 + 6/x)^4)/(Sqrt[613] + (6 - x
)^2/x^2)^2]*(Sqrt[613] + (6 - x)^2/x^2)*x^2*EllipticF[2*ArcTan[(6
 - x)/(613^(1/4)*x)], (613 + 91*Sqrt[613])/1226])/(12*613^(1/4)*S
qrt[9 - 6*x - 44*x^2 + 15*x^3 + 3*x^4])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-1296\int^{-\frac{1}{6}+\frac{1}{x}}\frac{1}{\sqrt{\frac{15116544x^4-76422528x^2+7150032}{(-36x-6)^4}}(-36x-6)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(3*x**4+15*x**3-44*x**2-6*x+9)**(1/2),x)
```

```
[Out] -1296*Integral(1/(sqrt((15116544*x**4 - 76422528*x**2 + 7150032)/
(-36*x - 6)**4)*(-36*x - 6)**2), (x, -1/6 + 1/x))
```

Mathematica [C] time = 0.202398, size = 826, normalized size = 6.35

$$-\frac{2F\left(\sin^{-1}\left(\sqrt{\frac{\left(x-\text{Root}\left[3\#1^4+15\#1^3-44\#1^2-6\#1+9\&,1\right]\right)\left(\text{Root}\left[3\#1^4+15\#1^3-44\#1^2-6\#1+9\&,2\right]-\text{Root}\left[3\#1^4+15\#1^3-44\#1^2-6\#1+9\&,4\right]\right)}{\left(x-\text{Root}\left[3\#1^4+15\#1^3-44\#1^2-6\#1+9\&,2\right]\right)\left(\text{Root}\left[3\#1^4+15\#1^3-44\#1^2-6\#1+9\&,1\right]-\text{Root}\left[3\#1^4+15\#1^3-44\#1^2-6\#1+9\&,4\right]\right)}}\right)|\ldots}{\sqrt{(3x^4+15x^3-44\ldots}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[9 - 6*x - 44*x^2 + 15*x^3 + 3*x^4],x]
```

```
[Out] (-2*EllipticF[ArcSin[Sqrt[((x - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3
 + 3*#1^4 & , 1, 0])*(Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4
& , 2, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4, 0])
)/((x - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0])*(Ro
ot[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 1, 0] - Root[9 - 6*#
1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4, 0]))]], ((Root[9 - 6*#1 - 4
4*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0] - Root[9 - 6*#1 - 44*#1^2 + 1
5*#1^3 + 3*#1^4 & , 3, 0])*(Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3
*#1^4 & , 1, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & ,
4, 0]))/((Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 1, 0] -
Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 3, 0])*(Root[9 - 6
*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0] - Root[9 - 6*#1 - 44*#
1^2 + 15*#1^3 + 3*#1^4 & , 4, 0]))]*Sqrt[(x - Root[9 - 6*#1 - 44*
#1^2 + 15*#1^3 + 3*#1^4 & , 1, 0])/(x - Root[9 - 6*#1 - 44*#1^2 +
 15*#1^3 + 3*#1^4 & , 2, 0])]*(x - Root[9 - 6*#1 - 44*#1^2 + 15*#
1^3 + 3*#1^4 & , 2, 0])^2*Sqrt[(x - Root[9 - 6*#1 - 44*#1^2 + 15*
#1^3 + 3*#1^4 & , 3, 0])/(x - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 +
 3*#1^4 & , 2, 0])]*Sqrt[(x - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 +
 3*#1^4 & , 4, 0])/(x - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^
4 & , 2, 0])])/Sqrt[(9 - 6*x - 44*x^2 + 15*x^3 + 3*x^4)*(Root[9 -
 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 1, 0] - Root[9 - 6*#1 - 44
*#1^2 + 15*#1^3 + 3*#1^4 & , 3, 0])*(Root[9 - 6*#1 - 44*#1^2 + 15
*#1^3 + 3*#1^4 & , 2, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*
#1^4 & , 4, 0])]
```

Maple [C] time = 0.911, size = 1182, normalized size = 9.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(3*x^4+15*x^3-44*x^2-6*x+9)^(1/2),x)
```

```
[Out] 2/3*(-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=4)+RootOf(3*_Z^4
+15*_Z^3-44*_Z^2-6*_Z+9,index=1))*((x-RootOf(3*_Z^4+15*_Z^3-44*_Z
^2-6*_Z+9,index=1))/(-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=
4)+RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=1))/(x-RootOf(3*_Z^
4+15*_Z^3-44*_Z^2-6*_Z+9,index=2))*(-RootOf(3*_Z^4+15*_Z^3-44*_Z^
2-6*_Z+9,index=4)+RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=2)))
^(1/2)*(x-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=2))^2*(-(x-R
ootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=3))/(-RootOf(3*_Z^4+15*
_Z^3-44*_Z^2-6*_Z+9,index=3)+RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9
,index=1))/(x-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=2))*(Roo
tOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=2)-RootOf(3*_Z^4+15*_Z^3-
44*_Z^2-6*_Z+9,index=1)))^(1/2)*(-(x-RootOf(3*_Z^4+15*_Z^3-44*_Z^
2-6*_Z+9,index=4))/(-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=4
)+RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=1))/(x-RootOf(3*_Z^4
+15*_Z^3-44*_Z^2-6*_Z+9,index=2))*(RootOf(3*_Z^4+15*_Z^3-44*_Z^2-
6*_Z+9,index=2)-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=1)))^(
1/2)/(RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=4)-RootOf(3*_Z^4
+15*_Z^3-44*_Z^2-6*_Z+9,index=2))/(RootOf(3*_Z^4+15*_Z^3-44*_Z^2-
6*_Z+9,index=2)-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=1))*3^
(1/2)/((x-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=1))*(x-RootO
f(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=2))*(x-RootOf(3*_Z^4+15*_Z^
3-44*_Z^2-6*_Z+9,index=3))*(x-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+
9,index=4)))^(1/2)*EllipticF(((x-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*
_Z+9,index=1))/(-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=4)+Ro
otOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=1))/(x-RootOf(3*_Z^4+15*
_Z^3-44*_Z^2-6*_Z+9,index=2))*(-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_
Z+9,index=4)+RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=2)))^(1/2
),((RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=2)-RootOf(3*_Z^4+1
5*_Z^3-44*_Z^2-6*_Z+9,index=3))*(-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6
*_Z+9,index=4)+RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=1))/(-R
ootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=3)+RootOf(3*_Z^4+15*_Z^
3-44*_Z^2-6*_Z+9,index=1))/(-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9
,index=4)+RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^4+15\,x^3-44\,x^2-6\,x+9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 + 15*x^3 - 44*x^2 - 6*x + 9),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 + 15*x^3 - 44*x^2 - 6*x + 9), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{3\,x^4+15\,x^3-44\,x^2-6\,x+9}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 + 15*x^3 - 44*x^2 - 6*x + 9),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(3*x^4 + 15*x^3 - 44*x^2 - 6*x + 9), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3x^4+15x^3-44x^2-6x+9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3*x**4+15*x**3-44*x**2-6*x+9)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 + 15*x**3 - 44*x**2 - 6*x + 9), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^4+15\,x^3-44\,x^2-6\,x+9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 + 15*x^3 - 44*x^2 - 6*x + 9),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 + 15*x^3 - 44*x^2 - 6*x + 9), x)
```

3.648 $\int \frac{1}{(9-6x-44x^2+15x^3+3x^4)^{3/2}}\,dx$

Optimal. Leaf size=444

$$-\frac{\left(176-23\left(1-\frac{6}{x}\right)^2\right)x^2}{51759\sqrt{3x^4+15x^3-44x^2-6x+9}}+\frac{\left(45401-3722\left(1-\frac{6}{x}\right)^2\right)\left(1-\frac{6}{x}\right)x^2}{31728267\sqrt{3x^4+15x^3-44x^2-6x+9}}$$
$$+\frac{3722\left(\left(\frac{6}{x}-1\right)^4-182\left(1-\frac{6}{x}\right)^2+613\right)\left(1-\frac{6}{x}\right)x^2}{31728267\left(\frac{(6-x)^2}{x^2}+\sqrt{613}\right)\sqrt{3x^4+15x^3-44x^2-6x+9}}$$
$$-\frac{\left(7444-145\sqrt{613}\right)\sqrt{\frac{\left(\frac{6}{x}-1\right)^4-182\left(1-\frac{6}{x}\right)^2+613}{\left(\frac{(6-x)^2}{x^2}+\sqrt{613}\right)^2}}\left(\frac{(6-x)^2}{x^2}+\sqrt{613}\right)x^2F\left(2\tan^{-1}\left(\frac{6-x}{\sqrt[4]{613}x}\right)|\frac{613+91\sqrt{613}}{1226}\right)}{207036\ 613^{3/4}\sqrt{3x^4+15x^3-44x^2-6x+9}}$$
$$+\frac{3722\sqrt{\frac{\left(\frac{6}{x}-1\right)^4-182\left(1-\frac{6}{x}\right)^2+613}{\left(\frac{(6-x)^2}{x^2}+\sqrt{613}\right)^2}}\left(\frac{(6-x)^2}{x^2}+\sqrt{613}\right)x^2E\left(2\tan^{-1}\left(\frac{6-x}{\sqrt[4]{613}x}\right)|\frac{613+91\sqrt{613}}{1226}\right)}{51759\ 613^{3/4}\sqrt{3x^4+15x^3-44x^2-6x+9}}$$

```
[Out] -((176 - 23*(1 - 6/x)^2)*x^2)/(51759*Sqrt[9 - 6*x - 44*x^2 + 15*x
^3 + 3*x^4]) + ((45401 - 3722*(1 - 6/x)^2)*(1 - 6/x)*x^2)/(317282
67*Sqrt[9 - 6*x - 44*x^2 + 15*x^3 + 3*x^4]) + (3722*(613 - 182*(1
 - 6/x)^2 + (-1 + 6/x)^4)*(1 - 6/x)*x^2)/(31728267*(Sqrt[613] + (
6 - x)^2/x^2)*Sqrt[9 - 6*x - 44*x^2 + 15*x^3 + 3*x^4]) + (3722*Sq
rt[(613 - 182*(1 - 6/x)^2 + (-1 + 6/x)^4)/(Sqrt[613] + (6 - x)^2/
x^2)^2]*(Sqrt[613] + (6 - x)^2/x^2)*x^2*EllipticE[2*ArcTan[(6 - x
)/(613^(1/4)*x)], (613 + 91*Sqrt[613])/1226])/(51759*613^(3/4)*Sq
rt[9 - 6*x - 44*x^2 + 15*x^3 + 3*x^4]) - ((7444 - 145*Sqrt[613])*
Sqrt[(613 - 182*(1 - 6/x)^2 + (-1 + 6/x)^4)/(Sqrt[613] + (6 - x)^
2/x^2)^2]*(Sqrt[613] + (6 - x)^2/x^2)*x^2*EllipticF[2*ArcTan[(6 -
 x)/(613^(1/4)*x)], (613 + 91*Sqrt[613])/1226])/(207036*613^(3/4)
*Sqrt[9 - 6*x - 44*x^2 + 15*x^3 + 3*x^4])
```

Rubi [A] time = 0.887108, antiderivative size = 444, normalized size of antiderivative = 1., number

of steps used = 10, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$

$$-\frac{\left(176-23\left(1-\frac{6}{x}\right)^2\right)x^2}{51759\sqrt{3x^4+15x^3-44x^2-6x+9}}+\frac{\left(45401-3722\left(1-\frac{6}{x}\right)^2\right)\left(1-\frac{6}{x}\right)x^2}{31728267\sqrt{3x^4+15x^3-44x^2-6x+9}}$$
$$+\frac{3722\left(\left(\frac{6}{x}-1\right)^4-182\left(1-\frac{6}{x}\right)^2+613\right)\left(1-\frac{6}{x}\right)x^2}{31728267\left(\frac{(6-x)^2}{x^2}+\sqrt{613}\right)\sqrt{3x^4+15x^3-44x^2-6x+9}}$$
$$-\frac{\left(7444-145\sqrt{613}\right)\sqrt{\frac{\left(\frac{6}{x}-1\right)^4-182\left(1-\frac{6}{x}\right)^2+613}{\left(\frac{(6-x)^2}{x^2}+\sqrt{613}\right)^2}}\left(\frac{(6-x)^2}{x^2}+\sqrt{613}\right)x^2F\left(2\tan^{-1}\left(\frac{6-x}{\sqrt[4]{613}x}\right)|\frac{613+91\sqrt{613}}{1226}\right)}{207036\ 613^{3/4}\sqrt{3x^4+15x^3-44x^2-6x+9}}$$
$$+\frac{3722\sqrt{\frac{\left(\frac{6}{x}-1\right)^4-182\left(1-\frac{6}{x}\right)^2+613}{\left(\frac{(6-x)^2}{x^2}+\sqrt{613}\right)^2}}\left(\frac{(6-x)^2}{x^2}+\sqrt{613}\right)x^2E\left(2\tan^{-1}\left(\frac{6-x}{\sqrt[4]{613}x}\right)|\frac{613+91\sqrt{613}}{1226}\right)}{51759\ 613^{3/4}\sqrt{3x^4+15x^3-44x^2-6x+9}}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[(9 - 6*x - 44*x^2 + 15*x^3 + 3*x^4)^(-3/2),x]
```

```
[Out] -((176 - 23*(1 - 6/x)^2)*x^2)/(51759*Sqrt[9 - 6*x - 44*x^2 + 15*x
^3 + 3*x^4]) + ((45401 - 3722*(1 - 6/x)^2)*(1 - 6/x)*x^2)/(317282
67*Sqrt[9 - 6*x - 44*x^2 + 15*x^3 + 3*x^4]) + (3722*(613 - 182*(1
 - 6/x)^2 + (-1 + 6/x)^4)*(1 - 6/x)*x^2)/(31728267*(Sqrt[613] + (
6 - x)^2/x^2)*Sqrt[9 - 6*x - 44*x^2 + 15*x^3 + 3*x^4]) + (3722*Sq
rt[(613 - 182*(1 - 6/x)^2 + (-1 + 6/x)^4)/(Sqrt[613] + (6 - x)^2/
x^2)^2]*(Sqrt[613] + (6 - x)^2/x^2)*x^2*EllipticE[2*ArcTan[(6 - x
)/(613^(1/4)*x)], (613 + 91*Sqrt[613])/1226])/(51759*613^(3/4)*Sq
rt[9 - 6*x - 44*x^2 + 15*x^3 + 3*x^4]) - ((7444 - 145*Sqrt[613])*
Sqrt[(613 - 182*(1 - 6/x)^2 + (-1 + 6/x)^4)/(Sqrt[613] + (6 - x)^
2/x^2)^2]*(Sqrt[613] + (6 - x)^2/x^2)*x^2*EllipticF[2*ArcTan[(6 -
 x)/(613^(1/4)*x)], (613 + 91*Sqrt[613])/1226])/(207036*613^(3/4)
*Sqrt[9 - 6*x - 44*x^2 + 15*x^3 + 3*x^4])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-1296\int^{-\frac{1}{6}+\frac{1}{x}}\frac{1}{\left(\frac{15116544x^4-76422528x^2+7150032}{(-36x-6)^4}\right)^{\frac{3}{2}}(-36x-6)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(3*x**4+15*x**3-44*x**2-6*x+9)**(3/2),x)
```

```
[Out] -1296*Integral(1/(((15116544*x**4 - 76422528*x**2 + 7150032)/(-36
*x - 6)**4)**(3/2)*(-36*x - 6)**2), (x, -1/6 + 1/x))
```

Mathematica [C] time = 6.06871, size = 5428, normalized size = 12.23

Result too large to show

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(9 - 6*x - 44*x^2 + 15*x^3 + 3*x^4)^(-3/2),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.025, size = 5427, normalized size = 12.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(3*x^4+15*x^3-44*x^2-6*x+9)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(3\,x^{4}+15\,x^{3}-44\,x^{2}-6\,x+9\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^4 + 15*x^3 - 44*x^2 - 6*x + 9)^(-3/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^4 + 15*x^3 - 44*x^2 - 6*x + 9)^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\left(3\,x^{4}+15\,x^{3}-44\,x^{2}-6\,x+9\right)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^4 + 15*x^3 - 44*x^2 - 6*x + 9)^(-3/2),x, algorithm="fricas")
```

```
[Out] integral((3*x^4 + 15*x^3 - 44*x^2 - 6*x + 9)^(-3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3x^4 + 15x^3 - 44x^2 - 6x + 9)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3*x**4+15*x**3-44*x**2-6*x+9)**(3/2),x)
```

```
[Out] Integral((3*x**4 + 15*x**3 - 44*x**2 - 6*x + 9)**(-3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3\,x^4 + 15\,x^3 - 44\,x^2 - 6\,x + 9)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^4 + 15*x^3 - 44*x^2 - 6*x + 9)^(-3/2),x, algorithm="giac")
```

```
[Out] integrate((3*x^4 + 15*x^3 - 44*x^2 - 6*x + 9)^(-3/2), x)
```

3.649 $\int \frac{\left(2\sqrt{3-x}+\frac{3}{\sqrt{1+x}}\right)^2}{x}\,dx$

Optimal. Leaf size=56

$$-4x+21\log(x)-9\log(x+1)+12\sin^{-1}\left(\frac{1-x}{2}\right)-24\sqrt{3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{x+1}}{\sqrt{3-x}}\right)$$

```
[Out] -4*x + 12*ArcSin[(1 - x)/2] - 24*Sqrt[3]*ArcTanh[(Sqrt[3]*Sqrt[1
+ x])/Sqrt[3 - x]] + 21*Log[x] - 9*Log[1 + x]
```

Rubi [A] time = 0.408798, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.37

$$-4x+21\log(x)-9\log(x+1)+12\sin^{-1}\left(\frac{1-x}{2}\right)-24\sqrt{3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{x+1}}{\sqrt{3-x}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(2*Sqrt[3 - x] + 3/Sqrt[1 + x])^2/x,x]
```

```
[Out] -4*x + 12*ArcSin[(1 - x)/2] - 24*Sqrt[3]*ArcTanh[(Sqrt[3]*Sqrt[1
+ x])/Sqrt[3 - x]] + 21*Log[x] - 9*Log[1 + x]
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*(3-x)**(1/2)+3/(1+x)**(1/2))**2/x,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.104082, size = 69, normalized size = 1.23

$$-12\sqrt{3}\log\left(\sqrt{-3x^2+6x+9}+x+3\right)-4x+3\left(7+4\sqrt{3}\right)\log(x)-9\log(x+1)+12\tan^{-1}\left(\frac{1-x}{\sqrt{-(x-3)(x+1)}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2*Sqrt[3 - x] + 3/Sqrt[1 + x])^2/x,x]
```

```
[Out] -4*x + 12*ArcTan[(1 - x)/Sqrt[-((-3 + x)*(1 + x))]] + 3*(7 + 4*Sq
rt[3])*Log[x] - 9*Log[1 + x] - 12*Sqrt[3]*Log[3 + x + Sqrt[9 + 6*
x - 3*x^2]]
```

Maple [A] time = 0.025, size = 76, normalized size = 1.4

$$-9\ln(1+x)+21\ln(x)+12\frac{\sqrt{3-x}\sqrt{1+x}}{\sqrt{-x^2+2x+3}}\left(-\arcsin(-1/2+x/2)-\sqrt{3}Artanh\left(1/3\frac{(3+x)\sqrt{3}}{\sqrt{-x^2+2x+3}}\right)\right)-4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*(3-x)^(1/2)+3/(1+x)^(1/2))^2/x,x)
```

```
[Out] -9*ln(1+x)+21*ln(x)+12*(3-x)^(1/2)*(1+x)^(1/2)/(-x^2+2*x+3)^(1/2)
*(-arcsin(-1/2+1/2*x)-3^(1/2)*arctanh(1/3*(3+x)*3^(1/2)/(-x^2+2*x
+3)^(1/2)))-4*x
```

Maple [A] time = 0.843976, size = 77, normalized size = 1.38

$$-12\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{-x^2+2x+3}}{|x|}+\frac{6}{|x|}+2\right)-4x+12\arcsin\left(-\frac{1}{2}x+\frac{1}{2}\right)-9\log(x+1)+21\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*sqrt(-x + 3) + 3/sqrt(x + 1))^2/x,x, algorithm="maxima")
```

```
[Out] -12*sqrt(3)*log(2*sqrt(3)*sqrt(-x^2 + 2*x + 3)/abs(x) + 6/abs(x)
+ 2) - 4*x + 12*arcsin(-1/2*x + 1/2) - 9*log(x + 1) + 21*log(x)
```

Fricas [A] time = 0.283814, size = 96, normalized size = 1.71

$$6\sqrt{3}\log\left(-\frac{\sqrt{3}(x+3)\sqrt{x+1}\sqrt{-x+3}+x^2-6x-9}{x^2}\right)-4x-12\arctan\left(\frac{x-1}{\sqrt{x+1}\sqrt{-x+3}}\right)-9\log(x+1)+21\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*sqrt(-x + 3) + 3/sqrt(x + 1))^2/x,x, algorithm="fricas")
```

```
[Out] 6*sqrt(3)*log(-(sqrt(3)*(x + 3)*sqrt(x + 1)*sqrt(-x + 3) + x^2 -
6*x - 9)/x^2) - 4*x - 12*arctan((x - 1)/(sqrt(x + 1)*sqrt(-x + 3)
)) - 9*log(x + 1) + 21*log(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(2\sqrt{-x+3}\sqrt{x+1}+3\right)^2}{x\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*(3-x)**(1/2)+3/(1+x)**(1/2))**2/x,x)
```

```
[Out] Integral((2*sqrt(-x + 3)*sqrt(x + 1) + 3)**2/(x*(x + 1)), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*sqrt(-x + 3) + 3/sqrt(x + 1))^2/x,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.650 $\int \frac{-1+x+x^2}{1+\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=65

$$\frac{1}{2}\sqrt{x^2+1}x+\sqrt{x^2+1}+\frac{\sqrt{x^2+1}}{x}-\log\left(\sqrt{x^2+1}+1\right)-x-\frac{1}{x}-\frac{1}{2}\sinh^{-1}(x)$$

```
[Out] -x^(-1) - x + Sqrt[1 + x^2] + Sqrt[1 + x^2]/x + (x*Sqrt[1 + x^2])
/2 - ArcSinh[x]/2 - Log[1 + Sqrt[1 + x^2]]
```

Rubi [A] time = 0.255223, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35

$$\frac{1}{2}\sqrt{x^2+1}x+\sqrt{x^2+1}+\frac{\sqrt{x^2+1}}{x}-\log\left(\sqrt{x^2+1}+1\right)-x-\frac{1}{x}-\frac{1}{2}\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + x + x^2)/(1 + Sqrt[1 + x^2]),x]
```

```
[Out] -x^(-1) - x + Sqrt[1 + x^2] + Sqrt[1 + x^2]/x + (x*Sqrt[1 + x^2])
/2 - ArcSinh[x]/2 - Log[1 + Sqrt[1 + x^2]]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2+x-1}{\sqrt{x^2+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+x-1)/(1+(x**2+1)**(1/2)),x)
```

```
[Out] Integral((x**2 + x - 1)/(sqrt(x**2 + 1) + 1), x)
```

Mathematica [A] time = 0.0436108, size = 49, normalized size = 0.75

$$\sqrt{x^2+1}\left(\frac{x}{2}+\frac{1}{x}+1\right)-\log\left(\sqrt{x^2+1}+1\right)-x-\frac{1}{x}-\frac{1}{2}\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + x + x^2)/(1 + Sqrt[1 + x^2]),x]
```

```
[Out] -x^(-1) - x + (1 + x^(-1) + x/2)*Sqrt[1 + x^2] - ArcSinh[x]/2 - L
og[1 + Sqrt[1 + x^2]]
```

Maple [A] time = 0.008, size = 56, normalized size = 0.9

$$-x-x^{-1}-\frac{x}{2}\sqrt{x^2+1}-\frac{Arcsinh(x)}{2}+\sqrt{x^2+1}-Artanh\left(\frac{1}{\sqrt{x^2+1}}\right)-\ln(x)+\frac{1}{x}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+x-1)/(1+(x^2+1)^(1/2)),x)
```

```
[Out] -x-1/x-1/2*x*(x^2+1)^(1/2)-1/2*arcsinh(x)+(x^2+1)^(1/2)-arctanh(1
/(x^2+1)^(1/2))-ln(x)+1/x*(x^2+1)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$2x-5\arctan\left(\frac{1}{2}x\right)+\int\frac{x^6+x^5-x^4}{3x^4+16x^2+(x^4+8x^2+16)\sqrt{x^2+1}+16}\,dx+\log\left(x^2+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + x - 1)/(sqrt(x^2 + 1) + 1),x, algorithm="maxima")
```

```
[Out] 2*x - 5*arctan(1/2*x) + integrate((x^6 + x^5 - x^4)/(3*x^4 + 16*x
^2 + (x^4 + 8*x^2 + 16)*sqrt(x^2 + 1) + 16), x) + log(x^2 + 4)
```

Fricas [A] time = 0.281266, size = 332, normalized size = 5.11

$$-\frac{4x^6+16x^5+5x^4+24x^3+5x^2+2\left(4x^4+3x^2\right)\log(x)+2\left(4x^4+3x^2-\left(4x^3+x\right)\sqrt{x^2+1}\right)\log\left(-x+\sqrt{x^2+1}+1\right)-\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + x - 1)/(sqrt(x^2 + 1) + 1),x, algorithm="fricas")
```

```
[Out] -1/2*(4*x^6 + 16*x^5 + 5*x^4 + 24*x^3 + 5*x^2 + 2*(4*x^4 + 3*x^2)
*log(x) + 2*(4*x^4 + 3*x^2 - (4*x^3 + x)*sqrt(x^2 + 1))*log(-x +
sqrt(x^2 + 1) + 1) - (4*x^4 + 3*x^2 - (4*x^3 + x)*sqrt(x^2 + 1))*
log(-x + sqrt(x^2 + 1)) - 2*(4*x^4 + 3*x^2 - (4*x^3 + x)*sqrt(x^2
 + 1))*log(-x + sqrt(x^2 + 1) - 1) - (4*x^5 + 16*x^4 + 3*x^3 + 16
*x^2 + 2*(4*x^3 + x)*log(x) + 4*x + 2)*sqrt(x^2 + 1) + 8*x + 2)/(
4*x^4 + 3*x^2 - (4*x^3 + x)*sqrt(x^2 + 1))
```

Sympy [A] time = 11.8625, size = 76, normalized size = 1.17

$$\frac{x\sqrt{x^2+1}}{2} - x + \frac{x}{\sqrt{x^2+1}} + \sqrt{x^2+1} - \log\left(1+\frac{1}{\sqrt{x^2+1}}\right) + \log\left(\frac{1}{\sqrt{x^2+1}}\right) - \frac{\operatorname{asinh}(x)}{2} - \frac{1}{x} + \frac{1}{x\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+x-1)/(1+(x**2+1)**(1/2)),x)
```

```
[Out] x*sqrt(x**2 + 1)/2 - x + x/sqrt(x**2 + 1) + sqrt(x**2 + 1) - log(
1 + 1/sqrt(x**2 + 1)) + log(1/sqrt(x**2 + 1)) - asinh(x)/2 - 1/x
+ 1/(x*sqrt(x**2 + 1))
```

GIAC/XCAS [A] time = 0.273024, size = 120, normalized size = 1.85

$$\frac{1}{2}\sqrt{x^2+1}(x+2) - x - \frac{2}{\left(x-\sqrt{x^2+1}\right)^2-1} - \frac{1}{x} + \frac{1}{2}\ln\left(-x+\sqrt{x^2+1}\right) - \ln\left(|x|\right) - \ln\left(\left|-x+\sqrt{x^2+1}+1\right|\right) + \ln\left(\left|-x+\sqrt{x^2+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + x - 1)/(sqrt(x^2 + 1) + 1),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(x^2 + 1)*(x + 2) - x - 2/((x - sqrt(x^2 + 1))^2 - 1) - 1
/x + 1/2*ln(-x + sqrt(x^2 + 1)) - ln(abs(x)) - ln(abs(-x + sqrt(x
^2 + 1) + 1)) + ln(abs(-x + sqrt(x^2 + 1) - 1))
```

3.651 $\int \frac{-1+x+x^2}{1+x+\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=53

$$\frac{1}{12}\left(2x^3+6x^2+\left(-2x^2-3x+4\right)\sqrt{x^2+1}-6\log\left(\sqrt{x^2+1}+1\right)-3\sinh^{-1}(x)\right)$$

```
[Out] (6*x^2 + 2*x^3 + (4 - 3*x - 2*x^2)*Sqrt[1 + x^2] - 3*ArcSinh[x] -
 6*Log[1 + Sqrt[1 + x^2]])/12
```

Rubi [A] time = 0.365495, antiderivative size = 101, normalized size of antiderivative = 1.91, number of steps used = 12, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286

$$\frac{x^3}{6}+\frac{x^2}{2}-\frac{1}{4}\sqrt{x^2+1}x-\frac{1}{6}\left(x^2+1\right)^{3/2}+\frac{1}{2\left(\sqrt{x^2+1}+x\right)}+\frac{1}{2}\log\left(\sqrt{x^2+1}+x\right)-\log\left(\sqrt{x^2+1}+x+1\right)+\frac{x}{2}-\frac{1}{4}\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + x + x^2)/(1 + x + Sqrt[1 + x^2]),x]
```

```
[Out] x/2 + x^2/2 + x^3/6 - (x*Sqrt[1 + x^2])/4 - (1 + x^2)^(3/2)/6 + 1
/(2*(x + Sqrt[1 + x^2])) - ArcSinh[x]/4 + Log[x + Sqrt[1 + x^2]]/
2 - Log[1 + x + Sqrt[1 + x^2]]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2+x-1}{x+\sqrt{x^2+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+x-1)/(1+x+(x**2+1)**(1/2)),x)
```

```
[Out] Integral((x**2 + x - 1)/(x + sqrt(x**2 + 1) + 1), x)
```

Mathematica [A] time = 0.0489116, size = 53, normalized size = 1.

$$\frac{1}{12}\left(2x^3+6x^2+\left(-2x^2-3x+4\right)\sqrt{x^2+1}-6\log\left(\sqrt{x^2+1}+1\right)-3\sinh^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + x + x^2)/(1 + x + Sqrt[1 + x^2]),x]
```

```
[Out] (6*x^2 + 2*x^3 + (4 - 3*x - 2*x^2)*Sqrt[1 + x^2] - 3*ArcSinh[x] -
 6*Log[1 + Sqrt[1 + x^2]])/12
```

Maple [A] time = 0.007, size = 58, normalized size = 1.1

$$\frac{x^2}{2} - \frac{\ln(x)}{2} + \frac{x^3}{6} - \frac{x}{4}\sqrt{x^2+1} - \frac{\mathit{Arcsinh}(x)}{4} + \frac{1}{2}\sqrt{x^2+1} - \frac{1}{2}\mathit{Artanh}\left(\frac{1}{\sqrt{x^2+1}}\right) - \frac{1}{6}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+x-1)/(1+x+(x^2+1)^(1/2)),x)
```

```
[Out] 1/2*x^2-1/2*ln(x)+1/6*x^3-1/4*x*(x^2+1)^(1/2)-1/4*arcsinh(x)+1/2*
(x^2+1)^(1/2)-1/2*arctanh(1/(x^2+1)^(1/2))-1/6*(x^2+1)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{4}x^2 - \frac{3}{56}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(4x+3)\right) + \frac{1}{4}x + \int \frac{x^4+x^3-x^2}{4x^5+12x^4+19x^3+19x^2+(4x^4+12x^3+17x^2+12x+4)\sqrt{x^2+1}+12x+4}\,dx - \frac{7}{16}\log\left(2x^2+3x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 + x - 1)/(x + sqrt(x^2 + 1) + 1),x, algorithm="maxima")
```

```
[Out] 1/4*x^2 - 3/56*sqrt(7)*arctan(1/7*sqrt(7)*(4*x + 3)) + 1/4*x + in
tegrate((x^4 + x^3 - x^2)/(4*x^5 + 12*x^4 + 19*x^3 + 19*x^2 + (4*
x^4 + 12*x^3 + 17*x^2 + 12*x + 4)*sqrt(x^2 + 1) + 12*x + 4), x) -
 7/16*log(2*x^2 + 3*x + 2)
```

Fricas [A] time = 0.280396, size = 311, normalized size = 5.87

$$\frac{16\,x^6+36\,x^5+33\,x^3-18\,x^2-6\left(4\,x^3+3\,x\right)\log\left(x\right)-6\left(4\,x^3-\left(4\,x^2+1\right)\sqrt{x^2+1}+3\,x\right)\log\left(-x+\sqrt{x^2+1}+1\right)+3\left(4\,x^3-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + x - 1)/(x + sqrt(x^2 + 1) + 1),x, algorithm="fricas")
```

```
[Out] 1/12*(16*x^6 + 36*x^5 + 33*x^3 - 18*x^2 - 6*(4*x^3 + 3*x)*log(x)
- 6*(4*x^3 - (4*x^2 + 1)*sqrt(x^2 + 1) + 3*x)*log(-x + sqrt(x^2 +
 1) + 1) + 3*(4*x^3 - (4*x^2 + 1)*sqrt(x^2 + 1) + 3*x)*log(-x + s
qrt(x^2 + 1)) + 6*(4*x^3 - (4*x^2 + 1)*sqrt(x^2 + 1) + 3*x)*log(-
x + sqrt(x^2 + 1) - 1) - (16*x^5 + 36*x^4 - 8*x^3 + 15*x^2 - 6*(4
*x^2 + 1)*log(x) - 12*x)*sqrt(x^2 + 1) + 3*x - 4)/(4*x^3 - (4*x^2
 + 1)*sqrt(x^2 + 1) + 3*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2+x-1}{x+\sqrt{x^2+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+x-1)/(1+x+(x**2+1)**(1/2)),x)
```

```
[Out] Integral((x**2 + x - 1)/(x + sqrt(x**2 + 1) + 1), x)
```

GIAC/XCAS [A] time = 0.270023, size = 108, normalized size = 2.04

$$\frac{1}{6}\,x^3+\frac{1}{2}\,x^2-\frac{1}{12}\,((2\,x+3)x-4)\sqrt{x^2+1}+\frac{1}{4}\ln\left(-x+\sqrt{x^2+1}\right) - \frac{1}{2}\ln\left(|x|\right)-\frac{1}{2}\ln\left(\left|-x+\sqrt{x^2+1}+1\right|\right)+\frac{1}{2}\ln\left(\left|-x+\sqrt{x^2+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + x - 1)/(x + sqrt(x^2 + 1) + 1),x, algorithm="giac")
```

```
[Out] 1/6*x^3 + 1/2*x^2 - 1/12*((2*x + 3)*x - 4)*sqrt(x^2 + 1) + 1/4*ln
(-x + sqrt(x^2 + 1)) - 1/2*ln(abs(x)) - 1/2*ln(abs(-x + sqrt(x^2
+ 1) + 1)) + 1/2*ln(abs(-x + sqrt(x^2 + 1) - 1))
```

3.652 $\int \frac{2\sqrt{-1+x}+x}{\sqrt{-1+x}x}\,dx$

Optimal. Leaf size=14

$$2\sqrt{x-1}+2\log(x)$$

```
[Out] 2*Sqrt[-1 + x] + 2*Log[x]
```

Rubi [A] time = 0.193232, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045

$$2\sqrt{x-1}+2\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(2*Sqrt[-1 + x] + x)/(Sqrt[-1 + x]*x),x]

[Out] 2*Sqrt[-1 + x] + 2*Log[x]
```

Rubi in Sympy [A] time = 11.3264, size = 12, normalized size = 0.86

$$2\sqrt{x-1}+2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x+2*(-1+x)**(1/2))/x/(-1+x)**(1/2),x)

[Out] 2*sqrt(x - 1) + 2*log(x)
```

Mathematica [A] time = 0.00777559, size = 14, normalized size = 1.

$$2\sqrt{x-1}+2\log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2*Sqrt[-1 + x] + x)/(Sqrt[-1 + x]*x),x]
```

```
[Out] 2*Sqrt[-1 + x] + 2*Log[x]
```

Maple [A] time = 0.002, size = 13, normalized size = 0.9

$$2\ln(x) + 2\sqrt{-1+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x+2*(-1+x)^(1/2))/x/(-1+x)^(1/2),x)
```

```
[Out] 2*ln(x)+2*(-1+x)^(1/2)
```

Maxima [A] time = 0.801496, size = 16, normalized size = 1.14

$$2\sqrt{x-1} + 2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 2*sqrt(x - 1))/(sqrt(x - 1)*x),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x - 1) + 2*log(x)
```

Fricas [A] time = 0.26912, size = 16, normalized size = 1.14

$$2\sqrt{x-1} + 2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 2*sqrt(x - 1))/(sqrt(x - 1)*x),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x - 1) + 2*log(x)
```

Sympy [A] time = 0.347121, size = 12, normalized size = 0.86

$$2\sqrt{x-1} + 2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x+2*(-1+x)**(1/2))/x/(-1+x)**(1/2),x)

[Out] 2*sqrt(x - 1) + 2*log(x)
```

GIAC/XCAS [A] time = 0.264567, size = 16, normalized size = 1.14

$$2\sqrt{x-1}+2\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 2*sqrt(x - 1))/(sqrt(x - 1)*x),x, algorithm="giac")

[Out] 2*sqrt(x - 1) + 2*ln(x)
```

3.653 $\int \left(a+c\sqrt{x}+bx^{2/3}\right)^2\,dx$

Optimal. Leaf size=61

$$a^2x+\frac{6}{5}abx^{5/3}+\frac{4}{3}acx^{3/2}+\frac{3}{7}b^2x^{7/3}+\frac{12}{13}bcx^{13/6}+\frac{c^2x^2}{2}$$

```
[Out] a^2*x + (4*a*c*x^(3/2))/3 + (6*a*b*x^(5/3))/5 + (c^2*x^2)/2 + (12
*b*c*x^(13/6))/13 + (3*b^2*x^(7/3))/7
```

Rubi [A] time = 0.279751, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$a^2x+\frac{6}{5}abx^{5/3}+\frac{4}{3}acx^{3/2}+\frac{3}{7}b^2x^{7/3}+\frac{12}{13}bcx^{13/6}+\frac{c^2x^2}{2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + c*Sqrt[x] + b*x^(2/3))^2,x]
```

```
[Out] a^2*x + (4*a*c*x^(3/2))/3 + (6*a*b*x^(5/3))/5 + (c^2*x^2)/2 + (12
*b*c*x^(13/6))/13 + (3*b^2*x^(7/3))/7
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{6abx^{\frac{5}{3}}}{5}+\frac{4acx^{\frac{3}{2}}}{3}+\frac{3b^2x^{\frac{7}{3}}}{7}+\frac{12bcx^{\frac{13}{6}}}{13}+c^2\int x\,dx+\int a^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b*x**(2/3)+c*x**(1/2))**2,x)
```

```
[Out] 6*a*b*x**(5/3)/5 + 4*a*c*x**(3/2)/3 + 3*b**2*x**(7/3)/7 + 12*b*c*
x**(13/6)/13 + c**2*Integral(x, x) + Integral(a**2, x)
```

Mathematica [A] time = 0.0289994, size = 61, normalized size = 1.

$$a^2x+\frac{6}{5}abx^{5/3}+\frac{4}{3}acx^{3/2}+\frac{3}{7}b^2x^{7/3}+\frac{12}{13}bcx^{13/6}+\frac{c^2x^2}{2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + c*Sqrt[x] + b*x^(2/3))^2,x]
```

```
[Out] a^2*x + (4*a*c*x^(3/2))/3 + (6*a*b*x^(5/3))/5 + (c^2*x^2)/2 + (12
*b*c*x^(13/6))/13 + (3*b^2*x^(7/3))/7
```

Maple [A] time = 0.003, size = 46, normalized size = 0.8

$$\frac{c^2x^2}{2} + 2\,c\left(\frac{6\,b}{13}x^{\frac{13}{6}} + 2/3\,ax^{3/2}\right) + a^2x + \frac{3\,b^2}{7}x^{\frac{7}{3}} + \frac{6\,ab}{5}x^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b*x^(2/3)+c*x^(1/2))^2,x)
```

```
[Out] 1/2*c^2*x^2+2*c*(6/13*b*x^(13/6)+2/3*a*x^(3/2))+a^2*x+3/7*b^2*x^(
7/3)+6/5*a*b*x^(5/3)
```

Maxima [A] time = 0.723466, size = 61, normalized size = 1.

$$\frac{3}{7}\,b^2x^{\frac{7}{3}} + \frac{12}{13}\,bcx^{\frac{13}{6}} + \frac{1}{2}\,c^2x^2 + a^2x + \frac{2}{15}\left(9\,bx^{\frac{5}{3}} + 10\,cx^{\frac{3}{2}}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^(2/3) + c*sqrt(x) + a)^2,x, algorithm="maxima")
```

```
[Out] 3/7*b^2*x^(7/3) + 12/13*b*c*x^(13/6) + 1/2*c^2*x^2 + a^2*x + 2/15
*(9*b*x^(5/3) + 10*c*x^(3/2))*a
```

Fricas [A] time = 0.261377, size = 58, normalized size = 0.95

$$\frac{3}{7}\,b^2x^{\frac{7}{3}} + \frac{12}{13}\,bcx^{\frac{13}{6}} + \frac{1}{2}\,c^2x^2 + \frac{6}{5}\,abx^{\frac{5}{3}} + \frac{4}{3}\,acx^{\frac{3}{2}} + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^(2/3) + c*sqrt(x) + a)^2,x, algorithm="fricas")
```

```
[Out] 3/7*b^2*x^(7/3) + 12/13*b*c*x^(13/6) + 1/2*c^2*x^2 + 6/5*a*b*x^(5
/3) + 4/3*a*c*x^(3/2) + a^2*x
```

Sympy [A] time = 1.22844, size = 60, normalized size = 0.98

$$a^2x + \frac{6abx^{\frac{5}{3}}}{5} + \frac{4acx^{\frac{3}{2}}}{3} + \frac{3b^2x^{\frac{7}{3}}}{7} + \frac{12bcx^{\frac{13}{6}}}{13} + \frac{c^2x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b*x**(2/3)+c*x**(1/2))**2,x)
```

```
[Out] a**2*x + 6*a*b*x**(5/3)/5 + 4*a*c*x**(3/2)/3 + 3*b**2*x**(7/3)/7
+ 12*b*c*x**(13/6)/13 + c**2*x**2/2
```

GIAC/XCAS [A] time = 0.268326, size = 58, normalized size = 0.95

$$\frac{3}{7}\,b^2x^{\frac{7}{3}} + \frac{12}{13}\,bcx^{\frac{13}{6}} + \frac{1}{2}\,c^2x^2 + \frac{6}{5}\,abx^{\frac{5}{3}} + \frac{4}{3}\,acx^{\frac{3}{2}} + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^(2/3) + c*sqrt(x) + a)^2,x, algorithm="giac")
```

```
[Out] 3/7*b^2*x^(7/3) + 12/13*b*c*x^(13/6) + 1/2*c^2*x^2 + 6/5*a*b*x^(5
/3) + 4/3*a*c*x^(3/2) + a^2*x
```

3.654 $\int \left(a+c\sqrt{x}+bx^{2/3}\right)^3\,dx$

Optimal. Leaf size=114

$$a^3x+\frac{9}{5}a^2bx^{5/3}+2a^2cx^{3/2}+\frac{9}{7}ab^2x^{7/3}+\frac{36}{13}abcx^{13/6}+\frac{3}{2}ac^2x^2+\frac{b^3x^3}{3}+\frac{18}{17}b^2cx^{17/6}+\frac{9}{8}bc^2x^{8/3}+\frac{2}{5}c^3x^{5/2}$$

```
[Out] a^3*x + 2*a^2*c*x^(3/2) + (9*a^2*b*x^(5/3))/5 + (3*a*c^2*x^2)/2 +
 (36*a*b*c*x^(13/6))/13 + (9*a*b^2*x^(7/3))/7 + (2*c^3*x^(5/2))/5
 + (9*b*c^2*x^(8/3))/8 + (18*b^2*c*x^(17/6))/17 + (b^3*x^3)/3
```

Rubi [A] time = 0.339489, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$a^3x+\frac{9}{5}a^2bx^{5/3}+2a^2cx^{3/2}+\frac{9}{7}ab^2x^{7/3}+\frac{36}{13}abcx^{13/6}+\frac{3}{2}ac^2x^2+\frac{b^3x^3}{3}+\frac{18}{17}b^2cx^{17/6}+\frac{9}{8}bc^2x^{8/3}+\frac{2}{5}c^3x^{5/2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + c*Sqrt[x] + b*x^(2/3))^3,x]
```

```
[Out] a^3*x + 2*a^2*c*x^(3/2) + (9*a^2*b*x^(5/3))/5 + (3*a*c^2*x^2)/2 +
 (36*a*b*c*x^(13/6))/13 + (9*a*b^2*x^(7/3))/7 + (2*c^3*x^(5/2))/5
 + (9*b*c^2*x^(8/3))/8 + (18*b^2*c*x^(17/6))/17 + (b^3*x^3)/3
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{9a^2bx^{\frac{5}{3}}}{5}+2a^2cx^{\frac{3}{2}}+\frac{9ab^2x^{\frac{7}{3}}}{7}+\frac{36abcx^{\frac{13}{6}}}{13}+3ac^2\int x\,dx+\frac{b^3x^3}{3}+\frac{18b^2cx^{\frac{17}{6}}}{17}+\frac{9bc^2x^{\frac{8}{3}}}{8}+\frac{2c^3x^{\frac{5}{2}}}{5}+\int a^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b*x**(2/3)+c*x**(1/2))**3,x)
```

```
[Out] 9*a**2*b*x**(5/3)/5 + 2*a**2*c*x**(3/2) + 9*a*b**2*x**(7/3)/7 + 3
6*a*b*c*x**(13/6)/13 + 3*a*c**2*Integral(x, x) + b**3*x**3/3 + 18
*b**2*c*x**(17/6)/17 + 9*b*c**2*x**(8/3)/8 + 2*c**3*x**(5/2)/5 +
Integral(a**3, x)
```

Mathematica [A] time = 0.056646, size = 114, normalized size = 1.

$$a^3x+\frac{9}{5}a^2bx^{5/3}+2a^2cx^{3/2}+\frac{9}{7}ab^2x^{7/3}+\frac{36}{13}abcx^{13/6}+\frac{3}{2}ac^2x^2+\frac{b^3x^3}{3}+\frac{18}{17}b^2cx^{17/6}+\frac{9}{8}bc^2x^{8/3}+\frac{2}{5}c^3x^{5/2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + c*Sqrt[x] + b*x^(2/3))^3,x]
```

```
[Out] a^3*x + 2*a^2*c*x^(3/2) + (9*a^2*b*x^(5/3))/5 + (3*a*c^2*x^2)/2 +
 (36*a*b*c*x^(13/6))/13 + (9*a*b^2*x^(7/3))/7 + (2*c^3*x^(5/2))/5
 + (9*b*c^2*x^(8/3))/8 + (18*b^2*c*x^(17/6))/17 + (b^3*x^3)/3
```

Maple [A] time = 0.004, size = 86, normalized size = 0.8

$$\frac{2c^3}{5}x^{\frac{5}{2}}+3c^2\left(3/8\,x^{8/3}b+1/2\,ax^2\right)+3c\left(\frac{6b^2}{17}x^{\frac{17}{6}}+\frac{12ab}{13}x^{\frac{13}{6}}+2/3\,a^2x^{3/2}\right)+a^3x+\frac{b^3x^3}{3}+\frac{9a^2b}{5}x^{\frac{5}{3}}+\frac{9ab^2}{7}x^{\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b*x^(2/3)+c*x^(1/2))^3,x)
```

```
[Out] 2/5*c^3*x^(5/2)+3*c^2*(3/8*x^(8/3)*b+1/2*a*x^2)+3*c*(6/17*b^2*x^(
17/6)+12/13*a*b*x^(13/6)+2/3*a^2*x^(3/2))+a^3*x+1/3*b^3*x^3+9/5*a
^2*b*x^(5/3)+9/7*a*b^2*x^(7/3)
```

Maxima [A] time = 0.722573, size = 115, normalized size = 1.01

$$\frac{1}{3}b^3x^3+\frac{18}{17}b^2cx^{\frac{17}{6}}+\frac{9}{8}bc^2x^{\frac{8}{3}}+\frac{2}{5}c^3x^{\frac{5}{2}}+a^3x+\frac{1}{5}\left(9bx^{\frac{5}{3}}+10cx^{\frac{3}{2}}\right)a^2+\frac{3}{182}\left(78b^2x^{\frac{7}{3}}+168bcx^{\frac{13}{6}}+91c^2x^2\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^(2/3) + c*sqrt(x) + a)^3,x, algorithm="maxima")
```

```
[Out] 1/3*b^3*x^3 + 18/17*b^2*c*x^(17/6) + 9/8*b*c^2*x^(8/3) + 2/5*c^3*
x^(5/2) + a^3*x + 1/5*(9*b*x^(5/3) + 10*c*x^(3/2))*a^2 + 3/182*(7
8*b^2*x^(7/3) + 168*b*c*x^(13/6) + 91*c^2*x^2)*a
```

Fricas [A] time = 0.265884, size = 123, normalized size = 1.08

$$\frac{1}{3}\,b^{3}x^{3}+\frac{18}{17}\,b^{2}cx^{\frac{17}{6}}+\frac{9}{7}\,ab^{2}x^{\frac{7}{3}}+\frac{36}{13}\,abcx^{\frac{13}{6}}+\frac{3}{2}\,ac^{2}x^{2}+a^{3}x+\frac{9}{40}\,\left(5\,bc^{2}x^{2}+8\,a^{2}bx\right)x^{\frac{2}{3}}+\frac{2}{5}\,\left(c^{3}x^{2}+5\,a^{2}cx\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^(2/3) + c*sqrt(x) + a)^3,x, algorithm="fricas")
```

```
[Out] 1/3*b^3*x^3 + 18/17*b^2*c*x^(17/6) + 9/7*a*b^2*x^(7/3) + 36/13*a*
b*c*x^(13/6) + 3/2*a*c^2*x^2 + a^3*x + 9/40*(5*b*c^2*x^2 + 8*a^2*
b*x)*x^(2/3) + 2/5*(c^3*x^2 + 5*a^2*c*x)*sqrt(x)
```

Sympy [A] time = 1.28097, size = 116, normalized size = 1.02

$$a^{3}x+\frac{9a^{2}bx^{\frac{5}{3}}}{5}+2a^{2}cx^{\frac{3}{2}}+\frac{9ab^{2}x^{\frac{7}{3}}}{7}+\frac{36abcx^{\frac{13}{6}}}{13}+\frac{3ac^{2}x^{2}}{2}+\frac{b^{3}x^{3}}{3}+\frac{18b^{2}cx^{\frac{17}{6}}}{17}+\frac{9bc^{2}x^{\frac{8}{3}}}{8}+\frac{2c^{3}x^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b*x**(2/3)+c*x**(1/2))**3,x)
```

```
[Out] a**3*x + 9*a**2*b*x**(5/3)/5 + 2*a**2*c*x**(3/2) + 9*a*b**2*x**(7
/3)/7 + 36*a*b*c*x**(13/6)/13 + 3*a*c**2*x**2/2 + b**3*x**3/3 + 1
8*b**2*c*x**(17/6)/17 + 9*b*c**2*x**(8/3)/8 + 2*c**3*x**(5/2)/5
```

GIAC/XCAS [A] time = 0.26422, size = 113, normalized size = 0.99

$$\frac{1}{3}\,b^{3}x^{3}+\frac{18}{17}\,b^{2}cx^{\frac{17}{6}}+\frac{9}{8}\,bc^{2}x^{\frac{8}{3}}+\frac{2}{5}\,c^{3}x^{\frac{5}{2}}+\frac{9}{7}\,ab^{2}x^{\frac{7}{3}}+\frac{36}{13}\,abcx^{\frac{13}{6}}+\frac{3}{2}\,ac^{2}x^{2}+\frac{9}{5}\,a^{2}bx^{\frac{5}{3}}+2\,a^{2}cx^{\frac{3}{2}}+a^{3}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^(2/3) + c*sqrt(x) + a)^3,x, algorithm="giac")
```

```
[Out] 1/3*b^3*x^3 + 18/17*b^2*c*x^(17/6) + 9/8*b*c^2*x^(8/3) + 2/5*c^3*
x^(5/2) + 9/7*a*b^2*x^(7/3) + 36/13*a*b*c*x^(13/6) + 3/2*a*c^2*x^
2 + 9/5*a^2*b*x^(5/3) + 2*a^2*c*x^(3/2) + a^3*x
```

3.655 $\int \frac{-1+x^2}{\sqrt{a-b+\frac{b}{x^2}}x^3}\,dx$

Optimal. Leaf size=58

$$\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{b}+\frac{\tanh^{-1}\left(\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

```
[Out] Sqrt[a - b*(1 - x^(-2))]/b + ArcTanh[Sqrt[a - b*(1 - x^(-2))]/Sqr
t[a - b]]/Sqrt[a - b]
```

Rubi [A] time = 0.206537, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217

$$\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{b}+\frac{\tanh^{-1}\left(\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + x^2)/(Sqrt[a - b + b/x^2]*x^3),x]
```

```
[Out] Sqrt[a - b*(1 - x^(-2))]/b + ArcTanh[Sqrt[a - b*(1 - x^(-2))]/Sqr
t[a - b]]/Sqrt[a - b]
```

Rubi in Sympy [A] time = 8.82937, size = 39, normalized size = 0.67

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{a-b+\frac{b}{x^2}}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}+\frac{\sqrt{a-b+\frac{b}{x^2}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2-1)/x**3/(a-b+b/x**2)**(1/2),x)
```

```
[Out] atanh(sqrt(a - b + b/x**2)/sqrt(a - b))/sqrt(a - b) + sqrt(a - b
+ b/x**2)/b
```

Mathematica [A] time = 0.121466, size = 109, normalized size = 1.88

$$\frac{\sqrt{a-b}\left(ax^2-bx^2+b\right)+bx\sqrt{ax^2-bx^2+b}\log\left(\sqrt{a-b}\sqrt{ax^2-bx^2+b}+ax-bx\right)}{bx^2\sqrt{a-b}\sqrt{a+b\left(\frac{1}{x^2}-1\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + x^2)/(Sqrt[a - b + b/x^2]*x^3),x]
```

```
[Out] (Sqrt[a - b]*(b + a*x^2 - b*x^2) + b*x*Sqrt[b + a*x^2 - b*x^2]*Lo
g[a*x - b*x + Sqrt[a - b]*Sqrt[b + a*x^2 - b*x^2]])/(Sqrt[a - b]*
b*Sqrt[a + b*(-1 + x^(-2))]*x^2)
```

Maple [B] time = 0.027, size = 102, normalized size = 1.8

$$\frac{1}{bx^2}\sqrt{ax^2-bx^2+b}\left(\ln\left(\sqrt{a-b}x+\sqrt{ax^2-bx^2+b}\right)xb+\sqrt{ax^2-bx^2+b}\sqrt{a-b}\right)\frac{1}{\sqrt{\frac{ax^2-bx^2+b}{x^2}}}\frac{1}{\sqrt{a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2-1)/x^3/(a-b+b/x^2)^(1/2),x)
```

```
[Out] (a*x^2-b*x^2+b)^(1/2)*(ln((a-b)^(1/2)*x+(a*x^2-b*x^2+b)^(1/2))*x*
b+(a*x^2-b*x^2+b)^(1/2)*(a-b)^(1/2))/((a*x^2-b*x^2+b)/x^2)^(1/2)/
x^2/(a-b)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 1)/(sqrt(a - b + b/x^2)*x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.283279, size = 1, normalized size = 0.02

$$\left[\frac{\sqrt{a-b}b\log\left(-2\,(a-b)x^2\sqrt{\frac{(a-b)x^2+b}{x^2}}-\left(2\,(a-b)x^2+b\right)\sqrt{a-b}\right)+2\,(a-b)\sqrt{\frac{(a-b)x^2+b}{x^2}}}{2\,(ab-b^2)},\right.$$
$$\left.-\frac{\sqrt{-a+b}b\arctan\left(\frac{\sqrt{-a+b}}{\sqrt{\frac{(a-b)x^2+b}{x^2}}}\right)-(a-b)\sqrt{\frac{(a-b)x^2+b}{x^2}}}{ab-b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 1)/(sqrt(a - b + b/x^2)*x^3),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a - b)*b*log(-2*(a - b)*x^2*sqrt(((a - b)*x^2 + b)/x^2
) - (2*(a - b)*x^2 + b)*sqrt(a - b)) + 2*(a - b)*sqrt(((a - b)*x^
2 + b)/x^2))/(a*b - b^2), -(sqrt(-a + b)*b*arctan(sqrt(-a + b)/sq
rt(((a - b)*x^2 + b)/x^2)) - (a - b)*sqrt(((a - b)*x^2 + b)/x^2))
/(a*b - b^2)]
```

Sympy [A] time = 7.3092, size = 53, normalized size = 0.91

$$-\begin{cases}-\frac{1}{2\sqrt{a}x^2} & \text{for } b=0\\ -\frac{\sqrt{a-b+\frac{b}{x^2}}}{b} & \text{otherwise}\end{cases}+\frac{\operatorname{asinh}\left(\frac{x\sqrt{\operatorname{polar\_lift}(a-b)}}{\sqrt{b}}\right)}{\sqrt{\operatorname{polar\_lift}(a-b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2-1)/x**3/(a-b+b/x**2)**(1/2),x)
```

```
[Out] -Piecewise((-1/(2*sqrt(a)*x**2), Eq(b, 0)), (-sqrt(a - b + b/x**2
)/b, True)) + asinh(x*sqrt(polar_lift(a - b))/sqrt(b))/sqrt(polar
_lift(a - b))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 - 1}{\sqrt{a - b + \frac{b}{x^2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 1)/(sqrt(a - b + b/x^2)*x^3),x, algorithm="giac")

[Out] integrate((x^2 - 1)/(sqrt(a - b + b/x^2)*x^3), x)
```

3.656 $\int \frac{-1+x^2}{\sqrt{a+b\left(-1+\frac{1}{x^2}\right)}x^3}\,dx$

Optimal. Leaf size=58

$$\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{b}+\frac{\tanh^{-1}\left(\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

```
[Out] Sqrt[a - b*(1 - x^(-2))]/b + ArcTanh[Sqrt[a - b*(1 - x^(-2))]/Sqr
t[a - b]]/Sqrt[a - b]
```

Rubi [A] time = 0.322338, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273

$$\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{b}+\frac{\tanh^{-1}\left(\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + x^2)/(Sqrt[a + b*(-1 + x^(-2))]*x^3),x]
```

```
[Out] Sqrt[a - b*(1 - x^(-2))]/b + ArcTanh[Sqrt[a - b*(1 - x^(-2))]/Sqr
t[a - b]]/Sqrt[a - b]
```

Rubi in Sympy [A] time = 10.4692, size = 39, normalized size = 0.67

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{a-b+\frac{b}{x^2}}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}+\frac{\sqrt{a-b+\frac{b}{x^2}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2-1)/x**3/(a+b*(-1+1/x**2))**(1/2),x)
```

```
[Out] atanh(sqrt(a - b + b/x**2)/sqrt(a - b))/sqrt(a - b) + sqrt(a - b
+ b/x**2)/b
```

Mathematica [A] time = 0.0289012, size = 109, normalized size = 1.88

$$\frac{\sqrt{a-b}\left(ax^2-bx^2+b\right)+bx\sqrt{ax^2-bx^2+b}\log\left(\sqrt{a-b}\sqrt{ax^2-bx^2+b}+ax-bx\right)}{bx^2\sqrt{a-b}\sqrt{a+b\left(\frac{1}{x^2}-1\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + x^2)/(Sqrt[a + b*(-1 + x^(-2))]*x^3),x]
```

```
[Out] (Sqrt[a - b]*(b + a*x^2 - b*x^2) + b*x*Sqrt[b + a*x^2 - b*x^2]*Lo
g[a*x - b*x + Sqrt[a - b]*Sqrt[b + a*x^2 - b*x^2]])/(Sqrt[a - b]*
b*Sqrt[a + b*(-1 + x^(-2))]*x^2)
```

Maple [B] time = 0.013, size = 102, normalized size = 1.8

$$\frac{1}{bx^2}\sqrt{ax^2-bx^2+b}\left(\ln\left(\sqrt{a-b}x+\sqrt{ax^2-bx^2+b}\right)xb+\sqrt{ax^2-bx^2+b}\sqrt{a-b}\right)\frac{1}{\sqrt{\frac{ax^2-bx^2+b}{x^2}}}\frac{1}{\sqrt{a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2-1)/x^3/(a+b*(-1+1/x^2))^(1/2),x)
```

```
[Out] (a*x^2-b*x^2+b)^(1/2)*(ln((a-b)^(1/2)*x+(a*x^2-b*x^2+b)^(1/2))*x*
b+(a*x^2-b*x^2+b)^(1/2)*(a-b)^(1/2))/((a*x^2-b*x^2+b)/x^2)^(1/2)/
x^2/(a-b)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 - 1)/(sqrt(b*(1/x^2 - 1) + a)*x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.28231, size = 1, normalized size = 0.02

$$\left[\frac{\sqrt{a-b}b\log\left(-2(a-b)x^2\sqrt{\frac{(a-b)x^2+b}{x^2}}-\left(2(a-b)x^2+b\right)\sqrt{a-b}\right)+2(a-b)\sqrt{\frac{(a-b)x^2+b}{x^2}}}{2(ab-b^2)},\right.$$
$$\left.-\frac{\sqrt{-a+b}b\arctan\left(\frac{\sqrt{-a+b}}{\sqrt{\frac{(a-b)x^2+b}{x^2}}}\right)-(a-b)\sqrt{\frac{(a-b)x^2+b}{x^2}}}{ab-b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 - 1)/(sqrt(b*(1/x^2 - 1) + a)*x^3),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a - b)*b*log(-2*(a - b)*x^2*sqrt(((a - b)*x^2 + b)/x^2
) - (2*(a - b)*x^2 + b)*sqrt(a - b)) + 2*(a - b)*sqrt(((a - b)*x^
2 + b)/x^2))/(a*b - b^2), -(sqrt(-a + b)*b*arctan(sqrt(-a + b)/sq
rt(((a - b)*x^2 + b)/x^2)) - (a - b)*sqrt(((a - b)*x^2 + b)/x^2))
/(a*b - b^2)]
```

Sympy [A] time = 28.2983, size = 53, normalized size = 0.91

$$-\begin{cases}-\frac{1}{2\sqrt{a}x^2} & \text{for } b=0\\ -\frac{\sqrt{a-b+\frac{b}{x^2}}}{b} & \text{otherwise}\end{cases}+\frac{\operatorname{asinh}\left(\frac{x\sqrt{\text{polar\_lift}(a-b)}}{\sqrt{b}}\right)}{\sqrt{\text{polar\_lift}(a-b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2-1)/x**3/(a+b*(-1+1/x**2))**(1/2),x)
```

```
[Out] -Piecewise((-1/(2*sqrt(a)*x**2), Eq(b, 0)), (-sqrt(a - b + b/x**2
)/b, True)) + asinh(x*sqrt(polar_lift(a - b))/sqrt(b))/sqrt(polar
_lift(a - b))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 - 1}{\sqrt{b\left(\frac{1}{x^2} - 1\right) + ax^3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 1)/(sqrt(b*(1/x^2 - 1) + a)*x^3),x, algorithm="giac")
```

```
[Out] integrate((x^2 - 1)/(sqrt(b*(1/x^2 - 1) + a)*x^3), x)
```

3.657 $\int \frac{1+x}{(4+x^2)\sqrt{9+x^2}}\,dx$

Optimal. Leaf size=53

$$\frac{\tan^{-1}\left(\frac{\sqrt{5}x}{2\sqrt{x^2+9}}\right)}{2\sqrt{5}}-\frac{\tanh^{-1}\left(\frac{\sqrt{x^2+9}}{\sqrt{5}}\right)}{\sqrt{5}}$$

```
[Out] ArcTan[(Sqrt[5]*x)/(2*Sqrt[9 + x^2])]/(2*Sqrt[5]) - ArcTanh[Sqrt[
9 + x^2]/Sqrt[5]]/Sqrt[5]
```

Rubi [A] time = 0.0967123, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$\frac{\tan^{-1}\left(\frac{\sqrt{5}x}{2\sqrt{x^2+9}}\right)}{2\sqrt{5}}-\frac{\tanh^{-1}\left(\frac{\sqrt{x^2+9}}{\sqrt{5}}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x)/((4 + x^2)*Sqrt[9 + x^2]),x]
```

```
[Out] ArcTan[(Sqrt[5]*x)/(2*Sqrt[9 + x^2])]/(2*Sqrt[5]) - ArcTanh[Sqrt[
9 + x^2]/Sqrt[5]]/Sqrt[5]
```

Rubi in Sympy [A] time = 5.92944, size = 48, normalized size = 0.91

$$\frac{\sqrt{5}\,\text{atan}\left(\frac{\sqrt{5}x}{2\sqrt{x^2+9}}\right)}{10}-\frac{\sqrt{5}\,\text{atanh}\left(\frac{\sqrt{5}\sqrt{x^2+9}}{5}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+x)/(x**2+4)/(x**2+9)**(1/2),x)
```

```
[Out] sqrt(5)*atan(sqrt(5)*x/(2*sqrt(x**2 + 9)))/10 - sqrt(5)*atanh(sqr
t(5)*sqrt(x**2 + 9)/5)/5
```

Mathematica [A] time = 0.0979993, size = 75, normalized size = 1.42

$$-\frac{-\log\left(x^2+4\right)+\log\left(x^2+2\sqrt{5}\sqrt{x^2+9}+14\right)+\tan^{-1}\left(\frac{18-8x^2}{9x^2+5\sqrt{5}\sqrt{x^2+9}x+36}\right)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x)/((4 + x^2)*Sqrt[9 + x^2]),x]
```

```
[Out] -(ArcTan[(18 - 8*x^2)/(36 + 9*x^2 + 5*Sqrt[5]*x*Sqrt[9 + x^2])] -
 Log[4 + x^2] + Log[14 + x^2 + 2*Sqrt[5]*Sqrt[9 + x^2]])/(2*Sqrt[
5])
```

Maple [A] time = 0.02, size = 39, normalized size = 0.7

$$\frac{\sqrt{5}}{10}\arctan\left(\frac{x\sqrt{5}}{2}\frac{1}{\sqrt{x^2+9}}\right)-\frac{\sqrt{5}}{5}Artanh\left(\frac{\sqrt{5}}{5}\sqrt{x^2+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+x)/(x^2+4)/(x^2+9)^(1/2),x)
```

```
[Out] 1/10*arctan(1/2*x*5^(1/2)/(x^2+9)^(1/2))*5^(1/2)-1/5*arctanh(1/5*
(x^2+9)^(1/2)*5^(1/2))*5^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1}{\sqrt{x^2+9}(x^2+4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 1)/(sqrt(x^2 + 9)*(x^2 + 4)),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)/(sqrt(x^2 + 9)*(x^2 + 4)), x)
```

Fricas [A] time = 0.296107, size = 254, normalized size = 4.79

$$\frac{1}{10}\sqrt{5}\left(2\arctan\left(-\frac{2\sqrt{5}}{\sqrt{5}x-\sqrt{5}\sqrt{x^2+9}-\sqrt{10\,x^2-10\sqrt{x^2+9}\left(x+\sqrt{5}\right)+10\sqrt{5}x+90}+5}\right)-2\arctan\left(-\frac{}{\sqrt{5}x-\sqrt{5}\sqrt{x^2+9}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 1)/(sqrt(x^2 + 9)*(x^2 + 4)),x, algorithm="fricas")
```

```
[Out] 1/10*sqrt(5)*(2*arctan(-2*sqrt(5)/(sqrt(5)*x - sqrt(5)*sqrt(x^2 +
 9) - sqrt(10*x^2 - 10*sqrt(x^2 + 9)*(x + sqrt(5)) + 10*sqrt(5)*x
 + 90) + 5)) - 2*arctan(-2*sqrt(5)/(sqrt(5)*x - sqrt(5)*sqrt(x^2
+ 9) - sqrt(10*x^2 - 10*sqrt(x^2 + 9)*(x - sqrt(5)) - 10*sqrt(5)*
x + 90) - 5)) + log(10*x^2 - 10*sqrt(x^2 + 9)*(x + sqrt(5)) + 10*
sqrt(5)*x + 90) - log(10*x^2 - 10*sqrt(x^2 + 9)*(x - sqrt(5)) - 1
0*sqrt(5)*x + 90))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1}{(x^2+4)\sqrt{x^2+9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+x)/(x**2+4)/(x**2+9)**(1/2),x)
```

```
[Out] Integral((x + 1)/((x**2 + 4)*sqrt(x**2 + 9)), x)
```

GIAC/XCAS [A] time = 0.298337, size = 528, normalized size = 9.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 1)/(sqrt(x^2 + 9)*(x^2 + 4)),x, algorithm="giac")
```

```
[Out] 1/40*(9*sqrt(5)*arctan(2/(sqrt(5) + 3)) + 9*sqrt(5)*arctan(2/(sqr
t(5) - 3)) + 49*sqrt(5)*ln(3/2*sqrt(5) + 9/2) - 49*sqrt(5)*ln(-3/
2*sqrt(5) + 9/2) - 15*arctan(2/(sqrt(5) + 3)) + 15*arctan(2/(sqrt
(5) - 3)) - 105*ln(3/2*sqrt(5) + 9/2) - 105*ln(-3/2*sqrt(5) + 9/2
))*sign(x) - 1/10*(7*sqrt(5) + 15)*ln((sqrt(9/x^2 + 1) - 3/x)^2 +
 1/2*(3*sqrt(5)*sign(x) + 7*sign(x))/sign(x))*sign(x)/(7*abs(sign
(x))*sign(x) + 3*sqrt(5)) + 1/10*(7*sqrt(5) - 15)*ln((sqrt(9/x^2
+ 1) - 3/x)^2 - 1/2*(3*sqrt(5)*sign(x) - 7*sign(x))/sign(x))*sign
(x)/(7*abs(sign(x))*sign(x) - 3*sqrt(5)) - 1/20*(5*(sqrt(5) + 3)*
abs(sign(x)) + 3*(3*sqrt(5) + 5)*sign(x))*arctan(2*sqrt(1/2)*(sqr
t(9/x^2 + 1) - 3/x)/sqrt((3*sqrt(5)*sign(x) + 7*sign(x))/sign(x))
)/(7*abs(sign(x))*sign(x) + 3*sqrt(5)) + 1/20*(5*(sqrt(5) - 3)*ab
s(sign(x)) + 3*(3*sqrt(5) - 5)*sign(x))*arctan(2*sqrt(1/2)*(sqrt(
9/x^2 + 1) - 3/x)/sqrt(-(3*sqrt(5)*sign(x) - 7*sign(x))/sign(x)))
/(7*abs(sign(x))*sign(x) - 3*sqrt(5))
```

3.658 $\int x\left(1+\sqrt{1-x^2}\right)\,dx$

Optimal. Leaf size=23

$$\frac{x^2}{2}-\frac{1}{3}\left(1-x^2\right)^{3/2}$$

```
[Out] x^2/2 - (1 - x^2)^(3/2)/3
```

Rubi [A] time = 0.0159755, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$\frac{x^2}{2}-\frac{1}{3}\left(1-x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(1 + Sqrt[1 - x^2]),x]
```

```
[Out] x^2/2 - (1 - x^2)^(3/2)/3
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(-x^2+1\right)^{\frac{3}{2}}}{3}+\int x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(1+(-x**2+1)**(1/2)),x)
```

```
[Out] -(-x**2 + 1)**(3/2)/3 + Integral(x, x)
```

Mathematica [A] time = 0.0188422, size = 23, normalized size = 1.

$$\frac{x^2}{2}-\frac{1}{3}\left(1-x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(1 + Sqrt[1 - x^2]),x]
```

```
[Out] x^2/2 - (1 - x^2)^(3/2)/3
```

Maple [A] time = 0.001, size = 18, normalized size = 0.8

$$\frac{x^2}{2} - \frac{1}{3}\left(-x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(1+(-x^2+1)^(1/2)),x)
```

```
[Out] 1/2*x^2-1/3*(-x^2+1)^(3/2)
```

Maxima [A] time = 0.718612, size = 23, normalized size = 1.

$$\frac{1}{2}x^2 - \frac{1}{3}\left(-x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(sqrt(-x^2 + 1) + 1),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 1/3*(-x^2 + 1)^(3/2)
```

Fricas [A] time = 0.264869, size = 70, normalized size = 3.04

$$\frac{2x^6 + 3\sqrt{-x^2+1}x^4 - 3x^4}{6\left(3x^2 - (x^2-4)\sqrt{-x^2+1} - 4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(sqrt(-x^2 + 1) + 1),x, algorithm="fricas")
```

```
[Out] 1/6*(2*x^6 + 3*sqrt(-x^2 + 1)*x^4 - 3*x^4)/(3*x^2 - (x^2 - 4)*sqr
t(-x^2 + 1) - 4)
```

Sympy [A] time = 0.459458, size = 27, normalized size = 1.17

$$\frac{x^2\sqrt{-x^2+1}}{3}+\frac{x^2}{2}-\frac{\sqrt{-x^2+1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(1+(-x**2+1)**(1/2)),x)
```

```
[Out] x**2*sqrt(-x**2 + 1)/3 + x**2/2 - sqrt(-x**2 + 1)/3
```

GIAC/XCAS [A] time = 0.261545, size = 24, normalized size = 1.04

$$\frac{1}{2}x^2-\frac{1}{3}\left(-x^2+1\right)^{\frac{3}{2}}-\frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(sqrt(-x^2 + 1) + 1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 1/3*(-x^2 + 1)^(3/2) - 1/2
```

3.659 $\int x\left(1+\sqrt{1-x}\sqrt{1+x}\right)\,dx$

Optimal. Leaf size=23

$$\frac{x^2}{2}-\frac{1}{3}\left(1-x^2\right)^{3/2}$$

```
[Out] x^2/2 - (1 - x^2)^(3/2)/3
```

Rubi [A] time = 0.0164698, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095

$$\frac{x^2}{2}-\frac{1}{3}\left(1-x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(1 + Sqrt[1 - x]*Sqrt[1 + x]),x]
```

```
[Out] x^2/2 - (1 - x^2)^(3/2)/3
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$2\int^{\sqrt{x+1}} x\left(x^2-1\right)\left(x\sqrt{-x^2+2}+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(1+(1-x)**(1/2)*(1+x)**(1/2)),x)
```

```
[Out] 2*Integral(x*(x**2 - 1)*(x*sqrt(-x**2 + 2) + 1), (x, sqrt(x + 1))
)
```

Mathematica [A] time = 0.00634142, size = 23, normalized size = 1.

$$\frac{x^2}{2}-\frac{1}{3}\left(1-x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(1 + Sqrt[1 - x]*Sqrt[1 + x]),x]
```

```
[Out] x^2/2 - (1 - x^2)^(3/2)/3
```

Maple [A] time = 0.002, size = 26, normalized size = 1.1

$$\frac{x^2-1}{3}\sqrt{1-x}\sqrt{1+x}+\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(1+(1-x)^(1/2)*(1+x)^(1/2)),x)
```

```
[Out] 1/3*(1-x)^(1/2)*(1+x)^(1/2)*(x^2-1)+1/2*x^2
```

Maxima [A] time = 0.799495, size = 23, normalized size = 1.

$$\frac{1}{2}x^2-\frac{1}{3}\left(-x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(sqrt(x + 1)*sqrt(-x + 1) + 1),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 1/3*(-x^2 + 1)^(3/2)
```

Fricas [A] time = 0.264758, size = 78, normalized size = 3.39

$$\frac{2\,x^6+3\,\sqrt{x+1}x^4\sqrt{-x+1}-3\,x^4}{6\left(3\,x^2-(x^2-4)\sqrt{x+1}\sqrt{-x+1}-4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(sqrt(x + 1)*sqrt(-x + 1) + 1),x, algorithm="fricas")
```

```
[Out] 1/6*(2*x^6 + 3*sqrt(x + 1)*x^4*sqrt(-x + 1) - 3*x^4)/(3*x^2 - (x^
2 - 4)*sqrt(x + 1)*sqrt(-x + 1) - 4)
```

Sympy [A] time = 3.76043, size = 63, normalized size = 2.74

$$\begin{cases} \frac{ix^2\sqrt{x^2-1}}{3} + \frac{x^2}{2} - \frac{i\sqrt{x^2-1}}{3} & \text{for } \left|x^2\right| > 1 \\ -\frac{x^2\sqrt{-x^2+1}}{3} + \frac{x^2}{2} + \frac{\sqrt{-x^2+1}}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(1+(1-x)**(1/2)*(1+x)**(1/2)),x)
```

```
[Out] Piecewise((I*x**2*sqrt(x**2 - 1)/3 + x**2/2 - I*sqrt(x**2 - 1)/3,
 Abs(x**2) > 1), (-x**2*sqrt(-x**2 + 1)/3 + x**2/2 + sqrt(-x**2 +
 1)/3, True))
```

GIAC/XCAS [A] time = 0.270317, size = 39, normalized size = 1.7

$$\frac{1}{3}(x+1)^{\frac{3}{2}}(x-1)\sqrt{-x+1}+\frac{1}{2}(x+1)^2-x-1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(sqrt(x + 1)*sqrt(-x + 1) + 1),x, algorithm="giac")
```

```
[Out] 1/3*(x + 1)^(3/2)*(x - 1)*sqrt(-x + 1) + 1/2*(x + 1)^2 - x - 1
```

3.660 $\int x\left(1+\frac{1}{\sqrt{2+x}\sqrt{3+x}}\right)\,dx$

Optimal. Leaf size=33

$$\frac{x^2}{2}+\sqrt{x+2}\sqrt{x+3}-5\sinh^{-1}\left(\sqrt{x+2}\right)$$

```
[Out] x^2/2 + Sqrt[2 + x]*Sqrt[3 + x] - 5*ArcSinh[Sqrt[2 + x]]
```

Rubi [A] time = 0.0464142, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21

$$\frac{x^2}{2}+\sqrt{x+2}\sqrt{x+3}-5\sinh^{-1}\left(\sqrt{x+2}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x*(1 + 1/(Sqrt[2 + x]*Sqrt[3 + x])),x]
```

```
[Out] x^2/2 + Sqrt[2 + x]*Sqrt[3 + x] - 5*ArcSinh[Sqrt[2 + x]]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$2\int^{\sqrt{x+2}}\frac{\left(x^2-2\right)\left(x\sqrt{x^2+1}+1\right)}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(1+1/(2+x)**(1/2)/(3+x)**(1/2)),x)
```

```
[Out] 2*Integral((x**2 - 2)*(x*sqrt(x**2 + 1) + 1)/sqrt(x**2 + 1), (x,
sqrt(x + 2)))
```

Mathematica [A] time = 0.0255826, size = 33, normalized size = 1.

$$\frac{x^2}{2}+\sqrt{x+2}\sqrt{x+3}-5\sinh^{-1}\left(\sqrt{x+2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(1 + 1/(Sqrt[2 + x]*Sqrt[3 + x])),x]
```

```
[Out] x^2/2 + Sqrt[2 + x]*Sqrt[3 + x] - 5*ArcSinh[Sqrt[2 + x]]
```

Maple [B] time = 0.016, size = 58, normalized size = 1.8

$$-\frac{1}{2}\sqrt{2+x}\sqrt{3+x}\left(-2\sqrt{x^2+5x+6}+5\ln\left(5/2+x+\sqrt{x^2+5x+6}\right)\right)\frac{1}{\sqrt{x^2+5x+6}}+\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(1+1/(2+x)^(1/2)/(3+x)^(1/2)),x)
```

```
[Out] -1/2*(2+x)^(1/2)*(3+x)^(1/2)*(-2*(x^2+5*x+6)^(1/2)+5*ln(5/2+x+(x^
2+5*x+6)^(1/2)))/(x^2+5*x+6)^(1/2)+1/2*x^2
```

Maxima [A] time = 0.756415, size = 49, normalized size = 1.48

$$\frac{1}{2}x^2+\sqrt{x^2+5x+6}-\frac{5}{2}\log\left(2x+2\sqrt{x^2+5x+6}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(1/(sqrt(x + 3)*sqrt(x + 2)) + 1),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + sqrt(x^2 + 5*x + 6) - 5/2*log(2*x + 2*sqrt(x^2 + 5*x +
6) + 5)
```

Fricas [A] time = 0.270809, size = 128, normalized size = 3.88

$$-\frac{4x^3-2\left(2x^2-4x-5\right)\sqrt{x+3}\sqrt{x+2}+2x^2-10\left(2\sqrt{x+3}\sqrt{x+2}-2x-5\right)\log\left(2\sqrt{x+3}\sqrt{x+2}-2x-5\right)-30x-23}{4\left(2\sqrt{x+3}\sqrt{x+2}-2x-5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(1/(sqrt(x + 3)*sqrt(x + 2)) + 1),x, algorithm="fricas")
```

```
[Out] -1/4*(4*x^3 - 2*(2*x^2 - 4*x - 5)*sqrt(x + 3)*sqrt(x + 2) + 2*x^2
 - 10*(2*sqrt(x + 3)*sqrt(x + 2) - 2*x - 5)*log(2*sqrt(x + 3)*sqr
```

```
t(x + 2) - 2*x - 5) - 30*x - 23)/(2*sqrt(x + 3)*sqrt(x + 2) - 2*x
 - 5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(\sqrt{x+2}\sqrt{x+3}+1\right)}{\sqrt{x+2}\sqrt{x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(1+1/(2+x)**(1/2)/(3+x)**(1/2)),x)
```

```
[Out] Integral(x*(sqrt(x + 2)*sqrt(x + 3) + 1)/(sqrt(x + 2)*sqrt(x + 3)
), x)
```

GIAC/XCAS [A] time = 0.293439, size = 54, normalized size = 1.64

$$\frac{1}{2}\,(x+3)^2+\sqrt{x+3}\sqrt{x+2}-3\,x+5\ln\left(\left|-\sqrt{x+3}+\sqrt{x+2}\right|\right)-9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(1/(sqrt(x + 3)*sqrt(x + 2)) + 1),x, algorithm="giac")
```

```
[Out] 1/2*(x + 3)^2 + sqrt(x + 3)*sqrt(x + 2) - 3*x + 5*ln(abs(-sqrt(x
+ 3) + sqrt(x + 2))) - 9
```

3.661 $\int \frac{x-\sqrt{x^6}}{x(1-x^4)}\,dx$

Optimal. Leaf size=45

$$\frac{\sqrt{x^6}\tan^{-1}(x)}{2x^3}-\frac{\sqrt{x^6}\tanh^{-1}(x)}{2x^3}+\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)$$

```
[Out] ArcTan[x]/2 + (Sqrt[x^6]*ArcTan[x])/(2*x^3) + ArcTanh[x]/2 - (Sqr
t[x^6]*ArcTanh[x])/(2*x^3)
```

Rubi [A] time = 0.252606, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{\sqrt{x^6}\tan^{-1}(x)}{2x^3}-\frac{\sqrt{x^6}\tanh^{-1}(x)}{2x^3}+\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(x - Sqrt[x^6])/(x*(1 - x^4)),x]
```

```
[Out] ArcTan[x]/2 + (Sqrt[x^6]*ArcTan[x])/(2*x^3) + ArcTanh[x]/2 - (Sqr
t[x^6]*ArcTanh[x])/(2*x^3)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{x}{2}+\frac{\sqrt{x^6}}{6}+\frac{\operatorname{atan}(x)}{2}+\int\frac{x-\sqrt{x^6}}{x}\,dx+\int\frac{-\frac{x}{4}+\frac{\sqrt{x^6}}{4}}{x-1}\,dx+\int\frac{-\frac{x}{4}+\frac{\sqrt{x^6}}{4}}{x+1}\,dx-\frac{\sqrt{x^6}}{2x^2}+\frac{\sqrt{x^6}\operatorname{atan}(x)}{2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x-(x**6)**(1/2))/x/(-x**4+1),x)
```

```
[Out] -x/2 + sqrt(x**6)/6 + atan(x)/2 + Integral((x - sqrt(x**6))/x, x)
 + Integral((-x/4 + sqrt(x**6)/4)/(x - 1), x) + Integral((-x/4 +
sqrt(x**6)/4)/(x + 1), x) - sqrt(x**6)/(2*x**2) + sqrt(x**6)*atan
(x)/(2*x**3)
```

Mathematica [A] time = 0.0916927, size = 0, normalized size = 0.

$$\int\frac{x-\sqrt{x^6}}{x\left(1-x^4\right)}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(x - Sqrt[x^6])/(x*(1 - x^4)),x]
```

```
[Out] Integrate[(x - Sqrt[x^6])/(x*(1 - x^4)), x]
```

Maple [A] time = 0.006, size = 35, normalized size = 0.8

$$\frac{\ln(-1+x)-\ln(1+x)+2\arctan(x)}{4x^3}\sqrt{x^6}+\frac{Artanh(x)}{2}+\frac{\arctan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x-(x^6)^(1/2))/x/(-x^4+1),x)
```

```
[Out] 1/4*(x^6)^(1/2)*(ln(-1+x)-ln(1+x)+2*arctan(x))/x^3+1/2*arctanh(x)
+1/2*arctan(x)
```

Maxima [A] time = 0.92148, size = 3, normalized size = 0.07

$$\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x - sqrt(x^6))/((x^4 - 1)*x),x, algorithm="maxima")
```

```
[Out] arctan(x)
```

Fricas [A] time = 0.265254, size = 3, normalized size = 0.07

$$\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x - sqrt(x^6))/((x^4 - 1)*x),x, algorithm="fricas")
```

```
[Out] arctan(x)
```

Sympy [A] time = 0.199121, size = 2, normalized size = 0.04

$$\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x-(x**6)**(1/2))/x/(-x**4+1),x)
```

```
[Out] atan(x)
```

GIAC/XCAS [A] time = 0.264492, size = 42, normalized size = 0.93

$$\frac{1}{2}\left(\operatorname{sign}(x)+1\right)\arctan(x)-\frac{1}{4}\left(\operatorname{sign}(x)-1\right)\ln\left(|x+1|\right)+\frac{1}{4}\left(\operatorname{sign}(x)-1\right)\ln\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x - sqrt(x^6))/((x^4 - 1)*x),x, algorithm="giac")
```

```
[Out] 1/2*(sign(x) + 1)*arctan(x) - 1/4*(sign(x) - 1)*ln(abs(x + 1)) +
1/4*(sign(x) - 1)*ln(abs(x - 1))
```

3.662 $\int \frac{1-\frac{\sqrt{x^6}}{x}}{1-x^4} \, dx$

Optimal. Leaf size=45

$$\frac{\sqrt{x^6}\tan^{-1}(x)}{2x^3}-\frac{\sqrt{x^6}\tanh^{-1}(x)}{2x^3}+\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)$$

```
[Out] ArcTan[x]/2 + (Sqrt[x^6]*ArcTan[x])/(2*x^3) + ArcTanh[x]/2 - (Sqr
t[x^6]*ArcTanh[x])/(2*x^3)
```

Rubi [A] time = 0.0958688, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{\sqrt{x^6}\tan^{-1}(x)}{2x^3}-\frac{\sqrt{x^6}\tanh^{-1}(x)}{2x^3}+\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - Sqrt[x^6]/x)/(1 - x^4),x]
```

```
[Out] ArcTan[x]/2 + (Sqrt[x^6]*ArcTan[x])/(2*x^3) + ArcTanh[x]/2 - (Sqr
t[x^6]*ArcTanh[x])/(2*x^3)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1-(x**6)**(1/2)/x)/(-x**4+1),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.0580209, size = 0, normalized size = 0.

$$\int \frac{1-\frac{\sqrt{x^6}}{x}}{1-x^4} \, dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(1 - Sqrt[x^6]/x)/(1 - x^4),x]
```

```
[Out] Integrate[(1 - Sqrt[x^6]/x)/(1 - x^4), x]
```

Maple [A] time = 0.004, size = 35, normalized size = 0.8

$$\frac{\ln(-1+x)-\ln(1+x)+2\arctan(x)}{4x^3}\sqrt{x^6}+\frac{Artanh(x)}{2}+\frac{\arctan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1-(x^6)^(1/2)/x)/(-x^4+1),x)
```

```
[Out] 1/4*(x^6)^(1/2)*(ln(-1+x)-ln(1+x)+2*arctan(x))/x^3+1/2*arctanh(x)
+1/2*arctan(x)
```

Maxima [A] time = 0.859317, size = 3, normalized size = 0.07

$$\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(x^6)/x - 1)/(x^4 - 1),x, algorithm="maxima")
```

```
[Out] arctan(x)
```

Fricas [A] time = 0.264862, size = 3, normalized size = 0.07

$$\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(x^6)/x - 1)/(x^4 - 1),x, algorithm="fricas")
```

```
[Out] arctan(x)
```

Sympy [A] time = 0.190615, size = 2, normalized size = 0.04

$$\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1-(x**6)**(1/2)/x)/(-x**4+1),x)
```

```
[Out] atan(x)
```

GIAC/XCAS [A] time = 0.264345, size = 42, normalized size = 0.93

$$\frac{1}{2}\left(\operatorname{sign}(x)+1\right)\arctan(x)-\frac{1}{4}\left(\operatorname{sign}(x)-1\right)\ln\left(|x+1|\right)+\frac{1}{4}\left(\operatorname{sign}(x)-1\right)\ln\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(x^6)/x - 1)/(x^4 - 1),x, algorithm="giac")
```

```
[Out] 1/2*(sign(x) + 1)*arctan(x) - 1/4*(sign(x) - 1)*ln(abs(x + 1)) +
1/4*(sign(x) - 1)*ln(abs(x - 1))
```

3.663 $\int \frac{x-\sqrt{x^6}}{x-x^5} dx$

Optimal. Leaf size=45

$$\frac{\sqrt{x^6}\tan^{-1}(x)}{2x^3}-\frac{\sqrt{x^6}\tanh^{-1}(x)}{2x^3}+\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)$$

```
[Out] ArcTan[x]/2 + (Sqrt[x^6]*ArcTan[x])/(2*x^3) + ArcTanh[x]/2 - (Sqr
t[x^6]*ArcTanh[x])/(2*x^3)
```

Rubi [A] time = 0.164941, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{\sqrt{x^6}\tan^{-1}(x)}{2x^3}-\frac{\sqrt{x^6}\tanh^{-1}(x)}{2x^3}+\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(x - Sqrt[x^6])/(x - x^5),x]
```

```
[Out] ArcTan[x]/2 + (Sqrt[x^6]*ArcTan[x])/(2*x^3) + ArcTanh[x]/2 - (Sqr
t[x^6]*ArcTanh[x])/(2*x^3)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{x}{2}+\frac{\sqrt{x^6}}{6}+\frac{\operatorname{atan}(x)}{2}+\int\frac{x-\sqrt{x^6}}{x}\,dx+\int\frac{-\frac{x}{4}+\frac{\sqrt{x^6}}{4}}{x-1}\,dx+\int\frac{-\frac{x}{4}+\frac{\sqrt{x^6}}{4}}{x+1}\,dx-\frac{\sqrt{x^6}}{2x^2}+\frac{\sqrt{x^6}\operatorname{atan}(x)}{2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x-(x**6)**(1/2))/(-x**5+x),x)
```

```
[Out] -x/2 + sqrt(x**6)/6 + atan(x)/2 + Integral((x - sqrt(x**6))/x, x)
 + Integral((-x/4 + sqrt(x**6)/4)/(x - 1), x) + Integral((-x/4 +
sqrt(x**6)/4)/(x + 1), x) - sqrt(x**6)/(2*x**2) + sqrt(x**6)*atan
(x)/(2*x**3)
```

Mathematica [A] time = 0.0557132, size = 0, normalized size = 0.

$$\int\frac{x-\sqrt{x^6}}{x-x^5}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(x - Sqrt[x^6])/(x - x^5),x]
```

```
[Out] Integrate[(x - Sqrt[x^6])/(x - x^5), x]
```

Maple [A] time = 0.005, size = 35, normalized size = 0.8

$$\frac{\ln(-1+x) - \ln(1+x) + 2\arctan(x)}{4x^3}\sqrt{x^6} + \frac{Artanh(x)}{2} + \frac{\arctan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x-(x^6)^(1/2))/(-x^5+x),x)
```

```
[Out] 1/4*(x^6)^(1/2)*(ln(-1+x)-ln(1+x)+2*arctan(x))/x^3+1/2*arctanh(x)
+1/2*arctan(x)
```

Maxima [A] time = 0.835025, size = 3, normalized size = 0.07

$$\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x - sqrt(x^6))/(x^5 - x),x, algorithm="maxima")
```

```
[Out] arctan(x)
```

Fricas [A] time = 0.273767, size = 3, normalized size = 0.07

$$\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x - sqrt(x^6))/(x^5 - x),x, algorithm="fricas")
```

```
[Out] arctan(x)
```

Sympy [A] time = 0.184984, size = 2, normalized size = 0.04

$$\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x-(x**6)**(1/2))/(-x**5+x),x)
```

```
[Out] atan(x)
```

GIAC/XCAS [A] time = 0.271519, size = 42, normalized size = 0.93

$$\frac{1}{2}\left(\operatorname{sign}(x)+1\right)\arctan(x)-\frac{1}{4}\left(\operatorname{sign}(x)-1\right)\ln\left(|x+1|\right)+\frac{1}{4}\left(\operatorname{sign}(x)-1\right)\ln\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x - sqrt(x^6))/(x^5 - x),x, algorithm="giac")
```

```
[Out] 1/2*(sign(x) + 1)*arctan(x) - 1/4*(sign(x) - 1)*ln(abs(x + 1)) +
1/4*(sign(x) - 1)*ln(abs(x - 1))
```

3.664 $\int \frac{x}{x+\sqrt{x^6}}\,dx$

Optimal. Leaf size=45

$$\frac{\sqrt{x^6}\tan^{-1}(x)}{2x^3}-\frac{\sqrt{x^6}\tanh^{-1}(x)}{2x^3}+\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)$$

```
[Out] ArcTan[x]/2 + (Sqrt[x^6]*ArcTan[x])/(2*x^3) + ArcTanh[x]/2 - (Sqr
t[x^6]*ArcTanh[x])/(2*x^3)
```

Rubi [A] time = 0.216194, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615

$$\frac{\sqrt{x^6}\tan^{-1}(x)}{2x^3}-\frac{\sqrt{x^6}\tanh^{-1}(x)}{2x^3}+\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[x/(x + Sqrt[x^6]),x]
```

```
[Out] ArcTan[x]/2 + (Sqrt[x^6]*ArcTan[x])/(2*x^3) + ArcTanh[x]/2 - (Sqr
t[x^6]*ArcTanh[x])/(2*x^3)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{x}{2}+\frac{\sqrt{x^6}}{6}+\frac{\operatorname{atan}(x)}{2}+\int\frac{x-\sqrt{x^6}}{x}\,dx+\int\frac{-\frac{x}{4}+\frac{\sqrt{x^6}}{4}}{x-1}\,dx+\int\frac{-\frac{x}{4}+\frac{\sqrt{x^6}}{4}}{x+1}\,dx-\frac{\sqrt{x^6}}{2x^2}+\frac{\sqrt{x^6}\operatorname{atan}(x)}{2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(x+(x**6)**(1/2)),x)
```

```
[Out] -x/2 + sqrt(x**6)/6 + atan(x)/2 + Integral((x - sqrt(x**6))/x, x)
 + Integral((-x/4 + sqrt(x**6)/4)/(x - 1), x) + Integral((-x/4 +
sqrt(x**6)/4)/(x + 1), x) - sqrt(x**6)/(2*x**2) + sqrt(x**6)*atan
(x)/(2*x**3)
```

Mathematica [A] time = 0.0681589, size = 0, normalized size = 0.

$$\int\frac{x}{x+\sqrt{x^6}}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[x/(x + Sqrt[x^6]),x]
```

```
[Out] Integrate[x/(x + Sqrt[x^6]), x]
```

Maple [A] time = 0.012, size = 27, normalized size = 0.6

$$1 \arctan\left(\sqrt{\frac{1}{x^3}\sqrt{x^6}}\,x\right)\frac{1}{\sqrt{\frac{1}{x^3}\sqrt{x^6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(x+(x^6)^(1/2)),x)
```

```
[Out] 1/((x^6)^(1/2)/x^3)^(1/2)*arctan(((x^6)^(1/2)/x^3)^(1/2)*x)
```

Maxima [A] time = 0.833021, size = 3, normalized size = 0.07

$$\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x + sqrt(x^6)),x, algorithm="maxima")
```

```
[Out] arctan(x)
```

Fricas [A] time = 0.26283, size = 3, normalized size = 0.07

$$\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x + sqrt(x^6)),x, algorithm="fricas")
```

```
[Out] arctan(x)
```

Sympy [A] time = 0.180629, size = 2, normalized size = 0.04

$$\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x+(x**6)**(1/2)),x)

[Out] atan(x)
```

GIAC/XCAS [A] time = 0.264695, size = 16, normalized size = 0.36

$$\frac{\arctan\left(x\sqrt{\operatorname{sign}(x)}\right)}{\sqrt{\operatorname{sign}(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x + sqrt(x^6)),x, algorithm="giac")

[Out] arctan(x*sqrt(sign(x)))/sqrt(sign(x))
```

3.665 $\int \frac{\sqrt{x}-\sqrt{x^3}}{x-x^3}\,dx$

Optimal. Leaf size=52

$$\frac{\sqrt{x^3}\tan^{-1}\left(\sqrt{x}\right)}{x^{3/2}}-\frac{\sqrt{x^3}\tanh^{-1}\left(\sqrt{x}\right)}{x^{3/2}}+\tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)$$

```
[Out] ArcTan[Sqrt[x]] + (Sqrt[x^3]*ArcTan[Sqrt[x]])/x^(3/2) + ArcTanh[S
qrt[x]] - (Sqrt[x^3]*ArcTanh[Sqrt[x]])/x^(3/2)
```

Rubi [A] time = 0.287317, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32

$$\frac{\sqrt{x^3}\tan^{-1}\left(\sqrt{x}\right)}{x^{3/2}}-\frac{\sqrt{x^3}\tanh^{-1}\left(\sqrt{x}\right)}{x^{3/2}}+\tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[x] - Sqrt[x^3])/(x - x^3),x]
```

```
[Out] ArcTan[Sqrt[x]] + (Sqrt[x^3]*ArcTan[Sqrt[x]])/x^(3/2) + ArcTanh[S
qrt[x]] - (Sqrt[x^3]*ArcTanh[Sqrt[x]])/x^(3/2)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\sqrt{x}+\frac{\sqrt{x^3}}{3}+\operatorname{atan}\left(\sqrt{x}\right)+2\int^{\sqrt{x}}\frac{x-\sqrt{x^6}}{x}\,dx-2\int^{\sqrt{x}}\frac{\frac{x}{4}-\frac{\sqrt{x^6}}{4}}{x-1}\,dx-2\int^{\sqrt{x}}\frac{\frac{x}{4}-\frac{\sqrt{x^6}}{4}}{x+1}\,dx-\frac{\sqrt{x^3}}{x}+\frac{\sqrt{x^3}\operatorname{atan}\left(\sqrt{x}\right)}{x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**(1/2)-(x**3)**(1/2))/(-x**3+x),x)
```

```
[Out] -sqrt(x) + sqrt(x**3)/3 + atan(sqrt(x)) + 2*Integral((x - sqrt(x*
*6))/x, (x, sqrt(x))) - 2*Integral((x/4 - sqrt(x**6)/4)/(x - 1),
(x, sqrt(x))) - 2*Integral((x/4 - sqrt(x**6)/4)/(x + 1), (x, sqrt
(x))) - sqrt(x**3)/x + sqrt(x**3)*atan(sqrt(x))/x**(3/2)
```

Mathematica [A] time = 0.166167, size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}-\sqrt{x^3}}{x-x^3}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(Sqrt[x] - Sqrt[x^3])/(x - x^3),x]
```

```
[Out] Integrate[(Sqrt[x] - Sqrt[x^3])/(x - x^3), x]
```

Maple [A] time = 0.008, size = 41, normalized size = 0.8

$$Artanh\left(\sqrt{x}\right)+\arctan\left(\sqrt{x}\right)+\frac{1}{2}\sqrt{x^3}\left(\ln\left(-1+\sqrt{x}\right)-\ln\left(1+\sqrt{x}\right)+2\arctan\left(\sqrt{x}\right)\right)x^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^(1/2)-(x^3)^(1/2))/(-x^3+x),x)
```

```
[Out] arctanh(x^(1/2))+arctan(x^(1/2))+1/2*(x^3)^(1/2)*(ln(-1+x^(1/2))-
ln(1+x^(1/2))+2*arctan(x^(1/2)))/x^(3/2)
```

Maxima [A] time = 0.852144, size = 58, normalized size = 1.12

$$2\arctan\left(\sqrt{x}\right)-\frac{1}{2}\log\left(4\sqrt{x}+4\right)+\frac{1}{2}\log\left(4\sqrt{x}-4\right)+\frac{1}{2}\log\left(\sqrt{x}+1\right)-\frac{1}{2}\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(x^3) - sqrt(x))/(x^3 - x),x, algorithm="maxima")
```

```
[Out] 2*arctan(sqrt(x)) - 1/2*log(4*sqrt(x) + 4) + 1/2*log(4*sqrt(x) -
4) + 1/2*log(sqrt(x) + 1) - 1/2*log(sqrt(x) - 1)
```

Fricas [A] time = 0.271974, size = 8, normalized size = 0.15

$$2\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(x^3) - sqrt(x))/(x^3 - x),x, algorithm="fricas")

[Out] 2*arctan(sqrt(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**(1/2)-(x**3)**(1/2))/(-x**3+x),x)

[Out] Timed out
```

GIAC/XCAS [A] time = 0.264907, size = 8, normalized size = 0.15

$$2 \arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(x^3) - sqrt(x))/(x^3 - x),x, algorithm="giac")

[Out] 2*arctan(sqrt(x))
```

3.666 $\int \frac{1}{\sqrt{x}+\sqrt{x^3}} dx$

Optimal. Leaf size=52

$$\frac{\sqrt{x^3}\tan^{-1}\left(\sqrt{x}\right)}{x^{3/2}}-\frac{\sqrt{x^3}\tanh^{-1}\left(\sqrt{x}\right)}{x^{3/2}}+\tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)$$

```
[Out] ArcTan[Sqrt[x]] + (Sqrt[x^3]*ArcTan[Sqrt[x]])/x^(3/2) + ArcTanh[S
qrt[x]] - (Sqrt[x^3]*ArcTanh[Sqrt[x]])/x^(3/2)
```

Rubi [A] time = 0.213682, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6

$$\frac{\sqrt{x^3}\tan^{-1}\left(\sqrt{x}\right)}{x^{3/2}}-\frac{\sqrt{x^3}\tanh^{-1}\left(\sqrt{x}\right)}{x^{3/2}}+\tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[x] + Sqrt[x^3])^(-1),x]
```

```
[Out] ArcTan[Sqrt[x]] + (Sqrt[x^3]*ArcTan[Sqrt[x]])/x^(3/2) + ArcTanh[S
qrt[x]] - (Sqrt[x^3]*ArcTanh[Sqrt[x]])/x^(3/2)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\sqrt{x}+\frac{\sqrt{x^3}}{3}+\operatorname{atan}\left(\sqrt{x}\right)+2\int^{\sqrt{x}}\frac{x-\sqrt{x^6}}{x}\,dx-2\int^{\sqrt{x}}\frac{\frac{x}{4}-\frac{\sqrt{x^6}}{4}}{x-1}\,dx-2\int^{\sqrt{x}}\frac{\frac{x}{4}-\frac{\sqrt{x^6}}{4}}{x+1}\,dx-\frac{\sqrt{x^3}}{x}+\frac{\sqrt{x^3}\operatorname{atan}\left(\sqrt{x}\right)}{x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**(1/2)+(x**3)**(1/2)),x)
```

```
[Out] -sqrt(x) + sqrt(x**3)/3 + atan(sqrt(x)) + 2*Integral((x - sqrt(x*
*6))/x, (x, sqrt(x))) - 2*Integral((x/4 - sqrt(x**6)/4)/(x - 1),
(x, sqrt(x))) - 2*Integral((x/4 - sqrt(x**6)/4)/(x + 1), (x, sqrt
(x))) - sqrt(x**3)/x + sqrt(x**3)*atan(sqrt(x))/x**(3/2)
```

Mathematica [A] time = 0.0863081, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x}+\sqrt{x^3}}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(Sqrt[x] + Sqrt[x^3])^(-1),x]
```

```
[Out] Integrate[(Sqrt[x] + Sqrt[x^3])^(-1), x]
```

Maple [A] time = 0.014, size = 30, normalized size = 0.6

$$2\,1\arctan\left(\sqrt{\frac{\sqrt{x^3}}{x^{3/2}}}\sqrt{x}\right)\frac{1}{\sqrt{\frac{\sqrt{x^3}}{x^{3/2}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^(1/2)+(x^3)^(1/2)),x)
```

```
[Out] 2/((x^3)^(1/2)/x^(3/2))^(1/2)*arctan(((x^3)^(1/2)/x^(3/2))^(1/2)*
x^(1/2))
```

Maxima [A] time = 1.16592, size = 8, normalized size = 0.15

$$2\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^3) + sqrt(x)),x, algorithm="maxima")
```

```
[Out] 2*arctan(sqrt(x))
```

Fricas [A] time = 0.269641, size = 8, normalized size = 0.15

$$2\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^3) + sqrt(x)),x, algorithm="fricas")
```

```
[Out] 2*arctan(sqrt(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x} + \sqrt{x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**(1/2)+(x**3)**(1/2)),x)
```

```
[Out] Integral(1/(sqrt(x) + sqrt(x**3)), x)
```

GIAC/XCAS [A] time = 0.266514, size = 8, normalized size = 0.15

$$2\,\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^3) + sqrt(x)),x, algorithm="giac")
```

```
[Out] 2*arctan(sqrt(x))
```

3.667 $\int \frac{1}{\sqrt{-1+x}+\sqrt{(-1+x)^3}}\,dx$

Optimal. Leaf size=68

$$\frac{\sqrt{(x-1)^3}\tan^{-1}\left(\sqrt{x-1}\right)}{(x-1)^{3/2}}+\tan^{-1}\left(\sqrt{x-1}\right)-\frac{\sqrt{(x-1)^3}\tanh^{-1}\left(\sqrt{x-1}\right)}{(x-1)^{3/2}}+\tanh^{-1}\left(\sqrt{x-1}\right)$$

```
[Out] ArcTan[Sqrt[-1 + x]] + (Sqrt[(-1 + x)^3]*ArcTan[Sqrt[-1 + x]])/(-
1 + x)^(3/2) + ArcTanh[Sqrt[-1 + x]] - (Sqrt[(-1 + x)^3]*ArcTanh[
Sqrt[-1 + x]])/(-1 + x)^(3/2)
```

Rubi [A] time = 0.255928, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474

$$\frac{\sqrt{(x-1)^3}\tan^{-1}\left(\sqrt{x-1}\right)}{(x-1)^{3/2}}+\tan^{-1}\left(\sqrt{x-1}\right)-\frac{\sqrt{(x-1)^3}\tanh^{-1}\left(\sqrt{x-1}\right)}{(x-1)^{3/2}}+\tanh^{-1}\left(\sqrt{x-1}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[-1 + x] + Sqrt[(-1 + x)^3])^(-1),x]
```

```
[Out] ArcTan[Sqrt[-1 + x]] + (Sqrt[(-1 + x)^3]*ArcTan[Sqrt[-1 + x]])/(-
1 + x)^(3/2) + ArcTanh[Sqrt[-1 + x]] - (Sqrt[(-1 + x)^3]*ArcTanh[
Sqrt[-1 + x]])/(-1 + x)^(3/2)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\sqrt{x-1}+\frac{\sqrt{(x-1)^3}}{3}+\operatorname{atan}\left(\sqrt{x-1}\right)+2\int^{\sqrt{x-1}}\frac{x-\sqrt{x^6}}{x}\,dx+2\int^{\sqrt{x-1}}\frac{-\frac{x}{4}+\frac{\sqrt{x^6}}{4}}{x-1}\,dx$$
$$+2\int^{\sqrt{x-1}}\frac{-\frac{x}{4}+\frac{\sqrt{x^6}}{4}}{x+1}\,dx+\frac{\sqrt{(x-1)^3}\operatorname{atan}\left(\sqrt{x-1}\right)}{(x-1)^{\frac{3}{2}}}+\frac{\sqrt{(x-1)^3}}{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/((-1+x)**(1/2)+((-1+x)**3)**(1/2)),x)
```

```
[Out] -sqrt(x - 1) + sqrt((x - 1)**3)/3 + atan(sqrt(x - 1)) + 2*Integra
l((x - sqrt(x**6))/x, (x, sqrt(x - 1))) + 2*Integral((-x/4 + sqrt
```

```
(x**6)/4)/(x - 1), (x, sqrt(x - 1))) + 2*Integral((-x/4 + sqrt(x*
*6)/4)/(x + 1), (x, sqrt(x - 1))) + sqrt((x - 1)**3)*atan(sqrt(x
- 1))/(x - 1)**(3/2) + sqrt((x - 1)**3)/(-x + 1)
```

Mathematica [A] time = 0.0381708, size = 51, normalized size = 0.75

$$\tan^{-1}\left(\sqrt{x-1}\right)+\tan^{-1}\left(\frac{\sqrt{(x-1)^3}}{x-1}\right)+\tanh^{-1}\left(\sqrt{x-1}\right)-\tanh^{-1}\left(\frac{\sqrt{(x-1)^3}}{x-1}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[-1 + x] + Sqrt[(-1 + x)^3])^(-1),x]
```

```
[Out] ArcTan[Sqrt[-1 + x]] + ArcTan[Sqrt[(-1 + x)^3]/(-1 + x)] + ArcTan
h[Sqrt[-1 + x]] - ArcTanh[Sqrt[(-1 + x)^3]/(-1 + x)]
```

Maple [A] time = 0.018, size = 40, normalized size = 0.6

$$2\,1\arctan\left(\sqrt{\frac{\sqrt{(-1+x)^3}}{(-1+x)^{3/2}}}\sqrt{-1+x}\right)\frac{1}{\sqrt{\frac{\sqrt{(-1+x)^3}}{(-1+x)^{3/2}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/((-1+x)^(1/2)+((-1+x)^3)^(1/2)),x)
```

```
[Out] 2/(((-1+x)^3)^(1/2)/(-1+x)^(3/2))^(1/2)*arctan((((-1+x)^3)^(1/2)/
(-1+x)^(3/2))^(1/2)*(-1+x)^(1/2))
```

Maxima [A] time = 1.41734, size = 11, normalized size = 0.16

$$2\arctan\left(\sqrt{x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt((x - 1)^3) + sqrt(x - 1)),x, algorithm="maxima")
```

```
[Out] 2*arctan(sqrt(x - 1))
```

Fricas [A] time = 0.268465, size = 11, normalized size = 0.16

$$2\arctan\left(\sqrt{x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt((x - 1)^3) + sqrt(x - 1)),x, algorithm="fricas")

[Out] 2*arctan(sqrt(x - 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x-1}+\sqrt{(x-1)^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((-1+x)**(1/2)+((-1+x)**3)**(1/2)),x)

[Out] Integral(1/(sqrt(x - 1) + sqrt((x - 1)**3)), x)
```

GIAC/XCAS [A] time = 0.263084, size = 11, normalized size = 0.16

$$2\arctan\left(\sqrt{x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt((x - 1)^3) + sqrt(x - 1)),x, algorithm="giac")

[Out] 2*arctan(sqrt(x - 1))
```

3.668 $\int \left(-\frac{3}{(4+5x)^2}-\frac{5+4x}{(4+5x)^2\sqrt{1-x^2}}\right) \, dx$

Optimal. Leaf size=31

$$\frac{\sqrt{1-x^2}}{5x+4}+\frac{3}{5(5x+4)}$$

```
[Out] 3/(5*(4 + 5*x)) + Sqrt[1 - x^2]/(4 + 5*x)
```

Rubi [A] time = 0.0580792, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029

$$\frac{\sqrt{1-x^2}}{5x+4}+\frac{3}{5(5x+4)}$$

Antiderivative was successfully verified.

```
[In]  Int[-3/(4 + 5*x)^2 - (5 + 4*x)/((4 + 5*x)^2*Sqrt[1 - x^2]),x]

[Out] 3/(5*(4 + 5*x)) + Sqrt[1 - x^2]/(4 + 5*x)
```

Rubi in Sympy [A] time = 2.61917, size = 19, normalized size = 0.61

$$\frac{\sqrt{-x^2+1}}{5x+4}+\frac{3}{5\,(5x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(-3/(4+5*x)**2+(-5-4*x)/(4+5*x)**2/(-x**2+1)**(1/2),x)

[Out] sqrt(-x**2 + 1)/(5*x + 4) + 3/(5*(5*x + 4))
```

Mathematica [A] time = 0.0532263, size = 23, normalized size = 0.74

$$\frac{5\sqrt{1-x^2}+3}{25x+20}$$

Antiderivative was successfully verified.

```
[In]  Integrate[-3/(4 + 5*x)^2 - (5 + 4*x)/((4 + 5*x)^2*Sqrt[1 - x^2]),x]
```

```
[Out] (3 + 5*Sqrt[1 - x^2])/(20 + 25*x)
```

Maple [A] time = 0.013, size = 32, normalized size = 1.

$$\frac{1}{5}\sqrt{-\left(x+\frac{4}{5}\right)^{2}+\frac{8\,x}{5}+\frac{41}{25}}\,\left(x+\frac{4}{5}\right)^{-1}+\frac{3}{20+25\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(-3/(4+5*x)^2+(-5-4*x)/(4+5*x)^2/(-x^2+1)^(1/2),x)
```

```
[Out] 1/5/(x+4/5)*(-(x+4/5)^2+8/5*x+41/25)^(1/2)+3/5/(4+5*x)
```

Maxima [A] time = 1.04935, size = 36, normalized size = 1.16

$$\frac{\sqrt{-x^2+1}}{5\,x+4}+\frac{3}{5\,(5\,x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(4*x + 5)/(sqrt(-x^2 + 1)*(5*x + 4)^2) - 3/(5*x + 4)^2,x, algorithm="ma
```

```
[Out] sqrt(-x^2 + 1)/(5*x + 4) + 3/5/(5*x + 4)
```

Fricas [A] time = 0.266149, size = 68, normalized size = 2.19

$$-\frac{20\,x^2-\sqrt{-x^2+1}(25\,x+12)+25\,x+12}{20\left(\sqrt{-x^2+1}(5\,x+4)-5\,x-4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(4*x + 5)/(sqrt(-x^2 + 1)*(5*x + 4)^2) - 3/(5*x + 4)^2,x, algorithm="fr
```

```
[Out] -1/20*(20*x^2 - sqrt(-x^2 + 1)*(25*x + 12) + 25*x + 12)/(sqrt(-x^
2 + 1)*(5*x + 4) - 5*x - 4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{4x}{25x^2\sqrt{-x^2+1}+40x\sqrt{-x^2+1}+16\sqrt{-x^2+1}}\,dx - \int \frac{3\sqrt{-x^2+1}}{25x^2\sqrt{-x^2+1}+40x\sqrt{-x^2+1}+16\sqrt{-x^2+1}}\,dx - \int \frac{5}{25x^2\sqrt{-x^2+1}+40x\sqrt{-x^2+1}+16\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-3/(4+5*x)**2+(-5-4*x)/(4+5*x)**2/(-x**2+1)**(1/2),x)
```

```
[Out] -Integral(4*x/(25*x**2*sqrt(-x**2 + 1) + 40*x*sqrt(-x**2 + 1) + 1
6*sqrt(-x**2 + 1)), x) - Integral(3*sqrt(-x**2 + 1)/(25*x**2*sqrt
(-x**2 + 1) + 40*x*sqrt(-x**2 + 1) + 16*sqrt(-x**2 + 1)), x) - In
tegral(5/(25*x**2*sqrt(-x**2 + 1) + 40*x*sqrt(-x**2 + 1) + 16*sqr
t(-x**2 + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{4\,x+5}{\sqrt{-x^2+1}(5\,x+4)^2} - \frac{3}{(5\,x+4)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(4*x + 5)/(sqrt(-x^2 + 1)*(5*x + 4)^2) - 3/(5*x + 4)^2,x, algorithm="gi
```

```
[Out] integrate(-(4*x + 5)/(sqrt(-x^2 + 1)*(5*x + 4)^2) - 3/(5*x + 4)^2
, x)
```

3.669 $\int \frac{-5-4x-3\sqrt{1-x^2}}{(4+5x)^2\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=31

$$\frac{\sqrt{1-x^2}}{5x+4}+\frac{3}{5(5x+4)}$$

```
[Out] 3/(5*(4 + 5*x)) + Sqrt[1 - x^2]/(4 + 5*x)
```

Rubi [A] time = 0.520675, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135

$$\frac{\sqrt{1-x^2}}{5x+4}+\frac{3}{5(5x+4)}$$

Antiderivative was successfully verified.

```
[In]  Int[(-5 - 4*x - 3*Sqrt[1 - x^2])/((4 + 5*x)^2*Sqrt[1 - x^2]),x]

[Out] 3/(5*(4 + 5*x)) + Sqrt[1 - x^2]/(4 + 5*x)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-5-4*x-3*(-x**2+1)**(1/2))/(4+5*x)**2/(-x**2+1)**(1/2),x)

[Out] Timed out
```

Mathematica [A] time = 0.0308358, size = 23, normalized size = 0.74

$$\frac{5\sqrt{1-x^2}+3}{25x+20}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-5 - 4*x - 3*Sqrt[1 - x^2])/((4 + 5*x)^2*Sqrt[1 - x^2]),x]
```

```
[Out] (3 + 5*Sqrt[1 - x^2])/(20 + 25*x)
```

Maple [A] time = 0.003, size = 32, normalized size = 1.

$$\frac{1}{5}\sqrt{-\left(x+\frac{4}{5}\right)^{2}+\frac{8x}{5}+\frac{41}{25}}\left(x+\frac{4}{5}\right)^{-1}+\frac{3}{20+25x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-5-4*x-3*(-x^2+1)^(1/2))/(4+5*x)^2/(-x^2+1)^(1/2),x)
```

```
[Out] 1/5/(x+4/5)*(-(x+4/5)^2+8/5*x+41/25)^(1/2)+3/5/(4+5*x)
```

Maxima [A] time = 0.795728, size = 34, normalized size = 1.1

$$\frac{5\sqrt{x+1}\sqrt{-x+1}+3}{5\,(5x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(4*x + 3*sqrt(-x^2 + 1) + 5)/(sqrt(-x^2 + 1)*(5*x + 4)^2),x, algorithm=
```

```
[Out] 1/5*(5*sqrt(x + 1)*sqrt(-x + 1) + 3)/(5*x + 4)
```

Fricas [A] time = 0.2666, size = 68, normalized size = 2.19

$$-\frac{20x^{2}-\sqrt{-x^{2}+1}(25x+12)+25x+12}{20\left(\sqrt{-x^{2}+1}(5x+4)-5x-4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(4*x + 3*sqrt(-x^2 + 1) + 5)/(sqrt(-x^2 + 1)*(5*x + 4)^2),x, algorithm=
```

```
[Out] -1/20*(20*x^2 - sqrt(-x^2 + 1)*(25*x + 12) + 25*x + 12)/(sqrt(-x^
2 + 1)*(5*x + 4) - 5*x - 4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{4x}{25x^2\sqrt{-x^2+1}+40x\sqrt{-x^2+1}+16\sqrt{-x^2+1}}\,dx - \int \frac{3\sqrt{-x^2+1}}{25x^2\sqrt{-x^2+1}+40x\sqrt{-x^2+1}+16\sqrt{-x^2+1}}\,dx - \int \frac{5}{25x^2\sqrt{-x^2+1}+40x\sqrt{-x^2+1}+16\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5-4*x-3*(-x**2+1)**(1/2))/(4+5*x)**2/(-x**2+1)**(1/2),x)
```

```
[Out] -Integral(4*x/(25*x**2*sqrt(-x**2 + 1) + 40*x*sqrt(-x**2 + 1) + 1
6*sqrt(-x**2 + 1)), x) - Integral(3*sqrt(-x**2 + 1)/(25*x**2*sqrt
(-x**2 + 1) + 40*x*sqrt(-x**2 + 1) + 16*sqrt(-x**2 + 1)), x) - In
tegral(5/(25*x**2*sqrt(-x**2 + 1) + 40*x*sqrt(-x**2 + 1) + 16*sqr
t(-x**2 + 1)), x)
```

GIAC/XCAS [A] time = 0.28688, size = 74, normalized size = 2.39

$$-\frac{1}{5}\, i\mathrm{sign}\left(\frac{1}{5\,x+4}\right) + \frac{\sqrt{\frac{8}{5\,x+4}+\frac{9}{(5\,x+4)^2}-1}}{5\,\mathrm{sign}\left(\frac{1}{5\,x+4}\right)} + \frac{3}{5\,(5\,x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(4*x + 3*sqrt(-x^2 + 1) + 5)/(sqrt(-x^2 + 1)*(5*x + 4)^2),x, algorithm=
```

```
[Out] -1/5*i*sign(1/(5*x + 4)) + 1/5*sqrt(8/(5*x + 4) + 9/(5*x + 4)^2 -
 1)/sign(1/(5*x + 4)) + 3/5/(5*x + 4)
```

3.670 $\int \frac{1}{(-5-4x)\sqrt{1-x^2}+3(1-x^2)}\,dx$

Optimal. Leaf size=31

$$\frac{\sqrt{1-x^2}}{5x+4}+\frac{3}{5(5x+4)}$$

```
[Out] 3/(5*(4 + 5*x)) + Sqrt[1 - x^2]/(4 + 5*x)
```

Rubi [A] time = 0.32459, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276

$$\frac{\sqrt{1-x^2}}{5x+4}+\frac{3}{5(5x+4)}$$

Antiderivative was successfully verified.

```
[In]  Int[((-5 - 4*x)*Sqrt[1 - x^2] + 3*(1 - x^2))^(-1),x]

[Out] 3/(5*(4 + 5*x)) + Sqrt[1 - x^2]/(4 + 5*x)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{-3x^2+(-4x-5)\sqrt{-x^2+1}+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-3*x**2+3+(-5-4*x)*(-x**2+1)**(1/2)),x)

[Out] Integral(1/(-3*x**2 + (-4*x - 5)*sqrt(-x**2 + 1) + 3), x)
```

Mathematica [A] time = 0.0412241, size = 23, normalized size = 0.74

$$\frac{5\sqrt{1-x^2}+3}{25x+20}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((-5 - 4*x)*Sqrt[1 - x^2] + 3*(1 - x^2))^(-1),x]
```

```
[Out] (3 + 5*Sqrt[1 - x^2])/(20 + 25*x)
```

Maple [B] time = 0.048, size = 81, normalized size = 2.6

$$\frac{3}{20+25x}-\frac{1}{2}\sqrt{-(1+x)^2+2+2x}+\frac{5}{9}\left(-\left(x+\frac{4}{5}\right)^2+\frac{8x}{5}+\frac{41}{25}\right)^{\frac{3}{2}}\left(x+\frac{4}{5}\right)^{-1}+\frac{5x}{9}\sqrt{-\left(x+\frac{4}{5}\right)^2+\frac{8x}{5}+\frac{41}{25}}+\frac{1}{18}\sqrt{-(-1+x)^2-2x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-3*x^2+3+(-5-4*x)*(-x^2+1)^(1/2)),x)
```

```
[Out] 3/5/(4+5*x)-1/2*(-(1+x)^2+2+2*x)^(1/2)+5/9/(x+4/5)*(-(x+4/5)^2+8/
5*x+41/25)^(3/2)+5/9*x*(-(x+4/5)^2+8/5*x+41/25)^(1/2)+1/18*(-(-1+
x)^2-2*x+2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{3x^2+\sqrt{-x^2+1}(4x+5)-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(3*x^2 + sqrt(-x^2 + 1)*(4*x + 5) - 3),x, algorithm="maxima")
```

```
[Out] -integrate(1/(3*x^2 + sqrt(-x^2 + 1)*(4*x + 5) - 3), x)
```

Fricas [A] time = 0.269386, size = 68, normalized size = 2.19

$$-\frac{20x^2-\sqrt{-x^2+1}(25x+12)+25x+12}{20\left(\sqrt{-x^2+1}(5x+4)-5x-4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/(3*x^2 + sqrt(-x^2 + 1)*(4*x + 5) - 3),x, algorithm="fricas")
```

```
[Out] -1/20*(20*x^2 - sqrt(-x^2 + 1)*(25*x + 12) + 25*x + 12)/(sqrt(-x^
2 + 1)*(5*x + 4) - 5*x - 4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{3x^2 + 4x\sqrt{-x^2+1} + 5\sqrt{-x^2+1} - 3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2+3+(-5-4*x)*(-x**2+1)**(1/2)),x)
```

```
[Out] -Integral(1/(3*x**2 + 4*x*sqrt(-x**2 + 1) + 5*sqrt(-x**2 + 1) - 3
), x)
```

GIAC/XCAS [A] time = 0.272032, size = 92, normalized size = 2.97

$$\frac{\frac{5\left(\sqrt{-x^2+1}-1\right)}{x} - 4}{4\left(\frac{5\left(\sqrt{-x^2+1}-1\right)}{x} - \frac{2\left(\sqrt{-x^2+1}-1\right)^2}{x^2} - 2\right)} + \frac{3}{5\,(5\,x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/(3*x^2 + sqrt(-x^2 + 1)*(4*x + 5) - 3),x, algorithm="giac")
```

```
[Out] 1/4*(5*(sqrt(-x^2 + 1) - 1)/x - 4)/(5*(sqrt(-x^2 + 1) - 1)/x - 2*
(sqrt(-x^2 + 1) - 1)^2/x^2 - 2) + 3/5/(5*x + 4)
```

3.671 $\int \frac{1}{3-3x^2-5\sqrt{1-x^2}-4x\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=31

$$\frac{\sqrt{1-x^2}}{5x+4}+\frac{3}{5(5x+4)}$$

```
[Out] 3/(5*(4 + 5*x)) + Sqrt[1 - x^2]/(4 + 5*x)
```

Rubi [A] time = 0.300407, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 8, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\frac{\sqrt{1-x^2}}{5x+4}+\frac{3}{5(5x+4)}$$

Antiderivative was successfully verified.

```
[In]  Int[(3 - 3*x^2 - 5*Sqrt[1 - x^2] - 4*x*Sqrt[1 - x^2])^(-1),x]
```

```
[Out] 3/(5*(4 + 5*x)) + Sqrt[1 - x^2]/(4 + 5*x)
```

Rubi in Sympy [A] time = 4.96378, size = 46, normalized size = 1.48

$$\frac{4+\frac{8\left(-\sqrt{-x^2+1}+1\right)}{x}}{8\left(1-\frac{4\left(\sqrt{-x^2+1}-1\right)}{x}+\frac{4\left(\sqrt{-x^2+1}-1\right)^2}{x^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(3-3*x**2-5*(-x**2+1)**(1/2)-4*x*(-x**2+1)**(1/2)),x)
```

```
[Out] (4 + 8*(-sqrt(-x**2 + 1) + 1)/x)/(8*(1 - 4*(sqrt(-x**2 + 1) - 1)/
x + 4*(sqrt(-x**2 + 1) - 1)**2/x**2))
```

Mathematica [A] time = 0.0285025, size = 23, normalized size = 0.74

$$\frac{5\sqrt{1-x^2}+3}{25x+20}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3 - 3*x^2 - 5*Sqrt[1 - x^2] - 4*x*Sqrt[1 - x^2])^(-1),x]
```

```
[Out] (3 + 5*Sqrt[1 - x^2])/(20 + 25*x)
```

Maple [B] time = 0.004, size = 81, normalized size = 2.6

$$\frac{3}{20+25x}-\frac{1}{2}\sqrt{-(1+x)^2+2+2x}+\frac{5}{9}\left(-\left(x+\frac{4}{5}\right)^2+\frac{8x}{5}+\frac{41}{25}\right)^{\frac{3}{2}}\left(x+\frac{4}{5}\right)^{-1}+\frac{5x}{9}\sqrt{-\left(x+\frac{4}{5}\right)^2+\frac{8x}{5}+\frac{41}{25}}+\frac{1}{18}\sqrt{-(-1+x)^2-2x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(3-3*x^2-5*(-x^2+1)^(1/2)-4*x*(-x^2+1)^(1/2)),x)
```

```
[Out] 3/5/(4+5*x)-1/2*(-(1+x)^2+2+2*x)^(1/2)+5/9/(x+4/5)*(-(x+4/5)^2+8/
5*x+41/25)^(3/2)+5/9*x*(-(x+4/5)^2+8/5*x+41/25)^(1/2)+1/18*(-(-1+
x)^2-2*x+2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{3x^2+4\sqrt{-x^2+1}x+5\sqrt{-x^2+1}-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/(3*x^2 + 4*sqrt(-x^2 + 1)*x + 5*sqrt(-x^2 + 1) - 3),x, algorithm="max
```

```
[Out] -integrate(1/(3*x^2 + 4*sqrt(-x^2 + 1)*x + 5*sqrt(-x^2 + 1) - 3),
 x)
```

Fricas [A] time = 0.266187, size = 68, normalized size = 2.19

$$-\frac{20x^2-\sqrt{-x^2+1}(25x+12)+25x+12}{20\left(\sqrt{-x^2+1}(5x+4)-5x-4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(3*x^2 + 4*sqrt(-x^2 + 1)*x + 5*sqrt(-x^2 + 1) - 3),x, algorithm="fri
```

```
[Out] -1/20*(20*x^2 - sqrt(-x^2 + 1)*(25*x + 12) + 25*x + 12)/(sqrt(-x^
2 + 1)*(5*x + 4) - 5*x - 4)
```

Sympy [F]   time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{3x^2 + 4x\sqrt{-x^2+1} + 5\sqrt{-x^2+1} - 3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3-3*x**2-5*(-x**2+1)**(1/2)-4*x*(-x**2+1)**(1/2)),x)
```

```
[Out] -Integral(1/(3*x**2 + 4*x*sqrt(-x**2 + 1) + 5*sqrt(-x**2 + 1) - 3
), x)
```

GIAC/XCAS [A]   time = 0.268847, size = 92, normalized size = 2.97

$$\frac{\frac{5\left(\sqrt{-x^2+1}-1\right)}{x} - 4}{4\left(\frac{5\left(\sqrt{-x^2+1}-1\right)}{x} - \frac{2\left(\sqrt{-x^2+1}-1\right)^2}{x^2} - 2\right)} + \frac{3}{5\,(5\,x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(3*x^2 + 4*sqrt(-x^2 + 1)*x + 5*sqrt(-x^2 + 1) - 3),x, algorithm="gia
```

```
[Out] 1/4*(5*(sqrt(-x^2 + 1) - 1)/x - 4)/(5*(sqrt(-x^2 + 1) - 1)/x - 2*
(sqrt(-x^2 + 1) - 1)^2/x^2 - 2) + 3/5/(5*x + 4)
```

3.672 $\int \frac{-1+\sqrt{1-x^2}}{\sqrt{1-x^2}\left(2+x-2\sqrt{1-x^2}\right)^2}\,dx$

Optimal. Leaf size=31

$$\frac{\sqrt{1-x^2}}{5x+4}+\frac{3}{5(5x+4)}$$

```
[Out] 3/(5*(4 + 5*x)) + Sqrt[1 - x^2]/(4 + 5*x)
```

Rubi [A] time = 1.24118, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 31, number of rules used = 14, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.326

$$\frac{\sqrt{1-x^2}}{5x+4}+\frac{3}{5(5x+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + Sqrt[1 - x^2])/(Sqrt[1 - x^2]*(2 + x - 2*Sqrt[1 - x^2])^2),x]
```

```
[Out] 3/(5*(4 + 5*x)) + Sqrt[1 - x^2]/(4 + 5*x)
```

Rubi in Sympy [A] time = 22.256, size = 15, normalized size = 0.48

$$\frac{1}{2\left(1-\frac{2\left(\sqrt{-x^2+1}-1\right)}{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-1+(-x**2+1)**(1/2))/(2+x-2*(-x**2+1)**(1/2))**2/(-
x**2+1)**(1/2),x)
```

```
[Out] 1/(2*(1 - 2*(sqrt(-x**2 + 1) - 1)/x))
```

Mathematica [A] time = 0.0435279, size = 23, normalized size = 0.74

$$\frac{5\sqrt{1-x^2}+3}{25x+20}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + Sqrt[1 - x^2])/(Sqrt[1 - x^2]*(2 + x - 2*Sqrt[1 - x^2])^2),x]
```

```
[Out] (3 + 5*Sqrt[1 - x^2])/(20 + 25*x)
```

Maple [A] time = 0.009, size = 32, normalized size = 1.

$$\frac{1}{5}\sqrt{-\left(x+\frac{4}{5}\right)^{2}+\frac{8x}{5}+\frac{41}{25}}\left(x+\frac{4}{5}\right)^{-1}+\frac{3}{20+25x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((-x^2+1)^(1/2)-1)/(2+x-2*(-x^2+1)^(1/2))^2/(-x^2+1)^(1/2),x)
```

```
[Out] 1/5/(x+4/5)*(-(x+4/5)^2+8/5*x+41/25)^(1/2)+3/5/(4+5*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{56}\sqrt{7}\log\left(\frac{3x-2\sqrt{7}-2}{3x+2\sqrt{7}-2}\right)-\int -\frac{100x^{7}+285x^{6}+264x^{5}+80x^{4}}{8\left(21x^{9}+278x^{8}+283x^{7}-2022x^{6}-3632x^{5}+2256x^{4}+7424x^{3}+1536x^{2}-8\left(9x^{8}+12x^{7}-101x^{6}-172x^{5}+284x^{4}\right.\right.}$$
$$-\frac{1}{24}\log(x+2)+\frac{1}{16}\log(x+1)-\frac{1}{48}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sqrt(-x^2 + 1) - 1)/(sqrt(-x^2 + 1)*(x - 2*sqrt(-
x^2 + 1) + 2)^2),x, algorithm="maxima")
```

```
[Out] -1/56*sqrt(7)*log((3*x - 2*sqrt(7) - 2)/(3*x + 2*sqrt(7) - 2)) -
integrate(-1/8*(100*x^7 + 285*x^6 + 264*x^5 + 80*x^4)/(21*x^9 + 2
78*x^8 + 283*x^7 - 2022*x^6 - 3632*x^5 + 2256*x^4 + 7424*x^3 + 15
36*x^2 - 8*(9*x^8 + 12*x^7 - 101*x^6 - 172*x^5 + 284*x^4 + 672*x^
3 + 64*x^2 - 512*x - 256)*sqrt(x + 1)*sqrt(-x + 1) - 4096*x - 204
8), x) - 1/24*log(x + 2) + 1/16*log(x + 1) - 1/48*log(x - 1)
```

Fricas [A] time = 0.26752, size = 68, normalized size = 2.19

$$-\frac{20\,x^2-\sqrt{-x^2+1}(25\,x+12)+25\,x+12}{20\left(\sqrt{-x^2+1}(5\,x+4)-5\,x-4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((sqrt(-x^2 + 1) - 1)/(sqrt(-x^2 + 1)*(x - 2*sqrt(-
x^2 + 1) + 2)^2),x, algorithm="fricas")
```

```
[Out] -1/20*(20*x^2 - sqrt(-x^2 + 1)*(25*x + 12) + 25*x + 12)/(sqrt(-x^
2 + 1)*(5*x + 4) - 5*x - 4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+(-x**2+1)**(1/2))/(2+x-2*(-x**2+1)**(1/2))**2/(-x**2+1)**(1/2),x
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.285564, size = 92, normalized size = 2.97

$$\frac{\frac{5\left(\sqrt{-x^2+1}-1\right)}{x}-4}{4\left(\frac{5\left(\sqrt{-x^2+1}-1\right)}{x}-\frac{2\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-2\right)}+\frac{3}{5\,(5\,x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((sqrt(-x^2 + 1) - 1)/(sqrt(-x^2 + 1)*(x - 2*sqrt(-
x^2 + 1) + 2)^2),x, algorithm="giac")
```

```
[Out] 1/4*(5*(sqrt(-x^2 + 1) - 1)/x - 4)/(5*(sqrt(-x^2 + 1) - 1)/x - 2*
(sqrt(-x^2 + 1) - 1)^2/x^2 - 2) + 3/5/(5*x + 4)
```

3.673 $\int \frac{a+bx^{-1+n}}{cx+dx^n}\,dx$

Optimal. Leaf size=43

$$\frac{b\log(x)}{d}-\frac{(bc-ad)\log\left(cx^{1-n}+d\right)}{cd(1-n)}$$

```
[Out] (b*Log[x])/d - ((b*c - a*d)*Log[d + c*x^(1 - n)])/(c*d*(1 - n))
```

Rubi [A] time = 0.167116, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19

$$\frac{b\log(x)}{d}-\frac{(bc-ad)\log\left(cx^{1-n}+d\right)}{cd(1-n)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^(-1 + n))/(c*x + d*x^n),x]
```

```
[Out] (b*Log[x])/d - ((b*c - a*d)*Log[d + c*x^(1 - n)])/(c*d*(1 - n))
```

Rubi in Sympy [A] time = 9.51961, size = 36, normalized size = 0.84

$$\frac{b\log\left(x^{-n+1}\right)}{d\left(-n+1\right)}+\frac{(ad-bc)\log\left(cx^{-n+1}+d\right)}{cd\left(-n+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b*x**(-1+n))/(c*x+d*x**n),x)
```

```
[Out] b*log(x**(-n + 1))/(d*(-n + 1)) + (a*d - b*c)*log(c*x**(-n + 1) +
 d)/(c*d*(-n + 1))
```

Mathematica [A] time = 0.0670188, size = 44, normalized size = 1.02

$$\frac{(bc-ad)\log\left(cx+dx^n\right)+\log(x)(adn-bc)}{cd(n-1)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^(-1 + n))/(c*x + d*x^n),x]
```

```
[Out] ((-(b*c) + a*d*n)*Log[x] + (b*c - a*d)*Log[c*x + d*x^n])/(c*d*(-1
 + n))
```

Maple [A] time = 0.025, size = 73, normalized size = 1.7

$$\frac{\ln(x)\,an}{c(-1+n)} - \frac{\ln(x)\,b}{d(-1+n)} - \frac{\ln\left(cx + d\mathrm{e}^{n\ln(x)}\right)a}{c(-1+n)} + \frac{\ln\left(cx + d\mathrm{e}^{n\ln(x)}\right)b}{d(-1+n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b*x^(-1+n))/(c*x+d*x^n),x)
```

```
[Out] 1/c/(-1+n)*ln(x)*a*n-1/d/(-1+n)*ln(x)*b-1/c/(-1+n)*ln(c*x+d*exp(n
*ln(x)))*a+1/d/(-1+n)*ln(c*x+d*exp(n*ln(x)))*b
```

Maxima [A] time = 0.740299, size = 115, normalized size = 2.67

$$b\left(\frac{\log(x)}{d} - \frac{n\log(x)}{d(n-1)} + \frac{\log\left(\frac{cx+dx^n}{d}\right)}{d(n-1)}\right) + a\left(\frac{n\log(x)}{c(n-1)} - \frac{\log\left(\frac{cx+dx^n}{d}\right)}{c(n-1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^(n - 1) + a)/(c*x + d*x^n),x, algorithm="maxima")
```

```
[Out] b*(log(x)/d - n*log(x)/(d*(n - 1)) + log((c*x + d*x^n)/d)/(d*(n -
 1))) + a*(n*log(x)/(c*(n - 1)) - log((c*x + d*x^n)/d)/(c*(n - 1)
))
```

Fricas [A] time = 0.297474, size = 59, normalized size = 1.37

$$\frac{(bc-ad)\log(cx+dx^n) + (adn-bc)\log(x)}{cdn-cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^(n - 1) + a)/(c*x + d*x^n),x, algorithm="fricas")
```

```
[Out] ((b*c - a*d)*log(c*x + d*x^n) + (a*d*n - b*c)*log(x))/(c*d*n - c*
d)
```

Sympy [A] time = 40.4087, size = 206, normalized size = 4.79

$$\begin{cases} \tilde{\infty}(a+b)\log(x) & \text{for } c=0 \wedge d=0 \wedge n=1 \\ \frac{-\frac{anx}{n^2x^n-nx^n}+\frac{bn^2x^n\log(x)}{n^2x^n-nx^n}-\frac{bnx^n\log(x)}{n^2x^n-nx^n}-\frac{bnx^n}{n^2x^n-nx^n}}{d} & \text{for } c=0 \\ \frac{\frac{an\log(x)}{n-1}-\frac{a\log(x)}{n-1}+\frac{bx^n}{nx-x}}{c} & \text{for } d=0 \\ \frac{(a+b)\log(x)}{c+d} & \text{for } n=1 \\ \frac{adn\log(x)}{cdn-cd}-\frac{ad\log\left(x+\frac{dx^n}{c}\right)}{cdn-cd}-\frac{bc\log(x)}{cdn-cd}+\frac{bc\log\left(x+\frac{dx^n}{c}\right)}{cdn-cd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b*x**(-1+n))/(c*x+d*x**n),x)
```

```
[Out] Piecewise((zoo*(a + b)*log(x), Eq(c, 0) & Eq(d, 0) & Eq(n, 1)), (
(-a*n*x/(n**2*x**n - n*x**n) + b*n**2*x**n*log(x)/(n**2*x**n - n*
x**n) - b*n*x**n*log(x)/(n**2*x**n - n*x**n) - b*n*x**n/(n**2*x**
n - n*x**n))/d, Eq(c, 0)), ((a*n*log(x)/(n - 1) - a*log(x)/(n - 1
) + b*x**n/(n*x - x))/c, Eq(d, 0)), ((a + b)*log(x)/(c + d), Eq(n
, 1)), (a*d*n*log(x)/(c*d*n - c*d) - a*d*log(x + d*x**n/c)/(c*d*n
 - c*d) - b*c*log(x)/(c*d*n - c*d) + b*c*log(x + d*x**n/c)/(c*d*n
 - c*d), True))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^{n-1}+a}{cx+dx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^(n - 1) + a)/(c*x + d*x^n),x, algorithm="giac")
```

```
[Out] integrate((b*x^(n - 1) + a)/(c*x + d*x^n), x)
```

3.674 $\int \frac{\sqrt{1+2x^2}}{1+\sqrt{1+2x^2}}\,dx$

Optimal. Leaf size=42

$$\frac{\sqrt{2x^2+1}}{2x}+x-\frac{1}{2x}-\frac{\sinh^{-1}\left(\sqrt{2}x\right)}{\sqrt{2}}$$

```
[Out] -1/(2*x) + x + Sqrt[1 + 2*x^2]/(2*x) - ArcSinh[Sqrt[2]*x]/Sqrt[2]
```

Rubi [A] time = 0.222706, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148

$$\frac{\sqrt{2x^2+1}}{2x}+x-\frac{1}{2x}-\frac{\sinh^{-1}\left(\sqrt{2}x\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 + 2*x^2]/(1 + Sqrt[1 + 2*x^2]),x]
```

```
[Out] -1/(2*x) + x + Sqrt[1 + 2*x^2]/(2*x) - ArcSinh[Sqrt[2]*x]/Sqrt[2]
```

Rubi in Sympy [A] time = 9.87343, size = 49, normalized size = 1.17

$$x-\frac{\sqrt{2}\log\left(\sqrt{2}x+\sqrt{2x^2+1}\right)}{2}-\frac{\sqrt{2}}{\sqrt{2}x+\sqrt{2x^2+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*x**2+1)**(1/2)/(1+(2*x**2+1)**(1/2)),x)
```

```
[Out] x - sqrt(2)*log(sqrt(2)*x + sqrt(2*x**2 + 1))/2 - sqrt(2)/(sqrt(2
)*x + sqrt(2*x**2 + 1) + 1)
```

Mathematica [A] time = 0.0227639, size = 42, normalized size = 1.

$$\frac{\sqrt{2x^2+1}}{2x}+x-\frac{1}{2x}-\frac{\sinh^{-1}\left(\sqrt{2}x\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 + 2*x^2]/(1 + Sqrt[1 + 2*x^2]),x]
```

```
[Out] -1/(2*x) + x + Sqrt[1 + 2*x^2]/(2*x) - ArcSinh[Sqrt[2]*x]/Sqrt[2]
```

Maple [A] time = 0.009, size = 45, normalized size = 1.1

$$x-\frac{1}{2x}+\frac{1}{2x}\left(2x^{2}+1\right)^{\frac{3}{2}}-x\sqrt{2x^{2}+1}-\frac{Arcsinh\left(\sqrt{2}x\right)\sqrt{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*x^2+1)^(1/2)/(1+(2*x^2+1)^(1/2)),x)
```

```
[Out] x-1/2/x+1/2/x*(2*x^2+1)^(3/2)-x*(2*x^2+1)^(1/2)-1/2*arcsinh(2^(1/
2)*x)*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$x-\int\frac{1}{\sqrt{2x^{2}+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(2*x^2 + 1)/(sqrt(2*x^2 + 1) + 1),x, algorithm="maxima")
```

```
[Out] x - integrate(1/(sqrt(2*x^2 + 1) + 1), x)
```

Fricas [A] time = 0.270421, size = 142, normalized size = 3.38

$$\frac{\left(\sqrt{2}\sqrt{2x^{2}+1}x-\sqrt{2}x\right)\log\left(-\frac{2x^{2}-\sqrt{2x^{2}+1}\left(\sqrt{2}x+1\right)+\sqrt{2}x+1}{\sqrt{2x^{2}+1}-1}\right)+2\sqrt{2x^{2}+1}\left(x^{2}-1\right)+2}{2\left(\sqrt{2x^{2}+1}x-x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(2*x^2 + 1)/(sqrt(2*x^2 + 1) + 1),x, algorithm="fricas")
```

```
[Out] 1/2*((sqrt(2)*sqrt(2*x^2 + 1)*x - sqrt(2)*x)*log(-(2*x^2 - sqrt(2
*x^2 + 1)*(sqrt(2)*x + 1) + sqrt(2)*x + 1)/(sqrt(2*x^2 + 1) - 1))
 + 2*sqrt(2*x^2 + 1)*(x^2 - 1) + 2)/(sqrt(2*x^2 + 1)*x - x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2x^2+1}}{\sqrt{2x^2+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x**2+1)**(1/2)/(1+(2*x**2+1)**(1/2)),x)
```

```
[Out] Integral(sqrt(2*x**2 + 1)/(sqrt(2*x**2 + 1) + 1), x)
```

GIAC/XCAS [A] time = 0.273292, size = 77, normalized size = 1.83

$$\frac{1}{2}\,\sqrt{2}\ln\left(-\sqrt{2}x+\sqrt{2\,x^2+1}\right)+x-\frac{\sqrt{2}}{\left(\sqrt{2}x-\sqrt{2\,x^2+1}\right)^2-1}-\frac{1}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(2*x^2 + 1)/(sqrt(2*x^2 + 1) + 1),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*ln(-sqrt(2)*x + sqrt(2*x^2 + 1)) + x - sqrt(2)/((sqrt
(2)*x - sqrt(2*x^2 + 1))^2 - 1) - 1/2/x
```

3.675 $\int \frac{\sqrt{-1+4x^2}}{x+\sqrt{-1+4x^2}}\,dx$

Optimal. Leaf size=65

$$-\frac{1}{3}\sqrt{4x^2-1}+\frac{\tanh^{-1}\left(\sqrt{3}\sqrt{4x^2-1}\right)}{3\sqrt{3}}+\frac{4x}{3}-\frac{\tanh^{-1}\left(\sqrt{3}x\right)}{3\sqrt{3}}$$

```
[Out] (4*x)/3 - Sqrt[-1 + 4*x^2]/3 - ArcTanh[Sqrt[3]*x]/(3*Sqrt[3]) + A
rcTanh[Sqrt[3]*Sqrt[-1 + 4*x^2]]/(3*Sqrt[3])
```

Rubi [A] time = 0.256962, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$-\frac{1}{3}\sqrt{4x^2-1}+\frac{\tanh^{-1}\left(\sqrt{3}\sqrt{4x^2-1}\right)}{3\sqrt{3}}+\frac{4x}{3}-\frac{\tanh^{-1}\left(\sqrt{3}x\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[-1 + 4*x^2]/(x + Sqrt[-1 + 4*x^2]),x]
```

```
[Out] (4*x)/3 - Sqrt[-1 + 4*x^2]/3 - ArcTanh[Sqrt[3]*x]/(3*Sqrt[3]) + A
rcTanh[Sqrt[3]*Sqrt[-1 + 4*x^2]]/(3*Sqrt[3])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{4x^2-1}}{x+\sqrt{4x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((4*x**2-1)**(1/2)/(x+(4*x**2-1)**(1/2)),x)
```

```
[Out] Integral(sqrt(4*x**2 - 1)/(x + sqrt(4*x**2 - 1)), x)
```

Mathematica [A] time = 0.0742754, size = 98, normalized size = 1.51

$$\frac{1}{18}\left(-6\sqrt{4x^2-1}+\sqrt{3}\log\left(-\sqrt{12x^2-3}-4\sqrt{3}x+3\right)+\sqrt{3}\log\left(-\sqrt{12x^2-3}+4\sqrt{3}x+3\right)+24x-2\sqrt{3}\log\left(3x+\sqrt{3}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[-1 + 4*x^2]/(x + Sqrt[-1 + 4*x^2]),x]
```

```
[Out] (24*x - 6*Sqrt[-1 + 4*x^2] - 2*Sqrt[3]*Log[Sqrt[3] + 3*x] + Sqrt[
3]*Log[3 - 4*Sqrt[3]*x - Sqrt[-3 + 12*x^2]] + Sqrt[3]*Log[3 + 4*S
qrt[3]*x - Sqrt[-3 + 12*x^2]])/18
```

Maple [B] time = 0.049, size = 262, normalized size = 4.

$$\frac{4x}{3}-\frac{\mathit{Artanh}\left(x\sqrt{3}\right)\sqrt{3}}{9}-\frac{1}{18}\sqrt{36\left(x-1/3\sqrt{3}\right)^{2}+24\left(x-1/3\sqrt{3}\right)\sqrt{3}+3}$$
$$-\frac{\sqrt{3}\sqrt{4}}{18}\ln\left(x\sqrt{4}+\sqrt{4\left(x-1/3\sqrt{3}\right)^{2}+\frac{8\sqrt{3}}{3}\left(x-\frac{\sqrt{3}}{3}\right)+\frac{1}{3}}\right)$$
$$+\frac{\sqrt{3}}{18}\mathit{Artanh}\left(\frac{3\sqrt{3}}{2}\left(\frac{2}{3}+\frac{8\sqrt{3}}{3}\left(x-\frac{\sqrt{3}}{3}\right)\right)\frac{1}{\sqrt{36\left(x-1/3\sqrt{3}\right)^{2}+24\left(x-1/3\sqrt{3}\right)\sqrt{3}+3}}\right)$$
$$-\frac{1}{18}\sqrt{36\left(x+1/3\sqrt{3}\right)^{2}-24\left(x+1/3\sqrt{3}\right)\sqrt{3}+3}$$
$$+\frac{\sqrt{3}\sqrt{4}}{18}\ln\left(x\sqrt{4}+\sqrt{4\left(x+1/3\sqrt{3}\right)^{2}-\frac{8\sqrt{3}}{3}\left(x+\frac{\sqrt{3}}{3}\right)+\frac{1}{3}}\right)$$
$$+\frac{\sqrt{3}}{18}\mathit{Artanh}\left(\frac{3\sqrt{3}}{2}\left(\frac{2}{3}-\frac{8\sqrt{3}}{3}\left(x+\frac{\sqrt{3}}{3}\right)\right)\frac{1}{\sqrt{36\left(x+1/3\sqrt{3}\right)^{2}-24\left(x+1/3\sqrt{3}\right)\sqrt{3}+3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((4*x^2-1)^(1/2)/(x+(4*x^2-1)^(1/2)),x)
```

```
[Out] 4/3*x-1/9*arctanh(x*3^(1/2))*3^(1/2)-1/18*(36*(x-1/3*3^(1/2))^2+2
4*(x-1/3*3^(1/2))*3^(1/2)+3)^(1/2)-1/18*3^(1/2)*ln(x*4^(1/2)+(4*(
x-1/3*3^(1/2))^2+8/3*(x-1/3*3^(1/2))*3^(1/2)+1/3)^(1/2))*4^(1/2)+
1/18*3^(1/2)*arctanh(3/2*(2/3+8/3*(x-1/3*3^(1/2))*3^(1/2))*3^(1/2
)/(36*(x-1/3*3^(1/2))^2+24*(x-1/3*3^(1/2))*3^(1/2)+3)^(1/2))-1/18
*(36*(x+1/3*3^(1/2))^2-24*(x+1/3*3^(1/2))*3^(1/2)+3)^(1/2)+1/18*3
^(1/2)*ln(x*4^(1/2)+(4*(x+1/3*3^(1/2))^2-8/3*(x+1/3*3^(1/2))*3^(1
/2)+1/3)^(1/2))*4^(1/2)+1/18*3^(1/2)*arctanh(3/2*(2/3-8/3*(x+1/3*
3^(1/2))*3^(1/2))*3^(1/2)/(36*(x+1/3*3^(1/2))^2-24*(x+1/3*3^(1/2)
)*3^(1/2)+3)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$x - \int \frac{x}{\sqrt{2\,x+1}\sqrt{2\,x-1}+x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(4*x^2 - 1)/(x + sqrt(4*x^2 - 1)),x, algorithm="maxima")
```

```
[Out] x - integrate(x/(sqrt(2*x + 1)*sqrt(2*x - 1) + x), x)
```

Fricas [A] time = 0.277461, size = 300, normalized size = 4.62

$$\frac{\left(2\,x-\sqrt{4\,x^2-1}\right)\log\left(-\frac{48\,x^3-\sqrt{3}\left(48\,x^4-14\,x^2+1\right)-\left(24\,x^2-4\,\sqrt{3}\left(6\,x^3-x\right)-3\right)\sqrt{4\,x^2-1}-12\,x}{24\,x^4-11\,x^2-4\,(3\,x^3-x)\sqrt{4\,x^2-1}+1}\right)+2\,x\log\left(\frac{\sqrt{3}\left(3\,x^2+1\right)-6\,x}{3\,x^2-1}\right)+2\,\sqrt{3}\left(12\,x^2-1\right)}{6\left(2\,\sqrt{3}x-\sqrt{3}\sqrt{4\,x^2-1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(4*x^2 - 1)/(x + sqrt(4*x^2 - 1)),x, algorithm="fricas")
```

```
[Out] 1/6*((2*x - sqrt(4*x^2 - 1))*log(-(48*x^3 - sqrt(3)*(48*x^4 - 14*
x^2 + 1) - (24*x^2 - 4*sqrt(3)*(6*x^3 - x) - 3)*sqrt(4*x^2 - 1) -
 12*x)/(24*x^4 - 11*x^2 - 4*(3*x^3 - x)*sqrt(4*x^2 - 1) + 1)) + 2
*x*log((sqrt(3)*(3*x^2 + 1) - 6*x)/(3*x^2 - 1)) + 2*sqrt(3)*(12*x
^2 - 1) - sqrt(4*x^2 - 1)*(12*sqrt(3)*x + log((sqrt(3)*(3*x^2 + 1
) - 6*x)/(3*x^2 - 1))))/(2*sqrt(3)*x - sqrt(3)*sqrt(4*x^2 - 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(2x-1)\,(2x+1)}}{x+\sqrt{4x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x**2-1)**(1/2)/(x+(4*x**2-1)**(1/2)),x)
```

```
[Out] Integral(sqrt((2*x - 1)*(2*x + 1))/(x + sqrt(4*x**2 - 1)), x)
```

GIAC/XCAS [A] time = 0.277628, size = 180, normalized size = 2.77

$$\frac{1}{18}\sqrt{3}\ln\left(\frac{\left|6x-2\sqrt{3}\right|}{\left|6x+2\sqrt{3}\right|}\right)-\frac{1}{18}\sqrt{3}\ln\left(-\frac{\left|-12x-4\sqrt{3}+6\sqrt{4x^2-1}+\frac{6}{2x-\sqrt{4x^2-1}}\right|}{2\left(6x-2\sqrt{3}-3\sqrt{4x^2-1}-\frac{3}{2x-\sqrt{4x^2-1}}\right)}\right)+\frac{4}{3}x-\frac{1}{3}\sqrt{4x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(4*x^2 - 1)/(x + sqrt(4*x^2 - 1)),x, algorithm="giac")
```

```
[Out] 1/18*sqrt(3)*ln(abs(6*x - 2*sqrt(3))/abs(6*x + 2*sqrt(3))) - 1/18
*sqrt(3)*ln(-1/2*abs(-12*x - 4*sqrt(3) + 6*sqrt(4*x^2 - 1) + 6/(2
*x - sqrt(4*x^2 - 1)))/(6*x - 2*sqrt(3) - 3*sqrt(4*x^2 - 1) - 3/(
2*x - sqrt(4*x^2 - 1)))) + 4/3*x - 1/3*sqrt(4*x^2 - 1)
```

3.676 $\int \frac{a+bx+cx^2}{(d+ex)^3\sqrt{-1+x^2}} dx$

Optimal. Leaf size=195

$$-\frac{\sqrt{x^2-1}\left(ae^2-bde+cd^2\right)}{2e\left(d^2-e^2\right)(d+ex)^2}-\frac{\tanh^{-1}\left(\frac{dx+e}{\sqrt{x^2-1}\sqrt{d^2-e^2}}\right)\left(-a\left(2d^2+e^2\right)+3bde-c\left(d^2+2e^2\right)\right)}{2\left(d^2-e^2\right)^{5/2}}+\frac{\sqrt{x^2-1}\left(c\left(d^3-4de^2\right)-e\left(3ade-b\left(d^2+2e^2\right)\right)\right)}{2e\left(d^2-e^2\right)^2(d+ex)}$$

```
[Out] -((c*d^2 - b*d*e + a*e^2)*Sqrt[-1 + x^2])/(2*e*(d^2 - e^2)*(d + e
*x)^2) + ((c*(d^3 - 4*d*e^2) - e*(3*a*d*e - b*(d^2 + 2*e^2)))*Sqr
t[-1 + x^2])/(2*e*(d^2 - e^2)^2*(d + e*x)) - ((3*b*d*e - a*(2*d^2
 + e^2) - c*(d^2 + 2*e^2))*ArcTanh[(e + d*x)/(Sqrt[d^2 - e^2]*Sqr
t[-1 + x^2])])/(2*(d^2 - e^2)^(5/2))
```

Rubi [A] time = 0.46931, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$

$$-\frac{\sqrt{x^2-1}\left(ae^2-bde+cd^2\right)}{2e\left(d^2-e^2\right)(d+ex)^2}-\frac{\tanh^{-1}\left(\frac{dx+e}{\sqrt{x^2-1}\sqrt{d^2-e^2}}\right)\left(-a\left(2d^2+e^2\right)+3bde-c\left(d^2+2e^2\right)\right)}{2\left(d^2-e^2\right)^{5/2}}+\frac{\sqrt{x^2-1}\left(c\left(d^3-4de^2\right)-e\left(3ade-b\left(d^2+2e^2\right)\right)\right)}{2e\left(d^2-e^2\right)^2(d+ex)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x + c*x^2)/((d + e*x)^3*Sqrt[-1 + x^2]),x]
```

```
[Out] -((c*d^2 - b*d*e + a*e^2)*Sqrt[-1 + x^2])/(2*e*(d^2 - e^2)*(d + e
*x)^2) + ((c*(d^3 - 4*d*e^2) - e*(3*a*d*e - b*(d^2 + 2*e^2)))*Sqr
t[-1 + x^2])/(2*e*(d^2 - e^2)^2*(d + e*x)) - ((3*b*d*e - a*(2*d^2
 + e^2) - c*(d^2 + 2*e^2))*ArcTanh[(e + d*x)/(Sqrt[d^2 - e^2]*Sqr
t[-1 + x^2])])/(2*(d^2 - e^2)^(5/2))
```

Rubi in Sympy [A] time = 39.1502, size = 170, normalized size = 0.87

$$-\frac{\left(2ad^2+ae^2-3bde+cd^2+2ce^2\right)\operatorname{atanh}\left(\frac{-dx-e}{\sqrt{d^2-e^2}\sqrt{x^2-1}}\right)}{2\left(d^2-e^2\right)^{\frac{5}{2}}}+\frac{\sqrt{x^2-1}\left(-3ade^2+bd^2e+2be^3+cd^3-4cde^2\right)}{2e(d+ex)\left(d^2-e^2\right)^2}-\frac{\sqrt{x^2-1}\left(ae^2-bde+cd^2\right)}{2e(d+ex)^2\left(d^2-e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**2+b*x+a)/(e*x+d)**3/(x**2-1)**(1/2),x)
```

```
[Out] -(2*a*d**2 + a*e**2 - 3*b*d*e + c*d**2 + 2*c*e**2)*atanh((-d*x -
e)/(sqrt(d**2 - e**2)*sqrt(x**2 - 1)))/(2*(d**2 - e**2)**(5/2)) +
 sqrt(x**2 - 1)*(-3*a*d*e**2 + b*d**2*e + 2*b*e**3 + c*d**3 - 4*c
*d*e**2)/(2*e*(d + e*x)*(d**2 - e**2)**2) - sqrt(x**2 - 1)*(a*e**
2 - b*d*e + c*d**2)/(2*e*(d + e*x)**2*(d**2 - e**2))
```

Mathematica [A] time = 0.437397, size = 240, normalized size = 1.23

$$\frac{1}{2}\left(-\frac{\log\left(-\sqrt{x^2-1}\sqrt{d^2-e^2}+dx+e\right)\left(a\left(2d^2+e^2\right)-3bde+c\left(d^2+2e^2\right)\right)}{(d-e)^2(d+e)^2\sqrt{d^2-e^2}}+\frac{\log(d+ex)\left(a\left(2d^2+e^2\right)-3bde+c\left(d^2+2e^2\right)\right)}{(d-e)^2(d+e)^2\sqrt{d^2-e^2}}+\frac{\sqrt{x^2-1}\left(ae\left(-4d^2-3dex+e^2\right)+b\left(2d^3+d^2ex+de^2+2e^3x\right)+cd\left(d^2x-3de-4e^2x\right)\right)}{\left(d^2-e^2\right)^2(d+ex)^2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x + c*x^2)/((d + e*x)^3*Sqrt[-1 + x^2]),x]
```

```
[Out] ((Sqrt[-1 + x^2]*(a*e*(-4*d^2 + e^2 - 3*d*e*x) + c*d*(-3*d*e + d^
2*x - 4*e^2*x) + b*(2*d^3 + d*e^2 + d^2*e*x + 2*e^3*x)))/((d^2 -
e^2)^2*(d + e*x)^2) + ((-3*b*d*e + a*(2*d^2 + e^2) + c*(d^2 + 2*e
^2))*Log[d + e*x])/((d - e)^2*(d + e)^2*Sqrt[d^2 - e^2]) - ((-3*b
*d*e + a*(2*d^2 + e^2) + c*(d^2 + 2*e^2))*Log[e + d*x - Sqrt[d^2
- e^2]*Sqrt[-1 + x^2]])/((d - e)^2*(d + e)^2*Sqrt[d^2 - e^2]))/2
```

Maple [B] time = 0.045, size = 1407, normalized size = 7.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^2+b*x+a)/(e*x+d)^3/(x^2-1)^(1/2),x)
```

```
[Out] -c/e^3/((d^2-e^2)/e^2)^(1/2)*ln((2*(d^2-e^2)/e^2-2*d/e*(x+d/e)+2*
((d^2-e^2)/e^2)^(1/2)*((x+d/e)^2-2*d/e*(x+d/e)+(d^2-e^2)/e^2)^(1/
```

```
2))/(x+d/e))-1/e/(d^2-e^2)/(x+d/e)*((x+d/e)^2-2*d/e*(x+d/e)+(d^2-
e^2)/e^2)^(1/2)*b+2/e^2/(d^2-e^2)/(x+d/e)*((x+d/e)^2-2*d/e*(x+d/e
)+(d^2-e^2)/e^2)^(1/2)*c*d-3/2/e^2*d/(d^2-e^2)/((d^2-e^2)/e^2)^(1
/2)*ln((2*(d^2-e^2)/e^2-2*d/e*(x+d/e)+2*((d^2-e^2)/e^2)^(1/2)*((x
+d/e)^2-2*d/e*(x+d/e)+(d^2-e^2)/e^2)^(1/2))/(x+d/e))*b+5/2/e^3*d^
2/(d^2-e^2)/((d^2-e^2)/e^2)^(1/2)*ln((2*(d^2-e^2)/e^2-2*d/e*(x+d/
e)+2*((d^2-e^2)/e^2)^(1/2)*((x+d/e)^2-2*d/e*(x+d/e)+(d^2-e^2)/e^2
)^(1/2))/(x+d/e))*c-1/2/e/(d^2-e^2)/(x+d/e)^2*((x+d/e)^2-2*d/e*(x
+d/e)+(d^2-e^2)/e^2)^(1/2)*a+1/2/e^2/(d^2-e^2)/(x+d/e)^2*((x+d/e)
^2-2*d/e*(x+d/e)+(d^2-e^2)/e^2)^(1/2)*b*d-1/2/e^3/(d^2-e^2)/(x+d/
e)^2*((x+d/e)^2-2*d/e*(x+d/e)+(d^2-e^2)/e^2)^(1/2)*c*d^2-3/2*d/(d
^2-e^2)^2/(x+d/e)*((x+d/e)^2-2*d/e*(x+d/e)+(d^2-e^2)/e^2)^(1/2)*a
+3/2/e*d^2/(d^2-e^2)^2/(x+d/e)*((x+d/e)^2-2*d/e*(x+d/e)+(d^2-e^2)
/e^2)^(1/2)*b-3/2/e^2*d^3/(d^2-e^2)^2/(x+d/e)*((x+d/e)^2-2*d/e*(x
+d/e)+(d^2-e^2)/e^2)^(1/2)*c-3/2/e*d^2/(d^2-e^2)^2/((d^2-e^2)/e^2
)^(1/2)*ln((2*(d^2-e^2)/e^2-2*d/e*(x+d/e)+2*((d^2-e^2)/e^2)^(1/2)
*((x+d/e)^2-2*d/e*(x+d/e)+(d^2-e^2)/e^2)^(1/2))/(x+d/e))*a+3/2/e^
2*d^3/(d^2-e^2)^2/((d^2-e^2)/e^2)^(1/2)*ln((2*(d^2-e^2)/e^2-2*d/e
*(x+d/e)+2*((d^2-e^2)/e^2)^(1/2)*((x+d/e)^2-2*d/e*(x+d/e)+(d^2-e^
2)/e^2)^(1/2))/(x+d/e))*b-3/2/e^3*d^4/(d^2-e^2)^2/((d^2-e^2)/e^2)
^(1/2)*ln((2*(d^2-e^2)/e^2-2*d/e*(x+d/e)+2*((d^2-e^2)/e^2)^(1/2)*
((x+d/e)^2-2*d/e*(x+d/e)+(d^2-e^2)/e^2)^(1/2))/(x+d/e))*c+1/2/e/(
d^2-e^2)/((d^2-e^2)/e^2)^(1/2)*ln((2*(d^2-e^2)/e^2-2*d/e*(x+d/e)+
2*((d^2-e^2)/e^2)^(1/2)*((x+d/e)^2-2*d/e*(x+d/e)+(d^2-e^2)/e^2)^(
1/2))/(x+d/e))*a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2 + b*x + a)/((e*x + d)^3*sqrt(x^2 - 1)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.309867, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2 + b*x + a)/((e*x + d)^3*sqrt(x^2 - 1)),x, algorithm="fricas")
```

```
[Out] [1/2*((2*b*d^3*e^2 - (4*a + 3*c)*d^2*e^3 + b*d*e^4 + a*e^5 + 2*(2
*c*d^4*e - (2*a + 5*c)*d^2*e^3 + 3*b*d*e^4 - a*e^5)*x^2 + (2*c*d^
```

```
5 + 2*b*d^4*e - (6*a + 7*c)*d^3*e^2 + 5*b*d^2*e^3 - (3*a + 4*c)*d
*e^4 + 2*b*e^5)*x)*sqrt(d^2 - e^2)*sqrt(x^2 - 1) + ((2*a + c)*d^4
*e^2 - 3*b*d^3*e^3 + (a + 2*c)*d^2*e^4 - 2*((2*a + c)*d^2*e^4 - 3
*b*d*e^5 + (a + 2*c)*e^6)*x^4 - 4*((2*a + c)*d^3*e^3 - 3*b*d^2*e^
4 + (a + 2*c)*d*e^5)*x^3 - (2*(2*a + c)*d^4*e^2 - 6*b*d^3*e^3 + 3
*c*d^2*e^4 + 3*b*d*e^5 - (a + 2*c)*e^6)*x^2 + 2*((2*a + c)*d^3*e^
3 - 3*b*d^2*e^4 + (a + 2*c)*d*e^5)*x + 2*(((2*a + c)*d^2*e^4 - 3*
b*d*e^5 + (a + 2*c)*e^6)*x^3 + 2*((2*a + c)*d^3*e^3 - 3*b*d^2*e^4
 + (a + 2*c)*d*e^5)*x^2 + ((2*a + c)*d^4*e^2 - 3*b*d^3*e^3 + (a +
 2*c)*d^2*e^4)*x)*sqrt(x^2 - 1))*log((d^3 - d*e^2 + (d^2*e - e^3)
*x + (e^2*x^2 + d*e*x + d^2 - e^2)*sqrt(d^2 - e^2) - (d^2*e - e^3
 + (e^2*x + d*e)*sqrt(d^2 - e^2))*sqrt(x^2 - 1))/(e*x^2 + d*x - (
e*x + d)*sqrt(x^2 - 1))) + (c*d^5 + b*d^4*e - (3*a + 4*c)*d^3*e^2
 + 2*b*d^2*e^3 - 2*(2*c*d^4*e - (2*a + 5*c)*d^2*e^3 + 3*b*d*e^4 -
 a*e^5)*x^3 - (2*c*d^5 + 2*b*d^4*e - (6*a + 7*c)*d^3*e^2 + 5*b*d^
2*e^3 - (3*a + 4*c)*d*e^4 + 2*b*e^5)*x^2 + 2*(c*d^4*e - b*d^3*e^2
 + (a - c)*d^2*e^3 + b*d*e^4 - a*e^5)*x)*sqrt(d^2 - e^2))/(2*((d^
4*e^4 - 2*d^2*e^6 + e^8)*x^3 + 2*(d^5*e^3 - 2*d^3*e^5 + d*e^7)*x^
2 + (d^6*e^2 - 2*d^4*e^4 + d^2*e^6)*x)*sqrt(d^2 - e^2)*sqrt(x^2 -
 1) + (d^6*e^2 - 2*d^4*e^4 + d^2*e^6 - 2*(d^4*e^4 - 2*d^2*e^6 + e
^8)*x^4 - 4*(d^5*e^3 - 2*d^3*e^5 + d*e^7)*x^3 - (2*d^6*e^2 - 5*d^
4*e^4 + 4*d^2*e^6 - e^8)*x^2 + 2*(d^5*e^3 - 2*d^3*e^5 + d*e^7)*x)
*sqrt(d^2 - e^2)), 1/2*((2*b*d^3*e^2 - (4*a + 3*c)*d^2*e^3 + b*d*
e^4 + a*e^5 + 2*(2*c*d^4*e - (2*a + 5*c)*d^2*e^3 + 3*b*d*e^4 - a*
e^5)*x^2 + (2*c*d^5 + 2*b*d^4*e - (6*a + 7*c)*d^3*e^2 + 5*b*d^2*e
^3 - (3*a + 4*c)*d*e^4 + 2*b*e^5)*x)*sqrt(-d^2 + e^2)*sqrt(x^2 -
1) + 2*((2*a + c)*d^4*e^2 - 3*b*d^3*e^3 + (a + 2*c)*d^2*e^4 - 2*(
(2*a + c)*d^2*e^4 - 3*b*d*e^5 + (a + 2*c)*e^6)*x^4 - 4*((2*a + c)
*d^3*e^3 - 3*b*d^2*e^4 + (a + 2*c)*d*e^5)*x^3 - (2*(2*a + c)*d^4*
e^2 - 6*b*d^3*e^3 + 3*c*d^2*e^4 + 3*b*d*e^5 - (a + 2*c)*e^6)*x^2
+ 2*((2*a + c)*d^3*e^3 - 3*b*d^2*e^4 + (a + 2*c)*d*e^5)*x + 2*(((
2*a + c)*d^2*e^4 - 3*b*d*e^5 + (a + 2*c)*e^6)*x^3 + 2*((2*a + c)*
d^3*e^3 - 3*b*d^2*e^4 + (a + 2*c)*d*e^5)*x^2 + ((2*a + c)*d^4*e^2
 - 3*b*d^3*e^3 + (a + 2*c)*d^2*e^4)*x)*sqrt(x^2 - 1))*arctan(-(sq
rt(-d^2 + e^2)*sqrt(x^2 - 1)*e - sqrt(-d^2 + e^2)*(e*x + d))/(d^2
 - e^2)) + (c*d^5 + b*d^4*e - (3*a + 4*c)*d^3*e^2 + 2*b*d^2*e^3 -
 2*(2*c*d^4*e - (2*a + 5*c)*d^2*e^3 + 3*b*d*e^4 - a*e^5)*x^3 - (2
*c*d^5 + 2*b*d^4*e - (6*a + 7*c)*d^3*e^2 + 5*b*d^2*e^3 - (3*a + 4
*c)*d*e^4 + 2*b*e^5)*x^2 + 2*(c*d^4*e - b*d^3*e^2 + (a - c)*d^2*e
^3 + b*d*e^4 - a*e^5)*x)*sqrt(-d^2 + e^2))/(2*((d^4*e^4 - 2*d^2*e
^6 + e^8)*x^3 + 2*(d^5*e^3 - 2*d^3*e^5 + d*e^7)*x^2 + (d^6*e^2 -
2*d^4*e^4 + d^2*e^6)*x)*sqrt(-d^2 + e^2)*sqrt(x^2 - 1) + (d^6*e^2
 - 2*d^4*e^4 + d^2*e^6 - 2*(d^4*e^4 - 2*d^2*e^6 + e^8)*x^4 - 4*(d
^5*e^3 - 2*d^3*e^5 + d*e^7)*x^3 - (2*d^6*e^2 - 5*d^4*e^4 + 4*d^2*
e^6 - e^8)*x^2 + 2*(d^5*e^3 - 2*d^3*e^5 + d*e^7)*x)*sqrt(-d^2 + e
^2))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + bx + cx^2}{\sqrt{(x-1)(x+1)}\,(d+ex)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**2+b*x+a)/(e*x+d)**3/(x**2-1)**(1/2),x)
```

```
[Out] Integral((a + b*x + c*x**2)/(sqrt((x - 1)*(x + 1))*(d + e*x)**3),
 x)
```

GIAC/XCAS [A] time = 0.2748, size = 724, normalized size = 3.71

$$\frac{\left(2\,ad^2+cd^2-3\,bde+ae^2+2\,ce^2\right)\arctan\left(-\frac{\left(x-\sqrt{x^2-1}\right)e+d}{\sqrt{-d^2+e^2}}\right)}{\left(d^4-2\,d^2e^2+e^4\right)\sqrt{-d^2+e^2}}+\frac{2\,cd^4\left(x-\sqrt{x^2-1}\right)^3e+2\,cd^5\left(x-\sqrt{x^2-1}\right)^2+2\,bd^4\left(x-\sqrt{x^2-1}\right)^2e-2\,ad^2\left(x-\sqrt{x^2-1}\right)^3e^3-5\,cd^2\left(x-\sqrt{x^2-1}\right)^3e^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x + a)/((e*x + d)^3*sqrt(x^2 - 1)),x, algorithm="giac")
```

```
[Out] (2*a*d^2 + c*d^2 - 3*b*d*e + a*e^2 + 2*c*e^2)*arctan(-((x - sqrt(
x^2 - 1))*e + d)/sqrt(-d^2 + e^2))/((d^4 - 2*d^2*e^2 + e^4)*sqrt(
-d^2 + e^2)) + (2*c*d^4*(x - sqrt(x^2 - 1))^3*e + 2*c*d^5*(x - sq
rt(x^2 - 1))^2 + 2*b*d^4*(x - sqrt(x^2 - 1))^2*e - 2*a*d^2*(x - s
qrt(x^2 - 1))^3*e^3 - 5*c*d^2*(x - sqrt(x^2 - 1))^3*e^3 - 6*a*d^3
*(x - sqrt(x^2 - 1))^2*e^2 - 7*c*d^3*(x - sqrt(x^2 - 1))^2*e^2 +
2*c*d^4*(x - sqrt(x^2 - 1))*e + 3*b*d*(x - sqrt(x^2 - 1))^3*e^4 +
 5*b*d^2*(x - sqrt(x^2 - 1))^2*e^3 + 4*b*d^3*(x - sqrt(x^2 - 1))*
e^2 - a*(x - sqrt(x^2 - 1))^3*e^5 - 3*a*d*(x - sqrt(x^2 - 1))^2*e
^4 - 4*c*d*(x - sqrt(x^2 - 1))^2*e^4 - 10*a*d^2*(x - sqrt(x^2 - 1
))*e^3 - 11*c*d^2*(x - sqrt(x^2 - 1))*e^3 + c*d^3*e^2 + 2*b*(x -
sqrt(x^2 - 1))^2*e^5 + 5*b*d*(x - sqrt(x^2 - 1))*e^4 + b*d^2*e^3
+ a*(x - sqrt(x^2 - 1))*e^5 - 3*a*d*e^4 - 4*c*d*e^4 + 2*b*e^5)/((
d^4*e^2 - 2*d^2*e^4 + e^6)*((x - sqrt(x^2 - 1))^2*e + 2*d*(x - sq
rt(x^2 - 1)) + e)^2)
```

3.677 $\int \frac{1+2x^8}{x(1+x^8)^{3/2}}\,dx$

Optimal. Leaf size=28

$$-\frac{1}{4\sqrt{x^8+1}}-\frac{1}{4}\tanh^{-1}\left(\sqrt{x^8+1}\right)$$

```
[Out] -1/(4*Sqrt[1 + x^8]) - ArcTanh[Sqrt[1 + x^8]]/4
```

Rubi [A] time = 0.054886, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$-\frac{1}{4\sqrt{x^8+1}}-\frac{1}{4}\tanh^{-1}\left(\sqrt{x^8+1}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + 2*x^8)/(x*(1 + x^8)^(3/2)),x]

[Out] -1/(4*Sqrt[1 + x^8]) - ArcTanh[Sqrt[1 + x^8]]/4
```

Rubi in Sympy [A] time = 3.70643, size = 24, normalized size = 0.86

$$-\frac{\operatorname{atanh}\left(\sqrt{x^8+1}\right)}{4}-\frac{1}{4\sqrt{x^8+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*x**8+1)/x/(x**8+1)**(3/2),x)

[Out] -atanh(sqrt(x**8 + 1))/4 - 1/(4*sqrt(x**8 + 1))
```

Mathematica [A] time = 0.0389295, size = 28, normalized size = 1.

$$-\frac{1}{4\sqrt{x^8+1}}-\frac{1}{4}\tanh^{-1}\left(\sqrt{x^8+1}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + 2*x^8)/(x*(1 + x^8)^(3/2)),x]

[Out] -1/(4*Sqrt[1 + x^8]) - ArcTanh[Sqrt[1 + x^8]]/4
```

Maple [A] time = 0.04, size = 29, normalized size = 1.

$$-\frac{1}{4}\frac{1}{\sqrt{x^8+1}}+\frac{1}{4}\ln\left(1\left(\sqrt{x^8+1}-1\right)\frac{1}{\sqrt{x^8}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*x^8+1)/x/(x^8+1)^(3/2),x)

[Out] -1/4/(x^8+1)^(1/2)+1/4*ln(((x^8+1)^(1/2)-1)/(x^8)^(1/2))
```

Maxima [A] time = 0.781892, size = 46, normalized size = 1.64

$$-\frac{1}{4\sqrt{x^8+1}}-\frac{1}{8}\log\left(\sqrt{x^8+1}+1\right)+\frac{1}{8}\log\left(\sqrt{x^8+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^8 + 1)/((x^8 + 1)^(3/2)*x),x, algorithm="maxima")

[Out] -1/4/sqrt(x^8 + 1) - 1/8*log(sqrt(x^8 + 1) + 1) + 1/8*log(sqrt(x^
8 + 1) - 1)
```

Fricas [A] time = 0.28572, size = 65, normalized size = 2.32

$$-\frac{\sqrt{x^8+1}\log\left(\sqrt{x^8+1}+1\right)-\sqrt{x^8+1}\log\left(\sqrt{x^8+1}-1\right)+2}{8\sqrt{x^8+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^8 + 1)/((x^8 + 1)^(3/2)*x),x, algorithm="fricas")

[Out] -1/8*(sqrt(x^8 + 1)*log(sqrt(x^8 + 1) + 1) - sqrt(x^8 + 1)*log(sq
rt(x^8 + 1) - 1) + 2)/sqrt(x^8 + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x**8+1)/x/(x**8+1)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.264582, size = 46, normalized size = 1.64

$$-\frac{1}{4\sqrt{x^8+1}}-\frac{1}{8}\ln\left(\sqrt{x^8+1}+1\right)+\frac{1}{8}\ln\left(\sqrt{x^8+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^8 + 1)/((x^8 + 1)^(3/2)*x),x, algorithm="giac")
```

```
[Out] -1/4/sqrt(x^8 + 1) - 1/8*ln(sqrt(x^8 + 1) + 1) + 1/8*ln(sqrt(x^8
+ 1) - 1)
```

3.678 $\int \frac{\sqrt{1+x^8}\left(1+2x^8\right)}{x+2x^9+x^{17}}\,dx$

Optimal. Leaf size=28

$$-\frac{1}{4\sqrt{x^8+1}}-\frac{1}{4}\tanh^{-1}\left(\sqrt{x^8+1}\right)$$

```
[Out] -1/(4*Sqrt[1 + x^8]) - ArcTanh[Sqrt[1 + x^8]]/4
```

Rubi [A] time = 0.0715885, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207

$$-\frac{1}{4\sqrt{x^8+1}}-\frac{1}{4}\tanh^{-1}\left(\sqrt{x^8+1}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[1 + x^8]*(1 + 2*x^8))/(x + 2*x^9 + x^17),x]

[Out] -1/(4*Sqrt[1 + x^8]) - ArcTanh[Sqrt[1 + x^8]]/4
```

Rubi in Sympy [A] time = 5.77778, size = 24, normalized size = 0.86

$$-\frac{\operatorname{atanh}\left(\sqrt{x^8+1}\right)}{4}-\frac{1}{4\sqrt{x^8+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*x**8+1)*(x**8+1)**(1/2)/(x**17+2*x**9+x),x)

[Out] -atanh(sqrt(x**8 + 1))/4 - 1/(4*sqrt(x**8 + 1))
```

Mathematica [A] time = 0.0211707, size = 28, normalized size = 1.

$$-\frac{1}{4\sqrt{x^8+1}}-\frac{1}{4}\tanh^{-1}\left(\sqrt{x^8+1}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[1 + x^8]*(1 + 2*x^8))/(x + 2*x^9 + x^17),x]
```

```
[Out] -1/(4*Sqrt[1 + x^8]) - ArcTanh[Sqrt[1 + x^8]]/4
```

Maple [A] time = 0.037, size = 29, normalized size = 1.

$$-\frac{1}{4}\frac{1}{\sqrt{x^8+1}}+\frac{1}{4}\ln\left(1\left(\sqrt{x^8+1}-1\right)\frac{1}{\sqrt{x^8}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*x^8+1)*(x^8+1)^(1/2)/(x^17+2*x^9+x),x)
```

```
[Out] -1/4/(x^8+1)^(1/2)+1/4*ln(((x^8+1)^(1/2)-1)/(x^8)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2\,x^8+1)\sqrt{x^8+1}}{x^{17}+2\,x^9+x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^8 + 1)*sqrt(x^8 + 1)/(x^17 + 2*x^9 + x),x, algorithm="maxima")
```

```
[Out] integrate((2*x^8 + 1)*sqrt(x^8 + 1)/(x^17 + 2*x^9 + x), x)
```

Fricas [A] time = 0.270203, size = 65, normalized size = 2.32

$$-\frac{\sqrt{x^8+1}\log\left(\sqrt{x^8+1}+1\right)-\sqrt{x^8+1}\log\left(\sqrt{x^8+1}-1\right)+2}{8\sqrt{x^8+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^8 + 1)*sqrt(x^8 + 1)/(x^17 + 2*x^9 + x),x, algorithm="fricas")
```

```
[Out] -1/8*(sqrt(x^8 + 1)*log(sqrt(x^8 + 1) + 1) - sqrt(x^8 + 1)*log(sq
rt(x^8 + 1) - 1) + 2)/sqrt(x^8 + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x**8+1)*(x**8+1)**(1/2)/(x**17+2*x**9+x),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.266928, size = 46, normalized size = 1.64

$$-\frac{1}{4\sqrt{x^8+1}}-\frac{1}{8}\ln\left(\sqrt{x^8+1}+1\right)+\frac{1}{8}\ln\left(\sqrt{x^8+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^8 + 1)*sqrt(x^8 + 1)/(x^17 + 2*x^9 + x),x, algorithm="giac")
```

```
[Out] -1/4/sqrt(x^8 + 1) - 1/8*ln(sqrt(x^8 + 1) + 1) + 1/8*ln(sqrt(x^8
+ 1) - 1)
```

3.679 $\int \left(1-9x^2+\frac{x}{\sqrt{1-9x^2}}\right)\,dx$

Optimal. Leaf size=22

$$-3x^3-\frac{1}{9}\sqrt{1-9x^2}+x$$

```
[Out] x - 3*x^3 - Sqrt[1 - 9*x^2]/9
```

Rubi [A] time = 0.0120048, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.05$

$$-3x^3-\frac{1}{9}\sqrt{1-9x^2}+x$$

Antiderivative was successfully verified.

```
[In] Int[1 - 9*x^2 + x/Sqrt[1 - 9*x^2],x]

[Out] x - 3*x^3 - Sqrt[1 - 9*x^2]/9
```

Rubi in Sympy [A] time = 1.14837, size = 17, normalized size = 0.77

$$-3x^3+x-\frac{\sqrt{-9x^2+1}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1-9*x**2+x/(-9*x**2+1)**(1/2),x)

[Out] -3*x**3 + x - sqrt(-9*x**2 + 1)/9
```

Mathematica [A] time = 0.0124547, size = 22, normalized size = 1.

$$-3x^3-\frac{1}{9}\sqrt{1-9x^2}+x$$

Antiderivative was successfully verified.

```
[In]  Integrate[1 - 9*x^2 + x/Sqrt[1 - 9*x^2],x]
```

```
[Out] x - 3*x^3 - Sqrt[1 - 9*x^2]/9
```

Maple [A] time = 0.003, size = 19, normalized size = 0.9

$$x - 3x^3 - \frac{1}{9}\sqrt{-9x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1-9*x^2+x/(-9*x^2+1)^(1/2),x)
```

```
[Out] x-3*x^3-1/9*(-9*x^2+1)^(1/2)
```

Maxima [A] time = 0.699712, size = 24, normalized size = 1.09

$$-3x^3 + x - \frac{1}{9}\sqrt{-9x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-9*x^2 + x/sqrt(-9*x^2 + 1) + 1,x, algorithm="maxima")
```

```
[Out] -3*x^3 + x - 1/9*sqrt(-9*x^2 + 1)
```

Fricas [A] time = 0.267895, size = 62, normalized size = 2.82

$$\frac{3x^3 + x^2 - (3x^3 - x)\sqrt{-9x^2 + 1} - x}{\sqrt{-9x^2 + 1} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-9*x^2 + x/sqrt(-9*x^2 + 1) + 1,x, algorithm="fricas")
```

```
[Out] (3*x^3 + x^2 - (3*x^3 - x)*sqrt(-9*x^2 + 1) - x)/(sqrt(-9*x^2 + 1
) - 1)
```

Sympy [A] time = 0.323442, size = 17, normalized size = 0.77

$$-3x^3 + x - \frac{\sqrt{-9x^2 + 1}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1-9*x**2+x/(-9*x**2+1)**(1/2),x)
```

```
[Out] -3*x**3 + x - sqrt(-9*x**2 + 1)/9
```

GIAC/XCAS [A] time = 0.264756, size = 24, normalized size = 1.09

$$-3\,x^3 + x - \frac{1}{9}\,\sqrt{-9\,x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-9*x^2 + x/sqrt(-9*x^2 + 1) + 1,x, algorithm="giac")
```

```
[Out] -3*x^3 + x - 1/9*sqrt(-9*x^2 + 1)
```

3.680 $\int \frac{x+(1-9x^2)^{3/2}}{\sqrt{1-9x^2}}\,dx$

Optimal. Leaf size=22

$$-3x^3-\frac{1}{9}\sqrt{1-9x^2}+x$$

```
[Out] x - 3*x^3 - Sqrt[1 - 9*x^2]/9
```

Rubi [A] time = 0.132968, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08

$$-3x^3-\frac{1}{9}\sqrt{1-9x^2}+x$$

Antiderivative was successfully verified.

```
[In]  Int[(x + (1 - 9*x^2)^(3/2))/Sqrt[1 - 9*x^2],x]

[Out] x - 3*x^3 - Sqrt[1 - 9*x^2]/9
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+\left(-9x^2+1\right)^{\frac{3}{2}}}{\sqrt{-9x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x+(-9*x**2+1)**(3/2))/(-9*x**2+1)**(1/2),x)

[Out] Integral((x + (-9*x**2 + 1)**(3/2))/sqrt(-9*x**2 + 1), x)
```

Mathematica [A] time = 0.00293968, size = 22, normalized size = 1.

$$-3x^3-\frac{1}{9}\sqrt{1-9x^2}+x$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x + (1 - 9*x^2)^(3/2))/Sqrt[1 - 9*x^2],x]
```

```
[Out] x - 3*x^3 - Sqrt[1 - 9*x^2]/9
```

Maple [A] time = 0.002, size = 19, normalized size = 0.9

$$x - 3x^3 - \frac{1}{9}\sqrt{-9x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x+(-9*x^2+1)^(3/2))/(-9*x^2+1)^(1/2),x)
```

```
[Out] x-3*x^3-1/9*(-9*x^2+1)^(1/2)
```

Maxima [A] time = 0.699771, size = 24, normalized size = 1.09

$$-3x^3 + x - \frac{1}{9}\sqrt{-9x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((-9*x^2 + 1)^(3/2) + x)/sqrt(-9*x^2 + 1),x, algorithm="maxima")
```

```
[Out] -3*x^3 + x - 1/9*sqrt(-9*x^2 + 1)
```

Fricas [A] time = 0.262978, size = 62, normalized size = 2.82

$$\frac{3x^3 + x^2 - \left(3x^3 - x\right)\sqrt{-9x^2 + 1} - x}{\sqrt{-9x^2 + 1} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((-9*x^2 + 1)^(3/2) + x)/sqrt(-9*x^2 + 1),x, algorithm="fricas")
```

```
[Out] (3*x^3 + x^2 - (3*x^3 - x)*sqrt(-9*x^2 + 1) - x)/(sqrt(-9*x^2 + 1
) - 1)
```

Sympy [A] time = 3.42675, size = 17, normalized size = 0.77

$$-3x^3 + x - \frac{\sqrt{-9x^2 + 1}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x+(-9*x**2+1)**(3/2))/(-9*x**2+1)**(1/2),x)
```

```
[Out] -3*x**3 + x - sqrt(-9*x**2 + 1)/9
```

GIAC/XCAS [A] time = 0.279793, size = 24, normalized size = 1.09

$$-3\,x^3 + x - \frac{1}{9}\,\sqrt{-9\,x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-9*x^2 + 1)^(3/2) + x)/sqrt(-9*x^2 + 1),x, algorithm="giac")
```

```
[Out] -3*x^3 + x - 1/9*sqrt(-9*x^2 + 1)
```

3.681 $\int \frac{(-3+2\sqrt{x})(-3\sqrt{x}+x)^{2/3}}{\sqrt{x}} \, dx$

Optimal. Leaf size=17

$$\frac{6}{5}\left(x-3\sqrt{x}\right)^{5/3}$$

```
[Out] (6*(-3*Sqrt[x] + x)^(5/3))/5
```

Rubi [A] time = 0.0845331, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071

$$\frac{6}{5}\left(x-3\sqrt{x}\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In]  Int[((-3 + 2*Sqrt[x])*(-3*Sqrt[x] + x)^(2/3))/Sqrt[x],x]

[Out] (6*(-3*Sqrt[x] + x)^(5/3))/5
```

Rubi in Sympy [A] time = 6.32127, size = 14, normalized size = 0.82

$$\frac{6\left(-3\sqrt{x}+x\right)^{\frac{5}{3}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x-3*x**(1/2))**(2/3)*(-3+2*x**(1/2))/x**(1/2),x)

[Out] 6*(-3*sqrt(x) + x)**(5/3)/5
```

Mathematica [A] time = 0.0212846, size = 17, normalized size = 1.

$$\frac{6}{5}\left(x-3\sqrt{x}\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((-3 + 2*Sqrt[x])*(-3*Sqrt[x] + x)^(2/3))/Sqrt[x],x]
```

```
[Out] (6*(-3*Sqrt[x] + x)^(5/3))/5
```

Maple [A] time = 0.013, size = 12, normalized size = 0.7

$$\frac{6}{5}\left(x-3\sqrt{x}\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x-3*x^(1/2))^(2/3)*(-3+2*x^(1/2))/x^(1/2),x)
```

```
[Out] 6/5*(x-3*x^(1/2))^(5/3)
```

Maxima [A] time = 0.946612, size = 24, normalized size = 1.41

$$\frac{6}{5}\left(x^{\frac{4}{3}}-3\,x^{\frac{5}{6}}\right)\left(\sqrt{x}-3\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - 3*sqrt(x))^(2/3)*(2*sqrt(x) - 3)/sqrt(x),x, algorithm="maxima")
```

```
[Out] 6/5*(x^(4/3) - 3*x^(5/6))*(sqrt(x) - 3)^(2/3)
```

Fricas [A] time = 0.305304, size = 15, normalized size = 0.88

$$\frac{6}{5}\left(x-3\sqrt{x}\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - 3*sqrt(x))^(2/3)*(2*sqrt(x) - 3)/sqrt(x),x, algorithm="fricas")
```

```
[Out] 6/5*(x - 3*sqrt(x))^(5/3)
```

Sympy [A] time = 3.90253, size = 36, normalized size = 2.12

$$-\frac{18\sqrt{x}\left(-3\sqrt{x}+x\right)^{\frac{2}{3}}}{5}+\frac{6x\left(-3\sqrt{x}+x\right)^{\frac{2}{3}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x-3*x**(1/2))**(2/3)*(-3+2*x**(1/2))/x**(1/2),x)
```

```
[Out] -18*sqrt(x)*(-3*sqrt(x) + x)**(2/3)/5 + 6*x*(-3*sqrt(x) + x)**(2/
3)/5
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x-3\sqrt{x}\right)^{\frac{2}{3}}\left(2\sqrt{x}-3\right)}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - 3*sqrt(x))^(2/3)*(2*sqrt(x) - 3)/sqrt(x),x, algorithm="giac")
```

```
[Out] integrate((x - 3*sqrt(x))^(2/3)*(2*sqrt(x) - 3)/sqrt(x), x)
```

3.682 $\int \frac{9-9\sqrt{x}+2x}{\sqrt[3]{-3\sqrt{x}+x}}\,dx$

Optimal. Leaf size=17

$$\frac{6}{5}\left(x-3\sqrt{x}\right)^{5/3}$$

```
[Out] (6*(-3*Sqrt[x] + x)^(5/3))/5
```

Rubi [A] time = 0.0720455, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$

$$\frac{6}{5}\left(x-3\sqrt{x}\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In]  Int[(9 - 9*Sqrt[x] + 2*x)/(-3*Sqrt[x] + x)^(1/3),x]
```

```
[Out] (6*(-3*Sqrt[x] + x)^(5/3))/5
```

Rubi in Sympy [A] time = 37.6662, size = 36, normalized size = 2.12

$$-\frac{18\sqrt{x}\left(-3\sqrt{x}+x\right)^{\frac{2}{3}}}{5}+\frac{6x\left(-3\sqrt{x}+x\right)^{\frac{2}{3}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((9+2*x-9*x**(1/2))/(x-3*x**(1/2))**(1/3),x)
```

```
[Out] -18*sqrt(x)*(-3*sqrt(x) + x)**(2/3)/5 + 6*x*(-3*sqrt(x) + x)**(2/
3)/5
```

Mathematica [A] time = 0.0157377, size = 17, normalized size = 1.

$$\frac{6}{5}\left(x-3\sqrt{x}\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(9 - 9*Sqrt[x] + 2*x)/(-3*Sqrt[x] + x)^(1/3),x]
```

```
[Out] (6*(-3*Sqrt[x] + x)^(5/3))/5
```

Maple [C] time = 0.123, size = 125, normalized size = 7.4

$$\frac{18\,3^{2/3}}{5}\sqrt[3]{-signum\left(-1+\frac{1}{3}\sqrt{x}\right)}\,x^{\frac{5}{6}}\,{}_2F_1\!\left(\frac{1}{3},\frac{5}{3};\frac{8}{3};\frac{1}{3}\sqrt{x}\right)\frac{1}{\sqrt[3]{signum\left(-1+\frac{1}{3}\sqrt{x}\right)}} + \frac{4\,3^{2/3}}{11}\sqrt[3]{-signum\left(-1+\frac{1}{3}\sqrt{x}\right)}\,x^{\frac{11}{6}}\,{}_2F_1\!\left(\frac{1}{3},\frac{11}{3};\frac{14}{3};\frac{1}{3}\sqrt{x}\right)\frac{1}{\sqrt[3]{signum\left(-1+\frac{1}{3}\sqrt{x}\right)}} - \frac{9\,3^{2/3}}{4}\sqrt[3]{-signum\left(-1+\frac{1}{3}\sqrt{x}\right)}\,x^{\frac{4}{3}}\,{}_2F_1\!\left(\frac{1}{3},\frac{8}{3};\frac{11}{3};\frac{1}{3}\sqrt{x}\right)\frac{1}{\sqrt[3]{signum\left(-1+\frac{1}{3}\sqrt{x}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((9+2*x-9*x^(1/2))/(x-3*x^(1/2))^(1/3),x)
```

```
[Out] 18/5*3^(2/3)/signum(-1+1/3*x^(1/2))^(1/3)*(-signum(-1+1/3*x^(1/2)
))^(1/3)*x^(5/6)*hypergeom([1/3,5/3],[8/3],1/3*x^(1/2))+4/11*3^(2
/3)/signum(-1+1/3*x^(1/2))^(1/3)*(-signum(-1+1/3*x^(1/2)))^(1/3)*
x^(11/6)*hypergeom([1/3,11/3],[14/3],1/3*x^(1/2))-9/4*3^(2/3)/sig
num(-1+1/3*x^(1/2))^(1/3)*(-signum(-1+1/3*x^(1/2)))^(1/3)*x^(4/3)
*hypergeom([1/3,8/3],[11/3],1/3*x^(1/2))
```

Maxima [A] time = 0.97544, size = 31, normalized size = 1.82

$$\frac{6\left(x^{\frac{11}{6}} - 6\,x^{\frac{4}{3}} + 9\,x^{\frac{5}{6}}\right)}{5\left(\sqrt{x}-3\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x - 9*sqrt(x) + 9)/(x - 3*sqrt(x))^(1/3),x, algorithm="maxima")
```

```
[Out] 6/5*(x^(11/6) - 6*x^(4/3) + 9*x^(5/6))/(sqrt(x) - 3)^(1/3)
```

Fricas [A] time = 0.301829, size = 15, normalized size = 0.88

$$\frac{6}{5}\left(x-3\sqrt{x}\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x - 9*sqrt(x) + 9)/(x - 3*sqrt(x))^(1/3),x, algorithm="fricas")
```

```
[Out] 6/5*(x - 3*sqrt(x))^(5/3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{-9\sqrt{x}+2x+9}{\sqrt[3]{-3\sqrt{x}+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9+2*x-9*x**(1/2))/(x-3*x**(1/2))**(1/3),x)
```

```
[Out] Integral((-9*sqrt(x) + 2*x + 9)/(-3*sqrt(x) + x)**(1/3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{2\,x-9\,\sqrt{x}+9}{\left(x-3\,\sqrt{x}\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x - 9*sqrt(x) + 9)/(x - 3*sqrt(x))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((2*x - 9*sqrt(x) + 9)/(x - 3*sqrt(x))^(1/3), x)
```

3.683 $\int \frac{1}{\sqrt{4-9x^2}}\,dx$

Optimal. Leaf size=10

$$\frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right)$$

```
[Out] ArcSin[(3*x)/2]/3
```

Rubi [A] time = 0.00580065, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[4 - 9*x^2],x]

[Out] ArcSin[(3*x)/2]/3
```

Rubi in Sympy [A] time = 0.54615, size = 7, normalized size = 0.7

$$\frac{\operatorname{asin}\left(\frac{3x}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-9*x**2+4)**(1/2),x)

[Out] asin(3*x/2)/3
```

Mathematica [A] time = 0.00670908, size = 10, normalized size = 1.

$$\frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[4 - 9*x^2],x]
```

```
[Out] ArcSin[(3*x)/2]/3
```

Maple [A] time = 0.005, size = 7, normalized size = 0.7

$$\frac{1}{3}\arcsin\left(\frac{3x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-9*x^2+4)^(1/2),x)
```

```
[Out] 1/3*arcsin(3/2*x)
```

Maple is followed here by **Maxima [A]** time = 0.798958, size = 8, normalized size = 0.8

$$\frac{1}{3}\arcsin\left(\frac{3}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-9*x^2 + 4),x, algorithm="maxima")
```

```
[Out] 1/3*arcsin(3/2*x)
```

Fricas [A] time = 0.264605, size = 26, normalized size = 2.6

$$-\frac{2}{3}\arctan\left(\frac{\sqrt{-9x^2+4}-2}{3x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-9*x^2 + 4),x, algorithm="fricas")
```

```
[Out] -2/3*arctan(1/3*(sqrt(-9*x^2 + 4) - 2)/x)
```

Sympy [A] time = 0.337213, size = 7, normalized size = 0.7

$$\frac{\operatorname{asin}\left(\frac{3x}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-9*x**2+4)**(1/2),x)
```

```
[Out] asin(3*x/2)/3
```

GIAC/XCAS [A] time = 0.272269, size = 8, normalized size = 0.8

$$\frac{1}{3} \arcsin\left(\frac{3}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-9*x^2 + 4),x, algorithm="giac")
```

```
[Out] 1/3*arcsin(3/2*x)
```

3.684 $\int \frac{1}{\sqrt{2-3x}\sqrt{2+3x}}\,dx$

Optimal. Leaf size=10

$$\frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right)$$

```
[Out] ArcSin[(3*x)/2]/3
```

Rubi [A] time = 0.0136073, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$\frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[2 - 3*x]*Sqrt[2 + 3*x]),x]
```

```
[Out] ArcSin[(3*x)/2]/3
```

Rubi in Sympy [A] time = 1.50161, size = 7, normalized size = 0.7

$$\frac{\operatorname{asin}\left(\frac{3x}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(2-3*x)**(1/2)/(2+3*x)**(1/2),x)
```

```
[Out] asin(3*x/2)/3
```

Mathematica [A] time = 0.00696571, size = 10, normalized size = 1.

$$\frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[2 - 3*x]*Sqrt[2 + 3*x]),x]
```

```
[Out] ArcSin[(3*x)/2]/3
```

Maple [B] time = 0.01, size = 34, normalized size = 3.4

$$\frac{1}{3}\sqrt{(2-3x)(2+3x)}\arcsin\left(\frac{3x}{2}\right)\frac{1}{\sqrt{2-3x}}\frac{1}{\sqrt{2+3x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(2-3*x)^(1/2)/(2+3*x)^(1/2),x)
```

```
[Out] 1/3*((2-3*x)*(2+3*x))^(1/2)/(2-3*x)^(1/2)/(2+3*x)^(1/2)*arcsin(3/
2*x)
```

Maxima [A] time = 0.8093, size = 8, normalized size = 0.8

$$\frac{1}{3}\arcsin\left(\frac{3}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(3*x + 2)*sqrt(-3*x + 2)),x, algorithm="maxima")
```

```
[Out] 1/3*arcsin(3/2*x)
```

Fricas [A] time = 0.265899, size = 34, normalized size = 3.4

$$-\frac{2}{3}\arctan\left(\frac{\sqrt{3x+2}\sqrt{-3x+2}-2}{3x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(3*x + 2)*sqrt(-3*x + 2)),x, algorithm="fricas")
```

```
[Out] -2/3*arctan(1/3*(sqrt(3*x + 2)*sqrt(-3*x + 2) - 2)/x)
```

Sympy [A] time = 3.8553, size = 51, normalized size = 5.1

$$\begin{cases} -\frac{2i\operatorname{acosh}\left(\frac{\sqrt{3}\sqrt{x+\frac{2}{3}}}{2}\right)}{3} & \text{for}\ \frac{3\left|x+\frac{2}{3}\right|}{4} > 1 \\ \frac{2\operatorname{asin}\left(\frac{\sqrt{3}\sqrt{x+\frac{2}{3}}}{2}\right)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(2-3*x)**(1/2)/(2+3*x)**(1/2),x)
```

```
[Out] Piecewise((-2*I*acosh(sqrt(3)*sqrt(x + 2/3)/2)/3, 3*Abs(x + 2/3)/
4 > 1), (2*asin(sqrt(3)*sqrt(x + 2/3)/2)/3, True))
```

GIAC/XCAS [A] time = 0.267658, size = 16, normalized size = 1.6

$$\frac{2}{3}\arcsin\left(\frac{1}{2}\sqrt{3x+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(3*x + 2)*sqrt(-3*x + 2)),x, algorithm="giac")
```

```
[Out] 2/3*arcsin(1/2*sqrt(3*x + 2))
```

3.685 $\int \frac{1}{\sqrt{(2-3x)(2+3x)}}\,dx$

Optimal. Leaf size=10

$$\frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right)$$

```
[Out] ArcSin[(3*x)/2]/3
```

Rubi [A] time = 0.00871314, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$\frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[(2 - 3*x)*(2 + 3*x)],x]

[Out] ArcSin[(3*x)/2]/3
```

Rubi in Sympy [A] time = 0.616473, size = 7, normalized size = 0.7

$$\frac{\operatorname{asin}\left(\frac{3x}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/((2-3*x)*(2+3*x))**(1/2),x)

[Out] asin(3*x/2)/3
```

Mathematica [A] time = 0.00640894, size = 10, normalized size = 1.

$$\frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[(2 - 3*x)*(2 + 3*x)],x]

[Out] ArcSin[(3*x)/2]/3
```

Maple [A] time = 0.008, size = 7, normalized size = 0.7

$$\frac{1}{3}\arcsin\left(\frac{3x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/((2-3*x)*(2+3*x))^(1/2),x)

[Out] 1/3*arcsin(3/2*x)
```

Maxima [A] time = 0.818011, size = 8, normalized size = 0.8

$$\frac{1}{3}\arcsin\left(\frac{3}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-(3*x + 2)*(3*x - 2)),x, algorithm="maxima")

[Out] 1/3*arcsin(3/2*x)
```

Fricas [A] time = 0.267666, size = 26, normalized size = 2.6

$$-\frac{2}{3}\arctan\left(\frac{\sqrt{-9x^2+4}-2}{3x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-(3*x + 2)*(3*x - 2)),x, algorithm="fricas")

[Out] -2/3*arctan(1/3*(sqrt(-9*x^2 + 4) - 2)/x)
```

Sympy [A] time = 4.13498, size = 7, normalized size = 0.7

$$\frac{\operatorname{asin}\left(\frac{3x}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((2-3*x)*(2+3*x))**(1/2),x)

[Out] asin(3*x/2)/3
```

GIAC/XCAS [A] time = 0.2718, size = 8, normalized size = 0.8

$$\frac{1}{3}\arcsin\left(\frac{3}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-(3*x + 2)*(3*x - 2)),x, algorithm="giac")

[Out] 1/3*arcsin(3/2*x)
```

3.686 $\int \frac{1}{\sqrt{15-2x-x^2}}\,dx$

Optimal. Leaf size=12

$$-\sin^{-1}\left(\frac{1}{4}(-x-1)\right)$$

```
[Out] -ArcSin[(-1 - x)/4]
```

Rubi [A] time = 0.016483, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$-\sin^{-1}\left(\frac{1}{4}(-x-1)\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[15 - 2*x - x^2],x]

[Out] -ArcSin[(-1 - x)/4]
```

Rubi in Sympy [A] time = 0.701885, size = 22, normalized size = 1.83

$$\operatorname{atan}\left(-\frac{-2x-2}{2\sqrt{-x^2-2x+15}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-x**2-2*x+15)**(1/2),x)

[Out] atan(-(-2*x - 2)/(2*sqrt(-x**2 - 2*x + 15)))
```

Mathematica [A] time = 0.00954637, size = 12, normalized size = 1.

$$-\sin^{-1}\left(\frac{1}{4}(-x-1)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[15 - 2*x - x^2],x]
```

```
[Out] -ArcSin[(-1 - x)/4]
```

Maple [A] time = 0.005, size = 7, normalized size = 0.6

$$\arcsin\left(\frac{1}{4}+\frac{x}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-x^2-2*x+15)^(1/2),x)
```

```
[Out] arcsin(1/4+1/4*x)
```

Maxima [A] time = 0.79663, size = 11, normalized size = 0.92

$$-\arcsin\left(-\frac{1}{4}\,x-\frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-x^2 - 2*x + 15),x, algorithm="maxima")
```

```
[Out] -arcsin(-1/4*x - 1/4)
```

Fricas [A] time = 0.266934, size = 23, normalized size = 1.92

$$\arctan\left(\frac{x+1}{\sqrt{-x^2-2\,x+15}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-x^2 - 2*x + 15),x, algorithm="fricas")
```

```
[Out] arctan((x + 1)/sqrt(-x^2 - 2*x + 15))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2 - 2x + 15}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-x**2-2*x+15)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-x**2 - 2*x + 15), x)
```

GIAC/XCAS [A] time = 0.268775, size = 8, normalized size = 0.67

$$\arcsin\left(\frac{1}{4}\, x + \frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-x^2 - 2*x + 15),x, algorithm="giac")
```

```
[Out] arcsin(1/4*x + 1/4)
```

3.687 $\int \frac{1}{\sqrt{3-x}\sqrt{5+x}}\,dx$

Optimal. Leaf size=12

$$-\sin^{-1}\left(\frac{1}{4}(-x-1)\right)$$

```
[Out] -ArcSin[(-1 - x)/4]
```

Rubi [A] time = 0.0234365, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$

$$-\sin^{-1}\left(\frac{1}{4}(-x-1)\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[3 - x]*Sqrt[5 + x]),x]
```

```
[Out] -ArcSin[(-1 - x)/4]
```

Rubi in Sympy [A] time = 1.51967, size = 22, normalized size = 1.83

$$\operatorname{atan}\left(-\frac{-2x-2}{2\sqrt{-x^2-2x+15}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(3-x)**(1/2)/(5+x)**(1/2),x)
```

```
[Out] atan(-(-2*x - 2)/(2*sqrt(-x**2 - 2*x + 15)))
```

Mathematica [B] time = 0.0190806, size = 45, normalized size = 3.75

$$\frac{2\sqrt{x-3}\sqrt{x+5}\sinh^{-1}\left(\frac{\sqrt{x-3}}{2\sqrt{2}}\right)}{\sqrt{-(x-3)(x+5)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[3 - x]*Sqrt[5 + x]),x]
```

```
[Out] (2*Sqrt[-3 + x]*Sqrt[5 + x]*ArcSinh[Sqrt[-3 + x]/(2*Sqrt[2])])/Sq
rt[-((-3 + x)*(5 + x))]
```

Maple [B] time = 0.009, size = 31, normalized size = 2.6

$$1\sqrt{(3-x)(5+x)}\arcsin\left(\frac{1}{4}+\frac{x}{4}\right)\frac{1}{\sqrt{3-x}}\frac{1}{\sqrt{5+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(3-x)^(1/2)/(5+x)^(1/2),x)
```

```
[Out] ((3-x)*(5+x))^(1/2)/(3-x)^(1/2)/(5+x)^(1/2)*arcsin(1/4+1/4*x)
```

Maxima [A] time = 0.819908, size = 11, normalized size = 0.92

$$-\arcsin\left(-\frac{1}{4}x-\frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x + 5)*sqrt(-x + 3)),x, algorithm="maxima")
```

```
[Out] -arcsin(-1/4*x - 1/4)
```

Fricas [A] time = 0.268892, size = 23, normalized size = 1.92

$$\arctan\left(\frac{x+1}{\sqrt{x+5}\sqrt{-x+3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x + 5)*sqrt(-x + 3)),x, algorithm="fricas")
```

```
[Out] arctan((x + 1)/(sqrt(x + 5)*sqrt(-x + 3)))
```

Sympy [A] time = 3.79521, size = 41, normalized size = 3.42

$$\begin{cases} -2i \operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+5}}{4}\right) & \text{for } \frac{|x+5|}{8} > 1 \\ 2 \operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+5}}{4}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3-x)**(1/2)/(5+x)**(1/2),x)
```

```
[Out] Piecewise((-2*I*acosh(sqrt(2)*sqrt(x + 5)/4), Abs(x + 5)/8 > 1),
(2*asin(sqrt(2)*sqrt(x + 5)/4), True))
```

GIAC/XCAS [A] time = 0.269612, size = 18, normalized size = 1.5

$$2 \arcsin\left(\frac{1}{4} \sqrt{2}\sqrt{x+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x + 5)*sqrt(-x + 3)),x, algorithm="giac")
```

```
[Out] 2*arcsin(1/4*sqrt(2)*sqrt(x + 5))
```

3.688 $\int \frac{1}{\sqrt{(3-x)(5+x)}}\,dx$

Optimal. Leaf size=12

$$-\sin^{-1}\left(\frac{1}{4}(-x-1)\right)$$

```
[Out] -ArcSin[(-1 - x)/4]
```

Rubi [A] time = 0.019837, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$-\sin^{-1}\left(\frac{1}{4}(-x-1)\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[(3 - x)*(5 + x)],x]

[Out] -ArcSin[(-1 - x)/4]
```

Rubi in Sympy [A] time = 0.772345, size = 22, normalized size = 1.83

$$\operatorname{atan}\left(-\frac{-2x-2}{2\sqrt{-x^2-2x+15}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/((3-x)*(5+x))**(1/2),x)

[Out] atan(-(-2*x - 2)/(2*sqrt(-x**2 - 2*x + 15)))
```

Mathematica [B] time = 0.0102718, size = 45, normalized size = 3.75

$$\frac{2\sqrt{x-3}\sqrt{x+5}\,\sinh^{-1}\left(\frac{\sqrt{x-3}}{2\sqrt{2}}\right)}{\sqrt{-(x-3)(x+5)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[(3 - x)*(5 + x)],x]
```

```
[Out] (2*Sqrt[-3 + x]*Sqrt[5 + x]*ArcSinh[Sqrt[-3 + x]/(2*Sqrt[2])])/Sq
rt[-((-3 + x)*(5 + x))]
```

Maple [A] time = 0.008, size = 7, normalized size = 0.6

$$\arcsin\left(\frac{1}{4} + \frac{x}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/((3-x)*(5+x))^(1/2),x)
```

```
[Out] arcsin(1/4+1/4*x)
```

Maxima [A] time = 0.765043, size = 11, normalized size = 0.92

$$-\arcsin\left(-\frac{1}{4}\,x - \frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-(x + 5)*(x - 3)),x, algorithm="maxima")
```

```
[Out] -arcsin(-1/4*x - 1/4)
```

Fricas [A] time = 0.27316, size = 23, normalized size = 1.92

$$\arctan\left(\frac{x + 1}{\sqrt{-x^2 - 2\,x + 15}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-(x + 5)*(x - 3)),x, algorithm="fricas")
```

```
[Out] arctan((x + 1)/sqrt(-x^2 - 2*x + 15))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(-x+3)(x+5)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((3-x)*(5+x))**(1/2),x)
```

```
[Out] Integral(1/sqrt((-x + 3)*(x + 5)), x)
```

GIAC/XCAS [A] time = 0.267677, size = 8, normalized size = 0.67

$$\arcsin\left(\frac{1}{4}\,x + \frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-(x + 5)*(x - 3)),x, algorithm="giac")
```

```
[Out] arcsin(1/4*x + 1/4)
```

3.689 $\int \frac{1}{\sqrt{-15-8x-x^2}}\,dx$

Optimal. Leaf size=4

$$\sin^{-1}(x+4)$$

```
[Out] ArcSin[4 + x]
```

Rubi [A] time = 0.0113972, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$\sin^{-1}(x+4)$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[-15 - 8*x - x^2],x]

[Out] ArcSin[4 + x]
```

Rubi in Sympy [A] time = 0.705083, size = 24, normalized size = 6.

$$\operatorname{atan}\left(-\frac{-2x-8}{2\sqrt{-x^2-8x-15}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-x**2-8*x-15)**(1/2),x)

[Out] atan(-(-2*x - 8)/(2*sqrt(-x**2 - 8*x - 15)))
```

Mathematica [A] time = 0.00934286, size = 4, normalized size = 1.

$$\sin^{-1}(x+4)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[-15 - 8*x - x^2],x]
```

```
[Out] ArcSin[4 + x]
```

Maple [A] time = 0.004, size = 5, normalized size = 1.3

$$\arcsin(4+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-x^2-8*x-15)^(1/2),x)

[Out] arcsin(4+x)
```

Maxima [A] time = 0.763192, size = 11, normalized size = 2.75

$$-\arcsin(-x-4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-x^2 - 8*x - 15),x, algorithm="maxima")

[Out] -arcsin(-x - 4)
```

Fricas [A] time = 0.272359, size = 23, normalized size = 5.75

$$\arctan\left(\frac{x+4}{\sqrt{-x^2-8\,x-15}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-x^2 - 8*x - 15),x, algorithm="fricas")

[Out] arctan((x + 4)/sqrt(-x^2 - 8*x - 15))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2-8x-15}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-x**2-8*x-15)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-x**2 - 8*x - 15), x)
```

GIAC/XCAS [A] time = 0.267783, size = 5, normalized size = 1.25

$$\arcsin(x + 4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-x^2 - 8*x - 15),x, algorithm="giac")
```

```
[Out] arcsin(x + 4)
```

3.690 $\int \frac{1}{\sqrt{-3-x}\sqrt{5+x}}\,dx$

Optimal. Leaf size=4

$$\sin^{-1}(x+4)$$

```
[Out] ArcSin[4 + x]
```

Rubi [A] time = 0.0197263, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$

$$\sin^{-1}(x+4)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[-3 - x]*Sqrt[5 + x]),x]

[Out] ArcSin[4 + x]
```

Rubi in Sympy [A] time = 1.52303, size = 24, normalized size = 6.

$$\operatorname{atan}\left(-\frac{-2x-8}{2\sqrt{-x^2-8x-15}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-3-x)**(1/2)/(5+x)**(1/2),x)

[Out] atan(-(-2*x - 8)/(2*sqrt(-x**2 - 8*x - 15)))
```

Mathematica [B] time = 0.0163908, size = 42, normalized size = 10.5

$$\frac{2\sqrt{x+3}\sqrt{x+5}\sinh^{-1}\left(\frac{\sqrt{x+3}}{\sqrt{2}}\right)}{\sqrt{-(x+3)(x+5)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[-3 - x]*Sqrt[5 + x]),x]
```

```
[Out] (2*Sqrt[3 + x]*Sqrt[5 + x]*ArcSinh[Sqrt[3 + x]/Sqrt[2]])/Sqrt[-((
3 + x)*(5 + x))]
```

Maple [B] time = 0.008, size = 29, normalized size = 7.3

$$\arcsin(4+x)\sqrt{(-3-x)(5+x)}\frac{1}{\sqrt{-3-x}}\frac{1}{\sqrt{5+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-3-x)^(1/2)/(5+x)^(1/2),x)
```

```
[Out] ((-3-x)*(5+x))^(1/2)/(-3-x)^(1/2)/(5+x)^(1/2)*arcsin(4+x)
```

Maxima [A] time = 0.754736, size = 11, normalized size = 2.75

$$-\arcsin(-x-4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x + 5)*sqrt(-x - 3)),x, algorithm="maxima")
```

```
[Out] -arcsin(-x - 4)
```

Fricas [A] time = 0.27047, size = 23, normalized size = 5.75

$$\arctan\left(\frac{x+4}{\sqrt{x+5}\sqrt{-x-3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x + 5)*sqrt(-x - 3)),x, algorithm="fricas")
```

```
[Out] arctan((x + 4)/(sqrt(x + 5)*sqrt(-x - 3)))
```

Sympy [A] time = 3.75162, size = 41, normalized size = 10.25

$$\begin{cases} -2i \operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+5}}{2}\right) & \text{for } \frac{|x+5|}{2} > 1 \\ 2 \operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+5}}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-3-x)**(1/2)/(5+x)**(1/2),x)
```

```
[Out] Piecewise((-2*I*acosh(sqrt(2)*sqrt(x + 5)/2), Abs(x + 5)/2 > 1),
(2*asin(sqrt(2)*sqrt(x + 5)/2), True))
```

GIAC/XCAS [A] time = 0.268886, size = 18, normalized size = 4.5

$$2 \arcsin\left(\frac{1}{2} \sqrt{2}\sqrt{x+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x + 5)*sqrt(-x - 3)),x, algorithm="giac")
```

```
[Out] 2*arcsin(1/2*sqrt(2)*sqrt(x + 5))
```

3.691 $\int \frac{1}{\sqrt{(-3-x)(5+x)}}\,dx$

Optimal. Leaf size=4

$$\sin^{-1}(x+4)$$

```
[Out] ArcSin[4 + x]
```

Rubi [A] time = 0.0146539, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$\sin^{-1}(x+4)$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[(-3 - x)*(5 + x)],x]

[Out] ArcSin[4 + x]
```

Rubi in Sympy [A] time = 0.774144, size = 24, normalized size = 6.

$$\operatorname{atan}\left(-\frac{-2x-8}{2\sqrt{-x^2-8x-15}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/((-3-x)*(5+x))**(1/2),x)

[Out] atan(-(-2*x - 8)/(2*sqrt(-x**2 - 8*x - 15)))
```

Mathematica [B] time = 0.00652221, size = 42, normalized size = 10.5

$$\frac{2\sqrt{x+3}\sqrt{x+5}\sinh^{-1}\left(\frac{\sqrt{x+3}}{\sqrt{2}}\right)}{\sqrt{-(x+3)(x+5)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[(-3 - x)*(5 + x)],x]
```

```
[Out] (2*Sqrt[3 + x]*Sqrt[5 + x]*ArcSinh[Sqrt[3 + x]/Sqrt[2]])/Sqrt[-((
3 + x)*(5 + x))]
```

Maple [A] time = 0.007, size = 5, normalized size = 1.3

$$\arcsin(4 + x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/((-3-x)*(5+x))^(1/2),x)
```

```
[Out] arcsin(4+x)
```

Maxima [A] time = 0.749723, size = 11, normalized size = 2.75

$$-\arcsin(-x - 4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-(x + 5)*(x + 3)),x, algorithm="maxima")
```

```
[Out] -arcsin(-x - 4)
```

Fricas [A] time = 0.272978, size = 23, normalized size = 5.75

$$\arctan\left(\frac{x + 4}{\sqrt{-x^2 - 8x - 15}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-(x + 5)*(x + 3)),x, algorithm="fricas")
```

```
[Out] arctan((x + 4)/sqrt(-x^2 - 8*x - 15))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(-x-3)(x+5)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((-3-x)*(5+x))**(1/2),x)
```

```
[Out] Integral(1/sqrt((-x - 3)*(x + 5)), x)
```

GIAC/XCAS [A] time = 0.267951, size = 5, normalized size = 1.25

$$\arcsin(x+4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-(x + 5)*(x + 3)),x, algorithm="giac")
```

```
[Out] arcsin(x + 4)
```

3.692 $\int \left(1-\sqrt{x}\right) \, dx$

Optimal. Leaf size=11

$$x-\frac{2x^{3/2}}{3}$$

```
[Out] x - (2*x^(3/2))/3
```

Rubi [A] time = 0.00568546, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.

$$x-\frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

```
[In]  Int[1 - Sqrt[x],x]

[Out] x - (2*x^(3/2))/3
```

Rubi in Sympy [A] time = 0.639337, size = 8, normalized size = 0.73

$$-\frac{2x^{\frac{3}{2}}}{3}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1-x**(1/2),x)

[Out] -2*x**(3/2)/3 + x
```

Mathematica [A] time = 0.00252499, size = 11, normalized size = 1.

$$x-\frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1 - Sqrt[x],x]
```

```
[Out] x - (2*x^(3/2))/3
```

Maple [A] time = 0.001, size = 8, normalized size = 0.7

$$x - \frac{2}{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1-x^(1/2),x)
```

```
[Out] x-2/3*x^(3/2)
```

Maxima [A] time = 0.680649, size = 9, normalized size = 0.82

$$-\frac{2}{3}x^{\frac{3}{2}} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sqrt(x) + 1,x, algorithm="maxima")
```

```
[Out] -2/3*x^(3/2) + x
```

Fricas [A] time = 0.26406, size = 9, normalized size = 0.82

$$-\frac{2}{3}x^{\frac{3}{2}} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sqrt(x) + 1,x, algorithm="fricas")
```

```
[Out] -2/3*x^(3/2) + x
```

Sympy [A] time = 0.06273, size = 8, normalized size = 0.73

$$-\frac{2x^{\frac{3}{2}}}{3} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1-x**(1/2),x)

[Out] -2*x**(3/2)/3 + x
```

GIAC/XCAS [A] time = 0.265063, size = 9, normalized size = 0.82

$$-\frac{2}{3}x^{\frac{3}{2}} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sqrt(x) + 1,x, algorithm="giac")

[Out] -2/3*x^(3/2) + x
```

3.693 $\int \frac{1-x}{1+\sqrt{x}} \, dx$

Optimal. Leaf size=11

$$x-\frac{2x^{3/2}}{3}$$

```
[Out] x - (2*x^(3/2))/3
```

Rubi [A] time = 0.0216977, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$x-\frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)/(1 + Sqrt[x]),x]
```

```
[Out] x - (2*x^(3/2))/3
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2x^{\frac{3}{2}}}{3}+2\int^{\sqrt{x}} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((1-x)/(1+x**(1/2)),x)
```

```
[Out] -2*x**(3/2)/3 + 2*Integral(x, (x, sqrt(x)))
```

Mathematica [A] time = 0.000793878, size = 11, normalized size = 1.

$$x-\frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - x)/(1 + Sqrt[x]),x]

[Out] x - (2*x^(3/2))/3
```

Maple [A] time = 0.002, size = 8, normalized size = 0.7

$$x - \frac{2}{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1-x)/(1+x^(1/2)),x)

[Out] x-2/3*x^(3/2)
```

Maxima [A] time = 0.675083, size = 9, normalized size = 0.82

$$-\frac{2}{3}x^{\frac{3}{2}} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x - 1)/(sqrt(x) + 1),x, algorithm="maxima")

[Out] -2/3*x^(3/2) + x
```

Fricas [A] time = 0.261059, size = 9, normalized size = 0.82

$$-\frac{2}{3}x^{\frac{3}{2}} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x - 1)/(sqrt(x) + 1),x, algorithm="fricas")

[Out] -2/3*x^(3/2) + x
```

Sympy [A] time = 2.91155, size = 22, normalized size = 2.

$$-\frac{2\sqrt{x}\,(x-1)}{3}-\frac{2\sqrt{x}}{3}+x-1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1-x)/(1+x**(1/2)),x)

[Out] -2*sqrt(x)*(x - 1)/3 - 2*sqrt(x)/3 + x - 1
```

GIAC/XCAS [A] time = 0.263503, size = 9, normalized size = 0.82

$$-\frac{2}{3}\,x^{\frac{3}{2}}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x - 1)/(sqrt(x) + 1),x, algorithm="giac")

[Out] -2/3*x^(3/2) + x
```

3.694 $\int \sqrt{\frac{1}{1-x^2}}\, dx$

Optimal. Leaf size=27

$$\sqrt{\frac{1}{1-x^2}}\sqrt{1-x^2}\sin^{-1}(x)$$

```
[Out] Sqrt[(1 - x^2)^(-1)]*Sqrt[1 - x^2]*ArcSin[x]
```

Rubi [A] time = 0.0278542, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\sqrt{\frac{1}{1-x^2}}\sqrt{1-x^2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[(1 - x^2)^(-1)],x]

[Out] Sqrt[(1 - x^2)^(-1)]*Sqrt[1 - x^2]*ArcSin[x]
```

Rubi in Sympy [A] time = 0.693494, size = 20, normalized size = 0.74

$$\sqrt{-x^2+1}\sqrt{\frac{1}{-x^2+1}}\,\mathrm{asin}\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1/(-x**2+1))**(1/2),x)

[Out] sqrt(-x**2 + 1)*sqrt(1/(-x**2 + 1))*asin(x)
```

Mathematica [A] time = 0.0115194, size = 27, normalized size = 1.

$$\sqrt{\frac{1}{1-x^2}}\sqrt{1-x^2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[(1 - x^2)^(-1)],x]

[Out] Sqrt[(1 - x^2)^(-1)]*Sqrt[1 - x^2]*ArcSin[x]
```

Maple [A] time = 0.008, size = 30, normalized size = 1.1

$$\sqrt{-\left(x^{2}-1\right)^{-1}}\sqrt{x^{2}-1}\ln\left(x+\sqrt{x^{2}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1/(-x^2+1))^(1/2),x)

[Out] (-1/(x^2-1))^(1/2)*(x^2-1)^(1/2)*ln(x+(x^2-1)^(1/2))
```

Maxima [A] time = 0.7899, size = 3, normalized size = 0.11

$$\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-1/(x^2 - 1)),x, algorithm="maxima")

[Out] arcsin(x)
```

Fricas [A] time = 0.268061, size = 42, normalized size = 1.56

$$2\,\arctan\left(\frac{\sqrt{-\frac{1}{x^{2}-1}}-1}{x\sqrt{-\frac{1}{x^{2}-1}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-1/(x^2 - 1)),x, algorithm="fricas")

[Out] 2*arctan((sqrt(-1/(x^2 - 1)) - 1)/(x*sqrt(-1/(x^2 - 1))))
```

Sympy [A] time = 2.9306, size = 7, normalized size = 0.26

$$\begin{cases} \operatorname{asin}(x) & \text{for } x > -1 \wedge x < 1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1/(-x**2+1))**(1/2),x)

[Out] Piecewise((asin(x), (x > -1) & (x < 1)))
```

GIAC/XCAS [A] time = 0.270197, size = 14, normalized size = 0.52

$$-\arcsin(x)\operatorname{sign}\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-1/(x^2 - 1)),x, algorithm="giac")

[Out] -arcsin(x)*sign(x^2 - 1)
```

3.695 $\int \sqrt{\frac{1+x^2}{1-x^4}} \, dx$

Optimal. Leaf size=27

$$\sqrt{\frac{1}{1-x^2}}\sqrt{1-x^2}\sin^{-1}(x)$$

```
[Out] Sqrt[(1 - x^2)^(-1)]*Sqrt[1 - x^2]*ArcSin[x]
```

Rubi [A] time = 0.0401559, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$\sqrt{\frac{1}{1-x^2}}\sqrt{1-x^2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[(1 + x^2)/(1 - x^4)],x]

[Out] Sqrt[(1 - x^2)^(-1)]*Sqrt[1 - x^2]*ArcSin[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{x^2+1}{-x^4+1}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((x**2+1)/(-x**4+1))**(1/2),x)

[Out] Integral(sqrt((x**2 + 1)/(-x**4 + 1)), x)
```

Mathematica [A] time = 0.00774935, size = 27, normalized size = 1.

$$\sqrt{\frac{1}{1-x^2}}\sqrt{1-x^2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[(1 + x^2)/(1 - x^4)],x]

[Out] Sqrt[(1 - x^2)^(-1)]*Sqrt[1 - x^2]*ArcSin[x]
```

Maple [A] time = 0.005, size = 30, normalized size = 1.1

$$\sqrt{-\left(x^2-1\right)^{-1}}\sqrt{x^2-1}\ln\left(x+\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((x^2+1)/(-x^4+1))^(1/2),x)

[Out] (-1/(x^2-1))^(1/2)*(x^2-1)^(1/2)*ln(x+(x^2-1)^(1/2))
```

Maxima [A] time = 0.792059, size = 3, normalized size = 0.11

$$\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-(x^2 + 1)/(x^4 - 1)),x, algorithm="maxima")

[Out] arcsin(x)
```

Fricas [A] time = 0.2736, size = 42, normalized size = 1.56

$$2\arctan\left(\frac{\sqrt{-\frac{1}{x^2-1}}-1}{x\sqrt{-\frac{1}{x^2-1}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-(x^2 + 1)/(x^4 - 1)),x, algorithm="fricas")

[Out] 2*arctan((sqrt(-1/(x^2 - 1)) - 1)/(x*sqrt(-1/(x^2 - 1))))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{x^2 + 1}{-x^4 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((x**2+1)/(-x**4+1))**(1/2),x)
```

```
[Out] Integral(sqrt((x**2 + 1)/(-x**4 + 1)), x)
```

GIAC/XCAS [A] time = 0.268806, size = 14, normalized size = 0.52

$$-\arcsin(x)\operatorname{sign}\left(x^2 - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-(x^2 + 1)/(x^4 - 1)),x, algorithm="giac")
```

```
[Out] -arcsin(x)*sign(x^2 - 1)
```

3.696 $\int \sqrt{\frac{1}{-1+x^2}}\,dx$

Optimal. Leaf size=33

$$\sqrt{\frac{1}{x^2-1}}\sqrt{x^2-1}\tanh^{-1}\left(\frac{x}{\sqrt{x^2-1}}\right)$$

```
[Out] Sqrt[(-1 + x^2)^(-1)]*Sqrt[-1 + x^2]*ArcTanh[x/Sqrt[-1 + x^2]]
```

Rubi [A] time = 0.0290945, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273

$$\sqrt{\frac{1}{x^2-1}}\sqrt{x^2-1}\tanh^{-1}\left(\frac{x}{\sqrt{x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[(-1 + x^2)^(-1)],x]
```

```
[Out] Sqrt[(-1 + x^2)^(-1)]*Sqrt[-1 + x^2]*ArcTanh[x/Sqrt[-1 + x^2]]
```

Rubi in Sympy [A] time = 0.717235, size = 29, normalized size = 0.88

$$\sqrt{x^2-1}\sqrt{\frac{1}{x^2-1}}\,\mathrm{atanh}\left(\frac{x}{\sqrt{x^2-1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1/(x**2-1))**(1/2),x)
```

```
[Out] sqrt(x**2 - 1)*sqrt(1/(x**2 - 1))*atanh(x/sqrt(x**2 - 1))
```

Mathematica [A] time = 0.0341611, size = 56, normalized size = 1.7

$$\frac{1}{2}\sqrt{\frac{1}{x^2-1}}\sqrt{x^2-1}\left(\log\left(\frac{x}{\sqrt{x^2-1}}+1\right)-\log\left(1-\frac{x}{\sqrt{x^2-1}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[(-1 + x^2)^(-1)],x]
```

```
[Out] (Sqrt[(-1 + x^2)^(-1)]*Sqrt[-1 + x^2]*(-Log[1 - x/Sqrt[-1 + x^2]]
 + Log[1 + x/Sqrt[-1 + x^2]]))/2
```

Maple [A] time = 0.005, size = 28, normalized size = 0.9

$$\sqrt{(x^2-1)^{-1}}\sqrt{x^2-1}\ln\left(x+\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1/(x^2-1))^(1/2),x)
```

```
[Out] (1/(x^2-1))^(1/2)*(x^2-1)^(1/2)*ln(x+(x^2-1)^(1/2))
```

Maxima [A] time = 0.677104, size = 19, normalized size = 0.58

$$\log\left(2\,x+2\,\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(x^2 - 1),x, algorithm="maxima")
```

```
[Out] log(2*x + 2*sqrt(x^2 - 1))
```

Fricas [A] time = 0.266237, size = 19, normalized size = 0.58

$$-\log\left(-x+\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(x^2 - 1),x, algorithm="fricas")
```

```
[Out] -log(-x + sqrt(x^2 - 1))
```

Sympy [A] time = 3.47764, size = 15, normalized size = 0.45

$$\begin{cases} \log\left(x+\sqrt{x^2-1}\right) & \text{for } x > -1 \wedge x < 1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1/(x**2-1))**(1/2),x)
```

```
[Out] Piecewise((log(x + sqrt(x**2 - 1)), (x > -1) & (x < 1)))
```

GIAC/XCAS [A] time = 0.265338, size = 20, normalized size = 0.61

$$-\ln\left(\left|-x+\sqrt{x^2-1}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(x^2 - 1),x, algorithm="giac")
```

```
[Out] -ln(abs(-x + sqrt(x^2 - 1)))
```

3.697 $\int \sqrt{\frac{1+x^2}{-1+x^4}}\,dx$

Optimal. Leaf size=33

$$\sqrt{\frac{1}{x^2-1}}\sqrt{x^2-1}\tanh^{-1}\left(\frac{x}{\sqrt{x^2-1}}\right)$$

```
[Out] Sqrt[(-1 + x^2)^(-1)]*Sqrt[-1 + x^2]*ArcTanh[x/Sqrt[-1 + x^2]]
```

Rubi [A] time = 0.0396453, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235

$$\sqrt{\frac{1}{x^2-1}}\sqrt{x^2-1}\tanh^{-1}\left(\frac{x}{\sqrt{x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[(1 + x^2)/(-1 + x^4)],x]

[Out] Sqrt[(-1 + x^2)^(-1)]*Sqrt[-1 + x^2]*ArcTanh[x/Sqrt[-1 + x^2]]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{x^2+1}{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((x**2+1)/(x**4-1))**(1/2),x)

[Out] Integral(sqrt((x**2 + 1)/(x**4 - 1)), x)
```

Mathematica [A] time = 0.00501797, size = 56, normalized size = 1.7

$$\frac{1}{2}\sqrt{\frac{1}{x^2-1}}\sqrt{x^2-1}\left(\log\left(\frac{x}{\sqrt{x^2-1}}+1\right)-\log\left(1-\frac{x}{\sqrt{x^2-1}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[(1 + x^2)/(-1 + x^4)],x]
```

```
[Out] (Sqrt[(-1 + x^2)^(-1)]*Sqrt[-1 + x^2]*(-Log[1 - x/Sqrt[-1 + x^2]]
 + Log[1 + x/Sqrt[-1 + x^2]]))/2
```

Maple [A] time = 0.005, size = 28, normalized size = 0.9

$$\sqrt{(x^2-1)^{-1}}\sqrt{x^2-1}\ln\left(x+\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((x^2+1)/(x^4-1))^(1/2),x)
```

```
[Out] (1/(x^2-1))^(1/2)*(x^2-1)^(1/2)*ln(x+(x^2-1)^(1/2))
```

Maxima [A] time = 0.71637, size = 19, normalized size = 0.58

$$\log\left(2\,x+2\,\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((x^2 + 1)/(x^4 - 1)),x, algorithm="maxima")
```

```
[Out] log(2*x + 2*sqrt(x^2 - 1))
```

Fricas [A] time = 0.266031, size = 31, normalized size = 0.94

$$-\log\left(-\sqrt{x^2-1}\left(\frac{x}{\sqrt{x^2-1}}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((x^2 + 1)/(x^4 - 1)),x, algorithm="fricas")
```

```
[Out] -log(-sqrt(x^2 - 1)*(x/sqrt(x^2 - 1) - 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{x^2+1}{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((x**2+1)/(x**4-1))**(1/2),x)
```

```
[Out] Integral(sqrt((x**2 + 1)/(x**4 - 1)), x)
```

GIAC/XCAS [A] time = 0.264541, size = 28, normalized size = 0.85

$$-\ln\left(\left|-x+\sqrt{x^2-1}\right|\right)\operatorname{sign}\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((x^2 + 1)/(x^4 - 1)),x, algorithm="giac")
```

```
[Out] -ln(abs(-x + sqrt(x^2 - 1)))*sign(x^2 - 1)
```

3.698 $\int \frac{1}{\sqrt{1-x}}\,dx$

Optimal. Leaf size=11

$$-2\sqrt{1-x}$$

```
[Out] -2*Sqrt[1 - x]
```

Rubi [A] time = 0.00565314, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$-2\sqrt{1-x}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[1 - x],x]

[Out] -2*Sqrt[1 - x]
```

Rubi in Sympy [A] time = 0.525788, size = 8, normalized size = 0.73

$$-2\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1-x)**(1/2),x)

[Out] -2*sqrt(-x + 1)
```

Mathematica [A] time = 0.00252851, size = 11, normalized size = 1.

$$-2\sqrt{1-x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[1 - x],x]
```

```
[Out] -2*Sqrt[1 - x]
```

Maple [A] time = 0.003, size = 10, normalized size = 0.9

$$-2\sqrt{1-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1-x)^(1/2),x)

[Out] -2*(1-x)^(1/2)
```

Maxima [A] time = 0.680677, size = 12, normalized size = 1.09

$$-2\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-x + 1),x, algorithm="maxima")

[Out] -2*sqrt(-x + 1)
```

Fricas [A] time = 0.265146, size = 12, normalized size = 1.09

$$-2\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-x + 1),x, algorithm="fricas")

[Out] -2*sqrt(-x + 1)
```

Sympy [A] time = 0.066209, size = 8, normalized size = 0.73

$$-2\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1-x)**(1/2),x)
```

```
[Out] -2*sqrt(-x + 1)
```

GIAC/XCAS [A] time = 0.261089, size = 12, normalized size = 1.09

$$-2\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-x + 1),x, algorithm="giac")
```

```
[Out] -2*sqrt(-x + 1)
```

3.699 $\int \frac{\sqrt{1+x}}{\sqrt{1-x^2}} \, dx$

Optimal. Leaf size=11

$$-2\sqrt{1-x}$$

```
[Out] -2*Sqrt[1 - x]
```

Rubi [A] time = 0.00611616, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$-2\sqrt{1-x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x]/Sqrt[1 - x^2],x]

[Out] -2*Sqrt[1 - x]
```

Rubi in Sympy [A] time = 1.25145, size = 8, normalized size = 0.73

$$-2\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((1+x)**(1/2)/(-x**2+1)**(1/2),x)

[Out] -2*sqrt(-x + 1)
```

Mathematica [B] time = 0.0123075, size = 23, normalized size = 2.09

$$\frac{2(x-1)\sqrt{x+1}}{\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x]/Sqrt[1 - x^2],x]
```

```
[Out] (2*(-1 + x)*Sqrt[1 + x])/Sqrt[1 - x^2]
```

Maple [B] time = 0.003, size = 20, normalized size = 1.8

$$2\frac{(-1+x)\sqrt{1+x}}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(1/2)/(-x^2+1)^(1/2),x)
```

```
[Out] 2*(-1+x)*(1+x)^(1/2)/(-x^2+1)^(1/2)
```

Maxima [A] time = 0.689151, size = 16, normalized size = 1.45

$$\frac{2(x-1)}{\sqrt{-x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x + 1)/sqrt(-x^2 + 1),x, algorithm="maxima")
```

```
[Out] 2*(x - 1)/sqrt(-x + 1)
```

Fricas [A] time = 0.261909, size = 28, normalized size = 2.55

$$\frac{2\left(x^2-1\right)}{\sqrt{-x^2+1}\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x + 1)/sqrt(-x^2 + 1),x, algorithm="fricas")
```

```
[Out] 2*(x^2 - 1)/(sqrt(-x^2 + 1)*sqrt(x + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x+1}}{\sqrt{-\left(x-1\right)\left(x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+x)**(1/2)/(-x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(x + 1)/sqrt(-(x - 1)*(x + 1)), x)
```

GIAC/XCAS [A] time = 0.265007, size = 20, normalized size = 1.82

$$2\sqrt{2} - 2\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x + 1)/sqrt(-x^2 + 1),x, algorithm="giac")
```

```
[Out] 2*sqrt(2) - 2*sqrt(-x + 1)
```

3.700 $\int \frac{1}{\sqrt{1+x}}\,dx$

Optimal. Leaf size=9

$$2\sqrt{x+1}$$

```
[Out] 2*Sqrt[1 + x]
```

Rubi [A] time = 0.00440617, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$2\sqrt{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 + x],x]
```

```
[Out] 2*Sqrt[1 + x]
```

Rubi in Sympy [A] time = 0.522944, size = 7, normalized size = 0.78

$$2\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(1+x)**(1/2),x)
```

```
[Out] 2*sqrt(x + 1)
```

Mathematica [A] time = 0.00155672, size = 9, normalized size = 1.

$$2\sqrt{x+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 + x],x]
```

```
[Out] 2*Sqrt[1 + x]
```

Maple [A] time = 0.003, size = 8, normalized size = 0.9

$$2\sqrt{1+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1+x)^(1/2),x)
```

```
[Out] 2*(1+x)^(1/2)
```

Maxima [A] time = 0.682816, size = 9, normalized size = 1.

$$2\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(x + 1),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x + 1)
```

Fricas [A] time = 0.259293, size = 9, normalized size = 1.

$$2\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(x + 1),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x + 1)
```

Sympy [A] time = 0.06467, size = 7, normalized size = 0.78

$$2\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1+x)**(1/2),x)

[Out] 2*sqrt(x + 1)
```

GIAC/XCAS [A] time = 0.265011, size = 9, normalized size = 1.

$$2\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(x + 1),x, algorithm="giac")

[Out] 2*sqrt(x + 1)
```

3.701 $\int \frac{\sqrt{1-x}}{\sqrt{1-x^2}} \, dx$

Optimal. Leaf size=9

$$2\sqrt{x+1}$$

```
[Out] 2*Sqrt[1 + x]
```

Rubi [A] time = 0.00546179, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$

$$2\sqrt{x+1}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 - x]/Sqrt[1 - x^2],x]
```

```
[Out] 2*Sqrt[1 + x]
```

Rubi in Sympy [A] time = 1.40581, size = 7, normalized size = 0.78

$$2\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1-x)**(1/2)/(-x**2+1)**(1/2),x)
```

```
[Out] 2*sqrt(x + 1)
```

Mathematica [B] time = 0.0125821, size = 25, normalized size = 2.78

$$\frac{2\sqrt{1-x}(x+1)}{\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 - x]/Sqrt[1 - x^2],x]
```

```
[Out] (2*Sqrt[1 - x]*(1 + x))/Sqrt[1 - x^2]
```

Maple [B] time = 0.003, size = 22, normalized size = 2.4

$$2\,\frac{(1+x)\,\sqrt{1-x}}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(1/2)/(-x^2+1)^(1/2),x)
```

```
[Out] 2*(1+x)*(1-x)^(1/2)/(-x^2+1)^(1/2)
```

Maxima [A] time = 0.688524, size = 9, normalized size = 1.

$$2\,\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-x + 1)/sqrt(-x^2 + 1),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x + 1)
```

Fricas [A] time = 0.267257, size = 31, normalized size = 3.44

$$-\frac{2\,\left(x^2-1\right)}{\sqrt{-x^2+1}\sqrt{-x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-x + 1)/sqrt(-x^2 + 1),x, algorithm="fricas")
```

```
[Out] -2*(x^2 - 1)/(sqrt(-x^2 + 1)*sqrt(-x + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x+1}}{\sqrt{-(x-1)\,(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1-x)**(1/2)/(-x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-x + 1)/sqrt(-(x - 1)*(x + 1)), x)
```

GIAC/XCAS [A] time = 0.264204, size = 18, normalized size = 2.

$$-2\sqrt{2}+2\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x + 1)/sqrt(-x^2 + 1),x, algorithm="giac")
```

```
[Out] -2*sqrt(2) + 2*sqrt(x + 1)
```

3.702 $\int \sqrt{1-x}\,dx$

Optimal. Leaf size=13

$$-\frac{2}{3}(1-x)^{3/2}$$

[Out] (-2*(1 - x)^(3/2))/3

Rubi [A] time = 0.00545891, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$-\frac{2}{3}(1-x)^{3/2}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 - x],x]

[Out] (-2*(1 - x)^(3/2))/3

Rubi in Sympy [A] time = 0.532459, size = 10, normalized size = 0.77

$$-\frac{2\left(-x+1\right)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate((1-x)**(1/2),x)

[Out] -2*(-x + 1)**(3/2)/3

Mathematica [A] time = 0.00247891, size = 13, normalized size = 1.

$$-\frac{2}{3}(1-x)^{3/2}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 - x],x]

```
[Out] (-2*(1 - x)^(3/2))/3
```

Maple [A] time = 0.002, size = 10, normalized size = 0.8

$$-\frac{2}{3}(1-x)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1-x)^(1/2),x)

[Out] -2/3*(1-x)^(3/2)
```

Maxima [A] time = 0.68239, size = 12, normalized size = 0.92

$$-\frac{2}{3}(-x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x + 1),x, algorithm="maxima")

[Out] -2/3*(-x + 1)^(3/2)
```

Fricas [A] time = 0.270825, size = 16, normalized size = 1.23

$$\frac{2}{3}(x-1)\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x + 1),x, algorithm="fricas")

[Out] 2/3*(x - 1)*sqrt(-x + 1)
```

Sympy [A] time = 0.068271, size = 10, normalized size = 0.77

$$-\frac{2(-x+1)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1-x)**(1/2),x)
```

```
[Out] -2*(-x + 1)**(3/2)/3
```

GIAC/XCAS [A] time = 0.263411, size = 12, normalized size = 0.92

$$-\frac{2}{3}\,(-x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x + 1),x, algorithm="giac")
```

```
[Out] -2/3*(-x + 1)^(3/2)
```

3.703 $\int \frac{\sqrt{1-x^2}}{\sqrt{1+x}}\,dx$

Optimal. Leaf size=13

$$-\frac{2}{3}(1-x)^{3/2}$$

```
[Out] (-2*(1 - x)^(3/2))/3
```

Rubi [A] time = 0.00614335, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$-\frac{2}{3}(1-x)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^2]/Sqrt[1 + x],x]

[Out] (-2*(1 - x)^(3/2))/3
```

Rubi in Sympy [A] time = 1.24227, size = 10, normalized size = 0.77

$$-\frac{2\left(-x+1\right)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-x**2+1)**(1/2)/(1+x)**(1/2),x)

[Out] -2*(-x + 1)**(3/2)/3
```

Mathematica [A] time = 0.0110106, size = 25, normalized size = 1.92

$$\frac{2(x-1)\sqrt{1-x^2}}{3\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 - x^2]/Sqrt[1 + x],x]
```

```
[Out] (2*(-1 + x)*Sqrt[1 - x^2])/(3*Sqrt[1 + x])
```

Maple [B] time = 0.003, size = 20, normalized size = 1.5

$$\frac{2x-2}{3}\sqrt{-x^2+1}\frac{1}{\sqrt{1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2+1)^(1/2)/(1+x)^(1/2),x)
```

```
[Out] 2/3*(-1+x)*(-x^2+1)^(1/2)/(1+x)^(1/2)
```

Maxima [A] time = 0.691604, size = 16, normalized size = 1.23

$$\frac{2}{3}(x-1)\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 1)/sqrt(x + 1),x, algorithm="maxima")
```

```
[Out] 2/3*(x - 1)*sqrt(-x + 1)
```

Fricas [A] time = 0.265076, size = 39, normalized size = 3.

$$-\frac{2\left(x^3-x^2-x+1\right)}{3\sqrt{-x^2+1}\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 1)/sqrt(x + 1),x, algorithm="fricas")
```

```
[Out] -2/3*(x^3 - x^2 - x + 1)/(sqrt(-x^2 + 1)*sqrt(x + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(x-1)(x+1)}}{\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+1)**(1/2)/(1+x)**(1/2),x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 1))/sqrt(x + 1), x)
```

GIAC/XCAS [A] time = 0.267156, size = 20, normalized size = 1.54

$$-\frac{2}{3}(-x+1)^{\frac{3}{2}}+\frac{4}{3}\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 1)/sqrt(x + 1),x, algorithm="giac")
```

```
[Out] -2/3*(-x + 1)^(3/2) + 4/3*sqrt(2)
```

3.704 $\int \sqrt{1+x}\,dx$

Optimal. Leaf size=11

$$\frac{2}{3}(x+1)^{3/2}$$

```
[Out] (2*(1 + x)^(3/2))/3
```

Rubi [A] time = 0.00429097, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{2}{3}(x+1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x],x]

[Out] (2*(1 + x)^(3/2))/3
```

Rubi in Sympy [A] time = 0.51193, size = 8, normalized size = 0.73

$$\frac{2\,(x+1)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((1+x)**(1/2),x)

[Out] 2*(x + 1)**(3/2)/3
```

Mathematica [A] time = 0.002221, size = 11, normalized size = 1.

$$\frac{2}{3}(x+1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x],x]
```

```
[Out] (2*(1 + x)^(3/2))/3
```

Maple [A] time = 0.003, size = 8, normalized size = 0.7

$$\frac{2}{3}(1+x)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+x)^(1/2),x)
```

```
[Out] 2/3*(1+x)^(3/2)
```

Maxima [A] time = 0.678469, size = 9, normalized size = 0.82

$$\frac{2}{3}(x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x + 1),x, algorithm="maxima")
```

```
[Out] 2/3*(x + 1)^(3/2)
```

Fricas [A] time = 0.267706, size = 9, normalized size = 0.82

$$\frac{2}{3}(x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x + 1),x, algorithm="fricas")
```

```
[Out] 2/3*(x + 1)^(3/2)
```

Sympy [A] time = 0.06315, size = 8, normalized size = 0.73

$$\frac{2(x+1)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+x)**(1/2),x)
```

```
[Out] 2*(x + 1)**(3/2)/3
```

GIAC/XCAS [A] time = 0.261984, size = 9, normalized size = 0.82

$$\frac{2}{3}(x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x + 1),x, algorithm="giac")
```

```
[Out] 2/3*(x + 1)^(3/2)
```

3.705 $\int \frac{\sqrt{1-x^2}}{\sqrt{1-x}} \, dx$

Optimal. Leaf size=11

$$\frac{2}{3}(x+1)^{3/2}$$

```
[Out] (2*(1 + x)^(3/2))/3
```

Rubi [A] time = 0.00532804, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$

$$\frac{2}{3}(x+1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^2]/Sqrt[1 - x],x]
```

```
[Out] (2*(1 + x)^(3/2))/3
```

Rubi in Sympy [A] time = 1.40369, size = 8, normalized size = 0.73

$$\frac{2\,(x+1)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-x**2+1)**(1/2)/(1-x)**(1/2),x)
```

```
[Out] 2*(x + 1)**(3/2)/3
```

Mathematica [B] time = 0.0120282, size = 27, normalized size = 2.45

$$\frac{2(x+1)\sqrt{1-x^2}}{3\sqrt{1-x}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 - x^2]/Sqrt[1 - x],x]
```

```
[Out] (2*(1 + x)*Sqrt[1 - x^2])/(3*Sqrt[1 - x])
```

Maple [B] time = 0.003, size = 22, normalized size = 2.

$$\frac{2+2x}{3}\sqrt{-x^2+1}\frac{1}{\sqrt{1-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2+1)^(1/2)/(1-x)^(1/2),x)
```

```
[Out] 2/3*(1+x)*(-x^2+1)^(1/2)/(1-x)^(1/2)
```

Maxima [A] time = 0.69087, size = 9, normalized size = 0.82

$$\frac{2}{3}(x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 1)/sqrt(-x + 1),x, algorithm="maxima")
```

```
[Out] 2/3*(x + 1)^(3/2)
```

Fricas [A] time = 0.263995, size = 39, normalized size = 3.55

$$-\frac{2\left(x^3+x^2-x-1\right)}{3\sqrt{-x^2+1}\sqrt{-x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 1)/sqrt(-x + 1),x, algorithm="fricas")
```

```
[Out] -2/3*(x^3 + x^2 - x - 1)/(sqrt(-x^2 + 1)*sqrt(-x + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(x-1)(x+1)}}{\sqrt{-x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+1)**(1/2)/(1-x)**(1/2),x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 1))/sqrt(-x + 1), x)
```

GIAC/XCAS [A] time = 0.264834, size = 18, normalized size = 1.64

$$\frac{2}{3}\,(x+1)^{\frac{3}{2}} - \frac{4}{3}\,\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 1)/sqrt(-x + 1),x, algorithm="giac")
```

```
[Out] 2/3*(x + 1)^(3/2) - 4/3*sqrt(2)
```

3.706 $\int \frac{\sqrt{2+3x}}{\sqrt{1+x}}\,dx$

Optimal. Leaf size=35

$$\sqrt{x+1}\sqrt{3x+2}-\frac{\sinh^{-1}\left(\sqrt{3x+2}\right)}{\sqrt{3}}$$

```
[Out] Sqrt[1 + x]*Sqrt[2 + 3*x] - ArcSinh[Sqrt[2 + 3*x]]/Sqrt[3]
```

Rubi [A] time = 0.0294906, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$

$$\sqrt{x+1}\sqrt{3x+2}-\frac{\sinh^{-1}\left(\sqrt{3x+2}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[2 + 3*x]/Sqrt[1 + x],x]
```

```
[Out] Sqrt[1 + x]*Sqrt[2 + 3*x] - ArcSinh[Sqrt[2 + 3*x]]/Sqrt[3]
```

Rubi in Sympy [A] time = 2.045, size = 31, normalized size = 0.89

$$\sqrt{x+1}\sqrt{3x+2}-\frac{\sqrt{3}\operatorname{asinh}\left(\sqrt{3x+2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2+3*x)**(1/2)/(1+x)**(1/2),x)
```

```
[Out] sqrt(x + 1)*sqrt(3*x + 2) - sqrt(3)*asinh(sqrt(3*x + 2))/3
```

Mathematica [A] time = 0.0253311, size = 45, normalized size = 1.29

$$\sqrt{x+1}\sqrt{3x+2}-\frac{\log\left(3\sqrt{x+1}+\sqrt{9x+6}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[2 + 3*x]/Sqrt[1 + x],x]
```

```
[Out] Sqrt[1 + x]*Sqrt[2 + 3*x] - Log[3*Sqrt[1 + x] + Sqrt[6 + 9*x]]/Sq
rt[3]
```

Maple [B] time = 0.008, size = 67, normalized size = 1.9

$$\sqrt{1+x}\sqrt{2+3x}-\frac{\sqrt{3}}{6}\sqrt{(1+x)(2+3x)}\ln\left(\frac{\sqrt{3}}{3}\left(\frac{5}{2}+3x\right)+\sqrt{3x^2+5x+2}\right)\frac{1}{\sqrt{1+x}}\frac{1}{\sqrt{2+3x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2+3*x)^(1/2)/(1+x)^(1/2),x)
```

```
[Out] (1+x)^(1/2)*(2+3*x)^(1/2)-1/6*((1+x)*(2+3*x))^(1/2)/(2+3*x)^(1/2)
/(1+x)^(1/2)*ln(1/3*(5/2+3*x)*3^(1/2)+(3*x^2+5*x+2)^(1/2))*3^(1/2
)
```

Maxima [A] time = 0.759216, size = 55, normalized size = 1.57

$$-\frac{1}{6}\sqrt{3}\log\left(2\sqrt{3}\sqrt{3x^2+5x+2}+6x+5\right)+\sqrt{3x^2+5x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(3*x + 2)/sqrt(x + 1),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*log(2*sqrt(3)*sqrt(3*x^2 + 5*x + 2) + 6*x + 5) + sqr
t(3*x^2 + 5*x + 2)
```

Fricas [A] time = 0.27843, size = 78, normalized size = 2.23

$$\frac{1}{12}\sqrt{3}\left(4\sqrt{3}\sqrt{3x+2}\sqrt{x+1}+\log\left(-12(6x+5)\sqrt{3x+2}\sqrt{x+1}+\sqrt{3}\left(72x^2+120x+49\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(3*x + 2)/sqrt(x + 1),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*(4*sqrt(3)*sqrt(3*x + 2)*sqrt(x + 1) + log(-12*(6*x
+ 5)*sqrt(3*x + 2)*sqrt(x + 1) + sqrt(3)*(72*x^2 + 120*x + 49)))
```

Sympy [A] time = 5.777, size = 97, normalized size = 2.77

$$\begin{cases} \frac{3(x+1)^{\frac{3}{2}}}{\sqrt{3x+2}} - \frac{\sqrt{x+1}}{\sqrt{3x+2}} - \frac{\sqrt{3}\operatorname{acosh}\left(\sqrt{3}\sqrt{x+1}\right)}{3} & \text{for } 3|x+1| > 1 \\ i\sqrt{-3x-2}\sqrt{x+1} + \frac{\sqrt{3}i\operatorname{asin}\left(\sqrt{3}\sqrt{x+1}\right)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2+3*x)**(1/2)/(1+x)**(1/2),x)
```

```
[Out] Piecewise((3*(x + 1)**(3/2)/sqrt(3*x + 2) - sqrt(x + 1)/sqrt(3*x
+ 2) - sqrt(3)*acosh(sqrt(3)*sqrt(x + 1))/3, 3*Abs(x + 1) > 1), (
I*sqrt(-3*x - 2)*sqrt(x + 1) + sqrt(3)*I*asin(sqrt(3)*sqrt(x + 1)
)/3, True))
```

GIAC/XCAS [A] time = 0.268893, size = 53, normalized size = 1.51

$$\frac{1}{3}\sqrt{3}\left(\sqrt{3x+3}\sqrt{3x+2} + \ln\left(\sqrt{3x+3} - \sqrt{3x+2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(3*x + 2)/sqrt(x + 1),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*(sqrt(3*x + 3)*sqrt(3*x + 2) + ln(sqrt(3*x + 3) - sqr
t(3*x + 2)))
```

3.707 $\int \frac{\sqrt{1-x}\sqrt{2+3x}}{\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=35

$$\sqrt{x+1}\sqrt{3x+2}-\frac{\sinh^{-1}\left(\sqrt{3x+2}\right)}{\sqrt{3}}$$

```
[Out] Sqrt[1 + x]*Sqrt[2 + 3*x] - ArcSinh[Sqrt[2 + 3*x]]/Sqrt[3]
```

Rubi [A] time = 0.0287898, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$\sqrt{x+1}\sqrt{3x+2}-\frac{\sinh^{-1}\left(\sqrt{3x+2}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[1 - x]*Sqrt[2 + 3*x])/Sqrt[1 - x^2],x]

[Out] Sqrt[1 + x]*Sqrt[2 + 3*x] - ArcSinh[Sqrt[2 + 3*x]]/Sqrt[3]
```

Rubi in Sympy [A] time = 2.95537, size = 31, normalized size = 0.89

$$\sqrt{x+1}\sqrt{3x+2}-\frac{\sqrt{3}\operatorname{asinh}\left(\sqrt{3x+2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1-x)**(1/2)*(2+3*x)**(1/2)/(-x**2+1)**(1/2),x)

[Out] sqrt(x + 1)*sqrt(3*x + 2) - sqrt(3)*asinh(sqrt(3*x + 2))/3
```

Mathematica [B] time = 0.0748978, size = 79, normalized size = 2.26

$$\frac{\sqrt{1-x}\left(3\sqrt{3x+2}(x+1)+\sqrt{3}\sqrt{-x-1}\tan^{-1}\left(\frac{\sqrt{3}\sqrt{-x-1}}{\sqrt{3x+2}}\right)\right)}{3\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[1 - x]*Sqrt[2 + 3*x])/Sqrt[1 - x^2],x]
```

```
[Out] (Sqrt[1 - x]*(3*(1 + x)*Sqrt[2 + 3*x] + Sqrt[3]*Sqrt[-1 - x]*ArcT
an[(Sqrt[3]*Sqrt[-1 - x])/Sqrt[2 + 3*x]]))/(3*Sqrt[1 - x^2])
```

Maple [B] time = 0.014, size = 86, normalized size = 2.5

$$\frac{1}{-6+6x}\sqrt{1-x}\sqrt{2+3x}\sqrt{-x^2+1}\left(\ln\left(\frac{5\sqrt{3}}{6}+x\sqrt{3}+\sqrt{3x^2+5x+2}\right)\sqrt{3}-6\sqrt{3x^2+5x+2}\right)\frac{1}{\sqrt{3x^2+5x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1-x)^(1/2)*(2+3*x)^(1/2)/(-x^2+1)^(1/2),x)
```

```
[Out] 1/6*(1-x)^(1/2)*(2+3*x)^(1/2)*(-x^2+1)^(1/2)*(ln(5/6*3^(1/2)+x*3^
(1/2)+(3*x^2+5*x+2)^(1/2))*3^(1/2)-6*(3*x^2+5*x+2)^(1/2))/(-1+x)/
(3*x^2+5*x+2)^(1/2)
```

Maxima [A] time = 0.780965, size = 55, normalized size = 1.57

$$-\frac{1}{6}\sqrt{3}\log\left(2\sqrt{3}\sqrt{3x^2+5x+2}+6x+5\right)+\sqrt{3x^2+5x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(3*x + 2)*sqrt(-x + 1)/sqrt(-x^2 + 1),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*log(2*sqrt(3)*sqrt(3*x^2 + 5*x + 2) + 6*x + 5) + sqr
t(3*x^2 + 5*x + 2)
```

Fricas [A] time = 0.302, size = 138, normalized size = 3.94

$$-\frac{\sqrt{3}\left(4\sqrt{3}\sqrt{-x^2+1}\sqrt{3x+2}\sqrt{-x+1}-(x-1)\log\left(-\frac{12\sqrt{-x^2+1}(6x+5)\sqrt{3x+2}\sqrt{-x+1}+\sqrt{3}(72x^3+48x^2-71x-49)}{x-1}\right)\right)}{12(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(3*x + 2)*sqrt(-x + 1)/sqrt(-x^2 + 1),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(3)*(4*sqrt(3)*sqrt(-x^2 + 1)*sqrt(3*x + 2)*sqrt(-x + 1
) - (x - 1)*log(-(12*sqrt(-x^2 + 1)*(6*x + 5)*sqrt(3*x + 2)*sqrt(
-x + 1) + sqrt(3)*(72*x^3 + 48*x^2 - 71*x - 49))/(x - 1)))/(x - 1
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x+1}\sqrt{3x+2}}{\sqrt{-(x-1)(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1-x)**(1/2)*(2+3*x)**(1/2)/(-x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-x + 1)*sqrt(3*x + 2)/sqrt(-(x - 1)*(x + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{3\,x+2}\sqrt{-x+1}}{\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(3*x + 2)*sqrt(-x + 1)/sqrt(-x^2 + 1),x, algorithm="giac")
```

```
[Out] integrate(sqrt(3*x + 2)*sqrt(-x + 1)/sqrt(-x^2 + 1), x)
```

3.708 $\int \frac{(1+x)^{3/2}}{(1-x)^{3/2}x}\,dx$

Optimal. Leaf size=43

$$\frac{4\sqrt{x+1}}{\sqrt{1-x}} - \sin^{-1}(x) - \tanh^{-1}\left(\sqrt{1-x}\sqrt{x+1}\right)$$

```
[Out] (4*Sqrt[1 + x])/Sqrt[1 - x] - ArcSin[x] - ArcTanh[Sqrt[1 - x]*Sqr
t[1 + x]]
```

Rubi [A] time = 0.0848861, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35

$$\frac{4\sqrt{x+1}}{\sqrt{1-x}} - \sin^{-1}(x) - \tanh^{-1}\left(\sqrt{1-x}\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x)^(3/2)/((1 - x)^(3/2)*x),x]
```

```
[Out] (4*Sqrt[1 + x])/Sqrt[1 - x] - ArcSin[x] - ArcTanh[Sqrt[1 - x]*Sqr
t[1 + x]]
```

Rubi in Sympy [A] time = 4.67878, size = 32, normalized size = 0.74

$$-\operatorname{asin}(x) - \operatorname{atanh}\left(\sqrt{-x+1}\sqrt{x+1}\right) + \frac{4\sqrt{x+1}}{\sqrt{-x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+x)**(3/2)/(1-x)**(3/2)/x,x)
```

```
[Out] -asin(x) - atanh(sqrt(-x + 1)*sqrt(x + 1)) + 4*sqrt(x + 1)/sqrt(-
x + 1)
```

Mathematica [B] time = 0.0890679, size = 101, normalized size = 2.35

$$-\frac{4\sqrt{1-x^2}}{x-1}+\log\left(1-\sqrt{x+1}\right)-\log\left(\sqrt{1-x}-\sqrt{x+1}+2\right)-\log\left(\sqrt{x+1}+1\right)+\log\left(\sqrt{1-x}+\sqrt{x+1}+2\right)-2\sin^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{2}}\right)$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(1 + x)^(3/2)/((1 - x)^(3/2)*x),x]
```

```
[Out] (-4*Sqrt[1 - x^2])/(-1 + x) - 2*ArcSin[Sqrt[1 + x]/Sqrt[2]] + Log
[1 - Sqrt[1 + x]] - Log[2 + Sqrt[1 - x] - Sqrt[1 + x]] - Log[1 +
Sqrt[1 + x]] + Log[2 + Sqrt[1 - x] + Sqrt[1 + x]]
```

Maple [A] time = 0.018, size = 70, normalized size = 1.6

$$\frac{1}{-1+x}\left(-Artanh\left(\frac{1}{\sqrt{-x^2+1}}\right)x-\arcsin(x)\,x+Artanh\left(\frac{1}{\sqrt{-x^2+1}}\right)+\arcsin(x)-4\sqrt{-x^2+1}\right)\sqrt{1-x}\sqrt{1+x}\frac{1}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+x)^(3/2)/(1-x)^(3/2)/x,x)
```

```
[Out] (-arctanh(1/(-x^2+1)^(1/2))*x-arcsin(x)*x+arctanh(1/(-x^2+1)^(1/2
))+arcsin(x)-4*(-x^2+1)^(1/2))*(1-x)^(1/2)*(1+x)^(1/2)/(-1+x)/(-x
^2+1)^(1/2)
```

Maxima [A] time = 0.767473, size = 72, normalized size = 1.67

$$\frac{4\,x}{\sqrt{-x^2+1}}+\frac{4}{\sqrt{-x^2+1}}-\arcsin(x)-\log\left(\frac{2\sqrt{-x^2+1}}{|x|}+\frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 1)^(3/2)/(x*(-x + 1)^(3/2)),x, algorithm="maxima")
```

```
[Out] 4*x/sqrt(-x^2 + 1) + 4/sqrt(-x^2 + 1) - arcsin(x) - log(2*sqrt(-x
^2 + 1)/abs(x) + 2/abs(x))
```

Fricas [A] time = 0.279741, size = 132, normalized size = 3.07

$$\frac{2\left(x+\sqrt{x+1}\sqrt{-x+1}-1\right)\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)+\left(x+\sqrt{x+1}\sqrt{-x+1}-1\right)\log\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)+8\,x}{x+\sqrt{x+1}\sqrt{-x+1}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 1)^(3/2)/(x*(-x + 1)^(3/2)),x, algorithm="fricas")
```

```
[Out] (2*(x + sqrt(x + 1)*sqrt(-x + 1) - 1)*arctan((sqrt(x + 1)*sqrt(-x
 + 1) - 1)/x) + (x + sqrt(x + 1)*sqrt(-x + 1) - 1)*log((sqrt(x +
1)*sqrt(-x + 1) - 1)/x) + 8*x)/(x + sqrt(x + 1)*sqrt(-x + 1) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x+1)^{\frac{3}{2}}}{x\,(-x+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+x)**(3/2)/(1-x)**(3/2)/x,x)
```

```
[Out] Integral((x + 1)**(3/2)/(x*(-x + 1)**(3/2)), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 1)^(3/2)/(x*(-x + 1)^(3/2)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.709 $\int \frac{(1+x)^3}{x(1-x^2)^{3/2}}\,dx$

Optimal. Leaf size=35

$$\frac{4(x+1)}{\sqrt{1-x^2}}-\tanh^{-1}\left(\sqrt{1-x^2}\right)-\sin^{-1}(x)$$

```
[Out] (4*(1 + x))/Sqrt[1 - x^2] - ArcSin[x] - ArcTanh[Sqrt[1 - x^2]]
```

Rubi [A] time = 0.12164, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$\frac{4(x+1)}{\sqrt{1-x^2}}-\tanh^{-1}\left(\sqrt{1-x^2}\right)-\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x)^3/(x*(1 - x^2)^(3/2)),x]

[Out] (4*(1 + x))/Sqrt[1 - x^2] - ArcSin[x] - ArcTanh[Sqrt[1 - x^2]]
```

Rubi in Sympy [A] time = 5.15534, size = 26, normalized size = 0.74

$$-\operatorname{asin}(x)-\operatorname{atanh}\left(\sqrt{-x^2+1}\right)+\frac{4\sqrt{-x^2+1}}{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+x)**3/x/(-x**2+1)**(3/2),x)

[Out] -asin(x) - atanh(sqrt(-x**2 + 1)) + 4*sqrt(-x**2 + 1)/(-x + 1)
```

Mathematica [A] time = 0.0435132, size = 41, normalized size = 1.17

$$-\frac{4\sqrt{1-x^2}}{x-1}-\log\left(\sqrt{1-x^2}+1\right)+\log(x)-\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x)^3/(x*(1 - x^2)^(3/2)),x]
```

```
[Out] (-4*Sqrt[1 - x^2])/(-1 + x) - ArcSin[x] + Log[x] - Log[1 + Sqrt[1
 - x^2]]
```

Maple [A] time = 0.011, size = 41, normalized size = 1.2

$$4\,\frac{1}{\sqrt{-x^2+1}}-\mathit{Artanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)+4\,\frac{x}{\sqrt{-x^2+1}}-\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+x)^3/x/(-x^2+1)^(3/2),x)
```

```
[Out] 4/(-x^2+1)^(1/2)-arctanh(1/(-x^2+1)^(1/2))+4*x/(-x^2+1)^(1/2)-arc
sin(x)
```

Maxima [A] time = 0.758425, size = 72, normalized size = 2.06

$$\frac{4\,x}{\sqrt{-x^2+1}}+\frac{4}{\sqrt{-x^2+1}}-\arcsin\left(x\right)-\log\left(\frac{2\,\sqrt{-x^2+1}}{|x|}+\frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 1)^3/((-x^2 + 1)^(3/2)*x),x, algorithm="maxima")
```

```
[Out] 4*x/sqrt(-x^2 + 1) + 4/sqrt(-x^2 + 1) - arcsin(x) - log(2*sqrt(-x
^2 + 1)/abs(x) + 2/abs(x))
```

Fricas [A] time = 0.27733, size = 105, normalized size = 3.

$$\frac{2\left(x+\sqrt{-x^2+1}-1\right)\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)+\left(x+\sqrt{-x^2+1}-1\right)\log\left(\frac{\sqrt{-x^2+1}-1}{x}\right)+8\,x}{x+\sqrt{-x^2+1}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 1)^3/((-x^2 + 1)^(3/2)*x),x, algorithm="fricas")
```

```
[Out] (2*(x + sqrt(-x^2 + 1) - 1)*arctan((sqrt(-x^2 + 1) - 1)/x) + (x +
 sqrt(-x^2 + 1) - 1)*log((sqrt(-x^2 + 1) - 1)/x) + 8*x)/(x + sqrt
(-x^2 + 1) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x+1)^3}{x\left(-(x-1)(x+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+x)**3/x/(-x**2+1)**(3/2),x)
```

```
[Out] Integral((x + 1)**3/(x*(-(x - 1)*(x + 1))**(3/2)), x)
```

GIAC/XCAS [A] time = 0.273532, size = 59, normalized size = 1.69

$$\frac{8}{\frac{\sqrt{-x^2+1}-1}{x}+1} - \arcsin(x) + \ln\left(-\frac{\sqrt{-x^2+1}-1}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 1)^3/((-x^2 + 1)^(3/2)*x),x, algorithm="giac")
```

```
[Out] 8/((sqrt(-x^2 + 1) - 1)/x + 1) - arcsin(x) + ln(-(sqrt(-x^2 + 1)
- 1)/abs(x))
```

3.710 $\int \frac{(1+ax)^{3/2}}{x(1-ax)^{3/2}} \, dx$

Optimal. Leaf size=51

$$\frac{4\sqrt{ax+1}}{\sqrt{1-ax}} - \sin^{-1}(ax) - \tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)$$

```
[Out] (4*Sqrt[1 + a*x])/Sqrt[1 - a*x] - ArcSin[a*x] - ArcTanh[Sqrt[1 -
a*x]*Sqrt[1 + a*x]]
```

Rubi [A] time = 0.15832, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$

$$\frac{4\sqrt{ax+1}}{\sqrt{1-ax}} - \sin^{-1}(ax) - \tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a*x)^(3/2)/(x*(1 - a*x)^(3/2)),x]
```

```
[Out] (4*Sqrt[1 + a*x])/Sqrt[1 - a*x] - ArcSin[a*x] - ArcTanh[Sqrt[1 -
a*x]*Sqrt[1 + a*x]]
```

Rubi in Sympy [A] time = 8.14738, size = 41, normalized size = 0.8

$$-\operatorname{asin}(ax) - \operatorname{atanh}\left(\sqrt{-ax+1}\sqrt{ax+1}\right) + \frac{4\sqrt{ax+1}}{\sqrt{-ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((a*x+1)**(3/2)/x/(-a*x+1)**(3/2),x)
```

```
[Out] -asin(a*x) - atanh(sqrt(-a*x + 1)*sqrt(a*x + 1)) + 4*sqrt(a*x + 1
)/sqrt(-a*x + 1)
```

Mathematica [C] time = 0.112888, size = 74, normalized size = 1.45

$$\frac{4\sqrt{1-a^2x^2}}{1-ax} - \log\left(\sqrt{1-a^2x^2}+1\right) - i\log\left(2\left(\sqrt{1-a^2x^2}-iax\right)\right) + \log(x)$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(1 + a*x)^(3/2)/(x*(1 - a*x)^(3/2)),x]
```

```
[Out] (4*Sqrt[1 - a^2*x^2])/(1 - a*x) + Log[x] - Log[1 + Sqrt[1 - a^2*x
^2]] - I*Log[2*((-I)*a*x + Sqrt[1 - a^2*x^2])]
```

Maple [C] time = 0.045, size = 130, normalized size = 2.6

$$\frac{csgn(a)}{ax-1}\left(-Artanh\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)csgn(a)\,xa-\arctan\left(csgn(a)\,xa\frac{1}{\sqrt{-a^2x^2+1}}\right)xa+Artanh\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)csgn(a)-4\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a*x+1)^(3/2)/x/(-a*x+1)^(3/2),x)
```

```
[Out] (-arctanh(1/(-a^2*x^2+1)^(1/2))*csgn(a)*x*a-arctan(csgn(a)*a*x/(-
a^2*x^2+1)^(1/2))*x*a+arctanh(1/(-a^2*x^2+1)^(1/2))*csgn(a)-4*(-a
^2*x^2+1)^(1/2)*csgn(a)+arctan(csgn(a)*a*x/(-a^2*x^2+1)^(1/2)))*c
sgn(a)*(-a*x+1)^(1/2)*(a*x+1)^(1/2)/(a*x-1)/(-a^2*x^2+1)^(1/2)
```

Maxima [A] time = 0.760395, size = 105, normalized size = 2.06

$$\frac{4\,ax}{\sqrt{-a^2x^2+1}}-\frac{a\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{\sqrt{a^2}}+\frac{4}{\sqrt{-a^2x^2+1}}-\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x + 1)^(3/2)/((-a*x + 1)^(3/2)*x),x, algorithm="maxima")
```

```
[Out] 4*a*x/sqrt(-a^2*x^2 + 1) - a*arcsin(a^2*x/sqrt(a^2))/sqrt(a^2) +
4/sqrt(-a^2*x^2 + 1) - log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)
)
```

Fricas [A] time = 0.290707, size = 166, normalized size = 3.25

$$\frac{8\,ax+2\left(ax+\sqrt{ax+1}\sqrt{-ax+1}-1\right)\arctan\left(\frac{\sqrt{ax+1}\sqrt{-ax+1}-1}{ax}\right)+\left(ax+\sqrt{ax+1}\sqrt{-ax+1}-1\right)\log\left(\frac{\sqrt{ax+1}\sqrt{-ax+1}-1}{x}\right)}{ax+\sqrt{ax+1}\sqrt{-ax+1}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a*x + 1)^(3/2)/((-a*x + 1)^(3/2)*x),x, algorithm="fricas")
```

```
[Out] (8*a*x + 2*(a*x + sqrt(a*x + 1)*sqrt(-a*x + 1) - 1)*arctan((sqrt(
a*x + 1)*sqrt(-a*x + 1) - 1)/(a*x)) + (a*x + sqrt(a*x + 1)*sqrt(-
a*x + 1) - 1)*log((sqrt(a*x + 1)*sqrt(-a*x + 1) - 1)/x))/(a*x + s
qrt(a*x + 1)*sqrt(-a*x + 1) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^{\frac{3}{2}}}{x(-ax+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a*x+1)**(3/2)/x/(-a*x+1)**(3/2),x)
```

```
[Out] Integral((a*x + 1)**(3/2)/(x*(-a*x + 1)**(3/2)), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a*x + 1)^(3/2)/((-a*x + 1)^(3/2)*x),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.711 $\int \frac{(1+ax)^3}{x(1-a^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=45

$$\frac{4(ax+1)}{\sqrt{1-a^2x^2}} - \tanh^{-1}\left(\sqrt{1-a^2x^2}\right) - \sin^{-1}(ax)$$

```
[Out] (4*(1 + a*x))/Sqrt[1 - a^2*x^2] - ArcSin[a*x] - ArcTanh[Sqrt[1 -
a^2*x^2]]
```

Rubi [A] time = 0.196041, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$

$$\frac{4(ax+1)}{\sqrt{1-a^2x^2}} - \tanh^{-1}\left(\sqrt{1-a^2x^2}\right) - \sin^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + a*x)^3/(x*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] (4*(1 + a*x))/Sqrt[1 - a^2*x^2] - ArcSin[a*x] - ArcTanh[Sqrt[1 -
a^2*x^2]]
```

Rubi in Sympy [A] time = 8.32408, size = 36, normalized size = 0.8

$$-\operatorname{asin}(ax) - \operatorname{atanh}\left(\sqrt{-a^2x^2+1}\right) + \frac{4\sqrt{-a^2x^2+1}}{-ax+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a*x+1)**3/x/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] -asin(a*x) - atanh(sqrt(-a**2*x**2 + 1)) + 4*sqrt(-a**2*x**2 + 1)
/(-a*x + 1)
```

Mathematica [A] time = 0.0780291, size = 51, normalized size = 1.13

$$-\frac{4\sqrt{1-a^2x^2}}{ax-1} - \log\left(\sqrt{1-a^2x^2}+1\right) - \sin^{-1}(ax) + \log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + a*x)^3/(x*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] (-4*Sqrt[1 - a^2*x^2])/(-1 + a*x) - ArcSin[a*x] + Log[x] - Log[1
+ Sqrt[1 - a^2*x^2]]
```

Maple [A] time = 0.015, size = 75, normalized size = 1.7

$$4\frac{1}{\sqrt{-a^2x^2+1}} - Artanh\left(\frac{1}{\sqrt{-a^2x^2+1}}\right) + 4\frac{ax}{\sqrt{-a^2x^2+1}} - a\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a*x+1)^3/x/(-a^2*x^2+1)^(3/2),x)
```

```
[Out] 4/(-a^2*x^2+1)^(1/2)-arctanh(1/(-a^2*x^2+1)^(1/2))+4*a*x/(-a^2*x^
2+1)^(1/2)-a/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))
```

Maxima [A] time = 0.755662, size = 105, normalized size = 2.33

$$\frac{4\,ax}{\sqrt{-a^2x^2+1}} - \frac{a\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{\sqrt{a^2}} + \frac{4}{\sqrt{-a^2x^2+1}} - \log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|} + \frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*x),x, algorithm="maxima")
```

```
[Out] 4*a*x/sqrt(-a^2*x^2 + 1) - a*arcsin(a^2*x/sqrt(a^2))/sqrt(a^2) +
4/sqrt(-a^2*x^2 + 1) - log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)
)
```

Fricas [A] time = 0.284308, size = 139, normalized size = 3.09

$$\frac{8\,ax + 2\left(ax + \sqrt{-a^2x^2+1} - 1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right) + \left(ax + \sqrt{-a^2x^2+1} - 1\right)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)}{ax + \sqrt{-a^2x^2+1} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*x),x, algorithm="fricas")
```

```
[Out] (8*a*x + 2*(a*x + sqrt(-a^2*x^2 + 1) - 1)*arctan((sqrt(-a^2*x^2 +
 1) - 1)/(a*x)) + (a*x + sqrt(-a^2*x^2 + 1) - 1)*log((sqrt(-a^2*x
^2 + 1) - 1)/x))/(a*x + sqrt(-a^2*x^2 + 1) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+1)^3}{x\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a*x+1)**3/x/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral((a*x + 1)**3/(x*(-(a*x - 1)*(a*x + 1))**(3/2)), x)
```

GIAC/XCAS [A] time = 0.278451, size = 117, normalized size = 2.6

$$-\frac{a\arcsin(ax)\operatorname{sign}(a)}{|a|}-\frac{a\ln\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2a\right|}{2a^2|x|}\right)}{|a|}+\frac{8a}{\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}-1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*x),x, algorithm="giac")
```

```
[Out] -a*arcsin(a*x)*sign(a)/abs(a) - a*ln(1/2*abs(-2*sqrt(-a^2*x^2 + 1
)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) + 8*a/(((sqrt(-a^2*x^2 + 1)*
abs(a) + a)/(a^2*x) - 1)*abs(a))
```

3.712 $\int \frac{1}{\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=2

$$\sin^{-1}(x)$$

```
[Out] ArcSin[x]
```

Rubi [A] time = 0.00441641, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[1 - x^2],x]

[Out] ArcSin[x]
```

Rubi in Sympy [A] time = 0.098879, size = 2, normalized size = 1.

$$\operatorname{asin}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-x**2+1)**(1/2),x)

[Out] asin(x)
```

Mathematica [A] time = 0.00629055, size = 2, normalized size = 1.

$$\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[1 - x^2],x]

[Out] ArcSin[x]
```

Maple [A] time = 0.003, size = 3, normalized size = 1.5

$$\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-x^2+1)^(1/2),x)
```

```
[Out] arcsin(x)
```

Maxima [A] time = 0.755059, size = 3, normalized size = 1.5

$$\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-x^2 + 1),x, algorithm="maxima")
```

```
[Out] arcsin(x)
```

Fricas [A] time = 0.267103, size = 24, normalized size = 12.

$$-2 \arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-x^2 + 1),x, algorithm="fricas")
```

```
[Out] -2*arctan((sqrt(-x^2 + 1) - 1)/x)
```

Sympy [A] time = 0.29004, size = 2, normalized size = 1.

$$\operatorname{asin}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-x**2+1)**(1/2),x)
```

```
[Out] asin(x)
```

GIAC/XCAS [A] time = 0.266818, size = 3, normalized size = 1.5

$$\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-x^2 + 1),x, algorithm="giac")
```

```
[Out] arcsin(x)
```

3.713 $\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^4}} \, dx$

Optimal. Leaf size=2

$$\sin^{-1}(x)$$

```
[Out] ArcSin[x]
```

Rubi [A] time = 0.00537891, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095

$$\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 + x^2]/Sqrt[1 - x^4],x]

[Out] ArcSin[x]
```

Rubi in Sympy [A] time = 1.29857, size = 2, normalized size = 1.

$$\operatorname{asin}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+1)**(1/2)/(-x**4+1)**(1/2),x)

[Out] asin(x)
```

Mathematica [B] time = 0.0169729, size = 32, normalized size = 16.

$$-\tan^{-1}\left(\frac{x\sqrt{x^2+1}\sqrt{1-x^4}}{x^4-1}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 + x^2]/Sqrt[1 - x^4],x]
```

```
[Out] -ArcTan[(x*Sqrt[1 + x^2]*Sqrt[1 - x^4])/(-1 + x^4)]
```

Maple [B] time = 0.017, size = 29, normalized size = 14.5

$$\arcsin(x)\sqrt{-x^4+1}\frac{1}{\sqrt{x^2+1}}\frac{1}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+1)^(1/2)/(-x^4+1)^(1/2),x)
```

```
[Out] 1/(x^2+1)^(1/2)*(-x^4+1)^(1/2)/(-x^2+1)^(1/2)*arcsin(x)
```

Maxima [A] time = 0.777977, size = 3, normalized size = 1.5

$$\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 1)/sqrt(-x^4 + 1),x, algorithm="maxima")
```

```
[Out] arcsin(x)
```

Fricas [A] time = 0.26998, size = 36, normalized size = 18.

$$-\arctan\left(\frac{\sqrt{-x^4+1}\sqrt{x^2+1}}{x^3+x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 1)/sqrt(-x^4 + 1),x, algorithm="fricas")
```

```
[Out] -arctan(sqrt(-x^4 + 1)*sqrt(x^2 + 1)/(x^3 + x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{x^2+1}}{\sqrt{-(x-1)(x+1)(x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+1)**(1/2)/(-x**4+1)**(1/2),x)
```

```
[Out] Integral(sqrt(x**2 + 1)/sqrt(-(x - 1)*(x + 1)*(x**2 + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+1}}{\sqrt{-x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 1)/sqrt(-x^4 + 1),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 + 1)/sqrt(-x^4 + 1), x)
```

3.714 $\int \frac{1}{\sqrt{1+x^2}} dx$

Optimal. Leaf size=2

$$\sinh^{-1}(x)$$

```
[Out] ArcSinh[x]
```

Rubi [A] time = 0.00396203, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[1 + x^2],x]

[Out] ArcSinh[x]
```

Rubi in Sympy [A] time = 0.088769, size = 2, normalized size = 1.

$$\operatorname{asinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**2+1)**(1/2),x)

[Out] asinh(x)
```

Mathematica [A] time = 0.00512549, size = 2, normalized size = 1.

$$\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[1 + x^2],x]

[Out] ArcSinh[x]
```

Maple [A] time = 0.003, size = 3, normalized size = 1.5

$$Arcsinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^2+1)^(1/2),x)

[Out] arcsinh(x)
```

Maxima [A] time = 0.798872, size = 3, normalized size = 1.5

$$\operatorname{arsinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(x^2 + 1),x, algorithm="maxima")

[Out] arcsinh(x)
```

Fricas [A] time = 0.266181, size = 19, normalized size = 9.5

$$-\log\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(x^2 + 1),x, algorithm="fricas")

[Out] -log(-x + sqrt(x^2 + 1))
```

Sympy [A] time = 0.280099, size = 2, normalized size = 1.

$$\operatorname{asinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**2+1)**(1/2),x)
```

```
[Out] asinh(x)
```

GIAC/XCAS [A] time = 0.263489, size = 19, normalized size = 9.5

$$-\ln\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(x^2 + 1),x, algorithm="giac")
```

```
[Out] -ln(-x + sqrt(x^2 + 1))
```

3.715 $\int \frac{\sqrt{1-x^2}}{\sqrt{1-x^4}} \, dx$

Optimal. Leaf size=2

$$\sinh^{-1}(x)$$

```
[Out] ArcSinh[x]
```

Rubi [A] time = 0.00472519, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$

$$\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 - x^2]/Sqrt[1 - x^4],x]

[Out] ArcSinh[x]
```

Rubi in Sympy [A] time = 1.51386, size = 2, normalized size = 1.

$$\operatorname{asinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2+1)**(1/2)/(-x**4+1)**(1/2),x)

[Out] asinh(x)
```

Mathematica [B] time = 0.0148152, size = 42, normalized size = 21.

$$\log\left(1-x^2\right)-\log\left(x^3+\sqrt{1-x^2}\sqrt{1-x^4}-x\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 - x^2]/Sqrt[1 - x^4],x]
```

```
[Out] Log[1 - x^2] - Log[-x + x^3 + Sqrt[1 - x^2]*Sqrt[1 - x^4]]
```

Maple [B] time = 0.011, size = 29, normalized size = 14.5

$$Arcsinh(x)\sqrt{-x^4+1}\frac{1}{\sqrt{-x^2+1}}\frac{1}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2+1)^(1/2)/(-x^4+1)^(1/2),x)
```

```
[Out] 1/(-x^2+1)^(1/2)/(x^2+1)^(1/2)*(-x^4+1)^(1/2)*arcsinh(x)
```

Maxima [A] time = 0.779472, size = 3, normalized size = 1.5

$$\operatorname{arsinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 1)/sqrt(-x^4 + 1),x, algorithm="maxima")
```

```
[Out] arcsinh(x)
```

Fricas [A] time = 0.265651, size = 109, normalized size = 54.5

$$-\frac{1}{2}\log\left(\frac{x^3+\sqrt{-x^4+1}\sqrt{-x^2+1}-x}{x^3-x}\right)+\frac{1}{2}\log\left(-\frac{x^3-\sqrt{-x^4+1}\sqrt{-x^2+1}-x}{x^3-x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 1)/sqrt(-x^4 + 1),x, algorithm="fricas")
```

```
[Out] -1/2*log((x^3 + sqrt(-x^4 + 1)*sqrt(-x^2 + 1) - x)/(x^3 - x)) + 1
/2*log(-(x^3 - sqrt(-x^4 + 1)*sqrt(-x^2 + 1) - x)/(x^3 - x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-\left(x-1\right)\left(x+1\right)}}{\sqrt{-\left(x-1\right)\left(x+1\right)\left(x^{2}+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+1)**(1/2)/(-x**4+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 1))/sqrt(-(x - 1)*(x + 1)*(x**2 + 1))
, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^{2}+1}}{\sqrt{-x^{4}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 1)/sqrt(-x^4 + 1),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^2 + 1)/sqrt(-x^4 + 1), x)
```

3.716 $\int \sqrt{1-x^2}\,dx$

Optimal. Leaf size=23

$$\frac{1}{2}\sqrt{1-x^2}x+\frac{1}{2}\sin^{-1}(x)$$

```
[Out] (x*Sqrt[1 - x^2])/2 + ArcSin[x]/2
```

Rubi [A] time = 0.010235, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{1}{2}\sqrt{1-x^2}x+\frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 - x^2],x]

[Out] (x*Sqrt[1 - x^2])/2 + ArcSin[x]/2
```

Rubi in Sympy [A] time = 0.618516, size = 15, normalized size = 0.65

$$\frac{x\sqrt{-x^2+1}}{2}+\frac{\operatorname{asin}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2+1)**(1/2),x)

[Out] x*sqrt(-x**2 + 1)/2 + asin(x)/2
```

Mathematica [A] time = 0.00860338, size = 20, normalized size = 0.87

$$\frac{1}{2}\left(\sqrt{1-x^2}x+\sin^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 - x^2],x]
```

```
[Out] (x*Sqrt[1 - x^2] + ArcSin[x])/2
```

Maple [A] time = 0.003, size = 18, normalized size = 0.8

$$\frac{\arcsin(x)}{2}+\frac{x}{2}\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2+1)^(1/2),x)
```

```
[Out] 1/2*arcsin(x)+1/2*x*(-x^2+1)^(1/2)
```

Maxima [A] time = 0.794323, size = 23, normalized size = 1.

$$\frac{1}{2}\sqrt{-x^2+1}x+\frac{1}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 1),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-x^2 + 1)*x + 1/2*arcsin(x)
```

Fricas [A] time = 0.264015, size = 109, normalized size = 4.74

$$-\frac{2x^3+2\left(x^2+2\sqrt{-x^2+1}-2\right)\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)-\left(x^3-2x\right)\sqrt{-x^2+1}-2x}{2\left(x^2+2\sqrt{-x^2+1}-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 1),x, algorithm="fricas")
```

```
[Out] -1/2*(2*x^3 + 2*(x^2 + 2*sqrt(-x^2 + 1) - 2)*arctan((sqrt(-x^2 +
1) - 1)/x) - (x^3 - 2*x)*sqrt(-x^2 + 1) - 2*x)/(x^2 + 2*sqrt(-x^2
 + 1) - 2)
```

Sympy [A] time = 0.448555, size = 15, normalized size = 0.65

$$\frac{x\sqrt{-x^2+1}}{2}+\frac{\operatorname{asin}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+1)**(1/2),x)
```

```
[Out] x*sqrt(-x**2 + 1)/2 + asin(x)/2
```

GIAC/XCAS [A] time = 0.263781, size = 23, normalized size = 1.

$$\frac{1}{2}\sqrt{-x^2+1}x+\frac{1}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 1),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-x^2 + 1)*x + 1/2*arcsin(x)
```

3.717 $\int \frac{\sqrt{1-x^4}}{\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=23

$$\frac{1}{2}\sqrt{1-x^2}x+\frac{1}{2}\sin^{-1}(x)$$

```
[Out] (x*Sqrt[1 - x^2])/2 + ArcSin[x]/2
```

Rubi [A] time = 0.011002, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{1}{2}\sqrt{1-x^2}x+\frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 - x^4]/Sqrt[1 + x^2],x]

[Out] (x*Sqrt[1 - x^2])/2 + ArcSin[x]/2
```

Rubi in Sympy [A] time = 1.39287, size = 15, normalized size = 0.65

$$\frac{x\sqrt{-x^2+1}}{2}+\frac{\operatorname{asin}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**4+1)**(1/2)/(x**2+1)**(1/2),x)

[Out] x*sqrt(-x**2 + 1)/2 + asin(x)/2
```

Mathematica [B] time = 0.0448667, size = 50, normalized size = 2.17

$$\frac{1}{2}\left(\frac{\sqrt{1-x^4}x}{\sqrt{x^2+1}}+\tan^{-1}\left(\frac{x\sqrt{x^2+1}}{\sqrt{1-x^4}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 - x^4]/Sqrt[1 + x^2],x]
```

```
[Out] ((x*Sqrt[1 - x^4])/Sqrt[1 + x^2] + ArcTan[(x*Sqrt[1 + x^2])/Sqrt[
1 - x^4]])/2
```

Maple [B] time = 0.011, size = 42, normalized size = 1.8

$$\frac{1}{2}\sqrt{-x^4+1}\left(x\sqrt{-x^2+1}+\arcsin(x)\right)\frac{1}{\sqrt{x^2+1}}\frac{1}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^4+1)^(1/2)/(x^2+1)^(1/2),x)
```

```
[Out] 1/2*(-x^4+1)^(1/2)/(x^2+1)^(1/2)*(x*(-x^2+1)^(1/2)+arcsin(x))/(-x
^2+1)^(1/2)
```

Maxima [A] time = 0.839288, size = 23, normalized size = 1.

$$\frac{1}{2}\sqrt{-x^2+1}x+\frac{1}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^4 + 1)/sqrt(x^2 + 1),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-x^2 + 1)*x + 1/2*arcsin(x)
```

Fricas [A] time = 0.268601, size = 81, normalized size = 3.52

$$\frac{\sqrt{-x^4+1}\sqrt{x^2+1}x-\left(x^2+1\right)\arctan\left(\frac{\sqrt{-x^4+1}\sqrt{x^2+1}}{x^3+x}\right)}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^4 + 1)/sqrt(x^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(-x^4 + 1)*sqrt(x^2 + 1)*x - (x^2 + 1)*arctan(sqrt(-x^4
+ 1)*sqrt(x^2 + 1)/(x^3 + x)))/(x^2 + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(x-1)(x+1)(x^2+1)}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**4+1)**(1/2)/(x**2+1)**(1/2),x)

[Out] Integral(sqrt(-(x - 1)*(x + 1)*(x**2 + 1))/sqrt(x**2 + 1), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^4+1}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^4 + 1)/sqrt(x^2 + 1),x, algorithm="giac")

[Out] integrate(sqrt(-x^4 + 1)/sqrt(x^2 + 1), x)
```

3.718 $\int \sqrt{1+x^2}\,dx$

Optimal. Leaf size=21

$$\frac{1}{2}\sqrt{x^2+1}x+\frac{1}{2}\sinh^{-1}(x)$$

```
[Out] (x*Sqrt[1 + x^2])/2 + ArcSinh[x]/2
```

Rubi [A] time = 0.00867122, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$\frac{1}{2}\sqrt{x^2+1}x+\frac{1}{2}\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^2],x]

[Out] (x*Sqrt[1 + x^2])/2 + ArcSinh[x]/2
```

Rubi in Sympy [A] time = 0.580111, size = 15, normalized size = 0.71

$$\frac{x\sqrt{x^2+1}}{2}+\frac{\operatorname{asinh}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((x**2+1)**(1/2),x)

[Out] x*sqrt(x**2 + 1)/2 + asinh(x)/2
```

Mathematica [A] time = 0.0067718, size = 18, normalized size = 0.86

$$\frac{1}{2}\left(\sqrt{x^2+1}x+\sinh^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^2],x]
```

```
[Out] (x*Sqrt[1 + x^2] + ArcSinh[x])/2
```

Maple [A] time = 0.002, size = 16, normalized size = 0.8

$$\frac{Arcsinh\,(x)}{2}+\frac{x}{2}\sqrt{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+1)^(1/2),x)
```

```
[Out] 1/2*arcsinh(x)+1/2*x*(x^2+1)^(1/2)
```

Maxima [A] time = 0.775904, size = 20, normalized size = 0.95

$$\frac{1}{2}\sqrt{x^2+1}x+\frac{1}{2}\,\mathrm{arsinh}\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 1),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(x^2 + 1)*x + 1/2*arcsinh(x)
```

Fricas [A] time = 0.262929, size = 105, normalized size = 5.

$$-\frac{2\,x^4+2\,x^2+\left(2\,x^2-2\,\sqrt{x^2+1}x+1\right)\log\left(-x+\sqrt{x^2+1}\right)-\left(2\,x^3+x\right)\sqrt{x^2+1}}{2\left(2\,x^2-2\,\sqrt{x^2+1}x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 1),x, algorithm="fricas")
```

```
[Out] -1/2*(2*x^4 + 2*x^2 + (2*x^2 - 2*sqrt(x^2 + 1)*x + 1)*log(-x + sq
rt(x^2 + 1)) - (2*x^3 + x)*sqrt(x^2 + 1))/(2*x^2 - 2*sqrt(x^2 + 1
)*x + 1)
```

Sympy [A] time = 0.447945, size = 15, normalized size = 0.71

$$\frac{x\sqrt{x^2+1}}{2}+\frac{\operatorname{asinh}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+1)**(1/2),x)
```

```
[Out] x*sqrt(x**2 + 1)/2 + asinh(x)/2
```

GIAC/XCAS [A] time = 0.267376, size = 34, normalized size = 1.62

$$\frac{1}{2}\sqrt{x^2+1}x-\frac{1}{2}\ln\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 1),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(x^2 + 1)*x - 1/2*ln(-x + sqrt(x^2 + 1))
```

3.719 $\int \frac{\sqrt{1-x^4}}{\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=21

$$\frac{1}{2}\sqrt{x^2+1}x+\frac{1}{2}\sinh^{-1}(x)$$

```
[Out] (x*Sqrt[1 + x^2])/2 + ArcSinh[x]/2
```

Rubi [A] time = 0.0090648, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13

$$\frac{1}{2}\sqrt{x^2+1}x+\frac{1}{2}\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 - x^4]/Sqrt[1 - x^2],x]

[Out] (x*Sqrt[1 + x^2])/2 + ArcSinh[x]/2
```

Rubi in Sympy [A] time = 1.60775, size = 15, normalized size = 0.71

$$\frac{x\sqrt{x^2+1}}{2}+\frac{\operatorname{asinh}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**4+1)**(1/2)/(-x**2+1)**(1/2),x)

[Out] x*sqrt(x**2 + 1)/2 + asinh(x)/2
```

Mathematica [B] time = 0.075347, size = 70, normalized size = 3.33

$$\frac{1}{2}\left(\log\left(1-x^2\right)+\frac{\sqrt{1-x^4}x}{\sqrt{1-x^2}}-\log\left(x^3+\sqrt{1-x^2}\sqrt{1-x^4}-x\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 - x^4]/Sqrt[1 - x^2],x]
```

```
[Out] ((x*Sqrt[1 - x^4])/Sqrt[1 - x^2] + Log[1 - x^2] - Log[-x + x^3 +
Sqrt[1 - x^2]*Sqrt[1 - x^4]])/2
```

Maple [B] time = 0.011, size = 47, normalized size = 2.2

$$-\frac{1}{2x^2-2}\sqrt{-x^4+1}\sqrt{-x^2+1}\left(x\sqrt{x^2+1}+Arcsinh\,(x)\right)\frac{1}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^4+1)^(1/2)/(-x^2+1)^(1/2),x)
```

```
[Out] -1/2*(-x^4+1)^(1/2)*(-x^2+1)^(1/2)*(x*(x^2+1)^(1/2)+arcsinh(x))/(
x^2-1)/(x^2+1)^(1/2)
```

Maxima [A] time = 0.801589, size = 20, normalized size = 0.95

$$\frac{1}{2}\sqrt{x^2+1}x+\frac{1}{2}\,\text{arsinh}\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^4 + 1)/sqrt(-x^2 + 1),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(x^2 + 1)*x + 1/2*arcsinh(x)
```

Fricas [A] time = 0.272544, size = 162, normalized size = 7.71

$$-\frac{2\sqrt{-x^4+1}\sqrt{-x^2+1}x+\left(x^2-1\right)\log\left(\frac{x^3+\sqrt{-x^4+1}\sqrt{-x^2+1}-x}{x^3-x}\right)-\left(x^2-1\right)\log\left(-\frac{x^3-\sqrt{-x^4+1}\sqrt{-x^2+1}-x}{x^3-x}\right)}{4\left(x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^4 + 1)/sqrt(-x^2 + 1),x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(-x^4 + 1)*sqrt(-x^2 + 1)*x + (x^2 - 1)*log((x^3 + sq
rt(-x^4 + 1)*sqrt(-x^2 + 1) - x)/(x^3 - x)) - (x^2 - 1)*log(-(x^3
```

```
- sqrt(-x^4 + 1)*sqrt(-x^2 + 1) - x)/(x^3 - x)))/(x^2 - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(x-1)(x+1)(x^2+1)}}{\sqrt{-(x-1)(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**4+1)**(1/2)/(-x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 1)*(x**2 + 1))/sqrt(-(x - 1)*(x + 1))
, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^4+1}}{\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^4 + 1)/sqrt(-x^2 + 1),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^4 + 1)/sqrt(-x^2 + 1), x)
```

3.720 $\int \left(\frac{a+b+cx^2}{d}\right)^m dx$

Optimal. Leaf size=49

$$\frac{dx\left(\frac{a+b}{d}+\frac{cx^2}{d}\right)^{m+1}{}_2F_1\left(1,m+\frac{3}{2};\frac{3}{2};-\frac{cx^2}{a+b}\right)}{a+b}$$

```
[Out] (d*x*((a + b)/d + (c*x^2)/d)^(1 + m)*Hypergeometric2F1[1, 3/2 + m
, 3/2, -((c*x^2)/(a + b))])/(a + b)
```

Rubi [A] time = 0.0401009, antiderivative size = 57, normalized size of antiderivative = 1.16, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214

$$x\left(\frac{cx^2}{a+b}+1\right)^{-m}\left(\frac{a+b}{d}+\frac{cx^2}{d}\right)^m{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};-\frac{cx^2}{a+b}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b + c*x^2)/d)^m,x]
```

```
[Out] (x*((a + b)/d + (c*x^2)/d)^m*Hypergeometric2F1[1/2, -m, 3/2, -((c
*x^2)/(a + b))])/(1 + (c*x^2)/(a + b))^m
```

Rubi in Sympy [A] time = 3.9821, size = 42, normalized size = 0.86

$$x\left(\frac{cx^2}{d}+\frac{a+b}{d}\right)^m\left(\frac{cx^2}{a+b}+1\right)^{-m}{}_2F_1\left(\begin{matrix}-m,\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|-\frac{cx^2}{a+b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((c*x**2+a+b)/d)**m,x)
```

```
[Out] x*(c*x**2/d + (a + b)/d)**m*(c*x**2/(a + b) + 1)**(-m)*hyper((-m,
 1/2), (3/2,), -c*x**2/(a + b))
```

Mathematica [A] time = 0.0283969, size = 53, normalized size = 1.08

$$x\left(\frac{cx^2}{a+b}+1\right)^{-m}\left(\frac{a+b+cx^2}{d}\right)^m{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};-\frac{cx^2}{a+b}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b + c*x^2)/d)^m,x]
```

```
[Out] (x*((a + b + c*x^2)/d)^m*Hypergeometric2F1[1/2, -m, 3/2, -((c*x^2
)/(a + b))])/(1 + (c*x^2)/(a + b))^m
```

Maple [F] time = 0.055, size = 0, normalized size = 0.

$$\int \left(\frac{c x^2 + a + b}{d}\right)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((c*x^2+a+b)/d)^m,x)
```

```
[Out] int(((c*x^2+a+b)/d)^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\frac{c x^2 + a + b}{d}\right)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((c*x^2 + a + b)/d)^m,x, algorithm="maxima")
```

```
[Out] integrate(((c*x^2 + a + b)/d)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\frac{c x^2 + a + b}{d}\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((c*x^2 + a + b)/d)^m,x, algorithm="fricas")
```

```
[Out] integral(((c*x^2 + a + b)/d)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\frac{a+b+cx^2}{d}\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((c*x**2+a+b)/d)**m,x)
```

```
[Out] Integral(((a + b + c*x**2)/d)**m, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\frac{cx^2+a+b}{d}\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((c*x^2 + a + b)/d)^m,x, algorithm="giac")
```

```
[Out] integrate(((c*x^2 + a + b)/d)^m, x)
```

3.721 $\int \frac{1}{x-\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=28

$$-\frac{x^2}{2}-\frac{1}{2}\sqrt{x^2+1}x-\frac{1}{2}\sinh^{-1}(x)$$

```
[Out] -x^2/2 - (x*Sqrt[1 + x^2])/2 - ArcSinh[x]/2
```

Rubi [A] time = 0.019533, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267

$$-\frac{x^2}{2}-\frac{1}{2}\sqrt{x^2+1}x-\frac{1}{2}\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(x - Sqrt[1 + x^2])^(-1),x]
```

```
[Out] -x^2/2 - (x*Sqrt[1 + x^2])/2 - ArcSinh[x]/2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{x\sqrt{x^2+1}}{2}-\frac{\operatorname{asinh}(x)}{2}-\int x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x-(x**2+1)**(1/2)),x)
```

```
[Out] -x*sqrt(x**2 + 1)/2 - asinh(x)/2 - Integral(x, x)
```

Mathematica [A] time = 0.0250543, size = 23, normalized size = 0.82

$$\frac{1}{2}\left(-x\left(\sqrt{x^2+1}+x\right)-\sinh^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x - Sqrt[1 + x^2])^(-1),x]

[Out] (-(x*(x + Sqrt[1 + x^2])) - ArcSinh[x])/2
```

Maple [A] time = 0.003, size = 21, normalized size = 0.8

$$-\frac{x^2}{2} - \frac{Arcsinh(x)}{2} - \frac{x}{2}\sqrt{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x-(x^2+1)^(1/2)),x)

[Out] -1/2*x^2-1/2*arcsinh(x)-1/2*x*(x^2+1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x - \sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x - sqrt(x^2 + 1)),x, algorithm="maxima")

[Out] integrate(1/(x - sqrt(x^2 + 1)), x)
```

Fricas [A] time = 0.266012, size = 88, normalized size = 3.14

$$\frac{x^2 + \left(2\,x^2 - 2\,\sqrt{x^2+1}x + 1\right)\log\left(-x + \sqrt{x^2+1}\right) - \sqrt{x^2+1}x}{2\left(2\,x^2 - 2\,\sqrt{x^2+1}x + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x - sqrt(x^2 + 1)),x, algorithm="fricas")

[Out] 1/2*(x^2 + (2*x^2 - 2*sqrt(x^2 + 1)*x + 1)*log(-x + sqrt(x^2 + 1)
) - sqrt(x^2 + 1)*x)/(2*x^2 - 2*sqrt(x^2 + 1)*x + 1)
```

Sympy [A] time = 1.46705, size = 58, normalized size = 2.07

$$-\frac{x\operatorname{asinh}(x)}{2x-2\sqrt{x^2+1}}+\frac{x}{2x-2\sqrt{x^2+1}}+\frac{\sqrt{x^2+1}\operatorname{asinh}(x)}{2x-2\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x-(x**2+1)**(1/2)),x)
```

```
[Out] -x*asinh(x)/(2*x - 2*sqrt(x**2 + 1)) + x/(2*x - 2*sqrt(x**2 + 1))
 + sqrt(x**2 + 1)*asinh(x)/(2*x - 2*sqrt(x**2 + 1))
```

GIAC/XCAS [A] time = 0.266995, size = 41, normalized size = 1.46

$$-\frac{1}{2}x^2-\frac{1}{2}\sqrt{x^2+1}x+\frac{1}{2}\ln\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x - sqrt(x^2 + 1)),x, algorithm="giac")
```

```
[Out] -1/2*x^2 - 1/2*sqrt(x^2 + 1)*x + 1/2*ln(-x + sqrt(x^2 + 1))
```

3.722 $\int \frac{1}{x-\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=37

$$\frac{1}{4}\log\left(1-2x^2\right)-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)-\frac{1}{2}\sin^{-1}(x)$$

```
[Out] -ArcSin[x]/2 - ArcTanh[x/Sqrt[1 - x^2]]/2 + Log[1 - 2*x^2]/4
```

Rubi [A] time = 0.0916905, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353

$$\frac{1}{4}\log\left(1-2x^2\right)-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)-\frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(x - Sqrt[1 - x^2])^(-1),x]

[Out] -ArcSin[x]/2 - ArcTanh[x/Sqrt[1 - x^2]]/2 + Log[1 - 2*x^2]/4
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x-\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x-(-x**2+1)**(1/2)),x)

[Out] Integral(1/(x - sqrt(-x**2 + 1)), x)
```

Mathematica [A] time = 0.0210904, size = 37, normalized size = 1.

$$\frac{1}{4}\log\left(1-2x^2\right)-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)-\frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x - Sqrt[1 - x^2])^(-1),x]

[Out] -ArcSin[x]/2 - ArcTanh[x/Sqrt[1 - x^2]]/2 + Log[1 - 2*x^2]/4
```

Maple [B] time = 0.047, size = 175, normalized size = 4.7

$$\frac{\ln\left(2x^2-1\right)}{4}+\frac{\sqrt{2}}{8}\sqrt{-4\left(x-1/2\sqrt{2}\right)^2-4\sqrt{2}\left(x-1/2\sqrt{2}\right)+2}-\frac{\arcsin\left(x\right)}{2}$$
$$-\frac{1}{4}Artanh\left(\sqrt{2}\left(1-\sqrt{2}\left(x-\frac{\sqrt{2}}{2}\right)\right)\frac{1}{\sqrt{-4\left(x-1/2\sqrt{2}\right)^2-4\sqrt{2}\left(x-1/2\sqrt{2}\right)+2}}\right)$$
$$-\frac{\sqrt{2}}{8}\sqrt{-4\left(x+1/2\sqrt{2}\right)^2+4\sqrt{2}\left(x+1/2\sqrt{2}\right)+2}$$
$$+\frac{1}{4}Artanh\left(\sqrt{2}\left(\sqrt{2}\left(x+\frac{\sqrt{2}}{2}\right)+1\right)\frac{1}{\sqrt{-4\left(x+1/2\sqrt{2}\right)^2+4\sqrt{2}\left(x+1/2\sqrt{2}\right)+2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x-(-x^2+1)^(1/2)),x)

[Out] 1/4*ln(2*x^2-1)+1/8*2^(1/2)*(-4*(x-1/2*2^(1/2))^2-4*2^(1/2)*(x-1/
2*2^(1/2))+2)^(1/2)-1/2*arcsin(x)-1/4*arctanh((1-2^(1/2)*(x-1/2*2
^(1/2)))*2^(1/2)/(-4*(x-1/2*2^(1/2))^2-4*2^(1/2)*(x-1/2*2^(1/2))+
2)^(1/2))-1/8*2^(1/2)*(-4*(x+1/2*2^(1/2))^2+4*2^(1/2)*(x+1/2*2^(1
/2))+2)^(1/2)+1/4*arctanh((2^(1/2)*(x+1/2*2^(1/2))+1)*2^(1/2)/(-4
*(x+1/2*2^(1/2))^2+4*2^(1/2)*(x+1/2*2^(1/2))+2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x-\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x - sqrt(-x^2 + 1)),x, algorithm="maxima")

[Out] integrate(1/(x - sqrt(-x^2 + 1)), x)
```

Fricas [A] time = 0.269286, size = 113, normalized size = 3.05

$$\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)+\frac{1}{4}\log\left(2x^2-1\right)+\frac{1}{4}\log\left(-\frac{x^2+\sqrt{-x^2+1}(x+1)-x-1}{x^2}\right)-\frac{1}{4}\log\left(-\frac{x^2-\sqrt{-x^2+1}(x-1)+x-1}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x - sqrt(-x^2 + 1)),x, algorithm="fricas")
```

```
[Out] arctan((sqrt(-x^2 + 1) - 1)/x) + 1/4*log(2*x^2 - 1) + 1/4*log(-(x
^2 + sqrt(-x^2 + 1)*(x + 1) - x - 1)/x^2) - 1/4*log(-(x^2 - sqrt(
-x^2 + 1)*(x - 1) + x - 1)/x^2)
```

Sympy [A] time = 0.37748, size = 17, normalized size = 0.46

$$\frac{\log\left(x-\sqrt{-x^2+1}\right)}{2}-\frac{\operatorname{asin}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x-(-x**2+1)**(1/2)),x)
```

```
[Out] log(x - sqrt(-x**2 + 1))/2 - asin(x)/2
```

GIAC/XCAS [A] time = 0.273892, size = 189, normalized size = 5.11

$$-\frac{1}{4}\pi\operatorname{sign}(x)-\frac{1}{2}\arctan\left(-\frac{x\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-1\right)}{2\left(\sqrt{-x^2+1}-1\right)}\right)+\frac{1}{4}\ln\left(\left|x+\frac{1}{2}\sqrt{2}\right|\right)+\frac{1}{4}\ln\left(\left|x-\frac{1}{2}\sqrt{2}\right|\right)-\frac{1}{4}\ln\left(\left|-\frac{x}{\sqrt{-x^2+1}-1}+\frac{\sqrt{-x^2+1}-1}{x}+2\right|\right)+\frac{1}{4}\ln\left(\left|-\frac{x}{\sqrt{-x^2+1}-1}+\frac{\sqrt{-x^2+1}-1}{x}-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x - sqrt(-x^2 + 1)),x, algorithm="giac")
```

```
[Out] -1/4*pi*sign(x) - 1/2*arctan(-1/2*x*((sqrt(-x^2 + 1) - 1)^2/x^2 -
 1)/(sqrt(-x^2 + 1) - 1)) + 1/4*ln(abs(x + 1/2*sqrt(2))) + 1/4*ln
(abs(x - 1/2*sqrt(2))) - 1/4*ln(abs(-x/(sqrt(-x^2 + 1) - 1) + (sq
rt(-x^2 + 1) - 1)/x + 2)) + 1/4*ln(abs(-x/(sqrt(-x^2 + 1) - 1) +
(sqrt(-x^2 + 1) - 1)/x - 2))
```

3.723 $\int \frac{1}{x-\sqrt{1+2x^2}}\,dx$

Optimal. Leaf size=40

$$-\frac{1}{2}\log\left(x^2+1\right)+\tanh^{-1}\left(\frac{x}{\sqrt{2x^2+1}}\right)-\sqrt{2}\sinh^{-1}\left(\sqrt{2}x\right)$$

```
[Out] -(Sqrt[2]*ArcSinh[Sqrt[2]*x]) + ArcTanh[x/Sqrt[1 + 2*x^2]] - Log[
1 + x^2]/2
```

Rubi [A] time = 0.090177, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$

$$-\frac{1}{2}\log\left(x^2+1\right)+\tanh^{-1}\left(\frac{x}{\sqrt{2x^2+1}}\right)-\sqrt{2}\sinh^{-1}\left(\sqrt{2}x\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(x - Sqrt[1 + 2*x^2])^(-1),x]
```

```
[Out] -(Sqrt[2]*ArcSinh[Sqrt[2]*x]) + ArcTanh[x/Sqrt[1 + 2*x^2]] - Log[
1 + x^2]/2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x-\sqrt{2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x-(2*x**2+1)**(1/2)),x)
```

```
[Out] Integral(1/(x - sqrt(2*x**2 + 1)), x)
```

Mathematica [A] time = 0.0466097, size = 74, normalized size = 1.85

$$\frac{1}{4}\left(-2\log\left(x^2+1\right)-\log\left(3x^2-2\sqrt{2x^2+1}x+1\right)+\log\left(3x^2+2\sqrt{2x^2+1}x+1\right)-4\sqrt{2}\sinh^{-1}\left(\sqrt{2}x\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x - Sqrt[1 + 2*x^2])^(-1),x]
```

```
[Out] (-4*Sqrt[2]*ArcSinh[Sqrt[2]*x] - 2*Log[1 + x^2] - Log[1 + 3*x^2 -
 2*x*Sqrt[1 + 2*x^2]] + Log[1 + 3*x^2 + 2*x*Sqrt[1 + 2*x^2]])/4
```

Maple [A] time = 0.012, size = 33, normalized size = 0.8

$$Artanh\left(x\frac{1}{\sqrt{2\,x^2+1}}\right)-\frac{\ln\left(x^2+1\right)}{2}-Arcsinh\left(\sqrt{2}x\right)\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x-(2*x^2+1)^(1/2)),x)
```

```
[Out] arctanh(x/(2*x^2+1)^(1/2))-1/2*ln(x^2+1)-arcsinh(2^(1/2)*x)*2^(1/
2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x-\sqrt{2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x - sqrt(2*x^2 + 1)),x, algorithm="maxima")
```

```
[Out] integrate(1/(x - sqrt(2*x^2 + 1)), x)
```

Fricas [A] time = 0.276397, size = 159, normalized size = 3.98

$$\sqrt{2}\log\left(-\frac{2\,x^2-\sqrt{2\,x^2+1}\left(\sqrt{2}x+1\right)+\sqrt{2}x+1}{\sqrt{2\,x^2+1}-1}\right)-\frac{1}{2}\,\log\left(x^2+1\right)-\frac{1}{2}\,\log\left(\frac{2\,x^2-\sqrt{2\,x^2+1}(x+1)+x+1}{x^2}\right)+\frac{1}{2}\,\log\left(\frac{2\,x^2+\sqrt{2\,x^2+1}(x-1)-x+1}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x - sqrt(2*x^2 + 1)),x, algorithm="fricas")
```

```
[Out] sqrt(2)*log(-(2*x^2 - sqrt(2*x^2 + 1)*(sqrt(2)*x + 1) + sqrt(2)*x
 + 1)/(sqrt(2*x^2 + 1) - 1)) - 1/2*log(x^2 + 1) - 1/2*log((2*x^2
- sqrt(2*x^2 + 1)*(x + 1) + x + 1)/x^2) + 1/2*log((2*x^2 + sqrt(2
*x^2 + 1)*(x - 1) - x + 1)/x^2)
```

Sympy [A] time = 0.550336, size = 27, normalized size = 0.68

$$-\log\left(x-\sqrt{2x^2+1}\right)-\sqrt{2}\operatorname{asinh}\left(\sqrt{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x-(2*x**2+1)**(1/2)),x)
```

```
[Out] -log(x - sqrt(2*x**2 + 1)) - sqrt(2)*asinh(sqrt(2)*x)
```

GIAC/XCAS [A] time = 0.268594, size = 119, normalized size = 2.98

$$\sqrt{2}\ln\left(-\sqrt{2}x+\sqrt{2\,x^2+1}\right)-\frac{1}{2}\ln\left(x^2+1\right)-\frac{1}{2}\ln\left(\frac{\left(\sqrt{2}x-\sqrt{2\,x^2+1}\right)^2-2\,\sqrt{2}+3}{\left(\sqrt{2}x-\sqrt{2\,x^2+1}\right)^2+2\,\sqrt{2}+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x - sqrt(2*x^2 + 1)),x, algorithm="giac")
```

```
[Out] sqrt(2)*ln(-sqrt(2)*x + sqrt(2*x^2 + 1)) - 1/2*ln(x^2 + 1) - 1/2*
ln(((sqrt(2)*x - sqrt(2*x^2 + 1))^2 - 2*sqrt(2) + 3)/((sqrt(2)*x
- sqrt(2*x^2 + 1))^2 + 2*sqrt(2) + 3))
```

3.724 $\int \frac{2x-x^3+x^2\sqrt{2-x^2}}{-2+2x^2}\,dx$

Optimal. Leaf size=54

$$-\frac{x^2}{4}+\frac{1}{4}\sqrt{2-x^2}\,x+\frac{1}{4}\log\left(1-x^2\right)-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{2-x^2}}\right)$$

```
[Out] -x^2/4 + (x*Sqrt[2 - x^2])/4 - ArcTanh[x/Sqrt[2 - x^2]]/2 + Log[1
 - x^2]/4
```

Rubi [A] time = 0.258306, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235

$$-\frac{x^2}{4}+\frac{1}{4}\sqrt{2-x^2}\,x+\frac{1}{4}\log\left(1-x^2\right)-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{2-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(2*x - x^3 + x^2*Sqrt[2 - x^2])/(-2 + 2*x^2),x]
```

```
[Out] -x^2/4 + (x*Sqrt[2 - x^2])/4 - ArcTanh[x/Sqrt[2 - x^2]]/2 + Log[1
 - x^2]/4
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*x-x**3+x**2*(-x**2+2)**(1/2))/(2*x**2-2),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.042192, size = 77, normalized size = 1.43

$$\frac{1}{4}\left(-x^2+\sqrt{2-x^2}\,x+\log\left(1-x^2\right)-\log\left(\sqrt{2-x^2}-x+2\right)+\log\left(\sqrt{2-x^2}+x+2\right)+\log(1-x)-\log(x+1)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2*x - x^3 + x^2*Sqrt[2 - x^2])/(-2 + 2*x^2),x]
```

```
[Out] (-x^2 + x*Sqrt[2 - x^2] + Log[1 - x] - Log[1 + x] + Log[1 - x^2]
- Log[2 - x + Sqrt[2 - x^2]] + Log[2 + x + Sqrt[2 - x^2]])/4
```

Maple [B] time = 0.019, size = 111, normalized size = 2.1

$$\frac{x}{4}\sqrt{-x^2+2}-\frac{1}{4}\sqrt{-(1+x)^2+3+2x}+\frac{1}{4}\mathit{Artanh}\left(\frac{4+2x}{2}\frac{1}{\sqrt{-(1+x)^2+3+2x}}\right)+\frac{1}{4}\sqrt{-(-1+x)^2-2x+3}-\frac{1}{4}\mathit{Artanh}\left(\frac{-2x+4}{2}\frac{1}{\sqrt{-(-1+x)^2-2x+3}}\right)-\frac{x^2}{4}+\frac{\ln(-1+x)}{4}+\frac{\ln(1+x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*x-x^3+x^2*(-x^2+2)^(1/2))/(2*x^2-2),x)
```

```
[Out] 1/4*x*(-x^2+2)^(1/2)-1/4*(-(1+x)^2+3+2*x)^(1/2)+1/4*arctanh(1/2*(
4+2*x)/(-(1+x)^2+3+2*x)^(1/2))+1/4*(-(-1+x)^2-2*x+3)^(1/2)-1/4*ar
ctanh(1/2*(-2*x+4)/(-(-1+x)^2-2*x+3)^(1/2))-1/4*x^2+1/4*ln(-1+x)+
1/4*ln(1+x)
```

Maxima [A] time = 0.766618, size = 127, normalized size = 2.35

$$-\frac{1}{4}x^2+\frac{1}{4}\sqrt{-x^2+2}x+\frac{1}{4}\log\left(x^2-1\right)+\frac{1}{4}\log\left(\frac{2\sqrt{-x^2+2}}{|2x+2|}+\frac{2}{|2x+2|}+1\right)-\frac{1}{4}\log\left(\frac{2\sqrt{-x^2+2}}{|2x-2|}+\frac{2}{|2x-2|}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/2*(x^3 - sqrt(-x^2 + 2)*x^2 - 2*x)/(x^2 - 1),x, algorithm="maxima")
```

```
[Out] -1/4*x^2 + 1/4*sqrt(-x^2 + 2)*x + 1/4*log(x^2 - 1) + 1/4*log(2*sq
rt(-x^2 + 2)/abs(2*x + 2) + 2/abs(2*x + 2) + 1) - 1/4*log(2*sqrt(
-x^2 + 2)/abs(2*x - 2) + 2/abs(2*x - 2) - 1)
```

Fricas [A] time = 0.281271, size = 90, normalized size = 1.67

$$-\frac{1}{4}x^2 + \frac{1}{4}\sqrt{-x^2+2}x + \frac{1}{4}\log\left(x^2-1\right) - \frac{1}{8}\log\left(-\frac{\sqrt{-x^2+2}x+1}{x^2}\right) + \frac{1}{8}\log\left(\frac{\sqrt{-x^2+2}x-1}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/2*(x^3 - sqrt(-x^2 + 2)*x^2 - 2*x)/(x^2 - 1),x, algorithm="fricas")
```

```
[Out] -1/4*x^2 + 1/4*sqrt(-x^2 + 2)*x + 1/4*log(x^2 - 1) - 1/8*log(-(sq
rt(-x^2 + 2)*x + 1)/x^2) + 1/8*log((sqrt(-x^2 + 2)*x - 1)/x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int\left(-\frac{2x}{x^2-1}\right)dx + \int\frac{x^3}{x^2-1}dx + \int\left(-\frac{x^2\sqrt{-x^2+2}}{x^2-1}\right)dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x-x**3+x**2*(-x**2+2)**(1/2))/(2*x**2-2),x)
```

```
[Out] -(Integral(-2*x/(x**2 - 1), x) + Integral(x**3/(x**2 - 1), x) + I
ntegral(-x**2*sqrt(-x**2 + 2)/(x**2 - 1), x))/2
```

GIAC/XCAS [A] time = 0.282264, size = 158, normalized size = 2.93

$$-\frac{1}{4}x^2 + \frac{1}{4}\sqrt{-x^2+2}x + \frac{1}{4}\ln\left(\left|x^2-1\right|\right) - \frac{1}{4}\ln\left(\left|\frac{x}{\sqrt{2}-\sqrt{-x^2+2}} - \frac{\sqrt{2}-\sqrt{-x^2+2}}{x} + 2\right|\right) + \frac{1}{4}\ln\left(\left|\frac{x}{\sqrt{2}-\sqrt{-x^2+2}} - \frac{\sqrt{2}-\sqrt{-x^2+2}}{x} - 2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/2*(x^3 - sqrt(-x^2 + 2)*x^2 - 2*x)/(x^2 - 1),x, algorithm="giac")
```

```
[Out] -1/4*x^2 + 1/4*sqrt(-x^2 + 2)*x + 1/4*ln(abs(x^2 - 1)) - 1/4*ln(a
bs(x/(sqrt(2) - sqrt(-x^2 + 2)) - (sqrt(2) - sqrt(-x^2 + 2))/x +
2)) + 1/4*ln(abs(x/(sqrt(2) - sqrt(-x^2 + 2)) - (sqrt(2) - sqrt(-
x^2 + 2))/x - 2))
```

3.725 $\int \frac{x\sqrt{2-x^2}}{x-\sqrt{2-x^2}}\,dx$

Optimal. Leaf size=60

$$-\frac{x^2}{4}+\frac{1}{4}\sqrt{2-x^2}x-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{2-x^2}}\right)+\frac{1}{4}\log(1-x)+\frac{1}{4}\log(x+1)$$

```
[Out] -x^2/4 + (x*Sqrt[2 - x^2])/4 - ArcTanh[x/Sqrt[2 - x^2]]/2 + Log[1
 - x]/4 + Log[1 + x]/4
```

Rubi [A] time = 0.518817, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233

$$-\frac{x^2}{4}+\frac{1}{4}\sqrt{2-x^2}x-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{2-x^2}}\right)+\frac{1}{4}\log(1-x)+\frac{1}{4}\log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[(x*Sqrt[2 - x^2])/(x - Sqrt[2 - x^2]),x]
```

```
[Out] -x^2/4 + (x*Sqrt[2 - x^2])/4 - ArcTanh[x/Sqrt[2 - x^2]]/2 + Log[1
 - x]/4 + Log[1 + x]/4
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{-x^2+2}}{x-\sqrt{-x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(-x**2+2)**(1/2)/(x-(-x**2+2)**(1/2)),x)
```

```
[Out] Integral(x*sqrt(-x**2 + 2)/(x - sqrt(-x**2 + 2)), x)
```

Mathematica [A] time = 0.0296679, size = 77, normalized size = 1.28

$$\frac{1}{4}\left(-x^2+\sqrt{2-x^2}x+\log\left(1-x^2\right)-\log\left(\sqrt{2-x^2}-x+2\right)+\log\left(\sqrt{2-x^2}+x+2\right)+\log(1-x)-\log(x+1)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*Sqrt[2 - x^2])/(x - Sqrt[2 - x^2]),x]
```

```
[Out] (-x^2 + x*Sqrt[2 - x^2] + Log[1 - x] - Log[1 + x] + Log[1 - x^2]
- Log[2 - x + Sqrt[2 - x^2]] + Log[2 + x + Sqrt[2 - x^2]])/4
```

Maple [B] time = 0.007, size = 111, normalized size = 1.9

$$\frac{x}{4}\sqrt{-x^2+2}-\frac{1}{4}\sqrt{-\left(1+x\right)^2+3+2\,x}+\frac{1}{4}\mathit{Artanh}\left(\frac{4+2\,x}{2}\frac{1}{\sqrt{-\left(1+x\right)^2+3+2\,x}}\right)+\frac{1}{4}\sqrt{-\left(-1+x\right)^2-2\,x+3}-\frac{1}{4}\mathit{Artanh}\left(\frac{-2\,x+4}{2}\frac{1}{\sqrt{-\left(-1+x\right)^2-2\,x+3}}\right)-\frac{x^2}{4}+\frac{\ln\left(-1+x\right)}{4}+\frac{\ln\left(1+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(-x^2+2)^(1/2)/(x-(-x^2+2)^(1/2)),x)
```

```
[Out] 1/4*x*(-x^2+2)^(1/2)-1/4*(-(1+x)^2+3+2*x)^(1/2)+1/4*arctanh(1/2*(
4+2*x)/(-(1+x)^2+3+2*x)^(1/2))+1/4*(-(-1+x)^2-2*x+3)^(1/2)-1/4*ar
ctanh(1/2*(-2*x+4)/(-(-1+x)^2-2*x+3)^(1/2))-1/4*x^2+1/4*ln(-1+x)+
1/4*ln(1+x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}\,x^2-\int -\frac{x^2}{x-\sqrt{-x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-x^2 + 2)*x/(x - sqrt(-x^2 + 2)),x, algorithm="maxima")
```

```
[Out] -1/2*x^2 - integrate(-x^2/(x - sqrt(-x^2 + 2)), x)
```

Fricas [A] time = 0.273947, size = 90, normalized size = 1.5

$$-\frac{1}{4}\,x^2+\frac{1}{4}\,\sqrt{-x^2+2}x+\frac{1}{4}\,\log\left(x^2-1\right)-\frac{1}{8}\,\log\left(-\frac{\sqrt{-x^2+2}x+1}{x^2}\right)+\frac{1}{8}\,\log\left(\frac{\sqrt{-x^2+2}x-1}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 2)*x/(x - sqrt(-x^2 + 2)),x, algorithm="fricas")
```

```
[Out] -1/4*x^2 + 1/4*sqrt(-x^2 + 2)*x + 1/4*log(x^2 - 1) - 1/8*log(-(sq
rt(-x^2 + 2)*x + 1)/x^2) + 1/8*log((sqrt(-x^2 + 2)*x - 1)/x^2)
```

Sympy [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{-x^2+2}}{x-\sqrt{-x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(-x**2+2)**(1/2)/(x-(-x**2+2)**(1/2)),x)
```

```
[Out] Integral(x*sqrt(-x**2 + 2)/(x - sqrt(-x**2 + 2)), x)
```

GIAC/XCAS [A]   time = 0.288509, size = 158, normalized size = 2.63

$$-\frac{1}{4}x^2+\frac{1}{4}\sqrt{-x^2+2}x+\frac{1}{4}\ln\left(\left|x^2-1\right|\right)-\frac{1}{4}\ln\left(\left|\frac{x}{\sqrt{2}-\sqrt{-x^2+2}}-\frac{\sqrt{2}-\sqrt{-x^2+2}}{x}+2\right|\right)+\frac{1}{4}\ln\left(\left|\frac{x}{\sqrt{2}-\sqrt{-x^2+2}}-\frac{\sqrt{2}-\sqrt{-x^2+2}}{x}-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 2)*x/(x - sqrt(-x^2 + 2)),x, algorithm="giac")
```

```
[Out] -1/4*x^2 + 1/4*sqrt(-x^2 + 2)*x + 1/4*ln(abs(x^2 - 1)) - 1/4*ln(a
bs(x/(sqrt(2) - sqrt(-x^2 + 2)) - (sqrt(2) - sqrt(-x^2 + 2))/x +
2)) + 1/4*ln(abs(x/(sqrt(2) - sqrt(-x^2 + 2)) - (sqrt(2) - sqrt(-
x^2 + 2))/x - 2))
```

3.726 $\int \frac{x}{-x+\sqrt{2x-x^2}}\,dx$

Optimal. Leaf size=51

$$-\frac{1}{2}\sqrt{2x-x^2}+\frac{1}{2}\tanh^{-1}\left(\sqrt{2x-x^2}\right)-\frac{x}{2}-\frac{1}{2}\log(1-x)$$

```
[Out] -x/2 - Sqrt[2*x - x^2]/2 + ArcTanh[Sqrt[2*x - x^2]]/2 - Log[1 - x
]/2
```

Rubi [A] time = 0.198276, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19

$$-\frac{1}{2}\sqrt{2x-x^2}+\frac{1}{2}\tanh^{-1}\left(\sqrt{2x-x^2}\right)-\frac{x}{2}-\frac{1}{2}\log(1-x)$$

Antiderivative was successfully verified.

```
[In]  Int[x/(-x + Sqrt[2*x - x^2]),x]
```

```
[Out] -x/2 - Sqrt[2*x - x^2]/2 + ArcTanh[Sqrt[2*x - x^2]]/2 - Log[1 - x
]/2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{-x+\sqrt{-x^2+2x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(-x+(-x**2+2*x)**(1/2)),x)
```

```
[Out] Integral(x/(-x + sqrt(-x**2 + 2*x)), x)
```

Mathematica [A] time = 0.0386552, size = 41, normalized size = 0.8

$$\frac{1}{2}\left(-x-\sqrt{-(x-2)x}-2\log(1-x)+\log\left(\sqrt{-(x-2)x}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(-x + Sqrt[2*x - x^2]),x]
```

```
[Out] (-x - Sqrt[-((-2 + x)*x)] - 2*Log[1 - x] + Log[1 + Sqrt[-((-2 + x
)*x)]])/2
```

Maple [A] time = 0.007, size = 38, normalized size = 0.8

$$-\frac{x}{2}-\frac{\ln\left(-1+x\right)}{2}-\frac{1}{2}\sqrt{-\left(-1+x\right)^{2}+1}+\frac{1}{2}\mathit{Artanh}\left(\frac{1}{\sqrt{-\left(-1+x\right)^{2}+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(-x+(-x^2+2*x)^(1/2)),x)
```

```
[Out] -1/2*x-1/2*ln(-1+x)-1/2*(-(-1+x)^2+1)^(1/2)+1/2*arctanh(1/(-(-1+x
)^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x}{x-\sqrt{-x^{2}+2\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x/(x - sqrt(-x^2 + 2*x)),x, algorithm="maxima")
```

```
[Out] -integrate(x/(x - sqrt(-x^2 + 2*x)), x)
```

Fricas [A] time = 0.272809, size = 89, normalized size = 1.75

$$-\frac{1}{2}\,x-\frac{1}{2}\,\sqrt{-x^{2}+2\,x}-\frac{1}{2}\,\log\left(x-1\right)+\frac{1}{2}\,\log\left(\frac{x+\sqrt{-x^{2}+2\,x}}{x}\right)-\frac{1}{2}\,\log\left(-\frac{x-\sqrt{-x^{2}+2\,x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x/(x - sqrt(-x^2 + 2*x)),x, algorithm="fricas")
```

```
[Out] -1/2*x - 1/2*sqrt(-x^2 + 2*x) - 1/2*log(x - 1) + 1/2*log((x + sqr
t(-x^2 + 2*x))/x) - 1/2*log(-(x - sqrt(-x^2 + 2*x))/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{x-\sqrt{-x^2+2x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(-x+(-x**2+2*x)**(1/2)),x)
```

```
[Out] -Integral(x/(x - sqrt(-x**2 + 2*x)), x)
```

GIAC/XCAS [A] time = 0.271647, size = 68, normalized size = 1.33

$$-\frac{1}{2}\,x-\frac{1}{2}\,\sqrt{-x^2+2\,x}-\frac{1}{2}\,\ln\left(-\frac{2\left(\sqrt{-x^2+2\,x}-1\right)}{|-2\,x+2|}\right)-\frac{1}{2}\,\ln\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x/(x - sqrt(-x^2 + 2*x)),x, algorithm="giac")
```

```
[Out] -1/2*x - 1/2*sqrt(-x^2 + 2*x) - 1/2*ln(-2*(sqrt(-x^2 + 2*x) - 1)/
abs(-2*x + 2)) - 1/2*ln(abs(x - 1))
```

3.727 $\int \frac{x+\sqrt{2x-x^2}}{2-2x}\,dx$

Optimal. Leaf size=51

$$-\frac{1}{2}\sqrt{2x-x^2}+\frac{1}{2}\tanh^{-1}\left(\sqrt{2x-x^2}\right)-\frac{x}{2}-\frac{1}{2}\log(1-x)$$

```
[Out] -x/2 - Sqrt[2*x - x^2]/2 + ArcTanh[Sqrt[2*x - x^2]]/2 - Log[1 - x
]/2
```

Rubi [A] time = 0.181128, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217

$$-\frac{1}{2}\sqrt{2x-x^2}+\frac{1}{2}\tanh^{-1}\left(\sqrt{2x-x^2}\right)-\frac{x}{2}-\frac{1}{2}\log(1-x)$$

Antiderivative was successfully verified.

```
[In]  Int[(x + Sqrt[2*x - x^2])/(2 - 2*x),x]
```

```
[Out] -x/2 - Sqrt[2*x - x^2]/2 + ArcTanh[Sqrt[2*x - x^2]]/2 - Log[1 - x
]/2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+\sqrt{-x^2+2x}}{-2x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x+(-x**2+2*x)**(1/2))/(2-2*x),x)
```

```
[Out] Integral((x + sqrt(-x**2 + 2*x))/(-2*x + 2), x)
```

Mathematica [A] time = 0.0208031, size = 41, normalized size = 0.8

$$\frac{1}{2}\left(-x-\sqrt{-(x-2)x}-2\log(1-x)+\log\left(\sqrt{-(x-2)x}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x + Sqrt[2*x - x^2])/(2 - 2*x),x]
```

```
[Out] (-x - Sqrt[-((-2 + x)*x)] - 2*Log[1 - x] + Log[1 + Sqrt[-((-2 + x
)*x)]])/2
```

Maple [A] time = 0.005, size = 38, normalized size = 0.8

$$-\frac{x}{2}-\frac{\ln\left(-1+x\right)}{2}-\frac{1}{2}\sqrt{-\left(-1+x\right)^{2}+1}+\frac{1}{2}\mathit{Artanh}\left(\frac{1}{\sqrt{-\left(-1+x\right)^{2}+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x+(-x^2+2*x)^(1/2))/(2-2*x),x)
```

```
[Out] -1/2*x-1/2*ln(-1+x)-1/2*(-(-1+x)^2+1)^(1/2)+1/2*arctanh(1/(-(-1+x
)^2+1)^(1/2))
```

Maxima [A] time = 0.771004, size = 73, normalized size = 1.43

$$-\frac{1}{2}\,x-\frac{1}{2}\,\sqrt{-x^{2}+2\,x}-\frac{1}{2}\,\log\left(x-1\right)+\frac{1}{2}\,\log\left(\frac{2\,\sqrt{-x^{2}+2\,x}}{\left|x-1\right|}+\frac{2}{\left|x-1\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/2*(x + sqrt(-x^2 + 2*x))/(x - 1),x, algorithm="maxima")
```

```
[Out] -1/2*x - 1/2*sqrt(-x^2 + 2*x) - 1/2*log(x - 1) + 1/2*log(2*sqrt(-
x^2 + 2*x)/abs(x - 1) + 2/abs(x - 1))
```

Fricas [A] time = 0.272148, size = 89, normalized size = 1.75

$$-\frac{1}{2}\,x-\frac{1}{2}\,\sqrt{-x^{2}+2\,x}-\frac{1}{2}\,\log\left(x-1\right)+\frac{1}{2}\,\log\left(\frac{x+\sqrt{-x^{2}+2\,x}}{x}\right)-\frac{1}{2}\,\log\left(-\frac{x-\sqrt{-x^{2}+2\,x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/2*(x + sqrt(-x^2 + 2*x))/(x - 1),x, algorithm="fricas")
```

```
[Out] -1/2*x - 1/2*sqrt(-x^2 + 2*x) - 1/2*log(x - 1) + 1/2*log((x + sqr
t(-x^2 + 2*x))/x) - 1/2*log(-(x - sqrt(-x^2 + 2*x))/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{x}{x-1}\,dx + \int \frac{\sqrt{-x^2+2x}}{x-1}\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x+(-x**2+2*x)**(1/2))/(2-2*x),x)
```

```
[Out] -(Integral(x/(x - 1), x) + Integral(sqrt(-x**2 + 2*x)/(x - 1), x)
)/2
```

GIAC/XCAS [A] time = 0.267155, size = 68, normalized size = 1.33

$$-\frac{1}{2}\,x - \frac{1}{2}\,\sqrt{-x^2 + 2\,x} - \frac{1}{2}\,\ln\left(-\frac{2\left(\sqrt{-x^2 + 2\,x} - 1\right)}{|-2\,x + 2|}\right) - \frac{1}{2}\,\ln\left(|x - 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/2*(x + sqrt(-x^2 + 2*x))/(x - 1),x, algorithm="giac")
```

```
[Out] -1/2*x - 1/2*sqrt(-x^2 + 2*x) - 1/2*ln(-2*(sqrt(-x^2 + 2*x) - 1)/
abs(-2*x + 2)) - 1/2*ln(abs(x - 1))
```

3.728 $\int \frac{\sqrt{2-x}\sqrt{x}+x}{2-2x}\,dx$

Optimal. Leaf size=51

$$-\frac{1}{2}\sqrt{2x-x^2}+\frac{1}{2}\tanh^{-1}\left(\sqrt{2x-x^2}\right)-\frac{x}{2}-\frac{1}{2}\log(1-x)$$

```
[Out] -x/2 - Sqrt[2*x - x^2]/2 + ArcTanh[Sqrt[2*x - x^2]]/2 - Log[1 - x
]/2
```

Rubi [A] time = 0.263505, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28

$$-\frac{1}{2}\sqrt{2x-x^2}+\frac{1}{2}\tanh^{-1}\left(\sqrt{2x-x^2}\right)-\frac{x}{2}-\frac{1}{2}\log(1-x)$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[2 - x]*Sqrt[x] + x)/(2 - 2*x),x]
```

```
[Out] -x/2 - Sqrt[2*x - x^2]/2 + ArcTanh[Sqrt[2*x - x^2]]/2 - Log[1 - x
]/2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\sqrt{x}\sqrt{-x+2}}{2}-\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x}}{2}\right)-\int^{\sqrt{x}}x\,dx-2\int^{\sqrt{x}}\frac{-\frac{x}{4}-\frac{\sqrt{-x^2+2}}{4}}{x+1}\,dx-2\int^{\sqrt{x}}\frac{\frac{x}{4}+\frac{\sqrt{-x^2+2}}{4}}{x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x+(2-x)**(1/2)*x**(1/2))/(2-2*x),x)
```

```
[Out] -sqrt(x)*sqrt(-x + 2)/2 - asin(sqrt(2)*sqrt(x)/2) - Integral(x, (
x, sqrt(x))) - 2*Integral((-x/4 - sqrt(-x**2 + 2)/4)/(x + 1), (x,
 sqrt(x))) - 2*Integral((x/4 + sqrt(-x**2 + 2)/4)/(x - 1), (x, sq
rt(x)))
```

Mathematica [A] time = 0.01792, size = 41, normalized size = 0.8

$$\frac{1}{2}\left(-x-\sqrt{-(x-2)x}-2\log(1-x)+\log\left(\sqrt{-(x-2)x}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[2 - x]*Sqrt[x] + x)/(2 - 2*x),x]
```

```
[Out] (-x - Sqrt[-((-2 + x)*x)] - 2*Log[1 - x] + Log[1 + Sqrt[-((-2 + x
)*x)]])/2
```

Maple [A] time = 0.01, size = 51, normalized size = 1.

$$-\frac{1}{2}\sqrt{2-x}\sqrt{x}\left(\sqrt{-x(x-2)}-Artanh\left(\frac{1}{\sqrt{-x(x-2)}}\right)\right)\frac{1}{\sqrt{-x(x-2)}}-\frac{x}{2}-\frac{\ln(-1+x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x+(2-x)^(1/2)*x^(1/2))/(2-2*x),x)
```

```
[Out] -1/2*(2-x)^(1/2)*x^(1/2)/(-x*(x-2))^(1/2)*((-x*(x-2))^(1/2)-arcta
nh(1/(-x*(x-2))^(1/2)))-1/2*x-1/2*ln(-1+x)
```

Maxima [A] time = 0.804424, size = 73, normalized size = 1.43

$$-\frac{1}{2}x-\frac{1}{2}\sqrt{-x^2+2x}-\frac{1}{2}\log(x-1)+\frac{1}{2}\log\left(\frac{2\sqrt{-x^2+2x}}{|x-1|}+\frac{2}{|x-1|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/2*(x + sqrt(x)*sqrt(-x + 2))/(x - 1),x, algorithm="maxima")
```

```
[Out] -1/2*x - 1/2*sqrt(-x^2 + 2*x) - 1/2*log(x - 1) + 1/2*log(2*sqrt(-
x^2 + 2*x)/abs(x - 1) + 2/abs(x - 1))
```

Fricas [A] time = 0.304433, size = 86, normalized size = 1.69

$$-\frac{1}{2}x-\frac{1}{2}\sqrt{x}\sqrt{-x+2}-\frac{1}{2}\log(x-1)+\frac{1}{2}\log\left(\frac{x+\sqrt{x}\sqrt{-x+2}}{x}\right)-\frac{1}{2}\log\left(-\frac{x-\sqrt{x}\sqrt{-x+2}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/2*(x + sqrt(x)*sqrt(-x + 2))/(x - 1),x, algorithm="fricas")
```

```
[Out] -1/2*x - 1/2*sqrt(x)*sqrt(-x + 2) - 1/2*log(x - 1) + 1/2*log((x +
 sqrt(x)*sqrt(-x + 2))/x) - 1/2*log(-(x - sqrt(x)*sqrt(-x + 2))/x
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{x}{x-1}\,dx + \int \frac{\sqrt{x}\sqrt{-x+2}}{x-1}\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(2-x)**(1/2)*x**(1/2))/(2-2*x),x)
```

```
[Out] -(Integral(x/(x - 1), x) + Integral(sqrt(x)*sqrt(-x + 2)/(x - 1),
 x))/2
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/2*(x + sqrt(x)*sqrt(-x + 2))/(x - 1),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.729 $\int \frac{\sqrt{x}}{\sqrt{2-x}-\sqrt{x}}\,dx$

Optimal. Leaf size=54

$$-\frac{x}{2}-\frac{1}{2}\sqrt{2-x}\sqrt{x}-\frac{1}{2}\log(1-x)+\frac{1}{2}\tanh^{-1}\left(\sqrt{2-x}\sqrt{x}\right)$$

```
[Out] -(Sqrt[2 - x]*Sqrt[x])/2 - x/2 + ArcTanh[Sqrt[2 - x]*Sqrt[x]]/2 -
 Log[1 - x]/2
```

Rubi [A] time = 0.11844, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$

$$-\frac{x}{2}-\frac{1}{2}\sqrt{2-x}\sqrt{x}-\frac{1}{2}\log(1-x)+\frac{1}{2}\tanh^{-1}\left(\sqrt{2-x}\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x]/(Sqrt[2 - x] - Sqrt[x]),x]
```

```
[Out] -(Sqrt[2 - x]*Sqrt[x])/2 - x/2 + ArcTanh[Sqrt[2 - x]*Sqrt[x]]/2 -
 Log[1 - x]/2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\sqrt{x}\sqrt{-x+2}}{2}-\frac{\log\left(-x+1\right)}{2}+\frac{\operatorname{atanh}\left(\sqrt{x}\sqrt{-x+2}\right)}{2}+\int\left(-\frac{1}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(1/2)/((2-x)**(1/2)-x**(1/2)),x)
```

```
[Out] -sqrt(x)*sqrt(-x + 2)/2 - log(-x + 1)/2 + atanh(sqrt(x)*sqrt(-x +
 2))/2 + Integral(-1/2, x)
```

Mathematica [A] time = 0.0488636, size = 86, normalized size = 1.59

$$\frac{1}{2}\left(-x-\sqrt{-(x-2)x}-\log\left(1-\sqrt{x}\right)+\log\left(\sqrt{2-x}-\sqrt{x}+2\right)+\log\left(\sqrt{x}+1\right)-\log\left(\sqrt{2-x}+\sqrt{x}+2\right)-\log(1-x)\right)$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[Sqrt[x]/(Sqrt[2 - x] - Sqrt[x]),x]
```

```
[Out] (-x - Sqrt[-((-2 + x)*x)] - Log[1 - Sqrt[x]] + Log[2 + Sqrt[2 - x
] - Sqrt[x]] + Log[1 + Sqrt[x]] - Log[2 + Sqrt[2 - x] + Sqrt[x]]
- Log[1 - x])/2
```

Maple [A] time = 0.009, size = 51, normalized size = 0.9

$$-\frac{1}{2}\sqrt{2-x}\sqrt{x}\left(\sqrt{-x(x-2)}-Artanh\left(\frac{1}{\sqrt{-x(x-2)}}\right)\right)\frac{1}{\sqrt{-x(x-2)}}-\frac{x}{2}-\frac{\ln(-1+x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(1/2)/((2-x)^(1/2)-x^(1/2)),x)
```

```
[Out] -1/2*(2-x)^(1/2)*x^(1/2)/(-x*(x-2))^(1/2)*((-x*(x-2))^(1/2)-arcta
nh(1/(-x*(x-2))^(1/2)))-1/2*x-1/2*ln(-1+x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{x}}{\sqrt{x}-\sqrt{-x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sqrt(x)/(sqrt(x) - sqrt(-x + 2)),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(x)/(sqrt(x) - sqrt(-x + 2)), x)
```

Fricas [A] time = 0.276113, size = 86, normalized size = 1.59

$$-\frac{1}{2}x-\frac{1}{2}\sqrt{x}\sqrt{-x+2}-\frac{1}{2}\log(x-1)+\frac{1}{2}\log\left(\frac{x+\sqrt{x}\sqrt{-x+2}}{x}\right)-\frac{1}{2}\log\left(-\frac{x-\sqrt{x}\sqrt{-x+2}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sqrt(x)/(sqrt(x) - sqrt(-x + 2)),x, algorithm="fricas")
```

```
[Out] -1/2*x - 1/2*sqrt(x)*sqrt(-x + 2) - 1/2*log(x - 1) + 1/2*log((x +
 sqrt(x)*sqrt(-x + 2))/x) - 1/2*log(-(x - sqrt(x)*sqrt(-x + 2))/x
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{-\sqrt{x}+\sqrt{-x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(1/2)/((2-x)**(1/2)-x**(1/2)),x)
```

```
[Out] Integral(sqrt(x)/(-sqrt(x) + sqrt(-x + 2)), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sqrt(x)/(sqrt(x) - sqrt(-x + 2)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.730 $\int \frac{\sqrt{\frac{x^2}{-1+x^2}}}{1+x^2} \, dx$

Optimal. Leaf size=52

$$\frac{\sqrt{-\frac{x^2}{1-x^2}}\sqrt{x^2-1}\tan^{-1}\left(\frac{\sqrt{x^2-1}}{\sqrt{2}}\right)}{\sqrt{2}x}$$

```
[Out] (Sqrt[-(x^2/(1 - x^2))]*Sqrt[-1 + x^2]*ArcTan[Sqrt[-1 + x^2]/Sqrt
[2]])/(Sqrt[2]*x)
```

Rubi [A] time = 0.177504, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174

$$\frac{\sqrt{-\frac{x^2}{1-x^2}}\sqrt{x^2-1}\tan^{-1}\left(\frac{\sqrt{x^2-1}}{\sqrt{2}}\right)}{\sqrt{2}x}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x^2/(-1 + x^2)]/(1 + x^2),x]
```

```
[Out] (Sqrt[-(x^2/(1 - x^2))]*Sqrt[-1 + x^2]*ArcTan[Sqrt[-1 + x^2]/Sqrt
[2]])/(Sqrt[2]*x)
```

Rubi in Sympy [A] time = 15.4958, size = 44, normalized size = 0.85

$$\frac{\sqrt{2}\sqrt{\frac{x^2}{x^2-1}}\sqrt{x^2-1}\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{x^2-1}}{2}\right)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2/(x**2-1))**(1/2)/(x**2+1),x)
```

```
[Out] sqrt(2)*sqrt(x**2/(x**2 - 1))*sqrt(x**2 - 1)*atan(sqrt(2)*sqrt(x*
*2 - 1)/2)/(2*x)
```

Mathematica [A] time = 0.0298416, size = 49, normalized size = 0.94

$$\frac{\sqrt{\frac{x^2}{x^2-1}}\sqrt{x^2-1}\tan^{-1}\left(\frac{\sqrt{x^2-1}}{\sqrt{2}}\right)}{\sqrt{2}x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[x^2/(-1 + x^2)]/(1 + x^2),x]
```

```
[Out] (Sqrt[x^2/(-1 + x^2)]*Sqrt[-1 + x^2]*ArcTan[Sqrt[-1 + x^2]/Sqrt[2
]])/(Sqrt[2]*x)
```

Maple [A] time = 0.016, size = 42, normalized size = 0.8

$$\frac{\sqrt{2}}{2\,x}\sqrt{\frac{x^2}{x^2-1}}\sqrt{x^2-1}\arctan\left(\frac{\sqrt{2}}{2}\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2/(x^2-1))^(1/2)/(x^2+1),x)
```

```
[Out] 1/2*(x^2/(x^2-1))^(1/2)/x*(x^2-1)^(1/2)*2^(1/2)*arctan(1/2*(x^2-1
)^(1/2)*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{\frac{x^2}{x^2-1}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2/(x^2 - 1))/(x^2 + 1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2/(x^2 - 1))/(x^2 + 1), x)
```

Fricas [A] time = 0.272557, size = 81, normalized size = 1.56

$$-\frac{1}{2}\sqrt{2}\arctan\left(-\frac{\sqrt{2}x^2-\sqrt{2}(x^2-1)\sqrt{\frac{x^2}{x^2-1}}}{2\left(x\sqrt{\frac{x^2}{x^2-1}}-x\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2/(x^2 - 1))/(x^2 + 1),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*arctan(-1/2*(sqrt(2)*x^2 - sqrt(2)*(x^2 - 1)*sqrt(x^
2/(x^2 - 1)))/(x*sqrt(x^2/(x^2 - 1)) - x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{\frac{x^2}{x^2-1}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2/(x**2-1))**(1/2)/(x**2+1),x)
```

```
[Out] Integral(sqrt(x**2/(x**2 - 1))/(x**2 + 1), x)
```

GIAC/XCAS [A] time = 0.281761, size = 55, normalized size = 1.06

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{x^2-1}\right)\mathrm{sign}\left(x^2-1\right)\mathrm{sign}\left(x\right)+\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}i\right)\mathrm{sign}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2/(x^2 - 1))/(x^2 + 1),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(x^2 - 1))*sign(x^2 - 1)*sign(
x) + 1/2*sqrt(2)*arctan(1/2*sqrt(2)*i)*sign(x)
```

3.731 $\int \frac{\sqrt{\frac{x^2}{-1+a+(1+a)x^2}}}{1+x^2}\,dx$

Optimal. Leaf size=68

$$\frac{\sqrt{-\frac{x^2}{-(a+1)x^2-a+1}}\sqrt{(a+1)x^2+a-1}\tan^{-1}\left(\frac{\sqrt{(a+1)x^2+a-1}}{\sqrt{2}}\right)}{\sqrt{2}x}$$

```
[Out] (Sqrt[-(x^2/(1 - a - (1 + a)*x^2))]*Sqrt[-1 + a + (1 + a)*x^2]*Ar
cTan[Sqrt[-1 + a + (1 + a)*x^2]/Sqrt[2]])/(Sqrt[2]*x)
```

Rubi [A] time = 0.335117, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{\sqrt{-\frac{x^2}{-(a+1)x^2-a+1}}\sqrt{(a+1)x^2+a-1}\tan^{-1}\left(\frac{\sqrt{(a+1)x^2+a-1}}{\sqrt{2}}\right)}{\sqrt{2}x}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x^2/(-1 + a + (1 + a)*x^2)]/(1 + x^2),x]
```

```
[Out] (Sqrt[-(x^2/(1 - a - (1 + a)*x^2))]*Sqrt[-1 + a + (1 + a)*x^2]*Ar
cTan[Sqrt[-1 + a + (1 + a)*x^2]/Sqrt[2]])/(Sqrt[2]*x)
```

Rubi in Sympy [A] time = 17.7705, size = 60, normalized size = 0.88

$$\frac{\sqrt{2}\sqrt{\frac{x^2}{a+x^2(a+1)-1}}\sqrt{a+x^2\,(a+1)-1}\,\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{a+x^2(a+1)-1}}{2}\right)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2/(-1+a+(1+a)*x**2))**(1/2)/(x**2+1),x)
```

```
[Out] sqrt(2)*sqrt(x**2/(a + x**2*(a + 1) - 1))*sqrt(a + x**2*(a + 1) -
 1)*atan(sqrt(2)*sqrt(a + x**2*(a + 1) - 1)/2)/(2*x)
```

Mathematica [A] time = 0.0714029, size = 67, normalized size = 0.99

$$\frac{\sqrt{ax^2+a+x^2-1}\sqrt{\frac{x^2}{(a+1)x^2+a-1}}\tan^{-1}\left(\frac{\sqrt{a(x^2+1)+x^2-1}}{\sqrt{2}}\right)}{\sqrt{2}x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[x^2/(-1 + a + (1 + a)*x^2)]/(1 + x^2),x]
```

```
[Out] (Sqrt[-1 + a + x^2 + a*x^2]*Sqrt[x^2/(-1 + a + (1 + a)*x^2)]*ArcT
an[Sqrt[-1 + x^2 + a*(1 + x^2)]/Sqrt[2]])/(Sqrt[2]*x)
```

Maple [A] time = 0.04, size = 60, normalized size = 0.9

$$\frac{\sqrt{2}}{2\,x}\sqrt{\frac{x^2}{ax^2+x^2+a-1}}\sqrt{ax^2+x^2+a-1}\,\arctan\left(\frac{\sqrt{2}}{2}\sqrt{ax^2+x^2+a-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2/(-1+a+(1+a)*x^2))^(1/2)/(x^2+1),x)
```

```
[Out] 1/2*(x^2/(a*x^2+x^2+a-1))^(1/2)/x*(a*x^2+x^2+a-1)^(1/2)*2^(1/2)*a
rctan(1/2*(a*x^2+x^2+a-1)^(1/2)*2^(1/2))
```

Maxima [A] time = 0.817198, size = 32, normalized size = 0.47

$$\frac{1}{2}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\sqrt{ax^2+x^2+a-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x^2/((a + 1)*x^2 + a - 1))/(x^2 + 1),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(a*x^2 + x^2 + a - 1))
```

Fricas [A] time = 0.280264, size = 57, normalized size = 0.84

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{\sqrt{2}\left((a+1)x^2+a-3\right)\sqrt{\frac{x^2}{(a+1)x^2+a-1}}}{4\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x^2/((a + 1)*x^2 + a - 1))/(x^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*arctan(1/4*sqrt(2)*((a + 1)*x^2 + a - 3)*sqrt(x^2/((a
 + 1)*x^2 + a - 1))/x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2/(-1+a+(1+a)*x**2))**(1/2)/(x**2+1),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.275341, size = 82, normalized size = 1.21

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{ax^2+x^2+a-1}\right)\operatorname{sign}\left(ax^2+x^2+a-1\right)\operatorname{sign}(x) - \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{a-1}\right)\operatorname{sign}(a-1)\operatorname{sign}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x^2/((a + 1)*x^2 + a - 1))/(x^2 + 1),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(a*x^2 + x^2 + a - 1))*sign(a*
x^2 + x^2 + a - 1)*sign(x) - 1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(
a - 1))*sign(a - 1)*sign(x)
```

3.732 $\int \frac{1}{((1+x)(-1+x^2))^{2/3}} \, dx$

Optimal. Leaf size=27

$$-\frac{3\left(1-x^2\right)}{2\left(-(x+1)\left(1-x^2\right)\right)^{2/3}}$$

```
[Out] (-3*(1 - x^2))/(2*(-((1 + x)*(1 - x^2)))^(2/3))
```

Rubi [A] time = 0.0680533, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$-\frac{3(1-x)(x+1)}{2\left(x^3+x^2-x-1\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[((1 + x)*(-1 + x^2))^(-2/3),x]

[Out] (-3*(1 - x)*(1 + x))/(2*(-1 - x + x^2 + x^3)^(2/3))
```

Rubi in Sympy [A] time = 1.87336, size = 24, normalized size = 0.89

$$-\frac{3\left(-x+1\right)\left(x+1\right)}{2\left(x^3+x^2-x-1\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/((1+x)*(x**2-1))**(2/3),x)

[Out] -3*(-x + 1)*(x + 1)/(2*(x**3 + x**2 - x - 1)**(2/3))
```

Mathematica [A] time = 0.0201148, size = 23, normalized size = 0.85

$$\frac{3(x-1)(x+1)}{2\left((x-1)(x+1)^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((1 + x)*(-1 + x^2))^(-2/3),x]

[Out] (3*(-1 + x)*(1 + x))/(2*((-1 + x)*(1 + x)^2)^(2/3))
```

Maple [A] time = 0.004, size = 20, normalized size = 0.7

$$\frac{(-3+3x)(1+x)}{2}\left((1+x)\left(x^2-1\right)\right)^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/((1+x)*(x^2-1))^(2/3),x)

[Out] 3/2*(-1+x)*(1+x)/((1+x)*(x^2-1))^(2/3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left((x^2-1)(x+1)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((x^2 - 1)*(x + 1))^(-2/3),x, algorithm="maxima")

[Out] integrate(((x^2 - 1)*(x + 1))^(-2/3), x)
```

Fricas [A] time = 0.262481, size = 27, normalized size = 1.

$$\frac{3\left(x^3+x^2-x-1\right)^{\frac{1}{3}}}{2(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((x^2 - 1)*(x + 1))^(-2/3),x, algorithm="fricas")

[Out] 3/2*(x^3 + x^2 - x - 1)^(1/3)/(x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\left(x+1\right)\left(x^{2}-1\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((1+x)*(x**2-1))**(2/3),x)
```

```
[Out] Integral(((x + 1)*(x**2 - 1))**(-2/3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\left(x^{2}-1\right)\left(x+1\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((x^2 - 1)*(x + 1))^(-2/3),x, algorithm="giac")
```

```
[Out] integrate(((x^2 - 1)*(x + 1))^(-2/3), x)
```

3.733 $\int \frac{-1+x^2}{(1+x^2)\sqrt{x(1+x^2)}}\,dx$

Optimal. Leaf size=14

$$-\frac{2x}{\sqrt{x\left(x^2+1\right)}}$$

```
[Out] (-2*x)/Sqrt[x*(1 + x^2)]
```

Rubi [A] time = 0.246295, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$-\frac{2x}{\sqrt{x\left(x^2+1\right)}}$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + x^2)/((1 + x^2)*Sqrt[x*(1 + x^2)]),x]
```

```
[Out] (-2*x)/Sqrt[x*(1 + x^2)]
```

Rubi in Sympy [A] time = 50.0794, size = 235, normalized size = 16.79

$$-\frac{\sqrt{2}\sqrt{x}\,(1+i)\left(x^2+1\right)\sqrt{-ix+1}E\left(\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x}(1+i)}{2}\right)\middle|-1\right)}{2\,(x+i)\,\sqrt{x^3+x}\sqrt{ix+1}} - \frac{\sqrt{2}\sqrt{x}\,(1-i)\left(x^2+1\right)\sqrt{ix+1}E\left(\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x}(-1+i)}{2}\right)\middle|-1\right)}{4\left(-\frac{x}{2}+\frac{i}{2}\right)\sqrt{x^3+x}\sqrt{-ix+1}} - \frac{ix\left(x^2+1\right)}{(x+i)\sqrt{x^3+x}} - \frac{2ix\left(x^2+1\right)}{(-2x+2i)\sqrt{x^3+x}} + \frac{\sqrt{\frac{x^2+1}{(x+1)^2}}\,(x+1)\sqrt{x^3+x}F\left(2\operatorname{atan}\left(\sqrt{x}\right)\middle|\frac{1}{2}\right)}{\sqrt{x}\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2-1)/(x**2+1)/(x*(x**2+1))**(1/2),x)
```

```
[Out] -sqrt(2)*sqrt(x)*(1 + I)*(x**2 + 1)*sqrt(-I*x + 1)*elliptic_e(asi
n(sqrt(2)*sqrt(x)*(1 + I)/2), -1)/(2*(x + I)*sqrt(x**3 + x)*sqrt(
I*x + 1)) - sqrt(2)*sqrt(x)*(1 - I)*(x**2 + 1)*sqrt(I*x + 1)*elli
ptic_e(asin(sqrt(2)*sqrt(x)*(-1 + I)/2), -1)/(4*(-x/2 + I/2)*sqrt
```

```
(x**3 + x)*sqrt(-I*x + 1)) - I*x*(x**2 + 1)/((x + I)*sqrt(x**3 +
x)) - 2*I*x*(x**2 + 1)/((-2*x + 2*I)*sqrt(x**3 + x)) + sqrt((x**2
 + 1)/(x + 1)**2)*(x + 1)*sqrt(x**3 + x)*elliptic_f(2*atan(sqrt(x
)), 1/2)/(sqrt(x)*(x**2 + 1))
```

Mathematica [A] time = 0.0196121, size = 12, normalized size = 0.86

$$-\frac{2x}{\sqrt{x^3+x}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + x^2)/((1 + x^2)*Sqrt[x*(1 + x^2)]),x]

[Out] (-2*x)/Sqrt[x + x^3]
```

Maple [A] time = 0.009, size = 13, normalized size = 0.9

$$-2\,\frac{x}{\sqrt{x\,(x^2+1)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2-1)/(x^2+1)/(x*(x^2+1))^(1/2),x)

[Out] -2*x/(x*(x^2+1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2-1}{\sqrt{(x^2+1)x}(x^2+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 1)/(sqrt((x^2 + 1)*x)*(x^2 + 1)),x, algorithm="maxima")

[Out] integrate((x^2 - 1)/(sqrt((x^2 + 1)*x)*(x^2 + 1)), x)
```

Fricas [A] time = 0.273522, size = 22, normalized size = 1.57

$$-\frac{2\sqrt{x^3+x}}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 - 1)/(sqrt((x^2 + 1)*x)*(x^2 + 1)),x, algorithm="fricas")
```

```
[Out] -2*sqrt(x^3 + x)/(x^2 + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x-1)(x+1)}{\sqrt{x(x^2+1)}(x^2+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)/(x**2+1)/(x*(x**2+1))**(1/2),x)
```

```
[Out] Integral((x - 1)*(x + 1)/(sqrt(x*(x**2 + 1))*(x**2 + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2-1}{\sqrt{(x^2+1)x}(x^2+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 - 1)/(sqrt((x^2 + 1)*x)*(x^2 + 1)),x, algorithm="giac")
```

```
[Out] integrate((x^2 - 1)/(sqrt((x^2 + 1)*x)*(x^2 + 1)), x)
```

3.734 $\int \frac{-1+x^2}{(1+x^2)\sqrt{x+x^3}}\,dx$

Optimal. Leaf size=12

$$-\frac{2x}{\sqrt{x^3+x}}$$

```
[Out] (-2*x)/Sqrt[x + x^3]
```

Rubi [A] time = 0.111015, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$-\frac{2x}{\sqrt{x^3+x}}$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + x^2)/((1 + x^2)*Sqrt[x + x^3]),x]

[Out] (-2*x)/Sqrt[x + x^3]
```

Rubi in Sympy [A] time = 12.4373, size = 15, normalized size = 1.25

$$-\frac{2\sqrt{x^3+x}}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2-1)/(x**2+1)/(x**3+x)**(1/2),x)

[Out] -2*sqrt(x**3 + x)/(x**2 + 1)
```

Mathematica [A] time = 0.0118042, size = 12, normalized size = 1.

$$-\frac{2x}{\sqrt{x^3+x}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + x^2)/((1 + x^2)*Sqrt[x + x^3]),x]

[Out] (-2*x)/Sqrt[x + x^3]
```

Maple [A] time = 0.006, size = 11, normalized size = 0.9

$$-2\,\frac{x}{\sqrt{x^3+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2-1)/(x^2+1)/(x^3+x)^(1/2),x)

[Out] -2*x/(x^3+x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2-1}{\sqrt{x^3+x}\,(x^2+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 1)/(sqrt(x^3 + x)*(x^2 + 1)),x, algorithm="maxima")

[Out] integrate((x^2 - 1)/(sqrt(x^3 + x)*(x^2 + 1)), x)
```

Fricas [A] time = 0.270438, size = 22, normalized size = 1.83

$$-\frac{2\,\sqrt{x^3+x}}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 1)/(sqrt(x^3 + x)*(x^2 + 1)),x, algorithm="fricas")

[Out] -2*sqrt(x^3 + x)/(x^2 + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x-1)(x+1)}{\sqrt{x(x^2+1)}(x^2+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2-1)/(x**2+1)/(x**3+x)**(1/2),x)
```

```
[Out] Integral((x - 1)*(x + 1)/(sqrt(x*(x**2 + 1))*(x**2 + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2-1}{\sqrt{x^3+x}(x^2+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 1)/(sqrt(x^3 + x)*(x^2 + 1)),x, algorithm="giac")
```

```
[Out] integrate((x^2 - 1)/(sqrt(x^3 + x)*(x^2 + 1)), x)
```

3.735 $\int \frac{\sqrt{\frac{(-1+x^2)^2}{x(1+x^2)}}}{1+x^2}\,dx$

Optimal. Leaf size=36

$$\frac{2x\sqrt{\frac{(1-x^2)^2}{x(x^2+1)}}}{1-x^2}$$

```
[Out] (2*x*Sqrt[(1 - x^2)^2/(x*(1 + x^2))])/(1 - x^2)
```

Rubi [A] time = 0.223266, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$\frac{2x\sqrt{\frac{(1-x^2)^2}{x(x^2+1)}}}{1-x^2}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[(-1 + x^2)^2/(x*(1 + x^2))]/(1 + x^2),x]
```

```
[Out] (2*x*Sqrt[(1 - x^2)^2/(x*(1 + x^2))])/(1 - x^2)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$i\int \frac{\sqrt{\frac{(x^2-1)^2}{x(x^2+1)}}}{-2x+2i}\,dx + \frac{i\int \frac{\sqrt{\frac{(x^2-1)^2}{x(x^2+1)}}}{x+i}\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((x**2-1)**2/x/(x**2+1))**(1/2)/(x**2+1),x)
```

```
[Out] I*Integral(sqrt((x**2 - 1)**2/(x*(x**2 + 1)))/(-2*x + 2*I), x) +
I*Integral(sqrt((x**2 - 1)**2/(x*(x**2 + 1)))/(x + I), x)/2
```

Mathematica [A] time = 0.0270965, size = 29, normalized size = 0.81

$$-\frac{2x\sqrt{\frac{(x^2-1)^2}{x^3+x}}}{x^2-1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[(-1 + x^2)^2/(x*(1 + x^2))]/(1 + x^2),x]

[Out] (-2*x*Sqrt[(-1 + x^2)^2/(x + x^3)])/(-1 + x^2)
```

Maple [A] time = 0.007, size = 34, normalized size = 0.9

$$-2\,\frac{x}{(-1+x)\,(1+x)}\sqrt{\frac{(x^2-1)^2}{x\,(x^2+1)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((x^2-1)^2/x/(x^2+1))^(1/2)/(x^2+1),x)

[Out] -2*x/(-1+x)/(1+x)*((x^2-1)^2/x/(x^2+1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{\frac{(x^2-1)^2}{(x^2+1)x}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((x^2 - 1)^2/((x^2 + 1)*x))/(x^2 + 1),x, algorithm="maxima")

[Out] integrate(sqrt((x^2 - 1)^2/((x^2 + 1)*x))/(x^2 + 1), x)
```

Fricas [A] time = 0.272272, size = 41, normalized size = 1.14

$$-\frac{2\,x\sqrt{\frac{x^4-2\,x^2+1}{x^3+x}}}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt((x^2 - 1)^2/((x^2 + 1)*x))/(x^2 + 1),x, algorithm="fricas")
```

```
[Out] -2*x*sqrt((x^4 - 2*x^2 + 1)/(x^3 + x))/(x^2 - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{(x-1)^2(x+1)^2}{x^3+x}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x**2-1)**2/x/(x**2+1))**(1/2)/(x**2+1),x)
```

```
[Out] Integral(sqrt((x - 1)**2*(x + 1)**2/(x**3 + x))/(x**2 + 1), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{(x^2-1)^2}{(x^2+1)x}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt((x^2 - 1)^2/((x^2 + 1)*x))/(x^2 + 1),x, algorithm="giac")
```

```
[Out] integrate(sqrt((x^2 - 1)^2/((x^2 + 1)*x))/(x^2 + 1), x)
```

3.736 $\int \frac{\sqrt{\frac{\left(-1+x^2\right)^2}{x+x^3}}}{1+x^2}\,dx$

Optimal. Leaf size=33

$$\frac{2x\sqrt{\frac{(1-x^2)^2}{x^3+x}}}{1-x^2}$$

```
[Out] (2*x*Sqrt[(1 - x^2)^2/(x + x^3)])/(1 - x^2)
```

Rubi [A] time = 0.303391, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$\frac{2x\sqrt{\frac{(1-x^2)^2}{x^3+x}}}{1-x^2}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[(-1 + x^2)^2/(x + x^3)]/(1 + x^2),x]

[Out] (2*x*Sqrt[(1 - x^2)^2/(x + x^3)])/(1 - x^2)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$i\int \frac{\sqrt{\frac{(x^2-1)^2}{x^3+x}}}{-2x+2i}\,dx + \frac{i\int \frac{\sqrt{\frac{\left(x^2-1\right)^2}{x^3+x}}}{x+i}\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((x**2-1)**2/(x**3+x))**(1/2)/(x**2+1),x)

[Out] I*Integral(sqrt((x**2 - 1)**2/(x**3 + x))/(-2*x + 2*I), x) + I*In
tegral(sqrt((x**2 - 1)**2/(x**3 + x))/(x + I), x)/2
```

Mathematica [A] time = 0.0123657, size = 29, normalized size = 0.88

$$-\frac{2x\sqrt{\frac{(x^2-1)^2}{x^3+x}}}{x^2-1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[(-1 + x^2)^2/(x + x^3)]/(1 + x^2),x]
```

```
[Out] (-2*x*Sqrt[(-1 + x^2)^2/(x + x^3)])/(-1 + x^2)
```

Maple [A] time = 0.007, size = 34, normalized size = 1.

$$-2\frac{x}{(-1+x)(1+x)}\sqrt{\frac{(x^2-1)^2}{x(x^2+1)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((x^2-1)^2/(x^3+x))^(1/2)/(x^2+1),x)
```

```
[Out] -2*x/(-1+x)/(1+x)*((x^2-1)^2/x/(x^2+1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{(x^2-1)^2}{x^3+x}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((x^2 - 1)^2/(x^3 + x))/(x^2 + 1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((x^2 - 1)^2/(x^3 + x))/(x^2 + 1), x)
```

Fricas [A] time = 0.277066, size = 41, normalized size = 1.24

$$-\frac{2\,x\sqrt{\frac{x^4-2\,x^2+1}{x^3+x}}}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((x^2 - 1)^2/(x^3 + x))/(x^2 + 1),x, algorithm="fricas")
```

```
[Out] -2*x*sqrt((x^4 - 2*x^2 + 1)/(x^3 + x))/(x^2 - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{(x-1)^2(x+1)^2}{x^3+x}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((x**2-1)**2/(x**3+x))**(1/2)/(x**2+1),x)
```

```
[Out] Integral(sqrt((x - 1)**2*(x + 1)**2/(x**3 + x))/(x**2 + 1), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{(x^2-1)^2}{x^3+x}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((x^2 - 1)^2/(x^3 + x))/(x^2 + 1),x, algorithm="giac")
```

```
[Out] integrate(sqrt((x^2 - 1)^2/(x^3 + x))/(x^2 + 1), x)
```

3.737 $\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=70

$$\frac{\sqrt{ax^2+b}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ax^2+b}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{d}x\sqrt{a+\frac{b}{x^2}}}$$

```
[Out] (Sqrt[b + a*x^2]*ArcTanh[(Sqrt[d]*Sqrt[b + a*x^2])/(Sqrt[a]*Sqrt[
c + d*x^2])])/(Sqrt[a]*Sqrt[d]*Sqrt[a + b/x^2]*x)
```

Rubi [A] time = 0.163251, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174

$$\frac{\sqrt{ax^2+b}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ax^2+b}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{d}x\sqrt{a+\frac{b}{x^2}}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[a + b/x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[b + a*x^2]*ArcTanh[(Sqrt[d]*Sqrt[b + a*x^2])/(Sqrt[a]*Sqrt[
c + d*x^2])])/(Sqrt[a]*Sqrt[d]*Sqrt[a + b/x^2]*x)
```

Rubi in Sympy [A] time = 8.69258, size = 63, normalized size = 0.9

$$\frac{x\sqrt{a+\frac{b}{x^2}}\operatorname{atanh}\left(\frac{\sqrt{d}\sqrt{ax^2+b}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{d}\sqrt{ax^2+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a+b/x**2)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] x*sqrt(a + b/x**2)*atanh(sqrt(d)*sqrt(a*x**2 + b)/(sqrt(a)*sqrt(c
 + d*x**2)))/(sqrt(a)*sqrt(d)*sqrt(a*x**2 + b))
```

Mathematica [A] time = 0.0796566, size = 88, normalized size = 1.26

$$\frac{\sqrt{ax^2+b}\log\left(2\sqrt{a}\sqrt{d}\sqrt{ax^2+b}\sqrt{c+dx^2}+ac+2adx^2+bd\right)}{2\sqrt{a}\sqrt{d}x\sqrt{a+\frac{b}{x^2}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[a + b/x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[b + a*x^2]*Log[a*c + b*d + 2*a*d*x^2 + 2*Sqrt[a]*Sqrt[d]*Sq
rt[b + a*x^2]*Sqrt[c + d*x^2]])/(2*Sqrt[a]*Sqrt[d]*Sqrt[a + b/x^2
]*x)
```

Maple [B] time = 0.072, size = 117, normalized size = 1.7

$$\frac{ax^2+b}{2x}\ln\left(\frac{1}{2}\left(2adx^2+2\sqrt{adx^4+acx^2+bdx^2+bc}\sqrt{ad}+ac+bd\right)\frac{1}{\sqrt{ad}}\right)\sqrt{dx^2+c}\frac{1}{\sqrt{\frac{ax^2+b}{x^2}}}\frac{1}{\sqrt{adx^4+acx^2+bdx^2+bc}}\frac{1}{\sqrt{ad}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a+b/x^2)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/2/((a*x^2+b)/x^2)^(1/2)/x*(a*x^2+b)*ln(1/2*(2*a*d*x^2+2*(a*d*x^
4+a*c*x^2+b*d*x^2+b*c)^(1/2)*(a*d)^(1/2)+a*c+b*d)/(a*d)^(1/2))*(d
*x^2+c)^(1/2)/(a*d*x^4+a*c*x^2+b*d*x^2+b*c)^(1/2)/(a*d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(d*x^2 + c)*sqrt(a + b/x^2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.287848, size = 1, normalized size = 0.01

$$\left[\frac{\sqrt{ad}\log\left(4\left(2\,a^{2}d^{2}x^{3}+\left(a^{2}cd+abd^{2}\right)x\right)\sqrt{dx^{2}+c}\sqrt{\frac{ax^{2}+b}{x^{2}}}+\left(8\,a^{2}d^{2}x^{4}+a^{2}c^{2}+6\,abcd+b^{2}d^{2}+8\left(a^{2}cd+abd^{2}\right)x^{2}\right)\sqrt{ad}\right)}{4\,ad},\right.$$
$$\left.-\frac{\sqrt{-ad}\arctan\left(\frac{\left(2\,adx^{2}+ac+bd\right)\sqrt{-ad}}{2\sqrt{dx^{2}+c}adx\sqrt{\frac{ax^{2}+b}{x^{2}}}}\right)}{2\,ad}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(d*x^2 + c)*sqrt(a + b/x^2)),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(a*d)*log(4*(2*a^2*d^2*x^3 + (a^2*c*d + a*b*d^2)*x)*sqrt
(d*x^2 + c)*sqrt((a*x^2 + b)/x^2) + (8*a^2*d^2*x^4 + a^2*c^2 + 6*
a*b*c*d + b^2*d^2 + 8*(a^2*c*d + a*b*d^2)*x^2)*sqrt(a*d))/(a*d),
-1/2*sqrt(-a*d)*arctan(1/2*(2*a*d*x^2 + a*c + b*d)*sqrt(-a*d)/(sq
rt(d*x^2 + c)*a*d*x*sqrt((a*x^2 + b)/x^2)))/(a*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+\frac{b}{x^{2}}}\sqrt{c+dx^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a+b/x**2)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b/x**2)*sqrt(c + d*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{dx^{2}+c}\sqrt{a+\frac{b}{x^{2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(d*x^2 + c)*sqrt(a + b/x^2)),x, algorithm="giac")

[Out] integrate(1/(sqrt(d*x^2 + c)*sqrt(a + b/x^2)), x)
```

3.738 $\int \frac{\sqrt{-2x^2+x^4}}{(-1+x^2)(2+x^2)}\,dx$

Optimal. Leaf size=83

$$\frac{2\sqrt{x^4-2x^2}\tan^{-1}\left(\frac{\sqrt{x^2-2}}{2}\right)}{3x\sqrt{x^2-2}}-\frac{\sqrt{x^4-2x^2}\tan^{-1}\left(\sqrt{x^2-2}\right)}{3x\sqrt{x^2-2}}$$

```
[Out] (2*Sqrt[-2*x^2 + x^4]*ArcTan[Sqrt[-2 + x^2]/2])/(3*x*Sqrt[-2 + x^
2]) - (Sqrt[-2*x^2 + x^4]*ArcTan[Sqrt[-2 + x^2]])/(3*x*Sqrt[-2 +
x^2])
```

Rubi [A] time = 0.311982, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179

$$\frac{2\sqrt{x^4-2x^2}\tan^{-1}\left(\frac{\sqrt{x^2-2}}{2}\right)}{3x\sqrt{x^2-2}}-\frac{\sqrt{x^4-2x^2}\tan^{-1}\left(\sqrt{x^2-2}\right)}{3x\sqrt{x^2-2}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[-2*x^2 + x^4]/((-1 + x^2)*(2 + x^2)),x]
```

```
[Out] (2*Sqrt[-2*x^2 + x^4]*ArcTan[Sqrt[-2 + x^2]/2])/(3*x*Sqrt[-2 + x^
2]) - (Sqrt[-2*x^2 + x^4]*ArcTan[Sqrt[-2 + x^2]])/(3*x*Sqrt[-2 +
x^2])
```

Rubi in Sympy [A] time = 24.3553, size = 70, normalized size = 0.84

$$\frac{2\sqrt{x^4-2x^2}\,\mathrm{atan}\left(\frac{\sqrt{x^2-2}}{2}\right)}{3x\sqrt{x^2-2}}-\frac{\sqrt{x^4-2x^2}\,\mathrm{atan}\left(\sqrt{x^2-2}\right)}{3x\sqrt{x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**4-2*x**2)**(1/2)/(x**2-1)/(x**2+2),x)
```

```
[Out] 2*sqrt(x**4 - 2*x**2)*atan(sqrt(x**2 - 2)/2)/(3*x*sqrt(x**2 - 2))
 - sqrt(x**4 - 2*x**2)*atan(sqrt(x**2 - 2))/(3*x*sqrt(x**2 - 2))
```

Mathematica [A] time = 0.0423178, size = 52, normalized size = 0.63

$$-\frac{x\sqrt{x^2-2}\left(2\tan^{-1}\left(\frac{2}{\sqrt{x^2-2}}\right)+\tan^{-1}\left(\sqrt{x^2-2}\right)\right)}{3\sqrt{x^2(x^2-2)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[-2*x^2 + x^4]/((-1 + x^2)*(2 + x^2)),x]
```

```
[Out] -(x*Sqrt[-2 + x^2]*(2*ArcTan[2/Sqrt[-2 + x^2]] + ArcTan[Sqrt[-2 +
 x^2]]))/(3*Sqrt[x^2*(-2 + x^2)])
```

Maple [A] time = 0.039, size = 63, normalized size = 0.8

$$\frac{1}{6x}\sqrt{x^4-2x^2}\left(\arctan\left((2+x)\frac{1}{\sqrt{x^2-2}}\right)-\arctan\left((x-2)\frac{1}{\sqrt{x^2-2}}\right)+4\arctan\left(1/2\sqrt{x^2-2}\right)\right)\frac{1}{\sqrt{x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^4-2*x^2)^(1/2)/(x^2-1)/(x^2+2),x)
```

```
[Out] 1/6*(x^4-2*x^2)^(1/2)*(arctan((2+x)/(x^2-2)^(1/2))-arctan((x-2)/(
x^2-2)^(1/2))+4*arctan(1/2*(x^2-2)^(1/2)))/x/(x^2-2)^(1/2)
```

Maxima [A] time = 0.828996, size = 31, normalized size = 0.37

$$\frac{2}{3}\arctan\left(\frac{1}{2}\sqrt{x^2-2}\right)-\frac{1}{3}\arctan\left(\sqrt{x^2-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x^4 - 2*x^2)/((x^2 + 2)*(x^2 - 1)),x, algorithm="maxima")
```

```
[Out] 2/3*arctan(1/2*sqrt(x^2 - 2)) - 1/3*arctan(sqrt(x^2 - 2))
```

Fricas [A] time = 0.285729, size = 122, normalized size = 1.47

$$\frac{1}{3}\arctan\left(\frac{x^3-\sqrt{x^4-2x^2}x-2x}{x^2-\sqrt{x^4-2x^2}}\right)-\frac{2}{3}\arctan\left(\frac{x^3-\sqrt{x^4-2x^2}x-2x}{2\left(x^2-\sqrt{x^4-2x^2}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 - 2*x^2)/((x^2 + 2)*(x^2 - 1)),x, algorithm="fricas")
```

```
[Out] 1/3*arctan((x^3 - sqrt(x^4 - 2*x^2)*x - 2*x)/(x^2 - sqrt(x^4 - 2*
x^2))) - 2/3*arctan(1/2*(x^3 - sqrt(x^4 - 2*x^2)*x - 2*x)/(x^2 -
sqrt(x^4 - 2*x^2)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2 (x^2 - 2)}}{(x - 1) (x + 1) (x^2 + 2)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**4-2*x**2)**(1/2)/(x**2-1)/(x**2+2),x)
```

```
[Out] Integral(sqrt(x**2*(x**2 - 2))/((x - 1)*(x + 1)*(x**2 + 2)), x)
```

GIAC/XCAS [A] time = 0.286085, size = 65, normalized size = 0.78

$$\frac{1}{3} \left(\arctan\left(\sqrt{2}i\right) - 2 \arctan\left(\frac{1}{2} \sqrt{2}i\right)\right) \operatorname{sign}(x) + \frac{1}{3} \left(2 \arctan\left(\frac{1}{2} \sqrt{x^2 - 2}\right) - \arctan\left(\sqrt{x^2 - 2}\right)\right) \operatorname{sign}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 - 2*x^2)/((x^2 + 2)*(x^2 - 1)),x, algorithm="giac")
```

```
[Out] 1/3*(arctan(sqrt(2)*i) - 2*arctan(1/2*sqrt(2)*i))*sign(x) + 1/3*(
2*arctan(1/2*sqrt(x^2 - 2)) - arctan(sqrt(x^2 - 2)))*sign(x)
```

3.739 $\int \frac{\sqrt{1-\frac{1}{\left(-1+x^2\right)^2}}}{2-x^2}\,dx$

Optimal. Leaf size=47

$$\frac{\left(1-x^2\right)\sqrt{1-\frac{1}{\left(1-x^2\right)^2}}\tan^{-1}\left(\sqrt{x^2-2}\right)}{x\sqrt{x^2-2}}$$

```
[Out] ((1 - x^2)*Sqrt[1 - (1 - x^2)^(-2)]*ArcTan[Sqrt[-2 + x^2]])/(x*Sq
rt[-2 + x^2])
```

Rubi [A] time = 0.784073, antiderivative size = 73, normalized size of antiderivative = 1.55, number of steps used = 13, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$

$$\frac{\left(1-x^2\right)\sqrt{x^4-2x^2}\sqrt{1-\frac{1}{\left(1-x^2\right)^2}}\tan^{-1}\left(\sqrt{x^2-2}\right)}{x\sqrt{x^2-2}\sqrt{\left(x^2-1\right)^2-1}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 - (-1 + x^2)^(-2)]/(2 - x^2),x]
```

```
[Out] ((1 - x^2)*Sqrt[-2*x^2 + x^4]*Sqrt[1 - (1 - x^2)^(-2)]*ArcTan[Sqr
t[-2 + x^2]])/(x*Sqrt[-2 + x^2]*Sqrt[-1 + (-1 + x^2)^2])
```

Rubi in Sympy [A] time = 35.4131, size = 61, normalized size = 1.3

$$\frac{\sqrt{1-\frac{1}{\left(x^2-1\right)^2}}\left(-x^2+1\right)\sqrt{x^4-2x^2}\,\operatorname{atan}\left(\sqrt{x^2-2}\right)}{x\sqrt{x^2-2}\sqrt{\left(x^2-1\right)^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1-1/(x**2-1)**2)**(1/2)/(-x**2+2),x)
```

```
[Out] sqrt(1 - 1/(x**2 - 1)**2)*(-x**2 + 1)*sqrt(x**4 - 2*x**2)*atan(sq
rt(x**2 - 2))/(x*sqrt(x**2 - 2)*sqrt((x**2 - 1)**2 - 1))
```

Mathematica [A] time = 0.0401975, size = 91, normalized size = 1.94

$$\frac{1}{2}\tan^{-1}\left(\frac{(x-1)(x+1)(x+2)\sqrt{\frac{x^2(x^2-2)}{(x^2-1)^2}}}{x\left(x^2-2\right)}\right)-\frac{1}{2}\tan^{-1}\left(\frac{(x-2)(x-1)(x+1)\sqrt{\frac{x^2(x^2-2)}{(x^2-1)^2}}}{x\left(x^2-2\right)}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 - (-1 + x^2)^(-2)]/(2 - x^2),x]
```

```
[Out] -ArcTan[((-2 + x)*(-1 + x)*(1 + x)*Sqrt[(x^2*(-2 + x^2))/(-1 + x^
2)^2])/(x*(-2 + x^2))]/2 + ArcTan[((-1 + x)*(1 + x)*(2 + x)*Sqrt[
(x^2*(-2 + x^2))/(-1 + x^2)^2])/(x*(-2 + x^2))]/2
```

Maple [A] time = 0.028, size = 63, normalized size = 1.3

$$\frac{x^2-1}{2\,x}\sqrt{\frac{x^2\left(x^2-2\right)}{\left(x^2-1\right)^2}}\left(\arctan\left((2+x)\frac{1}{\sqrt{x^2-2}}\right)-\arctan\left((x-2)\frac{1}{\sqrt{x^2-2}}\right)\right)\frac{1}{\sqrt{x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1-1/(x^2-1)^2)^(1/2)/(-x^2+2),x)
```

```
[Out] 1/2*(x^2*(x^2-2)/(x^2-1)^2)^(1/2)*(x^2-1)*(arctan((2+x)/(x^2-2)^(
1/2))-arctan((x-2)/(x^2-2)^(1/2)))/x/(x^2-2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\sqrt{-\frac{1}{\left(x^2-1\right)^2}+1}}{x^2-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sqrt(-1/(x^2 - 1)^2 + 1)/(x^2 - 2),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(-1/(x^2 - 1)^2 + 1)/(x^2 - 2), x)
```

Fricas [A] time = 0.279441, size = 107, normalized size = 2.28

$$\arctan\left(\frac{x^3 - (x^3 - x)\sqrt{\frac{x^4 - 2x^2}{x^4 - 2x^2 + 1}} - 2x}{x^2 - (x^2 - 1)\sqrt{\frac{x^4 - 2x^2}{x^4 - 2x^2 + 1}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sqrt(-1/(x^2 - 1)^2 + 1)/(x^2 - 2),x, algorithm="fricas")
```

```
[Out] arctan((x^3 - (x^3 - x)*sqrt((x^4 - 2*x^2)/(x^4 - 2*x^2 + 1)) - 2
*x)/(x^2 - (x^2 - 1)*sqrt((x^4 - 2*x^2)/(x^4 - 2*x^2 + 1))))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1-1/(x**2-1)**2)**(1/2)/(-x**2+2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.265688, size = 24, normalized size = 0.51

$$-\arctan\left(\sqrt{x^2 - 2}\right)\operatorname{sign}\left(x^3 - x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sqrt(-1/(x^2 - 1)^2 + 1)/(x^2 - 2),x, algorithm="giac")
```

```
[Out] -arctan(sqrt(x^2 - 2))*sign(x^3 - x)
```

3.740 $\int \frac{\sqrt{\frac{-2x^2+x^4}{\left(-1+x^2\right)^2}}}{2+x^2}\,dx$

Optimal. Leaf size=123

$$\frac{\left(1-x^2\right)\sqrt{-\frac{2x^2-x^4}{(1-x^2)^2}}\tan^{-1}\left(\sqrt{x^2-2}\right)}{3x\sqrt{x^2-2}}-\frac{2\left(1-x^2\right)\sqrt{-\frac{2x^2-x^4}{(1-x^2)^2}}\tan^{-1}\left(\frac{\sqrt{x^2-2}}{2}\right)}{3x\sqrt{x^2-2}}$$

```
[Out] (-2*(1 - x^2)*Sqrt[-((2*x^2 - x^4)/(1 - x^2)^2)]*ArcTan[Sqrt[-2 +
 x^2]/2])/(3*x*Sqrt[-2 + x^2]) + ((1 - x^2)*Sqrt[-((2*x^2 - x^4)/
(1 - x^2)^2)]*ArcTan[Sqrt[-2 + x^2]])/(3*x*Sqrt[-2 + x^2])
```

Rubi [A] time = 0.517798, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207

$$\frac{\left(1-x^2\right)\sqrt{-\frac{2x^2-x^4}{(1-x^2)^2}}\tan^{-1}\left(\sqrt{x^2-2}\right)}{3x\sqrt{x^2-2}}-\frac{2\left(1-x^2\right)\sqrt{-\frac{2x^2-x^4}{(1-x^2)^2}}\tan^{-1}\left(\frac{\sqrt{x^2-2}}{2}\right)}{3x\sqrt{x^2-2}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[(-2*x^2 + x^4)/(-1 + x^2)^2]/(2 + x^2),x]
```

```
[Out] (-2*(1 - x^2)*Sqrt[-((2*x^2 - x^4)/(1 - x^2)^2)]*ArcTan[Sqrt[-2 +
 x^2]/2])/(3*x*Sqrt[-2 + x^2]) + ((1 - x^2)*Sqrt[-((2*x^2 - x^4)/
(1 - x^2)^2)]*ArcTan[Sqrt[-2 + x^2]])/(3*x*Sqrt[-2 + x^2])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\sqrt{2}i\int\frac{\sqrt{\frac{x^4-2x^2}{(x^2-1)^2}}}{-4x+4\sqrt{2}i}\,dx+\frac{\sqrt{2}i\int\frac{\sqrt{\frac{x^4-2x^2}{\left(x^2-1\right)^2}}}{x+\sqrt{2}i}\,dx}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((x**4-2*x**2)/(x**2-1)**2)**(1/2)/(x**2+2),x)
```

```
[Out] sqrt(2)*I*Integral(sqrt((x**4 - 2*x**2)/(x**2 - 1)**2)/(-4*x + 4*
sqrt(2)*I), x) + sqrt(2)*I*Integral(sqrt((x**4 - 2*x**2)/(x**2 -
```

```
1)**2)/(x + sqrt(2)*I), x)/4
```

Mathematica [A] time = 0.0402558, size = 70, normalized size = 0.57

$$\frac{\sqrt{\frac{x^2(x^2-2)}{(x^2-1)^2}}\,(x^2-1)\left(2\tan^{-1}\left(\frac{\sqrt{x^2-2}}{2}\right)-\tan^{-1}\left(\sqrt{x^2-2}\right)\right)}{3x\sqrt{x^2-2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[(-2*x^2 + x^4)/(-1 + x^2)^2]/(2 + x^2),x]
```

```
[Out] (Sqrt[(x^2*(-2 + x^2))/(-1 + x^2)^2]*(-1 + x^2)*(2*ArcTan[Sqrt[-2
 + x^2]/2] - ArcTan[Sqrt[-2 + x^2]]))/(3*x*Sqrt[-2 + x^2])
```

Maple [A] time = 0.012, size = 75, normalized size = 0.6

$$\frac{x^2-1}{6\,x}\sqrt{\frac{x^2\,(x^2-2)}{(x^2-1)^2}}\left(\arctan\left((2+x)\frac{1}{\sqrt{x^2-2}}\right)-\arctan\left((x-2)\frac{1}{\sqrt{x^2-2}}\right)+4\,\arctan\left(1/2\,\sqrt{x^2-2}\right)\right)\frac{1}{\sqrt{x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((x^4-2*x^2)/(x^2-1)^2)^(1/2)/(x^2+2),x)
```

```
[Out] 1/6*(x^2*(x^2-2)/(x^2-1)^2)^(1/2)*(x^2-1)*(arctan((2+x)/(x^2-2)^(
1/2))-arctan((x-2)/(x^2-2)^(1/2))+4*arctan(1/2*(x^2-2)^(1/2)))/x/
(x^2-2)^(1/2)
```

Maxima [A] time = 0.815802, size = 31, normalized size = 0.25

$$\frac{2}{3}\,\arctan\left(\frac{1}{2}\,\sqrt{x^2-2}\right)-\frac{1}{3}\,\arctan\left(\sqrt{x^2-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((x^4 - 2*x^2)/(x^2 - 1)^2)/(x^2 + 2),x, algorithm="maxima")
```

```
[Out] 2/3*arctan(1/2*sqrt(x^2 - 2)) - 1/3*arctan(sqrt(x^2 - 2))
```

Fricas [A] time = 0.279032, size = 221, normalized size = 1.8

$$\frac{1}{3}\arctan\left(\frac{x^3-(x^3-x)\sqrt{\frac{x^4-2\,x^2}{x^4-2\,x^2+1}}-2\,x}{x^2-(x^2-1)\sqrt{\frac{x^4-2\,x^2}{x^4-2\,x^2+1}}}\right)-\frac{2}{3}\arctan\left(\frac{x^3-(x^3-x)\sqrt{\frac{x^4-2\,x^2}{x^4-2\,x^2+1}}-2\,x}{2\left(x^2-(x^2-1)\sqrt{\frac{x^4-2\,x^2}{x^4-2\,x^2+1}}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt((x^4 - 2*x^2)/(x^2 - 1)^2)/(x^2 + 2),x, algorithm="fricas")
```

```
[Out] 1/3*arctan((x^3 - (x^3 - x)*sqrt((x^4 - 2*x^2)/(x^4 - 2*x^2 + 1))
 - 2*x)/(x^2 - (x^2 - 1)*sqrt((x^4 - 2*x^2)/(x^4 - 2*x^2 + 1))))
- 2/3*arctan(1/2*(x^3 - (x^3 - x)*sqrt((x^4 - 2*x^2)/(x^4 - 2*x^2
 + 1)) - 2*x)/(x^2 - (x^2 - 1)*sqrt((x^4 - 2*x^2)/(x^4 - 2*x^2 +
1))))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((x**4-2*x**2)/(x**2-1)**2)**(1/2)/(x**2+2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.269075, size = 45, normalized size = 0.37

$$\frac{1}{3}\left(2\arctan\left(\frac{1}{2}\sqrt{x^2-2}\right)-\arctan\left(\sqrt{x^2-2}\right)\right)\operatorname{sign}\left(x^3-x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt((x^4 - 2*x^2)/(x^2 - 1)^2)/(x^2 + 2),x, algorithm="giac")
```

```
[Out] 1/3*(2*arctan(1/2*sqrt(x^2 - 2)) - arctan(sqrt(x^2 - 2)))*sign(x^
3 - x)
```

3.741 $\int \left(1+\frac{2x}{1+x^2}\right)^{5/2} dx$

Optimal. Leaf size=133

$$-\frac{(1-x)\sqrt{\frac{2x}{x^2+1}+1}(x+1)^3}{3\left(x^2+1\right)}-\frac{4}{3}(1-2x)\sqrt{\frac{2x}{x^2+1}+1}(x+1)-\frac{(3x+4)\left(x^2+1\right)\sqrt{\frac{2x}{x^2+1}+1}}{x+1}+\frac{5\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}\sinh^{-1}(x)}{x+1}$$

```
[Out] (-4*(1 - 2*x)*(1 + x)*Sqrt[1 + (2*x)/(1 + x^2)])/3 - ((1 - x)*(1
+ x)^3*Sqrt[1 + (2*x)/(1 + x^2)])/(3*(1 + x^2)) - ((4 + 3*x)*(1 +
 x^2)*Sqrt[1 + (2*x)/(1 + x^2)])/(1 + x) + (5*Sqrt[1 + x^2]*Sqrt[
1 + (2*x)/(1 + x^2)]*ArcSinh[x])/(1 + x)
```

Rubi [A] time = 0.194981, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$

$$-\frac{(1-x)\sqrt{\frac{2x}{x^2+1}+1}(x+1)^3}{3\left(x^2+1\right)}-\frac{4}{3}(1-2x)\sqrt{\frac{2x}{x^2+1}+1}(x+1)-\frac{(3x+4)\left(x^2+1\right)\sqrt{\frac{2x}{x^2+1}+1}}{x+1}+\frac{5\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}\sinh^{-1}(x)}{x+1}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + (2*x)/(1 + x^2))^(5/2),x]
```

```
[Out] (-4*(1 - 2*x)*(1 + x)*Sqrt[1 + (2*x)/(1 + x^2)])/3 - ((1 - x)*(1
+ x)^3*Sqrt[1 + (2*x)/(1 + x^2)])/(3*(1 + x^2)) - ((4 + 3*x)*(1 +
 x^2)*Sqrt[1 + (2*x)/(1 + x^2)])/(1 + x) + (5*Sqrt[1 + x^2]*Sqrt[
1 + (2*x)/(1 + x^2)]*ArcSinh[x])/(1 + x)
```

Rubi in Sympy [A] time = 9.30216, size = 112, normalized size = 0.84

$$-\frac{(-128x+64)(x+1)\sqrt{\frac{2x}{x^2+1}+1}}{48}-\frac{(-2x+2)(x+1)^3\sqrt{\frac{2x}{x^2+1}+1}}{6\left(x^2+1\right)}-\frac{(1152x+1536)\left(x^2+1\right)\sqrt{\frac{2x}{x^2+1}+1}}{384(x+1)}+\frac{5\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}\operatorname{asinh}(x)}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+2*x/(x**2+1))**(5/2),x)
```

```
[Out] -(-128*x + 64)*(x + 1)*sqrt(2*x/(x**2 + 1) + 1)/48 - (-2*x + 2)*(
x + 1)**3*sqrt(2*x/(x**2 + 1) + 1)/(6*(x**2 + 1)) - (1152*x + 153
6)*(x**2 + 1)*sqrt(2*x/(x**2 + 1) + 1)/(384*(x + 1)) + 5*sqrt(x**
2 + 1)*sqrt(2*x/(x**2 + 1) + 1)*asinh(x)/(x + 1)
```

Mathematica [A] time = 0.0841018, size = 64, normalized size = 0.48

$$\frac{(x+1)\left(3x^4-8x^3-18x^2+15\left(x^2+1\right)^{3/2}\sinh^{-1}(x)-12x-17\right)}{3\sqrt{\frac{(x+1)^2}{x^2+1}}\left(x^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + (2*x)/(1 + x^2))^(5/2),x]
```

```
[Out] ((1 + x)*(-17 - 12*x - 18*x^2 - 8*x^3 + 3*x^4 + 15*(1 + x^2)^(3/2
)*ArcSinh[x]))/(3*Sqrt[(1 + x)^2/(1 + x^2)]*(1 + x^2)^2)
```

Maple [A] time = 0.024, size = 62, normalized size = 0.5

$$\frac{x^2+1}{3\,(1+x)^5}\left(\frac{x^2+2\,x+1}{x^2+1}\right)^{\frac{5}{2}}\left(15\,Arcsinh(x)\left(x^2+1\right)^{3/2}+3\,x^4-8\,x^3-18\,x^2-12\,x-17\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+2*x/(x^2+1))^(5/2),x)
```

```
[Out] 1/3*((x^2+2*x+1)/(x^2+1))^(5/2)/(1+x)^5*(x^2+1)*(15*arcsinh(x)*(x
^2+1)^(3/2)+3*x^4-8*x^3-18*x^2-12*x-17)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(\frac{2\,x}{x^2+1}+1\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x/(x^2 + 1) + 1)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((2*x/(x^2 + 1) + 1)^(5/2), x)
```

Fricas [A] time = 0.302351, size = 169, normalized size = 1.27

$$-\frac{8\,x^{3}+8\,x^{2}+15\,\left(x^{3}+x^{2}+x+1\right)\,\log\left(-\frac{x\sqrt{\frac{x^{2}+2\,x+1}{x^{2}+1}}-x-1}{\sqrt{\frac{x^{2}+2\,x+1}{x^{2}+1}}}\right)-\left(3\,x^{4}-8\,x^{3}-18\,x^{2}-12\,x-17\right)\sqrt{\frac{x^{2}+2\,x+1}{x^{2}+1}}+8\,x+8}{3\,(x^{3}+x^{2}+x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x/(x^2 + 1) + 1)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(8*x^3 + 8*x^2 + 15*(x^3 + x^2 + x + 1)*log(-(x*sqrt((x^2 +
2*x + 1)/(x^2 + 1)) - x - 1)/sqrt((x^2 + 2*x + 1)/(x^2 + 1))) - (
3*x^4 - 8*x^3 - 18*x^2 - 12*x - 17)*sqrt((x^2 + 2*x + 1)/(x^2 + 1
)) + 8*x + 8)/(x^3 + x^2 + x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(\frac{2x}{x^{2}+1}+1\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+2*x/(x**2+1))**(5/2),x)
```

```
[Out] Integral((2*x/(x**2 + 1) + 1)**(5/2), x)
```

GIAC/XCAS [A] time = 0.270192, size = 116, normalized size = 0.87

$$\left(\sqrt{2}+5\ln\left(\sqrt{2}+1\right)\right)\operatorname{sign}(x+1)-5\ln\left(-x+\sqrt{x^{2}+1}\right)\operatorname{sign}(x+1)$$
$$+\frac{(((3\,x\operatorname{sign}(x+1)-8\operatorname{sign}(x+1))x-18\operatorname{sign}(x+1))x-12\operatorname{sign}(x+1))x-17\operatorname{sign}(x+1)}{3\left(x^{2}+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x/(x^2 + 1) + 1)^(5/2),x, algorithm="giac")
```

```
[Out] (sqrt(2) + 5*ln(sqrt(2) + 1))*sign(x + 1) - 5*ln(-x + sqrt(x^2 +
1))*sign(x + 1) + 1/3*((((3*x*sign(x + 1) - 8*sign(x + 1))*x - 18
*sign(x + 1))*x - 12*sign(x + 1))*x - 17*sign(x + 1))/(x^2 + 1)^(
3/2)
```

3.742 $\int \left(1+\frac{2x}{1+x^2}\right)^{3/2} dx$

Optimal. Leaf size=90

$$-(1-x)\sqrt{\frac{2x}{x^2+1}+1}(x+1)-\frac{x\left(x^2+1\right)\sqrt{\frac{2x}{x^2+1}+1}}{x+1}+\frac{3\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}\sinh^{-1}(x)}{x+1}$$

```
[Out] -((1 - x)*(1 + x)*Sqrt[1 + (2*x)/(1 + x^2)]) - (x*(1 + x^2)*Sqrt[
1 + (2*x)/(1 + x^2)])/(1 + x) + (3*Sqrt[1 + x^2]*Sqrt[1 + (2*x)/(
1 + x^2)]*ArcSinh[x])/(1 + x)
```

Rubi [A] time = 0.130463, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375

$$-(1-x)\sqrt{\frac{2x}{x^2+1}+1}(x+1)-\frac{x\left(x^2+1\right)\sqrt{\frac{2x}{x^2+1}+1}}{x+1}+\frac{3\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}\sinh^{-1}(x)}{x+1}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + (2*x)/(1 + x^2))^(3/2),x]
```

```
[Out] -((1 - x)*(1 + x)*Sqrt[1 + (2*x)/(1 + x^2)]) - (x*(1 + x^2)*Sqrt[
1 + (2*x)/(1 + x^2)])/(1 + x) + (3*Sqrt[1 + x^2]*Sqrt[1 + (2*x)/(
1 + x^2)]*ArcSinh[x])/(1 + x)
```

Rubi in Sympy [A] time = 6.73701, size = 76, normalized size = 0.84

$$-\frac{x\left(x^2+1\right)\sqrt{\frac{2x}{x^2+1}+1}}{x+1}-\frac{\left(-2x+2\right)\left(x+1\right)\sqrt{\frac{2x}{x^2+1}+1}}{2}+\frac{3\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}\operatorname{asinh}\left(x\right)}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+2*x/(x**2+1))**(3/2),x)
```

```
[Out] -x*(x**2 + 1)*sqrt(2*x/(x**2 + 1) + 1)/(x + 1) - (-2*x + 2)*(x +
1)*sqrt(2*x/(x**2 + 1) + 1)/2 + 3*sqrt(x**2 + 1)*sqrt(2*x/(x**2 +
 1) + 1)*asinh(x)/(x + 1)
```

Mathematica [A] time = 0.0343863, size = 44, normalized size = 0.49

$$\frac{\sqrt{\frac{(x+1)^2}{x^2+1}}\left(x^2+3\sqrt{x^2+1}\sinh^{-1}(x)-2x-1\right)}{x+1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + (2*x)/(1 + x^2))^(3/2),x]
```

```
[Out] (Sqrt[(1 + x)^2/(1 + x^2)]*(-1 - 2*x + x^2 + 3*Sqrt[1 + x^2]*ArcS
inh[x]))/(1 + x)
```

Maple [A] time = 0.014, size = 49, normalized size = 0.5

$$\frac{x^2+1}{(1+x)^3}\left(\frac{x^2+2\,x+1}{x^2+1}\right)^{\frac{3}{2}}\left(3\,Arcsinh\,(x)\,\sqrt{x^2+1}+x^2-2\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+2*x/(x^2+1))^(3/2),x)
```

```
[Out] ((x^2+2*x+1)/(x^2+1))^(3/2)/(1+x)^3*(x^2+1)*(3*arcsinh(x)*(x^2+1)
^(1/2)+x^2-2*x-1)
```

Maxima [A] time = 0.787713, size = 47, normalized size = 0.52

$$\frac{x^2}{\sqrt{x^2+1}}-\frac{2\,x}{\sqrt{x^2+1}}-\frac{1}{\sqrt{x^2+1}}+3\,\operatorname{arsinh}\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x/(x^2 + 1) + 1)^(3/2),x, algorithm="maxima")
```

```
[Out] x^2/sqrt(x^2 + 1) - 2*x/sqrt(x^2 + 1) - 1/sqrt(x^2 + 1) + 3*arcsi
nh(x)
```

Fricas [A] time = 0.269331, size = 123, normalized size = 1.37

$$-\frac{3\,(x+1)\log\left(-\frac{x\sqrt{\frac{x^2+2\,x+1}{x^2+1}}-x-1}{\sqrt{\frac{x^2+2\,x+1}{x^2+1}}}\right)-\left(x^2-2\,x-1\right)\sqrt{\frac{x^2+2\,x+1}{x^2+1}}+2\,x+2}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x/(x^2 + 1) + 1)^(3/2),x, algorithm="fricas")
```

```
[Out] -(3*(x + 1)*log(-(x*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) - x - 1)/sqrt
((x^2 + 2*x + 1)/(x^2 + 1))) - (x^2 - 2*x - 1)*sqrt((x^2 + 2*x +
1)/(x^2 + 1)) + 2*x + 2)/(x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(\frac{2x}{x^2+1}+1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+2*x/(x**2+1))**(3/2),x)
```

```
[Out] Integral((2*x/(x**2 + 1) + 1)**(3/2), x)
```

GIAC/XCAS [A] time = 0.263801, size = 90, normalized size = 1.

$$-\left(\sqrt{2}-3\ln\left(\sqrt{2}+1\right)\right)\mathrm{sign}\,(x+1)-3\ln\left(-x+\sqrt{x^2+1}\right)\mathrm{sign}\,(x+1)+\frac{(x\mathrm{sign}\,(x+1)-2\,\mathrm{sign}\,(x+1))x-\mathrm{sign}\,(x+1)}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x/(x^2 + 1) + 1)^(3/2),x, algorithm="giac")
```

```
[Out] -(sqrt(2) - 3*ln(sqrt(2) + 1))*sign(x + 1) - 3*ln(-x + sqrt(x^2 +
 1))*sign(x + 1) + ((x*sign(x + 1) - 2*sign(x + 1))*x - sign(x +
1))/sqrt(x^2 + 1)
```

3.743 $\int \sqrt{1+\frac{2x}{1+x^2}}\,dx$

Optimal. Leaf size=61

$$\frac{\sqrt{\frac{2x}{x^2+1}+1}\,(x^2+1)}{x+1}+\frac{\sqrt{\frac{2x}{x^2+1}+1}\sqrt{x^2+1}\sinh^{-1}(x)}{x+1}$$

```
[Out] ((1 + x^2)*Sqrt[1 + (2*x)/(1 + x^2)])/(1 + x) + (Sqrt[1 + x^2]*Sq
rt[1 + (2*x)/(1 + x^2)]*ArcSinh[x])/(1 + x)
```

Rubi [A] time = 0.0727453, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{\sqrt{\frac{2x}{x^2+1}+1}\,(x^2+1)}{x+1}+\frac{\sqrt{\frac{2x}{x^2+1}+1}\sqrt{x^2+1}\sinh^{-1}(x)}{x+1}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 + (2*x)/(1 + x^2)],x]
```

```
[Out] ((1 + x^2)*Sqrt[1 + (2*x)/(1 + x^2)])/(1 + x) + (Sqrt[1 + x^2]*Sq
rt[1 + (2*x)/(1 + x^2)]*ArcSinh[x])/(1 + x)
```

Rubi in Sympy [A] time = 4.69614, size = 49, normalized size = 0.8

$$\frac{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}\,\operatorname{asinh}(x)}{x+1}+\frac{(x^2+1)\sqrt{\frac{2x}{x^2+1}+1}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+2*x/(x**2+1))**(1/2),x)
```

```
[Out] sqrt(x**2 + 1)*sqrt(2*x/(x**2 + 1) + 1)*asinh(x)/(x + 1) + (x**2
+ 1)*sqrt(2*x/(x**2 + 1) + 1)/(x + 1)
```

Mathematica [A] time = 0.0264008, size = 40, normalized size = 0.66

$$\frac{\sqrt{\frac{(x+1)^2}{x^2+1}}\left(x^2+\sqrt{x^2+1}\sinh^{-1}(x)+1\right)}{x+1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 + (2*x)/(1 + x^2)],x]
```

```
[Out] (Sqrt[(1 + x)^2/(1 + x^2)]*(1 + x^2 + Sqrt[1 + x^2]*ArcSinh[x]))/
(1 + x)
```

Maple [A] time = 0.008, size = 42, normalized size = 0.7

$$\frac{1}{1+x}\sqrt{\frac{x^2+2\,x+1}{x^2+1}}\sqrt{x^2+1}\left(Arcsinh\,(x)+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+2*x/(x^2+1))^(1/2),x)
```

```
[Out] ((x^2+2*x+1)/(x^2+1))^(1/2)/(1+x)*(x^2+1)^(1/2)*(arcsinh(x)+(x^2+
1)^(1/2))
```

Maxima [A] time = 0.785498, size = 14, normalized size = 0.23

$$\sqrt{x^2+1}+\operatorname{arsinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(2*x/(x^2 + 1) + 1),x, algorithm="maxima")
```

```
[Out] sqrt(x^2 + 1) + arcsinh(x)
```

Fricas [A] time = 0.268413, size = 112, normalized size = 1.84

$$-\frac{(x+1)\log\left(-\frac{x\sqrt{\frac{x^2+2\,x+1}{x^2+1}}-x-1}{\sqrt{\frac{x^2+2\,x+1}{x^2+1}}}\right)-\left(x^2+1\right)\sqrt{\frac{x^2+2\,x+1}{x^2+1}}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(2*x/(x^2 + 1) + 1),x, algorithm="fricas")
```

```
[Out] -((x + 1)*log(-(x*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) - x - 1)/sqrt((
x^2 + 2*x + 1)/(x^2 + 1))) - (x^2 + 1)*sqrt((x^2 + 2*x + 1)/(x^2
+ 1)))/(x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{2x}{x^2+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+2*x/(x**2+1))**(1/2),x)
```

```
[Out] Integral(sqrt(2*x/(x**2 + 1) + 1), x)
```

GIAC/XCAS [A] time = 0.263335, size = 66, normalized size = 1.08

$$-\left(\sqrt{2}-\ln\left(\sqrt{2}+1\right)\right)\operatorname{sign}\left(x+1\right)-\ln\left(-x+\sqrt{x^2+1}\right)\operatorname{sign}\left(x+1\right)+\sqrt{x^2+1}\operatorname{sign}\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(2*x/(x^2 + 1) + 1),x, algorithm="giac")
```

```
[Out] -(sqrt(2) - ln(sqrt(2) + 1))*sign(x + 1) - ln(-x + sqrt(x^2 + 1))
*sign(x + 1) + sqrt(x^2 + 1)*sign(x + 1)
```

3.744 $\int \frac{1}{\sqrt{1+\frac{2x}{1+x^2}}}\,dx$

Optimal. Leaf size=109

$$\frac{x+1}{\sqrt{\frac{2x}{x^2+1}+1}}-\frac{(x+1)\sinh^{-1}(x)}{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}}-\frac{\sqrt{2}(x+1)\tanh^{-1}\left(\frac{1-x}{\sqrt{2}\sqrt{x^2+1}}\right)}{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}}$$

```
[Out] (1 + x)/Sqrt[1 + (2*x)/(1 + x^2)] - ((1 + x)*ArcSinh[x])/(Sqrt[1
+ x^2]*Sqrt[1 + (2*x)/(1 + x^2)]) - (Sqrt[2]*(1 + x)*ArcTanh[(1 -
 x)/(Sqrt[2]*Sqrt[1 + x^2])])/(Sqrt[1 + x^2]*Sqrt[1 + (2*x)/(1 +
x^2)])
```

Rubi [A] time = 0.164534, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$

$$\frac{x+1}{\sqrt{\frac{2x}{x^2+1}+1}}-\frac{(x+1)\sinh^{-1}(x)}{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}}-\frac{\sqrt{2}(x+1)\tanh^{-1}\left(\frac{1-x}{\sqrt{2}\sqrt{x^2+1}}\right)}{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[1 + (2*x)/(1 + x^2)],x]
```

```
[Out] (1 + x)/Sqrt[1 + (2*x)/(1 + x^2)] - ((1 + x)*ArcSinh[x])/(Sqrt[1
+ x^2]*Sqrt[1 + (2*x)/(1 + x^2)]) - (Sqrt[2]*(1 + x)*ArcTanh[(1 -
 x)/(Sqrt[2]*Sqrt[1 + x^2])])/(Sqrt[1 + x^2]*Sqrt[1 + (2*x)/(1 +
x^2)])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1+2*x/(x**2+1))**(1/2),x)
```

```
[Out] Exception raised: RecursionError
```

Mathematica [A] time = 0.0651665, size = 82, normalized size = 0.75

$$\frac{(x+1)\left(\sqrt{x^2+1}-\sqrt{2}\log\left(\sqrt{2}\sqrt{x^2+1}-x+1\right)+\sqrt{2}\log(x+1)-\sinh^{-1}(x)\right)}{\sqrt{\frac{(x+1)^2}{x^2+1}}\sqrt{x^2+1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[1 + (2*x)/(1 + x^2)],x]
```

```
[Out] ((1 + x)*(Sqrt[1 + x^2] - ArcSinh[x] + Sqrt[2]*Log[1 + x] - Sqrt[
2]*Log[1 - x + Sqrt[2]*Sqrt[1 + x^2]]))/(Sqrt[(1 + x)^2/(1 + x^2)
]*Sqrt[1 + x^2])
```

Maple [A] time = 0.047, size = 79, normalized size = 0.7

$$(1+x)\frac{1}{\sqrt{\frac{(1+x)^2}{x^2+1}}}+(1+x)\left(-Arcsinh(x)-\sqrt{2}Artanh\left(\frac{(2-2x)\sqrt{2}}{4}\frac{1}{\sqrt{(1+x)^2-2x}}\right)\right)\frac{1}{\sqrt{\frac{(1+x)^2}{x^2+1}}}\frac{1}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1+2*x/(x^2+1))^(1/2),x)
```

```
[Out] 1/((1+x)^2/(x^2+1))^(1/2)*(1+x)+(-arcsinh(x)-2^(1/2)*arctanh(1/4*
(2-2*x)*2^(1/2)/((1+x)^2-2*x)^(1/2)))/((1+x)^2/(x^2+1))^(1/2)/(x^
2+1)^(1/2)*(1+x)
```

Maxima [A] time = 0.780275, size = 46, normalized size = 0.42

$$\sqrt{2}\,\mathrm{arsinh}\left(\frac{x}{|x+1|}-\frac{1}{|x+1|}\right)+\sqrt{x^2+1}-\mathrm{arsinh}\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x/(x^2 + 1) + 1),x, algorithm="maxima")
```

```
[Out] sqrt(2)*arcsinh(x/abs(x + 1) - 1/abs(x + 1)) + sqrt(x^2 + 1) - ar
csinh(x)
```

Fricas [A] time = 0.307585, size = 290, normalized size = 2.66

$$\frac{\sqrt{2}(x+1)\log\left(\frac{2\,x^3+4\,x^2+\sqrt{2}\left(2\,x^2+3\,x+1\right)-\left(2\,x^3+2\,x^2+\sqrt{2}\left(2\,x^2+x+1\right)+3\,x+1\right)\sqrt{\frac{x^2+2\,x+1}{x^2+1}}+4\,x+2}{2\,x^3+4\,x^2-\left(2\,x^3+2\,x^2+x+1\right)\sqrt{\frac{x^2+2\,x+1}{x^2+1}}+2\,x}\right)+(x+1)\log\left(-\frac{x\sqrt{\frac{x^2+2\,x+1}{x^2+1}}-x-1}{\sqrt{\frac{x^2+2\,x+1}{x^2+1}}}\right)+\left(x^2+1\right)\sqrt{\ldots}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x/(x^2 + 1) + 1),x, algorithm="fricas")
```

```
[Out] (sqrt(2)*(x + 1)*log((2*x^3 + 4*x^2 + sqrt(2)*(2*x^2 + 3*x + 1) -
 (2*x^3 + 2*x^2 + sqrt(2)*(2*x^2 + x + 1) + 3*x + 1)*sqrt((x^2 +
2*x + 1)/(x^2 + 1)) + 4*x + 2)/(2*x^3 + 4*x^2 - (2*x^3 + 2*x^2 +
x + 1)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) + 2*x)) + (x + 1)*log(-(x*
sqrt((x^2 + 2*x + 1)/(x^2 + 1)) - x - 1)/sqrt((x^2 + 2*x + 1)/(x^
2 + 1))) + (x^2 + 1)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)))/(x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\frac{2x}{x^2+1}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1+2*x/(x**2+1))**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x/(x**2 + 1) + 1), x)
```

GIAC/XCAS [A] time = 0.307031, size = 119, normalized size = 1.09

$$\frac{\sqrt{2}\ln\left(\frac{\left|-2\,x-2\,\sqrt{2}+2\,\sqrt{x^2+1}-2\right|}{\left|-2\,x+2\,\sqrt{2}+2\,\sqrt{x^2+1}-2\right|}\right)}{\mathrm{sign}\left(x+1\right)}+\frac{\ln\left(-x+\sqrt{x^2+1}\right)}{\mathrm{sign}\left(x+1\right)}+\frac{\sqrt{x^2+1}}{\mathrm{sign}\left(x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x/(x^2 + 1) + 1),x, algorithm="giac")
```

```
[Out] sqrt(2)*ln(abs(-2*x - 2*sqrt(2) + 2*sqrt(x^2 + 1) - 2)/abs(-2*x +
 2*sqrt(2) + 2*sqrt(x^2 + 1) - 2))/sign(x + 1) + ln(-x + sqrt(x^2
 + 1))/sign(x + 1) + sqrt(x^2 + 1)/sign(x + 1)
```

3.745 $\int \frac{1}{\left(1+\frac{2x}{1+x^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=144

$$\frac{3(x+2)}{2\sqrt{\frac{2x}{x^2+1}+1}}-\frac{x^2+1}{2(x+1)\sqrt{\frac{2x}{x^2+1}+1}}-\frac{3(x+1)\sinh^{-1}(x)}{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}}-\frac{9(x+1)\tanh^{-1}\left(\frac{1-x}{\sqrt{2}\sqrt{x^2+1}}\right)}{2\sqrt{2}\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}}$$

```
[Out] (3*(2 + x))/(2*Sqrt[1 + (2*x)/(1 + x^2)]) - (1 + x^2)/(2*(1 + x)*
Sqrt[1 + (2*x)/(1 + x^2)]) - (3*(1 + x)*ArcSinh[x])/(Sqrt[1 + x^2
]*Sqrt[1 + (2*x)/(1 + x^2)]) - (9*(1 + x)*ArcTanh[(1 - x)/(Sqrt[2
]*Sqrt[1 + x^2])])/(2*Sqrt[2]*Sqrt[1 + x^2]*Sqrt[1 + (2*x)/(1 + x
^2)])
```

Rubi [A] time = 0.212711, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$\frac{3(x+2)}{2\sqrt{\frac{2x}{x^2+1}+1}}-\frac{x^2+1}{2(x+1)\sqrt{\frac{2x}{x^2+1}+1}}-\frac{3(x+1)\sinh^{-1}(x)}{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}}-\frac{9(x+1)\tanh^{-1}\left(\frac{1-x}{\sqrt{2}\sqrt{x^2+1}}\right)}{2\sqrt{2}\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + (2*x)/(1 + x^2))^(-3/2),x]
```

```
[Out] (3*(2 + x))/(2*Sqrt[1 + (2*x)/(1 + x^2)]) - (1 + x^2)/(2*(1 + x)*
Sqrt[1 + (2*x)/(1 + x^2)]) - (3*(1 + x)*ArcSinh[x])/(Sqrt[1 + x^2
]*Sqrt[1 + (2*x)/(1 + x^2)]) - (9*(1 + x)*ArcTanh[(1 - x)/(Sqrt[2
]*Sqrt[1 + x^2])])/(2*Sqrt[2]*Sqrt[1 + x^2]*Sqrt[1 + (2*x)/(1 + x
^2)])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1+2*x/(x**2+1))**(3/2),x)
```

```
[Out] Exception raised: RecursionError
```

Mathematica [A] time = 0.180511, size = 184, normalized size = 1.28

$$\frac{(x+1)\left(4\sqrt{x^2+1}x^2+18\sqrt{x^2+1}x+10\sqrt{x^2+1}-9\sqrt{2}x^2\log\left(\sqrt{2}\sqrt{x^2+1}-x+1\right)-18\sqrt{2}x\log\left(\sqrt{2}\sqrt{x^2+1}-x+1\right)-9\sqrt{2}\right.}{4\left(\frac{(x+1)^2}{x^2+1}\right)^{3/2}(x^2+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + (2*x)/(1 + x^2))^(-3/2),x]
```

```
[Out] ((1 + x)*(10*Sqrt[1 + x^2] + 18*x*Sqrt[1 + x^2] + 4*x^2*Sqrt[1 +
x^2] - 12*(1 + x)^2*ArcSinh[x] + 9*Sqrt[2]*(1 + x)^2*Log[1 + x] -
 9*Sqrt[2]*Log[1 - x + Sqrt[2]*Sqrt[1 + x^2]] - 18*Sqrt[2]*x*Log[
1 - x + Sqrt[2]*Sqrt[1 + x^2]] - 9*Sqrt[2]*x^2*Log[1 - x + Sqrt[2
]*Sqrt[1 + x^2]]))/(4*((1 + x)^2/(1 + x^2))^(3/2)*(1 + x^2)^(3/2)
)
```

Maple [A] time = 0.015, size = 217, normalized size = 1.5

$$-\frac{1+x}{8}\left(-\left(x^2+1\right)^{\frac{5}{2}}x+\left(x^2+1\right)^{\frac{3}{2}}x^3+\left(x^2+1\right)^{\frac{5}{2}}-\left(x^2+1\right)^{\frac{3}{2}}x^2-18\,Artanh\left(1/2\,\frac{(-1+x)\sqrt{2}}{\sqrt{x^2+1}}\right)\sqrt{2}x^2-5\,x\left(x^2+1\right)^{3/2}+6\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1+2*x/(x^2+1))^(3/2),x)
```

```
[Out] -1/8/((x^2+2*x+1)/(x^2+1))^(3/2)*(1+x)*(-(x^2+1)^(5/2)*x+(x^2+1)^
(3/2)*x^3+(x^2+1)^(5/2)-(x^2+1)^(3/2)*x^2-18*arctanh(1/2*(-1+x)*2
^(1/2)/(x^2+1)^(1/2))*2^(1/2)*x^2-5*x*(x^2+1)^(3/2)+6*(x^2+1)^(1/
2)*x^3+24*arcsinh(x)*x^2-36*arctanh(1/2*(-1+x)*2^(1/2)/(x^2+1)^(1
/2))*2^(1/2)*x-3*(x^2+1)^(3/2)-6*(x^2+1)^(1/2)*x^2+48*arcsinh(x)*
x-18*2^(1/2)*arctanh(1/2*(-1+x)*2^(1/2)/(x^2+1)^(1/2))-30*x*(x^2+
1)^(1/2)+24*arcsinh(x)-18*(x^2+1)^(1/2))/(x^2+1)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(\frac{2\,x}{x^2+1}+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x/(x^2 + 1) + 1)^(-3/2),x, algorithm="maxima")
```

```
[Out] integrate((2*x/(x^2 + 1) + 1)^(-3/2), x)
```

Fricas [A] time = 0.28165, size = 377, normalized size = 2.62

$$\frac{10\,x^3 + 9\,\sqrt{2}\left(x^3 + 3\,x^2 + 3\,x + 1\right)\log\left(\frac{4\,x^2 + 2\,\sqrt{2}\left(x^3 + 2\,x^2 + 2\,x + 1\right) - \left(4\,x^2 + \sqrt{2}\left(2\,x^3 + 2\,x^2 + 3\,x + 1\right) + 2\,x + 2\right)\sqrt{\frac{x^2+2\,x+1}{x^2+1}} + 6\,x + 2}{2\,x^3 + 4\,x^2 - \left(2\,x^3 + 2\,x^2 + x + 1\right)\sqrt{\frac{x^2+2\,x+1}{x^2+1}} + 2\,x}\right) + 30\,x^2 + 12\,\left(x^3 + 3\right.}{4\left(x^3 + 3\,x^2 + 3\,x + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x/(x^2 + 1) + 1)^(-3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(10*x^3 + 9*sqrt(2)*(x^3 + 3*x^2 + 3*x + 1)*log((4*x^2 + 2*sq
rt(2)*(x^3 + 2*x^2 + 2*x + 1) - (4*x^2 + sqrt(2)*(2*x^3 + 2*x^2 +
 3*x + 1) + 2*x + 2)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) + 6*x + 2)/(
2*x^3 + 4*x^2 - (2*x^3 + 2*x^2 + x + 1)*sqrt((x^2 + 2*x + 1)/(x^2
 + 1)) + 2*x)) + 30*x^2 + 12*(x^3 + 3*x^2 + 3*x + 1)*log(-(x*sqrt
((x^2 + 2*x + 1)/(x^2 + 1)) - x - 1)/sqrt((x^2 + 2*x + 1)/(x^2 +
1))) + 2*(2*x^4 + 9*x^3 + 7*x^2 + 9*x + 5)*sqrt((x^2 + 2*x + 1)/(
x^2 + 1)) + 30*x + 10)/(x^3 + 3*x^2 + 3*x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\frac{2x}{x^2+1} + 1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1+2*x/(x**2+1))**(3/2),x)
```

```
[Out] Integral((2*x/(x**2 + 1) + 1)**(-3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\frac{2\,x}{x^2+1} + 1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x/(x^2 + 1) + 1)^(-3/2),x, algorithm="giac")
```

```
[Out] integrate((2*x/(x^2 + 1) + 1)^(-3/2), x)
```

3.746 $\int \frac{\sqrt{1+\frac{2x}{1+x^2}}}{1+x^2}\,dx$

Optimal. Leaf size=28

$$-\frac{(1-x)\sqrt{\frac{2x}{x^2+1}+1}}{x+1}$$

```
[Out] -(((1 - x)*Sqrt[1 + (2*x)/(1 + x^2)])/(1 + x))
```

Rubi [A] time = 0.211871, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$-\frac{(1-x)\sqrt{\frac{2x}{x^2+1}+1}}{x+1}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 + (2*x)/(1 + x^2)]/(1 + x^2),x]
```

```
[Out] -(((1 - x)*Sqrt[1 + (2*x)/(1 + x^2)])/(1 + x))
```

Rubi in Sympy [A] time = 9.25262, size = 24, normalized size = 0.86

$$-\frac{(-2x+2)\sqrt{\frac{2x}{x^2+1}+1}}{2(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+2*x/(x**2+1))**(1/2)/(x**2+1),x)
```

```
[Out] -(-2*x + 2)*sqrt(2*x/(x**2 + 1) + 1)/(2*(x + 1))
```

Mathematica [A] time = 0.0213153, size = 25, normalized size = 0.89

$$\frac{(x-1)\sqrt{\frac{2x}{x^2+1}+1}}{x+1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 + (2*x)/(1 + x^2)]/(1 + x^2),x]
```

```
[Out] ((-1 + x)*Sqrt[1 + (2*x)/(1 + x^2)])/(1 + x)
```

Maple [A] time = 0.006, size = 28, normalized size = 1.

$$\frac{-1+x}{1+x}\sqrt{\frac{x^2+2\,x+1}{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+2*x/(x^2+1))^(1/2)/(x^2+1),x)
```

```
[Out] (-1+x)/(1+x)*((x^2+2*x+1)/(x^2+1))^(1/2)
```

Maxima [A] time = 0.78526, size = 26, normalized size = 0.93

$$\frac{x}{\sqrt{x^2+1}}-\frac{1}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(2*x/(x^2 + 1) + 1)/(x^2 + 1),x, algorithm="maxima")
```

```
[Out] x/sqrt(x^2 + 1) - 1/sqrt(x^2 + 1)
```

Fricas [A] time = 0.263292, size = 42, normalized size = 1.5

$$\frac{(x-1)\sqrt{\frac{x^2+2\,x+1}{x^2+1}}+x+1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(2*x/(x^2 + 1) + 1)/(x^2 + 1),x, algorithm="fricas")
```

```
[Out] ((x - 1)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) + x + 1)/(x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{(x+1)^2}{x^2+1}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+2*x/(x**2+1))**(1/2)/(x**2+1),x)
```

```
[Out] Integral(sqrt((x + 1)**2/(x**2 + 1))/(x**2 + 1), x)
```

GIAC/XCAS [A] time = 0.266658, size = 41, normalized size = 1.46

$$\sqrt{2}\operatorname{sign}(x+1) + \frac{x\operatorname{sign}(x+1) - \operatorname{sign}(x+1)}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(2*x/(x^2 + 1) + 1)/(x^2 + 1),x, algorithm="giac")
```

```
[Out] sqrt(2)*sign(x + 1) + (x*sign(x + 1) - sign(x + 1))/sqrt(x^2 + 1)
```

3.747 $\int x^2 \left(\frac{c}{a+bx^2}\right)^{3/2} dx$

Optimal. Leaf size=75

$$\frac{c\sqrt{a+bx^2}\sqrt{\frac{c}{a+bx^2}}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}-\frac{cx\sqrt{\frac{c}{a+bx^2}}}{b}$$

```
[Out] -((c*x*Sqrt[c/(a + b*x^2)])/b) + (c*Sqrt[c/(a + b*x^2)]*Sqrt[a +
b*x^2]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/b^(3/2)
```

Rubi [A] time = 0.237621, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$\frac{c\sqrt{a+bx^2}\sqrt{\frac{c}{a+bx^2}}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}-\frac{cx\sqrt{\frac{c}{a+bx^2}}}{b}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(c/(a + b*x^2))^(3/2),x]
```

```
[Out] -((c*x*Sqrt[c/(a + b*x^2)])/b) + (c*Sqrt[c/(a + b*x^2)]*Sqrt[a +
b*x^2]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/b^(3/2)
```

Rubi in Sympy [A] time = 7.50974, size = 63, normalized size = 0.84

$$-\frac{cx\sqrt{\frac{c}{a+bx^2}}}{b}+\frac{c\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(c/(b*x**2+a))**(3/2),x)
```

```
[Out] -c*x*sqrt(c/(a + b*x**2))/b + c*sqrt(c/(a + b*x**2))*sqrt(a + b*x
**2)*atanh(sqrt(b)*x/sqrt(a + b*x**2))/b**(3/2)
```

Mathematica [A] time = 0.0501516, size = 66, normalized size = 0.88

$$-\frac{c\sqrt{\frac{c}{a+bx^2}}\left(\sqrt{b}x-\sqrt{a+bx^2}\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(c/(a + b*x^2))^(3/2),x]
```

```
[Out] -((c*Sqrt[c/(a + b*x^2)]*(Sqrt[b]*x - Sqrt[a + b*x^2]*Log[b*x + S
qrt[b]*Sqrt[a + b*x^2]]))/b^(3/2))
```

Maple [A] time = 0.017, size = 60, normalized size = 0.8

$$-(bx^2+a)\left(\frac{c}{bx^2+a}\right)^{\frac{3}{2}}\left(xb^{\frac{3}{2}}-\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b\sqrt{bx^2+a}\right)b^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(c/(b*x^2+a))^(3/2),x)
```

```
[Out] -(c/(b*x^2+a))^(3/2)*(b*x^2+a)*(x*b^(3/2)-ln(x*b^(1/2)+(b*x^2+a)^
(1/2))*b*(b*x^2+a)^(1/2))/b^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*(c/(b*x^2 + a))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.285441, size = 1, normalized size = 0.01

$$\left[-\frac{2\,cx\sqrt{\frac{c}{bx^2+a}}-c\sqrt{\frac{c}{b}}\log\left(-2\,bcx^2-ac-2\left(b^2x^3+abx\right)\sqrt{\frac{c}{bx^2+a}}\sqrt{\frac{c}{b}}\right)}{2\,b},\ -\frac{cx\sqrt{\frac{c}{bx^2+a}}-c\sqrt{-\frac{c}{b}}\arctan\left(\frac{cx}{(bx^2+a)\sqrt{\frac{c}{bx^2+a}}\sqrt{-\frac{c}{b}}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*(c/(b*x^2 + a))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*c*x*sqrt(c/(b*x^2 + a)) - c*sqrt(c/b)*log(-2*b*c*x^2 - a
*c - 2*(b^2*x^3 + a*b*x)*sqrt(c/(b*x^2 + a))*sqrt(c/b)))/b, -(c*x
*sqrt(c/(b*x^2 + a)) - c*sqrt(-c/b)*arctan(c*x/((b*x^2 + a)*sqrt(
c/(b*x^2 + a))*sqrt(-c/b))))/b]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\left(\frac{c}{a+bx^2}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(c/(b*x**2+a))**(3/2),x)
```

```
[Out] Integral(x**2*(c/(a + b*x**2))**(3/2), x)
```

GIAC/XCAS [A] time = 0.28659, size = 96, normalized size = 1.28

$$-\left(\frac{cx\mathrm{sign}\left(bx^2+a\right)}{\sqrt{bcx^2+ac}b}+\frac{c\ln\left(\left|-\sqrt{bc}x+\sqrt{bcx^2+ac}\right|\right)\mathrm{sign}\left(bx^2+a\right)}{\sqrt{bc}b}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*(c/(b*x^2 + a))^(3/2),x, algorithm="giac")
```

```
[Out] -(c*x*sign(b*x^2 + a)/(sqrt(b*c*x^2 + a*c)*b) + c*ln(abs(-sqrt(b*
c)*x + sqrt(b*c*x^2 + a*c)))*sign(b*x^2 + a)/(sqrt(b*c)*b))*c
```

3.748 $\int x\left(\frac{c}{a+bx^2}\right)^{3/2}\,dx$

Optimal. Leaf size=21

$$-\frac{c\sqrt{\frac{c}{a+bx^2}}}{b}$$

```
[Out] -((c*Sqrt[c/(a + b*x^2)])/b)
```

Rubi [A] time = 0.0214014, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$-\frac{c\sqrt{\frac{c}{a+bx^2}}}{b}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(c/(a + b*x^2))^(3/2),x]

[Out] -((c*Sqrt[c/(a + b*x^2)])/b)
```

Rubi in Sympy [A] time = 2.13074, size = 15, normalized size = 0.71

$$-\frac{c\sqrt{\frac{c}{a+bx^2}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(c/(b*x**2+a))**(3/2),x)

[Out] -c*sqrt(c/(a + b*x**2))/b
```

Mathematica [A] time = 0.0082162, size = 21, normalized size = 1.

$$-\frac{c\sqrt{\frac{c}{a+bx^2}}}{b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(c/(a + b*x^2))^(3/2),x]
```

```
[Out] -((c*Sqrt[c/(a + b*x^2)])/b)
```

Maple [A] time = 0.005, size = 26, normalized size = 1.2

$$-\frac{bx^2+a}{b}\left(\frac{c}{bx^2+a}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(c/(b*x^2+a))^(3/2),x)
```

```
[Out] -(b*x^2+a)/b*(c/(b*x^2+a))^(3/2)
```

Maxima [A] time = 0.686796, size = 26, normalized size = 1.24

$$-\frac{c\sqrt{\frac{c}{bx^2+a}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(c/(b*x^2 + a))^(3/2),x, algorithm="maxima")
```

```
[Out] -c*sqrt(c/(b*x^2 + a))/b
```

Fricas [A] time = 0.273092, size = 26, normalized size = 1.24

$$-\frac{c\sqrt{\frac{c}{bx^2+a}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(c/(b*x^2 + a))^(3/2),x, algorithm="fricas")
```

```
[Out] -c*sqrt(c/(b*x^2 + a))/b
```

Sympy [A] time = 4.62353, size = 53, normalized size = 2.52

$$\begin{cases} -\frac{ac^{\frac{3}{2}}\left(\frac{1}{a+bx^2}\right)^{\frac{3}{2}}}{b} - c^{\frac{3}{2}}x^2\left(\frac{1}{a+bx^2}\right)^{\frac{3}{2}} & \text{for } b \neq 0 \\ \frac{x^2\left(\frac{c}{a}\right)^{\frac{3}{2}}}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(c/(b*x**2+a))**(3/2),x)
```

```
[Out] Piecewise((-a*c**(3/2)*(1/(a + b*x**2))**(3/2)/b - c**(3/2)*x**2*
(1/(a + b*x**2))**(3/2), Ne(b, 0)), (x**2*(c/a)**(3/2)/2, True))
```

GIAC/XCAS [A] time = 0.263225, size = 38, normalized size = 1.81

$$-\frac{c^2 \mathrm{sign}\left(bx^2 + a\right)}{\sqrt{bcx^2 + ac}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(c/(b*x^2 + a))^(3/2),x, algorithm="giac")
```

```
[Out] -c^2*sign(b*x^2 + a)/(sqrt(b*c*x^2 + a*c)*b)
```

3.749 $\int \left(\frac{c}{a+bx^2}\right)^{3/2} dx$

Optimal. Leaf size=21

$$\frac{cx\sqrt{\frac{c}{a+bx^2}}}{a}$$

```
[Out] (c*x*Sqrt[c/(a + b*x^2)])/a
```

Rubi [A] time = 0.0312931, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$\frac{cx\sqrt{\frac{c}{a+bx^2}}}{a}$$

Antiderivative was successfully verified.

```
[In]  Int[(c/(a + b*x^2))^(3/2),x]

[Out] (c*x*Sqrt[c/(a + b*x^2)])/a
```

Rubi in Sympy [A] time = 1.95366, size = 15, normalized size = 0.71

$$\frac{cx\sqrt{\frac{c}{a+bx^2}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c/(b*x**2+a))**(3/2),x)

[Out] c*x*sqrt(c/(a + b*x**2))/a
```

Mathematica [A] time = 0.0130355, size = 21, normalized size = 1.

$$\frac{cx\sqrt{\frac{c}{a+bx^2}}}{a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c/(a + b*x^2))^(3/2),x]
```

```
[Out] (c*x*Sqrt[c/(a + b*x^2)])/a
```

Maple [A] time = 0.003, size = 26, normalized size = 1.2

$$\frac{x\left(bx^2+a\right)}{a}\left(\frac{c}{bx^2+a}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c/(b*x^2+a))^(3/2),x)
```

```
[Out] (b*x^2+a)/a*x*(c/(b*x^2+a))^(3/2)
```

Maxima [A] time = 0.700135, size = 23, normalized size = 1.1

$$\frac{c^{\frac{3}{2}}x}{\sqrt{bx^2+a}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c/(b*x^2 + a))^(3/2),x, algorithm="maxima")
```

```
[Out] c^(3/2)*x/(sqrt(b*x^2 + a)*a)
```

Fricas [A] time = 0.303143, size = 26, normalized size = 1.24

$$\frac{cx\sqrt{\frac{c}{bx^2+a}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c/(b*x^2 + a))^(3/2),x, algorithm="fricas")
```

```
[Out] c*x*sqrt(c/(b*x^2 + a))/a
```

Sympy [A] time = 4.69849, size = 66, normalized size = 3.14

$$\begin{cases} c^{\frac{3}{2}}x\left(\frac{1}{a+bx^2}\right)^{\frac{3}{2}} + \frac{bc^{\frac{3}{2}}x^3\left(\frac{1}{a+bx^2}\right)^{\frac{3}{2}}}{a} & \text{for } a \neq 0 \\ -\frac{c^{\frac{3}{2}}x\left(\frac{1}{b}\right)^{\frac{3}{2}}\left(\frac{1}{x^2}\right)^{\frac{3}{2}}}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c/(b*x**2+a))**(3/2),x)
```

```
[Out] Piecewise((c**(3/2)*x*(1/(a + b*x**2))**(3/2) + b*c**(3/2)*x**3*(
1/(a + b*x**2))**(3/2)/a, Ne(a, 0)), (-c**(3/2)*x*(1/b)**(3/2)*(x
**(-2))**(3/2)/2, True))
```

GIAC/XCAS [A] time = 0.272286, size = 38, normalized size = 1.81

$$\frac{c^2 x \operatorname{sign}\left(bx^2 + a\right)}{\sqrt{bcx^2 + ac}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c/(b*x^2 + a))^(3/2),x, algorithm="giac")
```

```
[Out] c^2*x*sign(b*x^2 + a)/(sqrt(b*c*x^2 + a*c)*a)
```

3.750 $\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x}\,dx$

Optimal. Leaf size=73

$$\frac{c\sqrt{\frac{c}{a+bx^2}}}{a}-\frac{c\sqrt{a+bx^2}\sqrt{\frac{c}{a+bx^2}}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

```
[Out] (c*Sqrt[c/(a + b*x^2)])/a - (c*Sqrt[c/(a + b*x^2)]*Sqrt[a + b*x^2
]*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^(3/2)
```

Rubi [A] time = 0.238015, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263

$$\frac{c\sqrt{\frac{c}{a+bx^2}}}{a}-\frac{c\sqrt{a+bx^2}\sqrt{\frac{c}{a+bx^2}}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c/(a + b*x^2))^(3/2)/x,x]
```

```
[Out] (c*Sqrt[c/(a + b*x^2)])/a - (c*Sqrt[c/(a + b*x^2)]*Sqrt[a + b*x^2
]*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^(3/2)
```

Rubi in Sympy [A] time = 8.25011, size = 60, normalized size = 0.82

$$\frac{c\sqrt{\frac{c}{a+bx^2}}}{a}-\frac{c\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\,\operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c/(b*x**2+a))**(3/2)/x,x)
```

```
[Out] c*sqrt(c/(a + b*x**2))/a - c*sqrt(c/(a + b*x**2))*sqrt(a + b*x**2
)*atanh(sqrt(a + b*x**2)/sqrt(a))/a**(3/2)
```

Mathematica [A] time = 0.0667933, size = 75, normalized size = 1.03

$$\frac{c\sqrt{\frac{c}{a+bx^2}}\left(\log(x)\sqrt{a+bx^2}-\sqrt{a+bx^2}\log\left(\sqrt{a}\sqrt{a+bx^2}+a\right)+\sqrt{a}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c/(a + b*x^2))^(3/2)/x,x]
```

```
[Out] (c*Sqrt[c/(a + b*x^2)]*(Sqrt[a] + Sqrt[a + b*x^2]*Log[x] - Sqrt[a
 + b*x^2]*Log[a + Sqrt[a]*Sqrt[a + b*x^2]]))/a^(3/2)
```

Maple [A] time = 0.01, size = 64, normalized size = 0.9

$$-(bx^2+a)\left(\frac{c}{bx^2+a}\right)^{\frac{3}{2}}\left(\ln\left(2\,\frac{\sqrt{a}\sqrt{bx^2+a}+a}{x}\right)a\sqrt{bx^2+a}-a^{\frac{3}{2}}\right)a^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c/(b*x^2+a))^(3/2)/x,x)
```

```
[Out] -(c/(b*x^2+a))^(3/2)*(b*x^2+a)*(ln(2*(a^(1/2)*(b*x^2+a)^(1/2)+a)/
x)*a*(b*x^2+a)^(1/2)-a^(3/2))/a^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c/(b*x^2 + a))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.287349, size = 1, normalized size = 0.01

$$\left[\frac{c\sqrt{\frac{c}{a}}\log\left(-\frac{bcx^2+2\,ac-2\,(abx^2+a^2)\sqrt{\frac{c}{bx^2+a}}\sqrt{\frac{c}{a}}}{x^2}\right)+2\,c\sqrt{\frac{c}{bx^2+a}}}{2\,a}, -\frac{c\sqrt{-\frac{c}{a}}\arctan\left(\frac{c}{(bx^2+a)\sqrt{\frac{c}{bx^2+a}}\sqrt{-\frac{c}{a}}}\right)-c\sqrt{\frac{c}{bx^2+a}}}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c/(b*x^2 + a))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/2*(c*sqrt(c/a)*log(-(b*c*x^2 + 2*a*c - 2*(a*b*x^2 + a^2)*sqrt(
c/(b*x^2 + a))*sqrt(c/a))/x^2) + 2*c*sqrt(c/(b*x^2 + a)))/a, -(c*
sqrt(-c/a)*arctan(c/((b*x^2 + a)*sqrt(c/(b*x^2 + a))*sqrt(-c/a)))
 - c*sqrt(c/(b*x^2 + a)))/a]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{c}{a+bx^2}\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c/(b*x**2+a))**(3/2)/x,x)
```

```
[Out] Integral((c/(a + b*x**2))**(3/2)/x, x)
```

GIAC/XCAS [A] time = 0.272139, size = 88, normalized size = 1.21

$$c^3\left(\frac{\arctan\left(\frac{\sqrt{bcx^2+ac}}{\sqrt{-ac}}\right)}{\sqrt{-ac}ac}+\frac{1}{\sqrt{bcx^2+ac}ac}\right)\mathrm{sign}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c/(b*x^2 + a))^(3/2)/x,x, algorithm="giac")
```

```
[Out] c^3*(arctan(sqrt(b*c*x^2 + a*c)/sqrt(-a*c))/(sqrt(-a*c)*a*c) + 1/
(sqrt(b*c*x^2 + a*c)*a*c))*sign(b*x^2 + a)
```

3.751 $\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x^2} dx$

Optimal. Leaf size=48

$$-\frac{2bcx\sqrt{\frac{c}{a+bx^2}}}{a^2}-\frac{c\sqrt{\frac{c}{a+bx^2}}}{ax}$$

```
[Out] -((c*Sqrt[c/(a + b*x^2)])/(a*x)) - (2*b*c*x*Sqrt[c/(a + b*x^2)])/
a^2
```

Rubi [A] time = 0.192181, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$-\frac{2bcx\sqrt{\frac{c}{a+bx^2}}}{a^2}-\frac{c\sqrt{\frac{c}{a+bx^2}}}{ax}$$

Antiderivative was successfully verified.

```
[In]  Int[(c/(a + b*x^2))^(3/2)/x^2,x]
```

```
[Out] -((c*Sqrt[c/(a + b*x^2)])/(a*x)) - (2*b*c*x*Sqrt[c/(a + b*x^2)])/
a^2
```

Rubi in Sympy [A] time = 6.2987, size = 39, normalized size = 0.81

$$-\frac{c\sqrt{\frac{c}{a+bx^2}}}{ax}-\frac{2bcx\sqrt{\frac{c}{a+bx^2}}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c/(b*x**2+a))**(3/2)/x**2,x)
```

```
[Out] -c*sqrt(c/(a + b*x**2))/(a*x) - 2*b*c*x*sqrt(c/(a + b*x**2))/a**2
```

Mathematica [A] time = 0.0244448, size = 32, normalized size = 0.67

$$-\frac{c\left(a+2bx^2\right)\sqrt{\frac{c}{a+bx^2}}}{a^2x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c/(a + b*x^2))^(3/2)/x^2,x]
```

```
[Out] -((c*Sqrt[c/(a + b*x^2)]*(a + 2*b*x^2))/(a^2*x))
```

Maple [A]   time = 0.007, size = 37, normalized size = 0.8

$$-\frac{(bx^2+a)\,(2\,bx^2+a)}{a^2x}\left(\frac{c}{bx^2+a}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c/(b*x^2+a))^(3/2)/x^2,x)
```

```
[Out] -(b*x^2+a)*(2*b*x^2+a)*(c/(b*x^2+a))^(3/2)/a^2/x
```

Maxima [A]   time = 0.685968, size = 62, normalized size = 1.29

$$-\frac{2\,b^2c^{\frac{3}{2}}x^4+3\,abc^{\frac{3}{2}}x^2+a^2c^{\frac{3}{2}}}{(bx^2+a)^{\frac{3}{2}}a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c/(b*x^2 + a))^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] -(2*b^2*c^(3/2)*x^4 + 3*a*b*c^(3/2)*x^2 + a^2*c^(3/2))/((b*x^2 +
a)^(3/2)*a^2*x)
```

Fricas [A]   time = 0.278843, size = 43, normalized size = 0.9

$$-\frac{\left(2\,bcx^2+ac\right)\sqrt{\frac{c}{bx^2+a}}}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c/(b*x^2 + a))^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] -(2*b*c*x^2 + a*c)*sqrt(c/(b*x^2 + a))/(a^2*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{c}{a+bx^2}\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c/(b*x**2+a))**(3/2)/x**2,x)
```

```
[Out] Integral((c/(a + b*x**2))**(3/2)/x**2, x)
```

GIAC/XCAS [A] time = 0.296351, size = 109, normalized size = 2.27

$$-\left(\frac{bcx\mathrm{sign}\left(bx^2+a\right)}{\sqrt{bcx^2+ac}a^2} - \frac{2\sqrt{bc}c\mathrm{sign}\left(bx^2+a\right)}{\left(\left(\sqrt{bc}x-\sqrt{bcx^2+ac}\right)^2-ac\right)a}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c/(b*x^2 + a))^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] -(b*c*x*sign(b*x^2 + a)/(sqrt(b*c*x^2 + a*c)*a^2) - 2*sqrt(b*c)*c
*sign(b*x^2 + a)/(((sqrt(b*c)*x - sqrt(b*c*x^2 + a*c))^2 - a*c)*a
))*c
```

3.752 $\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=112

$$\frac{3bc\sqrt{a+bx^2}\sqrt{\frac{c}{a+bx^2}}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{3c\left(a+bx^2\right)\sqrt{\frac{c}{a+bx^2}}}{2a^2x^2}+\frac{c\sqrt{\frac{c}{a+bx^2}}}{ax^2}$$

```
[Out] (c*Sqrt[c/(a + b*x^2)])/(a*x^2) - (3*c*Sqrt[c/(a + b*x^2)]*(a + b
*x^2))/(2*a^2*x^2) + (3*b*c*Sqrt[c/(a + b*x^2)]*Sqrt[a + b*x^2]*A
rcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(5/2))
```

Rubi [A] time = 0.275438, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$

$$\frac{3bc\sqrt{a+bx^2}\sqrt{\frac{c}{a+bx^2}}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{3c\left(a+bx^2\right)\sqrt{\frac{c}{a+bx^2}}}{2a^2x^2}+\frac{c\sqrt{\frac{c}{a+bx^2}}}{ax^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c/(a + b*x^2))^(3/2)/x^3,x]
```

```
[Out] (c*Sqrt[c/(a + b*x^2)])/(a*x^2) - (3*c*Sqrt[c/(a + b*x^2)]*(a + b
*x^2))/(2*a^2*x^2) + (3*b*c*Sqrt[c/(a + b*x^2)]*Sqrt[a + b*x^2]*A
rcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(5/2))
```

Rubi in Sympy [A] time = 10.4834, size = 99, normalized size = 0.88

$$\frac{c\sqrt{\frac{c}{a+bx^2}}}{ax^2}-\frac{3c\sqrt{\frac{c}{a+bx^2}}\left(a+bx^2\right)}{2a^2x^2}+\frac{3bc\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\,\operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c/(b*x**2+a))**(3/2)/x**3,x)
```

```
[Out] c*sqrt(c/(a + b*x**2))/(a*x**2) - 3*c*sqrt(c/(a + b*x**2))*(a + b
*x**2)/(2*a**2*x**2) + 3*b*c*sqrt(c/(a + b*x**2))*sqrt(a + b*x**2
)*atanh(sqrt(a + b*x**2)/sqrt(a))/(2*a**(5/2))
```

Mathematica [A] time = 0.0890055, size = 99, normalized size = 0.88

$$-\frac{c\sqrt{\frac{c}{a+bx^2}}\left(\sqrt{a}\left(a+3bx^2\right)+3bx^2\log(x)\sqrt{a+bx^2}-3bx^2\sqrt{a+bx^2}\log\left(\sqrt{a}\sqrt{a+bx^2}+a\right)\right)}{2a^{5/2}x^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c/(a + b*x^2))^(3/2)/x^3,x]
```

```
[Out] -(c*Sqrt[c/(a + b*x^2)]*(Sqrt[a]*(a + 3*b*x^2) + 3*b*x^2*Sqrt[a +
 b*x^2]*Log[x] - 3*b*x^2*Sqrt[a + b*x^2]*Log[a + Sqrt[a]*Sqrt[a +
 b*x^2]]))/(2*a^(5/2)*x^2)
```

Maple [A] time = 0.01, size = 79, normalized size = 0.7

$$-\frac{bx^2+a}{2\,x^2}\left(\frac{c}{bx^2+a}\right)^{\frac{3}{2}}\left(3\,a^{3/2}x^2b-3\,\ln\left(2\,\frac{\sqrt{a}\sqrt{bx^2+a}+a}{x}\right)\sqrt{bx^2+a}x^2ab+a^{\frac{5}{2}}\right)a^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c/(b*x^2+a))^(3/2)/x^3,x)
```

```
[Out] -1/2*(c/(b*x^2+a))^(3/2)*(b*x^2+a)*(3*a^(3/2)*x^2*b-3*ln(2*(a^(1/
2)*(b*x^2+a)^(1/2)+a)/x)*(b*x^2+a)^(1/2)*x^2*a*b+a^(5/2))/a^(7/2)
/x^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c/(b*x^2 + a))^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.296741, size = 1, normalized size = 0.01

$$\left[\frac{3\,bcx^2\sqrt{\frac{c}{a}}\log\left(-\frac{bcx^2+2\,ac+2\,(abx^2+a^2)\sqrt{\frac{c}{bx^2+a}}\sqrt{\frac{c}{a}}}{x^2}\right)-2\,\left(3\,bcx^2+ac\right)\sqrt{\frac{c}{bx^2+a}}}{4\,a^2x^2},\ \frac{3\,bcx^2\sqrt{-\frac{c}{a}}\arctan\left(\frac{c}{(bx^2+a)\sqrt{\frac{c}{bx^2+a}}\sqrt{-\frac{c}{a}}}\right)-(3\,bc}{2\,a^2x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c/(b*x^2 + a))^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/4*(3*b*c*x^2*sqrt(c/a)*log(-(b*c*x^2 + 2*a*c + 2*(a*b*x^2 + a^
2)*sqrt(c/(b*x^2 + a))*sqrt(c/a))/x^2) - 2*(3*b*c*x^2 + a*c)*sqrt
(c/(b*x^2 + a)))/(a^2*x^2), 1/2*(3*b*c*x^2*sqrt(-c/a)*arctan(c/((
b*x^2 + a)*sqrt(c/(b*x^2 + a))*sqrt(-c/a))) - (3*b*c*x^2 + a*c)*s
qrt(c/(b*x^2 + a)))/(a^2*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(\frac{c}{a+bx^2}\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c/(b*x**2+a))**(3/2)/x**3,x)
```

```
[Out] Integral((c/(a + b*x**2))**(3/2)/x**3, x)
```

GIAC/XCAS [A] time = 0.272278, size = 135, normalized size = 1.21

$$-\frac{1}{2}\,bc^4\left(\frac{3\,\arctan\left(\frac{\sqrt{bcx^2+ac}}{\sqrt{-ac}}\right)}{\sqrt{-ac}a^2c^2}-\frac{3\,bcx^2+ac}{\left(\sqrt{bcx^2+ac}ac-(bcx^2+ac)^{\frac{3}{2}}\right)a^2c^2}\right)\operatorname{sign}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c/(b*x^2 + a))^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] -1/2*b*c^4*(3*arctan(sqrt(b*c*x^2 + a*c)/sqrt(-a*c))/(sqrt(-a*c)*
a^2*c^2) - (3*b*c*x^2 + a*c)/((sqrt(b*c*x^2 + a*c)*a*c - (b*c*x^2
```

```
+ a*c)^(3/2))*a^2*c^2))*sign(b*x^2 + a)
```

3.753 $\int x^2 \left(c \left(a+bx^2\right)^3\right)^{3/2} dx$

Optimal. Leaf size=254

$$-\frac{21a^6c\sqrt{c\left(a+bx^2\right)^3}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{1024b^{3/2}\left(a+bx^2\right)^{3/2}}+\frac{21a^5cx\sqrt{c\left(a+bx^2\right)^3}}{1024b\left(a+bx^2\right)}+\frac{21a^4cx^3\sqrt{c\left(a+bx^2\right)^3}}{512\left(a+bx^2\right)}+\frac{7}{128}a^3cx^3\sqrt{c\left(a+bx^2\right)^3}+\frac{21}{320}a^2cx^3\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3}+\frac{3}{40}acx^3\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3}+\frac{1}{12}cx^3\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3}$$

```
[Out] (7*a^3*c*x^3*Sqrt[c*(a + b*x^2)^3])/128 + (21*a^5*c*x*Sqrt[c*(a +
 b*x^2)^3])/(1024*b*(a + b*x^2)) + (21*a^4*c*x^3*Sqrt[c*(a + b*x^
2)^3])/(512*(a + b*x^2)) + (21*a^2*c*x^3*(a + b*x^2)*Sqrt[c*(a +
b*x^2)^3])/320 + (3*a*c*x^3*(a + b*x^2)^2*Sqrt[c*(a + b*x^2)^3])/
40 + (c*x^3*(a + b*x^2)^3*Sqrt[c*(a + b*x^2)^3])/12 - (21*a^6*c*S
qrt[c*(a + b*x^2)^3]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(1024*
b^(3/2)*(a + b*x^2)^(3/2))
```

Rubi [A] time = 0.471998, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263

$$-\frac{21a^6c\sqrt{c\left(a+bx^2\right)^3}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{1024b^{3/2}\left(a+bx^2\right)^{3/2}}+\frac{21a^5cx\sqrt{c\left(a+bx^2\right)^3}}{1024b\left(a+bx^2\right)}+\frac{21a^4cx^3\sqrt{c\left(a+bx^2\right)^3}}{512\left(a+bx^2\right)}+\frac{7}{128}a^3cx^3\sqrt{c\left(a+bx^2\right)^3}+\frac{21}{320}a^2cx^3\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3}+\frac{3}{40}acx^3\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3}+\frac{1}{12}cx^3\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(c*(a + b*x^2)^3)^(3/2),x]
```


```
[Out] (7*a^3*c*x^3*Sqrt[c*(a + b*x^2)^3])/128 + (21*a^5*c*x*Sqrt[c*(a +
 b*x^2)^3])/(1024*b*(a + b*x^2)) + (21*a^4*c*x^3*Sqrt[c*(a + b*x^
2)^3])/(512*(a + b*x^2)) + (21*a^2*c*x^3*(a + b*x^2)*Sqrt[c*(a +
b*x^2)^3])/320 + (3*a*c*x^3*(a + b*x^2)^2*Sqrt[c*(a + b*x^2)^3])/
40 + (c*x^3*(a + b*x^2)^3*Sqrt[c*(a + b*x^2)^3])/12 - (21*a^6*c*S
qrt[c*(a + b*x^2)^3]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(1024*
b^(3/2)*(a + b*x^2)^(3/2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \left(c \left(a+bx^2\right)^3\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(c*(b*x**2+a)**3)**(3/2),x)
```

```
[Out] Integral(x**2*(c*(a + b*x**2)**3)**(3/2), x)
```

Mathematica [A] time = 0.202246, size = 135, normalized size = 0.53

$$\frac{\left(c \left(a+bx^2\right)^3\right)^{3/2} \left(\sqrt{b}x\sqrt{a+bx^2} \left(315a^5+4910a^4bx^2+11432a^3b^2x^4+12144a^2b^3x^6+6272ab^4x^8+1280b^5x^{10}\right)-315a^6 \log\right.}{15360b^{3/2} \left(a+bx^2\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(c*(a + b*x^2)^3)^(3/2),x]
```

```
[Out] ((c*(a + b*x^2)^3)^(3/2)*(Sqrt[b]*x*Sqrt[a + b*x^2]*(315*a^5 + 49
10*a^4*b*x^2 + 11432*a^3*b^2*x^4 + 12144*a^2*b^3*x^6 + 6272*a*b^4
*x^8 + 1280*b^5*x^10) - 315*a^6*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]
]))/(15360*b^(3/2)*(a + b*x^2)^(9/2))
```

Maple [A] time = 0.048, size = 236, normalized size = 0.9

$$\frac{1}{15360 \left(bx^2+a\right)^3 bc} \left(c \left(bx^2+a\right)^3\right)^{\frac{3}{2}} \left(1280\, b^3x^7 \left(bcx^2+ac\right)^{5/2} \sqrt{bc}+3712\, b^2ax^5 \left(bcx^2+ac\right)^{5/2} \sqrt{bc}+3440\, a^2x^3 \left(bcx^2+ac\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(c*(b*x^2+a)^3)^(3/2),x)
```

```
[Out] 1/15360*(c*(b*x^2+a)^3)^(3/2)*(1280*b^3*x^7*(b*c*x^2+a*c)^(5/2)*(
b*c)^(1/2)+3712*b^2*a*x^5*(b*c*x^2+a*c)^(5/2)*(b*c)^(1/2)+3440*a^
2*x^3*(b*c*x^2+a*c)^(5/2)*b*(b*c)^(1/2)+840*a^3*x*(b*c*x^2+a*c)^(
5/2)*(b*c)^(1/2)-210*a^4*x*(b*c*x^2+a*c)^(3/2)*c*(b*c)^(1/2)-315*
a^5*c^2*x*(b*c*x^2+a*c)^(1/2)*(b*c)^(1/2)-315*a^6*c^3*ln((b*c*x+(
b*c*x^2+a*c)^(1/2)*(b*c)^(1/2))/(b*c)^(1/2)))/(b*x^2+a)^3/(c*(b*x
```

```
^2+a))^(3/2)/b/c/(b*c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((b*x^2 + a)^3*c)^(3/2)*x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.428264, size = 1, normalized size = 0.

$$\left[\frac{315\left(a^6bcx^2+a^7c\right)\sqrt{\frac{c}{b}}\log\left(-\frac{2\,b^2cx^4+3\,abcx^2+a^2c-2\,\sqrt{b^3cx^6+3\,ab^2cx^4+3\,a^2bcx^2+a^3c}\,bx\sqrt{\frac{c}{b}}}{bx^2+a}\right)+2\left(1280\,b^5cx^{11}+6272\,ab^4cx^9+12144\cdots\right.}{30720\left(b^2x^2+ab\right)}\right.$$

$$-\frac{315\left(a^6bcx^2+a^7c\right)\sqrt{-\frac{c}{b}}\arctan\left(\frac{bcx^3+acx}{\sqrt{b^3cx^6+3\,ab^2cx^4+3\,a^2bcx^2+a^3c}\sqrt{-\frac{c}{b}}}\right)-\left(1280\,b^5cx^{11}+6272\,ab^4cx^9+12144\,a^2b^3cx^7+11432\cdots\right.}{15360\left(b^2x^2+ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((b*x^2 + a)^3*c)^(3/2)*x^2,x, algorithm="fricas")
```

```
[Out] [1/30720*(315*(a^6*b*c*x^2 + a^7*c)*sqrt(c/b)*log(-(2*b^2*c*x^4 +
 3*a*b*c*x^2 + a^2*c - 2*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b
*c*x^2 + a^3*c)*b*x*sqrt(c/b))/(b*x^2 + a)) + 2*(1280*b^5*c*x^11
+ 6272*a*b^4*c*x^9 + 12144*a^2*b^3*c*x^7 + 11432*a^3*b^2*c*x^5 +
4910*a^4*b*c*x^3 + 315*a^5*c*x)*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 +
3*a^2*b*c*x^2 + a^3*c))/(b^2*x^2 + a*b), -1/15360*(315*(a^6*b*c*x
^2 + a^7*c)*sqrt(-c/b)*arctan((b*c*x^3 + a*c*x)/(sqrt(b^3*c*x^6 +
 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c)*sqrt(-c/b))) - (1280*b^5*
c*x^11 + 6272*a*b^4*c*x^9 + 12144*a^2*b^3*c*x^7 + 11432*a^3*b^2*c
*x^5 + 4910*a^4*b*c*x^3 + 315*a^5*c*x)*sqrt(b^3*c*x^6 + 3*a*b^2*c
*x^4 + 3*a^2*b*c*x^2 + a^3*c))/(b^2*x^2 + a*b)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(c*(b*x**2+a)**3)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.280046, size = 239, normalized size = 0.94

$$\frac{1}{15360}\left(\frac{315\,a^6c\ln\left(\left|-\sqrt{bc}x+\sqrt{bcx^2+ac}\right|\right)\,\mathrm{sign}\left(bx^2+a\right)}{\sqrt{bc}b}+\left(\frac{315\,a^5\mathrm{sign}\left(bx^2+a\right)}{b}+2\left(2455\,a^4\mathrm{sign}\left(bx^2+a\right)+4\left(1429\,a^3b\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((b*x^2 + a)^3*c)^(3/2)*x^2,x, algorithm="giac")
```

```
[Out] 1/15360*(315*a^6*c*ln(abs(-sqrt(b*c)*x + sqrt(b*c*x^2 + a*c)))*si
gn(b*x^2 + a)/(sqrt(b*c)*b) + (315*a^5*sign(b*x^2 + a)/b + 2*(245
5*a^4*sign(b*x^2 + a) + 4*(1429*a^3*b*sign(b*x^2 + a) + 2*(759*a^
2*b^2*sign(b*x^2 + a) + 8*(10*b^4*x^2*sign(b*x^2 + a) + 49*a*b^3*
sign(b*x^2 + a))*x^2)*x^2)*x^2)*x^2)*sqrt(b*c*x^2 + a*c)*x)*c
```

3.754 $\int x \left(c \left(a+bx^2\right)^3\right)^{3/2} dx$

Optimal. Leaf size=32

$$\frac{c\left(a+bx^2\right)^4 \sqrt{c\left(a+bx^2\right)^3}}{11b}$$

```
[Out] (c*(a + b*x^2)^4*Sqrt[c*(a + b*x^2)^3])/(11*b)
```

Rubi [A] time = 0.0314822, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$

$$\frac{c\left(a+bx^2\right)^4 \sqrt{c\left(a+bx^2\right)^3}}{11b}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(c*(a + b*x^2)^3)^(3/2),x]

[Out] (c*(a + b*x^2)^4*Sqrt[c*(a + b*x^2)^3])/(11*b)
```

Rubi in Sympy [A] time = 2.68851, size = 26, normalized size = 0.81

$$\frac{c\sqrt{c\left(a+bx^2\right)^3}\left(a+bx^2\right)^4}{11b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(c*(b*x**2+a)**3)**(3/2),x)

[Out] c*sqrt(c*(a + b*x**2)**3)*(a + b*x**2)**4/(11*b)
```

Mathematica [A] time = 0.0272466, size = 29, normalized size = 0.91

$$\frac{\left(a+bx^2\right)\left(c\left(a+bx^2\right)^3\right)^{3/2}}{11b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(c*(a + b*x^2)^3)^(3/2),x]
```

```
[Out] ((a + b*x^2)*(c*(a + b*x^2)^3)^(3/2))/(11*b)
```

Maple [A] time = 0.005, size = 26, normalized size = 0.8

$$\frac{bx^2+a}{11\,b}\left(c\left(bx^2+a\right)^3\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(c*(b*x^2+a)^3)^(3/2),x)
```

```
[Out] 1/11*(b*x^2+a)/b*(c*(b*x^2+a)^3)^(3/2)
```

Maxima [A] time = 0.706534, size = 95, normalized size = 2.97

$$\frac{\left(b^4c^{\frac{3}{2}}x^8+4\,ab^3c^{\frac{3}{2}}x^6+6\,a^2b^2c^{\frac{3}{2}}x^4+4\,a^3bc^{\frac{3}{2}}x^2+a^4c^{\frac{3}{2}}\right)\left(bx^2+a\right)^{\frac{3}{2}}}{11\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((b*x^2 + a)^3*c)^(3/2)*x,x, algorithm="maxima")
```

```
[Out] 1/11*(b^4*c^(3/2)*x^8 + 4*a*b^3*c^(3/2)*x^6 + 6*a^2*b^2*c^(3/2)*x
^4 + 4*a^3*b*c^(3/2)*x^2 + a^4*c^(3/2))*(b*x^2 + a)^(3/2)/b
```

Fricas [A] time = 0.291354, size = 117, normalized size = 3.66

$$\frac{\left(b^4cx^8+4\,ab^3cx^6+6\,a^2b^2cx^4+4\,a^3bcx^2+a^4c\right)\sqrt{b^3cx^6+3\,ab^2cx^4+3\,a^2bcx^2+a^3c}}{11\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((b*x^2 + a)^3*c)^(3/2)*x,x, algorithm="fricas")
```

```
[Out] 1/11*(b^4*c*x^8 + 4*a*b^3*c*x^6 + 6*a^2*b^2*c*x^4 + 4*a^3*b*c*x^2
 + a^4*c)*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c)
/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(c*(b*x**2+a)**3)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.27018, size = 450, normalized size = 14.06

$$1155 \left(bcx^2+ac\right)^{\frac{3}{2}} a^4 \mathrm{sign}\left(bx^2+a\right) - \frac{924 \left(5 \left(bcx^2+ac\right)^{\frac{3}{2}} ac - 3 \left(bcx^2+ac\right)^{\frac{5}{2}}\right) a^3 \mathrm{sign}\left(bx^2+a\right)}{c} + \frac{198 \left(35 \left(bcx^2+ac\right)^{\frac{3}{2}} a^2c^2 - 42 \left(bcx^2+ac\right)^{\frac{5}{2}} ac + 15 (b\right.}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((b*x^2 + a)^3*c)^(3/2)*x,x, algorithm="giac")
```

```
[Out] 1/3465*(1155*(b*c*x^2 + a*c)^(3/2)*a^4*sign(b*x^2 + a) - 924*(5*(
b*c*x^2 + a*c)^(3/2)*a*c - 3*(b*c*x^2 + a*c)^(5/2))*a^3*sign(b*x^
2 + a)/c + 198*(35*(b*c*x^2 + a*c)^(3/2)*a^2*c^2 - 42*(b*c*x^2 +
a*c)^(5/2)*a*c + 15*(b*c*x^2 + a*c)^(7/2))*a^2*sign(b*x^2 + a)/c^
2 - 44*(105*(b*c*x^2 + a*c)^(3/2)*a^3*c^3 - 189*(b*c*x^2 + a*c)^(
5/2)*a^2*c^2 + 135*(b*c*x^2 + a*c)^(7/2)*a*c - 35*(b*c*x^2 + a*c)
^(9/2))*a*sign(b*x^2 + a)/c^3 + (1155*(b*c*x^2 + a*c)^(3/2)*a^4*c
^4 - 2772*(b*c*x^2 + a*c)^(5/2)*a^3*c^3 + 2970*(b*c*x^2 + a*c)^(7
/2)*a^2*c^2 - 1540*(b*c*x^2 + a*c)^(9/2)*a*c + 315*(b*c*x^2 + a*c
)^(11/2))*sign(b*x^2 + a)/c^4)/b
```

3.755 $\int \left(c\left(a+bx^2\right)^3\right)^{3/2}\,dx$

Optimal. Leaf size=208

$$\frac{63a^5c\sqrt{c\left(a+bx^2\right)^3}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256\sqrt{b}\left(a+bx^2\right)^{3/2}}+\frac{63a^4cx\sqrt{c\left(a+bx^2\right)^3}}{256\left(a+bx^2\right)}+\frac{21}{128}a^3cx\sqrt{c\left(a+bx^2\right)^3}+\frac{21}{160}a^2cx\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3}+\frac{9}{80}acx\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3}+\frac{1}{10}cx\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3}$$

```
[Out] (21*a^3*c*x*Sqrt[c*(a + b*x^2)^3])/128 + (63*a^4*c*x*Sqrt[c*(a +
b*x^2)^3])/(256*(a + b*x^2)) + (21*a^2*c*x*(a + b*x^2)*Sqrt[c*(a
+ b*x^2)^3])/160 + (9*a*c*x*(a + b*x^2)^2*Sqrt[c*(a + b*x^2)^3])/
80 + (c*x*(a + b*x^2)^3*Sqrt[c*(a + b*x^2)^3])/10 + (63*a^5*c*Sqr
t[c*(a + b*x^2)^3]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(256*Sqr
t[b]*(a + b*x^2)^(3/2))
```

Rubi [A] time = 0.156294, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\frac{63a^5c\sqrt{c\left(a+bx^2\right)^3}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256\sqrt{b}\left(a+bx^2\right)^{3/2}}+\frac{63a^4cx\sqrt{c\left(a+bx^2\right)^3}}{256\left(a+bx^2\right)}+\frac{21}{128}a^3cx\sqrt{c\left(a+bx^2\right)^3}+\frac{21}{160}a^2cx\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3}+\frac{9}{80}acx\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3}+\frac{1}{10}cx\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(c*(a + b*x^2)^3)^(3/2),x]
```

```
[Out] (21*a^3*c*x*Sqrt[c*(a + b*x^2)^3])/128 + (63*a^4*c*x*Sqrt[c*(a +
b*x^2)^3])/(256*(a + b*x^2)) + (21*a^2*c*x*(a + b*x^2)*Sqrt[c*(a
+ b*x^2)^3])/160 + (9*a*c*x*(a + b*x^2)^2*Sqrt[c*(a + b*x^2)^3])/
80 + (c*x*(a + b*x^2)^3*Sqrt[c*(a + b*x^2)^3])/10 + (63*a^5*c*Sqr
t[c*(a + b*x^2)^3]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(256*Sqr
t[b]*(a + b*x^2)^(3/2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c\left(a+bx^2\right)^3\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*(b*x**2+a)**3)**(3/2),x)
```

```
[Out] Integral((c*(a + b*x**2)**3)**(3/2), x)
```

Mathematica [A] time = 0.111739, size = 124, normalized size = 0.6

$$\frac{\left(c\left(a+bx^2\right)^3\right)^{3/2}\left(315a^5\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)+\sqrt{b}x\sqrt{a+bx^2}\left(965a^4+1490a^3bx^2+1368a^2b^2x^4+656ab^3x^6+128b^4x^8\right)\right.}{1280\sqrt{b}\left(a+bx^2\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c*(a + b*x^2)^3)^(3/2),x]
```

```
[Out] ((c*(a + b*x^2)^3)^(3/2)*(Sqrt[b]*x*Sqrt[a + b*x^2]*(965*a^4 + 14
90*a^3*b*x^2 + 1368*a^2*b^2*x^4 + 656*a*b^3*x^6 + 128*b^4*x^8) +
315*a^5*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]]))/(1280*Sqrt[b]*(a + b
*x^2)^(9/2))
```

Maple [A] time = 0.01, size = 205, normalized size = 1.

$$\frac{1}{1280\,c\left(bx^2+a\right)^3}\left(c\left(bx^2+a\right)^3\right)^{\frac{3}{2}}\left(128\,b^2x^5\left(bcx^2+ac\right)^{5/2}\sqrt{bc}+400\,bax^3\left(bcx^2+ac\right)^{5/2}\sqrt{bc}+315\,a^5c^3\ln\left(\frac{bcx+\sqrt{bcx^2}}{\sqrt{bc}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*(b*x^2+a)^3)^(3/2),x)
```

```
[Out] 1/1280*(c*(b*x^2+a)^3)^(3/2)*(128*b^2*x^5*(b*c*x^2+a*c)^(5/2)*(b*
c)^(1/2)+400*b*a*x^3*(b*c*x^2+a*c)^(5/2)*(b*c)^(1/2)+315*a^5*c^3*
ln((b*c*x+(b*c*x^2+a*c)^(1/2)*(b*c)^(1/2))/(b*c)^(1/2))+440*a^2*x
*(b*c*x^2+a*c)^(5/2)*(b*c)^(1/2)+210*a^3*x*(b*c*x^2+a*c)^(3/2)*(b
*c)^(1/2)*c+315*a^4*c^2*x*(b*c*x^2+a*c)^(1/2)*(b*c)^(1/2))/(b*x^2
+a)^3/(c*(b*x^2+a))^(3/2)/(b*c)^(1/2)/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((b*x^2 + a)^3*c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.358238, size = 1, normalized size = 0.

$$\left[\frac{315\left(a^{5}bcx^{2}+a^{6}c\right)\sqrt{\frac{c}{b}}\log\left(-\frac{2\,b^{2}cx^{4}+3\,abcx^{2}+a^{2}c+2\,\sqrt{b^{3}cx^{6}+3\,ab^{2}cx^{4}+3\,a^{2}bcx^{2}+a^{3}c}\,bx\sqrt{\frac{c}{b}}}{bx^{2}+a}\right)+2\left(128\,b^{4}cx^{9}+656\,ab^{3}cx^{7}+1368\,a^{2}b\cdots\right.}{2560\left(bx^{2}+a\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((b*x^2 + a)^3*c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2560*(315*(a^5*b*c*x^2 + a^6*c)*sqrt(c/b)*log(-(2*b^2*c*x^4 +
3*a*b*c*x^2 + a^2*c + 2*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*
c*x^2 + a^3*c)*b*x*sqrt(c/b))/(b*x^2 + a)) + 2*(128*b^4*c*x^9 + 6
56*a*b^3*c*x^7 + 1368*a^2*b^2*c*x^5 + 1490*a^3*b*c*x^3 + 965*a^4*
c*x)*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c))/(b*
x^2 + a), 1/1280*(315*(a^5*b*c*x^2 + a^6*c)*sqrt(-c/b)*arctan((b*
c*x^3 + a*c*x)/(sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 +
a^3*c)*sqrt(-c/b))) + (128*b^4*c*x^9 + 656*a*b^3*c*x^7 + 1368*a^2
*b^2*c*x^5 + 1490*a^3*b*c*x^3 + 965*a^4*c*x)*sqrt(b^3*c*x^6 + 3*a
*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c))/(b*x^2 + a)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*(b*x**2+a)**3)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.280902, size = 207, normalized size = 1.

$$-\frac{1}{1280}\left(\frac{315\,a^5c\ln\left(\left|-\sqrt{bc}x+\sqrt{bcx^2+ac}\right|\right)\operatorname{sign}\left(bx^2+a\right)}{\sqrt{bc}}-\left(965\,a^4\operatorname{sign}\left(bx^2+a\right)+2\left(745\,a^3b\operatorname{sign}\left(bx^2+a\right)+4\left(171\,a^2b^2\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((b*x^2 + a)^3*c)^(3/2),x, algorithm="giac")
```

```
[Out] -1/1280*(315*a^5*c*ln(abs(-sqrt(b*c)*x + sqrt(b*c*x^2 + a*c)))*si
gn(b*x^2 + a)/sqrt(b*c) - (965*a^4*sign(b*x^2 + a) + 2*(745*a^3*b
*sign(b*x^2 + a) + 4*(171*a^2*b^2*sign(b*x^2 + a) + 2*(8*b^4*x^2*
sign(b*x^2 + a) + 41*a*b^3*sign(b*x^2 + a))*x^2)*x^2)*x^2)*sqrt(b
*c*x^2 + a*c)*x)*c
```

3.756 $\int \frac{\left(c\left(a+bx^2\right)^3\right)^{3/2}}{x}\,dx$

Optimal. Leaf size=194

$$-\frac{a^{9/2}c\sqrt{c\left(a+bx^2\right)^3}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\left(a+bx^2\right)^{3/2}}+\frac{a^4c\sqrt{c\left(a+bx^2\right)^3}}{a+bx^2}+\frac{1}{3}a^3c\sqrt{c\left(a+bx^2\right)^3}+\frac{1}{5}a^2c\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3}+\frac{1}{7}ac\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3}+\frac{1}{9}c\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3}$$

```
[Out] (a^3*c*Sqrt[c*(a + b*x^2)^3])/3 + (a^4*c*Sqrt[c*(a + b*x^2)^3])/(
a + b*x^2) + (a^2*c*(a + b*x^2)*Sqrt[c*(a + b*x^2)^3])/5 + (a*c*(
a + b*x^2)^2*Sqrt[c*(a + b*x^2)^3])/7 + (c*(a + b*x^2)^3*Sqrt[c*(
a + b*x^2)^3])/9 - (a^(9/2)*c*Sqrt[c*(a + b*x^2)^3]*ArcTanh[Sqrt[
a + b*x^2]/Sqrt[a]])/(a + b*x^2)^(3/2)
```

Rubi [A] time = 0.404341, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263

$$-\frac{a^{9/2}c\sqrt{c\left(a+bx^2\right)^3}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\left(a+bx^2\right)^{3/2}}+\frac{a^4c\sqrt{c\left(a+bx^2\right)^3}}{a+bx^2}+\frac{1}{3}a^3c\sqrt{c\left(a+bx^2\right)^3}+\frac{1}{5}a^2c\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3}+\frac{1}{7}ac\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3}+\frac{1}{9}c\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(c*(a + b*x^2)^3)^(3/2)/x,x]
```


```
[Out] (a^3*c*Sqrt[c*(a + b*x^2)^3])/3 + (a^4*c*Sqrt[c*(a + b*x^2)^3])/(
a + b*x^2) + (a^2*c*(a + b*x^2)*Sqrt[c*(a + b*x^2)^3])/5 + (a*c*(
a + b*x^2)^2*Sqrt[c*(a + b*x^2)^3])/7 + (c*(a + b*x^2)^3*Sqrt[c*(
a + b*x^2)^3])/9 - (a^(9/2)*c*Sqrt[c*(a + b*x^2)^3]*ArcTanh[Sqrt[
a + b*x^2]/Sqrt[a]])/(a + b*x^2)^(3/2)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c\left(a+bx^2\right)^3\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*(b*x**2+a)**3)**(3/2)/x,x)
```

```
[Out] Integral((c*(a + b*x**2)**3)**(3/2)/x, x)
```

Mathematica [A] time = 0.156014, size = 122, normalized size = 0.63

$$\frac{\left(c\left(a+bx^2\right)^3\right)^{3/2}\left(-315a^{9/2}\log\left(\sqrt{a}\sqrt{a+bx^2}+a\right)+315a^{9/2}\log(x)+\sqrt{a+bx^2}\left(563a^4+506a^3bx^2+408a^2b^2x^4+185ab^3x^{6}\right.\right.}{315\left(a+bx^2\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c*(a + b*x^2)^3)^(3/2)/x,x]
```

```
[Out] ((c*(a + b*x^2)^3)^(3/2)*(Sqrt[a + b*x^2]*(563*a^4 + 506*a^3*b*x^
2 + 408*a^2*b^2*x^4 + 185*a*b^3*x^6 + 35*b^4*x^8) + 315*a^(9/2)*L
og[x] - 315*a^(9/2)*Log[a + Sqrt[a]*Sqrt[a + b*x^2]]))/(315*(a +
b*x^2)^(9/2))
```

Maple [A] time = 0.021, size = 221, normalized size = 1.1

$$-\frac{1}{315\,c\left(bx^2+a\right)^3}\left(c\left(bx^2+a\right)^3\right)^{\frac{3}{2}}\left(-35\sqrt{ac}\left(bcx^2+ac\right)^{5/2}x^4b^2-115\sqrt{ac}\left(bcx^2+ac\right)^{5/2}x^2ab+315\,a^5c^3\ln\left(2\,\frac{\sqrt{ac}\sqrt{bcx^2}}{x}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*(b*x^2+a)^3)^(3/2)/x,x)
```

```
[Out] -1/315*(c*(b*x^2+a)^3)^(3/2)*(-35*(a*c)^(1/2)*(b*c*x^2+a*c)^(5/2)
*x^4*b^2-115*(a*c)^(1/2)*(b*c*x^2+a*c)^(5/2)*x^2*a*b+315*a^5*c^3*
ln(2*((a*c)^(1/2)*(b*c*x^2+a*c)^(1/2)+a*c)/x)+46*(a*c)^(1/2)*(b*c
*x^2+a*c)^(5/2)*a^2-105*a^3*(b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*c-315
*a^4*c^2*(b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)-189*a^2*(c*(b*x^2+a))^(5
/2)*(a*c)^(1/2))/(b*x^2+a)^3/(c*(b*x^2+a))^(3/2)/(a*c)^(1/2)/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((b*x^2 + a)^3*c)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.320006, size = 1, normalized size = 0.01

$$\left[\frac{315\,\left(a^4bcx^2 + a^5c\right)\sqrt{ac}\log\left(-\frac{b^2cx^4+3\,abcx^2+2\,a^2c-2\,\sqrt{b^3cx^6+3\,ab^2cx^4+3\,a^2bcx^2+a^3c}\sqrt{ac}}{bx^4+ax^2}\right) + 2\,\left(35\,b^4cx^8 + 185\,ab^3cx^6 + 408\,a^2b^2cx^4 \right.}{630\,(bx^2 + a)}\right.$$

$$-\frac{315\,\left(a^4bcx^2 + a^5c\right)\sqrt{-ac}\arctan\left(\frac{abcx^2+a^2c}{\sqrt{b^3cx^6+3\,ab^2cx^4+3\,a^2bcx^2+a^3c}\sqrt{-ac}}\right) - \left(35\,b^4cx^8 + 185\,ab^3cx^6 + 408\,a^2b^2cx^4 + 506\,a^3bcx^2 + \right.}{315\,(bx^2 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((b*x^2 + a)^3*c)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/630*(315*(a^4*b*c*x^2 + a^5*c)*sqrt(a*c)*log(-(b^2*c*x^4 + 3*a
*b*c*x^2 + 2*a^2*c - 2*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c
*x^2 + a^3*c)*sqrt(a*c))/(b*x^4 + a*x^2)) + 2*(35*b^4*c*x^8 + 185
*a*b^3*c*x^6 + 408*a^2*b^2*c*x^4 + 506*a^3*b*c*x^2 + 563*a^4*c)*s
qrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c))/(b*x^2 +
a), -1/315*(315*(a^4*b*c*x^2 + a^5*c)*sqrt(-a*c)*arctan((a*b*c*x^
2 + a^2*c)/(sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*
c)*sqrt(-a*c))) - (35*b^4*c*x^8 + 185*a*b^3*c*x^6 + 408*a^2*b^2*c
*x^4 + 506*a^3*b*c*x^2 + 563*a^4*c)*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^
4 + 3*a^2*b*c*x^2 + a^3*c))/(b*x^2 + a)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*(b*x**2+a)**3)**(3/2)/x,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.275613, size = 194, normalized size = 1.

$$\frac{1}{315}\left(\frac{315\,a^5c\arctan\left(\frac{\sqrt{bcx^2+ac}}{\sqrt{-ac}}\right)}{\sqrt{-ac}}+\frac{315\,\sqrt{bcx^2+ac}a^4c^{36}+105\,\left(bcx^2+ac\right)^{\frac{3}{2}}a^3c^{35}+63\,\left(bcx^2+ac\right)^{\frac{5}{2}}a^2c^{34}+45\,\left(bcx^2+ac\right)^{\frac{7}{2}}a\right.}{c^{36}}$$

$$+\,a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((b*x^2 + a)^3*c)^(3/2)/x,x, algorithm="giac")
```

```
[Out] 1/315*(315*a^5*c*arctan(sqrt(b*c*x^2 + a*c)/sqrt(-a*c))/sqrt(-a*c
) + (315*sqrt(b*c*x^2 + a*c)*a^4*c^36 + 105*(b*c*x^2 + a*c)^(3/2)
*a^3*c^35 + 63*(b*c*x^2 + a*c)^(5/2)*a^2*c^34 + 45*(b*c*x^2 + a*c
)^(7/2)*a*c^33 + 35*(b*c*x^2 + a*c)^(9/2)*c^32)/c^36)*c*sign(b*x^
2 + a)
```

3.757 $\int \frac{\left(c\left(a+bx^2\right)^3\right)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=209

$$\frac{315a^4\sqrt{b}c\sqrt{c\left(a+bx^2\right)^3}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128\left(a+bx^2\right)^{3/2}}+\frac{315a^3bcx\sqrt{c\left(a+bx^2\right)^3}}{128\left(a+bx^2\right)}+\frac{105}{64}a^2bcx\sqrt{c\left(a+bx^2\right)^3}+\frac{21}{16}abcx\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3}-\frac{c\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3}}{x}+\frac{9}{8}bcx\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3}$$

```
[Out] (105*a^2*b*c*x*Sqrt[c*(a + b*x^2)^3])/64 + (315*a^3*b*c*x*Sqrt[c*
(a + b*x^2)^3])/(128*(a + b*x^2)) + (21*a*b*c*x*(a + b*x^2)*Sqrt[
c*(a + b*x^2)^3])/16 + (9*b*c*x*(a + b*x^2)^2*Sqrt[c*(a + b*x^2)^
3])/8 - (c*(a + b*x^2)^3*Sqrt[c*(a + b*x^2)^3])/x + (315*a^4*Sqrt
[b]*c*Sqrt[c*(a + b*x^2)^3]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])
/(128*(a + b*x^2)^(3/2))
```

Rubi [A] time = 0.348095, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$

$$\frac{315a^4\sqrt{b}c\sqrt{c\left(a+bx^2\right)^3}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128\left(a+bx^2\right)^{3/2}}+\frac{315a^3bcx\sqrt{c\left(a+bx^2\right)^3}}{128\left(a+bx^2\right)}+\frac{105}{64}a^2bcx\sqrt{c\left(a+bx^2\right)^3}+\frac{21}{16}abcx\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3}-\frac{c\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3}}{x}+\frac{9}{8}bcx\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(c*(a + b*x^2)^3)^(3/2)/x^2,x]
```

```
[Out] (105*a^2*b*c*x*Sqrt[c*(a + b*x^2)^3])/64 + (315*a^3*b*c*x*Sqrt[c*
(a + b*x^2)^3])/(128*(a + b*x^2)) + (21*a*b*c*x*(a + b*x^2)*Sqrt[
c*(a + b*x^2)^3])/16 + (9*b*c*x*(a + b*x^2)^2*Sqrt[c*(a + b*x^2)^
3])/8 - (c*(a + b*x^2)^3*Sqrt[c*(a + b*x^2)^3])/x + (315*a^4*Sqrt
[b]*c*Sqrt[c*(a + b*x^2)^3]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])
/(128*(a + b*x^2)^(3/2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c\left(a+bx^2\right)^3\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*(b*x**2+a)**3)**(3/2)/x**2,x)
```

```
[Out] Integral((c*(a + b*x**2)**3)**(3/2)/x**2, x)
```

Mathematica [A] time = 0.124534, size = 122, normalized size = 0.58

$$\frac{\left(c\left(a+bx^2\right)^3\right)^{3/2}\left(315a^4\sqrt{b}x\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)+\sqrt{a+bx^2}\left(-128a^4+325a^3bx^2+210a^2b^2x^4+88ab^3x^6+16b^4x^8\right)\right)}{128x\left(a+bx^2\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c*(a + b*x^2)^3)^(3/2)/x^2,x]
```

```
[Out] ((c*(a + b*x^2)^3)^(3/2)*(Sqrt[a + b*x^2]*(-128*a^4 + 325*a^3*b*x
^2 + 210*a^2*b^2*x^4 + 88*a*b^3*x^6 + 16*b^4*x^8) + 315*a^4*Sqrt[
b]*x*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]]))/(128*x*(a + b*x^2)^(9/2
))
```

Maple [A] time = 0.017, size = 215, normalized size = 1.

$$\frac{1}{128\,c\left(bx^2+a\right)^3x}\left(c\left(bx^2+a\right)^3\right)^{\frac{3}{2}}\left(16\,b^2x^4\left(bcx^2+ac\right)^{5/2}\sqrt{bc}+315\,a^4bc^3\ln\left(\frac{bcx+\sqrt{bcx^2+ac}\sqrt{bc}}{\sqrt{bc}}\right)x+56\,bax^2\left(bcx^2+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*(b*x^2+a)^3)^(3/2)/x^2,x)
```

```
[Out] 1/128*(c*(b*x^2+a)^3)^(3/2)*(16*b^2*x^4*(b*c*x^2+a*c)^(5/2)*(b*c)
^(1/2)+315*a^4*b*c^3*ln((b*c*x+(b*c*x^2+a*c)^(1/2)*(b*c)^(1/2))/(
b*c)^(1/2))*x+56*b*a*x^2*(b*c*x^2+a*c)^(5/2)*(b*c)^(1/2)+210*a^2*
b*x^2*(b*c*x^2+a*c)^(3/2)*c*(b*c)^(1/2)+315*a^3*b*c^2*x^2*(b*c*x^
2+a*c)^(1/2)*(b*c)^(1/2)-128*a^2*(b*c*x^2+a*c)^(5/2)*(b*c)^(1/2))
/(b*x^2+a)^3/(c*(b*x^2+a))^(3/2)/c/x/(b*c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((b*x^2 + a)^3*c)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.344913, size = 1, normalized size = 0.

$$\left[\frac{315\left(a^4bcx^3+a^5cx\right)\sqrt{bc}\log\left(-\frac{2b^2cx^4+3abcx^2+a^2c+2\sqrt{b^3cx^6+3ab^2cx^4+3a^2bcx^2+a^3c}\sqrt{bc}x}{bx^2+a}\right)+2\left(16b^4cx^8+88ab^3cx^6+210a^2b^2cx\right.}{256\left(bx^3+ax\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((b*x^2 + a)^3*c)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/256*(315*(a^4*b*c*x^3 + a^5*c*x)*sqrt(b*c)*log(-(2*b^2*c*x^4 +
 3*a*b*c*x^2 + a^2*c + 2*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b
*c*x^2 + a^3*c)*sqrt(b*c)*x)/(b*x^2 + a)) + 2*(16*b^4*c*x^8 + 88*
a*b^3*c*x^6 + 210*a^2*b^2*c*x^4 + 325*a^3*b*c*x^2 - 128*a^4*c)*sq
rt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c))/(b*x^3 + a
*x), 1/128*(315*(a^4*b*c*x^3 + a^5*c*x)*sqrt(-b*c)*arctan((b^2*c*
x^3 + a*b*c*x)/(sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 +
a^3*c)*sqrt(-b*c))) + (16*b^4*c*x^8 + 88*a*b^3*c*x^6 + 210*a^2*b^
2*c*x^4 + 325*a^3*b*c*x^2 - 128*a^4*c)*sqrt(b^3*c*x^6 + 3*a*b^2*c
*x^4 + 3*a^2*b*c*x^2 + a^3*c))/(b*x^3 + a*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*(b*x**2+a)**3)**(3/2)/x**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.278366, size = 250, normalized size = 1.2

$$\frac{1}{256}\left(\frac{512\sqrt{bc}a^5c\text{sign}\left(bx^2+a\right)}{\left(\sqrt{bc}x-\sqrt{bcx^2+ac}\right)^2-ac}-315\sqrt{bc}a^4\ln\left(\left(\sqrt{bc}x-\sqrt{bcx^2+ac}\right)^2\right)\text{sign}\left(bx^2+a\right)+2\left(325a^3b\text{sign}\left(bx^2+a\right)+2\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((b*x^2 + a)^3*c)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/256*(512*sqrt(b*c)*a^5*c*sign(b*x^2 + a)/((sqrt(b*c)*x - sqrt(b
*c*x^2 + a*c))^2 - a*c) - 315*sqrt(b*c)*a^4*ln((sqrt(b*c)*x - sqr
t(b*c*x^2 + a*c))^2)*sign(b*x^2 + a) + 2*(325*a^3*b*sign(b*x^2 +
a) + 2*(105*a^2*b^2*sign(b*x^2 + a) + 4*(2*b^4*x^2*sign(b*x^2 + a
) + 11*a*b^3*sign(b*x^2 + a))*x^2)*x^2)*sqrt(b*c*x^2 + a*c)*x)*c
```

3.758 $\int \frac{\left(c\left(a+bx^2\right)^3\right)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=204

$$-\frac{9a^{7/2}bc\sqrt{c\left(a+bx^2\right)^3}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\left(a+bx^2\right)^{3/2}}+\frac{9a^3bc\sqrt{c\left(a+bx^2\right)^3}}{2\left(a+bx^2\right)}+\frac{3}{2}a^2bc\sqrt{c\left(a+bx^2\right)^3}+\frac{9}{10}abc\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3}-\frac{c\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3}}{2x^2}+\frac{9}{14}bc\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3}$$

```
[Out] (3*a^2*b*c*Sqrt[c*(a + b*x^2)^3])/2 + (9*a^3*b*c*Sqrt[c*(a + b*x^
2)^3])/(2*(a + b*x^2)) + (9*a*b*c*(a + b*x^2)*Sqrt[c*(a + b*x^2)^
3])/10 + (9*b*c*(a + b*x^2)^2*Sqrt[c*(a + b*x^2)^3])/14 - (c*(a +
 b*x^2)^3*Sqrt[c*(a + b*x^2)^3])/(2*x^2) - (9*a^(7/2)*b*c*Sqrt[c*
(a + b*x^2)^3]*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*(a + b*x^2)^(
3/2))
```

Rubi [A] time = 0.424296, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316

$$-\frac{9a^{7/2}bc\sqrt{c\left(a+bx^2\right)^3}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\left(a+bx^2\right)^{3/2}}+\frac{9a^3bc\sqrt{c\left(a+bx^2\right)^3}}{2\left(a+bx^2\right)}+\frac{3}{2}a^2bc\sqrt{c\left(a+bx^2\right)^3}+\frac{9}{10}abc\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3}-\frac{c\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3}}{2x^2}+\frac{9}{14}bc\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(c*(a + b*x^2)^3)^(3/2)/x^3,x]
```

```
[Out] (3*a^2*b*c*Sqrt[c*(a + b*x^2)^3])/2 + (9*a^3*b*c*Sqrt[c*(a + b*x^
2)^3])/(2*(a + b*x^2)) + (9*a*b*c*(a + b*x^2)*Sqrt[c*(a + b*x^2)^
3])/10 + (9*b*c*(a + b*x^2)^2*Sqrt[c*(a + b*x^2)^3])/14 - (c*(a +
 b*x^2)^3*Sqrt[c*(a + b*x^2)^3])/(2*x^2) - (9*a^(7/2)*b*c*Sqrt[c*
(a + b*x^2)^3]*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*(a + b*x^2)^(
3/2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c\left(a+bx^2\right)^3\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*(b*x**2+a)**3)**(3/2)/x**3,x)
```

```
[Out] Integral((c*(a + b*x**2)**3)**(3/2)/x**3, x)
```

Mathematica [A] time = 0.15492, size = 133, normalized size = 0.65

$$-\frac{\left(c\left(a+bx^2\right)^3\right)^{3/2}\left(-315a^{7/2}bx^2\log(x)+315a^{7/2}bx^2\log\left(\sqrt{a}\sqrt{a+bx^2}+a\right)+\sqrt{a+bx^2}\left(35a^4-388a^3bx^2-156a^2b^2x^4-5\right.\right.}{70x^2\left(a+bx^2\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c*(a + b*x^2)^3)^(3/2)/x^3,x]
```

```
[Out] -((c*(a + b*x^2)^3)^(3/2)*(Sqrt[a + b*x^2]*(35*a^4 - 388*a^3*b*x^
2 - 156*a^2*b^2*x^4 - 58*a*b^3*x^6 - 10*b^4*x^8) - 315*a^(7/2)*b*
x^2*Log[x] + 315*a^(7/2)*b*x^2*Log[a + Sqrt[a]*Sqrt[a + b*x^2]]))
/(70*x^2*(a + b*x^2)^(9/2))
```

Maple [A] time = 0.018, size = 238, normalized size = 1.2

$$-\frac{1}{70\left(bx^2+a\right)^3x^2c}\left(c\left(bx^2+a\right)^3\right)^{\frac{3}{2}}\left(-10\sqrt{ac}\left(bcx^2+ac\right)^{5/2}x^4b^2+315a^4bc^3\ln\left(2\frac{\sqrt{ac}\sqrt{bcx^2+ac}+ac}{x}\right)x^2+4\sqrt{ac}\left(bcx^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*(b*x^2+a)^3)^(3/2)/x^3,x)
```

```
[Out] -1/70*(c*(b*x^2+a)^3)^(3/2)*(-10*(a*c)^(1/2)*(b*c*x^2+a*c)^(5/2)*
x^4*b^2+315*a^4*b*c^3*ln(2*((a*c)^(1/2)*(b*c*x^2+a*c)^(1/2)+a*c)/
x)*x^2+4*(a*c)^(1/2)*(b*c*x^2+a*c)^(5/2)*x^2*a*b-105*a^2*b*(b*c*x
^2+a*c)^(3/2)*x^2*c*(a*c)^(1/2)-315*a^3*b*c^2*(b*c*x^2+a*c)^(1/2)
*x^2*(a*c)^(1/2)-42*a*b*(c*(b*x^2+a))^(5/2)*x^2*(a*c)^(1/2)+35*(a
*c)^(1/2)*(b*c*x^2+a*c)^(5/2)*a^2)/(b*x^2+a)^3/(c*(b*x^2+a))^(3/2
)/x^2/c/(a*c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((b*x^2 + a)^3*c)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.300757, size = 1, normalized size = 0.

$$\left[\frac{315\left(a^3b^2cx^4+a^4bcx^2\right)\sqrt{ac}\log\left(-\frac{b^2cx^4+3\,abcx^2+2\,a^2c-2\,\sqrt{b^3cx^6+3\,ab^2cx^4+3\,a^2bcx^2+a^3c}\sqrt{ac}}{bx^4+ax^2}\right)+2\left(10\,b^4cx^8+58\,ab^3cx^6+156\,a^2b^2\right.}{140\left(bx^4+ax^2\right)}\right.$$

$$-\frac{315\left(a^3b^2cx^4+a^4bcx^2\right)\sqrt{-ac}\arctan\left(\frac{abcx^2+a^2c}{\sqrt{b^3cx^6+3\,ab^2cx^4+3\,a^2bcx^2+a^3c}\sqrt{-ac}}\right)-\left(10\,b^4cx^8+58\,ab^3cx^6+156\,a^2b^2cx^4+388\,a^3bcx\right.}{70\left(bx^4+ax^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((b*x^2 + a)^3*c)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/140*(315*(a^3*b^2*c*x^4 + a^4*b*c*x^2)*sqrt(a*c)*log(-(b^2*c*x
^4 + 3*a*b*c*x^2 + 2*a^2*c - 2*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3
*a^2*b*c*x^2 + a^3*c)*sqrt(a*c))/(b*x^4 + a*x^2)) + 2*(10*b^4*c*x
^8 + 58*a*b^3*c*x^6 + 156*a^2*b^2*c*x^4 + 388*a^3*b*c*x^2 - 35*a^
4*c)*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c))/(b*
x^4 + a*x^2), -1/70*(315*(a^3*b^2*c*x^4 + a^4*b*c*x^2)*sqrt(-a*c)
*arctan((a*b*c*x^2 + a^2*c)/(sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a
^2*b*c*x^2 + a^3*c)*sqrt(-a*c))) - (10*b^4*c*x^8 + 58*a*b^3*c*x^6
 + 156*a^2*b^2*c*x^4 + 388*a^3*b*c*x^2 - 35*a^4*c)*sqrt(b^3*c*x^6
 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c))/(b*x^4 + a*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*(b*x**2+a)**3)**(3/2)/x**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.276985, size = 204, normalized size = 1.

$$\frac{1}{70}\left(\frac{315\,a^4c^2\arctan\left(\frac{\sqrt{bcx^2+ac}}{\sqrt{-ac}}\right)}{\sqrt{-ac}} - \frac{35\,\sqrt{bcx^2+ac}a^4c}{bx^2} + \frac{2\left(140\,\sqrt{bcx^2+ac}a^3c^{15} + 35\left(bcx^2+ac\right)^{\frac{3}{2}}a^2c^{14} + 14\left(bcx^2+ac\right)^{\frac{5}{2}}ac\right.}{c^{14}}\right.$$
$$\left.+\,a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((b*x^2 + a)^3*c)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/70*(315*a^4*c^2*arctan(sqrt(b*c*x^2 + a*c)/sqrt(-a*c))/sqrt(-a*
c) - 35*sqrt(b*c*x^2 + a*c)*a^4*c/(b*x^2) + 2*(140*sqrt(b*c*x^2 +
 a*c)*a^3*c^15 + 35*(b*c*x^2 + a*c)^(3/2)*a^2*c^14 + 14*(b*c*x^2
+ a*c)^(5/2)*a*c^13 + 5*(b*c*x^2 + a*c)^(7/2)*c^12)/c^14)*b*sign(
b*x^2 + a)
```

3.759 $\int \sqrt{x-x^2}F(x)\,dx$

Optimal. Leaf size=17

$$\mathrm{Int}\left(\sqrt{x-x^2}F(x),x\right)$$

```
[Out] CannotIntegrate[Sqrt[x - x^2]*F[x], x]
```

Rubi [A] time = 0.0571509, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.

$$\mathrm{Int}\left(\sqrt{x-x^2}F(x),x\right)$$

Verification is Not applicable to the result.

```
[In]  Int[Sqrt[x - x^2]*F[x],x]
```

```
[Out] Defer[Int][Sqrt[x - x^2]*F[x], x]
```

Rubi in Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{2\sqrt{-x^2+x}\int^{\sqrt{x}} x^2\sqrt{-x^2+1}F\left(x^2\right)\,dx}{\sqrt{x}\sqrt{-x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(F(x)*(-x**2+x)**(1/2),x)
```

```
[Out] 2*sqrt(-x**2 + x)*Integral(x**2*sqrt(-x**2 + 1)*F(x**2), (x, sqrt
(x)))/(sqrt(x)*sqrt(-x + 1))
```

Mathematica [A] time = 0.100756, size = 0, normalized size = 0.

$$\int \sqrt{x-x^2}F(x)\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[Sqrt[x - x^2]*F[x],x]
```

```
[Out] Integrate[Sqrt[x - x^2]*F[x], x]
```

Maple [A] time = 0.026, size = 0, normalized size = 0.

$$\int F(x)\sqrt{-x^2+x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(F(x)*(-x^2+x)^(1/2),x)
```

```
[Out] int(F(x)*(-x^2+x)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x^2+x}F(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + x)*F(x),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + x)*F(x), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + x)*F(x),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x\,(x-1)}F(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(F(x)*(-x**2+x)**(1/2),x)
```

```
[Out] Integral(sqrt(-x*(x - 1))*F(x), x)
```

GIAC/XCAS [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x^2+x}F(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + x)*F(x),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^2 + x)*F(x), x)
```

3.760 $\int \frac{F(x)}{\sqrt{x-x^2}}\,dx$

Optimal. Leaf size=17

$$\operatorname{Int}\left(\frac{F(x)}{\sqrt{x-x^2}},x\right)$$

```
[Out] CannotIntegrate[F[x]/Sqrt[x - x^2], x]
```

Rubi [A] time = 0.0601219, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.

$$\operatorname{Int}\left(\frac{F(x)}{\sqrt{x-x^2}},x\right)$$

Verification is Not applicable to the result.

```
[In]  Int[F[x]/Sqrt[x - x^2],x]
```

```
[Out] Defer[Int][F[x]/Sqrt[x - x^2], x]
```

Rubi in Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{2\sqrt{-x^2+x}\int^{\sqrt{x}}\frac{F(x^2)}{\sqrt{-x^2+1}}\,dx}{\sqrt{x}\sqrt{-x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(F(x)/(-x**2+x)**(1/2),x)
```

```
[Out] 2*sqrt(-x**2 + x)*Integral(F(x**2)/sqrt(-x**2 + 1), (x, sqrt(x)))
/(sqrt(x)*sqrt(-x + 1))
```

Mathematica [A] time = 0.109296, size = 0, normalized size = 0.

$$\int \frac{F(x)}{\sqrt{x-x^2}}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[F[x]/Sqrt[x - x^2],x]

[Out] Integrate[F[x]/Sqrt[x - x^2], x]
```

Maple [A] time = 0.028, size = 0, normalized size = 0.

$$\int F(x)\,\frac{1}{\sqrt{-x^2+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(F(x)/(-x^2+x)^(1/2),x)

[Out] int(F(x)/(-x^2+x)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{F(x)}{\sqrt{-x^2+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(F(x)/sqrt(-x^2 + x),x, algorithm="maxima")

[Out] integrate(F(x)/sqrt(-x^2 + x), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(F(x)/sqrt(-x^2 + x),x, algorithm="fricas")

[Out] Exception raised: TypeError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{F(x)}{\sqrt{-x\,(x-1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(F(x)/(-x**2+x)**(1/2),x)
```

```
[Out] Integral(F(x)/sqrt(-x*(x - 1)), x)
```

GIAC/XCAS [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{F(x)}{\sqrt{-x^2+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(F(x)/sqrt(-x^2 + x),x, algorithm="giac")
```

```
[Out] integrate(F(x)/sqrt(-x^2 + x), x)
```

3.761 $\int \sqrt{1-x}\sqrt{x}F(x)\,dx$

Optimal. Leaf size=17

$$\mathrm{Int}\left(\sqrt{x-x^2}F(x),x\right)$$

```
[Out] CannotIntegrate[Sqrt[x - x^2]*F[x], x]
```

Rubi [A] time = 0.173845, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.

$$\mathrm{Int}\left(\sqrt{1-x}\sqrt{x}F(x),x\right)$$

Verification is Not applicable to the result.

```
[In]  Int[Sqrt[1 - x]*Sqrt[x]*F[x],x]
```

```
[Out] Defer[Int][Sqrt[x - x^2]*F[x], x]
```

Rubi in Sympy [A] time = 0., size = 0, normalized size = 0.

$$2\int^{\sqrt{x}} x^2\sqrt{-x^2+1}F\left(x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(F(x)*(1-x)**(1/2)*x**(1/2),x)
```

```
[Out] 2*Integral(x**2*sqrt(-x**2 + 1)*F(x**2), (x, sqrt(x)))
```

Mathematica [A] time = 0.0320601, size = 0, normalized size = 0.

$$\int \sqrt{1-x}\sqrt{x}F(x)\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[Sqrt[1 - x]*Sqrt[x]*F[x],x]
```

```
[Out] Integrate[Sqrt[1 - x]*Sqrt[x]*F[x], x]
```

Maple [A] time = 0.025, size = 0, normalized size = 0.

$$\int F(x)\sqrt{1-x}\sqrt{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(F(x)*(1-x)^(1/2)*x^(1/2),x)
```

```
[Out] int(F(x)*(1-x)^(1/2)*x^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x}\sqrt{-x+1}F(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)*sqrt(-x + 1)*F(x),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)*sqrt(-x + 1)*F(x), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)*sqrt(-x + 1)*F(x),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x}\sqrt{-x+1}F(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(F(x)*(1-x)**(1/2)*x**(1/2),x)
```

```
[Out] Integral(sqrt(x)*sqrt(-x + 1)*F(x), x)
```

<u>GIAC/XCAS</u> [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x}\sqrt{-x+1}F(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)*sqrt(-x + 1)*F(x),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x)*sqrt(-x + 1)*F(x), x)
```

3.762 $\int \frac{F(x)}{\sqrt{1-x}\sqrt{x}}\,dx$

Optimal. Leaf size=17

$$\text{Int}\left(\frac{F(x)}{\sqrt{x-x^2}},x\right)$$

```
[Out] CannotIntegrate[F[x]/Sqrt[x - x^2], x]
```

Rubi [A] time = 0.19041, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$

$$\text{Int}\left(\frac{F(x)}{\sqrt{1-x}\sqrt{x}},x\right)$$

Verification is Not applicable to the result.

```
[In]  Int[F[x]/(Sqrt[1 - x]*Sqrt[x]),x]

[Out] Defer[Int][F[x]/Sqrt[x - x^2], x]
```

Rubi in Sympy [A] time = 0., size = 0, normalized size = 0.

$$2\int^{\sqrt{x}}\frac{F(x^2)}{\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(F(x)/(1-x)**(1/2)/x**(1/2),x)

[Out] 2*Integral(F(x**2)/sqrt(-x**2 + 1), (x, sqrt(x)))
```

Mathematica [A] time = 0.0329714, size = 0, normalized size = 0.

$$\int\frac{F(x)}{\sqrt{1-x}\sqrt{x}}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[F[x]/(Sqrt[1 - x]*Sqrt[x]),x]

[Out] Integrate[F[x]/(Sqrt[1 - x]*Sqrt[x]), x]
```

Maple [A] time = 0.025, size = 0, normalized size = 0.

$$\int F(x)\,\frac{1}{\sqrt{1-x}}\,\frac{1}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(F(x)/(1-x)^(1/2)/x^(1/2),x)

[Out] int(F(x)/(1-x)^(1/2)/x^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{F(x)}{\sqrt{x}\sqrt{-x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(F(x)/(sqrt(x)*sqrt(-x + 1)),x, algorithm="maxima")

[Out] integrate(F(x)/(sqrt(x)*sqrt(-x + 1)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(F(x)/(sqrt(x)*sqrt(-x + 1)),x, algorithm="fricas")

[Out] Exception raised: TypeError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{F(x)}{\sqrt{x}\sqrt{-x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(F(x)/(1-x)**(1/2)/x**(1/2),x)
```

```
[Out] Integral(F(x)/(sqrt(x)*sqrt(-x + 1)), x)
```

GIAC/XCAS [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{F(x)}{\sqrt{x}\sqrt{-x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(F(x)/(sqrt(x)*sqrt(-x + 1)),x, algorithm="giac")
```

```
[Out] integrate(F(x)/(sqrt(x)*sqrt(-x + 1)), x)
```

3.763 $\int f\left(\frac{a+bx}{x}\right)\,dx$

Optimal. Leaf size=11

$$\text{Int}\left(f\left(\frac{a}{x}+b\right),x\right)$$

```
[Out] CannotIntegrate[f[b + a/x], x]
```

Rubi [A] time = 0.0228253, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.

$$\text{Int}\left(f\left(\frac{a+bx}{x}\right),x\right)$$

Verification is Not applicable to the result.

```
[In]  Int[f[(a + b*x)/x],x]

[Out] Defer[Int][f[b + a/x], x]
```

Rubi in Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int F\left(\frac{a+bx}{x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(F((b*x+a)/x),x)

[Out] Integral(F((a + b*x)/x), x)
```

Mathematica [A] time = 0.00455912, size = 0, normalized size = 0.

$$\int f\left(\frac{a+bx}{x}\right)\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[f[(a + b*x)/x],x]
```

```
[Out] Integrate[f[(a + b*x)/x], x]
```

Maple [A] time = 0.005, size = 0, normalized size = 0.

$$\int f\left(\frac{bx+a}{x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(f((b*x+a)/x),x)
```

```
[Out] int(f((b*x+a)/x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int F\left(\frac{bx+a}{x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(F((b*x + a)/x),x, algorithm="maxima")
```

```
[Out] integrate(F((b*x + a)/x), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(F((b*x + a)/x),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int F\left(\frac{a+bx}{x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(F((b*x+a)/x),x)
```

```
[Out] Integral(F((a + b*x)/x), x)
```

GIAC/XCAS [A] time = 0., size = 0, normalized size = 0.

$$\int F\left(\frac{bx+a}{x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(F((b*x + a)/x),x, algorithm="giac")
```

```
[Out] integrate(F((b*x + a)/x), x)
```

3.764 $\int f\left(\frac{a+bx^2}{x^2}\right) dx$

Optimal. Leaf size=11

$$\text{Int}\left(f\left(\frac{a}{x^2}+b\right),x\right)$$

```
[Out] CannotIntegrate[f[b + a/x^2], x]
```

Rubi [A] time = 0.0220187, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$

$$\text{Int}\left(f\left(\frac{a+bx^2}{x^2}\right),x\right)$$

Verification is Not applicable to the result.

```
[In]  Int[f[(a + b*x^2)/x^2],x]

[Out] Defer[Int][f[b + a/x^2], x]
```

Rubi in Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int F\left(\frac{a}{x^2}+b\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(F((b*x**2+a)/x**2),x)

[Out] Integral(F(a/x**2 + b), x)
```

Mathematica [A] time = 0.00567234, size = 0, normalized size = 0.

$$\int f\left(\frac{a+bx^2}{x^2}\right) dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[f[(a + b*x^2)/x^2],x]

[Out] Integrate[f[(a + b*x^2)/x^2], x]
```

Maple [A] time = 0.008, size = 0, normalized size = 0.

$$\int f\left(\frac{bx^2+a}{x^2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(f((b*x^2+a)/x^2),x)

[Out] int(f((b*x^2+a)/x^2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int F\left(\frac{bx^2+a}{x^2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(F((b*x^2 + a)/x^2),x, algorithm="maxima")

[Out] integrate(F((b*x^2 + a)/x^2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(F((b*x^2 + a)/x^2),x, algorithm="fricas")

[Out] Exception raised: TypeError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int F\left(\frac{a+bx^2}{x^2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(F((b*x**2+a)/x**2),x)

[Out] Integral(F((a + b*x**2)/x**2), x)
```

GIAC/XCAS [A] time = 0., size = 0, normalized size = 0.

$$\int F\left(\frac{bx^2+a}{x^2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(F((b*x^2 + a)/x^2),x, algorithm="giac")

[Out] integrate(F((b*x^2 + a)/x^2), x)
```

3.765 $\int f\left(\frac{x}{a+bx}\right)\,dx$

Optimal. Leaf size=13

$$\mathrm{Int}\left(f\left(\frac{x}{a+bx}\right),x\right)$$

```
[Out] CannotIntegrate[f[x/(a + b*x)], x]
```

Rubi [A] time = 0.0151675, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.

$$\mathrm{Int}\left(f\left(\frac{x}{a+bx}\right),x\right)$$

Verification is Not applicable to the result.

```
[In] Int[f[x/(a + b*x)],x]

[Out] Defer[Int][f[x/(a + b*x)], x]
```

Rubi in Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int F\left(\frac{x}{a+bx}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(F(x/(b*x+a)),x)

[Out] Integral(F(x/(a + b*x)), x)
```

Mathematica [A] time = 0.00634846, size = 0, normalized size = 0.

$$\int f\left(\frac{x}{a+bx}\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[f[x/(a + b*x)],x]
```

```
[Out] Integrate[f[x/(a + b*x)], x]
```

Maple [A] time = 0., size = 0, normalized size = 0.

$$\int f\left(\frac{x}{bx+a}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f(x/(b*x+a)),x)
```

```
[Out] int(f(x/(b*x+a)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int F\left(\frac{x}{bx+a}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x/(b*x + a)),x, algorithm="maxima")
```

```
[Out] integrate(F(x/(b*x + a)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x/(b*x + a)),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int F\left(\frac{x}{a+bx}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(F(x/(b*x+a)),x)
```

```
[Out] Integral(F(x/(a + b*x)), x)
```

GIAC/XCAS [A] time = 0., size = 0, normalized size = 0.

$$\int F\left(\frac{x}{bx+a}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(F(x/(b*x + a)),x, algorithm="giac")
```

```
[Out] integrate(F(x/(b*x + a)), x)
```

3.766 $\int f\left(\frac{x^2}{a+bx^2}\right) dx$

Optimal. Leaf size=17

$$\text{Int}\left(f\left(\frac{x^2}{a+bx^2}\right),x\right)$$

```
[Out] CannotIntegrate[f[x^2/(a + b*x^2)], x]
```

Rubi [A] time = 0.0169517, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$

$$\text{Int}\left(f\left(\frac{x^2}{a+bx^2}\right),x\right)$$

Verification is Not applicable to the result.

```
[In]  Int[f[x^2/(a + b*x^2)],x]

[Out] Defer[Int][f[x^2/(a + b*x^2)], x]
```

Rubi in Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int F\left(\frac{x^2}{a+bx^2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(F(x**2/(b*x**2+a)),x)

[Out] Integral(F(x**2/(a + b*x**2)), x)
```

Mathematica [A] time = 0.00849107, size = 0, normalized size = 0.

$$\int f\left(\frac{x^2}{a+bx^2}\right) dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[f[x^2/(a + b*x^2)],x]

[Out] Integrate[f[x^2/(a + b*x^2)], x]
```

Maple [A] time = 0., size = 0, normalized size = 0.

$$\int f\left(\frac{x^2}{bx^2+a}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(f(x^2/(b*x^2+a)),x)

[Out] int(f(x^2/(b*x^2+a)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int F\left(\frac{x^2}{bx^2+a}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(F(x^2/(b*x^2 + a)),x, algorithm="maxima")

[Out] integrate(F(x^2/(b*x^2 + a)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(F(x^2/(b*x^2 + a)),x, algorithm="fricas")

[Out] Exception raised: TypeError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int F\left(\frac{x^2}{a+bx^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(F(x**2/(b*x**2+a)),x)
```

```
[Out] Integral(F(x**2/(a + b*x**2)), x)
```

GIAC/XCAS [A] time = 0., size = 0, normalized size = 0.

$$\int F\left(\frac{x^2}{bx^2+a}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(F(x^2/(b*x^2 + a)),x, algorithm="giac")
```

```
[Out] integrate(F(x^2/(b*x^2 + a)), x)
```

3.767 $\int f\left(\frac{x^2}{(a+bx)^2}\right) dx$

Optimal. Leaf size=15

$$\text{Int}\left(f\left(\frac{x^2}{(a+bx)^2}\right),x\right)$$

```
[Out] CannotIntegrate[f[x^2/(a + b*x)^2], x]
```

Rubi [A] time = 0.0164775, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$

$$\text{Int}\left(f\left(\frac{x^2}{(a+bx)^2}\right),x\right)$$

Verification is Not applicable to the result.

```
[In]  Int[f[x^2/(a + b*x)^2],x]

[Out] Defer[Int][f[x^2/(a + b*x)^2], x]
```

Rubi in Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int F\left(\frac{x^2}{(a+bx)^2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(F(x**2/(b*x+a)**2),x)

[Out] Integral(F(x**2/(a + b*x)**2), x)
```

Mathematica [A] time = 0.0118247, size = 0, normalized size = 0.

$$\int f\left(\frac{x^2}{(a+bx)^2}\right) dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[f[x^2/(a + b*x)^2],x]

[Out] Integrate[f[x^2/(a + b*x)^2], x]
```

Maple [A] time = 0.001, size = 0, normalized size = 0.

$$\int f\left(\frac{x^2}{(bx+a)^2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(f(x^2/(b*x+a)^2),x)

[Out] int(f(x^2/(b*x+a)^2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int F\left(\frac{x^2}{(bx+a)^2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(F(x^2/(b*x + a)^2),x, algorithm="maxima")

[Out] integrate(F(x^2/(b*x + a)^2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(F(x^2/(b*x + a)^2),x, algorithm="fricas")

[Out] Exception raised: TypeError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int F\left(\frac{x^2}{(a+bx)^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(F(x**2/(b*x+a)**2),x)
```

```
[Out] Integral(F(x**2/(a + b*x)**2), x)
```

GIAC/XCAS [A] time = 0., size = 0, normalized size = 0.

$$\int F\left(\frac{x^2}{(bx+a)^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(F(x^2/(b*x + a)^2),x, algorithm="giac")
```

```
[Out] integrate(F(x^2/(b*x + a)^2), x)
```

3.768 $\int f\left(\frac{x^4}{(a+bx^2)^2}\right) dx$

Optimal. Leaf size=17

$$\text{Int}\left(f\left(\frac{x^4}{(a+bx^2)^2}\right),x\right)$$

```
[Out] CannotIntegrate[f[x^4/(a + b*x^2)^2], x]
```

Rubi [A] time = 0.0174068, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$

$$\text{Int}\left(f\left(\frac{x^4}{(a+bx^2)^2}\right),x\right)$$

Verification is Not applicable to the result.

```
[In]  Int[f[x^4/(a + b*x^2)^2],x]
```

```
[Out] Defer[Int][f[x^4/(a + b*x^2)^2], x]
```

Rubi in Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int F\left(\frac{x^4}{(a+bx^2)^2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(F(x**4/(b*x**2+a)**2),x)
```

```
[Out] Integral(F(x**4/(a + b*x**2)**2), x)
```

Mathematica [A] time = 0.0121437, size = 0, normalized size = 0.

$$\int f\left(\frac{x^4}{(a+bx^2)^2}\right) dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[f[x^4/(a + b*x^2)^2],x]

[Out] Integrate[f[x^4/(a + b*x^2)^2], x]
```

Maple [A] time = 0., size = 0, normalized size = 0.

$$\int f\left(\frac{x^4}{(bx^2+a)^2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(f(x^4/(b*x^2+a)^2),x)

[Out] int(f(x^4/(b*x^2+a)^2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int F\left(\frac{x^4}{(bx^2+a)^2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(F(x^4/(b*x^2 + a)^2),x, algorithm="maxima")

[Out] integrate(F(x^4/(b*x^2 + a)^2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(F(x^4/(b*x^2 + a)^2),x, algorithm="fricas")

[Out] Exception raised: TypeError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int F\left(\frac{x^4}{(a+bx^2)^2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(F(x**4/(b*x**2+a)**2),x)
```

```
[Out] Integral(F(x**4/(a + b*x**2)**2), x)
```

GIAC/XCAS [A] time = 0., size = 0, normalized size = 0.

$$\int F\left(\frac{x^4}{(bx^2+a)^2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(F(x^4/(b*x^2 + a)^2),x, algorithm="giac")
```

```
[Out] integrate(F(x^4/(b*x^2 + a)^2), x)
```

3.769 $\int \frac{\sqrt{bx^2+\sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}}\,dx$

Optimal. Leaf size=47

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{\sqrt{a+b^2x^4}+bx^2}}\right)}{\sqrt{2}\sqrt{b}}$$

```
[Out] ArcTanh[(Sqrt[2]*Sqrt[b]*x)/Sqrt[b*x^2 + Sqrt[a + b^2*x^4]]]/(Sqr
t[2]*Sqrt[b])
```

Rubi [A] time = 0.173782, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{\sqrt{a+b^2x^4}+bx^2}}\right)}{\sqrt{2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[b*x^2 + Sqrt[a + b^2*x^4]]/Sqrt[a + b^2*x^4],x]
```

```
[Out] ArcTanh[(Sqrt[2]*Sqrt[b]*x)/Sqrt[b*x^2 + Sqrt[a + b^2*x^4]]]/(Sqr
t[2]*Sqrt[b])
```

Rubi in Sympy [A] time = 6.26988, size = 44, normalized size = 0.94

$$\frac{\sqrt{2}\operatorname{atanh}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{bx^2+\sqrt{a+b^2x^4}}}\right)}{2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+(b**2*x**4+a)**(1/2))**(1/2)/(b**2*x**4+a)**(1/2),x)
```

```
[Out] sqrt(2)*atanh(sqrt(2)*sqrt(b)*x/sqrt(b*x**2 + sqrt(a + b**2*x**4)
))/(2*sqrt(b))
```

Mathematica [A] time = 0.0830897, size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2 + \sqrt{a + b^2x^4}}}{\sqrt{a + b^2x^4}}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[Sqrt[b*x^2 + Sqrt[a + b^2*x^4]]/Sqrt[a + b^2*x^4],x]

[Out] Integrate[Sqrt[b*x^2 + Sqrt[a + b^2*x^4]]/Sqrt[a + b^2*x^4], x]
```

Maple [F] time = 0.05, size = 0, normalized size = 0.

$$\int 1\sqrt{bx^2 + \sqrt{b^2x^4 + a}}\frac{1}{\sqrt{b^2x^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x)

[Out] int((b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2 + \sqrt{b^2x^4 + a}}}{\sqrt{b^2x^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + sqrt(b^2*x^4 + a))/sqrt(b^2*x^4 + a),x, algorithm="maxima

[Out] integrate(sqrt(b*x^2 + sqrt(b^2*x^4 + a))/sqrt(b^2*x^4 + a), x)
```

Fricas [A] time = 1.52291, size = 1, normalized size = 0.02

$$\left[\frac{\sqrt{2}\log\left(4\,b^2x^4 + 4\,\sqrt{b^2x^4 + a}bx^2 + 2\left(\sqrt{2}b^{\frac{3}{2}}x^3 + \sqrt{2}\sqrt{b^2x^4 + a}\sqrt{b}x\right)\sqrt{bx^2 + \sqrt{b^2x^4 + a}} + a\right)}{4\,\sqrt{b}}, \frac{1}{2}\,\sqrt{2}\sqrt{-\frac{1}{b}}\arctan\left(\frac{\sqrt{2}\sqrt{bx^2 +}}{2\,bx}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + sqrt(b^2*x^4 + a))/sqrt(b^2*x^4 + a),x, algorithm="fricas
```

```
[Out] [1/4*sqrt(2)*log(4*b^2*x^4 + 4*sqrt(b^2*x^4 + a)*b*x^2 + 2*(sqrt(
2)*b^(3/2)*x^3 + sqrt(2)*sqrt(b^2*x^4 + a)*sqrt(b)*x)*sqrt(b*x^2
+ sqrt(b^2*x^4 + a)) + a)/sqrt(b), 1/2*sqrt(2)*sqrt(-1/b)*arctan(
1/2*sqrt(2)*sqrt(b*x^2 + sqrt(b^2*x^4 + a))/(b*x*sqrt(-1/b)))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2 + \sqrt{a + b^2x^4}}}{\sqrt{a + b^2x^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+(b**2*x**4+a)**(1/2))**(1/2)/(b**2*x**4+a)**(1/2),x)
```

```
[Out] Integral(sqrt(b*x**2 + sqrt(a + b**2*x**4))/sqrt(a + b**2*x**4),
x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2 + \sqrt{b^2x^4 + a}}}{\sqrt{b^2x^4 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + sqrt(b^2*x^4 + a))/sqrt(b^2*x^4 + a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + sqrt(b^2*x^4 + a))/sqrt(b^2*x^4 + a), x)
```

3.770 $\int \frac{\sqrt{-bx^2+\sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}}\,dx$

Optimal. Leaf size=48

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{\sqrt{a+b^2x^4}-bx^2}}\right)}{\sqrt{2}\sqrt{b}}$$

```
[Out] ArcTan[(Sqrt[2]*Sqrt[b]*x)/Sqrt[-(b*x^2) + Sqrt[a + b^2*x^4]]]/(S
qrt[2]*Sqrt[b])
```

Rubi [A] time = 0.177289, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{\sqrt{a+b^2x^4}-bx^2}}\right)}{\sqrt{2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[-(b*x^2) + Sqrt[a + b^2*x^4]]/Sqrt[a + b^2*x^4],x]
```

```
[Out] ArcTan[(Sqrt[2]*Sqrt[b]*x)/Sqrt[-(b*x^2) + Sqrt[a + b^2*x^4]]]/(S
qrt[2]*Sqrt[b])
```

Rubi in Sympy [A] time = 6.42141, size = 44, normalized size = 0.92

$$\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{-bx^2+\sqrt{a+b^2x^4}}}\right)}{2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-b*x**2+(b**2*x**4+a)**(1/2))**(1/2)/(b**2*x**4+a)**(1/2),x)
```

```
[Out] sqrt(2)*atan(sqrt(2)*sqrt(b)*x/sqrt(-b*x**2 + sqrt(a + b**2*x**4)
))/(2*sqrt(b))
```

Mathematica [A] time = 0.0797564, size = 0, normalized size = 0.

$$\int \frac{\sqrt{-bx^2 + \sqrt{a + b^2x^4}}}{\sqrt{a + b^2x^4}}\, dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[Sqrt[-(b*x^2) + Sqrt[a + b^2*x^4]]/Sqrt[a + b^2*x^4],x]

[Out] Integrate[Sqrt[-(b*x^2) + Sqrt[a + b^2*x^4]]/Sqrt[a + b^2*x^4], x
]
```

Maple [F] time = 0.052, size = 0, normalized size = 0.

$$\int 1\sqrt{-bx^2 + \sqrt{b^2x^4 + a}}\frac{1}{\sqrt{b^2x^4 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x)

[Out] int((-b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-bx^2 + \sqrt{b^2x^4 + a}}}{\sqrt{b^2x^4 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-b*x^2 + sqrt(b^2*x^4 + a))/sqrt(b^2*x^4 + a),x, algorithm="maxima

[Out] integrate(sqrt(-b*x^2 + sqrt(b^2*x^4 + a))/sqrt(b^2*x^4 + a), x)
```

Fricas [A] time = 1.61101, size = 1, normalized size = 0.02

$$\left[\frac{1}{4}\sqrt{2}\sqrt{-\frac{1}{b}}\log\left(4\,b^2x^4 - 4\,\sqrt{b^2x^4+a}bx^2 + 2\left(\sqrt{2}b^2x^3\sqrt{-\frac{1}{b}} - \sqrt{2}\sqrt{b^2x^4+a}bx\sqrt{-\frac{1}{b}}\right)\sqrt{-bx^2+\sqrt{b^2x^4+a}} + a\right), -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-bx^2+\sqrt{b^2x^4+a}}}{2\,\sqrt{b}x}\right)}{2\,\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-b*x^2 + sqrt(b^2*x^4 + a))/sqrt(b^2*x^4 + a),x, algorithm="fricas
```

```
[Out] [1/4*sqrt(2)*sqrt(-1/b)*log(4*b^2*x^4 - 4*sqrt(b^2*x^4 + a)*b*x^2
 + 2*(sqrt(2)*b^2*x^3*sqrt(-1/b) - sqrt(2)*sqrt(b^2*x^4 + a)*b*x*
sqrt(-1/b))*sqrt(-b*x^2 + sqrt(b^2*x^4 + a)) + a), -1/2*sqrt(2)*a
rctan(1/2*sqrt(2)*sqrt(-b*x^2 + sqrt(b^2*x^4 + a))/(sqrt(b)*x))/s
qrt(b)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-bx^2+\sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+(b**2*x**4+a)**(1/2))**(1/2)/(b**2*x**4+a)**(1/2),x)
```

```
[Out] Integral(sqrt(-b*x**2 + sqrt(a + b**2*x**4))/sqrt(a + b**2*x**4),
 x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-bx^2+\sqrt{b^2x^4+a}}}{\sqrt{b^2x^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-b*x^2 + sqrt(b^2*x^4 + a))/sqrt(b^2*x^4 + a),x, algorithm="giac"

[Out] integrate(sqrt(-b*x^2 + sqrt(b^2*x^4 + a))/sqrt(b^2*x^4 + a), x)
```

3.771 $\int \frac{\sqrt{2x^2+\sqrt{3+4x^4}}}{(c+dx)\sqrt{3+4x^4}}\,dx$

Optimal. Leaf size=169

$$\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\tan^{-1}\left(\frac{\sqrt{3}d+2icx}{\sqrt{\sqrt{3}-2ix^2}\sqrt{-\sqrt{3}d^2+2ic^2}}\right)}{\sqrt{-\sqrt{3}d^2+2ic^2}}-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\tanh^{-1}\left(\frac{\sqrt{3}d-2icx}{\sqrt{\sqrt{3}+2ix^2}\sqrt{\sqrt{3}d^2+2ic^2}}\right)}{\sqrt{\sqrt{3}d^2+2ic^2}}$$

```
[Out] ((1/2 - I/2)*ArcTan[(Sqrt[3]*d + (2*I)*c*x)/(Sqrt[(2*I)*c^2 - Sqr
t[3]*d^2]*Sqrt[Sqrt[3] - (2*I)*x^2])])/Sqrt[(2*I)*c^2 - Sqrt[3]*d
^2] - ((1/2 + I/2)*ArcTanh[(Sqrt[3]*d - (2*I)*c*x)/(Sqrt[(2*I)*c^
2 + Sqrt[3]*d^2]*Sqrt[Sqrt[3] + (2*I)*x^2])])/Sqrt[(2*I)*c^2 + Sq
rt[3]*d^2]
```

Rubi [A] time = 0.469406, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\tan^{-1}\left(\frac{\sqrt{3}d+2icx}{\sqrt{\sqrt{3}-2ix^2}\sqrt{-\sqrt{3}d^2+2ic^2}}\right)}{\sqrt{-\sqrt{3}d^2+2ic^2}}-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\tanh^{-1}\left(\frac{\sqrt{3}d-2icx}{\sqrt{\sqrt{3}+2ix^2}\sqrt{\sqrt{3}d^2+2ic^2}}\right)}{\sqrt{\sqrt{3}d^2+2ic^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[2*x^2 + Sqrt[3 + 4*x^4]]/((c + d*x)*Sqrt[3 + 4*x^4]),x]
```

```
[Out] ((1/2 - I/2)*ArcTan[(Sqrt[3]*d + (2*I)*c*x)/(Sqrt[(2*I)*c^2 - Sqr
t[3]*d^2]*Sqrt[Sqrt[3] - (2*I)*x^2])])/Sqrt[(2*I)*c^2 - Sqrt[3]*d
^2] - ((1/2 + I/2)*ArcTanh[(Sqrt[3]*d - (2*I)*c*x)/(Sqrt[(2*I)*c^
2 + Sqrt[3]*d^2]*Sqrt[Sqrt[3] + (2*I)*x^2])])/Sqrt[(2*I)*c^2 + Sq
rt[3]*d^2]
```

Rubi in Sympy [A] time = 19.8718, size = 146, normalized size = 0.86

$$-\frac{(1+i)\operatorname{atanh}\left(\frac{-2icx+\sqrt{3}d}{\sqrt{2ic^2+\sqrt{3}d^2}\sqrt{2ix^2+\sqrt{3}}}\right)}{2\sqrt{2ic^2+\sqrt{3}d^2}}-\frac{(1-i)\operatorname{atanh}\left(\frac{2icx+\sqrt{3}d}{\sqrt{-2ic^2+\sqrt{3}d^2}\sqrt{-2ix^2+\sqrt{3}}}\right)}{2\sqrt{-2ic^2+\sqrt{3}d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((2*x**2+(4*x**4+3)**(1/2))**(1/2)/(d*x+c)/(4*x**4+3)**(1/2),x)
```

```
[Out] -(1 + I)*atanh((-2*I*c*x + sqrt(3)*d)/(sqrt(2*I*c**2 + sqrt(3)*d*
*2)*sqrt(2*I*x**2 + sqrt(3))))/(2*sqrt(2*I*c**2 + sqrt(3)*d**2))
- (1 - I)*atanh((2*I*c*x + sqrt(3)*d)/(sqrt(-2*I*c**2 + sqrt(3)*d
**2)*sqrt(-2*I*x**2 + sqrt(3))))/(2*sqrt(-2*I*c**2 + sqrt(3)*d**2
))
```

Mathematica [A] time = 0.0951031, size = 0, normalized size = 0.

$$\int \frac{\sqrt{2x^2+\sqrt{3+4x^4}}}{(c+dx)\sqrt{3+4x^4}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[2*x^2 + Sqrt[3 + 4*x^4]]/((c + d*x)*Sqrt[3 + 4*x^4]),x]
```

```
[Out] Integrate[Sqrt[2*x^2 + Sqrt[3 + 4*x^4]]/((c + d*x)*Sqrt[3 + 4*x^4
]), x]
```

Maple [F] time = 0.052, size = 0, normalized size = 0.

$$\int \frac{1}{dx+c}\sqrt{2\,x^2+\sqrt{4\,x^4+3}}\frac{1}{\sqrt{4\,x^4+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)/(4*x^4+3)^(1/2),x)
```

```
[Out] int((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)/(4*x^4+3)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2\,x^2+\sqrt{4\,x^4+3}}}{\sqrt{4\,x^4+3}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(2*x^2 + sqrt(4*x^4 + 3))/(sqrt(4*x^4 + 3)*(d*x + c)),x, algorithm=
```

```
[Out] integrate(sqrt(2*x^2 + sqrt(4*x^4 + 3))/(sqrt(4*x^4 + 3)*(d*x + c
)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(2*x^2 + sqrt(4*x^4 + 3))/(sqrt(4*x^4 + 3)*(d*x + c)),x, algorithm=
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2x^2 + \sqrt{4x^4 + 3}}}{(c + dx)\sqrt{4x^4 + 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+(4*x**4+3)**(1/2))**(1/2)/(d*x+c)/(4*x**4+3)**(1/2),x)
```

```
[Out] Integral(sqrt(2*x**2 + sqrt(4*x**4 + 3))/((c + d*x)*sqrt(4*x**4 +
 3)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2\,x^2 + \sqrt{4\,x^4 + 3}}}{\sqrt{4\,x^4 + 3}(dx + c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(2*x^2 + sqrt(4*x^4 + 3))/(sqrt(4*x^4 + 3)*(d*x + c)),x, algorithm=
```

```
[Out] integrate(sqrt(2*x^2 + sqrt(4*x^4 + 3))/(sqrt(4*x^4 + 3)*(d*x + c
)), x)
```

3.772 $\int \frac{\sqrt{2x^2+\sqrt{3+4x^4}}}{(c+dx)^2\sqrt{3+4x^4}}\,dx$

Optimal. Leaf size=268

$$\frac{\left(\frac{1}{2}-\frac{i}{2}\right)d\sqrt{\sqrt{3}-2ix^2}}{\left(-\sqrt{3}d^2+2ic^2\right)(c+dx)}-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)d\sqrt{\sqrt{3}+2ix^2}}{\left(\sqrt{3}d^2+2ic^2\right)(c+dx)}+\frac{(1+i)c\tan^{-1}\left(\frac{\sqrt{3}d+2icx}{\sqrt{\sqrt{3}-2ix^2}\sqrt{-\sqrt{3}d^2+2ic^2}}\right)}{\left(-\sqrt{3}d^2+2ic^2\right)^{3/2}}+\frac{(1-i)c\tanh^{-1}\left(\frac{\sqrt{3}d-2icx}{\sqrt{\sqrt{3}+2ix^2}\sqrt{\sqrt{3}d^2+2ic^2}}\right)}{\left(\sqrt{3}d^2+2ic^2\right)^{3/2}}$$

```
[Out] ((1/2 - I/2)*d*Sqrt[Sqrt[3] - (2*I)*x^2])/(((2*I)*c^2 - Sqrt[3]*d
^2)*(c + d*x)) - ((1/2 + I/2)*d*Sqrt[Sqrt[3] + (2*I)*x^2])/(((2*I
)*c^2 + Sqrt[3]*d^2)*(c + d*x)) + ((1 + I)*c*ArcTan[(Sqrt[3]*d +
(2*I)*c*x)/(Sqrt[(2*I)*c^2 - Sqrt[3]*d^2]*Sqrt[Sqrt[3] - (2*I)*x^
2])])/((2*I)*c^2 - Sqrt[3]*d^2)^(3/2) + ((1 - I)*c*ArcTanh[(Sqrt[
3]*d - (2*I)*c*x)/(Sqrt[(2*I)*c^2 + Sqrt[3]*d^2]*Sqrt[Sqrt[3] + (
2*I)*x^2])])/((2*I)*c^2 + Sqrt[3]*d^2)^(3/2)
```

Rubi [A] time = 0.625938, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{\left(\frac{1}{2}-\frac{i}{2}\right)d\sqrt{\sqrt{3}-2ix^2}}{\left(-\sqrt{3}d^2+2ic^2\right)(c+dx)}-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)d\sqrt{\sqrt{3}+2ix^2}}{\left(\sqrt{3}d^2+2ic^2\right)(c+dx)}+\frac{(1+i)c\tan^{-1}\left(\frac{\sqrt{3}d+2icx}{\sqrt{\sqrt{3}-2ix^2}\sqrt{-\sqrt{3}d^2+2ic^2}}\right)}{\left(-\sqrt{3}d^2+2ic^2\right)^{3/2}}+\frac{(1-i)c\tanh^{-1}\left(\frac{\sqrt{3}d-2icx}{\sqrt{\sqrt{3}+2ix^2}\sqrt{\sqrt{3}d^2+2ic^2}}\right)}{\left(\sqrt{3}d^2+2ic^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[2*x^2 + Sqrt[3 + 4*x^4]]/((c + d*x)^2*Sqrt[3 + 4*x^4]),x]
```

```
[Out] ((1/2 - I/2)*d*Sqrt[Sqrt[3] - (2*I)*x^2])/(((2*I)*c^2 - Sqrt[3]*d
^2)*(c + d*x)) - ((1/2 + I/2)*d*Sqrt[Sqrt[3] + (2*I)*x^2])/(((2*I
)*c^2 + Sqrt[3]*d^2)*(c + d*x)) + ((1 + I)*c*ArcTan[(Sqrt[3]*d +
(2*I)*c*x)/(Sqrt[(2*I)*c^2 - Sqrt[3]*d^2]*Sqrt[Sqrt[3] - (2*I)*x^
2])])/((2*I)*c^2 - Sqrt[3]*d^2)^(3/2) + ((1 - I)*c*ArcTanh[(Sqrt[
3]*d - (2*I)*c*x)/(Sqrt[(2*I)*c^2 + Sqrt[3]*d^2]*Sqrt[Sqrt[3] + (
2*I)*x^2])])/((2*I)*c^2 + Sqrt[3]*d^2)^(3/2)
```

Rubi in Sympy [A] time = 26.9435, size = 231, normalized size = 0.86

$$\frac{c(1-i)\operatorname{atanh}\left(\frac{-2icx+\sqrt{3}d}{\sqrt{2ic^2+\sqrt{3}d^2}\sqrt{2ix^2+\sqrt{3}}}\right)}{\left(2ic^2+\sqrt{3}d^2\right)^{\frac{3}{2}}}+\frac{ic(1-i)\operatorname{atanh}\left(\frac{2icx+\sqrt{3}d}{\sqrt{-2ic^2+\sqrt{3}d^2}\sqrt{-2ix^2+\sqrt{3}}}\right)}{\left(-2ic^2+\sqrt{3}d^2\right)^{\frac{3}{2}}}-\frac{d(1+i)\sqrt{2ix^2+\sqrt{3}}}{2(c+dx)\left(2ic^2+\sqrt{3}d^2\right)}-\frac{d(1-i)\sqrt{-2ix^2+\sqrt{3}}}{2(c+dx)\left(-2ic^2+\sqrt{3}d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((2*x**2+(4*x**4+3)**(1/2))**(1/2)/(d*x+c)**2/(4*x**4+3)**(1/2),
```

```
[Out] c*(1 - I)*atanh((-2*I*c*x + sqrt(3)*d)/(sqrt(2*I*c**2 + sqrt(3)*d
**2)*sqrt(2*I*x**2 + sqrt(3))))/(2*I*c**2 + sqrt(3)*d**2)**(3/2)
+ I*c*(1 - I)*atanh((2*I*c*x + sqrt(3)*d)/(sqrt(-2*I*c**2 + sqrt(
3)*d**2)*sqrt(-2*I*x**2 + sqrt(3))))/(-2*I*c**2 + sqrt(3)*d**2)**
(3/2) - d*(1 + I)*sqrt(2*I*x**2 + sqrt(3))/(2*(c + d*x)*(2*I*c**2
 + sqrt(3)*d**2)) - d*(1 - I)*sqrt(-2*I*x**2 + sqrt(3))/(2*(c + d
*x)*(-2*I*c**2 + sqrt(3)*d**2))
```

Mathematica [A] time = 0.0979583, size = 0, normalized size = 0.

$$\int \frac{\sqrt{2x^2+\sqrt{3+4x^4}}}{(c+dx)^2\sqrt{3+4x^4}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[2*x^2 + Sqrt[3 + 4*x^4]]/((c + d*x)^2*Sqrt[3 + 4*x^4]),x]
```

```
[Out] Integrate[Sqrt[2*x^2 + Sqrt[3 + 4*x^4]]/((c + d*x)^2*Sqrt[3 + 4*x
^4]), x]
```

Maple [F] time = 0.039, size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)^2}\sqrt{2\,x^2+\sqrt{4\,x^4+3}}\frac{1}{\sqrt{4\,x^4+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)^2/(4*x^4+3)^(1/2),x)
```

```
[Out] int((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)^2/(4*x^4+3)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2\,x^2 + \sqrt{4\,x^4 + 3}}}{\sqrt{4\,x^4 + 3}(dx + c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(2*x^2 + sqrt(4*x^4 + 3))/(sqrt(4*x^4 + 3)*(d*x + c)^2),x, algorith
```

```
[Out] integrate(sqrt(2*x^2 + sqrt(4*x^4 + 3))/(sqrt(4*x^4 + 3)*(d*x + c
)^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(2*x^2 + sqrt(4*x^4 + 3))/(sqrt(4*x^4 + 3)*(d*x + c)^2),x, algorith
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2x^2 + \sqrt{4x^4 + 3}}}{(c + dx)^2\,\sqrt{4x^4 + 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+(4*x**4+3)**(1/2))**(1/2)/(d*x+c)**2/(4*x**4+3)**(1/2),x)
```

```
[Out] Integral(sqrt(2*x**2 + sqrt(4*x**4 + 3))/((c + d*x)**2*sqrt(4*x**
4 + 3)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2\,x^2+\sqrt{4\,x^4+3}}}{\sqrt{4\,x^4+3}(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(2*x^2 + sqrt(4*x^4 + 3))/(sqrt(4*x^4 + 3)*(d*x + c)^2),x, algorith
```

```
[Out] integrate(sqrt(2*x^2 + sqrt(4*x^4 + 3))/(sqrt(4*x^4 + 3)*(d*x + c
)^2), x)
```

3.773 $\int \frac{-4+x}{\left(1+\sqrt[3]{x}\right)\sqrt{x}}\,dx$

Optimal. Leaf size=41

$$\frac{6x^{7/6}}{7}-\frac{6x^{5/6}}{5}+2\sqrt{x}-30\sqrt[6]{x}+30\tan^{-1}\left(\sqrt[6]{x}\right)$$

```
[Out] -30*x^(1/6) + 2*Sqrt[x] - (6*x^(5/6))/5 + (6*x^(7/6))/7 + 30*ArcT
an[x^(1/6)]
```

Rubi [A] time = 0.0709741, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278

$$\frac{6x^{7/6}}{7}-\frac{6x^{5/6}}{5}+2\sqrt{x}-30\sqrt[6]{x}+30\tan^{-1}\left(\sqrt[6]{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(-4 + x)/((1 + x^(1/3))*Sqrt[x]),x]
```

```
[Out] -30*x^(1/6) + 2*Sqrt[x] - (6*x^(5/6))/5 + (6*x^(7/6))/7 + 30*ArcT
an[x^(1/6)]
```

Rubi in Sympy [A] time = 7.11213, size = 37, normalized size = 0.9

$$\frac{6x^{\frac{7}{6}}}{7}-\frac{6x^{\frac{5}{6}}}{5}-30\sqrt[6]{x}+2\sqrt{x}+30\,\mathrm{atan}\left(\sqrt[6]{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-4+x)/(1+x**(1/3))/x**(1/2),x)
```

```
[Out] 6*x**(7/6)/7 - 6*x**(5/6)/5 - 30*x**(1/6) + 2*sqrt(x) + 30*atan(x
**(1/6))
```

Mathematica [A] time = 0.0200002, size = 41, normalized size = 1.

$$\frac{6x^{7/6}}{7}-\frac{6x^{5/6}}{5}+2\sqrt{x}-30\sqrt[6]{x}+30\tan^{-1}\left(\sqrt[6]{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-4 + x)/((1 + x^(1/3))*Sqrt[x]),x]
```

```
[Out] -30*x^(1/6) + 2*Sqrt[x] - (6*x^(5/6))/5 + (6*x^(7/6))/7 + 30*ArcT
an[x^(1/6)]
```

Maple [A] time = 0.006, size = 28, normalized size = 0.7

$$-30\sqrt[6]{x}-\frac{6}{5}x^{\frac{5}{6}}+\frac{6}{7}x^{\frac{7}{6}}+30\,\arctan\left(\sqrt[6]{x}\right)+2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x-4)/(1+x^(1/3))/x^(1/2),x)
```

```
[Out] -30*x^(1/6)-6/5*x^(5/6)+6/7*x^(7/6)+30*arctan(x^(1/6))+2*x^(1/2)
```

Maxima [A] time = 0.755047, size = 36, normalized size = 0.88

$$\frac{6}{7}\,x^{\frac{7}{6}}-\frac{6}{5}\,x^{\frac{5}{6}}+2\sqrt{x}-30\,x^{\frac{1}{6}}+30\,\arctan\left(x^{\frac{1}{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - 4)/(sqrt(x)*(x^(1/3) + 1)),x, algorithm="maxima")
```

```
[Out] 6/7*x^(7/6) - 6/5*x^(5/6) + 2*sqrt(x) - 30*x^(1/6) + 30*arctan(x^
(1/6))
```

Fricas [A] time = 0.289274, size = 34, normalized size = 0.83

$$\frac{6}{7}\,(x-35)x^{\frac{1}{6}}-\frac{6}{5}\,x^{\frac{5}{6}}+2\sqrt{x}+30\,\arctan\left(x^{\frac{1}{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - 4)/(sqrt(x)*(x^(1/3) + 1)),x, algorithm="fricas")
```

```
[Out] 6/7*(x - 35)*x^(1/6) - 6/5*x^(5/6) + 2*sqrt(x) + 30*arctan(x^(1/6
))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-4}{\sqrt{x}\left(\sqrt[3]{x}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-4+x)/(1+x**(1/3))/x**(1/2),x)
```

```
[Out] Integral((x - 4)/(sqrt(x)*(x**(1/3) + 1)), x)
```

GIAC/XCAS [A] time = 0.260999, size = 36, normalized size = 0.88

$$\frac{6}{7}\,x^{\frac{7}{6}} - \frac{6}{5}\,x^{\frac{5}{6}} + 2\,\sqrt{x} - 30\,x^{\frac{1}{6}} + 30\,\arctan\left(x^{\frac{1}{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - 4)/(sqrt(x)*(x^(1/3) + 1)),x, algorithm="giac")
```

```
[Out] 6/7*x^(7/6) - 6/5*x^(5/6) + 2*sqrt(x) - 30*x^(1/6) + 30*arctan(x^
(1/6))
```

3.774 $\int \frac{1+\sqrt{x}}{x^{5/6}+x^{7/6}}\,dx$

Optimal. Leaf size=26

$$3\sqrt[3]{x}-3\log\left(\sqrt[3]{x}+1\right)+6\tan^{-1}\left(\sqrt[6]{x}\right)$$

```
[Out] 3*x^(1/3) + 6*ArcTan[x^(1/6)] - 3*Log[1 + x^(1/3)]
```

Rubi [A] time = 0.0605814, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$3\sqrt[3]{x}-3\log\left(\sqrt[3]{x}+1\right)+6\tan^{-1}\left(\sqrt[6]{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + Sqrt[x])/(x^(5/6) + x^(7/6)),x]
```

```
[Out] 3*x^(1/3) + 6*ArcTan[x^(1/6)] - 3*Log[1 + x^(1/3)]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-3\log\left(\sqrt[3]{x}+1\right)+6\,\mathrm{atan}\left(\sqrt[6]{x}\right)+6\int^{\sqrt[6]{x}} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+x**(1/2))/(x**(5/6)+x**(7/6)),x)
```

```
[Out] -3*log(x**(1/3) + 1) + 6*atan(x**(1/6)) + 6*Integral(x, (x, x**(1
/6)))
```

Mathematica [A] time = 0.0154289, size = 26, normalized size = 1.

$$3\sqrt[3]{x}-3\log\left(\sqrt[3]{x}+1\right)+6\tan^{-1}\left(\sqrt[6]{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + Sqrt[x])/(x^(5/6) + x^(7/6)),x]
```

```
[Out] 3*x^(1/3) + 6*ArcTan[x^(1/6)] - 3*Log[1 + x^(1/3)]
```

Maple [A] time = 0.005, size = 21, normalized size = 0.8

$$3\sqrt[3]{x}+6\arctan\left(\sqrt[6]{x}\right)-3\ln\left(1+\sqrt[3]{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x^(1/2))/(x^(5/6)+x^(7/6)),x)
```

```
[Out] 3*x^(1/3)+6*arctan(x^(1/6))-3*ln(1+x^(1/3))
```

Maxima [A] time = 0.755672, size = 27, normalized size = 1.04

$$3\,x^{\frac{1}{3}}+6\arctan\left(x^{\frac{1}{6}}\right)-3\log\left(x^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sqrt(x) + 1)/(x^(7/6) + x^(5/6)),x, algorithm="maxima")
```

```
[Out] 3*x^(1/3) + 6*arctan(x^(1/6)) - 3*log(x^(1/3) + 1)
```

Fricas [A] time = 0.278808, size = 27, normalized size = 1.04

$$3\,x^{\frac{1}{3}}+6\arctan\left(x^{\frac{1}{6}}\right)-3\log\left(x^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sqrt(x) + 1)/(x^(7/6) + x^(5/6)),x, algorithm="fricas")
```

```
[Out] 3*x^(1/3) + 6*arctan(x^(1/6)) - 3*log(x^(1/3) + 1)
```

Sympy [A] time = 26.8895, size = 24, normalized size = 0.92

$$3\sqrt[3]{x}-3\log\left(\sqrt[3]{x}+1\right)+6\operatorname{atan}\left(\sqrt[6]{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+x**(1/2))/(x**(5/6)+x**(7/6)),x)
```

```
[Out] 3*x**(1/3) - 3*log(x**(1/3) + 1) + 6*atan(x**(1/6))
```

GIAC/XCAS [A] time = 0.262336, size = 27, normalized size = 1.04

$$3\,x^{\frac{1}{3}} + 6\,\arctan\left(x^{\frac{1}{6}}\right) - 3\,\ln\left(x^{\frac{1}{3}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(x) + 1)/(x^(7/6) + x^(5/6)),x, algorithm="giac")
```

```
[Out] 3*x^(1/3) + 6*arctan(x^(1/6)) - 3*ln(x^(1/3) + 1)
```

3.775 $\int \frac{1+\sqrt{x}}{\left(1+\sqrt[3]{x}\right)\sqrt{x}}\,dx$

Optimal. Leaf size=42

$$\frac{3x^{2/3}}{2} - 3\sqrt[3]{x} + 6\sqrt[6]{x} + 3\log\left(\sqrt[3]{x}+1\right) - 6\tan^{-1}\left(\sqrt[6]{x}\right)$$

```
[Out] 6*x^(1/6) - 3*x^(1/3) + (3*x^(2/3))/2 - 6*ArcTan[x^(1/6)] + 3*Log
[1 + x^(1/3)]
```

Rubi [A] time = 0.241455, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273

$$\frac{3x^{2/3}}{2} - 3\sqrt[3]{x} + 6\sqrt[6]{x} + 3\log\left(\sqrt[3]{x}+1\right) - 6\tan^{-1}\left(\sqrt[6]{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + Sqrt[x])/((1 + x^(1/3))*Sqrt[x]),x]
```

```
[Out] 6*x^(1/6) - 3*x^(1/3) + (3*x^(2/3))/2 - 6*ArcTan[x^(1/6)] + 3*Log
[1 + x^(1/3)]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$2\int^{\sqrt{x}} \frac{\sqrt{x^2}+1}{\sqrt[3]{x^2}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+x**(1/2))/(1+x**(1/3))/x**(1/2),x)
```

```
[Out] 2*Integral((sqrt(x**2) + 1)/((x**2)**(1/3) + 1), (x, sqrt(x)))
```

Mathematica [A] time = 0.0183225, size = 42, normalized size = 1.

$$\frac{3x^{2/3}}{2} - 3\sqrt[3]{x} + 6\sqrt[6]{x} + 3\log\left(\sqrt[3]{x}+1\right) - 6\tan^{-1}\left(\sqrt[6]{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + Sqrt[x])/((1 + x^(1/3))*Sqrt[x]),x]
```

```
[Out] 6*x^(1/6) - 3*x^(1/3) + (3*x^(2/3))/2 - 6*ArcTan[x^(1/6)] + 3*Log
[1 + x^(1/3)]
```

Maple [A] time = 0.006, size = 48, normalized size = 1.1

$$\ln(1+x)+\frac{3}{2}x^{\frac{2}{3}}+2\ln\left(1+\sqrt[3]{x}\right)-\ln\left(x^{\frac{2}{3}}-\sqrt[3]{x}+1\right)-3\sqrt[3]{x}+6\sqrt[6]{x}-6\arctan\left(\sqrt[6]{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+x^(1/2))/(1+x^(1/3))/x^(1/2),x)
```

```
[Out] ln(1+x)+3/2*x^(2/3)+2*ln(1+x^(1/3))-ln(x^(2/3)-x^(1/3)+1)-3*x^(1/
3)+6*x^(1/6)-6*arctan(x^(1/6))
```

Maxima [A] time = 0.758112, size = 41, normalized size = 0.98

$$\frac{3}{2}x^{\frac{2}{3}}-3x^{\frac{1}{3}}+6x^{\frac{1}{6}}-6\arctan\left(x^{\frac{1}{6}}\right)+3\log\left(x^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sqrt(x) + 1)/(sqrt(x)*(x^(1/3) + 1)),x, algorithm="maxima")
```

```
[Out] 3/2*x^(2/3) - 3*x^(1/3) + 6*x^(1/6) - 6*arctan(x^(1/6)) + 3*log(x
^(1/3) + 1)
```

Fricas [A] time = 0.280949, size = 41, normalized size = 0.98

$$\frac{3}{2}x^{\frac{2}{3}}-3x^{\frac{1}{3}}+6x^{\frac{1}{6}}-6\arctan\left(x^{\frac{1}{6}}\right)+3\log\left(x^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sqrt(x) + 1)/(sqrt(x)*(x^(1/3) + 1)),x, algorithm="fricas")
```

```
[Out] 3/2*x^(2/3) - 3*x^(1/3) + 6*x^(1/6) - 6*arctan(x^(1/6)) + 3*log(x
^(1/3) + 1)
```

Sympy [A] time = 18.2063, size = 39, normalized size = 0.93

$$6\sqrt[6]{x} + \frac{3x^{\frac{2}{3}}}{2} - 3\sqrt[3]{x} + 3\log\left(\sqrt[3]{x} + 1\right) - 6\operatorname{atan}\left(\sqrt[6]{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+x**(1/2))/(1+x**(1/3))/x**(1/2),x)
```

```
[Out] 6*x**(1/6) + 3*x**(2/3)/2 - 3*x**(1/3) + 3*log(x**(1/3) + 1) - 6*
atan(x**(1/6))
```

GIAC/XCAS [A] time = 0.261293, size = 41, normalized size = 0.98

$$\frac{3}{2}\,x^{\frac{2}{3}} - 3\,x^{\frac{1}{3}} + 6\,x^{\frac{1}{6}} - 6\,\arctan\left(x^{\frac{1}{6}}\right) + 3\ln\left(x^{\frac{1}{3}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(x) + 1)/(sqrt(x)*(x^(1/3) + 1)),x, algorithm="giac")
```

```
[Out] 3/2*x^(2/3) - 3*x^(1/3) + 6*x^(1/6) - 6*arctan(x^(1/6)) + 3*ln(x^
(1/3) + 1)
```

3.776 $\int \frac{\sqrt{2+\frac{b}{x^2}}}{b+2x^2} \, dx$

Optimal. Leaf size=20

$$-\frac{\operatorname{csch}^{-1}\left(\frac{\sqrt{2}x}{\sqrt{b}}\right)}{\sqrt{b}}$$

```
[Out] -(ArcCsch[(Sqrt[2]*x)/Sqrt[b]]/Sqrt[b])
```

Rubi [A] time = 0.0293956, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$-\frac{\operatorname{csch}^{-1}\left(\frac{\sqrt{2}x}{\sqrt{b}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[2 + b/x^2]/(b + 2*x^2),x]

[Out] -(ArcCsch[(Sqrt[2]*x)/Sqrt[b]]/Sqrt[b])
```

Rubi in Sympy [A] time = 2.80731, size = 20, normalized size = 1.

$$-\frac{\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}}{2x}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2+b/x**2)**(1/2)/(2*x**2+b),x)

[Out] -asinh(sqrt(2)*sqrt(b)/(2*x))/sqrt(b)
```

Mathematica [B] time = 0.0424659, size = 54, normalized size = 2.7

$$\frac{x\sqrt{\frac{b}{x^2}+2}\left(\log(x)-\log\left(\sqrt{b}\sqrt{b+2x^2}+b\right)\right)}{\sqrt{b}\sqrt{b+2x^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[2 + b/x^2]/(b + 2*x^2),x]
```

```
[Out] (Sqrt[2 + b/x^2]*x*(Log[x] - Log[b + Sqrt[b]*Sqrt[b + 2*x^2]]))/(
Sqrt[b]*Sqrt[b + 2*x^2])
```

Maple [B] time = 0.015, size = 50, normalized size = 2.5

$$-x\sqrt{\frac{2x^{2}+b}{x^{2}}}\ln\left(2\frac{\sqrt{b}\sqrt{2x^{2}+b}+b}{x}\right)\frac{1}{\sqrt{b}}\frac{1}{\sqrt{2x^{2}+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2+b/x^2)^(1/2)/(2*x^2+b),x)
```

```
[Out] -((2*x^2+b)/x^2)^(1/2)*x/(2*x^2+b)^(1/2)/b^(1/2)*ln(2*(b^(1/2)*(2
*x^2+b)^(1/2)+b)/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b/x^2 + 2)/(2*x^2 + b),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.287161, size = 1, normalized size = 0.05

$$\left[\frac{\log\left(\frac{bx\sqrt{\frac{2x^{2}+b}{x^{2}}}-(x^{2}+b)\sqrt{b}}{x^{2}}\right)}{2\sqrt{b}}, \frac{\sqrt{-b}\arctan\left(\frac{\sqrt{-b}}{x\sqrt{\frac{2x^{2}+b}{x^{2}}}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b/x^2 + 2)/(2*x^2 + b),x, algorithm="fricas")
```

```
[Out] [1/2*log((b*x*sqrt((2*x^2 + b)/x^2) - (x^2 + b)*sqrt(b))/x^2)/sqr
t(b), sqrt(-b)*arctan(sqrt(-b)/(x*sqrt((2*x^2 + b)/x^2)))/b]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{b}{x^2} + 2}}{b + 2x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2+b/x**2)**(1/2)/(2*x**2+b),x)
```

```
[Out] Integral(sqrt(b/x**2 + 2)/(b + 2*x**2), x)
```

GIAC/XCAS [A] time = 0.263193, size = 59, normalized size = 2.95

$$\frac{\arctan\left(\frac{\sqrt{2\,x^2+b}}{\sqrt{-b}}\right)\operatorname{sign}(x)}{\sqrt{-b}} - \frac{\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)\operatorname{sign}(x)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b/x^2 + 2)/(2*x^2 + b),x, algorithm="giac")
```

```
[Out] arctan(sqrt(2*x^2 + b)/sqrt(-b))*sign(x)/sqrt(-b) - arctan(sqrt(b
)/sqrt(-b))*sign(x)/sqrt(-b)
```

3.777 $\int \frac{\sqrt{2-\frac{b}{x^2}}}{-b+2x^2}\,dx$

Optimal. Leaf size=20

$$-\frac{\csc^{-1}\left(\frac{\sqrt{2}x}{\sqrt{b}}\right)}{\sqrt{b}}$$

```
[Out] -(ArcCsc[(Sqrt[2]*x)/Sqrt[b]]/Sqrt[b])
```

Rubi [A] time = 0.0305389, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$-\frac{\csc^{-1}\left(\frac{\sqrt{2}x}{\sqrt{b}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[2 - b/x^2]/(-b + 2*x^2),x]

[Out] -(ArcCsc[(Sqrt[2]*x)/Sqrt[b]]/Sqrt[b])
```

Rubi in Sympy [A] time = 3.35642, size = 20, normalized size = 1.

$$-\frac{\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}}{2x}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2-b/x**2)**(1/2)/(2*x**2-b),x)

[Out] -asin(sqrt(2)*sqrt(b)/(2*x))/sqrt(b)
```

Mathematica [C] time = 0.0451672, size = 64, normalized size = 3.2

$$-\frac{ix\sqrt{2-\frac{b}{x^2}}\log\left(\frac{2\left(\sqrt{2x^2-b}-i\sqrt{b}\right)}{x}\right)}{\sqrt{b}\sqrt{2x^2-b}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[2 - b/x^2]/(-b + 2*x^2),x]
```

```
[Out] ((-I)*Sqrt[2 - b/x^2]*x*Log[(2*((-I)*Sqrt[b] + Sqrt[-b + 2*x^2]))
/x])/(Sqrt[b]*Sqrt[-b + 2*x^2])
```

Maple [B] time = 0.013, size = 62, normalized size = 3.1

$$-x\sqrt{\frac{2x^2-b}{x^2}}\ln\left(2\,\frac{\sqrt{-b}\sqrt{2x^2-b}-b}{x}\right)\frac{1}{\sqrt{-b}}\frac{1}{\sqrt{2x^2-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2-b/x^2)^(1/2)/(2*x^2-b),x)
```

```
[Out] -((2*x^2-b)/x^2)^(1/2)*x/(2*x^2-b)^(1/2)/(-b)^(1/2)*ln(2*((-b)^(1
/2)*(2*x^2-b)^(1/2)-b)/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-b/x^2 + 2)/(2*x^2 - b),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.287642, size = 1, normalized size = 0.05

$$\left[-\frac{\sqrt{-b}\log\left(-\frac{bx\sqrt{\frac{2x^2-b}{x^2}}+(x^2-b)\sqrt{-b}}{x^2}\right)}{2b}, -\frac{\arctan\left(\frac{\sqrt{b}}{x\sqrt{\frac{2x^2-b}{x^2}}}\right)}{\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-b/x^2 + 2)/(2*x^2 - b),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b)*log(-(b*x*sqrt((2*x^2 - b)/x^2) + (x^2 - b)*sqrt(-
b))/x^2)/b, -arctan(sqrt(b)/(x*sqrt((2*x^2 - b)/x^2)))/sqrt(b)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-\frac{b}{x^2}+2}}{-b+2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2-b/x**2)**(1/2)/(2*x**2-b),x)
```

```
[Out] Integral(sqrt(-b/x**2 + 2)/(-b + 2*x**2), x)
```

GIAC/XCAS [A] time = 0.266131, size = 54, normalized size = 2.7

$$\frac{\arctan\left(\frac{\sqrt{2\,x^2-b}}{\sqrt{b}}\right)\operatorname{sign}(x)}{\sqrt{b}}-\frac{\arctan\left(\frac{\sqrt{-b}}{\sqrt{b}}\right)\operatorname{sign}(x)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-b/x^2 + 2)/(2*x^2 - b),x, algorithm="giac")
```

```
[Out] arctan(sqrt(2*x^2 - b)/sqrt(b))*sign(x)/sqrt(b) - arctan(sqrt(-b)
/sqrt(b))*sign(x)/sqrt(b)
```

3.778 $\int \frac{\sqrt{a+\frac{c}{x^2}}}{d+ex}\,dx$

Optimal. Leaf size=121

$$-\frac{\sqrt{ad^2+ce^2}\tanh^{-1}\left(\frac{ad-\frac{ce}{x}}{\sqrt{a+\frac{c}{x^2}}\sqrt{ad^2+ce^2}}\right)}{de}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}}{x\sqrt{a+\frac{c}{x^2}}}\right)}{d}+\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+\frac{c}{x^2}}}{\sqrt{a}}\right)}{e}$$

```
[Out] (Sqrt[a]*ArcTanh[Sqrt[a + c/x^2]/Sqrt[a]])/e - (Sqrt[a*d^2 + c*e^
2]*ArcTanh[(a*d - (c*e)/x)/(Sqrt[a*d^2 + c*e^2]*Sqrt[a + c/x^2])]
)/(d*e) - (Sqrt[c]*ArcTanh[Sqrt[c]/(Sqrt[a + c/x^2]*x)])/d
```

Rubi [A] time = 0.466512, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$

$$-\frac{\sqrt{ad^2+ce^2}\tanh^{-1}\left(\frac{ad-\frac{ce}{x}}{\sqrt{a+\frac{c}{x^2}}\sqrt{ad^2+ce^2}}\right)}{de}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}}{x\sqrt{a+\frac{c}{x^2}}}\right)}{d}+\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+\frac{c}{x^2}}}{\sqrt{a}}\right)}{e}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + c/x^2]/(d + e*x),x]
```

```
[Out] (Sqrt[a]*ArcTanh[Sqrt[a + c/x^2]/Sqrt[a]])/e - (Sqrt[a*d^2 + c*e^
2]*ArcTanh[(a*d - (c*e)/x)/(Sqrt[a*d^2 + c*e^2]*Sqrt[a + c/x^2])]
)/(d*e) - (Sqrt[c]*ArcTanh[Sqrt[c]/(Sqrt[a + c/x^2]*x)])/d
```

Rubi in Sympy [A] time = 19.9262, size = 99, normalized size = 0.82

$$\frac{\sqrt{a}\operatorname{atanh}\left(\frac{\sqrt{a+\frac{c}{x^2}}}{\sqrt{a}}\right)}{e}-\frac{\sqrt{c}\operatorname{atanh}\left(\frac{\sqrt{c}}{x\sqrt{a+\frac{c}{x^2}}}\right)}{d}-\frac{\sqrt{ad^2+ce^2}\operatorname{atanh}\left(\frac{ad-\frac{ce}{x}}{\sqrt{a+\frac{c}{x^2}}\sqrt{ad^2+ce^2}}\right)}{de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+c/x**2)**(1/2)/(e*x+d),x)
```

```
[Out] sqrt(a)*atanh(sqrt(a + c/x**2)/sqrt(a))/e - sqrt(c)*atanh(sqrt(c)
/(x*sqrt(a + c/x**2)))/d - sqrt(a*d**2 + c*e**2)*atanh((a*d - c*e
```

```
/x)/(sqrt(a + c/x**2)*sqrt(a*d**2 + c*e**2)))/(d*e)
```

Mathematica [A] time = 0.163435, size = 173, normalized size = 1.43

$$\frac{x\sqrt{a+\frac{c}{x^2}}\left(\sqrt{ad^2+ce^2}\log\left(\sqrt{ax^2+c}\sqrt{ad^2+ce^2}-adx+ce\right)-\sqrt{ad^2+ce^2}\log(d+ex)+\sqrt{a}d\log\left(\sqrt{a}\sqrt{ax^2+c}+ax\right)-\sqrt{ce}\right.}{de\sqrt{ax^2+c}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + c/x^2]/(d + e*x),x]
```

```
[Out] (Sqrt[a + c/x^2]*x*(Sqrt[c]*e*Log[x] - Sqrt[a*d^2 + c*e^2]*Log[d
+ e*x] + Sqrt[a]*d*Log[a*x + Sqrt[a]*Sqrt[c + a*x^2]] - Sqrt[c]*e
*Log[c + Sqrt[c]*Sqrt[c + a*x^2]] + Sqrt[a*d^2 + c*e^2]*Log[c*e -
 a*d*x + Sqrt[a*d^2 + c*e^2]*Sqrt[c + a*x^2]]))/(d*e*Sqrt[c + a*x
^2])
```

Maple [B] time = 0.042, size = 247, normalized size = 2.

$$-\frac{x}{e^2d}\sqrt{\frac{ax^2+c}{x^2}}\left(\sqrt{c}\ln\left(2\,\frac{\sqrt{c}\sqrt{ax^2+c}+c}{x}\right)e^2\sqrt{\frac{ad^2+e^2c}{e^2}}-\sqrt{a}\ln\left(1\left(\sqrt{ax^2+c}\sqrt{a}+ax\right)\frac{1}{\sqrt{a}}\right)de\sqrt{\frac{ad^2+e^2c}{e^2}}-d^2\ln\left(2\,\frac{}{ex}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+c/x^2)^(1/2)/(e*x+d),x)
```

```
[Out] -((a*x^2+c)/x^2)^(1/2)*x*(c^(1/2)*ln(2*(c^(1/2)*(a*x^2+c)^(1/2)+c
)/x)*e^2*((a*d^2+c*e^2)/e^2)^(1/2)-a^(1/2)*ln(((a*x^2+c)^(1/2)*a^
(1/2)+a*x)/a^(1/2))*d*e*((a*d^2+c*e^2)/e^2)^(1/2)-d^2*ln(2*((a*x^
2+c)^(1/2)*((a*d^2+c*e^2)/e^2)^(1/2)*e-a*d*x+c*e)/(e*x+d))*a-ln(2
*((a*x^2+c)^(1/2)*((a*d^2+c*e^2)/e^2)^(1/2)*e-a*d*x+c*e)/(e*x+d))
*c*e^2)/(a*x^2+c)^(1/2)/d/e^2/((a*d^2+c*e^2)/e^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a + c/x^2)/(e*x + d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.03245, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a + c/x^2)/(e*x + d),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*d*log(-2*a*x^2 - 2*sqrt(a)*x^2*sqrt((a*x^2 + c)/x^2
) - c) + sqrt(c)*e*log(-(a*x^2 - 2*sqrt(c)*x*sqrt((a*x^2 + c)/x^2
) + 2*c)/x^2) + sqrt(a*d^2 + c*e^2)*log((2*a*c*d*e*x - a*c*d^2 -
2*c^2*e^2 - (2*a^2*d^2 + a*c*e^2)*x^2 + 2*(a*d*x^2 - c*e*x)*sqrt(
a*d^2 + c*e^2)*sqrt((a*x^2 + c)/x^2))/(e^2*x^2 + 2*d*e*x + d^2)))
/(d*e), 1/2*(2*sqrt(-a)*d*arctan(a/(sqrt(-a)*sqrt((a*x^2 + c)/x^2
))) + sqrt(c)*e*log(-(a*x^2 - 2*sqrt(c)*x*sqrt((a*x^2 + c)/x^2) +
 2*c)/x^2) + sqrt(a*d^2 + c*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*c
^2*e^2 - (2*a^2*d^2 + a*c*e^2)*x^2 + 2*(a*d*x^2 - c*e*x)*sqrt(a*d
^2 + c*e^2)*sqrt((a*x^2 + c)/x^2))/(e^2*x^2 + 2*d*e*x + d^2)))/(d
*e), 1/2*(sqrt(a)*d*log(-2*a*x^2 - 2*sqrt(a)*x^2*sqrt((a*x^2 + c)
/x^2) - c) + sqrt(c)*e*log(-(a*x^2 - 2*sqrt(c)*x*sqrt((a*x^2 + c)
/x^2) + 2*c)/x^2) - 2*sqrt(-a*d^2 - c*e^2)*arctan((a*d*x - c*e)/(
sqrt(-a*d^2 - c*e^2)*x*sqrt((a*x^2 + c)/x^2))))/(d*e), 1/2*(2*sqr
t(-a)*d*arctan(a/(sqrt(-a)*sqrt((a*x^2 + c)/x^2))) + sqrt(c)*e*lo
g(-(a*x^2 - 2*sqrt(c)*x*sqrt((a*x^2 + c)/x^2) + 2*c)/x^2) - 2*sqr
t(-a*d^2 - c*e^2)*arctan((a*d*x - c*e)/(sqrt(-a*d^2 - c*e^2)*x*sq
rt((a*x^2 + c)/x^2))))/(d*e), -1/2*(2*sqrt(-c)*e*arctan(c/(sqrt(-
c)*x*sqrt((a*x^2 + c)/x^2))) - sqrt(a)*d*log(-2*a*x^2 - 2*sqrt(a)
*x^2*sqrt((a*x^2 + c)/x^2) - c) - sqrt(a*d^2 + c*e^2)*log((2*a*c*
d*e*x - a*c*d^2 - 2*c^2*e^2 - (2*a^2*d^2 + a*c*e^2)*x^2 + 2*(a*d*
x^2 - c*e*x)*sqrt(a*d^2 + c*e^2)*sqrt((a*x^2 + c)/x^2))/(e^2*x^2
+ 2*d*e*x + d^2)))/(d*e), 1/2*(2*sqrt(-a)*d*arctan(a/(sqrt(-a)*sq
rt((a*x^2 + c)/x^2))) - 2*sqrt(-c)*e*arctan(c/(sqrt(-c)*x*sqrt((a
*x^2 + c)/x^2))) + sqrt(a*d^2 + c*e^2)*log((2*a*c*d*e*x - a*c*d^2
 - 2*c^2*e^2 - (2*a^2*d^2 + a*c*e^2)*x^2 + 2*(a*d*x^2 - c*e*x)*sq
rt(a*d^2 + c*e^2)*sqrt((a*x^2 + c)/x^2))/(e^2*x^2 + 2*d*e*x + d^2
)))/(d*e), -1/2*(2*sqrt(-c)*e*arctan(c/(sqrt(-c)*x*sqrt((a*x^2 +
c)/x^2))) - sqrt(a)*d*log(-2*a*x^2 - 2*sqrt(a)*x^2*sqrt((a*x^2 +
c)/x^2) - c) + 2*sqrt(-a*d^2 - c*e^2)*arctan((a*d*x - c*e)/(sqrt(
-a*d^2 - c*e^2)*x*sqrt((a*x^2 + c)/x^2))))/(d*e), (sqrt(-a)*d*arc
tan(a/(sqrt(-a)*sqrt((a*x^2 + c)/x^2))) - sqrt(-c)*e*arctan(c/(sq
rt(-c)*x*sqrt((a*x^2 + c)/x^2))) - sqrt(-a*d^2 - c*e^2)*arctan((a
*d*x - c*e)/(sqrt(-a*d^2 - c*e^2)*x*sqrt((a*x^2 + c)/x^2))))/(d*e
)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + \frac{c}{x^2}}}{d + ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+c/x**2)**(1/2)/(e*x+d),x)

[Out] Integral(sqrt(a + c/x**2)/(d + e*x), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a + c/x^2)/(e*x + d),x, algorithm="giac")

[Out] Exception raised: TypeError
```

3.779 $\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}{d+ex}\,dx$

Optimal. Leaf size=181

$$-\frac{\sqrt{ad^2-e(bd-ce)}\tanh^{-1}\left(\frac{2ad+\frac{bd-2ce}{x}-be}{2\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{ad^2-e(bd-ce)}}\right)}{de}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{d}+\frac{\sqrt{a}\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{e}$$

```
[Out] (Sqrt[a]*ArcTanh[(2*a + b/x)/(2*Sqrt[a]*Sqrt[a + c/x^2 + b/x])])/
e - (Sqrt[c]*ArcTanh[(b + (2*c)/x)/(2*Sqrt[c]*Sqrt[a + c/x^2 + b/
x])])/d - (Sqrt[a*d^2 - e*(b*d - c*e)]*ArcTanh[(2*a*d - b*e + (b*
d - 2*c*e)/x)/(2*Sqrt[a*d^2 - e*(b*d - c*e)]*Sqrt[a + c/x^2 + b/x
])])/(d*e)
```

Rubi [A] time = 0.733715, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292

$$-\frac{\sqrt{ad^2-e(bd-ce)}\tanh^{-1}\left(\frac{2ad+\frac{bd-2ce}{x}-be}{2\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{ad^2-e(bd-ce)}}\right)}{de}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{d}+\frac{\sqrt{a}\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{e}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + c/x^2 + b/x]/(d + e*x),x]
```



```
[Out] (Sqrt[a]*ArcTanh[(2*a + b/x)/(2*Sqrt[a]*Sqrt[a + c/x^2 + b/x])])/
e - (Sqrt[c]*ArcTanh[(b + (2*c)/x)/(2*Sqrt[c]*Sqrt[a + c/x^2 + b/
x])])/d - (Sqrt[a*d^2 - e*(b*d - c*e)]*ArcTanh[(2*a*d - b*e + (b*
d - 2*c*e)/x)/(2*Sqrt[a*d^2 - e*(b*d - c*e)]*Sqrt[a + c/x^2 + b/x
])])/(d*e)
```

Rubi in Sympy [A] time = 32.4875, size = 146, normalized size = 0.81

$$\frac{\sqrt{a}\operatorname{atanh}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{e}-\frac{\sqrt{c}\operatorname{atanh}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{d}-\frac{\sqrt{ad^2-bde+ce^2}\operatorname{atanh}\left(\frac{2ad-be+\frac{bd-2ce}{x}}{2\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{ad^2-bde+ce^2}}\right)}{de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+c/x**2+b/x)**(1/2)/(e*x+d),x)
```

```
[Out] sqrt(a)*atanh((2*a + b/x)/(2*sqrt(a)*sqrt(a + b/x + c/x**2)))/e -
 sqrt(c)*atanh((b + 2*c/x)/(2*sqrt(c)*sqrt(a + b/x + c/x**2)))/d
- sqrt(a*d**2 - b*d*e + c*e**2)*atanh((2*a*d - b*e + (b*d - 2*c*e
)/x)/(2*sqrt(a + b/x + c/x**2)*sqrt(a*d**2 - b*d*e + c*e**2)))/(d
*e)
```

Mathematica [A] time = 0.480269, size = 219, normalized size = 1.21

$$\frac{x\sqrt{a+\frac{bx+c}{x^2}}\left(-\log(d+ex)\sqrt{ad^2-bde+ce^2}+\sqrt{ad^2-bde+ce^2}\log\left(2\sqrt{x(ax+b)+c}\sqrt{ad^2-bde+ce^2}-2adx-bd+bex\right.\right.}{de\sqrt{x(ax+b)+c}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + c/x^2 + b/x]/(d + e*x),x]
```

```
[Out] (x*Sqrt[a + (c + b*x)/x^2]*(Sqrt[c]*e*Log[x] - Sqrt[a*d^2 - b*d*e
 + c*e^2]*Log[d + e*x] + Sqrt[a]*d*Log[b + 2*a*x + 2*Sqrt[a]*Sqrt
[c + x*(b + a*x)]] - Sqrt[c]*e*Log[2*c + b*x + 2*Sqrt[c]*Sqrt[c +
 x*(b + a*x)]] + Sqrt[a*d^2 - b*d*e + c*e^2]*Log[-(b*d) + 2*c*e -
 2*a*d*x + b*e*x + 2*Sqrt[a*d^2 - b*d*e + c*e^2]*Sqrt[c + x*(b +
a*x)]]))/(d*e*Sqrt[c + x*(b + a*x)])
```

Maple [B] time = 0.046, size = 385, normalized size = 2.1

$$-\frac{x}{e^2d}\sqrt{\frac{ax^2+bx+c}{x^2}}\left(\sqrt{c}\ln\left(\frac{1}{x}\left(2\,c+bx+2\,\sqrt{c}\sqrt{ax^2+bx+c}\right)\right)e^2\sqrt{\frac{ad^2-bde+e^2c}{e^2}}-\ln\left(\frac{1}{2}\left(2\,\sqrt{ax^2+bx+c}\sqrt{a}+2\,ax+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+c/x^2+b/x)^(1/2)/(e*x+d),x)
```

```
[Out] -((a*x^2+b*x+c)/x^2)^(1/2)*x*(c^(1/2)*ln((2*c+b*x+2*c^(1/2)*(a*x^
2+b*x+c)^(1/2))/x)*e^2*((a*d^2-b*d*e+c*e^2)/e^2)^(1/2)-ln(1/2*(2*
(a*x^2+b*x+c)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^(1/2)*d*e*((a*d^2
-b*d*e+c*e^2)/e^2)^(1/2)-d^2*ln((2*(a*x^2+b*x+c)^(1/2)*((a*d^2-b*
d*e+c*e^2)/e^2)^(1/2)*e-2*a*d*x+b*e*x-b*d+2*c*e)/(e*x+d))*a+ln((2
*(a*x^2+b*x+c)^(1/2)*((a*d^2-b*d*e+c*e^2)/e^2)^(1/2)*e-2*a*d*x+b*
e*x-b*d+2*c*e)/(e*x+d))*b*d*e-ln((2*(a*x^2+b*x+c)^(1/2)*((a*d^2-b
*d*e+c*e^2)/e^2)^(1/2)*e-2*a*d*x+b*e*x-b*d+2*c*e)/(e*x+d))*c*e^2)
/(a*x^2+b*x+c)^(1/2)/d/e^2/((a*d^2-b*d*e+c*e^2)/e^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a + b/x + c/x^2)/(e*x + d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 74.2548, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a + b/x + c/x^2)/(e*x + d),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*d*log(-8*a^2*x^2 - 8*a*b*x - b^2 - 4*a*c - 4*(2*a*x
^2 + b*x)*sqrt(a)*sqrt((a*x^2 + b*x + c)/x^2)) + sqrt(c)*e*log(-(
8*b*c*x + (b^2 + 4*a*c)*x^2 + 8*c^2 - 4*(b*x^2 + 2*c*x)*sqrt(c)*s
qrt((a*x^2 + b*x + c)/x^2))/x^2) + sqrt(a*d^2 - b*d*e + c*e^2)*lo
g((8*b*c*d*e - 8*c^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*a^2*d^2 - 8*a*b
*d*e + (b^2 + 4*a*c)*e^2)*x^2 - 2*(4*a*b*d^2 + 4*b*c*e^2 - (3*b^2
 + 4*a*c)*d*e)*x + 4*sqrt(a*d^2 - b*d*e + c*e^2)*((2*a*d - b*e)*x
^2 + (b*d - 2*c*e)*x)*sqrt((a*x^2 + b*x + c)/x^2))/(e^2*x^2 + 2*d
*e*x + d^2)))/(d*e), 1/2*(2*sqrt(-a)*d*arctan(1/2*(2*a*x + b)/(sq
rt(-a)*x*sqrt((a*x^2 + b*x + c)/x^2))) + sqrt(c)*e*log(-(8*b*c*x
+ (b^2 + 4*a*c)*x^2 + 8*c^2 - 4*(b*x^2 + 2*c*x)*sqrt(c)*sqrt((a*x
^2 + b*x + c)/x^2))/x^2) + sqrt(a*d^2 - b*d*e + c*e^2)*log((8*b*c
*d*e - 8*c^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*a^2*d^2 - 8*a*b*d*e + (
b^2 + 4*a*c)*e^2)*x^2 - 2*(4*a*b*d^2 + 4*b*c*e^2 - (3*b^2 + 4*a*c
)*d*e)*x + 4*sqrt(a*d^2 - b*d*e + c*e^2)*((2*a*d - b*e)*x^2 + (b*
d - 2*c*e)*x)*sqrt((a*x^2 + b*x + c)/x^2))/(e^2*x^2 + 2*d*e*x + d
^2)))/(d*e), 1/2*(sqrt(a)*d*log(-8*a^2*x^2 - 8*a*b*x - b^2 - 4*a*
```

```
c - 4*(2*a*x^2 + b*x)*sqrt(a)*sqrt((a*x^2 + b*x + c)/x^2)) + sqrt
(c)*e*log(-(8*b*c*x + (b^2 + 4*a*c)*x^2 + 8*c^2 - 4*(b*x^2 + 2*c*
x)*sqrt(c)*sqrt((a*x^2 + b*x + c)/x^2))/x^2) + 2*sqrt(-a*d^2 + b*
d*e - c*e^2)*arctan(-1/2*(b*d - 2*c*e + (2*a*d - b*e)*x)/(sqrt(-a
*d^2 + b*d*e - c*e^2)*x*sqrt((a*x^2 + b*x + c)/x^2)))))/(d*e), 1/2
*(2*sqrt(-a)*d*arctan(1/2*(2*a*x + b)/(sqrt(-a)*x*sqrt((a*x^2 + b
*x + c)/x^2))) + sqrt(c)*e*log(-(8*b*c*x + (b^2 + 4*a*c)*x^2 + 8*
c^2 - 4*(b*x^2 + 2*c*x)*sqrt(c)*sqrt((a*x^2 + b*x + c)/x^2))/x^2)
 + 2*sqrt(-a*d^2 + b*d*e - c*e^2)*arctan(-1/2*(b*d - 2*c*e + (2*a
*d - b*e)*x)/(sqrt(-a*d^2 + b*d*e - c*e^2)*x*sqrt((a*x^2 + b*x +
c)/x^2)))))/(d*e), -1/2*(2*sqrt(-c)*e*arctan(1/2*(b*x + 2*c)/(sqrt
(-c)*x*sqrt((a*x^2 + b*x + c)/x^2))) - sqrt(a)*d*log(-8*a^2*x^2 -
 8*a*b*x - b^2 - 4*a*c - 4*(2*a*x^2 + b*x)*sqrt(a)*sqrt((a*x^2 +
b*x + c)/x^2)) - sqrt(a*d^2 - b*d*e + c*e^2)*log((8*b*c*d*e - 8*c
^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*a^2*d^2 - 8*a*b*d*e + (b^2 + 4*a*
c)*e^2)*x^2 - 2*(4*a*b*d^2 + 4*b*c*e^2 - (3*b^2 + 4*a*c)*d*e)*x +
 4*sqrt(a*d^2 - b*d*e + c*e^2)*((2*a*d - b*e)*x^2 + (b*d - 2*c*e)
*x)*sqrt((a*x^2 + b*x + c)/x^2))/(e^2*x^2 + 2*d*e*x + d^2)))/(d*e
), 1/2*(2*sqrt(-a)*d*arctan(1/2*(2*a*x + b)/(sqrt(-a)*x*sqrt((a*x
^2 + b*x + c)/x^2))) - 2*sqrt(-c)*e*arctan(1/2*(b*x + 2*c)/(sqrt(
-c)*x*sqrt((a*x^2 + b*x + c)/x^2))) + sqrt(a*d^2 - b*d*e + c*e^2)
*log((8*b*c*d*e - 8*c^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*a^2*d^2 - 8*
a*b*d*e + (b^2 + 4*a*c)*e^2)*x^2 - 2*(4*a*b*d^2 + 4*b*c*e^2 - (3*
b^2 + 4*a*c)*d*e)*x + 4*sqrt(a*d^2 - b*d*e + c*e^2)*((2*a*d - b*e
)*x^2 + (b*d - 2*c*e)*x)*sqrt((a*x^2 + b*x + c)/x^2))/(e^2*x^2 +
2*d*e*x + d^2)))/(d*e), -1/2*(2*sqrt(-c)*e*arctan(1/2*(b*x + 2*c)
/(sqrt(-c)*x*sqrt((a*x^2 + b*x + c)/x^2))) - sqrt(a)*d*log(-8*a^2
*x^2 - 8*a*b*x - b^2 - 4*a*c - 4*(2*a*x^2 + b*x)*sqrt(a)*sqrt((a*
x^2 + b*x + c)/x^2)) - 2*sqrt(-a*d^2 + b*d*e - c*e^2)*arctan(-1/2
*(b*d - 2*c*e + (2*a*d - b*e)*x)/(sqrt(-a*d^2 + b*d*e - c*e^2)*x*
sqrt((a*x^2 + b*x + c)/x^2))))/(d*e), (sqrt(-a)*d*arctan(1/2*(2*a
*x + b)/(sqrt(-a)*x*sqrt((a*x^2 + b*x + c)/x^2))) - sqrt(-c)*e*ar
ctan(1/2*(b*x + 2*c)/(sqrt(-c)*x*sqrt((a*x^2 + b*x + c)/x^2))) +
sqrt(-a*d^2 + b*d*e - c*e^2)*arctan(-1/2*(b*d - 2*c*e + (2*a*d -
b*e)*x)/(sqrt(-a*d^2 + b*d*e - c*e^2)*x*sqrt((a*x^2 + b*x + c)/x^
2))))/(d*e)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + \frac{b}{x} + \frac{c}{x^2}}}{d + ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+c/x**2+b/x)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(sqrt(a + b/x + c/x**2)/(d + e*x), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a + b/x + c/x^2)/(e*x + d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.780 $\int \frac{\sqrt[6]{x}+\sqrt[5]{x^3}}{\sqrt{x}}\,dx$

Optimal. Leaf size=26

$$\frac{3x^{2/3}}{2}+\frac{10}{11}\sqrt[5]{x^3}\sqrt{x}$$

```
[Out] (3*x^(2/3))/2 + (10*Sqrt[x]*(x^3)^(1/5))/11
```

Rubi [A] time = 0.0149048, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$\frac{3x^{2/3}}{2}+\frac{10}{11}\sqrt[5]{x^3}\sqrt{x}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^(1/6) + (x^3)^(1/5))/Sqrt[x],x]
```

```
[Out] (3*x^(2/3))/2 + (10*Sqrt[x]*(x^3)^(1/5))/11
```

Rubi in Sympy [A] time = 2.43389, size = 22, normalized size = 0.85

$$\frac{3x^{\frac{2}{3}}}{2}+\frac{10\sqrt{x}\sqrt[5]{x^3}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**(1/6)+(x**3)**(1/5))/x**(1/2),x)
```

```
[Out] 3*x**(2/3)/2 + 10*sqrt(x)*(x**3)**(1/5)/11
```

Mathematica [A] time = 0.0157092, size = 26, normalized size = 1.

$$\frac{3x^{2/3}}{2}+\frac{10}{11}\sqrt[5]{x^3}\sqrt{x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^(1/6) + (x^3)^(1/5))/Sqrt[x],x]

[Out] (3*x^(2/3))/2 + (10*Sqrt[x]*(x^3)^(1/5))/11
```

Maple [A] time = 0.005, size = 17, normalized size = 0.7

$$\frac{3}{2}x^{\frac{2}{3}}+\frac{10}{11}\sqrt[5]{x^{3}}\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^(1/6)+(x^3)^(1/5))/x^(1/2),x)

[Out] 3/2*x^(2/3)+10/11*(x^3)^(1/5)*x^(1/2)
```

Maxima [A] time = 0.709692, size = 22, normalized size = 0.85

$$\frac{10}{11}\left(x^{3}\right)^{\frac{1}{5}}\sqrt{x}+\frac{3}{2}x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((x^3)^(1/5) + x^(1/6))/sqrt(x),x, algorithm="maxima")

[Out] 10/11*(x^3)^(1/5)*sqrt(x) + 3/2*x^(2/3)
```

Fricas [A] time = 0.266713, size = 26, normalized size = 1.

$$\frac{20\left(x^{3}\right)^{\frac{1}{5}}x^{\frac{5}{6}}+33\,x}{22\,x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((x^3)^(1/5) + x^(1/6))/sqrt(x),x, algorithm="fricas")

[Out] 1/22*(20*(x^3)^(1/5)*x^(5/6) + 33*x)/x^(1/3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**(1/6)+(x**3)**(1/5))/x**(1/2),x)

[Out] Timed out
```

GIAC/XCAS [A] time = 0.261164, size = 15, normalized size = 0.58

$$\frac{10}{11}\,x^{\frac{11}{10}} + \frac{3}{2}\,x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((x^3)^(1/5) + x^(1/6))/sqrt(x),x, algorithm="giac")

[Out] 10/11*x^(11/10) + 3/2*x^(2/3)
```

3.781 $\int \frac{2+x}{\sqrt{4x-x^2}} \, dx$

Optimal. Leaf size=26

$$-\sqrt{4x-x^2}-4\sin^{-1}\left(1-\frac{x}{2}\right)$$

```
[Out] -Sqrt[4*x - x^2] - 4*ArcSin[1 - x/2]
```

Rubi [A] time = 0.0312822, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$-\sqrt{4x-x^2}-4\sin^{-1}\left(1-\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + x)/Sqrt[4*x - x^2],x]

[Out] -Sqrt[4*x - x^2] - 4*ArcSin[1 - x/2]
```

Rubi in Sympy [A] time = 2.16481, size = 17, normalized size = 0.65

$$-\sqrt{-x^2+4x}+4\,\mathrm{asin}\left(\frac{x}{2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2+x)/(-x**2+4*x)**(1/2),x)

[Out] -sqrt(-x**2 + 4*x) + 4*asin(x/2 - 1)
```

Mathematica [A] time = 0.0284919, size = 45, normalized size = 1.73

$$\frac{(x-4)x+8\sqrt{x-4}\sqrt{x}\log\left(\sqrt{x-4}+\sqrt{x}\right)}{\sqrt{-(x-4)x}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + x)/Sqrt[4*x - x^2],x]
```

```
[Out] ((-4 + x)*x + 8*Sqrt[-4 + x]*Sqrt[x]*Log[Sqrt[-4 + x] + Sqrt[x]])
/Sqrt[-((-4 + x)*x)]
```

Maple [A] time = 0.009, size = 23, normalized size = 0.9

$$4 \arcsin\left(x/2-1\right)-\sqrt{-x^{2}+4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2+x)/(-x^2+4*x)^(1/2),x)
```

```
[Out] 4*arcsin(1/2*x-1)-(-x^2+4*x)^(1/2)
```

Maxima [A] time = 0.786281, size = 30, normalized size = 1.15

$$-\sqrt{-x^{2}+4\,x}-4 \arcsin\left(-\frac{1}{2}\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 2)/sqrt(-x^2 + 4*x),x, algorithm="maxima")
```

```
[Out] -sqrt(-x^2 + 4*x) - 4*arcsin(-1/2*x + 1)
```

Fricas [A] time = 0.264813, size = 43, normalized size = 1.65

$$-\sqrt{-x^{2}+4\,x}-8 \arctan\left(\frac{\sqrt{-x^{2}+4\,x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 2)/sqrt(-x^2 + 4*x),x, algorithm="fricas")
```

```
[Out] -sqrt(-x^2 + 4*x) - 8*arctan(sqrt(-x^2 + 4*x)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+2}{\sqrt{-x(x-4)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2+x)/(-x**2+4*x)**(1/2),x)
```

```
[Out] Integral((x + 2)/sqrt(-x*(x - 4)), x)
```

GIAC/XCAS [A] time = 0.266707, size = 30, normalized size = 1.15

$$-\sqrt{-x^2+4x}+4\arcsin\left(\frac{1}{2}x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 2)/sqrt(-x^2 + 4*x),x, algorithm="giac")
```

```
[Out] -sqrt(-x^2 + 4*x) + 4*arcsin(1/2*x - 1)
```

3.782 $\int \frac{3+x}{\sqrt[3]{6x+x^2}}\,dx$

Optimal. Leaf size=15

$$\frac{3}{4}\left(x^2+6x\right)^{2/3}$$

```
[Out] (3*(6*x + x^2)^(2/3))/4
```

Rubi [A] time = 0.0075356, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$\frac{3}{4}\left(x^2+6x\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In]  Int[(3 + x)/(6*x + x^2)^(1/3),x]

[Out] (3*(6*x + x^2)^(2/3))/4
```

Rubi in Sympy [A] time = 1.13665, size = 12, normalized size = 0.8

$$\frac{3\left(x^2+6x\right)^{\frac{2}{3}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3+x)/(x**2+6*x)**(1/3),x)

[Out] 3*(x**2 + 6*x)**(2/3)/4
```

Mathematica [A] time = 0.0170807, size = 13, normalized size = 0.87

$$\frac{3}{4}(x(x+6))^{2/3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3 + x)/(6*x + x^2)^(1/3),x]

[Out] (3*(x*(6 + x))^(2/3))/4
```

Maple [A] time = 0.005, size = 16, normalized size = 1.1

$$\frac{3x(x+6)}{4}\frac{1}{\sqrt[3]{x^2+6x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3+x)/(x^2+6*x)^(1/3),x)

[Out] 3/4*x*(x+6)/(x^2+6*x)^(1/3)
```

Maxima [A] time = 0.674427, size = 15, normalized size = 1.

$$\frac{3}{4}\left(x^2+6x\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 3)/(x^2 + 6*x)^(1/3),x, algorithm="maxima")

[Out] 3/4*(x^2 + 6*x)^(2/3)
```

Fricas [A] time = 0.259433, size = 15, normalized size = 1.

$$\frac{3}{4}\left(x^2+6x\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 3)/(x^2 + 6*x)^(1/3),x, algorithm="fricas")

[Out] 3/4*(x^2 + 6*x)^(2/3)
```

Sympy [A] time = 0.381422, size = 12, normalized size = 0.8

$$\frac{3\left(x^2+6x\right)^{\frac{2}{3}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3+x)/(x**2+6*x)**(1/3),x)
```

```
[Out] 3*(x**2 + 6*x)**(2/3)/4
```

GIAC/XCAS [A] time = 0.258715, size = 15, normalized size = 1.

$$\frac{3}{4}\left(x^2+6\,x\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 3)/(x^2 + 6*x)^(1/3),x, algorithm="giac")
```

```
[Out] 3/4*(x^2 + 6*x)^(2/3)
```

3.783 $\int \frac{4+x}{(6x-x^2)^{3/2}} \, dx$

Optimal. Leaf size=22

$$-\frac{12-7x}{9\sqrt{6x-x^2}}$$

```
[Out] -(12 - 7*x)/(9*Sqrt[6*x - x^2])
```

Rubi [A] time = 0.0183545, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059

$$-\frac{12-7x}{9\sqrt{6x-x^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(4 + x)/(6*x - x^2)^(3/2),x]

[Out] -(12 - 7*x)/(9*Sqrt[6*x - x^2])
```

Rubi in Sympy [A] time = 1.87237, size = 17, normalized size = 0.77

$$-\frac{-28x+48}{36\sqrt{-x^2+6x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((4+x)/(-x**2+6*x)**(3/2),x)

[Out] -(-28*x + 48)/(36*sqrt(-x**2 + 6*x))
```

Mathematica [A] time = 0.0249244, size = 19, normalized size = 0.86

$$\frac{7x-12}{9\sqrt{-(x-6)x}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(4 + x)/(6*x - x^2)^(3/2),x]

[Out] (-12 + 7*x)/(9*Sqrt[-((-6 + x)*x)])
```

Maple [A] time = 0.004, size = 23, normalized size = 1.1

$$-\frac{x(-6+x)(-12+7x)}{9}\left(-x^2+6x\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((4+x)/(-x^2+6*x)^(3/2),x)

[Out] -1/9*x*(-6+x)*(-12+7*x)/(-x^2+6*x)^(3/2)
```

Maxima [A] time = 0.739876, size = 38, normalized size = 1.73

$$\frac{7x}{9\sqrt{-x^2+6x}}-\frac{4}{3\sqrt{-x^2+6x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 4)/(-x^2 + 6*x)^(3/2),x, algorithm="maxima")

[Out] 7/9*x/sqrt(-x^2 + 6*x) - 4/3/sqrt(-x^2 + 6*x)
```

Fricas [A] time = 0.25948, size = 24, normalized size = 1.09

$$\frac{7x-12}{9\sqrt{-x^2+6x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 4)/(-x^2 + 6*x)^(3/2),x, algorithm="fricas")

[Out] 1/9*(7*x - 12)/sqrt(-x^2 + 6*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x + 4}{(-x(x - 6))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4+x)/(-x**2+6*x)**(3/2),x)
```

```
[Out] Integral((x + 4)/(-x*(x - 6))**(3/2), x)
```

GIAC/XCAS [A] time = 0.268446, size = 36, normalized size = 1.64

$$-\frac{\sqrt{-x^2 + 6x}(7x - 12)}{9(x^2 - 6x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 4)/(-x^2 + 6*x)^(3/2),x, algorithm="giac")
```

```
[Out] -1/9*sqrt(-x^2 + 6*x)*(7*x - 12)/(x^2 - 6*x)
```

3.784 $\int \frac{1}{(1+x)\sqrt{2x+x^2}}\,dx$

Optimal. Leaf size=12

$$\tan^{-1}\left(\sqrt{x^2+2x}\right)$$

```
[Out] ArcTan[Sqrt[2*x + x^2]]
```

Rubi [A] time = 0.0224967, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118

$$\tan^{-1}\left(\sqrt{x^2+2x}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/((1 + x)*Sqrt[2*x + x^2]),x]

[Out] ArcTan[Sqrt[2*x + x^2]]
```

Rubi in Sympy [A] time = 2.18074, size = 10, normalized size = 0.83

$$\operatorname{atan}\left(\sqrt{x^2+2x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1+x)/(x**2+2*x)**(1/2),x)

[Out] atan(sqrt(x**2 + 2*x))
```

Mathematica [B] time = 0.0343143, size = 37, normalized size = 3.08

$$\frac{2\sqrt{x}\sqrt{x+2}\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt{x+2}}\right)}{\sqrt{x(x+2)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((1 + x)*Sqrt[2*x + x^2]),x]
```

```
[Out] (2*Sqrt[x]*Sqrt[2 + x]*ArcTan[Sqrt[x]/Sqrt[2 + x]])/Sqrt[x*(2 + x
)]
```

Maple [A] time = 0.009, size = 13, normalized size = 1.1

$$-\arctan\left(\frac{1}{\sqrt{(1+x)^2-1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1+x)/(x^2+2*x)^(1/2),x)
```

```
[Out] -arctan(1/((1+x)^2-1)^(1/2))
```

Maxima [A] time = 0.76532, size = 12, normalized size = 1.

$$-\arcsin\left(\frac{1}{|x+1|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 + 2*x)*(x + 1)),x, algorithm="maxima")
```

```
[Out] -arcsin(1/abs(x + 1))
```

Fricas [A] time = 0.26409, size = 23, normalized size = 1.92

$$2\arctan\left(-x+\sqrt{x^2+2x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 + 2*x)*(x + 1)),x, algorithm="fricas")
```

```
[Out] 2*arctan(-x + sqrt(x^2 + 2*x) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x\,(x+2)}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1+x)/(x**2+2*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x*(x + 2))*(x + 1)), x)
```

GIAC/XCAS [A] time = 0.266872, size = 23, normalized size = 1.92

$$2\,\arctan\left(-x+\sqrt{x^2+2\,x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 + 2*x)*(x + 1)),x, algorithm="giac")
```

```
[Out] 2*arctan(-x + sqrt(x^2 + 2*x) - 1)
```

3.785 $\int \frac{1}{(1+2x)\sqrt{x+x^2}}\,dx$

Optimal. Leaf size=12

$$\tan^{-1}\left(2\sqrt{x^2+x}\right)$$

```
[Out] ArcTan[2*Sqrt[x + x^2]]
```

Rubi [A] time = 0.0241491, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118

$$\tan^{-1}\left(2\sqrt{x^2+x}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/((1 + 2*x)*Sqrt[x + x^2]),x]

[Out] ArcTan[2*Sqrt[x + x^2]]
```

Rubi in Sympy [A] time = 2.24703, size = 10, normalized size = 0.83

$$\operatorname{atan}\left(2\sqrt{x^2+x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1+2*x)/(x**2+x)**(1/2),x)

[Out] atan(2*sqrt(x**2 + x))
```

Mathematica [B] time = 0.026756, size = 37, normalized size = 3.08

$$\frac{2\sqrt{x}\sqrt{x+1}\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt{x+1}}\right)}{\sqrt{x(x+1)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((1 + 2*x)*Sqrt[x + x^2]),x]
```

```
[Out] (2*Sqrt[x]*Sqrt[1 + x]*ArcTan[Sqrt[x]/Sqrt[1 + x]])/Sqrt[x*(1 + x
)]
```

Maple [A] time = 0.009, size = 15, normalized size = 1.3

$$-\arctan\left(\frac{1}{\sqrt{4\,(x+1/2)^2-1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1+2*x)/(x^2+x)^(1/2),x)
```

```
[Out] -arctan(1/(4*(x+1/2)^2-1)^(1/2))
```

Maxima [A] time = 0.773137, size = 15, normalized size = 1.25

$$-\arcsin\left(\frac{1}{|2\,x+1|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 + x)*(2*x + 1)),x, algorithm="maxima")
```

```
[Out] -arcsin(1/abs(2*x + 1))
```

Fricas [A] time = 0.265038, size = 23, normalized size = 1.92

$$2\,\arctan\left(-2\,x+2\,\sqrt{x^2+x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 + x)*(2*x + 1)),x, algorithm="fricas")
```

```
[Out] 2*arctan(-2*x + 2*sqrt(x^2 + x) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x\left(x+1\right)}\left(2x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1+2*x)/(x**2+x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x*(x + 1))*(2*x + 1)), x)
```

GIAC/XCAS [A] time = 0.264439, size = 23, normalized size = 1.92

$$2\arctan\left(-2\,x+2\,\sqrt{x^2+x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 + x)*(2*x + 1)),x, algorithm="giac")
```

```
[Out] 2*arctan(-2*x + 2*sqrt(x^2 + x) - 1)
```

3.786 $\int \frac{-1+x}{\sqrt{2x-x^2}} \, dx$

Optimal. Leaf size=15

$$-\sqrt{2x-x^2}$$

```
[Out] -Sqrt[2*x - x^2]
```

Rubi [A] time = 0.00856658, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$

$$-\sqrt{2x-x^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + x)/Sqrt[2*x - x^2],x]

[Out] -Sqrt[2*x - x^2]
```

Rubi in Sympy [A] time = 1.32372, size = 10, normalized size = 0.67

$$-\sqrt{-x^2+2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-1+x)/(-x**2+2*x)**(1/2),x)

[Out] -sqrt(-x**2 + 2*x)
```

Mathematica [A] time = 0.0138485, size = 12, normalized size = 0.8

$$-\sqrt{-(x-2)x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + x)/Sqrt[2*x - x^2],x]
```

```
[Out] -Sqrt[-((-2 + x)*x)]
```

Maple [A] time = 0.004, size = 17, normalized size = 1.1

$$x(x-2)\frac{1}{\sqrt{-x^2+2x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-1+x)/(-x^2+2*x)^(1/2),x)
```

```
[Out] x*(x-2)/(-x^2+2*x)^(1/2)
```

Maxima [A] time = 0.695921, size = 18, normalized size = 1.2

$$-\sqrt{-x^2+2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - 1)/sqrt(-x^2 + 2*x),x, algorithm="maxima")
```

```
[Out] -sqrt(-x^2 + 2*x)
```

Fricas [A] time = 0.262983, size = 18, normalized size = 1.2

$$-\sqrt{-x^2+2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - 1)/sqrt(-x^2 + 2*x),x, algorithm="fricas")
```

```
[Out] -sqrt(-x^2 + 2*x)
```

Sympy [A] time = 0.30012, size = 10, normalized size = 0.67

$$-\sqrt{-x^2+2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-1+x)/(-x**2+2*x)**(1/2),x)
```

```
[Out] -sqrt(-x**2 + 2*x)
```

GIAC/XCAS [A] time = 0.263443, size = 18, normalized size = 1.2

$$-\sqrt{-x^2 + 2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - 1)/sqrt(-x^2 + 2*x),x, algorithm="giac")
```

```
[Out] -sqrt(-x^2 + 2*x)
```

3.787 $\int \frac{\sqrt{x-x^2}}{1+x}\,dx$

Optimal. Leaf size=54

$$\sqrt{x-x^2}+\sqrt{2}\tan^{-1}\left(\frac{1-3x}{2\sqrt{2}\sqrt{x-x^2}}\right)-\frac{3}{2}\sin^{-1}(1-2x)$$

```
[Out] Sqrt[x - x^2] - (3*ArcSin[1 - 2*x])/2 + Sqrt[2]*ArcTan[(1 - 3*x)/
(2*Sqrt[2]*Sqrt[x - x^2])]
```

Rubi [A] time = 0.114361, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353

$$\sqrt{x-x^2}+\sqrt{2}\tan^{-1}\left(\frac{1-3x}{2\sqrt{2}\sqrt{x-x^2}}\right)-\frac{3}{2}\sin^{-1}(1-2x)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x - x^2]/(1 + x),x]
```

```
[Out] Sqrt[x - x^2] - (3*ArcSin[1 - 2*x])/2 + Sqrt[2]*ArcTan[(1 - 3*x)/
(2*Sqrt[2]*Sqrt[x - x^2])]
```

Rubi in Sympy [A] time = 6.65732, size = 44, normalized size = 0.81

$$\sqrt{-x^2+x}+\frac{3\operatorname{asin}(2x-1)}{2}-\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}(3x-1)}{4\sqrt{-x^2+x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2+x)**(1/2)/(1+x),x)
```

```
[Out] sqrt(-x**2 + x) + 3*asin(2*x - 1)/2 - sqrt(2)*atan(sqrt(2)*(3*x -
 1)/(4*sqrt(-x**2 + x)))
```

Mathematica [B] time = 0.0817704, size = 120, normalized size = 2.22

$$\frac{\sqrt{-(x-1)x}\left(2\sqrt{x-1}\sqrt{x}-6\log\left(\sqrt{x-1}+\sqrt{x}\right)+\sqrt{2}\log\left(-3x-2\sqrt{2}\sqrt{x-1}\sqrt{x}+1\right)-\sqrt{2}\log\left(-3x+2\sqrt{2}\sqrt{x-1}\sqrt{x}+1\right)\right)}{2\sqrt{x-1}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[x - x^2]/(1 + x),x]
```

```
[Out] (Sqrt[-((-1 + x)*x)]*(2*Sqrt[-1 + x]*Sqrt[x] - 6*Log[Sqrt[-1 + x]
 + Sqrt[x]] + Sqrt[2]*Log[1 - 2*Sqrt[2]*Sqrt[-1 + x]*Sqrt[x] - 3*
x] - Sqrt[2]*Log[1 + 2*Sqrt[2]*Sqrt[-1 + x]*Sqrt[x] - 3*x]))/(2*S
qrt[-1 + x]*Sqrt[x])
```

Maple [A]   time = 0.01, size = 54, normalized size = 1.

$$\sqrt{-\left(1+x\right)^{2}+1+3\,x}+\frac{3\,\arcsin\left(2\,x-1\right)}{2}-\sqrt{2}\arctan\left(\frac{\left(3\,x-1\right)\sqrt{2}}{4}\frac{1}{\sqrt{-\left(1+x\right)^{2}+1+3\,x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2+x)^(1/2)/(1+x),x)
```

```
[Out] (-(1+x)^2+1+3*x)^(1/2)+3/2*arcsin(2*x-1)-2^(1/2)*arctan(1/4*(3*x-
1)*2^(1/2)/(-(1+x)^2+1+3*x)^(1/2))
```

Maxima [A]   time = 0.76602, size = 57, normalized size = 1.06

$$-\sqrt{2}\arcsin\left(\frac{3\,x}{|x+1|}-\frac{1}{|x+1|}\right)+\sqrt{-x^{2}+x}+\frac{3}{2}\,\arcsin\left(2\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + x)/(x + 1),x, algorithm="maxima")
```

```
[Out] -sqrt(2)*arcsin(3*x/abs(x + 1) - 1/abs(x + 1)) + sqrt(-x^2 + x) +
 3/2*arcsin(2*x - 1)
```

Fricas [A]   time = 0.274778, size = 66, normalized size = 1.22

$$2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-x^{2}+x}}{2\,x}\right)+\sqrt{-x^{2}+x}-3\,\arctan\left(\frac{\sqrt{-x^{2}+x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + x)/(x + 1),x, algorithm="fricas")
```

```
[Out] 2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-x^2 + x)/x) + sqrt(-x^2 + x) -
 3*arctan(sqrt(-x^2 + x)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x\,(x-1)}}{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+x)**(1/2)/(1+x),x)
```

```
[Out] Integral(sqrt(-x*(x - 1))/(x + 1), x)
```

GIAC/XCAS [A] time = 0.266554, size = 72, normalized size = 1.33

$$2\,\sqrt{2}\arctan\left(\frac{1}{4}\,\sqrt{2}\left(\frac{3\,\left(2\,\sqrt{-x^2+x}-1\right)}{2\,x-1}-1\right)\right)+\sqrt{-x^2+x}+\frac{3}{2}\,\arcsin\left(2\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + x)/(x + 1),x, algorithm="giac")
```

```
[Out] 2*sqrt(2)*arctan(1/4*sqrt(2)*(3*(2*sqrt(-x^2 + x) - 1)/(2*x - 1)
- 1)) + sqrt(-x^2 + x) + 3/2*arcsin(2*x - 1)
```

3.788 $\int \sqrt{\sqrt[4]{x}+x}\,dx$

Optimal. Leaf size=59

$$\frac{2}{3}\sqrt{x+\sqrt[4]{x}}x+\frac{1}{3}\sqrt{x+\sqrt[4]{x}}\sqrt[4]{x}-\frac{1}{3}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{x+\sqrt[4]{x}}}\right)$$

```
[Out] (x^(1/4)*Sqrt[x^(1/4) + x])/3 + (2*x*Sqrt[x^(1/4) + x])/3 - ArcTa
nh[Sqrt[x]/Sqrt[x^(1/4) + x]]/3
```

Rubi [A] time = 0.113001, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454

$$\frac{2}{3}\sqrt{x+\sqrt[4]{x}}x+\frac{1}{3}\sqrt{x+\sqrt[4]{x}}\sqrt[4]{x}-\frac{1}{3}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{x+\sqrt[4]{x}}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x^(1/4) + x],x]
```

```
[Out] (x^(1/4)*Sqrt[x^(1/4) + x])/3 + (2*x*Sqrt[x^(1/4) + x])/3 - ArcTa
nh[Sqrt[x]/Sqrt[x^(1/4) + x]]/3
```

Rubi in Sympy [A] time = 6.50314, size = 49, normalized size = 0.83

$$\frac{\sqrt[4]{x}\sqrt{\sqrt[4]{x}+x}}{3}+\frac{2x\sqrt{\sqrt[4]{x}+x}}{3}-\frac{\operatorname{atanh}\left(\frac{\sqrt{x}}{\sqrt{\sqrt[4]{x}+x}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**(1/4)+x)**(1/2),x)
```

```
[Out] x**(1/4)*sqrt(x**(1/4) + x)/3 + 2*x*sqrt(x**(1/4) + x)/3 - atanh(
sqrt(x)/sqrt(x**(1/4) + x))/3
```

Mathematica [A] time = 0.0404164, size = 57, normalized size = 0.97

$$\frac{3x^{5/4}-\sqrt{x^{3/4}+1}\sqrt[8]{x}\sinh^{-1}\left(x^{3/8}\right)+2x^2+\sqrt{x}}{3\sqrt{x+\sqrt[4]{x}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[x^(1/4) + x],x]
```

```
[Out] (Sqrt[x] + 3*x^(5/4) + 2*x^2 - Sqrt[1 + x^(3/4)]*x^(1/8)*ArcSinh[
x^(3/8)])/(3*Sqrt[x^(1/4) + x])
```

Maple [C] time = 0.108, size = 342, normalized size = 5.8

$$\frac{2x}{3}\sqrt{\sqrt[4]{x}+x}+\frac{1}{3}\sqrt[4]{x}\sqrt{\sqrt[4]{x}+x}$$
$$+\frac{-\frac{1}{2}-\frac{i}{2}\sqrt{3}}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}\sqrt{\frac{\frac{3}{2}+\frac{i}{2}\sqrt{3}}{\frac{1}{2}+\frac{i}{2}\sqrt{3}}\sqrt[4]{x}\left(1+\sqrt[4]{x}\right)^{-1}}\left(1+\sqrt[4]{x}\right)^{2}\sqrt{-\frac{1}{\frac{1}{2}-\frac{i}{2}\sqrt{3}}\left(\sqrt[4]{x}-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\left(1+\sqrt[4]{x}\right)^{-1}}\sqrt{-\frac{1}{\frac{1}{2}+\frac{i}{2}\sqrt{3}}\left(\sqrt[4]{x}-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^(1/4)+x)^(1/2),x)
```

```
[Out] 2/3*x*(x^(1/4)+x)^(1/2)+1/3*x^(1/4)*(x^(1/4)+x)^(1/2)+(-1/2-1/2*I
*3^(1/2))*((3/2+1/2*I*3^(1/2))*x^(1/4)/(1/2+1/2*I*3^(1/2))/(1+x^(
1/4)))^(1/2)*(1+x^(1/4))^2*(-(x^(1/4)-1/2+1/2*I*3^(1/2))/(1/2-1/2
*I*3^(1/2))/(1+x^(1/4)))^(1/2)*(-(x^(1/4)-1/2-1/2*I*3^(1/2))/(1/2
+1/2*I*3^(1/2))/(1+x^(1/4)))^(1/2)/(3/2+1/2*I*3^(1/2))/(x^(1/4)*(
1+x^(1/4))*(x^(1/4)-1/2+1/2*I*3^(1/2))*(x^(1/4)-1/2-1/2*I*3^(1/2)
))^(1/2)*(-EllipticF(((3/2+1/2*I*3^(1/2))*x^(1/4)/(1/2+1/2*I*3^(1
/2))/(1+x^(1/4)))^(1/2),((-3/2+1/2*I*3^(1/2))*(-1/2-1/2*I*3^(1/2)
)/(-1/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+EllipticPi(((
3/2+1/2*I*3^(1/2))*x^(1/4)/(1/2+1/2*I*3^(1/2))/(1+x^(1/4)))^(1/2)
,(1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)),((-3/2+1/2*I*3^(1/2))*(-
1/2-1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/
2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{x+x^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x + x^(1/4)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x + x^(1/4)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x + x^(1/4)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sqrt[4]{x} + x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**(1/4)+x)**(1/2),x)
```

```
[Out] Integral(sqrt(x**(1/4) + x), x)
```

GIAC/XCAS [A] time = 1.00642, size = 61, normalized size = 1.03

$$\frac{1}{3}\sqrt{x + x^{\frac{1}{4}}}x^{\frac{1}{4}}\left(2\,x^{\frac{3}{4}} + 1\right) - \frac{1}{6}\ln\left(\sqrt{\frac{1}{x^{\frac{3}{4}}} + 1} + 1\right) + \frac{1}{6}\ln\left(\left|\sqrt{\frac{1}{x^{\frac{3}{4}}} + 1} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x + x^(1/4)),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(x + x^(1/4))*x^(1/4)*(2*x^(3/4) + 1) - 1/6*ln(sqrt(1/x^(
3/4) + 1) + 1) + 1/6*ln(abs(sqrt(1/x^(3/4) + 1) - 1))
```

3.789 $\int \sqrt{x + x^{3/2}}\, dx$

Optimal. Leaf size=59

$$\frac{4\left(x^{3/2}+x\right)^{3/2}}{7\sqrt{x}}-\frac{16\left(x^{3/2}+x\right)^{3/2}}{35x}+\frac{32\left(x^{3/2}+x\right)^{3/2}}{105x^{3/2}}$$

```
[Out] (32*(x + x^(3/2))^(3/2))/(105*x^(3/2)) - (16*(x + x^(3/2))^(3/2))
/(35*x) + (4*(x + x^(3/2))^(3/2))/(7*Sqrt[x])
```

Rubi [A] time = 0.0867029, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273

$$\frac{4\left(x^{3/2}+x\right)^{3/2}}{7\sqrt{x}}-\frac{16\left(x^{3/2}+x\right)^{3/2}}{35x}+\frac{32\left(x^{3/2}+x\right)^{3/2}}{105x^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x + x^(3/2)],x]
```

```
[Out] (32*(x + x^(3/2))^(3/2))/(105*x^(3/2)) - (16*(x + x^(3/2))^(3/2))
/(35*x) + (4*(x + x^(3/2))^(3/2))/(7*Sqrt[x])
```

Rubi in Sympy [A] time = 4.63909, size = 51, normalized size = 0.86

$$-\frac{16\left(x^{\frac{3}{2}}+x\right)^{\frac{3}{2}}}{35x}+\frac{4\left(x^{\frac{3}{2}}+x\right)^{\frac{3}{2}}}{7\sqrt{x}}+\frac{32\left(x^{\frac{3}{2}}+x\right)^{\frac{3}{2}}}{105x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x+x**(3/2))**(1/2),x)
```

```
[Out] -16*(x**(3/2) + x)**(3/2)/(35*x) + 4*(x**(3/2) + x)**(3/2)/(7*sqr
t(x)) + 32*(x**(3/2) + x)**(3/2)/(105*x**(3/2))
```

Mathematica [A] time = 0.0208424, size = 41, normalized size = 0.69

$$\left(\frac{4x}{7}+\frac{4\sqrt{x}}{35}+\frac{32}{105\sqrt{x}}-\frac{16}{105}\right)\sqrt{\left(\sqrt{x}+1\right)x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[x + x^(3/2)],x]
```

```
[Out] (-16/105 + 32/(105*Sqrt[x]) + (4*Sqrt[x])/35 + (4*x)/7)*Sqrt[(1 +
 Sqrt[x])*x]
```

Maple [A] time = 0.012, size = 28, normalized size = 0.5

$$\frac{4}{105}\sqrt{x+x^{\frac{3}{2}}}\,\left(1+\sqrt{x}\right)\left(15\,x-12\,\sqrt{x}+8\right)\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x+x^(3/2))^(1/2),x)
```

```
[Out] 4/105*(x+x^(3/2))^(1/2)*(1+x^(1/2))*(15*x-12*x^(1/2)+8)/x^(1/2)
```

Maxima [A] time = 0.721673, size = 38, normalized size = 0.64

$$\frac{4}{7}\left(\sqrt{x}+1\right)^{\frac{7}{2}}-\frac{8}{5}\left(\sqrt{x}+1\right)^{\frac{5}{2}}+\frac{4}{3}\left(\sqrt{x}+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^(3/2) + x),x, algorithm="maxima")
```

```
[Out] 4/7*(sqrt(x) + 1)^(7/2) - 8/5*(sqrt(x) + 1)^(5/2) + 4/3*(sqrt(x)
+ 1)^(3/2)
```

Fricas [A] time = 0.300985, size = 41, normalized size = 0.69

$$\frac{4\left(15\,x^{2}+(3\,x+8)\sqrt{x}-4\,x\right)\sqrt{x^{\frac{3}{2}}+x}}{105\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^(3/2) + x),x, algorithm="fricas")
```

```
[Out] 4/105*(15*x^2 + (3*x + 8)*sqrt(x) - 4*x)*sqrt(x^(3/2) + x)/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^{\frac{3}{2}} + x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x+x**(3/2))**(1/2),x)
```

```
[Out] Integral(sqrt(x**(3/2) + x), x)
```

GIAC/XCAS [A] time = 0.281018, size = 39, normalized size = 0.66

$$\frac{4}{7}\left(\sqrt{x} + 1\right)^{\frac{7}{2}} - \frac{8}{5}\left(\sqrt{x} + 1\right)^{\frac{5}{2}} + \frac{4}{3}\left(\sqrt{x} + 1\right)^{\frac{3}{2}} - \frac{32}{105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^(3/2) + x),x, algorithm="giac")
```

```
[Out] 4/7*(sqrt(x) + 1)^(7/2) - 8/5*(sqrt(x) + 1)^(5/2) + 4/3*(sqrt(x)
+ 1)^(3/2) - 32/105
```

3.790 $\int x\sqrt{x+x^{3/2}}\,dx$

Optimal. Leaf size=94

$$\frac{4}{11}\sqrt{x}\left(x^{3/2}+x\right)^{3/2}+\frac{64\left(x^{3/2}+x\right)^{3/2}}{231\sqrt{x}}-\frac{256\left(x^{3/2}+x\right)^{3/2}}{1155x}+\frac{512\left(x^{3/2}+x\right)^{3/2}}{3465x^{3/2}}-\frac{32}{99}\left(x^{3/2}+x\right)^{3/2}$$

```
[Out] (-32*(x + x^(3/2))^(3/2))/99 + (512*(x + x^(3/2))^(3/2))/(3465*x^
(3/2)) - (256*(x + x^(3/2))^(3/2))/(1155*x) + (64*(x + x^(3/2))^(
3/2))/(231*Sqrt[x]) + (4*Sqrt[x]*(x + x^(3/2))^(3/2))/11
```

Rubi [A] time = 0.149267, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$\frac{4}{11}\sqrt{x}\left(x^{3/2}+x\right)^{3/2}+\frac{64\left(x^{3/2}+x\right)^{3/2}}{231\sqrt{x}}-\frac{256\left(x^{3/2}+x\right)^{3/2}}{1155x}+\frac{512\left(x^{3/2}+x\right)^{3/2}}{3465x^{3/2}}-\frac{32}{99}\left(x^{3/2}+x\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In]  Int[x*Sqrt[x + x^(3/2)],x]
```

```
[Out] (-32*(x + x^(3/2))^(3/2))/99 + (512*(x + x^(3/2))^(3/2))/(3465*x^
(3/2)) - (256*(x + x^(3/2))^(3/2))/(1155*x) + (64*(x + x^(3/2))^(
3/2))/(231*Sqrt[x]) + (4*Sqrt[x]*(x + x^(3/2))^(3/2))/11
```

Rubi in Sympy [A] time = 7.87215, size = 83, normalized size = 0.88

$$\frac{4\sqrt{x}\left(x^{\frac{3}{2}}+x\right)^{\frac{3}{2}}}{11}-\frac{32\left(x^{\frac{3}{2}}+x\right)^{\frac{3}{2}}}{99}-\frac{256\left(x^{\frac{3}{2}}+x\right)^{\frac{3}{2}}}{1155x}+\frac{64\left(x^{\frac{3}{2}}+x\right)^{\frac{3}{2}}}{231\sqrt{x}}+\frac{512\left(x^{\frac{3}{2}}+x\right)^{\frac{3}{2}}}{3465x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(x+x**(3/2))**(1/2),x)
```

```
[Out] 4*sqrt(x)*(x**(3/2) + x)**(3/2)/11 - 32*(x**(3/2) + x)**(3/2)/99
- 256*(x**(3/2) + x)**(3/2)/(1155*x) + 64*(x**(3/2) + x)**(3/2)/(
231*sqrt(x)) + 512*(x**(3/2) + x)**(3/2)/(3465*x**(3/2))
```

Mathematica [A] time = 0.021794, size = 51, normalized size = 0.54

$$\frac{4\sqrt{x^{3/2}+x}\left(315x^{5/2}-40x^{3/2}+35x^2+48x-64\sqrt{x}+128\right)}{3465\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*Sqrt[x + x^(3/2)],x]
```

```
[Out] (4*Sqrt[x + x^(3/2)]*(128 - 64*Sqrt[x] + 48*x - 40*x^(3/2) + 35*x
^2 + 315*x^(5/2)))/(3465*Sqrt[x])
```

Maple [A] time = 0.005, size = 38, normalized size = 0.4

$$\frac{4}{3465}\sqrt{x+x^{\frac{3}{2}}}\left(1+\sqrt{x}\right)\left(315\,x^2-280\,x^{3/2}+240\,x-192\,\sqrt{x}+128\right)\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(x+x^(3/2))^(1/2),x)
```

```
[Out] 4/3465*(x+x^(3/2))^(1/2)*(1+x^(1/2))*(315*x^2-280*x^(3/2)+240*x-1
92*x^(1/2)+128)/x^(1/2)
```

Maxima [A] time = 0.711294, size = 62, normalized size = 0.66

$$\frac{4}{11}\left(\sqrt{x}+1\right)^{\frac{11}{2}}-\frac{16}{9}\left(\sqrt{x}+1\right)^{\frac{9}{2}}+\frac{24}{7}\left(\sqrt{x}+1\right)^{\frac{7}{2}}-\frac{16}{5}\left(\sqrt{x}+1\right)^{\frac{5}{2}}+\frac{4}{3}\left(\sqrt{x}+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^(3/2) + x)*x,x, algorithm="maxima")
```

```
[Out] 4/11*(sqrt(x) + 1)^(11/2) - 16/9*(sqrt(x) + 1)^(9/2) + 24/7*(sqrt
(x) + 1)^(7/2) - 16/5*(sqrt(x) + 1)^(5/2) + 4/3*(sqrt(x) + 1)^(3/
2)
```

Fricas [A] time = 0.30126, size = 54, normalized size = 0.57

$$\frac{4\left(315\,x^3-40\,x^2+\left(35\,x^2+48\,x+128\right)\sqrt{x}-64\,x\right)\sqrt{x^{\frac{3}{2}}+x}}{3465\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^(3/2) + x)*x,x, algorithm="fricas")
```

```
[Out] 4/3465*(315*x^3 - 40*x^2 + (35*x^2 + 48*x + 128)*sqrt(x) - 64*x)*
sqrt(x^(3/2) + x)/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{x^{\frac{3}{2}}+x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(x+x**(3/2))**(1/2),x)
```

```
[Out] Integral(x*sqrt(x**(3/2) + x), x)
```

GIAC/XCAS [A] time = 0.268615, size = 63, normalized size = 0.67

$$\frac{4}{11}\left(\sqrt{x}+1\right)^{\frac{11}{2}}-\frac{16}{9}\left(\sqrt{x}+1\right)^{\frac{9}{2}}+\frac{24}{7}\left(\sqrt{x}+1\right)^{\frac{7}{2}}-\frac{16}{5}\left(\sqrt{x}+1\right)^{\frac{5}{2}}+\frac{4}{3}\left(\sqrt{x}+1\right)^{\frac{3}{2}}-\frac{512}{3465}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^(3/2) + x)*x,x, algorithm="giac")
```

```
[Out] 4/11*(sqrt(x) + 1)^(11/2) - 16/9*(sqrt(x) + 1)^(9/2) + 24/7*(sqrt
(x) + 1)^(7/2) - 16/5*(sqrt(x) + 1)^(5/2) + 4/3*(sqrt(x) + 1)^(3/
2) - 512/3465
```

3.791 $\int \left(1-x^2\right) \sqrt{\frac{1}{2-x^2}} \, dx$

Optimal. Leaf size=18

$$\frac{x}{2\sqrt{\frac{1}{2-x^2}}}$$

```
[Out] x/(2*Sqrt[(2 - x^2)^(-1)])
```

Rubi [A] time = 0.085749, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095

$$\frac{x}{2\sqrt{\frac{1}{2-x^2}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - x^2)*Sqrt[(2 - x^2)^(-1)],x]

[Out] x/(2*Sqrt[(2 - x^2)^(-1)])
```

Rubi in Sympy [A] time = 4.51291, size = 17, normalized size = 0.94

$$\frac{x\left(-x^2+2\right)\sqrt{\frac{1}{-x^2+2}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2+1)*(1/(-x**2+2))**(1/2),x)

[Out] x*(-x**2 + 2)*sqrt(1/(-x**2 + 2))/2
```

Mathematica [A] time = 0.0207967, size = 18, normalized size = 1.

$$\frac{x}{2\sqrt{\frac{1}{2-x^2}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - x^2)*Sqrt[(2 - x^2)^(-1)],x]
```

```
[Out] x/(2*Sqrt[(2 - x^2)^(-1)])
```

Maple [A] time = 0.008, size = 20, normalized size = 1.1

$$-\frac{x\left(x^{2}-2\right)}{2}\sqrt{-\left(x^{2}-2\right)^{-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2+1)*(1/(-x^2+2))^(1/2),x)
```

```
[Out] -1/2*(x^2-2)*x*(-1/(x^2-2))^(1/2)
```

Maxima [A] time = 0.776316, size = 16, normalized size = 0.89

$$\frac{1}{2}\sqrt{-x^{2}+2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 1)*sqrt(-1/(x^2 - 2)),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-x^2 + 2)*x
```

Fricas [A] time = 0.27434, size = 19, normalized size = 1.06

$$\frac{x}{2\sqrt{-\frac{1}{x^{2}-2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 1)*sqrt(-1/(x^2 - 2)),x, algorithm="fricas")
```

```
[Out] 1/2*x/sqrt(-1/(x^2 - 2))
```

Sympy [A] time = 1.89166, size = 26, normalized size = 1.44

$$-\frac{x^3\sqrt{\frac{1}{-x^2+2}}}{2}+x\sqrt{\frac{1}{-x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+1)*(1/(-x**2+2))**(1/2),x)
```

```
[Out] -x**3*sqrt(1/(-x**2 + 2))/2 + x*sqrt(1/(-x**2 + 2))
```

GIAC/XCAS [A] time = 0.269351, size = 24, normalized size = 1.33

$$-\frac{1}{2}\sqrt{-x^2+2}x\mathrm{sign}\left(x^2-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 1)*sqrt(-1/(x^2 - 2)),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-x^2 + 2)*x*sign(x^2 - 2)
```

3.792 $\int \sqrt{x^2 + x^3 - x^4}\, dx$

Optimal. Leaf size=107

$$-\frac{\sqrt{-x^4+x^3+x^2}(1-2x)}{8x}-\frac{\left(-x^2+x+1\right)\sqrt{-x^4+x^3+x^2}}{3x}-\frac{5\sqrt{-x^4+x^3+x^2}\sin^{-1}\left(\frac{1-2x}{\sqrt{5}}\right)}{16x\sqrt{-x^2+x+1}}$$

```
[Out] -((1 - 2*x)*Sqrt[x^2 + x^3 - x^4])/(8*x) - ((1 + x - x^2)*Sqrt[x^
2 + x^3 - x^4])/(3*x) - (5*Sqrt[x^2 + x^3 - x^4]*ArcSin[(1 - 2*x)
/Sqrt[5]])/(16*x*Sqrt[1 + x - x^2])
```

Rubi [A] time = 0.0637217, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$

$$-\frac{\sqrt{-x^4+x^3+x^2}(1-2x)}{8x}-\frac{\left(-x^2+x+1\right)\sqrt{-x^4+x^3+x^2}}{3x}-\frac{5\sqrt{-x^4+x^3+x^2}\sin^{-1}\left(\frac{1-2x}{\sqrt{5}}\right)}{16x\sqrt{-x^2+x+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x^2 + x^3 - x^4],x]
```

```
[Out] -((1 - 2*x)*Sqrt[x^2 + x^3 - x^4])/(8*x) - ((1 + x - x^2)*Sqrt[x^
2 + x^3 - x^4])/(3*x) - (5*Sqrt[x^2 + x^3 - x^4]*ArcSin[(1 - 2*x)
/Sqrt[5]])/(16*x*Sqrt[1 + x - x^2])
```

Rubi in Sympy [A] time = 5.45391, size = 94, normalized size = 0.88

$$-\frac{(-2x+1)\sqrt{-x^4+x^3+x^2}}{8x}-\frac{\left(-x^2+x+1\right)\sqrt{-x^4+x^3+x^2}}{3x}-\frac{5\sqrt{-x^4+x^3+x^2}\,\text{atan}\left(\frac{-2x+1}{2\sqrt{-x^2+x+1}}\right)}{16x\sqrt{-x^2+x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**4+x**3+x**2)**(1/2),x)
```

```
[Out] -(-2*x + 1)*sqrt(-x**4 + x**3 + x**2)/(8*x) - (-x**2 + x + 1)*sqr
t(-x**4 + x**3 + x**2)/(3*x) - 5*sqrt(-x**4 + x**3 + x**2)*atan((
-2*x + 1)/(2*sqrt(-x**2 + x + 1)))/(16*x*sqrt(-x**2 + x + 1))
```

Mathematica [A] time = 0.0438319, size = 82, normalized size = 0.77

$$\frac{\sqrt{-x^4+x^3+x^2}\left(2\sqrt{x^2-x-1}\left(8x^2-2x-11\right)-15\log\left(-2\sqrt{x^2-x-1}-2x+1\right)\right)}{48x\sqrt{x^2-x-1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[x^2 + x^3 - x^4],x]
```

```
[Out] (Sqrt[x^2 + x^3 - x^4]*(2*Sqrt[-1 - x + x^2]*(-11 - 2*x + 8*x^2)
- 15*Log[1 - 2*x - 2*Sqrt[-1 - x + x^2]]))/(48*x*Sqrt[-1 - x + x^
2])
```

Maple [A] time = 0.009, size = 81, normalized size = 0.8

$$\frac{1}{48\,x}\sqrt{-x^4+x^3+x^2}\left(-16\,\left(-x^2+x+1\right)^{3/2}+12\,x\sqrt{-x^2+x+1}+15\,\arcsin\left(1/5\,\left(2\,x-1\right)\sqrt{5}\right)-6\,\sqrt{-x^2+x+1}\right)\frac{1}{\sqrt{-x^2+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^4+x^3+x^2)^(1/2),x)
```

```
[Out] 1/48*(-x^4+x^3+x^2)^(1/2)*(-16*(-x^2+x+1)^(3/2)+12*x*(-x^2+x+1)^(
1/2)+15*arcsin(1/5*(2*x-1)*5^(1/2))-6*(-x^2+x+1)^(1/2))/x/(-x^2+x
+1)^(1/2)
```

Maxima [A] time = 0.794795, size = 69, normalized size = 0.64

$$-\frac{1}{3}\left(-x^2+x+1\right)^{\frac{3}{2}}+\frac{1}{4}\sqrt{-x^2+x+1}x-\frac{1}{8}\sqrt{-x^2+x+1}-\frac{5}{16}\arcsin\left(-\frac{1}{5}\sqrt{5}(2\,x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^4 + x^3 + x^2),x, algorithm="maxima")
```

```
[Out] -1/3*(-x^2 + x + 1)^(3/2) + 1/4*sqrt(-x^2 + x + 1)*x - 1/8*sqrt(-
x^2 + x + 1) - 5/16*arcsin(-1/5*sqrt(5)*(2*x - 1))
```

Fricas [A] time = 0.281575, size = 84, normalized size = 0.79

$$-\frac{15\,x\arctan\left(-\frac{x-\sqrt{-x^4+x^3+x^2}}{x^2}\right)-\sqrt{-x^4+x^3+x^2}\left(8\,x^2-2\,x-11\right)+11\,x}{24\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^4 + x^3 + x^2),x, algorithm="fricas")
```

```
[Out] -1/24*(15*x*arctan(-(x - sqrt(-x^4 + x^3 + x^2))/x^2) - sqrt(-x^4
 + x^3 + x^2)*(8*x^2 - 2*x - 11) + 11*x)/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-x^4+x^3+x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**4+x**3+x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-x**4 + x**3 + x**2), x)
```

GIAC/XCAS [A] time = 0.268193, size = 81, normalized size = 0.76

$$\frac{1}{48}\left(15\arcsin\left(\frac{1}{5}\sqrt{5}\right)+22\right)\mathrm{sign}\left(x\right)+\frac{5}{16}\arcsin\left(\frac{1}{5}\sqrt{5}(2\,x-1)\right)\mathrm{sign}\left(x\right)+\frac{1}{24}\left(2\left(4\,x\mathrm{sign}\left(x\right)-\mathrm{sign}\left(x\right)\right)x-11\,\mathrm{sign}\left(x\right)\right)\sqrt{-x^2+x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^4 + x^3 + x^2),x, algorithm="giac")
```

```
[Out] 1/48*(15*arcsin(1/5*sqrt(5)) + 22)*sign(x) + 5/16*arcsin(1/5*sqrt
(5)*(2*x - 1))*sign(x) + 1/24*(2*(4*x*sign(x) - sign(x))*x - 11*s
ign(x))*sqrt(-x^2 + x + 1)
```

3.793 $\int \frac{1}{\sqrt{(a^2+x^2)^3}}\,dx$

Optimal. Leaf size=25

$$\frac{x\left(a^2+x^2\right)}{a^2\sqrt{(a^2+x^2)^3}}$$

```
[Out] (x*(a^2 + x^2))/(a^2*Sqrt[(a^2 + x^2)^3])
```

Rubi [A] time = 0.0305376, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$\frac{x\left(a^2+x^2\right)}{a^2\sqrt{(a^2+x^2)^3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[(a^2 + x^2)^3],x]

[Out] (x*(a^2 + x^2))/(a^2*Sqrt[(a^2 + x^2)^3])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(a^2+x^2)^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/((a**2+x**2)**3)**(1/2),x)

[Out] Integral(1/sqrt((a**2 + x**2)**3), x)
```

Mathematica [A] time = 0.0380825, size = 25, normalized size = 1.

$$\frac{x\left(a^2+x^2\right)}{a^2\sqrt{(a^2+x^2)^3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[(a^2 + x^2)^3],x]
```

```
[Out] (x*(a^2 + x^2))/(a^2*Sqrt[(a^2 + x^2)^3])
```

Maple [A] time = 0.006, size = 24, normalized size = 1.

$$\frac{x\left(a^{2}+x^{2}\right)}{a^{2}}\frac{1}{\sqrt{\left(a^{2}+x^{2}\right)^{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/((a^2+x^2)^3)^(1/2),x)
```

```
[Out] x*(a^2+x^2)/a^2/((a^2+x^2)^3)^(1/2)
```

Maxima [A] time = 0.707499, size = 19, normalized size = 0.76

$$\frac{x}{\sqrt{a^{2}+x^{2}}a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt((a^2 + x^2)^3),x, algorithm="maxima")
```

```
[Out] x/(sqrt(a^2 + x^2)*a^2)
```

Fricas [A] time = 0.277155, size = 86, normalized size = 3.44

$$\frac{a^{4}+2\,a^{2}x^{2}+x^{4}+\sqrt{a^{6}+3\,a^{4}x^{2}+3\,a^{2}x^{4}+x^{6}}x}{a^{6}+2\,a^{4}x^{2}+a^{2}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt((a^2 + x^2)^3),x, algorithm="fricas")
```

```
[Out] (a^4 + 2*a^2*x^2 + x^4 + sqrt(a^6 + 3*a^4*x^2 + 3*a^2*x^4 + x^6)*
x)/(a^6 + 2*a^4*x^2 + a^2*x^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(a^2 + x^2)^3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a**2+x**2)**3)**(1/2),x)
```

```
[Out] Integral(1/sqrt((a**2 + x**2)**3), x)
```

GIAC/XCAS [A] time = 0.264094, size = 19, normalized size = 0.76

$$\frac{x}{\sqrt{a^2 + x^2}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt((a^2 + x^2)^3),x, algorithm="giac")
```

```
[Out] x/(sqrt(a^2 + x^2)*a^2)
```

3.794 $\int \frac{\sqrt{x}}{1+\sqrt{x}+x} \, dx$

Optimal. Leaf size=42

$$2\sqrt{x} - \log\left(x + \sqrt{x} + 1\right) - \frac{2\tan^{-1}\left(\frac{2\sqrt{x}+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] 2*Sqrt[x] - (2*ArcTan[(1 + 2*Sqrt[x])/Sqrt[3]])/Sqrt[3] - Log[1 +
 Sqrt[x] + x]
```

Rubi [A] time = 0.0632325, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375

$$2\sqrt{x} - \log\left(x + \sqrt{x} + 1\right) - \frac{2\tan^{-1}\left(\frac{2\sqrt{x}+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x]/(1 + Sqrt[x] + x),x]
```

```
[Out] 2*Sqrt[x] - (2*ArcTan[(1 + 2*Sqrt[x])/Sqrt[3]])/Sqrt[3] - Log[1 +
 Sqrt[x] + x]
```

Rubi in Sympy [A] time = 5.44955, size = 42, normalized size = 1.

$$2\sqrt{x} - \log\left(\sqrt{x} + x + 1\right) - \frac{2\sqrt{3}\,\text{atan}\left(\sqrt{3}\left(\frac{2\sqrt{x}}{3} + \frac{1}{3}\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(1/2)/(1+x+x**(1/2)),x)
```

```
[Out] 2*sqrt(x) - log(sqrt(x) + x + 1) - 2*sqrt(3)*atan(sqrt(3)*(2*sqrt
(x)/3 + 1/3))/3
```

Mathematica [A] time = 0.0169869, size = 42, normalized size = 1.

$$2\sqrt{x} - \log\left(x + \sqrt{x} + 1\right) - \frac{2\tan^{-1}\left(\frac{2\sqrt{x}+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[x]/(1 + Sqrt[x] + x),x]
```

```
[Out] 2*Sqrt[x] - (2*ArcTan[(1 + 2*Sqrt[x])/Sqrt[3]])/Sqrt[3] - Log[1 +
 Sqrt[x] + x]
```

Maple [A] time = 0.006, size = 34, normalized size = 0.8

$$-\ln\left(1+x+\sqrt{x}\right)-\frac{2\sqrt{3}}{3}\arctan\left(\frac{\sqrt{3}}{3}\left(1+2\sqrt{x}\right)\right)+2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(1/2)/(1+x+x^(1/2)),x)
```

```
[Out] -ln(1+x+x^(1/2))-2/3*arctan(1/3*(1+2*x^(1/2))*3^(1/2))*3^(1/2)+2*
x^(1/2)
```

Maxima [A] time = 0.787762, size = 45, normalized size = 1.07

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\sqrt{x}+1\right)\right)+2\sqrt{x}-\log\left(x+\sqrt{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/(x + sqrt(x) + 1),x, algorithm="maxima")
```

```
[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*sqrt(x) + 1)) + 2*sqrt(x) - lo
g(x + sqrt(x) + 1)
```

Fricas [A] time = 0.273461, size = 57, normalized size = 1.36

$$-\frac{1}{3}\sqrt{3}\left(\sqrt{3}\log\left(x+\sqrt{x}+1\right)-2\sqrt{3}\sqrt{x}+2\arctan\left(\frac{2}{3}\sqrt{3}\sqrt{x}+\frac{1}{3}\sqrt{3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/(x + sqrt(x) + 1),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*(sqrt(3)*log(x + sqrt(x) + 1) - 2*sqrt(3)*sqrt(x) +
2*arctan(2/3*sqrt(3)*sqrt(x) + 1/3*sqrt(3)))
```

Sympy [A] time = 1.9015, size = 46, normalized size = 1.1

$$2\sqrt{x} - \log\left(\sqrt{x} + x + 1\right) - \frac{2\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}\sqrt{x}}{3} + \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(1/2)/(1+x+x**(1/2)),x)
```

```
[Out] 2*sqrt(x) - log(sqrt(x) + x + 1) - 2*sqrt(3)*atan(2*sqrt(3)*sqrt(
x)/3 + sqrt(3)/3)/3
```

GIAC/XCAS [A] time = 0.262999, size = 45, normalized size = 1.07

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\sqrt{x} + 1\right)\right) + 2\sqrt{x} - \ln\left(x + \sqrt{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/(x + sqrt(x) + 1),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*sqrt(x) + 1)) + 2*sqrt(x) - ln
(x + sqrt(x) + 1)
```

3.795 $\int \frac{x}{1+\sqrt{x}+x}\,dx$

Optimal. Leaf size=32

$$x-2\sqrt{x}+\frac{4\tan^{-1}\left(\frac{2\sqrt{x}+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -2*Sqrt[x] + x + (4*ArcTan[(1 + 2*Sqrt[x])/Sqrt[3]])/Sqrt[3]
```

Rubi [A] time = 0.0500143, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$x-2\sqrt{x}+\frac{4\tan^{-1}\left(\frac{2\sqrt{x}+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(1 + Sqrt[x] + x),x]
```

```
[Out] -2*Sqrt[x] + x + (4*ArcTan[(1 + 2*Sqrt[x])/Sqrt[3]])/Sqrt[3]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-2\sqrt{x}+\frac{4\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2\sqrt{x}}{3}+\frac{1}{3}\right)\right)}{3}+2\int^{\sqrt{x}} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(1+x+x**(1/2)),x)
```

```
[Out] -2*sqrt(x) + 4*sqrt(3)*atan(sqrt(3)*(2*sqrt(x)/3 + 1/3))/3 + 2*In
tegral(x, (x, sqrt(x)))
```

Mathematica [A] time = 0.0102977, size = 32, normalized size = 1.

$$x-2\sqrt{x}+\frac{4\tan^{-1}\left(\frac{2\sqrt{x}+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(1 + Sqrt[x] + x),x]

[Out] -2*Sqrt[x] + x + (4*ArcTan[(1 + 2*Sqrt[x])/Sqrt[3]])/Sqrt[3]
```

Maple [A] time = 0.004, size = 26, normalized size = 0.8

$$x + \frac{4\sqrt{3}}{3}\arctan\left(\frac{\sqrt{3}}{3}\left(1 + 2\sqrt{x}\right)\right) - 2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(1+x+x^(1/2)),x)

[Out] x+4/3*arctan(1/3*(1+2*x^(1/2))*3^(1/2))*3^(1/2)-2*x^(1/2)
```

Maxima [A] time = 0.762577, size = 34, normalized size = 1.06

$$\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\sqrt{x} + 1\right)\right) + x - 2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x + sqrt(x) + 1),x, algorithm="maxima")

[Out] 4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*sqrt(x) + 1)) + x - 2*sqrt(x)
```

Fricas [A] time = 0.272774, size = 49, normalized size = 1.53

$$\frac{1}{3}\sqrt{3}\left(\sqrt{3}x - 2\sqrt{3}\sqrt{x} + 4\arctan\left(\frac{2}{3}\sqrt{3}\sqrt{x} + \frac{1}{3}\sqrt{3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x + sqrt(x) + 1),x, algorithm="fricas")

[Out] 1/3*sqrt(3)*(sqrt(3)*x - 2*sqrt(3)*sqrt(x) + 4*arctan(2/3*sqrt(3)
*sqrt(x) + 1/3*sqrt(3)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x} + x + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(1+x+x**(1/2)),x)
```

```
[Out] Integral(x/(sqrt(x) + x + 1), x)
```

GIAC/XCAS [A] time = 0.261984, size = 34, normalized size = 1.06

$$\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\sqrt{x} + 1\right)\right) + x - 2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x + sqrt(x) + 1),x, algorithm="giac")
```

```
[Out] 4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*sqrt(x) + 1)) + x - 2*sqrt(x)
```

3.796 $\int \frac{1}{\sqrt{x}\left(1+\sqrt{x}+x\right)^{7/2}}\,dx$

Optimal. Leaf size=76

$$\frac{512\left(2\sqrt{x}+1\right)}{405\sqrt{x+\sqrt{x}+1}}+\frac{64\left(2\sqrt{x}+1\right)}{135\left(x+\sqrt{x}+1\right)^{3/2}}+\frac{4\left(2\sqrt{x}+1\right)}{15\left(x+\sqrt{x}+1\right)^{5/2}}$$

```
[Out] (4*(1 + 2*Sqrt[x]))/(15*(1 + Sqrt[x] + x)^(5/2)) + (64*(1 + 2*Sqr
t[x]))/(135*(1 + Sqrt[x] + x)^(3/2)) + (512*(1 + 2*Sqrt[x]))/(405
*Sqrt[1 + Sqrt[x] + x])
```

Rubi [A] time = 0.049974, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{512\left(2\sqrt{x}+1\right)}{405\sqrt{x+\sqrt{x}+1}}+\frac{64\left(2\sqrt{x}+1\right)}{135\left(x+\sqrt{x}+1\right)^{3/2}}+\frac{4\left(2\sqrt{x}+1\right)}{15\left(x+\sqrt{x}+1\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[x]*(1 + Sqrt[x] + x)^(7/2)),x]
```

```
[Out] (4*(1 + 2*Sqrt[x]))/(15*(1 + Sqrt[x] + x)^(5/2)) + (64*(1 + 2*Sqr
t[x]))/(135*(1 + Sqrt[x] + x)^(3/2)) + (512*(1 + 2*Sqrt[x]))/(405
*Sqrt[1 + Sqrt[x] + x])
```

Rubi in Sympy [A] time = 2.48128, size = 65, normalized size = 0.86

$$\frac{64\left(2\sqrt{x}+1\right)}{135\left(\sqrt{x}+x+1\right)^{\frac{3}{2}}}+\frac{4\left(2\sqrt{x}+1\right)}{15\left(\sqrt{x}+x+1\right)^{\frac{5}{2}}}+\frac{256\left(4\sqrt{x}+2\right)}{405\sqrt{\sqrt{x}+x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**(1/2)/(1+x+x**(1/2))**(7/2),x)
```

```
[Out] 64*(2*sqrt(x) + 1)/(135*(sqrt(x) + x + 1)**(3/2)) + 4*(2*sqrt(x)
+ 1)/(15*(sqrt(x) + x + 1)**(5/2)) + 256*(4*sqrt(x) + 2)/(405*sqr
t(sqrt(x) + x + 1))
```

Mathematica [A] time = 0.0275061, size = 49, normalized size = 0.64

$$\frac{4\left(2\sqrt{x}+1\right)\left(256x^{3/2}+128x^2+432x+304\sqrt{x}+203\right)}{405\left(x+\sqrt{x}+1\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[x]*(1 + Sqrt[x] + x)^(7/2)),x]
```

```
[Out] (4*(1 + 2*Sqrt[x])*(203 + 304*Sqrt[x] + 432*x + 256*x^(3/2) + 128
*x^2))/(405*(1 + Sqrt[x] + x)^(5/2))
```

Maple [A] time = 0.003, size = 53, normalized size = 0.7

$$\frac{4}{15}\left(1+2\sqrt{x}\right)\left(1+x+\sqrt{x}\right)^{-\frac{5}{2}}+\frac{64}{135}\left(1+2\sqrt{x}\right)\left(1+x+\sqrt{x}\right)^{-\frac{3}{2}}+\frac{512}{405}\left(1+2\sqrt{x}\right)\frac{1}{\sqrt{1+x+\sqrt{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^(1/2)/(1+x+x^(1/2))^(7/2),x)
```

```
[Out] 4/15*(1+2*x^(1/2))/(1+x+x^(1/2))^(5/2)+64/135*(1+2*x^(1/2))/(1+x+
x^(1/2))^(3/2)+512/405*(1+2*x^(1/2))/(1+x+x^(1/2))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x+\sqrt{x}+1\right)^{\frac{7}{2}}\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((x + sqrt(x) + 1)^(7/2)*sqrt(x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((x + sqrt(x) + 1)^(7/2)*sqrt(x)), x)
```

Fricas [A] time = 0.300592, size = 128, normalized size = 1.68

$$-\frac{4\left(128\,x^5+272\,x^4+455\,x^3+232\,x^2-\left(256\,x^5+736\,x^4+1366\,x^3+1427\,x^2+839\,x+101\right)\sqrt{x}-128\,x-203\right)\sqrt{x+\sqrt{x}+1}}{405\left(x^6+3\,x^5+6\,x^4+7\,x^3+6\,x^2+3\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x + sqrt(x) + 1)^(7/2)*sqrt(x)),x, algorithm="fricas")
```

```
[Out] -4/405*(128*x^5 + 272*x^4 + 455*x^3 + 232*x^2 - (256*x^5 + 736*x^
4 + 1366*x^3 + 1427*x^2 + 839*x + 101)*sqrt(x) - 128*x - 203)*sqr
t(x + sqrt(x) + 1)/(x^6 + 3*x^5 + 6*x^4 + 7*x^3 + 6*x^2 + 3*x + 1
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**(1/2)/(1+x+x**(1/2))**(7/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.268933, size = 61, normalized size = 0.8

$$\frac{4\left(2\left(8\left(2\left(4\sqrt{x}\left(2\sqrt{x}+5\right)+35\right)\sqrt{x}+65\right)\sqrt{x}+355\right)\sqrt{x}+203\right)}{405\left(x+\sqrt{x}+1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x + sqrt(x) + 1)^(7/2)*sqrt(x)),x, algorithm="giac")
```

```
[Out] 4/405*(2*(8*(2*(4*sqrt(x)*(2*sqrt(x) + 5) + 35)*sqrt(x) + 65)*sqr
t(x) + 355)*sqrt(x) + 203)/(x + sqrt(x) + 1)^(5/2)
```

3.797 $\int \frac{-1+x}{1+\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=46

$$\frac{\sqrt{x^2+1}}{x}+\sqrt{x^2+1}-\log\left(\sqrt{x^2+1}+1\right)-\frac{1}{x}-\sinh^{-1}(x)$$

```
[Out] -x^(-1) + Sqrt[1 + x^2] + Sqrt[1 + x^2]/x - ArcSinh[x] - Log[1 +
Sqrt[1 + x^2]]
```

Rubi [A] time = 0.145907, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353

$$\frac{\sqrt{x^2+1}}{x}+\sqrt{x^2+1}-\log\left(\sqrt{x^2+1}+1\right)-\frac{1}{x}-\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x)/(1 + Sqrt[1 + x^2]),x]
```

```
[Out] -x^(-1) + Sqrt[1 + x^2] + Sqrt[1 + x^2]/x - ArcSinh[x] - Log[1 +
Sqrt[1 + x^2]]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-1}{\sqrt{x^2+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-1+x)/(1+(x**2+1)**(1/2)),x)
```

```
[Out] Integral((x - 1)/(sqrt(x**2 + 1) + 1), x)
```

Mathematica [A] time = 0.0405428, size = 39, normalized size = 0.85

$$\sqrt{x^2+1}\left(\frac{1}{x}+1\right)-\log\left(\sqrt{x^2+1}+1\right)-\frac{1}{x}-\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + x)/(1 + Sqrt[1 + x^2]),x]
```

```
[Out] -x^(-1) + (1 + x^(-1))*Sqrt[1 + x^2] - ArcSinh[x] - Log[1 + Sqrt[
1 + x^2]]
```

Maple [A] time = 0.005, size = 53, normalized size = 1.2

$$-x^{-1}+\sqrt{x^2+1}-Artanh\left(\frac{1}{\sqrt{x^2+1}}\right)-\ln(x)+\frac{1}{x}\left(x^2+1\right)^{\frac{3}{2}}-x\sqrt{x^2+1}-Arcsinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-1+x)/(1+(x^2+1)^(1/2)),x)
```

```
[Out] -1/x+(x^2+1)^(1/2)-arctanh(1/(x^2+1)^(1/2))-ln(x)+1/x*(x^2+1)^(3/
2)-x*(x^2+1)^(1/2)-arcsinh(x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{4}x^2-\frac{1}{2}x-\int\frac{x^3-x^2}{2\left(x^2+2\sqrt{x^2+1}+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - 1)/(sqrt(x^2 + 1) + 1),x, algorithm="maxima")
```

```
[Out] 1/4*x^2 - 1/2*x - integrate(1/2*(x^3 - x^2)/(x^2 + 2*sqrt(x^2 + 1
) + 2), x)
```

Fricas [A] time = 0.27678, size = 201, normalized size = 4.37

$$-\frac{2x^4+4x^2-\left(2x^3-2\sqrt{x^2+1}x^2+x\right)\log\left(2x^2-\sqrt{x^2+1}(2x+1)+x+1\right)+\left(2x^3+x\right)\log(x)+\left(2x^3-2\sqrt{x^2+1}x^2+x\right.}{2x^3-2\sqrt{x^2+1}x^2+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - 1)/(sqrt(x^2 + 1) + 1),x, algorithm="fricas")
```

```
[Out] -(2*x^4 + 4*x^2 - (2*x^3 - 2*sqrt(x^2 + 1)*x^2 + x)*log(2*x^2 - s
qrt(x^2 + 1)*(2*x + 1) + x + 1) + (2*x^3 + x)*log(x) + (2*x^3 - 2
*sqrt(x^2 + 1)*x^2 + x)*log(-x + sqrt(x^2 + 1) + 1) - (2*x^3 + 2*
x^2*log(x) + 3*x + 1)*sqrt(x^2 + 1) + x + 1)/(2*x^3 - 2*sqrt(x^2
+ 1)*x^2 + x)
```

Sympy [A] time = 7.15092, size = 61, normalized size = 1.33

$$\frac{x}{\sqrt{x^2+1}} + \sqrt{x^2+1} - \log\left(1 + \frac{1}{\sqrt{x^2+1}}\right) + \log\left(\frac{1}{\sqrt{x^2+1}}\right) - \operatorname{asinh}(x) - \frac{1}{x} + \frac{1}{x\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-1+x)/(1+(x**2+1)**(1/2)),x)
```

```
[Out] x/sqrt(x**2 + 1) + sqrt(x**2 + 1) - log(1 + 1/sqrt(x**2 + 1)) + l
og(1/sqrt(x**2 + 1)) - asinh(x) - 1/x + 1/(x*sqrt(x**2 + 1))
```

GIAC/XCAS [A] time = 0.266822, size = 107, normalized size = 2.33

$$\sqrt{x^2+1} - \frac{2}{\left(x - \sqrt{x^2+1}\right)^2 - 1} - \frac{1}{x} + \ln\left(-x + \sqrt{x^2+1}\right) - \ln(|x|) - \ln\left(\left|-x + \sqrt{x^2+1} + 1\right|\right) + \ln\left(\left|-x + \sqrt{x^2+1} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - 1)/(sqrt(x^2 + 1) + 1),x, algorithm="giac")
```

```
[Out] sqrt(x^2 + 1) - 2/((x - sqrt(x^2 + 1))^2 - 1) - 1/x + ln(-x + sqr
t(x^2 + 1)) - ln(abs(x)) - ln(abs(-x + sqrt(x^2 + 1) + 1)) + ln(a
bs(-x + sqrt(x^2 + 1) - 1))
```

3.798 $\int \frac{1}{(1+x)^{2/3}(-1+x^2)^{2/3}}\,dx$

Optimal. Leaf size=20

$$\frac{3\sqrt[3]{x^2-1}}{2(x+1)^{2/3}}$$

```
[Out] (3*(-1 + x^2)^(1/3))/(2*(1 + x)^(2/3))
```

Rubi [A] time = 0.0186093, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$

$$\frac{3\sqrt[3]{x^2-1}}{2(x+1)^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((1 + x)^(2/3)*(-1 + x^2)^(2/3)),x]
```

```
[Out] (3*(-1 + x^2)^(1/3))/(2*(1 + x)^(2/3))
```

Rubi in Sympy [A] time = 1.26123, size = 17, normalized size = 0.85

$$\frac{3\sqrt[3]{x^2-1}}{2\,(x+1)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1+x)**(2/3)/(x**2-1)**(2/3),x)
```

```
[Out] 3*(x**2 - 1)**(1/3)/(2*(x + 1)**(2/3))
```

Mathematica [A] time = 0.018686, size = 23, normalized size = 1.15

$$\frac{3(x-1)\sqrt[3]{x+1}}{2\,(x^2-1)^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((1 + x)^(2/3)*(-1 + x^2)^(2/3)),x]
```

```
[Out] (3*(-1 + x)*(1 + x)^(1/3))/(2*(-1 + x^2)^(2/3))
```

Maple [A] time = 0.004, size = 18, normalized size = 0.9

$$\frac{-3+3x}{2}\sqrt[3]{1+x}\left(x^{2}-1\right)^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1+x)^(2/3)/(x^2-1)^(2/3),x)
```

```
[Out] 3/2*(-1+x)*(1+x)^(1/3)/(x^2-1)^(2/3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^{2}-1)^{\frac{2}{3}}(x+1)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 - 1)^(2/3)*(x + 1)^(2/3)),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 - 1)^(2/3)*(x + 1)^(2/3)), x)
```

Fricas [A] time = 0.268034, size = 19, normalized size = 0.95

$$\frac{3\left(x^{2}-1\right)^{\frac{1}{3}}}{2\,(x+1)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 - 1)^(2/3)*(x + 1)^(2/3)),x, algorithm="fricas")
```

```
[Out] 3/2*(x^2 - 1)^(1/3)/(x + 1)^(2/3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\left(x-1\right)\left(x+1\right)\right)^{\frac{2}{3}}\left(x+1\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1+x)**(2/3)/(x**2-1)**(2/3),x)
```

```
[Out] Integral(1/(((x - 1)*(x + 1))**(2/3)*(x + 1)**(2/3)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^2-1\right)^{\frac{2}{3}}\left(x+1\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 - 1)^(2/3)*(x + 1)^(2/3)),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 - 1)^(2/3)*(x + 1)^(2/3)), x)
```

3.799 $\int \left(\left(1-x^6\right)^{2/3} + \frac{\left(1-x^6\right)^{2/3}}{x^6} \right) dx$

Optimal. Leaf size=35

$$\frac{1}{5}x\left(1-x^6\right)^{2/3} - \frac{\left(1-x^6\right)^{2/3}}{5x^5}$$

```
[Out] -(1 - x^6)^(2/3)/(5*x^5) + (x*(1 - x^6)^(2/3))/5
```

Rubi [C] time = 0.0298573, antiderivative size = 36, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074

$$x\,{}_2F_1\left(-\frac{2}{3},\frac{1}{6};\frac{7}{6};x^6\right) - \frac{{}_2F_1\left(-\frac{5}{6},-\frac{2}{3};\frac{1}{6};x^6\right)}{5x^5}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - x^6)^(2/3) + (1 - x^6)^(2/3)/x^6,x]
```

```
[Out] -Hypergeometric2F1[-5/6, -2/3, 1/6, x^6]/(5*x^5) + x*Hypergeometr
ic2F1[-2/3, 1/6, 7/6, x^6]
```

Rubi in Sympy [A] time = 1.44004, size = 31, normalized size = 0.89

$$x{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{6}\\ \frac{7}{6}\end{matrix}\middle| x^6\right) - \frac{{}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{5}{6}\\ \frac{1}{6}\end{matrix}\middle| x^6\right)}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**6+1)**(2/3)+(-x**6+1)**(2/3)/x**6,x)
```

```
[Out] x*hyper((-2/3, 1/6), (7/6,), x**6) - hyper((-2/3, -5/6), (1/6,),
x**6)/(5*x**5)
```

Mathematica [A] time = 0.0189683, size = 18, normalized size = 0.51

$$-\frac{\left(1-x^6\right)^{5/3}}{5x^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - x^6)^(2/3) + (1 - x^6)^(2/3)/x^6,x]
```

```
[Out] -(1 - x^6)^(5/3)/(5*x^5)
```

Maple [A] time = 0.012, size = 35, normalized size = 1.

$$\frac{(-1+x)(1+x)\left(x^{2}+x+1\right)\left(x^{2}-x+1\right)}{5\,x^{5}}\left(-x^{6}+1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^6+1)^(2/3)+(-x^6+1)^(2/3)/x^6,x)
```

```
[Out] 1/5*(-x^6+1)^(2/3)*(x^2-x+1)*(x^2+x+1)*(1+x)*(-1+x)/x^5
```

Maxima [A] time = 0.895435, size = 51, normalized size = 1.46

$$\frac{\left(x^{6}-1\right)\left(x^{2}+x+1\right)^{\frac{2}{3}}\left(-x^{2}+x-1\right)^{\frac{2}{3}}(x+1)^{\frac{2}{3}}(x-1)^{\frac{2}{3}}}{5\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^6 + 1)^(2/3) + (-x^6 + 1)^(2/3)/x^6,x, algorithm="maxima")
```

```
[Out] 1/5*(x^6 - 1)*(x^2 + x + 1)^(2/3)*(-x^2 + x - 1)^(2/3)*(x + 1)^(2
/3)*(x - 1)^(2/3)/x^5
```

Fricas [A] time = 0.28297, size = 26, normalized size = 0.74

$$\frac{\left(x^{6}-1\right)\left(-x^{6}+1\right)^{\frac{2}{3}}}{5\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^6 + 1)^(2/3) + (-x^6 + 1)^(2/3)/x^6,x, algorithm="fricas")
```

```
[Out] 1/5*(x^6 - 1)*(-x^6 + 1)^(2/3)/x^5
```

Sympy [A] time = 3.99027, size = 68, normalized size = 1.94

$$\frac{x\Gamma\left(\frac{1}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{2}{3},\frac{1}{6}\\ \frac{7}{6}\end{matrix}\middle|x^{6}e^{2i\pi}\right)}{6\Gamma\left(\frac{7}{6}\right)}+\frac{\Gamma\left(-\frac{5}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{5}{6},-\frac{2}{3}\\ \frac{1}{6}\end{matrix}\middle|x^{6}e^{2i\pi}\right)}{6x^{5}\Gamma\left(\frac{1}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**6+1)**(2/3)+(-x**6+1)**(2/3)/x**6,x)
```

```
[Out] x*gamma(1/6)*hyper((-2/3, 1/6), (7/6,), x**6*exp_polar(2*I*pi))/(
6*gamma(7/6)) + gamma(-5/6)*hyper((-5/6, -2/3), (1/6,), x**6*exp_
polar(2*I*pi))/(6*x**5*gamma(1/6))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int(-x^{6}+1)^{\frac{2}{3}}+\frac{(-x^{6}+1)^{\frac{2}{3}}}{x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^6 + 1)^(2/3) + (-x^6 + 1)^(2/3)/x^6,x, algorithm="giac")
```

```
[Out] integrate((-x^6 + 1)^(2/3) + (-x^6 + 1)^(2/3)/x^6, x)
```

3.800 $\int \frac{x^{-1+m}(2am+b(2m-n)x^n)}{2(a+bx^n)^{3/2}}\,dx$

Optimal. Leaf size=15

$$\frac{x^m}{\sqrt{a+bx^n}}$$

```
[Out] x^m/Sqrt[a + b*x^n]
```

Rubi [A] time = 0.0580868, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054

$$\frac{x^m}{\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^(-1 + m)*(2*a*m + b*(2*m - n)*x^n))/(2*(a + b*x^n)^(3/2)),x]
```

```
[Out] x^m/Sqrt[a + b*x^n]
```

Rubi in Sympy [A] time = 3.8424, size = 12, normalized size = 0.8

$$\frac{x^m}{\sqrt{a+bx^n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/2*x**(-1+m)*(2*a*m+b*(2*m-n)*x**n)/(a+b*x**n)**(3/2),x)
```

```
[Out] x**m/sqrt(a + b*x**n)
```

Mathematica [A] time = 0.0861637, size = 15, normalized size = 1.

$$\frac{x^m}{\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + m)*(2*a*m + b*(2*m - n)*x^n))/(2*(a + b*x^n)^(3/2)),x]

[Out] x^m/Sqrt[a + b*x^n]
```

Maple [F] time = 0.069, size = 0, normalized size = 0.

$$\int \frac{x^{-1+m} (2\,am + b\,(2\,m - n)\,x^n)}{2} (a + bx^n)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2*x^(-1+m)*(2*a*m+b*(2*m-n)*x^n)/(a+b*x^n)^(3/2),x)

[Out] int(1/2*x^(-1+m)*(2*a*m+b*(2*m-n)*x^n)/(a+b*x^n)^(3/2),x)
```

Maxima [A] time = 0.834525, size = 18, normalized size = 1.2

$$\frac{x^m}{\sqrt{bx^n + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*(b*(2*m - n)*x^n + 2*a*m)*x^(m - 1)/(b*x^n + a)^(3/2),x, algorithm=

[Out] x^m/sqrt(b*x^n + a)
```

Fricas [A] time = 0.29713, size = 22, normalized size = 1.47

$$\frac{xx^{m-1}}{\sqrt{bx^n + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*(b*(2*m - n)*x^n + 2*a*m)*x^(m - 1)/(b*x^n + a)^(3/2),x, algorithm=

[Out] x*x^(m - 1)/sqrt(b*x^n + a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x**(-1+m)*(2*a*m+b*(2*m-n)*x**n)/(a+b*x**n)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b(2\,m-n)x^n+2\,am)x^{m-1}}{2\,(bx^n+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*(b*(2*m - n)*x^n + 2*a*m)*x^(m - 1)/(b*x^n + a)^(3/2),x, algorithm=
```

```
[Out] integrate(1/2*(b*(2*m - n)*x^n + 2*a*m)*x^(m - 1)/(b*x^n + a)^(3/
2), x)
```

3.801 $\int \frac{x-2x^3}{\sqrt{2+3x}}\,dx$

Optimal. Leaf size=53

$$-\frac{4}{567}(3x+2)^{7/2}+\frac{8}{135}(3x+2)^{5/2}-\frac{10}{81}(3x+2)^{3/2}-\frac{4}{81}\sqrt{3x+2}$$

```
[Out] (-4*Sqrt[2 + 3*x])/81 - (10*(2 + 3*x)^(3/2))/81 + (8*(2 + 3*x)^(5
/2))/135 - (4*(2 + 3*x)^(7/2))/567
```

Rubi [A] time = 0.0502457, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118

$$-\frac{4}{567}(3x+2)^{7/2}+\frac{8}{135}(3x+2)^{5/2}-\frac{10}{81}(3x+2)^{3/2}-\frac{4}{81}\sqrt{3x+2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x - 2*x^3)/Sqrt[2 + 3*x],x]
```

```
[Out] (-4*Sqrt[2 + 3*x])/81 - (10*(2 + 3*x)^(3/2))/81 + (8*(2 + 3*x)^(5
/2))/135 - (4*(2 + 3*x)^(7/2))/567
```

Rubi in Sympy [A] time = 3.45853, size = 46, normalized size = 0.87

$$-\frac{4(3x+2)^{\frac{7}{2}}}{567}+\frac{8(3x+2)^{\frac{5}{2}}}{135}-\frac{10(3x+2)^{\frac{3}{2}}}{81}-\frac{4\sqrt{3x+2}}{81}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-2*x**3+x)/(2+3*x)**(1/2),x)
```

```
[Out] -4*(3*x + 2)**(7/2)/567 + 8*(3*x + 2)**(5/2)/135 - 10*(3*x + 2)**
(3/2)/81 - 4*sqrt(3*x + 2)/81
```

Mathematica [A] time = 0.0135814, size = 28, normalized size = 0.53

$$-\frac{2\sqrt{3x+2}\left(270x^3-216x^2-123x+164\right)}{2835}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x - 2*x^3)/Sqrt[2 + 3*x],x]
```

```
[Out] (-2*Sqrt[2 + 3*x]*(164 - 123*x - 216*x^2 + 270*x^3))/2835
```

Maple [A] time = 0.006, size = 25, normalized size = 0.5

$$-\frac{540 x^3 - 432 x^2 - 246 x + 328}{2835}\sqrt{2 + 3 x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-2*x^3+x)/(2+3*x)^(1/2),x)
```

```
[Out] -2/2835*(270*x^3-216*x^2-123*x+164)*(2+3*x)^(1/2)
```

Maxima [A] time = 0.691183, size = 50, normalized size = 0.94

$$-\frac{4}{567}(3 x + 2)^{\frac{7}{2}} + \frac{8}{135}(3 x + 2)^{\frac{5}{2}} - \frac{10}{81}(3 x + 2)^{\frac{3}{2}} - \frac{4}{81}\sqrt{3 x + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(2*x^3 - x)/sqrt(3*x + 2),x, algorithm="maxima")
```

```
[Out] -4/567*(3*x + 2)^(7/2) + 8/135*(3*x + 2)^(5/2) - 10/81*(3*x + 2)^
(3/2) - 4/81*sqrt(3*x + 2)
```

Fricas [A] time = 0.272769, size = 32, normalized size = 0.6

$$-\frac{2}{2835}\left(270 x^3 - 216 x^2 - 123 x + 164\right)\sqrt{3 x + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(2*x^3 - x)/sqrt(3*x + 2),x, algorithm="fricas")
```

```
[Out] -2/2835*(270*x^3 - 216*x^2 - 123*x + 164)*sqrt(3*x + 2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-2*x**3+x)/(2+3*x)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.263622, size = 50, normalized size = 0.94

$$-\frac{4}{567}\,(3\,x+2)^{\frac{7}{2}}+\frac{8}{135}\,(3\,x+2)^{\frac{5}{2}}-\frac{10}{81}\,(3\,x+2)^{\frac{3}{2}}-\frac{4}{81}\,\sqrt{3\,x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(2*x^3 - x)/sqrt(3*x + 2),x, algorithm="giac")
```

```
[Out] -4/567*(3*x + 2)^(7/2) + 8/135*(3*x + 2)^(5/2) - 10/81*(3*x + 2)^
(3/2) - 4/81*sqrt(3*x + 2)
```

3.802 $\int \frac{1}{\sqrt[4]{1+x}+\sqrt{1+x}}\,dx$

Optimal. Leaf size=31

$$2\sqrt{x+1}-4\sqrt[4]{x+1}+4\log\left(\sqrt[4]{x+1}+1\right)$$

```
[Out] -4*(1 + x)^(1/4) + 2*Sqrt[1 + x] + 4*Log[1 + (1 + x)^(1/4)]
```

Rubi [A] time = 0.034163, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235

$$2\sqrt{x+1}-4\sqrt[4]{x+1}+4\log\left(\sqrt[4]{x+1}+1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[((1 + x)^(1/4) + Sqrt[1 + x])^(-1),x]

[Out] -4*(1 + x)^(1/4) + 2*Sqrt[1 + x] + 4*Log[1 + (1 + x)^(1/4)]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-4\sqrt[4]{x+1}+4\log\left(\sqrt[4]{x+1}+1\right)+4\int^{\sqrt[4]{x+1}} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/((1+x)**(1/4)+(1+x)**(1/2)),x)

[Out] -4*(x + 1)**(1/4) + 4*log((x + 1)**(1/4) + 1) + 4*Integral(x, (x,
 (x + 1)**(1/4)))
```

Mathematica [A] time = 0.0143116, size = 31, normalized size = 1.

$$2\sqrt{x+1}-4\sqrt[4]{x+1}+4\log\left(\sqrt[4]{x+1}+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((1 + x)^(1/4) + Sqrt[1 + x])^(-1),x]

[Out] -4*(1 + x)^(1/4) + 2*Sqrt[1 + x] + 4*Log[1 + (1 + x)^(1/4)]
```

Maple [A] time = 0.011, size = 26, normalized size = 0.8

$$-4\sqrt[4]{1+x}+4\ln\left(1+\sqrt[4]{1+x}\right)+2\sqrt{1+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/((1+x)^(1/4)+(1+x)^(1/2)),x)

[Out] -4*(1+x)^(1/4)+4*ln(1+(1+x)^(1/4))+2*(1+x)^(1/2)
```

Maxima [A] time = 0.704884, size = 34, normalized size = 1.1

$$2\sqrt{x+1}-4\,(x+1)^{\frac{1}{4}}+4\,\log\left((x+1)^{\frac{1}{4}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x + 1) + (x + 1)^(1/4)),x, algorithm="maxima")

[Out] 2*sqrt(x + 1) - 4*(x + 1)^(1/4) + 4*log((x + 1)^(1/4) + 1)
```

Fricas [A] time = 0.271504, size = 34, normalized size = 1.1

$$2\sqrt{x+1}-4\,(x+1)^{\frac{1}{4}}+4\,\log\left((x+1)^{\frac{1}{4}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x + 1) + (x + 1)^(1/4)),x, algorithm="fricas")

[Out] 2*sqrt(x + 1) - 4*(x + 1)^(1/4) + 4*log((x + 1)^(1/4) + 1)
```

Sympy [A] time = 0.588333, size = 27, normalized size = 0.87

$$-4\sqrt[4]{x+1}+2\sqrt{x+1}+4\log\left(\sqrt[4]{x+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((1+x)**(1/4)+(1+x)**(1/2)),x)
```

```
[Out] -4*(x + 1)**(1/4) + 2*sqrt(x + 1) + 4*log((x + 1)**(1/4) + 1)
```

GIAC/XCAS [A] time = 0.262126, size = 34, normalized size = 1.1

$$2\sqrt{x+1}-4\,(x+1)^{\frac{1}{4}}+4\ln\left((x+1)^{\frac{1}{4}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x + 1) + (x + 1)^(1/4)),x, algorithm="giac")
```

```
[Out] 2*sqrt(x + 1) - 4*(x + 1)^(1/4) + 4*ln((x + 1)^(1/4) + 1)
```

3.803 $\int \frac{1+2x}{\sqrt{x+x^2}}\,dx$

Optimal. Leaf size=11

$$2\sqrt{x^2+x}$$

```
[Out] 2*Sqrt[x + x^2]
```

Rubi [A] time = 0.00654045, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$2\sqrt{x^2+x}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + 2*x)/Sqrt[x + x^2],x]

[Out] 2*Sqrt[x + x^2]
```

Rubi in Sympy [A] time = 1.05087, size = 8, normalized size = 0.73

$$2\sqrt{x^2+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+2*x)/(x**2+x)**(1/2),x)

[Out] 2*sqrt(x**2 + x)
```

Mathematica [A] time = 0.0138085, size = 11, normalized size = 1.

$$2\sqrt{x(x+1)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + 2*x)/Sqrt[x + x^2],x]
```

```
[Out] 2*Sqrt[x*(1 + x)]
```

Maple [A] time = 0.005, size = 14, normalized size = 1.3

$$2\frac{x(1+x)}{\sqrt{x^2+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+2*x)/(x^2+x)^(1/2),x)
```

```
[Out] 2*x*(1+x)/(x^2+x)^(1/2)
```

Maxima [A] time = 0.705094, size = 12, normalized size = 1.09

$$2\sqrt{x^2+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x + 1)/sqrt(x^2 + x),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x^2 + x)
```

Fricas [A] time = 0.263172, size = 57, normalized size = 5.18

$$-\frac{8x^2-2\sqrt{x^2+x}(4x+1)+6x-1}{2\left(2x-2\sqrt{x^2+x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x + 1)/sqrt(x^2 + x),x, algorithm="fricas")
```

```
[Out] -1/2*(8*x^2 - 2*sqrt(x^2 + x)*(4*x + 1) + 6*x - 1)/(2*x - 2*sqrt(
x^2 + x) + 1)
```

Sympy [A] time = 0.293673, size = 8, normalized size = 0.73

$$2\sqrt{x^2 + x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+2*x)/(x**2+x)**(1/2),x)
```

```
[Out] 2*sqrt(x**2 + x)
```

GIAC/XCAS [A] time = 0.261638, size = 12, normalized size = 1.09

$$2\sqrt{x^2 + x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x + 1)/sqrt(x^2 + x),x, algorithm="giac")
```

```
[Out] 2*sqrt(x^2 + x)
```

3.804 $\int \frac{1}{2\sqrt{x}(1+x)}\,dx$

Optimal. Leaf size=6

$$\tan^{-1}\left(\sqrt{x}\right)$$

```
[Out] ArcTan[Sqrt[x]]
```

Rubi [A] time = 0.00837332, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214

$$\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(2*Sqrt[x]*(1 + x)),x]

[Out] ArcTan[Sqrt[x]]
```

Rubi in Sympy [A] time = 1.03338, size = 5, normalized size = 0.83

$$\operatorname{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/2/(1+x)/x**(1/2),x)

[Out] atan(sqrt(x))
```

Mathematica [A] time = 0.00637886, size = 6, normalized size = 1.

$$\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(2*Sqrt[x]*(1 + x)),x]
```

```
[Out] ArcTan[Sqrt[x]]
```

Maple [A] time = 0.005, size = 5, normalized size = 0.8

$$\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2/(1+x)/x^(1/2),x)
```

```
[Out] arctan(x^(1/2))
```

Maxima [A] time = 0.774768, size = 5, normalized size = 0.83

$$\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/((x + 1)*sqrt(x)),x, algorithm="maxima")
```

```
[Out] arctan(sqrt(x))
```

Fricas [A] time = 0.270482, size = 5, normalized size = 0.83

$$\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/((x + 1)*sqrt(x)),x, algorithm="fricas")
```

```
[Out] arctan(sqrt(x))
```

Sympy [A] time = 3.30272, size = 32, normalized size = 5.33

$$\frac{\begin{cases} 2i \operatorname{acosh}\left(\frac{1}{\sqrt{x+1}}\right) & \text{for } \left|\frac{1}{x+1}\right| > 1 \\ -2 \operatorname{asin}\left(\frac{1}{\sqrt{x+1}}\right) & \text{otherwise} \end{cases}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/2/(1+x)/x**(1/2),x)
```

```
[Out] Piecewise((2*I*acosh(1/sqrt(x + 1)), Abs(1/(x + 1)) > 1), (-2*asi
n(1/sqrt(x + 1)), True))/2
```

GIAC/XCAS [A] time = 0.259751, size = 5, normalized size = 0.83

$$\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/2/((x + 1)*sqrt(x)),x, algorithm="giac")
```

```
[Out] arctan(sqrt(x))
```

3.805 $\int \frac{1}{x\sqrt{6x-x^2}}\,dx$

Optimal. Leaf size=20

$$-\frac{\sqrt{6x-x^2}}{3x}$$

```
[Out] -Sqrt[6*x - x^2]/(3*x)
```

Rubi [A] time = 0.0191091, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$

$$-\frac{\sqrt{6x-x^2}}{3x}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*Sqrt[6*x - x^2]),x]

[Out] -Sqrt[6*x - x^2]/(3*x)
```

Rubi in Sympy [A] time = 1.7664, size = 14, normalized size = 0.7

$$-\frac{\sqrt{-x^2+6x}}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(-x**2+6*x)**(1/2),x)

[Out] -sqrt(-x**2 + 6*x)/(3*x)
```

Mathematica [A] time = 0.0135887, size = 17, normalized size = 0.85

$$\frac{x-6}{3\sqrt{-(x-6)x}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*Sqrt[6*x - x^2]),x]

[Out] (-6 + x)/(3*Sqrt[-((-6 + x)*x)])
```

Maple [A] time = 0.004, size = 17, normalized size = 0.9

$$\frac{-6+x}{3}\frac{1}{\sqrt{-x^2+6\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(-x^2+6*x)^(1/2),x)

[Out] 1/3*(-6+x)/(-x^2+6*x)^(1/2)
```

Maxima [A] time = 0.783018, size = 22, normalized size = 1.1

$$-\frac{\sqrt{-x^2+6\,x}}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-x^2 + 6*x)*x),x, algorithm="maxima")

[Out] -1/3*sqrt(-x^2 + 6*x)/x
```

Fricas [A] time = 0.261602, size = 22, normalized size = 1.1

$$-\frac{\sqrt{-x^2+6\,x}}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-x^2 + 6*x)*x),x, algorithm="fricas")

[Out] -1/3*sqrt(-x^2 + 6*x)/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{-x\left(x-6\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(-x**2+6*x)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(-x*(x - 6))), x)
```

GIAC/XCAS [A] time = 0.26747, size = 34, normalized size = 1.7

$$\frac{2}{3\left(\frac{\sqrt{-x^{2}+6\,x}-3}{x-3}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-x^2 + 6*x)*x),x, algorithm="giac")
```

```
[Out] 2/3/((sqrt(-x^2 + 6*x) - 3)/(x - 3) - 1)
```

3.806 $\int \left(1+\sqrt{x}\right) \sqrt{x}\,dx$

Optimal. Leaf size=17

$$\frac{2x^{3/2}}{3}+\frac{x^2}{2}$$

```
[Out] (2*x^(3/2))/3 + x^2/2
```

Rubi [A] time = 0.00896272, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{2x^{3/2}}{3}+\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + Sqrt[x])*Sqrt[x],x]

[Out] (2*x^(3/2))/3 + x^2/2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{2x^{\frac{3}{2}}}{3}+\int x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(1/2)*(1+x**(1/2)),x)

[Out] 2*x**(3/2)/3 + Integral(x, x)
```

Mathematica [A] time = 0.00372876, size = 17, normalized size = 1.

$$\frac{2x^{3/2}}{3}+\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + Sqrt[x])*Sqrt[x],x]
```

```
[Out] (2*x^(3/2))/3 + x^2/2
```

Maple [A] time = 0.002, size = 12, normalized size = 0.7

$$\frac{2}{3}x^{\frac{3}{2}} + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(1/2)*(1+x^(1/2)),x)
```

```
[Out] 2/3*x^(3/2)+1/2*x^2
```

Maxima [A] time = 0.706601, size = 35, normalized size = 2.06

$$\frac{1}{2}\left(\sqrt{x}+1\right)^4 - \frac{4}{3}\left(\sqrt{x}+1\right)^3 + \left(\sqrt{x}+1\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)*(sqrt(x) + 1),x, algorithm="maxima")
```

```
[Out] 1/2*(sqrt(x) + 1)^4 - 4/3*(sqrt(x) + 1)^3 + (sqrt(x) + 1)^2
```

Fricas [A] time = 0.26268, size = 15, normalized size = 0.88

$$\frac{1}{2}x^2 + \frac{2}{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)*(sqrt(x) + 1),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 + 2/3*x^(3/2)
```

Sympy [A] time = 0.277931, size = 12, normalized size = 0.71

$$\frac{2x^{\frac{3}{2}}}{3} + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(1/2)*(1+x**(1/2)),x)
```

```
[Out] 2*x**(3/2)/3 + x**2/2
```

GIAC/XCAS [A] time = 0.261648, size = 15, normalized size = 0.88

$$\frac{1}{2}\,x^2 + \frac{2}{3}\,x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)*(sqrt(x) + 1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 + 2/3*x^(3/2)
```

3.807 $\int \frac{1-\sqrt{x}}{\sqrt[3]{x}}\,dx$

Optimal. Leaf size=19

$$\frac{3x^{2/3}}{2}-\frac{6x^{7/6}}{7}$$

```
[Out] (3*x^(2/3))/2 - (6*x^(7/6))/7
```

Rubi [A] time = 0.00994923, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$\frac{3x^{2/3}}{2}-\frac{6x^{7/6}}{7}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - Sqrt[x])/x^(1/3),x]

[Out] (3*x^(2/3))/2 - (6*x^(7/6))/7
```

Rubi in Sympy [A] time = 1.23819, size = 15, normalized size = 0.79

$$-\frac{6x^{\frac{7}{6}}}{7}+\frac{3x^{\frac{2}{3}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1-x**(1/2))/x**(1/3),x)

[Out] -6*x**(7/6)/7 + 3*x**(2/3)/2
```

Mathematica [A] time = 0.00578017, size = 19, normalized size = 1.

$$\frac{3x^{2/3}}{2}-\frac{6x^{7/6}}{7}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - Sqrt[x])/x^(1/3),x]

[Out] (3*x^(2/3))/2 - (6*x^(7/6))/7
```

Maple [A] time = 0.002, size = 12, normalized size = 0.6

$$\frac{3}{2}x^{\frac{2}{3}} - \frac{6}{7}x^{\frac{7}{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1-x^(1/2))/x^(1/3),x)

[Out] 3/2*x^(2/3)-6/7*x^(7/6)
```

Maxima [A] time = 0.67822, size = 15, normalized size = 0.79

$$-\frac{6}{7}x^{\frac{7}{6}} + \frac{3}{2}x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(sqrt(x) - 1)/x^(1/3),x, algorithm="maxima")

[Out] -6/7*x^(7/6) + 3/2*x^(2/3)
```

Fricas [A] time = 0.260389, size = 15, normalized size = 0.79

$$-\frac{6}{7}x^{\frac{7}{6}} + \frac{3}{2}x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(sqrt(x) - 1)/x^(1/3),x, algorithm="fricas")

[Out] -6/7*x^(7/6) + 3/2*x^(2/3)
```

Sympy [A] time = 1.31166, size = 15, normalized size = 0.79

$$-\frac{6x^{\frac{7}{6}}}{7}+\frac{3x^{\frac{2}{3}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1-x**(1/2))/x**(1/3),x)
```

```
[Out] -6*x**(7/6)/7 + 3*x**(2/3)/2
```

GIAC/XCAS [A] time = 0.261083, size = 15, normalized size = 0.79

$$-\frac{6}{7}\,x^{\frac{7}{6}}+\frac{3}{2}\,x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(sqrt(x) - 1)/x^(1/3),x, algorithm="giac")
```

```
[Out] -6/7*x^(7/6) + 3/2*x^(2/3)
```

3.808 $\int \frac{\sqrt{x}}{1+\sqrt[3]{x}}\,dx$

Optimal. Leaf size=41

$$\frac{6x^{7/6}}{7}-\frac{6x^{5/6}}{5}+2\sqrt{x}-6\sqrt[6]{x}+6\tan^{-1}\left(\sqrt[6]{x}\right)$$

```
[Out] -6*x^(1/6) + 2*Sqrt[x] - (6*x^(5/6))/5 + (6*x^(7/6))/7 + 6*ArcTan
[x^(1/6)]
```

Rubi [A] time = 0.0346666, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267

$$\frac{6x^{7/6}}{7}-\frac{6x^{5/6}}{5}+2\sqrt{x}-6\sqrt[6]{x}+6\tan^{-1}\left(\sqrt[6]{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x]/(1 + x^(1/3)),x]
```

```
[Out] -6*x^(1/6) + 2*Sqrt[x] - (6*x^(5/6))/5 + (6*x^(7/6))/7 + 6*ArcTan
[x^(1/6)]
```

Rubi in Sympy [A] time = 2.88017, size = 37, normalized size = 0.9

$$\frac{6x^{\frac{7}{6}}}{7}-\frac{6x^{\frac{5}{6}}}{5}-6\sqrt[6]{x}+2\sqrt{x}+6\,\text{atan}\left(\sqrt[6]{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(1/2)/(1+x**(1/3)),x)
```

```
[Out] 6*x**(7/6)/7 - 6*x**(5/6)/5 - 6*x**(1/6) + 2*sqrt(x) + 6*atan(x**
(1/6))
```

Mathematica [A] time = 0.0132777, size = 41, normalized size = 1.

$$\frac{6x^{7/6}}{7}-\frac{6x^{5/6}}{5}+2\sqrt{x}-6\sqrt[6]{x}+6\tan^{-1}\left(\sqrt[6]{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[x]/(1 + x^(1/3)),x]
```

```
[Out] -6*x^(1/6) + 2*Sqrt[x] - (6*x^(5/6))/5 + (6*x^(7/6))/7 + 6*ArcTan
[x^(1/6)]
```

Maple [A] time = 0.002, size = 28, normalized size = 0.7

$$-6\sqrt[6]{x}-\frac{6}{5}x^{\frac{5}{6}}+\frac{6}{7}x^{\frac{7}{6}}+6\arctan\left(\sqrt[6]{x}\right)+2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(1/2)/(1+x^(1/3)),x)
```

```
[Out] -6*x^(1/6)-6/5*x^(5/6)+6/7*x^(7/6)+6*arctan(x^(1/6))+2*x^(1/2)
```

Maxima [A] time = 0.755433, size = 36, normalized size = 0.88

$$\frac{6}{7}x^{\frac{7}{6}}-\frac{6}{5}x^{\frac{5}{6}}+2\sqrt{x}-6x^{\frac{1}{6}}+6\arctan\left(x^{\frac{1}{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/(x^(1/3) + 1),x, algorithm="maxima")
```

```
[Out] 6/7*x^(7/6) - 6/5*x^(5/6) + 2*sqrt(x) - 6*x^(1/6) + 6*arctan(x^(1
/6))
```

Fricas [A] time = 0.266904, size = 34, normalized size = 0.83

$$\frac{6}{7}(x-7)x^{\frac{1}{6}}-\frac{6}{5}x^{\frac{5}{6}}+2\sqrt{x}+6\arctan\left(x^{\frac{1}{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/(x^(1/3) + 1),x, algorithm="fricas")
```

```
[Out] 6/7*(x - 7)*x^(1/6) - 6/5*x^(5/6) + 2*sqrt(x) + 6*arctan(x^(1/6))
```

Sympy [A] time = 9.66581, size = 37, normalized size = 0.9

$$\frac{6x^{\frac{7}{6}}}{7} - \frac{6x^{\frac{5}{6}}}{5} - 6\sqrt[6]{x} + 2\sqrt{x} + 6\,\mathrm{atan}\left(\sqrt[6]{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(1/2)/(1+x**(1/3)),x)
```

```
[Out] 6*x**(7/6)/7 - 6*x**(5/6)/5 - 6*x**(1/6) + 2*sqrt(x) + 6*atan(x**
(1/6))
```

GIAC/XCAS [A] time = 0.262878, size = 36, normalized size = 0.88

$$\frac{6}{7}\,x^{\frac{7}{6}} - \frac{6}{5}\,x^{\frac{5}{6}} + 2\,\sqrt{x} - 6\,x^{\frac{1}{6}} + 6\,\arctan\left(x^{\frac{1}{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/(x^(1/3) + 1),x, algorithm="giac")
```

```
[Out] 6/7*x^(7/6) - 6/5*x^(5/6) + 2*sqrt(x) - 6*x^(1/6) + 6*arctan(x^(1
/6))
```

3.809 $\int \frac{\sqrt[3]{1+\sqrt{x}}}{x}\,dx$

Optimal. Leaf size=67

$$6\sqrt[3]{\sqrt{x}+1}+3\log\left(1-\sqrt[3]{\sqrt{x}+1}\right)-\frac{\log(x)}{2}-2\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{\sqrt{x}+1}+1}{\sqrt{3}}\right)$$

```
[Out] 6*(1 + Sqrt[x])^(1/3) - 2*Sqrt[3]*ArcTan[(1 + 2*(1 + Sqrt[x])^(1/
3))/Sqrt[3]] + 3*Log[1 - (1 + Sqrt[x])^(1/3)] - Log[x]/2
```

Rubi [A] time = 0.068901, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4

$$6\sqrt[3]{\sqrt{x}+1}+3\log\left(1-\sqrt[3]{\sqrt{x}+1}\right)-\frac{\log(x)}{2}-2\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{\sqrt{x}+1}+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + Sqrt[x])^(1/3)/x,x]
```

```
[Out] 6*(1 + Sqrt[x])^(1/3) - 2*Sqrt[3]*ArcTan[(1 + 2*(1 + Sqrt[x])^(1/
3))/Sqrt[3]] + 3*Log[1 - (1 + Sqrt[x])^(1/3)] - Log[x]/2
```

Rubi in Sympy [A] time = 2.38318, size = 63, normalized size = 0.94

$$6\sqrt[3]{\sqrt{x}+1}-\log\left(\sqrt{x}\right)+3\log\left(-\sqrt[3]{\sqrt{x}+1}+1\right)-2\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2\sqrt[3]{\sqrt{x}+1}}{3}+\frac{1}{3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+x**(1/2))**(1/3)/x,x)
```

```
[Out] 6*(sqrt(x) + 1)**(1/3) - log(sqrt(x)) + 3*log(-(sqrt(x) + 1)**(1/
3) + 1) - 2*sqrt(3)*atan(sqrt(3)*(2*(sqrt(x) + 1)**(1/3)/3 + 1/3)
)
```

Mathematica [C] time = 0.0267403, size = 51, normalized size = 0.76

$$\frac{-3\left(\frac{1}{\sqrt{x}}+1\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};-\frac{1}{\sqrt{x}}\right)+6\sqrt{x}+6}{\left(\sqrt{x}+1\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + Sqrt[x])^(1/3)/x,x]
```

```
[Out] (6 + 6*Sqrt[x] - 3*(1 + 1/Sqrt[x])^(2/3)*Hypergeometric2F1[2/3, 2
/3, 5/3, -(1/Sqrt[x])])/(1 + Sqrt[x])^(2/3)
```

Maple [A] time = 0.006, size = 64, normalized size = 1.

$$6\sqrt[3]{1+\sqrt{x}}-\ln\left(\left(1+\sqrt{x}\right)^{\frac{2}{3}}+\sqrt[3]{1+\sqrt{x}}+1\right)-2\,\arctan\left(1/3\,\left(1+2\sqrt[3]{1+\sqrt{x}}\right)\sqrt{3}\right)\sqrt{3}+2\,\ln\left(\sqrt[3]{1+\sqrt{x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+x^(1/2))^(1/3)/x,x)
```

```
[Out] 6*(1+x^(1/2))^(1/3)-ln((1+x^(1/2))^(2/3)+(1+x^(1/2))^(1/3)+1)-2*a
rctan(1/3*(1+2*(1+x^(1/2))^(1/3))*3^(1/2))*3^(1/2)+2*ln((1+x^(1/2
))^(1/3)-1)
```

Maxima [A] time = 0.78451, size = 85, normalized size = 1.27

$$-2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\sqrt{x}+1\right)^{\frac{1}{3}}+1\right)\right)+6\left(\sqrt{x}+1\right)^{\frac{1}{3}}$$
$$-\log\left(\left(\sqrt{x}+1\right)^{\frac{2}{3}}+\left(\sqrt{x}+1\right)^{\frac{1}{3}}+1\right)+2\log\left(\left(\sqrt{x}+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(x) + 1)^(1/3)/x,x, algorithm="maxima")
```

```
[Out] -2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(sqrt(x) + 1)^(1/3) + 1)) + 6*(s
qrt(x) + 1)^(1/3) - log((sqrt(x) + 1)^(2/3) + (sqrt(x) + 1)^(1/3)
 + 1) + 2*log((sqrt(x) + 1)^(1/3) - 1)
```

Fricas [A] time = 0.287226, size = 85, normalized size = 1.27

$$-2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\sqrt{x}+1\right)^{\frac{1}{3}}+1\right)\right)+6\left(\sqrt{x}+1\right)^{\frac{1}{3}}-\log\left(\left(\sqrt{x}+1\right)^{\frac{2}{3}}+\left(\sqrt{x}+1\right)^{\frac{1}{3}}+1\right)+2\log\left(\left(\sqrt{x}+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(x) + 1)^(1/3)/x,x, algorithm="fricas")
```

```
[Out] -2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(sqrt(x) + 1)^(1/3) + 1)) + 6*(s
qrt(x) + 1)^(1/3) - log((sqrt(x) + 1)^(2/3) + (sqrt(x) + 1)^(1/3)
 + 1) + 2*log((sqrt(x) + 1)^(1/3) - 1)
```

Sympy [A] time = 3.86289, size = 39, normalized size = 0.58

$$-\frac{2\sqrt[6]{x}\left(-\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{e^{i\pi}}{\sqrt{x}}\right)}{\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+x**(1/2))**(1/3)/x,x)
```

```
[Out] -2*x**(1/6)*gamma(-1/3)*hyper((-1/3, -1/3), (2/3,), exp_polar(I*p
i)/sqrt(x))/gamma(2/3)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(x) + 1)^(1/3)/x,x, algorithm="giac")
```

```
[Out] Timed out
```

3.810 $\int \left(1-\sqrt{x}\right) \, dx$

Optimal. Leaf size=11

$$x-\frac{2x^{3/2}}{3}$$

```
[Out] x - (2*x^(3/2))/3
```

Rubi [A] time = 0.0056749, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$

$$x-\frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

```
[In]  Int[1 - Sqrt[x],x]

[Out] x - (2*x^(3/2))/3
```

Rubi in Sympy [A] time = 0.635257, size = 8, normalized size = 0.73

$$-\frac{2x^{\frac{3}{2}}}{3}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1-x**(1/2),x)

[Out] -2*x**(3/2)/3 + x
```

Mathematica [A] time = 0.00148184, size = 11, normalized size = 1.

$$x-\frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1 - Sqrt[x],x]
```

```
[Out] x - (2*x^(3/2))/3
```

Maple [A] time = 0., size = 8, normalized size = 0.7

$$x-\frac{2}{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1-x^(1/2),x)
```

```
[Out] x-2/3*x^(3/2)
```

Maxima [A] time = 0.684185, size = 9, normalized size = 0.82

$$-\frac{2}{3}x^{\frac{3}{2}}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sqrt(x) + 1,x, algorithm="maxima")
```

```
[Out] -2/3*x^(3/2) + x
```

Fricas [A] time = 0.259731, size = 9, normalized size = 0.82

$$-\frac{2}{3}x^{\frac{3}{2}}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sqrt(x) + 1,x, algorithm="fricas")
```

```
[Out] -2/3*x^(3/2) + x
```

Sympy [A] time = 0.068311, size = 8, normalized size = 0.73

$$-\frac{2x^{\frac{3}{2}}}{3}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1-x**(1/2),x)

[Out] -2*x**(3/2)/3 + x
```

GIAC/XCAS [A] time = 0.279556, size = 9, normalized size = 0.82

$$-\frac{2}{3}\,x^{\frac{3}{2}}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sqrt(x) + 1,x, algorithm="giac")

[Out] -2/3*x^(3/2) + x
```

3.811 $\int \left(1-\sqrt[4]{x}\right) dx$

Optimal. Leaf size=11

$$x-\frac{4x^{5/4}}{5}$$

```
[Out] x - (4*x^(5/4))/5
```

Rubi [A] time = 0.00552035, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.

$$x-\frac{4x^{5/4}}{5}$$

Antiderivative was successfully verified.

```
[In]  Int[1 - x^(1/4),x]

[Out] x - (4*x^(5/4))/5
```

Rubi in Sympy [A] time = 0.633267, size = 8, normalized size = 0.73

$$-\frac{4x^{\frac{5}{4}}}{5}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1-x**(1/4),x)

[Out] -4*x**(5/4)/5 + x
```

Mathematica [A] time = 0.00276401, size = 11, normalized size = 1.

$$x-\frac{4x^{5/4}}{5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1 - x^(1/4),x]
```

```
[Out] x - (4*x^(5/4))/5
```

Maple [A] time = 0.002, size = 8, normalized size = 0.7

$$x - \frac{4}{5}x^{\frac{5}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1-x^(1/4),x)
```

```
[Out] x-4/5*x^(5/4)
```

Maxima [A] time = 0.702502, size = 9, normalized size = 0.82

$$-\frac{4}{5}x^{\frac{5}{4}} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x^(1/4) + 1,x, algorithm="maxima")
```

```
[Out] -4/5*x^(5/4) + x
```

Fricas [A] time = 0.265185, size = 9, normalized size = 0.82

$$-\frac{4}{5}x^{\frac{5}{4}} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x^(1/4) + 1,x, algorithm="fricas")
```

```
[Out] -4/5*x^(5/4) + x
```

Sympy [A] time = 0.068443, size = 8, normalized size = 0.73

$$-\frac{4x^{\frac{5}{4}}}{5} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1-x**(1/4),x)
```

```
[Out] -4*x**(5/4)/5 + x
```

GIAC/XCAS [A] time = 0.259644, size = 9, normalized size = 0.82

$$-\frac{4}{5}\,x^{\frac{5}{4}} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x^(1/4) + 1,x, algorithm="giac")
```

```
[Out] -4/5*x^(5/4) + x
```

3.812 $\int \frac{1-\sqrt{x}}{1+\sqrt[4]{x}}\,dx$

Optimal. Leaf size=11

$$x-\frac{4x^{5/4}}{5}$$

```
[Out] x - (4*x^(5/4))/5
```

Rubi [A] time = 0.00621247, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053

$$x-\frac{4x^{5/4}}{5}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - Sqrt[x])/(1 + x^(1/4)),x]
```

```
[Out] x - (4*x^(5/4))/5
```

Rubi in Sympy [A] time = 1.19604, size = 8, normalized size = 0.73

$$-\frac{4x^{\frac{5}{4}}}{5}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1-x**(1/2))/(1+x**(1/4)),x)
```

```
[Out] -4*x**(5/4)/5 + x
```

Mathematica [A] time = 0.000650205, size = 11, normalized size = 1.

$$x-\frac{4x^{5/4}}{5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - Sqrt[x])/(1 + x^(1/4)),x]
```

```
[Out] x - (4*x^(5/4))/5
```

Maple [C] time = 0.013, size = 46, normalized size = 4.2

$$-\frac{4}{5}x^{\frac{5}{4}} + x + 2\ln\left(1 + \sqrt[4]{x}\right) - \ln\left(1 - x\right) - \ln\left(-1 + \sqrt{x}\right) + \ln\left(1 + \sqrt{x}\right) + 2\ln\left(\sqrt[4]{x} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1-x^(1/2))/(1+x^(1/4)),x)
```

```
[Out] -4/5*x^(5/4)+x+2*ln(1+x^(1/4))-ln(1-x)-ln(-1+x^(1/2))+ln(1+x^(1/2
))+2*ln(x^(1/4)-1)
```

Maxima [A] time = 0.705103, size = 9, normalized size = 0.82

$$-\frac{4}{5}x^{\frac{5}{4}} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(sqrt(x) - 1)/(x^(1/4) + 1),x, algorithm="maxima")
```

```
[Out] -4/5*x^(5/4) + x
```

Fricas [A] time = 0.267476, size = 9, normalized size = 0.82

$$-\frac{4}{5}x^{\frac{5}{4}} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(sqrt(x) - 1)/(x^(1/4) + 1),x, algorithm="fricas")
```

```
[Out] -4/5*x^(5/4) + x
```

Sympy [A] time = 12.2177, size = 8, normalized size = 0.73

$$-\frac{4x^{\frac{5}{4}}}{5}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1-x**(1/2))/(1+x**(1/4)),x)
```

```
[Out] -4*x**(5/4)/5 + x
```

GIAC/XCAS [A] time = 0.265398, size = 9, normalized size = 0.82

$$-\frac{4}{5}x^{\frac{5}{4}}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(sqrt(x) - 1)/(x^(1/4) + 1),x, algorithm="giac")
```

```
[Out] -4/5*x^(5/4) + x
```

3.813 $\int \frac{1}{\sqrt{(a+bx)(c+dx)}}\,dx$

Optimal. Leaf size=61

$$\frac{\tanh^{-1}\left(\frac{ad+bc+2bdx}{2\sqrt{b}\sqrt{d}\sqrt{x(ad+bc)+ac+bdx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

```
[Out] ArcTanh[(b*c + a*d + 2*b*d*x)/(2*Sqrt[b]*Sqrt[d]*Sqrt[a*c + (b*c
+ a*d)*x + b*d*x^2])]/(Sqrt[b]*Sqrt[d])
```

Rubi [A] time = 0.0583457, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{\tanh^{-1}\left(\frac{ad+bc+2bdx}{2\sqrt{b}\sqrt{d}\sqrt{x(ad+bc)+ac+bdx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[(a + b*x)*(c + d*x)],x]
```

```
[Out] ArcTanh[(b*c + a*d + 2*b*d*x)/(2*Sqrt[b]*Sqrt[d]*Sqrt[a*c + (b*c
+ a*d)*x + b*d*x^2])]/(Sqrt[b]*Sqrt[d])
```

Rubi in Sympy [A] time = 2.43263, size = 58, normalized size = 0.95

$$\frac{\operatorname{atanh}\left(\frac{ad+bc+2bdx}{2\sqrt{b}\sqrt{d}\sqrt{ac+bdx^2+x(ad+bc)}}\right)}{\sqrt{b}\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/((b*x+a)*(d*x+c))**(1/2),x)
```

```
[Out] atanh((a*d + b*c + 2*b*d*x)/(2*sqrt(b)*sqrt(d)*sqrt(a*c + b*d*x**
2 + x*(a*d + b*c))))/(sqrt(b)*sqrt(d))
```

Mathematica [A] time = 0.0489312, size = 87, normalized size = 1.43

$$\frac{\sqrt{a+bx}\sqrt{c+dx}\log\left(2\sqrt{b}\sqrt{d}\sqrt{a+bx}\sqrt{c+dx}+ad+bc+2bdx\right)}{\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[(a + b*x)*(c + d*x)],x]
```

```
[Out] (Sqrt[a + b*x]*Sqrt[c + d*x]*Log[b*c + a*d + 2*b*d*x + 2*Sqrt[b]*
Sqrt[d]*Sqrt[a + b*x]*Sqrt[c + d*x]])/(Sqrt[b]*Sqrt[d]*Sqrt[(a +
b*x)*(c + d*x)])
```

Maple [A] time = 0.012, size = 49, normalized size = 0.8

$$1\ln\left(1\left(\frac{ad}{2}+\frac{bc}{2}+bdx\right)\frac{1}{\sqrt{bd}}+\sqrt{ac+(ad+bc)x+bdx^2}\right)\frac{1}{\sqrt{bd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/((b*x+a)*(d*x+c))^(1/2),x)
```

```
[Out] ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(a*c+(a*d+b*c)*x+b*d*x^2)^
(1/2))/(b*d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt((b*x + a)*(d*x + c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.2842, size = 1, normalized size = 0.02

$$\left[\frac{\log\left(4\left(2\,b^2d^2x+b^2cd+abd^2\right)\sqrt{bdx^2+ac+(bc+ad)x}+\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2+8\left(b^2cd+abd^2\right)x\right)\sqrt{bd}\right)}{2\sqrt{bd}}, \text{arc}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt((b*x + a)*(d*x + c)),x, algorithm="fricas")
```

```
[Out] [1/2*log(4*(2*b^2*d^2*x + b^2*c*d + a*b*d^2)*sqrt(b*d*x^2 + a*c +
 (b*c + a*d)*x) + (8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2
+ 8*(b^2*c*d + a*b*d^2)*x)*sqrt(b*d))/sqrt(b*d), arctan(1/2*(2*b*
d*x + b*c + a*d)*sqrt(-b*d)/(sqrt(b*d*x^2 + a*c + (b*c + a*d)*x)*
b*d))/sqrt(-b*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x+a)*(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.295161, size = 92, normalized size = 1.51

$$-\frac{\sqrt{bd}\ln\left(\left|-2\left(\sqrt{bd}x-\sqrt{bdx^2+bcx+adx+ac}\right)bd-\sqrt{bd}bc-\sqrt{bd}ad\right|\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt((b*x + a)*(d*x + c)),x, algorithm="giac")
```

```
[Out] -sqrt(b*d)*ln(abs(-2*(sqrt(b*d)*x - sqrt(b*d*x^2 + b*c*x + a*d*x
+ a*c))*b*d - sqrt(b*d)*b*c - sqrt(b*d)*a*d))/(b*d)
```

3.814 $\int \frac{1}{\sqrt{(a+bx)(c-dx)}} dx$

Optimal. Leaf size=65

$$-\frac{\tan^{-1}\left(\frac{-ad+bc-2bdx}{2\sqrt{b}\sqrt{d}\sqrt{x(bc-ad)+ac-bdx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

```
[Out] -(ArcTan[(b*c - a*d - 2*b*d*x)/(2*Sqrt[b]*Sqrt[d]*Sqrt[a*c + (b*c
 - a*d)*x - b*d*x^2])]/(Sqrt[b]*Sqrt[d]))
```

Rubi [A] time = 0.0585131, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188

$$-\frac{\tan^{-1}\left(\frac{-ad+bc-2bdx}{2\sqrt{b}\sqrt{d}\sqrt{x(bc-ad)+ac-bdx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[(a + b*x)*(c - d*x)],x]
```

```
[Out] -(ArcTan[(b*c - a*d - 2*b*d*x)/(2*Sqrt[b]*Sqrt[d]*Sqrt[a*c + (b*c
 - a*d)*x - b*d*x^2])]/(Sqrt[b]*Sqrt[d]))
```

Rubi in Sympy [A] time = 2.49379, size = 60, normalized size = 0.92

$$-\frac{\operatorname{atan}\left(\frac{-ad+bc-2bdx}{2\sqrt{b}\sqrt{d}\sqrt{ac-bdx^2+x(-ad+bc)}}\right)}{\sqrt{b}\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/((b*x+a)*(-d*x+c))**(1/2),x)
```

```
[Out] -atan((-a*d + b*c - 2*b*d*x)/(2*sqrt(b)*sqrt(d)*sqrt(a*c - b*d*x*
*2 + x*(-a*d + b*c))))/(sqrt(b)*sqrt(d))
```

Mathematica [C] time = 0.12729, size = 99, normalized size = 1.52

$$\frac{i\sqrt{a+bx}\sqrt{c-dx}\log\left(2\sqrt{a+bx}\sqrt{c-dx}-\frac{i(ad-bc+2bdx)}{\sqrt{b}\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c-dx)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[(a + b*x)*(c - d*x)],x]
```

```
[Out] (I*Sqrt[a + b*x]*Sqrt[c - d*x]*Log[2*Sqrt[a + b*x]*Sqrt[c - d*x]
- (I*(-(b*c) + a*d + 2*b*d*x))/(Sqrt[b]*Sqrt[d])])/(Sqrt[b]*Sqrt[
d]*Sqrt[(a + b*x)*(c - d*x)])
```

Maple [A] time = 0.017, size = 55, normalized size = 0.9

$$1\arctan\left(1\sqrt{bd}\left(x-\frac{-ad+bc}{2\,bd}\right)\frac{1}{\sqrt{ac+(-ad+bc)\,x-bdx^2}}\right)\frac{1}{\sqrt{bd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/((b*x+a)*(-d*x+c))^(1/2),x)
```

```
[Out] 1/(b*d)^(1/2)*arctan((b*d)^(1/2)*(x-1/2*(-a*d+b*c)/b/d)/(a*c+(-a*
d+b*c)*x-b*d*x^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-(b*x + a)*(d*x - c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.279448, size = 1, normalized size = 0.02

$$\left[\frac{\log\left(4\left(2\,b^2d^2x - b^2cd + abd^2\right)\sqrt{-bdx^2 + ac + (bc - ad)x} + \left(8\,b^2d^2x^2 + b^2c^2 - 6\,abcd + a^2d^2 - 8\left(b^2cd - abd^2\right)x\right)\sqrt{-bd}\right)}{2\sqrt{-bd}},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-(b*x + a)*(d*x - c)),x, algorithm="fricas")
```

```
[Out] [1/2*log(4*(2*b^2*d^2*x - b^2*c*d + a*b*d^2)*sqrt(-b*d*x^2 + a*c
+ (b*c - a*d)*x) + (8*b^2*d^2*x^2 + b^2*c^2 - 6*a*b*c*d + a^2*d^2
 - 8*(b^2*c*d - a*b*d^2)*x)*sqrt(-b*d))/sqrt(-b*d), arctan(1/2*(2
*b*d*x - b*c + a*d)*sqrt(b*d)/(sqrt(-b*d*x^2 + a*c + (b*c - a*d)*
x)*b*d))/sqrt(b*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x+a)*(-d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.29075, size = 80, normalized size = 1.23

$$-\frac{\ln\left(\left|bc - ad + 2\sqrt{-bd}\left(\sqrt{-bd}x - \sqrt{-bdx^2 + bcx - adx + ac}\right)\right|\right)}{\sqrt{-bd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-(b*x + a)*(d*x - c)),x, algorithm="giac")
```

```
[Out] -ln(abs(b*c - a*d + 2*sqrt(-b*d)*(sqrt(-b*d)*x - sqrt(-b*d*x^2 +
b*c*x - a*d*x + a*c))))/sqrt(-b*d)
```

3.815 $\int \frac{1}{\sqrt{x}(1-x^2)} dx$

Optimal. Leaf size=13

$$\tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)$$

```
[Out] ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]
```

Rubi [A] time = 0.0174192, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[x]*(1 - x^2)),x]
```

```
[Out] ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]
```

Rubi in Sympy [A] time = 1.90019, size = 12, normalized size = 0.92

$$\operatorname{atan}\left(\sqrt{x}\right)+\operatorname{atanh}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-x**2+1)/x**(1/2),x)
```

```
[Out] atan(sqrt(x)) + atanh(sqrt(x))
```

Mathematica [B] time = 0.00840691, size = 33, normalized size = 2.54

$$-\frac{1}{2}\log\left(1-\sqrt{x}\right)+\frac{1}{2}\log\left(\sqrt{x}+1\right)+\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[x]*(1 - x^2)),x]
```

```
[Out] ArcTan[Sqrt[x]] - Log[1 - Sqrt[x]]/2 + Log[1 + Sqrt[x]]/2
```

Maple [A] time = 0.006, size = 10, normalized size = 0.8

$$\arctan\left(\sqrt{x}\right) + Artanh\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-x^2+1)/x^(1/2),x)
```

```
[Out] arctan(x^(1/2))+arctanh(x^(1/2))
```

Maxima [A] time = 0.809102, size = 28, normalized size = 2.15

$$\arctan\left(\sqrt{x}\right) + \frac{1}{2}\log\left(\sqrt{x}+1\right) - \frac{1}{2}\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((x^2 - 1)*sqrt(x)),x, algorithm="maxima")
```

```
[Out] arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(sqrt(x) - 1)
```

Fricas [A] time = 0.283119, size = 28, normalized size = 2.15

$$\arctan\left(\sqrt{x}\right) + \frac{1}{2}\log\left(\sqrt{x}+1\right) - \frac{1}{2}\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((x^2 - 1)*sqrt(x)),x, algorithm="fricas")
```

```
[Out] arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(sqrt(x) - 1)
```

Sympy [A] time = 1.31764, size = 26, normalized size = 2.

$$-\frac{\log\left(\sqrt{x}-1\right)}{2} + \frac{\log\left(\sqrt{x}+1\right)}{2} + \operatorname{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-x**2+1)/x**(1/2),x)
```

```
[Out] -log(sqrt(x) - 1)/2 + log(sqrt(x) + 1)/2 + atan(sqrt(x))
```

GIAC/XCAS [A] time = 0.263804, size = 30, normalized size = 2.31

$$\arctan\left(\sqrt{x}\right) + \frac{1}{2}\ln\left(\sqrt{x}+1\right) - \frac{1}{2}\ln\left(\left|\sqrt{x}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((x^2 - 1)*sqrt(x)),x, algorithm="giac")
```

```
[Out] arctan(sqrt(x)) + 1/2*ln(sqrt(x) + 1) - 1/2*ln(abs(sqrt(x) - 1))
```

3.816 $\int \frac{\sqrt{x}}{x-x^3} \, dx$

Optimal. Leaf size=13

$$\tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)$$

```
[Out] ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]
```

Rubi [A] time = 0.0191388, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(x - x^3),x]
```

```
[Out] ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]
```

Rubi in Sympy [A] time = 2.37322, size = 12, normalized size = 0.92

$$\operatorname{atan}\left(\sqrt{x}\right)+\operatorname{atanh}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**(1/2)/(-x**3+x),x)
```

```
[Out] atan(sqrt(x)) + atanh(sqrt(x))
```

Mathematica [B] time = 0.00756152, size = 33, normalized size = 2.54

$$-\frac{1}{2}\log\left(1-\sqrt{x}\right)+\frac{1}{2}\log\left(\sqrt{x}+1\right)+\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(x - x^3),x]
```

```
[Out] ArcTan[Sqrt[x]] - Log[1 - Sqrt[x]]/2 + Log[1 + Sqrt[x]]/2
```

Maple [A] time = 0.005, size = 10, normalized size = 0.8

$$\arctan\left(\sqrt{x}\right) + \mathit{Artanh}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(-x^3+x),x)
```

```
[Out] arctan(x^(1/2))+arctanh(x^(1/2))
```

Maxima [A] time = 0.794672, size = 28, normalized size = 2.15

$$\arctan\left(\sqrt{x}\right) + \frac{1}{2}\log\left(\sqrt{x}+1\right) - \frac{1}{2}\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sqrt(x)/(x^3 - x),x, algorithm="maxima")
```

```
[Out] arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(sqrt(x) - 1)
```

Fricas [A] time = 0.28008, size = 28, normalized size = 2.15

$$\arctan\left(\sqrt{x}\right) + \frac{1}{2}\log\left(\sqrt{x}+1\right) - \frac{1}{2}\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sqrt(x)/(x^3 - x),x, algorithm="fricas")
```

```
[Out] arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(sqrt(x) - 1)
```

Sympy [A] time = 164.965, size = 26, normalized size = 2.

$$-\frac{\log\left(\sqrt{x}-1\right)}{2} + \frac{\log\left(\sqrt{x}+1\right)}{2} + \operatorname{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(1/2)/(-x**3+x),x)
```

```
[Out] -log(sqrt(x) - 1)/2 + log(sqrt(x) + 1)/2 + atan(sqrt(x))
```

GIAC/XCAS [A] time = 0.268558, size = 30, normalized size = 2.31

$$\arctan\left(\sqrt{x}\right) + \frac{1}{2}\ln\left(\sqrt{x} + 1\right) - \frac{1}{2}\ln\left(\left|\sqrt{x} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sqrt(x)/(x^3 - x),x, algorithm="giac")
```

```
[Out] arctan(sqrt(x)) + 1/2*ln(sqrt(x) + 1) - 1/2*ln(abs(sqrt(x) - 1))
```

3.817 $\int \frac{x}{2-\sqrt{3}+\left(1+\sqrt{3}\right)x+x^2}\,dx$

Optimal. Leaf size=72

$$\frac{1}{2}\log\left(x^2+\left(1+\sqrt{3}\right)x-\sqrt{3}+2\right)+\sqrt{\frac{1}{23}\left(13+8\sqrt{3}\right)}\tanh^{-1}\left(\frac{2x+\sqrt{3}+1}{\sqrt{2\left(3\sqrt{3}-2\right)}}\right)$$

```
[Out] Sqrt[(13 + 8*Sqrt[3])/23]*ArcTanh[(1 + Sqrt[3] + 2*x)/Sqrt[2*(-2
+ 3*Sqrt[3])]] + Log[2 - Sqrt[3] + (1 + Sqrt[3])*x + x^2]/2
```

Rubi [A] time = 0.201693, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16

$$\frac{1}{2}\log\left(x^2+\left(1+\sqrt{3}\right)x-\sqrt{3}+2\right)+\sqrt{\frac{1}{23}\left(13+8\sqrt{3}\right)}\tanh^{-1}\left(\frac{2x+\sqrt{3}+1}{\sqrt{2\left(3\sqrt{3}-2\right)}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x/(2 - Sqrt[3] + (1 + Sqrt[3])*x + x^2),x]
```

```
[Out] Sqrt[(13 + 8*Sqrt[3])/23]*ArcTanh[(1 + Sqrt[3] + 2*x)/Sqrt[2*(-2
+ 3*Sqrt[3])]] + Log[2 - Sqrt[3] + (1 + Sqrt[3])*x + x^2]/2
```

Rubi in Sympy [A] time = 4.72877, size = 78, normalized size = 1.08

$$\frac{\log\left(x^2+x\left(1+\sqrt{3}\right)-\sqrt{3}+2\right)}{2}+\frac{\sqrt{2}\left(\frac{1}{2}+\frac{\sqrt{3}}{2}\right)\operatorname{atanh}\left(\frac{\sqrt{2}\left(x+\frac{1}{2}+\frac{\sqrt{3}}{2}\right)}{\sqrt{-2+3\sqrt{3}}}\right)}{\sqrt{-2+3\sqrt{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(2+x**2-3**(1/2)+x*(1+3**(1/2))),x)
```

```
[Out] log(x**2 + x*(1 + sqrt(3)) - sqrt(3) + 2)/2 + sqrt(2)*(1/2 + sqrt
(3)/2)*atanh(sqrt(2)*(x + 1/2 + sqrt(3)/2)/sqrt(-2 + 3*sqrt(3)))/
```

```
sqrt(-2 + 3*sqrt(3))
```

Mathematica [A] time = 0.160082, size = 72, normalized size = 1.

$$\frac{1}{2}\log\left(x^2+\sqrt{3}x+x-\sqrt{3}+2\right)+\frac{\left(1+\sqrt{3}\right)\tanh^{-1}\left(\frac{2x+\sqrt{3}+1}{\sqrt{6\sqrt{3}-4}}\right)}{\sqrt{6\sqrt{3}-4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(2 - Sqrt[3] + (1 + Sqrt[3])*x + x^2),x]
```

```
[Out] ((1 + Sqrt[3])*ArcTanh[(1 + Sqrt[3] + 2*x)/Sqrt[-4 + 6*Sqrt[3]]])
/Sqrt[-4 + 6*Sqrt[3]] + Log[2 - Sqrt[3] + x + Sqrt[3]*x + x^2]/2
```

Maple [A] time = 0.023, size = 82, normalized size = 1.1

$$\frac{\ln\left(x\sqrt{3}+x^2-\sqrt{3}+x+2\right)}{2}+\frac{1}{\sqrt{-4+6\sqrt{3}}}\mathit{Artanh}\left(\frac{1+2x+\sqrt{3}}{\sqrt{-4+6\sqrt{3}}}\right)+\frac{\sqrt{3}}{\sqrt{-4+6\sqrt{3}}}\mathit{Artanh}\left(\frac{1+2x+\sqrt{3}}{\sqrt{-4+6\sqrt{3}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(2+x^2-3^(1/2)+x*(1+3^(1/2))),x)
```

```
[Out] 1/2*ln(x*3^(1/2)+x^2-3^(1/2)+x+2)+1/(-4+6*3^(1/2))^(1/2)*arctanh(
(1+2*x+3^(1/2))/(-4+6*3^(1/2))^(1/2))+1/(-4+6*3^(1/2))^(1/2)*arct
anh((1+2*x+3^(1/2))/(-4+6*3^(1/2))^(1/2))*3^(1/2)
```

Maxima [A] time = 0.828038, size = 104, normalized size = 1.44

$$-\frac{\left(\sqrt{3}+1\right)\log\left(\frac{2x+\sqrt{3}-\sqrt{6\sqrt{3}-4}+1}{2x+\sqrt{3}+\sqrt{6\sqrt{3}-4}+1}\right)}{2\sqrt{6\sqrt{3}-4}}+\frac{1}{2}\log\left(x^2+x\left(\sqrt{3}+1\right)-\sqrt{3}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2 + x*(sqrt(3) + 1) - sqrt(3) + 2),x, algorithm="maxima")
```

```
[Out] -1/2*(sqrt(3) + 1)*log((2*x + sqrt(3) - sqrt(6*sqrt(3) - 4) + 1)/
(2*x + sqrt(3) + sqrt(6*sqrt(3) - 4) + 1))/sqrt(6*sqrt(3) - 4) +
1/2*log(x^2 + x*(sqrt(3) + 1) - sqrt(3) + 2)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2 + x*(sqrt(3) + 1) - sqrt(3) + 2),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 4.22278, size = 168, normalized size = 2.33

$$\left(\frac{1}{2} - \frac{\sqrt{11 + 64\sqrt{3}}}{2\left(-31 + 12\sqrt{3}\right)}\right)\log\left(x - \frac{-521 + 287\sqrt{3}}{11 + 64\sqrt{3}} + \frac{\left(\frac{1}{2} - \frac{\sqrt{11+64\sqrt{3}}}{2\left(-31+12\sqrt{3}\right)}\right)\left(269 + 459\sqrt{3}\right)}{214 + 139\sqrt{3}}\right) + \left(\frac{\sqrt{11 + 64\sqrt{3}}}{2\left(-31 + 12\sqrt{3}\right)} + \frac{1}{2}\right)\log\left(x + \frac{\left(269 + 459\sqrt{3}\right)\left(\frac{\sqrt{11+64\sqrt{3}}}{2\left(-31+12\sqrt{3}\right)} + \frac{1}{2}\right)}{214 + 139\sqrt{3}} - \frac{-521 + 287\sqrt{3}}{11 + 64\sqrt{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(2+x**2-3**(1/2)+x*(1+3**(1/2))),x)
```

```
[Out] (1/2 - sqrt(11 + 64*sqrt(3))/(2*(-31 + 12*sqrt(3))))*log(x - (-52
1 + 287*sqrt(3))/(11 + 64*sqrt(3)) + (1/2 - sqrt(11 + 64*sqrt(3))
/(2*(-31 + 12*sqrt(3))))*(269 + 459*sqrt(3))/(214 + 139*sqrt(3)))
 + (sqrt(11 + 64*sqrt(3))/(2*(-31 + 12*sqrt(3))) + 1/2)*log(x + (
269 + 459*sqrt(3))*(sqrt(11 + 64*sqrt(3))/(2*(-31 + 12*sqrt(3)))
+ 1/2)/(214 + 139*sqrt(3)) - (-521 + 287*sqrt(3))/(11 + 64*sqrt(3
)))
```

GIAC/XCAS [A] time = 0.285437, size = 108, normalized size = 1.5

$$-\frac{\left(\sqrt{3}+1\right)\ln\left(\frac{\left|2\,x+\sqrt{3}-\sqrt{6\,\sqrt{3}-4}+1\right|}{\left|2\,x+\sqrt{3}+\sqrt{6\,\sqrt{3}-4}+1\right|}\right)}{2\,\sqrt{6\,\sqrt{3}-4}}+\frac{1}{2}\,\ln\left(\left|x^2+x\left(\sqrt{3}+1\right)-\sqrt{3}+2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2 + x*(sqrt(3) + 1) - sqrt(3) + 2),x, algorithm="giac")
```

```
[Out] -1/2*(sqrt(3) + 1)*ln(abs(2*x + sqrt(3) - sqrt(6*sqrt(3) - 4) + 1
)/abs(2*x + sqrt(3) + sqrt(6*sqrt(3) - 4) + 1))/sqrt(6*sqrt(3) -
4) + 1/2*ln(abs(x^2 + x*(sqrt(3) + 1) - sqrt(3) + 2))
```

3.818 $\int \sqrt{x^2+x^3}\,dx$

Optimal. Leaf size=37

$$\frac{2\left(x^3+x^2\right)^{3/2}}{5x^2}-\frac{4\left(x^3+x^2\right)^{3/2}}{15x^3}$$

```
[Out] (-4*(x^2 + x^3)^(3/2))/(15*x^3) + (2*(x^2 + x^3)^(3/2))/(5*x^2)
```

Rubi [A] time = 0.0455966, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{2\left(x^3+x^2\right)^{3/2}}{5x^2}-\frac{4\left(x^3+x^2\right)^{3/2}}{15x^3}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x^2 + x^3],x]

[Out] (-4*(x^2 + x^3)^(3/2))/(15*x^3) + (2*(x^2 + x^3)^(3/2))/(5*x^2)
```

Rubi in Sympy [A] time = 3.03249, size = 32, normalized size = 0.86

$$\frac{2\left(x^3+x^2\right)^{\frac{3}{2}}}{5x^2}-\frac{4\left(x^3+x^2\right)^{\frac{3}{2}}}{15x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**3+x**2)**(1/2),x)

[Out] 2*(x**3 + x**2)**(3/2)/(5*x**2) - 4*(x**3 + x**2)**(3/2)/(15*x**3
)
```

Mathematica [A] time = 0.0110324, size = 23, normalized size = 0.62

$$\frac{2\left(x^2(x+1)\right)^{3/2}(3x-2)}{15x^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[x^2 + x^3],x]

[Out] (2*(x^2*(1 + x))^(3/2)*(-2 + 3*x))/(15*x^3)
```

Maple [A] time = 0.003, size = 23, normalized size = 0.6

$$\frac{(2+2x)(3x-2)}{15x}\sqrt{x^3+x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^3+x^2)^(1/2),x)

[Out] 2/15*(1+x)*(3*x-2)*(x^3+x^2)^(1/2)/x
```

Maxima [A] time = 0.721156, size = 20, normalized size = 0.54

$$\frac{2}{15}\left(3x^2+x-2\right)\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^3 + x^2),x, algorithm="maxima")

[Out] 2/15*(3*x^2 + x - 2)*sqrt(x + 1)
```

Fricas [A] time = 0.265747, size = 30, normalized size = 0.81

$$\frac{2\sqrt{x^3+x^2}\left(3x^2+x-2\right)}{15x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^3 + x^2),x, algorithm="fricas")

[Out] 2/15*sqrt(x^3 + x^2)*(3*x^2 + x - 2)/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^3 + x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3+x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(x**3 + x**2), x)
```

GIAC/XCAS [A] time = 0.262637, size = 32, normalized size = 0.86

$$\frac{2}{15}\left(3\,(x+1)^{\frac{5}{2}} - 5\,(x+1)^{\frac{3}{2}}\right)\operatorname{sign}(x) + \frac{4}{15}\operatorname{sign}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^3 + x^2),x, algorithm="giac")
```

```
[Out] 2/15*(3*(x + 1)^(5/2) - 5*(x + 1)^(3/2))*sign(x) + 4/15*sign(x)
```

3.819 $\int \frac{1}{(1+x)\sqrt{2x+x^2}}\,dx$

Optimal. Leaf size=12

$$\tan^{-1}\left(\sqrt{x^2+2x}\right)$$

```
[Out] ArcTan[Sqrt[2*x + x^2]]
```

Rubi [A] time = 0.022977, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$

$$\tan^{-1}\left(\sqrt{x^2+2x}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/((1 + x)*Sqrt[2*x + x^2]),x]

[Out] ArcTan[Sqrt[2*x + x^2]]
```

Rubi in Sympy [A] time = 2.1984, size = 10, normalized size = 0.83

$$\operatorname{atan}\left(\sqrt{x^2+2x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1+x)/(x**2+2*x)**(1/2),x)

[Out] atan(sqrt(x**2 + 2*x))
```

Mathematica [B] time = 0.0332174, size = 37, normalized size = 3.08

$$\frac{2\sqrt{x}\sqrt{x+2}\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt{x+2}}\right)}{\sqrt{x(x+2)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((1 + x)*Sqrt[2*x + x^2]),x]
```

```
[Out] (2*Sqrt[x]*Sqrt[2 + x]*ArcTan[Sqrt[x]/Sqrt[2 + x]])/Sqrt[x*(2 + x
)]
```

Maple [A] time = 0., size = 13, normalized size = 1.1

$$-\arctan\left(\frac{1}{\sqrt{(1+x)^2-1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1+x)/(x^2+2*x)^(1/2),x)
```

```
[Out] -arctan(1/((1+x)^2-1)^(1/2))
```

Maxima [A] time = 0.785636, size = 12, normalized size = 1.

$$-\arcsin\left(\frac{1}{|x+1|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 + 2*x)*(x + 1)),x, algorithm="maxima")
```

```
[Out] -arcsin(1/abs(x + 1))
```

Fricas [A] time = 0.267443, size = 23, normalized size = 1.92

$$2\arctan\left(-x+\sqrt{x^2+2x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 + 2*x)*(x + 1)),x, algorithm="fricas")
```

```
[Out] 2*arctan(-x + sqrt(x^2 + 2*x) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x(x+2)}(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1+x)/(x**2+2*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x*(x + 2))*(x + 1)), x)
```

GIAC/XCAS [A] time = 0.26816, size = 23, normalized size = 1.92

$$2\arctan\left(-x+\sqrt{x^2+2x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 + 2*x)*(x + 1)),x, algorithm="giac")
```

```
[Out] 2*arctan(-x + sqrt(x^2 + 2*x) - 1)
```

3.820 $\int \sqrt{1-\sqrt{x}-x}\sqrt{x}\,dx$

Optimal. Leaf size=95

$$-\frac{1}{2}\sqrt{x}\left(-x-\sqrt{x}+1\right)^{3/2}+\frac{5}{12}\left(-x-\sqrt{x}+1\right)^{3/2}+\frac{9}{32}\left(2\sqrt{x}+1\right)\sqrt{-x-\sqrt{x}+1}+\frac{45}{64}\sin^{-1}\left(\frac{2\sqrt{x}+1}{\sqrt{5}}\right)$$

```
[Out] (9*(1 + 2*Sqrt[x])*Sqrt[1 - Sqrt[x] - x])/32 + (5*(1 - Sqrt[x] -
x)^(3/2))/12 - ((1 - Sqrt[x] - x)^(3/2)*Sqrt[x])/2 + (45*ArcSin[(
1 + 2*Sqrt[x])/Sqrt[5]])/64
```

Rubi [A] time = 0.111151, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273

$$-\frac{1}{2}\sqrt{x}\left(-x-\sqrt{x}+1\right)^{3/2}+\frac{5}{12}\left(-x-\sqrt{x}+1\right)^{3/2}+\frac{9}{32}\left(2\sqrt{x}+1\right)\sqrt{-x-\sqrt{x}+1}+\frac{45}{64}\sin^{-1}\left(\frac{2\sqrt{x}+1}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 - Sqrt[x] - x]*Sqrt[x],x]
```

```
[Out] (9*(1 + 2*Sqrt[x])*Sqrt[1 - Sqrt[x] - x])/32 + (5*(1 - Sqrt[x] -
x)^(3/2))/12 - ((1 - Sqrt[x] - x)^(3/2)*Sqrt[x])/2 + (45*ArcSin[(
1 + 2*Sqrt[x])/Sqrt[5]])/64
```

Rubi in Sympy [A] time = 6.77057, size = 87, normalized size = 0.92

$$-\frac{\sqrt{x}\left(-\sqrt{x}-x+1\right)^{\frac{3}{2}}}{2}+\frac{9\left(2\sqrt{x}+1\right)\sqrt{-\sqrt{x}-x+1}}{32}+\frac{5\left(-\sqrt{x}-x+1\right)^{\frac{3}{2}}}{12}+\frac{45\,\mathrm{atan}\left(-\frac{-2\sqrt{x}-1}{2\sqrt{-\sqrt{x}-x+1}}\right)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(1/2)*(1-x-x**(1/2))**(1/2),x)
```

```
[Out] -sqrt(x)*(-sqrt(x) - x + 1)**(3/2)/2 + 9*(2*sqrt(x) + 1)*sqrt(-sq
rt(x) - x + 1)/32 + 5*(-sqrt(x) - x + 1)**(3/2)/12 + 45*atan(-(-2
*sqrt(x) - 1)/(2*sqrt(-sqrt(x) - x + 1)))/64
```

Mathematica [A] time = 0.0483693, size = 60, normalized size = 0.63

$$\frac{1}{96}\sqrt{-x-\sqrt{x}+1}\left(48x^{3/2}+8x-34\sqrt{x}+67\right)-\frac{45}{64}\sin^{-1}\left(\frac{-2\sqrt{x}-1}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 - Sqrt[x] - x]*Sqrt[x],x]
```

```
[Out] (Sqrt[1 - Sqrt[x] - x]*(67 - 34*Sqrt[x] + 8*x + 48*x^(3/2)))/96 -
 (45*ArcSin[(-1 - 2*Sqrt[x])/Sqrt[5]])/64
```

Maple [A] time = 0.005, size = 67, normalized size = 0.7

$$-\frac{1}{2}\left(1-x-\sqrt{x}\right)^{\frac{3}{2}}\sqrt{x}+\frac{5}{12}\left(1-x-\sqrt{x}\right)^{\frac{3}{2}}-\frac{9}{32}\left(-2\sqrt{x}-1\right)\sqrt{1-x-\sqrt{x}}+\frac{45}{64}\arcsin\left(\frac{2\sqrt{5}}{5}\left(\sqrt{x}+\frac{1}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(1/2)*(1-x-x^(1/2))^(1/2),x)
```

```
[Out] -1/2*(1-x-x^(1/2))^(3/2)*x^(1/2)+5/12*(1-x-x^(1/2))^(3/2)-9/32*(-
2*x^(1/2)-1)*(1-x-x^(1/2))^(1/2)+45/64*arcsin(2/5*5^(1/2)*(x^(1/2
)+1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x}\sqrt{-x-\sqrt{x}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)*sqrt(-x - sqrt(x) + 1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)*sqrt(-x - sqrt(x) + 1), x)
```

Fricas [A] time = 1.22884, size = 89, normalized size = 0.94

$$\frac{1}{96}\left(2\left(24x-17\right)\sqrt{x}+8x+67\right)\sqrt{-x-\sqrt{x}+1}+\frac{45}{128}\arctan\left(\frac{8x+8\sqrt{x}-3}{4\sqrt{-x-\sqrt{x}+1}\left(2\sqrt{x}+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)*sqrt(-x - sqrt(x) + 1),x, algorithm="fricas")
```

```
[Out] 1/96*(2*(24*x - 17)*sqrt(x) + 8*x + 67)*sqrt(-x - sqrt(x) + 1) +
45/128*arctan(1/4*(8*x + 8*sqrt(x) - 3)/(sqrt(-x - sqrt(x) + 1)*(
2*sqrt(x) + 1)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x}\sqrt{-\sqrt{x} - x + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(1/2)*(1-x-x**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(x)*sqrt(-sqrt(x) - x + 1), x)
```

GIAC/XCAS [A] time = 0.268708, size = 69, normalized size = 0.73

$$\frac{1}{96}\left(2\left(4\sqrt{x}\left(6\sqrt{x} + 1\right) - 17\right)\sqrt{x} + 67\right)\sqrt{-x - \sqrt{x} + 1} + \frac{45}{64}\arcsin\left(\frac{1}{5}\sqrt{5}\left(2\sqrt{x} + 1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)*sqrt(-x - sqrt(x) + 1),x, algorithm="giac")
```

```
[Out] 1/96*(2*(4*sqrt(x)*(6*sqrt(x) + 1) - 17)*sqrt(x) + 67)*sqrt(-x -
sqrt(x) + 1) + 45/64*arcsin(1/5*sqrt(5)*(2*sqrt(x) + 1))
```

3.821 $\int \sqrt[3]{1+\sqrt{-3+x}}\,dx$

Optimal. Leaf size=35

$$\frac{6}{7}\left(\sqrt{x-3}+1\right)^{7/3}-\frac{3}{2}\left(\sqrt{x-3}+1\right)^{4/3}$$

```
[Out] (-3*(1 + Sqrt[-3 + x])^(4/3))/2 + (6*(1 + Sqrt[-3 + x])^(7/3))/7
```

Rubi [A] time = 0.0256892, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$\frac{6}{7}\left(\sqrt{x-3}+1\right)^{7/3}-\frac{3}{2}\left(\sqrt{x-3}+1\right)^{4/3}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + Sqrt[-3 + x])^(1/3),x]

[Out] (-3*(1 + Sqrt[-3 + x])^(4/3))/2 + (6*(1 + Sqrt[-3 + x])^(7/3))/7
```

Rubi in Sympy [A] time = 1.19683, size = 29, normalized size = 0.83

$$\frac{6\left(\sqrt{x-3}+1\right)^{\frac{7}{3}}}{7}-\frac{3\left(\sqrt{x-3}+1\right)^{\frac{4}{3}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+(-3+x)**(1/2))**(1/3),x)

[Out] 6*(sqrt(x - 3) + 1)**(7/3)/7 - 3*(sqrt(x - 3) + 1)**(4/3)/2
```

Mathematica [A] time = 0.0128329, size = 28, normalized size = 0.8

$$\frac{3}{14}\left(\sqrt{x-3}+1\right)^{4/3}\left(4\sqrt{x-3}-3\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + Sqrt[-3 + x])^(1/3),x]
```

```
[Out] (3*(1 + Sqrt[-3 + x])^(4/3)*(-3 + 4*Sqrt[-3 + x]))/14
```

Maple [A] time = 0.004, size = 24, normalized size = 0.7

$$-\frac{3}{2}\left(1+\sqrt{-3+x}\right)^{\frac{4}{3}}+\frac{6}{7}\left(1+\sqrt{-3+x}\right)^{\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+(-3+x)^(1/2))^(1/3),x)
```

```
[Out] -3/2*(1+(-3+x)^(1/2))^(4/3)+6/7*(1+(-3+x)^(1/2))^(7/3)
```

Maxima [A] time = 0.692064, size = 31, normalized size = 0.89

$$\frac{6}{7}\left(\sqrt{x-3}+1\right)^{\frac{7}{3}}-\frac{3}{2}\left(\sqrt{x-3}+1\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(x - 3) + 1)^(1/3),x, algorithm="maxima")
```

```
[Out] 6/7*(sqrt(x - 3) + 1)^(7/3) - 3/2*(sqrt(x - 3) + 1)^(4/3)
```

Fricas [A] time = 0.268696, size = 28, normalized size = 0.8

$$\frac{3}{14}\left(4\,x+\sqrt{x-3}-15\right)\left(\sqrt{x-3}+1\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(x - 3) + 1)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/14*(4*x + sqrt(x - 3) - 15)*(sqrt(x - 3) + 1)^(1/3)
```

Sympy [A] time = 3.62622, size = 184, normalized size = 5.26

$$\frac{12\left(x-3\right)^{\frac{7}{2}}\sqrt[3]{\sqrt{x-3}+1}}{14\left(x-3\right)^{\frac{5}{2}}+14\left(x-3\right)^{2}}-\frac{6\left(x-3\right)^{\frac{5}{2}}\sqrt[3]{\sqrt{x-3}+1}}{14\left(x-3\right)^{\frac{5}{2}}+14\left(x-3\right)^{2}}+\frac{9\left(x-3\right)^{\frac{5}{2}}}{14\left(x-3\right)^{\frac{5}{2}}+14\left(x-3\right)^{2}}+\frac{15\left(x-3\right)^{3}\sqrt[3]{\sqrt{x-3}+1}}{14\left(x-3\right)^{\frac{5}{2}}+14\left(x-3\right)^{2}}-\frac{9\left(x-3\right)^{2}\sqrt[3]{\sqrt{x-3}+1}}{14\left(x-3\right)^{\frac{5}{2}}+14\left(x-3\right)^{2}}+\frac{9\left(x-3\right)^{2}}{14\left(x-3\right)^{\frac{5}{2}}+14\left(x-3\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+(-3+x)**(1/2))**(1/3),x)
```

```
[Out] 12*(x - 3)**(7/2)*(sqrt(x - 3) + 1)**(1/3)/(14*(x - 3)**(5/2) + 1
4*(x - 3)**2) - 6*(x - 3)**(5/2)*(sqrt(x - 3) + 1)**(1/3)/(14*(x
- 3)**(5/2) + 14*(x - 3)**2) + 9*(x - 3)**(5/2)/(14*(x - 3)**(5/2
) + 14*(x - 3)**2) + 15*(x - 3)**3*(sqrt(x - 3) + 1)**(1/3)/(14*(
x - 3)**(5/2) + 14*(x - 3)**2) - 9*(x - 3)**2*(sqrt(x - 3) + 1)**
(1/3)/(14*(x - 3)**(5/2) + 14*(x - 3)**2) + 9*(x - 3)**2/(14*(x -
 3)**(5/2) + 14*(x - 3)**2)
```

GIAC/XCAS [A] time = 0.259502, size = 31, normalized size = 0.89

$$\frac{6}{7}\left(\sqrt{x-3}+1\right)^{\frac{7}{3}}-\frac{3}{2}\left(\sqrt{x-3}+1\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(x - 3) + 1)^(1/3),x, algorithm="giac")
```

```
[Out] 6/7*(sqrt(x - 3) + 1)^(7/3) - 3/2*(sqrt(x - 3) + 1)^(4/3)
```

3.822 $\int \frac{1}{\sqrt{3+\sqrt{-1+2x}}}\,dx$

Optimal. Leaf size=37

$$\frac{2}{3}\left(\sqrt{2x-1}+3\right)^{3/2}-6\sqrt{\sqrt{2x-1}+3}$$

```
[Out] -6*Sqrt[3 + Sqrt[-1 + 2*x]] + (2*(3 + Sqrt[-1 + 2*x])^(3/2))/3
```

Rubi [A] time = 0.0313455, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{2}{3}\left(\sqrt{2x-1}+3\right)^{3/2}-6\sqrt{\sqrt{2x-1}+3}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[3 + Sqrt[-1 + 2*x]],x]
```

```
[Out] -6*Sqrt[3 + Sqrt[-1 + 2*x]] + (2*(3 + Sqrt[-1 + 2*x])^(3/2))/3
```

Rubi in Sympy [A] time = 1.37624, size = 31, normalized size = 0.84

$$\frac{2\left(\sqrt{2x-1}+3\right)^{\frac{3}{2}}}{3}-6\sqrt{\sqrt{2x-1}+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(3+(-1+2*x)**(1/2))**(1/2),x)
```

```
[Out] 2*(sqrt(2*x - 1) + 3)**(3/2)/3 - 6*sqrt(sqrt(2*x - 1) + 3)
```

Mathematica [A] time = 0.0140015, size = 30, normalized size = 0.81

$$\frac{2}{3}\left(\sqrt{2x-1}-6\right)\sqrt{\sqrt{2x-1}+3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[3 + Sqrt[-1 + 2*x]],x]

[Out] (2*(-6 + Sqrt[-1 + 2*x])*Sqrt[3 + Sqrt[-1 + 2*x]])/3
```

Maple [A] time = 0.006, size = 28, normalized size = 0.8

$$\frac{2}{3}\left(3+\sqrt{2x-1}\right)^{\frac{3}{2}}-6\sqrt{3+\sqrt{2x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(3+(2*x-1)^(1/2))^(1/2),x)

[Out] 2/3*(3+(2*x-1)^(1/2))^(3/2)-6*(3+(2*x-1)^(1/2))^(1/2)
```

Maxima [A] time = 0.719549, size = 36, normalized size = 0.97

$$\frac{2}{3}\left(\sqrt{2x-1}+3\right)^{\frac{3}{2}}-6\sqrt{\sqrt{2x-1}+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(sqrt(2*x - 1) + 3),x, algorithm="maxima")

[Out] 2/3*(sqrt(2*x - 1) + 3)^(3/2) - 6*sqrt(sqrt(2*x - 1) + 3)
```

Fricas [A] time = 0.265742, size = 30, normalized size = 0.81

$$\frac{2}{3}\sqrt{\sqrt{2x-1}+3}\left(\sqrt{2x-1}-6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(sqrt(2*x - 1) + 3),x, algorithm="fricas")

[Out] 2/3*sqrt(sqrt(2*x - 1) + 3)*(sqrt(2*x - 1) - 6)
```

Sympy [A] time = 3.49664, size = 265, normalized size = 7.16

$$-\frac{6\sqrt{6}\left(x-\frac{1}{2}\right)^{\frac{5}{2}}\sqrt{\sqrt{2}\sqrt{x-\frac{1}{2}}+3}}{3\sqrt{6}\left(x-\frac{1}{2}\right)^{\frac{5}{2}}+9\sqrt{3}\left(x-\frac{1}{2}\right)^{2}}+\frac{36\sqrt{2}\left(x-\frac{1}{2}\right)^{\frac{5}{2}}}{3\sqrt{6}\left(x-\frac{1}{2}\right)^{\frac{5}{2}}+9\sqrt{3}\left(x-\frac{1}{2}\right)^{2}}+\frac{4\sqrt{3}\left(x-\frac{1}{2}\right)^{3}\sqrt{\sqrt{2}\sqrt{x-\frac{1}{2}}+3}}{3\sqrt{6}\left(x-\frac{1}{2}\right)^{\frac{5}{2}}+9\sqrt{3}\left(x-\frac{1}{2}\right)^{2}}-\frac{36\sqrt{3}\left(x-\frac{1}{2}\right)^{2}\sqrt{\sqrt{2}\sqrt{x-\frac{1}{2}}+3}}{3\sqrt{6}\left(x-\frac{1}{2}\right)^{\frac{5}{2}}+9\sqrt{3}\left(x-\frac{1}{2}\right)^{2}}+\frac{108\left(x-\frac{1}{2}\right)^{2}}{3\sqrt{6}\left(x-\frac{1}{2}\right)^{\frac{5}{2}}+9\sqrt{3}\left(x-\frac{1}{2}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3+(-1+2*x)**(1/2))**(1/2),x)
```

```
[Out] -6*sqrt(6)*(x - 1/2)**(5/2)*sqrt(sqrt(2)*sqrt(x - 1/2) + 3)/(3*sq
rt(6)*(x - 1/2)**(5/2) + 9*sqrt(3)*(x - 1/2)**2) + 36*sqrt(2)*(x
- 1/2)**(5/2)/(3*sqrt(6)*(x - 1/2)**(5/2) + 9*sqrt(3)*(x - 1/2)**
2) + 4*sqrt(3)*(x - 1/2)**3*sqrt(sqrt(2)*sqrt(x - 1/2) + 3)/(3*sq
rt(6)*(x - 1/2)**(5/2) + 9*sqrt(3)*(x - 1/2)**2) - 36*sqrt(3)*(x
- 1/2)**2*sqrt(sqrt(2)*sqrt(x - 1/2) + 3)/(3*sqrt(6)*(x - 1/2)**(
5/2) + 9*sqrt(3)*(x - 1/2)**2) + 108*(x - 1/2)**2/(3*sqrt(6)*(x -
 1/2)**(5/2) + 9*sqrt(3)*(x - 1/2)**2)
```

GIAC/XCAS [A] time = 0.262192, size = 43, normalized size = 1.16

$$\frac{2}{3}\left(\sqrt{2x-1}+3\right)^{\frac{3}{2}}+4\sqrt{3}-6\sqrt{\sqrt{2x-1}+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(sqrt(2*x - 1) + 3),x, algorithm="giac")
```

```
[Out] 2/3*(sqrt(2*x - 1) + 3)^(3/2) + 4*sqrt(3) - 6*sqrt(sqrt(2*x - 1)
+ 3)
```

3.823 $\int \frac{\sqrt{1-x}}{1+\sqrt{x}}\,dx$

Optimal. Leaf size=29

$$-\sqrt{1-x}\left(2-\sqrt{x}\right)-\sin^{-1}\left(\sqrt{x}\right)$$

```
[Out] -((2 - Sqrt[x])*Sqrt[1 - x]) - ArcSin[Sqrt[x]]
```

Rubi [A] time = 0.0908416, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21

$$-\sqrt{1-x}\left(2-\sqrt{x}\right)-\sin^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 - x]/(1 + Sqrt[x]),x]

[Out] -((2 - Sqrt[x])*Sqrt[1 - x]) - ArcSin[Sqrt[x]]
```

Rubi in Sympy [A] time = 4.26201, size = 27, normalized size = 0.93

$$-\sqrt{-x+1}-\operatorname{asin}\left(\sqrt{x}\right)-\frac{(-x+1)^{\frac{3}{2}}}{\sqrt{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1-x)**(1/2)/(1+x**(1/2)),x)

[Out] -sqrt(-x + 1) - asin(sqrt(x)) - (-x + 1)**(3/2)/(sqrt(x) + 1)
```

Mathematica [A] time = 0.0172564, size = 26, normalized size = 0.9

$$\left(\sqrt{x}-2\right)\sqrt{1-x}-\sin^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 - x]/(1 + Sqrt[x]),x]
```

```
[Out] (-2 + Sqrt[x])*Sqrt[1 - x] - ArcSin[Sqrt[x]]
```

Maple [B] time = 0.01, size = 48, normalized size = 1.7

$$-\frac{1}{2}\sqrt{1-x}\sqrt{x}\left(-2\sqrt{-x\left(-1+x\right)}+\arcsin\left(2\,x-1\right)\right)\frac{1}{\sqrt{-x\left(-1+x\right)}}-2\sqrt{1-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1-x)^(1/2)/(1+x^(1/2)),x)
```

```
[Out] -1/2*(1-x)^(1/2)*x^(1/2)*(-2*(-x*(-1+x))^(1/2)+arcsin(2*x-1))/(-x
*(-1+x))^(1/2)-2*(1-x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-x+1}}{\sqrt{x}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x + 1)/(sqrt(x) + 1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x + 1)/(sqrt(x) + 1), x)
```

Fricas [A] time = 0.269627, size = 45, normalized size = 1.55

$$\sqrt{x}\sqrt{-x+1}-2\sqrt{-x+1}+\arctan\left(\frac{\sqrt{-x+1}}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x + 1)/(sqrt(x) + 1),x, algorithm="fricas")
```

```
[Out] sqrt(x)*sqrt(-x + 1) - 2*sqrt(-x + 1) + arctan(sqrt(-x + 1)/sqrt(
x))
```

Sympy [A] time = 6.20318, size = 32, normalized size = 1.1

$$i\sqrt{x}\sqrt{x-1} - 2i\sqrt{x-1} + i\,\mathrm{asinh}\left(\sqrt{x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1-x)**(1/2)/(1+x**(1/2)),x)

[Out] I*sqrt(x)*sqrt(x - 1) - 2*I*sqrt(x - 1) + I*asinh(sqrt(x - 1))
```

GIAC/XCAS [A] time = 0.265805, size = 39, normalized size = 1.34

$$\sqrt{x}\sqrt{-x+1} - 2\sqrt{-x+1} + \arcsin\left(\sqrt{-x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x + 1)/(sqrt(x) + 1),x, algorithm="giac")

[Out] sqrt(x)*sqrt(-x + 1) - 2*sqrt(-x + 1) + arcsin(sqrt(-x + 1))
```

3.824 $\int \frac{\sqrt{1-x}}{1-\sqrt{x}} \, dx$

Optimal. Leaf size=25

$$\sin^{-1}\left(\sqrt{x}\right)-\left(\sqrt{x}+2\right)\sqrt{1-x}$$

```
[Out] -((2 + Sqrt[x])*Sqrt[1 - x]) + ArcSin[Sqrt[x]]
```

Rubi [A] time = 0.0905389, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19

$$\sin^{-1}\left(\sqrt{x}\right)-\left(\sqrt{x}+2\right)\sqrt{1-x}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 - x]/(1 - Sqrt[x]),x]

[Out] -((2 + Sqrt[x])*Sqrt[1 - x]) + ArcSin[Sqrt[x]]
```

Rubi in Sympy [A] time = 5.61758, size = 26, normalized size = 1.04

$$-\sqrt{-x+1}+\operatorname{asin}\left(\sqrt{x}\right)-\frac{\left(-x+1\right)^{\frac{3}{2}}}{-\sqrt{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1-x)**(1/2)/(1-x**(1/2)),x)

[Out] -sqrt(-x + 1) + asin(sqrt(x)) - (-x + 1)**(3/2)/(-sqrt(x) + 1)
```

Mathematica [A] time = 0.0203013, size = 26, normalized size = 1.04

$$\sqrt{1-x}\left(-\sqrt{x}-2\right)+\sin^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 - x]/(1 - Sqrt[x]),x]
```

```
[Out] (-2 - Sqrt[x])*Sqrt[1 - x] + ArcSin[Sqrt[x]]
```

Maple [B] time = 0.004, size = 48, normalized size = 1.9

$$-2\sqrt{1-x}+\frac{1}{2}\sqrt{1-x}\sqrt{x}\left(-2\sqrt{-x(-1+x)}+\arcsin(2x-1)\right)\frac{1}{\sqrt{-x(-1+x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1-x)^(1/2)/(1-x^(1/2)),x)
```

```
[Out] -2*(1-x)^(1/2)+1/2*(1-x)^(1/2)*x^(1/2)*(-2*(-x*(-1+x))^(1/2)+arcs
in(2*x-1))/(-x*(-1+x))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\sqrt{-x+1}}{\sqrt{x}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sqrt(-x + 1)/(sqrt(x) - 1),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(-x + 1)/(sqrt(x) - 1), x)
```

Fricas [A] time = 0.264293, size = 49, normalized size = 1.96

$$-\sqrt{x}\sqrt{-x+1}-2\sqrt{-x+1}-\arctan\left(\frac{\sqrt{-x+1}}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sqrt(-x + 1)/(sqrt(x) - 1),x, algorithm="fricas")
```

```
[Out] -sqrt(x)*sqrt(-x + 1) - 2*sqrt(-x + 1) - arctan(sqrt(-x + 1)/sqrt
(x))
```

Sympy [A] time = 8.6973, size = 87, normalized size = 3.48

$$2\left(\begin{cases} -\sqrt{-x+1} + \frac{i\operatorname{acosh}\left(\sqrt{-x+1}\right)}{2} - \frac{i(-x+1)^{\frac{3}{2}}}{2\sqrt{-x}} + \frac{i\sqrt{-x+1}}{2\sqrt{-x}} & \text{for } |x-1| > 1 \\ \frac{\sqrt{x}\sqrt{-x+1}}{2} - \sqrt{-x+1} + \frac{\operatorname{asin}\left(\sqrt{-x+1}\right)}{2} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1-x)**(1/2)/(1-x**(1/2)),x)
```

```
[Out] 2*Piecewise((-sqrt(-x + 1) + I*acosh(sqrt(-x + 1))/2 - I*(-x + 1)
**(3/2)/(2*sqrt(-x)) + I*sqrt(-x + 1)/(2*sqrt(-x)), Abs(x - 1) >
1), (sqrt(x)*sqrt(-x + 1)/2 - sqrt(-x + 1) + asin(sqrt(-x + 1))/2
, True))
```

GIAC/XCAS [A] time = 0.265383, size = 43, normalized size = 1.72

$$-\sqrt{x}\sqrt{-x+1} - 2\sqrt{-x+1} - \arcsin\left(\sqrt{-x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sqrt(-x + 1)/(sqrt(x) - 1),x, algorithm="giac")
```

```
[Out] -sqrt(x)*sqrt(-x + 1) - 2*sqrt(-x + 1) - arcsin(sqrt(-x + 1))
```

3.825 $\int \frac{x}{x-\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=21

$$-\frac{x^3}{3}-\frac{1}{3}\left(x^2+1\right)^{3/2}$$

```
[Out] -x^3/3 - (1 + x^2)^(3/2)/3
```

Rubi [A] time = 0.0403636, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$-\frac{x^3}{3}-\frac{1}{3}\left(x^2+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(x - Sqrt[1 + x^2]),x]
```

```
[Out] -x^3/3 - (1 + x^2)^(3/2)/3
```

Rubi in Sympy [A] time = 2.57951, size = 15, normalized size = 0.71

$$-\frac{x^3}{3}-\frac{\left(x^2+1\right)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(x-(x**2+1)**(1/2)),x)
```

```
[Out] -x**3/3 - (x**2 + 1)**(3/2)/3
```

Mathematica [A] time = 0.0216363, size = 21, normalized size = 1.

$$\frac{1}{3}\left(-x^3-\left(x^2+1\right)^{3/2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(x - Sqrt[1 + x^2]),x]
```

```
[Out] (-x^3 - (1 + x^2)^(3/2))/3
```

Maple [A] time = 0.004, size = 16, normalized size = 0.8

$$-\frac{x^3}{3} - \frac{1}{3}\left(x^2 + 1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(x-(x^2+1)^(1/2)),x)
```

```
[Out] -1/3*x^3-1/3*(x^2+1)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{x - \sqrt{x^2 + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x - sqrt(x^2 + 1)),x, algorithm="maxima")
```

```
[Out] integrate(x/(x - sqrt(x^2 + 1)), x)
```

Fricas [A] time = 0.26072, size = 77, normalized size = 3.67

$$\frac{6\,x^4 + 6\,x^2 - 3\left(2\,x^3 + x\right)\sqrt{x^2 + 1} + 1}{3\left(4\,x^3 - (4\,x^2 + 1)\sqrt{x^2 + 1} + 3\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x - sqrt(x^2 + 1)),x, algorithm="fricas")
```

```
[Out] 1/3*(6*x^4 + 6*x^2 - 3*(2*x^3 + x)*sqrt(x^2 + 1) + 1)/(4*x^3 - (4
*x^2 + 1)*sqrt(x^2 + 1) + 3*x)
```

Sympy [A] time = 1.49251, size = 56, normalized size = 2.67

$$\frac{2x^2}{3x-3\sqrt{x^2+1}}-\frac{x\sqrt{x^2+1}}{3x-3\sqrt{x^2+1}}+\frac{1}{3x-3\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x-(x**2+1)**(1/2)),x)
```

```
[Out] 2*x**2/(3*x - 3*sqrt(x**2 + 1)) - x*sqrt(x**2 + 1)/(3*x - 3*sqrt(
x**2 + 1)) + 1/(3*x - 3*sqrt(x**2 + 1))
```

GIAC/XCAS [A] time = 0.261396, size = 20, normalized size = 0.95

$$-\frac{1}{3}x^3-\frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x - sqrt(x^2 + 1)),x, algorithm="giac")
```

```
[Out] -1/3*x^3 - 1/3*(x^2 + 1)^(3/2)
```

3.826 $\int \frac{x}{x-\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=65

$$\frac{\sqrt{1-x^2}}{2}-\frac{\tanh^{-1}\left(\sqrt{2}\sqrt{1-x^2}\right)}{2\sqrt{2}}+\frac{x}{2}-\frac{\tanh^{-1}\left(\sqrt{2}x\right)}{2\sqrt{2}}$$

```
[Out] x/2 + Sqrt[1 - x^2]/2 - ArcTanh[Sqrt[2]*x]/(2*Sqrt[2]) - ArcTanh[
Sqrt[2]*Sqrt[1 - x^2]]/(2*Sqrt[2])
```

Rubi [A] time = 0.124664, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316

$$\frac{\sqrt{1-x^2}}{2}-\frac{\tanh^{-1}\left(\sqrt{2}\sqrt{1-x^2}\right)}{2\sqrt{2}}+\frac{x}{2}-\frac{\tanh^{-1}\left(\sqrt{2}x\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(x - Sqrt[1 - x^2]),x]
```

```
[Out] x/2 + Sqrt[1 - x^2]/2 - ArcTanh[Sqrt[2]*x]/(2*Sqrt[2]) - ArcTanh[
Sqrt[2]*Sqrt[1 - x^2]]/(2*Sqrt[2])
```

Rubi in Sympy [A] time = 7.54676, size = 49, normalized size = 0.75

$$\frac{x}{2}+\frac{\sqrt{-x^2+1}}{2}-\frac{\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}x\right)}{4}-\frac{\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}\sqrt{-x^2+1}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(x-(-x**2+1)**(1/2)),x)
```

```
[Out] x/2 + sqrt(-x**2 + 1)/2 - sqrt(2)*atanh(sqrt(2)*x)/4 - sqrt(2)*at
anh(sqrt(2)*sqrt(-x**2 + 1))/4
```

Mathematica [A] time = 0.0682169, size = 95, normalized size = 1.46

$$\frac{1}{8}\left(4\sqrt{1-x^2}-\sqrt{2}\log\left(\sqrt{2-2x^2}-\sqrt{2}x+2\right)-\sqrt{2}\log\left(\sqrt{2-2x^2}+\sqrt{2}x+2\right)+4x+2\sqrt{2}\log\left(\sqrt{2}-2x\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(x - Sqrt[1 - x^2]),x]
```

```
[Out] (4*x + 4*Sqrt[1 - x^2] + 2*Sqrt[2]*Log[Sqrt[2] - 2*x] - Sqrt[2]*L
og[2 - Sqrt[2]*x + Sqrt[2 - 2*x^2]] - Sqrt[2]*Log[2 + Sqrt[2]*x +
 Sqrt[2 - 2*x^2]])/8
```

Maple [B] time = 0.013, size = 175, normalized size = 2.7

$$\frac{x}{2}-\frac{\mathit{Artanh}\left(\sqrt{2}x\right)\sqrt{2}}{4}+\frac{1}{8}\sqrt{-4\left(x-1/2\sqrt{2}\right)^{2}-4\sqrt{2}\left(x-1/2\sqrt{2}\right)+2}$$
$$-\frac{\sqrt{2}}{8}\mathit{Artanh}\left(\sqrt{2}\left(1-\sqrt{2}\left(x-\frac{\sqrt{2}}{2}\right)\right)\frac{1}{\sqrt{-4\left(x-1/2\sqrt{2}\right)^{2}-4\sqrt{2}\left(x-1/2\sqrt{2}\right)+2}}\right)$$
$$+\frac{1}{8}\sqrt{-4\left(x+1/2\sqrt{2}\right)^{2}+4\sqrt{2}\left(x+1/2\sqrt{2}\right)+2}$$
$$-\frac{\sqrt{2}}{8}\mathit{Artanh}\left(\sqrt{2}\left(\sqrt{2}\left(x+\frac{\sqrt{2}}{2}\right)+1\right)\frac{1}{\sqrt{-4\left(x+1/2\sqrt{2}\right)^{2}+4\sqrt{2}\left(x+1/2\sqrt{2}\right)+2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(x-(-x^2+1)^(1/2)),x)
```

```
[Out] 1/2*x-1/4*arctanh(2^(1/2)*x)*2^(1/2)+1/8*(-4*(x-1/2*2^(1/2))^2-4*
2^(1/2)*(x-1/2*2^(1/2))+2)^(1/2)-1/8*2^(1/2)*arctanh((1-2^(1/2)*(
x-1/2*2^(1/2)))*2^(1/2)/(-4*(x-1/2*2^(1/2))^2-4*2^(1/2)*(x-1/2*2^
(1/2))+2)^(1/2))+1/8*(-4*(x+1/2*2^(1/2))^2+4*2^(1/2)*(x+1/2*2^(1/
2))+2)^(1/2)-1/8*2^(1/2)*arctanh((2^(1/2)*(x+1/2*2^(1/2))+1)*2^(1
/2)/(-4*(x+1/2*2^(1/2))^2+4*2^(1/2)*(x+1/2*2^(1/2))+2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{x-\sqrt{-x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x - sqrt(-x^2 + 1)),x, algorithm="maxima")
```

```
[Out] integrate(x/(x - sqrt(-x^2 + 1)), x)
```

Fricas [A] time = 0.26939, size = 282, normalized size = 4.34

$$\frac{\left(\sqrt{-x^2+1}-1\right)\log\left(-\frac{8\,x^4-4\,x^2-\sqrt{2}\left(6\,x^4-5\,x^2+2\right)+2\left(2\,x^2-\sqrt{2}\left(2\,x^2-1\right)\right)\sqrt{-x^2+1}}{2\,x^4-5\,x^2+2\left(2\,x^2-1\right)\sqrt{-x^2+1}+2}\right)-2\,\sqrt{2}\left(x^2+x\right)+\sqrt{-x^2+1}\left(2\,\sqrt{2}x+\log\left(\frac{\sqrt{2}\left(2\,x^2+1\right)-}{2\,x^2-1}\right.\right.}{4\left(\sqrt{2}\sqrt{-x^2+1}-\sqrt{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x - sqrt(-x^2 + 1)),x, algorithm="fricas")
```

```
[Out] 1/4*((sqrt(-x^2 + 1) - 1)*log(-(8*x^4 - 4*x^2 - sqrt(2)*(6*x^4 -
5*x^2 + 2) + 2*(2*x^2 - sqrt(2)*(2*x^2 - 1))*sqrt(-x^2 + 1))/(2*x
^4 - 5*x^2 + 2*(2*x^2 - 1)*sqrt(-x^2 + 1) + 2)) - 2*sqrt(2)*(x^2
+ x) + sqrt(-x^2 + 1)*(2*sqrt(2)*x + log((sqrt(2)*(2*x^2 + 1) - 4
*x)/(2*x^2 - 1))) - log((sqrt(2)*(2*x^2 + 1) - 4*x)/(2*x^2 - 1)))
/(sqrt(2)*sqrt(-x^2 + 1) - sqrt(2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{x-\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x-(-x**2+1)**(1/2)),x)
```

```
[Out] Integral(x/(x - sqrt(-x**2 + 1)), x)
```

GIAC/XCAS [A] time = 0.292831, size = 142, normalized size = 2.18

$$\frac{1}{8}\,\sqrt{2}\ln\left(\frac{\left|4\,x-2\,\sqrt{2}\right|}{\left|4\,x+2\,\sqrt{2}\right|}\right)-\frac{1}{8}\,\sqrt{2}\ln\left(\frac{\left|-4\,\sqrt{2}+\frac{2\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-6\right|}{\left|4\,\sqrt{2}+\frac{2\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-6\right|}\right)+\frac{1}{2}\,x+\frac{1}{2}\,\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x - sqrt(-x^2 + 1)),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*ln(abs(4*x - 2*sqrt(2))/abs(4*x + 2*sqrt(2))) - 1/8*s
qrt(2)*ln(abs(-4*sqrt(2) + 2*(sqrt(-x^2 + 1) - 1)^2/x^2 - 6)/abs(
4*sqrt(2) + 2*(sqrt(-x^2 + 1) - 1)^2/x^2 - 6)) + 1/2*x + 1/2*sqrt
(-x^2 + 1)
```

3.827 $\int \frac{x}{x-\sqrt{1+2x^2}}\,dx$

Optimal. Leaf size=31

$$-\sqrt{2x^2+1}+\tan^{-1}\left(\sqrt{2x^2+1}\right)-x+\tan^{-1}(x)$$

```
[Out] -x - Sqrt[1 + 2*x^2] + ArcTan[x] + ArcTan[Sqrt[1 + 2*x^2]]
```

Rubi [A] time = 0.0979391, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316

$$-\sqrt{2x^2+1}+\tan^{-1}\left(\sqrt{2x^2+1}\right)-x+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[x/(x - Sqrt[1 + 2*x^2]),x]

[Out] -x - Sqrt[1 + 2*x^2] + ArcTan[x] + ArcTan[Sqrt[1 + 2*x^2]]
```

Rubi in Sympy [A] time = 6.94545, size = 26, normalized size = 0.84

$$-x-\sqrt{2x^2+1}+\operatorname{atan}(x)+\operatorname{atan}\left(\sqrt{2x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(x-(2*x**2+1)**(1/2)),x)

[Out] -x - sqrt(2*x**2 + 1) + atan(x) + atan(sqrt(2*x**2 + 1))
```

Mathematica [C] time = 0.0649585, size = 101, normalized size = 3.26

$$\frac{1}{4}\left(-4\sqrt{2x^2+1}+2i\log\left(x^2+1\right)-i\log\left(3x^2-2\sqrt{2x^2+1}x+1\right)\right.$$
$$\left.-i\log\left(3x^2+2\sqrt{2x^2+1}x+1\right)-4\tan^{-1}\left(\frac{1}{\sqrt{2x^2+1}}\right)-4x+4\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(x - Sqrt[1 + 2*x^2]),x]
```

```
[Out] (-4*x - 4*Sqrt[1 + 2*x^2] + 4*ArcTan[x] - 4*ArcTan[1/Sqrt[1 + 2*x
^2]] + (2*I)*Log[1 + x^2] - I*Log[1 + 3*x^2 - 2*x*Sqrt[1 + 2*x^2]
] - I*Log[1 + 3*x^2 + 2*x*Sqrt[1 + 2*x^2]])/4
```

Maple [A] time = 0.01, size = 28, normalized size = 0.9

$$-x + \arctan(x) + \arctan\left(\sqrt{2x^2+1}\right) - \sqrt{2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(x-(2*x^2+1)^(1/2)),x)
```

```
[Out] -x+arctan(x)+arctan((2*x^2+1)^(1/2))-(2*x^2+1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{x - \sqrt{2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x - sqrt(2*x^2 + 1)),x, algorithm="maxima")
```

```
[Out] integrate(x/(x - sqrt(2*x^2 + 1)), x)
```

Fricas [A] time = 0.264969, size = 103, normalized size = 3.32

$$-\frac{2x^2 + \left(\sqrt{2x^2+1} - 1\right)\arctan\left(-\frac{x^2 - \sqrt{2x^2+1} + 1}{x^2}\right) + \sqrt{2x^2+1}(x - \arctan(x)) - x + \arctan(x)}{\sqrt{2x^2+1} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x - sqrt(2*x^2 + 1)),x, algorithm="fricas")
```

```
[Out] -(2*x^2 + (sqrt(2*x^2 + 1) - 1)*arctan(-(x^2 - sqrt(2*x^2 + 1) +
1)/x^2) + sqrt(2*x^2 + 1)*(x - arctan(x)) - x + arctan(x))/(sqrt(
```

```
2*x^2 + 1) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{x - \sqrt{2x^2 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x-(2*x**2+1)**(1/2)),x)
```

```
[Out] Integral(x/(x - sqrt(2*x**2 + 1)), x)
```

GIAC/XCAS [A] time = 0.269298, size = 85, normalized size = 2.74

$$-\frac{1}{2}\,\pi - x - \sqrt{2\,x^2 + 1} + \arctan\left(x\right) + \arctan\left(-\frac{\left(\sqrt{2}x - \sqrt{2\,x^2 + 1}\right)^2 + 1}{2\,\left(\sqrt{2}x - \sqrt{2\,x^2 + 1}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x - sqrt(2*x^2 + 1)),x, algorithm="giac")
```

```
[Out] -1/2*pi - x - sqrt(2*x^2 + 1) + arctan(x) + arctan(-1/2*((sqrt(2)
*x - sqrt(2*x^2 + 1))^2 + 1)/(sqrt(2)*x - sqrt(2*x^2 + 1)))
```

3.828 $\int \sqrt{x}\sqrt{\sqrt{x}+x}\,dx$

Optimal. Leaf size=82

$$\frac{1}{2}\sqrt{x}\left(x+\sqrt{x}\right)^{3/2}-\frac{5}{12}\left(x+\sqrt{x}\right)^{3/2}+\frac{5}{32}\left(2\sqrt{x}+1\right)\sqrt{x+\sqrt{x}}-\frac{5}{32}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{x+\sqrt{x}}}\right)$$

```
[Out] (5*(1 + 2*Sqrt[x])*Sqrt[Sqrt[x] + x])/32 - (5*(Sqrt[x] + x)^(3/2)
)/12 + (Sqrt[x]*(Sqrt[x] + x)^(3/2))/2 - (5*ArcTanh[Sqrt[x]/Sqrt[
Sqrt[x] + x]])/32
```

Rubi [A] time = 0.0859314, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353

$$\frac{1}{2}\sqrt{x}\left(x+\sqrt{x}\right)^{3/2}-\frac{5}{12}\left(x+\sqrt{x}\right)^{3/2}+\frac{5}{32}\left(2\sqrt{x}+1\right)\sqrt{x+\sqrt{x}}-\frac{5}{32}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{x+\sqrt{x}}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x]*Sqrt[Sqrt[x] + x],x]
```

```
[Out] (5*(1 + 2*Sqrt[x])*Sqrt[Sqrt[x] + x])/32 - (5*(Sqrt[x] + x)^(3/2)
)/12 + (Sqrt[x]*(Sqrt[x] + x)^(3/2))/2 - (5*ArcTanh[Sqrt[x]/Sqrt[
Sqrt[x] + x]])/32
```

Rubi in Sympy [A] time = 4.77879, size = 71, normalized size = 0.87

$$\frac{\sqrt{x}\left(\sqrt{x}+x\right)^{\frac{3}{2}}}{2}-\frac{5\left(\sqrt{x}+x\right)^{\frac{3}{2}}}{12}+\frac{5\sqrt{\sqrt{x}+x}\left(2\sqrt{x}+1\right)}{32}-\frac{5\,\mathrm{atanh}\left(\frac{\sqrt{x}}{\sqrt{\sqrt{x}+x}}\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(1/2)*(x+x**(1/2))**(1/2),x)
```

```
[Out] sqrt(x)*(sqrt(x) + x)**(3/2)/2 - 5*(sqrt(x) + x)**(3/2)/12 + 5*sq
rt(sqrt(x) + x)*(2*sqrt(x) + 1)/32 - 5*atanh(sqrt(x)/sqrt(sqrt(x)
 + x))/32
```

Mathematica [A] time = 0.0395397, size = 62, normalized size = 0.76

$$\frac{1}{96}\sqrt{x+\sqrt{x}}\left(48x^{3/2}+8x-10\sqrt{x}+15\right)-\frac{5}{64}\log\left(2\sqrt{x}+2\sqrt{x+\sqrt{x}}+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[x]*Sqrt[Sqrt[x] + x],x]
```

```
[Out] (Sqrt[Sqrt[x] + x]*(15 - 10*Sqrt[x] + 8*x + 48*x^(3/2)))/96 - (5*
Log[1 + 2*Sqrt[x] + 2*Sqrt[Sqrt[x] + x]])/64
```

Maple [A] time = 0.005, size = 54, normalized size = 0.7

$$\frac{1}{2}\sqrt{x}\left(x+\sqrt{x}\right)^{\frac{3}{2}}-\frac{5}{12}\left(x+\sqrt{x}\right)^{\frac{3}{2}}+\frac{5}{32}\left(1+2\sqrt{x}\right)\sqrt{x+\sqrt{x}}-\frac{5}{64}\ln\left(\frac{1}{2}+\sqrt{x}+\sqrt{x+\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(1/2)*(x+x^(1/2))^(1/2),x)
```

```
[Out] 1/2*x^(1/2)*(x+x^(1/2))^(3/2)-5/12*(x+x^(1/2))^(3/2)+5/32*(1+2*x^
(1/2))*(x+x^(1/2))^(1/2)-5/64*ln(1/2+x^(1/2)+(x+x^(1/2))^(1/2))
```

Maxima [A] time = 0.750809, size = 180, normalized size = 2.2

$$\frac{\frac{15\left(\sqrt{x}+1\right)^{\frac{7}{2}}}{x^{\frac{7}{4}}}-\frac{55\left(\sqrt{x}+1\right)^{\frac{5}{2}}}{x^{\frac{5}{4}}}+\frac{73\left(\sqrt{x}+1\right)^{\frac{3}{2}}}{x^{\frac{3}{4}}}+\frac{15\sqrt{\sqrt{x}+1}}{x^{\frac{1}{4}}}}{96\left(\frac{\left(\sqrt{x}+1\right)^{4}}{x^{2}}-\frac{4\left(\sqrt{x}+1\right)^{3}}{x^{\frac{3}{2}}}+\frac{6\left(\sqrt{x}+1\right)^{2}}{x}-\frac{4\left(\sqrt{x}+1\right)}{\sqrt{x}}+1\right)}-\frac{5}{64}\log\left(\frac{\sqrt{\sqrt{x}+1}}{x^{\frac{1}{4}}}+1\right)+\frac{5}{64}\log\left(\frac{\sqrt{\sqrt{x}+1}}{x^{\frac{1}{4}}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x + sqrt(x))*sqrt(x),x, algorithm="maxima")
```

```
[Out] 1/96*(15*(sqrt(x) + 1)^(7/2)/x^(7/4) - 55*(sqrt(x) + 1)^(5/2)/x^(
5/4) + 73*(sqrt(x) + 1)^(3/2)/x^(3/4) + 15*sqrt(sqrt(x) + 1)/x^(1
/4))/((sqrt(x) + 1)^4/x^2 - 4*(sqrt(x) + 1)^3/x^(3/2) + 6*(sqrt(x
) + 1)^2/x - 4*(sqrt(x) + 1)/sqrt(x) + 1) - 5/64*log(sqrt(sqrt(x)
 + 1)/x^(1/4) + 1) + 5/64*log(sqrt(sqrt(x) + 1)/x^(1/4) - 1)
```

Fricas [A] time = 0.701001, size = 73, normalized size = 0.89

$$\frac{1}{96}\left(2\,(24\,x-5)\sqrt{x}+8\,x+15\right)\sqrt{x+\sqrt{x}}+\frac{5}{128}\,\log\left(4\,\sqrt{x+\sqrt{x}}\left(2\,\sqrt{x}+1\right)-8\,x-8\,\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x + sqrt(x))*sqrt(x),x, algorithm="fricas")
```

```
[Out] 1/96*(2*(24*x - 5)*sqrt(x) + 8*x + 15)*sqrt(x + sqrt(x)) + 5/128*
log(4*sqrt(x + sqrt(x))*(2*sqrt(x) + 1) - 8*x - 8*sqrt(x) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{x}\sqrt{\sqrt{x}+x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(1/2)*(x+x**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(x)*sqrt(sqrt(x) + x), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x + sqrt(x))*sqrt(x),x, algorithm="giac")
```

```
[Out] Timed out
```

3.829 $\int \frac{1+\sqrt[3]{x}}{1+\sqrt{x}} dx$

Optimal. Leaf size=74

$$\frac{6x^{5/6}}{5} + 2\sqrt{x} - 3\sqrt[3]{x} - 4\log\left(\sqrt[6]{x}+1\right) - \log\left(\sqrt[3]{x}-\sqrt[6]{x}+1\right) - 2\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[6]{x}}{\sqrt{3}}\right)$$

```
[Out] -3*x^(1/3) + 2*Sqrt[x] + (6*x^(5/6))/5 - 2*Sqrt[3]*ArcTan[(1 - 2*
x^(1/6))/Sqrt[3]] - 4*Log[1 + x^(1/6)] - Log[1 - x^(1/6) + x^(1/3
)]
```

Rubi [A] time = 0.182749, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471

$$\frac{6x^{5/6}}{5} + 2\sqrt{x} - 3\sqrt[3]{x} - 4\log\left(\sqrt[6]{x}+1\right) - \log\left(\sqrt[3]{x}-\sqrt[6]{x}+1\right) - 2\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[6]{x}}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^(1/3))/(1 + Sqrt[x]),x]
```

```
[Out] -3*x^(1/3) + 2*Sqrt[x] + (6*x^(5/6))/5 - 2*Sqrt[3]*ArcTan[(1 - 2*
x^(1/6))/Sqrt[3]] - 4*Log[1 + x^(1/6)] - Log[1 - x^(1/6) + x^(1/3
)]
```

Rubi in Sympy [A] time = 6.92872, size = 65, normalized size = 0.88

$$\frac{6x^{\frac{5}{6}}}{5} - 3\sqrt[3]{x} + 2\sqrt{x} - 3\log\left(\sqrt[6]{x}+1\right) - \log\left(\sqrt{x}+1\right) + 2\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2\sqrt[6]{x}}{3} - \frac{1}{3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+x**(1/3))/(1+x**(1/2)),x)
```

```
[Out] 6*x**(5/6)/5 - 3*x**(1/3) + 2*sqrt(x) - 3*log(x**(1/6) + 1) - log
(sqrt(x) + 1) + 2*sqrt(3)*atan(sqrt(3)*(2*x**(1/6)/3 - 1/3))
```

Mathematica [A] time = 0.0294826, size = 82, normalized size = 1.11

$$\frac{6x^{5/6}}{5}+2\sqrt{x}-3\sqrt[3]{x}-2\log\left(\sqrt[6]{x}+1\right)+\log\left(\sqrt[3]{x}-\sqrt[6]{x}+1\right)-2\log\left(\sqrt{x}+1\right)+2\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[6]{x}-1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^(1/3))/(1 + Sqrt[x]),x]
```

```
[Out] -3*x^(1/3) + 2*Sqrt[x] + (6*x^(5/6))/5 + 2*Sqrt[3]*ArcTan[(-1 + 2
*x^(1/6))/Sqrt[3]] - 2*Log[1 + x^(1/6)] + Log[1 - x^(1/6) + x^(1/
3)] - 2*Log[1 + Sqrt[x]]
```

Maple [A] time = 0.011, size = 56, normalized size = 0.8

$$\frac{6}{5}x^{\frac{5}{6}}+2\sqrt{x}-3\sqrt[3]{x}-4\ln\left(1+\sqrt[6]{x}\right)-\ln\left(1-\sqrt[6]{x}+\sqrt[3]{x}\right)+2\sqrt{3}\arctan\left(1/3\left(2\sqrt[6]{x}-1\right)\sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+x^(1/3))/(1+x^(1/2)),x)
```

```
[Out] 6/5*x^(5/6)+2*x^(1/2)-3*x^(1/3)-4*ln(1+x^(1/6))-ln(1-x^(1/6)+x^(1
/3))+2*3^(1/2)*arctan(1/3*(2*x^(1/6)-1)*3^(1/2))
```

Maxima [A] time = 0.773578, size = 74, normalized size = 1.

$$2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{6}}-1\right)\right)+\frac{6}{5}x^{\frac{5}{6}}+2\sqrt{x}-3x^{\frac{1}{3}}-\log\left(x^{\frac{1}{3}}-x^{\frac{1}{6}}+1\right)-4\log\left(x^{\frac{1}{6}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^(1/3) + 1)/(sqrt(x) + 1),x, algorithm="maxima")
```

```
[Out] 2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/6) - 1)) + 6/5*x^(5/6) + 2*s
qrt(x) - 3*x^(1/3) - log(x^(1/3) - x^(1/6) + 1) - 4*log(x^(1/6) +
 1)
```

Fricas [A] time = 0.273009, size = 74, normalized size = 1.

$$2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{6}}-1\right)\right)+\frac{6}{5}x^{\frac{5}{6}}+2\sqrt{x}-3x^{\frac{1}{3}}-\log\left(x^{\frac{1}{3}}-x^{\frac{1}{6}}+1\right)-4\log\left(x^{\frac{1}{6}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^(1/3) + 1)/(sqrt(x) + 1),x, algorithm="fricas")
```

```
[Out] 2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/6) - 1)) + 6/5*x^(5/6) + 2*s
qrt(x) - 3*x^(1/3) - log(x^(1/3) - x^(1/6) + 1) - 4*log(x^(1/6) +
 1)
```

Sympy [A] time = 3.83023, size = 155, normalized size = 2.09

$$\frac{16x^{\frac{5}{6}}\Gamma\left(\frac{8}{3}\right)}{5\Gamma\left(\frac{11}{3}\right)}-\frac{8\sqrt[3]{x}\,\Gamma\left(\frac{8}{3}\right)}{\Gamma\left(\frac{11}{3}\right)}+2\sqrt{x}-2\log\left(\sqrt{x}+1\right)-\frac{16e^{\frac{10i\pi}{3}}\log\left(-\sqrt[6]{x}e^{\frac{i\pi}{3}}+1\right)\Gamma\left(\frac{8}{3}\right)}{3\Gamma\left(\frac{11}{3}\right)}-\frac{16\log\left(-\sqrt[6]{x}e^{i\pi}+1\right)\Gamma\left(\frac{8}{3}\right)}{3\Gamma\left(\frac{11}{3}\right)}-\frac{16e^{\frac{2i\pi}{3}}\log\left(-\sqrt[6]{x}e^{\frac{5i\pi}{3}}+1\right)\Gamma\left(\frac{8}{3}\right)}{3\Gamma\left(\frac{11}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+x**(1/3))/(1+x**(1/2)),x)
```

```
[Out] 16*x**(5/6)*gamma(8/3)/(5*gamma(11/3)) - 8*x**(1/3)*gamma(8/3)/ga
mma(11/3) + 2*sqrt(x) - 2*log(sqrt(x) + 1) - 16*exp(10*I*pi/3)*lo
g(-x**(1/6)*exp_polar(I*pi/3) + 1)*gamma(8/3)/(3*gamma(11/3)) - 1
6*log(-x**(1/6)*exp_polar(I*pi) + 1)*gamma(8/3)/(3*gamma(11/3)) -
 16*exp(2*I*pi/3)*log(-x**(1/6)*exp_polar(5*I*pi/3) + 1)*gamma(8/
3)/(3*gamma(11/3))
```

GIAC/XCAS [A] time = 0.276657, size = 74, normalized size = 1.

$$2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{6}}-1\right)\right)+\frac{6}{5}x^{\frac{5}{6}}+2\sqrt{x}-3x^{\frac{1}{3}}-\ln\left(x^{\frac{1}{3}}-x^{\frac{1}{6}}+1\right)-4\ln\left(x^{\frac{1}{6}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^(1/3) + 1)/(sqrt(x) + 1),x, algorithm="giac")
```

```
[Out] 2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/6) - 1)) + 6/5*x^(5/6) + 2*s
qrt(x) - 3*x^(1/3) - ln(x^(1/3) - x^(1/6) + 1) - 4*ln(x^(1/6) + 1
)
```

3.830 $\int \frac{1+\sqrt[3]{x}}{1+\sqrt[4]{x}}\,dx$

Optimal. Leaf size=115

$$\frac{12x^{13/12}}{13}-\frac{6x^{5/6}}{5}+\frac{4x^{3/4}}{3}+\frac{12x^{7/12}}{7}-2\sqrt{x}-3\sqrt[3]{x}+4\sqrt[4]{x}+12\sqrt[12]{x}-8\log\left(\sqrt[12]{x}+1\right)-2\log\left(\sqrt[6]{x}-\sqrt[12]{x}+1\right)+4\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[12]{x}}{\sqrt{3}}\right)$$

```
[Out] 12*x^(1/12) + 4*x^(1/4) - 3*x^(1/3) - 2*Sqrt[x] + (12*x^(7/12))/7
 + (4*x^(3/4))/3 - (6*x^(5/6))/5 + (12*x^(13/12))/13 + 4*Sqrt[3]*
ArcTan[(1 - 2*x^(1/12))/Sqrt[3]] - 8*Log[1 + x^(1/12)] - 2*Log[1
- x^(1/12) + x^(1/6)]
```

Rubi [A] time = 0.243356, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529

$$\frac{12x^{13/12}}{13}-\frac{6x^{5/6}}{5}+\frac{4x^{3/4}}{3}+\frac{12x^{7/12}}{7}-2\sqrt{x}-3\sqrt[3]{x}+4\sqrt[4]{x}+12\sqrt[12]{x}-8\log\left(\sqrt[12]{x}+1\right)-2\log\left(\sqrt[6]{x}-\sqrt[12]{x}+1\right)+4\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[12]{x}}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^(1/3))/(1 + x^(1/4)),x]
```

```
[Out] 12*x^(1/12) + 4*x^(1/4) - 3*x^(1/3) - 2*Sqrt[x] + (12*x^(7/12))/7
 + (4*x^(3/4))/3 - (6*x^(5/6))/5 + (12*x^(13/12))/13 + 4*Sqrt[3]*
ArcTan[(1 - 2*x^(1/12))/Sqrt[3]] - 8*Log[1 + x^(1/12)] - 2*Log[1
- x^(1/12) + x^(1/6)]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{12x^{\frac{13}{12}}}{13}+\frac{12x^{\frac{7}{12}}}{7}+12\sqrt[12]{x}-\frac{6x^{\frac{5}{6}}}{5}+\frac{4x^{\frac{3}{4}}}{3}+4\sqrt[4]{x}-3\sqrt[3]{x}-6\log\left(\sqrt[12]{x}+1\right)-2\log\left(\sqrt[4]{x}+1\right)-4\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2\sqrt[12]{x}}{3}-\frac{1}{3}\right)\right)-4\int^{\sqrt[4]{x}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+x**(1/3))/(1+x**(1/4)),x)
```

```
[Out] 12*x**(13/12)/13 + 12*x**(7/12)/7 + 12*x**(1/12) - 6*x**(5/6)/5 +
 4*x**(3/4)/3 + 4*x**(1/4) - 3*x**(1/3) - 6*log(x**(1/12) + 1) -
2*log(x**(1/4) + 1) - 4*sqrt(3)*atan(sqrt(3)*(2*x**(1/12)/3 - 1/3
)) - 4*Integral(x, (x, x**(1/4)))
```

Mathematica [A] time = 0.0364198, size = 125, normalized size = 1.09

$$\frac{12x^{13/12}}{13}-\frac{6x^{5/6}}{5}+\frac{4x^{3/4}}{3}+\frac{12x^{7/12}}{7}-2\sqrt{x}-3\sqrt[3]{x}+4\sqrt[4]{x}+12\sqrt[12]{x}-4\log\left(\sqrt[12]{x}+1\right)+2\log\left(\sqrt[6]{x}-\sqrt[12]{x}+1\right)-4\log\left(\sqrt[4]{x}+1\right)-4\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[12]{x}-1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^(1/3))/(1 + x^(1/4)),x]
```

```
[Out] 12*x^(1/12) + 4*x^(1/4) - 3*x^(1/3) - 2*Sqrt[x] + (12*x^(7/12))/7
 + (4*x^(3/4))/3 - (6*x^(5/6))/5 + (12*x^(13/12))/13 - 4*Sqrt[3]*
ArcTan[(-1 + 2*x^(1/12))/Sqrt[3]] - 4*Log[1 + x^(1/12)] + 2*Log[1
 - x^(1/12) + x^(1/6)] - 4*Log[1 + x^(1/4)]
```

Maple [A] time = 0.01, size = 81, normalized size = 0.7

$$\frac{12}{13}x^{\frac{13}{12}}-\frac{6}{5}x^{\frac{5}{6}}+\frac{4}{3}x^{\frac{3}{4}}+\frac{12}{7}x^{\frac{7}{12}}-2\sqrt{x}-3\sqrt[3]{x}+4\sqrt[4]{x}+12x^{1/12}-8\ln\left(1+x^{1/12}\right)-2\ln\left(1-x^{1/12}+\sqrt[6]{x}\right)-4\sqrt{3}\arctan\left(1/3\left(2x^{1/12}-1\right)\sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+x^(1/3))/(1+x^(1/4)),x)
```

```
[Out] 12/13*x^(13/12)-6/5*x^(5/6)+4/3*x^(3/4)+12/7*x^(7/12)-2*x^(1/2)-3
*x^(1/3)+4*x^(1/4)+12*x^(1/12)-8*ln(1+x^(1/12))-2*ln(1-x^(1/12)+x
^(1/6))-4*3^(1/2)*arctan(1/3*(2*x^(1/12)-1)*3^(1/2))
```

Maxima [A] time = 0.787992, size = 108, normalized size = 0.94

$$-4\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{12}}-1\right)\right)+\frac{12}{13}x^{\frac{13}{12}}-\frac{6}{5}x^{\frac{5}{6}}+\frac{4}{3}x^{\frac{3}{4}}+\frac{12}{7}x^{\frac{7}{12}}-2\sqrt{x}-3x^{\frac{1}{3}}+4x^{\frac{1}{4}}+12x^{\frac{1}{12}}-2\log\left(x^{\frac{1}{6}}-x^{\frac{1}{12}}+1\right)-8\log\left(x^{\frac{1}{12}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^(1/3) + 1)/(x^(1/4) + 1),x, algorithm="maxima")
```

```
[Out] -4*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/12) - 1)) + 12/13*x^(13/12)
 - 6/5*x^(5/6) + 4/3*x^(3/4) + 12/7*x^(7/12) - 2*sqrt(x) - 3*x^(1
/3) + 4*x^(1/4) + 12*x^(1/12) - 2*log(x^(1/6) - x^(1/12) + 1) - 8
*log(x^(1/12) + 1)
```

Fricas [A] time = 0.278469, size = 105, normalized size = 0.91

$$-4\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{12}}-1\right)\right)+\frac{12}{13}(x+13)x^{\frac{1}{12}}-\frac{6}{5}x^{\frac{5}{6}}+\frac{4}{3}x^{\frac{3}{4}}+\frac{12}{7}x^{\frac{7}{12}}-2\sqrt{x}-3x^{\frac{1}{3}}+4x^{\frac{1}{4}}-2\log\left(x^{\frac{1}{6}}-x^{\frac{1}{12}}+1\right)-8\log\left(x^{\frac{1}{12}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^(1/3) + 1)/(x^(1/4) + 1),x, algorithm="fricas")
```

```
[Out] -4*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/12) - 1)) + 12/13*(x + 13)*
x^(1/12) - 6/5*x^(5/6) + 4/3*x^(3/4) + 12/7*x^(7/12) - 2*sqrt(x)
- 3*x^(1/3) + 4*x^(1/4) - 2*log(x^(1/6) - x^(1/12) + 1) - 8*log(x
^(1/12) + 1)
```

Sympy [A] time = 12.1051, size = 223, normalized size = 1.94

$$\frac{64x^{\frac{13}{12}}\Gamma\left(\frac{16}{3}\right)}{13\Gamma\left(\frac{19}{3}\right)}+\frac{64x^{\frac{7}{12}}\Gamma\left(\frac{16}{3}\right)}{7\Gamma\left(\frac{19}{3}\right)}+\frac{64\sqrt[12]{x}\Gamma\left(\frac{16}{3}\right)}{\Gamma\left(\frac{19}{3}\right)}-\frac{32x^{\frac{5}{6}}\Gamma\left(\frac{16}{3}\right)}{5\Gamma\left(\frac{19}{3}\right)}+\frac{4x^{\frac{3}{4}}}{3}+4\sqrt[4]{x}-\frac{16\sqrt[3]{x}\Gamma\left(\frac{16}{3}\right)}{\Gamma\left(\frac{19}{3}\right)}-2\sqrt{x}-4\log\left(\sqrt[4]{x}+1\right)+\frac{64e^{\frac{5i\pi}{3}}\log\left(-\sqrt[12]{x}e^{\frac{i\pi}{3}}+1\right)\Gamma\left(\frac{16}{3}\right)}{3\Gamma\left(\frac{19}{3}\right)}-\frac{64\log\left(-\sqrt[12]{x}e^{i\pi}+1\right)\Gamma\left(\frac{16}{3}\right)}{3\Gamma\left(\frac{19}{3}\right)}+\frac{64e^{\frac{i\pi}{3}}\log\left(-\sqrt[12]{x}e^{\frac{5i\pi}{3}}+1\right)\Gamma\left(\frac{16}{3}\right)}{3\Gamma\left(\frac{19}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+x**(1/3))/(1+x**(1/4)),x)
```

```
[Out] 64*x**(13/12)*gamma(16/3)/(13*gamma(19/3)) + 64*x**(7/12)*gamma(1
6/3)/(7*gamma(19/3)) + 64*x**(1/12)*gamma(16/3)/gamma(19/3) - 32*
x**(5/6)*gamma(16/3)/(5*gamma(19/3)) + 4*x**(3/4)/3 + 4*x**(1/4)
- 16*x**(1/3)*gamma(16/3)/gamma(19/3) - 2*sqrt(x) - 4*log(x**(1/4
) + 1) + 64*exp(5*I*pi/3)*log(-x**(1/12)*exp_polar(I*pi/3) + 1)*g
amma(16/3)/(3*gamma(19/3)) - 64*log(-x**(1/12)*exp_polar(I*pi) +
1)*gamma(16/3)/(3*gamma(19/3)) + 64*exp(I*pi/3)*log(-x**(1/12)*ex
p_polar(5*I*pi/3) + 1)*gamma(16/3)/(3*gamma(19/3))
```

GIAC/XCAS [A] time = 0.271114, size = 108, normalized size = 0.94

$$-4\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{12}}-1\right)\right)+\frac{12}{13}x^{\frac{13}{12}}-\frac{6}{5}x^{\frac{5}{6}}+\frac{4}{3}x^{\frac{3}{4}}+\frac{12}{7}x^{\frac{7}{12}}-2\sqrt{x}-3x^{\frac{1}{3}}+4x^{\frac{1}{4}}+12x^{\frac{1}{12}}-2\ln\left(x^{\frac{1}{6}}-x^{\frac{1}{12}}+1\right)-8\ln\left(x^{\frac{1}{12}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^(1/3) + 1)/(x^(1/4) + 1),x, algorithm="giac")
```

```
[Out] -4*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/12) - 1)) + 12/13*x^(13/12)
 - 6/5*x^(5/6) + 4/3*x^(3/4) + 12/7*x^(7/12) - 2*sqrt(x) - 3*x^(1
/3) + 4*x^(1/4) + 12*x^(1/12) - 2*ln(x^(1/6) - x^(1/12) + 1) - 8*
ln(x^(1/12) + 1)
```

3.831 $\int \frac{x^2}{-1+x^2+\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=4

$$x + \sin^{-1}(x)$$

```
[Out] x + ArcSin[x]
```

Rubi [A] time = 0.0717603, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$x + \sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(-1 + x^2 + Sqrt[1 - x^2]),x]

[Out] x + ArcSin[x]
```

Rubi in Sympy [A] time = 4.15297, size = 3, normalized size = 0.75

$$x + \operatorname{asin}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(-1+x**2+(-x**2+1)**(1/2)),x)

[Out] x + asin(x)
```

Mathematica [A] time = 0.0195708, size = 4, normalized size = 1.

$$x + \sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(-1 + x^2 + Sqrt[1 - x^2]),x]

[Out] x + ArcSin[x]
```

Maple [B] time = 0.012, size = 51, normalized size = 12.8

$$x+\frac{\ln\left(-1+x\right)}{2}-\frac{\ln\left(1+x\right)}{2}+\mathit{Artanh}\left(x\right)+\frac{1}{2}\sqrt{-\left(1+x\right)^{2}+2+2\,x}+\arcsin\left(x\right)-\frac{1}{2}\sqrt{-\left(-1+x\right)^{2}-2\,x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(-1+x^2+(-x^2+1)^(1/2)),x)
```

```
[Out] x+1/2*ln(-1+x)-1/2*ln(1+x)+arctanh(x)+1/2*(-(1+x)^2+2+2*x)^(1/2)+
arcsin(x)-1/2*(-(-1+x)^2-2*x+2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2}}{x^{2}+\sqrt{-x^{2}+1}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x^2 + sqrt(-x^2 + 1) - 1),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(x^2 + sqrt(-x^2 + 1) - 1), x)
```

Fricas [A] time = 0.266583, size = 27, normalized size = 6.75

$$x-2\,\arctan\left(\frac{\sqrt{-x^{2}+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x^2 + sqrt(-x^2 + 1) - 1),x, algorithm="fricas")
```

```
[Out] x - 2*arctan((sqrt(-x^2 + 1) - 1)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2}}{x^{2}+\sqrt{-x^{2}+1}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(-1+x**2+(-x**2+1)**(1/2)),x)

[Out] Integral(x**2/(x**2 + sqrt(-x**2 + 1) - 1), x)
```

GIAC/XCAS [A] time = 0.26808, size = 5, normalized size = 1.25

$$x + \arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x^2 + sqrt(-x^2 + 1) - 1),x, algorithm="giac")

[Out] x + arcsin(x)
```

3.832 $\int \sqrt{\frac{1+x}{x}}\,dx$

Optimal. Leaf size=22

$$\sqrt{\frac{1}{x}+1}x+\tanh^{-1}\left(\sqrt{\frac{1}{x}+1}\right)$$

```
[Out] Sqrt[1 + x^(-1)]*x + ArcTanh[Sqrt[1 + x^(-1)]]
```

Rubi [A] time = 0.0292551, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454

$$\sqrt{\frac{1}{x}+1}x+\tanh^{-1}\left(\sqrt{\frac{1}{x}+1}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[(1 + x)/x],x]

[Out] Sqrt[1 + x^(-1)]*x + ArcTanh[Sqrt[1 + x^(-1)]]
```

Rubi in Sympy [A] time = 1.59993, size = 19, normalized size = 0.86

$$x\sqrt{1+\frac{1}{x}}+\operatorname{atanh}\left(\sqrt{1+\frac{1}{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((1+x)/x)**(1/2),x)

[Out] x*sqrt(1 + 1/x) + atanh(sqrt(1 + 1/x))
```

Mathematica [A] time = 0.0226183, size = 34, normalized size = 1.55

$$\sqrt{\frac{1}{x}+1}x+\frac{1}{2}\log\left(\left(2\sqrt{\frac{1}{x}+1}+2\right)x+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[(1 + x)/x],x]
```

```
[Out] Sqrt[1 + x^(-1)]*x + Log[1 + (2 + 2*Sqrt[1 + x^(-1)])*x]/2
```

Maple [B] time = 0.005, size = 41, normalized size = 1.9

$$\frac{x}{2}\sqrt{\frac{1+x}{x}}\left(2\sqrt{x^2+x}+\ln\left(\frac{1}{2}+x+\sqrt{x^2+x}\right)\right)\frac{1}{\sqrt{x(1+x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((1+x)/x)^(1/2),x)
```

```
[Out] 1/2*((1+x)/x)^(1/2)*x*(2*(x^2+x)^(1/2)+ln(1/2+x+(x^2+x)^(1/2)))/(
x*(1+x))^(1/2)
```

Maxima [A] time = 0.698558, size = 68, normalized size = 3.09

$$\frac{\sqrt{\frac{x+1}{x}}}{\frac{x+1}{x}-1}+\frac{1}{2}\log\left(\sqrt{\frac{x+1}{x}}+1\right)-\frac{1}{2}\log\left(\sqrt{\frac{x+1}{x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((x + 1)/x),x, algorithm="maxima")
```

```
[Out] sqrt((x + 1)/x)/((x + 1)/x - 1) + 1/2*log(sqrt((x + 1)/x) + 1) -
1/2*log(sqrt((x + 1)/x) - 1)
```

Fricas [A] time = 0.26469, size = 54, normalized size = 2.45

$$x\sqrt{\frac{x+1}{x}}+\frac{1}{2}\log\left(\sqrt{\frac{x+1}{x}}+1\right)-\frac{1}{2}\log\left(\sqrt{\frac{x+1}{x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((x + 1)/x),x, algorithm="fricas")
```

```
[Out] x*sqrt((x + 1)/x) + 1/2*log(sqrt((x + 1)/x) + 1) - 1/2*log(sqrt((
x + 1)/x) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{x+1}{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((1+x)/x)**(1/2),x)
```

```
[Out] Integral(sqrt((x + 1)/x), x)
```

GIAC/XCAS [A] time = 0.267479, size = 42, normalized size = 1.91

$$-\frac{1}{2}\ln\left(\left|-2\,x+2\,\sqrt{x^2+x}-1\right|\right)\operatorname{sign}(x)+\sqrt{x^2+x}\operatorname{sign}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((x + 1)/x),x, algorithm="giac")
```

```
[Out] -1/2*ln(abs(-2*x + 2*sqrt(x^2 + x) - 1))*sign(x) + sqrt(x^2 + x)*
sign(x)
```

3.833 $\int \sqrt{\frac{1-x}{x}}\,dx$

Optimal. Leaf size=24

$$\sqrt{\frac{1}{x}-1}x-\tan^{-1}\left(\sqrt{\frac{1}{x}-1}\right)$$

```
[Out] Sqrt[-1 + x^(-1)]*x - ArcTan[Sqrt[-1 + x^(-1)]]
```

Rubi [A] time = 0.0293924, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385

$$\sqrt{\frac{1}{x}-1}x-\tan^{-1}\left(\sqrt{\frac{1}{x}-1}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[(1 - x)/x],x]

[Out] Sqrt[-1 + x^(-1)]*x - ArcTan[Sqrt[-1 + x^(-1)]]
```

Rubi in Sympy [A] time = 1.60218, size = 19, normalized size = 0.79

$$x\sqrt{-1+\frac{1}{x}}-\operatorname{atan}\left(\sqrt{-1+\frac{1}{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((1-x)/x)**(1/2),x)

[Out] x*sqrt(-1 + 1/x) - atan(sqrt(-1 + 1/x))
```

Mathematica [A] time = 0.0229767, size = 40, normalized size = 1.67

$$\sqrt{\frac{1}{x}-1}x-\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{\frac{1}{x}-1}(2x-1)}{2(x-1)}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[(1 - x)/x],x]
```

```
[Out] Sqrt[-1 + x^(-1)]*x - ArcTan[(Sqrt[-1 + x^(-1)]*(-1 + 2*x))/(2*(-
1 + x))]/2
```

Maple [A] time = 0.008, size = 40, normalized size = 1.7

$$\frac{x}{2}\sqrt{-\frac{-1+x}{x}}\left(2\sqrt{-x^{2}+x}+\arcsin\left(2x-1\right)\right)\frac{1}{\sqrt{-x\left(-1+x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((1-x)/x)^(1/2),x)
```

```
[Out] 1/2*(-(-1+x)/x)^(1/2)*x*(2*(-x^2+x)^(1/2)+arcsin(2*x-1))/(-x*(-1+
x))^(1/2)
```

Maxima [A] time = 0.767678, size = 50, normalized size = 2.08

$$-\frac{\sqrt{-\frac{x-1}{x}}}{\frac{x-1}{x}-1}-\arctan\left(\sqrt{-\frac{x-1}{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-(x - 1)/x),x, algorithm="maxima")
```

```
[Out] -sqrt(-(x - 1)/x)/((x - 1)/x - 1) - arctan(sqrt(-(x - 1)/x))
```

Fricas [A] time = 0.269147, size = 35, normalized size = 1.46

$$x\sqrt{-\frac{x-1}{x}}-\arctan\left(\sqrt{-\frac{x-1}{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-(x - 1)/x),x, algorithm="fricas")
```

```
[Out] x*sqrt(-(x - 1)/x) - arctan(sqrt(-(x - 1)/x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{-x+1}{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((1-x)/x)**(1/2),x)
```

```
[Out] Integral(sqrt((-x + 1)/x), x)
```

GIAC/XCAS [A] time = 0.266369, size = 38, normalized size = 1.58

$$\frac{1}{4}\,\pi \operatorname{sign}\left(x\right) + \frac{1}{2}\,\arcsin\left(2\,x - 1\right)\operatorname{sign}\left(x\right) + \sqrt{-x^{2} + x}\operatorname{sign}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-(x - 1)/x),x, algorithm="giac")
```

```
[Out] 1/4*pi*sign(x) + 1/2*arcsin(2*x - 1)*sign(x) + sqrt(-x^2 + x)*sig
n(x)
```

3.834 $\int \sqrt{\frac{-1+x}{x}}\,dx$

Optimal. Leaf size=24

$$\sqrt{x-1}\sqrt{x}-\sinh^{-1}\left(\sqrt{x-1}\right)$$

```
[Out] Sqrt[-1 + x]*Sqrt[x] - ArcSinh[Sqrt[-1 + x]]
```

Rubi [A] time = 0.0349092, antiderivative size = 28, normalized size of antiderivative = 1.17, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454

$$\sqrt{-\frac{1-x}{x}}x-\tanh^{-1}\left(\sqrt{-\frac{1-x}{x}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[(-1 + x)/x],x]
```

```
[Out] Sqrt[-((1 - x)/x)]*x - ArcTanh[Sqrt[-((1 - x)/x)]]
```

Rubi in Sympy [A] time = 1.90904, size = 19, normalized size = 0.79

$$x\sqrt{1-\frac{1}{x}}-\operatorname{atanh}\left(\sqrt{1-\frac{1}{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((-1+x)/x)**(1/2),x)
```

```
[Out] x*sqrt(1 - 1/x) - atanh(sqrt(1 - 1/x))
```

Mathematica [A] time = 0.0222932, size = 30, normalized size = 1.25

$$\sqrt{x-1}\sqrt{x}-\log\left(\sqrt{x-1}+\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[(-1 + x)/x],x]
```

```
[Out] Sqrt[-1 + x]*Sqrt[x] - Log[Sqrt[-1 + x] + Sqrt[x]]
```

Maple [B] time = 0.008, size = 45, normalized size = 1.9

$$-\frac{x}{2}\sqrt{\frac{-1+x}{x}}\left(-2\sqrt{x^2-x}+\ln\left(x-\frac{1}{2}+\sqrt{x^2-x}\right)\right)\frac{1}{\sqrt{x(-1+x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((-1+x)/x)^(1/2),x)
```

```
[Out] -1/2*((-1+x)/x)^(1/2)*x*(-2*(x^2-x)^(1/2)+ln(x-1/2+(x^2-x)^(1/2))
)/(x*(-1+x))^(1/2)
```

Maxima [A] time = 0.718419, size = 69, normalized size = 2.88

$$-\frac{\sqrt{\frac{x-1}{x}}}{\frac{x-1}{x}-1}-\frac{1}{2}\log\left(\sqrt{\frac{x-1}{x}}+1\right)+\frac{1}{2}\log\left(\sqrt{\frac{x-1}{x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((x - 1)/x),x, algorithm="maxima")
```

```
[Out] -sqrt((x - 1)/x)/((x - 1)/x - 1) - 1/2*log(sqrt((x - 1)/x) + 1) +
 1/2*log(sqrt((x - 1)/x) - 1)
```

Fricas [A] time = 0.268847, size = 54, normalized size = 2.25

$$x\sqrt{\frac{x-1}{x}}-\frac{1}{2}\log\left(\sqrt{\frac{x-1}{x}}+1\right)+\frac{1}{2}\log\left(\sqrt{\frac{x-1}{x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((x - 1)/x),x, algorithm="fricas")
```

```
[Out] x*sqrt((x - 1)/x) - 1/2*log(sqrt((x - 1)/x) + 1) + 1/2*log(sqrt((
x - 1)/x) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{x-1}{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((-1+x)/x)**(1/2),x)
```

```
[Out] Integral(sqrt((x - 1)/x), x)
```

GIAC/XCAS [A] time = 0.267937, size = 47, normalized size = 1.96

$$\frac{1}{2}\ln\left(\left|-2\,x+2\,\sqrt{x^2-x}+1\right|\right)\,\mathrm{sign}\left(x\right)+\sqrt{x^2-x}\mathrm{sign}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((x - 1)/x),x, algorithm="giac")
```

```
[Out] 1/2*ln(abs(-2*x + 2*sqrt(x^2 - x) + 1))*sign(x) + sqrt(x^2 - x)*s
ign(x)
```

3.835 $\int \frac{\sqrt{\frac{1+x}{x}}}{x} \, dx$

Optimal. Leaf size=24

$$2\tanh^{-1}\left(\sqrt{\frac{1}{x}+1}\right)-2\sqrt{\frac{1}{x}+1}$$

```
[Out] -2*Sqrt[1 + x^(-1)] + 2*ArcTanh[Sqrt[1 + x^(-1)]]
```

Rubi [A] time = 0.0420477, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$2\tanh^{-1}\left(\sqrt{\frac{1}{x}+1}\right)-2\sqrt{\frac{1}{x}+1}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[(1 + x)/x]/x,x]

[Out] -2*Sqrt[1 + x^(-1)] + 2*ArcTanh[Sqrt[1 + x^(-1)]]
```

Rubi in Sympy [A] time = 2.63613, size = 20, normalized size = 0.83

$$-2\sqrt{1+\frac{1}{x}}+2\operatorname{atanh}\left(\sqrt{1+\frac{1}{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((1+x)/x)**(1/2)/x,x)

[Out] -2*sqrt(1 + 1/x) + 2*atanh(sqrt(1 + 1/x))
```

Mathematica [A] time = 0.011041, size = 30, normalized size = 1.25

$$\log\left(\left(2\sqrt{\frac{1}{x}+1}+2\right)x+1\right)-2\sqrt{\frac{1}{x}+1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[(1 + x)/x]/x,x]
```

```
[Out] -2*Sqrt[1 + x^(-1)] + Log[1 + (2 + 2*Sqrt[1 + x^(-1)])*x]
```

Maple [B] time = 0.008, size = 60, normalized size = 2.5

$$-\frac{1}{x}\sqrt{\frac{1+x}{x}}\left(2\left(x^2+x\right)^{3/2}-2x^2\sqrt{x^2+x}-\ln\left(\frac{1}{2}+x+\sqrt{x^2+x}\right)x^2\right)\frac{1}{\sqrt{x(1+x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((1+x)/x)^(1/2)/x,x)
```

```
[Out] -((1+x)/x)^(1/2)/x*(2*(x^2+x)^(3/2)-2*x^2*(x^2+x)^(1/2)-ln(1/2+x+
(x^2+x)^(1/2))*x^2)/(x*(1+x))^(1/2)
```

Maxima [A] time = 0.708732, size = 51, normalized size = 2.12

$$-2\sqrt{\frac{x+1}{x}}+\log\left(\sqrt{\frac{x+1}{x}}+1\right)-\log\left(\sqrt{\frac{x+1}{x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((x + 1)/x)/x,x, algorithm="maxima")
```

```
[Out] -2*sqrt((x + 1)/x) + log(sqrt((x + 1)/x) + 1) - log(sqrt((x + 1)/
x) - 1)
```

Fricas [A] time = 0.276383, size = 51, normalized size = 2.12

$$-2\sqrt{\frac{x+1}{x}}+\log\left(\sqrt{\frac{x+1}{x}}+1\right)-\log\left(\sqrt{\frac{x+1}{x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((x + 1)/x)/x,x, algorithm="fricas")
```

```
[Out] -2*sqrt((x + 1)/x) + log(sqrt((x + 1)/x) + 1) - log(sqrt((x + 1)/
x) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{1+\frac{1}{x}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((1+x)/x)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(1 + 1/x)/x, x)
```

GIAC/XCAS [A] time = 0.273743, size = 51, normalized size = 2.12

$$-\ln\left(\left|-2\,x+2\,\sqrt{x^2+x}-1\right|\right)\,\mathrm{sign}\left(x\right)+\frac{2\,\mathrm{sign}\left(x\right)}{x-\sqrt{x^2+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((x + 1)/x)/x,x, algorithm="giac")
```

```
[Out] -ln(abs(-2*x + 2*sqrt(x^2 + x) - 1))*sign(x) + 2*sign(x)/(x - sqr
t(x^2 + x))
```

3.836 $\int \sqrt{\frac{x}{1+x}}\,dx$

Optimal. Leaf size=22

$$\sqrt{x}\sqrt{x+1}-\sinh^{-1}\left(\sqrt{x}\right)$$

```
[Out] Sqrt[x]*Sqrt[1 + x] - ArcSinh[Sqrt[x]]
```

Rubi [A] time = 0.0176576, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364

$$\sqrt{x}\sqrt{x+1}-\sinh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x/(1 + x)],x]

[Out] Sqrt[x]*Sqrt[1 + x] - ArcSinh[Sqrt[x]]
```

Rubi in Sympy [A] time = 1.88413, size = 24, normalized size = 1.09

$$\frac{\sqrt{\frac{x}{x+1}}}{-\frac{x}{x+1}+1}-\operatorname{atanh}\left(\sqrt{\frac{x}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x/(1+x))**(1/2),x)

[Out] sqrt(x/(x + 1))/(-x/(x + 1) + 1) - atanh(sqrt(x/(x + 1)))
```

Mathematica [A] time = 0.0279902, size = 42, normalized size = 1.91

$$\frac{\sqrt{\frac{x}{x+1}}\left(\sqrt{x}(x+1)-\sqrt{x+1}\sinh^{-1}\left(\sqrt{x}\right)\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[x/(1 + x)],x]
```

```
[Out] (Sqrt[x/(1 + x)]*(Sqrt[x]*(1 + x) - Sqrt[1 + x]*ArcSinh[Sqrt[x]])
)/Sqrt[x]
```

Maple [B] time = 0.004, size = 45, normalized size = 2.1

$$\frac{1+x}{2}\sqrt{\frac{x}{1+x}}\left(2\sqrt{x^2+x}-\ln\left(\frac{1}{2}+x+\sqrt{x^2+x}\right)\right)\frac{1}{\sqrt{x(1+x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x/(1+x))^(1/2),x)
```

```
[Out] 1/2*(x/(1+x))^(1/2)*(1+x)*(2*(x^2+x)^(1/2)-ln(1/2+x+(x^2+x)^(1/2)
))/(x*(1+x))^(1/2)
```

Maxima [A] time = 0.687948, size = 69, normalized size = 3.14

$$-\frac{\sqrt{\frac{x}{x+1}}}{\frac{x}{x+1}-1}-\frac{1}{2}\log\left(\sqrt{\frac{x}{x+1}}+1\right)+\frac{1}{2}\log\left(\sqrt{\frac{x}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x/(x + 1)),x, algorithm="maxima")
```

```
[Out] -sqrt(x/(x + 1))/(x/(x + 1) - 1) - 1/2*log(sqrt(x/(x + 1)) + 1) +
 1/2*log(sqrt(x/(x + 1)) - 1)
```

Fricas [A] time = 0.300103, size = 57, normalized size = 2.59

$$(x+1)\sqrt{\frac{x}{x+1}}-\frac{1}{2}\log\left(\sqrt{\frac{x}{x+1}}+1\right)+\frac{1}{2}\log\left(\sqrt{\frac{x}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x/(x + 1)),x, algorithm="fricas")
```

```
[Out] (x + 1)*sqrt(x/(x + 1)) - 1/2*log(sqrt(x/(x + 1)) + 1) + 1/2*log(
sqrt(x/(x + 1)) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{x}{x+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x/(1+x))**(1/2),x)
```

```
[Out] Integral(sqrt(x/(x + 1)), x)
```

GIAC/XCAS [A] time = 0.27227, size = 47, normalized size = 2.14

$$\frac{1}{2}\ln\left(\left|-2\,x+2\,\sqrt{x^2+x}-1\right|\right)\operatorname{sign}\left(x+1\right)+\sqrt{x^2+x}\operatorname{sign}\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x/(x + 1)),x, algorithm="giac")
```

```
[Out] 1/2*ln(abs(-2*x + 2*sqrt(x^2 + x) - 1))*sign(x + 1) + sqrt(x^2 +
x)*sign(x + 1)
```

3.837 $\int \frac{1}{\sqrt{\frac{-1-x}{x}}}\,dx$

Optimal. Leaf size=29

$$\tan^{-1}\left(\sqrt{-\frac{x+1}{x}}\right)-x\sqrt{-\frac{x+1}{x}}$$

```
[Out] -(x*Sqrt[-((1 + x)/x)]) + ArcTan[Sqrt[-((1 + x)/x)]]
```

Rubi [A] time = 0.033125, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385

$$\tan^{-1}\left(\sqrt{-\frac{x+1}{x}}\right)-x\sqrt{-\frac{x+1}{x}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[(-1 - x)/x],x]

[Out] -(x*Sqrt[-((1 + x)/x)]) + ArcTan[Sqrt[-((1 + x)/x)]]
```

Rubi in Sympy [A] time = 1.74025, size = 22, normalized size = 0.76

$$-x\sqrt{-1-\frac{1}{x}}+\operatorname{atan}\left(\sqrt{-1-\frac{1}{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/((-1-x)/x)**(1/2),x)

[Out] -x*sqrt(-1 - 1/x) + atan(sqrt(-1 - 1/x))
```

Mathematica [A] time = 0.0235898, size = 43, normalized size = 1.48

$$\frac{\sqrt{x}(x+1)-\sqrt{x+1}\sinh^{-1}\left(\sqrt{x}\right)}{\sqrt{x}\sqrt{-\frac{x+1}{x}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[(-1 - x)/x],x]
```

```
[Out] (Sqrt[x]*(1 + x) - Sqrt[1 + x]*ArcSinh[Sqrt[x]])/(Sqrt[x]*Sqrt[-(
(1 + x)/x)])
```

Maple [A]   time = 0.008, size = 44, normalized size = 1.5

$$\frac{1+x}{2}\left(2\sqrt{-x^2-x}+\arcsin\left(1+2\,x\right)\right)\frac{1}{\sqrt{-\frac{1+x}{x}}}\frac{1}{\sqrt{-x\left(1+x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/((-1-x)/x)^(1/2),x)
```

```
[Out] 1/2*(1+x)*(2*(-x^2-x)^(1/2)+arcsin(1+2*x))/(-(1+x)/x)^(1/2)/(-x*(
1+x))^(1/2)
```

Maxima [A]   time = 0.763151, size = 47, normalized size = 1.62

$$-\frac{\sqrt{-\frac{x+1}{x}}}{\frac{x+1}{x}-1}+\arctan\left(\sqrt{-\frac{x+1}{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-(x + 1)/x),x, algorithm="maxima")
```

```
[Out] -sqrt(-(x + 1)/x)/((x + 1)/x - 1) + arctan(sqrt(-(x + 1)/x))
```

Fricas [A]   time = 0.268014, size = 34, normalized size = 1.17

$$-x\sqrt{-\frac{x+1}{x}}+\arctan\left(\sqrt{-\frac{x+1}{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-(x + 1)/x),x, algorithm="fricas")
```

```
[Out] -x*sqrt(-(x + 1)/x) + arctan(sqrt(-(x + 1)/x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\frac{-x-1}{x}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((-1-x)/x)**(1/2),x)
```

```
[Out] Integral(1/sqrt((-x - 1)/x), x)
```

GIAC/XCAS [A] time = 0.27004, size = 47, normalized size = 1.62

$$\frac{1}{4}\,\pi\operatorname{sign}\left(x\right) - \frac{\arcsin\left(2\,x + 1\right)}{2\,\operatorname{sign}\left(x\right)} - \frac{\sqrt{-x^{2} - x}}{\operatorname{sign}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-(x + 1)/x),x, algorithm="giac")
```

```
[Out] 1/4*pi*sign(x) - 1/2*arcsin(2*x + 1)/sign(x) - sqrt(-x^2 - x)/sig
n(x)
```

3.838 $\int \sqrt{(4-x)x}\,dx$

Optimal. Leaf size=33

$$-\frac{1}{2}\sqrt{4x-x^2}(2-x)-2\sin^{-1}\left(1-\frac{x}{2}\right)$$

```
[Out] -((2 - x)*Sqrt[4*x - x^2])/2 - 2*ArcSin[1 - x/2]
```

Rubi [A] time = 0.0270427, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364

$$-\frac{1}{2}\sqrt{4x-x^2}(2-x)-2\sin^{-1}\left(1-\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[(4 - x)*x],x]
```

```
[Out] -((2 - x)*Sqrt[4*x - x^2])/2 - 2*ArcSin[1 - x/2]
```

Rubi in Sympy [A] time = 0.953432, size = 24, normalized size = 0.73

$$-\frac{(-2x+4)\sqrt{-x^2+4x}}{4}+2\,\text{asin}\left(\frac{x}{2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((4-x)*x)**(1/2),x)
```

```
[Out] -(-2*x + 4)*sqrt(-x**2 + 4*x)/4 + 2*asin(x/2 - 1)
```

Mathematica [A] time = 0.0427216, size = 45, normalized size = 1.36

$$\frac{1}{2}\sqrt{-(x-4)x}\left(x-\frac{8\log\left(\sqrt{x-4}+\sqrt{x}\right)}{\sqrt{x-4}\sqrt{x}}-2\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[(4 - x)*x],x]
```

```
[Out] (Sqrt[-((-4 + x)*x)]*(-2 + x - (8*Log[Sqrt[-4 + x] + Sqrt[x]])/(S
qrt[-4 + x]*Sqrt[x])))/2
```

Maple [A] time = 0.01, size = 28, normalized size = 0.9

$$-\frac{-2\,x+4}{4}\sqrt{-x^{2}+4\,x}+2\,\arcsin\left(x/2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((4-x)*x)^(1/2),x)
```

```
[Out] -1/4*(-2*x+4)*(-x^2+4*x)^(1/2)+2*arcsin(1/2*x-1)
```

Maxima [A] time = 0.752189, size = 49, normalized size = 1.48

$$\frac{1}{2}\,\sqrt{-x^{2}+4\,x}x-\sqrt{-x^{2}+4\,x}-2\,\arcsin\left(-\frac{1}{2}\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-(x - 4)*x),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-x^2 + 4*x)*x - sqrt(-x^2 + 4*x) - 2*arcsin(-1/2*x + 1)
```

Fricas [A] time = 0.265156, size = 47, normalized size = 1.42

$$\frac{1}{2}\,\sqrt{-x^{2}+4\,x}(x-2)-4\,\arctan\left(\frac{\sqrt{-x^{2}+4\,x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-(x - 4)*x),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(-x^2 + 4*x)*(x - 2) - 4*arctan(sqrt(-x^2 + 4*x)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x(-x+4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((4-x)*x)**(1/2),x)
```

```
[Out] Integral(sqrt(x*(-x + 4)), x)
```

GIAC/XCAS [A] time = 0.260836, size = 34, normalized size = 1.03

$$\frac{1}{2}\sqrt{-x^2+4\,x}(x-2)+2\,\arcsin\left(\frac{1}{2}\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-(x - 4)*x),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-x^2 + 4*x)*(x - 2) + 2*arcsin(1/2*x - 1)
```

3.839 $\int \frac{1}{\sqrt{(1-x)x}}\,dx$

Optimal. Leaf size=8

$$-\sin^{-1}(1-2x)$$

```
[Out] -ArcSin[1 - 2*x]
```

Rubi [A] time = 0.0115754, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$

$$-\sin^{-1}(1-2x)$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[(1 - x)*x],x]

[Out] -ArcSin[1 - 2*x]
```

Rubi in Sympy [A] time = 0.644821, size = 5, normalized size = 0.62

$$\operatorname{asin}(2x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/((1-x)*x)**(1/2),x)

[Out] asin(2*x - 1)
```

Mathematica [B] time = 0.0119987, size = 38, normalized size = 4.75

$$\frac{2\sqrt{x-1}\sqrt{x}\log\left(\sqrt{x-1}+\sqrt{x}\right)}{\sqrt{-(x-1)x}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[(1 - x)*x],x]
```

```
[Out] (2*Sqrt[-1 + x]*Sqrt[x]*Log[Sqrt[-1 + x] + Sqrt[x]])/Sqrt[-((-1 +
 x)*x)]
```

Maple [A] time = 0.006, size = 7, normalized size = 0.9

$$\arcsin(2x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1-x)*x)^(1/2),x)
```

```
[Out] arcsin(2*x-1)
```

Maxima [A] time = 0.760251, size = 8, normalized size = 1.

$$\arcsin(2x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sqrt(-(x - 1)*x),x, algorithm="maxima")
```

```
[Out] arcsin(2*x - 1)
```

Fricas [A] time = 0.2745, size = 22, normalized size = 2.75

$$-2\arctan\left(\frac{\sqrt{-x^2+x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sqrt(-(x - 1)*x),x, algorithm="fricas")
```

```
[Out] -2*arctan(sqrt(-x^2 + x)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x(-x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((1-x)*x)**(1/2),x)

[Out] Integral(1/sqrt(x*(-x + 1)), x)
```

GIAC/XCAS [A] time = 0.263701, size = 8, normalized size = 1.

$$\arcsin(2x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-(x - 1)*x),x, algorithm="giac")

[Out] arcsin(2*x - 1)
```

3.840 $\int \frac{x}{(x(2+x))^{3/2}}\,dx$

Optimal. Leaf size=13

$$\frac{x}{\sqrt{x^2+2x}}$$

```
[Out] x/Sqrt[2*x + x^2]
```

Rubi [A] time = 0.0252502, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\frac{x}{\sqrt{x^2+2x}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(x*(2 + x))^(3/2),x]

[Out] x/Sqrt[2*x + x^2]
```

Rubi in Sympy [A] time = 1.63904, size = 10, normalized size = 0.77

$$\frac{x}{\sqrt{x^2+2x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(x*(2+x))**(3/2),x)

[Out] x/sqrt(x**2 + 2*x)
```

Mathematica [A] time = 0.00911312, size = 11, normalized size = 0.85

$$\frac{x}{\sqrt{x(x+2)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(x*(2 + x))^(3/2),x]
```

```
[Out] x/Sqrt[x*(2 + x)]
```

Maple [A] time = 0.006, size = 15, normalized size = 1.2

$$x^{2} (2 + x) (x (2 + x))^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(x*(2+x))^(3/2),x)
```

```
[Out] x^2*(2+x)/(x*(2+x))^(3/2)
```

Maxima [A] time = 0.680424, size = 15, normalized size = 1.15

$$\frac{x}{\sqrt{x^{2} + 2\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((x + 2)*x)^(3/2),x, algorithm="maxima")
```

```
[Out] x/sqrt(x^2 + 2*x)
```

Fricas [A] time = 0.266967, size = 24, normalized size = 1.85

$$\frac{2}{x - \sqrt{x^{2} + 2\,x} + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((x + 2)*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/(x - sqrt(x^2 + 2*x) + 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(x (x + 2))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x*(2+x))**(3/2),x)
```

```
[Out] Integral(x/(x*(x + 2))**(3/2), x)
```

GIAC/XCAS [A] time = 0.261805, size = 22, normalized size = 1.69

$$\frac{2}{x - \sqrt{(x+2)x} + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((x + 2)*x)^(3/2),x, algorithm="giac")
```

```
[Out] 2/(x - sqrt((x + 2)*x) + 2)
```

3.841 $\int \frac{\sqrt{1+\frac{1}{x}}}{1-x^2} dx$

Optimal. Leaf size=22

$$\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{x}+1}}{\sqrt{2}}\right)$$

```
[Out] Sqrt[2]*ArcTanh[Sqrt[1 + x^(-1)]/Sqrt[2]]
```

Rubi [A] time = 0.0994283, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263

$$\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{x}+1}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 + x^(-1)]/(1 - x^2),x]
```

```
[Out] Sqrt[2]*ArcTanh[Sqrt[1 + x^(-1)]/Sqrt[2]]
```

Rubi in Sympy [A] time = 6.16368, size = 20, normalized size = 0.91

$$\sqrt{2}\,\text{atanh}\left(\frac{\sqrt{2}\sqrt{1+\frac{1}{x}}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+1/x)**(1/2)/(-x**2+1),x)
```

```
[Out] sqrt(2)*atanh(sqrt(2)*sqrt(1 + 1/x)/2)
```

Mathematica [A] time = 0.0327234, size = 38, normalized size = 1.73

$$\frac{\log\left(\left(2\sqrt{2}\sqrt{\frac{1}{x}+1}+3\right)x+1\right)-\log(1-x)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 + x^(-1)]/(1 - x^2),x]
```

```
[Out] (-Log[1 - x] + Log[1 + (3 + 2*Sqrt[2]*Sqrt[1 + x^(-1)])*x])/Sqrt[
2]
```

Maple [B] time = 0.019, size = 41, normalized size = 1.9

$$\frac{\sqrt{2}x}{2}\sqrt{\frac{1+x}{x}}Artanh\left(\frac{(3x+1)\sqrt{2}}{4}\frac{1}{\sqrt{x^2+x}}\right)\frac{1}{\sqrt{x(1+x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+1/x)^(1/2)/(-x^2+1),x)
```

```
[Out] 1/2*((1+x)/x)^(1/2)*x/(x*(1+x))^(1/2)*2^(1/2)*arctanh(1/4*(3*x+1)
*2^(1/2)/(x^2+x)^(1/2))
```

Maxima [A] time = 0.777649, size = 54, normalized size = 2.45

$$-\frac{1}{2}\sqrt{2}\log\left(-\frac{2\left(\sqrt{2}-\sqrt{\frac{x+1}{x}}\right)}{2\sqrt{2}+2\sqrt{\frac{x+1}{x}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sqrt(1/x + 1)/(x^2 - 1),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*log(-2*(sqrt(2) - sqrt((x + 1)/x))/((2*sqrt(2)) + 2*
sqrt((x + 1)/x)))
```

Fricas [A] time = 0.27427, size = 45, normalized size = 2.05

$$\frac{1}{2}\sqrt{2}\log\left(-\frac{2\sqrt{2}x\sqrt{\frac{x+1}{x}}+3x+1}{x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sqrt(1/x + 1)/(x^2 - 1),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*log(-(2*sqrt(2)*x*sqrt((x + 1)/x) + 3*x + 1)/(x - 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\sqrt{1+\frac{1}{x}}}{x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+1/x)**(1/2)/(-x**2+1),x)
```

```
[Out] -Integral(sqrt(1 + 1/x)/(x**2 - 1), x)
```

GIAC/XCAS [A] time = 0.290333, size = 99, normalized size = 4.5

$$\frac{1}{2}\sqrt{2}\ln\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)\mathrm{sign}(x)-\frac{1}{2}\sqrt{2}\ln\left(\frac{\left|-2x-2\sqrt{2}+2\sqrt{x^2+x}+2\right|}{\left|-2x+2\sqrt{2}+2\sqrt{x^2+x}+2\right|}\right)\mathrm{sign}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sqrt(1/x + 1)/(x^2 - 1),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*ln((sqrt(2) - 1)/(sqrt(2) + 1))*sign(x) - 1/2*sqrt(2)
*ln(abs(-2*x - 2*sqrt(2) + 2*sqrt(x^2 + x) + 2)/abs(-2*x + 2*sqrt
(2) + 2*sqrt(x^2 + x) + 2))*sign(x)
```

3.842 $\int \frac{1}{1+\sqrt{5}-x^2+\sqrt{5}x^2} \, dx$

Optimal. Leaf size=24

$$\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}x\right)$$

```
[Out] ArcTan[Sqrt[(3 - Sqrt[5])/2]*x]/2
```

Rubi [A] time = 0.0524535, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$

$$\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}x\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + Sqrt[5] - x^2 + Sqrt[5]*x^2)^(-1),x]
```

```
[Out] ArcTan[Sqrt[(3 - Sqrt[5])/2]*x]/2
```

Rubi in Sympy [A] time = 2.00668, size = 42, normalized size = 1.75

$$\frac{\operatorname{atan}\left(\frac{x\sqrt{-1+\sqrt{5}}}{\sqrt{1+\sqrt{5}}}\right)}{\sqrt{-1+\sqrt{5}}\sqrt{1+\sqrt{5}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1-x**2+5**(1/2)+x**2*5**(1/2)),x)
```

```
[Out] atan(x*sqrt(-1 + sqrt(5))/sqrt(1 + sqrt(5)))/(sqrt(-1 + sqrt(5))*
sqrt(1 + sqrt(5)))
```

Mathematica [C] time = 0.0357357, size = 39, normalized size = 1.62

$$\frac{1}{4}i\log\left(-2ix+\sqrt{5}+1\right)-\frac{1}{4}i\log\left(2ix+\sqrt{5}+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + Sqrt[5] - x^2 + Sqrt[5]*x^2)^(-1),x]
```

```
[Out] (I/4)*Log[1 + Sqrt[5] - (2*I)*x] - (I/4)*Log[1 + Sqrt[5] + (2*I)*
x]
```

Maple [B] time = 0.018, size = 32, normalized size = 1.3

$$4\,\frac{1}{\left(\sqrt{5}-1\right)\left(2\,\sqrt{5}+2\right)}\arctan\left(4\,\frac{x}{2\,\sqrt{5}+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1-x^2+5^(1/2)+5^(1/2)*x^2),x)
```

```
[Out] 4/(5^(1/2)-1)/(2*5^(1/2)+2)*arctan(4*x/(2*5^(1/2)+2))
```

Maxima [A] time = 0.761452, size = 15, normalized size = 0.62

$$\frac{1}{2}\arctan\left(\frac{1}{2}\,x\left(\sqrt{5}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(5)*x^2 - x^2 + sqrt(5) + 1),x, algorithm="maxima")
```

```
[Out] 1/2*arctan(1/2*x*(sqrt(5) - 1))
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(5)*x^2 - x^2 + sqrt(5) + 1),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 0.611955, size = 14, normalized size = 0.58

$$\frac{\operatorname{atan}\left(x\left(-\frac{1}{2}+\frac{\sqrt{5}}{2}\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1-x**2+5**(1/2)+x**2*5**(1/2)),x)

[Out] atan(x*(-1/2 + sqrt(5)/2))/2
```

GIAC/XCAS [A] time = 0.262188, size = 18, normalized size = 0.75

$$\frac{1}{2}\arctan\left(\frac{2x}{\sqrt{5}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(5)*x^2 - x^2 + sqrt(5) + 1),x, algorithm="giac")

[Out] 1/2*arctan(2*x/(sqrt(5) + 1))
```

3.843 $\int \sqrt{(b-x)(-a+x)}\,dx$

Optimal. Leaf size=71

$$-\frac{1}{4}(a+b-2x)\sqrt{x(a+b)-ab-x^2}-\frac{1}{8}(a-b)^2\tan^{-1}\left(\frac{a+b-2x}{2\sqrt{x(a+b)-ab-x^2}}\right)$$

```
[Out] -((a + b - 2*x)*Sqrt[-(a*b) + (a + b)*x - x^2])/4 - ((a - b)^2*Ar
cTan[(a + b - 2*x)/(2*Sqrt[-(a*b) + (a + b)*x - x^2])])/8
```

Rubi [A] time = 0.0581787, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267

$$-\frac{1}{4}(a+b-2x)\sqrt{x(a+b)-ab-x^2}-\frac{1}{8}(a-b)^2\tan^{-1}\left(\frac{a+b-2x}{2\sqrt{x(a+b)-ab-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[(b - x)*(-a + x)],x]
```

```
[Out] -((a + b - 2*x)*Sqrt[-(a*b) + (a + b)*x - x^2])/4 - ((a - b)^2*Ar
cTan[(a + b - 2*x)/(2*Sqrt[-(a*b) + (a + b)*x - x^2])])/8
```

Rubi in Sympy [A] time = 2.01007, size = 56, normalized size = 0.79

$$-\frac{(a-b)^2\operatorname{atan}\left(\frac{a+b-2x}{2\sqrt{-ab-x^2+x(a+b)}}\right)}{8}-\frac{(a+b-2x)\sqrt{-ab-x^2+x(a+b)}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((b-x)*(-a+x))**(1/2),x)
```

```
[Out] -(a - b)**2*atan((a + b - 2*x)/(2*sqrt(-a*b - x**2 + x*(a + b))))
/8 - (a + b - 2*x)*sqrt(-a*b - x**2 + x*(a + b))/4
```

Mathematica [A] time = 0.195257, size = 84, normalized size = 1.18

$$\frac{1}{8}\sqrt{(a-x)(x-b)}\left(-2(a+b-2x)-\frac{(a-b)^2\tan^{-1}\left(\frac{a+b-2x}{2\sqrt{x-a}\sqrt{b-x}}\right)}{\sqrt{x-a}\sqrt{b-x}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[(b - x)*(-a + x)],x]
```

```
[Out] (Sqrt[(a - x)*(-b + x)]*(-2*(a + b - 2*x) - ((a - b)^2*ArcTan[(a
+ b - 2*x)/(2*Sqrt[b - x]*Sqrt[-a + x])])/(Sqrt[b - x]*Sqrt[-a +
x])))/8
```

Maple [A] time = 0.022, size = 122, normalized size = 1.7

$$-\frac{a+b-2\,x}{4}\sqrt{-ab+(a+b)\,x-x^2}-\frac{ab}{4}\arctan\left(1\left(x-\frac{a}{2}-\frac{b}{2}\right)\frac{1}{\sqrt{-ab+(a+b)\,x-x^2}}\right)$$
$$+\frac{a^2}{8}\arctan\left(1\left(x-\frac{a}{2}-\frac{b}{2}\right)\frac{1}{\sqrt{-ab+(a+b)\,x-x^2}}\right)$$
$$+\frac{b^2}{8}\arctan\left(1\left(x-\frac{a}{2}-\frac{b}{2}\right)\frac{1}{\sqrt{-ab+(a+b)\,x-x^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((b-x)*(-a+x))^(1/2),x)
```

```
[Out] -1/4*(a+b-2*x)*(-a*b+(a+b)*x-x^2)^(1/2)-1/4*arctan((x-1/2*a-1/2*b
)/(-a*b+(a+b)*x-x^2)^(1/2))*a*b+1/8*arctan((x-1/2*a-1/2*b)/(-a*b+
(a+b)*x-x^2)^(1/2))*a^2+1/8*arctan((x-1/2*a-1/2*b)/(-a*b+(a+b)*x-
x^2)^(1/2))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-(a - x)*(b - x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.281503, size = 88, normalized size = 1.24

$$\frac{1}{8}\left(a^{2}-2\,ab+b^{2}\right)\arctan\left(-\frac{a+b-2\,x}{2\,\sqrt{-ab+(a+b)x-x^{2}}}\right)-\frac{1}{4}\,\sqrt{-ab+(a+b)x-x^{2}}(a+b-2\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-(a - x)*(b - x)),x, algorithm="fricas")
```

```
[Out] 1/8*(a^2 - 2*a*b + b^2)*arctan(-1/2*(a + b - 2*x)/sqrt(-a*b + (a
+ b)*x - x^2)) - 1/4*sqrt(-a*b + (a + b)*x - x^2)*(a + b - 2*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{(-a+x)\,(b-x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((b-x)*(-a+x))**(1/2),x)
```

```
[Out] Integral(sqrt((-a + x)*(b - x)), x)
```

GIAC/XCAS [A] time = 0.265396, size = 82, normalized size = 1.15

$$\frac{1}{8}\left(a^{2}-2\,ab+b^{2}\right)\arcsin\left(\frac{a+b-2\,x}{a-b}\right)\,\mathrm{sign}\left(-a+b\right)-\frac{1}{4}\,\sqrt{-ab+ax+bx-x^{2}}(a+b-2\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-(a - x)*(b - x)),x, algorithm="giac")
```

```
[Out] 1/8*(a^2 - 2*a*b + b^2)*arcsin((a + b - 2*x)/(a - b))*sign(-a + b
) - 1/4*sqrt(-a*b + a*x + b*x - x^2)*(a + b - 2*x)
```

3.844 $\int \frac{1}{\sqrt{(b-x)(-a+x)}}\,dx$

Optimal. Leaf size=32

$$-\tan^{-1}\left(\frac{a+b-2x}{2\sqrt{x(a+b)-ab-x^2}}\right)$$

```
[Out] -ArcTan[(a + b - 2*x)/(2*Sqrt[-(a*b) + (a + b)*x - x^2])]
```

Rubi [A] time = 0.0304909, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$-\tan^{-1}\left(\frac{a+b-2x}{2\sqrt{x(a+b)-ab-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[(b - x)*(-a + x)],x]
```

```
[Out] -ArcTan[(a + b - 2*x)/(2*Sqrt[-(a*b) + (a + b)*x - x^2])]
```

Rubi in Sympy [A] time = 1.20588, size = 26, normalized size = 0.81

$$-\operatorname{atan}\left(\frac{a+b-2x}{2\sqrt{-ab-x^2+x\,(a+b)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/((b-x)*(-a+x))**(1/2),x)
```

```
[Out] -atan((a + b - 2*x)/(2*sqrt(-a*b - x**2 + x*(a + b))))
```

Mathematica [A] time = 0.039802, size = 64, normalized size = 2.

$$-\frac{\sqrt{x-a}\sqrt{b-x}\tan^{-1}\left(\frac{a+b-2x}{2\sqrt{x-a}\sqrt{b-x}}\right)}{\sqrt{(a-x)(x-b)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[(b - x)*(-a + x)],x]
```

```
[Out] -((Sqrt[b - x]*Sqrt[-a + x]*ArcTan[(a + b - 2*x)/(2*Sqrt[b - x]*S
qrt[-a + x])])/Sqrt[(a - x)*(-b + x)])
```

Maple [A] time = 0.006, size = 28, normalized size = 0.9

$$\arctan\left(1\left(x-\frac{a}{2}-\frac{b}{2}\right)\frac{1}{\sqrt{-ab+(a+b)\,x-x^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/((b-x)*(-a+x))^(1/2),x)
```

```
[Out] arctan((x-1/2*a-1/2*b)/(-a*b+(a+b)*x-x^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-(a - x)*(b - x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.270571, size = 35, normalized size = 1.09

$$\arctan\left(-\frac{a+b-2\,x}{2\,\sqrt{-ab+(a+b)x-x^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-(a - x)*(b - x)),x, algorithm="fricas")
```

```
[Out] arctan(-1/2*(a + b - 2*x)/sqrt(-a*b + (a + b)*x - x^2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(-a+x)\,(b-x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b-x)*(-a+x))**(1/2),x)
```

```
[Out] Integral(1/sqrt((-a + x)*(b - x)), x)
```

GIAC/XCAS [A] time = 0.278803, size = 30, normalized size = 0.94

$$\arcsin\left(\frac{a+b-2\,x}{a-b}\right)\mathrm{sign}\left(-a+b\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-(a - x)*(b - x)),x, algorithm="giac")
```

```
[Out] arcsin((a + b - 2*x)/(a - b))*sign(-a + b)
```

3.845 $\int \sqrt{(1-x^2)(3+x^2)}\,dx$

Optimal. Leaf size=48

$$\frac{1}{3}\sqrt{-x^4-2x^2+3}x+\frac{4F\left(\sin^{-1}(x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}-\frac{2E\left(\sin^{-1}(x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}$$

```
[Out] (x*Sqrt[3 - 2*x^2 - x^4])/3 - (2*EllipticE[ArcSin[x], -1/3])/Sqrt
[3] + (4*EllipticF[ArcSin[x], -1/3])/Sqrt[3]
```

Rubi [A] time = 0.140866, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353

$$\frac{1}{3}\sqrt{-x^4-2x^2+3}x+\frac{4F\left(\sin^{-1}(x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}-\frac{2E\left(\sin^{-1}(x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[(1 - x^2)*(3 + x^2)],x]
```

```
[Out] (x*Sqrt[3 - 2*x^2 - x^4])/3 - (2*EllipticE[ArcSin[x], -1/3])/Sqrt
[3] + (4*EllipticF[ArcSin[x], -1/3])/Sqrt[3]
```

Rubi in Sympy [A] time = 12.1868, size = 49, normalized size = 1.02

$$\frac{x\sqrt{-x^4-2x^2+3}}{3}-\frac{2\sqrt{3}E\left(\operatorname{asin}(x)\middle|-\frac{1}{3}\right)}{3}+\frac{4\sqrt{3}F\left(\operatorname{asin}(x)\middle|-\frac{1}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((-x**2+1)*(x**2+3))**(1/2),x)
```

```
[Out] x*sqrt(-x**4 - 2*x**2 + 3)/3 - 2*sqrt(3)*elliptic_e(asin(x), -1/3
)/3 + 4*sqrt(3)*elliptic_f(asin(x), -1/3)/3
```

Mathematica [C] time = 0.0951991, size = 59, normalized size = 1.23

$$\frac{1}{3}\left(\sqrt{-x^4-2x^2+3}x-4iF\left(i\sinh^{-1}\left(\frac{x}{\sqrt{3}}\right)\middle|-3\right)-2iE\left(i\sinh^{-1}\left(\frac{x}{\sqrt{3}}\right)\middle|-3\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[(1 - x^2)*(3 + x^2)],x]
```

```
[Out] (x*Sqrt[3 - 2*x^2 - x^4] - (2*I)*EllipticE[I*ArcSinh[x/Sqrt[3]],
-3] - (4*I)*EllipticF[I*ArcSinh[x/Sqrt[3]], -3])/3
```

Maple [B] time = 0.016, size = 114, normalized size = 2.4

$$\frac{x}{3}\sqrt{-x^4-2x^2+3}+\frac{2\,EllipticF\left(x,i/3\sqrt{3}\right)}{3}\sqrt{-x^2+1}\sqrt{3x^2+9}\frac{1}{\sqrt{-x^4-2x^2+3}}+\frac{2\,EllipticF\left(x,i/3\sqrt{3}\right)-2\,EllipticE\left(x,i/3\sqrt{3}\right)}{3}\sqrt{-x^2+1}\sqrt{3x^2+9}\frac{1}{\sqrt{-x^4-2x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((-x^2+1)*(x^2+3))^(1/2),x)
```

```
[Out] 1/3*x*(-x^4-2*x^2+3)^(1/2)+2/3*(-x^2+1)^(1/2)*(3*x^2+9)^(1/2)/(-x
^4-2*x^2+3)^(1/2)*EllipticF(x,1/3*I*3^(1/2))+2/3*(-x^2+1)^(1/2)*(
3*x^2+9)^(1/2)/(-x^4-2*x^2+3)^(1/2)*(EllipticF(x,1/3*I*3^(1/2))-E
llipticE(x,1/3*I*3^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-(x^2+3)(x^2-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-(x^2 + 3)*(x^2 - 1)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-(x^2 + 3)*(x^2 - 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{-x^4-2x^2+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-(x^2 + 3)*(x^2 - 1)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 - 2*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\left(-x^{2}+1\right)\left(x^{2}+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((-x**2+1)*(x**2+3))**(1/2),x)
```

```
[Out] Integral(sqrt((-x**2 + 1)*(x**2 + 3)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-(x^{2}+3)(x^{2}-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-(x^2 + 3)*(x^2 - 1)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-(x^2 + 3)*(x^2 - 1)), x)
```

3.846 $\int \frac{1}{\sqrt{(1-x^2)(3+x^2)}}\,dx$

Optimal. Leaf size=12

$$\frac{F\left(\sin^{-1}(x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}$$

```
[Out] EllipticF[ArcSin[x], -1/3]/Sqrt[3]
```

Rubi [A] time = 0.0383122, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$\frac{F\left(\sin^{-1}(x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[(1 - x^2)*(3 + x^2)],x]
```

```
[Out] EllipticF[ArcSin[x], -1/3]/Sqrt[3]
```

Rubi in Sympy [A] time = 3.23253, size = 14, normalized size = 1.17

$$\frac{\sqrt{3}F\left(\operatorname{asin}(x)\middle|-\frac{1}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/((-x**2+1)*(x**2+3))**(1/2),x)
```

```
[Out] sqrt(3)*elliptic_f(asin(x), -1/3)/3
```

Mathematica [C] time = 0.0249868, size = 18, normalized size = 1.5

$$-iF\left(i\sinh^{-1}\left(\frac{x}{\sqrt{3}}\right)\middle|-3\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[(1 - x^2)*(3 + x^2)],x]
```

```
[Out] (-I)*EllipticF[I*ArcSinh[x/Sqrt[3]], -3]
```

Maple [B] time = 0.01, size = 43, normalized size = 3.6

$$\frac{\mathit{EllipticF}\left(x, \frac{i}{3}\sqrt{3}\right)}{3}\sqrt{-x^2+1}\sqrt{3\,x^2+9}\frac{1}{\sqrt{-x^4-2\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/((-x^2+1)*(x^2+3))^(1/2),x)
```

```
[Out] 1/3*(-x^2+1)^(1/2)*(3*x^2+9)^(1/2)/(-x^4-2*x^2+3)^(1/2)*EllipticF
(x,1/3*I*3^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-(x^2+3)(x^2-1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-(x^2 + 3)*(x^2 - 1)),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-(x^2 + 3)*(x^2 - 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{-x^4-2\,x^2+3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-(x^2 + 3)*(x^2 - 1)),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(-x^4 - 2*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(-x^2+1)(x^2+3)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((-x**2+1)*(x**2+3))**(1/2),x)
```

```
[Out] Integral(1/sqrt((-x**2 + 1)*(x**2 + 3)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-(x^2+3)(x^2-1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-(x^2 + 3)*(x^2 - 1)),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-(x^2 + 3)*(x^2 - 1)), x)
```

3.847 $\int \frac{1}{\sqrt{ax+bx^2}} dx$

Optimal. Leaf size=28

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

```
[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]
```

Rubi [A] time = 0.0218977, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[a*x + b*x^2],x]

[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]
```

Rubi in Sympy [A] time = 1.16208, size = 26, normalized size = 0.93

$$\frac{2\,\text{atanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a*x)**(1/2),x)

[Out] 2*atanh(sqrt(b)*x/sqrt(a*x + b*x**2))/sqrt(b)
```

Mathematica [A] time = 0.0294724, size = 56, normalized size = 2.

$$\frac{2\sqrt{x}\sqrt{a+bx}\log\left(\sqrt{b}\sqrt{a+bx}+b\sqrt{x}\right)}{\sqrt{b}\sqrt{x(a+bx)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[a*x + b*x^2],x]
```

```
[Out] (2*Sqrt[x]*Sqrt[a + b*x]*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[a + b*x]])/
(Sqrt[b]*Sqrt[x*(a + b*x)])
```

Maple [A] time = 0.005, size = 29, normalized size = 1.

$$1\ln\left(1\left(\frac{a}{2}+bx\right)\frac{1}{\sqrt{b}}+\sqrt{bx^2+ax}\right)\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a*x)^(1/2),x)
```

```
[Out] ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))/b^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(b*x^2 + a*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.272986, size = 1, normalized size = 0.04

$$\left[\frac{\log\left((2\,bx+a)\sqrt{b}+2\,\sqrt{bx^2+ax}b\right)}{\sqrt{b}},\frac{2\,\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx}\right)}{\sqrt{-b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(b*x^2 + a*x),x, algorithm="fricas")
```

```
[Out] [log((2*b*x + a)*sqrt(b) + 2*sqrt(b*x^2 + a*x)*b)/sqrt(b), 2*arct
an(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x))/sqrt(-b)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax + bx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*x + b*x**2), x)
```

GIAC/XCAS [A] time = 0.273711, size = 47, normalized size = 1.68

$$-\frac{\ln\left(\left|-2\left(\sqrt{b}x - \sqrt{bx^2 + ax}\right)\sqrt{b} - a\right|\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(b*x^2 + a*x),x, algorithm="giac")
```

```
[Out] -ln(abs(-2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) - a))/sqrt(b)
```

3.848 $\int \frac{1}{\sqrt{x(a+bx)}}\,dx$

Optimal. Leaf size=28

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

```
[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]
```

Rubi [A] time = 0.0252943, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[x*(a + b*x)],x]

[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]
```

Rubi in Sympy [A] time = 1.25238, size = 26, normalized size = 0.93

$$\frac{2\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x*(b*x+a))**(1/2),x)

[Out] 2*atanh(sqrt(b)*x/sqrt(a*x + b*x**2))/sqrt(b)
```

Mathematica [A] time = 0.00978284, size = 56, normalized size = 2.

$$\frac{2\sqrt{x}\sqrt{a+bx}\log\left(\sqrt{b}\sqrt{a+bx}+b\sqrt{x}\right)}{\sqrt{b}\sqrt{x(a+bx)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[x*(a + b*x)],x]
```

```
[Out] (2*Sqrt[x]*Sqrt[a + b*x]*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[a + b*x]])/
(Sqrt[b]*Sqrt[x*(a + b*x)])
```

Maple [A]   time = 0.007, size = 29, normalized size = 1.

$$1 \ln \left(1 \left(\frac{a}{2} + bx\right) \frac{1}{\sqrt{b}} + \sqrt{bx^2 + ax}\right) \frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x*(b*x+a))^(1/2),x)
```

```
[Out] ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))/b^(1/2)
```

Maxima [F]   time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt((b*x + a)*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A]   time = 0.272221, size = 1, normalized size = 0.04

$$\left[\frac{\log\left((2\,bx + a)\sqrt{b} + 2\,\sqrt{bx^2 + ax}b\right)}{\sqrt{b}}, \frac{2\,\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx}\right)}{\sqrt{-b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt((b*x + a)*x),x, algorithm="fricas")
```

```
[Out] [log((2*b*x + a)*sqrt(b) + 2*sqrt(b*x^2 + a*x)*b)/sqrt(b), 2*arct
an(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x))/sqrt(-b)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x*(b*x+a))**(1/2),x)
```

```
[Out] Integral(1/sqrt(x*(a + b*x)), x)
```

GIAC/XCAS [A] time = 0.273528, size = 47, normalized size = 1.68

$$-\frac{\ln\left(\left|-2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}-a\right|\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt((b*x + a)*x),x, algorithm="giac")
```

```
[Out] -ln(abs(-2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) - a))/sqrt(b)
```

3.849 $\int \frac{1}{\sqrt{\left(b+\frac{a}{x}\right)x^2}}\,dx$

Optimal. Leaf size=28

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

```
[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]
```

Rubi [A] time = 0.0271822, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[(b + a/x)*x^2],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]
```

Rubi in Sympy [A] time = 1.24677, size = 26, normalized size = 0.93

$$\frac{2\,\text{atanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/((a/x+b)*x**2)**(1/2),x)
```

```
[Out] 2*atanh(sqrt(b)*x/sqrt(a*x + b*x**2))/sqrt(b)
```

Mathematica [A] time = 0.00963597, size = 56, normalized size = 2.

$$\frac{2\sqrt{x}\sqrt{a+bx}\log\left(\sqrt{b}\sqrt{a+bx}+b\sqrt{x}\right)}{\sqrt{b}\sqrt{x(a+bx)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[(b + a/x)*x^2],x]
```

```
[Out] (2*Sqrt[x]*Sqrt[a + b*x]*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[a + b*x]])/
(Sqrt[b]*Sqrt[x*(a + b*x)])
```

Maple [A] time = 0.005, size = 29, normalized size = 1.

$$1\ln\left(1\left(\frac{a}{2}+bx\right)\frac{1}{\sqrt{b}}+\sqrt{bx^2+ax}\right)\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/((b+a/x)*x^2)^(1/2),x)
```

```
[Out] ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))/b^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt((b + a/x)*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.272706, size = 1, normalized size = 0.04

$$\left[\frac{\log\left((2\,bx+a)\sqrt{b}+2\,\sqrt{bx^2+ax}b\right)}{\sqrt{b}},\ \frac{2\,\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx}\right)}{\sqrt{-b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt((b + a/x)*x^2),x, algorithm="fricas")
```

```
[Out] [log((2*b*x + a)*sqrt(b) + 2*sqrt(b*x^2 + a*x)*b)/sqrt(b), 2*arct
an(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x))/sqrt(-b)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2\left(\frac{a}{x}+b\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a/x+b)*x**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**2*(a/x + b)), x)
```

GIAC/XCAS [A] time = 0.274774, size = 47, normalized size = 1.68

$$-\frac{\ln\left(\left|-2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}-a\right|\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt((b + a/x)*x^2),x, algorithm="giac")
```

```
[Out] -ln(abs(-2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) - a))/sqrt(b)
```

3.850 $\int \frac{1}{\sqrt{\left(\frac{a}{x^2}+\frac{b}{x}\right)x^3}}\,dx$

Optimal. Leaf size=28

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

```
[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]
```

Rubi [A] time = 0.028026, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(a/x^2 + b/x)*x^3],x]

[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]
```

Rubi in Sympy [A] time = 1.24455, size = 26, normalized size = 0.93

$$\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/((a/x**2+b/x)*x**3)**(1/2),x)

[Out] 2*atanh(sqrt(b)*x/sqrt(a*x + b*x**2))/sqrt(b)
```

Mathematica [A] time = 0.0101054, size = 56, normalized size = 2.

$$\frac{2\sqrt{x}\sqrt{a+bx}\log\left(\sqrt{b}\sqrt{a+bx}+b\sqrt{x}\right)}{\sqrt{b}\sqrt{x(a+bx)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[(a/x^2 + b/x)*x^3],x]
```

```
[Out] (2*Sqrt[x]*Sqrt[a + b*x]*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[a + b*x]])/
(Sqrt[b]*Sqrt[x*(a + b*x)])
```

Maple [A] time = 0.005, size = 29, normalized size = 1.

$$1\ln\left(1\left(\frac{a}{2}+bx\right)\frac{1}{\sqrt{b}}+\sqrt{bx^2+ax}\right)\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/((a/x^2+b/x)*x^3)^(1/2),x)
```

```
[Out] ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))/b^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(x^3*(b/x + a/x^2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.271477, size = 1, normalized size = 0.04

$$\left[\frac{\log\left((2\,bx+a)\sqrt{b}+2\,\sqrt{bx^2+ax}b\right)}{\sqrt{b}},\ \frac{2\,\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx}\right)}{\sqrt{-b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(x^3*(b/x + a/x^2)),x, algorithm="fricas")
```

```
[Out] [log((2*b*x + a)*sqrt(b) + 2*sqrt(b*x^2 + a*x)*b)/sqrt(b), 2*arct
an(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x))/sqrt(-b)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^3\left(\frac{a}{x^2}+\frac{b}{x}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a/x**2+b/x)*x**3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**3*(a/x**2 + b/x)), x)
```

GIAC/XCAS [A] time = 0.272813, size = 47, normalized size = 1.68

$$-\frac{\ln\left(\left|-2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}-a\right|\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(x^3*(b/x + a/x^2)),x, algorithm="giac")
```

```
[Out] -ln(abs(-2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) - a))/sqrt(b)
```

3.851 $\int \frac{1}{\sqrt{\frac{ax^2+bx^3}{x}}}\,dx$

Optimal. Leaf size=28

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

```
[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]
```

Rubi [A] time = 0.0261503, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[(a*x^2 + b*x^3)/x],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]
```

Rubi in Sympy [A] time = 1.25523, size = 26, normalized size = 0.93

$$\frac{2\,\text{atanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/((b*x**3+a*x**2)/x)**(1/2),x)
```

```
[Out] 2*atanh(sqrt(b)*x/sqrt(a*x + b*x**2))/sqrt(b)
```

Mathematica [A] time = 0.009827, size = 56, normalized size = 2.

$$\frac{2\sqrt{x}\sqrt{a+bx}\log\left(\sqrt{b}\sqrt{a+bx}+b\sqrt{x}\right)}{\sqrt{b}\sqrt{x(a+bx)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[(a*x^2 + b*x^3)/x],x]
```

```
[Out] (2*Sqrt[x]*Sqrt[a + b*x]*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[a + b*x]])/
(Sqrt[b]*Sqrt[x*(a + b*x)])
```

Maple [A] time = 0.006, size = 29, normalized size = 1.

$$1\ln\left(1\left(\frac{a}{2}+bx\right)\frac{1}{\sqrt{b}}+\sqrt{bx^2+ax}\right)\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/((b*x^3+a*x^2)/x)^(1/2),x)
```

```
[Out] ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))/b^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt((b*x^3 + a*x^2)/x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.271562, size = 1, normalized size = 0.04

$$\left[\frac{\log\left((2\,bx+a)\sqrt{b}+2\,\sqrt{bx^2+ax}b\right)}{\sqrt{b}},\frac{2\,\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx}\right)}{\sqrt{-b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt((b*x^3 + a*x^2)/x),x, algorithm="fricas")
```

```
[Out] [log((2*b*x + a)*sqrt(b) + 2*sqrt(b*x^2 + a*x)*b)/sqrt(b), 2*arct
an(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x))/sqrt(-b)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\frac{ax^2+bx^3}{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x**3+a*x**2)/x)**(1/2),x)
```

```
[Out] Integral(1/sqrt((a*x**2 + b*x**3)/x), x)
```

GIAC/XCAS [A] time = 0.271942, size = 47, normalized size = 1.68

$$-\frac{\ln\left(\left|-2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}-a\right|\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt((b*x^3 + a*x^2)/x),x, algorithm="giac")
```

```
[Out] -ln(abs(-2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) - a))/sqrt(b)
```

3.852 $\int \frac{1}{\sqrt{\frac{ax^3+bx^4}{x^2}}}\,dx$

Optimal. Leaf size=28

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

```
[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]
```

Rubi [A] time = 0.0269787, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[(a*x^3 + b*x^4)/x^2],x]

[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]
```

Rubi in Sympy [A] time = 1.25201, size = 26, normalized size = 0.93

$$\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{b}\,x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/((b*x**4+a*x**3)/x**2)**(1/2),x)

[Out] 2*atanh(sqrt(b)*x/sqrt(a*x + b*x**2))/sqrt(b)
```

Mathematica [A] time = 0.0102977, size = 56, normalized size = 2.

$$\frac{2\sqrt{x}\sqrt{a+bx}\log\left(\sqrt{b}\sqrt{a+bx}+b\sqrt{x}\right)}{\sqrt{b}\sqrt{x(a+bx)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[(a*x^3 + b*x^4)/x^2],x]
```

```
[Out] (2*Sqrt[x]*Sqrt[a + b*x]*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[a + b*x]])/
(Sqrt[b]*Sqrt[x*(a + b*x)])
```

Maple [A] time = 0.005, size = 29, normalized size = 1.

$$1\ln\left(1\left(\frac{a}{2}+bx\right)\frac{1}{\sqrt{b}}+\sqrt{bx^2+ax}\right)\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/((b*x^4+a*x^3)/x^2)^(1/2),x)
```

```
[Out] ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))/b^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt((b*x^4 + a*x^3)/x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.283338, size = 1, normalized size = 0.04

$$\left[\frac{\log\left((2\,bx+a)\sqrt{b}+2\,\sqrt{bx^2+ax}b\right)}{\sqrt{b}},\,\frac{2\,\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx}\right)}{\sqrt{-b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt((b*x^4 + a*x^3)/x^2),x, algorithm="fricas")
```

```
[Out] [log((2*b*x + a)*sqrt(b) + 2*sqrt(b*x^2 + a*x)*b)/sqrt(b), 2*arct
an(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x))/sqrt(-b)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\frac{ax^3+bx^4}{x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x**4+a*x**3)/x**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt((a*x**3 + b*x**4)/x**2), x)
```

GIAC/XCAS [A] time = 0.275986, size = 47, normalized size = 1.68

$$-\frac{\ln\left(\left|-2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}-a\right|\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt((b*x^4 + a*x^3)/x^2),x, algorithm="giac")
```

```
[Out] -ln(abs(-2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) - a))/sqrt(b)
```

3.853 $\int \frac{1}{\sqrt{acx+bcx^2}}\,dx$

Optimal. Leaf size=40

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c}x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\sqrt{c}}$$

```
[Out] (2*ArcTanh[(Sqrt[b]*Sqrt[c]*x)/Sqrt[a*c*x + b*c*x^2]])/(Sqrt[b]*S
qrt[c])
```

Rubi [A] time = 0.0363069, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c}x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[a*c*x + b*c*x^2],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*Sqrt[c]*x)/Sqrt[a*c*x + b*c*x^2]])/(Sqrt[b]*S
qrt[c])
```

Rubi in Sympy [A] time = 2.10681, size = 39, normalized size = 0.98

$$\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{b}\sqrt{c}x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*c*x**2+a*c*x)**(1/2),x)
```

```
[Out] 2*atanh(sqrt(b)*sqrt(c)*x/sqrt(a*c*x + b*c*x**2))/(sqrt(b)*sqrt(c
))
```

Mathematica [A] time = 0.0230961, size = 57, normalized size = 1.42

$$\frac{2\sqrt{x}\sqrt{a+bx}\log\left(\sqrt{b}\sqrt{a+bx}+b\sqrt{x}\right)}{\sqrt{b}\sqrt{cx(a+bx)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[a*c*x + b*c*x^2],x]
```

```
[Out] (2*Sqrt[x]*Sqrt[a + b*x]*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[a + b*x]])/
(Sqrt[b]*Sqrt[c*x*(a + b*x)])
```

Maple [A] time = 0.005, size = 37, normalized size = 0.9

$$1\ln\left(1\left(\frac{ac}{2}+bcx\right)\frac{1}{\sqrt{bc}}+\sqrt{bcx^2+acx}\right)\frac{1}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*c*x^2+a*c*x)^(1/2),x)
```

```
[Out] ln((1/2*a*c+b*c*x)/(b*c)^(1/2)+(b*c*x^2+a*c*x)^(1/2))/(b*c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(b*c*x^2 + a*c*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.27476, size = 1, normalized size = 0.02

$$\left[\frac{\log\left(\sqrt{bc}(2\,bx+a)+2\,\sqrt{bcx^2+acx}b\right)}{\sqrt{bc}},\ \frac{2\,\arctan\left(\frac{\sqrt{bcx^2+acx}\sqrt{-bc}}{bcx}\right)}{\sqrt{-bc}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(b*c*x^2 + a*c*x),x, algorithm="fricas")
```

```
[Out] [log(sqrt(b*c)*(2*b*x + a) + 2*sqrt(b*c*x^2 + a*c*x)*b)/sqrt(b*c)
, 2*arctan(sqrt(b*c*x^2 + a*c*x)*sqrt(-b*c)/(b*c*x))/sqrt(-b*c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{acx + bcx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*c*x**2+a*c*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*c*x + b*c*x**2), x)
```

GIAC/XCAS [A] time = 0.283997, size = 68, normalized size = 1.7

$$-\frac{\sqrt{bc}\ln\left(\left|-2\left(\sqrt{bc}x - \sqrt{bcx^2 + acx}\right)b - \sqrt{bc}a\right|\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(b*c*x^2 + a*c*x),x, algorithm="giac")
```

```
[Out] -sqrt(b*c)*ln(abs(-2*(sqrt(b*c)*x - sqrt(b*c*x^2 + a*c*x))*b - sq
rt(b*c)*a))/(b*c)
```

3.854 $\int \frac{1}{\sqrt{c(ax+bx^2)}}\,dx$

Optimal. Leaf size=40

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c}x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\sqrt{c}}$$

```
[Out] (2*ArcTanh[(Sqrt[b]*Sqrt[c]*x)/Sqrt[a*c*x + b*c*x^2]])/(Sqrt[b]*S
qrt[c])
```

Rubi [A] time = 0.0363187, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c}x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[c*(a*x + b*x^2)],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*Sqrt[c]*x)/Sqrt[a*c*x + b*c*x^2]])/(Sqrt[b]*S
qrt[c])
```

Rubi in Sympy [A] time = 2.17579, size = 39, normalized size = 0.98

$$\frac{2\,\text{atanh}\left(\frac{\sqrt{b}\sqrt{c}x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c*(b*x**2+a*x))**(1/2),x)
```

```
[Out] 2*atanh(sqrt(b)*sqrt(c)*x/sqrt(a*c*x + b*c*x**2))/(sqrt(b)*sqrt(c
))
```

Mathematica [A] time = 0.0104209, size = 57, normalized size = 1.42

$$\frac{2\sqrt{x}\sqrt{a+bx}\log\left(\sqrt{b}\sqrt{a+bx}+b\sqrt{x}\right)}{\sqrt{b}\sqrt{cx(a+bx)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[c*(a*x + b*x^2)],x]
```

```
[Out] (2*Sqrt[x]*Sqrt[a + b*x]*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[a + b*x]])/
(Sqrt[b]*Sqrt[c*x*(a + b*x)])
```

Maple [A] time = 0.007, size = 37, normalized size = 0.9

$$1\ln\left(1\left(\frac{ac}{2}+bcx\right)\frac{1}{\sqrt{bc}}+\sqrt{bcx^2+acx}\right)\frac{1}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c*(b*x^2+a*x))^(1/2),x)
```

```
[Out] ln((1/2*a*c+b*c*x)/(b*c)^(1/2)+(b*c*x^2+a*c*x)^(1/2))/(b*c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt((b*x^2 + a*x)*c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.275092, size = 1, normalized size = 0.02

$$\left[\frac{\log\left(\sqrt{bc}(2\,bx+a)+2\,\sqrt{bcx^2+acx}b\right)}{\sqrt{bc}}, \frac{2\,\arctan\left(\frac{\sqrt{bcx^2+acx}\sqrt{-bc}}{bcx}\right)}{\sqrt{-bc}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt((b*x^2 + a*x)*c),x, algorithm="fricas")
```

```
[Out] [log(sqrt(b*c)*(2*b*x + a) + 2*sqrt(b*c*x^2 + a*c*x)*b)/sqrt(b*c)
, 2*arctan(sqrt(b*c*x^2 + a*c*x)*sqrt(-b*c)/(b*c*x))/sqrt(-b*c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{c\,(ax+bx^2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*(b*x**2+a*x))**(1/2),x)
```

```
[Out] Integral(1/sqrt(c*(a*x + b*x**2)), x)
```

GIAC/XCAS [A] time = 0.320606, size = 68, normalized size = 1.7

$$-\frac{\sqrt{bc}\ln\left(\left|-2\left(\sqrt{bc}x-\sqrt{bcx^2+acx}\right)b-\sqrt{bc}a\right|\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt((b*x^2 + a*x)*c),x, algorithm="giac")
```

```
[Out] -sqrt(b*c)*ln(abs(-2*(sqrt(b*c)*x - sqrt(b*c*x^2 + a*c*x))*b - sq
rt(b*c)*a))/(b*c)
```

3.855 $\int \frac{1}{\sqrt{cx(a+bx)}}\,dx$

Optimal. Leaf size=40

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{cx}}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\sqrt{c}}$$

```
[Out] (2*ArcTanh[(Sqrt[b]*Sqrt[c]*x)/Sqrt[a*c*x + b*c*x^2]])/(Sqrt[b]*S
qrt[c])
```

Rubi [A] time = 0.0358339, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{cx}}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[c*x*(a + b*x)],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*Sqrt[c]*x)/Sqrt[a*c*x + b*c*x^2]])/(Sqrt[b]*S
qrt[c])
```

Rubi in Sympy [A] time = 2.19871, size = 39, normalized size = 0.98

$$\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{b}\sqrt{cx}}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c*x*(b*x+a))**(1/2),x)
```

```
[Out] 2*atanh(sqrt(b)*sqrt(c)*x/sqrt(a*c*x + b*c*x**2))/(sqrt(b)*sqrt(c
))
```

Mathematica [A] time = 0.00941646, size = 57, normalized size = 1.42

$$\frac{2\sqrt{x}\sqrt{a+bx}\log\left(\sqrt{b}\sqrt{a+bx}+b\sqrt{x}\right)}{\sqrt{b}\sqrt{cx(a+bx)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[c*x*(a + b*x)],x]
```

```
[Out] (2*Sqrt[x]*Sqrt[a + b*x]*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[a + b*x]])/
(Sqrt[b]*Sqrt[c*x*(a + b*x)])
```

Maple [A] time = 0.005, size = 37, normalized size = 0.9

$$1\ln\left(1\left(\frac{ac}{2}+bcx\right)\frac{1}{\sqrt{bc}}+\sqrt{bcx^2+acx}\right)\frac{1}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c*x*(b*x+a))^(1/2),x)
```

```
[Out] ln((1/2*a*c+b*c*x)/(b*c)^(1/2)+(b*c*x^2+a*c*x)^(1/2))/(b*c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt((b*x + a)*c*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.283085, size = 1, normalized size = 0.02

$$\left[\frac{\log\left(\sqrt{bc}(2\,bx+a)+2\,\sqrt{bcx^2+acx}b\right)}{\sqrt{bc}},\ \frac{2\,\arctan\left(\frac{\sqrt{bcx^2+acx}\sqrt{-bc}}{bcx}\right)}{\sqrt{-bc}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt((b*x + a)*c*x),x, algorithm="fricas")
```

```
[Out] [log(sqrt(b*c)*(2*b*x + a) + 2*sqrt(b*c*x^2 + a*c*x)*b)/sqrt(b*c)
, 2*arctan(sqrt(b*c*x^2 + a*c*x)*sqrt(-b*c)/(b*c*x))/sqrt(-b*c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx\left(a+bx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x*(b*x+a))**(1/2),x)
```

```
[Out] Integral(1/sqrt(c*x*(a + b*x)), x)
```

GIAC/XCAS [A] time = 0.279879, size = 68, normalized size = 1.7

$$-\frac{\sqrt{bc}\ln\left(\left|-2\left(\sqrt{bc}x-\sqrt{bcx^2+acx}\right)b-\sqrt{bc}a\right|\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt((b*x + a)*c*x),x, algorithm="giac")
```

```
[Out] -sqrt(b*c)*ln(abs(-2*(sqrt(b*c)*x - sqrt(b*c*x^2 + a*c*x))*b - sq
rt(b*c)*a))/(b*c)
```

3.856 $\int \frac{1}{\sqrt{c\left(b+\frac{a}{x}\right)x^2}}\,dx$

Optimal. Leaf size=40

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\sqrt{c}}$$

```
[Out] (2*ArcTanh[(Sqrt[b]*Sqrt[c]*x)/Sqrt[a*c*x + b*c*x^2]])/(Sqrt[b]*S
qrt[c])
```

Rubi [A] time = 0.0393585, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[c*(b + a/x)*x^2],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*Sqrt[c]*x)/Sqrt[a*c*x + b*c*x^2]])/(Sqrt[b]*S
qrt[c])
```

Rubi in Sympy [A] time = 2.20507, size = 39, normalized size = 0.98

$$\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c*(a/x+b)*x**2)**(1/2),x)
```

```
[Out] 2*atanh(sqrt(b)*sqrt(c)*x/sqrt(a*c*x + b*c*x**2))/(sqrt(b)*sqrt(c
))
```

Mathematica [A] time = 0.0100913, size = 57, normalized size = 1.42

$$\frac{2\sqrt{x}\sqrt{a+bx}\log\left(\sqrt{b}\sqrt{a+bx}+b\sqrt{x}\right)}{\sqrt{b}\sqrt{cx(a+bx)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[c*(b + a/x)*x^2],x]
```

```
[Out] (2*Sqrt[x]*Sqrt[a + b*x]*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[a + b*x]])/
(Sqrt[b]*Sqrt[c*x*(a + b*x)])
```

Maple [A] time = 0.006, size = 37, normalized size = 0.9

$$1\ln\left(1\left(\frac{ac}{2}+bcx\right)\frac{1}{\sqrt{bc}}+\sqrt{bcx^2+acx}\right)\frac{1}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c*(b+a/x)*x^2)^(1/2),x)
```

```
[Out] ln((1/2*a*c+b*c*x)/(b*c)^(1/2)+(b*c*x^2+a*c*x)^(1/2))/(b*c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt((b + a/x)*c*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.274425, size = 1, normalized size = 0.02

$$\left[\frac{\log\left(\sqrt{bc}(2\,bx+a)+2\,\sqrt{bcx^2+acx}b\right)}{\sqrt{bc}},\ \frac{2\ \arctan\left(\frac{\sqrt{bcx^2+acx}\sqrt{-bc}}{bcx}\right)}{\sqrt{-bc}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt((b + a/x)*c*x^2),x, algorithm="fricas")
```

```
[Out] [log(sqrt(b*c)*(2*b*x + a) + 2*sqrt(b*c*x^2 + a*c*x)*b)/sqrt(b*c)
, 2*arctan(sqrt(b*c*x^2 + a*c*x)*sqrt(-b*c)/(b*c*x))/sqrt(-b*c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2\left(\frac{a}{x}+b\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*(a/x+b)*x**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(c*x**2*(a/x + b)), x)
```

GIAC/XCAS [A] time = 0.275217, size = 68, normalized size = 1.7

$$-\frac{\sqrt{bc}\ln\left(\left|-2\left(\sqrt{bc}x-\sqrt{bcx^2+acx}\right)b-\sqrt{bc}a\right|\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt((b + a/x)*c*x^2),x, algorithm="giac")
```

```
[Out] -sqrt(b*c)*ln(abs(-2*(sqrt(b*c)*x - sqrt(b*c*x^2 + a*c*x))*b - sq
rt(b*c)*a))/(b*c)
```

3.857 $\int \sqrt{1-x^2+x\sqrt{-1+x^2}}\,dx$

Optimal. Leaf size=63

$$\frac{1}{4}\sqrt{-x^2+\sqrt{x^2-1}x+1}\left(\sqrt{x^2-1}+3x\right)+\frac{3\sin^{-1}\left(x-\sqrt{x^2-1}\right)}{4\sqrt{2}}$$

```
[Out] ((3*x + Sqrt[-1 + x^2])*Sqrt[1 - x^2 + x*Sqrt[-1 + x^2]])/4 + (3*
ArcSin[x - Sqrt[-1 + x^2]])/(4*Sqrt[2])
```

Rubi [F] time = 0.0462596, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$

$$\text{Int}\left(\sqrt{1-x^2+x\sqrt{-1+x^2}},x\right)$$

Verification is Not applicable to the result.

```
[In]  Int[Sqrt[1 - x^2 + x*Sqrt[-1 + x^2]],x]
```

```
[Out] Defer[Int][Sqrt[1 - x^2 + x*Sqrt[-1 + x^2]], x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x^2+x\sqrt{x^2-1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1-x**2+x*(x**2-1)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(-x**2 + x*sqrt(x**2 - 1) + 1), x)
```

Mathematica [A] time = 0.0277576, size = 0, normalized size = 0.

$$\int \sqrt{1-x^2+x\sqrt{-1+x^2}}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[Sqrt[1 - x^2 + x*Sqrt[-1 + x^2]],x]
```

```
[Out] Integrate[Sqrt[1 - x^2 + x*Sqrt[-1 + x^2]], x]
```

Maple [F] time = 0.011, size = 0, normalized size = 0.

$$\int \sqrt{1 - x^2 + x\sqrt{x^2 - 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1-x^2+x*(x^2-1)^(1/2))^(1/2),x)
```

```
[Out] int((1-x^2+x*(x^2-1)^(1/2))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x^2 + \sqrt{x^2 - 1}x + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + sqrt(x^2 - 1)*x + 1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + sqrt(x^2 - 1)*x + 1), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + sqrt(x^2 - 1)*x + 1),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x^2 + x\sqrt{x^2 - 1} + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1-x**2+x*(x**2-1)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(-x**2 + x*sqrt(x**2 - 1) + 1), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x^2 + \sqrt{x^2 - 1}x + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + sqrt(x^2 - 1)*x + 1),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^2 + sqrt(x^2 - 1)*x + 1), x)
```

3.858 $\int \frac{\sqrt{-x+\sqrt{x}\sqrt{1+x}}}{\sqrt{1+x}}\,dx$

Optimal. Leaf size=66

$$\frac{1}{2}\left(\sqrt{x}+3\sqrt{x+1}\right)\sqrt{\sqrt{x}\sqrt{x+1}-x}-\frac{3\sin^{-1}\left(\sqrt{x}-\sqrt{x+1}\right)}{2\sqrt{2}}$$

```
[Out] ((Sqrt[x] + 3*Sqrt[1 + x])*Sqrt[-x + Sqrt[x]*Sqrt[1 + x]])/2 - (3
*ArcSin[Sqrt[x] - Sqrt[1 + x]])/(2*Sqrt[2])
```

Rubi [F] time = 0.229311, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.

$$\operatorname{Int}\left(\frac{\sqrt{-x+\sqrt{x}\sqrt{1+x}}}{\sqrt{1+x}},x\right)$$

Verification is Not applicable to the result.

```
[In]  Int[Sqrt[-x + Sqrt[x]*Sqrt[1 + x]]/Sqrt[1 + x],x]
```

```
[Out] 2*Defer[Subst][Defer[Int][Sqrt[1 - x^2 + x*Sqrt[-1 + x^2]], x], x
, Sqrt[1 + x]]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$2\int^{\sqrt{x+1}}\sqrt{-x^2+x\sqrt{x^2-1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x+x**(1/2)*(1+x)**(1/2))**(1/2)/(1+x)**(1/2),x)
```

```
[Out] 2*Integral(sqrt(-x**2 + x*sqrt(x**2 - 1) + 1), (x, sqrt(x + 1)))
```

Mathematica [B] time = 0.703212, size = 180, normalized size = 2.73

$$-\frac{(x+1)\left(2x-2\sqrt{x+1}\sqrt{x}+1\right)^2\left(2\sqrt{\sqrt{x}\sqrt{x+1}-x}\left(-2x+2\sqrt{x+1}\sqrt{x}-3\right)+3\sqrt{-4x+4\sqrt{x+1}\sqrt{x}-2}\log\left(2\sqrt{\sqrt{x}\sqrt{x+1}\right.\right.}{4\left(\sqrt{x+1}-\sqrt{x}\right)^3\left(x-\sqrt{x+1}\sqrt{x}+1\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[-x + Sqrt[x]*Sqrt[1 + x]]/Sqrt[1 + x],x]
```

```
[Out] -((1 + x)*(1 + 2*x - 2*Sqrt[x]*Sqrt[1 + x])^2*(2*Sqrt[-x + Sqrt[x
]*Sqrt[1 + x]]*(-3 - 2*x + 2*Sqrt[x]*Sqrt[1 + x]) + 3*Sqrt[-2 - 4
*x + 4*Sqrt[x]*Sqrt[1 + x]]*Log[2*Sqrt[-x + Sqrt[x]*Sqrt[1 + x]]
+ Sqrt[-2 - 4*x + 4*Sqrt[x]*Sqrt[1 + x]]]))/(4*(-Sqrt[x] + Sqrt[1
 + x])^3*(1 + x - Sqrt[x]*Sqrt[1 + x])^2)
```

Maple [F] time = 0.031, size = 0, normalized size = 0.

$$\int 1\sqrt{-x+\sqrt{x}\sqrt{1+x}}\frac{1}{\sqrt{1+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x+x^(1/2)*(1+x)^(1/2))^(1/2)/(1+x)^(1/2),x)
```

```
[Out] int((-x+x^(1/2)*(1+x)^(1/2))^(1/2)/(1+x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sqrt{x+1}\sqrt{x}-x}}{\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(sqrt(x + 1)*sqrt(x) - x)/sqrt(x + 1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sqrt(x + 1)*sqrt(x) - x)/sqrt(x + 1), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(sqrt(x + 1)*sqrt(x) - x)/sqrt(x + 1),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sqrt{x}\sqrt{x+1}-x}}{\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x+x**(1/2)*(1+x)**(1/2))**(1/2)/(1+x)**(1/2),x)
```

```
[Out] Integral(sqrt(sqrt(x)*sqrt(x + 1) - x)/sqrt(x + 1), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sqrt{x+1}\sqrt{x}-x}}{\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(sqrt(x + 1)*sqrt(x) - x)/sqrt(x + 1),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sqrt(x + 1)*sqrt(x) - x)/sqrt(x + 1), x)
```

3.859 $\int -\frac{x+2\sqrt{1+x^2}}{x+x^3+\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=78

$$\sqrt{2\left(\sqrt{5}-1\right)}\tanh^{-1}\left(\sqrt{2+\sqrt{5}}\left(\sqrt{x^2+1}+x\right)\right)-\sqrt{2\left(1+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\sqrt{5}-2}\left(\sqrt{x^2+1}+x\right)\right)$$

```
[Out] -(Sqrt[2*(1 + Sqrt[5])]*ArcTan[Sqrt[-2 + Sqrt[5]]*(x + Sqrt[1 + x
^2])]) + Sqrt[2*(-1 + Sqrt[5])]*ArcTanh[Sqrt[2 + Sqrt[5]]*(x + Sq
rt[1 + x^2])]
```

Rubi [B] time = 1.12049, antiderivative size = 319, normalized size of antiderivative = 4.09, number of steps used = 25, number of rules used = 12, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.387

$$-\sqrt{\frac{2}{5}\left(\sqrt{5}-1\right)}\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\sqrt{x^2+1}\right)-\sqrt{\frac{2}{5\left(\sqrt{5}-1\right)}}\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\sqrt{x^2+1}\right)$$
$$+\sqrt{\frac{2}{5}\left(1+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{x^2+1}\right)-\sqrt{\frac{2}{5\left(1+\sqrt{5}\right)}}\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{x^2+1}\right)$$
$$-\sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)-2\sqrt{\frac{2}{5\left(1+\sqrt{5}\right)}}\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)$$
$$+\sqrt{\frac{1}{10}\left(\sqrt{5}-1\right)}\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)-2\sqrt{\frac{2}{5\left(\sqrt{5}-1\right)}}\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)$$

Antiderivative was successfully verified.

```
[In]  Int[-((x + 2*Sqrt[1 + x^2])/(x + x^3 + Sqrt[1 + x^2])),x]
```

```
[Out] -2*Sqrt[2/(5*(1 + Sqrt[5]))]*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x] - Sq
rt[(1 + Sqrt[5])/10]*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x] - Sqrt[2/(5*
(-1 + Sqrt[5]))]*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*Sqrt[1 + x^2]] - S
qrt[(2*(-1 + Sqrt[5]))/5]*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*Sqrt[1 +
x^2]] - 2*Sqrt[2/(5*(-1 + Sqrt[5]))]*ArcTanh[Sqrt[2/(-1 + Sqrt[5]
)]*x] + Sqrt[(-1 + Sqrt[5])/10]*ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*x]
 - Sqrt[2/(5*(1 + Sqrt[5]))]*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*Sqrt[1
 + x^2]] + Sqrt[(2*(1 + Sqrt[5]))/5]*ArcTanh[Sqrt[2/(1 + Sqrt[5])
]*Sqrt[1 + x^2]]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{-x-2\sqrt{x^2+1}}{x^3+x+\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x-2*(x**2+1)**(1/2))/(x+x**3+(x**2+1)**(1/2)),x)

[Out] Integral((-x - 2*sqrt(x**2 + 1))/(x**3 + x + sqrt(x**2 + 1)), x)
```

Mathematica [F] time = 0.122551, size = 34, normalized size = 0.44

$$-\int \frac{2\sqrt{x^2+1}+x}{x^3+\sqrt{x^2+1}+x}\,dx$$

Antiderivative was successfully verified.

```
[In]  Integrate[-((x + 2*Sqrt[1 + x^2])/(x + x^3 + Sqrt[1 + x^2])),x]

[Out] -Integrate[(x + 2*Sqrt[1 + x^2])/(x + x^3 + Sqrt[1 + x^2]), x]
```

Maple [B] time = 0.194, size = 438, normalized size = 5.6

$$
\begin{aligned}
&-\frac{\sqrt{5}}{\sqrt{2\sqrt{5}+2}}\arctan\left(2\frac{x}{\sqrt{2\sqrt{5}+2}}\right)-\frac{1}{\sqrt{2\sqrt{5}+2}}\arctan\left(2\frac{x}{\sqrt{2\sqrt{5}+2}}\right)\\
&-\frac{\sqrt{5}}{\sqrt{-2+2\sqrt{5}}}\mathit{Artanh}\left(2\frac{x}{\sqrt{-2+2\sqrt{5}}}\right)+\frac{1}{\sqrt{-2+2\sqrt{5}}}\mathit{Artanh}\left(2\frac{x}{\sqrt{-2+2\sqrt{5}}}\right)\\
&-\frac{1}{2}\sqrt{x^2+1}-\frac{x}{2}-\frac{1}{2\sqrt{-2+\sqrt{5}}}\arctan\left(\frac{1}{\sqrt{-2+\sqrt{5}}}\left(\sqrt{x^2+1}-x\right)\right)\\
&+\frac{\sqrt{5}}{2\sqrt{-2+\sqrt{5}}}\arctan\left(\frac{1}{\sqrt{-2+\sqrt{5}}}\left(\sqrt{x^2+1}-x\right)\right)\\
&+\frac{1}{2\sqrt{2+\sqrt{5}}}\mathit{Artanh}\left(\frac{1}{\sqrt{2+\sqrt{5}}}\left(\sqrt{x^2+1}-x\right)\right)\\
&+\frac{\sqrt{5}}{2\sqrt{2+\sqrt{5}}}\mathit{Artanh}\left(\frac{1}{\sqrt{2+\sqrt{5}}}\left(\sqrt{x^2+1}-x\right)\right)\\
&+\frac{3\sqrt{5}}{10\sqrt{2+\sqrt{5}}}\arctan\left(\frac{1}{\sqrt{2+\sqrt{5}}}\left(\sqrt{x^2+1}-x\right)\right)\\
&+\frac{1}{2\sqrt{2+\sqrt{5}}}\arctan\left(\frac{1}{\sqrt{2+\sqrt{5}}}\left(\sqrt{x^2+1}-x\right)\right)\\
&+\frac{3\sqrt{5}}{10\sqrt{-2+\sqrt{5}}}\mathit{Artanh}\left(\frac{1}{\sqrt{-2+\sqrt{5}}}\left(\sqrt{x^2+1}-x\right)\right)\\
&-\frac{1}{2\sqrt{-2+\sqrt{5}}}\mathit{Artanh}\left(\frac{1}{\sqrt{-2+\sqrt{5}}}\left(\sqrt{x^2+1}-x\right)\right)\\
&+\frac{1}{2}\left(\sqrt{x^2+1}-x\right)^{-1}-\frac{2\sqrt{2+\sqrt{5}}\sqrt{5}}{5}\arctan\left(\frac{1}{\sqrt{2+\sqrt{5}}}\left(\sqrt{x^2+1}-x\right)\right)\\
&+\frac{2\sqrt{-2+\sqrt{5}}\sqrt{5}}{5}\mathit{Artanh}\left(\frac{1}{\sqrt{-2+\sqrt{5}}}\left(\sqrt{x^2+1}-x\right)\right)
\end{aligned}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x-2*(x^2+1)^(1/2))/(x+x^3+(x^2+1)^(1/2)),x)
```

```
[Out] -5^(1/2)/(2*5^(1/2)+2)^(1/2)*arctan(2*x/(2*5^(1/2)+2)^(1/2))-1/(2
*5^(1/2)+2)^(1/2)*arctan(2*x/(2*5^(1/2)+2)^(1/2))-5^(1/2)/(-2+2*5
^(1/2))^(1/2)*arctanh(2*x/(-2+2*5^(1/2))^(1/2))+1/(-2+2*5^(1/2))^
(1/2)*arctanh(2*x/(-2+2*5^(1/2))^(1/2))-1/2*(x^2+1)^(1/2)-1/2*x-1
/2/(-2+5^(1/2))^(1/2)*arctan(((x^2+1)^(1/2)-x)/(-2+5^(1/2))^(1/2)
)+1/2*5^(1/2)/(-2+5^(1/2))^(1/2)*arctan(((x^2+1)^(1/2)-x)/(-2+5^(
1/2))^(1/2))+1/2/(2+5^(1/2))^(1/2)*arctanh(((x^2+1)^(1/2)-x)/(2+5
^(1/2))^(1/2))+1/2*5^(1/2)/(2+5^(1/2))^(1/2)*arctanh(((x^2+1)^(1/
2)-x)/(2+5^(1/2))^(1/2))+3/10*5^(1/2)/(2+5^(1/2))^(1/2)*arctan(((
```

```
x^2+1)^(1/2)-x)/(2+5^(1/2))^(1/2))+1/2/(2+5^(1/2))^(1/2)*arctan((
(x^2+1)^(1/2)-x)/(2+5^(1/2))^(1/2))+3/10*5^(1/2)/(-2+5^(1/2))^(1/
2)*arctanh(((x^2+1)^(1/2)-x)/(-2+5^(1/2))^(1/2))-1/2/(-2+5^(1/2))
^(1/2)*arctanh(((x^2+1)^(1/2)-x)/(-2+5^(1/2))^(1/2))+1/2/((x^2+1)
^(1/2)-x)-2/5*(2+5^(1/2))^(1/2)*5^(1/2)*arctan(((x^2+1)^(1/2)-x)/
(2+5^(1/2))^(1/2))+2/5*(-2+5^(1/2))^(1/2)*5^(1/2)*arctanh(((x^2+1
)^(1/2)-x)/(-2+5^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-x-\frac{1}{2}\arctan\left(x\right)+\int\frac{2\,x^{6}+3\,x^{4}-x^{2}-1}{2\left(x^{6}+2\,x^{4}+2\,x^{2}+2\left(x^{3}+x\right)\sqrt{x^{2}+1}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x + 2*sqrt(x^2 + 1))/(x^3 + x + sqrt(x^2 + 1)),x, algorithm="maxima")
```

```
[Out] -x - 1/2*arctan(x) + integrate(1/2*(2*x^6 + 3*x^4 - x^2 - 1)/(x^6
 + 2*x^4 + 2*x^2 + 2*(x^3 + x)*sqrt(x^2 + 1) + 1), x)
```

Fricas [A] time = 0.326752, size = 448, normalized size = 5.74

$$\frac{1}{4}\sqrt{2}\left(4\sqrt{\sqrt{5}+1}\arctan\left(\frac{\left(\sqrt{5}x-\sqrt{x^{2}+1}\left(\sqrt{5}-1\right)-x\right)\sqrt{\sqrt{5}+1}}{2\left(\sqrt{2}\sqrt{x^{2}+1}x-\sqrt{2}(x^{2}+1)-\sqrt{4\,x^{4}+4\,x^{2}+\sqrt{5}(2\,x^{2}+1)-2\left(2\,x^{3}+\sqrt{5}x+x\right)\sqrt{x^{2}+1}+1}\right)}\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x + 2*sqrt(x^2 + 1))/(x^3 + x + sqrt(x^2 + 1)),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*(4*sqrt(sqrt(5) + 1)*arctan(1/2*(sqrt(5)*x - sqrt(x^2
 + 1)*(sqrt(5) - 1) - x)*sqrt(sqrt(5) + 1)/(sqrt(2)*sqrt(x^2 + 1)
*x - sqrt(2)*(x^2 + 1) - sqrt(4*x^4 + 4*x^2 + sqrt(5)*(2*x^2 + 1)
 - 2*(2*x^3 + sqrt(5)*x + x)*sqrt(x^2 + 1) + 1))) + 4*sqrt(sqrt(5
) + 1)*arctan(sqrt(sqrt(5) + 1)/(sqrt(2)*x + sqrt(2*x^2 + sqrt(5)
 + 1))) - sqrt(sqrt(5) - 1)*log(-2*sqrt(2)*sqrt(x^2 + 1)*x + 2*sq
rt(2)*(x^2 + 1) + (sqrt(5)*x - sqrt(x^2 + 1)*(sqrt(5) + 1) + x)*s
qrt(sqrt(5) - 1)) + sqrt(sqrt(5) - 1)*log(-2*sqrt(2)*sqrt(x^2 + 1
)*x + 2*sqrt(2)*(x^2 + 1) - (sqrt(5)*x - sqrt(x^2 + 1)*(sqrt(5) +
 1) + x)*sqrt(sqrt(5) - 1)) - sqrt(sqrt(5) - 1)*log(sqrt(2)*x + s
qrt(sqrt(5) - 1)) + sqrt(sqrt(5) - 1)*log(sqrt(2)*x - sqrt(sqrt(5
) - 1)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{x^3 + x + \sqrt{x^2 + 1}}\,dx - \int \frac{2\sqrt{x^2 + 1}}{x^3 + x + \sqrt{x^2 + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x-2*(x**2+1)**(1/2))/(x+x**3+(x**2+1)**(1/2)),x)
```

```
[Out] -Integral(x/(x**3 + x + sqrt(x**2 + 1)), x) - Integral(2*sqrt(x**
2 + 1)/(x**3 + x + sqrt(x**2 + 1)), x)
```

GIAC/XCAS [A] time = 0.384355, size = 294, normalized size = 3.77

$$-\frac{1}{2}\sqrt{2\sqrt{5}+2}\arctan\left(-\frac{x-\sqrt{x^2+1}+\frac{1}{x-\sqrt{x^2+1}}}{\sqrt{2\sqrt{5}-2}}\right)-\frac{1}{2}\sqrt{2\sqrt{5}+2}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)$$
$$+\frac{1}{4}\sqrt{2\sqrt{5}-2}\ln\left(-x+\sqrt{x^2+1}+\sqrt{2\sqrt{5}+2}-\frac{1}{x-\sqrt{x^2+1}}\right)$$
$$-\frac{1}{4}\sqrt{2\sqrt{5}-2}\ln\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)+\frac{1}{4}\sqrt{2\sqrt{5}-2}\ln\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)$$
$$-\frac{1}{4}\sqrt{2\sqrt{5}-2}\ln\left(\left|-x+\sqrt{x^2+1}-\sqrt{2\sqrt{5}+2}-\frac{1}{x-\sqrt{x^2+1}}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x + 2*sqrt(x^2 + 1))/(x^3 + x + sqrt(x^2 + 1)),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2*sqrt(5) + 2)*arctan(-(x - sqrt(x^2 + 1) + 1/(x - sqrt
(x^2 + 1)))/sqrt(2*sqrt(5) - 2)) - 1/2*sqrt(2*sqrt(5) + 2)*arctan
(x/sqrt(1/2*sqrt(5) + 1/2)) + 1/4*sqrt(2*sqrt(5) - 2)*ln(-x + sqr
t(x^2 + 1) + sqrt(2*sqrt(5) + 2) - 1/(x - sqrt(x^2 + 1))) - 1/4*s
qrt(2*sqrt(5) - 2)*ln(abs(x + sqrt(1/2*sqrt(5) - 1/2))) + 1/4*sqr
t(2*sqrt(5) - 2)*ln(abs(x - sqrt(1/2*sqrt(5) - 1/2))) - 1/4*sqrt(
2*sqrt(5) - 2)*ln(abs(-x + sqrt(x^2 + 1) - sqrt(2*sqrt(5) + 2) -
1/(x - sqrt(x^2 + 1))))
```

3.860 $\int \frac{1+2x}{(1+x^2)\sqrt{2+2x+x^2}}\,dx$

Optimal. Leaf size=126

$$-\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\frac{2\sqrt{5}-\left(5+\sqrt{5}\right)x}{\sqrt{10\left(1+\sqrt{5}\right)}\sqrt{x^2+2x+2}}\right)-\sqrt{\frac{1}{2}\left(\sqrt{5}-1\right)}\tanh^{-1}\left(\frac{\left(5-\sqrt{5}\right)x+2\sqrt{5}}{\sqrt{10\left(\sqrt{5}-1\right)}\sqrt{x^2+2x+2}}\right)$$

```
[Out] -(Sqrt[(1 + Sqrt[5])/2]*ArcTan[(2*Sqrt[5] - (5 + Sqrt[5])*x)/(Sqr
t[10*(1 + Sqrt[5])]*Sqrt[2 + 2*x + x^2])]) - Sqrt[(-1 + Sqrt[5])/
2]*ArcTanh[(2*Sqrt[5] + (5 - Sqrt[5])*x)/(Sqrt[10*(-1 + Sqrt[5])]
*Sqrt[2 + 2*x + x^2])]
```

Rubi [A] time = 0.354784, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16

$$-\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\frac{2\sqrt{5}-\left(5+\sqrt{5}\right)x}{\sqrt{10\left(1+\sqrt{5}\right)}\sqrt{x^2+2x+2}}\right)-\sqrt{\frac{1}{2}\left(\sqrt{5}-1\right)}\tanh^{-1}\left(\frac{\left(5-\sqrt{5}\right)x+2\sqrt{5}}{\sqrt{10\left(\sqrt{5}-1\right)}\sqrt{x^2+2x+2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + 2*x)/((1 + x^2)*Sqrt[2 + 2*x + x^2]),x]
```

```
[Out] -(Sqrt[(1 + Sqrt[5])/2]*ArcTan[(2*Sqrt[5] - (5 + Sqrt[5])*x)/(Sqr
t[10*(1 + Sqrt[5])]*Sqrt[2 + 2*x + x^2])]) - Sqrt[(-1 + Sqrt[5])/
2]*ArcTanh[(2*Sqrt[5] + (5 - Sqrt[5])*x)/(Sqrt[10*(-1 + Sqrt[5])]
*Sqrt[2 + 2*x + x^2])]
```

Rubi in Sympy [A] time = 13.0668, size = 141, normalized size = 1.12

$$\frac{\sqrt{10}\left(2\sqrt{5}+10\right)\operatorname{atan}\left(\frac{\sqrt{10}\left(x\left(\sqrt{5}+5\right)-2\sqrt{5}\right)}{10\sqrt{1+\sqrt{5}}\sqrt{x^2+2x+2}}\right)}{20\sqrt{1+\sqrt{5}}}-\frac{\sqrt{10}\left(-2\sqrt{5}+10\right)\operatorname{atanh}\left(\frac{\sqrt{10}\left(x\left(-\sqrt{5}+5\right)+2\sqrt{5}\right)}{10\sqrt{-1+\sqrt{5}}\sqrt{x^2+2x+2}}\right)}{20\sqrt{-1+\sqrt{5}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+2*x)/(x**2+1)/(x**2+2*x+2)**(1/2),x)
```

```
[Out] sqrt(10)*(2*sqrt(5) + 10)*atan(sqrt(10)*(x*(sqrt(5) + 5) - 2*sqrt
(5))/(10*sqrt(1 + sqrt(5))*sqrt(x**2 + 2*x + 2)))/(20*sqrt(1 + sq
rt(5))) - sqrt(10)*(-2*sqrt(5) + 10)*atanh(sqrt(10)*(x*(-sqrt(5)
+ 5) + 2*sqrt(5))/(10*sqrt(-1 + sqrt(5))*sqrt(x**2 + 2*x + 2)))/(
20*sqrt(-1 + sqrt(5)))
```

Mathematica [C] time = 0.688957, size = 433, normalized size = 3.44

$$\frac{1}{4}\Bigg(i\Big(\left(\sqrt{1-2i}-\sqrt{1+2i}\right)\log\left(x^2+1\right)$$

$$-\sqrt{1-2i}\log\left((3-2i)x^2+2\sqrt{1-2i}\sqrt{x^2+2x+2}x+4\sqrt{1-2i}\sqrt{x^2+2x+2}+(8-4i)x+(7-4i)\right)$$

$$+\sqrt{1+2i}\log\left((3+2i)x^2+2\sqrt{1+2i}\sqrt{x^2+2x+2}x+4\sqrt{1+2i}\sqrt{x^2+2x+2}+(8+4i)x+(7+4i)\right)\Big)$$

$$+2\sqrt{1+2i}\tan^{-1}\left(\frac{(-1+4i)x^3+\left(5\sqrt{1+2i}\sqrt{x^2+2x+2}-(2-13i)\right)x^2+(1+i)\left(5\sqrt{1+2i}\sqrt{x^2+2x+2}+(9+5i)\right)x+5i\sqrt{1\cdots}}{(-3-8i)x^3+(4-11i)x^2+(2+2i)x+(4+14i)}\right.$$

$$+2i\sqrt{1-2i}\tanh^{-1}\left(\frac{(1+4i)x^3+\left((2+13i)-5\sqrt{1-2i}\sqrt{x^2+2x+2}\right)x^2+(1+i)\left(5i\sqrt{1-2i}\sqrt{x^2+2x+2}+(5+9i)\right)x+5i\sqrt{\cdots}}{(8+3i)x^3+(11-4i)x^2-(2+2i)x-(14+4i)}\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + 2*x)/((1 + x^2)*Sqrt[2 + 2*x + x^2]),x]
```

```
[Out] (2*Sqrt[1 + 2*I]*ArcTan[((8 + 8*I) - (1 - 4*I)*x^3 + (5*I)*Sqrt[1
 + 2*I]*Sqrt[2 + 2*x + x^2] + x^2*((-2 + 13*I) + 5*Sqrt[1 + 2*I]*
Sqrt[2 + 2*x + x^2]) + (1 + I)*x*((9 + 5*I) + 5*Sqrt[1 + 2*I]*Sqr
t[2 + 2*x + x^2]))/((4 + 14*I) + (2 + 2*I)*x + (4 - 11*I)*x^2 - (
3 + 8*I)*x^3)] + (2*I)*Sqrt[1 - 2*I]*ArcTanh[((-8 + 8*I) + (1 + 4
*I)*x^3 + (5*I)*Sqrt[1 - 2*I]*Sqrt[2 + 2*x + x^2] + x^2*((2 + 13*
I) - 5*Sqrt[1 - 2*I]*Sqrt[2 + 2*x + x^2]) + (1 + I)*x*((5 + 9*I)
+ (5*I)*Sqrt[1 - 2*I]*Sqrt[2 + 2*x + x^2]))/((-14 - 4*I) - (2 + 2
*I)*x + (11 - 4*I)*x^2 + (8 + 3*I)*x^3)] + I*((Sqrt[1 - 2*I] - Sq
rt[1 + 2*I])*Log[1 + x^2] - Sqrt[1 - 2*I]*Log[(7 - 4*I) + (8 - 4*
I)*x + (3 - 2*I)*x^2 + 4*Sqrt[1 - 2*I]*Sqrt[2 + 2*x + x^2] + 2*Sq
rt[1 - 2*I]*x*Sqrt[2 + 2*x + x^2]] + Sqrt[1 + 2*I]*Log[(7 + 4*I)
+ (8 + 4*I)*x + (3 + 2*I)*x^2 + 4*Sqrt[1 + 2*I]*Sqrt[2 + 2*x + x^
2] + 2*Sqrt[1 + 2*I]*x*Sqrt[2 + 2*x + x^2]]))/4
```

Maple [B] time = 0.143, size = 753, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+2*x)/(x^2+1)/(x^2+2*x+2)^(1/2),x)
```

```
[Out] -1/2*(10*(-1/2*5^(1/2)+1/2+x)^2/(-1/2*5^(1/2)-1/2-x)^2-2*5^(1/2)*
(-1/2*5^(1/2)+1/2+x)^2/(-1/2*5^(1/2)-1/2-x)^2+10+2*5^(1/2))^(1/2)
*(5*arctan(1/80*(-1/2*5^(1/2)+1/2+x)/(-1/2*5^(1/2)-1/2-x)*(-5+5^(
1/2))*(-22+10*5^(1/2))^(1/2)*((5-5^(1/2))*(2*(-1/2*5^(1/2)+1/2+x)
^2/(-1/2*5^(1/2)-1/2-x)^2+5^(1/2)+3))^(1/2)*(11*5^(1/2)*(-1/2*5^(
1/2)+1/2+x)^2/(-1/2*5^(1/2)-1/2-x)^2+25*(-1/2*5^(1/2)+1/2+x)^2/(-
1/2*5^(1/2)-1/2-x)^2+4*5^(1/2)+10)/((-1/2*5^(1/2)+1/2+x)^4/(-1/2*
5^(1/2)-1/2-x)^4+3*(-1/2*5^(1/2)+1/2+x)^2/(-1/2*5^(1/2)-1/2-x)^2+
1))*(-10+10*5^(1/2))^(1/2)*(-22+10*5^(1/2))^(1/2)+3*arctan(1/80*(
-1/2*5^(1/2)+1/2+x)/(-1/2*5^(1/2)-1/2-x)*(-5+5^(1/2))*(-22+10*5^(
1/2))^(1/2)*((5-5^(1/2))*(2*(-1/2*5^(1/2)+1/2+x)^2/(-1/2*5^(1/2)-
1/2-x)^2+5^(1/2)+3))^(1/2)*(11*5^(1/2)*(-1/2*5^(1/2)+1/2+x)^2/(-1
/2*5^(1/2)-1/2-x)^2+25*(-1/2*5^(1/2)+1/2+x)^2/(-1/2*5^(1/2)-1/2-x
)^2+4*5^(1/2)+10)/((-1/2*5^(1/2)+1/2+x)^4/(-1/2*5^(1/2)-1/2-x)^4+
3*(-1/2*5^(1/2)+1/2+x)^2/(-1/2*5^(1/2)-1/2-x)^2+1))*5^(1/2)*(-10+
10*5^(1/2))^(1/2)*(-22+10*5^(1/2))^(1/2)+20*arctanh((10*(-1/2*5^(
1/2)+1/2+x)^2/(-1/2*5^(1/2)-1/2-x)^2-2*5^(1/2)*(-1/2*5^(1/2)+1/2+
x)^2/(-1/2*5^(1/2)-1/2-x)^2+10+2*5^(1/2))^(1/2)/(-10+10*5^(1/2))^
(1/2))*5^(1/2)-60*arctanh((10*(-1/2*5^(1/2)+1/2+x)^2/(-1/2*5^(1/2
)-1/2-x)^2-2*5^(1/2)*(-1/2*5^(1/2)+1/2+x)^2/(-1/2*5^(1/2)-1/2-x)^
2+10+2*5^(1/2))^(1/2)/(-10+10*5^(1/2))^(1/2)))/(-2*(5^(1/2)*(-1/2
*5^(1/2)+1/2+x)^2/(-1/2*5^(1/2)-1/2-x)^2-5*(-1/2*5^(1/2)+1/2+x)^2
/(-1/2*5^(1/2)-1/2-x)^2-5^(1/2)-5)/(1+(-1/2*5^(1/2)+1/2+x)/(-1/2*
5^(1/2)-1/2-x))^2)^(1/2)/(1+(-1/2*5^(1/2)+1/2+x)/(-1/2*5^(1/2)-1/
2-x))/(-5+5^(1/2))/(-10+10*5^(1/2))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x+1}{\sqrt{x^2+2x+2}(x^2+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x + 1)/(sqrt(x^2 + 2*x + 2)*(x^2 + 1)),x, algorithm="maxima")
```

```
[Out] integrate((2*x + 1)/(sqrt(x^2 + 2*x + 2)*(x^2 + 1)), x)
```

Fricas [A] time = 0.31703, size = 1114, normalized size = 8.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x + 1)/(sqrt(x^2 + 2*x + 2)*(x^2 + 1)),x, algorithm="fricas")
```

```
[Out] 1/2*(5^(1/4)*(sqrt(5) - 1)*log(-1/5*(70*x^2 + 2*5^(1/4)*(sqrt(5)*
(7*x + 11) - 15*x - 25)*sqrt((sqrt(5) - 5)/(sqrt(5) - 3)) - 15*sq
rt(5)*(2*x^2 + 2*x + 3) - 2*sqrt(x^2 + 2*x + 2)*(5^(1/4)*(7*sqrt(
5) - 15)*sqrt((sqrt(5) - 5)/(sqrt(5) - 3)) - 15*sqrt(5)*x + 35*x)
 - 5*sqrt(5)*(3*sqrt(5) - 7) + 70*x + 105)/(3*sqrt(5) - 7)) - 5^(
1/4)*(sqrt(5) - 1)*log(-1/5*(70*x^2 - 2*5^(1/4)*(sqrt(5)*(7*x + 1
1) - 15*x - 25)*sqrt((sqrt(5) - 5)/(sqrt(5) - 3)) - 15*sqrt(5)*(2
*x^2 + 2*x + 3) + 2*sqrt(x^2 + 2*x + 2)*(5^(1/4)*(7*sqrt(5) - 15)
*sqrt((sqrt(5) - 5)/(sqrt(5) - 3)) + 15*sqrt(5)*x - 35*x) - 5*sqr
t(5)*(3*sqrt(5) - 7) + 70*x + 105)/(3*sqrt(5) - 7)) - 8*5^(1/4)*a
rctan(((sqrt(5) - 1)*sqrt((sqrt(5) - 5)/(sqrt(5) - 3)) + 5^(1/4)*
(sqrt(5) - 1))/(sqrt(1/5)*(sqrt(5) - 1)*sqrt(-(70*x^2 - 2*5^(1/4)
*(sqrt(5)*(7*x + 11) - 15*x - 25)*sqrt((sqrt(5) - 5)/(sqrt(5) - 3
)) - 15*sqrt(5)*(2*x^2 + 2*x + 3) + 2*sqrt(x^2 + 2*x + 2)*(5^(1/4
)*(7*sqrt(5) - 15)*sqrt((sqrt(5) - 5)/(sqrt(5) - 3)) + 15*sqrt(5)
*x - 35*x) - 5*sqrt(5)*(3*sqrt(5) - 7) + 70*x + 105)/(3*sqrt(5) -
 7))*sqrt((sqrt(5) - 5)/(sqrt(5) - 3)) + sqrt(x^2 + 2*x + 2)*(sqr
t(5) - 1)*sqrt((sqrt(5) - 5)/(sqrt(5) - 3)) - (sqrt(5)*x - x)*sqr
t((sqrt(5) - 5)/(sqrt(5) - 3)) + 2*5^(1/4))) + 8*5^(1/4)*arctan((
(sqrt(5) - 1)*sqrt((sqrt(5) - 5)/(sqrt(5) - 3)) - 5^(1/4)*(sqrt(5
) - 1))/(sqrt(1/5)*(sqrt(5) - 1)*sqrt(-(70*x^2 + 2*5^(1/4)*(sqrt(
5)*(7*x + 11) - 15*x - 25)*sqrt((sqrt(5) - 5)/(sqrt(5) - 3)) - 15
*sqrt(5)*(2*x^2 + 2*x + 3) - 2*sqrt(x^2 + 2*x + 2)*(5^(1/4)*(7*sq
rt(5) - 15)*sqrt((sqrt(5) - 5)/(sqrt(5) - 3)) - 15*sqrt(5)*x + 35
*x) - 5*sqrt(5)*(3*sqrt(5) - 7) + 70*x + 105)/(3*sqrt(5) - 7))*sq
rt((sqrt(5) - 5)/(sqrt(5) - 3)) + sqrt(x^2 + 2*x + 2)*(sqrt(5) -
1)*sqrt((sqrt(5) - 5)/(sqrt(5) - 3)) - (sqrt(5)*x - x)*sqrt((sqrt
(5) - 5)/(sqrt(5) - 3)) - 2*5^(1/4))))/((sqrt(5) - 1)*sqrt((sqrt(
5) - 5)/(sqrt(5) - 3)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x+1}{(x^2+1)\sqrt{x^2+2x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x**2+1)/(x**2+2*x+2)**(1/2),x)
```

```
[Out] Integral((2*x + 1)/((x**2 + 1)*sqrt(x**2 + 2*x + 2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x+1}{\sqrt{x^2+2x+2}\,(x^2+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x + 1)/(sqrt(x^2 + 2*x + 2)*(x^2 + 1)),x, algorithm="giac")
```

```
[Out] integrate((2*x + 1)/(sqrt(x^2 + 2*x + 2)*(x^2 + 1)), x)
```

3.861 $\int \frac{1}{(1+x^4)\sqrt{-x^2+\sqrt{1+x^4}}}\,dx$

Optimal. Leaf size=22

$$\tan^{-1}\left(\frac{x}{\sqrt{\sqrt{x^4+1}-x^2}}\right)$$

```
[Out] ArcTan[x/Sqrt[-x^2 + Sqrt[1 + x^4]]]
```

Rubi [A] time = 0.101277, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074

$$\tan^{-1}\left(\frac{x}{\sqrt{\sqrt{x^4+1}-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/((1 + x^4)*Sqrt[-x^2 + Sqrt[1 + x^4]]),x]
```

```
[Out] ArcTan[x/Sqrt[-x^2 + Sqrt[1 + x^4]]]
```

Rubi in Sympy [A] time = 4.33165, size = 17, normalized size = 0.77

$$\operatorname{atan}\left(\frac{x}{\sqrt{-x^2+\sqrt{x^4+1}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**4+1)/(-x**2+(x**4+1)**(1/2))**(1/2),x)
```

```
[Out] atan(x/sqrt(-x**2 + sqrt(x**4 + 1)))
```

Mathematica [A] time = 1.37299, size = 24, normalized size = 1.09

$$\cot^{-1}\left(\frac{\sqrt{\sqrt{x^4+1}-x^2}}{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((1 + x^4)*Sqrt[-x^2 + Sqrt[1 + x^4]]),x]
```

```
[Out] ArcCot[Sqrt[-x^2 + Sqrt[1 + x^4]]/x]
```

Maple [F] time = 0.04, size = 0, normalized size = 0.

$$\int \frac{1}{x^4+1}\frac{1}{\sqrt{-x^2+\sqrt{x^4+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x)
```

```
[Out] int(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^4+1)\sqrt{-x^2+\sqrt{x^4+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^4 + 1)*sqrt(-x^2 + sqrt(x^4 + 1))),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 + 1)*sqrt(-x^2 + sqrt(x^4 + 1))), x)
```

Fricas [A] time = 0.787091, size = 77, normalized size = 3.5

$$-\frac{1}{4}\arctan\left(\frac{4\left(2\,x^3-\sqrt{x^4+1}x\right)\sqrt{-x^2+\sqrt{x^4+1}}}{9\,x^4-8\,\sqrt{x^4+1}x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^4 + 1)*sqrt(-x^2 + sqrt(x^4 + 1))),x, algorithm="fricas")
```

```
[Out] -1/4*arctan(4*(2*x^3 - sqrt(x^4 + 1)*x)*sqrt(-x^2 + sqrt(x^4 + 1)
)/(9*x^4 - 8*sqrt(x^4 + 1)*x^2 + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2+\sqrt{x^4+1}}\,(x^4+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+1)/(-x**2+(x**4+1)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-x**2 + sqrt(x**4 + 1))*(x**4 + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^4+1)\sqrt{-x^2+\sqrt{x^4+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((x^4 + 1)*sqrt(-x^2 + sqrt(x^4 + 1))),x, algorithm="giac")
```

```
[Out] integrate(1/((x^4 + 1)*sqrt(-x^2 + sqrt(x^4 + 1))), x)
```

3.862 $\int \frac{1}{(a+bx^4)\sqrt{cx^2+d\sqrt{a+bx^4}}}\,dx$

Optimal. Leaf size=40

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d\sqrt{a+bx^4}+cx^2}}\right)}{a\sqrt{c}}$$

```
[Out] ArcTanh[(Sqrt[c]*x)/Sqrt[c*x^2 + d*Sqrt[a + b*x^4]]]/(a*Sqrt[c])
```

Rubi [A] time = 0.220544, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d\sqrt{a+bx^4}+cx^2}}\right)}{a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^4)*Sqrt[c*x^2 + d*Sqrt[a + b*x^4]]),x]
```

```
[Out] ArcTanh[(Sqrt[c]*x)/Sqrt[c*x^2 + d*Sqrt[a + b*x^4]]]/(a*Sqrt[c])
```

Rubi in Sympy [A] time = 6.40103, size = 34, normalized size = 0.85

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{c}x}{\sqrt{cx^2+d\sqrt{a+bx^4}}}\right)}{a\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**4+a)/(c*x**2+d*(b*x**4+a)**(1/2))**(1/2),x)
```

```
[Out] atanh(sqrt(c)*x/sqrt(c*x**2 + d*sqrt(a + b*x**4)))/(a*sqrt(c))
```

Mathematica [A] time = 0.16237, size = 50, normalized size = 1.25

$$\frac{\sqrt{-\frac{1}{c}}\cot^{-1}\left(\frac{\sqrt{-\frac{1}{c}}\sqrt{d\sqrt{a+bx^4}+cx^2}}{x}\right)}{a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b*x^4)*Sqrt[c*x^2 + d*Sqrt[a + b*x^4]]),x]
```

```
[Out] (Sqrt[-c^(-1)]*ArcCot[(Sqrt[-c^(-1)]*Sqrt[c*x^2 + d*Sqrt[a + b*x^
4]])/x])/a
```

Maple [F] time = 0.035, size = 0, normalized size = 0.

$$\int \frac{1}{bx^4+a}\frac{1}{\sqrt{cx^2+d\sqrt{bx^4+a}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^4+a)/(c*x^2+d*(b*x^4+a)^(1/2))^(1/2),x)
```

```
[Out] int(1/(b*x^4+a)/(c*x^2+d*(b*x^4+a)^(1/2))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^4+a)\sqrt{cx^2+\sqrt{bx^4+a}\,d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^4 + a)*sqrt(c*x^2 + sqrt(b*x^4 + a)*d)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)*sqrt(c*x^2 + sqrt(b*x^4 + a)*d)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^4 + a)*sqrt(c*x^2 + sqrt(b*x^4 + a)*d)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx^4)\sqrt{cx^2+d\sqrt{a+bx^4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**4+a)/(c*x**2+d*(b*x**4+a)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**4)*sqrt(c*x**2 + d*sqrt(a + b*x**4))), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^4+a)\sqrt{cx^2+\sqrt{bx^4+a}\,d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^4 + a)*sqrt(c*x^2 + sqrt(b*x^4 + a)*d)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)*sqrt(c*x^2 + sqrt(b*x^4 + a)*d)), x)
```

3.863 $\int \frac{1}{(a+bx^4)\sqrt{-cx^2+d\sqrt{a+bx^4}}}\,dx$

Optimal. Leaf size=41

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d\sqrt{a+bx^4}-cx^2}}\right)}{a\sqrt{c}}$$

```
[Out] ArcTan[(Sqrt[c]*x)/Sqrt[-(c*x^2) + d*Sqrt[a + b*x^4]]]/(a*Sqrt[c]
)
```

Rubi [A] time = 0.239726, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d\sqrt{a+bx^4}-cx^2}}\right)}{a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^4)*Sqrt[-(c*x^2) + d*Sqrt[a + b*x^4]]),x]
```

```
[Out] ArcTan[(Sqrt[c]*x)/Sqrt[-(c*x^2) + d*Sqrt[a + b*x^4]]]/(a*Sqrt[c]
)
```

Rubi in Sympy [A] time = 6.46754, size = 34, normalized size = 0.83

$$\frac{\operatorname{atan}\left(\frac{\sqrt{c}x}{\sqrt{-cx^2+d\sqrt{a+bx^4}}}\right)}{a\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(b*x**4+a)/(-c*x**2+d*(b*x**4+a)**(1/2))**(1/2),x)
```

```
[Out] atan(sqrt(c)*x/sqrt(-c*x**2 + d*sqrt(a + b*x**4)))/(a*sqrt(c))
```

Mathematica [A] time = 0.152236, size = 47, normalized size = 1.15

$$\frac{\sqrt{\frac{1}{c}}\cot^{-1}\left(\frac{\sqrt{\frac{1}{c}}\sqrt{d\sqrt{a+bx^4}-cx^2}}{x}\right)}{a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b*x^4)*Sqrt[-(c*x^2) + d*Sqrt[a + b*x^4]]),x]
```

```
[Out] (Sqrt[c^(-1)]*ArcCot[(Sqrt[c^(-1)]*Sqrt[-(c*x^2) + d*Sqrt[a + b*x
^4]])/x])/a
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int \frac{1}{bx^4+a}\frac{1}{\sqrt{-cx^2+d\sqrt{bx^4+a}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^4+a)/(-c*x^2+d*(b*x^4+a)^(1/2))^(1/2),x)
```

```
[Out] int(1/(b*x^4+a)/(-c*x^2+d*(b*x^4+a)^(1/2))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^4+a)\sqrt{-cx^2+\sqrt{bx^4+a}\,d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^4 + a)*sqrt(-c*x^2 + sqrt(b*x^4 + a)*d)),x, algorithm="maxima"
```

```
[Out] integrate(1/((b*x^4 + a)*sqrt(-c*x^2 + sqrt(b*x^4 + a)*d)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^4 + a)*sqrt(-c*x^2 + sqrt(b*x^4 + a)*d)),x, algorithm="fricas"
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + bx^4)\sqrt{-cx^2 + d\sqrt{a + bx^4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)/(-c*x**2+d*(b*x**4+a)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**4)*sqrt(-c*x**2 + d*sqrt(a + b*x**4))), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^4 + a)\sqrt{-cx^2 + \sqrt{bx^4 + a}d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^4 + a)*sqrt(-c*x^2 + sqrt(b*x^4 + a)*d)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)*sqrt(-c*x^2 + sqrt(b*x^4 + a)*d)), x)
```

3.864 $\int \frac{x}{\sqrt{a+bc^4+4bc^3dx+6bc^2d^2x^2+4bcd^3x^3+bd^4x^4}}\,dx$

Optimal. Leaf size=184

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{a+bd^4\left(\frac{c}{d}+x\right)^4}}\right)}{2\sqrt{b}d^2}-\frac{c\left(\sqrt{a}+\sqrt{b}d^2\left(\frac{c}{d}+x\right)^2\right)\sqrt{\frac{a+bd^4\left(\frac{c}{d}+x\right)^4}{\left(\sqrt{a}+\sqrt{b}d^2\left(\frac{c}{d}+x\right)^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}d^2\sqrt{a+bd^4\left(\frac{c}{d}+x\right)^4}}$$

```
[Out] ArcTanh[(Sqrt[b]*d^2*(c/d + x)^2)/Sqrt[a + b*d^4*(c/d + x)^4]]/(2
*Sqrt[b]*d^2) - (c*(Sqrt[a] + Sqrt[b]*d^2*(c/d + x)^2)*Sqrt[(a +
b*d^4*(c/d + x)^4)/(Sqrt[a] + Sqrt[b]*d^2*(c/d + x)^2)^2]*Ellipti
cF[2*ArcTan[(b^(1/4)*(c + d*x))/a^(1/4)], 1/2])/(2*a^(1/4)*b^(1/4
)*d^2*Sqrt[a + b*d^4*(c/d + x)^4])
```

Rubi [A] time = 0.419961, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a+b(c+dx)^4}}\right)}{2\sqrt{b}d^2}-\frac{c\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)\sqrt{\frac{a+b(c+dx)^4}{\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}d^2\sqrt{a+b(c+dx)^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + b*c^4 + 4*b*c^3*d*x + 6*b*c^2*d^2*x^2 + 4*b*c*d^3*x^3 + b*d^4*x^4],x]
```

```
[Out] ArcTanh[(Sqrt[b]*(c + d*x)^2)/Sqrt[a + b*(c + d*x)^4]]/(2*Sqrt[b]
*d^2) - (c*(Sqrt[a] + Sqrt[b]*(c + d*x)^2)*Sqrt[(a + b*(c + d*x)^
4)/(Sqrt[a] + Sqrt[b]*(c + d*x)^2)^2]*EllipticF[2*ArcTan[(b^(1/4)
*(c + d*x))/a^(1/4)], 1/2])/(2*a^(1/4)*b^(1/4)*d^2*Sqrt[a + b*(c
+ d*x)^4])
```

Rubi in Sympy [A] time = 31.5441, size = 160, normalized size = 0.87

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{b}d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{a+bd^4\left(\frac{c}{d}+x\right)^4}}\right)}{2\sqrt{b}d^2}-\frac{c\sqrt{\frac{a+bd^4\left(\frac{c}{d}+x\right)^4}{\left(\sqrt{a}+\sqrt{b}d^2\left(\frac{c}{d}+x\right)^2\right)^2}}\left(\sqrt{a}+\sqrt{b}d^2\left(\frac{c}{d}+x\right)^2\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}d\left(\frac{c}{d}+x\right)}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}d^2\sqrt{a+bd^4\left(\frac{c}{d}+x\right)^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x/(b*d**4*x**4+4*b*c*d**3*x**3+6*b*c**2*d**2*x**2+4*b*c**3*d*x+
```

```
[Out] atanh(sqrt(b)*d**2*(c/d + x)**2/sqrt(a + b*d**4*(c/d + x)**4))/(2
*sqrt(b)*d**2) - c*sqrt((a + b*d**4*(c/d + x)**4)/(sqrt(a) + sqrt
(b)*d**2*(c/d + x)**2)**2)*(sqrt(a) + sqrt(b)*d**2*(c/d + x)**2)*
elliptic_f(2*atan(b**(1/4)*d*(c/d + x)/a**(1/4)), 1/2)/(2*a**(1/4
)*b**(1/4)*d**2*sqrt(a + b*d**4*(c/d + x)**4))
```

Mathematica [C] time = 0.88718, size = 330, normalized size = 1.79

$$\frac{\sqrt[4]{-1}\sqrt{2}\sqrt{-\frac{i\left(\sqrt[4]{-1}\sqrt[4]{a}+\sqrt[4]{b}(c+dx)\right)}{\sqrt[4]{-1}\sqrt[4]{a}-\sqrt[4]{b}(c+dx)}}\left(\sqrt{b}(c+dx)^2+i\sqrt{a}\right)\left(\left(\sqrt[4]{-1}\sqrt[4]{a}-\sqrt[4]{b}c\right)F\left(\sin^{-1}\left(\sqrt{-\frac{i\left(\sqrt[4]{b}(c+dx)+\sqrt[4]{-1}\sqrt[4]{a}\right)}{\sqrt[4]{-1}\sqrt[4]{a}-\sqrt[4]{b}(c+dx)}}\right)\middle|-1\right)-2\sqrt[4]{-1}}{\sqrt[4]{a}\sqrt{b}d^2\sqrt{\frac{\sqrt{b}(c+dx)^2+i\sqrt{a}}{\left(\sqrt[4]{-1}\sqrt[4]{a}-\sqrt[4]{b}(c+dx)\right)^2}}\sqrt{a+b(c+dx)^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a + b*c^4 + 4*b*c^3*d*x + 6*b*c^2*d^2*x^2 + 4*b*c*d^3*x^3 + b*d^
```

```
[Out] ((-1)^(1/4)*Sqrt[2]*Sqrt[((-I)*((-1)^(1/4)*a^(1/4) + b^(1/4)*(c +
 d*x)))/((-1)^(1/4)*a^(1/4) - b^(1/4)*(c + d*x))]*(I*Sqrt[a] + Sq
rt[b]*(c + d*x)^2)*(((-1)^(1/4)*a^(1/4) - b^(1/4)*c)*EllipticF[Ar
cSin[Sqrt[((-I)*((-1)^(1/4)*a^(1/4) + b^(1/4)*(c + d*x)))/((-1)^(
1/4)*a^(1/4) - b^(1/4)*(c + d*x))]], -1] - 2*(-1)^(1/4)*a^(1/4)*E
llipticPi[-I, ArcSin[Sqrt[((-I)*((-1)^(1/4)*a^(1/4) + b^(1/4)*(c
+ d*x)))/((-1)^(1/4)*a^(1/4) - b^(1/4)*(c + d*x))]], -1]))/(a^(1/
4)*Sqrt[b]*d^2*Sqrt[(I*Sqrt[a] + Sqrt[b]*(c + d*x)^2)/((-1)^(1/4)
*a^(1/4) - b^(1/4)*(c + d*x))^2]*Sqrt[a + b*(c + d*x)^4])
```

Maple [C] time = 0.349, size = 1528, normalized size = 8.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*d^4*x^4+4*b*c*d^3*x^3+6*b*c^2*d^2*x^2+4*b*c^3*d*x+b*c^4+a)^(1/2),x)
```

```
[Out] 2*((1/b*(-a*b^3)^(1/4)-c)/d-(-I/b*(-a*b^3)^(1/4)-c)/d)*(((-I/b*(-
a*b^3)^(1/4)-c)/d-(I/b*(-a*b^3)^(1/4)-c)/d)*(x-(1/b*(-a*b^3)^(1/4
)-c)/d)/((-I/b*(-a*b^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/(x-(
I/b*(-a*b^3)^(1/4)-c)/d))^(1/2)*(x-(I/b*(-a*b^3)^(1/4)-c)/d)^2*((
```

```
(I/b*(-a*b^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)*(x-(-1/b*(-a*b
^3)^(1/4)-c)/d)/((-1/b*(-a*b^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)
/d)/(x-(I/b*(-a*b^3)^(1/4)-c)/d))^(1/2)*(((I/b*(-a*b^3)^(1/4)-c)/
d-(1/b*(-a*b^3)^(1/4)-c)/d)*(x-(-I/b*(-a*b^3)^(1/4)-c)/d)/((-I/b*
(-a*b^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/(x-(I/b*(-a*b^3)^(1
/4)-c)/d))^(1/2)/((-I/b*(-a*b^3)^(1/4)-c)/d-(I/b*(-a*b^3)^(1/4)-c
)/d)/((I/b*(-a*b^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/(b*d^4*(
x-(1/b*(-a*b^3)^(1/4)-c)/d)*(x-(I/b*(-a*b^3)^(1/4)-c)/d)*(x-(-1/b
*(-a*b^3)^(1/4)-c)/d)*(x-(-I/b*(-a*b^3)^(1/4)-c)/d))^(1/2)*((I/b*
(-a*b^3)^(1/4)-c)/d*EllipticF((((-I/b*(-a*b^3)^(1/4)-c)/d-(I/b*(-
a*b^3)^(1/4)-c)/d)*(x-(1/b*(-a*b^3)^(1/4)-c)/d)/((-I/b*(-a*b^3)^(
1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/(x-(I/b*(-a*b^3)^(1/4)-c)/d))
^(1/2),(((I/b*(-a*b^3)^(1/4)-c)/d-(-1/b*(-a*b^3)^(1/4)-c)/d)*((1/
b*(-a*b^3)^(1/4)-c)/d-(-I/b*(-a*b^3)^(1/4)-c)/d)/((1/b*(-a*b^3)^(
1/4)-c)/d-(-1/b*(-a*b^3)^(1/4)-c)/d)/((I/b*(-a*b^3)^(1/4)-c)/d-(-
I/b*(-a*b^3)^(1/4)-c)/d))^(1/2))+((1/b*(-a*b^3)^(1/4)-c)/d-(I/b*(
-a*b^3)^(1/4)-c)/d)*EllipticPi((((-I/b*(-a*b^3)^(1/4)-c)/d-(I/b*(
-a*b^3)^(1/4)-c)/d)*(x-(1/b*(-a*b^3)^(1/4)-c)/d)/((-I/b*(-a*b^3)^
(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/(x-(I/b*(-a*b^3)^(1/4)-c)/d)
)^(1/2),((-I/b*(-a*b^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/((-I
/b*(-a*b^3)^(1/4)-c)/d-(I/b*(-a*b^3)^(1/4)-c)/d),(((I/b*(-a*b^3)^
(1/4)-c)/d-(-1/b*(-a*b^3)^(1/4)-c)/d)*((1/b*(-a*b^3)^(1/4)-c)/d-(
-I/b*(-a*b^3)^(1/4)-c)/d)/((1/b*(-a*b^3)^(1/4)-c)/d-(-1/b*(-a*b^3
)^(1/4)-c)/d)/((I/b*(-a*b^3)^(1/4)-c)/d-(-I/b*(-a*b^3)^(1/4)-c)/d
))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{bd^4x^4 + 4\,bcd^3x^3 + 6\,bc^2d^2x^2 + 4\,bc^3dx + bc^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sqrt(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x + b*c^
```

```
[Out] integrate(x/sqrt(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*
b*c^3*d*x + b*c^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x}{\sqrt{bd^4x^4 + 4\,bcd^3x^3 + 6\,bc^2d^2x^2 + 4\,bc^3dx + bc^4 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sqrt(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x + b*c^
```

```
[Out] integral(x/sqrt(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b
*c^3*d*x + b*c^4 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a + bc^4 + 4bc^3dx + 6bc^2d^2x^2 + 4bcd^3x^3 + bd^4x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*d**4*x**4+4*b*c*d**3*x**3+6*b*c**2*d**2*x**2+4*b*c**3*d*x+b*c**
```

```
[Out] Integral(x/sqrt(a + b*c**4 + 4*b*c**3*d*x + 6*b*c**2*d**2*x**2 +
4*b*c*d**3*x**3 + b*d**4*x**4), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{bd^4x^4 + 4\,bcd^3x^3 + 6\,bc^2d^2x^2 + 4\,bc^3dx + bc^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sqrt(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x + b*c^
```

```
[Out] integrate(x/sqrt(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*
b*c^3*d*x + b*c^4 + a), x)
```

3.865 $\int \frac{1}{\sqrt{a+bc^4+4bc^3dx+6bc^2d^2x^2+4bcd^3x^3+bd^4x^4}}\,dx$

Optimal. Leaf size=131

$$\frac{\left(\sqrt{a}+\sqrt{b}d^2\left(\frac{c}{d}+x\right)^2\right)\sqrt{\frac{a+bd^4\left(\frac{c}{d}+x\right)^4}{\left(\sqrt{a}+\sqrt{b}d^2\left(\frac{c}{d}+x\right)^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}d\sqrt{a+bd^4\left(\frac{c}{d}+x\right)^4}}$$

```
[Out] ((Sqrt[a] + Sqrt[b]*d^2*(c/d + x)^2)*Sqrt[(a + b*d^4*(c/d + x)^4)
/(Sqrt[a] + Sqrt[b]*d^2*(c/d + x)^2)^2]*EllipticF[2*ArcTan[(b^(1/
4)*(c + d*x))/a^(1/4)], 1/2])/(2*a^(1/4)*b^(1/4)*d*Sqrt[a + b*d^4
*(c/d + x)^4])
```

Rubi [A] time = 0.202158, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.041

$$\frac{\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)\sqrt{\frac{a+b(c+dx)^4}{\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}d\sqrt{a+b(c+dx)^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*c^4 + 4*b*c^3*d*x + 6*b*c^2*d^2*x^2 + 4*b*c*d^3*x^3 + b*d^4*x^4]
```

```
[Out] ((Sqrt[a] + Sqrt[b]*(c + d*x)^2)*Sqrt[(a + b*(c + d*x)^4)/(Sqrt[a
] + Sqrt[b]*(c + d*x)^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*(c + d*x)
)/a^(1/4)], 1/2])/(2*a^(1/4)*b^(1/4)*d*Sqrt[a + b*(c + d*x)^4])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(b*d**4*x**4+4*b*c*d**3*x**3+6*b*c**2*d**2*x**2+4*b*c**3*d*x+
```

```
[Out] Timed out
```

Mathematica [C] time = 0.0975455, size = 90, normalized size = 0.69

$$-\frac{i\sqrt{\frac{a+b(c+dx)^4}{a}}F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}(c+dx)\right)\middle|-1\right)}{d\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{a+b(c+dx)^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*c^4 + 4*b*c^3*d*x + 6*b*c^2*d^2*x^2 + 4*b*c*d^3*x^3 + b*d^
```

```
[Out] ((-I)*Sqrt[(a + b*(c + d*x)^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqr
t[b])/Sqrt[a]]*(c + d*x)], -1])/(Sqrt[(I*Sqrt[b])/Sqrt[a]]*d*Sqrt
[a + b*(c + d*x)^4])
```

Maple [C] time = 0.036, size = 1036, normalized size = 7.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*d^4*x^4+4*b*c*d^3*x^3+6*b*c^2*d^2*x^2+4*b*c^3*d*x+b*c^4+a)^(1/2),x)
```

```
[Out] 2*((1/b*(-a*b^3)^(1/4)-c)/d-(-I/b*(-a*b^3)^(1/4)-c)/d)*(((-I/b*(-
a*b^3)^(1/4)-c)/d-(I/b*(-a*b^3)^(1/4)-c)/d)*(x-(1/b*(-a*b^3)^(1/4
)-c)/d)/((-I/b*(-a*b^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/(x-(
I/b*(-a*b^3)^(1/4)-c)/d))^(1/2)*(x-(I/b*(-a*b^3)^(1/4)-c)/d)^2*((
(I/b*(-a*b^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)*(x-(-1/b*(-a*b
^3)^(1/4)-c)/d)/((-1/b*(-a*b^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)
/d)/(x-(I/b*(-a*b^3)^(1/4)-c)/d))^(1/2)*(((I/b*(-a*b^3)^(1/4)-c)/
d-(1/b*(-a*b^3)^(1/4)-c)/d)*(x-(-I/b*(-a*b^3)^(1/4)-c)/d)/((-I/b*
(-a*b^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/(x-(I/b*(-a*b^3)^(1
/4)-c)/d))^(1/2)/((-I/b*(-a*b^3)^(1/4)-c)/d-(I/b*(-a*b^3)^(1/4)-c
)/d)/((I/b*(-a*b^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/(b*d^4*(
x-(1/b*(-a*b^3)^(1/4)-c)/d)*(x-(I/b*(-a*b^3)^(1/4)-c)/d)*(x-(-1/b
*(-a*b^3)^(1/4)-c)/d)*(x-(-I/b*(-a*b^3)^(1/4)-c)/d))^(1/2)*Ellipt
icF((((-I/b*(-a*b^3)^(1/4)-c)/d-(I/b*(-a*b^3)^(1/4)-c)/d)*(x-(1/b
*(-a*b^3)^(1/4)-c)/d)/((-I/b*(-a*b^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1
/4)-c)/d)/(x-(I/b*(-a*b^3)^(1/4)-c)/d))^(1/2),(((I/b*(-a*b^3)^(1/
4)-c)/d-(-1/b*(-a*b^3)^(1/4)-c)/d)*((1/b*(-a*b^3)^(1/4)-c)/d-(-I/
b*(-a*b^3)^(1/4)-c)/d)/((1/b*(-a*b^3)^(1/4)-c)/d-(-1/b*(-a*b^3)^(
1/4)-c)/d)/((I/b*(-a*b^3)^(1/4)-c)/d-(-I/b*(-a*b^3)^(1/4)-c)/d))^
(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bd^4x^4 + 4\,bcd^3x^3 + 6\,bc^2d^2x^2 + 4\,bc^3dx + bc^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sqrt(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x + b*c^
```

```
[Out] integrate(1/sqrt(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*
b*c^3*d*x + b*c^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{bd^4x^4 + 4\,bcd^3x^3 + 6\,bc^2d^2x^2 + 4\,bc^3dx + bc^4 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sqrt(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x + b*c^
```

```
[Out] integral(1/sqrt(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b
*c^3*d*x + b*c^4 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + bc^4 + 4bc^3dx + 6bc^2d^2x^2 + 4bcd^3x^3 + bd^4x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*d**4*x**4+4*b*c*d**3*x**3+6*b*c**2*d**2*x**2+4*b*c**3*d*x+b*c**
```

```
[Out] Integral(1/sqrt(a + b*c**4 + 4*b*c**3*d*x + 6*b*c**2*d**2*x**2 +
4*b*c*d**3*x**3 + b*d**4*x**4), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bd^4x^4 + 4\,bcd^3x^3 + 6\,bc^2d^2x^2 + 4\,bc^3dx + bc^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sqrt(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x + b*c^
```

```
[Out] integrate(1/sqrt(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*
b*c^3*d*x + b*c^4 + a), x)
```

3.866 $\int \frac{a-cx^4}{\sqrt{a+bx^2+cx^4}(ad+aex^2+cdx^4)}\,dx$

Optimal. Leaf size=54

$$\frac{\tanh^{-1}\left(\frac{x\sqrt{bd-ae}}{\sqrt{d}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{d}\sqrt{bd-ae}}$$

```
[Out] ArcTanh[(Sqrt[b*d - a*e]*x)/(Sqrt[d]*Sqrt[a + b*x^2 + c*x^4])]/(S
qrt[d]*Sqrt[b*d - a*e])
```

Rubi [A] time = 0.403541, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.047

$$\frac{\tanh^{-1}\left(\frac{x\sqrt{bd-ae}}{\sqrt{d}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{d}\sqrt{bd-ae}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - c*x^4)/(Sqrt[a + b*x^2 + c*x^4]*(a*d + a*e*x^2 + c*d*x^4)),x]
```

```
[Out] ArcTanh[(Sqrt[b*d - a*e]*x)/(Sqrt[d]*Sqrt[a + b*x^2 + c*x^4])]/(S
qrt[d]*Sqrt[b*d - a*e])
```

Rubi in Sympy [A] time = 24.6843, size = 48, normalized size = 0.89

$$\frac{\operatorname{atan}\left(\frac{x\sqrt{ae-bd}}{\sqrt{d}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{d}\sqrt{ae-bd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-c*x**4+a)/(c*d*x**4+a*e*x**2+a*d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] atan(x*sqrt(a*e - b*d)/(sqrt(d)*sqrt(a + b*x**2 + c*x**4)))/(sqrt
(d)*sqrt(a*e - b*d))
```

Mathematica [C] time = 1.24306, size = 419, normalized size = 7.76

$$\frac{i\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\left(-\left(\frac{\left(b+\sqrt{b^2-4ac}\right)d}{ae-\sqrt{a}\sqrt{ae^2-4cd^2}};i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)-\left(\frac{\left(b+\sqrt{b^2-4ac}\right)d}{ae+\sqrt{a}\sqrt{ae^2-4cd^2}};i\sinh^{-}\right.\right.}{\sqrt{2}d\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - c*x^4)/(Sqrt[a + b*x^2 + c*x^4]*(a*d + a*e*x^2 + c*d*x^4)),x]
```

```
[Out] (I*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])
]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(EllipticF[I*ArcSin
h[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a
*c])/(b - Sqrt[b^2 - 4*a*c])] - EllipticPi[((b + Sqrt[b^2 - 4*a*c
])*d)/(a*e - Sqrt[a]*Sqrt[-4*c*d^2 + a*e^2]), I*ArcSinh[Sqrt[2]*S
qrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - S
qrt[b^2 - 4*a*c])] - EllipticPi[((b + Sqrt[b^2 - 4*a*c])*d)/(a*e
+ Sqrt[a]*Sqrt[-4*c*d^2 + a*e^2]), I*ArcSinh[Sqrt[2]*Sqrt[c/(b +
Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4
*a*c])]))/(Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*d*Sqrt[a + b*x
^2 + c*x^4])
```

Maple [C] time = 0.057, size = 514, normalized size = 9.5

$$-\frac{\sqrt{2}}{4\,d}\sqrt{4-2\,\frac{\left(-b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\sqrt{4+2\,\frac{\left(b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}EllipticF\left(\frac{\sqrt{2}x}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{-4\,ac+b^2}\right)},\frac{1}{2}\sqrt{-4+2\,\frac{b\left(b\right.}{}}\right.$$

$$-\frac{a}{4\,d}\sum_{\_alpha=RootOf\left(cd\_Z^4+ae\_Z^2+ad\right)}\frac{-\_alpha^2e-2\,d}{\_alpha\,\left(2\,\_alpha^2cd+ae\right)}\left(-1Artanh\left(\frac{2\,\_alpha^2cx^2+b\_alpha^2+bx^2+2\,a}{2}\frac{1}{\sqrt{\frac{\_alpha^2(-}{d}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c*x^4+a)/(c*d*x^4+a*e*x^2+a*d)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/4/d*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c
+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2
)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)
^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1
/4*a/d*sum((-_alpha^2*e-2*d)/_alpha/(2*_alpha^2*c*d+a*e)*(-1/(_al
pha^2/d*(-a*e+b*d))^(1/2)*arctanh(1/2*(2*_alpha^2*c*x^2+_alpha^2*
```

```
b+b*x^2+2*a)/(_alpha^2/d*(-a*e+b*d))^(1/2)/(c*x^4+b*x^2+a)^(1/2))
+1/a/d*2^(1/2)*_alpha*(_alpha^2*c*d+a*e)/((-b+(-4*a*c+b^2)^(1/2))
/a)^(1/2)*(2+b*x^2/a-1/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)*(2+b*x^2/a
+1/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)*Elliptic
Pi(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(_alpha^2*
(-4*a*c+b^2)^(1/2)*c*d+_alpha^2*b*c*d+(-4*a*c+b^2)^(1/2)*a*e+a*b*
e)/a/c/d,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))/a)^(1/2))),_alpha=RootOf(_Z^4*c*d+_Z^2*a*e+a*d))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{cx^4 - a}{(cdx^4 + aex^2 + ad)\sqrt{cx^4 + bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(c*x^4 - a)/((c*d*x^4 + a*e*x^2 + a*d)*sqrt(c*x^4 + b*x^2 + a)),x, algo
```

```
[Out] -integrate((c*x^4 - a)/((c*d*x^4 + a*e*x^2 + a*d)*sqrt(c*x^4 + b*
x^2 + a)), x)
```

Fricas [A] time = 12.9547, size = 1, normalized size = 0.02

$$\left[\frac{\log\left(-\frac{4\left(\left(bcd^3-acd^2e\right)x^5+\left(2\,b^2d^3-3\,abd^2e+a^2de^2\right)x^3+\left(abd^3-a^2d^2e\right)x\right)\sqrt{cx^4+bx^2+a}+\left(c^2d^2x^8+2\left(4\,bcd^2-3\,acde\right)x^6-\left(8\,abde-a^2e^2-2\left(4\,b^2+ac\right)d^2\right)x}{c^2d^2x^8+2\,acdex^6+2\,a^2dex^2+\left(2\,acd^2+a^2e^2\right)x^4+a^2d^2}\right.}{4\sqrt{bd^2-ade}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(c*x^4 - a)/((c*d*x^4 + a*e*x^2 + a*d)*sqrt(c*x^4 + b*x^2 + a)),x, algo
```

```
[Out] [1/4*log(-(4*((b*c*d^3 - a*c*d^2*e)*x^5 + (2*b^2*d^3 - 3*a*b*d^2*
e + a^2*d*e^2)*x^3 + (a*b*d^3 - a^2*d^2*e)*x)*sqrt(c*x^4 + b*x^2
+ a) + (c^2*d^2*x^8 + 2*(4*b*c*d^2 - 3*a*c*d*e)*x^6 - (8*a*b*d*e
- a^2*e^2 - 2*(4*b^2 + a*c)*d^2)*x^4 + a^2*d^2 + 2*(4*a*b*d^2 - 3
*a^2*d*e)*x^2)*sqrt(b*d^2 - a*d*e))/(c^2*d^2*x^8 + 2*a*c*d*e*x^6
+ 2*a^2*d*e*x^2 + (2*a*c*d^2 + a^2*e^2)*x^4 + a^2*d^2))/sqrt(b*d^
2 - a*d*e), 1/2*arctan(2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-b*d^2 + a*
d*e)*x/(c*d*x^4 + (2*b*d - a*e)*x^2 + a*d))/sqrt(-b*d^2 + a*d*e)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*x**4+a)/(c*d*x**4+a*e*x**2+a*d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(c*x^4 - a)/((c*d*x^4 + a*e*x^2 + a*d)*sqrt(c*x^4 + b*x^2 + a)),x, algo
```

```
[Out] Exception raised: TypeError
```

3.867 $\int \frac{a-cx^4}{\sqrt{a-bx^2+cx^4}(ad+aex^2+cdx^4)}\,dx$

Optimal. Leaf size=53

$$\frac{\tan^{-1}\left(\frac{x\sqrt{ae+bd}}{\sqrt{d}\sqrt{a-bx^2+cx^4}}\right)}{\sqrt{d}\sqrt{ae+bd}}$$

```
[Out] ArcTan[(Sqrt[b*d + a*e]*x)/(Sqrt[d]*Sqrt[a - b*x^2 + c*x^4])]/(Sq
rt[d]*Sqrt[b*d + a*e])
```

Rubi [A] time = 0.418844, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045

$$\frac{\tan^{-1}\left(\frac{x\sqrt{ae+bd}}{\sqrt{d}\sqrt{a-bx^2+cx^4}}\right)}{\sqrt{d}\sqrt{ae+bd}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - c*x^4)/(Sqrt[a - b*x^2 + c*x^4]*(a*d + a*e*x^2 + c*d*x^4)),x]
```

```
[Out] ArcTan[(Sqrt[b*d + a*e]*x)/(Sqrt[d]*Sqrt[a - b*x^2 + c*x^4])]/(Sq
rt[d]*Sqrt[b*d + a*e])
```

Rubi in Sympy [A] time = 25.7817, size = 48, normalized size = 0.91

$$\frac{\operatorname{atan}\left(\frac{x\sqrt{ae+bd}}{\sqrt{d}\sqrt{a-bx^2+cx^4}}\right)}{\sqrt{d}\sqrt{ae+bd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-c*x**4+a)/(c*d*x**4+a*e*x**2+a*d)/(c*x**4-b*x**2+a)**(1/2),x)
```

```
[Out] atan(x*sqrt(a*e + b*d)/(sqrt(d)*sqrt(a - b*x**2 + c*x**4)))/(sqrt
(d)*sqrt(a*e + b*d))
```

Mathematica [C] time = 1.17526, size = 416, normalized size = 7.85

$$\frac{i\sqrt{\frac{4cx^2}{\sqrt{b^2-4ac}-b}+2}\sqrt{1-\frac{2cx^2}{\sqrt{b^2-4ac}+b}}\left(-\left(\frac{\left(b-\sqrt{b^2-4ac}\right)d}{\sqrt{a}\sqrt{ae^2-4cd^2}-ae};i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{\sqrt{b^2-4ac}-b}}x\right)|\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)-\left(\frac{\left(\sqrt{b^2-4ac}-b\right)d}{ae+\sqrt{a}\sqrt{ae^2-4cd^2}};i\sinh^{-1}\right.\right.}{2d\sqrt{\frac{c}{\sqrt{b^2-4ac}-b}}\sqrt{a-bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a - c*x^4)/(Sqrt[a - b*x^2 + c*x^4]*(a*d + a*e*x^2 + c*d*x^4)),x]
```

```
[Out] ((I/2)*Sqrt[2 + (4*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 - (2*c
*x^2)/(b + Sqrt[b^2 - 4*a*c])]*(EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[
c/(-b + Sqrt[b^2 - 4*a*c])]*x], (b - Sqrt[b^2 - 4*a*c])/(b + Sqrt
[b^2 - 4*a*c])] - EllipticPi[((b - Sqrt[b^2 - 4*a*c])*d)/(-(a*e)
+ Sqrt[a]*Sqrt[-4*c*d^2 + a*e^2]), I*ArcSinh[Sqrt[2]*Sqrt[c/(-b +
 Sqrt[b^2 - 4*a*c])]*x], (b - Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 -
4*a*c])] - EllipticPi[((-b + Sqrt[b^2 - 4*a*c])*d)/(a*e + Sqrt[a]
*Sqrt[-4*c*d^2 + a*e^2]), I*ArcSinh[Sqrt[2]*Sqrt[c/(-b + Sqrt[b^2
 - 4*a*c])]*x], (b - Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c])])
)/(Sqrt[c/(-b + Sqrt[b^2 - 4*a*c])]*d*Sqrt[a - b*x^2 + c*x^4])
```

Maple [C] time = 0.067, size = 517, normalized size = 9.8

$$-\frac{\sqrt{2}}{4d}\sqrt{4-2\frac{\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+2\frac{\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}EllipticF\left(\frac{\sqrt{2}x}{2}\sqrt{\frac{1}{a}\left(b+\sqrt{-4ac+b^2}\right)},\frac{1}{2}\sqrt{-4-2\frac{b\left(-b\right.}{}}\right.$$

$$-\frac{a}{4d}\sum_{\_alpha=RootOf\left(\_Z^4cd+\_Z^2ae+ad\right)}\frac{-\_alpha^2e-2d}{\_alpha\left(2\_alpha^2cd+ae\right)}\left(-1Artanh\left(\frac{2\_alpha^2cx^2-b\_alpha^2-bx^2+2a}{2}\frac{1}{\sqrt{-\frac{\_alpha^2(}{a}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-c*x^4+a)/(c*d*x^4+a*e*x^2+a*d)/(c*x^4-b*x^2+a)^(1/2),x)
```

```
[Out] -1/4/d*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(b+(-4*a*c+b
^2)^(1/2))/a*x^2)^(1/2)*(4+2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)
/(c*x^4-b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((b+(-4*a*c+b^2)^(
1/2))/a)^(1/2),1/2*(-4-2*b*(-b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/
4*a/d*sum((-_alpha^2*e-2*d)/_alpha/(2*_alpha^2*c*d+a*e)*(-1/(-_al
pha^2*(a*e+b*d)/d)^(1/2)*arctanh(1/2*(2*_alpha^2*c*x^2-_alpha^2*b
-b*x^2+2*a)/(-_alpha^2*(a*e+b*d)/d)^(1/2)/(c*x^4-b*x^2+a)^(1/2))+
```

```
1/a/d*2^(1/2)*_alpha*(_alpha^2*c*d+a*e)/((b+(-4*a*c+b^2)^(1/2))/a
)^(1/2)*(2-b*x^2/a-1/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)*(2-b*x^2/a+1
/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4-b*x^2+a)^(1/2)*EllipticPi
(1/2*x*2^(1/2)*((b+(-4*a*c+b^2)^(1/2))/a)^(1/2),-1/2*(-_alpha^2*(
-4*a*c+b^2)^(1/2)*c*d+_alpha^2*b*c*d-(-4*a*c+b^2)^(1/2)*a*e+a*b*e
)/a/c/d,(-1/2*(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))/a)^(1/2))),_alpha=RootOf(_Z^4*c*d+_Z^2*a*e+a*d))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{cx^4 - a}{(cdx^4 + aex^2 + ad)\sqrt{cx^4 - bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(c*x^4 - a)/((c*d*x^4 + a*e*x^2 + a*d)*sqrt(c*x^4 - b*x^2 + a)),x, algo
```

```
[Out] -integrate((c*x^4 - a)/((c*d*x^4 + a*e*x^2 + a*d)*sqrt(c*x^4 - b*
x^2 + a)), x)
```

Fricas [A] time = 12.8586, size = 1, normalized size = 0.02

$$\left[\frac{\log\left(\frac{4\,\left(\left(bcd^3+acd^2e\right)x^5-\left(2\,b^2d^3+3\,abd^2e+a^2de^2\right)x^3+\left(abd^3+a^2d^2e\right)x\right)\sqrt{cx^4-bx^2+a}-\left(c^2d^2x^8-2\,\left(4\,bcd^2+3\,acde\right)x^6+\left(8\,abde+a^2e^2+2\,\left(4\,b^2+ac\right)d^2\right)x^4+}{c^2d^2x^8+2\,acdex^6+2\,a^2dex^2+(2\,acd^2+a^2e^2)x^4+a^2d^2}\right.}{4\,\sqrt{-bd^2-ade}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(c*x^4 - a)/((c*d*x^4 + a*e*x^2 + a*d)*sqrt(c*x^4 - b*x^2 + a)),x, algo
```

```
[Out] [1/4*log((4*((b*c*d^3 + a*c*d^2*e)*x^5 - (2*b^2*d^3 + 3*a*b*d^2*e
 + a^2*d*e^2)*x^3 + (a*b*d^3 + a^2*d^2*e)*x)*sqrt(c*x^4 - b*x^2 +
 a) - (c^2*d^2*x^8 - 2*(4*b*c*d^2 + 3*a*c*d*e)*x^6 + (8*a*b*d*e +
 a^2*e^2 + 2*(4*b^2 + a*c)*d^2)*x^4 + a^2*d^2 - 2*(4*a*b*d^2 + 3*
a^2*d*e)*x^2)*sqrt(-b*d^2 - a*d*e))/(c^2*d^2*x^8 + 2*a*c*d*e*x^6
+ 2*a^2*d*e*x^2 + (2*a*c*d^2 + a^2*e^2)*x^4 + a^2*d^2))/sqrt(-b*d
^2 - a*d*e), 1/2*arctan(2*sqrt(c*x^4 - b*x^2 + a)*sqrt(b*d^2 + a*
d*e)*x/(c*d*x^4 - (2*b*d + a*e)*x^2 + a*d))/sqrt(b*d^2 + a*d*e)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*x**4+a)/(c*d*x**4+a*e*x**2+a*d)/(c*x**4-b*x**2+a)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(c*x^4 - a)/((c*d*x^4 + a*e*x^2 + a*d)*sqrt(c*x^4 - b*x^2 + a)),x, algo
```

```
[Out] Exception raised: TypeError
```

3.868 $\int \frac{1}{\sqrt{5-2x+x^2}(8+x^3)}\,dx$

Optimal. Leaf size=84

$$-\frac{\tan^{-1}\left(\frac{1-x}{\sqrt{3}\sqrt{x^2-2x+5}}\right)}{4\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{7-3x}{\sqrt{13}\sqrt{x^2-2x+5}}\right)}{12\sqrt{13}}+\frac{1}{12}\tanh^{-1}\left(\sqrt{x^2-2x+5}\right)$$

```
[Out] -ArcTan[(1 - x)/(Sqrt[3]*Sqrt[5 - 2*x + x^2])]/(4*Sqrt[3]) - ArcT
anh[(7 - 3*x)/(Sqrt[13]*Sqrt[5 - 2*x + x^2])]/(12*Sqrt[13]) + Arc
Tanh[Sqrt[5 - 2*x + x^2]]/12
```

Rubi [A] time = 0.268241, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4

$$-\frac{\tan^{-1}\left(\frac{1-x}{\sqrt{3}\sqrt{x^2-2x+5}}\right)}{4\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{7-3x}{\sqrt{13}\sqrt{x^2-2x+5}}\right)}{12\sqrt{13}}+\frac{1}{12}\tanh^{-1}\left(\sqrt{x^2-2x+5}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[5 - 2*x + x^2]*(8 + x^3)),x]
```

```
[Out] -ArcTan[(1 - x)/(Sqrt[3]*Sqrt[5 - 2*x + x^2])]/(4*Sqrt[3]) - ArcT
anh[(7 - 3*x)/(Sqrt[13]*Sqrt[5 - 2*x + x^2])]/(12*Sqrt[13]) + Arc
Tanh[Sqrt[5 - 2*x + x^2]]/12
```

Rubi in Sympy [A] time = 24.9973, size = 78, normalized size = 0.93

$$\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}(2x-2)}{6\sqrt{x^2-2x+5}}\right)}{12}-\frac{\sqrt{13}\,\mathrm{atanh}\left(\frac{\sqrt{13}(-6x+14)}{26\sqrt{x^2-2x+5}}\right)}{156}+\frac{\mathrm{atanh}\left(\sqrt{x^2-2x+5}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**3+8)/(x**2-2*x+5)**(1/2),x)
```

```
[Out] sqrt(3)*atan(sqrt(3)*(2*x - 2)/(6*sqrt(x**2 - 2*x + 5)))/12 - sqr
t(13)*atanh(sqrt(13)*(-6*x + 14)/(26*sqrt(x**2 - 2*x + 5)))/156 +
 atanh(sqrt(x**2 - 2*x + 5))/12
```

Mathematica [A] time = 0.185958, size = 154, normalized size = 1.83

$$\frac{1}{312}\left(-13\log\left(\left(x^2-2x+4\right)^2\right)+13\log\left(\left(x^2-2x+4\right)\left(x^2+2\sqrt{x^2-2x+5}-2x+6\right)\right)\right.$$
$$-2\sqrt{13}\log\left(\sqrt{13}\sqrt{x^2-2x+5}-3x+7\right)$$
$$\left.-26\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\left(x^2-\left(\sqrt{x^2-2x+5}+2\right)x+\sqrt{x^2-2x+5}+4\right)}{2x^2-4x+11}\right)+2\sqrt{13}\log(x+2)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[5 - 2*x + x^2]*(8 + x^3)),x]
```

```
[Out] (-26*Sqrt[3]*ArcTan[(Sqrt[3]*(4 + x^2 + Sqrt[5 - 2*x + x^2] - x*(
2 + Sqrt[5 - 2*x + x^2])))/(11 - 4*x + 2*x^2)] + 2*Sqrt[13]*Log[2
 + x] - 13*Log[(4 - 2*x + x^2)^2] + 13*Log[(4 - 2*x + x^2)*(6 - 2
*x + x^2 + 2*Sqrt[5 - 2*x + x^2])] - 2*Sqrt[13]*Log[7 - 3*x + Sqr
t[13]*Sqrt[5 - 2*x + x^2]])/312
```

Maple [A] time = 0.031, size = 69, normalized size = 0.8

$$-\frac{\sqrt{13}}{156}\mathit{Artanh}\left(\frac{(14-6x)\sqrt{13}}{26}\frac{1}{\sqrt{(2+x)^2-6x+1}}\right)$$
$$+\frac{1}{12}\mathit{Artanh}\left(\sqrt{x^2-2x+5}\right)+\frac{\sqrt{3}}{12}\arctan\left(\frac{\sqrt{3}(2x-2)}{6}\frac{1}{\sqrt{x^2-2x+5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^3+8)/(x^2-2*x+5)^(1/2),x)
```

```
[Out] -1/156*13^(1/2)*arctanh(1/26*(14-6*x)*13^(1/2)/((2+x)^2-6*x+1)^(1
/2))+1/12*arctanh((x^2-2*x+5)^(1/2))+1/12*3^(1/2)*arctan(1/6*3^(1
/2)/(x^2-2*x+5)^(1/2)*(2*x-2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(x^3+8)\sqrt{x^2-2x+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^3 + 8)*sqrt(x^2 - 2*x + 5)),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 8)*sqrt(x^2 - 2*x + 5)), x)
```

Fricas [A] time = 0.302154, size = 262, normalized size = 3.12

$$\frac{1}{936}\sqrt{13}\sqrt{3}\left(\sqrt{13}\sqrt{3}\log\left(x^2-\sqrt{x^2-2x+5}(x-2)-3x+6\right)-\sqrt{13}\sqrt{3}\log\left(x^2-\sqrt{x^2-2x+5}x-x+4\right)+6\sqrt{13}\arctan\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^3 + 8)*sqrt(x^2 - 2*x + 5)),x, algorithm="fricas")
```

```
[Out] 1/936*sqrt(13)*sqrt(3)*(sqrt(13)*sqrt(3)*log(x^2 - sqrt(x^2 - 2*x
 + 5)*(x - 2) - 3*x + 6) - sqrt(13)*sqrt(3)*log(x^2 - sqrt(x^2 -
2*x + 5)*x - x + 4) + 6*sqrt(13)*arctan(-1/3*sqrt(3)*(x - 2) + 1/
3*sqrt(3)*sqrt(x^2 - 2*x + 5)) - 6*sqrt(13)*arctan(-1/3*sqrt(3)*x
 + 1/3*sqrt(3)*sqrt(x^2 - 2*x + 5)) + 2*sqrt(3)*log((sqrt(13)*(x^
2 + x + 11) - sqrt(x^2 - 2*x + 5)*(sqrt(13)*(x + 2) + 13) + 13*x
+ 26)/(x^2 - sqrt(x^2 - 2*x + 5)*(x + 2) + x - 2)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x+2)\left(x^2-2x+4\right)\sqrt{x^2-2x+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**3+8)/(x**2-2*x+5)**(1/2),x)
```

```
[Out] Integral(1/((x + 2)*(x**2 - 2*x + 4)*sqrt(x**2 - 2*x + 5)), x)
```

GIAC/XCAS [A] time = 0.286713, size = 221, normalized size = 2.63

$$-\frac{1}{12}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(x-\sqrt{x^2-2x+5}\right)\right)+\frac{1}{12}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(x-\sqrt{x^2-2x+5}-2\right)\right)+\frac{1}{156}\sqrt{13}\ln\left(\frac{\left|-2x-2\sqrt{13}+2\sqrt{x^2-2x+5}-4\right|}{\left|-2x+2\sqrt{13}+2\sqrt{x^2-2x+5}-4\right|}\right)+\frac{1}{24}\ln\left(\left(x-\sqrt{x^2-2x+5}\right)^2-4x+4\sqrt{x^2-2x+5}+7\right)-\frac{1}{24}\ln\left(\left(x-\sqrt{x^2-2x+5}\right)^2+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^3 + 8)*sqrt(x^2 - 2*x + 5)),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*arctan(-1/3*sqrt(3)*(x - sqrt(x^2 - 2*x + 5))) + 1/
12*sqrt(3)*arctan(-1/3*sqrt(3)*(x - sqrt(x^2 - 2*x + 5) - 2)) + 1
/156*sqrt(13)*ln(abs(-2*x - 2*sqrt(13) + 2*sqrt(x^2 - 2*x + 5) -
4)/abs(-2*x + 2*sqrt(13) + 2*sqrt(x^2 - 2*x + 5) - 4)) + 1/24*ln(
(x - sqrt(x^2 - 2*x + 5))^2 - 4*x + 4*sqrt(x^2 - 2*x + 5) + 7) -
1/24*ln((x - sqrt(x^2 - 2*x + 5))^2 + 3)
```

3.869 $\int \sqrt{\frac{x^2}{1+x^2}}\,dx$

Optimal. Leaf size=20

$$\frac{\sqrt{x^2}\sqrt{x^2+1}}{x}$$

```
[Out] (Sqrt[x^2]*Sqrt[1 + x^2])/x
```

Rubi [A] time = 0.0108797, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{\sqrt{x^2}\sqrt{x^2+1}}{x}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x^2/(1 + x^2)],x]

[Out] (Sqrt[x^2]*Sqrt[1 + x^2])/x
```

Rubi in Sympy [A] time = 3.40483, size = 15, normalized size = 0.75

$$\frac{\sqrt{x^2+1}\sqrt{x^2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2/(x**2+1))**(1/2),x)

[Out] sqrt(x**2 + 1)*sqrt(x**2)/x
```

Mathematica [A] time = 0.00926511, size = 17, normalized size = 0.85

$$\frac{x}{\sqrt{\frac{x^2}{x^2+1}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[x^2/(1 + x^2)],x]
```

```
[Out] x/Sqrt[x^2/(1 + x^2)]
```

Maple [A] time = 0.005, size = 23, normalized size = 1.2

$$\frac{x^2+1}{x}\sqrt{\frac{x^2}{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2/(x^2+1))^(1/2),x)
```

```
[Out] (x^2+1)/x*(x^2/(x^2+1))^(1/2)
```

Maxima [A] time = 0.757597, size = 9, normalized size = 0.45

$$\sqrt{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2/(x^2 + 1)),x, algorithm="maxima")
```

```
[Out] sqrt(x^2 + 1)
```

Fricas [A] time = 0.262364, size = 30, normalized size = 1.5

$$\frac{(x^2+1)\sqrt{\frac{x^2}{x^2+1}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2/(x^2 + 1)),x, algorithm="fricas")
```

```
[Out] (x^2 + 1)*sqrt(x^2/(x^2 + 1))/x
```

Sympy [A] time = 1.8012, size = 36, normalized size = 1.8

$$x\sqrt{x^2}\sqrt{\frac{1}{x^2+1}}+\frac{\sqrt{x^2}\sqrt{\frac{1}{x^2+1}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2/(x**2+1))**(1/2),x)
```

```
[Out] x*sqrt(x**2)*sqrt(1/(x**2 + 1)) + sqrt(x**2)*sqrt(1/(x**2 + 1))/x
```

GIAC/XCAS [A] time = 0.258509, size = 20, normalized size = 1.

$$\sqrt{x^2+1}\,\mathrm{sign}\left(x\right)-\mathrm{sign}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2/(x^2 + 1)),x, algorithm="giac")
```

```
[Out] sqrt(x^2 + 1)*sign(x) - sign(x)
```

3.870 $\int \sqrt{\frac{x^n}{1+x^n}}\,dx$

Optimal. Leaf size=46

$$\frac{2x\sqrt{x^n}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(1+\frac{2}{n}\right);\frac{1}{2}\left(3+\frac{2}{n}\right);-x^n\right)}{n+2}$$

```
[Out] (2*x*Sqrt[x^n]*Hypergeometric2F1[1/2, (1 + 2/n)/2, (3 + 2/n)/2, -
x^n])/(2 + n)
```

Rubi [A] time = 0.0376636, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{2x\sqrt{x^n}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(1+\frac{2}{n}\right);\frac{1}{2}\left(3+\frac{2}{n}\right);-x^n\right)}{n+2}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x^n/(1 + x^n)],x]
```

```
[Out] (2*x*Sqrt[x^n]*Hypergeometric2F1[1/2, (1 + 2/n)/2, (3 + 2/n)/2, -
x^n])/(2 + n)
```

Rubi in Sympy [A] time = 4.21582, size = 41, normalized size = 0.89

$$\frac{2x^{-\frac{n}{2}}x^{\frac{n}{2}+1}\sqrt{x^n}\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{n+2}{2n}\\\frac{3}{2}+\frac{1}{n}\end{matrix}\middle|-x^n\right)}{n+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**n/(1+x**n))**(1/2),x)
```

```
[Out] 2*x**(-n/2)*x**(n/2 + 1)*sqrt(x**n)*hyper((1/2, (n + 2)/(2*n)), (
3/2 + 1/n,), -x**n)/(n + 2)
```

Mathematica [A] time = 0.0320127, size = 38, normalized size = 0.83

$$\frac{2x\sqrt{x^n}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}+\frac{1}{n};\frac{3}{2}+\frac{1}{n};-x^n\right)}{n+2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[x^n/(1 + x^n)],x]
```

```
[Out] (2*x*Sqrt[x^n]*Hypergeometric2F1[1/2, 1/2 + n^(-1), 3/2 + n^(-1),
 -x^n])/(2 + n)
```

Maple [F] time = 0.027, size = 0, normalized size = 0.

$$\int \sqrt{\frac{x^n}{1+x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^n/(1+x^n))^(1/2),x)
```

```
[Out] int((x^n/(1+x^n))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{x^n}{x^n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^n/(x^n + 1)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^n/(x^n + 1)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^n/(x^n + 1)),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{x^n}{x^n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**n/(1+x**n))**(1/2),x)

[Out] Integral(sqrt(x**n/(x**n + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{x^n}{x^n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^n/(x^n + 1)),x, algorithm="giac")

[Out] integrate(sqrt(x^n/(x^n + 1)), x)
```

3.871 $\int \frac{ef-efx^2}{(ad+bdx+adx^2)\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx$

Optimal. Leaf size=88

$$\frac{ef\tan^{-1}\left(\frac{x(4a^2-2ac+b^2)+abx^2+ab}{2a\sqrt{2a-c}\sqrt{ax^4+a+bx^3+bx+cx^2}}\right)}{ad\sqrt{2a-c}}$$

```
[Out] (e*f*ArcTan[(a*b + (4*a^2 + b^2 - 2*a*c)*x + a*b*x^2)/(2*a*Sqrt[2
*a - c]*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4])])/(a*Sqrt[2*a - c]
*d)
```

Rubi [A] time = 0.417129, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 52, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.019

$$\frac{ef\tan^{-1}\left(\frac{x(4a^2-2ac+b^2)+abx^2+ab}{2a\sqrt{2a-c}\sqrt{ax^4+a+bx^3+bx+cx^2}}\right)}{ad\sqrt{2a-c}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*f - e*f*x^2)/((a*d + b*d*x + a*d*x^2)*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4]),x]
```

```
[Out] (e*f*ArcTan[(a*b + (4*a^2 + b^2 - 2*a*c)*x + a*b*x^2)/(2*a*Sqrt[2
*a - c]*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4])])/(a*Sqrt[2*a - c]
*d)
```

Rubi in Sympy [A] time = 33.8519, size = 76, normalized size = 0.86

$$\frac{ef\,\mathrm{atan}\left(\frac{abx^2+ab+x(4a^2-2ac+b^2)}{2a\sqrt{2a-c}\sqrt{ax^4+a+bx^3+bx+cx^2}}\right)}{ad\sqrt{2a-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-e*f*x**2+e*f)/(a*d*x**2+b*d*x+a*d)/(a*x**4+b*x**3+c*x**2+b*x+
```

```
[Out] e*f*atan((a*b*x**2 + a*b + x*(4*a**2 - 2*a*c + b**2))/(2*a*sqrt(2
*a - c)*sqrt(a*x**4 + a + b*x**3 + b*x + c*x**2)))/(a*d*sqrt(2*a
- c))
```

Mathematica [C] time = 6.32351, size = 13884, normalized size = 157.77

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*f - e*f*x^2)/((a*d + b*d*x + a*d*x^2)*Sqrt[a + b*x + c*x^2 + b*x^3 + a
```

```
[Out] Result too large to show
```

Maple [C] time = 0.181, size = 242984, normalized size = 2761.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e*f*x^2+e*f)/(a*d*x^2+b*d*x+a*d)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{efx^2 - ef}{\sqrt{ax^4 + bx^3 + cx^2 + bx + a}(adx^2 + bdx + ad)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(e*f*x^2 - e*f)/(sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*(a*d*x^2 + b*d*x
```

```
[Out] -integrate((e*f*x^2 - e*f)/(sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)
*(a*d*x^2 + b*d*x + a*d)), x)
```

Fricas [A] time = 3.41962, size = 1, normalized size = 0.01

$$\left[\frac{ef\log\left(\frac{4\sqrt{ax^4+bx^3+cx^2+bx+a}\left(2a^3b-a^2bc+\left(2a^3b-a^2bc\right)x^2+\left(8a^4+2a^2b^2+2a^2c^2-\left(8a^3+ab^2\right)c\right)x\right)+\left(2ab^3x^3+2ab^3x-\left(8a^4-a^2b^2-4a^3c\right)x^4-8a^4+a^2b\right.}{a^2x^4+2abx^3+2abx+(2a^2+b^2)x^2+a^2}\right.}{2a\sqrt{-2a+c}d}\right.$$
$$\left.-\frac{ef\arctan\left(\frac{2\sqrt{ax^4+bx^3+cx^2+bx+a}\sqrt{2a-c}a}{abx^2+ab+(4a^2+b^2-2ac)x}\right)}{\sqrt{2a-c}ad}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(e*f*x^2 - e*f)/(sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*(a*d*x^2 + b*d*x
```

```
[Out] [1/2*e*f*log((4*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*(2*a^3*b -
a^2*b*c + (2*a^3*b - a^2*b*c)*x^2 + (8*a^4 + 2*a^2*b^2 + 2*a^2*c^
2 - (8*a^3 + a*b^2)*c)*x) + (2*a*b^3*x^3 + 2*a*b^3*x - (8*a^4 - a
^2*b^2 - 4*a^3*c)*x^4 - 8*a^4 + a^2*b^2 + 4*a^3*c + (16*a^4 + 10*
a^2*b^2 + b^4 + 8*a^2*c^2 - 4*(6*a^3 + a*b^2)*c)*x^2)*sqrt(-2*a +
 c))/(a^2*x^4 + 2*a*b*x^3 + 2*a*b*x + (2*a^2 + b^2)*x^2 + a^2))/(
a*sqrt(-2*a + c)*d), -e*f*arctan(2*sqrt(a*x^4 + b*x^3 + c*x^2 + b
*x + a)*sqrt(2*a - c)*a/(a*b*x^2 + a*b + (4*a^2 + b^2 - 2*a*c)*x)
)/(sqrt(2*a - c)*a*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{ef\left(\int\frac{x^2}{ax^2\sqrt{ax^4+a+bx^3+bx+cx^2}+a\sqrt{ax^4+a+bx^3+bx+cx^2}+bx\sqrt{ax^4+a+bx^3+bx+cx^2}}\,dx+\int\left(-\frac{1}{ax^2\sqrt{ax^4+a+bx^3+bx+cx^2}+a\sqrt{ax^4+a+bx^3+bx+cx^2}+b\right.\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*f*x**2+e*f)/(a*d*x**2+b*d*x+a*d)/(a*x**4+b*x**3+c*x**2+b*x+a)**(
```

```
[Out] -e*f*(Integral(x**2/(a*x**2*sqrt(a*x**4 + a + b*x**3 + b*x + c*x*
*2) + a*sqrt(a*x**4 + a + b*x**3 + b*x + c*x**2) + b*x*sqrt(a*x**
4 + a + b*x**3 + b*x + c*x**2)), x) + Integral(-1/(a*x**2*sqrt(a*
x**4 + a + b*x**3 + b*x + c*x**2) + a*sqrt(a*x**4 + a + b*x**3 +
b*x + c*x**2) + b*x*sqrt(a*x**4 + a + b*x**3 + b*x + c*x**2)), x)
)/d
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{efx^2 - ef}{\sqrt{ax^4 + bx^3 + cx^2 + bx + a}(adx^2 + bdx + ad)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(e*f*x^2 - e*f)/(sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*(a*d*x^2 + b*d*x
```

```
[Out] integrate(-(e*f*x^2 - e*f)/(sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)
*(a*d*x^2 + b*d*x + a*d)), x)
```

3.872 $\int \frac{ef-efx^2}{(-ad+bdx-adx^2)\sqrt{-a+bx+cx^2+bx^3-ax^4}}\,dx$

Optimal. Leaf size=88

$$\frac{ef\tanh^{-1}\left(\frac{-x(4a^2+2ac+b^2)+abx^2+ab}{2a\sqrt{2a+c}\sqrt{-ax^4-a+bx^3+bx+cx^2}}\right)}{ad\sqrt{2a+c}}$$

```
[Out] (e*f*ArcTanh[(a*b - (4*a^2 + b^2 + 2*a*c)*x + a*b*x^2)/(2*a*Sqrt[
2*a + c]*Sqrt[-a + b*x + c*x^2 + b*x^3 - a*x^4])])/(a*Sqrt[2*a +
c]*d)
```

Rubi [A] time = 0.555088, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 57, $\frac{\text{number of rules}}{\text{integrand size}} = 0.018$

$$\frac{ef\tanh^{-1}\left(\frac{-x(4a^2+2ac+b^2)+abx^2+ab}{2a\sqrt{2a+c}\sqrt{-ax^4-a+bx^3+bx+cx^2}}\right)}{ad\sqrt{2a+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*f - e*f*x^2)/((-(a*d) + b*d*x - a*d*x^2)*Sqrt[-a + b*x + c*x^2 + b*x^3 - a*x
```

```
[Out] (e*f*ArcTanh[(a*b - (4*a^2 + b^2 + 2*a*c)*x + a*b*x^2)/(2*a*Sqrt[
2*a + c]*Sqrt[-a + b*x + c*x^2 + b*x^3 - a*x^4])])/(a*Sqrt[2*a +
c]*d)
```

Rubi in Sympy [A] time = 49.3683, size = 78, normalized size = 0.89

$$-\frac{ef\,\mathrm{atanh}\left(\frac{-abx^2-ab+x(4a^2+2ac+b^2)}{2a\sqrt{2a+c}\sqrt{-ax^4-a+bx^3+bx+cx^2}}\right)}{ad\sqrt{2a+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-e*f*x**2+e*f)/(-a*d*x**2+b*d*x-a*d)/(-a*x**4+b*x**3+c*x**2+b*
a)**(1/2),x)
```

```
[Out] -e*f*atanh((-a*b*x**2 - a*b + x*(4*a**2 + 2*a*c + b**2))/(2*a*sqr
t(2*a + c)*sqrt(-a*x**4 - a + b*x**3 + b*x + c*x**2)))/(a*d*sqrt(
2*a + c))
```

Mathematica [C] time = 6.33964, size = 15147, normalized size = 172.12

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*f - e*f*x^2)/((-(a*d) + b*d*x - a*d*x^2)*Sqrt[-a + b*x + c*x^2 + b*x^
```

```
[Out] Result too large to show
```

Maple [C] time = 0.174, size = 269221, normalized size = 3059.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e*f*x^2+e*f)/(-a*d*x^2+b*d*x-a*d)/(-a*x^4+b*x^3+c*x^2+b*x-
a)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{efx^2 - ef}{\sqrt{-ax^4 + bx^3 + cx^2 + bx - a}(adx^2 - bdx + ad)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x^2 - e*f)/(sqrt(-a*x^4 + b*x^3 + c*x^2 + b*x - a)*(a*d*x^2 - b*d*x
```

```
[Out] integrate((e*f*x^2 - e*f)/(sqrt(-a*x^4 + b*x^3 + c*x^2 + b*x - a)
*(a*d*x^2 - b*d*x + a*d)), x)
```

Fricas [A] time = 3.41688, size = 1, normalized size = 0.01

$$\left[\frac{ef\log\left(-\frac{4\sqrt{-ax^4+bx^3+cx^2+bx-a}\left(2a^3b+a^2bc+\left(2a^3b+a^2bc\right)x^2-\left(8a^4+2a^2b^2+2a^2c^2+\left(8a^3+ab^2\right)c\right)x\right)-\left(2ab^3x^3+2ab^3x+\left(8a^4-a^2b^2+4a^3c\right)x^4+8a^4-\cdots\right.}{a^2x^4-2abx^3-2abx+(2a^2+b^2)x^2+a^2}\right.}{2\sqrt{2a+c}ad}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x^2 - e*f)/(sqrt(-a*x^4 + b*x^3 + c*x^2 + b*x - a)*(a*d*x^2 - b*d*x
```

```
[Out] [1/2*e*f*log(-(4*sqrt(-a*x^4 + b*x^3 + c*x^2 + b*x - a)*(2*a^3*b
+ a^2*b*c + (2*a^3*b + a^2*b*c)*x^2 - (8*a^4 + 2*a^2*b^2 + 2*a^2*
c^2 + (8*a^3 + a*b^2)*c)*x) - (2*a*b^3*x^3 + 2*a*b^3*x + (8*a^4 -
 a^2*b^2 + 4*a^3*c)*x^4 + 8*a^4 - a^2*b^2 + 4*a^3*c - (16*a^4 + 1
0*a^2*b^2 + b^4 + 8*a^2*c^2 + 4*(6*a^3 + a*b^2)*c)*x^2)*sqrt(2*a
+ c))/(a^2*x^4 - 2*a*b*x^3 - 2*a*b*x + (2*a^2 + b^2)*x^2 + a^2))/
(sqrt(2*a + c)*a*d), e*f*arctan(2*sqrt(-a*x^4 + b*x^3 + c*x^2 + b
*x - a)*a*sqrt(-2*a - c)/(a*b*x^2 + a*b - (4*a^2 + b^2 + 2*a*c)*x
))/(a*sqrt(-2*a - c)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{ef\left(\int \frac{x^2}{ax^2\sqrt{-ax^4-a+bx^3+bx+cx^2}+a\sqrt{-ax^4-a+bx^3+bx+cx^2}-bx\sqrt{-ax^4-a+bx^3+bx+cx^2}}\,dx+\int\left(-\frac{1}{ax^2\sqrt{-ax^4-a+bx^3+bx+cx^2}+a\sqrt{-ax^4-a+bx^3+bx+}\right.\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*f*x**2+e*f)/(-a*d*x**2+b*d*x-a*d)/(-a*x**4+b*x**3+c*x**2+b*x-
a)**(1/2),x)
```

```
[Out] e*f*(Integral(x**2/(a*x**2*sqrt(-a*x**4 - a + b*x**3 + b*x + c*x*
*2) + a*sqrt(-a*x**4 - a + b*x**3 + b*x + c*x**2) - b*x*sqrt(-a*x
**4 - a + b*x**3 + b*x + c*x**2)), x) + Integral(-1/(a*x**2*sqrt(
-a*x**4 - a + b*x**3 + b*x + c*x**2) + a*sqrt(-a*x**4 - a + b*x**
3 + b*x + c*x**2) - b*x*sqrt(-a*x**4 - a + b*x**3 + b*x + c*x**2)
), x))/d
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{efx^2 - ef}{\sqrt{-ax^4 + bx^3 + cx^2 + bx - a}(adx^2 - bdx + ad)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x^2 - e*f)/(sqrt(-a*x^4 + b*x^3 + c*x^2 + b*x - a)*(a*d*x^2 - b*d*x
```

```
[Out] integrate((e*f*x^2 - e*f)/(sqrt(-a*x^4 + b*x^3 + c*x^2 + b*x - a)
*(a*d*x^2 - b*d*x + a*d)), x)
```

3.873 $\int \frac{\sqrt{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}{x\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx$

Optimal. Leaf size=46

$$\frac{\sqrt{2}b\sinh^{-1}\left(\frac{b\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] (Sqrt[2]*b*ArcSinh[(a*x + b*Sqrt[-(a/b^2) + (a^2*x^2)/b^2])/Sqrt[
a]])/Sqrt[a]
```

Rubi [A] time = 0.970811, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 59, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034

$$\frac{\sqrt{2}b\sinh^{-1}\left(\frac{b\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2]]/(x*Sqrt[-
(a/b^2) + (a^2*x^2)/b^2]),x]
```

```
[Out] (Sqrt[2]*b*ArcSinh[(a*x + b*Sqrt[-(a/b^2) + (a^2*x^2)/b^2])/Sqrt[
a]])/Sqrt[a]
```

Rubi in Sympy [A] time = 17.8563, size = 41, normalized size = 0.89

$$\frac{\sqrt{2}b\,\text{asinh}\left(\frac{ax+b\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((a*x**2+b*x*(-a/b**2+a**2*x**2/b**2)**(1/2))**(1/2)/x/(-
a/b**2+a**2*x**2/b**2)**(1/2),x)
```

```
[Out] sqrt(2)*b*asinh((a*x + b*sqrt(a**2*x**2/b**2 - a/b**2))/sqrt(a))/
sqrt(a)
```

Mathematica [B] time = 1.1849, size = 199, normalized size = 4.33

$$-\frac{x\sqrt{ax\left(b\sqrt{\frac{a(ax^2-1)}{b^2}}+ax\right)}\left(bx\sqrt{\frac{a(ax^2-1)}{b^2}}+ax^2-1\right)\left(\log\left(1-\frac{\sqrt{ax\left(b\sqrt{\frac{a\left(ax^2-1\right)}{b^2}}+ax\right)}}{\sqrt{2}ax}\right)-\log\left(\frac{\sqrt{ax\left(b\sqrt{\frac{a\left(ax^2-1\right)}{b^2}}+ax\right)}}{\sqrt{2}ax}+1\right)\right)}{\sqrt{2}\sqrt{\frac{a(ax^2-1)}{b^2}}\left(x\left(b\sqrt{\frac{a(ax^2-1)}{b^2}}+ax\right)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2]]/(x*Sqrt[-
(a/b^2) + (a^2*x^2)/b^2]),x]
```

```
[Out] -((x*Sqrt[a*x*(a*x + b*Sqrt[(a*(-1 + a*x^2))/b^2])]*(-1 + a*x^2 +
 b*x*Sqrt[(a*(-1 + a*x^2))/b^2])*(Log[1 - Sqrt[a*x*(a*x + b*Sqrt[
(a*(-1 + a*x^2))/b^2])]/(Sqrt[2]*a*x)] - Log[1 + Sqrt[a*x*(a*x +
b*Sqrt[(a*(-1 + a*x^2))/b^2])]/(Sqrt[2]*a*x)]))/(Sqrt[2]*Sqrt[(a*
(-1 + a*x^2))/b^2]*(x*(a*x + b*Sqrt[(a*(-1 + a*x^2))/b^2]))^(3/2)
))
```

Maple [F] time = 0.05, size = 0, normalized size = 0.

$$\int \frac{1}{x}\sqrt{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\frac{1}{\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2))^(1/2)/x/(-a/b^2+a^2*x^2/b^2)^(1/2),
```

```
[Out] int((a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2))^(1/2)/x/(-a/b^2+a^2*x^
2/b^2)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax^2 + \sqrt{\frac{a^2x^2}{b^2} - \frac{a}{b^2}}bx}}{\sqrt{\frac{a^2x^2}{b^2} - \frac{a}{b^2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(a*x^2 + sqrt(a^2*x^2/b^2 - a/b^2)*b*x)/(sqrt(a^2*x^2/b^2 - a/b^2)
```

```
[Out] integrate(sqrt(a*x^2 + sqrt(a^2*x^2/b^2 - a/b^2)*b*x)/(sqrt(a^2*x
^2/b^2 - a/b^2)*x), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(a*x^2 + sqrt(a^2*x^2/b^2 - a/b^2)*b*x)/(sqrt(a^2*x^2/b^2 - a/b^2)
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x\left(ax + b\sqrt{\frac{a^2x^2}{b^2} - \frac{a}{b^2}}\right)}}{x\sqrt{\frac{a(ax^2-1)}{b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a*x**2+b*x*(-a/b**2+a**2*x**2/b**2)**(1/2))**(1/2)/x/(-
a/b**2+a**2*x**2/b**2)**(1/2),x)
```

```
[Out] Integral(sqrt(x*(a*x + b*sqrt(a**2*x**2/b**2 - a/b**2)))/(x*sqrt(
a*(a*x**2 - 1)/b**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax^2 + \sqrt{\frac{a^2x^2}{b^2} - \frac{a}{b^2}}bx}}{\sqrt{\frac{a^2x^2}{b^2} - \frac{a}{b^2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(a*x^2 + sqrt(a^2*x^2/b^2 - a/b^2)*b*x)/(sqrt(a^2*x^2/b^2 - a/b^2)
```

```
[Out] integrate(sqrt(a*x^2 + sqrt(a^2*x^2/b^2 - a/b^2)*b*x)/(sqrt(a^2*x
^2/b^2 - a/b^2)*x), x)
```

3.874 $\int \frac{\sqrt{-ax^2+bx\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}{x\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx$

Optimal. Leaf size=46

$$\frac{\sqrt{2}b\sin^{-1}\left(\frac{ax-b\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] (Sqrt[2]*b*ArcSin[(a*x - b*Sqrt[a/b^2 + (a^2*x^2)/b^2])/Sqrt[a]])
/Sqrt[a]
```

Rubi [A] time = 0.973359, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 58, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034

$$\frac{\sqrt{2}b\sin^{-1}\left(\frac{ax-b\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-(a*x^2) + b*x*Sqrt[a/b^2 + (a^2*x^2)/b^2]]/(x*Sqrt[a/b^2 + (a^2*x^2)/b
```

```
[Out] (Sqrt[2]*b*ArcSin[(a*x - b*Sqrt[a/b^2 + (a^2*x^2)/b^2])/Sqrt[a]])
/Sqrt[a]
```

Rubi in Sympy [A] time = 17.5592, size = 42, normalized size = 0.91

$$-\frac{\sqrt{2}b\,\text{asin}\left(\frac{-ax+b\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-a*x**2+b*x*(a/b**2+a**2*x**2/b**2)**(1/2))**(1/2)/x/(a/b**2+a
```

```
[Out] -sqrt(2)*b*asin((-a*x + b*sqrt(a**2*x**2/b**2 + a/b**2))/sqrt(a))
/sqrt(a)
```

Mathematica [B] time = 1.12716, size = 213, normalized size = 4.63

$$\frac{b^2\sqrt{\frac{a(ax^2+1)}{b^2}}\sqrt{ax\left(ax-b\sqrt{\frac{a(ax^2+1)}{b^2}}\right)}\sqrt{x\left(b\sqrt{\frac{a(ax^2+1)}{b^2}}-ax\right)}\left(\log\left(1-\frac{\sqrt{ax\left(ax-b\sqrt{\frac{a(ax^2+1)}{b^2}}\right)}}{\sqrt{2}ax}\right)-\log\left(\frac{\sqrt{ax\left(ax-b\sqrt{\frac{a(ax^2+1)}{b^2}}\right)}}{\sqrt{2}ax}+1\right.\right.}{\sqrt{2}a^2x\left(bx\sqrt{\frac{a(ax^2+1)}{b^2}}-ax^2-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-(a*x^2) + b*x*Sqrt[a/b^2 + (a^2*x^2)/b^2]]/(x*Sqrt[a/b^2 + (a^2*
```

```
[Out] (b^2*Sqrt[(a*(1 + a*x^2))/b^2]*Sqrt[a*x*(a*x - b*Sqrt[(a*(1 + a*x
^2))/b^2])]*Sqrt[x*(-(a*x) + b*Sqrt[(a*(1 + a*x^2))/b^2])]*(Log[1
 - Sqrt[a*x*(a*x - b*Sqrt[(a*(1 + a*x^2))/b^2])]/(Sqrt[2]*a*x)] -
 Log[1 + Sqrt[a*x*(a*x - b*Sqrt[(a*(1 + a*x^2))/b^2])]/(Sqrt[2]*a
*x)]))/(Sqrt[2]*a^2*x*(-1 - a*x^2 + b*x*Sqrt[(a*(1 + a*x^2))/b^2]
))
```

Maple [F] time = 0.048, size = 0, normalized size = 0.

$$\int \frac{1}{x}\sqrt{-ax^2+bx\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\frac{1}{\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*x^2+b*x*(a/b^2+a^2*x^2/b^2)^(1/2))^(1/2)/x/(a/b^2+a^2*x^2/b^2)^(1/2),x
```

```
[Out] int((-a*x^2+b*x*(a/b^2+a^2*x^2/b^2)^(1/2))^(1/2)/x/(a/b^2+a^2*x^2
/b^2)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-ax^2+\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}bx}}{\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-a*x^2 + sqrt(a^2*x^2/b^2 + a/b^2)*b*x)/(sqrt(a^2*x^2/b^2 + a/b^2
```

```
[Out] integrate(sqrt(-a*x^2 + sqrt(a^2*x^2/b^2 + a/b^2)*b*x)/(sqrt(a^2*
x^2/b^2 + a/b^2)*x), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-a*x^2 + sqrt(a^2*x^2/b^2 + a/b^2)*b*x)/(sqrt(a^2*x^2/b^2 + a/b^2
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x\left(ax - b\sqrt{\frac{a^2x^2}{b^2} + \frac{a}{b^2}}\right)}}{x\sqrt{\frac{a(ax^2+1)}{b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x**2+b*x*(a/b**2+a**2*x**2/b**2)**(1/2))**(1/2)/x/(a/b**2+a**2*x
```

```
[Out] Integral(sqrt(-x*(a*x - b*sqrt(a**2*x**2/b**2 + a/b**2)))/(x*sqrt
(a*(a*x**2 + 1)/b**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-ax^2 + \sqrt{\frac{a^2x^2}{b^2} + \frac{a}{b^2}}bx}}{\sqrt{\frac{a^2x^2}{b^2} + \frac{a}{b^2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-a*x^2 + sqrt(a^2*x^2/b^2 + a/b^2)*b*x)/(sqrt(a^2*x^2/b^2 + a/b^2
```

```
[Out] integrate(sqrt(-a*x^2 + sqrt(a^2*x^2/b^2 + a/b^2)*b*x)/(sqrt(a^2*
x^2/b^2 + a/b^2)*x), x)
```

3.875 $\int \frac{\sqrt{x\left(ax+b\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}}{x\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx$

Optimal. Leaf size=46

$$\frac{\sqrt{2}b\sinh^{-1}\left(\frac{b\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] (Sqrt[2]*b*ArcSinh[(a*x + b*Sqrt[-(a/b^2) + (a^2*x^2)/b^2])/Sqrt[
a]])/Sqrt[a]
```

Rubi [A] time = 1.83898, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 58, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.052

$$\frac{\sqrt{2}b\sinh^{-1}\left(\frac{b\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x*(a*x + b*Sqrt[-(a/b^2) + (a^2*x^2)/b^2])]/(x*Sqrt[-
(a/b^2) + (a^2*x^2)/b^2]),x]
```

```
[Out] (Sqrt[2]*b*ArcSinh[(a*x + b*Sqrt[-(a/b^2) + (a^2*x^2)/b^2])/Sqrt[
a]])/Sqrt[a]
```

Rubi in Sympy [A] time = 25.9554, size = 41, normalized size = 0.89

$$\frac{\sqrt{2}b\operatorname{asinh}\left(\frac{ax+b\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((x*(a*x+(-a/b**2+a**2*x**2/b**2)**(1/2)*b))**(1/2)/x/(-
a/b**2+a**2*x**2/b**2)**(1/2),x)
```

```
[Out] sqrt(2)*b*asinh((a*x + b*sqrt(a**2*x**2/b**2 - a/b**2))/sqrt(a))/
sqrt(a)
```

Mathematica [B] time = 0.698278, size = 199, normalized size = 4.33

$$-\frac{x\sqrt{ax\left(b\sqrt{\frac{a(ax^2-1)}{b^2}}+ax\right)}\left(bx\sqrt{\frac{a(ax^2-1)}{b^2}}+ax^2-1\right)\left(\log\left(1-\frac{\sqrt{ax\left(b\sqrt{\frac{a(ax^2-1)}{b^2}}+ax\right)}}{\sqrt{2}ax}\right)-\log\left(\frac{\sqrt{ax\left(b\sqrt{\frac{a(ax^2-1)}{b^2}}+ax\right)}}{\sqrt{2}ax}+1\right)\right)}{\sqrt{2}\sqrt{\frac{a(ax^2-1)}{b^2}}\left(x\left(b\sqrt{\frac{a(ax^2-1)}{b^2}}+ax\right)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x*(a*x + b*Sqrt[-(a/b^2) + (a^2*x^2)/b^2])]/(x*Sqrt[-
(a/b^2) + (a^2*x^2)/b^2]),x]
```

```
[Out] -((x*Sqrt[a*x*(a*x + b*Sqrt[(a*(-1 + a*x^2))/b^2])]*(-1 + a*x^2 +
 b*x*Sqrt[(a*(-1 + a*x^2))/b^2])*(Log[1 - Sqrt[a*x*(a*x + b*Sqrt[
(a*(-1 + a*x^2))/b^2])]/(Sqrt[2]*a*x)] - Log[1 + Sqrt[a*x*(a*x +
b*Sqrt[(a*(-1 + a*x^2))/b^2])]/(Sqrt[2]*a*x)]))/(Sqrt[2]*Sqrt[(a*
(-1 + a*x^2))/b^2]*(x*(a*x + b*Sqrt[(a*(-1 + a*x^2))/b^2]))^(3/2)
))
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int \frac{1}{x}\sqrt{x\left(ax+b\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}\frac{1}{\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*x+b*(-a/b^2+a^2*x^2/b^2)^(1/2)))^(1/2)/x/(-a/b^2+a^2*x^2/b^2)^(1/2),
```

```
[Out] int((x*(a*x+b*(-a/b^2+a^2*x^2/b^2)^(1/2)))^(1/2)/x/(-a/b^2+a^2*x^
2/b^2)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\left(ax+\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}\,b\right)x}}{\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt((a*x + sqrt(a^2*x^2/b^2 - a/b^2)*b)*x)/(sqrt(a^2*x^2/b^2 - a/b^2)
```

```
[Out] integrate(sqrt((a*x + sqrt(a^2*x^2/b^2 - a/b^2)*b)*x)/(sqrt(a^2*x
^2/b^2 - a/b^2)*x), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt((a*x + sqrt(a^2*x^2/b^2 - a/b^2)*b)*x)/(sqrt(a^2*x^2/b^2 - a/b^2)
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(a*x+(-a/b**2+a**2*x**2/b**2)**(1/2)*b))**(1/2)/x/(-
a/b**2+a**2*x**2/b**2)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\left(ax + \sqrt{\frac{a^2x^2}{b^2} - \frac{a}{b^2}}b\right)x}}{\sqrt{\frac{a^2x^2}{b^2} - \frac{a}{b^2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt((a*x + sqrt(a^2*x^2/b^2 - a/b^2)*b)*x)/(sqrt(a^2*x^2/b^2 - a/b^2)
```

```
[Out] integrate(sqrt((a*x + sqrt(a^2*x^2/b^2 - a/b^2)*b)*x)/(sqrt(a^2*x
^2/b^2 - a/b^2)*x), x)
```

3.876 $\int \frac{\sqrt{x\left(-ax+b\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}}{x\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx$

Optimal. Leaf size=46

$$\frac{\sqrt{2}b\sin^{-1}\left(\frac{ax-b\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] (Sqrt[2]*b*ArcSin[(a*x - b*Sqrt[a/b^2 + (a^2*x^2)/b^2])/Sqrt[a]])
/Sqrt[a]
```

Rubi [A] time = 1.84009, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 57, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053

$$\frac{\sqrt{2}b\sin^{-1}\left(\frac{ax-b\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x*(-(a*x) + b*Sqrt[a/b^2 + (a^2*x^2)/b^2])]/(x*Sqrt[a/b^2 + (a^2*x^2)/b
```

```
[Out] (Sqrt[2]*b*ArcSin[(a*x - b*Sqrt[a/b^2 + (a^2*x^2)/b^2])/Sqrt[a]])
/Sqrt[a]
```

Rubi in Sympy [A] time = 26.3129, size = 42, normalized size = 0.91

$$-\frac{\sqrt{2}b\,\mathrm{asin}\left(\frac{-ax+b\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((x*(-a*x+(a/b**2+a**2*x**2/b**2)**(1/2)*b))**(1/2)/x/(a/b**2+a*
```

```
[Out] -sqrt(2)*b*asin((-a*x + b*sqrt(a**2*x**2/b**2 + a/b**2))/sqrt(a))
/sqrt(a)
```

Mathematica [B] time = 0.320454, size = 213, normalized size = 4.63

$$\frac{b^2\sqrt{\frac{a(ax^2+1)}{b^2}}\sqrt{ax\left(ax-b\sqrt{\frac{a(ax^2+1)}{b^2}}\right)}\sqrt{x\left(b\sqrt{\frac{a(ax^2+1)}{b^2}}-ax\right)}\left(\log\left(1-\frac{\sqrt{ax\left(ax-b\sqrt{\frac{a(ax^2+1)}{b^2}}\right)}}{\sqrt{2}ax}\right)-\log\left(\frac{\sqrt{ax\left(ax-b\sqrt{\frac{a(ax^2+1)}{b^2}}\right)}}{\sqrt{2}ax}+1\right)\right)}{\sqrt{2}a^2x\left(bx\sqrt{\frac{a(ax^2+1)}{b^2}}-ax^2-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x*(-(a*x) + b*Sqrt[a/b^2 + (a^2*x^2)/b^2])]/(x*Sqrt[a/b^2 + (a^2*
```

```
[Out] (b^2*Sqrt[(a*(1 + a*x^2))/b^2]*Sqrt[a*x*(a*x - b*Sqrt[(a*(1 + a*x
^2))/b^2])]*Sqrt[x*(-(a*x) + b*Sqrt[(a*(1 + a*x^2))/b^2])]*(Log[1
 - Sqrt[a*x*(a*x - b*Sqrt[(a*(1 + a*x^2))/b^2])]/(Sqrt[2]*a*x)] -
 Log[1 + Sqrt[a*x*(a*x - b*Sqrt[(a*(1 + a*x^2))/b^2])]/(Sqrt[2]*a
*x)]))/(Sqrt[2]*a^2*x*(-1 - a*x^2 + b*x*Sqrt[(a*(1 + a*x^2))/b^2]
))
```

Maple [F] time = 0.035, size = 0, normalized size = 0.

$$\int \frac{1}{x}\sqrt{x\left(-ax+b\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}\frac{1}{\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(-a*x+b*(a/b^2+a^2*x^2/b^2)^(1/2)))^(1/2)/x/(a/b^2+a^2*x^2/b^2)^(1/2),x
```

```
[Out] int((x*(-a*x+b*(a/b^2+a^2*x^2/b^2)^(1/2)))^(1/2)/x/(a/b^2+a^2*x^2
/b^2)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-\left(ax-\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}b\right)x}}{\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-(a*x - sqrt(a^2*x^2/b^2 + a/b^2)*b)*x)/(sqrt(a^2*x^2/b^2 + a/b^2
```

```
[Out] integrate(sqrt(-(a*x - sqrt(a^2*x^2/b^2 + a/b^2)*b)*x)/(sqrt(a^2*
x^2/b^2 + a/b^2)*x), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-(a*x - sqrt(a^2*x^2/b^2 + a/b^2)*b)*x)/(sqrt(a^2*x^2/b^2 + a/b^2
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(-a*x+(a/b**2+a**2*x**2/b**2)**(1/2)*b))**(1/2)/x/(a/b**2+a**2*x*
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-\left(ax - \sqrt{\frac{a^2x^2}{b^2} + \frac{a}{b^2}}b\right)x}}{\sqrt{\frac{a^2x^2}{b^2} + \frac{a}{b^2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-(a*x - sqrt(a^2*x^2/b^2 + a/b^2)*b)*x)/(sqrt(a^2*x^2/b^2 + a/b^2
```

```
[Out] integrate(sqrt(-(a*x - sqrt(a^2*x^2/b^2 + a/b^2)*b)*x)/(sqrt(a^2*
x^2/b^2 + a/b^2)*x), x)
```

3.877 $\int \frac{-\sqrt{-4+x}-4\sqrt{-1+x}+\sqrt{-4+x}x+\sqrt{-1+x}x}{\left(1+\sqrt{-4+x}+\sqrt{-1+x}\right)(4-5x+x^2)}\,dx$

Optimal. Leaf size=19

$$2\log\left(\sqrt{x-4}+\sqrt{x-1}+1\right)$$

```
[Out] 2*Log[1 + Sqrt[-4 + x] + Sqrt[-1 + x]]
```

Rubi [A] time = 0.817248, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 66, $\frac{\text{number of rules}}{\text{integrand size}} = 0.03$

$$2\log\left(\sqrt{x-4}+\sqrt{x-1}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-Sqrt[-4 + x] - 4*Sqrt[-1 + x] + Sqrt[-4 + x]*x + Sqrt[-
1 + x]*x)/((1 + Sqrt[-4 + x] + Sqrt[-1 + x])*(4 - 5*x + x^2)),x]
```

```
[Out] 2*Log[1 + Sqrt[-4 + x] + Sqrt[-1 + x]]
```

Rubi in Sympy [A] time = 17.864, size = 17, normalized size = 0.89

$$2\log\left(\sqrt{x-4}+\sqrt{x-1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-(-4+x)**(1/2)+x*(-4+x)**(1/2)-4*(-1+x)**(1/2)+x*(-
1+x)**(1/2))/(x**2-5*x+4)/(1+(-4+x)**(1/2)+(-1+x)**(1/2)),x)
```

```
[Out] 2*log(sqrt(x - 4) + sqrt(x - 1) + 1)
```

Mathematica [B] time = 0.0747189, size = 75, normalized size = 3.95

$$\frac{1}{2}\log\left(-5x-4\sqrt{x-4}\sqrt{x-1}+17\right)+\frac{1}{2}\log\left(-2x-2\sqrt{x-4}\sqrt{x-1}+5\right)\\ -\tanh^{-1}\left(\sqrt{x-4}\right)+\tanh^{-1}\left(\frac{\sqrt{x-1}}{2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-Sqrt[-4 + x] - 4*Sqrt[-1 + x] + Sqrt[-4 + x]*x + Sqrt[-
1 + x]*x)/((1 + Sqrt[-4 + x] + Sqrt[-1 + x])*(4 - 5*x + x^2)),x]
```

```
[Out] -ArcTanh[Sqrt[-4 + x]] + ArcTanh[Sqrt[-1 + x]/2] + Log[17 - 4*Sqr
t[-4 + x]*Sqrt[-1 + x] - 5*x]/2 + Log[5 - 2*Sqrt[-4 + x]*Sqrt[-1
+ x] - 2*x]/2
```

Maple [B] time = 0.074, size = 147, normalized size = 7.7

$$\frac{\ln(-5+x)}{2}+\frac{1}{2}\ln\left(-1+\sqrt{x-4}\right)-\frac{1}{2}\ln\left(1+\sqrt{x-4}\right)+\frac{1}{2}\ln\left(\sqrt{-1+x}+2\right)-\frac{1}{2}\ln\left(\sqrt{-1+x}-2\right)+\frac{7}{4}\sqrt{x-4}\sqrt{-1+x}Artanh\left(\frac{5x-17}{4}\frac{1}{\sqrt{x^2-5x+4}}\right)\frac{1}{\sqrt{x^2-5x+4}}+\frac{1}{4}\sqrt{x-4}\sqrt{-1+x}\left(2\ln\left(-5/2+x+\sqrt{x^2-5x+4}\right)-5\,Artanh\left(1/4\,\frac{5x-17}{\sqrt{x^2-5x+4}}\right)\right)\frac{1}{\sqrt{x^2-5x+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-(x-4)^(1/2)+x*(x-4)^(1/2)-4*(-1+x)^(1/2)+x*(-1+x)^(1/2))/(x^2-
5*x+4)/(1+(x-4)^(1/2)+(-1+x)^(1/2)),x)
```

```
[Out] 1/2*ln(-5+x)+1/2*ln(-1+(x-4)^(1/2))-1/2*ln(1+(x-4)^(1/2))+1/2*ln(
(-1+x)^(1/2)+2)-1/2*ln((-1+x)^(1/2)-2)+7/4*(x-4)^(1/2)*(-1+x)^(1/
2)/(x^2-5*x+4)^(1/2)*arctanh(1/4*(5*x-17)/(x^2-5*x+4)^(1/2))+1/4*
(x-4)^(1/2)*(-1+x)^(1/2)*(2*ln(-5/2+x+(x^2-5*x+4)^(1/2))-5*arctan
h(1/4*(5*x-17)/(x^2-5*x+4)^(1/2)))/(x^2-5*x+4)^(1/2)
```

Maxima [A] time = 0.771392, size = 127, normalized size = 6.68

$$\frac{1}{2}\log(x-1)+\frac{1}{2}\log\left(\frac{2x^2+2\left((x-1)\sqrt{x-4}+2x-6\right)\sqrt{x-1}+2(2x-3)\sqrt{x-4}-7x+3}{2\left((x-1)\sqrt{x-4}+2x-6\right)}\right)+\frac{1}{2}\log\left(\frac{(x-1)\sqrt{x-4}+2x-6}{x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(x - 1)*x + sqrt(x - 4)*x - 4*sqrt(x - 1) - sqrt(x - 4))/((x^2 - 5*x
```

```
[Out] 1/2*log(x - 1) + 1/2*log(1/2*(2*x^2 + 2*((x - 1)*sqrt(x - 4) + 2*
x - 6)*sqrt(x - 1) + 2*(2*x - 3)*sqrt(x - 4) - 7*x + 3)/((x - 1)*
sqrt(x - 4) + 2*x - 6)) + 1/2*log(((x - 1)*sqrt(x - 4) + 2*x - 6)
/(x - 1))
```

Fricas [A] time = 0.308698, size = 130, normalized size = 6.84

$$-\frac{1}{2}\log\left(-(4x-11)\sqrt{x-1}\sqrt{x-4}+4x^2-21x+23\right)+\frac{1}{2}\log\left(\sqrt{x-1}\sqrt{x-4}-x+7\right)+\frac{1}{2}\log(x-5)+\frac{1}{2}\log\left(\sqrt{x-1}+2\right)-\frac{1}{2}\log\left(\sqrt{x-1}-2\right)-\frac{1}{2}\log\left(\sqrt{x-4}+1\right)+\frac{1}{2}\log\left(\sqrt{x-4}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sqrt(x - 1)*x + sqrt(x - 4)*x - 4*sqrt(x - 1) - sqrt(x - 4))/((x^2 - 5*x
```

```
[Out] -1/2*log(-(4*x - 11)*sqrt(x - 1)*sqrt(x - 4) + 4*x^2 - 21*x + 23)
 + 1/2*log(sqrt(x - 1)*sqrt(x - 4) - x + 7) + 1/2*log(x - 5) + 1/
2*log(sqrt(x - 1) + 2) - 1/2*log(sqrt(x - 1) - 2) - 1/2*log(sqrt(
x - 4) + 1) + 1/2*log(sqrt(x - 4) - 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(-4+x)**(1/2)+x*(-4+x)**(1/2)-4*(-1+x)**(1/2)+x*(-1+x)**(1/2))/(x*
5*x+4)/(1+(-4+x)**(1/2)+(-1+x)**(1/2)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.37642, size = 81, normalized size = 4.26

$$\ln\left(\sqrt{x-1}+2\right)-\ln\left(\left|-\sqrt{x-1}+\sqrt{x-4}\right|\right)-\ln\left(\left|-\sqrt{x-1}+\sqrt{x-4}-1\right|\right)+\ln\left(\left|-\sqrt{x-1}+\sqrt{x-4}-3\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sqrt(x - 1)*x + sqrt(x - 4)*x - 4*sqrt(x - 1) - sqrt(x - 4))/((x^2 - 5*x
```

```
[Out] ln(sqrt(x - 1) + 2) - ln(abs(-sqrt(x - 1) + sqrt(x - 4))) - ln(ab
s(-sqrt(x - 1) + sqrt(x - 4) - 1)) + ln(abs(-sqrt(x - 1) + sqrt(x
 - 4) - 3))
```

3.878 $\int \frac{1}{x(3+3x+x^2)\sqrt[3]{3+3x+3x^2+x^3}}\,dx$

Optimal. Leaf size=123

$$\frac{\log\left(1-\frac{\sqrt[3]{3}(x+1)}{\sqrt[3]{(x+1)^3+2}}\right)}{3\sqrt[3]{3}}-\frac{\log\left(\frac{3^{2/3}(x+1)^2}{((x+1)^3+2)^{2/3}}+\frac{\sqrt[3]{3}(x+1)}{\sqrt[3]{(x+1)^3+2}}+1\right)}{6\sqrt[3]{3}}-\frac{\tan^{-1}\left(\frac{2(x+1)}{\sqrt[6]{3}\sqrt[3]{(x+1)^3+2}}+\frac{1}{\sqrt{3}}\right)}{3^{5/6}}$$

```
[Out] -(ArcTan[1/Sqrt[3] + (2*(1 + x))/(3^(1/6)*(2 + (1 + x)^3)^(1/3))]
/3^(5/6)) + Log[1 - (3^(1/3)*(1 + x))/(2 + (1 + x)^3)^(1/3)]/(3*3
^(1/3)) - Log[1 + (3^(2/3)*(1 + x)^2)/(2 + (1 + x)^3)^(2/3) + (3^
(1/3)*(1 + x))/(2 + (1 + x)^3)^(1/3)]/(6*3^(1/3))
```

Rubi [A] time = 0.261326, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.29$

$$\frac{\log\left(1-\frac{\sqrt[3]{3}(x+1)}{\sqrt[3]{(x+1)^3+2}}\right)}{3\sqrt[3]{3}}-\frac{\log\left(\frac{3^{2/3}(x+1)^2}{((x+1)^3+2)^{2/3}}+\frac{\sqrt[3]{3}(x+1)}{\sqrt[3]{(x+1)^3+2}}+1\right)}{6\sqrt[3]{3}}-\frac{\tan^{-1}\left(\frac{2(x+1)}{\sqrt[6]{3}\sqrt[3]{(x+1)^3+2}}+\frac{1}{\sqrt{3}}\right)}{3^{5/6}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(3 + 3*x + x^2)*(3 + 3*x + 3*x^2 + x^3)^(1/3)),x]
```

```
[Out] -(ArcTan[1/Sqrt[3] + (2*(1 + x))/(3^(1/6)*(2 + (1 + x)^3)^(1/3))]
/3^(5/6)) + Log[1 - (3^(1/3)*(1 + x))/(2 + (1 + x)^3)^(1/3)]/(3*3
^(1/3)) - Log[1 + (3^(2/3)*(1 + x)^2)/(2 + (1 + x)^3)^(2/3) + (3^
(1/3)*(1 + x))/(2 + (1 + x)^3)^(1/3)]/(6*3^(1/3))
```

Rubi in Sympy [A] time = 16.1275, size = 116, normalized size = 0.94

$$\frac{3^{\frac{2}{3}}\log\left(-\frac{\sqrt[3]{3}(x+1)}{\sqrt[3]{(x+1)^3+2}}+1\right)}{9}-\frac{3^{\frac{2}{3}}\log\left(\frac{3^{\frac{2}{3}}(x+1)^2}{\left((x+1)^3+2\right)^{\frac{2}{3}}}+\frac{\sqrt[3]{3}(x+1)}{\sqrt[3]{(x+1)^3+2}}+1\right)}{18}-\frac{\sqrt[6]{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2\sqrt[3]{3}(x+1)}{3\sqrt[3]{(x+1)^3+2}}+\frac{1}{3}\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(x**2+3*x+3)/(x**3+3*x**2+3*x+3)**(1/3),x)
```

```
[Out] 3**(2/3)*log(-3**(1/3)*(x + 1)/((x + 1)**3 + 2)**(1/3) + 1)/9 - 3
**(2/3)*log(3**(2/3)*(x + 1)**2/((x + 1)**3 + 2)**(2/3) + 3**(1/3
)*(x + 1)/((x + 1)**3 + 2)**(1/3) + 1)/18 - 3**(1/6)*atan(sqrt(3)
*(2*3**(1/3)*(x + 1)/(3*((x + 1)**3 + 2)**(1/3)) + 1/3))/3
```

Mathematica [A] time = 0.0800684, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(3+3x+x^2\right)\sqrt[3]{3+3x+3x^2+x^3}}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[1/(x*(3 + 3*x + x^2)*(3 + 3*x + 3*x^2 + x^3)^(1/3)),x]
```

```
[Out] Integrate[1/(x*(3 + 3*x + x^2)*(3 + 3*x + 3*x^2 + x^3)^(1/3)), x]
```

Maple [F] time = 0.217, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(x^2+3x+3\right)}\frac{1}{\sqrt[3]{x^3+3x^2+3x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(x^2+3*x+3)/(x^3+3*x^2+3*x+3)^(1/3),x)
```

```
[Out] int(1/x/(x^2+3*x+3)/(x^3+3*x^2+3*x+3)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^3+3x^2+3x+3\right)^{\frac{1}{3}}\left(x^2+3x+3\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((x^3 + 3*x^2 + 3*x + 3)^(1/3)*(x^2 + 3*x + 3)*x),x, algorithm="maxima
```

```
[Out] integrate(1/((x^3 + 3*x^2 + 3*x + 3)^(1/3)*(x^2 + 3*x + 3)*x), x)
```

Fricas [A] time = 7.25573, size = 489, normalized size = 3.98

$$\frac{1}{54} \cdot 3^{\frac{1}{6}} \left(2\sqrt{3}\log\left(\frac{3\cdot 3^{\frac{2}{3}}\left(x^3+3x^2+3x+3\right)^{\frac{2}{3}}(x+1)+2\cdot 3^{\frac{1}{3}}\left(x^3+3x^2+3x\right)-9\left(x^3+3x^2+3x+3\right)^{\frac{1}{3}}\left(x^2+2x+1\right)}{x^3+3x^2+3x}\right) - \sqrt{3}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((x^3 + 3*x^2 + 3*x + 3)^(1/3)*(x^2 + 3*x + 3)*x),x, algorithm="fricas
```

```
[Out] 1/54*3^(1/6)*(2*sqrt(3)*log((3*3^(2/3)*(x^3 + 3*x^2 + 3*x + 3)^(2
/3)*(x + 1) + 2*3^(1/3)*(x^3 + 3*x^2 + 3*x) - 9*(x^3 + 3*x^2 + 3*
x + 3)^(1/3)*(x^2 + 2*x + 1))/(x^3 + 3*x^2 + 3*x)) - sqrt(3)*log(
(3^(2/3)*(31*x^6 + 186*x^5 + 465*x^4 + 666*x^3 + 603*x^2 + 324*x
+ 81) + 9*3^(1/3)*(5*x^5 + 25*x^4 + 50*x^3 + 54*x^2 + 33*x + 9)*(
x^3 + 3*x^2 + 3*x + 3)^(1/3) + 9*(7*x^4 + 28*x^3 + 42*x^2 + 30*x
+ 9)*(x^3 + 3*x^2 + 3*x + 3)^(2/3))/(x^6 + 6*x^5 + 15*x^4 + 18*x^
3 + 9*x^2)) + 6*arctan(-1/3*(2*3^(5/6)*(x^3 + 3*x^2 + 3*x) - 9*sq
rt(3)*(x^3 + 3*x^2 + 3*x + 3)^(1/3)*(x^2 + 2*x + 1) - 18*3^(1/6)*
(x^3 + 3*x^2 + 3*x + 3)^(2/3)*(x + 1))/(2*3^(1/3)*(x^3 + 3*x^2 +
3*x) + 9*(x^3 + 3*x^2 + 3*x + 3)^(1/3)*(x^2 + 2*x + 1))))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(x^2+3x+3\right)\sqrt[3]{x^3+3x^2+3x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(x**2+3*x+3)/(x**3+3*x**2+3*x+3)**(1/3),x)
```

```
[Out] Integral(1/(x*(x**2 + 3*x + 3)*(x**3 + 3*x**2 + 3*x + 3)**(1/3)),
 x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^3+3x^2+3x+3\right)^{\frac{1}{3}}\left(x^2+3x+3\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((x^3 + 3*x^2 + 3*x + 3)^(1/3)*(x^2 + 3*x + 3)*x),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 3*x^2 + 3*x + 3)^(1/3)*(x^2 + 3*x + 3)*x), x)
```

3.879 $\int \frac{1-x^2}{(1-x+x^2)(1-x^3)^{2/3}}\,dx$

Optimal. Leaf size=103

$$-\frac{\log\left(-x^3+2(1-x)^3+1\right)}{2\,2^{2/3}}+\frac{3\log\left(\sqrt[3]{1-x^3}+\sqrt[3]{2}(1-x)\right)}{2\,2^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}}$$

```
[Out] (Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]
])/2^(2/3) - Log[1 + 2*(1 - x)^3 - x^3]/(2*2^(2/3)) + (3*Log[2^(1
/3)*(1 - x) + (1 - x^3)^(1/3)])/(2*2^(2/3))
```

Rubi [F] time = 0.807854, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$

$$\text{Int}\left(\frac{1-x^2}{(1-x+x^2)\,(1-x^3)^{2/3}},x\right)$$

Verification is Not applicable to the result.

```
[In]  Int[(1 - x^2)/((1 - x + x^2)*(1 - x^3)^(2/3)),x]
```

```
[Out] -(x*Hypergeometric2F1[1/3, 2/3, 4/3, x^3]) - (1 + I*Sqrt[3])*Defe
r[Int][1/((-1 - I*Sqrt[3] + 2*x)*(1 - x^3)^(2/3)), x] - (1 - I*Sq
rt[3])*Defer[Int][1/((-1 + I*Sqrt[3] + 2*x)*(1 - x^3)^(2/3)), x]
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2+1)/(x**2-x+1)/(-x**3+1)**(2/3),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.119075, size = 0, normalized size = 0.

$$\int \frac{1-x^2}{(1-x+x^2)(1-x^3)^{2/3}}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(1 - x^2)/((1 - x + x^2)*(1 - x^3)^(2/3)),x]
```

```
[Out] Integrate[(1 - x^2)/((1 - x + x^2)*(1 - x^3)^(2/3)), x]
```

Maple [F] time = 0.187, size = 0, normalized size = 0.

$$\int \frac{-x^2+1}{x^2-x+1}\left(-x^3+1\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2+1)/(x^2-x+1)/(-x^3+1)^(2/3),x)
```

```
[Out] int((-x^2+1)/(x^2-x+1)/(-x^3+1)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2-1}{(-x^3+1)^{\frac{2}{3}}(x^2-x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 1)/((-x^3 + 1)^(2/3)*(x^2 - x + 1)),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 1)/((-x^3 + 1)^(2/3)*(x^2 - x + 1)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 1)/((-x^3 + 1)^(2/3)*(x^2 - x + 1)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]   time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{x^2 \left(-x^3+1\right)^{\frac{2}{3}} - x \left(-x^3+1\right)^{\frac{2}{3}} + \left(-x^3+1\right)^{\frac{2}{3}}}\, dx - \int \left(-\frac{1}{x^2 \left(-x^3+1\right)^{\frac{2}{3}} - x \left(-x^3+1\right)^{\frac{2}{3}} + \left(-x^3+1\right)^{\frac{2}{3}}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+1)/(x**2-x+1)/(-x**3+1)**(2/3),x)
```

```
[Out] -Integral(x**2/(x**2*(-x**3 + 1)**(2/3) - x*(-x**3 + 1)**(2/3) +
(-x**3 + 1)**(2/3)), x) - Integral(-1/(x**2*(-x**3 + 1)**(2/3) -
x*(-x**3 + 1)**(2/3) + (-x**3 + 1)**(2/3)), x)
```

GIAC/XCAS [F]   time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2-1}{\left(-x^3+1\right)^{\frac{2}{3}} \left(x^2-x+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 1)/((-x^3 + 1)^(2/3)*(x^2 - x + 1)),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 1)/((-x^3 + 1)^(2/3)*(x^2 - x + 1)), x)
```

3.880 $\int \frac{x^2}{\sqrt{-1+x^4}(1+x^4)}\,dx$

Optimal. Leaf size=49

$$-\frac{1}{4}\tan^{-1}\left(\frac{x^2+1}{x\sqrt{x^4-1}}\right)-\frac{1}{4}\tanh^{-1}\left(\frac{1-x^2}{x\sqrt{x^4-1}}\right)$$

```
[Out] -ArcTan[(1 + x^2)/(x*Sqrt[-1 + x^4])]/4 - ArcTanh[(1 - x^2)/(x*Sq
rt[-1 + x^4])]/4
```

Rubi [C] time = 0.240622, antiderivative size = 47, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$\left(\frac{1}{8}+\frac{i}{8}\right)\tanh^{-1}\left(\frac{(1+i)x}{\sqrt{x^4-1}}\right)-\left(\frac{1}{8}+\frac{i}{8}\right)\tan^{-1}\left(\frac{(1+i)x}{\sqrt{x^4-1}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(Sqrt[-1 + x^4]*(1 + x^4)),x]
```

```
[Out] (-1/8 - I/8)*ArcTan[((1 + I)*x)/Sqrt[-1 + x^4]] + (1/8 + I/8)*Arc
Tanh[((1 + I)*x)/Sqrt[-1 + x^4]]
```

Rubi in Sympy [A] time = 72.0828, size = 197, normalized size = 4.02

$$\frac{\left(\frac{1}{4}-\frac{i}{4}\right)\sqrt{-x^4+1}F\left(\operatorname{asin}(x)|-1\right)}{\sqrt{x^4-1}}+\frac{\left(\frac{1}{4}+\frac{i}{4}\right)\sqrt{-x^4+1}F\left(\operatorname{asin}(x)|-1\right)}{\sqrt{x^4-1}}+\frac{\left(\frac{1}{4}-\frac{i}{4}\right)\sqrt{x^4-1}F\left(\operatorname{asin}(x)|-1\right)}{\sqrt{-x^4+1}}+\frac{\left(\frac{1}{4}+\frac{i}{4}\right)\sqrt{x^4-1}F\left(\operatorname{asin}(x)|-1\right)}{\sqrt{-x^4+1}}-\frac{i\sqrt{x^4-1}\,\Pi\left(-i;\operatorname{asin}(x)|-1\right)}{2\sqrt{-x^2+1}\sqrt{x^2+1}}-\frac{(1-i)^2\sqrt{x^4-1}\,\Pi\left(i;\operatorname{asin}(x)|-1\right)}{4\sqrt{-x^2+1}\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(x**4+1)/(x**4-1)**(1/2),x)
```

```
[Out] (1/4 - I/4)*sqrt(-x**4 + 1)*elliptic_f(asin(x), -1)/sqrt(x**4 - 1
) + (1/4 + I/4)*sqrt(-x**4 + 1)*elliptic_f(asin(x), -1)/sqrt(x**4
 - 1) + (1/4 - I/4)*sqrt(x**4 - 1)*elliptic_f(asin(x), -1)/sqrt(-
x**4 + 1) + (1/4 + I/4)*sqrt(x**4 - 1)*elliptic_f(asin(x), -1)/sq
rt(-x**4 + 1) - I*sqrt(x**4 - 1)*elliptic_pi(-I, asin(x), -1)/(2*
sqrt(-x**2 + 1)*sqrt(x**2 + 1)) - (1 - I)**2*sqrt(x**4 - 1)*ellip
```

```
tic_pi(I, asin(x), -1)/(4*sqrt(-x**2 + 1)*sqrt(x**2 + 1))
```

Mathematica [C] time = 0.187742, size = 114, normalized size = 2.33

$$-\frac{7x^3F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};x^4,-x^4\right)}{3\sqrt{x^4-1}\,(x^4+1)\left(2x^4\left(2F_1\left(\frac{7}{4};\frac{1}{2},2;\frac{11}{4};x^4,-x^4\right)-F_1\left(\frac{7}{4};\frac{3}{2},1;\frac{11}{4};x^4,-x^4\right)\right)-7F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};x^4,-x^4\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^2/(Sqrt[-1 + x^4]*(1 + x^4)),x]
```

```
[Out] (-7*x^3*AppellF1[3/4, 1/2, 1, 7/4, x^4, -x^4])/(3*Sqrt[-1 + x^4]*
(1 + x^4)*(-7*AppellF1[3/4, 1/2, 1, 7/4, x^4, -x^4] + 2*x^4*(2*Ap
pellF1[7/4, 1/2, 2, 11/4, x^4, -x^4] - AppellF1[7/4, 3/2, 1, 11/4
, x^4, -x^4])))
```

Maple [B] time = 0.026, size = 88, normalized size = 1.8

$$\frac{1}{8}\arctan\left(\frac{1}{x}\sqrt{x^4-1}+1\right)-\frac{1}{8}\arctan\left(-\frac{1}{x}\sqrt{x^4-1}+1\right)$$
$$+\frac{1}{16}\ln\left(1\left(\frac{x^4-1}{2\,x^2}+\frac{1}{x}\sqrt{x^4-1}+1\right)\left(\frac{x^4-1}{2\,x^2}-\frac{1}{x}\sqrt{x^4-1}+1\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(x^4+1)/(x^4-1)^(1/2),x)
```

```
[Out] 1/8*arctan((x^4-1)^(1/2)/x+1)-1/8*arctan(-(x^4-1)^(1/2)/x+1)+1/16
*ln((1/2*(x^4-1)/x^2+(x^4-1)^(1/2)/x+1)/(1/2*(x^4-1)/x^2-(x^4-1)^
(1/2)/x+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2}{(x^4+1)\sqrt{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((x^4 + 1)*sqrt(x^4 - 1)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((x^4 + 1)*sqrt(x^4 - 1)), x)
```

Fricas [A] time = 0.321909, size = 92, normalized size = 1.88

$$\frac{1}{4} \arctan\left(\frac{x^3 + \sqrt{x^4 - 1}x^2 - x}{x^3 + x + \sqrt{x^4 - 1}}\right) + \frac{1}{8} \log\left(\frac{x^4 + 2x^2 + 2\sqrt{x^4 - 1}x - 1}{x^4 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((x^4 + 1)*sqrt(x^4 - 1)),x, algorithm="fricas")
```

```
[Out] 1/4*arctan((x^3 + sqrt(x^4 - 1)*x^2 - x)/(x^3 + x + sqrt(x^4 - 1)
)) + 1/8*log((x^4 + 2*x^2 + 2*sqrt(x^4 - 1)*x - 1)/(x^4 + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{(x-1)(x+1)(x^2+1)}(x^4+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(x**4+1)/(x**4-1)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt((x - 1)*(x + 1)*(x**2 + 1))*(x**4 + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(x^4+1)\sqrt{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((x^4 + 1)*sqrt(x^4 - 1)),x, algorithm="giac")
```

```
[Out] integrate(x^2/((x^4 + 1)*sqrt(x^4 - 1)), x)
```

3.881 $\int \frac{a-cx^4}{(ae+cdx^2)(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=80

$$\frac{\tan^{-1}\left(\frac{x\sqrt{ae^2-bde+cd^2}}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{d}\sqrt{e}\sqrt{ae^2-bde+cd^2}}$$

```
[Out] ArcTan[(Sqrt[c*d^2 - b*d*e + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a +
b*x^2 + c*x^4])]/(Sqrt[d]*Sqrt[e]*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Rubi [A] time = 0.700983, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$

$$\frac{\tan^{-1}\left(\frac{x\sqrt{ae^2-bde+cd^2}}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{d}\sqrt{e}\sqrt{ae^2-bde+cd^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - c*x^4)/((a*e + c*d*x^2)*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] ArcTan[(Sqrt[c*d^2 - b*d*e + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a +
b*x^2 + c*x^4])]/(Sqrt[d]*Sqrt[e]*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-c*x**4+a)/(c*d*x**2+a*e)/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),
```

```
[Out] Timed out
```

Mathematica [C] time = 0.770264, size = 383, normalized size = 4.79

$$\frac{i\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\left(-\left(\frac{\left(b+\sqrt{b^2-4ac}\right)d}{2ae};i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)-\left(\frac{\left(b+\sqrt{b^2-4ac}\right)e}{2cd};i\sinh^{-1}\left(\sqrt{2}\right.\right.\right.}{\sqrt{2}de\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a - c*x^4)/((a*e + c*d*x^2)*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (I*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])
]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(EllipticF[I*ArcSin
h[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a
*c])/(b - Sqrt[b^2 - 4*a*c])] - EllipticPi[((b + Sqrt[b^2 - 4*a*c
])*d)/(2*a*e), I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*
x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - EllipticPi
[((b + Sqrt[b^2 - 4*a*c])*e)/(2*c*d), I*ArcSinh[Sqrt[2]*Sqrt[c/(b
 + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2
- 4*a*c])]))/(Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*d*e*Sqrt[a
+ b*x^2 + c*x^4])
```

Maple [C] time = 0.059, size = 555, normalized size = 6.9

$$-\frac{\sqrt{2}}{4\,de}\sqrt{4-2\,\frac{\left(-b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\sqrt{4+2\,\frac{\left(b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\mathit{EllipticF}\left(\frac{\sqrt{2}x}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{-4\,ac+b^2}\right)},\frac{1}{2}\sqrt{-4+2\,\frac{b\left(b\right.}{}}\right.$$

$$+\frac{\sqrt{2}}{de}\sqrt{1+\frac{bx^2}{2\,a}-\frac{x^2}{2\,a}\sqrt{-4\,ac+b^2}}\sqrt{1+\frac{bx^2}{2\,a}+\frac{x^2}{2\,a}\sqrt{-4\,ac+b^2}}\mathit{EllipticPi}\left(\frac{\sqrt{2}x}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{-4\,ac+b^2}\right)},-2\,\frac{ae}{\left(-b+\sqrt{-4\,ac+\right.}}\right.$$

$$+\frac{\sqrt{2}}{de}\sqrt{1+\frac{bx^2}{2\,a}-\frac{x^2}{2\,a}\sqrt{-4\,ac+b^2}}\sqrt{1+\frac{bx^2}{2\,a}+\frac{x^2}{2\,a}\sqrt{-4\,ac+b^2}}\mathit{EllipticPi}\left(\frac{\sqrt{2}x}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{-4\,ac+b^2}\right)},-2\,\frac{cd}{\left(-b+\sqrt{-4\,ac+\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-c*x^4+a)/(c*d*x^2+a*e)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/4/d/e*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a
*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1
/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^
2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))
+1/e/d*2^(1/2)/(-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b*x^2/a
-1/2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b*x^2/a+1/2/a*x^2*(-4
*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(
1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),-2/(-b+(-4*a*c+b^2)^(1/2))
*a*e/d,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/((-b+(-4*a*c
+b^2)^(1/2))/a)^(1/2))+1/d/e*2^(1/2)/(-b/a+1/a*(-4*a*c+b^2)^(1/2)
)^(1/2)*(1+1/2*b*x^2/a-1/2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2
```

```
*b*x^2/a+1/2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2
)*EllipticPi(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),-2/(
-b+(-4*a*c+b^2)^(1/2))*c*d/e,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2
)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{cx^4 - a}{\sqrt{cx^4 + bx^2 + a}(cdx^2 + ae)(ex^2 + d)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(c*x^4 - a)/(sqrt(c*x^4 + b*x^2 + a)*(c*d*x^2 + a*e)*(e*x^2 + d)),x, al
```

```
[Out] -integrate((c*x^4 - a)/(sqrt(c*x^4 + b*x^2 + a)*(c*d*x^2 + a*e)*(
e*x^2 + d)), x)
```

Fricas [A] time = 44.3741, size = 1, normalized size = 0.01

$$\left[\frac{\log\left(-\frac{\left(c^2d^2e^2x^8-2\left(3\,c^2d^3e-4\,bcd^2e^2+3\,acde^3\right)x^6+a^2d^2e^2+\left(c^2d^4-8\,bcd^3e-8\,abde^3+a^2e^4+4\left(2\,b^2+ac\right)d^2e^2\right)x^4-2\left(3\,acd^3e-4\,abd^2e^2+3\,a^2de^3\right)x^2\right)\sqrt{-c}}{c^2d^2e^2x^8+2\left(c^2d^3e+acde^3\right)x^6+a^2d^2e^2+(}\right.}{4\sqrt{-cd^3e+b}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(c*x^4 - a)/(sqrt(c*x^4 + b*x^2 + a)*(c*d*x^2 + a*e)*(e*x^2 + d)),x, al
```

```
[Out] [1/4*log(-((c^2*d^2*e^2*x^8 - 2*(3*c^2*d^3*e - 4*b*c*d^2*e^2 + 3*
a*c*d*e^3)*x^6 + a^2*d^2*e^2 + (c^2*d^4 - 8*b*c*d^3*e - 8*a*b*d*e
^3 + a^2*e^4 + 4*(2*b^2 + a*c)*d^2*e^2)*x^4 - 2*(3*a*c*d^3*e - 4*
a*b*d^2*e^2 + 3*a^2*d*e^3)*x^2)*sqrt(-c*d^3*e + b*d^2*e^2 - a*d*e
^3) - 4*((c^2*d^4*e^2 - b*c*d^3*e^3 + a*c*d^2*e^4)*x^5 - (c^2*d^5
*e - 3*b*c*d^4*e^2 - 3*a*b*d^2*e^4 + a^2*d*e^5 + 2*(b^2 + a*c)*d^
3*e^3)*x^3 + (a*c*d^4*e^2 - a*b*d^3*e^3 + a^2*d^2*e^4)*x)*sqrt(c*
x^4 + b*x^2 + a))/(c^2*d^2*e^2*x^8 + 2*(c^2*d^3*e + a*c*d*e^3)*x^
6 + a^2*d^2*e^2 + (c^2*d^4 + 4*a*c*d^2*e^2 + a^2*e^4)*x^4 + 2*(a*
c*d^3*e + a^2*d*e^3)*x^2))/sqrt(-c*d^3*e + b*d^2*e^2 - a*d*e^3),
1/2*arctan(2*sqrt(c*d^3*e - b*d^2*e^2 + a*d*e^3)*sqrt(c*x^4 + b*x
^2 + a)*x/(c*d*e*x^4 + a*d*e - (c*d^2 - 2*b*d*e + a*e^2)*x^2))/sq
rt(c*d^3*e - b*d^2*e^2 + a*d*e^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*x**4+a)/(c*d*x**2+a*e)/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(c*x^4 - a)/(sqrt(c*x^4 + b*x^2 + a)*(c*d*x^2 + a*e)*(e*x^2 + d)),x, al
```

```
[Out] Exception raised: TypeError
```

3.882 $\int \left(x+\frac{1-x^2}{1+x}\right) dx$

Optimal. Leaf size=1

$$x$$

```
[Out] x
```

Rubi [A] time = 0.00524836, antiderivative size = 1, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.

$$x$$

Antiderivative was successfully verified.

```
[In]  Int[x + (1 - x^2)/(1 + x),x]

[Out] x
```

Rubi in Sympy [A] time = 1.68149, size = 0, normalized size = 0.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x+(-x**2+1)/(1+x),x)

[Out] x
```

Mathematica [A] time = 0.000289905, size = 1, normalized size = 1.

$$x$$

Antiderivative was successfully verified.

```
[In]  Integrate[x + (1 - x^2)/(1 + x),x]

[Out] x
```

Maple [A] time = 0.001, size = 2, normalized size = 2.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x+(-x^2+1)/(1+x),x)

[Out] x
```

Maxima [A] time = 0.694714, size = 1, normalized size = 1.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x - (x^2 - 1)/(x + 1),x, algorithm="maxima")

[Out] x
```

Fricas [A] time = 0.24318, size = 1, normalized size = 1.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x - (x^2 - 1)/(x + 1),x, algorithm="fricas")

[Out] x
```

Sympy [A] time = 0.064693, size = 0, normalized size = 0.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x+(-x**2+1)/(1+x),x)
```

```
[Out] x
```

GIAC/XCAS [A] time = 0.27403, size = 1, normalized size = 1.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x - (x^2 - 1)/(x + 1),x, algorithm="giac")
```

```
[Out] x
```

3.883 $\int \frac{1}{\frac{1}{x}+\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=122

$$-\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{-\frac{-\sqrt{3}+i}{\sqrt{3}+i}}\sqrt{1-x^2}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt{-\frac{-\sqrt{3}+i}{\sqrt{3}+i}}\,x}{\sqrt{1-x^2}}\right)}{\sqrt{3}}+\sin^{-1}(x)$$

```
[Out] ArcSin[x] - ArcTan[(1 - 2*x^2)/Sqrt[3]]/Sqrt[3] - ArcTan[x/(Sqrt[
-((I - Sqrt[3])/(I + Sqrt[3]))]*Sqrt[1 - x^2])]/Sqrt[3] - ArcTan[
(Sqrt[-((I - Sqrt[3])/(I + Sqrt[3]))]*x)/Sqrt[1 - x^2]]/Sqrt[3]
```

Rubi [A] time = 0.335202, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529

$$-\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{-\frac{-\sqrt{3}+i}{\sqrt{3}+i}}\sqrt{1-x^2}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt{-\frac{-\sqrt{3}+i}{\sqrt{3}+i}}\,x}{\sqrt{1-x^2}}\right)}{\sqrt{3}}+\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(x^(-1) + Sqrt[1 - x^2])^(-1),x]
```


```
[Out] ArcSin[x] - ArcTan[(1 - 2*x^2)/Sqrt[3]]/Sqrt[3] - ArcTan[x/(Sqrt[
-((I - Sqrt[3])/(I + Sqrt[3]))]*Sqrt[1 - x^2])]/Sqrt[3] - ArcTan[
(Sqrt[-((I - Sqrt[3])/(I + Sqrt[3]))]*x)/Sqrt[1 - x^2]]/Sqrt[3]
```

Rubi in Sympy [A] time = 52.7528, size = 112, normalized size = 0.92

$$\frac{2\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{1}{3}+\frac{\sqrt{-x^2+1}-1}{3x}\right)\right)}{3}-\frac{2\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{1}{3}-\frac{\sqrt{-x^2+1}-1}{x}-\frac{2\left(\sqrt{-x^2+1}-1\right)^2}{3x^2}-\frac{\left(\sqrt{-x^2+1}-1\right)^3}{3x^3}\right)\right)}{3}-2\,\mathrm{atan}\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1/x+(-x**2+1)**(1/2)),x)
```

```
[Out] 2*sqrt(3)*atan(sqrt(3)*(1/3 + (sqrt(-x**2 + 1) - 1)/(3*x)))/3 - 2
*sqrt(3)*atan(sqrt(3)*(1/3 - (sqrt(-x**2 + 1) - 1)/x - 2*(sqrt(-x
**2 + 1) - 1)**2/(3*x**2) - (sqrt(-x**2 + 1) - 1)**3/(3*x**3)))/3
 - 2*atan((sqrt(-x**2 + 1) - 1)/x)
```

Mathematica [B] time = 6.67332, size = 2681, normalized size = 21.98

Result too large to show

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(x^(-1) + Sqrt[1 - x^2])^(-1),x]
```

```
[Out] ((1 + x*Sqrt[1 - x^2])*ArcSin[x])/(x*(x^(-1) + Sqrt[1 - x^2])) +
((-I + Sqrt[3])*(1 + x*Sqrt[1 - x^2])*ArcTan[(x*(7*I - Sqrt[3] +
(8*I)*Sqrt[3]*x + (7*I)*x^2 + Sqrt[3]*x^2))/(-6*I + 2*Sqrt[3] + 3
*x - (11*I)*Sqrt[3]*x - (18*I)*x^2 - 2*Sqrt[3]*x^2 - 3*x^3 - (3*I
)*Sqrt[3]*x^3 - (2*I)*Sqrt[2*(1 - I*Sqrt[3])]*Sqrt[1 - x^2] - (2*
I)*Sqrt[6*(1 - I*Sqrt[3])]*x*Sqrt[1 - x^2] - (2*I)*Sqrt[2*(1 - I*
Sqrt[3])]*x^2*Sqrt[1 - x^2])])/(2*Sqrt[6*(1 - I*Sqrt[3])]*x*(x^(-
1) + Sqrt[1 - x^2])) - ((-I + Sqrt[3])*(1 + x*Sqrt[1 - x^2])*ArcT
an[(x*(7*I - Sqrt[3] - (8*I)*Sqrt[3]*x + (7*I)*x^2 + Sqrt[3]*x^2)
)/(6*I - 2*Sqrt[3] + 3*x - (11*I)*Sqrt[3]*x + (18*I)*x^2 + 2*Sqrt
[3]*x^2 - 3*x^3 - (3*I)*Sqrt[3]*x^3 + (2*I)*Sqrt[2*(1 - I*Sqrt[3]
)]*Sqrt[1 - x^2] - (2*I)*Sqrt[6*(1 - I*Sqrt[3])]*x*Sqrt[1 - x^2]
+ (2*I)*Sqrt[2*(1 - I*Sqrt[3])]*x^2*Sqrt[1 - x^2])])/(2*Sqrt[6*(1
 - I*Sqrt[3])]*x*(x^(-1) + Sqrt[1 - x^2])) - ((I + Sqrt[3])*(1 +
x*Sqrt[1 - x^2])*ArcTan[(x*(-7*I - Sqrt[3] - (8*I)*Sqrt[3]*x - (7
*I)*x^2 + Sqrt[3]*x^2))/(-6*I - 2*Sqrt[3] - 3*x - (11*I)*Sqrt[3]*
x - (18*I)*x^2 + 2*Sqrt[3]*x^2 + 3*x^3 - (3*I)*Sqrt[3]*x^3 - (2*I
)*Sqrt[2*(1 + I*Sqrt[3])]*Sqrt[1 - x^2] - (2*I)*Sqrt[6*(1 + I*Sqr
t[3])]*x*Sqrt[1 - x^2] - (2*I)*Sqrt[2*(1 + I*Sqrt[3])]*x^2*Sqrt[1
 - x^2])])/(2*Sqrt[6*(1 + I*Sqrt[3])]*x*(x^(-1) + Sqrt[1 - x^2]))
 + ((I + Sqrt[3])*(1 + x*Sqrt[1 - x^2])*ArcTan[(x*(-7*I - Sqrt[3]
 + (8*I)*Sqrt[3]*x - (7*I)*x^2 + Sqrt[3]*x^2))/(6*I + 2*Sqrt[3] -
 3*x - (11*I)*Sqrt[3]*x + (18*I)*x^2 - 2*Sqrt[3]*x^2 + 3*x^3 - (3
*I)*Sqrt[3]*x^3 + (2*I)*Sqrt[2*(1 + I*Sqrt[3])]*Sqrt[1 - x^2] - (
2*I)*Sqrt[6*(1 + I*Sqrt[3])]*x*Sqrt[1 - x^2] + (2*I)*Sqrt[2*(1 +
I*Sqrt[3])]*x^2*Sqrt[1 - x^2])])/(2*Sqrt[6*(1 + I*Sqrt[3])]*x*(x^
(-1) + Sqrt[1 - x^2])) + ((I/4)*(-I + Sqrt[3])*(1 + x*Sqrt[1 - x^
2])*Log[(-I + Sqrt[3] - 2*x)^2*(I + Sqrt[3] - 2*x)^2])/(Sqrt[6*(1
 - I*Sqrt[3])]*x*(x^(-1) + Sqrt[1 - x^2])) - ((I/4)*(I + Sqrt[3])
*(1 + x*Sqrt[1 - x^2])*Log[(-I + Sqrt[3] - 2*x)^2*(I + Sqrt[3] -
2*x)^2])/(Sqrt[6*(1 + I*Sqrt[3])]*x*(x^(-1) + Sqrt[1 - x^2])) - (
(I/4)*(-I + Sqrt[3])*(1 + x*Sqrt[1 - x^2])*Log[(-I + Sqrt[3] + 2*
x)^2*(I + Sqrt[3] + 2*x)^2])/(Sqrt[6*(1 - I*Sqrt[3])]*x*(x^(-1) +
 Sqrt[1 - x^2])) + ((I/4)*(I + Sqrt[3])*(1 + x*Sqrt[1 - x^2])*Log
```

```
[(-I + Sqrt[3] + 2*x)^2*(I + Sqrt[3] + 2*x)^2])/(Sqrt[6*(1 + I*Sq
rt[3])]*x*(x^(-1) + Sqrt[1 - x^2])) - ((I/2)*(1 + x*Sqrt[1 - x^2]
)*Log[-1/2 - (I/2)*Sqrt[3] + x^2])/(Sqrt[3]*x*(x^(-1) + Sqrt[1 -
x^2])) + ((I/2)*(1 + x*Sqrt[1 - x^2])*Log[-1/2 + (I/2)*Sqrt[3] +
x^2])/(Sqrt[3]*x*(x^(-1) + Sqrt[1 - x^2])) - ((I/4)*(-I + Sqrt[3]
)*(1 + x*Sqrt[1 - x^2])*Log[3*I + Sqrt[3] - 3*x - (5*I)*Sqrt[3]*x
 + (10*I)*x^2 + 3*x^3 - (3*I)*Sqrt[3]*x^3 + I*x^4 - Sqrt[3]*x^4 +
 (2*I)*Sqrt[2*(1 - I*Sqrt[3])]*Sqrt[1 - x^2] - (3*I)*Sqrt[6*(1 -
I*Sqrt[3])]*x*Sqrt[1 - x^2] + (5*I)*Sqrt[2*(1 - I*Sqrt[3])]*x^2*S
qrt[1 - x^2] - I*Sqrt[6*(1 - I*Sqrt[3])]*x^3*Sqrt[1 - x^2]])/(Sqr
t[6*(1 - I*Sqrt[3])]*x*(x^(-1) + Sqrt[1 - x^2])) + ((I/4)*(-I + S
qrt[3])*(1 + x*Sqrt[1 - x^2])*Log[3*I + Sqrt[3] + 3*x + (5*I)*Sqr
t[3]*x + (10*I)*x^2 - 3*x^3 + (3*I)*Sqrt[3]*x^3 + I*x^4 - Sqrt[3]
*x^4 + (2*I)*Sqrt[2*(1 - I*Sqrt[3])]*Sqrt[1 - x^2] + (3*I)*Sqrt[6
*(1 - I*Sqrt[3])]*x*Sqrt[1 - x^2] + (5*I)*Sqrt[2*(1 - I*Sqrt[3])]
*x^2*Sqrt[1 - x^2] + I*Sqrt[6*(1 - I*Sqrt[3])]*x^3*Sqrt[1 - x^2]]
)/(Sqrt[6*(1 - I*Sqrt[3])]*x*(x^(-1) + Sqrt[1 - x^2])) - ((I/4)*(
I + Sqrt[3])*(1 + x*Sqrt[1 - x^2])*Log[-3*I + Sqrt[3] + 3*x - (5*
I)*Sqrt[3]*x - (10*I)*x^2 - 3*x^3 - (3*I)*Sqrt[3]*x^3 - I*x^4 - S
qrt[3]*x^4 - (2*I)*Sqrt[2*(1 + I*Sqrt[3])]*Sqrt[1 - x^2] - (3*I)*
Sqrt[6*(1 + I*Sqrt[3])]*x*Sqrt[1 - x^2] - (5*I)*Sqrt[2*(1 + I*Sqr
t[3])]*x^2*Sqrt[1 - x^2] - I*Sqrt[6*(1 + I*Sqrt[3])]*x^3*Sqrt[1 -
 x^2]])/(Sqrt[6*(1 + I*Sqrt[3])]*x*(x^(-1) + Sqrt[1 - x^2])) + ((
I/4)*(I + Sqrt[3])*(1 + x*Sqrt[1 - x^2])*Log[-3*I + Sqrt[3] - 3*x
 + (5*I)*Sqrt[3]*x - (10*I)*x^2 + 3*x^3 + (3*I)*Sqrt[3]*x^3 - I*x
^4 - Sqrt[3]*x^4 - (2*I)*Sqrt[2*(1 + I*Sqrt[3])]*Sqrt[1 - x^2] +
(3*I)*Sqrt[6*(1 + I*Sqrt[3])]*x*Sqrt[1 - x^2] - (5*I)*Sqrt[2*(1 +
 I*Sqrt[3])]*x^2*Sqrt[1 - x^2] + I*Sqrt[6*(1 + I*Sqrt[3])]*x^3*Sq
rt[1 - x^2]])/(Sqrt[6*(1 + I*Sqrt[3])]*x*(x^(-1) + Sqrt[1 - x^2])
)
```

Maple [B] time = 0.047, size = 234, normalized size = 1.9

$$\frac{i}{6}\sqrt{3}\ln\left(\frac{1}{x^2}\left(\sqrt{-x^2+1}-1\right)^2+\frac{i\sqrt{3}+1}{x}\left(\sqrt{-x^2+1}-1\right)-1\right)$$
$$-\frac{i}{6}\sqrt{3}\ln\left(\frac{1}{x^2}\left(\sqrt{-x^2+1}-1\right)^2+\frac{1-i\sqrt{3}}{x}\left(\sqrt{-x^2+1}-1\right)-1\right)-2\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$
$$+\frac{i}{6}\sqrt{3}\ln\left(\frac{1}{x^2}\left(\sqrt{-x^2+1}-1\right)^2+\frac{-1+i\sqrt{3}}{x}\left(\sqrt{-x^2+1}-1\right)-1\right)$$
$$-\frac{i}{6}\sqrt{3}\ln\left(\frac{1}{x^2}\left(\sqrt{-x^2+1}-1\right)^2+\frac{-i\sqrt{3}-1}{x}\left(\sqrt{-x^2+1}-1\right)-1\right)+\frac{\sqrt{3}}{3}\arctan\left(\frac{\left(2x^2-1\right)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(1/(1/x+(-x^2+1)^(1/2)),x)
```

```
[Out] 1/6*I*3^(1/2)*ln(((-x^2+1)^(1/2)-1)^2/x^2+(I*3^(1/2)+1)*((-x^2+1)
^(1/2)-1)/x-1)-1/6*I*3^(1/2)*ln(((-x^2+1)^(1/2)-1)^2/x^2+(1-I*3^(
1/2))*((-x^2+1)^(1/2)-1)/x-1)-2*arctan(((-x^2+1)^(1/2)-1)/x)+1/6*
I*3^(1/2)*ln(((-x^2+1)^(1/2)-1)^2/x^2+(-1+I*3^(1/2))*((-x^2+1)^(1
/2)-1)/x-1)-1/6*I*3^(1/2)*ln(((-x^2+1)^(1/2)-1)^2/x^2+(-I*3^(1/2)
-1)*((-x^2+1)^(1/2)-1)/x-1)+1/3*3^(1/2)*arctan(1/3*(2*x^2-1)*3^(1
/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2+1}+\frac{1}{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-x^2 + 1) + 1/x),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^2 + 1) + 1/x), x)
```

Fricas [A] time = 0.260602, size = 151, normalized size = 1.24

$$-\frac{1}{3}\sqrt{3}\left(2\sqrt{3}\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)-\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)+\arctan\left(-\frac{2\sqrt{3}\left(2x^2-1\right)\sqrt{-x^2+1}+\sqrt{3}\left(2x^4-5x^2+2\right)}{3\left(2x^3-\left(x^3-2x\right)\sqrt{-x^2+1}-2x\right)}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-x^2 + 1) + 1/x),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*(2*sqrt(3)*arctan((sqrt(-x^2 + 1) - 1)/x) - arctan(1
/3*sqrt(3)*(2*x^2 - 1)) + arctan(-1/3*(2*sqrt(3)*(2*x^2 - 1)*sqrt
(-x^2 + 1) + sqrt(3)*(2*x^4 - 5*x^2 + 2))/(2*x^3 - (x^3 - 2*x)*sq
rt(-x^2 + 1) - 2*x)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{x\sqrt{-x^2+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1/x+(-x**2+1)**(1/2)),x)
```

```
[Out] Integral(x/(x*sqrt(-x**2 + 1) + 1), x)
```

GIAC/XCAS [A] time = 0.274593, size = 261, normalized size = 2.14

$$\frac{1}{2}\,\pi\operatorname{sign}\left(x\right) - \frac{1}{6}\,\sqrt{3}\left(\pi\operatorname{sign}\left(x\right) + 2\,\arctan\left(-\frac{\sqrt{3}x\left(\frac{\sqrt{-x^{2}+1}-1}{x} + \frac{\left(\sqrt{-x^{2}+1}-1\right)^{2}}{x^{2}} - 1\right)}{3\left(\sqrt{-x^{2}+1}-1\right)}\right)\right) - \frac{1}{6}\,\sqrt{3}\left(\pi\operatorname{sign}\left(x\right) + 2\,\arctan\left(\frac{\sqrt{3}x\left(\frac{\sqrt{-x^{2}+1}-1}{x} - \frac{\left(\sqrt{-x^{2}+1}-1\right)^{2}}{x^{2}} + 1\right)}{3\left(\sqrt{-x^{2}+1}-1\right)}\right)\right) + \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^{2}-1\right)\right) + \arctan\left(-\frac{x\left(\frac{\left(\sqrt{-x^{2}+1}-1\right)^{2}}{x^{2}} - 1\right)}{2\left(\sqrt{-x^{2}+1}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-x^2 + 1) + 1/x),x, algorithm="giac")
```

```
[Out] 1/2*pi*sign(x) - 1/6*sqrt(3)*(pi*sign(x) + 2*arctan(-1/3*sqrt(3)*
x*((sqrt(-x^2 + 1) - 1)/x + (sqrt(-x^2 + 1) - 1)^2/x^2 - 1)/(sqrt
(-x^2 + 1) - 1))) - 1/6*sqrt(3)*(pi*sign(x) + 2*arctan(1/3*sqrt(3
)*x*((sqrt(-x^2 + 1) - 1)/x - (sqrt(-x^2 + 1) - 1)^2/x^2 + 1)/(sq
rt(-x^2 + 1) - 1))) + 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1))
 + arctan(-1/2*x*((sqrt(-x^2 + 1) - 1)^2/x^2 - 1)/(sqrt(-x^2 + 1)
 - 1))
```

3.884 $\int \frac{x\sqrt{1-x^2}}{x-x^3+\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=122

$$-\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{-\frac{-\sqrt{3}+i}{\sqrt{3}+i}}\sqrt{1-x^2}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt{-\frac{-\sqrt{3}+i}{\sqrt{3}+i}}\,x}{\sqrt{1-x^2}}\right)}{\sqrt{3}}+\sin^{-1}(x)$$

```
[Out] ArcSin[x] - ArcTan[(1 - 2*x^2)/Sqrt[3]]/Sqrt[3] - ArcTan[x/(Sqrt[
-((I - Sqrt[3])/(I + Sqrt[3]))]*Sqrt[1 - x^2])]/Sqrt[3] - ArcTan[
(Sqrt[-((I - Sqrt[3])/(I + Sqrt[3]))]*x)/Sqrt[1 - x^2]]/Sqrt[3]
```

Rubi [A] time = 0.775428, antiderivative size = 149, normalized size of antiderivative = 1.22, number of steps used = 13, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.303

$$-\frac{x^2}{2}-\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{-\frac{-\sqrt{3}+i}{\sqrt{3}+i}}\sqrt{1-x^2}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt{-\frac{-\sqrt{3}+i}{\sqrt{3}+i}}\,x}{\sqrt{1-x^2}}\right)}{\sqrt{3}}+\frac{1}{4}(1-x)^2+\frac{1}{4}(x+1)^2+\sin^{-1}(x)$$

Warning: Unable to verify antiderivative.

```
[In]  Int[(x*Sqrt[1 - x^2])/(x - x^3 + Sqrt[1 - x^2]),x]
```

```
[Out] (1 - x)^2/4 - x^2/2 + (1 + x)^2/4 + ArcSin[x] - ArcTan[(1 - 2*x^2
)/Sqrt[3]]/Sqrt[3] - ArcTan[x/(Sqrt[-((I - Sqrt[3])/(I + Sqrt[3])
)]*Sqrt[1 - x^2])]/Sqrt[3] - ArcTan[(Sqrt[-((I - Sqrt[3])/(I + Sq
rt[3]))]*x)/Sqrt[1 - x^2]]/Sqrt[3]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{-x^2+1}}{-x^3+x+\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(-x**2+1)**(1/2)/(x-x**3+(-x**2+1)**(1/2)),x)
```

```
[Out] Integral(x*sqrt(-x**2 + 1)/(-x**3 + x + sqrt(-x**2 + 1)), x)
```

Mathematica [B] time = 6.56211, size = 2155, normalized size = 17.66

Result too large to show

Antiderivative was successfully verified.

```
[In]  Integrate[(x*Sqrt[1 - x^2])/(x - x^3 + Sqrt[1 - x^2]),x]
```

```
[Out] ArcSin[x] + ((-I + Sqrt[3])*ArcTan[(x*(7*I - Sqrt[3] + (8*I)*Sqrt
[3]*x + (7*I)*x^2 + Sqrt[3]*x^2))/(-6*I + 2*Sqrt[3] + 3*x - (11*I
)*Sqrt[3]*x - (18*I)*x^2 - 2*Sqrt[3]*x^2 - 3*x^3 - (3*I)*Sqrt[3]*
x^3 - (2*I)*Sqrt[2*(1 - I*Sqrt[3])]*Sqrt[1 - x^2] - (2*I)*Sqrt[6*
(1 - I*Sqrt[3])]*x*Sqrt[1 - x^2] - (2*I)*Sqrt[2*(1 - I*Sqrt[3])]*
x^2*Sqrt[1 - x^2])])/(2*Sqrt[6*(1 - I*Sqrt[3])]) - ((-I + Sqrt[3]
)*ArcTan[(x*(7*I - Sqrt[3] - (8*I)*Sqrt[3]*x + (7*I)*x^2 + Sqrt[3
]*x^2))/(6*I - 2*Sqrt[3] + 3*x - (11*I)*Sqrt[3]*x + (18*I)*x^2 +
2*Sqrt[3]*x^2 - 3*x^3 - (3*I)*Sqrt[3]*x^3 + (2*I)*Sqrt[2*(1 - I*S
qrt[3])]*Sqrt[1 - x^2] - (2*I)*Sqrt[6*(1 - I*Sqrt[3])]*x*Sqrt[1 -
 x^2] + (2*I)*Sqrt[2*(1 - I*Sqrt[3])]*x^2*Sqrt[1 - x^2])])/(2*Sqr
t[6*(1 - I*Sqrt[3])]) - ((I + Sqrt[3])*ArcTan[(x*(-7*I - Sqrt[3]
- (8*I)*Sqrt[3]*x - (7*I)*x^2 + Sqrt[3]*x^2))/(-6*I - 2*Sqrt[3] -
 3*x - (11*I)*Sqrt[3]*x - (18*I)*x^2 + 2*Sqrt[3]*x^2 + 3*x^3 - (3
*I)*Sqrt[3]*x^3 - (2*I)*Sqrt[2*(1 + I*Sqrt[3])]*Sqrt[1 - x^2] - (
2*I)*Sqrt[6*(1 + I*Sqrt[3])]*x*Sqrt[1 - x^2] - (2*I)*Sqrt[2*(1 +
I*Sqrt[3])]*x^2*Sqrt[1 - x^2])])/(2*Sqrt[6*(1 + I*Sqrt[3])]) + ((
I + Sqrt[3])*ArcTan[(x*(-7*I - Sqrt[3] + (8*I)*Sqrt[3]*x - (7*I)*
x^2 + Sqrt[3]*x^2))/(6*I + 2*Sqrt[3] - 3*x - (11*I)*Sqrt[3]*x + (
18*I)*x^2 - 2*Sqrt[3]*x^2 + 3*x^3 - (3*I)*Sqrt[3]*x^3 + (2*I)*Sqr
t[2*(1 + I*Sqrt[3])]*Sqrt[1 - x^2] - (2*I)*Sqrt[6*(1 + I*Sqrt[3])
]*x*Sqrt[1 - x^2] + (2*I)*Sqrt[2*(1 + I*Sqrt[3])]*x^2*Sqrt[1 - x^
2])])/(2*Sqrt[6*(1 + I*Sqrt[3])]) + ((I/4)*(-I + Sqrt[3])*Log[(-I
 + Sqrt[3] - 2*x)^2*(I + Sqrt[3] - 2*x)^2])/Sqrt[6*(1 - I*Sqrt[3]
)] - ((I/4)*(I + Sqrt[3])*Log[(-I + Sqrt[3] - 2*x)^2*(I + Sqrt[3]
 - 2*x)^2])/Sqrt[6*(1 + I*Sqrt[3])] - ((I/4)*(-I + Sqrt[3])*Log[(
-I + Sqrt[3] + 2*x)^2*(I + Sqrt[3] + 2*x)^2])/Sqrt[6*(1 - I*Sqrt[
3])] + ((I/4)*(I + Sqrt[3])*Log[(-I + Sqrt[3] + 2*x)^2*(I + Sqrt[
3] + 2*x)^2])/Sqrt[6*(1 + I*Sqrt[3])] - ((I/2)*Log[-1/2 - (I/2)*S
qrt[3] + x^2])/Sqrt[3] + ((I/2)*Log[-1/2 + (I/2)*Sqrt[3] + x^2])/
Sqrt[3] - ((I/4)*(-I + Sqrt[3])*Log[3*I + Sqrt[3] - 3*x - (5*I)*S
qrt[3]*x + (10*I)*x^2 + 3*x^3 - (3*I)*Sqrt[3]*x^3 + I*x^4 - Sqrt[
3]*x^4 + (2*I)*Sqrt[2*(1 - I*Sqrt[3])]*Sqrt[1 - x^2] - (3*I)*Sqrt
[6*(1 - I*Sqrt[3])]*x*Sqrt[1 - x^2] + (5*I)*Sqrt[2*(1 - I*Sqrt[3]
)]*x^2*Sqrt[1 - x^2] - I*Sqrt[6*(1 - I*Sqrt[3])]*x^3*Sqrt[1 - x^2
]])/Sqrt[6*(1 - I*Sqrt[3])] + ((I/4)*(-I + Sqrt[3])*Log[3*I + Sqr
t[3] + 3*x + (5*I)*Sqrt[3]*x + (10*I)*x^2 - 3*x^3 + (3*I)*Sqrt[3]
*x^3 + I*x^4 - Sqrt[3]*x^4 + (2*I)*Sqrt[2*(1 - I*Sqrt[3])]*Sqrt[1
 - x^2] + (3*I)*Sqrt[6*(1 - I*Sqrt[3])]*x*Sqrt[1 - x^2] + (5*I)*S
qrt[2*(1 - I*Sqrt[3])]*x^2*Sqrt[1 - x^2] + I*Sqrt[6*(1 - I*Sqrt[3
])]*x^3*Sqrt[1 - x^2]])/Sqrt[6*(1 - I*Sqrt[3])] - ((I/4)*(I + Sqr
t[3])*Log[-3*I + Sqrt[3] + 3*x - (5*I)*Sqrt[3]*x - (10*I)*x^2 - 3
```

```
*x^3 - (3*I)*Sqrt[3]*x^3 - I*x^4 - Sqrt[3]*x^4 - (2*I)*Sqrt[2*(1
+ I*Sqrt[3])]*Sqrt[1 - x^2] - (3*I)*Sqrt[6*(1 + I*Sqrt[3])]*x*Sqr
t[1 - x^2] - (5*I)*Sqrt[2*(1 + I*Sqrt[3])]*x^2*Sqrt[1 - x^2] - I*
Sqrt[6*(1 + I*Sqrt[3])]*x^3*Sqrt[1 - x^2]])/Sqrt[6*(1 + I*Sqrt[3]
)] + ((I/4)*(I + Sqrt[3])*Log[-3*I + Sqrt[3] - 3*x + (5*I)*Sqrt[3
]*x - (10*I)*x^2 + 3*x^3 + (3*I)*Sqrt[3]*x^3 - I*x^4 - Sqrt[3]*x^
4 - (2*I)*Sqrt[2*(1 + I*Sqrt[3])]*Sqrt[1 - x^2] + (3*I)*Sqrt[6*(1
 + I*Sqrt[3])]*x*Sqrt[1 - x^2] - (5*I)*Sqrt[2*(1 + I*Sqrt[3])]*x^
2*Sqrt[1 - x^2] + I*Sqrt[6*(1 + I*Sqrt[3])]*x^3*Sqrt[1 - x^2]])/S
qrt[6*(1 + I*Sqrt[3])]
```

Maple [B] time = 0.082, size = 234, normalized size = 1.9

$$\frac{i}{6}\sqrt{3}\ln\left(\frac{1}{x^2}\left(\sqrt{-x^2+1}-1\right)^2+\frac{i\sqrt{3}+1}{x}\left(\sqrt{-x^2+1}-1\right)-1\right)$$
$$-\frac{i}{6}\sqrt{3}\ln\left(\frac{1}{x^2}\left(\sqrt{-x^2+1}-1\right)^2+\frac{1-i\sqrt{3}}{x}\left(\sqrt{-x^2+1}-1\right)-1\right)-2\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$
$$+\frac{i}{6}\sqrt{3}\ln\left(\frac{1}{x^2}\left(\sqrt{-x^2+1}-1\right)^2+\frac{-1+i\sqrt{3}}{x}\left(\sqrt{-x^2+1}-1\right)-1\right)$$
$$-\frac{i}{6}\sqrt{3}\ln\left(\frac{1}{x^2}\left(\sqrt{-x^2+1}-1\right)^2+\frac{-i\sqrt{3}-1}{x}\left(\sqrt{-x^2+1}-1\right)-1\right)+\frac{\sqrt{3}}{3}\arctan\left(\frac{\left(2x^2-1\right)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(-x^2+1)^(1/2)/(x-x^3+(-x^2+1)^(1/2)),x)
```

```
[Out] 1/6*I*3^(1/2)*ln(((-x^2+1)^(1/2)-1)^2/x^2+(I*3^(1/2)+1)*((-x^2+1)
^(1/2)-1)/x-1)-1/6*I*3^(1/2)*ln(((-x^2+1)^(1/2)-1)^2/x^2+(1-I*3^(
1/2))*((-x^2+1)^(1/2)-1)/x-1)-2*arctan(((-x^2+1)^(1/2)-1)/x)+1/6*
I*3^(1/2)*ln(((-x^2+1)^(1/2)-1)^2/x^2+(-1+I*3^(1/2))*((-x^2+1)^(1
/2)-1)/x-1)-1/6*I*3^(1/2)*ln(((-x^2+1)^(1/2)-1)^2/x^2+(-I*3^(1/2)
-1)*((-x^2+1)^(1/2)-1)/x-1)+1/3*3^(1/2)*arctan(1/3*(2*x^2-1)*3^(1
/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}x^2+\int -\frac{x^4-x^2}{x^3-x-\sqrt{x+1}\sqrt{-x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sqrt(-x^2 + 1)*x/(x^3 - x - sqrt(-x^2 + 1)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + integrate(-(x^4 - x^2)/(x^3 - x - sqrt(x + 1)*sqrt(-x +
 1)), x)
```

Fricas [A] time = 0.268776, size = 151, normalized size = 1.24

$$-\frac{1}{3}\sqrt{3}\left(2\sqrt{3}\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)-\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)+\arctan\left(-\frac{2\sqrt{3}\left(2x^2-1\right)\sqrt{-x^2+1}+\sqrt{3}\left(2x^4-5x^2+2\right)}{3\left(2x^3-\left(x^3-2x\right)\sqrt{-x^2+1}-2x\right)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sqrt(-x^2 + 1)*x/(x^3 - x - sqrt(-x^2 + 1)),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*(2*sqrt(3)*arctan((sqrt(-x^2 + 1) - 1)/x) - arctan(1
/3*sqrt(3)*(2*x^2 - 1)) + arctan(-1/3*(2*sqrt(3)*(2*x^2 - 1)*sqrt
(-x^2 + 1) + sqrt(3)*(2*x^4 - 5*x^2 + 2))/(2*x^3 - (x^3 - 2*x)*sq
rt(-x^2 + 1) - 2*x)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x**2+1)**(1/2)/(x-x**3+(-x**2+1)**(1/2)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.294657, size = 261, normalized size = 2.14

$$\frac{1}{2}\,\pi\,\mathrm{sign}\left(x\right) - \frac{1}{6}\,\sqrt{3}\left(\pi\,\mathrm{sign}\left(x\right) + 2\,\arctan\left(-\frac{\sqrt{3}x\left(\frac{\sqrt{-x^2+1}-1}{x} + \frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2} - 1\right)}{3\left(\sqrt{-x^2+1}-1\right)}\right)\right) - \frac{1}{6}\,\sqrt{3}\left(\pi\,\mathrm{sign}\left(x\right) + 2\,\arctan\left(\frac{\sqrt{3}x\left(\frac{\sqrt{-x^2+1}-1}{x} - \frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2} + 1\right)}{3\left(\sqrt{-x^2+1}-1\right)}\right)\right) + \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^2 - 1\right)\right) + \arctan\left(-\frac{x\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2} - 1\right)}{2\left(\sqrt{-x^2+1}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sqrt(-x^2 + 1)*x/(x^3 - x - sqrt(-x^2 + 1)),x, algorithm="giac")
```

```
[Out] 1/2*pi*sign(x) - 1/6*sqrt(3)*(pi*sign(x) + 2*arctan(-1/3*sqrt(3)*
x*((sqrt(-x^2 + 1) - 1)/x + (sqrt(-x^2 + 1) - 1)^2/x^2 - 1)/(sqrt
(-x^2 + 1) - 1))) - 1/6*sqrt(3)*(pi*sign(x) + 2*arctan(1/3*sqrt(3
)*x*((sqrt(-x^2 + 1) - 1)/x - (sqrt(-x^2 + 1) - 1)^2/x^2 + 1)/(sq
rt(-x^2 + 1) - 1))) + 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1))
 + arctan(-1/2*x*((sqrt(-x^2 + 1) - 1)^2/x^2 - 1)/(sqrt(-x^2 + 1)
 - 1))
```

3.885 $\int \left(1+x+x^2+x^3\right)^{-n} \left(1-x^4\right)^n \, dx$

Optimal. Leaf size=34

$$-\frac{(1-x)\left(x^3+x^2+x+1\right)^{-n}\left(1-x^4\right)^n}{n+1}$$

```
[Out] -(((1 - x)*(1 - x^4)^n)/((1 + n)*(1 + x + x^2 + x^3)^n))
```

Rubi [F] time = 0.105439, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.

$$\text{Int}\left(\left(1+x+x^2+x^3\right)^{-n}\left(1-x^4\right)^n, x\right)$$

Verification is Not applicable to the result.

```
[In]  Int[(1 - x^4)^n/(1 + x + x^2 + x^3)^n,x]
```

```
[Out] Defer[Int][(1 - x^4)^n/(1 + x + x^2 + x^3)^n, x]
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: GeneratorsError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**4+1)**n/((x**3+x**2+x+1)**n),x)
```

```
[Out] Exception raised: GeneratorsError
```

Mathematica [A] time = 0.0238496, size = 31, normalized size = 0.91

$$\frac{(x-1)\left(x^3+x^2+x+1\right)^{-n}\left(1-x^4\right)^n}{n+1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - x^4)^n/(1 + x + x^2 + x^3)^n,x]
```

```
[Out] ((-1 + x)*(1 - x^4)^n)/((1 + n)*(1 + x + x^2 + x^3)^n)
```

Maple [A] time = 0.003, size = 32, normalized size = 0.9

$$\frac{(-1+x)\left(-x^4+1\right)^n}{(1+n)\left(x^3+x^2+x+1\right)^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^4+1)^n/((x^3+x^2+x+1)^n),x)
```

```
[Out] (-1+x)/(1+n)*(-x^4+1)^n/((x^3+x^2+x+1)^n)
```

Maxima [A] time = 0.776766, size = 22, normalized size = 0.65

$$\frac{(x-1)(-x+1)^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^4 + 1)^n/(x^3 + x^2 + x + 1)^n,x, algorithm="maxima")
```

```
[Out] (x - 1)*(-x + 1)^n/(n + 1)
```

Fricas [A] time = 0.29238, size = 42, normalized size = 1.24

$$\frac{\left(-x^4+1\right)^n(x-1)}{\left(x^3+x^2+x+1\right)^n(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^4 + 1)^n/(x^3 + x^2 + x + 1)^n,x, algorithm="fricas")
```

```
[Out] (-x^4 + 1)^n*(x - 1)/((x^3 + x^2 + x + 1)^n*(n + 1))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**4+1)**n/((x**3+x**2+x+1)**n),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.254906, size = 39, normalized size = 1.15

$$\frac{xe^{(n\ln(-x+1))} - e^{(n\ln(-x+1))}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^4 + 1)^n/(x^3 + x^2 + x + 1)^n,x, algorithm="giac")
```

```
[Out] (x*e^(n*ln(-x + 1)) - e^(n*ln(-x + 1)))/(n + 1)
```

4 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Mathematica/Rubi followed by one for Maple. The following are links to the source code.

The following are the listing of the above functions.

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)
(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)
```

```
ExpnType[expn_] :=
  If[AtomQ[expn], 1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]

SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]
HypergeometricFunctionQ[func_] := MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
AppellFunctionQ[func_] := MemberQ[{AppellF1},func]
```

```
# File: GradeAntiderivative.mpl Original version thanks to Albert Rich emailed on 03/21/2017
#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);
  #This check below actually is not needed, since I only call this grading only for
  #passed integrals. i.e. I check for "F" before calling this.

  if not type(result,freeof('int')) then
     return "F";
  end if;

  if ExpnType_result<=ExpnType_optimal then
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
         if leaf_count_result<=2*leaf_count_optimal then
            return "A";
         else
            return "B";
         end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      return "C";
  end if
end proc:
```

```
# is_contains_complex(result) takes expressions and returns true if it contains "I"
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1 else
    max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1 else
      max(2,ExpnType(op(1,expn))) end if else
    max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' or op(0,expn)='integrate' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:
```

```
ElementaryFunctionQ := proc(func)
  member(func,[exp,log,ln, sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[erf,erfc,erfi,FresnelS,FresnelC,Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u) else
  apply(op(0,u),op(2..nops(u),u))
  end if
end proc:
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```