Computer algebra independent integration tests

`1_Algebraic_functions/1.3_Miscellaneous/1.3.1Rationalfunctions`

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Contents

1 Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from Albert Rich Rubi web site.

1.1 Listing of CAS systems tested

The following systems were tested at this time.

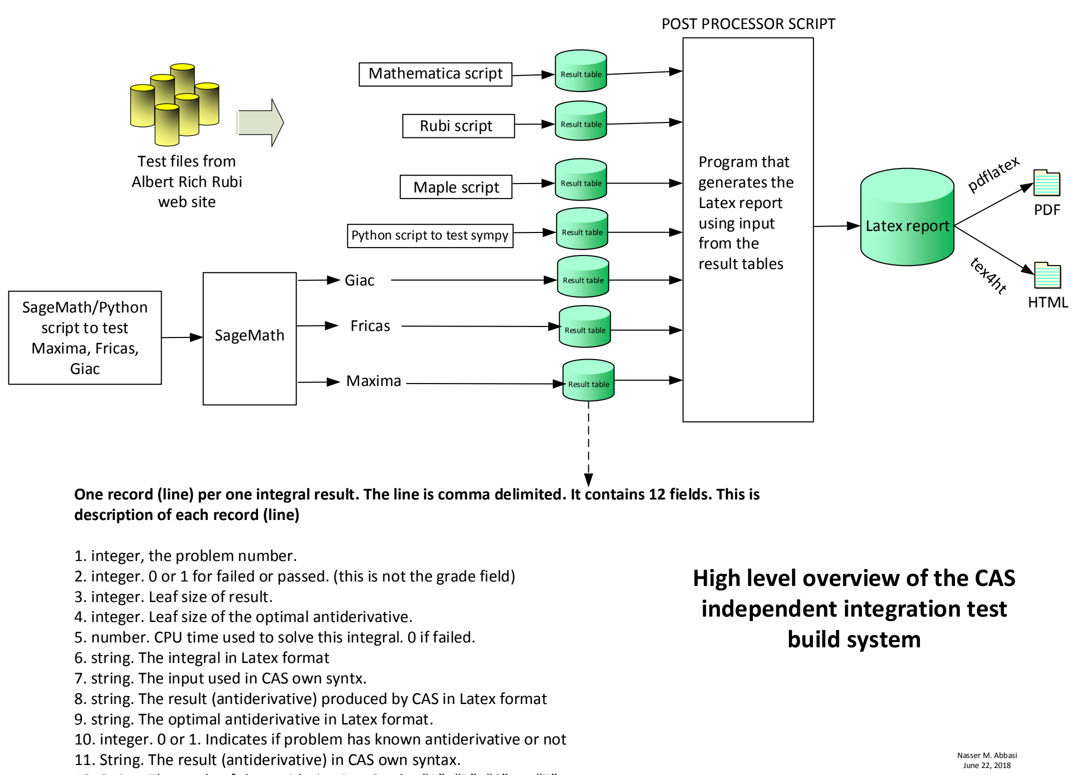
1. Mathematica 11.3 (64 bit).
2. Rubi 4.15.2 in Mathematica 11.3.
3. Rubi in Sympy (Version 1.3) under Python 3.7.0 using Anaconda distribution.
4. Maple 2018.1 (64 bit).
5. Maxima 5.41 Using Lisp ECL 16.1.2.
6. Fricas 1.3.4.
7. Sympy 1.3 under Python 3.7.0 using Anaconda distribution.
8. Giac/Xcas 1.4.9.

Maxima, Fricas and Giac/Xcas were called from inside SageMath version 8.3. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python. Rubi in Sympy was also called directly using sympy 1.3 in python.

1.2 Design of the test system

The following diagram gives a high level view of the current test build system.



1.3 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.4 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.5 Important notes about some of the results

Important note about Maxima results Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

Important note about FriCAS and Giac/XCAS results There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

Important note about finding leaf size of antiderivative For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-express

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems implement a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.6 Grading of results

The table below summarizes the grading of each CAS system.

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 99.39 (491)	% 0.61 (3)
Rubi in Sympy	% 65.38 (323)	% 34.62 (171)
Mathematica	% 100. (494)	% 0. (0)
Maple	% 98.99 (489)	% 1.01 (5)
Maxima	% 73.28 (362)	% 26.72 (132)
Fricas	% 81.78 (404)	% 18.22 (90)
Sympy	% 86.84 (429)	% 13.16 (65)
Giac	% 81.98 (405)	% 18.02 (89)

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

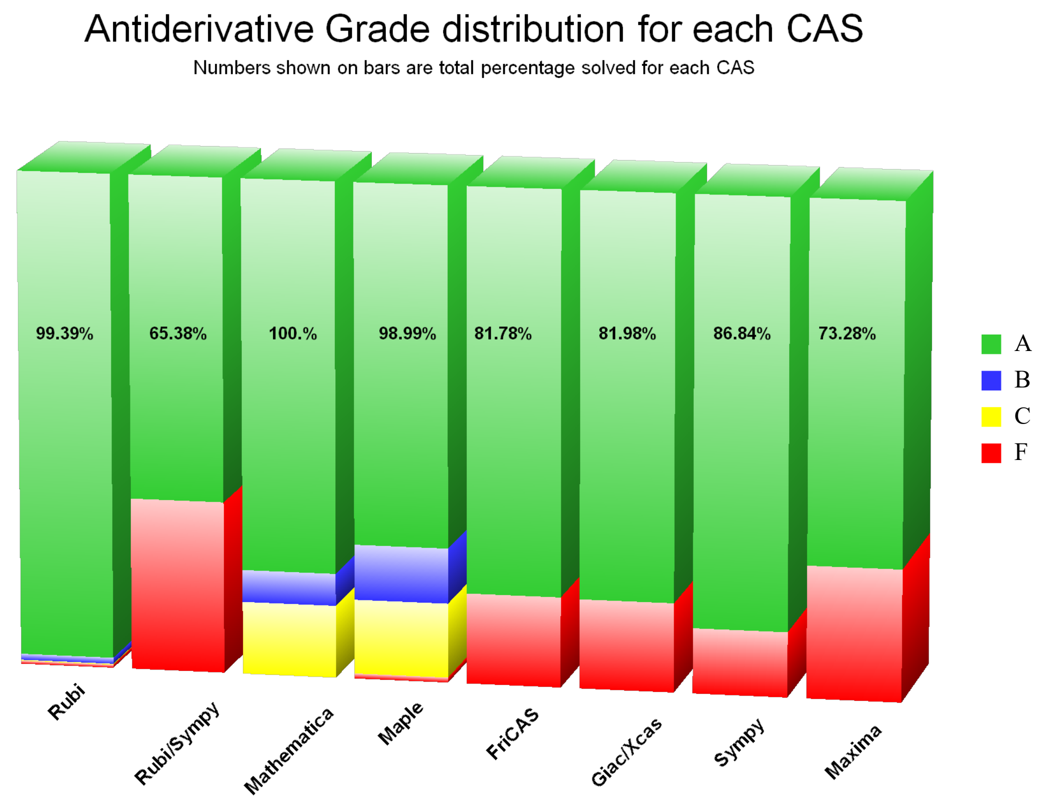
grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

Grading is currently implemented only for for Mathematica, Rubi and Maple results. For all other CAS systems (Maxima, Fricas, Sympy, Giac, Rubi in sympy), the grading function is not yet implemented. For these systems, a grade of A is assigned if the integrate command completes successfully and a grade of F otherwise.

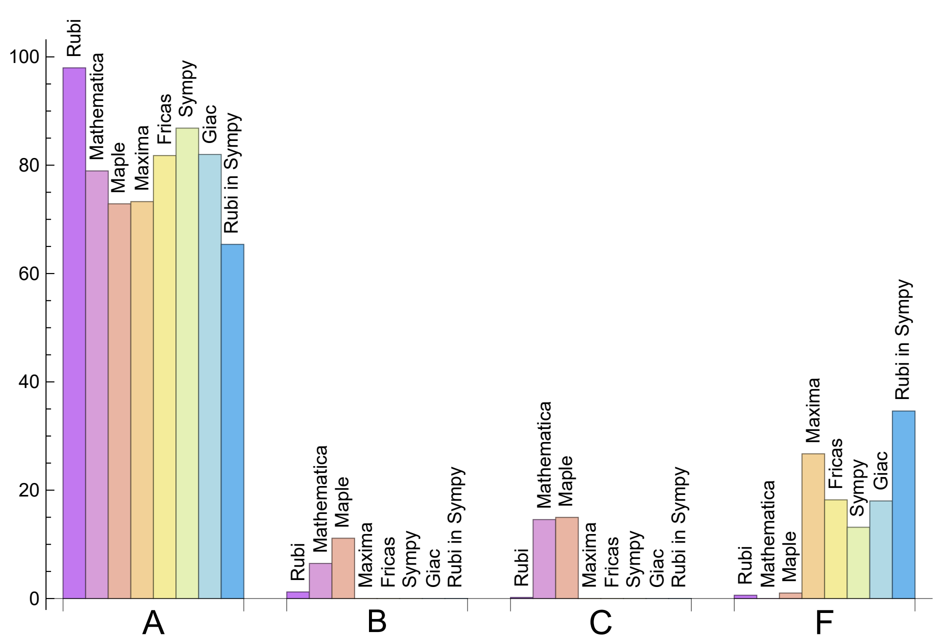
Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	97.98	1.21	0.2	0.61
Rubi in Sympy	65.38	0.	0.	34.62
Mathematica	78.95	6.48	14.57	0.
Maple	72.87	11.13	14.98	1.01
Maxima	73.28	0.	0.	26.72
Fricas	81.78	0.	0.	18.22
Sympy	86.84	0.	0.	13.16
Giac	81.98	0.	0.	18.02

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.7 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.47	119.42	1.04	33.	1.
Rubi in Sympy	28.2	67.42	0.92	26.	0.86
Mathematica	0.1	80.63	1.21	36.5	1.
Maple	0.01	208.61	6.06	34.	0.91
Maxima	0.84	119.82	2.49	35.	1.18
Fricas	0.28	108.66	1.73	28.	1.22
Sympy	2.83	100.48	2.12	39.	0.89
Giac	0.27	88.72	2.38	38.	1.23

1.8 list of integrals that has no closed form antiderivative

{}

1.9 list of integrals not solved by each system

Not solved by Rubi {393, 493, 494}

Not solved by Rubi in Sympy {1, 5, 10, 11, 15, 16, 17, 18, 19, 20, 33, 34, 35, 37, 38, 40, 41, 42, 44, 48, 54, 60, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 74, 75, 76, 77, 78, 98, 103, 109, 115, 116, 117, 118, 119, 123, 124, 125, 126, 130, 131, 132, 133, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 163, 171, 177, 178, 219, 220, 227, 230, 233, 234, 235, 236, 237, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 258, 263, 265, 266, 267, 269, 277, 279, 285, 290, 291, 293, 294, 295, 296, 299, 300, 301, 303, 304, 306, 308, 311, 315, 318, 320, 321, 332, 334, 339, 363, 364, 366, 368, 375, 378, 379, 391, 393, 399, 400, 405, 406, 407, 412, 413, 414, 427, 428, 429, 430, 444, 448, 462, 463, 466, 473, 474, 476, 478, 481, 482, 483, 486, 487, 493, 494}

Not solved by Mathematica {}

Not solved by Maple {29, 30, 31, 32, 176}

Not solved by Maxima {1, 12, 13, 14, 26, 27, 29, 30, 31, 32, 37, 38, 43, 44, 49, 50, 55, 56, 61, 62, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 120, 121, 122, 127, 128, 129, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 163, 172, 179, 180, 181, 183, 184, 186, 187, 227, 228, 250, 251, 252, 253, 254, 255, 256, 257, 334, 337, 338, 340, 341, 342, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 459, 489, 490, 491}

Not solved by Fricas {19, 20, 29, 30, 31, 32, 49, 50, 55, 56, 61, 62, 103, 104, 105, 107, 108, 109, 110, 111, 112, 114, 115, 127, 128, 129, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 227, 250, 251, 252, 253, 254, 255, 256, 334, 337, 338, 341, 342, 368, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 398, 399, 400, 401, 402, 403, 405, 406, 407, 408, 409, 410, 412, 413, 414, 483}

Not solved by Sympy {2, 18, 19, 20, 28, 29, 30, 31, 100, 101, 102, 108, 109, 114, 115, 122, 129, 136, 137, 139, 140, 141, 142, 163, 169, 170, 171, 173, 174, 175, 176, 180, 181, 182, 183, 185, 186, 190, 208, 209, 216, 217, 218, 227, 234, 235, 236, 237, 238, 239, 240, 241, 337, 338, 341, 342, 398, 399, 400, 405, 406, 407, 412, 413, 414}

Not solved by Giac {12, 13, 14, 18, 29, 30, 31, 32, 37, 38, 43, 44, 49, 50, 55, 56, 61, 62, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 120, 121, 122, 127, 128, 129, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 167, 171, 174, 176, 227, 238, 239, 240, 241, 250, 251, 252, 253, 254, 255, 256, 257, 339, 387, 388, 389, 390, 391, 392, 393, 399, 491, 493}

1.10 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Rubi in Sympy {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {154, 155, 156, 157}

Mathematica {31, 32}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Rubi in Sympy Verification phase not implemented yet.

2 detailed summary tables of results

2.1 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	143	43	0	105	60	73	0
normalized size	1	1.	1.86	0.56	0.	1.36	0.78	0.95	0.
time (sec)	N/A	0.131	0.07	0.009	0.	0.287	1.755	0.28	0.

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	23	46	34	58	0	104	80
normalized size	1	1.	0.77	1.53	1.13	1.93	0.	3.47	2.67
time (sec)	N/A	0.036	0.019	0.002	0.784	0.301	0.	0.264	19.648

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	98	292	1	107	131	8
normalized size	1	1.	1.	7.	20.86	0.07	7.64	9.36	0.57
time (sec)	N/A	0.017	0.002	0.002	0.788	0.252	0.15	0.26	15.695

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	65	134	1	66	86	8
normalized size	1	1.	1.	4.64	9.57	0.07	4.71	6.14	0.57
time (sec)	N/A	0.017	0.002	0.001	0.776	0.249	0.126	0.262	15.622

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	32	42	1	32	42	0
normalized size	1	1.	1.	0.91	1.2	0.03	0.91	1.2	0.
time (sec)	N/A	0.018	0.	0.001	0.758	0.238	0.085	0.26	0.

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	32	32	26	16	12
normalized size	1	1.	1.	0.93	2.29	2.29	1.86	1.14	0.86
time (sec)	N/A	0.019	0.005	0.006	0.76	0.273	1.253	0.26	15.719

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	77	77	61	16	12
normalized size	1	1.	1.	0.93	5.5	5.5	4.36	1.14	0.86
time (sec)	N/A	0.018	0.007	0.004	0.779	0.274	1.866	0.26	15.686

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	122	122	97	16	12
normalized size	1	1.	1.	0.93	8.71	8.71	6.93	1.14	0.86
time (sec)	N/A	0.018	0.005	0.004	0.788	0.269	2.701	0.259	15.688

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	159	295	275	1	175	224	83
normalized size	1	1.	1.89	3.51	3.27	0.01	2.08	2.67	0.99
time (sec)	N/A	0.248	0.035	0.001	0.794	0.253	0.195	0.258	34.021

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	82	84	126	1	87	112	0
normalized size	1	1.	1.46	1.5	2.25	0.02	1.55	2.	0.
time (sec)	N/A	0.139	0.016	0.001	0.767	0.265	0.127	0.26	0.

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	29	38	1	31	38	0
normalized size	1	1.	1.	0.91	1.19	0.03	0.97	1.19	0.
time (sec)	N/A	0.017	0.	0.001	0.771	0.279	0.091	0.259	0.

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	63	57	0	351	53	0	184
normalized size	1	1.	0.34	0.3	0.	1.87	0.28	0.	0.98
time (sec)	N/A	0.652	0.026	0.004	0.	0.276	1.178	0.	74.946

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	112	136	0	610	192	0	228
normalized size	1	1.	0.46	0.56	0.	2.49	0.78	0.	0.93
time (sec)	N/A	0.612	0.095	0.013	0.	0.29	5.623	0.	92.298

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	149	276	0	1160	474	0	280
normalized size	1	1.	0.49	0.9	0.	3.8	1.55	0.	0.92
time (sec)	N/A	0.76	0.142	0.023	0.	0.288	23.604	0.	112.989

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	361	361	653	861	622	1	1018	1	0
normalized size	1	1.	1.81	2.39	1.72	0.	2.82	0.	0.
time (sec)	N/A	1.798	0.47	0.002	0.779	0.257	0.657	0.258	0.

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	241	188	243	1	345	467	0
normalized size	1	1.	1.25	0.97	1.26	0.01	1.79	2.42	0.
time (sec)	N/A	0.606	0.146	0.001	0.776	0.268	0.31	0.263	0.

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	76	51	68	1	63	73	0
normalized size	1	1.	1.36	0.91	1.21	0.02	1.12	1.3	0.
time (sec)	N/A	0.046	0.	0.001	0.768	0.278	0.11	0.263	0.

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	80	87	151	151	0	0	0
normalized size	1	1.	0.93	1.01	1.76	1.76	0.	0.	0.
time (sec)	N/A	0.147	0.088	0.012	0.784	5.575	0.	0.	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	F(-1)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	232	398	2830	0	0	1	0
normalized size	1	1.	0.99	1.7	12.09	0.	0.	0.	0.
time (sec)	N/A	0.888	1.33	0.066	0.865	0.	0.	0.28	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	F(-1)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	495	495	490	1076	14857	0	0	1	0
normalized size	1	1.	0.99	2.17	30.01	0.	0.	0.	0.
time (sec)	N/A	3.587	2.831	0.047	1.389	0.	0.	0.38	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	26	26	19	27	31
normalized size	1	1.	1.	0.8	1.04	1.04	0.76	1.08	1.24
time (sec)	N/A	0.033	0.009	0.006	0.855	0.32	0.255	0.262	128.344

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	26	34	34	24	35	34
normalized size	1	1.	1.	0.84	1.1	1.1	0.77	1.13	1.1
time (sec)	N/A	0.043	0.01	0.01	0.859	0.321	0.291	0.261	32.079

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	11	11	8	11	8
normalized size	1	1.	1.	0.9	1.1	1.1	0.8	1.1	0.8
time (sec)	N/A	0.005	0.001	0.001	0.77	0.298	0.072	0.261	1.55

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	29	38	35	19	41	24
normalized size	1	1.	1.	1.04	1.36	1.25	0.68	1.46	0.86
time (sec)	N/A	0.033	0.008	0.014	0.765	0.317	1.274	0.26	14.178

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	27	24	15	32	19
normalized size	1	1.	1.	0.95	1.23	1.09	0.68	1.45	0.86
time (sec)	N/A	0.03	0.007	0.007	0.778	0.31	0.493	0.26	156.994

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	61	62	0	1	564	84	54
normalized size	1	1.	0.98	1.	0.	0.02	9.1	1.35	0.87
time (sec)	N/A	0.112	0.132	0.013	0.	0.305	6.673	0.264	20.222

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	89	91	0	120	20	151	109
normalized size	1	1.	0.77	0.79	0.	1.04	0.17	1.31	0.95
time (sec)	N/A	0.133	0.046	0.007	0.	0.328	0.357	0.263	25.01

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	23	22	0	24	22
normalized size	1	1.	1.	1.06	1.44	1.38	0.	1.5	1.38
time (sec)	N/A	0.011	0.004	0.003	0.8	0.336	0.	0.261	1.904

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	53	0	0	0	0	0	53
normalized size	1	1.	0.96	0.	0.	0.	0.	0.	0.96
time (sec)	N/A	0.052	0.044	0.053	0.	0.	0.	0.	8.255

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	59	61	0	0	0	0	0	53
normalized size	1	1.11	1.15	0.	0.	0.	0.	0.	1.
time (sec)	N/A	0.058	0.047	0.053	0.	0.	0.	0.	9.851

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	438	0	0	0	0	0	117
normalized size	1	1.	3.32	0.	0.	0.	0.	0.	0.89
time (sec)	N/A	0.392	4.46	0.03	0.	0.	0.	0.	29.702

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	35	44	196	0	0	0	34	0	34
normalized size	1	1.26	5.6	0.	0.	0.	0.97	0.	0.97
time (sec)	N/A	0.024	0.304	0.048	0.	0.	68.839	0.	3.717

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	285	392	502	1	299	374	0
normalized size	1	1.	1.06	1.45	1.86	0.	1.11	1.39	0.
time (sec)	N/A	1.049	0.064	0.002	0.789	0.243	0.257	0.262	0.

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	171	231	277	1	180	224	0
normalized size	1	1.	1.	1.35	1.62	0.01	1.05	1.31	0.
time (sec)	N/A	0.193	0.032	0.001	0.782	0.23	0.187	0.259	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	92	84	127	1	95	112	0
normalized size	1	1.	1.	0.91	1.38	0.01	1.03	1.22	0.
time (sec)	N/A	0.089	0.014	0.001	0.768	0.246	0.133	0.263	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	29	38	1	31	38	31
normalized size	1	1.	1.	0.91	1.19	0.03	0.97	1.19	0.97
time (sec)	N/A	0.017	0.	0.001	0.766	0.228	0.081	0.262	3.246

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	529	529	71	64	0	1222	88	0	0
normalized size	1	1.	0.13	0.12	0.	2.31	0.17	0.	0.
time (sec)	N/A	1.998	0.045	0.056	0.	0.3	3.144	0.	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	746	746	182	232	0	4350	427	0	0
normalized size	1	1.	0.24	0.31	0.	5.83	0.57	0.	0.
time (sec)	N/A	4.064	0.173	0.045	0.	0.347	20.224	0.	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	345	500	517	1	366	436	262
normalized size	1	1.	1.17	1.69	1.75	0.	1.24	1.48	0.89
time (sec)	N/A	1.027	0.092	0.003	0.786	0.255	0.288	0.26	111.56

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	207	288	289	1	218	252	0
normalized size	1	1.	1.02	1.42	1.42	0.	1.07	1.24	0.
time (sec)	N/A	0.254	0.046	0.002	0.788	0.231	0.208	0.261	0.

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	109	100	136	1	112	122	0
normalized size	1	1.	1.02	0.93	1.27	0.01	1.05	1.14	0.
time (sec)	N/A	0.108	0.021	0.001	0.767	0.236	0.143	0.26	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	34	45	1	36	41	0
normalized size	1	1.	1.	0.92	1.22	0.03	0.97	1.11	0.
time (sec)	N/A	0.021	0.	0.001	0.797	0.229	0.095	0.261	0.

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	71	67	0	1505	122	0	148
normalized size	1	1.	0.46	0.44	0.	9.84	0.8	0.	0.97
time (sec)	N/A	0.578	0.038	0.093	0.	0.282	5.017	0.	78.714

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	342	342	234	288	0	5785	580	0	0
normalized size	1	1.	0.68	0.84	0.	16.92	1.7	0.	0.
time (sec)	N/A	1.437	0.282	0.051	0.	0.417	44.809	0.	0.

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	96	85	113	1	94	113	94
normalized size	1	1.	1.	0.89	1.18	0.01	0.98	1.18	0.98
time (sec)	N/A	0.063	0.002	0.003	0.774	0.256	0.111	0.263	47.027

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	65	86	1	71	86	71
normalized size	1	1.	1.	0.88	1.16	0.01	0.96	1.16	0.96
time (sec)	N/A	0.047	0.002	0.002	0.773	0.289	0.095	0.257	34.932

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	59	1	49	59	49
normalized size	1	1.	1.	0.83	1.09	0.02	0.91	1.09	0.91
time (sec)	N/A	0.033	0.002	0.001	0.762	0.239	0.079	0.26	23.629

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	26	1	19	26	0
normalized size	1	1.	1.	0.87	1.13	0.04	0.83	1.13	0.
time (sec)	N/A	0.01	0.	0.001	0.777	0.256	0.059	0.263	0.

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	45	41	0	0	41	0	382
normalized size	1	1.	0.17	0.15	0.	0.	0.15	0.	1.43
time (sec)	N/A	0.868	0.014	0.007	0.	0.	3.041	0.	65.258

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	357	357	113	83	0	0	71	0	439
normalized size	1	1.	0.32	0.23	0.	0.	0.2	0.	1.23
time (sec)	N/A	1.016	0.027	0.012	0.	0.	3.391	0.	80.7

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	97	78	104	1	94	104	94
normalized size	1	1.	1.	0.8	1.07	0.01	0.97	1.07	0.97
time (sec)	N/A	0.059	0.002	0.002	0.803	0.227	0.105	0.259	41.489

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	77	1	66	77	66
normalized size	1	1.	1.	0.84	1.12	0.01	0.96	1.12	0.96
time (sec)	N/A	0.042	0.001	0.002	0.789	0.228	0.093	0.26	31.77

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	38	50	1	42	50	42
normalized size	1	1.	1.	0.84	1.11	0.02	0.93	1.11	0.93
time (sec)	N/A	0.03	0.002	0.001	0.785	0.223	0.077	0.26	24.129

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	23	1	19	23	0
normalized size	1	1.	1.	0.86	1.1	0.05	0.9	1.1	0.
time (sec)	N/A	0.009	0.	0.001	0.809	0.237	0.059	0.26	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	47	41	0	0	36	0	345
normalized size	1	1.	0.2	0.18	0.	0.	0.15	0.	1.47
time (sec)	N/A	0.704	0.024	0.007	0.	0.	2.769	0.	62.878

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	108	79	0	0	71	0	393
normalized size	1	1.	0.34	0.25	0.	0.	0.22	0.	1.24
time (sec)	N/A	0.871	0.04	0.013	0.	0.	3.39	0.	77.583

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	104	85	113	1	100	113	100
normalized size	1	1.	1.	0.82	1.09	0.01	0.96	1.09	0.96
time (sec)	N/A	0.067	0.003	0.002	0.808	0.224	0.134	0.262	74.742

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	76	65	86	1	73	86	73
normalized size	1	1.	1.	0.86	1.13	0.01	0.96	1.13	0.96
time (sec)	N/A	0.05	0.002	0.002	0.806	0.237	0.111	0.262	61.257

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	45	59	1	49	59	49
normalized size	1	1.	1.	0.87	1.13	0.02	0.94	1.13	0.94
time (sec)	N/A	0.035	0.002	0.002	0.8	0.224	0.086	0.26	50.015

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	32	1	27	32	0
normalized size	1	1.	1.	0.83	1.07	0.03	0.9	1.07	0.
time (sec)	N/A	0.012	0.	0.	0.8	0.233	0.063	0.261	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	55	49	0	0	41	0	372
normalized size	1	1.	0.21	0.19	0.	0.	0.16	0.	1.41
time (sec)	N/A	1.147	0.016	0.007	0.	0.	3.046	0.	89.264

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	366	366	128	96	0	0	76	0	428
normalized size	1	1.	0.35	0.26	0.	0.	0.21	0.	1.17
time (sec)	N/A	1.414	0.031	0.012	0.	0.	3.841	0.	103.641

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	164	799	1	185	220	0
normalized size	1	1.	1.	11.71	57.07	0.07	13.21	15.71	0.
time (sec)	N/A	0.035	0.003	0.002	0.814	0.239	0.233	0.26	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	109	308	1	114	146	0
normalized size	1	1.	1.	7.79	22.	0.07	8.14	10.43	0.
time (sec)	N/A	0.032	0.002	0.002	0.812	0.228	0.172	0.261	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	B	B	B	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	61	61	54	72	1	60	72	0
normalized size	1	4.36	4.36	3.86	5.14	0.07	4.29	5.14	0.
time (sec)	N/A	0.034	0.	0.001	0.795	0.237	0.105	0.259	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	62	62	49	16	0
normalized size	1	1.	1.	0.93	4.43	4.43	3.5	1.14	0.
time (sec)	N/A	0.035	0.006	0.006	0.832	0.271	1.684	0.259	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	136	136	109	16	0
normalized size	1	1.	1.	0.93	9.71	9.71	7.79	1.14	0.
time (sec)	N/A	0.033	0.005	0.004	0.824	0.247	3.076	0.258	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	211	211	168	16	0
normalized size	1	1.	1.	0.93	15.07	15.07	12.	1.14	0.
time (sec)	N/A	0.033	0.006	0.004	0.82	0.271	5.72	0.261	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	31	41	41	29	42	0
normalized size	1	1.	1.	0.82	1.08	1.08	0.76	1.11	0.
time (sec)	N/A	0.049	0.011	0.01	0.909	0.257	0.319	0.263	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	84	65	86	1	80	86	0
normalized size	1	1.	1.	0.77	1.02	0.01	0.95	1.02	0.
time (sec)	N/A	0.331	0.002	0.003	0.816	0.241	0.12	0.26	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	50	66	1	60	66	0
normalized size	1	1.	1.	0.79	1.05	0.02	0.95	1.05	0.
time (sec)	N/A	0.293	0.003	0.002	0.811	0.256	0.101	0.261	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	35	46	1	41	46	0
normalized size	1	1.	1.	0.8	1.05	0.02	0.93	1.05	0.
time (sec)	N/A	0.233	0.001	0.002	0.819	0.23	0.085	0.26	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	26	1	22	26	22
normalized size	1	1.	1.	0.8	1.04	0.04	0.88	1.04	0.88
time (sec)	N/A	0.01	0.	0.001	0.824	0.23	0.065	0.261	1.918

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	62	42	73	88	63	84	0
normalized size	1	1.	2.	1.35	2.35	2.84	2.03	2.71	0.
time (sec)	N/A	0.049	0.024	0.015	0.885	0.254	0.322	0.262	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	103	84	120	266	104	131	0
normalized size	1	1.	1.16	0.94	1.35	2.99	1.17	1.47	0.
time (sec)	N/A	0.123	0.088	0.031	0.895	0.26	3.853	0.263	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	137	126	161	408	134	151	0
normalized size	1	1.	0.85	0.78	1.	2.53	0.83	0.94	0.
time (sec)	N/A	0.244	0.155	0.035	0.917	0.274	4.311	0.262	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	B	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	205	132	116	162	324	296	181	0
normalized size	1	2.25	1.45	1.27	1.78	3.56	3.25	1.99	0.
time (sec)	N/A	0.265	0.146	0.028	0.888	0.291	4.001	0.269	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	73	127	0	1	209	104	0
normalized size	1	1.	0.94	1.63	0.	0.01	2.68	1.33	0.
time (sec)	N/A	0.14	0.085	0.011	0.	0.258	2.843	0.263	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	54	89	0	1	153	73	49
normalized size	1	1.	1.08	1.78	0.	0.02	3.06	1.46	0.98
time (sec)	N/A	0.092	0.046	0.004	0.	0.28	2.182	0.263	13.093

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	38	54	0	1	124	58	36
normalized size	1	1.	0.93	1.32	0.	0.02	3.02	1.41	0.88
time (sec)	N/A	0.053	0.022	0.004	0.	0.257	0.645	0.267	7.569

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	28	0	1	54	23	17
normalized size	1	1.	1.	1.33	0.	0.05	2.57	1.1	0.81
time (sec)	N/A	0.02	0.006	0.002	0.	0.274	0.444	0.26	2.476

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	48	72	0	1	738	84	53
normalized size	1	1.	0.81	1.22	0.	0.02	12.51	1.42	0.9
time (sec)	N/A	0.082	0.057	0.008	0.	0.274	5.978	0.261	10.961

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	81	123	0	1	1620	158	76
normalized size	1	1.	1.03	1.56	0.	0.01	20.51	2.	0.96
time (sec)	N/A	0.191	0.075	0.011	0.	0.278	15.447	0.266	24.022

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	106	198	0	1	3284	263	112
normalized size	1	1.	0.88	1.64	0.	0.01	27.14	2.17	0.93
time (sec)	N/A	0.277	0.225	0.012	0.	0.295	24.795	0.265	37.117

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	34	0	1	61	32	27
normalized size	1	1.	1.	1.1	0.	0.03	1.97	1.03	0.87
time (sec)	N/A	0.037	0.015	0.008	0.	0.259	0.632	0.262	4.631

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	60	86	0	1	117	88	49
normalized size	1	1.	0.95	1.37	0.	0.02	1.86	1.4	0.78
time (sec)	N/A	0.065	0.042	0.005	0.	0.275	2.956	0.263	6.905

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	75	147	0	1	257	139	78
normalized size	1	1.	0.82	1.62	0.	0.01	2.82	1.53	0.86
time (sec)	N/A	0.094	0.111	0.007	0.	0.297	6.45	0.26	9.713

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	42	89	1	92	41	31
normalized size	1	1.	1.	1.2	2.54	0.03	2.63	1.17	0.89
time (sec)	N/A	0.067	0.024	0.012	0.888	0.285	0.552	0.261	10.251

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	24	14	24	14	7
normalized size	1	1.	1.	1.1	2.4	1.4	2.4	1.4	0.7
time (sec)	N/A	0.009	0.006	0.005	0.903	0.252	0.385	0.26	1.627

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	31	59	69	74	56	55	26
normalized size	1	1.	0.84	1.59	1.86	2.	1.51	1.49	0.7
time (sec)	N/A	0.024	0.019	0.008	0.907	0.254	2.312	0.263	2.461

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	52	94	155	207	146	99	49
normalized size	1	1.	0.87	1.57	2.58	3.45	2.43	1.65	0.82
time (sec)	N/A	0.037	0.025	0.005	0.902	0.258	4.928	0.261	3.207

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	32	26	34	30	22	36	7
normalized size	1	1.	3.2	2.6	3.4	3.	2.2	3.6	0.7
time (sec)	N/A	0.009	0.008	0.01	0.816	0.272	0.4	0.262	1.686

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	45	52	76	115	53	76	26
normalized size	1	1.	1.15	1.33	1.95	2.95	1.36	1.95	0.67
time (sec)	N/A	0.029	0.033	0.014	0.804	0.258	2.279	0.265	2.6

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	65	78	165	297	141	119	49
normalized size	1	1.	1.02	1.22	2.58	4.64	2.2	1.86	0.77
time (sec)	N/A	0.049	0.045	0.015	0.826	0.269	4.881	0.263	3.413

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	15	12	15	15	10	18	3
normalized size	1	1.	3.75	3.	3.75	3.75	2.5	4.5	0.75
time (sec)	N/A	0.006	0.004	0.007	0.805	0.257	0.178	0.262	0.343

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	26	24	34	53	22	36	17
normalized size	1	1.	0.96	0.89	1.26	1.96	0.81	1.33	0.63
time (sec)	N/A	0.017	0.03	0.013	0.797	0.274	0.237	0.262	1.379

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	37	36	59	96	44	53	34
normalized size	1	1.	0.82	0.8	1.31	2.13	0.98	1.18	0.76
time (sec)	N/A	0.03	0.029	0.013	0.807	0.266	0.339	0.262	1.661

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	64	64	73	73	58	84	0
normalized size	1	1.	1.08	1.08	1.24	1.24	0.98	1.42	0.
time (sec)	N/A	0.115	0.037	0.002	0.797	0.272	1.272	0.262	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	11	12	15	15	10	15	10
normalized size	1	1.	1.1	1.2	1.5	1.5	1.	1.5	1.
time (sec)	N/A	0.029	0.011	0.003	0.802	0.289	0.148	0.265	5.572

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	51	35	49	47	0	31	34
normalized size	1	1.	1.16	0.8	1.11	1.07	0.	0.7	0.77
time (sec)	N/A	0.066	0.033	0.006	0.89	0.28	0.	0.266	8.278

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	55	152	0	100	0	74	54
normalized size	1	1.	0.82	2.27	0.	1.49	0.	1.1	0.81
time (sec)	N/A	0.121	0.075	0.022	0.	0.276	0.	0.281	14.127

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	51	146	0	369	0	95	54
normalized size	1	1.	0.81	2.32	0.	5.86	0.	1.51	0.86
time (sec)	N/A	0.101	0.066	0.013	0.	0.269	0.	0.274	11.261

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	132	108	0	0	238	0	0
normalized size	1	1.	0.56	0.46	0.	0.	1.02	0.	0.
time (sec)	N/A	0.699	0.076	0.007	0.	0.	8.967	0.	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	81	74	0	0	158	0	204
normalized size	1	1.	0.39	0.35	0.	0.	0.75	0.	0.97
time (sec)	N/A	0.473	0.045	0.006	0.	0.	3.167	0.	52.289

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	79	72	0	0	83	0	175
normalized size	1	1.	0.44	0.4	0.	0.	0.46	0.	0.97
time (sec)	N/A	0.332	0.037	0.004	0.	0.	2.214	0.	39.679

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	116	71	0	161	26	0	134
normalized size	1	1.	0.83	0.51	0.	1.15	0.19	0.	0.96
time (sec)	N/A	0.225	0.054	0.003	0.	0.268	0.695	0.	30.099

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	238	119	105	0	0	559	0	219
normalized size	1	1.06	0.53	0.47	0.	0.	2.5	0.	0.98
time (sec)	N/A	0.609	0.077	0.009	0.	0.	52.142	0.	74.278

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-2)	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	314	312	173	144	0	0	0	0	314
normalized size	1	0.99	0.55	0.46	0.	0.	0.	0.	1.
time (sec)	N/A	1.091	0.141	0.014	0.	0.	0.	0.	133.834

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	393	393	244	217	0	0	0	0	0
normalized size	1	1.	0.62	0.55	0.	0.	0.	0.	0.
time (sec)	N/A	1.211	0.23	0.017	0.	0.	0.	0.	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	106	97	0	0	374	0	338
normalized size	1	1.	0.3	0.27	0.	0.	1.05	0.	0.95
time (sec)	N/A	0.83	0.067	0.019	0.	0.	11.154	0.	91.503

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	318	106	97	0	0	274	0	301
normalized size	1	1.	0.33	0.31	0.	0.	0.86	0.	0.95
time (sec)	N/A	0.654	0.053	0.005	0.	0.	8.282	0.	80.479

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	104	95	0	0	131	0	250
normalized size	1	1.	0.4	0.36	0.	0.	0.5	0.	0.96
time (sec)	N/A	0.543	0.043	0.006	0.	0.	2.924	0.	67.394

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	161	94	0	215	26	0	202
normalized size	1	1.	0.73	0.43	0.	0.97	0.12	0.	0.91
time (sec)	N/A	0.373	0.136	0.004	0.	0.264	0.838	0.	52.5

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-2)	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	393	393	163	139	0	0	0	0	360
normalized size	1	1.	0.41	0.35	0.	0.	0.	0.	0.92
time (sec)	N/A	0.985	0.106	0.011	0.	0.	0.	0.	115.583

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	496	496	238	188	0	0	0	0	0
normalized size	1	1.	0.48	0.38	0.	0.	0.	0.	0.
time (sec)	N/A	1.923	0.195	0.017	0.	0.	0.	0.	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	195	264	259	1	199	296	0
normalized size	1	1.	1.59	2.15	2.11	0.01	1.62	2.41	0.
time (sec)	N/A	0.424	0.047	0.002	0.82	0.226	0.23	0.26	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	114	138	161	1	114	173	0
normalized size	1	1.	0.95	1.15	1.34	0.01	0.95	1.44	0.
time (sec)	N/A	0.119	0.026	0.002	0.799	0.239	0.164	0.259	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	66	63	88	1	65	88	0
normalized size	1	1.	0.92	0.88	1.22	0.01	0.9	1.22	0.
time (sec)	N/A	0.06	0.014	0.001	0.803	0.26	0.107	0.263	0.

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	23	30	1	22	30	0
normalized size	1	1.	1.	0.88	1.15	0.04	0.85	1.15	0.
time (sec)	N/A	0.012	0.	0.001	0.803	0.248	0.067	0.262	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	57	49	0	617	66	0	73
normalized size	1	1.	0.64	0.55	0.	6.93	0.74	0.	0.82
time (sec)	N/A	0.175	0.023	0.023	0.	0.271	2.28	0.	31.268

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	150	158	0	2630	292	0	143
normalized size	1	1.	0.89	0.93	0.	15.56	1.73	0.	0.85
time (sec)	N/A	0.627	0.08	0.029	0.	0.287	14.985	0.	58.933

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	254	398	0	5361	0	0	226
normalized size	1	1.	1.01	1.58	0.	21.27	0.	0.	0.9
time (sec)	N/A	1.414	0.188	0.045	0.	0.286	0.	0.	120.835

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	204	267	246	1	212	300	0
normalized size	1	1.	0.97	1.27	1.17	0.	1.01	1.43	0.
time (sec)	N/A	0.483	0.048	0.002	0.815	0.226	0.249	0.26	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	130	143	153	1	128	180	0
normalized size	1	1.	0.97	1.07	1.14	0.01	0.96	1.34	0.
time (sec)	N/A	0.295	0.032	0.002	0.802	0.232	0.177	0.259	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	75	66	80	1	70	92	0
normalized size	1	1.	0.95	0.84	1.01	0.01	0.89	1.16	0.
time (sec)	N/A	0.164	0.015	0.002	0.805	0.226	0.114	0.259	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	28	36	1	29	36	0
normalized size	1	1.	1.	0.8	1.03	0.03	0.83	1.03	0.
time (sec)	N/A	0.027	0.002	0.001	0.809	0.239	0.065	0.261	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	59	50	0	0	155	0	99
normalized size	1	1.	0.51	0.43	0.	0.	1.34	0.	0.85
time (sec)	N/A	0.241	0.028	0.004	0.	0.	8.243	0.	43.048

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	166	162	0	0	539	0	184
normalized size	1	1.	0.72	0.7	0.	0.	2.33	0.	0.8
time (sec)	N/A	0.74	0.099	0.025	0.	0.	45.002	0.	82.061

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-2)	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	284	405	0	0	0	0	292
normalized size	1	1.	0.81	1.16	0.	0.	0.	0.	0.84
time (sec)	N/A	1.513	0.208	0.036	0.	0.	0.	0.	161.775

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	204	267	246	1	219	300	0
normalized size	1	1.	0.97	1.27	1.17	0.	1.04	1.43	0.
time (sec)	N/A	0.505	0.051	0.002	0.788	0.23	0.257	0.26	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	132	143	153	1	134	180	0
normalized size	1	1.	0.96	1.04	1.11	0.01	0.97	1.3	0.
time (sec)	N/A	0.324	0.03	0.002	0.803	0.237	0.177	0.262	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	73	66	80	1	73	92	0
normalized size	1	1.	0.92	0.84	1.01	0.01	0.92	1.16	0.
time (sec)	N/A	0.19	0.019	0.001	0.803	0.228	0.116	0.26	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	28	36	1	29	36	0
normalized size	1	1.	1.	0.8	1.03	0.03	0.83	1.03	0.
time (sec)	N/A	0.026	0.002	0.001	0.782	0.236	0.072	0.259	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	61	52	0	0	172	0	85
normalized size	1	1.	0.62	0.53	0.	0.	1.74	0.	0.86
time (sec)	N/A	0.22	0.029	0.004	0.	0.	11.916	0.	54.805

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	182	160	0	0	559	0	180
normalized size	1	1.	0.81	0.71	0.	0.	2.48	0.	0.8
time (sec)	N/A	0.536	0.102	0.015	0.	0.	49.138	0.	78.192

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	545	545	99	93	0	0	0	0	0
normalized size	1	1.	0.18	0.17	0.	0.	0.	0.	0.
time (sec)	N/A	4.493	0.095	0.014	0.	0.	0.	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	487	487	99	93	0	0	0	0	0
normalized size	1	1.	0.2	0.19	0.	0.	0.	0.	0.
time (sec)	N/A	3.088	0.078	0.005	0.	0.	0.	0.	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-2)	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	334	334	97	93	0	0	167	0	0
normalized size	1	1.	0.29	0.28	0.	0.	0.5	0.	0.
time (sec)	N/A	2.268	0.067	0.006	0.	0.	81.725	0.	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	469	469	95	91	0	0	0	0	0
normalized size	1	1.	0.2	0.19	0.	0.	0.	0.	0.
time (sec)	N/A	2.964	0.068	0.005	0.	0.	0.	0.	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	522	522	99	90	0	0	0	0	0
normalized size	1	1.	0.19	0.17	0.	0.	0.	0.	0.
time (sec)	N/A	3.523	0.09	0.005	0.	0.	0.	0.	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	563	563	157	134	0	0	0	0	0
normalized size	1	1.	0.28	0.24	0.	0.	0.	0.	0.
time (sec)	N/A	3.91	0.147	0.013	0.	0.	0.	0.	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	645	640	163	133	0	0	0	0	0
normalized size	1	0.99	0.25	0.21	0.	0.	0.	0.	0.
time (sec)	N/A	5.037	0.195	0.011	0.	0.	0.	0.	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-2)	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	395	395	61	56	0	0	70	0	0
normalized size	1	1.	0.15	0.14	0.	0.	0.18	0.	0.
time (sec)	N/A	3.49	0.024	0.009	0.	0.	0.607	0.	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-2)	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	377	377	61	56	0	0	65	0	0
normalized size	1	1.	0.16	0.15	0.	0.	0.17	0.	0.
time (sec)	N/A	2.742	0.02	0.007	0.	0.	0.711	0.	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-2)	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	361	361	61	56	0	0	61	0	0
normalized size	1	1.	0.17	0.16	0.	0.	0.17	0.	0.
time (sec)	N/A	1.905	0.02	0.007	0.	0.	0.673	0.	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-2)	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	59	56	0	0	48	0	0
normalized size	1	1.	0.24	0.23	0.	0.	0.19	0.	0.
time (sec)	N/A	1.323	0.019	0.008	0.	0.	0.485	0.	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-2)	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	361	361	57	54	0	0	61	0	0
normalized size	1	1.	0.16	0.15	0.	0.	0.17	0.	0.
time (sec)	N/A	2.005	0.019	0.007	0.	0.	0.692	0.	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-2)	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	377	377	62	53	0	0	65	0	0
normalized size	1	1.	0.16	0.14	0.	0.	0.17	0.	0.
time (sec)	N/A	2.734	0.018	0.007	0.	0.	0.689	0.	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-2)	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	415	415	103	75	0	0	82	0	0
normalized size	1	1.	0.25	0.18	0.	0.	0.2	0.	0.
time (sec)	N/A	3.066	0.029	0.013	0.	0.	1.063	0.	0.

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-2)	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	448	448	109	74	0	0	70	0	0
normalized size	1	1.	0.24	0.17	0.	0.	0.16	0.	0.
time (sec)	N/A	3.689	0.029	0.012	0.	0.	0.81	0.	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-2)	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1064	1064	167	122	0	0	112	0	0
normalized size	1	1.	0.16	0.11	0.	0.	0.11	0.	0.
time (sec)	N/A	8.291	0.058	0.016	0.	0.	1.189	0.	0.

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-2)	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1005	1005	167	122	0	0	112	0	0
normalized size	1	1.	0.17	0.12	0.	0.	0.11	0.	0.
time (sec)	N/A	8.294	0.04	0.016	0.	0.	1.201	0.	0.

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-2)	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	675	675	167	122	0	0	112	0	0
normalized size	1	1.	0.25	0.18	0.	0.	0.17	0.	0.
time (sec)	N/A	5.068	0.055	0.016	0.	0.	1.173	0.	0.

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-2)	A	F	F(-1)
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	682	682	167	122	0	0	104	0	0
normalized size	1	1.	0.24	0.18	0.	0.	0.15	0.	0.
time (sec)	N/A	4.233	0.043	0.015	0.	0.	0.872	0.	0.

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-2)	A	F	F(-1)
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	850	850	167	122	0	0	112	0	0
normalized size	1	1.	0.2	0.14	0.	0.	0.13	0.	0.
time (sec)	N/A	5.221	0.054	0.015	0.	0.	1.182	0.	0.

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-2)	A	F	F(-1)
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	873	873	167	122	0	0	112	0	0
normalized size	1	1.	0.19	0.14	0.	0.	0.13	0.	0.
time (sec)	N/A	6.346	0.042	0.014	0.	0.	1.207	0.	0.

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-2)	A	F	F(-1)
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	986	986	167	122	0	0	112	0	0
normalized size	1	1.	0.17	0.12	0.	0.	0.11	0.	0.
time (sec)	N/A	7.463	0.054	0.015	0.	0.	1.192	0.	0.

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	28	28	22	28	0
normalized size	1	1.	1.	0.88	1.12	1.12	0.88	1.12	0.
time (sec)	N/A	0.034	0.003	0.001	0.808	0.242	0.141	0.26	0.

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	79	114	142	142	90	493	0
normalized size	1	1.	0.84	1.21	1.51	1.51	0.96	5.24	0.
time (sec)	N/A	0.234	0.062	0.005	0.82	0.248	1.683	0.264	0.

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	172	155	18	1	175	208	10
normalized size	1	1.	11.47	10.33	1.2	0.07	11.67	13.87	0.67
time (sec)	N/A	0.015	0.01	0.	0.807	0.232	0.273	0.261	3.164

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	182	157	211	1	182	211	12
normalized size	1	1.	10.11	8.72	11.72	0.06	10.11	11.72	0.67
time (sec)	N/A	0.024	0.009	0.003	0.804	0.236	0.274	0.26	12.872

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	186	157	211	1	185	211	12
normalized size	1	1.	10.33	8.72	11.72	0.06	10.28	11.72	0.67
time (sec)	N/A	0.025	0.01	0.003	0.814	0.229	0.288	0.262	12.339

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	230	0	354	0	275	0
normalized size	1	1.	1.	10.95	0.	16.86	0.	13.1	0.
time (sec)	N/A	0.07	0.059	0.062	0.	0.286	0.	0.39	0.

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	9	9	14	14	8	15	8
normalized size	1	1.	0.9	0.9	1.4	1.4	0.8	1.5	0.8
time (sec)	N/A	0.01	0.006	0.001	0.805	0.25	1.074	0.26	3.224

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	18	12	24	15
normalized size	1	1.	1.	0.93	1.2	1.2	0.8	1.6	1.
time (sec)	N/A	0.065	0.009	0.007	0.791	0.25	1.216	0.262	11.159

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	18	12	20	15
normalized size	1	1.	1.	0.93	1.2	1.2	0.8	1.33	1.
time (sec)	N/A	0.063	0.011	0.006	0.81	0.247	1.278	0.261	10.284

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	18	63	31	29	0	15
normalized size	1	1.	1.	1.2	4.2	2.07	1.93	0.	1.
time (sec)	N/A	0.062	0.017	0.022	0.817	0.267	4.519	0.	11.065

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	14	177	18	109	87	18	14
normalized size	1	1.	0.93	11.8	1.2	7.27	5.8	1.2	0.93
time (sec)	N/A	0.01	0.034	0.	0.808	0.263	17.678	0.263	3.153

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	16	197	109	109	0	20	15
normalized size	1	1.	0.89	10.94	6.06	6.06	0.	1.11	0.83
time (sec)	N/A	0.015	0.048	0.021	0.833	0.258	0.	0.264	12.589

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	16	197	109	109	0	20	15
normalized size	1	1.	0.89	10.94	6.06	6.06	0.	1.11	0.83
time (sec)	N/A	0.014	0.06	0.019	0.814	0.278	0.	0.267	12.802

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	127	203	826	193	0	0	0
normalized size	1	1.	6.05	9.67	39.33	9.19	0.	0.	0.
time (sec)	N/A	0.069	0.08	0.09	0.859	0.462	0.	0.	0.

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	17	24	0	35	46	55	14
normalized size	1	1.	0.89	1.26	0.	1.84	2.42	2.89	0.74
time (sec)	N/A	0.015	0.033	0.	0.	0.267	1.675	0.266	3.504

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	31	47	43	0	73	20
normalized size	1	1.	1.	1.15	1.74	1.59	0.	2.7	0.74
time (sec)	N/A	0.019	0.051	0.006	0.93	0.275	0.	0.269	11.5

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	C	A	C	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	116	27	142	47	45	0	0	102
normalized size	1	4.3	1.	5.26	1.74	1.67	0.	0.	3.78
time (sec)	N/A	0.222	0.029	0.173	0.921	0.274	0.	0.	32.265

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	33	47	46	0	78	22
normalized size	1	1.	1.	1.14	1.62	1.59	0.	2.69	0.76
time (sec)	N/A	0.02	0.053	0.006	0.925	0.282	0.	0.274	11.231

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	31	0	53	57	0	0	26
normalized size	1	1.	0.86	0.	1.47	1.58	0.	0.	0.72
time (sec)	N/A	0.145	0.109	0.113	1.132	0.274	0.	0.	12.878

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	35	35	29	35	0
normalized size	1	1.	1.	0.84	1.09	1.09	0.91	1.09	0.
time (sec)	N/A	0.037	0.003	0.002	0.794	0.245	0.149	0.26	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	14	14	8	14	0
normalized size	1	1.	1.	0.92	1.17	1.17	0.67	1.17	0.
time (sec)	N/A	0.035	0.001	0.001	0.816	0.244	0.158	0.263	0.

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	32	0	1	124	42	37
normalized size	1	1.	1.	0.76	0.	0.02	2.95	1.	0.88
time (sec)	N/A	0.068	0.025	0.006	0.	0.256	0.813	0.266	34.432

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	23	26	0	51	0	132	20
normalized size	1	1.	0.92	1.04	0.	2.04	0.	5.28	0.8
time (sec)	N/A	0.018	0.049	0.006	0.	0.297	0.	0.289	7.834

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	21	34	0	49	0	100	19
normalized size	1	1.	0.88	1.42	0.	2.04	0.	4.17	0.79
time (sec)	N/A	0.016	0.052	0.006	0.	0.27	0.	0.27	7.362

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	24	26	53	47	0	104	20
normalized size	1	1.	0.96	1.04	2.12	1.88	0.	4.16	0.8
time (sec)	N/A	0.019	0.051	0.005	0.917	0.286	0.	0.27	12.587

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	0	38	0	80	15
normalized size	1	1.	1.	1.05	0.	1.9	0.	4.	0.75
time (sec)	N/A	0.014	0.03	0.005	0.	0.274	0.	0.266	4.493

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	26	0	35	73	55	14
normalized size	1	1.	1.	1.37	0.	1.84	3.84	2.89	0.74
time (sec)	N/A	0.013	0.039	0.005	0.	0.271	42.401	0.272	4.023

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	23	42	36	0	63	17
normalized size	1	1.	1.	1.05	1.91	1.64	0.	2.86	0.77
time (sec)	N/A	0.016	0.035	0.004	0.913	0.281	0.	0.275	6.37

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	21	23	0	43	0	90	17
normalized size	1	1.	0.95	1.05	0.	1.95	0.	4.09	0.77
time (sec)	N/A	0.015	0.035	0.004	0.	0.271	0.	0.275	7.303

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	19	28	0	41	53	63	15
normalized size	1	1.	0.9	1.33	0.	1.95	2.52	3.	0.71
time (sec)	N/A	0.015	0.038	0.005	0.	0.279	2.426	0.277	6.416

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	22	28	43	42	56	63	124
normalized size	1	1.	0.92	1.17	1.79	1.75	2.33	2.62	5.17
time (sec)	N/A	0.018	0.041	0.004	0.912	0.271	18.302	0.275	40.988

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	23	43	39	53	63	31
normalized size	1	1.	1.	1.05	1.95	1.77	2.41	2.86	1.41
time (sec)	N/A	0.017	0.036	0.004	0.895	0.281	17.763	0.266	17.106

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	21	23	43	43	0	90	17
normalized size	1	1.	0.95	1.05	1.95	1.95	0.	4.09	0.77
time (sec)	N/A	0.015	0.021	0.004	0.865	0.268	0.	0.272	4.937

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	19	28	43	41	53	63	15
normalized size	1	1.	0.9	1.33	2.05	1.95	2.52	3.	0.71
time (sec)	N/A	0.015	0.03	0.004	0.907	0.276	2.405	0.267	4.429

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	143	25686	26	1	1771	1	17
normalized size	1	1.	6.81	1223.14	1.24	0.05	84.33	0.05	0.81
time (sec)	N/A	0.043	0.243	0.006	0.789	0.236	1.172	0.268	14.439

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	18	5596	24	1	469	670	15
normalized size	1	1.	0.9	279.8	1.2	0.05	23.45	33.5	0.75
time (sec)	N/A	0.023	0.047	0.004	0.814	0.239	0.447	0.267	19.124

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	18	5596	595	1	469	670	15
normalized size	1	1.	0.95	294.53	31.32	0.05	24.68	35.26	0.79
time (sec)	N/A	0.026	0.02	0.002	0.821	0.235	0.408	0.26	15.275

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	127	2185	19	1	483	656	12
normalized size	1	1.	7.94	136.56	1.19	0.06	30.19	41.	0.75
time (sec)	N/A	0.02	0.106	0.004	0.81	0.234	0.411	0.26	5.268

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	98	89	119	1	97	119	12
normalized size	1	1.	6.12	5.56	7.44	0.06	6.06	7.44	0.75
time (sec)	N/A	0.014	0.006	0.002	0.819	0.23	0.162	0.261	10.471

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	98	89	18	1	97	119	12
normalized size	1	1.	6.53	5.93	1.2	0.07	6.47	7.93	0.8
time (sec)	N/A	0.014	0.004	0.002	0.839	0.236	0.182	0.261	12.06

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	115	2205	22	1	484	659	14
normalized size	1	1.	6.39	122.5	1.22	0.06	26.89	36.61	0.78
time (sec)	N/A	0.021	0.111	0.004	0.803	0.231	0.432	0.262	7.85

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	98	89	20	1	97	119	10
normalized size	1	1.	5.76	5.24	1.18	0.06	5.71	7.	0.59
time (sec)	N/A	0.015	0.006	0.003	0.791	0.256	0.19	0.258	8.

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	98	89	119	1	97	119	10
normalized size	1	1.	5.44	4.94	6.61	0.06	5.39	6.61	0.56
time (sec)	N/A	0.016	0.005	0.003	0.793	0.229	0.188	0.26	8.207

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	98	89	119	1	97	119	10
normalized size	1	1.	7.	6.36	8.5	0.07	6.93	8.5	0.71
time (sec)	N/A	0.014	0.004	0.003	0.793	0.226	0.188	0.259	6.15

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	115	2205	618	1	484	659	14
normalized size	1	1.	6.39	122.5	34.33	0.06	26.89	36.61	0.78
time (sec)	N/A	0.033	0.024	0.003	0.802	0.242	0.398	0.261	4.597

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	98	89	119	1	97	119	10
normalized size	1	1.	5.76	5.24	7.	0.06	5.71	7.	0.59
time (sec)	N/A	0.018	0.004	0.002	0.791	0.23	0.182	0.261	6.208

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	98	89	119	1	97	119	10
normalized size	1	1.	5.44	4.94	6.61	0.06	5.39	6.61	0.56
time (sec)	N/A	0.02	0.004	0.002	0.792	0.24	0.183	0.258	6.175

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	98	89	119	1	97	119	10
normalized size	1	1.	7.	6.36	8.5	0.07	6.93	8.5	0.71
time (sec)	N/A	0.013	0.004	0.002	0.818	0.233	0.157	0.258	4.277

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	80	67	89	89	70	89	22
normalized size	1	1.	2.86	2.39	3.18	3.18	2.5	3.18	0.79
time (sec)	N/A	0.026	0.008	0.001	0.816	0.256	0.16	0.259	2.463

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	108	325	252	252	194	281	26
normalized size	1	1.	3.48	10.48	8.13	8.13	6.26	9.06	0.84
time (sec)	N/A	0.037	0.057	0.001	0.822	0.255	0.238	0.26	2.982

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	31	70	65	0	41	26
normalized size	1	1.	1.	0.91	2.06	1.91	0.	1.21	0.76
time (sec)	N/A	0.019	0.03	0.002	0.991	0.291	0.	0.261	2.348

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	73	33	73	70	0	115	29
normalized size	1	1.	2.09	0.94	2.09	2.	0.	3.29	0.83
time (sec)	N/A	0.02	0.066	0.003	0.968	0.273	0.	0.27	2.385

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	93	78	104	104	87	104	22
normalized size	1	1.	3.1	2.6	3.47	3.47	2.9	3.47	0.73
time (sec)	N/A	0.03	0.009	0.005	0.804	0.257	0.174	0.259	4.072

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	140	618	378	378	314	393	26
normalized size	1	1.	4.52	19.94	12.19	12.19	10.13	12.68	0.84
time (sec)	N/A	0.049	0.075	0.004	0.814	0.268	0.309	0.259	6.824

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	98	81	108	108	90	108	29
normalized size	1	1.	2.88	2.38	3.18	3.18	2.65	3.18	0.85
time (sec)	N/A	0.031	0.012	0.003	0.791	0.248	0.18	0.261	4.682

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	146	646	390	390	321	402	32
normalized size	1	1.	3.56	15.76	9.51	9.51	7.83	9.8	0.78
time (sec)	N/A	0.052	0.087	0.003	0.809	0.248	0.328	0.262	6.777

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	244	1523	390	390	323	417	36
normalized size	1	1.	5.3	33.11	8.48	8.48	7.02	9.07	0.78
time (sec)	N/A	0.05	0.136	0.004	0.81	0.251	0.347	0.261	9.874

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	248	4284	1044	1044	930	1253	39
normalized size	1	1.	5.28	91.15	22.21	22.21	19.79	26.66	0.83
time (sec)	N/A	0.101	0.221	0.004	0.818	0.256	0.676	0.265	21.764

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	36	31	73	65	0	41	26
normalized size	1	1.	1.06	0.91	2.15	1.91	0.	1.21	0.76
time (sec)	N/A	0.02	0.034	0.003	0.999	0.29	0.	0.259	2.505

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	42	37	96	77	0	49	32
normalized size	1	1.	0.95	0.84	2.18	1.75	0.	1.11	0.73
time (sec)	N/A	0.021	0.037	0.004	0.993	0.28	0.	0.26	3.324

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	49	43	112	97	0	57	39
normalized size	1	1.	0.98	0.86	2.24	1.94	0.	1.14	0.78
time (sec)	N/A	0.021	0.058	0.003	1.018	0.275	0.	0.26	3.157

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	33	30	23	1	29	39	0
normalized size	1	1.	1.74	1.58	1.21	0.05	1.53	2.05	0.
time (sec)	N/A	0.012	0.002	0.001	0.785	0.231	0.073	0.26	0.

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	21	18	19	1	17	23	0
normalized size	1	1.	1.31	1.12	1.19	0.06	1.06	1.44	0.
time (sec)	N/A	0.01	0.002	0.001	0.841	0.237	0.064	0.258	0.

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	B	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	96	96	87	116	1	94	116	94
normalized size	1	2.91	2.91	2.64	3.52	0.03	2.85	3.52	2.85
time (sec)	N/A	0.318	0.009	0.002	0.831	0.233	0.133	0.257	27.512

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	B	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	96	96	87	116	1	94	116	94
normalized size	1	2.91	2.91	2.64	3.52	0.03	2.85	3.52	2.85
time (sec)	N/A	0.266	0.008	0.002	0.897	0.231	0.158	0.259	27.909

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	16	77	56	16	12
normalized size	1	1.	1.	0.93	1.14	5.5	4.	1.14	0.86
time (sec)	N/A	0.009	0.008	0.002	0.811	0.245	0.667	0.26	2.939

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	18	12	19	12
normalized size	1	1.	1.	0.93	1.2	1.2	0.8	1.27	0.8
time (sec)	N/A	0.01	0.007	0.002	0.849	0.251	0.158	0.261	3.952

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	20	14	20	20	14	24	12
normalized size	1	1.	1.18	0.82	1.18	1.18	0.82	1.41	0.71
time (sec)	N/A	0.01	0.009	0.002	0.823	0.247	0.202	0.263	14.967

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	23	28	31	31	19	32	34
normalized size	1	1.	0.57	0.7	0.78	0.78	0.48	0.8	0.85
time (sec)	N/A	0.143	0.101	0.014	0.842	0.254	111.918	0.263	64.648

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	605	605	98	2105	0	0	0	0	0
normalized size	1	1.	0.16	3.48	0.	0.	0.	0.	0.
time (sec)	N/A	10.699	0.118	0.071	0.	0.	0.	0.	0.

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	55	82	0	112	58	78	53
normalized size	1	1.	0.87	1.3	0.	1.78	0.92	1.24	0.84
time (sec)	N/A	0.132	0.04	0.036	0.	0.264	0.255	0.275	37.282

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	30	51	20	18	12
normalized size	1	1.	1.	0.93	2.14	3.64	1.43	1.29	0.86
time (sec)	N/A	0.061	0.011	0.009	0.837	0.266	0.195	0.259	34.368

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	24	27	43	62	29	31	0
normalized size	1	1.	0.86	0.96	1.54	2.21	1.04	1.11	0.
time (sec)	N/A	0.051	0.02	0.01	0.831	0.265	0.232	0.26	0.

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	28	30	51	51	34	38	53
normalized size	1	1.	0.47	0.51	0.86	0.86	0.58	0.64	0.9
time (sec)	N/A	0.131	0.019	0.01	0.855	0.258	0.548	0.263	35.486

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	41	15	15	8	15	8
normalized size	1	1.	1.	3.73	1.36	1.36	0.73	1.36	0.73
time (sec)	N/A	0.008	0.011	0.013	0.857	0.259	0.332	0.259	6.795

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	B	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	205	132	116	162	324	272	181	0
normalized size	1	2.25	1.45	1.27	1.78	3.56	2.99	1.99	0.
time (sec)	N/A	0.345	0.153	0.026	0.894	0.275	3.891	0.266	0.

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	23	26	59	54	0	155	0
normalized size	1	1.	0.92	1.04	2.36	2.16	0.	6.2	0.
time (sec)	N/A	0.03	0.138	0.011	0.994	0.506	0.	0.415	0.

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	21	24	53	53	0	131	0
normalized size	1	1.	0.91	1.04	2.3	2.3	0.	5.7	0.
time (sec)	N/A	0.023	0.064	0.011	0.939	0.294	0.	0.469	0.

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	21	24	53	53	0	131	0
normalized size	1	1.	0.91	1.04	2.3	2.3	0.	5.7	0.
time (sec)	N/A	0.021	0.059	0.009	0.901	0.291	0.	0.328	0.

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	19	22	50	50	0	128	0
normalized size	1	1.	0.9	1.05	2.38	2.38	0.	6.1	0.
time (sec)	N/A	0.016	0.053	0.009	0.891	0.291	0.	0.321	0.

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	17	20	45	45	0	0	17
normalized size	1	1.	0.89	1.05	2.37	2.37	0.	0.	0.89
time (sec)	N/A	0.022	0.04	0.007	0.923	0.29	0.	0.	17.551

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	21	24	49	49	0	0	0
normalized size	1	1.	0.91	1.04	2.13	2.13	0.	0.	0.
time (sec)	N/A	0.021	0.072	0.009	0.901	0.32	0.	0.	0.

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	21	24	49	49	0	0	0
normalized size	1	1.	0.91	1.04	2.13	2.13	0.	0.	0.
time (sec)	N/A	0.021	0.078	0.008	0.907	0.357	0.	0.	0.

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	21	24	49	49	0	0	0
normalized size	1	1.	0.91	1.04	2.13	2.13	0.	0.	0.
time (sec)	N/A	0.021	0.096	0.009	0.942	0.353	0.	0.	0.

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	83	74	99	122	97	99	0
normalized size	1	1.	0.86	0.76	1.02	1.26	1.	1.02	0.
time (sec)	N/A	0.265	0.057	0.012	0.904	0.274	0.723	0.264	0.

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	78	69	92	115	92	92	0
normalized size	1	1.	0.87	0.77	1.02	1.28	1.02	1.02	0.
time (sec)	N/A	0.236	0.041	0.009	0.914	0.274	0.684	0.262	0.

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	72	62	82	105	78	82	0
normalized size	1	1.	0.94	0.81	1.06	1.36	1.01	1.06	0.
time (sec)	N/A	0.237	0.049	0.008	0.915	0.276	0.807	0.261	0.

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	69	57	76	96	75	76	0
normalized size	1	1.	0.96	0.79	1.06	1.33	1.04	1.06	0.
time (sec)	N/A	0.187	0.037	0.005	0.914	0.272	0.731	0.263	0.

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	65	56	74	89	75	74	0
normalized size	1	1.	0.92	0.79	1.04	1.25	1.06	1.04	0.
time (sec)	N/A	0.166	0.028	0.006	0.896	0.276	0.663	0.263	0.

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	69	60	80	99	78	81	0
normalized size	1	1.	0.92	0.8	1.07	1.32	1.04	1.08	0.
time (sec)	N/A	0.292	0.032	0.01	0.919	0.274	0.901	0.263	0.

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	78	65	86	116	87	88	0
normalized size	1	1.	0.93	0.77	1.02	1.38	1.04	1.05	0.
time (sec)	N/A	0.32	0.057	0.012	0.893	0.276	0.943	0.262	0.

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	82	70	93	136	94	95	0
normalized size	1	1.	0.9	0.77	1.02	1.49	1.03	1.04	0.
time (sec)	N/A	0.331	0.102	0.012	0.918	0.276	1.016	0.264	0.

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	307	307	109	74	0	0	61	0	0
normalized size	1	1.	0.36	0.24	0.	0.	0.2	0.	0.
time (sec)	N/A	1.303	0.033	0.01	0.	0.	3.407	0.	0.

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	101	67	0	0	53	0	0
normalized size	1	1.	0.38	0.25	0.	0.	0.2	0.	0.
time (sec)	N/A	1.046	0.027	0.009	0.	0.	2.979	0.	0.

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	94	62	0	0	48	0	0
normalized size	1	1.	0.41	0.27	0.	0.	0.21	0.	0.
time (sec)	N/A	0.95	0.025	0.009	0.	0.	2.838	0.	0.

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	90	58	0	0	46	0	216
normalized size	1	1.	0.45	0.29	0.	0.	0.23	0.	1.09
time (sec)	N/A	0.56	0.022	0.007	0.	0.	2.851	0.	84.751

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	101	67	0	0	60	0	253
normalized size	1	1.	0.41	0.27	0.	0.	0.24	0.	1.03
time (sec)	N/A	1.194	0.028	0.012	0.	0.	30.542	0.	115.025

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	109	72	0	0	65	0	284
normalized size	1	1.	0.39	0.26	0.	0.	0.23	0.	1.01
time (sec)	N/A	1.303	0.029	0.012	0.	0.	9.647	0.	133.822

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	116	77	0	0	70	0	318
normalized size	1	1.	0.37	0.24	0.	0.	0.22	0.	1.
time (sec)	N/A	1.467	0.03	0.015	0.	0.	6.68	0.	137.984

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	87	75	0	112	44	0	14
normalized size	1	1.	4.58	3.95	0.	5.89	2.32	0.	0.74
time (sec)	N/A	0.175	0.072	0.428	0.	0.264	4.119	0.	53.019

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	57	34	45	63	36	46	0
normalized size	1	1.	1.33	0.79	1.05	1.47	0.84	1.07	0.
time (sec)	N/A	0.115	0.037	0.013	0.903	0.275	0.421	0.262	0.

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	20	20	14	24	14
normalized size	1	1.	1.	0.94	1.18	1.18	0.82	1.41	0.82
time (sec)	N/A	0.053	0.009	0.011	0.997	0.258	0.271	0.263	9.158

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	28	28	20	32	20
normalized size	1	1.	1.	0.88	1.12	1.12	0.8	1.28	0.8
time (sec)	N/A	0.052	0.009	0.01	0.824	0.259	0.277	0.26	12.61

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	20	19	24	32	14	38	14
normalized size	1	1.	0.91	0.86	1.09	1.45	0.64	1.73	0.64
time (sec)	N/A	0.042	0.013	0.011	0.944	0.245	0.225	0.26	4.404

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	38	22	28	28	26	30	26
normalized size	1	1.	1.41	0.81	1.04	1.04	0.96	1.11	0.96
time (sec)	N/A	0.074	0.017	0.008	0.982	0.257	0.313	0.265	8.094

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	23	23	15	23	0
normalized size	1	1.	1.	0.86	1.1	1.1	0.71	1.1	0.
time (sec)	N/A	0.025	0.007	0.004	0.898	0.257	0.136	0.26	0.

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	31	31	22	31	22
normalized size	1	1.	1.	0.89	1.15	1.15	0.81	1.15	0.81
time (sec)	N/A	0.049	0.012	0.006	0.865	0.273	0.203	0.26	36.65

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	26	34	34	26	39	0
normalized size	1	1.	1.	0.67	0.87	0.87	0.67	1.	0.
time (sec)	N/A	0.045	0.01	0.012	0.8	0.263	0.655	0.266	0.

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	23	19	24	24	17	26	0
normalized size	1	1.	1.05	0.86	1.09	1.09	0.77	1.18	0.
time (sec)	N/A	0.032	0.006	0.003	0.814	0.262	0.114	0.259	0.

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	18	14	18	0
normalized size	1	1.	1.	0.93	1.2	1.2	0.93	1.2	0.
time (sec)	N/A	0.04	0.007	0.004	0.905	0.253	0.17	0.258	0.

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	24	21	27	35	17	30	7
normalized size	1	1.	2.	1.75	2.25	2.92	1.42	2.5	0.58
time (sec)	N/A	0.04	0.016	0.011	0.79	0.248	0.187	0.265	15.438

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	28	30	19	31	0
normalized size	1	1.	1.	0.88	1.12	1.2	0.76	1.24	0.
time (sec)	N/A	0.044	0.01	0.011	0.825	0.252	0.201	0.261	0.

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	15	15	10	15	10
normalized size	1	1.	1.	0.92	1.15	1.15	0.77	1.15	0.77
time (sec)	N/A	0.041	0.011	0.007	0.865	0.248	0.243	0.261	17.337

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	31	47	84	36	41	46
normalized size	1	1.	1.	0.89	1.34	2.4	1.03	1.17	1.31
time (sec)	N/A	0.057	0.026	0.008	0.886	0.249	0.364	0.262	23.983

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	23	23	17	27	17
normalized size	1	1.	1.	0.78	1.	1.	0.74	1.17	0.74
time (sec)	N/A	0.055	0.009	0.011	0.806	0.254	0.302	0.262	11.425

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	26	26	19	27	19
normalized size	1	1.	1.	0.87	1.13	1.13	0.83	1.17	0.83
time (sec)	N/A	0.064	0.013	0.007	0.801	0.248	0.212	0.261	30.173

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	25	24	31	43	20	31	20
normalized size	1	1.	0.86	0.83	1.07	1.48	0.69	1.07	0.69
time (sec)	N/A	0.033	0.015	0.007	0.899	0.245	0.264	0.261	7.915

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	42	55	93	49	58	46
normalized size	1	1.	1.	0.95	1.25	2.11	1.11	1.32	1.05
time (sec)	N/A	0.422	0.038	0.013	0.889	0.254	0.606	0.263	148.204

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	41	51	62	51	51	48
normalized size	1	1.	1.	0.89	1.11	1.35	1.11	1.11	1.04
time (sec)	N/A	0.257	0.035	0.009	0.876	0.261	0.588	0.261	47.228

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	34	59	97	39	59	0
normalized size	1	1.	1.	1.03	1.79	2.94	1.18	1.79	0.
time (sec)	N/A	0.303	0.032	0.015	0.794	0.265	0.515	0.263	0.

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	12	15	15	10	15	10
normalized size	1	1.	1.	0.71	0.88	0.88	0.59	0.88	0.59
time (sec)	N/A	0.023	0.01	0.01	0.898	0.263	0.401	0.262	5.121

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	22	21	27	27	34	27	0
normalized size	1	1.	0.92	0.88	1.12	1.12	1.42	1.12	0.
time (sec)	N/A	0.042	0.015	0.003	0.879	0.258	1.295	0.26	0.

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	23	18	23	23	17	27	10
normalized size	1	1.	1.53	1.2	1.53	1.53	1.13	1.8	0.67
time (sec)	N/A	0.085	0.009	0.012	0.88	0.276	0.291	0.262	34.099

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	18	23	23	19	23	19
normalized size	1	1.	1.	0.9	1.15	1.15	0.95	1.15	0.95
time (sec)	N/A	0.041	0.015	0.009	0.889	0.261	0.405	0.262	8.126

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	33	28	36	59	29	81	29
normalized size	1	1.	0.89	0.76	0.97	1.59	0.78	2.19	0.78
time (sec)	N/A	0.078	0.039	0.011	0.888	0.256	0.382	0.263	34.322

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	22	28	35	26	28	26
normalized size	1	1.	1.	0.85	1.08	1.35	1.	1.08	1.
time (sec)	N/A	0.031	0.015	0.003	0.892	0.263	0.171	0.259	7.057

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	14	22	10	15	10
normalized size	1	1.	1.	0.92	1.17	1.83	0.83	1.25	0.83
time (sec)	N/A	0.036	0.007	0.007	0.787	0.246	0.19	0.262	8.435

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	23	23	15	27	0
normalized size	1	1.	1.	0.86	1.1	1.1	0.71	1.29	0.
time (sec)	N/A	0.055	0.011	0.011	0.796	0.25	0.285	0.262	0.

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	27	46	17	27	17
normalized size	1	1.	1.	0.95	1.23	2.09	0.77	1.23	0.77
time (sec)	N/A	0.033	0.014	0.007	0.873	0.249	0.235	0.261	9.507

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	27	43	19	27	20
normalized size	1	1.	1.	0.88	1.12	1.79	0.79	1.12	0.83
time (sec)	N/A	0.035	0.017	0.007	0.881	0.247	0.241	0.261	12.894

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	34	45	59	39	45	39
normalized size	1	1.	1.	0.94	1.25	1.64	1.08	1.25	1.08
time (sec)	N/A	0.085	0.025	0.005	0.867	0.252	0.515	0.261	17.354

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	28	36	36	29	36	29
normalized size	1	1.	1.	0.76	0.97	0.97	0.78	0.97	0.78
time (sec)	N/A	0.08	0.012	0.005	0.894	0.252	0.481	0.262	17.107

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	22	28	28	22	31	0
normalized size	1	1.	1.	0.76	0.97	0.97	0.76	1.07	0.
time (sec)	N/A	0.039	0.009	0.009	0.803	0.25	0.214	0.265	0.

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	20	20	14	22	0
normalized size	1	1.	1.	0.84	1.05	1.05	0.74	1.16	0.
time (sec)	N/A	0.036	0.006	0.003	0.801	0.245	0.13	0.261	0.

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	35	46	57	46	46	42
normalized size	1	1.	1.	0.85	1.12	1.39	1.12	1.12	1.02
time (sec)	N/A	0.063	0.025	0.004	0.881	0.249	0.231	0.262	22.93

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	39	32	42	42	34	42	0
normalized size	1	1.	0.95	0.78	1.02	1.02	0.83	1.02	0.
time (sec)	N/A	0.058	0.011	0.006	0.874	0.255	0.237	0.261	0.

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	24	21	27	27	24	31	0
normalized size	1	1.	0.8	0.7	0.9	0.9	0.8	1.03	0.
time (sec)	N/A	0.102	0.018	0.011	0.801	0.253	0.328	0.262	0.

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	28	36	36	31	41	0
normalized size	1	1.	1.	0.8	1.03	1.03	0.89	1.17	0.
time (sec)	N/A	0.074	0.013	0.013	0.797	0.255	0.324	0.262	0.

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	32	25	32	46	27	46	0
normalized size	1	1.	0.94	0.74	0.94	1.35	0.79	1.35	0.
time (sec)	N/A	0.089	0.025	0.013	0.804	0.254	0.336	0.262	0.

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	35	45	68	36	45	39
normalized size	1	1.	1.	0.83	1.07	1.62	0.86	1.07	0.93
time (sec)	N/A	0.045	0.031	0.007	0.871	0.253	0.292	0.263	8.942

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	38	50	50	46	50	46
normalized size	1	1.	1.	0.78	1.02	1.02	0.94	1.02	0.94
time (sec)	N/A	0.29	0.023	0.009	0.887	0.255	0.627	0.262	82.318

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	20	26	26	24	30	0
normalized size	1	1.	1.	0.69	0.9	0.9	0.83	1.03	0.
time (sec)	N/A	0.094	0.012	0.011	0.815	0.257	0.309	0.258	0.

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	43	34	45	45	41	46	0
normalized size	1	1.	0.93	0.74	0.98	0.98	0.89	1.	0.
time (sec)	N/A	0.083	0.026	0.008	0.89	0.255	0.315	0.262	0.

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	14	15	19	23	10	20	0
normalized size	1	1.	0.88	0.94	1.19	1.44	0.62	1.25	0.
time (sec)	N/A	0.05	0.012	0.008	0.787	0.252	0.186	0.263	0.

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	14	16	23	23	15	16	17
normalized size	1	1.	0.67	0.76	1.1	1.1	0.71	0.76	0.81
time (sec)	N/A	0.04	0.004	0.007	0.807	0.239	0.166	0.259	13.196

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	31	31	19	41	0
normalized size	1	1.	1.	1.07	2.07	2.07	1.27	2.73	0.
time (sec)	N/A	0.037	0.019	0.008	0.797	0.242	0.274	0.263	0.

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	29	38	51	3	39	0
normalized size	1	1.	1.	0.94	1.23	1.65	0.1	1.26	0.
time (sec)	N/A	0.089	0.022	0.008	0.883	0.259	0.27	0.262	0.

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	26	26	19	30	19
normalized size	1	1.	1.	0.8	1.04	1.04	0.76	1.2	0.76
time (sec)	N/A	0.062	0.01	0.01	0.815	0.275	0.297	0.26	13.789

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	29	25	32	49	20	35	0
normalized size	1	1.	0.97	0.83	1.07	1.63	0.67	1.17	0.
time (sec)	N/A	0.061	0.026	0.01	0.798	0.253	0.194	0.26	0.

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	23	23	17	24	17
normalized size	1	1.	1.	0.78	1.	1.	0.74	1.04	0.74
time (sec)	N/A	0.059	0.009	0.007	0.871	0.256	0.303	0.262	8.875

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	93	73	104	211	88	100	0
normalized size	1	1.	0.9	0.71	1.01	2.05	0.85	0.97	0.
time (sec)	N/A	0.942	0.072	0.017	0.887	0.264	1.663	0.264	0.

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	30	28	34	49	24	34	27
normalized size	1	1.	0.91	0.85	1.03	1.48	0.73	1.03	0.82
time (sec)	N/A	0.037	0.018	0.007	0.884	0.256	0.281	0.261	11.737

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	39	59	27	41	29
normalized size	1	1.	1.	0.85	1.18	1.79	0.82	1.24	0.88
time (sec)	N/A	0.083	0.037	0.011	0.885	0.254	0.359	0.261	16.797

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	31	26	17	35	0
normalized size	1	1.	1.	0.96	1.24	1.04	0.68	1.4	0.
time (sec)	N/A	0.054	0.009	0.01	0.801	0.252	0.178	0.26	0.

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	32	42	42	36	42	36
normalized size	1	1.	1.	0.89	1.17	1.17	1.	1.17	1.
time (sec)	N/A	0.217	0.026	0.005	0.897	0.253	0.547	0.265	35.115

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	22	28	45	22	28	22
normalized size	1	1.	1.	0.76	0.97	1.55	0.76	0.97	0.76
time (sec)	N/A	0.193	0.027	0.013	0.884	0.252	0.51	0.261	30.839

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	36	47	66	46	49	42
normalized size	1	1.	1.	0.78	1.02	1.43	1.	1.07	0.91
time (sec)	N/A	0.1	0.046	0.01	0.903	0.255	0.379	0.262	21.526

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	26	26	19	24	24	17	27	0
normalized size	1	1.18	1.18	0.86	1.09	1.09	0.77	1.23	0.
time (sec)	N/A	0.03	0.008	0.008	0.792	0.251	0.173	0.261	0.

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	20	20	15	24	15
normalized size	1	1.	1.	0.94	1.18	1.18	0.88	1.41	0.88
time (sec)	N/A	0.056	0.01	0.01	0.799	0.252	0.287	0.259	13.59

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	19	24	14	22	14
normalized size	1	1.	1.	1.07	1.36	1.71	1.	1.57	1.
time (sec)	N/A	0.048	0.006	0.01	0.792	0.247	0.237	0.26	7.065

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	25	18	23	23	17	27	0
normalized size	1	1.	1.32	0.95	1.21	1.21	0.89	1.42	0.
time (sec)	N/A	0.054	0.011	0.011	0.789	0.255	0.274	0.262	0.

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	23	23	19	23	19
normalized size	1	1.	1.	0.78	1.	1.	0.83	1.	0.83
time (sec)	N/A	0.056	0.014	0.006	0.892	0.255	0.494	0.262	22.507

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	57	51	68	86	68	72	0
normalized size	1	1.	0.9	0.81	1.08	1.37	1.08	1.14	0.
time (sec)	N/A	0.167	0.041	0.016	0.893	0.26	1.085	0.262	0.

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	86	67	54	80	155	63	80	0
normalized size	1	1.25	0.97	0.78	1.16	2.25	0.91	1.16	0.
time (sec)	N/A	0.199	0.075	0.02	0.875	0.266	0.566	0.262	0.

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	26	12	18	12
normalized size	1	1.	1.	0.82	1.06	1.53	0.71	1.06	0.71
time (sec)	N/A	0.042	0.009	0.009	0.884	0.253	0.209	0.26	6.983

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	23	23	15	23	15
normalized size	1	1.	1.	0.95	1.21	1.21	0.79	1.21	0.79
time (sec)	N/A	0.042	0.008	0.003	0.923	0.246	0.184	0.26	9.739

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	9	9	7	38	7
normalized size	1	1.	1.	0.89	1.	1.	0.78	4.22	0.78
time (sec)	N/A	0.104	0.01	0.006	0.901	0.256	0.37	0.264	34.798

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	16	16	12	16	12
normalized size	1	1.	1.	1.08	1.33	1.33	1.	1.33	1.
time (sec)	N/A	0.024	0.009	0.003	0.81	0.247	0.148	0.264	21.666

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	58	41	69	104	99	74	75
normalized size	1	1.	0.89	0.63	1.06	1.6	1.52	1.14	1.15
time (sec)	N/A	0.123	0.059	0.01	0.892	0.256	2.003	0.26	24.109

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	29	38	62	24	38	32
normalized size	1	1.	1.	1.04	1.36	2.21	0.86	1.36	1.14
time (sec)	N/A	0.046	0.016	0.009	0.9	0.253	0.301	0.26	18.268

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	35	35	29	35	29
normalized size	1	1.	1.	0.84	1.09	1.09	0.91	1.09	0.91
time (sec)	N/A	0.063	0.037	0.004	0.902	0.283	0.18	0.259	5.004

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	41	39	24	31	31	26	36	32
normalized size	1	1.32	1.26	0.77	1.	1.	0.84	1.16	1.03
time (sec)	N/A	0.088	0.011	0.013	0.817	0.258	0.543	0.261	21.913

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	30	35	19	31	19
normalized size	1	1.	1.	0.96	1.25	1.46	0.79	1.29	0.79
time (sec)	N/A	0.315	0.014	0.01	0.893	0.275	0.355	0.261	85.723

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	19	32	47	20	26	22
normalized size	1	1.	1.	0.83	1.39	2.04	0.87	1.13	0.96
time (sec)	N/A	0.041	0.019	0.007	0.894	0.253	0.322	0.261	15.598

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	19	32	47	20	26	0
normalized size	1	1.	1.	0.83	1.39	2.04	0.87	1.13	0.
time (sec)	N/A	0.072	0.009	0.004	0.903	0.246	0.306	0.258	0.

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	18	20	10	18	10
normalized size	1	1.	1.	1.08	1.38	1.54	0.77	1.38	0.77
time (sec)	N/A	0.055	0.01	0.006	0.798	0.247	0.209	0.258	22.148

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	193	236	0	0	138	302	0
normalized size	1	1.	0.94	1.15	0.	0.	0.67	1.47	0.
time (sec)	N/A	0.56	0.186	0.009	0.	0.	4.954	0.267	0.

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	41	51	72	44	51	41
normalized size	1	1.	1.	0.91	1.13	1.6	0.98	1.13	0.91
time (sec)	N/A	0.111	0.042	0.013	0.924	0.267	0.398	0.262	23.412

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	44	41	70	101	46	57	48
normalized size	1	1.	0.75	0.69	1.19	1.71	0.78	0.97	0.81
time (sec)	N/A	0.111	0.035	0.019	0.879	0.261	0.559	0.259	16.458

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	234	616	0	0	0	1	221
normalized size	1	1.	1.12	2.95	0.	0.	0.	0.	1.06
time (sec)	N/A	0.87	0.425	0.073	0.	0.	0.	1.025	60.401

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	245	633	0	0	0	1	235
normalized size	1	1.	1.09	2.83	0.	0.	0.	0.	1.05
time (sec)	N/A	0.668	0.454	0.039	0.	0.	0.	1.041	61.612

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	57	81	88	190	0	0
normalized size	1	1.	1.	1.02	1.45	1.57	3.39	0.	0.
time (sec)	N/A	0.11	0.043	0.009	0.818	0.296	4.043	0.	0.

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	73	87	0	1	1355	115	82
normalized size	1	1.	0.76	0.91	0.	0.01	14.11	1.2	0.85
time (sec)	N/A	0.203	0.056	0.009	0.	0.292	22.451	0.264	23.694

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	228	336	0	0	0	432	231
normalized size	1	1.	0.86	1.27	0.	0.	0.	1.64	0.88
time (sec)	N/A	0.961	0.17	0.008	0.	0.	0.	0.271	113.591

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	417	417	370	422	0	0	0	575	381
normalized size	1	1.	0.89	1.01	0.	0.	0.	1.38	0.91
time (sec)	N/A	1.177	0.486	0.019	0.	0.	0.	0.294	148.362

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	24	21	27	35	19	28	10
normalized size	1	1.	1.5	1.31	1.69	2.19	1.19	1.75	0.62
time (sec)	N/A	0.029	0.014	0.01	0.831	0.243	0.224	0.259	4.714

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	27	24	31	46	20	41	12
normalized size	1	1.	1.42	1.26	1.63	2.42	1.05	2.16	0.63
time (sec)	N/A	0.044	0.018	0.014	0.804	0.255	0.28	0.261	8.291

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	85	90	101	159	92	104	85
normalized size	1	1.	0.88	0.93	1.04	1.64	0.95	1.07	0.88
time (sec)	N/A	0.184	0.083	0.019	0.906	0.261	1.158	0.264	25.717

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	18	12	18	12
normalized size	1	1.	1.	0.93	1.2	1.2	0.8	1.2	0.8
time (sec)	N/A	0.19	0.012	0.006	0.902	0.263	0.264	0.264	32.741

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	15	15	10	15	10
normalized size	1	1.	1.	0.92	1.15	1.15	0.77	1.15	0.77
time (sec)	N/A	0.167	0.01	0.005	0.89	0.255	0.274	0.261	28.978

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	25	32	32	29	32	29
normalized size	1	1.	1.	0.86	1.1	1.1	1.	1.1	1.
time (sec)	N/A	0.217	0.025	0.006	0.884	0.256	0.526	0.261	40.59

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	19	23	10	19	10
normalized size	1	1.	1.	1.07	1.36	1.64	0.71	1.36	0.71
time (sec)	N/A	0.03	0.013	0.007	0.884	0.248	0.304	0.259	9.025

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	14	16	7	15	7
normalized size	1	1.	1.	0.92	1.17	1.33	0.58	1.25	0.58
time (sec)	N/A	0.034	0.005	0.008	0.813	0.25	0.175	0.261	4.454

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	12	12	8	14	8
normalized size	1	1.	1.	0.91	1.09	1.09	0.73	1.27	0.73
time (sec)	N/A	0.048	0.007	0.007	0.877	0.29	0.391	0.263	7.836

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	14	18	8	15	8
normalized size	1	1.	1.	0.92	1.17	1.5	0.67	1.25	0.67
time (sec)	N/A	0.043	0.006	0.007	0.813	0.25	0.169	0.262	7.

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	16	16	12	19	12
normalized size	1	1.	1.	1.08	1.33	1.33	1.	1.58	1.
time (sec)	N/A	0.038	0.005	0.008	0.817	0.251	0.199	0.259	6.113

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	20	20	15	22	15
normalized size	1	1.	1.	0.94	1.18	1.18	0.88	1.29	0.88
time (sec)	N/A	0.056	0.007	0.007	0.891	0.254	0.295	0.259	8.409

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	18	18	10	19	10
normalized size	1	1.	1.	1.08	1.38	1.38	0.77	1.46	0.77
time (sec)	N/A	0.057	0.013	0.008	0.902	0.255	0.366	0.263	8.377

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	22	21	27	35	20	30	20
normalized size	1	1.	0.79	0.75	0.96	1.25	0.71	1.07	0.71
time (sec)	N/A	0.036	0.021	0.011	0.802	0.253	0.236	0.26	6.785

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	35	50	26	58	26
normalized size	1	1.	1.	0.84	1.09	1.56	0.81	1.81	0.81
time (sec)	N/A	0.064	0.025	0.011	0.812	0.249	0.317	0.262	8.42

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	28	20	26	26	19	27	19
normalized size	1	1.	1.22	0.87	1.13	1.13	0.83	1.17	0.83
time (sec)	N/A	0.064	0.012	0.007	0.885	0.254	0.31	0.264	9.049

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	22	21	27	49	20	63	20
normalized size	1	1.	0.92	0.88	1.12	2.04	0.83	2.62	0.83
time (sec)	N/A	0.061	0.022	0.009	0.907	0.259	0.336	0.263	9.071

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	40	53	50	44	53	46
normalized size	1	1.	1.	0.82	1.08	1.02	0.9	1.08	0.94
time (sec)	N/A	0.258	0.023	0.01	0.883	0.277	0.635	0.26	54.723

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	19	20	26	26	19	30	19
normalized size	1	1.	0.76	0.8	1.04	1.04	0.76	1.2	0.76
time (sec)	N/A	0.096	0.013	0.01	0.805	0.262	0.309	0.262	10.398

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	54	48	63	99	63	81	60
normalized size	1	1.	0.9	0.8	1.05	1.65	1.05	1.35	1.
time (sec)	N/A	0.457	0.09	0.014	0.901	0.295	0.762	0.264	164.427

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	12	12	5	12	0
normalized size	1	1.	1.	0.91	1.09	1.09	0.45	1.09	0.
time (sec)	N/A	0.012	0.001	0.001	0.832	0.247	0.072	0.26	0.

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	22	28	28	22	31	0
normalized size	1	1.	1.	0.76	0.97	0.97	0.76	1.07	0.
time (sec)	N/A	0.036	0.009	0.008	0.825	0.253	0.213	0.261	0.

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	37	50	70	37	50	34
normalized size	1	1.	1.	0.82	1.11	1.56	0.82	1.11	0.76
time (sec)	N/A	0.053	0.02	0.009	0.89	0.258	0.325	0.26	13.982

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	29	38	70	29	39	0
normalized size	1	1.	1.	0.91	1.19	2.19	0.91	1.22	0.
time (sec)	N/A	0.464	0.029	0.013	0.898	0.327	0.784	0.264	0.

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	159	110	178	257	146	165	136
normalized size	1	1.	1.07	0.74	1.2	1.74	0.99	1.11	0.92
time (sec)	N/A	0.272	0.102	0.017	0.875	0.296	1.795	0.273	40.943

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	F(-2)	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	101	79	134	0	61	124	0
normalized size	1	1.	0.9	0.71	1.2	0.	0.54	1.11	0.
time (sec)	N/A	0.233	0.078	0.01	0.878	0.	3.129	0.262	0.

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	16	16	10	19	10
normalized size	1	1.	1.	0.93	1.14	1.14	0.71	1.36	0.71
time (sec)	N/A	0.038	0.005	0.007	0.796	0.27	0.188	0.259	4.67

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	14	14	8	16	8
normalized size	1	1.	1.	0.92	1.17	1.17	0.67	1.33	0.67
time (sec)	N/A	0.046	0.007	0.008	0.805	0.259	0.186	0.26	7.054

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	20	28	14	23	14
normalized size	1	1.	1.	0.94	1.18	1.65	0.82	1.35	0.82
time (sec)	N/A	0.036	0.006	0.01	0.825	0.27	0.214	0.259	7.941

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	20	20	14	23	14
normalized size	1	1.	1.	0.94	1.18	1.18	0.82	1.35	0.82
time (sec)	N/A	0.045	0.007	0.008	0.791	0.262	0.185	0.259	12.582

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	22	34	15	31	19
normalized size	1	1.	1.	0.94	1.22	1.89	0.83	1.72	1.06
time (sec)	N/A	0.072	0.007	0.009	0.888	0.258	0.23	0.26	9.387

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	14	19	7	15	7
normalized size	1	1.	1.	1.1	1.4	1.9	0.7	1.5	0.7
time (sec)	N/A	0.038	0.008	0.008	0.801	0.287	0.165	0.26	7.471

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	31	41	41	36	45	0
normalized size	1	1.	1.	0.74	0.98	0.98	0.86	1.07	0.
time (sec)	N/A	0.061	0.01	0.011	0.816	0.256	0.339	0.26	0.

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	41	51	62	51	51	48
normalized size	1	1.	1.	0.89	1.11	1.35	1.11	1.11	1.04
time (sec)	N/A	0.246	0.034	0.001	0.883	0.259	0.608	0.26	47.856

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	20	20	14	23	14
normalized size	1	1.	1.	0.84	1.05	1.05	0.74	1.21	0.74
time (sec)	N/A	0.11	0.005	0.008	0.804	0.276	0.21	0.262	14.709

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	31	41	41	34	43	0
normalized size	1	1.	1.	0.78	1.02	1.02	0.85	1.08	0.
time (sec)	N/A	0.062	0.009	0.009	0.8	0.267	0.262	0.261	0.

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	26	34	34	27	36	0
normalized size	1	1.	1.	0.79	1.03	1.03	0.82	1.09	0.
time (sec)	N/A	0.058	0.007	0.008	0.83	0.267	0.241	0.262	0.

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	21	27	27	20	30	20
normalized size	1	1.	1.	0.81	1.04	1.04	0.77	1.15	0.77
time (sec)	N/A	0.044	0.007	0.008	0.815	0.263	0.231	0.263	10.956

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	23	23	17	26	17
normalized size	1	1.	1.	0.86	1.1	1.1	0.81	1.24	0.81
time (sec)	N/A	0.023	0.005	0.007	0.816	0.263	0.212	0.26	4.018

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	23	23	15	26	15
normalized size	1	1.	1.	0.86	1.1	1.1	0.71	1.24	0.71
time (sec)	N/A	0.029	0.004	0.007	0.805	0.281	0.202	0.26	6.877

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	22	28	28	24	32	24
normalized size	1	1.	1.	0.81	1.04	1.04	0.89	1.19	0.89
time (sec)	N/A	0.046	0.007	0.009	0.82	0.271	0.311	0.264	12.101

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	27	35	41	31	39	31
normalized size	1	1.	1.	0.79	1.03	1.21	0.91	1.15	0.91
time (sec)	N/A	0.078	0.006	0.012	0.813	0.274	0.355	0.264	16.261

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	32	42	53	36	46	37
normalized size	1	1.	1.	0.78	1.02	1.29	0.88	1.12	0.9
time (sec)	N/A	0.085	0.007	0.013	0.808	0.259	0.399	0.262	16.934

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	37	49	59	41	53	44
normalized size	1	1.	1.	0.77	1.02	1.23	0.85	1.1	0.92
time (sec)	N/A	0.099	0.008	0.013	0.789	0.272	0.441	0.263	17.492

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	61	54	0	0	41	0	158
normalized size	1	1.	0.39	0.34	0.	0.	0.26	0.	1.01
time (sec)	N/A	0.344	0.026	0.013	0.	0.	0.666	0.	38.692

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	61	56	0	0	41	0	163
normalized size	1	1.	0.39	0.36	0.	0.	0.26	0.	1.04
time (sec)	N/A	0.267	0.025	0.013	0.	0.	0.666	0.	39.029

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	61	54	0	0	39	0	236
normalized size	1	1.	0.32	0.29	0.	0.	0.21	0.	1.26
time (sec)	N/A	0.575	0.019	0.009	0.	0.	0.592	0.	83.554

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	61	56	0	0	39	0	236
normalized size	1	1.	0.32	0.3	0.	0.	0.21	0.	1.26
time (sec)	N/A	0.379	0.019	0.009	0.	0.	0.601	0.	86.226

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	663	663	63	69	0	0	133	0	0
normalized size	1	1.	0.1	0.1	0.	0.	0.2	0.	0.
time (sec)	N/A	3.075	0.053	0.095	0.	0.	11.21	0.	0.

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	663	663	63	69	0	0	133	0	216
normalized size	1	1.	0.1	0.1	0.	0.	0.2	0.	0.33
time (sec)	N/A	2.694	0.043	0.002	0.	0.	11.29	0.	117.049

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	F	C	C	F	F(-2)	A	F	F(-1)
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	0	95	67	0	0	42	0	0
normalized size	1	0.	0.57	0.4	0.	0.	0.25	0.	0.
time (sec)	N/A	0.676	0.096	0.235	0.	0.	6.133	0.	0.

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	322	314	0	0	384	420	306
normalized size	1	1.	1.01	0.98	0.	0.	1.2	1.31	0.96
time (sec)	N/A	0.583	0.541	0.011	0.	0.	13.53	0.274	85.503

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	243	292	0	0	277	385	272
normalized size	1	1.	0.84	1.	0.	0.	0.95	1.32	0.93
time (sec)	N/A	0.467	0.199	0.005	0.	0.	10.086	0.271	79.4

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	184	151	0	0	124	290	207
normalized size	1	1.	0.84	0.69	0.	0.	0.57	1.32	0.95
time (sec)	N/A	0.36	0.101	0.005	0.	0.	2.915	0.269	60.161

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	134	128	0	142	20	242	172
normalized size	1	1.	0.72	0.69	0.	0.77	0.11	1.31	0.93
time (sec)	N/A	0.226	0.033	0.002	0.	0.26	0.385	0.264	47.674

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	416	416	404	433	0	0	0	520	379
normalized size	1	1.	0.97	1.04	0.	0.	0.	1.25	0.91
time (sec)	N/A	0.946	0.286	0.012	0.	0.	0.	0.292	133.876

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	552	552	524	866	0	0	0	0	0
normalized size	1	1.	0.95	1.57	0.	0.	0.	0.	0.
time (sec)	N/A	1.822	1.377	0.016	0.	0.	0.	0.	0.

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	680	680	738	1201	0	0	0	1	0
normalized size	1	1.	1.09	1.77	0.	0.	0.	0.	0.
time (sec)	N/A	2.133	1.967	0.019	0.	0.	0.	0.365	0.

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	347	390	0	0	350	462	333
normalized size	1	1.	0.99	1.12	0.	0.	1.	1.32	0.95
time (sec)	N/A	0.713	0.726	0.008	0.	0.	22.832	0.27	145.379

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	321	362	0	0	318	436	299
normalized size	1	1.	1.	1.12	0.	0.	0.99	1.35	0.93
time (sec)	N/A	0.58	0.654	0.007	0.	0.	13.061	0.271	98.979

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	224	188	0	0	155	325	231
normalized size	1	1.	0.93	0.78	0.	0.	0.64	1.35	0.96
time (sec)	N/A	0.424	0.352	0.006	0.	0.	4.999	0.27	82.947

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	183	143	0	213	39	262	190
normalized size	1	1.	0.91	0.71	0.	1.05	0.19	1.3	0.94
time (sec)	N/A	0.255	0.242	0.002	0.	0.282	2.018	0.262	55.095

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	855	855	558	1126	0	0	0	1038	0
normalized size	1	1.	0.65	1.32	0.	0.	0.	1.21	0.
time (sec)	N/A	1.992	0.722	0.038	0.	0.	0.	0.322	0.

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1141	1141	807	1644	0	0	0	1	0
normalized size	1	1.	0.71	1.44	0.	0.	0.	0.	0.
time (sec)	N/A	3.953	1.733	0.032	0.	0.	0.	0.362	0.

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1384	1384	996	2133	0	0	0	1	0
normalized size	1	1.	0.72	1.54	0.	0.	0.	0.	0.
time (sec)	N/A	4.67	2.436	0.037	0.	0.	0.	0.43	0.

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	394	394	388	470	0	0	413	525	386
normalized size	1	1.	0.98	1.19	0.	0.	1.05	1.33	0.98
time (sec)	N/A	0.781	0.835	0.01	0.	0.	30.204	0.276	121.822

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	360	358	419	0	0	374	481	343
normalized size	1	1.	0.99	1.16	0.	0.	1.04	1.34	0.95
time (sec)	N/A	0.697	0.601	0.007	0.	0.	16.814	0.274	119.628

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	249	222	0	0	192	351	257
normalized size	1	1.	0.94	0.83	0.	0.	0.72	1.32	0.97
time (sec)	N/A	0.521	0.353	0.007	0.	0.	8.866	0.272	90.524

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	200	158	0	293	63	275	207
normalized size	1	1.	0.91	0.72	0.	1.34	0.29	1.26	0.95
time (sec)	N/A	0.314	0.181	0.006	0.	0.301	5.048	0.267	59.989

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1352	1352	835	2106	0	0	0	1	0
normalized size	1	1.	0.62	1.56	0.	0.	0.	0.	0.
time (sec)	N/A	3.21	1.522	0.035	0.	0.	0.	0.389	0.

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1830	1830	1115	2781	0	0	0	1	0
normalized size	1	1.	0.61	1.52	0.	0.	0.	0.	0.
time (sec)	N/A	6.584	3.036	0.042	0.	0.	0.	0.41	0.

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2204	2204	1338	3352	0	0	0	1	0
normalized size	1	1.	0.61	1.52	0.	0.	0.	0.	0.
time (sec)	N/A	7.568	5.713	0.048	0.	0.	0.	0.531	0.

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	33	29	38	43	34	38	31
normalized size	1	1.	1.03	0.91	1.19	1.34	1.06	1.19	0.97
time (sec)	N/A	0.046	0.016	0.002	0.889	0.273	0.209	0.259	5.621

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	33	29	38	43	34	38	31
normalized size	1	1.	1.03	0.91	1.19	1.34	1.06	1.19	0.97
time (sec)	N/A	0.045	0.008	0.002	0.882	0.269	0.226	0.261	6.696

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	31	29	35	42	36	35	32
normalized size	1	1.	0.97	0.91	1.09	1.31	1.12	1.09	1.
time (sec)	N/A	0.041	0.015	0.007	0.874	0.275	0.22	0.261	5.835

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	31	29	35	42	36	35	32
normalized size	1	1.	0.97	0.91	1.09	1.31	1.12	1.09	1.
time (sec)	N/A	0.044	0.007	0.002	0.911	0.27	0.235	0.262	7.997

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	42	29	51	65	39	59	46
normalized size	1	1.	0.93	0.64	1.13	1.44	0.87	1.31	1.02
time (sec)	N/A	0.045	0.035	0.003	0.896	0.272	0.206	0.262	4.656

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	42	29	51	65	39	59	46
normalized size	1	1.	0.93	0.64	1.13	1.44	0.87	1.31	1.02
time (sec)	N/A	0.033	0.008	0.003	0.918	0.276	0.228	0.26	6.59

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	23	18	23	23	15	20	5
normalized size	1	1.	3.83	3.	3.83	3.83	2.5	3.33	0.83
time (sec)	N/A	0.008	0.005	0.008	0.79	0.267	0.16	0.258	2.062

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	23	18	23	23	15	26	15
normalized size	1	1.	1.1	0.86	1.1	1.1	0.71	1.24	0.71
time (sec)	N/A	0.013	0.004	0.002	0.841	0.269	0.177	0.264	1.93

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	11	10	12	32	22	12	8
normalized size	1	1.	0.85	0.77	0.92	2.46	1.69	0.92	0.62
time (sec)	N/A	0.008	0.004	0.001	0.783	0.263	0.321	0.259	1.921

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	B	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	81	11	58	77	32	22	77	63
normalized size	1	6.23	0.85	4.46	5.92	2.46	1.69	5.92	4.85
time (sec)	N/A	0.031	0.002	0.004	0.797	0.266	0.834	0.259	5.528

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	78	67	89	109	85	92	71
normalized size	1	1.	1.13	0.97	1.29	1.58	1.23	1.33	1.03
time (sec)	N/A	0.233	0.024	0.015	0.861	0.286	0.721	0.263	45.504

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	78	67	89	109	85	92	71
normalized size	1	1.	1.13	0.97	1.29	1.58	1.23	1.33	1.03
time (sec)	N/A	0.243	0.008	0.003	0.861	0.285	0.736	0.261	47.45

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	31	21	27	27	20	28	0
normalized size	1	1.	1.29	0.88	1.12	1.12	0.83	1.17	0.
time (sec)	N/A	0.041	0.009	0.004	0.796	0.273	0.124	0.261	0.

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	25	21	27	27	19	28	0
normalized size	1	1.	0.96	0.81	1.04	1.04	0.73	1.08	0.
time (sec)	N/A	0.035	0.007	0.003	0.793	0.269	0.116	0.26	0.

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	22	16	20	1	15	20	0
normalized size	1	1.	1.29	0.94	1.18	0.06	0.88	1.18	0.
time (sec)	N/A	0.019	0.	0.001	0.783	0.248	0.069	0.258	0.

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	22	20	27	1	15	27	0
normalized size	1	1.	0.92	0.83	1.12	0.04	0.62	1.12	0.
time (sec)	N/A	0.017	0.002	0.001	0.791	0.249	0.075	0.257	0.

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	23	30	34	20	31	20
normalized size	1	1.	1.09	1.05	1.36	1.55	0.91	1.41	0.91
time (sec)	N/A	0.055	0.008	0.01	0.859	0.281	0.281	0.261	7.06

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	27	16	20	20	15	24	15
normalized size	1	1.	1.59	0.94	1.18	1.18	0.88	1.41	0.88
time (sec)	N/A	0.041	0.01	0.01	0.797	0.28	0.265	0.259	5.614

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	18	18	12	22	14
normalized size	1	1.	1.	0.74	0.95	0.95	0.63	1.16	0.74
time (sec)	N/A	0.05	0.006	0.007	0.788	0.259	0.178	0.259	8.205

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	17	11	14	14	8	20	14
normalized size	1	1.	1.42	0.92	1.17	1.17	0.67	1.67	1.17
time (sec)	N/A	0.009	0.005	0.002	0.801	0.266	0.159	0.261	8.121

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	11	11	14	14	8	22	14
normalized size	1	1.	1.1	1.1	1.4	1.4	0.8	2.2	1.4
time (sec)	N/A	0.01	0.009	0.002	0.789	0.26	1.112	0.261	12.469

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	20	20	15	24	15
normalized size	1	1.	1.	0.94	1.18	1.18	0.88	1.41	0.88
time (sec)	N/A	0.044	0.009	0.01	0.795	0.264	0.252	0.26	6.573

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	23	23	17	24	17
normalized size	1	1.	1.	0.78	1.	1.	0.74	1.04	0.74
time (sec)	N/A	0.048	0.007	0.007	0.872	0.263	0.272	0.261	6.974

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	18	10	18	10
normalized size	1	1.	1.	0.93	1.2	1.2	0.67	1.2	0.67
time (sec)	N/A	0.01	0.008	0.002	0.8	0.267	0.194	0.261	5.06

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	23	23	20	27	20
normalized size	1	1.	1.	0.86	1.1	1.1	0.95	1.29	0.95
time (sec)	N/A	0.048	0.009	0.01	0.785	0.271	0.276	0.26	12.617

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	9	14	12	12	7	12	7
normalized size	1	1.	0.9	1.4	1.2	1.2	0.7	1.2	0.7
time (sec)	N/A	0.016	0.009	0.007	0.791	0.251	0.162	0.259	6.909

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	18	23	23	20	27	20
normalized size	1	1.	1.	0.72	0.92	0.92	0.8	1.08	0.8
time (sec)	N/A	0.047	0.008	0.01	0.793	0.258	0.299	0.264	9.823

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	19	19	12	19	12
normalized size	1	1.	1.	1.07	1.36	1.36	0.86	1.36	0.86
time (sec)	N/A	0.044	0.006	0.003	0.871	0.265	0.197	0.261	7.141

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	14	30	15	22	17	15	14
normalized size	1	1.	1.08	2.31	1.15	1.69	1.31	1.15	1.08
time (sec)	N/A	0.008	0.01	0.017	0.788	0.245	0.368	0.26	8.477

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	25	32	35	34	20	36	0
normalized size	1	1.	0.96	1.23	1.35	1.31	0.77	1.38	0.
time (sec)	N/A	0.07	0.011	0.004	0.789	0.272	1.241	0.263	0.

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	4	5	5	5	3	7	3
normalized size	1	1.	0.67	0.83	0.83	0.83	0.5	1.17	0.5
time (sec)	N/A	0.025	0.001	0.001	0.782	0.252	0.084	0.261	6.117

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	27	34	19	36	20
normalized size	1	1.	1.	0.95	1.35	1.7	0.95	1.8	1.
time (sec)	N/A	0.052	0.007	0.009	0.782	0.254	0.23	0.259	8.09

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	40	28	36	36	31	38	31
normalized size	1	1.	1.05	0.74	0.95	0.95	0.82	1.	0.82
time (sec)	N/A	0.07	0.017	0.008	0.881	0.253	0.307	0.266	6.351

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	18	16	20	20	14	22	0
normalized size	1	1.	1.06	0.94	1.18	1.18	0.82	1.29	0.
time (sec)	N/A	0.024	0.005	0.003	0.784	0.251	0.118	0.258	0.

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	36	22	28	28	22	30	22
normalized size	1	1.	1.16	0.71	0.9	0.9	0.71	0.97	0.71
time (sec)	N/A	0.036	0.008	0.007	0.871	0.253	0.318	0.261	5.006

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	23	20	17	23	17
normalized size	1	1.	1.	0.84	1.21	1.05	0.89	1.21	0.89
time (sec)	N/A	0.053	0.008	0.008	0.789	0.254	0.258	0.266	8.065

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	9	9	12	12	8	15	8
normalized size	1	1.	0.82	0.82	1.09	1.09	0.73	1.36	0.73
time (sec)	N/A	0.022	0.005	0.002	0.801	0.249	0.165	0.261	3.474

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	13	16	16	14	16	14
normalized size	1	1.	1.	0.72	0.89	0.89	0.78	0.89	0.78
time (sec)	N/A	0.04	0.011	0.009	0.865	0.258	0.427	0.259	15.173

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	44	39	62	112	46	70	41
normalized size	1	1.	0.96	0.85	1.35	2.43	1.	1.52	0.89
time (sec)	N/A	0.06	0.027	0.016	0.78	0.275	0.508	0.261	7.235

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	10	14	11	11	7	12	7
normalized size	1	1.	0.83	1.17	0.92	0.92	0.58	1.	0.58
time (sec)	N/A	0.007	0.002	0.003	0.775	0.259	0.122	0.26	1.7

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	27	28	31	46	20	34	12
normalized size	1	1.	1.29	1.33	1.48	2.19	0.95	1.62	0.57
time (sec)	N/A	0.011	0.012	0.013	0.805	0.265	0.219	0.26	1.184

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	20	28	12	20	12
normalized size	1	1.	1.	0.84	1.05	1.47	0.63	1.05	0.63
time (sec)	N/A	0.016	0.012	0.009	0.869	0.26	0.204	0.26	3.287

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	11	11	7	12	7
normalized size	1	1.	1.	0.9	1.1	1.1	0.7	1.2	0.7
time (sec)	N/A	0.006	0.001	0.001	0.782	0.268	0.062	0.263	1.08

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	14	14	20	14	5
normalized size	1	1.	1.	1.1	1.4	1.4	2.	1.4	0.5
time (sec)	N/A	0.009	0.003	0.005	0.871	0.266	0.244	0.26	1.548

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	16	0	1	53	20	22
normalized size	1	1.	1.	0.67	0.	0.04	2.21	0.83	0.92
time (sec)	N/A	0.019	0.007	0.001	0.	0.259	0.3	0.259	2.476

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	17	22	22	26	22	22
normalized size	1	1.	1.	0.89	1.16	1.16	1.37	1.16	1.16
time (sec)	N/A	0.032	0.009	0.004	0.872	0.271	0.212	0.259	1.405

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	22	1	17	22	17
normalized size	1	1.	1.	0.77	1.	0.05	0.77	1.	0.77
time (sec)	N/A	0.024	0.001	0.001	0.817	0.236	0.067	0.259	4.292

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	22	1	15	22	0
normalized size	1	1.	1.	0.77	1.	0.05	0.68	1.	0.
time (sec)	N/A	0.02	0.001	0.001	0.81	0.23	0.065	0.258	0.

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	19	20	10	19	0
normalized size	1	1.	1.	0.94	1.19	1.25	0.62	1.19	0.
time (sec)	N/A	0.014	0.002	0.005	0.82	0.253	0.108	0.261	0.

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	31	41	46	39	41	34
normalized size	1	1.	1.	0.84	1.11	1.24	1.05	1.11	0.92
time (sec)	N/A	0.05	0.019	0.006	0.888	0.253	0.233	0.26	6.371

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	16	1	10	16	10
normalized size	1	1.	1.	0.93	1.14	0.07	0.71	1.14	0.71
time (sec)	N/A	0.012	0.001	0.001	0.806	0.224	0.06	0.257	1.566

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	12	12	7	12	0
normalized size	1	1.	1.	0.91	1.09	1.09	0.64	1.09	0.
time (sec)	N/A	0.011	0.001	0.001	0.807	0.254	0.063	0.261	0.

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	26	26	20	27	20
normalized size	1	1.	1.	0.8	1.04	1.04	0.8	1.08	0.8
time (sec)	N/A	0.032	0.009	0.008	0.889	0.261	0.287	0.262	4.815

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	26	26	19	27	19
normalized size	1	1.	1.	0.8	1.04	1.04	0.76	1.08	0.76
time (sec)	N/A	0.031	0.008	0.006	0.884	0.267	0.261	0.26	5.124

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	26	26	19	27	19
normalized size	1	1.	1.	0.8	1.04	1.04	0.76	1.08	0.76
time (sec)	N/A	0.053	0.009	0.007	0.881	0.282	0.265	0.261	5.401

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	7	8	9	16	5	11	5
normalized size	1	1.	0.78	0.89	1.	1.78	0.56	1.22	0.56
time (sec)	N/A	0.007	0.005	0.005	0.776	0.255	0.124	0.259	3.01

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	22	30	20	22	20
normalized size	1	1.	1.	0.77	1.	1.36	0.91	1.	0.91
time (sec)	N/A	0.032	0.02	0.01	0.858	0.265	0.411	0.26	8.636

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	31	41	47	44	41	37
normalized size	1	1.	1.	0.84	1.11	1.27	1.19	1.11	1.
time (sec)	N/A	0.053	0.021	0.006	0.855	0.265	0.253	0.262	6.567

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	20	26	14	22	0
normalized size	1	1.	1.	0.93	1.33	1.73	0.93	1.47	0.
time (sec)	N/A	0.013	0.003	0.008	0.781	0.263	0.169	0.261	0.

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	20	10	19	0
normalized size	1	1.	1.	0.93	1.2	1.33	0.67	1.27	0.
time (sec)	N/A	0.011	0.001	0.005	0.803	0.265	0.125	0.26	0.

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	18	15	8	18	12
normalized size	1	1.	1.	1.09	1.64	1.36	0.73	1.64	1.09
time (sec)	N/A	0.041	0.005	0.006	0.793	0.257	0.189	0.261	6.694

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	19	18	23	1	17	23	0
normalized size	1	1.	0.86	0.82	1.05	0.05	0.77	1.05	0.
time (sec)	N/A	0.018	0.002	0.001	0.782	0.246	0.059	0.261	0.

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	26	1	19	12	7
normalized size	1	1.	1.	0.91	2.36	0.09	1.73	1.09	0.64
time (sec)	N/A	0.007	0.002	0.001	0.778	0.224	0.066	0.258	1.056

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	16	14	18	18	12	19	0
normalized size	1	1.	1.23	1.08	1.38	1.38	0.92	1.46	0.
time (sec)	N/A	0.026	0.006	0.003	0.782	0.261	0.123	0.261	0.

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	19	24	14	22	14
normalized size	1	1.	1.	0.94	1.19	1.5	0.88	1.38	0.88
time (sec)	N/A	0.025	0.004	0.009	0.772	0.26	0.194	0.263	3.269

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	31	26	24	34	53	22	36	22
normalized size	1	1.24	1.04	0.96	1.36	2.12	0.88	1.44	0.88
time (sec)	N/A	0.027	0.031	0.013	0.781	0.276	0.236	0.26	15.594

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	18	16	20	20	12	22	0
normalized size	1	1.	1.06	0.94	1.18	1.18	0.71	1.29	0.
time (sec)	N/A	0.024	0.005	0.003	0.781	0.253	0.108	0.26	0.

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	22	27	15	23	0
normalized size	1	1.	1.	0.94	1.22	1.5	0.83	1.28	0.
time (sec)	N/A	0.016	0.002	0.008	0.781	0.25	0.13	0.26	0.

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	F(-2)	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	28	19	0	14	19	0
normalized size	1	1.	1.	1.56	1.06	0.	0.78	1.06	0.
time (sec)	N/A	0.031	0.001	0.001	0.869	0.	0.066	0.258	0.

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	46	62	31	34	19
normalized size	1	1.	1.	0.96	2.	2.7	1.35	1.48	0.83
time (sec)	N/A	0.037	0.018	0.009	0.794	0.249	0.249	0.261	7.274

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	12	13	16	20	10	43	10
normalized size	1	1.	0.75	0.81	1.	1.25	0.62	2.69	0.62
time (sec)	N/A	0.037	0.011	0.006	0.876	0.249	0.243	0.265	4.705

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	30	26	34	34	24	35	0
normalized size	1	1.	1.03	0.9	1.17	1.17	0.83	1.21	0.
time (sec)	N/A	0.043	0.013	0.003	0.795	0.25	0.139	0.26	0.

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	16	16	10	16	0
normalized size	1	1.	1.	0.81	1.	1.	0.62	1.	0.
time (sec)	N/A	0.012	0.001	0.001	0.785	0.245	0.073	0.259	0.

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	23	34	14	16	14
normalized size	1	1.	1.	0.94	1.35	2.	0.82	0.94	0.82
time (sec)	N/A	0.045	0.019	0.009	0.855	0.263	0.308	0.261	5.937

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	43	0	1	151	65	54
normalized size	1	1.	1.	0.91	0.	0.02	3.21	1.38	1.15
time (sec)	N/A	0.128	0.04	0.009	0.	0.283	0.83	0.261	8.438

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	61	61	0	1	294	81	68
normalized size	1	1.	1.07	1.07	0.	0.02	5.16	1.42	1.19
time (sec)	N/A	0.159	0.048	0.009	0.	0.276	1.046	0.26	11.858

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	39	308	0	630	24	0	185
normalized size	1	1.	0.21	1.64	0.	3.35	0.13	0.	0.98
time (sec)	N/A	0.397	0.046	0.047	0.	0.289	1.766	0.	36.341

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	27	28	88	88	61	41	60
normalized size	1	1.	0.45	0.47	1.47	1.47	1.02	0.68	1.
time (sec)	N/A	0.232	0.023	0.012	0.799	0.256	0.823	0.26	57.896

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	F	A	C	A	A	A	F	F(-1)
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	27	250	88	88	61	0	0
normalized size	1	0.	1.	9.26	3.26	3.26	2.26	0.	0.
time (sec)	N/A	0.549	0.014	0.04	0.813	0.258	1.067	0.	0.

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	F	A	B	A	A	A	A	F(-1)
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	27	112	88	88	61	150	0
normalized size	1	0.	1.	4.15	3.26	3.26	2.26	5.56	0.
time (sec)	N/A	0.787	0.014	0.024	0.811	0.277	0.952	0.262	0.

2.2 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [412] had the largest ratio of [0.8235]

Table 1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	2	1.	24	0.083
2	A	3	3	1.	29	0.103
3	A	2	2	1.	29	0.069
4	A	2	2	1.	29	0.069
5	A	1	0	1.	27	0.
6	A	2	2	1.	29	0.069
7	A	2	2	1.	29	0.069
8	A	2	2	1.	29	0.069
9	A	3	2	1.	27	0.074
10	A	3	2	1.	27	0.074
11	A	1	0	1.	25	0.
12	A	7	7	1.	27	0.259
13	A	8	8	1.	27	0.296
14	A	9	8	1.	27	0.296
15	A	3	2	1.	46	0.043
16	A	3	2	1.	46	0.043
17	A	1	0	1.	44	0.
18	A	2	1	1.	46	0.022
19	A	2	1	1.	46	0.022
20	A	2	1	1.	46	0.022
21	A	5	4	1.	11	0.364
22	A	5	4	1.	17	0.235
23	A	2	2	1.	7	0.286
24	A	3	2	1.	13	0.154
25	A	5	5	1.	11	0.454
26	A	7	7	1.	16	0.438
27	A	6	6	1.	9	0.667
28	A	2	2	1.	7	0.286
29	A	3	3	1.	13	0.231
30	A	3	3	1.11	11	0.273
31	A	3	3	1.	16	0.188
32	A	2	2	1.26	9	0.222
33	A	3	2	1.	29	0.069
34	A	2	1	1.	29	0.034
35	A	2	1	1.	29	0.034
36	A	1	0	1.	27	0.
37	A	10	6	1.	29	0.207
38	A	11	7	1.	29	0.241
39	A	3	2	1.	32	0.062
40	A	2	1	1.	32	0.031
41	A	2	1	1.	32	0.031
42	A	1	0	1.	30	0.
43	A	4	3	1.	32	0.094
44	A	5	4	1.	32	0.125
45	A	2	1	1.	17	0.059
46	A	2	1	1.	17	0.059

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
47	A	2	1	1.	17	0.059
48	A	1	0	1.	15	0.
49	A	16	9	1.	17	0.529
50	A	18	11	1.	17	0.647
51	A	2	1	1.	17	0.059
52	A	2	1	1.	17	0.059
53	A	2	1	1.	17	0.059
54	A	1	0	1.	15	0.
55	A	15	9	1.	17	0.529
56	A	17	11	1.	17	0.647
57	A	2	1	1.	22	0.045
58	A	2	1	1.	22	0.045
59	A	2	1	1.	22	0.045
60	A	1	0	1.	20	0.
61	A	16	9	1.	22	0.409
62	A	18	11	1.	22	0.5
63	A	2	2	1.	51	0.039
64	A	2	2	1.	51	0.039
65	B	1	0	4.36	49	0.
66	A	2	2	1.	51	0.039
67	A	2	2	1.	51	0.039
68	A	2	2	1.	51	0.039
69	A	6	4	1.	13	0.308
70	A	3	2	1.	19	0.105
71	A	3	2	1.	19	0.105
72	A	3	2	1.	19	0.105
73	A	1	0	1.	17	0.
74	A	5	2	1.	19	0.105
75	A	7	3	1.	19	0.158
76	A	10	3	1.	19	0.158
77	B	15	7	2.25	17	0.412
78	A	6	5	1.	15	0.333
79	A	6	5	1.	15	0.333
80	A	4	4	1.	13	0.308
81	A	2	2	1.	11	0.182
82	A	6	6	1.	15	0.4
83	A	7	6	1.	15	0.4
84	A	7	6	1.	15	0.4
85	A	2	2	1.	13	0.154
86	A	3	3	1.	13	0.231
87	A	4	3	1.	13	0.231
88	A	2	2	1.	19	0.105
89	A	2	2	1.	11	0.182
90	A	3	3	1.	11	0.273
91	A	4	3	1.	11	0.273
92	A	2	2	1.	13	0.154
93	A	3	3	1.	13	0.231

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
94	A	4	3	1.	13	0.231
95	A	2	2	1.	11	0.182
96	A	3	3	1.	11	0.273
97	A	4	3	1.	11	0.273
98	A	3	2	1.	15	0.133
99	A	4	3	1.	13	0.231
100	A	4	4	1.	17	0.235
101	A	4	4	1.	19	0.21
102	A	4	4	1.	17	0.235
103	A	11	9	1.	17	0.529
104	A	9	8	1.	17	0.471
105	A	7	7	1.	15	0.467
106	A	7	7	1.	13	0.538
107	A	11	9	1.06	17	0.529
108	A	11	9	0.99	17	0.529
109	A	11	9	1.	17	0.529
110	A	16	11	1.	17	0.647
111	A	14	10	1.	17	0.588
112	A	14	10	1.	15	0.667
113	A	10	7	1.	13	0.538
114	A	18	12	1.	17	0.706
115	A	18	12	1.	17	0.706
116	A	3	2	1.	22	0.091
117	A	2	1	1.	22	0.045
118	A	2	1	1.	22	0.045
119	A	1	0	1.	20	0.
120	A	4	3	1.	22	0.136
121	A	5	4	1.	22	0.182
122	A	6	5	1.	22	0.227
123	A	2	1	1.	24	0.042
124	A	2	1	1.	24	0.042
125	A	2	1	1.	24	0.042
126	A	2	1	1.	22	0.045
127	A	8	7	1.	24	0.292
128	A	10	9	1.	24	0.375
129	A	12	10	1.	24	0.417
130	A	2	1	1.	26	0.038
131	A	2	1	1.	26	0.038
132	A	2	1	1.	26	0.038
133	A	2	1	1.	24	0.042
134	A	9	8	1.	26	0.308
135	A	11	10	1.	26	0.385
136	A	14	5	1.	46	0.109
137	A	14	5	1.	46	0.109
138	A	8	3	1.	46	0.065
139	A	14	5	1.	44	0.114
140	A	14	5	1.	42	0.119

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
141	A	14	5	1.	46	0.109
142	A	14	5	0.99	46	0.109
143	A	14	6	1.	26	0.231
144	A	14	6	1.	26	0.231
145	A	14	6	1.	26	0.231
146	A	8	4	1.	26	0.154
147	A	14	6	1.	24	0.25
148	A	14	6	1.	22	0.273
149	A	14	6	1.	26	0.231
150	A	14	6	1.	26	0.231
151	A	23	7	1.	26	0.269
152	A	23	7	1.	26	0.269
153	A	14	6	1.	26	0.231
154	A	17	5	1.	26	0.192
155	A	23	7	1.	26	0.269
156	A	23	7	1.	26	0.269
157	A	23	7	1.	26	0.269
158	A	2	1	1.	52	0.019
159	A	4	3	1.	52	0.058
160	A	1	1	1.	18	0.056
161	A	1	1	1.	23	0.043
162	A	1	1	1.	23	0.043
163	A	3	3	1.	29	0.103
164	A	1	1	1.	18	0.056
165	A	4	3	1.	20	0.15
166	A	4	3	1.	20	0.15
167	A	4	3	1.	22	0.136
168	A	1	1	1.	18	0.056
169	A	1	1	1.	23	0.043
170	A	1	1	1.	23	0.043
171	A	3	3	1.	29	0.103
172	A	1	1	1.	18	0.056
173	A	1	1	1.	25	0.04
174	C	7	3	4.3	38	0.079
175	A	1	1	1.	27	0.037
176	A	1	1	1.	31	0.032
177	A	2	1	1.	54	0.019
178	A	3	1	1.	54	0.019
179	A	5	4	1.	54	0.074
180	A	1	1	1.	30	0.033
181	A	1	1	1.	29	0.034
182	A	1	1	1.	28	0.036
183	A	1	1	1.	21	0.048
184	A	1	1	1.	20	0.05
185	A	1	1	1.	21	0.048
186	A	1	1	1.	26	0.038
187	A	1	1	1.	25	0.04

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
188	A	1	1	1.	26	0.038
189	A	1	1	1.	24	0.042
190	A	1	1	1.	24	0.042
191	A	1	1	1.	23	0.043
192	A	1	1	1.	30	0.033
193	A	1	1	1.	29	0.034
194	A	1	1	1.	28	0.036
195	A	1	1	1.	21	0.048
196	A	1	1	1.	21	0.048
197	A	1	1	1.	20	0.05
198	A	1	1	1.	26	0.038
199	A	1	1	1.	25	0.04
200	A	1	1	1.	26	0.038
201	A	1	1	1.	22	0.045
202	A	1	1	1.	24	0.042
203	A	1	1	1.	23	0.043
204	A	1	1	1.	23	0.043
205	A	1	1	1.	19	0.053
206	A	2	1	1.	22	0.045
207	A	2	1	1.	23	0.043
208	A	2	1	1.	22	0.045
209	A	2	1	1.	23	0.043
210	A	2	1	1.	24	0.042
211	A	2	1	1.	25	0.04
212	A	2	1	1.	31	0.032
213	A	2	1	1.	32	0.031
214	A	2	1	1.	35	0.029
215	A	2	1	1.	36	0.028
216	A	2	1	1.	24	0.042
217	A	2	1	1.	31	0.032
218	A	2	1	1.	35	0.029
219	A	1	1	1.	22	0.045
220	A	1	1	1.	18	0.056
221	B	3	2	2.91	26	0.077
222	B	2	1	2.91	28	0.036
223	A	1	1	1.	18	0.056
224	A	1	1	1.	20	0.05
225	A	1	1	1.	21	0.048
226	A	3	3	1.	52	0.058
227	A	9	5	1.	38	0.132
228	A	3	2	1.	32	0.062
229	A	4	3	1.	33	0.091
230	A	3	2	1.	34	0.059
231	A	6	6	1.	43	0.14
232	A	1	1	1.	16	0.062
233	B	15	7	2.25	25	0.28
234	A	1	1	1.	56	0.018

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
235	A	1	1	1.	51	0.02
236	A	1	1	1.	49	0.02
237	A	1	1	1.	46	0.022
238	A	1	1	1.	48	0.021
239	A	1	1	1.	49	0.02
240	A	1	1	1.	48	0.021
241	A	1	1	1.	48	0.021
242	A	10	5	1.	35	0.143
243	A	10	5	1.	35	0.143
244	A	10	5	1.	35	0.143
245	A	10	5	1.	33	0.152
246	A	10	5	1.	32	0.156
247	A	13	6	1.	35	0.171
248	A	13	6	1.	35	0.171
249	A	13	6	1.	35	0.171
250	A	13	6	1.	35	0.171
251	A	13	6	1.	35	0.171
252	A	11	6	1.	33	0.182
253	A	9	5	1.	32	0.156
254	A	13	6	1.	35	0.171
255	A	13	6	1.	35	0.171
256	A	13	6	1.	35	0.171
257	A	2	2	1.	40	0.05
258	A	6	5	1.	20	0.25
259	A	3	2	1.	20	0.1
260	A	3	2	1.	20	0.1
261	A	2	1	1.	16	0.062
262	A	5	4	1.	22	0.182
263	A	3	2	1.	21	0.095
264	A	6	5	1.	26	0.192
265	A	2	1	1.	20	0.05
266	A	2	1	1.	11	0.091
267	A	4	3	1.	22	0.136
268	A	3	2	1.	21	0.095
269	A	3	2	1.	25	0.08
270	A	5	4	1.	22	0.182
271	A	5	5	1.	31	0.161
272	A	3	2	1.	21	0.095
273	A	4	3	1.	33	0.091
274	A	4	4	1.	14	0.286
275	A	7	6	1.	33	0.182
276	A	7	6	1.	29	0.207
277	A	6	3	1.	44	0.068
278	A	3	2	1.	15	0.133
279	A	5	4	1.	15	0.267
280	A	4	3	1.	18	0.167
281	A	3	2	1.	20	0.1

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
282	A	5	4	1.	26	0.154
283	A	3	2	1.	13	0.154
284	A	3	2	1.	18	0.111
285	A	2	1	1.	26	0.038
286	A	5	5	1.	19	0.263
287	A	5	5	1.	24	0.208
288	A	8	6	1.	20	0.3
289	A	8	6	1.	18	0.333
290	A	5	3	1.	19	0.158
291	A	3	2	1.	13	0.154
292	A	6	5	1.	22	0.227
293	A	6	5	1.	24	0.208
294	A	2	1	1.	29	0.034
295	A	2	1	1.	30	0.033
296	A	2	1	1.	19	0.053
297	A	4	4	1.	16	0.25
298	A	10	5	1.	36	0.139
299	A	2	1	1.	21	0.048
300	A	5	4	1.	16	0.25
301	A	2	1	1.	24	0.042
302	A	2	1	1.	21	0.048
303	A	2	1	1.	24	0.042
304	A	6	5	1.	26	0.192
305	A	3	2	1.	25	0.08
306	A	2	1	1.	29	0.034
307	A	6	5	1.	20	0.25
308	A	14	9	1.	32	0.281
309	A	4	4	1.	23	0.174
310	A	6	5	1.	26	0.192
311	A	4	3	1.	26	0.115
312	A	8	4	1.	25	0.16
313	A	6	3	1.	23	0.13
314	A	7	6	1.	23	0.261
315	A	5	3	1.18	20	0.15
316	A	3	2	1.	25	0.08
317	A	3	2	1.	22	0.091
318	A	2	1	1.	24	0.042
319	A	6	5	1.	24	0.208
320	A	6	5	1.	43	0.116
321	A	7	5	1.25	50	0.1
322	A	3	2	1.	16	0.125
323	A	6	5	1.	15	0.333
324	A	6	4	1.	20	0.2
325	A	3	2	1.	24	0.083
326	A	5	3	1.	27	0.111
327	A	5	5	1.	26	0.192
328	A	3	2	1.	16	0.125

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
329	A	11	8	1.32	22	0.364
330	A	5	4	1.	24	0.167
331	A	4	3	1.	26	0.115
332	A	5	3	1.	36	0.083
333	A	4	3	1.	26	0.115
334	A	10	8	1.	20	0.4
335	A	6	6	1.	27	0.222
336	A	7	7	1.	20	0.35
337	A	8	7	1.	25	0.28
338	A	8	7	1.	22	0.318
339	A	2	1	1.	18	0.056
340	A	5	4	1.	20	0.2
341	A	10	8	1.	20	0.4
342	A	16	11	1.	20	0.55
343	A	3	2	1.	14	0.143
344	A	2	2	1.	20	0.1
345	A	14	8	1.	20	0.4
346	A	4	3	1.	26	0.115
347	A	4	3	1.	24	0.125
348	A	6	4	1.	30	0.133
349	A	4	4	1.	21	0.19
350	A	2	1	1.	15	0.067
351	A	4	3	1.	18	0.167
352	A	3	2	1.	22	0.091
353	A	3	2	1.	16	0.125
354	A	6	5	1.	16	0.312
355	A	3	2	1.	25	0.08
356	A	3	2	1.	19	0.105
357	A	2	1	1.	23	0.043
358	A	5	4	1.	23	0.174
359	A	5	4	1.	21	0.19
360	A	10	6	1.	28	0.214
361	A	2	1	1.	24	0.042
362	A	6	5	1.	26	0.192
363	A	2	1	1.	14	0.071
364	A	5	3	1.	16	0.188
365	A	5	5	1.	16	0.312
366	A	7	5	1.	43	0.116
367	A	17	12	1.	26	0.462
368	A	18	12	1.	16	0.75
369	A	3	2	1.	15	0.133
370	A	4	3	1.	15	0.2
371	A	3	2	1.	17	0.118
372	A	4	3	1.	15	0.2
373	A	4	3	1.	15	0.2
374	A	4	3	1.	18	0.167
375	A	3	2	1.	20	0.1

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
376	A	7	6	1.	29	0.207
377	A	4	3	1.	22	0.136
378	A	6	4	1.	16	0.25
379	A	6	4	1.	16	0.25
380	A	5	4	1.	14	0.286
381	A	4	3	1.	12	0.25
382	A	4	3	1.	16	0.188
383	A	6	5	1.	16	0.312
384	A	4	3	1.	16	0.188
385	A	4	3	1.	16	0.188
386	A	4	3	1.	16	0.188
387	A	8	5	1.	17	0.294
388	A	8	5	1.	19	0.263
389	A	8	5	1.	15	0.333
390	A	8	5	1.	17	0.294
391	A	16	10	1.	23	0.435
392	A	17	10	1.	21	0.476
393	F	0	0	N/A	0	N/A
394	A	15	10	1.	17	0.588
395	A	13	9	1.	17	0.529
396	A	13	9	1.	15	0.6
397	A	9	6	1.	9	0.667
398	A	17	11	1.	17	0.647
399	A	17	11	1.	17	0.647
400	A	17	11	1.	17	0.647
401	A	16	12	1.	17	0.706
402	A	14	10	1.	17	0.588
403	A	14	10	1.	15	0.667
404	A	10	7	1.	9	0.778
405	A	31	13	1.	17	0.765
406	A	31	13	1.	17	0.765
407	A	31	13	1.	17	0.765
408	A	15	11	1.	17	0.647
409	A	15	10	1.	17	0.588
410	A	15	10	1.	15	0.667
411	A	11	7	1.	9	0.778
412	A	46	14	1.	17	0.824
413	A	46	14	1.	17	0.824
414	A	46	14	1.	17	0.824
415	A	4	4	1.	14	0.286
416	A	5	5	1.	13	0.385
417	A	4	4	1.	16	0.25
418	A	5	5	1.	18	0.278
419	A	3	2	1.	14	0.143
420	A	4	3	1.	16	0.188
421	A	2	2	1.	11	0.182
422	A	1	0	1.	17	0.

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
423	A	1	1	1.	11	0.091
424	B	1	0	6.23	73	0.
425	A	11	7	1.	13	0.538
426	A	13	8	1.	19	0.421
427	A	3	2	1.	15	0.133
428	A	3	2	1.	11	0.182
429	A	1	0	1.	11	0.
430	A	1	0	1.	11	0.
431	A	5	4	1.	16	0.25
432	A	2	1	1.	16	0.062
433	A	4	3	1.	15	0.2
434	A	1	1	1.	15	0.067
435	A	1	1	1.	20	0.05
436	A	3	2	1.	15	0.133
437	A	6	5	1.	13	0.385
438	A	1	1	1.	22	0.045
439	A	3	2	1.	18	0.111
440	A	3	3	1.	20	0.15
441	A	3	2	1.	16	0.125
442	A	6	5	1.	17	0.294
443	A	1	1	1.	17	0.059
444	A	4	2	1.	25	0.08
445	A	4	4	1.	20	0.2
446	A	3	2	1.	18	0.111
447	A	6	5	1.	15	0.333
448	A	2	1	1.	11	0.091
449	A	5	5	1.	13	0.385
450	A	3	2	1.	20	0.1
451	A	2	1	1.	16	0.062
452	A	4	3	1.	16	0.188
453	A	2	1	1.	16	0.062
454	A	1	1	1.	9	0.111
455	A	2	2	1.	7	0.286
456	A	2	2	1.	11	0.182
457	A	1	1	1.	7	0.143
458	A	1	1	1.	9	0.111
459	A	1	1	1.	9	0.111
460	A	2	2	1.	10	0.2
461	A	2	1	1.	13	0.077
462	A	2	1	1.	11	0.091
463	A	2	1	1.	14	0.071
464	A	4	4	1.	16	0.25
465	A	2	1	1.	7	0.143
466	A	2	1	1.	11	0.091
467	A	5	5	1.	13	0.385
468	A	5	5	1.	13	0.385
469	A	5	4	1.	14	0.286

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
470	A	1	1	1.	13	0.077
471	A	3	2	1.	20	0.1
472	A	4	4	1.	18	0.222
473	A	2	1	1.	12	0.083
474	A	2	1	1.	10	0.1
475	A	3	2	1.	16	0.125
476	A	2	1	1.	11	0.091
477	A	1	1	1.	7	0.143
478	A	2	1	1.	15	0.067
479	A	2	1	1.	12	0.083
480	A	4	3	1.24	16	0.188
481	A	2	1	1.	11	0.091
482	A	2	1	1.	17	0.059
483	A	2	1	1.	29	0.034
484	A	2	1	1.	18	0.056
485	A	3	2	1.	14	0.143
486	A	2	1	1.	24	0.042
487	A	2	1	1.	11	0.091
488	A	3	2	1.	18	0.111
489	A	3	3	1.	15	0.2
490	A	3	3	1.	16	0.188
491	A	10	7	1.	15	0.467
492	A	5	2	1.	50	0.04
493	F	0	0	N/A	0	N/A
494	F	0	0	N/A	0	N/A

3 Listing of integrals

3.1 $\int \frac{1}{2\sqrt{3}b^{3/2}-9bx+9x^3}\,dx$

Optimal. Leaf size=77

$$\frac{1}{3\sqrt{3}\sqrt{b}\left(\sqrt{3}\sqrt{b}-3x\right)}-\frac{\log\left(\sqrt{b}-\sqrt{3}x\right)}{27b}+\frac{\log\left(2\sqrt{b}+\sqrt{3}x\right)}{27b}$$

```
[Out] 1/(3*Sqrt[3]*Sqrt[b]*(Sqrt[3]*Sqrt[b] - 3*x)) - Log[Sqrt[b] - Sqr
t[3]*x]/(27*b) + Log[2*Sqrt[b] + Sqrt[3]*x]/(27*b)
```

Rubi [A] time = 0.130636, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$\frac{1}{3\sqrt{3}\sqrt{b}\left(\sqrt{3}\sqrt{b}-3x\right)}-\frac{\log\left(\sqrt{b}-\sqrt{3}x\right)}{27b}+\frac{\log\left(2\sqrt{b}+\sqrt{3}x\right)}{27b}$$

Antiderivative was successfully verified.

```
[In]  Int[(2*Sqrt[3]*b^(3/2) - 9*b*x + 9*x^3)^(-1),x]
```

```
[Out] 1/(3*Sqrt[3]*Sqrt[b]*(Sqrt[3]*Sqrt[b] - 3*x)) - Log[Sqrt[b] - Sqr
t[3]*x]/(27*b) + Log[2*Sqrt[b] + Sqrt[3]*x]/(27*b)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-9*b*x+9*x**3+2*b**(3/2)*3**(1/2)),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.0697358, size = 143, normalized size = 1.86

$$-\frac{\left(3x-\sqrt{3}\sqrt{b}\right)\left(2\sqrt{3}\sqrt{b}+3x\right)\left(\left(3x-\sqrt{3}\sqrt{b}\right)\log\left(3x-\sqrt{3}\sqrt{b}\right)+\left(\sqrt{3}\sqrt{b}-3x\right)\log\left(2\sqrt{3}\sqrt{b}+3x\right)+3\sqrt{3}\sqrt{b}\right)}{81b\left(2\sqrt{3}b^{3/2}-9bx+9x^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2*Sqrt[3]*b^(3/2) - 9*b*x + 9*x^3)^(-1),x]
```

```
[Out] -((-(Sqrt[3]*Sqrt[b]) + 3*x)*(2*Sqrt[3]*Sqrt[b] + 3*x)*(3*Sqrt[3]
*Sqrt[b] + (-(Sqrt[3]*Sqrt[b]) + 3*x)*Log[-(Sqrt[3]*Sqrt[b]) + 3*
x] + (Sqrt[3]*Sqrt[b] - 3*x)*Log[2*Sqrt[3]*Sqrt[b] + 3*x]))/(81*b
*(2*Sqrt[3]*b^(3/2) - 9*b*x + 9*x^3))
```

Maple [C] time = 0.009, size = 43, normalized size = 0.6

$$\frac{1}{9}\sum_{\_R=RootOf\left(-9\_Z\,b+9\_Z^3+2\,b^{3/2}\sqrt{3}\right)}\frac{\ln\left(x-\_R\right)}{3\_R^2-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-9*b*x+9*x^3+2*b^(3/2)*3^(1/2)),x)
```

```
[Out] 1/9*sum(1/(3*_R^2-b)*ln(x-_R),_R=RootOf(-9*_Z*b+9*_Z^3+2*b^(3/2)*
3^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{9\,x^3+2\,\sqrt{3}b^{\frac{3}{2}}-9\,bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(9*x^3 + 2*sqrt(3)*b^(3/2) - 9*b*x),x, algorithm="maxima")
```

```
[Out] integrate(1/(9*x^3 + 2*sqrt(3)*b^(3/2) - 9*b*x), x)
```

Fricas [A] time = 0.286915, size = 105, normalized size = 1.36

$$\frac{\left(\sqrt{3}b-3\,\sqrt{b}x\right)\log\left(\sqrt{3}\sqrt{b}x+2\,b\right)-\left(\sqrt{3}b-3\,\sqrt{b}x\right)\log\left(\sqrt{3}\sqrt{b}x-b\right)+3\,\sqrt{3}b}{27\left(\sqrt{3}b^2-3\,b^{\frac{3}{2}}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(9*x^3 + 2*sqrt(3)*b^(3/2) - 9*b*x),x, algorithm="fricas")
```

```
[Out] 1/27*((sqrt(3)*b - 3*sqrt(b)*x)*log(sqrt(3)*sqrt(b)*x + 2*b) - (s
qrt(3)*b - 3*sqrt(b)*x)*log(sqrt(3)*sqrt(b)*x - b) + 3*sqrt(3)*b)
/(sqrt(3)*b^2 - 3*b^(3/2)*x)
```

Sympy [A] time = 1.7554, size = 60, normalized size = 0.78

$$-\frac{3\sqrt{3}}{81\sqrt{b}x-27\sqrt{3}b}+\frac{-\frac{\log\left(-\frac{\sqrt{3}\sqrt{b}}{3}+x\right)}{27}+\frac{\log\left(\frac{2\sqrt{3}\sqrt{b}}{3}+x\right)}{27}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-9*b*x+9*x**3+2*b**(3/2)*3**(1/2)),x)
```

```
[Out] -3*sqrt(3)/(81*sqrt(b)*x - 27*sqrt(3)*b) + (-log(-sqrt(3)*sqrt(b)
/3 + x)/27 + log(2*sqrt(3)*sqrt(b)/3 + x)/27)/b
```

GIAC/XCAS [A] time = 0.280025, size = 73, normalized size = 0.95

$$\frac{\ln\left(\left|\sqrt{3}x + 2\sqrt{b}\right|\right)}{27\,b} - \frac{\ln\left(\left|-\sqrt{3}x + \sqrt{b}\right|\right)}{27\,b} - \frac{1}{9\left(\sqrt{3}x - \sqrt{b}\right)\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x^3 + 2*sqrt(3)*b^(3/2) - 9*b*x),x, algorithm="giac")
```

```
[Out] 1/27*ln(abs(sqrt(3)*x + 2*sqrt(b)))/b - 1/27*ln(abs(-sqrt(3)*x +
sqrt(b)))/b - 1/9/((sqrt(3)*x - sqrt(b))*sqrt(b))
```

3.2 $\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^p\,dx$

Optimal. Leaf size=30

$$\frac{\left(\frac{a}{b}+x\right)\left(b^3\left(\frac{a}{b}+x\right)^3\right)^p}{3p+1}$$

```
[Out] ((a/b + x)*(b^3*(a/b + x)^3)^p)/(1 + 3*p)
```

Rubi [A] time = 0.0364867, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$

$$\frac{(a+bx)\left((a+bx)^3\right)^p}{b(3p+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^p,x]

[Out] ((a + b*x)*((a + b*x)^3)^p)/(b*(1 + 3*p))
```

Rubi in Sympy [A] time = 19.6477, size = 80, normalized size = 2.67

$$\frac{\left(3a^2b+3ab^2x\right)^{-3p}\left(3a^2b+3ab^2x\right)^{3p+1}\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^p}{3ab^2\left(3p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3)**p,x)

[Out] (3*a**2*b + 3*a*b**2*x)**(-3*p)*(3*a**2*b + 3*a*b**2*x)**(3*p + 1
)*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3)**p/(3*a*b**2*(3
*p + 1))
```

Mathematica [A] time = 0.0189052, size = 23, normalized size = 0.77

$$\frac{(a+bx)\left((a+bx)^3\right)^p}{3bp+b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^p,x]

[Out] ((a + b*x)*((a + b*x)^3)^p)/(b + 3*b*p)
```

Maple [A] time = 0.002, size = 46, normalized size = 1.5

$$\frac{(bx+a)\left(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3\right)^p}{b\,(1+3\,p)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^p,x)

[Out] (b*x+a)/b/(1+3*p)*(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^p
```

Maxima [A] time = 0.783747, size = 34, normalized size = 1.13

$$\frac{(bx+a)(bx+a)^{3\,p}}{b(3\,p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)^p,x, algorithm="maxima")

[Out] (b*x + a)*(b*x + a)^(3*p)/(b*(3*p + 1))
```

Fricas [A] time = 0.301408, size = 58, normalized size = 1.93

$$\frac{(bx+a)\left(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3\right)^p}{3\,bp+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)^p,x, algorithm="fricas")

[Out] (b*x + a)*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)^p/(3*b*p + b)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3)**p,x)

[Out] Exception raised: TypeError
```

GIAC/XCAS [A] time = 0.263777, size = 104, normalized size = 3.47

$$\frac{bxe^{\left(p\ln\left(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3\right)\right)}+ae^{\left(p\ln\left(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3\right)\right)}}{3\,bp+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)^p,x, algorithm="giac")

[Out] (b*x*e^(p*ln(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)) + a*e^(p*l
n(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)))/(3*b*p + b)
```

3.3 $\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^3\,dx$

Optimal. Leaf size=14

$$\frac{(a+bx)^{10}}{10b}$$

```
[Out] (a + b*x)^10/(10*b)
```

Rubi [A] time = 0.0169009, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069

$$\frac{(a+bx)^{10}}{10b}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^3,x]
```

```
[Out] (a + b*x)^10/(10*b)
```

Rubi in Sympy [A] time = 15.6953, size = 8, normalized size = 0.57

$$\frac{(a+bx)^{10}}{10b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3)**3,x)
```

```
[Out] (a + b*x)**10/(10*b)
```

Mathematica [A] time = 0.00151992, size = 14, normalized size = 1.

$$\frac{(a+bx)^{10}}{10b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^3,x]
```

```
[Out] (a + b*x)^10/(10*b)
```

Maple [B] time = 0.002, size = 98, normalized size = 7.

$$\frac{b^9x^{10}}{10}+ab^8x^9+\frac{9\,a^2b^7x^8}{2}+12\,a^3b^6x^7+21\,a^4b^5x^6+\frac{126\,a^5b^4x^5}{5}+21\,a^6b^3x^4+12\,a^7b^2x^3+\frac{9\,a^8bx^2}{2}+a^9x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x)
```

```
[Out] 1/10*b^9*x^10+a*b^8*x^9+9/2*a^2*b^7*x^8+12*a^3*b^6*x^7+21*a^4*b^5
*x^6+126/5*a^5*b^4*x^5+21*a^6*b^3*x^4+12*a^7*b^2*x^3+9/2*a^8*b*x^
2+a^9*x
```

Maxima [A] time = 0.788025, size = 292, normalized size = 20.86

$$\frac{1}{10}\,b^9x^{10} + ab^8x^9 + \frac{27}{8}\,a^2b^7x^8 + \frac{27}{7}\,a^3b^6x^7 + \frac{27}{4}\,a^6b^3x^4 + a^9x + \frac{3}{4}\left(b^3x^4 + 4\,ab^2x^3 + 6\,a^2bx^2\right)a^6 + \frac{9}{10}\left(5\,b^3x^6 + 18\,ab^2x^5\right)a^4b^2 + \frac{3}{70}\left(10\,b^6x^7 + 70\,ab^5x^6 + 126\,a^2b^4x^5 + 210\,a^4b^2x^3 + 21\left(4\,b^3x^5 + 15\,ab^2x^4\right)a^2b\right)a^3 + \frac{9}{56}\left(7\,b^6x^8 + 48\,ab^5x^7 + 84\,a^2b^4x^6\right)a^2b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)^3,x, algorithm="maxima")
```

```
[Out] 1/10*b^9*x^10 + a*b^8*x^9 + 27/8*a^2*b^7*x^8 + 27/7*a^3*b^6*x^7 +
 27/4*a^6*b^3*x^4 + a^9*x + 3/4*(b^3*x^4 + 4*a*b^2*x^3 + 6*a^2*b*
x^2)*a^6 + 9/10*(5*b^3*x^6 + 18*a*b^2*x^5)*a^4*b^2 + 3/70*(10*b^6
*x^7 + 70*a*b^5*x^6 + 126*a^2*b^4*x^5 + 210*a^4*b^2*x^3 + 21*(4*b
^3*x^5 + 15*a*b^2*x^4)*a^2*b)*a^3 + 9/56*(7*b^6*x^8 + 48*a*b^5*x^
7 + 84*a^2*b^4*x^6)*a^2*b
```

Fricas [A] time = 0.252483, size = 1, normalized size = 0.07

$$\frac{1}{10}x^{10}b^9 + x^9b^8a + \frac{9}{2}x^8b^7a^2 + 12x^7b^6a^3 + 21x^6b^5a^4 + \frac{126}{5}x^5b^4a^5 + 21x^4b^3a^6 + 12x^3b^2a^7 + \frac{9}{2}x^2ba^8 + xa^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)^3,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*b^9 + x^9*b^8*a + 9/2*x^8*b^7*a^2 + 12*x^7*b^6*a^3 + 21
*x^6*b^5*a^4 + 126/5*x^5*b^4*a^5 + 21*x^4*b^3*a^6 + 12*x^3*b^2*a^
7 + 9/2*x^2*b*a^8 + x*a^9
```

Sympy [A] time = 0.150073, size = 107, normalized size = 7.64

$$a^9x + \frac{9a^8bx^2}{2} + 12a^7b^2x^3 + 21a^6b^3x^4 + \frac{126a^5b^4x^5}{5} + 21a^4b^5x^6 + 12a^3b^6x^7 + \frac{9a^2b^7x^8}{2} + ab^8x^9 + \frac{b^9x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3)**3,x)
```

```
[Out] a**9*x + 9*a**8*b*x**2/2 + 12*a**7*b**2*x**3 + 21*a**6*b**3*x**4
+ 126*a**5*b**4*x**5/5 + 21*a**4*b**5*x**6 + 12*a**3*b**6*x**7 +
9*a**2*b**7*x**8/2 + a*b**8*x**9 + b**9*x**10/10
```

GIAC/XCAS [A] time = 0.259515, size = 131, normalized size = 9.36

$$\frac{1}{10}\,b^9x^{10} + ab^8x^9 + \frac{9}{2}\,a^2b^7x^8 + 12\,a^3b^6x^7 + 21\,a^4b^5x^6 + \frac{126}{5}\,a^5b^4x^5 + 21\,a^6b^3x^4 + 12\,a^7b^2x^3 + \frac{9}{2}\,a^8bx^2 + a^9x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)^3,x, algorithm="giac")
```

```
[Out] 1/10*b^9*x^10 + a*b^8*x^9 + 9/2*a^2*b^7*x^8 + 12*a^3*b^6*x^7 + 21
*a^4*b^5*x^6 + 126/5*a^5*b^4*x^5 + 21*a^6*b^3*x^4 + 12*a^7*b^2*x^
3 + 9/2*a^8*b*x^2 + a^9*x
```

3.4 $\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^2\,dx$

Optimal. Leaf size=14

$$\frac{(a+bx)^7}{7b}$$

```
[Out] (a + b*x)^7/(7*b)
```

Rubi [A] time = 0.0169498, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$

$$\frac{(a+bx)^7}{7b}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^2,x]

[Out] (a + b*x)^7/(7*b)
```

Rubi in Sympy [A] time = 15.6221, size = 8, normalized size = 0.57

$$\frac{(a+bx)^7}{7b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3)**2,x)

[Out] (a + b*x)**7/(7*b)
```

Mathematica [A] time = 0.00204501, size = 14, normalized size = 1.

$$\frac{(a+bx)^7}{7b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^2,x]

[Out] (a + b*x)^7/(7*b)
```

Maple [B] time = 0.001, size = 65, normalized size = 4.6

$$\frac{b^6x^7}{7}+ab^5x^6+3\,a^2b^4x^5+5\,a^3b^3x^4+5\,a^4b^2x^3+3\,a^5bx^2+a^6x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x)
```

```
[Out] 1/7*b^6*x^7+a*b^5*x^6+3*a^2*b^4*x^5+5*a^3*b^3*x^4+5*a^4*b^2*x^3+3
*a^5*b*x^2+a^6*x
```

Maxima [A] time = 0.77603, size = 134, normalized size = 9.57

$$\frac{1}{7}\,b^6x^7 + ab^5x^6 + \frac{9}{5}\,a^2b^4x^5 + 3\,a^4b^2x^3 + a^6x + \frac{1}{2}\,\left(b^3x^4 + 4\,ab^2x^3 + 6\,a^2bx^2\right)a^3 + \frac{3}{10}\,\left(4\,b^3x^5 + 15\,ab^2x^4\right)a^2b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)^2,x, algorithm="maxima")
```

```
[Out] 1/7*b^6*x^7 + a*b^5*x^6 + 9/5*a^2*b^4*x^5 + 3*a^4*b^2*x^3 + a^6*x
 + 1/2*(b^3*x^4 + 4*a*b^2*x^3 + 6*a^2*b*x^2)*a^3 + 3/10*(4*b^3*x^
5 + 15*a*b^2*x^4)*a^2*b
```

Fricas [A] time = 0.249373, size = 1, normalized size = 0.07

$$\frac{1}{7}x^7b^6 + x^6b^5a + 3x^5b^4a^2 + 5x^4b^3a^3 + 5x^3b^2a^4 + 3x^2ba^5 + xa^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)^2,x, algorithm="fricas")
```

```
[Out] 1/7*x^7*b^6 + x^6*b^5*a + 3*x^5*b^4*a^2 + 5*x^4*b^3*a^3 + 5*x^3*b
^2*a^4 + 3*x^2*b*a^5 + x*a^6
```

Sympy [A] time = 0.125764, size = 66, normalized size = 4.71

$$a^6x + 3a^5bx^2 + 5a^4b^2x^3 + 5a^3b^3x^4 + 3a^2b^4x^5 + ab^5x^6 + \frac{b^6x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3)**2,x)
```

```
[Out] a**6*x + 3*a**5*b*x**2 + 5*a**4*b**2*x**3 + 5*a**3*b**3*x**4 + 3*
a**2*b**4*x**5 + a*b**5*x**6 + b**6*x**7/7
```

GIAC/XCAS [A] time = 0.261963, size = 86, normalized size = 6.14

$$\frac{1}{7}\,b^6x^7 + ab^5x^6 + 3\,a^2b^4x^5 + 5\,a^3b^3x^4 + 5\,a^4b^2x^3 + 3\,a^5bx^2 + a^6x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)^2,x, algorithm="giac")
```

```
[Out] 1/7*b^6*x^7 + a*b^5*x^6 + 3*a^2*b^4*x^5 + 5*a^3*b^3*x^4 + 5*a^4*b
^2*x^3 + 3*a^5*b*x^2 + a^6*x
```

3.5 $\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)\,dx$

Optimal. Leaf size=35

$$a^3x+\frac{3}{2}a^2bx^2+ab^2x^3+\frac{b^3x^4}{4}$$

```
[Out] a^3*x + (3*a^2*b*x^2)/2 + a*b^2*x^3 + (b^3*x^4)/4
```

Rubi [A] time = 0.0181635, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.

$$a^3x+\frac{3}{2}a^2bx^2+ab^2x^3+\frac{b^3x^4}{4}$$

Antiderivative was successfully verified.

```
[In]  Int[a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3,x]

[Out] a^3*x + (3*a^2*b*x^2)/2 + a*b^2*x^3 + (b^3*x^4)/4
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$3a^2b\int x\,dx+ab^2x^3+\frac{b^3x^4}{4}+\int a^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3,x)

[Out] 3*a**2*b*Integral(x, x) + a*b**2*x**3 + b**3*x**4/4 + Integral(a*
*3, x)
```

Mathematica [A] time = 0.0000883153, size = 35, normalized size = 1.

$$a^3x+\frac{3}{2}a^2bx^2+ab^2x^3+\frac{b^3x^4}{4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3,x]

[Out] a^3*x + (3*a^2*b*x^2)/2 + a*b^2*x^3 + (b^3*x^4)/4
```

Maple [A] time = 0.001, size = 32, normalized size = 0.9

$$a^3x+\frac{3\,a^2bx^2}{2}+ab^2x^3+\frac{b^3x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3,x)
```

```
[Out] a^3*x+3/2*a^2*b*x^2+a*b^2*x^3+1/4*b^3*x^4
```

Maxima [A] time = 0.757947, size = 42, normalized size = 1.2

$$\frac{1}{4}\,b^3x^4 + ab^2x^3 + \frac{3}{2}\,a^2bx^2 + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3,x, algorithm="maxima")
```

```
[Out] 1/4*b^3*x^4 + a*b^2*x^3 + 3/2*a^2*b*x^2 + a^3*x
```

Fricas [A] time = 0.23837, size = 1, normalized size = 0.03

$$\frac{1}{4}x^4b^3 + x^3b^2a + \frac{3}{2}x^2ba^2 + xa^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3,x, algorithm="fricas")
```

```
[Out] 1/4*x^4*b^3 + x^3*b^2*a + 3/2*x^2*b*a^2 + x*a^3
```

Sympy [A] time = 0.084519, size = 32, normalized size = 0.91

$$a^3x + \frac{3a^2bx^2}{2} + ab^2x^3 + \frac{b^3x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3,x)
```

```
[Out] a**3*x + 3*a**2*b*x**2/2 + a*b**2*x**3 + b**3*x**4/4
```

GIAC/XCAS [A] time = 0.259653, size = 42, normalized size = 1.2

$$\frac{1}{4}\,b^3x^4 + ab^2x^3 + \frac{3}{2}\,a^2bx^2 + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3,x, algorithm="giac")
```

```
[Out] 1/4*b^3*x^4 + a*b^2*x^3 + 3/2*a^2*b*x^2 + a^3*x
```

3.6 $\int \frac{1}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx$

Optimal. Leaf size=14

$$-\frac{1}{2b(a+bx)^2}$$

```
[Out] -1/(2*b*(a + b*x)^2)
```

Rubi [A] time = 0.0191987, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$

$$-\frac{1}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(-1),x]

[Out] -1/(2*b*(a + b*x)^2)
```

Rubi in Sympy [A] time = 15.719, size = 12, normalized size = 0.86

$$-\frac{1}{2b\left(a+bx\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3),x)

[Out] -1/(2*b*(a + b*x)**2)
```

Mathematica [A] time = 0.00467367, size = 14, normalized size = 1.

$$-\frac{1}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(-1),x]

[Out] -1/(2*b*(a + b*x)^2)
```

Maple [A] time = 0.006, size = 13, normalized size = 0.9

$$-\frac{1}{2\,b\left(bx+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3),x)
```

```
[Out] -1/2/b/(b*x+a)^2
```

Maxima [A] time = 0.759675, size = 32, normalized size = 2.29

$$-\frac{1}{2\,(b^3x^2+2\,ab^2x+a^2b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3),x, algorithm="maxima")
```

```
[Out] -1/2/(b^3*x^2 + 2*a*b^2*x + a^2*b)
```

Fricas [A] time = 0.273123, size = 32, normalized size = 2.29

$$-\frac{1}{2\,(b^3x^2+2\,ab^2x+a^2b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3),x, algorithm="fricas")
```

```
[Out] -1/2/(b^3*x^2 + 2*a*b^2*x + a^2*b)
```

Sympy [A] time = 1.2534, size = 26, normalized size = 1.86

$$-\frac{1}{2a^2b+4ab^2x+2b^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3),x)
```

```
[Out] -1/(2*a**2*b + 4*a*b**2*x + 2*b**3*x**2)
```

GIAC/XCAS [A] time = 0.259695, size = 16, normalized size = 1.14

$$-\frac{1}{2\,(bx+a)^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3),x, algorithm="giac")
```

```
[Out] -1/2/((b*x + a)^2*b)
```

3.7 $\int \frac{1}{(a^3+3a^2bx+3ab^2x^2+b^3x^3)^2} dx$

Optimal. Leaf size=14

$$-\frac{1}{5b(a+bx)^5}$$

```
[Out] -1/(5*b*(a + b*x)^5)
```

Rubi [A] time = 0.0178925, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069

$$-\frac{1}{5b(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(-2),x]

[Out] -1/(5*b*(a + b*x)^5)
```

Rubi in Sympy [A] time = 15.686, size = 12, normalized size = 0.86

$$-\frac{1}{5b\left(a+bx\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3)**2,x)

[Out] -1/(5*b*(a + b*x)**5)
```

Mathematica [A] time = 0.00681724, size = 14, normalized size = 1.

$$-\frac{1}{5b(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(-2),x]

[Out] -1/(5*b*(a + b*x)^5)
```

Maple [A] time = 0.004, size = 13, normalized size = 0.9

$$-\frac{1}{5\,b\left(bx+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x)
```

```
[Out] -1/5/b/(b*x+a)^5
```

Maxima [A] time = 0.778562, size = 77, normalized size = 5.5

$$-\frac{1}{5\,(b^6x^5 + 5\,ab^5x^4 + 10\,a^2b^4x^3 + 10\,a^3b^3x^2 + 5\,a^4b^2x + a^5b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)^(-2),x, algorithm="maxima")
```

```
[Out] -1/5/(b^6*x^5 + 5*a*b^5*x^4 + 10*a^2*b^4*x^3 + 10*a^3*b^3*x^2 + 5
*a^4*b^2*x + a^5*b)
```

Fricas [A] time = 0.273852, size = 77, normalized size = 5.5

$$-\frac{1}{5\,(b^6x^5 + 5\,ab^5x^4 + 10\,a^2b^4x^3 + 10\,a^3b^3x^2 + 5\,a^4b^2x + a^5b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)^(-2),x, algorithm="fricas")
```

```
[Out] -1/5/(b^6*x^5 + 5*a*b^5*x^4 + 10*a^2*b^4*x^3 + 10*a^3*b^3*x^2 + 5
*a^4*b^2*x + a^5*b)
```

Sympy [A] time = 1.86628, size = 61, normalized size = 4.36

$$-\frac{1}{5a^5b + 25a^4b^2x + 50a^3b^3x^2 + 50a^2b^4x^3 + 25ab^5x^4 + 5b^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3)**2,x)
```

```
[Out] -1/(5*a**5*b + 25*a**4*b**2*x + 50*a**3*b**3*x**2 + 50*a**2*b**4*
x**3 + 25*a*b**5*x**4 + 5*b**6*x**5)
```

GIAC/XCAS [A] time = 0.259782, size = 16, normalized size = 1.14

$$-\frac{1}{5\,(bx+a)^5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)^(-2),x, algorithm="giac")
```

```
[Out] -1/5/((b*x + a)^5*b)
```

3.8 $\int \frac{1}{(a^3+3a^2bx+3ab^2x^2+b^3x^3)^3}\,dx$

Optimal. Leaf size=14

$$-\frac{1}{8b(a+bx)^8}$$

```
[Out] -1/(8*b*(a + b*x)^8)
```

Rubi [A] time = 0.0184617, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069

$$-\frac{1}{8b(a+bx)^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(-3),x]

[Out] -1/(8*b*(a + b*x)^8)
```

Rubi in Sympy [A] time = 15.6879, size = 12, normalized size = 0.86

$$-\frac{1}{8b\,(a+bx)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3)**3,x)

[Out] -1/(8*b*(a + b*x)**8)
```

Mathematica [A] time = 0.00506053, size = 14, normalized size = 1.

$$-\frac{1}{8b(a+bx)^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(-3),x]

[Out] -1/(8*b*(a + b*x)^8)
```

Maple [A] time = 0.004, size = 13, normalized size = 0.9

$$-\frac{1}{8\,b\,(bx+a)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x)
```

```
[Out] -1/8/b/(b*x+a)^8
```

Maxima [A] time = 0.787831, size = 122, normalized size = 8.71

$$-\frac{1}{8\left(b^9x^8 + 8\,ab^8x^7 + 28\,a^2b^7x^6 + 56\,a^3b^6x^5 + 70\,a^4b^5x^4 + 56\,a^5b^4x^3 + 28\,a^6b^3x^2 + 8\,a^7b^2x + a^8b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)^(-3),x, algorithm="maxima")
```

```
[Out] -1/8/(b^9*x^8 + 8*a*b^8*x^7 + 28*a^2*b^7*x^6 + 56*a^3*b^6*x^5 + 7
0*a^4*b^5*x^4 + 56*a^5*b^4*x^3 + 28*a^6*b^3*x^2 + 8*a^7*b^2*x + a
^8*b)
```

Fricas [A] time = 0.268892, size = 122, normalized size = 8.71

$$-\frac{1}{8\left(b^9x^8 + 8\,ab^8x^7 + 28\,a^2b^7x^6 + 56\,a^3b^6x^5 + 70\,a^4b^5x^4 + 56\,a^5b^4x^3 + 28\,a^6b^3x^2 + 8\,a^7b^2x + a^8b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)^(-3),x, algorithm="fricas")
```

```
[Out] -1/8/(b^9*x^8 + 8*a*b^8*x^7 + 28*a^2*b^7*x^6 + 56*a^3*b^6*x^5 + 7
0*a^4*b^5*x^4 + 56*a^5*b^4*x^3 + 28*a^6*b^3*x^2 + 8*a^7*b^2*x + a
^8*b)
```

Sympy [A] time = 2.70059, size = 97, normalized size = 6.93

$$-\frac{1}{8a^8b + 64a^7b^2x + 224a^6b^3x^2 + 448a^5b^4x^3 + 560a^4b^5x^4 + 448a^3b^6x^5 + 224a^2b^7x^6 + 64ab^8x^7 + 8b^9x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3)**3,x)
```

```
[Out] -1/(8*a**8*b + 64*a**7*b**2*x + 224*a**6*b**3*x**2 + 448*a**5*b**
4*x**3 + 560*a**4*b**5*x**4 + 448*a**3*b**6*x**5 + 224*a**2*b**7*
x**6 + 64*a*b**8*x**7 + 8*b**9*x**8)
```

GIAC/XCAS [A] time = 0.259044, size = 16, normalized size = 1.14

$$-\frac{1}{8\,(bx + a)^8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)^(-3),x, algorithm="giac")
```

```
[Out] -1/8/((b*x + a)^8*b)
```

3.9 $\int \left(3ab + 3b^2x + 3bcx^2 + c^2x^3\right)^3 dx$

Optimal. Leaf size=84

$$-\frac{3b\left(b^2-3ac\right)(b+cx)^7}{7c^4}+\frac{3b^2\left(b^2-3ac\right)^2(b+cx)^4}{4c^4}-\frac{b^3x\left(b^2-3ac\right)^3}{c^3}+\frac{(b+cx)^{10}}{10c^4}$$

```
[Out] -((b^3*(b^2 - 3*a*c)^3*x)/c^3) + (3*b^2*(b^2 - 3*a*c)^2*(b + c*x)
^4)/(4*c^4) - (3*b*(b^2 - 3*a*c)*(b + c*x)^7)/(7*c^4) + (b + c*x)
^10/(10*c^4)
```

Rubi [A] time = 0.248093, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$

$$-\frac{3b\left(b^2-3ac\right)(b+cx)^7}{7c^4}+\frac{3b^2\left(b^2-3ac\right)^2(b+cx)^4}{4c^4}-\frac{b^3x\left(b^2-3ac\right)^3}{c^3}+\frac{(b+cx)^{10}}{10c^4}$$

Antiderivative was successfully verified.

```
[In]  Int[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^3,x]
```

```
[Out] -((b^3*(b^2 - 3*a*c)^3*x)/c^3) + (3*b^2*(b^2 - 3*a*c)^2*(b + c*x)
^4)/(4*c^4) - (3*b*(b^2 - 3*a*c)*(b + c*x)^7)/(7*c^4) + (b + c*x)
^10/(10*c^4)
```

Rubi in Sympy [A] time = 34.0212, size = 83, normalized size = 0.99

$$-\frac{b^3(b+cx)\left(-3ac+b^2\right)^3}{c^4}+\frac{3b^2(b+cx)^4\left(-3ac+b^2\right)^2}{4c^4}-\frac{3b(b+cx)^7\left(-3ac+b^2\right)}{7c^4}+\frac{(b+cx)^{10}}{10c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c**2*x**3+3*b*c*x**2+3*b**2*x+3*a*b)**3,x)
```

```
[Out] -b**3*(b + c*x)*(-3*a*c + b**2)**3/c**4 + 3*b**2*(b + c*x)**4*(-3
*a*c + b**2)**2/(4*c**4) - 3*b*(b + c*x)**7*(-3*a*c + b**2)/(7*c*
*4) + (b + c*x)**10/(10*c**4)
```

Mathematica [A] time = 0.0346586, size = 159, normalized size = 1.89

$$27a^3b^3x+\frac{81}{2}a^2b^4x^2+\frac{27}{4}b^2x^4\left(a^2c^2+6ab^2c+b^4\right)+\frac{9}{7}bc^3x^7\left(ac+9b^2\right)+9b^2c^2x^6\left(ac+2b^2\right)$$
$$+\frac{27}{5}b^3cx^5\left(5ac+3b^2\right)+27ab^3x^3\left(ac+b^2\right)+\frac{9}{2}b^2c^4x^8+bc^5x^9+\frac{c^6x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^3,x]
```

```
[Out] 27*a^3*b^3*x + (81*a^2*b^4*x^2)/2 + 27*a*b^3*(b^2 + a*c)*x^3 + (2
7*b^2*(b^4 + 6*a*b^2*c + a^2*c^2)*x^4)/4 + (27*b^3*c*(3*b^2 + 5*a
*c)*x^5)/5 + 9*b^2*c^2*(2*b^2 + a*c)*x^6 + (9*b*c^3*(9*b^2 + a*c)
*x^7)/7 + (9*b^2*c^4*x^8)/2 + b*c^5*x^9 + (c^6*x^10)/10
```

Maple [B] time = 0.001, size = 295, normalized size = 3.5

$$\frac{c^{6}x^{10}}{10}+bc^{5}x^{9}+\frac{9\,b^{2}c^{4}x^{8}}{2}+\frac{\left(3\,abc^{4}+63\,b^{3}c^{3}+c^{2}\left(6\,abc^{2}+18\,b^{3}c\right)\right)x^{7}}{7}+\frac{\left(18\,ab^{2}c^{3}+45\,b^{4}c^{2}+3\,bc\left(6\,abc^{2}+18\,b^{3}c\right)+c^{2}\left(18\,ab^{2}c+9\,b^{4}\right)\right)x^{6}}{6}+\frac{\left(63\,ab^{3}c^{2}+3\,b^{2}\left(6\,abc^{2}+18\,b^{3}c\right)+3\,bc\left(18\,ab^{2}c+9\,b^{4}\right)\right)x^{5}}{5}+\frac{\left(3\,ab\left(6\,abc^{2}+18\,b^{3}c\right)+3\,b^{2}\left(18\,ab^{2}c+9\,b^{4}\right)+54\,ab^{4}c+9\,a^{2}b^{2}c^{2}\right)x^{4}}{4}+\frac{\left(3\,ab\left(18\,ab^{2}c+9\,b^{4}\right)+54\,ab^{5}+27\,a^{2}b^{3}c\right)x^{3}}{3}+\frac{81\,a^{2}b^{4}x^{2}}{2}+27\,a^{3}b^{3}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x)
```

```
[Out] 1/10*c^6*x^10+b*c^5*x^9+9/2*b^2*c^4*x^8+1/7*(3*a*b*c^4+63*b^3*c^3
+c^2*(6*a*b*c^2+18*b^3*c))*x^7+1/6*(18*a*b^2*c^3+45*b^4*c^2+3*b*c
*(6*a*b*c^2+18*b^3*c)+c^2*(18*a*b^2*c+9*b^4))*x^6+1/5*(63*a*b^3*c
^2+3*b^2*(6*a*b*c^2+18*b^3*c)+3*b*c*(18*a*b^2*c+9*b^4))*x^5+1/4*(
3*a*b*(6*a*b*c^2+18*b^3*c)+3*b^2*(18*a*b^2*c+9*b^4)+54*a*b^4*c+9*
a^2*b^2*c^2)*x^4+1/3*(3*a*b*(18*a*b^2*c+9*b^4)+54*a*b^5+27*a^2*b^
3*c)*x^3+81/2*a^2*b^4*x^2+27*a^3*b^3*x
```

Maxima [A] time = 0.794495, size = 275, normalized size = 3.27

$$\frac{1}{10}\,c^{6}x^{10}+bc^{5}x^{9}+\frac{27}{8}\,b^{2}c^{4}x^{8}+\frac{27}{7}\,b^{3}c^{3}x^{7}+\frac{27}{4}\,b^{6}x^{4}+27\,a^{3}b^{3}x+\frac{27}{4}\left(c^{2}x^{4}+4\,bcx^{3}+6\,b^{2}x^{2}\right)a^{2}b^{2}+\frac{9}{10}\left(5\,c^{2}x^{6}+18\,bcx^{5}\right)b^{4}+\frac{9}{70}\left(10\,c^{4}x^{7}+70\,bc^{3}x^{6}+126\,b^{2}c^{2}x^{5}+210\,b^{4}x^{3}+21\left(4\,c^{2}x^{5}+15\,bcx^{4}\right)b^{2}\right)ab+\frac{9}{56}\left(7\,c^{4}x^{8}+48\,bc^{3}x^{7}+84\,b^{2}c^{2}x^{6}\right)b^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c^2*x^3 + 3*b*c*x^2 + 3*b^2*x + 3*a*b)^3,x, algorithm="maxima")
```

```
[Out] 1/10*c^6*x^10 + b*c^5*x^9 + 27/8*b^2*c^4*x^8 + 27/7*b^3*c^3*x^7 +
 27/4*b^6*x^4 + 27*a^3*b^3*x + 27/4*(c^2*x^4 + 4*b*c*x^3 + 6*b^2*
x^2)*a^2*b^2 + 9/10*(5*c^2*x^6 + 18*b*c*x^5)*b^4 + 9/70*(10*c^4*x
^7 + 70*b*c^3*x^6 + 126*b^2*c^2*x^5 + 210*b^4*x^3 + 21*(4*c^2*x^5
 + 15*b*c*x^4)*b^2)*a*b + 9/56*(7*c^4*x^8 + 48*b*c^3*x^7 + 84*b^2
*c^2*x^6)*b^2
```

Fricas [A] time = 0.253282, size = 1, normalized size = 0.01

$$\frac{1}{10}x^{10}c^{6}+x^{9}c^{5}b+\frac{9}{2}x^{8}c^{4}b^{2}+\frac{81}{7}x^{7}c^{3}b^{3}+\frac{9}{7}x^{7}c^{4}ba+18x^{6}c^{2}b^{4}+9x^{6}c^{3}b^{2}a+\frac{81}{5}x^{5}cb^{5}+27x^{5}c^{2}b^{3}a+\frac{27}{4}x^{4}b^{6}+\frac{81}{2}x^{4}cb^{4}a+\frac{27}{4}x^{4}c^{2}b^{2}a^{2}+27x^{3}b^{5}a+27x^{3}cb^{3}a^{2}+\frac{81}{2}x^{2}b^{4}a^{2}+27xb^{3}a^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c^2*x^3 + 3*b*c*x^2 + 3*b^2*x + 3*a*b)^3,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*c^6 + x^9*c^5*b + 9/2*x^8*c^4*b^2 + 81/7*x^7*c^3*b^3 +
9/7*x^7*c^4*b*a + 18*x^6*c^2*b^4 + 9*x^6*c^3*b^2*a + 81/5*x^5*c*b
```

```
^5 + 27*x^5*c^2*b^3*a + 27/4*x^4*b^6 + 81/2*x^4*c*b^4*a + 27/4*x^
4*c^2*b^2*a^2 + 27*x^3*b^5*a + 27*x^3*c*b^3*a^2 + 81/2*x^2*b^4*a^
2 + 27*x*b^3*a^3
```

Sympy [A] time = 0.194612, size = 175, normalized size = 2.08

$$27a^3b^3x + \frac{81a^2b^4x^2}{2} + \frac{9b^2c^4x^8}{2} + bc^5x^9 + \frac{c^6x^{10}}{10} + x^7\left(\frac{9abc^4}{7} + \frac{81b^3c^3}{7}\right) + x^6\left(9ab^2c^3 + 18b^4c^2\right) + x^5\left(27ab^3c^2 + \frac{81b^5c}{5}\right) + x^4\left(\frac{27a^2b^2c^2}{4} + \frac{81ab^4c}{2} + \frac{27b^6}{4}\right) + x^3\left(27a^2b^3c + 27ab^5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c**2*x**3+3*b*c*x**2+3*b**2*x+3*a*b)**3,x)
```

```
[Out] 27*a**3*b**3*x + 81*a**2*b**4*x**2/2 + 9*b**2*c**4*x**8/2 + b*c**
5*x**9 + c**6*x**10/10 + x**7*(9*a*b*c**4/7 + 81*b**3*c**3/7) + x
**6*(9*a*b**2*c**3 + 18*b**4*c**2) + x**5*(27*a*b**3*c**2 + 81*b*
*5*c/5) + x**4*(27*a**2*b**2*c**2/4 + 81*a*b**4*c/2 + 27*b**6/4)
+ x**3*(27*a**2*b**3*c + 27*a*b**5)
```

GIAC/XCAS [A] time = 0.258455, size = 224, normalized size = 2.67

$$\frac{1}{10}\,c^6x^{10} + bc^5x^9 + \frac{9}{2}\,b^2c^4x^8 + \frac{81}{7}\,b^3c^3x^7 + \frac{9}{7}\,abc^4x^7 + 18\,b^4c^2x^6 + 9\,ab^2c^3x^6 + \frac{81}{5}\,b^5cx^5 + 27\,ab^3c^2x^5 + \frac{27}{4}\,b^6x^4 + \frac{81}{2}\,ab^4cx^4 + \frac{27}{4}\,a^2b^2c^2x^4 + 27\,ab^5x^3 + 27\,a^2b^3cx^3 + \frac{81}{2}\,a^2b^4x^2 + 27\,a^3b^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c^2*x^3 + 3*b*c*x^2 + 3*b^2*x + 3*a*b)^3,x, algorithm="giac")
```

```
[Out] 1/10*c^6*x^10 + b*c^5*x^9 + 9/2*b^2*c^4*x^8 + 81/7*b^3*c^3*x^7 +
9/7*a*b*c^4*x^7 + 18*b^4*c^2*x^6 + 9*a*b^2*c^3*x^6 + 81/5*b^5*c*x
^5 + 27*a*b^3*c^2*x^5 + 27/4*b^6*x^4 + 81/2*a*b^4*c*x^4 + 27/4*a^
2*b^2*c^2*x^4 + 27*a*b^5*x^3 + 27*a^2*b^3*c*x^3 + 81/2*a^2*b^4*x^
2 + 27*a^3*b^3*x
```

3.10 $\int \left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2\,dx$

Optimal. Leaf size=56

$$-\frac{b\left(b^2-3ac\right)(b+cx)^4}{2c^3}+\frac{b^2x\left(b^2-3ac\right)^2}{c^2}+\frac{(b+cx)^7}{7c^3}$$

```
[Out] (b^2*(b^2 - 3*a*c)^2*x)/c^2 - (b*(b^2 - 3*a*c)*(b + c*x)^4)/(2*c^
3) + (b + c*x)^7/(7*c^3)
```

Rubi [A] time = 0.138858, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074

$$-\frac{b\left(b^2-3ac\right)(b+cx)^4}{2c^3}+\frac{b^2x\left(b^2-3ac\right)^2}{c^2}+\frac{(b+cx)^7}{7c^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^2,x]
```

```
[Out] (b^2*(b^2 - 3*a*c)^2*x)/c^2 - (b*(b^2 - 3*a*c)*(b + c*x)^4)/(2*c^
3) + (b + c*x)^7/(7*c^3)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{b(b+cx)^4\left(-3ac+b^2\right)}{2c^3}+\frac{(b+cx)^7}{7c^3}+\frac{\int^{\frac{b}{c}+x}\left(81ab^4c-27b^6\right)^2\,dx}{729b^6c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c**2*x**3+3*b*c*x**2+3*b**2*x+3*a*b)**2,x)
```

```
[Out] -b*(b + c*x)**4*(-3*a*c + b**2)/(2*c**3) + (b + c*x)**7/(7*c**3)
+ Integral((81*a*b**4*c - 27*b**6)**2, (x, b/c + x))/(729*b**6*c*
*2)
```

Mathematica [A] time = 0.0160593, size = 82, normalized size = 1.46

$$9a^2b^2x+9ab^3x^2+\frac{3}{2}bcx^4\left(ac+3b^2\right)+3b^2x^3\left(2ac+b^2\right)+3b^2c^2x^5+bc^3x^6+\frac{c^4x^7}{7}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^2,x]
```

```
[Out] 9*a^2*b^2*x + 9*a*b^3*x^2 + 3*b^2*(b^2 + 2*a*c)*x^3 + (3*b*c*(3*b
^2 + a*c)*x^4)/2 + 3*b^2*c^2*x^5 + b*c^3*x^6 + (c^4*x^7)/7
```

Maple [A] time = 0.001, size = 84, normalized size = 1.5

$$\frac{c^4x^7}{7}+bc^3x^6+3\,b^2c^2x^5+\frac{\left(6\,abc^2+18\,b^3c\right)x^4}{4}+\frac{\left(18\,ab^2c+9\,b^4\right)x^3}{3}+9\,ab^3x^2+9\,a^2b^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x)
```

```
[Out] 1/7*c^4*x^7+b*c^3*x^6+3*b^2*c^2*x^5+1/4*(6*a*b*c^2+18*b^3*c)*x^4+
1/3*(18*a*b^2*c+9*b^4)*x^3+9*a*b^3*x^2+9*a^2*b^2*x
```

Maxima [A] time = 0.767112, size = 126, normalized size = 2.25

$$\frac{1}{7}\,c^4x^7 + bc^3x^6 + \frac{9}{5}\,b^2c^2x^5 + 3\,b^4x^3 + 9\,a^2b^2x + \frac{3}{2}\,\left(c^2x^4 + 4\,bcx^3 + 6\,b^2x^2\right)ab + \frac{3}{10}\,\left(4\,c^2x^5 + 15\,bcx^4\right)b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c^2*x^3 + 3*b*c*x^2 + 3*b^2*x + 3*a*b)^2,x, algorithm="maxima")
```

```
[Out] 1/7*c^4*x^7 + b*c^3*x^6 + 9/5*b^2*c^2*x^5 + 3*b^4*x^3 + 9*a^2*b^2
*x + 3/2*(c^2*x^4 + 4*b*c*x^3 + 6*b^2*x^2)*a*b + 3/10*(4*c^2*x^5
+ 15*b*c*x^4)*b^2
```

Fricas [A] time = 0.264854, size = 1, normalized size = 0.02

$$\frac{1}{7}x^7c^4 + x^6c^3b + 3x^5c^2b^2 + \frac{9}{2}x^4cb^3 + \frac{3}{2}x^4c^2ba + 3x^3b^4 + 6x^3cb^2a + 9x^2b^3a + 9xb^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c^2*x^3 + 3*b*c*x^2 + 3*b^2*x + 3*a*b)^2,x, algorithm="fricas")
```

```
[Out] 1/7*x^7*c^4 + x^6*c^3*b + 3*x^5*c^2*b^2 + 9/2*x^4*c*b^3 + 3/2*x^4
*c^2*b*a + 3*x^3*b^4 + 6*x^3*c*b^2*a + 9*x^2*b^3*a + 9*x*b^2*a^2
```

Sympy [A] time = 0.127391, size = 87, normalized size = 1.55

$$9a^2b^2x + 9ab^3x^2 + 3b^2c^2x^5 + bc^3x^6 + \frac{c^4x^7}{7} + x^4\left(\frac{3abc^2}{2} + \frac{9b^3c}{2}\right) + x^3\left(6ab^2c + 3b^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((c**2*x**3+3*b*c*x**2+3*b**2*x+3*a*b)**2,x)
```

```
[Out] 9*a**2*b**2*x + 9*a*b**3*x**2 + 3*b**2*c**2*x**5 + b*c**3*x**6 +
c**4*x**7/7 + x**4*(3*a*b*c**2/2 + 9*b**3*c/2) + x**3*(6*a*b**2*c
 + 3*b**4)
```

GIAC/XCAS [A] time = 0.26037, size = 112, normalized size = 2.

$$\frac{1}{7}\,c^4x^7 + bc^3x^6 + 3\,b^2c^2x^5 + \frac{9}{2}\,b^3cx^4 + \frac{3}{2}\,abc^2x^4 + 3\,b^4x^3 + 6\,ab^2cx^3 + 9\,ab^3x^2 + 9\,a^2b^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c^2*x^3 + 3*b*c*x^2 + 3*b^2*x + 3*a*b)^2,x, algorithm="giac")
```

```
[Out] 1/7*c^4*x^7 + b*c^3*x^6 + 3*b^2*c^2*x^5 + 9/2*b^3*c*x^4 + 3/2*a*b
*c^2*x^4 + 3*b^4*x^3 + 6*a*b^2*c*x^3 + 9*a*b^3*x^2 + 9*a^2*b^2*x
```

3.11 $\int \left(3ab + 3b^2x + 3bcx^2 + c^2x^3\right)\,dx$

Optimal. Leaf size=32

$$3abx + \frac{3b^2x^2}{2} + bcx^3 + \frac{c^2x^4}{4}$$

```
[Out] 3*a*b*x + (3*b^2*x^2)/2 + b*c*x^3 + (c^2*x^4)/4
```

Rubi [A] time = 0.0170615, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$

$$3abx + \frac{3b^2x^2}{2} + bcx^3 + \frac{c^2x^4}{4}$$

Antiderivative was successfully verified.

```
[In]  Int[3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3,x]

[Out] 3*a*b*x + (3*b^2*x^2)/2 + b*c*x^3 + (c^2*x^4)/4
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$3abx + 3b^2 \int x\,dx + bcx^3 + \frac{c^2x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(c**2*x**3+3*b*c*x**2+3*b**2*x+3*a*b,x)

[Out] 3*a*b*x + 3*b**2*Integral(x, x) + b*c*x**3 + c**2*x**4/4
```

Mathematica [A] time = 0.0000655965, size = 32, normalized size = 1.

$$3abx + \frac{3b^2x^2}{2} + bcx^3 + \frac{c^2x^4}{4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3,x]

[Out] 3*a*b*x + (3*b^2*x^2)/2 + b*c*x^3 + (c^2*x^4)/4
```

Maple [A] time = 0.001, size = 29, normalized size = 0.9

$$3\,abx + \frac{3\,b^2x^2}{2} + bcx^3 + \frac{c^2x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b,x)
```

```
[Out] 3*a*b*x+3/2*b^2*x^2+b*c*x^3+1/4*c^2*x^4
```

Maxima [A] time = 0.77087, size = 38, normalized size = 1.19

$$\frac{1}{4}\,c^2x^4 + bcx^3 + \frac{3}{2}\,b^2x^2 + 3\,abx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c^2*x^3 + 3*b*c*x^2 + 3*b^2*x + 3*a*b,x, algorithm="maxima")
```

```
[Out] 1/4*c^2*x^4 + b*c*x^3 + 3/2*b^2*x^2 + 3*a*b*x
```

Fricas [A] time = 0.2788, size = 1, normalized size = 0.03

$$\frac{1}{4}x^4c^2 + x^3cb + \frac{3}{2}x^2b^2 + 3xba$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c^2*x^3 + 3*b*c*x^2 + 3*b^2*x + 3*a*b,x, algorithm="fricas")
```

```
[Out] 1/4*x^4*c^2 + x^3*c*b + 3/2*x^2*b^2 + 3*x*b*a
```

Sympy [A] time = 0.090594, size = 31, normalized size = 0.97

$$3abx + \frac{3b^2x^2}{2} + bcx^3 + \frac{c^2x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(c**2*x**3+3*b*c*x**2+3*b**2*x+3*a*b,x)
```

```
[Out] 3*a*b*x + 3*b**2*x**2/2 + b*c*x**3 + c**2*x**4/4
```

GIAC/XCAS [A] time = 0.25942, size = 38, normalized size = 1.19

$$\frac{1}{4}\,c^2x^4 + bcx^3 + \frac{3}{2}\,b^2x^2 + 3\,abx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c^2*x^3 + 3*b*c*x^2 + 3*b^2*x + 3*a*b,x, algorithm="giac")
```

```
[Out] 1/4*c^2*x^4 + b*c*x^3 + 3/2*b^2*x^2 + 3*a*b*x
```

3.12 $\int \frac{1}{3ab+3b^2x+3bcx^2+c^2x^3}\,dx$

Optimal. Leaf size=188

$$-\frac{\log\left(\sqrt[3]{b}c\sqrt[3]{b^2-3ac}\left(\frac{b}{c}+x\right)+b^{2/3}\left(b^2-3ac\right)^{2/3}+c^2\left(\frac{b}{c}+x\right)^2\right)}{6b^{2/3}\left(b^2-3ac\right)^{2/3}}+\frac{\log\left(-\sqrt[3]{b}\sqrt[3]{b^2-3ac}+b+cx\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}-\frac{\tan^{-1}\left(\frac{\frac{2(b+cx)}{\sqrt[3]{b^2-3ac}}+\sqrt[3]{b}}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}b^{2/3}\left(b^2-3ac\right)^{2/3}}$$

```
[Out] -(ArcTan[(b^(1/3) + (2*(b + c*x))/(b^2 - 3*a*c)^(1/3))/(Sqrt[3]*b
^(1/3))]/(Sqrt[3]*b^(2/3)*(b^2 - 3*a*c)^(2/3))) + Log[b - b^(1/3)
*(b^2 - 3*a*c)^(1/3) + c*x]/(3*b^(2/3)*(b^2 - 3*a*c)^(2/3)) - Log
[b^(2/3)*(b^2 - 3*a*c)^(2/3) + b^(1/3)*c*(b^2 - 3*a*c)^(1/3)*(b/c
 + x) + c^2*(b/c + x)^2]/(6*b^(2/3)*(b^2 - 3*a*c)^(2/3))
```

Rubi [A] time = 0.652216, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$

$$\frac{\log\left(\sqrt[3]{b}\left(b^{2/3}-\sqrt[3]{b^2-3ac}\right)+cx\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}-\frac{\log\left(\sqrt[3]{b}\sqrt[3]{b^2-3ac}(b+cx)+b^{2/3}\left(b^2-3ac\right)^{2/3}+(b+cx)^2\right)}{6b^{2/3}\left(b^2-3ac\right)^{2/3}}-\frac{\tan^{-1}\left(\frac{\frac{2(b+cx)}{\sqrt[3]{b^2-3ac}}+\sqrt[3]{b}}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}b^{2/3}\left(b^2-3ac\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^(-1),x]
```

```
[Out] -(ArcTan[(b^(1/3) + (2*(b + c*x))/(b^2 - 3*a*c)^(1/3))/(Sqrt[3]*b
^(1/3))]/(Sqrt[3]*b^(2/3)*(b^2 - 3*a*c)^(2/3))) + Log[b^(1/3)*(b^
(2/3) - (b^2 - 3*a*c)^(1/3)) + c*x]/(3*b^(2/3)*(b^2 - 3*a*c)^(2/3
)) - Log[b^(2/3)*(b^2 - 3*a*c)^(2/3) + b^(1/3)*(b^2 - 3*a*c)^(1/3
)*(b + c*x) + (b + c*x)^2]/(6*b^(2/3)*(b^2 - 3*a*c)^(2/3))
```

Rubi in Sympy [A] time = 74.9459, size = 184, normalized size = 0.98

$$\frac{\log\left(\sqrt[3]{b}\sqrt[3]{-3ac+b^2}-b-cx\right)}{3b^{\frac{2}{3}}\left(-3ac+b^2\right)^{\frac{2}{3}}}-\frac{\log\left(9b^{\frac{8}{3}}\left(-3ac+b^2\right)^{\frac{2}{3}}+b^{\frac{7}{3}}\left(9b+9cx\right)\sqrt[3]{-3ac+b^2}+9b^2\left(b+cx\right)^2\right)}{6b^{\frac{2}{3}}\left(-3ac+b^2\right)^{\frac{2}{3}}}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{b}}{3}+\frac{\frac{2b}{3}+\frac{2cx}{3}}{\sqrt[3]{-3ac+b^2}}\right)}{\sqrt[3]{b}}\right)}{3b^{\frac{2}{3}}\left(-3ac+b^2\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c**2*x**3+3*b*c*x**2+3*b**2*x+3*a*b),x)
```

```
[Out] log(b**(1/3)*(-3*a*c + b**2)**(1/3) - b - c*x)/(3*b**(2/3)*(-3*a*
c + b**2)**(2/3)) - log(9*b**(8/3)*(-3*a*c + b**2)**(2/3) + b**(7
```

```
/3)*(9*b + 9*c*x)*(-3*a*c + b**2)**(1/3) + 9*b**2*(b + c*x)**2)/(
6*b**(2/3)*(-3*a*c + b**2)**(2/3)) - sqrt(3)*atan(sqrt(3)*(b**(1/
3)/3 + (2*b/3 + 2*c*x/3)/(-3*a*c + b**2)**(1/3))/b**(1/3))/(3*b**
(2/3)*(-3*a*c + b**2)**(2/3))
```

Mathematica [C] time = 0.0264543, size = 63, normalized size = 0.34

$$\frac{1}{3}\text{RootSum}\left[\#1^3c^2+3\#1^2bc+3\#1b^2+3ab\&,\frac{\log(x-\#1)}{\#1^2c^2+2\#1bc+b^2}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^(-1),x]
```

```
[Out] RootSum[3*a*b + 3*b^2*#1 + 3*b*c*#1^2 + c^2*#1^3 & , Log[x - #1]/
(b^2 + 2*b*c*#1 + c^2*#1^2) & ]/3
```

Maple [C] time = 0.004, size = 57, normalized size = 0.3

$$\frac{1}{3}\sum_{\_R=RootOf\left(c^2\_Z^3+3\,bc\_Z^2+3\,b^2\_Z+3\,ab\right)}\frac{\ln(x-\_R)}{\_R^2c^2+2\,\_R\,bc+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b),x)
```

```
[Out] 1/3*sum(1/(_R^2*c^2+2*_R*b*c+b^2)*ln(x-_R),_R=RootOf(_Z^3*c^2+3*_
Z^2*b*c+3*_Z*b^2+3*a*b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{c^2x^3+3\,bcx^2+3\,b^2x+3\,ab}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c^2*x^3 + 3*b*c*x^2 + 3*b^2*x + 3*a*b),x, algorithm="maxima")
```

```
[Out] integrate(1/(c^2*x^3 + 3*b*c*x^2 + 3*b^2*x + 3*a*b), x)
```

Fricas [A] time = 0.276251, size = 351, normalized size = 1.87

$$-\frac{\sqrt{3}\left(\sqrt{3}\log\left(b^6-6\,ab^4c+9\,a^2b^2c^2+\left(b^6-6\,ab^4c+9\,a^2b^2c^2\right)^{\frac{2}{3}}\left(c^2x^2+2\,bcx+b^2\right)+\left(b^6-6\,ab^4c+9\,a^2b^2c^2\right)^{\frac{1}{3}}\left(b^4-3\,ab^2c\right.\right.\right.}{18\,(b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c^2*x^3 + 3*b*c*x^2 + 3*b^2*x + 3*a*b),x, algorithm="fricas")
```

```
[Out] -1/18*sqrt(3)*(sqrt(3)*log(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2 + (b^6
 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*(c^2*x^2 + 2*b*c*x + b^2) + (
```

```
b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(1/3)*(b^4 - 3*a*b^2*c + (b^3*c
- 3*a*b*c^2)*x)) - 2*sqrt(3)*log(-b^3 + 3*a*b*c + (b^6 - 6*a*b^4*
c + 9*a^2*b^2*c^2)^(1/3)*(c*x + b)) - 6*arctan(-1/3*(2*sqrt(3)*(b
^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(1/3)*(c*x + b) + sqrt(3)*(b^3 -
3*a*b*c))/(b^3 - 3*a*b*c)))/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(1/
3)
```

Sympy [A] time = 1.17777, size = 53, normalized size = 0.28

$$\operatorname{RootSum}\left(t^{3}\left(243a^{2}b^{2}c^{2}-162ab^{4}c+27b^{6}\right)-1,\left(t\mapsto t\log\left(x+\frac{9tabc-3tb^{3}+b}{c}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c**2*x**3+3*b*c*x**2+3*b**2*x+3*a*b),x)
```

```
[Out] RootSum(_t**3*(243*a**2*b**2*c**2 - 162*a*b**4*c + 27*b**6) - 1,
Lambda(_t, _t*log(x + (9*_t*a*b*c - 3*_t*b**3 + b)/c)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{c^2x^3 + 3\,bcx^2 + 3\,b^2x + 3\,ab}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^3 + 3*b*c*x^2 + 3*b^2*x + 3*a*b),x, algorithm="giac")
```

```
[Out] integrate(1/(c^2*x^3 + 3*b*c*x^2 + 3*b^2*x + 3*a*b), x)
```

3.13 $\int \frac{1}{(3ab+3b^2x+3bcx^2+c^2x^3)^2}\,dx$

Optimal. Leaf size=245

$$-\frac{c\left(\frac{b}{c}+x\right)}{3b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)}+\frac{c\log\left(\sqrt[3]{b}c\sqrt[3]{b^2-3ac}\left(\frac{b}{c}+x\right)+b^{2/3}\left(b^2-3ac\right)^{2/3}+c^2\left(\frac{b}{c}+x\right)^2\right)}{9b^{5/3}\left(b^2-3ac\right)^{5/3}}-\frac{2c\log\left(-\sqrt[3]{b}\sqrt[3]{b^2-3ac}+b+cx\right)}{9b^{5/3}\left(b^2-3ac\right)^{5/3}}+\frac{2c\tan^{-1}\left(\frac{\frac{2(b+cx)}{\sqrt[3]{b^2-3ac}}+\sqrt[3]{b}}{\sqrt{3}\sqrt[3]{b}}\right)}{3\sqrt{3}b^{5/3}\left(b^2-3ac\right)^{5/3}}$$

```
[Out] -(c*(b/c + x))/(3*b*(b^2 - 3*a*c)*(3*a*b + 3*b^2*x + 3*b*c*x^2 +
c^2*x^3)) + (2*c*ArcTan[(b^(1/3) + (2*(b + c*x))/(b^2 - 3*a*c)^(1
/3))/(Sqrt[3]*b^(1/3))])/(3*Sqrt[3]*b^(5/3)*(b^2 - 3*a*c)^(5/3))
- (2*c*Log[b - b^(1/3)*(b^2 - 3*a*c)^(1/3) + c*x])/(9*b^(5/3)*(b^
2 - 3*a*c)^(5/3)) + (c*Log[b^(2/3)*(b^2 - 3*a*c)^(2/3) + b^(1/3)*
c*(b^2 - 3*a*c)^(1/3)*(b/c + x) + c^2*(b/c + x)^2])/(9*b^(5/3)*(b
^2 - 3*a*c)^(5/3))
```

Rubi [A] time = 0.611663, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$

$$-\frac{b+cx}{3b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)}-\frac{2c\log\left(\sqrt[3]{b}\left(b^{2/3}-\sqrt[3]{b^2-3ac}\right)+cx\right)}{9b^{5/3}\left(b^2-3ac\right)^{5/3}}+\frac{c\log\left(\sqrt[3]{b}\sqrt[3]{b^2-3ac}(b+cx)+b^{2/3}\left(b^2-3ac\right)^{2/3}+(b+cx)^2\right)}{9b^{5/3}\left(b^2-3ac\right)^{5/3}}+\frac{2c\tan^{-1}\left(\frac{\frac{2(b+cx)}{\sqrt[3]{b^2-3ac}}+\sqrt[3]{b}}{\sqrt{3}\sqrt[3]{b}}\right)}{3\sqrt{3}b^{5/3}\left(b^2-3ac\right)^{5/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^(-2),x]
```

```
[Out] -(b + c*x)/(3*b*(b^2 - 3*a*c)*(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*
x^3)) + (2*c*ArcTan[(b^(1/3) + (2*(b + c*x))/(b^2 - 3*a*c)^(1/3))
/(Sqrt[3]*b^(1/3))])/(3*Sqrt[3]*b^(5/3)*(b^2 - 3*a*c)^(5/3)) - (2
*c*Log[b^(1/3)*(b^(2/3) - (b^2 - 3*a*c)^(1/3)) + c*x])/(9*b^(5/3)
*(b^2 - 3*a*c)^(5/3)) + (c*Log[b^(2/3)*(b^2 - 3*a*c)^(2/3) + b^(1
/3)*(b^2 - 3*a*c)^(1/3)*(b + c*x) + (b + c*x)^2])/(9*b^(5/3)*(b^2
 - 3*a*c)^(5/3))
```

Rubi in Sympy [A] time = 92.2977, size = 228, normalized size = 0.93

$$-\frac{c(b+cx)}{3b\left(-3ac+b^2\right)\left(b\left(3ac-b^2\right)+(b+cx)^3\right)}-\frac{2c\log\left(\sqrt[3]{b}\sqrt[3]{-3ac+b^2}-b-cx\right)}{9b^{\frac{5}{3}}\left(-3ac+b^2\right)^{\frac{5}{3}}}+\frac{c\log\left(9b^{\frac{8}{3}}\left(-3ac+b^2\right)^{\frac{2}{3}}+b^{\frac{7}{3}}(9b+9cx)\sqrt[3]{-3ac+b^2}+9b^2(b+cx)^2\right)}{9b^{\frac{5}{3}}\left(-3ac+b^2\right)^{\frac{5}{3}}}+\frac{2\sqrt{3}c\,\mathrm{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{b}}{3}+\frac{\frac{2b}{3}+\frac{2cx}{3}}{\sqrt[3]{-3ac+b^2}}\right)}{\sqrt[3]{b}}\right)}{9b^{\frac{5}{3}}\left(-3ac+b^2\right)^{\frac{5}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c**2*x**3+3*b*c*x**2+3*b**2*x+3*a*b)**2,x)
```

```
[Out] -c*(b + c*x)/(3*b*(-3*a*c + b**2)*(b*(3*a*c - b**2) + (b + c*x)**
3)) - 2*c*log(b**(1/3)*(-3*a*c + b**2)**(1/3) - b - c*x)/(9*b**(5
/3)*(-3*a*c + b**2)**(5/3)) + c*log(9*b**(8/3)*(-3*a*c + b**2)**(
2/3) + b**(7/3)*(9*b + 9*c*x)*(-3*a*c + b**2)**(1/3) + 9*b**2*(b
+ c*x)**2)/(9*b**(5/3)*(-3*a*c + b**2)**(5/3)) + 2*sqrt(3)*c*atan
(sqrt(3)*(b**(1/3)/3 + (2*b/3 + 2*c*x/3)/(-3*a*c + b**2)**(1/3))/
b**(1/3))/(9*b**(5/3)*(-3*a*c + b**2)**(5/3))
```

Mathematica [C] time = 0.095482, size = 112, normalized size = 0.46

$$-\frac{2c\text{RootSum}\left[\#1^3c^2+3\#1^2bc+3\#1b^2+3ab\&,\frac{\log(x-\#1)}{\#1^2c^2+2\#1bc+b^2}\&\right]+\frac{3(b+cx)}{3ab+x(3b^2+3bcx+c^2x^2)}}{9\left(b^3-3abc\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^(-2),x]
```

```
[Out] -((3*(b + c*x))/(3*a*b + x*(3*b^2 + 3*b*c*x + c^2*x^2)) + 2*c*Roo
tSum[3*a*b + 3*b^2*#1 + 3*b*c*#1^2 + c^2*#1^3 & , Log[x - #1]/(b^
2 + 2*b*c*#1 + c^2*#1^2) & ])/(9*(b^3 - 3*a*b*c))
```

Maple [C] time = 0.013, size = 136, normalized size = 0.6

$$\frac{1}{c^2x^3+3\,bcx^2+3\,b^2x+3\,ab}\left(\frac{cx}{3\,b\left(3\,ac-b^2\right)}+\frac{1}{9\,ac-3\,b^2}\right)+\frac{2\,c}{9\,b\left(3\,ac-b^2\right)}\sum_{\_R=RootOf\left(\_Z^3c^2+3\_Z^2bc+3\_Z\,b^2+3\,ab\right)}\frac{\ln\left(x-\_R\right)}{\_R^2c^2+2\,\_R\,bc+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x)
```

```
[Out] (1/3*c/b/(3*a*c-b^2)*x+1/3/(3*a*c-b^2))/(c^2*x^3+3*b*c*x^2+3*b^2*
x+3*a*b)+2/9*c/b/(3*a*c-b^2)*sum(1/(_R^2*c^2+2*_R*b*c+b^2)*ln(x-_
R),_R=RootOf(_Z^3*c^2+3*_Z^2*b*c+3*_Z*b^2+3*a*b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2\,c\int\frac{1}{c^2x^3+3\,bcx^2+3\,b^2x+3\,ab}\,dx}{3\left(b^3-3\,abc\right)}-\frac{cx+b}{3\left(3\,ab^4-9\,a^2b^2c+\left(b^3c^2-3\,abc^3\right)x^3+3\left(b^4c-3\,ab^2c^2\right)x^2+3\left(b^5-3\,ab^3c\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c^2*x^3 + 3*b*c*x^2 + 3*b^2*x + 3*a*b)^(-2),x, algorithm="maxima")
```

```
[Out] -2/3*c*integrate(1/(c^2*x^3 + 3*b*c*x^2 + 3*b^2*x + 3*a*b), x)/(b
^3 - 3*a*b*c) - 1/3*(c*x + b)/(3*a*b^4 - 9*a^2*b^2*c + (b^3*c^2 -
 3*a*b*c^3)*x^3 + 3*(b^4*c - 3*a*b^2*c^2)*x^2 + 3*(b^5 - 3*a*b^3*
c)*x)
```

Fricas [A] time = 0.290114, size = 610, normalized size = 2.49

$$\frac{\sqrt{3}\left(\sqrt{3}\left(c^3x^3 + 3\,bc^2x^2 + 3\,b^2cx + 3\,abc\right)\log\left(b^6 - 6\,ab^4c + 9\,a^2b^2c^2 + \left(b^6 - 6\,ab^4c + 9\,a^2b^2c^2\right)^{\frac{2}{3}}\left(c^2x^2 + 2\,bcx + b^2\right) + \left(b^6 - \right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^3 + 3*b*c*x^2 + 3*b^2*x + 3*a*b)^(-2),x, algorithm="fricas")
```

```
[Out] 1/27*sqrt(3)*(sqrt(3)*(c^3*x^3 + 3*b*c^2*x^2 + 3*b^2*c*x + 3*a*b*
c)*log(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2 + (b^6 - 6*a*b^4*c + 9*a^2
*b^2*c^2)^(2/3)*(c^2*x^2 + 2*b*c*x + b^2) + (b^6 - 6*a*b^4*c + 9*
a^2*b^2*c^2)^(1/3)*(b^4 - 3*a*b^2*c + (b^3*c - 3*a*b*c^2)*x)) - 2
*sqrt(3)*(c^3*x^3 + 3*b*c^2*x^2 + 3*b^2*c*x + 3*a*b*c)*log(-b^3 +
 3*a*b*c + (b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(1/3)*(c*x + b)) - 6
*(c^3*x^3 + 3*b*c^2*x^2 + 3*b^2*c*x + 3*a*b*c)*arctan(-1/3*(2*sqr
t(3)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(1/3)*(c*x + b) + sqrt(3)*
(b^3 - 3*a*b*c))/(b^3 - 3*a*b*c)) - 3*sqrt(3)*(b^6 - 6*a*b^4*c +
9*a^2*b^2*c^2)^(1/3)*(c*x + b))/((b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2
)^(1/3)*(3*a*b^4 - 9*a^2*b^2*c + (b^3*c^2 - 3*a*b*c^3)*x^3 + 3*(b
^4*c - 3*a*b^2*c^2)*x^2 + 3*(b^5 - 3*a*b^3*c)*x))
```

Sympy [A] time = 5.62329, size = 192, normalized size = 0.78

$$\frac{b + cx}{27a^2b^2c - 9ab^4 + x^3\left(9abc^3 - 3b^3c^2\right) + x^2\left(27ab^2c^2 - 9b^4c\right) + x\left(27ab^3c - 9b^5\right)}$$
$$+\operatorname{RootSum}\left(t^3\left(177147a^5b^5c^5 - 295245a^4b^7c^4 + 196830a^3b^9c^3 - 65610a^2b^{11}c^2 + 10935ab^{13}c - 729b^{15}\right) - 8c^3, \left(t \mapsto t\log\left(x\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c**2*x**3+3*b*c*x**2+3*b**2*x+3*a*b)**2,x)
```

```
[Out] (b + c*x)/(27*a**2*b**2*c - 9*a*b**4 + x**3*(9*a*b*c**3 - 3*b**3*
c**2) + x**2*(27*a*b**2*c**2 - 9*b**4*c) + x*(27*a*b**3*c - 9*b**
5)) + RootSum(_t**3*(177147*a**5*b**5*c**5 - 295245*a**4*b**7*c**
4 + 196830*a**3*b**9*c**3 - 65610*a**2*b**11*c**2 + 10935*a*b**13
*c - 729*b**15) - 8*c**3, Lambda(_t, _t*log(x + (81*_t*a**2*b**2*
c**2 - 54*_t*a*b**4*c + 9*_t*b**6 + 2*b*c)/(2*c**2))))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c^2x^3 + 3\,bcx^2 + 3\,b^2x + 3\,ab\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^3 + 3*b*c*x^2 + 3*b^2*x + 3*a*b)^(-2),x, algorithm="giac")
```

```
[Out] integrate((c^2*x^3 + 3*b*c*x^2 + 3*b^2*x + 3*a*b)^(-2), x)
```

3.14 $\int \frac{1}{(3ab+3b^2x+3bcx^2+c^2x^3)^3}\,dx$

Optimal. Leaf size=305

$$\frac{5c^2\left(\frac{b}{c}+x\right)}{18b^2(b^2-3ac)^2(3ab+3b^2x+3bcx^2+c^2x^3)} - \frac{c\left(\frac{b}{c}+x\right)}{6b(b^2-3ac)(3ab+3b^2x+3bcx^2+c^2x^3)^2} + \frac{5c^2\log\left(-\sqrt[3]{b}\sqrt[3]{b^2-3ac}+b+cx\right)}{27b^{8/3}(b^2-3ac)^{8/3}} - \frac{5c^2\log\left(\sqrt[3]{b}c\sqrt[3]{b^2-3ac}\left(\frac{b}{c}+x\right)+b^{2/3}(b^2-3ac)^{2/3}+c^2\left(\frac{b}{c}+x\right)^2\right)}{54b^{8/3}(b^2-3ac)^{8/3}} - \frac{5c^2\tan^{-1}\left(\frac{\frac{2(b+cx)}{\sqrt[3]{b^2-3ac}}+\sqrt[3]{b}}{\sqrt{3}\sqrt[3]{b}}\right)}{9\sqrt{3}b^{8/3}(b^2-3ac)^{8/3}}$$

```
[Out] -(c*(b/c + x))/(6*b*(b^2 - 3*a*c)*(3*a*b + 3*b^2*x + 3*b*c*x^2 +
c^2*x^3)^2) + (5*c^2*(b/c + x))/(18*b^2*(b^2 - 3*a*c)^2*(3*a*b +
3*b^2*x + 3*b*c*x^2 + c^2*x^3)) - (5*c^2*ArcTan[(b^(1/3) + (2*(b
+ c*x))/(b^2 - 3*a*c)^(1/3))/(Sqrt[3]*b^(1/3))])/(9*Sqrt[3]*b^(8/
3)*(b^2 - 3*a*c)^(8/3)) + (5*c^2*Log[b - b^(1/3)*(b^2 - 3*a*c)^(1
/3) + c*x])/(27*b^(8/3)*(b^2 - 3*a*c)^(8/3)) - (5*c^2*Log[b^(2/3)
*(b^2 - 3*a*c)^(2/3) + b^(1/3)*c*(b^2 - 3*a*c)^(1/3)*(b/c + x) +
c^2*(b/c + x)^2])/(54*b^(8/3)*(b^2 - 3*a*c)^(8/3))
```

Rubi [A] time = 0.759839, antiderivative size = 305, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$

$$\frac{5c(b+cx)}{18b^2(b^2-3ac)^2(3ab+3b^2x+3bcx^2+c^2x^3)} - \frac{b+cx}{6b(b^2-3ac)(3ab+3b^2x+3bcx^2+c^2x^3)^2} + \frac{5c^2\log\left(\sqrt[3]{b}\left(b^{2/3}-\sqrt[3]{b^2-3ac}\right)+cx\right)}{27b^{8/3}(b^2-3ac)^{8/3}} - \frac{5c^2\log\left(\sqrt[3]{b}\sqrt[3]{b^2-3ac}(b+cx)+b^{2/3}(b^2-3ac)^{2/3}+(b+cx)^2\right)}{54b^{8/3}(b^2-3ac)^{8/3}} - \frac{5c^2\tan^{-1}\left(\frac{\frac{2(b+cx)}{\sqrt[3]{b^2-3ac}}+\sqrt[3]{b}}{\sqrt{3}\sqrt[3]{b}}\right)}{9\sqrt{3}b^{8/3}(b^2-3ac)^{8/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^(-3),x]
```

```
[Out] -(b + c*x)/(6*b*(b^2 - 3*a*c)*(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*
x^3)^2) + (5*c*(b + c*x))/(18*b^2*(b^2 - 3*a*c)^2*(3*a*b + 3*b^2*
x + 3*b*c*x^2 + c^2*x^3)) - (5*c^2*ArcTan[(b^(1/3) + (2*(b + c*x)
)/(b^2 - 3*a*c)^(1/3))/(Sqrt[3]*b^(1/3))])/(9*Sqrt[3]*b^(8/3)*(b^
2 - 3*a*c)^(8/3)) + (5*c^2*Log[b^(1/3)*(b^(2/3) - (b^2 - 3*a*c)^(
1/3)) + c*x])/(27*b^(8/3)*(b^2 - 3*a*c)^(8/3)) - (5*c^2*Log[b^(2/
3)*(b^2 - 3*a*c)^(2/3) + b^(1/3)*(b^2 - 3*a*c)^(1/3)*(b + c*x) +
(b + c*x)^2])/(54*b^(8/3)*(b^2 - 3*a*c)^(8/3))
```

Rubi in Sympy [A] time = 112.989, size = 280, normalized size = 0.92

$$-\frac{c^2(b+cx)}{6b(-3ac+b^2)\left(b(3ac-b^2)+(b+cx)^3\right)^2}+\frac{5c^2(b+cx)}{18b^2(-3ac+b^2)^2\left(b(3ac-b^2)+(b+cx)^3\right)}+\frac{5c^2\log\left(\sqrt[3]{b}\sqrt[3]{-3ac+b^2}-b-cx\right)}{27b^{\frac{8}{3}}(-3ac+b^2)^{\frac{8}{3}}}-\frac{5c^2\log\left(9b^{\frac{8}{3}}(-3ac+b^2)^{\frac{2}{3}}+b^{\frac{7}{3}}(9b+9cx)\sqrt[3]{-3ac+b^2}+9b^2(b+cx)^2\right)}{54b^{\frac{8}{3}}(-3ac+b^2)^{\frac{8}{3}}}-\frac{5\sqrt{3}c^2\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{b}}{3}+\frac{\frac{2b}{3}+\frac{2cx}{3}}{\sqrt[3]{-3ac+b^2}}\right)}{\sqrt[3]{b}}\right)}{27b^{\frac{8}{3}}(-3ac+b^2)^{\frac{8}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c**2*x**3+3*b*c*x**2+3*b**2*x+3*a*b)**3,x)
```

```
[Out] -c**2*(b + c*x)/(6*b*(-3*a*c + b**2)*(b*(3*a*c - b**2) + (b + c*x
)**3)**2) + 5*c**2*(b + c*x)/(18*b**2*(-3*a*c + b**2)**2*(b*(3*a*
c - b**2) + (b + c*x)**3)) + 5*c**2*log(b**(1/3)*(-3*a*c + b**2)*
*(1/3) - b - c*x)/(27*b**(8/3)*(-3*a*c + b**2)**(8/3)) - 5*c**2*l
og(9*b**(8/3)*(-3*a*c + b**2)**(2/3) + b**(7/3)*(9*b + 9*c*x)*(-3
*a*c + b**2)**(1/3) + 9*b**2*(b + c*x)**2)/(54*b**(8/3)*(-3*a*c +
 b**2)**(8/3)) - 5*sqrt(3)*c**2*atan(sqrt(3)*(b**(1/3)/3 + (2*b/3
 + 2*c*x/3)/(-3*a*c + b**2)**(1/3))/b**(1/3))/(27*b**(8/3)*(-3*a*
c + b**2)**(8/3))
```

Mathematica [C] time = 0.142461, size = 149, normalized size = 0.49

$$\frac{10c^2\text{RootSum}\left[\#1^3c^2+3\#1^2bc+3\#1b^2+3ab\&,\frac{\log(x-\#1)}{\#1^2c^2+2\#1bc+b^2}\&\right]-\frac{3(b+cx)\left(-3bc\left(8a+5cx^2\right)+3b^3-15b^2cx-5c^3x^3\right)}{\left(3ab+x\left(3b^2+3bcx+c^2x^2\right)\right)^2}}{54\left(b^3-3abc\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^(-3),x]
```

```
[Out] ((-3*(b + c*x)*(3*b^3 - 15*b^2*c*x - 5*c^3*x^3 - 3*b*c*(8*a + 5*c
*x^2)))/(3*a*b + x*(3*b^2 + 3*b*c*x + c^2*x^2))^2 + 10*c^2*RootSu
m[3*a*b + 3*b^2*#1 + 3*b*c*#1^2 + c^2*#1^3 & , Log[x - #1]/(b^2 +
 2*b*c*#1 + c^2*#1^2) & ])/(54*(b^3 - 3*a*b*c)^2)
```

Maple [C] time = 0.023, size = 276, normalized size = 0.9

$$\frac{1}{(c^2x^3+3\,bcx^2+3\,b^2x+3\,ab)^2}\left(\frac{5\,c^4x^4}{18\,b^2\,(9\,a^2c^2-6\,ab^2c+b^4)}+\frac{10\,c^3x^3}{9\,b\,(9\,a^2c^2-6\,ab^2c+b^4)}+\frac{5\,c^2x^2}{27\,a^2c^2-18\,ab^2c+3\,b^4}+\frac{(4}{3\,b\,(9\,a}\right.$$
$$+\frac{5\,c^2}{27\,b^2\,(9\,a^2c^2-6\,ab^2c+b^4)}\sum_{\_R=RootOf(\_Z^3c^2+3\_Z^2bc+3\_Z\,b^2+3\,ab)}\frac{\ln(x-\_R)}{\_R^2c^2+2\_Rbc+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x)
```

```
[Out] (5/18*c^4/b^2/(9*a^2*c^2-6*a*b^2*c+b^4)*x^4+10/9/b*c^3/(9*a^2*c^2
-6*a*b^2*c+b^4)*x^3+5/3*c^2/(9*a^2*c^2-6*a*b^2*c+b^4)*x^2+2/3/b*(
2*a*c+b^2)*c/(9*a^2*c^2-6*a*b^2*c+b^4)*x+1/6*(8*a*c-b^2)/(9*a^2*c
^2-6*a*b^2*c+b^4))/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2+5/27*c^2/b
^2/(9*a^2*c^2-6*a*b^2*c+b^4)*sum(1/(_R^2*c^2+2*_R*b*c+b^2)*ln(x-_
R),_R=RootOf(_Z^3*c^2+3*_Z^2*b*c+3*_Z*b^2+3*a*b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{5\,c^2 \int \frac{1}{c^2x^3+3\,bcx^2+3\,b^2x+3\,ab}\,dx}{9\,(b^6-6\,ab^4c+9\,a^2b^2c^2)} + \frac{5\,c^4x^4+20\,bc^3x^3+30\,b^2c^2x^2 \cdots}{18\,(9\,a^2b^8-54\,a^3b^6c+81\,a^4b^4c^2+(b^6c^4-6\,ab^4c^5+9\,a^2b^2c^6)x^6+6\,(b^7c^3-6\,ab^5c^4+9\,a^2b^3c^5)x^5+15\,(b^8c^2-6\,ab^6c^3+9\,\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^3 + 3*b*c*x^2 + 3*b^2*x + 3*a*b)^(-3),x, algorithm="maxima")
```

```
[Out] 5/9*c^2*integrate(1/(c^2*x^3 + 3*b*c*x^2 + 3*b^2*x + 3*a*b), x)/(
b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2) + 1/18*(5*c^4*x^4 + 20*b*c^3*x^3
 + 30*b^2*c^2*x^2 - 3*b^4 + 24*a*b^2*c + 12*(b^3*c + 2*a*b*c^2)*x
)/(9*a^2*b^8 - 54*a^3*b^6*c + 81*a^4*b^4*c^2 + (b^6*c^4 - 6*a*b^4
*c^5 + 9*a^2*b^2*c^6)*x^6 + 6*(b^7*c^3 - 6*a*b^5*c^4 + 9*a^2*b^3*
c^5)*x^5 + 15*(b^8*c^2 - 6*a*b^6*c^3 + 9*a^2*b^4*c^4)*x^4 + 6*(3*
b^9*c - 17*a*b^7*c^2 + 21*a^2*b^5*c^3 + 9*a^3*b^3*c^4)*x^3 + 9*(b
^10 - 4*a*b^8*c - 3*a^2*b^6*c^2 + 18*a^3*b^4*c^3)*x^2 + 18*(a*b^9
 - 6*a^2*b^7*c + 9*a^3*b^5*c^2)*x)
```

Fricas [A] time = 0.288221, size = 1160, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^3 + 3*b*c*x^2 + 3*b^2*x + 3*a*b)^(-3),x, algorithm="fricas")
```

```
[Out] -1/162*sqrt(3)*(5*sqrt(3)*(c^6*x^6 + 6*b*c^5*x^5 + 15*b^2*c^4*x^4
 + 18*a*b^3*c^2*x + 9*a^2*b^2*c^2 + 6*(3*b^3*c^3 + a*b*c^4)*x^3 +
 9*(b^4*c^2 + 2*a*b^2*c^3)*x^2)*log(b^6 - 6*a*b^4*c + 9*a^2*b^2*c
^2 + (b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*(c^2*x^2 + 2*b*c*x +
 b^2) + (b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(1/3)*(b^4 - 3*a*b^2*c
+ (b^3*c - 3*a*b*c^2)*x)) - 10*sqrt(3)*(c^6*x^6 + 6*b*c^5*x^5 + 1
5*b^2*c^4*x^4 + 18*a*b^3*c^2*x + 9*a^2*b^2*c^2 + 6*(3*b^3*c^3 + a
*b*c^4)*x^3 + 9*(b^4*c^2 + 2*a*b^2*c^3)*x^2)*log(-b^3 + 3*a*b*c +
 (b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(1/3)*(c*x + b)) - 30*(c^6*x^6
 + 6*b*c^5*x^5 + 15*b^2*c^4*x^4 + 18*a*b^3*c^2*x + 9*a^2*b^2*c^2
+ 6*(3*b^3*c^3 + a*b*c^4)*x^3 + 9*(b^4*c^2 + 2*a*b^2*c^3)*x^2)*ar
ctan(-1/3*(2*sqrt(3)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(1/3)*(c*x
 + b) + sqrt(3)*(b^3 - 3*a*b*c))/(b^3 - 3*a*b*c)) - 3*sqrt(3)*(5*
c^4*x^4 + 20*b*c^3*x^3 + 30*b^2*c^2*x^2 - 3*b^4 + 24*a*b^2*c + 12
*(b^3*c + 2*a*b*c^2)*x)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(1/3))/
((9*a^2*b^8 - 54*a^3*b^6*c + 81*a^4*b^4*c^2 + (b^6*c^4 - 6*a*b^4*
c^5 + 9*a^2*b^2*c^6)*x^6 + 6*(b^7*c^3 - 6*a*b^5*c^4 + 9*a^2*b^3*c
^5)*x^5 + 15*(b^8*c^2 - 6*a*b^6*c^3 + 9*a^2*b^4*c^4)*x^4 + 6*(3*b
^9*c - 17*a*b^7*c^2 + 21*a^2*b^5*c^3 + 9*a^3*b^3*c^4)*x^3 + 9*(b^
10 - 4*a*b^8*c - 3*a^2*b^6*c^2 + 18*a^3*b^4*c^3)*x^2 + 18*(a*b^9
- 6*a^2*b^7*c + 9*a^3*b^5*c^2)*x)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^
2)^(1/3))
```

Sympy [A] time = 23.6042, size = 474, normalized size = 1.55

$$\frac{24ab^2c}{1458a^4b^4c^2 - 972a^3b^6c + 162a^2b^8 + x^6\left(162a^2b^2c^6 - 108ab^4c^5 + 18b^6c^4\right) + x^5\left(972a^2b^3c^5 - 648ab^5c^4 + 108b^7c^3\right) + x^4\left(2430a\right.}$$
$$+\mathrm{RootSum}\left(t^3\left(129140163a^8b^8c^8 - 344373768a^7b^{10}c^7 + 401769396a^6b^{12}c^6 - 267846264a^5b^{14}c^5 + 111602610a^4b^{16}c^4 - 2976\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c**2*x**3+3*b*c*x**2+3*b**2*x+3*a*b)**3,x)
```

```
[Out] (24*a*b**2*c - 3*b**4 + 30*b**2*c**2*x**2 + 20*b*c**3*x**3 + 5*c*
*4*x**4 + x*(24*a*b*c**2 + 12*b**3*c))/(1458*a**4*b**4*c**2 - 972
*a**3*b**6*c + 162*a**2*b**8 + x**6*(162*a**2*b**2*c**6 - 108*a*b
**4*c**5 + 18*b**6*c**4) + x**5*(972*a**2*b**3*c**5 - 648*a*b**5*
c**4 + 108*b**7*c**3) + x**4*(2430*a**2*b**4*c**4 - 1620*a*b**6*c
**3 + 270*b**8*c**2) + x**3*(972*a**3*b**3*c**4 + 2268*a**2*b**5*
c**3 - 1836*a*b**7*c**2 + 324*b**9*c) + x**2*(2916*a**3*b**4*c**3
 - 486*a**2*b**6*c**2 - 648*a*b**8*c + 162*b**10) + x*(2916*a**3*
b**5*c**2 - 1944*a**2*b**7*c + 324*a*b**9)) + RootSum(_t**3*(1291
40163*a**8*b**8*c**8 - 344373768*a**7*b**10*c**7 + 401769396*a**6
*b**12*c**6 - 267846264*a**5*b**14*c**5 + 111602610*a**4*b**16*c*
*4 - 29760696*a**3*b**18*c**3 + 4960116*a**2*b**20*c**2 - 472392*
a*b**22*c + 19683*b**24) - 125*c**6, Lambda(_t, _t*log(x + (729*_
t*a**3*b**3*c**3 - 729*_t*a**2*b**5*c**2 + 243*_t*a*b**7*c - 27*_
t*b**9 + 5*b*c**2)/(5*c**3))))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c^2x^3 + 3\,bcx^2 + 3\,b^2x + 3\,ab\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c^2*x^3 + 3*b*c*x^2 + 3*b^2*x + 3*a*b)^(-3),x, algorithm="giac")
```

```
[Out] integrate((c^2*x^3 + 3*b*c*x^2 + 3*b^2*x + 3*a*b)^(-3), x)
```

3.15 $\int \left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^3\,dx$

Optimal. Leaf size=361

$$\frac{3df(a+bx)^8\left(5a^2d^2f^2-5abdf(cf+de)+b^2\left(c^2f^2+3cdef+d^2e^2\right)\right)}{8b^7}+\frac{(a+bx)^7(-2adf+bcf+bde)\left(10a^2d^2f^2-10abdf(cf+de)+b^2\left(c^2f^2+8cdef+d^2e^2\right)\right)}{7b^7}+\frac{(a+bx)^6(bc-ad)(be-af)\left(5a^2d^2f^2-5abdf(cf+de)+b^2\left(c^2f^2+3cdef+d^2e^2\right)\right)}{2b^7}+\frac{d^2f^2(a+bx)^9(-2adf+bcf+bde)}{3b^7}+\frac{3(a+bx)^5(bc-ad)^2(be-af)^2(-2adf+bcf+bde)}{5b^7}+\frac{(a+bx)^4(bc-ad)^3(be-af)^3}{4b^7}+\frac{d^3f^3(a+bx)^{10}}{10b^7}$$

```
[Out] ((b*c - a*d)^3*(b*e - a*f)^3*(a + b*x)^4)/(4*b^7) + (3*(b*c - a*d
)^2*(b*e - a*f)^2*(b*d*e + b*c*f - 2*a*d*f)*(a + b*x)^5)/(5*b^7)
+ ((b*c - a*d)*(b*e - a*f)*(5*a^2*d^2*f^2 - 5*a*b*d*f*(d*e + c*f)
 + b^2*(d^2*e^2 + 3*c*d*e*f + c^2*f^2))*(a + b*x)^6)/(2*b^7) + ((
b*d*e + b*c*f - 2*a*d*f)*(10*a^2*d^2*f^2 - 10*a*b*d*f*(d*e + c*f)
 + b^2*(d^2*e^2 + 8*c*d*e*f + c^2*f^2))*(a + b*x)^7)/(7*b^7) + (3
*d*f*(5*a^2*d^2*f^2 - 5*a*b*d*f*(d*e + c*f) + b^2*(d^2*e^2 + 3*c*
d*e*f + c^2*f^2))*(a + b*x)^8)/(8*b^7) + (d^2*f^2*(b*d*e + b*c*f
- 2*a*d*f)*(a + b*x)^9)/(3*b^7) + (d^3*f^3*(a + b*x)^10)/(10*b^7)
```

Rubi [A] time = 1.79802, antiderivative size = 361, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043

$$\frac{3df(a+bx)^8\left(5a^2d^2f^2-5abdf(cf+de)+b^2\left(c^2f^2+3cdef+d^2e^2\right)\right)}{8b^7}+\frac{(a+bx)^7(-2adf+bcf+bde)\left(10a^2d^2f^2-10abdf(cf+de)+b^2\left(c^2f^2+8cdef+d^2e^2\right)\right)}{7b^7}+\frac{(a+bx)^6(bc-ad)(be-af)\left(5a^2d^2f^2-5abdf(cf+de)+b^2\left(c^2f^2+3cdef+d^2e^2\right)\right)}{2b^7}+\frac{d^2f^2(a+bx)^9(-2adf+bcf+bde)}{3b^7}+\frac{3(a+bx)^5(bc-ad)^2(be-af)^2(-2adf+bcf+bde)}{5b^7}+\frac{(a+bx)^4(bc-ad)^3(be-af)^3}{4b^7}+\frac{d^3f^3(a+bx)^{10}}{10b^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*f*x^3)^3
```



```
[Out] ((b*c - a*d)^3*(b*e - a*f)^3*(a + b*x)^4)/(4*b^7) + (3*(b*c - a*d
)^2*(b*e - a*f)^2*(b*d*e + b*c*f - 2*a*d*f)*(a + b*x)^5)/(5*b^7)
+ ((b*c - a*d)*(b*e - a*f)*(5*a^2*d^2*f^2 - 5*a*b*d*f*(d*e + c*f)
 + b^2*(d^2*e^2 + 3*c*d*e*f + c^2*f^2))*(a + b*x)^6)/(2*b^7) + ((
b*d*e + b*c*f - 2*a*d*f)*(10*a^2*d^2*f^2 - 10*a*b*d*f*(d*e + c*f)
 + b^2*(d^2*e^2 + 8*c*d*e*f + c^2*f^2))*(a + b*x)^7)/(7*b^7) + (3
*d*f*(5*a^2*d^2*f^2 - 5*a*b*d*f*(d*e + c*f) + b^2*(d^2*e^2 + 3*c*
d*e*f + c^2*f^2))*(a + b*x)^8)/(8*b^7) + (d^2*f^2*(b*d*e + b*c*f
- 2*a*d*f)*(a + b*x)^9)/(3*b^7) + (d^3*f^3*(a + b*x)^10)/(10*b^7)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x**2+b*d*f*x**
```

```
[Out] Timed out
```

Mathematica [A] time = 0.470189, size = 653, normalized size = 1.81

$$a^3c^3e^3x+\frac{3}{8}bdfx^8\left(a^2d^2f^2+3abdf(cf+de)+b^2\left(c^2f^2+3cdef+d^2e^2\right)\right)$$
$$+acex^3\left(a^2\left(c^2f^2+3cdef+d^2e^2\right)+3abce(cf+de)+b^2c^2e^2\right)+\frac{3}{2}a^2c^2e^2x^2(acf+ade+bce)$$
$$+\frac{1}{7}x^7\left(a^3d^3f^3+9a^2bd^2f^2(cf+de)+9ab^2df\left(c^2f^2+3cdef+d^2e^2\right)+b^3\left(c^3f^3+9c^2def^2+9cd^2e^2f+d^3e^3\right)\right)$$
$$+\frac{1}{2}x^6\left(a^3d^2f^2(cf+de)+3a^2bdf\left(c^2f^2+3cdef+d^2e^2\right)+ab^2\left(c^3f^3+9c^2def^2+9cd^2e^2f+d^3e^3\right)\right.$$
$$\left.+b^3ce\left(c^2f^2+3cdef+d^2e^2\right)\right)+\frac{3}{5}x^5\left(a^3df\left(c^2f^2+3cdef+d^2e^2\right)\right.$$
$$\left.+a^2b\left(c^3f^3+9c^2def^2+9cd^2e^2f+d^3e^3\right)+3ab^2ce\left(c^2f^2+3cdef+d^2e^2\right)+b^3c^2e^2(cf+de)\right)$$
$$+\frac{1}{4}x^4\left(a^3\left(c^3f^3+9c^2def^2+9cd^2e^2f+d^3e^3\right)+9a^2bce\left(c^2f^2+3cdef+d^2e^2\right)+9ab^2c^2e^2(cf+de)+b^3c^3e^3\right)$$
$$+\frac{1}{3}b^2d^2f^2x^9(adf+bcf+bde)+\frac{1}{10}b^3d^3f^3x^{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*f
```

```
[Out] a^3*c^3*e^3*x + (3*a^2*c^2*e^2*(b*c*e + a*d*e + a*c*f)*x^2)/2 + a
*c*e*(b^2*c^2*e^2 + 3*a*b*c*e*(d*e + c*f) + a^2*(d^2*e^2 + 3*c*d*
e*f + c^2*f^2))*x^3 + ((b^3*c^3*e^3 + 9*a*b^2*c^2*e^2*(d*e + c*f)
 + 9*a^2*b*c*e*(d^2*e^2 + 3*c*d*e*f + c^2*f^2) + a^3*(d^3*e^3 + 9
*c*d^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f^3))*x^4)/4 + (3*(b^3*c^2*e^2
*(d*e + c*f) + 3*a*b^2*c*e*(d^2*e^2 + 3*c*d*e*f + c^2*f^2) + a^3*
d*f*(d^2*e^2 + 3*c*d*e*f + c^2*f^2) + a^2*b*(d^3*e^3 + 9*c*d^2*e^
2*f + 9*c^2*d*e*f^2 + c^3*f^3))*x^5)/5 + ((a^3*d^2*f^2*(d*e + c*f
) + b^3*c*e*(d^2*e^2 + 3*c*d*e*f + c^2*f^2) + 3*a^2*b*d*f*(d^2*e^
2 + 3*c*d*e*f + c^2*f^2) + a*b^2*(d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2
*d*e*f^2 + c^3*f^3))*x^6)/2 + ((a^3*d^3*f^3 + 9*a^2*b*d^2*f^2*(d*
e + c*f) + 9*a*b^2*d*f*(d^2*e^2 + 3*c*d*e*f + c^2*f^2) + b^3*(d^3
*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f^3))*x^7)/7 + (3*b*d*
f*(a^2*d^2*f^2 + 3*a*b*d*f*(d*e + c*f) + b^2*(d^2*e^2 + 3*c*d*e*f
 + c^2*f^2))*x^8)/8 + (b^2*d^2*f^2*(b*d*e + b*c*f + a*d*f)*x^9)/3
 + (b^3*d^3*f^3*x^10)/10
```

Maple [B] time = 0.002, size = 861, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^3,x)
```

```
[Out] 1/10*b^3*d^3*f^3*x^10+1/3*(a*d*f+b*c*f+b*d*e)*b^2*d^2*f^2*x^9+1/8
*((a*c*f+a*d*e+b*c*e)*b^2*d^2*f^2+2*(a*d*f+b*c*f+b*d*e)^2*b*d*f+b
*d*f*(2*(a*c*f+a*d*e+b*c*e)*b*d*f+(a*d*f+b*c*f+b*d*e)^2))*x^8+1/7
*(a*c*e*b^2*d^2*f^2+2*(a*c*f+a*d*e+b*c*e)*(a*d*f+b*c*f+b*d*e)*b*d
*f+(a*d*f+b*c*f+b*d*e)*(2*(a*c*f+a*d*e+b*c*e)*b*d*f+(a*d*f+b*c*f+
b*d*e)^2)+b*d*f*(2*a*c*e*b*d*f+2*(a*c*f+a*d*e+b*c*e)*(a*d*f+b*c*f
+b*d*e)))*x^7+1/6*(2*a*c*e*(a*d*f+b*c*f+b*d*e)*b*d*f+(a*c*f+a*d*e
+b*c*e)*(2*(a*c*f+a*d*e+b*c*e)*b*d*f+(a*d*f+b*c*f+b*d*e)^2)+(a*d*
f+b*c*f+b*d*e)*(2*a*c*e*b*d*f+2*(a*c*f+a*d*e+b*c*e)*(a*d*f+b*c*f+
b*d*e))+b*d*f*(2*a*c*e*(a*d*f+b*c*f+b*d*e)+(a*c*f+a*d*e+b*c*e)^2)
```

```
)*x^6+1/5*(a*c*e*(2*(a*c*f+a*d*e+b*c*e)*b*d*f+(a*d*f+b*c*f+b*d*e)
^2)+(a*c*f+a*d*e+b*c*e)*(2*a*c*e*b*d*f+2*(a*c*f+a*d*e+b*c*e)*(a*d
*f+b*c*f+b*d*e))+(a*d*f+b*c*f+b*d*e)*(2*a*c*e*(a*d*f+b*c*f+b*d*e)
+(a*c*f+a*d*e+b*c*e)^2)+2*b*d*f*a*c*e*(a*c*f+a*d*e+b*c*e))*x^5+1/
4*(a*c*e*(2*a*c*e*b*d*f+2*(a*c*f+a*d*e+b*c*e)*(a*d*f+b*c*f+b*d*e)
)+(a*c*f+a*d*e+b*c*e)*(2*a*c*e*(a*d*f+b*c*f+b*d*e)+(a*c*f+a*d*e+b
*c*e)^2)+2*(a*d*f+b*c*f+b*d*e)*a*c*e*(a*c*f+a*d*e+b*c*e)+b*d*f*a^
2*c^2*e^2)*x^4+1/3*(a*c*e*(2*a*c*e*(a*d*f+b*c*f+b*d*e)+(a*c*f+a*d
*e+b*c*e)^2)+2*(a*c*f+a*d*e+b*c*e)^2*a*c*e+(a*d*f+b*c*f+b*d*e)*a^
2*c^2*e^2)*x^3+3/2*a^2*c^2*e^2*(a*c*f+a*d*e+b*c*e)*x^2+a^3*c^3*e^
3*x
```

Maxima [A] time = 0.778591, size = 622, normalized size = 1.72

$$\frac{1}{10}b^3d^3f^3x^{10}+\frac{1}{3}(bde+bcf+adf)b^2d^2f^2x^9$$
$$+\frac{3}{8}(bde+bcf+adf)^2bdfx^8+a^3c^3e^3x+\frac{1}{7}(bde+bcf+adf)^3x^7$$
$$+\frac{1}{4}\left(3\,bdfx^4+4\,(bde+bcf+adf)x^3+6\,(bce+ade+acf)x^2\right)a^2c^2e^2+\frac{1}{4}(bce+ade+acf)^3x^4$$
$$+\frac{1}{70}\left(30\,b^2d^2f^2x^7+70\,(bde+bcf+adf)bdfx^6+42\,(bde+bcf+adf)^2x^5+70\,(bce+ade+acf)^2x^3+21\left(4\,bdfx^5+5\,(bd\right.\right.$$
$$+\frac{1}{10}\left(5\,bdfx^6+6\,(bde+(bc+ad)f)x^5\right)(bce+ade+acf)^2$$
$$+\frac{1}{56}\left(21\,b^2d^2f^2x^8+48\left(b^2d^2ef+\left(b^2cd+abd^2\right)f^2\right)x^7+28\left(b^2d^2e^2+2\left(b^2cd+abd^2\right)ef+\left(b^2c^2+2\,abcd+a^2d^2\right)f^2\right)x^6\right)(b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*f*x^3 + a*c*e + (b*d*e + b*c*f + a*d*f)*x^2 + (b*c*e + a*d*e + a*c*f
```

```
[Out] 1/10*b^3*d^3*f^3*x^10 + 1/3*(b*d*e + b*c*f + a*d*f)*b^2*d^2*f^2*x
^9 + 3/8*(b*d*e + b*c*f + a*d*f)^2*b*d*f*x^8 + a^3*c^3*e^3*x + 1/
7*(b*d*e + b*c*f + a*d*f)^3*x^7 + 1/4*(3*b*d*f*x^4 + 4*(b*d*e + b
*c*f + a*d*f)*x^3 + 6*(b*c*e + a*d*e + a*c*f)*x^2)*a^2*c^2*e^2 +
1/4*(b*c*e + a*d*e + a*c*f)^3*x^4 + 1/70*(30*b^2*d^2*f^2*x^7 + 70
*(b*d*e + b*c*f + a*d*f)*b*d*f*x^6 + 42*(b*d*e + b*c*f + a*d*f)^2
*x^5 + 70*(b*c*e + a*d*e + a*c*f)^2*x^3 + 21*(4*b*d*f*x^5 + 5*(b*
d*e + (b*c + a*d)*f)*x^4)*(b*c*e + a*d*e + a*c*f))*a*c*e + 1/10*(
5*b*d*f*x^6 + 6*(b*d*e + (b*c + a*d)*f)*x^5)*(b*c*e + a*d*e + a*c
*f)^2 + 1/56*(21*b^2*d^2*f^2*x^8 + 48*(b^2*d^2*e*f + (b^2*c*d + a
*b*d^2)*f^2)*x^7 + 28*(b^2*d^2*e^2 + 2*(b^2*c*d + a*b*d^2)*e*f +
(b^2*c^2 + 2*a*b*c*d + a^2*d^2)*f^2)*x^6)*(b*c*e + a*d*e + a*c*f)
```

Fricas [A] time = 0.256996, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*f*x^3 + a*c*e + (b*d*e + b*c*f + a*d*f)*x^2 + (b*c*e + a*d*e + a*c*f
```

```
[Out] 1/10*x^10*f^3*d^3*b^3 + 1/3*x^9*f^2*e*d^3*b^3 + 1/3*x^9*f^3*d^2*c
*b^3 + 1/3*x^9*f^3*d^3*b^2*a + 3/8*x^8*f*e^2*d^3*b^3 + 9/8*x^8*f^
2*e*d^2*c*b^3 + 3/8*x^8*f^3*d*c^2*b^3 + 9/8*x^8*f^2*e*d^3*b^2*a +
 9/8*x^8*f^3*d^2*c*b^2*a + 3/8*x^8*f^3*d^3*b*a^2 + 1/7*x^7*e^3*d^
3*b^3 + 9/7*x^7*f*e^2*d^2*c*b^3 + 9/7*x^7*f^2*e*d*c^2*b^3 + 1/7*x
^7*f^3*c^3*b^3 + 9/7*x^7*f*e^2*d^3*b^2*a + 27/7*x^7*f^2*e*d^2*c*b
^2*a + 9/7*x^7*f^3*d*c^2*b^2*a + 9/7*x^7*f^2*e*d^3*b*a^2 + 9/7*x^
7*f^3*d^2*c*b*a^2 + 1/7*x^7*f^3*d^3*a^3 + 1/2*x^6*e^3*d^2*c*b^3 +
 3/2*x^6*f*e^2*d*c^2*b^3 + 1/2*x^6*f^2*e*c^3*b^3 + 1/2*x^6*e^3*d^
3*b^2*a + 9/2*x^6*f*e^2*d^2*c*b^2*a + 9/2*x^6*f^2*e*d*c^2*b^2*a +
 1/2*x^6*f^3*c^3*b^2*a + 3/2*x^6*f*e^2*d^3*b*a^2 + 9/2*x^6*f^2*e*
```

```
d^2*c*b*a^2 + 3/2*x^6*f^3*d*c^2*b*a^2 + 1/2*x^6*f^2*e*d^3*a^3 + 1
/2*x^6*f^3*d^2*c*a^3 + 3/5*x^5*e^3*d*c^2*b^3 + 3/5*x^5*f*e^2*c^3*
b^3 + 9/5*x^5*e^3*d^2*c*b^2*a + 27/5*x^5*f*e^2*d*c^2*b^2*a + 9/5*
x^5*f^2*e*c^3*b^2*a + 3/5*x^5*e^3*d^3*b*a^2 + 27/5*x^5*f*e^2*d^2*
c*b*a^2 + 27/5*x^5*f^2*e*d*c^2*b*a^2 + 3/5*x^5*f^3*c^3*b*a^2 + 3/
5*x^5*f*e^2*d^3*a^3 + 9/5*x^5*f^2*e*d^2*c*a^3 + 3/5*x^5*f^3*d*c^2
*a^3 + 1/4*x^4*e^3*c^3*b^3 + 9/4*x^4*e^3*d*c^2*b^2*a + 9/4*x^4*f*
e^2*c^3*b^2*a + 9/4*x^4*e^3*d^2*c*b*a^2 + 27/4*x^4*f*e^2*d*c^2*b*
a^2 + 9/4*x^4*f^2*e*c^3*b*a^2 + 1/4*x^4*e^3*d^3*a^3 + 9/4*x^4*f*e
^2*d^2*c*a^3 + 9/4*x^4*f^2*e*d*c^2*a^3 + 1/4*x^4*f^3*c^3*a^3 + x^
3*e^3*c^3*b^2*a + 3*x^3*e^3*d*c^2*b*a^2 + 3*x^3*f*e^2*c^3*b*a^2 +
 x^3*e^3*d^2*c*a^3 + 3*x^3*f*e^2*d*c^2*a^3 + x^3*f^2*e*c^3*a^3 +
3/2*x^2*e^3*c^3*b*a^2 + 3/2*x^2*e^3*d*c^2*a^3 + 3/2*x^2*f*e^2*c^3
*a^3 + x*e^3*c^3*a^3
```

Sympy [A] time = 0.65747, size = 1018, normalized size = 2.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x**2+b*d*f*x**3)**3
```

```
[Out] a**3*c**3*e**3*x + b**3*d**3*f**3*x**10/10 + x**9*(a*b**2*d**3*f*
*3/3 + b**3*c*d**2*f**3/3 + b**3*d**3*e*f**2/3) + x**8*(3*a**2*b*
d**3*f**3/8 + 9*a*b**2*c*d**2*f**3/8 + 9*a*b**2*d**3*e*f**2/8 + 3
*b**3*c**2*d*f**3/8 + 9*b**3*c*d**2*e*f**2/8 + 3*b**3*d**3*e**2*f
/8) + x**7*(a**3*d**3*f**3/7 + 9*a**2*b*c*d**2*f**3/7 + 9*a**2*b*
d**3*e*f**2/7 + 9*a*b**2*c**2*d*f**3/7 + 27*a*b**2*c*d**2*e*f**2/
7 + 9*a*b**2*d**3*e**2*f/7 + b**3*c**3*f**3/7 + 9*b**3*c**2*d*e*f
**2/7 + 9*b**3*c*d**2*e**2*f/7 + b**3*d**3*e**3/7) + x**6*(a**3*c
*d**2*f**3/2 + a**3*d**3*e*f**2/2 + 3*a**2*b*c**2*d*f**3/2 + 9*a*
*2*b*c*d**2*e*f**2/2 + 3*a**2*b*d**3*e**2*f/2 + a*b**2*c**3*f**3/
2 + 9*a*b**2*c**2*d*e*f**2/2 + 9*a*b**2*c*d**2*e**2*f/2 + a*b**2*
d**3*e**3/2 + b**3*c**3*e*f**2/2 + 3*b**3*c**2*d*e**2*f/2 + b**3*
c*d**2*e**3/2) + x**5*(3*a**3*c**2*d*f**3/5 + 9*a**3*c*d**2*e*f**
2/5 + 3*a**3*d**3*e**2*f/5 + 3*a**2*b*c**3*f**3/5 + 27*a**2*b*c**
2*d*e*f**2/5 + 27*a**2*b*c*d**2*e**2*f/5 + 3*a**2*b*d**3*e**3/5 +
 9*a*b**2*c**3*e*f**2/5 + 27*a*b**2*c**2*d*e**2*f/5 + 9*a*b**2*c*
d**2*e**3/5 + 3*b**3*c**3*e**2*f/5 + 3*b**3*c**2*d*e**3/5) + x**4
*(a**3*c**3*f**3/4 + 9*a**3*c**2*d*e*f**2/4 + 9*a**3*c*d**2*e**2*
f/4 + a**3*d**3*e**3/4 + 9*a**2*b*c**3*e*f**2/4 + 27*a**2*b*c**2*
d*e**2*f/4 + 9*a**2*b*c*d**2*e**3/4 + 9*a*b**2*c**3*e**2*f/4 + 9*
a*b**2*c**2*d*e**3/4 + b**3*c**3*e**3/4) + x**3*(a**3*c**3*e*f**2
 + 3*a**3*c**2*d*e**2*f + a**3*c*d**2*e**3 + 3*a**2*b*c**3*e**2*f
 + 3*a**2*b*c**2*d*e**3 + a*b**2*c**3*e**3) + x**2*(3*a**3*c**3*e
**2*f/2 + 3*a**3*c**2*d*e**3/2 + 3*a**2*b*c**3*e**3/2)
```

GIAC/XCAS [A] time = 0.258286, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*f*x^3 + a*c*e + (b*d*e + b*c*f + a*d*f)*x^2 + (b*c*e + a*d*e + a*c*f
```

```
[Out] Done
```

3.16 $\int \left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^2 dx$

Optimal. Leaf size=193

$$\frac{(a+bx)^5\left(6a^2d^2f^2-6abdf(cf+de)+b^2\left(c^2f^2+4cdef+d^2e^2\right)\right)}{5b^5}+\frac{df(a+bx)^6(-2adf+bcf+bde)}{3b^5}+\frac{(a+bx)^4(bc-ad)(be-af)(-2adf+bcf+bde)}{2b^5}+\frac{(a+bx)^3(bc-ad)^2(be-af)^2}{3b^5}+\frac{d^2f^2(a+bx)^7}{7b^5}$$

```
[Out] ((b*c - a*d)^2*(b*e - a*f)^2*(a + b*x)^3)/(3*b^5) + ((b*c - a*d)*
(b*e - a*f)*(b*d*e + b*c*f - 2*a*d*f)*(a + b*x)^4)/(2*b^5) + ((6*
a^2*d^2*f^2 - 6*a*b*d*f*(d*e + c*f) + b^2*(d^2*e^2 + 4*c*d*e*f +
c^2*f^2))*(a + b*x)^5)/(5*b^5) + (d*f*(b*d*e + b*c*f - 2*a*d*f)*(
a + b*x)^6)/(3*b^5) + (d^2*f^2*(a + b*x)^7)/(7*b^5)
```

Rubi [A] time = 0.605631, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$

$$\frac{(a+bx)^5\left(6a^2d^2f^2-6abdf(cf+de)+b^2\left(c^2f^2+4cdef+d^2e^2\right)\right)}{5b^5}+\frac{df(a+bx)^6(-2adf+bcf+bde)}{3b^5}+\frac{(a+bx)^4(bc-ad)(be-af)(-2adf+bcf+bde)}{2b^5}+\frac{(a+bx)^3(bc-ad)^2(be-af)^2}{3b^5}+\frac{d^2f^2(a+bx)^7}{7b^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*f*x^3)^2
```

```
[Out] ((b*c - a*d)^2*(b*e - a*f)^2*(a + b*x)^3)/(3*b^5) + ((b*c - a*d)*
(b*e - a*f)*(b*d*e + b*c*f - 2*a*d*f)*(a + b*x)^4)/(2*b^5) + ((6*
a^2*d^2*f^2 - 6*a*b*d*f*(d*e + c*f) + b^2*(d^2*e^2 + 4*c*d*e*f +
c^2*f^2))*(a + b*x)^5)/(5*b^5) + (d*f*(b*d*e + b*c*f - 2*a*d*f)*(
a + b*x)^6)/(3*b^5) + (d^2*f^2*(a + b*x)^7)/(7*b^5)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$2ace\left(acf+ade+bce\right)\int x\,dx+\frac{b^2d^2f^2x^7}{7}+\frac{bdfx^6\left(adf+bcf+bde\right)}{3}+c^2e^2\int a^2\,dx+x^5\left(\frac{a^2d^2f^2}{5}+\frac{4abcdf^2}{5}+\frac{4abd^2ef}{5}+\frac{b^2c^2f^2}{5}+\frac{4b^2cdef}{5}+\frac{b^2d^2e^2}{5}\right)+x^4\left(\frac{a^2cdf^2}{2}+\frac{a^2d^2ef}{2}+\frac{abc^2f^2}{2}+2abcdef+\frac{abd^2e^2}{2}+\frac{b^2c^2ef}{2}+\frac{b^2cde^2}{2}\right)+x^3\left(\frac{a^2c^2f^2}{3}+\frac{4a^2cdef}{3}+\frac{a^2d^2e^2}{3}+\frac{4abc^2ef}{3}+\frac{4abcde^2}{3}+\frac{b^2c^2e^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x**2+b*d*f*x**
```

```
[Out] 2*a*c*e*(a*c*f + a*d*e + b*c*e)*Integral(x, x) + b**2*d**2*f**2*x
**7/7 + b*d*f*x**6*(a*d*f + b*c*f + b*d*e)/3 + c**2*e**2*Integral
(a**2, x) + x**5*(a**2*d**2*f**2/5 + 4*a*b*c*d*f**2/5 + 4*a*b*d**
2*e*f/5 + b**2*c**2*f**2/5 + 4*b**2*c*d*e*f/5 + b**2*d**2*e**2/5)
 + x**4*(a**2*c*d*f**2/2 + a**2*d**2*e*f/2 + a*b*c**2*f**2/2 + 2*
```

```
a*b*c*d*e*f + a*b*d**2*e**2/2 + b**2*c**2*e*f/2 + b**2*c*d*e**2/2
) + x**3*(a**2*c**2*f**2/3 + 4*a**2*c*d*e*f/3 + a**2*d**2*e**2/3
+ 4*a*b*c**2*e*f/3 + 4*a*b*c*d*e**2/3 + b**2*c**2*e**2/3)
```

Mathematica [A] time = 0.146409, size = 241, normalized size = 1.25

$$\frac{1}{5}x^5\left(a^2d^2f^2+4abdf(cf+de)+b^2\left(c^2f^2+4cdef+d^2e^2\right)\right)+\frac{1}{2}x^4\left(a^2df(cf+de)+ab\left(c^2f^2+4cdef+d^2e^2\right)+b^2ce(cf+de)\right)+\frac{1}{3}x^3\left(a^2\left(c^2f^2+4cdef+d^2e^2\right)+4abce(cf+de)+b^2c^2e^2\right)+a^2c^2e^2x+\frac{1}{3}bdfx^6(adf+bcf+bde)+acex^2(acf+ade+bce)+\frac{1}{7}b^2d^2f^2x^7$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*f
```

```
[Out] a^2*c^2*e^2*x + a*c*e*(b*c*e + a*d*e + a*c*f)*x^2 + ((b^2*c^2*e^2
 + 4*a*b*c*e*(d*e + c*f) + a^2*(d^2*e^2 + 4*c*d*e*f + c^2*f^2))*x
^3)/3 + ((b^2*c*e*(d*e + c*f) + a^2*d*f*(d*e + c*f) + a*b*(d^2*e^
2 + 4*c*d*e*f + c^2*f^2))*x^4)/2 + ((a^2*d^2*f^2 + 4*a*b*d*f*(d*e
 + c*f) + b^2*(d^2*e^2 + 4*c*d*e*f + c^2*f^2))*x^5)/5 + (b*d*f*(b
*d*e + b*c*f + a*d*f)*x^6)/3 + (b^2*d^2*f^2*x^7)/7
```

Maple [A] time = 0.001, size = 188, normalized size = 1.

$$\frac{b^2d^2f^2x^7}{7}+\frac{(adf+bcf+bde)\,bdfx^6}{3}+\frac{\left(2\,(acf+ade+bce)\,bdf+(adf+bcf+bde)^2\right)x^5}{5}+\frac{(2\,acebdf+2\,(acf+ade+bce)\,(adf+bcf+bde))\,x^4}{4}+\frac{\left(2\,ace\,(adf+bcf+bde)+(acf+ade+bce)^2\right)x^3}{3}+ace\,(acf+ade+bce)\,x^2+a^2c^2e^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^2,x)
```

```
[Out] 1/7*b^2*d^2*f^2*x^7+1/3*(a*d*f+b*c*f+b*d*e)*b*d*f*x^6+1/5*(2*(a*c
*f+a*d*e+b*c*e)*b*d*f+(a*d*f+b*c*f+b*d*e)^2)*x^5+1/4*(2*a*c*e*b*d
*f+2*(a*c*f+a*d*e+b*c*e)*(a*d*f+b*c*f+b*d*e))*x^4+1/3*(2*a*c*e*(a
*d*f+b*c*f+b*d*e)+(a*c*f+a*d*e+b*c*e)^2)*x^3+a*c*e*(a*c*f+a*d*e+b
*c*e)*x^2+a^2*c^2*e^2*x
```

Maxima [A] time = 0.776301, size = 243, normalized size = 1.26

$$\frac{1}{7}\,b^2d^2f^2x^7+\frac{1}{3}\,(bde+bcf+adf)bdfx^6+a^2c^2e^2x+\frac{1}{5}\,(bde+bcf+adf)^2x^5+\frac{1}{3}\,(bce+ade+acf)^2x^3+\frac{1}{6}\left(3\,bdfx^4+4\,(bde+bcf+adf)x^3+6\,(bce+ade+acf)x^2\right)ace+\frac{1}{10}\left(4\,bdfx^5+5\,(bde+(bc+ad)f)x^4\right)(bce+ade+acf)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*f*x^3 + a*c*e + (b*d*e + b*c*f + a*d*f)*x^2 + (b*c*e + a*d*e + a*c*f
```

```
[Out] 1/7*b^2*d^2*f^2*x^7 + 1/3*(b*d*e + b*c*f + a*d*f)*b*d*f*x^6 + a^2
*c^2*e^2*x + 1/5*(b*d*e + b*c*f + a*d*f)^2*x^5 + 1/3*(b*c*e + a*d
*e + a*c*f)^2*x^3 + 1/6*(3*b*d*f*x^4 + 4*(b*d*e + b*c*f + a*d*f)*
x^3 + 6*(b*c*e + a*d*e + a*c*f)*x^2)*a*c*e + 1/10*(4*b*d*f*x^5 +
5*(b*d*e + (b*c + a*d)*f)*x^4)*(b*c*e + a*d*e + a*c*f)
```

Fricas [A] time = 0.267591, size = 1, normalized size = 0.01

$$\frac{1}{7}x^7f^2d^2b^2 + \frac{1}{3}x^6fed^2b^2 + \frac{1}{3}x^6f^2dcb^2 + \frac{1}{3}x^6f^2d^2ba + \frac{1}{5}x^5e^2d^2b^2 + \frac{4}{5}x^5fedcb^2 + \frac{1}{5}x^5f^2c^2b^2 + \frac{4}{5}x^5fed^2ba + \frac{4}{5}x^5f^2dcba + \frac{1}{5}x^5f^2d^2a^2 + \frac{1}{2}x^4e^2dcb^2 + \frac{1}{2}x^4fec^2b^2 + \frac{1}{2}x^4e^2d^2ba + 2x^4fedcba + \frac{1}{2}x^4f^2c^2ba + \frac{1}{2}x^4fed^2a^2 + \frac{1}{2}x^4f^2dca^2 + \frac{1}{3}x^3e^2c^2b^2 + \frac{4}{3}x^3e^2dcba + \frac{4}{3}x^3fec^2ba + \frac{1}{3}x^3e^2d^2a^2 + \frac{4}{3}x^3fedca^2 + \frac{1}{3}x^3f^2c^2a^2 + x^2e^2c^2ba + x^2e^2dca^2 + x^2fec^2a^2 + xe^2c^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*f*x^3 + a*c*e + (b*d*e + b*c*f + a*d*f)*x^2 + (b*c*e + a*d*e + a*c*f
```

```
[Out] 1/7*x^7*f^2*d^2*b^2 + 1/3*x^6*f*e*d^2*b^2 + 1/3*x^6*f^2*d*c*b^2 +
 1/3*x^6*f^2*d^2*b*a + 1/5*x^5*e^2*d^2*b^2 + 4/5*x^5*f*e*d*c*b^2
+ 1/5*x^5*f^2*c^2*b^2 + 4/5*x^5*f*e*d^2*b*a + 4/5*x^5*f^2*d*c*b*a
 + 1/5*x^5*f^2*d^2*a^2 + 1/2*x^4*e^2*d*c*b^2 + 1/2*x^4*f*e*c^2*b^
2 + 1/2*x^4*e^2*d^2*b*a + 2*x^4*f*e*d*c*b*a + 1/2*x^4*f^2*c^2*b*a
 + 1/2*x^4*f*e*d^2*a^2 + 1/2*x^4*f^2*d*c*a^2 + 1/3*x^3*e^2*c^2*b^
2 + 4/3*x^3*e^2*d*c*b*a + 4/3*x^3*f*e*c^2*b*a + 1/3*x^3*e^2*d^2*a
^2 + 4/3*x^3*f*e*d*c*a^2 + 1/3*x^3*f^2*c^2*a^2 + x^2*e^2*c^2*b*a
+ x^2*e^2*d*c*a^2 + x^2*f*e*c^2*a^2 + x*e^2*c^2*a^2
```

Sympy [A] time = 0.309959, size = 345, normalized size = 1.79

$$a^2c^2e^2x + \frac{b^2d^2f^2x^7}{7} + x^6\left(\frac{abd^2f^2}{3} + \frac{b^2cdf^2}{3} + \frac{b^2d^2ef}{3}\right) + x^5\left(\frac{a^2d^2f^2}{5} + \frac{4abcdf^2}{5} + \frac{4abd^2ef}{5} + \frac{b^2c^2f^2}{5} + \frac{4b^2cdef}{5} + \frac{b^2d^2e^2}{5}\right) + x^4\left(\frac{a^2cdf^2}{2} + \frac{a^2d^2ef}{2} + \frac{abc^2f^2}{2} + 2abcdef + \frac{abd^2e^2}{2} + \frac{b^2c^2ef}{2} + \frac{b^2cde^2}{2}\right) + x^3\left(\frac{a^2c^2f^2}{3} + \frac{4a^2cdef}{3} + \frac{a^2d^2e^2}{3} + \frac{4abc^2ef}{3} + \frac{4abcde^2}{3} + \frac{b^2c^2e^2}{3}\right) + x^2\left(a^2c^2ef + a^2cde^2 + abc^2e^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x**2+b*d*f*x**3)**2
```

```
[Out] a**2*c**2*e**2*x + b**2*d**2*f**2*x**7/7 + x**6*(a*b*d**2*f**2/3
+ b**2*c*d*f**2/3 + b**2*d**2*e*f/3) + x**5*(a**2*d**2*f**2/5 + 4
*a*b*c*d*f**2/5 + 4*a*b*d**2*e*f/5 + b**2*c**2*f**2/5 + 4*b**2*c*
d*e*f/5 + b**2*d**2*e**2/5) + x**4*(a**2*c*d*f**2/2 + a**2*d**2*e
*f/2 + a*b*c**2*f**2/2 + 2*a*b*c*d*e*f + a*b*d**2*e**2/2 + b**2*c
**2*e*f/2 + b**2*c*d*e**2/2) + x**3*(a**2*c**2*f**2/3 + 4*a**2*c*
d*e*f/3 + a**2*d**2*e**2/3 + 4*a*b*c**2*e*f/3 + 4*a*b*c*d*e**2/3
+ b**2*c**2*e**2/3) + x**2*(a**2*c**2*e*f + a**2*c*d*e**2 + a*b*c
**2*e**2)
```

GIAC/XCAS [A] time = 0.26279, size = 467, normalized size = 2.42

$$\frac{1}{7}\,b^2d^2f^2x^7+\frac{1}{3}\,b^2cdf^2x^6+\frac{1}{3}\,abd^2f^2x^6+\frac{1}{3}\,b^2d^2fx^6e+\frac{1}{5}\,b^2c^2f^2x^5+\frac{4}{5}\,abcdf^2x^5+\frac{1}{5}\,a^2d^2f^2x^5+\frac{4}{5}\,b^2cdfx^5e+\frac{4}{5}\,abd^2fx^5e+\frac{1}{2}\,abc^2f^2x^4+\frac{1}{2}\,a^2cdf^2x^4+\frac{1}{5}\,b^2d^2x^5e^2+\frac{1}{2}\,b^2c^2fx^4e+2\,abcdfx^4e+\frac{1}{2}\,a^2d^2fx^4e+\frac{1}{3}\,a^2c^2f^2x^3+\frac{1}{2}\,b^2cdx^4e^2+\frac{1}{2}\,abd^2x^4e^2+\frac{4}{3}\,abc^2fx^3e+\frac{4}{3}\,a^2cdfx^3e+\frac{1}{3}\,b^2c^2x^3e^2+\frac{4}{3}\,abcdx^3e^2+\frac{1}{3}\,a^2d^2x^3e^2+a^2c^2fx^2e+abc^2x^2e^2+a^2cdx^2e^2+a^2c^2xe^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*f*x^3 + a*c*e + (b*d*e + b*c*f + a*d*f)*x^2 + (b*c*e + a*d*e + a*c*f
```

```
[Out] 1/7*b^2*d^2*f^2*x^7 + 1/3*b^2*c*d*f^2*x^6 + 1/3*a*b*d^2*f^2*x^6 +
 1/3*b^2*d^2*f*x^6*e + 1/5*b^2*c^2*f^2*x^5 + 4/5*a*b*c*d*f^2*x^5
+ 1/5*a^2*d^2*f^2*x^5 + 4/5*b^2*c*d*f*x^5*e + 4/5*a*b*d^2*f*x^5*e
 + 1/2*a*b*c^2*f^2*x^4 + 1/2*a^2*c*d*f^2*x^4 + 1/5*b^2*d^2*x^5*e^
2 + 1/2*b^2*c^2*f*x^4*e + 2*a*b*c*d*f*x^4*e + 1/2*a^2*d^2*f*x^4*e
 + 1/3*a^2*c^2*f^2*x^3 + 1/2*b^2*c*d*x^4*e^2 + 1/2*a*b*d^2*x^4*e^
2 + 4/3*a*b*c^2*f*x^3*e + 4/3*a^2*c*d*f*x^3*e + 1/3*b^2*c^2*x^3*e
^2 + 4/3*a*b*c*d*x^3*e^2 + 1/3*a^2*d^2*x^3*e^2 + a^2*c^2*f*x^2*e
+ a*b*c^2*x^2*e^2 + a^2*c*d*x^2*e^2 + a^2*c^2*x*e^2
```

3.17 $\int \left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)\,dx$

Optimal. Leaf size=56

$$\frac{1}{3}x^3(adf+bcf+bde)+\frac{1}{2}x^2(acf+ade+bce)+acex+\frac{1}{4}bdfx^4$$

```
[Out] a*c*e*x + ((b*c*e + a*d*e + a*c*f)*x^2)/2 + ((b*d*e + b*c*f + a*d
*f)*x^3)/3 + (b*d*f*x^4)/4
```

Rubi [A] time = 0.0455521, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.

$$\frac{1}{3}x^3(adf+bcf+bde)+\frac{1}{2}x^2(acf+ade+bce)+acex+\frac{1}{4}bdfx^4$$

Antiderivative was successfully verified.

```
[In] Int[a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*f*x^3,x]
```

```
[Out] a*c*e*x + ((b*c*e + a*d*e + a*c*f)*x^2)/2 + ((b*d*e + b*c*f + a*d
*f)*x^3)/3 + (b*d*f*x^4)/4
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{bdfx^4}{4}+ce\int a\,dx+x^3\left(\frac{adf}{3}+\frac{bcf}{3}+\frac{bde}{3}\right)+(acf+ade+bce)\int x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x**2+b*d*f*x**3
```

```
[Out] b*d*f*x**4/4 + c*e*Integral(a, x) + x**3*(a*d*f/3 + b*c*f/3 + b*d
*e/3) + (a*c*f + a*d*e + b*c*e)*Integral(x, x)
```

Mathematica [A] time = 0.0000892753, size = 76, normalized size = 1.36

$$acex+\frac{1}{2}acfx^2+\frac{1}{2}adex^2+\frac{1}{3}adfx^3+\frac{1}{2}bcex^2+\frac{1}{3}bcfx^3+\frac{1}{3}bdex^3+\frac{1}{4}bdfx^4$$

Antiderivative was successfully verified.

```
[In] Integrate[a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*f*x
```

```
[Out] a*c*e*x + (b*c*e*x^2)/2 + (a*d*e*x^2)/2 + (a*c*f*x^2)/2 + (b*d*e*
x^3)/3 + (b*c*f*x^3)/3 + (a*d*f*x^3)/3 + (b*d*f*x^4)/4
```

Maple [A] time = 0.001, size = 51, normalized size = 0.9

$$acex+\frac{(acf+ade+bce)\,x^2}{2}+\frac{(adf+bcf+bde)\,x^3}{3}+\frac{bdfx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3,x)
```

```
[Out] a*c*e*x+1/2*(a*c*f+a*d*e+b*c*e)*x^2+1/3*(a*d*f+b*c*f+b*d*e)*x^3+1
/4*b*d*f*x^4
```

Maxima [A] time = 0.76833, size = 68, normalized size = 1.21

$$\frac{1}{4}\,bdfx^4 + acex + \frac{1}{3}\,(bde + bcf + adf)x^3 + \frac{1}{2}\,(bce + ade + acf)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*d*f*x^3 + a*c*e + (b*d*e + b*c*f + a*d*f)*x^2 + (b*c*e + a*d*e + a*c*f)
```

```
[Out] 1/4*b*d*f*x^4 + a*c*e*x + 1/3*(b*d*e + b*c*f + a*d*f)*x^3 + 1/2*(
b*c*e + a*d*e + a*c*f)*x^2
```

Fricas [A] time = 0.278045, size = 1, normalized size = 0.02

$$\frac{1}{4}x^4fdb + \frac{1}{3}x^3edb + \frac{1}{3}x^3fcb + \frac{1}{3}x^3fda + \frac{1}{2}x^2ecb + \frac{1}{2}x^2eda + \frac{1}{2}x^2fca + xeca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*d*f*x^3 + a*c*e + (b*d*e + b*c*f + a*d*f)*x^2 + (b*c*e + a*d*e + a*c*f)
```

```
[Out] 1/4*x^4*f*d*b + 1/3*x^3*e*d*b + 1/3*x^3*f*c*b + 1/3*x^3*f*d*a + 1
/2*x^2*e*c*b + 1/2*x^2*e*d*a + 1/2*x^2*f*c*a + x*e*c*a
```

Sympy [A] time = 0.109789, size = 63, normalized size = 1.12

$$acex + \frac{bdfx^4}{4} + x^3\left(\frac{adf}{3} + \frac{bcf}{3} + \frac{bde}{3}\right) + x^2\left(\frac{acf}{2} + \frac{ade}{2} + \frac{bce}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x**2+b*d*f*x**3,x)
```

```
[Out] a*c*e*x + b*d*f*x**4/4 + x**3*(a*d*f/3 + b*c*f/3 + b*d*e/3) + x**
2*(a*c*f/2 + a*d*e/2 + b*c*e/2)
```

GIAC/XCAS [A] time = 0.262889, size = 73, normalized size = 1.3

$$\frac{1}{4}\,bdfx^4 + \frac{1}{3}\,(bcf + adf + bde)x^3 + acxe + \frac{1}{2}\,(acf + bce + ade)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*d*f*x^3 + a*c*e + (b*d*e + b*c*f + a*d*f)*x^2 + (b*c*e + a*d*e + a*c*f)
```

```
[Out] 1/4*b*d*f*x^4 + 1/3*(b*c*f + a*d*f + b*d*e)*x^3 + a*c*x*e + 1/2*(
a*c*f + b*c*e + a*d*e)*x^2
```

3.18 $\int \frac{1}{ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3}\,dx$

Optimal. Leaf size=86

$$\frac{b\log(a+bx)}{(bc-ad)(be-af)}-\frac{d\log(c+dx)}{(bc-ad)(de-cf)}+\frac{f\log(e+fx)}{(be-af)(de-cf)}$$

```
[Out] (b*Log[a + b*x])/((b*c - a*d)*(b*e - a*f)) - (d*Log[c + d*x])/((b
*c - a*d)*(d*e - c*f)) + (f*Log[e + f*x])/((b*e - a*f)*(d*e - c*f
))
```

Rubi [A] time = 0.146877, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.022

$$\frac{b\log(a+bx)}{(bc-ad)(be-af)}-\frac{d\log(c+dx)}{(bc-ad)(de-cf)}+\frac{f\log(e+fx)}{(be-af)(de-cf)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*f*x^3)^(
1),x]
```

```
[Out] (b*Log[a + b*x])/((b*c - a*d)*(b*e - a*f)) - (d*Log[c + d*x])/((b
*c - a*d)*(d*e - c*f)) + (f*Log[e + f*x])/((b*e - a*f)*(d*e - c*f
))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x**2+b*d*f*x
```

```
[Out] Timed out
```

Mathematica [A] time = 0.0879275, size = 80, normalized size = 0.93

$$\frac{b\log(a+bx)(cf-de)+d(be-af)\log(c+dx)+f(ad-bc)\log(e+fx)}{(bc-ad)(be-af)(cf-de)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*f*
1),x]
```

```
[Out] (b*(-(d*e) + c*f)*Log[a + b*x] + d*(b*e - a*f)*Log[c + d*x] + (-(
b*c) + a*d)*f*Log[e + f*x])/((b*c - a*d)*(b*e - a*f)*(-(d*e) + c*
f))
```

Maple [A] time = 0.012, size = 87, normalized size = 1.

$$\frac{f\ln(fx+e)}{(cf-de)(fa-be)}-\frac{d\ln(dx+c)}{(cf-de)(ad-bc)}+\frac{b\ln(bx+a)}{(fa-be)(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3),x)
```

```
[Out] f/(c*f-d*e)/(a*f-b*e)*ln(f*x+e)-d/(c*f-d*e)/(a*d-b*c)*ln(d*x+c)+b
/(a*f-b*e)/(a*d-b*c)*ln(b*x+a)
```

Maxima [A] time = 0.783575, size = 151, normalized size = 1.76

$$\frac{b\log\left(bx+a\right)}{(b^2c-abd)e-(abc-a^2d)f}-\frac{d\log\left(dx+c\right)}{(bcd-ad^2)e-(bc^2-acd)f}+\frac{f\log\left(fx+e\right)}{bde^2+acf^2-(bc+ad)ef}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*d*f*x^3 + a*c*e + (b*d*e + b*c*f + a*d*f)*x^2 + (b*c*e + a*d*e + a*c
```

```
[Out] b*log(b*x + a)/((b^2*c - a*b*d)*e - (a*b*c - a^2*d)*f) - d*log(d*
x + c)/((b*c*d - a*d^2)*e - (b*c^2 - a*c*d)*f) + f*log(f*x + e)/(
b*d*e^2 + a*c*f^2 - (b*c + a*d)*e*f)
```

Fricas [A] time = 5.57496, size = 151, normalized size = 1.76

$$\frac{(bc-ad)f\log\left(fx+e\right)+(bde-bcf)\log\left(bx+a\right)-(bde-adf)\log\left(dx+c\right)}{(b^2cd-abd^2)e^2-(b^2c^2-a^2d^2)ef+(abc^2-a^2cd)f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*d*f*x^3 + a*c*e + (b*d*e + b*c*f + a*d*f)*x^2 + (b*c*e + a*d*e + a*c
```

```
[Out] ((b*c - a*d)*f*log(f*x + e) + (b*d*e - b*c*f)*log(b*x + a) - (b*d
*e - a*d*f)*log(d*x + c))/((b^2*c*d - a*b*d^2)*e^2 - (b^2*c^2 - a
^2*d^2)*e*f + (a*b*c^2 - a^2*c*d)*f^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x**2+b*d*f*x**3),
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{bdfx^3+ace+(bde+bcf+adf)x^2+(bce+ade+acf)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*d*f*x^3 + a*c*e + (b*d*e + b*c*f + a*d*f)*x^2 + (b*c*e + a*d*e + a*c
```

```
[Out] integrate(1/(b*d*f*x^3 + a*c*e + (b*d*e + b*c*f + a*d*f)*x^2 + (b
*c*e + a*d*e + a*c*f)*x), x)
```

3.19 $\int \frac{1}{(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3)^2}\,dx$

Optimal. Leaf size=234

$$-\frac{b^3}{(a+bx)(bc-ad)^2(be-af)^2}-\frac{2b^3\log(a+bx)(-2adf+bcf+bde)}{(bc-ad)^3(be-af)^3}-\frac{d^3}{(c+dx)(bc-ad)^2(de-cf)^2}+\frac{2d^3\log(c+dx)(adf-2bcf+bde)}{(bc-ad)^3(de-cf)^3}-\frac{f^3}{(e+fx)(be-af)^2(de-cf)^2}+\frac{2f^3\log(e+fx)(-adf-bcf+2bde)}{(be-af)^3(de-cf)^3}$$

```
[Out] -(b^3/((b*c - a*d)^2*(b*e - a*f)^2*(a + b*x))) - d^3/((b*c - a*d)
^2*(d*e - c*f)^2*(c + d*x)) - f^3/((b*e - a*f)^2*(d*e - c*f)^2*(e
 + f*x)) - (2*b^3*(b*d*e + b*c*f - 2*a*d*f)*Log[a + b*x])/((b*c -
 a*d)^3*(b*e - a*f)^3) + (2*d^3*(b*d*e - 2*b*c*f + a*d*f)*Log[c +
 d*x])/((b*c - a*d)^3*(d*e - c*f)^3) + (2*f^3*(2*b*d*e - b*c*f -
a*d*f)*Log[e + f*x])/((b*e - a*f)^3*(d*e - c*f)^3)
```

Rubi [A] time = 0.888164, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.022$

$$-\frac{b^3}{(a+bx)(bc-ad)^2(be-af)^2}-\frac{2b^3\log(a+bx)(-2adf+bcf+bde)}{(bc-ad)^3(be-af)^3}-\frac{d^3}{(c+dx)(bc-ad)^2(de-cf)^2}+\frac{2d^3\log(c+dx)(adf-2bcf+bde)}{(bc-ad)^3(de-cf)^3}-\frac{f^3}{(e+fx)(be-af)^2(de-cf)^2}+\frac{2f^3\log(e+fx)(-adf-bcf+2bde)}{(be-af)^3(de-cf)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*f*x^3)^(
2),x]
```

```
[Out] -(b^3/((b*c - a*d)^2*(b*e - a*f)^2*(a + b*x))) - d^3/((b*c - a*d)
^2*(d*e - c*f)^2*(c + d*x)) - f^3/((b*e - a*f)^2*(d*e - c*f)^2*(e
 + f*x)) - (2*b^3*(b*d*e + b*c*f - 2*a*d*f)*Log[a + b*x])/((b*c -
 a*d)^3*(b*e - a*f)^3) + (2*d^3*(b*d*e - 2*b*c*f + a*d*f)*Log[c +
 d*x])/((b*c - a*d)^3*(d*e - c*f)^3) + (2*f^3*(2*b*d*e - b*c*f -
a*d*f)*Log[e + f*x])/((b*e - a*f)^3*(d*e - c*f)^3)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x**2+b*d*f*x
```

```
[Out] Timed out
```

Mathematica [A] time = 1.33016, size = 232, normalized size = 0.99

$$-\frac{b^3}{(a+bx)(bc-ad)^2(be-af)^2}-\frac{2b^3\log(a+bx)(-2adf+bcf+bde)}{(bc-ad)^3(be-af)^3}-\frac{d^3}{(c+dx)(bc-ad)^2(de-cf)^2}-\frac{2d^3\log(c+dx)(adf-2bcf+bde)}{(bc-ad)^3(cf-de)^3}-\frac{f^3}{(e+fx)(be-af)^2(de-cf)^2}-\frac{2f^3\log(e+fx)(adf+bcf-2bde)}{(be-af)^3(de-cf)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*f*
2),x]
```

```
[Out] -(b^3/((b*c - a*d)^2*(b*e - a*f)^2*(a + b*x))) - d^3/((b*c - a*d)
^2*(d*e - c*f)^2*(c + d*x)) - f^3/((b*e - a*f)^2*(d*e - c*f)^2*(e
 + f*x)) - (2*b^3*(b*d*e + b*c*f - 2*a*d*f)*Log[a + b*x])/((b*c -
 a*d)^3*(b*e - a*f)^3) - (2*d^3*(b*d*e - 2*b*c*f + a*d*f)*Log[c +
 d*x])/((b*c - a*d)^3*(-(d*e) + c*f)^3) - (2*f^3*(-2*b*d*e + b*c*
f + a*d*f)*Log[e + f*x])/((b*e - a*f)^3*(d*e - c*f)^3)
```

Maple [A] time = 0.066, size = 398, normalized size = 1.7

$$-\frac{f^3}{(cf-de)^2(fa-be)^2(fx+e)}-2\frac{f^4\ln(fx+e)ad}{(cf-de)^3(fa-be)^3}-2\frac{f^4\ln(fx+e)bc}{(cf-de)^3(fa-be)^3}+4\frac{f^3\ln(fx+e)bde}{(cf-de)^3(fa-be)^3}-\frac{d^3}{(cf-de)^2(ad-bc)^2(dx+c)}+2\frac{d^4\ln(dx+c)af}{(cf-de)^3(ad-bc)^3}-4\frac{d^3\ln(dx+c)bcf}{(cf-de)^3(ad-bc)^3}+2\frac{d^4\ln(dx+c)be}{(cf-de)^3(ad-bc)^3}-\frac{b^3}{(fa-be)^2(ad-bc)^2(bx+a)}+4\frac{b^3\ln(bx+a)adf}{(fa-be)^3(ad-bc)^3}-2\frac{b^4\ln(bx+a)cf}{(fa-be)^3(ad-bc)^3}-2\frac{b^4\ln(bx+a)de}{(fa-be)^3(ad-bc)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^2,x)
```

```
[Out] -f^3/(c*f-d*e)^2/(a*f-b*e)^2/(f*x+e)-2*f^4/(c*f-d*e)^3/(a*f-b*e)^
3*ln(f*x+e)*a*d-2*f^4/(c*f-d*e)^3/(a*f-b*e)^3*ln(f*x+e)*b*c+4*f^3
/(c*f-d*e)^3/(a*f-b*e)^3*ln(f*x+e)*b*d*e-d^3/(c*f-d*e)^2/(a*d-b*c
)^2/(d*x+c)+2*d^4/(c*f-d*e)^3/(a*d-b*c)^3*ln(d*x+c)*a*f-4*d^3/(c*
f-d*e)^3/(a*d-b*c)^3*ln(d*x+c)*b*c*f+2*d^4/(c*f-d*e)^3/(a*d-b*c)^
3*ln(d*x+c)*b*e-b^3/(a*f-b*e)^2/(a*d-b*c)^2/(b*x+a)+4*b^3/(a*f-b*
e)^3/(a*d-b*c)^3*ln(b*x+a)*a*d*f-2*b^4/(a*f-b*e)^3/(a*d-b*c)^3*ln
(b*x+a)*c*f-2*b^4/(a*f-b*e)^3/(a*d-b*c)^3*ln(b*x+a)*d*e
```

Maxima [A] time = 0.86502, size = 2830, normalized size = 12.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*f*x^3 + a*c*e + (b*d*e + b*c*f + a*d*f)*x^2 + (b*c*e + a*d*e + a*c*f
2),x, algorithm="maxima")
```

```
[Out] -2*(b^4*d*e + (b^4*c - 2*a*b^3*d)*f)*log(b*x + a)/((b^6*c^3 - 3*a
*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*e^3 - 3*(a*b^5*c^3 -
3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 - a^4*b^2*d^3)*e^2*f + 3*(a^2*b
```

```
^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 - a^5*b*d^3)*e*f^2 - (
a^3*b^3*c^3 - 3*a^4*b^2*c^2*d + 3*a^5*b*c*d^2 - a^6*d^3)*f^3) + 2
*(b*d^4*e - (2*b*c*d^3 - a*d^4)*f)*log(d*x + c)/((b^3*c^3*d^3 - 3
*a*b^2*c^2*d^4 + 3*a^2*b*c*d^5 - a^3*d^6)*e^3 - 3*(b^3*c^4*d^2 -
3*a*b^2*c^3*d^3 + 3*a^2*b*c^2*d^4 - a^3*c*d^5)*e^2*f + 3*(b^3*c^5
*d - 3*a*b^2*c^4*d^2 + 3*a^2*b*c^3*d^3 - a^3*c^2*d^4)*e*f^2 - (b^
3*c^6 - 3*a*b^2*c^5*d + 3*a^2*b*c^4*d^2 - a^3*c^3*d^3)*f^3) + 2*(
2*b*d*e*f^3 - (b*c + a*d)*f^4)*log(f*x + e)/(b^3*d^3*e^6 + a^3*c^
3*f^6 - 3*(b^3*c*d^2 + a*b^2*d^3)*e^5*f + 3*(b^3*c^2*d + 3*a*b^2*
c*d^2 + a^2*b*d^3)*e^4*f^2 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c
*d^2 + a^3*d^3)*e^3*f^3 + 3*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^
2)*e^2*f^4 - 3*(a^2*b*c^3 + a^3*c^2*d)*e*f^5) - ((b^3*c*d^2 + a*b
^2*d^3)*e^3 - 2*(b^3*c^2*d + a^2*b*d^3)*e^2*f + (b^3*c^3 + a^3*d^
3)*e*f^2 + (a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2)*f^3 + 2*(b^3*d
^3*e^2*f - (b^3*c*d^2 + a*b^2*d^3)*e*f^2 + (b^3*c^2*d - a*b^2*c*d
^2 + a^2*b*d^3)*f^3)*x^2 + (2*b^3*d^3*e^3 - (b^3*c*d^2 + a*b^2*d^
3)*e^2*f - (b^3*c^2*d + a^2*b*d^3)*e*f^2 + (2*b^3*c^3 - a*b^2*c^2
*d - a^2*b*c*d^2 + 2*a^3*d^3)*f^3)*x)/((a*b^4*c^3*d^2 - 2*a^2*b^3
*c^2*d^3 + a^3*b^2*c*d^4)*e^5 - 2*(a*b^4*c^4*d - a^2*b^3*c^3*d^2
- a^3*b^2*c^2*d^3 + a^4*b*c*d^4)*e^4*f + (a*b^4*c^5 + 2*a^2*b^3*c
^4*d - 6*a^3*b^2*c^3*d^2 + 2*a^4*b*c^2*d^3 + a^5*c*d^4)*e^3*f^2 -
 2*(a^2*b^3*c^5 - a^3*b^2*c^4*d - a^4*b*c^3*d^2 + a^5*c^2*d^3)*e^
2*f^3 + (a^3*b^2*c^5 - 2*a^4*b*c^4*d + a^5*c^3*d^2)*e*f^4 + ((b^5
*c^2*d^3 - 2*a*b^4*c*d^4 + a^2*b^3*d^5)*e^4*f - 2*(b^5*c^3*d^2 -
a*b^4*c^2*d^3 - a^2*b^3*c*d^4 + a^3*b^2*d^5)*e^3*f^2 + (b^5*c^4*d
 + 2*a*b^4*c^3*d^2 - 6*a^2*b^3*c^2*d^3 + 2*a^3*b^2*c*d^4 + a^4*b*
d^5)*e^2*f^3 - 2*(a*b^4*c^4*d - a^2*b^3*c^3*d^2 - a^3*b^2*c^2*d^3
 + a^4*b*c*d^4)*e*f^4 + (a^2*b^3*c^4*d - 2*a^3*b^2*c^3*d^2 + a^4*
b*c^2*d^3)*f^5)*x^3 + ((b^5*c^2*d^3 - 2*a*b^4*c*d^4 + a^2*b^3*d^5
)*e^5 - (b^5*c^3*d^2 - a*b^4*c^2*d^3 - a^2*b^3*c*d^4 + a^3*b^2*d^
5)*e^4*f - (b^5*c^4*d - 2*a*b^4*c^3*d^2 + 2*a^2*b^3*c^2*d^3 - 2*a
^3*b^2*c*d^4 + a^4*b*d^5)*e^3*f^2 + (b^5*c^5 + a*b^4*c^4*d - 2*a^
2*b^3*c^3*d^2 - 2*a^3*b^2*c^2*d^3 + a^4*b*c*d^4 + a^5*d^5)*e^2*f^
3 - (2*a*b^4*c^5 - a^2*b^3*c^4*d - 2*a^3*b^2*c^3*d^2 - a^4*b*c^2*
d^3 + 2*a^5*c*d^4)*e*f^4 + (a^2*b^3*c^5 - a^3*b^2*c^4*d - a^4*b*c
^3*d^2 + a^5*c^2*d^3)*f^5)*x^2 + ((b^5*c^3*d^2 - a*b^4*c^2*d^3 -
a^2*b^3*c*d^4 + a^3*b^2*d^5)*e^5 - (2*b^5*c^4*d - a*b^4*c^3*d^2 -
 2*a^2*b^3*c^2*d^3 - a^3*b^2*c*d^4 + 2*a^4*b*d^5)*e^4*f + (b^5*c^
5 + a*b^4*c^4*d - 2*a^2*b^3*c^3*d^2 - 2*a^3*b^2*c^2*d^3 + a^4*b*c
*d^4 + a^5*d^5)*e^3*f^2 - (a*b^4*c^5 - 2*a^2*b^3*c^4*d + 2*a^3*b^
2*c^3*d^2 - 2*a^4*b*c^2*d^3 + a^5*c*d^4)*e^2*f^3 - (a^2*b^3*c^5 -
 a^3*b^2*c^4*d - a^4*b*c^3*d^2 + a^5*c^2*d^3)*e*f^4 + (a^3*b^2*c^
5 - 2*a^4*b*c^4*d + a^5*c^3*d^2)*f^5)*x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*f*x^3 + a*c*e + (b*d*e + b*c*f + a*d*f)*x^2 + (b*c*e + a*d*e + a*c*f
2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x**2+b*d*f*x**3)*
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.280415, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*f*x^3 + a*c*e + (b*d*e + b*c*f + a*d*f)*x^2 + (b*c*e + a*d*e + a*c*f
2),x, algorithm="giac")
```

```
[Out] Done
```

3.20 $\int \frac{1}{(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3)^3}\,dx$

Optimal. Leaf size=495

$$\frac{3f^5\log(e+fx)\left(2a^2d^2f^2-abdf(7de-3cf)+b^2\left(2c^2f^2-7cdef+7d^2e^2\right)\right)}{(be-af)^5(de-cf)^5} - \frac{3d^5\log(c+dx)\left(2a^2d^2f^2+abdf(3de-7cf)+b^2\left(7c^2f^2-7cdef+2d^2e^2\right)\right)}{(bc-ad)^5(de-cf)^5} + \frac{3b^5\log(a+bx)\left(7a^2d^2f^2-7abdf(cf+de)+b^2\left(2c^2f^2+3cdef+2d^2e^2\right)\right)}{(bc-ad)^5(be-af)^5} + \frac{3b^5(-2adf+bcf+bde)}{(a+bx)(bc-ad)^4(be-af)^4} - \frac{b^5}{2(a+bx)^2(bc-ad)^3(be-af)^3} + \frac{3d^5(adf-2bcf+bde)}{(c+dx)(bc-ad)^4(de-cf)^4} + \frac{d^5}{2(c+dx)^2(bc-ad)^3(de-cf)^3} - \frac{3f^5(-adf-bcf+2bde)}{(e+fx)(be-af)^4(de-cf)^4} - \frac{f^5}{2(e+fx)^2(be-af)^3(de-cf)^3}$$

```
[Out] -b^5/(2*(b*c - a*d)^3*(b*e - a*f)^3*(a + b*x)^2) + (3*b^5*(b*d*e
+ b*c*f - 2*a*d*f))/((b*c - a*d)^4*(b*e - a*f)^4*(a + b*x)) + d^5
/(2*(b*c - a*d)^3*(d*e - c*f)^3*(c + d*x)^2) + (3*d^5*(b*d*e - 2*
b*c*f + a*d*f))/((b*c - a*d)^4*(d*e - c*f)^4*(c + d*x)) - f^5/(2*
(b*e - a*f)^3*(d*e - c*f)^3*(e + f*x)^2) - (3*f^5*(2*b*d*e - b*c*
f - a*d*f))/((b*e - a*f)^4*(d*e - c*f)^4*(e + f*x)) + (3*b^5*(7*a
^2*d^2*f^2 - 7*a*b*d*f*(d*e + c*f) + b^2*(2*d^2*e^2 + 3*c*d*e*f +
 2*c^2*f^2))*Log[a + b*x])/((b*c - a*d)^5*(b*e - a*f)^5) - (3*d^5
*(2*a^2*d^2*f^2 + a*b*d*f*(3*d*e - 7*c*f) + b^2*(2*d^2*e^2 - 7*c*
d*e*f + 7*c^2*f^2))*Log[c + d*x])/((b*c - a*d)^5*(d*e - c*f)^5) +
 (3*f^5*(2*a^2*d^2*f^2 - a*b*d*f*(7*d*e - 3*c*f) + b^2*(7*d^2*e^2
 - 7*c*d*e*f + 2*c^2*f^2))*Log[e + f*x])/((b*e - a*f)^5*(d*e - c*
f)^5)
```

Rubi [A] time = 3.58718, antiderivative size = 495, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.022$

$$\frac{3f^5\log(e+fx)\left(2a^2d^2f^2-abdf(7de-3cf)+b^2\left(2c^2f^2-7cdef+7d^2e^2\right)\right)}{(be-af)^5(de-cf)^5} - \frac{3d^5\log(c+dx)\left(2a^2d^2f^2+abdf(3de-7cf)+b^2\left(7c^2f^2-7cdef+2d^2e^2\right)\right)}{(bc-ad)^5(de-cf)^5} + \frac{3b^5\log(a+bx)\left(7a^2d^2f^2-7abdf(cf+de)+b^2\left(2c^2f^2+3cdef+2d^2e^2\right)\right)}{(bc-ad)^5(be-af)^5} + \frac{3b^5(-2adf+bcf+bde)}{(a+bx)(bc-ad)^4(be-af)^4} - \frac{b^5}{2(a+bx)^2(bc-ad)^3(be-af)^3} + \frac{3d^5(adf-2bcf+bde)}{(c+dx)(bc-ad)^4(de-cf)^4} + \frac{d^5}{2(c+dx)^2(bc-ad)^3(de-cf)^3} - \frac{3f^5(-adf-bcf+2bde)}{(e+fx)(be-af)^4(de-cf)^4} - \frac{f^5}{2(e+fx)^2(be-af)^3(de-cf)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*f*x^3)^(
3),x]
```

```
[Out] -b^5/(2*(b*c - a*d)^3*(b*e - a*f)^3*(a + b*x)^2) + (3*b^5*(b*d*e
+ b*c*f - 2*a*d*f))/((b*c - a*d)^4*(b*e - a*f)^4*(a + b*x)) + d^5
/(2*(b*c - a*d)^3*(d*e - c*f)^3*(c + d*x)^2) + (3*d^5*(b*d*e - 2*
b*c*f + a*d*f))/((b*c - a*d)^4*(d*e - c*f)^4*(c + d*x)) - f^5/(2*
(b*e - a*f)^3*(d*e - c*f)^3*(e + f*x)^2) - (3*f^5*(2*b*d*e - b*c*
f - a*d*f))/((b*e - a*f)^4*(d*e - c*f)^4*(e + f*x)) + (3*b^5*(7*a
^2*d^2*f^2 - 7*a*b*d*f*(d*e + c*f) + b^2*(2*d^2*e^2 + 3*c*d*e*f +
```

```
 2*c^2*f^2))*Log[a + b*x])/((b*c - a*d)^5*(b*e - a*f)^5) - (3*d^5
*(2*a^2*d^2*f^2 + a*b*d*f*(3*d*e - 7*c*f) + b^2*(2*d^2*e^2 - 7*c*
d*e*f + 7*c^2*f^2))*Log[c + d*x])/((b*c - a*d)^5*(d*e - c*f)^5) +
 (3*f^5*(2*a^2*d^2*f^2 - a*b*d*f*(7*d*e - 3*c*f) + b^2*(7*d^2*e^2
 - 7*c*d*e*f + 2*c^2*f^2))*Log[e + f*x])/((b*e - a*f)^5*(d*e - c*
f)^5)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x**2+b*d*f*x
```

```
[Out] Timed out
```

Mathematica [A] time = 2.8311, size = 490, normalized size = 0.99

$$\frac{1}{2}\left(\frac{6f^5\log(e+fx)\left(2a^2d^2f^2+abdf(3cf-7de)+b^2\left(2c^2f^2-7cdef+7d^2e^2\right)\right)}{(be-af)^5(de-cf)^5}+\frac{6d^5\log(c+dx)\left(2a^2d^2f^2+abdf(3de-7cf)+b^2\left(7c^2f^2-7cdef+2d^2e^2\right)\right)}{(bc-ad)^5(cf-de)^5}+\frac{6b^5\log(a+bx)\left(7a^2d^2f^2-7abdf(cf+de)+b^2\left(2c^2f^2+3cdef+2d^2e^2\right)\right)}{(bc-ad)^5(be-af)^5}+\frac{6b^5(-2adf+bcf+bde)}{(a+bx)(bc-ad)^4(be-af)^4}-\frac{b^5}{(a+bx)^2(bc-ad)^3(be-af)^3}+\frac{6d^5(adf-2bcf+bde)}{(c+dx)(bc-ad)^4(de-cf)^4}-\frac{d^5}{(c+dx)^2(bc-ad)^3(cf-de)^3}+\frac{6f^5(adf+bcf-2bde)}{(e+fx)(be-af)^4(de-cf)^4}-\frac{f^5}{(e+fx)^2(be-af)^3(de-cf)^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*f*
3),x]
```

```
[Out] (-(b^5/((b*c - a*d)^3*(b*e - a*f)^3*(a + b*x)^2)) + (6*b^5*(b*d*e
 + b*c*f - 2*a*d*f))/((b*c - a*d)^4*(b*e - a*f)^4*(a + b*x)) - d^
5/((b*c - a*d)^3*(-(d*e) + c*f)^3*(c + d*x)^2) + (6*d^5*(b*d*e -
2*b*c*f + a*d*f))/((b*c - a*d)^4*(d*e - c*f)^4*(c + d*x)) - f^5/(
(b*e - a*f)^3*(d*e - c*f)^3*(e + f*x)^2) + (6*f^5*(-2*b*d*e + b*c
*f + a*d*f))/((b*e - a*f)^4*(d*e - c*f)^4*(e + f*x)) + (6*b^5*(7*
a^2*d^2*f^2 - 7*a*b*d*f*(d*e + c*f) + b^2*(2*d^2*e^2 + 3*c*d*e*f
+ 2*c^2*f^2))*Log[a + b*x])/((b*c - a*d)^5*(b*e - a*f)^5) + (6*d^
5*(2*a^2*d^2*f^2 + a*b*d*f*(3*d*e - 7*c*f) + b^2*(2*d^2*e^2 - 7*c
*d*e*f + 7*c^2*f^2))*Log[c + d*x])/((b*c - a*d)^5*(-(d*e) + c*f)^
5) + (6*f^5*(2*a^2*d^2*f^2 + a*b*d*f*(-7*d*e + 3*c*f) + b^2*(7*d^
2*e^2 - 7*c*d*e*f + 2*c^2*f^2))*Log[e + f*x])/((b*e - a*f)^5*(d*e
 - c*f)^5))/2
```

Maple [B] time = 0.047, size = 1076, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^3,x)
```

```
[Out] -21*b^6/(a*f-b*e)^5/(a*d-b*c)^5*ln(b*x+a)*a*c*d*f^2+21*d^6/(c*f-d
*e)^5/(a*d-b*c)^5*ln(d*x+c)*a*b*c*f^2+9*f^7/(c*f-d*e)^5/(a*f-b*e)
^5*ln(f*x+e)*a*b*c*d-9*d^7/(c*f-d*e)^5/(a*d-b*c)^5*ln(d*x+c)*a*b*
e*f-6*f^5/(c*f-d*e)^4/(a*f-b*e)^4/(f*x+e)*b*d*e+21*f^5/(c*f-d*e)^
5/(a*f-b*e)^5*ln(f*x+e)*b^2*d^2*e^2-6*d^5/(c*f-d*e)^4/(a*d-b*c)^4
/(d*x+c)*b*c*f-21*d^5/(c*f-d*e)^5/(a*d-b*c)^5*ln(d*x+c)*b^2*f^2*c
^2-6*b^5/(a*f-b*e)^4/(a*d-b*c)^4/(b*x+a)*a*d*f+21*b^5/(a*f-b*e)^5
/(a*d-b*c)^5*ln(b*x+a)*a^2*d^2*f^2+6*b^7/(a*f-b*e)^5/(a*d-b*c)^5*
ln(b*x+a)*f^2*c^2+6*b^7/(a*f-b*e)^5/(a*d-b*c)^5*ln(b*x+a)*d^2*e^2
+6*f^7/(c*f-d*e)^5/(a*f-b*e)^5*ln(f*x+e)*a^2*d^2+6*f^7/(c*f-d*e)^
5/(a*f-b*e)^5*ln(f*x+e)*b^2*c^2+3*d^6/(c*f-d*e)^4/(a*d-b*c)^4/(d*
x+c)*a*f+3*d^6/(c*f-d*e)^4/(a*d-b*c)^4/(d*x+c)*b*e-6*d^7/(c*f-d*e
)^5/(a*d-b*c)^5*ln(d*x+c)*a^2*f^2-6*d^7/(c*f-d*e)^5/(a*d-b*c)^5*l
n(d*x+c)*b^2*e^2+3*b^6/(a*f-b*e)^4/(a*d-b*c)^4/(b*x+a)*c*f+3*b^6/
(a*f-b*e)^4/(a*d-b*c)^4/(b*x+a)*d*e+3*f^6/(c*f-d*e)^4/(a*f-b*e)^4
/(f*x+e)*a*d+21*d^6/(c*f-d*e)^5/(a*d-b*c)^5*ln(d*x+c)*b^2*c*e*f+9
*b^7/(a*f-b*e)^5/(a*d-b*c)^5*ln(b*x+a)*c*d*e*f+3*f^6/(c*f-d*e)^4/
(a*f-b*e)^4/(f*x+e)*b*c-21*f^6/(c*f-d*e)^5/(a*f-b*e)^5*ln(f*x+e)*
b^2*c*d*e-21*b^6/(a*f-b*e)^5/(a*d-b*c)^5*ln(b*x+a)*a*d^2*e*f-21*f
^6/(c*f-d*e)^5/(a*f-b*e)^5*ln(f*x+e)*a*b*d^2*e-1/2*f^5/(c*f-d*e)^
3/(a*f-b*e)^3/(f*x+e)^2-1/2*b^5/(a*f-b*e)^3/(a*d-b*c)^3/(b*x+a)^2
+1/2*d^5/(c*f-d*e)^3/(a*d-b*c)^3/(d*x+c)^2
```

<u>Maxima</u> [A] time = 1.38902, size = 14857, normalized size = 30.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*f*x^3 + a*c*e + (b*d*e + b*c*f + a*d*f)*x^2 + (b*c*e + a*d*e + a*c*f
3),x, algorithm="maxima")
```

```
[Out] 3*(2*b^7*d^2*e^2 + (3*b^7*c*d - 7*a*b^6*d^2)*e*f + (2*b^7*c^2 - 7
*a*b^6*c*d + 7*a^2*b^5*d^2)*f^2)*log(b*x + a)/((b^10*c^5 - 5*a*b^
9*c^4*d + 10*a^2*b^8*c^3*d^2 - 10*a^3*b^7*c^2*d^3 + 5*a^4*b^6*c*d
^4 - a^5*b^5*d^5)*e^5 - 5*(a*b^9*c^5 - 5*a^2*b^8*c^4*d + 10*a^3*b
^7*c^3*d^2 - 10*a^4*b^6*c^2*d^3 + 5*a^5*b^5*c*d^4 - a^6*b^4*d^5)*
e^4*f + 10*(a^2*b^8*c^5 - 5*a^3*b^7*c^4*d + 10*a^4*b^6*c^3*d^2 -
10*a^5*b^5*c^2*d^3 + 5*a^6*b^4*c*d^4 - a^7*b^3*d^5)*e^3*f^2 - 10*
(a^3*b^7*c^5 - 5*a^4*b^6*c^4*d + 10*a^5*b^5*c^3*d^2 - 10*a^6*b^4*
c^2*d^3 + 5*a^7*b^3*c*d^4 - a^8*b^2*d^5)*e^2*f^3 + 5*(a^4*b^6*c^5
 - 5*a^5*b^5*c^4*d + 10*a^6*b^4*c^3*d^2 - 10*a^7*b^3*c^2*d^3 + 5*
a^8*b^2*c*d^4 - a^9*b*d^5)*e*f^4 - (a^5*b^5*c^5 - 5*a^6*b^4*c^4*d
 + 10*a^7*b^3*c^3*d^2 - 10*a^8*b^2*c^2*d^3 + 5*a^9*b*c*d^4 - a^10
*d^5)*f^5) - 3*(2*b^2*d^7*e^2 - (7*b^2*c*d^6 - 3*a*b*d^7)*e*f + (
7*b^2*c^2*d^5 - 7*a*b*c*d^6 + 2*a^2*d^7)*f^2)*log(d*x + c)/((b^5*
c^5*d^5 - 5*a*b^4*c^4*d^6 + 10*a^2*b^3*c^3*d^7 - 10*a^3*b^2*c^2*d
^8 + 5*a^4*b*c*d^9 - a^5*d^10)*e^5 - 5*(b^5*c^6*d^4 - 5*a*b^4*c^5
*d^5 + 10*a^2*b^3*c^4*d^6 - 10*a^3*b^2*c^3*d^7 + 5*a^4*b*c^2*d^8
- a^5*c*d^9)*e^4*f + 10*(b^5*c^7*d^3 - 5*a*b^4*c^6*d^4 + 10*a^2*b
^3*c^5*d^5 - 10*a^3*b^2*c^4*d^6 + 5*a^4*b*c^3*d^7 - a^5*c^2*d^8)*
e^3*f^2 - 10*(b^5*c^8*d^2 - 5*a*b^4*c^7*d^3 + 10*a^2*b^3*c^6*d^4
- 10*a^3*b^2*c^5*d^5 + 5*a^4*b*c^4*d^6 - a^5*c^3*d^7)*e^2*f^3 + 5
*(b^5*c^9*d - 5*a*b^4*c^8*d^2 + 10*a^2*b^3*c^7*d^3 - 10*a^3*b^2*c
^6*d^4 + 5*a^4*b*c^5*d^5 - a^5*c^4*d^6)*e*f^4 - (b^5*c^10 - 5*a*b
^4*c^9*d + 10*a^2*b^3*c^8*d^2 - 10*a^3*b^2*c^7*d^3 + 5*a^4*b*c^6*
d^4 - a^5*c^5*d^5)*f^5) + 3*(7*b^2*d^2*e^2*f^5 - 7*(b^2*c*d + a*b
*d^2)*e*f^6 + (2*b^2*c^2 + 3*a*b*c*d + 2*a^2*d^2)*f^7)*log(f*x +
e)/(b^5*d^5*e^10 + a^5*c^5*f^10 - 5*(b^5*c*d^4 + a*b^4*d^5)*e^9*f
 + 5*(2*b^5*c^2*d^3 + 5*a*b^4*c*d^4 + 2*a^2*b^3*d^5)*e^8*f^2 - 10
*(b^5*c^3*d^2 + 5*a*b^4*c^2*d^3 + 5*a^2*b^3*c*d^4 + a^3*b^2*d^5)*
e^7*f^3 + 5*(b^5*c^4*d + 10*a*b^4*c^3*d^2 + 20*a^2*b^3*c^2*d^3 +
10*a^3*b^2*c*d^4 + a^4*b*d^5)*e^6*f^4 - (b^5*c^5 + 25*a*b^4*c^4*d
 + 100*a^2*b^3*c^3*d^2 + 100*a^3*b^2*c^2*d^3 + 25*a^4*b*c*d^4 + a
^5*d^5)*e^5*f^5 + 5*(a*b^4*c^5 + 10*a^2*b^3*c^4*d + 20*a^3*b^2*c^
```

```
3*d^2 + 10*a^4*b*c^2*d^3 + a^5*c*d^4)*e^4*f^6 - 10*(a^2*b^3*c^5 +
 5*a^3*b^2*c^4*d + 5*a^4*b*c^3*d^2 + a^5*c^2*d^3)*e^3*f^7 + 5*(2*
a^3*b^2*c^5 + 5*a^4*b*c^4*d + 2*a^5*c^3*d^2)*e^2*f^8 - 5*(a^4*b*c
^5 + a^5*c^4*d)*e*f^9) - 1/2*((b^7*c^3*d^4 - 7*a*b^6*c^2*d^5 - 7*
a^2*b^5*c*d^6 + a^3*b^4*d^7)*e^7 - (4*b^7*c^4*d^3 - 21*a*b^6*c^3*
d^4 - 26*a^2*b^5*c^2*d^5 - 21*a^3*b^4*c*d^6 + 4*a^4*b^3*d^7)*e^6*
f + 2*(3*b^7*c^5*d^2 - 7*a*b^6*c^4*d^3 - 26*a^2*b^5*c^3*d^4 - 26*
a^3*b^4*c^2*d^5 - 7*a^4*b^3*c*d^6 + 3*a^5*b^2*d^7)*e^5*f^2 - 2*(2
*b^7*c^6*d + 7*a*b^6*c^5*d^2 - 39*a^2*b^5*c^4*d^3 - 39*a^4*b^3*c^
2*d^5 + 7*a^5*b^2*c*d^6 + 2*a^6*b*d^7)*e^4*f^3 + (b^7*c^7 + 21*a*
b^6*c^6*d - 52*a^2*b^5*c^5*d^2 - 52*a^5*b^2*c^2*d^5 + 21*a^6*b*c*
d^6 + a^7*d^7)*e^3*f^4 - (7*a*b^6*c^7 - 26*a^2*b^5*c^6*d + 52*a^3
*b^4*c^5*d^2 - 78*a^4*b^3*c^4*d^3 + 52*a^5*b^2*c^3*d^4 - 26*a^6*b
*c^2*d^5 + 7*a^7*c*d^6)*e^2*f^5 - 7*(a^2*b^5*c^7 - 3*a^3*b^4*c^6*
d + 2*a^4*b^3*c^5*d^2 + 2*a^5*b^2*c^4*d^3 - 3*a^6*b*c^3*d^4 + a^7
*c^2*d^5)*e*f^6 + (a^3*b^4*c^7 - 4*a^4*b^3*c^6*d + 6*a^5*b^2*c^5*
d^2 - 4*a^6*b*c^4*d^3 + a^7*c^3*d^4)*f^7) - 6*(2*b^7*d^7*e^5*f^2 -
 5*(b^7*c*d^6 + a*b^6*d^7)*e^4*f^3 + 2*(b^7*c^2*d^5 + 8*a*b^6*c*d
^6 + a^2*b^5*d^7)*e^3*f^4 + 2*(b^7*c^3*d^4 - 6*a*b^6*c^2*d^5 - 6*
a^2*b^5*c*d^6 + a^3*b^4*d^7)*e^2*f^5 - (5*b^7*c^4*d^3 - 16*a*b^6*
c^3*d^4 + 12*a^2*b^5*c^2*d^5 - 16*a^3*b^4*c*d^6 + 5*a^4*b^3*d^7)*
e*f^6 + (2*b^7*c^5*d^2 - 5*a*b^6*c^4*d^3 + 2*a^2*b^5*c^3*d^4 + 2*
a^3*b^4*c^2*d^5 - 5*a^4*b^3*c*d^6 + 2*a^5*b^2*d^7)*f^7)*x^5 - 3*(
8*b^7*d^7*e^6*f - 14*(b^7*c*d^6 + a*b^6*d^7)*e^5*f^2 - (7*b^7*c^2
*d^5 - 34*a*b^6*c*d^6 + 7*a^2*b^5*d^7)*e^4*f^3 + 2*(7*b^7*c^3*d^4
 + 3*a*b^6*c^2*d^5 + 3*a^2*b^5*c*d^6 + 7*a^3*b^4*d^7)*e^3*f^4 - (
7*b^7*c^4*d^3 - 6*a*b^6*c^3*d^4 + 78*a^2*b^5*c^2*d^5 - 6*a^3*b^4*
c*d^6 + 7*a^4*b^3*d^7)*e^2*f^5 - 2*(7*b^7*c^5*d^2 - 17*a*b^6*c^4*
d^3 - 3*a^2*b^5*c^3*d^4 - 3*a^3*b^4*c^2*d^5 - 17*a^4*b^3*c*d^6 +
7*a^5*b^2*d^7)*e*f^6 + (8*b^7*c^6*d - 14*a*b^6*c^5*d^2 - 7*a^2*b^
5*c^4*d^3 + 14*a^3*b^4*c^3*d^4 - 7*a^4*b^3*c^2*d^5 - 14*a^5*b^2*c
*d^6 + 8*a^6*b*d^7)*f^7)*x^4 - 2*(6*b^7*d^7*e^7 + 3*(b^7*c*d^6 +
a*b^6*d^7)*e^6*f - (37*b^7*c^2*d^5 + 28*a*b^6*c*d^6 + 37*a^2*b^5*
d^7)*e^5*f^2 + (19*b^7*c^3*d^4 + 86*a*b^6*c^2*d^5 + 86*a^2*b^5*c*
d^6 + 19*a^3*b^4*d^7)*e^4*f^3 + (19*b^7*c^4*d^3 - 68*a*b^6*c^3*d^
4 - 52*a^2*b^5*c^2*d^5 - 68*a^3*b^4*c*d^6 + 19*a^4*b^3*d^7)*e^3*f
^4 - (37*b^7*c^5*d^2 - 86*a*b^6*c^4*d^3 + 52*a^2*b^5*c^3*d^4 + 52
*a^3*b^4*c^2*d^5 - 86*a^4*b^3*c*d^6 + 37*a^5*b^2*d^7)*e^2*f^5 + (
3*b^7*c^6*d - 28*a*b^6*c^5*d^2 + 86*a^2*b^5*c^4*d^3 - 68*a^3*b^4*
c^3*d^4 + 86*a^4*b^3*c^2*d^5 - 28*a^5*b^2*c*d^6 + 3*a^6*b*d^7)*e*
f^6 + (6*b^7*c^7 + 3*a*b^6*c^6*d - 37*a^2*b^5*c^5*d^2 + 19*a^3*b^
4*c^4*d^3 + 19*a^4*b^3*c^3*d^4 - 37*a^5*b^2*c^2*d^5 + 3*a^6*b*c*d
^6 + 6*a^7*d^7)*f^7)*x^3 - (18*(b^7*c*d^6 + a*b^6*d^7)*e^7 - (37*
b^7*c^2*d^5 + 34*a*b^6*c*d^6 + 37*a^2*b^5*d^7)*e^6*f - 3*(b^7*c^3
*d^4 - 3*a*b^6*c^2*d^5 - 3*a^2*b^5*c*d^6 + a^3*b^4*d^7)*e^5*f^2 +
 (32*b^7*c^4*d^3 + a*b^6*c^3*d^4 + 234*a^2*b^5*c^2*d^5 + a^3*b^4*
c*d^6 + 32*a^4*b^3*d^7)*e^4*f^3 - (3*b^7*c^5*d^2 - a*b^6*c^4*d^3
+ 208*a^2*b^5*c^3*d^4 + 208*a^3*b^4*c^2*d^5 - a^4*b^3*c*d^6 + 3*a
^5*b^2*d^7)*e^3*f^4 - (37*b^7*c^6*d - 9*a*b^6*c^5*d^2 - 234*a^2*b
^5*c^4*d^3 + 208*a^3*b^4*c^3*d^4 - 234*a^4*b^3*c^2*d^5 - 9*a^5*b^
2*c*d^6 + 37*a^6*b*d^7)*e^2*f^5 + (18*b^7*c^7 - 34*a*b^6*c^6*d +
9*a^2*b^5*c^5*d^2 + a^3*b^4*c^4*d^3 + a^4*b^3*c^3*d^4 + 9*a^5*b^2
*c^2*d^5 - 34*a^6*b*c*d^6 + 18*a^7*d^7)*e*f^6 + (18*a*b^6*c^7 - 3
7*a^2*b^5*c^6*d - 3*a^3*b^4*c^5*d^2 + 32*a^4*b^3*c^4*d^3 - 3*a^5*
b^2*c^3*d^4 - 37*a^6*b*c^2*d^5 + 18*a^7*c*d^6)*f^7)*x^2 - 2*(2*(b
^7*c^2*d^5 + 7*a*b^6*c*d^6 + a^2*b^5*d^7)*e^7 - 3*(2*b^7*c^3*d^4
+ 11*a*b^6*c^2*d^5 + 11*a^2*b^5*c*d^6 + 2*a^3*b^4*d^7)*e^6*f + (4
*b^7*c^4*d^3 + 17*a*b^6*c^3*d^4 + 78*a^2*b^5*c^2*d^5 + 17*a^3*b^4
*c*d^6 + 4*a^4*b^3*d^7)*e^5*f^2 + 2*(2*b^7*c^5*d^2 - 4*a*b^6*c^4*
d^3 - 13*a^2*b^5*c^3*d^4 - 13*a^3*b^4*c^2*d^5 - 4*a^4*b^3*c*d^6 +
 2*a^5*b^2*d^7)*e^4*f^3 - (6*b^7*c^6*d - 17*a*b^6*c^5*d^2 + 26*a^
2*b^5*c^4*d^3 + 26*a^4*b^3*c^2*d^5 - 17*a^5*b^2*c*d^6 + 6*a^6*b*d
^7)*e^3*f^4 + (2*b^7*c^7 - 33*a*b^6*c^6*d + 78*a^2*b^5*c^5*d^2 -
26*a^3*b^4*c^4*d^3 - 26*a^4*b^3*c^3*d^4 + 78*a^5*b^2*c^2*d^5 - 33
*a^6*b*c*d^6 + 2*a^7*d^7)*e^2*f^5 + (14*a*b^6*c^7 - 33*a^2*b^5*c^
6*d + 17*a^3*b^4*c^5*d^2 - 8*a^4*b^3*c^4*d^3 + 17*a^5*b^2*c^3*d^4
 - 33*a^6*b*c^2*d^5 + 14*a^7*c*d^6)*e*f^6 + 2*(a^2*b^5*c^7 - 3*a^
3*b^4*c^6*d + 2*a^4*b^3*c^5*d^2 + 2*a^5*b^2*c^4*d^3 - 3*a^6*b*c^3
*d^4 + a^7*c^2*d^5)*f^7)*x)/((a^2*b^8*c^6*d^4 - 4*a^3*b^7*c^5*d^5
 + 6*a^4*b^6*c^4*d^6 - 4*a^5*b^5*c^3*d^7 + a^6*b^4*c^2*d^8)*e^10
- 4*(a^2*b^8*c^7*d^3 - 3*a^3*b^7*c^6*d^4 + 2*a^4*b^6*c^5*d^5 + 2*
a^5*b^5*c^4*d^6 - 3*a^6*b^4*c^3*d^7 + a^7*b^3*c^2*d^8)*e^9*f + 2*
```

```
(3*a^2*b^8*c^8*d^2 - 4*a^3*b^7*c^7*d^3 - 11*a^4*b^6*c^6*d^4 + 24*
a^5*b^5*c^5*d^5 - 11*a^6*b^4*c^4*d^6 - 4*a^7*b^3*c^3*d^7 + 3*a^8*
b^2*c^2*d^8)*e^8*f^2 - 4*(a^2*b^8*c^9*d + 2*a^3*b^7*c^8*d^2 - 12*
a^4*b^6*c^7*d^3 + 9*a^5*b^5*c^6*d^4 + 9*a^6*b^4*c^5*d^5 - 12*a^7*
b^3*c^4*d^6 + 2*a^8*b^2*c^3*d^7 + a^9*b*c^2*d^8)*e^7*f^3 + (a^2*b
^8*c^10 + 12*a^3*b^7*c^9*d - 22*a^4*b^6*c^8*d^2 - 36*a^5*b^5*c^7*
d^3 + 90*a^6*b^4*c^6*d^4 - 36*a^7*b^3*c^5*d^5 - 22*a^8*b^2*c^4*d^
6 + 12*a^9*b*c^3*d^7 + a^10*c^2*d^8)*e^6*f^4 - 4*(a^3*b^7*c^10 +
2*a^4*b^6*c^9*d - 12*a^5*b^5*c^8*d^2 + 9*a^6*b^4*c^7*d^3 + 9*a^7*
b^3*c^6*d^4 - 12*a^8*b^2*c^5*d^5 + 2*a^9*b*c^4*d^6 + a^10*c^3*d^7
)*e^5*f^5 + 2*(3*a^4*b^6*c^10 - 4*a^5*b^5*c^9*d - 11*a^6*b^4*c^8*
d^2 + 24*a^7*b^3*c^7*d^3 - 11*a^8*b^2*c^6*d^4 - 4*a^9*b*c^5*d^5 +
 3*a^10*c^4*d^6)*e^4*f^6 - 4*(a^5*b^5*c^10 - 3*a^6*b^4*c^9*d + 2*
a^7*b^3*c^8*d^2 + 2*a^8*b^2*c^7*d^3 - 3*a^9*b*c^6*d^4 + a^10*c^5*
d^5)*e^3*f^7 + (a^6*b^4*c^10 - 4*a^7*b^3*c^9*d + 6*a^8*b^2*c^8*d^
2 - 4*a^9*b*c^7*d^3 + a^10*c^6*d^4)*e^2*f^8 + ((b^10*c^4*d^6 - 4*
a*b^9*c^3*d^7 + 6*a^2*b^8*c^2*d^8 - 4*a^3*b^7*c*d^9 + a^4*b^6*d^1
0)*e^8*f^2 - 4*(b^10*c^5*d^5 - 3*a*b^9*c^4*d^6 + 2*a^2*b^8*c^3*d^
7 + 2*a^3*b^7*c^2*d^8 - 3*a^4*b^6*c*d^9 + a^5*b^5*d^10)*e^7*f^3 +
 2*(3*b^10*c^6*d^4 - 4*a*b^9*c^5*d^5 - 11*a^2*b^8*c^4*d^6 + 24*a^
3*b^7*c^3*d^7 - 11*a^4*b^6*c^2*d^8 - 4*a^5*b^5*c*d^9 + 3*a^6*b^4*
d^10)*e^6*f^4 - 4*(b^10*c^7*d^3 + 2*a*b^9*c^6*d^4 - 12*a^2*b^8*c^
5*d^5 + 9*a^3*b^7*c^4*d^6 + 9*a^4*b^6*c^3*d^7 - 12*a^5*b^5*c^2*d^
8 + 2*a^6*b^4*c*d^9 + a^7*b^3*d^10)*e^5*f^5 + (b^10*c^8*d^2 + 12*
a*b^9*c^7*d^3 - 22*a^2*b^8*c^6*d^4 - 36*a^3*b^7*c^5*d^5 + 90*a^4*
b^6*c^4*d^6 - 36*a^5*b^5*c^3*d^7 - 22*a^6*b^4*c^2*d^8 + 12*a^7*b^
3*c*d^9 + a^8*b^2*d^10)*e^4*f^6 - 4*(a*b^9*c^8*d^2 + 2*a^2*b^8*c^
7*d^3 - 12*a^3*b^7*c^6*d^4 + 9*a^4*b^6*c^5*d^5 + 9*a^5*b^5*c^4*d^
6 - 12*a^6*b^4*c^3*d^7 + 2*a^7*b^3*c^2*d^8 + a^8*b^2*c*d^9)*e^3*f
^7 + 2*(3*a^2*b^8*c^8*d^2 - 4*a^3*b^7*c^7*d^3 - 11*a^4*b^6*c^6*d^
4 + 24*a^5*b^5*c^5*d^5 - 11*a^6*b^4*c^4*d^6 - 4*a^7*b^3*c^3*d^7 +
 3*a^8*b^2*c^2*d^8)*e^2*f^8 - 4*(a^3*b^7*c^8*d^2 - 3*a^4*b^6*c^7*
d^3 + 2*a^5*b^5*c^6*d^4 + 2*a^6*b^4*c^5*d^5 - 3*a^7*b^3*c^4*d^6 +
 a^8*b^2*c^3*d^7)*e*f^9 + (a^4*b^6*c^8*d^2 - 4*a^5*b^5*c^7*d^3 +
6*a^6*b^4*c^6*d^4 - 4*a^7*b^3*c^5*d^5 + a^8*b^2*c^4*d^6)*f^10)*x^
6 + 2*((b^10*c^4*d^6 - 4*a*b^9*c^3*d^7 + 6*a^2*b^8*c^2*d^8 - 4*a^
3*b^7*c*d^9 + a^4*b^6*d^10)*e^9*f - 3*(b^10*c^5*d^5 - 3*a*b^9*c^4
*d^6 + 2*a^2*b^8*c^3*d^7 + 2*a^3*b^7*c^2*d^8 - 3*a^4*b^6*c*d^9 +
a^5*b^5*d^10)*e^8*f^2 + 2*(b^10*c^6*d^4 - 9*a^2*b^8*c^4*d^6 + 16*
a^3*b^7*c^3*d^7 - 9*a^4*b^6*c^2*d^8 + a^6*b^4*d^10)*e^7*f^3 + 2*(
b^10*c^7*d^3 - 5*a*b^9*c^6*d^4 + 9*a^2*b^8*c^5*d^5 - 5*a^3*b^7*c^
4*d^6 - 5*a^4*b^6*c^3*d^7 + 9*a^5*b^5*c^2*d^8 - 5*a^6*b^4*c*d^9 +
 a^7*b^3*d^10)*e^6*f^4 - 3*(b^10*c^8*d^2 - 6*a^2*b^8*c^6*d^4 + 8*
a^3*b^7*c^5*d^5 - 6*a^4*b^6*c^4*d^6 + 8*a^5*b^5*c^3*d^7 - 6*a^6*b
^4*c^2*d^8 + a^8*b^2*d^10)*e^5*f^5 + (b^10*c^9*d + 9*a*b^9*c^8*d^
2 - 18*a^2*b^8*c^7*d^3 - 10*a^3*b^7*c^6*d^4 + 18*a^4*b^6*c^5*d^5
+ 18*a^5*b^5*c^4*d^6 - 10*a^6*b^4*c^3*d^7 - 18*a^7*b^3*c^2*d^8 +
9*a^8*b^2*c*d^9 + a^9*b*d^10)*e^4*f^6 - 2*(2*a*b^9*c^9*d + 3*a^2*
b^8*c^8*d^2 - 16*a^3*b^7*c^7*d^3 + 5*a^4*b^6*c^6*d^4 + 12*a^5*b^5
*c^5*d^5 + 5*a^6*b^4*c^4*d^6 - 16*a^7*b^3*c^3*d^7 + 3*a^8*b^2*c^2
*d^8 + 2*a^9*b*c*d^9)*e^3*f^7 + 6*(a^2*b^8*c^9*d - a^3*b^7*c^8*d^
2 - 3*a^4*b^6*c^7*d^3 + 3*a^5*b^5*c^6*d^4 + 3*a^6*b^4*c^5*d^5 - 3
*a^7*b^3*c^4*d^6 - a^8*b^2*c^3*d^7 + a^9*b*c^2*d^8)*e^2*f^8 - (4*
a^3*b^7*c^9*d - 9*a^4*b^6*c^8*d^2 + 10*a^6*b^4*c^6*d^4 - 9*a^8*b^
2*c^4*d^6 + 4*a^9*b*c^3*d^7)*e*f^9 + (a^4*b^6*c^9*d - 3*a^5*b^5*c
^8*d^2 + 2*a^6*b^4*c^7*d^3 + 2*a^7*b^3*c^6*d^4 - 3*a^8*b^2*c^5*d^
5 + a^9*b*c^4*d^6)*f^10)*x^5 + ((b^10*c^4*d^6 - 4*a*b^9*c^3*d^7 +
 6*a^2*b^8*c^2*d^8 - 4*a^3*b^7*c*d^9 + a^4*b^6*d^10)*e^10 - 3*(3*
b^10*c^6*d^4 - 8*a*b^9*c^5*d^5 + 5*a^2*b^8*c^4*d^6 + 5*a^4*b^6*c^
2*d^8 - 8*a^5*b^5*c*d^9 + 3*a^6*b^4*d^10)*e^8*f^2 + 4*(4*b^10*c^7
*d^3 - 5*a*b^9*c^6*d^4 - 9*a^2*b^8*c^5*d^5 + 10*a^3*b^7*c^4*d^6 +
 10*a^4*b^6*c^3*d^7 - 9*a^5*b^5*c^2*d^8 - 5*a^6*b^4*c*d^9 + 4*a^7
*b^3*d^10)*e^7*f^3 - (9*b^10*c^8*d^2 + 20*a*b^9*c^7*d^3 - 90*a^2*
b^8*c^6*d^4 + 36*a^3*b^7*c^5*d^5 + 50*a^4*b^6*c^4*d^6 + 36*a^5*b^
5*c^3*d^7 - 90*a^6*b^4*c^2*d^8 + 20*a^7*b^3*c*d^9 + 9*a^8*b^2*d^1
0)*e^6*f^4 + 12*(2*a*b^9*c^8*d^2 - 3*a^2*b^8*c^7*d^3 - 3*a^3*b^7*
c^6*d^4 + 4*a^4*b^6*c^5*d^5 + 4*a^5*b^5*c^4*d^6 - 3*a^6*b^4*c^3*d
^7 - 3*a^7*b^3*c^2*d^8 + 2*a^8*b^2*c*d^9)*e^5*f^5 + (b^10*c^10 -
15*a^2*b^8*c^8*d^2 + 40*a^3*b^7*c^7*d^3 - 50*a^4*b^6*c^6*d^4 + 48
*a^5*b^5*c^5*d^5 - 50*a^6*b^4*c^4*d^6 + 40*a^7*b^3*c^3*d^7 - 15*a
^8*b^2*c^2*d^8 + a^10*d^10)*e^4*f^6 - 4*(a*b^9*c^10 - 10*a^4*b^6*
c^7*d^3 + 9*a^5*b^5*c^6*d^4 + 9*a^6*b^4*c^5*d^5 - 10*a^7*b^3*c^4*
```

```
d^6 + a^10*c*d^9)*e^3*f^7 + 3*(2*a^2*b^8*c^10 - 5*a^4*b^6*c^8*d^2
 - 12*a^5*b^5*c^7*d^3 + 30*a^6*b^4*c^6*d^4 - 12*a^7*b^3*c^5*d^5 -
 5*a^8*b^2*c^4*d^6 + 2*a^10*c^2*d^8)*e^2*f^8 - 4*(a^3*b^7*c^10 -
6*a^5*b^5*c^8*d^2 + 5*a^6*b^4*c^7*d^3 + 5*a^7*b^3*c^6*d^4 - 6*a^8
*b^2*c^5*d^5 + a^10*c^3*d^7)*e*f^9 + (a^4*b^6*c^10 - 9*a^6*b^4*c^
8*d^2 + 16*a^7*b^3*c^7*d^3 - 9*a^8*b^2*c^6*d^4 + a^10*c^4*d^6)*f^
10)*x^4 + 2*((b^10*c^5*d^5 - 3*a*b^9*c^4*d^6 + 2*a^2*b^8*c^3*d^7
+ 2*a^3*b^7*c^2*d^8 - 3*a^4*b^6*c*d^9 + a^5*b^5*d^10)*e^10 - (3*b
^10*c^6*d^4 - 8*a*b^9*c^5*d^5 + 5*a^2*b^8*c^4*d^6 + 5*a^4*b^6*c^2
*d^8 - 8*a^5*b^5*c*d^9 + 3*a^6*b^4*d^10)*e^9*f + (2*b^10*c^7*d^3
- 5*a*b^9*c^6*d^4 + 3*a^2*b^8*c^5*d^5 + 3*a^5*b^5*c^2*d^8 - 5*a^6
*b^4*c*d^9 + 2*a^7*b^3*d^10)*e^8*f^2 + 2*(b^10*c^8*d^2 - 16*a^3*b
^7*c^5*d^5 + 30*a^4*b^6*c^4*d^6 - 16*a^5*b^5*c^3*d^7 + a^8*b^2*d^
10)*e^7*f^3 - (3*b^10*c^9*d + 5*a*b^9*c^8*d^2 - 60*a^3*b^7*c^6*d^
4 + 52*a^4*b^6*c^5*d^5 + 52*a^5*b^5*c^4*d^6 - 60*a^6*b^4*c^3*d^7
+ 5*a^8*b^2*c*d^9 + 3*a^9*b*d^10)*e^6*f^4 + (b^10*c^10 + 8*a*b^9*
c^9*d + 3*a^2*b^8*c^8*d^2 - 32*a^3*b^7*c^7*d^3 - 52*a^4*b^6*c^6*d
^4 + 144*a^5*b^5*c^5*d^5 - 52*a^6*b^4*c^4*d^6 - 32*a^7*b^3*c^3*d^
7 + 3*a^8*b^2*c^2*d^8 + 8*a^9*b*c*d^9 + a^10*d^10)*e^5*f^5 - (3*a
*b^9*c^10 + 5*a^2*b^8*c^9*d - 60*a^4*b^6*c^7*d^3 + 52*a^5*b^5*c^6
*d^4 + 52*a^6*b^4*c^5*d^5 - 60*a^7*b^3*c^4*d^6 + 5*a^9*b*c^2*d^8
+ 3*a^10*c*d^9)*e^4*f^6 + 2*(a^2*b^8*c^10 - 16*a^5*b^5*c^7*d^3 +
30*a^6*b^4*c^6*d^4 - 16*a^7*b^3*c^5*d^5 + a^10*c^2*d^8)*e^3*f^7 +
 (2*a^3*b^7*c^10 - 5*a^4*b^6*c^9*d + 3*a^5*b^5*c^8*d^2 + 3*a^8*b^
2*c^5*d^5 - 5*a^9*b*c^4*d^6 + 2*a^10*c^3*d^7)*e^2*f^8 - (3*a^4*b^
6*c^10 - 8*a^5*b^5*c^9*d + 5*a^6*b^4*c^8*d^2 + 5*a^8*b^2*c^6*d^4
- 8*a^9*b*c^5*d^5 + 3*a^10*c^4*d^6)*e*f^9 + (a^5*b^5*c^10 - 3*a^6
*b^4*c^9*d + 2*a^7*b^3*c^8*d^2 + 2*a^8*b^2*c^7*d^3 - 3*a^9*b*c^6*
d^4 + a^10*c^5*d^5)*f^10)*x^3 + ((b^10*c^6*d^4 - 9*a^2*b^8*c^4*d^
6 + 16*a^3*b^7*c^3*d^7 - 9*a^4*b^6*c^2*d^8 + a^6*b^4*d^10)*e^10 -
 4*(b^10*c^7*d^3 - 6*a^2*b^8*c^5*d^5 + 5*a^3*b^7*c^4*d^6 + 5*a^4*
b^6*c^3*d^7 - 6*a^5*b^5*c^2*d^8 + a^7*b^3*d^10)*e^9*f + 3*(2*b^10
*c^8*d^2 - 5*a^2*b^8*c^6*d^4 - 12*a^3*b^7*c^5*d^5 + 30*a^4*b^6*c^
4*d^6 - 12*a^5*b^5*c^3*d^7 - 5*a^6*b^4*c^2*d^8 + 2*a^8*b^2*d^10)*
e^8*f^2 - 4*(b^10*c^9*d - 10*a^3*b^7*c^6*d^4 + 9*a^4*b^6*c^5*d^5
+ 9*a^5*b^5*c^4*d^6 - 10*a^6*b^4*c^3*d^7 + a^9*b*d^10)*e^7*f^3 +
(b^10*c^10 - 15*a^2*b^8*c^8*d^2 + 40*a^3*b^7*c^7*d^3 - 50*a^4*b^6
*c^6*d^4 + 48*a^5*b^5*c^5*d^5 - 50*a^6*b^4*c^4*d^6 + 40*a^7*b^3*c
^3*d^7 - 15*a^8*b^2*c^2*d^8 + a^10*d^10)*e^6*f^4 + 12*(2*a^2*b^8*
c^9*d - 3*a^3*b^7*c^8*d^2 - 3*a^4*b^6*c^7*d^3 + 4*a^5*b^5*c^6*d^4
 + 4*a^6*b^4*c^5*d^5 - 3*a^7*b^3*c^4*d^6 - 3*a^8*b^2*c^3*d^7 + 2*
a^9*b*c^2*d^8)*e^5*f^5 - (9*a^2*b^8*c^10 + 20*a^3*b^7*c^9*d - 90*
a^4*b^6*c^8*d^2 + 36*a^5*b^5*c^7*d^3 + 50*a^6*b^4*c^6*d^4 + 36*a^
7*b^3*c^5*d^5 - 90*a^8*b^2*c^4*d^6 + 20*a^9*b*c^3*d^7 + 9*a^10*c^
2*d^8)*e^4*f^6 + 4*(4*a^3*b^7*c^10 - 5*a^4*b^6*c^9*d - 9*a^5*b^5*
c^8*d^2 + 10*a^6*b^4*c^7*d^3 + 10*a^7*b^3*c^6*d^4 - 9*a^8*b^2*c^5
*d^5 - 5*a^9*b*c^4*d^6 + 4*a^10*c^3*d^7)*e^3*f^7 - 3*(3*a^4*b^6*c
^10 - 8*a^5*b^5*c^9*d + 5*a^6*b^4*c^8*d^2 + 5*a^8*b^2*c^6*d^4 - 8
*a^9*b*c^5*d^5 + 3*a^10*c^4*d^6)*e^2*f^8 + (a^6*b^4*c^10 - 4*a^7*
b^3*c^9*d + 6*a^8*b^2*c^8*d^2 - 4*a^9*b*c^7*d^3 + a^10*c^6*d^4)*f
^10)*x^2 + 2*((a*b^9*c^6*d^4 - 3*a^2*b^8*c^5*d^5 + 2*a^3*b^7*c^4*
d^6 + 2*a^4*b^6*c^3*d^7 - 3*a^5*b^5*c^2*d^8 + a^6*b^4*c*d^9)*e^10
 - (4*a*b^9*c^7*d^3 - 9*a^2*b^8*c^6*d^4 + 10*a^4*b^6*c^4*d^6 - 9*
a^6*b^4*c^2*d^8 + 4*a^7*b^3*c*d^9)*e^9*f + 6*(a*b^9*c^8*d^2 - a^2
*b^8*c^7*d^3 - 3*a^3*b^7*c^6*d^4 + 3*a^4*b^6*c^5*d^5 + 3*a^5*b^5*
c^4*d^6 - 3*a^6*b^4*c^3*d^7 - a^7*b^3*c^2*d^8 + a^8*b^2*c*d^9)*e^
8*f^2 - 2*(2*a*b^9*c^9*d + 3*a^2*b^8*c^8*d^2 - 16*a^3*b^7*c^7*d^3
 + 5*a^4*b^6*c^6*d^4 + 12*a^5*b^5*c^5*d^5 + 5*a^6*b^4*c^4*d^6 - 1
6*a^7*b^3*c^3*d^7 + 3*a^8*b^2*c^2*d^8 + 2*a^9*b*c*d^9)*e^7*f^3 +
(a*b^9*c^10 + 9*a^2*b^8*c^9*d - 18*a^3*b^7*c^8*d^2 - 10*a^4*b^6*c
^7*d^3 + 18*a^5*b^5*c^6*d^4 + 18*a^6*b^4*c^5*d^5 - 10*a^7*b^3*c^4
*d^6 - 18*a^8*b^2*c^3*d^7 + 9*a^9*b*c^2*d^8 + a^10*c*d^9)*e^6*f^4
 - 3*(a^2*b^8*c^10 - 6*a^4*b^6*c^8*d^2 + 8*a^5*b^5*c^7*d^3 - 6*a^
6*b^4*c^6*d^4 + 8*a^7*b^3*c^5*d^5 - 6*a^8*b^2*c^4*d^6 + a^10*c^2*
d^8)*e^5*f^5 + 2*(a^3*b^7*c^10 - 5*a^4*b^6*c^9*d + 9*a^5*b^5*c^8*
d^2 - 5*a^6*b^4*c^7*d^3 - 5*a^7*b^3*c^6*d^4 + 9*a^8*b^2*c^5*d^5 -
 5*a^9*b*c^4*d^6 + a^10*c^3*d^7)*e^4*f^6 + 2*(a^4*b^6*c^10 - 9*a^
6*b^4*c^8*d^2 + 16*a^7*b^3*c^7*d^3 - 9*a^8*b^2*c^6*d^4 + a^10*c^4
*d^6)*e^3*f^7 - 3*(a^5*b^5*c^10 - 3*a^6*b^4*c^9*d + 2*a^7*b^3*c^8
*d^2 + 2*a^8*b^2*c^7*d^3 - 3*a^9*b*c^6*d^4 + a^10*c^5*d^5)*e^2*f^
8 + (a^6*b^4*c^10 - 4*a^7*b^3*c^9*d + 6*a^8*b^2*c^8*d^2 - 4*a^9*b
*c^7*d^3 + a^10*c^6*d^4)*e*f^9)*x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*f*x^3 + a*c*e + (b*d*e + b*c*f + a*d*f)*x^2 + (b*c*e + a*d*e + a*c*f
3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x**2+b*d*f*x**3)*
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.380396, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*f*x^3 + a*c*e + (b*d*e + b*c*f + a*d*f)*x^2 + (b*c*e + a*d*e + a*c*f
3),x, algorithm="giac")
```

```
[Out] Done
```

3.21 $\int \frac{1}{1+x+x^2+x^3} \, dx$

Optimal. Leaf size=25

$$-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2}\log(x+1)+\frac{1}{2}\tan^{-1}(x)$$

```
[Out] ArcTan[x]/2 + Log[1 + x]/2 - Log[1 + x^2]/4
```

Rubi [A] time = 0.0331365, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364

$$-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2}\log(x+1)+\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x + x^2 + x^3)^(-1),x]
```

```
[Out] ArcTan[x]/2 + Log[1 + x]/2 - Log[1 + x^2]/4
```

Rubi in Sympy [A] time = 128.344, size = 31, normalized size = 1.24

$$3\log(3x+5)-\frac{\log\left(-6x+9\left(x+\frac{1}{3}\right)^2+8\right)}{24}+\frac{23\,\text{atan}\,(x)}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**3+x**2+x+1),x)
```

```
[Out] 3*log(3*x + 5) - log(-6*x + 9*(x + 1/3)**2 + 8)/24 + 23*atan(x)/3
6
```

Mathematica [A] time = 0.00869618, size = 25, normalized size = 1.

$$-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2}\log(x+1)+\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x + x^2 + x^3)^(-1),x]
```

```
[Out] ArcTan[x]/2 + Log[1 + x]/2 - Log[1 + x^2]/4
```

Maple [A] time = 0.006, size = 20, normalized size = 0.8

$$\frac{\arctan(x)}{2}+\frac{\ln(1+x)}{2}-\frac{\ln\left(x^2+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^3+x^2+x+1),x)
```

```
[Out] 1/2*arctan(x)+1/2*ln(1+x)-1/4*ln(x^2+1)
```

Maxima [A] time = 0.855483, size = 26, normalized size = 1.04

$$\frac{1}{2}\arctan(x) - \frac{1}{4}\log\left(x^2+1\right) + \frac{1}{2}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^3 + x^2 + x + 1),x, algorithm="maxima")
```

```
[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(x + 1)
```

Fricas [A] time = 0.319696, size = 26, normalized size = 1.04

$$\frac{1}{2}\arctan(x) - \frac{1}{4}\log\left(x^2+1\right) + \frac{1}{2}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^3 + x^2 + x + 1),x, algorithm="fricas")
```

```
[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(x + 1)
```

Sympy [A] time = 0.255215, size = 19, normalized size = 0.76

$$\frac{\log(x+1)}{2} - \frac{\log\left(x^2+1\right)}{4} + \frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**3+x**2+x+1),x)
```

```
[Out] log(x + 1)/2 - log(x**2 + 1)/4 + atan(x)/2
```

GIAC/XCAS [A] time = 0.26215, size = 27, normalized size = 1.08

$$\frac{1}{2}\arctan(x) - \frac{1}{4}\ln\left(x^2+1\right) + \frac{1}{2}\ln(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^3 + x^2 + x + 1),x, algorithm="giac")
```

```
[Out] 1/2*arctan(x) - 1/4*ln(x^2 + 1) + 1/2*ln(abs(x + 1))
```

3.22 $\int \frac{1}{-1+4x-4x^2+16x^3} \, dx$

Optimal. Leaf size=31

$$-\frac{1}{10}\log\left(4x^2+1\right)+\frac{1}{5}\log(1-4x)-\frac{1}{10}\tan^{-1}(2x)$$

```
[Out] -ArcTan[2*x]/10 + Log[1 - 4*x]/5 - Log[1 + 4*x^2]/10
```

Rubi [A] time = 0.0429689, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235

$$-\frac{1}{10}\log\left(4x^2+1\right)+\frac{1}{5}\log(1-4x)-\frac{1}{10}\tan^{-1}(2x)$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + 4*x - 4*x^2 + 16*x^3)^(-1),x]

[Out] -ArcTan[2*x]/10 + Log[1 - 4*x]/5 - Log[1 + 4*x^2]/10
```

Rubi in Sympy [A] time = 32.0786, size = 34, normalized size = 1.1

$$\frac{\log\left(-6x+\frac{3}{2}\right)}{5}-\frac{\log\left(24x+144\left(x-\frac{1}{12}\right)^2+35\right)}{60}-\frac{23\,\text{atan}\,(2x)}{180}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(16*x**3-4*x**2+4*x-1),x)

[Out] log(-6*x + 3/2)/5 - log(24*x + 144*(x - 1/12)**2 + 35)/60 - 23*at
an(2*x)/180
```

Mathematica [A] time = 0.0104558, size = 31, normalized size = 1.

$$-\frac{1}{10}\log\left(4x^2+1\right)+\frac{1}{5}\log(1-4x)-\frac{1}{10}\tan^{-1}(2x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + 4*x - 4*x^2 + 16*x^3)^(-1),x]

[Out] -ArcTan[2*x]/10 + Log[1 - 4*x]/5 - Log[1 + 4*x^2]/10
```

Maple [A] time = 0.01, size = 26, normalized size = 0.8

$$\frac{\ln\left(-1+4\,x\right)}{5}-\frac{\ln\left(4\,x^2+1\right)}{10}-\frac{\arctan\left(2\,x\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(16*x^3-4*x^2+4*x-1),x)
```

```
[Out] 1/5*ln(-1+4*x)-1/10*ln(4*x^2+1)-1/10*arctan(2*x)
```

Maxima [A] time = 0.858651, size = 34, normalized size = 1.1

$$-\frac{1}{10}\arctan\left(2x\right)-\frac{1}{10}\log\left(4x^2+1\right)+\frac{1}{5}\log\left(4x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(16*x^3 - 4*x^2 + 4*x - 1),x, algorithm="maxima")
```

```
[Out] -1/10*arctan(2*x) - 1/10*log(4*x^2 + 1) + 1/5*log(4*x - 1)
```

Fricas [A] time = 0.321349, size = 34, normalized size = 1.1

$$-\frac{1}{10}\arctan\left(2x\right)-\frac{1}{10}\log\left(4x^2+1\right)+\frac{1}{5}\log\left(4x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(16*x^3 - 4*x^2 + 4*x - 1),x, algorithm="fricas")
```

```
[Out] -1/10*arctan(2*x) - 1/10*log(4*x^2 + 1) + 1/5*log(4*x - 1)
```

Sympy [A] time = 0.290941, size = 24, normalized size = 0.77

$$\frac{\log\left(x-\frac{1}{4}\right)}{5}-\frac{\log\left(x^2+\frac{1}{4}\right)}{10}-\frac{\operatorname{atan}\left(2x\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(16*x**3-4*x**2+4*x-1),x)
```

```
[Out] log(x - 1/4)/5 - log(x**2 + 1/4)/10 - atan(2*x)/10
```

GIAC/XCAS [A] time = 0.261065, size = 35, normalized size = 1.13

$$-\frac{1}{10}\arctan\left(2x\right)-\frac{1}{10}\ln\left(4x^2+1\right)+\frac{1}{5}\ln\left(|4x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(16*x^3 - 4*x^2 + 4*x - 1),x, algorithm="giac")
```

```
[Out] -1/10*arctan(2*x) - 1/10*ln(4*x^2 + 1) + 1/5*ln(abs(4*x - 1))
```

3.23 $\int \frac{1}{dx^3} \, dx$

Optimal. Leaf size=10

$$-\frac{1}{2dx^2}$$

```
[Out] -1/(2*d*x^2)
```

Rubi [A] time = 0.00546883, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286

$$-\frac{1}{2dx^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(d*x^3),x]

[Out] -1/(2*d*x^2)
```

Rubi in Sympy [A] time = 1.54964, size = 8, normalized size = 0.8

$$-\frac{1}{2dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/d/x**3,x)

[Out] -1/(2*d*x**2)
```

Mathematica [A] time = 0.00056349, size = 10, normalized size = 1.

$$-\frac{1}{2dx^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(d*x^3),x]

[Out] -1/(2*d*x^2)
```

Maple [A] time = 0.001, size = 9, normalized size = 0.9

$$-\frac{1}{2\,dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/d/x^3,x)

[Out] -1/2/d/x^2
```

Maxima [A] time = 0.769606, size = 11, normalized size = 1.1

$$-\frac{1}{2\,dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(d*x^3),x, algorithm="maxima")
```

```
[Out] -1/2/(d*x^2)
```

Fricas [A] time = 0.298204, size = 11, normalized size = 1.1

$$-\frac{1}{2\,dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(d*x^3),x, algorithm="fricas")
```

```
[Out] -1/2/(d*x^2)
```

Sympy [A] time = 0.071728, size = 8, normalized size = 0.8

$$-\frac{1}{2dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/d/x**3,x)
```

```
[Out] -1/(2*d*x**2)
```

GIAC/XCAS [A] time = 0.260774, size = 11, normalized size = 1.1

$$-\frac{1}{2\,dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(d*x^3),x, algorithm="giac")
```

```
[Out] -1/2/(d*x^2)
```

3.24 $\int \frac{1}{cx^2+dx^3}\,dx$

Optimal. Leaf size=28

$$-\frac{d\log(x)}{c^2}+\frac{d\log(c+dx)}{c^2}-\frac{1}{cx}$$

```
[Out] -(1/(c*x)) - (d*Log[x])/c^2 + (d*Log[c + d*x])/c^2
```

Rubi [A] time = 0.0327931, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$-\frac{d\log(x)}{c^2}+\frac{d\log(c+dx)}{c^2}-\frac{1}{cx}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2 + d*x^3)^(-1),x]

[Out] -(1/(c*x)) - (d*Log[x])/c^2 + (d*Log[c + d*x])/c^2
```

Rubi in Sympy [A] time = 14.1775, size = 24, normalized size = 0.86

$$-\frac{1}{cx}-\frac{d\log(x)}{c^2}+\frac{d\log(c+dx)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(d*x**3+c*x**2),x)

[Out] -1/(c*x) - d*log(x)/c**2 + d*log(c + d*x)/c**2
```

Mathematica [A] time = 0.00777943, size = 28, normalized size = 1.

$$-\frac{d\log(x)}{c^2}+\frac{d\log(c+dx)}{c^2}-\frac{1}{cx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2 + d*x^3)^(-1),x]

[Out] -(1/(c*x)) - (d*Log[x])/c^2 + (d*Log[c + d*x])/c^2
```

Maple [A] time = 0.014, size = 29, normalized size = 1.

$$-\frac{1}{cx}-\frac{d\ln(x)}{c^2}+\frac{d\ln(dx+c)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x^3+c*x^2),x)
```

```
[Out] -1/c/x-d*ln(x)/c^2+d*ln(d*x+c)/c^2
```

Maxima [A] time = 0.765276, size = 38, normalized size = 1.36

$$\frac{d\log\left(dx+c\right)}{c^2}-\frac{d\log\left(x\right)}{c^2}-\frac{1}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(d*x^3 + c*x^2),x, algorithm="maxima")
```

```
[Out] d*log(d*x + c)/c^2 - d*log(x)/c^2 - 1/(c*x)
```

Fricas [A] time = 0.317002, size = 35, normalized size = 1.25

$$\frac{dx\log\left(dx+c\right)-dx\log\left(x\right)-c}{c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(d*x^3 + c*x^2),x, algorithm="fricas")
```

```
[Out] (d*x*log(d*x + c) - d*x*log(x) - c)/(c^2*x)
```

Sympy [A] time = 1.27418, size = 19, normalized size = 0.68

$$-\frac{1}{cx}+\frac{d\left(-\log\left(x\right)+\log\left(\frac{c}{d}+x\right)\right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(d*x**3+c*x**2),x)
```

```
[Out] -1/(c*x) + d*(-log(x) + log(c/d + x))/c**2
```

GIAC/XCAS [A] time = 0.259568, size = 41, normalized size = 1.46

$$\frac{d\ln\left(|dx+c|\right)}{c^2}-\frac{d\ln\left(|x|\right)}{c^2}-\frac{1}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(d*x^3 + c*x^2),x, algorithm="giac")
```

```
[Out] d*ln(abs(d*x + c))/c^2 - d*ln(abs(x))/c^2 - 1/(c*x)
```

3.25 $\int \frac{1}{bx+dx^3} \, dx$

Optimal. Leaf size=22

$$\frac{\log(x)}{b} - \frac{\log\left(b + dx^2\right)}{2b}$$

```
[Out] Log[x]/b - Log[b + d*x^2]/(2*b)
```

Rubi [A] time = 0.0296154, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$

$$\frac{\log(x)}{b} - \frac{\log\left(b + dx^2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x + d*x^3)^(-1),x]

[Out] Log[x]/b - Log[b + d*x^2]/(2*b)
```

Rubi in Sympy [A] time = 156.994, size = 19, normalized size = 0.86

$$\frac{\log\left(x^2\right)}{2b} - \frac{\log\left(b + dx^2\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(d*x**3+b*x),x)

[Out] log(x**2)/(2*b) - log(b + d*x**2)/(2*b)
```

Mathematica [A] time = 0.00749272, size = 22, normalized size = 1.

$$\frac{\log(x)}{b} - \frac{\log\left(b + dx^2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x + d*x^3)^(-1),x]

[Out] Log[x]/b - Log[b + d*x^2]/(2*b)
```

Maple [A] time = 0.007, size = 21, normalized size = 1.

$$\frac{\ln(x)}{b} - \frac{\ln\left(dx^2 + b\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(d*x^3+b*x),x)
```

```
[Out] ln(x)/b-1/2*ln(d*x^2+b)/b
```

Maxima [A] time = 0.778088, size = 27, normalized size = 1.23

$$-\frac{\log\left(dx^2+b\right)}{2\,b}+\frac{\log\left(x\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(d*x^3 + b*x),x, algorithm="maxima")
```

```
[Out] -1/2*log(d*x^2 + b)/b + log(x)/b
```

Fricas [A] time = 0.310416, size = 24, normalized size = 1.09

$$-\frac{\log\left(dx^2+b\right)-2\,\log\left(x\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(d*x^3 + b*x),x, algorithm="fricas")
```

```
[Out] -1/2*(log(d*x^2 + b) - 2*log(x))/b
```

Sympy [A] time = 0.49307, size = 15, normalized size = 0.68

$$\frac{\log\left(x\right)}{b}-\frac{\log\left(\frac{b}{d}+x^2\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(d*x**3+b*x),x)
```

```
[Out] log(x)/b - log(b/d + x**2)/(2*b)
```

GIAC/XCAS [A] time = 0.260396, size = 32, normalized size = 1.45

$$\frac{\ln\left(x^2\right)}{2\,b}-\frac{\ln\left(\left|dx^2+b\right|\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(d*x^3 + b*x),x, algorithm="giac")
```

```
[Out] 1/2*ln(x^2)/b - 1/2*ln(abs(d*x^2 + b))/b
```

3.26 $\int \frac{1}{bx+cx^2+dx^3}\,dx$

Optimal. Leaf size=62

$$\frac{c\tanh^{-1}\left(\frac{c+2dx}{\sqrt{c^2-4bd}}\right)}{b\sqrt{c^2-4bd}}-\frac{\log\left(b+cx+dx^2\right)}{2b}+\frac{\log(x)}{b}$$

```
[Out] (c*ArcTanh[(c + 2*d*x)/Sqrt[c^2 - 4*b*d]])/(b*Sqrt[c^2 - 4*b*d])
+ Log[x]/b - Log[b + c*x + d*x^2]/(2*b)
```

Rubi [A] time = 0.112362, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438

$$\frac{c\tanh^{-1}\left(\frac{c+2dx}{\sqrt{c^2-4bd}}\right)}{b\sqrt{c^2-4bd}}-\frac{\log\left(b+cx+dx^2\right)}{2b}+\frac{\log(x)}{b}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x + c*x^2 + d*x^3)^(-1),x]
```

```
[Out] (c*ArcTanh[(c + 2*d*x)/Sqrt[c^2 - 4*b*d]])/(b*Sqrt[c^2 - 4*b*d])
+ Log[x]/b - Log[b + c*x + d*x^2]/(2*b)
```

Rubi in Sympy [A] time = 20.222, size = 54, normalized size = 0.87

$$\frac{c\,\operatorname{atanh}\left(\frac{c+2dx}{\sqrt{-4bd+c^2}}\right)}{b\sqrt{-4bd+c^2}}+\frac{\log(x)}{b}-\frac{\log\left(b+cx+dx^2\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(d*x**3+c*x**2+b*x),x)
```

```
[Out] c*atanh((c + 2*d*x)/sqrt(-4*b*d + c**2))/(b*sqrt(-4*b*d + c**2))
+ log(x)/b - log(b + c*x + d*x**2)/(2*b)
```

Mathematica [A] time = 0.131815, size = 61, normalized size = 0.98

$$-\frac{\frac{2c\tan^{-1}\left(\frac{c+2dx}{\sqrt{4bd-c^2}}\right)}{\sqrt{4bd-c^2}}+\log(b+x(c+dx))-2\log(x)}{2b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x + c*x^2 + d*x^3)^(-1),x]
```

```
[Out] -((2*c*ArcTan[(c + 2*d*x)/Sqrt[-c^2 + 4*b*d]])/Sqrt[-c^2 + 4*b*d]
 - 2*Log[x] + Log[b + x*(c + d*x)])/(2*b)
```

Maple [A] time = 0.013, size = 62, normalized size = 1.

$$-\frac{\ln\left(dx^2+cx+b\right)}{2\,b}-\frac{c}{b}\arctan\left((2\,dx+c)\frac{1}{\sqrt{4\,bd-c^2}}\right)\frac{1}{\sqrt{4\,bd-c^2}}+\frac{\ln(x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(d*x^3+c*x^2+b*x),x)
```

```
[Out] -1/2*ln(d*x^2+c*x+b)/b-1/b*c/(4*b*d-c^2)^(1/2)*arctan((2*d*x+c)/(
4*b*d-c^2)^(1/2))+ln(x)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(d*x^3 + c*x^2 + b*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.304896, size = 1, normalized size = 0.02

$$\left[\frac{c\log\left(\frac{c^3-4\,bcd+2\,(c^2d-4\,bd^2)\,x+(2\,d^2x^2+2\,cdx+c^2-2\,bd)\,\sqrt{c^2-4\,bd}}{dx^2+cx+b}\right)-\sqrt{c^2-4\,bd}\left(\log\left(dx^2+cx+b\right)-2\,\log\left(x\right)\right)}{2\,\sqrt{c^2-4\,bd}b},\right.$$
$$\left.-\frac{2\,c\arctan\left(-\frac{\sqrt{-c^2+4\,bd}(2\,dx+c)}{c^2-4\,bd}\right)+\sqrt{-c^2+4\,bd}\left(\log\left(dx^2+cx+b\right)-2\,\log\left(x\right)\right)}{2\,\sqrt{-c^2+4\,bd}b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(d*x^3 + c*x^2 + b*x),x, algorithm="fricas")
```

```
[Out] [1/2*(c*log((c^3 - 4*b*c*d + 2*(c^2*d - 4*b*d^2)*x + (2*d^2*x^2 +
 2*c*d*x + c^2 - 2*b*d)*sqrt(c^2 - 4*b*d))/(d*x^2 + c*x + b)) - s
qrt(c^2 - 4*b*d)*(log(d*x^2 + c*x + b) - 2*log(x)))/(sqrt(c^2 - 4
*b*d)*b), -1/2*(2*c*arctan(-sqrt(-c^2 + 4*b*d)*(2*d*x + c)/(c^2 -
 4*b*d)) + sqrt(-c^2 + 4*b*d)*(log(d*x^2 + c*x + b) - 2*log(x)))/
(sqrt(-c^2 + 4*b*d)*b)]
```

Sympy [A] time = 6.67278, size = 564, normalized size = 9.1

$$\left(-\frac{c\sqrt{-4bd+c^2}}{2b\,(4bd-c^2)}-\frac{1}{2b}\right)\log\left(x+\frac{24b^4d^2\left(-\frac{c\sqrt{-4bd+c^2}}{2b(4bd-c^2)}-\frac{1}{2b}\right)^2-14b^3c^2d\left(-\frac{c\sqrt{-4bd+c^2}}{2b(4bd-c^2)}-\frac{1}{2b}\right)^2-12b^3d^2\left(-\frac{c\sqrt{-4bd+c^2}}{2b(4bd-c^2)}-\frac{1}{2b}\right)+2b^2c^4\left(-\frac{c\sqrt{-4bd+}}{2b(4bd-}\right.}{9bcd^2-2c^3d}\right.$$
$$+\left(\frac{c\sqrt{-4bd+c^2}}{2b\,(4bd-c^2)}-\frac{1}{2b}\right)\log\left(x+\frac{24b^4d^2\left(\frac{c\sqrt{-4bd+c^2}}{2b(4bd-c^2)}-\frac{1}{2b}\right)^2-14b^3c^2d\left(\frac{c\sqrt{-4bd+c^2}}{2b(4bd-c^2)}-\frac{1}{2b}\right)^2-12b^3d^2\left(\frac{c\sqrt{-4bd+c^2}}{2b(4bd-c^2)}-\frac{1}{2b}\right)+2b^2c^4\left(\frac{c\sqrt{-4bd+c^2}}{2b(4bd-c^2)}-\frac{1}{2b}\right.}{9bcd^2-2c^3d}\right.$$
$$+\frac{\log\left(x\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(d*x**3+c*x**2+b*x),x)
```

```
[Out] (-c*sqrt(-4*b*d + c**2)/(2*b*(4*b*d - c**2)) - 1/(2*b))*log(x + (
24*b**4*d**2*(-c*sqrt(-4*b*d + c**2)/(2*b*(4*b*d - c**2)) - 1/(2*
b))**2 - 14*b**3*c**2*d*(-c*sqrt(-4*b*d + c**2)/(2*b*(4*b*d - c**
2)) - 1/(2*b))**2 - 12*b**3*d**2*(-c*sqrt(-4*b*d + c**2)/(2*b*(4*
b*d - c**2)) - 1/(2*b)) + 2*b**2*c**4*(-c*sqrt(-4*b*d + c**2)/(2*
b*(4*b*d - c**2)) - 1/(2*b))**2 + 3*b**2*c**2*d*(-c*sqrt(-4*b*d +
 c**2)/(2*b*(4*b*d - c**2)) - 1/(2*b)) - 12*b**2*d**2 + 11*b*c**2
*d - 2*c**4)/(9*b*c*d**2 - 2*c**3*d)) + (c*sqrt(-4*b*d + c**2)/(2
*b*(4*b*d - c**2)) - 1/(2*b))*log(x + (24*b**4*d**2*(c*sqrt(-4*b*
d + c**2)/(2*b*(4*b*d - c**2)) - 1/(2*b))**2 - 14*b**3*c**2*d*(c*
sqrt(-4*b*d + c**2)/(2*b*(4*b*d - c**2)) - 1/(2*b))**2 - 12*b**3*
d**2*(c*sqrt(-4*b*d + c**2)/(2*b*(4*b*d - c**2)) - 1/(2*b)) + 2*b
**2*c**4*(c*sqrt(-4*b*d + c**2)/(2*b*(4*b*d - c**2)) - 1/(2*b))**
2 + 3*b**2*c**2*d*(c*sqrt(-4*b*d + c**2)/(2*b*(4*b*d - c**2)) - 1
/(2*b)) - 12*b**2*d**2 + 11*b*c**2*d - 2*c**4)/(9*b*c*d**2 - 2*c*
*3*d)) + log(x)/b
```

GIAC/XCAS [A] time = 0.263704, size = 84, normalized size = 1.35

$$-\frac{c \arctan\left(\frac{2 \, dx+c}{\sqrt{-c^2+4 \, bd}}\right)}{\sqrt{-c^2 + 4 \, bd}b} - \frac{\ln\left(dx^2 + cx + b\right)}{2 \, b} + \frac{\ln\left(|x|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(d*x^3 + c*x^2 + b*x),x, algorithm="giac")
```

```
[Out] -c*arctan((2*d*x + c)/sqrt(-c^2 + 4*b*d))/(sqrt(-c^2 + 4*b*d)*b)
- 1/2*ln(d*x^2 + c*x + b)/b + ln(abs(x))/b
```

3.27 $\int \frac{1}{a+dx^3}\,dx$

Optimal. Leaf size=115

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{d}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{d}x\right)}{3a^{2/3}\sqrt[3]{d}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{d}}$$

```
[Out] -(ArcTan[(a^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(2
/3)*d^(1/3))) + Log[a^(1/3) + d^(1/3)*x]/(3*a^(2/3)*d^(1/3)) - Lo
g[a^(2/3) - a^(1/3)*d^(1/3)*x + d^(2/3)*x^2]/(6*a^(2/3)*d^(1/3))
```

Rubi [A] time = 0.132862, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{d}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{d}x\right)}{3a^{2/3}\sqrt[3]{d}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{d}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + d*x^3)^(-1),x]
```

```
[Out] -(ArcTan[(a^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(2
/3)*d^(1/3))) + Log[a^(1/3) + d^(1/3)*x]/(3*a^(2/3)*d^(1/3)) - Lo
g[a^(2/3) - a^(1/3)*d^(1/3)*x + d^(2/3)*x^2]/(6*a^(2/3)*d^(1/3))
```

Rubi in Sympy [A] time = 25.01, size = 109, normalized size = 0.95

$$\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{d}x\right)}{3a^{\frac{2}{3}}\sqrt[3]{d}}-\frac{\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{d}x+d^{\frac{2}{3}}x^2\right)}{6a^{\frac{2}{3}}\sqrt[3]{d}}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{d}x}{3}\right)}{\sqrt[3]{a}}\right)}{3a^{\frac{2}{3}}\sqrt[3]{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(d*x**3+a),x)
```

```
[Out] log(a**(1/3) + d**(1/3)*x)/(3*a**(2/3)*d**(1/3)) - log(a**(2/3) -
 a**(1/3)*d**(1/3)*x + d**(2/3)*x**2)/(6*a**(2/3)*d**(1/3)) - sqr
t(3)*atan(sqrt(3)*(a**(1/3)/3 - 2*d**(1/3)*x/3)/a**(1/3))/(3*a**(
2/3)*d**(1/3))
```

Mathematica [A] time = 0.0461809, size = 89, normalized size = 0.77

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{d}x+d^{2/3}x^2\right)-2\log\left(\sqrt[3]{a}+\sqrt[3]{d}x\right)+2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{6a^{2/3}\sqrt[3]{d}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + d*x^3)^(-1),x]
```

```
[Out] -(2*Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*Log[a
^(1/3) + d^(1/3)*x] + Log[a^(2/3) - a^(1/3)*d^(1/3)*x + d^(2/3)*x
^2])/(6*a^(2/3)*d^(1/3))
```

Maple [A] time = 0.007, size = 91, normalized size = 0.8

$$\frac{1}{3\,d}\ln\left(x+\sqrt[3]{\frac{a}{d}}\right)\left(\frac{a}{d}\right)^{-\frac{2}{3}}-\frac{1}{6\,d}\ln\left(x^{2}-x\sqrt[3]{\frac{a}{d}}+\left(\frac{a}{d}\right)^{\frac{2}{3}}\right)\left(\frac{a}{d}\right)^{-\frac{2}{3}}+\frac{\sqrt{3}}{3\,d}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{d}}}-1\right)\right)\left(\frac{a}{d}\right)^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(d*x^3+a),x)
```

```
[Out] 1/3/d/(a/d)^(2/3)*ln(x+(a/d)^(1/3))-1/6/d/(a/d)^(2/3)*ln(x^2-x*(a
/d)^(1/3)+(a/d)^(2/3))+1/3/d/(a/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/
2)*(2/(a/d)^(1/3)*x-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(d*x^3 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.328105, size = 120, normalized size = 1.04

$$-\frac{\sqrt{3}\left(\sqrt{3}\log\left(\left(a^{2}d\right)^{\frac{2}{3}}x^{2}-\left(a^{2}d\right)^{\frac{1}{3}}ax+a^{2}\right)-2\,\sqrt{3}\log\left(\left(a^{2}d\right)^{\frac{1}{3}}x+a\right)-6\,\arctan\left(\frac{2\,\sqrt{3}\left(a^{2}d\right)^{\frac{1}{3}}x-\sqrt{3}a}{3\,a}\right)\right)}{18\,\left(a^{2}d\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(d*x^3 + a),x, algorithm="fricas")
```

```
[Out] -1/18*sqrt(3)*(sqrt(3)*log((a^2*d)^(2/3)*x^2 - (a^2*d)^(1/3)*a*x
+ a^2) - 2*sqrt(3)*log((a^2*d)^(1/3)*x + a) - 6*arctan(1/3*(2*sqr
t(3)*(a^2*d)^(1/3)*x - sqrt(3)*a)/a))/(a^2*d)^(1/3)
```

Sympy [A] time = 0.356894, size = 20, normalized size = 0.17

$$\operatorname{RootSum}\left(27t^{3}a^{2}d-1,\left(t\mapsto t\log\left(3ta+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(d*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**2*d - 1, Lambda(_t, _t*log(3*_t*a + x)))
```

GIAC/XCAS [A] time = 0.26287, size = 151, normalized size = 1.31

$$-\frac{\left(-\frac{a}{d}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,a}+\frac{\sqrt{3}\left(-ad^{2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{d}\right)^{\frac{1}{3}}}\right)}{3\,ad}+\frac{\left(-ad^{2}\right)^{\frac{1}{3}}\ln\left(x^{2}+x\left(-\frac{a}{d}\right)^{\frac{1}{3}}+\left(-\frac{a}{d}\right)^{\frac{2}{3}}\right)}{6\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(d*x^3 + a),x, algorithm="giac")
```

```
[Out] -1/3*(-a/d)^(1/3)*ln(abs(x - (-a/d)^(1/3)))/a + 1/3*sqrt(3)*(-a*d
^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/d)^(1/3))/(-a/d)^(1/3))/(
a*d) + 1/6*(-a*d^2)^(1/3)*ln(x^2 + x*(-a/d)^(1/3) + (-a/d)^(2/3))
/(a*d)
```

3.28 $\int (dx^3)^n \, dx$

Optimal. Leaf size=16

$$\frac{x (dx^3)^n}{3n + 1}$$

```
[Out] (x*(d*x^3)^n)/(1 + 3*n)
```

Rubi [A] time = 0.0111188, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286

$$\frac{x (dx^3)^n}{3n + 1}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x^3)^n,x]

[Out] (x*(d*x^3)^n)/(1 + 3*n)
```

Rubi in Sympy [A] time = 1.90445, size = 22, normalized size = 1.38

$$\frac{x^{-3n} x^{3n+1} (dx^3)^n}{3n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**3)**n,x)

[Out] x**(-3*n)*x**(3*n + 1)*(d*x**3)**n/(3*n + 1)
```

Mathematica [A] time = 0.00350797, size = 16, normalized size = 1.

$$\frac{x (dx^3)^n}{3n + 1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x^3)^n,x]

[Out] (x*(d*x^3)^n)/(1 + 3*n)
```

Maple [A] time = 0.003, size = 17, normalized size = 1.1

$$\frac{x (dx^3)^n}{1 + 3\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^3)^n,x)
```

```
[Out] x*(d*x^3)^n/(1+3*n)
```

Maxima [A] time = 0.800202, size = 23, normalized size = 1.44

$$\frac{d^n x x^{3\,n}}{3\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3)^n,x, algorithm="maxima")

[Out] d^n*x*x^(3*n)/(3*n + 1)
```

Fricas [A] time = 0.33561, size = 22, normalized size = 1.38

$$\frac{\left(dx^3\right)^n x}{3\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3)^n,x, algorithm="fricas")

[Out] (d*x^3)^n*x/(3*n + 1)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**3)**n,x)

[Out] Exception raised: TypeError
```

GIAC/XCAS [A] time = 0.26067, size = 24, normalized size = 1.5

$$\frac{x e^{\left(n \ln\left(dx^3\right)\right)}}{3\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3)^n,x, algorithm="giac")

[Out] x*e^(n*ln(d*x^3))/(3*n + 1)
```

3.29 $\int \left(cx^2+dx^3\right)^n\,dx$

Optimal. Leaf size=55

$$\frac{x\left(\frac{dx}{c}+1\right)^{-n}\left(cx^2+dx^3\right)^n\,{}_2F_1\left(-n,2n+1;2(n+1);-\frac{dx}{c}\right)}{2n+1}$$

```
[Out] (x*(c*x^2 + d*x^3)^n*Hypergeometric2F1[-n, 1 + 2*n, 2*(1 + n), -(
(d*x)/c)])/((1 + 2*n)*(1 + (d*x)/c)^n)
```

Rubi [A] time = 0.0521924, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$\frac{x\left(\frac{dx}{c}+1\right)^{-n}\left(cx^2+dx^3\right)^n\,{}_2F_1\left(-n,2n+1;2(n+1);-\frac{dx}{c}\right)}{2n+1}$$

Antiderivative was successfully verified.

```
[In]  Int[(c*x^2 + d*x^3)^n,x]
```

```
[Out] (x*(c*x^2 + d*x^3)^n*Hypergeometric2F1[-n, 1 + 2*n, 2*(1 + n), -(
(d*x)/c)])/((1 + 2*n)*(1 + (d*x)/c)^n)
```

Rubi in Sympy [A] time = 8.25506, size = 53, normalized size = 0.96

$$\frac{x^{-2n}x^{2n+1}\left(1+\frac{dx}{c}\right)^{-n}\left(cx^2+dx^3\right)^n\,{}_2F_1\left(\left.\begin{matrix}-n,2n+1\\2n+2\end{matrix}\right|-\frac{dx}{c}\right)}{2n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**3+c*x**2)**n,x)
```

```
[Out] x**(-2*n)*x**(2*n + 1)*(1 + d*x/c)**(-n)*(c*x**2 + d*x**3)**n*hyp
er((-n, 2*n + 1), (2*n + 2,), -d*x/c)/(2*n + 1)
```

Mathematica [A] time = 0.0435314, size = 53, normalized size = 0.96

$$\frac{x\left(x^2(c+dx)\right)^n\left(\frac{dx}{c}+1\right)^{-n}\,{}_2F_1\left(-n,2n+1;2n+2;-\frac{dx}{c}\right)}{2n+1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c*x^2 + d*x^3)^n,x]
```

```
[Out] (x*(x^2*(c + d*x))^n*Hypergeometric2F1[-n, 1 + 2*n, 2 + 2*n, -((d
*x)/c)])/((1 + 2*n)*(1 + (d*x)/c)^n)
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int \left(dx^3+cx^2\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^3+c*x^2)^n,x)
```

```
[Out] int((d*x^3+c*x^2)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(dx^3 + cx^2\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c*x^2)^n,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c*x^2)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(dx^3 + cx^2\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c*x^2)^n,x, algorithm="fricas")
```

```
[Out] integral((d*x^3 + c*x^2)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2 + dx^3\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**3+c*x**2)**n,x)
```

```
[Out] Integral((c*x**2 + d*x**3)**n, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(dx^3 + cx^2\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c*x^2)^n,x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c*x^2)^n, x)
```

3.30 $\int \left(bx+dx^3\right)^n\,dx$

Optimal. Leaf size=53

$$\frac{x\left(b+dx^2\right)\left(bx+dx^3\right)^n\,{}_2F_1\left(1,\frac{3(n+1)}{2};\frac{n+3}{2};-\frac{dx^2}{b}\right)}{b(n+1)}$$

```
[Out] (x*(b + d*x^2)*(b*x + d*x^3)^n*Hypergeometric2F1[1, (3*(1 + n))/2
, (3 + n)/2, -((d*x^2)/b)])/(b*(1 + n))
```

Rubi [A] time = 0.0579224, antiderivative size = 59, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273

$$\frac{x\left(\frac{dx^2}{b}+1\right)^{-n}\left(bx+dx^3\right)^n\,{}_2F_1\left(-n,\frac{n+1}{2};\frac{n+3}{2};-\frac{dx^2}{b}\right)}{n+1}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x + d*x^3)^n,x]
```

```
[Out] (x*(b*x + d*x^3)^n*Hypergeometric2F1[-n, (1 + n)/2, (3 + n)/2, -(
(d*x^2)/b)])/((1 + n)*(1 + (d*x^2)/b)^n)
```

Rubi in Sympy [A] time = 9.85145, size = 53, normalized size = 1.

$$\frac{x^{-n}x^{n+1}\left(1+\frac{dx^2}{b}\right)^{-n}\left(bx+dx^3\right)^n\,{}_2F_1\left(\begin{matrix}-n,\frac{n}{2}+\frac{1}{2}\\ \frac{n}{2}+\frac{3}{2}\end{matrix}\middle|-\frac{dx^2}{b}\right)}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**3+b*x)**n,x)
```

```
[Out] x**(-n)*x**(n + 1)*(1 + d*x**2/b)**(-n)*(b*x + d*x**3)**n*hyper((
-n, n/2 + 1/2), (n/2 + 3/2,), -d*x**2/b)/(n + 1)
```

Mathematica [A] time = 0.0469738, size = 61, normalized size = 1.15

$$\frac{x\left(x\left(b+dx^2\right)\right)^n\left(\frac{dx^2}{b}+1\right)^{-n}\,{}_2F_1\left(-n,\frac{n+1}{2};\frac{n+1}{2}+1;-\frac{dx^2}{b}\right)}{n+1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x + d*x^3)^n,x]
```

```
[Out] (x*(x*(b + d*x^2))^n*Hypergeometric2F1[-n, (1 + n)/2, 1 + (1 + n)
/2, -((d*x^2)/b)])/((1 + n)*(1 + (d*x^2)/b)^n)
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int \left(dx^3+bx\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^3+b*x)^n,x)
```

```
[Out] int((d*x^3+b*x)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dx^3 + bx)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + b*x)^n,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + b*x)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((dx^3 + bx)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + b*x)^n,x, algorithm="fricas")
```

```
[Out] integral((d*x^3 + b*x)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (bx + dx^3)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**3+b*x)**n,x)
```

```
[Out] Integral((b*x + d*x**3)**n, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int (dx^3 + bx)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + b*x)^n,x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + b*x)^n, x)
```

3.31 $\int \left(bx+cx^2+dx^3\right)^n\,dx$

Optimal. Leaf size=132

$$\frac{x\left(\frac{2dx}{c-\sqrt{c^2-4bd}}+1\right)^{-n}\left(\frac{2dx}{\sqrt{c^2-4bd}+c}+1\right)^{-n}\left(bx+cx^2+dx^3\right)^n F_1\left(n+1;-n,-n;n+2;-\frac{2dx}{c-\sqrt{c^2-4bd}},-\frac{2dx}{c+\sqrt{c^2-4bd}}\right)}{n+1}$$

```
[Out] (x*(b*x + c*x^2 + d*x^3)^n*AppellF1[1 + n, -n, -n, 2 + n, (-2*d*x
)/(c - Sqrt[c^2 - 4*b*d]), (-2*d*x)/(c + Sqrt[c^2 - 4*b*d])])/((1
 + n)*(1 + (2*d*x)/(c - Sqrt[c^2 - 4*b*d]))^n*(1 + (2*d*x)/(c + S
qrt[c^2 - 4*b*d]))^n)
```

Rubi [A] time = 0.391994, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188

$$\frac{x\left(\frac{2dx}{c-\sqrt{c^2-4bd}}+1\right)^{-n}\left(\frac{2dx}{\sqrt{c^2-4bd}+c}+1\right)^{-n}\left(bx+cx^2+dx^3\right)^n F_1\left(n+1;-n,-n;n+2;-\frac{2dx}{c-\sqrt{c^2-4bd}},-\frac{2dx}{c+\sqrt{c^2-4bd}}\right)}{n+1}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x + c*x^2 + d*x^3)^n,x]
```

```
[Out] (x*(b*x + c*x^2 + d*x^3)^n*AppellF1[1 + n, -n, -n, 2 + n, (-2*d*x
)/(c - Sqrt[c^2 - 4*b*d]), (-2*d*x)/(c + Sqrt[c^2 - 4*b*d])])/((1
 + n)*(1 + (2*d*x)/(c - Sqrt[c^2 - 4*b*d]))^n*(1 + (2*d*x)/(c + S
qrt[c^2 - 4*b*d]))^n)
```

Rubi in Sympy [A] time = 29.7021, size = 117, normalized size = 0.89

$$\frac{x^{-n}x^{n+1}\left(\frac{2dx}{c-\sqrt{-4bd+c^2}}+1\right)^{-n}\left(\frac{2dx}{c+\sqrt{-4bd+c^2}}+1\right)^{-n}\left(bx+cx^2+dx^3\right)^n \operatorname{appellf}_1\left(n+1,-n,-n,n+2,-\frac{2dx}{c-\sqrt{-4bd+c^2}},-\frac{2dx}{c+\sqrt{-4bd+c^2}}\right)}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**3+c*x**2+b*x)**n,x)
```

```
[Out] x**(-n)*x**(n + 1)*(2*d*x/(c - sqrt(-4*b*d + c**2)) + 1)**(-n)*(2
*d*x/(c + sqrt(-4*b*d + c**2)) + 1)**(-n)*(b*x + c*x**2 + d*x**3)
**n*appellf1(n + 1, -n, -n, n + 2, -2*d*x/(c - sqrt(-4*b*d + c**2
)), -2*d*x/(c + sqrt(-4*b*d + c**2)))/(n + 1)
```

Mathematica [B] time = 4.45954, size = 438, normalized size = 3.32

$$\frac{d2^{-n-1}(n+2)x^2\left(\sqrt{c^2-4bd}+c\right)\left(x\left(c-\sqrt{c^2-4bd}\right)+2b\right)^2\left(\frac{c-\sqrt{c^2-4bd}}{2d}+x\right)^{-n}\Big(}{(n+1)\left(\sqrt{c^2-4bd}-c\right)\left(\sqrt{c^2-4bd}+c+2dx\right)\left(nx\left(\left(\sqrt{c^2-4bd}-c\right)F_1\left(n+2;1-n,-n;n+3;-\frac{2dx}{c+\sqrt{c^2-4bd}},\frac{2dx}{\sqrt{c^2-4bd}-c}\right)-\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(b*x + c*x^2 + d*x^3)^n,x]
```

```
[Out] (2^(-1 - n)*d*(c + Sqrt[c^2 - 4*b*d])*(2 + n)*x^2*((c - Sqrt[c^2
- 4*b*d] + 2*d*x)/d)^(1 + n)*(2*b + (c - Sqrt[c^2 - 4*b*d])*x)^2*
```

```
(x*(b + x*(c + d*x)))^(-1 + n)*AppellF1[1 + n, -n, -n, 2 + n, (-2
*d*x)/(c + Sqrt[c^2 - 4*b*d]), (2*d*x)/(-c + Sqrt[c^2 - 4*b*d])])
/((-c + Sqrt[c^2 - 4*b*d])*(1 + n)*((c - Sqrt[c^2 - 4*b*d])/(2*d)
 + x)^n*(c + Sqrt[c^2 - 4*b*d] + 2*d*x)*(-2*b*(2 + n)*AppellF1[1
+ n, -n, -n, 2 + n, (-2*d*x)/(c + Sqrt[c^2 - 4*b*d]), (2*d*x)/(-c
 + Sqrt[c^2 - 4*b*d])] + n*x*((-c + Sqrt[c^2 - 4*b*d])*AppellF1[2
 + n, 1 - n, -n, 3 + n, (-2*d*x)/(c + Sqrt[c^2 - 4*b*d]), (2*d*x)
/(-c + Sqrt[c^2 - 4*b*d])] - (c + Sqrt[c^2 - 4*b*d])*AppellF1[2 +
 n, -n, 1 - n, 3 + n, (-2*d*x)/(c + Sqrt[c^2 - 4*b*d]), (2*d*x)/(
-c + Sqrt[c^2 - 4*b*d])])))
```

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$\int \left(d x^{3}+c x^{2}+b x\right)^{n} d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^3+c*x^2+b*x)^n,x)

[Out] int((d*x^3+c*x^2+b*x)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d x^{3}+c x^{2}+b x\right)^{n} d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c*x^2 + b*x)^n,x, algorithm="maxima")

[Out] integrate((d*x^3 + c*x^2 + b*x)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(d x^{3}+c x^{2}+b x\right)^{n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c*x^2 + b*x)^n,x, algorithm="fricas")

[Out] integral((d*x^3 + c*x^2 + b*x)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**3+c*x**2+b*x)**n,x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int (dx^3 + cx^2 + bx)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c*x^2 + b*x)^n,x, algorithm="giac")

[Out] integrate((d*x^3 + c*x^2 + b*x)^n, x)
```

3.32 $\int \left(a+dx^3\right)^n \, dx$

Optimal. Leaf size=35

$$\frac{x \left(a+dx^3\right)^{n+1} {}_2F_1\left(1,n+\frac{4}{3};\frac{4}{3};-\frac{dx^3}{a}\right)}{a}$$

```
[Out] (x*(a + d*x^3)^(1 + n)*Hypergeometric2F1[1, 4/3 + n, 4/3, -((d*x^
3)/a)])/a
```

Rubi [A] time = 0.0244163, antiderivative size = 44, normalized size of antiderivative = 1.26, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$x \left(a+dx^3\right)^n \left(\frac{dx^3}{a}+1\right)^{-n} {}_2F_1\left(\frac{1}{3},-n;\frac{4}{3};-\frac{dx^3}{a}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(a + d*x^3)^n,x]
```

```
[Out] (x*(a + d*x^3)^n*Hypergeometric2F1[1/3, -n, 4/3, -((d*x^3)/a)])/(
1 + (d*x^3)/a)^n
```

Rubi in Sympy [A] time = 3.71689, size = 34, normalized size = 0.97

$$x \left(1+\frac{dx^3}{a}\right)^{-n} \left(a+dx^3\right)^n {}_2F_1\left(\begin{matrix} -n, \frac{1}{3} \\ \frac{4}{3} \end{matrix}\middle| -\frac{dx^3}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**3+a)**n,x)
```

```
[Out] x*(1 + d*x**3/a)**(-n)*(a + d*x**3)**n*hyper((-n, 1/3), (4/3,), -
d*x**3/a)
```

Mathematica [C] time = 0.304042, size = 196, normalized size = 5.6

$$\frac{2^{-n} \left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{d}x\right) \left(\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{d}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}\right)^{-n} \left(\frac{i\left(\frac{\sqrt[3]{d}x}{\sqrt[3]{a}}+1\right)}{\sqrt{3}+3i}\right)^{-n} \left(a+dx^3\right)^n F_1\left(n+1;-n,-n;n+2;-\frac{i\left(\sqrt[3]{d}x+(-1)^{2/3}\sqrt[3]{a}\right)}{\sqrt{3}\sqrt[3]{a}},\frac{-\frac{2i\sqrt[3]{d}x}{\sqrt[3]{a}}+ \ldots}{3i+\sqrt{3}}\right)}{\sqrt[3]{d}(n+1)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + d*x^3)^n,x]
```

```
[Out] (((-1)^(2/3)*a^(1/3) + d^(1/3)*x)*(a + d*x^3)^n*AppellF1[1 + n, -
n, -n, 2 + n, ((-I)*((-1)^(2/3)*a^(1/3) + d^(1/3)*x))/(Sqrt[3]*a^
(1/3)), (I + Sqrt[3] - ((2*I)*d^(1/3)*x)/a^(1/3))/(3*I + Sqrt[3])
])/(2^n*d^(1/3)*(1 + n)*((a^(1/3) + (-1)^(2/3)*d^(1/3)*x)/((1 + (
-1)^(1/3))*a^(1/3)))^n*((I*(1 + (d^(1/3)*x)/a^(1/3)))/(3*I + Sqrt
[3]))^n)
```

Maple [F] time = 0.048, size = 0, normalized size = 0.

$$\int \left(d x^{3}+a\right)^{n}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^3+a)^n,x)

[Out] int((d*x^3+a)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(dx^{3}+a\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + a)^n,x, algorithm="maxima")

[Out] integrate((d*x^3 + a)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(dx^{3}+a\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + a)^n,x, algorithm="fricas")

[Out] integral((d*x^3 + a)^n, x)
```

Sympy [A] time = 68.8391, size = 34, normalized size = 0.97

$$\frac{a^{n}x\,\Gamma\left(\frac{1}{3}\right)\,{}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},-n\\\frac{4}{3}\end{matrix}\middle|\frac{dx^{3}e^{i\pi}}{a}\right)}{3\,\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**3+a)**n,x)

[Out] a**n*x*gamma(1/3)*hyper((1/3, -n), (4/3,), d*x**3*exp_polar(I*pi)
/a)/(3*gamma(4/3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(dx^{3}+a\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + a)^n,x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + a)^n, x)
```

3.33 $\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^4\,dx$

Optimal. Leaf size=270

$$\frac{2}{9}c^2\left(48a^2d^4+120ac^3d^2+35c^6\right)\left(\frac{c}{d}+x\right)^9-\frac{8}{11}c^3d^2\left(12ad^2+7c^3\right)\left(\frac{c}{d}+x\right)^{11}+\frac{4}{13}cd^4\left(4ad^2+7c^3\right)\left(\frac{c}{d}+x\right)^{13}+\frac{4c^3\left(4ad^2+c^3\right)^2\left(4ad^2+7c^3\right)\left(\frac{c}{d}+x\right)^5}{5d^4}-\frac{8c^5\left(4ad^2+c^3\right)^3\left(\frac{c}{d}+x\right)^3}{3d^6}-\frac{8c^4\left(4ad^2+c^3\right)\left(12ad^2+7c^3\right)\left(\frac{c}{d}+x\right)^7}{7d^2}+\frac{c^4x\left(4ad^2+c^3\right)^4}{d^8}-\frac{8}{15}c^2d^6\left(\frac{c}{d}+x\right)^{15}+\frac{1}{17}d^8\left(\frac{c}{d}+x\right)^{17}$$

```
[Out] (c^4*(c^3 + 4*a*d^2)^4*x)/d^8 - (8*c^5*(c^3 + 4*a*d^2)^3*(c/d + x
)^3)/(3*d^6) + (4*c^3*(c^3 + 4*a*d^2)^2*(7*c^3 + 4*a*d^2)*(c/d +
x)^5)/(5*d^4) - (8*c^4*(c^3 + 4*a*d^2)*(7*c^3 + 12*a*d^2)*(c/d +
x)^7)/(7*d^2) + (2*c^2*(35*c^6 + 120*a*c^3*d^2 + 48*a^2*d^4)*(c/d
 + x)^9)/9 - (8*c^3*d^2*(7*c^3 + 12*a*d^2)*(c/d + x)^11)/11 + (4*
c*d^4*(7*c^3 + 4*a*d^2)*(c/d + x)^13)/13 - (8*c^2*d^6*(c/d + x)^1
5)/15 + (d^8*(c/d + x)^17)/17
```

Rubi [A] time = 1.04918, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}=0.069$

$$\frac{2c^2\left(48a^2d^4+120ac^3d^2+35c^6\right)(c+dx)^9}{9d^9}+\frac{4c\left(4ad^2+7c^3\right)(c+dx)^{13}}{13d^9}-\frac{8c^3\left(12ad^2+7c^3\right)(c+dx)^{11}}{11d^9}+\frac{4c^3\left(4ad^2+c^3\right)^2\left(4ad^2+7c^3\right)(c+dx)^5}{5d^9}-\frac{8c^5\left(4ad^2+c^3\right)^3(c+dx)^3}{3d^9}-\frac{8c^4\left(4ad^2+c^3\right)\left(12ad^2+7c^3\right)(c+dx)^7}{7d^9}+\frac{c^4x\left(4ad^2+c^3\right)^4}{d^8}-\frac{8c^2(c+dx)^{15}}{15d^9}+\frac{(c+dx)^{17}}{17d^9}$$

Antiderivative was successfully verified.

```
[In]  Int[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^4,x]
```

```
[Out] (c^4*(c^3 + 4*a*d^2)^4*x)/d^8 - (8*c^5*(c^3 + 4*a*d^2)^3*(c + d*x
)^3)/(3*d^9) + (4*c^3*(c^3 + 4*a*d^2)^2*(7*c^3 + 4*a*d^2)*(c + d*
x)^5)/(5*d^9) - (8*c^4*(c^3 + 4*a*d^2)*(7*c^3 + 12*a*d^2)*(c + d*
x)^7)/(7*d^9) + (2*c^2*(35*c^6 + 120*a*c^3*d^2 + 48*a^2*d^4)*(c +
 d*x)^9)/(9*d^9) - (8*c^3*(7*c^3 + 12*a*d^2)*(c + d*x)^11)/(11*d^
9) + (4*c*(7*c^3 + 4*a*d^2)*(c + d*x)^13)/(13*d^9) - (8*c^2*(c +
d*x)^15)/(15*d^9) + (c + d*x)^17/(17*d^9)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{8c^5\left(4ad^2+c^3\right)^3\left(\frac{c}{d}+x\right)^3}{3d^6}-\frac{8c^4\left(4ad^2+c^3\right)\left(12ad^2+7c^3\right)\left(\frac{c}{d}+x\right)^7}{7d^2}-\frac{8c^3d^2\left(12ad^2+7c^3\right)\left(\frac{c}{d}+x\right)^{11}}{11}+\frac{4c^3\left(4ad^2+c^3\right)^2\left(4ad^2+7c^3\right)\left(\frac{c}{d}+x\right)^5}{5d^4}-\frac{8c^2d^6\left(\frac{c}{d}+x\right)^{15}}{15}+\frac{2c^2\left(\frac{c}{d}+x\right)^9\left(48a^2d^4+120ac^3d^2+35c^6\right)}{9}+\frac{4cd^4\left(4ad^2+7c^3\right)\left(\frac{c}{d}+x\right)^{13}}{13}+\frac{d^8\left(\frac{c}{d}+x\right)^{17}}{17}+\frac{\left(4ad^2+c^3\right)^4\int^{\frac{c}{d}+x}c^{16}\,dx}{c^{12}d^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**4,x)
```

```
[Out] -8*c**5*(4*a*d**2 + c**3)**3*(c/d + x)**3/(3*d**6) - 8*c**4*(4*a*
d**2 + c**3)*(12*a*d**2 + 7*c**3)*(c/d + x)**7/(7*d**2) - 8*c**3*
d**2*(12*a*d**2 + 7*c**3)*(c/d + x)**11/11 + 4*c**3*(4*a*d**2 + c
**3)**2*(4*a*d**2 + 7*c**3)*(c/d + x)**5/(5*d**4) - 8*c**2*d**6*(
c/d + x)**15/15 + 2*c**2*(c/d + x)**9*(48*a**2*d**4 + 120*a*c**3*
d**2 + 35*c**6)/9 + 4*c*d**4*(4*a*d**2 + 7*c**3)*(c/d + x)**13/13
 + d**8*(c/d + x)**17/17 + (4*a*d**2 + c**3)**4*Integral(c**16, (
x, c/d + x))/(c**12*d**8)
```

Mathematica [A] time = 0.0636101, size = 285, normalized size = 1.06

$$256a^4c^4x+\frac{1024}{3}a^3c^5x^3+256a^3c^4dx^4+512a^2c^5dx^6+\frac{256}{5}a^2c^3x^5\left(ad^2+6c^3\right)+\frac{32}{9}c^2x^9\left(3a^2d^4+120ac^3d^2+8c^6\right)+\frac{64}{11}c^3d^2x^{11}\left(15ad^2+28c^3\right)+96ac^3dx^8\left(ad^2+4c^3\right)+\frac{16}{13}cd^4x^{13}\left(ad^2+70c^3\right)+\frac{256}{5}c^4dx^{10}\left(5ad^2+2c^3\right)+\frac{256}{7}ac^4x^7\left(9ad^2+4c^3\right)+\frac{16}{3}c^2d^3x^{12}\left(3ad^2+28c^3\right)+32c^3d^5x^{14}+\frac{112}{15}c^2d^6x^{15}+cd^7x^{16}+\frac{d^8x^{17}}{17}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^4,x]
```

```
[Out] 256*a^4*c^4*x + (1024*a^3*c^5*x^3)/3 + 256*a^3*c^4*d*x^4 + (256*a
^2*c^3*(6*c^3 + a*d^2)*x^5)/5 + 512*a^2*c^5*d*x^6 + (256*a*c^4*(4
*c^3 + 9*a*d^2)*x^7)/7 + 96*a*c^3*d*(4*c^3 + a*d^2)*x^8 + (32*c^2
*(8*c^6 + 120*a*c^3*d^2 + 3*a^2*d^4)*x^9)/9 + (256*c^4*d*(2*c^3 +
 5*a*d^2)*x^10)/5 + (64*c^3*d^2*(28*c^3 + 15*a*d^2)*x^11)/11 + (1
6*c^2*d^3*(28*c^3 + 3*a*d^2)*x^12)/3 + (16*c*d^4*(70*c^3 + a*d^2)
*x^13)/13 + 32*c^3*d^5*x^14 + (112*c^2*d^6*x^15)/15 + c*d^7*x^16
+ (d^8*x^17)/17
```

Maple [A] time = 0.002, size = 392, normalized size = 1.5

$$\frac{d^8x^{17}}{17}+cd^7x^{16}+\frac{112\,c^2d^6x^{15}}{15}+32\,c^3d^5x^{14}+\frac{\left(2\,\left(8\,acd^2+16\,c^4\right)d^4+1088\,c^4d^4\right)x^{13}}{13}+\frac{\left(64\,ac^2d^5+16\,\left(8\,acd^2+16\,c^4\right)cd^3+1536\,c^5d^3\right)x^{12}}{12}+\frac{\left(576\,ac^3d^4+48\,\left(8\,acd^2+16\,c^4\right)c^2d^2+1024\,c^6d^2\right)x^{11}}{11}+\frac{\left(2048\,ac^4d^3+64\,\left(8\,acd^2+16\,c^4\right)c^3d\right)x^{10}}{10}+\frac{\left(32\,a^2c^2d^4+3584\,ac^5d^2+\left(8\,acd^2+16\,c^4\right)^2\right)x^9}{9}+\frac{\left(256\,a^2c^3d^3+2048\,ac^6d+64\,ac^2d\left(8\,acd^2+16\,c^4\right)\right)x^8}{8}+\frac{\left(1792\,a^2c^4d^2+64\,ac^3\left(8\,acd^2+16\,c^4\right)\right)x^7}{7}+512\,a^2c^5dx^6+\frac{\left(32\,a^2c^2\left(8\,acd^2+16\,c^4\right)+1024\,a^2c^6\right)x^5}{5}+256\,a^3c^4dx^4+\frac{1024\,a^3c^5x^3}{3}+256\,a^4c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^4,x)
```

```
[Out] 1/17*d^8*x^17+c*d^7*x^16+112/15*c^2*d^6*x^15+32*c^3*d^5*x^14+1/13
*(2*(8*a*c*d^2+16*c^4)*d^4+1088*c^4*d^4)*x^13+1/12*(64*a*c^2*d^5+
16*(8*a*c*d^2+16*c^4)*c*d^3+1536*c^5*d^3)*x^12+1/11*(576*a*c^3*d^
4+48*(8*a*c*d^2+16*c^4)*c^2*d^2+1024*c^6*d^2)*x^11+1/10*(2048*a*c
^4*d^3+64*(8*a*c*d^2+16*c^4)*c^3*d)*x^10+1/9*(32*a^2*c^2*d^4+3584
*a*c^5*d^2+(8*a*c*d^2+16*c^4)^2)*x^9+1/8*(256*a^2*c^3*d^3+2048*a*
```

```
c^6*d+64*a*c^2*d*(8*a*c*d^2+16*c^4))*x^8+1/7*(1792*a^2*c^4*d^2+64
*a*c^3*(8*a*c*d^2+16*c^4))*x^7+512*a^2*c^5*d*x^6+1/5*(32*a^2*c^2*
(8*a*c*d^2+16*c^4)+1024*a^2*c^6)*x^5+256*a^3*c^4*d*x^4+1024/3*a^3
*c^5*x^3+256*a^4*c^4*x
```

Maxima [A] time = 0.789407, size = 502, normalized size = 1.86

$$
\begin{aligned}
&\frac{1}{17}\,d^8x^{17} + cd^7x^{16} + \frac{32}{5}\,c^2d^6x^{15} + \frac{128}{7}\,c^3d^5x^{14} + \frac{256}{13}\,c^4d^4x^{13} + \frac{256}{9}\,c^8x^9 \\
&+ 256\,a^4c^4x + \frac{256}{15}\left(3\,d^2x^5 + 15\,cdx^4 + 20\,c^2x^3\right)a^3c^3 + \frac{256}{55}\left(5\,d^2x^{11} + 22\,cdx^{10}\right)c^6 \\
&+ \frac{32}{105}\left(35\,d^4x^9 + 315\,cd^3x^8 + 720\,c^2d^2x^7 + 1008\,c^4x^5 + 120\left(3\,d^2x^7 + 14\,cdx^6\right)c^2\right)a^2c^2 \\
&+ \frac{32}{143}\left(33\,d^4x^{13} + 286\,cd^3x^{12} + 624\,c^2d^2x^{11}\right)c^4 \\
&+ \frac{16}{15015}\left(1155\,d^6x^{13} + 15015\,cd^5x^{12} + 65520\,c^2d^4x^{11} + 96096\,c^3d^3x^{10} + 137280\,c^6x^7 + 40040\left(2\,d^2x^9 + 9\,cdx^8\right)c^4 + 364\left(45\,d\right.\right. \\
&+ \frac{16}{1365}\left(91\,d^6x^{15} + 1170\,cd^5x^{14} + 5040\,c^2d^4x^{13} + 7280\,c^3d^3x^{12}\right)c^2
\end{aligned}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c)^4,x, algorithm="maxima")
```

```
[Out] 1/17*d^8*x^17 + c*d^7*x^16 + 32/5*c^2*d^6*x^15 + 128/7*c^3*d^5*x^
14 + 256/13*c^4*d^4*x^13 + 256/9*c^8*x^9 + 256*a^4*c^4*x + 256/15
*(3*d^2*x^5 + 15*c*d*x^4 + 20*c^2*x^3)*a^3*c^3 + 256/55*(5*d^2*x^
11 + 22*c*d*x^10)*c^6 + 32/105*(35*d^4*x^9 + 315*c*d^3*x^8 + 720*
c^2*d^2*x^7 + 1008*c^4*x^5 + 120*(3*d^2*x^7 + 14*c*d*x^6)*c^2)*a^
2*c^2 + 32/143*(33*d^4*x^13 + 286*c*d^3*x^12 + 624*c^2*d^2*x^11)*
c^4 + 16/15015*(1155*d^6*x^13 + 15015*c*d^5*x^12 + 65520*c^2*d^4*
x^11 + 96096*c^3*d^3*x^10 + 137280*c^6*x^7 + 40040*(2*d^2*x^9 + 9
*c*d*x^8)*c^4 + 364*(45*d^4*x^11 + 396*c*d^3*x^10 + 880*c^2*d^2*x
^9)*c^2)*a*c + 16/1365*(91*d^6*x^15 + 1170*c*d^5*x^14 + 5040*c^2*
d^4*x^13 + 7280*c^3*d^3*x^12)*c^2
```

Fricas [A] time = 0.243362, size = 1, normalized size = 0.

$$
\begin{aligned}
&\frac{1}{17}x^{17}d^8 + x^{16}d^7c + \frac{112}{15}x^{15}d^6c^2 + 32x^{14}d^5c^3 + \frac{1120}{13}x^{13}d^4c^4 + \frac{16}{13}x^{13}d^6ca + \frac{448}{3}x^{12}d^3c^5 \\
&+ 16x^{12}d^5c^2a + \frac{1792}{11}x^{11}d^2c^6 + \frac{960}{11}x^{11}d^4c^3a + \frac{512}{5}x^{10}dc^7 + 256x^{10}d^3c^4a + \frac{256}{9}x^9c^8 \\
&+ \frac{1280}{3}x^9d^2c^5a + \frac{32}{3}x^9d^4c^2a^2 + 384x^8dc^6a + 96x^8d^3c^3a^2 + \frac{1024}{7}x^7c^7a + \frac{2304}{7}x^7d^2c^4a^2 \\
&+ 512x^6dc^5a^2 + \frac{1536}{5}x^5c^6a^2 + \frac{256}{5}x^5d^2c^3a^3 + 256x^4dc^4a^3 + \frac{1024}{3}x^3c^5a^3 + 256xc^4a^4
\end{aligned}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c)^4,x, algorithm="fricas")
```

```
[Out] 1/17*x^17*d^8 + x^16*d^7*c + 112/15*x^15*d^6*c^2 + 32*x^14*d^5*c^
3 + 1120/13*x^13*d^4*c^4 + 16/13*x^13*d^6*c*a + 448/3*x^12*d^3*c^
5 + 16*x^12*d^5*c^2*a + 1792/11*x^11*d^2*c^6 + 960/11*x^11*d^4*c^
3*a + 512/5*x^10*d*c^7 + 256*x^10*d^3*c^4*a + 256/9*x^9*c^8 + 128
0/3*x^9*d^2*c^5*a + 32/3*x^9*d^4*c^2*a^2 + 384*x^8*d*c^6*a + 96*x
^8*d^3*c^3*a^2 + 1024/7*x^7*c^7*a + 2304/7*x^7*d^2*c^4*a^2 + 512*
x^6*d*c^5*a^2 + 1536/5*x^5*c^6*a^2 + 256/5*x^5*d^2*c^3*a^3 + 256*
x^4*d*c^4*a^3 + 1024/3*x^3*c^5*a^3 + 256*x*c^4*a^4
```

Sympy [A] time = 0.257015, size = 299, normalized size = 1.11

$$256a^4c^4x + \frac{1024a^3c^5x^3}{3} + 256a^3c^4dx^4 + 512a^2c^5dx^6 + 32c^3d^5x^{14} + \frac{112c^2d^6x^{15}}{15} + cd^7x^{16} + \frac{d^8x^{17}}{17} + x^{13}\left(\frac{16acd^6}{13} + \frac{1120c^4d^4}{13}\right) + x^{12}\left(16ac^2d^5 + \frac{448c^5d^3}{3}\right) + x^{11}\left(\frac{960ac^3d^4}{11} + \frac{1792c^6d^2}{11}\right) + x^{10}\left(256ac^4d^3 + \frac{512c^7d}{5}\right) + x^9\left(\frac{32a^2c^2d^4}{3} + \frac{1280ac^5d^2}{3} + \frac{256c^8}{9}\right) + x^8\left(96a^2c^3d^3 + 384ac^6d\right) + x^7\left(\frac{2304a^2c^4d^2}{7} + \frac{1024ac^7}{7}\right) + x^5\left(\frac{256a^3c^3d^2}{5} + \frac{1536a^2c^6}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**4,x)
```

```
[Out] 256*a**4*c**4*x + 1024*a**3*c**5*x**3/3 + 256*a**3*c**4*d*x**4 +
512*a**2*c**5*d*x**6 + 32*c**3*d**5*x**14 + 112*c**2*d**6*x**15/1
5 + c*d**7*x**16 + d**8*x**17/17 + x**13*(16*a*c*d**6/13 + 1120*c
**4*d**4/13) + x**12*(16*a*c**2*d**5 + 448*c**5*d**3/3) + x**11*(
960*a*c**3*d**4/11 + 1792*c**6*d**2/11) + x**10*(256*a*c**4*d**3
+ 512*c**7*d/5) + x**9*(32*a**2*c**2*d**4/3 + 1280*a*c**5*d**2/3
+ 256*c**8/9) + x**8*(96*a**2*c**3*d**3 + 384*a*c**6*d) + x**7*(2
304*a**2*c**4*d**2/7 + 1024*a*c**7/7) + x**5*(256*a**3*c**3*d**2/
5 + 1536*a**2*c**6/5)
```

GIAC/XCAS [A] time = 0.261679, size = 374, normalized size = 1.39

$$\frac{1}{17}d^8x^{17} + cd^7x^{16} + \frac{112}{15}c^2d^6x^{15} + 32c^3d^5x^{14} + \frac{1120}{13}c^4d^4x^{13} + \frac{16}{13}acd^6x^{13} + \frac{448}{3}c^5d^3x^{12} + 16ac^2d^5x^{12} + \frac{1792}{11}c^6d^2x^{11} + \frac{960}{11}ac^3d^4x^{11} + \frac{512}{5}c^7dx^{10} + 256ac^4d^3x^{10} + \frac{256}{9}c^8x^9 + \frac{1280}{3}ac^5d^2x^9 + \frac{32}{3}a^2c^2d^4x^9 + 384ac^6dx^8 + 96a^2c^3d^3x^8 + \frac{1024}{7}ac^7x^7 + \frac{2304}{7}a^2c^4d^2x^7 + 512a^2c^5dx^6 + \frac{1536}{5}a^2c^6x^5 + \frac{256}{5}a^3c^3d^2x^5 + 256a^3c^4dx^4 + \frac{1024}{3}a^3c^5x^3 + 256a^4c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c)^4,x, algorithm="giac")
```

```
[Out] 1/17*d^8*x^17 + c*d^7*x^16 + 112/15*c^2*d^6*x^15 + 32*c^3*d^5*x^1
4 + 1120/13*c^4*d^4*x^13 + 16/13*a*c*d^6*x^13 + 448/3*c^5*d^3*x^1
2 + 16*a*c^2*d^5*x^12 + 1792/11*c^6*d^2*x^11 + 960/11*a*c^3*d^4*x
^11 + 512/5*c^7*d*x^10 + 256*a*c^4*d^3*x^10 + 256/9*c^8*x^9 + 128
0/3*a*c^5*d^2*x^9 + 32/3*a^2*c^2*d^4*x^9 + 384*a*c^6*d*x^8 + 96*a
^2*c^3*d^3*x^8 + 1024/7*a*c^7*x^7 + 2304/7*a^2*c^4*d^2*x^7 + 512*
a^2*c^5*d*x^6 + 1536/5*a^2*c^6*x^5 + 256/5*a^3*c^3*d^2*x^5 + 256*
a^3*c^4*d*x^4 + 1024/3*a^3*c^5*x^3 + 256*a^4*c^4*x
```

3.34 $\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^3 \, dx$

Optimal. Leaf size=171

$$64a^3c^3x+64a^2c^4x^3+48a^2c^3dx^4+64ac^4dx^6+\frac{4}{3}cd^2x^9\left(ad^2+20c^3\right)+\frac{32}{7}c^3x^7\left(9ad^2+2c^3\right)+12c^2dx^8\left(ad^2+2c^3\right)+\frac{48}{5}ac^2x^5\left(ad^2+4c^3\right)+16c^3d^3x^{10}+\frac{60}{11}c^2d^4x^{11}+cd^5x^{12}+\frac{d^6x^{13}}{13}$$

```
[Out] 64*a^3*c^3*x + 64*a^2*c^4*x^3 + 48*a^2*c^3*d*x^4 + (48*a*c^2*(4*c
^3 + a*d^2)*x^5)/5 + 64*a*c^4*d*x^6 + (32*c^3*(2*c^3 + 9*a*d^2)*x
^7)/7 + 12*c^2*d*(2*c^3 + a*d^2)*x^8 + (4*c*d^2*(20*c^3 + a*d^2)*
x^9)/3 + 16*c^3*d^3*x^10 + (60*c^2*d^4*x^11)/11 + c*d^5*x^12 + (d
^6*x^13)/13
```

Rubi [A] time = 0.193106, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034

$$64a^3c^3x+64a^2c^4x^3+48a^2c^3dx^4+64ac^4dx^6+\frac{4}{3}cd^2x^9\left(ad^2+20c^3\right)+\frac{32}{7}c^3x^7\left(9ad^2+2c^3\right)+12c^2dx^8\left(ad^2+2c^3\right)+\frac{48}{5}ac^2x^5\left(ad^2+4c^3\right)+16c^3d^3x^{10}+\frac{60}{11}c^2d^4x^{11}+cd^5x^{12}+\frac{d^6x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In]  Int[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^3,x]
```

```
[Out] 64*a^3*c^3*x + 64*a^2*c^4*x^3 + 48*a^2*c^3*d*x^4 + (48*a*c^2*(4*c
^3 + a*d^2)*x^5)/5 + 64*a*c^4*d*x^6 + (32*c^3*(2*c^3 + 9*a*d^2)*x
^7)/7 + 12*c^2*d*(2*c^3 + a*d^2)*x^8 + (4*c*d^2*(20*c^3 + a*d^2)*
x^9)/3 + 16*c^3*d^3*x^10 + (60*c^2*d^4*x^11)/11 + c*d^5*x^12 + (d
^6*x^13)/13
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.0321813, size = 171, normalized size = 1.

$$64a^3c^3x+64a^2c^4x^3+48a^2c^3dx^4+64ac^4dx^6+\frac{4}{3}cd^2x^9\left(ad^2+20c^3\right)+\frac{32}{7}c^3x^7\left(9ad^2+2c^3\right)+12c^2dx^8\left(ad^2+2c^3\right)+\frac{48}{5}ac^2x^5\left(ad^2+4c^3\right)+16c^3d^3x^{10}+\frac{60}{11}c^2d^4x^{11}+cd^5x^{12}+\frac{d^6x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^3,x]
```

```
[Out] 64*a^3*c^3*x + 64*a^2*c^4*x^3 + 48*a^2*c^3*d*x^4 + (48*a*c^2*(4*c
^3 + a*d^2)*x^5)/5 + 64*a*c^4*d*x^6 + (32*c^3*(2*c^3 + 9*a*d^2)*x
```

```
^7)/7 + 12*c^2*d*(2*c^3 + a*d^2)*x^8 + (4*c*d^2*(20*c^3 + a*d^2)*
x^9)/3 + 16*c^3*d^3*x^10 + (60*c^2*d^4*x^11)/11 + c*d^5*x^12 + (d
^6*x^13)/13
```

Maple [A] time = 0.001, size = 231, normalized size = 1.4

$$\frac{d^6x^{13}}{13} + cd^5x^{12} + \frac{60\,c^2d^4x^{11}}{11} + 16\,c^3d^3x^{10} + \frac{\left(4\,acd^4 + 224\,c^4d^2 + d^2\left(8\,acd^2 + 16\,c^4\right)\right)x^9}{9} + \frac{\left(64\,ac^2d^3 + 128\,c^5d + 4\,cd\left(8\,acd^2 + 16\,c^4\right)\right)x^8}{8} + \frac{\left(256\,ac^3d^2 + 4\,c^2\left(8\,acd^2 + 16\,c^4\right)\right)x^7}{7} + 64\,ac^4dx^6 + \frac{\left(4\,ac\left(8\,acd^2 + 16\,c^4\right) + 128\,ac^5 + 16\,a^2c^2d^2\right)x^5}{5} + 48\,a^2c^3dx^4 + 64\,a^2c^4x^3 + 64\,a^3c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^3,x)
```

```
[Out] 1/13*d^6*x^13+c*d^5*x^12+60/11*c^2*d^4*x^11+16*c^3*d^3*x^10+1/9*(
4*a*c*d^4+224*c^4*d^2+d^2*(8*a*c*d^2+16*c^4))*x^9+1/8*(64*a*c^2*d
^3+128*c^5*d+4*c*d*(8*a*c*d^2+16*c^4))*x^8+1/7*(256*a*c^3*d^2+4*c
^2*(8*a*c*d^2+16*c^4))*x^7+64*a*c^4*d*x^6+1/5*(4*a*c*(8*a*c*d^2+1
6*c^4)+128*a*c^5+16*a^2*c^2*d^2)*x^5+48*a^2*c^3*d*x^4+64*a^2*c^4*
x^3+64*a^3*c^3*x
```

Maxima [A] time = 0.7818, size = 277, normalized size = 1.62

$$\frac{1}{13}\,d^6x^{13} + cd^5x^{12} + \frac{48}{11}\,c^2d^4x^{11} + \frac{32}{5}\,c^3d^3x^{10} + \frac{64}{7}\,c^6x^7 + 64\,a^3c^3x + \frac{16}{5}\left(3\,d^2x^5 + 15\,cdx^4 + 20\,c^2x^3\right)a^2c^2 + \frac{8}{3}\left(2\,d^2x^9 + 9\,cdx^8\right)c^4 + \frac{4}{105}\left(35\,d^4x^9 + 315\,cd^3x^8 + 720\,c^2d^2x^7 + 1008\,c^4x^5 + 120\left(3\,d^2x^7 + 14\,cdx^6\right)c^2\right)ac + \frac{4}{165}\left(45\,d^4x^{11} + 396\,cd^3x^{10} + 880\,c^2d^2x^9\right)c^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c)^3,x, algorithm="maxima")
```

```
[Out] 1/13*d^6*x^13 + c*d^5*x^12 + 48/11*c^2*d^4*x^11 + 32/5*c^3*d^3*x^
10 + 64/7*c^6*x^7 + 64*a^3*c^3*x + 16/5*(3*d^2*x^5 + 15*c*d*x^4 +
 20*c^2*x^3)*a^2*c^2 + 8/3*(2*d^2*x^9 + 9*c*d*x^8)*c^4 + 4/105*(3
5*d^4*x^9 + 315*c*d^3*x^8 + 720*c^2*d^2*x^7 + 1008*c^4*x^5 + 120*
(3*d^2*x^7 + 14*c*d*x^6)*c^2)*a*c + 4/165*(45*d^4*x^11 + 396*c*d^
3*x^10 + 880*c^2*d^2*x^9)*c^2
```

Fricas [A] time = 0.229538, size = 1, normalized size = 0.01

$$\frac{1}{13}x^{13}d^6 + x^{12}d^5c + \frac{60}{11}x^{11}d^4c^2 + 16x^{10}d^3c^3 + \frac{80}{3}x^9d^2c^4 + \frac{4}{3}x^9d^4ca + 24x^8dc^5 + 12x^8d^3c^2a + \frac{64}{7}x^7c^6 + \frac{288}{7}x^7d^2c^3a + 64x^6dc^4a + \frac{192}{5}x^5c^5a + \frac{48}{5}x^5d^2c^2a^2 + 48x^4dc^3a^2 + 64x^3c^4a^2 + 64xc^3a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c)^3,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*d^6 + x^12*d^5*c + 60/11*x^11*d^4*c^2 + 16*x^10*d^3*c^3
 + 80/3*x^9*d^2*c^4 + 4/3*x^9*d^4*c*a + 24*x^8*d*c^5 + 12*x^8*d^3
*c^2*a + 64/7*x^7*c^6 + 288/7*x^7*d^2*c^3*a + 64*x^6*d*c^4*a + 19
2/5*x^5*c^5*a + 48/5*x^5*d^2*c^2*a^2 + 48*x^4*d*c^3*a^2 + 64*x^3*
c^4*a^2 + 64*x*c^3*a^3
```

Sympy [A] time = 0.187082, size = 180, normalized size = 1.05

$$64a^3c^3x + 64a^2c^4x^3 + 48a^2c^3dx^4 + 64ac^4dx^6 + 16c^3d^3x^{10} + \frac{60c^2d^4x^{11}}{11} + cd^5x^{12} + \frac{d^6x^{13}}{13} + x^9\left(\frac{4acd^4}{3} + \frac{80c^4d^2}{3}\right) + x^8\left(12ac^2d^3 + 24c^5d\right) + x^7\left(\frac{288ac^3d^2}{7} + \frac{64c^6}{7}\right) + x^5\left(\frac{48a^2c^2d^2}{5} + \frac{192ac^5}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**3,x)
```

```
[Out] 64*a**3*c**3*x + 64*a**2*c**4*x**3 + 48*a**2*c**3*d*x**4 + 64*a*c
**4*d*x**6 + 16*c**3*d**3*x**10 + 60*c**2*d**4*x**11/11 + c*d**5*
x**12 + d**6*x**13/13 + x**9*(4*a*c*d**4/3 + 80*c**4*d**2/3) + x*
*8*(12*a*c**2*d**3 + 24*c**5*d) + x**7*(288*a*c**3*d**2/7 + 64*c*
*6/7) + x**5*(48*a**2*c**2*d**2/5 + 192*a*c**5/5)
```

GIAC/XCAS [A] time = 0.25945, size = 224, normalized size = 1.31

$$\frac{1}{13}d^6x^{13} + cd^5x^{12} + \frac{60}{11}c^2d^4x^{11} + 16\,c^3d^3x^{10} + \frac{80}{3}c^4d^2x^9 + \frac{4}{3}acd^4x^9 + 24\,c^5dx^8 + 12\,ac^2d^3x^8 + \frac{64}{7}c^6x^7 + \frac{288}{7}ac^3d^2x^7 + 64\,ac^4dx^6 + \frac{192}{5}ac^5x^5 + \frac{48}{5}a^2c^2d^2x^5 + 48\,a^2c^3dx^4 + 64\,a^2c^4x^3 + 64\,a^3c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c)^3,x, algorithm="giac")
```

```
[Out] 1/13*d^6*x^13 + c*d^5*x^12 + 60/11*c^2*d^4*x^11 + 16*c^3*d^3*x^10
 + 80/3*c^4*d^2*x^9 + 4/3*a*c*d^4*x^9 + 24*c^5*d*x^8 + 12*a*c^2*d
^3*x^8 + 64/7*c^6*x^7 + 288/7*a*c^3*d^2*x^7 + 64*a*c^4*d*x^6 + 19
2/5*a*c^5*x^5 + 48/5*a^2*c^2*d^2*x^5 + 48*a^2*c^3*d*x^4 + 64*a^2*
c^4*x^3 + 64*a^3*c^3*x
```

3.35 $\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^2 dx$

Optimal. Leaf size=92

$$16a^2c^2x+\frac{8}{5}cx^5\left(ad^2+2c^3\right)+\frac{32}{3}ac^3x^3+8ac^2dx^4+\frac{16}{3}c^3dx^6+\frac{24}{7}c^2d^2x^7+cd^3x^8+\frac{d^4x^9}{9}$$

```
[Out] 16*a^2*c^2*x + (32*a*c^3*x^3)/3 + 8*a*c^2*d*x^4 + (8*c*(2*c^3 + a
*d^2)*x^5)/5 + (16*c^3*d*x^6)/3 + (24*c^2*d^2*x^7)/7 + c*d^3*x^8
+ (d^4*x^9)/9
```

Rubi [A] time = 0.0887364, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$

$$16a^2c^2x+\frac{8}{5}cx^5\left(ad^2+2c^3\right)+\frac{32}{3}ac^3x^3+8ac^2dx^4+\frac{16}{3}c^3dx^6+\frac{24}{7}c^2d^2x^7+cd^3x^8+\frac{d^4x^9}{9}$$

Antiderivative was successfully verified.

```
[In]  Int[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^2,x]
```

```
[Out] 16*a^2*c^2*x + (32*a*c^3*x^3)/3 + 8*a*c^2*d*x^4 + (8*c*(2*c^3 + a
*d^2)*x^5)/5 + (16*c^3*d*x^6)/3 + (24*c^2*d^2*x^7)/7 + c*d^3*x^8
+ (d^4*x^9)/9
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.0138229, size = 92, normalized size = 1.

$$16a^2c^2x+\frac{8}{5}cx^5\left(ad^2+2c^3\right)+\frac{32}{3}ac^3x^3+8ac^2dx^4+\frac{16}{3}c^3dx^6+\frac{24}{7}c^2d^2x^7+cd^3x^8+\frac{d^4x^9}{9}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^2,x]
```

```
[Out] 16*a^2*c^2*x + (32*a*c^3*x^3)/3 + 8*a*c^2*d*x^4 + (8*c*(2*c^3 + a
*d^2)*x^5)/5 + (16*c^3*d*x^6)/3 + (24*c^2*d^2*x^7)/7 + c*d^3*x^8
+ (d^4*x^9)/9
```

Maple [A] time = 0.001, size = 84, normalized size = 0.9

$$\frac{d^4x^9}{9}+cd^3x^8+\frac{24\,c^2d^2x^7}{7}+\frac{16\,c^3dx^6}{3}+\frac{\left(8\,acd^2+16\,c^4\right)x^5}{5}+8\,ac^2dx^4+\frac{32\,ac^3x^3}{3}+16\,a^2c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x)
```

```
[Out] 1/9*d^4*x^9+c*d^3*x^8+24/7*c^2*d^2*x^7+16/3*c^3*d*x^6+1/5*(8*a*c*
d^2+16*c^4)*x^5+8*a*c^2*d*x^4+32/3*a*c^3*x^3+16*a^2*c^2*x
```

Maxima [A] time = 0.768284, size = 127, normalized size = 1.38

$$\frac{1}{9}d^4x^9 + cd^3x^8 + \frac{16}{7}c^2d^2x^7 + \frac{16}{5}c^4x^5 + 16a^2c^2x + \frac{8}{15}\left(3d^2x^5 + 15cdx^4 + 20c^2x^3\right)ac + \frac{8}{21}\left(3d^2x^7 + 14cdx^6\right)c^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c)^2,x, algorithm="maxima")
```

```
[Out] 1/9*d^4*x^9 + c*d^3*x^8 + 16/7*c^2*d^2*x^7 + 16/5*c^4*x^5 + 16*a^
2*c^2*x + 8/15*(3*d^2*x^5 + 15*c*d*x^4 + 20*c^2*x^3)*a*c + 8/21*(
3*d^2*x^7 + 14*c*d*x^6)*c^2
```

Fricas [A] time = 0.246444, size = 1, normalized size = 0.01

$$\frac{1}{9}x^9d^4 + x^8d^3c + \frac{24}{7}x^7d^2c^2 + \frac{16}{3}x^6dc^3 + \frac{16}{5}x^5c^4 + \frac{8}{5}x^5d^2ca + 8x^4dc^2a + \frac{32}{3}x^3c^3a + 16xc^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c)^2,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*d^4 + x^8*d^3*c + 24/7*x^7*d^2*c^2 + 16/3*x^6*d*c^3 + 16/
5*x^5*c^4 + 8/5*x^5*d^2*c*a + 8*x^4*d*c^2*a + 32/3*x^3*c^3*a + 16
*x*c^2*a^2
```

Sympy [A] time = 0.132949, size = 95, normalized size = 1.03

$$16a^2c^2x + \frac{32ac^3x^3}{3} + 8ac^2dx^4 + \frac{16c^3dx^6}{3} + \frac{24c^2d^2x^7}{7} + cd^3x^8 + \frac{d^4x^9}{9} + x^5\left(\frac{8acd^2}{5} + \frac{16c^4}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**2,x)
```

```
[Out] 16*a**2*c**2*x + 32*a*c**3*x**3/3 + 8*a*c**2*d*x**4 + 16*c**3*d*x
**6/3 + 24*c**2*d**2*x**7/7 + c*d**3*x**8 + d**4*x**9/9 + x**5*(8
*a*c*d**2/5 + 16*c**4/5)
```

GIAC/XCAS [A] time = 0.262998, size = 112, normalized size = 1.22

$$\frac{1}{9}d^4x^9 + cd^3x^8 + \frac{24}{7}c^2d^2x^7 + \frac{16}{3}c^3dx^6 + \frac{16}{5}c^4x^5 + \frac{8}{5}acd^2x^5 + 8ac^2dx^4 + \frac{32}{3}ac^3x^3 + 16a^2c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c)^2,x, algorithm="giac")
```

```
[Out] 1/9*d^4*x^9 + c*d^3*x^8 + 24/7*c^2*d^2*x^7 + 16/3*c^3*d*x^6 + 16/
5*c^4*x^5 + 8/5*a*c*d^2*x^5 + 8*a*c^2*d*x^4 + 32/3*a*c^3*x^3 + 16
*a^2*c^2*x
```

3.36 $\int \left(4ac + 4c^2x^2 + 4cdx^3 + d^2x^4\right) \, dx$

Optimal. Leaf size=32

$$4acx + \frac{4c^2x^3}{3} + cdx^4 + \frac{d^2x^5}{5}$$

```
[Out] 4*a*c*x + (4*c^2*x^3)/3 + c*d*x^4 + (d^2*x^5)/5
```

Rubi [A] time = 0.0173725, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$

$$4acx + \frac{4c^2x^3}{3} + cdx^4 + \frac{d^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In]  Int[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4,x]

[Out] 4*a*c*x + (4*c^2*x^3)/3 + c*d*x^4 + (d^2*x^5)/5
```

Rubi in Sympy [A] time = 3.24629, size = 31, normalized size = 0.97

$$4acx + \frac{4c^2x^3}{3} + cdx^4 + \frac{d^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c,x)

[Out] 4*a*c*x + 4*c**2*x**3/3 + c*d*x**4 + d**2*x**5/5
```

Mathematica [A] time = 0.0000665565, size = 32, normalized size = 1.

$$4acx + \frac{4c^2x^3}{3} + cdx^4 + \frac{d^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4,x]

[Out] 4*a*c*x + (4*c^2*x^3)/3 + c*d*x^4 + (d^2*x^5)/5
```

Maple [A] time = 0.001, size = 29, normalized size = 0.9

$$4\,acx + \frac{4\,c^2x^3}{3} + cdx^4 + \frac{d^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c,x)
```

```
[Out] 4*a*c*x+4/3*c^2*x^3+c*d*x^4+1/5*d^2*x^5
```

Maxima [A] time = 0.765837, size = 38, normalized size = 1.19

$$\frac{1}{5}\,d^2x^5 + cdx^4 + \frac{4}{3}\,c^2x^3 + 4\,acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c,x, algorithm="maxima")
```

```
[Out] 1/5*d^2*x^5 + c*d*x^4 + 4/3*c^2*x^3 + 4*a*c*x
```

Fricas [A] time = 0.228305, size = 1, normalized size = 0.03

$$\frac{1}{5}x^5d^2 + x^4dc + \frac{4}{3}x^3c^2 + 4xca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c,x, algorithm="fricas")
```

```
[Out] 1/5*x^5*d^2 + x^4*d*c + 4/3*x^3*c^2 + 4*x*c*a
```

Sympy [A] time = 0.080779, size = 31, normalized size = 0.97

$$4acx + \frac{4c^2x^3}{3} + cdx^4 + \frac{d^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c,x)
```

```
[Out] 4*a*c*x + 4*c**2*x**3/3 + c*d*x**4 + d**2*x**5/5
```

GIAC/XCAS [A] time = 0.261961, size = 38, normalized size = 1.19

$$\frac{1}{5}\,d^2x^5 + cdx^4 + \frac{4}{3}\,c^2x^3 + 4\,acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c,x, algorithm="giac")
```

```
[Out] 1/5*d^2*x^5 + c*d*x^4 + 4/3*c^2*x^3 + 4*a*c*x
```

3.37 $\int \frac{1}{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx$

Optimal. Leaf size=529

$$-\frac{d\log\left(\sqrt{c}\sqrt{4ad^2+c^3}-\sqrt{2}\sqrt[4]{c}d\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}\left(\frac{c}{d}+x\right)+d^2\left(\frac{c}{d}+x\right)^2\right)}{4\sqrt{2}c^{3/4}\sqrt{4ad^2+c^3}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}+\frac{d\log\left(\sqrt{c}\sqrt{4ad^2+c^3}+\sqrt{2}\sqrt[4]{c}d\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}\left(\frac{c}{d}+x\right)+d^2\left(\frac{c}{d}+x\right)^2\right)}{4\sqrt{2}c^{3/4}\sqrt{4ad^2+c^3}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}-\frac{d\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}+\sqrt{2}c+\sqrt{2}dx}{\sqrt[4]{c}\sqrt{c^{3/2}-\sqrt{4ad^2+c^3}}}\right)}{2\sqrt{2}c^{3/4}\sqrt{4ad^2+c^3}\sqrt{c^{3/2}-\sqrt{4ad^2+c^3}}}+\frac{d\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}-\sqrt{2}(c+dx)}{\sqrt[4]{c}\sqrt{c^{3/2}-\sqrt{4ad^2+c^3}}}\right)}{2\sqrt{2}c^{3/4}\sqrt{4ad^2+c^3}\sqrt{c^{3/2}-\sqrt{4ad^2+c^3}}}$$

```
[Out] -(d*ArcTanh[(Sqrt[2]*c + c^(1/4)*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^
2]] + Sqrt[2]*d*x)/(c^(1/4)*Sqrt[c^(3/2) - Sqrt[c^3 + 4*a*d^2]])]
)/(2*Sqrt[2]*c^(3/4)*Sqrt[c^3 + 4*a*d^2]*Sqrt[c^(3/2) - Sqrt[c^3
+ 4*a*d^2]]) + (d*ArcTanh[(c^(1/4)*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*
d^2]] - Sqrt[2]*(c + d*x))/(c^(1/4)*Sqrt[c^(3/2) - Sqrt[c^3 + 4*a
*d^2]])])/(2*Sqrt[2]*c^(3/4)*Sqrt[c^3 + 4*a*d^2]*Sqrt[c^(3/2) - S
qrt[c^3 + 4*a*d^2]]) - (d*Log[Sqrt[c]*Sqrt[c^3 + 4*a*d^2] - Sqrt[
2]*c^(1/4)*d*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]]*(c/d + x) + d^2*
(c/d + x)^2])/(4*Sqrt[2]*c^(3/4)*Sqrt[c^3 + 4*a*d^2]*Sqrt[c^(3/2)
 + Sqrt[c^3 + 4*a*d^2]]) + (d*Log[Sqrt[c]*Sqrt[c^3 + 4*a*d^2] + S
qrt[2]*c^(1/4)*d*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]]*(c/d + x) +
d^2*(c/d + x)^2])/(4*Sqrt[2]*c^(3/4)*Sqrt[c^3 + 4*a*d^2]*Sqrt[c^(
3/2) + Sqrt[c^3 + 4*a*d^2]])
```

Rubi [A] time = 1.99819, antiderivative size = 529, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207

$$-\frac{d\log\left(\sqrt{c}\sqrt{4ad^2+c^3}-\sqrt{2}\sqrt[4]{c}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}(c+dx)+(c+dx)^2\right)}{4\sqrt{2}c^{3/4}\sqrt{4ad^2+c^3}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}+\frac{d\log\left(\sqrt{c}\sqrt{4ad^2+c^3}+\sqrt{2}\sqrt[4]{c}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}(c+dx)+(c+dx)^2\right)}{4\sqrt{2}c^{3/4}\sqrt{4ad^2+c^3}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}-\frac{d\tanh^{-1}\left(\frac{\sqrt[4]{c}\left(\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}+\sqrt{2}c^{3/4}\right)+\sqrt{2}dx}{\sqrt[4]{c}\sqrt{c^{3/2}-\sqrt{4ad^2+c^3}}}\right)}{2\sqrt{2}c^{3/4}\sqrt{4ad^2+c^3}\sqrt{c^{3/2}-\sqrt{4ad^2+c^3}}}+\frac{d\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}-\sqrt{2}(c+dx)}{\sqrt[4]{c}\sqrt{c^{3/2}-\sqrt{4ad^2+c^3}}}\right)}{2\sqrt{2}c^{3/4}\sqrt{4ad^2+c^3}\sqrt{c^{3/2}-\sqrt{4ad^2+c^3}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(-1),x]
```

```
[Out] -(d*ArcTanh[(c^(1/4)*(Sqrt[2]*c^(3/4) + Sqrt[c^(3/2) + Sqrt[c^3 +
 4*a*d^2]]) + Sqrt[2]*d*x)/(c^(1/4)*Sqrt[c^(3/2) - Sqrt[c^3 + 4*a
*d^2]])])/(2*Sqrt[2]*c^(3/4)*Sqrt[c^3 + 4*a*d^2]*Sqrt[c^(3/2) - S
qrt[c^3 + 4*a*d^2]]) + (d*ArcTanh[(c^(1/4)*Sqrt[c^(3/2) + Sqrt[c^
3 + 4*a*d^2]] - Sqrt[2]*(c + d*x))/(c^(1/4)*Sqrt[c^(3/2) - Sqrt[c
^3 + 4*a*d^2]])])/(2*Sqrt[2]*c^(3/4)*Sqrt[c^3 + 4*a*d^2]*Sqrt[c^(
3/2) - Sqrt[c^3 + 4*a*d^2]]) - (d*Log[Sqrt[c]*Sqrt[c^3 + 4*a*d^2]
 - Sqrt[2]*c^(1/4)*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]]*(c + d*x)
+ (c + d*x)^2])/(4*Sqrt[2]*c^(3/4)*Sqrt[c^3 + 4*a*d^2]*Sqrt[c^(3/
2) + Sqrt[c^3 + 4*a*d^2]]) + (d*Log[Sqrt[c]*Sqrt[c^3 + 4*a*d^2] +
 Sqrt[2]*c^(1/4)*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]]*(c + d*x) +
(c + d*x)^2])/(4*Sqrt[2]*c^(3/4)*Sqrt[c^3 + 4*a*d^2]*Sqrt[c^(3/2)
 + Sqrt[c^3 + 4*a*d^2]])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.0451106, size = 71, normalized size = 0.13

$$\frac{1}{4}\text{RootSum}\left[\#1^4d^2+4\#1^3cd+4\#1^2c^2+4ac\&,\frac{\log(x-\#1)}{\#1^3d^2+3\#1^2cd+2\#1c^2}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(-1),x]
```

```
[Out] RootSum[4*a*c + 4*c^2*#1^2 + 4*c*d*#1^3 + d^2*#1^4 & , Log[x - #1
]/(2*c^2*#1 + 3*c*d*#1^2 + d^2*#1^3) & ]/4
```

Maple [C] time = 0.056, size = 64, normalized size = 0.1

$$\frac{1}{4}\sum_{\_R=RootOf\left(d^2\_Z^4+4\,cd\_Z^3+4\,c^2\_Z^2+4\,ac\right)}\frac{\ln\left(x-\_R\right)}{\_R^3d^2+3\,\_R^2cd+2\,\_R\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c),x)
```

```
[Out] 1/4*sum(1/(_R^3*d^2+3*_R^2*c*d+2*_R*c^2)*ln(x-_R),_R=RootOf(_Z^4*
d^2+4*_Z^3*c*d+4*_Z^2*c^2+4*a*c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{d^2x^4+4\,cdx^3+4\,c^2x^2+4\,ac}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c),x, algorithm="maxima")
```

```
[Out] integrate(1/(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c), x)
```

Fricas [A] time = 0.299792, size = 1222, normalized size = 2.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(-(2*(a*c^3 + 4*a^2*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2
 + 16*a^3*c^3*d^4)) + 1)/(a*c^3 + 4*a^2*d^2))*log(d^2*x + c*d + (
2*a*c*d^2 + (a*c^7 + 4*a^2*c^4*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*
d^2 + 16*a^3*c^3*d^4)))*sqrt(-(2*(a*c^3 + 4*a^2*d^2)*sqrt(-d^2/(a
*c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^4)) + 1)/(a*c^3 + 4*a^2*d^2))
) - 1/8*sqrt(-(2*(a*c^3 + 4*a^2*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6
*d^2 + 16*a^3*c^3*d^4)) + 1)/(a*c^3 + 4*a^2*d^2))*log(d^2*x + c*d
 - (2*a*c*d^2 + (a*c^7 + 4*a^2*c^4*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*
c^6*d^2 + 16*a^3*c^3*d^4)))*sqrt(-(2*(a*c^3 + 4*a^2*d^2)*sqrt(-d^
2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^4)) + 1)/(a*c^3 + 4*a^2*d
^2))) + 1/8*sqrt((2*(a*c^3 + 4*a^2*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*
c^6*d^2 + 16*a^3*c^3*d^4)) - 1)/(a*c^3 + 4*a^2*d^2))*log(d^2*x +
c*d + (2*a*c*d^2 - (a*c^7 + 4*a^2*c^4*d^2)*sqrt(-d^2/(a*c^9 + 8*a
^2*c^6*d^2 + 16*a^3*c^3*d^4)))*sqrt((2*(a*c^3 + 4*a^2*d^2)*sqrt(-
d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^4)) - 1)/(a*c^3 + 4*a^2
*d^2))) - 1/8*sqrt((2*(a*c^3 + 4*a^2*d^2)*sqrt(-d^2/(a*c^9 + 8*a^
2*c^6*d^2 + 16*a^3*c^3*d^4)) - 1)/(a*c^3 + 4*a^2*d^2))*log(d^2*x
+ c*d - (2*a*c*d^2 - (a*c^7 + 4*a^2*c^4*d^2)*sqrt(-d^2/(a*c^9 + 8
*a^2*c^6*d^2 + 16*a^3*c^3*d^4)))*sqrt((2*(a*c^3 + 4*a^2*d^2)*sqrt
(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^4)) - 1)/(a*c^3 + 4*a
^2*d^2)))
```

Sympy [A] time = 3.1435, size = 88, normalized size = 0.17

$$\operatorname{RootSum}\left(t^{4}\left(16384a^{3}c^{3}d^{2}+4096a^{2}c^{6}\right)+128t^{2}ac^{3}+1,\left(t\mapsto t\log\left(x+\frac{-1024t^{3}a^{2}c^{4}d^{2}-256t^{3}ac^{7}+16tacd^{2}-4tc^{4}+cd}{d^{2}}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c),x)
```

```
[Out] RootSum(_t**4*(16384*a**3*c**3*d**2 + 4096*a**2*c**6) + 128*_t**2
*a*c**3 + 1, Lambda(_t, _t*log(x + (-1024*_t**3*a**2*c**4*d**2 -
256*_t**3*a*c**7 + 16*_t*a*c*d**2 - 4*_t*c**4 + c*d)/d**2)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{d^{2}x^{4}+4\,cdx^{3}+4\,c^{2}x^{2}+4\,ac}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c),x, algorithm="giac")
```

```
[Out] integrate(1/(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c), x)
```

3.38 $\int \frac{1}{(4ac+4c^2x^2+4cdx^3+d^2x^4)^2}\,dx$

Optimal. Leaf size=746

$$-\frac{d\left(-c^{3/2}\sqrt{4ad^2+c^3}+12ad^2+c^3\right)\log\left(\sqrt{c}\sqrt{4ad^2+c^3}-\sqrt{2}\sqrt[4]{c}d\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}\left(\frac{c}{d}+x\right)+d^2\left(\frac{c}{d}+x\right)^2\right)}{64\sqrt{2}ac^{7/4}\left(4ad^2+c^3\right)^{3/2}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}$$
$$+\frac{d\left(-c^{3/2}\sqrt{4ad^2+c^3}+12ad^2+c^3\right)\log\left(\sqrt{c}\sqrt{4ad^2+c^3}+\sqrt{2}\sqrt[4]{c}d\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}\left(\frac{c}{d}+x\right)+d^2\left(\frac{c}{d}+x\right)^2\right)}{64\sqrt{2}ac^{7/4}\left(4ad^2+c^3\right)^{3/2}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}$$
$$-\frac{d\left(c^{3/2}\sqrt{4ad^2+c^3}+12ad^2+c^3\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}+\sqrt{2}c+\sqrt{2}dx}{\sqrt[4]{c}\sqrt{c^{3/2}-\sqrt{4ad^2+c^3}}}\right)}{32\sqrt{2}ac^{7/4}\left(4ad^2+c^3\right)^{3/2}\sqrt{c^{3/2}-\sqrt{4ad^2+c^3}}}$$
$$+\frac{d\left(c^{3/2}\sqrt{4ad^2+c^3}+12ad^2+c^3\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}-\sqrt{2}(c+dx)}{\sqrt[4]{c}\sqrt{c^{3/2}-\sqrt{4ad^2+c^3}}}\right)}{32\sqrt{2}ac^{7/4}\left(4ad^2+c^3\right)^{3/2}\sqrt{c^{3/2}-\sqrt{4ad^2+c^3}}}$$
$$-\frac{\left(\frac{c}{d}+x\right)\left(-4ad^2+c^3-cd^2\left(\frac{c}{d}+x\right)^2\right)}{16ac\left(4ad^2+c^3\right)\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)}$$

```
[Out] -((c/d + x)*(c^3 - 4*a*d^2 - c*d^2*(c/d + x)^2))/(16*a*c*(c^3 + 4
*a*d^2)*(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)) - (d*(c^3 + 12
*a*d^2 + c^(3/2)*Sqrt[c^3 + 4*a*d^2])*ArcTanh[(Sqrt[2]*c + c^(1/4
)*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]] + Sqrt[2]*d*x)/(c^(1/4)*Sqr
t[c^(3/2) - Sqrt[c^3 + 4*a*d^2]])])/(32*Sqrt[2]*a*c^(7/4)*(c^3 +
4*a*d^2)^(3/2)*Sqrt[c^(3/2) - Sqrt[c^3 + 4*a*d^2]]) + (d*(c^3 + 1
2*a*d^2 + c^(3/2)*Sqrt[c^3 + 4*a*d^2])*ArcTanh[(c^(1/4)*Sqrt[c^(3
/2) + Sqrt[c^3 + 4*a*d^2]] - Sqrt[2]*(c + d*x))/(c^(1/4)*Sqrt[c^(
3/2) - Sqrt[c^3 + 4*a*d^2]])])/(32*Sqrt[2]*a*c^(7/4)*(c^3 + 4*a*d
^2)^(3/2)*Sqrt[c^(3/2) - Sqrt[c^3 + 4*a*d^2]]) - (d*(c^3 + 12*a*d
^2 - c^(3/2)*Sqrt[c^3 + 4*a*d^2])*Log[Sqrt[c]*Sqrt[c^3 + 4*a*d^2]
 - Sqrt[2]*c^(1/4)*d*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]]*(c/d + x
) + d^2*(c/d + x)^2])/(64*Sqrt[2]*a*c^(7/4)*(c^3 + 4*a*d^2)^(3/2)
*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]]) + (d*(c^3 + 12*a*d^2 - c^(3
/2)*Sqrt[c^3 + 4*a*d^2])*Log[Sqrt[c]*Sqrt[c^3 + 4*a*d^2] + Sqrt[2
]*c^(1/4)*d*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]]*(c/d + x) + d^2*(
c/d + x)^2])/(64*Sqrt[2]*a*c^(7/4)*(c^3 + 4*a*d^2)^(3/2)*Sqrt[c^(
3/2) + Sqrt[c^3 + 4*a*d^2]])
```

Rubi [A] time = 4.06436, antiderivative size = 746, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$

$$-\frac{d\left(-c^{3/2}\sqrt{4ad^2+c^3}+12ad^2+c^3\right)\log\left(\sqrt{c}\sqrt{4ad^2+c^3}-\sqrt{2}\sqrt[4]{c}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}(c+dx)+(c+dx)^2\right)}{64\sqrt{2}ac^{7/4}\left(4ad^2+c^3\right)^{3/2}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}$$
$$+\frac{d\left(-c^{3/2}\sqrt{4ad^2+c^3}+12ad^2+c^3\right)\log\left(\sqrt{c}\sqrt{4ad^2+c^3}+\sqrt{2}\sqrt[4]{c}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}(c+dx)+(c+dx)^2\right)}{64\sqrt{2}ac^{7/4}\left(4ad^2+c^3\right)^{3/2}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}$$
$$-\frac{d\left(c^{3/2}\sqrt{4ad^2+c^3}+12ad^2+c^3\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\left(\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}+\sqrt{2}c^{3/4}\right)+\sqrt{2}dx}{\sqrt[4]{c}\sqrt{c^{3/2}-\sqrt{4ad^2+c^3}}}\right)}{32\sqrt{2}ac^{7/4}\left(4ad^2+c^3\right)^{3/2}\sqrt{c^{3/2}-\sqrt{4ad^2+c^3}}}$$
$$+\frac{d\left(c^{3/2}\sqrt{4ad^2+c^3}+12ad^2+c^3\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}-\sqrt{2}(c+dx)}{\sqrt[4]{c}\sqrt{c^{3/2}-\sqrt{4ad^2+c^3}}}\right)}{32\sqrt{2}ac^{7/4}\left(4ad^2+c^3\right)^{3/2}\sqrt{c^{3/2}-\sqrt{4ad^2+c^3}}}$$
$$-\frac{(c+dx)\left(-4ad^2+c^3-c(c+dx)^2\right)}{16acd\left(4ad^2+c^3\right)\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(-2),x]
```

```
[Out] -((c + d*x)*(c^3 - 4*a*d^2 - c*(c + d*x)^2))/(16*a*c*d*(c^3 + 4*a
*d^2)*(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)) - (d*(c^3 + 12*a
*d^2 + c^(3/2)*Sqrt[c^3 + 4*a*d^2])*ArcTanh[(c^(1/4)*(Sqrt[2]*c^(
3/4) + Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]]) + Sqrt[2]*d*x)/(c^(1/
4)*Sqrt[c^(3/2) - Sqrt[c^3 + 4*a*d^2]])])/(32*Sqrt[2]*a*c^(7/4)*(
c^3 + 4*a*d^2)^(3/2)*Sqrt[c^(3/2) - Sqrt[c^3 + 4*a*d^2]]) + (d*(c
^3 + 12*a*d^2 + c^(3/2)*Sqrt[c^3 + 4*a*d^2])*ArcTanh[(c^(1/4)*Sqr
t[c^(3/2) + Sqrt[c^3 + 4*a*d^2]] - Sqrt[2]*(c + d*x))/(c^(1/4)*Sq
rt[c^(3/2) - Sqrt[c^3 + 4*a*d^2]])])/(32*Sqrt[2]*a*c^(7/4)*(c^3 +
 4*a*d^2)^(3/2)*Sqrt[c^(3/2) - Sqrt[c^3 + 4*a*d^2]]) - (d*(c^3 +
12*a*d^2 - c^(3/2)*Sqrt[c^3 + 4*a*d^2])*Log[Sqrt[c]*Sqrt[c^3 + 4*
a*d^2] - Sqrt[2]*c^(1/4)*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]]*(c +
 d*x) + (c + d*x)^2])/(64*Sqrt[2]*a*c^(7/4)*(c^3 + 4*a*d^2)^(3/2)
*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]]) + (d*(c^3 + 12*a*d^2 - c^(3
/2)*Sqrt[c^3 + 4*a*d^2])*Log[Sqrt[c]*Sqrt[c^3 + 4*a*d^2] + Sqrt[2
]*c^(1/4)*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]]*(c + d*x) + (c + d*
x)^2])/(64*Sqrt[2]*a*c^(7/4)*(c^3 + 4*a*d^2)^(3/2)*Sqrt[c^(3/2) +
 Sqrt[c^3 + 4*a*d^2]])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**2,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.173404, size = 182, normalized size = 0.24

$$\frac{\text{RootSum}\left[\#1^4 d^2+4\#1^3 cd+4\#1^2 c^2+4ac\&,\frac{\#1^2 cd^2\log(x-\#1)+12ad^2\log(x-\#1)+2c^3\log(x-\#1)+2\#1c^2d\log(x-\#1)}{\#1^3 d^2+3\#1^2 cd+2\#1 c^2}\&\right]+\frac{4(c+dx)(4ad+cx(2c}{4ac+x^2(2c+dx)}}{64ac\left(4ad^2+c^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(-2),x]
```

```
[Out] ((4*(c + d*x)*(4*a*d + c*x*(2*c + d*x)))/(4*a*c + x^2*(2*c + d*x)
^2) + RootSum[4*a*c + 4*c^2*#1^2 + 4*c*d*#1^3 + d^2*#1^4 & , (2*c
^3*Log[x - #1] + 12*a*d^2*Log[x - #1] + 2*c^2*d*Log[x - #1]*#1 +
c*d^2*Log[x - #1]*#1^2)/(2*c^2*#1 + 3*c*d*#1^2 + d^2*#1^3) & ])/(
64*a*c*(c^3 + 4*a*d^2))
```

Maple [C] time = 0.045, size = 232, normalized size = 0.3

$$\frac{1}{d^2x^4+4\,cdx^3+4\,c^2x^2+4\,ac}\left(\frac{d^2x^3}{16\,a\,(4\,ad^2+c^3)}+\frac{3\,cdx^2}{16\,a\,(4\,ad^2+c^3)}+\frac{\left(2\,ad^2+c^3\right)x}{8\,\left(4\,ad^2+c^3\right)ac}+\frac{d}{16\,ad^2+4\,c^3}\right)+\frac{1}{64\,ac}\sum_{\_R=RootOf\left(\_Z^4d^2+4\_Z^3cd+4\_Z^2c^2+4\,ac\right)}\frac{\left(\_R^2cd^2+2\_R\,c^2d+12\,ad^2+2\,c^3\right)\ln\left(x-\_R\right)}{\left(4\,ad^2+c^3\right)\left(\_R^3d^2+3\_R^2cd+2\_R\,c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x)
```

```
[Out] (1/16*d^2/a/(4*a*d^2+c^3)*x^3+3/16/a*c*d/(4*a*d^2+c^3)*x^2+1/8/c*
(2*a*d^2+c^3)/(4*a*d^2+c^3)/a*x+1/4*d/(4*a*d^2+c^3))/(d^2*x^4+4*c
*d*x^3+4*c^2*x^2+4*a*c)+1/64/a/c*sum((_R^2*c*d^2+2*_R*c^2*d+12*a*
d^2+2*c^3)/(4*a*d^2+c^3)/(_R^3*d^2+3*_R^2*c*d+2*_R*c^2)*ln(x-_R),
_R=RootOf(_Z^4*d^2+4*_Z^3*c*d+4*_Z^2*c^2+4*a*c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{cd^2x^3 + 3\,c^2dx^2 + 4\,acd + 2\left(c^3 + 2\,ad^2\right)x}{16\left(4\,a^2c^5 + 16\,a^3c^2d^2 + (ac^4d^2 + 4\,a^2cd^4)x^4 + 4\,(ac^5d + 4\,a^2c^2d^3)x^3 + 4\,(ac^6 + 4\,a^2c^3d^2)x^2\right)} + \frac{\int \frac{cd^2x^2 + 2\,c^2dx + 2\,c^3 + 12\,ad^2}{d^2x^4 + 4\,cdx^3 + 4\,c^2x^2 + 4\,ac}\,dx}{16\left(ac^4 + 4\,a^2cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c)^(-2),x, algorithm="maxima")
```

```
[Out] 1/16*(c*d^2*x^3 + 3*c^2*d*x^2 + 4*a*c*d + 2*(c^3 + 2*a*d^2)*x)/(4
*a^2*c^5 + 16*a^3*c^2*d^2 + (a*c^4*d^2 + 4*a^2*c*d^4)*x^4 + 4*(a*
c^5*d + 4*a^2*c^2*d^3)*x^3 + 4*(a*c^6 + 4*a^2*c^3*d^2)*x^2) + 1/1
6*integrate((c*d^2*x^2 + 2*c^2*d*x + 2*c^3 + 12*a*d^2)/(d^2*x^4 +
 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c), x)/(a*c^4 + 4*a^2*c*d^2)
```

Fricas [A] time = 0.347202, size = 4350, normalized size = 5.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c)^(-2),x, algorithm="fricas")
```

```
[Out] 1/64*(4*c*d^2*x^3 + 12*c^2*d*x^2 + 16*a*c*d + (4*a^2*c^5 + 16*a^3
*c^2*d^2 + (a*c^4*d^2 + 4*a^2*c*d^4)*x^4 + 4*(a*c^5*d + 4*a^2*c^2
*d^3)*x^3 + 4*(a*c^6 + 4*a^2*c^3*d^2)*x^2)*sqrt(-(c^6 + 15*a*c^3*
d^2 + 60*a^2*d^4 + 2*(a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4
+ 64*a^6*c^2*d^6)*sqrt(-(25*c^6*d^6 + 360*a*c^3*d^8 + 1296*a^2*d^
10)/(a^3*c^25 + 24*a^4*c^22*d^2 + 240*a^5*c^19*d^4 + 1280*a^6*c^1
6*d^6 + 3840*a^7*c^13*d^8 + 6144*a^8*c^10*d^10 + 4096*a^9*c^7*d^1
2)))/(a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6
))*log(5*c^7*d^3 + 81*a*c^4*d^5 + 324*a^2*c*d^7 + (5*c^6*d^4 + 81
*a*c^3*d^6 + 324*a^2*d^8)*x + (5*a^2*c^8*d^4 + 96*a^3*c^5*d^6 + 4
32*a^4*c^2*d^8 + (a^3*c^19 + 20*a^4*c^16*d^2 + 144*a^5*c^13*d^4 +
 448*a^6*c^10*d^6 + 512*a^7*c^7*d^8)*sqrt(-(25*c^6*d^6 + 360*a*c^
3*d^8 + 1296*a^2*d^10)/(a^3*c^25 + 24*a^4*c^22*d^2 + 240*a^5*c^19
*d^4 + 1280*a^6*c^16*d^6 + 3840*a^7*c^13*d^8 + 6144*a^8*c^10*d^10
 + 4096*a^9*c^7*d^12)))*sqrt(-(c^6 + 15*a*c^3*d^2 + 60*a^2*d^4 +
2*(a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6)*s
qrt(-(25*c^6*d^6 + 360*a*c^3*d^8 + 1296*a^2*d^10)/(a^3*c^25 + 24*
a^4*c^22*d^2 + 240*a^5*c^19*d^4 + 1280*a^6*c^16*d^6 + 3840*a^7*c^
13*d^8 + 6144*a^8*c^10*d^10 + 4096*a^9*c^7*d^12)))/(a^3*c^11 + 12
*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6))) - (4*a^2*c^5 +
16*a^3*c^2*d^2 + (a*c^4*d^2 + 4*a^2*c*d^4)*x^4 + 4*(a*c^5*d + 4*a
^2*c^2*d^3)*x^3 + 4*(a*c^6 + 4*a^2*c^3*d^2)*x^2)*sqrt(-(c^6 + 15*
a*c^3*d^2 + 60*a^2*d^4 + 2*(a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^
5*d^4 + 64*a^6*c^2*d^6)*sqrt(-(25*c^6*d^6 + 360*a*c^3*d^8 + 1296*
a^2*d^10)/(a^3*c^25 + 24*a^4*c^22*d^2 + 240*a^5*c^19*d^4 + 1280*a
^6*c^16*d^6 + 3840*a^7*c^13*d^8 + 6144*a^8*c^10*d^10 + 4096*a^9*c
^7*d^12)))/(a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c
^2*d^6))*log(5*c^7*d^3 + 81*a*c^4*d^5 + 324*a^2*c*d^7 + (5*c^6*d^
4 + 81*a*c^3*d^6 + 324*a^2*d^8)*x - (5*a^2*c^8*d^4 + 96*a^3*c^5*d
^6 + 432*a^4*c^2*d^8 + (a^3*c^19 + 20*a^4*c^16*d^2 + 144*a^5*c^13
*d^4 + 448*a^6*c^10*d^6 + 512*a^7*c^7*d^8)*sqrt(-(25*c^6*d^6 + 36
```

```
0*a*c^3*d^8 + 1296*a^2*d^10)/(a^3*c^25 + 24*a^4*c^22*d^2 + 240*a^
5*c^19*d^4 + 1280*a^6*c^16*d^6 + 3840*a^7*c^13*d^8 + 6144*a^8*c^1
0*d^10 + 4096*a^9*c^7*d^12)))*sqrt(-(c^6 + 15*a*c^3*d^2 + 60*a^2*
d^4 + 2*(a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*
d^6)*sqrt(-(25*c^6*d^6 + 360*a*c^3*d^8 + 1296*a^2*d^10)/(a^3*c^25
 + 24*a^4*c^22*d^2 + 240*a^5*c^19*d^4 + 1280*a^6*c^16*d^6 + 3840*
a^7*c^13*d^8 + 6144*a^8*c^10*d^10 + 4096*a^9*c^7*d^12)))/(a^3*c^1
1 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6))) + (4*a^2*
c^5 + 16*a^3*c^2*d^2 + (a*c^4*d^2 + 4*a^2*c*d^4)*x^4 + 4*(a*c^5*d
 + 4*a^2*c^2*d^3)*x^3 + 4*(a*c^6 + 4*a^2*c^3*d^2)*x^2)*sqrt(-(c^6
 + 15*a*c^3*d^2 + 60*a^2*d^4 - 2*(a^3*c^11 + 12*a^4*c^8*d^2 + 48*
a^5*c^5*d^4 + 64*a^6*c^2*d^6)*sqrt(-(25*c^6*d^6 + 360*a*c^3*d^8 +
 1296*a^2*d^10)/(a^3*c^25 + 24*a^4*c^22*d^2 + 240*a^5*c^19*d^4 +
1280*a^6*c^16*d^6 + 3840*a^7*c^13*d^8 + 6144*a^8*c^10*d^10 + 4096
*a^9*c^7*d^12)))/(a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64
*a^6*c^2*d^6))*log(5*c^7*d^3 + 81*a*c^4*d^5 + 324*a^2*c*d^7 + (5*
c^6*d^4 + 81*a*c^3*d^6 + 324*a^2*d^8)*x + (5*a^2*c^8*d^4 + 96*a^3
*c^5*d^6 + 432*a^4*c^2*d^8 - (a^3*c^19 + 20*a^4*c^16*d^2 + 144*a^
5*c^13*d^4 + 448*a^6*c^10*d^6 + 512*a^7*c^7*d^8)*sqrt(-(25*c^6*d^
6 + 360*a*c^3*d^8 + 1296*a^2*d^10)/(a^3*c^25 + 24*a^4*c^22*d^2 +
240*a^5*c^19*d^4 + 1280*a^6*c^16*d^6 + 3840*a^7*c^13*d^8 + 6144*a
^8*c^10*d^10 + 4096*a^9*c^7*d^12)))*sqrt(-(c^6 + 15*a*c^3*d^2 + 6
0*a^2*d^4 - 2*(a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^
6*c^2*d^6)*sqrt(-(25*c^6*d^6 + 360*a*c^3*d^8 + 1296*a^2*d^10)/(a^
3*c^25 + 24*a^4*c^22*d^2 + 240*a^5*c^19*d^4 + 1280*a^6*c^16*d^6 +
 3840*a^7*c^13*d^8 + 6144*a^8*c^10*d^10 + 4096*a^9*c^7*d^12)))/(a
^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6))) - (
4*a^2*c^5 + 16*a^3*c^2*d^2 + (a*c^4*d^2 + 4*a^2*c*d^4)*x^4 + 4*(a
*c^5*d + 4*a^2*c^2*d^3)*x^3 + 4*(a*c^6 + 4*a^2*c^3*d^2)*x^2)*sqrt
(-(c^6 + 15*a*c^3*d^2 + 60*a^2*d^4 - 2*(a^3*c^11 + 12*a^4*c^8*d^2
 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6)*sqrt(-(25*c^6*d^6 + 360*a*c^3
*d^8 + 1296*a^2*d^10)/(a^3*c^25 + 24*a^4*c^22*d^2 + 240*a^5*c^19*
d^4 + 1280*a^6*c^16*d^6 + 3840*a^7*c^13*d^8 + 6144*a^8*c^10*d^10
+ 4096*a^9*c^7*d^12)))/(a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^
4 + 64*a^6*c^2*d^6))*log(5*c^7*d^3 + 81*a*c^4*d^5 + 324*a^2*c*d^7
 + (5*c^6*d^4 + 81*a*c^3*d^6 + 324*a^2*d^8)*x - (5*a^2*c^8*d^4 +
96*a^3*c^5*d^6 + 432*a^4*c^2*d^8 - (a^3*c^19 + 20*a^4*c^16*d^2 +
144*a^5*c^13*d^4 + 448*a^6*c^10*d^6 + 512*a^7*c^7*d^8)*sqrt(-(25*
c^6*d^6 + 360*a*c^3*d^8 + 1296*a^2*d^10)/(a^3*c^25 + 24*a^4*c^22*
d^2 + 240*a^5*c^19*d^4 + 1280*a^6*c^16*d^6 + 3840*a^7*c^13*d^8 +
6144*a^8*c^10*d^10 + 4096*a^9*c^7*d^12)))*sqrt(-(c^6 + 15*a*c^3*d
^2 + 60*a^2*d^4 - 2*(a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 +
 64*a^6*c^2*d^6)*sqrt(-(25*c^6*d^6 + 360*a*c^3*d^8 + 1296*a^2*d^1
0)/(a^3*c^25 + 24*a^4*c^22*d^2 + 240*a^5*c^19*d^4 + 1280*a^6*c^16
*d^6 + 3840*a^7*c^13*d^8 + 6144*a^8*c^10*d^10 + 4096*a^9*c^7*d^12
)))/(a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6)
)) + 8*(c^3 + 2*a*d^2)*x)/(4*a^2*c^5 + 16*a^3*c^2*d^2 + (a*c^4*d^
2 + 4*a^2*c*d^4)*x^4 + 4*(a*c^5*d + 4*a^2*c^2*d^3)*x^3 + 4*(a*c^6
 + 4*a^2*c^3*d^2)*x^2)
```

Sympy [A] time = 20.2243, size = 427, normalized size = 0.57

$$\frac{4acd+3c^2dx^2+cd^2x^3+x\left(4ad^2+2c^3\right)}{256a^3c^2d^2+64a^2c^5+x^4\left(64a^2cd^4+16ac^4d^2\right)+x^3\left(256a^2c^2d^3+64ac^5d\right)+x^2\left(256a^2c^3d^2+64ac^6\right)}$$

$$+\text{RootSum}\left(t^4\left(1073741824a^9c^7d^6+805306368a^8c^{10}d^4+201326592a^7c^{13}d^2+16777216a^6c^{16}\right)+t^2\left(491520a^5c^5d^4+122880\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**2,x)
```

```
[Out] (4*a*c*d + 3*c**2*d*x**2 + c*d**2*x**3 + x*(4*a*d**2 + 2*c**3))/(
256*a**3*c**2*d**2 + 64*a**2*c**5 + x**4*(64*a**2*c*d**4 + 16*a*c
**4*d**2) + x**3*(256*a**2*c**2*d**3 + 64*a*c**5*d) + x**2*(256*a
**2*c**3*d**2 + 64*a*c**6)) + RootSum(_t**4*(1073741824*a**9*c**7
*d**6 + 805306368*a**8*c**10*d**4 + 201326592*a**7*c**13*d**2 + 1
6777216*a**6*c**16) + _t**2*(491520*a**5*c**5*d**4 + 122880*a**4*
```

```
c**8*d**2 + 8192*a**3*c**11) + 81*a**2*d**4 + 18*a*c**3*d**2 + c*
*6, Lambda(_t, _t*log(x + (-67108864*_t**3*a**7*c**7*d**8 - 58720
256*_t**3*a**6*c**10*d**6 - 18874368*_t**3*a**5*c**13*d**4 - 2621
440*_t**3*a**4*c**16*d**2 - 131072*_t**3*a**3*c**19 + 27648*_t*a*
*4*c**2*d**8 - 9216*_t*a**3*c**5*d**6 - 5440*_t*a**2*c**8*d**4 -
736*_t*a*c**11*d**2 - 32*_t*c**14 + 324*a**2*c*d**7 + 81*a*c**4*d
**5 + 5*c**7*d**3)/(324*a**2*d**8 + 81*a*c**3*d**6 + 5*c**6*d**4)
)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d^2x^4 + 4\,cdx^3 + 4\,c^2x^2 + 4\,ac)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c)^(-2),x, algorithm="giac")
```

```
[Out] integrate((d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c)^(-2), x)
```

3.39 $\int \left(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4\right)^4 dx$

Optimal. Leaf size=295

$$\frac{1}{24}e^4\left(65536a^2e^6+20992ad^4e^3+601d^8\right)\left(\frac{d}{4e}+x\right)^9+\frac{\left(256ae^3+5d^4\right)^2\left(256ae^3+59d^4\right)\left(\frac{d}{4e}+x\right)^5}{5120}$$
$$+\frac{64}{13}e^8\left(256ae^3+59d^4\right)\left(\frac{d}{4e}+x\right)^{13}+\frac{x\left(256ae^3+5d^4\right)^4}{1048576e^4}$$
$$-\frac{72}{11}d^2e^6\left(256ae^3+17d^4\right)\left(\frac{d}{4e}+x\right)^{11}-\frac{9}{224}d^2e^2\left(256ae^3+5d^4\right)\left(256ae^3+17d^4\right)\left(\frac{d}{4e}+x\right)^7$$
$$-\frac{d^2\left(256ae^3+5d^4\right)^3\left(\frac{d}{4e}+x\right)^3}{8192e^2}-\frac{2048}{5}d^2e^{10}\left(\frac{d}{4e}+x\right)^{15}+\frac{4096}{17}e^{12}\left(\frac{d}{4e}+x\right)^{17}$$

```
[Out] ((5*d^4 + 256*a*e^3)^4*x)/(1048576*e^4) - (d^2*(5*d^4 + 256*a*e^3
)^3*(d/(4*e) + x)^3)/(8192*e^2) + ((5*d^4 + 256*a*e^3)^2*(59*d^4
+ 256*a*e^3)*(d/(4*e) + x)^5)/5120 - (9*d^2*e^2*(5*d^4 + 256*a*e^
3)*(17*d^4 + 256*a*e^3)*(d/(4*e) + x)^7)/224 + (e^4*(601*d^8 + 20
992*a*d^4*e^3 + 65536*a^2*e^6)*(d/(4*e) + x)^9)/24 - (72*d^2*e^6*
(17*d^4 + 256*a*e^3)*(d/(4*e) + x)^11)/11 + (64*e^8*(59*d^4 + 256
*a*e^3)*(d/(4*e) + x)^13)/13 - (2048*d^2*e^10*(d/(4*e) + x)^15)/5
 + (4096*e^12*(d/(4*e) + x)^17)/17
```

Rubi [A] time = 1.02707, antiderivative size = 295, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$

$$\frac{\left(65536a^2e^6+20992ad^4e^3+601d^8\right)(d+4ex)^9}{6291456e^5}+\frac{\left(256ae^3+59d^4\right)(d+4ex)^{13}}{13631488e^5}$$
$$+\frac{\left(256ae^3+5d^4\right)^2\left(256ae^3+59d^4\right)(d+4ex)^5}{5242880e^5}+\frac{x\left(256ae^3+5d^4\right)^4}{1048576e^4}$$
$$-\frac{9d^2\left(256ae^3+17d^4\right)(d+4ex)^{11}}{5767168e^5}-\frac{9d^2\left(256ae^3+5d^4\right)\left(256ae^3+17d^4\right)(d+4ex)^7}{3670016e^5}$$
$$-\frac{d^2\left(256ae^3+5d^4\right)^3(d+4ex)^3}{524288e^5}-\frac{d^2(d+4ex)^{15}}{2621440e^5}+\frac{(d+4ex)^{17}}{71303168e^5}$$

Antiderivative was successfully verified.

```
[In]  Int[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^4,x]
```

```
[Out] ((5*d^4 + 256*a*e^3)^4*x)/(1048576*e^4) - (d^2*(5*d^4 + 256*a*e^3
)^3*(d + 4*e*x)^3)/(524288*e^5) + ((5*d^4 + 256*a*e^3)^2*(59*d^4
+ 256*a*e^3)*(d + 4*e*x)^5)/(5242880*e^5) - (9*d^2*(5*d^4 + 256*a
*e^3)*(17*d^4 + 256*a*e^3)*(d + 4*e*x)^7)/(3670016*e^5) + ((601*d
^8 + 20992*a*d^4*e^3 + 65536*a^2*e^6)*(d + 4*e*x)^9)/(6291456*e^5
) - (9*d^2*(17*d^4 + 256*a*e^3)*(d + 4*e*x)^11)/(5767168*e^5) + (
(59*d^4 + 256*a*e^3)*(d + 4*e*x)^13)/(13631488*e^5) - (d^2*(d + 4
*e*x)^15)/(2621440*e^5) + (d + 4*e*x)^17/(71303168*e^5)
```

Rubi in Sympy [A] time = 111.56, size = 262, normalized size = 0.89

$$-\frac{2048d^2e^{10}\left(\frac{d}{4e}+x\right)^{15}}{5}-\frac{72d^2e^6\left(256ae^3+17d^4\right)\left(\frac{d}{4e}+x\right)^{11}}{11}-\frac{9d^2e^2\left(\frac{d}{4e}+x\right)^7\left(65536a^2e^6+5632ad^4e^3+85d^8\right)}{224}-\frac{d^2\left(256ae^3+5d^4\right)^3\left(\frac{d}{4e}+x\right)^3}{8192e^2}+\frac{4096e^{12}\left(\frac{d}{4e}+x\right)^{17}}{17}+\frac{64e^8\left(256ae^3+59d^4\right)\left(\frac{d}{4e}+x\right)^{13}}{13}+\frac{e^4\left(\frac{d}{4e}+x\right)^9\left(65536a^2e^6+20992ad^4e^3+601d^8\right)}{24}+\frac{\left(256ae^3+5d^4\right)^2\left(256ae^3+59d^4\right)\left(\frac{d}{4e}+x\right)^5}{5120}+\frac{\left(256ae^3+5d^4\right)^4\left(\frac{d}{4e}+x\right)}{1048576e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2)**4,x)
```

```
[Out] -2048*d**2*e**10*(d/(4*e) + x)**15/5 - 72*d**2*e**6*(256*a*e**3 +
 17*d**4)*(d/(4*e) + x)**11/11 - 9*d**2*e**2*(d/(4*e) + x)**7*(65
536*a**2*e**6 + 5632*a*d**4*e**3 + 85*d**8)/224 - d**2*(256*a*e**
3 + 5*d**4)**3*(d/(4*e) + x)**3/(8192*e**2) + 4096*e**12*(d/(4*e)
 + x)**17/17 + 64*e**8*(256*a*e**3 + 59*d**4)*(d/(4*e) + x)**13/1
3 + e**4*(d/(4*e) + x)**9*(65536*a**2*e**6 + 20992*a*d**4*e**3 +
601*d**8)/24 + (256*a*e**3 + 5*d**4)**2*(256*a*e**3 + 59*d**4)*(d
/(4*e) + x)**5/5120 + (256*a*e**3 + 5*d**4)**4*(d/(4*e) + x)/(104
8576*e**4)
```

Mathematica [A] time = 0.0919206, size = 345, normalized size = 1.17

$$4096a^4e^8x-1024a^3d^3e^6x^2+8ade^2x^4\left(512a^2e^6-d^8\right)+128a^2d^6e^4x^3-4de^3x^8\left(-1536a^2e^6+192ad^4e^3+d^8\right)+\frac{128}{3}e^4x^9\left(64a^2e^6-32ad^4e^3+d^8\right)-\frac{32}{7}d^2e^2x^7\left(-768a^2e^6-24ad^4e^3+d^8\right)+\frac{1}{5}x^5\left(16384a^3e^9-6144a^2d^4e^6+d^{12}\right)+\frac{2048}{13}e^8x^{13}\left(8ae^3-d^4\right)-512de^7x^{12}\left(d^4-8ae^3\right)+\frac{128}{5}d^3e^5x^{10}\left(40ae^3+3d^4\right)-128ad^3e^4x^6\left(8ae^3-d^4\right)+\frac{128}{11}d^2e^6x^{11}\left(384ae^3-13d^4\right)+1024d^3e^9x^{14}+\frac{8192}{5}d^2e^{10}x^{15}+1024de^{11}x^{16}+\frac{4096e^{12}x^{17}}{17}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^4,x]
```

```
[Out] 4096*a^4*e^8*x - 1024*a^3*d^3*e^6*x^2 + 128*a^2*d^6*e^4*x^3 + 8*a
*d*e^2*(-d^8 + 512*a^2*e^6)*x^4 + ((d^12 - 6144*a^2*d^4*e^6 + 163
84*a^3*e^9)*x^5)/5 - 128*a*d^3*e^4*(-d^4 + 8*a*e^3)*x^6 - (32*d^2
*e^2*(d^8 - 24*a*d^4*e^3 - 768*a^2*e^6)*x^7)/7 - 4*d*e^3*(d^8 + 1
92*a*d^4*e^3 - 1536*a^2*e^6)*x^8 + (128*e^4*(d^8 - 32*a*d^4*e^3 +
 64*a^2*e^6)*x^9)/3 + (128*d^3*e^5*(3*d^4 + 40*a*e^3)*x^10)/5 + (
128*d^2*e^6*(-13*d^4 + 384*a*e^3)*x^11)/11 - 512*d*e^7*(d^4 - 8*a
*e^3)*x^12 + (2048*e^8*(-d^4 + 8*a*e^3)*x^13)/13 + 1024*d^3*e^9*x
^14 + (8192*d^2*e^10*x^15)/5 + 1024*d*e^11*x^16 + (4096*e^12*x^17
)/17
```

Maple [A] time = 0.003, size = 500, normalized size = 1.7

$$\frac{4096 e^{12}x^{17}}{17}+1024 de^{11}x^{16}+\frac{8192 d^{2}e^{10}x^{15}}{5}+1024 d^{3}e^{9}x^{14}+\frac{\left(16384 ae^{5}-2048 d^{4}e^{2}\right) e^{6}x^{13}}{13}+\frac{\left(16384 ae^{10}d+256 \left(128 ae^{5}-16 d^{4}e^{2}\right) de^{5}-2048 d^{5}e^{7}\right) x^{12}}{12}+\frac{\left(384 d^{6}e^{6}+32768 ae^{9}d^{2}+128 \left(128 ae^{5}-16 d^{4}e^{2}\right) d^{2}e^{4}\right) x^{11}}{11}+\frac{\left(14336 ad^{3}e^{8}+256 d^{7}e^{5}-32 \left(128 ae^{5}-16 d^{4}e^{2}\right) d^{3}e^{3}\right) x^{10}}{10}+\frac{\left(8192 a^{2}e^{10}-8192 ad^{4}e^{7}+128 d^{8}e^{4}+\left(128 ae^{5}-16 d^{4}e^{2}\right)^{2}\right) x^{9}}{9}+\frac{\left(16384 a^{2}e^{9}d-2048 ad^{5}e^{6}-32 d^{9}e^{3}+256 ae^{4}d \left(128 ae^{5}-16 d^{4}e^{2}\right)\right) x^{8}}{8}+\frac{\left(24576 a^{2}e^{8}d^{2}+512 ad^{6}e^{5}+2 d^{6} \left(128 ae^{5}-16 d^{4}e^{2}\right)\right) x^{7}}{7}+\frac{\left(-2048 a^{2}e^{7}d^{3}-32 ad^{3}e^{2} \left(128 ae^{5}-16 d^{4}e^{2}\right)+256 d^{7}ae^{4}\right) x^{6}}{6}+\frac{\left(128 a^{2}e^{4} \left(128 ae^{5}-16 d^{4}e^{2}\right)-4096 a^{2}d^{4}e^{6}+d^{12}\right) x^{5}}{5}+\frac{\left(16384 a^{3}e^{8}d-32 ad^{9}e^{2}\right) x^{4}}{4}+128 a^{2}e^{4}d^{6}x^{3}-1024 a^{3}e^{6}d^{3}x^{2}+4096 a^{4}e^{8}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^4,x)
```

```
[Out] 4096/17*e^12*x^17+1024*d*e^11*x^16+8192/5*d^2*e^10*x^15+1024*d^3*
e^9*x^14+128/13*(128*a*e^5-16*d^4*e^2)*e^6*x^13+1/12*(16384*a*e^1
0*d+256*(128*a*e^5-16*d^4*e^2)*d*e^5-2048*d^5*e^7)*x^12+1/11*(384
*d^6*e^6+32768*a*e^9*d^2+128*(128*a*e^5-16*d^4*e^2)*d^2*e^4)*x^11
+1/10*(14336*a*d^3*e^8+256*d^7*e^5-32*(128*a*e^5-16*d^4*e^2)*d^3*
e^3)*x^10+1/9*(8192*a^2*e^10-8192*a*d^4*e^7+128*d^8*e^4+(128*a*e^
5-16*d^4*e^2)^2)*x^9+1/8*(16384*a^2*e^9*d-2048*a*d^5*e^6-32*d^9*e
^3+256*a*e^4*d*(128*a*e^5-16*d^4*e^2))*x^8+1/7*(24576*a^2*e^8*d^2
+512*a*d^6*e^5+2*d^6*(128*a*e^5-16*d^4*e^2))*x^7+1/6*(-2048*a^2*e
^7*d^3-32*a*d^3*e^2*(128*a*e^5-16*d^4*e^2)+256*d^7*a*e^4)*x^6+1/5
*(128*a^2*e^4*(128*a*e^5-16*d^4*e^2)-4096*a^2*d^4*e^6+d^12)*x^5+1
/4*(16384*a^3*d*e^8-32*a*d^9*e^2)*x^4+128*a^2*e^4*d^6*x^3-1024*a^
3*e^6*d^3*x^2+4096*a^4*e^8*x
```

Maxima [A] time = 0.785978, size = 517, normalized size = 1.75

$$\frac{4096}{17} e^{12}x^{17}+1024 de^{11}x^{16}+\frac{8192}{5} d^{2}e^{10}x^{15}+\frac{8192}{7} d^{3}e^{9}x^{14}+\frac{4096}{13} d^{4}e^{8}x^{13}+\frac{1}{5} d^{12}x^{5}+4096 a^{4}e^{8}x-\frac{4}{7} \left(7 e^{3}x^{8}+8 de^{2}x^{7}\right) d^{9}+\frac{1024}{5} \left(16 e^{3}x^{5}+20 de^{2}x^{4}-5 d^{3}x^{2}\right) a^{3}e^{6}+\frac{128}{165} \left(45 e^{6}x^{11}+99 de^{5}x^{10}+55 d^{2}e^{4}x^{9}\right) d^{6}+\frac{128}{105} \left(2240 e^{6}x^{9}+5040 de^{5}x^{8}+2880 d^{2}e^{4}x^{7}+105 d^{6}x^{3}-168 \left(5 e^{3}x^{6}+6 de^{2}x^{5}\right) d^{3}\right) a^{2}e^{4}-\frac{512}{1001} \left(286 e^{9}x^{14}+924 de^{8}x^{13}+1001 d^{2}e^{7}x^{12}+364 d^{3}e^{6}x^{11}\right) d^{3}+\frac{8}{15015} \left(2365440 e^{9}x^{13}+7687680 de^{8}x^{12}+8386560 d^{2}e^{7}x^{11}+3075072 d^{3}e^{6}x^{10}-15015 d^{9}x^{4}+34320 \left(6 e^{3}x^{7}+7 de^{2}x^{6}\right) d^{6} \cdot\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2)^4,x, algorithm="maxima")
```

```
[Out] 4096/17*e^12*x^17 + 1024*d*e^11*x^16 + 8192/5*d^2*e^10*x^15 + 819
2/7*d^3*e^9*x^14 + 4096/13*d^4*e^8*x^13 + 1/5*d^12*x^5 + 4096*a^4
```

```
*e^8*x - 4/7*(7*e^3*x^8 + 8*d*e^2*x^7)*d^9 + 1024/5*(16*e^3*x^5 +
 20*d*e^2*x^4 - 5*d^3*x^2)*a^3*e^6 + 128/165*(45*e^6*x^11 + 99*d*
e^5*x^10 + 55*d^2*e^4*x^9)*d^6 + 128/105*(2240*e^6*x^9 + 5040*d*e
^5*x^8 + 2880*d^2*e^4*x^7 + 105*d^6*x^3 - 168*(5*e^3*x^6 + 6*d*e^
2*x^5)*d^3)*a^2*e^4 - 512/1001*(286*e^9*x^14 + 924*d*e^8*x^13 + 1
001*d^2*e^7*x^12 + 364*d^3*e^6*x^11)*d^3 + 8/15015*(2365440*e^9*x
^13 + 7687680*d*e^8*x^12 + 8386560*d^2*e^7*x^11 + 3075072*d^3*e^6
*x^10 - 15015*d^9*x^4 + 34320*(6*e^3*x^7 + 7*d*e^2*x^6)*d^6 - 320
32*(36*e^6*x^10 + 80*d*e^5*x^9 + 45*d^2*e^4*x^8)*d^3)*a*e^2
```

Fricas [A] time = 0.254619, size = 1, normalized size = 0.

$$\frac{4096}{17}x^{17}e^{12} + 1024x^{16}e^{11}d + \frac{8192}{5}x^{15}e^{10}d^2 + 1024x^{14}e^9d^3 - \frac{2048}{13}x^{13}e^8d^4 + \frac{16384}{13}x^{13}e^{11}a - 512x^{12}e^7d^5 + 4096x^{12}e^{10}da - \frac{1664}{11}x^{11}e^6d^6 + \frac{49152}{11}x^{11}e^9d^2a + \frac{384}{5}x^{10}e^5d^7 + 1024x^{10}e^8d^3a + \frac{128}{3}x^9e^4d^8 - \frac{4096}{3}x^9e^7d^4a + \frac{8192}{3}x^9e^{10}a^2 - 4x^8e^3d^9 - 768x^8e^6d^5a + 6144x^8e^9da^2 - \frac{32}{7}x^7e^2d^{10} + \frac{768}{7}x^7e^5d^6a + \frac{24576}{7}x^7e^8d^2a^2 + 128x^6e^4d^7a - 1024x^6e^7d^3a^2 + \frac{1}{5}x^5d^{12} - \frac{6144}{5}x^5e^6d^4a^2 + \frac{16384}{5}x^5e^9a^3 - 8x^4e^2d^9a + 4096x^4e^8da^3 + 128x^3e^4d^6a^2 - 1024x^2e^6d^3a^3 + 4096xe^8a^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2)^4,x, algorithm="fricas")
```

```
[Out] 4096/17*x^17*e^12 + 1024*x^16*e^11*d + 8192/5*x^15*e^10*d^2 + 102
4*x^14*e^9*d^3 - 2048/13*x^13*e^8*d^4 + 16384/13*x^13*e^11*a - 51
2*x^12*e^7*d^5 + 4096*x^12*e^10*d*a - 1664/11*x^11*e^6*d^6 + 4915
2/11*x^11*e^9*d^2*a + 384/5*x^10*e^5*d^7 + 1024*x^10*e^8*d^3*a +
128/3*x^9*e^4*d^8 - 4096/3*x^9*e^7*d^4*a + 8192/3*x^9*e^10*a^2 -
4*x^8*e^3*d^9 - 768*x^8*e^6*d^5*a + 6144*x^8*e^9*d*a^2 - 32/7*x^7
*e^2*d^10 + 768/7*x^7*e^5*d^6*a + 24576/7*x^7*e^8*d^2*a^2 + 128*x
^6*e^4*d^7*a - 1024*x^6*e^7*d^3*a^2 + 1/5*x^5*d^12 - 6144/5*x^5*e
^6*d^4*a^2 + 16384/5*x^5*e^9*a^3 - 8*x^4*e^2*d^9*a + 4096*x^4*e^8
*d*a^3 + 128*x^3*e^4*d^6*a^2 - 1024*x^2*e^6*d^3*a^3 + 4096*x*e^8*
a^4
```

Sympy [A] time = 0.287537, size = 366, normalized size = 1.24

$$4096a^4e^8x - 1024a^3d^3e^6x^2 + 128a^2d^6e^4x^3 + 1024d^3e^9x^{14} + \frac{8192d^2e^{10}x^{15}}{5} + 1024de^{11}x^{16} + \frac{4096e^{12}x^{17}}{17} + x^{13}\left(\frac{16384ae^{11}}{13} - \frac{2048d^4e^8}{13}\right) + x^{12}\left(4096ade^{10} - 512d^5e^7\right) + x^{11}\left(\frac{49152ad^2e^9}{11} - \frac{1664d^6e^6}{11}\right) + x^{10}\left(1024ad^3e^8 + \frac{384d^7e^5}{5}\right) + x^9\left(\frac{8192a^2e^{10}}{3} - \frac{4096ad^4e^7}{3} + \frac{128d^8e^4}{3}\right) + x^8\left(6144a^2de^9 - 768ad^5e^6 - 4d^9e^3\right) + x^7\left(\frac{24576a^2d^2e^8}{7} + \frac{768ad^6e^5}{7} - \frac{32d^{10}e^2}{7}\right) + x^6\left(-1024a^2d^3e^7 + 128ad^7e^4\right) + x^5\left(\frac{16384a^3e^9}{5} - \frac{6144a^2d^4e^6}{5} + \frac{d^{12}}{5}\right) + x^4\left(4096a^3de^8 - 8ad^9e^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2)**4,x)
```

```
[Out] 4096*a**4*e**8*x - 1024*a**3*d**3*e**6*x**2 + 128*a**2*d**6*e**4*
x**3 + 1024*d**3*e**9*x**14 + 8192*d**2*e**10*x**15/5 + 1024*d*e*
```

```
*11*x**16 + 4096*e**12*x**17/17 + x**13*(16384*a*e**11/13 - 2048*
d**4*e**8/13) + x**12*(4096*a*d*e**10 - 512*d**5*e**7) + x**11*(4
9152*a*d**2*e**9/11 - 1664*d**6*e**6/11) + x**10*(1024*a*d**3*e**
8 + 384*d**7*e**5/5) + x**9*(8192*a**2*e**10/3 - 4096*a*d**4*e**7
/3 + 128*d**8*e**4/3) + x**8*(6144*a**2*d*e**9 - 768*a*d**5*e**6
- 4*d**9*e**3) + x**7*(24576*a**2*d**2*e**8/7 + 768*a*d**6*e**5/7
 - 32*d**10*e**2/7) + x**6*(-1024*a**2*d**3*e**7 + 128*a*d**7*e**
4) + x**5*(16384*a**3*e**9/5 - 6144*a**2*d**4*e**6/5 + d**12/5) +
 x**4*(4096*a**3*d*e**8 - 8*a*d**9*e**2)
```

GIAC/XCAS [A] time = 0.260194, size = 436, normalized size = 1.48

$$\frac{4096}{17}\,x^{17}e^{12} + 1024\,dx^{16}e^{11} + \frac{8192}{5}\,d^2x^{15}e^{10} + 1024\,d^3x^{14}e^9 - \frac{2048}{13}\,d^4x^{13}e^8 - 512\,d^5x^{12}e^7 - \frac{1664}{11}\,d^6x^{11}e^6 + \frac{384}{5}\,d^7x^{10}e^5 + \frac{128}{3}\,d^8x^9e^4 - 4\,d^9x^8e^3 - \frac{32}{7}\,d^{10}x^7e^2 + \frac{1}{5}\,d^{12}x^5 + \frac{16384}{13}\,ax^{13}e^{11} + 4096\,adx^{12}e^{10} + \frac{49152}{11}\,ad^2x^{11}e^9 + 1024\,ad^3x^{10}e^8 - \frac{4096}{3}\,ad^4x^9e^7 - 768\,ad^5x^8e^6 + \frac{768}{7}\,ad^6x^7e^5 + 128\,ad^7x^6e^4 - 8\,ad^9x^4e^2 + \frac{8192}{3}\,a^2x^9e^{10} + 6144\,a^2dx^8e^9 + \frac{24576}{7}\,a^2d^2x^7e^8 - 1024\,a^2d^3x^6e^7 - \frac{6144}{5}\,a^2d^4x^5e^6 + 128\,a^2d^6x^3e^4 + \frac{16384}{5}\,a^3x^5e^9 + 4096\,a^3dx^4e^8 - 1024\,a^3d^3x^2e^6 + 4096\,a^4xe^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2)^4,x, algorithm="giac")
```

```
[Out] 4096/17*x^17*e^12 + 1024*d*x^16*e^11 + 8192/5*d^2*x^15*e^10 + 102
4*d^3*x^14*e^9 - 2048/13*d^4*x^13*e^8 - 512*d^5*x^12*e^7 - 1664/1
1*d^6*x^11*e^6 + 384/5*d^7*x^10*e^5 + 128/3*d^8*x^9*e^4 - 4*d^9*x
^8*e^3 - 32/7*d^10*x^7*e^2 + 1/5*d^12*x^5 + 16384/13*a*x^13*e^11
+ 4096*a*d*x^12*e^10 + 49152/11*a*d^2*x^11*e^9 + 1024*a*d^3*x^10*
e^8 - 4096/3*a*d^4*x^9*e^7 - 768*a*d^5*x^8*e^6 + 768/7*a*d^6*x^7*
e^5 + 128*a*d^7*x^6*e^4 - 8*a*d^9*x^4*e^2 + 8192/3*a^2*x^9*e^10 +
 6144*a^2*d*x^8*e^9 + 24576/7*a^2*d^2*x^7*e^8 - 1024*a^2*d^3*x^6*
e^7 - 6144/5*a^2*d^4*x^5*e^6 + 128*a^2*d^6*x^3*e^4 + 16384/5*a^3*
x^5*e^9 + 4096*a^3*d*x^4*e^8 - 1024*a^3*d^3*x^2*e^6 + 4096*a^4*x*
e^8
```

3.40 $\int \left(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4\right)^3 dx$

Optimal. Leaf size=203

$$512a^3e^6x - \frac{1}{4}dx^4\left(d^8 - 1536a^2e^6\right) - 96a^2d^3e^4x^2 + 8ad^6e^2x^3 - \frac{128}{3}e^5x^9\left(d^4 - 4ae^3\right) - 24de^4x^8\left(d^4 - 16ae^3\right) - \frac{384}{5}ae^4x^5\left(d^4 - 4ae^3\right) + 4d^3e^2x^6\left(d^4 - 16ae^3\right) + \frac{24}{7}d^2e^3x^7\left(64ae^3 + d^4\right) + 32d^3e^6x^{10} + \frac{1536}{11}d^2e^7x^{11} + 128de^8x^{12} + \frac{512e^9x^{13}}{13}$$

```
[Out] 512*a^3*e^6*x - 96*a^2*d^3*e^4*x^2 + 8*a*d^6*e^2*x^3 - (d*(d^8 -
1536*a^2*e^6)*x^4)/4 - (384*a*e^4*(d^4 - 4*a*e^3)*x^5)/5 + 4*d^3*
e^2*(d^4 - 16*a*e^3)*x^6 + (24*d^2*e^3*(d^4 + 64*a*e^3)*x^7)/7 -
24*d*e^4*(d^4 - 16*a*e^3)*x^8 - (128*e^5*(d^4 - 4*a*e^3)*x^9)/3 +
 32*d^3*e^6*x^10 + (1536*d^2*e^7*x^11)/11 + 128*d*e^8*x^12 + (512
*e^9*x^13)/13
```

Rubi [A] time = 0.254319, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$

$$512a^3e^6x - \frac{1}{4}dx^4\left(d^8 - 1536a^2e^6\right) - 96a^2d^3e^4x^2 + 8ad^6e^2x^3 - \frac{128}{3}e^5x^9\left(d^4 - 4ae^3\right) - 24de^4x^8\left(d^4 - 16ae^3\right) - \frac{384}{5}ae^4x^5\left(d^4 - 4ae^3\right) + 4d^3e^2x^6\left(d^4 - 16ae^3\right) + \frac{24}{7}d^2e^3x^7\left(64ae^3 + d^4\right) + 32d^3e^6x^{10} + \frac{1536}{11}d^2e^7x^{11} + 128de^8x^{12} + \frac{512e^9x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In]  Int[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^3,x]
```

```
[Out] 512*a^3*e^6*x - 96*a^2*d^3*e^4*x^2 + 8*a*d^6*e^2*x^3 - (d*(d^8 -
1536*a^2*e^6)*x^4)/4 - (384*a*e^4*(d^4 - 4*a*e^3)*x^5)/5 + 4*d^3*
e^2*(d^4 - 16*a*e^3)*x^6 + (24*d^2*e^3*(d^4 + 64*a*e^3)*x^7)/7 -
24*d*e^4*(d^4 - 16*a*e^3)*x^8 - (128*e^5*(d^4 - 4*a*e^3)*x^9)/3 +
 32*d^3*e^6*x^10 + (1536*d^2*e^7*x^11)/11 + 128*d*e^8*x^12 + (512
*e^9*x^13)/13
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.046186, size = 207, normalized size = 1.02

$$512a^3e^6x - \frac{1}{4}dx^4\left(d^8 - 1536a^2e^6\right) - 96a^2d^3e^4x^2 + 8ad^6e^2x^3 + \frac{128}{3}e^5x^9\left(4ae^3 - d^4\right) - 24de^4x^8\left(d^4 - 16ae^3\right) + \frac{384}{5}ae^4x^5\left(4ae^3 - d^4\right) + 4d^3e^2x^6\left(d^4 - 16ae^3\right) + \frac{24}{7}d^2e^3x^7\left(64ae^3 + d^4\right) + 32d^3e^6x^{10} + \frac{1536}{11}d^2e^7x^{11} + 128de^8x^{12} + \frac{512e^9x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^3,x]
```

```
[Out] 512*a^3*e^6*x - 96*a^2*d^3*e^4*x^2 + 8*a*d^6*e^2*x^3 - (d*(d^8 -
1536*a^2*e^6)*x^4)/4 + (384*a*e^4*(-d^4 + 4*a*e^3)*x^5)/5 + 4*d^3
*e^2*(d^4 - 16*a*e^3)*x^6 + (24*d^2*e^3*(d^4 + 64*a*e^3)*x^7)/7 -
 24*d*e^4*(d^4 - 16*a*e^3)*x^8 + (128*e^5*(-d^4 + 4*a*e^3)*x^9)/3
 + 32*d^3*e^6*x^10 + (1536*d^2*e^7*x^11)/11 + 128*d*e^8*x^12 + (5
12*e^9*x^13)/13
```

Maple [A] time = 0.002, size = 288, normalized size = 1.4

$$\frac{512\,e^{9}x^{13}}{13}+128\,de^{8}x^{12}+\frac{1536\,d^{2}e^{7}x^{11}}{11}+32\,d^{3}e^{6}x^{10}+\frac{\left(512\,ae^{8}-256\,d^{4}e^{5}+8\,e^{3}\left(128\,ae^{5}-16\,d^{4}e^{2}\right)\right)x^{9}}{9}+\frac{\left(2048\,ae^{7}d-64\,d^{5}e^{4}+8\,e^{2}d\left(128\,ae^{5}-16\,d^{4}e^{2}\right)\right)x^{8}}{8}+\frac{\left(1536\,ae^{6}d^{2}+24\,d^{6}e^{3}\right)x^{7}}{7}+\frac{\left(-256\,ae^{5}d^{3}-d^{3}\left(128\,ae^{5}-16\,d^{4}e^{2}\right)+8\,e^{2}d^{7}\right)x^{6}}{6}+\frac{\left(8\,ae^{2}\left(128\,ae^{5}-16\,d^{4}e^{2}\right)-256\,d^{4}ae^{4}+512\,e^{7}a^{2}\right)x^{5}}{5}+\frac{\left(1536\,a^{2}e^{6}d-d^{9}\right)x^{4}}{4}+8\,ad^{6}e^{2}x^{3}-96\,a^{2}d^{3}e^{4}x^{2}+512\,a^{3}e^{6}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^3,x)
```

```
[Out] 512/13*e^9*x^13+128*d*e^8*x^12+1536/11*d^2*e^7*x^11+32*d^3*e^6*x^
10+1/9*(512*a*e^8-256*d^4*e^5+8*e^3*(128*a*e^5-16*d^4*e^2))*x^9+1
/8*(2048*a*e^7*d-64*d^5*e^4+8*e^2*d*(128*a*e^5-16*d^4*e^2))*x^8+1
/7*(1536*a*d^2*e^6+24*d^6*e^3)*x^7+1/6*(-256*a*e^5*d^3-d^3*(128*a
*e^5-16*d^4*e^2)+8*e^2*d^7)*x^6+1/5*(8*a*e^2*(128*a*e^5-16*d^4*e^
2)-256*d^4*a*e^4+512*e^7*a^2)*x^5+1/4*(1536*a^2*d*e^6-d^9)*x^4+8*
a*d^6*e^2*x^3-96*a^2*d^3*e^4*x^2+512*a^3*e^6*x
```

Maxima [A] time = 0.788304, size = 289, normalized size = 1.42

$$\frac{512}{13}\,e^{9}x^{13}+128\,de^{8}x^{12}+\frac{1536}{11}\,d^{2}e^{7}x^{11}+\frac{256}{5}\,d^{3}e^{6}x^{10}-\frac{1}{4}\,d^{9}x^{4}+512\,a^{3}e^{6}x+\frac{4}{7}\left(6\,e^{3}x^{7}+7\,de^{2}x^{6}\right)d^{6}+\frac{96}{5}\left(16\,e^{3}x^{5}+20\,de^{2}x^{4}-5\,d^{3}x^{2}\right)a^{2}e^{4}-\frac{8}{15}\left(36\,e^{6}x^{10}+80\,de^{5}x^{9}+45\,d^{2}e^{4}x^{8}\right)d^{3}+\frac{8}{105}\left(2240\,e^{6}x^{9}+5040\,de^{5}x^{8}+2880\,d^{2}e^{4}x^{7}+105\,d^{6}x^{3}-168\left(5\,e^{3}x^{6}+6\,de^{2}x^{5}\right)d^{3}\right)ae^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2)^3,x, algorithm="maxima")
```

```
[Out] 512/13*e^9*x^13 + 128*d*e^8*x^12 + 1536/11*d^2*e^7*x^11 + 256/5*d
^3*e^6*x^10 - 1/4*d^9*x^4 + 512*a^3*e^6*x + 4/7*(6*e^3*x^7 + 7*d*
e^2*x^6)*d^6 + 96/5*(16*e^3*x^5 + 20*d*e^2*x^4 - 5*d^3*x^2)*a^2*e
^4 - 8/15*(36*e^6*x^10 + 80*d*e^5*x^9 + 45*d^2*e^4*x^8)*d^3 + 8/1
05*(2240*e^6*x^9 + 5040*d*e^5*x^8 + 2880*d^2*e^4*x^7 + 105*d^6*x^
3 - 168*(5*e^3*x^6 + 6*d*e^2*x^5)*d^3)*a*e^2
```

Fricas [A] time = 0.231084, size = 1, normalized size = 0.

$$\frac{512}{13}x^{13}e^9+128x^{12}e^8d+\frac{1536}{11}x^{11}e^7d^2+32x^{10}e^6d^3-\frac{128}{3}x^9e^5d^4+\frac{512}{3}x^9e^8a-24x^8e^4d^5+384x^8e^7da+\frac{24}{7}x^7e^3d^6+\frac{1536}{7}x^7e^6d^2a+4x^6e^2d^7-64x^6e^5d^3a-\frac{384}{5}x^5e^4d^4a+\frac{1536}{5}x^5e^7a^2-\frac{1}{4}x^4d^9+384x^4e^6da^2+8x^3e^2d^6a-96x^2e^4d^3a^2+512xe^6a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2)^3,x, algorithm="fricas")
```

```
[Out] 512/13*x^13*e^9 + 128*x^12*e^8*d + 1536/11*x^11*e^7*d^2 + 32*x^10
*e^6*d^3 - 128/3*x^9*e^5*d^4 + 512/3*x^9*e^8*a - 24*x^8*e^4*d^5 +
 384*x^8*e^7*d*a + 24/7*x^7*e^3*d^6 + 1536/7*x^7*e^6*d^2*a + 4*x^
6*e^2*d^7 - 64*x^6*e^5*d^3*a - 384/5*x^5*e^4*d^4*a + 1536/5*x^5*e
^7*a^2 - 1/4*x^4*d^9 + 384*x^4*e^6*d*a^2 + 8*x^3*e^2*d^6*a - 96*x
^2*e^4*d^3*a^2 + 512*x*e^6*a^3
```

Sympy [A] time = 0.208052, size = 218, normalized size = 1.07

$$512a^3e^6x-96a^2d^3e^4x^2+8ad^6e^2x^3+32d^3e^6x^{10}+\frac{1536d^2e^7x^{11}}{11}+128de^8x^{12}+\frac{512e^9x^{13}}{13}+x^9\left(\frac{512ae^8}{3}-\frac{128d^4e^5}{3}\right)+x^8\left(384ade^7-24d^5e^4\right)+x^7\left(\frac{1536ad^2e^6}{7}+\frac{24d^6e^3}{7}\right)+x^6\left(-64ad^3e^5+4d^7e^2\right)+x^5\left(\frac{1536a^2e^7}{5}-\frac{384ad^4e^4}{5}\right)+x^4\left(384a^2de^6-\frac{d^9}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2)**3,x)
```

```
[Out] 512*a**3*e**6*x - 96*a**2*d**3*e**4*x**2 + 8*a*d**6*e**2*x**3 + 3
2*d**3*e**6*x**10 + 1536*d**2*e**7*x**11/11 + 128*d*e**8*x**12 +
512*e**9*x**13/13 + x**9*(512*a*e**8/3 - 128*d**4*e**5/3) + x**8*
(384*a*d*e**7 - 24*d**5*e**4) + x**7*(1536*a*d**2*e**6/7 + 24*d**
6*e**3/7) + x**6*(-64*a*d**3*e**5 + 4*d**7*e**2) + x**5*(1536*a**
2*e**7/5 - 384*a*d**4*e**4/5) + x**4*(384*a**2*d*e**6 - d**9/4)
```

GIAC/XCAS [A] time = 0.261037, size = 252, normalized size = 1.24

$$\frac{512}{13}x^{13}e^9+128\,dx^{12}e^8+\frac{1536}{11}d^2x^{11}e^7+32\,d^3x^{10}e^6-\frac{128}{3}d^4x^9e^5-24\,d^5x^8e^4+\frac{24}{7}d^6x^7e^3+4\,d^7x^6e^2-\frac{1}{4}d^9x^4+\frac{512}{3}ax^9e^8+384\,adx^8e^7+\frac{1536}{7}ad^2x^7e^6-64\,ad^3x^6e^5-\frac{384}{5}ad^4x^5e^4+8\,ad^6x^3e^2+\frac{1536}{5}a^2x^5e^7+384\,a^2dx^4e^6-96\,a^2d^3x^2e^4+512\,a^3xe^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2)^3,x, algorithm="giac")
```

```
[Out] 512/13*x^13*e^9 + 128*d*x^12*e^8 + 1536/11*d^2*x^11*e^7 + 32*d^3*
x^10*e^6 - 128/3*d^4*x^9*e^5 - 24*d^5*x^8*e^4 + 24/7*d^6*x^7*e^3
+ 4*d^7*x^6*e^2 - 1/4*d^9*x^4 + 512/3*a*x^9*e^8 + 384*a*d*x^8*e^7
 + 1536/7*a*d^2*x^7*e^6 - 64*a*d^3*x^6*e^5 - 384/5*a*d^4*x^5*e^4
+ 8*a*d^6*x^3*e^2 + 1536/5*a^2*x^5*e^7 + 384*a^2*d*x^4*e^6 - 96*a
^2*d^3*x^2*e^4 + 512*a^3*x*e^6
```

3.41 $\int \left(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4\right)^2 dx$

Optimal. Leaf size=107

$$64a^2e^4x - \frac{16}{5}e^2x^5\left(d^4 - 8ae^3\right) - 8ad^3e^2x^2 + 32ade^4x^4 + \frac{d^6x^3}{3} - \frac{8}{3}d^3e^3x^6 + \frac{64}{7}d^2e^4x^7 + 16de^5x^8 + \frac{64e^6x^9}{9}$$

```
[Out] 64*a^2*e^4*x - 8*a*d^3*e^2*x^2 + (d^6*x^3)/3 + 32*a*d*e^4*x^4 - (
16*e^2*(d^4 - 8*a*e^3)*x^5)/5 - (8*d^3*e^3*x^6)/3 + (64*d^2*e^4*x
^7)/7 + 16*d*e^5*x^8 + (64*e^6*x^9)/9
```

Rubi [A] time = 0.108091, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$

$$64a^2e^4x - \frac{16}{5}e^2x^5\left(d^4 - 8ae^3\right) - 8ad^3e^2x^2 + 32ade^4x^4 + \frac{d^6x^3}{3} - \frac{8}{3}d^3e^3x^6 + \frac{64}{7}d^2e^4x^7 + 16de^5x^8 + \frac{64e^6x^9}{9}$$

Antiderivative was successfully verified.

```
[In]  Int[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^2,x]
```

```
[Out] 64*a^2*e^4*x - 8*a*d^3*e^2*x^2 + (d^6*x^3)/3 + 32*a*d*e^4*x^4 - (
16*e^2*(d^4 - 8*a*e^3)*x^5)/5 - (8*d^3*e^3*x^6)/3 + (64*d^2*e^4*x
^7)/7 + 16*d*e^5*x^8 + (64*e^6*x^9)/9
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.0213755, size = 109, normalized size = 1.02

$$64a^2e^4x + \frac{16}{5}e^2x^5\left(8ae^3 - d^4\right) - 8ad^3e^2x^2 + 32ade^4x^4 + \frac{d^6x^3}{3} - \frac{8}{3}d^3e^3x^6 + \frac{64}{7}d^2e^4x^7 + 16de^5x^8 + \frac{64e^6x^9}{9}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^2,x]
```

```
[Out] 64*a^2*e^4*x - 8*a*d^3*e^2*x^2 + (d^6*x^3)/3 + 32*a*d*e^4*x^4 + (
16*e^2*(-d^4 + 8*a*e^3)*x^5)/5 - (8*d^3*e^3*x^6)/3 + (64*d^2*e^4*
x^7)/7 + 16*d*e^5*x^8 + (64*e^6*x^9)/9
```

Maple [A] time = 0.001, size = 100, normalized size = 0.9

$$\frac{64\,e^6x^9}{9} + 16\,de^5x^8 + \frac{64\,d^2e^4x^7}{7} - \frac{8\,d^3e^3x^6}{3} + \frac{\left(128\,ae^5 - 16\,d^4e^2\right)x^5}{5} + 32\,ade^4x^4 + \frac{d^6x^3}{3} - 8\,ad^3e^2x^2 + 64\,a^2e^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x)
```

```
[Out] 64/9*e^6*x^9+16*d*e^5*x^8+64/7*d^2*e^4*x^7-8/3*d^3*e^3*x^6+1/5*(1
28*a*e^5-16*d^4*e^2)*x^5+32*a*d*e^4*x^4+1/3*d^6*x^3-8*a*d^3*e^2*x
^2+64*a^2*e^4*x
```

Maxima [A] time = 0.767343, size = 136, normalized size = 1.27

$$\frac{64}{9}e^6x^9 + 16\,de^5x^8 + \frac{64}{7}d^2e^4x^7 + \frac{1}{3}d^6x^3 + 64\,a^2e^4x - \frac{8}{15}\left(5\,e^3x^6 + 6\,de^2x^5\right)d^3 + \frac{8}{5}\left(16\,e^3x^5 + 20\,de^2x^4 - 5\,d^3x^2\right)ae^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2)^2,x, algorithm="maxima")
```

```
[Out] 64/9*e^6*x^9 + 16*d*e^5*x^8 + 64/7*d^2*e^4*x^7 + 1/3*d^6*x^3 + 64
*a^2*e^4*x - 8/15*(5*e^3*x^6 + 6*d*e^2*x^5)*d^3 + 8/5*(16*e^3*x^5
 + 20*d*e^2*x^4 - 5*d^3*x^2)*a*e^2
```

Fricas [A] time = 0.236083, size = 1, normalized size = 0.01

$$\frac{64}{9}x^9e^6 + 16x^8e^5d + \frac{64}{7}x^7e^4d^2 - \frac{8}{3}x^6e^3d^3 - \frac{16}{5}x^5e^2d^4 + \frac{128}{5}x^5e^5a + 32x^4e^4da + \frac{1}{3}x^3d^6 - 8x^2e^2d^3a + 64xe^4a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2)^2,x, algorithm="fricas")
```

```
[Out] 64/9*x^9*e^6 + 16*x^8*e^5*d + 64/7*x^7*e^4*d^2 - 8/3*x^6*e^3*d^3
- 16/5*x^5*e^2*d^4 + 128/5*x^5*e^5*a + 32*x^4*e^4*d*a + 1/3*x^3*d
^6 - 8*x^2*e^2*d^3*a + 64*x*e^4*a^2
```

Sympy [A] time = 0.143078, size = 112, normalized size = 1.05

$$64a^2e^4x - 8ad^3e^2x^2 + 32ade^4x^4 + \frac{d^6x^3}{3} - \frac{8d^3e^3x^6}{3} + \frac{64d^2e^4x^7}{7} + 16de^5x^8 + \frac{64e^6x^9}{9} + x^5\left(\frac{128ae^5}{5} - \frac{16d^4e^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2)**2,x)
```

```
[Out] 64*a**2*e**4*x - 8*a*d**3*e**2*x**2 + 32*a*d*e**4*x**4 + d**6*x**
3/3 - 8*d**3*e**3*x**6/3 + 64*d**2*e**4*x**7/7 + 16*d*e**5*x**8 +
 64*e**6*x**9/9 + x**5*(128*a*e**5/5 - 16*d**4*e**2/5)
```

GIAC/XCAS [A] time = 0.26019, size = 122, normalized size = 1.14

$$\frac{64}{9}x^9e^6 + 16\,dx^8e^5 + \frac{64}{7}d^2x^7e^4 - \frac{8}{3}d^3x^6e^3 - \frac{16}{5}d^4x^5e^2 + \frac{1}{3}d^6x^3 + \frac{128}{5}ax^5e^5 + 32\,adx^4e^4 - 8\,ad^3x^2e^2 + 64\,a^2xe^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2)^2,x, algorithm="giac")
```

```
[Out] 64/9*x^9*e^6 + 16*d*x^8*e^5 + 64/7*d^2*x^7*e^4 - 8/3*d^3*x^6*e^3
- 16/5*d^4*x^5*e^2 + 1/3*d^6*x^3 + 128/5*a*x^5*e^5 + 32*a*d*x^4*e
^4 - 8*a*d^3*x^2*e^2 + 64*a^2*x*e^4
```

3.42 $\int \left(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4\right)\, dx$

Optimal. Leaf size=37

$$8ae^2x - \frac{d^3x^2}{2} + 2de^2x^4 + \frac{8e^3x^5}{5}$$

```
[Out] 8*a*e^2*x - (d^3*x^2)/2 + 2*d*e^2*x^4 + (8*e^3*x^5)/5
```

Rubi [A] time = 0.0205413, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.

$$8ae^2x - \frac{d^3x^2}{2} + 2de^2x^4 + \frac{8e^3x^5}{5}$$

Antiderivative was successfully verified.

```
[In]  Int[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4,x]

[Out] 8*a*e^2*x - (d^3*x^2)/2 + 2*d*e^2*x^4 + (8*e^3*x^5)/5
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$8ae^2x - d^3\int x\, dx + 2de^2x^4 + \frac{8e^3x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2,x)

[Out] 8*a*e**2*x - d**3*Integral(x, x) + 2*d*e**2*x**4 + 8*e**3*x**5/5
```

Mathematica [A] time = 0.0000697563, size = 37, normalized size = 1.

$$8ae^2x - \frac{d^3x^2}{2} + 2de^2x^4 + \frac{8e^3x^5}{5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4,x]

[Out] 8*a*e^2*x - (d^3*x^2)/2 + 2*d*e^2*x^4 + (8*e^3*x^5)/5
```

Maple [A] time = 0.001, size = 34, normalized size = 0.9

$$8\,ae^2x - \frac{d^3x^2}{2} + 2\,de^2x^4 + \frac{8\,e^3x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2,x)
```

```
[Out] 8*a*e^2*x-1/2*d^3*x^2+2*d*e^2*x^4+8/5*e^3*x^5
```

Maxima [A] time = 0.796685, size = 45, normalized size = 1.22

$$\frac{8}{5}e^3x^5 + 2\,de^2x^4 - \frac{1}{2}\,d^3x^2 + 8\,ae^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2,x, algorithm="maxima")
```

```
[Out] 8/5*e^3*x^5 + 2*d*e^2*x^4 - 1/2*d^3*x^2 + 8*a*e^2*x
```

Fricas [A] time = 0.229299, size = 1, normalized size = 0.03

$$\frac{8}{5}x^5e^3 + 2x^4e^2d - \frac{1}{2}x^2d^3 + 8xe^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2,x, algorithm="fricas")
```

```
[Out] 8/5*x^5*e^3 + 2*x^4*e^2*d - 1/2*x^2*d^3 + 8*x*e^2*a
```

Sympy [A] time = 0.094644, size = 36, normalized size = 0.97

$$8ae^2x - \frac{d^3x^2}{2} + 2de^2x^4 + \frac{8e^3x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2,x)
```

```
[Out] 8*a*e**2*x - d**3*x**2/2 + 2*d*e**2*x**4 + 8*e**3*x**5/5
```

GIAC/XCAS [A] time = 0.26142, size = 41, normalized size = 1.11

$$\frac{8}{5}x^5e^3 + 2\,dx^4e^2 - \frac{1}{2}\,d^3x^2 + 8\,axe^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2,x, algorithm="giac")
```

```
[Out] 8/5*x^5*e^3 + 2*d*x^4*e^2 - 1/2*d^3*x^2 + 8*a*x*e^2
```

3.43 $\int \frac{1}{8ae^2-d^3x+8de^2x^3+8e^3x^4}\,dx$

Optimal. Leaf size=153

$$\frac{2\tanh^{-1}\left(\frac{d+4ex}{\sqrt{3d^2-2\sqrt{d^4-64ae^3}}}\right)}{\sqrt{d^4-64ae^3}\sqrt{3d^2-2\sqrt{d^4-64ae^3}}}-\frac{2\tanh^{-1}\left(\frac{d+4ex}{\sqrt{2\sqrt{d^4-64ae^3}+3d^2}}\right)}{\sqrt{d^4-64ae^3}\sqrt{2\sqrt{d^4-64ae^3}+3d^2}}$$

```
[Out] (2*ArcTanh[(d + 4*e*x)/Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]]])/(Sq
rt[d^4 - 64*a*e^3]*Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]]) - (2*Arc
Tanh[(d + 4*e*x)/Sqrt[3*d^2 + 2*Sqrt[d^4 - 64*a*e^3]]])/(Sqrt[d^4
 - 64*a*e^3]*Sqrt[3*d^2 + 2*Sqrt[d^4 - 64*a*e^3]])
```

Rubi [A] time = 0.577814, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$

$$\frac{2\tanh^{-1}\left(\frac{d+4ex}{\sqrt{3d^2-2\sqrt{d^4-64ae^3}}}\right)}{\sqrt{d^4-64ae^3}\sqrt{3d^2-2\sqrt{d^4-64ae^3}}}-\frac{2\tanh^{-1}\left(\frac{d+4ex}{\sqrt{2\sqrt{d^4-64ae^3}+3d^2}}\right)}{\sqrt{d^4-64ae^3}\sqrt{2\sqrt{d^4-64ae^3}+3d^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^(-1),x]
```

```
[Out] (2*ArcTanh[(d + 4*e*x)/Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]]])/(Sq
rt[d^4 - 64*a*e^3]*Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]]) - (2*Arc
Tanh[(d + 4*e*x)/Sqrt[3*d^2 + 2*Sqrt[d^4 - 64*a*e^3]]])/(Sqrt[d^4
 - 64*a*e^3]*Sqrt[3*d^2 + 2*Sqrt[d^4 - 64*a*e^3]])
```

Rubi in Sympy [A] time = 78.7143, size = 148, normalized size = 0.97

$$-\frac{2\,\mathrm{atanh}\left(\frac{4e\left(\frac{d}{4e}+x\right)}{\sqrt{3d^2+2\sqrt{-64ae^3+d^4}}}\right)}{\sqrt{3d^2+2\sqrt{-64ae^3+d^4}}\sqrt{-64ae^3+d^4}}+\frac{2\,\mathrm{atanh}\left(\frac{4e\left(\frac{d}{4e}+x\right)}{\sqrt{3d^2-2\sqrt{-64ae^3+d^4}}}\right)}{\sqrt{3d^2-2\sqrt{-64ae^3+d^4}}\sqrt{-64ae^3+d^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2),x)
```

```
[Out] -2*atanh(4*e*(d/(4*e) + x)/sqrt(3*d**2 + 2*sqrt(-64*a*e**3 + d**4
)))/(sqrt(3*d**2 + 2*sqrt(-64*a*e**3 + d**4))*sqrt(-64*a*e**3 + d
**4)) + 2*atanh(4*e*(d/(4*e) + x)/sqrt(3*d**2 - 2*sqrt(-64*a*e**3
 + d**4)))/(sqrt(3*d**2 - 2*sqrt(-64*a*e**3 + d**4))*sqrt(-64*a*e
**3 + d**4))
```

Mathematica [C] time = 0.0381324, size = 71, normalized size = 0.46

$$-\mathrm{RootSum}\left[8\#1^4e^3+8\#1^3de^2-\#1d^3+8ae^2\&,\frac{\log(x-\#1)}{-32\#1^3e^3-24\#1^2de^2+d^3}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^(-1),x]
```

```
[Out] -RootSum[8*a*e^2 - d^3*#1 + 8*d*e^2*#1^3 + 8*e^3*#1^4 & , Log[x -
 #1]/(d^3 - 24*d*e^2*#1^2 - 32*e^3*#1^3) & ]
```

Maple [C] time = 0.093, size = 67, normalized size = 0.4

$$\sum_{\_R=RootOf\left(8\,e^3\_Z^4+8\,e^2d\_Z^3-d^3\_Z+8\,ae^2\right)}\frac{\ln\left(x-\_R\right)}{32\,\_R^3e^3+24\,\_R^2de^2-d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2),x)
```

```
[Out] sum(1/(32*_R^3*e^3+24*_R^2*d*e^2-d^3)*ln(x-_R),_R=RootOf(8*_Z^4*e
^3+8*_Z^3*d*e^2-_Z*d^3+8*a*e^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{8\,e^3x^4+8\,de^2x^3-d^3x+8\,ae^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2),x, algorithm="maxima")
```

```
[Out] integrate(1/(8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2), x)
```

Fricas [A] time = 0.281848, size = 1505, normalized size = 9.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2),x, algorithm="fricas")
```

```
[Out] -sqrt((3*d^2 + 2*(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)/sqrt(25*d
^12 + 960*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194304*a^3*e^9))/(5*d^
8 - 64*a*d^4*e^3 - 16384*a^2*e^6))*log(8*e*x + 2*(2*d^4 - 128*a*e
^3 - 3*(5*d^10 - 64*a*d^6*e^3 - 16384*a^2*d^2*e^6)/sqrt(25*d^12 +
 960*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194304*a^3*e^9))*sqrt((3*d^
2 + 2*(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)/sqrt(25*d^12 + 960*a
*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194304*a^3*e^9))/(5*d^8 - 64*a*d^
4*e^3 - 16384*a^2*e^6)) + 2*d) + sqrt((3*d^2 + 2*(5*d^8 - 64*a*d^
4*e^3 - 16384*a^2*e^6)/sqrt(25*d^12 + 960*a*d^8*e^3 - 98304*a^2*d
^4*e^6 - 4194304*a^3*e^9))/(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)
)*log(8*e*x - 2*(2*d^4 - 128*a*e^3 - 3*(5*d^10 - 64*a*d^6*e^3 - 1
6384*a^2*d^2*e^6)/sqrt(25*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^4*e^
6 - 4194304*a^3*e^9))*sqrt((3*d^2 + 2*(5*d^8 - 64*a*d^4*e^3 - 163
84*a^2*e^6)/sqrt(25*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 41
94304*a^3*e^9))/(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)) + 2*d) -
sqrt((3*d^2 - 2*(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)/sqrt(25*d^
12 + 960*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194304*a^3*e^9))/(5*d^8
 - 64*a*d^4*e^3 - 16384*a^2*e^6))*log(8*e*x + 2*(2*d^4 - 128*a*e^
3 + 3*(5*d^10 - 64*a*d^6*e^3 - 16384*a^2*d^2*e^6)/sqrt(25*d^12 +
960*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194304*a^3*e^9))*sqrt((3*d^2
 - 2*(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)/sqrt(25*d^12 + 960*a*
d^8*e^3 - 98304*a^2*d^4*e^6 - 4194304*a^3*e^9))/(5*d^8 - 64*a*d^4
*e^3 - 16384*a^2*e^6)) + 2*d) + sqrt((3*d^2 - 2*(5*d^8 - 64*a*d^4
```

```
*e^3 - 16384*a^2*e^6)/sqrt(25*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^
4*e^6 - 4194304*a^3*e^9))/(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6))
*log(8*e*x - 2*(2*d^4 - 128*a*e^3 + 3*(5*d^10 - 64*a*d^6*e^3 - 16
384*a^2*d^2*e^6)/sqrt(25*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^4*e^6
 - 4194304*a^3*e^9))*sqrt((3*d^2 - 2*(5*d^8 - 64*a*d^4*e^3 - 1638
4*a^2*e^6)/sqrt(25*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 419
4304*a^3*e^9))/(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)) + 2*d)
```

Sympy [A] time = 5.0173, size = 122, normalized size = 0.8

$$\text{RootSum}\left(t^4\left(1048576a^3e^9 - 12288a^2d^4e^6 - 384ad^8e^3 + 5d^{12}\right) + t^2\left(384ad^2e^3 - 6d^6\right) + 1, \left(t \mapsto t\log\left(x + \frac{-49152t^3a^2d^2e^6}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2),x)
```

```
[Out] RootSum(_t**4*(1048576*a**3*e**9 - 12288*a**2*d**4*e**6 - 384*a*d
**8*e**3 + 5*d**12) + _t**2*(384*a*d**2*e**3 - 6*d**6) + 1, Lambd
a(_t, _t*log(x + (-49152*_t**3*a**2*d**2*e**6 - 192*_t**3*a*d**6*
e**3 + 15*_t**3*d**10 + 256*_t*a*e**3 - 13*_t*d**4 + 2*d)/(8*e)))
)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{8\,e^3x^4 + 8\,de^2x^3 - d^3x + 8\,ae^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2),x, algorithm="giac")
```

```
[Out] integrate(1/(8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2), x)
```

3.44 $\int \frac{1}{(8ae^2-d^3x+8de^2x^3+8e^3x^4)^2}\,dx$

Optimal. Leaf size=342

$$\frac{2e\left(\frac{d}{4e}+x\right)\left(-256ae^3+13d^4-48d^2e^2\left(\frac{d}{4e}+x\right)^2\right)}{(-16384a^2e^6-64ad^4e^3+5d^8)\,(8ae^2-d^3x+8de^2x^3+8e^3x^4)}-\frac{24e\left(-d^2\sqrt{d^4-64ae^3}+128ae^3+d^4\right)\tanh^{-1}\left(\frac{d+4ex}{\sqrt{3d^2-2\sqrt{d^4-64ae^3}}}\right)}{(d^4-64ae^3)^{3/2}\,(256ae^3+5d^4)\sqrt{3d^2-2\sqrt{d^4-64ae^3}}}+\frac{24e\left(d^2\sqrt{d^4-64ae^3}+128ae^3+d^4\right)\tanh^{-1}\left(\frac{d+4ex}{\sqrt{2\sqrt{d^4-64ae^3}+3d^2}}\right)}{(d^4-64ae^3)^{3/2}\,(256ae^3+5d^4)\sqrt{2\sqrt{d^4-64ae^3}+3d^2}}$$

```
[Out] (2*e*(d/(4*e) + x)*(13*d^4 - 256*a*e^3 - 48*d^2*e^2*(d/(4*e) + x)
^2))/((5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)*(8*a*e^2 - d^3*x + 8
*d*e^2*x^3 + 8*e^3*x^4)) - (24*e*(d^4 + 128*a*e^3 - d^2*Sqrt[d^4
- 64*a*e^3])*ArcTanh[(d + 4*e*x)/Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e
^3]]])/((d^4 - 64*a*e^3)^(3/2)*(5*d^4 + 256*a*e^3)*Sqrt[3*d^2 - 2
*Sqrt[d^4 - 64*a*e^3]]) + (24*e*(d^4 + 128*a*e^3 + d^2*Sqrt[d^4 -
 64*a*e^3])*ArcTanh[(d + 4*e*x)/Sqrt[3*d^2 + 2*Sqrt[d^4 - 64*a*e^
3]]])/((d^4 - 64*a*e^3)^(3/2)*(5*d^4 + 256*a*e^3)*Sqrt[3*d^2 + 2*
Sqrt[d^4 - 64*a*e^3]])
```

Rubi [A] time = 1.43676, antiderivative size = 342, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{(d+4ex)\left(-256ae^3+13d^4-3d^2(d+4ex)^2\right)}{2\,(-16384a^2e^6-64ad^4e^3+5d^8)\,(8ae^2-d^3x+8de^2x^3+8e^3x^4)}-\frac{24e\left(-d^2\sqrt{d^4-64ae^3}+128ae^3+d^4\right)\tanh^{-1}\left(\frac{d+4ex}{\sqrt{3d^2-2\sqrt{d^4-64ae^3}}}\right)}{(d^4-64ae^3)^{3/2}\,(256ae^3+5d^4)\sqrt{3d^2-2\sqrt{d^4-64ae^3}}}+\frac{24e\left(d^2\sqrt{d^4-64ae^3}+128ae^3+d^4\right)\tanh^{-1}\left(\frac{d+4ex}{\sqrt{2\sqrt{d^4-64ae^3}+3d^2}}\right)}{(d^4-64ae^3)^{3/2}\,(256ae^3+5d^4)\sqrt{2\sqrt{d^4-64ae^3}+3d^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^(-2),x]
```

```
[Out] ((d + 4*e*x)*(13*d^4 - 256*a*e^3 - 3*d^2*(d + 4*e*x)^2))/(2*(5*d^
8 - 64*a*d^4*e^3 - 16384*a^2*e^6)*(8*a*e^2 - d^3*x + 8*d*e^2*x^3
+ 8*e^3*x^4)) - (24*e*(d^4 + 128*a*e^3 - d^2*Sqrt[d^4 - 64*a*e^3]
)*ArcTanh[(d + 4*e*x)/Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]]])/((d^
4 - 64*a*e^3)^(3/2)*(5*d^4 + 256*a*e^3)*Sqrt[3*d^2 - 2*Sqrt[d^4 -
 64*a*e^3]]) + (24*e*(d^4 + 128*a*e^3 + d^2*Sqrt[d^4 - 64*a*e^3])
*ArcTanh[(d + 4*e*x)/Sqrt[3*d^2 + 2*Sqrt[d^4 - 64*a*e^3]]])/((d^4
 - 64*a*e^3)^(3/2)*(5*d^4 + 256*a*e^3)*Sqrt[3*d^2 + 2*Sqrt[d^4 -
64*a*e^3]])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2)**2,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.281656, size = 234, normalized size = 0.68

$$\frac{48e^2\text{RootSum}\left[8\#1^4e^3+8\#1^3de^2-\#1d^3+8ae^2\&,\frac{2\#1^2d^2e\log(x-\#1)+32ae^2\log(x-\#1)+\#1d^3\log(x-\#1)}{32\#1^3e^3+24\#1^2de^2-d^3}\&\right]}{16384a^2e^6+64ad^4e^3-5d^8}+\frac{(d+4ex)\left(-128ae^3+5d^4-12d^3ex-24d^2e^2x^2\right)}{(d^4-64ae^3)(256ae^3+5d^4)(8ae^2-d^3x+8de^2x^3+8e^3x^4)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^(-2),x]
```

```
[Out] ((d + 4*e*x)*(5*d^4 - 128*a*e^3 - 12*d^3*e*x - 24*d^2*e^2*x^2))/(
(d^4 - 64*a*e^3)*(5*d^4 + 256*a*e^3)*(8*a*e^2 - d^3*x + 8*d*e^2*x
^3 + 8*e^3*x^4)) + (48*e^2*RootSum[8*a*e^2 - d^3*#1 + 8*d*e^2*#1^
3 + 8*e^3*#1^4 & , (32*a*e^2*Log[x - #1] + d^3*Log[x - #1]*#1 + 2
*d^2*e*Log[x - #1]*#1^2)/(-d^3 + 24*d*e^2*#1^2 + 32*e^3*#1^3) & ]
)/(-5*d^8 + 64*a*d^4*e^3 + 16384*a^2*e^6)
```

Maple [C] time = 0.051, size = 288, normalized size = 0.8

$$1\left(12\frac{d^2e^3x^3}{(256e^3a+5d^4)(64e^3a-d^4)}+9\frac{d^3e^2x^2}{(256e^3a+5d^4)(64e^3a-d^4)}+\frac{ex}{256e^3a+5d^4}+\frac{d(128e^3a-5d^4)}{131072a^2e^6+512ad^4e^3-40d^8}\right)\Big($$
$$+48e^2\sum_{\_R=RootOf(8\_Z^4e^3+8\_Z^3de^2-\_Zd^3+8ae^2)}\frac{(2d^2e\_R^2+d^3\_R+32ae^2)\ln(x-\_R)}{(256e^3a+5d^4)(64e^3a-d^4)(32\_R^3e^3+24\_R^2de^2-d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x)
```

```
[Out] (12*d^2*e^3/(256*a*e^3+5*d^4)/(64*a*e^3-d^4)*x^3+9*d^3*e^2/(256*a
*e^3+5*d^4)/(64*a*e^3-d^4)*x^2+e/(256*a*e^3+5*d^4)*x+1/8*d*(128*a
*e^3-5*d^4)/(16384*a^2*e^6+64*a*d^4*e^3-5*d^8))/(e^3*x^4+d*e^2*x^
3-1/8*d^3*x+a*e^2)+48*e^2*sum((2*_R^2*d^2*e+_R*d^3+32*a*e^2)/(256
*a*e^3+5*d^4)/(64*a*e^3-d^4)/(32*_R^3*e^3+24*_R^2*d*e^2-d^3)*ln(x
-_R),_R=RootOf(8*_Z^4*e^3+8*_Z^3*d*e^2-_Z*d^3+8*a*e^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{48e^2\int\frac{2d^2ex^2+d^3x+32ae^2}{8e^3x^4+8de^2x^3-d^3x+8ae^2}\,dx}{5d^8-64ad^4e^3-16384a^2e^6}$$
$$-\frac{96d^2e^3x^3+72d^3e^2x^2-5d^5+128ade^3-8\left(d^4e-64ae^4\right)x}{40ad^8e^2-512a^2d^4e^5-131072a^3e^8+8(5d^8e^3-64ad^4e^6-16384a^2e^9)x^4+8(5d^9e^2-64ad^5e^5-16384a^2de^8)x^3-(5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2)^(-2),x, algorithm="maxima"
```

```
[Out] -48*e^2*integrate((2*d^2*e*x^2 + d^3*x + 32*a*e^2)/(8*e^3*x^4 + 8
*d*e^2*x^3 - d^3*x + 8*a*e^2), x)/(5*d^8 - 64*a*d^4*e^3 - 16384*a
^2*e^6) - (96*d^2*e^3*x^3 + 72*d^3*e^2*x^2 - 5*d^5 + 128*a*d*e^3
```

```
- 8*(d^4*e - 64*a*e^4)*x)/(40*a*d^8*e^2 - 512*a^2*d^4*e^5 - 13107
2*a^3*e^8 + 8*(5*d^8*e^3 - 64*a*d^4*e^6 - 16384*a^2*e^9)*x^4 + 8*
(5*d^9*e^2 - 64*a*d^5*e^5 - 16384*a^2*d*e^8)*x^3 - (5*d^11 - 64*a
*d^7*e^3 - 16384*a^2*d^3*e^6)*x)
```

Fricas [A] time = 0.41681, size = 5785, normalized size = 16.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2)^(-2),x, algorithm="fricas"
```

```
[Out] -(96*d^2*e^3*x^3 + 72*d^3*e^2*x^2 - 5*d^5 + 128*a*d*e^3 + 12*sqrt
(2)*(40*a*d^8*e^2 - 512*a^2*d^4*e^5 - 131072*a^3*e^8 + 8*(5*d^8*e
^3 - 64*a*d^4*e^6 - 16384*a^2*e^9)*x^4 + 8*(5*d^9*e^2 - 64*a*d^5*
e^5 - 16384*a^2*d*e^8)*x^3 - (5*d^11 - 64*a*d^7*e^3 - 16384*a^2*d
^3*e^6)*x)*sqrt((d^10*e^2 + 160*a*d^6*e^5 + 40960*a^2*d^2*e^8 + (
125*d^24 - 4800*a*d^20*e^3 - 1167360*a^2*d^16*e^6 + 31195136*a^3*
d^12*e^9 + 3825205248*a^4*d^8*e^12 - 51539607552*a^5*d^4*e^15 - 4
398046511104*a^6*e^18)*sqrt((d^8*e^4 + 512*a*d^4*e^7 + 65536*a^2*
e^10)/(15625*d^36 + 1800000*a*d^32*e^3 - 115200000*a^2*d^28*e^6 -
 21135360000*a^3*d^24*e^9 - 150994944000*a^4*d^20*e^12 + 78082505
441280*a^5*d^16*e^15 + 2744381022928896*a^6*d^12*e^18 - 709316941
31085312*a^7*d^8*e^21 - 5188146770730811392*a^8*d^4*e^24 - 737869
76294838206464*a^9*e^27)))/(125*d^24 - 4800*a*d^20*e^3 - 1167360*
a^2*d^16*e^6 + 31195136*a^3*d^12*e^9 + 3825205248*a^4*d^8*e^12 -
51539607552*a^5*d^4*e^15 - 4398046511104*a^6*e^18))*log(884736*a*
d^5*e^6 + 226492416*a^2*d*e^9 + 3538944*(a*d^4*e^7 + 256*a^2*e^10
)*x + 13824*sqrt(2)*(d^16*e^2 - 128*a*d^12*e^5 - 61440*a^2*d^8*e^
8 + 8388608*a^3*d^4*e^11 - 268435456*a^4*e^14 - (125*d^30 + 59200
*a*d^26*e^3 - 3624960*a^2*d^22*e^6 - 566493184*a^3*d^18*e^9 + 197
97114880*a^4*d^14*e^12 + 1906965479424*a^5*d^10*e^15 - 3078632557
7728*a^6*d^6*e^18 - 2251799813685248*a^7*d^2*e^21)*sqrt((d^8*e^4
+ 512*a*d^4*e^7 + 65536*a^2*e^10)/(15625*d^36 + 1800000*a*d^32*e^
3 - 115200000*a^2*d^28*e^6 - 21135360000*a^3*d^24*e^9 - 150994944
000*a^4*d^20*e^12 + 78082505441280*a^5*d^16*e^15 + 27443810229288
96*a^6*d^12*e^18 - 70931694131085312*a^7*d^8*e^21 - 5188146770730
811392*a^8*d^4*e^24 - 73786976294838206464*a^9*e^27)))*sqrt((d^10
*e^2 + 160*a*d^6*e^5 + 40960*a^2*d^2*e^8 + (125*d^24 - 4800*a*d^2
0*e^3 - 1167360*a^2*d^16*e^6 + 31195136*a^3*d^12*e^9 + 3825205248
*a^4*d^8*e^12 - 51539607552*a^5*d^4*e^15 - 4398046511104*a^6*e^18
)*sqrt((d^8*e^4 + 512*a*d^4*e^7 + 65536*a^2*e^10)/(15625*d^36 + 1
800000*a*d^32*e^3 - 115200000*a^2*d^28*e^6 - 21135360000*a^3*d^24
*e^9 - 150994944000*a^4*d^20*e^12 + 78082505441280*a^5*d^16*e^15
+ 2744381022928896*a^6*d^12*e^18 - 70931694131085312*a^7*d^8*e^21
 - 5188146770730811392*a^8*d^4*e^24 - 73786976294838206464*a^9*e^
27)))/(125*d^24 - 4800*a*d^20*e^3 - 1167360*a^2*d^16*e^6 + 311951
36*a^3*d^12*e^9 + 3825205248*a^4*d^8*e^12 - 51539607552*a^5*d^4*e
^15 - 4398046511104*a^6*e^18))) - 12*sqrt(2)*(40*a*d^8*e^2 - 512*
a^2*d^4*e^5 - 131072*a^3*e^8 + 8*(5*d^8*e^3 - 64*a*d^4*e^6 - 1638
4*a^2*e^9)*x^4 + 8*(5*d^9*e^2 - 64*a*d^5*e^5 - 16384*a^2*d*e^8)*x
^3 - (5*d^11 - 64*a*d^7*e^3 - 16384*a^2*d^3*e^6)*x)*sqrt((d^10*e^
2 + 160*a*d^6*e^5 + 40960*a^2*d^2*e^8 + (125*d^24 - 4800*a*d^20*e
^3 - 1167360*a^2*d^16*e^6 + 31195136*a^3*d^12*e^9 + 3825205248*a^
4*d^8*e^12 - 51539607552*a^5*d^4*e^15 - 4398046511104*a^6*e^18)*s
qrt((d^8*e^4 + 512*a*d^4*e^7 + 65536*a^2*e^10)/(15625*d^36 + 1800
000*a*d^32*e^3 - 115200000*a^2*d^28*e^6 - 21135360000*a^3*d^24*e^
9 - 150994944000*a^4*d^20*e^12 + 78082505441280*a^5*d^16*e^15 + 2
744381022928896*a^6*d^12*e^18 - 70931694131085312*a^7*d^8*e^21 -
5188146770730811392*a^8*d^4*e^24 - 73786976294838206464*a^9*e^27)
))/(125*d^24 - 4800*a*d^20*e^3 - 1167360*a^2*d^16*e^6 + 31195136*
a^3*d^12*e^9 + 3825205248*a^4*d^8*e^12 - 51539607552*a^5*d^4*e^15
 - 4398046511104*a^6*e^18))*log(884736*a*d^5*e^6 + 226492416*a^2*
d*e^9 + 3538944*(a*d^4*e^7 + 256*a^2*e^10)*x - 13824*sqrt(2)*(d^1
6*e^2 - 128*a*d^12*e^5 - 61440*a^2*d^8*e^8 + 8388608*a^3*d^4*e^11
 - 268435456*a^4*e^14 - (125*d^30 + 59200*a*d^26*e^3 - 3624960*a^
2*d^22*e^6 - 566493184*a^3*d^18*e^9 + 19797114880*a^4*d^14*e^12 +
```

```
1906965479424*a^5*d^10*e^15 - 30786325577728*a^6*d^6*e^18 - 2251
799813685248*a^7*d^2*e^21)*sqrt((d^8*e^4 + 512*a*d^4*e^7 + 65536*
a^2*e^10)/(15625*d^36 + 1800000*a*d^32*e^3 - 115200000*a^2*d^28*e
^6 - 21135360000*a^3*d^24*e^9 - 150994944000*a^4*d^20*e^12 + 7808
2505441280*a^5*d^16*e^15 + 2744381022928896*a^6*d^12*e^18 - 70931
694131085312*a^7*d^8*e^21 - 518814677073081l392*a^8*d^4*e^24 - 73
786976294838206464*a^9*e^27)))*sqrt((d^10*e^2 + 160*a*d^6*e^5 + 4
0960*a^2*d^2*e^8 + (125*d^24 - 4800*a*d^20*e^3 - 1167360*a^2*d^16
*e^6 + 31195136*a^3*d^12*e^9 + 3825205248*a^4*d^8*e^12 - 51539607
552*a^5*d^4*e^15 - 4398046511104*a^6*e^18)*sqrt((d^8*e^4 + 512*a*
d^4*e^7 + 65536*a^2*e^10)/(15625*d^36 + 1800000*a*d^32*e^3 - 1152
00000*a^2*d^28*e^6 - 21135360000*a^3*d^24*e^9 - 150994944000*a^4*
d^20*e^12 + 78082505441280*a^5*d^16*e^15 + 2744381022928896*a^6*d
^12*e^18 - 70931694131085312*a^7*d^8*e^21 - 5188146770730811392*a
^8*d^4*e^24 - 73786976294838206464*a^9*e^27)))/(125*d^24 - 4800*a
*d^20*e^3 - 1167360*a^2*d^16*e^6 + 31195136*a^3*d^12*e^9 + 382520
5248*a^4*d^8*e^12 - 51539607552*a^5*d^4*e^15 - 4398046511104*a^6*
e^18))) + 12*sqrt(2)*(40*a*d^8*e^2 - 512*a^2*d^4*e^5 - 131072*a^3
*e^8 + 8*(5*d^8*e^3 - 64*a*d^4*e^6 - 16384*a^2*e^9)*x^4 + 8*(5*d^
9*e^2 - 64*a*d^5*e^5 - 16384*a^2*d*e^8)*x^3 - (5*d^11 - 64*a*d^7*
e^3 - 16384*a^2*d^3*e^6)*x)*sqrt((d^10*e^2 + 160*a*d^6*e^5 + 4096
0*a^2*d^2*e^8 - (125*d^24 - 4800*a*d^20*e^3 - 1167360*a^2*d^16*e^
6 + 31195136*a^3*d^12*e^9 + 3825205248*a^4*d^8*e^12 - 51539607552
*a^5*d^4*e^15 - 4398046511104*a^6*e^18)*sqrt((d^8*e^4 + 512*a*d^4
*e^7 + 65536*a^2*e^10)/(15625*d^36 + 1800000*a*d^32*e^3 - 1152000
00*a^2*d^28*e^6 - 21135360000*a^3*d^24*e^9 - 150994944000*a^4*d^2
0*e^12 + 78082505441280*a^5*d^16*e^15 + 2744381022928896*a^6*d^12
*e^18 - 70931694131085312*a^7*d^8*e^21 - 5188146770730811392*a^8*
d^4*e^24 - 73786976294838206464*a^9*e^27)))/(125*d^24 - 4800*a*d^
20*e^3 - 1167360*a^2*d^16*e^6 + 31195136*a^3*d^12*e^9 + 382520524
8*a^4*d^8*e^12 - 51539607552*a^5*d^4*e^15 - 4398046511104*a^6*e^1
8))*log(884736*a*d^5*e^6 + 226492416*a^2*d*e^9 + 3538944*(a*d^4*e
^7 + 256*a^2*e^10)*x + 13824*sqrt(2)*(d^16*e^2 - 128*a*d^12*e^5 -
 61440*a^2*d^8*e^8 + 8388608*a^3*d^4*e^11 - 268435456*a^4*e^14 +
(125*d^30 + 59200*a*d^26*e^3 - 3624960*a^2*d^22*e^6 - 566493184*a
^3*d^18*e^9 + 19797114880*a^4*d^14*e^12 + 1906965479424*a^5*d^10*
e^15 - 30786325577728*a^6*d^6*e^18 - 2251799813685248*a^7*d^2*e^2
1)*sqrt((d^8*e^4 + 512*a*d^4*e^7 + 65536*a^2*e^10)/(15625*d^36 +
1800000*a*d^32*e^3 - 115200000*a^2*d^28*e^6 - 21135360000*a^3*d^2
4*e^9 - 150994944000*a^4*d^20*e^12 + 78082505441280*a^5*d^16*e^15
 + 2744381022928896*a^6*d^12*e^18 - 70931694131085312*a^7*d^8*e^2
1 - 5188146770730811392*a^8*d^4*e^24 - 73786976294838206464*a^9*e
^27)))*sqrt((d^10*e^2 + 160*a*d^6*e^5 + 40960*a^2*d^2*e^8 - (125*
d^24 - 4800*a*d^20*e^3 - 1167360*a^2*d^16*e^6 + 31195136*a^3*d^12
*e^9 + 3825205248*a^4*d^8*e^12 - 51539607552*a^5*d^4*e^15 - 43980
46511104*a^6*e^18)*sqrt((d^8*e^4 + 512*a*d^4*e^7 + 65536*a^2*e^10
)/(15625*d^36 + 1800000*a*d^32*e^3 - 115200000*a^2*d^28*e^6 - 211
35360000*a^3*d^24*e^9 - 150994944000*a^4*d^20*e^12 + 780825054412
80*a^5*d^16*e^15 + 2744381022928896*a^6*d^12*e^18 - 7093169413108
5312*a^7*d^8*e^21 - 5188146770730811392*a^8*d^4*e^24 - 7378697629
4838206464*a^9*e^27)))/(125*d^24 - 4800*a*d^20*e^3 - 1167360*a^2*
d^16*e^6 + 31195136*a^3*d^12*e^9 + 3825205248*a^4*d^8*e^12 - 5153
9607552*a^5*d^4*e^15 - 4398046511104*a^6*e^18))) - 12*sqrt(2)*(40
*a*d^8*e^2 - 512*a^2*d^4*e^5 - 131072*a^3*e^8 + 8*(5*d^8*e^3 - 64
*a*d^4*e^6 - 16384*a^2*e^9)*x^4 + 8*(5*d^9*e^2 - 64*a*d^5*e^5 - 1
6384*a^2*d*e^8)*x^3 - (5*d^11 - 64*a*d^7*e^3 - 16384*a^2*d^3*e^6)
*x)*sqrt((d^10*e^2 + 160*a*d^6*e^5 + 40960*a^2*d^2*e^8 - (125*d^2
4 - 4800*a*d^20*e^3 - 1167360*a^2*d^16*e^6 + 31195136*a^3*d^12*e^
9 + 3825205248*a^4*d^8*e^12 - 51539607552*a^5*d^4*e^15 - 43980465
11104*a^6*e^18)*sqrt((d^8*e^4 + 512*a*d^4*e^7 + 65536*a^2*e^10)/(
15625*d^36 + 1800000*a*d^32*e^3 - 115200000*a^2*d^28*e^6 - 211353
60000*a^3*d^24*e^9 - 150994944000*a^4*d^20*e^12 + 78082505441280*
a^5*d^16*e^15 + 2744381022928896*a^6*d^12*e^18 - 7093169413108531
2*a^7*d^8*e^21 - 5188146770730811392*a^8*d^4*e^24 - 7378697629483
8206464*a^9*e^27)))/(125*d^24 - 4800*a*d^20*e^3 - 1167360*a^2*d^1
6*e^6 + 31195136*a^3*d^12*e^9 + 3825205248*a^4*d^8*e^12 - 5153960
7552*a^5*d^4*e^15 - 4398046511104*a^6*e^18))*log(884736*a*d^5*e^6
 + 226492416*a^2*d*e^9 + 3538944*(a*d^4*e^7 + 256*a^2*e^10)*x - 1
3824*sqrt(2)*(d^16*e^2 - 128*a*d^12*e^5 - 61440*a^2*d^8*e^8 + 838
8608*a^3*d^4*e^11 - 268435456*a^4*e^14 + (125*d^30 + 59200*a*d^26
*e^3 - 3624960*a^2*d^22*e^6 - 566493184*a^3*d^18*e^9 + 1979711488
0*a^4*d^14*e^12 + 1906965479424*a^5*d^10*e^15 - 30786325577728*a^
```

```
6*d^6*e^18 - 2251799813685248*a^7*d^2*e^21)*sqrt((d^8*e^4 + 512*a
*d^4*e^7 + 65536*a^2*e^10)/(15625*d^36 + 1800000*a*d^32*e^3 - 115
200000*a^2*d^28*e^6 - 21135360000*a^3*d^24*e^9 - 150994944000*a^4
*d^20*e^12 + 78082505441280*a^5*d^16*e^15 + 2744381022928896*a^6*
d^12*e^18 - 70931694131085312*a^7*d^8*e^21 - 5188146770730811392*
a^8*d^4*e^24 - 73786976294838206464*a^9*e^27)))*sqrt((d^10*e^2 +
160*a*d^6*e^5 + 40960*a^2*d^2*e^8 - (125*d^24 - 4800*a*d^20*e^3 -
 1167360*a^2*d^16*e^6 + 31195136*a^3*d^12*e^9 + 3825205248*a^4*d^
8*e^12 - 51539607552*a^5*d^4*e^15 - 4398046511104*a^6*e^18)*sqrt(
(d^8*e^4 + 512*a*d^4*e^7 + 65536*a^2*e^10)/(15625*d^36 + 1800000*
a*d^32*e^3 - 115200000*a^2*d^28*e^6 - 21135360000*a^3*d^24*e^9 -
150994944000*a^4*d^20*e^12 + 78082505441280*a^5*d^16*e^15 + 27443
81022928896*a^6*d^12*e^18 - 70931694131085312*a^7*d^8*e^21 - 5188
146770730811392*a^8*d^4*e^24 - 73786976294838206464*a^9*e^27)))/(
125*d^24 - 4800*a*d^20*e^3 - 1167360*a^2*d^16*e^6 + 31195136*a^3*
d^12*e^9 + 3825205248*a^4*d^8*e^12 - 51539607552*a^5*d^4*e^15 - 4
398046511104*a^6*e^18))) - 8*(d^4*e - 64*a*e^4)*x)/(40*a*d^8*e^2
- 512*a^2*d^4*e^5 - 131072*a^3*e^8 + 8*(5*d^8*e^3 - 64*a*d^4*e^6
- 16384*a^2*e^9)*x^4 + 8*(5*d^9*e^2 - 64*a*d^5*e^5 - 16384*a^2*d*
e^8)*x^3 - (5*d^11 - 64*a*d^7*e^3 - 16384*a^2*d^3*e^6)*x)
```

Sympy [A] time = 44.8092, size = 580, normalized size = 1.7

$$\frac{128ade^3 - 5d^5 + 72d^3e^2x^2 + 96d^2e^3x^3 + x\left(512ae^4 - 8d^4e\right)}{131072a^3e^8 + 512a^2d^4e^5 - 40ad^8e^2 + x^4\left(131072a^2e^9 + 512ad^4e^6 - 40d^8e^3\right) + x^3\left(131072a^2de^8 + 512ad^5e^5 - 40d^9e^2\right) + x\left(-1\right.}$$

$$+\mathrm{RootSum}\left(t^4\left(1152921504606846976a^9e^{27} - 40532396646334464a^8d^4e^{24} - 791648371998720a^7d^8e^{21} + 44324062494720a^6\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2)**2,x)
```

```
[Out] (128*a*d*e**3 - 5*d**5 + 72*d**3*e**2*x**2 + 96*d**2*e**3*x**3 +
x*(512*a*e**4 - 8*d**4*e))/(131072*a**3*e**8 + 512*a**2*d**4*e**5
 - 40*a*d**8*e**2 + x**4*(131072*a**2*e**9 + 512*a*d**4*e**6 - 40
*d**8*e**3) + x**3*(131072*a**2*d*e**8 + 512*a*d**5*e**5 - 40*d**
9*e**2) + x*(-16384*a**2*d**3*e**6 - 64*a*d**7*e**3 + 5*d**11)) +
 RootSum(_t**4*(1152921504606846976*a**9*e**27 - 4053239664633446
4*a**8*d**4*e**24 - 791648371998720*a**7*d**8*e**21 + 44324062494
720*a**6*d**12*e**18 - 96636764160*a**5*d**16*e**15 - 15250489344
*a**4*d**20*e**12 + 163577856*a**3*d**24*e**9 + 1290240*a**2*d**2
8*e**6 - 28800*a*d**32*e**3 + 125*d**36) + _t**2*(6184752906240*a
**5*d**2*e**17 - 265751101440*a**4*d**6*e**14 + 3548381184*a**3*d
**10*e**11 - 12976128*a**2*d**14*e**8 + 18432*a*d**18*e**5 - 576*
d**22*e**2) + 84934656*a**2*e**10, Lambda(_t, _t*log(x + (-225179
9813685248*_t**3*a**7*d**2*e**21 - 30786325577728*_t**3*a**6*d**6
*e**18 + 1906965479424*_t**3*a**5*d**10*e**15 + 19797114880*_t**3
*a**4*d**14*e**12 - 566493184*_t**3*a**3*d**18*e**9 - 3624960*_t*
*3*a**2*d**22*e**6 + 59200*_t**3*a*d**26*e**3 + 125*_t**3*d**30 +
 77309411328*_t*a**4*e**14 - 8455716864*_t*a**3*d**4*e**11 - 1769
4720*_t*a**2*d**8*e**8 - 156672*_t*a*d**12*e**5 - 576*_t*d**16*e*
*2 + 56623104*a**2*d*e**9 + 221184*a*d**5*e**6)/(226492416*a**2*e
**10 + 884736*a*d**4*e**7))))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(8\,e^3x^4 + 8\,de^2x^3 - d^3x + 8\,ae^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2)^(-2),x, algorithm="giac")
```

```
[Out] integrate((8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2)^(-2), x)
```

3.45 $\int \left(8+8x-x^3+8x^4\right)^4 \, dx$

Optimal. Leaf size=96

$$\frac{4096x^{17}}{17}-128x^{16}+\frac{128x^{15}}{5}+1168x^{14}+\frac{10241x^{13}}{13}-448x^{12}+\frac{25312x^{11}}{11}+\frac{21488x^{10}}{5}+1408x^9+1376x^8+6784x^7+7168x^6+\frac{14336x^5}{5}+3584x^4+8192x^3+8192x^2+4096x$$

```
[Out] 4096*x + 8192*x^2 + 8192*x^3 + 3584*x^4 + (14336*x^5)/5 + 7168*x^
6 + 6784*x^7 + 1376*x^8 + 1408*x^9 + (21488*x^10)/5 + (25312*x^11
)/11 - 448*x^12 + (10241*x^13)/13 + 1168*x^14 + (128*x^15)/5 - 12
8*x^16 + (4096*x^17)/17
```

Rubi [A] time = 0.0626047, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059

$$\frac{4096x^{17}}{17}-128x^{16}+\frac{128x^{15}}{5}+1168x^{14}+\frac{10241x^{13}}{13}-448x^{12}+\frac{25312x^{11}}{11}+\frac{21488x^{10}}{5}+1408x^9+1376x^8+6784x^7+7168x^6+\frac{14336x^5}{5}+3584x^4+8192x^3+8192x^2+4096x$$

Antiderivative was successfully verified.

```
[In]  Int[(8 + 8*x - x^3 + 8*x^4)^4,x]
```

```
[Out] 4096*x + 8192*x^2 + 8192*x^3 + 3584*x^4 + (14336*x^5)/5 + 7168*x^
6 + 6784*x^7 + 1376*x^8 + 1408*x^9 + (21488*x^10)/5 + (25312*x^11
)/11 - 448*x^12 + (10241*x^13)/13 + 1168*x^14 + (128*x^15)/5 - 12
8*x^16 + (4096*x^17)/17
```

Rubi in Sympy [A] time = 47.027, size = 94, normalized size = 0.98

$$\frac{4096x^{17}}{17}-128x^{16}+\frac{128x^{15}}{5}+1168x^{14}+\frac{10241x^{13}}{13}-448x^{12}+\frac{25312x^{11}}{11}+\frac{21488x^{10}}{5}+1408x^9+1376x^8+6784x^7+7168x^6+\frac{14336x^5}{5}+3584x^4+8192x^3+8192x^2+4096x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((8*x**4-x**3+8*x+8)**4,x)
```

```
[Out] 4096*x**17/17 - 128*x**16 + 128*x**15/5 + 1168*x**14 + 10241*x**1
3/13 - 448*x**12 + 25312*x**11/11 + 21488*x**10/5 + 1408*x**9 + 1
376*x**8 + 6784*x**7 + 7168*x**6 + 14336*x**5/5 + 3584*x**4 + 819
2*x**3 + 8192*x**2 + 4096*x
```

Mathematica [A] time = 0.00233268, size = 96, normalized size = 1.

$$\frac{4096x^{17}}{17}-128x^{16}+\frac{128x^{15}}{5}+1168x^{14}+\frac{10241x^{13}}{13}-448x^{12}+\frac{25312x^{11}}{11}+\frac{21488x^{10}}{5}+1408x^9+1376x^8+6784x^7+7168x^6+\frac{14336x^5}{5}+3584x^4+8192x^3+8192x^2+4096x$$

Antiderivative was successfully verified.

```
[In]  Integrate[(8 + 8*x - x^3 + 8*x^4)^4,x]
```

```
[Out] 4096*x + 8192*x^2 + 8192*x^3 + 3584*x^4 + (14336*x^5)/5 + 7168*x^
6 + 6784*x^7 + 1376*x^8 + 1408*x^9 + (21488*x^10)/5 + (25312*x^11
)/11 - 448*x^12 + (10241*x^13)/13 + 1168*x^14 + (128*x^15)/5 - 12
8*x^16 + (4096*x^17)/17
```

Maple [A] time = 0.003, size = 85, normalized size = 0.9

$$4096\,x+8192\,x^{2}+8192\,x^{3}+3584\,x^{4}+\frac{14336\,x^{5}}{5}+7168\,x^{6}+6784\,x^{7}+1376\,x^{8}+1408\,x^{9}+\frac{21488\,x^{10}}{5}+\frac{25312\,x^{11}}{11}-448\,x^{12}+\frac{10241\,x^{13}}{13}+1168\,x^{14}+\frac{128\,x^{15}}{5}-128\,x^{16}+\frac{4096\,x^{17}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((8*x^4-x^3+8*x+8)^4,x)
```

```
[Out] 4096*x+8192*x^2+8192*x^3+3584*x^4+14336/5*x^5+7168*x^6+6784*x^7+1
376*x^8+1408*x^9+21488/5*x^10+25312/11*x^11-448*x^12+10241/13*x^1
3+1168*x^14+128/5*x^15-128*x^16+4096/17*x^17
```

Maxima [A] time = 0.77424, size = 113, normalized size = 1.18

$$\frac{4096}{17}\,x^{17}-128\,x^{16}+\frac{128}{5}\,x^{15}+1168\,x^{14}+\frac{10241}{13}\,x^{13}-448\,x^{12}+\frac{25312}{11}\,x^{11}+\frac{21488}{5}\,x^{10}+1408\,x^{9}+1376\,x^{8}+6784\,x^{7}+7168\,x^{6}+\frac{14336}{5}\,x^{5}+3584\,x^{4}+8192\,x^{3}+8192\,x^{2}+4096\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((8*x^4 - x^3 + 8*x + 8)^4,x, algorithm="maxima")
```

```
[Out] 4096/17*x^17 - 128*x^16 + 128/5*x^15 + 1168*x^14 + 10241/13*x^13
- 448*x^12 + 25312/11*x^11 + 21488/5*x^10 + 1408*x^9 + 1376*x^8 +
 6784*x^7 + 7168*x^6 + 14336/5*x^5 + 3584*x^4 + 8192*x^3 + 8192*x
^2 + 4096*x
```

Fricas [A] time = 0.255569, size = 1, normalized size = 0.01

$$\frac{4096}{17}x^{17}-128x^{16}+\frac{128}{5}x^{15}+1168x^{14}+\frac{10241}{13}x^{13}-448x^{12}+\frac{25312}{11}x^{11}+\frac{21488}{5}x^{10}+1408x^{9}+1376x^{8}+6784x^{7}+7168x^{6}+\frac{14336}{5}x^{5}+3584x^{4}+8192x^{3}+8192x^{2}+4096x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((8*x^4 - x^3 + 8*x + 8)^4,x, algorithm="fricas")
```

```
[Out] 4096/17*x^17 - 128*x^16 + 128/5*x^15 + 1168*x^14 + 10241/13*x^13
- 448*x^12 + 25312/11*x^11 + 21488/5*x^10 + 1408*x^9 + 1376*x^8 +
 6784*x^7 + 7168*x^6 + 14336/5*x^5 + 3584*x^4 + 8192*x^3 + 8192*x
^2 + 4096*x
```

Sympy [A] time = 0.110818, size = 94, normalized size = 0.98

$$\frac{4096x^{17}}{17} - 128x^{16} + \frac{128x^{15}}{5} + 1168x^{14} + \frac{10241x^{13}}{13} - 448x^{12} + \frac{25312x^{11}}{11} + \frac{21488x^{10}}{5} + 1408x^9 + 1376x^8 + 6784x^7 + 7168x^6 + \frac{14336x^5}{5} + 3584x^4 + 8192x^3 + 8192x^2 + 4096x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((8*x**4-x**3+8*x+8)**4,x)
```

```
[Out] 4096*x**17/17 - 128*x**16 + 128*x**15/5 + 1168*x**14 + 10241*x**1
3/13 - 448*x**12 + 25312*x**11/11 + 21488*x**10/5 + 1408*x**9 + 1
376*x**8 + 6784*x**7 + 7168*x**6 + 14336*x**5/5 + 3584*x**4 + 819
2*x**3 + 8192*x**2 + 4096*x
```

GIAC/XCAS [A] time = 0.263269, size = 113, normalized size = 1.18

$$\frac{4096}{17}\,x^{17} - 128\,x^{16} + \frac{128}{5}\,x^{15} + 1168\,x^{14} + \frac{10241}{13}\,x^{13} - 448\,x^{12} + \frac{25312}{11}\,x^{11} + \frac{21488}{5}\,x^{10} + 1408\,x^9 + 1376\,x^8 + 6784\,x^7 + 7168\,x^6 + \frac{14336}{5}\,x^5 + 3584\,x^4 + 8192\,x^3 + 8192\,x^2 + 4096\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((8*x^4 - x^3 + 8*x + 8)^4,x, algorithm="giac")
```

```
[Out] 4096/17*x^17 - 128*x^16 + 128/5*x^15 + 1168*x^14 + 10241/13*x^13
- 448*x^12 + 25312/11*x^11 + 21488/5*x^10 + 1408*x^9 + 1376*x^8 +
 6784*x^7 + 7168*x^6 + 14336/5*x^5 + 3584*x^4 + 8192*x^3 + 8192*x
^2 + 4096*x
```

3.46 $\int \left(8+8x-x^3+8x^4\right)^3 \, dx$

Optimal. Leaf size=74

$$\frac{512x^{13}}{13}-16x^{12}+\frac{24x^{11}}{11}+\frac{307x^{10}}{2}+128x^9-45x^8+\frac{1560x^7}{7}+480x^6+\frac{1152x^5}{5}+80x^4+512x^3+768x^2+512x$$

```
[Out] 512*x + 768*x^2 + 512*x^3 + 80*x^4 + (1152*x^5)/5 + 480*x^6 + (15
60*x^7)/7 - 45*x^8 + 128*x^9 + (307*x^10)/2 + (24*x^11)/11 - 16*x
^12 + (512*x^13)/13
```

Rubi [A] time = 0.047089, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$

$$\frac{512x^{13}}{13}-16x^{12}+\frac{24x^{11}}{11}+\frac{307x^{10}}{2}+128x^9-45x^8+\frac{1560x^7}{7}+480x^6+\frac{1152x^5}{5}+80x^4+512x^3+768x^2+512x$$

Antiderivative was successfully verified.

```
[In]  Int[(8 + 8*x - x^3 + 8*x^4)^3,x]
```

```
[Out] 512*x + 768*x^2 + 512*x^3 + 80*x^4 + (1152*x^5)/5 + 480*x^6 + (15
60*x^7)/7 - 45*x^8 + 128*x^9 + (307*x^10)/2 + (24*x^11)/11 - 16*x
^12 + (512*x^13)/13
```

Rubi in Sympy [A] time = 34.9319, size = 71, normalized size = 0.96

$$\frac{512x^{13}}{13}-16x^{12}+\frac{24x^{11}}{11}+\frac{307x^{10}}{2}+128x^9-45x^8+\frac{1560x^7}{7}+480x^6+\frac{1152x^5}{5}+80x^4+512x^3+768x^2+512x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((8*x**4-x**3+8*x+8)**3,x)
```

```
[Out] 512*x**13/13 - 16*x**12 + 24*x**11/11 + 307*x**10/2 + 128*x**9 -
45*x**8 + 1560*x**7/7 + 480*x**6 + 1152*x**5/5 + 80*x**4 + 512*x*
*3 + 768*x**2 + 512*x
```

Mathematica [A] time = 0.0015436, size = 74, normalized size = 1.

$$\frac{512x^{13}}{13}-16x^{12}+\frac{24x^{11}}{11}+\frac{307x^{10}}{2}+128x^9-45x^8+\frac{1560x^7}{7}+480x^6+\frac{1152x^5}{5}+80x^4+512x^3+768x^2+512x$$

Antiderivative was successfully verified.

```
[In]  Integrate[(8 + 8*x - x^3 + 8*x^4)^3,x]
```

```
[Out] 512*x + 768*x^2 + 512*x^3 + 80*x^4 + (1152*x^5)/5 + 480*x^6 + (15
60*x^7)/7 - 45*x^8 + 128*x^9 + (307*x^10)/2 + (24*x^11)/11 - 16*x
^12 + (512*x^13)/13
```

Maple [A] time = 0.002, size = 65, normalized size = 0.9

$$512\,x+768\,x^{2}+512\,x^{3}+80\,x^{4}+\frac{1152\,x^{5}}{5}+480\,x^{6}+\frac{1560\,x^{7}}{7}-45\,x^{8}+128\,x^{9}+\frac{307\,x^{10}}{2}+\frac{24\,x^{11}}{11}-16\,x^{12}+\frac{512\,x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((8*x^4-x^3+8*x+8)^3,x)
```

```
[Out] 512*x+768*x^2+512*x^3+80*x^4+1152/5*x^5+480*x^6+1560/7*x^7-45*x^8
+128*x^9+307/2*x^10+24/11*x^11-16*x^12+512/13*x^13
```

Maxima [A] time = 0.772563, size = 86, normalized size = 1.16

$$\frac{512}{13}\,x^{13}-16\,x^{12}+\frac{24}{11}\,x^{11}+\frac{307}{2}\,x^{10}+128\,x^{9}-45\,x^{8}+\frac{1560}{7}\,x^{7}+480\,x^{6}+\frac{1152}{5}\,x^{5}+80\,x^{4}+512\,x^{3}+768\,x^{2}+512\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((8*x^4 - x^3 + 8*x + 8)^3,x, algorithm="maxima")
```

```
[Out] 512/13*x^13 - 16*x^12 + 24/11*x^11 + 307/2*x^10 + 128*x^9 - 45*x^
8 + 1560/7*x^7 + 480*x^6 + 1152/5*x^5 + 80*x^4 + 512*x^3 + 768*x^
2 + 512*x
```

Fricas [A] time = 0.288966, size = 1, normalized size = 0.01

$$\frac{512}{13}x^{13}-16x^{12}+\frac{24}{11}x^{11}+\frac{307}{2}x^{10}+128x^{9}-45x^{8}+\frac{1560}{7}x^{7}+480x^{6}+\frac{1152}{5}x^{5}+80x^{4}+512x^{3}+768x^{2}+512x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((8*x^4 - x^3 + 8*x + 8)^3,x, algorithm="fricas")
```

```
[Out] 512/13*x^13 - 16*x^12 + 24/11*x^11 + 307/2*x^10 + 128*x^9 - 45*x^
8 + 1560/7*x^7 + 480*x^6 + 1152/5*x^5 + 80*x^4 + 512*x^3 + 768*x^
2 + 512*x
```

Sympy [A] time = 0.095243, size = 71, normalized size = 0.96

$$\frac{512x^{13}}{13}-16x^{12}+\frac{24x^{11}}{11}+\frac{307x^{10}}{2}+128x^{9}-45x^{8}+\frac{1560x^{7}}{7}+480x^{6}+\frac{1152x^{5}}{5}+80x^{4}+512x^{3}+768x^{2}+512x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((8*x**4-x**3+8*x+8)**3,x)
```

```
[Out] 512*x**13/13 - 16*x**12 + 24*x**11/11 + 307*x**10/2 + 128*x**9 -
45*x**8 + 1560*x**7/7 + 480*x**6 + 1152*x**5/5 + 80*x**4 + 512*x*
*3 + 768*x**2 + 512*x
```

GIAC/XCAS [A] time = 0.257434, size = 86, normalized size = 1.16

$$\frac{512}{13}\,x^{13} - 16\,x^{12} + \frac{24}{11}\,x^{11} + \frac{307}{2}\,x^{10} + 128\,x^{9} - 45\,x^{8} + \frac{1560}{7}\,x^{7} + 480\,x^{6} + \frac{1152}{5}\,x^{5} + 80\,x^{4} + 512\,x^{3} + 768\,x^{2} + 512\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((8*x^4 - x^3 + 8*x + 8)^3,x, algorithm="giac")
```

```
[Out] 512/13*x^13 - 16*x^12 + 24/11*x^11 + 307/2*x^10 + 128*x^9 - 45*x^
8 + 1560/7*x^7 + 480*x^6 + 1152/5*x^5 + 80*x^4 + 512*x^3 + 768*x^
2 + 512*x
```

3.47 $\int \left(8+8x-x^3+8x^4\right)^2 \, dx$

Optimal. Leaf size=54

$$\frac{64x^9}{9}-2x^8+\frac{x^7}{7}+\frac{64x^6}{3}+\frac{112x^5}{5}-4x^4+\frac{64x^3}{3}+64x^2+64x$$

```
[Out] 64*x + 64*x^2 + (64*x^3)/3 - 4*x^4 + (112*x^5)/5 + (64*x^6)/3 + x
^7/7 - 2*x^8 + (64*x^9)/9
```

Rubi [A] time = 0.0327499, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059

$$\frac{64x^9}{9}-2x^8+\frac{x^7}{7}+\frac{64x^6}{3}+\frac{112x^5}{5}-4x^4+\frac{64x^3}{3}+64x^2+64x$$

Antiderivative was successfully verified.

```
[In]  Int[(8 + 8*x - x^3 + 8*x^4)^2,x]
```

```
[Out] 64*x + 64*x^2 + (64*x^3)/3 - 4*x^4 + (112*x^5)/5 + (64*x^6)/3 + x
^7/7 - 2*x^8 + (64*x^9)/9
```

Rubi in Sympy [A] time = 23.6292, size = 49, normalized size = 0.91

$$\frac{64x^9}{9}-2x^8+\frac{x^7}{7}+\frac{64x^6}{3}+\frac{112x^5}{5}-4x^4+\frac{64x^3}{3}+64x^2+64x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((8*x**4-x**3+8*x+8)**2,x)
```

```
[Out] 64*x**9/9 - 2*x**8 + x**7/7 + 64*x**6/3 + 112*x**5/5 - 4*x**4 + 6
4*x**3/3 + 64*x**2 + 64*x
```

Mathematica [A] time = 0.00172663, size = 54, normalized size = 1.

$$\frac{64x^9}{9}-2x^8+\frac{x^7}{7}+\frac{64x^6}{3}+\frac{112x^5}{5}-4x^4+\frac{64x^3}{3}+64x^2+64x$$

Antiderivative was successfully verified.

```
[In]  Integrate[(8 + 8*x - x^3 + 8*x^4)^2,x]
```

```
[Out] 64*x + 64*x^2 + (64*x^3)/3 - 4*x^4 + (112*x^5)/5 + (64*x^6)/3 + x
^7/7 - 2*x^8 + (64*x^9)/9
```

Maple [A] time = 0.001, size = 45, normalized size = 0.8

$$64\,x+64\,{x}^{2}+{\frac{64\,{x}^{3}}{3}}-4\,{x}^{4}+{\frac{112\,{x}^{5}}{5}}+{\frac{64\,{x}^{6}}{3}}+{\frac{{x}^{7}}{7}}-2\,{x}^{8}+{\frac{64\,{x}^{9}}{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((8*x^4-x^3+8*x+8)^2,x)
```

```
[Out] 64*x+64*x^2+64/3*x^3-4*x^4+112/5*x^5+64/3*x^6+1/7*x^7-2*x^8+64/9*
x^9
```

Maxima [A] time = 0.76249, size = 59, normalized size = 1.09

$$\frac{64}{9}\,x^9 - 2\,x^8 + \frac{1}{7}\,x^7 + \frac{64}{3}\,x^6 + \frac{112}{5}\,x^5 - 4\,x^4 + \frac{64}{3}\,x^3 + 64\,x^2 + 64\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((8*x^4 - x^3 + 8*x + 8)^2,x, algorithm="maxima")
```

```
[Out] 64/9*x^9 - 2*x^8 + 1/7*x^7 + 64/3*x^6 + 112/5*x^5 - 4*x^4 + 64/3*
x^3 + 64*x^2 + 64*x
```

Fricas [A] time = 0.239183, size = 1, normalized size = 0.02

$$\frac{64}{9}x^9 - 2x^8 + \frac{1}{7}x^7 + \frac{64}{3}x^6 + \frac{112}{5}x^5 - 4x^4 + \frac{64}{3}x^3 + 64x^2 + 64x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((8*x^4 - x^3 + 8*x + 8)^2,x, algorithm="fricas")
```

```
[Out] 64/9*x^9 - 2*x^8 + 1/7*x^7 + 64/3*x^6 + 112/5*x^5 - 4*x^4 + 64/3*
x^3 + 64*x^2 + 64*x
```

Sympy [A] time = 0.079356, size = 49, normalized size = 0.91

$$\frac{64x^9}{9} - 2x^8 + \frac{x^7}{7} + \frac{64x^6}{3} + \frac{112x^5}{5} - 4x^4 + \frac{64x^3}{3} + 64x^2 + 64x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((8*x**4-x**3+8*x+8)**2,x)
```

```
[Out] 64*x**9/9 - 2*x**8 + x**7/7 + 64*x**6/3 + 112*x**5/5 - 4*x**4 + 6
4*x**3/3 + 64*x**2 + 64*x
```

GIAC/XCAS [A] time = 0.259669, size = 59, normalized size = 1.09

$$\frac{64}{9}\,x^9 - 2\,x^8 + \frac{1}{7}\,x^7 + \frac{64}{3}\,x^6 + \frac{112}{5}\,x^5 - 4\,x^4 + \frac{64}{3}\,x^3 + 64\,x^2 + 64\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((8*x^4 - x^3 + 8*x + 8)^2,x, algorithm="giac")
```

```
[Out] 64/9*x^9 - 2*x^8 + 1/7*x^7 + 64/3*x^6 + 112/5*x^5 - 4*x^4 + 64/3*
x^3 + 64*x^2 + 64*x
```

3.48 $\int \left(8+8x-x^3+8x^4\right)\,dx$

Optimal. Leaf size=23

$$\frac{8x^5}{5}-\frac{x^4}{4}+4x^2+8x$$

```
[Out] 8*x + 4*x^2 - x^4/4 + (8*x^5)/5
```

Rubi [A] time = 0.00993579, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.$

$$\frac{8x^5}{5}-\frac{x^4}{4}+4x^2+8x$$

Antiderivative was successfully verified.

```
[In]  Int[8 + 8*x - x^3 + 8*x^4,x]

[Out] 8*x + 4*x^2 - x^4/4 + (8*x^5)/5
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{8x^5}{5}-\frac{x^4}{4}+8x+8\int x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(8*x**4-x**3+8*x+8,x)

[Out] 8*x**5/5 - x**4/4 + 8*x + 8*Integral(x, x)
```

Mathematica [A] time = 0.0000591969, size = 23, normalized size = 1.

$$\frac{8x^5}{5}-\frac{x^4}{4}+4x^2+8x$$

Antiderivative was successfully verified.

```
[In]  Integrate[8 + 8*x - x^3 + 8*x^4,x]

[Out] 8*x + 4*x^2 - x^4/4 + (8*x^5)/5
```

Maple [A] time = 0.001, size = 20, normalized size = 0.9

$$8x+4x^2-\frac{x^4}{4}+\frac{8x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(8*x^4-x^3+8*x+8,x)
```

```
[Out] 8*x+4*x^2-1/4*x^4+8/5*x^5
```

Maxima [A] time = 0.777118, size = 26, normalized size = 1.13

$$\frac{8}{5}\,x^5 - \frac{1}{4}\,x^4 + 4\,x^2 + 8\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(8*x^4 - x^3 + 8*x + 8,x, algorithm="maxima")
```

```
[Out] 8/5*x^5 - 1/4*x^4 + 4*x^2 + 8*x
```

Fricas [A] time = 0.255837, size = 1, normalized size = 0.04

$$\frac{8}{5}x^5 - \frac{1}{4}x^4 + 4x^2 + 8x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(8*x^4 - x^3 + 8*x + 8,x, algorithm="fricas")
```

```
[Out] 8/5*x^5 - 1/4*x^4 + 4*x^2 + 8*x
```

Sympy [A] time = 0.059239, size = 19, normalized size = 0.83

$$\frac{8x^5}{5} - \frac{x^4}{4} + 4x^2 + 8x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(8*x**4-x**3+8*x+8,x)
```

```
[Out] 8*x**5/5 - x**4/4 + 4*x**2 + 8*x
```

GIAC/XCAS [A] time = 0.263249, size = 26, normalized size = 1.13

$$\frac{8}{5}\,x^5 - \frac{1}{4}\,x^4 + 4\,x^2 + 8\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(8*x^4 - x^3 + 8*x + 8,x, algorithm="giac")
```

```
[Out] 8/5*x^5 - 1/4*x^4 + 4*x^2 + 8*x
```

3.49 $\int \frac{1}{8+8x-x^3+8x^4}\,dx$

Optimal. Leaf size=268

$$-\frac{1}{24}\sqrt{\frac{67\sqrt{29}-109}{1218}}\log\left(\left(\frac{4}{x}+1\right)^2-\sqrt{6\left(1+\sqrt{29}\right)}\left(\frac{4}{x}+1\right)+3\sqrt{29}\right)+\frac{1}{24}\sqrt{\frac{67\sqrt{29}-109}{1218}}\log\left(\left(\frac{4}{x}+1\right)^2+\sqrt{6\left(1+\sqrt{29}\right)}\left(\frac{4}{x}+1\right)+3\sqrt{29}\right)-\frac{\tan^{-1}\left(\frac{3-\left(\frac{4}{x}+1\right)^2}{6\sqrt{7}}\right)}{12\sqrt{7}}-\frac{1}{12}\sqrt{\frac{109+67\sqrt{29}}{1218}}\tan^{-1}\left(\frac{\frac{8}{x}-\sqrt{6\left(1+\sqrt{29}\right)}+2}{\sqrt{6\left(\sqrt{29}-1\right)}}\right)-\frac{1}{12}\sqrt{\frac{109+67\sqrt{29}}{1218}}\tan^{-1}\left(\frac{\frac{8}{x}+\sqrt{6\left(1+\sqrt{29}\right)}+2}{\sqrt{6\left(\sqrt{29}-1\right)}}\right)$$

```
[Out] -ArcTan[(3 - (1 + 4/x)^2)/(6*Sqrt[7])]/(12*Sqrt[7]) - (Sqrt[(109
+ 67*Sqrt[29])/1218]*ArcTan[(2 - Sqrt[6*(1 + Sqrt[29])] + 8/x)/Sq
rt[6*(-1 + Sqrt[29])]])/12 - (Sqrt[(109 + 67*Sqrt[29])/1218]*ArcT
an[(2 + Sqrt[6*(1 + Sqrt[29])] + 8/x)/Sqrt[6*(-1 + Sqrt[29])]])/1
2 - (Sqrt[(-109 + 67*Sqrt[29])/1218]*Log[3*Sqrt[29] - Sqrt[6*(1 +
 Sqrt[29])]*(1 + 4/x) + (1 + 4/x)^2])/24 + (Sqrt[(-109 + 67*Sqrt[
29])/1218]*Log[3*Sqrt[29] + Sqrt[6*(1 + Sqrt[29])]*(1 + 4/x) + (1
 + 4/x)^2])/24
```

Rubi [A] time = 0.868381, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529

$$-\frac{1}{24}\sqrt{\frac{67\sqrt{29}-109}{1218}}\log\left(\left(\frac{4}{x}+1\right)^2-\sqrt{6\left(1+\sqrt{29}\right)}\left(\frac{4}{x}+1\right)+3\sqrt{29}\right)+\frac{1}{24}\sqrt{\frac{67\sqrt{29}-109}{1218}}\log\left(\left(\frac{4}{x}+1\right)^2+\sqrt{6\left(1+\sqrt{29}\right)}\left(\frac{4}{x}+1\right)+3\sqrt{29}\right)-\frac{\tan^{-1}\left(\frac{3-\left(\frac{4}{x}+1\right)^2}{6\sqrt{7}}\right)}{12\sqrt{7}}-\frac{1}{12}\sqrt{\frac{109+67\sqrt{29}}{1218}}\tan^{-1}\left(\frac{\frac{8}{x}-\sqrt{6\left(1+\sqrt{29}\right)}+2}{\sqrt{6\left(\sqrt{29}-1\right)}}\right)-\frac{1}{12}\sqrt{\frac{109+67\sqrt{29}}{1218}}\tan^{-1}\left(\frac{\frac{8}{x}+\sqrt{6\left(1+\sqrt{29}\right)}+2}{\sqrt{6\left(\sqrt{29}-1\right)}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(8 + 8*x - x^3 + 8*x^4)^(-1),x]
```


```
[Out] -ArcTan[(3 - (1 + 4/x)^2)/(6*Sqrt[7])]/(12*Sqrt[7]) - (Sqrt[(109
+ 67*Sqrt[29])/1218]*ArcTan[(2 - Sqrt[6*(1 + Sqrt[29])] + 8/x)/Sq
rt[6*(-1 + Sqrt[29])]])/12 - (Sqrt[(109 + 67*Sqrt[29])/1218]*ArcT
an[(2 + Sqrt[6*(1 + Sqrt[29])] + 8/x)/Sqrt[6*(-1 + Sqrt[29])]])/1
2 - (Sqrt[(-109 + 67*Sqrt[29])/1218]*Log[3*Sqrt[29] - Sqrt[6*(1 +
 Sqrt[29])]*(1 + 4/x) + (1 + 4/x)^2])/24 + (Sqrt[(-109 + 67*Sqrt[
29])/1218]*Log[3*Sqrt[29] + Sqrt[6*(1 + Sqrt[29])]*(1 + 4/x) + (1
 + 4/x)^2])/24
```

Rubi in Sympy [A] time = 65.2582, size = 382, normalized size = 1.43

$$\frac{\sqrt{174}\left(-96\sqrt{29}+32\right)\log\left(\sqrt{6}\left(-\frac{1}{16}-\frac{1}{4x}\right)\sqrt{1+\sqrt{29}}+\left(\frac{1}{4}+\frac{1}{x}\right)^2+\frac{3\sqrt{29}}{16}\right)}{133632\sqrt{1+\sqrt{29}}} - \frac{\sqrt{174}\left(-96\sqrt{29}+32\right)\log\left(\sqrt{6}\left(\frac{1}{16}+\frac{1}{4x}\right)\sqrt{1+\sqrt{29}}+\left(\frac{1}{4}+\frac{1}{x}\right)^2+\frac{3\sqrt{29}}{16}\right)}{133632\sqrt{1+\sqrt{29}}} + \frac{\sqrt{7}\operatorname{atan}\left(\sqrt{7}\left(\frac{8\left(\frac{1}{4}+\frac{1}{x}\right)^2}{21}-\frac{1}{14}\right)\right)}{84} - \frac{\sqrt{29}\left(16\sqrt{6}\sqrt{1+\sqrt{29}}-\frac{\sqrt{6}\sqrt{1+\sqrt{29}}\left(-192\sqrt{29}+64\right)}{8}\right)\operatorname{atan}\left(\frac{\sqrt{6}\left(\frac{1}{3}+\frac{\sqrt{6+6\sqrt{29}}}{6}+\frac{4}{3x}\right)}{\sqrt{-1+\sqrt{29}}}\right)}{16704\sqrt{-1+\sqrt{29}}\sqrt{1+\sqrt{29}}} - \frac{\sqrt{29}\left(16\sqrt{6}\sqrt{1+\sqrt{29}}-\frac{\sqrt{6}\sqrt{1+\sqrt{29}}\left(-192\sqrt{29}+64\right)}{8}\right)\operatorname{atan}\left(\frac{\sqrt{6}\left(-\frac{\sqrt{6+6\sqrt{29}}}{6}+\frac{1}{3}+\frac{4}{3x}\right)}{\sqrt{-1+\sqrt{29}}}\right)}{16704\sqrt{-1+\sqrt{29}}\sqrt{1+\sqrt{29}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(8*x**4-x**3+8*x+8),x)
```

```
[Out] sqrt(174)*(-96*sqrt(29) + 32)*log(sqrt(6)*(-1/16 - 1/(4*x))*sqrt(
1 + sqrt(29)) + (1/4 + 1/x)**2 + 3*sqrt(29)/16)/(133632*sqrt(1 +
sqrt(29))) - sqrt(174)*(-96*sqrt(29) + 32)*log(sqrt(6)*(1/16 + 1/
(4*x))*sqrt(1 + sqrt(29)) + (1/4 + 1/x)**2 + 3*sqrt(29)/16)/(1336
32*sqrt(1 + sqrt(29))) + sqrt(7)*atan(sqrt(7)*(8*(1/4 + 1/x)**2/2
1 - 1/14))/84 - sqrt(29)*(16*sqrt(6)*sqrt(1 + sqrt(29)) - sqrt(6)
*sqrt(1 + sqrt(29))*(-192*sqrt(29) + 64)/8)*atan(sqrt(6)*(1/3 + s
qrt(6 + 6*sqrt(29))/6 + 4/(3*x))/sqrt(-1 + sqrt(29)))/(16704*sqrt
(-1 + sqrt(29))*sqrt(1 + sqrt(29))) - sqrt(29)*(16*sqrt(6)*sqrt(1
 + sqrt(29)) - sqrt(6)*sqrt(1 + sqrt(29))*(-192*sqrt(29) + 64)/8)
*atan(sqrt(6)*(-sqrt(6 + 6*sqrt(29))/6 + 1/3 + 4/(3*x))/sqrt(-1 +
 sqrt(29)))/(16704*sqrt(-1 + sqrt(29))*sqrt(1 + sqrt(29)))
```

Mathematica [C] time = 0.0137583, size = 45, normalized size = 0.17

$$\text{RootSum}\left[8\#1^4-\#1^3+8\#1+8\&,\frac{\log(x-\#1)}{32\#1^3-3\#1^2+8}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[(8 + 8*x - x^3 + 8*x^4)^(-1),x]
```

```
[Out] RootSum[8 + 8*#1 - #1^3 + 8*#1^4 & , Log[x - #1]/(8 - 3*#1^2 + 32
*#1^3) & ]
```

Maple [C] time = 0.007, size = 41, normalized size = 0.2

$$\sum_{\_R=RootOf\left(8\_Z^4-\_Z^3+8\_Z+8\right)}\frac{\ln(x-\_R)}{32\_R^3-3\_R^2+8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(8*x^4-x^3+8*x+8),x)
```

```
[Out] sum(1/(32*_R^3-3*_R^2+8)*ln(x-_R),_R=RootOf(8*_Z^4-_Z^3+8*_Z+8))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{8\,x^4 - x^3 + 8\,x + 8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(8*x^4 - x^3 + 8*x + 8),x, algorithm="maxima")
```

```
[Out] integrate(1/(8*x^4 - x^3 + 8*x + 8), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(8*x^4 - x^3 + 8*x + 8),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 3.04107, size = 41, normalized size = 0.15

$$\operatorname{RootSum}\left(66298176t^4 + 74088t^2 + 4095t + 64, \left(t \mapsto t\log\left(\frac{35914274424t^3}{2109763} - \frac{1504863360t^2}{2109763} + \frac{102851343t}{2109763} + x + \frac{6055613}{16878104}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(8*x**4-x**3+8*x+8),x)
```

```
[Out] RootSum(66298176*_t**4 + 74088*_t**2 + 4095*_t + 64, Lambda(_t, _
t*log(35914274424*_t**3/2109763 - 1504863360*_t**2/2109763 + 1028
51343*_t/2109763 + x + 6055613/16878104)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{8\,x^4 - x^3 + 8\,x + 8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(8*x^4 - x^3 + 8*x + 8),x, algorithm="giac")
```

```
[Out] integrate(1/(8*x^4 - x^3 + 8*x + 8), x)
```

3.50 $\int \frac{1}{(8+8x-x^3+8x^4)^2}\,dx$

Optimal. Leaf size=357

$$-\frac{29\left(\frac{4}{x}+1\right)^2+207}{336\left(\left(\frac{4}{x}+1\right)^4-6\left(\frac{4}{x}+1\right)^2+261\right)}+\frac{5\left(199\left(\frac{4}{x}+1\right)^2+5157\right)\left(\frac{4}{x}+1\right)}{87696\left(\left(\frac{4}{x}+1\right)^4-6\left(\frac{4}{x}+1\right)^2+261\right)}-\frac{\sqrt{\frac{45923327\sqrt{29}-180983329}{1218}}\log\left(\left(\frac{4}{x}+1\right)^2-\sqrt{6\left(1+\sqrt{29}\right)}\left(\frac{4}{x}+1\right)+3\sqrt{29}\right)}{175392}+\frac{\sqrt{\frac{45923327\sqrt{29}-180983329}{1218}}\log\left(\left(\frac{4}{x}+1\right)^2+\sqrt{6\left(1+\sqrt{29}\right)}\left(\frac{4}{x}+1\right)+3\sqrt{29}\right)}{175392}-\frac{17\tan^{-1}\left(\frac{3-\left(\frac{4}{x}+1\right)^2}{6\sqrt{7}}\right)}{1008\sqrt{7}}-\frac{\sqrt{\frac{180983329+45923327\sqrt{29}}{1218}}\tan^{-1}\left(\frac{\frac{8}{x}-\sqrt{6\left(1+\sqrt{29}\right)}+2}{\sqrt{6\left(\sqrt{29}-1\right)}}\right)}{87696}-\frac{\sqrt{\frac{180983329+45923327\sqrt{29}}{1218}}\tan^{-1}\left(\frac{\frac{8}{x}+\sqrt{6\left(1+\sqrt{29}\right)}+2}{\sqrt{6\left(\sqrt{29}-1\right)}}\right)}{87696}$$

```
[Out] -(207 + 29*(1 + 4/x)^2)/(336*(261 - 6*(1 + 4/x)^2 + (1 + 4/x)^4))
 + (5*(5157 + 199*(1 + 4/x)^2)*(1 + 4/x))/(87696*(261 - 6*(1 + 4/
x)^2 + (1 + 4/x)^4)) - (17*ArcTan[(3 - (1 + 4/x)^2)/(6*Sqrt[7])])
/(1008*Sqrt[7]) - (Sqrt[(180983329 + 45923327*Sqrt[29])/1218]*Arc
Tan[(2 - Sqrt[6*(1 + Sqrt[29])] + 8/x)/Sqrt[6*(-1 + Sqrt[29])]])/
87696 - (Sqrt[(180983329 + 45923327*Sqrt[29])/1218]*ArcTan[(2 + S
qrt[6*(1 + Sqrt[29])] + 8/x)/Sqrt[6*(-1 + Sqrt[29])]])/87696 - (S
qrt[(-180983329 + 45923327*Sqrt[29])/1218]*Log[3*Sqrt[29] - Sqrt[
6*(1 + Sqrt[29])]*(1 + 4/x) + (1 + 4/x)^2])/175392 + (Sqrt[(-1809
83329 + 45923327*Sqrt[29])/1218]*Log[3*Sqrt[29] + Sqrt[6*(1 + Sqr
t[29])]*(1 + 4/x) + (1 + 4/x)^2])/175392
```

Rubi [A] time = 1.01584, antiderivative size = 357, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 11, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.647

$$-\frac{29\left(\frac{4}{x}+1\right)^2+207}{336\left(\left(\frac{4}{x}+1\right)^4-6\left(\frac{4}{x}+1\right)^2+261\right)}+\frac{5\left(199\left(\frac{4}{x}+1\right)^2+5157\right)\left(\frac{4}{x}+1\right)}{87696\left(\left(\frac{4}{x}+1\right)^4-6\left(\frac{4}{x}+1\right)^2+261\right)}-\frac{\sqrt{\frac{45923327\sqrt{29}-180983329}{1218}}\log\left(\left(\frac{4}{x}+1\right)^2-\sqrt{6\left(1+\sqrt{29}\right)}\left(\frac{4}{x}+1\right)+3\sqrt{29}\right)}{175392}+\frac{\sqrt{\frac{45923327\sqrt{29}-180983329}{1218}}\log\left(\left(\frac{4}{x}+1\right)^2+\sqrt{6\left(1+\sqrt{29}\right)}\left(\frac{4}{x}+1\right)+3\sqrt{29}\right)}{175392}-\frac{17\tan^{-1}\left(\frac{3-\left(\frac{4}{x}+1\right)^2}{6\sqrt{7}}\right)}{1008\sqrt{7}}-\frac{\sqrt{\frac{180983329+45923327\sqrt{29}}{1218}}\tan^{-1}\left(\frac{\frac{8}{x}-\sqrt{6\left(1+\sqrt{29}\right)}+2}{\sqrt{6\left(\sqrt{29}-1\right)}}\right)}{87696}-\frac{\sqrt{\frac{180983329+45923327\sqrt{29}}{1218}}\tan^{-1}\left(\frac{\frac{8}{x}+\sqrt{6\left(1+\sqrt{29}\right)}+2}{\sqrt{6\left(\sqrt{29}-1\right)}}\right)}{87696}$$

Antiderivative was successfully verified.

```
[In]  Int[(8 + 8*x - x^3 + 8*x^4)^(-2),x]
```

```
[Out] -(207 + 29*(1 + 4/x)^2)/(336*(261 - 6*(1 + 4/x)^2 + (1 + 4/x)^4))
 + (5*(5157 + 199*(1 + 4/x)^2)*(1 + 4/x))/(87696*(261 - 6*(1 + 4/
x)^2 + (1 + 4/x)^4)) - (17*ArcTan[(3 - (1 + 4/x)^2)/(6*Sqrt[7])])
/(1008*Sqrt[7]) - (Sqrt[(180983329 + 45923327*Sqrt[29])/1218]*Arc
Tan[(2 - Sqrt[6*(1 + Sqrt[29])] + 8/x)/Sqrt[6*(-1 + Sqrt[29])]])/
87696 - (Sqrt[(180983329 + 45923327*Sqrt[29])/1218]*ArcTan[(2 + S
qrt[6*(1 + Sqrt[29])] + 8/x)/Sqrt[6*(-1 + Sqrt[29])]])/87696 - (S
qrt[(-180983329 + 45923327*Sqrt[29])/1218]*Log[3*Sqrt[29] - Sqrt[
```

```
6*(1 + Sqrt[29])]*(1 + 4/x) + (1 + 4/x)^2])/175392 + (Sqrt[(-1809
83329 + 45923327*Sqrt[29])/1218]*Log[3*Sqrt[29] + Sqrt[6*(1 + Sqr
t[29])]*(1 + 4/x) + (1 + 4/x)^2])/175392
```

Rubi in Sympy [A] time = 80.7003, size = 439, normalized size = 1.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(8*x**4-x**3+8*x+8)**2,x)
```

```
[Out] (1/4 + 1/x)*(168397063675565562187575490014276708478061109979725
5466843635712*(1/4 + 1/x)**3 + 202361205745395814464538179425368749
922307734818631544626741248 0*(1/4 + 1/x)**2 + 215758737834318376
55283109658079203273751579716152335668934082 56 - 447991101462985
55810227283892385026521745061595149812555432591 36/x)/(850460316022
9511043070376306508480193356987992796927570739 2*(8388608*(1/4 + 1
/x)**4 - 3145728*(1/4 + 1/x)**2 + 8552448)) + sqrt(174)*(-8553320
0594269374091198329733247884310754318316241938680486297 6*sqrt(29)
 + 164011723475681042338940711628488252528111601907336860572791 60
32)*log(sqrt(6)*(-1/16 - 1/(4*x))*sqrt(1 + sqrt(29)) + (1/4 + 1/x
)**2 + 3*sqrt(29)/16)/(1163762820615976980180562459229533414603 44
9363920321561736213272985 6*sqrt(1 + sqrt(29))) - sqrt(174)*(-8553
3200594269374091198329733247884310754318316241938680486297 6*sqrt(
29) + 16401172347568104233894071162848825252811160190733686057279
16032)*log(sqrt(6)*(1/16 + 1/(4*x))*sqrt(1 + sqrt(29)) + (1/4 + 1
/x)**2 + 3*sqrt(29)/16)/(1163762820615976980180562459229533414603
44936392032156173621327298 56*sqrt(1 + sqrt(29))) + 17*sqrt(7)*ata
n(sqrt(7)*(8*(1/4 + 1/x)**2/21 - 1/14))/7056 - sqrt(29)*(-sqrt(6)
*sqrt(1 + sqrt(29))*(-171066401188538748182396659466495768621508 6
36632483877360972595 2*sqrt(29) + 328023446951362084677881423256 97
65050562232038146737211455832064)/8 + 82005861737840521169470355 8
14244126264055800953668430286395801 6*sqrt(6)*sqrt(1 + sqrt(29)))*
atan(sqrt(6)*(1/3 + sqrt(6 + 6*sqrt(29))/6 + 4/(3*x))/sqrt(-1 + s
qrt(29)))/(145470352576997122522570307403691676825431170490040195
2170266591232*sqrt(-1 + sqrt(29))*sqrt(1 + sqrt(29))) - sqrt(29)*
(-sqrt(6)*sqrt(1 + sqrt(29))*(-17106640118853874818239665946649 57
6862150863663248387736097259 52*sqrt(29) + 3280234469513620846778 8
14232569765050562232038146737211455832064)/8 + 820058617378405211
69470355814244126264055800953668430286395801 6*sqrt(6)*sqrt(1 + sq
rt(29)))*atan(sqrt(6)*(-sqrt(6 + 6*sqrt(29))/6 + 1/3 + 4/(3*x))/s
qrt(-1 + sqrt(29)))/(145470352576997122522570307403691676825431 17
0490040195217026659123 2*sqrt(-1 + sqrt(29))*sqrt(1 + sqrt(29)))
```

Mathematica [C] time = 0.0270747, size = 113, normalized size = 0.32

$$\frac{\text{RootSum}\left[8\#1^4-\#1^3+8\#1+8\&,\frac{392\#1^2\log(x-\#1)-1097\#1\log(x-\#1)+2243\log(x-\#1)}{32\#1^3-3\#1^2+8}\&\right]}{21924}+\frac{784x^3-1146x^2+1539x+544}{43848\left(8x^4-x^3+8x+8\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(8 + 8*x - x^3 + 8*x^4)^(-2),x]
```

```
[Out] (544 + 1539*x - 1146*x^2 + 784*x^3)/(43848*(8 + 8*x - x^3 + 8*x^4
)) + RootSum[8 + 8*#1 - #1^3 + 8*#1^4 & , (2243*Log[x - #1] - 109
7*Log[x - #1]*#1 + 392*Log[x - #1]*#1^2)/(8 - 3*#1^2 + 32*#1^3) &
 ]/21924
```

Maple [C] time = 0.012, size = 83, normalized size = 0.2

$$1\left(\frac{7x^3}{3132}-\frac{191x^2}{58464}+\frac{57x}{12992}+\frac{17}{10962}\right)\left(x^4-\frac{x^3}{8}+x+1\right)^{-1}+\frac{1}{21924}\sum_{\_R=RootOf(8\_Z^4-\_Z^3+8\_Z+8)}\frac{(392\_R^2-1097\_R+2243)\ln(x-\_R)}{32\_R^3-3\_R^2+8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(8*x^4-x^3+8*x+8)^2,x)
```

```
[Out] (7/3132*x^3-191/58464*x^2+57/12992*x+17/10962)/(x^4-1/8*x^3+x+1)+
1/21924*sum((392*_R^2-1097*_R+2243)/(32*_R^3-3*_R^2+8)*ln(x-_R),_
R=RootOf(8*_Z^4-_Z^3+8*_Z+8))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{784x^3-1146x^2+1539x+544}{43848(8x^4-x^3+8x+8)}+\frac{1}{21924}\int\frac{392x^2-1097x+2243}{8x^4-x^3+8x+8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((8*x^4 - x^3 + 8*x + 8)^(-2),x, algorithm="maxima")
```

```
[Out] 1/43848*(784*x^3 - 1146*x^2 + 1539*x + 544)/(8*x^4 - x^3 + 8*x +
8) + 1/21924*integrate((392*x^2 - 1097*x + 2243)/(8*x^4 - x^3 + 8
*x + 8), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((8*x^4 - x^3 + 8*x + 8)^(-2),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 3.39146, size = 71, normalized size = 0.2

$$\frac{784x^3-1146x^2+1539x+544}{350784x^4-43848x^3+350784x+350784}+\operatorname{RootSum}\left(56213386274315096064t^4+2228162991905088t^2+6447137250645t+4563337216,\left(t\mapsto t\log\left(\frac{777231320984}{84352082}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(8*x**4-x**3+8*x+8)**2,x)
```

```
[Out] (784*x**3 - 1146*x**2 + 1539*x + 544)/(350784*x**4 - 43848*x**3 +
 350784*x + 350784) + RootSum(56213386274315096064*_t**4 + 222816
2991905088*_t**2 + 6447137250645*_t + 4563337216, Lambda(_t, _t*l
```

```
og(77723132098413320679499673 2416*_t**3/843520820693366087 8927 -
12535959053974646848290969 60*_t**2/843520820693366087 8927 + 90007
2466443173277115848*_t/22797860018739623 9971 + x + 33397908111320
2533090737/6748166565546928703 1416)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(8x^4 - x^3 + 8x + 8)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((8*x^4 - x^3 + 8*x + 8)^(-2),x, algorithm="giac")
```

```
[Out] integrate((8*x^4 - x^3 + 8*x + 8)^(-2), x)
```

3.51 $\int \left(1+4x+4x^2+4x^4\right)^4 \, dx$

Optimal. Leaf size=97

$$\frac{256x^{17}}{17}+\frac{1024x^{15}}{15}+\frac{512x^{14}}{7}+\frac{1792x^{13}}{13}+256x^{12}+\frac{3328x^{11}}{11}+384x^{10} +\frac{4192x^9}{9}+448x^8+\frac{2752x^7}{7}+\frac{992x^6}{3}+\frac{1136x^5}{5}+112x^4+\frac{112x^3}{3}+8x^2+x$$

```
[Out] x + 8*x^2 + (112*x^3)/3 + 112*x^4 + (1136*x^5)/5 + (992*x^6)/3 +
(2752*x^7)/7 + 448*x^8 + (4192*x^9)/9 + 384*x^10 + (3328*x^11)/11
 + 256*x^12 + (1792*x^13)/13 + (512*x^14)/7 + (1024*x^15)/15 + (2
56*x^17)/17
```

Rubi [A] time = 0.0587508, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059

$$\frac{256x^{17}}{17}+\frac{1024x^{15}}{15}+\frac{512x^{14}}{7}+\frac{1792x^{13}}{13}+256x^{12}+\frac{3328x^{11}}{11}+384x^{10} +\frac{4192x^9}{9}+448x^8+\frac{2752x^7}{7}+\frac{992x^6}{3}+\frac{1136x^5}{5}+112x^4+\frac{112x^3}{3}+8x^2+x$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + 4*x + 4*x^2 + 4*x^4)^4,x]
```

```
[Out] x + 8*x^2 + (112*x^3)/3 + 112*x^4 + (1136*x^5)/5 + (992*x^6)/3 +
(2752*x^7)/7 + 448*x^8 + (4192*x^9)/9 + 384*x^10 + (3328*x^11)/11
 + 256*x^12 + (1792*x^13)/13 + (512*x^14)/7 + (1024*x^15)/15 + (2
56*x^17)/17
```

Rubi in Sympy [A] time = 41.4893, size = 94, normalized size = 0.97

$$\frac{256x^{17}}{17}+\frac{1024x^{15}}{15}+\frac{512x^{14}}{7}+\frac{1792x^{13}}{13}+256x^{12}+\frac{3328x^{11}}{11}+384x^{10} +\frac{4192x^9}{9}+448x^8+\frac{2752x^7}{7}+\frac{992x^6}{3}+\frac{1136x^5}{5}+112x^4+\frac{112x^3}{3}+8x^2+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((4*x**4+4*x**2+4*x+1)**4,x)
```

```
[Out] 256*x**17/17 + 1024*x**15/15 + 512*x**14/7 + 1792*x**13/13 + 256*
x**12 + 3328*x**11/11 + 384*x**10 + 4192*x**9/9 + 448*x**8 + 2752
*x**7/7 + 992*x**6/3 + 1136*x**5/5 + 112*x**4 + 112*x**3/3 + 8*x*
*2 + x
```

Mathematica [A] time = 0.00248531, size = 97, normalized size = 1.

$$\frac{256x^{17}}{17}+\frac{1024x^{15}}{15}+\frac{512x^{14}}{7}+\frac{1792x^{13}}{13}+256x^{12}+\frac{3328x^{11}}{11}+384x^{10} +\frac{4192x^9}{9}+448x^8+\frac{2752x^7}{7}+\frac{992x^6}{3}+\frac{1136x^5}{5}+112x^4+\frac{112x^3}{3}+8x^2+x$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + 4*x + 4*x^2 + 4*x^4)^4,x]
```

```
[Out] x + 8*x^2 + (112*x^3)/3 + 112*x^4 + (1136*x^5)/5 + (992*x^6)/3 +
(2752*x^7)/7 + 448*x^8 + (4192*x^9)/9 + 384*x^10 + (3328*x^11)/11
 + 256*x^12 + (1792*x^13)/13 + (512*x^14)/7 + (1024*x^15)/15 + (2
56*x^17)/17
```

Maple [A] time = 0.002, size = 78, normalized size = 0.8

$$x+8x^2+\frac{112x^3}{3}+112x^4+\frac{1136x^5}{5}+\frac{992x^6}{3}+\frac{2752x^7}{7}+448x^8+\frac{4192x^9}{9}+384x^{10}+\frac{3328x^{11}}{11}+256x^{12}+\frac{1792x^{13}}{13}+\frac{512x^{14}}{7}+\frac{1024x^{15}}{15}+\frac{256x^{17}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((4*x^4+4*x^2+4*x+1)^4,x)
```

```
[Out] x+8*x^2+112/3*x^3+112*x^4+1136/5*x^5+992/3*x^6+2752/7*x^7+448*x^8
+4192/9*x^9+384*x^10+3328/11*x^11+256*x^12+1792/13*x^13+512/7*x^1
4+1024/15*x^15+256/17*x^17
```

Maxima [A] time = 0.80259, size = 104, normalized size = 1.07

$$\frac{256}{17}x^{17}+\frac{1024}{15}x^{15}+\frac{512}{7}x^{14}+\frac{1792}{13}x^{13}+256x^{12}+\frac{3328}{11}x^{11}+384x^{10}+\frac{4192}{9}x^9+448x^8+\frac{2752}{7}x^7+\frac{992}{3}x^6+\frac{1136}{5}x^5+112x^4+\frac{112}{3}x^3+8x^2+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^4 + 4*x^2 + 4*x + 1)^4,x, algorithm="maxima")
```

```
[Out] 256/17*x^17 + 1024/15*x^15 + 512/7*x^14 + 1792/13*x^13 + 256*x^12
 + 3328/11*x^11 + 384*x^10 + 4192/9*x^9 + 448*x^8 + 2752/7*x^7 +
992/3*x^6 + 1136/5*x^5 + 112*x^4 + 112/3*x^3 + 8*x^2 + x
```

Fricas [A] time = 0.227086, size = 1, normalized size = 0.01

$$\frac{256}{17}x^{17}+\frac{1024}{15}x^{15}+\frac{512}{7}x^{14}+\frac{1792}{13}x^{13}+256x^{12}+\frac{3328}{11}x^{11}+384x^{10}+\frac{4192}{9}x^9+448x^8+\frac{2752}{7}x^7+\frac{992}{3}x^6+\frac{1136}{5}x^5+112x^4+\frac{112}{3}x^3+8x^2+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^4 + 4*x^2 + 4*x + 1)^4,x, algorithm="fricas")
```

```
[Out] 256/17*x^17 + 1024/15*x^15 + 512/7*x^14 + 1792/13*x^13 + 256*x^12
 + 3328/11*x^11 + 384*x^10 + 4192/9*x^9 + 448*x^8 + 2752/7*x^7 +
992/3*x^6 + 1136/5*x^5 + 112*x^4 + 112/3*x^3 + 8*x^2 + x
```

Sympy [A] time = 0.10541, size = 94, normalized size = 0.97

$$\frac{256x^{17}}{17} + \frac{1024x^{15}}{15} + \frac{512x^{14}}{7} + \frac{1792x^{13}}{13} + 256x^{12} + \frac{3328x^{11}}{11} + 384x^{10} + \frac{4192x^9}{9} + 448x^8 + \frac{2752x^7}{7} + \frac{992x^6}{3} + \frac{1136x^5}{5} + 112x^4 + \frac{112x^3}{3} + 8x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x**4+4*x**2+4*x+1)**4,x)
```

```
[Out] 256*x**17/17 + 1024*x**15/15 + 512*x**14/7 + 1792*x**13/13 + 256*
x**12 + 3328*x**11/11 + 384*x**10 + 4192*x**9/9 + 448*x**8 + 2752
*x**7/7 + 992*x**6/3 + 1136*x**5/5 + 112*x**4 + 112*x**3/3 + 8*x*
*2 + x
```

GIAC/XCAS [A] time = 0.259013, size = 104, normalized size = 1.07

$$\frac{256}{17}\,x^{17} + \frac{1024}{15}\,x^{15} + \frac{512}{7}\,x^{14} + \frac{1792}{13}\,x^{13} + 256\,x^{12} + \frac{3328}{11}\,x^{11} + 384\,x^{10} + \frac{4192}{9}\,x^9 + 448\,x^8 + \frac{2752}{7}\,x^7 + \frac{992}{3}\,x^6 + \frac{1136}{5}\,x^5 + 112\,x^4 + \frac{112}{3}\,x^3 + 8\,x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^4 + 4*x^2 + 4*x + 1)^4,x, algorithm="giac")
```

```
[Out] 256/17*x^17 + 1024/15*x^15 + 512/7*x^14 + 1792/13*x^13 + 256*x^12
 + 3328/11*x^11 + 384*x^10 + 4192/9*x^9 + 448*x^8 + 2752/7*x^7 +
992/3*x^6 + 1136/5*x^5 + 112*x^4 + 112/3*x^3 + 8*x^2 + x
```

3.52 $\int (1+4x+4x^2+4x^4)^3 \, dx$

Optimal. Leaf size=69

$$\frac{64x^{13}}{13}+\frac{192x^{11}}{11}+\frac{96x^{10}}{5}+\frac{80x^9}{3}+48x^8+\frac{352x^7}{7}+48x^6+\frac{252x^5}{5}+40x^4+20x^3+6x^2+x$$

```
[Out] x + 6*x^2 + 20*x^3 + 40*x^4 + (252*x^5)/5 + 48*x^6 + (352*x^7)/7
+ 48*x^8 + (80*x^9)/3 + (96*x^10)/5 + (192*x^11)/11 + (64*x^13)/1
3
```

Rubi [A] time = 0.0415652, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$

$$\frac{64x^{13}}{13}+\frac{192x^{11}}{11}+\frac{96x^{10}}{5}+\frac{80x^9}{3}+48x^8+\frac{352x^7}{7}+48x^6+\frac{252x^5}{5}+40x^4+20x^3+6x^2+x$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + 4*x + 4*x^2 + 4*x^4)^3,x]
```

```
[Out] x + 6*x^2 + 20*x^3 + 40*x^4 + (252*x^5)/5 + 48*x^6 + (352*x^7)/7
+ 48*x^8 + (80*x^9)/3 + (96*x^10)/5 + (192*x^11)/11 + (64*x^13)/1
3
```

Rubi in Sympy [A] time = 31.7697, size = 66, normalized size = 0.96

$$\frac{64x^{13}}{13}+\frac{192x^{11}}{11}+\frac{96x^{10}}{5}+\frac{80x^9}{3}+48x^8+\frac{352x^7}{7}+48x^6+\frac{252x^5}{5}+40x^4+20x^3+6x^2+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((4*x**4+4*x**2+4*x+1)**3,x)
```

```
[Out] 64*x**13/13 + 192*x**11/11 + 96*x**10/5 + 80*x**9/3 + 48*x**8 + 3
52*x**7/7 + 48*x**6 + 252*x**5/5 + 40*x**4 + 20*x**3 + 6*x**2 + x
```

Mathematica [A] time = 0.00142392, size = 69, normalized size = 1.

$$\frac{64x^{13}}{13}+\frac{192x^{11}}{11}+\frac{96x^{10}}{5}+\frac{80x^9}{3}+48x^8+\frac{352x^7}{7}+48x^6+\frac{252x^5}{5}+40x^4+20x^3+6x^2+x$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + 4*x + 4*x^2 + 4*x^4)^3,x]
```

```
[Out] x + 6*x^2 + 20*x^3 + 40*x^4 + (252*x^5)/5 + 48*x^6 + (352*x^7)/7
+ 48*x^8 + (80*x^9)/3 + (96*x^10)/5 + (192*x^11)/11 + (64*x^13)/1
3
```

Maple [A] time = 0.002, size = 58, normalized size = 0.8

$$x+6\,x^2+20\,x^3+40\,x^4+\frac{252\,x^5}{5}+48\,x^6+\frac{352\,x^7}{7}+48\,x^8+\frac{80\,x^9}{3}+\frac{96\,x^{10}}{5}+\frac{192\,x^{11}}{11}+\frac{64\,x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((4*x^4+4*x^2+4*x+1)^3,x)
```

```
[Out] x+6*x^2+20*x^3+40*x^4+252/5*x^5+48*x^6+352/7*x^7+48*x^8+80/3*x^9+
96/5*x^10+192/11*x^11+64/13*x^13
```

Maxima [A] time = 0.788934, size = 77, normalized size = 1.12

$$\frac{64}{13}\,x^{13} + \frac{192}{11}\,x^{11} + \frac{96}{5}\,x^{10} + \frac{80}{3}\,x^{9} + 48\,x^{8} + \frac{352}{7}\,x^{7} + 48\,x^{6} + \frac{252}{5}\,x^{5} + 40\,x^{4} + 20\,x^{3} + 6\,x^{2} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((4*x^4 + 4*x^2 + 4*x + 1)^3,x, algorithm="maxima")
```

```
[Out] 64/13*x^13 + 192/11*x^11 + 96/5*x^10 + 80/3*x^9 + 48*x^8 + 352/7*
x^7 + 48*x^6 + 252/5*x^5 + 40*x^4 + 20*x^3 + 6*x^2 + x
```

Fricas [A] time = 0.227525, size = 1, normalized size = 0.01

$$\frac{64}{13}x^{13} + \frac{192}{11}x^{11} + \frac{96}{5}x^{10} + \frac{80}{3}x^{9} + 48x^{8} + \frac{352}{7}x^{7} + 48x^{6} + \frac{252}{5}x^{5} + 40x^{4} + 20x^{3} + 6x^{2} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((4*x^4 + 4*x^2 + 4*x + 1)^3,x, algorithm="fricas")
```

```
[Out] 64/13*x^13 + 192/11*x^11 + 96/5*x^10 + 80/3*x^9 + 48*x^8 + 352/7*
x^7 + 48*x^6 + 252/5*x^5 + 40*x^4 + 20*x^3 + 6*x^2 + x
```

Sympy [A] time = 0.093272, size = 66, normalized size = 0.96

$$\frac{64x^{13}}{13} + \frac{192x^{11}}{11} + \frac{96x^{10}}{5} + \frac{80x^{9}}{3} + 48x^{8} + \frac{352x^{7}}{7} + 48x^{6} + \frac{252x^{5}}{5} + 40x^{4} + 20x^{3} + 6x^{2} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((4*x**4+4*x**2+4*x+1)**3,x)
```

```
[Out] 64*x**13/13 + 192*x**11/11 + 96*x**10/5 + 80*x**9/3 + 48*x**8 + 3
52*x**7/7 + 48*x**6 + 252*x**5/5 + 40*x**4 + 20*x**3 + 6*x**2 + x
```

GIAC/XCAS [A] time = 0.259721, size = 77, normalized size = 1.12

$$\frac{64}{13}\,x^{13} + \frac{192}{11}\,x^{11} + \frac{96}{5}\,x^{10} + \frac{80}{3}\,x^{9} + 48\,x^{8} + \frac{352}{7}\,x^{7} + 48\,x^{6} + \frac{252}{5}\,x^{5} + 40\,x^{4} + 20\,x^{3} + 6\,x^{2} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((4*x^4 + 4*x^2 + 4*x + 1)^3,x, algorithm="giac")
```

```
[Out] 64/13*x^13 + 192/11*x^11 + 96/5*x^10 + 80/3*x^9 + 48*x^8 + 352/7*
x^7 + 48*x^6 + 252/5*x^5 + 40*x^4 + 20*x^3 + 6*x^2 + x
```

3.53 $\int \left(1+4x+4x^2+4x^4\right)^2 \, dx$

Optimal. Leaf size=45

$$\frac{16x^9}{9}+\frac{32x^7}{7}+\frac{16x^6}{3}+\frac{24x^5}{5}+8x^4+8x^3+4x^2+x$$

```
[Out] x + 4*x^2 + 8*x^3 + 8*x^4 + (24*x^5)/5 + (16*x^6)/3 + (32*x^7)/7
+ (16*x^9)/9
```

Rubi [A] time = 0.0297536, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$

$$\frac{16x^9}{9}+\frac{32x^7}{7}+\frac{16x^6}{3}+\frac{24x^5}{5}+8x^4+8x^3+4x^2+x$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + 4*x + 4*x^2 + 4*x^4)^2,x]
```

```
[Out] x + 4*x^2 + 8*x^3 + 8*x^4 + (24*x^5)/5 + (16*x^6)/3 + (32*x^7)/7
+ (16*x^9)/9
```

Rubi in Sympy [A] time = 24.129, size = 42, normalized size = 0.93

$$\frac{16x^9}{9}+\frac{32x^7}{7}+\frac{16x^6}{3}+\frac{24x^5}{5}+8x^4+8x^3+4x^2+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((4*x**4+4*x**2+4*x+1)**2,x)
```

```
[Out] 16*x**9/9 + 32*x**7/7 + 16*x**6/3 + 24*x**5/5 + 8*x**4 + 8*x**3 +
 4*x**2 + x
```

Mathematica [A] time = 0.0018479, size = 45, normalized size = 1.

$$\frac{16x^9}{9}+\frac{32x^7}{7}+\frac{16x^6}{3}+\frac{24x^5}{5}+8x^4+8x^3+4x^2+x$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + 4*x + 4*x^2 + 4*x^4)^2,x]
```

```
[Out] x + 4*x^2 + 8*x^3 + 8*x^4 + (24*x^5)/5 + (16*x^6)/3 + (32*x^7)/7
+ (16*x^9)/9
```

Maple [A] time = 0.001, size = 38, normalized size = 0.8

$$x+4\,x^2+8\,x^3+8\,x^4+\frac{24\,x^5}{5}+\frac{16\,x^6}{3}+\frac{32\,x^7}{7}+\frac{16\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((4*x^4+4*x^2+4*x+1)^2,x)
```

```
[Out] x+4*x^2+8*x^3+8*x^4+24/5*x^5+16/3*x^6+32/7*x^7+16/9*x^9
```

Maxima [A] time = 0.785279, size = 50, normalized size = 1.11

$$\frac{16}{9}\,x^{9} + \frac{32}{7}\,x^{7} + \frac{16}{3}\,x^{6} + \frac{24}{5}\,x^{5} + 8\,x^{4} + 8\,x^{3} + 4\,x^{2} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^4 + 4*x^2 + 4*x + 1)^2,x, algorithm="maxima")
```

```
[Out] 16/9*x^9 + 32/7*x^7 + 16/3*x^6 + 24/5*x^5 + 8*x^4 + 8*x^3 + 4*x^2
 + x
```

Fricas [A] time = 0.222767, size = 1, normalized size = 0.02

$$\frac{16}{9}x^9 + \frac{32}{7}x^7 + \frac{16}{3}x^6 + \frac{24}{5}x^5 + 8x^4 + 8x^3 + 4x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^4 + 4*x^2 + 4*x + 1)^2,x, algorithm="fricas")
```

```
[Out] 16/9*x^9 + 32/7*x^7 + 16/3*x^6 + 24/5*x^5 + 8*x^4 + 8*x^3 + 4*x^2
 + x
```

Sympy [A] time = 0.076658, size = 42, normalized size = 0.93

$$\frac{16x^9}{9} + \frac{32x^7}{7} + \frac{16x^6}{3} + \frac{24x^5}{5} + 8x^4 + 8x^3 + 4x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x**4+4*x**2+4*x+1)**2,x)
```

```
[Out] 16*x**9/9 + 32*x**7/7 + 16*x**6/3 + 24*x**5/5 + 8*x**4 + 8*x**3 +
 4*x**2 + x
```

GIAC/XCAS [A] time = 0.259937, size = 50, normalized size = 1.11

$$\frac{16}{9}\,x^{9} + \frac{32}{7}\,x^{7} + \frac{16}{3}\,x^{6} + \frac{24}{5}\,x^{5} + 8\,x^{4} + 8\,x^{3} + 4\,x^{2} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^4 + 4*x^2 + 4*x + 1)^2,x, algorithm="giac")
```

```
[Out] 16/9*x^9 + 32/7*x^7 + 16/3*x^6 + 24/5*x^5 + 8*x^4 + 8*x^3 + 4*x^2
 + x
```

3.54 $\int \left(1+4x+4x^2+4x^4\right)\,dx$

Optimal. Leaf size=21

$$\frac{4x^5}{5}+\frac{4x^3}{3}+2x^2+x$$

```
[Out] x + 2*x^2 + (4*x^3)/3 + (4*x^5)/5
```

Rubi [A] time = 0.00926799, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.

$$\frac{4x^5}{5}+\frac{4x^3}{3}+2x^2+x$$

Antiderivative was successfully verified.

```
[In] Int[1 + 4*x + 4*x^2 + 4*x^4,x]

[Out] x + 2*x^2 + (4*x^3)/3 + (4*x^5)/5
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{4x^5}{5}+\frac{4x^3}{3}+x+4\int x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(4*x**4+4*x**2+4*x+1,x)

[Out] 4*x**5/5 + 4*x**3/3 + x + 4*Integral(x, x)
```

Mathematica [A] time = 0.000057277, size = 21, normalized size = 1.

$$\frac{4x^5}{5}+\frac{4x^3}{3}+2x^2+x$$

Antiderivative was successfully verified.

```
[In] Integrate[1 + 4*x + 4*x^2 + 4*x^4,x]

[Out] x + 2*x^2 + (4*x^3)/3 + (4*x^5)/5
```

Maple [A] time = 0.001, size = 18, normalized size = 0.9

$$x+2\,x^2+\frac{4\,x^3}{3}+\frac{4\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(4*x^4+4*x^2+4*x+1,x)
```

```
[Out] x+2*x^2+4/3*x^3+4/5*x^5
```

Maxima [A] time = 0.808889, size = 23, normalized size = 1.1

$$\frac{4}{5}\,x^5+\frac{4}{3}\,x^3+2\,x^2+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(4*x^4 + 4*x^2 + 4*x + 1,x, algorithm="maxima")
```

```
[Out] 4/5*x^5 + 4/3*x^3 + 2*x^2 + x
```

Fricas [A] time = 0.237061, size = 1, normalized size = 0.05

$$\frac{4}{5}x^5+\frac{4}{3}x^3+2x^2+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(4*x^4 + 4*x^2 + 4*x + 1,x, algorithm="fricas")
```

```
[Out] 4/5*x^5 + 4/3*x^3 + 2*x^2 + x
```

Sympy [A] time = 0.059024, size = 19, normalized size = 0.9

$$\frac{4x^5}{5}+\frac{4x^3}{3}+2x^2+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(4*x**4+4*x**2+4*x+1,x)
```

```
[Out] 4*x**5/5 + 4*x**3/3 + 2*x**2 + x
```

GIAC/XCAS [A] time = 0.259678, size = 23, normalized size = 1.1

$$\frac{4}{5}\,x^5+\frac{4}{3}\,x^3+2\,x^2+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(4*x^4 + 4*x^2 + 4*x + 1,x, algorithm="giac")
```

```
[Out] 4/5*x^5 + 4/3*x^3 + 2*x^2 + x
```

3.55 $\int \frac{1}{1+4x+4x^2+4x^4}\,dx$

Optimal. Leaf size=234

$$-\frac{1}{4}\sqrt{\frac{1}{5}\left(\sqrt{5}-2\right)}\log\left(\left(\frac{1}{x}+1\right)^2-\sqrt{2\left(1+\sqrt{5}\right)}\left(\frac{1}{x}+1\right)+\sqrt{5}\right)$$
$$+\frac{1}{4}\sqrt{\frac{1}{5}\left(\sqrt{5}-2\right)}\log\left(\left(\frac{1}{x}+1\right)^2+\sqrt{2\left(1+\sqrt{5}\right)}\left(\frac{1}{x}+1\right)+\sqrt{5}\right)+\frac{1}{2}\tan^{-1}\left(\frac{1}{2}\left(\left(\frac{1}{x}+1\right)^2-1\right)\right)$$
$$-\frac{1}{2}\sqrt{\frac{1}{5}\left(2+\sqrt{5}\right)}\tan^{-1}\left(\frac{\frac{2}{x}-\sqrt{2\left(1+\sqrt{5}\right)}+2}{\sqrt{2\left(\sqrt{5}-1\right)}}\right)-\frac{1}{2}\sqrt{\frac{1}{5}\left(2+\sqrt{5}\right)}\tan^{-1}\left(\frac{\frac{2}{x}+\sqrt{2\left(1+\sqrt{5}\right)}+2}{\sqrt{2\left(\sqrt{5}-1\right)}}\right)$$

```
[Out] ArcTan[(-1 + (1 + x^(-1))^2)/2]/2 - (Sqrt[(2 + Sqrt[5])/5]*ArcTan
[(2 - Sqrt[2*(1 + Sqrt[5])] + 2/x)/Sqrt[2*(-1 + Sqrt[5])]])/2 - (
Sqrt[(2 + Sqrt[5])/5]*ArcTan[(2 + Sqrt[2*(1 + Sqrt[5])] + 2/x)/Sq
rt[2*(-1 + Sqrt[5])]])/2 - (Sqrt[(-2 + Sqrt[5])/5]*Log[Sqrt[5] -
Sqrt[2*(1 + Sqrt[5])]*(1 + x^(-1)) + (1 + x^(-1))^2])/4 + (Sqrt[(
-2 + Sqrt[5])/5]*Log[Sqrt[5] + Sqrt[2*(1 + Sqrt[5])]*(1 + x^(-1))
 + (1 + x^(-1))^2])/4
```

Rubi [A] time = 0.703752, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$

$$-\frac{1}{4}\sqrt{\frac{1}{5}\left(\sqrt{5}-2\right)}\log\left(\left(\frac{1}{x}+1\right)^2-\sqrt{2\left(1+\sqrt{5}\right)}\left(\frac{1}{x}+1\right)+\sqrt{5}\right)$$
$$+\frac{1}{4}\sqrt{\frac{1}{5}\left(\sqrt{5}-2\right)}\log\left(\left(\frac{1}{x}+1\right)^2+\sqrt{2\left(1+\sqrt{5}\right)}\left(\frac{1}{x}+1\right)+\sqrt{5}\right)+\frac{1}{2}\tan^{-1}\left(\frac{1}{2}\left(\left(\frac{1}{x}+1\right)^2-1\right)\right)$$
$$-\frac{1}{2}\sqrt{\frac{1}{5}\left(2+\sqrt{5}\right)}\tan^{-1}\left(\frac{\frac{2}{x}-\sqrt{2\left(1+\sqrt{5}\right)}+2}{\sqrt{2\left(\sqrt{5}-1\right)}}\right)-\frac{1}{2}\sqrt{\frac{1}{5}\left(2+\sqrt{5}\right)}\tan^{-1}\left(\frac{\frac{2}{x}+\sqrt{2\left(1+\sqrt{5}\right)}+2}{\sqrt{2\left(\sqrt{5}-1\right)}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + 4*x + 4*x^2 + 4*x^4)^(-1),x]
```

```
[Out] ArcTan[(-1 + (1 + x^(-1))^2)/2]/2 - (Sqrt[(2 + Sqrt[5])/5]*ArcTan
[(2 - Sqrt[2*(1 + Sqrt[5])] + 2/x)/Sqrt[2*(-1 + Sqrt[5])]])/2 - (
Sqrt[(2 + Sqrt[5])/5]*ArcTan[(2 + Sqrt[2*(1 + Sqrt[5])] + 2/x)/Sq
rt[2*(-1 + Sqrt[5])]])/2 - (Sqrt[(-2 + Sqrt[5])/5]*Log[Sqrt[5] -
Sqrt[2*(1 + Sqrt[5])]*(1 + x^(-1)) + (1 + x^(-1))^2])/4 + (Sqrt[(
-2 + Sqrt[5])/5]*Log[Sqrt[5] + Sqrt[2*(1 + Sqrt[5])]*(1 + x^(-1))
 + (1 + x^(-1))^2])/4
```

Rubi in Sympy [A] time = 62.878, size = 345, normalized size = 1.47

$$\frac{\sqrt{10}\left(-8\sqrt{5}+8\right)\log\left(\sqrt{2}\left(-1-\frac{1}{x}\right)\sqrt{1+\sqrt{5}}+\left(1+\frac{1}{x}\right)^2+\sqrt{5}\right)}{320\sqrt{1+\sqrt{5}}} - \frac{\sqrt{10}\left(-8\sqrt{5}+8\right)\log\left(\left(1+\frac{1}{x}\right)^2+\sqrt{2}\sqrt{1+\sqrt{5}}\left(1+\frac{1}{x}\right)+\sqrt{5}\right)}{320\sqrt{1+\sqrt{5}}} - \frac{\sqrt{5}\left(-\frac{\sqrt{2}\sqrt{1+\sqrt{5}}\left(-16\sqrt{5}+16\right)}{2}+16\sqrt{2}\sqrt{1+\sqrt{5}}\right)\operatorname{atan}\left(\frac{\sqrt{2}\left(1+\frac{\sqrt{2+2\sqrt{5}}}{2}+\frac{1}{x}\right)}{\sqrt{-1+\sqrt{5}}}\right)}{160\sqrt{-1+\sqrt{5}}\sqrt{1+\sqrt{5}}} - \frac{\sqrt{5}\left(-\frac{\sqrt{2}\sqrt{1+\sqrt{5}}\left(-16\sqrt{5}+16\right)}{2}+16\sqrt{2}\sqrt{1+\sqrt{5}}\right)\operatorname{atan}\left(\frac{\sqrt{2}\left(-\frac{\sqrt{2+2\sqrt{5}}}{2}+1+\frac{1}{x}\right)}{\sqrt{-1+\sqrt{5}}}\right)}{160\sqrt{-1+\sqrt{5}}\sqrt{1+\sqrt{5}}} + \frac{\operatorname{atan}\left(\frac{\left(1+\frac{1}{x}\right)^2}{2}-\frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   rubi_integrate(1/(4*x**4+4*x**2+4*x+1),x)
```

```
[Out] sqrt(10)*(-8*sqrt(5) + 8)*log(sqrt(2)*(-1 - 1/x)*sqrt(1 + sqrt(5)
) + (1 + 1/x)**2 + sqrt(5))/(320*sqrt(1 + sqrt(5))) - sqrt(10)*(-
8*sqrt(5) + 8)*log((1 + 1/x)**2 + sqrt(2)*sqrt(1 + sqrt(5))*(1 +
1/x) + sqrt(5))/(320*sqrt(1 + sqrt(5))) - sqrt(5)*(-sqrt(2)*sqrt(
1 + sqrt(5))*(-16*sqrt(5) + 16)/2 + 16*sqrt(2)*sqrt(1 + sqrt(5)))
*atan(sqrt(2)*(1 + sqrt(2 + 2*sqrt(5))/2 + 1/x)/sqrt(-1 + sqrt(5)
))/(160*sqrt(-1 + sqrt(5))*sqrt(1 + sqrt(5))) - sqrt(5)*(-sqrt(2)
*sqrt(1 + sqrt(5))*(-16*sqrt(5) + 16)/2 + 16*sqrt(2)*sqrt(1 + sqr
t(5)))*atan(sqrt(2)*(-sqrt(2 + 2*sqrt(5))/2 + 1 + 1/x)/sqrt(-1 +
sqrt(5)))/(160*sqrt(-1 + sqrt(5))*sqrt(1 + sqrt(5))) + atan((1 +
1/x)**2/2 - 1/2)/2
```

Mathematica [C] time = 0.0236653, size = 47, normalized size = 0.2

$$\frac{1}{4}\text{RootSum}\left[4\#1^4+4\#1^2+4\#1+1\&,\frac{\log(x-\#1)}{4\#1^3+2\#1+1}\&\right]$$

Antiderivative was successfully verified.

```
[In]   Integrate[(1 + 4*x + 4*x^2 + 4*x^4)^(-1),x]
```

```
[Out] RootSum[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , Log[x - #1]/(1 + 2*#1 + 4*
#1^3) & ]/4
```

Maple [C] time = 0.007, size = 41, normalized size = 0.2

$$\frac{1}{4}\sum_{\_R=RootOf\left(4\_Z^4+4\_Z^2+4\_Z+1\right)}\frac{\ln\left(x-\_R\right)}{4\_R^3+2\_R+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(1/(4*x^4+4*x^2+4*x+1),x)
```

```
[Out] 1/4*sum(1/(4*_R^3+2*_R+1)*ln(x-_R),_R=RootOf(4*_Z^4+4*_Z^2+4*_Z+1
))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{4\,x^4+4\,x^2+4\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(4*x^4 + 4*x^2 + 4*x + 1),x, algorithm="maxima")
```

```
[Out] integrate(1/(4*x^4 + 4*x^2 + 4*x + 1), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(4*x^4 + 4*x^2 + 4*x + 1),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 2.76878, size = 36, normalized size = 0.15

$$\operatorname{RootSum}\left(1280t^4+288t^2+32t+1,\left(t\mapsto t\log\left(-240t^3+10t^2-54t+x-\frac{27}{8}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(4*x**4+4*x**2+4*x+1),x)
```

```
[Out] RootSum(1280*_t**4 + 288*_t**2 + 32*_t + 1, Lambda(_t, _t*log(-24
0*_t**3 + 10*_t**2 - 54*_t + x - 27/8)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{4\,x^4+4\,x^2+4\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(4*x^4 + 4*x^2 + 4*x + 1),x, algorithm="giac")
```

```
[Out] integrate(1/(4*x^4 + 4*x^2 + 4*x + 1), x)
```

3.56 $\int \frac{1}{(1+4x+4x^2+4x^4)^2}\,dx$

Optimal. Leaf size=317

$$-\frac{17-\left(\frac{1}{x}+1\right)^2}{2\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)}+\frac{\left(59-17\left(\frac{1}{x}+1\right)^2\right)\left(\frac{1}{x}+1\right)}{10\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)}$$
$$+\frac{1}{40}\sqrt{\frac{1}{10}\left(2665\sqrt{5}-5959\right)}\log\left(\left(\frac{1}{x}+1\right)^2-\sqrt{2\left(1+\sqrt{5}\right)}\left(\frac{1}{x}+1\right)+\sqrt{5}\right)$$
$$-\frac{1}{40}\sqrt{\frac{1}{10}\left(2665\sqrt{5}-5959\right)}\log\left(\left(\frac{1}{x}+1\right)^2+\sqrt{2\left(1+\sqrt{5}\right)}\left(\frac{1}{x}+1\right)+\sqrt{5}\right)$$
$$+\frac{7}{4}\tan^{-1}\left(\frac{1}{2}\left(\left(\frac{1}{x}+1\right)^2-1\right)\right)-\frac{1}{20}\sqrt{\frac{1}{10}\left(5959+2665\sqrt{5}\right)}\tan^{-1}\left(\frac{\frac{2}{x}-\sqrt{2\left(1+\sqrt{5}\right)}+2}{\sqrt{2\left(\sqrt{5}-1\right)}}\right)$$
$$-\frac{1}{20}\sqrt{\frac{1}{10}\left(5959+2665\sqrt{5}\right)}\tan^{-1}\left(\frac{\frac{2}{x}+\sqrt{2\left(1+\sqrt{5}\right)}+2}{\sqrt{2\left(\sqrt{5}-1\right)}}\right)$$

```
[Out] -(17 - (1 + x^(-1))^2)/(2*(5 - 2*(1 + x^(-1))^2 + (1 + x^(-1))^4)
) + ((59 - 17*(1 + x^(-1))^2)*(1 + x^(-1)))/(10*(5 - 2*(1 + x^(-1
))^2 + (1 + x^(-1))^4)) + (7*ArcTan[(-1 + (1 + x^(-1))^2)/2])/4 -
 (Sqrt[(5959 + 2665*Sqrt[5])/10]*ArcTan[(2 - Sqrt[2*(1 + Sqrt[5])
] + 2/x)/Sqrt[2*(-1 + Sqrt[5])]])/20 - (Sqrt[(5959 + 2665*Sqrt[5]
)/10]*ArcTan[(2 + Sqrt[2*(1 + Sqrt[5])] + 2/x)/Sqrt[2*(-1 + Sqrt[
5])]])/20 + (Sqrt[(-5959 + 2665*Sqrt[5])/10]*Log[Sqrt[5] - Sqrt[2
*(1 + Sqrt[5])]*(1 + x^(-1)) + (1 + x^(-1))^2])/40 - (Sqrt[(-5959
 + 2665*Sqrt[5])/10]*Log[Sqrt[5] + Sqrt[2*(1 + Sqrt[5])]*(1 + x^(
-1)) + (1 + x^(-1))^2])/40
```

Rubi [A] time = 0.870629, antiderivative size = 317, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 11, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.647

$$-\frac{17-\left(\frac{1}{x}+1\right)^2}{2\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)}+\frac{\left(59-17\left(\frac{1}{x}+1\right)^2\right)\left(\frac{1}{x}+1\right)}{10\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)}$$
$$+\frac{1}{40}\sqrt{\frac{1}{10}\left(2665\sqrt{5}-5959\right)}\log\left(\left(\frac{1}{x}+1\right)^2-\sqrt{2\left(1+\sqrt{5}\right)}\left(\frac{1}{x}+1\right)+\sqrt{5}\right)$$
$$-\frac{1}{40}\sqrt{\frac{1}{10}\left(2665\sqrt{5}-5959\right)}\log\left(\left(\frac{1}{x}+1\right)^2+\sqrt{2\left(1+\sqrt{5}\right)}\left(\frac{1}{x}+1\right)+\sqrt{5}\right)$$
$$+\frac{7}{4}\tan^{-1}\left(\frac{1}{2}\left(\left(\frac{1}{x}+1\right)^2-1\right)\right)-\frac{1}{20}\sqrt{\frac{1}{10}\left(5959+2665\sqrt{5}\right)}\tan^{-1}\left(\frac{\frac{2}{x}-\sqrt{2\left(1+\sqrt{5}\right)}+2}{\sqrt{2\left(\sqrt{5}-1\right)}}\right)$$
$$-\frac{1}{20}\sqrt{\frac{1}{10}\left(5959+2665\sqrt{5}\right)}\tan^{-1}\left(\frac{\frac{2}{x}+\sqrt{2\left(1+\sqrt{5}\right)}+2}{\sqrt{2\left(\sqrt{5}-1\right)}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + 4*x + 4*x^2 + 4*x^4)^(-2),x]
```


```
[Out] -(17 - (1 + x^(-1))^2)/(2*(5 - 2*(1 + x^(-1))^2 + (1 + x^(-1))^4)
) + ((59 - 17*(1 + x^(-1))^2)*(1 + x^(-1)))/(10*(5 - 2*(1 + x^(-1
```

```
))^2 + (1 + x^(-1))^4)) + (7*ArcTan[(-1 + (1 + x^(-1))^2)/2])/4 -
 (Sqrt[(5959 + 2665*Sqrt[5])/10]*ArcTan[(2 - Sqrt[2*(1 + Sqrt[5])
] + 2/x)/Sqrt[2*(-1 + Sqrt[5])]])/20 - (Sqrt[(5959 + 2665*Sqrt[5]
)/10]*ArcTan[(2 + Sqrt[2*(1 + Sqrt[5])] + 2/x)/Sqrt[2*(-1 + Sqrt[
5])]])/20 + (Sqrt[(-5959 + 2665*Sqrt[5])/10]*Log[Sqrt[5] - Sqrt[2
*(1 + Sqrt[5])]*(1 + x^(-1)) + (1 + x^(-1))^2])/40 - (Sqrt[(-5959
 + 2665*Sqrt[5])/10]*Log[Sqrt[5] + Sqrt[2*(1 + Sqrt[5])]*(1 + x^(
-1)) + (1 + x^(-1))^2])/40
```

Rubi in Sympy [A] time = 77.5827, size = 393, normalized size = 1.24

$$\frac{\left(1+\frac{1}{x}\right)\left(16441391146758956776003993 6\left(1+\frac{1}{x}\right)^{3}-164413911467589567760039936\left(1+\frac{1}{x}\right)^{2}+29014219670751100192 94\cdots\right.}{377789318629571617095680\left(256\left(1+\frac{1}{x}\right)^{4}-512\left(1+\frac{1}{x}\right)^{2}+1280\right)}$$

$$+\frac{\sqrt{10}\left(-130563988518379950868267008\sqrt{5}+294977899985969518628306944\right)\log\left(\sqrt{2}\left(-1-\frac{1}{x}\right)\sqrt{1+\sqrt{5}}+\left(1+\frac{1}{x}\right)^{2}+\sqrt{\cdots}\right.}{1934281311383406679529881600\sqrt{1+\sqrt{5}}}$$

$$-\frac{\sqrt{10}\left(-130563988518379950868267008\sqrt{5}+294977899985969518628306944\right)\log\left(\left(1+\frac{1}{x}\right)^{2}+\sqrt{2}\sqrt{1+\sqrt{5}}\left(1+\frac{1}{x}\right)+\sqrt{5}\right)\cdots}{1934281311383406679529881600\sqrt{1+\sqrt{5}}}$$

$$-\frac{\sqrt{5}\left(-\frac{\sqrt{2}\sqrt{1+\sqrt{5}}\left(-261127977036759901736534016\sqrt{5}+589955799971939037256613888\right)}{2}+589955799971939037256613888\sqrt{2}\sqrt{1+\sqrt{5}}\right)\operatorname{atan}\left(\cdots\right.}{967140655691703339764940800\sqrt{-1+\sqrt{5}}\sqrt{1+\sqrt{5}}}$$

$$-\frac{\sqrt{5}\left(-\frac{\sqrt{2}\sqrt{1+\sqrt{5}}\left(-261127977036759901736534016\sqrt{5}+589955799971939037256613888\right)}{2}+589955799971939037256613888\sqrt{2}\sqrt{1+\sqrt{5}}\right)\operatorname{atan}\left(\cdots\right.}{967140655691703339764940800\sqrt{-1+\sqrt{5}}\sqrt{1+\sqrt{5}}}$$

$$+\frac{7\operatorname{atan}\left(\frac{\left(1+\frac{1}{x}\right)^{2}}{2}-\frac{1}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(4*x**4+4*x**2+4*x+1)**2,x)
```

```
[Out] (1 + 1/x)*(164413911467589567760039936*(1 + 1/x)**3 - 16441391146
7589567760039936*(1 + 1/x)**2 + 290142196707511001929482240 - 280
470790150593968531832832/x)/(377789318629571617095680*(256*(1 + 1
/x)**4 - 512*(1 + 1/x)**2 + 1280)) + sqrt(10)*(-13056398851837995
0868267008*sqrt(5) + 294977899985969518628306944)*log(sqrt(2)*(-1
 - 1/x)*sqrt(1 + sqrt(5)) + (1 + 1/x)**2 + sqrt(5))/(193428131138
3406679529881600*sqrt(1 + sqrt(5))) - sqrt(10)*(-1305639885183799
50868267008*sqrt(5) + 294977899985969518628306944)*log((1 + 1/x)*
*2 + sqrt(2)*sqrt(1 + sqrt(5))*(1 + 1/x) + sqrt(5))/(193428131138
3406679529881600*sqrt(1 + sqrt(5))) - sqrt(5)*(-sqrt(2)*sqrt(1 +
sqrt(5))*(-261127977036759901736534016*sqrt(5) + 5899557999719390
37256613888)/2 + 589955799971939037256613888*sqrt(2)*sqrt(1 + sqr
t(5)))*atan(sqrt(2)*(1 + sqrt(2 + 2*sqrt(5))/2 + 1/x)/sqrt(-1 + s
qrt(5)))/(967140655691703339764940800*sqrt(-1 + sqrt(5))*sqrt(1 +
 sqrt(5))) - sqrt(5)*(-sqrt(2)*sqrt(1 + sqrt(5))*(-26112797703675
9901736534016*sqrt(5) + 589955799971939037256613888)/2 + 58995579
9971939037256613888*sqrt(2)*sqrt(1 + sqrt(5)))*atan(sqrt(2)*(-sqr
t(2 + 2*sqrt(5))/2 + 1 + 1/x)/sqrt(-1 + sqrt(5)))/(96714065569170
3339764940800*sqrt(-1 + sqrt(5))*sqrt(1 + sqrt(5))) + 7*atan((1 +
 1/x)**2/2 - 1/2)/4
```

Mathematica [C] time = 0.0399579, size = 108, normalized size = 0.34

$$\frac{1}{40}\left(\text{RootSum}\left[4\#1^4+4\#1^2+4\#1+1\&,\frac{18\#1^2\log(x-\#1)-16\#1\log(x-\#1)+27\log(x-\#1)}{4\#1^3+2\#1+1}\&\right]+\frac{72x^3-32x^2+84x+38}{4x^4+4x^2+4x+1}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + 4*x + 4*x^2 + 4*x^4)^(-2),x]
```

```
[Out] ((38 + 84*x - 32*x^2 + 72*x^3)/(1 + 4*x + 4*x^2 + 4*x^4) + RootSu
m[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , (27*Log[x - #1] - 16*Log[x - #1]
*#1 + 18*Log[x - #1]*#1^2)/(1 + 2*#1 + 4*#1^3) & ])/40
```

Maple [C] time = 0.013, size = 79, normalized size = 0.3

$$1\left(\frac{9x^3}{20}-\frac{x^2}{5}+\frac{21x}{40}+\frac{19}{80}\right)\left(x^4+x^2+x+\frac{1}{4}\right)^{-1}+\frac{1}{40}\sum_{\_R=RootOf(4\_Z^4+4\_Z^2+4\_Z+1)}\frac{(18\_R^2-16\_R+27)\ln(x-\_R)}{4\_R^3+2\_R+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(4*x^4+4*x^2+4*x+1)^2,x)
```

```
[Out] (9/20*x^3-1/5*x^2+21/40*x+19/80)/(x^4+x^2+x+1/4)+1/40*sum((18*_R^
2-16*_R+27)/(4*_R^3+2*_R+1)*ln(x-_R),_R=RootOf(4*_Z^4+4*_Z^2+4*_Z
+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{36x^3-16x^2+42x+19}{20(4x^4+4x^2+4x+1)}+\frac{1}{10}\int\frac{18x^2-16x+27}{4x^4+4x^2+4x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^4 + 4*x^2 + 4*x + 1)^(-2),x, algorithm="maxima")
```

```
[Out] 1/20*(36*x^3 - 16*x^2 + 42*x + 19)/(4*x^4 + 4*x^2 + 4*x + 1) + 1/
10*integrate((18*x^2 - 16*x + 27)/(4*x^4 + 4*x^2 + 4*x + 1), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^4 + 4*x^2 + 4*x + 1)^(-2),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 3.39001, size = 71, normalized size = 0.22

$$\frac{36x^3-16x^2+42x+19}{80x^4+80x^2+80x+20} + \mathrm{RootSum}\left(64000t^4+193344t^2-1064t+29, \left(t \mapsto t\log\left(-\frac{17084544000t^3}{541735337}-\frac{188086000t^2}{541735337}-\frac{51568487224t}{541735337}+x-\frac{71080995}{541735337}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(4*x**4+4*x**2+4*x+1)**2,x)
```

```
[Out] (36*x**3 - 16*x**2 + 42*x + 19)/(80*x**4 + 80*x**2 + 80*x + 20) +
 RootSum(64000*_t**4 + 193344*_t**2 - 1064*_t + 29, Lambda(_t, _t
*log(-17084544000*_t**3/541735337 - 188086000*_t**2/541735337 - 5
1568487224*_t/541735337 + x - 71080995/541735337)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(4x^4+4x^2+4x+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^4 + 4*x^2 + 4*x + 1)^(-2),x, algorithm="giac")
```

```
[Out] integrate((4*x^4 + 4*x^2 + 4*x + 1)^(-2), x)
```

3.57 $\int \left(8+24x+8x^2-15x^3+8x^4\right)^4 \, dx$

Optimal. Leaf size=104

$$\frac{4096x^{17}}{17}-1920x^{16}+\frac{102784x^{15}}{15}-\frac{75504x^{14}}{7}-\frac{12095x^{13}}{13}+31128x^{12}-\frac{331040x^{11}}{11}-\frac{169584x^{10}}{5}+\frac{641152x^9}{9}+36384x^8-\frac{566912x^7}{7}-30720x^6+\frac{538624x^5}{5}+139776x^4+\frac{237568x^3}{3}+24576x^2+4096x$$

```
[Out] 4096*x + 24576*x^2 + (237568*x^3)/3 + 139776*x^4 + (538624*x^5)/5
 - 30720*x^6 - (566912*x^7)/7 + 36384*x^8 + (641152*x^9)/9 - (169
584*x^10)/5 - (331040*x^11)/11 + 31128*x^12 - (12095*x^13)/13 - (
75504*x^14)/7 + (102784*x^15)/15 - 1920*x^16 + (4096*x^17)/17
```

Rubi [A] time = 0.0668608, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045

$$\frac{4096x^{17}}{17}-1920x^{16}+\frac{102784x^{15}}{15}-\frac{75504x^{14}}{7}-\frac{12095x^{13}}{13}+31128x^{12}-\frac{331040x^{11}}{11}-\frac{169584x^{10}}{5}+\frac{641152x^9}{9}+36384x^8-\frac{566912x^7}{7}-30720x^6+\frac{538624x^5}{5}+139776x^4+\frac{237568x^3}{3}+24576x^2+4096x$$

Antiderivative was successfully verified.

```
[In]  Int[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^4,x]
```

```
[Out] 4096*x + 24576*x^2 + (237568*x^3)/3 + 139776*x^4 + (538624*x^5)/5
 - 30720*x^6 - (566912*x^7)/7 + 36384*x^8 + (641152*x^9)/9 - (169
584*x^10)/5 - (331040*x^11)/11 + 31128*x^12 - (12095*x^13)/13 - (
75504*x^14)/7 + (102784*x^15)/15 - 1920*x^16 + (4096*x^17)/17
```

Rubi in Sympy [A] time = 74.7417, size = 100, normalized size = 0.96

$$\frac{4096x^{17}}{17}-1920x^{16}+\frac{102784x^{15}}{15}-\frac{75504x^{14}}{7}-\frac{12095x^{13}}{13}+31128x^{12}-\frac{331040x^{11}}{11}-\frac{169584x^{10}}{5}+\frac{641152x^9}{9}+36384x^8-\frac{566912x^7}{7}-30720x^6+\frac{538624x^5}{5}+139776x^4+\frac{237568x^3}{3}+24576x^2+4096x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((8*x**4-15*x**3+8*x**2+24*x+8)**4,x)
```

```
[Out] 4096*x**17/17 - 1920*x**16 + 102784*x**15/15 - 75504*x**14/7 - 12
095*x**13/13 + 31128*x**12 - 331040*x**11/11 - 169584*x**10/5 + 6
41152*x**9/9 + 36384*x**8 - 566912*x**7/7 - 30720*x**6 + 538624*x
**5/5 + 139776*x**4 + 237568*x**3/3 + 24576*x**2 + 4096*x
```

Mathematica [A] time = 0.00282705, size = 104, normalized size = 1.

$$\frac{4096x^{17}}{17}-1920x^{16}+\frac{102784x^{15}}{15}-\frac{75504x^{14}}{7}-\frac{12095x^{13}}{13}+31128x^{12}-\frac{331040x^{11}}{11}-\frac{169584x^{10}}{5}+\frac{641152x^9}{9}+36384x^8-\frac{566912x^7}{7}-30720x^6+\frac{538624x^5}{5}+139776x^4+\frac{237568x^3}{3}+24576x^2+4096x$$

Antiderivative was successfully verified.

```
[In]  Integrate[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^4,x]
```

```
[Out] 4096*x + 24576*x^2 + (237568*x^3)/3 + 139776*x^4 + (538624*x^5)/5
 - 30720*x^6 - (566912*x^7)/7 + 36384*x^8 + (641152*x^9)/9 - (169
584*x^10)/5 - (331040*x^11)/11 + 31128*x^12 - (12095*x^13)/13 - (
75504*x^14)/7 + (102784*x^15)/15 - 1920*x^16 + (4096*x^17)/17
```

Maple [A] time = 0.002, size = 85, normalized size = 0.8

$$4096\,x+24576\,x^2+\frac{237568\,x^3}{3}+139776\,x^4+\frac{538624\,x^5}{5}-30720\,x^6-\frac{566912\,x^7}{7}+36384\,x^8+\frac{641152\,x^9}{9}-\frac{169584\,x^{10}}{5}-\frac{331040\,x^{11}}{11}+31128\,x^{12}-\frac{12095\,x^{13}}{13}-\frac{75504\,x^{14}}{7}+\frac{102784\,x^{15}}{15}-1920\,x^{16}+\frac{4096\,x^{17}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((8*x^4-15*x^3+8*x^2+24*x+8)^4,x)
```

```
[Out] 4096*x+24576*x^2+237568/3*x^3+139776*x^4+538624/5*x^5-30720*x^6-5
66912/7*x^7+36384*x^8+641152/9*x^9-169584/5*x^10-331040/11*x^11+3
1128*x^12-12095/13*x^13-75504/7*x^14+102784/15*x^15-1920*x^16+409
6/17*x^17
```

Maxima [A] time = 0.807568, size = 113, normalized size = 1.09

$$\frac{4096}{17}\,x^{17}-1920\,x^{16}+\frac{102784}{15}\,x^{15}-\frac{75504}{7}\,x^{14}-\frac{12095}{13}\,x^{13}+31128\,x^{12}-\frac{331040}{11}\,x^{11}-\frac{169584}{5}\,x^{10}+\frac{641152}{9}\,x^{9}+36384\,x^{8}-\frac{566912}{7}\,x^{7}-30720\,x^{6}+\frac{538624}{5}\,x^{5}+139776\,x^{4}+\frac{237568}{3}\,x^{3}+24576\,x^{2}+4096\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)^4,x, algorithm="maxima")
```

```
[Out] 4096/17*x^17 - 1920*x^16 + 102784/15*x^15 - 75504/7*x^14 - 12095/
13*x^13 + 31128*x^12 - 331040/11*x^11 - 169584/5*x^10 + 641152/9*
x^9 + 36384*x^8 - 566912/7*x^7 - 30720*x^6 + 538624/5*x^5 + 13977
6*x^4 + 237568/3*x^3 + 24576*x^2 + 4096*x
```

Fricas [A] time = 0.2239, size = 1, normalized size = 0.01

$$\frac{4096}{17}x^{17}-1920x^{16}+\frac{102784}{15}x^{15}-\frac{75504}{7}x^{14}-\frac{12095}{13}x^{13}+31128x^{12}-\frac{331040}{11}x^{11}-\frac{169584}{5}x^{10}+\frac{641152}{9}x^{9}+36384x^{8}-\frac{566912}{7}x^{7}-30720x^{6}+\frac{538624}{5}x^{5}+139776x^{4}+\frac{237568}{3}x^{3}+24576x^{2}+4096x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)^4,x, algorithm="fricas")
```

```
[Out] 4096/17*x^17 - 1920*x^16 + 102784/15*x^15 - 75504/7*x^14 - 12095/
13*x^13 + 31128*x^12 - 331040/11*x^11 - 169584/5*x^10 + 641152/9*
x^9 + 36384*x^8 - 566912/7*x^7 - 30720*x^6 + 538624/5*x^5 + 13977
6*x^4 + 237568/3*x^3 + 24576*x^2 + 4096*x
```

Sympy [A] time = 0.134125, size = 100, normalized size = 0.96

$$\frac{4096x^{17}}{17} - 1920x^{16} + \frac{102784x^{15}}{15} - \frac{75504x^{14}}{7} - \frac{12095x^{13}}{13} + 31128x^{12} - \frac{331040x^{11}}{11} - \frac{169584x^{10}}{5} + \frac{641152x^{9}}{9} + 36384x^{8} - \frac{566912x^{7}}{7} - 30720x^{6} + \frac{538624x^{5}}{5} + 139776x^{4} + \frac{237568x^{3}}{3} + 24576x^{2} + 4096x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((8*x**4-15*x**3+8*x**2+24*x+8)**4,x)
```

```
[Out] 4096*x**17/17 - 1920*x**16 + 102784*x**15/15 - 75504*x**14/7 - 12
095*x**13/13 + 31128*x**12 - 331040*x**11/11 - 169584*x**10/5 + 6
41152*x**9/9 + 36384*x**8 - 566912*x**7/7 - 30720*x**6 + 538624*x
**5/5 + 139776*x**4 + 237568*x**3/3 + 24576*x**2 + 4096*x
```

GIAC/XCAS [A] time = 0.262244, size = 113, normalized size = 1.09

$$\frac{4096}{17}\,x^{17} - 1920\,x^{16} + \frac{102784}{15}\,x^{15} - \frac{75504}{7}\,x^{14} - \frac{12095}{13}\,x^{13} + 31128\,x^{12} - \frac{331040}{11}\,x^{11} - \frac{169584}{5}\,x^{10} + \frac{641152}{9}\,x^{9} + 36384\,x^{8} - \frac{566912}{7}\,x^{7} - 30720\,x^{6} + \frac{538624}{5}\,x^{5} + 139776\,x^{4} + \frac{237568}{3}\,x^{3} + 24576\,x^{2} + 4096\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)^4,x, algorithm="giac")
```

```
[Out] 4096/17*x^17 - 1920*x^16 + 102784/15*x^15 - 75504/7*x^14 - 12095/
13*x^13 + 31128*x^12 - 331040/11*x^11 - 169584/5*x^10 + 641152/9*
x^9 + 36384*x^8 - 566912/7*x^7 - 30720*x^6 + 538624/5*x^5 + 13977
6*x^4 + 237568/3*x^3 + 24576*x^2 + 4096*x
```

3.58 $\int \left(8+24x+8x^2-15x^3+8x^4\right)^3 dx$

Optimal. Leaf size=76

$$\frac{512x^{13}}{13}-240x^{12}+\frac{6936x^{11}}{11}-\frac{4527x^{10}}{10}-\frac{2936x^9}{3}+2097x^8+\frac{5528x^7}{7}-2976x^6-\frac{384x^5}{5}+5040x^4+5120x^3+2304x^2+512x$$

```
[Out] 512*x + 2304*x^2 + 5120*x^3 + 5040*x^4 - (384*x^5)/5 - 2976*x^6 +
 (5528*x^7)/7 + 2097*x^8 - (2936*x^9)/3 - (4527*x^10)/10 + (6936*
x^11)/11 - 240*x^12 + (512*x^13)/13
```

Rubi [A] time = 0.0500937, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$

$$\frac{512x^{13}}{13}-240x^{12}+\frac{6936x^{11}}{11}-\frac{4527x^{10}}{10}-\frac{2936x^9}{3}+2097x^8+\frac{5528x^7}{7}-2976x^6-\frac{384x^5}{5}+5040x^4+5120x^3+2304x^2+512x$$

Antiderivative was successfully verified.

```
[In]  Int[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^3,x]
```

```
[Out] 512*x + 2304*x^2 + 5120*x^3 + 5040*x^4 - (384*x^5)/5 - 2976*x^6 +
 (5528*x^7)/7 + 2097*x^8 - (2936*x^9)/3 - (4527*x^10)/10 + (6936*
x^11)/11 - 240*x^12 + (512*x^13)/13
```

Rubi in Sympy [A] time = 61.2572, size = 73, normalized size = 0.96

$$\frac{512x^{13}}{13}-240x^{12}+\frac{6936x^{11}}{11}-\frac{4527x^{10}}{10}-\frac{2936x^9}{3}+2097x^8+\frac{5528x^7}{7}-2976x^6-\frac{384x^5}{5}+5040x^4+5120x^3+2304x^2+512x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((8*x**4-15*x**3+8*x**2+24*x+8)**3,x)
```

```
[Out] 512*x**13/13 - 240*x**12 + 6936*x**11/11 - 4527*x**10/10 - 2936*x
**9/3 + 2097*x**8 + 5528*x**7/7 - 2976*x**6 - 384*x**5/5 + 5040*x
**4 + 5120*x**3 + 2304*x**2 + 512*x
```

Mathematica [A] time = 0.00154104, size = 76, normalized size = 1.

$$\frac{512x^{13}}{13}-240x^{12}+\frac{6936x^{11}}{11}-\frac{4527x^{10}}{10}-\frac{2936x^9}{3}+2097x^8+\frac{5528x^7}{7}-2976x^6-\frac{384x^5}{5}+5040x^4+5120x^3+2304x^2+512x$$

Antiderivative was successfully verified.

```
[In]  Integrate[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^3,x]
```

```
[Out] 512*x + 2304*x^2 + 5120*x^3 + 5040*x^4 - (384*x^5)/5 - 2976*x^6 +
 (5528*x^7)/7 + 2097*x^8 - (2936*x^9)/3 - (4527*x^10)/10 + (6936*
x^11)/11 - 240*x^12 + (512*x^13)/13
```

Maple [A] time = 0.002, size = 65, normalized size = 0.9

$$512\,x+2304\,x^{2}+5120\,x^{3}+5040\,x^{4}-\frac{384\,x^{5}}{5}-2976\,x^{6}+\frac{5528\,x^{7}}{7}+2097\,x^{8}-\frac{2936\,x^{9}}{3}-\frac{4527\,x^{10}}{10}+\frac{6936\,x^{11}}{11}-240\,x^{12}+\frac{512\,x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((8*x^4-15*x^3+8*x^2+24*x+8)^3,x)
```

```
[Out] 512*x+2304*x^2+5120*x^3+5040*x^4-384/5*x^5-2976*x^6+5528/7*x^7+20
97*x^8-2936/3*x^9-4527/10*x^10+6936/11*x^11-240*x^12+512/13*x^13
```

Maxima [A] time = 0.806273, size = 86, normalized size = 1.13

$$\frac{512}{13}\,x^{13}-240\,x^{12}+\frac{6936}{11}\,x^{11}-\frac{4527}{10}\,x^{10}-\frac{2936}{3}\,x^{9}+2097\,x^{8}+\frac{5528}{7}\,x^{7}-2976\,x^{6}-\frac{384}{5}\,x^{5}+5040\,x^{4}+5120\,x^{3}+2304\,x^{2}+512\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)^3,x, algorithm="maxima")
```

```
[Out] 512/13*x^13 - 240*x^12 + 6936/11*x^11 - 4527/10*x^10 - 2936/3*x^9
 + 2097*x^8 + 5528/7*x^7 - 2976*x^6 - 384/5*x^5 + 5040*x^4 + 5120
*x^3 + 2304*x^2 + 512*x
```

Fricas [A] time = 0.236993, size = 1, normalized size = 0.01

$$\frac{512}{13}x^{13}-240x^{12}+\frac{6936}{11}x^{11}-\frac{4527}{10}x^{10}-\frac{2936}{3}x^{9}+2097x^{8}+\frac{5528}{7}x^{7}-2976x^{6}-\frac{384}{5}x^{5}+5040x^{4}+5120x^{3}+2304x^{2}+512x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)^3,x, algorithm="fricas")
```

```
[Out] 512/13*x^13 - 240*x^12 + 6936/11*x^11 - 4527/10*x^10 - 2936/3*x^9
 + 2097*x^8 + 5528/7*x^7 - 2976*x^6 - 384/5*x^5 + 5040*x^4 + 5120
*x^3 + 2304*x^2 + 512*x
```

Sympy [A] time = 0.110758, size = 73, normalized size = 0.96

$$\frac{512x^{13}}{13}-240x^{12}+\frac{6936x^{11}}{11}-\frac{4527x^{10}}{10}-\frac{2936x^{9}}{3}+2097x^{8}+\frac{5528x^{7}}{7}-2976x^{6}-\frac{384x^{5}}{5}+5040x^{4}+5120x^{3}+2304x^{2}+512x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((8*x**4-15*x**3+8*x**2+24*x+8)**3,x)
```

```
[Out] 512*x**13/13 - 240*x**12 + 6936*x**11/11 - 4527*x**10/10 - 2936*x
**9/3 + 2097*x**8 + 5528*x**7/7 - 2976*x**6 - 384*x**5/5 + 5040*x
**4 + 5120*x**3 + 2304*x**2 + 512*x
```

GIAC/XCAS [A] time = 0.262088, size = 86, normalized size = 1.13

$$\frac{512}{13}x^{13} - 240x^{12} + \frac{6936}{11}x^{11} - \frac{4527}{10}x^{10} - \frac{2936}{3}x^9 + 2097x^8 + \frac{5528}{7}x^7 - 2976x^6 - \frac{384}{5}x^5 + 5040x^4 + 5120x^3 + 2304x^2 + 512x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)^3,x, algorithm="giac")
```

```
[Out] 512/13*x^13 - 240*x^12 + 6936/11*x^11 - 4527/10*x^10 - 2936/3*x^9
 + 2097*x^8 + 5528/7*x^7 - 2976*x^6 - 384/5*x^5 + 5040*x^4 + 5120
*x^3 + 2304*x^2 + 512*x
```

3.59 $\int \left(8+24x+8x^2-15x^3+8x^4\right)^2 \, dx$

Optimal. Leaf size=52

$$\frac{64x^9}{9}-30x^8+\frac{353x^7}{7}+24x^6-\frac{528x^5}{5}+36x^4+\frac{704x^3}{3}+192x^2+64x$$

```
[Out] 64*x + 192*x^2 + (704*x^3)/3 + 36*x^4 - (528*x^5)/5 + 24*x^6 + (3
53*x^7)/7 - 30*x^8 + (64*x^9)/9
```

Rubi [A] time = 0.0353111, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045

$$\frac{64x^9}{9}-30x^8+\frac{353x^7}{7}+24x^6-\frac{528x^5}{5}+36x^4+\frac{704x^3}{3}+192x^2+64x$$

Antiderivative was successfully verified.

```
[In] Int[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^2,x]
```

```
[Out] 64*x + 192*x^2 + (704*x^3)/3 + 36*x^4 - (528*x^5)/5 + 24*x^6 + (3
53*x^7)/7 - 30*x^8 + (64*x^9)/9
```

Rubi in Sympy [A] time = 50.0146, size = 49, normalized size = 0.94

$$\frac{64x^9}{9}-30x^8+\frac{353x^7}{7}+24x^6-\frac{528x^5}{5}+36x^4+\frac{704x^3}{3}+192x^2+64x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((8*x**4-15*x**3+8*x**2+24*x+8)**2,x)
```

```
[Out] 64*x**9/9 - 30*x**8 + 353*x**7/7 + 24*x**6 - 528*x**5/5 + 36*x**4
 + 704*x**3/3 + 192*x**2 + 64*x
```

Mathematica [A] time = 0.00175255, size = 52, normalized size = 1.

$$\frac{64x^9}{9}-30x^8+\frac{353x^7}{7}+24x^6-\frac{528x^5}{5}+36x^4+\frac{704x^3}{3}+192x^2+64x$$

Antiderivative was successfully verified.

```
[In] Integrate[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^2,x]
```

```
[Out] 64*x + 192*x^2 + (704*x^3)/3 + 36*x^4 - (528*x^5)/5 + 24*x^6 + (3
53*x^7)/7 - 30*x^8 + (64*x^9)/9
```

Maple [A] time = 0.002, size = 45, normalized size = 0.9

$$64\,x+192\,x^2+\frac{704\,x^3}{3}+36\,x^4-\frac{528\,x^5}{5}+24\,x^6+\frac{353\,x^7}{7}-30\,x^8+\frac{64\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((8*x^4-15*x^3+8*x^2+24*x+8)^2,x)
```

```
[Out] 64*x+192*x^2+704/3*x^3+36*x^4-528/5*x^5+24*x^6+353/7*x^7-30*x^8+6
4/9*x^9
```

Maxima [A] time = 0.800497, size = 59, normalized size = 1.13

$$\frac{64}{9}x^9 - 30x^8 + \frac{353}{7}x^7 + 24x^6 - \frac{528}{5}x^5 + 36x^4 + \frac{704}{3}x^3 + 192x^2 + 64x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)^2,x, algorithm="maxima")
```

```
[Out] 64/9*x^9 - 30*x^8 + 353/7*x^7 + 24*x^6 - 528/5*x^5 + 36*x^4 + 704
/3*x^3 + 192*x^2 + 64*x
```

Fricas [A] time = 0.224085, size = 1, normalized size = 0.02

$$\frac{64}{9}x^9 - 30x^8 + \frac{353}{7}x^7 + 24x^6 - \frac{528}{5}x^5 + 36x^4 + \frac{704}{3}x^3 + 192x^2 + 64x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)^2,x, algorithm="fricas")
```

```
[Out] 64/9*x^9 - 30*x^8 + 353/7*x^7 + 24*x^6 - 528/5*x^5 + 36*x^4 + 704
/3*x^3 + 192*x^2 + 64*x
```

Sympy [A] time = 0.085883, size = 49, normalized size = 0.94

$$\frac{64x^9}{9} - 30x^8 + \frac{353x^7}{7} + 24x^6 - \frac{528x^5}{5} + 36x^4 + \frac{704x^3}{3} + 192x^2 + 64x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((8*x**4-15*x**3+8*x**2+24*x+8)**2,x)
```

```
[Out] 64*x**9/9 - 30*x**8 + 353*x**7/7 + 24*x**6 - 528*x**5/5 + 36*x**4
 + 704*x**3/3 + 192*x**2 + 64*x
```

GIAC/XCAS [A] time = 0.260312, size = 59, normalized size = 1.13

$$\frac{64}{9}x^9 - 30x^8 + \frac{353}{7}x^7 + 24x^6 - \frac{528}{5}x^5 + 36x^4 + \frac{704}{3}x^3 + 192x^2 + 64x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)^2,x, algorithm="giac")
```

```
[Out] 64/9*x^9 - 30*x^8 + 353/7*x^7 + 24*x^6 - 528/5*x^5 + 36*x^4 + 704
/3*x^3 + 192*x^2 + 64*x
```

3.60 $\int (8 + 24x + 8x^2 - 15x^3 + 8x^4) \, dx$

Optimal. Leaf size=30

$$\frac{8x^5}{5} - \frac{15x^4}{4} + \frac{8x^3}{3} + 12x^2 + 8x$$

```
[Out] 8*x + 12*x^2 + (8*x^3)/3 - (15*x^4)/4 + (8*x^5)/5
```

Rubi [A] time = 0.0121648, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$

$$\frac{8x^5}{5} - \frac{15x^4}{4} + \frac{8x^3}{3} + 12x^2 + 8x$$

Antiderivative was successfully verified.

```
[In]  Int[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4,x]

[Out] 8*x + 12*x^2 + (8*x^3)/3 - (15*x^4)/4 + (8*x^5)/5
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{8x^5}{5} - \frac{15x^4}{4} + \frac{8x^3}{3} + 8x + 24\int x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(8*x**4-15*x**3+8*x**2+24*x+8,x)

[Out] 8*x**5/5 - 15*x**4/4 + 8*x**3/3 + 8*x + 24*Integral(x, x)
```

Mathematica [A] time = 0.0000659165, size = 30, normalized size = 1.

$$\frac{8x^5}{5} - \frac{15x^4}{4} + \frac{8x^3}{3} + 12x^2 + 8x$$

Antiderivative was successfully verified.

```
[In]  Integrate[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4,x]

[Out] 8*x + 12*x^2 + (8*x^3)/3 - (15*x^4)/4 + (8*x^5)/5
```

Maple [A] time = 0., size = 25, normalized size = 0.8

$$8\,x + 12\,x^2 + \frac{8\,x^3}{3} - \frac{15\,x^4}{4} + \frac{8\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(8*x^4-15*x^3+8*x^2+24*x+8,x)
```

```
[Out] 8*x+12*x^2+8/3*x^3-15/4*x^4+8/5*x^5
```

Maxima [A] time = 0.800406, size = 32, normalized size = 1.07

$$\frac{8}{5}x^5 - \frac{15}{4}x^4 + \frac{8}{3}x^3 + 12x^2 + 8x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8,x, algorithm="maxima")
```

```
[Out] 8/5*x^5 - 15/4*x^4 + 8/3*x^3 + 12*x^2 + 8*x
```

Fricas [A] time = 0.232534, size = 1, normalized size = 0.03

$$\frac{8}{5}x^5 - \frac{15}{4}x^4 + \frac{8}{3}x^3 + 12x^2 + 8x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8,x, algorithm="fricas")
```

```
[Out] 8/5*x^5 - 15/4*x^4 + 8/3*x^3 + 12*x^2 + 8*x
```

Sympy [A] time = 0.062927, size = 27, normalized size = 0.9

$$\frac{8x^5}{5} - \frac{15x^4}{4} + \frac{8x^3}{3} + 12x^2 + 8x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(8*x**4-15*x**3+8*x**2+24*x+8,x)
```

```
[Out] 8*x**5/5 - 15*x**4/4 + 8*x**3/3 + 12*x**2 + 8*x
```

GIAC/XCAS [A] time = 0.261141, size = 32, normalized size = 1.07

$$\frac{8}{5}x^5 - \frac{15}{4}x^4 + \frac{8}{3}x^3 + 12x^2 + 8x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8,x, algorithm="giac")
```

```
[Out] 8/5*x^5 - 15/4*x^4 + 8/3*x^3 + 12*x^2 + 8*x
```

3.61 $\int \frac{1}{8+24x+8x^2-15x^3+8x^4}\,dx$

Optimal. Leaf size=263

$$\frac{1}{4}\sqrt{\frac{3}{13}}\tan^{-1}\left(\frac{-5x^2+12x+8}{\sqrt{39}x^2}\right)-\frac{1}{8}\sqrt{\frac{235\sqrt{517}-5167}{40326}}\log\left(\left(\frac{4}{x}+3\right)^2-\sqrt{2\left(19+\sqrt{517}\right)}\left(\frac{4}{x}+3\right)+\sqrt{517}\right)+\frac{1}{8}\sqrt{\frac{235\sqrt{517}-5167}{40326}}\log\left(\left(\frac{4}{x}+3\right)^2+\sqrt{2\left(19+\sqrt{517}\right)}\left(\frac{4}{x}+3\right)+\sqrt{517}\right)-\frac{1}{4}\sqrt{\frac{5167+235\sqrt{517}}{40326}}\tan^{-1}\left(\frac{\frac{8}{x}-\sqrt{2\left(19+\sqrt{517}\right)}+6}{\sqrt{2\left(\sqrt{517}-19\right)}}\right)-\frac{1}{4}\sqrt{\frac{5167+235\sqrt{517}}{40326}}\tan^{-1}\left(\frac{\frac{8}{x}+\sqrt{2\left(19+\sqrt{517}\right)}+6}{\sqrt{2\left(\sqrt{517}-19\right)}}\right)$$

```
[Out] -(Sqrt[(5167 + 235*Sqrt[517])/40326]*ArcTan[(6 - Sqrt[2*(19 + Sqr
t[517])] + 8/x)/Sqrt[2*(-19 + Sqrt[517])]])/4 - (Sqrt[(5167 + 235
*Sqrt[517])/40326]*ArcTan[(6 + Sqrt[2*(19 + Sqrt[517])] + 8/x)/Sq
rt[2*(-19 + Sqrt[517])]])/4 + (Sqrt[3/13]*ArcTan[(8 + 12*x - 5*x^
2)/(Sqrt[39]*x^2)])/4 - (Sqrt[(-5167 + 235*Sqrt[517])/40326]*Log[
Sqrt[517] - Sqrt[2*(19 + Sqrt[517])]*(3 + 4/x) + (3 + 4/x)^2])/8
+ (Sqrt[(-5167 + 235*Sqrt[517])/40326]*Log[Sqrt[517] + Sqrt[2*(19
 + Sqrt[517])]*(3 + 4/x) + (3 + 4/x)^2])/8
```

Rubi [A] time = 1.1466, antiderivative size = 263, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409

$$\frac{1}{4}\sqrt{\frac{3}{13}}\tan^{-1}\left(\frac{-5x^2+12x+8}{\sqrt{39}x^2}\right)-\frac{1}{8}\sqrt{\frac{235\sqrt{517}-5167}{40326}}\log\left(\left(\frac{4}{x}+3\right)^2-\sqrt{2\left(19+\sqrt{517}\right)}\left(\frac{4}{x}+3\right)+\sqrt{517}\right)+\frac{1}{8}\sqrt{\frac{235\sqrt{517}-5167}{40326}}\log\left(\left(\frac{4}{x}+3\right)^2+\sqrt{2\left(19+\sqrt{517}\right)}\left(\frac{4}{x}+3\right)+\sqrt{517}\right)-\frac{1}{4}\sqrt{\frac{5167+235\sqrt{517}}{40326}}\tan^{-1}\left(\frac{\frac{8}{x}-\sqrt{2\left(19+\sqrt{517}\right)}+6}{\sqrt{2\left(\sqrt{517}-19\right)}}\right)-\frac{1}{4}\sqrt{\frac{5167+235\sqrt{517}}{40326}}\tan^{-1}\left(\frac{\frac{8}{x}+\sqrt{2\left(19+\sqrt{517}\right)}+6}{\sqrt{2\left(\sqrt{517}-19\right)}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-1),x]
```

```
[Out] -(Sqrt[(5167 + 235*Sqrt[517])/40326]*ArcTan[(6 - Sqrt[2*(19 + Sqr
t[517])] + 8/x)/Sqrt[2*(-19 + Sqrt[517])]])/4 - (Sqrt[(5167 + 235
*Sqrt[517])/40326]*ArcTan[(6 + Sqrt[2*(19 + Sqrt[517])] + 8/x)/Sq
rt[2*(-19 + Sqrt[517])]])/4 + (Sqrt[3/13]*ArcTan[(8 + 12*x - 5*x^
2)/(Sqrt[39]*x^2)])/4 - (Sqrt[(-5167 + 235*Sqrt[517])/40326]*Log[
```

```
Sqrt[517] - Sqrt[2*(19 + Sqrt[517])]*(3 + 4/x) + (3 + 4/x)^2])/8
+ (Sqrt[(-5167 + 235*Sqrt[517])/40326]*Log[Sqrt[517] + Sqrt[2*(19
 + Sqrt[517])]*(3 + 4/x) + (3 + 4/x)^2])/8
```

Rubi in Sympy [A] time = 89.2643, size = 372, normalized size = 1.41

$$\frac{\sqrt{1034}\left(-32\sqrt{517}+288\right)\log\left(\sqrt{2}\left(-\frac{3}{16}-\frac{1}{4x}\right)\sqrt{19+\sqrt{517}}+\left(\frac{3}{4}+\frac{1}{x}\right)^{2}+\frac{\sqrt{517}}{16}\right)}{264704\sqrt{19+\sqrt{517}}} - \frac{\sqrt{1034}\left(-32\sqrt{517}+288\right)\log\left(\sqrt{2}\left(\frac{3}{16}+\frac{1}{4x}\right)\sqrt{19+\sqrt{517}}+\left(\frac{3}{4}+\frac{1}{x}\right)^{2}+\frac{\sqrt{517}}{16}\right)}{264704\sqrt{19+\sqrt{517}}} + \frac{\sqrt{39}\operatorname{atan}\left(\sqrt{39}\left(\frac{8\left(\frac{3}{4}+\frac{1}{x}\right)^{2}}{39}-\frac{19}{78}\right)\right)}{52} - \frac{\sqrt{517}\left(-\frac{\sqrt{2}\sqrt{19+\sqrt{517}}\left(-64\sqrt{517}+576\right)}{8}+144\sqrt{2}\sqrt{19+\sqrt{517}}\right)\operatorname{atan}\left(\frac{\sqrt{2}\left(3+\frac{\sqrt{38+2\sqrt{517}}}{2}+\frac{4}{x}\right)}{\sqrt{-19+\sqrt{517}}}\right)}{33088\sqrt{-19+\sqrt{517}}\sqrt{19+\sqrt{517}}} - \frac{\sqrt{517}\left(-\frac{\sqrt{2}\sqrt{19+\sqrt{517}}\left(-64\sqrt{517}+576\right)}{8}+144\sqrt{2}\sqrt{19+\sqrt{517}}\right)\operatorname{atan}\left(\frac{\sqrt{2}\left(-\frac{\sqrt{38+2\sqrt{517}}}{2}+3+\frac{4}{x}\right)}{\sqrt{-19+\sqrt{517}}}\right)}{33088\sqrt{-19+\sqrt{517}}\sqrt{19+\sqrt{517}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(8*x**4-15*x**3+8*x**2+24*x+8),x)
```

```
[Out] sqrt(1034)*(-32*sqrt(517) + 288)*log(sqrt(2)*(-3/16 - 1/(4*x))*sq
rt(19 + sqrt(517)) + (3/4 + 1/x)**2 + sqrt(517)/16)/(264704*sqrt(
19 + sqrt(517))) - sqrt(1034)*(-32*sqrt(517) + 288)*log(sqrt(2)*(
3/16 + 1/(4*x))*sqrt(19 + sqrt(517)) + (3/4 + 1/x)**2 + sqrt(517)
/16)/(264704*sqrt(19 + sqrt(517))) + sqrt(39)*atan(sqrt(39)*(8*(3
/4 + 1/x)**2/39 - 19/78))/52 - sqrt(517)*(-sqrt(2)*sqrt(19 + sqrt
(517))*(-64*sqrt(517) + 576)/8 + 144*sqrt(2)*sqrt(19 + sqrt(517))
)*atan(sqrt(2)*(3 + sqrt(38 + 2*sqrt(517))/2 + 4/x)/sqrt(-19 + sq
rt(517)))/(33088*sqrt(-19 + sqrt(517))*sqrt(19 + sqrt(517))) - sq
rt(517)*(-sqrt(2)*sqrt(19 + sqrt(517))*(-64*sqrt(517) + 576)/8 +
144*sqrt(2)*sqrt(19 + sqrt(517)))*atan(sqrt(2)*(-sqrt(38 + 2*sqrt
(517))/2 + 3 + 4/x)/sqrt(-19 + sqrt(517)))/(33088*sqrt(-19 + sqrt
(517))*sqrt(19 + sqrt(517)))
```

Mathematica [C] time = 0.0158968, size = 55, normalized size = 0.21

$$\text{RootSum}\left[8\#1^4-15\#1^3+8\#1^2+24\#1+8\&,\frac{\log(x-\#1)}{32\#1^3-45\#1^2+16\#1+24}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-1),x]
```

```
[Out] RootSum[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , Log[x - #1]/(24
 + 16*#1 - 45*#1^2 + 32*#1^3) & ]
```

Maple [C] time = 0.007, size = 49, normalized size = 0.2

$$\sum_{\_R=RootOf\left(8\_Z^4-15\_Z^3+8\_Z^2+24\_Z+8\right)}\frac{\ln(x-\_R)}{32\_R^3-45\_R^2+16\_R+24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(8*x^4-15*x^3+8*x^2+24*x+8),x)
```

```
[Out] sum(1/(32*_R^3-45*_R^2+16*_R+24)*ln(x-_R),_R=RootOf(8*_Z^4-15*_Z^
3+8*_Z^2+24*_Z+8))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{8\,x^4 - 15\,x^3 + 8\,x^2 + 24\,x + 8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8),x, algorithm="maxima")
```

```
[Out] integrate(1/(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 3.04575, size = 41, normalized size = 0.16

$$\operatorname{RootSum}\left(50326848t^4 + 765960t^2 + 12753t + 64, \left(t \mapsto t\log\left(\frac{100785893208t^3}{4758335} - \frac{1430512512t^2}{4758335} + \frac{72982352521t}{223641745} + x + \frac{227}{178}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(8*x**4-15*x**3+8*x**2+24*x+8),x)
```

```
[Out] RootSum(50326848*_t**4 + 765960*_t**2 + 12753*_t + 64, Lambda(_t,
 _t*log(100785893208*_t**3/4758335 - 1430512512*_t**2/4758335 + 7
2982352521*_t/223641745 + x + 2270349121/1789133960)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{8\,x^4 - 15\,x^3 + 8\,x^2 + 24\,x + 8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8),x, algorithm="giac")
```

```
[Out] integrate(1/(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8), x)
```

3.62 $\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^2}\,dx$

Optimal. Leaf size=366

$$\frac{73}{208}\sqrt{\frac{3}{13}}\tan^{-1}\left(\frac{-5x^2+12x+8}{\sqrt{39}x^2}\right)-\frac{3\left(3359-107\left(\frac{4}{x}+3\right)^2\right)}{208\left(\left(\frac{4}{x}+3\right)^4-38\left(\frac{4}{x}+3\right)^2+517\right)}$$
$$+\frac{\left(3327931-129631\left(\frac{4}{x}+3\right)^2\right)\left(\frac{4}{x}+3\right)}{322608\left(\left(\frac{4}{x}+3\right)^4-38\left(\frac{4}{x}+3\right)^2+517\right)}$$
$$-\frac{\sqrt{\frac{2623170438295\sqrt{517}-59644114671451}{40326}}\log\left(\left(\frac{4}{x}+3\right)^2-\sqrt{2\left(19+\sqrt{517}\right)}\left(\frac{4}{x}+3\right)+\sqrt{517}\right)}{645216}$$
$$+\frac{\sqrt{\frac{2623170438295\sqrt{517}-59644114671451}{40326}}\log\left(\left(\frac{4}{x}+3\right)^2+\sqrt{2\left(19+\sqrt{517}\right)}\left(\frac{4}{x}+3\right)+\sqrt{517}\right)}{645216}$$
$$-\frac{\sqrt{\frac{19+\sqrt{517}}{40326}}\left(1678181+74897\sqrt{517}\right)\tan^{-1}\left(\frac{\frac{8}{x}-\sqrt{2\left(19+\sqrt{517}\right)}+6}{\sqrt{2\left(\sqrt{517}-19\right)}}\right)}{645216}$$
$$-\frac{\sqrt{\frac{19+\sqrt{517}}{40326}}\left(1678181+74897\sqrt{517}\right)\tan^{-1}\left(\frac{\frac{8}{x}+\sqrt{2\left(19+\sqrt{517}\right)}+6}{\sqrt{2\left(\sqrt{517}-19\right)}}\right)}{645216}$$

```
[Out] (-3*(3359 - 107*(3 + 4/x)^2))/(208*(517 - 38*(3 + 4/x)^2 + (3 + 4
/x)^4)) + ((3327931 - 129631*(3 + 4/x)^2)*(3 + 4/x))/(322608*(517
 - 38*(3 + 4/x)^2 + (3 + 4/x)^4)) - (Sqrt[(19 + Sqrt[517])/40326]
*(1678181 + 74897*Sqrt[517])*ArcTan[(6 - Sqrt[2*(19 + Sqrt[517])]
 + 8/x)/Sqrt[2*(-19 + Sqrt[517])]])/645216 - (Sqrt[(19 + Sqrt[517
])/40326]*(1678181 + 74897*Sqrt[517])*ArcTan[(6 + Sqrt[2*(19 + Sq
rt[517])] + 8/x)/Sqrt[2*(-19 + Sqrt[517])]])/645216 + (73*Sqrt[3/
13]*ArcTan[(8 + 12*x - 5*x^2)/(Sqrt[39]*x^2)])/208 - (Sqrt[(-5964
4114671451 + 2623170438295*Sqrt[517])/40326]*Log[Sqrt[517] - Sqrt
[2*(19 + Sqrt[517])]*(3 + 4/x) + (3 + 4/x)^2])/645216 + (Sqrt[(-5
9644114671451 + 2623170438295*Sqrt[517])/40326]*Log[Sqrt[517] + S
qrt[2*(19 + Sqrt[517])]*(3 + 4/x) + (3 + 4/x)^2])/645216
```

Rubi [A] time = 1.414, antiderivative size = 366, normalized size of antiderivative = 1., number of

steps used = 18, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$\frac{73}{208}\sqrt{\frac{3}{13}}\tan^{-1}\left(\frac{-5x^2+12x+8}{\sqrt{39}x^2}\right)-\frac{3\left(3359-107\left(\frac{4}{x}+3\right)^2\right)}{208\left(\left(\frac{4}{x}+3\right)^4-38\left(\frac{4}{x}+3\right)^2+517\right)}+\frac{\left(3327931-129631\left(\frac{4}{x}+3\right)^2\right)\left(\frac{4}{x}+3\right)}{322608\left(\left(\frac{4}{x}+3\right)^4-38\left(\frac{4}{x}+3\right)^2+517\right)}-\frac{\sqrt{\frac{2623170438295\sqrt{517}-59644114671451}{40326}}\log\left(\left(\frac{4}{x}+3\right)^2-\sqrt{2\left(19+\sqrt{517}\right)}\left(\frac{4}{x}+3\right)+\sqrt{517}\right)}{645216}+\frac{\sqrt{\frac{2623170438295\sqrt{517}-59644114671451}{40326}}\log\left(\left(\frac{4}{x}+3\right)^2+\sqrt{2\left(19+\sqrt{517}\right)}\left(\frac{4}{x}+3\right)+\sqrt{517}\right)}{645216}-\frac{\sqrt{\frac{19+\sqrt{517}}{40326}}\left(1678181+74897\sqrt{517}\right)\tan^{-1}\left(\frac{\frac{8}{x}-\sqrt{2\left(19+\sqrt{517}\right)}+6}{\sqrt{2\left(\sqrt{517}-19\right)}}\right)}{645216}-\frac{\sqrt{\frac{19+\sqrt{517}}{40326}}\left(1678181+74897\sqrt{517}\right)\tan^{-1}\left(\frac{\frac{8}{x}+\sqrt{2\left(19+\sqrt{517}\right)}+6}{\sqrt{2\left(\sqrt{517}-19\right)}}\right)}{645216}$$

Antiderivative was successfully verified.

```
[In]  Int[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-2),x]
```

```
[Out] (-3*(3359 - 107*(3 + 4/x)^2))/(208*(517 - 38*(3 + 4/x)^2 + (3 + 4
/x)^4)) + ((3327931 - 129631*(3 + 4/x)^2)*(3 + 4/x))/(322608*(517
 - 38*(3 + 4/x)^2 + (3 + 4/x)^4)) - (Sqrt[(19 + Sqrt[517])/40326]
*(1678181 + 74897*Sqrt[517])*ArcTan[(6 - Sqrt[2*(19 + Sqrt[517])]
 + 8/x)/Sqrt[2*(-19 + Sqrt[517])]])/645216 - (Sqrt[(19 + Sqrt[517
])/40326]*(1678181 + 74897*Sqrt[517])*ArcTan[(6 + Sqrt[2*(19 + Sq
rt[517])] + 8/x)/Sqrt[2*(-19 + Sqrt[517])]])/645216 + (73*Sqrt[3/
13]*ArcTan[(8 + 12*x - 5*x^2)/(Sqrt[39]*x^2)])/208 - (Sqrt[(-5964
4114671451 + 2623170438295*Sqrt[517])/40326]*Log[Sqrt[517] - Sqrt
[2*(19 + Sqrt[517])]*(3 + 4/x) + (3 + 4/x)^2])/645216 + (Sqrt[(-5
9644114671451 + 2623170438295*Sqrt[517])/40326]*Log[Sqrt[517] + S
qrt[2*(19 + Sqrt[517])]*(3 + 4/x) + (3 + 4/x)^2])/645216
```

Rubi in Sympy [A] time = 103.641, size = 428, normalized size = 1.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(8*x**4-15*x**3+8*x**2+24*x+8)**2,x)
```

```
[Out] (3/4 + 1/x)*(2082535753785622854384726344981544029013408509987009
7399325491462144*(3/4 + 1/x)**3 - 2232486456774371309555972849279
7338330276633138014460288875349671936*(3/4 + 1/x)**2 + 1480229314
4679335809984057739169174254798104165732379761527079567360 - 2802
4567081169544823174040534403879281470814168612404253710507573248/
x)/(2649263679538686150666915865659161512892006307978911135721062
4*(8388608*(3/4 + 1/x)**4 - 19922944*(3/4 + 1/x)**2 + 16941056))
+ sqrt(1034)*(-80616585805589561896755838925731607838920136393776
8084146489393152*sqrt(517) + 180633700393620701194252921377656241
73828969573340021378974320951296)*log(sqrt(2)*(-3/16 - 1/(4*x))*s
qrt(19 + sqrt(517)) + (3/4 + 1/x)**2 + sqrt(517)/16)/(71810118966
12949799179625444387253136471712130531836859083858959663104*sqrt(
19 + sqrt(517))) - sqrt(1034)*(-806165858055895618967558389257316
```

```
0783892013639377680841464893931 52*sqrt(517) + 1806337003936207011
94252921377656241738289695733400213789743209 51296)*log(sqrt(2)*(3
/16 + 1/(4*x))*sqrt(19 + sqrt(517)) + (3/4 + 1/x)**2 + sqrt(517)/
16)/(718101189661294979917962544438725313647171213053183685908385
8959663104*sqrt(19 + sqrt(517))) + 73*sqrt(39)*atan(sqrt(39)*(8*(
3/4 + 1/x)**2/39 - 19/78))/2704 - sqrt(517)*(-sqrt(2)*sqrt(19 + s
qrt(517))*(-161233171611179123793511677851463215677840272787553 61
68292978786304*sqrt(517) + 361267400787241402388505842755312483 47
657939146680042757948641902592)/8 + 90316850196810350597126460688
828120869144847866700106894871604 75648*sqrt(2)*sqrt(19 + sqrt(517
)))*atan(sqrt(2)*(3 + sqrt(38 + 2*sqrt(517))/2 + 4/x)/sqrt(-19 +
sqrt(517)))/(8976264870766187248974531805484066420589640163164796
07385482369957888*sqrt(-19 + sqrt(517))*sqrt(19 + sqrt(517))) - s
qrt(517)*(-sqrt(2)*sqrt(19 + sqrt(517))*(-161233171611179123793 51
1677851463215677840272787553616829297878 6304*sqrt(517) + 36126740
0787241402388505842755312483476579391466800427579486419 02592)/8 +
 903168501968103505971264606888281208691448478667001068948716 0475
648*sqrt(2)*sqrt(19 + sqrt(517)))*atan(sqrt(2)*(-sqrt(38 + 2*sqrt
(517))/2 + 3 + 4/x)/sqrt(-19 + sqrt(517)))/(89762648707661872489 7
453180548406642058964016316479607385482369957888*sqrt(-19 + sqrt(
517))*sqrt(19 + sqrt(517)))
```

Mathematica [C] time = 0.0306118, size = 128, normalized size = 0.35

$$\frac{\text{RootSum}\left[8\#1^4-15\#1^3+8\#1^2+24\#1+8\&,\frac{19640\#1^2\log(x-\#1)-57489\#1\log(x-\#1)+74897\log(x-\#1)}{32\#1^3-45\#1^2+16\#1+24}\&\right]}{80652}+\frac{39280x^3-94314x^2+89033x+72888}{161304\left(8x^4-15x^3+8x^2+24x+8\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-2),x]
```

```
[Out] (72888 + 89033*x - 94314*x^2 + 39280*x^3)/(161304*(8 + 24*x + 8*x
^2 - 15*x^3 + 8*x^4)) + RootSum[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*
#1^4 & , (74897*Log[x - #1] - 57489*Log[x - #1]*#1 + 19640*Log[x
- #1]*#1^2)/(24 + 16*#1 - 45*#1^2 + 32*#1^3) & ]/80652
```

Maple [C] time = 0.012, size = 96, normalized size = 0.3

$$1\left(\frac{2455\,x^3}{80652}-\frac{1429\,x^2}{19552}+\frac{89033\,x}{1290432}+\frac{3037}{53768}\right)\left(x^4-\frac{15\,x^3}{8}+x^2+3\,x+1\right)^{-1}+\frac{1}{80652}\sum_{\_R=RootOf\left(8\_Z^4-15\_Z^3+8\_Z^2+24\_Z+8\right)}\frac{\left(19640\_R^2-57489\_R+74897\right)\ln\left(x-\_R\right)}{32\_R^3-45\_R^2+16\_R+24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(8*x^4-15*x^3+8*x^2+24*x+8)^2,x)
```

```
[Out] (2455/80652*x^3-1429/19552*x^2+89033/1290432*x+3037/53768)/(x^4-1
5/8*x^3+x^2+3*x+1)+1/80652*sum((19640*_R^2-57489*_R+74897)/(32*_R
^3-45*_R^2+16*_R+24)*ln(x-_R),_R=RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*
_Z+8))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{39280\,x^3-94314\,x^2+89033\,x+72888}{161304\left(8\,x^4-15\,x^3+8\,x^2+24\,x+8\right)}+\frac{1}{80652}\int\frac{19640\,x^2-57489\,x+74897}{8\,x^4-15\,x^3+8\,x^2+24\,x+8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)^(-2),x, algorithm="maxima")
```

```
[Out] 1/161304*(39280*x^3 - 94314*x^2 + 89033*x + 72888)/(8*x^4 - 15*x^
3 + 8*x^2 + 24*x + 8) + 1/80652*integrate((19640*x^2 - 57489*x +
74897)/(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)^(-2),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 3.84116, size = 76, normalized size = 0.21

$$\frac{39280x^3 - 94314x^2 + 89033x + 72888}{1290432x^4 - 2419560x^3 + 1290432x^2 + 3871296x + 1290432} + \operatorname{RootSum}\left(1991678427489244336128t^4 + 566107340871621893 76t^2 + 20948104645409331t + 1938464112640, \left(t \mapsto t\log\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(8*x**4-15*x**3+8*x**2+24*x+8)**2,x)
```

```
[Out] (39280*x**3 - 94314*x**2 + 89033*x + 72888)/(1290432*x**4 - 24195
60*x**3 + 1290432*x**2 + 3871296*x + 1290432) + RootSum(199167842
7489244336128*_t**4 + 566107340871621893 76*_t**2 + 20948104645409
331*_t + 1938464112640, Lambda(_t, _t*log(-7050777423939663884532
545458302322744 32*_t**3/50310177134331359960511301071755 + 126981
4758239899452601522679045806 08*_t**2/503101771343313599605113010 7
1755 - 20040865325746858989799932658629535256*_t/503101771343313 5
9960511301071755 + x - 1814809597582050015741649548874985 9/241488
8502447905278104542451444 24)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(8x^4 - 15x^3 + 8x^2 + 24x + 8)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)^(-2),x, algorithm="giac")
```

```
[Out] integrate((8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)^(-2), x)
```

3.63 $\int \left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)^3\,dx$

Optimal. Leaf size=14

$$\frac{(a+bx)^{16}}{16b}$$

```
[Out] (a + b*x)^16/(16*b)
```

Rubi [A] time = 0.0351473, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}} = 0.039$

$$\frac{(a+bx)^{16}}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*x^5)^3

[Out] (a + b*x)^16/(16*b)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((b**5*x**5+5*a*b**4*x**4+10*a**2*b**3*x**3+10*a**3*b**2*x**2+5*

[Out] Timed out
```

Mathematica [A] time = 0.00270194, size = 14, normalized size = 1.

$$\frac{(a+bx)^{16}}{16b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*

[Out] (a + b*x)^16/(16*b)
```

Maple [B] time = 0.002, size = 164, normalized size = 11.7

$$\frac{b^{15}x^{16}}{16}+ab^{14}x^{15}+\frac{15\,a^2b^{13}x^{14}}{2}+35\,a^3b^{12}x^{13}+\frac{455\,a^4b^{11}x^{12}}{4}+273\,a^5b^{10}x^{11}+\frac{1001\,a^6b^9x^{10}}{2}+715\,a^7b^8x^9+\frac{6435\,a^8b^7x^8}{8}+715\,a^9b^6x^7+\frac{1001\,a^{10}b^5x^6}{2}+273\,a^{11}b^4x^5+\frac{455\,a^{12}b^3x^4}{4}+35\,a^{13}b^2x^3+\frac{15\,a^{14}bx^2}{2}+a^{15}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)^3,x)
```

```
[Out] 1/16*b^15*x^16+a*b^14*x^15+15/2*a^2*b^13*x^14+35*a^3*b^12*x^13+45
5/4*a^4*b^11*x^12+273*a^5*b^10*x^11+1001/2*a^6*b^9*x^10+715*a^7*b
^8*x^9+6435/8*a^8*b^7*x^8+715*a^9*b^6*x^7+1001/2*a^10*b^5*x^6+273
*a^11*b^4*x^5+455/4*a^12*b^3*x^4+35*a^13*b^2*x^3+15/2*a^14*b*x^2+
a^15*x
```

Maxima [A] time = 0.814304, size = 799, normalized size = 57.07

$$\frac{1}{16}\,b^{15}x^{16} + ab^{14}x^{15} + \frac{75}{14}\,a^2b^{13}x^{14} + \frac{125}{13}\,a^3b^{12}x^{13} + 100\,a^6b^9x^{10} + \frac{1000}{7}\,a^9b^6x^7 + \frac{125}{4}\,a^{12}b^3x^4 + a^{15}x + \frac{1}{2}\left(b^5x^6 + 6\,ab^4x^5 + 15\,a^2b^3x^4 + 20\,a^3b^2x^3 + 15\,a^4bx^2\right)a^{10} + \frac{25}{56}\left(21\,b^5x^8 + 120\,ab^4x^7 + 280\,a^2b^3x^6 + 336\,a^3b^2x^5\right)a^8b^2 + \frac{5}{3}\left(18\,b^5x^{10} + 100\,ab^4x^9 + 225\,a^2b^3x^8\right)a^6b^4 + \frac{25}{11}\left(11\,b^5x^{12} + 60\,ab^4x^{11}\right)a^4b^6 + \frac{1}{462}\left(126\,b^{10}x^{11} + 1386\,ab^9x^{10} + 3850\,a^2b^8x^9 + 19800\,a^4b^6x^7 + 27720\,a^6b^4x^5 + 11550\,a^8b^2x^3 + 330\left(6\,b^5x^7 + 35\,ab^4x^6 + 84\right.\right. + \frac{5}{308}\left(77\,b^{10}x^{12} + 840\,ab^9x^{11} + 4158\,a^2b^8x^{10} + 12320\,a^3b^7x^9 + 23100\,a^4b^6x^8 + 26400\,a^5b^5x^7 + 15400\,a^6b^4x^6\right)a^4b + \frac{5}{429}\left(198\,b^{10}x^{13} + 2145\,ab^9x^{12} + 10530\,a^2b^8x^{11} + 25740\,a^3b^7x^{10} + 28600\,a^4b^6x^9\right)a^3b^2 + \frac{5}{182}\left(78\,b^{10}x^{14} + 840\,ab^9x^{13} + 2275\,a^2b^8x^{12}\right)a^2b^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^5*x^5 + 5*a*b^4*x^4 + 10*a^2*b^3*x^3 + 10*a^3*b^2*x^2 + 5*a^4*b*x + a
```

```
[Out] 1/16*b^15*x^16 + a*b^14*x^15 + 75/14*a^2*b^13*x^14 + 125/13*a^3*b
^12*x^13 + 100*a^6*b^9*x^10 + 1000/7*a^9*b^6*x^7 + 125/4*a^12*b^3
*x^4 + a^15*x + 1/2*(b^5*x^6 + 6*a*b^4*x^5 + 15*a^2*b^3*x^4 + 20*
a^3*b^2*x^3 + 15*a^4*b*x^2)*a^10 + 25/56*(21*b^5*x^8 + 120*a*b^4*
x^7 + 280*a^2*b^3*x^6 + 336*a^3*b^2*x^5)*a^8*b^2 + 5/3*(18*b^5*x^
10 + 100*a*b^4*x^9 + 225*a^2*b^3*x^8)*a^6*b^4 + 25/11*(11*b^5*x^1
2 + 60*a*b^4*x^11)*a^4*b^6 + 1/462*(126*b^10*x^11 + 1386*a*b^9*x^
10 + 3850*a^2*b^8*x^9 + 19800*a^4*b^6*x^7 + 27720*a^6*b^4*x^5 + 1
1550*a^8*b^2*x^3 + 330*(6*b^5*x^7 + 35*a*b^4*x^6 + 84*a^2*b^3*x^5
 + 105*a^3*b^2*x^4)*a^4*b + 165*(21*b^5*x^8 + 120*a*b^4*x^7 + 280
*a^2*b^3*x^6)*a^3*b^2 + 385*(8*b^5*x^9 + 45*a*b^4*x^8)*a^2*b^3)*a
^5 + 5/308*(77*b^10*x^12 + 840*a*b^9*x^11 + 4158*a^2*b^8*x^10 + 1
2320*a^3*b^7*x^9 + 23100*a^4*b^6*x^8 + 26400*a^5*b^5*x^7 + 15400*
a^6*b^4*x^6)*a^4*b + 5/429*(198*b^10*x^13 + 2145*a*b^9*x^12 + 105
30*a^2*b^8*x^11 + 25740*a^3*b^7*x^10 + 28600*a^4*b^6*x^9)*a^3*b^2
 + 5/182*(78*b^10*x^14 + 840*a*b^9*x^13 + 2275*a^2*b^8*x^12)*a^2*
b^3
```

Fricas [A] time = 0.239415, size = 1, normalized size = 0.07

$$\frac{1}{16}x^{16}b^{15} + x^{15}b^{14}a + \frac{15}{2}x^{14}b^{13}a^2 + 35x^{13}b^{12}a^3 + \frac{455}{4}x^{12}b^{11}a^4 + 273x^{11}b^{10}a^5 + \frac{1001}{2}x^{10}b^9a^6 + 715x^9b^8a^7 + \frac{6435}{8}x^8b^7a^8 + 715x^7b^6a^9 + \frac{1001}{2}x^6b^5a^{10} + 273x^5b^4a^{11} + \frac{455}{4}x^4b^3a^{12} + 35x^3b^2a^{13} + \frac{15}{2}x^2ba^{14} + xa^{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^5*x^5 + 5*a*b^4*x^4 + 10*a^2*b^3*x^3 + 10*a^3*b^2*x^2 + 5*a^4*b*x + a
```

```
[Out] 1/16*x^16*b^15 + x^15*b^14*a + 15/2*x^14*b^13*a^2 + 35*x^13*b^12*
a^3 + 455/4*x^12*b^11*a^4 + 273*x^11*b^10*a^5 + 1001/2*x^10*b^9*a
^6 + 715*x^9*b^8*a^7 + 6435/8*x^8*b^7*a^8 + 715*x^7*b^6*a^9 + 100
1/2*x^6*b^5*a^10 + 273*x^5*b^4*a^11 + 455/4*x^4*b^3*a^12 + 35*x^3
*b^2*a^13 + 15/2*x^2*b*a^14 + x*a^15
```

Sympy [A] time = 0.233298, size = 185, normalized size = 13.21

$$a^{15}x + \frac{15a^{14}bx^2}{2} + 35a^{13}b^2x^3 + \frac{455a^{12}b^3x^4}{4} + 273a^{11}b^4x^5 + \frac{1001a^{10}b^5x^6}{2} + 715a^9b^6x^7 + \frac{6435a^8b^7x^8}{8} + 715a^7b^8x^9 + \frac{1001a^6b^9x^{10}}{2} + 273a^5b^{10}x^{11} + \frac{455a^4b^{11}x^{12}}{4} + 35a^3b^{12}x^{13} + \frac{15a^2b^{13}x^{14}}{2} + ab^{14}x^{15} + \frac{b^{15}x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**5*x**5+5*a*b**4*x**4+10*a**2*b**3*x**3+10*a**3*b**2*x**2+5*a**4*
```

```
[Out] a**15*x + 15*a**14*b*x**2/2 + 35*a**13*b**2*x**3 + 455*a**12*b**3
*x**4/4 + 273*a**11*b**4*x**5 + 1001*a**10*b**5*x**6/2 + 715*a**9
*b**6*x**7 + 6435*a**8*b**7*x**8/8 + 715*a**7*b**8*x**9 + 1001*a*
*6*b**9*x**10/2 + 273*a**5*b**10*x**11 + 455*a**4*b**11*x**12/4 +
 35*a**3*b**12*x**13 + 15*a**2*b**13*x**14/2 + a*b**14*x**15 + b*
*15*x**16/16
```

GIAC/XCAS [A] time = 0.259938, size = 220, normalized size = 15.71

$$\frac{1}{16}\,b^{15}x^{16} + ab^{14}x^{15} + \frac{15}{2}\,a^2b^{13}x^{14} + 35\,a^3b^{12}x^{13} + \frac{455}{4}\,a^4b^{11}x^{12} + 273\,a^5b^{10}x^{11} + \frac{1001}{2}\,a^6b^9x^{10} + 715\,a^7b^8x^9 + \frac{6435}{8}\,a^8b^7x^8 + 715\,a^9b^6x^7 + \frac{1001}{2}\,a^{10}b^5x^6 + 273\,a^{11}b^4x^5 + \frac{455}{4}\,a^{12}b^3x^4 + 35\,a^{13}b^2x^3 + \frac{15}{2}\,a^{14}bx^2 + a^{15}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^5*x^5 + 5*a*b^4*x^4 + 10*a^2*b^3*x^3 + 10*a^3*b^2*x^2 + 5*a^4*b*x + 
```

```
[Out] 1/16*b^15*x^16 + a*b^14*x^15 + 15/2*a^2*b^13*x^14 + 35*a^3*b^12*x
^13 + 455/4*a^4*b^11*x^12 + 273*a^5*b^10*x^11 + 1001/2*a^6*b^9*x^
10 + 715*a^7*b^8*x^9 + 6435/8*a^8*b^7*x^8 + 715*a^9*b^6*x^7 + 100
1/2*a^10*b^5*x^6 + 273*a^11*b^4*x^5 + 455/4*a^12*b^3*x^4 + 35*a^1
3*b^2*x^3 + 15/2*a^14*b*x^2 + a^15*x
```

3.64 $\int \left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)^2\,dx$

Optimal. Leaf size=14

$$\frac{(a+bx)^{11}}{11b}$$

```
[Out] (a + b*x)^11/(11*b)
```

Rubi [A] time = 0.0317289, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}} = 0.039$

$$\frac{(a+bx)^{11}}{11b}$$

Antiderivative was successfully verified.

```
[In] Int[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*x^5)^2
```

```
[Out] (a + b*x)^11/(11*b)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((b**5*x**5+5*a*b**4*x**4+10*a**2*b**3*x**3+10*a**3*b**2*x**2+5*
```

```
[Out] Timed out
```

Mathematica [A] time = 0.00157208, size = 14, normalized size = 1.

$$\frac{(a+bx)^{11}}{11b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*:
```

```
[Out] (a + b*x)^11/(11*b)
```

Maple [B] time = 0.002, size = 109, normalized size = 7.8

$$\frac{b^{10}x^{11}}{11}+ab^9x^{10}+5\,a^2b^8x^9+15\,a^3b^7x^8+30\,b^6a^4x^7+42\,a^5b^5x^6 \\ +42\,a^6b^4x^5+30\,a^7b^3x^4+15\,a^8b^2x^3+5\,a^9bx^2+a^{10}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)^2,x)
```

```
[Out] 1/11*b^10*x^11+a*b^9*x^10+5*a^2*b^8*x^9+15*a^3*b^7*x^8+30*b^6*a^4
*x^7+42*a^5*b^5*x^6+42*a^6*b^4*x^5+30*a^7*b^3*x^4+15*a^8*b^2*x^3+
5*a^9*b*x^2+a^10*x
```

Maxima [A] time = 0.812423, size = 308, normalized size = 22.

$$\frac{1}{11}b^{10}x^{11} + ab^9x^{10} + \frac{25}{9}a^2b^8x^9 + \frac{100}{7}a^4b^6x^7 + 20\,a^6b^4x^5 + \frac{25}{3}a^8b^2x^3 + a^{10}x + \frac{1}{3}\left(b^5x^6 + 6\,ab^4x^5 + 15\,a^2b^3x^4 + 20\,a^3b^2x^3 + 15\,a^4bx^2\right)a^5 + \frac{5}{21}\left(6\,b^5x^7 + 35\,ab^4x^6 + 84\,a^2b^3x^5 + 105\,a^3b^2x^4\right)a^4b + \frac{5}{42}\left(21\,b^5x^8 + 120\,ab^4x^7 + 280\,a^2b^3x^6\right)a^3b^2 + \frac{5}{18}\left(8\,b^5x^9 + 45\,ab^4x^8\right)a^2b^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^5*x^5 + 5*a*b^4*x^4 + 10*a^2*b^3*x^3 + 10*a^3*b^2*x^2 + 5*a^4*b*x + a
```

```
[Out] 1/11*b^10*x^11 + a*b^9*x^10 + 25/9*a^2*b^8*x^9 + 100/7*a^4*b^6*x^
7 + 20*a^6*b^4*x^5 + 25/3*a^8*b^2*x^3 + a^10*x + 1/3*(b^5*x^6 + 6
*a*b^4*x^5 + 15*a^2*b^3*x^4 + 20*a^3*b^2*x^3 + 15*a^4*b*x^2)*a^5
+ 5/21*(6*b^5*x^7 + 35*a*b^4*x^6 + 84*a^2*b^3*x^5 + 105*a^3*b^2*x
^4)*a^4*b + 5/42*(21*b^5*x^8 + 120*a*b^4*x^7 + 280*a^2*b^3*x^6)*a
^3*b^2 + 5/18*(8*b^5*x^9 + 45*a*b^4*x^8)*a^2*b^3
```

Fricas [A] time = 0.228044, size = 1, normalized size = 0.07

$$\frac{1}{11}x^{11}b^{10} + x^{10}b^9a + 5x^9b^8a^2 + 15x^8b^7a^3 + 30x^7b^6a^4 + 42x^6b^5a^5 + 42x^5b^4a^6 + 30x^4b^3a^7 + 15x^3b^2a^8 + 5x^2ba^9 + xa^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^5*x^5 + 5*a*b^4*x^4 + 10*a^2*b^3*x^3 + 10*a^3*b^2*x^2 + 5*a^4*b*x + a
```

```
[Out] 1/11*x^11*b^10 + x^10*b^9*a + 5*x^9*b^8*a^2 + 15*x^8*b^7*a^3 + 30
*x^7*b^6*a^4 + 42*x^6*b^5*a^5 + 42*x^5*b^4*a^6 + 30*x^4*b^3*a^7 +
 15*x^3*b^2*a^8 + 5*x^2*b*a^9 + x*a^10
```

Sympy [A] time = 0.171653, size = 114, normalized size = 8.14

$$a^{10}x + 5a^9bx^2 + 15a^8b^2x^3 + 30a^7b^3x^4 + 42a^6b^4x^5 + 42a^5b^5x^6 + 30a^4b^6x^7 + 15a^3b^7x^8 + 5a^2b^8x^9 + ab^9x^{10} + \frac{b^{10}x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**5*x**5+5*a*b**4*x**4+10*a**2*b**3*x**3+10*a**3*b**2*x**2+5*a**4*
```

```
[Out] a**10*x + 5*a**9*b*x**2 + 15*a**8*b**2*x**3 + 30*a**7*b**3*x**4 +
 42*a**6*b**4*x**5 + 42*a**5*b**5*x**6 + 30*a**4*b**6*x**7 + 15*a
**3*b**7*x**8 + 5*a**2*b**8*x**9 + a*b**9*x**10 + b**10*x**11/11
```

GIAC/XCAS [A] time = 0.261002, size = 146, normalized size = 10.43

$$\frac{1}{11}b^{10}x^{11} + ab^9x^{10} + 5a^2b^8x^9 + 15a^3b^7x^8 + 30a^4b^6x^7 + 42a^5b^5x^6 + 42a^6b^4x^5 + 30a^7b^3x^4 + 15a^8b^2x^3 + 5a^9bx^2 + a^{10}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^5*x^5 + 5*a*b^4*x^4 + 10*a^2*b^3*x^3 + 10*a^3*b^2*x^2 + 5*a^4*b*x + a
```

```
[Out] 1/11*b^10*x^11 + a*b^9*x^10 + 5*a^2*b^8*x^9 + 15*a^3*b^7*x^8 + 30
*a^4*b^6*x^7 + 42*a^5*b^5*x^6 + 42*a^6*b^4*x^5 + 30*a^7*b^3*x^4 +
 15*a^8*b^2*x^3 + 5*a^9*b*x^2 + a^10*x
```

3.65 $\int \left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)\,dx$

Optimal. Leaf size=14

$$\frac{(a+bx)^6}{6b}$$

```
[Out] (a + b*x)^6/(6*b)
```

Rubi [B] time = 0.0343188, antiderivative size = 61, normalized size of antiderivative = 4.36, number of steps used = 1, number of rules used = 0, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.

$$a^5x+\frac{5}{2}a^4bx^2+\frac{10}{3}a^3b^2x^3+\frac{5}{2}a^2b^3x^4+ab^4x^5+\frac{b^5x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*x^5,x]
```

```
[Out] a^5*x + (5*a^4*b*x^2)/2 + (10*a^3*b^2*x^3)/3 + (5*a^2*b^3*x^4)/2
+ a*b^4*x^5 + (b^5*x^6)/6
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$5a^4b\int x\,dx+\frac{10a^3b^2x^3}{3}+\frac{5a^2b^3x^4}{2}+ab^4x^5+\frac{b^5x^6}{6}+\int a^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(b**5*x**5+5*a*b**4*x**4+10*a**2*b**3*x**3+10*a**3*b**2*x**2+5*a
```

```
[Out] 5*a**4*b*Integral(x, x) + 10*a**3*b**2*x**3/3 + 5*a**2*b**3*x**4/
2 + a*b**4*x**5 + b**5*x**6/6 + Integral(a**5, x)
```

Mathematica [B] time = 0.0000892753, size = 61, normalized size = 4.36

$$a^5x+\frac{5}{2}a^4bx^2+\frac{10}{3}a^3b^2x^3+\frac{5}{2}a^2b^3x^4+ab^4x^5+\frac{b^5x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*x
```

```
[Out] a^5*x + (5*a^4*b*x^2)/2 + (10*a^3*b^2*x^3)/3 + (5*a^2*b^3*x^4)/2
+ a*b^4*x^5 + (b^5*x^6)/6
```

Maple [B] time = 0.001, size = 54, normalized size = 3.9

$$a^5x+\frac{5\,a^4bx^2}{2}+\frac{10\,a^3b^2x^3}{3}+\frac{5\,a^2b^3x^4}{2}+ab^4x^5+\frac{b^5x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5,x)
```

```
[Out] a^5*x+5/2*a^4*b*x^2+10/3*a^3*b^2*x^3+5/2*a^2*b^3*x^4+a*b^4*x^5+1/
6*b^5*x^6
```

Maxima [A] time = 0.795292, size = 72, normalized size = 5.14

$$\frac{1}{6}\,b^5x^6+ab^4x^5+\frac{5}{2}\,a^2b^3x^4+\frac{10}{3}\,a^3b^2x^3+\frac{5}{2}\,a^4bx^2+a^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b^5*x^5 + 5*a*b^4*x^4 + 10*a^2*b^3*x^3 + 10*a^3*b^2*x^2 + 5*a^4*b*x + a
```

```
[Out] 1/6*b^5*x^6 + a*b^4*x^5 + 5/2*a^2*b^3*x^4 + 10/3*a^3*b^2*x^3 + 5/
2*a^4*b*x^2 + a^5*x
```

Fricas [A] time = 0.23727, size = 1, normalized size = 0.07

$$\frac{1}{6}x^6b^5+x^5b^4a+\frac{5}{2}x^4b^3a^2+\frac{10}{3}x^3b^2a^3+\frac{5}{2}x^2ba^4+xa^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b^5*x^5 + 5*a*b^4*x^4 + 10*a^2*b^3*x^3 + 10*a^3*b^2*x^2 + 5*a^4*b*x + a
```

```
[Out] 1/6*x^6*b^5 + x^5*b^4*a + 5/2*x^4*b^3*a^2 + 10/3*x^3*b^2*a^3 + 5/
2*x^2*b*a^4 + x*a^5
```

Sympy [A] time = 0.105178, size = 60, normalized size = 4.29

$$a^5x+\frac{5a^4bx^2}{2}+\frac{10a^3b^2x^3}{3}+\frac{5a^2b^3x^4}{2}+ab^4x^5+\frac{b^5x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b**5*x**5+5*a*b**4*x**4+10*a**2*b**3*x**3+10*a**3*b**2*x**2+5*a**4*b
```

```
[Out] a**5*x + 5*a**4*b*x**2/2 + 10*a**3*b**2*x**3/3 + 5*a**2*b**3*x**4
/2 + a*b**4*x**5 + b**5*x**6/6
```

GIAC/XCAS [A] time = 0.25914, size = 72, normalized size = 5.14

$$\frac{1}{6}\,b^5x^6+ab^4x^5+\frac{5}{2}\,a^2b^3x^4+\frac{10}{3}\,a^3b^2x^3+\frac{5}{2}\,a^4bx^2+a^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b^5*x^5 + 5*a*b^4*x^4 + 10*a^2*b^3*x^3 + 10*a^3*b^2*x^2 + 5*a^4*b*x + a
```

```
[Out] 1/6*b^5*x^6 + a*b^4*x^5 + 5/2*a^2*b^3*x^4 + 10/3*a^3*b^2*x^3 + 5/
2*a^4*b*x^2 + a^5*x
```

3.66 $\int \frac{1}{a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5} dx$

Optimal. Leaf size=14

$$-\frac{1}{4b(a+bx)^4}$$

```
[Out] -1/(4*b*(a + b*x)^4)
```

Rubi [A] time = 0.035304, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.039

$$-\frac{1}{4b(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*x^5)^(
1),x]
```

```
[Out] -1/(4*b*(a + b*x)^4)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(b**5*x**5+5*a*b**4*x**4+10*a**2*b**3*x**3+10*a**3*b**2*x**2+
```

```
[Out] Timed out
```

Mathematica [A] time = 0.0060112, size = 14, normalized size = 1.

$$-\frac{1}{4b(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*
1),x]
```

```
[Out] -1/(4*b*(a + b*x)^4)
```

Maple [A] time = 0.006, size = 13, normalized size = 0.9

$$-\frac{1}{4\,b\,(bx+a)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5),x)
```

```
[Out] -1/4/b/(b*x+a)^4
```

Maxima [A] time = 0.832262, size = 62, normalized size = 4.43

$$-\frac{1}{4\,(b^5x^4 + 4\,ab^4x^3 + 6\,a^2b^3x^2 + 4\,a^3b^2x + a^4b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^5*x^5 + 5*a*b^4*x^4 + 10*a^2*b^3*x^3 + 10*a^3*b^2*x^2 + 5*a^4*b*x
```

```
[Out] -1/4/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b
)
```

Fricas [A] time = 0.270872, size = 62, normalized size = 4.43

$$-\frac{1}{4\,(b^5x^4 + 4\,ab^4x^3 + 6\,a^2b^3x^2 + 4\,a^3b^2x + a^4b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^5*x^5 + 5*a*b^4*x^4 + 10*a^2*b^3*x^3 + 10*a^3*b^2*x^2 + 5*a^4*b*x
```

```
[Out] -1/4/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b
)
```

Sympy [A] time = 1.68389, size = 49, normalized size = 3.5

$$-\frac{1}{4a^4b + 16a^3b^2x + 24a^2b^3x^2 + 16ab^4x^3 + 4b^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**5*x**5+5*a*b**4*x**4+10*a**2*b**3*x**3+10*a**3*b**2*x**2+5*a**
```

```
[Out] -1/(4*a**4*b + 16*a**3*b**2*x + 24*a**2*b**3*x**2 + 16*a*b**4*x**
3 + 4*b**5*x**4)
```

GIAC/XCAS [A] time = 0.259136, size = 16, normalized size = 1.14

$$-\frac{1}{4\,(bx + a)^4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^5*x^5 + 5*a*b^4*x^4 + 10*a^2*b^3*x^3 + 10*a^3*b^2*x^2 + 5*a^4*b*x
```

```
[Out] -1/4/((b*x + a)^4*b)
```

3.67 $\int \frac{1}{(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5)^2}\,dx$

Optimal. Leaf size=14

$$-\frac{1}{9b(a+bx)^9}$$

```
[Out] -1/(9*b*(a + b*x)^9)
```

Rubi [A] time = 0.0330936, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.039

$$-\frac{1}{9b(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*x^5)^(
2),x]

[Out] -1/(9*b*(a + b*x)^9)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(b**5*x**5+5*a*b**4*x**4+10*a**2*b**3*x**3+10*a**3*b**2*x**2+

[Out] Timed out
```

Mathematica [A] time = 0.00541347, size = 14, normalized size = 1.

$$-\frac{1}{9b(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*
2),x]

[Out] -1/(9*b*(a + b*x)^9)
```

Maple [A] time = 0.004, size = 13, normalized size = 0.9

$$-\frac{1}{9\,b\,(bx+a)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)^2,x)
```

```
[Out] -1/9/b/(b*x+a)^9
```

Maxima [A] time = 0.823871, size = 136, normalized size = 9.71

$$-\frac{1}{9\,(b^{10}x^9+9\,ab^9x^8+36\,a^2b^8x^7+84\,a^3b^7x^6+126\,a^4b^6x^5+126\,a^5b^5x^4+84\,a^6b^4x^3+36\,a^7b^3x^2+9\,a^8b^2x+a^9b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^5*x^5 + 5*a*b^4*x^4 + 10*a^2*b^3*x^3 + 10*a^3*b^2*x^2 + 5*a^4*b*x + a
2),x, algorithm="maxima")
```

```
[Out] -1/9/(b^10*x^9 + 9*a*b^9*x^8 + 36*a^2*b^8*x^7 + 84*a^3*b^7*x^6 +
126*a^4*b^6*x^5 + 126*a^5*b^5*x^4 + 84*a^6*b^4*x^3 + 36*a^7*b^3*x
^2 + 9*a^8*b^2*x + a^9*b)
```

Fricas [A] time = 0.24729, size = 136, normalized size = 9.71

$$-\frac{1}{9\,(b^{10}x^9+9\,ab^9x^8+36\,a^2b^8x^7+84\,a^3b^7x^6+126\,a^4b^6x^5+126\,a^5b^5x^4+84\,a^6b^4x^3+36\,a^7b^3x^2+9\,a^8b^2x+a^9b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^5*x^5 + 5*a*b^4*x^4 + 10*a^2*b^3*x^3 + 10*a^3*b^2*x^2 + 5*a^4*b*x + a
2),x, algorithm="fricas")
```

```
[Out] -1/9/(b^10*x^9 + 9*a*b^9*x^8 + 36*a^2*b^8*x^7 + 84*a^3*b^7*x^6 +
126*a^4*b^6*x^5 + 126*a^5*b^5*x^4 + 84*a^6*b^4*x^3 + 36*a^7*b^3*x
^2 + 9*a^8*b^2*x + a^9*b)
```

Sympy [A] time = 3.07586, size = 109, normalized size = 7.79

$$-\frac{1}{9a^9b+81a^8b^2x+324a^7b^3x^2+756a^6b^4x^3+1134a^5b^5x^4+1134a^4b^6x^5+756a^3b^7x^6+324a^2b^8x^7+81ab^9x^8+9b^{10}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**5*x**5+5*a*b**4*x**4+10*a**2*b**3*x**3+10*a**3*b**2*x**2+5*a**
```

```
[Out] -1/(9*a**9*b + 81*a**8*b**2*x + 324*a**7*b**3*x**2 + 756*a**6*b**
4*x**3 + 1134*a**5*b**5*x**4 + 1134*a**4*b**6*x**5 + 756*a**3*b**
7*x**6 + 324*a**2*b**8*x**7 + 81*a*b**9*x**8 + 9*b**10*x**9)
```

GIAC/XCAS [A] time = 0.258385, size = 16, normalized size = 1.14

$$-\frac{1}{9\,(bx+a)^9b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^5*x^5 + 5*a*b^4*x^4 + 10*a^2*b^3*x^3 + 10*a^3*b^2*x^2 + 5*a^4*b*x + a
2),x, algorithm="giac")
```

```
[Out] -1/9/((b*x + a)^9*b)
```

3.68 $\int \frac{1}{(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5)^3} \, dx$

Optimal. Leaf size=14

$$-\frac{1}{14b(a+bx)^{14}}$$

```
[Out] -1/(14*b*(a + b*x)^14)
```

Rubi [A] time = 0.033027, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}} = 0.039$

$$-\frac{1}{14b(a+bx)^{14}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*x^5)^(
3),x]
```

```
[Out] -1/(14*b*(a + b*x)^14)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(b**5*x**5+5*a*b**4*x**4+10*a**2*b**3*x**3+10*a**3*b**2*x**2+
```

```
[Out] Timed out
```

Mathematica [A] time = 0.00618847, size = 14, normalized size = 1.

$$-\frac{1}{14b(a+bx)^{14}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*
3),x]
```

```
[Out] -1/(14*b*(a + b*x)^14)
```

Maple [A] time = 0.004, size = 13, normalized size = 0.9

$$-\frac{1}{14\,b\,(bx+a)^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)^3,x)
```

```
[Out] -1/14/b/(b*x+a)^14
```

Maxima [A] time = 0.820401, size = 211, normalized size = 15.07

$$-\frac{1}{14\left(b^{15}x^{14}+14\,ab^{14}x^{13}+91\,a^{2}b^{13}x^{12}+364\,a^{3}b^{12}x^{11}+1001\,a^{4}b^{11}x^{10}+2002\,a^{5}b^{10}x^{9}+3003\,a^{6}b^{9}x^{8}+3432\,a^{7}b^{8}x^{7}+3003\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^5*x^5 + 5*a*b^4*x^4 + 10*a^2*b^3*x^3 + 10*a^3*b^2*x^2 + 5*a^4*b*x + 
3),x, algorithm="maxima")
```

```
[Out] -1/14/(b^15*x^14 + 14*a*b^14*x^13 + 91*a^2*b^13*x^12 + 364*a^3*b^
12*x^11 + 1001*a^4*b^11*x^10 + 2002*a^5*b^10*x^9 + 3003*a^6*b^9*x
^8 + 3432*a^7*b^8*x^7 + 3003*a^8*b^7*x^6 + 2002*a^9*b^6*x^5 + 100
1*a^10*b^5*x^4 + 364*a^11*b^4*x^3 + 91*a^12*b^3*x^2 + 14*a^13*b^2
*x + a^14*b)
```

Fricas [A] time = 0.271264, size = 211, normalized size = 15.07

$$-\frac{1}{14\left(b^{15}x^{14}+14\,ab^{14}x^{13}+91\,a^{2}b^{13}x^{12}+364\,a^{3}b^{12}x^{11}+1001\,a^{4}b^{11}x^{10}+2002\,a^{5}b^{10}x^{9}+3003\,a^{6}b^{9}x^{8}+3432\,a^{7}b^{8}x^{7}+3003\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^5*x^5 + 5*a*b^4*x^4 + 10*a^2*b^3*x^3 + 10*a^3*b^2*x^2 + 5*a^4*b*x + 
3),x, algorithm="fricas")
```

```
[Out] -1/14/(b^15*x^14 + 14*a*b^14*x^13 + 91*a^2*b^13*x^12 + 364*a^3*b^
12*x^11 + 1001*a^4*b^11*x^10 + 2002*a^5*b^10*x^9 + 3003*a^6*b^9*x
^8 + 3432*a^7*b^8*x^7 + 3003*a^8*b^7*x^6 + 2002*a^9*b^6*x^5 + 100
1*a^10*b^5*x^4 + 364*a^11*b^4*x^3 + 91*a^12*b^3*x^2 + 14*a^13*b^2
*x + a^14*b)
```

Sympy [A] time = 5.72028, size = 168, normalized size = 12.

$$-\frac{1}{14a^{14}b+196a^{13}b^{2}x+1274a^{12}b^{3}x^{2}+5096a^{11}b^{4}x^{3}+14014a^{10}b^{5}x^{4}+28028a^{9}b^{6}x^{5}+42042a^{8}b^{7}x^{6}+48048a^{7}b^{8}x^{7}+42042a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**5*x**5+5*a*b**4*x**4+10*a**2*b**3*x**3+10*a**3*b**2*x**2+5*a**
```

```
[Out] -1/(14*a**14*b + 196*a**13*b**2*x + 1274*a**12*b**3*x**2 + 5096*a
**11*b**4*x**3 + 14014*a**10*b**5*x**4 + 28028*a**9*b**6*x**5 + 4
2042*a**8*b**7*x**6 + 48048*a**7*b**8*x**7 + 42042*a**6*b**9*x**8
 + 28028*a**5*b**10*x**9 + 14014*a**4*b**11*x**10 + 5096*a**3*b**
12*x**11 + 1274*a**2*b**13*x**12 + 196*a*b**14*x**13 + 14*b**15*x
**14)
```

GIAC/XCAS [A] time = 0.260633, size = 16, normalized size = 1.14

$$-\frac{1}{14\,(bx+a)^{14}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^5*x^5 + 5*a*b^4*x^4 + 10*a^2*b^3*x^3 + 10*a^3*b^2*x^2 + 5*a^4*b*x + a
3),x, algorithm="giac")
```

```
[Out] -1/14/((b*x + a)^14*b)
```

3.69 $\int \frac{1}{1+x^2+x^3+x^5} \, dx$

Optimal. Leaf size=38

$$\frac{1}{4}\log\left(x^2+1\right)-\frac{1}{3}\log\left(x^2-x+1\right)+\frac{1}{6}\log(x+1)+\frac{1}{2}\tan^{-1}(x)$$

```
[Out] ArcTan[x]/2 + Log[1 + x]/6 + Log[1 + x^2]/4 - Log[1 - x + x^2]/3
```

Rubi [A] time = 0.0489702, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308

$$\frac{1}{4}\log\left(x^2+1\right)-\frac{1}{3}\log\left(x^2-x+1\right)+\frac{1}{6}\log(x+1)+\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^2 + x^3 + x^5)^(-1),x]
```

```
[Out] ArcTan[x]/2 + Log[1 + x]/6 + Log[1 + x^2]/4 - Log[1 - x + x^2]/3
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^5+x^3+x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**5+x**3+x**2+1),x)
```

```
[Out] Integral(1/(x**5 + x**3 + x**2 + 1), x)
```

Mathematica [A] time = 0.0107985, size = 38, normalized size = 1.

$$\frac{1}{4}\log\left(x^2+1\right)-\frac{1}{3}\log\left(x^2-x+1\right)+\frac{1}{6}\log(x+1)+\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^2 + x^3 + x^5)^(-1),x]
```

```
[Out] ArcTan[x]/2 + Log[1 + x]/6 + Log[1 + x^2]/4 - Log[1 - x + x^2]/3
```

Maple [A] time = 0.01, size = 31, normalized size = 0.8

$$\frac{\arctan(x)}{2}+\frac{\ln(1+x)}{6}+\frac{\ln\left(x^2+1\right)}{4}-\frac{\ln\left(x^2-x+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^5+x^3+x^2+1),x)
```

```
[Out] 1/2*arctan(x)+1/6*ln(1+x)+1/4*ln(x^2+1)-1/3*ln(x^2-x+1)
```

Maxima [A] time = 0.909293, size = 41, normalized size = 1.08

$$\frac{1}{2}\arctan(x) - \frac{1}{3}\log\left(x^2 - x + 1\right) + \frac{1}{4}\log\left(x^2 + 1\right) + \frac{1}{6}\log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^5 + x^3 + x^2 + 1),x, algorithm="maxima")
```

```
[Out] 1/2*arctan(x) - 1/3*log(x^2 - x + 1) + 1/4*log(x^2 + 1) + 1/6*log
(x + 1)
```

Fricas [A] time = 0.25712, size = 41, normalized size = 1.08

$$\frac{1}{2}\arctan(x) - \frac{1}{3}\log\left(x^2 - x + 1\right) + \frac{1}{4}\log\left(x^2 + 1\right) + \frac{1}{6}\log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^5 + x^3 + x^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/2*arctan(x) - 1/3*log(x^2 - x + 1) + 1/4*log(x^2 + 1) + 1/6*log
(x + 1)
```

Sympy [A] time = 0.318706, size = 29, normalized size = 0.76

$$\frac{\log(x + 1)}{6} + \frac{\log\left(x^2 + 1\right)}{4} - \frac{\log\left(x^2 - x + 1\right)}{3} + \frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**5+x**3+x**2+1),x)
```

```
[Out] log(x + 1)/6 + log(x**2 + 1)/4 - log(x**2 - x + 1)/3 + atan(x)/2
```

GIAC/XCAS [A] time = 0.26338, size = 42, normalized size = 1.11

$$\frac{1}{2}\arctan(x) - \frac{1}{3}\ln\left(x^2 - x + 1\right) + \frac{1}{4}\ln\left(x^2 + 1\right) + \frac{1}{6}\ln(|x + 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^5 + x^3 + x^2 + 1),x, algorithm="giac")
```

```
[Out] 1/2*arctan(x) - 1/3*ln(x^2 - x + 1) + 1/4*ln(x^2 + 1) + 1/6*ln(ab
s(x + 1))
```

3.70 $\int \left(3-19x^2+32x^4-16x^6\right)^4 \, dx$

Optimal. Leaf size=84

$$\frac{65536x^{25}}{25}-\frac{524288x^{23}}{23}+\frac{1884160x^{21}}{21}-\frac{4014080x^{19}}{19}+\frac{5633536x^{17}}{17}-\frac{1094656x^{15}}{3}+\frac{3764416x^{13}}{13}-\frac{1841600x^{11}}{11}+\frac{634321x^9}{9}-\frac{149700x^7}{7}+4590x^5-684x^3+81x$$

```
[Out] 81*x - 684*x^3 + 4590*x^5 - (149700*x^7)/7 + (634321*x^9)/9 - (18
41600*x^11)/11 + (3764416*x^13)/13 - (1094656*x^15)/3 + (5633536*
x^17)/17 - (4014080*x^19)/19 + (1884160*x^21)/21 - (524288*x^23)/
23 + (65536*x^25)/25
```

Rubi [A] time = 0.331028, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$\frac{65536x^{25}}{25}-\frac{524288x^{23}}{23}+\frac{1884160x^{21}}{21}-\frac{4014080x^{19}}{19}+\frac{5633536x^{17}}{17}-\frac{1094656x^{15}}{3}+\frac{3764416x^{13}}{13}-\frac{1841600x^{11}}{11}+\frac{634321x^9}{9}-\frac{149700x^7}{7}+4590x^5-684x^3+81x$$

Antiderivative was successfully verified.

```
[In]  Int[(3 - 19*x^2 + 32*x^4 - 16*x^6)^4,x]
```

```
[Out] 81*x - 684*x^3 + 4590*x^5 - (149700*x^7)/7 + (634321*x^9)/9 - (18
41600*x^11)/11 + (3764416*x^13)/13 - (1094656*x^15)/3 + (5633536*
x^17)/17 - (4014080*x^19)/19 + (1884160*x^21)/21 - (524288*x^23)/
23 + (65536*x^25)/25
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-16*x**6+32*x**4-19*x**2+3)**4,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.00243667, size = 84, normalized size = 1.

$$\frac{65536x^{25}}{25}-\frac{524288x^{23}}{23}+\frac{1884160x^{21}}{21}-\frac{4014080x^{19}}{19}+\frac{5633536x^{17}}{17}-\frac{1094656x^{15}}{3}+\frac{3764416x^{13}}{13}-\frac{1841600x^{11}}{11}+\frac{634321x^9}{9}-\frac{149700x^7}{7}+4590x^5-684x^3+81x$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3 - 19*x^2 + 32*x^4 - 16*x^6)^4,x]
```

```
[Out] 81*x - 684*x^3 + 4590*x^5 - (149700*x^7)/7 + (634321*x^9)/9 - (18
41600*x^11)/11 + (3764416*x^13)/13 - (1094656*x^15)/3 + (5633536*
x^17)/17 - (4014080*x^19)/19 + (1884160*x^21)/21 - (524288*x^23)/
```

```
23 + (65536*x^25)/25
```

Maple [A] time = 0.003, size = 65, normalized size = 0.8

$$81\,x-684\,x^{3}+4590\,x^{5}-\frac{149700\,x^{7}}{7}+\frac{634321\,x^{9}}{9}-\frac{1841600\,x^{11}}{11}+\frac{3764416\,x^{13}}{13}-\frac{1094656\,x^{15}}{3}+\frac{5633536\,x^{17}}{17}-\frac{4014080\,x^{19}}{19}+\frac{1884160\,x^{21}}{21}-\frac{524288\,x^{23}}{23}+\frac{65536\,x^{25}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-16*x^6+32*x^4-19*x^2+3)^4,x)
```

```
[Out] 81*x-684*x^3+4590*x^5-149700/7*x^7+634321/9*x^9-1841600/11*x^11+3
764416/13*x^13-1094656/3*x^15+5633536/17*x^17-4014080/19*x^19+188
4160/21*x^21-524288/23*x^23+65536/25*x^25
```

Maxima [A] time = 0.815946, size = 86, normalized size = 1.02

$$\frac{65536}{25}\,x^{25}-\frac{524288}{23}\,x^{23}+\frac{1884160}{21}\,x^{21}-\frac{4014080}{19}\,x^{19}+\frac{5633536}{17}\,x^{17}-\frac{1094656}{3}\,x^{15}+\frac{3764416}{13}\,x^{13}-\frac{1841600}{11}\,x^{11}+\frac{634321}{9}\,x^{9}-\frac{149700}{7}\,x^{7}+4590\,x^{5}-684\,x^{3}+81\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((16*x^6 - 32*x^4 + 19*x^2 - 3)^4,x, algorithm="maxima")
```

```
[Out] 65536/25*x^25 - 524288/23*x^23 + 1884160/21*x^21 - 4014080/19*x^1
9 + 5633536/17*x^17 - 1094656/3*x^15 + 3764416/13*x^13 - 1841600/
11*x^11 + 634321/9*x^9 - 149700/7*x^7 + 4590*x^5 - 684*x^3 + 81*x
```

Fricas [A] time = 0.240843, size = 1, normalized size = 0.01

$$\frac{65536}{25}x^{25}-\frac{524288}{23}x^{23}+\frac{1884160}{21}x^{21}-\frac{4014080}{19}x^{19}+\frac{5633536}{17}x^{17}-\frac{1094656}{3}x^{15}+\frac{3764416}{13}x^{13}-\frac{1841600}{11}x^{11}+\frac{634321}{9}x^{9}-\frac{149700}{7}x^{7}+4590x^{5}-684x^{3}+81x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((16*x^6 - 32*x^4 + 19*x^2 - 3)^4,x, algorithm="fricas")
```

```
[Out] 65536/25*x^25 - 524288/23*x^23 + 1884160/21*x^21 - 4014080/19*x^1
9 + 5633536/17*x^17 - 1094656/3*x^15 + 3764416/13*x^13 - 1841600/
11*x^11 + 634321/9*x^9 - 149700/7*x^7 + 4590*x^5 - 684*x^3 + 81*x
```

Sympy [A] time = 0.120213, size = 80, normalized size = 0.95

$$\frac{65536x^{25}}{25}-\frac{524288x^{23}}{23}+\frac{1884160x^{21}}{21}-\frac{4014080x^{19}}{19}+\frac{5633536x^{17}}{17}-\frac{1094656x^{15}}{3}+\frac{3764416x^{13}}{13}-\frac{1841600x^{11}}{11}+\frac{634321x^{9}}{9}-\frac{149700x^{7}}{7}+4590x^{5}-684x^{3}+81x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-16*x**6+32*x**4-19*x**2+3)**4,x)
```

```
[Out] 65536*x**25/25 - 524288*x**23/23 + 1884160*x**21/21 - 4014080*x**
19/19 + 5633536*x**17/17 - 1094656*x**15/3 + 3764416*x**13/13 - 1
841600*x**11/11 + 634321*x**9/9 - 149700*x**7/7 + 4590*x**5 - 684
*x**3 + 81*x
```

GIAC/XCAS [A] time = 0.259657, size = 86, normalized size = 1.02

$$\frac{65536}{25}x^{25} - \frac{524288}{23}x^{23} + \frac{1884160}{21}x^{21} - \frac{4014080}{19}x^{19} + \frac{5633536}{17}x^{17} - \frac{1094656}{3}x^{15} + \frac{3764416}{13}x^{13} - \frac{1841600}{11}x^{11} + \frac{634321}{9}x^{9} - \frac{149700}{7}x^{7} + 4590\,x^{5} - 684\,x^{3} + 81\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((16*x^6 - 32*x^4 + 19*x^2 - 3)^4,x, algorithm="giac")
```

```
[Out] 65536/25*x^25 - 524288/23*x^23 + 1884160/21*x^21 - 4014080/19*x^1
9 + 5633536/17*x^17 - 1094656/3*x^15 + 3764416/13*x^13 - 1841600/
11*x^11 + 634321/9*x^9 - 149700/7*x^7 + 4590*x^5 - 684*x^3 + 81*x
```

3.71 $\int \left(3-19x^2+32x^4-16x^6\right)^3 \, dx$

Optimal. Leaf size=63

$$-\frac{4096x^{19}}{19}+\frac{24576x^{17}}{17}-\frac{21248x^{15}}{5}+\frac{93440x^{13}}{13}-\frac{84912x^{11}}{11}+\frac{16448x^9}{3}-2605x^7+\frac{4113x^5}{5}-171x^3+27x$$

```
[Out] 27*x - 171*x^3 + (4113*x^5)/5 - 2605*x^7 + (16448*x^9)/3 - (84912
*x^11)/11 + (93440*x^13)/13 - (21248*x^15)/5 + (24576*x^17)/17 -
(4096*x^19)/19
```

Rubi [A] time = 0.292921, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$-\frac{4096x^{19}}{19}+\frac{24576x^{17}}{17}-\frac{21248x^{15}}{5}+\frac{93440x^{13}}{13}-\frac{84912x^{11}}{11}+\frac{16448x^9}{3}-2605x^7+\frac{4113x^5}{5}-171x^3+27x$$

Antiderivative was successfully verified.

```
[In]  Int[(3 - 19*x^2 + 32*x^4 - 16*x^6)^3,x]
```

```
[Out] 27*x - 171*x^3 + (4113*x^5)/5 - 2605*x^7 + (16448*x^9)/3 - (84912
*x^11)/11 + (93440*x^13)/13 - (21248*x^15)/5 + (24576*x^17)/17 -
(4096*x^19)/19
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-16*x**6+32*x**4-19*x**2+3)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.00329678, size = 63, normalized size = 1.

$$-\frac{4096x^{19}}{19}+\frac{24576x^{17}}{17}-\frac{21248x^{15}}{5}+\frac{93440x^{13}}{13}-\frac{84912x^{11}}{11}+\frac{16448x^9}{3}-2605x^7+\frac{4113x^5}{5}-171x^3+27x$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3 - 19*x^2 + 32*x^4 - 16*x^6)^3,x]
```

```
[Out] 27*x - 171*x^3 + (4113*x^5)/5 - 2605*x^7 + (16448*x^9)/3 - (84912
*x^11)/11 + (93440*x^13)/13 - (21248*x^15)/5 + (24576*x^17)/17 -
(4096*x^19)/19
```

Maple [A] time = 0.002, size = 50, normalized size = 0.8

$$27\,x-171\,x^3+\frac{4113\,x^5}{5}-2605\,x^7+\frac{16448\,x^9}{3}-\frac{84912\,x^{11}}{11}+\frac{93440\,x^{13}}{13}-\frac{21248\,x^{15}}{5}+\frac{24576\,x^{17}}{17}-\frac{4096\,x^{19}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-16*x^6+32*x^4-19*x^2+3)^3,x)
```

```
[Out] 27*x-171*x^3+4113/5*x^5-2605*x^7+16448/3*x^9-84912/11*x^11+93440/
13*x^13-21248/5*x^15+24576/17*x^17-4096/19*x^19
```

Maxima [A] time = 0.811283, size = 66, normalized size = 1.05

$$-\frac{4096}{19}\,x^{19}+\frac{24576}{17}\,x^{17}-\frac{21248}{5}\,x^{15}+\frac{93440}{13}\,x^{13}-\frac{84912}{11}\,x^{11}+\frac{16448}{3}\,x^{9}-2605\,x^{7}+\frac{4113}{5}\,x^{5}-171\,x^{3}+27\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(16*x^6 - 32*x^4 + 19*x^2 - 3)^3,x, algorithm="maxima")
```

```
[Out] -4096/19*x^19 + 24576/17*x^17 - 21248/5*x^15 + 93440/13*x^13 - 84
912/11*x^11 + 16448/3*x^9 - 2605*x^7 + 4113/5*x^5 - 171*x^3 + 27*
x
```

Fricas [A] time = 0.25633, size = 1, normalized size = 0.02

$$-\frac{4096}{19}x^{19}+\frac{24576}{17}x^{17}-\frac{21248}{5}x^{15}+\frac{93440}{13}x^{13}-\frac{84912}{11}x^{11}+\frac{16448}{3}x^{9}-2605x^{7}+\frac{4113}{5}x^{5}-171x^{3}+27x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(16*x^6 - 32*x^4 + 19*x^2 - 3)^3,x, algorithm="fricas")
```

```
[Out] -4096/19*x^19 + 24576/17*x^17 - 21248/5*x^15 + 93440/13*x^13 - 84
912/11*x^11 + 16448/3*x^9 - 2605*x^7 + 4113/5*x^5 - 171*x^3 + 27*
x
```

Sympy [A] time = 0.10067, size = 60, normalized size = 0.95

$$-\frac{4096x^{19}}{19}+\frac{24576x^{17}}{17}-\frac{21248x^{15}}{5}+\frac{93440x^{13}}{13}-\frac{84912x^{11}}{11}+\frac{16448x^{9}}{3}-2605x^{7}+\frac{4113x^{5}}{5}-171x^{3}+27x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-16*x**6+32*x**4-19*x**2+3)**3,x)
```

```
[Out] -4096*x**19/19 + 24576*x**17/17 - 21248*x**15/5 + 93440*x**13/13
- 84912*x**11/11 + 16448*x**9/3 - 2605*x**7 + 4113*x**5/5 - 171*x
**3 + 27*x
```

GIAC/XCAS [A] time = 0.261095, size = 66, normalized size = 1.05

$$-\frac{4096}{19}\,x^{19}+\frac{24576}{17}\,x^{17}-\frac{21248}{5}\,x^{15}+\frac{93440}{13}\,x^{13}-\frac{84912}{11}\,x^{11}+\frac{16448}{3}\,x^{9}-2605\,x^{7}+\frac{4113}{5}\,x^{5}-171\,x^{3}+27\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(16*x^6 - 32*x^4 + 19*x^2 - 3)^3,x, algorithm="giac")
```

```
[Out] -4096/19*x^19 + 24576/17*x^17 - 21248/5*x^15 + 93440/13*x^13 - 84
912/11*x^11 + 16448/3*x^9 - 2605*x^7 + 4113/5*x^5 - 171*x^3 + 27*
x
```

3.72 $\int \left(3-19x^2+32x^4-16x^6\right)^2\,dx$

Optimal. Leaf size=44

$$\frac{256x^{13}}{13}-\frac{1024x^{11}}{11}+\frac{544x^9}{3}-\frac{1312x^7}{7}+\frac{553x^5}{5}-38x^3+9x$$

```
[Out] 9*x - 38*x^3 + (553*x^5)/5 - (1312*x^7)/7 + (544*x^9)/3 - (1024*x
^11)/11 + (256*x^13)/13
```

Rubi [A] time = 0.233078, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$\frac{256x^{13}}{13}-\frac{1024x^{11}}{11}+\frac{544x^9}{3}-\frac{1312x^7}{7}+\frac{553x^5}{5}-38x^3+9x$$

Antiderivative was successfully verified.

```
[In]  Int[(3 - 19*x^2 + 32*x^4 - 16*x^6)^2,x]
```

```
[Out] 9*x - 38*x^3 + (553*x^5)/5 - (1312*x^7)/7 + (544*x^9)/3 - (1024*x
^11)/11 + (256*x^13)/13
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-16x^6+32x^4-19x^2+3\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-16*x**6+32*x**4-19*x**2+3)**2,x)
```

```
[Out] Integral((-16*x**6 + 32*x**4 - 19*x**2 + 3)**2, x)
```

Mathematica [A] time = 0.0011097, size = 44, normalized size = 1.

$$\frac{256x^{13}}{13}-\frac{1024x^{11}}{11}+\frac{544x^9}{3}-\frac{1312x^7}{7}+\frac{553x^5}{5}-38x^3+9x$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3 - 19*x^2 + 32*x^4 - 16*x^6)^2,x]
```

```
[Out] 9*x - 38*x^3 + (553*x^5)/5 - (1312*x^7)/7 + (544*x^9)/3 - (1024*x
^11)/11 + (256*x^13)/13
```

Maple [A] time = 0.002, size = 35, normalized size = 0.8

$$9\,x-38\,x^3+\frac{553\,x^5}{5}-\frac{1312\,x^7}{7}+\frac{544\,x^9}{3}-\frac{1024\,x^{11}}{11}+\frac{256\,x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-16*x^6+32*x^4-19*x^2+3)^2,x)
```

```
[Out] 9*x-38*x^3+553/5*x^5-1312/7*x^7+544/3*x^9-1024/11*x^11+256/13*x^1
3
```

Maxima [A] time = 0.818507, size = 46, normalized size = 1.05

$$\frac{256}{13}\,x^{13}-\frac{1024}{11}\,x^{11}+\frac{544}{3}\,x^{9}-\frac{1312}{7}\,x^{7}+\frac{553}{5}\,x^{5}-38\,x^{3}+9\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((16*x^6 - 32*x^4 + 19*x^2 - 3)^2,x, algorithm="maxima")
```

```
[Out] 256/13*x^13 - 1024/11*x^11 + 544/3*x^9 - 1312/7*x^7 + 553/5*x^5 -
 38*x^3 + 9*x
```

Fricas [A] time = 0.230174, size = 1, normalized size = 0.02

$$\frac{256}{13}x^{13}-\frac{1024}{11}x^{11}+\frac{544}{3}x^{9}-\frac{1312}{7}x^{7}+\frac{553}{5}x^{5}-38x^{3}+9x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((16*x^6 - 32*x^4 + 19*x^2 - 3)^2,x, algorithm="fricas")
```

```
[Out] 256/13*x^13 - 1024/11*x^11 + 544/3*x^9 - 1312/7*x^7 + 553/5*x^5 -
 38*x^3 + 9*x
```

Sympy [A] time = 0.085475, size = 41, normalized size = 0.93

$$\frac{256x^{13}}{13}-\frac{1024x^{11}}{11}+\frac{544x^{9}}{3}-\frac{1312x^{7}}{7}+\frac{553x^{5}}{5}-38x^{3}+9x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-16*x**6+32*x**4-19*x**2+3)**2,x)
```

```
[Out] 256*x**13/13 - 1024*x**11/11 + 544*x**9/3 - 1312*x**7/7 + 553*x**
5/5 - 38*x**3 + 9*x
```

GIAC/XCAS [A] time = 0.260387, size = 46, normalized size = 1.05

$$\frac{256}{13}\,x^{13}-\frac{1024}{11}\,x^{11}+\frac{544}{3}\,x^{9}-\frac{1312}{7}\,x^{7}+\frac{553}{5}\,x^{5}-38\,x^{3}+9\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((16*x^6 - 32*x^4 + 19*x^2 - 3)^2,x, algorithm="giac")
```

```
[Out] 256/13*x^13 - 1024/11*x^11 + 544/3*x^9 - 1312/7*x^7 + 553/5*x^5 -
 38*x^3 + 9*x
```

3.73 $\int \left(3-19x^2+32x^4-16x^6\right)\,dx$

Optimal. Leaf size=25

$$-\frac{16x^7}{7}+\frac{32x^5}{5}-\frac{19x^3}{3}+3x$$

```
[Out] 3*x - (19*x^3)/3 + (32*x^5)/5 - (16*x^7)/7
```

Rubi [A] time = 0.0101575, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.

$$-\frac{16x^7}{7}+\frac{32x^5}{5}-\frac{19x^3}{3}+3x$$

Antiderivative was successfully verified.

```
[In]  Int[3 - 19*x^2 + 32*x^4 - 16*x^6,x]

[Out] 3*x - (19*x^3)/3 + (32*x^5)/5 - (16*x^7)/7
```

Rubi in Sympy [A] time = 1.91822, size = 22, normalized size = 0.88

$$-\frac{16x^7}{7}+\frac{32x^5}{5}-\frac{19x^3}{3}+3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(-16*x**6+32*x**4-19*x**2+3,x)

[Out] -16*x**7/7 + 32*x**5/5 - 19*x**3/3 + 3*x
```

Mathematica [A] time = 0.0000582369, size = 25, normalized size = 1.

$$-\frac{16x^7}{7}+\frac{32x^5}{5}-\frac{19x^3}{3}+3x$$

Antiderivative was successfully verified.

```
[In]  Integrate[3 - 19*x^2 + 32*x^4 - 16*x^6,x]

[Out] 3*x - (19*x^3)/3 + (32*x^5)/5 - (16*x^7)/7
```

Maple [A] time = 0.001, size = 20, normalized size = 0.8

$$3\,x-\frac{19\,x^3}{3}+\frac{32\,x^5}{5}-\frac{16\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(-16*x^6+32*x^4-19*x^2+3,x)
```

```
[Out] 3*x-19/3*x^3+32/5*x^5-16/7*x^7
```

Maxima [A] time = 0.823775, size = 26, normalized size = 1.04

$$-\frac{16}{7}\,x^{7} + \frac{32}{5}\,x^{5} - \frac{19}{3}\,x^{3} + 3\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-16*x^6 + 32*x^4 - 19*x^2 + 3,x, algorithm="maxima")
```

```
[Out] -16/7*x^7 + 32/5*x^5 - 19/3*x^3 + 3*x
```

Fricas [A] time = 0.229822, size = 1, normalized size = 0.04

$$-\frac{16}{7}x^{7} + \frac{32}{5}x^{5} - \frac{19}{3}x^{3} + 3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-16*x^6 + 32*x^4 - 19*x^2 + 3,x, algorithm="fricas")
```

```
[Out] -16/7*x^7 + 32/5*x^5 - 19/3*x^3 + 3*x
```

Sympy [A] time = 0.064875, size = 22, normalized size = 0.88

$$-\frac{16x^{7}}{7} + \frac{32x^{5}}{5} - \frac{19x^{3}}{3} + 3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-16*x**6+32*x**4-19*x**2+3,x)
```

```
[Out] -16*x**7/7 + 32*x**5/5 - 19*x**3/3 + 3*x
```

GIAC/XCAS [A] time = 0.260544, size = 26, normalized size = 1.04

$$-\frac{16}{7}\,x^{7} + \frac{32}{5}\,x^{5} - \frac{19}{3}\,x^{3} + 3\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-16*x^6 + 32*x^4 - 19*x^2 + 3,x, algorithm="giac")
```

```
[Out] -16/7*x^7 + 32/5*x^5 - 19/3*x^3 + 3*x
```

3.74 $\int \frac{1}{3-19x^2+32x^4-16x^6}\,dx$

Optimal. Leaf size=31

$$\frac{1}{3}\tanh^{-1}(x)+\frac{1}{3}\tanh^{-1}(2x)-\frac{\tanh^{-1}\left(\frac{2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] ArcTanh[x]/3 + ArcTanh[2*x]/3 - ArcTanh[(2*x)/Sqrt[3]]/Sqrt[3]
```

Rubi [A] time = 0.0486934, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$\frac{1}{3}\tanh^{-1}(x)+\frac{1}{3}\tanh^{-1}(2x)-\frac{\tanh^{-1}\left(\frac{2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(3 - 19*x^2 + 32*x^4 - 16*x^6)^(-1),x]
```

```
[Out] ArcTanh[x]/3 + ArcTanh[2*x]/3 - ArcTanh[(2*x)/Sqrt[3]]/Sqrt[3]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{-16x^6+32x^4-19x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-16*x**6+32*x**4-19*x**2+3),x)
```

```
[Out] Integral(1/(-16*x**6 + 32*x**4 - 19*x**2 + 3), x)
```

Mathematica [A] time = 0.0243145, size = 62, normalized size = 2.

$$\frac{1}{6}\left(-\log\left(2x^2-3x+1\right)+\log\left(2x^2+3x+1\right)+\sqrt{3}\log\left(\sqrt{3}-2x\right)-\sqrt{3}\log\left(2x+\sqrt{3}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3 - 19*x^2 + 32*x^4 - 16*x^6)^(-1),x]
```

```
[Out] (Sqrt[3]*Log[Sqrt[3] - 2*x] - Sqrt[3]*Log[Sqrt[3] + 2*x] - Log[1
- 3*x + 2*x^2] + Log[1 + 3*x + 2*x^2])/6
```

Maple [A] time = 0.015, size = 42, normalized size = 1.4

$$-\frac{\ln(-1+x)}{6}-\frac{\sqrt{3}}{3}\mathit{Artanh}\left(\frac{2x\sqrt{3}}{3}\right)+\frac{\ln(1+x)}{6}-\frac{\ln(2x-1)}{6}+\frac{\ln(1+2x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-16*x^6+32*x^4-19*x^2+3),x)
```

```
[Out] -1/6*ln(-1+x)-1/3*arctanh(2/3*x*3^(1/2))*3^(1/2)+1/6*ln(1+x)-1/6*
ln(2*x-1)+1/6*ln(1+2*x)
```

Maxima [A] time = 0.884587, size = 73, normalized size = 2.35

$$\frac{1}{6}\sqrt{3}\log\left(\frac{2x-\sqrt{3}}{2x+\sqrt{3}}\right)+\frac{1}{6}\log(2x+1)-\frac{1}{6}\log(2x-1)+\frac{1}{6}\log(x+1)-\frac{1}{6}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(16*x^6 - 32*x^4 + 19*x^2 - 3),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*log((2*x - sqrt(3))/(2*x + sqrt(3))) + 1/6*log(2*x +
1) - 1/6*log(2*x - 1) + 1/6*log(x + 1) - 1/6*log(x - 1)
```

Fricas [A] time = 0.254331, size = 88, normalized size = 2.84

$$\frac{1}{18}\sqrt{3}\left(\sqrt{3}\log\left(2x^2+3x+1\right)-\sqrt{3}\log\left(2x^2-3x+1\right)+3\log\left(\frac{\sqrt{3}\left(4x^2+3\right)-12x}{4x^2-3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(16*x^6 - 32*x^4 + 19*x^2 - 3),x, algorithm="fricas")
```

```
[Out] 1/18*sqrt(3)*(sqrt(3)*log(2*x^2 + 3*x + 1) - sqrt(3)*log(2*x^2 -
3*x + 1) + 3*log((sqrt(3)*(4*x^2 + 3) - 12*x)/(4*x^2 - 3)))
```

Sympy [A] time = 0.321591, size = 63, normalized size = 2.03

$$\frac{\sqrt{3}\log\left(x-\frac{\sqrt{3}}{2}\right)}{6}-\frac{\sqrt{3}\log\left(x+\frac{\sqrt{3}}{2}\right)}{6}-\frac{\log\left(x^2-\frac{3x}{2}+\frac{1}{2}\right)}{6}+\frac{\log\left(x^2+\frac{3x}{2}+\frac{1}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-16*x**6+32*x**4-19*x**2+3),x)
```

```
[Out] sqrt(3)*log(x - sqrt(3)/2)/6 - sqrt(3)*log(x + sqrt(3)/2)/6 - log
(x**2 - 3*x/2 + 1/2)/6 + log(x**2 + 3*x/2 + 1/2)/6
```

GIAC/XCAS [A] time = 0.262427, size = 84, normalized size = 2.71

$$\frac{1}{6}\sqrt{3}\ln\left(\frac{\left|8x-4\sqrt{3}\right|}{\left|8x+4\sqrt{3}\right|}\right)+\frac{1}{6}\ln\left(\left|2x+1\right|\right)-\frac{1}{6}\ln\left(\left|2x-1\right|\right)+\frac{1}{6}\ln\left(\left|x+1\right|\right)-\frac{1}{6}\ln\left(\left|x-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(16*x^6 - 32*x^4 + 19*x^2 - 3),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*ln(abs(8*x - 4*sqrt(3))/abs(8*x + 4*sqrt(3))) + 1/6*l
n(abs(2*x + 1)) - 1/6*ln(abs(2*x - 1)) + 1/6*ln(abs(x + 1)) - 1/6
*ln(abs(x - 1))
```

3.75 $\int \frac{1}{(3-19x^2+32x^4-16x^6)^2} \, dx$

Optimal. Leaf size=89

$$\frac{2x}{3(3-4x^2)}+\frac{1}{18(1-2x)}+\frac{1}{36(1-x)}-\frac{1}{36(x+1)}-\frac{1}{18(2x+1)}+\frac{67}{54}\tanh^{-1}(x)-\frac{7}{27}\tanh^{-1}(2x)-\frac{5\tanh^{-1}\left(\frac{2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] 1/(18*(1 - 2*x)) + 1/(36*(1 - x)) - 1/(36*(1 + x)) - 1/(18*(1 + 2
*x)) + (2*x)/(3*(3 - 4*x^2)) + (67*ArcTanh[x])/54 - (7*ArcTanh[2*
x])/27 - (5*ArcTanh[(2*x)/Sqrt[3]])/(3*Sqrt[3])
```

Rubi [A] time = 0.122869, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$\frac{2x}{3(3-4x^2)}+\frac{1}{18(1-2x)}+\frac{1}{36(1-x)}-\frac{1}{36(x+1)}-\frac{1}{18(2x+1)}+\frac{67}{54}\tanh^{-1}(x)-\frac{7}{27}\tanh^{-1}(2x)-\frac{5\tanh^{-1}\left(\frac{2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(3 - 19*x^2 + 32*x^4 - 16*x^6)^(-2),x]
```

```
[Out] 1/(18*(1 - 2*x)) + 1/(36*(1 - x)) - 1/(36*(1 + x)) - 1/(18*(1 + 2
*x)) + (2*x)/(3*(3 - 4*x^2)) + (67*ArcTanh[x])/54 - (7*ArcTanh[2*
x])/27 - (5*ArcTanh[(2*x)/Sqrt[3]])/(3*Sqrt[3])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-16*x**6+32*x**4-19*x**2+3)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.0878126, size = 103, normalized size = 1.16

$$\frac{1}{108}\left(-\frac{6x\left(80x^4-104x^2+27\right)}{16x^6-32x^4+19x^2-3}+14\log(1-2x)+30\sqrt{3}\log\left(\sqrt{3}-2x\right)-67\log(1-x)+67\log(x+1)-14\log(2x+1)-30\sqrt{3}\log\left(2x+\sqrt{3}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3 - 19*x^2 + 32*x^4 - 16*x^6)^(-2),x]
```

```
[Out] ((-6*x*(27 - 104*x^2 + 80*x^4))/(-3 + 19*x^2 - 32*x^4 + 16*x^6) +
 14*Log[1 - 2*x] + 30*Sqrt[3]*Log[Sqrt[3] - 2*x] - 67*Log[1 - x]
```

```
+ 67*Log[1 + x] - 14*Log[1 + 2*x] - 30*Sqrt[3]*Log[Sqrt[3] + 2*x]
)/108
```

Maple [A] time = 0.031, size = 84, normalized size = 0.9

$$-\frac{1}{-36+36\,x}-\frac{67\,\ln\left(-1+x\right)}{108}-\frac{x}{6}\left(x^{2}-\frac{3}{4}\right)^{-1}-\frac{5\,\sqrt{3}}{9}Artanh\left(\frac{2\,x\sqrt{3}}{3}\right)-\frac{1}{36+36\,x}+\frac{67\,\ln\left(1+x\right)}{108}-\frac{1}{36\,x-18}+\frac{7\,\ln\left(2\,x-1\right)}{54}-\frac{1}{18+36\,x}-\frac{7\,\ln\left(1+2\,x\right)}{54}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-16*x^6+32*x^4-19*x^2+3)^2,x)
```

```
[Out] -1/36/(-1+x)-67/108*ln(-1+x)-1/6*x/(x^2-3/4)-5/9*arctanh(2/3*x*3^
(1/2))*3^(1/2)-1/36/(1+x)+67/108*ln(1+x)-1/18/(2*x-1)+7/54*ln(2*x
-1)-1/18/(1+2*x)-7/54*ln(1+2*x)
```

Maxima [A] time = 0.894588, size = 120, normalized size = 1.35

$$\frac{5}{18}\,\sqrt{3}\log\left(\frac{2\,x-\sqrt{3}}{2\,x+\sqrt{3}}\right)-\frac{80\,x^{5}-104\,x^{3}+27\,x}{18\,\left(16\,x^{6}-32\,x^{4}+19\,x^{2}-3\right)}-\frac{7}{54}\,\log\left(2\,x+1\right)+\frac{7}{54}\,\log\left(2\,x-1\right)+\frac{67}{108}\,\log\left(x+1\right)-\frac{67}{108}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((16*x^6 - 32*x^4 + 19*x^2 - 3)^(-2),x, algorithm="maxima")
```

```
[Out] 5/18*sqrt(3)*log((2*x - sqrt(3))/(2*x + sqrt(3))) - 1/18*(80*x^5
- 104*x^3 + 27*x)/(16*x^6 - 32*x^4 + 19*x^2 - 3) - 7/54*log(2*x +
 1) + 7/54*log(2*x - 1) + 67/108*log(x + 1) - 67/108*log(x - 1)
```

Fricas [A] time = 0.259509, size = 266, normalized size = 2.99

$$-\frac{\sqrt{3}\left(14\,\sqrt{3}\left(16\,x^{6}-32\,x^{4}+19\,x^{2}-3\right)\,\log\left(2\,x+1\right)-14\,\sqrt{3}\left(16\,x^{6}-32\,x^{4}+19\,x^{2}-3\right)\,\log\left(2\,x-1\right)-67\,\sqrt{3}\left(16\,x^{6}-32\,x^{4}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((16*x^6 - 32*x^4 + 19*x^2 - 3)^(-2),x, algorithm="fricas")
```

```
[Out] -1/324*sqrt(3)*(14*sqrt(3)*(16*x^6 - 32*x^4 + 19*x^2 - 3)*log(2*x
 + 1) - 14*sqrt(3)*(16*x^6 - 32*x^4 + 19*x^2 - 3)*log(2*x - 1) -
67*sqrt(3)*(16*x^6 - 32*x^4 + 19*x^2 - 3)*log(x + 1) + 67*sqrt(3)
*(16*x^6 - 32*x^4 + 19*x^2 - 3)*log(x - 1) - 90*(16*x^6 - 32*x^4
+ 19*x^2 - 3)*log((sqrt(3)*(4*x^2 + 3) - 12*x)/(4*x^2 - 3)) + 6*s
qrt(3)*(80*x^5 - 104*x^3 + 27*x))/(16*x^6 - 32*x^4 + 19*x^2 - 3)
```

Sympy [A] time = 3.85285, size = 104, normalized size = 1.17

$$-\frac{80x^{5}-104x^{3}+27x}{288x^{6}-576x^{4}+342x^{2}-54}-\frac{67\log\left(x-1\right)}{108}+\frac{7\log\left(x-\frac{1}{2}\right)}{54}-\frac{7\log\left(x+\frac{1}{2}\right)}{54}+\frac{67\log\left(x+1\right)}{108}+\frac{5\sqrt{3}\log\left(x-\frac{\sqrt{3}}{2}\right)}{18}-\frac{5\sqrt{3}\log\left(x+\frac{\sqrt{3}}{2}\right)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-16*x**6+32*x**4-19*x**2+3)**2,x)
```

```
[Out] -(80*x**5 - 104*x**3 + 27*x)/(288*x**6 - 576*x**4 + 342*x**2 - 54
) - 67*log(x - 1)/108 + 7*log(x - 1/2)/54 - 7*log(x + 1/2)/54 + 6
7*log(x + 1)/108 + 5*sqrt(3)*log(x - sqrt(3)/2)/18 - 5*sqrt(3)*lo
g(x + sqrt(3)/2)/18
```

GIAC/XCAS [A] time = 0.263432, size = 131, normalized size = 1.47

$$\frac{5}{18}\sqrt{3}\ln\left(\frac{\left|8\,x-4\,\sqrt{3}\right|}{\left|8\,x+4\,\sqrt{3}\right|}\right)-\frac{80\,x^{5}-104\,x^{3}+27\,x}{18\,(16\,x^{6}-32\,x^{4}+19\,x^{2}-3)}-\frac{7}{54}\ln\left(\left|2\,x+1\right|\right)+\frac{7}{54}\ln\left(\left|2\,x-1\right|\right)+\frac{67}{108}\ln\left(\left|x+1\right|\right)-\frac{67}{108}\ln\left(\left|x-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((16*x^6 - 32*x^4 + 19*x^2 - 3)^(-2),x, algorithm="giac")
```

```
[Out] 5/18*sqrt(3)*ln(abs(8*x - 4*sqrt(3))/abs(8*x + 4*sqrt(3))) - 1/18
*(80*x^5 - 104*x^3 + 27*x)/(16*x^6 - 32*x^4 + 19*x^2 - 3) - 7/54*
ln(abs(2*x + 1)) + 7/54*ln(abs(2*x - 1)) + 67/108*ln(abs(x + 1))
- 67/108*ln(abs(x - 1))
```

3.76 $\int \frac{1}{(3-19x^2+32x^4-16x^6)^3}\,dx$

Optimal. Leaf size=161

$$\frac{5x}{3\left(3-4x^2\right)}-\frac{2x}{3\left(3-4x^2\right)^2}-\frac{7}{108(1-2x)}+\frac{67}{432(1-x)}-\frac{67}{432(x+1)}+\frac{7}{108(2x+1)}+\frac{1}{108(1-2x)^2}+\frac{1}{432(1-x)^2}-\frac{1}{432(x+1)^2}-\frac{1}{108(2x+1)^2}+\frac{3913}{648}\tanh^{-1}(x)+\frac{67}{162}\tanh^{-1}(2x)-4\sqrt{3}\tanh^{-1}\left(\frac{2x}{\sqrt{3}}\right)+\frac{5\tanh^{-1}\left(\frac{2x}{\sqrt{3}}\right)}{6\sqrt{3}}$$

```
[Out] 1/(108*(1 - 2*x)^2) - 7/(108*(1 - 2*x)) + 1/(432*(1 - x)^2) + 67/
(432*(1 - x)) - 1/(432*(1 + x)^2) - 67/(432*(1 + x)) - 1/(108*(1
+ 2*x)^2) + 7/(108*(1 + 2*x)) - (2*x)/(3*(3 - 4*x^2)^2) + (5*x)/(
3*(3 - 4*x^2)) + (3913*ArcTanh[x])/648 + (67*ArcTanh[2*x])/162 +
(5*ArcTanh[(2*x)/Sqrt[3]])/(6*Sqrt[3]) - 4*Sqrt[3]*ArcTanh[(2*x)/
Sqrt[3]]
```

Rubi [A] time = 0.243963, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$\frac{5x}{3\left(3-4x^2\right)}-\frac{2x}{3\left(3-4x^2\right)^2}-\frac{7}{108(1-2x)}+\frac{67}{432(1-x)}-\frac{67}{432(x+1)}+\frac{7}{108(2x+1)}+\frac{1}{108(1-2x)^2}+\frac{1}{432(1-x)^2}-\frac{1}{432(x+1)^2}-\frac{1}{108(2x+1)^2}+\frac{3913}{648}\tanh^{-1}(x)+\frac{67}{162}\tanh^{-1}(2x)-4\sqrt{3}\tanh^{-1}\left(\frac{2x}{\sqrt{3}}\right)+\frac{5\tanh^{-1}\left(\frac{2x}{\sqrt{3}}\right)}{6\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(3 - 19*x^2 + 32*x^4 - 16*x^6)^(-3),x]
```

```
[Out] 1/(108*(1 - 2*x)^2) - 7/(108*(1 - 2*x)) + 1/(432*(1 - x)^2) + 67/
(432*(1 - x)) - 1/(432*(1 + x)^2) - 67/(432*(1 + x)) - 1/(108*(1
+ 2*x)^2) + 7/(108*(1 + 2*x)) - (2*x)/(3*(3 - 4*x^2)^2) + (5*x)/(
3*(3 - 4*x^2)) + (3913*ArcTanh[x])/648 + (67*ArcTanh[2*x])/162 +
(5*ArcTanh[(2*x)/Sqrt[3]])/(6*Sqrt[3]) - 4*Sqrt[3]*ArcTanh[(2*x)/
Sqrt[3]]
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-16*x**6+32*x**4-19*x**2+3)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.155048, size = 137, normalized size = 0.85

$$\frac{\frac{36x\left(80x^4-104x^2+27\right)}{\left(-16x^6+32x^4-19x^2+3\right)^2}-\frac{6x\left(2288x^4-2384x^2+345\right)}{16x^6-32x^4+19x^2-3}-268\log(1-2x)+2412\sqrt{3}\log\left(\sqrt{3}-2x\right)-3913\log(1-x)+3913\log(x+1)+}{1296}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3 - 19*x^2 + 32*x^4 - 16*x^6)^(-3),x]
```

```
[Out] ((36*x*(27 - 104*x^2 + 80*x^4))/(3 - 19*x^2 + 32*x^4 - 16*x^6)^2
- (6*x*(345 - 2384*x^2 + 2288*x^4))/(-3 + 19*x^2 - 32*x^4 + 16*x^
6) - 268*Log[1 - 2*x] + 2412*Sqrt[3]*Log[Sqrt[3] - 2*x] - 3913*Lo
g[1 - x] + 3913*Log[1 + x] + 268*Log[1 + 2*x] - 2412*Sqrt[3]*Log[
Sqrt[3] + 2*x])/1296
```

Maple [A] time = 0.035, size = 126, normalized size = 0.8

$$\frac{1}{432\,(-1+x)^2}-\frac{67}{-432+432\,x}-\frac{3913\,\ln(-1+x)}{1296}+64\,\frac{1}{(4\,x^2-3)^2}\left(-\frac{5\,x^3}{48}+\frac{13\,x}{192}\right)-\frac{67\,\sqrt{3}}{18}Artanh\left(\frac{2\,x\sqrt{3}}{3}\right)-\frac{1}{432\,(1+x)^2}-\frac{67}{432+432\,x}+\frac{3913\,\ln(1+x)}{1296}+\frac{1}{108\,(2\,x-1)^2}+\frac{7}{216\,x-108}-\frac{67\,\ln(2\,x-1)}{324}-\frac{1}{108\,(1+2\,x)^2}+\frac{7}{108+216\,x}+\frac{67\,\ln(1+2\,x)}{324}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-16*x^6+32*x^4-19*x^2+3)^3,x)
```

```
[Out] 1/432/(-1+x)^2-67/432/(-1+x)-3913/1296*ln(-1+x)+64*(-5/48*x^3+13/
192*x)/(4*x^2-3)^2-67/18*arctanh(2/3*x*3^(1/2))*3^(1/2)-1/432/(1+
x)^2-67/432/(1+x)+3913/1296*ln(1+x)+1/108/(2*x-1)^2+7/108/(2*x-1)
-67/324*ln(2*x-1)-1/108/(1+2*x)^2+7/108/(1+2*x)+67/324*ln(1+2*x)
```

Maxima [A] time = 0.917111, size = 161, normalized size = 1.

$$\frac{67}{36}\sqrt{3}\log\left(\frac{2\,x-\sqrt{3}}{2\,x+\sqrt{3}}\right)-\frac{36608\,x^{11}-111360\,x^9+125280\,x^7-63680\,x^5+14331\,x^3-1197\,x}{216\,(256\,x^{12}-1024\,x^{10}+1632\,x^8-1312\,x^6+553\,x^4-114\,x^2+9)}+\frac{67}{324}\log(2\,x+1)-\frac{67}{324}\log(2\,x-1)+\frac{3913}{1296}\log(x+1)-\frac{3913}{1296}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(16*x^6 - 32*x^4 + 19*x^2 - 3)^3,x, algorithm="maxima")
```

```
[Out] 67/36*sqrt(3)*log((2*x - sqrt(3))/(2*x + sqrt(3))) - 1/216*(36608
*x^11 - 111360*x^9 + 125280*x^7 - 63680*x^5 + 14331*x^3 - 1197*x)
/(256*x^12 - 1024*x^10 + 1632*x^8 - 1312*x^6 + 553*x^4 - 114*x^2
+ 9) + 67/324*log(2*x + 1) - 67/324*log(2*x - 1) + 3913/1296*log(
x + 1) - 3913/1296*log(x - 1)
```

Fricas [A] time = 0.274147, size = 408, normalized size = 2.53

$$\sqrt{3}\Big(268\,\sqrt{3}\big(256\,x^{12}-1024\,x^{10}+1632\,x^8-1312\,x^6+553\,x^4-114\,x^2+9\big)\log(2\,x+1)-268\,\sqrt{3}\big(256\,x^{12}-1024\,x^{10}+1632$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(16*x^6 - 32*x^4 + 19*x^2 - 3)^3,x, algorithm="fricas")
```

```
[Out] 1/3888*sqrt(3)*(268*sqrt(3)*(256*x^12 - 1024*x^10 + 1632*x^8 - 13
12*x^6 + 553*x^4 - 114*x^2 + 9)*log(2*x + 1) - 268*sqrt(3)*(256*x
^12 - 1024*x^10 + 1632*x^8 - 1312*x^6 + 553*x^4 - 114*x^2 + 9)*lo
g(2*x - 1) + 3913*sqrt(3)*(256*x^12 - 1024*x^10 + 1632*x^8 - 1312
*x^6 + 553*x^4 - 114*x^2 + 9)*log(x + 1) - 3913*sqrt(3)*(256*x^12
 - 1024*x^10 + 1632*x^8 - 1312*x^6 + 553*x^4 - 114*x^2 + 9)*log(x
 - 1) + 7236*(256*x^12 - 1024*x^10 + 1632*x^8 - 1312*x^6 + 553*x^
4 - 114*x^2 + 9)*log((sqrt(3)*(4*x^2 + 3) - 12*x)/(4*x^2 - 3)) -
6*sqrt(3)*(36608*x^11 - 111360*x^9 + 125280*x^7 - 63680*x^5 + 143
31*x^3 - 1197*x))/(256*x^12 - 1024*x^10 + 1632*x^8 - 1312*x^6 + 5
53*x^4 - 114*x^2 + 9)
```

Sympy [A] time = 4.31092, size = 134, normalized size = 0.83

$$-\frac{36608x^{11}-111360x^{9}+125280x^{7}-63680x^{5}+14331x^{3}-1197x}{55296x^{12}-221184x^{10}+352512x^{8}-283392x^{6}+119448x^{4}-24624x^{2}+1944}-\frac{3913\log(x-1)}{1296}-\frac{67\log\left(x-\frac{1}{2}\right)}{324}+\frac{67\log\left(x+\frac{1}{2}\right)}{324}+\frac{3913\log(x+1)}{1296}+\frac{67\sqrt{3}\log\left(x-\frac{\sqrt{3}}{2}\right)}{36}-\frac{67\sqrt{3}\log\left(x+\frac{\sqrt{3}}{2}\right)}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-16*x**6+32*x**4-19*x**2+3)**3,x)
```

```
[Out] -(36608*x**11 - 111360*x**9 + 125280*x**7 - 63680*x**5 + 14331*x*
*3 - 1197*x)/(55296*x**12 - 221184*x**10 + 352512*x**8 - 283392*x
**6 + 119448*x**4 - 24624*x**2 + 1944) - 3913*log(x - 1)/1296 - 6
7*log(x - 1/2)/324 + 67*log(x + 1/2)/324 + 3913*log(x + 1)/1296 +
 67*sqrt(3)*log(x - sqrt(3)/2)/36 - 67*sqrt(3)*log(x + sqrt(3)/2)
/36
```

GIAC/XCAS [A] time = 0.26227, size = 151, normalized size = 0.94

$$\frac{67}{36}\sqrt{3}\ln\left(\frac{|8x-4\sqrt{3}|}{|8x+4\sqrt{3}|}\right)-\frac{36608x^{11}-111360x^{9}+125280x^{7}-63680x^{5}+14331x^{3}-1197x}{216\,(16x^{6}-32x^{4}+19x^{2}-3)^{2}}+\frac{67}{324}\ln(|2x+1|)-\frac{67}{324}\ln(|2x-1|)+\frac{3913}{1296}\ln(|x+1|)-\frac{3913}{1296}\ln(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(16*x^6 - 32*x^4 + 19*x^2 - 3)^3,x, algorithm="giac")
```

```
[Out] 67/36*sqrt(3)*ln(abs(8*x - 4*sqrt(3))/abs(8*x + 4*sqrt(3))) - 1/2
16*(36608*x^11 - 111360*x^9 + 125280*x^7 - 63680*x^5 + 14331*x^3
- 1197*x)/(16*x^6 - 32*x^4 + 19*x^2 - 3)^2 + 67/324*ln(abs(2*x +
1)) - 67/324*ln(abs(2*x - 1)) + 3913/1296*ln(abs(x + 1)) - 3913/1
296*ln(abs(x - 1))
```

3.77 $\int \frac{1}{(-1+7x^2-7x^4+x^6)^2} \, dx$

Optimal. Leaf size=91

$$\frac{x}{32(1-x^2)} + \frac{(99-17x^2)\,x}{128(x^4-6x^2+1)} + \frac{5}{32}\tanh^{-1}(x) + \frac{1}{512}\left(3\sqrt{2}-4\right)\tanh^{-1}\left(\left(\sqrt{2}-1\right)x\right) + \frac{1}{512}\left(4+3\sqrt{2}\right)\tanh^{-1}\left(\left(1+\sqrt{2}\right)x\right)$$

```
[Out] x/(32*(1 - x^2)) + (x*(99 - 17*x^2))/(128*(1 - 6*x^2 + x^4)) + (5
*ArcTanh[x])/32 + ((-4 + 3*Sqrt[2])*ArcTanh[(-1 + Sqrt[2])*x])/51
2 + ((4 + 3*Sqrt[2])*ArcTanh[(1 + Sqrt[2])*x])/512
```

Rubi [B] time = 0.264653, antiderivative size = 205, normalized size of antiderivative = 2.25, number of steps used = 15, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$

$$-\frac{41-17x}{256(-x^2+2x+1)} + \frac{17x+41}{256(-x^2-2x+1)} + \frac{1}{64(1-x)} - \frac{1}{64(x+1)} + \frac{1}{512}\left(2-7\sqrt{2}\right)\log\left(-x-\sqrt{2}+1\right) + \frac{1}{512}\left(2+7\sqrt{2}\right)\log\left(-x+\sqrt{2}+1\right) - \frac{1}{512}\left(2-7\sqrt{2}\right)\log\left(x-\sqrt{2}+1\right) - \frac{1}{512}\left(2+7\sqrt{2}\right)\log\left(x+\sqrt{2}+1\right) - \frac{17\tanh^{-1}\left(\frac{1-x}{\sqrt{2}}\right)}{256\sqrt{2}} + \frac{5}{32}\tanh^{-1}(x) + \frac{17\tanh^{-1}\left(\frac{x+1}{\sqrt{2}}\right)}{256\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + 7*x^2 - 7*x^4 + x^6)^(-2),x]
```

```
[Out] 1/(64*(1 - x)) - 1/(64*(1 + x)) + (41 + 17*x)/(256*(1 - 2*x - x^2
)) - (41 - 17*x)/(256*(1 + 2*x - x^2)) - (17*ArcTanh[(1 - x)/Sqrt
[2]])/(256*Sqrt[2]) + (5*ArcTanh[x])/32 + (17*ArcTanh[(1 + x)/Sqr
t[2]])/(256*Sqrt[2]) + ((2 - 7*Sqrt[2])*Log[1 - Sqrt[2] - x])/512
 + ((2 + 7*Sqrt[2])*Log[1 + Sqrt[2] - x])/512 - ((2 - 7*Sqrt[2])*
Log[1 - Sqrt[2] + x])/512 - ((2 + 7*Sqrt[2])*Log[1 + Sqrt[2] + x]
)/512
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**6-7*x**4+7*x**2-1)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.146309, size = 132, normalized size = 1.45

$$\frac{-\frac{8x(21x^4-140x^2+103)}{x^6-7x^4+7x^2-1} - 80\log(1-x) - \left(4+3\sqrt{2}\right)\log\left(-x+\sqrt{2}-1\right) + \left(4-3\sqrt{2}\right)\log\left(-x+\sqrt{2}+1\right) + 80\log(x+1) + \left(4+3\right.}{1024}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + 7*x^2 - 7*x^4 + x^6)^(-2),x]
```

```
[Out] ((-8*x*(103 - 140*x^2 + 21*x^4))/(-1 + 7*x^2 - 7*x^4 + x^6) - 80*
Log[1 - x] - (4 + 3*Sqrt[2])*Log[-1 + Sqrt[2] - x] + (4 - 3*Sqrt[
2])*Log[1 + Sqrt[2] - x] + 80*Log[1 + x] + (4 + 3*Sqrt[2])*Log[-1
 + Sqrt[2] + x] + (-4 + 3*Sqrt[2])*Log[1 + Sqrt[2] + x])/1024
```

Maple [A] time = 0.028, size = 116, normalized size = 1.3

$$\frac{1}{128\,x^2-256\,x-128}\left(-\frac{17\,x}{2}+\frac{41}{2}\right)+\frac{\ln\left(x^2-2\,x-1\right)}{256}+\frac{3\,\sqrt{2}}{512}\mathit{Artanh}\left(\frac{(2\,x-2)\,\sqrt{2}}{4}\right)-\frac{1}{-64+64\,x}-\frac{5\,\ln(-1+x)}{64}-\frac{1}{64+64\,x}+\frac{5\,\ln(1+x)}{64}-\frac{1}{128\,x^2+256\,x-128}\left(\frac{17\,x}{2}+\frac{41}{2}\right)-\frac{\ln\left(x^2+2\,x-1\right)}{256}+\frac{3\,\sqrt{2}}{512}\mathit{Artanh}\left(\frac{(2+2\,x)\,\sqrt{2}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^6-7*x^4+7*x^2-1)^2,x)
```

```
[Out] 1/128*(-17/2*x+41/2)/(x^2-2*x-1)+1/256*ln(x^2-2*x-1)+3/512*2^(1/2
)*arctanh(1/4*(2*x-2)*2^(1/2))-1/64/(-1+x)-5/64*ln(-1+x)-1/64/(1+
x)+5/64*ln(1+x)-1/128*(17/2*x+41/2)/(x^2+2*x-1)-1/256*ln(x^2+2*x-
1)+3/512*2^(1/2)*arctanh(1/4*(2+2*x)*2^(1/2))
```

Maxima [A] time = 0.888377, size = 162, normalized size = 1.78

$$-\frac{3}{1024}\,\sqrt{2}\log\left(\frac{2\left(x-\sqrt{2}+1\right)}{2\,x+2\,\sqrt{2}+2}\right)-\frac{3}{1024}\,\sqrt{2}\log\left(\frac{2\left(x-\sqrt{2}-1\right)}{2\,x+2\,\sqrt{2}-2}\right)-\frac{21\,x^5-140\,x^3+103\,x}{128\,(x^6-7\,x^4+7\,x^2-1)}-\frac{1}{256}\,\log\left(x^2+2\,x-1\right)+\frac{1}{256}\,\log\left(x^2-2\,x-1\right)+\frac{5}{64}\,\log\left(x+1\right)-\frac{5}{64}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^6 - 7*x^4 + 7*x^2 - 1)^(-2),x, algorithm="maxima")
```

```
[Out] -3/1024*sqrt(2)*log(2*(x - sqrt(2) + 1)/((2*sqrt(2)) + 2*x + 2))
- 3/1024*sqrt(2)*log(2*(x - sqrt(2) - 1)/((2*sqrt(2)) + 2*x - 2))
 - 1/128*(21*x^5 - 140*x^3 + 103*x)/(x^6 - 7*x^4 + 7*x^2 - 1) - 1
/256*log(x^2 + 2*x - 1) + 1/256*log(x^2 - 2*x - 1) + 5/64*log(x +
 1) - 5/64*log(x - 1)
```

Fricas [A] time = 0.291253, size = 324, normalized size = 3.56

$$-\frac{\sqrt{2}\Big(2\,\sqrt{2}(x^6-7\,x^4+7\,x^2-1)\,\log\left(x^2+2\,x-1\right)-2\,\sqrt{2}(x^6-7\,x^4+7\,x^2-1)\,\log\left(x^2-2\,x-1\right)-40\,\sqrt{2}(x^6-7\,x^4+7\,x^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^6 - 7*x^4 + 7*x^2 - 1)^(-2),x, algorithm="fricas")
```

```
[Out] -1/1024*sqrt(2)*(2*sqrt(2)*(x^6 - 7*x^4 + 7*x^2 - 1)*log(x^2 + 2*
x - 1) - 2*sqrt(2)*(x^6 - 7*x^4 + 7*x^2 - 1)*log(x^2 - 2*x - 1) -
 40*sqrt(2)*(x^6 - 7*x^4 + 7*x^2 - 1)*log(x + 1) + 40*sqrt(2)*(x^
6 - 7*x^4 + 7*x^2 - 1)*log(x - 1) - 3*(x^6 - 7*x^4 + 7*x^2 - 1)*l
```

```
og((sqrt(2)*(x^2 + 2*x + 3) + 4*x + 4)/(x^2 + 2*x - 1)) - 3*(x^6
- 7*x^4 + 7*x^2 - 1)*log((sqrt(2)*(x^2 - 2*x + 3) + 4*x - 4)/(x^2
 - 2*x - 1)) + 4*sqrt(2)*(21*x^5 - 140*x^3 + 103*x))/(x^6 - 7*x^4
 + 7*x^2 - 1)
```

Sympy [A] time = 4.00064, size = 296, normalized size = 3.25

$$-\frac{21x^5-140x^3+103x}{128x^6-896x^4+896x^2-128}-\frac{5\log(x-1)}{64}+\frac{5\log(x+1)}{64}+\left(-\frac{1}{256}+\frac{3\sqrt{2}}{1024}\right)\log\left(x-\frac{8071264001}{202624020}-\frac{471550901878784\left(-\frac{1}{256}+\frac{3\sqrt{2}}{1024}\right)^3}{2979765}+\frac{1299552375287054336\left(-\frac{1}{256}+\frac{3\sqrt{2}}{1024}\right)^5}{50656005}+\frac{8071264001\sqrt{2}}{270165360}\right)+\left(-\frac{3\sqrt{2}}{1024}-\frac{1}{256}\right)\log\left(x-\frac{8071264001\sqrt{2}}{270165360}-\frac{8071264001}{202624020}+\frac{1299552375287054336\left(-\frac{3\sqrt{2}}{1024}-\frac{1}{256}\right)^5}{50656005}-\frac{471550901878784\left(-\frac{3\sqrt{2}}{1024}-\frac{1}{256}\right)^3}{2979765}\right)+\left(-\frac{3\sqrt{2}}{1024}+\frac{1}{256}\right)\log\left(x-\frac{8071264001\sqrt{2}}{270165360}+\frac{1299552375287054336\left(-\frac{3\sqrt{2}}{1024}+\frac{1}{256}\right)^5}{50656005}-\frac{471550901878784\left(-\frac{3\sqrt{2}}{1024}+\frac{1}{256}\right)^3}{2979765}+\frac{8071264001}{202624020}\right)+\left(\frac{1}{256}+\frac{3\sqrt{2}}{1024}\right)\log\left(x-\frac{471550901878784\left(\frac{1}{256}+\frac{3\sqrt{2}}{1024}\right)^3}{2979765}+\frac{1299552375287054336\left(\frac{1}{256}+\frac{3\sqrt{2}}{1024}\right)^5}{50656005}+\frac{8071264001}{202624020}+\frac{8071264001\sqrt{2}}{270165360}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**6-7*x**4+7*x**2-1)**2,x)
```

```
[Out] -(21*x**5 - 140*x**3 + 103*x)/(128*x**6 - 896*x**4 + 896*x**2 - 1
28) - 5*log(x - 1)/64 + 5*log(x + 1)/64 + (-1/256 + 3*sqrt(2)/102
4)*log(x - 8071264001/202624020 - 471550901878784*(-1/256 + 3*sqr
t(2)/1024)**3/2979765 + 1299552375287054336*(-1/256 + 3*sqrt(2)/1
024)**5/50656005 + 8071264001*sqrt(2)/270165360) + (-3*sqrt(2)/10
24 - 1/256)*log(x - 8071264001*sqrt(2)/270165360 - 8071264001/202
624020 + 1299552375287054336*(-3*sqrt(2)/1024 - 1/256)**5/5065600
5 - 471550901878784*(-3*sqrt(2)/1024 - 1/256)**3/2979765) + (-3*s
qrt(2)/1024 + 1/256)*log(x - 8071264001*sqrt(2)/270165360 + 12995
52375287054336*(-3*sqrt(2)/1024 + 1/256)**5/50656005 - 4715509018
78784*(-3*sqrt(2)/1024 + 1/256)**3/2979765 + 8071264001/202624020
) + (1/256 + 3*sqrt(2)/1024)*log(x - 471550901878784*(1/256 + 3*s
qrt(2)/1024)**3/2979765 + 1299552375287054336*(1/256 + 3*sqrt(2)/
1024)**5/50656005 + 8071264001/202624020 + 8071264001*sqrt(2)/270
165360)
```

GIAC/XCAS [A] time = 0.269489, size = 181, normalized size = 1.99

$$-\frac{3}{1024}\sqrt{2}\ln\left(\frac{\left|2x-2\sqrt{2}+2\right|}{\left|2x+2\sqrt{2}+2\right|}\right)-\frac{3}{1024}\sqrt{2}\ln\left(\frac{\left|2x-2\sqrt{2}-2\right|}{\left|2x+2\sqrt{2}-2\right|}\right)-\frac{21x^5-140x^3+103x}{128(x^6-7x^4+7x^2-1)}-\frac{1}{256}\ln\left(\left|x^2+2x-1\right|\right)+\frac{1}{256}\ln\left(\left|x^2-2x-1\right|\right)+\frac{5}{64}\ln\left(\left|x+1\right|\right)-\frac{5}{64}\ln\left(\left|x-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^6 - 7*x^4 + 7*x^2 - 1)^(-2),x, algorithm="giac")
```

```
[Out] -3/1024*sqrt(2)*ln(abs(2*x - 2*sqrt(2) + 2)/abs(2*x + 2*sqrt(2) +
 2)) - 3/1024*sqrt(2)*ln(abs(2*x - 2*sqrt(2) - 2)/abs(2*x + 2*sqr
t(2) - 2)) - 1/128*(21*x^5 - 140*x^3 + 103*x)/(x^6 - 7*x^4 + 7*x^
2 - 1) - 1/256*ln(abs(x^2 + 2*x - 1)) + 1/256*ln(abs(x^2 - 2*x -
1)) + 5/64*ln(abs(x + 1)) - 5/64*ln(abs(x - 1))
```

3.78 $\int \frac{x^3}{c+(a+bx)^2}\,dx$

Optimal. Leaf size=78

$$\frac{(3a^2-c)\log\left((a+bx)^2+c\right)}{2b^4}-\frac{a(a^2-3c)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b^4\sqrt{c}}+\frac{(a+bx)^2}{2b^4}-\frac{3ax}{b^3}$$

```
[Out] (-3*a*x)/b^3 + (a + b*x)^2/(2*b^4) - (a*(a^2 - 3*c)*ArcTan[(a + b
*x)/Sqrt[c]])/(b^4*Sqrt[c]) + ((3*a^2 - c)*Log[c + (a + b*x)^2])/
(2*b^4)
```

Rubi [A] time = 0.139936, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{(3a^2-c)\log\left((a+bx)^2+c\right)}{2b^4}-\frac{a(a^2-3c)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b^4\sqrt{c}}+\frac{(a+bx)^2}{2b^4}-\frac{3ax}{b^3}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/(c + (a + b*x)^2),x]
```

```
[Out] (-3*a*x)/b^3 + (a + b*x)^2/(2*b^4) - (a*(a^2 - 3*c)*ArcTan[(a + b
*x)/Sqrt[c]])/(b^4*Sqrt[c]) + ((3*a^2 - c)*Log[c + (a + b*x)^2])/
(2*b^4)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a(3a+3bx)}{b^4}-\frac{a(a^2-3c)\operatorname{atan}\left(\frac{a+bx}{\sqrt{c}}\right)}{b^4\sqrt{c}}+\frac{(3a^2-c)\log\left(c+(a+bx)^2\right)}{2b^4}+\frac{\int^{a+bx}x\,dx}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(c+(b*x+a)**2),x)
```

```
[Out] -a*(3*a + 3*b*x)/b**4 - a*(a**2 - 3*c)*atan((a + b*x)/sqrt(c))/(b
**4*sqrt(c)) + (3*a**2 - c)*log(c + (a + b*x)**2)/(2*b**4) + Inte
gral(x, (x, a + b*x))/b**4
```

Mathematica [A] time = 0.0848409, size = 73, normalized size = 0.94

$$\frac{-\frac{2(a^3-3ac)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}}+(3a^2-c)\log\left(a^2+2abx+b^2x^2+c\right)+bx(bx-4a)}{2b^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/(c + (a + b*x)^2),x]
```

```
[Out] (b*x*(-4*a + b*x) - (2*(a^3 - 3*a*c)*ArcTan[(a + b*x)/Sqrt[c]])/S
qrt[c] + (3*a^2 - c)*Log[a^2 + c + 2*a*b*x + b^2*x^2])/(2*b^4)
```

Maple [A] time = 0.011, size = 127, normalized size = 1.6

$$\frac{x^2}{2\,b^2} - 2\,\frac{ax}{b^3} + \frac{3\,\ln\left(b^2x^2 + 2\,abx + a^2 + c\right)\,a^2}{2\,b^4} - \frac{\ln\left(b^2x^2 + 2\,abx + a^2 + c\right)\,c}{2\,b^4} - \frac{a^3}{b^4}\arctan\left(\frac{2\,b^2x + 2\,ab}{2\,b}\frac{1}{\sqrt{c}}\right)\frac{1}{\sqrt{c}} + 3\,\frac{\sqrt{c}a}{b^4}\arctan\left(1/2\,\frac{2\,b^2x + 2\,ab}{b\sqrt{c}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^3/(c+(b*x+a)^2),x)
```

```
[Out] 1/2/b^2*x^2-2*a*x/b^3+3/2/b^4*ln(b^2*x^2+2*a*b*x+a^2+c)*a^2-1/2/b
^4*ln(b^2*x^2+2*a*b*x+a^2+c)*c-1/b^4/c^(1/2)*arctan(1/2*(2*b^2*x+
2*a*b)/b/c^(1/2))*a^3+3/b^4*c^(1/2)*arctan(1/2*(2*b^2*x+2*a*b)/b/
c^(1/2))*a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x^3/((b*x + a)^2 + c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.257958, size = 1, normalized size = 0.01

$$\left[-\frac{\left(a^3 - 3\,ac\right)\log\left(\frac{2\,bcx + 2\,ac + \left(b^2x^2 + 2\,abx + a^2 - c\right)\sqrt{-c}}{b^2x^2 + 2\,abx + a^2 + c}\right) - \left(b^2x^2 - 4\,abx + \left(3\,a^2 - c\right)\log\left(b^2x^2 + 2\,abx + a^2 + c\right)\right)\sqrt{-c}}{2\,b^4\sqrt{-c}},\ -\frac{2\left(a^3 - 3\,ac\right)\arctan\left(\frac{bx+a}{\sqrt{c}}\right) - \left(b^2x^2 - 4\,abx + \left(3\,a^2 - c\right)\log\left(b^2x^2 + 2\,abx + a^2 + c\right)\right)\sqrt{c}}{2\,b^4\sqrt{c}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x^3/((b*x + a)^2 + c),x, algorithm="fricas")
```

```
[Out] [-1/2*((a^3 - 3*a*c)*log((2*b*c*x + 2*a*c + (b^2*x^2 + 2*a*b*x +
a^2 - c)*sqrt(-c))/(b^2*x^2 + 2*a*b*x + a^2 + c)) - (b^2*x^2 - 4*
a*b*x + (3*a^2 - c)*log(b^2*x^2 + 2*a*b*x + a^2 + c))*sqrt(-c))/(
b^4*sqrt(-c)), -1/2*(2*(a^3 - 3*a*c)*arctan((b*x + a)/sqrt(c)) -
(b^2*x^2 - 4*a*b*x + (3*a^2 - c)*log(b^2*x^2 + 2*a*b*x + a^2 + c)
)*sqrt(c))/(b^4*sqrt(c))]
```

Sympy [A] time = 2.8432, size = 209, normalized size = 2.68

$$-\frac{2ax}{b^3} + \left(-\frac{a\sqrt{-c}\left(a^2 - 3c\right)}{2b^4c} + \frac{3a^2 - c}{2b^4}\right)\log\left(x + \frac{a^4 - 2b^4c\left(-\frac{a\sqrt{-c}(a^2-3c)}{2b^4c} + \frac{3a^2-c}{2b^4}\right) - c^2}{a^3b - 3abc}\right) + \left(\frac{a\sqrt{-c}\left(a^2 - 3c\right)}{2b^4c} + \frac{3a^2 - c}{2b^4}\right)\log\left(x + \frac{a^4 - 2b^4c\left(\frac{a\sqrt{-c}(a^2-3c)}{2b^4c} + \frac{3a^2-c}{2b^4}\right) - c^2}{a^3b - 3abc}\right) + \frac{x^2}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(c+(b*x+a)**2),x)
```

```
[Out] -2*a*x/b**3 + (-a*sqrt(-c)*(a**2 - 3*c)/(2*b**4*c) + (3*a**2 - c)
/(2*b**4))*log(x + (a**4 - 2*b**4*c*(-a*sqrt(-c)*(a**2 - 3*c)/(2*
b**4*c) + (3*a**2 - c)/(2*b**4)) - c**2)/(a**3*b - 3*a*b*c)) + (a
*sqrt(-c)*(a**2 - 3*c)/(2*b**4*c) + (3*a**2 - c)/(2*b**4))*log(x
+ (a**4 - 2*b**4*c*(a*sqrt(-c)*(a**2 - 3*c)/(2*b**4*c) + (3*a**2
- c)/(2*b**4)) - c**2)/(a**3*b - 3*a*b*c)) + x**2/(2*b**2)
```

GIAC/XCAS [A] time = 0.262915, size = 104, normalized size = 1.33

$$\frac{\left(3\,a^{2}-c\right)\ln\left(b^{2}x^{2}+2\,abx+a^{2}+c\right)}{2\,b^{4}}-\frac{\left(a^{3}-3\,ac\right)\arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{b^{4}\sqrt{c}}+\frac{b^{2}x^{2}-4\,abx}{2\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((b*x + a)^2 + c),x, algorithm="giac")
```

```
[Out] 1/2*(3*a^2 - c)*ln(b^2*x^2 + 2*a*b*x + a^2 + c)/b^4 - (a^3 - 3*a*
c)*arctan((b*x + a)/sqrt(c))/(b^4*sqrt(c)) + 1/2*(b^2*x^2 - 4*a*b
*x)/b^4
```

3.79 $\int \frac{x^2}{c+(a+bx)^2}\,dx$

Optimal. Leaf size=50

$$\frac{(a^2-c)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b^3\sqrt{c}}-\frac{a\log\left((a+bx)^2+c\right)}{b^3}+\frac{x}{b^2}$$

```
[Out] x/b^2 + ((a^2 - c)*ArcTan[(a + b*x)/Sqrt[c]])/(b^3*Sqrt[c]) - (a*
Log[c + (a + b*x)^2])/b^3
```

Rubi [A] time = 0.0918802, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{(a^2-c)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b^3\sqrt{c}}-\frac{a\log\left((a+bx)^2+c\right)}{b^3}+\frac{x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(c + (a + b*x)^2),x]
```

```
[Out] x/b^2 + ((a^2 - c)*ArcTan[(a + b*x)/Sqrt[c]])/(b^3*Sqrt[c]) - (a*
Log[c + (a + b*x)^2])/b^3
```

Rubi in Sympy [A] time = 13.093, size = 49, normalized size = 0.98

$$-\frac{a\log\left(c+(a+bx)^2\right)}{b^3}+\frac{a}{b^3}+\frac{x}{b^2}+\frac{(a^2-c)\operatorname{atan}\left(\frac{a+bx}{\sqrt{c}}\right)}{b^3\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**2/(c+(b*x+a)**2),x)
```

```
[Out] -a*log(c + (a + b*x)**2)/b**3 + a/b**3 + x/b**2 + (a**2 - c)*atan
((a + b*x)/sqrt(c))/(b**3*sqrt(c))
```

Mathematica [A] time = 0.0464113, size = 54, normalized size = 1.08

$$\frac{-a\log\left(a^2+2abx+b^2x^2+c\right)+\frac{(a^2-c)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}}+bx}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(c + (a + b*x)^2),x]
```

```
[Out] (b*x + ((a^2 - c)*ArcTan[(a + b*x)/Sqrt[c]])/Sqrt[c] - a*Log[a^2
+ c + 2*a*b*x + b^2*x^2])/b^3
```

Maple [A] time = 0.004, size = 89, normalized size = 1.8

$$\frac{x}{b^2}-\frac{a\ln\left(b^2x^2+2\,abx+a^2+c\right)}{b^3}+\frac{a^2}{b^3}\arctan\left(\frac{2\,b^2x+2\,ab}{2\,b}\frac{1}{\sqrt{c}}\right)\frac{1}{\sqrt{c}}-\frac{1}{b^3}\sqrt{c}\arctan\left(\frac{2\,b^2x+2\,ab}{2\,b}\frac{1}{\sqrt{c}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(c+(b*x+a)^2),x)
```

```
[Out] x/b^2-1/b^3*a*ln(b^2*x^2+2*a*b*x+a^2+c)+1/b^3/c^(1/2)*arctan(1/2*
(2*b^2*x+2*a*b)/b/c^(1/2))*a^2-1/b^3*c^(1/2)*arctan(1/2*(2*b^2*x+
2*a*b)/b/c^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((b*x + a)^2 + c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.279845, size = 1, normalized size = 0.02

$$\left[-\frac{\left(a^2-c\right)\log\left(-\frac{2\,bcx+2\,ac-\left(b^2x^2+2\,abx+a^2-c\right)\sqrt{-c}}{b^2x^2+2\,abx+a^2+c}\right)-2\left(bx-a\log\left(b^2x^2+2\,abx+a^2+c\right)\right)\sqrt{-c}}{2\,b^3\sqrt{-c}},\ \frac{\left(a^2-c\right)\arctan\left(\frac{bx+a}{\sqrt{c}}\right)+(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((b*x + a)^2 + c),x, algorithm="fricas")
```

```
[Out] [-1/2*((a^2 - c)*log(-(2*b*c*x + 2*a*c - (b^2*x^2 + 2*a*b*x + a^2
 - c)*sqrt(-c))/(b^2*x^2 + 2*a*b*x + a^2 + c)) - 2*(b*x - a*log(b
^2*x^2 + 2*a*b*x + a^2 + c))*sqrt(-c))/(b^3*sqrt(-c)), ((a^2 - c)
*arctan((b*x + a)/sqrt(c)) + (b*x - a*log(b^2*x^2 + 2*a*b*x + a^2
 + c))*sqrt(c))/(b^3*sqrt(c))]
```

Sympy [A] time = 2.18159, size = 153, normalized size = 3.06

$$\left(-\frac{a}{b^3}-\frac{\sqrt{-c}\left(a^2-c\right)}{2b^3c}\right)\log\left(x+\frac{a^3+ac+2b^3c\left(-\frac{a}{b^3}-\frac{\sqrt{-c}\left(a^2-c\right)}{2b^3c}\right)}{a^2b-bc}\right)+\left(-\frac{a}{b^3}+\frac{\sqrt{-c}\left(a^2-c\right)}{2b^3c}\right)\log\left(x+\frac{a^3+ac+2b^3c\left(-\frac{a}{b^3}+\frac{\sqrt{-c}\left(a^2-c\right)}{2b^3c}\right)}{a^2b-bc}\right)+\frac{x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(c+(b*x+a)**2),x)
```

```
[Out] (-a/b**3 - sqrt(-c)*(a**2 - c)/(2*b**3*c))*log(x + (a**3 + a*c +
2*b**3*c*(-a/b**3 - sqrt(-c)*(a**2 - c)/(2*b**3*c)))/(a**2*b - b*
c)) + (-a/b**3 + sqrt(-c)*(a**2 - c)/(2*b**3*c))*log(x + (a**3 +
a*c + 2*b**3*c*(-a/b**3 + sqrt(-c)*(a**2 - c)/(2*b**3*c)))/(a**2*
b - b*c)) + x/b**2
```

GIAC/XCAS [A] time = 0.26322, size = 73, normalized size = 1.46

$$\frac{x}{b^2} - \frac{a\ln\left(b^2x^2 + 2\,abx + a^2 + c\right)}{b^3} + \frac{\left(a^2 - c\right)\arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{b^3\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((b*x + a)^2 + c),x, algorithm="giac")
```

```
[Out] x/b^2 - a*ln(b^2*x^2 + 2*a*b*x + a^2 + c)/b^3 + (a^2 - c)*arctan(
(b*x + a)/sqrt(c))/(b^3*sqrt(c))
```

3.80 $\int \frac{x}{c+(a+bx)^2}\,dx$

Optimal. Leaf size=41

$$\frac{\log\left((a+bx)^2+c\right)}{2b^2}-\frac{a\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b^2\sqrt{c}}$$

```
[Out] -((a*ArcTan[(a + b*x)/Sqrt[c]])/(b^2*Sqrt[c])) + Log[c + (a + b*x
)^2]/(2*b^2)
```

Rubi [A] time = 0.0525262, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308

$$\frac{\log\left((a+bx)^2+c\right)}{2b^2}-\frac{a\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b^2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(c + (a + b*x)^2),x]
```

```
[Out] -((a*ArcTan[(a + b*x)/Sqrt[c]])/(b^2*Sqrt[c])) + Log[c + (a + b*x
)^2]/(2*b^2)
```

Rubi in Sympy [A] time = 7.56939, size = 36, normalized size = 0.88

$$-\frac{a\,\mathrm{atan}\left(\frac{a+bx}{\sqrt{c}}\right)}{b^2\sqrt{c}}+\frac{\log\left(c+(a+bx)^2\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(c+(b*x+a)**2),x)
```

```
[Out] -a*atan((a + b*x)/sqrt(c))/(b**2*sqrt(c)) + log(c + (a + b*x)**2)
/(2*b**2)
```

Mathematica [A] time = 0.0222516, size = 38, normalized size = 0.93

$$\frac{\log\left((a+bx)^2+c\right)-\frac{2a\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}}}{2b^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(c + (a + b*x)^2),x]
```

```
[Out] ((-2*a*ArcTan[(a + b*x)/Sqrt[c]])/Sqrt[c] + Log[c + (a + b*x)^2])
/(2*b^2)
```

Maple [A] time = 0.004, size = 54, normalized size = 1.3

$$\frac{\ln\left(b^2x^2+2\,abx+a^2+c\right)}{2\,b^2}-\frac{a}{b^2}\arctan\left(\frac{2\,b^2x+2\,ab}{2\,b}\frac{1}{\sqrt{c}}\right)\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(c+(b*x+a)^2),x)
```

```
[Out] 1/2/b^2*ln(b^2*x^2+2*a*b*x+a^2+c)-a/b^2/c^(1/2)*arctan(1/2*(2*b^2
*x+2*a*b)/b/c^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((b*x + a)^2 + c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.256959, size = 1, normalized size = 0.02

$$\left[\frac{a \log\left(-\frac{2\,bcx+2\,ac-\left(b^2x^2+2\,abx+a^2-c\right)\sqrt{-c}}{b^2x^2+2\,abx+a^2+c}\right)+\sqrt{-c}\log\left(b^2x^2+2\,abx+a^2+c\right)}{2\,b^2\sqrt{-c}},\right.$$
$$\left.-\frac{2\,a\arctan\left(\frac{bx+a}{\sqrt{c}}\right)-\sqrt{c}\log\left(b^2x^2+2\,abx+a^2+c\right)}{2\,b^2\sqrt{c}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((b*x + a)^2 + c),x, algorithm="fricas")
```

```
[Out] [1/2*(a*log(-(2*b*c*x + 2*a*c - (b^2*x^2 + 2*a*b*x + a^2 - c)*sqr
t(-c))/(b^2*x^2 + 2*a*b*x + a^2 + c)) + sqrt(-c)*log(b^2*x^2 + 2*
a*b*x + a^2 + c))/(b^2*sqrt(-c)), -1/2*(2*a*arctan((b*x + a)/sqrt
(c)) - sqrt(c)*log(b^2*x^2 + 2*a*b*x + a^2 + c))/(b^2*sqrt(c))]
```

Sympy [A] time = 0.64546, size = 124, normalized size = 3.02

$$\left(-\frac{a\sqrt{-c}}{2b^2c}+\frac{1}{2b^2}\right)\log\left(x+\frac{a^2-2b^2c\left(-\frac{a\sqrt{-c}}{2b^2c}+\frac{1}{2b^2}\right)+c}{ab}\right)$$
$$+\left(\frac{a\sqrt{-c}}{2b^2c}+\frac{1}{2b^2}\right)\log\left(x+\frac{a^2-2b^2c\left(\frac{a\sqrt{-c}}{2b^2c}+\frac{1}{2b^2}\right)+c}{ab}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c+(b*x+a)**2),x)
```

```
[Out] (-a*sqrt(-c)/(2*b**2*c) + 1/(2*b**2))*log(x + (a**2 - 2*b**2*c*(-
a*sqrt(-c)/(2*b**2*c) + 1/(2*b**2)) + c)/(a*b)) + (a*sqrt(-c)/(2*
b**2*c) + 1/(2*b**2))*log(x + (a**2 - 2*b**2*c*(a*sqrt(-c)/(2*b**
2*c) + 1/(2*b**2)) + c)/(a*b))
```

GIAC/XCAS [A] time = 0.267374, size = 58, normalized size = 1.41

$$-\frac{a \arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{b^2\sqrt{c}} + \frac{\ln\left(b^2x^2 + 2\,abx + a^2 + c\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((b*x + a)^2 + c),x, algorithm="giac")
```

```
[Out] -a*arctan((b*x + a)/sqrt(c))/(b^2*sqrt(c)) + 1/2*ln(b^2*x^2 + 2*a
*b*x + a^2 + c)/b^2
```

3.81 $\int \frac{1}{c+(a+bx)^2}\,dx$

Optimal. Leaf size=21

$$\frac{\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b\sqrt{c}}$$

```
[Out] ArcTan[(a + b*x)/Sqrt[c]]/(b*Sqrt[c])
```

Rubi [A] time = 0.020165, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + (a + b*x)^2)^(-1),x]

[Out] ArcTan[(a + b*x)/Sqrt[c]]/(b*Sqrt[c])
```

Rubi in Sympy [A] time = 2.47644, size = 17, normalized size = 0.81

$$\frac{\operatorname{atan}\left(\frac{a+bx}{\sqrt{c}}\right)}{b\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c+(b*x+a)**2),x)

[Out] atan((a + b*x)/sqrt(c))/(b*sqrt(c))
```

Mathematica [A] time = 0.00574561, size = 21, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + (a + b*x)^2)^(-1),x]

[Out] ArcTan[(a + b*x)/Sqrt[c]]/(b*Sqrt[c])
```

Maple [A] time = 0.002, size = 28, normalized size = 1.3

$$\frac{1}{b}\arctan\left(\frac{2\,b^2x+2\,ab}{2\,b}\frac{1}{\sqrt{c}}\right)\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c+(b*x+a)^2),x)
```

```
[Out] 1/b/c^(1/2)*arctan(1/2*(2*b^2*x+2*a*b)/b/c^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x + a)^2 + c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.273865, size = 1, normalized size = 0.05

$$\left[\frac{\log\left(\frac{2\,bcx+2\,ac+\left(b^2x^2+2\,abx+a^2-c\right)\sqrt{-c}}{b^2x^2+2\,abx+a^2+c}\right)}{2\,b\sqrt{-c}}, \frac{\arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{b\sqrt{c}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x + a)^2 + c),x, algorithm="fricas")
```

```
[Out] [1/2*log((2*b*c*x + 2*a*c + (b^2*x^2 + 2*a*b*x + a^2 - c)*sqrt(-c
))/(b^2*x^2 + 2*a*b*x + a^2 + c))/(b*sqrt(-c)), arctan((b*x + a)/
sqrt(c))/(b*sqrt(c))]
```

Sympy [A] time = 0.443774, size = 54, normalized size = 2.57

$$\frac{-\frac{\sqrt{-\frac{1}{c}}\log\left(x+\frac{a-c\sqrt{-\frac{1}{c}}}{b}\right)}{2}+\frac{\sqrt{-\frac{1}{c}}\log\left(x+\frac{a+c\sqrt{-\frac{1}{c}}}{b}\right)}{2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c+(b*x+a)**2),x)
```

```
[Out] (-sqrt(-1/c)*log(x + (a - c*sqrt(-1/c))/b)/2 + sqrt(-1/c)*log(x +
 (a + c*sqrt(-1/c))/b)/2)/b
```

GIAC/XCAS [A] time = 0.26049, size = 23, normalized size = 1.1

$$\frac{\arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{b\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x + a)^2 + c),x, algorithm="giac")
```

```
[Out] arctan((b*x + a)/sqrt(c))/(b*sqrt(c))
```

3.82 $\int \frac{1}{x(c+(a+bx)^2)} dx$

Optimal. Leaf size=59

$$-\frac{\log\left((a+bx)^2+c\right)}{2\left(a^2+c\right)}-\frac{a\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}\left(a^2+c\right)}+\frac{\log(x)}{a^2+c}$$

```
[Out] -((a*ArcTan[(a + b*x)/Sqrt[c]])/(Sqrt[c]*(a^2 + c))) + Log[x]/(a^
2 + c) - Log[c + (a + b*x)^2]/(2*(a^2 + c))
```

Rubi [A] time = 0.082148, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4

$$-\frac{\log\left((a+bx)^2+c\right)}{2\left(a^2+c\right)}-\frac{a\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}\left(a^2+c\right)}+\frac{\log(x)}{a^2+c}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(c + (a + b*x)^2)),x]
```

```
[Out] -((a*ArcTan[(a + b*x)/Sqrt[c]])/(Sqrt[c]*(a^2 + c))) + Log[x]/(a^
2 + c) - Log[c + (a + b*x)^2]/(2*(a^2 + c))
```

Rubi in Sympy [A] time = 10.9615, size = 53, normalized size = 0.9

$$-\frac{a\,\mathrm{atan}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}\left(a^2+c\right)}+\frac{\log\left(-bx\right)}{a^2+c}-\frac{\log\left(c+(a+bx)^2\right)}{2\left(a^2+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(c+(b*x+a)**2),x)
```

```
[Out] -a*atan((a + b*x)/sqrt(c))/(sqrt(c)*(a**2 + c)) + log(-b*x)/(a**2
 + c) - log(c + (a + b*x)**2)/(2*(a**2 + c))
```

Mathematica [A] time = 0.0572488, size = 48, normalized size = 0.81

$$-\frac{\log\left((a+bx)^2+c\right)+\frac{2a\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}}-2\log(bx)}{2\left(a^2+c\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(c + (a + b*x)^2)),x]
```

```
[Out] -((2*a*ArcTan[(a + b*x)/Sqrt[c]])/Sqrt[c] - 2*Log[b*x] + Log[c +
(a + b*x)^2])/(2*(a^2 + c))
```

Maple [A] time = 0.008, size = 72, normalized size = 1.2

$$-\frac{\ln\left(b^2x^2+2\,abx+a^2+c\right)}{2\,a^2+2\,c}-\frac{a}{a^2+c}\arctan\left(\frac{2\,b^2x+2\,ab}{2\,b}\frac{1}{\sqrt{c}}\right)\frac{1}{\sqrt{c}}+\frac{\ln(x)}{a^2+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(c+(b*x+a)^2),x)
```

```
[Out] -1/2/(a^2+c)*ln(b^2*x^2+2*a*b*x+a^2+c)-1/(a^2+c)*a/c^(1/2)*arctan
(1/2*(2*b^2*x+2*a*b)/b/c^(1/2))+ln(x)/(a^2+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(((b*x + a)^2 + c)*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.274189, size = 1, normalized size = 0.02

$$\left[\frac{a \log\left(-\frac{2\,bcx+2\,ac-\left(b^{2}x^{2}+2\,abx+a^{2}-c\right)\sqrt{-c}}{b^{2}x^{2}+2\,abx+a^{2}+c}\right) - \sqrt{-c}\left(\log\left(b^{2}x^{2}+2\,abx+a^{2}+c\right) - 2\,\log\left(x\right)\right)}{2\,\left(a^{2}+c\right)\sqrt{-c}}, -\frac{2\,a\arctan\left(\frac{bx+a}{\sqrt{c}}\right) + \sqrt{c}\left(\log\left(b^{2}x^{2}+2\,abx+a^{2}+c\right) - 2\,\log\left(x\right)\right)}{2\,\left(a^{2}+c\right)\sqrt{c}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(((b*x + a)^2 + c)*x),x, algorithm="fricas")
```

```
[Out] [1/2*(a*log(-(2*b*c*x + 2*a*c - (b^2*x^2 + 2*a*b*x + a^2 - c)*sqr
t(-c))/(b^2*x^2 + 2*a*b*x + a^2 + c)) - sqrt(-c)*(log(b^2*x^2 + 2
*a*b*x + a^2 + c) - 2*log(x)))/((a^2 + c)*sqrt(-c)), -1/2*(2*a*ar
ctan((b*x + a)/sqrt(c)) + sqrt(c)*(log(b^2*x^2 + 2*a*b*x + a^2 +
c) - 2*log(x)))/((a^2 + c)*sqrt(c))]
```

Sympy [A] time = 5.9779, size = 738, normalized size = 12.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(c+(b*x+a)**2),x)
```

```
[Out] (-a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2 + c)))*log(x + (-4*a**
6*c*(-a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2 + c)))**2 + 4*a**4
*c**2*(-a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2 + c)))**2 - 6*a*
*4*c*(-a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2 + c))) + 20*a**2*
c**3*(-a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2 + c)))**2 - 12*a*
*2*c**2*(-a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2 + c))) + 10*a*
*2*c + 12*c**4*(-a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2 + c)))*
*2 - 6*c**3*(-a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2 + c))) - 6
*c**2)/(a**3*b + 9*a*b*c)) + (a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(2*
(a**2 + c)))*log(x + (-4*a**6*c*(a*sqrt(-c)/(2*c*(a**2 + c)) - 1/
(2*(a**2 + c)))**2 + 4*a**4*c**2*(a*sqrt(-c)/(2*c*(a**2 + c)) - 1
```

```
/(2*(a**2 + c)))**2 - 6*a**4*c*(a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(
2*(a**2 + c))) + 20*a**2*c**3*(a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(2
*(a**2 + c)))**2 - 12*a**2*c**2*(a*sqrt(-c)/(2*c*(a**2 + c)) - 1/
(2*(a**2 + c))) + 10*a**2*c + 12*c**4*(a*sqrt(-c)/(2*c*(a**2 + c)
) - 1/(2*(a**2 + c)))**2 - 6*c**3*(a*sqrt(-c)/(2*c*(a**2 + c)) -
1/(2*(a**2 + c))) - 6*c**2)/(a**3*b + 9*a*b*c)) + log(x + (-4*a**
6*c/(a**2 + c)**2 + 4*a**4*c**2/(a**2 + c)**2 - 6*a**4*c/(a**2 +
c) + 20*a**2*c**3/(a**2 + c)**2 - 12*a**2*c**2/(a**2 + c) + 10*a*
*2*c + 12*c**4/(a**2 + c)**2 - 6*c**3/(a**2 + c) - 6*c**2)/(a**3*
b + 9*a*b*c))/(a**2 + c)
```

GIAC/XCAS [A] time = 0.26141, size = 84, normalized size = 1.42

$$-\frac{a\arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{(a^2+c)\sqrt{c}} - \frac{\ln\left(b^2x^2+2\,abx+a^2+c\right)}{2\,(a^2+c)} + \frac{\ln\left(|x|\right)}{a^2+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(((b*x + a)^2 + c)*x),x, algorithm="giac")
```

```
[Out] -a*arctan((b*x + a)/sqrt(c))/((a^2 + c)*sqrt(c)) - 1/2*ln(b^2*x^2
 + 2*a*b*x + a^2 + c)/(a^2 + c) + ln(abs(x))/(a^2 + c)
```

3.83 $\int \frac{1}{x^2(c+(a+bx)^2)}\,dx$

Optimal. Leaf size=79

$$-\frac{2ab\log(x)}{(a^2+c)^2}+\frac{ab\log\left((a+bx)^2+c\right)}{(a^2+c)^2}+\frac{b\left(a^2-c\right)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}\,(a^2+c)^2}-\frac{1}{x\,(a^2+c)}$$

```
[Out] -(1/((a^2 + c)*x)) + (b*(a^2 - c)*ArcTan[(a + b*x)/Sqrt[c]])/(Sqr
t[c]*(a^2 + c)^2) - (2*a*b*Log[x])/(a^2 + c)^2 + (a*b*Log[c + (a
+ b*x)^2])/(a^2 + c)^2
```

Rubi [A] time = 0.191214, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4

$$-\frac{2ab\log(x)}{(a^2+c)^2}+\frac{ab\log\left((a+bx)^2+c\right)}{(a^2+c)^2}+\frac{b\left(a^2-c\right)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}\,(a^2+c)^2}-\frac{1}{x\,(a^2+c)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(c + (a + b*x)^2)),x]
```

```
[Out] -(1/((a^2 + c)*x)) + (b*(a^2 - c)*ArcTan[(a + b*x)/Sqrt[c]])/(Sqr
t[c]*(a^2 + c)^2) - (2*a*b*Log[x])/(a^2 + c)^2 + (a*b*Log[c + (a
+ b*x)^2])/(a^2 + c)^2
```

Rubi in Sympy [A] time = 24.0221, size = 76, normalized size = 0.96

$$-\frac{2ab\log(-bx)}{(a^2+c)^2}+\frac{ab\log\left(c+(a+bx)^2\right)}{(a^2+c)^2}+\frac{b\left(a^2-c\right)\operatorname{atan}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}\,(a^2+c)^2}-\frac{1}{x\,(a^2+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(c+(b*x+a)**2),x)
```

```
[Out] -2*a*b*log(-b*x)/(a**2 + c)**2 + a*b*log(c + (a + b*x)**2)/(a**2
+ c)**2 + b*(a**2 - c)*atan((a + b*x)/sqrt(c))/(sqrt(c)*(a**2 + c
)**2) - 1/(x*(a**2 + c))
```

Mathematica [A] time = 0.0748831, size = 81, normalized size = 1.03

$$\frac{bx\left(a^2-c\right)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)-\sqrt{c}\left(-abx\log\left(a^2+2abx+b^2x^2+c\right)+a^2+2abx\log(x)+c\right)}{\sqrt{c}x\,(a^2+c)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(c + (a + b*x)^2)),x]
```

```
[Out] (b*(a^2 - c)*x*ArcTan[(a + b*x)/Sqrt[c]] - Sqrt[c]*(a^2 + c + 2*a
*b*x*Log[x] - a*b*x*Log[a^2 + c + 2*a*b*x + b^2*x^2]))/(Sqrt[c]*(
a^2 + c)^2*x)
```

Maple [A] time = 0.011, size = 123, normalized size = 1.6

$$\frac{ab\ln\left(b^2x^2+2\,abx+a^2+c\right)}{\left(a^2+c\right)^2}+\frac{a^2b}{\left(a^2+c\right)^2}\arctan\left(\frac{2\,b^2x+2\,ab}{2\,b}\frac{1}{\sqrt{c}}\right)\frac{1}{\sqrt{c}}-\frac{b}{\left(a^2+c\right)^2}\sqrt{c}\arctan\left(\frac{2\,b^2x+2\,ab}{2\,b}\frac{1}{\sqrt{c}}\right)-\frac{1}{\left(a^2+c\right)x}-2\,\frac{ab\ln(x)}{\left(a^2+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(c+(b*x+a)^2),x)
```

```
[Out] b/(a^2+c)^2*a*ln(b^2*x^2+2*a*b*x+a^2+c)+b/(a^2+c)^2/c^(1/2)*arcta
n(1/2*(2*b^2*x+2*a*b)/b/c^(1/2))*a^2-b/(a^2+c)^2*c^(1/2)*arctan(1
/2*(2*b^2*x+2*a*b)/b/c^(1/2))-1/(a^2+c)/x-2*a*b*ln(x)/(a^2+c)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(((b*x + a)^2 + c)*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.278085, size = 1, normalized size = 0.01

$$\left[-\frac{\left(a^2b-bc\right)x\log\left(-\frac{2\,bcx+2\,ac-\left(b^2x^2+2\,abx+a^2-c\right)\sqrt{-c}}{b^2x^2+2\,abx+a^2+c}\right)-2\left(abx\log\left(b^2x^2+2\,abx+a^2+c\right)-2\,abx\log(x)-a^2-c\right)\sqrt{-c}}{2\left(a^4+2\,a^2c+c^2\right)\sqrt{-c}x},\ -\frac{(c}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(((b*x + a)^2 + c)*x^2),x, algorithm="fricas")
```

```
[Out] [-1/2*((a^2*b - b*c)*x*log(-(2*b*c*x + 2*a*c - (b^2*x^2 + 2*a*b*x
 + a^2 - c)*sqrt(-c))/(b^2*x^2 + 2*a*b*x + a^2 + c)) - 2*(a*b*x*l
og(b^2*x^2 + 2*a*b*x + a^2 + c) - 2*a*b*x*log(x) - a^2 - c)*sqrt(
-c))/((a^4 + 2*a^2*c + c^2)*sqrt(-c)*x), ((a^2*b - b*c)*x*arctan(
(b*x + a)/sqrt(c)) + (a*b*x*log(b^2*x^2 + 2*a*b*x + a^2 + c) - 2*
a*b*x*log(x) - a^2 - c)*sqrt(c))/((a^4 + 2*a^2*c + c^2)*sqrt(c)*x
)]
```

Sympy [A] time = 15.4473, size = 1620, normalized size = 20.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(c+(b*x+a)**2),x)
```

```
[Out] -2*a*b*log(x + (-16*a**13*b**2*c/(a**2 + c)**4 + 48*a**11*b**2*c*
*2/(a**2 + c)**4 + 352*a**9*b**2*c**3/(a**2 + c)**4 - 20*a**9*b**
```

```
2*c/(a**2 + c)**2 + 608*a**7*b**2*c**4/(a**2 + c)**4 - 64*a**7*b*
*2*c**2/(a**2 + c)**2 + 432*a**5*b**2*c**5/(a**2 + c)**4 - 72*a**
5*b**2*c**3/(a**2 + c)**2 + 36*a**5*b**2*c + 112*a**3*b**2*c**6/(
a**2 + c)**4 - 32*a**3*b**2*c**4/(a**2 + c)**2 - 88*a**3*b**2*c**
2 - 4*a*b**2*c**5/(a**2 + c)**2 + 4*a*b**2*c**3)/(a**6*b**3 + 33*
a**4*b**3*c - 33*a**2*b**3*c**2 - b**3*c**3))/(a**2 + c)**2 + (a*
b/(a**2 + c)**2 - b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c
**2)))*log(x + (-4*a**11*c*(a*b/(a**2 + c)**2 - b*sqrt(-c)*(a**2
- c)/(2*c*(a**4 + 2*a**2*c + c**2)))**2 + 12*a**9*c**2*(a*b/(a**2
 + c)**2 - b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2)))*
*2 + 10*a**8*b*c*(a*b/(a**2 + c)**2 - b*sqrt(-c)*(a**2 - c)/(2*c*
(a**4 + 2*a**2*c + c**2))) + 88*a**7*c**3*(a*b/(a**2 + c)**2 - b*
sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2)))**2 + 32*a**6*
b*c**2*(a*b/(a**2 + c)**2 - b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*
a**2*c + c**2))) + 36*a**5*b**2*c + 152*a**5*c**4*(a*b/(a**2 + c)
**2 - b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2)))**2 +
36*a**4*b*c**3*(a*b/(a**2 + c)**2 - b*sqrt(-c)*(a**2 - c)/(2*c*(a
**4 + 2*a**2*c + c**2))) - 88*a**3*b**2*c**2 + 108*a**3*c**5*(a*b
/(a**2 + c)**2 - b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c*
*2)))**2 + 16*a**2*b*c**4*(a*b/(a**2 + c)**2 - b*sqrt(-c)*(a**2 -
 c)/(2*c*(a**4 + 2*a**2*c + c**2))) + 4*a*b**2*c**3 + 28*a*c**6*(
a*b/(a**2 + c)**2 - b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c +
 c**2)))**2 + 2*b*c**5*(a*b/(a**2 + c)**2 - b*sqrt(-c)*(a**2 - c)
/(2*c*(a**4 + 2*a**2*c + c**2))))/(a**6*b**3 + 33*a**4*b**3*c - 3
3*a**2*b**3*c**2 - b**3*c**3)) + (a*b/(a**2 + c)**2 + b*sqrt(-c)*
(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2)))*log(x + (-4*a**11*c*(a
*b/(a**2 + c)**2 + b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c +
c**2)))**2 + 12*a**9*c**2*(a*b/(a**2 + c)**2 + b*sqrt(-c)*(a**2 -
 c)/(2*c*(a**4 + 2*a**2*c + c**2)))**2 + 10*a**8*b*c*(a*b/(a**2 +
 c)**2 + b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2))) +
88*a**7*c**3*(a*b/(a**2 + c)**2 + b*sqrt(-c)*(a**2 - c)/(2*c*(a**
4 + 2*a**2*c + c**2)))**2 + 32*a**6*b*c**2*(a*b/(a**2 + c)**2 + b
*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2))) + 36*a**5*b*
*2*c + 152*a**5*c**4*(a*b/(a**2 + c)**2 + b*sqrt(-c)*(a**2 - c)/(
2*c*(a**4 + 2*a**2*c + c**2)))**2 + 36*a**4*b*c**3*(a*b/(a**2 + c
)**2 + b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2))) - 88
*a**3*b**2*c**2 + 108*a**3*c**5*(a*b/(a**2 + c)**2 + b*sqrt(-c)*(
a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2)))**2 + 16*a**2*b*c**4*(a*
b/(a**2 + c)**2 + b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c
**2))) + 4*a*b**2*c**3 + 28*a*c**6*(a*b/(a**2 + c)**2 + b*sqrt(-c
)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2)))**2 + 2*b*c**5*(a*b/(
a**2 + c)**2 + b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2
))))/(a**6*b**3 + 33*a**4*b**3*c - 33*a**2*b**3*c**2 - b**3*c**3)
) - 1/(x*(a**2 + c))
```

GIAC/XCAS [A] time = 0.265838, size = 158, normalized size = 2.

$$\frac{ab\ln\left(b^2x^2 + 2\,abx + a^2 + c\right)}{a^4 + 2\,a^2c + c^2} - \frac{2\,ab\ln\left(|x|\right)}{a^4 + 2\,a^2c + c^2} + \frac{\left(a^2b^2 - b^2c\right)\arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{(a^4 + 2\,a^2c + c^2)b\sqrt{c}} - \frac{1}{(a^2 + c)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(((b*x + a)^2 + c)*x^2),x, algorithm="giac")
```

```
[Out] a*b*ln(b^2*x^2 + 2*a*b*x + a^2 + c)/(a^4 + 2*a^2*c + c^2) - 2*a*b
*ln(abs(x))/(a^4 + 2*a^2*c + c^2) + (a^2*b^2 - b^2*c)*arctan((b*x
 + a)/sqrt(c))/((a^4 + 2*a^2*c + c^2)*b*sqrt(c)) - 1/((a^2 + c)*x
)
```

3.84 $\int \frac{1}{x^3(c+(a+bx)^2)} dx$

Optimal. Leaf size=121

$$\frac{b^2\left(3a^2-c\right)\log(x)}{\left(a^2+c\right)^3}-\frac{b^2\left(3a^2-c\right)\log\left((a+bx)^2+c\right)}{2\left(a^2+c\right)^3}-\frac{ab^2\left(a^2-3c\right)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}\left(a^2+c\right)^3}+\frac{2ab}{x\left(a^2+c\right)^2}-\frac{1}{2x^2\left(a^2+c\right)}$$

```
[Out] -1/(2*(a^2 + c)*x^2) + (2*a*b)/((a^2 + c)^2*x) - (a*b^2*(a^2 - 3*
c)*ArcTan[(a + b*x)/Sqrt[c]])/(Sqrt[c]*(a^2 + c)^3) + (b^2*(3*a^2
 - c)*Log[x])/(a^2 + c)^3 - (b^2*(3*a^2 - c)*Log[c + (a + b*x)^2]
)/(2*(a^2 + c)^3)
```

Rubi [A] time = 0.276778, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4

$$\frac{b^2\left(3a^2-c\right)\log(x)}{\left(a^2+c\right)^3}-\frac{b^2\left(3a^2-c\right)\log\left((a+bx)^2+c\right)}{2\left(a^2+c\right)^3}-\frac{ab^2\left(a^2-3c\right)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}\left(a^2+c\right)^3}+\frac{2ab}{x\left(a^2+c\right)^2}-\frac{1}{2x^2\left(a^2+c\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*(c + (a + b*x)^2)),x]
```

```
[Out] -1/(2*(a^2 + c)*x^2) + (2*a*b)/((a^2 + c)^2*x) - (a*b^2*(a^2 - 3*
c)*ArcTan[(a + b*x)/Sqrt[c]])/(Sqrt[c]*(a^2 + c)^3) + (b^2*(3*a^2
 - c)*Log[x])/(a^2 + c)^3 - (b^2*(3*a^2 - c)*Log[c + (a + b*x)^2]
)/(2*(a^2 + c)^3)
```

Rubi in Sympy [A] time = 37.1168, size = 112, normalized size = 0.93

$$-\frac{ab^2\left(a^2-3c\right)\operatorname{atan}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}\left(a^2+c\right)^3}+\frac{2ab}{x\left(a^2+c\right)^2}+\frac{b^2\left(3a^2-c\right)\log\left(-bx\right)}{\left(a^2+c\right)^3}-\frac{b^2\left(3a^2-c\right)\log\left(c+(a+bx)^2\right)}{2\left(a^2+c\right)^3}-\frac{1}{2x^2\left(a^2+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(c+(b*x+a)**2),x)
```

```
[Out] -a*b**2*(a**2 - 3*c)*atan((a + b*x)/sqrt(c))/(sqrt(c)*(a**2 + c)*
*3) + 2*a*b/(x*(a**2 + c)**2) + b**2*(3*a**2 - c)*log(-b*x)/(a**2
 + c)**3 - b**2*(3*a**2 - c)*log(c + (a + b*x)**2)/(2*(a**2 + c)*
*3) - 1/(2*x**2*(a**2 + c))
```

Mathematica [A] time = 0.224982, size = 106, normalized size = 0.88

$$-\frac{b^2\left(3a^2-c\right)\log\left(a^2+2abx+b^2x^2+c\right)+2b^2\left(c-3a^2\right)\log(x)+\frac{2ab^2\left(a^2-3c\right)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}}+\frac{\left(a^2+c\right)\left(a^2-4abx+c\right)}{x^2}}{2\left(a^2+c\right)^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*(c + (a + b*x)^2)),x]
```

```
[Out] -(((a^2 + c)*(a^2 + c - 4*a*b*x))/x^2 + (2*a*b^2*(a^2 - 3*c)*ArcT
an[(a + b*x)/Sqrt[c]])/Sqrt[c] + 2*b^2*(-3*a^2 + c)*Log[x] + b^2*
(3*a^2 - c)*Log[a^2 + c + 2*a*b*x + b^2*x^2])/(2*(a^2 + c)^3)
```

Maple [A] time = 0.012, size = 198, normalized size = 1.6

$$-\frac{3\,b^2\ln\left(b^2x^2+2\,abx+a^2+c\right)a^2}{2\,\left(a^2+c\right)^3}+\frac{b^2\ln\left(b^2x^2+2\,abx+a^2+c\right)c}{2\,\left(a^2+c\right)^3} -\frac{a^3b^2}{\left(a^2+c\right)^3}\arctan\left(\frac{2\,b^2x+2\,ab}{2\,b}\frac{1}{\sqrt{c}}\right)\frac{1}{\sqrt{c}}+3\,\frac{b^2\sqrt{c}a}{\left(a^2+c\right)^3}\arctan\left(1/2\,\frac{2\,b^2x+2\,ab}{b\sqrt{c}}\right) -\frac{1}{\left(2\,a^2+2\,c\right)x^2}+3\,\frac{b^2\ln\left(x\right)a^2}{\left(a^2+c\right)^3}-\frac{b^2\ln\left(x\right)c}{\left(a^2+c\right)^3}+2\,\frac{ab}{\left(a^2+c\right)^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(c+(b*x+a)^2),x)
```

```
[Out] -3/2*b^2/(a^2+c)^3*ln(b^2*x^2+2*a*b*x+a^2+c)*a^2+1/2*b^2/(a^2+c)^
3*ln(b^2*x^2+2*a*b*x+a^2+c)*c-b^2/(a^2+c)^3/c^(1/2)*arctan(1/2*(2
*b^2*x+2*a*b)/b/c^(1/2))*a^3+3*b^2/(a^2+c)^3*c^(1/2)*arctan(1/2*(
2*b^2*x+2*a*b)/b/c^(1/2))*a-1/2/(a^2+c)/x^2+3*b^2/(a^2+c)^3*ln(x)
*a^2-b^2/(a^2+c)^3*ln(x)*c+2*a*b/(a^2+c)^2/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(((b*x + a)^2 + c)*x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.295395, size = 1, normalized size = 0.01

$$\left[-\frac{\left(a^3b^2-3\,ab^2c\right)x^2\log\left(\frac{2\,bcx+2\,ac+\left(b^2x^2+2\,abx+a^2-c\right)\sqrt{-c}}{b^2x^2+2\,abx+a^2+c}\right)+\left(a^4+\left(3\,a^2b^2-b^2c\right)x^2\log\left(b^2x^2+2\,abx+a^2+c\right)-2\left(3\,a^2b^2-\right.\right.}{2\left(a^6+3\,a^4c+3\,a^2c^2+c^3\right)\sqrt{-c}x^2}\right.$$

$$-\frac{2\left(a^3b^2-3\,ab^2c\right)x^2\arctan\left(\frac{bx+a}{\sqrt{c}}\right)+\left(a^4+\left(3\,a^2b^2-b^2c\right)x^2\log\left(b^2x^2+2\,abx+a^2+c\right)-2\left(3\,a^2b^2-b^2c\right)x^2\log\left(x\right)+2\,a^2\right.}{2\left(a^6+3\,a^4c+3\,a^2c^2+c^3\right)\sqrt{c}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(((b*x + a)^2 + c)*x^3),x, algorithm="fricas")
```

```
[Out] [-1/2*((a^3*b^2 - 3*a*b^2*c)*x^2*log((2*b*c*x + 2*a*c + (b^2*x^2
+ 2*a*b*x + a^2 - c)*sqrt(-c))/(b^2*x^2 + 2*a*b*x + a^2 + c)) + (
a^4 + (3*a^2*b^2 - b^2*c)*x^2*log(b^2*x^2 + 2*a*b*x + a^2 + c) -
```

```
2*(3*a^2*b^2 - b^2*c)*x^2*log(x) + 2*a^2*c + c^2 - 4*(a^3*b + a*b
*c)*x)*sqrt(-c))/((a^6 + 3*a^4*c + 3*a^2*c^2 + c^3)*sqrt(-c)*x^2)
, -1/2*(2*(a^3*b^2 - 3*a*b^2*c)*x^2*arctan((b*x + a)/sqrt(c)) + (
a^4 + (3*a^2*b^2 - b^2*c)*x^2*log(b^2*x^2 + 2*a*b*x + a^2 + c) -
2*(3*a^2*b^2 - b^2*c)*x^2*log(x) + 2*a^2*c + c^2 - 4*(a^3*b + a*b
*c)*x)*sqrt(c))/((a^6 + 3*a^4*c + 3*a^2*c^2 + c^3)*sqrt(c)*x^2)]
```

Sympy [A] time = 24.7947, size = 3284, normalized size = 27.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(c+(b*x+a)**2),x)
```

```
[Out] b**2*(3*a**2 - c)*log(x + (-4*a**16*b**4*c*(3*a**2 - c)**2/(a**2
+ c)**6 + 24*a**14*b**4*c**2*(3*a**2 - c)**2/(a**2 + c)**6 + 216*
a**12*b**4*c**3*(3*a**2 - c)**2/(a**2 + c)**6 - 14*a**12*b**4*c*(
3*a**2 - c)/(a**2 + c)**3 + 568*a**10*b**4*c**4*(3*a**2 - c)**2/(
a**2 + c)**6 - 44*a**10*b**4*c**2*(3*a**2 - c)/(a**2 + c)**3 + 72
0*a**8*b**4*c**5*(3*a**2 - c)**2/(a**2 + c)**6 - 42*a**8*b**4*c**
3*(3*a**2 - c)/(a**2 + c)**3 + 78*a**8*b**4*c + 456*a**6*b**4*c**
6*(3*a**2 - c)**2/(a**2 + c)**6 - 8*a**6*b**4*c**4*(3*a**2 - c)/(
a**2 + c)**3 - 464*a**6*b**4*c**2 + 104*a**4*b**4*c**7*(3*a**2 -
c)**2/(a**2 + c)**6 - 2*a**4*b**4*c**5*(3*a**2 - c)/(a**2 + c)**3
 + 380*a**4*b**4*c**3 - 24*a**2*b**4*c**8*(3*a**2 - c)**2/(a**2 +
 c)**6 - 12*a**2*b**4*c**6*(3*a**2 - c)/(a**2 + c)**3 - 96*a**2*b
**4*c**4 - 12*b**4*c**9*(3*a**2 - c)**2/(a**2 + c)**6 - 6*b**4*c*
*7*(3*a**2 - c)/(a**2 + c)**3 + 6*b**4*c**5)/(a**9*b**5 + 72*a**7
*b**5*c - 270*a**5*b**5*c**2 + 144*a**3*b**5*c**3 - 27*a*b**5*c**
4))/(a**2 + c)**3 + (-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3
*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)*
*3))*log(x + (-4*a**16*c*(-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**
6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2
+ c)**3))**2 + 24*a**14*c**2*(-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*
(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a
**2 + c)**3))**2 - 14*a**12*b**2*c*(-a*b**2*sqrt(-c)*(a**2 - 3*c)
/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)
/(2*(a**2 + c)**3)) + 216*a**12*c**3*(-a*b**2*sqrt(-c)*(a**2 - 3*
c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 -
c)/(2*(a**2 + c)**3))**2 - 44*a**10*b**2*c**2*(-a*b**2*sqrt(-c)*(
a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(
3*a**2 - c)/(2*(a**2 + c)**3)) + 568*a**10*c**4*(-a*b**2*sqrt(-c)
*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2
*(3*a**2 - c)/(2*(a**2 + c)**3))**2 + 78*a**8*b**4*c - 42*a**8*b*
*2*c**3*(-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*
a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3)) + 720*a
**8*c**5*(-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3
*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3))**2 - 4
64*a**6*b**4*c**2 - 8*a**6*b**2*c**4*(-a*b**2*sqrt(-c)*(a**2 - 3*
c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 -
c)/(2*(a**2 + c)**3)) + 456*a**6*c**6*(-a*b**2*sqrt(-c)*(a**2 - 3
*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 -
 c)/(2*(a**2 + c)**3))**2 + 380*a**4*b**4*c**3 - 2*a**4*b**2*c**5
*(-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c*
*2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3)) + 104*a**4*c**
7*(-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c
**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3))**2 - 96*a**2*
b**4*c**4 - 12*a**2*b**2*c**6*(-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c
*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(
a**2 + c)**3)) - 24*a**2*c**8*(-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c
*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(
a**2 + c)**3))**2 + 6*b**4*c**5 - 6*b**2*c**7*(-a*b**2*sqrt(-c)*(
a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(
3*a**2 - c)/(2*(a**2 + c)**3)) - 12*c**9*(-a*b**2*sqrt(-c)*(a**2
- 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**
2 - c)/(2*(a**2 + c)**3))**2)/(a**9*b**5 + 72*a**7*b**5*c - 270*a
```

```
**5*b**5*c**2 + 144*a**3*b**5*c**3 - 27*a*b**5*c**4)) + (a*b**2*s
qrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3))
 - b**2*(3*a**2 - c)/(2*(a**2 + c)**3))*log(x + (-4*a**16*c*(a*b*
*2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c*
*3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3))**2 + 24*a**14*c**2*(a
*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 +
 c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3))**2 - 14*a**12*b**2
*c*(a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c
**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3)) + 216*a**12*c
**3*(a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*
c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3))**2 - 44*a**1
0*b**2*c**2*(a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c +
 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3)) + 56
8*a**10*c**4*(a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c
+ 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3))**2
+ 78*a**8*b**4*c - 42*a**8*b**2*c**3*(a*b**2*sqrt(-c)*(a**2 - 3*c
)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c
)/(2*(a**2 + c)**3)) + 720*a**8*c**5*(a*b**2*sqrt(-c)*(a**2 - 3*c
)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c
)/(2*(a**2 + c)**3))**2 - 464*a**6*b**4*c**2 - 8*a**6*b**2*c**4*(
a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2
+ c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3)) + 456*a**6*c**6*(
a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2
+ c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3))**2 + 380*a**4*b**
4*c**3 - 2*a**4*b**2*c**5*(a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**
6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2
+ c)**3)) + 104*a**4*c**7*(a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**
6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2
+ c)**3))**2 - 96*a**2*b**4*c**4 - 12*a**2*b**2*c**6*(a*b**2*sqrt
(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) -
b**2*(3*a**2 - c)/(2*(a**2 + c)**3)) - 24*a**2*c**8*(a*b**2*sqrt(
-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b
**2*(3*a**2 - c)/(2*(a**2 + c)**3))**2 + 6*b**4*c**5 - 6*b**2*c**
7*(a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c*
*2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3)) - 12*c**9*(a*b
**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c
**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3))**2)/(a**9*b**5 + 72*
a**7*b**5*c - 270*a**5*b**5*c**2 + 144*a**3*b**5*c**3 - 27*a*b**5
*c**4)) + (-a**2 + 4*a*b*x - c)/(x**2*(2*a**4 + 4*a**2*c + 2*c**2
))
```

GIAC/XCAS [A] time = 0.2646, size = 263, normalized size = 2.17

$$-\frac{\left(3\,a^2b^2-b^2c\right)\ln\left(b^2x^2+2\,abx+a^2+c\right)}{2\left(a^6+3\,a^4c+3\,a^2c^2+c^3\right)}+\frac{\left(3\,a^2b^2-b^2c\right)\ln\left(|x|\right)}{a^6+3\,a^4c+3\,a^2c^2+c^3}-\frac{\left(a^3b^3-3\,ab^3c\right)\arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{\left(a^6+3\,a^4c+3\,a^2c^2+c^3\right)b\sqrt{c}}-\frac{a^4+2\,a^2c+c^2-4\left(a^3b+abc\right)x}{2\left(a^2+c\right)^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(((b*x + a)^2 + c)*x^3),x, algorithm="giac")
```

```
[Out] -1/2*(3*a^2*b^2 - b^2*c)*ln(b^2*x^2 + 2*a*b*x + a^2 + c)/(a^6 + 3
*a^4*c + 3*a^2*c^2 + c^3) + (3*a^2*b^2 - b^2*c)*ln(abs(x))/(a^6 +
 3*a^4*c + 3*a^2*c^2 + c^3) - (a^3*b^3 - 3*a*b^3*c)*arctan((b*x +
 a)/sqrt(c))/((a^6 + 3*a^4*c + 3*a^2*c^2 + c^3)*b*sqrt(c)) - 1/2*
(a^4 + 2*a^2*c + c^2 - 4*(a^3*b + a*b*c)*x)/((a^2 + c)^3*x^2)
```

3.85 $\int \frac{1}{a+b(c+dx)^2}\,dx$

Optimal. Leaf size=31

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d}$$

```
[Out] ArcTan[(Sqrt[b]*(c + d*x))/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*d)
```

Rubi [A] time = 0.0371622, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(c + d*x)^2)^(-1),x]

[Out] ArcTan[(Sqrt[b]*(c + d*x))/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*d)
```

Rubi in Sympy [A] time = 4.63101, size = 27, normalized size = 0.87

$$\frac{\operatorname{atan}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(a+b*(d*x+c)**2),x)

[Out] atan(sqrt(b)*(c + d*x)/sqrt(a))/(sqrt(a)*sqrt(b)*d)
```

Mathematica [A] time = 0.0149925, size = 31, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(c + d*x)^2)^(-1),x]

[Out] ArcTan[(Sqrt[b]*(c + d*x))/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*d)
```

Maple [A] time = 0.008, size = 34, normalized size = 1.1

$$\frac{1}{d}\arctan\left(\frac{2\,bd^2x+2\,bcd}{2\,d}\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a+b*(d*x+c)^2),x)
```

```
[Out] 1/d/(a*b)^(1/2)*arctan(1/2*(2*b*d^2*x+2*b*c*d)/d/(a*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((d*x + c)^2*b + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.258972, size = 1, normalized size = 0.03

$$\left[\frac{\log\left(\frac{2\,abdx+2\,abc+\left(bd^2x^2+2\,bcdx+bc^2-a\right)\sqrt{-ab}}{bd^2x^2+2\,bcdx+bc^2+a}\right)}{2\,\sqrt{-ab}d}, \frac{\arctan\left(\frac{\sqrt{ab}(dx+c)}{a}\right)}{\sqrt{ab}d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((d*x + c)^2*b + a),x, algorithm="fricas")
```

```
[Out] [1/2*log((2*a*b*d*x + 2*a*b*c + (b*d^2*x^2 + 2*b*c*d*x + b*c^2 -
a)*sqrt(-a*b))/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a))/(sqrt(-a*b)*d
), arctan(sqrt(a*b)*(d*x + c)/a)/(sqrt(a*b)*d)]
```

Sympy [A] time = 0.631541, size = 61, normalized size = 1.97

$$\frac{-\frac{\sqrt{-\frac{1}{ab}}\log\left(x+\frac{-a\sqrt{-\frac{1}{ab}}+c}{d}\right)}{2}+\frac{\sqrt{-\frac{1}{ab}}\log\left(x+\frac{a\sqrt{-\frac{1}{ab}}+c}{d}\right)}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a+b*(d*x+c)**2),x)
```

```
[Out] (-sqrt(-1/(a*b))*log(x + (-a*sqrt(-1/(a*b)) + c)/d)/2 + sqrt(-1/(
a*b))*log(x + (a*sqrt(-1/(a*b)) + c)/d)/2)/d
```

GIAC/XCAS [A] time = 0.262387, size = 32, normalized size = 1.03

$$\frac{\arctan\left(\frac{bdx+bc}{\sqrt{ab}}\right)}{\sqrt{ab}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((d*x + c)^2*b + a),x, algorithm="giac")
```

```
[Out] arctan((b*d*x + b*c)/sqrt(a*b))/(sqrt(a*b)*d)
```

3.86 $\int \frac{1}{(a+b(c+dx)^2)^2}\,dx$

Optimal. Leaf size=63

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}d}+\frac{c+dx}{2ad\,(a+b(c+dx)^2)}$$

```
[Out] (c + d*x)/(2*a*d*(a + b*(c + d*x)^2)) + ArcTan[(Sqrt[b]*(c + d*x)
)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b]*d)
```

Rubi [A] time = 0.0652337, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}d}+\frac{c+dx}{2ad\,(a+b(c+dx)^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*(c + d*x)^2)^(-2),x]
```

```
[Out] (c + d*x)/(2*a*d*(a + b*(c + d*x)^2)) + ArcTan[(Sqrt[b]*(c + d*x)
)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b]*d)
```

Rubi in Sympy [A] time = 6.90537, size = 49, normalized size = 0.78

$$\frac{c+dx}{2ad\left(a+b\left(c+dx\right)^2\right)}+\frac{\operatorname{atan}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}\sqrt{b}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a+b*(d*x+c)**2)**2,x)
```

```
[Out] (c + d*x)/(2*a*d*(a + b*(c + d*x)**2)) + atan(sqrt(b)*(c + d*x)/s
qrt(a))/(2*a**(3/2)*sqrt(b)*d)
```

Mathematica [A] time = 0.0421942, size = 60, normalized size = 0.95

$$\frac{\frac{\sqrt{a}(c+dx)}{a+b(c+dx)^2}+\frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{\sqrt{b}}}{2a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*(c + d*x)^2)^(-2),x]
```

```
[Out] ((Sqrt[a]*(c + d*x))/(a + b*(c + d*x)^2) + ArcTan[(Sqrt[b]*(c + d
*x))/Sqrt[a]]/Sqrt[b])/(2*a^(3/2)*d)
```

Maple [A] time = 0.005, size = 86, normalized size = 1.4

$$\frac{2\,bd^2x+2\,bcd}{4\,abd^2\,(bd^2x^2+2\,bcdx+c^2b+a)}+\frac{1}{2\,ad}\arctan\left(\frac{2\,bd^2x+2\,bcd}{2\,d}\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a+b*(d*x+c)^2)^2,x)
```

```
[Out] 1/4*(2*b*d^2*x+2*b*c*d)/a/b/d^2/(b*d^2*x^2+2*b*c*d*x+b*c^2+a)+1/2
/d/a/(a*b)^(1/2)*arctan(1/2*(2*b*d^2*x+2*b*c*d)/d/(a*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((d*x + c)^2*b + a)^(-2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.274856, size = 1, normalized size = 0.02

$$\left[\frac{\left(bd^2x^2 + 2\,bcdx + bc^2 + a\right)\log\left(\frac{2\,abdx+2\,abc+\left(bd^2x^2+2\,bcdx+bc^2-a\right)\sqrt{-ab}}{bd^2x^2+2\,bcdx+bc^2+a}\right) + 2\,\sqrt{-ab}(dx + c)}{4\left(abd^3x^2 + 2\,abcd^2x + (abc^2 + a^2)d\right)\sqrt{-ab}}, \frac{\left(bd^2x^2 + 2\,bcdx + bc^2 + a\right)\text{arcta}}{2\left(abd^3x^2 + 2\,abcd^2x\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((d*x + c)^2*b + a)^(-2),x, algorithm="fricas")
```

```
[Out] [1/4*((b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)*log((2*a*b*d*x + 2*a*b*
c + (b*d^2*x^2 + 2*b*c*d*x + b*c^2 - a)*sqrt(-a*b))/(b*d^2*x^2 +
2*b*c*d*x + b*c^2 + a)) + 2*sqrt(-a*b)*(d*x + c))/((a*b*d^3*x^2 +
 2*a*b*c*d^2*x + (a*b*c^2 + a^2)*d)*sqrt(-a*b)), 1/2*((b*d^2*x^2
+ 2*b*c*d*x + b*c^2 + a)*arctan(sqrt(a*b)*(d*x + c)/a) + sqrt(a*b
)*(d*x + c))/((a*b*d^3*x^2 + 2*a*b*c*d^2*x + (a*b*c^2 + a^2)*d)*s
qrt(a*b))]
```

Sympy [A] time = 2.95645, size = 117, normalized size = 1.86

$$\frac{c + dx}{2a^2d + 2abc^2d + 4abcd^2x + 2abd^3x^2} + \frac{-\frac{\sqrt{-\frac{1}{a^3b}}\log\left(x+\frac{-a^2\sqrt{-\frac{1}{a^3b}}+c}{d}\right)}{4} + \frac{\sqrt{-\frac{1}{a^3b}}\log\left(x+\frac{a^2\sqrt{-\frac{1}{a^3b}}+c}{d}\right)}{4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a+b*(d*x+c)**2)**2,x)
```

```
[Out] (c + d*x)/(2*a**2*d + 2*a*b*c**2*d + 4*a*b*c*d**2*x + 2*a*b*d**3*
x**2) + (-sqrt(-1/(a**3*b))*log(x + (-a**2*sqrt(-1/(a**3*b)) + c)
/d)/4 + sqrt(-1/(a**3*b))*log(x + (a**2*sqrt(-1/(a**3*b)) + c)/d)
/4)/d
```

GIAC/XCAS [A] time = 0.263163, size = 88, normalized size = 1.4

$$\frac{\arctan\left(\frac{bdx+bc}{\sqrt{ab}}\right)}{2\sqrt{ab}ad} + \frac{dx+c}{2\,(bd^2x^2 + 2\,bcdx + bc^2 + a)ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((d*x + c)^2*b + a)^(-2),x, algorithm="giac")
```

```
[Out] 1/2*arctan((b*d*x + b*c)/sqrt(a*b))/(sqrt(a*b)*a*d) + 1/2*(d*x +
c)/((b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)*a*d)
```

3.87 $\int \frac{1}{(a+b(c+dx)^2)^3}\,dx$

Optimal. Leaf size=91

$$\frac{3\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}d}+\frac{3(c+dx)}{8a^2d\left(a+b(c+dx)^2\right)}+\frac{c+dx}{4ad\left(a+b(c+dx)^2\right)^2}$$

```
[Out] (c + d*x)/(4*a*d*(a + b*(c + d*x)^2)^2) + (3*(c + d*x))/(8*a^2*d*
(a + b*(c + d*x)^2)) + (3*ArcTan[(Sqrt[b]*(c + d*x))/Sqrt[a]])/(8
*a^(5/2)*Sqrt[b]*d)
```

Rubi [A] time = 0.093635, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$\frac{3\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}d}+\frac{3(c+dx)}{8a^2d\left(a+b(c+dx)^2\right)}+\frac{c+dx}{4ad\left(a+b(c+dx)^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*(c + d*x)^2)^(-3),x]
```

```
[Out] (c + d*x)/(4*a*d*(a + b*(c + d*x)^2)^2) + (3*(c + d*x))/(8*a^2*d*
(a + b*(c + d*x)^2)) + (3*ArcTan[(Sqrt[b]*(c + d*x))/Sqrt[a]])/(8
*a^(5/2)*Sqrt[b]*d)
```

Rubi in Sympy [A] time = 9.71274, size = 78, normalized size = 0.86

$$\frac{c+dx}{4ad\left(a+b\left(c+dx\right)^2\right)^2}+\frac{3\left(c+dx\right)}{8a^2d\left(a+b\left(c+dx\right)^2\right)}+\frac{3\,\mathrm{atan}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{8a^{\frac{5}{2}}\sqrt{b}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a+b*(d*x+c)**2)**3,x)
```

```
[Out] (c + d*x)/(4*a*d*(a + b*(c + d*x)**2)**2) + 3*(c + d*x)/(8*a**2*d
*(a + b*(c + d*x)**2)) + 3*atan(sqrt(b)*(c + d*x)/sqrt(a))/(8*a**
(5/2)*sqrt(b)*d)
```

Mathematica [A] time = 0.111092, size = 75, normalized size = 0.82

$$\frac{\frac{\sqrt{a}(c+dx)\left(5a+3b(c+dx)^2\right)}{\left(a+b(c+dx)^2\right)^2}+\frac{3\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{\sqrt{b}}}{8a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*(c + d*x)^2)^(-3),x]
```

```
[Out] ((Sqrt[a]*(c + d*x)*(5*a + 3*b*(c + d*x)^2))/(a + b*(c + d*x)^2)^
2 + (3*ArcTan[(Sqrt[b]*(c + d*x))/Sqrt[a]])/Sqrt[b])/(8*a^(5/2)*d
)
```

Maple [A] time = 0.007, size = 147, normalized size = 1.6

$$\frac{2\,bd^2x+2\,bcd}{8\,abd^2\left(bd^2x^2+2\,bcdx+c^2b+a\right)^2}+\frac{3\,x}{8\,a^2\left(bd^2x^2+2\,bcdx+c^2b+a\right)}+\frac{3\,c}{8\,a^2d\left(bd^2x^2+2\,bcdx+c^2b+a\right)}+\frac{3}{8\,a^2d}\arctan\left(\frac{2\,bd^2x+2\,bcd}{2\,d}\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a+b*(d*x+c)^2)^3,x)
```

```
[Out] 1/8*(2*b*d^2*x+2*b*c*d)/a/b/d^2/(b*d^2*x^2+2*b*c*d*x+b*c^2+a)^2+3
/8/a^2/(b*d^2*x^2+2*b*c*d*x+b*c^2+a)*x+3/8/a^2/d/(b*d^2*x^2+2*b*c
*d*x+b*c^2+a)*c+3/8/a^2/d/(a*b)^(1/2)*arctan(1/2*(2*b*d^2*x+2*b*c
*d)/d/(a*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((d*x + c)^2*b + a)^(-3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.297156, size = 1, normalized size = 0.01

$$\left[\frac{3\left(b^2d^4x^4+4\,b^2cd^3x^3+b^2c^4+2\left(3\,b^2c^2+ab\right)d^2x^2+2\,abc^2+4\left(b^2c^3+abc\right)dx+a^2\right)\log\left(\frac{2\,abdx+2\,abc+\left(bd^2x^2+2\,bcdx+bc^2-a\right)}{bd^2x^2+2\,bcdx+bc^2+a}\right.}{16\left(a^2b^2d^5x^4+4\,a^2b^2cd^4x^3+2\left(3\,a^2b^2c^2+a^3b\right)d^3x^2+4\left(a^2b^2c^3+a^3bc\right)d^2x+\left(a^2\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((d*x + c)^2*b + a)^(-3),x, algorithm="fricas")
```

```
[Out] [1/16*(3*(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + b^2*c^4 + 2*(3*b^2*c^2
+ a*b)*d^2*x^2 + 2*a*b*c^2 + 4*(b^2*c^3 + a*b*c)*d*x + a^2)*log((
2*a*b*d*x + 2*a*b*c + (b*d^2*x^2 + 2*b*c*d*x + b*c^2 - a)*sqrt(-a
*b))/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)) + 2*(3*b*d^3*x^3 + 9*b*
c*d^2*x^2 + 3*b*c^3 + (9*b*c^2 + 5*a)*d*x + 5*a*c)*sqrt(-a*b))/((
a^2*b^2*d^5*x^4 + 4*a^2*b^2*c*d^4*x^3 + 2*(3*a^2*b^2*c^2 + a^3*b)
*d^3*x^2 + 4*(a^2*b^2*c^3 + a^3*b*c)*d^2*x + (a^2*b^2*c^4 + 2*a^3
*b*c^2 + a^4)*d)*sqrt(-a*b)), 1/8*(3*(b^2*d^4*x^4 + 4*b^2*c*d^3*x
^3 + b^2*c^4 + 2*(3*b^2*c^2 + a*b)*d^2*x^2 + 2*a*b*c^2 + 4*(b^2*c
^3 + a*b*c)*d*x + a^2)*arctan(sqrt(a*b)*(d*x + c)/a) + (3*b*d^3*x
^3 + 9*b*c*d^2*x^2 + 3*b*c^3 + (9*b*c^2 + 5*a)*d*x + 5*a*c)*sqrt(
a*b))/((a^2*b^2*d^5*x^4 + 4*a^2*b^2*c*d^4*x^3 + 2*(3*a^2*b^2*c^2
+ a^3*b)*d^3*x^2 + 4*(a^2*b^2*c^3 + a^3*b*c)*d^2*x + (a^2*b^2*c^4
 + 2*a^3*b*c^2 + a^4)*d)*sqrt(a*b))]
```

Sympy [A] time = 6.44952, size = 257, normalized size = 2.82

$$\frac{5ac+3bc^3+9bcd^2x^2+3bd^3x^3+x\left(5ad+9bc^2d\right)}{8a^4d+16a^3bc^2d+8a^2b^2c^4d+32a^2b^2cd^4x^3+8a^2b^2d^5x^4+x^2\left(16a^3bd^3+48a^2b^2c^2d^3\right)+x\left(32a^3bcd^2+32a^2b^2c^3d^2\right)}+\frac{-\frac{3\sqrt{-\frac{1}{a^5b}}\log\left(x+\frac{-3a^3\sqrt{-\frac{1}{a^5b}}+3c}{3d}\right)}{16}+\frac{3\sqrt{-\frac{1}{a^5b}}\log\left(x+\frac{3a^3\sqrt{-\frac{1}{a^5b}}+3c}{3d}\right)}{16}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a+b*(d*x+c)**2)**3,x)
```

```
[Out] (5*a*c + 3*b*c**3 + 9*b*c*d**2*x**2 + 3*b*d**3*x**3 + x*(5*a*d +
9*b*c**2*d))/(8*a**4*d + 16*a**3*b*c**2*d + 8*a**2*b**2*c**4*d +
32*a**2*b**2*c*d**4*x**3 + 8*a**2*b**2*d**5*x**4 + x**2*(16*a**3*
b*d**3 + 48*a**2*b**2*c**2*d**3) + x*(32*a**3*b*c*d**2 + 32*a**2*
b**2*c**3*d**2)) + (-3*sqrt(-1/(a**5*b))*log(x + (-3*a**3*sqrt(-1
/(a**5*b)) + 3*c)/(3*d))/16 + 3*sqrt(-1/(a**5*b))*log(x + (3*a**3
*sqrt(-1/(a**5*b)) + 3*c)/(3*d))/16)/d
```

GIAC/XCAS [A] time = 0.260209, size = 139, normalized size = 1.53

$$\frac{3\arctan\left(\frac{bdx+bc}{\sqrt{ab}}\right)}{8\sqrt{ab}a^2d}+\frac{3\,bd^3x^3+9\,bcd^2x^2+9\,bc^2dx+3\,bc^3+5\,adx+5\,ac}{8\left(bd^2x^2+2\,bcdx+bc^2+a\right)^2a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((d*x + c)^2*b + a)^(-3),x, algorithm="giac")
```

```
[Out] 3/8*arctan((b*d*x + b*c)/sqrt(a*b))/(sqrt(a*b)*a^2*d) + 1/8*(3*b*
d^3*x^3 + 9*b*c*d^2*x^2 + 9*b*c^2*d*x + 3*b*c^3 + 5*a*d*x + 5*a*c
)/((b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)^2*a^2*d)
```

3.88 $\int \frac{1}{\sqrt{-a}+b(c+dx)^2}\,dx$

Optimal. Leaf size=35

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt[4]{-a}}\right)}{\sqrt[4]{-a}\sqrt{b}d}$$

```
[Out] ArcTan[(Sqrt[b]*(c + d*x))/(-a)^(1/4)]/((-a)^(1/4)*Sqrt[b]*d)
```

Rubi [A] time = 0.0666464, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt[4]{-a}}\right)}{\sqrt[4]{-a}\sqrt{b}d}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[-a] + b*(c + d*x)^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[b]*(c + d*x))/(-a)^(1/4)]/((-a)^(1/4)*Sqrt[b]*d)
```

Rubi in Sympy [A] time = 10.251, size = 31, normalized size = 0.89

$$\frac{\operatorname{atan}\left(\frac{\sqrt{b}(c+dx)}{\sqrt[4]{-a}}\right)}{\sqrt{b}d\sqrt[4]{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*(d*x+c)**2+(-a)**(1/2)),x)
```

```
[Out] atan(sqrt(b)*(c + d*x)/(-a)**(1/4))/(sqrt(b)*d*(-a)**(1/4))
```

Mathematica [A] time = 0.0241034, size = 35, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt[4]{-a}}\right)}{\sqrt[4]{-a}\sqrt{b}d}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[-a] + b*(c + d*x)^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[b]*(c + d*x))/(-a)^(1/4)]/((-a)^(1/4)*Sqrt[b]*d)
```

Maple [A] time = 0.012, size = 42, normalized size = 1.2

$$\frac{1}{d}\arctan\left(\frac{2\,bd^2x+2\,bcd}{2\,d}\frac{1}{\sqrt{\sqrt{-a}b}}\right)\frac{1}{\sqrt{\sqrt{-a}b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(1/(b*(d*x+c)^2+(-a)^(1/2)),x)
```

```
[Out] 1/d/((-a)^(1/2)*b)^(1/2)*arctan(1/2*(2*b*d^2*x+2*b*c*d)/d/((-a)^(
1/2)*b)^(1/2))
```

Maxima [A] time = 0.888472, size = 89, normalized size = 2.54

$$\frac{\log\left(\frac{bd^2x+bcd-\sqrt{-\sqrt{-a}b}d}{bd^2x+bcd+\sqrt{-\sqrt{-a}b}d}\right)}{2\sqrt{-\sqrt{-a}b}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(1/((d*x + c)^2*b + sqrt(-a)),x, algorithm="maxima")
```

```
[Out] 1/2*log((b*d^2*x + b*c*d - sqrt(-sqrt(-a)*b)*d)/(b*d^2*x + b*c*d
+ sqrt(-sqrt(-a)*b)*d))/(sqrt(-sqrt(-a)*b)*d)
```

Fricas [A] time = 0.285339, size = 1, normalized size = 0.03

$$\left[\frac{\sqrt{\frac{\sqrt{-a}}{ab}}\log\left(\frac{bd^2x^2+2\,bcdx+bc^2+2\,(bdx+bc)\sqrt{-a}\sqrt{\frac{\sqrt{-a}}{ab}}-\sqrt{-a}}{bd^2x^2+2\,bcdx+bc^2+\sqrt{-a}}\right)}{2\,d},\ \frac{\sqrt{-\frac{\sqrt{-a}}{ab}}\arctan\left(\frac{dx+c}{\sqrt{-a}\sqrt{-\frac{\sqrt{-a}}{ab}}}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(1/((d*x + c)^2*b + sqrt(-a)),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(sqrt(-a)/(a*b))*log((b*d^2*x^2 + 2*b*c*d*x + b*c^2 + 2*
(b*d*x + b*c)*sqrt(-a)*sqrt(sqrt(-a)/(a*b)) - sqrt(-a))/(b*d^2*x^
2 + 2*b*c*d*x + b*c^2 + sqrt(-a)))/d, sqrt(-sqrt(-a)/(a*b))*arcta
n((d*x + c)/(sqrt(-a)*sqrt(-sqrt(-a)/(a*b))))/d]
```

Sympy [A] time = 0.552409, size = 92, normalized size = 2.63

$$\frac{-\frac{\sqrt{-\frac{1}{b\sqrt{-a}}}\log\left(x+\frac{c-\sqrt{-a}\sqrt{-\frac{1}{b\sqrt{-a}}}}{d}\right)}{2}+\frac{\sqrt{-\frac{1}{b\sqrt{-a}}}\log\left(x+\frac{c+\sqrt{-a}\sqrt{-\frac{1}{b\sqrt{-a}}}}{d}\right)}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(1/(b*(d*x+c)**2+(-a)**(1/2)),x)
```

```
[Out] (-sqrt(-1/(b*sqrt(-a)))*log(x + (c - sqrt(-a)*sqrt(-1/(b*sqrt(-a)
)))/d)/2 + sqrt(-1/(b*sqrt(-a)))*log(x + (c + sqrt(-a)*sqrt(-1/(b
*sqrt(-a))))/d)/2)/d
```

GIAC/XCAS [A] time = 0.260656, size = 41, normalized size = 1.17

$$\frac{\arctan\left(\frac{bdx+bc}{(-a)^{\frac{1}{4}}\sqrt{b}}\right)}{(-a)^{\frac{1}{4}}\sqrt{b}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((d*x + c)^2*b + sqrt(-a)),x, algorithm="giac")
```

```
[Out] arctan((b*d*x + b*c)/((-a)^(1/4)*sqrt(b)))/((-a)^(1/4)*sqrt(b)*d)
```

3.89 $\int \frac{1}{1+(c+dx)^2} \, dx$

Optimal. Leaf size=10

$$\frac{\tan^{-1}(c+dx)}{d}$$

```
[Out] ArcTan[c + d*x]/d
```

Rubi [A] time = 0.00874482, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{\tan^{-1}(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + (c + d*x)^2)^(-1),x]

[Out] ArcTan[c + d*x]/d
```

Rubi in Sympy [A] time = 1.62704, size = 7, normalized size = 0.7

$$\frac{\operatorname{atan}(c+dx)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1+(d*x+c)**2),x)

[Out] atan(c + d*x)/d
```

Mathematica [A] time = 0.00631198, size = 10, normalized size = 1.

$$\frac{\tan^{-1}(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + (c + d*x)^2)^(-1),x]

[Out] ArcTan[c + d*x]/d
```

Maple [A] time = 0.005, size = 11, normalized size = 1.1

$$\frac{\arctan(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1+(d*x+c)^2),x)
```

```
[Out] arctan(d*x+c)/d
```

Maxima [A] time = 0.902752, size = 24, normalized size = 2.4

$$\frac{\arctan\left(\frac{d^2x+cd}{d}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((d*x + c)^2 + 1),x, algorithm="maxima")
```

```
[Out] arctan((d^2*x + c*d)/d)/d
```

Fricas [A] time = 0.252228, size = 14, normalized size = 1.4

$$\frac{\arctan\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((d*x + c)^2 + 1),x, algorithm="fricas")
```

```
[Out] arctan(d*x + c)/d
```

Sympy [A] time = 0.385454, size = 24, normalized size = 2.4

$$\frac{-\frac{i\log\left(x+\frac{c-i}{d}\right)}{2}+\frac{i\log\left(x+\frac{c+i}{d}\right)}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1+(d*x+c)**2),x)
```

```
[Out] (-I*log(x + (c - I)/d)/2 + I*log(x + (c + I)/d)/2)/d
```

GIAC/XCAS [A] time = 0.259758, size = 14, normalized size = 1.4

$$\frac{\arctan\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((d*x + c)^2 + 1),x, algorithm="giac")
```

```
[Out] arctan(d*x + c)/d
```

3.90 $\int \frac{1}{(1+(c+dx)^2)^2} dx$

Optimal. Leaf size=37

$$\frac{c+dx}{2d\left((c+dx)^2+1\right)}+\frac{\tan^{-1}(c+dx)}{2d}$$

```
[Out] (c + d*x)/(2*d*(1 + (c + d*x)^2)) + ArcTan[c + d*x]/(2*d)
```

Rubi [A] time = 0.0237709, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273

$$\frac{c+dx}{2d\left((c+dx)^2+1\right)}+\frac{\tan^{-1}(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + (c + d*x)^2)^(-2),x]
```

```
[Out] (c + d*x)/(2*d*(1 + (c + d*x)^2)) + ArcTan[c + d*x]/(2*d)
```

Rubi in Sympy [A] time = 2.46138, size = 26, normalized size = 0.7

$$\frac{c+dx}{2d\left((c+dx)^2+1\right)}+\frac{\operatorname{atan}(c+dx)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(1+(d*x+c)**2)**2,x)
```

```
[Out] (c + d*x)/(2*d*((c + d*x)**2 + 1)) + atan(c + d*x)/(2*d)
```

Mathematica [A] time = 0.0194258, size = 31, normalized size = 0.84

$$\frac{\frac{c+dx}{(c+dx)^2+1}+\tan^{-1}(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + (c + d*x)^2)^(-2),x]
```

```
[Out] ((c + d*x)/(1 + (c + d*x)^2) + ArcTan[c + d*x])/(2*d)
```

Maple [A] time = 0.008, size = 59, normalized size = 1.6

$$\frac{2\,d^2x+2\,cd}{4\,d^2\left(d^2x^2+2\,cdx+c^2+1\right)}+\frac{1}{2\,d}\arctan\left(\frac{2\,d^2x+2\,cd}{2\,d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+(d*x+c)^2)^2,x)
```

```
[Out] 1/4*(2*d^2*x+2*c*d)/d^2/(d^2*x^2+2*c*d*x+c^2+1)+1/2/d*arctan(1/2*
(2*d^2*x+2*c*d)/d)
```

Maxima [A] time = 0.907047, size = 69, normalized size = 1.86

$$\frac{dx+c}{2\,(d^3x^2+2\,cd^2x+(c^2+1)d)}+\frac{\arctan\left(\frac{d^2x+cd}{d}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((d*x + c)^2 + 1)^(-2),x, algorithm="maxima")
```

```
[Out] 1/2*(d*x + c)/(d^3*x^2 + 2*c*d^2*x + (c^2 + 1)*d) + 1/2*arctan((d
^2*x + c*d)/d)/d
```

Fricas [A] time = 0.254473, size = 74, normalized size = 2.

$$\frac{dx+\left(d^2x^2+2\,cdx+c^2+1\right)\arctan\left(dx+c\right)+c}{2\,(d^3x^2+2\,cd^2x+(c^2+1)d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((d*x + c)^2 + 1)^(-2),x, algorithm="fricas")
```

```
[Out] 1/2*(d*x + (d^2*x^2 + 2*c*d*x + c^2 + 1)*arctan(d*x + c) + c)/(d^
3*x^2 + 2*c*d^2*x + (c^2 + 1)*d)
```

Sympy [A] time = 2.31239, size = 56, normalized size = 1.51

$$\frac{c+dx}{2c^2d+4cd^2x+2d^3x^2+2d}+\frac{-\frac{i\log\left(x+\frac{c-i}{d}\right)}{4}+\frac{i\log\left(x+\frac{c+i}{d}\right)}{4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1+(d*x+c)**2)**2,x)
```

```
[Out] (c + d*x)/(2*c**2*d + 4*c*d**2*x + 2*d**3*x**2 + 2*d) + (-I*log(x
 + (c - I)/d)/4 + I*log(x + (c + I)/d)/4)/d
```

GIAC/XCAS [A] time = 0.263369, size = 55, normalized size = 1.49

$$\frac{\arctan\left(dx+c\right)}{2\,d}+\frac{dx+c}{2\,(d^2x^2+2\,cdx+c^2+1)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((d*x + c)^2 + 1)^(-2),x, algorithm="giac")
```

```
[Out] 1/2*arctan(d*x + c)/d + 1/2*(d*x + c)/((d^2*x^2 + 2*c*d*x + c^2 +
 1)*d)
```

3.91 $\int \frac{1}{(1+(c+dx)^2)^3}\,dx$

Optimal. Leaf size=60

$$\frac{3(c+dx)}{8d\left((c+dx)^2+1\right)}+\frac{c+dx}{4d\left((c+dx)^2+1\right)^2}+\frac{3\tan^{-1}(c+dx)}{8d}$$

```
[Out] (c + d*x)/(4*d*(1 + (c + d*x)^2)^2) + (3*(c + d*x))/(8*d*(1 + (c
+ d*x)^2)) + (3*ArcTan[c + d*x])/(8*d)
```

Rubi [A] time = 0.0371075, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273

$$\frac{3(c+dx)}{8d\left((c+dx)^2+1\right)}+\frac{c+dx}{4d\left((c+dx)^2+1\right)^2}+\frac{3\tan^{-1}(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + (c + d*x)^2)^(-3),x]
```

```
[Out] (c + d*x)/(4*d*(1 + (c + d*x)^2)^2) + (3*(c + d*x))/(8*d*(1 + (c
+ d*x)^2)) + (3*ArcTan[c + d*x])/(8*d)
```

Rubi in Sympy [A] time = 3.20673, size = 49, normalized size = 0.82

$$\frac{3\left(c+dx\right)}{8d\left(\left(c+dx\right)^2+1\right)}+\frac{c+dx}{4d\left(\left(c+dx\right)^2+1\right)^2}+\frac{3\operatorname{atan}\left(c+dx\right)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1+(d*x+c)**2)**3,x)
```

```
[Out] 3*(c + d*x)/(8*d*((c + d*x)**2 + 1)) + (c + d*x)/(4*d*((c + d*x)*
*2 + 1)**2) + 3*atan(c + d*x)/(8*d)
```

Mathematica [A] time = 0.0247267, size = 52, normalized size = 0.87

$$\frac{\frac{3(c+dx)}{(c+dx)^2+1}+\frac{2(c+dx)}{((c+dx)^2+1)^2}+3\tan^{-1}(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + (c + d*x)^2)^(-3),x]
```

```
[Out] ((2*(c + d*x))/(1 + (c + d*x)^2)^2 + (3*(c + d*x))/(1 + (c + d*x)
^2) + 3*ArcTan[c + d*x])/(8*d)
```

Maple [A] time = 0.005, size = 94, normalized size = 1.6

$$\frac{2\,d^2x+2\,cd}{8\,d^2\left(d^2x^2+2\,cdx+c^2+1\right)^2}+\frac{6\,d^2x+6\,cd}{16\,d^2\left(d^2x^2+2\,cdx+c^2+1\right)}+\frac{3}{8\,d}\arctan\left(\frac{2\,d^2x+2\,cd}{2\,d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1+(d*x+c)^2)^3,x)
```

```
[Out] 1/8*(2*d^2*x+2*c*d)/d^2/(d^2*x^2+2*c*d*x+c^2+1)^2+3/16*(2*d^2*x+2
*c*d)/d^2/(d^2*x^2+2*c*d*x+c^2+1)+3/8/d*arctan(1/2*(2*d^2*x+2*c*d
)/d)
```

Maxima [A] time = 0.902447, size = 155, normalized size = 2.58

$$\frac{3\,d^3x^3+9\,cd^2x^2+3\,c^3+\left(9\,c^2+5\right)dx+5\,c}{8\,(d^5x^4+4\,cd^4x^3+2\,(3\,c^2+1)d^3x^2+4\,(c^3+c)d^2x+(c^4+2\,c^2+1)d)}+\frac{3\,\arctan\left(\frac{d^2x+cd}{d}\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((d*x + c)^2 + 1)^(-3),x, algorithm="maxima")
```

```
[Out] 1/8*(3*d^3*x^3 + 9*c*d^2*x^2 + 3*c^3 + (9*c^2 + 5)*d*x + 5*c)/(d^
5*x^4 + 4*c*d^4*x^3 + 2*(3*c^2 + 1)*d^3*x^2 + 4*(c^3 + c)*d^2*x +
 (c^4 + 2*c^2 + 1)*d) + 3/8*arctan((d^2*x + c*d)/d)/d
```

Fricas [A] time = 0.257938, size = 207, normalized size = 3.45

$$\frac{3\,d^3x^3+9\,cd^2x^2+3\,c^3+\left(9\,c^2+5\right)dx+3\,\left(d^4x^4+4\,cd^3x^3+2\,\left(3\,c^2+1\right)d^2x^2+c^4+4\,\left(c^3+c\right)dx+2\,c^2+1\right)\arctan\left(dx+c\right)+}{8\,(d^5x^4+4\,cd^4x^3+2\,(3\,c^2+1)d^3x^2+4\,(c^3+c)d^2x+(c^4+2\,c^2+1)d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((d*x + c)^2 + 1)^(-3),x, algorithm="fricas")
```

```
[Out] 1/8*(3*d^3*x^3 + 9*c*d^2*x^2 + 3*c^3 + (9*c^2 + 5)*d*x + 3*(d^4*x
^4 + 4*c*d^3*x^3 + 2*(3*c^2 + 1)*d^2*x^2 + c^4 + 4*(c^3 + c)*d*x
+ 2*c^2 + 1)*arctan(d*x + c) + 5*c)/(d^5*x^4 + 4*c*d^4*x^3 + 2*(3
*c^2 + 1)*d^3*x^2 + 4*(c^3 + c)*d^2*x + (c^4 + 2*c^2 + 1)*d)
```

Sympy [A] time = 4.92775, size = 146, normalized size = 2.43

$$\frac{3c^3+9cd^2x^2+5c+3d^3x^3+x\left(9c^2d+5d\right)}{8c^4d+16c^2d+32cd^4x^3+8d^5x^4+8d+x^2\left(48c^2d^3+16d^3\right)+x\left(32c^3d^2+32cd^2\right)}+\frac{-\frac{3i\log\left(x+\frac{3c-3i}{3d}\right)}{16}+\frac{3i\log\left(x+\frac{3c+3i}{3d}\right)}{16}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1+(d*x+c)**2)**3,x)
```

```
[Out] (3*c**3 + 9*c*d**2*x**2 + 5*c + 3*d**3*x**3 + x*(9*c**2*d + 5*d))
/(8*c**4*d + 16*c**2*d + 32*c*d**4*x**3 + 8*d**5*x**4 + 8*d + x**
2*(48*c**2*d**3 + 16*d**3) + x*(32*c**3*d**2 + 32*c*d**2)) + (-3*
I*log(x + (3*c - 3*I)/(3*d))/16 + 3*I*log(x + (3*c + 3*I)/(3*d))/
16)/d
```

GIAC/XCAS [A] time = 0.26095, size = 99, normalized size = 1.65

$$\frac{3\,\arctan\left(dx+c\right)}{8\,d}+\frac{3\,d^{3}x^{3}+9\,cd^{2}x^{2}+9\,c^{2}dx+3\,c^{3}+5\,dx+5\,c}{8\,{\left(d^{2}x^{2}+2\,cdx+c^{2}+1\right)}^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((d*x + c)^2 + 1)^(-3),x, algorithm="giac")
```

```
[Out] 3/8*arctan(d*x + c)/d + 1/8*(3*d^3*x^3 + 9*c*d^2*x^2 + 9*c^2*d*x
+ 3*c^3 + 5*d*x + 5*c)/((d^2*x^2 + 2*c*d*x + c^2 + 1)^2*d)
```

3.92 $\int \frac{1}{1-(c+dx)^2} \, dx$

Optimal. Leaf size=10

$$\frac{\tanh^{-1}(c+dx)}{d}$$

```
[Out] ArcTanh[c + d*x]/d
```

Rubi [A] time = 0.00909456, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{\tanh^{-1}(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - (c + d*x)^2)^(-1),x]

[Out] ArcTanh[c + d*x]/d
```

Rubi in Sympy [A] time = 1.68552, size = 7, normalized size = 0.7

$$\frac{\operatorname{atanh}(c+dx)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1-(d*x+c)**2),x)

[Out] atanh(c + d*x)/d
```

Mathematica [B] time = 0.00802069, size = 32, normalized size = 3.2

$$\frac{\log(c+dx+1)}{2d} - \frac{\log(-c-dx+1)}{2d}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - (c + d*x)^2)^(-1),x]

[Out] -Log[1 - c - d*x]/(2*d) + Log[1 + c + d*x]/(2*d)
```

Maple [B] time = 0.01, size = 26, normalized size = 2.6

$$-\frac{\ln(dx+c-1)}{2d} + \frac{\ln(dx+c+1)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1-(d*x+c)^2),x)
```

```
[Out] -1/2/d*ln(d*x+c-1)+1/2/d*ln(d*x+c+1)
```

Maxima [A] time = 0.816423, size = 34, normalized size = 3.4

$$\frac{\log(dx+c+1)}{2d}-\frac{\log(dx+c-1)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((d*x + c)^2 - 1),x, algorithm="maxima")
```

```
[Out] 1/2*log(d*x + c + 1)/d - 1/2*log(d*x + c - 1)/d
```

Fricas [A] time = 0.272084, size = 30, normalized size = 3.

$$\frac{\log(dx+c+1)-\log(dx+c-1)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((d*x + c)^2 - 1),x, algorithm="fricas")
```

```
[Out] 1/2*(log(d*x + c + 1) - log(d*x + c - 1))/d
```

Sympy [A] time = 0.399521, size = 22, normalized size = 2.2

$$-\frac{\frac{\log\left(x+\frac{c-1}{d}\right)}{2}-\frac{\log\left(x+\frac{c+1}{d}\right)}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1-(d*x+c)**2),x)
```

```
[Out] -(log(x + (c - 1)/d)/2 - log(x + (c + 1)/d)/2)/d
```

GIAC/XCAS [A] time = 0.2624, size = 36, normalized size = 3.6

$$\frac{\ln(|dx+c+1|)}{2d}-\frac{\ln(|dx+c-1|)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((d*x + c)^2 - 1),x, algorithm="giac")
```

```
[Out] 1/2*ln(abs(d*x + c + 1))/d - 1/2*ln(abs(d*x + c - 1))/d
```

3.93 $\int \frac{1}{(1-(c+dx)^2)^2} dx$

Optimal. Leaf size=39

$$\frac{c+dx}{2d\left(1-(c+dx)^2\right)}+\frac{\tanh^{-1}(c+dx)}{2d}$$

```
[Out] (c + d*x)/(2*d*(1 - (c + d*x)^2)) + ArcTanh[c + d*x]/(2*d)
```

Rubi [A] time = 0.0294724, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$\frac{c+dx}{2d\left(1-(c+dx)^2\right)}+\frac{\tanh^{-1}(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - (c + d*x)^2)^(-2),x]
```

```
[Out] (c + d*x)/(2*d*(1 - (c + d*x)^2)) + ArcTanh[c + d*x]/(2*d)
```

Rubi in Sympy [A] time = 2.59983, size = 26, normalized size = 0.67

$$\frac{c+dx}{2d\left(-(c+dx)^2+1\right)}+\frac{\operatorname{atanh}(c+dx)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(1-(d*x+c)**2)**2,x)
```

```
[Out] (c + d*x)/(2*d*(-(c + d*x)**2 + 1)) + atanh(c + d*x)/(2*d)
```

Mathematica [A] time = 0.0325388, size = 45, normalized size = 1.15

$$\frac{-\frac{2(c+dx)}{(c+dx)^2-1}-\log(-c-dx+1)+\log(c+dx+1)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - (c + d*x)^2)^(-2),x]
```

```
[Out] ((-2*(c + d*x))/(-1 + (c + d*x)^2) - Log[1 - c - d*x] + Log[1 + c
 + d*x])/(4*d)
```

Maple [A] time = 0.014, size = 52, normalized size = 1.3

$$-\frac{1}{4d(dx+c-1)}-\frac{\ln(dx+c-1)}{4d}-\frac{1}{4d(dx+c+1)}+\frac{\ln(dx+c+1)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1-(d*x+c)^2)^2,x)
```

```
[Out] -1/4/d/(d*x+c-1)-1/4/d*ln(d*x+c-1)-1/4/d/(d*x+c+1)+1/4/d*ln(d*x+c
+1)
```

Maxima [A] time = 0.803937, size = 76, normalized size = 1.95

$$-\frac{dx+c}{2\,(d^3x^2+2\,cd^2x+(c^2-1)d)}+\frac{\log(dx+c+1)}{4\,d}-\frac{\log(dx+c-1)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((d*x + c)^2 - 1)^(-2),x, algorithm="maxima")
```

```
[Out] -1/2*(d*x + c)/(d^3*x^2 + 2*c*d^2*x + (c^2 - 1)*d) + 1/4*log(d*x
+ c + 1)/d - 1/4*log(d*x + c - 1)/d
```

Fricas [A] time = 0.25824, size = 115, normalized size = 2.95

$$-\frac{2\,dx-\left(d^2x^2+2\,cdx+c^2-1\right)\log(dx+c+1)+\left(d^2x^2+2\,cdx+c^2-1\right)\log(dx+c-1)+2\,c}{4\,(d^3x^2+2\,cd^2x+(c^2-1)d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((d*x + c)^2 - 1)^(-2),x, algorithm="fricas")
```

```
[Out] -1/4*(2*d*x - (d^2*x^2 + 2*c*d*x + c^2 - 1)*log(d*x + c + 1) + (d
^2*x^2 + 2*c*d*x + c^2 - 1)*log(d*x + c - 1) + 2*c)/(d^3*x^2 + 2*
c*d^2*x + (c^2 - 1)*d)
```

Sympy [A] time = 2.27859, size = 53, normalized size = 1.36

$$-\frac{c+dx}{2c^2d+4cd^2x+2d^3x^2-2d}+\frac{-\frac{\log\left(x+\frac{c-1}{d}\right)}{4}+\frac{\log\left(x+\frac{c+1}{d}\right)}{4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1-(d*x+c)**2)**2,x)
```

```
[Out] -(c + d*x)/(2*c**2*d + 4*c*d**2*x + 2*d**3*x**2 - 2*d) + (-log(x
+ (c - 1)/d)/4 + log(x + (c + 1)/d)/4)/d
```

GIAC/XCAS [A] time = 0.264893, size = 76, normalized size = 1.95

$$\frac{\ln(|dx+c+1|)}{4\,d}-\frac{\ln(|dx+c-1|)}{4\,d}-\frac{dx+c}{2\,(d^2x^2+2\,cdx+c^2-1)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((d*x + c)^2 - 1)^(-2),x, algorithm="giac")
```

```
[Out] 1/4*ln(abs(d*x + c + 1))/d - 1/4*ln(abs(d*x + c - 1))/d - 1/2*(d*
x + c)/((d^2*x^2 + 2*c*d*x + c^2 - 1)*d)
```

3.94 $\int \frac{1}{(1-(c+dx)^2)^3}\,dx$

Optimal. Leaf size=64

$$\frac{3(c+dx)}{8d\left(1-(c+dx)^2\right)}+\frac{c+dx}{4d\left(1-(c+dx)^2\right)^2}+\frac{3\tanh^{-1}(c+dx)}{8d}$$

```
[Out] (c + d*x)/(4*d*(1 - (c + d*x)^2)^2) + (3*(c + d*x))/(8*d*(1 - (c
+ d*x)^2)) + (3*ArcTanh[c + d*x])/(8*d)
```

Rubi [A] time = 0.0485389, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$\frac{3(c+dx)}{8d\left(1-(c+dx)^2\right)}+\frac{c+dx}{4d\left(1-(c+dx)^2\right)^2}+\frac{3\tanh^{-1}(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - (c + d*x)^2)^(-3),x]
```

```
[Out] (c + d*x)/(4*d*(1 - (c + d*x)^2)^2) + (3*(c + d*x))/(8*d*(1 - (c
+ d*x)^2)) + (3*ArcTanh[c + d*x])/(8*d)
```

Rubi in Sympy [A] time = 3.41297, size = 49, normalized size = 0.77

$$\frac{3\,(c+dx)}{8d\left(-(c+dx)^2+1\right)}+\frac{c+dx}{4d\left(-(c+dx)^2+1\right)^2}+\frac{3\,\text{atanh}\,(c+dx)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1-(d*x+c)**2)**3,x)
```

```
[Out] 3*(c + d*x)/(8*d*(-(c + d*x)**2 + 1)) + (c + d*x)/(4*d*(-(c + d*x
)**2 + 1)**2) + 3*atanh(c + d*x)/(8*d)
```

Mathematica [A] time = 0.0449493, size = 65, normalized size = 1.02

$$\frac{-\frac{6(c+dx)}{(c+dx)^2-1}+\frac{4(c+dx)}{((c+dx)^2-1)^2}-3\log(-c-dx+1)+3\log(c+dx+1)}{16d}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - (c + d*x)^2)^(-3),x]
```

```
[Out] ((4*(c + d*x))/(-1 + (c + d*x)^2)^2 - (6*(c + d*x))/(-1 + (c + d*
x)^2) - 3*Log[1 - c - d*x] + 3*Log[1 + c + d*x])/(16*d)
```

Maple [A] time = 0.015, size = 78, normalized size = 1.2

$$\frac{1}{16\,d\,(dx+c-1)^2}-\frac{3}{16\,d\,(dx+c-1)}-\frac{3\,\ln(dx+c-1)}{16\,d}-\frac{1}{16\,d\,(dx+c+1)^2}-\frac{3}{16\,d\,(dx+c+1)}+\frac{3\,\ln(dx+c+1)}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1-(d*x+c)^2)^3,x)
```

```
[Out] 1/16/d/(d*x+c-1)^2-3/16/d/(d*x+c-1)-3/16/d*ln(d*x+c-1)-1/16/d/(d*
x+c+1)^2-3/16/d/(d*x+c+1)+3/16/d*ln(d*x+c+1)
```

Maxima [A] time = 0.826349, size = 165, normalized size = 2.58

$$-\frac{3\,d^3x^3+9\,cd^2x^2+3\,c^3+\left(9\,c^2-5\right)dx-5\,c}{8\,(d^5x^4+4\,cd^4x^3+2\,(3\,c^2-1)d^3x^2+4\,(c^3-c)d^2x+(c^4-2\,c^2+1)d)}+\frac{3\,\log\left(dx+c+1\right)}{16\,d}-\frac{3\,\log\left(dx+c-1\right)}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((d*x + c)^2 - 1)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(3*d^3*x^3 + 9*c*d^2*x^2 + 3*c^3 + (9*c^2 - 5)*d*x - 5*c)/(d
^5*x^4 + 4*c*d^4*x^3 + 2*(3*c^2 - 1)*d^3*x^2 + 4*(c^3 - c)*d^2*x
+ (c^4 - 2*c^2 + 1)*d) + 3/16*log(d*x + c + 1)/d - 3/16*log(d*x +
 c - 1)/d
```

Fricas [A] time = 0.268806, size = 297, normalized size = 4.64

$$-\frac{6\,d^3x^3+18\,cd^2x^2+6\,c^3+2\left(9\,c^2-5\right)dx-3\left(d^4x^4+4\,cd^3x^3+2\left(3\,c^2-1\right)d^2x^2+c^4+4\left(c^3-c\right)dx-2\,c^2+1\right)\log\left(dx+c\right.}{16\,(d^5x^4+4\,cd^4x^3+2\,(3\,c^2-1)d^3x^2+4\,(c^3-c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((d*x + c)^2 - 1)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(6*d^3*x^3 + 18*c*d^2*x^2 + 6*c^3 + 2*(9*c^2 - 5)*d*x - 3*(
d^4*x^4 + 4*c*d^3*x^3 + 2*(3*c^2 - 1)*d^2*x^2 + c^4 + 4*(c^3 - c)
*d*x - 2*c^2 + 1)*log(d*x + c + 1) + 3*(d^4*x^4 + 4*c*d^3*x^3 + 2
*(3*c^2 - 1)*d^2*x^2 + c^4 + 4*(c^3 - c)*d*x - 2*c^2 + 1)*log(d*x
 + c - 1) - 10*c)/(d^5*x^4 + 4*c*d^4*x^3 + 2*(3*c^2 - 1)*d^3*x^2
+ 4*(c^3 - c)*d^2*x + (c^4 - 2*c^2 + 1)*d)
```

Sympy [A] time = 4.88074, size = 141, normalized size = 2.2

$$-\frac{3c^3+9cd^2x^2-5c+3d^3x^3+x\left(9c^2d-5d\right)}{8c^4d-16c^2d+32cd^4x^3+8d^5x^4+8d+x^2\left(48c^2d^3-16d^3\right)+x\left(32c^3d^2-32cd^2\right)}-\frac{\frac{3\log\left(x+\frac{3c-3}{3d}\right)}{16}-\frac{3\log\left(x+\frac{3c+3}{3d}\right)}{16}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1-(d*x+c)**2)**3,x)
```

```
[Out] -(3*c**3 + 9*c*d**2*x**2 - 5*c + 3*d**3*x**3 + x*(9*c**2*d - 5*d)
)/(8*c**4*d - 16*c**2*d + 32*c*d**4*x**3 + 8*d**5*x**4 + 8*d + x*
*2*(48*c**2*d**3 - 16*d**3) + x*(32*c**3*d**2 - 32*c*d**2)) - (3*
log(x + (3*c - 3)/(3*d))/16 - 3*log(x + (3*c + 3)/(3*d))/16)/d
```

GIAC/XCAS [A] time = 0.263148, size = 119, normalized size = 1.86

$$\frac{3\ln\left(|dx+c+1|\right)}{16\,d}-\frac{3\ln\left(|dx+c-1|\right)}{16\,d}-\frac{3\,d^{3}x^{3}+9\,cd^{2}x^{2}+9\,c^{2}dx+3\,c^{3}-5\,dx-5\,c}{8\,(d^{2}x^{2}+2\,cdx+c^{2}-1)^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((d*x + c)^2 - 1)^3,x, algorithm="giac")
```

```
[Out] 3/16*ln(abs(d*x + c + 1))/d - 3/16*ln(abs(d*x + c - 1))/d - 1/8*(
3*d^3*x^3 + 9*c*d^2*x^2 + 9*c^2*d*x + 3*c^3 - 5*d*x - 5*c)/((d^2*
x^2 + 2*c*d*x + c^2 - 1)^2*d)
```

3.95 $\int \frac{1}{1-(1+x)^2} \, dx$

Optimal. Leaf size=4

$$\tanh^{-1}(x+1)$$

```
[Out] ArcTanh[1 + x]
```

Rubi [A] time = 0.00592961, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\tanh^{-1}(x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - (1 + x)^2)^(-1),x]
```

```
[Out] ArcTanh[1 + x]
```

Rubi in Sympy [A] time = 0.342818, size = 3, normalized size = 0.75

$$\operatorname{atanh}(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1-(1+x)**2),x)
```

```
[Out] atanh(x + 1)
```

Mathematica [B] time = 0.00360685, size = 15, normalized size = 3.75

$$\frac{1}{2}\log(x+2) - \frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - (1 + x)^2)^(-1),x]
```

```
[Out] -Log[x]/2 + Log[2 + x]/2
```

Maple [B] time = 0.007, size = 12, normalized size = 3.

$$\frac{\ln(2+x)}{2} - \frac{\ln(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1-(1+x)^2),x)
```

```
[Out] 1/2*ln(2+x)-1/2*ln(x)
```

Maxima [A] time = 0.805231, size = 15, normalized size = 3.75

$$\frac{1}{2}\log(x+2) - \frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((x + 1)^2 - 1),x, algorithm="maxima")

[Out] 1/2*log(x + 2) - 1/2*log(x)
```

Fricas [A] time = 0.257159, size = 15, normalized size = 3.75

$$\frac{1}{2}\log(x+2) - \frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((x + 1)^2 - 1),x, algorithm="fricas")

[Out] 1/2*log(x + 2) - 1/2*log(x)
```

Sympy [A] time = 0.178387, size = 10, normalized size = 2.5

$$-\frac{\log(x)}{2} + \frac{\log(x+2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1-(1+x)**2),x)

[Out] -log(x)/2 + log(x + 2)/2
```

GIAC/XCAS [A] time = 0.261982, size = 18, normalized size = 4.5

$$\frac{1}{2}\ln(|x+2|) - \frac{1}{2}\ln(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((x + 1)^2 - 1),x, algorithm="giac")

[Out] 1/2*ln(abs(x + 2)) - 1/2*ln(abs(x))
```

3.96 $\int \frac{1}{(1-(1+x)^2)^2}\,dx$

Optimal. Leaf size=27

$$\frac{x+1}{2\left(1-(x+1)^2\right)}+\frac{1}{2}\tanh^{-1}(x+1)$$

```
[Out] (1 + x)/(2*(1 - (1 + x)^2)) + ArcTanh[1 + x]/2
```

Rubi [A] time = 0.0165959, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273

$$\frac{x+1}{2\left(1-(x+1)^2\right)}+\frac{1}{2}\tanh^{-1}(x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - (1 + x)^2)^(-2),x]
```

```
[Out] (1 + x)/(2*(1 - (1 + x)^2)) + ArcTanh[1 + x]/2
```

Rubi in Sympy [A] time = 1.37929, size = 17, normalized size = 0.63

$$\frac{x+1}{2\left(-(x+1)^2+1\right)}+\frac{\operatorname{atanh}(x+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1-(1+x)**2)**2,x)
```

```
[Out] (x + 1)/(2*(-(x + 1)**2 + 1)) + atanh(x + 1)/2
```

Mathematica [A] time = 0.0295108, size = 26, normalized size = 0.96

$$\frac{1}{4}\left(-\frac{2(x+1)}{x(x+2)}-\log(x)+\log(x+2)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - (1 + x)^2)^(-2),x]
```

```
[Out] ((-2*(1 + x))/(x*(2 + x)) - Log[x] + Log[2 + x])/4
```

Maple [A] time = 0.013, size = 24, normalized size = 0.9

$$-\frac{1}{4\,x}-\frac{1}{8+4\,x}-\frac{\ln(x)}{4}+\frac{\ln(2+x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1-(1+x)^2)^2,x)
```

```
[Out] -1/4/x-1/4/(2+x)-1/4*ln(x)+1/4*ln(2+x)
```

Maxima [A] time = 0.797471, size = 34, normalized size = 1.26

$$-\frac{x+1}{2\,(x^2+2\,x)}+\frac{1}{4}\,\log\left(x+2\right)-\frac{1}{4}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((x + 1)^2 - 1)^(-2),x, algorithm="maxima")
```

```
[Out] -1/2*(x + 1)/(x^2 + 2*x) + 1/4*log(x + 2) - 1/4*log(x)
```

Fricas [A] time = 0.273721, size = 53, normalized size = 1.96

$$\frac{\left(x^2+2\,x\right)\,\log\left(x+2\right)-\left(x^2+2\,x\right)\,\log\left(x\right)-2\,x-2}{4\,(x^2+2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((x + 1)^2 - 1)^(-2),x, algorithm="fricas")
```

```
[Out] 1/4*((x^2 + 2*x)*log(x + 2) - (x^2 + 2*x)*log(x) - 2*x - 2)/(x^2
+ 2*x)
```

Sympy [A] time = 0.236917, size = 22, normalized size = 0.81

$$-\frac{x+1}{2x^2+4x}-\frac{\log\left(x\right)}{4}+\frac{\log\left(x+2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1-(1+x)**2)**2,x)
```

```
[Out] -(x + 1)/(2*x**2 + 4*x) - log(x)/4 + log(x + 2)/4
```

GIAC/XCAS [A] time = 0.261958, size = 36, normalized size = 1.33

$$-\frac{x+1}{2\,(x^2+2\,x)}+\frac{1}{4}\,\ln\left(|x+2|\right)-\frac{1}{4}\,\ln\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((x + 1)^2 - 1)^(-2),x, algorithm="giac")
```

```
[Out] -1/2*(x + 1)/(x^2 + 2*x) + 1/4*ln(abs(x + 2)) - 1/4*ln(abs(x))
```

3.97 $\int \frac{1}{(1-(1+x)^2)^3} \, dx$

Optimal. Leaf size=45

$$\frac{3(x+1)}{8(1-(x+1)^2)} + \frac{x+1}{4(1-(x+1)^2)^2} + \frac{3}{8}\tanh^{-1}(x+1)$$

```
[Out] (1 + x)/(4*(1 - (1 + x)^2)^2) + (3*(1 + x))/(8*(1 - (1 + x)^2)) +
 (3*ArcTanh[1 + x])/8
```

Rubi [A] time = 0.0301542, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$

$$\frac{3(x+1)}{8(1-(x+1)^2)} + \frac{x+1}{4(1-(x+1)^2)^2} + \frac{3}{8}\tanh^{-1}(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - (1 + x)^2)^(-3),x]
```

```
[Out] (1 + x)/(4*(1 - (1 + x)^2)^2) + (3*(1 + x))/(8*(1 - (1 + x)^2)) +
 (3*ArcTanh[1 + x])/8
```

Rubi in Sympy [A] time = 1.66059, size = 34, normalized size = 0.76

$$\frac{3(x+1)}{8\left(-(x+1)^2+1\right)} + \frac{x+1}{4\left(-(x+1)^2+1\right)^2} + \frac{3\,\mathrm{atanh}\,(x+1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(1-(1+x)**2)**3,x)
```

```
[Out] 3*(x + 1)/(8*(-(x + 1)**2 + 1)) + (x + 1)/(4*(-(x + 1)**2 + 1)**2
) + 3*atanh(x + 1)/8
```

Mathematica [A] time = 0.0289405, size = 37, normalized size = 0.82

$$\frac{1}{16}\left(\frac{1}{x^2} - \frac{3}{x} - \frac{3}{x+2} - \frac{1}{(x+2)^2} - 3\log(x) + 3\log(x+2)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - (1 + x)^2)^(-3),x]
```

```
[Out] (x^(-2) - 3/x - (2 + x)^(-2) - 3/(2 + x) - 3*Log[x] + 3*Log[2 + x
])/16
```

Maple [A] time = 0.013, size = 36, normalized size = 0.8

$$-\frac{1}{16\,(2+x)^2} - \frac{3}{32+16\,x} + \frac{3\,\ln(2+x)}{16} + \frac{1}{16\,x^2} - \frac{3}{16\,x} - \frac{3\,\ln(x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1-(1+x)^2)^3,x)
```

```
[Out] -1/16/(2+x)^2-3/16/(2+x)+3/16*ln(2+x)+1/16/x^2-3/16/x-3/16*ln(x)
```

Maxima [A] time = 0.806942, size = 59, normalized size = 1.31

$$-\frac{3\,x^3+9\,x^2+4\,x-2}{8\,(x^4+4\,x^3+4\,x^2)}+\frac{3}{16}\,\log\left(x+2\right)-\frac{3}{16}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((x + 1)^2 - 1)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(3*x^3 + 9*x^2 + 4*x - 2)/(x^4 + 4*x^3 + 4*x^2) + 3/16*log(x
 + 2) - 3/16*log(x)
```

Fricas [A] time = 0.265981, size = 96, normalized size = 2.13

$$-\frac{6\,x^3+18\,x^2-3\,\left(x^4+4\,x^3+4\,x^2\right)\,\log\left(x+2\right)+3\,\left(x^4+4\,x^3+4\,x^2\right)\,\log\left(x\right)+8\,x-4}{16\,(x^4+4\,x^3+4\,x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((x + 1)^2 - 1)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(6*x^3 + 18*x^2 - 3*(x^4 + 4*x^3 + 4*x^2)*log(x + 2) + 3*(x
^4 + 4*x^3 + 4*x^2)*log(x) + 8*x - 4)/(x^4 + 4*x^3 + 4*x^2)
```

Sympy [A] time = 0.339256, size = 44, normalized size = 0.98

$$-\frac{3\log\left(x\right)}{16}+\frac{3\log\left(x+2\right)}{16}-\frac{3x^3+9x^2+4x-2}{8x^4+32x^3+32x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1-(1+x)**2)**3,x)
```

```
[Out] -3*log(x)/16 + 3*log(x + 2)/16 - (3*x**3 + 9*x**2 + 4*x - 2)/(8*x
**4 + 32*x**3 + 32*x**2)
```

GIAC/XCAS [A] time = 0.261994, size = 53, normalized size = 1.18

$$-\frac{3\,x^3+9\,x^2+4\,x-2}{8\,(x^2+2\,x)^2}+\frac{3}{16}\ln\left(|x+2|\right)-\frac{3}{16}\ln\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((x + 1)^2 - 1)^3,x, algorithm="giac")
```

```
[Out] -1/8*(3*x^3 + 9*x^2 + 4*x - 2)/(x^2 + 2*x)^2 + 3/16*ln(abs(x + 2)
) - 3/16*ln(abs(x))
```

3.98 $\int \frac{\left(1+(a+bx)^2\right)^2}{x} \, dx$

Optimal. Leaf size=59

$$\frac{1}{2}\left(a^2+2\right)(a+bx)^2+a\left(a^2+2\right)bx+\left(a^2+1\right)^2\log(x)+\frac{1}{4}(a+bx)^4+\frac{1}{3}a(a+bx)^3$$

```
[Out] a*(2 + a^2)*b*x + ((2 + a^2)*(a + b*x)^2)/2 + (a*(a + b*x)^3)/3 +
 (a + b*x)^4/4 + (1 + a^2)^2*Log[x]
```

Rubi [A] time = 0.115344, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$\frac{1}{2}\left(a^2+2\right)(a+bx)^2+a\left(a^2+2\right)bx+\left(a^2+1\right)^2\log(x)+\frac{1}{4}(a+bx)^4+\frac{1}{3}a(a+bx)^3$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + (a + b*x)^2)^2/x,x]
```

```
[Out] a*(2 + a^2)*b*x + ((2 + a^2)*(a + b*x)^2)/2 + (a*(a + b*x)^3)/3 +
 (a + b*x)^4/4 + (1 + a^2)^2*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a(a+bx)^3}{3}+\frac{(a+bx)^4}{4}+\left(a^2+1\right)^2\log(-bx)+\left(a^2+2\right)\int^{a+bx} x\,dx+\frac{\left(a^2+2\right)\int^{a+bx} a^3\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+(b*x+a)**2)**2/x,x)
```

```
[Out] a*(a + b*x)**3/3 + (a + b*x)**4/4 + (a**2 + 1)**2*log(-b*x) + (a*
*2 + 2)*Integral(x, (x, a + b*x)) + (a**2 + 2)*Integral(a**3, (x,
 a + b*x))/a**2
```

Mathematica [A] time = 0.0374015, size = 64, normalized size = 1.08

$$\frac{1}{2}\left(a^2+2\right)(a+bx)^2+a\left(a^2+2\right)(a+bx)+\left(a^2+1\right)^2\log(bx)+\frac{1}{4}(a+bx)^4+\frac{1}{3}a(a+bx)^3$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + (a + b*x)^2)^2/x,x]
```

```
[Out] a*(2 + a^2)*(a + b*x) + ((2 + a^2)*(a + b*x)^2)/2 + (a*(a + b*x)^
3)/3 + (a + b*x)^4/4 + (1 + a^2)^2*Log[b*x]
```

Maple [A] time = 0.002, size = 64, normalized size = 1.1

$$\frac{b^4x^4}{4}+\frac{4\,ab^3x^3}{3}+3\,a^2b^2x^2+4\,a^3bx+b^2x^2+4\,abx+a^4\ln(x)+2\,a^2\ln(x)+\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+(b*x+a)^2)^2/x,x)
```

```
[Out] 1/4*b^4*x^4+4/3*a*b^3*x^3+3*a^2*b^2*x^2+4*a^3*b*x+b^2*x^2+4*a*b*x
+a^4*ln(x)+2*a^2*ln(x)+ln(x)
```

Maxima [A] time = 0.797313, size = 73, normalized size = 1.24

$$\frac{1}{4} b^4 x^4 + \frac{4}{3} a b^3 x^3 + \left(3 a^2 + 1\right) b^2 x^2 + 4 \left(a^3 + a\right) b x + \left(a^4 + 2 a^2 + 1\right) \log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((b*x + a)^2 + 1)^2/x,x, algorithm="maxima")
```

```
[Out] 1/4*b^4*x^4 + 4/3*a*b^3*x^3 + (3*a^2 + 1)*b^2*x^2 + 4*(a^3 + a)*b
*x + (a^4 + 2*a^2 + 1)*log(x)
```

Fricas [A] time = 0.271526, size = 73, normalized size = 1.24

$$\frac{1}{4} b^4 x^4 + \frac{4}{3} a b^3 x^3 + \left(3 a^2 + 1\right) b^2 x^2 + 4 \left(a^3 + a\right) b x + \left(a^4 + 2 a^2 + 1\right) \log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((b*x + a)^2 + 1)^2/x,x, algorithm="fricas")
```

```
[Out] 1/4*b^4*x^4 + 4/3*a*b^3*x^3 + (3*a^2 + 1)*b^2*x^2 + 4*(a^3 + a)*b
*x + (a^4 + 2*a^2 + 1)*log(x)
```

Sympy [A] time = 1.27172, size = 58, normalized size = 0.98

$$\frac{4ab^3x^3}{3} + \frac{b^4x^4}{4} + x^2 \left(3a^2b^2 + b^2\right) + x \left(4a^3b + 4ab\right) + \left(a^2 + 1\right)^2 \log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+(b*x+a)**2)**2/x,x)
```

```
[Out] 4*a*b**3*x**3/3 + b**4*x**4/4 + x**2*(3*a**2*b**2 + b**2) + x*(4*
a**3*b + 4*a*b) + (a**2 + 1)**2*log(x)
```

GIAC/XCAS [A] time = 0.262096, size = 84, normalized size = 1.42

$$\frac{1}{4} b^4 x^4 + \frac{4}{3} a b^3 x^3 + 3 a^2 b^2 x^2 + 4 a^3 b x + b^2 x^2 + 4 a b x + \left(a^4 + 2 a^2 + 1\right) \ln\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((b*x + a)^2 + 1)^2/x,x, algorithm="giac")
```

```
[Out] 1/4*b^4*x^4 + 4/3*a*b^3*x^3 + 3*a^2*b^2*x^2 + 4*a^3*b*x + b^2*x^2
 + 4*a*b*x + (a^4 + 2*a^2 + 1)*ln(abs(x))
```

3.99 $\int \frac{x^2}{1+(-1+x)^2} \, dx$

Optimal. Leaf size=10

$$x + \log\left((x-1)^2 + 1\right)$$

```
[Out] x + Log[1 + (-1 + x)^2]
```

Rubi [A] time = 0.0287703, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$x + \log\left((x-1)^2 + 1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(1 + (-1 + x)^2),x]

[Out] x + Log[1 + (-1 + x)^2]
```

Rubi in Sympy [A] time = 5.57177, size = 10, normalized size = 1.

$$x + \log\left((x-1)^2 + 1\right) - 1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(1+(-1+x)**2),x)

[Out] x + log((x - 1)**2 + 1) - 1
```

Mathematica [A] time = 0.0106058, size = 11, normalized size = 1.1

$$\log\left(x^2 - 2x + 2\right) + x$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(1 + (-1 + x)^2),x]

[Out] x + Log[2 - 2*x + x^2]
```

Maple [A] time = 0.003, size = 12, normalized size = 1.2

$$x + \ln\left(x^2 - 2\,x + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(1+(-1+x)^2),x)

[Out] x+ln(x^2-2*x+2)
```

Maxima [A] time = 0.801978, size = 15, normalized size = 1.5

$$x + \log\left(x^2 - 2x + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((x - 1)^2 + 1),x, algorithm="maxima")
```

```
[Out] x + log(x^2 - 2*x + 2)
```

Fricas [A] time = 0.289087, size = 15, normalized size = 1.5

$$x + \log\left(x^2 - 2x + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((x - 1)^2 + 1),x, algorithm="fricas")
```

```
[Out] x + log(x^2 - 2*x + 2)
```

Sympy [A] time = 0.147849, size = 10, normalized size = 1.

$$x + \log\left(x^2 - 2x + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(1+(-1+x)**2),x)
```

```
[Out] x + log(x**2 - 2*x + 2)
```

GIAC/XCAS [A] time = 0.265363, size = 15, normalized size = 1.5

$$x + \ln\left(x^2 - 2x + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((x - 1)^2 + 1),x, algorithm="giac")
```

```
[Out] x + ln(x^2 - 2*x + 2)
```

3.100 $\int \frac{x^2}{\sqrt{1-(1+x)^2}} \, dx$

Optimal. Leaf size=44

$$-\frac{1}{2}\sqrt{1-(x+1)^2}x+\frac{3}{2}\sqrt{1-(x+1)^2}+\frac{3}{2}\sin^{-1}(x+1)$$

```
[Out] (3*Sqrt[1 - (1 + x)^2])/2 - (x*Sqrt[1 - (1 + x)^2])/2 + (3*ArcSin
[1 + x])/2
```

Rubi [A] time = 0.0657623, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235

$$-\frac{1}{2}\sqrt{1-(x+1)^2}x+\frac{3}{2}\sqrt{1-(x+1)^2}+\frac{3}{2}\sin^{-1}(x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/Sqrt[1 - (1 + x)^2],x]
```

```
[Out] (3*Sqrt[1 - (1 + x)^2])/2 - (x*Sqrt[1 - (1 + x)^2])/2 + (3*ArcSin
[1 + x])/2
```

Rubi in Sympy [A] time = 8.27844, size = 34, normalized size = 0.77

$$-\frac{x\sqrt{-(x+1)^2+1}}{2}+\frac{3\sqrt{-(x+1)^2+1}}{2}+\frac{3\operatorname{asin}(x+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(1-(1+x)**2)**(1/2),x)
```

```
[Out] -x*sqrt(-(x + 1)**2 + 1)/2 + 3*sqrt(-(x + 1)**2 + 1)/2 + 3*asin(x
 + 1)/2
```

Mathematica [A] time = 0.0327615, size = 51, normalized size = 1.16

$$\frac{x\left(x^2-x-6\right)+6\sqrt{x}\sqrt{x+2}\sinh^{-1}\left(\frac{\sqrt{x}}{\sqrt{2}}\right)}{2\sqrt{-x(x+2)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/Sqrt[1 - (1 + x)^2],x]
```

```
[Out] (x*(-6 - x + x^2) + 6*Sqrt[x]*Sqrt[2 + x]*ArcSinh[Sqrt[x]/Sqrt[2]
])/(2*Sqrt[-(x*(2 + x))])
```

Maple [A] time = 0.006, size = 35, normalized size = 0.8

$$-\frac{x}{2}\sqrt{-x^2-2\,x}+\frac{3}{2}\sqrt{-x^2-2\,x}+\frac{3\arcsin(1+x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(1-(1+x)^2)^(1/2),x)
```

```
[Out] -1/2*x*(-x^2-2*x)^(1/2)+3/2*(-x^2-2*x)^(1/2)+3/2*arcsin(1+x)
```

Maxima [A] time = 0.890398, size = 49, normalized size = 1.11

$$-\frac{1}{2}\sqrt{-x^2-2\,x}x+\frac{3}{2}\sqrt{-x^2-2\,x}-\frac{3}{2}\arcsin\left(-x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt(-(x + 1)^2 + 1),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(-x^2 - 2*x)*x + 3/2*sqrt(-x^2 - 2*x) - 3/2*arcsin(-x -
1)
```

Fricas [A] time = 0.280235, size = 47, normalized size = 1.07

$$-\frac{1}{2}\sqrt{-x^2-2\,x}(x-3)-3\arctan\left(\frac{\sqrt{-x^2-2\,x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt(-(x + 1)^2 + 1),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(-x^2 - 2*x)*(x - 3) - 3*arctan(sqrt(-x^2 - 2*x)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2}{\sqrt{-x\left(x+2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(1-(1+x)**2)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(-x*(x + 2)), x)
```

GIAC/XCAS [A] time = 0.266137, size = 31, normalized size = 0.7

$$-\frac{1}{2}\sqrt{-(x+1)^2+1}(x-3)+\frac{3}{2}\arcsin\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt(-(x + 1)^2 + 1),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-(x + 1)^2 + 1)*(x - 3) + 3/2*arcsin(x + 1)
```

3.101 $\int \frac{x^2}{\sqrt{1-(a+bx)^2}}\, dx$

Optimal. Leaf size=67

$$\frac{\left(2a^2+1\right)\sin^{-1}(a+bx)}{2b^3}+\frac{3a\sqrt{1-(a+bx)^2}}{2b^3}-\frac{x\sqrt{1-(a+bx)^2}}{2b^2}$$

```
[Out] (3*a*Sqrt[1 - (a + b*x)^2])/(2*b^3) - (x*Sqrt[1 - (a + b*x)^2])/(
2*b^2) + ((1 + 2*a^2)*ArcSin[a + b*x])/(2*b^3)
```

Rubi [A] time = 0.121378, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21

$$\frac{\left(2a^2+1\right)\sin^{-1}(a+bx)}{2b^3}+\frac{3a\sqrt{1-(a+bx)^2}}{2b^3}-\frac{x\sqrt{1-(a+bx)^2}}{2b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/Sqrt[1 - (a + b*x)^2],x]
```

```
[Out] (3*a*Sqrt[1 - (a + b*x)^2])/(2*b^3) - (x*Sqrt[1 - (a + b*x)^2])/(
2*b^2) + ((1 + 2*a^2)*ArcSin[a + b*x])/(2*b^3)
```

Rubi in Sympy [A] time = 14.1268, size = 54, normalized size = 0.81

$$\frac{3a\sqrt{-(a+bx)^2+1}}{2b^3}-\frac{x\sqrt{-(a+bx)^2+1}}{2b^2}+\frac{\left(a^2+\frac{1}{2}\right)\operatorname{asin}(a+bx)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(1-(b*x+a)**2)**(1/2),x)
```

```
[Out] 3*a*sqrt(-(a + b*x)**2 + 1)/(2*b**3) - x*sqrt(-(a + b*x)**2 + 1)/
(2*b**2) + (a**2 + 1/2)*asin(a + b*x)/b**3
```

Mathematica [A] time = 0.074997, size = 55, normalized size = 0.82

$$\frac{\sqrt{-a^2-2abx-b^2x^2+1}(3a-bx)+\left(2a^2+1\right)\sin^{-1}(a+bx)}{2b^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/Sqrt[1 - (a + b*x)^2],x]
```

```
[Out] ((3*a - b*x)*Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2] + (1 + 2*a^2)*ArcS
in[a + b*x])/(2*b^3)
```

Maple [B] time = 0.022, size = 152, normalized size = 2.3

$$-\frac{x}{2\,b^2}\sqrt{-b^2x^2-2\,abx-a^2+1}+\frac{3\,a}{2\,b^3}\sqrt{-b^2x^2-2\,abx-a^2+1}+\frac{a^2}{b^2}\arctan\left(1\sqrt{b^2}\left(\frac{a}{b}+x\right)\frac{1}{\sqrt{-b^2x^2-2\,abx-a^2+1}}\right)\frac{1}{\sqrt{b^2}}+\frac{1}{2\,b^2}\arctan\left(1\sqrt{b^2}\left(\frac{a}{b}+x\right)\frac{1}{\sqrt{-b^2x^2-2\,abx-a^2+1}}\right)\frac{1}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(1-(b*x+a)^2)^(1/2),x)
```

```
[Out] -1/2*x/b^2*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+3/2*a/b^3*(-b^2*x^2-2*a
*b*x-a^2+1)^(1/2)+a^2/b^2/(b^2)^(1/2)*arctan((b^2)^(1/2)*(a/b+x)/
(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))+1/2/b^2/(b^2)^(1/2)*arctan((b^2)^
(1/2)*(a/b+x)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt(-(b*x + a)^2 + 1),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.275648, size = 100, normalized size = 1.49

$$\frac{\left(2\,a^2+1\right)\arctan\left(\frac{bx+a}{\sqrt{-b^2x^2-2\,abx-a^2+1}}\right)-\sqrt{-b^2x^2-2\,abx-a^2+1}(bx-3\,a)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt(-(b*x + a)^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/2*((2*a^2 + 1)*arctan((b*x + a)/sqrt(-b^2*x^2 - 2*a*b*x - a^2 +
 1)) - sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x - 3*a))/b^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2}{\sqrt{-(a+bx-1)(a+bx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(1-(b*x+a)**2)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(-(a + b*x - 1)*(a + b*x + 1)), x)
```

GIAC/XCAS [A] time = 0.280923, size = 74, normalized size = 1.1

$$-\frac{1}{2}\sqrt{-(bx+a)^2+1}\left(\frac{x}{b^2}-\frac{3a}{b^3}\right)-\frac{\left(2a^2+1\right)\arcsin\left(-bx-a\right)\operatorname{sign}\left(b\right)}{2b^2|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt(-(b*x + a)^2 + 1),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-(b*x + a)^2 + 1)*(x/b^2 - 3*a/b^3) - 1/2*(2*a^2 + 1)*a
rcsin(-b*x - a)*sign(b)/(b^2*abs(b))
```

3.102 $\int \frac{x^2}{\sqrt{1+(a+bx)^2}}\,dx$

Optimal. Leaf size=63

$$-\frac{(1-2a^2)\sinh^{-1}(a+bx)}{2b^3}-\frac{3a\sqrt{(a+bx)^2+1}}{2b^3}+\frac{x\sqrt{(a+bx)^2+1}}{2b^2}$$

```
[Out] (-3*a*Sqrt[1 + (a + b*x)^2])/(2*b^3) + (x*Sqrt[1 + (a + b*x)^2])/
(2*b^2) - ((1 - 2*a^2)*ArcSinh[a + b*x])/(2*b^3)
```

Rubi [A] time = 0.100849, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$

$$-\frac{(1-2a^2)\sinh^{-1}(a+bx)}{2b^3}-\frac{3a\sqrt{(a+bx)^2+1}}{2b^3}+\frac{x\sqrt{(a+bx)^2+1}}{2b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/Sqrt[1 + (a + b*x)^2],x]
```

```
[Out] (-3*a*Sqrt[1 + (a + b*x)^2])/(2*b^3) + (x*Sqrt[1 + (a + b*x)^2])/
(2*b^2) - ((1 - 2*a^2)*ArcSinh[a + b*x])/(2*b^3)
```

Rubi in Sympy [A] time = 11.2609, size = 54, normalized size = 0.86

$$-\frac{3a\sqrt{(a+bx)^2+1}}{2b^3}+\frac{x\sqrt{(a+bx)^2+1}}{2b^2}-\frac{\left(-a^2+\frac{1}{2}\right)\operatorname{asinh}(a+bx)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(1+(b*x+a)**2)**(1/2),x)
```

```
[Out] -3*a*sqrt((a + b*x)**2 + 1)/(2*b**3) + x*sqrt((a + b*x)**2 + 1)/(
2*b**2) - (-a**2 + 1/2)*asinh(a + b*x)/b**3
```

Mathematica [A] time = 0.0655677, size = 51, normalized size = 0.81

$$\frac{\sqrt{a^2+2abx+b^2x^2+1}(bx-3a)+(2a^2-1)\sinh^{-1}(a+bx)}{2b^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/Sqrt[1 + (a + b*x)^2],x]
```

```
[Out] ((-3*a + b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] + (-1 + 2*a^2)*Ar
cSinh[a + b*x])/(2*b^3)
```

Maple [B] time = 0.013, size = 146, normalized size = 2.3

$$\frac{x}{2\,b^2}\sqrt{b^2x^2+2\,abx+a^2+1}-\frac{3\,a}{2\,b^3}\sqrt{b^2x^2+2\,abx+a^2+1}+\frac{a^2}{b^2}\ln\left((b^2x+ab)\frac{1}{\sqrt{b^2}}+\sqrt{b^2x^2+2\,abx+a^2+1}\right)\frac{1}{\sqrt{b^2}}-\frac{1}{2\,b^2}\ln\left((b^2x+ab)\frac{1}{\sqrt{b^2}}+\sqrt{b^2x^2+2\,abx+a^2+1}\right)\frac{1}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(1+(b*x+a)^2)^(1/2),x)
```

```
[Out] 1/2*x/b^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-3/2*a/b^3*(b^2*x^2+2*a*b*
x+a^2+1)^(1/2)+a^2/b^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*
x+a^2+1)^(1/2))/(b^2)^(1/2)-1/2/b^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b
^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt((b*x + a)^2 + 1),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.269363, size = 369, normalized size = 5.86

$$-\frac{8\,b^4x^4-2\left(17\,a^2-4\right)b^2x^2-10\,a^4-4\left(9\,a^3+4\,a\right)bx-17\,a^2+4\left(2\left(2\,a^2-1\right)b^2x^2+4\,a^4+4\left(2\,a^3-a\right)bx-2\sqrt{b^2x^2+2\,a\cdots}\right.}{8\left(2\,b^5x^2+4\,ab^4x\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt((b*x + a)^2 + 1),x, algorithm="fricas")
```

```
[Out] -1/8*(8*b^4*x^4 - 2*(17*a^2 - 4)*b^2*x^2 - 10*a^4 - 4*(9*a^3 + 4*
a)*b*x - 17*a^2 + 4*(2*(2*a^2 - 1)*b^2*x^2 + 4*a^4 + 4*(2*a^3 - a
)*b*x - 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(2*a^3 + (2*a^2 - 1)*
b*x - a) - 1)*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) -
 2*(4*b^3*x^3 - 4*a*b^2*x^2 - 5*a^3 - (13*a^2 - 2)*b*x - 6*a)*sqr
t(b^2*x^2 + 2*a*b*x + a^2 + 1))/(2*b^5*x^2 + 4*a*b^4*x + (2*a^2 +
 1)*b^3 - 2*(b^4*x + a*b^3)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{a^2+2abx+b^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(1+(b*x+a)**2)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1), x)
```

GIAC/XCAS [A] time = 0.274188, size = 95, normalized size = 1.51

$$\frac{1}{2}\sqrt{(bx+a)^2+1}\left(\frac{x}{b^2}-\frac{3a}{b^3}\right)-\frac{\left(2a^2-1\right)\ln\left(-ab-\left(x|b|-\sqrt{(bx+a)^2+1}\right)|b|\right)}{2b^2|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt((b*x + a)^2 + 1),x, algorithm="giac")
```

```
[Out] 1/2*sqrt((b*x + a)^2 + 1)*(x/b^2 - 3*a/b^3) - 1/2*(2*a^2 - 1)*ln(
-a*b - (x*abs(b) - sqrt((b*x + a)^2 + 1))*abs(b))/(b^2*abs(b))
```

3.103 $\int \frac{x^3}{a+b(c+dx)^3}\,dx$

Optimal. Leaf size=234

$$-\frac{\left(3\sqrt[3]{a}b^{2/3}c^2+a+bc^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}b^{4/3}d^4}+\frac{\left(3\sqrt[3]{a}b^{2/3}c^2+a+bc^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}b^{4/3}d^4}+\frac{\left(-3\sqrt[3]{a}b^{2/3}c^2+a+bc^3\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{4/3}d^4}-\frac{c\log\left(a+b(c+dx)^3\right)}{bd^4}+\frac{x}{bd^3}$$

```
[Out] x/(b*d^3) + ((a - 3*a^(1/3)*b^(2/3)*c^2 + b*c^3)*ArcTan[(a^(1/3)
- 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(4/
3)*d^4) - ((a + 3*a^(1/3)*b^(2/3)*c^2 + b*c^3)*Log[a^(1/3) + b^(1
/3)*(c + d*x)])/(3*a^(2/3)*b^(4/3)*d^4) + ((a + 3*a^(1/3)*b^(2/3)
*c^2 + b*c^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(
c + d*x)^2])/(6*a^(2/3)*b^(4/3)*d^4) - (c*Log[a + b*(c + d*x)^3])
/(b*d^4)
```

Rubi [A] time = 0.698876, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$

$$-\frac{\left(3\sqrt[3]{a}b^{2/3}c^2+a+bc^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}b^{4/3}d^4}+\frac{\left(3\sqrt[3]{a}b^{2/3}c^2+a+bc^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}b^{4/3}d^4}+\frac{\left(-3\sqrt[3]{a}b^{2/3}c^2+a+bc^3\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{4/3}d^4}-\frac{c\log\left(a+b(c+dx)^3\right)}{bd^4}+\frac{x}{bd^3}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/(a + b*(c + d*x)^3),x]
```

```
[Out] x/(b*d^3) + ((a - 3*a^(1/3)*b^(2/3)*c^2 + b*c^3)*ArcTan[(a^(1/3)
- 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(4/
3)*d^4) - ((a + 3*a^(1/3)*b^(2/3)*c^2 + b*c^3)*Log[a^(1/3) + b^(1
/3)*(c + d*x)])/(3*a^(2/3)*b^(4/3)*d^4) + ((a + 3*a^(1/3)*b^(2/3)
*c^2 + b*c^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(
c + d*x)^2])/(6*a^(2/3)*b^(4/3)*d^4) - (c*Log[a + b*(c + d*x)^3])
/(b*d^4)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int^{c+dx}\frac{1}{b}\,dx}{d^4}-\frac{c\log\left(a+b\left(c+dx\right)^3\right)}{bd^4}+\frac{\sqrt{3}\left(-3\sqrt[3]{a}b^{\frac{2}{3}}c^2+a+bc^3\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}+\sqrt[3]{b}\left(-\frac{2c}{3}-\frac{2dx}{3}\right)\right)}{\sqrt[3]{a}}\right)}{3a^{\frac{2}{3}}b^{\frac{4}{3}}d^4}-\frac{\left(3\sqrt[3]{a}b^{\frac{2}{3}}c^2+a+bc^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\left(c+dx\right)\right)}{3a^{\frac{2}{3}}b^{\frac{4}{3}}d^4}+\frac{\left(3\sqrt[3]{a}b^{\frac{2}{3}}c^2+a+bc^3\right)\log\left(a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{b}\left(-c-dx\right)+b^{\frac{2}{3}}\left(c+dx\right)^2\right)}{6a^{\frac{2}{3}}b^{\frac{4}{3}}d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(a+b*(d*x+c)**3),x)
```

```
[Out] Integral(1/b, (x, c + d*x))/d**4 - c*log(a + b*(c + d*x)**3)/(b*d
**4) + sqrt(3)*(-3*a**(1/3)*b**(2/3)*c**2 + a + b*c**3)*atan(sqrt
(3)*(a**(1/3)/3 + b**(1/3)*(-2*c/3 - 2*d*x/3))/a**(1/3))/(3*a**(2
/3)*b**(4/3)*d**4) - (3*a**(1/3)*b**(2/3)*c**2 + a + b*c**3)*log(
a**(1/3) + b**(1/3)*(c + d*x))/(3*a**(2/3)*b**(4/3)*d**4) + (3*a*
*(1/3)*b**(2/3)*c**2 + a + b*c**3)*log(a**(2/3) + a**(1/3)*b**(1/
3)*(-c - d*x) + b**(2/3)*(c + d*x)**2)/(6*a**(2/3)*b**(4/3)*d**4)
```

Mathematica [C] time = 0.0759547, size = 132, normalized size = 0.56

$$-\frac{\text{RootSum}\left[\#1^3bd^3+3\#1^2bcd^2+3\#1bc^2d+a+bc^3\&,\frac{3\#1^2bcd^2\log(x-\#1)+a\log(x-\#1)+bc^3\log(x-\#1)+3\#1bc^2d\log(x-\#1)}{\#1^2d^2+2\#1cd+c^2}\&\right]-3bdx}{3b^2d^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/(a + b*(c + d*x)^3),x]
```

```
[Out] -(-3*b*d*x + RootSum[a + b*c^3 + 3*b*c^2*d*#1 + 3*b*c*d^2*#1^2 +
b*d^3*#1^3 & , (a*Log[x - #1] + b*c^3*Log[x - #1] + 3*b*c^2*d*Log
[x - #1]*#1 + 3*b*c*d^2*Log[x - #1]*#1^2)/(c^2 + 2*c*d*#1 + d^2*#
1^2) & ])/(3*b^2*d^4)
```

Maple [C] time = 0.007, size = 108, normalized size = 0.5

$$\frac{x}{bd^3}+\frac{1}{3\,b^2d^4}\sum_{\_R=RootOf\left(bd^3\_Z^3+3\,bcd^2\_Z^2+3\,c^2db\_Z+bc^3+a\right)}\frac{\left(-3\_R^2bcd^2-3\_Rbc^2d-bc^3-a\right)\ln(x-\_R)}{d^2\_R^2+2\,cd\_R+c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(a+b*(d*x+c)^3),x)
```

```
[Out] x/b/d^3+1/3/b^2/d^4*sum((-3*_R^2*b*c*d^2-3*_R*b*c^2*d-b*c^3-a)/(_
R^2*d^2+2*_R*c*d+c^2)*ln(x-_R),_R=RootOf(_Z^3*b*d^3+3*_Z^2*b*c*d^
2+3*_Z*b*c^2*d+b*c^3+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{x}{bd^3}-\frac{\int\frac{3\,bcd^2x^2+3\,bc^2dx+bc^3+a}{bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a}\,dx}{bd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((d*x + c)^3*b + a),x, algorithm="maxima")
```

```
[Out] x/(b*d^3) - integrate((3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)/(
b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a), x)/(b*d^3)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((d*x + c)^3*b + a),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 8.96674, size = 238, normalized size = 1.02

$$\operatorname{RootSum}\left(27t^3a^2b^4d^{12} + 81t^2a^2b^3cd^8 + t\left(54a^2b^2c^2d^4 - 27ab^3c^5d^4\right) + a^3 + 3a^2bc^3 + 3ab^2c^6 + b^3c^9, \left(t \mapsto t\log\left(x + \frac{-27t^2a\right.\right.\right.$$

$$+ \frac{x}{bd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(a+b*(d*x+c)**3),x)
```

```
[Out] RootSum(27*_t**3*a**2*b**4*d**12 + 81*_t**2*a**2*b**3*c*d**8 + _t
*(54*a**2*b**2*c**2*d**4 - 27*a*b**3*c**5*d**4) + a**3 + 3*a**2*b
*c**3 + 3*a*b**2*c**6 + b**3*c**9, Lambda(_t, _t*log(x + (-27*_t*
*2*a**2*b**3*c**2*d**8 - 3*_t*a**3*b*d**4 - 60*_t*a**2*b**2*c**3*
d**4 - 3*_t*a*b**3*c**6*d**4 - 2*a**3*c - 12*a**2*b*c**4 - 9*a*b*
*2*c**7 + b**3*c**10)/(a**3*d + 3*a**2*b*c**3*d - 24*a*b**2*c**6*
d + b**3*c**9*d)))) + x/(b*d**3)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(dx+c)^3b+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((d*x + c)^3*b + a),x, algorithm="giac")
```

```
[Out] integrate(x^3/((d*x + c)^3*b + a), x)
```

3.104 $\int \frac{x^2}{a+b(c+dx)^3}\,dx$

Optimal. Leaf size=210

$$\frac{c\left(2\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}b^{2/3}d^3}-\frac{c\left(2\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}b^{2/3}d^3}+\frac{c\left(2\sqrt[3]{a}-\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}d^3}+\frac{\log\left(a+b(c+dx)^3\right)}{3bd^3}$$

```
[Out] (c*(2*a^(1/3) - b^(1/3)*c)*ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))
/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(2/3)*d^3) + (c*(2*a^(1/3
) + b^(1/3)*c)*Log[a^(1/3) + b^(1/3)*(c + d*x)])/(3*a^(2/3)*b^(2/
3)*d^3) - (c*(2*a^(1/3) + b^(1/3)*c)*Log[a^(2/3) - a^(1/3)*b^(1/3
)*(c + d*x) + b^(2/3)*(c + d*x)^2])/(6*a^(2/3)*b^(2/3)*d^3) + Log
[a + b*(c + d*x)^3]/(3*b*d^3)
```

Rubi [A] time = 0.4734, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471

$$\frac{c\left(2\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}b^{2/3}d^3}-\frac{c\left(2\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}b^{2/3}d^3}+\frac{c\left(2\sqrt[3]{a}-\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}d^3}+\frac{\log\left(a+b(c+dx)^3\right)}{3bd^3}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(a + b*(c + d*x)^3),x]
```

```
[Out] (c*(2*a^(1/3) - b^(1/3)*c)*ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))
/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(2/3)*d^3) + (c*(2*a^(1/3
) + b^(1/3)*c)*Log[a^(1/3) + b^(1/3)*(c + d*x)])/(3*a^(2/3)*b^(2/
3)*d^3) - (c*(2*a^(1/3) + b^(1/3)*c)*Log[a^(2/3) - a^(1/3)*b^(1/3
)*(c + d*x) + b^(2/3)*(c + d*x)^2])/(6*a^(2/3)*b^(2/3)*d^3) + Log
[a + b*(c + d*x)^3]/(3*b*d^3)
```

Rubi in Sympy [A] time = 52.2893, size = 204, normalized size = 0.97

$$\frac{\log\left(a+b\left(c+dx\right)^3\right)}{3bd^3}+\frac{\sqrt{3}c\left(2\sqrt[3]{a}-\sqrt[3]{b}c\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}+\sqrt[3]{b}\left(-\frac{2c}{3}-\frac{2dx}{3}\right)\right)}{\sqrt[3]{a}}\right)}{3a^{\frac{2}{3}}b^{\frac{2}{3}}d^3}+\frac{c\left(2\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\left(c+dx\right)\right)}{3a^{\frac{2}{3}}b^{\frac{2}{3}}d^3}-\frac{c\left(2\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{b}\left(-c-dx\right)+b^{\frac{2}{3}}\left(c+dx\right)^2\right)}{6a^{\frac{2}{3}}b^{\frac{2}{3}}d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(a+b*(d*x+c)**3),x)
```

```
[Out] log(a + b*(c + d*x)**3)/(3*b*d**3) + sqrt(3)*c*(2*a**(1/3) - b**(
1/3)*c)*atan(sqrt(3)*(a**(1/3)/3 + b**(1/3)*(-2*c/3 - 2*d*x/3))/a
**(1/3))/(3*a**(2/3)*b**(2/3)*d**3) + c*(2*a**(1/3) + b**(1/3)*c)
*log(a**(1/3) + b**(1/3)*(c + d*x))/(3*a**(2/3)*b**(2/3)*d**3) -
```

```
c*(2*a**(1/3) + b**(1/3)*c)*log(a**(2/3) + a**(1/3)*b**(1/3)*(-c
- d*x) + b**(2/3)*(c + d*x)**2)/(6*a**(2/3)*b**(2/3)*d**3)
```

Mathematica [C] time = 0.0452165, size = 81, normalized size = 0.39

$$\frac{\text{RootSum}\left[\#1^3bd^3+3\#1^2bcd^2+3\#1bc^2d+a+bc^3\&,\frac{\#1^2\log(x-\#1)}{\#1^2d^2+2\#1cd+c^2}\&\right]}{3bd}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(a + b*(c + d*x)^3),x]
```

```
[Out] RootSum[a + b*c^3 + 3*b*c^2*d*#1 + 3*b*c*d^2*#1^2 + b*d^3*#1^3 &
, (Log[x - #1]*#1^2)/(c^2 + 2*c*d*#1 + d^2*#1^2) & ]/(3*b*d)
```

Maple [C] time = 0.006, size = 74, normalized size = 0.4

$$\frac{1}{3\,bd}\sum_{\_R=RootOf\left(\_Z^3bd^3+3\_Z^2bcd^2+3\_Z\,bc^2d+bc^3+a\right)}\frac{\_R^2\ln\left(x-\_R\right)}{d^2\_R^2+2\,cd\_R+c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(a+b*(d*x+c)^3),x)
```

```
[Out] 1/3/b/d*sum(_R^2/(_R^2*d^2+2*_R*c*d+c^2)*ln(x-_R),_R=RootOf(_Z^3*
b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2}{(dx+c)^3b+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((d*x + c)^3*b + a),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((d*x + c)^3*b + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((d*x + c)^3*b + a),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 3.16654, size = 158, normalized size = 0.75

$$\operatorname{RootSum}\left(27t^3a^2b^3d^9 - 27t^2a^2b^2d^6 + t\left(9a^2bd^3 - 18ab^2c^3d^3\right) - a^2 - 2abc^3 - b^2c^6, \left(t \mapsto t\log\left(x + \frac{18t^2a^2b^2d^6 - 12ta^2bd^3}{8a}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(a+b*(d*x+c)**3),x)
```

```
[Out] RootSum(27*_t**3*a**2*b**3*d**9 - 27*_t**2*a**2*b**2*d**6 + _t*(9
*a**2*b*d**3 - 18*a*b**2*c**3*d**3) - a**2 - 2*a*b*c**3 - b**2*c*
*6, Lambda(_t, _t*log(x + (18*_t**2*a**2*b**2*d**6 - 12*_t*a**2*b
*d**3 - 3*_t*a*b**2*c**3*d**3 + 2*a**2 + a*b*c**3 - b**2*c**6)/(8
*a*b*c**2*d - b**2*c**5*d))))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(dx+c)^3 b + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((d*x + c)^3*b + a),x, algorithm="giac")
```

```
[Out] integrate(x^2/((d*x + c)^3*b + a), x)
```

3.105 $\int \frac{x}{a+b(c+dx)^3}\,dx$

Optimal. Leaf size=180

$$-\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}b^{2/3}d^2}+\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}b^{2/3}d^2}-\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}d^2}$$

```
[Out] -(((a^(1/3) - b^(1/3)*c)*ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(
Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(2/3)*d^2)) - ((a^(1/3) + b
^(1/3)*c)*Log[a^(1/3) + b^(1/3)*(c + d*x)])/(3*a^(2/3)*b^(2/3)*d^
2) + ((a^(1/3) + b^(1/3)*c)*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*
x) + b^(2/3)*(c + d*x)^2])/(6*a^(2/3)*b^(2/3)*d^2)
```

Rubi [A] time = 0.332399, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467

$$-\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}b^{2/3}d^2}+\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}b^{2/3}d^2}-\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}d^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(a + b*(c + d*x)^3),x]
```

```
[Out] -(((a^(1/3) - b^(1/3)*c)*ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(
Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(2/3)*d^2)) - ((a^(1/3) + b
^(1/3)*c)*Log[a^(1/3) + b^(1/3)*(c + d*x)])/(3*a^(2/3)*b^(2/3)*d^
2) + ((a^(1/3) + b^(1/3)*c)*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*
x) + b^(2/3)*(c + d*x)^2])/(6*a^(2/3)*b^(2/3)*d^2)
```

Rubi in Sympy [A] time = 39.6786, size = 175, normalized size = 0.97

$$-\frac{\sqrt{3}\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}+\sqrt[3]{b}\left(-\frac{2c}{3}-\frac{2dx}{3}\right)\right)}{\sqrt[3]{a}}\right)}{3a^{\frac{2}{3}}b^{\frac{2}{3}}d^2}-\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\left(c+dx\right)\right)}{3a^{\frac{2}{3}}b^{\frac{2}{3}}d^2}+\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{b}\left(-c-dx\right)+b^{\frac{2}{3}}\left(c+dx\right)^2\right)}{6a^{\frac{2}{3}}b^{\frac{2}{3}}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(a+b*(d*x+c)**3),x)
```

```
[Out] -sqrt(3)*(a**(1/3) - b**(1/3)*c)*atan(sqrt(3)*(a**(1/3)/3 + b**(1
/3)*(-2*c/3 - 2*d*x/3))/a**(1/3))/(3*a**(2/3)*b**(2/3)*d**2) - (a
**(1/3) + b**(1/3)*c)*log(a**(1/3) + b**(1/3)*(c + d*x))/(3*a**(2
/3)*b**(2/3)*d**2) + (a**(1/3) + b**(1/3)*c)*log(a**(2/3) + a**(1
/3)*b**(1/3)*(-c - d*x) + b**(2/3)*(c + d*x)**2)/(6*a**(2/3)*b**(
2/3)*d**2)
```

Mathematica [C] time = 0.0369369, size = 79, normalized size = 0.44

$$\frac{\text{RootSum}\left[\#1^3bd^3 + 3\#1^2bcd^2 + 3\#1bc^2d + a + bc^3\&, \frac{\#1\log(x-\#1)}{\#1^2d^2+2\#1cd+c^2}\&\right]}{3bd}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(a + b*(c + d*x)^3),x]
```

```
[Out] RootSum[a + b*c^3 + 3*b*c^2*d*#1 + 3*b*c*d^2*#1^2 + b*d^3*#1^3 &
, (Log[x - #1]*#1)/(c^2 + 2*c*d*#1 + d^2*#1^2) & ]/(3*b*d)
```

Maple [C] time = 0.004, size = 72, normalized size = 0.4

$$\frac{1}{3\,bd}\sum_{\_R=RootOf(\_Z^3bd^3+3\_Z^2bcd^2+3\_Z\,bc^2d+bc^3+a)}\frac{\_R\ln(x-\_R)}{d^2\_R^2+2\,cd\_R+c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(a+b*(d*x+c)^3),x)
```

```
[Out] 1/3/b/d*sum(_R/(_R^2*d^2+2*_R*c*d+c^2)*ln(x-_R),_R=RootOf(_Z^3*b*
d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{(dx+c)^3b+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((d*x + c)^3*b + a),x, algorithm="maxima")
```

```
[Out] integrate(x/((d*x + c)^3*b + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((d*x + c)^3*b + a),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 2.21394, size = 83, normalized size = 0.46

$$\text{RootSum}\left(27t^3a^2b^2d^6 - 9tabcd^2 + a + bc^3, \left(t \mapsto t\log\left(x + \frac{9t^2a^2bd^4 + 3tabc^2d^2 - ac - bc^4}{ad - bc^3d}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(a+b*(d*x+c)**3),x)
```

```
[Out] RootSum(27*_t**3*a**2*b**2*d**6 - 9*_t*a*b*c*d**2 + a + b*c**3, L
ambda(_t, _t*log(x + (9*_t**2*a**2*b*d**4 + 3*_t*a*b*c**2*d**2 -
a*c - b*c**4)/(a*d - b*c**3*d))))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(dx+c)^3 b + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((d*x + c)^3*b + a),x, algorithm="giac")
```

```
[Out] integrate(x/((d*x + c)^3*b + a), x)
```

3.106 $\int \frac{1}{a+b(c+dx)^3} dx$

Optimal. Leaf size=140

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}\sqrt[3]{b}d}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}d}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}d}$$

```
[Out] -(ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))]/(Sqrt
[3]*a^(2/3)*b^(1/3)*d)) + Log[a^(1/3) + b^(1/3)*(c + d*x)]/(3*a^(
2/3)*b^(1/3)*d) - Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/
3)*(c + d*x)^2]/(6*a^(2/3)*b^(1/3)*d)
```

Rubi [A] time = 0.224768, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}\sqrt[3]{b}d}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}d}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}d}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*(c + d*x)^3)^(-1),x]
```

```
[Out] -(ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))]/(Sqrt
[3]*a^(2/3)*b^(1/3)*d)) + Log[a^(1/3) + b^(1/3)*(c + d*x)]/(3*a^(
2/3)*b^(1/3)*d) - Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/
3)*(c + d*x)^2]/(6*a^(2/3)*b^(1/3)*d)
```

Rubi in Sympy [A] time = 30.0985, size = 134, normalized size = 0.96

$$\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{\frac{2}{3}}\sqrt[3]{b}d}-\frac{\log\left(a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{b}\,(-c-dx)+b^{\frac{2}{3}}\,(c+dx)^2\right)}{6a^{\frac{2}{3}}\sqrt[3]{b}d}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}+\sqrt[3]{b}\left(-\frac{2c}{3}-\frac{2dx}{3}\right)\right)}{\sqrt[3]{a}}\right)}{3a^{\frac{2}{3}}\sqrt[3]{b}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a+b*(d*x+c)**3),x)
```

```
[Out] log(a**(1/3) + b**(1/3)*(c + d*x))/(3*a**(2/3)*b**(1/3)*d) - log(
a**(2/3) + a**(1/3)*b**(1/3)*(-c - d*x) + b**(2/3)*(c + d*x)**2)/
(6*a**(2/3)*b**(1/3)*d) - sqrt(3)*atan(sqrt(3)*(a**(1/3)/3 + b**(
1/3)*(-2*c/3 - 2*d*x/3))/a**(1/3))/(3*a**(2/3)*b**(1/3)*d)
```

Mathematica [A] time = 0.0543155, size = 116, normalized size = 0.83

$$\frac{-\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)+2\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)+2\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{b}(c+dx)-\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{6a^{2/3}\sqrt[3]{b}d}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*(c + d*x)^3)^(-1),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(-a^(1/3) + 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/
3))] + 2*Log[a^(1/3) + b^(1/3)*(c + d*x)] - Log[a^(2/3) - a^(1/3)
*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])/(6*a^(2/3)*b^(1/3)*d)
```

Maple [C] time = 0.003, size = 71, normalized size = 0.5

$$\frac{1}{3\,bd}\sum_{\_R=RootOf\left(\_Z^3bd^3+3\_Z^2bcd^2+3\_Z\,bc^2d+bc^3+a\right)}\frac{\ln\left(x-\_R\right)}{d^2\_R^2+2\,cd\_R+c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a+b*(d*x+c)^3),x)
```

```
[Out] 1/3/b/d*sum(1/(_R^2*d^2+2*_R*c*d+c^2)*ln(x-_R),_R=RootOf(_Z^3*b*d
^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(dx+c)^3b+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((d*x + c)^3*b + a),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x + c)^3*b + a), x)
```

Fricas [A] time = 0.26823, size = 161, normalized size = 1.15

$$-\frac{\sqrt{3}\left(\sqrt{3}\log\left(a^2+\left(d^2x^2+2\,cdx+c^2\right)\left(a^2b\right)^{\frac{2}{3}}-\left(a^2b\right)^{\frac{1}{3}}(adx+ac)\right)-2\,\sqrt{3}\log\left(\left(a^2b\right)^{\frac{1}{3}}(dx+c)+a\right)-6\,\arctan\left(\frac{2\,\sqrt{3}\left(a^2b\right)^{\frac{1}{3}}\cdots}{\cdots}\right.\cdots}{18\,\left(a^2b\right)^{\frac{1}{3}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((d*x + c)^3*b + a),x, algorithm="fricas")
```

```
[Out] -1/18*sqrt(3)*(sqrt(3)*log(a^2 + (d^2*x^2 + 2*c*d*x + c^2)*(a^2*b
)^(2/3) - (a^2*b)^(1/3)*(a*d*x + a*c)) - 2*sqrt(3)*log((a^2*b)^(1
/3)*(d*x + c) + a) - 6*arctan(1/3*(2*sqrt(3)*(a^2*b)^(1/3)*(d*x +
 c) - sqrt(3)*a)/a))/((a^2*b)^(1/3)*d)
```

Sympy [A] time = 0.695232, size = 26, normalized size = 0.19

$$\frac{\operatorname{RootSum}\left(27t^3a^2b-1,\left(t\mapsto t\log\left(x+\frac{3ta+c}{d}\right)\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a+b*(d*x+c)**3),x)
```

```
[Out] RootSum(27*_t**3*a**2*b - 1, Lambda(_t, _t*log(x + (3*_t*a + c)/d
)))/d
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)^3 b + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((d*x + c)^3*b + a),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)^3*b + a), x)
```

3.107 $\int \frac{1}{x(a+b(c+dx)^3)}\,dx$

Optimal. Leaf size=224

$$-\frac{\left(2\sqrt[3]{a}-\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}c+b^{2/3}c^2\right)}+\frac{\sqrt[3]{b}c\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}c+b^{2/3}c^2\right)}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)}+\frac{\log(x)}{a+bc^3}$$

```
[Out] (b^(1/3)*c*ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3
))])/(Sqrt[3]*a^(2/3)*(a^(2/3) - a^(1/3)*b^(1/3)*c + b^(2/3)*c^2)
) + Log[x]/(a + b*c^3) - Log[a^(1/3) + b^(1/3)*(c + d*x)]/(3*a^(2
/3)*(a^(1/3) + b^(1/3)*c)) - ((2*a^(1/3) - b^(1/3)*c)*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])/(6*a^(2/3)*(
a^(2/3) - a^(1/3)*b^(1/3)*c + b^(2/3)*c^2))
```

Rubi [A] time = 0.609398, antiderivative size = 238, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529

$$-\frac{\sqrt[3]{b}c\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}\left(a+bc^3\right)}+\frac{\sqrt[3]{b}c\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\left(a+bc^3\right)}+\frac{\sqrt[3]{b}c\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\left(a+bc^3\right)}-\frac{\log\left(a+b(c+dx)^3\right)}{3\left(a+bc^3\right)}+\frac{\log(x)}{a+bc^3}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(a + b*(c + d*x)^3)),x]
```

```
[Out] (b^(1/3)*c*(a^(1/3) + b^(1/3)*c)*ArcTan[(a^(1/3) - 2*b^(1/3)*(c +
 d*x))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*(a + b*c^3)) + Log[x]
/(a + b*c^3) + (b^(1/3)*c*(a^(1/3) - b^(1/3)*c)*Log[a^(1/3) + b^(
1/3)*(c + d*x)])/(3*a^(2/3)*(a + b*c^3)) - (b^(1/3)*c*(a^(1/3) -
b^(1/3)*c)*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c +
 d*x)^2])/(6*a^(2/3)*(a + b*c^3)) - Log[a + b*(c + d*x)^3]/(3*(a
+ b*c^3))
```

Rubi in Sympy [A] time = 74.278, size = 219, normalized size = 0.98

$$\frac{\log(-dx)}{a+bc^3}-\frac{\log\left(a+b\left(c+dx\right)^3\right)}{3\left(a+bc^3\right)}+\frac{\sqrt[3]{b}c\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\left(c+dx\right)\right)}{3a^{\frac{2}{3}}\left(a+bc^3\right)}-\frac{\sqrt[3]{b}c\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}\left(c+dx\right)+b^{\frac{2}{3}}\left(c+dx\right)^2\right)}{6a^{\frac{2}{3}}\left(a+bc^3\right)}+\frac{\sqrt{3}\sqrt[3]{b}c\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}(c+dx)}{3}\right)}{\sqrt[3]{a}}\right)}{3a^{\frac{2}{3}}\left(a+bc^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(a+b*(d*x+c)**3),x)
```

```
[Out] log(-d*x)/(a + b*c**3) - log(a + b*(c + d*x)**3)/(3*(a + b*c**3))
 + b**(1/3)*c*(a**(1/3) - b**(1/3)*c)*log(a**(1/3) + b**(1/3)*(c
+ d*x))/(3*a**(2/3)*(a + b*c**3)) - b**(1/3)*c*(a**(1/3) - b**(1/
3)*c)*log(a**(2/3) - a**(1/3)*b**(1/3)*(c + d*x) + b**(2/3)*(c +
d*x)**2)/(6*a**(2/3)*(a + b*c**3)) + sqrt(3)*b**(1/3)*c*(a**(1/3)
 + b**(1/3)*c)*atan(sqrt(3)*(a**(1/3)/3 - 2*b**(1/3)*(c + d*x)/3)
/a**(1/3))/(3*a**(2/3)*(a + b*c**3))
```

Mathematica [C] time = 0.0765582, size = 119, normalized size = 0.53

$$-\frac{\text{RootSum}\left[\#1^3bd^3+3\#1^2bcd^2+3\#1bc^2d+a+bc^3\&,\frac{\#1^2d^2\log(x-\#1)+3c^2\log(x-\#1)+3\#1cd\log(x-\#1)}{\#1^2d^2+2\#1cd+c^2}\&\right]-3\log(x)}{3\left(a+bc^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(a + b*(c + d*x)^3)),x]
```

```
[Out] -(-3*Log[x] + RootSum[a + b*c^3 + 3*b*c^2*d*#1 + 3*b*c*d^2*#1^2 +
 b*d^3*#1^3 & , (3*c^2*Log[x - #1] + 3*c*d*Log[x - #1]*#1 + d^2*L
og[x - #1]*#1^2)/(c^2 + 2*c*d*#1 + d^2*#1^2) & ])/(3*(a + b*c^3))
```

Maple [C] time = 0.009, size = 105, normalized size = 0.5

$$\frac{\ln(x)}{bc^3+a}-\frac{1}{3\,bc^3+3\,a}\sum_{\_R=RootOf\left(\_Z^3bd^3+3\_Z^2bcd^2+3\_Z\,bc^2d+bc^3+a\right)}\frac{\left(d^2\_R^2+3\,cd\_R+3\,c^2\right)\ln(x-\_R)}{d^2\_R^2+2\,cd\_R+c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(a+b*(d*x+c)^3),x)
```

```
[Out] ln(x)/(b*c^3+a)-1/3/(b*c^3+a)*sum((_R^2*d^2+3*_R*c*d+3*c^2)/(_R^2
*d^2+2*_R*c*d+c^2)*ln(x-_R),_R=RootOf(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3
*_Z*b*c^2*d+b*c^3+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{bd\int\frac{d^2x^2+3\,cdx+3\,c^2}{bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a}\,dx}{bc^3+a}+\frac{\log(x)}{bc^3+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(((d*x + c)^3*b + a)*x),x, algorithm="maxima")
```

```
[Out] -b*d*integrate((d^2*x^2 + 3*c*d*x + 3*c^2)/(b*d^3*x^3 + 3*b*c*d^2
*x^2 + 3*b*c^2*d*x + b*c^3 + a), x)/(b*c^3 + a) + log(x)/(b*c^3 +
 a)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(((d*x + c)^3*b + a)*x),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 52.1419, size = 559, normalized size = 2.5

$$\operatorname{RootSum}\left(t^3\left(27a^3+27a^2bc^3\right)+27t^2a^2+9ta+1,\left(t\mapsto t\log\left(x+\frac{-432t^3a^6-837t^3a^5bc^3-405t^3a^4b^2c^6-27t^3a^3b^3c^9-27\right.$$

$$+\frac{\log\left(x+\frac{-\frac{432a^6}{\left(a+bc^3\right)^3}-\frac{837a^5bc^3}{\left(a+bc^3\right)^3}+\frac{144a^5}{\left(a+bc^3\right)^2}-\frac{405a^4b^2c^6}{\left(a+bc^3\right)^3}+\frac{270a^4bc^3}{\left(a+bc^3\right)^2}+\frac{240a^4}{a+bc^3}-\frac{27a^3b^3c^9}{\left(a+bc^3\right)^3}+\frac{108a^3b^2c^6}{\left(a+bc^3\right)^2}-\frac{261a^3bc^3}{a+bc^3}+48a^3-\frac{27a^2b^4c^{12}}{\left(a+bc^3\right)^3}-\frac{18a^2b^3c^9}{\left(a+bc^3\right)^2}-\frac{27a^2b^2c^6}{a+bc^3}+60a^2bc^3-12}{64a^2bc^2d+11ab^2c^5d+b^3c^8d}\right.}{a+bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(a+b*(d*x+c)**3),x)
```

```
[Out] RootSum(_t**3*(27*a**3 + 27*a**2*b*c**3) + 27*_t**2*a**2 + 9*_t*a
 + 1, Lambda(_t, _t*log(x + (-432*_t**3*a**6 - 837*_t**3*a**5*b*c
**3 - 405*_t**3*a**4*b**2*c**6 - 27*_t**3*a**3*b**3*c**9 - 27*_t*
*3*a**2*b**4*c**12 + 144*_t**2*a**5 + 270*_t**2*a**4*b*c**3 + 108
*_t**2*a**3*b**2*c**6 - 18*_t**2*a**2*b**3*c**9 + 240*_t*a**4 - 2
61*_t*a**3*b*c**3 - 27*_t*a**2*b**2*c**6 - 12*_t*a*b**3*c**9 + 48
*a**3 + 60*a**2*b*c**3 + 12*a*b**2*c**6)/(64*a**2*b*c**2*d + 11*a
*b**2*c**5*d + b**3*c**8*d)))) + log(x + (-432*a**6/(a + b*c**3)*
*3 - 837*a**5*b*c**3/(a + b*c**3)**3 + 144*a**5/(a + b*c**3)**2 -
 405*a**4*b**2*c**6/(a + b*c**3)**3 + 270*a**4*b*c**3/(a + b*c**3
)**2 + 240*a**4/(a + b*c**3) - 27*a**3*b**3*c**9/(a + b*c**3)**3
+ 108*a**3*b**2*c**6/(a + b*c**3)**2 - 261*a**3*b*c**3/(a + b*c**
3) + 48*a**3 - 27*a**2*b**4*c**12/(a + b*c**3)**3 - 18*a**2*b**3*
c**9/(a + b*c**3)**2 - 27*a**2*b**2*c**6/(a + b*c**3) + 60*a**2*b
*c**3 - 12*a*b**3*c**9/(a + b*c**3) + 12*a*b**2*c**6)/(64*a**2*b*
c**2*d + 11*a*b**2*c**5*d + b**3*c**8*d))/(a + b*c**3)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left((dx+c)^3b+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(((d*x + c)^3*b + a)*x),x, algorithm="giac")
```

```
[Out] integrate(1/(((d*x + c)^3*b + a)*x), x)
```

3.108 $\int \frac{1}{x^2(a+b(c+dx)^3)}\,dx$

Optimal. Leaf size=314

$$-\frac{\sqrt[3]{b}d\left(\sqrt[3]{a}\left(a-2bc^3\right)-\sqrt[3]{b}c\left(2a-bc^3\right)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}\left(a+bc^3\right)^2}+\frac{\sqrt[3]{b}d\left(\sqrt[3]{a}\left(a-2bc^3\right)-\sqrt[3]{b}c\left(2a-bc^3\right)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\left(a+bc^3\right)^2}+\frac{\sqrt[3]{b}d\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)^3\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\left(a+bc^3\right)^2}-\frac{1}{x\left(a+bc^3\right)}-\frac{3bc^2d\log(x)}{\left(a+bc^3\right)^2}+\frac{bc^2d\log\left(a+b(c+dx)^3\right)}{\left(a+bc^3\right)^2}$$

```
[Out] -(1/((a + b*c^3)*x)) + (b^(1/3)*(a^(1/3) - b^(1/3)*c)*(a^(1/3) +
b^(1/3)*c)^3*d*ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^
(1/3))])/(Sqrt[3]*a^(2/3)*(a + b*c^3)^2) - (3*b*c^2*d*Log[x])/(a
+ b*c^3)^2 + (b^(1/3)*(a^(1/3)*(a - 2*b*c^3) - b^(1/3)*c*(2*a - b
*c^3))*d*Log[a^(1/3) + b^(1/3)*(c + d*x)])/(3*a^(2/3)*(a + b*c^3)
^2) - (b^(1/3)*(a^(1/3)*(a - 2*b*c^3) - b^(1/3)*c*(2*a - b*c^3))*
d*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])
/(6*a^(2/3)*(a + b*c^3)^2) + (b*c^2*d*Log[a + b*(c + d*x)^3])/(a
+ b*c^3)^2
```

Rubi [A] time = 1.09058, antiderivative size = 312, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$

$$\frac{b^{2/3}d\left(-\frac{\sqrt[3]{a}\left(a-2bc^3\right)}{\sqrt[3]{b}}+2ac-bc^4\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}\left(a+bc^3\right)^2}+\frac{\sqrt[3]{b}d\left(\sqrt[3]{a}\left(a-2bc^3\right)-\sqrt[3]{b}c\left(2a-bc^3\right)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\left(a+bc^3\right)^2}+\frac{\sqrt[3]{b}d\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)^3\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\left(a+bc^3\right)^2}-\frac{1}{x\left(a+bc^3\right)}-\frac{3bc^2d\log(x)}{\left(a+bc^3\right)^2}+\frac{bc^2d\log\left(a+b(c+dx)^3\right)}{\left(a+bc^3\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(a + b*(c + d*x)^3)),x]
```

```
[Out] -(1/((a + b*c^3)*x)) + (b^(1/3)*(a^(1/3) - b^(1/3)*c)*(a^(1/3) +
b^(1/3)*c)^3*d*ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^
(1/3))])/(Sqrt[3]*a^(2/3)*(a + b*c^3)^2) - (3*b*c^2*d*Log[x])/(a
+ b*c^3)^2 + (b^(1/3)*(a^(1/3)*(a - 2*b*c^3) - b^(1/3)*c*(2*a - b
*c^3))*d*Log[a^(1/3) + b^(1/3)*(c + d*x)])/(3*a^(2/3)*(a + b*c^3)
^2) + (b^(2/3)*(2*a*c - b*c^4 - (a^(1/3)*(a - 2*b*c^3))/b^(1/3))*
d*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])
/(6*a^(2/3)*(a + b*c^3)^2) + (b*c^2*d*Log[a + b*(c + d*x)^3])/(a
+ b*c^3)^2
```

Rubi in Sympy [A] time = 133.834, size = 314, normalized size = 1.

$$-\frac{3bc^2d\log(-dx)}{(a+bc^3)^2}+\frac{bc^2d\log\left(a+b(c+dx)^3\right)}{(a+bc^3)^2}-\frac{1}{x(a+bc^3)}+\frac{\sqrt[3]{b}d\left(\sqrt[3]{a}(a-2bc^3)-\sqrt[3]{b}c(2a-bc^3)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{\frac{2}{3}}(a+bc^3)^2}-\frac{\sqrt[3]{b}d\left(\sqrt[3]{a}(a-2bc^3)-\sqrt[3]{b}c(2a-bc^3)\right)\log\left(a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{b}(-c-dx)+b^{\frac{2}{3}}(c+dx)^2\right)}{6a^{\frac{2}{3}}(a+bc^3)^2}+\frac{\sqrt{3}\sqrt[3]{b}d\left(a^{\frac{4}{3}}-2\sqrt[3]{a}bc^3+2a\sqrt[3]{b}c-b^{\frac{4}{3}}c^4\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}+\sqrt[3]{b}\left(-\frac{2c}{3}-\frac{2dx}{3}\right)\right)}{\sqrt[3]{a}}\right)}{3a^{\frac{2}{3}}(a+bc^3)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(a+b*(d*x+c)**3),x)
```

```
[Out] -3*b*c**2*d*log(-d*x)/(a + b*c**3)**2 + b*c**2*d*log(a + b*(c + d
*x)**3)/(a + b*c**3)**2 - 1/(x*(a + b*c**3)) + b**(1/3)*d*(a**(1/
3)*(a - 2*b*c**3) - b**(1/3)*c*(2*a - b*c**3))*log(a**(1/3) + b**
(1/3)*(c + d*x))/(3*a**(2/3)*(a + b*c**3)**2) - b**(1/3)*d*(a**(1
/3)*(a - 2*b*c**3) - b**(1/3)*c*(2*a - b*c**3))*log(a**(2/3) + a*
*(1/3)*b**(1/3)*(-c - d*x) + b**(2/3)*(c + d*x)**2)/(6*a**(2/3)*(
a + b*c**3)**2) + sqrt(3)*b**(1/3)*d*(a**(4/3) - 2*a**(1/3)*b*c**
3 + 2*a*b**(1/3)*c - b**(4/3)*c**4)*atan(sqrt(3)*(a**(1/3)/3 + b*
*(1/3)*(-2*c/3 - 2*d*x/3))/a**(1/3))/(3*a**(2/3)*(a + b*c**3)**2)
```

Mathematica [C] time = 0.141238, size = 173, normalized size = 0.55

$$\frac{dx\text{RootSum}\left[\#1^3bd^3+3\#1^2bcd^2+3\#1bc^2d+a+bc^3\&,\frac{3\#1^2bc^2d^2\log(x-\#1)-3ac\log(x-\#1)-\#1ad\log(x-\#1)+6bc^4\log(x-\#1)+8\#1bc^3d\log}{\#1^2d^2+2\#1cd+c^2}\right.}{3x(a+bc^3)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(a + b*(c + d*x)^3)),x]
```

```
[Out] (-3*(a + b*c^3 + 3*b*c^2*d*x*Log[x]) + d*x*RootSum[a + b*c^3 + 3*
b*c^2*d*#1 + 3*b*c*d^2*#1^2 + b*d^3*#1^3 & , (-3*a*c*Log[x - #1]
+ 6*b*c^4*Log[x - #1] - a*d*Log[x - #1]*#1 + 8*b*c^3*d*Log[x - #1
]*#1 + 3*b*c^2*d^2*Log[x - #1]*#1^2)/(c^2 + 2*c*d*#1 + d^2*#1^2)
& ])/(3*(a + b*c^3)^2*x)
```

Maple [C] time = 0.014, size = 144, normalized size = 0.5

$$-\frac{1}{(bc^3+a)x}-3\frac{c^2db\ln(x)}{(bc^3+a)^2}+\frac{d}{3(bc^3+a)^2}\sum_{\_R=RootOf(\_Z^3bd^3+3\_Z^2bcd^2+3\_Zbc^2d+bc^3+a)}\frac{\left(3\_R^2bc^2d^2+8\_Rbc^3d+6bc^4-\_Rad-3ac\right)\ln(x-\_R)}{d^2\_R^2+2cd\_R+c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(a+b*(d*x+c)^3),x)
```

```
[Out] -1/(b*c^3+a)/x-3*b*c^2*d*ln(x)/(b*c^3+a)^2+1/3*d/(b*c^3+a)^2*sum(
(3*_R^2*b*c^2*d^2+8*_R*b*c^3*d+6*b*c^4-_R*a*d-3*a*c)/(_R^2*d^2+2*
_R*c*d+c^2)*ln(x-_R),_R=RootOf(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c
^2*d+b*c^3+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{3\,bc^2d\log\left(x\right)}{b^2c^6+2\,abc^3+a^2}+\frac{bd^2\int\frac{3\,bc^2d^2x^2+6\,bc^4+\left(8\,bc^3-a\right)dx-3\,ac}{bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a}\,dx}{b^2c^6+2\,abc^3+a^2}-\frac{1}{\left(bc^3+a\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(((d*x + c)^3*b + a)*x^2),x, algorithm="maxima")
```

```
[Out] -3*b*c^2*d*log(x)/(b^2*c^6 + 2*a*b*c^3 + a^2) + b*d^2*integrate((
3*b*c^2*d^2*x^2 + 6*b*c^4 + (8*b*c^3 - a)*d*x - 3*a*c)/(b*d^3*x^3
 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a), x)/(b^2*c^6 + 2*a*b*
c^3 + a^2) - 1/((b*c^3 + a)*x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(((d*x + c)^3*b + a)*x^2),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(a+b*(d*x+c)**3),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(\left(dx+c\right)^3b+a\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(((d*x + c)^3*b + a)*x^2),x, algorithm="giac")
```

```
[Out] integrate(1/(((d*x + c)^3*b + a)*x^2), x)
```

3.109 $\int \frac{1}{x^3(a+b(c+dx)^3)}\,dx$

Optimal. Leaf size=393

$$\frac{b^{2/3}d^2\left(-3a^{2/3}\sqrt[3]{b}c+a+bc^3\right)\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)^3\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\left(a+bc^3\right)^3}-\frac{b^{2/3}d^2\left(6a^{4/3}b^{2/3}c^2+a^2-3\sqrt[3]{a}b^{5/3}c^5-7abc^3+b^2c^6\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\left(a+bc^3\right)^3}+\frac{b^{2/3}d^2\left(6a^{4/3}b^{2/3}c^2+a^2-3\sqrt[3]{a}b^{5/3}c^5-7abc^3+b^2c^6\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}\left(a+bc^3\right)^3}-\frac{3bcd^2\log(x)\left(a-2bc^3\right)}{\left(a+bc^3\right)^3}+\frac{bcd^2\left(a-2bc^3\right)\log\left(a+b(c+dx)^3\right)}{\left(a+bc^3\right)^3}-\frac{1}{2x^2\left(a+bc^3\right)}+\frac{3bc^2d}{x\left(a+bc^3\right)^2}$$

```
[Out] -1/(2*(a + b*c^3)*x^2) + (3*b*c^2*d)/((a + b*c^3)^2*x) + (b^(2/3)
*(a^(1/3) + b^(1/3)*c)^3*(a - 3*a^(2/3)*b^(1/3)*c + b*c^3)*d^2*Ar
cTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]
*a^(2/3)*(a + b*c^3)^3) - (3*b*c*(a - 2*b*c^3)*d^2*Log[x])/(a + b
*c^3)^3 - (b^(2/3)*(a^2 + 6*a^(4/3)*b^(2/3)*c^2 - 7*a*b*c^3 - 3*a
^(1/3)*b^(5/3)*c^5 + b^2*c^6)*d^2*Log[a^(1/3) + b^(1/3)*(c + d*x)
])/(3*a^(2/3)*(a + b*c^3)^3) + (b^(2/3)*(a^2 + 6*a^(4/3)*b^(2/3)*
c^2 - 7*a*b*c^3 - 3*a^(1/3)*b^(5/3)*c^5 + b^2*c^6)*d^2*Log[a^(2/3
) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])/(6*a^(2/3)*
(a + b*c^3)^3) + (b*c*(a - 2*b*c^3)*d^2*Log[a + b*(c + d*x)^3])/(
a + b*c^3)^3
```

Rubi [A] time = 1.21127, antiderivative size = 393, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$

$$\frac{b^{2/3}d^2\left(-3a^{2/3}\sqrt[3]{b}c+a+bc^3\right)\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)^3\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\left(a+bc^3\right)^3}-\frac{b^{2/3}d^2\left(6a^{4/3}b^{2/3}c^2+a^2-3\sqrt[3]{a}b^{5/3}c^5-7abc^3+b^2c^6\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\left(a+bc^3\right)^3}+\frac{b^{2/3}d^2\left(6a^{4/3}b^{2/3}c^2+a^2-3\sqrt[3]{a}b^{5/3}c^5-7abc^3+b^2c^6\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}\left(a+bc^3\right)^3}-\frac{3bcd^2\log(x)\left(a-2bc^3\right)}{\left(a+bc^3\right)^3}+\frac{bcd^2\left(a-2bc^3\right)\log\left(a+b(c+dx)^3\right)}{\left(a+bc^3\right)^3}-\frac{1}{2x^2\left(a+bc^3\right)}+\frac{3bc^2d}{x\left(a+bc^3\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*(a + b*(c + d*x)^3)),x]
```


```
[Out] -1/(2*(a + b*c^3)*x^2) + (3*b*c^2*d)/((a + b*c^3)^2*x) + (b^(2/3)
*(a^(1/3) + b^(1/3)*c)^3*(a - 3*a^(2/3)*b^(1/3)*c + b*c^3)*d^2*Ar
cTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]
*a^(2/3)*(a + b*c^3)^3) - (3*b*c*(a - 2*b*c^3)*d^2*Log[x])/(a + b
*c^3)^3 - (b^(2/3)*(a^2 + 6*a^(4/3)*b^(2/3)*c^2 - 7*a*b*c^3 - 3*a
^(1/3)*b^(5/3)*c^5 + b^2*c^6)*d^2*Log[a^(1/3) + b^(1/3)*(c + d*x)
])/(3*a^(2/3)*(a + b*c^3)^3) + (b^(2/3)*(a^2 + 6*a^(4/3)*b^(2/3)*
c^2 - 7*a*b*c^3 - 3*a^(1/3)*b^(5/3)*c^5 + b^2*c^6)*d^2*Log[a^(2/3
) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])/(6*a^(2/3)*
(a + b*c^3)^3) + (b*c*(a - 2*b*c^3)*d^2*Log[a + b*(c + d*x)^3])/(
a + b*c^3)^3
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(a+b*(d*x+c)**3),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.23049, size = 244, normalized size = 0.62

$$-\frac{2d^2x^2\text{RootSum}\left[\#1^3bd^3+3\#1^2bcd^2+3\#1bc^2d+a+bc^3\&,\frac{-3\#1^2abcd^2\log(x-\#1)+6\#1^2b^2c^4d^2\log(x-\#1)+a^2\log(x-\#1)-16abc^3\log(x-\#1)}{\#1^2d^2+2\#1cd}\right.}{6x^2(a+}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*(a + b*(c + d*x)^3)),x]
```

```
[Out] -(3*(a + b*c^3)*(a + b*c^2*(c - 6*d*x)) + 18*b*c*(a - 2*b*c^3)*d^
2*x^2*Log[x] + 2*d^2*x^2*RootSum[a + b*c^3 + 3*b*c^2*d*#1 + 3*b*c
*d^2*#1^2 + b*d^3*#1^3 & , (a^2*Log[x - #1] - 16*a*b*c^3*Log[x -
#1] + 10*b^2*c^6*Log[x - #1] - 12*a*b*c^2*d*Log[x - #1]*#1 + 15*b
^2*c^5*d*Log[x - #1]*#1 - 3*a*b*c*d^2*Log[x - #1]*#1^2 + 6*b^2*c^
4*d^2*Log[x - #1]*#1^2)/(c^2 + 2*c*d*#1 + d^2*#1^2) & ])/(6*(a +
b*c^3)^3*x^2)
```

Maple [C] time = 0.017, size = 217, normalized size = 0.6

$$-\frac{1}{(2\,bc^3+2\,a)\,x^2}+3\,\frac{c^2db}{(bc^3+a)^2\,x}+6\,\frac{c^4d^2b^2\ln(x)}{(bc^3+a)^3}-3\,\frac{bcd^2\ln(x)\,a}{(bc^3+a)^3}+\frac{d^2}{3\,(bc^3+a)^3}\sum_{\_R=RootOf(\_Z^3bd^3+3\_Z^2bcd^2+3\_Z\,bc^2d+bc^3+a)}\frac{(-6\_R^2b^2c^4d^2-15\_R\,b^2c^5d-10\,b^2c^6+3\_R^2abcd^2+12\_R\,abc^2d+}{d^2\_R^2+2\,cd\_R+c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(a+b*(d*x+c)^3),x)
```

```
[Out] -1/2/(b*c^3+a)/x^2+3*b*c^2*d/(b*c^3+a)^2/x+6*b^2*c^4*d^2/(b*c^3+a
)^3*ln(x)-3*b*c*d^2/(b*c^3+a)^3*ln(x)*a+1/3/(b*c^3+a)^3*d^2*sum((
-6*_R^2*b^2*c^4*d^2-15*_R*b^2*c^5*d-10*b^2*c^6+3*_R^2*a*b*c*d^2+1
2*_R*a*b*c^2*d+16*a*b*c^3-a^2)/(_R^2*d^2+2*_R*c*d+c^2)*ln(x-_R),_
R=RootOf(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{bd^3\int\frac{10\,b^2c^6-16\,abc^3+3\,(2\,b^2c^4-abc)\,d^2x^2+3\,(5\,b^2c^5-4\,abc^2)\,dx+a^2}{bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a}\,dx}{b^3c^9+3\,ab^2c^6+3\,a^2bc^3+a^3}+\frac{3\,(2\,b^2c^4-abc)\,d^2\log(x)}{b^3c^9+3\,ab^2c^6+3\,a^2bc^3+a^3}+\frac{6\,bc^2dx-bc^3-a}{2\,(b^2c^6+2\,abc^3+a^2)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(((d*x + c)^3*b + a)*x^3),x, algorithm="maxima")
```

```
[Out] -b*d^3*integrate((10*b^2*c^6 - 16*a*b*c^3 + 3*(2*b^2*c^4 - a*b*c)
*d^2*x^2 + 3*(5*b^2*c^5 - 4*a*b*c^2)*d*x + a^2)/(b*d^3*x^3 + 3*b*
c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a), x)/(b^3*c^9 + 3*a*b^2*c^6 +
 3*a^2*b*c^3 + a^3) + 3*(2*b^2*c^4 - a*b*c)*d^2*log(x)/(b^3*c^9 +
 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3) + 1/2*(6*b*c^2*d*x - b*c^3 - a)
/((b^2*c^6 + 2*a*b*c^3 + a^2)*x^2)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(((d*x + c)^3*b + a)*x^3),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(a+b*(d*x+c)**3),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left((dx+c)^3 b + a\right) x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(((d*x + c)^3*b + a)*x^3),x, algorithm="giac")
```

```
[Out] integrate(1/(((d*x + c)^3*b + a)*x^3), x)
```

3.110 $\int \frac{x^3}{a+b(c+dx)^4}\,dx$

Optimal. Leaf size=356

$$-\frac{c\left(3\sqrt{a}-\sqrt{b}c^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}d^4}+\frac{c\left(3\sqrt{a}-\sqrt{b}c^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}d^4}+\frac{c\left(3\sqrt{a}+\sqrt{b}c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}d^4}-\frac{c\left(3\sqrt{a}+\sqrt{b}c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}b^{3/4}d^4}+\frac{3c^2\tan^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}d^4}+\frac{\log\left(a+b(c+dx)^4\right)}{4bd^4}$$

```
[Out] (3*c^2*ArcTan[(Sqrt[b]*(c + d*x)^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]*
d^4) + (c*(3*Sqrt[a] + Sqrt[b]*c^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*(
c + d*x))/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)*d^4) - (c*(3*Sqrt[
a] + Sqrt[b]*c^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)]
)/(2*Sqrt[2]*a^(3/4)*b^(3/4)*d^4) - (c*(3*Sqrt[a] - Sqrt[b]*c^2)*
Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*
x)^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)*d^4) + (c*(3*Sqrt[a] - Sqrt[b]*
c^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c
 + d*x)^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)*d^4) + Log[a + b*(c + d*x)
^4]/(4*b*d^4)
```

Rubi [A] time = 0.83028, antiderivative size = 356, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 11, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.647

$$-\frac{c\left(3\sqrt{a}-\sqrt{b}c^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}d^4}+\frac{c\left(3\sqrt{a}-\sqrt{b}c^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}d^4}+\frac{c\left(3\sqrt{a}+\sqrt{b}c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}d^4}-\frac{c\left(3\sqrt{a}+\sqrt{b}c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}b^{3/4}d^4}+\frac{3c^2\tan^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}d^4}+\frac{\log\left(a+b(c+dx)^4\right)}{4bd^4}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/(a + b*(c + d*x)^4),x]
```

```
[Out] (3*c^2*ArcTan[(Sqrt[b]*(c + d*x)^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]*
d^4) + (c*(3*Sqrt[a] + Sqrt[b]*c^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*(
c + d*x))/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)*d^4) - (c*(3*Sqrt[
a] + Sqrt[b]*c^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)]
)/(2*Sqrt[2]*a^(3/4)*b^(3/4)*d^4) - (c*(3*Sqrt[a] - Sqrt[b]*c^2)*
Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*
x)^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)*d^4) + (c*(3*Sqrt[a] - Sqrt[b]*
c^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c
 + d*x)^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)*d^4) + Log[a + b*(c + d*x)
^4]/(4*b*d^4)
```

Rubi in Sympy [A] time = 91.5026, size = 338, normalized size = 0.95

$$\frac{\log\left(a+b\left(c+dx\right)^4\right)}{4bd^4}+\frac{3c^2\operatorname{atan}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}d^4}-\frac{\sqrt{2}c\left(3\sqrt{a}-\sqrt{b}c^2\right)\log\left(\sqrt{2}\sqrt[4]{a}b^{\frac{3}{4}}\left(-c-dx\right)+\sqrt{a}\sqrt{b}+b\left(c+dx\right)^2\right)}{8a^{\frac{3}{4}}b^{\frac{3}{4}}d^4}+\frac{\sqrt{2}c\left(3\sqrt{a}-\sqrt{b}c^2\right)\log\left(\sqrt{2}\sqrt[4]{a}b^{\frac{3}{4}}\left(c+dx\right)+\sqrt{a}\sqrt{b}+b\left(c+dx\right)^2\right)}{8a^{\frac{3}{4}}b^{\frac{3}{4}}d^4}+\frac{\sqrt{2}c\left(3\sqrt{a}+\sqrt{b}c^2\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}(-c-dx)}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}b^{\frac{3}{4}}d^4}-\frac{\sqrt{2}c\left(3\sqrt{a}+\sqrt{b}c^2\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}b^{\frac{3}{4}}d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(a+b*(d*x+c)**4),x)
```

```
[Out] log(a + b*(c + d*x)**4)/(4*b*d**4) + 3*c**2*atan(sqrt(b)*(c + d*x
)**2/sqrt(a))/(2*sqrt(a)*sqrt(b)*d**4) - sqrt(2)*c*(3*sqrt(a) - s
qrt(b)*c**2)*log(sqrt(2)*a**(1/4)*b**(3/4)*(-c - d*x) + sqrt(a)*s
qrt(b) + b*(c + d*x)**2)/(8*a**(3/4)*b**(3/4)*d**4) + sqrt(2)*c*(
3*sqrt(a) - sqrt(b)*c**2)*log(sqrt(2)*a**(1/4)*b**(3/4)*(c + d*x)
 + sqrt(a)*sqrt(b) + b*(c + d*x)**2)/(8*a**(3/4)*b**(3/4)*d**4) +
 sqrt(2)*c*(3*sqrt(a) + sqrt(b)*c**2)*atan(1 + sqrt(2)*b**(1/4)*(
-c - d*x)/a**(1/4))/(4*a**(3/4)*b**(3/4)*d**4) - sqrt(2)*c*(3*sqr
t(a) + sqrt(b)*c**2)*atan(1 + sqrt(2)*b**(1/4)*(c + d*x)/a**(1/4)
)/(4*a**(3/4)*b**(3/4)*d**4)
```

Mathematica [C] time = 0.0665021, size = 106, normalized size = 0.3

$$\frac{\text{RootSum}\left[\#1^4bd^4+4\#1^3bcd^3+6\#1^2bc^2d^2+4\#1bc^3d+a+bc^4\&,\frac{\#1^3\log(x-\#1)}{\#1^3d^3+3\#1^2cd^2+3\#1c^2d+c^3}\&\right]}{4bd}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/(a + b*(c + d*x)^4),x]
```

```
[Out] RootSum[a + b*c^4 + 4*b*c^3*d*#1 + 6*b*c^2*d^2*#1^2 + 4*b*c*d^3*#
1^3 + b*d^4*#1^4 & , (Log[x - #1]*#1^3)/(c^3 + 3*c^2*d*#1 + 3*c*d
^2*#1^2 + d^3*#1^3) & ]/(4*b*d)
```

Maple [C] time = 0.019, size = 97, normalized size = 0.3

$$\frac{1}{4\,bd}\sum_{\_R=RootOf\left(bd^4\_Z^4+4\,bcd^3\_Z^3+6\,bc^2d^2\_Z^2+4\,bc^3d\_Z+bc^4+a\right)}\frac{\_R^3\ln\left(x-\_R\right)}{d^3\_R^3+3\,cd^2\_R^2+3\,c^2d\_R+c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(a+b*(d*x+c)^4),x)
```

```
[Out] 1/4/b/d*sum(_R^3/(_R^3*d^3+3*_R^2*c*d^2+3*_R*c^2*d+c^3)*ln(x-_R),
_R=RootOf(_Z^4*b*d^4+4*_Z^3*b*c*d^3+6*_Z^2*b*c^2*d^2+4*_Z*b*c^3*d
+b*c^4+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(dx+c)^4 b + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((d*x + c)^4*b + a),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((d*x + c)^4*b + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((d*x + c)^4*b + a),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 11.1541, size = 374, normalized size = 1.05

$$\mathrm{RootSum}\left(256t^4a^3b^4d^{16} - 256t^3a^3b^3d^{12} + t^2\left(96a^3b^2d^8 + 480a^2b^3c^4d^8\right) + t\left(-16a^3bd^4 + 192a^2b^2c^4d^4 - 48ab^3c^8d^4\right) + a^3 + 3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(a+b*(d*x+c)**4),x)
```

```
[Out] RootSum(256*_t**4*a**3*b**4*d**16 - 256*_t**3*a**3*b**3*d**12 + _
t**2*(96*a**3*b**2*d**8 + 480*a**2*b**3*c**4*d**8) + _t*(-16*a**3
*b*d**4 + 192*a**2*b**2*c**4*d**4 - 48*a*b**3*c**8*d**4) + a**3 +
 3*a**2*b*c**4 + 3*a*b**2*c**8 + b**3*c**12, Lambda(_t, _t*log(x
+ (-1728*_t**3*a**4*b**3*d**12 - 960*_t**3*a**3*b**4*c**4*d**12 +
 1296*_t**2*a**4*b**2*d**8 + 2016*_t**2*a**3*b**3*c**4*d**8 - 48*
_t**2*a**2*b**4*c**8*d**8 - 324*_t*a**4*b*d**4 - 4716*_t*a**3*b**
2*c**4*d**4 - 1452*_t*a**2*b**3*c**8*d**4 - 4*_t*a*b**4*c**12*d**
4 + 27*a**4 - 390*a**3*b*c**4 - 444*a**2*b**2*c**8 - 26*a*b**3*c*
*12 + b**4*c**16)/(729*a**3*b*c**3*d - 1053*a**2*b**2*c**7*d - 11
7*a*b**3*c**11*d + b**4*c**15*d))))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(dx+c)^4 b + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((d*x + c)^4*b + a),x, algorithm="giac")
```

```
[Out] integrate(x^3/((d*x + c)^4*b + a), x)
```

3.111 $\int \frac{x^2}{a+b(c+dx)^4}\,dx$

Optimal. Leaf size=318

$$\frac{\left(\sqrt{a}-\sqrt{b}c^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}d^3}-\frac{\left(\sqrt{a}-\sqrt{b}c^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}d^3}-\frac{\left(\sqrt{a}+\sqrt{b}c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}d^3}+\frac{\left(\sqrt{a}+\sqrt{b}c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}b^{3/4}d^3}-\frac{c\tan^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d^3}$$

```
[Out] -((c*ArcTan[(Sqrt[b]*(c + d*x)^2)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]*d^3)
) - ((Sqrt[a] + Sqrt[b]*c^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*(c + d*x
))/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)*d^3) + ((Sqrt[a] + Sqrt[b
]*c^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/(2*Sqrt[2
]*a^(3/4)*b^(3/4)*d^3) + ((Sqrt[a] - Sqrt[b]*c^2)*Log[Sqrt[a] - S
qrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/(4*Sqrt[
2]*a^(3/4)*b^(3/4)*d^3) - ((Sqrt[a] - Sqrt[b]*c^2)*Log[Sqrt[a] +
Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/(4*Sqrt
[2]*a^(3/4)*b^(3/4)*d^3)
```

Rubi [A] time = 0.654407, antiderivative size = 318, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.588

$$\frac{\left(\sqrt{a}-\sqrt{b}c^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}d^3}-\frac{\left(\sqrt{a}-\sqrt{b}c^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}d^3}-\frac{\left(\sqrt{a}+\sqrt{b}c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}d^3}+\frac{\left(\sqrt{a}+\sqrt{b}c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}b^{3/4}d^3}-\frac{c\tan^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d^3}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(a + b*(c + d*x)^4),x]
```

```
[Out] -((c*ArcTan[(Sqrt[b]*(c + d*x)^2)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]*d^3)
) - ((Sqrt[a] + Sqrt[b]*c^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*(c + d*x
))/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)*d^3) + ((Sqrt[a] + Sqrt[b
]*c^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/(2*Sqrt[2
]*a^(3/4)*b^(3/4)*d^3) + ((Sqrt[a] - Sqrt[b]*c^2)*Log[Sqrt[a] - S
qrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/(4*Sqrt[
2]*a^(3/4)*b^(3/4)*d^3) - ((Sqrt[a] - Sqrt[b]*c^2)*Log[Sqrt[a] +
Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/(4*Sqrt
[2]*a^(3/4)*b^(3/4)*d^3)
```

Rubi in Sympy [A] time = 80.4785, size = 301, normalized size = 0.95

$$-\frac{c\operatorname{atan}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d^3}+\frac{\sqrt{2}\left(\sqrt{a}-\sqrt{b}c^2\right)\log\left(\sqrt{2}\sqrt[4]{a}b^{\frac{3}{4}}(-c-dx)+\sqrt{a}\sqrt{b}+b(c+dx)^2\right)}{8a^{\frac{3}{4}}b^{\frac{3}{4}}d^3}-\frac{\sqrt{2}\left(\sqrt{a}-\sqrt{b}c^2\right)\log\left(\sqrt{2}\sqrt[4]{a}b^{\frac{3}{4}}(c+dx)+\sqrt{a}\sqrt{b}+b(c+dx)^2\right)}{8a^{\frac{3}{4}}b^{\frac{3}{4}}d^3}-\frac{\sqrt{2}\left(\sqrt{a}+\sqrt{b}c^2\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}(-c-dx)}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}b^{\frac{3}{4}}d^3}+\frac{\sqrt{2}\left(\sqrt{a}+\sqrt{b}c^2\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}b^{\frac{3}{4}}d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(a+b*(d*x+c)**4),x)
```

```
[Out] -c*atan(sqrt(b)*(c + d*x)**2/sqrt(a))/(sqrt(a)*sqrt(b)*d**3) + sq
rt(2)*(sqrt(a) - sqrt(b)*c**2)*log(sqrt(2)*a**(1/4)*b**(3/4)*(-c
- d*x) + sqrt(a)*sqrt(b) + b*(c + d*x)**2)/(8*a**(3/4)*b**(3/4)*d
**3) - sqrt(2)*(sqrt(a) - sqrt(b)*c**2)*log(sqrt(2)*a**(1/4)*b**(
3/4)*(c + d*x) + sqrt(a)*sqrt(b) + b*(c + d*x)**2)/(8*a**(3/4)*b*
*(3/4)*d**3) - sqrt(2)*(sqrt(a) + sqrt(b)*c**2)*atan(1 + sqrt(2)*
b**(1/4)*(-c - d*x)/a**(1/4))/(4*a**(3/4)*b**(3/4)*d**3) + sqrt(2
)*(sqrt(a) + sqrt(b)*c**2)*atan(1 + sqrt(2)*b**(1/4)*(c + d*x)/a*
*(1/4))/(4*a**(3/4)*b**(3/4)*d**3)
```

Mathematica [C] time = 0.0534919, size = 106, normalized size = 0.33

$$\frac{\text{RootSum}\left[\#1^4bd^4+4\#1^3bcd^3+6\#1^2bc^2d^2+4\#1bc^3d+a+bc^4\&,\frac{\#1^2\log(x-\#1)}{\#1^3d^3+3\#1^2cd^2+3\#1c^2d+c^3}\&\right]}{4bd}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(a + b*(c + d*x)^4),x]
```

```
[Out] RootSum[a + b*c^4 + 4*b*c^3*d*#1 + 6*b*c^2*d^2*#1^2 + 4*b*c*d^3*#
1^3 + b*d^4*#1^4 & , (Log[x - #1]*#1^2)/(c^3 + 3*c^2*d*#1 + 3*c*d
^2*#1^2 + d^3*#1^3) & ]/(4*b*d)
```

Maple [C] time = 0.005, size = 97, normalized size = 0.3

$$\frac{1}{4bd}\sum_{\_R=RootOf\left(\_Z^4bd^4+4\_Z^3bcd^3+6\_Z^2bc^2d^2+4\_Zbc^3d+bc^4+a\right)}\frac{\_R^2\ln(x-\_R)}{d^3\_R^3+3cd^2\_R^2+3c^2d\_R+c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(a+b*(d*x+c)^4),x)
```

```
[Out] 1/4/b/d*sum(_R^2/(_R^3*d^3+3*_R^2*c*d^2+3*_R*c^2*d+c^3)*ln(x-_R),
_R=RootOf(_Z^4*b*d^4+4*_Z^3*b*c*d^3+6*_Z^2*b*c^2*d^2+4*_Z*b*c^3*d
+b*c^4+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2}{(dx+c)^4b+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((d*x + c)^4*b + a),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((d*x + c)^4*b + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((d*x + c)^4*b + a),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 8.28211, size = 274, normalized size = 0.86

$$\mathrm{RootSum}\left(256t^4a^3b^3d^{12} + 192t^2a^2b^2c^2d^6 + t\left(-32a^2bcd^3 + 32ab^2c^5d^3\right) + a^2 + 2abc^4 + b^2c^8, \left(t \mapsto t\log\left(x + \frac{64t^3a^4b^2d^9 + 4\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(a+b*(d*x+c)**4),x)
```

```
[Out] RootSum(256*_t**4*a**3*b**3*d**12 + 192*_t**2*a**2*b**2*c**2*d**6
 + _t*(-32*a**2*b*c*d**3 + 32*a*b**2*c**5*d**3) + a**2 + 2*a*b*c*
*4 + b**2*c**8, Lambda(_t, _t*log(x + (64*_t**3*a**4*b**2*d**9 +
448*_t**3*a**3*b**3*c**4*d**9 + 160*_t**2*a**3*b**2*c**3*d**6 - 3
2*_t**2*a**2*b**3*c**7*d**6 + 60*_t*a**3*b*c**2*d**3 + 256*_t*a**
2*b**2*c**6*d**3 + 4*_t*a*b**3*c**10*d**3 - 5*a**3*c - 9*a**2*b*c
**5 - 3*a*b**2*c**9 + b**3*c**13)/(a**3*d - 33*a**2*b*c**4*d - 33
*a*b**2*c**8*d + b**3*c**12*d))))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(dx+c)^4b+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((d*x + c)^4*b + a),x, algorithm="giac")
```

```
[Out] integrate(x^2/((d*x + c)^4*b + a), x)
```

3.112 $\int \frac{x}{a+b(c+dx)^4} \, dx$

Optimal. Leaf size=261

$$\frac{c\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d^2}-\frac{c\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d^2}+\frac{c\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}d^2}-\frac{c\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}d^2}+\frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}d^2}$$

```
[Out] ArcTan[(Sqrt[b]*(c + d*x)^2)/Sqrt[a]]/(2*Sqrt[a]*Sqrt[b]*d^2) + (
c*ArcTan[1 - (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/(2*Sqrt[2]*a^(
3/4)*b^(1/4)*d^2) - (c*ArcTan[1 + (Sqrt[2]*b^(1/4)*(c + d*x))/a^(
1/4)])/(2*Sqrt[2]*a^(3/4)*b^(1/4)*d^2) + (c*Log[Sqrt[a] - Sqrt[2]
*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/(4*Sqrt[2]*a^(
3/4)*b^(1/4)*d^2) - (c*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*(c +
 d*x) + Sqrt[b]*(c + d*x)^2])/(4*Sqrt[2]*a^(3/4)*b^(1/4)*d^2)
```

Rubi [A] time = 0.543396, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667

$$\frac{c\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d^2}-\frac{c\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d^2}+\frac{c\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}d^2}-\frac{c\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}d^2}+\frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}d^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(a + b*(c + d*x)^4),x]
```

```
[Out] ArcTan[(Sqrt[b]*(c + d*x)^2)/Sqrt[a]]/(2*Sqrt[a]*Sqrt[b]*d^2) + (
c*ArcTan[1 - (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/(2*Sqrt[2]*a^(
3/4)*b^(1/4)*d^2) - (c*ArcTan[1 + (Sqrt[2]*b^(1/4)*(c + d*x))/a^(
1/4)])/(2*Sqrt[2]*a^(3/4)*b^(1/4)*d^2) + (c*Log[Sqrt[a] - Sqrt[2]
*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/(4*Sqrt[2]*a^(
3/4)*b^(1/4)*d^2) - (c*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*(c +
 d*x) + Sqrt[b]*(c + d*x)^2])/(4*Sqrt[2]*a^(3/4)*b^(1/4)*d^2)
```

Rubi in Sympy [A] time = 67.3942, size = 250, normalized size = 0.96

$$\frac{\operatorname{atan}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}d^2}+\frac{\sqrt{2}c\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\left(-c-dx\right)+\sqrt{a}+\sqrt{b}\left(c+dx\right)^2\right)}{8a^{\frac{3}{4}}\sqrt[4]{b}d^2}-\frac{\sqrt{2}c\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\left(c+dx\right)+\sqrt{a}+\sqrt{b}\left(c+dx\right)^2\right)}{8a^{\frac{3}{4}}\sqrt[4]{b}d^2}+\frac{\sqrt{2}c\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}(-c-dx)}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}\sqrt[4]{b}d^2}-\frac{\sqrt{2}c\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}\sqrt[4]{b}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(a+b*(d*x+c)**4),x)
```

```
[Out] atan(sqrt(b)*(c + d*x)**2/sqrt(a))/(2*sqrt(a)*sqrt(b)*d**2) + sqr
t(2)*c*log(sqrt(2)*a**(1/4)*b**(1/4)*(-c - d*x) + sqrt(a) + sqrt(
b)*(c + d*x)**2)/(8*a**(3/4)*b**(1/4)*d**2) - sqrt(2)*c*log(sqrt(
2)*a**(1/4)*b**(1/4)*(c + d*x) + sqrt(a) + sqrt(b)*(c + d*x)**2)/
(8*a**(3/4)*b**(1/4)*d**2) + sqrt(2)*c*atan(1 + sqrt(2)*b**(1/4)*
(-c - d*x)/a**(1/4))/(4*a**(3/4)*b**(1/4)*d**2) - sqrt(2)*c*atan(
1 + sqrt(2)*b**(1/4)*(c + d*x)/a**(1/4))/(4*a**(3/4)*b**(1/4)*d**
2)
```

Mathematica [C] time = 0.0430668, size = 104, normalized size = 0.4

$$\frac{\text{RootSum}\left[\#1^4bd^4+4\#1^3bcd^3+6\#1^2bc^2d^2+4\#1bc^3d+a+bc^4\&,\frac{\#1\log(x-\#1)}{\#1^3d^3+3\#1^2cd^2+3\#1c^2d+c^3}\&\right]}{4bd}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(a + b*(c + d*x)^4),x]
```

```
[Out] RootSum[a + b*c^4 + 4*b*c^3*d*#1 + 6*b*c^2*d^2*#1^2 + 4*b*c*d^3*#
1^3 + b*d^4*#1^4 & , (Log[x - #1]*#1)/(c^3 + 3*c^2*d*#1 + 3*c*d^2
*#1^2 + d^3*#1^3) & ]/(4*b*d)
```

Maple [C] time = 0.006, size = 95, normalized size = 0.4

$$\frac{1}{4\,bd}\sum_{\_R=RootOf\left(\_Z^4bd^4+4\_Z^3bcd^3+6\_Z^2bc^2d^2+4\_Z\,bc^3d+bc^4+a\right)}\frac{\_R\ln(x-\_R)}{d^3\_R^3+3\,cd^2\_R^2+3\,c^2d\_R+c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(a+b*(d*x+c)^4),x)
```

```
[Out] 1/4/b/d*sum(_R/(_R^3*d^3+3*_R^2*c*d^2+3*_R*c^2*d+c^3)*ln(x-_R),_R
=RootOf(_Z^4*b*d^4+4*_Z^3*b*c*d^3+6*_Z^2*b*c^2*d^2+4*_Z*b*c^3*d+b
*c^4+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{(dx+c)^4b+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((d*x + c)^4*b + a),x, algorithm="maxima")
```

```
[Out] integrate(x/((d*x + c)^4*b + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((d*x + c)^4*b + a),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 2.92449, size = 131, normalized size = 0.5

$$\mathrm{RootSum}\left(256t^4a^3b^2d^8 + 32t^2a^2bd^4 - 16tabc^2d^2 + a + bc^4, \left(t \mapsto t\log\left(x + \frac{128t^3a^3bd^6 + 16t^2a^2bc^2d^4 + 8ta^2d^2 + 4tabc^4d^2}{4acd - bc^5d}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(a+b*(d*x+c)**4),x)
```

```
[Out] RootSum(256*_t**4*a**3*b**2*d**8 + 32*_t**2*a**2*b*d**4 - 16*_t*a
*b*c**2*d**2 + a + b*c**4, Lambda(_t, _t*log(x + (128*_t**3*a**3*
b*d**6 + 16*_t**2*a**2*b*c**2*d**4 + 8*_t*a**2*d**2 + 4*_t*a*b*c*
*4*d**2 - a*c**2 - b*c**6)/(4*a*c*d - b*c**5*d))))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(dx+c)^4 b + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((d*x + c)^4*b + a),x, algorithm="giac")
```

```
[Out] integrate(x/((d*x + c)^4*b + a), x)
```

3.113 $\int \frac{1}{a+b(c+dx)^4}\,dx$

Optimal. Leaf size=221

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}d}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}d}$$

```
[Out] -ArcTan[1 - (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)]/(2*Sqrt[2]*a^(3/
4)*b^(1/4)*d) + ArcTan[1 + (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)]/(
2*Sqrt[2]*a^(3/4)*b^(1/4)*d) - Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1
/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2]/(4*Sqrt[2]*a^(3/4)*b^(1/4)*d
) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c
+ d*x)^2]/(4*Sqrt[2]*a^(3/4)*b^(1/4)*d)
```

Rubi [A] time = 0.3728, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}d}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}d}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*(c + d*x)^4)^(-1),x]
```

```
[Out] -ArcTan[1 - (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)]/(2*Sqrt[2]*a^(3/
4)*b^(1/4)*d) + ArcTan[1 + (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)]/(
2*Sqrt[2]*a^(3/4)*b^(1/4)*d) - Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1
/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2]/(4*Sqrt[2]*a^(3/4)*b^(1/4)*d
) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c
+ d*x)^2]/(4*Sqrt[2]*a^(3/4)*b^(1/4)*d)
```

Rubi in Sympy [A] time = 52.4997, size = 202, normalized size = 0.91

$$-\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,(-c-dx)+\sqrt{a}+\sqrt{b}\,(c+dx)^2\right)}{8a^{\frac{3}{4}}\sqrt[4]{b}d}+\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,(c+dx)+\sqrt{a}+\sqrt{b}\,(c+dx)^2\right)}{8a^{\frac{3}{4}}\sqrt[4]{b}d}-\frac{\sqrt{2}\,\mathrm{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}(-c-dx)}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}\sqrt[4]{b}d}+\frac{\sqrt{2}\,\mathrm{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}\sqrt[4]{b}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a+b*(d*x+c)**4),x)
```

```
[Out] -sqrt(2)*log(sqrt(2)*a**(1/4)*b**(1/4)*(-c - d*x) + sqrt(a) + sqr
t(b)*(c + d*x)**2)/(8*a**(3/4)*b**(1/4)*d) + sqrt(2)*log(sqrt(2)*
```

```
a**(1/4)*b**(1/4)*(c + d*x) + sqrt(a) + sqrt(b)*(c + d*x)**2)/(8*
a**(3/4)*b**(1/4)*d) - sqrt(2)*atan(1 + sqrt(2)*b**(1/4)*(-c - d*
x)/a**(1/4))/(4*a**(3/4)*b**(1/4)*d) + sqrt(2)*atan(1 + sqrt(2)*b
**(1/4)*(c + d*x)/a**(1/4))/(4*a**(3/4)*b**(1/4)*d)
```

Mathematica [A] time = 0.135701, size = 161, normalized size = 0.73

$$\frac{-\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)+\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)-2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)+}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*(c + d*x)^4)^(-1),x]
```

```
[Out] (-2*ArcTan[1 - (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)] + 2*ArcTan[1
+ (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)] - Log[Sqrt[a] - Sqrt[2]*a^
(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2] + Log[Sqrt[a] + Sq
rt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/(4*Sqrt[2
]*a^(3/4)*b^(1/4)*d)
```

Maple [C] time = 0.004, size = 94, normalized size = 0.4

$$\frac{1}{4\,bd}\sum_{\_R=RootOf\left(\_Z^4bd^4+4\_Z^3bcd^3+6\_Z^2bc^2d^2+4\_Z\,bc^3d+bc^4+a\right)}\frac{\ln\left(x-\_R\right)}{d^3\_R^3+3\,cd^2\_R^2+3\,c^2d\_R+c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a+b*(d*x+c)^4),x)
```

```
[Out] 1/4/b/d*sum(1/(_R^3*d^3+3*_R^2*c*d^2+3*_R*c^2*d+c^3)*ln(x-_R),_R=
RootOf(_Z^4*b*d^4+4*_Z^3*b*c*d^3+6*_Z^2*b*c^2*d^2+4*_Z*b*c^3*d+b*
c^4+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(dx+c)^4b+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((d*x + c)^4*b + a),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x + c)^4*b + a), x)
```

Fricas [A] time = 0.264368, size = 215, normalized size = 0.97

$$-\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}\arctan\left(\frac{ad\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}}{dx+d\sqrt{\frac{a^2d^2\sqrt{-\frac{1}{a^3bd^4}}+d^2x^2+2\,cdx+c^2}{d^2}}+c}\right)+\frac{1}{4}\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}\log\left(ad\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}+dx+c\right)-\frac{1}{4}\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}\log\left(-ad\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}+dx+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((d*x + c)^4*b + a),x, algorithm="fricas")
```

```
[Out] -(-1/(a^3*b*d^4))^(1/4)*arctan(a*d*(-1/(a^3*b*d^4))^(1/4)/(d*x +
d*sqrt((a^2*d^2*sqrt(-1/(a^3*b*d^4)) + d^2*x^2 + 2*c*d*x + c^2)/d
^2) + c)) + 1/4*(-1/(a^3*b*d^4))^(1/4)*log(a*d*(-1/(a^3*b*d^4))^(
1/4) + d*x + c) - 1/4*(-1/(a^3*b*d^4))^(1/4)*log(-a*d*(-1/(a^3*b*
d^4))^(1/4) + d*x + c)
```

Sympy [A] time = 0.838331, size = 26, normalized size = 0.12

$$\frac{\operatorname{RootSum}\left(256t^4a^3b+1,\left(t\mapsto t\log\left(x+\frac{4ta+c}{d}\right)\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a+b*(d*x+c)**4),x)
```

```
[Out] RootSum(256*_t**4*a**3*b + 1, Lambda(_t, _t*log(x + (4*_t*a + c)/
d)))/d
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)^4b+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((d*x + c)^4*b + a),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)^4*b + a), x)
```

3.114 $\int \frac{1}{x(a+b(c+dx)^4)}\,dx$

Optimal. Leaf size=393

$$-\frac{\sqrt[4]{b}c\left(\sqrt{a}-\sqrt{b}c^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\left(a+bc^4\right)}+\frac{\sqrt[4]{b}c\left(\sqrt{a}-\sqrt{b}c^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\left(a+bc^4\right)}+\frac{\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(a+bc^4\right)}-\frac{\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\left(a+bc^4\right)}-\frac{\log\left(a+b(c+dx)^4\right)}{4\left(a+bc^4\right)}+\frac{\log(x)}{a+bc^4}-\frac{\sqrt{b}c^2\tan^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(a+bc^4\right)}$$

```
[Out] -(Sqrt[b]*c^2*ArcTan[(Sqrt[b]*(c + d*x)^2)/Sqrt[a]])/(2*Sqrt[a]*(
a + b*c^4)) + (b^(1/4)*c*(Sqrt[a] + Sqrt[b]*c^2)*ArcTan[1 - (Sqrt
[2]*b^(1/4)*(c + d*x))/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(a + b*c^4))
- (b^(1/4)*c*(Sqrt[a] + Sqrt[b]*c^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*
(c + d*x))/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(a + b*c^4)) + Log[x]/(a
+ b*c^4) - (b^(1/4)*c*(Sqrt[a] - Sqrt[b]*c^2)*Log[Sqrt[a] - Sqrt[
2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/(4*Sqrt[2]*a
^(3/4)*(a + b*c^4)) + (b^(1/4)*c*(Sqrt[a] - Sqrt[b]*c^2)*Log[Sqrt
[a] + Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/(
4*Sqrt[2]*a^(3/4)*(a + b*c^4)) - Log[a + b*(c + d*x)^4]/(4*(a + b
*c^4))
```

Rubi [A] time = 0.98461, antiderivative size = 393, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 12, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.706

$$-\frac{\sqrt[4]{b}c\left(\sqrt{a}-\sqrt{b}c^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\left(a+bc^4\right)}+\frac{\sqrt[4]{b}c\left(\sqrt{a}-\sqrt{b}c^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\left(a+bc^4\right)}+\frac{\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(a+bc^4\right)}-\frac{\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\left(a+bc^4\right)}-\frac{\log\left(a+b(c+dx)^4\right)}{4\left(a+bc^4\right)}+\frac{\log(x)}{a+bc^4}-\frac{\sqrt{b}c^2\tan^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(a+bc^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(a + b*(c + d*x)^4)),x]
```


```
[Out] -(Sqrt[b]*c^2*ArcTan[(Sqrt[b]*(c + d*x)^2)/Sqrt[a]])/(2*Sqrt[a]*(
a + b*c^4)) + (b^(1/4)*c*(Sqrt[a] + Sqrt[b]*c^2)*ArcTan[1 - (Sqrt
[2]*b^(1/4)*(c + d*x))/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(a + b*c^4))
- (b^(1/4)*c*(Sqrt[a] + Sqrt[b]*c^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*
(c + d*x))/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(a + b*c^4)) + Log[x]/(a
+ b*c^4) - (b^(1/4)*c*(Sqrt[a] - Sqrt[b]*c^2)*Log[Sqrt[a] - Sqrt[
2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/(4*Sqrt[2]*a
^(3/4)*(a + b*c^4)) + (b^(1/4)*c*(Sqrt[a] - Sqrt[b]*c^2)*Log[Sqrt
[a] + Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/(
4*Sqrt[2]*a^(3/4)*(a + b*c^4)) - Log[a + b*(c + d*x)^4]/(4*(a + b
*c^4))
```

Rubi in Sympy [A] time = 115.583, size = 360, normalized size = 0.92

$$\frac{\log(-dx)}{a+bc^4}-\frac{\log\left(a+b(c+dx)^4\right)}{4(a+bc^4)}-\frac{\sqrt{b}c^2\operatorname{atan}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}(a+bc^4)}-\frac{\sqrt{2}\sqrt[4]{b}c\left(\sqrt{a}-\sqrt{b}c^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}b^{\frac{3}{4}}(c+dx)+\sqrt{a}\sqrt{b}+b(c+dx)^2\right)}{8a^{\frac{3}{4}}(a+bc^4)}+\frac{\sqrt{2}\sqrt[4]{b}c\left(\sqrt{a}-\sqrt{b}c^2\right)\log\left(\sqrt{2}\sqrt[4]{a}b^{\frac{3}{4}}(c+dx)+\sqrt{a}\sqrt{b}+b(c+dx)^2\right)}{8a^{\frac{3}{4}}(a+bc^4)}+\frac{\sqrt{2}\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}c^2\right)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}(a+bc^4)}-\frac{\sqrt{2}\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}c^2\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}(a+bc^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(a+b*(d*x+c)**4),x)
```

```
[Out] log(-d*x)/(a + b*c**4) - log(a + b*(c + d*x)**4)/(4*(a + b*c**4))
 - sqrt(b)*c**2*atan(sqrt(b)*(c + d*x)**2/sqrt(a))/(2*sqrt(a)*(a
+ b*c**4)) - sqrt(2)*b**(1/4)*c*(sqrt(a) - sqrt(b)*c**2)*log(-sqr
t(2)*a**(1/4)*b**(3/4)*(c + d*x) + sqrt(a)*sqrt(b) + b*(c + d*x)*
*2)/(8*a**(3/4)*(a + b*c**4)) + sqrt(2)*b**(1/4)*c*(sqrt(a) - sqr
t(b)*c**2)*log(sqrt(2)*a**(1/4)*b**(3/4)*(c + d*x) + sqrt(a)*sqrt
(b) + b*(c + d*x)**2)/(8*a**(3/4)*(a + b*c**4)) + sqrt(2)*b**(1/4
)*c*(sqrt(a) + sqrt(b)*c**2)*atan(1 - sqrt(2)*b**(1/4)*(c + d*x)/
a**(1/4))/(4*a**(3/4)*(a + b*c**4)) - sqrt(2)*b**(1/4)*c*(sqrt(a)
 + sqrt(b)*c**2)*atan(1 + sqrt(2)*b**(1/4)*(c + d*x)/a**(1/4))/(4
*a**(3/4)*(a + b*c**4))
```

Mathematica [C] time = 0.105736, size = 163, normalized size = 0.41

$$-\frac{\text{RootSum}\left[\#1^4bd^4+4\#1^3bcd^3+6\#1^2bc^2d^2+4\#1bc^3d+a+bc^4\&,\frac{\#1^3d^3\log(x-\#1)+4\#1^2cd^2\log(x-\#1)+4c^3\log(x-\#1)+6\#1c^2d\log(x-\#1)}{\#1^3d^3+3\#1^2cd^2+3\#1c^2d+c^3}\&\right]-4\log(x)}{4(a+bc^4)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(a + b*(c + d*x)^4)),x]
```

```
[Out] -(-4*Log[x] + RootSum[a + b*c^4 + 4*b*c^3*d*#1 + 6*b*c^2*d^2*#1^2
 + 4*b*c*d^3*#1^3 + b*d^4*#1^4 & , (4*c^3*Log[x - #1] + 6*c^2*d*L
og[x - #1]*#1 + 4*c*d^2*Log[x - #1]*#1^2 + d^3*Log[x - #1]*#1^3)/
(c^3 + 3*c^2*d*#1 + 3*c*d^2*#1^2 + d^3*#1^3) & ])/(4*(a + b*c^4))
```

Maple [C] time = 0.011, size = 139, normalized size = 0.4

$$\frac{\ln(x)}{bc^4+a}-\frac{1}{4bc^4+4a}\sum_{\_R=RootOf\left(\_Z^4bd^4+4\_Z^3bcd^3+6\_Z^2bc^2d^2+4\_Zbc^3d+bc^4+a\right)}\frac{\left(\_R^3d^3+4\_R^2cd^2+6\_Rc^2d+4c^3\right)\ln(x-\_R)}{\_R^3d^3+3\_R^2cd^2+3\_Rc^2d+c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(a+b*(d*x+c)^4),x)
```

```
[Out] ln(x)/(b*c^4+a)-1/4/(b*c^4+a)*sum((_R^3*d^3+4*_R^2*c*d^2+6*_R*c^2
*d+4*c^3)/(_R^3*d^3+3*_R^2*c*d^2+3*_R*c^2*d+c^3)*ln(x-_R),_R=Root
Of(_Z^4*b*d^4+4*_Z^3*b*c*d^3+6*_Z^2*b*c^2*d^2+4*_Z*b*c^3*d+b*c^4+
a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{bd\int \frac{d^3x^3+4\,cd^2x^2+6\,c^2dx+4\,c^3}{bd^4x^4+4\,bcd^3x^3+6\,bc^2d^2x^2+4\,bc^3dx+bc^4+a}\,dx}{bc^4+a}+\frac{\log\left(x\right)}{bc^4+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((((d*x + c)^4*b + a)*x),x, algorithm="maxima")
```

```
[Out] -b*d*integrate((d^3*x^3 + 4*c*d^2*x^2 + 6*c^2*d*x + 4*c^3)/(b*d^4
*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x + b*c^4 + a)
, x)/(b*c^4 + a) + log(x)/(b*c^4 + a)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((((d*x + c)^4*b + a)*x),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(a+b*(d*x+c)**4),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left((dx+c)^4b+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((((d*x + c)^4*b + a)*x),x, algorithm="giac")
```

```
[Out] integrate(1/((((d*x + c)^4*b + a)*x), x)
```

3.115 $\int \frac{1}{x^2(a+b(c+dx)^4)}\,dx$

Optimal. Leaf size=496

$$-\frac{\sqrt[4]{b}d\left(\sqrt{a}\left(a-3bc^4\right)-\sqrt{b}c^2\left(3a-bc^4\right)\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\left(a+bc^4\right)^2}+\frac{\sqrt[4]{b}d\left(\sqrt{a}\left(a-3bc^4\right)-\sqrt{b}c^2\left(3a-bc^4\right)\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\left(a+bc^4\right)^2}+\frac{\sqrt[4]{b}d\left(\sqrt{a}\left(a-3bc^4\right)+\sqrt{b}c^2\left(3a-bc^4\right)\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(a+bc^4\right)^2}-\frac{\sqrt[4]{b}d\left(\sqrt{a}\left(a-3bc^4\right)+\sqrt{b}c^2\left(3a-bc^4\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\left(a+bc^4\right)^2}-\frac{\sqrt{b}cd\left(a-bc^4\right)\tan^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{\sqrt{a}\left(a+bc^4\right)^2}-\frac{1}{x\left(a+bc^4\right)}-\frac{4bc^3d\log(x)}{\left(a+bc^4\right)^2}+\frac{bc^3d\log\left(a+b(c+dx)^4\right)}{\left(a+bc^4\right)^2}$$

```
[Out] -(1/((a + b*c^4)*x)) - (Sqrt[b]*c*(a - b*c^4)*d*ArcTan[(Sqrt[b]*(
c + d*x)^2)/Sqrt[a]])/(Sqrt[a]*(a + b*c^4)^2) + (b^(1/4)*(Sqrt[a]
*(a - 3*b*c^4) + Sqrt[b]*c^2*(3*a - b*c^4))*d*ArcTan[1 - (Sqrt[2]
*b^(1/4)*(c + d*x))/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(a + b*c^4)^2) -
 (b^(1/4)*(Sqrt[a]*(a - 3*b*c^4) + Sqrt[b]*c^2*(3*a - b*c^4))*d*A
rcTan[1 + (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/(2*Sqrt[2]*a^(3/4
)*(a + b*c^4)^2) - (4*b*c^3*d*Log[x])/(a + b*c^4)^2 - (b^(1/4)*(S
qrt[a]*(a - 3*b*c^4) - Sqrt[b]*c^2*(3*a - b*c^4))*d*Log[Sqrt[a] -
 Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/(4*Sqr
t[2]*a^(3/4)*(a + b*c^4)^2) + (b^(1/4)*(Sqrt[a]*(a - 3*b*c^4) - S
qrt[b]*c^2*(3*a - b*c^4))*d*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)
*(c + d*x) + Sqrt[b]*(c + d*x)^2])/(4*Sqrt[2]*a^(3/4)*(a + b*c^4)
^2) + (b*c^3*d*Log[a + b*(c + d*x)^4])/(a + b*c^4)^2
```

Rubi [A] time = 1.92344, antiderivative size = 496, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 12, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.706$

$$-\frac{\sqrt[4]{b}d\left(\sqrt{a}\left(a-3bc^4\right)-\sqrt{b}c^2\left(3a-bc^4\right)\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\left(a+bc^4\right)^2}+\frac{\sqrt[4]{b}d\left(\sqrt{a}\left(a-3bc^4\right)-\sqrt{b}c^2\left(3a-bc^4\right)\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\left(a+bc^4\right)^2}+\frac{\sqrt[4]{b}d\left(\sqrt{a}\left(a-3bc^4\right)+\sqrt{b}c^2\left(3a-bc^4\right)\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(a+bc^4\right)^2}-\frac{\sqrt[4]{b}d\left(\sqrt{a}\left(a-3bc^4\right)+\sqrt{b}c^2\left(3a-bc^4\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\left(a+bc^4\right)^2}-\frac{\sqrt{b}cd\left(a-bc^4\right)\tan^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{\sqrt{a}\left(a+bc^4\right)^2}-\frac{1}{x\left(a+bc^4\right)}-\frac{4bc^3d\log(x)}{\left(a+bc^4\right)^2}+\frac{bc^3d\log\left(a+b(c+dx)^4\right)}{\left(a+bc^4\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(a + b*(c + d*x)^4)),x]
```


```
[Out] -(1/((a + b*c^4)*x)) - (Sqrt[b]*c*(a - b*c^4)*d*ArcTan[(Sqrt[b]*(
c + d*x)^2)/Sqrt[a]])/(Sqrt[a]*(a + b*c^4)^2) + (b^(1/4)*(Sqrt[a]
*(a - 3*b*c^4) + Sqrt[b]*c^2*(3*a - b*c^4))*d*ArcTan[1 - (Sqrt[2]
```

```
*b^(1/4)*(c + d*x))/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(a + b*c^4)^2) -
 (b^(1/4)*(Sqrt[a]*(a - 3*b*c^4) + Sqrt[b]*c^2*(3*a - b*c^4))*d*A
rcTan[1 + (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/(2*Sqrt[2]*a^(3/4
)*(a + b*c^4)^2) - (4*b*c^3*d*Log[x])/(a + b*c^4)^2 - (b^(1/4)*(S
qrt[a]*(a - 3*b*c^4) - Sqrt[b]*c^2*(3*a - b*c^4))*d*Log[Sqrt[a] -
 Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/(4*Sqr
t[2]*a^(3/4)*(a + b*c^4)^2) + (b^(1/4)*(Sqrt[a]*(a - 3*b*c^4) - S
qrt[b]*c^2*(3*a - b*c^4))*d*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)
*(c + d*x) + Sqrt[b]*(c + d*x)^2])/(4*Sqrt[2]*a^(3/4)*(a + b*c^4)
^2) + (b*c^3*d*Log[a + b*(c + d*x)^4])/(a + b*c^4)^2
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(a+b*(d*x+c)**4),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.19518, size = 238, normalized size = 0.48

$$\frac{dx\text{RootSum}\left[\#1^4bd^4+4\#1^3bcd^3+6\#1^2bc^2d^2+4\#1bc^3d+a+bc^4\&,\frac{4\#1^3bc^3d^3\log(x-\#1)-\#1^2ad^2\log(x-\#1)+15\#1^2bc^4d^2\log(x-\#1)-6}{\#1^3d^3+3\#1}\right.}{4x\left(a+bc^4\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(a + b*(c + d*x)^4)),x]
```

```
[Out] (-4*(a + b*c^4 + 4*b*c^3*d*x*Log[x]) + d*x*RootSum[a + b*c^4 + 4*
b*c^3*d*#1 + 6*b*c^2*d^2*#1^2 + 4*b*c*d^3*#1^3 + b*d^4*#1^4 & , (
-6*a*c^2*Log[x - #1] + 10*b*c^6*Log[x - #1] - 4*a*c*d*Log[x - #1]
*#1 + 20*b*c^5*d*Log[x - #1]*#1 - a*d^2*Log[x - #1]*#1^2 + 15*b*c
^4*d^2*Log[x - #1]*#1^2 + 4*b*c^3*d^3*Log[x - #1]*#1^3)/(c^3 + 3*
c^2*d*#1 + 3*c*d^2*#1^2 + d^3*#1^3) & ])/(4*(a + b*c^4)^2*x)
```

Maple [C] time = 0.017, size = 188, normalized size = 0.4

$$-\frac{1}{\left(bc^4+a\right)x}-4\frac{bc^3d\ln(x)}{\left(bc^4+a\right)^2}+\frac{d}{4\left(bc^4+a\right)^2}\sum_{\_R=RootOf\left(\_Z^4bd^4+4\_Z^3bcd^3+6\_Z^2bc^2d^2+4\_Zbc^3d+bc^4+a\right)}\frac{\left(4bd^3c^3\_R^3+d^2\left(15bc^4-a\right)\_R^2+4cd\left(5bc^4-a\right)\_R+1\right.}{\_R^3d^3+3\_R^2cd^2+3\_Rc^2d+c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(a+b*(d*x+c)^4),x)
```

```
[Out] -1/(b*c^4+a)/x-4*b*c^3*d*ln(x)/(b*c^4+a)^2+1/4*d/(b*c^4+a)^2*sum(
(4*b*d^3*c^3*_R^3+d^2*(15*b*c^4-a)*_R^2+4*c*d*(5*b*c^4-a)*_R+10*b
*c^6-6*a*c^2)/(_R^3*d^3+3*_R^2*c*d^2+3*_R*c^2*d+c^3)*ln(x-_R),_R=
RootOf(_Z^4*b*d^4+4*_Z^3*b*c*d^3+6*_Z^2*b*c^2*d^2+4*_Z*b*c^3*d+b*
c^4+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{4\,bc^3d\log\left(x\right)}{b^2c^8+2\,abc^4+a^2}+\frac{bd^2\int\frac{4\,bc^3d^3x^3+10\,bc^6+\left(15\,bc^4-a\right)d^2x^2-6\,ac^2+4\,\left(5\,bc^5-ac\right)dx}{bd^4x^4+4\,bcd^3x^3+6\,bc^2d^2x^2+4\,bc^3dx+bc^4+a}\,dx}{b^2c^8+2\,abc^4+a^2}-\frac{1}{\left(bc^4+a\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(((d*x + c)^4*b + a)*x^2),x, algorithm="maxima")
```

```
[Out] -4*b*c^3*d*log(x)/(b^2*c^8 + 2*a*b*c^4 + a^2) + b*d^2*integrate((
4*b*c^3*d^3*x^3 + 10*b*c^6 + (15*b*c^4 - a)*d^2*x^2 - 6*a*c^2 + 4
*(5*b*c^5 - a*c)*d*x)/(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^
2 + 4*b*c^3*d*x + b*c^4 + a), x)/(b^2*c^8 + 2*a*b*c^4 + a^2) - 1/
((b*c^4 + a)*x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(((d*x + c)^4*b + a)*x^2),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(a+b*(d*x+c)**4),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(\left(dx+c\right)^4b+a\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(((d*x + c)^4*b + a)*x^2),x, algorithm="giac")
```

```
[Out] integrate(1/(((d*x + c)^4*b + a)*x^2), x)
```

3.116 $\int \left(a+8x-8x^2+4x^3-x^4\right)^4 \, dx$

Optimal. Leaf size=123

$$-\frac{2}{9}\left(-3a^2+6a+37\right)(x-1)^9+\frac{4}{13}(3-a)(x-1)^{13}-\frac{8}{11}(3a+5)(x-1)^{11}+\frac{8}{7}(a+3)(3a+5)(x-1)^7+\frac{4}{5}(3-a)(a+3)^2(x-1)^5-\frac{8}{3}(a+3)^3(x-1)^3+(a+3)^4x+\frac{1}{17}(x-1)^{17}+\frac{8}{15}(x-1)^{15}$$

```
[Out] (-8*(3 + a)^3*(-1 + x)^3)/3 + (4*(3 - a)*(3 + a)^2*(-1 + x)^5)/5
+ (8*(3 + a)*(5 + 3*a)*(-1 + x)^7)/7 - (2*(37 + 6*a - 3*a^2)*(-1
+ x)^9)/9 - (8*(5 + 3*a)*(-1 + x)^11)/11 + (4*(3 - a)*(-1 + x)^13
)/13 + (8*(-1 + x)^15)/15 + (-1 + x)^17/17 + (3 + a)^4*x
```

Rubi [A] time = 0.424485, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{2}{9}\left(-3a^2+6a+37\right)(1-x)^9-\frac{4}{13}(3-a)(1-x)^{13}+\frac{8}{11}(3a+5)(1-x)^{11}-\frac{8}{7}(a+3)(3a+5)(1-x)^7-\frac{4}{5}(3-a)(a+3)^2(1-x)^5+\frac{8}{3}(a+3)^3(1-x)^3+(a+3)^4x-\frac{1}{17}(1-x)^{17}-\frac{8}{15}(1-x)^{15}$$

Antiderivative was successfully verified.

```
[In] Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x]
```

```
[Out] (8*(3 + a)^3*(1 - x)^3)/3 - (4*(3 - a)*(3 + a)^2*(1 - x)^5)/5 - (
8*(3 + a)*(5 + 3*a)*(1 - x)^7)/7 + (2*(37 + 6*a - 3*a^2)*(1 - x)^
9)/9 + (8*(5 + 3*a)*(1 - x)^11)/11 - (4*(3 - a)*(1 - x)^13)/13 -
(8*(1 - x)^15)/15 - (1 - x)^17/17 + (3 + a)^4*x
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$81x+\frac{4(-a+3)(a+3)^2(x-1)^5}{5}+\left(-\frac{4a}{13}+\frac{12}{13}\right)(x-1)^{13}-\frac{8(a+3)^3(x-1)^3}{3}+\frac{8(a+3)(3a+5)(x-1)^7}{7}+(a+6)\left(a^2+6a+18\right)\int^{x-1} a\,dx-\left(\frac{24a}{11}+\frac{40}{11}\right)(x-1)^{11}+\frac{(x-1)^{17}}{17}+\frac{8(x-1)^{15}}{15}-(x-1)^9\left(-\frac{2a^2}{3}+\frac{4a}{3}+\frac{74}{9}\right)-81$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-x**4+4*x**3-8*x**2+a+8*x)**4,x)
```

```
[Out] 81*x + 4*(-a + 3)*(a + 3)**2*(x - 1)**5/5 + (-4*a/13 + 12/13)*(x
- 1)**13 - 8*(a + 3)**3*(x - 1)**3/3 + 8*(a + 3)*(3*a + 5)*(x - 1
)**7/7 + (a + 6)*(a**2 + 6*a + 18)*Integral(a, (x, x - 1)) - (24*
a/11 + 40/11)*(x - 1)**11 + (x - 1)**17/17 + 8*(x - 1)**15/15 - (
x - 1)**9*(-2*a**2/3 + 4*a/3 + 74/9) - 81
```

Mathematica [A] time = 0.0474637, size = 195, normalized size = 1.59

$$a^4x + 16a^3x^2 + \frac{2}{9}\left(3a^2 - 1536a + 20480\right)x^9 - 6\left(a^2 - 128a + 896\right)x^8 + \frac{64}{7}\left(3a^2 - 140a + 512\right)x^7 - \frac{16}{3}\left(15a^2 - 288a + 512\right)x^6 + 4a\left(a^2 - 48a + 128\right)x^4 - \frac{32}{3}(a-12)a^2x^3 - \frac{4}{5}\left(a^3 - 192a^2 + 1536a - 1024\right)x^5 - \frac{4}{13}(a-640)x^{13} + \frac{4}{3}(3a-464)x^{12} - \frac{32}{11}(9a-524)x^{11} + \frac{16}{5}(35a-928)x^{10} + \frac{x^{17}}{17} - x^{16} + \frac{128x^{15}}{15} - 48x^{14}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x]
```

```
[Out] a^4*x + 16*a^3*x^2 - (32*(-12 + a)*a^2*x^3)/3 + 4*a*(128 - 48*a +
 a^2)*x^4 - (4*(-1024 + 1536*a - 192*a^2 + a^3)*x^5)/5 - (16*(512
 - 288*a + 15*a^2)*x^6)/3 + (64*(512 - 140*a + 3*a^2)*x^7)/7 - 6*
(896 - 128*a + a^2)*x^8 + (2*(20480 - 1536*a + 3*a^2)*x^9)/9 + (1
6*(-928 + 35*a)*x^10)/5 - (32*(-524 + 9*a)*x^11)/11 + (4*(-464 +
3*a)*x^12)/3 - (4*(-640 + a)*x^13)/13 - 48*x^14 + (128*x^15)/15 -
 x^16 + x^17/17
```

Maple [B] time = 0.002, size = 264, normalized size = 2.2

$$\frac{x^{17}}{17} - x^{16} + \frac{128\,x^{15}}{15} - 48\,x^{14} + \frac{(-4\,a+2560)\,x^{13}}{13} + \frac{(48\,a-7424)\,x^{12}}{12} + \frac{(-288\,a+16768)\,x^{11}}{11} + \frac{(1120\,a-29696)\,x^{10}}{10} + \frac{\left(2\,a^2-2560\,a+24576+(-2\,a+128)^2\right)x^9}{9} + \frac{\left(-16\,a^2+3584\,a-10240+2\,(8\,a-128)(-2\,a+128)\right)x^8}{8} + \frac{\left(64\,a^2-2560\,a+2\,(-16\,a+64)(-2\,a+128)+(8\,a-128)^2\right)x^7}{7} + \frac{\left(-160\,a^2+32\,a(-2\,a+128)+2\,(-16\,a+64)(8\,a-128)\right)x^6}{6} + \frac{\left(2\,a^2(-2\,a+128)+32\,a(8\,a-128)+(-16\,a+64)^2\right)x^5}{5} + \frac{\left(2\,a^2(8\,a-128)+32\,a(-16\,a+64)\right)x^4}{4} + \frac{\left(2\,a^2(-16\,a+64)+256\,a^2\right)x^3}{3} + 16\,x^2a^3 + a^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^4+4*x^3-8*x^2+a+8*x)^4,x)
```

```
[Out] 1/17*x^17-x^16+128/15*x^15-48*x^14+1/13*(-4*a+2560)*x^13+1/12*(48
*a-7424)*x^12+1/11*(-288*a+16768)*x^11+1/10*(1120*a-29696)*x^10+1
/9*(2*a^2-2560*a+24576+(-2*a+128)^2)*x^9+1/8*(-16*a^2+3584*a-1024
0+2*(8*a-128)*(-2*a+128))*x^8+1/7*(64*a^2-2560*a+2*(-16*a+64)*(-2
*a+128)+(8*a-128)^2)*x^7+1/6*(-160*a^2+32*a*(-2*a+128)+2*(-16*a+6
4)*(8*a-128))*x^6+1/5*(2*a^2*(-2*a+128)+32*a*(8*a-128)+(-16*a+64)
^2)*x^5+1/4*(2*a^2*(8*a-128)+32*a*(-16*a+64))*x^4+1/3*(2*a^2*(-16
*a+64)+256*a^2)*x^3+16*x^2*a^3+a^4*x
```

Maxima [A] time = 0.819532, size = 259, normalized size = 2.11

$$\frac{1}{17}x^{17} - x^{16} + \frac{128}{15}x^{15} - 48x^{14} + \frac{2560}{13}x^{13} - \frac{1856}{3}x^{12} + \frac{16768}{11}x^{11} - \frac{14848}{5}x^{10} + \frac{40960}{9}x^9 - 5376x^8 + \frac{32768}{7}x^7 - \frac{8192}{3}x^6 + a^4x + \frac{4096}{5}x^5 - \frac{4}{15}\left(3x^5 - 15x^4 + 40x^3 - 60x^2\right)a^3 + \frac{2}{105}\left(35x^9 - 315x^8 + 1440x^7 - 4200x^6 + 8064x^5 - 10080x^4 + 6720x^3\right)a^2 - \frac{4}{2145}\left(165x^{13} - 2145x^{12} + 14040x^{11} - 60060x^{10} + 183040x^9 - 411840x^8 + 686400x^7 - 823680x^6 + 658944x^5 - 27456\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 - 4*x^3 + 8*x^2 - a - 8*x)^4,x, algorithm="maxima")
```

```
[Out] 1/17*x^17 - x^16 + 128/15*x^15 - 48*x^14 + 2560/13*x^13 - 1856/3*
x^12 + 16768/11*x^11 - 14848/5*x^10 + 40960/9*x^9 - 5376*x^8 + 32
768/7*x^7 - 8192/3*x^6 + a^4*x + 4096/5*x^5 - 4/15*(3*x^5 - 15*x^
4 + 40*x^3 - 60*x^2)*a^3 + 2/105*(35*x^9 - 315*x^8 + 1440*x^7 - 4
200*x^6 + 8064*x^5 - 10080*x^4 + 6720*x^3)*a^2 - 4/2145*(165*x^13
 - 2145*x^12 + 14040*x^11 - 60060*x^10 + 183040*x^9 - 411840*x^8
+ 686400*x^7 - 823680*x^6 + 658944*x^5 - 274560*x^4)*a
```

Fricas [A] time = 0.226096, size = 1, normalized size = 0.01

$$\frac{1}{17}x^{17} - x^{16} + \frac{128}{15}x^{15} - 48x^{14} - \frac{4}{13}x^{13}a + \frac{2560}{13}x^{13} + 4x^{12}a - \frac{1856}{3}x^{12} - \frac{288}{11}x^{11}a + \frac{16768}{11}x^{11} + 112x^{10}a + \frac{2}{3}x^9a^2 - \frac{14848}{5}x^{10} - \frac{1024}{3}x^9a - 6x^8a^2 + \frac{40960}{9}x^9 + 768x^8a + \frac{192}{7}x^7a^2 - 5376x^8 - 1280x^7a - 80x^6a^2 - \frac{4}{5}x^5a^3 + \frac{32768}{7}x^7 + 1536x^6a + \frac{768}{5}x^5a^2 + 4x^4a^3 - \frac{8192}{3}x^6 - \frac{6144}{5}x^5a - 192x^4a^2 - \frac{32}{3}x^3a^3 + \frac{4096}{5}x^5 + 512x^4a + 128x^3a^2 + 16x^2a^3 + xa^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 - 4*x^3 + 8*x^2 - a - 8*x)^4,x, algorithm="fricas")
```

```
[Out] 1/17*x^17 - x^16 + 128/15*x^15 - 48*x^14 - 4/13*x^13*a + 2560/13*
x^13 + 4*x^12*a - 1856/3*x^12 - 288/11*x^11*a + 16768/11*x^11 + 1
12*x^10*a + 2/3*x^9*a^2 - 14848/5*x^10 - 1024/3*x^9*a - 6*x^8*a^2
 + 40960/9*x^9 + 768*x^8*a + 192/7*x^7*a^2 - 5376*x^8 - 1280*x^7*
a - 80*x^6*a^2 - 4/5*x^5*a^3 + 32768/7*x^7 + 1536*x^6*a + 768/5*x
^5*a^2 + 4*x^4*a^3 - 8192/3*x^6 - 6144/5*x^5*a - 192*x^4*a^2 - 32
/3*x^3*a^3 + 4096/5*x^5 + 512*x^4*a + 128*x^3*a^2 + 16*x^2*a^3 +
x*a^4
```

Sympy [A] time = 0.229779, size = 199, normalized size = 1.62

$$a^4x + 16a^3x^2 + \frac{x^{17}}{17} - x^{16} + \frac{128x^{15}}{15} - 48x^{14} + x^{13}\left(-\frac{4a}{13} + \frac{2560}{13}\right) + x^{12}\left(4a - \frac{1856}{3}\right) + x^{11}\left(-\frac{288a}{11} + \frac{16768}{11}\right) + x^{10}\left(112a - \frac{14848}{5}\right) + x^9\left(\frac{2a^2}{3} - \frac{1024a}{3} + \frac{40960}{9}\right) + x^8\left(-6a^2 + 768a - 5376\right) + x^7\left(\frac{192a^2}{7} - 1280a + \frac{32768}{7}\right) + x^6\left(-80a^2 + 1536a - \frac{8192}{3}\right) + x^5\left(-\frac{4a^3}{5} + \frac{768a^2}{5} - \frac{6144a}{5} + \frac{4096}{5}\right) + x^4\left(4a^3 - 192a^2 + 512a\right) + x^3\left(-\frac{32a^3}{3} + 128a^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**4+4*x**3-8*x**2+a+8*x)**4,x)
```

```
[Out] a**4*x + 16*a**3*x**2 + x**17/17 - x**16 + 128*x**15/15 - 48*x**1
4 + x**13*(-4*a/13 + 2560/13) + x**12*(4*a - 1856/3) + x**11*(-28
8*a/11 + 16768/11) + x**10*(112*a - 14848/5) + x**9*(2*a**2/3 - 1
024*a/3 + 40960/9) + x**8*(-6*a**2 + 768*a - 5376) + x**7*(192*a*
*2/7 - 1280*a + 32768/7) + x**6*(-80*a**2 + 1536*a - 8192/3) + x*
*5*(-4*a**3/5 + 768*a**2/5 - 6144*a/5 + 4096/5) + x**4*(4*a**3 -
192*a**2 + 512*a) + x**3*(-32*a**3/3 + 128*a**2)
```

GIAC/XCAS [A] time = 0.259512, size = 296, normalized size = 2.41

$$\frac{1}{17}x^{17} - x^{16} + \frac{128}{15}x^{15} - \frac{4}{13}ax^{13} - 48x^{14} + 4ax^{12} + \frac{2560}{13}x^{13} - \frac{288}{11}ax^{11} - \frac{1856}{3}x^{12} + \frac{2}{3}a^2x^9 + 112ax^{10} + \frac{16768}{11}x^{11} - 6a^2x^8 - \frac{1024}{3}ax^9 - \frac{14848}{5}x^{10} + \frac{192}{7}a^2x^7 + 768ax^8 + \frac{40960}{9}x^9 - \frac{4}{5}a^3x^5 - 80a^2x^6 - 1280ax^7 - 5376x^8 + 4a^3x^4 + \frac{768}{5}a^2x^5 + 1536ax^6 + \frac{32768}{7}x^7 - \frac{32}{3}a^3x^3 - 192a^2x^4 - \frac{6144}{5}ax^5 - \frac{8192}{3}x^6 + a^4x + 16a^3x^2 + 128a^2x^3 + 512ax^4 + \frac{4096}{5}x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 - 4*x^3 + 8*x^2 - a - 8*x)^4,x, algorithm="giac")
```

```
[Out] 1/17*x^17 - x^16 + 128/15*x^15 - 4/13*a*x^13 - 48*x^14 + 4*a*x^12
 + 2560/13*x^13 - 288/11*a*x^11 - 1856/3*x^12 + 2/3*a^2*x^9 + 112
*a*x^10 + 16768/11*x^11 - 6*a^2*x^8 - 1024/3*a*x^9 - 14848/5*x^10
 + 192/7*a^2*x^7 + 768*a*x^8 + 40960/9*x^9 - 4/5*a^3*x^5 - 80*a^2
*x^6 - 1280*a*x^7 - 5376*x^8 + 4*a^3*x^4 + 768/5*a^2*x^5 + 1536*a
*x^6 + 32768/7*x^7 - 32/3*a^3*x^3 - 192*a^2*x^4 - 6144/5*a*x^5 -
8192/3*x^6 + a^4*x + 16*a^3*x^2 + 128*a^2*x^3 + 512*a*x^4 + 4096/
5*x^5
```

3.117 $\int \left(a+8x-8x^2+4x^3-x^4\right)^3\,dx$

Optimal. Leaf size=120

$$a^3x-\frac{3}{5}\left(a^2-128a+512\right)x^5+\left(3a^2-96a+128\right)x^4+12a^2x^2-\frac{1}{3}(256-a)x^9+3(64-a)x^8-\frac{32}{7}(70-3a)x^7+8(48-5a)x^6+8(8-a)ax^3-\frac{x^{13}}{13}+x^{12}-\frac{72x^{11}}{11}+28x^{10}$$

```
[Out] a^3*x + 12*a^2*x^2 + 8*(8 - a)*a*x^3 + (128 - 96*a + 3*a^2)*x^4 -
 (3*(512 - 128*a + a^2)*x^5)/5 + 8*(48 - 5*a)*x^6 - (32*(70 - 3*a
)*x^7)/7 + 3*(64 - a)*x^8 - ((256 - a)*x^9)/3 + 28*x^10 - (72*x^1
1)/11 + x^12 - x^13/13
```

Rubi [A] time = 0.119372, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045

$$a^3x-\frac{3}{5}\left(a^2-128a+512\right)x^5+\left(3a^2-96a+128\right)x^4+12a^2x^2-\frac{1}{3}(256-a)x^9+3(64-a)x^8-\frac{32}{7}(70-3a)x^7+8(48-5a)x^6+8(8-a)ax^3-\frac{x^{13}}{13}+x^{12}-\frac{72x^{11}}{11}+28x^{10}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]
```

```
[Out] a^3*x + 12*a^2*x^2 + 8*(8 - a)*a*x^3 + (128 - 96*a + 3*a^2)*x^4 -
 (3*(512 - 128*a + a^2)*x^5)/5 + 8*(48 - 5*a)*x^6 - (32*(70 - 3*a
)*x^7)/7 + 3*(64 - a)*x^8 - ((256 - a)*x^9)/3 + 28*x^10 - (72*x^1
1)/11 + x^12 - x^13/13
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**4+4*x**3-8*x**2+a+8*x)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.0255542, size = 114, normalized size = 0.95

$$a^3x-\frac{3}{5}\left(a^2-128a+512\right)x^5+\left(3a^2-96a+128\right)x^4+12a^2x^2+\frac{1}{3}(a-256)x^9-3(a-64)x^8+\frac{32}{7}(3a-70)x^7-8(5a-48)x^6-8(a-8)ax^3-\frac{x^{13}}{13}+x^{12}-\frac{72x^{11}}{11}+28x^{10}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]
```

```
[Out] a^3*x + 12*a^2*x^2 - 8*(-8 + a)*a*x^3 + (128 - 96*a + 3*a^2)*x^4
- (3*(512 - 128*a + a^2)*x^5)/5 - 8*(-48 + 5*a)*x^6 + (32*(-70 +
3*a)*x^7)/7 - 3*(-64 + a)*x^8 + ((-256 + a)*x^9)/3 + 28*x^10 - (7
```

```
2*x^11)/11 + x^12 - x^13/13
```

Maple [A] time = 0.002, size = 138, normalized size = 1.2

$$-\frac{x^{13}}{13}+x^{12}-\frac{72\,x^{11}}{11}+28\,x^{10}+\frac{(3\,a-768)\,x^{9}}{9}+\frac{(-24\,a+1536)\,x^{8}}{8}+\frac{(96\,a-2240)\,x^{7}}{7}+\frac{(-240\,a+2304)\,x^{6}}{6}+\frac{\left(a\,(-2\,a+128)+256\,a-1536-a^{2}\right)\,x^{5}}{5}+\frac{\left(a\,(8\,a-128)-256\,a+512+4\,a^{2}\right)\,x^{4}}{4}+\frac{\left(a\,(-16\,a+64)+128\,a-8\,a^{2}\right)\,x^{3}}{3}+12\,a^{2}x^{2}+a^{3}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^4+4*x^3-8*x^2+a+8*x)^3,x)
```

```
[Out] -1/13*x^13+x^12-72/11*x^11+28*x^10+1/9*(3*a-768)*x^9+1/8*(-24*a+1
536)*x^8+1/7*(96*a-2240)*x^7+1/6*(-240*a+2304)*x^6+1/5*(a*(-2*a+1
28)+256*a-1536-a^2)*x^5+1/4*(a*(8*a-128)-256*a+512+4*a^2)*x^4+1/3
*(a*(-16*a+64)+128*a-8*a^2)*x^3+12*a^2*x^2+a^3*x
```

Maxima [A] time = 0.798777, size = 161, normalized size = 1.34

$$-\frac{1}{13}\,x^{13}+x^{12}-\frac{72}{11}\,x^{11}+28\,x^{10}-\frac{256}{3}\,x^{9}+192\,x^{8}-320\,x^{7}+384\,x^{6}-\frac{1536}{5}\,x^{5}+a^{3}x+128\,x^{4}-\frac{1}{5}\left(3\,x^{5}-15\,x^{4}+40\,x^{3}-60\,x^{2}\right)a^{2}+\frac{1}{105}\left(35\,x^{9}-315\,x^{8}+1440\,x^{7}-4200\,x^{6}+8064\,x^{5}-10080\,x^{4}+6720\,x^{3}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 4*x^3 + 8*x^2 - a - 8*x)^3,x, algorithm="maxima")
```

```
[Out] -1/13*x^13 + x^12 - 72/11*x^11 + 28*x^10 - 256/3*x^9 + 192*x^8 -
320*x^7 + 384*x^6 - 1536/5*x^5 + a^3*x + 128*x^4 - 1/5*(3*x^5 - 1
5*x^4 + 40*x^3 - 60*x^2)*a^2 + 1/105*(35*x^9 - 315*x^8 + 1440*x^7
 - 4200*x^6 + 8064*x^5 - 10080*x^4 + 6720*x^3)*a
```

Fricas [A] time = 0.239023, size = 1, normalized size = 0.01

$$-\frac{1}{13}x^{13}+x^{12}-\frac{72}{11}x^{11}+28x^{10}+\frac{1}{3}x^{9}a-\frac{256}{3}x^{9}-3x^{8}a+192x^{8}+\frac{96}{7}x^{7}a-320x^{7}-40x^{6}a-\frac{3}{5}x^{5}a^{2}+384x^{6}+\frac{384}{5}x^{5}a+3x^{4}a^{2}-\frac{1536}{5}x^{5}-96x^{4}a-8x^{3}a^{2}+128x^{4}+64x^{3}a+12x^{2}a^{2}+xa^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 4*x^3 + 8*x^2 - a - 8*x)^3,x, algorithm="fricas")
```

```
[Out] -1/13*x^13 + x^12 - 72/11*x^11 + 28*x^10 + 1/3*x^9*a - 256/3*x^9
- 3*x^8*a + 192*x^8 + 96/7*x^7*a - 320*x^7 - 40*x^6*a - 3/5*x^5*a
^2 + 384*x^6 + 384/5*x^5*a + 3*x^4*a^2 - 1536/5*x^5 - 96*x^4*a -
8*x^3*a^2 + 128*x^4 + 64*x^3*a + 12*x^2*a^2 + x*a^3
```

Sympy [A] time = 0.164499, size = 114, normalized size = 0.95

$$a^3x + 12a^2x^2 - \frac{x^{13}}{13} + x^{12} - \frac{72x^{11}}{11} + 28x^{10} + x^9\left(\frac{a}{3} - \frac{256}{3}\right) + x^8(-3a + 192) + x^7\left(\frac{96a}{7} - 320\right) + x^6(-40a + 384) + x^5\left(-\frac{3a^2}{5} + \frac{384a}{5} - \frac{1536}{5}\right) + x^4\left(3a^2 - 96a + 128\right) + x^3\left(-8a^2 + 64a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**4+4*x**3-8*x**2+a+8*x)**3,x)
```

```
[Out] a**3*x + 12*a**2*x**2 - x**13/13 + x**12 - 72*x**11/11 + 28*x**10
 + x**9*(a/3 - 256/3) + x**8*(-3*a + 192) + x**7*(96*a/7 - 320) +
 x**6*(-40*a + 384) + x**5*(-3*a**2/5 + 384*a/5 - 1536/5) + x**4*
(3*a**2 - 96*a + 128) + x**3*(-8*a**2 + 64*a)
```

GIAC/XCAS [A] time = 0.258632, size = 173, normalized size = 1.44

$$-\frac{1}{13}x^{13} + x^{12} - \frac{72}{11}x^{11} + \frac{1}{3}ax^9 + 28x^{10} - 3ax^8 - \frac{256}{3}x^9 + \frac{96}{7}ax^7 + 192x^8 - \frac{3}{5}a^2x^5 - 40ax^6 - 320x^7 + 3a^2x^4 + \frac{384}{5}ax^5 + 384x^6 - 8a^2x^3 - 96ax^4 - \frac{1536}{5}x^5 + a^3x + 12a^2x^2 + 64ax^3 + 128x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 4*x^3 + 8*x^2 - a - 8*x)^3,x, algorithm="giac")
```

```
[Out] -1/13*x^13 + x^12 - 72/11*x^11 + 1/3*a*x^9 + 28*x^10 - 3*a*x^8 -
256/3*x^9 + 96/7*a*x^7 + 192*x^8 - 3/5*a^2*x^5 - 40*a*x^6 - 320*x
^7 + 3*a^2*x^4 + 384/5*a*x^5 + 384*x^6 - 8*a^2*x^3 - 96*a*x^4 - 1
536/5*x^5 + a^3*x + 12*a^2*x^2 + 64*a*x^3 + 128*x^4
```

3.118 $\int \left(a+8x-8x^2+4x^3-x^4\right)^2\,dx$

Optimal. Leaf size=72

$$a^2x+\frac{2}{5}(64-a)x^5-2(16-a)x^4+\frac{16}{3}(4-a)x^3+8ax^2+\frac{x^9}{9}-x^8+\frac{32x^7}{7}-\frac{40x^6}{3}$$

```
[Out] a^2*x + 8*a*x^2 + (16*(4 - a)*x^3)/3 - 2*(16 - a)*x^4 + (2*(64 -
a)*x^5)/5 - (40*x^6)/3 + (32*x^7)/7 - x^8 + x^9/9
```

Rubi [A] time = 0.0600135, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045

$$a^2x+\frac{2}{5}(64-a)x^5-2(16-a)x^4+\frac{16}{3}(4-a)x^3+8ax^2+\frac{x^9}{9}-x^8+\frac{32x^7}{7}-\frac{40x^6}{3}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

```
[Out] a^2*x + 8*a*x^2 + (16*(4 - a)*x^3)/3 - 2*(16 - a)*x^4 + (2*(64 -
a)*x^5)/5 - (40*x^6)/3 + (32*x^7)/7 - x^8 + x^9/9
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**4+4*x**3-8*x**2+a+8*x)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.0140277, size = 66, normalized size = 0.92

$$a^2x-\frac{2}{5}(a-64)x^5+2(a-16)x^4-\frac{16}{3}(a-4)x^3+8ax^2+\frac{x^9}{9}-x^8+\frac{32x^7}{7}-\frac{40x^6}{3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

```
[Out] a^2*x + 8*a*x^2 - (16*(-4 + a)*x^3)/3 + 2*(-16 + a)*x^4 - (2*(-64
 + a)*x^5)/5 - (40*x^6)/3 + (32*x^7)/7 - x^8 + x^9/9
```

Maple [A] time = 0.001, size = 63, normalized size = 0.9

$$\frac{x^9}{9}-x^8+\frac{32\,x^7}{7}-\frac{40\,x^6}{3}+\frac{(-2\,a+128)\,x^5}{5}+\frac{(8\,a-128)\,x^4}{4}+\frac{(-16\,a+64)\,x^3}{3}+8\,ax^2+a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((-x^4+4*x^3-8*x^2+a+8*x)^2,x)
```

```
[Out] 1/9*x^9-x^8+32/7*x^7-40/3*x^6+1/5*(-2*a+128)*x^5+1/4*(8*a-128)*x^
4+1/3*(-16*a+64)*x^3+8*a*x^2+a^2*x
```

Maxima [A] time = 0.802978, size = 88, normalized size = 1.22

$$\frac{1}{9}\,x^9 - x^8 + \frac{32}{7}\,x^7 - \frac{40}{3}\,x^6 + \frac{128}{5}\,x^5 - 32\,x^4 + a^2x + \frac{64}{3}\,x^3 - \frac{2}{15}\left(3\,x^5 - 15\,x^4 + 40\,x^3 - 60\,x^2\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((x^4 - 4*x^3 + 8*x^2 - a - 8*x)^2,x, algorithm="maxima")
```

```
[Out] 1/9*x^9 - x^8 + 32/7*x^7 - 40/3*x^6 + 128/5*x^5 - 32*x^4 + a^2*x
+ 64/3*x^3 - 2/15*(3*x^5 - 15*x^4 + 40*x^3 - 60*x^2)*a
```

Fricas [A] time = 0.259592, size = 1, normalized size = 0.01

$$\frac{1}{9}x^9 - x^8 + \frac{32}{7}x^7 - \frac{40}{3}x^6 - \frac{2}{5}x^5a + \frac{128}{5}x^5 + 2x^4a - 32x^4 - \frac{16}{3}x^3a + \frac{64}{3}x^3 + 8x^2a + xa^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((x^4 - 4*x^3 + 8*x^2 - a - 8*x)^2,x, algorithm="fricas")
```

```
[Out] 1/9*x^9 - x^8 + 32/7*x^7 - 40/3*x^6 - 2/5*x^5*a + 128/5*x^5 + 2*x
^4*a - 32*x^4 - 16/3*x^3*a + 64/3*x^3 + 8*x^2*a + x*a^2
```

Sympy [A] time = 0.106636, size = 65, normalized size = 0.9

$$a^2x + 8ax^2 + \frac{x^9}{9} - x^8 + \frac{32x^7}{7} - \frac{40x^6}{3} + x^5\left(-\frac{2a}{5} + \frac{128}{5}\right) + x^4\left(2a - 32\right) + x^3\left(-\frac{16a}{3} + \frac{64}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((-x**4+4*x**3-8*x**2+a+8*x)**2,x)
```

```
[Out] a**2*x + 8*a*x**2 + x**9/9 - x**8 + 32*x**7/7 - 40*x**6/3 + x**5*
(-2*a/5 + 128/5) + x**4*(2*a - 32) + x**3*(-16*a/3 + 64/3)
```

GIAC/XCAS [A] time = 0.262786, size = 88, normalized size = 1.22

$$\frac{1}{9}\,x^9 - x^8 + \frac{32}{7}\,x^7 - \frac{2}{5}\,ax^5 - \frac{40}{3}\,x^6 + 2\,ax^4 + \frac{128}{5}\,x^5 - \frac{16}{3}\,ax^3 - 32\,x^4 + a^2x + 8\,ax^2 + \frac{64}{3}\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((x^4 - 4*x^3 + 8*x^2 - a - 8*x)^2,x, algorithm="giac")
```

```
[Out] 1/9*x^9 - x^8 + 32/7*x^7 - 2/5*a*x^5 - 40/3*x^6 + 2*a*x^4 + 128/5
*x^5 - 16/3*a*x^3 - 32*x^4 + a^2*x + 8*a*x^2 + 64/3*x^3
```

3.119 $\int \left(a+8x-8x^2+4x^3-x^4\right)\,dx$

Optimal. Leaf size=26

$$ax-\frac{x^5}{5}+x^4-\frac{8x^3}{3}+4x^2$$

```
[Out] a*x + 4*x^2 - (8*x^3)/3 + x^4 - x^5/5
```

Rubi [A] time = 0.0115341, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.

$$ax-\frac{x^5}{5}+x^4-\frac{8x^3}{3}+4x^2$$

Antiderivative was successfully verified.

```
[In] Int[a + 8*x - 8*x^2 + 4*x^3 - x^4,x]

[Out] a*x + 4*x^2 - (8*x^3)/3 + x^4 - x^5/5
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{x^5}{5}+x^4-\frac{8x^3}{3}+\int a\,dx+8\int x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(-x**4+4*x**3-8*x**2+a+8*x,x)

[Out] -x**5/5 + x**4 - 8*x**3/3 + Integral(a, x) + 8*Integral(x, x)
```

Mathematica [A] time = 0.0000591969, size = 26, normalized size = 1.

$$ax-\frac{x^5}{5}+x^4-\frac{8x^3}{3}+4x^2$$

Antiderivative was successfully verified.

```
[In] Integrate[a + 8*x - 8*x^2 + 4*x^3 - x^4,x]

[Out] a*x + 4*x^2 - (8*x^3)/3 + x^4 - x^5/5
```

Maple [A] time = 0.001, size = 23, normalized size = 0.9

$$ax+4\,x^2-\frac{8\,x^3}{3}+x^4-\frac{x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^4+4*x^3-8*x^2+a+8*x,x)
```

```
[Out] a*x+4*x^2-8/3*x^3+x^4-1/5*x^5
```

Maxima [A] time = 0.802742, size = 30, normalized size = 1.15

$$-\frac{1}{5}x^5 + x^4 - \frac{8}{3}x^3 + ax + 4x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x^4 + 4*x^3 - 8*x^2 + a + 8*x,x, algorithm="maxima")
```

```
[Out] -1/5*x^5 + x^4 - 8/3*x^3 + a*x + 4*x^2
```

Fricas [A] time = 0.247975, size = 1, normalized size = 0.04

$$-\frac{1}{5}x^5 + x^4 - \frac{8}{3}x^3 + 4x^2 + xa$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x^4 + 4*x^3 - 8*x^2 + a + 8*x,x, algorithm="fricas")
```

```
[Out] -1/5*x^5 + x^4 - 8/3*x^3 + 4*x^2 + x*a
```

Sympy [A] time = 0.066972, size = 22, normalized size = 0.85

$$ax - \frac{x^5}{5} + x^4 - \frac{8x^3}{3} + 4x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x**4+4*x**3-8*x**2+a+8*x,x)
```

```
[Out] a*x - x**5/5 + x**4 - 8*x**3/3 + 4*x**2
```

GIAC/XCAS [A] time = 0.261776, size = 30, normalized size = 1.15

$$-\frac{1}{5}x^5 + x^4 - \frac{8}{3}x^3 + ax + 4x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x^4 + 4*x^3 - 8*x^2 + a + 8*x,x, algorithm="giac")
```

```
[Out] -1/5*x^5 + x^4 - 8/3*x^3 + a*x + 4*x^2
```

3.120 $\int \frac{1}{a+8x-8x^2+4x^3-x^4}\,dx$

Optimal. Leaf size=89

$$\frac{\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{2\sqrt{a+4}\sqrt{\sqrt{a+4}+1}}-\frac{\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{2\sqrt{a+4}\sqrt{1-\sqrt{a+4}}}$$

```
[Out] -ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4 + a]]]/(2*Sqrt[4 + a]*Sqrt[1 - S
qrt[4 + a]]) + ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]]/(2*Sqrt[4 +
 a]*Sqrt[1 + Sqrt[4 + a]])
```

Rubi [A] time = 0.17455, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$\frac{\tan^{-1}\left(\frac{1-x}{\sqrt{1-\sqrt{a+4}}}\right)}{2\sqrt{a+4}\sqrt{1-\sqrt{a+4}}}-\frac{\tan^{-1}\left(\frac{1-x}{\sqrt{\sqrt{a+4}+1}}\right)}{2\sqrt{a+4}\sqrt{\sqrt{a+4}+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-1),x]
```

```
[Out] ArcTan[(1 - x)/Sqrt[1 - Sqrt[4 + a]]]/(2*Sqrt[4 + a]*Sqrt[1 - Sqr
t[4 + a]]) - ArcTan[(1 - x)/Sqrt[1 + Sqrt[4 + a]]]/(2*Sqrt[4 + a]
*Sqrt[1 + Sqrt[4 + a]])
```

Rubi in Sympy [A] time = 31.2683, size = 73, normalized size = 0.82

$$\frac{\operatorname{atan}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{2\sqrt{a+4}\sqrt{\sqrt{a+4}+1}}-\frac{\operatorname{atan}\left(\frac{x-1}{\sqrt{-\sqrt{a+4}+1}}\right)}{2\sqrt{a+4}\sqrt{-\sqrt{a+4}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-x**4+4*x**3-8*x**2+a+8*x),x)
```

```
[Out] atan((x - 1)/sqrt(sqrt(a + 4) + 1))/(2*sqrt(a + 4)*sqrt(sqrt(a +
4) + 1)) - atan((x - 1)/sqrt(-sqrt(a + 4) + 1))/(2*sqrt(a + 4)*sq
rt(-sqrt(a + 4) + 1))
```

Mathematica [C] time = 0.023153, size = 57, normalized size = 0.64

$$-\frac{1}{4}\text{RootSum}\left[-\#1^4+4\#1^3-8\#1^2+8\#1+a\&,\frac{\log(x-\#1)}{\#1^3-3\#1^2+4\#1-2}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-1),x]
```

```
[Out] -RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 & , Log[x - #1]/(-2 +
4*#1 - 3*#1^2 + #1^3) & ]/4
```

Maple [C] time = 0.023, size = 49, normalized size = 0.6

$$-\frac{1}{4}\left(\sum_{\_R=RootOf\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)}\frac{\ln\left(x-\_R\right)}{\_R^3-3\_R^2+4\_R-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(1/(-x^4+4*x^3-8*x^2+a+8*x),x)
```

```
[Out] -1/4*sum(1/(_R^3-3*_R^2+4*_R-2)*ln(x-_R),_R=RootOf(_Z^4-4*_Z^3+8*
_Z^2-8*_Z-a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{1}{x^4-4x^3+8x^2-a-8x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(-1/(x^4 - 4*x^3 + 8*x^2 - a - 8*x),x, algorithm="maxima")
```

```
[Out] -integrate(1/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)
```

Fricas [A] time = 0.270787, size = 617, normalized size = 6.93

$$\frac{1}{4}\sqrt{\frac{\frac{a^2+7a+12}{\sqrt{a^3+10a^2+33a+36}}+1}{a^2+7a+12}}\log\left(\left(a-\frac{a^2+7a+12}{\sqrt{a^3+10a^2+33a+36}}+4\right)\sqrt{\frac{\frac{a^2+7a+12}{\sqrt{a^3+10a^2+33a+36}}+1}{a^2+7a+12}}+x-1\right)$$
$$-\frac{1}{4}\sqrt{\frac{\frac{a^2+7a+12}{\sqrt{a^3+10a^2+33a+36}}+1}{a^2+7a+12}}\log\left(-\left(a-\frac{a^2+7a+12}{\sqrt{a^3+10a^2+33a+36}}+4\right)\sqrt{\frac{\frac{a^2+7a+12}{\sqrt{a^3+10a^2+33a+36}}+1}{a^2+7a+12}}+x-1\right)$$
$$+\frac{1}{4}\sqrt{-\frac{\frac{a^2+7a+12}{\sqrt{a^3+10a^2+33a+36}}-1}{a^2+7a+12}}\log\left(\left(a+\frac{a^2+7a+12}{\sqrt{a^3+10a^2+33a+36}}+4\right)\sqrt{-\frac{\frac{a^2+7a+12}{\sqrt{a^3+10a^2+33a+36}}-1}{a^2+7a+12}}+x-1\right)$$
$$-\frac{1}{4}\sqrt{-\frac{\frac{a^2+7a+12}{\sqrt{a^3+10a^2+33a+36}}-1}{a^2+7a+12}}\log\left(-\left(a+\frac{a^2+7a+12}{\sqrt{a^3+10a^2+33a+36}}+4\right)\sqrt{-\frac{\frac{a^2+7a+12}{\sqrt{a^3+10a^2+33a+36}}-1}{a^2+7a+12}}+x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(-1/(x^4 - 4*x^3 + 8*x^2 - a - 8*x),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(((a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) + 1)/(a
^2 + 7*a + 12))*log((a - (a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*
a + 36) + 4)*sqrt(((a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36
) + 1)/(a^2 + 7*a + 12)) + x - 1) - 1/4*sqrt(((a^2 + 7*a + 12)/sq
rt(a^3 + 10*a^2 + 33*a + 36) + 1)/(a^2 + 7*a + 12))*log(-(a - (a^
2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) + 4)*sqrt(((a^2 + 7*
```

```
a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) + 1)/(a^2 + 7*a + 12)) + x
 - 1) + 1/4*sqrt(-((a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36
) - 1)/(a^2 + 7*a + 12))*log((a + (a^2 + 7*a + 12)/sqrt(a^3 + 10*
a^2 + 33*a + 36) + 4)*sqrt(-((a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 +
 33*a + 36) - 1)/(a^2 + 7*a + 12)) + x - 1) - 1/4*sqrt(-((a^2 + 7
*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) - 1)/(a^2 + 7*a + 12))*lo
g(-(a + (a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) + 4)*sqrt
(-((a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) - 1)/(a^2 + 7*
a + 12)) + x - 1)
```

Sympy [A] time = 2.27999, size = 66, normalized size = 0.74

$$-\operatorname{RootSum}\left(t^4\left(256a^3+2816a^2+10240a+12288\right)+t^2(-32a-128)-1,\left(t\mapsto t\log\left(64t^3a^2+448t^3a+768t^3-4ta-20t\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-x**4+4*x**3-8*x**2+a+8*x),x)
```

```
[Out] -RootSum(_t**4*(256*a**3 + 2816*a**2 + 10240*a + 12288) + _t**2*(
-32*a - 128) - 1, Lambda(_t, _t*log(64*_t**3*a**2 + 448*_t**3*a +
 768*_t**3 - 4*_t*a - 20*_t + x - 1)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{x^4-4x^3+8x^2-a-8x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(x^4 - 4*x^3 + 8*x^2 - a - 8*x),x, algorithm="giac")
```

```
[Out] integrate(-1/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)
```

3.121 $\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^2}\,dx$

Optimal. Leaf size=169

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{4\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)}-\frac{\left(3a+\sqrt{a+4}+10\right)\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{8(a+3)(a+4)^{3/2}\sqrt{1-\sqrt{a+4}}}+\frac{\left(3a-\sqrt{a+4}+10\right)\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{8(a+3)(a+4)^{3/2}\sqrt{\sqrt{a+4}+1}}$$

```
[Out] ((5 + a + (-1 + x)^2)*(-1 + x))/(4*(12 + 7*a + a^2)*(3 + a - 2*(-
1 + x)^2 - (-1 + x)^4)) - ((10 + 3*a + Sqrt[4 + a])*ArcTan[(-1 +
x)/Sqrt[1 - Sqrt[4 + a]]])/(8*(3 + a)*(4 + a)^(3/2)*Sqrt[1 - Sqrt
[4 + a]]) + ((10 + 3*a - Sqrt[4 + a])*ArcTan[(-1 + x)/Sqrt[1 + Sq
rt[4 + a]]])/(8*(3 + a)*(4 + a)^(3/2)*Sqrt[1 + Sqrt[4 + a]])
```

Rubi [A] time = 0.627456, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$-\frac{(1-x)\left(a+(x-1)^2+5\right)}{4\left(a^2+7a+12\right)\left(a-(1-x)^4-2(1-x)^2+3\right)}+\frac{\left(3a+\sqrt{a+4}+10\right)\tan^{-1}\left(\frac{1-x}{\sqrt{1-\sqrt{a+4}}}\right)}{8(a+3)(a+4)^{3/2}\sqrt{1-\sqrt{a+4}}}-\frac{\left(3a-\sqrt{a+4}+10\right)\tan^{-1}\left(\frac{1-x}{\sqrt{\sqrt{a+4}+1}}\right)}{8(a+3)(a+4)^{3/2}\sqrt{\sqrt{a+4}+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-2),x]
```

```
[Out] -((5 + a + (-1 + x)^2)*(1 - x))/(4*(12 + 7*a + a^2)*(3 + a - 2*(1
 - x)^2 - (1 - x)^4)) + ((10 + 3*a + Sqrt[4 + a])*ArcTan[(1 - x)/
Sqrt[1 - Sqrt[4 + a]]])/(8*(3 + a)*(4 + a)^(3/2)*Sqrt[1 - Sqrt[4
+ a]]) - ((10 + 3*a - Sqrt[4 + a])*ArcTan[(1 - x)/Sqrt[1 + Sqrt[4
 + a]]])/(8*(3 + a)*(4 + a)^(3/2)*Sqrt[1 + Sqrt[4 + a]])
```

Rubi in Sympy [A] time = 58.9326, size = 143, normalized size = 0.85

$$\frac{(x-1)\left(2a+2(x-1)^2+10\right)}{8(a+3)(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)}+\frac{\left(3a-\sqrt{a+4}+10\right)\operatorname{atan}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{8(a+3)(a+4)^{\frac{3}{2}}\sqrt{\sqrt{a+4}+1}}-\frac{\left(3a+\sqrt{a+4}+10\right)\operatorname{atan}\left(\frac{x-1}{\sqrt{-\sqrt{a+4}+1}}\right)}{8(a+3)(a+4)^{\frac{3}{2}}\sqrt{-\sqrt{a+4}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-x**4+4*x**3-8*x**2+a+8*x)**2,x)
```

```
[Out] (x - 1)*(2*a + 2*(x - 1)**2 + 10)/(8*(a + 3)*(a + 4)*(a - (x - 1)
**4 - 2*(x - 1)**2 + 3)) + (3*a - sqrt(a + 4) + 10)*atan((x - 1)/
sqrt(sqrt(a + 4) + 1))/(8*(a + 3)*(a + 4)**(3/2)*sqrt(sqrt(a + 4)
 + 1)) - (3*a + sqrt(a + 4) + 10)*atan((x - 1)/sqrt(-sqrt(a + 4)
+ 1))/(8*(a + 3)*(a + 4)**(3/2)*sqrt(-sqrt(a + 4) + 1))
```

Mathematica [C] time = 0.0804479, size = 150, normalized size = 0.89

$$\frac{(x-1)\left(a+x^2-2x+6\right)}{4(a+3)(a+4)\left(a-x\left(x^3-4x^2+8x-8\right)\right)} - \frac{\text{RootSum}\left[-\#1^4+4\#1^3-8\#1^2+8\#1+a\&,\frac{\#1^2\log(x-\#1)+3a\log(x-\#1)-2\#1\log(x-\#1)+12\log(x-\#1)}{\#1^3-3\#1^2+4\#1-2}\&\right]}{16\left(a^2+7a+12\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-2),x]
```

```
[Out] ((-1 + x)*(6 + a - 2*x + x^2))/(4*(3 + a)*(4 + a)*(a - x*(-8 + 8*
x - 4*x^2 + x^3))) - RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 &
, (12*Log[x - #1] + 3*a*Log[x - #1] - 2*Log[x - #1]*#1 + Log[x -
#1]*#1^2)/(-2 + 4*#1 - 3*#1^2 + #1^3) & ]/(16*(12 + 7*a + a^2))
```

Maple [C] time = 0.029, size = 158, normalized size = 0.9

$$\frac{1}{x^4-4x^3+8x^2-a-8x}\left(-\frac{x^3}{4a^2+28a+48}+\frac{3x^2}{4a^2+28a+48}-\frac{(8+a)x}{4a^2+28a+48}+\frac{6+a}{4a^2+28a+48}\right) + \frac{1}{16}\sum_{\_R=RootOf(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a)}\frac{\left(-\_R^2+2\_R-3a-12\right)\ln(x-\_R)}{\left(\_R^3-3\_R^2+4\_R-2\right)(4+a)(3+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-x^4+4*x^3-8*x^2+a+8*x)^2,x)
```

```
[Out] (-1/4/(a^2+7*a+12)*x^3+3/4/(a^2+7*a+12)*x^2-1/4*(8+a)/(a^2+7*a+12
)*x+1/4*(6+a)/(a^2+7*a+12))/(x^4-4*x^3+8*x^2-a-8*x)+1/16*sum((-_R
^2+2*_R-3*a-12)/(_R^3-3*_R^2+4*_R-2)/(4+a)/(3+a)*ln(x-_R),_R=Root
Of(_Z^4-4*_Z^3+8*_Z^2-8*_Z-a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{x^3+(a+8)x-3x^2-a-6}{4\left((a^2+7a+12)x^4-4(a^2+7a+12)x^3-a^3+8(a^2+7a+12)x^2-7a^2-8(a^2+7a+12)x-12a\right)} - \frac{\int\frac{x^2+3a-2x+12}{x^4-4x^3+8x^2-a-8x}\,dx}{4\left(a^2+7a+12\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 - 4*x^3 + 8*x^2 - a - 8*x)^(-2),x, algorithm="maxima")
```

```
[Out] -1/4*(x^3 + (a + 8)*x - 3*x^2 - a - 6)/((a^2 + 7*a + 12)*x^4 - 4*
(a^2 + 7*a + 12)*x^3 - a^3 + 8*(a^2 + 7*a + 12)*x^2 - 7*a^2 - 8*(
a^2 + 7*a + 12)*x - 12*a) - 1/4*integrate((x^2 + 3*a - 2*x + 12)/
(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)/(a^2 + 7*a + 12)
```

Fricas [A] time = 0.286674, size = 2630, normalized size = 15.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 - 4*x^3 + 8*x^2 - a - 8*x)^(-2),x, algorithm="fricas")
```

```
[Out] -1/16*(4*x^3 - ((a^2 + 7*a + 12)*x^4 - 4*(a^2 + 7*a + 12)*x^3 - a
^3 + 8*(a^2 + 7*a + 12)*x^2 - 7*a^2 - 8*(a^2 + 7*a + 12)*x - 12*a
)*sqrt((15*a^2 + (a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3
024*a + 1728)*sqrt((81*a^2 + 558*a + 961)/(a^9 + 30*a^8 + 399*a^7
 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 111105*a^3 + 156492*a^2 + 1
28304*a + 46656)) + 105*a + 184)/(a^6 + 21*a^5 + 183*a^4 + 847*a^
3 + 2196*a^2 + 3024*a + 1728))*log(-81*a^2 + (81*a^2 + 567*a + 99
2)*x + (27*a^4 + 408*a^3 + 2309*a^2 - 2*(2*a^7 + 49*a^6 + 513*a^5
 + 2975*a^4 + 10321*a^3 + 21420*a^2 + 24624*a + 12096)*sqrt((81*a
^2 + 558*a + 961)/(a^9 + 30*a^8 + 399*a^7 + 3088*a^6 + 15327*a^5
+ 50598*a^4 + 111105*a^3 + 156492*a^2 + 128304*a + 46656)) + 5800
*a + 5456)*sqrt((15*a^2 + (a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 219
6*a^2 + 3024*a + 1728)*sqrt((81*a^2 + 558*a + 961)/(a^9 + 30*a^8
+ 399*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 111105*a^3 + 15649
2*a^2 + 128304*a + 46656)) + 105*a + 184)/(a^6 + 21*a^5 + 183*a^4
 + 847*a^3 + 2196*a^2 + 3024*a + 1728)) - 567*a - 992) + ((a^2 +
7*a + 12)*x^4 - 4*(a^2 + 7*a + 12)*x^3 - a^3 + 8*(a^2 + 7*a + 12)
*x^2 - 7*a^2 - 8*(a^2 + 7*a + 12)*x - 12*a)*sqrt((15*a^2 + (a^6 +
 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728)*sqrt((81*
a^2 + 558*a + 961)/(a^9 + 30*a^8 + 399*a^7 + 3088*a^6 + 15327*a^5
 + 50598*a^4 + 111105*a^3 + 156492*a^2 + 128304*a + 46656)) + 105
*a + 184)/(a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a +
 1728))*log(-81*a^2 + (81*a^2 + 567*a + 992)*x - (27*a^4 + 408*a^
3 + 2309*a^2 - 2*(2*a^7 + 49*a^6 + 513*a^5 + 2975*a^4 + 10321*a^3
 + 21420*a^2 + 24624*a + 12096)*sqrt((81*a^2 + 558*a + 961)/(a^9
+ 30*a^8 + 399*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 111105*a^
3 + 156492*a^2 + 128304*a + 46656)) + 5800*a + 5456)*sqrt((15*a^2
 + (a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728)*
sqrt((81*a^2 + 558*a + 961)/(a^9 + 30*a^8 + 399*a^7 + 3088*a^6 +
15327*a^5 + 50598*a^4 + 111105*a^3 + 156492*a^2 + 128304*a + 4665
6)) + 105*a + 184)/(a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 +
 3024*a + 1728)) - 567*a - 992) - ((a^2 + 7*a + 12)*x^4 - 4*(a^2
+ 7*a + 12)*x^3 - a^3 + 8*(a^2 + 7*a + 12)*x^2 - 7*a^2 - 8*(a^2 +
 7*a + 12)*x - 12*a)*sqrt((15*a^2 - (a^6 + 21*a^5 + 183*a^4 + 847
*a^3 + 2196*a^2 + 3024*a + 1728)*sqrt((81*a^2 + 558*a + 961)/(a^9
 + 30*a^8 + 399*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 111105*a
^3 + 156492*a^2 + 128304*a + 46656)) + 105*a + 184)/(a^6 + 21*a^5
 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728))*log(-81*a^2 + (
81*a^2 + 567*a + 992)*x + (27*a^4 + 408*a^3 + 2309*a^2 + 2*(2*a^7
 + 49*a^6 + 513*a^5 + 2975*a^4 + 10321*a^3 + 21420*a^2 + 24624*a
+ 12096)*sqrt((81*a^2 + 558*a + 961)/(a^9 + 30*a^8 + 399*a^7 + 30
88*a^6 + 15327*a^5 + 50598*a^4 + 111105*a^3 + 156492*a^2 + 128304
*a + 46656)) + 5800*a + 5456)*sqrt((15*a^2 - (a^6 + 21*a^5 + 183*
a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728)*sqrt((81*a^2 + 558*a +
961)/(a^9 + 30*a^8 + 399*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 +
 111105*a^3 + 156492*a^2 + 128304*a + 46656)) + 105*a + 184)/(a^6
 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728)) - 567*
a - 992) + ((a^2 + 7*a + 12)*x^4 - 4*(a^2 + 7*a + 12)*x^3 - a^3 +
 8*(a^2 + 7*a + 12)*x^2 - 7*a^2 - 8*(a^2 + 7*a + 12)*x - 12*a)*sq
rt((15*a^2 - (a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*
a + 1728)*sqrt((81*a^2 + 558*a + 961)/(a^9 + 30*a^8 + 399*a^7 + 3
088*a^6 + 15327*a^5 + 50598*a^4 + 111105*a^3 + 156492*a^2 + 12830
4*a + 46656)) + 105*a + 184)/(a^6 + 21*a^5 + 183*a^4 + 847*a^3 +
2196*a^2 + 3024*a + 1728))*log(-81*a^2 + (81*a^2 + 567*a + 992)*x
 - (27*a^4 + 408*a^3 + 2309*a^2 + 2*(2*a^7 + 49*a^6 + 513*a^5 + 2
975*a^4 + 10321*a^3 + 21420*a^2 + 24624*a + 12096)*sqrt((81*a^2 +
 558*a + 961)/(a^9 + 30*a^8 + 399*a^7 + 3088*a^6 + 15327*a^5 + 50
598*a^4 + 111105*a^3 + 156492*a^2 + 128304*a + 46656)) + 5800*a +
 5456)*sqrt((15*a^2 - (a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^
2 + 3024*a + 1728)*sqrt((81*a^2 + 558*a + 961)/(a^9 + 30*a^8 + 39
9*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 111105*a^3 + 156492*a^
2 + 128304*a + 46656)) + 105*a + 184)/(a^6 + 21*a^5 + 183*a^4 + 8
47*a^3 + 2196*a^2 + 3024*a + 1728)) - 567*a - 992) + 4*(a + 8)*x
- 12*x^2 - 4*a - 24)/((a^2 + 7*a + 12)*x^4 - 4*(a^2 + 7*a + 12)*x
^3 - a^3 + 8*(a^2 + 7*a + 12)*x^2 - 7*a^2 - 8*(a^2 + 7*a + 12)*x
- 12*a)
```

Sympy [A] time = 14.9847, size = 292, normalized size = 1.73

$$-\frac{-a+x^3-3x^2+x(a+8)-6}{-4a^3-28a^2-48a+x^4(4a^2+28a+48)+x^3(-16a^2-112a-192)+x^2(32a^2+224a+384)+x(-32a^2-224a-384)}$$

$$+\mathrm{RootSum}\Big(t^4\left(65536a^9+2162688a^8+31653888a^7+269680640a^6+1473773568a^5+5357174784a^4+12952010752a^3+20\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-x**4+4*x**3-8*x**2+a+8*x)**2,x)
```

```
[Out] -(-a + x**3 - 3*x**2 + x*(a + 8) - 6)/(-4*a**3 - 28*a**2 - 48*a +
 x**4*(4*a**2 + 28*a + 48) + x**3*(-16*a**2 - 112*a - 192) + x**2
*(32*a**2 + 224*a + 384) + x*(-32*a**2 - 224*a - 384)) + RootSum(
_t**4*(65536*a**9 + 2162688*a**8 + 31653888*a**7 + 269680640*a**6
 + 1473773568*a**5 + 5357174784*a**4 + 12952010752*a**3 + 2008232
7552*a**2 + 18119393280*a + 7247757312) + _t**2*(-7680*a**5 - 145
920*a**4 - 1107968*a**3 - 4202496*a**2 - 7962624*a - 6029312) - 8
1*a**2 - 576*a - 1024, Lambda(_t, _t*log(x + (-16384*_t**3*a**7 -
 401408*_t**3*a**6 - 4202496*_t**3*a**5 - 24371200*_t**3*a**4 - 8
4549632*_t**3*a**3 - 175472640*_t**3*a**2 - 201719808*_t**3*a - 9
9090432*_t**3 + 432*_t*a**4 + 7488*_t*a**3 + 47024*_t*a**2 + 1280
96*_t*a + 128512*_t - 81*a**2 - 567*a - 992)/(81*a**2 + 567*a + 9
92))))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^4-4x^3+8x^2-a-8x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 - 4*x^3 + 8*x^2 - a - 8*x)^(-2),x, algorithm="giac")
```

```
[Out] integrate((x^4 - 4*x^3 + 8*x^2 - a - 8*x)^(-2), x)
```

3.122 $\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^3}\,dx$

Optimal. Leaf size=252

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{8\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^2}-\frac{3\left(7a^2+\left(4\sqrt{a+4}+47\right)a+14\sqrt{a+4}+80\right)\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{64(a+3)^2(a+4)^{5/2}\sqrt{1-\sqrt{a+4}}}-\frac{3\left(-\frac{7a^2+47a+80}{\sqrt{a+4}}+4a+14\right)\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{64(a+3)^2(a+4)^2\sqrt{\sqrt{a+4}+1}}+\frac{(x-1)\left(6(2a+7)(x-1)^2+(a+6)(7a+25)\right)}{32(a+3)^2(a+4)^2\left(a-(x-1)^4-2(x-1)^2+3\right)}$$

```
[Out] ((5 + a + (-1 + x)^2)*(-1 + x))/(8*(12 + 7*a + a^2)*(3 + a - 2*(-
1 + x)^2 - (-1 + x)^4)^2) + (((6 + a)*(25 + 7*a) + 6*(7 + 2*a)*(-
1 + x)^2)*(-1 + x))/(32*(3 + a)^2*(4 + a)^2*(3 + a - 2*(-1 + x)^2
 - (-1 + x)^4)) - (3*(80 + 7*a^2 + 14*Sqrt[4 + a] + a*(47 + 4*Sqr
t[4 + a]))*ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4 + a]]])/(64*(3 + a)^2*
(4 + a)^(5/2)*Sqrt[1 - Sqrt[4 + a]]) - (3*(14 + 4*a - (80 + 47*a
+ 7*a^2)/Sqrt[4 + a])*ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]])/(64
*(3 + a)^2*(4 + a)^2*Sqrt[1 + Sqrt[4 + a]])
```

Rubi [A] time = 1.41356, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227

$$-\frac{(1-x)\left(a+(x-1)^2+5\right)}{8\left(a^2+7a+12\right)\left(a-(1-x)^4-2(1-x)^2+3\right)^2}+\frac{3\left(7a^2+\left(4\sqrt{a+4}+47\right)a+14\sqrt{a+4}+80\right)\tan^{-1}\left(\frac{1-x}{\sqrt{1-\sqrt{a+4}}}\right)}{64(a+3)^2(a+4)^{5/2}\sqrt{1-\sqrt{a+4}}}+\frac{3\left(-\frac{7a^2+47a+80}{\sqrt{a+4}}+4a+14\right)\tan^{-1}\left(\frac{1-x}{\sqrt{\sqrt{a+4}+1}}\right)}{64(a+3)^2(a+4)^2\sqrt{\sqrt{a+4}+1}}-\frac{(1-x)\left(6(2a+7)(1-x)^2+(a+6)(7a+25)\right)}{32(a+3)^2(a+4)^2\left(a-(1-x)^4-2(1-x)^2+3\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-3),x]
```

```
[Out] -(((6 + a)*(25 + 7*a) + 6*(7 + 2*a)*(1 - x)^2)*(1 - x))/(32*(3 +
a)^2*(4 + a)^2*(3 + a - 2*(1 - x)^2 - (1 - x)^4)) - ((5 + a + (-1
 + x)^2)*(1 - x))/(8*(12 + 7*a + a^2)*(3 + a - 2*(1 - x)^2 - (1 -
 x)^4)^2) + (3*(80 + 7*a^2 + 14*Sqrt[4 + a] + a*(47 + 4*Sqrt[4 +
a]))*ArcTan[(1 - x)/Sqrt[1 - Sqrt[4 + a]]])/(64*(3 + a)^2*(4 + a)
^(5/2)*Sqrt[1 - Sqrt[4 + a]]) + (3*(14 + 4*a - (80 + 47*a + 7*a^2
)/Sqrt[4 + a])*ArcTan[(1 - x)/Sqrt[1 + Sqrt[4 + a]]])/(64*(3 + a)
^2*(4 + a)^2*Sqrt[1 + Sqrt[4 + a]])
```

Rubi in Sympy [A] time = 120.835, size = 226, normalized size = 0.9

$$\frac{(x-1)\left(2a+2(x-1)^2+10\right)}{16(a+3)(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)^2}+\frac{(x-1)\left(28a^2+268a+(48a+168)(x-1)^2+600\right)}{128(a+3)^2(a+4)^2\left(a-(x-1)^4-2(x-1)^2+3\right)}+\frac{3\left(7a^2+47a-2\sqrt{a+4}(2a+7)+80\right)\operatorname{atan}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{64(a+3)^2(a+4)^{\frac{5}{2}}\sqrt{\sqrt{a+4}+1}}-\frac{3\left(7a^2+47a+2\sqrt{a+4}(2a+7)+80\right)\operatorname{atan}\left(\frac{x-1}{\sqrt{-\sqrt{a+4}+1}}\right)}{64(a+3)^2(a+4)^{\frac{5}{2}}\sqrt{-\sqrt{a+4}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-x**4+4*x**3-8*x**2+a+8*x)**3,x)
```

```
[Out] (x - 1)*(2*a + 2*(x - 1)**2 + 10)/(16*(a + 3)*(a + 4)*(a - (x - 1
)**4 - 2*(x - 1)**2 + 3)**2) + (x - 1)*(28*a**2 + 268*a + (48*a +
 168)*(x - 1)**2 + 600)/(128*(a + 3)**2*(a + 4)**2*(a - (x - 1)**
4 - 2*(x - 1)**2 + 3)) + 3*(7*a**2 + 47*a - 2*sqrt(a + 4)*(2*a +
7) + 80)*atan((x - 1)/sqrt(sqrt(a + 4) + 1))/(64*(a + 3)**2*(a +
4)**(5/2)*sqrt(sqrt(a + 4) + 1)) - 3*(7*a**2 + 47*a + 2*sqrt(a +
4)*(2*a + 7) + 80)*atan((x - 1)/sqrt(-sqrt(a + 4) + 1))/(64*(a +
3)**2*(a + 4)**(5/2)*sqrt(-sqrt(a + 4) + 1))
```

Mathematica [C] time = 0.187523, size = 254, normalized size = 1.01

$$\frac{1}{128}\left(-\frac{3\text{RootSum}\left[-\#1^4+4\#1^3-8\#1^2+8\#1+a\&,\frac{4\#1^2 a\log(x-\#1)+14\#1^2\log(x-\#1)+7a^2\log(x-\#1)+55a\log(x-\#1)-8\#1a\log(x-\#1)+108}{\#1^3-3\#1^2+4\#1-2}\right.}{\left(a^2+7a+12\right)^2}+\frac{4(x-1)\left(7a^2+a\left(12x^2-24x+79\right)+6\left(7x^2-14x+32\right)\right)}{(a+3)^2(a+4)^2\left(a-x\left(x^3-4x^2+8x-8\right)\right)}+\frac{16(x-1)\left(a+x^2-2x+6\right)}{(a+3)(a+4)\left(a-x\left(x^3-4x^2+8x-8\right)\right)^2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-3),x]
```

```
[Out] ((16*(-1 + x)*(6 + a - 2*x + x^2))/((3 + a)*(4 + a)*(a - x*(-8 +
8*x - 4*x^2 + x^3))^2) + (4*(-1 + x)*(7*a^2 + 6*(32 - 14*x + 7*x^
2) + a*(79 - 24*x + 12*x^2)))/((3 + a)^2*(4 + a)^2*(a - x*(-8 + 8
*x - 4*x^2 + x^3))) - (3*RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^
4 & , (108*Log[x - #1] + 55*a*Log[x - #1] + 7*a^2*Log[x - #1] - 2
8*Log[x - #1]*#1 - 8*a*Log[x - #1]*#1 + 14*Log[x - #1]*#1^2 + 4*a
*Log[x - #1]*#1^2)/(-2 + 4*#1 - 3*#1^2 + #1^3) & ])/(12 + 7*a + a
^2)^2)/128
```

Maple [C] time = 0.045, size = 398, normalized size = 1.6

$$-\frac{1}{\left(x^4-4x^3+8x^2-a-8x\right)^2}\left(\frac{(6a+21)x^7}{16a^4+224a^3+1168a^2+2688a+2304}-\frac{(147+42a)x^6}{\left(16a^2+128a+256\right)\left(a^2+6a+9\right)}+\frac{(7}{32a^4+448}\right.$$

$$-\frac{3}{128}\sum_{\_R=RootOf\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)}\frac{\left(108+2(7+2a)\_R^2+4(-2a-7)\_R+7a^2+55a\right)\ln(x-\_R)}{\left(\_R^3-3\_R^2+4\_R-2\right)\left(a^3+10a^2+33a+36\right)(4+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-x^4+4*x^3-8*x^2+a+8*x)^3,x)
```

```
[Out] -(3/16*(7+2*a)/(a^4+14*a^3+73*a^2+168*a+144)*x^7-21/16*(7+2*a)/(a
^2+8*a+16)/(a^2+6*a+9)*x^6+1/32*(7*a^2+343*a+1116)/(a^4+14*a^3+73
*a^2+168*a+144)*x^5-5/32*(7*a^2+175*a+528)/(a^4+14*a^3+73*a^2+168
*a+144)*x^4+1/16*(34*a^2+679*a+1968)/(a^4+14*a^3+73*a^2+168*a+144
)*x^3-1/16*(32*a^2+623*a+1800)/(a^4+14*a^3+73*a^2+168*a+144)*x^2-
1/32*(11*a^3+107*a^2-84*a-1152)/(a^4+14*a^3+73*a^2+168*a+144)*x+1
/32*(11*a^3+131*a^2+408*a+288)/(a^4+14*a^3+73*a^2+168*a+144))/(x^
4-4*x^3+8*x^2-a-8*x)^2-3/128*sum((108+2*(7+2*a)*_R^2+4*(-2*a-7)*_
R+7*a^2+55*a)/(_R^3-3*_R^2+4*_R-2)/(a^3+10*a^2+33*a+36)/(4+a)*ln(
x-_R),_R=RootOf(_Z^4-4*_Z^3+8*_Z^2-8*_Z-a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{6\,(2\,a+7)x^7-42\,(2\,a+\cdots}{32\,((a^4+14\,a^3+73\,a^2+168\,a+144)x^8-8\,(a^4+14\,a^3+73\,a^2+168\,a+144)x^7+32\,(a^4+14\,a^3+73\,a^2+168\,a+144)x^6+\cdots}-\frac{3\int\frac{2\,(2\,a+7)x^2+7\,a^2-4\,(2\,a+7)x+55\,a+108}{x^4-4\,x^3+8\,x^2-a-8\,x}\,dx}{32\,(a^4+14\,a^3+73\,a^2+168\,a+144)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(x^4 - 4*x^3 + 8*x^2 - a - 8*x)^3,x, algorithm="maxima")
```

```
[Out] -1/32*(6*(2*a + 7)*x^7 - 42*(2*a + 7)*x^6 + (7*a^2 + 343*a + 1116
)*x^5 - 5*(7*a^2 + 175*a + 528)*x^4 + 2*(34*a^2 + 679*a + 1968)*x
^3 + 11*a^3 - 2*(32*a^2 + 623*a + 1800)*x^2 + 131*a^2 - (11*a^3 +
 107*a^2 - 84*a - 1152)*x + 408*a + 288)/((a^4 + 14*a^3 + 73*a^2
+ 168*a + 144)*x^8 - 8*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^7
+ 32*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^6 + a^6 - 80*(a^4 +
14*a^3 + 73*a^2 + 168*a + 144)*x^5 + 14*a^5 - 2*(a^5 - 50*a^4 - 8
23*a^3 - 4504*a^2 - 10608*a - 9216)*x^4 + 73*a^4 + 8*(a^5 - 2*a^4
 - 151*a^3 - 1000*a^2 - 2544*a - 2304)*x^3 + 168*a^3 - 16*(a^5 +
10*a^4 + 17*a^3 - 124*a^2 - 528*a - 576)*x^2 + 144*a^2 + 16*(a^5
+ 14*a^4 + 73*a^3 + 168*a^2 + 144*a)*x) - 3/32*integrate((2*(2*a
+ 7)*x^2 + 7*a^2 - 4*(2*a + 7)*x + 55*a + 108)/(x^4 - 4*x^3 + 8*x
^2 - a - 8*x), x)/(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)
```

Fricas [A] time = 0.286467, size = 5361, normalized size = 21.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(x^4 - 4*x^3 + 8*x^2 - a - 8*x)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(24*(2*a + 7)*x^7 - 168*(2*a + 7)*x^6 + 4*(7*a^2 + 343*a +
 1116)*x^5 - 20*(7*a^2 + 175*a + 528)*x^4 + 8*(34*a^2 + 679*a + 1
968)*x^3 + 44*a^3 - 8*(32*a^2 + 623*a + 1800)*x^2 - 3*((a^4 + 14*
a^3 + 73*a^2 + 168*a + 144)*x^8 - 8*(a^4 + 14*a^3 + 73*a^2 + 168*
a + 144)*x^7 + 32*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^6 + a^6
 - 80*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^5 + 14*a^5 - 2*(a^5
 - 50*a^4 - 823*a^3 - 4504*a^2 - 10608*a - 9216)*x^4 + 73*a^4 + 8
*(a^5 - 2*a^4 - 151*a^3 - 1000*a^2 - 2544*a - 2304)*x^3 + 168*a^3
 - 16*(a^5 + 10*a^4 + 17*a^3 - 124*a^2 - 528*a - 576)*x^2 + 144*a
^2 + 16*(a^5 + 14*a^4 + 73*a^3 + 168*a^2 + 144*a)*x)*sqrt((105*a^
4 + 1470*a^3 + 7749*a^2 + (a^10 + 35*a^9 + 550*a^8 + 5110*a^7 + 3
1085*a^6 + 129367*a^5 + 373020*a^4 + 735840*a^3 + 950400*a^2 + 72
5760*a + 248832)*sqrt((2401*a^4 + 33124*a^3 + 171966*a^2 + 398164
*a + 346921)/(a^15 + 50*a^14 + 1165*a^13 + 16780*a^12 + 167090*a^
11 + 1218460*a^10 + 6722130*a^9 + 28570320*a^8 + 94320045*a^7 + 2
```

```
41870050*a^6 + 477857313*a^5 + 714317940*a^4 + 782071200*a^3 + 59
2064640*a^2 + 277136640*a + 60466176)) + 18228*a + 16144)/(a^10 +
 35*a^9 + 550*a^8 + 5110*a^7 + 31085*a^6 + 129367*a^5 + 373020*a^
4 + 735840*a^3 + 950400*a^2 + 725760*a + 248832))*log(-64827*a^4
- 907578*a^3 - 4780647*a^2 + 27*(2401*a^4 + 33614*a^3 + 177061*a^
2 + 415884*a + 367536)*x + 27*(343*a^7 + 8981*a^6 + 100811*a^5 +
628887*a^4 + 2354874*a^3 + 5293208*a^2 - (11*a^12 + 462*a^11 + 88
81*a^10 + 103320*a^9 + 810205*a^8 + 4511542*a^7 + 18292039*a^6 +
54410692*a^5 + 117844800*a^4 + 181238400*a^3 + 187875072*a^2 + 11
7863424*a + 33841152)*sqrt((2401*a^4 + 33124*a^3 + 171966*a^2 + 3
98164*a + 346921)/(a^15 + 50*a^14 + 1165*a^13 + 16780*a^12 + 1670
90*a^11 + 1218460*a^10 + 6722130*a^9 + 28570320*a^8 + 94320045*a^
7 + 241870050*a^6 + 477857313*a^5 + 714317940*a^4 + 782071200*a^3
 + 592064640*a^2 + 277136640*a + 60466176)) + 6613472*a + 3543424
)*sqrt((105*a^4 + 1470*a^3 + 7749*a^2 + (a^10 + 35*a^9 + 550*a^8
+ 5110*a^7 + 31085*a^6 + 129367*a^5 + 373020*a^4 + 735840*a^3 + 9
50400*a^2 + 725760*a + 248832)*sqrt((2401*a^4 + 33124*a^3 + 17196
6*a^2 + 398164*a + 346921)/(a^15 + 50*a^14 + 1165*a^13 + 16780*a^
12 + 167090*a^11 + 1218460*a^10 + 6722130*a^9 + 28570320*a^8 + 94
320045*a^7 + 241870050*a^6 + 477857313*a^5 + 714317940*a^4 + 7820
71200*a^3 + 592064640*a^2 + 277136640*a + 60466176)) + 18228*a +
16144)/(a^10 + 35*a^9 + 550*a^8 + 5110*a^7 + 31085*a^6 + 129367*a
^5 + 373020*a^4 + 735840*a^3 + 950400*a^2 + 725760*a + 248832)) -
 11228868*a - 9923472) + 3*((a^4 + 14*a^3 + 73*a^2 + 168*a + 144)
*x^8 - 8*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^7 + 32*(a^4 + 14
*a^3 + 73*a^2 + 168*a + 144)*x^6 + a^6 - 80*(a^4 + 14*a^3 + 73*a^
2 + 168*a + 144)*x^5 + 14*a^5 - 2*(a^5 - 50*a^4 - 823*a^3 - 4504*
a^2 - 10608*a - 9216)*x^4 + 73*a^4 + 8*(a^5 - 2*a^4 - 151*a^3 - 1
000*a^2 - 2544*a - 2304)*x^3 + 168*a^3 - 16*(a^5 + 10*a^4 + 17*a^
3 - 124*a^2 - 528*a - 576)*x^2 + 144*a^2 + 16*(a^5 + 14*a^4 + 73*
a^3 + 168*a^2 + 144*a)*x)*sqrt((105*a^4 + 1470*a^3 + 7749*a^2 + (
a^10 + 35*a^9 + 550*a^8 + 5110*a^7 + 31085*a^6 + 129367*a^5 + 373
020*a^4 + 735840*a^3 + 950400*a^2 + 725760*a + 248832)*sqrt((2401
*a^4 + 33124*a^3 + 171966*a^2 + 398164*a + 346921)/(a^15 + 50*a^1
4 + 1165*a^13 + 16780*a^12 + 167090*a^11 + 1218460*a^10 + 6722130
*a^9 + 28570320*a^8 + 94320045*a^7 + 241870050*a^6 + 477857313*a^
5 + 714317940*a^4 + 782071200*a^3 + 592064640*a^2 + 277136640*a +
 60466176)) + 18228*a + 16144)/(a^10 + 35*a^9 + 550*a^8 + 5110*a^
7 + 31085*a^6 + 129367*a^5 + 373020*a^4 + 735840*a^3 + 950400*a^2
 + 725760*a + 248832))*log(-64827*a^4 - 907578*a^3 - 4780647*a^2
+ 27*(2401*a^4 + 33614*a^3 + 177061*a^2 + 415884*a + 367536)*x -
27*(343*a^7 + 8981*a^6 + 100811*a^5 + 628887*a^4 + 2354874*a^3 +
5293208*a^2 - (11*a^12 + 462*a^11 + 8881*a^10 + 103320*a^9 + 8102
05*a^8 + 4511542*a^7 + 18292039*a^6 + 54410692*a^5 + 117844800*a^
4 + 181238400*a^3 + 187875072*a^2 + 117863424*a + 33841152)*sqrt(
(2401*a^4 + 33124*a^3 + 171966*a^2 + 398164*a + 346921)/(a^15 + 5
0*a^14 + 1165*a^13 + 16780*a^12 + 167090*a^11 + 1218460*a^10 + 67
22130*a^9 + 28570320*a^8 + 94320045*a^7 + 241870050*a^6 + 4778573
13*a^5 + 714317940*a^4 + 782071200*a^3 + 592064640*a^2 + 27713664
0*a + 60466176)) + 6613472*a + 3543424)*sqrt((105*a^4 + 1470*a^3
+ 7749*a^2 + (a^10 + 35*a^9 + 550*a^8 + 5110*a^7 + 31085*a^6 + 12
9367*a^5 + 373020*a^4 + 735840*a^3 + 950400*a^2 + 725760*a + 2488
32)*sqrt((2401*a^4 + 33124*a^3 + 171966*a^2 + 398164*a + 346921)/
(a^15 + 50*a^14 + 1165*a^13 + 16780*a^12 + 167090*a^11 + 1218460*
a^10 + 6722130*a^9 + 28570320*a^8 + 94320045*a^7 + 241870050*a^6
+ 477857313*a^5 + 714317940*a^4 + 782071200*a^3 + 592064640*a^2 +
 277136640*a + 60466176)) + 18228*a + 16144)/(a^10 + 35*a^9 + 550
*a^8 + 5110*a^7 + 31085*a^6 + 129367*a^5 + 373020*a^4 + 735840*a^
3 + 950400*a^2 + 725760*a + 248832)) - 11228868*a - 9923472) - 3*
((a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^8 - 8*(a^4 + 14*a^3 + 73
*a^2 + 168*a + 144)*x^7 + 32*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144
)*x^6 + a^6 - 80*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^5 + 14*a
^5 - 2*(a^5 - 50*a^4 - 823*a^3 - 4504*a^2 - 10608*a - 9216)*x^4 +
 73*a^4 + 8*(a^5 - 2*a^4 - 151*a^3 - 1000*a^2 - 2544*a - 2304)*x^
3 + 168*a^3 - 16*(a^5 + 10*a^4 + 17*a^3 - 124*a^2 - 528*a - 576)*
x^2 + 144*a^2 + 16*(a^5 + 14*a^4 + 73*a^3 + 168*a^2 + 144*a)*x)*s
qrt((105*a^4 + 1470*a^3 + 7749*a^2 - (a^10 + 35*a^9 + 550*a^8 + 5
110*a^7 + 31085*a^6 + 129367*a^5 + 373020*a^4 + 735840*a^3 + 9504
00*a^2 + 725760*a + 248832)*sqrt((2401*a^4 + 33124*a^3 + 171966*a
^2 + 398164*a + 346921)/(a^15 + 50*a^14 + 1165*a^13 + 16780*a^12
+ 167090*a^11 + 1218460*a^10 + 6722130*a^9 + 28570320*a^8 + 94320
045*a^7 + 241870050*a^6 + 477857313*a^5 + 714317940*a^4 + 7820712
```

```
00*a^3 + 592064640*a^2 + 277136640*a + 60466176)) + 18228*a + 161
44)/(a^10 + 35*a^9 + 550*a^8 + 5110*a^7 + 31085*a^6 + 129367*a^5
+ 373020*a^4 + 735840*a^3 + 950400*a^2 + 725760*a + 248832))*log(
-64827*a^4 - 907578*a^3 - 4780647*a^2 + 27*(2401*a^4 + 33614*a^3
+ 177061*a^2 + 415884*a + 367536)*x + 27*(343*a^7 + 8981*a^6 + 10
0811*a^5 + 628887*a^4 + 2354874*a^3 + 5293208*a^2 + (11*a^12 + 46
2*a^11 + 8881*a^10 + 103320*a^9 + 810205*a^8 + 4511542*a^7 + 1829
2039*a^6 + 54410692*a^5 + 117844800*a^4 + 181238400*a^3 + 1878750
72*a^2 + 117863424*a + 33841152)*sqrt((2401*a^4 + 33124*a^3 + 171
966*a^2 + 398164*a + 346921)/(a^15 + 50*a^14 + 1165*a^13 + 16780*
a^12 + 167090*a^11 + 1218460*a^10 + 6722130*a^9 + 28570320*a^8 +
94320045*a^7 + 241870050*a^6 + 477857313*a^5 + 714317940*a^4 + 78
2071200*a^3 + 592064640*a^2 + 277136640*a + 60466176)) + 6613472*
a + 3543424)*sqrt((105*a^4 + 1470*a^3 + 7749*a^2 - (a^10 + 35*a^9
 + 550*a^8 + 5110*a^7 + 31085*a^6 + 129367*a^5 + 373020*a^4 + 735
840*a^3 + 950400*a^2 + 725760*a + 248832)*sqrt((2401*a^4 + 33124*
a^3 + 171966*a^2 + 398164*a + 346921)/(a^15 + 50*a^14 + 1165*a^13
 + 16780*a^12 + 167090*a^11 + 1218460*a^10 + 6722130*a^9 + 285703
20*a^8 + 94320045*a^7 + 241870050*a^6 + 477857313*a^5 + 714317940
*a^4 + 782071200*a^3 + 592064640*a^2 + 277136640*a + 60466176)) +
 18228*a + 16144)/(a^10 + 35*a^9 + 550*a^8 + 5110*a^7 + 31085*a^6
 + 129367*a^5 + 373020*a^4 + 735840*a^3 + 950400*a^2 + 725760*a +
 248832)) - 11228868*a - 9923472) + 3*((a^4 + 14*a^3 + 73*a^2 + 1
68*a + 144)*x^8 - 8*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^7 + 3
2*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^6 + a^6 - 80*(a^4 + 14*
a^3 + 73*a^2 + 168*a + 144)*x^5 + 14*a^5 - 2*(a^5 - 50*a^4 - 823*
a^3 - 4504*a^2 - 10608*a - 9216)*x^4 + 73*a^4 + 8*(a^5 - 2*a^4 -
151*a^3 - 1000*a^2 - 2544*a - 2304)*x^3 + 168*a^3 - 16*(a^5 + 10*
a^4 + 17*a^3 - 124*a^2 - 528*a - 576)*x^2 + 144*a^2 + 16*(a^5 + 1
4*a^4 + 73*a^3 + 168*a^2 + 144*a)*x)*sqrt((105*a^4 + 1470*a^3 + 7
749*a^2 - (a^10 + 35*a^9 + 550*a^8 + 5110*a^7 + 31085*a^6 + 12936
7*a^5 + 373020*a^4 + 735840*a^3 + 950400*a^2 + 725760*a + 248832)
*sqrt((2401*a^4 + 33124*a^3 + 171966*a^2 + 398164*a + 346921)/(a^
15 + 50*a^14 + 1165*a^13 + 16780*a^12 + 167090*a^11 + 1218460*a^1
0 + 6722130*a^9 + 28570320*a^8 + 94320045*a^7 + 241870050*a^6 + 4
77857313*a^5 + 714317940*a^4 + 782071200*a^3 + 592064640*a^2 + 27
7136640*a + 60466176)) + 18228*a + 16144)/(a^10 + 35*a^9 + 550*a^
8 + 5110*a^7 + 31085*a^6 + 129367*a^5 + 373020*a^4 + 735840*a^3 +
 950400*a^2 + 725760*a + 248832))*log(-64827*a^4 - 907578*a^3 - 4
780647*a^2 + 27*(2401*a^4 + 33614*a^3 + 177061*a^2 + 415884*a + 3
67536)*x - 27*(343*a^7 + 8981*a^6 + 100811*a^5 + 628887*a^4 + 235
4874*a^3 + 5293208*a^2 + (11*a^12 + 462*a^11 + 8881*a^10 + 103320
*a^9 + 810205*a^8 + 4511542*a^7 + 18292039*a^6 + 54410692*a^5 + 1
17844800*a^4 + 181238400*a^3 + 187875072*a^2 + 117863424*a + 3384
1152)*sqrt((2401*a^4 + 33124*a^3 + 171966*a^2 + 398164*a + 346921
)/(a^15 + 50*a^14 + 1165*a^13 + 16780*a^12 + 167090*a^11 + 121846
0*a^10 + 6722130*a^9 + 28570320*a^8 + 94320045*a^7 + 241870050*a^
6 + 477857313*a^5 + 714317940*a^4 + 782071200*a^3 + 592064640*a^2
 + 277136640*a + 60466176)) + 6613472*a + 3543424)*sqrt((105*a^4
+ 1470*a^3 + 7749*a^2 - (a^10 + 35*a^9 + 550*a^8 + 5110*a^7 + 310
85*a^6 + 129367*a^5 + 373020*a^4 + 735840*a^3 + 950400*a^2 + 7257
60*a + 248832)*sqrt((2401*a^4 + 33124*a^3 + 171966*a^2 + 398164*a
 + 346921)/(a^15 + 50*a^14 + 1165*a^13 + 16780*a^12 + 167090*a^11
 + 1218460*a^10 + 6722130*a^9 + 28570320*a^8 + 94320045*a^7 + 241
870050*a^6 + 477857313*a^5 + 714317940*a^4 + 782071200*a^3 + 5920
64640*a^2 + 277136640*a + 60466176)) + 18228*a + 16144)/(a^10 + 3
5*a^9 + 550*a^8 + 5110*a^7 + 31085*a^6 + 129367*a^5 + 373020*a^4
+ 735840*a^3 + 950400*a^2 + 725760*a + 248832)) - 11228868*a - 99
23472) + 524*a^2 - 4*(11*a^3 + 107*a^2 - 84*a - 1152)*x + 1632*a
+ 1152)/((a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^8 - 8*(a^4 + 14*
a^3 + 73*a^2 + 168*a + 144)*x^7 + 32*(a^4 + 14*a^3 + 73*a^2 + 168
*a + 144)*x^6 + a^6 - 80*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^
5 + 14*a^5 - 2*(a^5 - 50*a^4 - 823*a^3 - 4504*a^2 - 10608*a - 921
6)*x^4 + 73*a^4 + 8*(a^5 - 2*a^4 - 151*a^3 - 1000*a^2 - 2544*a -
2304)*x^3 + 168*a^3 - 16*(a^5 + 10*a^4 + 17*a^3 - 124*a^2 - 528*a
 - 576)*x^2 + 144*a^2 + 16*(a^5 + 14*a^4 + 73*a^3 + 168*a^2 + 144
*a)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-x**4+4*x**3-8*x**2+a+8*x)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\left(x^4 - 4\,x^3 + 8\,x^2 - a - 8\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(x^4 - 4*x^3 + 8*x^2 - a - 8*x)^3,x, algorithm="giac")
```

```
[Out] integrate(-1/(x^4 - 4*x^3 + 8*x^2 - a - 8*x)^3, x)
```

3.123 $\int x\left(a+8x-8x^2+4x^3-x^4\right)^4\,dx$

Optimal. Leaf size=210

$$\frac{a^4x^2}{2}+\frac{32a^3x^3}{3}+\frac{1}{5}\left(3a^2-1536a+20480\right)x^{10}-\frac{16}{3}\left(a^2-128a+896\right)x^9-\frac{32}{7}\left(15a^2-288a+512\right)x^7+\frac{16}{5}a\left(a^2-48a+128\right)x^5+8(12-a)a^2x^4+\frac{2}{3}\left(-a^3+192a^2-1536a+1024\right)x^6+\frac{2}{7}(640-a)x^{14}-\frac{16}{13}(464-3a)x^{13}+\frac{8}{3}(524-9a)x^{12}-\frac{32}{11}(928-35a)x^{11}+8(128-3a)(4-a)x^8+\frac{x^{18}}{18}-\frac{16x^{17}}{17}+8x^{16}-\frac{224x^{15}}{5}$$

```
[Out] (a^4*x^2)/2 + (32*a^3*x^3)/3 + 8*(12 - a)*a^2*x^4 + (16*a*(128 -
48*a + a^2)*x^5)/5 + (2*(1024 - 1536*a + 192*a^2 - a^3)*x^6)/3 -
(32*(512 - 288*a + 15*a^2)*x^7)/7 + 8*(128 - 3*a)*(4 - a)*x^8 - (
16*(896 - 128*a + a^2)*x^9)/3 + ((20480 - 1536*a + 3*a^2)*x^10)/5
 - (32*(928 - 35*a)*x^11)/11 + (8*(524 - 9*a)*x^12)/3 - (16*(464
- 3*a)*x^13)/13 + (2*(640 - a)*x^14)/7 - (224*x^15)/5 + 8*x^16 -
(16*x^17)/17 + x^18/18
```

Rubi [A] time = 0.483287, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042

$$\frac{a^4x^2}{2}+\frac{32a^3x^3}{3}+\frac{1}{5}\left(3a^2-1536a+20480\right)x^{10}-\frac{16}{3}\left(a^2-128a+896\right)x^9-\frac{32}{7}\left(15a^2-288a+512\right)x^7+\frac{16}{5}a\left(a^2-48a+128\right)x^5+8(12-a)a^2x^4+\frac{2}{3}\left(-a^3+192a^2-1536a+1024\right)x^6+\frac{2}{7}(640-a)x^{14}-\frac{16}{13}(464-3a)x^{13}+\frac{8}{3}(524-9a)x^{12}-\frac{32}{11}(928-35a)x^{11}+8(128-3a)(4-a)x^8+\frac{x^{18}}{18}-\frac{16x^{17}}{17}+8x^{16}-\frac{224x^{15}}{5}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x]
```

```
[Out] (a^4*x^2)/2 + (32*a^3*x^3)/3 + 8*(12 - a)*a^2*x^4 + (16*a*(128 -
48*a + a^2)*x^5)/5 + (2*(1024 - 1536*a + 192*a^2 - a^3)*x^6)/3 -
(32*(512 - 288*a + 15*a^2)*x^7)/7 + 8*(128 - 3*a)*(4 - a)*x^8 - (
16*(896 - 128*a + a^2)*x^9)/3 + ((20480 - 1536*a + 3*a^2)*x^10)/5
 - (32*(928 - 35*a)*x^11)/11 + (8*(524 - 9*a)*x^12)/3 - (16*(464
- 3*a)*x^13)/13 + (2*(640 - a)*x^14)/7 - (224*x^15)/5 + 8*x^16 -
(16*x^17)/17 + x^18/18
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(-x**4+4*x**3-8*x**2+a+8*x)**4,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.048177, size = 204, normalized size = 0.97

$$\frac{a^4x^2}{2}+\frac{32a^3x^3}{3}+\frac{1}{5}\left(3a^2-1536a+20480\right)x^{10}-\frac{16}{3}\left(a^2-128a+896\right)x^9+8\left(3a^2-140a+512\right)x^8-\frac{32}{7}\left(15a^2-288a+512\right)x^7+\frac{16}{5}a\left(a^2-48a+128\right)x^5-8(a-12)a^2x^4-\frac{2}{3}\left(a^3-192a^2+1536a-1024\right)x^6-\frac{2}{7}(a-640)x^{14}+\frac{16}{13}(3a-464)x^{13}-\frac{8}{3}(9a-524)x^{12}+\frac{32}{11}(35a-928)x^{11}+\frac{x^{18}}{18}-\frac{16x^{17}}{17}+8x^{16}-\frac{224x^{15}}{5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x]
```

```
[Out] (a^4*x^2)/2 + (32*a^3*x^3)/3 - 8*(-12 + a)*a^2*x^4 + (16*a*(128 -
 48*a + a^2)*x^5)/5 - (2*(-1024 + 1536*a - 192*a^2 + a^3)*x^6)/3
- (32*(512 - 288*a + 15*a^2)*x^7)/7 + 8*(512 - 140*a + 3*a^2)*x^8
 - (16*(896 - 128*a + a^2)*x^9)/3 + ((20480 - 1536*a + 3*a^2)*x^1
0)/5 + (32*(-928 + 35*a)*x^11)/11 - (8*(-524 + 9*a)*x^12)/3 + (16
*(-464 + 3*a)*x^13)/13 - (2*(-640 + a)*x^14)/7 - (224*x^15)/5 + 8
*x^16 - (16*x^17)/17 + x^18/18
```

Maple [A] time = 0.002, size = 267, normalized size = 1.3

$$\frac{x^{18}}{18}-\frac{16\,x^{17}}{17}+8\,x^{16}-\frac{224\,x^{15}}{5}+\frac{(-4\,a+2560)\,x^{14}}{14}+\frac{(48\,a-7424)\,x^{13}}{13}+\frac{(-288\,a+16768)\,x^{12}}{12}+\frac{(1120\,a-29696)\,x^{11}}{11}+\frac{\left(2\,a^2-2560\,a+24576+(-2\,a+128)^2\right)x^{10}}{10}+\frac{\left(-16\,a^2+3584\,a-10240+2\,(8\,a-128)\,(-2\,a+128)\right)x^9}{9}+\frac{\left(64\,a^2-2560\,a+2\,(-16\,a+64)\,(-2\,a+128)+(8\,a-128)^2\right)x^8}{8}+\frac{\left(-160\,a^2+32\,a\,(-2\,a+128)+2\,(-16\,a+64)\,(8\,a-128)\right)x^7}{7}+\frac{\left(2\,a^2\,(-2\,a+128)+32\,a\,(8\,a-128)+(-16\,a+64)^2\right)x^6}{6}+\frac{\left(2\,a^2\,(8\,a-128)+32\,a\,(-16\,a+64)\right)x^5}{5}+\frac{\left(2\,a^2\,(-16\,a+64)+256\,a^2\right)x^4}{4}+\frac{32\,a^3x^3}{3}+\frac{a^4x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(-x^4+4*x^3-8*x^2+a+8*x)^4,x)
```

```
[Out] 1/18*x^18-16/17*x^17+8*x^16-224/5*x^15+1/14*(-4*a+2560)*x^14+1/13
*(48*a-7424)*x^13+1/12*(-288*a+16768)*x^12+1/11*(1120*a-29696)*x^
11+1/10*(2*a^2-2560*a+24576+(-2*a+128)^2)*x^10+1/9*(-16*a^2+3584*
a-10240+2*(8*a-128)*(-2*a+128))*x^9+1/8*(64*a^2-2560*a+2*(-16*a+6
4)*(-2*a+128)+(8*a-128)^2)*x^8+1/7*(-160*a^2+32*a*(-2*a+128)+2*(-
16*a+64)*(8*a-128))*x^7+1/6*(2*a^2*(-2*a+128)+32*a*(8*a-128)+(-16
*a+64)^2)*x^6+1/5*(2*a^2*(8*a-128)+32*a*(-16*a+64))*x^5+1/4*(2*a^
2*(-16*a+64)+256*a^2)*x^4+32/3*a^3*x^3+1/2*a^4*x^2
```

Maxima [A] time = 0.815219, size = 246, normalized size = 1.17

$$\frac{1}{18}\,x^{18}-\frac{16}{17}\,x^{17}+8\,x^{16}-\frac{2}{7}\,(a-640)x^{14}-\frac{224}{5}\,x^{15}+\frac{16}{13}\,(3\,a-464)x^{13}-\frac{8}{3}\,(9\,a-524)x^{12}+\frac{32}{11}\,(35\,a-928)x^{11}+\frac{1}{5}\,\left(3\,a^2-1536\,a+20480\right)x^{10}-\frac{16}{3}\,\left(a^2-128\,a+896\right)x^9+8\,\left(3\,a^2-140\,a+512\right)x^8-\frac{32}{7}\,\left(15\,a^2-288\,a+512\right)x^7-\frac{2}{3}\,\left(a^3-192\,a^2+1536\,a-1024\right)x^6+\frac{1}{2}\,a^4x^2+\frac{32}{3}\,a^3x^3+\frac{16}{5}\,\left(a^3-48\,a^2+128\,a\right)x^5-8\,\left(a^3-12\,a^2\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 - 4*x^3 + 8*x^2 - a - 8*x)^4*x,x, algorithm="maxima")
```

```
[Out] 1/18*x^18 - 16/17*x^17 + 8*x^16 - 2/7*(a - 640)*x^14 - 224/5*x^15
 + 16/13*(3*a - 464)*x^13 - 8/3*(9*a - 524)*x^12 + 32/11*(35*a -
928)*x^11 + 1/5*(3*a^2 - 1536*a + 20480)*x^10 - 16/3*(a^2 - 128*a
 + 896)*x^9 + 8*(3*a^2 - 140*a + 512)*x^8 - 32/7*(15*a^2 - 288*a
+ 512)*x^7 - 2/3*(a^3 - 192*a^2 + 1536*a - 1024)*x^6 + 1/2*a^4*x^
2 + 32/3*a^3*x^3 + 16/5*(a^3 - 48*a^2 + 128*a)*x^5 - 8*(a^3 - 12*
a^2)*x^4
```

Fricas [A] time = 0.225836, size = 1, normalized size = 0.

$$\frac{1}{18}x^{18}-\frac{16}{17}x^{17}+8x^{16}-\frac{224}{5}x^{15}-\frac{2}{7}x^{14}a+\frac{1280}{7}x^{14}+\frac{48}{13}x^{13}a-\frac{7424}{13}x^{13}-24x^{12}a+\frac{4192}{3}x^{12}+\frac{1120}{11}x^{11}a+\frac{3}{5}x^{10}a^2-\frac{29696}{11}x^{11}-\frac{1536}{5}x^{10}a-\frac{16}{3}x^9a^2+4096x^{10}+\frac{2048}{3}x^9a+24x^8a^2-\frac{14336}{3}x^9-1120x^8a-\frac{480}{7}x^7a^2-\frac{2}{3}x^6a^3+4096x^8+\frac{9216}{7}x^7a+128x^6a^2+\frac{16}{5}x^5a^3-\frac{16384}{7}x^7-1024x^6a-\frac{768}{5}x^5a^2-8x^4a^3+\frac{2048}{3}x^6+\frac{2048}{5}x^5a+96x^4a^2+\frac{32}{3}x^3a^3+\frac{1}{2}x^2a^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 - 4*x^3 + 8*x^2 - a - 8*x)^4*x,x, algorithm="fricas")
```

```
[Out] 1/18*x^18 - 16/17*x^17 + 8*x^16 - 224/5*x^15 - 2/7*x^14*a + 1280/
7*x^14 + 48/13*x^13*a - 7424/13*x^13 - 24*x^12*a + 4192/3*x^12 +
1120/11*x^11*a + 3/5*x^10*a^2 - 29696/11*x^11 - 1536/5*x^10*a - 1
6/3*x^9*a^2 + 4096*x^10 + 2048/3*x^9*a + 24*x^8*a^2 - 14336/3*x^9
 - 1120*x^8*a - 480/7*x^7*a^2 - 2/3*x^6*a^3 + 4096*x^8 + 9216/7*x
^7*a + 128*x^6*a^2 + 16/5*x^5*a^3 - 16384/7*x^7 - 1024*x^6*a - 76
8/5*x^5*a^2 - 8*x^4*a^3 + 2048/3*x^6 + 2048/5*x^5*a + 96*x^4*a^2
+ 32/3*x^3*a^3 + 1/2*x^2*a^4
```

Sympy [A] time = 0.249164, size = 212, normalized size = 1.01

$$\frac{a^4x^2}{2}+\frac{32a^3x^3}{3}+\frac{x^{18}}{18}-\frac{16x^{17}}{17}+8x^{16}-\frac{224x^{15}}{5}+x^{14}\left(-\frac{2a}{7}+\frac{1280}{7}\right)+x^{13}\left(\frac{48a}{13}-\frac{7424}{13}\right)+x^{12}\left(-24a+\frac{4192}{3}\right)+x^{11}\left(\frac{1120a}{11}-\frac{29696}{11}\right)+x^{10}\left(\frac{3a^2}{5}-\frac{1536a}{5}+4096\right)+x^9\left(-\frac{16a^2}{3}+\frac{2048a}{3}-\frac{14336}{3}\right)+x^8\left(24a^2-1120a+4096\right)+x^7\left(-\frac{480a^2}{7}+\frac{9216a}{7}-\frac{16384}{7}\right)+x^6\left(-\frac{2a^3}{3}+128a^2-1024a+\frac{2048}{3}\right)+x^5\left(\frac{16a^3}{5}-\frac{768a^2}{5}+\frac{2048a}{5}\right)+x^4\left(-8a^3+96a^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(-x**4+4*x**3-8*x**2+a+8*x)**4,x)
```

```
[Out] a**4*x**2/2 + 32*a**3*x**3/3 + x**18/18 - 16*x**17/17 + 8*x**16 -
 224*x**15/5 + x**14*(-2*a/7 + 1280/7) + x**13*(48*a/13 - 7424/13
) + x**12*(-24*a + 4192/3) + x**11*(1120*a/11 - 29696/11) + x**10
*(3*a**2/5 - 1536*a/5 + 4096) + x**9*(-16*a**2/3 + 2048*a/3 - 143
36/3) + x**8*(24*a**2 - 1120*a + 4096) + x**7*(-480*a**2/7 + 9216
*a/7 - 16384/7) + x**6*(-2*a**3/3 + 128*a**2 - 1024*a + 2048/3) +
 x**5*(16*a**3/5 - 768*a**2/5 + 2048*a/5) + x**4*(-8*a**3 + 96*a*
*2)
```

GIAC/XCAS [A] time = 0.259783, size = 300, normalized size = 1.43

$$\frac{1}{18}x^{18} - \frac{16}{17}x^{17} + 8x^{16} - \frac{2}{7}ax^{14} - \frac{224}{5}x^{15} + \frac{48}{13}ax^{13} + \frac{1280}{7}x^{14} - 24ax^{12} - \frac{7424}{13}x^{13} + \frac{3}{5}a^2x^{10} + \frac{1120}{11}ax^{11} + \frac{4192}{3}x^{12} - \frac{16}{3}a^2x^9 - \frac{1536}{5}ax^{10} - \frac{29696}{11}x^{11} + 24a^2x^8 + \frac{2048}{3}ax^9 + 4096x^{10} - \frac{2}{3}a^3x^6 - \frac{480}{7}a^2x^7 - 1120ax^8 - \frac{14336}{3}x^9 + \frac{16}{5}a^3x^5 + 128a^2x^6 + \frac{9216}{7}ax^7 + 4096x^8 - 8a^3x^4 - \frac{768}{5}a^2x^5 - 1024ax^6 - \frac{16384}{7}x^7 + \frac{1}{2}a^4x^2 + \frac{32}{3}a^3x^3 + 96a^2x^4 + \frac{2048}{5}ax^5 + \frac{2048}{3}x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 - 4*x^3 + 8*x^2 - a - 8*x)^4*x,x, algorithm="giac")
```

```
[Out] 1/18*x^18 - 16/17*x^17 + 8*x^16 - 2/7*a*x^14 - 224/5*x^15 + 48/13
*a*x^13 + 1280/7*x^14 - 24*a*x^12 - 7424/13*x^13 + 3/5*a^2*x^10 +
 1120/11*a*x^11 + 4192/3*x^12 - 16/3*a^2*x^9 - 1536/5*a*x^10 - 29
696/11*x^11 + 24*a^2*x^8 + 2048/3*a*x^9 + 4096*x^10 - 2/3*a^3*x^6
 - 480/7*a^2*x^7 - 1120*a*x^8 - 14336/3*x^9 + 16/5*a^3*x^5 + 128*
a^2*x^6 + 9216/7*a*x^7 + 4096*x^8 - 8*a^3*x^4 - 768/5*a^2*x^5 - 1
024*a*x^6 - 16384/7*x^7 + 1/2*a^4*x^2 + 32/3*a^3*x^3 + 96*a^2*x^4
 + 2048/5*a*x^5 + 2048/3*x^6
```

3.124 $\int x\left(a+8x-8x^2+4x^3-x^4\right)^3\,dx$

Optimal. Leaf size=134

$$\frac{a^3x^2}{2}-\frac{1}{2}\left(a^2-128a+512\right)x^6+\frac{4}{5}\left(3a^2-96a+128\right)x^5+8a^2x^3-\frac{3}{10}(256-a)x^{10}+\frac{8}{3}(64-a)x^9-4(70-3a)x^8+\frac{48}{7}(48-5a)x^7+6(8-a)ax^4-\frac{x^{14}}{14}+\frac{12x^{13}}{13}-6x^{12}+\frac{280x^{11}}{11}$$

```
[Out] (a^3*x^2)/2 + 8*a^2*x^3 + 6*(8 - a)*a*x^4 + (4*(128 - 96*a + 3*a^
2)*x^5)/5 - ((512 - 128*a + a^2)*x^6)/2 + (48*(48 - 5*a)*x^7)/7 -
 4*(70 - 3*a)*x^8 + (8*(64 - a)*x^9)/3 - (3*(256 - a)*x^10)/10 +
(280*x^11)/11 - 6*x^12 + (12*x^13)/13 - x^14/14
```

Rubi [A] time = 0.294727, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042

$$\frac{a^3x^2}{2}-\frac{1}{2}\left(a^2-128a+512\right)x^6+\frac{4}{5}\left(3a^2-96a+128\right)x^5+8a^2x^3-\frac{3}{10}(256-a)x^{10}+\frac{8}{3}(64-a)x^9-4(70-3a)x^8+\frac{48}{7}(48-5a)x^7+6(8-a)ax^4-\frac{x^{14}}{14}+\frac{12x^{13}}{13}-6x^{12}+\frac{280x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]
```

```
[Out] (a^3*x^2)/2 + 8*a^2*x^3 + 6*(8 - a)*a*x^4 + (4*(128 - 96*a + 3*a^
2)*x^5)/5 - ((512 - 128*a + a^2)*x^6)/2 + (48*(48 - 5*a)*x^7)/7 -
 4*(70 - 3*a)*x^8 + (8*(64 - a)*x^9)/3 - (3*(256 - a)*x^10)/10 +
(280*x^11)/11 - 6*x^12 + (12*x^13)/13 - x^14/14
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(-x**4+4*x**3-8*x**2+a+8*x)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.0319769, size = 130, normalized size = 0.97

$$\frac{a^3x^2}{2}+\frac{1}{2}\left(-a^2+128a-512\right)x^6+\frac{4}{5}\left(3a^2-96a+128\right)x^5+8a^2x^3+\frac{3}{10}(a-256)x^{10}-\frac{8}{3}(a-64)x^9+4(3a-70)x^8-\frac{48}{7}(5a-48)x^7-6(a-8)ax^4-\frac{x^{14}}{14}+\frac{12x^{13}}{13}-6x^{12}+\frac{280x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]
```

```
[Out] (a^3*x^2)/2 + 8*a^2*x^3 - 6*(-8 + a)*a*x^4 + (4*(128 - 96*a + 3*a
^2)*x^5)/5 + ((-512 + 128*a - a^2)*x^6)/2 - (48*(-48 + 5*a)*x^7)/
7 + 4*(-70 + 3*a)*x^8 - (8*(-64 + a)*x^9)/3 + (3*(-256 + a)*x^10)
```

```
/10 + (280*x^11)/11 - 6*x^12 + (12*x^13)/13 - x^14/14
```

Maple [A] time = 0.002, size = 143, normalized size = 1.1

$$-\frac{x^{14}}{14}+\frac{12\,x^{13}}{13}-6\,x^{12}+\frac{280\,x^{11}}{11}+\frac{(3\,a-768)\,x^{10}}{10}+\frac{(-24\,a+1536)\,x^{9}}{9}+\frac{(96\,a-2240)\,x^{8}}{8}+\frac{(-240\,a+2304)\,x^{7}}{7}+\frac{\left(a\,(-2\,a+128)+256\,a-1536-a^{2}\right)\,x^{6}}{6}+\frac{\left(a\,(8\,a-128)-256\,a+512+4\,a^{2}\right)\,x^{5}}{5}+\frac{\left(a\,(-16\,a+64)+128\,a-8\,a^{2}\right)\,x^{4}}{4}+8\,a^{2}x^{3}+\frac{x^{2}a^{3}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(-x^4+4*x^3-8*x^2+a+8*x)^3,x)
```

```
[Out] -1/14*x^14+12/13*x^13-6*x^12+280/11*x^11+1/10*(3*a-768)*x^10+1/9*
(-24*a+1536)*x^9+1/8*(96*a-2240)*x^8+1/7*(-240*a+2304)*x^7+1/6*(a
*(-2*a+128)+256*a-1536-a^2)*x^6+1/5*(a*(8*a-128)-256*a+512+4*a^2)
*x^5+1/4*(a*(-16*a+64)+128*a-8*a^2)*x^4+8*a^2*x^3+1/2*x^2*a^3
```

Maxima [A] time = 0.801518, size = 153, normalized size = 1.14

$$-\frac{1}{14}\,x^{14}+\frac{12}{13}\,x^{13}-6\,x^{12}+\frac{3}{10}\,(a-256)x^{10}+\frac{280}{11}\,x^{11}-\frac{8}{3}\,(a-64)x^{9}+4\,(3\,a-70)x^{8}-\frac{48}{7}\,(5\,a-48)x^{7}-\frac{1}{2}\,\left(a^{2}-128\,a+512\right)x^{6}+\frac{4}{5}\,\left(3\,a^{2}-96\,a+128\right)x^{5}+\frac{1}{2}\,a^{3}x^{2}+8\,a^{2}x^{3}-6\,\left(a^{2}-8\,a\right)x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 4*x^3 + 8*x^2 - a - 8*x)^3*x,x, algorithm="maxima")
```

```
[Out] -1/14*x^14 + 12/13*x^13 - 6*x^12 + 3/10*(a - 256)*x^10 + 280/11*x
^11 - 8/3*(a - 64)*x^9 + 4*(3*a - 70)*x^8 - 48/7*(5*a - 48)*x^7 -
 1/2*(a^2 - 128*a + 512)*x^6 + 4/5*(3*a^2 - 96*a + 128)*x^5 + 1/2
*a^3*x^2 + 8*a^2*x^3 - 6*(a^2 - 8*a)*x^4
```

Fricas [A] time = 0.232488, size = 1, normalized size = 0.01

$$-\frac{1}{14}x^{14}+\frac{12}{13}x^{13}-6x^{12}+\frac{280}{11}x^{11}+\frac{3}{10}x^{10}a-\frac{384}{5}x^{10}-\frac{8}{3}x^{9}a+\frac{512}{3}x^{9}+12x^{8}a-280x^{8}-\frac{240}{7}x^{7}a-\frac{1}{2}x^{6}a^{2}+\frac{2304}{7}x^{7}+64x^{6}a+\frac{12}{5}x^{5}a^{2}-256x^{6}-\frac{384}{5}x^{5}a-6x^{4}a^{2}+\frac{512}{5}x^{5}+48x^{4}a+8x^{3}a^{2}+\frac{1}{2}x^{2}a^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 4*x^3 + 8*x^2 - a - 8*x)^3*x,x, algorithm="fricas")
```

```
[Out] -1/14*x^14 + 12/13*x^13 - 6*x^12 + 280/11*x^11 + 3/10*x^10*a - 38
4/5*x^10 - 8/3*x^9*a + 512/3*x^9 + 12*x^8*a - 280*x^8 - 240/7*x^7
*a - 1/2*x^6*a^2 + 2304/7*x^7 + 64*x^6*a + 12/5*x^5*a^2 - 256*x^6
 - 384/5*x^5*a - 6*x^4*a^2 + 512/5*x^5 + 48*x^4*a + 8*x^3*a^2 + 1
/2*x^2*a^3
```

Sympy [A] time = 0.177258, size = 128, normalized size = 0.96

$$\frac{a^{3}x^{2}}{2} + 8a^{2}x^{3} - \frac{x^{14}}{14} + \frac{12x^{13}}{13} - 6x^{12} + \frac{280x^{11}}{11} + x^{10}\left(\frac{3a}{10} - \frac{384}{5}\right) + x^{9}\left(-\frac{8a}{3} + \frac{512}{3}\right) + x^{8}\left(12a - 280\right) + x^{7}\left(-\frac{240a}{7} + \frac{2304}{7}\right) + x^{6}\left(-\frac{a^{2}}{2} + 64a - 256\right) + x^{5}\left(\frac{12a^{2}}{5} - \frac{384a}{5} + \frac{512}{5}\right) + x^{4}\left(-6a^{2} + 48a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(-x**4+4*x**3-8*x**2+a+8*x)**3,x)
```

```
[Out] a**3*x**2/2 + 8*a**2*x**3 - x**14/14 + 12*x**13/13 - 6*x**12 + 28
0*x**11/11 + x**10*(3*a/10 - 384/5) + x**9*(-8*a/3 + 512/3) + x**
8*(12*a - 280) + x**7*(-240*a/7 + 2304/7) + x**6*(-a**2/2 + 64*a
- 256) + x**5*(12*a**2/5 - 384*a/5 + 512/5) + x**4*(-6*a**2 + 48*
a)
```

GIAC/XCAS [A] time = 0.258707, size = 180, normalized size = 1.34

$$-\frac{1}{14}x^{14} + \frac{12}{13}x^{13} - 6x^{12} + \frac{3}{10}ax^{10} + \frac{280}{11}x^{11} - \frac{8}{3}ax^{9} - \frac{384}{5}x^{10} + 12ax^{8} + \frac{512}{3}x^{9} - \frac{1}{2}a^{2}x^{6} - \frac{240}{7}ax^{7} - 280x^{8} + \frac{12}{5}a^{2}x^{5} + 64ax^{6} + \frac{2304}{7}x^{7} - 6a^{2}x^{4} - \frac{384}{5}ax^{5} - 256x^{6} + \frac{1}{2}a^{3}x^{2} + 8a^{2}x^{3} + 48ax^{4} + \frac{512}{5}x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 4*x^3 + 8*x^2 - a - 8*x)^3*x,x, algorithm="giac")
```

```
[Out] -1/14*x^14 + 12/13*x^13 - 6*x^12 + 3/10*a*x^10 + 280/11*x^11 - 8/
3*a*x^9 - 384/5*x^10 + 12*a*x^8 + 512/3*x^9 - 1/2*a^2*x^6 - 240/7
*a*x^7 - 280*x^8 + 12/5*a^2*x^5 + 64*a*x^6 + 2304/7*x^7 - 6*a^2*x
^4 - 384/5*a*x^5 - 256*x^6 + 1/2*a^3*x^2 + 8*a^2*x^3 + 48*a*x^4 +
 512/5*x^5
```

3.125 $\int x\left(a+8x-8x^2+4x^3-x^4\right)^2\,dx$

Optimal. Leaf size=79

$$\frac{a^2x^2}{2}+\frac{1}{3}(64-a)x^6-\frac{8}{5}(16-a)x^5+4(4-a)x^4+\frac{16ax^3}{3}+\frac{x^{10}}{10}-\frac{8x^9}{9}+4x^8-\frac{80x^7}{7}$$

```
[Out] (a^2*x^2)/2 + (16*a*x^3)/3 + 4*(4 - a)*x^4 - (8*(16 - a)*x^5)/5 +
 ((64 - a)*x^6)/3 - (80*x^7)/7 + 4*x^8 - (8*x^9)/9 + x^10/10
```

Rubi [A] time = 0.16401, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$

$$\frac{a^2x^2}{2}+\frac{1}{3}(64-a)x^6-\frac{8}{5}(16-a)x^5+4(4-a)x^4+\frac{16ax^3}{3}+\frac{x^{10}}{10}-\frac{8x^9}{9}+4x^8-\frac{80x^7}{7}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

```
[Out] (a^2*x^2)/2 + (16*a*x^3)/3 + 4*(4 - a)*x^4 - (8*(16 - a)*x^5)/5 +
 ((64 - a)*x^6)/3 - (80*x^7)/7 + 4*x^8 - (8*x^9)/9 + x^10/10
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(-x**4+4*x**3-8*x**2+a+8*x)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.0151128, size = 75, normalized size = 0.95

$$\frac{a^2x^2}{2}+\frac{1}{3}(64-a)x^6+\frac{8}{5}(a-16)x^5-4(a-4)x^4+\frac{16ax^3}{3}+\frac{x^{10}}{10}-\frac{8x^9}{9}+4x^8-\frac{80x^7}{7}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

```
[Out] (a^2*x^2)/2 + (16*a*x^3)/3 - 4*(-4 + a)*x^4 + (8*(-16 + a)*x^5)/5
 + ((64 - a)*x^6)/3 - (80*x^7)/7 + 4*x^8 - (8*x^9)/9 + x^10/10
```

Maple [A] time = 0.002, size = 66, normalized size = 0.8

$$\frac{x^{10}}{10}-\frac{8\,x^9}{9}+4\,x^8-\frac{80\,x^7}{7}+\frac{(-2\,a+128)\,x^6}{6}+\frac{(8\,a-128)\,x^5}{5}+\frac{(-16\,a+64)\,x^4}{4}+\frac{16\,ax^3}{3}+\frac{a^2x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(-x^4+4*x^3-8*x^2+a+8*x)^2,x)
```

```
[Out] 1/10*x^10-8/9*x^9+4*x^8-80/7*x^7+1/6*(-2*a+128)*x^6+1/5*(8*a-128)
*x^5+1/4*(-16*a+64)*x^4+16/3*a*x^3+1/2*a^2*x^2
```

Maxima [A] time = 0.805447, size = 80, normalized size = 1.01

$$\frac{1}{10}\,x^{10} - \frac{8}{9}\,x^{9} + 4\,x^{8} - \frac{1}{3}\,(a-64)x^{6} - \frac{80}{7}\,x^{7} + \frac{8}{5}\,(a-16)x^{5} - 4\,(a-4)x^{4} + \frac{1}{2}\,a^{2}x^{2} + \frac{16}{3}\,ax^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 - 4*x^3 + 8*x^2 - a - 8*x)^2*x,x, algorithm="maxima")
```

```
[Out] 1/10*x^10 - 8/9*x^9 + 4*x^8 - 1/3*(a - 64)*x^6 - 80/7*x^7 + 8/5*(
a - 16)*x^5 - 4*(a - 4)*x^4 + 1/2*a^2*x^2 + 16/3*a*x^3
```

Fricas [A] time = 0.226312, size = 1, normalized size = 0.01

$$\frac{1}{10}x^{10} - \frac{8}{9}x^{9} + 4x^{8} - \frac{80}{7}x^{7} - \frac{1}{3}x^{6}a + \frac{64}{3}x^{6} + \frac{8}{5}x^{5}a - \frac{128}{5}x^{5} - 4x^{4}a + 16x^{4} + \frac{16}{3}x^{3}a + \frac{1}{2}x^{2}a^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 - 4*x^3 + 8*x^2 - a - 8*x)^2*x,x, algorithm="fricas")
```

```
[Out] 1/10*x^10 - 8/9*x^9 + 4*x^8 - 80/7*x^7 - 1/3*x^6*a + 64/3*x^6 + 8
/5*x^5*a - 128/5*x^5 - 4*x^4*a + 16*x^4 + 16/3*x^3*a + 1/2*x^2*a^
2
```

Sympy [A] time = 0.113957, size = 70, normalized size = 0.89

$$\frac{a^{2}x^{2}}{2} + \frac{16ax^{3}}{3} + \frac{x^{10}}{10} - \frac{8x^{9}}{9} + 4x^{8} - \frac{80x^{7}}{7} + x^{6}\left(-\frac{a}{3} + \frac{64}{3}\right) + x^{5}\left(\frac{8a}{5} - \frac{128}{5}\right) + x^{4}\left(-4a + 16\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(-x**4+4*x**3-8*x**2+a+8*x)**2,x)
```

```
[Out] a**2*x**2/2 + 16*a*x**3/3 + x**10/10 - 8*x**9/9 + 4*x**8 - 80*x**
7/7 + x**6*(-a/3 + 64/3) + x**5*(8*a/5 - 128/5) + x**4*(-4*a + 16
)
```

GIAC/XCAS [A] time = 0.259232, size = 92, normalized size = 1.16

$$\frac{1}{10}\,x^{10} - \frac{8}{9}\,x^{9} + 4\,x^{8} - \frac{1}{3}\,ax^{6} - \frac{80}{7}\,x^{7} + \frac{8}{5}\,ax^{5} + \frac{64}{3}\,x^{6} - 4\,ax^{4} - \frac{128}{5}\,x^{5} + \frac{1}{2}\,a^{2}x^{2} + \frac{16}{3}\,ax^{3} + 16\,x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 - 4*x^3 + 8*x^2 - a - 8*x)^2*x,x, algorithm="giac")
```

```
[Out] 1/10*x^10 - 8/9*x^9 + 4*x^8 - 1/3*a*x^6 - 80/7*x^7 + 8/5*a*x^5 +
64/3*x^6 - 4*a*x^4 - 128/5*x^5 + 1/2*a^2*x^2 + 16/3*a*x^3 + 16*x^
4
```

3.126 $\int x\left(a+8x-8x^2+4x^3-x^4\right)\,dx$

Optimal. Leaf size=35

$$\frac{ax^2}{2}-\frac{x^6}{6}+\frac{4x^5}{5}-2x^4+\frac{8x^3}{3}$$

```
[Out] (a*x^2)/2 + (8*x^3)/3 - 2*x^4 + (4*x^5)/5 - x^6/6
```

Rubi [A] time = 0.0268965, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045

$$\frac{ax^2}{2}-\frac{x^6}{6}+\frac{4x^5}{5}-2x^4+\frac{8x^3}{3}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]

[Out] (a*x^2)/2 + (8*x^3)/3 - 2*x^4 + (4*x^5)/5 - x^6/6
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(a-x^4+4x^3-8x^2+8x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(-x**4+4*x**3-8*x**2+a+8*x),x)

[Out] Integral(x*(a - x**4 + 4*x**3 - 8*x**2 + 8*x), x)
```

Mathematica [A] time = 0.00213301, size = 35, normalized size = 1.

$$\frac{ax^2}{2}-\frac{x^6}{6}+\frac{4x^5}{5}-2x^4+\frac{8x^3}{3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]

[Out] (a*x^2)/2 + (8*x^3)/3 - 2*x^4 + (4*x^5)/5 - x^6/6
```

Maple [A] time = 0.001, size = 28, normalized size = 0.8

$$\frac{ax^2}{2}+\frac{8\,x^3}{3}-2\,x^4+\frac{4\,x^5}{5}-\frac{x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(-x^4+4*x^3-8*x^2+a+8*x),x)
```

```
[Out] 1/2*a*x^2+8/3*x^3-2*x^4+4/5*x^5-1/6*x^6
```

Maxima [A] time = 0.808736, size = 36, normalized size = 1.03

$$-\frac{1}{6}x^6 + \frac{4}{5}x^5 - 2x^4 + \frac{1}{2}ax^2 + \frac{8}{3}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 4*x^3 + 8*x^2 - a - 8*x)*x,x, algorithm="maxima")
```

```
[Out] -1/6*x^6 + 4/5*x^5 - 2*x^4 + 1/2*a*x^2 + 8/3*x^3
```

Fricas [A] time = 0.2392, size = 1, normalized size = 0.03

$$-\frac{1}{6}x^6 + \frac{4}{5}x^5 - 2x^4 + \frac{8}{3}x^3 + \frac{1}{2}x^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 4*x^3 + 8*x^2 - a - 8*x)*x,x, algorithm="fricas")
```

```
[Out] -1/6*x^6 + 4/5*x^5 - 2*x^4 + 8/3*x^3 + 1/2*x^2*a
```

Sympy [A] time = 0.064578, size = 29, normalized size = 0.83

$$\frac{ax^2}{2} - \frac{x^6}{6} + \frac{4x^5}{5} - 2x^4 + \frac{8x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(-x**4+4*x**3-8*x**2+a+8*x),x)
```

```
[Out] a*x**2/2 - x**6/6 + 4*x**5/5 - 2*x**4 + 8*x**3/3
```

GIAC/XCAS [A] time = 0.261089, size = 36, normalized size = 1.03

$$-\frac{1}{6}x^6 + \frac{4}{5}x^5 - 2x^4 + \frac{1}{2}ax^2 + \frac{8}{3}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 4*x^3 + 8*x^2 - a - 8*x)*x,x, algorithm="giac")
```

```
[Out] -1/6*x^6 + 4/5*x^5 - 2*x^4 + 1/2*a*x^2 + 8/3*x^3
```

3.127 $\int \frac{x}{a+8x-8x^2+4x^3-x^4}\,dx$

Optimal. Leaf size=116

$$-\frac{\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{2\sqrt{a+4}\sqrt{1-\sqrt{a+4}}}+\frac{\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{2\sqrt{a+4}\sqrt{\sqrt{a+4}+1}}+\frac{\tanh^{-1}\left(\frac{(x-1)^2+1}{\sqrt{a+4}}\right)}{2\sqrt{a+4}}$$

```
[Out] -ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4 + a]]]/(2*Sqrt[4 + a]*Sqrt[1 - S
qrt[4 + a]]) + ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]]/(2*Sqrt[4 +
 a]*Sqrt[1 + Sqrt[4 + a]]) + ArcTanh[(1 + (-1 + x)^2)/Sqrt[4 + a]
]/(2*Sqrt[4 + a])
```

Rubi [A] time = 0.240668, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292

$$\frac{\tan^{-1}\left(\frac{1-x}{\sqrt{1-\sqrt{a+4}}}\right)}{2\sqrt{a+4}\sqrt{1-\sqrt{a+4}}}-\frac{\tan^{-1}\left(\frac{1-x}{\sqrt{\sqrt{a+4}+1}}\right)}{2\sqrt{a+4}\sqrt{\sqrt{a+4}+1}}+\frac{\tanh^{-1}\left(\frac{(x-1)^2+1}{\sqrt{a+4}}\right)}{2\sqrt{a+4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]
```

```
[Out] ArcTan[(1 - x)/Sqrt[1 - Sqrt[4 + a]]]/(2*Sqrt[4 + a]*Sqrt[1 - Sqr
t[4 + a]]) - ArcTan[(1 - x)/Sqrt[1 + Sqrt[4 + a]]]/(2*Sqrt[4 + a]
*Sqrt[1 + Sqrt[4 + a]]) + ArcTanh[(1 + (-1 + x)^2)/Sqrt[4 + a]]/(
2*Sqrt[4 + a])
```

Rubi in Sympy [A] time = 43.0483, size = 99, normalized size = 0.85

$$-\frac{\operatorname{atanh}\left(\frac{-(x-1)^2-1}{\sqrt{a+4}}\right)}{2\sqrt{a+4}}+\frac{\operatorname{atan}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{2\sqrt{a+4}\sqrt{\sqrt{a+4}+1}}-\frac{\operatorname{atan}\left(\frac{x-1}{\sqrt{-\sqrt{a+4}+1}}\right)}{2\sqrt{a+4}\sqrt{-\sqrt{a+4}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(-x**4+4*x**3-8*x**2+a+8*x),x)
```

```
[Out] -atanh((-(x - 1)**2 - 1)/sqrt(a + 4))/(2*sqrt(a + 4)) + atan((x -
 1)/sqrt(sqrt(a + 4) + 1))/(2*sqrt(a + 4)*sqrt(sqrt(a + 4) + 1))
- atan((x - 1)/sqrt(-sqrt(a + 4) + 1))/(2*sqrt(a + 4)*sqrt(-sqrt(
a + 4) + 1))
```

Mathematica [C] time = 0.0283873, size = 59, normalized size = 0.51

$$-\frac{1}{4}\text{RootSum}\left[-\#1^4+4\#1^3-8\#1^2+8\#1+a\&,\frac{\#1\log(x-\#1)}{\#1^3-3\#1^2+4\#1-2}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]
```

```
[Out] -RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 & , (Log[x - #1]*#1)/(
-2 + 4*#1 - 3*#1^2 + #1^3) & ]/4
```

Maple [C] time = 0.004, size = 50, normalized size = 0.4

$$-\frac{1}{4}\sum_{\_R=RootOf(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a)}\frac{\_R\ln(x-\_R)}{\_R^3-3\_R^2+4\_R-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^4+4*x^3-8*x^2+a+8*x),x)
```

```
[Out] -1/4*sum(_R/(_R^3-3*_R^2+4*_R-2)*ln(x-_R),_R=RootOf(_Z^4-4*_Z^3+8
*_Z^2-8*_Z-a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x}{x^4-4x^3+8x^2-a-8x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x/(x^4 - 4*x^3 + 8*x^2 - a - 8*x),x, algorithm="maxima")
```

```
[Out] -integrate(x/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x/(x^4 - 4*x^3 + 8*x^2 - a - 8*x),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 8.24331, size = 155, normalized size = 1.34

$$-\operatorname{RootSum}\left(t^4\left(256a^3+2816a^2+10240a+12288\right)+t^2\left(-32a^2-256a-512\right)+t\left(-16a-64\right)+a,\left(t\mapsto t\log\left(x+\frac{-128t^3}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**4+4*x**3-8*x**2+a+8*x),x)
```

```
[Out] -RootSum(_t**4*(256*a**3 + 2816*a**2 + 10240*a + 12288) + _t**2*(
-32*a**2 - 256*a - 512) + _t*(-16*a - 64) + a, Lambda(_t, _t*log(
x + (-128*_t**3*a**4 - 1728*_t**3*a**3 - 8640*_t**3*a**2 - 18944*
_t**3*a - 15360*_t**3 + 48*_t**2*a**3 + 464*_t**2*a**2 + 1472*_t*
*2*a + 1536*_t**2 + 8*_t*a**3 + 88*_t*a**2 + 312*_t*a + 352*_t -
a**2 - 2*a)/(4*a**2 + 21*a + 28))))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x}{x^4 - 4x^3 + 8x^2 - a - 8x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x/(x^4 - 4*x^3 + 8*x^2 - a - 8*x),x, algorithm="giac")
```

```
[Out] integrate(-x/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)
```

3.128 $\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^2}\,dx$

Optimal. Leaf size=231

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{4\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)}+\frac{(x-1)^2+1}{4(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)}-\frac{\left(3a+\sqrt{a+4}+10\right)\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{8(a+3)(a+4)^{3/2}\sqrt{1-\sqrt{a+4}}}+\frac{\left(3a-\sqrt{a+4}+10\right)\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{8(a+3)(a+4)^{3/2}\sqrt{\sqrt{a+4}+1}}+\frac{\tanh^{-1}\left(\frac{(x-1)^2+1}{\sqrt{a+4}}\right)}{4(a+4)^{3/2}}$$

```
[Out] (1 + (-1 + x)^2)/(4*(4 + a)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4))
+ ((5 + a + (-1 + x)^2)*(-1 + x))/(4*(12 + 7*a + a^2)*(3 + a - 2*
(-1 + x)^2 - (-1 + x)^4)) - ((10 + 3*a + Sqrt[4 + a])*ArcTan[(-1
+ x)/Sqrt[1 - Sqrt[4 + a]]])/(8*(3 + a)*(4 + a)^(3/2)*Sqrt[1 - Sq
rt[4 + a]]) + ((10 + 3*a - Sqrt[4 + a])*ArcTan[(-1 + x)/Sqrt[1 +
Sqrt[4 + a]]])/(8*(3 + a)*(4 + a)^(3/2)*Sqrt[1 + Sqrt[4 + a]]) +
ArcTanh[(1 + (-1 + x)^2)/Sqrt[4 + a]]/(4*(4 + a)^(3/2))
```

Rubi [A] time = 0.739641, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375

$$-\frac{(1-x)\left(a+(x-1)^2+5\right)}{4\left(a^2+7a+12\right)\left(a-(1-x)^4-2(1-x)^2+3\right)}+\frac{(x-1)^2+1}{4(a+4)\left(a-(1-x)^4-2(1-x)^2+3\right)}+\frac{\left(3a+\sqrt{a+4}+10\right)\tan^{-1}\left(\frac{1-x}{\sqrt{1-\sqrt{a+4}}}\right)}{8(a+3)(a+4)^{3/2}\sqrt{1-\sqrt{a+4}}}-\frac{\left(3a-\sqrt{a+4}+10\right)\tan^{-1}\left(\frac{1-x}{\sqrt{\sqrt{a+4}+1}}\right)}{8(a+3)(a+4)^{3/2}\sqrt{\sqrt{a+4}+1}}+\frac{\tanh^{-1}\left(\frac{(x-1)^2+1}{\sqrt{a+4}}\right)}{4(a+4)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

```
[Out] (1 + (-1 + x)^2)/(4*(4 + a)*(3 + a - 2*(1 - x)^2 - (1 - x)^4)) -
((5 + a + (-1 + x)^2)*(1 - x))/(4*(12 + 7*a + a^2)*(3 + a - 2*(1
- x)^2 - (1 - x)^4)) + ((10 + 3*a + Sqrt[4 + a])*ArcTan[(1 - x)/S
qrt[1 - Sqrt[4 + a]]])/(8*(3 + a)*(4 + a)^(3/2)*Sqrt[1 - Sqrt[4 +
 a]]) - ((10 + 3*a - Sqrt[4 + a])*ArcTan[(1 - x)/Sqrt[1 + Sqrt[4
+ a]]])/(8*(3 + a)*(4 + a)^(3/2)*Sqrt[1 + Sqrt[4 + a]]) + ArcTanh
[(1 + (-1 + x)^2)/Sqrt[4 + a]]/(4*(4 + a)^(3/2))
```

Rubi in Sympy [A] time = 82.0614, size = 184, normalized size = 0.8

$$-\frac{\operatorname{atanh}\left(\frac{-(x-1)^2-1}{\sqrt{a+4}}\right)}{4\left(a+4\right)^{\frac{3}{2}}}+\frac{(x-1)\left(2a+(2a+10)(x-1)+2(x-1)^3+2(x-1)^2+10\right)}{8(a+3)(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)}+\frac{\left(3a-\sqrt{a+4}+10\right)\operatorname{atan}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{8(a+3)(a+4)^{\frac{3}{2}}\sqrt{\sqrt{a+4}+1}}-\frac{\left(3a+\sqrt{a+4}+10\right)\operatorname{atan}\left(\frac{x-1}{\sqrt{-\sqrt{a+4}+1}}\right)}{8(a+3)(a+4)^{\frac{3}{2}}\sqrt{-\sqrt{a+4}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(-x**4+4*x**3-8*x**2+a+8*x)**2,x)
```

```
[Out] -atanh((-(x - 1)**2 - 1)/sqrt(a + 4))/(4*(a + 4)**(3/2)) + (x - 1
)*(2*a + (2*a + 10)*(x - 1) + 2*(x - 1)**3 + 2*(x - 1)**2 + 10)/(
8*(a + 3)*(a + 4)*(a - (x - 1)**4 - 2*(x - 1)**2 + 3)) + (3*a - s
qrt(a + 4) + 10)*atan((x - 1)/sqrt(sqrt(a + 4) + 1))/(8*(a + 3)*(
```

```
a + 4)**(3/2)*sqrt(sqrt(a + 4) + 1)) - (3*a + sqrt(a + 4) + 10)*a
tan((x - 1)/sqrt(-sqrt(a + 4) + 1))/(8*(a + 3)*(a + 4)**(3/2)*sqr
t(-sqrt(a + 4) + 1))
```

Mathematica [C] time = 0.0986098, size = 166, normalized size = 0.72

$$\frac{ax^2 - ax + a + x^3 + 2x}{4(a+3)(a+4)\,(a - x\,(x^3 - 4x^2 + 8x - 8))} - \frac{\text{RootSum}\left[-\#1^4 + 4\#1^3 - 8\#1^2 + 8\#1 + a\&, \frac{\#1^2 \log(x-\#1)+2\#1 a \log(x-\#1)+a \log(x-\#1)+4\#1 \log(x-\#1)+6 \log(x-\#1)}{\#1^3-3\#1^2+4\#1-2}\&\right]}{16\,(a^2 + 7a + 12)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

```
[Out] (a + 2*x - a*x + a*x^2 + x^3)/(4*(3 + a)*(4 + a)*(a - x*(-8 + 8*x
 - 4*x^2 + x^3))) - RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 & ,
 (6*Log[x - #1] + a*Log[x - #1] + 4*Log[x - #1]*#1 + 2*a*Log[x -
#1]*#1 + Log[x - #1]*#1^2)/(-2 + 4*#1 - 3*#1^2 + #1^3) & ]/(16*(1
2 + 7*a + a^2))
```

Maple [C] time = 0.025, size = 162, normalized size = 0.7

$$\frac{1}{x^4 - 4\,x^3 + 8\,x^2 - a - 8\,x}\left(-\frac{x^3}{4\,a^2 + 28\,a + 48} - \frac{ax^2}{4\,a^2 + 28\,a + 48} + \frac{(a-2)\,x}{4\,a^2 + 28\,a + 48} - \frac{a}{4\,a^2 + 28\,a + 48}\right) + \frac{1}{16}\sum_{\_R = RootOf(\_Z^4 - 4\_Z^3 + 8\_Z^2 - 8\_Z - a)} \frac{\left(-6 - \_R^2 + 2\,(-a-2)\,\_R - a\right)\ln(x - \_R)}{(a^2 + 7\,a + 12)\,(\_R^3 - 3\,\_R^2 + 4\,\_R - 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(-x^4+4*x^3-8*x^2+a+8*x)^2,x)
```

```
[Out] (-1/4/(a^2+7*a+12)*x^3-1/4*a/(a^2+7*a+12)*x^2+1/4*(a-2)/(a^2+7*a+
12)*x-1/4*a/(a^2+7*a+12))/(x^4-4*x^3+8*x^2-a-8*x)+1/16*sum((-6-_R
^2+2*(-a-2)*_R-a)/(a^2+7*a+12)/(_R^3-3*_R^2+4*_R-2)*ln(x-_R),_R=R
ootOf(_Z^4-4*_Z^3+8*_Z^2-8*_Z-a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{ax^2 + x^3 - (a-2)x + a}{4\left((a^2 + 7\,a + 12)x^4 - 4\,(a^2 + 7\,a + 12)x^3 - a^3 + 8\,(a^2 + 7\,a + 12)x^2 - 7\,a^2 - 8\,(a^2 + 7\,a + 12)x - 12\,a\right)} - \frac{\int \frac{2\,(a+2)x + x^2 + a + 6}{x^4 - 4\,x^3 + 8\,x^2 - a - 8\,x}\,dx}{4\,(a^2 + 7\,a + 12)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^4 - 4*x^3 + 8*x^2 - a - 8*x)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(a*x^2 + x^3 - (a - 2)*x + a)/((a^2 + 7*a + 12)*x^4 - 4*(a^2
 + 7*a + 12)*x^3 - a^3 + 8*(a^2 + 7*a + 12)*x^2 - 7*a^2 - 8*(a^2
+ 7*a + 12)*x - 12*a) - 1/4*integrate((2*(a + 2)*x + x^2 + a + 6)
/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)/(a^2 + 7*a + 12)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^4 - 4*x^3 + 8*x^2 - a - 8*x)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 45.0025, size = 539, normalized size = 2.33

$$-\frac{ax^2+a+x^3+x(-a+2)}{-4a^3-28a^2-48a+x^4(4a^2+28a+48)+x^3(-16a^2-112a-192)+x^2(32a^2+224a+384)+x(-32a^2-224a-384)}$$
$$+\mathrm{RootSum}\Big(t^4\left(65536a^9+2162688a^8+31653888a^7+269680640a^6+1473773568a^5+5357174784a^4+12952010752a^3+20\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(-x**4+4*x**3-8*x**2+a+8*x)**2,x)
```

```
[Out] -(a*x**2 + a + x**3 + x*(-a + 2))/(-4*a**3 - 28*a**2 - 48*a + x**
4*(4*a**2 + 28*a + 48) + x**3*(-16*a**2 - 112*a - 192) + x**2*(32
*a**2 + 224*a + 384) + x*(-32*a**2 - 224*a - 384)) + RootSum(_t**
4*(65536*a**9 + 2162688*a**8 + 31653888*a**7 + 269680640*a**6 + 1
473773568*a**5 + 5357174784*a**4 + 12952010752*a**3 + 20082327552
*a**2 + 18119393280*a + 7247757312) + _t**2*(-2048*a**6 - 50688*a
**5 - 520704*a**4 - 2842624*a**3 - 8699904*a**2 - 14155776*a - 95
68256) + _t*(1152*a**4 + 17792*a**3 + 102912*a**2 + 264192*a + 25
3952) + 16*a**3 - 57*a**2 - 984*a - 2064, Lambda(_t, _t*log(x + (
98304*_t**3*a**12 + 3948544*_t**3*a**11 + 72196096*_t**3*a**10 +
793837568*_t**3*a**9 + 5839372288*_t**3*a**8 + 30226464768*_t**3*
a**7 + 112668450816*_t**3*a**6 + 303864643584*_t**3*a**5 + 586157
391872*_t**3*a**4 + 784017129472*_t**3*a**3 + 683648483328*_t**3*
a**2 + 343136010240*_t**3*a + 72477573120*_t**3 + 30208*_t**2*a**
10 + 986624*_t**2*a**9 + 14420992*_t**2*a**8 + 124156928*_t**2*a*
*7 + 696815104*_t**2*a**6 + 2661758464*_t**2*a**5 + 7001485312*_t
**2*a**4 + 12506562560*_t**2*a**3 + 14494924800*_t**2*a**2 + 9820
569600*_t**2*a + 2944401408*_t**2 - 1536*_t*a**9 - 52048*_t*a**8
- 757040*_t*a**7 - 6200656*_t*a**6 - 31380496*_t*a**5 - 100736416
*_t*a**4 - 200813696*_t*a**3 - 228144640*_t*a**2 - 114632704*_t*a
 - 2490368*_t + 248*a**7 + 6797*a**6 + 71132*a**5 + 369745*a**4 +
 987758*a**3 + 1128896*a**2 - 129568*a - 956416)/(576*a**7 + 1098
5*a**6 + 88746*a**5 + 396609*a**4 + 1076268*a**3 + 1826304*a**2 +
 1867776*a + 917504))))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(x^4-4x^3+8x^2-a-8x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^4 - 4*x^3 + 8*x^2 - a - 8*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x/(x^4 - 4*x^3 + 8*x^2 - a - 8*x)^2, x)
```

3.129 $\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^3}\,dx$

Optimal. Leaf size=349

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{8\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^2}-\frac{3\left(7a^2+\left(4\sqrt{a+4}+47\right)a+14\sqrt{a+4}+80\right)\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{64(a+3)^2(a+4)^{5/2}\sqrt{1-\sqrt{a+4}}}-\frac{3\left(-\frac{7a^2+47a+80}{\sqrt{a+4}}+4a+14\right)\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{64(a+3)^2(a+4)^2\sqrt{\sqrt{a+4}+1}}+\frac{3\left((x-1)^2+1\right)}{16(a+4)^2\left(a-(x-1)^4-2(x-1)^2+3\right)}+\frac{(x-1)^2+1}{8(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)^2}+\frac{(x-1)\left(6(2a+7)(x-1)^2+(a+6)(7a+25)\right)}{32(a+3)^2(a+4)^2\left(a-(x-1)^4-2(x-1)^2+3\right)}+\frac{3\tanh^{-1}\left(\frac{(x-1)^2+1}{\sqrt{a+4}}\right)}{16(a+4)^{5/2}}$$

```
[Out] (1 + (-1 + x)^2)/(8*(4 + a)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)^2
) + (3*(1 + (-1 + x)^2))/(16*(4 + a)^2*(3 + a - 2*(-1 + x)^2 - (-
1 + x)^4)) + ((5 + a + (-1 + x)^2)*(-1 + x))/(8*(12 + 7*a + a^2)*
(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)^2) + (((6 + a)*(25 + 7*a) + 6
*(7 + 2*a)*(-1 + x)^2)*(-1 + x))/(32*(3 + a)^2*(4 + a)^2*(3 + a -
 2*(-1 + x)^2 - (-1 + x)^4)) - (3*(80 + 7*a^2 + 14*Sqrt[4 + a] +
a*(47 + 4*Sqrt[4 + a]))*ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4 + a]]])/(
64*(3 + a)^2*(4 + a)^(5/2)*Sqrt[1 - Sqrt[4 + a]]) - (3*(14 + 4*a
- (80 + 47*a + 7*a^2)/Sqrt[4 + a])*ArcTan[(-1 + x)/Sqrt[1 + Sqrt[
4 + a]]])/(64*(3 + a)^2*(4 + a)^2*Sqrt[1 + Sqrt[4 + a]]) + (3*Arc
Tanh[(1 + (-1 + x)^2)/Sqrt[4 + a]])/(16*(4 + a)^(5/2))
```

Rubi [A] time = 1.51275, antiderivative size = 349, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$

$$-\frac{(1-x)\left(a+(x-1)^2+5\right)}{8\left(a^2+7a+12\right)\left(a-(1-x)^4-2(1-x)^2+3\right)^2}+\frac{3\left(7a^2+\left(4\sqrt{a+4}+47\right)a+14\sqrt{a+4}+80\right)\tan^{-1}\left(\frac{1-x}{\sqrt{1-\sqrt{a+4}}}\right)}{64(a+3)^2(a+4)^{5/2}\sqrt{1-\sqrt{a+4}}}+\frac{3\left(-\frac{7a^2+47a+80}{\sqrt{a+4}}+4a+14\right)\tan^{-1}\left(\frac{1-x}{\sqrt{\sqrt{a+4}+1}}\right)}{64(a+3)^2(a+4)^2\sqrt{\sqrt{a+4}+1}}+\frac{3\left((x-1)^2+1\right)}{16(a+4)^2\left(a-(1-x)^4-2(1-x)^2+3\right)}+\frac{(x-1)^2+1}{8(a+4)\left(a-(1-x)^4-2(1-x)^2+3\right)^2}-\frac{(1-x)\left(6(2a+7)(1-x)^2+(a+6)(7a+25)\right)}{32(a+3)^2(a+4)^2\left(a-(1-x)^4-2(1-x)^2+3\right)}+\frac{3\tanh^{-1}\left(\frac{(x-1)^2+1}{\sqrt{a+4}}\right)}{16(a+4)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]
```

```
[Out] (1 + (-1 + x)^2)/(8*(4 + a)*(3 + a - 2*(1 - x)^2 - (1 - x)^4)^2)
+ (3*(1 + (-1 + x)^2))/(16*(4 + a)^2*(3 + a - 2*(1 - x)^2 - (1 -
x)^4)) - (((6 + a)*(25 + 7*a) + 6*(7 + 2*a)*(1 - x)^2)*(1 - x))/(
32*(3 + a)^2*(4 + a)^2*(3 + a - 2*(1 - x)^2 - (1 - x)^4)) - ((5 +
 a + (-1 + x)^2)*(1 - x))/(8*(12 + 7*a + a^2)*(3 + a - 2*(1 - x)^
2 - (1 - x)^4)^2) + (3*(80 + 7*a^2 + 14*Sqrt[4 + a] + a*(47 + 4*S
qrt[4 + a]))*ArcTan[(1 - x)/Sqrt[1 - Sqrt[4 + a]]])/(64*(3 + a)^2
```

```
*(4 + a)^(5/2)*Sqrt[1 - Sqrt[4 + a]]) + (3*(14 + 4*a - (80 + 47*a
 + 7*a^2)/Sqrt[4 + a])*ArcTan[(1 - x)/Sqrt[1 + Sqrt[4 + a]]])/(64
*(3 + a)^2*(4 + a)^2*Sqrt[1 + Sqrt[4 + a]]) + (3*ArcTanh[(1 + (-1
 + x)^2)/Sqrt[4 + a]])/(16*(4 + a)^(5/2))
```

Rubi in Sympy [A] time = 161.775, size = 292, normalized size = 0.84

$$-\frac{3\operatorname{atanh}\left(\frac{-(x-1)^2-1}{\sqrt{a+4}}\right)}{16(a+4)^{\frac{5}{2}}}+\frac{(x-1)\left(2a+(2a+10)(x-1)+2(x-1)^3+2(x-1)^2+10\right)}{16(a+3)(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)^2}$$
$$+\frac{(x-1)\left(28a^2+268a+(40a+136)(x-1)^3+(48a+168)(x-1)^2+(x-1)\left(24a^2+224a+488\right)+600\right)}{128(a+3)^2(a+4)^2\left(a-(x-1)^4-2(x-1)^2+3\right)}$$
$$+\frac{3\left(7a^2+47a-2\sqrt{a+4}(2a+7)+80\right)\operatorname{atan}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{64(a+3)^2(a+4)^{\frac{5}{2}}\sqrt{\sqrt{a+4}+1}}$$
$$-\frac{3\left(7a^2+47a+2\sqrt{a+4}(2a+7)+80\right)\operatorname{atan}\left(\frac{x-1}{\sqrt{-\sqrt{a+4}+1}}\right)}{64(a+3)^2(a+4)^{\frac{5}{2}}\sqrt{-\sqrt{a+4}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   rubi_integrate(x/(-x**4+4*x**3-8*x**2+a+8*x)**3,x)
```

```
[Out] -3*atanh((-(x - 1)**2 - 1)/sqrt(a + 4))/(16*(a + 4)**(5/2)) + (x
- 1)*(2*a + (2*a + 10)*(x - 1) + 2*(x - 1)**3 + 2*(x - 1)**2 + 10
)/(16*(a + 3)*(a + 4)*(a - (x - 1)**4 - 2*(x - 1)**2 + 3)**2) + (
x - 1)*(28*a**2 + 268*a + (40*a + 136)*(x - 1)**3 + (48*a + 168)*
(x - 1)**2 + (x - 1)*(24*a**2 + 224*a + 488) + 600)/(128*(a + 3)*
*2*(a + 4)**2*(a - (x - 1)**4 - 2*(x - 1)**2 + 3)) + 3*(7*a**2 +
47*a - 2*sqrt(a + 4)*(2*a + 7) + 80)*atan((x - 1)/sqrt(sqrt(a + 4
) + 1))/(64*(a + 3)**2*(a + 4)**(5/2)*sqrt(sqrt(a + 4) + 1)) - 3*
(7*a**2 + 47*a + 2*sqrt(a + 4)*(2*a + 7) + 80)*atan((x - 1)/sqrt(
-sqrt(a + 4) + 1))/(64*(a + 3)**2*(a + 4)**(5/2)*sqrt(-sqrt(a + 4
) + 1))
```

Mathematica [C] time = 0.208436, size = 284, normalized size = 0.81

$$\frac{1}{128}\left(-\frac{3\text{RootSum}\left[-\#1^4+4\#1^3-8\#1^2+8\#1+a\&,\frac{4\#1^2a\log(x-\#1)+14\#1^2\log(x-\#1)+3a^2\log(x-\#1)+4\#1a^2\log(x-\#1)+31a\log(x-\#1)+16\#}{\#1^3-3\#1^2+4\#1-2}\right.}{\left(a^2+7a+12\right)^2}\right.$$
$$+\frac{4\left(a^2\left(6x^2-5x+5\right)+a\left(12x^3+31x-7\right)+6\left(7x^3-12x^2+28x-14\right)\right)}{(a+3)^2(a+4)^2\left(a-x\left(x^3-4x^2+8x-8\right)\right)}$$
$$\left.+\frac{16\left(ax^2-ax+a+x^3+2x\right)}{(a+3)(a+4)\left(a-x\left(x^3-4x^2+8x-8\right)\right)^2}\right)$$

Antiderivative was successfully verified.

```
[In]   Integrate[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]
```

```
[Out] ((16*(a + 2*x - a*x + a*x^2 + x^3))/((3 + a)*(4 + a)*(a - x*(-8 +
 8*x - 4*x^2 + x^3))^2) + (4*(a^2*(5 - 5*x + 6*x^2) + 6*(-14 + 28
*x - 12*x^2 + 7*x^3) + a*(-7 + 31*x + 12*x^3)))/((3 + a)^2*(4 + a
)^2*(a - x*(-8 + 8*x - 4*x^2 + x^3))) - (3*RootSum[a + 8*#1 - 8*#
1^2 + 4*#1^3 - #1^4 & , (72*Log[x - #1] + 31*a*Log[x - #1] + 3*a^
2*Log[x - #1] + 8*Log[x - #1]*#1 + 16*a*Log[x - #1]*#1 + 4*a^2*Lo
g[x - #1]*#1 + 14*Log[x - #1]*#1^2 + 4*a*Log[x - #1]*#1^2)/(-2 +
```

```
4*#1 - 3*#1^2 + #1^3) & ])/(12 + 7*a + a^2)^2)/128
```

Maple [C] time = 0.036, size = 405, normalized size = 1.2

$$-\frac{1}{(x^4-4x^3+8x^2-a-8x)^2}\left(\frac{(6a+21)x^7}{16a^4+224a^3+1168a^2+2688a+2304}+\frac{(3a^2-24a-120)x^6}{16a^4+224a^3+1168a^2+2688a+2304}-\frac{}{32a^4}\right.$$

$$-\frac{3}{128}\sum_{\_R=RootOf(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a)}\frac{(72+2(7+2a)\_R^2+4(a^2+4a+2)\_R+3a^2+31a)\ln(x-\_R)}{(a^4+14a^3+73a^2+168a+144)(\_R^3-3\_R^2+4\_R-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(-x^4+4*x^3-8*x^2+a+8*x)^3,x)
```

```
[Out] -(3/16*(7+2*a)/(a^4+14*a^3+73*a^2+168*a+144)*x^7+3/16*(a^2-8*a-40
)/(a^4+14*a^3+73*a^2+168*a+144)*x^6-1/32*(29*a^2-127*a-792)/(a^4+
14*a^3+73*a^2+168*a+144)*x^5+1/32*(73*a^2-227*a-1668)/(a^4+14*a^3
+73*a^2+168*a+144)*x^4-1/16*(62*a^2-103*a-1104)/(a^4+14*a^3+73*a^
2+168*a+144)*x^3-1/16*(5*a^3-26*a^2+140*a+1008)/(a^4+14*a^3+73*a^
2+168*a+144)*x^2+3/32*(3*a^3-17*a^2-40*a+192)/(a^4+14*a^3+73*a^2+
168*a+144)*x-3/32*a*(3*a^2+7*a-12)/(a^4+14*a^3+73*a^2+168*a+144))
/(x^4-4*x^3+8*x^2-a-8*x)^2-3/128*sum((72+2*(7+2*a)*_R^2+4*(a^2+4*
a+2)*_R+3*a^2+31*a)/(a^4+14*a^3+73*a^2+168*a+144)/(_R^3-3*_R^2+4*
_R-2)*ln(x-_R),_R=RootOf(_Z^4-4*_Z^3+8*_Z^2-8*_Z-a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{6(2a+7)x^7+6(a^2-8a}{32((a^4+14a^3+73a^2+168a+144)x^8-8(a^4+14a^3+73a^2+168a+144)x^7+32(a^4+14a^3+73a^2+168a+144)x^6+}$$

$$-\frac{3\int\frac{2(2a+7)x^2+3a^2+4(a^2+4a+2)x+31a+72}{x^4-4x^3+8x^2-a-8x}\,dx}{32(a^4+14a^3+73a^2+168a+144)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x/(x^4 - 4*x^3 + 8*x^2 - a - 8*x)^3,x, algorithm="maxima")
```

```
[Out] -1/32*(6*(2*a + 7)*x^7 + 6*(a^2 - 8*a - 40)*x^6 - (29*a^2 - 127*a
 - 792)*x^5 + (73*a^2 - 227*a - 1668)*x^4 - 2*(62*a^2 - 103*a - 1
104)*x^3 - 9*a^3 - 2*(5*a^3 - 26*a^2 + 140*a + 1008)*x^2 - 21*a^2
 + 3*(3*a^3 - 17*a^2 - 40*a + 192)*x + 36*a)/((a^4 + 14*a^3 + 73*
a^2 + 168*a + 144)*x^8 - 8*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*
x^7 + 32*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^6 + a^6 - 80*(a^
4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^5 + 14*a^5 - 2*(a^5 - 50*a^4
 - 823*a^3 - 4504*a^2 - 10608*a - 9216)*x^4 + 73*a^4 + 8*(a^5 - 2
*a^4 - 151*a^3 - 1000*a^2 - 2544*a - 2304)*x^3 + 168*a^3 - 16*(a^
5 + 10*a^4 + 17*a^3 - 124*a^2 - 528*a - 576)*x^2 + 144*a^2 + 16*(
a^5 + 14*a^4 + 73*a^3 + 168*a^2 + 144*a)*x) - 3/32*integrate((2*(
2*a + 7)*x^2 + 3*a^2 + 4*(a^2 + 4*a + 2)*x + 31*a + 72)/(x^4 - 4*
x^3 + 8*x^2 - a - 8*x), x)/(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x/(x^4 - 4*x^3 + 8*x^2 - a - 8*x)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(-x**4+4*x**3-8*x**2+a+8*x)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x}{(x^4 - 4x^3 + 8x^2 - a - 8x)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x/(x^4 - 4*x^3 + 8*x^2 - a - 8*x)^3,x, algorithm="giac")
```

```
[Out] integrate(-x/(x^4 - 4*x^3 + 8*x^2 - a - 8*x)^3, x)
```

3.130 $\int x^2 \left(a+8x-8x^2+4x^3-x^4\right)^4 \, dx$

Optimal. Leaf size=210

$$\frac{a^4x^3}{3}+8a^3x^4+\frac{2}{11}\left(3a^2-1536a+20480\right)x^{11}-\frac{24}{5}\left(a^2-128a+896\right)x^{10}$$
$$-4\left(15a^2-288a+512\right)x^8+\frac{8}{3}a\left(a^2-48a+128\right)x^6+\frac{32}{5}(12-a)a^2x^5$$
$$+\frac{4}{7}\left(-a^3+192a^2-1536a+1024\right)x^7+\frac{4}{15}(640-a)x^{15}-\frac{8}{7}(464-3a)x^{14}+\frac{32}{13}(524-9a)x^{13}$$
$$-\frac{8}{3}(928-35a)x^{12}+\frac{64}{9}(128-3a)(4-a)x^9+\frac{x^{19}}{19}-\frac{8x^{18}}{9}+\frac{128x^{17}}{17}-42x^{16}$$

```
[Out] (a^4*x^3)/3 + 8*a^3*x^4 + (32*(12 - a)*a^2*x^5)/5 + (8*a*(128 - 4
8*a + a^2)*x^6)/3 + (4*(1024 - 1536*a + 192*a^2 - a^3)*x^7)/7 - 4
*(512 - 288*a + 15*a^2)*x^8 + (64*(128 - 3*a)*(4 - a)*x^9)/9 - (2
4*(896 - 128*a + a^2)*x^10)/5 + (2*(20480 - 1536*a + 3*a^2)*x^11)
/11 - (8*(928 - 35*a)*x^12)/3 + (32*(524 - 9*a)*x^13)/13 - (8*(46
4 - 3*a)*x^14)/7 + (4*(640 - a)*x^15)/15 - 42*x^16 + (128*x^17)/1
7 - (8*x^18)/9 + x^19/19
```

Rubi [A] time = 0.504945, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038

$$\frac{a^4x^3}{3}+8a^3x^4+\frac{2}{11}\left(3a^2-1536a+20480\right)x^{11}-\frac{24}{5}\left(a^2-128a+896\right)x^{10}$$
$$-4\left(15a^2-288a+512\right)x^8+\frac{8}{3}a\left(a^2-48a+128\right)x^6+\frac{32}{5}(12-a)a^2x^5$$
$$+\frac{4}{7}\left(-a^3+192a^2-1536a+1024\right)x^7+\frac{4}{15}(640-a)x^{15}-\frac{8}{7}(464-3a)x^{14}+\frac{32}{13}(524-9a)x^{13}$$
$$-\frac{8}{3}(928-35a)x^{12}+\frac{64}{9}(128-3a)(4-a)x^9+\frac{x^{19}}{19}-\frac{8x^{18}}{9}+\frac{128x^{17}}{17}-42x^{16}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x]
```

```
[Out] (a^4*x^3)/3 + 8*a^3*x^4 + (32*(12 - a)*a^2*x^5)/5 + (8*a*(128 - 4
8*a + a^2)*x^6)/3 + (4*(1024 - 1536*a + 192*a^2 - a^3)*x^7)/7 - 4
*(512 - 288*a + 15*a^2)*x^8 + (64*(128 - 3*a)*(4 - a)*x^9)/9 - (2
4*(896 - 128*a + a^2)*x^10)/5 + (2*(20480 - 1536*a + 3*a^2)*x^11)
/11 - (8*(928 - 35*a)*x^12)/3 + (32*(524 - 9*a)*x^13)/13 - (8*(46
4 - 3*a)*x^14)/7 + (4*(640 - a)*x^15)/15 - 42*x^16 + (128*x^17)/1
7 - (8*x^18)/9 + x^19/19
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(-x**4+4*x**3-8*x**2+a+8*x)**4,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.0509707, size = 204, normalized size = 0.97

$$\frac{a^4x^3}{3}+8a^3x^4+\frac{2}{11}\left(3a^2-1536a+20480\right)x^{11}-\frac{24}{5}\left(a^2-128a+896\right)x^{10}+\frac{64}{9}\left(3a^2-140a+512\right)x^9-4\left(15a^2-288a+512\right)x^8+\frac{8}{3}a\left(a^2-48a+128\right)x^6-\frac{32}{5}(a-12)a^2x^5-\frac{4}{7}\left(a^3-192a^2+1536a-1024\right)x^7-\frac{4}{15}(a-640)x^{15}+\frac{8}{7}(3a-464)x^{14}-\frac{32}{13}(9a-524)x^{13}+\frac{8}{3}(35a-928)x^{12}+\frac{x^{19}}{19}-\frac{8x^{18}}{9}+\frac{128x^{17}}{17}-42x^{16}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x]
```

```
[Out] (a^4*x^3)/3 + 8*a^3*x^4 - (32*(-12 + a)*a^2*x^5)/5 + (8*a*(128 -
48*a + a^2)*x^6)/3 - (4*(-1024 + 1536*a - 192*a^2 + a^3)*x^7)/7 -
 4*(512 - 288*a + 15*a^2)*x^8 + (64*(512 - 140*a + 3*a^2)*x^9)/9
- (24*(896 - 128*a + a^2)*x^10)/5 + (2*(20480 - 1536*a + 3*a^2)*x
^11)/11 + (8*(-928 + 35*a)*x^12)/3 - (32*(-524 + 9*a)*x^13)/13 +
(8*(-464 + 3*a)*x^14)/7 - (4*(-640 + a)*x^15)/15 - 42*x^16 + (128
*x^17)/17 - (8*x^18)/9 + x^19/19
```

Maple [A] time = 0.002, size = 267, normalized size = 1.3

$$\frac{x^{19}}{19}-\frac{8\,x^{18}}{9}+\frac{128\,x^{17}}{17}-42\,x^{16}+\frac{(-4\,a+2560)\,x^{15}}{15}+\frac{(48\,a-7424)\,x^{14}}{14}+\frac{(-288\,a+16768)\,x^{13}}{13}+\frac{(1120\,a-29696)\,x^{12}}{12}+\frac{\left(2\,a^2-2560\,a+24576+(-2\,a+128)^2\right)\,x^{11}}{11}+\frac{\left(-16\,a^2+3584\,a-10240+2\,(8\,a-128)\,(-2\,a+128)\right)\,x^{10}}{10}+\frac{\left(64\,a^2-2560\,a+2\,(-16\,a+64)\,(-2\,a+128)+(8\,a-128)^2\right)\,x^9}{9}+\frac{\left(-160\,a^2+32\,a\,(-2\,a+128)+2\,(-16\,a+64)\,(8\,a-128)\right)\,x^8}{8}+\frac{\left(2\,a^2\,(-2\,a+128)+32\,a\,(8\,a-128)+(-16\,a+64)^2\right)\,x^7}{7}+\frac{\left(2\,a^2\,(8\,a-128)+32\,a\,(-16\,a+64)\right)\,x^6}{6}+\frac{\left(2\,a^2\,(-16\,a+64)+256\,a^2\right)\,x^5}{5}+8\,a^3x^4+\frac{a^4x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^4,x)
```

```
[Out] 1/19*x^19-8/9*x^18+128/17*x^17-42*x^16+1/15*(-4*a+2560)*x^15+1/14
*(48*a-7424)*x^14+1/13*(-288*a+16768)*x^13+1/12*(1120*a-29696)*x^
12+1/11*(2*a^2-2560*a+24576+(-2*a+128)^2)*x^11+1/10*(-16*a^2+3584
*a-10240+2*(8*a-128)*(-2*a+128))*x^10+1/9*(64*a^2-2560*a+2*(-16*a
+64)*(-2*a+128)+(8*a-128)^2)*x^9+1/8*(-160*a^2+32*a*(-2*a+128)+2*
(-16*a+64)*(8*a-128))*x^8+1/7*(2*a^2*(-2*a+128)+32*a*(8*a-128)+(-
16*a+64)^2)*x^7+1/6*(2*a^2*(8*a-128)+32*a*(-16*a+64))*x^6+1/5*(2*
a^2*(-16*a+64)+256*a^2)*x^5+8*a^3*x^4+1/3*a^4*x^3
```

Maxima [A] time = 0.787546, size = 246, normalized size = 1.17

$$\frac{1}{19}x^{19} - \frac{8}{9}x^{18} + \frac{128}{17}x^{17} - \frac{4}{15}(a-640)x^{15} - 42x^{16} + \frac{8}{7}(3a-464)x^{14} - \frac{32}{13}(9a-524)x^{13} + \frac{8}{3}(35a-928)x^{12} + \frac{2}{11}\left(3a^2-1536a+20480\right)x^{11} - \frac{24}{5}\left(a^2-128a+896\right)x^{10} + \frac{64}{9}\left(3a^2-140a+512\right)x^9 - 4\left(15a^2-288a+512\right)x^8 - \frac{4}{7}\left(a^3-192a^2+1536a-1024\right)x^7 + \frac{1}{3}a^4x^3 + 8a^3x^4 + \frac{8}{3}\left(a^3-48a^2+128a\right)x^6 - \frac{32}{5}\left(a^3-12a^2\right)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 - 4*x^3 + 8*x^2 - a - 8*x)^4*x^2,x, algorithm="maxima")
```

```
[Out] 1/19*x^19 - 8/9*x^18 + 128/17*x^17 - 4/15*(a - 640)*x^15 - 42*x^1
6 + 8/7*(3*a - 464)*x^14 - 32/13*(9*a - 524)*x^13 + 8/3*(35*a - 9
28)*x^12 + 2/11*(3*a^2 - 1536*a + 20480)*x^11 - 24/5*(a^2 - 128*a
 + 896)*x^10 + 64/9*(3*a^2 - 140*a + 512)*x^9 - 4*(15*a^2 - 288*a
 + 512)*x^8 - 4/7*(a^3 - 192*a^2 + 1536*a - 1024)*x^7 + 1/3*a^4*x
^3 + 8*a^3*x^4 + 8/3*(a^3 - 48*a^2 + 128*a)*x^6 - 32/5*(a^3 - 12*
a^2)*x^5
```

Fricas [A] time = 0.230268, size = 1, normalized size = 0.

$$\frac{1}{19}x^{19} - \frac{8}{9}x^{18} + \frac{128}{17}x^{17} - 42x^{16} - \frac{4}{15}x^{15}a + \frac{512}{3}x^{15} + \frac{24}{7}x^{14}a - \frac{3712}{7}x^{14} - \frac{288}{13}x^{13}a + \frac{16768}{13}x^{13} + \frac{280}{3}x^{12}a + \frac{6}{11}x^{11}a^2 - \frac{7424}{3}x^{12} - \frac{3072}{11}x^{11}a - \frac{24}{5}x^{10}a^2 + \frac{40960}{11}x^{11} + \frac{3072}{5}x^{10}a + \frac{64}{3}x^9a^2 - \frac{21504}{5}x^{10} - \frac{8960}{9}x^9a - 60x^8a^2 - \frac{4}{7}x^7a^3 + \frac{32768}{9}x^9 + 1152x^8a + \frac{768}{7}x^7a^2 + \frac{8}{3}x^6a^3 - 2048x^8 - \frac{6144}{7}x^7a - 128x^6a^2 - \frac{32}{5}x^5a^3 + \frac{4096}{7}x^7 + \frac{1024}{3}x^6a + \frac{384}{5}x^5a^2 + 8x^4a^3 + \frac{1}{3}x^3a^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 - 4*x^3 + 8*x^2 - a - 8*x)^4*x^2,x, algorithm="fricas")
```

```
[Out] 1/19*x^19 - 8/9*x^18 + 128/17*x^17 - 42*x^16 - 4/15*x^15*a + 512/
3*x^15 + 24/7*x^14*a - 3712/7*x^14 - 288/13*x^13*a + 16768/13*x^1
3 + 280/3*x^12*a + 6/11*x^11*a^2 - 7424/3*x^12 - 3072/11*x^11*a -
 24/5*x^10*a^2 + 40960/11*x^11 + 3072/5*x^10*a + 64/3*x^9*a^2 - 2
1504/5*x^10 - 8960/9*x^9*a - 60*x^8*a^2 - 4/7*x^7*a^3 + 32768/9*x
^9 + 1152*x^8*a + 768/7*x^7*a^2 + 8/3*x^6*a^3 - 2048*x^8 - 6144/7
*x^7*a - 128*x^6*a^2 - 32/5*x^5*a^3 + 4096/7*x^7 + 1024/3*x^6*a +
 384/5*x^5*a^2 + 8*x^4*a^3 + 1/3*x^3*a^4
```

Sympy [A] time = 0.257307, size = 219, normalized size = 1.04

$$\frac{a^4x^3}{3} + 8a^3x^4 + \frac{x^{19}}{19} - \frac{8x^{18}}{9} + \frac{128x^{17}}{17} - 42x^{16} + x^{15}\left(-\frac{4a}{15} + \frac{512}{3}\right) + x^{14}\left(\frac{24a}{7} - \frac{3712}{7}\right) + x^{13}\left(-\frac{288a}{13} + \frac{16768}{13}\right) + x^{12}\left(\frac{280a}{3} - \frac{7424}{3}\right) + x^{11}\left(\frac{6a^2}{11} - \frac{3072a}{11} + \frac{40960}{11}\right) + x^{10}\left(-\frac{24a^2}{5} + \frac{3072a}{5} - \frac{21504}{5}\right) + x^9\left(\frac{64a^2}{3} - \frac{8960a}{9} + \frac{32768}{9}\right) + x^8\left(-60a^2 + 1152a - 2048\right) + x^7\left(-\frac{4a^3}{7} + \frac{768a^2}{7} - \frac{6144a}{7} + \frac{4096}{7}\right) + x^6\left(\frac{8a^3}{3} - 128a^2 + \frac{1024a}{3}\right) + x^5\left(-\frac{32a^3}{5} + \frac{384a^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(-x**4+4*x**3-8*x**2+a+8*x)**4,x)
```

```
[Out] a**4*x**3/3 + 8*a**3*x**4 + x**19/19 - 8*x**18/9 + 128*x**17/17 -
 42*x**16 + x**15*(-4*a/15 + 512/3) + x**14*(24*a/7 - 3712/7) + x
**13*(-288*a/13 + 16768/13) + x**12*(280*a/3 - 7424/3) + x**11*(6
*a**2/11 - 3072*a/11 + 40960/11) + x**10*(-24*a**2/5 + 3072*a/5 -
 21504/5) + x**9*(64*a**2/3 - 8960*a/9 + 32768/9) + x**8*(-60*a**
2 + 1152*a - 2048) + x**7*(-4*a**3/7 + 768*a**2/7 - 6144*a/7 + 40
96/7) + x**6*(8*a**3/3 - 128*a**2 + 1024*a/3) + x**5*(-32*a**3/5
+ 384*a**2/5)
```

GIAC/XCAS [A] time = 0.25987, size = 300, normalized size = 1.43

$$\frac{1}{19}\,x^{19} - \frac{8}{9}\,x^{18} + \frac{128}{17}\,x^{17} - \frac{4}{15}\,ax^{15} - 42\,x^{16} + \frac{24}{7}\,ax^{14} + \frac{512}{3}\,x^{15} - \frac{288}{13}\,ax^{13} - \frac{3712}{7}\,x^{14} + \frac{6}{11}\,a^{2}x^{11} + \frac{280}{3}\,ax^{12} + \frac{16768}{13}\,x^{13} - \frac{24}{5}\,a^{2}x^{10} - \frac{3072}{11}\,ax^{11} - \frac{7424}{3}\,x^{12} + \frac{64}{3}\,a^{2}x^{9} + \frac{3072}{5}\,ax^{10} + \frac{40960}{11}\,x^{11} - \frac{4}{7}\,a^{3}x^{7} - 60\,a^{2}x^{8} - \frac{8960}{9}\,ax^{9} - \frac{21504}{5}\,x^{10} + \frac{8}{3}\,a^{3}x^{6} + \frac{768}{7}\,a^{2}x^{7} + 1152\,ax^{8} + \frac{32768}{9}\,x^{9} - \frac{32}{5}\,a^{3}x^{5} - 128\,a^{2}x^{6} - \frac{6144}{7}\,ax^{7} - 2048\,x^{8} + \frac{1}{3}\,a^{4}x^{3} + 8\,a^{3}x^{4} + \frac{384}{5}\,a^{2}x^{5} + \frac{1024}{3}\,ax^{6} + \frac{4096}{7}\,x^{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 - 4*x^3 + 8*x^2 - a - 8*x)^4*x^2,x, algorithm="giac")
```

```
[Out] 1/19*x^19 - 8/9*x^18 + 128/17*x^17 - 4/15*a*x^15 - 42*x^16 + 24/7
*a*x^14 + 512/3*x^15 - 288/13*a*x^13 - 3712/7*x^14 + 6/11*a^2*x^1
1 + 280/3*a*x^12 + 16768/13*x^13 - 24/5*a^2*x^10 - 3072/11*a*x^11
 - 7424/3*x^12 + 64/3*a^2*x^9 + 3072/5*a*x^10 + 40960/11*x^11 - 4
/7*a^3*x^7 - 60*a^2*x^8 - 8960/9*a*x^9 - 21504/5*x^10 + 8/3*a^3*x
^6 + 768/7*a^2*x^7 + 1152*a*x^8 + 32768/9*x^9 - 32/5*a^3*x^5 - 12
8*a^2*x^6 - 6144/7*a*x^7 - 2048*x^8 + 1/3*a^4*x^3 + 8*a^3*x^4 + 3
84/5*a^2*x^5 + 1024/3*a*x^6 + 4096/7*x^7
```

3.131 $\int x^2 \left(a+8x-8x^2+4x^3-x^4\right)^3 \, dx$

Optimal. Leaf size=138

$$\frac{a^3x^3}{3}-\frac{3}{7}\left(a^2-128a+512\right)x^7+\frac{2}{3}\left(3a^2-96a+128\right)x^6+6a^2x^4-\frac{3}{11}(256-a)x^{11}+\frac{12}{5}(64-a)x^{10}-\frac{32}{9}(70-3a)x^9+6(48-5a)x^8+\frac{24}{5}(8-a)ax^5-\frac{x^{15}}{15}+\frac{6x^{14}}{7}-\frac{72x^{13}}{13}+\frac{70x^{12}}{3}$$

```
[Out] (a^3*x^3)/3 + 6*a^2*x^4 + (24*(8 - a)*a*x^5)/5 + (2*(128 - 96*a +
 3*a^2)*x^6)/3 - (3*(512 - 128*a + a^2)*x^7)/7 + 6*(48 - 5*a)*x^8
 - (32*(70 - 3*a)*x^9)/9 + (12*(64 - a)*x^10)/5 - (3*(256 - a)*x^
11)/11 + (70*x^12)/3 - (72*x^13)/13 + (6*x^14)/7 - x^15/15
```

Rubi [A] time = 0.323994, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038

$$\frac{a^3x^3}{3}-\frac{3}{7}\left(a^2-128a+512\right)x^7+\frac{2}{3}\left(3a^2-96a+128\right)x^6+6a^2x^4-\frac{3}{11}(256-a)x^{11}+\frac{12}{5}(64-a)x^{10}-\frac{32}{9}(70-3a)x^9+6(48-5a)x^8+\frac{24}{5}(8-a)ax^5-\frac{x^{15}}{15}+\frac{6x^{14}}{7}-\frac{72x^{13}}{13}+\frac{70x^{12}}{3}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]
```

```
[Out] (a^3*x^3)/3 + 6*a^2*x^4 + (24*(8 - a)*a*x^5)/5 + (2*(128 - 96*a +
 3*a^2)*x^6)/3 - (3*(512 - 128*a + a^2)*x^7)/7 + 6*(48 - 5*a)*x^8
 - (32*(70 - 3*a)*x^9)/9 + (12*(64 - a)*x^10)/5 - (3*(256 - a)*x^
11)/11 + (70*x^12)/3 - (72*x^13)/13 + (6*x^14)/7 - x^15/15
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(-x**4+4*x**3-8*x**2+a+8*x)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.0301622, size = 132, normalized size = 0.96

$$\frac{a^3x^3}{3}-\frac{3}{7}\left(a^2-128a+512\right)x^7+\frac{2}{3}\left(3a^2-96a+128\right)x^6+6a^2x^4+\frac{3}{11}(a-256)x^{11}-\frac{12}{5}(a-64)x^{10}+\frac{32}{9}(3a-70)x^9-6(5a-48)x^8-\frac{24}{5}(a-8)ax^5-\frac{x^{15}}{15}+\frac{6x^{14}}{7}-\frac{72x^{13}}{13}+\frac{70x^{12}}{3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]
```

```
[Out] (a^3*x^3)/3 + 6*a^2*x^4 - (24*(-8 + a)*a*x^5)/5 + (2*(128 - 96*a
+ 3*a^2)*x^6)/3 - (3*(512 - 128*a + a^2)*x^7)/7 - 6*(-48 + 5*a)*x
^8 + (32*(-70 + 3*a)*x^9)/9 - (12*(-64 + a)*x^10)/5 + (3*(-256 +
```

```
a)*x^11)/11 + (70*x^12)/3 - (72*x^13)/13 + (6*x^14)/7 - x^15/15
```

Maple [A] time = 0.002, size = 143, normalized size = 1.

$$-\frac{x^{15}}{15}+\frac{6\,x^{14}}{7}-\frac{72\,x^{13}}{13}+\frac{70\,x^{12}}{3}+\frac{(3\,a-768)\,x^{11}}{11}+\frac{(-24\,a+1536)\,x^{10}}{10}+\frac{(96\,a-2240)\,x^{9}}{9}+\frac{(-240\,a+2304)\,x^{8}}{8}+\frac{\left(a\,(-2\,a+128)+256\,a-1536-a^{2}\right)\,x^{7}}{7}+\frac{\left(a\,(8\,a-128)-256\,a+512+4\,a^{2}\right)\,x^{6}}{6}+\frac{\left(a\,(-16\,a+64)+128\,a-8\,a^{2}\right)\,x^{5}}{5}+6\,a^{2}x^{4}+\frac{a^{3}x^{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^3,x)
```

```
[Out] -1/15*x^15+6/7*x^14-72/13*x^13+70/3*x^12+1/11*(3*a-768)*x^11+1/10
*(-24*a+1536)*x^10+1/9*(96*a-2240)*x^9+1/8*(-240*a+2304)*x^8+1/7*
(a*(-2*a+128)+256*a-1536-a^2)*x^7+1/6*(a*(8*a-128)-256*a+512+4*a^
2)*x^6+1/5*(a*(-16*a+64)+128*a-8*a^2)*x^5+6*a^2*x^4+1/3*a^3*x^3
```

Maxima [A] time = 0.803474, size = 153, normalized size = 1.11

$$-\frac{1}{15}\,x^{15}+\frac{6}{7}\,x^{14}-\frac{72}{13}\,x^{13}+\frac{3}{11}\,(a-256)x^{11}+\frac{70}{3}\,x^{12}-\frac{12}{5}\,(a-64)x^{10}+\frac{32}{9}\,(3\,a-70)x^{9}-6\,(5\,a-48)x^{8}-\frac{3}{7}\,\left(a^{2}-128\,a+512\right)x^{7}+\frac{2}{3}\,\left(3\,a^{2}-96\,a+128\right)x^{6}+\frac{1}{3}\,a^{3}x^{3}+6\,a^{2}x^{4}-\frac{24}{5}\,\left(a^{2}-8\,a\right)x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 4*x^3 + 8*x^2 - a - 8*x)^3*x^2,x, algorithm="maxima")
```

```
[Out] -1/15*x^15 + 6/7*x^14 - 72/13*x^13 + 3/11*(a - 256)*x^11 + 70/3*x
^12 - 12/5*(a - 64)*x^10 + 32/9*(3*a - 70)*x^9 - 6*(5*a - 48)*x^8
 - 3/7*(a^2 - 128*a + 512)*x^7 + 2/3*(3*a^2 - 96*a + 128)*x^6 + 1
/3*a^3*x^3 + 6*a^2*x^4 - 24/5*(a^2 - 8*a)*x^5
```

Fricas [A] time = 0.237487, size = 1, normalized size = 0.01

$$-\frac{1}{15}x^{15}+\frac{6}{7}x^{14}-\frac{72}{13}x^{13}+\frac{70}{3}x^{12}+\frac{3}{11}x^{11}a-\frac{768}{11}x^{11}-\frac{12}{5}x^{10}a+\frac{768}{5}x^{10}+\frac{32}{3}x^{9}a-\frac{2240}{9}x^{9}-30x^{8}a-\frac{3}{7}x^{7}a^{2}+288x^{8}+\frac{384}{7}x^{7}a+2x^{6}a^{2}-\frac{1536}{7}x^{7}-64x^{6}a-\frac{24}{5}x^{5}a^{2}+\frac{256}{3}x^{6}+\frac{192}{5}x^{5}a+6x^{4}a^{2}+\frac{1}{3}x^{3}a^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 4*x^3 + 8*x^2 - a - 8*x)^3*x^2,x, algorithm="fricas")
```

```
[Out] -1/15*x^15 + 6/7*x^14 - 72/13*x^13 + 70/3*x^12 + 3/11*x^11*a - 76
8/11*x^11 - 12/5*x^10*a + 768/5*x^10 + 32/3*x^9*a - 2240/9*x^9 -
30*x^8*a - 3/7*x^7*a^2 + 288*x^8 + 384/7*x^7*a + 2*x^6*a^2 - 1536
/7*x^7 - 64*x^6*a - 24/5*x^5*a^2 + 256/3*x^6 + 192/5*x^5*a + 6*x^
4*a^2 + 1/3*x^3*a^3
```

Sympy [A] time = 0.176813, size = 134, normalized size = 0.97

$$\frac{a^3x^3}{3}+6a^2x^4-\frac{x^{15}}{15}+\frac{6x^{14}}{7}-\frac{72x^{13}}{13}+\frac{70x^{12}}{3}+x^{11}\left(\frac{3a}{11}-\frac{768}{11}\right)+x^{10}\left(-\frac{12a}{5}+\frac{768}{5}\right)+x^9\left(\frac{32a}{3}-\frac{2240}{9}\right)+x^8(-30a+288)+x^7\left(-\frac{3a^2}{7}+\frac{384a}{7}-\frac{1536}{7}\right)+x^6\left(2a^2-64a+\frac{256}{3}\right)+x^5\left(-\frac{24a^2}{5}+\frac{192a}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(-x**4+4*x**3-8*x**2+a+8*x)**3,x)
```

```
[Out] a**3*x**3/3 + 6*a**2*x**4 - x**15/15 + 6*x**14/7 - 72*x**13/13 +
70*x**12/3 + x**11*(3*a/11 - 768/11) + x**10*(-12*a/5 + 768/5) +
x**9*(32*a/3 - 2240/9) + x**8*(-30*a + 288) + x**7*(-3*a**2/7 + 3
84*a/7 - 1536/7) + x**6*(2*a**2 - 64*a + 256/3) + x**5*(-24*a**2/
5 + 192*a/5)
```

GIAC/XCAS [A] time = 0.261684, size = 180, normalized size = 1.3

$$-\frac{1}{15}x^{15}+\frac{6}{7}x^{14}-\frac{72}{13}x^{13}+\frac{3}{11}ax^{11}+\frac{70}{3}x^{12}-\frac{12}{5}ax^{10}-\frac{768}{11}x^{11}+\frac{32}{3}ax^9+\frac{768}{5}x^{10}-\frac{3}{7}a^2x^7-30ax^8-\frac{2240}{9}x^9+2a^2x^6+\frac{384}{7}ax^7+288x^8-\frac{24}{5}a^2x^5-64ax^6-\frac{1536}{7}x^7+\frac{1}{3}a^3x^3+6a^2x^4+\frac{192}{5}ax^5+\frac{256}{3}x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 4*x^3 + 8*x^2 - a - 8*x)^3*x^2,x, algorithm="giac")
```

```
[Out] -1/15*x^15 + 6/7*x^14 - 72/13*x^13 + 3/11*a*x^11 + 70/3*x^12 - 12
/5*a*x^10 - 768/11*x^11 + 32/3*a*x^9 + 768/5*x^10 - 3/7*a^2*x^7 -
 30*a*x^8 - 2240/9*x^9 + 2*a^2*x^6 + 384/7*a*x^7 + 288*x^8 - 24/5
*a^2*x^5 - 64*a*x^6 - 1536/7*x^7 + 1/3*a^3*x^3 + 6*a^2*x^4 + 192/
5*a*x^5 + 256/3*x^6
```

3.132 $\int x^2 \left(a+8x-8x^2+4x^3-x^4\right)^2 \, dx$

Optimal. Leaf size=79

$$\frac{a^2x^3}{3}+\frac{2}{7}(64-a)x^7-\frac{4}{3}(16-a)x^6+\frac{16}{5}(4-a)x^5+4ax^4+\frac{x^{11}}{11}-\frac{4x^{10}}{5}+\frac{32x^9}{9}-10x^8$$

```
[Out] (a^2*x^3)/3 + 4*a*x^4 + (16*(4 - a)*x^5)/5 - (4*(16 - a)*x^6)/3 +
 (2*(64 - a)*x^7)/7 - 10*x^8 + (32*x^9)/9 - (4*x^10)/5 + x^11/11
```

Rubi [A] time = 0.18989, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$

$$\frac{a^2x^3}{3}+\frac{2}{7}(64-a)x^7-\frac{4}{3}(16-a)x^6+\frac{16}{5}(4-a)x^5+4ax^4+\frac{x^{11}}{11}-\frac{4x^{10}}{5}+\frac{32x^9}{9}-10x^8$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

```
[Out] (a^2*x^3)/3 + 4*a*x^4 + (16*(4 - a)*x^5)/5 - (4*(16 - a)*x^6)/3 +
 (2*(64 - a)*x^7)/7 - 10*x^8 + (32*x^9)/9 - (4*x^10)/5 + x^11/11
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(-x**4+4*x**3-8*x**2+a+8*x)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.0191206, size = 73, normalized size = 0.92

$$\frac{a^2x^3}{3}-\frac{2}{7}(a-64)x^7+\frac{4}{3}(a-16)x^6-\frac{16}{5}(a-4)x^5+4ax^4+\frac{x^{11}}{11}-\frac{4x^{10}}{5}+\frac{32x^9}{9}-10x^8$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

```
[Out] (a^2*x^3)/3 + 4*a*x^4 - (16*(-4 + a)*x^5)/5 + (4*(-16 + a)*x^6)/3
 - (2*(-64 + a)*x^7)/7 - 10*x^8 + (32*x^9)/9 - (4*x^10)/5 + x^11/
11
```

Maple [A] time = 0.001, size = 66, normalized size = 0.8

$$\frac{x^{11}}{11}-\frac{4\,x^{10}}{5}+\frac{32\,x^9}{9}-10\,x^8+\frac{(-2\,a+128)\,x^7}{7}+\frac{(8\,a-128)\,x^6}{6}+\frac{(-16\,a+64)\,x^5}{5}+4\,ax^4+\frac{a^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^2,x)
```

```
[Out] 1/11*x^11-4/5*x^10+32/9*x^9-10*x^8+1/7*(-2*a+128)*x^7+1/6*(8*a-12
8)*x^6+1/5*(-16*a+64)*x^5+4*a*x^4+1/3*a^2*x^3
```

Maxima [A] time = 0.803248, size = 80, normalized size = 1.01

$$\frac{1}{11}\,x^{11} - \frac{4}{5}\,x^{10} + \frac{32}{9}\,x^{9} - \frac{2}{7}\,(a-64)x^{7} - 10\,x^{8} + \frac{4}{3}\,(a-16)x^{6} - \frac{16}{5}\,(a-4)x^{5} + \frac{1}{3}\,a^{2}x^{3} + 4\,ax^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 - 4*x^3 + 8*x^2 - a - 8*x)^2*x^2,x, algorithm="maxima")
```

```
[Out] 1/11*x^11 - 4/5*x^10 + 32/9*x^9 - 2/7*(a - 64)*x^7 - 10*x^8 + 4/3
*(a - 16)*x^6 - 16/5*(a - 4)*x^5 + 1/3*a^2*x^3 + 4*a*x^4
```

Fricas [A] time = 0.227586, size = 1, normalized size = 0.01

$$\frac{1}{11}x^{11} - \frac{4}{5}x^{10} + \frac{32}{9}x^{9} - 10x^{8} - \frac{2}{7}x^{7}a + \frac{128}{7}x^{7} + \frac{4}{3}x^{6}a - \frac{64}{3}x^{6} - \frac{16}{5}x^{5}a + \frac{64}{5}x^{5} + 4x^{4}a + \frac{1}{3}x^{3}a^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 - 4*x^3 + 8*x^2 - a - 8*x)^2*x^2,x, algorithm="fricas")
```

```
[Out] 1/11*x^11 - 4/5*x^10 + 32/9*x^9 - 10*x^8 - 2/7*x^7*a + 128/7*x^7
+ 4/3*x^6*a - 64/3*x^6 - 16/5*x^5*a + 64/5*x^5 + 4*x^4*a + 1/3*x^
3*a^2
```

Sympy [A] time = 0.11609, size = 73, normalized size = 0.92

$$\frac{a^{2}x^{3}}{3} + 4ax^{4} + \frac{x^{11}}{11} - \frac{4x^{10}}{5} + \frac{32x^{9}}{9} - 10x^{8} + x^{7}\left(-\frac{2a}{7} + \frac{128}{7}\right) + x^{6}\left(\frac{4a}{3} - \frac{64}{3}\right) + x^{5}\left(-\frac{16a}{5} + \frac{64}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(-x**4+4*x**3-8*x**2+a+8*x)**2,x)
```

```
[Out] a**2*x**3/3 + 4*a*x**4 + x**11/11 - 4*x**10/5 + 32*x**9/9 - 10*x*
*8 + x**7*(-2*a/7 + 128/7) + x**6*(4*a/3 - 64/3) + x**5*(-16*a/5
+ 64/5)
```

GIAC/XCAS [A] time = 0.259915, size = 92, normalized size = 1.16

$$\frac{1}{11}\,x^{11} - \frac{4}{5}\,x^{10} + \frac{32}{9}\,x^{9} - \frac{2}{7}\,ax^{7} - 10\,x^{8} + \frac{4}{3}\,ax^{6} + \frac{128}{7}\,x^{7} - \frac{16}{5}\,ax^{5} - \frac{64}{3}\,x^{6} + \frac{1}{3}\,a^{2}x^{3} + 4\,ax^{4} + \frac{64}{5}\,x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 - 4*x^3 + 8*x^2 - a - 8*x)^2*x^2,x, algorithm="giac")
```

```
[Out] 1/11*x^11 - 4/5*x^10 + 32/9*x^9 - 2/7*a*x^7 - 10*x^8 + 4/3*a*x^6
+ 128/7*x^7 - 16/5*a*x^5 - 64/3*x^6 + 1/3*a^2*x^3 + 4*a*x^4 + 64/
5*x^5
```

3.133 $\int x^2 \left(a + 8x - 8x^2 + 4x^3 - x^4\right) \, dx$

Optimal. Leaf size=35

$$\frac{ax^3}{3} - \frac{x^7}{7} + \frac{2x^6}{3} - \frac{8x^5}{5} + 2x^4$$

```
[Out] (a*x^3)/3 + 2*x^4 - (8*x^5)/5 + (2*x^6)/3 - x^7/7
```

Rubi [A] time = 0.0260389, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$

$$\frac{ax^3}{3} - \frac{x^7}{7} + \frac{2x^6}{3} - \frac{8x^5}{5} + 2x^4$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]

[Out] (a*x^3)/3 + 2*x^4 - (8*x^5)/5 + (2*x^6)/3 - x^7/7
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \left(a - x^4 + 4x^3 - 8x^2 + 8x\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**2*(-x**4+4*x**3-8*x**2+a+8*x),x)

[Out] Integral(x**2*(a - x**4 + 4*x**3 - 8*x**2 + 8*x), x)
```

Mathematica [A] time = 0.00207125, size = 35, normalized size = 1.

$$\frac{ax^3}{3} - \frac{x^7}{7} + \frac{2x^6}{3} - \frac{8x^5}{5} + 2x^4$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]

[Out] (a*x^3)/3 + 2*x^4 - (8*x^5)/5 + (2*x^6)/3 - x^7/7
```

Maple [A] time = 0.001, size = 28, normalized size = 0.8

$$\frac{ax^3}{3} + 2x^4 - \frac{8x^5}{5} + \frac{2x^6}{3} - \frac{x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-x^4+4*x^3-8*x^2+a+8*x),x)
```

```
[Out] 1/3*a*x^3+2*x^4-8/5*x^5+2/3*x^6-1/7*x^7
```

Maxima [A] time = 0.782415, size = 36, normalized size = 1.03

$$-\frac{1}{7}x^7 + \frac{2}{3}x^6 - \frac{8}{5}x^5 + \frac{1}{3}ax^3 + 2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x^4 - 4*x^3 + 8*x^2 - a - 8*x)*x^2,x, algorithm="maxima")
```

```
[Out] -1/7*x^7 + 2/3*x^6 - 8/5*x^5 + 1/3*a*x^3 + 2*x^4
```

Fricas [A] time = 0.236225, size = 1, normalized size = 0.03

$$-\frac{1}{7}x^7 + \frac{2}{3}x^6 - \frac{8}{5}x^5 + 2x^4 + \frac{1}{3}x^3a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x^4 - 4*x^3 + 8*x^2 - a - 8*x)*x^2,x, algorithm="fricas")
```

```
[Out] -1/7*x^7 + 2/3*x^6 - 8/5*x^5 + 2*x^4 + 1/3*x^3*a
```

Sympy [A] time = 0.072376, size = 29, normalized size = 0.83

$$\frac{ax^3}{3} - \frac{x^7}{7} + \frac{2x^6}{3} - \frac{8x^5}{5} + 2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(-x**4+4*x**3-8*x**2+a+8*x),x)
```

```
[Out] a*x**3/3 - x**7/7 + 2*x**6/3 - 8*x**5/5 + 2*x**4
```

GIAC/XCAS [A] time = 0.259225, size = 36, normalized size = 1.03

$$-\frac{1}{7}x^7 + \frac{2}{3}x^6 - \frac{8}{5}x^5 + \frac{1}{3}ax^3 + 2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 4*x^3 + 8*x^2 - a - 8*x)*x^2,x, algorithm="giac")
```

```
[Out] -1/7*x^7 + 2/3*x^6 - 8/5*x^5 + 1/3*a*x^3 + 2*x^4
```

3.134 $\int \frac{x^2}{a+8x-8x^2+4x^3-x^4} \, dx$

Optimal. Leaf size=99

$$-\frac{\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{2\sqrt{1-\sqrt{a+4}}}-\frac{\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{2\sqrt{\sqrt{a+4}+1}}+\frac{\tanh^{-1}\left(\frac{(x-1)^2+1}{\sqrt{a+4}}\right)}{\sqrt{a+4}}$$

```
[Out] -ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4 + a]]]/(2*Sqrt[1 - Sqrt[4 + a]])
 - ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]]/(2*Sqrt[1 + Sqrt[4 + a]
]) + ArcTanh[(1 + (-1 + x)^2)/Sqrt[4 + a]]/Sqrt[4 + a]
```

Rubi [A] time = 0.220096, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$

$$\frac{\tan^{-1}\left(\frac{1-x}{\sqrt{1-\sqrt{a+4}}}\right)}{2\sqrt{1-\sqrt{a+4}}}+\frac{\tan^{-1}\left(\frac{1-x}{\sqrt{\sqrt{a+4}+1}}\right)}{2\sqrt{\sqrt{a+4}+1}}+\frac{\tanh^{-1}\left(\frac{(x-1)^2+1}{\sqrt{a+4}}\right)}{\sqrt{a+4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]
```

```
[Out] ArcTan[(1 - x)/Sqrt[1 - Sqrt[4 + a]]]/(2*Sqrt[1 - Sqrt[4 + a]]) +
 ArcTan[(1 - x)/Sqrt[1 + Sqrt[4 + a]]]/(2*Sqrt[1 + Sqrt[4 + a]])
+ ArcTanh[(1 + (-1 + x)^2)/Sqrt[4 + a]]/Sqrt[4 + a]
```

Rubi in Sympy [A] time = 54.8052, size = 85, normalized size = 0.86

$$-\frac{\operatorname{atan}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{2\sqrt{\sqrt{a+4}+1}}-\frac{\operatorname{atan}\left(\frac{x-1}{\sqrt{-\sqrt{a+4}+1}}\right)}{2\sqrt{-\sqrt{a+4}+1}}-\frac{\operatorname{atanh}\left(\frac{-(x-1)^2-1}{\sqrt{a+4}}\right)}{\sqrt{a+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(-x**4+4*x**3-8*x**2+a+8*x),x)
```

```
[Out] -atan((x - 1)/sqrt(sqrt(a + 4) + 1))/(2*sqrt(sqrt(a + 4) + 1)) -
atan((x - 1)/sqrt(-sqrt(a + 4) + 1))/(2*sqrt(-sqrt(a + 4) + 1)) -
 atanh((-(x - 1)**2 - 1)/sqrt(a + 4))/sqrt(a + 4)
```

Mathematica [C] time = 0.028795, size = 61, normalized size = 0.62

$$-\frac{1}{4}\text{RootSum}\left[-\#1^4+4\#1^3-8\#1^2+8\#1+a\&,\frac{\#1^2\log(x-\#1)}{\#1^3-3\#1^2+4\#1-2}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]
```

```
[Out] -RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 & , (Log[x - #1]*#1^2)
/(-2 + 4*#1 - 3*#1^2 + #1^3) & ]/4
```

Maple [C] time = 0.004, size = 52, normalized size = 0.5

$$-\frac{1}{4}\sum_{\_R=RootOf\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)}\frac{\_R^2\ln\left(x-\_R\right)}{\_R^3-3\_R^2+4\_R-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(-x^4+4*x^3-8*x^2+a+8*x),x)
```

```
[Out] -1/4*sum(_R^2/(_R^3-3*_R^2+4*_R-2)*ln(x-_R),_R=RootOf(_Z^4-4*_Z^3
+8*_Z^2-8*_Z-a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x^2}{x^4-4x^3+8x^2-a-8x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x^2/(x^4 - 4*x^3 + 8*x^2 - a - 8*x),x, algorithm="maxima")
```

```
[Out] -integrate(x^2/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x^2/(x^4 - 4*x^3 + 8*x^2 - a - 8*x),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 11.916, size = 172, normalized size = 1.74

$$-\operatorname{RootSum}\left(t^4\left(256a^3+2816a^2+10240a+12288\right)+t^2\left(-160a^2-1152a-2048\right)+t\left(-32a^2-256a-512\right)-a^2,\left(t\mapsto t\,l\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(-x**4+4*x**3-8*x**2+a+8*x),x)
```

```
[Out] -RootSum(_t**4*(256*a**3 + 2816*a**2 + 10240*a + 12288) + _t**2*(
-160*a**2 - 1152*a - 2048) + _t*(-32*a**2 - 256*a - 512) - a**2,
Lambda(_t, _t*log(x + (-64*_t**3*a**4 - 448*_t**3*a**3 - 256*_t**
3*a**2 + 3584*_t**3*a + 6144*_t**3 - 224*_t**2*a**3 - 2208*_t**2*
a**2 - 7168*_t**2*a - 7680*_t**2 + 56*_t*a**3 + 400*_t*a**2 + 864
*_t*a + 512*_t + 5*a**3 + 34*a**2 + 56*a)/(a**3 + 60*a**2 + 320*a
 + 448))))
```

<u>GIAC/XCAS</u> [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2}{x^4 - 4\,x^3 + 8\,x^2 - a - 8\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x^2/(x^4 - 4*x^3 + 8*x^2 - a - 8*x),x, algorithm="giac")
```

```
[Out] integrate(-x^2/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)
```

3.135 $\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^2}\,dx$

Optimal. Leaf size=225

$$\frac{(a+4)\left((x-1)^2+2\right)(x-1)}{4\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)}+\frac{(x-1)^2+1}{2(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)}-\frac{\left(a+\sqrt{a+4}+4\right)\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{8(a+3)(a+4)\sqrt{1-\sqrt{a+4}}}-\frac{\left(a-\sqrt{a+4}+4\right)\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{8(a+3)(a+4)\sqrt{\sqrt{a+4}+1}}+\frac{\tanh^{-1}\left(\frac{(x-1)^2+1}{\sqrt{a+4}}\right)}{2(a+4)^{3/2}}$$

```
[Out] (1 + (-1 + x)^2)/(2*(4 + a)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4))
+ ((4 + a)*(2 + (-1 + x)^2)*(-1 + x))/(4*(12 + 7*a + a^2)*(3 + a
- 2*(-1 + x)^2 - (-1 + x)^4)) - ((4 + a + Sqrt[4 + a])*ArcTan[(-1
 + x)/Sqrt[1 - Sqrt[4 + a]]])/(8*(3 + a)*(4 + a)*Sqrt[1 - Sqrt[4
+ a]]) - ((4 + a - Sqrt[4 + a])*ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 +
 a]]])/(8*(3 + a)*(4 + a)*Sqrt[1 + Sqrt[4 + a]]) + ArcTanh[(1 + (
-1 + x)^2)/Sqrt[4 + a]]/(2*(4 + a)^(3/2))
```

Rubi [A] time = 0.535527, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385

$$\frac{(x-1)^2+1}{2(a+4)\left(a-(1-x)^4-2(1-x)^2+3\right)}-\frac{\left((x-1)^2+2\right)(1-x)}{4(a+3)\left(a-(1-x)^4-2(1-x)^2+3\right)}+\frac{\left(a+\sqrt{a+4}+4\right)\tan^{-1}\left(\frac{1-x}{\sqrt{1-\sqrt{a+4}}}\right)}{8(a+3)(a+4)\sqrt{1-\sqrt{a+4}}}+\frac{\left(a-\sqrt{a+4}+4\right)\tan^{-1}\left(\frac{1-x}{\sqrt{\sqrt{a+4}+1}}\right)}{8(a+3)(a+4)\sqrt{\sqrt{a+4}+1}}+\frac{\tanh^{-1}\left(\frac{(x-1)^2+1}{\sqrt{a+4}}\right)}{2(a+4)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

```
[Out] (1 + (-1 + x)^2)/(2*(4 + a)*(3 + a - 2*(1 - x)^2 - (1 - x)^4)) -
((2 + (-1 + x)^2)*(1 - x))/(4*(3 + a)*(3 + a - 2*(1 - x)^2 - (1 -
 x)^4)) + ((4 + a + Sqrt[4 + a])*ArcTan[(1 - x)/Sqrt[1 - Sqrt[4 +
 a]]])/(8*(3 + a)*(4 + a)*Sqrt[1 - Sqrt[4 + a]]) + ((4 + a - Sqrt
[4 + a])*ArcTan[(1 - x)/Sqrt[1 + Sqrt[4 + a]]])/(8*(3 + a)*(4 + a
)*Sqrt[1 + Sqrt[4 + a]]) + ArcTanh[(1 + (-1 + x)^2)/Sqrt[4 + a]]/
(2*(4 + a)^(3/2))
```

Rubi in Sympy [A] time = 78.1917, size = 180, normalized size = 0.8

$$-\frac{\operatorname{atanh}\left(\frac{-(x-1)^2-1}{\sqrt{a+4}}\right)}{2\,(a+4)^{\frac{3}{2}}}+\frac{(x-1)\left(4a+(2a+8)\,(x-1)^2+(4a+20)\,(x-1)+4\,(x-1)^3+16\right)}{8\,(a+3)\,(a+4)\left(a-(x-1)^4-2\,(x-1)^2+3\right)}+\frac{\left(-\sqrt{a+4}+1\right)\operatorname{atan}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{8\,(a+3)\,\sqrt{a+4}\sqrt{\sqrt{a+4}+1}}-\frac{\left(\sqrt{a+4}+1\right)\operatorname{atan}\left(\frac{x-1}{\sqrt{-\sqrt{a+4}+1}}\right)}{8\,(a+3)\,\sqrt{a+4}\sqrt{-\sqrt{a+4}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(-x**4+4*x**3-8*x**2+a+8*x)**2,x)
```

```
[Out] -atanh((-(x - 1)**2 - 1)/sqrt(a + 4))/(2*(a + 4)**(3/2)) + (x - 1
)*(4*a + (2*a + 8)*(x - 1)**2 + (4*a + 20)*(x - 1) + 4*(x - 1)**3
 + 16)/(8*(a + 3)*(a + 4)*(a - (x - 1)**4 - 2*(x - 1)**2 + 3)) +
(-sqrt(a + 4) + 1)*atan((x - 1)/sqrt(sqrt(a + 4) + 1))/(8*(a + 3)
```

```
*sqrt(a + 4)*sqrt(sqrt(a + 4) + 1)) - (sqrt(a + 4) + 1)*atan((x -
 1)/sqrt(-sqrt(a + 4) + 1))/(8*(a + 3)*sqrt(a + 4)*sqrt(-sqrt(a +
 4) + 1))
```

Mathematica [C] time = 0.102175, size = 182, normalized size = 0.81

$$\frac{a(x^3-x^2+x+1)+2x(2x^2-3x+4)}{4(a+3)(a+4)(a-x(x^3-4x^2+8x-8))} - \frac{\text{RootSum}\left[-\#1^4+4\#1^3-8\#1^2+8\#1+a\&,\frac{\#1^2 a\log(x-\#1)+4\#1^2\log(x-\#1)+2\#1 a\log(x-\#1)-a\log(x-\#1)+4\#1\log(x-\#1)}{\#1^3-3\#1^2+4\#1-2}\&\right]}{16(a^2+7a+12)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

```
[Out] (2*x*(4 - 3*x + 2*x^2) + a*(1 + x - x^2 + x^3))/(4*(3 + a)*(4 + a
)*(a - x*(-8 + 8*x - 4*x^2 + x^3))) - RootSum[a + 8*#1 - 8*#1^2 +
 4*#1^3 - #1^4 & , (-(a*Log[x - #1]) + 4*Log[x - #1]*#1 + 2*a*Log
[x - #1]*#1 + 4*Log[x - #1]*#1^2 + a*Log[x - #1]*#1^2)/(-2 + 4*#1
 - 3*#1^2 + #1^3) & ]/(16*(12 + 7*a + a^2))
```

Maple [C] time = 0.015, size = 160, normalized size = 0.7

$$\frac{1}{x^4-4x^3+8x^2-a-8x}\left(-\frac{x^3}{12+4a}+\frac{(6+a)x^2}{(16+4a)(3+a)}-\frac{(8+a)x}{(16+4a)(3+a)}-\frac{a}{(16+4a)(3+a)}\right)+\frac{1}{16}\sum_{\_R=RootOf(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a)}\frac{((-a-4)\_R^2+2(-a-2)\_R+a)\ln(x-\_R)}{(\_R^3-3\_R^2+4\_R-2)(4+a)(3+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^2,x)
```

```
[Out] (-1/4/(3+a)*x^3+1/4*(6+a)/(4+a)/(3+a)*x^2-1/4*(8+a)/(4+a)/(3+a)*x
-1/4*a/(4+a)/(3+a))/(x^4-4*x^3+8*x^2-a-8*x)+1/16*sum(((-a-4)*_R^2
+2*(-a-2)*_R+a)/(_R^3-3*_R^2+4*_R-2)/(4+a)/(3+a)*ln(x-_R),_R=Root
Of(_Z^4-4*_Z^3+8*_Z^2-8*_Z-a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{(a+4)x^3-(a+6)x^2+(a+8)x+a}{4((a^2+7a+12)x^4-4(a^2+7a+12)x^3-a^3+8(a^2+7a+12)x^2-7a^2-8(a^2+7a+12)x-12a)} - \frac{\int \frac{(a+4)x^2+2(a+2)x-a}{x^4-4x^3+8x^2-a-8x}\,dx}{4(a^2+7a+12)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x^4 - 4*x^3 + 8*x^2 - a - 8*x)^2,x, algorithm="maxima")
```

```
[Out] -1/4*((a + 4)*x^3 - (a + 6)*x^2 + (a + 8)*x + a)/((a^2 + 7*a + 12
)*x^4 - 4*(a^2 + 7*a + 12)*x^3 - a^3 + 8*(a^2 + 7*a + 12)*x^2 - 7
*a^2 - 8*(a^2 + 7*a + 12)*x - 12*a) - 1/4*integrate(((a + 4)*x^2
+ 2*(a + 2)*x - a)/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)/(a^2 + 7*a
 + 12)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4 - 4*x^3 + 8*x^2 - a - 8*x)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 49.1375, size = 559, normalized size = 2.48

$$-\frac{a+x^3(a+4)+x^2(-a-6)+x(a+8)}{-4a^3-28a^2-48a+x^4(4a^2+28a+48)+x^3(-16a^2-112a-192)+x^2(32a^2+224a+384)+x(-32a^2-224a-384)}$$
$$+\mathrm{RootSum}\Big(t^4\left(65536a^9+2162688a^8+31653888a^7+269680640a^6+1473773568a^5+5357174784a^4+12952010752a^3+20\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(-x**4+4*x**3-8*x**2+a+8*x)**2,x)
```

```
[Out] -(a + x**3*(a + 4) + x**2*(-a - 6) + x*(a + 8))/(-4*a**3 - 28*a**
2 - 48*a + x**4*(4*a**2 + 28*a + 48) + x**3*(-16*a**2 - 112*a - 1
92) + x**2*(32*a**2 + 224*a + 384) + x*(-32*a**2 - 224*a - 384))
+ RootSum(_t**4*(65536*a**9 + 2162688*a**8 + 31653888*a**7 + 2696
80640*a**6 + 1473773568*a**5 + 5357174784*a**4 + 12952010752*a**3
 + 20082327552*a**2 + 18119393280*a + 7247757312) + _t**2*(-9728*
a**6 - 209408*a**5 - 1878016*a**4 - 8986624*a**3 - 24215552*a**2
- 34865152*a - 20971520) + _t*(256*a**5 + 5888*a**4 + 53248*a**3
+ 237568*a**2 + 524288*a + 458752) - a**4 + 144*a**3 + 1024*a**2
+ 1792*a, Lambda(_t, _t*log(x + (4096*_t**3*a**12 - 61440*_t**3*a
**11 - 5480448*_t**3*a**10 - 111403008*_t**3*a**9 - 1227173888*_t
**3*a**8 - 8682876928*_t**3*a**7 - 42187440128*_t**3*a**6 - 14463
0284288*_t**3*a**5 - 350972280832*_t**3*a**4 - 591750234112*_t**3
*a**3 - 660716126208*_t**3*a**2 - 439848271872*_t**3*a - 13227157
0944*_t**3 - 28672*_t**2*a**10 - 993280*_t**2*a**9 - 15400960*_t*
*2*a**8 - 140742656*_t**2*a**7 - 839462912*_t**2*a**6 - 341442764
8*_t**2*a**5 - 9590087680*_t**2*a**4 - 18363547648*_t**2*a**3 - 2
2938255360*_t**2*a**2 - 16873684992*_t**2*a - 5549064192*_t**2 -
848*_t*a**9 - 6096*_t*a**8 + 174608*_t*a**7 + 3323792*_t*a**6 + 2
6276224*_t*a**5 + 119009280*_t*a**4 + 332017664*_t*a**3 + 5664972
80*_t*a**2 + 544112640*_t*a + 225837056*_t + 11*a**8 + 958*a**7 +
 17419*a**6 + 142964*a**5 + 632632*a**4 + 1567552*a**3 + 2049792*
a**2 + 1100800*a)/(a**8 + 870*a**7 + 18289*a**6 + 165176*a**5 + 8
24560*a**4 + 2452288*a**3 + 4340224*a**2 + 4229120*a + 1748992)))
)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(x^4-4\,x^3+8\,x^2-a-8\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x^4 - 4*x^3 + 8*x^2 - a - 8*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2/(x^4 - 4*x^3 + 8*x^2 - a - 8*x)^2, x)
```

3.136 $\int \frac{x^4}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx$

Optimal. Leaf size=545

$$-\frac{\sqrt[3]{-1}\left(3\sqrt[3]{a}c^{2/3}+2\sqrt[3]{-1}b\right)\tan^{-1}\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{3\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{5/6}b^2c^{2/3}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}-\frac{\left(2b-3\sqrt[3]{a}c^{2/3}\right)\tan^{-1}\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}\right)}{9\sqrt{3}a^{5/6}b^2c^{2/3}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}-\frac{(-1)^{2/3}\left(3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}+2b\right)\tan^{-1}\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}+4b}}\right)}{3\sqrt{3}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2a^{5/6}b^2c^{2/3}\sqrt{3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}+4b}}-\frac{\log\left(3a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{18a^{2/3}b^2\sqrt[3]{c}}+\frac{\log\left(-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{6\left(1+\sqrt[3]{-1}\right)^2a^{2/3}b^2\sqrt[3]{c}}+\frac{\sqrt[3]{-1}\log\left(3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{18a^{2/3}b^2\sqrt[3]{c}}$$

```
[Out] -((-1)^(1/3)*(2*(-1)^(1/3)*b + 3*a^(1/3)*c^(2/3))*ArcTan[(3*(-1)^
(1/3)*a^(2/3)*c^(1/3) - 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*(-1)
^(2/3)*a^(1/3)*c^(2/3)]])]/(3*Sqrt[3]*(1 + (-1)^(1/3))^2*a^(5/6)*
b^2*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)]*c^(2/3)) - ((2*b - 3
*a^(1/3)*c^(2/3))*ArcTan[(3*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqr
t[a]*Sqrt[4*b - 3*a^(1/3)*c^(2/3)]])]/(9*Sqrt[3]*a^(5/6)*b^2*Sqrt
[4*b - 3*a^(1/3)*c^(2/3)]*c^(2/3)) - ((-1)^(2/3)*(2*b + 3*(-1)^(1
/3)*a^(1/3)*c^(2/3))*ArcTan[(3*(-1)^(2/3)*a^(2/3)*c^(1/3) + 2*b*x
)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)]])]/(3
*Sqrt[3]*(1 - (-1)^(1/3))*(1 + (-1)^(1/3))^2*a^(5/6)*b^2*Sqrt[4*b
 + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)]*c^(2/3)) - Log[3*a + 3*a^(2/3)*c
^(1/3)*x + b*x^2]/(18*a^(2/3)*b^2*c^(1/3)) + Log[3*a - 3*(-1)^(1/
3)*a^(2/3)*c^(1/3)*x + b*x^2]/(6*(1 + (-1)^(1/3))^2*a^(2/3)*b^2*c
^(1/3)) + ((-1)^(1/3)*Log[3*a + 3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x +
b*x^2])/(18*a^(2/3)*b^2*c^(1/3))
```

Rubi [A] time = 4.49273, antiderivative size = 545, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.109

$$-\frac{\sqrt[3]{-1}\left(3\sqrt[3]{a}c^{2/3}+2\sqrt[3]{-1}b\right)\tan^{-1}\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{3\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{5/6}b^2c^{2/3}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}-\frac{\left(2b-3\sqrt[3]{a}c^{2/3}\right)\tan^{-1}\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}\right)}{9\sqrt{3}a^{5/6}b^2c^{2/3}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}-\frac{(-1)^{2/3}\left(3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}+2b\right)\tan^{-1}\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}+4b}}\right)}{3\sqrt{3}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2a^{5/6}b^2c^{2/3}\sqrt{3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}+4b}}-\frac{\log\left(3a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{18a^{2/3}b^2\sqrt[3]{c}}+\frac{\log\left(-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{6\left(1+\sqrt[3]{-1}\right)^2a^{2/3}b^2\sqrt[3]{c}}+\frac{\sqrt[3]{-1}\log\left(3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{18a^{2/3}b^2\sqrt[3]{c}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6),x]
```

```
[Out] -((-1)^(1/3)*(2*(-1)^(1/3)*b + 3*a^(1/3)*c^(2/3))*ArcTan[(3*(-1)^
(1/3)*a^(2/3)*c^(1/3) - 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*(-1)
^(2/3)*a^(1/3)*c^(2/3)])])/(3*Sqrt[3]*(1 + (-1)^(1/3))^2*a^(5/6)*
b^2*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)]*c^(2/3)) - ((2*b - 3
*a^(1/3)*c^(2/3))*ArcTan[(3*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqr
t[a]*Sqrt[4*b - 3*a^(1/3)*c^(2/3)])])/(9*Sqrt[3]*a^(5/6)*b^2*Sqrt
[4*b - 3*a^(1/3)*c^(2/3)]*c^(2/3)) - ((-1)^(2/3)*(2*b + 3*(-1)^(1
/3)*a^(1/3)*c^(2/3))*ArcTan[(3*(-1)^(2/3)*a^(2/3)*c^(1/3) + 2*b*x
)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)])])/(3
*Sqrt[3]*(1 - (-1)^(1/3))*(1 + (-1)^(1/3))^2*a^(5/6)*b^2*Sqrt[4*b
 + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)]*c^(2/3)) - Log[3*a + 3*a^(2/3)*c
^(1/3)*x + b*x^2]/(18*a^(2/3)*b^2*c^(1/3)) + Log[3*a - 3*(-1)^(1/
3)*a^(2/3)*c^(1/3)*x + b*x^2]/(6*(1 + (-1)^(1/3))^2*a^(2/3)*b^2*c
^(1/3)) + ((-1)^(1/3)*Log[3*a + 3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x +
b*x^2])/(18*a^(2/3)*b^2*c^(1/3))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**4/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*
```

```
[Out] Timed out
```

Mathematica [C] time = 0.0948964, size = 99, normalized size = 0.18

$$\frac{1}{3}\text{RootSum}\left[\#1^6 b^3+9\#1^4 a b^2+27\#1^3 a^2 c+27\#1^2 a^2 b+27 a^3\&,\frac{\#1^3 \log (x-\#1)}{2\#1^4 b^3+12\#1^2 a b^2+27\#1 a^2 c+18 a^2 b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6),x]
```

```
[Out] RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^2*#1^4 + b
^3*#1^6 & , (Log[x - #1]*#1^3)/(18*a^2*b + 27*a^2*c*#1 + 12*a*b^2
*#1^2 + 2*b^3*#1^4) & ]/3
```

Maple [C] time = 0.014, size = 93, normalized size = 0.2

$$\frac{1}{3}\sum_{\_R=RootOf\left(b^3\_Z^6+9\,ab^2\_Z^4+27\,a^2c\_Z^3+27\,a^2b\_Z^2+27\,a^3\right)}\frac{\_R^4\ln(x-\_R)}{2\,\_R^5b^3+12\,\_R^3ab^2+27\,\_R^2a^2c+18\,\_R\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x)
```

```
[Out] 1/3*sum(_R^4/(2*_R^5*b^3+12*_R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)
*ln(x-_R),_R=RootOf(_Z^6*b^3+9*_Z^4*a*b^2+27*_Z^3*a^2*c+27*_Z^2*a
^2*b+27*a^3))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{b^3x^6+9\,ab^2x^4+27\,a^2cx^3+27\,a^2bx^2+27\,a^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3),x, a
```

```
[Out] integrate(x^4/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^
2 + 27*a^3), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3),x, a
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*a**3)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{b^3x^6 + 9\,ab^2x^4 + 27\,a^2cx^3 + 27\,a^2bx^2 + 27\,a^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3),x, a
```

```
[Out] integrate(x^4/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^
2 + 27*a^3), x)
```

3.137 $\int \frac{x^3}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx$

Optimal. Leaf size=487

$$\frac{\log\left(3a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{54a^{4/3}bc^{2/3}}-\frac{(-1)^{2/3}\log\left(-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{18\left(1+\sqrt[3]{-1}\right)^2a^{4/3}bc^{2/3}}+\frac{(-1)^{2/3}\log\left(3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{54a^{4/3}bc^{2/3}}-\frac{\tan^{-1}\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{3\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{7/6}b\sqrt[3]{c}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}-\frac{\tan^{-1}\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}\right)}{9\sqrt{3}a^{7/6}b\sqrt[3]{c}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}+\frac{\sqrt[3]{-1}\tan^{-1}\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}+4b}}\right)}{3\sqrt{3}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2a^{7/6}b\sqrt[3]{c}\sqrt{3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}+4b}}$$

```
[Out] -ArcTan[(3*(-1)^(1/3)*a^(2/3)*c^(1/3) - 2*b*x)/(Sqrt[3]*Sqrt[a]*S
qrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)])]/(3*Sqrt[3]*(1 + (-1)^(1
/3))^2*a^(7/6)*b*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)]*c^(1/3)
) - ArcTan[(3*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b
- 3*a^(1/3)*c^(2/3)])]/(9*Sqrt[3]*a^(7/6)*b*Sqrt[4*b - 3*a^(1/3)*
c^(2/3)]*c^(1/3)) + ((-1)^(1/3)*ArcTan[(3*(-1)^(2/3)*a^(2/3)*c^(1
/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(
2/3)])])/(3*Sqrt[3]*(1 - (-1)^(1/3))*(1 + (-1)^(1/3))^2*a^(7/6)*b
*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)]*c^(1/3)) + Log[3*a + 3*
a^(2/3)*c^(1/3)*x + b*x^2]/(54*a^(4/3)*b*c^(2/3)) - ((-1)^(2/3)*L
og[3*a - 3*(-1)^(1/3)*a^(2/3)*c^(1/3)*x + b*x^2])/(18*(1 + (-1)^(
1/3))^2*a^(4/3)*b*c^(2/3)) + ((-1)^(2/3)*Log[3*a + 3*(-1)^(2/3)*a
^(2/3)*c^(1/3)*x + b*x^2])/(54*a^(4/3)*b*c^(2/3))
```

Rubi [A] time = 3.088, antiderivative size = 487, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.109

$$\frac{\log\left(3a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{54a^{4/3}bc^{2/3}}-\frac{(-1)^{2/3}\log\left(-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{18\left(1+\sqrt[3]{-1}\right)^2a^{4/3}bc^{2/3}}+\frac{(-1)^{2/3}\log\left(3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{54a^{4/3}bc^{2/3}}-\frac{\tan^{-1}\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{3\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{7/6}b\sqrt[3]{c}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}-\frac{\tan^{-1}\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}\right)}{9\sqrt{3}a^{7/6}b\sqrt[3]{c}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}+\frac{\sqrt[3]{-1}\tan^{-1}\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}+4b}}\right)}{3\sqrt{3}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2a^{7/6}b\sqrt[3]{c}\sqrt{3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}+4b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6),x]
```

```
[Out] -ArcTan[(3*(-1)^(1/3)*a^(2/3)*c^(1/3) - 2*b*x)/(Sqrt[3]*Sqrt[a]*S
qrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)])]/(3*Sqrt[3]*(1 + (-1)^(1
/3))^2*a^(7/6)*b*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)]*c^(1/3)
) - ArcTan[(3*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b
- 3*a^(1/3)*c^(2/3)])]/(9*Sqrt[3]*a^(7/6)*b*Sqrt[4*b - 3*a^(1/3)*
c^(2/3)]*c^(1/3)) + ((-1)^(1/3)*ArcTan[(3*(-1)^(2/3)*a^(2/3)*c^(1
/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(
2/3)])])/(3*Sqrt[3]*(1 - (-1)^(1/3))*(1 + (-1)^(1/3))^2*a^(7/6)*b
*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)]*c^(1/3)) + Log[3*a + 3*
a^(2/3)*c^(1/3)*x + b*x^2]/(54*a^(4/3)*b*c^(2/3)) - ((-1)^(2/3)*L
```

```
og[3*a - 3*(-1)^(1/3)*a^(2/3)*c^(1/3)*x + b*x^2])/(18*(1 + (-1)^(
1/3))^2*a^(4/3)*b*c^(2/3)) + ((-1)^(2/3)*Log[3*a + 3*(-1)^(2/3)*a
^(2/3)*c^(1/3)*x + b*x^2])/(54*a^(4/3)*b*c^(2/3))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**3/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*
```

```
[Out] Timed out
```

Mathematica [C] time = 0.0784563, size = 99, normalized size = 0.2

$$\frac{1}{3}\text{RootSum}\left[\#1^6b^3+9\#1^4ab^2+27\#1^3a^2c+27\#1^2a^2b+27a^3\&,\frac{\#1^2\log(x-\#1)}{2\#1^4b^3+12\#1^2ab^2+27\#1a^2c+18a^2b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6),x]
```

```
[Out] RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^2*#1^4 + b
^3*#1^6 & , (Log[x - #1]*#1^2)/(18*a^2*b + 27*a^2*c*#1 + 12*a*b^2
*#1^2 + 2*b^3*#1^4) & ]/3
```

Maple [C] time = 0.005, size = 93, normalized size = 0.2

$$\frac{1}{3}\sum_{\_R=RootOf\left(b^3\_Z^6+9\,ab^2\_Z^4+27\,a^2c\_Z^3+27\,a^2b\_Z^2+27\,a^3\right)}\frac{\_R^3\ln(x-\_R)}{2\,\_R^5b^3+12\,\_R^3ab^2+27\,\_R^2a^2c+18\,\_R\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x)
```

```
[Out] 1/3*sum(_R^3/(2*_R^5*b^3+12*_R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)
*ln(x-_R),_R=RootOf(_Z^6*b^3+9*_Z^4*a*b^2+27*_Z^3*a^2*c+27*_Z^2*a
^2*b+27*a^3))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^3}{b^3x^6+9\,ab^2x^4+27\,a^2cx^3+27\,a^2bx^2+27\,a^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3),x, a
```

```
[Out] integrate(x^3/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^
2 + 27*a^3), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3),x, a
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*a**3)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{b^3x^6 + 9\,ab^2x^4 + 27\,a^2cx^3 + 27\,a^2bx^2 + 27\,a^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3),x, a
```

```
[Out] integrate(x^3/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^
2 + 27*a^3), x)
```

3.138 $\int \frac{x^2}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx$

Optimal. Leaf size=334

$$\frac{2(-1)^{2/3}\tan^{-1}\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{9\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{11/6}c^{2/3}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}+\frac{2\tan^{-1}\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}\right)}{27\sqrt{3}a^{11/6}c^{2/3}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}+\frac{2(-1)^{2/3}\tan^{-1}\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}+4b}}\right)}{9\sqrt{3}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2a^{11/6}c^{2/3}\sqrt{3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}+4b}}$$

```
[Out] (2*(-1)^(2/3)*ArcTan[(3*(-1)^(1/3)*a^(2/3)*c^(1/3) - 2*b*x)/(Sqrt
[3]*Sqrt[a]*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)])])/(9*Sqrt[3
]*(1 + (-1)^(1/3))^2*a^(11/6)*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(
2/3)]*c^(2/3)) + (2*ArcTan[(3*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*S
qrt[a]*Sqrt[4*b - 3*a^(1/3)*c^(2/3)])])/(27*Sqrt[3]*a^(11/6)*Sqrt
[4*b - 3*a^(1/3)*c^(2/3)]*c^(2/3)) + (2*(-1)^(2/3)*ArcTan[(3*(-1)
^(2/3)*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b + 3*(-1
)^(1/3)*a^(1/3)*c^(2/3)])])/(9*Sqrt[3]*(1 - (-1)^(1/3))*(1 + (-1)
^(1/3))^2*a^(11/6)*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)]*c^(2/
3))
```

Rubi [A] time = 2.26823, antiderivative size = 334, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$

$$\frac{2(-1)^{2/3}\tan^{-1}\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{9\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{11/6}c^{2/3}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}+\frac{2\tan^{-1}\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}\right)}{27\sqrt{3}a^{11/6}c^{2/3}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}+\frac{2(-1)^{2/3}\tan^{-1}\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}+4b}}\right)}{9\sqrt{3}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2a^{11/6}c^{2/3}\sqrt{3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}+4b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6),x]
```


```
[Out] (2*(-1)^(2/3)*ArcTan[(3*(-1)^(1/3)*a^(2/3)*c^(1/3) - 2*b*x)/(Sqrt
[3]*Sqrt[a]*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)])])/(9*Sqrt[3
]*(1 + (-1)^(1/3))^2*a^(11/6)*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(
2/3)]*c^(2/3)) + (2*ArcTan[(3*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*S
qrt[a]*Sqrt[4*b - 3*a^(1/3)*c^(2/3)])])/(27*Sqrt[3]*a^(11/6)*Sqrt
[4*b - 3*a^(1/3)*c^(2/3)]*c^(2/3)) + (2*(-1)^(2/3)*ArcTan[(3*(-1)
^(2/3)*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b + 3*(-1
)^(1/3)*a^(1/3)*c^(2/3)])])/(9*Sqrt[3]*(1 - (-1)^(1/3))*(1 + (-1)
^(1/3))^2*a^(11/6)*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)]*c^(2/
3))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**2/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*
```

```
[Out] Timed out
```

Mathematica [C] time = 0.0670611, size = 97, normalized size = 0.29

$$\frac{1}{3}\text{RootSum}\left[\#1^6b^3+9\#1^4ab^2+27\#1^3a^2c+27\#1^2a^2b+27a^3\&,\frac{\#1\log(x-\#1)}{2\#1^4b^3+12\#1^2ab^2+27\#1a^2c+18a^2b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6),x]
```

```
[Out] RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^2*#1^4 + b
^3*#1^6 & , (Log[x - #1]*#1)/(18*a^2*b + 27*a^2*c*#1 + 12*a*b^2*#
1^2 + 2*b^3*#1^4) & ]/3
```

Maple [C] time = 0.006, size = 93, normalized size = 0.3

$$\frac{1}{3}\sum_{\_R=RootOf\left(b^3\_Z^6+9\,ab^2\_Z^4+27\,a^2c\_Z^3+27\,a^2b\_Z^2+27\,a^3\right)}\frac{\_R^2\ln(x-\_R)}{2\_R^5b^3+12\_R^3ab^2+27\_R^2a^2c+18\_Ra^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x)
```

```
[Out] 1/3*sum(_R^2/(2*_R^5*b^3+12*_R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)
*ln(x-_R),_R=RootOf(_Z^6*b^3+9*_Z^4*a*b^2+27*_Z^3*a^2*c+27*_Z^2*a
^2*b+27*a^3))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2}{b^3x^6+9\,ab^2x^4+27\,a^2cx^3+27\,a^2bx^2+27\,a^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3),x, a
```

```
[Out] integrate(x^2/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^
2 + 27*a^3), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3),x, a
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 81.7248, size = 167, normalized size = 0.5

$$\text{RootSum}\left(t^6\left(282429536481a^{12}c^6 - 669462604992a^{11}b^3c^4\right) - 129140163t^4a^8c^4 + 19683t^2a^4c^2 - 1, \left(t \mapsto t\log\left(x + \frac{627621\cdots}{\cdots}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*a**3)
```

```
[Out] RootSum(_t**6*(282429536481*a**12*c**6 - 669462604992*a**11*b**3*
c**4) - 129140163*_t**4*a**8*c**4 + 19683*_t**2*a**4*c**2 - 1, La
mbda(_t, _t*log(x + (62762119218*_t**5*a**11*c**6 - 148769467776*
_t**5*a**10*b**3*c**4 - 387420489*_t**4*a**9*c**5 + 918330048*_t*
*4*a**8*b**3*c**3 - 23914845*_t**3*a**7*c**4 - 11337408*_t**3*a**
6*b**3*c**2 + 177147*_t**2*a**5*c**3 + 2187*_t*a**3*c**2 - 18*a*c
)/(8*b**2))))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{b^3x^6 + 9\,ab^2x^4 + 27\,a^2cx^3 + 27\,a^2bx^2 + 27\,a^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3),x, a
```

```
[Out] integrate(x^2/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^
2 + 27*a^3), x)
```

3.139 $\int \frac{x}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx$

Optimal. Leaf size=469

$$-\frac{\log\left(3a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{162a^{7/3}c^{2/3}}+\frac{(-1)^{2/3}\log\left(-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{54\left(1+\sqrt[3]{-1}\right)^2a^{7/3}c^{2/3}}-\frac{(-1)^{2/3}\log\left(3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{162a^{7/3}c^{2/3}}-\frac{\tan^{-1}\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{9\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{13/6}\sqrt[3]{c}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}-\frac{\tan^{-1}\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}\right)}{27\sqrt{3}a^{13/6}\sqrt[3]{c}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}+\frac{\sqrt[3]{-1}\tan^{-1}\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}+4b}}\right)}{9\sqrt{3}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2a^{13/6}\sqrt[3]{c}\sqrt{3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}+4b}}$$

```
[Out] -ArcTan[(3*(-1)^(1/3)*a^(2/3)*c^(1/3) - 2*b*x)/(Sqrt[3]*Sqrt[a]*S
qrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)])]/(9*Sqrt[3]*(1 + (-1)^(1
/3))^2*a^(13/6)*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)]*c^(1/3))
 - ArcTan[(3*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b -
 3*a^(1/3)*c^(2/3)])]/(27*Sqrt[3]*a^(13/6)*Sqrt[4*b - 3*a^(1/3)*c
^(2/3)]*c^(1/3)) + ((-1)^(1/3)*ArcTan[(3*(-1)^(2/3)*a^(2/3)*c^(1/
3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2
/3)])])/(9*Sqrt[3]*(1 - (-1)^(1/3))*(1 + (-1)^(1/3))^2*a^(13/6)*S
qrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)]*c^(1/3)) - Log[3*a + 3*a^
(2/3)*c^(1/3)*x + b*x^2]/(162*a^(7/3)*c^(2/3)) + ((-1)^(2/3)*Log[
3*a - 3*(-1)^(1/3)*a^(2/3)*c^(1/3)*x + b*x^2])/(54*(1 + (-1)^(1/3
))^2*a^(7/3)*c^(2/3)) - ((-1)^(2/3)*Log[3*a + 3*(-1)^(2/3)*a^(2/3
)*c^(1/3)*x + b*x^2])/(162*a^(7/3)*c^(2/3))
```

Rubi [A] time = 2.96379, antiderivative size = 469, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114

$$-\frac{\log\left(3a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{162a^{7/3}c^{2/3}}+\frac{(-1)^{2/3}\log\left(-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{54\left(1+\sqrt[3]{-1}\right)^2a^{7/3}c^{2/3}}-\frac{(-1)^{2/3}\log\left(3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{162a^{7/3}c^{2/3}}-\frac{\tan^{-1}\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{9\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{13/6}\sqrt[3]{c}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}-\frac{\tan^{-1}\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}\right)}{27\sqrt{3}a^{13/6}\sqrt[3]{c}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}+\frac{\sqrt[3]{-1}\tan^{-1}\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}+4b}}\right)}{9\sqrt{3}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2a^{13/6}\sqrt[3]{c}\sqrt{3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}+4b}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6),x]
```


```
[Out] -ArcTan[(3*(-1)^(1/3)*a^(2/3)*c^(1/3) - 2*b*x)/(Sqrt[3]*Sqrt[a]*S
qrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)])]/(9*Sqrt[3]*(1 + (-1)^(1
/3))^2*a^(13/6)*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)]*c^(1/3))
 - ArcTan[(3*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b -
 3*a^(1/3)*c^(2/3)])]/(27*Sqrt[3]*a^(13/6)*Sqrt[4*b - 3*a^(1/3)*c
^(2/3)]*c^(1/3)) + ((-1)^(1/3)*ArcTan[(3*(-1)^(2/3)*a^(2/3)*c^(1/
3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2
/3)])])/(9*Sqrt[3]*(1 - (-1)^(1/3))*(1 + (-1)^(1/3))^2*a^(13/6)*S
qrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)]*c^(1/3)) - Log[3*a + 3*a^
(2/3)*c^(1/3)*x + b*x^2]/(162*a^(7/3)*c^(2/3)) + ((-1)^(2/3)*Log[
```

```
3*a - 3*(-1)^(1/3)*a^(2/3)*c^(1/3)*x + b*x^2])/(54*(1 + (-1)^(1/3
))^2*a^(7/3)*c^(2/3)) - ((-1)^(2/3)*Log[3*a + 3*(-1)^(2/3)*a^(2/3
)*c^(1/3)*x + b*x^2])/(162*a^(7/3)*c^(2/3))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*a**
```

```
[Out] Timed out
```

Mathematica [C] time = 0.0683781, size = 95, normalized size = 0.2

$$\frac{1}{3}\text{RootSum}\left[\#1^6b^3+9\#1^4ab^2+27\#1^3a^2c+27\#1^2a^2b+27a^3\&,\frac{\log(x-\#1)}{2\#1^4b^3+12\#1^2ab^2+27\#1a^2c+18a^2b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6),x]
```

```
[Out] RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^2*#1^4 + b
^3*#1^6 & , Log[x - #1]/(18*a^2*b + 27*a^2*c*#1 + 12*a*b^2*#1^2 +
 2*b^3*#1^4) & ]/3
```

Maple [C] time = 0.005, size = 91, normalized size = 0.2

$$\frac{1}{3}\sum_{\_R=RootOf\left(b^3\_Z^6+9\,ab^2\_Z^4+27\,a^2c\_Z^3+27\,a^2b\_Z^2+27\,a^3\right)}\frac{\_R\ln(x-\_R)}{2\,\_R^5b^3+12\,\_R^3ab^2+27\,\_R^2a^2c+18\,\_R\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x)
```

```
[Out] 1/3*sum(_R/(2*_R^5*b^3+12*_R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)*l
n(x-_R),_R=RootOf(_Z^6*b^3+9*_Z^4*a*b^2+27*_Z^3*a^2*c+27*_Z^2*a^2
*b+27*a^3))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{b^3x^6+9\,ab^2x^4+27\,a^2cx^3+27\,a^2bx^2+27\,a^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3),x, alg
```

```
[Out] integrate(x/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2
+ 27*a^3), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3),x, alg
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*a**3),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{b^3x^6 + 9\,ab^2x^4 + 27\,a^2cx^3 + 27\,a^2bx^2 + 27\,a^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3),x, alg
```

```
[Out] integrate(x/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2
+ 27*a^3), x)
```

3.140 $\int \frac{1}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx$

Optimal. Leaf size=522

$$-\frac{\sqrt[3]{-1}\left(3\sqrt[3]{a}c^{2/3}+2\sqrt[3]{-1}b\right)\tan^{-1}\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{27\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{17/6}c^{2/3}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}-\frac{\left(2b-3\sqrt[3]{a}c^{2/3}\right)\tan^{-1}\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}\right)}{81\sqrt{3}a^{17/6}c^{2/3}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}-\frac{\left(2(-1)^{2/3}b-3\sqrt[3]{a}c^{2/3}\right)\tan^{-1}\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}+4b}}\right)}{27\sqrt{3}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2a^{17/6}c^{2/3}\sqrt{3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}+4b}}+\frac{\log\left(3a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{162a^{8/3}\sqrt[3]{c}}-\frac{\log\left(-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{54\left(1+\sqrt[3]{-1}\right)^2a^{8/3}\sqrt[3]{c}}-\frac{\sqrt[3]{-1}\log\left(3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{162a^{8/3}\sqrt[3]{c}}$$

```
[Out] -((-1)^(1/3)*(2*(-1)^(1/3)*b + 3*a^(1/3)*c^(2/3))*ArcTan[(3*(-1)^
(1/3)*a^(2/3)*c^(1/3) - 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*(-1)
^(2/3)*a^(1/3)*c^(2/3)])])/(27*Sqrt[3]*(1 + (-1)^(1/3))^2*a^(17/6
)*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)]*c^(2/3)) - ((2*b - 3*a
^(1/3)*c^(2/3))*ArcTan[(3*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[
a]*Sqrt[4*b - 3*a^(1/3)*c^(2/3)])])/(81*Sqrt[3]*a^(17/6)*Sqrt[4*b
 - 3*a^(1/3)*c^(2/3)]*c^(2/3)) - ((2*(-1)^(2/3)*b - 3*a^(1/3)*c^(
2/3))*ArcTan[(3*(-1)^(2/3)*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt
[a]*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)])])/(27*Sqrt[3]*(1 -
(-1)^(1/3))*(1 + (-1)^(1/3))^2*a^(17/6)*Sqrt[4*b + 3*(-1)^(1/3)*a
^(1/3)*c^(2/3)]*c^(2/3)) + Log[3*a + 3*a^(2/3)*c^(1/3)*x + b*x^2]
/(162*a^(8/3)*c^(1/3)) - Log[3*a - 3*(-1)^(1/3)*a^(2/3)*c^(1/3)*x
 + b*x^2]/(54*(1 + (-1)^(1/3))^2*a^(8/3)*c^(1/3)) - ((-1)^(1/3)*L
og[3*a + 3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x + b*x^2])/(162*a^(8/3)*c^
(1/3))
```

Rubi [A] time = 3.52345, antiderivative size = 522, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.119$

$$-\frac{\sqrt[3]{-1}\left(3\sqrt[3]{a}c^{2/3}+2\sqrt[3]{-1}b\right)\tan^{-1}\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{27\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{17/6}c^{2/3}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}-\frac{\left(2b-3\sqrt[3]{a}c^{2/3}\right)\tan^{-1}\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}\right)}{81\sqrt{3}a^{17/6}c^{2/3}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}-\frac{\left(2(-1)^{2/3}b-3\sqrt[3]{a}c^{2/3}\right)\tan^{-1}\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}+4b}}\right)}{27\sqrt{3}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2a^{17/6}c^{2/3}\sqrt{3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}+4b}}+\frac{\log\left(3a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{162a^{8/3}\sqrt[3]{c}}-\frac{\log\left(-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{54\left(1+\sqrt[3]{-1}\right)^2a^{8/3}\sqrt[3]{c}}-\frac{\sqrt[3]{-1}\log\left(3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{162a^{8/3}\sqrt[3]{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6)^(-
1),x]
```

```
[Out] -((-1)^(1/3)*(2*(-1)^(1/3)*b + 3*a^(1/3)*c^(2/3))*ArcTan[(3*(-1)^
(1/3)*a^(2/3)*c^(1/3) - 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*(-1)
^(2/3)*a^(1/3)*c^(2/3)])])/(27*Sqrt[3]*(1 + (-1)^(1/3))^2*a^(17/6
)*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)]*c^(2/3)) - ((2*b - 3*a
^(1/3)*c^(2/3))*ArcTan[(3*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[
a]*Sqrt[4*b - 3*a^(1/3)*c^(2/3)])])/(81*Sqrt[3]*a^(17/6)*Sqrt[4*b
 - 3*a^(1/3)*c^(2/3)]*c^(2/3)) - ((2*(-1)^(2/3)*b - 3*a^(1/3)*c^(
2/3))*ArcTan[(3*(-1)^(2/3)*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt
[a]*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)])])/(27*Sqrt[3]*(1 -
(-1)^(1/3))*(1 + (-1)^(1/3))^2*a^(17/6)*Sqrt[4*b + 3*(-1)^(1/3)*a
^(1/3)*c^(2/3)]*c^(2/3)) + Log[3*a + 3*a^(2/3)*c^(1/3)*x + b*x^2]
/(162*a^(8/3)*c^(1/3)) - Log[3*a - 3*(-1)^(1/3)*a^(2/3)*c^(1/3)*x
 + b*x^2]/(54*(1 + (-1)^(1/3))^2*a^(8/3)*c^(1/3)) - ((-1)^(1/3)*L
og[3*a + 3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x + b*x^2])/(162*a^(8/3)*c^
(1/3))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*a**
```

```
[Out] Timed out
```

Mathematica [C] time = 0.0898899, size = 99, normalized size = 0.19

$$\frac{1}{3}\text{RootSum}\left[\#1^6b^3+9\#1^4ab^2+27\#1^3a^2c+27\#1^2a^2b+27a^3\&,\frac{\log(x-\#1)}{2\#1^5b^3+12\#1^3ab^2+27\#1^2a^2c+18\#1a^2b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6)^(-
1),x]
```

```
[Out] RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^2*#1^4 + b
^3*#1^6 & , Log[x - #1]/(18*a^2*b*#1 + 27*a^2*c*#1^2 + 12*a*b^2*#
1^3 + 2*b^3*#1^5) & ]/3
```

Maple [C] time = 0.005, size = 90, normalized size = 0.2

$$\frac{1}{3}\sum_{\_R=RootOf\left(b^3\_Z^6+9\,ab^2\_Z^4+27\,a^2c\_Z^3+27\,a^2b\_Z^2+27\,a^3\right)}\frac{\ln(x-\_R)}{2\_R^5b^3+12\_R^3ab^2+27\_R^2a^2c+18\_R\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x)
```

```
[Out] 1/3*sum(1/(2*_R^5*b^3+12*_R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)*ln
(x-_R),_R=RootOf(_Z^6*b^3+9*_Z^4*a*b^2+27*_Z^3*a^2*c+27*_Z^2*a^2*
b+27*a^3))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{b^3x^6+9\,ab^2x^4+27\,a^2cx^3+27\,a^2bx^2+27\,a^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3),x, alg
```

```
[Out] integrate(1/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2
+ 27*a^3), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3),x, alg
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*a**3),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b^3x^6 + 9\,ab^2x^4 + 27\,a^2cx^3 + 27\,a^2bx^2 + 27\,a^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3),x, alg
```

```
[Out] integrate(1/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2
+ 27*a^3), x)
```

3.141 $\int \frac{1}{x(27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6)}\,dx$

Optimal. Leaf size=563

$$-\frac{\left(3\sqrt[3]{a}-\frac{b}{c^{2/3}}\right)\log\left(3a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{486a^{10/3}}-\frac{\left(6\sqrt[3]{a}c^{2/3}+i\sqrt{3}b+b\right)\log\left(-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{972a^{10/3}c^{2/3}}-\frac{\left(3\sqrt[3]{a}-\frac{(-1)^{2/3}b}{c^{2/3}}\right)\log\left(3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{486a^{10/3}}+\frac{\left(b-(-1)^{2/3}\sqrt[3]{a}c^{2/3}\right)\tan^{-1}\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{9\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{19/6}\sqrt[3]{c}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}+\frac{\left(b-\sqrt[3]{a}c^{2/3}\right)\tan^{-1}\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}\right)}{27\sqrt{3}a^{19/6}\sqrt[3]{c}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}+\frac{(-1)^{2/3}\left((-1)^{2/3}b-\sqrt[3]{a}c^{2/3}\right)\tan^{-1}\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}+4b}}\right)}{9\sqrt{3}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2a^{19/6}\sqrt[3]{c}\sqrt{3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}+4b}}+\frac{\log(x)}{27a^3}$$

```
[Out] ((b - (-1)^(2/3)*a^(1/3)*c^(2/3))*ArcTan[(3*(-1)^(1/3)*a^(2/3)*c^
(1/3) - 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c
^(2/3)])])/(9*Sqrt[3]*(1 + (-1)^(1/3))^2*a^(19/6)*Sqrt[4*b - 3*(-
1)^(2/3)*a^(1/3)*c^(2/3)]*c^(1/3)) + ((b - a^(1/3)*c^(2/3))*ArcTa
n[(3*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*a^(1/
3)*c^(2/3)])])/(27*Sqrt[3]*a^(19/6)*Sqrt[4*b - 3*a^(1/3)*c^(2/3)]
*c^(1/3)) + ((-1)^(2/3)*((-1)^(2/3)*b - a^(1/3)*c^(2/3))*ArcTan[(
3*(-1)^(2/3)*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b +
 3*(-1)^(1/3)*a^(1/3)*c^(2/3)])])/(9*Sqrt[3]*(1 - (-1)^(1/3))*(1
+ (-1)^(1/3))^2*a^(19/6)*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)]
*c^(1/3)) + Log[x]/(27*a^3) - ((3*a^(1/3) - b/c^(2/3))*Log[3*a +
3*a^(2/3)*c^(1/3)*x + b*x^2])/(486*a^(10/3)) - ((b + I*Sqrt[3]*b
+ 6*a^(1/3)*c^(2/3))*Log[3*a - 3*(-1)^(1/3)*a^(2/3)*c^(1/3)*x + b
*x^2])/(972*a^(10/3)*c^(2/3)) - ((3*a^(1/3) - ((-1)^(2/3)*b)/c^(2
/3))*Log[3*a + 3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x + b*x^2])/(486*a^(1
0/3))
```

Rubi [A] time = 3.91037, antiderivative size = 563, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.109$

$$-\frac{\left(3\sqrt[3]{a}-\frac{b}{c^{2/3}}\right)\log\left(3a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{486a^{10/3}}-\frac{\left(6\sqrt[3]{a}c^{2/3}+i\sqrt{3}b+b\right)\log\left(-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{972a^{10/3}c^{2/3}}-\frac{\left(3\sqrt[3]{a}-\frac{(-1)^{2/3}b}{c^{2/3}}\right)\log\left(3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{486a^{10/3}}+\frac{\left(b-(-1)^{2/3}\sqrt[3]{a}c^{2/3}\right)\tan^{-1}\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{9\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{19/6}\sqrt[3]{c}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}+\frac{\left(b-\sqrt[3]{a}c^{2/3}\right)\tan^{-1}\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}\right)}{27\sqrt{3}a^{19/6}\sqrt[3]{c}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}+\frac{(-1)^{2/3}\left((-1)^{2/3}b-\sqrt[3]{a}c^{2/3}\right)\tan^{-1}\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}+4b}}\right)}{9\sqrt{3}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2a^{19/6}\sqrt[3]{c}\sqrt{3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}+4b}}+\frac{\log(x)}{27a^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6)),x]
```

```
[Out] ((b - (-1)^(2/3)*a^(1/3)*c^(2/3))*ArcTan[(3*(-1)^(1/3)*a^(2/3)*c^
(1/3) - 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c
^(2/3)])])/(9*Sqrt[3]*(1 + (-1)^(1/3))^2*a^(19/6)*Sqrt[4*b - 3*(-
1)^(2/3)*a^(1/3)*c^(2/3)]*c^(1/3)) + ((b - a^(1/3)*c^(2/3))*ArcTa
n[(3*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*a^(1/
3)*c^(2/3)])])/(27*Sqrt[3]*a^(19/6)*Sqrt[4*b - 3*a^(1/3)*c^(2/3)]
*c^(1/3)) + ((-1)^(2/3)*((-1)^(2/3)*b - a^(1/3)*c^(2/3))*ArcTan[(
3*(-1)^(2/3)*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b +
 3*(-1)^(1/3)*a^(1/3)*c^(2/3)])])/(9*Sqrt[3]*(1 - (-1)^(1/3))*(1
+ (-1)^(1/3))^2*a^(19/6)*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)]
*c^(1/3)) + Log[x]/(27*a^3) - ((3*a^(1/3) - b/c^(2/3))*Log[3*a +
3*a^(2/3)*c^(1/3)*x + b*x^2])/(486*a^(10/3)) - ((b + I*Sqrt[3]*b
+ 6*a^(1/3)*c^(2/3))*Log[3*a - 3*(-1)^(1/3)*a^(2/3)*c^(1/3)*x + b
*x^2])/(972*a^(10/3)*c^(2/3)) - ((3*a^(1/3) - ((-1)^(2/3)*b)/c^(2
/3))*Log[3*a + 3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x + b*x^2])/(486*a^(1
0/3))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/x/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*a
```

```
[Out] Timed out
```

Mathematica [C] time = 0.147325, size = 157, normalized size = 0.28

$$-\frac{\text{RootSum}\left[\#1^6 b^3+9\#1^4 ab^2+27\#1^3 a^2 c+27\#1^2 a^2 b+27a^3\&,\frac{\#1^4 b^3 \log(x-\#1)+9\#1^2 ab^2 \log(x-\#1)+27a^2 b \log(x-\#1)+27\#1 a^2 c \log(x-\#1}{2\#1^4 b^3+12\#1^2 ab^2+27\#1 a^2 c+18a^2 b}\right.}{81a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6)),x]
```

```
[Out] -(-3*Log[x] + RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*
a*b^2*#1^4 + b^3*#1^6 & , (27*a^2*b*Log[x - #1] + 27*a^2*c*Log[x
- #1]*#1 + 9*a*b^2*Log[x - #1]*#1^2 + b^3*Log[x - #1]*#1^4)/(18*a
^2*b + 27*a^2*c*#1 + 12*a*b^2*#1^2 + 2*b^3*#1^4) & ])/(81*a^3)
```

Maple [C] time = 0.013, size = 134, normalized size = 0.2

$$-\frac{1}{81\,a^3}\sum_{\_R=RootOf\left(b^3\_Z^6+9\,ab^2\_Z^4+27\,a^2c\_Z^3+27\,a^2b\_Z^2+27\,a^3\right)}\frac{\left(\_R^5b^3+9\,\_R^3ab^2+27\,\_R^2a^2c+27\,\_R\,a^2b\right)\ln(x-\_R)}{2\,\_R^5b^3+12\,\_R^3ab^2+27\,\_R^2a^2c+18\,\_R\,a^2b}+\frac{\ln(x)}{27\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x)
```

```
[Out] -1/81/a^3*sum((_R^5*b^3+9*_R^3*a*b^2+27*_R^2*a^2*c+27*_R*a^2*b)/(
2*_R^5*b^3+12*_R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)*ln(x-_R),_R=R
```

```
ootOf(_Z^6*b^3+9*_Z^4*a*b^2+27*_Z^3*a^2*c+27*_Z^2*a^2*b+27*a^3))+
1/27*ln(x)/a^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{b^3x^5+9\,ab^2x^3+27\,a^2cx^2+27\,a^2bx}{b^3x^6+9\,ab^2x^4+27\,a^2cx^3+27\,a^2bx^2+27\,a^3}\,dx}{27\,a^3}+\frac{\log\left(x\right)}{27\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3)*x),x,
```

```
[Out] -1/27*integrate((b^3*x^5 + 9*a*b^2*x^3 + 27*a^2*c*x^2 + 27*a^2*b*
x)/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3)
, x)/a^3 + 1/27*log(x)/a^3
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3)*x),x,
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*a**3),
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b^3x^6+9\,ab^2x^4+27\,a^2cx^3+27\,a^2bx^2+27\,a^3\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3)*x),x,
```

```
[Out] integrate(1/((b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2
 + 27*a^3)*x), x)
```

3.142 $\int \frac{1}{x^2(27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6)}\,dx$

Optimal. Leaf size=645

$$\frac{\left(9a^{2/3}c^{4/3}+12\sqrt[3]{-1}\sqrt[3]{a}bc^{2/3}+2(-1)^{2/3}b^2\right)\tan^{-1}\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{81\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{23/6}c^{2/3}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}+\frac{\left(9a^{2/3}c^{4/3}-12\sqrt[3]{a}bc^{2/3}+2b^2\right)\tan^{-1}\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}\right)}{243\sqrt{3}a^{23/6}c^{2/3}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}+\frac{(-1)^{2/3}\left(9(-1)^{2/3}a^{2/3}c^{4/3}+12\sqrt[3]{-1}\sqrt[3]{a}bc^{2/3}+2b^2\right)\tan^{-1}\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}+4b}}\right)}{81\sqrt{3}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2a^{23/6}c^{2/3}\sqrt{3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}+4b}}-\frac{\left(2b-3\sqrt[3]{a}c^{2/3}\right)\log\left(3a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{486a^{11/3}\sqrt[3]{c}}+\frac{\left(2b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}\right)\log\left(-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{162\left(1+\sqrt[3]{-1}\right)^2a^{11/3}\sqrt[3]{c}}+\frac{\sqrt[3]{-1}\left(3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}+2b\right)\log\left(3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{486a^{11/3}\sqrt[3]{c}}-\frac{1}{27a^3x}$$

```
[Out] -1/(27*a^3*x) + ((2*(-1)^(2/3)*b^2 + 12*(-1)^(1/3)*a^(1/3)*b*c^(2
/3) + 9*a^(2/3)*c^(4/3))*ArcTan[(3*(-1)^(1/3)*a^(2/3)*c^(1/3) - 2
*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)])]
)/(81*Sqrt[3]*(1 + (-1)^(1/3))^2*a^(23/6)*Sqrt[4*b - 3*(-1)^(2/3)
*a^(1/3)*c^(2/3)]*c^(2/3)) + ((2*b^2 - 12*a^(1/3)*b*c^(2/3) + 9*a
^(2/3)*c^(4/3))*ArcTan[(3*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[
a]*Sqrt[4*b - 3*a^(1/3)*c^(2/3)])])/(243*Sqrt[3]*a^(23/6)*Sqrt[4*
b - 3*a^(1/3)*c^(2/3)]*c^(2/3)) + ((-1)^(2/3)*(2*b^2 + 12*(-1)^(1
/3)*a^(1/3)*b*c^(2/3) + 9*(-1)^(2/3)*a^(2/3)*c^(4/3))*ArcTan[(3*(
-1)^(2/3)*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b + 3*
(-1)^(1/3)*a^(1/3)*c^(2/3)])])/(81*Sqrt[3]*(1 - (-1)^(1/3))*(1 +
(-1)^(1/3))^2*a^(23/6)*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)]*c
^(2/3)) - ((2*b - 3*a^(1/3)*c^(2/3))*Log[3*a + 3*a^(2/3)*c^(1/3)*
x + b*x^2])/(486*a^(11/3)*c^(1/3)) + ((2*b - 3*(-1)^(2/3)*a^(1/3)
*c^(2/3))*Log[3*a - 3*(-1)^(1/3)*a^(2/3)*c^(1/3)*x + b*x^2])/(162
*(1 + (-1)^(1/3))^2*a^(11/3)*c^(1/3)) + ((-1)^(1/3)*(2*b + 3*(-1)
^(1/3)*a^(1/3)*c^(2/3))*Log[3*a + 3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x
+ b*x^2])/(486*a^(11/3)*c^(1/3))
```

Rubi [A] time = 5.03742, antiderivative size = 640, normalized size of antiderivative = 0.99, number of steps used = 14, number of rules used = 5, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.109

$$\frac{\left(9a^{2/3}c^{4/3}+12\sqrt[3]{-1}\sqrt[3]{a}bc^{2/3}+2(-1)^{2/3}b^2\right)\tan^{-1}\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{81\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{23/6}c^{2/3}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}
+\frac{\left(9a^{2/3}c^{4/3}-12\sqrt[3]{a}bc^{2/3}+2b^2\right)\tan^{-1}\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}\right)}{243\sqrt{3}a^{23/6}c^{2/3}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}
+\frac{\left(-9\sqrt[3]{-1}a^{2/3}c^{4/3}-12\sqrt[3]{a}bc^{2/3}+2(-1)^{2/3}b^2\right)\tan^{-1}\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}+4b}}\right)}{81\sqrt{3}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2a^{23/6}c^{2/3}\sqrt{3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}+4b}}
-\frac{\left(2b-3\sqrt[3]{a}c^{2/3}\right)\log\left(3a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{486a^{11/3}\sqrt[3]{c}}
+\frac{\left(2b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}\right)\log\left(-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{162\left(1+\sqrt[3]{-1}\right)^2a^{11/3}\sqrt[3]{c}}
+\frac{\sqrt[3]{-1}\left(3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}+2b\right)\log\left(3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+3a+bx^2\right)}{486a^{11/3}\sqrt[3]{c}}-\frac{1}{27a^3x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6)),x]
```

```
[Out] -1/(27*a^3*x) + ((2*(-1)^(2/3)*b^2 + 12*(-1)^(1/3)*a^(1/3)*b*c^(2
/3) + 9*a^(2/3)*c^(4/3))*ArcTan[(3*(-1)^(1/3)*a^(2/3)*c^(1/3) - 2
*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)])]
)/(81*Sqrt[3]*(1 + (-1)^(1/3))^2*a^(23/6)*Sqrt[4*b - 3*(-1)^(2/3)
*a^(1/3)*c^(2/3)]*c^(2/3)) + ((2*b^2 - 12*a^(1/3)*b*c^(2/3) + 9*a
^(2/3)*c^(4/3))*ArcTan[(3*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[
a]*Sqrt[4*b - 3*a^(1/3)*c^(2/3)])])/(243*Sqrt[3]*a^(23/6)*Sqrt[4*
b - 3*a^(1/3)*c^(2/3)]*c^(2/3)) + ((2*(-1)^(2/3)*b^2 - 12*a^(1/3)
*b*c^(2/3) - 9*(-1)^(1/3)*a^(2/3)*c^(4/3))*ArcTan[(3*(-1)^(2/3)*a
^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b + 3*(-1)^(1/3)*
a^(1/3)*c^(2/3)])])/(81*Sqrt[3]*(1 - (-1)^(1/3))*(1 + (-1)^(1/3))
^2*a^(23/6)*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)]*c^(2/3)) - (
(2*b - 3*a^(1/3)*c^(2/3))*Log[3*a + 3*a^(2/3)*c^(1/3)*x + b*x^2])
/(486*a^(11/3)*c^(1/3)) + ((2*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3))*L
og[3*a - 3*(-1)^(1/3)*a^(2/3)*c^(1/3)*x + b*x^2])/(162*(1 + (-1)^
(1/3))^2*a^(11/3)*c^(1/3)) + ((-1)^(1/3)*(2*b + 3*(-1)^(1/3)*a^(1
/3)*c^(2/3))*Log[3*a + 3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x + b*x^2])/(
486*a^(11/3)*c^(1/3))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/x**2/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+2
```

```
[Out] Timed out
```

Mathematica [C] time = 0.194832, size = 163, normalized size = 0.25

$$-\frac{x\text{RootSum}\left[\#1^6b^3+9\#1^4ab^2+27\#1^3a^2c+27\#1^2a^2b+27a^3\&,\frac{\#1^4b^3\log(x-\#1)+9\#1^2ab^2\log(x-\#1)+27a^2b\log(x-\#1)+27\#1a^2c\log(x-\#}{2\#1^5b^3+12\#1^3ab^2+27\#1^2a^2c+18\#1a^2b}\right.}{81a^3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6)),
```

```
[Out] -(3 + x*RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^2*
#1^4 + b^3*#1^6 & , (27*a^2*b*Log[x - #1] + 27*a^2*c*Log[x - #1]*
#1 + 9*a*b^2*Log[x - #1]*#1^2 + b^3*Log[x - #1]*#1^4)/(18*a^2*b*#
1 + 27*a^2*c*#1^2 + 12*a*b^2*#1^3 + 2*b^3*#1^5) & ])/(81*a^3*x)
```

Maple [C] time = 0.011, size = 133, normalized size = 0.2

$$\frac{1}{81\,a^3}\sum_{\_R=RootOf\left(b^3\_Z^6+9\,ab^2\_Z^4+27\,a^2c\_Z^3+27\,a^2b\_Z^2+27\,a^3\right)}\frac{\left(-\_R^4b^3-9\,\_R^2ab^2-27\,\_R\,a^2c-27\,a^2b\right)\ln\left(x-\_R\right)}{2\,\_R^5b^3+12\,\_R^3ab^2+27\,\_R^2a^2c+18\,\_R\,a^2b}-\frac{1}{27\,a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x)
```

```
[Out] 1/81/a^3*sum((-_R^4*b^3-9*_R^2*a*b^2-27*_R*a^2*c-27*a^2*b)/(2*_R^
5*b^3+12*_R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)*ln(x-_R),_R=RootOf
(_Z^6*b^3+9*_Z^4*a*b^2+27*_Z^3*a^2*c+27*_Z^2*a^2*b+27*a^3))-1/27/
a^3/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int\frac{b^3x^4+9\,ab^2x^2+27\,a^2cx+27\,a^2b}{b^3x^6+9\,ab^2x^4+27\,a^2cx^3+27\,a^2bx^2+27\,a^3}\,dx}{27\,a^3}-\frac{1}{27\,a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3)*x^2),
```

```
[Out] -1/27*integrate((b^3*x^4 + 9*a*b^2*x^2 + 27*a^2*c*x + 27*a^2*b)/(
b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3), x)
/a^3 - 1/27/(a^3*x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3)*x^2),
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*a**
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b^3x^6 + 9\,ab^2x^4 + 27\,a^2cx^3 + 27\,a^2bx^2 + 27\,a^3)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3)*x^2),
```

```
[Out] integrate(1/((b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2
 + 27*a^3)*x^2), x)
```

3.143 $\int \frac{x^5}{216+108x^2+324x^3+18x^4+x^6}\,dx$

Optimal. Leaf size=395

$$\frac{1}{216}\left(36+2^{2/3}\sqrt[3]{3}\left(1+i\sqrt{3}\right)\right)\log\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)$$

$$+\frac{1}{108}\left(18-(-2)^{2/3}\sqrt[3]{3}\right)\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)+\frac{1}{108}\left(18-2^{2/3}\sqrt[3]{3}\right)\log\left(x^2+3\,2^{2/3}\sqrt[3]{3}x+6\right)-\frac{\sqrt[3]{-2}\left(1+\sqrt[3]{-2}3^{2/3}\right)\tan^{-1}\left(\cdots\right)}{3^{5/6}\sqrt{8+9i\sqrt[3]{2}\sqrt[6]{3}+\cdots}}$$

```
[Out] -(((-2)^(1/3)*(1 + (-2)^(1/3)*3^(2/3))*ArcTan[(3*(-2)^(2/3)*3^(1/
3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(3^(5/6)*Sqrt[8 +
(9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3)])) + ((3/2)^(1/6)*(1 -
(-3)^(2/3)*2^(1/3))*ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/S
qrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/((1 + (-1)^(1/3))^2*Sqrt[4 -
3*(-3)^(2/3)*2^(1/3)]) - ((1 - 2^(1/3)*3^(2/3))*ArcTanh[(2^(1/6)*
(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(2^(1
/6)*3^(5/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)]) + ((36 + 2^(2/3)*3^(1/3
)*(1 + I*Sqrt[3]))*Log[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2])/216 + (
(18 - (-2)^(2/3)*3^(1/3))*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/
108 + ((18 - 2^(2/3)*3^(1/3))*Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2])
/108
```

Rubi [A] time = 3.4896, antiderivative size = 395, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$\frac{1}{216}\left(36+2^{2/3}\sqrt[3]{3}\left(1+i\sqrt{3}\right)\right)\log\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)$$

$$+\frac{1}{108}\left(18-(-2)^{2/3}\sqrt[3]{3}\right)\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)+\frac{1}{108}\left(18-2^{2/3}\sqrt[3]{3}\right)\log\left(x^2+3\,2^{2/3}\sqrt[3]{3}x+6\right)-\frac{\sqrt[3]{-2}\left(1+\sqrt[3]{-2}3^{2/3}\right)\tan^{-1}\left(\cdots\right)}{3^{5/6}\sqrt{8+9i\sqrt[3]{2}\sqrt[6]{3}+\cdots}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

```
[Out] -(((-2)^(1/3)*(1 + (-2)^(1/3)*3^(2/3))*ArcTan[(3*(-2)^(2/3)*3^(1/
3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(3^(5/6)*Sqrt[8 +
(9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3)])) + ((3/2)^(1/6)*(1 -
(-3)^(2/3)*2^(1/3))*ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/S
qrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/((1 + (-1)^(1/3))^2*Sqrt[4 -
3*(-3)^(2/3)*2^(1/3)]) - ((1 - 2^(1/3)*3^(2/3))*ArcTanh[(2^(1/6)*
(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(2^(1
/6)*3^(5/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)]) + ((36 + 2^(2/3)*3^(1/3
)*(1 + I*Sqrt[3]))*Log[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2])/216 + (
(18 - (-2)^(2/3)*3^(1/3))*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/
108 + ((18 - 2^(2/3)*3^(1/3))*Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2])
/108
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.0236538, size = 61, normalized size = 0.15

$$\frac{1}{6}\text{RootSum}\left[\#1^6+18\#1^4+324\#1^3+108\#1^2+216\&,\frac{\#1^4\log(x-\#1)}{\#1^4+12\#1^2+162\#1+36}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

```
[Out] RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , (Log[x - #
1]*#1^4)/(36 + 162*#1 + 12*#1^2 + #1^4) & ]/6
```

Maple [C] time = 0.009, size = 56, normalized size = 0.1

$$\frac{1}{6}\sum_{\_R=RootOf(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216)}\frac{\_R^5\ln(x-\_R)}{\_R^5+12\_R^3+162\_R^2+36\_R}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(x^6+18*x^4+324*x^3+108*x^2+216),x)
```

```
[Out] 1/6*sum(_R^5/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^
6+18*_Z^4+324*_Z^3+108*_Z^2+216))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^5}{x^6+18x^4+324x^3+108x^2+216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216),x, algorithm="maxima")
```

```
[Out] integrate(x^5/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 0.606971, size = 70, normalized size = 0.18

$$\operatorname{RootSum}\left(72662865048t^6 - 72662865048t^5 + 24163559388t^4 - 2646786132t^3 - 6626610t^2 - 4374t - 1, \left(t \mapsto t\log\left(-\frac{8923\ldots}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

```
[Out] RootSum(72662865048*_t**6 - 72662865048*_t**5 + 24163559388*_t**4
 - 2646786132*_t**3 - 6626610*_t**2 - 4374*_t - 1, Lambda(_t, _t*
log(-89236417131047376*_t**5/833243797 + 89301949532998128*_t**4/
833243797 - 29740560281805852*_t**3/833243797 + 192466080408420*_
t**2/49014341 + 5867255361684*_t/833243797 + x + 5365044886/24997
31391)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{x^6 + 18\,x^4 + 324\,x^3 + 108\,x^2 + 216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216),x, algorithm="giac")
```

```
[Out] integrate(x^5/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

3.144 $\int \frac{x^4}{216+108x^2+324x^3+18x^4+x^6}\,dx$

Optimal. Leaf size=377

$$\frac{\log\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}{6\,2^{2/3}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^2}+\frac{\sqrt[3]{-\frac{1}{3}}\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}{18\,2^{2/3}}-\frac{\log\left(x^2+3\,2^{2/3}\sqrt[3]{3}x+6\right)}{18\,2^{2/3}\sqrt[3]{3}}+\frac{(-1)^{2/3}\left(3(-3)^{2/3}-2^{2/3}\right)\tan^{-1}\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{9\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^2\sqrt{2\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}+\frac{\left(9-(-2)^{2/3}\sqrt[3]{3}\right)\tan^{-1}\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{27\sqrt{3\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)}}-\frac{\left(9-2^{2/3}\sqrt[3]{3}\right)\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(3\sqrt[3]{2}3^{2/3}-4\right)}}\right)}{27\sqrt{6\left(3\sqrt[3]{2}3^{2/3}-4\right)}}$$

```
[Out] ((-1)^(2/3)*(3*(-3)^(2/3) - 2^(2/3))*ArcTan[(3*(-3)^(1/3)*2^(2/3)
 - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(9*3^(1/6)*(1 + (-1)
^(1/3))^2*Sqrt[2*(4 - 3*(-3)^(2/3)*2^(1/3))]) + ((9 - (-2)^(2/3)*
3^(1/3))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(
1/3)*3^(2/3))]])/(27*Sqrt[3*(8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3
)*3^(2/3))]) - ((9 - 2^(2/3)*3^(1/3))*ArcTanh[(2^(1/6)*(3*3^(1/3)
 + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(27*Sqrt[6*(-4
+ 3*2^(1/3)*3^(2/3))]) + Log[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2]/(6
*2^(2/3)*3^(1/3)*(1 + (-1)^(1/3))^2) + ((-1/3)^(1/3)*Log[6 + 3*(-
2)^(2/3)*3^(1/3)*x + x^2])/(18*2^(2/3)) - Log[6 + 3*2^(2/3)*3^(1/
3)*x + x^2]/(18*2^(2/3)*3^(1/3))
```

Rubi [A] time = 2.74215, antiderivative size = 377, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$\frac{\log\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}{6\,2^{2/3}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^2}+\frac{\sqrt[3]{-\frac{1}{3}}\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}{18\,2^{2/3}}-\frac{\log\left(x^2+3\,2^{2/3}\sqrt[3]{3}x+6\right)}{18\,2^{2/3}\sqrt[3]{3}}+\frac{(-1)^{2/3}\left(3(-3)^{2/3}-2^{2/3}\right)\tan^{-1}\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{9\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^2\sqrt{2\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}-\frac{\left((-2)^{2/3}-3\,3^{2/3}\right)\tan^{-1}\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{27\sqrt[6]{3}\sqrt{8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}}}+\frac{\left(2^{2/3}-3\,3^{2/3}\right)\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(3\sqrt[3]{2}3^{2/3}-4\right)}}\right)}{27\sqrt[6]{3}\sqrt{2\left(3\sqrt[3]{2}3^{2/3}-4\right)}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

```
[Out] ((-1)^(2/3)*(3*(-3)^(2/3) - 2^(2/3))*ArcTan[(3*(-3)^(1/3)*2^(2/3)
 - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(9*3^(1/6)*(1 + (-1)
^(1/3))^2*Sqrt[2*(4 - 3*(-3)^(2/3)*2^(1/3))]) - (((-2)^(2/3) - 3*
```

```
3^(2/3))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(
1/3)*3^(2/3))]])/(27*3^(1/6)*Sqrt[8 + (9*I)*2^(1/3)*3^(1/6) + 3*2
^(1/3)*3^(2/3)]) + ((2^(2/3) - 3*3^(2/3))*ArcTanh[(2^(1/6)*(3*3^(
1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(27*3^(1/6)
*Sqrt[2*(-4 + 3*2^(1/3)*3^(2/3))]) + Log[6 - 3*(-3)^(1/3)*2^(2/3)
*x + x^2]/(6*2^(2/3)*3^(1/3)*(1 + (-1)^(1/3))^2) + ((-1/3)^(1/3)*
Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(18*2^(2/3)) - Log[6 + 3*2
^(2/3)*3^(1/3)*x + x^2]/(18*2^(2/3)*3^(1/3))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.0203644, size = 61, normalized size = 0.16

$$\frac{1}{6}\text{RootSum}\left[\#1^6+18\#1^4+324\#1^3+108\#1^2+216\&,\frac{\#1^3\log(x-\#1)}{\#1^4+12\#1^2+162\#1+36}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

```
[Out] RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , (Log[x - #
1]*#1^3)/(36 + 162*#1 + 12*#1^2 + #1^4) & ]/6
```

Maple [C] time = 0.007, size = 56, normalized size = 0.2

$$\frac{1}{6}\sum_{\_R=RootOf\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\_R^4\ln(x-\_R)}{\_R^5+12\_R^3+162\_R^2+36\_R}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(x^6+18*x^4+324*x^3+108*x^2+216),x)
```

```
[Out] 1/6*sum(_R^4/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^
6+18*_Z^4+324*_Z^3+108*_Z^2+216))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^4}{x^6+18\,x^4+324\,x^3+108\,x^2+216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 0.710731, size = 65, normalized size = 0.17

$$\text{RootSum}\left(15695178850368t^6 - 2066242608t^4 + 1845163152t^3 - 1180980t^2 - 1944t - 1, \left(t \mapsto t\log\left(\frac{614714526178551746}{57121295165}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

```
[Out] RootSum(15695178850368*_t**6 - 2066242608*_t**4 + 1845163152*_t**
3 - 1180980*_t**2 - 1944*_t - 1, Lambda(_t, _t*log(61471452617855
1746208*_t**5/57121295165 - 1270857362386176*_t**4/57121295165 -
80483053187684376*_t**3/57121295165 + 72431318325103884*_t**2/571
21295165 - 45358602689088*_t/57121295165 + x - 44532180783/571212
95165)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{x^6 + 18\,x^4 + 324\,x^3 + 108\,x^2 + 216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216),x, algorithm="giac")
```

```
[Out] integrate(x^4/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

3.145 $\int \frac{x^3}{216+108x^2+324x^3+18x^4+x^6}\,dx$

Optimal. Leaf size=361

$$-\frac{(-1)^{2/3}\log\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}{36\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}+\frac{(-1)^{2/3}\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}{108\sqrt[3]{2}3^{2/3}}+\frac{\log\left(x^2+3\,2^{2/3}\sqrt[3]{3}x+6\right)}{108\sqrt[3]{2}3^{2/3}}-\frac{\tan^{-1}\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{6\sqrt[6]{2}3^{5/6}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}+\frac{\sqrt[3]{-1}\tan^{-1}\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{9\,2^{2/3}3^{5/6}\sqrt{8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}}}+\frac{\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(3\sqrt[3]{2}3^{2/3}-4\right)}}\right)}{18\sqrt[6]{2}3^{5/6}\sqrt{3\sqrt[3]{2}3^{2/3}-4}}$$

```
[Out] -ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*2^(
1/3))]]/(6*2^(1/6)*3^(5/6)*(1 + (-1)^(1/3))^2*Sqrt[4 - 3*(-3)^(2/
3)*2^(1/3)]) + ((-1)^(1/3)*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sq
rt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(9*2^(2/3)*3^(5/6)*Sqrt[8 + (9
*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3)]) + ArcTanh[(2^(1/6)*(3*3
^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]]/(18*2^(1/6
)*3^(5/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)]) - ((-1)^(2/3)*Log[6 - 3*(
-3)^(1/3)*2^(2/3)*x + x^2])/(36*2^(1/3)*3^(2/3)*(1 + (-1)^(1/3))^
2) + ((-1)^(2/3)*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(108*2^(1
/3)*3^(2/3)) + Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(108*2^(1/3)*3^
(2/3))
```

Rubi [A] time = 1.90455, antiderivative size = 361, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$-\frac{(-1)^{2/3}\log\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}{36\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}+\frac{(-1)^{2/3}\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}{108\sqrt[3]{2}3^{2/3}}+\frac{\log\left(x^2+3\,2^{2/3}\sqrt[3]{3}x+6\right)}{108\sqrt[3]{2}3^{2/3}}-\frac{\tan^{-1}\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{6\sqrt[6]{2}3^{5/6}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}+\frac{\sqrt[3]{-1}\tan^{-1}\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{9\,2^{2/3}3^{5/6}\sqrt{8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}}}+\frac{\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(3\sqrt[3]{2}3^{2/3}-4\right)}}\right)}{18\sqrt[6]{2}3^{5/6}\sqrt{3\sqrt[3]{2}3^{2/3}-4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

```
[Out] -ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*2^(
1/3))]]/(6*2^(1/6)*3^(5/6)*(1 + (-1)^(1/3))^2*Sqrt[4 - 3*(-3)^(2/
3)*2^(1/3)]) + ((-1)^(1/3)*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sq
rt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(9*2^(2/3)*3^(5/6)*Sqrt[8 + (9
*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3)]) + ArcTanh[(2^(1/6)*(3*3
^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]]/(18*2^(1/6
)*3^(5/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)]) - ((-1)^(2/3)*Log[6 - 3*(
```

```
-3)^(1/3)*2^(2/3)*x + x^2])/(36*2^(1/3)*3^(2/3)*(1 + (-1)^(1/3))^
2) + ((-1)^(2/3)*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(108*2^(1
/3)*3^(2/3)) + Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(108*2^(1/3)*3^
(2/3))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.0195321, size = 61, normalized size = 0.17

$$\frac{1}{6}\text{RootSum}\left[\#1^6+18\#1^4+324\#1^3+108\#1^2+216\&,\frac{\#1^2\log(x-\#1)}{\#1^4+12\#1^2+162\#1+36}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

```
[Out] RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , (Log[x - #
1]*#1^2)/(36 + 162*#1 + 12*#1^2 + #1^4) & ]/6
```

Maple [C] time = 0.007, size = 56, normalized size = 0.2

$$\frac{1}{6}\sum_{\_R=RootOf\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\_R^3\ln(x-\_R)}{\_R^5+12\_R^3+162\_R^2+36\_R}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(x^6+18*x^4+324*x^3+108*x^2+216),x)
```

```
[Out] 1/6*sum(_R^3/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^
6+18*_Z^4+324*_Z^3+108*_Z^2+216))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^3}{x^6+18x^4+324x^3+108x^2+216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 0.672815, size = 61, normalized size = 0.17

$$\operatorname{RootSum}\left(3390158631679488t^6 - 74384733888t^4 - 1332145440t^3 - 1417176t^2 - 1, \left(t \mapsto t\log\left(-\frac{8482372214243328t^5}{415817} + \right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

```
[Out] RootSum(3390158631679488*_t**6 - 74384733888*_t**4 - 1332145440*_
t**3 - 1417176*_t**2 - 1, Lambda(_t, _t*log(-8482372214243328*_t*
*5/415817 + 2216055910930560*_t**4/415817 - 2062546612992*_t**3/4
15817 - 57027208896*_t**2/415817 - 416583756*_t/415817 + x - 8993
8/415817)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{x^6 + 18\,x^4 + 324\,x^3 + 108\,x^2 + 216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216),x, algorithm="giac")
```

```
[Out] integrate(x^3/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

3.146 $\int \frac{x^2}{216+108x^2+324x^3+18x^4+x^6}\,dx$

Optimal. Leaf size=248

$$\frac{(-1)^{2/3}\tan^{-1}\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{27\ 2^{5/6}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}+\frac{(-1)^{2/3}\tan^{-1}\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{81\sqrt[3]{2}\sqrt[6]{3}\sqrt{8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}}}-\frac{\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(3\sqrt[3]{2}3^{2/3}-4\right)}}\right)}{81\ 2^{5/6}\sqrt[6]{3}\sqrt{3\sqrt[3]{2}3^{2/3}-4}}$$

```
[Out] ((-1)^(2/3)*ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3
)^(2/3)*2^(1/3))]])/(27*2^(5/6)*3^(1/6)*(1 + (-1)^(1/3))^2*Sqrt[4
 - 3*(-3)^(2/3)*2^(1/3)]) + ((-1)^(2/3)*ArcTan[(3*(-2)^(2/3)*3^(1
/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(81*2^(1/3)*3^(1/
6)*Sqrt[8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3)]) - ArcTanh
[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3)
)]]/(81*2^(5/6)*3^(1/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)])
```

Rubi [A] time = 1.32345, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$\frac{(-1)^{2/3}\tan^{-1}\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{27\ 2^{5/6}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}+\frac{(-1)^{2/3}\tan^{-1}\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{81\sqrt[3]{2}\sqrt[6]{3}\sqrt{8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}}}-\frac{\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(3\sqrt[3]{2}3^{2/3}-4\right)}}\right)}{81\ 2^{5/6}\sqrt[6]{3}\sqrt{3\sqrt[3]{2}3^{2/3}-4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

```
[Out] ((-1)^(2/3)*ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3
)^(2/3)*2^(1/3))]])/(27*2^(5/6)*3^(1/6)*(1 + (-1)^(1/3))^2*Sqrt[4
 - 3*(-3)^(2/3)*2^(1/3)]) + ((-1)^(2/3)*ArcTan[(3*(-2)^(2/3)*3^(1
/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(81*2^(1/3)*3^(1/
6)*Sqrt[8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3)]) - ArcTanh
[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3)
)]]/(81*2^(5/6)*3^(1/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.019311, size = 59, normalized size = 0.24

$$\frac{1}{6}\text{RootSum}\left[\#1^6+18\#1^4+324\#1^3+108\#1^2+216\&,\frac{\#1\log(x-\#1)}{\#1^4+12\#1^2+162\#1+36}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

```
[Out] RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , (Log[x - #
1]*#1)/(36 + 162*#1 + 12*#1^2 + #1^4) & ]/6
```

Maple [C] time = 0.008, size = 56, normalized size = 0.2

$$\frac{1}{6}\sum_{\_R=RootOf\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\_R^2\ln\left(x-\_R\right)}{\_R^5+12\_R^3+162\_R^2+36\_R}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x)
```

```
[Out] 1/6*sum(_R^2/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^
6+18*_Z^4+324*_Z^3+108*_Z^2+216))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2}{x^6+18\,x^4+324\,x^3+108\,x^2+216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 0.485465, size = 48, normalized size = 0.19

$$\operatorname{RootSum}\left(732274264442769408t^6-2677850419968t^4+2834352t^2-1,\left(t\mapsto t\log\left(10170475895038464t^5-52317262834\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

```
[Out] RootSum(732274264442769408*_t**6 - 2677850419968*_t**4 + 2834352*
_t**2 - 1, Lambda(_t, _t*log(10170475895038464*_t**5 - 5231726283
456*_t**4 - 31809932496*_t**3 + 19131876*_t**2 + 19683*_t + x - 2
7/2)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{x^6 + 18\,x^4 + 324\,x^3 + 108\,x^2 + 216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216),x, algorithm="giac")
```

```
[Out] integrate(x^2/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

3.147 $\int \frac{x}{216+108x^2+324x^3+18x^4+x^6}\,dx$

Optimal. Leaf size=361

$$\frac{(-1)^{2/3}\log\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}{216\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}-\frac{(-1)^{2/3}\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}{648\sqrt[3]{2}3^{2/3}}-\frac{\log\left(x^2+3\,2^{2/3}\sqrt[3]{3}x+6\right)}{648\sqrt[3]{2}3^{2/3}}-\frac{\tan^{-1}\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{36\sqrt[6]{2}3^{5/6}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}+\frac{\sqrt[3]{-1}\tan^{-1}\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{54\,2^{2/3}3^{5/6}\sqrt{8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}}}+\frac{\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(3\sqrt[3]{2}3^{2/3}-4\right)}}\right)}{108\sqrt[6]{2}3^{5/6}\sqrt{3\sqrt[3]{2}3^{2/3}-4}}$$

```
[Out] -ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*2^(
1/3))]]/(36*2^(1/6)*3^(5/6)*(1 + (-1)^(1/3))^2*Sqrt[4 - 3*(-3)^(2
/3)*2^(1/3)]) + ((-1)^(1/3)*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/S
qrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(54*2^(2/3)*3^(5/6)*Sqrt[8 +
(9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3)]) + ArcTanh[(2^(1/6)*(3
*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]]/(108*2^(
1/6)*3^(5/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)]) + ((-1)^(2/3)*Log[6 -
3*(-3)^(1/3)*2^(2/3)*x + x^2])/(216*2^(1/3)*3^(2/3)*(1 + (-1)^(1/
3))^2) - ((-1)^(2/3)*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(648*
2^(1/3)*3^(2/3)) - Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(648*2^(1/3
)*3^(2/3))
```

Rubi [A] time = 2.0054, antiderivative size = 361, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{(-1)^{2/3}\log\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}{216\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}-\frac{(-1)^{2/3}\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}{648\sqrt[3]{2}3^{2/3}}-\frac{\log\left(x^2+3\,2^{2/3}\sqrt[3]{3}x+6\right)}{648\sqrt[3]{2}3^{2/3}}-\frac{\tan^{-1}\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{36\sqrt[6]{2}3^{5/6}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}+\frac{\sqrt[3]{-1}\tan^{-1}\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{54\,2^{2/3}3^{5/6}\sqrt{8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}}}+\frac{\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(3\sqrt[3]{2}3^{2/3}-4\right)}}\right)}{108\sqrt[6]{2}3^{5/6}\sqrt{3\sqrt[3]{2}3^{2/3}-4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```


```
[Out] -ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*2^(
1/3))]]/(36*2^(1/6)*3^(5/6)*(1 + (-1)^(1/3))^2*Sqrt[4 - 3*(-3)^(2
/3)*2^(1/3)]) + ((-1)^(1/3)*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/S
qrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(54*2^(2/3)*3^(5/6)*Sqrt[8 +
(9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3)]) + ArcTanh[(2^(1/6)*(3
*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]]/(108*2^(
1/6)*3^(5/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)]) + ((-1)^(2/3)*Log[6 -
```

```
3*(-3)^(1/3)*2^(2/3)*x + x^2])/(216*2^(1/3)*3^(2/3)*(1 + (-1)^(1/
3))^2) - ((-1)^(2/3)*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(648*
2^(1/3)*3^(2/3)) - Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(648*2^(1/3
)*3^(2/3))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.0186764, size = 57, normalized size = 0.16

$$\frac{1}{6}\text{RootSum}\left[\#1^6+18\#1^4+324\#1^3+108\#1^2+216\&,\frac{\log(x-\#1)}{\#1^4+12\#1^2+162\#1+36}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

```
[Out] RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , Log[x - #1
]/(36 + 162*#1 + 12*#1^2 + #1^4) & ]/6
```

Maple [C] time = 0.007, size = 54, normalized size = 0.2

$$\frac{1}{6}\sum_{\_R=RootOf\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\_R\ln(x-\_R)}{\_R^5+12\_R^3+162\_R^2+36\_R}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(x^6+18*x^4+324*x^3+108*x^2+216),x)
```

```
[Out] 1/6*sum(_R/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+
18*_Z^4+324*_Z^3+108*_Z^2+216))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{x^6+18x^4+324x^3+108x^2+216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216),x, algorithm="maxima")
```

```
[Out] integrate(x/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 0.691747, size = 61, normalized size = 0.17

$$\operatorname{RootSum}\left(158171241119638192128t^6 - 96402615118848t^4 + 287743415040t^3 - 51018336t^2 - 1, \left(t \mapsto t\log\left(\frac{65418399445}{41}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

```
[Out] RootSum(158171241119638192128*_t**6 - 96402615118848*_t**4 + 2877
43415040*_t**3 - 51018336*_t**2 - 1, Lambda(_t, _t*log(6541839944
5721140961280*_t**5/415817 + 2480926457425102848*_t**4/415817 - 3
9451802929737984*_t**3/415817 + 118071997444800*_t**2/415817 - 16
745884920*_t/415817 + x - 268790/415817)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{x^6 + 18\,x^4 + 324\,x^3 + 108\,x^2 + 216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216),x, algorithm="giac")
```

```
[Out] integrate(x/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

3.148 $\int \frac{1}{216+108x^2+324x^3+18x^4+x^6}\,dx$

Optimal. Leaf size=377

$$-\frac{\log\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}{216\ 2^{2/3}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^2}-\frac{\sqrt[3]{-\frac{1}{3}}\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}{648\ 2^{2/3}}+\frac{\log\left(x^2+3\ 2^{2/3}\sqrt[3]{3}x+6\right)}{648\ 2^{2/3}\sqrt[3]{3}}+\frac{(-1)^{2/3}\left(3(-3)^{2/3}-2^{2/3}\right)\tan^{-1}\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{324\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^2\sqrt{2\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}+\frac{\left(9-(-2)^{2/3}\sqrt[3]{3}\right)\tan^{-1}\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{972\sqrt{3\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)}}-\frac{\left(9-2^{2/3}\sqrt[3]{3}\right)\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(3\sqrt[3]{2}3^{2/3}-4\right)}}\right)}{972\sqrt{6\left(3\sqrt[3]{2}3^{2/3}-4\right)}}$$

```
[Out] ((-1)^(2/3)*(3*(-3)^(2/3) - 2^(2/3))*ArcTan[(3*(-3)^(1/3)*2^(2/3)
 - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(324*3^(1/6)*(1 + (-
1)^(1/3))^2*Sqrt[2*(4 - 3*(-3)^(2/3)*2^(1/3))]) + ((9 - (-2)^(2/3
)*3^(1/3))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)
^(1/3)*3^(2/3))]])/(972*Sqrt[3*(8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(
1/3)*3^(2/3))]) - ((9 - 2^(2/3)*3^(1/3))*ArcTanh[(2^(1/6)*(3*3^(1
/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(972*Sqrt[6*
(-4 + 3*2^(1/3)*3^(2/3))]) - Log[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2
]/(216*2^(2/3)*3^(1/3)*(1 + (-1)^(1/3))^2) - ((-1/3)^(1/3)*Log[6
+ 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(648*2^(2/3)) + Log[6 + 3*2^(2/3
)*3^(1/3)*x + x^2]/(648*2^(2/3)*3^(1/3))
```

Rubi [A] time = 2.73424, antiderivative size = 377, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$

$$-\frac{\log\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}{216\ 2^{2/3}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^2}-\frac{\sqrt[3]{-\frac{1}{3}}\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}{648\ 2^{2/3}}+\frac{\log\left(x^2+3\ 2^{2/3}\sqrt[3]{3}x+6\right)}{648\ 2^{2/3}\sqrt[3]{3}}+\frac{(-1)^{2/3}\left(3(-3)^{2/3}-2^{2/3}\right)\tan^{-1}\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{324\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^2\sqrt{2\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}-\frac{\left((-2)^{2/3}-3\ 3^{2/3}\right)\tan^{-1}\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{972\sqrt[6]{3}\sqrt{8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}}}+\frac{\left(2^{2/3}-3\ 3^{2/3}\right)\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(3\sqrt[3]{2}3^{2/3}-4\right)}}\right)}{972\sqrt[6]{3}\sqrt{2\left(3\sqrt[3]{2}3^{2/3}-4\right)}}$$

Antiderivative was successfully verified.

```
[In]  Int[(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^(-1),x]
```

```
[Out] ((-1)^(2/3)*(3*(-3)^(2/3) - 2^(2/3))*ArcTan[(3*(-3)^(1/3)*2^(2/3)
 - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(324*3^(1/6)*(1 + (-
1)^(1/3))^2*Sqrt[2*(4 - 3*(-3)^(2/3)*2^(1/3))]) - (((-2)^(2/3) -
```

```
3*3^(2/3))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)
^(1/3)*3^(2/3))]])/(972*3^(1/6)*Sqrt[8 + (9*I)*2^(1/3)*3^(1/6) +
3*2^(1/3)*3^(2/3)]) + ((2^(2/3) - 3*3^(2/3))*ArcTanh[(2^(1/6)*(3*
3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(972*3^(
1/6)*Sqrt[2*(-4 + 3*2^(1/3)*3^(2/3))]) - Log[6 - 3*(-3)^(1/3)*2^(
2/3)*x + x^2]/(216*2^(2/3)*3^(1/3)*(1 + (-1)^(1/3))^2) - ((-1/3)^
(1/3)*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(648*2^(2/3)) + Log[
6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(648*2^(2/3)*3^(1/3))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.0180006, size = 62, normalized size = 0.16

$$\frac{1}{6}\text{RootSum}\left[\#1^6+18\#1^4+324\#1^3+108\#1^2+216\&,\frac{\log(x-\#1)}{\#1^5+12\#1^3+162\#1^2+36\#1}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^(-1),x]
```

```
[Out] RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , Log[x - #1
]/(36*#1 + 162*#1^2 + 12*#1^3 + #1^5) & ]/6
```

Maple [C] time = 0.007, size = 53, normalized size = 0.1

$$\frac{1}{6}\sum_{\_R=RootOf(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216)}\frac{\ln(x-\_R)}{\_R^5+12\_R^3+162\_R^2+36\_R}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^6+18*x^4+324*x^3+108*x^2+216),x)
```

```
[Out] 1/6*sum(1/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+1
8*_Z^4+324*_Z^3+108*_Z^2+216))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x^6+18x^4+324x^3+108x^2+216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 0.689366, size = 65, normalized size = 0.17

$$\operatorname{RootSum}\left(34164988081841849499648t^6 - 3470494144278528t^4 - 86087932019712t^3 - 1530550080t^2 + 69984t - 1, \left(t \mapsto t\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

```
[Out] RootSum(34164988081841849499648*_t**6 - 3470494144278528*_t**4 -
86087932019712*_t**3 - 1530550080*_t**2 + 69984*_t - 1, Lambda(_t
, _t*log(18590444669910961141057378 7136*_t**5/57121295165 + 63773
012532679173827665 92*_t**4/57121295165 - 18904636002388564311552*
_t**3/57121295165 - 4690805529151817 23968*_t**2/57121295165 - 243
58640509989936*_t/57121295165 + x + 152427895956/57121295165)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^6 + 18\,x^4 + 324\,x^3 + 108\,x^2 + 216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216),x, algorithm="giac")
```

```
[Out] integrate(1/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

3.149 $\int \frac{1}{x(216+108x^2+324x^3+18x^4+x^6)}\,dx$

Optimal. Leaf size=415

$$-\frac{\left(36+2^{2/3}\sqrt[3]{3}\left(1+i\sqrt{3}\right)\right)\log\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}{46656}-\frac{\left(18-(-2)^{2/3}\sqrt[3]{3}\right)\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}{23328}-\frac{\left(18-2^{2/3}\sqrt[3]{3}\right)\log\left(x^2+3\,2^{2/3}\sqrt[3]{3}x+6\right)}{23328}+\frac{\log(x)}{216}+\frac{(-1)^{2/3}\left((-2)^{2/3}-2\,3^{2/3}\right)\tan^{-1}\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{216\sqrt[3]{2}3^{5/6}\sqrt{8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}}}-\frac{(-1)^{2/3}\left(\sqrt[3]{-3}+3\sqrt[3]{2}\right)\tan^{-1}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{216\sqrt[6]{6}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}-\frac{\left(1-\sqrt[3]{2}3^{2/3}\right)\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(3\sqrt[3]{2}3^{2/3}-4\right)}}\right)}{216\sqrt[6]{2}3^{5/6}\sqrt{3\sqrt[3]{2}3^{2/3}-4}}$$

```
[Out] ((-1)^(2/3)*((-2)^(2/3) - 2*3^(2/3))*ArcTan[(3*(-2)^(2/3)*3^(1/3)
 + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(216*2^(1/3)*3^(5/6)
*Sqrt[8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3)]) - ((-1)^(2/
3)*((-3)^(1/3) + 3*2^(1/3))*ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/
3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(216*6^(1/6)*(1 + (-1
)^(1/3))^2*Sqrt[4 - 3*(-3)^(2/3)*2^(1/3)]) - ((1 - 2^(1/3)*3^(2/3
))*ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/
3)*3^(2/3))]])/(216*2^(1/6)*3^(5/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)])
 + Log[x]/216 - ((36 + 2^(2/3)*3^(1/3)*(1 + I*Sqrt[3]))*Log[6 - 3
*(-3)^(1/3)*2^(2/3)*x + x^2])/46656 - ((18 - (-2)^(2/3)*3^(1/3))*
Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/23328 - ((18 - 2^(2/3)*3^(
1/3))*Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2])/23328
```

Rubi [A] time = 3.06564, antiderivative size = 415, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$-\frac{\left(36+2^{2/3}\sqrt[3]{3}\left(1+i\sqrt{3}\right)\right)\log\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}{46656}-\frac{\left(18-(-2)^{2/3}\sqrt[3]{3}\right)\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}{23328}-\frac{\left(18-2^{2/3}\sqrt[3]{3}\right)\log\left(x^2+3\,2^{2/3}\sqrt[3]{3}x+6\right)}{23328}+\frac{\log(x)}{216}+\frac{(-1)^{2/3}\left((-2)^{2/3}-2\,3^{2/3}\right)\tan^{-1}\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{216\sqrt[3]{2}3^{5/6}\sqrt{8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}}}-\frac{(-1)^{2/3}\left(\sqrt[3]{-3}+3\sqrt[3]{2}\right)\tan^{-1}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{216\sqrt[6]{6}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}-\frac{\left(1-\sqrt[3]{2}3^{2/3}\right)\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(3\sqrt[3]{2}3^{2/3}-4\right)}}\right)}{216\sqrt[6]{2}3^{5/6}\sqrt{3\sqrt[3]{2}3^{2/3}-4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)),x]
```

```
[Out] ((-1)^(2/3)*((-2)^(2/3) - 2*3^(2/3))*ArcTan[(3*(-2)^(2/3)*3^(1/3)
 + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(216*2^(1/3)*3^(5/6)
```

```
*Sqrt[8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3)]) - ((-1)^(2/
3)*((-3)^(1/3) + 3*2^(1/3))*ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/
3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(216*6^(1/6)*(1 + (-1
)^(1/3))^2*Sqrt[4 - 3*(-3)^(2/3)*2^(1/3)]) - ((1 - 2^(1/3)*3^(2/3
))*ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/
3)*3^(2/3))]])/(216*2^(1/6)*3^(5/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)])
 + Log[x]/216 - ((36 + 2^(2/3)*3^(1/3)*(1 + I*Sqrt[3]))*Log[6 - 3
*(-3)^(1/3)*2^(2/3)*x + x^2])/46656 - ((18 - (-2)^(2/3)*3^(1/3))*
Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/23328 - ((18 - 2^(2/3)*3^(
1/3))*Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2])/23328
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.0288183, size = 103, normalized size = 0.25

$$\frac{\log(x)}{216} - \frac{\text{RootSum}\left[\#1^6 + 18\#1^4 + 324\#1^3 + 108\#1^2 + 216\&, \frac{\#1^4\log(x-\#1)+18\#1^2\log(x-\#1)+324\#1\log(x-\#1)+108\log(x-\#1)}{\#1^4+12\#1^2+162\#1+36}\&\right]}{1296}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)),x]
```

```
[Out] Log[x]/216 - RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 &
 , (108*Log[x - #1] + 324*Log[x - #1]*#1 + 18*Log[x - #1]*#1^2 +
Log[x - #1]*#1^4)/(36 + 162*#1 + 12*#1^2 + #1^4) & ]/1296
```

Maple [C] time = 0.013, size = 75, normalized size = 0.2

$$-\frac{1}{1296}\sum_{\_R=RootOf(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216)}\frac{(\_R^5+18\_R^3+324\_R^2+108\_R)\ln(x-\_R)}{\_R^5+12\_R^3+162\_R^2+36\_R}+\frac{\ln(x)}{216}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(x^6+18*x^4+324*x^3+108*x^2+216),x)
```

```
[Out] -1/1296*sum((_R^5+18*_R^3+324*_R^2+108*_R)/(_R^5+12*_R^3+162*_R^2
+36*_R)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^4+324*_Z^3+108*_Z^2+216))+1
/216*ln(x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{216}\int\frac{x^5+18x^3+324x^2+108x}{x^6+18x^4+324x^3+108x^2+216}\,dx+\frac{1}{216}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)*x),x, algorithm="maxima")
```

```
[Out] -1/216*integrate((x^5 + 18*x^3 + 324*x^2 + 108*x)/(x^6 + 18*x^4 +
 324*x^3 + 108*x^2 + 216), x) + 1/216*log(x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)*x),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 1.06293, size = 82, normalized size = 0.2

$$\frac{\log(x)}{216} + \operatorname{RootSum}\left(737963742567783949192396 8t^6 + 34164988081841849499648t^5 + 52598809250685370368t^4 + 26673506015311\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

```
[Out] log(x)/216 + RootSum(7379637425677839491923968*_t**6 + 3416498808
1841849499648*_t**5 + 52598809250685370368*_t**4 + 26673506015311
872*_t**3 - 309171116160*_t**2 + 944784*_t - 1, Lambda(_t, _t*log
(814557009966881793678336211511929736056012 8*_t**6/14342579930905
2440063 + 9770687667708063810873582575647457 28*_t**5/143425799309
052440063 - 1165295266088512642884009715390615 38816*_t**4/1434257
99309052440063 - 23935979498524220254250144071076 6592*_t**3/14342
5799309052440063 - 1366783126381370944398873414182 40*_t**2/143425
799309052440063 + 15631155690676637957354136 96*_t/143425799309052
440063 + x - 316444631507523619004 4/143425799309052440063)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^6 + 18\,x^4 + 324\,x^3 + 108\,x^2 + 216)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)*x),x, algorithm="giac")
```

```
[Out] integrate(1/((x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)*x), x)
```

3.150 $\int \frac{1}{x^2(216+108x^2+324x^3+18x^4+x^6)}\,dx$

Optimal. Leaf size=448

$$-\frac{(-1)^{2/3}\left(9+\sqrt[3]{-3}2^{2/3}\right)\log\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}{1296\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}+\frac{\left(3(-6)^{2/3}+2\sqrt[3]{-2}\right)\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}{7776\sqrt[3]{3}}-\frac{\left(2^{2/3}-3\ 3^{2/3}\right)\log\left(x^2+3\ 2^{2/3}\sqrt[3]{3}x+6\right)}{3888\sqrt[3]{6}}-\frac{1}{216x}-\frac{\left(27\sqrt[3]{-6}-(-2)^{2/3}+12\ 3^{2/3}\right)\tan^{-1}\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{5832\sqrt[6]{3}\sqrt{8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}}}-\frac{(-1)^{2/3}\left(6(-6)^{2/3}+27\sqrt[3]{-3}-\sqrt[3]{2}\right)\tan^{-1}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{1944\sqrt[6]{6}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}-\frac{\left(\sqrt[3]{2}+27\sqrt[3]{3}-6\ 6^{2/3}\right)\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(3\sqrt[3]{2}3^{2/3}-4\right)}}\right)}{5832\sqrt[6]{6}\sqrt{3\sqrt[3]{2}3^{2/3}-4}}$$

```
[Out] -1/(216*x) - ((27*(-6)^(1/3) - (-2)^(2/3) + 12*3^(2/3))*ArcTan[(3
*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(
5832*3^(1/6)*Sqrt[8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3)])
 - ((-1)^(2/3)*(6*(-6)^(2/3) + 27*(-3)^(1/3) - 2^(1/3))*ArcTan[(2
^(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/
3))]])/(1944*6^(1/6)*(1 + (-1)^(1/3))^2*Sqrt[4 - 3*(-3)^(2/3)*2^(
1/3)]) - ((2^(1/3) + 27*3^(1/3) - 6*6^(2/3))*ArcTanh[(2^(1/6)*(3*
3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(5832*6^
(1/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)]) - ((-1)^(2/3)*(9 + (-3)^(1/3)
*2^(2/3))*Log[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2])/(1296*2^(1/3)*3^
(2/3)*(1 + (-1)^(1/3))^2) + ((3*(-6)^(2/3) + 2*(-2)^(1/3))*Log[6
+ 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(7776*3^(1/3)) - ((2^(2/3) - 3*3
^(2/3))*Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2])/(3888*6^(1/3))
```

Rubi [A] time = 3.68911, antiderivative size = 448, normalized size of antiderivative = 1., number of

steps used = 14, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$-\frac{(-1)^{2/3}\left(9+\sqrt[3]{-3}2^{2/3}\right)\log\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}{1296\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}+\frac{\left(3(-6)^{2/3}+2\sqrt[3]{-2}\right)\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}{7776\sqrt[3]{3}}-\frac{\left(2^{2/3}-3\,3^{2/3}\right)\log\left(x^2+3\,2^{2/3}\sqrt[3]{3}x+6\right)}{3888\sqrt[3]{6}}-\frac{1}{216x}-\frac{\left(27\sqrt[3]{-6}-(-2)^{2/3}+12\,3^{2/3}\right)\tan^{-1}\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{5832\sqrt[6]{3}\sqrt{8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}}}-\frac{(-1)^{2/3}\left(6(-6)^{2/3}+27\sqrt[3]{-3}-\sqrt[3]{2}\right)\tan^{-1}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{1944\sqrt[6]{6}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}-\frac{\left(\sqrt[3]{2}+27\sqrt[3]{3}-6\,6^{2/3}\right)\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(3\sqrt[3]{2}3^{2/3}-4\right)}}\right)}{5832\sqrt[6]{6}\sqrt{3\sqrt[3]{2}3^{2/3}-4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)),x]
```

```
[Out] -1/(216*x) - ((27*(-6)^(1/3) - (-2)^(2/3) + 12*3^(2/3))*ArcTan[(3
*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(
5832*3^(1/6)*Sqrt[8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3)])
 - ((-1)^(2/3)*(6*(-6)^(2/3) + 27*(-3)^(1/3) - 2^(1/3))*ArcTan[(2
^(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/
3))]])/(1944*6^(1/6)*(1 + (-1)^(1/3))^2*Sqrt[4 - 3*(-3)^(2/3)*2^(
1/3)]) - ((2^(1/3) + 27*3^(1/3) - 6*6^(2/3))*ArcTanh[(2^(1/6)*(3*
3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(5832*6^
(1/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)]) - ((-1)^(2/3)*(9 + (-3)^(1/3)
*2^(2/3))*Log[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2])/(1296*2^(1/3)*3^
(2/3)*(1 + (-1)^(1/3))^2) + ((3*(-6)^(2/3) + 2*(-2)^(1/3))*Log[6
+ 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(7776*3^(1/3)) - ((2^(2/3) - 3*3
^(2/3))*Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2])/(3888*6^(1/3))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.0294103, size = 109, normalized size = 0.24

$$-\frac{\text{RootSum}\left[\#1^6+18\#1^4+324\#1^3+108\#1^2+216\&,\frac{\#1^4\log(x-\#1)+18\#1^2\log(x-\#1)+324\#1\log(x-\#1)+108\log(x-\#1)}{\#1^5+12\#1^3+162\#1^2+36\#1}\&\right]}{1296}-\frac{1}{216x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)),x]
```

```
[Out] -1/(216*x) - RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 &
 , (108*Log[x - #1] + 324*Log[x - #1]*#1 + 18*Log[x - #1]*#1^2 +
Log[x - #1]*#1^4)/(36*#1 + 162*#1^2 + 12*#1^3 + #1^5) & ]/1296
```

Maple [C] time = 0.012, size = 74, normalized size = 0.2

$$\frac{1}{1296}\sum_{\_R=RootOf\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\left(-\_R^4-18\_R^2-324\_R-108\right)\ln\left(x-\_R\right)}{\_R^5+12\_R^3+162\_R^2+36\_R}-\frac{1}{216x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x)
```

```
[Out] 1/1296*sum((-_R^4-18*_R^2-324*_R-108)/(_R^5+12*_R^3+162*_R^2+36*_
R)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^4+324*_Z^3+108*_Z^2+216))-1/216/
x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{216\,x}-\frac{1}{216}\int\frac{x^4+18\,x^2+324\,x+108}{x^6+18\,x^4+324\,x^3+108\,x^2+216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)*x^2),x, algorithm="maxima")
```

```
[Out] -1/216/x - 1/216*integrate((x^4 + 18*x^2 + 324*x + 108)/(x^6 + 18
*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)*x^2),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 0.810258, size = 70, normalized size = 0.16

$$\operatorname{RootSum}\left(1594001683946413330255577088t^6+379161202646033163878 4t^4-8643672699589509120t^3-10942820851968\right.$$
$$-\frac{1}{216x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

```
[Out] RootSum(1594001683946413330255577088*_t**6 + 379161202646033163877
84*_t**4 - 8643672699589509120*_t**3 - 10942820851968*_t**2 - 839
808*_t - 1, Lambda(_t, _t*log(-4987553276190249600329356123691446
8028416*_t**5/12350449784703991795 + 126254898724316203880059752O
0497664*_t**4/12350449784703991795 - 1186376926075737712385507988
52644864*_t**3/12350449784703991795 + 270486324927832147818193778
754816*_t**2/12350449784703991795 + 273914194897479402961199352*_
t/12350449784703991795 + x - 12798926329353908292/123504497847039
91795))) - 1/(216*x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^6 + 18\,x^4 + 324\,x^3 + 108\,x^2 + 216)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)*x^2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)*x^2), x)
```

3.151 $\int \frac{x^8}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$

Optimal. Leaf size=1064

result too large to display

```
[Out] -((-1/3)^(1/3)*(9*(6 + (-3)^(1/3)*2^(2/3)) + (2 - 2^(2/3)*(6*(-6)
^(2/3) + 27*(-3)^(1/3)))*x))/(162*2^(2/3)*(1 + (-1)^(1/3))^4*(4 -
 3*(-3)^(2/3)*2^(1/3))*(6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2)) - ((-1
/3)^(1/3)*(9*(6 - (-2)^(2/3)*3^(1/3)) + (2 + 27*(-2)^(2/3)*3^(1/3
) + 12*(-2)^(1/3)*3^(2/3))*x))/(729*2^(2/3)*(8 + (9*I)*2^(1/3)*3^
(1/6) + 3*2^(1/3)*3^(2/3))*(6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2)) +
(9*(6 - 2^(2/3)*3^(1/3)) + (2 + 2^(2/3)*(27*3^(1/3) - 6*6^(2/3)))
*x)/(1458*2^(2/3)*3^(1/3)*(4 - 3*2^(1/3)*3^(2/3))*(6 + 3*2^(2/3)*
3^(1/3)*x + x^2)) - ((I/162)*((-2)^(2/3) + 6*3^(2/3))*ArcTan[(3*(
-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(2^
(5/6)*3^(1/3)*(1 + (-1)^(1/3))^5*Sqrt[4 + 3*(-2)^(1/3)*3^(2/3)])
- ((-1)^(1/3)*(2 + 27*(-2)^(2/3)*3^(1/3) + 12*(-2)^(1/3)*3^(2/3))
*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(
2/3))]])/(162*2^(1/6)*3^(5/6)*(1 - (-1)^(1/3))^2*(1 + (-1)^(1/3))
^4*(4 + 3*(-2)^(1/3)*3^(2/3))^(3/2)) - ((-1)^(1/3)*(6*(-6)^(2/3)
+ 27*(-3)^(1/3) - 2^(1/3))*ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3
)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(81*Sqrt[2]*3^(5/6)*(1
 + (-1)^(1/3))^4*(4 - 3*(-3)^(2/3)*2^(1/3))^(3/2)) + ((I*2^(2/3)
- 9*3^(1/6) - (3*I)*3^(2/3))*ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1
/3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(162*2^(5/6)*3^(1/3)
*(1 + (-1)^(1/3))^5*Sqrt[4 - 3*(-3)^(2/3)*2^(1/3)]) - ((1 + 3*2^(
1/3)*3^(2/3))*ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-
4 + 3*2^(1/3)*3^(2/3))]])/(1458*2^(1/6)*3^(5/6)*Sqrt[-4 + 3*2^(1/
3)*3^(2/3)]) + ((2^(1/3) + 27*3^(1/3) - 6*6^(2/3))*ArcTanh[(2^(1/
6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(8
1*Sqrt[2]*3^(5/6)*(1 - (-1)^(1/3))^2*(1 + (-1)^(1/3))^4*(-4 + 3*2
^(1/3)*3^(2/3))^(3/2)) - Log[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2]/(9
72*2^(1/3)*3^(2/3)*(1 + (-1)^(1/3))^4) + ((I/972)*Log[6 + 3*(-2)^
(2/3)*3^(1/3)*x + x^2])/(2^(1/3)*3^(1/6)*(1 + (-1)^(1/3))^5) - Lo
g[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(8748*2^(1/3)*3^(2/3))
```

Rubi [A] time = 8.29129, antiderivative size = 1064, normalized size of antiderivative = 1., number

of steps used = 23, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$

$$-\frac{\sqrt[3]{-\frac{1}{3}}\left(\left(2+27(-2)^{2/3}\sqrt[3]{3}+12\sqrt[3]{-2}3^{2/3}\right)x+9\left(6-(-2)^{2/3}\sqrt[3]{3}\right)\right)}{729\ 2^{2/3}\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}$$
$$-\frac{\sqrt[3]{-1}\left(2+27(-2)^{2/3}\sqrt[3]{3}+12\sqrt[3]{-2}3^{2/3}\right)\tan^{-1}\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{162\sqrt[6]{2}3^{5/6}\left(1-\sqrt[3]{-1}\right)^2\left(1+\sqrt[3]{-1}\right)^4\left(4+3\sqrt[3]{-2}3^{2/3}\right)^{3/2}}$$
$$-\frac{i\left((-2)^{2/3}+6\ 3^{2/3}\right)\tan^{-1}\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{162\ 2^{5/6}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^5\sqrt{4+3\sqrt[3]{-2}3^{2/3}}}$$
$$-\frac{\left(9\sqrt[6]{3}-i\left(2^{2/3}-3\ 3^{2/3}\right)\right)\tan^{-1}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{162\ 2^{5/6}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^5\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}$$
$$-\frac{\sqrt[3]{-1}\left(6(-6)^{2/3}+27\sqrt[3]{-3}-\sqrt[3]{2}\right)\tan^{-1}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{81\sqrt{2}3^{5/6}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)^{3/2}}$$
$$+\frac{\left(\sqrt[3]{2}+27\sqrt[3]{3}-6\ 6^{2/3}\right)\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{81\sqrt{2}3^{5/6}\left(1-\sqrt[3]{-1}\right)^2\left(1+\sqrt[3]{-1}\right)^4\left(-4+3\sqrt[3]{2}3^{2/3}\right)^{3/2}}$$
$$-\frac{\left(1+3\sqrt[3]{2}3^{2/3}\right)\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{1458\sqrt[6]{2}3^{5/6}\sqrt{-4+3\sqrt[3]{2}3^{2/3}}}-\frac{\log\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}{972\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^4}$$
$$+\frac{i\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}{972\sqrt[3]{2}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^5}-\frac{\log\left(x^2+3\ 2^{2/3}\sqrt[3]{3}x+6\right)}{8748\sqrt[3]{2}3^{2/3}}$$
$$-\frac{\sqrt[3]{-\frac{1}{3}}\left(\left(2-3\ 2^{2/3}\left(2(-6)^{2/3}+9\sqrt[3]{-3}\right)\right)x+9\left(6+\sqrt[3]{-3}2^{2/3}\right)\right)}{162\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}$$
$$+\frac{\left(2+2^{2/3}\left(27\sqrt[3]{3}-6\ 6^{2/3}\right)\right)x+9\left(6-2^{2/3}\sqrt[3]{3}\right)}{1458\ 2^{2/3}\sqrt[3]{3}\left(4-3\sqrt[3]{2}3^{2/3}\right)\left(x^2+3\ 2^{2/3}\sqrt[3]{3}x+6\right)}$$

Antiderivative was successfully verified.

```
[In]   Int[x^8/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] -((-1/3)^(1/3)*(9*(6 + (-3)^(1/3)*2^(2/3)) + (2 - 3*2^(2/3)*(2*(-
6)^(2/3) + 9*(-3)^(1/3)))*x))/(162*2^(2/3)*(1 + (-1)^(1/3))^4*(4
- 3*(-3)^(2/3)*2^(1/3))*(6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2)) - ((-
1/3)^(1/3)*(9*(6 - (-2)^(2/3)*3^(1/3)) + (2 + 27*(-2)^(2/3)*3^(1/
3) + 12*(-2)^(1/3)*3^(2/3))*x))/(729*2^(2/3)*(8 + (9*I)*2^(1/3)*3
^(1/6) + 3*2^(1/3)*3^(2/3))*(6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2)) +
```

```
(9*(6 - 2^(2/3)*3^(1/3)) + (2 + 2^(2/3)*(27*3^(1/3) - 6*6^(2/3))
)*x)/(1458*2^(2/3)*3^(1/3)*(4 - 3*2^(1/3)*3^(2/3))*(6 + 3*2^(2/3)
*3^(1/3)*x + x^2)) - ((I/162)*((-2)^(2/3) + 6*3^(2/3))*ArcTan[(3*
(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(2
^(5/6)*3^(1/3)*(1 + (-1)^(1/3))^5*Sqrt[4 + 3*(-2)^(1/3)*3^(2/3)])
 - ((-1)^(1/3)*(2 + 27*(-2)^(2/3)*3^(1/3) + 12*(-2)^(1/3)*3^(2/3)
)*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^
(2/3))]])/(162*2^(1/6)*3^(5/6)*(1 - (-1)^(1/3))^2*(1 + (-1)^(1/3)
)^4*(4 + 3*(-2)^(1/3)*3^(2/3))^(3/2)) - ((-1)^(1/3)*(6*(-6)^(2/3)
 + 27*(-3)^(1/3) - 2^(1/3))*ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/
3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(81*Sqrt[2]*3^(5/6)*(
1 + (-1)^(1/3))^4*(4 - 3*(-3)^(2/3)*2^(1/3))^(3/2)) - ((9*3^(1/6)
 - I*(2^(2/3) - 3*3^(2/3)))*ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/
3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(162*2^(5/6)*3^(1/3)*
(1 + (-1)^(1/3))^5*Sqrt[4 - 3*(-3)^(2/3)*2^(1/3)]) - ((1 + 3*2^(1
/3)*3^(2/3))*ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4
 + 3*2^(1/3)*3^(2/3))]])/(1458*2^(1/6)*3^(5/6)*Sqrt[-4 + 3*2^(1/3
)*3^(2/3)]) + ((2^(1/3) + 27*3^(1/3) - 6*6^(2/3))*ArcTanh[(2^(1/6
)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(81
*Sqrt[2]*3^(5/6)*(1 - (-1)^(1/3))^2*(1 + (-1)^(1/3))^4*(-4 + 3*2^
(1/3)*3^(2/3))^(3/2)) - Log[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2]/(97
2*2^(1/3)*3^(2/3)*(1 + (-1)^(1/3))^4) + ((I/972)*Log[6 + 3*(-2)^(
2/3)*3^(1/3)*x + x^2])/(2^(1/3)*3^(1/6)*(1 + (-1)^(1/3))^5) - Log
[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(8748*2^(1/3)*3^(2/3))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**8/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.0581649, size = 167, normalized size = 0.16

$$\frac{-9x^5-203x^4-11610x^3-3990x^2+324x-7884}{34182\left(x^6+18x^4+324x^3+108x^2+216\right)}-\frac{\text{RootSum}\left[\#1^6+18\#1^4+324\#1^3+108\#1^2+216\&,\frac{9\#1^4\log(x-\#1)+406\#1^3\log(x-\#1)+324\#1^2\log(x-\#1)-96\#1\log(x-\#1)+324\log(x-\#1)}{\#1^5+12\#1^3+162\#1^2+36\#1}\&\right]}{205092}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^8/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] (-7884 + 324*x - 3990*x^2 - 11610*x^3 - 203*x^4 - 9*x^5)/(34182*(
216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)) - RootSum[216 + 108*#1^2
 + 324*#1^3 + 18*#1^4 + #1^6 & , (324*Log[x - #1] - 96*Log[x - #1
]*#1 + 324*Log[x - #1]*#1^2 + 406*Log[x - #1]*#1^3 + 9*Log[x - #1
]*#1^4)/(36*#1 + 162*#1^2 + 12*#1^3 + #1^5) & ]/205092
```

Maple [C] time = 0.016, size = 122, normalized size = 0.1

$$\frac{1}{x^6+18x^4+324x^3+108x^2+216}\left(-\frac{x^5}{3798}-\frac{203x^4}{34182}-\frac{215x^3}{633}-\frac{665x^2}{5697}+\frac{2x}{211}-\frac{146}{633}\right)+\frac{1}{205092}\sum_{\_R=RootOf(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216)}\frac{\left(-9\_R^4-406\_R^3-324\_R^2+96\_R-324\right)\ln(x-\_R)}{\_R^5+12\_R^3+162\_R^2+36\_R}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^8/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x)
```

```
[Out] (-1/3798*x^5-203/34182*x^4-215/633*x^3-665/5697*x^2+2/211*x-146/6
33)/(x^6+18*x^4+324*x^3+108*x^2+216)+1/205092*sum((-9*_R^4-406*_R
^3-324*_R^2+96*_R-324)/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=
RootOf(_Z^6+18*_Z^4+324*_Z^3+108*_Z^2+216))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{9\,x^5+203\,x^4+11610\,x^3+3990\,x^2-324\,x+7884}{34182\,(x^6+18\,x^4+324\,x^3+108\,x^2+216)}-\frac{1}{34182}\int\frac{9\,x^4+406\,x^3+324\,x^2-96\,x+324}{x^6+18\,x^4+324\,x^3+108\,x^2+216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2,x, algorithm="maxima")
```

```
[Out] -1/34182*(9*x^5 + 203*x^4 + 11610*x^3 + 3990*x^2 - 324*x + 7884)/
(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216) - 1/34182*integrate((9*x
^4 + 406*x^3 + 324*x^2 - 96*x + 324)/(x^6 + 18*x^4 + 324*x^3 + 10
8*x^2 + 216), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 1.18898, size = 112, normalized size = 0.11

$$\operatorname{RootSum}\left(852560170529641874151233606645 76t^6+5010519153338543456870 4t^4+48885748051277486016t^3+86544778\right.$$
$$-\frac{9x^5+203x^4+11610x^3+3990x^2-324x+7884}{34182x^6+615276x^4+11074968x^3+3691656x^2+7383312}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**8/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)
```

```
[Out] RootSum(85256017052964187415123360664576*_t**6 + 5010519153338543
4568704*_t**4 + 48885748051277486016*_t**3 + 865447782603408*_t**
2 + 3220532460*_t + 4513, Lambda(_t, _t*log(35492036204084174404 1
191931354834874665907640 32*_t**5/356900697070792948475845 - 19474
1600672188370868268096310170223 08224*_t**4/71380139414158589695 16
9 + 207799630765451322338945827649033 96544*_t**3/3569006970707929
48475845 + 20265219154367004972162198012037344*_t**2/356900697070
792948475845 + 27519246894921053204907514537 2*_t/3569006970707929
48475845 + x + 129028519129217728962 2012/107070209121237884542753
5))) - (9*x**5 + 203*x**4 + 11610*x**3 + 3990*x**2 - 324*x + 7884
)/(34182*x**6 + 615276*x**4 + 11074968*x**3 + 3691656*x**2 + 7383
312)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^8}{\left(x^6+18\,x^4+324\,x^3+108\,x^2+216\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2,x, algorithm="giac")
```

```
[Out] integrate(x^8/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2, x)
```

3.152 $\int \frac{x^7}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$

Optimal. Leaf size=1005

result too large to display

```
[Out] -(2*(2*(-1)^(1/3)*3^(2/3) + 9*6^(1/3)) - 9*((-2)^(2/3) + 2*(-1)^(
1/3)*3^(2/3))*x)/(972*2^(2/3)*(1 + (-1)^(1/3))^4*(4 - 3*(-3)^(2/3
)*2^(1/3))*(6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2)) - ((-6)^(1/3)*(9*(
-2)^(1/3) + 2*3^(1/3)) - 9*(1 + (-2)^(1/3)*3^(2/3))*x)/(4374*(8 +
 (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))*(6 + 3*(-2)^(2/3)*3^(
1/3)*x + x^2)) + (2*(2 - 3*2^(1/3)*3^(2/3)) - 3*(6 - 2^(2/3)*3^(1
/3))*x)/(2916*2^(2/3)*3^(1/3)*(4 - 3*2^(1/3)*3^(2/3))*(6 + 3*2^(2
/3)*3^(1/3)*x + x^2)) + ((9*I + 3^(1/3)*((2*I)*2^(2/3) - 9*3^(1/6
) + 2*2^(2/3)*Sqrt[3]))*ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[
6*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(5832*(1 + (-1)^(1/3))^5*Sqrt[2*(
4 - 3*(-3)^(2/3)*2^(1/3))]) + ((1 + (-2)^(1/3)*3^(2/3))*ArcTan[(3
*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(
54*Sqrt[6]*(1 - (-1)^(1/3))^2*(1 + (-1)^(1/3))^4*(4 + 3*(-2)^(1/3
)*3^(2/3))^(3/2)) + ((9*3^(1/6) + I*(4*2^(2/3) - 3*3^(2/3)))*ArcT
an[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))
]])/(1944*3^(2/3)*(1 + (-1)^(1/3))^5*Sqrt[2*(4 + 3*(-2)^(1/3)*3^(
2/3))]) - ((-1)^(1/3)*((-3)^(1/3) + 3*2^(1/3))*ArcTan[(2^(1/6)*(3
*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(5
4*Sqrt[2]*3^(5/6)*(1 + (-1)^(1/3))^4*(4 - 3*(-3)^(2/3)*2^(1/3))^(
3/2)) + ((1 - 2^(1/3)*3^(2/3))*ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1
/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(54*Sqrt[6]*(1 - (-1)^
(1/3))^2*(1 + (-1)^(1/3))^4*(-4 + 3*2^(1/3)*3^(2/3))^(3/2)) + ((2
*2^(2/3) + 3*3^(2/3))*ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/S
qrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(26244*3^(1/6)*Sqrt[2*(-4 + 3*2
^(1/3)*3^(2/3))]) + ((I/648)*Log[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2
])/(2^(2/3)*3^(5/6)*(1 + (-1)^(1/3))^5) - ((I + Sqrt[3])*Log[6 +
3*(-2)^(2/3)*3^(1/3)*x + x^2])/(1296*2^(2/3)*3^(5/6)*(1 + (-1)^(1
/3))^5) - Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(17496*2^(2/3)*3^(1/
3))
```

Rubi [A] time = 8.2942, antiderivative size = 1005, normalized size of antiderivative = 1., number of

steps used = 23, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$

$$\frac{2\left(2-3\sqrt[3]{2}3^{2/3}\right)-3\left(6-2^{2/3}\sqrt[3]{3}\right)x}{2916\,2^{2/3}\sqrt[3]{3}\left(4-3\sqrt[3]{2}3^{2/3}\right)\left(x^2+3\,2^{2/3}\sqrt[3]{3}x+6\right)}+\frac{\left(9i+\sqrt[3]{3}\left(2i2^{2/3}-9\sqrt[6]{3}+2\,2^{2/3}\sqrt{3}\right)\right)\tan^{-1}\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{5832\left(1+\sqrt[3]{-1}\right)^5\sqrt{2\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}-\frac{\left(9i-\sqrt[3]{3}\left(4i2^{2/3}+9\sqrt[6]{3}\right)\right)\tan^{-1}\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{5832\left(1+\sqrt[3]{-1}\right)^5\sqrt{2\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}+\frac{\left(1+\sqrt[3]{-2}3^{2/3}\right)\tan^{-1}\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{54\sqrt{6}\left(1-\sqrt[3]{-1}\right)^2\left(1+\sqrt[3]{-1}\right)^4\left(4+3\sqrt[3]{-2}3^{2/3}\right)^{3/2}}-\frac{\sqrt[3]{-1}\left(\sqrt[3]{-3}+3\sqrt[3]{2}\right)\tan^{-1}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{54\sqrt{2}3^{5/6}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)^{3/2}}+\frac{\left(2\,2^{2/3}+3\,3^{2/3}\right)\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{26244\sqrt[6]{3}\sqrt{2\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}+\frac{\left(1-\sqrt[3]{2}3^{2/3}\right)\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{54\sqrt{6}\left(1-\sqrt[3]{-1}\right)^2\left(1+\sqrt[3]{-1}\right)^4\left(-4+3\sqrt[3]{2}3^{2/3}\right)^{3/2}}+\frac{i\log\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}{648\,2^{2/3}3^{5/6}\left(1+\sqrt[3]{-1}\right)^5}-\frac{\left(i+\sqrt{3}\right)\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}{1296\,2^{2/3}3^{5/6}\left(1+\sqrt[3]{-1}\right)^5}-\frac{\log\left(x^2+3\,2^{2/3}\sqrt[3]{3}x+6\right)}{17496\,2^{2/3}\sqrt[3]{3}}-\frac{2\left(2\sqrt[3]{-1}3^{2/3}+9\sqrt[6]{6}\right)-9\left((-2)^{2/3}+2\sqrt[3]{-1}3^{2/3}\right)x}{972\,2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}-\frac{\sqrt[3]{-6}\left(9\sqrt[3]{-2}+2\sqrt[3]{3}\right)-9\left(1+\sqrt[3]{-2}3^{2/3}\right)x}{4374\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}$$

Antiderivative was successfully verified.

```
[In]   Int[x^7/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] -(2*(2*(-1)^(1/3)*3^(2/3) + 9*6^(1/3)) - 9*((-2)^(2/3) + 2*(-1)^(
1/3)*3^(2/3))*x)/(972*2^(2/3)*(1 + (-1)^(1/3))^4*(4 - 3*(-3)^(2/3
)*2^(1/3))*(6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2)) - ((-6)^(1/3)*(9*(
-2)^(1/3) + 2*3^(1/3)) - 9*(1 + (-2)^(1/3)*3^(2/3))*x)/(4374*(8 +
 (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))*(6 + 3*(-2)^(2/3)*3^(
1/3)*x + x^2)) + (2*(2 - 3*2^(1/3)*3^(2/3)) - 3*(6 - 2^(2/3)*3^(1
/3))*x)/(2916*2^(2/3)*3^(1/3)*(4 - 3*2^(1/3)*3^(2/3))*(6 + 3*2^(2
/3)*3^(1/3)*x + x^2)) + ((9*I + 3^(1/3)*((2*I)*2^(2/3) - 9*3^(1/6
) + 2*2^(2/3)*Sqrt[3]))*ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[
6*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(5832*(1 + (-1)^(1/3))^5*Sqrt[2*(
4 - 3*(-3)^(2/3)*2^(1/3))]) + ((1 + (-2)^(1/3)*3^(2/3))*ArcTan[(3
*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(
```

```
54*Sqrt[6]*(1 - (-1)^(1/3))^2*(1 + (-1)^(1/3))^4*(4 + 3*(-2)^(1/3
)*3^(2/3))^(3/2)) - ((9*I - 3^(1/3)*((4*I)*2^(2/3) + 9*3^(1/6)))*
ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2
/3))]])/(5832*(1 + (-1)^(1/3))^5*Sqrt[2*(4 + 3*(-2)^(1/3)*3^(2/3)
)]) - ((-1)^(1/3)*((-3)^(1/3) + 3*2^(1/3))*ArcTan[(2^(1/6)*(3*(-3
)^(1/3) - 2^(1/3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(54*Sq
rt[2]*3^(5/6)*(1 + (-1)^(1/3))^4*(4 - 3*(-3)^(2/3)*2^(1/3))^(3/2)
) + ((1 - 2^(1/3)*3^(2/3))*ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*
x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(54*Sqrt[6]*(1 - (-1)^(1/3
))^2*(1 + (-1)^(1/3))^4*(-4 + 3*2^(1/3)*3^(2/3))^(3/2)) + ((2*2^(
2/3) + 3*3^(2/3))*ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[
3*(-4 + 3*2^(1/3)*3^(2/3))]])/(26244*3^(1/6)*Sqrt[2*(-4 + 3*2^(1/
3)*3^(2/3))]) + ((I/648)*Log[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2])/(
2^(2/3)*3^(5/6)*(1 + (-1)^(1/3))^5) - ((I + Sqrt[3])*Log[6 + 3*(-
2)^(2/3)*3^(1/3)*x + x^2])/(1296*2^(2/3)*3^(5/6)*(1 + (-1)^(1/3))
^5) - Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(17496*2^(2/3)*3^(1/3))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.0396603, size = 167, normalized size = 0.17

$$\frac{\text{RootSum}\left[\#1^6+18\#1^4+324\#1^3+108\#1^2+216\&,\frac{73\#1^4\log(x-\#1)-36\#1^3\log(x-\#1)+96\#1^2\log(x-\#1)-216\#1\log(x-\#1)+96\log(x-\#1)}{\#1^5+12\#1^3+162\#1^2+36\#1}\&\right]}{410184}+\frac{73x^5-18x^4+908x^3+432x^2-96x+648}{68364\left(x^6+18x^4+324x^3+108x^2+216\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^7/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] (648 - 96*x + 432*x^2 + 908*x^3 - 18*x^4 + 73*x^5)/(68364*(216 +
108*x^2 + 324*x^3 + 18*x^4 + x^6)) + RootSum[216 + 108*#1^2 + 324
*#1^3 + 18*#1^4 + #1^6 & , (96*Log[x - #1] - 216*Log[x - #1]*#1 +
 96*Log[x - #1]*#1^2 - 36*Log[x - #1]*#1^3 + 73*Log[x - #1]*#1^4)
/(36*#1 + 162*#1^2 + 12*#1^3 + #1^5) & ]/410184
```

Maple [C] time = 0.016, size = 122, normalized size = 0.1

$$\frac{1}{x^6+18x^4+324x^3+108x^2+216}\left(\frac{73x^5}{68364}-\frac{x^4}{3798}+\frac{227x^3}{17091}+\frac{4x^2}{633}-\frac{8x}{5697}+\frac{2}{211}\right)+\frac{1}{410184}\sum_{\_R=RootOf(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216)}\frac{\left(73\_R^4-36\_R^3+96\_R^2-216\_R+96\right)\ln(x-\_R)}{\_R^5+12\_R^3+162\_R^2+36\_R}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^7/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x)
```

```
[Out] (73/68364*x^5-1/3798*x^4+227/17091*x^3+4/633*x^2-8/5697*x+2/211)/
(x^6+18*x^4+324*x^3+108*x^2+216)+1/410184*sum((73*_R^4-36*_R^3+96
*_R^2-216*_R+96)/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf
(_Z^6+18*_Z^4+324*_Z^3+108*_Z^2+216))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{73\,x^5-18\,x^4+908\,x^3+432\,x^2-96\,x+648}{68364\,(x^6+18\,x^4+324\,x^3+108\,x^2+216)}+\frac{1}{68364}\int\frac{73\,x^4-36\,x^3+96\,x^2-216\,x+96}{x^6+18\,x^4+324\,x^3+108\,x^2+216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2,x, algorithm="maxima")
```

```
[Out] 1/68364*(73*x^5 - 18*x^4 + 908*x^3 + 432*x^2 - 96*x + 648)/(x^6 +
 18*x^4 + 324*x^3 + 108*x^2 + 216) + 1/68364*integrate((73*x^4 -
36*x^3 + 96*x^2 - 216*x + 96)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 +
 216), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 1.20115, size = 112, normalized size = 0.11

$$\operatorname{RootSum}\left(589289589870088463413332668913549312t^6 - 539640290266075248405737472t^4 + 92182638168509682392064t\right.$$
$$+\frac{73x^5-18x^4+908x^3+432x^2-96x+648}{68364x^6+1230552x^4+22149936x^3+7383312x^2+14766624}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**7/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)
```

```
[Out] RootSum(589289589870088463413332668913549312*_t**6 - 539640290266
075248405737472*_t**4 + 92182638168509682392064*_t**3 - 553241442
069170496*_t**2 - 3759837842016*_t - 7197829, Lambda(_t, _t*log(4
29960276397274477140037433051607461110184385010259993231 36*_t**5/
1542060097910520444906943803032 37521 - 425847662595081946846897 15
474422251405157209835847680*_t**4/154206009791052044490694380303 2
37521 - 37512446128849588150108369449323754078317341082112*_t**3/
1542060097910520444906943803032 37521 + 71520375940216752676388907
155316724819202221 44*_t**2/1542060097910520444906943803032 37521 -
 44227546998835297723830291794974310524032*_t/1542060097910520444
9069438030 3237521 + x - 17457334903667604773413256958302485 5/1542
06009791052044490694380303237521))) + (73*x**5 - 18*x**4 + 908*x*
*3 + 432*x**2 - 96*x + 648)/(68364*x**6 + 1230552*x**4 + 22149936
*x**3 + 7383312*x**2 + 14766624)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{\left(x^6 + 18\,x^4 + 324\,x^3 + 108\,x^2 + 216\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2,x, algorithm="giac")
```

```
[Out] integrate(x^7/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2, x)
```

3.153 $\int \frac{x^6}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$

Optimal. Leaf size=675

$$\frac{\sqrt[3]{6}\left(9+\sqrt[3]{-3}2^{2/3}\right)x+9(-2)^{2/3}}{2916\,2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}+\frac{\sqrt[3]{-1}3^{2/3}\left(2+3\sqrt[3]{-2}3^{2/3}\right)x+9\,2^{2/3}}{13122\,2^{2/3}\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}+\frac{3\,2^{2/3}\sqrt[3]{3}-\left(2-3\sqrt[3]{2}3^{2/3}\right)x}{8748\,2^{2/3}\sqrt[3]{3}\left(4-3\sqrt[3]{2}3^{2/3}\right)\left(x^2+3\,2^{2/3}\sqrt[3]{3}x+6\right)}+\frac{\log\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}{5832\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^4}-\frac{i\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}{5832\sqrt[3]{2}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^5}+\frac{\log\left(x^2+3\,2^{2/3}\sqrt[3]{3}x+6\right)}{52488\sqrt[3]{2}3^{2/3}}+\frac{\sqrt[3]{-1}\left(3(-3)^{2/3}-2^{2/3}\right)\tan^{-1}\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{486\,6^{5/6}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)^{3/2}}+\frac{\left(3(-3)^{2/3}+\sqrt[3]{-1}2^{2/3}\right)\tan^{-1}\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{486\,6^{5/6}\left(1-\sqrt[3]{-1}\right)^2\left(1+\sqrt[3]{-1}\right)^4\left(4+3\sqrt[3]{-2}3^{2/3}\right)^{3/2}}-\frac{\left(2^{2/3}-3\,3^{2/3}\right)\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(3\sqrt[3]{2}3^{2/3}-4\right)}}\right)}{486\,6^{5/6}\left(1-\sqrt[3]{-1}\right)^2\left(1+\sqrt[3]{-1}\right)^4\left(3\sqrt[3]{2}3^{2/3}-4\right)^{3/2}}$$

```
[Out] (9*(-2)^(2/3) + 6^(1/3)*(9 + (-3)^(1/3)*2^(2/3))*x)/(2916*2^(2/3)
*(1 + (-1)^(1/3))^4*(4 - 3*(-3)^(2/3)*2^(1/3))*(6 - 3*(-3)^(1/3)*
2^(2/3)*x + x^2)) + (9*2^(2/3) + (-1)^(1/3)*3^(2/3)*(2 + 3*(-2)^(
1/3)*3^(2/3))*x)/(13122*2^(2/3)*(8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^
(1/3)*3^(2/3))*(6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2)) + (3*2^(2/3)*3
^(1/3) - (2 - 3*2^(1/3)*3^(2/3))*x)/(8748*2^(2/3)*3^(1/3)*(4 - 3*
2^(1/3)*3^(2/3))*(6 + 3*2^(2/3)*3^(1/3)*x + x^2)) + ((-1)^(1/3)*(
3*(-3)^(2/3) - 2^(2/3))*ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[
6*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(486*6^(5/6)*(1 + (-1)^(1/3))^4*(
4 - 3*(-3)^(2/3)*2^(1/3))^(3/2)) + ((3*(-3)^(2/3) + (-1)^(1/3)*2^
(2/3))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/
3)*3^(2/3))]])/(486*6^(5/6)*(1 - (-1)^(1/3))^2*(1 + (-1)^(1/3))^4
*(4 + 3*(-2)^(1/3)*3^(2/3))^(3/2)) - ((2^(2/3) - 3*3^(2/3))*ArcTa
nh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/
3))]])/(486*6^(5/6)*(1 - (-1)^(1/3))^2*(1 + (-1)^(1/3))^4*(-4 + 3
*2^(1/3)*3^(2/3))^(3/2)) + Log[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2]/
(5832*2^(1/3)*3^(2/3)*(1 + (-1)^(1/3))^4) - ((I/5832)*Log[6 + 3*(
-2)^(2/3)*3^(1/3)*x + x^2])/(2^(1/3)*3^(1/6)*(1 + (-1)^(1/3))^5)
+ Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(52488*2^(1/3)*3^(2/3))
```

Rubi [A] time = 5.06837, antiderivative size = 675, normalized size of antiderivative = 1., number of

steps used = 14, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$\frac{\sqrt[3]{6}\left(9+\sqrt[3]{-3}2^{2/3}\right)x+9(-2)^{2/3}}{2916\,2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}+\frac{\sqrt[3]{-1}3^{2/3}\left(2+3\sqrt[3]{-2}3^{2/3}\right)x+9\,2^{2/3}}{13122\,2^{2/3}\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}+\frac{3\,2^{2/3}\sqrt[3]{3}-\left(2-3\sqrt[3]{2}3^{2/3}\right)x}{8748\,2^{2/3}\sqrt[3]{3}\left(4-3\sqrt[3]{2}3^{2/3}\right)\left(x^2+3\,2^{2/3}\sqrt[3]{3}x+6\right)}+\frac{\log\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}{5832\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^4}-\frac{i\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}{5832\sqrt[3]{2}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^5}+\frac{\log\left(x^2+3\,2^{2/3}\sqrt[3]{3}x+6\right)}{52488\sqrt[3]{2}3^{2/3}}+\frac{\sqrt[3]{-1}\left(3(-3)^{2/3}-2^{2/3}\right)\tan^{-1}\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{486\,6^{5/6}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)^{3/2}}+\frac{\left(3(-3)^{2/3}+\sqrt[3]{-1}2^{2/3}\right)\tan^{-1}\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{486\,6^{5/6}\left(1-\sqrt[3]{-1}\right)^2\left(1+\sqrt[3]{-1}\right)^4\left(4+3\sqrt[3]{-2}3^{2/3}\right)^{3/2}}-\frac{\left(2^{2/3}-3\,3^{2/3}\right)\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(3\sqrt[3]{2}3^{2/3}-4\right)}}\right)}{486\,6^{5/6}\left(1-\sqrt[3]{-1}\right)^2\left(1+\sqrt[3]{-1}\right)^4\left(3\sqrt[3]{2}3^{2/3}-4\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^6/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] (9*(-2)^(2/3) + 6^(1/3)*(9 + (-3)^(1/3)*2^(2/3))*x)/(2916*2^(2/3)
*(1 + (-1)^(1/3))^4*(4 - 3*(-3)^(2/3)*2^(1/3))*(6 - 3*(-3)^(1/3)*
2^(2/3)*x + x^2)) + (9*2^(2/3) + (-1)^(1/3)*3^(2/3)*(2 + 3*(-2)^(
1/3)*3^(2/3))*x)/(13122*2^(2/3)*(8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^
(1/3)*3^(2/3))*(6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2)) + (3*2^(2/3)*3
^(1/3) - (2 - 3*2^(1/3)*3^(2/3))*x)/(8748*2^(2/3)*3^(1/3)*(4 - 3*
2^(1/3)*3^(2/3))*(6 + 3*2^(2/3)*3^(1/3)*x + x^2)) + ((-1)^(1/3)*(
3*(-3)^(2/3) - 2^(2/3))*ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[
6*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(486*6^(5/6)*(1 + (-1)^(1/3))^4*(
4 - 3*(-3)^(2/3)*2^(1/3))^(3/2)) + ((3*(-3)^(2/3) + (-1)^(1/3)*2^
(2/3))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/
3)*3^(2/3))]])/(486*6^(5/6)*(1 - (-1)^(1/3))^2*(1 + (-1)^(1/3))^4
*(4 + 3*(-2)^(1/3)*3^(2/3))^(3/2)) - ((2^(2/3) - 3*3^(2/3))*ArcTa
nh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/
3))]])/(486*6^(5/6)*(1 - (-1)^(1/3))^2*(1 + (-1)^(1/3))^4*(-4 + 3
*2^(1/3)*3^(2/3))^(3/2)) + Log[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2]/
(5832*2^(1/3)*3^(2/3)*(1 + (-1)^(1/3))^4) - ((I/5832)*Log[6 + 3*(
-2)^(2/3)*3^(1/3)*x + x^2])/(2^(1/3)*3^(1/6)*(1 + (-1)^(1/3))^5)
+ Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(52488*2^(1/3)*3^(2/3))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.0547693, size = 167, normalized size = 0.25

$$\frac{-3x^5+73x^4-72x^3-64x^2+108x-96}{68364\left(x^6+18x^4+324x^3+108x^2+216\right)}-\frac{\text{RootSum}\left[\#1^6+18\#1^4+324\#1^3+108\#1^2+216\&,\frac{3\#1^4\log(x-\#1)-146\#1^3\log(x-\#1)+108\#1^2\log(x-\#1)-32\#1\log(x-\#1)+108\log(x-\#1)}{\#1^5+12\#1^3+162\#1^2+36\#1}\right.}{410184}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^6/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] (-96 + 108*x - 64*x^2 - 72*x^3 + 73*x^4 - 3*x^5)/(68364*(216 + 10
8*x^2 + 324*x^3 + 18*x^4 + x^6)) - RootSum[216 + 108*#1^2 + 324*#
1^3 + 18*#1^4 + #1^6 & , (108*Log[x - #1] - 32*Log[x - #1]*#1 + 1
08*Log[x - #1]*#1^2 - 146*Log[x - #1]*#1^3 + 3*Log[x - #1]*#1^4)/
(36*#1 + 162*#1^2 + 12*#1^3 + #1^5) & ]/410184
```

Maple [C] time = 0.016, size = 122, normalized size = 0.2

$$\frac{1}{x^6+18x^4+324x^3+108x^2+216}\left(-\frac{x^5}{22788}+\frac{73x^4}{68364}-\frac{2x^3}{1899}-\frac{16x^2}{17091}+\frac{x}{633}-\frac{8}{5697}\right)+\frac{1}{410184}\sum_{\_R=RootOf\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\left(-3\_R^4+146\_R^3-108\_R^2+32\_R-108\right)\ln\left(x-\_R\right)}{\_R^5+12\_R^3+162\_R^2+36\_R}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^6/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x)
```

```
[Out] (-1/22788*x^5+73/68364*x^4-2/1899*x^3-16/17091*x^2+1/633*x-8/5697
)/(x^6+18*x^4+324*x^3+108*x^2+216)+1/410184*sum((-3*_R^4+146*_R^3
-108*_R^2+32*_R-108)/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=Ro
otOf(_Z^6+18*_Z^4+324*_Z^3+108*_Z^2+216))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{3x^5-73x^4+72x^3+64x^2-108x+96}{68364\left(x^6+18x^4+324x^3+108x^2+216\right)}-\frac{1}{68364}\int\frac{3x^4-146x^3+108x^2-32x+108}{x^6+18x^4+324x^3+108x^2+216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2,x, algorithm="maxima")
```

```
[Out] -1/68364*(3*x^5 - 73*x^4 + 72*x^3 + 64*x^2 - 108*x + 96)/(x^6 + 1
8*x^4 + 324*x^3 + 108*x^2 + 216) - 1/68364*integrate((3*x^4 - 146
*x^3 + 108*x^2 - 32*x + 108)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 +
216), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 1.17311, size = 112, normalized size = 0.17

$$\operatorname{RootSum}\left(397770473162309712803999551516645785 6t^6 - 1010314319415295961050951680t^4 - 2016822447709395715123\right.$$
$$-\frac{3x^5 - 73x^4 + 72x^3 + 64x^2 - 108x + 96}{68364x^6 + 1230552x^4 + 22149936x^3 + 7383312x^2 + 14766624}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**6/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)
```

```
[Out] RootSum(3977704731623097128039995515166457856*_t**6 - 10103143194
15295961050951680*_t**4 - 20168224477093957151232*_t**3 - 1125828
56818899648*_t**2 - 50648453064*_t - 880007, Lambda(_t, _t*log(-2
73655567090018991570649941414395560986199688040644608*_t**5/49797
8553961399002675733956 95 + 118370084701960460853086462307643542 92
805044570112*_t**4/497978553961399002675733956 95 - 10570581900446
717266374077482873315047787008*_t**3/497978553961399002675733956
95 - 155254741156946987238756321879278932 3392*_t**2/49797855396139
900267573395695 - 12542923791159140826909003250295928*_t/49797855
396139900267573395695 + x - 23066533870320322410834348296/4979785
5396139900267573395695))) - (3*x**5 - 73*x**4 + 72*x**3 + 64*x**2
 - 108*x + 96)/(68364*x**6 + 1230552*x**4 + 22149936*x**3 + 73833
12*x**2 + 14766624)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\left(x^6 + 18\,x^4 + 324\,x^3 + 108\,x^2 + 216\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2,x, algorithm="giac")
```

```
[Out] integrate(x^6/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2, x)
```

3.154 $\int \frac{x^5}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$

Optimal. Leaf size=682

$$\frac{\sqrt[3]{-\frac{1}{3}}\left(4-\sqrt[3]{-3}2^{2/3}x\right)}{1944\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}+\frac{\sqrt[3]{-\frac{1}{3}}\left((-2)^{2/3}\sqrt[3]{3}x+4\right)}{8748\ 2^{2/3}\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}-\frac{2^{2/3}\sqrt[3]{3}x+4}{17496\ 2^{2/3}\sqrt[3]{3}\left(4-3\sqrt[3]{2}3^{2/3}\right)\left(x^2+3\ 2^{2/3}\sqrt[3]{3}x+6\right)}+\frac{\tan^{-1}\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{4374\sqrt{3}\left(8-9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)^{3/2}}-\frac{\tan^{-1}\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{4374\ 2^{5/6}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^4\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}-\frac{\tan^{-1}\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{4374\sqrt{3}\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)^{3/2}}-\frac{i\tan^{-1}\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{1458\ 2^{5/6}3^{2/3}\left(1+\sqrt[3]{-1}\right)^5\sqrt{4+3\sqrt[3]{-2}3^{2/3}}}-\frac{\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(3\sqrt[3]{2}3^{2/3}-4\right)}}\right)}{39366\ 2^{5/6}\sqrt[6]{3}\sqrt{3\sqrt[3]{2}3^{2/3}-4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(3\sqrt[3]{2}3^{2/3}-4\right)}}\right)}{8748\sqrt{6}\left(3\sqrt[3]{2}3^{2/3}-4\right)^{3/2}}$$

```
[Out] ((-1/3)^(1/3)*(4 - (-3)^(1/3)*2^(2/3)*x))/(1944*2^(2/3)*(1 + (-1)
^(1/3))^4*(4 - 3*(-3)^(2/3)*2^(1/3))*(6 - 3*(-3)^(1/3)*2^(2/3)*x
+ x^2)) + ((-1/3)^(1/3)*(4 + (-2)^(2/3)*3^(1/3)*x))/(8748*2^(2/3)
*(8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))*(6 + 3*(-2)^(2/3
)*3^(1/3)*x + x^2)) - (4 + 2^(2/3)*3^(1/3)*x)/(17496*2^(2/3)*3^(1
/3)*(4 - 3*2^(1/3)*3^(2/3))*(6 + 3*2^(2/3)*3^(1/3)*x + x^2)) - Ar
cTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*2^(1/3
))]]/(4374*2^(5/6)*3^(1/6)*(1 + (-1)^(1/3))^4*Sqrt[4 - 3*(-3)^(2/
3)*2^(1/3)]) + ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*
(-3)^(2/3)*2^(1/3))]]/(4374*Sqrt[3]*(8 - (9*I)*2^(1/3)*3^(1/6) +
3*2^(1/3)*3^(2/3))^(3/2)) - ((I/1458)*ArcTan[(3*(-2)^(2/3)*3^(1/3
) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(2^(5/6)*3^(2/3)*(1
 + (-1)^(1/3))^5*Sqrt[4 + 3*(-2)^(1/3)*3^(2/3)]) - ArcTan[(3*(-2)
^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]]/(4374*S
qrt[3]*(8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))^(3/2)) - A
rcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3
^(2/3))]]/(8748*Sqrt[6]*(-4 + 3*2^(1/3)*3^(2/3))^(3/2)) - ArcTanh
[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3)
)]]/(39366*2^(5/6)*3^(1/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)])
```

Rubi [A] time = 4.23325, antiderivative size = 682, normalized size of antiderivative = 1., number of

steps used = 17, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$

$$\frac{\sqrt[3]{-\frac{1}{3}}\left(4-\sqrt[3]{-3}2^{2/3}x\right)}{1944\,2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}$$
$$+\frac{\sqrt[3]{-\frac{1}{3}}\left((-2)^{2/3}\sqrt[3]{3}x+4\right)}{8748\,2^{2/3}\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}$$
$$-\frac{2^{2/3}\sqrt[3]{3}x+4}{17496\,2^{2/3}\sqrt[3]{3}\left(4-3\sqrt[3]{2}3^{2/3}\right)\left(x^2+3\,2^{2/3}\sqrt[3]{3}x+6\right)}$$
$$+\frac{\tan^{-1}\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{4374\sqrt{3}\left(8-9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)^{3/2}}-\frac{\tan^{-1}\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{4374\,2^{5/6}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^4\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}$$
$$-\frac{\tan^{-1}\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{4374\sqrt{3}\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)^{3/2}}-\frac{i\tan^{-1}\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{1458\,2^{5/6}3^{2/3}\left(1+\sqrt[3]{-1}\right)^5\sqrt{4+3\sqrt[3]{-2}3^{2/3}}}$$
$$-\frac{\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(3\sqrt[3]{2}3^{2/3}-4\right)}}\right)}{39366\,2^{5/6}\sqrt[6]{3}\sqrt{3\sqrt[3]{2}3^{2/3}-4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(3\sqrt[3]{2}3^{2/3}-4\right)}}\right)}{8748\sqrt{6}\left(3\sqrt[3]{2}3^{2/3}-4\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[x^5/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] ((-1/3)^(1/3)*(4 - (-3)^(1/3)*2^(2/3)*x))/(1944*2^(2/3)*(1 + (-1)
^(1/3))^4*(4 - 3*(-3)^(2/3)*2^(1/3))*(6 - 3*(-3)^(1/3)*2^(2/3)*x
+ x^2)) + ((-1/3)^(1/3)*(4 + (-2)^(2/3)*3^(1/3)*x))/(8748*2^(2/3)
*(8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))*(6 + 3*(-2)^(2/3
)*3^(1/3)*x + x^2)) - (4 + 2^(2/3)*3^(1/3)*x)/(17496*2^(2/3)*3^(1
/3)*(4 - 3*2^(1/3)*3^(2/3))*(6 + 3*2^(2/3)*3^(1/3)*x + x^2)) - Ar
cTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*2^(1/3
))]]/(4374*2^(5/6)*3^(1/6)*(1 + (-1)^(1/3))^4*Sqrt[4 - 3*(-3)^(2/
3)*2^(1/3)]) + ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*
(-3)^(2/3)*2^(1/3))]]/(4374*Sqrt[3]*(8 - (9*I)*2^(1/3)*3^(1/6) +
3*2^(1/3)*3^(2/3))^(3/2)) - ((I/1458)*ArcTan[(3*(-2)^(2/3)*3^(1/3
) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(2^(5/6)*3^(2/3)*(1
 + (-1)^(1/3))^5*Sqrt[4 + 3*(-2)^(1/3)*3^(2/3)]) - ArcTan[(3*(-2)
^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]]/(4374*S
qrt[3]*(8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))^(3/2)) - A
rcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3
^(2/3))]]/(8748*Sqrt[6]*(-4 + 3*2^(1/3)*3^(2/3))^(3/2)) - ArcTanh
[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3)
)]]/(39366*2^(5/6)*3^(1/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.0427513, size = 167, normalized size = 0.24

$$\frac{\text{RootSum}\left[\#1^6+18\#1^4+324\#1^3+108\#1^2+216\&,\frac{4\#1^4\log(x-\#1)-54\#1^3\log(x-\#1)+2043\#1^2\log(x-\#1)-324\#1\log(x-\#1)+144\log(x-\#1)}{\#1^5+12\#1^3+162\#1^2+36\#1}\&\right]}{3691656}+\frac{4x^5-27x^4+729x^3+648x^2-144x+972}{615276\left(x^6+18x^4+324x^3+108x^2+216\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] (972 - 144*x + 648*x^2 + 729*x^3 - 27*x^4 + 4*x^5)/(615276*(216 +
 108*x^2 + 324*x^3 + 18*x^4 + x^6)) + RootSum[216 + 108*#1^2 + 32
4*#1^3 + 18*#1^4 + #1^6 & , (144*Log[x - #1] - 324*Log[x - #1]*#1
 + 2043*Log[x - #1]*#1^2 - 54*Log[x - #1]*#1^3 + 4*Log[x - #1]*#1
^4)/(36*#1 + 162*#1^2 + 12*#1^3 + #1^5) & ]/3691656
```

Maple [C] time = 0.015, size = 122, normalized size = 0.2

$$\frac{1}{x^6+18x^4+324x^3+108x^2+216}\left(\frac{x^5}{153819}-\frac{x^4}{22788}+\frac{x^3}{844}+\frac{2x^2}{1899}-\frac{4x}{17091}+\frac{1}{633}\right)+\frac{1}{3691656}\sum_{\_R=RootOf\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\left(4\_R^4-54\_R^3+2043\_R^2-324\_R+144\right)\ln\left(x-\_R\right)}{\_R^5+12\_R^3+162\_R^2+36\_R}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x)
```

```
[Out] (1/153819*x^5-1/22788*x^4+1/844*x^3+2/1899*x^2-4/17091*x+1/633)/(
x^6+18*x^4+324*x^3+108*x^2+216)+1/3691656*sum((4*_R^4-54*_R^3+204
3*_R^2-324*_R+144)/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=Root
Of(_Z^6+18*_Z^4+324*_Z^3+108*_Z^2+216))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{4x^5-27x^4+729x^3+648x^2-144x+972}{615276\left(x^6+18x^4+324x^3+108x^2+216\right)}+\frac{1}{615276}\int\frac{4x^4-54x^3+2043x^2-324x+144}{x^6+18x^4+324x^3+108x^2+216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2,x, algorithm="maxima")
```

```
[Out] 1/615276*(4*x^5 - 27*x^4 + 729*x^3 + 648*x^2 - 144*x + 972)/(x^6
+ 18*x^4 + 324*x^3 + 108*x^2 + 216) + 1/615276*integrate((4*x^4 -
 54*x^3 + 2043*x^2 - 324*x + 144)/(x^6 + 18*x^4 + 324*x^3 + 108*x
^2 + 216), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 0.872339, size = 104, normalized size = 0.15

$$\text{RootSum}\left(274938951049788473490124490008305567006720t^6 - 13187181892269500888629831925760t^4 + 1212091770477677\right.$$

$$+ \frac{4x^5 - 27x^4 + 729x^3 + 648x^2 - 144x + 972}{615276x^6 + 11074968x^4 + 199349424x^3 + 66449808x^2 + 132899616}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)
```

```
[Out] RootSum(274938951049788473490124490008305567006720*_t**6 - 1318718
18922695008886298319257600*_t**4 + 1212091770477677644800*_t**2 - 397
53025, Lambda(_t, _t*log(947842259001288723909832054550209950242045
45952*_t**5/61864539719962655 - 2434586468177756076396548894808145
592*_t**4/9811980923071 - 4168255647506750043178731077966745600*_t*
*3/61864539719962655 + 1202687744266432861646227200*_t**2/981198092
3071 + 2161426184888597936684280*_t/61864539719962655 + x - 308574
300024117/39247923692284))) + (4*x**5 - 27*x**4 + 729*x**3 + 648*
x**2 - 144*x + 972)/(615276*x**6 + 11074968*x**4 + 199349424*x**3
 + 66449808*x**2 + 132899616)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\left(x^6 + 18\,x^4 + 324\,x^3 + 108\,x^2 + 216\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2,x, algorithm="giac")
```

```
[Out] integrate(x^5/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2, x)
```

3.155 $\int \frac{x^4}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$

Optimal. Leaf size=850

result too large to display

```
[Out] ((-1/3)^(1/3)*(3*(-3)^(1/3)*2^(2/3) - 2*x))/(5832*2^(2/3)*(1 + (-
1)^(1/3))^4*(4 - 3*(-3)^(2/3)*2^(1/3))*(6 - 3*(-3)^(1/3)*2^(2/3)*
x + x^2)) - ((-1/3)^(1/3)*(3*(-2)^(2/3)*3^(1/3) + 2*x))/(26244*2^
(2/3)*(8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))*(6 + 3*(-2)
^(2/3)*3^(1/3)*x + x^2)) - (3*3^(1/3) + 2^(1/3)*x)/(52488*(9*2^(1
/3) - 4*3^(1/3))*(6 + 3*2^(2/3)*3^(1/3)*x + x^2)) + ((-1)^(1/3)*A
rcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*2^(1/
3))]])/(729*2^(2/3)*3^(5/6)*(1 + (-1)^(1/3))^4*(8 - (9*I)*2^(1/3)
*3^(1/6) + 3*2^(1/3)*3^(2/3))^(3/2)) - ((-1)^(1/3)*ArcTan[(3*(-2)
^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(2916*
2^(1/6)*3^(5/6)*(1 - (-1)^(1/3))^2*(1 + (-1)^(1/3))^4*(4 + 3*(-2)
^(1/3)*3^(2/3))^(3/2)) - ((I + Sqrt[3])*ArcTan[(3*(-2)^(2/3)*3^(1
/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(11664*2^(1/6)*3^
(1/3)*(1 + (-1)^(1/3))^5*Sqrt[4 + 3*(-2)^(1/3)*3^(2/3)]) - ((I/58
32)*ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[3*(4 - 3*(-3
)^(2/3)*2^(1/3))]])/(2^(1/6)*3^(1/3)*(1 + (-1)^(1/3))^5*Sqrt[4 -
3*(-3)^(2/3)*2^(1/3)]) + ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x)
)/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]]/(26244*2^(1/6)*3^(5/6)*(-4 +
3*2^(1/3)*3^(2/3))^(3/2)) + ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)
*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]]/(52488*2^(1/6)*3^(5/6)*Sqr
t[-4 + 3*2^(1/3)*3^(2/3)]) - Log[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2
]/(34992*2^(1/3)*3^(2/3)*(1 + (-1)^(1/3))^4) + ((I/34992)*Log[6 +
 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(2^(1/3)*3^(1/6)*(1 + (-1)^(1/3))
^5) - Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(314928*2^(1/3)*3^(2/3))
```

Rubi [A] time = 5.22123, antiderivative size = 850, normalized size of antiderivative = 1., number of

steps used = 23, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$

$$\frac{\sqrt[3]{-\frac{1}{3}}\left(3\sqrt[3]{-3}2^{2/3}-2x\right)}{5832\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}+\frac{\sqrt[3]{-1}\tan^{-1}\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{729\ 2^{2/3}3^{5/6}\left(1+\sqrt[3]{-1}\right)^4\left(8-9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)^{3/2}}-\frac{\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{11664\sqrt[6]{2}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^5\sqrt{4+3\sqrt[3]{-2}3^{2/3}}}-\frac{\sqrt[3]{-1}\tan^{-1}\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{2916\sqrt[6]{2}3^{5/6}\left(1-\sqrt[3]{-1}\right)^2\left(1+\sqrt[3]{-1}\right)^4\left(4+3\sqrt[3]{-2}3^{2/3}\right)^{3/2}}-\frac{i\tan^{-1}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{5832\sqrt[6]{2}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^5\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}+\frac{\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{52488\sqrt[6]{2}3^{5/6}\sqrt{-4+3\sqrt[3]{2}3^{2/3}}}+\frac{\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{26244\sqrt[6]{2}3^{5/6}\left(-4+3\sqrt[3]{2}3^{2/3}\right)^{3/2}}-\frac{\log\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}{34992\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^4}+\frac{i\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}{34992\sqrt[3]{2}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^5}-\frac{\log\left(x^2+3\ 2^{2/3}\sqrt[3]{3}x+6\right)}{314928\sqrt[3]{2}3^{2/3}}-\frac{\sqrt[3]{-\frac{1}{3}}\left(2x+3(-2)^{2/3}\sqrt[3]{3}\right)}{26244\ 2^{2/3}\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}-\frac{\sqrt[3]{2}x+3\sqrt[3]{3}}{52488\left(9\sqrt[3]{2}-4\sqrt[3]{3}\right)\left(x^2+3\ 2^{2/3}\sqrt[3]{3}x+6\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[x^4/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] ((-1/3)^(1/3)*(3*(-3)^(1/3)*2^(2/3) - 2*x))/(5832*2^(2/3)*(1 + (-
1)^(1/3))^4*(4 - 3*(-3)^(2/3)*2^(1/3))*(6 - 3*(-3)^(1/3)*2^(2/3)*
x + x^2)) - ((-1/3)^(1/3)*(3*(-2)^(2/3)*3^(1/3) + 2*x))/(26244*2^
(2/3)*(8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))*(6 + 3*(-2)
^(2/3)*3^(1/3)*x + x^2)) - (3*3^(1/3) + 2^(1/3)*x)/(52488*(9*2^(1
/3) - 4*3^(1/3))*(6 + 3*2^(2/3)*3^(1/3)*x + x^2)) + ((-1)^(1/3)*A
rcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*2^(1/
3))]])/(729*2^(2/3)*3^(5/6)*(1 + (-1)^(1/3))^4*(8 - (9*I)*2^(1/3)
*3^(1/6) + 3*2^(1/3)*3^(2/3))^(3/2)) - ((-1)^(1/3)*ArcTan[(3*(-2)
^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(2916*
2^(1/6)*3^(5/6)*(1 - (-1)^(1/3))^2*(1 + (-1)^(1/3))^4*(4 + 3*(-2)
^(1/3)*3^(2/3))^(3/2)) - ((I + Sqrt[3])*ArcTan[(3*(-2)^(2/3)*3^(1
/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(11664*2^(1/6)*3^
```

```
(1/3)*(1 + (-1)^(1/3))^5*Sqrt[4 + 3*(-2)^(1/3)*3^(2/3)]) - ((I/58
32)*ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[3*(4 - 3*(-3
)^(2/3)*2^(1/3))]])/(2^(1/6)*3^(1/3)*(1 + (-1)^(1/3))^5*Sqrt[4 -
3*(-3)^(2/3)*2^(1/3)]) + ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x)
)/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]]/(26244*2^(1/6)*3^(5/6)*(-4 +
3*2^(1/3)*3^(2/3))^(3/2)) + ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)
*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]]/(52488*2^(1/6)*3^(5/6)*Sqr
t[-4 + 3*2^(1/3)*3^(2/3)]) - Log[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2
]/(34992*2^(1/3)*3^(2/3)*(1 + (-1)^(1/3))^4) + ((I/34992)*Log[6 +
 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(2^(1/3)*3^(1/6)*(1 + (-1)^(1/3))
^5) - Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(314928*2^(1/3)*3^(2/3))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.0543798, size = 167, normalized size = 0.2

$$\frac{-9x^5+8x^4-216x^3-1458x^2+324x-288}{1230552\,(x^6+18x^4+324x^3+108x^2+216)}-\frac{\text{RootSum}\left[\#1^6+18\#1^4+324\#1^3+108\#1^2+216\&,\frac{9\#1^4\log(x-\#1)-16\#1^3\log(x-\#1)+324\#1^2\log(x-\#1)-2628\#1\log(x-\#1)+324\log(x-\#1)}{\#1^5+12\#1^3+162\#1^2+36\#1}\&\right]}{7383312}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] (-288 + 324*x - 1458*x^2 - 216*x^3 + 8*x^4 - 9*x^5)/(1230552*(216
 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)) - RootSum[216 + 108*#1^2 +
324*#1^3 + 18*#1^4 + #1^6 & , (324*Log[x - #1] - 2628*Log[x - #1]
*#1 + 324*Log[x - #1]*#1^2 - 16*Log[x - #1]*#1^3 + 9*Log[x - #1]*
#1^4)/(36*#1 + 162*#1^2 + 12*#1^3 + #1^5) & ]/7383312
```

Maple [C] time = 0.015, size = 122, normalized size = 0.1

$$\frac{1}{x^6+18\,x^4+324\,x^3+108\,x^2+216}\left(-\frac{x^5}{136728}+\frac{x^4}{153819}-\frac{x^3}{5697}-\frac{x^2}{844}+\frac{x}{3798}-\frac{4}{17091}\right)+\frac{1}{7383312}\sum_{\_R=RootOf(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216)}\frac{\left(-9\_R^4+16\_R^3-324\_R^2+2628\_R-324\right)\ln(x-\_R)}{\_R^5+12\_R^3+162\_R^2+36\_R}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x)
```

```
[Out] (-1/136728*x^5+1/153819*x^4-1/5697*x^3-1/844*x^2+1/3798*x-4/17091
)/(x^6+18*x^4+324*x^3+108*x^2+216)+1/7383312*sum((-9*_R^4+16*_R^3
-324*_R^2+2628*_R-324)/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=
RootOf(_Z^6+18*_Z^4+324*_Z^3+108*_Z^2+216))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{9\,x^5-8\,x^4+216\,x^3+1458\,x^2-324\,x+288}{1230552\,(x^6+18\,x^4+324\,x^3+108\,x^2+216)}-\frac{1}{1230552}\int\frac{9\,x^4-16\,x^3+324\,x^2-2628\,x+324}{x^6+18\,x^4+324\,x^3+108\,x^2+216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2,x, algorithm="maxima")
```

```
[Out] -1/1230552*(9*x^5 - 8*x^4 + 216*x^3 + 1458*x^2 - 324*x + 288)/(x^
6 + 18*x^4 + 324*x^3 + 108*x^2 + 216) - 1/1230552*integrate((9*x^
4 - 16*x^3 + 324*x^2 - 2628*x + 324)/(x^6 + 18*x^4 + 324*x^3 + 10
8*x^2 + 216), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 1.18209, size = 112, normalized size = 0.13

$$\mathrm{RootSum}\Big(185583791958607219605834030755606257729536t^6-1309367357962223565522033377280t^4+43563364870522$$

$$-\frac{9x^5-8x^4+216x^3+1458x^2-324x+288}{1230552x^6+22149936x^4+398698848x^3+132899616x^2+265799232}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)
```

```
[Out] RootSum(185583791958607219605834030755606257729536*_t**6 - 130936
7357962223565522033377280*_t**4 + 435633648705229474466 6112*_t**3
 - 4052982845480387328*_t**2 + 303890718384*_t - 880007, Lambda(_
t, _t*log(3908346265795559347684104470733356597641295275928063469
1584*_t**5/497978553961399002675733956695 + 883697934622378553891 2
8176014147111023968044625756 16*_t**4/49797855396139900267573395 69
5 - 26493058134830853258884424959713469570680506 7776*_t**3/497978
553961399002675733 95695 + 886135333547363185201515109826158376250
624*_t**2/497978553961399002675733 95695 - 682321479574909906511 39
463585560193 6*_t/497978553961399002675733 95695 + x - 21375560770 8
464862242915195 68/4979785539613990026757339 5695))) - (9*x**5 - 8*
x**4 + 216*x**3 + 1458*x**2 - 324*x + 288)/(1230552*x**6 + 221499
36*x**4 + 398698848*x**3 + 132899616*x**2 + 265799232)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^4}{(x^6+18\,x^4+324\,x^3+108\,x^2+216)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2,x, algorithm="giac")

[Out] integrate(x^4/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2, x)
```

3.156 $\int \frac{x^3}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$

Optimal. Leaf size=873

result too large to display

```
[Out] ((-6)^(1/3)*(2*(-3)^(1/3) + 9*2^(1/3)) - 3*x)/(157464*(8 - (9*I)*
2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))*(6 - 3*(-3)^(1/3)*2^(2/3)*x
+ x^2)) - ((-6)^(1/3)*(9*(-2)^(1/3) + 2*3^(1/3)) + 3*x)/(157464*(
8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))*(6 + 3*(-2)^(2/3)*
3^(1/3)*x + x^2)) - (2*2^(1/3) - 3*6^(2/3) - 3^(1/3)*x)/(104976*(
9*2^(1/3) - 4*3^(1/3))*(6 + 3*2^(2/3)*3^(1/3)*x + x^2)) + ArcTan[
(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*2^(1/3))]]/
(26244*Sqrt[3]*(8 - (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))^(3
/2)) - ((9*I - 3^(1/3)*((2*I)*2^(2/3) + 9*3^(1/6) + 2*2^(2/3)*Sqr
t[3]))*ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/
3)*2^(1/3))]])/(209952*(1 + (-1)^(1/3))^5*Sqrt[2*(4 - 3*(-3)^(2/3
)*2^(1/3))]) - ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*
(-2)^(1/3)*3^(2/3))]]/(26244*Sqrt[3]*(8 + (9*I)*2^(1/3)*3^(1/6) +
 3*2^(1/3)*3^(2/3))^(3/2)) + ((9*I + 3^(1/3)*((4*I)*2^(2/3) - 9*3
^(1/6)))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(
1/3)*3^(2/3))]])/(209952*(1 + (-1)^(1/3))^5*Sqrt[2*(4 + 3*(-2)^(1
/3)*3^(2/3))]) - ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3
*(-4 + 3*2^(1/3)*3^(2/3))]]/(52488*Sqrt[6]*(-4 + 3*2^(1/3)*3^(2/3
))^(3/2)) + ((2*2^(2/3) - 3*3^(2/3))*ArcTanh[(2^(1/6)*(3*3^(1/3)
+ 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(944784*3^(1/6)*
Sqrt[2*(-4 + 3*2^(1/3)*3^(2/3))]) - ((I/23328)*Log[6 - 3*(-3)^(1/
3)*2^(2/3)*x + x^2])/(2^(2/3)*3^(5/6)*(1 + (-1)^(1/3))^5) + ((I +
 Sqrt[3])*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(46656*2^(2/3)*3
^(5/6)*(1 + (-1)^(1/3))^5) + Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(
629856*2^(2/3)*3^(1/3))
```

Rubi [A] time = 6.34642, antiderivative size = 873, normalized size of antiderivative = 1., number of

steps used = 23, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$

$$\begin{aligned}
&\frac{\sqrt[3]{-6}\left(2\sqrt[3]{-3}+9\sqrt[3]{2}\right)-3x}{157464\left(8-9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}\\
&+\frac{\left(9\sqrt[6]{3}+i\left(2\,2^{2/3}-2i2^{2/3}\sqrt{3}-3\,3^{2/3}\right)\right)\tan^{-1}\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{69984\,3^{2/3}\left(1+\sqrt[3]{-1}\right)^5\sqrt{2\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\\
&+\frac{\tan^{-1}\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{26244\sqrt{3}\left(8-9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)^{3/2}}-\frac{\left(9\sqrt[6]{3}-i\left(4\,2^{2/3}+3\,3^{2/3}\right)\right)\tan^{-1}\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{69984\,3^{2/3}\left(1+\sqrt[3]{-1}\right)^5\sqrt{2\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\\
&-\frac{\tan^{-1}\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{26244\sqrt{3}\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)^{3/2}}+\frac{\left(2\,2^{2/3}-3\,3^{2/3}\right)\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{944784\sqrt[6]{3}\sqrt{2\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\\
&-\frac{\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{52488\sqrt{6}\left(-4+3\sqrt[3]{2}3^{2/3}\right)^{3/2}}-\frac{i\log\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}{23328\,2^{2/3}3^{5/6}\left(1+\sqrt[3]{-1}\right)^5}\\
&+\frac{\left(i+\sqrt{3}\right)\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}{46656\,2^{2/3}3^{5/6}\left(1+\sqrt[3]{-1}\right)^5}+\frac{\log\left(x^2+3\,2^{2/3}\sqrt[3]{3}x+6\right)}{629856\,2^{2/3}\sqrt[3]{3}}\\
&-\frac{3x+\sqrt[3]{-6}\left(9\sqrt[3]{-2}+2\sqrt[3]{3}\right)}{157464\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}\\
&-\frac{-\sqrt[3]{3}x-3\,6^{2/3}+2\sqrt[3]{2}}{104976\left(9\sqrt[3]{2}-4\sqrt[3]{3}\right)\left(x^2+3\,2^{2/3}\sqrt[3]{3}x+6\right)}
\end{aligned}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[x^3/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] ((-6)^(1/3)*(2*(-3)^(1/3) + 9*2^(1/3)) - 3*x)/(157464*(8 - (9*I)*
2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))*(6 - 3*(-3)^(1/3)*2^(2/3)*x
+ x^2)) - ((-6)^(1/3)*(9*(-2)^(1/3) + 2*3^(1/3)) + 3*x)/(157464*(
8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))*(6 + 3*(-2)^(2/3)*
3^(1/3)*x + x^2)) - (2*2^(1/3) - 3*6^(2/3) - 3^(1/3)*x)/(104976*(
9*2^(1/3) - 4*3^(1/3))*(6 + 3*2^(2/3)*3^(1/3)*x + x^2)) + ArcTan[
(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*2^(1/3))]]/
(26244*Sqrt[3]*(8 - (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))^(3
/2)) + ((9*3^(1/6) + I*(2*2^(2/3) - (2*I)*2^(2/3)*Sqrt[3] - 3*3^(
2/3)))*ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/
3)*2^(1/3))]])/(69984*3^(2/3)*(1 + (-1)^(1/3))^5*Sqrt[2*(4 - 3*(-
3)^(2/3)*2^(1/3))]) - ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*
(4 + 3*(-2)^(1/3)*3^(2/3))]]/(26244*Sqrt[3]*(8 + (9*I)*2^(1/3)*3^
(1/6) + 3*2^(1/3)*3^(2/3))^(3/2)) - ((9*3^(1/6) - I*(4*2^(2/3) +
3*3^(2/3)))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2
)^(1/3)*3^(2/3))]])/(69984*3^(2/3)*(1 + (-1)^(1/3))^5*Sqrt[2*(4 +
 3*(-2)^(1/3)*3^(2/3))]) - ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*
x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]]/(52488*Sqrt[6]*(-4 + 3*2^(1
/3)*3^(2/3))^(3/2)) + ((2*2^(2/3) - 3*3^(2/3))*ArcTanh[(2^(1/6)*(
3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(94478
```

```
4*3^(1/6)*Sqrt[2*(-4 + 3*2^(1/3)*3^(2/3))]) - ((I/23328)*Log[6 -
3*(-3)^(1/3)*2^(2/3)*x + x^2])/(2^(2/3)*3^(5/6)*(1 + (-1)^(1/3))^
5) + ((I + Sqrt[3])*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(46656
*2^(2/3)*3^(5/6)*(1 + (-1)^(1/3))^5) + Log[6 + 3*2^(2/3)*3^(1/3)*
x + x^2]/(629856*2^(2/3)*3^(1/3))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)

[Out] Timed out
```

Mathematica [C] time = 0.0422749, size = 167, normalized size = 0.19

$$\frac{\text{RootSum}\left[\#1^6+18\#1^4+324\#1^3+108\#1^2+216\&,\frac{2\#1^4\log(x-\#1)-27\#1^3\log(x-\#1)+72\#1^2\log(x-\#1)-162\#1\log(x-\#1)+1971\log(x-\#1)}{\#1^5+12\#1^3+162\#1^2+36\#1}\&\right]}{11074968}+\frac{4x^5-27x^4+96x^3+648x^2-3942x+972}{3691656\left(x^6+18x^4+324x^3+108x^2+216\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]

[Out] (972 - 3942*x + 648*x^2 + 96*x^3 - 27*x^4 + 4*x^5)/(3691656*(216
+ 108*x^2 + 324*x^3 + 18*x^4 + x^6)) + RootSum[216 + 108*#1^2 + 3
24*#1^3 + 18*#1^4 + #1^6 & , (1971*Log[x - #1] - 162*Log[x - #1]*
#1 + 72*Log[x - #1]*#1^2 - 27*Log[x - #1]*#1^3 + 2*Log[x - #1]*#1
^4)/(36*#1 + 162*#1^2 + 12*#1^3 + #1^5) & ]/11074968
```

Maple [C] time = 0.014, size = 122, normalized size = 0.1

$$\frac{1}{x^6+18x^4+324x^3+108x^2+216}\left(\frac{x^5}{922914}-\frac{x^4}{136728}+\frac{4x^3}{153819}+\frac{x^2}{5697}-\frac{73x}{68364}+\frac{1}{3798}\right)+\frac{1}{11074968}\sum_{\_R=RootOf\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\left(2\_R^4-27\_R^3+72\_R^2-162\_R+1971\right)\ln\left(x-\_R\right)}{\_R^5+12\_R^3+162\_R^2+36\_R}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x)

[Out] (1/922914*x^5-1/136728*x^4+4/153819*x^3+1/5697*x^2-73/68364*x+1/3
798)/(x^6+18*x^4+324*x^3+108*x^2+216)+1/11074968*sum((2*_R^4-27*_
R^3+72*_R^2-162*_R+1971)/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_
R=RootOf(_Z^6+18*_Z^4+324*_Z^3+108*_Z^2+216))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{4x^5-27x^4+96x^3+648x^2-3942x+972}{3691656\left(x^6+18x^4+324x^3+108x^2+216\right)}+\frac{1}{1845828}\int\frac{2x^4-27x^3+72x^2-162x+1971}{x^6+18x^4+324x^3+108x^2+216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2,x, algorithm="maxima")
```

```
[Out] 1/3691656*(4*x^5 - 27*x^4 + 96*x^3 + 648*x^2 - 3942*x + 972)/(x^6
 + 18*x^4 + 324*x^3 + 108*x^2 + 216) + 1/1845828*integrate((2*x^4
 - 27*x^3 + 72*x^2 - 162*x + 1971)/(x^6 + 18*x^4 + 324*x^3 + 108*
x^2 + 216), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 1.20717, size = 112, normalized size = 0.13

$$\operatorname{RootSum}\left(1282755170017893101915524820582750453426552832t^6 - 906388465775544244426251149770752t^4 - 4300873\right.$$
$$+ \frac{4x^5 - 27x^4 + 96x^3 + 648x^2 - 3942x + 972}{3691656x^6 + 66449808x^4 + 1196096544x^3 + 398698848x^2 + 797397696}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)
```

```
[Out] RootSum(1282755170017893101915524820582750453426552832*_t**6 - 90
6388465775544244426251149770752*_t**4 - 4300873166389987741684137
984*_t**3 - 717000908921644962816*_t**2 + 135354162312576*_t - 71
97829, Lambda(_t, _t*log(172579355928104499014095565978918829956O
4001083339368041361480613888*_t**5/1542060097910520444906943803O3
237521 + 238960740062098552437635885357265220718195632456058768 40
52992*_t**4/154206009791052044490694380303237521 - 1228607216088 3
2839307117159488782600789969921934883880 96*_t**3/1542060097910520
444906943803032 37521 - 594905535739591731611254960135279097541565
58410752*_t**2/154206009791052044490694380303237521 - 17520149679
83669111236706419771375300482720 0*_t/1542060097910520444906943803
03237521 + x + 7664229887072296150558552870408873 32/1542060097910
520444906943803032375 21))) + (4*x**5 - 27*x**4 + 96*x**3 + 648*x*
*2 - 3942*x + 972)/(3691656*x**6 + 66449808*x**4 + 1196096544*x**
3 + 398698848*x**2 + 797397696)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(x^6 + 18\,x^4 + 324\,x^3 + 108\,x^2 + 216)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2, x)
```

3.157 $\int \frac{x^2}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$

Optimal. Leaf size=986

result too large to display

```
[Out] -(27*((-2)^(2/3) + 2*(-1)^(1/3)*3^(2/3)) - 6^(1/3)*(9 + (-3)^(1/3
)*2^(2/3))*x)/(104976*2^(2/3)*(1 + (-1)^(1/3))^4*(4 - 3*(-3)^(2/3
)*2^(1/3))*(6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2)) - (27*2^(2/3)*(1 +
 (-2)^(1/3)*3^(2/3)) - (-1)^(1/3)*3^(2/3)*(2 + 3*(-2)^(1/3)*3^(2/
3))*x)/(472392*2^(2/3)*(8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(
2/3))*(6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2)) + (9*(6 - 2^(2/3)*3^(1/
3)) - (2 - 3*2^(1/3)*3^(2/3))*x)/(314928*2^(2/3)*3^(1/3)*(4 - 3*2
^(1/3)*3^(2/3))*(6 + 3*2^(2/3)*3^(1/3)*x + x^2)) - ((1 + I*Sqrt[3
] + 3*2^(1/3)*3^(2/3))*ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6
*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(8748*2^(2/3)*3^(5/6)*(1 + (-1)^(1
/3))^4*(8 - (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))^(3/2)) + (
(3*(-3)^(2/3) + (-1)^(1/3)*2^(2/3))*ArcTan[(3*(-2)^(2/3)*3^(1/3)
+ 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(17496*6^(5/6)*(1 - (
-1)^(1/3))^2*(1 + (-1)^(1/3))^4*(4 + 3*(-2)^(1/3)*3^(2/3))^(3/2))
 + ((I + Sqrt[3])*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 +
 3*(-2)^(1/3)*3^(2/3))]])/(34992*2^(1/6)*3^(1/3)*(1 + (-1)^(1/3))
^5*Sqrt[4 + 3*(-2)^(1/3)*3^(2/3)]) + ((I/17496)*ArcTan[(2^(1/6)*(
3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(
2^(1/6)*3^(1/3)*(1 + (-1)^(1/3))^5*Sqrt[4 - 3*(-3)^(2/3)*2^(1/3)]
) - ((2^(2/3) - 3*3^(2/3))*ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*
x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(17496*6^(5/6)*(1 - (-1)^(
1/3))^2*(1 + (-1)^(1/3))^4*(-4 + 3*2^(1/3)*3^(2/3))^(3/2)) - ArcT
anh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2
/3))]]/(157464*2^(1/6)*3^(5/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)]) + ((
I + Sqrt[3])*Log[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2])/(419904*2^(1/
3)*3^(1/6)*(1 + (-1)^(1/3))^5) - ((I/209952)*Log[6 + 3*(-2)^(2/3)
*3^(1/3)*x + x^2])/(2^(1/3)*3^(1/6)*(1 + (-1)^(1/3))^5) + Log[6 +
 3*2^(2/3)*3^(1/3)*x + x^2]/(1889568*2^(1/3)*3^(2/3))
```

Rubi [A] time = 7.46294, antiderivative size = 986, normalized size of antiderivative = 1., number of

steps used = 23, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$

$$-\frac{27\left((-2)^{2/3}+2\sqrt[3]{-1}3^{2/3}\right)-\sqrt[3]{6}\left(9+\sqrt[3]{-3}2^{2/3}\right)x}{104976\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}$$
$$-\frac{\left(1+i\sqrt{3}+3\sqrt[3]{2}3^{2/3}\right)\tan^{-1}\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{8748\ 2^{2/3}3^{5/6}\left(1+\sqrt[3]{-1}\right)^4\left(8-9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)^{3/2}}$$
$$+\frac{\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{34992\sqrt[6]{2}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^5\sqrt{4+3\sqrt[3]{-2}3^{2/3}}}$$
$$+\frac{\left(3(-3)^{2/3}+\sqrt[3]{-1}2^{2/3}\right)\tan^{-1}\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{17496\ 6^{5/6}\left(1-\sqrt[3]{-1}\right)^2\left(1+\sqrt[3]{-1}\right)^4\left(4+3\sqrt[3]{-2}3^{2/3}\right)^{3/2}}$$
$$+\frac{i\tan^{-1}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{17496\sqrt[6]{2}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^5\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}-\frac{\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{157464\sqrt[6]{2}3^{5/6}\sqrt{-4+3\sqrt[3]{2}3^{2/3}}}$$
$$-\frac{\left(2^{2/3}-3\ 3^{2/3}\right)\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{17496\ 6^{5/6}\left(1-\sqrt[3]{-1}\right)^2\left(1+\sqrt[3]{-1}\right)^4\left(-4+3\sqrt[3]{2}3^{2/3}\right)^{3/2}}$$
$$+\frac{\left(i+\sqrt{3}\right)\log\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}{419904\sqrt[3]{2}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^5}-\frac{i\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}{209952\sqrt[3]{2}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^5}$$
$$+\frac{\log\left(x^2+3\ 2^{2/3}\sqrt[3]{3}x+6\right)}{1889568\sqrt[3]{2}3^{2/3}}-\frac{27\ 2^{2/3}\left(1+\sqrt[3]{-2}3^{2/3}\right)-\sqrt[3]{-1}3^{2/3}\left(2+3\sqrt[3]{-2}3^{2/3}\right)x}{472392\ 2^{2/3}\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}$$
$$+\frac{9\left(6-2^{2/3}\sqrt[3]{3}\right)-\left(2-3\sqrt[3]{2}3^{2/3}\right)x}{314928\ 2^{2/3}\sqrt[3]{3}\left(4-3\sqrt[3]{2}3^{2/3}\right)\left(x^2+3\ 2^{2/3}\sqrt[3]{3}x+6\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[x^2/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] -(27*((-2)^(2/3) + 2*(-1)^(1/3)*3^(2/3)) - 6^(1/3)*(9 + (-3)^(1/3
)*2^(2/3))*x)/(104976*2^(2/3)*(1 + (-1)^(1/3))^4*(4 - 3*(-3)^(2/3
)*2^(1/3))*(6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2)) - (27*2^(2/3)*(1 +
 (-2)^(1/3)*3^(2/3)) - (-1)^(1/3)*3^(2/3)*(2 + 3*(-2)^(1/3)*3^(2/
3))*x)/(472392*2^(2/3)*(8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(
2/3))*(6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2)) + (9*(6 - 2^(2/3)*3^(1/
3)) - (2 - 3*2^(1/3)*3^(2/3))*x)/(314928*2^(2/3)*3^(1/3)*(4 - 3*2
^(1/3)*3^(2/3))*(6 + 3*2^(2/3)*3^(1/3)*x + x^2)) - ((1 + I*Sqrt[3
] + 3*2^(1/3)*3^(2/3))*ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6
*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(8748*2^(2/3)*3^(5/6)*(1 + (-1)^(1
/3))^4*(8 - (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))^(3/2)) + (
(3*(-3)^(2/3) + (-1)^(1/3)*2^(2/3))*ArcTan[(3*(-2)^(2/3)*3^(1/3)
+ 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(17496*6^(5/6)*(1 - (
-1)^(1/3))^2*(1 + (-1)^(1/3))^4*(4 + 3*(-2)^(1/3)*3^(2/3))^(3/2))
```

```
 + ((I + Sqrt[3])*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 +
 3*(-2)^(1/3)*3^(2/3))]])/(34992*2^(1/6)*3^(1/3)*(1 + (-1)^(1/3))
^5*Sqrt[4 + 3*(-2)^(1/3)*3^(2/3)]) + ((I/17496)*ArcTan[(2^(1/6)*(
3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(
2^(1/6)*3^(1/3)*(1 + (-1)^(1/3))^5*Sqrt[4 - 3*(-3)^(2/3)*2^(1/3)]
) - ((2^(2/3) - 3*3^(2/3))*ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*
x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(17496*6^(5/6)*(1 - (-1)^(
1/3))^2*(1 + (-1)^(1/3))^4*(-4 + 3*2^(1/3)*3^(2/3))^(3/2)) - ArcT
anh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2
/3))]]/(157464*2^(1/6)*3^(5/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)]) + ((
I + Sqrt[3])*Log[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2])/(419904*2^(1/
3)*3^(1/6)*(1 + (-1)^(1/3))^5) - ((I/209952)*Log[6 + 3*(-2)^(2/3)
*3^(1/3)*x + x^2])/(2^(1/3)*3^(1/6)*(1 + (-1)^(1/3))^5) + Log[6 +
 3*2^(2/3)*3^(1/3)*x + x^2]/(1889568*2^(1/3)*3^(2/3))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.0543622, size = 167, normalized size = 0.17

$$\frac{-9x^5+8x^4-216x^3-2724x^2+324x-7884}{7383312\left(x^6+18x^4+324x^3+108x^2+216\right)} - \frac{\text{RootSum}\left[\#1^6+18\#1^4+324\#1^3+108\#1^2+216\&, \frac{9\#1^4\log(x-\#1)-16\#1^3\log(x-\#1)+324\#1^2\log(x-\#1)+2436\#1\log(x-\#1)+324\log(x-\#1)}{\#1^5+12\#1^3+162\#1^2+36\#1}\right.}{44299872}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] (-7884 + 324*x - 2724*x^2 - 216*x^3 + 8*x^4 - 9*x^5)/(7383312*(21
6 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)) - RootSum[216 + 108*#1^2 +
 324*#1^3 + 18*#1^4 + #1^6 & , (324*Log[x - #1] + 2436*Log[x - #1
]*#1 + 324*Log[x - #1]*#1^2 - 16*Log[x - #1]*#1^3 + 9*Log[x - #1]
*#1^4)/(36*#1 + 162*#1^2 + 12*#1^3 + #1^5) & ]/44299872
```

Maple [C] time = 0.015, size = 122, normalized size = 0.1

$$\frac{1}{x^6+18\,x^4+324\,x^3+108\,x^2+216}\left(-\frac{x^5}{820368}+\frac{x^4}{922914}-\frac{x^3}{34182}-\frac{227\,x^2}{615276}+\frac{x}{22788}-\frac{73}{68364}\right)+\frac{1}{44299872}\sum_{\_R=RootOf\left(\_Z^6+18\,\_Z^4+324\,\_Z^3+108\,\_Z^2+216\right)}\frac{\left(-9\,\_R^4+16\,\_R^3-324\,\_R^2-2436\,\_R-324\right)\ln\left(x-\_R\right)}{\_R^5+12\,\_R^3+162\,\_R^2+36\,\_R}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x)
```

```
[Out] (-1/820368*x^5+1/922914*x^4-1/34182*x^3-227/615276*x^2+1/22788*x-
73/68364)/(x^6+18*x^4+324*x^3+108*x^2+216)+1/44299872*sum((-9*_R^
```

```
4+16*_R^3-324*_R^2-2436*_R-324)/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(
x-_R),_R=RootOf(_Z^6+18*_Z^4+324*_Z^3+108*_Z^2+216))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{9\,x^{5}-8\,x^{4}+216\,x^{3}+2724\,x^{2}-324\,x+7884}{7383312\,(x^{6}+18\,x^{4}+324\,x^{3}+108\,x^{2}+216)}-\frac{1}{7383312}\int\frac{9\,x^{4}-16\,x^{3}+324\,x^{2}+2436\,x+324}{x^{6}+18\,x^{4}+324\,x^{3}+108\,x^{2}+216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2,x, algorithm="maxima")
```

```
[Out] -1/7383312*(9*x^5 - 8*x^4 + 216*x^3 + 2724*x^2 - 324*x + 7884)/(x
^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216) - 1/7383312*integrate((9*x
^4 - 16*x^3 + 324*x^2 + 2436*x + 324)/(x^6 + 18*x^4 + 324*x^3 + 1
08*x^2 + 216), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 1.19159, size = 112, normalized size = 0.11

$$\operatorname{RootSum}\left(865859739762077843792979253893356556062923161 6t^{6}+1090680958717701682488386456125 44t^{4}-4926557\right.$$

$$-\frac{9x^{5}-8x^{4}+216x^{3}+2724x^{2}-324x+7884}{7383312x^{6}+132899616x^{4}+2392193088x^{3}+797397696x^{2}+1594795392}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)
```

```
[Out] RootSum(8658597397620778437929792538933565560629231616*_t**6 + 10
906809587177016824883864561254 4*_t**4 - 492655707593366915713499 1
36*_t**3 + 40378331745144603648*_t**2 - 695635011360*_t + 4513, L
ambda(_t, _t*log(1014425315618041811131612870398593498518816196 53
631712165888*_t**5/356900697070792948475845 - 1497965500823593351
127094349719750889670502100 50048*_t**4/356900697070792948475845 +
 122240975445827281850589877776867078361723699 2*_t**3/35690069707
0792948475845 - 577505552425159572302290193855826145382 4*_t**2/35
6900697070792948475845 + 9616524220026026576560393047043 2*_t/7138
0139414158589695169 + x - 170591523411296981205455 84/107070209121
2378845427535))) - (9*x**5 - 8*x**4 + 216*x**3 + 2724*x**2 - 324*
x + 7884)/(7383312*x**6 + 132899616*x**4 + 2392193088*x**3 + 7973
97696*x**2 + 1594795392)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^{2}}{(x^{6}+18\,x^{4}+324\,x^{3}+108\,x^{2}+216)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2, x)
```

3.158 $\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{c+dx}\,dx$

Optimal. Leaf size=25

$$a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}$$

```
[Out] a^2*x + (2*a*b*x^3)/3 + (b^2*x^5)/5
```

Rubi [A] time = 0.0341374, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 52, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.019

$$a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/(c + d*x
```

```
[Out] a^2*x + (2*a*b*x^3)/3 + (b^2*x^5)/5
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{2abx^3}{3}+\frac{b^2x^5}{5}+\int a^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((b**2*d*x**5+b**2*c*x**4+2*a*b*d*x**3+2*a*b*c*x**2+a**2*d*x+a**
```

```
[Out] 2*a*b*x**3/3 + b**2*x**5/5 + Integral(a**2, x)
```

Mathematica [A] time = 0.00261426, size = 25, normalized size = 1.

$$a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/(
```

```
[Out] a^2*x + (2*a*b*x^3)/3 + (b^2*x^5)/5
```

Maple [A] time = 0.001, size = 22, normalized size = 0.9

$$a^2x+\frac{2\,abx^3}{3}+\frac{b^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x+c),x)
```

```
[Out] a^2*x+2/3*a*b*x^3+1/5*b^2*x^5
```

Maxima [A] time = 0.808451, size = 28, normalized size = 1.12

$$\frac{1}{5}\,b^2x^5+\frac{2}{3}\,abx^3+a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5 + b^2*c*x^4 + 2*a*b*d*x^3 + 2*a*b*c*x^2 + a^2*d*x + a^2*c)/(
```

```
[Out] 1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

Fricas [A] time = 0.241715, size = 28, normalized size = 1.12

$$\frac{1}{5}\,b^2x^5+\frac{2}{3}\,abx^3+a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5 + b^2*c*x^4 + 2*a*b*d*x^3 + 2*a*b*c*x^2 + a^2*d*x + a^2*c)/(
```

```
[Out] 1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

Sympy [A] time = 0.140783, size = 22, normalized size = 0.88

$$a^2x+\frac{2abx^3}{3}+\frac{b^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*d*x**5+b**2*c*x**4+2*a*b*d*x**3+2*a*b*c*x**2+a**2*d*x+a**2*c)/
```

```
[Out] a**2*x + 2*a*b*x**3/3 + b**2*x**5/5
```

GIAC/XCAS [A] time = 0.260389, size = 28, normalized size = 1.12

$$\frac{1}{5}\,b^2x^5+\frac{2}{3}\,abx^3+a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5 + b^2*c*x^4 + 2*a*b*d*x^3 + 2*a*b*c*x^2 + a^2*d*x + a^2*c)/(
```

```
[Out] 1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

3.159 $\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(c+dx)^2}\,dx$

Optimal. Leaf size=94

$$\frac{\left(ad^2+bc^2\right)^2\log(c+dx)}{d^5}-\frac{bcx\left(2ad^2+bc^2\right)}{d^4}+\frac{bx^2\left(2ad^2+bc^2\right)}{2d^3}-\frac{b^2cx^3}{3d^2}+\frac{b^2x^4}{4d}$$

```
[Out] -((b*c*(b*c^2 + 2*a*d^2)*x)/d^4) + (b*(b*c^2 + 2*a*d^2)*x^2)/(2*d
^3) - (b^2*c*x^3)/(3*d^2) + (b^2*x^4)/(4*d) + ((b*c^2 + a*d^2)^2*
Log[c + d*x])/d^5
```

Rubi [A] time = 0.234127, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 52, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.058

$$\frac{\left(ad^2+bc^2\right)^2\log(c+dx)}{d^5}-\frac{bcx\left(2ad^2+bc^2\right)}{d^4}+\frac{bx^2\left(2ad^2+bc^2\right)}{2d^3}-\frac{b^2cx^3}{3d^2}+\frac{b^2x^4}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/(c + d*x
```

```
[Out] -((b*c*(b*c^2 + 2*a*d^2)*x)/d^4) + (b*(b*c^2 + 2*a*d^2)*x^2)/(2*d
^3) - (b^2*c*x^3)/(3*d^2) + (b^2*x^4)/(4*d) + ((b*c^2 + a*d^2)^2*
Log[c + d*x])/d^5
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{b^2cx^3}{3d^2}+\frac{b^2x^4}{4d}+\frac{b\left(2ad^2+bc^2\right)\int x\,dx}{d^3}-\frac{b\left(2ad^2+bc^2\right)\int c\,dx}{d^4}+\frac{\left(ad^2+bc^2\right)^2\log(c+dx)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((b**2*d*x**5+b**2*c*x**4+2*a*b*d*x**3+2*a*b*c*x**2+a**2*d*x+a**
```

```
[Out] -b**2*c*x**3/(3*d**2) + b**2*x**4/(4*d) + b*(2*a*d**2 + b*c**2)*I
ntegral(x, x)/d**3 - b*(2*a*d**2 + b*c**2)*Integral(c, x)/d**4 +
(a*d**2 + b*c**2)**2*log(c + d*x)/d**5
```

Mathematica [A] time = 0.0617426, size = 79, normalized size = 0.84

$$\frac{12\left(ad^2+bc^2\right)^2\log(c+dx)+bdx\left(12ad^2(dx-2c)+b\left(-12c^3+6c^2dx-4cd^2x^2+3d^3x^3\right)\right)}{12d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/(
```

```
[Out] (b*d*x*(12*a*d^2*(-2*c + d*x) + b*(-12*c^3 + 6*c^2*d*x - 4*c*d^2*
x^2 + 3*d^3*x^3)) + 12*(b*c^2 + a*d^2)^2*Log[c + d*x])/(12*d^5)
```

Maple [A] time = 0.005, size = 114, normalized size = 1.2

$$\frac{b^2x^4}{4\,d}-\frac{b^2cx^3}{3\,d^2}+\frac{bx^2a}{d}+\frac{b^2x^2c^2}{2\,d^3}-2\,\frac{abcx}{d^2}-\frac{b^2c^3x}{d^4}+\frac{\ln(dx+c)\,a^2}{d}+2\,\frac{\ln(dx+c)\,abc^2}{d^3}+\frac{\ln(dx+c)\,b^2c^4}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x+c)^2,x)
```

```
[Out] 1/4*b^2*x^4/d-1/3*b^2*c*x^3/d^2+b/d*x^2*a+1/2*b^2/d^3*x^2*c^2-2*b
/d^2*a*c*x-b^2/d^4*c^3*x+1/d*ln(d*x+c)*a^2+2/d^3*ln(d*x+c)*a*b*c^
2+1/d^5*ln(d*x+c)*b^2*c^4
```

Maxima [A] time = 0.819737, size = 142, normalized size = 1.51

$$\frac{3\,b^2d^3x^4 - 4\,b^2cd^2x^3 + 6\,\left(b^2c^2d + 2\,abd^3\right)x^2 - 12\,\left(b^2c^3 + 2\,abcd^2\right)x}{12\,d^4} + \frac{\left(b^2c^4 + 2\,abc^2d^2 + a^2d^4\right)\log\left(dx + c\right)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5 + b^2*c*x^4 + 2*a*b*d*x^3 + 2*a*b*c*x^2 + a^2*d*x + a^2*c)/(
```

```
[Out] 1/12*(3*b^2*d^3*x^4 - 4*b^2*c*d^2*x^3 + 6*(b^2*c^2*d + 2*a*b*d^3)
*x^2 - 12*(b^2*c^3 + 2*a*b*c*d^2)*x)/d^4 + (b^2*c^4 + 2*a*b*c^2*d
^2 + a^2*d^4)*log(d*x + c)/d^5
```

Fricas [A] time = 0.248021, size = 142, normalized size = 1.51

$$\frac{3\,b^2d^4x^4 - 4\,b^2cd^3x^3 + 6\,\left(b^2c^2d^2 + 2\,abd^4\right)x^2 - 12\,\left(b^2c^3d + 2\,abcd^3\right)x + 12\,\left(b^2c^4 + 2\,abc^2d^2 + a^2d^4\right)\log\left(dx + c\right)}{12\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5 + b^2*c*x^4 + 2*a*b*d*x^3 + 2*a*b*c*x^2 + a^2*d*x + a^2*c)/(
```

```
[Out] 1/12*(3*b^2*d^4*x^4 - 4*b^2*c*d^3*x^3 + 6*(b^2*c^2*d^2 + 2*a*b*d^
4)*x^2 - 12*(b^2*c^3*d + 2*a*b*c*d^3)*x + 12*(b^2*c^4 + 2*a*b*c^2
*d^2 + a^2*d^4)*log(d*x + c))/d^5
```

Sympy [A] time = 1.68272, size = 90, normalized size = 0.96

$$-\frac{b^2cx^3}{3d^2} + \frac{b^2x^4}{4d} + \frac{x^2\left(2abd^2 + b^2c^2\right)}{2d^3} - \frac{x\left(2abcd^2 + b^2c^3\right)}{d^4} + \frac{\left(ad^2 + bc^2\right)^2\log\left(c + dx\right)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*d*x**5+b**2*c*x**4+2*a*b*d*x**3+2*a*b*c*x**2+a**2*d*x+a**2*c)/
```

```
[Out] -b**2*c*x**3/(3*d**2) + b**2*x**4/(4*d) + x**2*(2*a*b*d**2 + b**2
*c**2)/(2*d**3) - x*(2*a*b*c*d**2 + b**2*c**3)/d**4 + (a*d**2 + b
*c**2)**2*log(c + d*x)/d**5
```

GIAC/XCAS [A] time = 0.26362, size = 493, normalized size = 5.24

$$-\frac{1}{12}\,b^{2}d\left(\frac{(dx+c)^{4}\left(\frac{20\,c}{dx+c}-\frac{60\,c^{2}}{(dx+c)^{2}}+\frac{120\,c^{3}}{(dx+c)^{3}}-3\right)}{d^{6}}+\frac{60\,c^{4}\ln\left(\frac{|dx+c|}{(dx+c)^{2}|d|}\right)}{d^{6}}-\frac{12\,c^{5}}{(dx+c)d^{6}}\right)-\frac{1}{3}\,b^{2}c\left(\frac{(dx+c)^{3}\left(\frac{6\,c}{dx+c}-\frac{18\,c^{2}}{(dx+c)^{2}}-1\right)}{d^{5}}-\frac{12\,c^{3}\ln\left(\frac{|dx+c|}{(dx+c)^{2}|d|}\right)}{d^{5}}+\frac{3\,c^{4}}{(dx+c)d^{5}}\right)-abd\left(\frac{(dx+c)^{2}\left(\frac{6\,c}{dx+c}-1\right)}{d^{4}}+\frac{6\,c^{2}\ln\left(\frac{|dx+c|}{(dx+c)^{2}|d|}\right)}{d^{4}}-\frac{2\,c^{3}}{(dx+c)d^{4}}\right)+2\,abc\left(\frac{2\,c\ln\left(\frac{|dx+c|}{(dx+c)^{2}|d|}\right)}{d^{3}}+\frac{dx+c}{d^{3}}-\frac{c^{2}}{(dx+c)d^{3}}\right)-a^{2}\left(\frac{\ln\left(\frac{|dx+c|}{(dx+c)^{2}|d|}\right)}{d}-\frac{c}{(dx+c)d}\right)-\frac{a^{2}c}{(dx+c)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5 + b^2*c*x^4 + 2*a*b*d*x^3 + 2*a*b*c*x^2 + a^2*d*x + a^2*c)/(
```

```
[Out] -1/12*b^2*d*((d*x + c)^4*(20*c/(d*x + c) - 60*c^2/(d*x + c)^2 + 1
20*c^3/(d*x + c)^3 - 3)/d^6 + 60*c^4*ln(abs(d*x + c)/((d*x + c)^2
*abs(d)))/d^6 - 12*c^5/((d*x + c)*d^6)) - 1/3*b^2*c*((d*x + c)^3*
(6*c/(d*x + c) - 18*c^2/(d*x + c)^2 - 1)/d^5 - 12*c^3*ln(abs(d*x
+ c)/((d*x + c)^2*abs(d)))/d^5 + 3*c^4/((d*x + c)*d^5)) - a*b*d*(
(d*x + c)^2*(6*c/(d*x + c) - 1)/d^4 + 6*c^2*ln(abs(d*x + c)/((d*x
 + c)^2*abs(d)))/d^4 - 2*c^3/((d*x + c)*d^4)) + 2*a*b*c*(2*c*ln(a
bs(d*x + c)/((d*x + c)^2*abs(d)))/d^3 + (d*x + c)/d^3 - c^2/((d*x
 + c)*d^3)) - a^2*(ln(abs(d*x + c)/((d*x + c)^2*abs(d)))/d - c/((
d*x + c)*d)) - a^2*c/((d*x + c)*d)
```

3.160 $\int (b+2cx)\left(bx+cx^2\right)^{13}\,dx$

Optimal. Leaf size=15

$$\frac{1}{14}\left(bx+cx^2\right)^{14}$$

```
[Out] (b*x + c*x^2)^14/14
```

Rubi [A] time = 0.0153022, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$

$$\frac{1}{14}\left(bx+cx^2\right)^{14}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x)*(b*x + c*x^2)^13,x]
```

```
[Out] (b*x + c*x^2)^14/14
```

Rubi in Sympy [A] time = 3.16395, size = 10, normalized size = 0.67

$$\frac{\left(bx+cx^2\right)^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((2*c*x+b)*(c*x**2+b*x)**13,x)
```

```
[Out] (b*x + c*x**2)**14/14
```

Mathematica [B] time = 0.00974348, size = 172, normalized size = 11.47

$$\frac{b^{14}x^{14}}{14}+b^{13}cx^{15}+\frac{13}{2}b^{12}c^2x^{16}+26b^{11}c^3x^{17}+\frac{143}{2}b^{10}c^4x^{18}+143b^9c^5x^{19}+\frac{429}{2}b^8c^6x^{20}+\frac{1716}{7}b^7c^7x^{21}+\frac{429}{2}b^6c^8x^{22}+143b^5c^9x^{23}+\frac{143}{2}b^4c^{10}x^{24}+26b^3c^{11}x^{25}+\frac{13}{2}b^2c^{12}x^{26}+bc^{13}x^{27}+\frac{c^{14}x^{28}}{14}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x)*(b*x + c*x^2)^13,x]
```

```
[Out] (b^14*x^14)/14 + b^13*c*x^15 + (13*b^12*c^2*x^16)/2 + 26*b^11*c^3
*x^17 + (143*b^10*c^4*x^18)/2 + 143*b^9*c^5*x^19 + (429*b^8*c^6*x
^20)/2 + (1716*b^7*c^7*x^21)/7 + (429*b^6*c^8*x^22)/2 + 143*b^5*c
^9*x^23 + (143*b^4*c^10*x^24)/2 + 26*b^3*c^11*x^25 + (13*b^2*c^12
*x^26)/2 + b*c^13*x^27 + (c^14*x^28)/14
```

Maple [B] time = 0., size = 155, normalized size = 10.3

$$\frac{c^{14}x^{28}}{14}+bc^{13}x^{27}+\frac{13\,b^2c^{12}x^{26}}{2}+26\,b^3c^{11}x^{25}+\frac{143\,b^4c^{10}x^{24}}{2}+143\,b^5c^9x^{23}+\frac{429\,b^6c^8x^{22}}{2}+\frac{1716\,b^7c^7x^{21}}{7}+\frac{429\,b^8c^6x^{20}}{2}+143\,b^9c^5x^{19}+\frac{143\,b^{10}c^4x^{18}}{2}+26\,b^{11}c^3x^{17}+\frac{13\,b^{12}c^2x^{16}}{2}+b^{13}cx^{15}+\frac{b^{14}x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*c*x+b)*(c*x^2+b*x)^13,x)
```

```
[Out] 1/14*c^14*x^28+b*c^13*x^27+13/2*b^2*c^12*x^26+26*b^3*c^11*x^25+14
3/2*b^4*c^10*x^24+143*b^5*c^9*x^23+429/2*b^6*c^8*x^22+1716/7*b^7*
c^7*x^21+429/2*b^8*c^6*x^20+143*b^9*c^5*x^19+143/2*b^10*c^4*x^18+
26*b^11*c^3*x^17+13/2*b^12*c^2*x^16+b^13*c*x^15+1/14*b^14*x^14
```

Maxima [A] time = 0.807024, size = 18, normalized size = 1.2

$$\frac{1}{14}\left(cx^2+bx\right)^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^13*(2*c*x + b),x, algorithm="maxima")
```

```
[Out] 1/14*(c*x^2 + b*x)^14
```

Fricas [A] time = 0.231559, size = 1, normalized size = 0.07

$$\frac{1}{14}x^{28}c^{14}+x^{27}c^{13}b+\frac{13}{2}x^{26}c^{12}b^2+26x^{25}c^{11}b^3+\frac{143}{2}x^{24}c^{10}b^4+143x^{23}c^9b^5+\frac{429}{2}x^{22}c^8b^6+\frac{1716}{7}x^{21}c^7b^7+\frac{429}{2}x^{20}c^6b^8+143x^{19}c^5b^9+\frac{143}{2}x^{18}c^4b^{10}+26x^{17}c^3b^{11}+\frac{13}{2}x^{16}c^2b^{12}+x^{15}cb^{13}+\frac{1}{14}x^{14}b^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^13*(2*c*x + b),x, algorithm="fricas")
```

```
[Out] 1/14*x^28*c^14 + x^27*c^13*b + 13/2*x^26*c^12*b^2 + 26*x^25*c^11*
b^3 + 143/2*x^24*c^10*b^4 + 143*x^23*c^9*b^5 + 429/2*x^22*c^8*b^6
 + 1716/7*x^21*c^7*b^7 + 429/2*x^20*c^6*b^8 + 143*x^19*c^5*b^9 +
143/2*x^18*c^4*b^10 + 26*x^17*c^3*b^11 + 13/2*x^16*c^2*b^12 + x^1
5*c*b^13 + 1/14*x^14*b^14
```

Sympy [A] time = 0.272978, size = 175, normalized size = 11.67

$$\frac{b^{14}x^{14}}{14}+b^{13}cx^{15}+\frac{13b^{12}c^2x^{16}}{2}+26b^{11}c^3x^{17}+\frac{143b^{10}c^4x^{18}}{2}+143b^9c^5x^{19}+\frac{429b^8c^6x^{20}}{2}+\frac{1716b^7c^7x^{21}}{7}+\frac{429b^6c^8x^{22}}{2}+143b^5c^9x^{23}+\frac{143b^4c^{10}x^{24}}{2}+26b^3c^{11}x^{25}+\frac{13b^2c^{12}x^{26}}{2}+bc^{13}x^{27}+\frac{c^{14}x^{28}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x+b)*(c*x**2+b*x)**13,x)
```

```
[Out] b**14*x**14/14 + b**13*c*x**15 + 13*b**12*c**2*x**16/2 + 26*b**11
*c**3*x**17 + 143*b**10*c**4*x**18/2 + 143*b**9*c**5*x**19 + 429*
b**8*c**6*x**20/2 + 1716*b**7*c**7*x**21/7 + 429*b**6*c**8*x**22/
2 + 143*b**5*c**9*x**23 + 143*b**4*c**10*x**24/2 + 26*b**3*c**11*
x**25 + 13*b**2*c**12*x**26/2 + b*c**13*x**27 + c**14*x**28/14
```

GIAC/XCAS [A] time = 0.261044, size = 208, normalized size = 13.87

$$\frac{1}{14}\,c^{14}x^{28}+bc^{13}x^{27}+\frac{13}{2}\,b^{2}c^{12}x^{26}+26\,b^{3}c^{11}x^{25}+\frac{143}{2}\,b^{4}c^{10}x^{24}+143\,b^{5}c^{9}x^{23}+\frac{429}{2}\,b^{6}c^{8}x^{22}+\frac{1716}{7}\,b^{7}c^{7}x^{21}+\frac{429}{2}\,b^{8}c^{6}x^{20}+143\,b^{9}c^{5}x^{19}+\frac{143}{2}\,b^{10}c^{4}x^{18}+26\,b^{11}c^{3}x^{17}+\frac{13}{2}\,b^{12}c^{2}x^{16}+b^{13}cx^{15}+\frac{1}{14}\,b^{14}x^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^13*(2*c*x + b),x, algorithm="giac")
```

```
[Out] 1/14*c^14*x^28 + b*c^13*x^27 + 13/2*b^2*c^12*x^26 + 26*b^3*c^11*x
^25 + 143/2*b^4*c^10*x^24 + 143*b^5*c^9*x^23 + 429/2*b^6*c^8*x^22
 + 1716/7*b^7*c^7*x^21 + 429/2*b^8*c^6*x^20 + 143*b^9*c^5*x^19 +
143/2*b^10*c^4*x^18 + 26*b^11*c^3*x^17 + 13/2*b^12*c^2*x^16 + b^1
3*c*x^15 + 1/14*b^14*x^14
```

3.161 $\int x^{14} \left(b+2 c x^2\right) \left(b x+c x^3\right)^{13} \, dx$

Optimal. Leaf size=18

$$\frac{1}{28} x^{14} \left(b x+c x^3\right)^{14}$$

```
[Out] (x^14*(b*x + c*x^3)^14)/28
```

Rubi [A] time = 0.0240358, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043

$$\frac{1}{28} x^{14} \left(b x+c x^3\right)^{14}$$

Antiderivative was successfully verified.

```
[In] Int[x^14*(b + 2*c*x^2)*(b*x + c*x^3)^13,x]
```

```
[Out] (x^14*(b*x + c*x^3)^14)/28
```

Rubi in Sympy [A] time = 12.8721, size = 12, normalized size = 0.67

$$\frac{x^{28} \left(b+c x^2\right)^{14}}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**14*(2*c*x**2+b)*(c*x**3+b*x)**13,x)
```

```
[Out] x**28*(b + c*x**2)**14/28
```

Mathematica [B] time = 0.00945518, size = 182, normalized size = 10.11

$$\frac{b^{14} x^{28}}{28}+\frac{1}{2} b^{13} c x^{30}+\frac{13}{4} b^{12} c^2 x^{32}+13 b^{11} c^3 x^{34}+\frac{143}{4} b^{10} c^4 x^{36}+\frac{143}{2} b^9 c^5 x^{38}+\frac{429}{4} b^8 c^6 x^{40}+\frac{858}{7} b^7 c^7 x^{42}+\frac{429}{4} b^6 c^8 x^{44}+\frac{143}{2} b^5 c^9 x^{46}+\frac{143}{4} b^4 c^{10} x^{48}+13 b^3 c^{11} x^{50}+\frac{13}{4} b^2 c^{12} x^{52}+\frac{1}{2} b c^{13} x^{54}+\frac{c^{14} x^{56}}{28}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14*(b + 2*c*x^2)*(b*x + c*x^3)^13,x]
```

```
[Out] (b^14*x^28)/28 + (b^13*c*x^30)/2 + (13*b^12*c^2*x^32)/4 + 13*b^11
*c^3*x^34 + (143*b^10*c^4*x^36)/4 + (143*b^9*c^5*x^38)/2 + (429*b
^8*c^6*x^40)/4 + (858*b^7*c^7*x^42)/7 + (429*b^6*c^8*x^44)/4 + (1
43*b^5*c^9*x^46)/2 + (143*b^4*c^10*x^48)/4 + 13*b^3*c^11*x^50 + (
13*b^2*c^12*x^52)/4 + (b*c^13*x^54)/2 + (c^14*x^56)/28
```

Maple [B] time = 0.003, size = 157, normalized size = 8.7

$$\frac{c^{14}x^{56}}{28}+\frac{bc^{13}x^{54}}{2}+\frac{13\,b^2c^{12}x^{52}}{4}+13\,b^3c^{11}x^{50}+\frac{143\,b^4c^{10}x^{48}}{4}+\frac{143\,b^5c^9x^{46}}{2}+\frac{429\,b^6c^8x^{44}}{4}+\frac{858\,b^7c^7x^{42}}{7}+\frac{429\,b^8c^6x^{40}}{4}+\frac{143\,b^9c^5x^{38}}{2}+\frac{143\,b^{10}c^4x^{36}}{4}+13\,b^{11}c^3x^{34}+\frac{13\,b^{12}c^2x^{32}}{4}+\frac{b^{13}cx^{30}}{2}+\frac{b^{14}x^{28}}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^14*(2*c*x^2+b)*(c*x^3+b*x)^13,x)
```

```
[Out] 1/28*c^14*x^56+1/2*b*c^13*x^54+13/4*b^2*c^12*x^52+13*b^3*c^11*x^5
0+143/4*b^4*c^10*x^48+143/2*b^5*c^9*x^46+429/4*b^6*c^8*x^44+858/7
*b^7*c^7*x^42+429/4*b^8*c^6*x^40+143/2*b^9*c^5*x^38+143/4*b^10*c^
4*x^36+13*b^11*c^3*x^34+13/4*b^12*c^2*x^32+1/2*b^13*c*x^30+1/28*b
^14*x^28
```

Maxima [A] time = 0.804003, size = 211, normalized size = 11.72

$$\frac{1}{28}\,c^{14}x^{56}+\frac{1}{2}\,bc^{13}x^{54}+\frac{13}{4}\,b^{2}c^{12}x^{52}+13\,b^{3}c^{11}x^{50}+\frac{143}{4}\,b^{4}c^{10}x^{48}+\frac{143}{2}\,b^{5}c^{9}x^{46}+\frac{429}{4}\,b^{6}c^{8}x^{44}+\frac{858}{7}\,b^{7}c^{7}x^{42}+\frac{429}{4}\,b^{8}c^{6}x^{40}+\frac{143}{2}\,b^{9}c^{5}x^{38}+\frac{143}{4}\,b^{10}c^{4}x^{36}+13\,b^{11}c^{3}x^{34}+\frac{13}{4}\,b^{12}c^{2}x^{32}+\frac{1}{2}\,b^{13}cx^{30}+\frac{1}{28}\,b^{14}x^{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^3 + b*x)^13*(2*c*x^2 + b)*x^14,x, algorithm="maxima")
```

```
[Out] 1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 13/4*b^2*c^12*x^52 + 13*b^3*c^
11*x^50 + 143/4*b^4*c^10*x^48 + 143/2*b^5*c^9*x^46 + 429/4*b^6*c^
8*x^44 + 858/7*b^7*c^7*x^42 + 429/4*b^8*c^6*x^40 + 143/2*b^9*c^5*
x^38 + 143/4*b^10*c^4*x^36 + 13*b^11*c^3*x^34 + 13/4*b^12*c^2*x^3
2 + 1/2*b^13*c*x^30 + 1/28*b^14*x^28
```

Fricas [A] time = 0.236299, size = 1, normalized size = 0.06

$$\frac{1}{28}x^{56}c^{14}+\frac{1}{2}x^{54}c^{13}b+\frac{13}{4}x^{52}c^{12}b^{2}+13x^{50}c^{11}b^{3}+\frac{143}{4}x^{48}c^{10}b^{4}+\frac{143}{2}x^{46}c^{9}b^{5}+\frac{429}{4}x^{44}c^{8}b^{6}+\frac{858}{7}x^{42}c^{7}b^{7}+\frac{429}{4}x^{40}c^{6}b^{8}+\frac{143}{2}x^{38}c^{5}b^{9}+\frac{143}{4}x^{36}c^{4}b^{10}+13x^{34}c^{3}b^{11}+\frac{13}{4}x^{32}c^{2}b^{12}+\frac{1}{2}x^{30}cb^{13}+\frac{1}{28}x^{28}b^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^3 + b*x)^13*(2*c*x^2 + b)*x^14,x, algorithm="fricas")
```

```
[Out] 1/28*x^56*c^14 + 1/2*x^54*c^13*b + 13/4*x^52*c^12*b^2 + 13*x^50*c
^11*b^3 + 143/4*x^48*c^10*b^4 + 143/2*x^46*c^9*b^5 + 429/4*x^44*c
^8*b^6 + 858/7*x^42*c^7*b^7 + 429/4*x^40*c^6*b^8 + 143/2*x^38*c^5
*b^9 + 143/4*x^36*c^4*b^10 + 13*x^34*c^3*b^11 + 13/4*x^32*c^2*b^1
2 + 1/2*x^30*c*b^13 + 1/28*x^28*b^14
```

Sympy [A] time = 0.274313, size = 182, normalized size = 10.11

$$\frac{b^{14}x^{28}}{28}+\frac{b^{13}cx^{30}}{2}+\frac{13b^{12}c^{2}x^{32}}{4}+13b^{11}c^{3}x^{34}+\frac{143b^{10}c^{4}x^{36}}{4}+\frac{143b^{9}c^{5}x^{38}}{2}+\frac{429b^{8}c^{6}x^{40}}{4}+\frac{858b^{7}c^{7}x^{42}}{7}+\frac{429b^{6}c^{8}x^{44}}{4}+\frac{143b^{5}c^{9}x^{46}}{2}+\frac{143b^{4}c^{10}x^{48}}{4}+13b^{3}c^{11}x^{50}+\frac{13b^{2}c^{12}x^{52}}{4}+\frac{bc^{13}x^{54}}{2}+\frac{c^{14}x^{56}}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**14*(2*c*x**2+b)*(c*x**3+b*x)**13,x)
```

```
[Out] b**14*x**28/28 + b**13*c*x**30/2 + 13*b**12*c**2*x**32/4 + 13*b**
11*c**3*x**34 + 143*b**10*c**4*x**36/4 + 143*b**9*c**5*x**38/2 +
429*b**8*c**6*x**40/4 + 858*b**7*c**7*x**42/7 + 429*b**6*c**8*x**
44/4 + 143*b**5*c**9*x**46/2 + 143*b**4*c**10*x**48/4 + 13*b**3*c
**11*x**50 + 13*b**2*c**12*x**52/4 + b*c**13*x**54/2 + c**14*x**5
6/28
```

GIAC/XCAS [A] time = 0.259954, size = 211, normalized size = 11.72

$$\frac{1}{28}c^{14}x^{56} + \frac{1}{2}bc^{13}x^{54} + \frac{13}{4}b^2c^{12}x^{52} + 13\,b^3c^{11}x^{50} + \frac{143}{4}b^4c^{10}x^{48} + \frac{143}{2}b^5c^9x^{46} + \frac{429}{4}b^6c^8x^{44} + \frac{858}{7}b^7c^7x^{42} + \frac{429}{4}b^8c^6x^{40} + \frac{143}{2}b^9c^5x^{38} + \frac{143}{4}b^{10}c^4x^{36} + 13\,b^{11}c^3x^{34} + \frac{13}{4}b^{12}c^2x^{32} + \frac{1}{2}b^{13}cx^{30} + \frac{1}{28}b^{14}x^{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^3 + b*x)^13*(2*c*x^2 + b)*x^14,x, algorithm="giac")
```

```
[Out] 1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 13/4*b^2*c^12*x^52 + 13*b^3*c^
11*x^50 + 143/4*b^4*c^10*x^48 + 143/2*b^5*c^9*x^46 + 429/4*b^6*c^
8*x^44 + 858/7*b^7*c^7*x^42 + 429/4*b^8*c^6*x^40 + 143/2*b^9*c^5*
x^38 + 143/4*b^10*c^4*x^36 + 13*b^11*c^3*x^34 + 13/4*b^12*c^2*x^3
2 + 1/2*b^13*c*x^30 + 1/28*b^14*x^28
```

3.162 $\int x^{28} \left(b+2cx^3\right) \left(bx+cx^4\right)^{13} \, dx$

Optimal. Leaf size=18

$$\frac{1}{42}x^{28}\left(bx+cx^4\right)^{14}$$

```
[Out] (x^28*(b*x + c*x^4)^14)/42
```

Rubi [A] time = 0.0247324, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$

$$\frac{1}{42}x^{28}\left(bx+cx^4\right)^{14}$$

Antiderivative was successfully verified.

```
[In]  Int[x^28*(b + 2*c*x^3)*(b*x + c*x^4)^13,x]
```

```
[Out] (x^28*(b*x + c*x^4)^14)/42
```

Rubi in Sympy [A] time = 12.3393, size = 12, normalized size = 0.67

$$\frac{x^{42}\left(b+cx^3\right)^{14}}{42}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**28*(2*c*x**3+b)*(c*x**4+b*x)**13,x)
```

```
[Out] x**42*(b + c*x**3)**14/42
```

Mathematica [B] time = 0.0100759, size = 186, normalized size = 10.33

$$\frac{b^{14}x^{42}}{42}+\frac{1}{3}b^{13}cx^{45}+\frac{13}{6}b^{12}c^2x^{48}+\frac{26}{3}b^{11}c^3x^{51}+\frac{143}{6}b^{10}c^4x^{54}+\frac{143}{3}b^9c^5x^{57}+\frac{143}{2}b^8c^6x^{60}+\frac{572}{7}b^7c^7x^{63}+\frac{143}{2}b^6c^8x^{66}+\frac{143}{3}b^5c^9x^{69}+\frac{143}{6}b^4c^{10}x^{72}+\frac{26}{3}b^3c^{11}x^{75}+\frac{13}{6}b^2c^{12}x^{78}+\frac{1}{3}bc^{13}x^{81}+\frac{c^{14}x^{84}}{42}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^28*(b + 2*c*x^3)*(b*x + c*x^4)^13,x]
```

```
[Out] (b^14*x^42)/42 + (b^13*c*x^45)/3 + (13*b^12*c^2*x^48)/6 + (26*b^1
1*c^3*x^51)/3 + (143*b^10*c^4*x^54)/6 + (143*b^9*c^5*x^57)/3 + (1
43*b^8*c^6*x^60)/2 + (572*b^7*c^7*x^63)/7 + (143*b^6*c^8*x^66)/2
+ (143*b^5*c^9*x^69)/3 + (143*b^4*c^10*x^72)/6 + (26*b^3*c^11*x^7
5)/3 + (13*b^2*c^12*x^78)/6 + (b*c^13*x^81)/3 + (c^14*x^84)/42
```

Maple [B] time = 0.003, size = 157, normalized size = 8.7

$$\frac{c^{14}x^{84}}{42}+\frac{bc^{13}x^{81}}{3}+\frac{13\,b^2c^{12}x^{78}}{6}+\frac{26\,b^3c^{11}x^{75}}{3}+\frac{143\,b^4c^{10}x^{72}}{6}+\frac{143\,b^5c^9x^{69}}{3}+\frac{143\,b^6c^8x^{66}}{2}+\frac{572\,b^7c^7x^{63}}{7}+\frac{143\,b^8c^6x^{60}}{2}+\frac{143\,b^9c^5x^{57}}{3}+\frac{143\,b^{10}c^4x^{54}}{6}+\frac{26\,b^{11}c^3x^{51}}{3}+\frac{13\,b^{12}c^2x^{48}}{6}+\frac{b^{13}cx^{45}}{3}+\frac{b^{14}x^{42}}{42}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^28*(2*c*x^3+b)*(c*x^4+b*x)^13,x)
```

```
[Out] 1/42*c^14*x^84+1/3*b*c^13*x^81+13/6*b^2*c^12*x^78+26/3*b^3*c^11*x
^75+143/6*b^4*c^10*x^72+143/3*b^5*c^9*x^69+143/2*b^6*c^8*x^66+572
/7*b^7*c^7*x^63+143/2*b^8*c^6*x^60+143/3*b^9*c^5*x^57+143/6*b^10*
c^4*x^54+26/3*b^11*c^3*x^51+13/6*b^12*c^2*x^48+1/3*b^13*c*x^45+1/
42*b^14*x^42
```

Maxima [A] time = 0.813703, size = 211, normalized size = 11.72

$$\frac{1}{42}\,c^{14}x^{84}+\frac{1}{3}\,bc^{13}x^{81}+\frac{13}{6}\,b^{2}c^{12}x^{78}+\frac{26}{3}\,b^{3}c^{11}x^{75}+\frac{143}{6}\,b^{4}c^{10}x^{72}+\frac{143}{3}\,b^{5}c^{9}x^{69}+\frac{143}{2}\,b^{6}c^{8}x^{66}+\frac{572}{7}\,b^{7}c^{7}x^{63}+\frac{143}{2}\,b^{8}c^{6}x^{60}+\frac{143}{3}\,b^{9}c^{5}x^{57}+\frac{143}{6}\,b^{10}c^{4}x^{54}+\frac{26}{3}\,b^{11}c^{3}x^{51}+\frac{13}{6}\,b^{12}c^{2}x^{48}+\frac{1}{3}\,b^{13}cx^{45}+\frac{1}{42}\,b^{14}x^{42}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x)^13*(2*c*x^3 + b)*x^28,x, algorithm="maxima")
```

```
[Out] 1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 13/6*b^2*c^12*x^78 + 26/3*b^3*
c^11*x^75 + 143/6*b^4*c^10*x^72 + 143/3*b^5*c^9*x^69 + 143/2*b^6*
c^8*x^66 + 572/7*b^7*c^7*x^63 + 143/2*b^8*c^6*x^60 + 143/3*b^9*c^
5*x^57 + 143/6*b^10*c^4*x^54 + 26/3*b^11*c^3*x^51 + 13/6*b^12*c^2
*x^48 + 1/3*b^13*c*x^45 + 1/42*b^14*x^42
```

Fricas [A] time = 0.229106, size = 1, normalized size = 0.06

$$\frac{1}{42}x^{84}c^{14}+\frac{1}{3}x^{81}c^{13}b+\frac{13}{6}x^{78}c^{12}b^{2}+\frac{26}{3}x^{75}c^{11}b^{3}+\frac{143}{6}x^{72}c^{10}b^{4}+\frac{143}{3}x^{69}c^{9}b^{5}+\frac{143}{2}x^{66}c^{8}b^{6}+\frac{572}{7}x^{63}c^{7}b^{7}+\frac{143}{2}x^{60}c^{6}b^{8}+\frac{143}{3}x^{57}c^{5}b^{9}+\frac{143}{6}x^{54}c^{4}b^{10}+\frac{26}{3}x^{51}c^{3}b^{11}+\frac{13}{6}x^{48}c^{2}b^{12}+\frac{1}{3}x^{45}cb^{13}+\frac{1}{42}x^{42}b^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x)^13*(2*c*x^3 + b)*x^28,x, algorithm="fricas")
```

```
[Out] 1/42*x^84*c^14 + 1/3*x^81*c^13*b + 13/6*x^78*c^12*b^2 + 26/3*x^75
*c^11*b^3 + 143/6*x^72*c^10*b^4 + 143/3*x^69*c^9*b^5 + 143/2*x^66
*c^8*b^6 + 572/7*x^63*c^7*b^7 + 143/2*x^60*c^6*b^8 + 143/3*x^57*c
^5*b^9 + 143/6*x^54*c^4*b^10 + 26/3*x^51*c^3*b^11 + 13/6*x^48*c^2
*b^12 + 1/3*x^45*c*b^13 + 1/42*x^42*b^14
```

Sympy [A] time = 0.287735, size = 185, normalized size = 10.28

$$\frac{b^{14}x^{42}}{42}+\frac{b^{13}cx^{45}}{3}+\frac{13b^{12}c^{2}x^{48}}{6}+\frac{26b^{11}c^{3}x^{51}}{3}+\frac{143b^{10}c^{4}x^{54}}{6}+\frac{143b^{9}c^{5}x^{57}}{3}+\frac{143b^{8}c^{6}x^{60}}{2}+\frac{572b^{7}c^{7}x^{63}}{7}+\frac{143b^{6}c^{8}x^{66}}{2}+\frac{143b^{5}c^{9}x^{69}}{3}+\frac{143b^{4}c^{10}x^{72}}{6}+\frac{26b^{3}c^{11}x^{75}}{3}+\frac{13b^{2}c^{12}x^{78}}{6}+\frac{bc^{13}x^{81}}{3}+\frac{c^{14}x^{84}}{42}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x**28*(2*c*x**3+b)*(c*x**4+b*x)**13,x)
```

```
[Out] b**14*x**42/42 + b**13*c*x**45/3 + 13*b**12*c**2*x**48/6 + 26*b**
11*c**3*x**51/3 + 143*b**10*c**4*x**54/6 + 143*b**9*c**5*x**57/3
+ 143*b**8*c**6*x**60/2 + 572*b**7*c**7*x**63/7 + 143*b**6*c**8*x
**66/2 + 143*b**5*c**9*x**69/3 + 143*b**4*c**10*x**72/6 + 26*b**3
*c**11*x**75/3 + 13*b**2*c**12*x**78/6 + b*c**13*x**81/3 + c**14*
x**84/42
```

GIAC/XCAS [A] time = 0.261564, size = 211, normalized size = 11.72

$$\frac{1}{42}c^{14}x^{84} + \frac{1}{3}bc^{13}x^{81} + \frac{13}{6}b^2c^{12}x^{78} + \frac{26}{3}b^3c^{11}x^{75} + \frac{143}{6}b^4c^{10}x^{72} + \frac{143}{3}b^5c^9x^{69} + \frac{143}{2}b^6c^8x^{66} + \frac{572}{7}b^7c^7x^{63} + \frac{143}{2}b^8c^6x^{60} + \frac{143}{3}b^9c^5x^{57} + \frac{143}{6}b^{10}c^4x^{54} + \frac{26}{3}b^{11}c^3x^{51} + \frac{13}{6}b^{12}c^2x^{48} + \frac{1}{3}b^{13}cx^{45} + \frac{1}{42}b^{14}x^{42}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x)^13*(2*c*x^3 + b)*x^28,x, algorithm="giac")
```

```
[Out] 1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 13/6*b^2*c^12*x^78 + 26/3*b^3*
c^11*x^75 + 143/6*b^4*c^10*x^72 + 143/3*b^5*c^9*x^69 + 143/2*b^6*
c^8*x^66 + 572/7*b^7*c^7*x^63 + 143/2*b^8*c^6*x^60 + 143/3*b^9*c^
5*x^57 + 143/6*b^10*c^4*x^54 + 26/3*b^11*c^3*x^51 + 13/6*b^12*c^2
*x^48 + 1/3*b^13*c*x^45 + 1/42*b^14*x^42
```

3.163 $\int x^{14(-1+n)}(b+2cx^n)\left(bx+cx^{1+n}\right)^{13}\,dx$

Optimal. Leaf size=21

$$\frac{x^{14n}(b+cx^n)^{14}}{14n}$$

```
[Out] (x^(14*n)*(b + c*x^n)^14)/(14*n)
```

Rubi [A] time = 0.0699924, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$

$$\frac{x^{14n}(b+cx^n)^{14}}{14n}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(14*(-1 + n))*(b + 2*c*x^n)*(b*x + c*x^(1 + n))^13,x]
```

```
[Out] (x^(14*n)*(b + c*x^n)^14)/(14*n)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(-14+14*n)*(b+2*c*x**n)*(b*x+c*x**(1+n))**13,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.059345, size = 21, normalized size = 1.

$$\frac{x^{14n}(b+cx^n)^{14}}{14n}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(14*(-1 + n))*(b + 2*c*x^n)*(b*x + c*x^(1 + n))^13,x]
```

```
[Out] (x^(14*n)*(b + c*x^n)^14)/(14*n)
```

Maple [B] time = 0.062, size = 230, normalized size = 11.

$$\frac{c^{14}(x^n)^{28}}{14\,n}+\frac{bc^{13}(x^n)^{27}}{n}+\frac{13\,c^{12}(x^n)^{26}b^2}{2\,n}+26\,\frac{b^3c^{11}(x^n)^{25}}{n}+\frac{143\,c^{10}(x^n)^{24}b^4}{2\,n}+143\,\frac{b^5c^9(x^n)^{23}}{n}+\frac{429\,c^8(x^n)^{22}b^6}{2\,n}+\frac{1716\,b^7c^7(x^n)^{21}}{7\,n}+\frac{429\,c^6(x^n)^{20}b^8}{2\,n}+143\,\frac{b^9c^5(x^n)^{19}}{n}+\frac{143\,c^4(x^n)^{18}b^{10}}{2\,n}+26\,\frac{b^{11}c^3(x^n)^{17}}{n}+\frac{13\,c^2(x^n)^{16}b^{12}}{2\,n}+\frac{b^{13}c(x^n)^{15}}{n}+\frac{(x^n)^{14}b^{14}}{14\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(-14+14*n)*(b+2*c*x^n)*(b*x+c*x^(1+n))^13,x)
```

```
[Out] 1/14*c^14/n*(x^n)^28+b*c^13/n*(x^n)^27+13/2*c^12/n*(x^n)^26*b^2+2
6*c^11*b^3/n*(x^n)^25+143/2*c^10/n*(x^n)^24*b^4+143*b^5*c^9/n*(x^
n)^23+429/2*c^8/n*(x^n)^22*b^6+1716/7*b^7*c^7/n*(x^n)^21+429/2*c^
6/n*(x^n)^20*b^8+143*c^5*b^9/n*(x^n)^19+143/2*c^4/n*(x^n)^18*b^10
+26*b^11*c^3/n*(x^n)^17+13/2*c^2/n*(x^n)^16*b^12+b^13*c/n*(x^n)^1
5+1/14/n*(x^n)^14*b^14
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x + c*x^(n + 1))^13*(2*c*x^n + b)*x^(14*n - 14),x, algorithm="maxima
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.286106, size = 354, normalized size = 16.86

$$\frac{b^{14}x^{14}x^{14\,n+14} + 14\,b^{13}cx^{13}x^{15\,n+15} + 91\,b^{12}c^2x^{12}x^{16\,n+16} + 364\,b^{11}c^3x^{11}x^{17\,n+17} + 1001\,b^{10}c^4x^{10}x^{18\,n+18} + 2002\,b^9c^5x^9x^{19\,n+19}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x + c*x^(n + 1))^13*(2*c*x^n + b)*x^(14*n - 14),x, algorithm="fricas
```

```
[Out] 1/14*(b^14*x^14*x^(14*n + 14) + 14*b^13*c*x^13*x^(15*n + 15) + 91
*b^12*c^2*x^12*x^(16*n + 16) + 364*b^11*c^3*x^11*x^(17*n + 17) +
1001*b^10*c^4*x^10*x^(18*n + 18) + 2002*b^9*c^5*x^9*x^(19*n + 19)
 + 3003*b^8*c^6*x^8*x^(20*n + 20) + 3432*b^7*c^7*x^7*x^(21*n + 21
) + 3003*b^6*c^8*x^6*x^(22*n + 22) + 2002*b^5*c^9*x^5*x^(23*n + 2
3) + 1001*b^4*c^10*x^4*x^(24*n + 24) + 364*b^3*c^11*x^3*x^(25*n +
 25) + 91*b^2*c^12*x^2*x^(26*n + 26) + 14*b*c^13*x*x^(27*n + 27)
+ c^14*x^(28*n + 28))/(n*x^28)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(-14+14*n)*(b+2*c*x**n)*(b*x+c*x**(1+n))**13,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.389785, size = 275, normalized size = 13.1

$$\frac{c^{14}e^{(28\,n\ln(x))} + 14\,bc^{13}e^{(27\,n\ln(x))} + 91\,b^2c^{12}e^{(26\,n\ln(x))} + 364\,b^3c^{11}e^{(25\,n\ln(x))} + 1001\,b^4c^{10}e^{(24\,n\ln(x))} + 2002\,b^5c^9e^{(23\,n\ln(x))} + 3003}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x + c*x^(n + 1))^13*(2*c*x^n + b)*x^(14*n - 14),x, algorithm="giac")
```

```
[Out] 1/14*(c^14*e^(28*n*ln(x)) + 14*b*c^13*e^(27*n*ln(x)) + 91*b^2*c^1
2*e^(26*n*ln(x)) + 364*b^3*c^11*e^(25*n*ln(x)) + 1001*b^4*c^10*e^
(24*n*ln(x)) + 2002*b^5*c^9*e^(23*n*ln(x)) + 3003*b^6*c^8*e^(22*n
*ln(x)) + 3432*b^7*c^7*e^(21*n*ln(x)) + 3003*b^8*c^6*e^(20*n*ln(x
)) + 2002*b^9*c^5*e^(19*n*ln(x)) + 1001*b^10*c^4*e^(18*n*ln(x)) +
 364*b^11*c^3*e^(17*n*ln(x)) + 91*b^12*c^2*e^(16*n*ln(x)) + 14*b^
13*c*e^(15*n*ln(x)) + b^14*e^(14*n*ln(x)))/n
```

3.164 $\int \frac{b+2cx}{bx+cx^2}\,dx$

Optimal. Leaf size=10

$$\log\left(bx+cx^2\right)$$

```
[Out] Log[b*x + c*x^2]
```

Rubi [A] time = 0.00952077, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$

$$\log\left(bx+cx^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x)/(b*x + c*x^2),x]

[Out] Log[b*x + c*x^2]
```

Rubi in Sympy [A] time = 3.22386, size = 8, normalized size = 0.8

$$\log\left(bx+cx^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((2*c*x+b)/(c*x**2+b*x),x)

[Out] log(b*x + c*x**2)
```

Mathematica [A] time = 0.00619487, size = 9, normalized size = 0.9

$$\log(b+cx)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x)/(b*x + c*x^2),x]

[Out] Log[x] + Log[b + c*x]
```

Maple [A] time = 0.001, size = 9, normalized size = 0.9

$$\ln\left(x\left(cx+b\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)/(c*x^2+b*x),x)

[Out] ln(x*(c*x+b))
```

Maxima [A] time = 0.804943, size = 14, normalized size = 1.4

$$\log \left(cx^2 + bx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x + b)/(c*x^2 + b*x),x, algorithm="maxima")

[Out] log(c*x^2 + b*x)
```

Fricas [A] time = 0.250039, size = 14, normalized size = 1.4

$$\log \left(cx^2 + bx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x + b)/(c*x^2 + b*x),x, algorithm="fricas")

[Out] log(c*x^2 + b*x)
```

Sympy [A] time = 1.0744, size = 8, normalized size = 0.8

$$\log \left(bx + cx^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x+b)/(c*x**2+b*x),x)

[Out] log(b*x + c*x**2)
```

GIAC/XCAS [A] time = 0.260005, size = 15, normalized size = 1.5

$$\ln \left(|cx + b|\right) + \ln \left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x + b)/(c*x^2 + b*x),x, algorithm="giac")

[Out] ln(abs(c*x + b)) + ln(abs(x))
```

3.165 $\int \frac{b+2cx^2}{bx+cx^3}\,dx$

Optimal. Leaf size=15

$$\frac{1}{2}\log\left(b+cx^2\right)+\log(x)$$

```
[Out] Log[x] + Log[b + c*x^2]/2
```

Rubi [A] time = 0.0652369, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15

$$\frac{1}{2}\log\left(b+cx^2\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(b + 2*c*x^2)/(b*x + c*x^3),x]

[Out] Log[x] + Log[b + c*x^2]/2
```

Rubi in Sympy [A] time = 11.1592, size = 15, normalized size = 1.

$$\frac{\log\left(x^2\right)}{2}+\frac{\log\left(b+cx^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*c*x**2+b)/(c*x**3+b*x),x)

[Out] log(x**2)/2 + log(b + c*x**2)/2
```

Mathematica [A] time = 0.00881329, size = 15, normalized size = 1.

$$\frac{1}{2}\log\left(b+cx^2\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b + 2*c*x^2)/(b*x + c*x^3),x]

[Out] Log[x] + Log[b + c*x^2]/2
```

Maple [A] time = 0.007, size = 14, normalized size = 0.9

$$\frac{\ln\left(cx^2+b\right)}{2}+\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*c*x^2+b)/(c*x^3+b*x),x)
```

```
[Out] 1/2*ln(c*x^2+b)+ln(x)
```

Maxima [A] time = 0.790944, size = 18, normalized size = 1.2

$$\frac{1}{2}\log\left(cx^2+b\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2 + b)/(c*x^3 + b*x),x, algorithm="maxima")
```

```
[Out] 1/2*log(c*x^2 + b) + log(x)
```

Fricas [A] time = 0.250094, size = 18, normalized size = 1.2

$$\frac{1}{2}\log\left(cx^2+b\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2 + b)/(c*x^3 + b*x),x, algorithm="fricas")
```

```
[Out] 1/2*log(c*x^2 + b) + log(x)
```

Sympy [A] time = 1.21628, size = 12, normalized size = 0.8

$$\log\left(x\right)+\frac{\log\left(\frac{b}{c}+x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x**2+b)/(c*x**3+b*x),x)
```

```
[Out] log(x) + log(b/c + x**2)/2
```

GIAC/XCAS [A] time = 0.26215, size = 24, normalized size = 1.6

$$\frac{1}{2}\ln\left(x^2\right)+\frac{1}{2}\ln\left(\left|cx^2+b\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2 + b)/(c*x^3 + b*x),x, algorithm="giac")
```

```
[Out] 1/2*ln(x^2) + 1/2*ln(abs(c*x^2 + b))
```

3.166 $\int \frac{b+2cx^3}{bx+cx^4} \, dx$

Optimal. Leaf size=15

$$\frac{1}{3}\log\left(b+cx^3\right)+\log(x)$$

```
[Out] Log[x] + Log[b + c*x^3]/3
```

Rubi [A] time = 0.0630632, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$

$$\frac{1}{3}\log\left(b+cx^3\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(b + 2*c*x^3)/(b*x + c*x^4),x]

[Out] Log[x] + Log[b + c*x^3]/3
```

Rubi in Sympy [A] time = 10.2843, size = 15, normalized size = 1.

$$\frac{\log\left(x^3\right)}{3}+\frac{\log\left(b+cx^3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*c*x**3+b)/(c*x**4+b*x),x)

[Out] log(x**3)/3 + log(b + c*x**3)/3
```

Mathematica [A] time = 0.0113956, size = 15, normalized size = 1.

$$\frac{1}{3}\log\left(b+cx^3\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b + 2*c*x^3)/(b*x + c*x^4),x]

[Out] Log[x] + Log[b + c*x^3]/3
```

Maple [A] time = 0.006, size = 14, normalized size = 0.9

$$\frac{\ln\left(cx^3+b\right)}{3}+\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*c*x^3+b)/(c*x^4+b*x),x)
```

```
[Out] 1/3*ln(c*x^3+b)+ln(x)
```

Maxima [A] time = 0.809738, size = 18, normalized size = 1.2

$$\frac{1}{3}\log\left(cx^3+b\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x^3 + b)/(c*x^4 + b*x),x, algorithm="maxima")
```

```
[Out] 1/3*log(c*x^3 + b) + log(x)
```

Fricas [A] time = 0.246781, size = 18, normalized size = 1.2

$$\frac{1}{3}\log\left(cx^3+b\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x^3 + b)/(c*x^4 + b*x),x, algorithm="fricas")
```

```
[Out] 1/3*log(c*x^3 + b) + log(x)
```

Sympy [A] time = 1.27795, size = 12, normalized size = 0.8

$$\log\left(x\right)+\frac{\log\left(\frac{b}{c}+x^3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x**3+b)/(c*x**4+b*x),x)
```

```
[Out] log(x) + log(b/c + x**3)/3
```

GIAC/XCAS [A] time = 0.261345, size = 20, normalized size = 1.33

$$\frac{1}{3}\ln\left(\left|cx^3+b\right|\right)+\ln\left(\left|x\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x^3 + b)/(c*x^4 + b*x),x, algorithm="giac")
```

```
[Out] 1/3*ln(abs(c*x^3 + b)) + ln(abs(x))
```

3.167 $\int \frac{b+2cx^n}{bx+cx^{1+n}}\,dx$

Optimal. Leaf size=15

$$\frac{\log(b+cx^n)}{n}+\log(x)$$

```
[Out] Log[x] + Log[b + c*x^n]/n
```

Rubi [A] time = 0.0623564, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$\frac{\log(b+cx^n)}{n}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x^n)/(b*x + c*x^(1 + n)),x]

[Out] Log[x] + Log[b + c*x^n]/n
```

Rubi in Sympy [A] time = 11.0648, size = 15, normalized size = 1.

$$\frac{\log(x^n)}{n}+\frac{\log(b+cx^n)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((b+2*c*x**n)/(b*x+c*x**(1+n)),x)

[Out] log(x**n)/n + log(b + c*x**n)/n
```

Mathematica [A] time = 0.0167562, size = 15, normalized size = 1.

$$\frac{\log(b+cx^n)}{n}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x^n)/(b*x + c*x^(1 + n)),x]

[Out] Log[x] + Log[b + c*x^n]/n
```

Maple [A] time = 0.022, size = 18, normalized size = 1.2

$$\ln(x)+\frac{\ln\left(ce^{n\ln(x)}+b\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b+2*c*x^n)/(b*x+c*x^(1+n)),x)
```

```
[Out] ln(x)+1/n*ln(c*exp(n*ln(x))+b)
```

Maxima [A] time = 0.817206, size = 63, normalized size = 4.2

$$b\left(\frac{\log(x)}{b}-\frac{\log\left(\frac{cx^n+b}{c}\right)}{bn}\right)+\frac{2\log\left(\frac{cx^n+b}{c}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x^n + b)/(b*x + c*x^(n + 1)),x, algorithm="maxima")
```

```
[Out] b*(log(x)/b - log((c*x^n + b)/c)/(b*n)) + 2*log((c*x^n + b)/c)/n
```

Fricas [A] time = 0.267056, size = 31, normalized size = 2.07

$$\frac{(n-1)\log(x)+\log\left(bx+cx^{n+1}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x^n + b)/(b*x + c*x^(n + 1)),x, algorithm="fricas")
```

```
[Out] ((n - 1)*log(x) + log(b*x + c*x^(n + 1)))/n
```

Sympy [A] time = 4.51914, size = 29, normalized size = 1.93

$$\begin{cases} \log(x) & \text{for } c=0 \wedge (c=0 \vee n=0) \\ \frac{(b+2c)\log(x)}{b+c} & \text{for } n=0 \\ \log(x)+\frac{\log\left(\frac{b}{c}+x^n\right)}{n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b+2*c*x**n)/(b*x+c*x**(1+n)),x)
```

```
[Out] Piecewise((log(x), Eq(c, 0) & (Eq(c, 0) | Eq(n, 0))), ((b + 2*c)*
log(x)/(b + c), Eq(n, 0)), (log(x) + log(b/c + x**n)/n, True))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2\,cx^n+b}{bx+cx^{n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x^n + b)/(b*x + c*x^(n + 1)),x, algorithm="giac")
```

```
[Out] integrate((2*c*x^n + b)/(b*x + c*x^(n + 1)), x)
```

3.168 $\int \frac{b+2cx}{(bx+cx^2)^8}\,dx$

Optimal. Leaf size=15

$$-\frac{1}{7\left(bx+cx^2\right)^7}$$

```
[Out] -1/(7*(b*x + c*x^2)^7)
```

Rubi [A] time = 0.0103982, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056

$$-\frac{1}{7\left(bx+cx^2\right)^7}$$

Antiderivative was successfully verified.

```
[In]  Int[(b + 2*c*x)/(b*x + c*x^2)^8,x]

[Out] -1/(7*(b*x + c*x^2)^7)
```

Rubi in Sympy [A] time = 3.15308, size = 14, normalized size = 0.93

$$-\frac{1}{7\left(bx+cx^2\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*c*x+b)/(c*x**2+b*x)**8,x)

[Out] -1/(7*(b*x + c*x**2)**7)
```

Mathematica [A] time = 0.033746, size = 14, normalized size = 0.93

$$-\frac{1}{7x^7(b+cx)^7}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b + 2*c*x)/(b*x + c*x^2)^8,x]

[Out] -1/(7*x^7*(b + c*x)^7)
```

Maple [B] time = 0., size = 177, normalized size = 11.8

$$-\frac{1}{7\,b^7x^7}-132\,\frac{c^6}{b^{13}x}+66\,\frac{c^5}{b^{12}x^2}-30\,\frac{c^4}{b^{11}x^3}+12\,\frac{c^3}{b^{10}x^4}-4\,\frac{c^2}{b^9x^5}+\frac{c}{b^8x^6}+132\,\frac{c^7}{b^{13}\,(cx+b)}+66\,\frac{c^7}{b^{12}\,(cx+b)^2}+30\,\frac{c^7}{b^{11}\,(cx+b)^3}+12\,\frac{c^7}{b^{10}\,(cx+b)^4}+4\,\frac{c^7}{b^9\,(cx+b)^5}+\frac{c^7}{b^8\,(cx+b)^6}+\frac{c^7}{7\,b^7\,(cx+b)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*c*x+b)/(c*x^2+b*x)^8,x)
```

```
[Out] -1/7/b^7/x^7-132/b^13*c^6/x+66/b^12*c^5/x^2-30/b^11*c^4/x^3+12/b^
10*c^3/x^4-4/b^9*c^2/x^5+1/b^8*c/x^6+132*c^7/b^13/(c*x+b)+66*c^7/
b^12/(c*x+b)^2+30/b^11*c^7/(c*x+b)^3+12*c^7/b^10/(c*x+b)^4+4/b^9*
c^7/(c*x+b)^5+c^7/b^8/(c*x+b)^6+1/7*c^7/b^7/(c*x+b)^7
```

Maxima [A] time = 0.808308, size = 18, normalized size = 1.2

$$-\frac{1}{7\,(cx^2+bx)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x + b)/(c*x^2 + b*x)^8,x, algorithm="maxima")
```

```
[Out] -1/7/(c*x^2 + b*x)^7
```

Fricas [A] time = 0.26313, size = 109, normalized size = 7.27

$$-\frac{1}{7\,(c^7x^{14}+7\,bc^6x^{13}+21\,b^2c^5x^{12}+35\,b^3c^4x^{11}+35\,b^4c^3x^{10}+21\,b^5c^2x^9+7\,b^6cx^8+b^7x^7)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x + b)/(c*x^2 + b*x)^8,x, algorithm="fricas")
```

```
[Out] -1/7/(c^7*x^14 + 7*b*c^6*x^13 + 21*b^2*c^5*x^12 + 35*b^3*c^4*x^11
 + 35*b^4*c^3*x^10 + 21*b^5*c^2*x^9 + 7*b^6*c*x^8 + b^7*x^7)
```

Sympy [A] time = 17.6781, size = 87, normalized size = 5.8

$$-\frac{1}{7b^7x^7+49b^6cx^8+147b^5c^2x^9+245b^4c^3x^{10}+245b^3c^4x^{11}+147b^2c^5x^{12}+49bc^6x^{13}+7c^7x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x+b)/(c*x**2+b*x)**8,x)
```

```
[Out] -1/(7*b**7*x**7 + 49*b**6*c*x**8 + 147*b**5*c**2*x**9 + 245*b**4*
c**3*x**10 + 245*b**3*c**4*x**11 + 147*b**2*c**5*x**12 + 49*b*c**
6*x**13 + 7*c**7*x**14)
```

GIAC/XCAS [A] time = 0.263093, size = 18, normalized size = 1.2

$$-\frac{1}{7\,(cx^2+bx)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x + b)/(c*x^2 + b*x)^8,x, algorithm="giac")
```

```
[Out] -1/7/(c*x^2 + b*x)^7
```

3.169 $\int \frac{b+2cx^2}{x^7(bx+cx^3)^8} \, dx$

Optimal. Leaf size=18

$$-\frac{1}{14x^7 (bx + cx^3)^7}$$

```
[Out] -1/(14*x^7*(b*x + c*x^3)^7)
```

Rubi [A] time = 0.0145864, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$

$$-\frac{1}{14x^7 (bx + cx^3)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x^2)/(x^7*(b*x + c*x^3)^8),x]
```

```
[Out] -1/(14*x^7*(b*x + c*x^3)^7)
```

Rubi in Sympy [A] time = 12.5887, size = 15, normalized size = 0.83

$$-\frac{1}{14x^{14} (b + cx^2)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((2*c*x**2+b)/x**7/(c*x**3+b*x)**8,x)
```

```
[Out] -1/(14*x**14*(b + c*x**2)**7)
```

Mathematica [A] time = 0.0482352, size = 16, normalized size = 0.89

$$-\frac{1}{14x^{14} (b + cx^2)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x^2)/(x^7*(b*x + c*x^3)^8),x]
```

```
[Out] -1/(14*x^14*(b + c*x^2)^7)
```

Maple [B] time = 0.021, size = 197, normalized size = 10.9

$$-\frac{c^8}{2\,b^{13}}\left(-\frac{b^6}{7\,c\,(cx^2+b)^7} - 66\,\frac{b}{c\,(cx^2+b)^2} - \frac{b^5}{c\,(cx^2+b)^6} - 30\,\frac{b^2}{c\,(cx^2+b)^3} - 12\,\frac{b^3}{c\,(cx^2+b)^4} - 4\,\frac{b^4}{c\,(cx^2+b)^5} - 132\,\frac{1}{(cx^2+b)\,c}\right.$$
$$-\frac{1}{14\,b^7x^{14}} - 66\,\frac{c^6}{b^{13}x^2} + 33\,\frac{c^5}{b^{12}x^4} - 15\,\frac{c^4}{b^{11}x^6} + 6\,\frac{c^3}{b^{10}x^8} - 2\,\frac{c^2}{b^9x^{10}} + \frac{c}{2\,b^8x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*c*x^2+b)/x^7/(c*x^3+b*x)^8,x)
```

```
[Out] -1/2*c^8/b^13*(-1/7*b^6/c/(c*x^2+b)^7-66*b/c/(c*x^2+b)^2-b^5/c/(c
*x^2+b)^6-30*b^2/c/(c*x^2+b)^3-12*b^3/c/(c*x^2+b)^4-4*b^4/c/(c*x^
2+b)^5-132/(c*x^2+b)/c)-1/14/b^7/x^14-66/b^13*c^6/x^2+33/b^12*c^5
/x^4-15/b^11*c^4/x^6+6/b^10*c^3/x^8-2/b^9*c^2/x^10+1/2/b^8*c/x^12
```

Maxima [A] time = 0.832899, size = 109, normalized size = 6.06

$$-\frac{1}{14\,(c^7x^{28} + 7\,bc^6x^{26} + 21\,b^2c^5x^{24} + 35\,b^3c^4x^{22} + 35\,b^4c^3x^{20} + 21\,b^5c^2x^{18} + 7\,b^6cx^{16} + b^7x^{14})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x^2 + b)/((c*x^3 + b*x)^8*x^7),x, algorithm="maxima")
```

```
[Out] -1/14/(c^7*x^28 + 7*b*c^6*x^26 + 21*b^2*c^5*x^24 + 35*b^3*c^4*x^2
2 + 35*b^4*c^3*x^20 + 21*b^5*c^2*x^18 + 7*b^6*c*x^16 + b^7*x^14)
```

Fricas [A] time = 0.258191, size = 109, normalized size = 6.06

$$-\frac{1}{14\,(c^7x^{28} + 7\,bc^6x^{26} + 21\,b^2c^5x^{24} + 35\,b^3c^4x^{22} + 35\,b^4c^3x^{20} + 21\,b^5c^2x^{18} + 7\,b^6cx^{16} + b^7x^{14})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x^2 + b)/((c*x^3 + b*x)^8*x^7),x, algorithm="fricas")
```

```
[Out] -1/14/(c^7*x^28 + 7*b*c^6*x^26 + 21*b^2*c^5*x^24 + 35*b^3*c^4*x^2
2 + 35*b^4*c^3*x^20 + 21*b^5*c^2*x^18 + 7*b^6*c*x^16 + b^7*x^14)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x**2+b)/x**7/(c*x**3+b*x)**8,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.264415, size = 20, normalized size = 1.11

$$-\frac{1}{14\,(cx^4 + bx^2)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x^2 + b)/((c*x^3 + b*x)^8*x^7),x, algorithm="giac")
```

```
[Out] -1/14/(c*x^4 + b*x^2)^7
```

3.170 $\int \frac{b+2cx^3}{x^{14}(bx+cx^4)^8}\,dx$

Optimal. Leaf size=18

$$-\frac{1}{21x^{14}\left(bx+cx^4\right)^7}$$

```
[Out] -1/(21*x^14*(b*x + c*x^4)^7)
```

Rubi [A] time = 0.0144082, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043

$$-\frac{1}{21x^{14}\left(bx+cx^4\right)^7}$$

Antiderivative was successfully verified.

```
[In]  Int[(b + 2*c*x^3)/(x^14*(b*x + c*x^4)^8),x]

[Out] -1/(21*x^14*(b*x + c*x^4)^7)
```

Rubi in Sympy [A] time = 12.8017, size = 15, normalized size = 0.83

$$-\frac{1}{21x^{21}\left(b+cx^3\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*c*x**3+b)/x**14/(c*x**4+b*x)**8,x)

[Out] -1/(21*x**21*(b + c*x**3)**7)
```

Mathematica [A] time = 0.0598535, size = 16, normalized size = 0.89

$$-\frac{1}{21x^{21}\left(b+cx^3\right)^7}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b + 2*c*x^3)/(x^14*(b*x + c*x^4)^8),x]

[Out] -1/(21*x^21*(b + c*x^3)^7)
```

Maple [B] time = 0.019, size = 197, normalized size = 10.9

$$-\frac{c^8}{3\,b^{13}}\left(-\frac{b^6}{7\,c\left(cx^3+b\right)^7}-66\,\frac{b}{c\left(cx^3+b\right)^2}-\frac{b^5}{c\left(cx^3+b\right)^6}-30\,\frac{b^2}{c\left(cx^3+b\right)^3}-12\,\frac{b^3}{c\left(cx^3+b\right)^4}-4\,\frac{b^4}{c\left(cx^3+b\right)^5}-132\,\frac{1}{c\left(cx^3+b\right)}\right.$$
$$-\frac{1}{21\,b^7x^{21}}-44\,\frac{c^6}{b^{13}x^3}+22\,\frac{c^5}{b^{12}x^6}-10\,\frac{c^4}{b^{11}x^9}+4\,\frac{c^3}{b^{10}x^{12}}-\frac{4\,c^2}{3\,b^9x^{15}}+\frac{c}{3\,b^8x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*c*x^3+b)/x^14/(c*x^4+b*x)^8,x)
```

```
[Out] -1/3*c^8/b^13*(-1/7*b^6/c/(c*x^3+b)^7-66*b/c/(c*x^3+b)^2-b^5/c/(c
*x^3+b)^6-30*b^2/c/(c*x^3+b)^3-12*b^3/c/(c*x^3+b)^4-4*b^4/c/(c*x^
3+b)^5-132/c/(c*x^3+b))-1/21/b^7/x^21-44/b^13*c^6/x^3+22/b^12*c^5
/x^6-10/b^11*c^4/x^9+4/b^10*c^3/x^12-4/3/b^9*c^2/x^15+1/3/b^8*c/x
^18
```

Maxima [A] time = 0.813995, size = 109, normalized size = 6.06

$$-\frac{1}{21\left(c^{7}x^{42}+7\,bc^{6}x^{39}+21\,b^{2}c^{5}x^{36}+35\,b^{3}c^{4}x^{33}+35\,b^{4}c^{3}x^{30}+21\,b^{5}c^{2}x^{27}+7\,b^{6}cx^{24}+b^{7}x^{21}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x^3 + b)/((c*x^4 + b*x)^8*x^14),x, algorithm="maxima")
```

```
[Out] -1/21/(c^7*x^42 + 7*b*c^6*x^39 + 21*b^2*c^5*x^36 + 35*b^3*c^4*x^3
3 + 35*b^4*c^3*x^30 + 21*b^5*c^2*x^27 + 7*b^6*c*x^24 + b^7*x^21)
```

Fricas [A] time = 0.277717, size = 109, normalized size = 6.06

$$-\frac{1}{21\left(c^{7}x^{42}+7\,bc^{6}x^{39}+21\,b^{2}c^{5}x^{36}+35\,b^{3}c^{4}x^{33}+35\,b^{4}c^{3}x^{30}+21\,b^{5}c^{2}x^{27}+7\,b^{6}cx^{24}+b^{7}x^{21}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x^3 + b)/((c*x^4 + b*x)^8*x^14),x, algorithm="fricas")
```

```
[Out] -1/21/(c^7*x^42 + 7*b*c^6*x^39 + 21*b^2*c^5*x^36 + 35*b^3*c^4*x^3
3 + 35*b^4*c^3*x^30 + 21*b^5*c^2*x^27 + 7*b^6*c*x^24 + b^7*x^21)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x**3+b)/x**14/(c*x**4+b*x)**8,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.266883, size = 20, normalized size = 1.11

$$-\frac{1}{21\left(cx^{6}+bx^{3}\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x^3 + b)/((c*x^4 + b*x)^8*x^14),x, algorithm="giac")
```

```
[Out] -1/21/(c*x^6 + b*x^3)^7
```

3.171 $\int \frac{x^{-7(-1+n)}(b+2cx^n)}{(bx+cx^{1+n})^8}\,dx$

Optimal. Leaf size=21

$$-\frac{x^{-7n}}{7n\left(b+cx^n\right)^7}$$

```
[Out] -1/(7*n*x^(7*n)*(b + c*x^n)^7)
```

Rubi [A] time = 0.069093, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$

$$-\frac{x^{-7n}}{7n\left(b+cx^n\right)^7}$$

Antiderivative was successfully verified.

```
[In]  Int[(b + 2*c*x^n)/(x^(7*(-1 + n))*(b*x + c*x^(1 + n))^8),x]
```

```
[Out] -1/(7*n*x^(7*n)*(b + c*x^n)^7)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b+2*c*x**n)/(x**(-7+7*n))/(b*x+c*x**(1+n))**8,x)
```

```
[Out] Timed out
```

Mathematica [B] time = 0.0802652, size = 127, normalized size = 6.05

$$-\frac{x^{-7n}\left(b^{14}+1716b^7c^7x^{7n}+12012b^6c^8x^{8n}+36036b^5c^9x^{9n}+60060b^4c^{10}x^{10n}+60060b^3c^{11}x^{11n}+36036b^2c^{12}x^{12n}+12012bc\right.}{7b^{14}n\left(b+cx^n\right)^7}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b + 2*c*x^n)/(x^(7*(-1 + n))*(b*x + c*x^(1 + n))^8),x]
```

```
[Out] -(b^14 + 1716*b^7*c^7*x^(7*n) + 12012*b^6*c^8*x^(8*n) + 36036*b^5
*c^9*x^(9*n) + 60060*b^4*c^10*x^(10*n) + 60060*b^3*c^11*x^(11*n)
+ 36036*b^2*c^12*x^(12*n) + 12012*b*c^13*x^(13*n) + 1716*c^14*x^(
14*n))/(7*b^14*n*x^(7*n)*(b + c*x^n)^7)
```

Maple [B] time = 0.09, size = 203, normalized size = 9.7

$$-132\,\frac{c^6}{b^{13}nx^n}+66\,\frac{c^5}{b^{12}n\left(x^n\right)^2}-30\,\frac{c^4}{b^{11}n\left(x^n\right)^3}+12\,\frac{c^3}{b^{10}n\left(x^n\right)^4}-4\,\frac{c^2}{b^9n\left(x^n\right)^5}+\frac{c}{b^8n\left(x^n\right)^6}-\frac{1}{7\,b^7n\left(x^n\right)^7}$$
$$+\frac{c^7\left(924\,\left(x^n\right)^6c^6+6006\,bc^5\left(x^n\right)^5+16380\,b^2c^4\left(x^n\right)^4+24024\,b^3c^3\left(x^n\right)^3+20020\,b^4c^2\left(x^n\right)^2+9009\,b^5cx^n+1716\,b^6\right)}{7\,b^{13}n\left(b+cx^n\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b+2*c*x^n)/(x^(-7+7*n))/(b*x+c*x^(1+n))^8,x)
```

```
[Out] -132/b^13*c^6/n/(x^n)+66/b^12*c^5/n/(x^n)^2-30/b^11*c^4/n/(x^n)^3
+12/b^10*c^3/n/(x^n)^4-4/b^9*c^2/n/(x^n)^5+1/b^8*c/n/(x^n)^6-1/7/
b^7/n/(x^n)^7+1/7*c^7*(924*(x^n)^6*c^6+6006*b*c^5*(x^n)^5+16380*b
^2*c^4*(x^n)^4+24024*b^3*c^3*(x^n)^3+20020*b^4*c^2*(x^n)^2+9009*b
^5*c*x^n+1716*b^6)/b^13/n/(b+c*x^n)^7
```

Maxima [A] time = 0.858851, size = 826, normalized size = 39.33

$$-\frac{1}{105}b\left(\frac{360360\,c^{13}x^{13\,n}+2342340\,bc^{12}x^{12\,n}+6426420\,b^{2}c^{11}x^{11\,n}+9579570\,b^{3}c^{10}x^{10\,n}+8270262\,b^{4}c^{9}x^{9\,n}+4018014\,b^{5}c^{8}x^{8}}{b^{14}c^{7}nx^{14\,n}+7\,b^{15}c^{6}nx^{13\,n}+21\,b^{16}c^{5}nx^{12\,n}+35\,b^{17}c^{4}n}\right.$$

$$+\frac{1}{105}c\left(\frac{360360\,c^{12}x^{12\,n}+2342340\,bc^{11}x^{11\,n}+6426420\,b^{2}c^{10}x^{10\,n}+9579570\,b^{3}c^{9}x^{9\,n}+8270262\,b^{4}c^{8}x^{8\,n}+4018014\,b^{5}c^{7}x^{7\,n}}{b^{13}c^{7}nx^{13\,n}+7\,b^{14}c^{6}nx^{12\,n}+21\,b^{15}c^{5}nx^{11\,n}+35\,b^{16}c^{4}nx^{10\,n}+3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^n + b)/((b*x + c*x^(n + 1))^8*x^(7*n - 7)),x, algorithm="maxima"
```

```
[Out] -1/105*b*((360360*c^13*x^(13*n) + 2342340*b*c^12*x^(12*n) + 64264
20*b^2*c^11*x^(11*n) + 9579570*b^3*c^10*x^(10*n) + 8270262*b^4*c^
9*x^(9*n) + 4018014*b^5*c^8*x^(8*n) + 934362*b^6*c^7*x^(7*n) + 45
045*b^7*c^6*x^(6*n) - 5005*b^8*c^5*x^(5*n) + 1001*b^9*c^4*x^(4*n)
 - 273*b^10*c^3*x^(3*n) + 91*b^11*c^2*x^(2*n) - 35*b^12*c*x^n + 1
5*b^13)/(b^14*c^7*n*x^(14*n) + 7*b^15*c^6*n*x^(13*n) + 21*b^16*c^
5*n*x^(12*n) + 35*b^17*c^4*n*x^(11*n) + 35*b^18*c^3*n*x^(10*n) +
21*b^19*c^2*n*x^(9*n) + 7*b^20*c*n*x^(8*n) + b^21*n*x^(7*n)) + 36
0360*c^7*log(x)/b^15 - 360360*c^7*log((c*x^n + b)/c)/(b^15*n)) +
1/105*c*((360360*c^12*x^(12*n) + 2342340*b*c^11*x^(11*n) + 642642
0*b^2*c^10*x^(10*n) + 9579570*b^3*c^9*x^(9*n) + 8270262*b^4*c^8*x
^(8*n) + 4018014*b^5*c^7*x^(7*n) + 934362*b^6*c^6*x^(6*n) + 45045
*b^7*c^5*x^(5*n) - 5005*b^8*c^4*x^(4*n) + 1001*b^9*c^3*x^(3*n) -
273*b^10*c^2*x^(2*n) + 91*b^11*c*x^n - 35*b^12)/(b^13*c^7*n*x^(13
*n) + 7*b^14*c^6*n*x^(12*n) + 21*b^15*c^5*n*x^(11*n) + 35*b^16*c^
4*n*x^(10*n) + 35*b^17*c^3*n*x^(9*n) + 21*b^18*c^2*n*x^(8*n) + 7*
b^19*c*n*x^(7*n) + b^20*n*x^(6*n)) + 360360*c^6*log(x)/b^14 - 360
360*c^6*log((c*x^n + b)/c)/(b^14*n))
```

Fricas [A] time = 0.461512, size = 193, normalized size = 9.19

$$-\frac{x^{14}}{7\,(b^{7}nx^{7}x^{7\,n+7}+7\,b^{6}cnx^{6}x^{8\,n+8}+21\,b^{5}c^{2}nx^{5}x^{9\,n+9}+35\,b^{4}c^{3}nx^{4}x^{10\,n+10}+35\,b^{3}c^{4}nx^{3}x^{11\,n+11}+21\,b^{2}c^{5}nx^{2}x^{12\,n+12}+7\,bc^{6}nx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^n + b)/((b*x + c*x^(n + 1))^8*x^(7*n - 7)),x, algorithm="fricas"
```

```
[Out] -1/7*x^14/(b^7*n*x^7*x^(7*n + 7) + 7*b^6*c*n*x^6*x^(8*n + 8) + 21
*b^5*c^2*n*x^5*x^(9*n + 9) + 35*b^4*c^3*n*x^4*x^(10*n + 10) + 35*
b^3*c^4*n*x^3*x^(11*n + 11) + 21*b^2*c^5*n*x^2*x^(12*n + 12) + 7*
b*c^6*n*x*x^(13*n + 13) + c^7*n*x^(14*n + 14))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b+2*c*x**n)/(x**(-7+7*n))/(b*x+c*x**(1+n))**8,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2\,cx^n+b}{\left(bx+cx^{n+1}\right)^8 x^{7\,n-7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^n + b)/((b*x + c*x^(n + 1))^8*x^(7*n - 7)),x, algorithm="giac")
```

```
[Out] integrate((2*c*x^n + b)/((b*x + c*x^(n + 1))^8*x^(7*n - 7)), x)
```

3.172 $\int (b+2cx)\left(bx+cx^2\right)^p\,dx$

Optimal. Leaf size=19

$$\frac{\left(bx+cx^2\right)^{p+1}}{p+1}$$

```
[Out] (b*x + c*x^2)^(1 + p)/(1 + p)
```

Rubi [A] time = 0.015045, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056

$$\frac{\left(bx+cx^2\right)^{p+1}}{p+1}$$

Antiderivative was successfully verified.

```
[In]  Int[(b + 2*c*x)*(b*x + c*x^2)^p,x]

[Out] (b*x + c*x^2)^(1 + p)/(1 + p)
```

Rubi in Sympy [A] time = 3.504, size = 14, normalized size = 0.74

$$\frac{\left(bx+cx^2\right)^{p+1}}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*c*x+b)*(c*x**2+b*x)**p,x)

[Out] (b*x + c*x**2)**(p + 1)/(p + 1)
```

Mathematica [A] time = 0.0331781, size = 17, normalized size = 0.89

$$\frac{(x(b+cx))^{p+1}}{p+1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b + 2*c*x)*(b*x + c*x^2)^p,x]

[Out] (x*(b + c*x))^(1 + p)/(1 + p)
```

Maple [A] time = 0., size = 24, normalized size = 1.3

$$\frac{x\,(cx+b)\left(cx^2+bx\right)^p}{1+p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*c*x+b)*(c*x^2+b*x)^p,x)
```

```
[Out] x*(c*x+b)/(1+p)*(c*x^2+b*x)^p
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x + b)*(c*x^2 + b*x)^p,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.266805, size = 35, normalized size = 1.84

$$\frac{\left(cx^2 + bx\right)\left(cx^2 + bx\right)^p}{p + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x + b)*(c*x^2 + b*x)^p,x, algorithm="fricas")
```

```
[Out] (c*x^2 + b*x)*(c*x^2 + b*x)^p/(p + 1)
```

Sympy [A] time = 1.67537, size = 46, normalized size = 2.42

$$\begin{cases} \frac{bx(bx+cx^2)^p}{p+1} + \frac{cx^2(bx+cx^2)^p}{p+1} & \text{for } p \neq -1 \\ \log(x) + \log\left(\frac{b}{c} + x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x+b)*(c*x**2+b*x)**p,x)
```

```
[Out] Piecewise((b*x*(b*x + c*x**2)**p/(p + 1) + c*x**2*(b*x + c*x**2)*
*p/(p + 1), Ne(p, -1)), (log(x) + log(b/c + x), True))
```

GIAC/XCAS [A] time = 0.265626, size = 55, normalized size = 2.89

$$\frac{cx^2 e^{\left(p \ln\left(cx^2 + bx\right)\right)} + bx e^{\left(p \ln\left(cx^2 + bx\right)\right)}}{p + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x + b)*(c*x^2 + b*x)^p,x, algorithm="giac")
```

```
[Out] (c*x^2*e^(p*ln(c*x^2 + b*x)) + b*x*e^(p*ln(c*x^2 + b*x)))/(p + 1)
```

3.173 $\int x^{1+p}\left(b+2cx^2\right)\left(bx+cx^3\right)^p\,dx$

Optimal. Leaf size=27

$$\frac{x^{p+1}\left(bx+cx^3\right)^{p+1}}{2(p+1)}$$

```
[Out] (x^(1 + p)*(b*x + c*x^3)^(1 + p))/(2*(1 + p))
```

Rubi [A] time = 0.0186476, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.04$

$$\frac{x^{p+1}\left(bx+cx^3\right)^{p+1}}{2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1 + p)*(b + 2*c*x^2)*(b*x + c*x^3)^p,x]

[Out] (x^(1 + p)*(b*x + c*x^3)^(1 + p))/(2*(1 + p))
```

Rubi in Sympy [A] time = 11.5, size = 20, normalized size = 0.74

$$\frac{x^{p+1}\left(bx+cx^3\right)^{p+1}}{2\,(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**(1+p)*(2*c*x**2+b)*(c*x**3+b*x)**p,x)

[Out] x**(p + 1)*(b*x + c*x**3)**(p + 1)/(2*(p + 1))
```

Mathematica [A] time = 0.0508187, size = 27, normalized size = 1.

$$\frac{x^{p+1}\left(x\left(b+cx^2\right)\right)^{p+1}}{2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1 + p)*(b + 2*c*x^2)*(b*x + c*x^3)^p,x]

[Out] (x^(1 + p)*(x*(b + c*x^2))^(1 + p))/(2*(1 + p))
```

Maple [A] time = 0.006, size = 31, normalized size = 1.2

$$\frac{x^{2+p}\left(cx^2+b\right)\left(cx^3+bx\right)^p}{2+2p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1+p)*(2*c*x^2+b)*(c*x^3+b*x)^p,x)
```

```
[Out] 1/2*x^(2+p)*(c*x^2+b)/(1+p)*(c*x^3+b*x)^p
```

Maxima [A] time = 0.929879, size = 47, normalized size = 1.74

$$\frac{\left(cx^4+bx^2\right)e^{\left(p\log\left(cx^2+b\right)+2\,p\log\left(x\right)\right)}}{2\,(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2 + b)*(c*x^3 + b*x)^p*x^(p + 1),x, algorithm="maxima")
```

```
[Out] 1/2*(c*x^4 + b*x^2)*e^(p*log(c*x^2 + b) + 2*p*log(x))/(p + 1)
```

Fricas [A] time = 0.275371, size = 43, normalized size = 1.59

$$\frac{\left(cx^3+bx\right)\left(cx^3+bx\right)^p x^{p+1}}{2\,(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2 + b)*(c*x^3 + b*x)^p*x^(p + 1),x, algorithm="fricas")
```

```
[Out] 1/2*(c*x^3 + b*x)*(c*x^3 + b*x)^p*x^(p + 1)/(p + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(1+p)*(2*c*x**2+b)*(c*x**3+b*x)**p,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.268733, size = 73, normalized size = 2.7

$$\frac{cx^3e^{\left(p\ln\left(cx^2+b\right)+2\,p\ln\left(x\right)+\ln\left(x\right)\right)}+bxe^{\left(p\ln\left(cx^2+b\right)+2\,p\ln\left(x\right)+\ln\left(x\right)\right)}}{2\,(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2 + b)*(c*x^3 + b*x)^p*x^(p + 1),x, algorithm="giac")
```

```
[Out] 1/2*(c*x^3*e^(p*ln(c*x^2 + b) + 2*p*ln(x) + ln(x)) + b*x*e^(p*ln(
c*x^2 + b) + 2*p*ln(x) + ln(x)))/(p + 1)
```

3.174 $\int \left(bx^{1+p}\left(bx+cx^3\right)^p+2cx^{3+p}\left(bx+cx^3\right)^p\right)\,dx$

Optimal. Leaf size=27

$$\frac{x^{p+1}\left(bx+cx^3\right)^{p+1}}{2(p+1)}$$

```
[Out] (x^(1 + p)*(b*x + c*x^3)^(1 + p))/(2*(1 + p))
```

Rubi [C] time = 0.221907, antiderivative size = 116, normalized size of antiderivative = 4.3, number of steps used = 7, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079

$$\frac{bx^{p+2}\left(bx+cx^3\right)^p\left(\frac{cx^2}{b}+1\right)^{-p}{}_2F_1\left(-p,p+1;p+2;-\frac{cx^2}{b}\right)}{2(p+1)}+\frac{cx^{p+4}\left(bx+cx^3\right)^p\left(\frac{cx^2}{b}+1\right)^{-p}{}_2F_1\left(-p,p+2;p+3;-\frac{cx^2}{b}\right)}{p+2}$$

Antiderivative was successfully verified.

```
[In]  Int[b*x^(1 + p)*(b*x + c*x^3)^p + 2*c*x^(3 + p)*(b*x + c*x^3)^p,x]
```

```
[Out] (b*x^(2 + p)*(b*x + c*x^3)^p*Hypergeometric2F1[-p, 1 + p, 2 + p,
-((c*x^2)/b)])/(2*(1 + p)*(1 + (c*x^2)/b)^p) + (c*x^(4 + p)*(b*x
+ c*x^3)^p*Hypergeometric2F1[-p, 2 + p, 3 + p, -((c*x^2)/b)])/((2
 + p)*(1 + (c*x^2)/b)^p)
```

Rubi in Sympy [A] time = 32.2655, size = 102, normalized size = 3.78

$$\frac{bx^{-p}x^{2p+2}\left(1+\frac{cx^2}{b}\right)^{-p}\left(bx+cx^3\right)^p{}_2F_1\left(\begin{matrix}-p,p+1\\p+2\end{matrix}\middle|-\frac{cx^2}{b}\right)}{2\left(p+1\right)}+\frac{cx^{-p}x^{2p+4}\left(1+\frac{cx^2}{b}\right)^{-p}\left(bx+cx^3\right)^p{}_2F_1\left(\begin{matrix}-p,p+2\\p+3\end{matrix}\middle|-\frac{cx^2}{b}\right)}{p+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(b*x**(1+p)*(c*x**3+b*x)**p+2*c*x**(3+p)*(c*x**3+b*x)**p,x)
```

```
[Out] b*x**(-p)*x**(2*p + 2)*(1 + c*x**2/b)**(-p)*(b*x + c*x**3)**p*hyp
er((-p, p + 1), (p + 2,), -c*x**2/b)/(2*(p + 1)) + c*x**(-p)*x**(
2*p + 4)*(1 + c*x**2/b)**(-p)*(b*x + c*x**3)**p*hyper((-p, p + 2)
, (p + 3,), -c*x**2/b)/(p + 2)
```

Mathematica [A] time = 0.0291373, size = 27, normalized size = 1.

$$\frac{x^{p+1}\left(x\left(b+cx^2\right)\right)^{p+1}}{2(p+1)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[b*x^(1 + p)*(b*x + c*x^3)^p + 2*c*x^(3 + p)*(b*x + c*x^3)^p,x]
```

```
[Out] (x^(1 + p)*(x*(b + c*x^2))^(1 + p))/(2*(1 + p))
```

Maple [C] time = 0.173, size = 142, normalized size = 5.3

$$\frac{x\left(cx^2+b\right)x^{1+p}}{2+2p}\mathrm{e}^{-\frac{p\left(i\left(\mathrm{csgn}\left(ix\left(cx^2+b\right)\right)\right)^3\pi-i\left(\mathrm{csgn}\left(ix\left(cx^2+b\right)\right)\right)^2\mathrm{csgn}(ix)\pi-i\left(\mathrm{csgn}\left(ix\left(cx^2+b\right)\right)\right)^2\mathrm{csgn}\left(i\left(cx^2+b\right)\right)\pi+i\,\mathrm{csgn}\left(ix\left(cx^2+b\right)\right)\mathrm{csgn}(ix)\mathrm{csgn}\left(i\left(cx^2+b\right)\right)\pi-2\ln\left(cx^2+b\right)}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(b*x^(1+p)*(c*x^3+b*x)^p+2*c*x^(3+p)*(c*x^3+b*x)^p,x)
```

```
[Out] 1/2*(c*x^2+b)*x*x^(1+p)/(1+p)*exp(-1/2*p*(I*csgn(I*x*(c*x^2+b))^3
*Pi-I*csgn(I*x*(c*x^2+b))^2*csgn(I*x)*Pi-I*csgn(I*x*(c*x^2+b))^2*
csgn(I*(c*x^2+b))*Pi+I*csgn(I*x*(c*x^2+b))*csgn(I*x)*csgn(I*(c*x^
2+b))*Pi-2*ln(c*x^2+b)-2*ln(x)))
```

Maxima [A] time = 0.920708, size = 47, normalized size = 1.74

$$\frac{\left(cx^4+bx^2\right)e^{\left(p\log\left(cx^2+b\right)+2\,p\log(x)\right)}}{2\,(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(2*(c*x^3 + b*x)^p*c*x^(p + 3) + (c*x^3 + b*x)^p*b*x^(p + 1),x, algorithm
```

```
[Out] 1/2*(c*x^4 + b*x^2)*e^(p*log(c*x^2 + b) + 2*p*log(x))/(p + 1)
```

Fricas [A] time = 0.273761, size = 45, normalized size = 1.67

$$\frac{\left(cx^2+b\right)\left(cx^3+bx\right)^p x^{p+3}}{2\,(p+1)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(2*(c*x^3 + b*x)^p*c*x^(p + 3) + (c*x^3 + b*x)^p*b*x^(p + 1),x, algorithm
```

```
[Out] 1/2*(c*x^2 + b)*(c*x^3 + b*x)^p*x^(p + 3)/((p + 1)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(b*x**(1+p)*(c*x**3+b*x)**p+2*c*x**(3+p)*(c*x**3+b*x)**p,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int 2\left(cx^3+bx\right)^p cx^{p+3}+\left(cx^3+bx\right)^p bx^{p+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*(c*x^3 + b*x)^p*c*x^(p + 3) + (c*x^3 + b*x)^p*b*x^(p + 1),x, algorithm
```

```
[Out] integrate(2*(c*x^3 + b*x)^p*c*x^(p + 3) + (c*x^3 + b*x)^p*b*x^(p
+ 1), x)
```

3.175 $\int x^{2(1+p)} \left(b+2 c x^3\right) \left(b x+c x^4\right)^p \, dx$

Optimal. Leaf size=29

$$\frac{x^{2(p+1)} \left(b x+c x^4\right)^{p+1}}{3(p+1)}$$

```
[Out] (x^(2*(1 + p))*(b*x + c*x^4)^(1 + p))/(3*(1 + p))
```

Rubi [A] time = 0.020133, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037

$$\frac{x^{2(p+1)} \left(b x+c x^4\right)^{p+1}}{3(p+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(2*(1 + p))*(b + 2*c*x^3)*(b*x + c*x^4)^p,x]

[Out] (x^(2*(1 + p))*(b*x + c*x^4)^(1 + p))/(3*(1 + p))
```

Rubi in Sympy [A] time = 11.2307, size = 22, normalized size = 0.76

$$\frac{x^{2p+2} \left(b x+c x^4\right)^{p+1}}{3 \left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(2+2*p)*(2*c*x**3+b)*(c*x**4+b*x)**p,x)

[Out] x**(2*p + 2)*(b*x + c*x**4)**(p + 1)/(3*(p + 1))
```

Mathematica [A] time = 0.0530708, size = 29, normalized size = 1.

$$\frac{x^{2p+2} \left(x \left(b+c x^3\right)\right)^{p+1}}{3(p+1)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(2*(1 + p))*(b + 2*c*x^3)*(b*x + c*x^4)^p,x]

[Out] (x^(2 + 2*p)*(x*(b + c*x^3))^(1 + p))/(3*(1 + p))
```

Maple [A] time = 0.006, size = 33, normalized size = 1.1

$$\frac{x^{3+2\,p} \left(c x^3+b\right) \left(c x^4+b x\right)^p}{3+3\,p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(2+2*p)*(2*c*x^3+b)*(c*x^4+b*x)^p,x)
```

```
[Out] 1/3*x^(3+2*p)*(c*x^3+b)/(1+p)*(c*x^4+b*x)^p
```

Maxima [A] time = 0.924804, size = 47, normalized size = 1.62

$$\frac{\left(cx^{6}+bx^{3}\right)e^{\left(p\log\left(cx^{3}+b\right)+3\,p\log\left(x\right)\right)}}{3\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3 + b)*(c*x^4 + b*x)^p*x^(2*p + 2),x, algorithm="maxima")
```

```
[Out] 1/3*(c*x^6 + b*x^3)*e^(p*log(c*x^3 + b) + 3*p*log(x))/(p + 1)
```

Fricas [A] time = 0.282326, size = 46, normalized size = 1.59

$$\frac{\left(cx^{4}+bx\right)\left(cx^{4}+bx\right)^{p}x^{2\,p+2}}{3\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3 + b)*(c*x^4 + b*x)^p*x^(2*p + 2),x, algorithm="fricas")
```

```
[Out] 1/3*(c*x^4 + b*x)*(c*x^4 + b*x)^p*x^(2*p + 2)/(p + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2+2*p)*(2*c*x**3+b)*(c*x**4+b*x)**p,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.273842, size = 78, normalized size = 2.69

$$\frac{cx^{4}e^{\left(p\ln\left(cx^{3}+b\right)+3\,p\ln\left(x\right)+2\,\ln\left(x\right)\right)}+bxe^{\left(p\ln\left(cx^{3}+b\right)+3\,p\ln\left(x\right)+2\,\ln\left(x\right)\right)}}{3\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3 + b)*(c*x^4 + b*x)^p*x^(2*p + 2),x, algorithm="giac")
```

```
[Out] 1/3*(c*x^4*e^(p*ln(c*x^3 + b) + 3*p*ln(x) + 2*ln(x)) + b*x*e^(p*l
n(c*x^3 + b) + 3*p*ln(x) + 2*ln(x)))/(p + 1)
```

3.176 $\int x^{(-1+n)(1+p)}\left(b+2cx^n\right)\left(bx+cx^{1+n}\right)^p\,dx$

Optimal. Leaf size=36

$$\frac{x^{-(1-n)(p+1)}\left(bx+cx^{n+1}\right)^{p+1}}{n(p+1)}$$

```
[Out] (b*x + c*x^(1 + n))^(1 + p)/(n*(1 + p)*x^((1 - n)*(1 + p)))
```

Rubi [A] time = 0.144703, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032

$$\frac{x^{-(1-n)(p+1)}\left(bx+cx^{n+1}\right)^{p+1}}{n(p+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^((-1 + n)*(1 + p))*(b + 2*c*x^n)*(b*x + c*x^(1 + n))^p,x]

[Out] (b*x + c*x^(1 + n))^(1 + p)/(n*(1 + p)*x^((1 - n)*(1 + p)))
```

Rubi in Sympy [A] time = 12.8781, size = 26, normalized size = 0.72

$$\frac{x^{(n-1)(p+1)}\left(bx+cx^{n+1}\right)^{p+1}}{n\,(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**((-1+n)*(1+p))*(b+2*c*x**n)*(b*x+c*x**(1+n))**p,x)

[Out] x**((n - 1)*(p + 1))*(b*x + c*x**(n + 1))**(p + 1)/(n*(p + 1))
```

Mathematica [A] time = 0.10897, size = 31, normalized size = 0.86

$$\frac{x^{(n-1)(p+1)}\left(x\left(b+cx^n\right)\right)^{p+1}}{n(p+1)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^((-1 + n)*(1 + p))*(b + 2*c*x^n)*(b*x + c*x^(1 + n))^p,x]

[Out] (x^((-1 + n)*(1 + p))*(x*(b + c*x^n))^(1 + p))/(n*(1 + p))
```

Maple [F] time = 0.113, size = 0, normalized size = 0.

$$\int x^{(-1+n)(1+p)}\left(b+2\,cx^n\right)\left(bx+cx^{1+n}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^((-1+n)*(1+p))*(b+2*c*x^n)*(b*x+c*x^(1+n))^p,x)
```

```
[Out] int(x^((-1+n)*(1+p))*(b+2*c*x^n)*(b*x+c*x^(1+n))^p,x)
```

Maxima [A] time = 1.13246, size = 53, normalized size = 1.47

$$\frac{\left(cx^{2n} + bx^n\right)e^{(np\log(x)+p\log(cx^n+b))}}{n(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^n + b)*(b*x + c*x^(n + 1))^p*x^((n - 1)*(p + 1)),x, algorithm="ma
```

```
[Out] (c*x^(2*n) + b*x^n)*e^(n*p*log(x) + p*log(c*x^n + b))/(n*(p + 1))
```

Fricas [A] time = 0.273908, size = 57, normalized size = 1.58

$$\frac{\left(bx + cx^{n+1}\right)\left(bx + cx^{n+1}\right)^p x^{(n-1)p+n-1}}{np + n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^n + b)*(b*x + c*x^(n + 1))^p*x^((n - 1)*(p + 1)),x, algorithm="fr
```

```
[Out] (b*x + c*x^(n + 1))*(b*x + c*x^(n + 1))^p*x^((n - 1)*p + n - 1)/(
n*p + n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**((-1+n)*(1+p))*(b+2*c*x**n)*(b*x+c*x**(1+n))**p,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int (2\,cx^n + b)\left(bx + cx^{n+1}\right)^p x^{(n-1)(p+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^n + b)*(b*x + c*x^(n + 1))^p*x^((n - 1)*(p + 1)),x, algorithm="gi
```

```
[Out] integrate((2*c*x^n + b)*(b*x + c*x^(n + 1))^p*x^((n - 1)*(p + 1))
, x)
```

3.177 $\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{a+bx^2} dx$

Optimal. Leaf size=32

$$acx + \frac{1}{2}adx^2 + \frac{1}{3}bcx^3 + \frac{1}{4}bdx^4$$

```
[Out] a*c*x + (a*d*x^2)/2 + (b*c*x^3)/3 + (b*d*x^4)/4
```

Rubi [A] time = 0.0366457, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 54, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.019

$$acx + \frac{1}{2}adx^2 + \frac{1}{3}bcx^3 + \frac{1}{4}bdx^4$$

Antiderivative was successfully verified.

```
[In] Int[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/(a + b*x

[Out] a*c*x + (a*d*x^2)/2 + (b*c*x^3)/3 + (b*d*x^4)/4
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$ad \int x\,dx + \frac{bcx^3}{3} + \frac{bdx^4}{4} + c \int a\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((b**2*d*x**5+b**2*c*x**4+2*a*b*d*x**3+2*a*b*c*x**2+a**2*d*x+a**

[Out] a*d*Integral(x, x) + b*c*x**3/3 + b*d*x**4/4 + c*Integral(a, x)
```

Mathematica [A] time = 0.00309232, size = 32, normalized size = 1.

$$acx + \frac{1}{2}adx^2 + \frac{1}{3}bcx^3 + \frac{1}{4}bdx^4$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/(

[Out] a*c*x + (a*d*x^2)/2 + (b*c*x^3)/3 + (b*d*x^4)/4
```

Maple [A] time = 0.002, size = 27, normalized size = 0.8

$$acx + \frac{adx^2}{2} + \frac{bcx^3}{3} + \frac{bdx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^2+a),x)
```

```
[Out] a*c*x+1/2*a*d*x^2+1/3*b*c*x^3+1/4*b*d*x^4
```

Maxima [A] time = 0.793782, size = 35, normalized size = 1.09

$$\frac{1}{4}\,bdx^4 + \frac{1}{3}\,bcx^3 + \frac{1}{2}\,adx^2 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5 + b^2*c*x^4 + 2*a*b*d*x^3 + 2*a*b*c*x^2 + a^2*d*x + a^2*c)/(b
```

```
[Out] 1/4*b*d*x^4 + 1/3*b*c*x^3 + 1/2*a*d*x^2 + a*c*x
```

Fricas [A] time = 0.244604, size = 35, normalized size = 1.09

$$\frac{1}{4}\,bdx^4 + \frac{1}{3}\,bcx^3 + \frac{1}{2}\,adx^2 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5 + b^2*c*x^4 + 2*a*b*d*x^3 + 2*a*b*c*x^2 + a^2*d*x + a^2*c)/(b
```

```
[Out] 1/4*b*d*x^4 + 1/3*b*c*x^3 + 1/2*a*d*x^2 + a*c*x
```

Sympy [A] time = 0.149388, size = 29, normalized size = 0.91

$$acx + \frac{adx^2}{2} + \frac{bcx^3}{3} + \frac{bdx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*d*x**5+b**2*c*x**4+2*a*b*d*x**3+2*a*b*c*x**2+a**2*d*x+a**2*c)/
```

```
[Out] a*c*x + a*d*x**2/2 + b*c*x**3/3 + b*d*x**4/4
```

GIAC/XCAS [A] time = 0.259875, size = 35, normalized size = 1.09

$$\frac{1}{4}\,bdx^4 + \frac{1}{3}\,bcx^3 + \frac{1}{2}\,adx^2 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5 + b^2*c*x^4 + 2*a*b*d*x^3 + 2*a*b*c*x^2 + a^2*d*x + a^2*c)/(b
```

```
[Out] 1/4*b*d*x^4 + 1/3*b*c*x^3 + 1/2*a*d*x^2 + a*c*x
```

3.178 $\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=12

$$cx+\frac{dx^2}{2}$$

```
[Out] c*x + (d*x^2)/2
```

Rubi [A] time = 0.0351876, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 1, integrand size = 54, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.019

$$cx+\frac{dx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/(a + b*x
```

```
[Out] c*x + (d*x^2)/2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$d\int x\,dx+\int c\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((b**2*d*x**5+b**2*c*x**4+2*a*b*d*x**3+2*a*b*c*x**2+a**2*d*x+a**
```

```
[Out] d*Integral(x, x) + Integral(c, x)
```

Mathematica [A] time = 0.0011929, size = 12, normalized size = 1.

$$cx+\frac{dx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/(
```

```
[Out] c*x + (d*x^2)/2
```

Maple [A] time = 0.001, size = 11, normalized size = 0.9

$$cx+\frac{dx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^2+a)^2,x)
```

```
[Out] c*x+1/2*d*x^2
```

Maxima [A] time = 0.815606, size = 14, normalized size = 1.17

$$\frac{1}{2}\,dx^2 + cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5 + b^2*c*x^4 + 2*a*b*d*x^3 + 2*a*b*c*x^2 + a^2*d*x + a^2*c)/(b
```

```
[Out] 1/2*d*x^2 + c*x
```

Fricas [A] time = 0.243901, size = 14, normalized size = 1.17

$$\frac{1}{2}\,dx^2 + cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5 + b^2*c*x^4 + 2*a*b*d*x^3 + 2*a*b*c*x^2 + a^2*d*x + a^2*c)/(b
```

```
[Out] 1/2*d*x^2 + c*x
```

Sympy [A] time = 0.15824, size = 8, normalized size = 0.67

$$cx + \frac{dx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*d*x**5+b**2*c*x**4+2*a*b*d*x**3+2*a*b*c*x**2+a**2*d*x+a**2*c)/
```

```
[Out] c*x + d*x**2/2
```

GIAC/XCAS [A] time = 0.263331, size = 14, normalized size = 1.17

$$\frac{1}{2}\,dx^2 + cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5 + b^2*c*x^4 + 2*a*b*d*x^3 + 2*a*b*c*x^2 + a^2*d*x + a^2*c)/(b
```

```
[Out] 1/2*d*x^2 + c*x
```

3.179 $\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=42

$$\frac{c\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}+\frac{d\log\left(a+bx^2\right)}{2b}$$

```
[Out] (c*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]) + (d*Log[a + b*
x^2])/(2*b)
```

Rubi [A] time = 0.0679989, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 54, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074

$$\frac{c\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}+\frac{d\log\left(a+bx^2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/(a + b*x
```

```
[Out] (c*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]) + (d*Log[a + b*
x^2])/(2*b)
```

Rubi in Sympy [A] time = 34.4324, size = 37, normalized size = 0.88

$$\frac{d\log\left(a+bx^2\right)}{2b}+\frac{c\,\mathrm{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((b**2*d*x**5+b**2*c*x**4+2*a*b*d*x**3+2*a*b*c*x**2+a**2*d*x+a**
```

```
[Out] d*log(a + b*x**2)/(2*b) + c*atan(sqrt(b)*x/sqrt(a))/(sqrt(a)*sqrt
(b))
```

Mathematica [A] time = 0.0247769, size = 42, normalized size = 1.

$$\frac{c\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}+\frac{d\log\left(a+bx^2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/(
```

```
[Out] (c*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]) + (d*Log[a + b*
x^2])/(2*b)
```

Maple [A] time = 0.006, size = 32, normalized size = 0.8

$$\frac{d\ln\left(bx^2+a\right)}{2\,b}+c\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^2+a)^3,x)
```

```
[Out] 1/2*d*ln(b*x^2+a)/b+c/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5 + b^2*c*x^4 + 2*a*b*d*x^3 + 2*a*b*c*x^2 + a^2*d*x + a^2*c)/(
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.2561, size = 1, normalized size = 0.02

$$\left[\frac{bc\log\left(\frac{2\,abx+\left(bx^2-a\right)\sqrt{-ab}}{bx^2+a}\right)+\sqrt{-ab}d\log\left(bx^2+a\right)}{2\,\sqrt{-ab}b}, \frac{2\,bc\arctan\left(\frac{\sqrt{ab}x}{a}\right)+\sqrt{ab}d\log\left(bx^2+a\right)}{2\,\sqrt{ab}b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5 + b^2*c*x^4 + 2*a*b*d*x^3 + 2*a*b*c*x^2 + a^2*d*x + a^2*c)/(
```

```
[Out] [1/2*(b*c*log((2*a*b*x + (b*x^2 - a)*sqrt(-a*b))/(b*x^2 + a)) + s
qrt(-a*b)*d*log(b*x^2 + a))/(sqrt(-a*b)*b), 1/2*(2*b*c*arctan(sqr
t(a*b)*x/a) + sqrt(a*b)*d*log(b*x^2 + a))/(sqrt(a*b)*b)]
```

Sympy [A] time = 0.813131, size = 124, normalized size = 2.95

$$\left(\frac{d}{2b}-\frac{c\sqrt{-ab^3}}{2ab^2}\right)\log\left(x+\frac{2ab\left(\frac{d}{2b}-\frac{c\sqrt{-ab^3}}{2ab^2}\right)-ad}{bc}\right)+\left(\frac{d}{2b}+\frac{c\sqrt{-ab^3}}{2ab^2}\right)\log\left(x+\frac{2ab\left(\frac{d}{2b}+\frac{c\sqrt{-ab^3}}{2ab^2}\right)-ad}{bc}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*d*x**5+b**2*c*x**4+2*a*b*d*x**3+2*a*b*c*x**2+a**2*d*x+a**2*c)/
```

```
[Out] (d/(2*b) - c*sqrt(-a*b**3)/(2*a*b**2))*log(x + (2*a*b*(d/(2*b) -
c*sqrt(-a*b**3)/(2*a*b**2)) - a*d)/(b*c)) + (d/(2*b) + c*sqrt(-a*
b**3)/(2*a*b**2))*log(x + (2*a*b*(d/(2*b) + c*sqrt(-a*b**3)/(2*a*
b**2)) - a*d)/(b*c))
```

GIAC/XCAS [A] time = 0.265637, size = 42, normalized size = 1.

$$\frac{c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}+\frac{d\ln\left(bx^2+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*d*x^5 + b^2*c*x^4 + 2*a*b*d*x^3 + 2*a*b*c*x^2 + a^2*d*x + a^2*c)/(
```

```
[Out] c*arctan(b*x/sqrt(a*b))/sqrt(a*b) + 1/2*d*ln(b*x^2 + a)/b
```

3.180 $\int \left(b+2cx+3dx^2\right) \left(a+bx+cx^2+dx^3\right)^n \, dx$

Optimal. Leaf size=25

$$\frac{\left(a+bx+cx^2+dx^3\right)^{n+1}}{n+1}$$

```
[Out] (a + b*x + c*x^2 + d*x^3)^(1 + n)/(1 + n)
```

Rubi [A] time = 0.01828, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033

$$\frac{\left(a+bx+cx^2+dx^3\right)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In]  Int[(b + 2*c*x + 3*d*x^2)*(a + b*x + c*x^2 + d*x^3)^n,x]

[Out] (a + b*x + c*x^2 + d*x^3)^(1 + n)/(1 + n)
```

Rubi in Sympy [A] time = 7.83432, size = 20, normalized size = 0.8

$$\frac{\left(a+bx+cx^2+dx^3\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*d*x**2+2*c*x+b)*(d*x**3+c*x**2+b*x+a)**n,x)

[Out] (a + b*x + c*x**2 + d*x**3)**(n + 1)/(n + 1)
```

Mathematica [A] time = 0.0489968, size = 23, normalized size = 0.92

$$\frac{(a+x(b+x(c+dx)))^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b + 2*c*x + 3*d*x^2)*(a + b*x + c*x^2 + d*x^3)^n,x]

[Out] (a + x*(b + x*(c + d*x)))^(1 + n)/(1 + n)
```

Maple [A] time = 0.006, size = 26, normalized size = 1.

$$\frac{\left(dx^3+cx^2+bx+a\right)^{1+n}}{1+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x+a)^n,x)
```

```
[Out] (d*x^3+c*x^2+b*x+a)^(1+n)/(1+n)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2 + 2*c*x + b)*(d*x^3 + c*x^2 + b*x + a)^n,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.29664, size = 51, normalized size = 2.04

$$\frac{\left(dx^3 + cx^2 + bx + a\right)\left(dx^3 + cx^2 + bx + a\right)^n}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2 + 2*c*x + b)*(d*x^3 + c*x^2 + b*x + a)^n,x, algorithm="fricas")
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^n/(n + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*d*x**2+2*c*x+b)*(d*x**3+c*x**2+b*x+a)**n,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.289081, size = 132, normalized size = 5.28

$$\frac{dx^3 e^{\left(n\ln\left(dx^3+cx^2+bx+a\right)\right)} + cx^2 e^{\left(n\ln\left(dx^3+cx^2+bx+a\right)\right)} + bxe^{\left(n\ln\left(dx^3+cx^2+bx+a\right)\right)} + ae^{\left(n\ln\left(dx^3+cx^2+bx+a\right)\right)}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2 + 2*c*x + b)*(d*x^3 + c*x^2 + b*x + a)^n,x, algorithm="giac")
```

```
[Out] (d*x^3*e^(n*ln(d*x^3 + c*x^2 + b*x + a)) + c*x^2*e^(n*ln(d*x^3 +
c*x^2 + b*x + a)) + b*x*e^(n*ln(d*x^3 + c*x^2 + b*x + a)) + a*e^(
n*ln(d*x^3 + c*x^2 + b*x + a)))/(n + 1)
```

3.181 $\int (b+2cx+3dx^2)\left(bx+cx^2+dx^3\right)^n\,dx$

Optimal. Leaf size=24

$$\frac{\left(bx+cx^2+dx^3\right)^{n+1}}{n+1}$$

```
[Out] (b*x + c*x^2 + d*x^3)^(1 + n)/(1 + n)
```

Rubi [A] time = 0.0160103, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034

$$\frac{\left(bx+cx^2+dx^3\right)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In]  Int[(b + 2*c*x + 3*d*x^2)*(b*x + c*x^2 + d*x^3)^n,x]

[Out] (b*x + c*x^2 + d*x^3)^(1 + n)/(1 + n)
```

Rubi in Sympy [A] time = 7.36209, size = 19, normalized size = 0.79

$$\frac{\left(bx+cx^2+dx^3\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*d*x**2+2*c*x+b)*(d*x**3+c*x**2+b*x)**n,x)

[Out] (b*x + c*x**2 + d*x**3)**(n + 1)/(n + 1)
```

Mathematica [A] time = 0.051956, size = 21, normalized size = 0.88

$$\frac{(x(b+x(c+dx)))^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b + 2*c*x + 3*d*x^2)*(b*x + c*x^2 + d*x^3)^n,x]

[Out] (x*(b + x*(c + d*x)))^(1 + n)/(1 + n)
```

Maple [A] time = 0.006, size = 34, normalized size = 1.4

$$\frac{x\left(dx^2+cx+b\right)\left(dx^3+cx^2+bx\right)^n}{1+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x)^n,x)
```

```
[Out] x*(d*x^2+c*x+b)/(1+n)*(d*x^3+c*x^2+b*x)^n
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2 + 2*c*x + b)*(d*x^3 + c*x^2 + b*x)^n,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.269827, size = 49, normalized size = 2.04

$$\frac{\left(dx^3 + cx^2 + bx\right)\left(dx^3 + cx^2 + bx\right)^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2 + 2*c*x + b)*(d*x^3 + c*x^2 + b*x)^n,x, algorithm="fricas")
```

```
[Out] (d*x^3 + c*x^2 + b*x)*(d*x^3 + c*x^2 + b*x)^n/(n + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*d*x**2+2*c*x+b)*(d*x**3+c*x**2+b*x)**n,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.270205, size = 100, normalized size = 4.17

$$\frac{dx^3 e^{\left(n\ln\left(dx^3+cx^2+bx\right)\right)} + cx^2 e^{\left(n\ln\left(dx^3+cx^2+bx\right)\right)} + bxe^{\left(n\ln\left(dx^3+cx^2+bx\right)\right)}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2 + 2*c*x + b)*(d*x^3 + c*x^2 + b*x)^n,x, algorithm="giac")
```

```
[Out] (d*x^3*e^(n*ln(d*x^3 + c*x^2 + b*x)) + c*x^2*e^(n*ln(d*x^3 + c*x^
2 + b*x)) + b*x*e^(n*ln(d*x^3 + c*x^2 + b*x)))/(n + 1)
```

3.182 $\int x^n \left(b+cx+dx^2\right)^n \left(b+2cx+3dx^2\right)\,dx$

Optimal. Leaf size=25

$$\frac{x^{n+1}\left(b+cx+dx^2\right)^{n+1}}{n+1}$$

```
[Out] (x^(1 + n)*(b + c*x + d*x^2)^(1 + n))/(1 + n)
```

Rubi [A] time = 0.0186243, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036

$$\frac{x^{n+1}\left(b+cx+dx^2\right)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In]  Int[x^n*(b + c*x + d*x^2)^n*(b + 2*c*x + 3*d*x^2),x]

[Out] (x^(1 + n)*(b + c*x + d*x^2)^(1 + n))/(1 + n)
```

Rubi in Sympy [A] time = 12.5869, size = 20, normalized size = 0.8

$$\frac{x^{n+1}\left(b+cx+dx^2\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**n*(d*x**2+c*x+b)**n*(3*d*x**2+2*c*x+b),x)

[Out] x**(n + 1)*(b + c*x + d*x**2)**(n + 1)/(n + 1)
```

Mathematica [A] time = 0.0505583, size = 24, normalized size = 0.96

$$\frac{x^{n+1}(b+x(c+dx))^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^n*(b + c*x + d*x^2)^n*(b + 2*c*x + 3*d*x^2),x]

[Out] (x^(1 + n)*(b + x*(c + d*x))^(1 + n))/(1 + n)
```

Maple [A] time = 0.005, size = 26, normalized size = 1.

$$\frac{x^{1+n}\left(dx^2+cx+b\right)^{1+n}}{1+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^n*(d*x^2+c*x+b)^n*(3*d*x^2+2*c*x+b),x)
```

```
[Out] x^(1+n)*(d*x^2+c*x+b)^(1+n)/(1+n)
```

Maxima [A] time = 0.917034, size = 53, normalized size = 2.12

$$\frac{\left(dx^3+cx^2+bx\right)e^{\left(n\log\left(dx^2+cx+b\right)+n\log(x)\right)}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2 + 2*c*x + b)*(d*x^2 + c*x + b)^n*x^n,x, algorithm="maxima")
```

```
[Out] (d*x^3 + c*x^2 + b*x)*e^(n*log(d*x^2 + c*x + b) + n*log(x))/(n +
1)
```

Fricas [A] time = 0.285959, size = 47, normalized size = 1.88

$$\frac{\left(dx^3+cx^2+bx\right)\left(dx^2+cx+b\right)^n x^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2 + 2*c*x + b)*(d*x^2 + c*x + b)^n*x^n,x, algorithm="fricas")
```

```
[Out] (d*x^3 + c*x^2 + b*x)*(d*x^2 + c*x + b)^n*x^n/(n + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**n*(d*x**2+c*x+b)**n*(3*d*x**2+2*c*x+b),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.270098, size = 104, normalized size = 4.16

$$\frac{dx^3e^{\left(n\ln\left(dx^2+cx+b\right)+n\ln(x)\right)}+cx^2e^{\left(n\ln\left(dx^2+cx+b\right)+n\ln(x)\right)}+bxe^{\left(n\ln\left(dx^2+cx+b\right)+n\ln(x)\right)}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2 + 2*c*x + b)*(d*x^2 + c*x + b)^n*x^n,x, algorithm="giac")
```

```
[Out] (d*x^3*e^(n*ln(d*x^2 + c*x + b) + n*ln(x)) + c*x^2*e^(n*ln(d*x^2
+ c*x + b) + n*ln(x)) + b*x*e^(n*ln(d*x^2 + c*x + b) + n*ln(x)))/
(n + 1)
```

3.183 $\int \left(b+3dx^2\right)\left(a+bx+dx^3\right)^n\,dx$

Optimal. Leaf size=20

$$\frac{\left(a+bx+dx^3\right)^{n+1}}{n+1}$$

```
[Out] (a + b*x + d*x^3)^(1 + n)/(1 + n)
```

Rubi [A] time = 0.0138082, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048

$$\frac{\left(a+bx+dx^3\right)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In]  Int[(b + 3*d*x^2)*(a + b*x + d*x^3)^n,x]

[Out] (a + b*x + d*x^3)^(1 + n)/(1 + n)
```

Rubi in Sympy [A] time = 4.49274, size = 15, normalized size = 0.75

$$\frac{\left(a+bx+dx^3\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*d*x**2+b)*(d*x**3+b*x+a)**n,x)

[Out] (a + b*x + d*x**3)**(n + 1)/(n + 1)
```

Mathematica [A] time = 0.030185, size = 20, normalized size = 1.

$$\frac{\left(a+bx+dx^3\right)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b + 3*d*x^2)*(a + b*x + d*x^3)^n,x]

[Out] (a + b*x + d*x^3)^(1 + n)/(1 + n)
```

Maple [A] time = 0.005, size = 21, normalized size = 1.1

$$\frac{\left(dx^3+bx+a\right)^{1+n}}{1+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*d*x^2+b)*(d*x^3+b*x+a)^n,x)
```

```
[Out] (d*x^3+b*x+a)^(1+n)/(1+n)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*d*x^2 + b)*(d*x^3 + b*x + a)^n,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.273885, size = 38, normalized size = 1.9

$$\frac{\left(dx^3+bx+a\right)\left(dx^3+bx+a\right)^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*d*x^2 + b)*(d*x^3 + b*x + a)^n,x, algorithm="fricas")
```

```
[Out] (d*x^3 + b*x + a)*(d*x^3 + b*x + a)^n/(n + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*d*x**2+b)*(d*x**3+b*x+a)**n,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.26588, size = 80, normalized size = 4.

$$\frac{dx^3e^{\left(n\ln\left(dx^3+bx+a\right)\right)}+bxe^{\left(n\ln\left(dx^3+bx+a\right)\right)}+ae^{\left(n\ln\left(dx^3+bx+a\right)\right)}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*d*x^2 + b)*(d*x^3 + b*x + a)^n,x, algorithm="giac")
```

```
[Out] (d*x^3*e^(n*ln(d*x^3 + b*x + a)) + b*x*e^(n*ln(d*x^3 + b*x + a))
+ a*e^(n*ln(d*x^3 + b*x + a)))/(n + 1)
```

3.184 $\int \left(b+3 d x^2\right) \left(b x+d x^3\right)^n \, dx$

Optimal. Leaf size=19

$$\frac{\left(b x+d x^3\right)^{n+1}}{n+1}$$

```
[Out] (b*x + d*x^3)^(1 + n)/(1 + n)
```

Rubi [A] time = 0.0133596, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05

$$\frac{\left(b x+d x^3\right)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In]  Int[(b + 3*d*x^2)*(b*x + d*x^3)^n,x]

[Out] (b*x + d*x^3)^(1 + n)/(1 + n)
```

Rubi in Sympy [A] time = 4.02258, size = 14, normalized size = 0.74

$$\frac{\left(b x+d x^3\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*d*x**2+b)*(d*x**3+b*x)**n,x)

[Out] (b*x + d*x**3)**(n + 1)/(n + 1)
```

Mathematica [A] time = 0.039293, size = 19, normalized size = 1.

$$\frac{\left(x \left(b+d x^2\right)\right)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b + 3*d*x^2)*(b*x + d*x^3)^n,x]

[Out] (x*(b + d*x^2))^(1 + n)/(1 + n)
```

Maple [A] time = 0.005, size = 26, normalized size = 1.4

$$\frac{x \left(d x^2+b\right) \left(d x^3+b x\right)^n}{1+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*d*x^2+b)*(d*x^3+b*x)^n,x)
```

```
[Out] x*(d*x^2+b)/(1+n)*(d*x^3+b*x)^n
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*d*x^2 + b)*(d*x^3 + b*x)^n,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.271494, size = 35, normalized size = 1.84

$$\frac{\left(dx^3+bx\right)\left(dx^3+bx\right)^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*d*x^2 + b)*(d*x^3 + b*x)^n,x, algorithm="fricas")
```

```
[Out] (d*x^3 + b*x)*(d*x^3 + b*x)^n/(n + 1)
```

Sympy [A] time = 42.4005, size = 73, normalized size = 3.84

$$\begin{cases} \frac{bx\left(bx+dx^3\right)^n}{n+1}+\frac{dx^3\left(bx+dx^3\right)^n}{n+1} & \text{for } n\neq -1 \\ \log(x)+\log\left(-i\sqrt{b}\sqrt{\frac{1}{d}}+x\right)+\log\left(i\sqrt{b}\sqrt{\frac{1}{d}}+x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*d*x**2+b)*(d*x**3+b*x)**n,x)
```

```
[Out] Piecewise((b*x*(b*x + d*x**3)**n/(n + 1) + d*x**3*(b*x + d*x**3)*
*n/(n + 1), Ne(n, -1)), (log(x) + log(-I*sqrt(b)*sqrt(1/d) + x) +
 log(I*sqrt(b)*sqrt(1/d) + x), True))
```

GIAC/XCAS [A] time = 0.271794, size = 55, normalized size = 2.89

$$\frac{dx^3e^{\left(n\ln\left(dx^3+bx\right)\right)}+bxe^{\left(n\ln\left(dx^3+bx\right)\right)}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*d*x^2 + b)*(d*x^3 + b*x)^n,x, algorithm="giac")
```

```
[Out] (d*x^3*e^(n*ln(d*x^3 + b*x)) + b*x*e^(n*ln(d*x^3 + b*x)))/(n + 1)
```

3.185 $\int x^n (b+dx^2)^n (b+3dx^2)\, dx$

Optimal. Leaf size=22

$$\frac{x^{n+1}(b+dx^2)^{n+1}}{n+1}$$

```
[Out] (x^(1 + n)*(b + d*x^2)^(1 + n))/(1 + n)
```

Rubi [A] time = 0.0159144, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$

$$\frac{x^{n+1}(b+dx^2)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^n*(b + d*x^2)^n*(b + 3*d*x^2),x]

[Out] (x^(1 + n)*(b + d*x^2)^(1 + n))/(1 + n)
```

Rubi in Sympy [A] time = 6.37048, size = 17, normalized size = 0.77

$$\frac{x^{n+1}(b+dx^2)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**n*(d*x**2+b)**n*(3*d*x**2+b),x)

[Out] x**(n + 1)*(b + d*x**2)**(n + 1)/(n + 1)
```

Mathematica [A] time = 0.0347575, size = 22, normalized size = 1.

$$\frac{x^{n+1}(b+dx^2)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^n*(b + d*x^2)^n*(b + 3*d*x^2),x]

[Out] (x^(1 + n)*(b + d*x^2)^(1 + n))/(1 + n)
```

Maple [A] time = 0.004, size = 23, normalized size = 1.1

$$\frac{x^{1+n}(dx^2+b)^{1+n}}{1+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n*(d*x^2+b)^n*(3*d*x^2+b),x)
```

```
[Out] x^(1+n)*(d*x^2+b)^(1+n)/(1+n)
```

Maxima [A] time = 0.913368, size = 42, normalized size = 1.91

$$\frac{\left(dx^3 + bx\right) e^{\left(n \log\left(dx^2 + b\right) + n \log(x)\right)}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*d*x^2 + b)*(d*x^2 + b)^n*x^n,x, algorithm="maxima")
```

```
[Out] (d*x^3 + b*x)*e^(n*log(d*x^2 + b) + n*log(x))/(n + 1)
```

Fricas [A] time = 0.281103, size = 36, normalized size = 1.64

$$\frac{\left(dx^3 + bx\right) \left(dx^2 + b\right)^n x^n}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*d*x^2 + b)*(d*x^2 + b)^n*x^n,x, algorithm="fricas")
```

```
[Out] (d*x^3 + b*x)*(d*x^2 + b)^n*x^n/(n + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**n*(d*x**2+b)**n*(3*d*x**2+b),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.27466, size = 63, normalized size = 2.86

$$\frac{dx^3 e^{\left(n\ln\left(dx^2 + b\right) + n\ln(x)\right)} + bx e^{\left(n\ln\left(dx^2 + b\right) + n\ln(x)\right)}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*d*x^2 + b)*(d*x^2 + b)^n*x^n,x, algorithm="giac")
```

```
[Out] (d*x^3*e^(n*ln(d*x^2 + b) + n*ln(x)) + b*x*e^(n*ln(d*x^2 + b) + n
*ln(x)))/(n + 1)
```

3.186 $\int (2cx + 3dx^2) (a + cx^2 + dx^3)^n dx$

Optimal. Leaf size=22

$$\frac{(a + cx^2 + dx^3)^{n+1}}{n + 1}$$

```
[Out] (a + c*x^2 + d*x^3)^(1 + n)/(1 + n)
```

Rubi [A] time = 0.0148789, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$

$$\frac{(a + cx^2 + dx^3)^{n+1}}{n + 1}$$

Antiderivative was successfully verified.

```
[In] Int[(2*c*x + 3*d*x^2)*(a + c*x^2 + d*x^3)^n,x]
```

```
[Out] (a + c*x^2 + d*x^3)^(1 + n)/(1 + n)
```

Rubi in Sympy [A] time = 7.3025, size = 17, normalized size = 0.77

$$\frac{(a + cx^2 + dx^3)^{n+1}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((3*d*x**2+2*c*x)*(d*x**3+c*x**2+a)**n,x)
```

```
[Out] (a + c*x**2 + d*x**3)**(n + 1)/(n + 1)
```

Mathematica [A] time = 0.034506, size = 21, normalized size = 0.95

$$\frac{(a + x^2(c + dx))^{n+1}}{n + 1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*c*x + 3*d*x^2)*(a + c*x^2 + d*x^3)^n,x]
```

```
[Out] (a + x^2*(c + d*x))^(1 + n)/(1 + n)
```

Maple [A] time = 0.004, size = 23, normalized size = 1.1

$$\frac{(dx^3 + cx^2 + a)^{1+n}}{1 + n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*d*x^2+2*c*x)*(d*x^3+c*x^2+a)^n,x)
```

```
[Out] (d*x^3+c*x^2+a)^(1+n)/(1+n)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2 + 2*c*x)*(d*x^3 + c*x^2 + a)^n,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.270943, size = 43, normalized size = 1.95

$$\frac{\left(dx^3 + cx^2 + a\right)\left(dx^3 + cx^2 + a\right)^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2 + 2*c*x)*(d*x^3 + c*x^2 + a)^n,x, algorithm="fricas")
```

```
[Out] (d*x^3 + c*x^2 + a)*(d*x^3 + c*x^2 + a)^n/(n + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*d*x**2+2*c*x)*(d*x**3+c*x**2+a)**n,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.275085, size = 90, normalized size = 4.09

$$\frac{dx^3 e^{\left(n\ln\left(dx^3+cx^2+a\right)\right)} + cx^2 e^{\left(n\ln\left(dx^3+cx^2+a\right)\right)} + ae^{\left(n\ln\left(dx^3+cx^2+a\right)\right)}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2 + 2*c*x)*(d*x^3 + c*x^2 + a)^n,x, algorithm="giac")
```

```
[Out] (d*x^3*e^(n*ln(d*x^3 + c*x^2 + a)) + c*x^2*e^(n*ln(d*x^3 + c*x^2
+ a)) + a*e^(n*ln(d*x^3 + c*x^2 + a)))/(n + 1)
```

3.187 $\int \left(2cx+3dx^2\right)\left(cx^2+dx^3\right)^n\,dx$

Optimal. Leaf size=21

$$\frac{\left(cx^2+dx^3\right)^{n+1}}{n+1}$$

```
[Out] (c*x^2 + d*x^3)^(1 + n)/(1 + n)
```

Rubi [A] time = 0.0145531, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04

$$\frac{\left(cx^2+dx^3\right)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(2*c*x + 3*d*x^2)*(c*x^2 + d*x^3)^n,x]

[Out] (c*x^2 + d*x^3)^(1 + n)/(1 + n)
```

Rubi in Sympy [A] time = 6.41581, size = 15, normalized size = 0.71

$$\frac{\left(cx^2+dx^3\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((3*d*x**2+2*c*x)*(d*x**3+c*x**2)**n,x)

[Out] (c*x**2 + d*x**3)**(n + 1)/(n + 1)
```

Mathematica [A] time = 0.0381455, size = 19, normalized size = 0.9

$$\frac{\left(x^2(c+dx)\right)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*c*x + 3*d*x^2)*(c*x^2 + d*x^3)^n,x]

[Out] (x^2*(c + d*x))^(1 + n)/(1 + n)
```

Maple [A] time = 0.005, size = 28, normalized size = 1.3

$$\frac{\left(dx^3+cx^2\right)^n x^2(dx+c)}{1+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*d*x^2+2*c*x)*(d*x^3+c*x^2)^n,x)
```

```
[Out] (d*x^3+c*x^2)^n*x^2*(d*x+c)/(1+n)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2 + 2*c*x)*(d*x^3 + c*x^2)^n,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.278936, size = 41, normalized size = 1.95

$$\frac{\left(dx^3 + cx^2\right)\left(dx^3 + cx^2\right)^n}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2 + 2*c*x)*(d*x^3 + c*x^2)^n,x, algorithm="fricas")
```

```
[Out] (d*x^3 + c*x^2)*(d*x^3 + c*x^2)^n/(n + 1)
```

Sympy [A] time = 2.42592, size = 53, normalized size = 2.52

$$\begin{cases} \frac{cx^2\left(cx^2+dx^3\right)^n}{n+1} + \frac{dx^3\left(cx^2+dx^3\right)^n}{n+1} & \text{for}\ n \neq -1 \\ 2\log\left(x\right) + \log\left(\frac{c}{d} + x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x**2+2*c*x)*(d*x**3+c*x**2)**n,x)
```

```
[Out] Piecewise((c*x**2*(c*x**2 + d*x**3)**n/(n + 1) + d*x**3*(c*x**2 +
 d*x**3)**n/(n + 1), Ne(n, -1)), (2*log(x) + log(c/d + x), True))
```

GIAC/XCAS [A] time = 0.277027, size = 63, normalized size = 3.

$$\frac{dx^3e^{\left(n\ln\left(dx^3+cx^2\right)\right)} + cx^2e^{\left(n\ln\left(dx^3+cx^2\right)\right)}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2 + 2*c*x)*(d*x^3 + c*x^2)^n,x, algorithm="giac")
```

```
[Out] (d*x^3*e^(n*ln(d*x^3 + c*x^2)) + c*x^2*e^(n*ln(d*x^3 + c*x^2)))/(
n + 1)
```

3.188 $\int x^n \left(cx+dx^2\right)^n \left(2cx+3dx^2\right)\,dx$

Optimal. Leaf size=24

$$\frac{x^{n+1}\left(cx+dx^2\right)^{n+1}}{n+1}$$

```
[Out] (x^(1 + n)*(c*x + d*x^2)^(1 + n))/(1 + n)
```

Rubi [A] time = 0.0175274, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$

$$\frac{x^{n+1}\left(cx+dx^2\right)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In]  Int[x^n*(c*x + d*x^2)^n*(2*c*x + 3*d*x^2),x]
```

```
[Out] (x^(1 + n)*(c*x + d*x^2)^(1 + n))/(1 + n)
```

Rubi in Sympy [A] time = 40.9879, size = 124, normalized size = 5.17

$$\frac{cx^{-2n-1}x^{n+1}x^{2n+2}\left(1+\frac{dx}{c}\right)^{-n}\left(cx+dx^2\right)^n {}_2F_1\left(\begin{matrix}-n,2n+2\\2n+3\end{matrix}\middle|-\frac{dx}{c}\right)}{n+1} + \frac{3dx^{-2n-2}x^{n+2}x^{2n+3}\left(1+\frac{dx}{c}\right)^{-n}\left(cx+dx^2\right)^n {}_2F_1\left(\begin{matrix}-n,2n+3\\2n+4\end{matrix}\middle|-\frac{dx}{c}\right)}{2n+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**n*(d*x**2+c*x)**n*(3*d*x**2+2*c*x),x)
```

```
[Out] c*x**(-2*n - 1)*x**(n + 1)*x**(2*n + 2)*(1 + d*x/c)**(-n)*(c*x +
d*x**2)**n*hyper((-n, 2*n + 2), (2*n + 3,), -d*x/c)/(n + 1) + 3*d
*x**(-2*n - 2)*x**(n + 2)*x**(2*n + 3)*(1 + d*x/c)**(-n)*(c*x + d
*x**2)**n*hyper((-n, 2*n + 3), (2*n + 4,), -d*x/c)/(2*n + 3)
```

Mathematica [A] time = 0.0413796, size = 22, normalized size = 0.92

$$\frac{x^{n+1}(x(c+dx))^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^n*(c*x + d*x^2)^n*(2*c*x + 3*d*x^2),x]
```

```
[Out] (x^(1 + n)*(x*(c + d*x))^(1 + n))/(1 + n)
```

Maple [A] time = 0.004, size = 28, normalized size = 1.2

$$\frac{\left(dx^2+cx\right)^n x^{2+n}(dx+c)}{1+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^n*(d*x^2+c*x)^n*(3*d*x^2+2*c*x),x)
```

```
[Out] (d*x^2+c*x)^n*x^(2+n)*(d*x+c)/(1+n)
```

Maxima [A] time = 0.911642, size = 43, normalized size = 1.79

$$\frac{\left(dx^3 + cx^2\right)e^{(n\log(dx+c)+2\,n\log(x))}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2 + 2*c*x)*(d*x^2 + c*x)^n*x^n,x, algorithm="maxima")
```

```
[Out] (d*x^3 + c*x^2)*e^(n*log(d*x + c) + 2*n*log(x))/(n + 1)
```

Fricas [A] time = 0.271137, size = 42, normalized size = 1.75

$$\frac{\left(dx^3 + cx^2\right)\left(dx^2 + cx\right)^n x^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2 + 2*c*x)*(d*x^2 + c*x)^n*x^n,x, algorithm="fricas")
```

```
[Out] (d*x^3 + c*x^2)*(d*x^2 + c*x)^n*x^n/(n + 1)
```

Sympy [A] time = 18.3019, size = 56, normalized size = 2.33

$$\begin{cases} \frac{cx^2x^n\left(cx+dx^2\right)^n}{n+1} + \frac{dx^3x^n\left(cx+dx^2\right)^n}{n+1} & \text{for } n \neq -1 \\ 2\log(x) + \log\left(\frac{c}{d} + x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x**n*(d*x**2+c*x)**n*(3*d*x**2+2*c*x),x)
```

```
[Out] Piecewise((c*x**2*x**n*(c*x + d*x**2)**n/(n + 1) + d*x**3*x**n*(c
*x + d*x**2)**n/(n + 1), Ne(n, -1)), (2*log(x) + log(c/d + x), Tr
ue))
```

GIAC/XCAS [A] time = 0.274939, size = 63, normalized size = 2.62

$$\frac{dx^3e^{(n\ln(dx+c)+2\,n\ln(x))} + cx^2e^{(n\ln(dx+c)+2\,n\ln(x))}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2 + 2*c*x)*(d*x^2 + c*x)^n*x^n,x, algorithm="giac")
```

```
[Out] (d*x^3*e^(n*ln(d*x + c) + 2*n*ln(x)) + c*x^2*e^(n*ln(d*x + c) + 2
*n*ln(x)))/(n + 1)
```

3.189 $\int x^{2n}(c+dx)^n \left(2cx+3dx^2\right)\,dx$

Optimal. Leaf size=22

$$\frac{x^{2(n+1)}(c+dx)^{n+1}}{n+1}$$

```
[Out] (x^(2*(1 + n))*(c + d*x)^(1 + n))/(1 + n)
```

Rubi [A] time = 0.0166558, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$

$$\frac{x^{2(n+1)}(c+dx)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2*n)*(c + d*x)^n*(2*c*x + 3*d*x^2),x]
```

```
[Out] (x^(2*(1 + n))*(c + d*x)^(1 + n))/(1 + n)
```

Rubi in Sympy [A] time = 17.1062, size = 31, normalized size = 1.41

$$\frac{xx^{2n+1}(c+dx)^{n+1}(9n+6)}{3(n+1)(3n+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**(2*n)*(d*x+c)**n*(3*d*x**2+2*c*x),x)
```

```
[Out] x*x**(2*n + 1)*(c + d*x)**(n + 1)*(9*n + 6)/(3*(n + 1)*(3*n + 2))
```

Mathematica [A] time = 0.0357936, size = 22, normalized size = 1.

$$\frac{x^{2n+2}(c+dx)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2*n)*(c + d*x)^n*(2*c*x + 3*d*x^2),x]
```

```
[Out] (x^(2 + 2*n)*(c + d*x)^(1 + n))/(1 + n)
```

Maple [A] time = 0.004, size = 23, normalized size = 1.1

$$\frac{x^{2+2\,n}(dx+c)^{1+n}}{1+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n)*(d*x+c)^n*(3*d*x^2+2*c*x),x)
```

```
[Out] x^(2+2*n)*(d*x+c)^(1+n)/(1+n)
```

Maxima [A] time = 0.895376, size = 43, normalized size = 1.95

$$\frac{\left(dx^3 + cx^2\right) e^{(n \log(dx+c)+2\, n \log(x))}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2 + 2*c*x)*(d*x + c)^n*x^(2*n),x, algorithm="maxima")
```

```
[Out] (d*x^3 + c*x^2)*e^(n*log(d*x + c) + 2*n*log(x))/(n + 1)
```

Fricas [A] time = 0.281125, size = 39, normalized size = 1.77

$$\frac{\left(dx^3 + cx^2\right) (dx + c)^n x^{2\,n}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2 + 2*c*x)*(d*x + c)^n*x^(2*n),x, algorithm="fricas")
```

```
[Out] (d*x^3 + c*x^2)*(d*x + c)^n*x^(2*n)/(n + 1)
```

Sympy [A] time = 17.7628, size = 53, normalized size = 2.41

$$\begin{cases} \frac{cx^2x^{2n}(c+dx)^n}{n+1} + \frac{dx^3x^{2n}(c+dx)^n}{n+1} & \text{for}\ n \neq -1 \\ 2\log(x) + \log\left(\frac{c}{d} + x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(2*n)*(d*x+c)**n*(3*d*x**2+2*c*x),x)
```

```
[Out] Piecewise((c*x**2*x**(2*n)*(c + d*x)**n/(n + 1) + d*x**3*x**(2*n)
*(c + d*x)**n/(n + 1), Ne(n, -1)), (2*log(x) + log(c/d + x), True
))
```

GIAC/XCAS [A] time = 0.266437, size = 63, normalized size = 2.86

$$\frac{dx^3 e^{(n\ln(dx+c)+2\, n\ln(x))} + cx^2 e^{(n\ln(dx+c)+2\, n\ln(x))}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2 + 2*c*x)*(d*x + c)^n*x^(2*n),x, algorithm="giac")
```

```
[Out] (d*x^3*e^(n*ln(d*x + c) + 2*n*ln(x)) + c*x^2*e^(n*ln(d*x + c) + 2
*n*ln(x)))/(n + 1)
```

3.190 $\int x(2c+3dx)\left(a+cx^2+dx^3\right)^n\,dx$

Optimal. Leaf size=22

$$\frac{\left(a+cx^2+dx^3\right)^{n+1}}{n+1}$$

```
[Out] (a + c*x^2 + d*x^3)^(1 + n)/(1 + n)
```

Rubi [A] time = 0.0153291, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042

$$\frac{\left(a+cx^2+dx^3\right)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(2*c + 3*d*x)*(a + c*x^2 + d*x^3)^n,x]
```

```
[Out] (a + c*x^2 + d*x^3)^(1 + n)/(1 + n)
```

Rubi in Sympy [A] time = 4.93662, size = 17, normalized size = 0.77

$$\frac{\left(a+cx^2+dx^3\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(3*d*x+2*c)*(d*x**3+c*x**2+a)**n,x)
```

```
[Out] (a + c*x**2 + d*x**3)**(n + 1)/(n + 1)
```

Mathematica [A] time = 0.0210146, size = 21, normalized size = 0.95

$$\frac{\left(a+x^2(c+dx)\right)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(2*c + 3*d*x)*(a + c*x^2 + d*x^3)^n,x]
```

```
[Out] (a + x^2*(c + d*x))^(1 + n)/(1 + n)
```

Maple [A] time = 0.004, size = 23, normalized size = 1.1

$$\frac{\left(dx^3+cx^2+a\right)^{1+n}}{1+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(3*d*x+2*c)*(d*x^3+c*x^2+a)^n,x)
```

```
[Out] (d*x^3+c*x^2+a)^(1+n)/(1+n)
```

Maxima [A] time = 0.864532, size = 43, normalized size = 1.95

$$\frac{\left(dx^3 + cx^2 + a\right)\left(dx^3 + cx^2 + a\right)^n}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x + 2*c)*(d*x^3 + c*x^2 + a)^n*x,x, algorithm="maxima")
```

```
[Out] (d*x^3 + c*x^2 + a)*(d*x^3 + c*x^2 + a)^n/(n + 1)
```

Fricas [A] time = 0.267711, size = 43, normalized size = 1.95

$$\frac{\left(dx^3 + cx^2 + a\right)\left(dx^3 + cx^2 + a\right)^n}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x + 2*c)*(d*x^3 + c*x^2 + a)^n*x,x, algorithm="fricas")
```

```
[Out] (d*x^3 + c*x^2 + a)*(d*x^3 + c*x^2 + a)^n/(n + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*d*x+2*c)*(d*x**3+c*x**2+a)**n,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.272057, size = 90, normalized size = 4.09

$$\frac{dx^3 e^{\left(n\ln\left(dx^3+cx^2+a\right)\right)} + cx^2 e^{\left(n\ln\left(dx^3+cx^2+a\right)\right)} + ae^{\left(n\ln\left(dx^3+cx^2+a\right)\right)}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x + 2*c)*(d*x^3 + c*x^2 + a)^n*x,x, algorithm="giac")
```

```
[Out] (d*x^3*e^(n*ln(d*x^3 + c*x^2 + a)) + c*x^2*e^(n*ln(d*x^3 + c*x^2
+ a)) + a*e^(n*ln(d*x^3 + c*x^2 + a)))/(n + 1)
```

3.191 $\int x(2c+3dx)\left(cx^2+dx^3\right)^n\,dx$

Optimal. Leaf size=21

$$\frac{\left(cx^2+dx^3\right)^{n+1}}{n+1}$$

```
[Out] (c*x^2 + d*x^3)^(1 + n)/(1 + n)
```

Rubi [A] time = 0.0146223, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043

$$\frac{\left(cx^2+dx^3\right)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(2*c + 3*d*x)*(c*x^2 + d*x^3)^n,x]

[Out] (c*x^2 + d*x^3)^(1 + n)/(1 + n)
```

Rubi in Sympy [A] time = 4.42921, size = 15, normalized size = 0.71

$$\frac{\left(cx^2+dx^3\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(3*d*x+2*c)*(d*x**3+c*x**2)**n,x)

[Out] (c*x**2 + d*x**3)**(n + 1)/(n + 1)
```

Mathematica [A] time = 0.0303184, size = 19, normalized size = 0.9

$$\frac{\left(x^2(c+dx)\right)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(2*c + 3*d*x)*(c*x^2 + d*x^3)^n,x]

[Out] (x^2*(c + d*x))^(1 + n)/(1 + n)
```

Maple [A] time = 0.004, size = 28, normalized size = 1.3

$$\frac{\left(dx^3+cx^2\right)^n x^2\,(dx+c)}{1+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(3*d*x+2*c)*(d*x^3+c*x^2)^n,x)
```

```
[Out] (d*x^3+c*x^2)^n*x^2*(d*x+c)/(1+n)
```

Maxima [A] time = 0.906612, size = 43, normalized size = 2.05

$$\frac{\left(dx^3+cx^2\right)e^{(n\log(dx+c)+2\,n\log(x))}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*d*x + 2*c)*(d*x^3 + c*x^2)^n*x,x, algorithm="maxima")
```

```
[Out] (d*x^3 + c*x^2)*e^(n*log(d*x + c) + 2*n*log(x))/(n + 1)
```

Fricas [A] time = 0.275777, size = 41, normalized size = 1.95

$$\frac{\left(dx^3+cx^2\right)\left(dx^3+cx^2\right)^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*d*x + 2*c)*(d*x^3 + c*x^2)^n*x,x, algorithm="fricas")
```

```
[Out] (d*x^3 + c*x^2)*(d*x^3 + c*x^2)^n/(n + 1)
```

Sympy [A] time = 2.40536, size = 53, normalized size = 2.52

$$\begin{cases} \frac{cx^2(cx^2+dx^3)^n}{n+1}+\frac{dx^3(cx^2+dx^3)^n}{n+1} & \text{for } n\neq -1 \\ 2\log(x)+\log\left(\frac{c}{d}+x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(3*d*x+2*c)*(d*x**3+c*x**2)**n,x)
```

```
[Out] Piecewise((c*x**2*(c*x**2 + d*x**3)**n/(n + 1) + d*x**3*(c*x**2 +
 d*x**3)**n/(n + 1), Ne(n, -1)), (2*log(x) + log(c/d + x), True))
```

GIAC/XCAS [A] time = 0.266766, size = 63, normalized size = 3.

$$\frac{dx^3e^{\left(n\ln\left(dx^3+cx^2\right)\right)}+cx^2e^{\left(n\ln\left(dx^3+cx^2\right)\right)}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*d*x + 2*c)*(d*x^3 + c*x^2)^n*x,x, algorithm="giac")
```

```
[Out] (d*x^3*e^(n*ln(d*x^3 + c*x^2)) + c*x^2*e^(n*ln(d*x^3 + c*x^2)))/(
n + 1)
```

3.192 $\int (b+2cx+3dx^2)(a+bx+cx^2+dx^3)^7\,dx$

Optimal. Leaf size=21

$$\frac{1}{8}(a+bx+cx^2+dx^3)^8$$

```
[Out] (a + b*x + c*x^2 + d*x^3)^8/8
```

Rubi [A] time = 0.0432703, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$

$$\frac{1}{8}(a+bx+cx^2+dx^3)^8$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x + 3*d*x^2)*(a + b*x + c*x^2 + d*x^3)^7,x]
```

```
[Out] (a + b*x + c*x^2 + d*x^3)^8/8
```

Rubi in Sympy [A] time = 14.4395, size = 17, normalized size = 0.81

$$\frac{(a+bx+cx^2+dx^3)^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((3*d*x**2+2*c*x+b)*(d*x**3+c*x**2+b*x+a)**7,x)
```

```
[Out] (a + b*x + c*x**2 + d*x**3)**8/8
```

Mathematica [B] time = 0.242511, size = 143, normalized size = 6.81

$$\frac{1}{8}x(b+x(c+dx))\left(8a^7+28a^6x(b+x(c+dx))+56a^5x^2(b+x(c+dx))^2+70a^4x^3(b+x(c+dx))^3+56a^3x^4(b+x(c+dx))^4+28a^2x^5(b+x(c+dx))^5+8ax^6(b+x(c+dx))^6+x^7(b+x(c+dx))^7\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x + 3*d*x^2)*(a + b*x + c*x^2 + d*x^3)^7,x]
```

```
[Out] (x*(b + x*(c + d*x))*(8*a^7 + 28*a^6*x*(b + x*(c + d*x)) + 56*a^5
*x^2*(b + x*(c + d*x))^2 + 70*a^4*x^3*(b + x*(c + d*x))^3 + 56*a^
3*x^4*(b + x*(c + d*x))^4 + 28*a^2*x^5*(b + x*(c + d*x))^5 + 8*a*
x^6*(b + x*(c + d*x))^6 + x^7*(b + x*(c + d*x))^7))/8
```

Maple [B] time = 0.006, size = 25686, normalized size = 1223.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x+a)^7,x)
```

```
[Out] result too large to display
```

Maxima [A] time = 0.788848, size = 26, normalized size = 1.24

$$\frac{1}{8} \left(dx^3 + cx^2 + bx + a\right)^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c*x^2 + b*x + a)^7*(3*d*x^2 + 2*c*x + b),x, algorithm="maxima")
```

```
[Out] 1/8*(d*x^3 + c*x^2 + b*x + a)^8
```

Fricas [A] time = 0.235679, size = 1, normalized size = 0.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c*x^2 + b*x + a)^7*(3*d*x^2 + 2*c*x + b),x, algorithm="fricas")
```

```
[Out] 1/8*x^24*d^8 + x^23*d^7*c + 7/2*x^22*d^6*c^2 + x^22*d^7*b + 7*x^2
1*d^5*c^3 + 7*x^21*d^6*c*b + x^21*d^7*a + 35/4*x^20*d^4*c^4 + 21*
x^20*d^5*c^2*b + 7/2*x^20*d^6*b^2 + 7*x^20*d^6*c*a + 7*x^19*d^3*c
^5 + 35*x^19*d^4*c^3*b + 21*x^19*d^5*c*b^2 + 21*x^19*d^5*c^2*a +
7*x^19*d^6*b*a + 7/2*x^18*d^2*c^6 + 35*x^18*d^3*c^4*b + 105/2*x^1
8*d^4*c^2*b^2 + 7*x^18*d^5*b^3 + 35*x^18*d^4*c^3*a + 42*x^18*d^5*
c*b*a + 7/2*x^18*d^6*a^2 + x^17*d*c^7 + 21*x^17*d^2*c^5*b + 70*x^
17*d^3*c^3*b^2 + 35*x^17*d^4*c*b^3 + 35*x^17*d^3*c^4*a + 105*x^17
*d^4*c^2*b*a + 21*x^17*d^5*b^2*a + 21*x^17*d^5*c*a^2 + 1/8*x^16*c
^8 + 7*x^16*d*c^6*b + 105/2*x^16*d^2*c^4*b^2 + 70*x^16*d^3*c^2*b^
3 + 35/4*x^16*d^4*b^4 + 21*x^16*d^2*c^5*a + 140*x^16*d^3*c^3*b*a
+ 105*x^16*d^4*c*b^2*a + 105/2*x^16*d^4*c^2*a^2 + 21*x^16*d^5*b*a
^2 + x^15*c^7*b + 21*x^15*d*c^5*b^2 + 70*x^15*d^2*c^3*b^3 + 35*x^
15*d^3*c*b^4 + 7*x^15*d*c^6*a + 105*x^15*d^2*c^4*b*a + 210*x^15*d
^3*c^2*b^2*a + 35*x^15*d^4*b^3*a + 70*x^15*d^3*c^3*a^2 + 105*x^15
*d^4*c*b*a^2 + 7*x^15*d^5*a^3 + 7/2*x^14*c^6*b^2 + 35*x^14*d*c^4*
b^3 + 105/2*x^14*d^2*c^2*b^4 + 7*x^14*d^3*b^5 + x^14*c^7*a + 42*x
^14*d*c^5*b*a + 210*x^14*d^2*c^3*b^2*a + 140*x^14*d^3*c*b^3*a + 1
05/2*x^14*d^2*c^4*a^2 + 210*x^14*d^3*c^2*b*a^2 + 105/2*x^14*d^4*b
^2*a^2 + 35*x^14*d^4*c*a^3 + 7*x^13*c^5*b^3 + 35*x^13*d*c^3*b^4 +
 21*x^13*d^2*c*b^5 + 7*x^13*c^6*b*a + 105*x^13*d*c^4*b^2*a + 210*
x^13*d^2*c^2*b^3*a + 35*x^13*d^3*b^4*a + 21*x^13*d*c^5*a^2 + 210*
x^13*d^2*c^3*b*a^2 + 210*x^13*d^3*c*b^2*a^2 + 70*x^13*d^3*c^2*a^3
 + 35*x^13*d^4*b*a^3 + 35/4*x^12*c^4*b^4 + 21*x^12*d*c^2*b^5 + 7/
2*x^12*d^2*b^6 + 21*x^12*c^5*b^2*a + 140*x^12*d*c^3*b^3*a + 105*x
^12*d^2*c*b^4*a + 7/2*x^12*c^6*a^2 + 105*x^12*d*c^4*b*a^2 + 315*x
^12*d^2*c^2*b^2*a^2 + 70*x^12*d^3*b^3*a^2 + 70*x^12*d^2*c^3*a^3 +
 140*x^12*d^3*c*b*a^3 + 35/4*x^12*d^4*a^4 + 7*x^11*c^3*b^5 + 7*x^
11*d*c*b^6 + 35*x^11*c^4*b^3*a + 105*x^11*d*c^2*b^4*a + 21*x^11*d
^2*b^5*a + 21*x^11*c^5*b*a^2 + 210*x^11*d*c^3*b^2*a^2 + 210*x^11*
d^2*c*b^3*a^2 + 35*x^11*d*c^4*a^3 + 210*x^11*d^2*c^2*b*a^3 + 70*x
^11*d^3*b^2*a^3 + 35*x^11*d^3*c*a^4 + 7/2*x^10*c^2*b^6 + x^10*d*b
^7 + 35*x^10*c^3*b^4*a + 42*x^10*d*c*b^5*a + 105/2*x^10*c^4*b^2*a
^2 + 210*x^10*d*c^2*b^3*a^2 + 105/2*x^10*d^2*b^4*a^2 + 7*x^10*c^5
*a^3 + 140*x^10*d*c^3*b*a^3 + 210*x^10*d^2*c*b^2*a^3 + 105/2*x^10
*d^2*c^2*a^4 + 35*x^10*d^3*b*a^4 + x^9*c*b^7 + 21*x^9*c^2*b^5*a +
 7*x^9*d*b^6*a + 70*x^9*c^3*b^3*a^2 + 105*x^9*d*c*b^4*a^2 + 35*x^
9*c^4*b*a^3 + 210*x^9*d*c^2*b^2*a^3 + 70*x^9*d^2*b^3*a^3 + 35*x^9
*d*c^3*a^4 + 105*x^9*d^2*c*b*a^4 + 7*x^9*d^3*a^5 + 1/8*x^8*b^8 +
```

```
7*x^8*c*b^6*a + 105/2*x^8*c^2*b^4*a^2 + 21*x^8*d*b^5*a^2 + 70*x^8
*c^3*b^2*a^3 + 140*x^8*d*c*b^3*a^3 + 35/4*x^8*c^4*a^4 + 105*x^8*d
*c^2*b*a^4 + 105/2*x^8*d^2*b^2*a^4 + 21*x^8*d^2*c*a^5 + x^7*b^7*a
 + 21*x^7*c*b^5*a^2 + 70*x^7*c^2*b^3*a^3 + 35*x^7*d*b^4*a^3 + 35*
x^7*c^3*b*a^4 + 105*x^7*d*c*b^2*a^4 + 21*x^7*d*c^2*a^5 + 21*x^7*d
^2*b*a^5 + 7/2*x^6*b^6*a^2 + 35*x^6*c*b^4*a^3 + 105/2*x^6*c^2*b^2
*a^4 + 35*x^6*d*b^3*a^4 + 7*x^6*c^3*a^5 + 42*x^6*d*c*b*a^5 + 7/2*
x^6*d^2*a^6 + 7*x^5*b^5*a^3 + 35*x^5*c*b^3*a^4 + 21*x^5*c^2*b*a^5
 + 21*x^5*d*b^2*a^5 + 7*x^5*d*c*a^6 + 35/4*x^4*b^4*a^4 + 21*x^4*c
*b^2*a^5 + 7/2*x^4*c^2*a^6 + 7*x^4*d*b*a^6 + 7*x^3*b^3*a^5 + 7*x^
3*c*b*a^6 + x^3*d*a^7 + 7/2*x^2*b^2*a^6 + x^2*c*a^7 + x*b*a^7
```

Sympy [A] time = 1.17226, size = 1771, normalized size = 84.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*d*x**2+2*c*x+b)*(d*x**3+c*x**2+b*x+a)**7,x)
```

```
[Out] a**7*b*x + c*d**7*x**23 + d**8*x**24/8 + x**22*(b*d**7 + 7*c**2*d
**6/2) + x**21*(a*d**7 + 7*b*c*d**6 + 7*c**3*d**5) + x**20*(7*a*c
*d**6 + 7*b**2*d**6/2 + 21*b*c**2*d**5 + 35*c**4*d**4/4) + x**19*
(7*a*b*d**6 + 21*a*c**2*d**5 + 21*b**2*c*d**5 + 35*b*c**3*d**4 +
7*c**5*d**3) + x**18*(7*a**2*d**6/2 + 42*a*b*c*d**5 + 35*a*c**3*d
**4 + 7*b**3*d**5 + 105*b**2*c**2*d**4/2 + 35*b*c**4*d**3 + 7*c**
6*d**2/2) + x**17*(21*a**2*c*d**5 + 21*a*b**2*d**5 + 105*a*b*c**2
*d**4 + 35*a*c**4*d**3 + 35*b**3*c*d**4 + 70*b**2*c**3*d**3 + 21*
b*c**5*d**2 + c**7*d) + x**16*(21*a**2*b*d**5 + 105*a**2*c**2*d**
4/2 + 105*a*b**2*c*d**4 + 140*a*b*c**3*d**3 + 21*a*c**5*d**2 + 35
*b**4*d**4/4 + 70*b**3*c**2*d**3 + 105*b**2*c**4*d**2/2 + 7*b*c**
6*d + c**8/8) + x**15*(7*a**3*d**5 + 105*a**2*b*c*d**4 + 70*a**2*
c**3*d**3 + 35*a*b**3*d**4 + 210*a*b**2*c**2*d**3 + 105*a*b*c**4*
d**2 + 7*a*c**6*d + 35*b**4*c*d**3 + 70*b**3*c**3*d**2 + 21*b**2*
c**5*d + b*c**7) + x**14*(35*a**3*c*d**4 + 105*a**2*b**2*d**4/2 +
 210*a**2*b*c**2*d**3 + 105*a**2*c**4*d**2/2 + 140*a*b**3*c*d**3
+ 210*a*b**2*c**3*d**2 + 42*a*b*c**5*d + a*c**7 + 7*b**5*d**3 + 1
05*b**4*c**2*d**2/2 + 35*b**3*c**4*d + 7*b**2*c**6/2) + x**13*(35
*a**3*b*d**4 + 70*a**3*c**2*d**3 + 210*a**2*b**2*c*d**3 + 210*a**
2*b*c**3*d**2 + 21*a**2*c**5*d + 35*a*b**4*d**3 + 210*a*b**3*c**2
*d**2 + 105*a*b**2*c**4*d + 7*a*b*c**6 + 21*b**5*c*d**2 + 35*b**4
*c**3*d + 7*b**3*c**5) + x**12*(35*a**4*d**4/4 + 140*a**3*b*c*d**
3 + 70*a**3*c**3*d**2 + 70*a**2*b**3*d**3 + 315*a**2*b**2*c**2*d*
*2 + 105*a**2*b*c**4*d + 7*a**2*c**6/2 + 105*a*b**4*c*d**2 + 140*
a*b**3*c**3*d + 21*a*b**2*c**5 + 7*b**6*d**2/2 + 21*b**5*c**2*d +
 35*b**4*c**4/4) + x**11*(35*a**4*c*d**3 + 70*a**3*b**2*d**3 + 21
0*a**3*b*c**2*d**2 + 35*a**3*c**4*d + 210*a**2*b**3*c*d**2 + 210*
a**2*b**2*c**3*d + 21*a**2*b*c**5 + 21*a*b**5*d**2 + 105*a*b**4*c
**2*d + 35*a*b**3*c**4 + 7*b**6*c*d + 7*b**5*c**3) + x**10*(35*a*
*4*b*d**3 + 105*a**4*c**2*d**2/2 + 210*a**3*b**2*c*d**2 + 140*a**
3*b*c**3*d + 7*a**3*c**5 + 105*a**2*b**4*d**2/2 + 210*a**2*b**3*c
**2*d + 105*a**2*b**2*c**4/2 + 42*a*b**5*c*d + 35*a*b**4*c**3 + b
**7*d + 7*b**6*c**2/2) + x**9*(7*a**5*d**3 + 105*a**4*b*c*d**2 +
35*a**4*c**3*d + 70*a**3*b**3*d**2 + 210*a**3*b**2*c**2*d + 35*a*
*3*b*c**4 + 105*a**2*b**4*c*d + 70*a**2*b**3*c**3 + 7*a*b**6*d +
21*a*b**5*c**2 + b**7*c) + x**8*(21*a**5*c*d**2 + 105*a**4*b**2*d
**2/2 + 105*a**4*b*c**2*d + 35*a**4*c**4/4 + 140*a**3*b**3*c*d +
70*a**3*b**2*c**3 + 21*a**2*b**5*d + 105*a**2*b**4*c**2/2 + 7*a*b
**6*c + b**8/8) + x**7*(21*a**5*b*d**2 + 21*a**5*c**2*d + 105*a**
4*b**2*c*d + 35*a**4*b*c**3 + 35*a**3*b**4*d + 70*a**3*b**3*c**2
+ 21*a**2*b**5*c + a*b**7) + x**6*(7*a**6*d**2/2 + 42*a**5*b*c*d
+ 7*a**5*c**3 + 35*a**4*b**3*d + 105*a**4*b**2*c**2/2 + 35*a**3*b
**4*c + 7*a**2*b**6/2) + x**5*(7*a**6*c*d + 21*a**5*b**2*d + 21*a
**5*b*c**2 + 35*a**4*b**3*c + 7*a**3*b**5) + x**4*(7*a**6*b*d + 7
*a**6*c**2/2 + 21*a**5*b**2*c + 35*a**4*b**4/4) + x**3*(a**7*d +
7*a**6*b*c + 7*a**5*b**3) + x**2*(a**7*c + 7*a**6*b**2/2)
```

GIAC/XCAS [A] time = 0.26797, size = 1, normalized size = 0.05

$$\text{Done}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c*x^2 + b*x + a)^7*(3*d*x^2 + 2*c*x + b),x, algorithm="giac")
```

```
[Out] Done
```

3.193 $\int \left(b+2cx+3dx^2\right)\left(bx+cx^2+dx^3\right)^7\,dx$

Optimal. Leaf size=20

$$\frac{1}{8}\left(bx+cx^2+dx^3\right)^8$$

```
[Out] (b*x + c*x^2 + d*x^3)^8/8
```

Rubi [A] time = 0.0229924, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034

$$\frac{1}{8}\left(bx+cx^2+dx^3\right)^8$$

Antiderivative was successfully verified.

```
[In]  Int[(b + 2*c*x + 3*d*x^2)*(b*x + c*x^2 + d*x^3)^7,x]
```

```
[Out] (b*x + c*x^2 + d*x^3)^8/8
```

Rubi in Sympy [A] time = 19.1243, size = 15, normalized size = 0.75

$$\frac{x^8\left(b+cx+dx^2\right)^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*d*x**2+2*c*x+b)*(d*x**3+c*x**2+b*x)**7,x)
```

```
[Out] x**8*(b + c*x + d*x**2)**8/8
```

Mathematica [A] time = 0.0467751, size = 18, normalized size = 0.9

$$\frac{1}{8}x^8(b+x(c+dx))^8$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b + 2*c*x + 3*d*x^2)*(b*x + c*x^2 + d*x^3)^7,x]
```

```
[Out] (x^8*(b + x*(c + d*x))^8)/8
```

Maple [B] time = 0.004, size = 5596, normalized size = 279.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x)^7,x)
```

```
[Out] result too large to display
```

Maxima [A] time = 0.813918, size = 24, normalized size = 1.2

$$\frac{1}{8}\left(dx^3+cx^2+bx\right)^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3 + c*x^2 + b*x)^7*(3*d*x^2 + 2*c*x + b),x, algorithm="maxima")
```

```
[Out] 1/8*(d*x^3 + c*x^2 + b*x)^8
```

Fricas [A] time = 0.238978, size = 1, normalized size = 0.05

$$\frac{1}{8}x^{24}d^8+x^{23}d^7c+\frac{7}{2}x^{22}d^6c^2+x^{22}d^7b+7x^{21}d^5c^3+7x^{21}d^6cb+\frac{35}{4}x^{20}d^4c^4+21x^{20}d^5c^2b+\frac{7}{2}x^{20}d^6b^2+7x^{19}d^3c^5+35x^{19}d^4c^3b+21x^{19}d^5cb^2+\frac{7}{2}x^{18}d^2c^6+35x^{18}d^3c^4b+\frac{105}{2}x^{18}d^4c^2b^2+7x^{18}d^5b^3+x^{17}dc^7+21x^{17}d^2c^5b+70x^{17}d^3c^3b^2+35x^{17}d^4cb^3+\frac{1}{8}x^{16}c^8+7x^{16}dc^6b+\frac{105}{2}x^{16}d^2c^4b^2+70x^{16}d^3c^2b^3+\frac{35}{4}x^{16}d^4b^4+x^{15}c^7b+21x^{15}dc^5b^2+70x^{15}d^2c^3b^3+35x^{15}d^3cb^4+\frac{7}{2}x^{14}c^6b^2+35x^{14}dc^4b^3+\frac{105}{2}x^{14}d^2c^2b^4+7x^{14}d^3b^5+7x^{13}c^5b^3+35x^{13}dc^3b^4+21x^{13}d^2cb^5+\frac{35}{4}x^{12}c^4b^4+21x^{12}dc^2b^5+\frac{7}{2}x^{12}d^2b^6+7x^{11}c^3b^5+7x^{11}dcb^6+\frac{7}{2}x^{10}c^2b^6+x^{10}db^7+x^9cb^7+\frac{1}{8}x^8b^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3 + c*x^2 + b*x)^7*(3*d*x^2 + 2*c*x + b),x, algorithm="fricas")
```

```
[Out] 1/8*x^24*d^8 + x^23*d^7*c + 7/2*x^22*d^6*c^2 + x^22*d^7*b + 7*x^2
1*d^5*c^3 + 7*x^21*d^6*c*b + 35/4*x^20*d^4*c^4 + 21*x^20*d^5*c^2*
b + 7/2*x^20*d^6*b^2 + 7*x^19*d^3*c^5 + 35*x^19*d^4*c^3*b + 21*x^
19*d^5*c*b^2 + 7/2*x^18*d^2*c^6 + 35*x^18*d^3*c^4*b + 105/2*x^18*
d^4*c^2*b^2 + 7*x^18*d^5*b^3 + x^17*d*c^7 + 21*x^17*d^2*c^5*b + 7
0*x^17*d^3*c^3*b^2 + 35*x^17*d^4*c*b^3 + 1/8*x^16*c^8 + 7*x^16*d*
c^6*b + 105/2*x^16*d^2*c^4*b^2 + 70*x^16*d^3*c^2*b^3 + 35/4*x^16*
d^4*b^4 + x^15*c^7*b + 21*x^15*d*c^5*b^2 + 70*x^15*d^2*c^3*b^3 +
35*x^15*d^3*c*b^4 + 7/2*x^14*c^6*b^2 + 35*x^14*d*c^4*b^3 + 105/2*
x^14*d^2*c^2*b^4 + 7*x^14*d^3*b^5 + 7*x^13*c^5*b^3 + 35*x^13*d*c^
3*b^4 + 21*x^13*d^2*c*b^5 + 35/4*x^12*c^4*b^4 + 21*x^12*d*c^2*b^5
 + 7/2*x^12*d^2*b^6 + 7*x^11*c^3*b^5 + 7*x^11*d*c*b^6 + 7/2*x^10*
c^2*b^6 + x^10*d*b^7 + x^9*c*b^7 + 1/8*x^8*b^8
```

Sympy [A] time = 0.447144, size = 469, normalized size = 23.45

$$\frac{b^8x^8}{8}+b^7cx^9+cd^7x^{23}+\frac{d^8x^{24}}{8}+x^{22}\left(bd^7+\frac{7c^2d^6}{2}\right)+x^{21}\left(7bcd^6+7c^3d^5\right)+x^{20}\left(\frac{7b^2d^6}{2}+21bc^2d^5+\frac{35c^4d^4}{4}\right)+x^{19}\left(21b^2cd^5+35bc^3d^4+7c^5d^3\right)+x^{18}\left(7b^3d^5+\frac{105b^2c^2d^4}{2}+35bc^4d^3+\frac{7c^6d^2}{2}\right)+x^{17}\left(35b^3cd^4+70b^2c^3d^3+21bc^5d^2+c^7d\right)+x^{16}\left(\frac{35b^4d^4}{4}+70b^3c^2d^3+\frac{105b^2c^4d^2}{2}+7bc^6d+\frac{c^8}{8}\right)+x^{15}\left(35b^4cd^3+70b^3c^3d^2+21b^2c^5d+bc^7\right)+x^{14}\left(7b^5d^3+\frac{105b^4c^2d^2}{2}+35b^3c^4d+\frac{7b^2c^6}{2}\right)+x^{13}\left(21b^5cd^2+35b^4c^3d+7b^3c^5\right)+x^{12}\left(\frac{7b^6d^2}{2}+21b^5c^2d+\frac{35b^4c^4}{4}\right)+x^{11}\left(7b^6cd+7b^5c^3\right)+x^{10}\left(b^7d+\frac{7b^6c^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*d*x**2+2*c*x+b)*(d*x**3+c*x**2+b*x)**7,x)
```

```
[Out] b**8*x**8/8 + b**7*c*x**9 + c*d**7*x**23 + d**8*x**24/8 + x**22*(
b*d**7 + 7*c**2*d**6/2) + x**21*(7*b*c*d**6 + 7*c**3*d**5) + x**2
0*(7*b**2*d**6/2 + 21*b*c**2*d**5 + 35*c**4*d**4/4) + x**19*(21*b
**2*c*d**5 + 35*b*c**3*d**4 + 7*c**5*d**3) + x**18*(7*b**3*d**5 +
 105*b**2*c**2*d**4/2 + 35*b*c**4*d**3 + 7*c**6*d**2/2) + x**17*(
35*b**3*c*d**4 + 70*b**2*c**3*d**3 + 21*b*c**5*d**2 + c**7*d) + x
**16*(35*b**4*d**4/4 + 70*b**3*c**2*d**3 + 105*b**2*c**4*d**2/2 +
 7*b*c**6*d + c**8/8) + x**15*(35*b**4*c*d**3 + 70*b**3*c**3*d**2
 + 21*b**2*c**5*d + b*c**7) + x**14*(7*b**5*d**3 + 105*b**4*c**2*
d**2/2 + 35*b**3*c**4*d + 7*b**2*c**6/2) + x**13*(21*b**5*c*d**2
+ 35*b**4*c**3*d + 7*b**3*c**5) + x**12*(7*b**6*d**2/2 + 21*b**5*
c**2*d + 35*b**4*c**4/4) + x**11*(7*b**6*c*d + 7*b**5*c**3) + x**
10*(b**7*d + 7*b**6*c**2/2)
```

GIAC/XCAS [A] time = 0.266695, size = 670, normalized size = 33.5

$$\frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{7}{2}c^2d^6x^{22} + bd^7x^{22} + 7c^3d^5x^{21} + 7bcd^6x^{21} + \frac{35}{4}c^4d^4x^{20} + 21bc^2d^5x^{20} + \frac{7}{2}b^2d^6x^{20} + 7c^5d^3x^{19} + 35bc^3d^4x^{19} + 21b^2cd^5x^{19} + \frac{7}{2}c^6d^2x^{18} + 35bc^4d^3x^{18} + \frac{105}{2}b^2c^2d^4x^{18} + 7b^3d^5x^{18} + c^7dx^{17} + 21bc^5d^2x^{17} + 70b^2c^3d^3x^{17} + 35b^3cd^4x^{17} + \frac{1}{8}c^8x^{16} + 7bc^6dx^{16} + \frac{105}{2}b^2c^4d^2x^{16} + 70b^3c^2d^3x^{16} + \frac{35}{4}b^4d^4x^{16} + bc^7x^{15} + 21b^2c^5dx^{15} + 70b^3c^3d^2x^{15} + 35b^4cd^3x^{15} + \frac{7}{2}b^2c^6x^{14} + 35b^3c^4dx^{14} + \frac{105}{2}b^4c^2d^2x^{14} + 7b^5d^3x^{14} + 7b^3c^5x^{13} + 35b^4c^3dx^{13} + 21b^5cd^2x^{13} + \frac{35}{4}b^4c^4x^{12} + 21b^5c^2dx^{12} + \frac{7}{2}b^6d^2x^{12} + 7b^5c^3x^{11} + 7b^6cdx^{11} + \frac{7}{2}b^6c^2x^{10} + b^7dx^{10} + b^7cx^9 + \frac{1}{8}b^8x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c*x^2 + b*x)^7*(3*d*x^2 + 2*c*x + b),x, algorithm="giac")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + b*d^7*x^22 + 7*c^3
*d^5*x^21 + 7*b*c*d^6*x^21 + 35/4*c^4*d^4*x^20 + 21*b*c^2*d^5*x^2
0 + 7/2*b^2*d^6*x^20 + 7*c^5*d^3*x^19 + 35*b*c^3*d^4*x^19 + 21*b^
2*c*d^5*x^19 + 7/2*c^6*d^2*x^18 + 35*b*c^4*d^3*x^18 + 105/2*b^2*c
^2*d^4*x^18 + 7*b^3*d^5*x^18 + c^7*d*x^17 + 21*b*c^5*d^2*x^17 + 7
0*b^2*c^3*d^3*x^17 + 35*b^3*c*d^4*x^17 + 1/8*c^8*x^16 + 7*b*c^6*d
*x^16 + 105/2*b^2*c^4*d^2*x^16 + 70*b^3*c^2*d^3*x^16 + 35/4*b^4*d
^4*x^16 + b*c^7*x^15 + 21*b^2*c^5*d*x^15 + 70*b^3*c^3*d^2*x^15 +
35*b^4*c*d^3*x^15 + 7/2*b^2*c^6*x^14 + 35*b^3*c^4*d*x^14 + 105/2*
b^4*c^2*d^2*x^14 + 7*b^5*d^3*x^14 + 7*b^3*c^5*x^13 + 35*b^4*c^3*d
*x^13 + 21*b^5*c*d^2*x^13 + 35/4*b^4*c^4*x^12 + 21*b^5*c^2*d*x^12
 + 7/2*b^6*d^2*x^12 + 7*b^5*c^3*x^11 + 7*b^6*c*d*x^11 + 7/2*b^6*c
^2*x^10 + b^7*d*x^10 + b^7*c*x^9 + 1/8*b^8*x^8
```

3.194 $\int x^7 \left(b+cx+dx^2\right)^7 \left(b+2cx+3dx^2\right)\, dx$

Optimal. Leaf size=19

$$\frac{1}{8}x^8 \left(b+cx+dx^2\right)^8$$

```
[Out] (x^8*(b + c*x + d*x^2)^8)/8
```

Rubi [A] time = 0.0260799, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$

$$\frac{1}{8}x^8 \left(b+cx+dx^2\right)^8$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(b + c*x + d*x^2)^7*(b + 2*c*x + 3*d*x^2),x]

[Out] (x^8*(b + c*x + d*x^2)^8)/8
```

Rubi in Sympy [A] time = 15.2746, size = 15, normalized size = 0.79

$$\frac{x^8 \left(b+cx+dx^2\right)^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**7*(d*x**2+c*x+b)**7*(3*d*x**2+2*c*x+b),x)

[Out] x**8*(b + c*x + d*x**2)**8/8
```

Mathematica [A] time = 0.0199906, size = 18, normalized size = 0.95

$$\frac{1}{8}x^8(b+x(c+dx))^8$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(b + c*x + d*x^2)^7*(b + 2*c*x + 3*d*x^2),x]

[Out] (x^8*(b + x*(c + d*x))^8)/8
```

Maple [B] time = 0.002, size = 5596, normalized size = 294.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(d*x^2+c*x+b)^7*(3*d*x^2+2*c*x+b),x)

[Out] result too large to display
```

Maxima [A] time = 0.821377, size = 595, normalized size = 31.32

$$\frac{1}{8}\,d^8x^{24}+cd^7x^{23}+\frac{1}{2}\,\left(7\,c^2d^6+2\,bd^7\right)x^{22}+7\,\left(c^3d^5+bcd^6\right)x^{21}+\frac{7}{4}\,\left(5\,c^4d^4+12\,bc^2d^5+2\,b^2d^6\right)x^{20}+7\,\left(c^5d^3+5\,bc^3d^4+3\,b^2cd^5\right)x^{19}+\frac{7}{2}\,\left(c^6d^2+10\,bc^4d^3+15\,b^2c^2d^4+2\,b^3d^5\right)x^{18}+\left(c^7d+21\,bc^5d^2+70\,b^2c^3d^3+35\,b^3cd^4\right)x^{17}+b^7cx^9+\frac{1}{8}\,\left(c^8+56\,bc^6d+420\,b^2c^4d^2+560\,b^3c^2d^3+70\,b^4d^4\right)x^{16}+\frac{1}{8}\,b^8x^8+\left(bc^7+21\,b^2c^5d+70\,b^3c^3d^2+35\,b^4cd^3\right)x^{15}+\frac{7}{2}\,\left(b^2c^6+10\,b^3c^4d+15\,b^4c^2d^2+2\,b^5d^3\right)x^{14}+7\,\left(b^3c^5+5\,b^4c^3d+3\,b^5cd^2\right)x^{13}+\frac{7}{4}\,\left(5\,b^4c^4+12\,b^5c^2d+2\,b^6d^2\right)x^{12}+7\,\left(b^5c^3+b^6cd\right)x^{11}+\frac{1}{2}\,\left(7\,b^6c^2+2\,b^7d\right)x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2 + 2*c*x + b)*(d*x^2 + c*x + b)^7*x^7,x, algorithm="maxima")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 1/2*(7*c^2*d^6 + 2*b*d^7)*x^22 + 7*(c
^3*d^5 + b*c*d^6)*x^21 + 7/4*(5*c^4*d^4 + 12*b*c^2*d^5 + 2*b^2*d^
6)*x^20 + 7*(c^5*d^3 + 5*b*c^3*d^4 + 3*b^2*c*d^5)*x^19 + 7/2*(c^6
*d^2 + 10*b*c^4*d^3 + 15*b^2*c^2*d^4 + 2*b^3*d^5)*x^18 + (c^7*d +
 21*b*c^5*d^2 + 70*b^2*c^3*d^3 + 35*b^3*c*d^4)*x^17 + b^7*c*x^9 +
 1/8*(c^8 + 56*b*c^6*d + 420*b^2*c^4*d^2 + 560*b^3*c^2*d^3 + 70*b
^4*d^4)*x^16 + 1/8*b^8*x^8 + (b*c^7 + 21*b^2*c^5*d + 70*b^3*c^3*d
^2 + 35*b^4*c*d^3)*x^15 + 7/2*(b^2*c^6 + 10*b^3*c^4*d + 15*b^4*c^
2*d^2 + 2*b^5*d^3)*x^14 + 7*(b^3*c^5 + 5*b^4*c^3*d + 3*b^5*c*d^2)
*x^13 + 7/4*(5*b^4*c^4 + 12*b^5*c^2*d + 2*b^6*d^2)*x^12 + 7*(b^5*
c^3 + b^6*c*d)*x^11 + 1/2*(7*b^6*c^2 + 2*b^7*d)*x^10
```

Fricas [A] time = 0.235008, size = 1, normalized size = 0.05

$$\frac{1}{8}x^{24}d^8+x^{23}d^7c+\frac{7}{2}x^{22}d^6c^2+x^{22}d^7b+7x^{21}d^5c^3+7x^{21}d^6cb+\frac{35}{4}x^{20}d^4c^4+21x^{20}d^5c^2b+\frac{7}{2}x^{20}d^6b^2+7x^{19}d^3c^5+35x^{19}d^4c^3b+21x^{19}d^5cb^2+\frac{7}{2}x^{18}d^2c^6+35x^{18}d^3c^4b+\frac{105}{2}x^{18}d^4c^2b^2+7x^{18}d^5b^3+x^{17}dc^7+21x^{17}d^2c^5b+70x^{17}d^3c^3b^2+35x^{17}d^4cb^3+\frac{1}{8}x^{16}c^8+7x^{16}dc^6b+\frac{105}{2}x^{16}d^2c^4b^2+70x^{16}d^3c^2b^3+\frac{35}{4}x^{16}d^4b^4+x^{15}c^7b+21x^{15}dc^5b^2+70x^{15}d^2c^3b^3+35x^{15}d^3cb^4+\frac{7}{2}x^{14}c^6b^2+35x^{14}dc^4b^3+\frac{105}{2}x^{14}d^2c^2b^4+7x^{14}d^3b^5+7x^{13}c^5b^3+35x^{13}dc^3b^4+21x^{13}d^2cb^5+\frac{35}{4}x^{12}c^4b^4+21x^{12}dc^2b^5+\frac{7}{2}x^{12}d^2b^6+7x^{11}c^3b^5+7x^{11}dcb^6+\frac{7}{2}x^{10}c^2b^6+x^{10}db^7+x^9cb^7+\frac{1}{8}x^8b^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2 + 2*c*x + b)*(d*x^2 + c*x + b)^7*x^7,x, algorithm="fricas")
```

```
[Out] 1/8*x^24*d^8 + x^23*d^7*c + 7/2*x^22*d^6*c^2 + x^22*d^7*b + 7*x^2
1*d^5*c^3 + 7*x^21*d^6*c*b + 35/4*x^20*d^4*c^4 + 21*x^20*d^5*c^2*
b + 7/2*x^20*d^6*b^2 + 7*x^19*d^3*c^5 + 35*x^19*d^4*c^3*b + 21*x^
19*d^5*c*b^2 + 7/2*x^18*d^2*c^6 + 35*x^18*d^3*c^4*b + 105/2*x^18*
d^4*c^2*b^2 + 7*x^18*d^5*b^3 + x^17*d*c^7 + 21*x^17*d^2*c^5*b + 7
0*x^17*d^3*c^3*b^2 + 35*x^17*d^4*c*b^3 + 1/8*x^16*c^8 + 7*x^16*d*
c^6*b + 105/2*x^16*d^2*c^4*b^2 + 70*x^16*d^3*c^2*b^3 + 35/4*x^16*
d^4*b^4 + x^15*c^7*b + 21*x^15*d*c^5*b^2 + 70*x^15*d^2*c^3*b^3 +
35*x^15*d^3*c*b^4 + 7/2*x^14*c^6*b^2 + 35*x^14*d*c^4*b^3 + 105/2*
x^14*d^2*c^2*b^4 + 7*x^14*d^3*b^5 + 7*x^13*c^5*b^3 + 35*x^13*d*c^
3*b^4 + 21*x^13*d^2*c*b^5 + 35/4*x^12*c^4*b^4 + 21*x^12*d*c^2*b^5
```

```
 + 7/2*x^12*d^2*b^6 + 7*x^11*c^3*b^5 + 7*x^11*d*c*b^6 + 7/2*x^10*
c^2*b^6 + x^10*d*b^7 + x^9*c*b^7 + 1/8*x^8*b^8
```

Sympy [A] time = 0.408204, size = 469, normalized size = 24.68

$$\frac{b^8x^8}{8}+b^7cx^9+cd^7x^{23}+\frac{d^8x^{24}}{8}+x^{22}\left(bd^7+\frac{7c^2d^6}{2}\right)+x^{21}\left(7bcd^6+7c^3d^5\right)+x^{20}\left(\frac{7b^2d^6}{2}+21bc^2d^5+\frac{35c^4d^4}{4}\right)+x^{19}\left(21b^2cd^5+35bc^3d^4+7c^5d^3\right)+x^{18}\left(7b^3d^5+\frac{105b^2c^2d^4}{2}+35bc^4d^3+\frac{7c^6d^2}{2}\right)+x^{17}\left(35b^3cd^4+70b^2c^3d^3+21bc^5d^2+c^7d\right)+x^{16}\left(\frac{35b^4d^4}{4}+70b^3c^2d^3+\frac{105b^2c^4d^2}{2}+7bc^6d+\frac{c^8}{8}\right)+x^{15}\left(35b^4cd^3+70b^3c^3d^2+21b^2c^5d+bc^7\right)+x^{14}\left(7b^5d^3+\frac{105b^4c^2d^2}{2}+35b^3c^4d+\frac{7b^2c^6}{2}\right)+x^{13}\left(21b^5cd^2+35b^4c^3d+7b^3c^5\right)+x^{12}\left(\frac{7b^6d^2}{2}+21b^5c^2d+\frac{35b^4c^4}{4}\right)+x^{11}\left(7b^6cd+7b^5c^3\right)+x^{10}\left(b^7d+\frac{7b^6c^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**7*(d*x**2+c*x+b)**7*(3*d*x**2+2*c*x+b),x)
```

```
[Out] b**8*x**8/8 + b**7*c*x**9 + c*d**7*x**23 + d**8*x**24/8 + x**22*(
b*d**7 + 7*c**2*d**6/2) + x**21*(7*b*c*d**6 + 7*c**3*d**5) + x**2
0*(7*b**2*d**6/2 + 21*b*c**2*d**5 + 35*c**4*d**4/4) + x**19*(21*b
**2*c*d**5 + 35*b*c**3*d**4 + 7*c**5*d**3) + x**18*(7*b**3*d**5 +
 105*b**2*c**2*d**4/2 + 35*b*c**4*d**3 + 7*c**6*d**2/2) + x**17*(
35*b**3*c*d**4 + 70*b**2*c**3*d**3 + 21*b*c**5*d**2 + c**7*d) + x
**16*(35*b**4*d**4/4 + 70*b**3*c**2*d**3 + 105*b**2*c**4*d**2/2 +
 7*b*c**6*d + c**8/8) + x**15*(35*b**4*c*d**3 + 70*b**3*c**3*d**2
 + 21*b**2*c**5*d + b*c**7) + x**14*(7*b**5*d**3 + 105*b**4*c**2*
d**2/2 + 35*b**3*c**4*d + 7*b**2*c**6/2) + x**13*(21*b**5*c*d**2
+ 35*b**4*c**3*d + 7*b**3*c**5) + x**12*(7*b**6*d**2/2 + 21*b**5*
c**2*d + 35*b**4*c**4/4) + x**11*(7*b**6*c*d + 7*b**5*c**3) + x**
10*(b**7*d + 7*b**6*c**2/2)
```

GIAC/XCAS [A] time = 0.260222, size = 670, normalized size = 35.26

$$\frac{1}{8}d^8x^{24}+cd^7x^{23}+\frac{7}{2}c^2d^6x^{22}+bd^7x^{22}+7c^3d^5x^{21}+7bcd^6x^{21}+\frac{35}{4}c^4d^4x^{20}+21bc^2d^5x^{20}+\frac{7}{2}b^2d^6x^{20}+7c^5d^3x^{19}+35bc^3d^4x^{19}+21b^2cd^5x^{19}+\frac{7}{2}c^6d^2x^{18}+35bc^4d^3x^{18}+\frac{105}{2}b^2c^2d^4x^{18}+7b^3d^5x^{18}+c^7dx^{17}+21bc^5d^2x^{17}+70b^2c^3d^3x^{17}+35b^3cd^4x^{17}+\frac{1}{8}c^8x^{16}+7bc^6dx^{16}+\frac{105}{2}b^2c^4d^2x^{16}+70b^3c^2d^3x^{16}+\frac{35}{4}b^4d^4x^{16}+bc^7x^{15}+21b^2c^5dx^{15}+70b^3c^3d^2x^{15}+35b^4cd^3x^{15}+\frac{7}{2}b^2c^6x^{14}+35b^3c^4dx^{14}+\frac{105}{2}b^4c^2d^2x^{14}+7b^5d^3x^{14}+7b^3c^5x^{13}+35b^4c^3dx^{13}+21b^5cd^2x^{13}+\frac{35}{4}b^4c^4x^{12}+21b^5c^2dx^{12}+\frac{7}{2}b^6d^2x^{12}+7b^5c^3x^{11}+7b^6cdx^{11}+\frac{7}{2}b^6c^2x^{10}+b^7dx^{10}+b^7cx^9+\frac{1}{8}b^8x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*d*x^2 + 2*c*x + b)*(d*x^2 + c*x + b)^7*x^7,x, algorithm="giac")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + b*d^7*x^22 + 7*c^3
*d^5*x^21 + 7*b*c*d^6*x^21 + 35/4*c^4*d^4*x^20 + 21*b*c^2*d^5*x^2
0 + 7/2*b^2*d^6*x^20 + 7*c^5*d^3*x^19 + 35*b*c^3*d^4*x^19 + 21*b^
```

```
2*c*d^5*x^19 + 7/2*c^6*d^2*x^18 + 35*b*c^4*d^3*x^18 + 105/2*b^2*c
^2*d^4*x^18 + 7*b^3*d^5*x^18 + c^7*d*x^17 + 21*b*c^5*d^2*x^17 + 7
0*b^2*c^3*d^3*x^17 + 35*b^3*c*d^4*x^17 + 1/8*c^8*x^16 + 7*b*c^6*d
*x^16 + 105/2*b^2*c^4*d^2*x^16 + 70*b^3*c^2*d^3*x^16 + 35/4*b^4*d
^4*x^16 + b*c^7*x^15 + 21*b^2*c^5*d*x^15 + 70*b^3*c^3*d^2*x^15 +
35*b^4*c*d^3*x^15 + 7/2*b^2*c^6*x^14 + 35*b^3*c^4*d*x^14 + 105/2*
b^4*c^2*d^2*x^14 + 7*b^5*d^3*x^14 + 7*b^3*c^5*x^13 + 35*b^4*c^3*d
*x^13 + 21*b^5*c*d^2*x^13 + 35/4*b^4*c^4*x^12 + 21*b^5*c^2*d*x^12
 + 7/2*b^6*d^2*x^12 + 7*b^5*c^3*x^11 + 7*b^6*c*d*x^11 + 7/2*b^6*c
^2*x^10 + b^7*d*x^10 + b^7*c*x^9 + 1/8*b^8*x^8
```

3.195 $\int (b + 3dx^2) (a + bx + dx^3)^7 \, dx$

Optimal. Leaf size=16

$$\frac{1}{8} (a + bx + dx^3)^8$$

```
[Out] (a + b*x + d*x^3)^8/8
```

Rubi [A] time = 0.0196703, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$

$$\frac{1}{8} (a + bx + dx^3)^8$$

Antiderivative was successfully verified.

```
[In]  Int[(b + 3*d*x^2)*(a + b*x + d*x^3)^7,x]

[Out] (a + b*x + d*x^3)^8/8
```

Rubi in Sympy [A] time = 5.26819, size = 12, normalized size = 0.75

$$\frac{(a + bx + dx^3)^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*d*x**2+b)*(d*x**3+b*x+a)**7,x)

[Out] (a + b*x + d*x**3)**8/8
```

Mathematica [B] time = 0.105953, size = 127, normalized size = 7.94

$$\frac{1}{8}x (b + dx^2) \left(8a^7 + 28a^6x (b + dx^2) + 56a^5x^2 (b + dx^2)^2 + 70a^4x^3 (b + dx^2)^3 + 56a^3x^4 (b + dx^2)^4 + 28a^2x^5 (b + dx^2)^5 + 8ax^6 (b + dx^2)^6 + x^7 (b + dx^2)^7\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b + 3*d*x^2)*(a + b*x + d*x^3)^7,x]

[Out] (x*(b + d*x^2)*(8*a^7 + 28*a^6*x*(b + d*x^2) + 56*a^5*x^2*(b + d*
x^2)^2 + 70*a^4*x^3*(b + d*x^2)^3 + 56*a^3*x^4*(b + d*x^2)^4 + 28
*a^2*x^5*(b + d*x^2)^5 + 8*a*x^6*(b + d*x^2)^6 + x^7*(b + d*x^2)^
7))/8
```

Maple [B] time = 0.004, size = 2185, normalized size = 136.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*d*x^2+b)*(d*x^3+b*x+a)^7,x)
```

```
[Out] 1/8*d^8*x^24+b*d^7*x^22+d^7*a*x^21+7/2*b^2*d^6*x^20+7*b*a*d^6*x^1
9+1/18*(21*b^3*d^5+3*d*(6*a^2*d^5+15*b^3*d^4+d*(2*(3*a^2*d+b^3)*d
^3+18*b^3*d^3+9*a^2*d^4)))*x^18+21*b^2*a*d^5*x^17+1/16*(b*(6*a^2*
d^5+15*b^3*d^4+d*(2*(3*a^2*d+b^3)*d^3+18*b^3*d^3+9*a^2*d^4))+3*d*
(30*a^2*b*d^4+b*(2*(3*a^2*d+b^3)*d^3+18*b^3*d^3+9*a^2*d^4)+d*(42*
a^2*b*d^3+6*(3*a^2*d+b^3)*b*d^2+9*b^4*d^2)))*x^16+1/15*(105*b^3*a
*d^4+3*d*(a*(2*(3*a^2*d+b^3)*d^3+18*b^3*d^3+9*a^2*d^4)+60*b^3*a*d
^3+d*(2*a^3*d^3+54*a*b^3*d^2+6*(3*a^2*d+b^3)*a*d^2)))*x^15+1/14*(
b*(30*a^2*b*d^4+b*(2*(3*a^2*d+b^3)*d^3+18*b^3*d^3+9*a^2*d^4)+d*(4
2*a^2*b*d^3+6*(3*a^2*d+b^3)*b*d^2+9*b^4*d^2))+3*d*(60*a^2*b^2*d^3
+b*(42*a^2*b*d^3+6*(3*a^2*d+b^3)*b*d^2+9*b^4*d^2)+d*(72*a^2*b^2*d
^2+6*(3*a^2*d+b^3)*b^2*d)))*x^14+1/13*(b*(a*(2*(3*a^2*d+b^3)*d^3+
18*b^3*d^3+9*a^2*d^4)+60*b^3*a*d^3+d*(2*a^3*d^3+54*a*b^3*d^2+6*(3
*a^2*d+b^3)*a*d^2))+3*d*(a*(42*a^2*b*d^3+6*(3*a^2*d+b^3)*b*d^2+9*
b^4*d^2)+b*(2*a^3*d^3+54*a*b^3*d^2+6*(3*a^2*d+b^3)*a*d^2)+d*(24*a
^3*b*d^2+18*a*b^4*d+12*(3*a^2*d+b^3)*a*b*d)))*x^13+1/12*(b*(60*a^
2*b^2*d^3+b*(42*a^2*b*d^3+6*(3*a^2*d+b^3)*b*d^2+9*b^4*d^2)+d*(72*
a^2*b^2*d^2+6*(3*a^2*d+b^3)*b^2*d))+3*d*(a*(2*a^3*d^3+54*a*b^3*d^
2+6*(3*a^2*d+b^3)*a*d^2)+b*(72*a^2*b^2*d^2+6*(3*a^2*d+b^3)*b^2*d)
+d*(6*a^4*d^2+54*a^2*b^3*d+(3*a^2*d+b^3)^2)))*x^12+1/11*(b*(a*(42
*a^2*b*d^3+6*(3*a^2*d+b^3)*b*d^2+9*b^4*d^2)+b*(2*a^3*d^3+54*a*b^3
*d^2+6*(3*a^2*d+b^3)*a*d^2)+d*(24*a^3*b*d^2+18*a*b^4*d+12*(3*a^2*
d+b^3)*a*b*d))+3*d*(a*(72*a^2*b^2*d^2+6*(3*a^2*d+b^3)*b^2*d)+b*(2
4*a^3*b*d^2+18*a*b^4*d+12*(3*a^2*d+b^3)*a*b*d)+d*(42*a^3*b^2*d+6*
a*b^2*(3*a^2*d+b^3))))*x^11+1/10*(b*(a*(2*a^3*d^3+54*a*b^3*d^2+6*
(3*a^2*d+b^3)*a*d^2)+b*(72*a^2*b^2*d^2+6*(3*a^2*d+b^3)*b^2*d)+d*(
6*a^4*d^2+54*a^2*b^3*d+(3*a^2*d+b^3)^2))+3*d*(a*(24*a^3*b*d^2+18*
a*b^4*d+12*(3*a^2*d+b^3)*a*b*d)+b*(6*a^4*d^2+54*a^2*b^3*d+(3*a^2*
d+b^3)^2)+d*(12*a^4*b*d+6*a^2*b*(3*a^2*d+b^3)+9*a^2*b^4)))*x^10+1
/9*(b*(a*(72*a^2*b^2*d^2+6*(3*a^2*d+b^3)*b^2*d)+b*(24*a^3*b*d^2+1
8*a*b^4*d+12*(3*a^2*d+b^3)*a*b*d)+d*(42*a^3*b^2*d+6*a*b^2*(3*a^2*
d+b^3)))+3*d*(a*(6*a^4*d^2+54*a^2*b^3*d+(3*a^2*d+b^3)^2)+b*(42*a^
3*b^2*d+6*a*b^2*(3*a^2*d+b^3))+d*(2*a^3*(3*a^2*d+b^3)+18*a^3*b^3)
))*x^9+1/8*(b*(a*(24*a^3*b*d^2+18*a*b^4*d+12*(3*a^2*d+b^3)*a*b*d)
+b*(6*a^4*d^2+54*a^2*b^3*d+(3*a^2*d+b^3)^2)+d*(12*a^4*b*d+6*a^2*b
*(3*a^2*d+b^3)+9*a^2*b^4))+3*d*(a*(42*a^3*b^2*d+6*a*b^2*(3*a^2*d+
b^3))+b*(12*a^4*b*d+6*a^2*b*(3*a^2*d+b^3)+9*a^2*b^4)+15*d*a^4*b^2
))*x^8+1/7*(b*(a*(6*a^4*d^2+54*a^2*b^3*d+(3*a^2*d+b^3)^2)+b*(42*a
^3*b^2*d+6*a*b^2*(3*a^2*d+b^3))+d*(2*a^3*(3*a^2*d+b^3)+18*a^3*b^3
))+3*d*(a*(12*a^4*b*d+6*a^2*b*(3*a^2*d+b^3)+9*a^2*b^4)+b*(2*a^3*(
3*a^2*d+b^3)+18*a^3*b^3)+6*d*a^5*b))*x^7+1/6*(b*(a*(42*a^3*b^2*d+
6*a*b^2*(3*a^2*d+b^3))+b*(12*a^4*b*d+6*a^2*b*(3*a^2*d+b^3)+9*a^2*
b^4)+15*d*a^4*b^2)+3*d*(a*(2*a^3*(3*a^2*d+b^3)+18*a^3*b^3)+15*b^3
*a^4+d*a^6))*x^6+1/5*(b*(a*(12*a^4*b*d+6*a^2*b*(3*a^2*d+b^3)+9*a^
2*b^4)+b*(2*a^3*(3*a^2*d+b^3)+18*a^3*b^3)+6*d*a^5*b)+63*d*a^5*b^2
)*x^5+1/4*(b*(a*(2*a^3*(3*a^2*d+b^3)+18*a^3*b^3)+15*b^3*a^4+d*a^6
)+21*d*a^6*b)*x^4+1/3*(3*a^7*d+21*a^5*b^3)*x^3+7/2*b^2*a^6*x^2+b*
a^7*x
```

Maxima [A] time = 0.810246, size = 19, normalized size = 1.19

$$\frac{1}{8}\left(dx^3 + bx + a\right)^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + b*x + a)^7*(3*d*x^2 + b),x, algorithm="maxima")
```

```
[Out] 1/8*(d*x^3 + b*x + a)^8
```

Fricas [A] time = 0.233856, size = 1, normalized size = 0.06

$$\frac{1}{8}x^{24}d^8 + x^{22}d^7b + x^{21}d^7a + \frac{7}{2}x^{20}d^6b^2 + 7x^{19}d^6ba + 7x^{18}d^5b^3 + \frac{7}{2}x^{18}d^6a^2 + 21x^{17}d^5b^2a + \frac{35}{4}x^{16}d^4b^4 + 21x^{16}d^5ba^2 + 35x^{15}d^4b^3a + 7x^{15}d^5a^3 + 7x^{14}d^3b^5 + \frac{105}{2}x^{14}d^4b^2a^2 + 35x^{13}d^3b^4a + 35x^{13}d^4ba^3 + \frac{7}{2}x^{12}d^2b^6 + 70x^{12}d^3b^3a^2 + \frac{35}{4}x^{12}d^4a^4 + 21x^{11}d^2b^5a + 70x^{11}d^3b^2a^3 + x^{10}db^7 + \frac{105}{2}x^{10}d^2b^4a^2 + 35x^{10}d^3ba^4 + 7x^9db^6a + 70x^9d^2b^3a^3 + 7x^9d^3a^5 + \frac{1}{8}x^8b^8 + 21x^8db^5a^2 + \frac{105}{2}x^8d^2b^2a^4 + x^7b^7a + 35x^7db^4a^3 + 21x^7d^2ba^5 + \frac{7}{2}x^6b^6a^2 + 35x^6db^3a^4 + \frac{7}{2}x^6d^2a^6 + 7x^5b^5a^3 + 21x^5db^2a^5 + \frac{35}{4}x^4b^4a^4 + 7x^4dba^6 + 7x^3b^3a^5 + x^3da^7 + \frac{7}{2}x^2b^2a^6 + xba^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + b*x + a)^7*(3*d*x^2 + b),x, algorithm="fricas")
```

```
[Out] 1/8*x^24*d^8 + x^22*d^7*b + x^21*d^7*a + 7/2*x^20*d^6*b^2 + 7*x^1
9*d^6*b*a + 7*x^18*d^5*b^3 + 7/2*x^18*d^6*a^2 + 21*x^17*d^5*b^2*a
 + 35/4*x^16*d^4*b^4 + 21*x^16*d^5*b*a^2 + 35*x^15*d^4*b^3*a + 7*
x^15*d^5*a^3 + 7*x^14*d^3*b^5 + 105/2*x^14*d^4*b^2*a^2 + 35*x^13*
d^3*b^4*a + 35*x^13*d^4*b*a^3 + 7/2*x^12*d^2*b^6 + 70*x^12*d^3*b^
3*a^2 + 35/4*x^12*d^4*a^4 + 21*x^11*d^2*b^5*a + 70*x^11*d^3*b^2*a
^3 + x^10*d*b^7 + 105/2*x^10*d^2*b^4*a^2 + 35*x^10*d^3*b*a^4 + 7*
x^9*d*b^6*a + 70*x^9*d^2*b^3*a^3 + 7*x^9*d^3*a^5 + 1/8*x^8*b^8 +
21*x^8*d*b^5*a^2 + 105/2*x^8*d^2*b^2*a^4 + x^7*b^7*a + 35*x^7*d*b
^4*a^3 + 21*x^7*d^2*b*a^5 + 7/2*x^6*b^6*a^2 + 35*x^6*d*b^3*a^4 +
7/2*x^6*d^2*a^6 + 7*x^5*b^5*a^3 + 21*x^5*d*b^2*a^5 + 35/4*x^4*b^4
*a^4 + 7*x^4*d*b*a^6 + 7*x^3*b^3*a^5 + x^3*d*a^7 + 7/2*x^2*b^2*a^
6 + x*b*a^7
```

Sympy [A] time = 0.410978, size = 483, normalized size = 30.19

$$a^7bx + \frac{7a^6b^2x^2}{2} + 21ab^2d^5x^{17} + 7abd^6x^{19} + ad^7x^{21} + \frac{7b^2d^6x^{20}}{2} + bd^7x^{22} + \frac{d^8x^{24}}{8} + x^{18}\left(\frac{7a^2d^6}{2} + 7b^3d^5\right) + x^{16}\left(21a^2bd^5 + \frac{35b^4d^4}{4}\right) + x^{15}\left(7a^3d^5 + 35ab^3d^4\right) + x^{14}\left(\frac{105a^2b^2d^4}{2} + 7b^5d^3\right) + x^{13}\left(35a^3bd^4 + 35ab^4d^3\right) + x^{12}\left(\frac{35a^4d^4}{4} + 70a^2b^3d^3 + \frac{7b^6d^2}{2}\right) + x^{11}\left(70a^3b^2d^3 + 21ab^5d^2\right) + x^{10}\left(35a^4bd^3 + \frac{105a^2b^4d^2}{2} + b^7d\right) + x^9\left(7a^5d^3 + 70a^3b^3d^2 + 7ab^6d\right) + x^8\left(\frac{105a^4b^2d^2}{2} + 21a^2b^5d + \frac{b^8}{8}\right) + x^7\left(21a^5bd^2 + 35a^3b^4d + ab^7\right) + x^6\left(\frac{7a^6d^2}{2} + 35a^4b^3d + \frac{7a^2b^6}{2}\right) + x^5\left(21a^5b^2d + 7a^3b^5\right) + x^4\left(7a^6bd + \frac{35a^4b^4}{4}\right) + x^3\left(a^7d + 7a^5b^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*d*x**2+b)*(d*x**3+b*x+a)**7,x)
```

```
[Out] a**7*b*x + 7*a**6*b**2*x**2/2 + 21*a*b**2*d**5*x**17 + 7*a*b*d**6
*x**19 + a*d**7*x**21 + 7*b**2*d**6*x**20/2 + b*d**7*x**22 + d**8
*x**24/8 + x**18*(7*a**2*d**6/2 + 7*b**3*d**5) + x**16*(21*a**2*b
*d**5 + 35*b**4*d**4/4) + x**15*(7*a**3*d**5 + 35*a*b**3*d**4) +
x**14*(105*a**2*b**2*d**4/2 + 7*b**5*d**3) + x**13*(35*a**3*b*d**
4 + 35*a*b**4*d**3) + x**12*(35*a**4*d**4/4 + 70*a**2*b**3*d**3 +
 7*b**6*d**2/2) + x**11*(70*a**3*b**2*d**3 + 21*a*b**5*d**2) + x*
*10*(35*a**4*b*d**3 + 105*a**2*b**4*d**2/2 + b**7*d) + x**9*(7*a*
*5*d**3 + 70*a**3*b**3*d**2 + 7*a*b**6*d) + x**8*(105*a**4*b**2*d
**2/2 + 21*a**2*b**5*d + b**8/8) + x**7*(21*a**5*b*d**2 + 35*a**3
*b**4*d + a*b**7) + x**6*(7*a**6*d**2/2 + 35*a**4*b**3*d + 7*a**2
```

```
*b**6/2) + x**5*(21*a**5*b**2*d + 7*a**3*b**5) + x**4*(7*a**6*b*d
 + 35*a**4*b**4/4) + x**3*(a**7*d + 7*a**5*b**3)
```

GIAC/XCAS [A] time = 0.26009, size = 656, normalized size = 41.

$$\frac{1}{8}d^8x^{24} + bd^7x^{22} + ad^7x^{21} + \frac{7}{2}b^2d^6x^{20} + 7\,abd^6x^{19} + 7\,b^3d^5x^{18} + \frac{7}{2}a^2d^6x^{18} + 21\,ab^2d^5x^{17} + \frac{35}{4}b^4d^4x^{16} + 21\,a^2bd^5x^{16} + 35\,ab^3d^4x^{15} + 7\,a^3d^5x^{15} + 7\,b^5d^3x^{14} + \frac{105}{2}a^2b^2d^4x^{14} + 35\,ab^4d^3x^{13} + 35\,a^3bd^4x^{13} + \frac{7}{2}b^6d^2x^{12} + 70\,a^2b^3d^3x^{12} + \frac{35}{4}a^4d^4x^{12} + 21\,ab^5d^2x^{11} + 70\,a^3b^2d^3x^{11} + b^7dx^{10} + \frac{105}{2}a^2b^4d^2x^{10} + 35\,a^4bd^3x^{10} + 7\,ab^6dx^9 + 70\,a^3b^3d^2x^9 + 7\,a^5d^3x^9 + \frac{1}{8}b^8x^8 + 21\,a^2b^5dx^8 + \frac{105}{2}a^4b^2d^2x^8 + ab^7x^7 + 35\,a^3b^4dx^7 + 21\,a^5bd^2x^7 + \frac{7}{2}a^2b^6x^6 + 35\,a^4b^3dx^6 + \frac{7}{2}a^6d^2x^6 + 7\,a^3b^5x^5 + 21\,a^5b^2dx^5 + \frac{35}{4}a^4b^4x^4 + 7\,a^6bdx^4 + 7\,a^5b^3x^3 + a^7dx^3 + \frac{7}{2}a^6b^2x^2 + a^7bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + b*x + a)^7*(3*d*x^2 + b),x, algorithm="giac")
```

```
[Out] 1/8*d^8*x^24 + b*d^7*x^22 + a*d^7*x^21 + 7/2*b^2*d^6*x^20 + 7*a*b
*d^6*x^19 + 7*b^3*d^5*x^18 + 7/2*a^2*d^6*x^18 + 21*a*b^2*d^5*x^17
 + 35/4*b^4*d^4*x^16 + 21*a^2*b*d^5*x^16 + 35*a*b^3*d^4*x^15 + 7*
a^3*d^5*x^15 + 7*b^5*d^3*x^14 + 105/2*a^2*b^2*d^4*x^14 + 35*a*b^4
*d^3*x^13 + 35*a^3*b*d^4*x^13 + 7/2*b^6*d^2*x^12 + 70*a^2*b^3*d^3
*x^12 + 35/4*a^4*d^4*x^12 + 21*a*b^5*d^2*x^11 + 70*a^3*b^2*d^3*x^
11 + b^7*d*x^10 + 105/2*a^2*b^4*d^2*x^10 + 35*a^4*b*d^3*x^10 + 7*
a*b^6*d*x^9 + 70*a^3*b^3*d^2*x^9 + 7*a^5*d^3*x^9 + 1/8*b^8*x^8 +
21*a^2*b^5*d*x^8 + 105/2*a^4*b^2*d^2*x^8 + a*b^7*x^7 + 35*a^3*b^4
*d*x^7 + 21*a^5*b*d^2*x^7 + 7/2*a^2*b^6*x^6 + 35*a^4*b^3*d*x^6 +
7/2*a^6*d^2*x^6 + 7*a^3*b^5*x^5 + 21*a^5*b^2*d*x^5 + 35/4*a^4*b^4
*x^4 + 7*a^6*b*d*x^4 + 7*a^5*b^3*x^3 + a^7*d*x^3 + 7/2*a^6*b^2*x^
2 + a^7*b*x
```

3.196 $\int x^7 \left(b+dx^2\right)^7 \left(b+3dx^2\right)\,dx$

Optimal. Leaf size=16

$$\frac{1}{8}x^8\left(b+dx^2\right)^8$$

```
[Out] (x^8*(b + d*x^2)^8)/8
```

Rubi [A] time = 0.0143733, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$

$$\frac{1}{8}x^8\left(b+dx^2\right)^8$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(b + d*x^2)^7*(b + 3*d*x^2),x]
```

```
[Out] (x^8*(b + d*x^2)^8)/8
```

Rubi in Sympy [A] time = 10.4714, size = 12, normalized size = 0.75

$$\frac{x^8\left(b+dx^2\right)^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**7*(d*x**2+b)**7*(3*d*x**2+b),x)
```

```
[Out] x**8*(b + d*x**2)**8/8
```

Mathematica [B] time = 0.00633022, size = 98, normalized size = 6.12

$$\frac{b^8x^8}{8}+b^7dx^{10}+\frac{7}{2}b^6d^2x^{12}+7b^5d^3x^{14}+\frac{35}{4}b^4d^4x^{16}+7b^3d^5x^{18}+\frac{7}{2}b^2d^6x^{20}+bd^7x^{22}+\frac{d^8x^{24}}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(b + d*x^2)^7*(b + 3*d*x^2),x]
```

```
[Out] (b^8*x^8)/8 + b^7*d*x^10 + (7*b^6*d^2*x^12)/2 + 7*b^5*d^3*x^14 +
(35*b^4*d^4*x^16)/4 + 7*b^3*d^5*x^18 + (7*b^2*d^6*x^20)/2 + b*d^7
*x^22 + (d^8*x^24)/8
```

Maple [B] time = 0.002, size = 89, normalized size = 5.6

$$\frac{d^8x^{24}}{8}+bd^7x^{22}+\frac{7\,b^2d^6x^{20}}{2}+7\,b^3d^5x^{18}+\frac{35\,b^4d^4x^{16}}{4}+7\,b^5d^3x^{14}+\frac{7\,b^6d^2x^{12}}{2}+db^7x^{10}+\frac{b^8x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(d*x^2+b)^7*(3*d*x^2+b),x)
```

```
[Out] 1/8*d^8*x^24+b*d^7*x^22+7/2*b^2*d^6*x^20+7*b^3*d^5*x^18+35/4*b^4*
d^4*x^16+7*b^5*d^3*x^14+7/2*b^6*d^2*x^12+d*b^7*x^10+1/8*b^8*x^8
```

Maxima [A] time = 0.819165, size = 119, normalized size = 7.44

$$\frac{1}{8}d^8x^{24} + bd^7x^{22} + \frac{7}{2}b^2d^6x^{20} + 7b^3d^5x^{18} + \frac{35}{4}b^4d^4x^{16} + 7b^5d^3x^{14} + \frac{7}{2}b^6d^2x^{12} + b^7dx^{10} + \frac{1}{8}b^8x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*d*x^2 + b)*(d*x^2 + b)^7*x^7,x, algorithm="maxima")
```

```
[Out] 1/8*d^8*x^24 + b*d^7*x^22 + 7/2*b^2*d^6*x^20 + 7*b^3*d^5*x^18 + 3
5/4*b^4*d^4*x^16 + 7*b^5*d^3*x^14 + 7/2*b^6*d^2*x^12 + b^7*d*x^10
 + 1/8*b^8*x^8
```

Fricas [A] time = 0.230134, size = 1, normalized size = 0.06

$$\frac{1}{8}x^{24}d^8 + x^{22}d^7b + \frac{7}{2}x^{20}d^6b^2 + 7x^{18}d^5b^3 + \frac{35}{4}x^{16}d^4b^4 + 7x^{14}d^3b^5 + \frac{7}{2}x^{12}d^2b^6 + x^{10}db^7 + \frac{1}{8}x^8b^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*d*x^2 + b)*(d*x^2 + b)^7*x^7,x, algorithm="fricas")
```

```
[Out] 1/8*x^24*d^8 + x^22*d^7*b + 7/2*x^20*d^6*b^2 + 7*x^18*d^5*b^3 + 3
5/4*x^16*d^4*b^4 + 7*x^14*d^3*b^5 + 7/2*x^12*d^2*b^6 + x^10*d*b^7
 + 1/8*x^8*b^8
```

Sympy [A] time = 0.161812, size = 97, normalized size = 6.06

$$\frac{b^8x^8}{8} + b^7dx^{10} + \frac{7b^6d^2x^{12}}{2} + 7b^5d^3x^{14} + \frac{35b^4d^4x^{16}}{4} + 7b^3d^5x^{18} + \frac{7b^2d^6x^{20}}{2} + bd^7x^{22} + \frac{d^8x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**7*(d*x**2+b)**7*(3*d*x**2+b),x)
```

```
[Out] b**8*x**8/8 + b**7*d*x**10 + 7*b**6*d**2*x**12/2 + 7*b**5*d**3*x*
*14 + 35*b**4*d**4*x**16/4 + 7*b**3*d**5*x**18 + 7*b**2*d**6*x**2
0/2 + b*d**7*x**22 + d**8*x**24/8
```

GIAC/XCAS [A] time = 0.260601, size = 119, normalized size = 7.44

$$\frac{1}{8}d^8x^{24} + bd^7x^{22} + \frac{7}{2}b^2d^6x^{20} + 7b^3d^5x^{18} + \frac{35}{4}b^4d^4x^{16} + 7b^5d^3x^{14} + \frac{7}{2}b^6d^2x^{12} + b^7dx^{10} + \frac{1}{8}b^8x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*d*x^2 + b)*(d*x^2 + b)^7*x^7,x, algorithm="giac")
```

```
[Out] 1/8*d^8*x^24 + b*d^7*x^22 + 7/2*b^2*d^6*x^20 + 7*b^3*d^5*x^18 + 3
5/4*b^4*d^4*x^16 + 7*b^5*d^3*x^14 + 7/2*b^6*d^2*x^12 + b^7*d*x^10
 + 1/8*b^8*x^8
```

3.197 $\int (b + 3dx^2) (bx + dx^3)^7 \, dx$

Optimal. Leaf size=15

$$\frac{1}{8}(bx + dx^3)^8$$

```
[Out] (b*x + d*x^3)^8/8
```

Rubi [A] time = 0.0135343, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.05$

$$\frac{1}{8}(bx + dx^3)^8$$

Antiderivative was successfully verified.

```
[In] Int[(b + 3*d*x^2)*(b*x + d*x^3)^7,x]
```

```
[Out] (b*x + d*x^3)^8/8
```

Rubi in Sympy [A] time = 12.0598, size = 12, normalized size = 0.8

$$\frac{x^8 (b + dx^2)^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((3*d*x**2+b)*(d*x**3+b*x)**7,x)
```

```
[Out] x**8*(b + d*x**2)**8/8
```

Mathematica [B] time = 0.00441641, size = 98, normalized size = 6.53

$$\frac{b^8x^8}{8} + b^7dx^{10} + \frac{7}{2}b^6d^2x^{12} + 7b^5d^3x^{14} + \frac{35}{4}b^4d^4x^{16} + 7b^3d^5x^{18} + \frac{7}{2}b^2d^6x^{20} + bd^7x^{22} + \frac{d^8x^{24}}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 3*d*x^2)*(b*x + d*x^3)^7,x]
```

```
[Out] (b^8*x^8)/8 + b^7*d*x^10 + (7*b^6*d^2*x^12)/2 + 7*b^5*d^3*x^14 +
(35*b^4*d^4*x^16)/4 + 7*b^3*d^5*x^18 + (7*b^2*d^6*x^20)/2 + b*d^7
*x^22 + (d^8*x^24)/8
```

Maple [B] time = 0.002, size = 89, normalized size = 5.9

$$\frac{d^8x^{24}}{8} + bd^7x^{22} + \frac{7\,b^2d^6x^{20}}{2} + 7\,b^3d^5x^{18} + \frac{35\,b^4d^4x^{16}}{4} + 7\,b^5d^3x^{14} + \frac{7\,b^6d^2x^{12}}{2} + db^7x^{10} + \frac{b^8x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*d*x^2+b)*(d*x^3+b*x)^7,x)
```

```
[Out] 1/8*d^8*x^24+b*d^7*x^22+7/2*b^2*d^6*x^20+7*b^3*d^5*x^18+35/4*b^4*
d^4*x^16+7*b^5*d^3*x^14+7/2*b^6*d^2*x^12+d*b^7*x^10+1/8*b^8*x^8
```

Maxima [A] time = 0.839403, size = 18, normalized size = 1.2

$$\frac{1}{8}\left(dx^3 + bx\right)^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + b*x)^7*(3*d*x^2 + b),x, algorithm="maxima")
```

```
[Out] 1/8*(d*x^3 + b*x)^8
```

Fricas [A] time = 0.236185, size = 1, normalized size = 0.07

$$\frac{1}{8}x^{24}d^8 + x^{22}d^7b + \frac{7}{2}x^{20}d^6b^2 + 7x^{18}d^5b^3 + \frac{35}{4}x^{16}d^4b^4 + 7x^{14}d^3b^5 + \frac{7}{2}x^{12}d^2b^6 + x^{10}db^7 + \frac{1}{8}x^8b^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + b*x)^7*(3*d*x^2 + b),x, algorithm="fricas")
```

```
[Out] 1/8*x^24*d^8 + x^22*d^7*b + 7/2*x^20*d^6*b^2 + 7*x^18*d^5*b^3 + 3
5/4*x^16*d^4*b^4 + 7*x^14*d^3*b^5 + 7/2*x^12*d^2*b^6 + x^10*d*b^7
 + 1/8*x^8*b^8
```

Sympy [A] time = 0.181643, size = 97, normalized size = 6.47

$$\frac{b^8x^8}{8} + b^7dx^{10} + \frac{7b^6d^2x^{12}}{2} + 7b^5d^3x^{14} + \frac{35b^4d^4x^{16}}{4} + 7b^3d^5x^{18} + \frac{7b^2d^6x^{20}}{2} + bd^7x^{22} + \frac{d^8x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*d*x**2+b)*(d*x**3+b*x)**7,x)
```

```
[Out] b**8*x**8/8 + b**7*d*x**10 + 7*b**6*d**2*x**12/2 + 7*b**5*d**3*x*
*14 + 35*b**4*d**4*x**16/4 + 7*b**3*d**5*x**18 + 7*b**2*d**6*x**2
0/2 + b*d**7*x**22 + d**8*x**24/8
```

GIAC/XCAS [A] time = 0.261197, size = 119, normalized size = 7.93

$$\frac{1}{8}d^8x^{24} + bd^7x^{22} + \frac{7}{2}b^2d^6x^{20} + 7b^3d^5x^{18} + \frac{35}{4}b^4d^4x^{16} + 7b^5d^3x^{14} + \frac{7}{2}b^6d^2x^{12} + b^7dx^{10} + \frac{1}{8}b^8x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + b*x)^7*(3*d*x^2 + b),x, algorithm="giac")
```

```
[Out] 1/8*d^8*x^24 + b*d^7*x^22 + 7/2*b^2*d^6*x^20 + 7*b^3*d^5*x^18 + 3
5/4*b^4*d^4*x^16 + 7*b^5*d^3*x^14 + 7/2*b^6*d^2*x^12 + b^7*d*x^10
 + 1/8*b^8*x^8
```

3.198 $\int \left(2cx+3dx^2\right) \left(a+cx^2+dx^3\right)^7 \, dx$

Optimal. Leaf size=18

$$\frac{1}{8} \left(a+cx^2+dx^3\right)^8$$

```
[Out] (a + c*x^2 + d*x^3)^8/8
```

Rubi [A] time = 0.021451, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038

$$\frac{1}{8} \left(a+cx^2+dx^3\right)^8$$

Antiderivative was successfully verified.

```
[In] Int[(2*c*x + 3*d*x^2)*(a + c*x^2 + d*x^3)^7,x]

[Out] (a + c*x^2 + d*x^3)^8/8
```

Rubi in Sympy [A] time = 7.84985, size = 14, normalized size = 0.78

$$\frac{\left(a+cx^2+dx^3\right)^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((3*d*x**2+2*c*x)*(d*x**3+c*x**2+a)**7,x)

[Out] (a + c*x**2 + d*x**3)**8/8
```

Mathematica [B] time = 0.110865, size = 115, normalized size = 6.39

$$\frac{1}{8}x^2(c+dx)\left(8a^7+28a^6x^2(c+dx)+56a^5x^4(c+dx)^2+70a^4x^6(c+dx)^3+56a^3x^8(c+dx)^4+28a^2x^{10}(c+dx)^5+8ax^{12}(c+dx)^6+x^{14}(c+dx)^7\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*c*x + 3*d*x^2)*(a + c*x^2 + d*x^3)^7,x]

[Out] (x^2*(c + d*x)*(8*a^7 + 28*a^6*x^2*(c + d*x) + 56*a^5*x^4*(c + d*
x)^2 + 70*a^4*x^6*(c + d*x)^3 + 56*a^3*x^8*(c + d*x)^4 + 28*a^2*x
^10*(c + d*x)^5 + 8*a*x^12*(c + d*x)^6 + x^14*(c + d*x)^7))/8
```

Maple [B] time = 0.004, size = 2205, normalized size = 122.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*d*x^2+2*c*x)*(d*x^3+c*x^2+a)^7,x)
```

```
[Out] 1/8*d^8*x^24+c*d^7*x^23+7/2*c^2*d^6*x^22+1/21*(42*c^3*d^5+3*d*(a*
d^6+15*c^3*d^4+d*(2*(3*a*d^2+c^3)*d^3+18*c^3*d^3)))*x^21+1/20*(2*
c*(a*d^6+15*c^3*d^4+d*(2*(3*a*d^2+c^3)*d^3+18*c^3*d^3))+3*d*(6*a*
c*d^5+c*(2*(3*a*d^2+c^3)*d^3+18*c^3*d^3)+d*(12*a*c*d^4+6*(3*a*d^2
+c^3)*c*d^2+9*c^4*d^2)))*x^20+1/19*(2*c*(6*a*c*d^5+c*(2*(3*a*d^2+
c^3)*d^3+18*c^3*d^3)+d*(12*a*c*d^4+6*(3*a*d^2+c^3)*c*d^2+9*c^4*d^
2))+3*d*(15*a*c^2*d^4+c*(12*a*c*d^4+6*(3*a*d^2+c^3)*c*d^2+9*c^4*d
^2)+d*(42*a*c^2*d^3+6*(3*a*d^2+c^3)*c^2*d)))*x^19+1/18*(2*c*(15*a
*c^2*d^4+c*(12*a*c*d^4+6*(3*a*d^2+c^3)*c*d^2+9*c^4*d^2)+d*(42*a*c
^2*d^3+6*(3*a*d^2+c^3)*c^2*d))+3*d*(a*(2*(3*a*d^2+c^3)*d^3+18*c^3
*d^3)+c*(42*a*c^2*d^3+6*(3*a*d^2+c^3)*c^2*d)+d*(6*a^2*d^4+54*a*c^
3*d^2+(3*a*d^2+c^3)^2)))*x^18+1/17*(2*c*(a*(2*(3*a*d^2+c^3)*d^3+1
8*c^3*d^3)+c*(42*a*c^2*d^3+6*(3*a*d^2+c^3)*c^2*d)+d*(6*a^2*d^4+54
*a*c^3*d^2+(3*a*d^2+c^3)^2))+3*d*(a*(12*a*c*d^4+6*(3*a*d^2+c^3)*c
*d^2+9*c^4*d^2)+c*(6*a^2*d^4+54*a*c^3*d^2+(3*a*d^2+c^3)^2)+d*(24*
a^2*c*d^3+18*a*c^4*d+12*a*c*d*(3*a*d^2+c^3))))*x^17+1/16*(2*c*(a*
(12*a*c*d^4+6*(3*a*d^2+c^3)*c*d^2+9*c^4*d^2)+c*(6*a^2*d^4+54*a*c^
3*d^2+(3*a*d^2+c^3)^2)+d*(24*a^2*c*d^3+18*a*c^4*d+12*a*c*d*(3*a*d
^2+c^3)))+3*d*(a*(42*a*c^2*d^3+6*(3*a*d^2+c^3)*c^2*d)+c*(24*a^2*c
*d^3+18*a*c^4*d+12*a*c*d*(3*a*d^2+c^3))+d*(72*a^2*c^2*d^2+6*a*c^2
*(3*a*d^2+c^3))))*x^16+1/15*(2*c*(a*(42*a*c^2*d^3+6*(3*a*d^2+c^3)
*c^2*d)+c*(24*a^2*c*d^3+18*a*c^4*d+12*a*c*d*(3*a*d^2+c^3))+d*(72*
a^2*c^2*d^2+6*a*c^2*(3*a*d^2+c^3)))+3*d*(a*(6*a^2*d^4+54*a*c^3*d^
2+(3*a*d^2+c^3)^2)+c*(72*a^2*c^2*d^2+6*a*c^2*(3*a*d^2+c^3))+d*(2*
a^3*d^3+54*a^2*c^3*d+6*a^2*d*(3*a*d^2+c^3))))*x^15+1/14*(2*c*(a*(
6*a^2*d^4+54*a*c^3*d^2+(3*a*d^2+c^3)^2)+c*(72*a^2*c^2*d^2+6*a*c^2
*(3*a*d^2+c^3))+d*(2*a^3*d^3+54*a^2*c^3*d+6*a^2*d*(3*a*d^2+c^3)))
+3*d*(a*(24*a^2*c*d^3+18*a*c^4*d+12*a*c*d*(3*a*d^2+c^3))+c*(2*a^3
*d^3+54*a^2*c^3*d+6*a^2*d*(3*a*d^2+c^3))+d*(42*a^3*c*d^2+6*a^2*c*
(3*a*d^2+c^3)+9*a^2*c^4)))*x^14+1/13*(2*c*(a*(24*a^2*c*d^3+18*a*c
^4*d+12*a*c*d*(3*a*d^2+c^3))+c*(2*a^3*d^3+54*a^2*c^3*d+6*a^2*d*(3
*a*d^2+c^3))+d*(42*a^3*c*d^2+6*a^2*c*(3*a*d^2+c^3)+9*a^2*c^4))+3*
d*(a*(72*a^2*c^2*d^2+6*a*c^2*(3*a*d^2+c^3))+c*(42*a^3*c*d^2+6*a^2
*c*(3*a*d^2+c^3)+9*a^2*c^4)+60*a^3*c^2*d^2))*x^13+1/12*(2*c*(a*(7
2*a^2*c^2*d^2+6*a*c^2*(3*a*d^2+c^3))+c*(42*a^3*c*d^2+6*a^2*c*(3*a
*d^2+c^3)+9*a^2*c^4)+60*a^3*c^2*d^2)+3*d*(a*(2*a^3*d^3+54*a^2*c^3
*d+6*a^2*d*(3*a*d^2+c^3))+60*a^3*c^3*d+d*(2*a^3*(3*a*d^2+c^3)+18*
a^3*c^3+9*a^4*d^2)))*x^12+1/11*(2*c*(a*(2*a^3*d^3+54*a^2*c^3*d+6*
a^2*d*(3*a*d^2+c^3))+60*a^3*c^3*d+d*(2*a^3*(3*a*d^2+c^3)+18*a^3*c
^3+9*a^4*d^2))+3*d*(a*(42*a^3*c*d^2+6*a^2*c*(3*a*d^2+c^3)+9*a^2*c
^4)+c*(2*a^3*(3*a*d^2+c^3)+18*a^3*c^3+9*a^4*d^2)+30*d^2*a^4*c))*x
^11+1/10*(2*c*(a*(42*a^3*c*d^2+6*a^2*c*(3*a*d^2+c^3)+9*a^2*c^4)+c
*(2*a^3*(3*a*d^2+c^3)+18*a^3*c^3+9*a^4*d^2)+30*d^2*a^4*c)+315*d^2
*a^4*c^2)*x^10+1/9*(210*c^3*a^4*d+3*d*(a*(2*a^3*(3*a*d^2+c^3)+18*
a^3*c^3+9*a^4*d^2)+15*c^3*a^4+6*a^5*d^2))*x^9+1/8*(2*c*(a*(2*a^3*
(3*a*d^2+c^3)+18*a^3*c^3+9*a^4*d^2)+15*c^3*a^4+6*a^5*d^2)+126*d^2
*a^5*c)*x^8+21*c^2*a^5*d*x^7+1/6*(21*a^6*d^2+42*a^5*c^3)*x^6+7*c*
d*a^6*x^5+7/2*c^2*a^6*x^4+d*a^7*x^3+c*a^7*x^2
```

Maxima [A] time = 0.802602, size = 22, normalized size = 1.22

$$\frac{1}{8}\left(dx^3 + cx^2 + a\right)^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c*x^2 + a)^7*(3*d*x^2 + 2*c*x),x, algorithm="maxima")
```

```
[Out] 1/8*(d*x^3 + c*x^2 + a)^8
```

Fricas [A] time = 0.230782, size = 1, normalized size = 0.06

$$\frac{1}{8}x^{24}d^8+x^{23}d^7c+\frac{7}{2}x^{22}d^6c^2+7x^{21}d^5c^3+x^{21}d^7a+\frac{35}{4}x^{20}d^4c^4+7x^{20}d^6ca+7x^{19}d^3c^5+21x^{19}d^5c^2a+\frac{7}{2}x^{18}d^2c^6+35x^{18}d^4c^3a+\frac{7}{2}x^{18}d^6a^2+x^{17}dc^7+35x^{17}d^3c^4a+21x^{17}d^5ca^2+\frac{1}{8}x^{16}c^8+21x^{16}d^2c^5a+\frac{105}{2}x^{16}d^4c^2a^2+7x^{15}dc^6a+70x^{15}d^3c^3a^2+7x^{15}d^5a^3+x^{14}c^7a+\frac{105}{2}x^{14}d^2c^4a^2+35x^{14}d^4ca^3+21x^{13}dc^5a^2+70x^{13}d^3c^2a^3+\frac{7}{2}x^{12}c^6a^2+70x^{12}d^2c^3a^3+\frac{35}{4}x^{12}d^4a^4+35x^{11}dc^4a^3+35x^{11}d^3ca^4+7x^{10}c^5a^3+\frac{105}{2}x^{10}d^2c^2a^4+35x^9dc^3a^4+7x^9d^3a^5+\frac{35}{4}x^8c^4a^4+21x^8d^2ca^5+21x^7dc^2a^5+7x^6c^3a^5+\frac{7}{2}x^6d^2a^6+7x^5dca^6+\frac{7}{2}x^4c^2a^6+x^3da^7+x^2ca^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3 + c*x^2 + a)^7*(3*d*x^2 + 2*c*x),x, algorithm="fricas")
```

```
[Out] 1/8*x^24*d^8 + x^23*d^7*c + 7/2*x^22*d^6*c^2 + 7*x^21*d^5*c^3 + x
^21*d^7*a + 35/4*x^20*d^4*c^4 + 7*x^20*d^6*c*a + 7*x^19*d^3*c^5 +
 21*x^19*d^5*c^2*a + 7/2*x^18*d^2*c^6 + 35*x^18*d^4*c^3*a + 7/2*x
^18*d^6*a^2 + x^17*d*c^7 + 35*x^17*d^3*c^4*a + 21*x^17*d^5*c*a^2
+ 1/8*x^16*c^8 + 21*x^16*d^2*c^5*a + 105/2*x^16*d^4*c^2*a^2 + 7*x
^15*d*c^6*a + 70*x^15*d^3*c^3*a^2 + 7*x^15*d^5*a^3 + x^14*c^7*a +
 105/2*x^14*d^2*c^4*a^2 + 35*x^14*d^4*c*a^3 + 21*x^13*d*c^5*a^2 +
 70*x^13*d^3*c^2*a^3 + 7/2*x^12*c^6*a^2 + 70*x^12*d^2*c^3*a^3 + 3
5/4*x^12*d^4*a^4 + 35*x^11*d*c^4*a^3 + 35*x^11*d^3*c*a^4 + 7*x^10
*c^5*a^3 + 105/2*x^10*d^2*c^2*a^4 + 35*x^9*d*c^3*a^4 + 7*x^9*d^3*
a^5 + 35/4*x^8*c^4*a^4 + 21*x^8*d^2*c*a^5 + 21*x^7*d*c^2*a^5 + 7*
x^6*c^3*a^5 + 7/2*x^6*d^2*a^6 + 7*x^5*d*c*a^6 + 7/2*x^4*c^2*a^6 +
 x^3*d*a^7 + x^2*c*a^7
```

Sympy [A] time = 0.431975, size = 484, normalized size = 26.89

$$a^7cx^2+a^7dx^3+\frac{7a^6c^2x^4}{2}+7a^6cdx^5+21a^5c^2dx^7+\frac{7c^2d^6x^{22}}{2}+cd^7x^{23}+\frac{d^8x^{24}}{8}+x^{21}\left(ad^7+7c^3d^5\right)+x^{20}\left(7acd^6+\frac{35c^4d^4}{4}\right)+x^{19}\left(21ac^2d^5+7c^5d^3\right)+x^{18}\left(\frac{7a^2d^6}{2}+35ac^3d^4+\frac{7c^6d^2}{2}\right)+x^{17}\left(21a^2cd^5+35ac^4d^3+c^7d\right)+x^{16}\left(\frac{105a^2c^2d^4}{2}+21ac^5d^2+\frac{c^8}{8}\right)+x^{15}\left(7a^3d^5+70a^2c^3d^3+7ac^6d\right)+x^{14}\left(35a^3cd^4+\frac{105a^2c^4d^2}{2}+ac^7\right)+x^{13}\left(70a^3c^2d^3+21a^2c^5d\right)+x^{12}\left(\frac{35a^4d^4}{4}+70a^3c^3d^2+\frac{7a^2c^6}{2}\right)+x^{11}\left(35a^4cd^3+35a^3c^4d\right)+x^{10}\left(\frac{105a^4c^2d^2}{2}+7a^3c^5\right)+x^9\left(7a^5d^3+35a^4c^3d\right)+x^8\left(21a^5cd^2+\frac{35a^4c^4}{4}\right)+x^6\left(\frac{7a^6d^2}{2}+7a^5c^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x**2+2*c*x)*(d*x**3+c*x**2+a)**7,x)
```

```
[Out] a**7*c*x**2 + a**7*d*x**3 + 7*a**6*c**2*x**4/2 + 7*a**6*c*d*x**5
+ 21*a**5*c**2*d*x**7 + 7*c**2*d**6*x**22/2 + c*d**7*x**23 + d**8
*x**24/8 + x**21*(a*d**7 + 7*c**3*d**5) + x**20*(7*a*c*d**6 + 35*
c**4*d**4/4) + x**19*(21*a*c**2*d**5 + 7*c**5*d**3) + x**18*(7*a*
*2*d**6/2 + 35*a*c**3*d**4 + 7*c**6*d**2/2) + x**17*(21*a**2*c*d*
*5 + 35*a*c**4*d**3 + c**7*d) + x**16*(105*a**2*c**2*d**4/2 + 21*
a*c**5*d**2 + c**8/8) + x**15*(7*a**3*d**5 + 70*a**2*c**3*d**3 +
7*a*c**6*d) + x**14*(35*a**3*c*d**4 + 105*a**2*c**4*d**2/2 + a*c*
*7) + x**13*(70*a**3*c**2*d**3 + 21*a**2*c**5*d) + x**12*(35*a**4
*d**4/4 + 70*a**3*c**3*d**2 + 7*a**2*c**6/2) + x**11*(35*a**4*c*d
**3 + 35*a**3*c**4*d) + x**10*(105*a**4*c**2*d**2/2 + 7*a**3*c**5
```

```
) + x**9*(7*a**5*d**3 + 35*a**4*c**3*d) + x**8*(21*a**5*c*d**2 +
35*a**4*c**4/4) + x**6*(7*a**6*d**2/2 + 7*a**5*c**3)
```

GIAC/XCAS [A] time = 0.261561, size = 659, normalized size = 36.61

$$\frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{7}{2}c^2d^6x^{22} + 7c^3d^5x^{21} + ad^7x^{21} + \frac{35}{4}c^4d^4x^{20} + 7acd^6x^{20} + 7c^5d^3x^{19} + 21ac^2d^5x^{19} + \frac{7}{2}c^6d^2x^{18} + 35ac^3d^4x^{18} + \frac{7}{2}a^2d^6x^{18} + c^7dx^{17} + 35ac^4d^3x^{17} + 21a^2cd^5x^{17} + \frac{1}{8}c^8x^{16} + 21ac^5d^2x^{16} + \frac{105}{2}a^2c^2d^4x^{16} + 7ac^6dx^{15} + 70a^2c^3d^3x^{15} + 7a^3d^5x^{15} + ac^7x^{14} + \frac{105}{2}a^2c^4d^2x^{14} + 35a^3cd^4x^{14} + 21a^2c^5dx^{13} + 70a^3c^2d^3x^{13} + \frac{7}{2}a^2c^6x^{12} + 70a^3c^3d^2x^{12} + \frac{35}{4}a^4d^4x^{12} + 35a^3c^4dx^{11} + 35a^4cd^3x^{11} + 7a^3c^5x^{10} + \frac{105}{2}a^4c^2d^2x^{10} + 35a^4c^3dx^9 + 7a^5d^3x^9 + \frac{35}{4}a^4c^4x^8 + 21a^5cd^2x^8 + 21a^5c^2dx^7 + 7a^5c^3x^6 + \frac{7}{2}a^6d^2x^6 + 7a^6cdx^5 + \frac{7}{2}a^6c^2x^4 + a^7dx^3 + a^7cx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3 + c*x^2 + a)^7*(3*d*x^2 + 2*c*x),x, algorithm="giac")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + a
*d^7*x^21 + 35/4*c^4*d^4*x^20 + 7*a*c*d^6*x^20 + 7*c^5*d^3*x^19 +
 21*a*c^2*d^5*x^19 + 7/2*c^6*d^2*x^18 + 35*a*c^3*d^4*x^18 + 7/2*a
^2*d^6*x^18 + c^7*d*x^17 + 35*a*c^4*d^3*x^17 + 21*a^2*c*d^5*x^17
+ 1/8*c^8*x^16 + 21*a*c^5*d^2*x^16 + 105/2*a^2*c^2*d^4*x^16 + 7*a
*c^6*d*x^15 + 70*a^2*c^3*d^3*x^15 + 7*a^3*d^5*x^15 + a*c^7*x^14 +
 105/2*a^2*c^4*d^2*x^14 + 35*a^3*c*d^4*x^14 + 21*a^2*c^5*d*x^13 +
 70*a^3*c^2*d^3*x^13 + 7/2*a^2*c^6*x^12 + 70*a^3*c^3*d^2*x^12 + 3
5/4*a^4*d^4*x^12 + 35*a^3*c^4*d*x^11 + 35*a^4*c*d^3*x^11 + 7*a^3*
c^5*x^10 + 105/2*a^4*c^2*d^2*x^10 + 35*a^4*c^3*d*x^9 + 7*a^5*d^3*
x^9 + 35/4*a^4*c^4*x^8 + 21*a^5*c*d^2*x^8 + 21*a^5*c^2*d*x^7 + 7*
a^5*c^3*x^6 + 7/2*a^6*d^2*x^6 + 7*a^6*c*d*x^5 + 7/2*a^6*c^2*x^4 +
 a^7*d*x^3 + a^7*c*x^2
```

3.199 $\int (2cx+3dx^2)(cx^2+dx^3)^7\,dx$

Optimal. Leaf size=17

$$\frac{1}{8}(cx^2+dx^3)^8$$

```
[Out] (c*x^2 + d*x^3)^8/8
```

Rubi [A] time = 0.0149723, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04

$$\frac{1}{8}(cx^2+dx^3)^8$$

Antiderivative was successfully verified.

```
[In]  Int[(2*c*x + 3*d*x^2)*(c*x^2 + d*x^3)^7,x]
```

```
[Out] (c*x^2 + d*x^3)^8/8
```

Rubi in Sympy [A] time = 7.9997, size = 10, normalized size = 0.59

$$\frac{x^{16}(c+dx)^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*d*x**2+2*c*x)*(d*x**3+c*x**2)**7,x)
```

```
[Out] x**16*(c + d*x)**8/8
```

Mathematica [B] time = 0.00607104, size = 98, normalized size = 5.76

$$\frac{c^8x^{16}}{8}+c^7dx^{17}+\frac{7}{2}c^6d^2x^{18}+7c^5d^3x^{19}+\frac{35}{4}c^4d^4x^{20}+7c^3d^5x^{21}+\frac{7}{2}c^2d^6x^{22}+cd^7x^{23}+\frac{d^8x^{24}}{8}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2*c*x + 3*d*x^2)*(c*x^2 + d*x^3)^7,x]
```

```
[Out] (c^8*x^16)/8 + c^7*d*x^17 + (7*c^6*d^2*x^18)/2 + 7*c^5*d^3*x^19 +
 (35*c^4*d^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/2 + c*d^
7*x^23 + (d^8*x^24)/8
```

Maple [B] time = 0.003, size = 89, normalized size = 5.2

$$\frac{d^8x^{24}}{8}+cd^7x^{23}+\frac{7\,c^2d^6x^{22}}{2}+7\,c^3d^5x^{21}+\frac{35\,c^4d^4x^{20}}{4}+7\,c^5d^3x^{19}+\frac{7\,c^6d^2x^{18}}{2}+c^7dx^{17}+\frac{c^8x^{16}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*d*x^2+2*c*x)*(d*x^3+c*x^2)^7,x)
```

```
[Out] 1/8*d^8*x^24+c*d^7*x^23+7/2*c^2*d^6*x^22+7*c^3*d^5*x^21+35/4*c^4*
d^4*x^20+7*c^5*d^3*x^19+7/2*c^6*d^2*x^18+c^7*d*x^17+1/8*c^8*x^16
```

Maxima [A] time = 0.791369, size = 20, normalized size = 1.18

$$\frac{1}{8}\left(dx^3+cx^2\right)^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3 + c*x^2)^7*(3*d*x^2 + 2*c*x),x, algorithm="maxima")
```

```
[Out] 1/8*(d*x^3 + c*x^2)^8
```

Fricas [A] time = 0.255988, size = 1, normalized size = 0.06

$$\frac{1}{8}x^{24}d^8+x^{23}d^7c+\frac{7}{2}x^{22}d^6c^2+7x^{21}d^5c^3+\frac{35}{4}x^{20}d^4c^4+7x^{19}d^3c^5+\frac{7}{2}x^{18}d^2c^6+x^{17}dc^7+\frac{1}{8}x^{16}c^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3 + c*x^2)^7*(3*d*x^2 + 2*c*x),x, algorithm="fricas")
```

```
[Out] 1/8*x^24*d^8 + x^23*d^7*c + 7/2*x^22*d^6*c^2 + 7*x^21*d^5*c^3 + 3
5/4*x^20*d^4*c^4 + 7*x^19*d^3*c^5 + 7/2*x^18*d^2*c^6 + x^17*d*c^7
 + 1/8*x^16*c^8
```

Sympy [A] time = 0.190356, size = 97, normalized size = 5.71

$$\frac{c^8x^{16}}{8}+c^7dx^{17}+\frac{7c^6d^2x^{18}}{2}+7c^5d^3x^{19}+\frac{35c^4d^4x^{20}}{4}+7c^3d^5x^{21}+\frac{7c^2d^6x^{22}}{2}+cd^7x^{23}+\frac{d^8x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*d*x**2+2*c*x)*(d*x**3+c*x**2)**7,x)
```

```
[Out] c**8*x**16/8 + c**7*d*x**17 + 7*c**6*d**2*x**18/2 + 7*c**5*d**3*x
**19 + 35*c**4*d**4*x**20/4 + 7*c**3*d**5*x**21 + 7*c**2*d**6*x**
22/2 + c*d**7*x**23 + d**8*x**24/8
```

GIAC/XCAS [A] time = 0.258411, size = 119, normalized size = 7.

$$\frac{1}{8}d^8x^{24}+cd^7x^{23}+\frac{7}{2}c^2d^6x^{22}+7c^3d^5x^{21}+\frac{35}{4}c^4d^4x^{20}+7c^5d^3x^{19}+\frac{7}{2}c^6d^2x^{18}+c^7dx^{17}+\frac{1}{8}c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c*x^2)^7*(3*d*x^2 + 2*c*x),x, algorithm="giac")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 3
5/4*c^4*d^4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17
 + 1/8*c^8*x^16
```

3.200 $\int x^7 \left(cx+dx^2\right)^7 \left(2cx+3dx^2\right)\,dx$

Optimal. Leaf size=18

$$\frac{1}{8}x^8\left(cx+dx^2\right)^8$$

```
[Out] (x^8*(c*x + d*x^2)^8)/8
```

Rubi [A] time = 0.0163828, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038

$$\frac{1}{8}x^8\left(cx+dx^2\right)^8$$

Antiderivative was successfully verified.

```
[In]  Int[x^7*(c*x + d*x^2)^7*(2*c*x + 3*d*x^2),x]
```

```
[Out] (x^8*(c*x + d*x^2)^8)/8
```

Rubi in Sympy [A] time = 8.20718, size = 10, normalized size = 0.56

$$\frac{x^{16}\left(c+dx\right)^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7*(d*x**2+c*x)**7*(3*d*x**2+2*c*x),x)
```

```
[Out] x**16*(c + d*x)**8/8
```

Mathematica [B] time = 0.00469767, size = 98, normalized size = 5.44

$$\frac{c^8x^{16}}{8}+c^7dx^{17}+\frac{7}{2}c^6d^2x^{18}+7c^5d^3x^{19}+\frac{35}{4}c^4d^4x^{20}+7c^3d^5x^{21}+\frac{7}{2}c^2d^6x^{22}+cd^7x^{23}+\frac{d^8x^{24}}{8}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^7*(c*x + d*x^2)^7*(2*c*x + 3*d*x^2),x]
```

```
[Out] (c^8*x^16)/8 + c^7*d*x^17 + (7*c^6*d^2*x^18)/2 + 7*c^5*d^3*x^19 +
 (35*c^4*d^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/2 + c*d^
7*x^23 + (d^8*x^24)/8
```

Maple [B] time = 0.003, size = 89, normalized size = 4.9

$$\frac{d^8x^{24}}{8}+cd^7x^{23}+\frac{7\,c^2d^6x^{22}}{2}+7\,c^3d^5x^{21}+\frac{35\,c^4d^4x^{20}}{4}+7\,c^5d^3x^{19}+\frac{7\,c^6d^2x^{18}}{2}+c^7dx^{17}+\frac{c^8x^{16}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^7*(d*x^2+c*x)^7*(3*d*x^2+2*c*x),x)
```

```
[Out] 1/8*d^8*x^24+c*d^7*x^23+7/2*c^2*d^6*x^22+7*c^3*d^5*x^21+35/4*c^4*
d^4*x^20+7*c^5*d^3*x^19+7/2*c^6*d^2*x^18+c^7*d*x^17+1/8*c^8*x^16
```

Maxima [A] time = 0.793371, size = 119, normalized size = 6.61

$$\frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{7}{2}c^2d^6x^{22} + 7c^3d^5x^{21} + \frac{35}{4}c^4d^4x^{20} + 7c^5d^3x^{19} + \frac{7}{2}c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2 + 2*c*x)*(d*x^2 + c*x)^7*x^7,x, algorithm="maxima")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 3
5/4*c^4*d^4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17
 + 1/8*c^8*x^16
```

Fricas [A] time = 0.229343, size = 1, normalized size = 0.06

$$\frac{1}{8}x^{24}d^8 + x^{23}d^7c + \frac{7}{2}x^{22}d^6c^2 + 7x^{21}d^5c^3 + \frac{35}{4}x^{20}d^4c^4 + 7x^{19}d^3c^5 + \frac{7}{2}x^{18}d^2c^6 + x^{17}dc^7 + \frac{1}{8}x^{16}c^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2 + 2*c*x)*(d*x^2 + c*x)^7*x^7,x, algorithm="fricas")
```

```
[Out] 1/8*x^24*d^8 + x^23*d^7*c + 7/2*x^22*d^6*c^2 + 7*x^21*d^5*c^3 + 3
5/4*x^20*d^4*c^4 + 7*x^19*d^3*c^5 + 7/2*x^18*d^2*c^6 + x^17*d*c^7
 + 1/8*x^16*c^8
```

Sympy [A] time = 0.187838, size = 97, normalized size = 5.39

$$\frac{c^8x^{16}}{8} + c^7dx^{17} + \frac{7c^6d^2x^{18}}{2} + 7c^5d^3x^{19} + \frac{35c^4d^4x^{20}}{4} + 7c^3d^5x^{21} + \frac{7c^2d^6x^{22}}{2} + cd^7x^{23} + \frac{d^8x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**7*(d*x**2+c*x)**7*(3*d*x**2+2*c*x),x)
```

```
[Out] c**8*x**16/8 + c**7*d*x**17 + 7*c**6*d**2*x**18/2 + 7*c**5*d**3*x
**19 + 35*c**4*d**4*x**20/4 + 7*c**3*d**5*x**21 + 7*c**2*d**6*x**
22/2 + c*d**7*x**23 + d**8*x**24/8
```

GIAC/XCAS [A] time = 0.259762, size = 119, normalized size = 6.61

$$\frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{7}{2}c^2d^6x^{22} + 7c^3d^5x^{21} + \frac{35}{4}c^4d^4x^{20} + 7c^5d^3x^{19} + \frac{7}{2}c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2 + 2*c*x)*(d*x^2 + c*x)^7*x^7,x, algorithm="giac")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 3
5/4*c^4*d^4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17
 + 1/8*c^8*x^16
```

3.201 $\int x^{14}(c+dx)^7 \left(2cx+3dx^2\right)\, dx$

Optimal. Leaf size=14

$$\frac{1}{8}x^{16}(c+dx)^8$$

```
[Out] (x^16*(c + d*x)^8)/8
```

Rubi [A] time = 0.0143048, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045

$$\frac{1}{8}x^{16}(c+dx)^8$$

Antiderivative was successfully verified.

```
[In] Int[x^14*(c + d*x)^7*(2*c*x + 3*d*x^2),x]
```

```
[Out] (x^16*(c + d*x)^8)/8
```

Rubi in Sympy [A] time = 6.1496, size = 10, normalized size = 0.71

$$\frac{x^{16}\,(c+dx)^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**14*(d*x+c)**7*(3*d*x**2+2*c*x),x)
```

```
[Out] x**16*(c + d*x)**8/8
```

Mathematica [B] time = 0.00407434, size = 98, normalized size = 7.

$$\frac{c^8x^{16}}{8}+c^7dx^{17}+\frac{7}{2}c^6d^2x^{18}+7c^5d^3x^{19}+\frac{35}{4}c^4d^4x^{20}+7c^3d^5x^{21}+\frac{7}{2}c^2d^6x^{22}+cd^7x^{23}+\frac{d^8x^{24}}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14*(c + d*x)^7*(2*c*x + 3*d*x^2),x]
```

```
[Out] (c^8*x^16)/8 + c^7*d*x^17 + (7*c^6*d^2*x^18)/2 + 7*c^5*d^3*x^19 +
 (35*c^4*d^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/2 + c*d^
7*x^23 + (d^8*x^24)/8
```

Maple [B] time = 0.003, size = 89, normalized size = 6.4

$$\frac{d^8x^{24}}{8}+cd^7x^{23}+\frac{7\,c^2d^6x^{22}}{2}+7\,c^3d^5x^{21}+\frac{35\,c^4d^4x^{20}}{4}+7\,c^5d^3x^{19}+\frac{7\,c^6d^2x^{18}}{2}+c^7dx^{17}+\frac{c^8x^{16}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14*(d*x+c)^7*(3*d*x^2+2*c*x),x)
```

```
[Out] 1/8*d^8*x^24+c*d^7*x^23+7/2*c^2*d^6*x^22+7*c^3*d^5*x^21+35/4*c^4*
d^4*x^20+7*c^5*d^3*x^19+7/2*c^6*d^2*x^18+c^7*d*x^17+1/8*c^8*x^16
```

Maxima [A] time = 0.793029, size = 119, normalized size = 8.5

$$\frac{1}{8}\,d^8x^{24} + cd^7x^{23} + \frac{7}{2}\,c^2d^6x^{22} + 7\,c^3d^5x^{21} + \frac{35}{4}\,c^4d^4x^{20} + 7\,c^5d^3x^{19} + \frac{7}{2}\,c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}\,c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*d*x^2 + 2*c*x)*(d*x + c)^7*x^14,x, algorithm="maxima")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 3
5/4*c^4*d^4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17
 + 1/8*c^8*x^16
```

Fricas [A] time = 0.226264, size = 1, normalized size = 0.07

$$\frac{1}{8}x^{24}d^8 + x^{23}d^7c + \frac{7}{2}x^{22}d^6c^2 + 7x^{21}d^5c^3 + \frac{35}{4}x^{20}d^4c^4 + 7x^{19}d^3c^5 + \frac{7}{2}x^{18}d^2c^6 + x^{17}dc^7 + \frac{1}{8}x^{16}c^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*d*x^2 + 2*c*x)*(d*x + c)^7*x^14,x, algorithm="fricas")
```

```
[Out] 1/8*x^24*d^8 + x^23*d^7*c + 7/2*x^22*d^6*c^2 + 7*x^21*d^5*c^3 + 3
5/4*x^20*d^4*c^4 + 7*x^19*d^3*c^5 + 7/2*x^18*d^2*c^6 + x^17*d*c^7
 + 1/8*x^16*c^8
```

Sympy [A] time = 0.188062, size = 97, normalized size = 6.93

$$\frac{c^8x^{16}}{8} + c^7dx^{17} + \frac{7c^6d^2x^{18}}{2} + 7c^5d^3x^{19} + \frac{35c^4d^4x^{20}}{4} + 7c^3d^5x^{21} + \frac{7c^2d^6x^{22}}{2} + cd^7x^{23} + \frac{d^8x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**14*(d*x+c)**7*(3*d*x**2+2*c*x),x)
```

```
[Out] c**8*x**16/8 + c**7*d*x**17 + 7*c**6*d**2*x**18/2 + 7*c**5*d**3*x
**19 + 35*c**4*d**4*x**20/4 + 7*c**3*d**5*x**21 + 7*c**2*d**6*x**
22/2 + c*d**7*x**23 + d**8*x**24/8
```

GIAC/XCAS [A] time = 0.25902, size = 119, normalized size = 8.5

$$\frac{1}{8}\,d^8x^{24} + cd^7x^{23} + \frac{7}{2}\,c^2d^6x^{22} + 7\,c^3d^5x^{21} + \frac{35}{4}\,c^4d^4x^{20} + 7\,c^5d^3x^{19} + \frac{7}{2}\,c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}\,c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*d*x^2 + 2*c*x)*(d*x + c)^7*x^14,x, algorithm="giac")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 3
5/4*c^4*d^4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17
 + 1/8*c^8*x^16
```

3.202 $\int x(2c+3dx)\left(a+cx^2+dx^3\right)^7\,dx$

Optimal. Leaf size=18

$$\frac{1}{8}\left(a+cx^2+dx^3\right)^8$$

```
[Out] (a + c*x^2 + d*x^3)^8/8
```

Rubi [A] time = 0.0327743, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042

$$\frac{1}{8}\left(a+cx^2+dx^3\right)^8$$

Antiderivative was successfully verified.

```
[In] Int[x*(2*c + 3*d*x)*(a + c*x^2 + d*x^3)^7,x]

[Out] (a + c*x^2 + d*x^3)^8/8
```

Rubi in Sympy [A] time = 4.59659, size = 14, normalized size = 0.78

$$\frac{\left(a+cx^2+dx^3\right)^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x*(3*d*x+2*c)*(d*x**3+c*x**2+a)**7,x)

[Out] (a + c*x**2 + d*x**3)**8/8
```

Mathematica [B] time = 0.0237575, size = 115, normalized size = 6.39

$$\frac{1}{8}x^2(c+dx)\left(8a^7+28a^6x^2(c+dx)+56a^5x^4(c+dx)^2+70a^4x^6(c+dx)^3+56a^3x^8(c+dx)^4+28a^2x^{10}(c+dx)^5+8ax^{12}(c+dx)^6+x^{14}(c+dx)^7\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(2*c + 3*d*x)*(a + c*x^2 + d*x^3)^7,x]

[Out] (x^2*(c + d*x)*(8*a^7 + 28*a^6*x^2*(c + d*x) + 56*a^5*x^4*(c + d*
x)^2 + 70*a^4*x^6*(c + d*x)^3 + 56*a^3*x^8*(c + d*x)^4 + 28*a^2*x
^10*(c + d*x)^5 + 8*a*x^12*(c + d*x)^6 + x^14*(c + d*x)^7))/8
```

Maple [B] time = 0.003, size = 2205, normalized size = 122.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(3*d*x+2*c)*(d*x^3+c*x^2+a)^7,x)
```

```
[Out] 1/8*d^8*x^24+c*d^7*x^23+7/2*c^2*d^6*x^22+1/21*(42*c^3*d^5+3*d*(a*
d^6+15*c^3*d^4+d*(2*(3*a*d^2+c^3)*d^3+18*c^3*d^3)))*x^21+1/20*(2*
c*(a*d^6+15*c^3*d^4+d*(2*(3*a*d^2+c^3)*d^3+18*c^3*d^3))+3*d*(6*a*
c*d^5+c*(2*(3*a*d^2+c^3)*d^3+18*c^3*d^3)+d*(12*a*c*d^4+6*(3*a*d^2
+c^3)*c*d^2+9*c^4*d^2)))*x^20+1/19*(2*c*(6*a*c*d^5+c*(2*(3*a*d^2+
c^3)*d^3+18*c^3*d^3)+d*(12*a*c*d^4+6*(3*a*d^2+c^3)*c*d^2+9*c^4*d^
2))+3*d*(15*a*c^2*d^4+c*(12*a*c*d^4+6*(3*a*d^2+c^3)*c*d^2+9*c^4*d
^2)+d*(42*a*c^2*d^3+6*(3*a*d^2+c^3)*c^2*d)))*x^19+1/18*(2*c*(15*a
*c^2*d^4+c*(12*a*c*d^4+6*(3*a*d^2+c^3)*c*d^2+9*c^4*d^2)+d*(42*a*c
^2*d^3+6*(3*a*d^2+c^3)*c^2*d))+3*d*(a*(2*(3*a*d^2+c^3)*d^3+18*c^3
*d^3)+c*(42*a*c^2*d^3+6*(3*a*d^2+c^3)*c^2*d)+d*(6*a^2*d^4+54*a*c^
3*d^2+(3*a*d^2+c^3)^2)))*x^18+1/17*(2*c*(a*(2*(3*a*d^2+c^3)*d^3+1
8*c^3*d^3)+c*(42*a*c^2*d^3+6*(3*a*d^2+c^3)*c^2*d)+d*(6*a^2*d^4+54
*a*c^3*d^2+(3*a*d^2+c^3)^2))+3*d*(a*(12*a*c*d^4+6*(3*a*d^2+c^3)*c
*d^2+9*c^4*d^2)+c*(6*a^2*d^4+54*a*c^3*d^2+(3*a*d^2+c^3)^2)+d*(24*
a^2*c*d^3+18*a*c^4*d+12*a*c*d*(3*a*d^2+c^3))))*x^17+1/16*(2*c*(a*
(12*a*c*d^4+6*(3*a*d^2+c^3)*c*d^2+9*c^4*d^2)+c*(6*a^2*d^4+54*a*c^
3*d^2+(3*a*d^2+c^3)^2)+d*(24*a^2*c*d^3+18*a*c^4*d+12*a*c*d*(3*a*d
^2+c^3)))+3*d*(a*(42*a*c^2*d^3+6*(3*a*d^2+c^3)*c^2*d)+c*(24*a^2*c
*d^3+18*a*c^4*d+12*a*c*d*(3*a*d^2+c^3))+d*(72*a^2*c^2*d^2+6*a*c^2
*(3*a*d^2+c^3))))*x^16+1/15*(2*c*(a*(42*a*c^2*d^3+6*(3*a*d^2+c^3)
*c^2*d)+c*(24*a^2*c*d^3+18*a*c^4*d+12*a*c*d*(3*a*d^2+c^3))+d*(72*
a^2*c^2*d^2+6*a*c^2*(3*a*d^2+c^3)))+3*d*(a*(6*a^2*d^4+54*a*c^3*d^
2+(3*a*d^2+c^3)^2)+c*(72*a^2*c^2*d^2+6*a*c^2*(3*a*d^2+c^3))+d*(2*
a^3*d^3+54*a^2*c^3*d+6*a^2*d*(3*a*d^2+c^3))))*x^15+1/14*(2*c*(a*(
6*a^2*d^4+54*a*c^3*d^2+(3*a*d^2+c^3)^2)+c*(72*a^2*c^2*d^2+6*a*c^2
*(3*a*d^2+c^3))+d*(2*a^3*d^3+54*a^2*c^3*d+6*a^2*d*(3*a*d^2+c^3)))
+3*d*(a*(24*a^2*c*d^3+18*a*c^4*d+12*a*c*d*(3*a*d^2+c^3))+c*(2*a^3
*d^3+54*a^2*c^3*d+6*a^2*d*(3*a*d^2+c^3))+d*(42*a^3*c*d^2+6*a^2*c*
(3*a*d^2+c^3)+9*a^2*c^4)))*x^14+1/13*(2*c*(a*(24*a^2*c*d^3+18*a*c
^4*d+12*a*c*d*(3*a*d^2+c^3))+c*(2*a^3*d^3+54*a^2*c^3*d+6*a^2*d*(3
*a*d^2+c^3))+d*(42*a^3*c*d^2+6*a^2*c*(3*a*d^2+c^3)+9*a^2*c^4))+3*
d*(a*(72*a^2*c^2*d^2+6*a*c^2*(3*a*d^2+c^3))+c*(42*a^3*c*d^2+6*a^2
*c*(3*a*d^2+c^3)+9*a^2*c^4)+60*a^3*c^2*d^2))*x^13+1/12*(2*c*(a*(7
2*a^2*c^2*d^2+6*a*c^2*(3*a*d^2+c^3))+c*(42*a^3*c*d^2+6*a^2*c*(3*a
*d^2+c^3)+9*a^2*c^4)+60*a^3*c^2*d^2)+3*d*(a*(2*a^3*d^3+54*a^2*c^3
*d+6*a^2*d*(3*a*d^2+c^3))+60*a^3*c^3*d+d*(2*a^3*(3*a*d^2+c^3)+18*
a^3*c^3+9*a^4*d^2)))*x^12+1/11*(2*c*(a*(2*a^3*d^3+54*a^2*c^3*d+6*
a^2*d*(3*a*d^2+c^3))+60*a^3*c^3*d+d*(2*a^3*(3*a*d^2+c^3)+18*a^3*c
^3+9*a^4*d^2))+3*d*(a*(42*a^3*c*d^2+6*a^2*c*(3*a*d^2+c^3)+9*a^2*c
^4)+c*(2*a^3*(3*a*d^2+c^3)+18*a^3*c^3+9*a^4*d^2)+30*d^2*a^4*c))*x
^11+1/10*(2*c*(a*(42*a^3*c*d^2+6*a^2*c*(3*a*d^2+c^3)+9*a^2*c^4)+c
*(2*a^3*(3*a*d^2+c^3)+18*a^3*c^3+9*a^4*d^2)+30*d^2*a^4*c)+315*d^2
*a^4*c^2)*x^10+1/9*(210*c^3*a^4*d+3*d*(a*(2*a^3*(3*a*d^2+c^3)+18*
a^3*c^3+9*a^4*d^2)+15*c^3*a^4+6*a^5*d^2))*x^9+1/8*(2*c*(a*(2*a^3*
(3*a*d^2+c^3)+18*a^3*c^3+9*a^4*d^2)+15*c^3*a^4+6*a^5*d^2)+126*d^2
*a^5*c)*x^8+21*c^2*a^5*d*x^7+1/6*(21*a^6*d^2+42*a^5*c^3)*x^6+7*c*
d*a^6*x^5+7/2*c^2*a^6*x^4+d*a^7*x^3+c*a^7*x^2
```

Maxima [A] time = 0.802213, size = 618, normalized size = 34.33

$$\frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{7}{2}c^2d^6x^{22} + \left(7c^3d^5 + ad^7\right)x^{21} + \frac{7}{4}\left(5c^4d^4 + 4acd^6\right)x^{20} + 7\left(c^5d^3 + 3ac^2d^5\right)x^{19} + \frac{7}{2}\left(c^6d^2 + 10ac^3d^4 + a^2d^6\right)x^{18} + \left(c^7d + 35ac^4d^3 + 21a^2cd^5\right)x^{17} + \frac{1}{8}\left(c^8 + 168ac^5d^2 + 420a^2c^2d^4\right)x^{16} + 7\left(ac^6d + 10a^2c^3d^3 + a^3d^5\right)x^{15} + 21a^5c^2dx^7 + \frac{1}{2}\left(2ac^7 + 105a^2c^4d^2 + 70a^3cd^4\right)x^{14} + 7\left(3a^2c^5d + 10a^3c^2d^3\right)x^{13} + 7a^6cdx^5 + \frac{7}{4}\left(2a^2c^6 + 40a^3c^3d^2 + 5a^4d^4\right)x^{12} + \frac{7}{2}a^6c^2x^4 + 35\left(a^3c^4d + a^4cd^3\right)x^{11} + a^7dx^3 + \frac{7}{2}\left(2a^3c^5 + 15a^4c^2d^2\right)x^{10} + a^7cx^2 + 7\left(5a^4c^3d + a^5d^3\right)x^9 + \frac{7}{4}\left(5a^4c^4 + 12a^5cd^2\right)x^8 + \frac{7}{2}\left(2a^5c^3 + a^6d^2\right)x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3 + c*x^2 + a)^7*(3*d*x + 2*c)*x,x, algorithm="maxima")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + (7*c^3*d^5 + a*d^7
)*x^21 + 7/4*(5*c^4*d^4 + 4*a*c*d^6)*x^20 + 7*(c^5*d^3 + 3*a*c^2*
d^5)*x^19 + 7/2*(c^6*d^2 + 10*a*c^3*d^4 + a^2*d^6)*x^18 + (c^7*d
+ 35*a*c^4*d^3 + 21*a^2*c*d^5)*x^17 + 1/8*(c^8 + 168*a*c^5*d^2 +
420*a^2*c^2*d^4)*x^16 + 7*(a*c^6*d + 10*a^2*c^3*d^3 + a^3*d^5)*x^
15 + 21*a^5*c^2*d*x^7 + 1/2*(2*a*c^7 + 105*a^2*c^4*d^2 + 70*a^3*c
*d^4)*x^14 + 7*(3*a^2*c^5*d + 10*a^3*c^2*d^3)*x^13 + 7*a^6*c*d*x^
5 + 7/4*(2*a^2*c^6 + 40*a^3*c^3*d^2 + 5*a^4*d^4)*x^12 + 7/2*a^6*c
^2*x^4 + 35*(a^3*c^4*d + a^4*c*d^3)*x^11 + a^7*d*x^3 + 7/2*(2*a^3
*c^5 + 15*a^4*c^2*d^2)*x^10 + a^7*c*x^2 + 7*(5*a^4*c^3*d + a^5*d^
3)*x^9 + 7/4*(5*a^4*c^4 + 12*a^5*c*d^2)*x^8 + 7/2*(2*a^5*c^3 + a^
6*d^2)*x^6
```

Fricas [A] time = 0.241809, size = 1, normalized size = 0.06

$$\frac{1}{8}x^{24}d^8 + x^{23}d^7c + \frac{7}{2}x^{22}d^6c^2 + 7x^{21}d^5c^3 + x^{21}d^7a + \frac{35}{4}x^{20}d^4c^4 + 7x^{20}d^6ca + 7x^{19}d^3c^5 + 21x^{19}d^5c^2a + \frac{7}{2}x^{18}d^2c^6 + 35x^{18}d^4c^3a + \frac{7}{2}x^{18}d^6a^2 + x^{17}dc^7 + 35x^{17}d^3c^4a + 21x^{17}d^5ca^2 + \frac{1}{8}x^{16}c^8 + 21x^{16}d^2c^5a + \frac{105}{2}x^{16}d^4c^2a^2 + 7x^{15}dc^6a + 70x^{15}d^3c^3a^2 + 7x^{15}d^5a^3 + x^{14}c^7a + \frac{105}{2}x^{14}d^2c^4a^2 + 35x^{14}d^4ca^3 + 21x^{13}dc^5a^2 + 70x^{13}d^3c^2a^3 + \frac{7}{2}x^{12}c^6a^2 + 70x^{12}d^2c^3a^3 + \frac{35}{4}x^{12}d^4a^4 + 35x^{11}dc^4a^3 + 35x^{11}d^3ca^4 + 7x^{10}c^5a^3 + \frac{105}{2}x^{10}d^2c^2a^4 + 35x^9dc^3a^4 + 7x^9d^3a^5 + \frac{35}{4}x^8c^4a^4 + 21x^8d^2ca^5 + 21x^7dc^2a^5 + 7x^6c^3a^5 + \frac{7}{2}x^6d^2a^6 + 7x^5dca^6 + \frac{7}{2}x^4c^2a^6 + x^3da^7 + x^2ca^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3 + c*x^2 + a)^7*(3*d*x + 2*c)*x,x, algorithm="fricas")
```

```
[Out] 1/8*x^24*d^8 + x^23*d^7*c + 7/2*x^22*d^6*c^2 + 7*x^21*d^5*c^3 + x
^21*d^7*a + 35/4*x^20*d^4*c^4 + 7*x^20*d^6*c*a + 7*x^19*d^3*c^5 +
 21*x^19*d^5*c^2*a + 7/2*x^18*d^2*c^6 + 35*x^18*d^4*c^3*a + 7/2*x
^18*d^6*a^2 + x^17*d*c^7 + 35*x^17*d^3*c^4*a + 21*x^17*d^5*c*a^2
+ 1/8*x^16*c^8 + 21*x^16*d^2*c^5*a + 105/2*x^16*d^4*c^2*a^2 + 7*x
^15*d*c^6*a + 70*x^15*d^3*c^3*a^2 + 7*x^15*d^5*a^3 + x^14*c^7*a +
 105/2*x^14*d^2*c^4*a^2 + 35*x^14*d^4*c*a^3 + 21*x^13*d*c^5*a^2 +
 70*x^13*d^3*c^2*a^3 + 7/2*x^12*c^6*a^2 + 70*x^12*d^2*c^3*a^3 + 3
5/4*x^12*d^4*a^4 + 35*x^11*d*c^4*a^3 + 35*x^11*d^3*c*a^4 + 7*x^10
*c^5*a^3 + 105/2*x^10*d^2*c^2*a^4 + 35*x^9*d*c^3*a^4 + 7*x^9*d^3*
a^5 + 35/4*x^8*c^4*a^4 + 21*x^8*d^2*c*a^5 + 21*x^7*d*c^2*a^5 + 7*
x^6*c^3*a^5 + 7/2*x^6*d^2*a^6 + 7*x^5*d*c*a^6 + 7/2*x^4*c^2*a^6 +
 x^3*d*a^7 + x^2*c*a^7
```

Sympy [A] time = 0.3982, size = 484, normalized size = 26.89

$$a^7cx^2 + a^7dx^3 + \frac{7a^6c^2x^4}{2} + 7a^6cdx^5 + 21a^5c^2dx^7 + \frac{7c^2d^6x^{22}}{2} + cd^7x^{23} + \frac{d^8x^{24}}{8} + x^{21}\left(ad^7 + 7c^3d^5\right) + x^{20}\left(7acd^6 + \frac{35c^4d^4}{4}\right) + x^{19}\left(21ac^2d^5 + 7c^5d^3\right) + x^{18}\left(\frac{7a^2d^6}{2} + 35ac^3d^4 + \frac{7c^6d^2}{2}\right) + x^{17}\left(21a^2cd^5 + 35ac^4d^3 + c^7d\right) + x^{16}\left(\frac{105a^2c^2d^4}{2} + 21ac^5d^2 + \frac{c^8}{8}\right) + x^{15}\left(7a^3d^5 + 70a^2c^3d^3 + 7ac^6d\right) + x^{14}\left(35a^3cd^4 + \frac{105a^2c^4d^2}{2} + ac^7\right) + x^{13}\left(70a^3c^2d^3 + 21a^2c^5d\right) + x^{12}\left(\frac{35a^4d^4}{4} + 70a^3c^3d^2 + \frac{7a^2c^6}{2}\right) + x^{11}\left(35a^4cd^3 + 35a^3c^4d\right) + x^{10}\left(\frac{105a^4c^2d^2}{2} + 7a^3c^5\right) + x^9\left(7a^5d^3 + 35a^4c^3d\right) + x^8\left(21a^5cd^2 + \frac{35a^4c^4}{4}\right) + x^6\left(\frac{7a^6d^2}{2} + 7a^5c^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x*(3*d*x+2*c)*(d*x**3+c*x**2+a)**7,x)
```

```
[Out] a**7*c*x**2 + a**7*d*x**3 + 7*a**6*c**2*x**4/2 + 7*a**6*c*d*x**5
+ 21*a**5*c**2*d*x**7 + 7*c**2*d**6*x**22/2 + c*d**7*x**23 + d**8
*x**24/8 + x**21*(a*d**7 + 7*c**3*d**5) + x**20*(7*a*c*d**6 + 35*
c**4*d**4/4) + x**19*(21*a*c**2*d**5 + 7*c**5*d**3) + x**18*(7*a*
*2*d**6/2 + 35*a*c**3*d**4 + 7*c**6*d**2/2) + x**17*(21*a**2*c*d*
*5 + 35*a*c**4*d**3 + c**7*d) + x**16*(105*a**2*c**2*d**4/2 + 21*
a*c**5*d**2 + c**8/8) + x**15*(7*a**3*d**5 + 70*a**2*c**3*d**3 +
7*a*c**6*d) + x**14*(35*a**3*c*d**4 + 105*a**2*c**4*d**2/2 + a*c*
*7) + x**13*(70*a**3*c**2*d**3 + 21*a**2*c**5*d) + x**12*(35*a**4
*d**4/4 + 70*a**3*c**3*d**2 + 7*a**2*c**6/2) + x**11*(35*a**4*c*d
**3 + 35*a**3*c**4*d) + x**10*(105*a**4*c**2*d**2/2 + 7*a**3*c**5
) + x**9*(7*a**5*d**3 + 35*a**4*c**3*d) + x**8*(21*a**5*c*d**2 +
35*a**4*c**4/4) + x**6*(7*a**6*d**2/2 + 7*a**5*c**3)
```

GIAC/XCAS [A] time = 0.261057, size = 659, normalized size = 36.61

$$\frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{7}{2}c^2d^6x^{22} + 7c^3d^5x^{21} + ad^7x^{21} + \frac{35}{4}c^4d^4x^{20} + 7acd^6x^{20} + 7c^5d^3x^{19} + 21ac^2d^5x^{19} + \frac{7}{2}c^6d^2x^{18} + 35ac^3d^4x^{18} + \frac{7}{2}a^2d^6x^{18} + c^7dx^{17} + 35ac^4d^3x^{17} + 21a^2cd^5x^{17} + \frac{1}{8}c^8x^{16} + 21ac^5d^2x^{16} + \frac{105}{2}a^2c^2d^4x^{16} + 7ac^6dx^{15} + 70a^2c^3d^3x^{15} + 7a^3d^5x^{15} + ac^7x^{14} + \frac{105}{2}a^2c^4d^2x^{14} + 35a^3cd^4x^{14} + 21a^2c^5dx^{13} + 70a^3c^2d^3x^{13} + \frac{7}{2}a^2c^6x^{12} + 70a^3c^3d^2x^{12} + \frac{35}{4}a^4d^4x^{12} + 35a^3c^4dx^{11} + 35a^4cd^3x^{11} + 7a^3c^5x^{10} + \frac{105}{2}a^4c^2d^2x^{10} + 35a^4c^3dx^9 + 7a^5d^3x^9 + \frac{35}{4}a^4c^4x^8 + 21a^5cd^2x^8 + 21a^5c^2dx^7 + 7a^5c^3x^6 + \frac{7}{2}a^6d^2x^6 + 7a^6cdx^5 + \frac{7}{2}a^6c^2x^4 + a^7dx^3 + a^7cx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((d*x^3 + c*x^2 + a)^7*(3*d*x + 2*c)*x,x, algorithm="giac")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + a
*d^7*x^21 + 35/4*c^4*d^4*x^20 + 7*a*c*d^6*x^20 + 7*c^5*d^3*x^19 +
 21*a*c^2*d^5*x^19 + 7/2*c^6*d^2*x^18 + 35*a*c^3*d^4*x^18 + 7/2*a
^2*d^6*x^18 + c^7*d*x^17 + 35*a*c^4*d^3*x^17 + 21*a^2*c*d^5*x^17
+ 1/8*c^8*x^16 + 21*a*c^5*d^2*x^16 + 105/2*a^2*c^2*d^4*x^16 + 7*a
*c^6*d*x^15 + 70*a^2*c^3*d^3*x^15 + 7*a^3*d^5*x^15 + a*c^7*x^14 +
 105/2*a^2*c^4*d^2*x^14 + 35*a^3*c*d^4*x^14 + 21*a^2*c^5*d*x^13 +
 70*a^3*c^2*d^3*x^13 + 7/2*a^2*c^6*x^12 + 70*a^3*c^3*d^2*x^12 + 3
5/4*a^4*d^4*x^12 + 35*a^3*c^4*d*x^11 + 35*a^4*c*d^3*x^11 + 7*a^3*
c^5*x^10 + 105/2*a^4*c^2*d^2*x^10 + 35*a^4*c^3*d*x^9 + 7*a^5*d^3*
x^9 + 35/4*a^4*c^4*x^8 + 21*a^5*c*d^2*x^8 + 21*a^5*c^2*d*x^7 + 7*
```

```
a^5*c^3*x^6 + 7/2*a^6*d^2*x^6 + 7*a^6*c*d*x^5 + 7/2*a^6*c^2*x^4 +
 a^7*d*x^3 + a^7*c*x^2
```

3.203 $\int x(2c+3dx)\left(cx^2+dx^3\right)^7\,dx$

Optimal. Leaf size=17

$$\frac{1}{8}\left(cx^2+dx^3\right)^8$$

```
[Out] (c*x^2 + d*x^3)^8/8
```

Rubi [A] time = 0.0182729, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043

$$\frac{1}{8}\left(cx^2+dx^3\right)^8$$

Antiderivative was successfully verified.

```
[In]  Int[x*(2*c + 3*d*x)*(c*x^2 + d*x^3)^7,x]
```

```
[Out] (c*x^2 + d*x^3)^8/8
```

Rubi in Sympy [A] time = 6.20772, size = 10, normalized size = 0.59

$$\frac{x^{16}\left(c+dx\right)^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(3*d*x+2*c)*(d*x**3+c*x**2)**7,x)
```

```
[Out] x**16*(c + d*x)**8/8
```

Mathematica [B] time = 0.00413834, size = 98, normalized size = 5.76

$$\frac{c^8x^{16}}{8}+c^7dx^{17}+\frac{7}{2}c^6d^2x^{18}+7c^5d^3x^{19}+\frac{35}{4}c^4d^4x^{20}+7c^3d^5x^{21}+\frac{7}{2}c^2d^6x^{22}+cd^7x^{23}+\frac{d^8x^{24}}{8}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(2*c + 3*d*x)*(c*x^2 + d*x^3)^7,x]
```

```
[Out] (c^8*x^16)/8 + c^7*d*x^17 + (7*c^6*d^2*x^18)/2 + 7*c^5*d^3*x^19 +
 (35*c^4*d^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/2 + c*d^
7*x^23 + (d^8*x^24)/8
```

Maple [B] time = 0.002, size = 89, normalized size = 5.2

$$\frac{d^8x^{24}}{8}+cd^7x^{23}+\frac{7\,c^2d^6x^{22}}{2}+7\,c^3d^5x^{21}+\frac{35\,c^4d^4x^{20}}{4}+7\,c^5d^3x^{19}+\frac{7\,c^6d^2x^{18}}{2}+c^7dx^{17}+\frac{c^8x^{16}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(3*d*x+2*c)*(d*x^3+c*x^2)^7,x)
```

```
[Out] 1/8*d^8*x^24+c*d^7*x^23+7/2*c^2*d^6*x^22+7*c^3*d^5*x^21+35/4*c^4*
d^4*x^20+7*c^5*d^3*x^19+7/2*c^6*d^2*x^18+c^7*d*x^17+1/8*c^8*x^16
```

Maxima [A] time = 0.790929, size = 119, normalized size = 7.

$$\frac{1}{8}\,d^{8}x^{24} + cd^{7}x^{23} + \frac{7}{2}\,c^{2}d^{6}x^{22} + 7\,c^{3}d^{5}x^{21} + \frac{35}{4}\,c^{4}d^{4}x^{20} + 7\,c^{5}d^{3}x^{19} + \frac{7}{2}\,c^{6}d^{2}x^{18} + c^{7}dx^{17} + \frac{1}{8}\,c^{8}x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c*x^2)^7*(3*d*x + 2*c)*x,x, algorithm="maxima")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 3
5/4*c^4*d^4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17
 + 1/8*c^8*x^16
```

Fricas [A] time = 0.230048, size = 1, normalized size = 0.06

$$\frac{1}{8}x^{24}d^{8} + x^{23}d^{7}c + \frac{7}{2}x^{22}d^{6}c^{2} + 7x^{21}d^{5}c^{3} + \frac{35}{4}x^{20}d^{4}c^{4} + 7x^{19}d^{3}c^{5} + \frac{7}{2}x^{18}d^{2}c^{6} + x^{17}dc^{7} + \frac{1}{8}x^{16}c^{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c*x^2)^7*(3*d*x + 2*c)*x,x, algorithm="fricas")
```

```
[Out] 1/8*x^24*d^8 + x^23*d^7*c + 7/2*x^22*d^6*c^2 + 7*x^21*d^5*c^3 + 3
5/4*x^20*d^4*c^4 + 7*x^19*d^3*c^5 + 7/2*x^18*d^2*c^6 + x^17*d*c^7
 + 1/8*x^16*c^8
```

Sympy [A] time = 0.181567, size = 97, normalized size = 5.71

$$\frac{c^{8}x^{16}}{8} + c^{7}dx^{17} + \frac{7c^{6}d^{2}x^{18}}{2} + 7c^{5}d^{3}x^{19} + \frac{35c^{4}d^{4}x^{20}}{4} + 7c^{3}d^{5}x^{21} + \frac{7c^{2}d^{6}x^{22}}{2} + cd^{7}x^{23} + \frac{d^{8}x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(3*d*x+2*c)*(d*x**3+c*x**2)**7,x)
```

```
[Out] c**8*x**16/8 + c**7*d*x**17 + 7*c**6*d**2*x**18/2 + 7*c**5*d**3*x
**19 + 35*c**4*d**4*x**20/4 + 7*c**3*d**5*x**21 + 7*c**2*d**6*x**
22/2 + c*d**7*x**23 + d**8*x**24/8
```

GIAC/XCAS [A] time = 0.261373, size = 119, normalized size = 7.

$$\frac{1}{8}\,d^{8}x^{24} + cd^{7}x^{23} + \frac{7}{2}\,c^{2}d^{6}x^{22} + 7\,c^{3}d^{5}x^{21} + \frac{35}{4}\,c^{4}d^{4}x^{20} + 7\,c^{5}d^{3}x^{19} + \frac{7}{2}\,c^{6}d^{2}x^{18} + c^{7}dx^{17} + \frac{1}{8}\,c^{8}x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c*x^2)^7*(3*d*x + 2*c)*x,x, algorithm="giac")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 3
5/4*c^4*d^4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17
 + 1/8*c^8*x^16
```

3.204 $\int x^8(2c+3dx)\left(cx+dx^2\right)^7\,dx$

Optimal. Leaf size=18

$$\frac{1}{8}x^8\left(cx+dx^2\right)^8$$

```
[Out] (x^8*(c*x + d*x^2)^8)/8
```

Rubi [A] time = 0.0197897, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043

$$\frac{1}{8}x^8\left(cx+dx^2\right)^8$$

Antiderivative was successfully verified.

```
[In]  Int[x^8*(2*c + 3*d*x)*(c*x + d*x^2)^7,x]
```

```
[Out] (x^8*(c*x + d*x^2)^8)/8
```

Rubi in Sympy [A] time = 6.17499, size = 10, normalized size = 0.56

$$\frac{x^{16}\left(c+dx\right)^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**8*(3*d*x+2*c)*(d*x**2+c*x)**7,x)
```

```
[Out] x**16*(c + d*x)**8/8
```

Mathematica [B] time = 0.00403115, size = 98, normalized size = 5.44

$$\frac{c^8x^{16}}{8}+c^7dx^{17}+\frac{7}{2}c^6d^2x^{18}+7c^5d^3x^{19}+\frac{35}{4}c^4d^4x^{20}+7c^3d^5x^{21}+\frac{7}{2}c^2d^6x^{22}+cd^7x^{23}+\frac{d^8x^{24}}{8}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^8*(2*c + 3*d*x)*(c*x + d*x^2)^7,x]
```

```
[Out] (c^8*x^16)/8 + c^7*d*x^17 + (7*c^6*d^2*x^18)/2 + 7*c^5*d^3*x^19 +
 (35*c^4*d^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/2 + c*d^
7*x^23 + (d^8*x^24)/8
```

Maple [B] time = 0.002, size = 89, normalized size = 4.9

$$\frac{d^8x^{24}}{8}+cd^7x^{23}+\frac{7\,c^2d^6x^{22}}{2}+7\,c^3d^5x^{21}+\frac{35\,c^4d^4x^{20}}{4}+7\,c^5d^3x^{19}+\frac{7\,c^6d^2x^{18}}{2}+c^7dx^{17}+\frac{c^8x^{16}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^8*(3*d*x+2*c)*(d*x^2+c*x)^7,x)
```

```
[Out] 1/8*d^8*x^24+c*d^7*x^23+7/2*c^2*d^6*x^22+7*c^3*d^5*x^21+35/4*c^4*
d^4*x^20+7*c^5*d^3*x^19+7/2*c^6*d^2*x^18+c^7*d*x^17+1/8*c^8*x^16
```

Maxima [A] time = 0.792115, size = 119, normalized size = 6.61

$$\frac{1}{8}\,d^8x^{24}+cd^7x^{23}+\frac{7}{2}\,c^2d^6x^{22}+7\,c^3d^5x^{21}+\frac{35}{4}\,c^4d^4x^{20}+7\,c^5d^3x^{19}+\frac{7}{2}\,c^6d^2x^{18}+c^7dx^{17}+\frac{1}{8}\,c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c*x)^7*(3*d*x + 2*c)*x^8,x, algorithm="maxima")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 3
5/4*c^4*d^4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17
 + 1/8*c^8*x^16
```

Fricas [A] time = 0.239792, size = 1, normalized size = 0.06

$$\frac{1}{8}x^{24}d^8+x^{23}d^7c+\frac{7}{2}x^{22}d^6c^2+7x^{21}d^5c^3+\frac{35}{4}x^{20}d^4c^4+7x^{19}d^3c^5+\frac{7}{2}x^{18}d^2c^6+x^{17}dc^7+\frac{1}{8}x^{16}c^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c*x)^7*(3*d*x + 2*c)*x^8,x, algorithm="fricas")
```

```
[Out] 1/8*x^24*d^8 + x^23*d^7*c + 7/2*x^22*d^6*c^2 + 7*x^21*d^5*c^3 + 3
5/4*x^20*d^4*c^4 + 7*x^19*d^3*c^5 + 7/2*x^18*d^2*c^6 + x^17*d*c^7
 + 1/8*x^16*c^8
```

Sympy [A] time = 0.182808, size = 97, normalized size = 5.39

$$\frac{c^8x^{16}}{8}+c^7dx^{17}+\frac{7c^6d^2x^{18}}{2}+7c^5d^3x^{19}+\frac{35c^4d^4x^{20}}{4}+7c^3d^5x^{21}+\frac{7c^2d^6x^{22}}{2}+cd^7x^{23}+\frac{d^8x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**8*(3*d*x+2*c)*(d*x**2+c*x)**7,x)
```

```
[Out] c**8*x**16/8 + c**7*d*x**17 + 7*c**6*d**2*x**18/2 + 7*c**5*d**3*x
**19 + 35*c**4*d**4*x**20/4 + 7*c**3*d**5*x**21 + 7*c**2*d**6*x**
22/2 + c*d**7*x**23 + d**8*x**24/8
```

GIAC/XCAS [A] time = 0.258424, size = 119, normalized size = 6.61

$$\frac{1}{8}\,d^8x^{24}+cd^7x^{23}+\frac{7}{2}\,c^2d^6x^{22}+7\,c^3d^5x^{21}+\frac{35}{4}\,c^4d^4x^{20}+7\,c^5d^3x^{19}+\frac{7}{2}\,c^6d^2x^{18}+c^7dx^{17}+\frac{1}{8}\,c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c*x)^7*(3*d*x + 2*c)*x^8,x, algorithm="giac")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 3
5/4*c^4*d^4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17
 + 1/8*c^8*x^16
```

3.205 $\int x^{15}(c+dx)^7(2c+3dx)\,dx$

Optimal. Leaf size=14

$$\frac{1}{8}x^{16}(c+dx)^8$$

```
[Out] (x^16*(c + d*x)^8)/8
```

Rubi [A] time = 0.0131977, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$

$$\frac{1}{8}x^{16}(c+dx)^8$$

Antiderivative was successfully verified.

```
[In]  Int[x^15*(c + d*x)^7*(2*c + 3*d*x),x]
```

```
[Out] (x^16*(c + d*x)^8)/8
```

Rubi in Sympy [A] time = 4.27716, size = 10, normalized size = 0.71

$$\frac{x^{16}\left(c+dx\right)^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**15*(d*x+c)**7*(3*d*x+2*c),x)
```

```
[Out] x**16*(c + d*x)**8/8
```

Mathematica [B] time = 0.00386987, size = 98, normalized size = 7.

$$\frac{c^8x^{16}}{8}+c^7dx^{17}+\frac{7}{2}c^6d^2x^{18}+7c^5d^3x^{19}+\frac{35}{4}c^4d^4x^{20}+7c^3d^5x^{21}+\frac{7}{2}c^2d^6x^{22}+cd^7x^{23}+\frac{d^8x^{24}}{8}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^15*(c + d*x)^7*(2*c + 3*d*x),x]
```

```
[Out] (c^8*x^16)/8 + c^7*d*x^17 + (7*c^6*d^2*x^18)/2 + 7*c^5*d^3*x^19 +
 (35*c^4*d^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/2 + c*d^
7*x^23 + (d^8*x^24)/8
```

Maple [B] time = 0.002, size = 89, normalized size = 6.4

$$\frac{d^8x^{24}}{8}+cd^7x^{23}+\frac{7\,c^2d^6x^{22}}{2}+7\,c^3d^5x^{21}+\frac{35\,c^4d^4x^{20}}{4}+7\,c^5d^3x^{19}+\frac{7\,c^6d^2x^{18}}{2}+c^7dx^{17}+\frac{c^8x^{16}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^15*(d*x+c)^7*(3*d*x+2*c),x)
```

```
[Out] 1/8*d^8*x^24+c*d^7*x^23+7/2*c^2*d^6*x^22+7*c^3*d^5*x^21+35/4*c^4*
d^4*x^20+7*c^5*d^3*x^19+7/2*c^6*d^2*x^18+c^7*d*x^17+1/8*c^8*x^16
```

Maxima [A] time = 0.818444, size = 119, normalized size = 8.5

$$\frac{1}{8}\,d^8x^{24} + cd^7x^{23} + \frac{7}{2}\,c^2d^6x^{22} + 7\,c^3d^5x^{21} + \frac{35}{4}\,c^4d^4x^{20} + 7\,c^5d^3x^{19} + \frac{7}{2}\,c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}\,c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*d*x + 2*c)*(d*x + c)^7*x^15,x, algorithm="maxima")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 3
5/4*c^4*d^4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17
 + 1/8*c^8*x^16
```

Fricas [A] time = 0.232574, size = 1, normalized size = 0.07

$$\frac{1}{8}x^{24}d^8 + x^{23}d^7c + \frac{7}{2}x^{22}d^6c^2 + 7x^{21}d^5c^3 + \frac{35}{4}x^{20}d^4c^4 + 7x^{19}d^3c^5 + \frac{7}{2}x^{18}d^2c^6 + x^{17}dc^7 + \frac{1}{8}x^{16}c^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*d*x + 2*c)*(d*x + c)^7*x^15,x, algorithm="fricas")
```

```
[Out] 1/8*x^24*d^8 + x^23*d^7*c + 7/2*x^22*d^6*c^2 + 7*x^21*d^5*c^3 + 3
5/4*x^20*d^4*c^4 + 7*x^19*d^3*c^5 + 7/2*x^18*d^2*c^6 + x^17*d*c^7
 + 1/8*x^16*c^8
```

Sympy [A] time = 0.157136, size = 97, normalized size = 6.93

$$\frac{c^8x^{16}}{8} + c^7dx^{17} + \frac{7c^6d^2x^{18}}{2} + 7c^5d^3x^{19} + \frac{35c^4d^4x^{20}}{4} + 7c^3d^5x^{21} + \frac{7c^2d^6x^{22}}{2} + cd^7x^{23} + \frac{d^8x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**15*(d*x+c)**7*(3*d*x+2*c),x)
```

```
[Out] c**8*x**16/8 + c**7*d*x**17 + 7*c**6*d**2*x**18/2 + 7*c**5*d**3*x
**19 + 35*c**4*d**4*x**20/4 + 7*c**3*d**5*x**21 + 7*c**2*d**6*x**
22/2 + c*d**7*x**23 + d**8*x**24/8
```

GIAC/XCAS [A] time = 0.257889, size = 119, normalized size = 8.5

$$\frac{1}{8}\,d^8x^{24} + cd^7x^{23} + \frac{7}{2}\,c^2d^6x^{22} + 7\,c^3d^5x^{21} + \frac{35}{4}\,c^4d^4x^{20} + 7\,c^5d^3x^{19} + \frac{7}{2}\,c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}\,c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*d*x + 2*c)*(d*x + c)^7*x^15,x, algorithm="giac")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 3
5/4*c^4*d^4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17
 + 1/8*c^8*x^16
```

3.206 $\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^4\right)\,dx$

Optimal. Leaf size=28

$$\frac{1}{160}x^5(2a+bx)^5+ax+\frac{bx^2}{2}$$

```
[Out] a*x + (b*x^2)/2 + (x^5*(2*a + b*x)^5)/160
```

Rubi [A] time = 0.0263356, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045

$$\frac{1}{160}x^5(2a+bx)^5+ax+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x)*(1 + (a*x + (b*x^2)/2)^4),x]

[Out] a*x + (b*x^2)/2 + (x^5*(2*a + b*x)^5)/160
```

Rubi in Sympy [A] time = 2.4626, size = 22, normalized size = 0.79

$$ax+\frac{bx^2}{2}+\frac{\left(ax+\frac{bx^2}{2}\right)^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x+a)*(1+(a*x+1/2*b*x**2)**4),x)

[Out] a*x + b*x**2/2 + (a*x + b*x**2/2)**5/5
```

Mathematica [B] time = 0.00773175, size = 80, normalized size = 2.86

$$\frac{a^5x^5}{5}+\frac{1}{2}a^4bx^6+\frac{1}{2}a^3b^2x^7+\frac{1}{4}a^2b^3x^8+\frac{1}{16}ab^4x^9+ax+\frac{b^5x^{10}}{160}+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x)*(1 + (a*x + (b*x^2)/2)^4),x]

[Out] a*x + (b*x^2)/2 + (a^5*x^5)/5 + (a^4*b*x^6)/2 + (a^3*b^2*x^7)/2 +
 (a^2*b^3*x^8)/4 + (a*b^4*x^9)/16 + (b^5*x^10)/160
```

Maple [B] time = 0.001, size = 67, normalized size = 2.4

$$\frac{b^5x^{10}}{160}+\frac{ab^4x^9}{16}+\frac{a^2b^3x^8}{4}+\frac{a^3b^2x^7}{2}+\frac{a^4bx^6}{2}+\frac{a^5x^5}{5}+\frac{bx^2}{2}+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x+a)*(1+(a*x+1/2*b*x^2)^4),x)
```

```
[Out] 1/160*b^5*x^10+1/16*a*b^4*x^9+1/4*a^2*b^3*x^8+1/2*a^3*b^2*x^7+1/2
*a^4*b*x^6+1/5*a^5*x^5+1/2*b*x^2+a*x
```

Maxima [A] time = 0.815985, size = 89, normalized size = 3.18

$$\frac{1}{160}\,b^5x^{10} + \frac{1}{16}\,ab^4x^9 + \frac{1}{4}\,a^2b^3x^8 + \frac{1}{2}\,a^3b^2x^7 + \frac{1}{2}\,a^4bx^6 + \frac{1}{5}\,a^5x^5 + \frac{1}{2}\,bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/16*((b*x^2 + 2*a*x)^4 + 16)*(b*x + a),x, algorithm="maxima")
```

```
[Out] 1/160*b^5*x^10 + 1/16*a*b^4*x^9 + 1/4*a^2*b^3*x^8 + 1/2*a^3*b^2*x
^7 + 1/2*a^4*b*x^6 + 1/5*a^5*x^5 + 1/2*b*x^2 + a*x
```

Fricas [A] time = 0.256412, size = 89, normalized size = 3.18

$$\frac{1}{160}\,b^5x^{10} + \frac{1}{16}\,ab^4x^9 + \frac{1}{4}\,a^2b^3x^8 + \frac{1}{2}\,a^3b^2x^7 + \frac{1}{2}\,a^4bx^6 + \frac{1}{5}\,a^5x^5 + \frac{1}{2}\,bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/16*((b*x^2 + 2*a*x)^4 + 16)*(b*x + a),x, algorithm="fricas")
```

```
[Out] 1/160*b^5*x^10 + 1/16*a*b^4*x^9 + 1/4*a^2*b^3*x^8 + 1/2*a^3*b^2*x
^7 + 1/2*a^4*b*x^6 + 1/5*a^5*x^5 + 1/2*b*x^2 + a*x
```

Sympy [A] time = 0.160112, size = 70, normalized size = 2.5

$$\frac{a^5x^5}{5} + \frac{a^4bx^6}{2} + \frac{a^3b^2x^7}{2} + \frac{a^2b^3x^8}{4} + \frac{ab^4x^9}{16} + ax + \frac{b^5x^{10}}{160} + \frac{bx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x+a)*(1+(a*x+1/2*b*x**2)**4),x)
```

```
[Out] a**5*x**5/5 + a**4*b*x**6/2 + a**3*b**2*x**7/2 + a**2*b**3*x**8/4
 + a*b**4*x**9/16 + a*x + b**5*x**10/160 + b*x**2/2
```

GIAC/XCAS [A] time = 0.258726, size = 89, normalized size = 3.18

$$\frac{1}{160}\,b^5x^{10} + \frac{1}{16}\,ab^4x^9 + \frac{1}{4}\,a^2b^3x^8 + \frac{1}{2}\,a^3b^2x^7 + \frac{1}{2}\,a^4bx^6 + \frac{1}{5}\,a^5x^5 + \frac{1}{2}\,bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/16*((b*x^2 + 2*a*x)^4 + 16)*(b*x + a),x, algorithm="giac")
```

```
[Out] 1/160*b^5*x^10 + 1/16*a*b^4*x^9 + 1/4*a^2*b^3*x^8 + 1/2*a^3*b^2*x
^7 + 1/2*a^4*b*x^6 + 1/5*a^5*x^5 + 1/2*b*x^2 + a*x
```

3.207 $\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^4\right)\,dx$

Optimal. Leaf size=31

$$\frac{1}{5}\left(ax+\frac{bx^2}{2}+c\right)^5+ax+\frac{bx^2}{2}$$

```
[Out] a*x + (b*x^2)/2 + (c + a*x + (b*x^2)/2)^5/5
```

Rubi [A] time = 0.0367936, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043

$$\frac{1}{160}\left(2ax+bx^2+2c\right)^5+ax+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x)*(1 + (c + a*x + (b*x^2)/2)^4),x]

[Out] a*x + (b*x^2)/2 + (2*c + 2*a*x + b*x^2)^5/160
```

Rubi in Sympy [A] time = 2.98185, size = 26, normalized size = 0.84

$$ax+\frac{bx^2}{2}+c+\frac{\left(ax+\frac{bx^2}{2}+c\right)^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x+a)*(1+(c+a*x+1/2*b*x**2)**4),x)

[Out] a*x + b*x**2/2 + c + (a*x + b*x**2/2 + c)**5/5
```

Mathematica [B] time = 0.056645, size = 108, normalized size = 3.48

$$\frac{1}{160}x(2a+bx)\left(16a^4x^4+32a^3bx^5+24a^2b^2x^6+8ab^3x^7+80c^3x(2a+bx)+40c^2x^2(2a+bx)^2+10cx^3(2a+bx)^3+b^4x^8+80c^4+80\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x)*(1 + (c + a*x + (b*x^2)/2)^4),x]

[Out] (x*(2*a + b*x)*(80 + 80*c^4 + 16*a^4*x^4 + 32*a^3*b*x^5 + 24*a^2*
b^2*x^6 + 8*a*b^3*x^7 + b^4*x^8 + 80*c^3*x*(2*a + b*x) + 40*c^2*x
^2*(2*a + b*x)^2 + 10*c*x^3*(2*a + b*x)^3))/160
```

Maple [B] time = 0.001, size = 325, normalized size = 10.5

$$\frac{b^5x^{10}}{160}+\frac{ab^4x^9}{16}+\frac{x^8}{8}\left(\frac{a^2b^3}{2}+b\left(\frac{\left(a^2+bc\right)b^2}{2}+a^2b^2\right)\right)+\frac{x^7}{7}\left(a\left(\frac{\left(a^2+bc\right)b^2}{2}+a^2b^2\right)+b\left(ab^2c+2\left(a^2+bc\right)ab\right)\right)+\frac{x^6}{6}\left(a\left(ab^2c+2\left(a^2+bc\right)ab\right)+b\left(\frac{b^2c^2}{2}+4a^2bc+\left(a^2+bc\right)^2\right)\right)+\frac{x^5}{5}\left(a\left(\frac{b^2c^2}{2}+4a^2bc+\left(a^2+bc\right)^2\right)+b\left(2abc^2+4ac\left(a^2+bc\right)\right)\right)+\frac{\left(a\left(2abc^2+4ac\left(a^2+bc\right)\right)+b\left(2c^2\left(a^2+bc\right)+4a^2c^2\right)\right)x^4}{4}+\frac{\left(a\left(2c^2\left(a^2+bc\right)+4a^2c^2\right)+4abc^3\right)x^3}{3}+\frac{\left(4a^2c^3+b\left(c^4+1\right)\right)x^2}{2}+a\left(c^4+1\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x+a)*(1+(c+a*x+1/2*b*x^2)^4),x)
```

```
[Out] 1/160*b^5*x^10+1/16*a*b^4*x^9+1/8*(1/2*a^2*b^3+b*(1/2*(a^2+b*c)*b
^2+a^2*b^2))*x^8+1/7*(a*(1/2*(a^2+b*c)*b^2+a^2*b^2)+b*(a*b^2*c+2*
(a^2+b*c)*a*b))*x^7+1/6*(a*(a*b^2*c+2*(a^2+b*c)*a*b)+b*(1/2*b^2*c
^2+4*a^2*b*c+(a^2+b*c)^2))*x^6+1/5*(a*(1/2*b^2*c^2+4*a^2*b*c+(a^2
+b*c)^2)+b*(2*a*b*c^2+4*a*c*(a^2+b*c)))*x^5+1/4*(a*(2*a*b*c^2+4*a
*c*(a^2+b*c))+b*(2*c^2*(a^2+b*c)+4*a^2*c^2))*x^4+1/3*(a*(2*c^2*(a
^2+b*c)+4*a^2*c^2)+4*a*b*c^3)*x^3+1/2*(4*a^2*c^3+b*(c^4+1))*x^2+a
*(c^4+1)*x
```

Maxima [A] time = 0.821986, size = 252, normalized size = 8.13

$$\frac{1}{160}b^5x^{10}+\frac{1}{16}ab^4x^9+\frac{1}{16}\left(4a^2b^3+b^4c\right)x^8+\frac{1}{2}\left(a^3b^2+ab^3c\right)x^7+\frac{1}{4}\left(2a^4b+6a^2b^2c+b^3c^2\right)x^6+\frac{1}{10}\left(2a^5+20a^3bc+15ab^2c^2\right)x^5+\frac{1}{2}\left(2a^4c+6a^2bc^2+b^2c^3\right)x^4+2\left(a^3c^2+abc^3\right)x^3+\frac{1}{2}\left(4a^2c^3+bc^4+b\right)x^2+\left(ac^4+a\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/16*((b*x^2 + 2*a*x + 2*c)^4 + 16)*(b*x + a),x, algorithm="maxima")
```

```
[Out] 1/160*b^5*x^10 + 1/16*a*b^4*x^9 + 1/16*(4*a^2*b^3 + b^4*c)*x^8 +
1/2*(a^3*b^2 + a*b^3*c)*x^7 + 1/4*(2*a^4*b + 6*a^2*b^2*c + b^3*c^
2)*x^6 + 1/10*(2*a^5 + 20*a^3*b*c + 15*a*b^2*c^2)*x^5 + 1/2*(2*a^
4*c + 6*a^2*b*c^2 + b^2*c^3)*x^4 + 2*(a^3*c^2 + a*b*c^3)*x^3 + 1/
2*(4*a^2*c^3 + b*c^4 + b)*x^2 + (a*c^4 + a)*x
```

Fricas [A] time = 0.255034, size = 252, normalized size = 8.13

$$\frac{1}{160}b^5x^{10}+\frac{1}{16}ab^4x^9+\frac{1}{16}\left(4a^2b^3+b^4c\right)x^8+\frac{1}{2}\left(a^3b^2+ab^3c\right)x^7+\frac{1}{4}\left(2a^4b+6a^2b^2c+b^3c^2\right)x^6+\frac{1}{10}\left(2a^5+20a^3bc+15ab^2c^2\right)x^5+\frac{1}{2}\left(2a^4c+6a^2bc^2+b^2c^3\right)x^4+2\left(a^3c^2+abc^3\right)x^3+\frac{1}{2}\left(4a^2c^3+bc^4+b\right)x^2+\left(ac^4+a\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/16*((b*x^2 + 2*a*x + 2*c)^4 + 16)*(b*x + a),x, algorithm="fricas")
```

```
[Out] 1/160*b^5*x^10 + 1/16*a*b^4*x^9 + 1/16*(4*a^2*b^3 + b^4*c)*x^8 +
1/2*(a^3*b^2 + a*b^3*c)*x^7 + 1/4*(2*a^4*b + 6*a^2*b^2*c + b^3*c^
2)*x^6 + 1/10*(2*a^5 + 20*a^3*b*c + 15*a*b^2*c^2)*x^5 + 1/2*(2*a^
4*c + 6*a^2*b*c^2 + b^2*c^3)*x^4 + 2*(a^3*c^2 + a*b*c^3)*x^3 + 1/
2*(4*a^2*c^3 + b*c^4 + b)*x^2 + (a*c^4 + a)*x
```

Sympy [A] time = 0.237916, size = 194, normalized size = 6.26

$$\frac{ab^4x^9}{16}+\frac{b^5x^{10}}{160}+x^8\left(\frac{a^2b^3}{4}+\frac{b^4c}{16}\right)+x^7\left(\frac{a^3b^2}{2}+\frac{ab^3c}{2}\right)+x^6\left(\frac{a^4b}{2}+\frac{3a^2b^2c}{2}+\frac{b^3c^2}{4}\right)+x^5\left(\frac{a^5}{5}+2a^3bc+\frac{3ab^2c^2}{2}\right)+x^4\left(a^4c+3a^2bc^2+\frac{b^2c^3}{2}\right)+x^3\left(2a^3c^2+2abc^3\right)+x^2\left(2a^2c^3+\frac{bc^4}{2}+\frac{b}{2}\right)+x\left(ac^4+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x+a)*(1+(c+a*x+1/2*b*x**2)**4),x)
```

```
[Out] a*b**4*x**9/16 + b**5*x**10/160 + x**8*(a**2*b**3/4 + b**4*c/16)
+ x**7*(a**3*b**2/2 + a*b**3*c/2) + x**6*(a**4*b/2 + 3*a**2*b**2*
c/2 + b**3*c**2/4) + x**5*(a**5/5 + 2*a**3*b*c + 3*a*b**2*c**2/2)
 + x**4*(a**4*c + 3*a**2*b*c**2 + b**2*c**3/2) + x**3*(2*a**3*c**
2 + 2*a*b*c**3) + x**2*(2*a**2*c**3 + b*c**4/2 + b/2) + x*(a*c**4
 + a)
```

GIAC/XCAS [A] time = 0.259592, size = 281, normalized size = 9.06

$$\frac{1}{160}\,b^5x^{10}+\frac{1}{16}\,ab^4x^9+\frac{1}{4}\,a^2b^3x^8+\frac{1}{16}\,b^4cx^8+\frac{1}{2}\,a^3b^2x^7+\frac{1}{2}\,ab^3cx^7+\frac{1}{2}\,a^4bx^6+\frac{3}{2}\,a^2b^2cx^6+\frac{1}{4}\,b^3c^2x^6+\frac{1}{5}\,a^5x^5+2\,a^3bcx^5+\frac{3}{2}\,ab^2c^2x^5+a^4cx^4+3\,a^2bc^2x^4+\frac{1}{2}\,b^2c^3x^4+2\,a^3c^2x^3+2\,abc^3x^3+2\,a^2c^3x^2+\frac{1}{2}\,bc^4x^2+ac^4x+\frac{1}{2}\,bx^2+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/16*((b*x^2 + 2*a*x + 2*c)^4 + 16)*(b*x + a),x, algorithm="giac")
```

```
[Out] 1/160*b^5*x^10 + 1/16*a*b^4*x^9 + 1/4*a^2*b^3*x^8 + 1/16*b^4*c*x^
8 + 1/2*a^3*b^2*x^7 + 1/2*a*b^3*c*x^7 + 1/2*a^4*b*x^6 + 3/2*a^2*b
^2*c*x^6 + 1/4*b^3*c^2*x^6 + 1/5*a^5*x^5 + 2*a^3*b*c*x^5 + 3/2*a*
b^2*c^2*x^5 + a^4*c*x^4 + 3*a^2*b*c^2*x^4 + 1/2*b^2*c^3*x^4 + 2*a
^3*c^2*x^3 + 2*a*b*c^3*x^3 + 2*a^2*c^3*x^2 + 1/2*b*c^4*x^2 + a*c^
4*x + 1/2*b*x^2 + a*x
```

3.208 $\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^n\right)\,dx$

Optimal. Leaf size=34

$$\frac{\left(ax+\frac{bx^2}{2}\right)^{n+1}}{n+1}+ax+\frac{bx^2}{2}$$

```
[Out] a*x + (b*x^2)/2 + (a*x + (b*x^2)/2)^(1 + n)/(1 + n)
```

Rubi [A] time = 0.019103, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045

$$\frac{\left(ax+\frac{bx^2}{2}\right)^{n+1}}{n+1}+ax+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x)*(1 + (a*x + (b*x^2)/2)^n),x]

[Out] a*x + (b*x^2)/2 + (a*x + (b*x^2)/2)^(1 + n)/(1 + n)
```

Rubi in Sympy [A] time = 2.34797, size = 26, normalized size = 0.76

$$ax+\frac{bx^2}{2}+\frac{\left(ax+\frac{bx^2}{2}\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x+a)*(1+(a*x+1/2*b*x**2)**n),x)

[Out] a*x + b*x**2/2 + (a*x + b*x**2/2)**(n + 1)/(n + 1)
```

Mathematica [A] time = 0.0301894, size = 34, normalized size = 1.

$$\frac{x(2a+bx)\left(\left(ax+\frac{bx^2}{2}\right)^n+n+1\right)}{2(n+1)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x)*(1 + (a*x + (b*x^2)/2)^n),x]

[Out] (x*(2*a + b*x)*(1 + n + (a*x + (b*x^2)/2)^n))/(2*(1 + n))
```

Maple [A] time = 0.002, size = 31, normalized size = 0.9

$$ax+\frac{bx^2}{2}+\frac{1}{1+n}\left(ax+\frac{bx^2}{2}\right)^{1+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x+a)*(1+(a*x+1/2*b*x^2)^n),x)

[Out] a*x+1/2*b*x^2+(a*x+1/2*b*x^2)^(1+n)/(1+n)
```

Maxima [A] time = 0.991402, size = 70, normalized size = 2.06

$$\frac{1}{2}\,bx^2+ax+\frac{\left(bx^2+2\,ax\right)e^{\left(n\log(bx+2\,a)+n\log(x)\right)}}{2^{n+1}n+2^{n+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x + a)*((1/2*b*x^2 + a*x)^n + 1),x, algorithm="maxima")

[Out] 1/2*b*x^2 + a*x + (b*x^2 + 2*a*x)*e^(n*log(b*x + 2*a) + n*log(x))
/(2^(n + 1)*n + 2^(n + 1))
```

Fricas [A] time = 0.291046, size = 65, normalized size = 1.91

$$\frac{(bn+b)x^2+\left(bx^2+2\,ax\right)\left(\frac{1}{2}\,bx^2+ax\right)^n+2\,(an+a)x}{2\,(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x + a)*((1/2*b*x^2 + a*x)^n + 1),x, algorithm="fricas")

[Out] 1/2*((b*n + b)*x^2 + (b*x^2 + 2*a*x)*(1/2*b*x^2 + a*x)^n + 2*(a*n
 + a)*x)/(n + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x+a)*(1+(a*x+1/2*b*x**2)**n),x)

[Out] Timed out
```

GIAC/XCAS [A] time = 0.261112, size = 41, normalized size = 1.21

$$\frac{1}{2}\,bx^2+ax+\frac{\left(\frac{1}{2}\,bx^2+ax\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x + a)*((1/2*b*x^2 + a*x)^n + 1),x, algorithm="giac")

[Out] 1/2*b*x^2 + a*x + (1/2*b*x^2 + a*x)^(n + 1)/(n + 1)
```

3.209 $\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^n\right)\,dx$

Optimal. Leaf size=35

$$\frac{\left(ax+\frac{bx^2}{2}+c\right)^{n+1}}{n+1}+ax+\frac{bx^2}{2}$$

```
[Out] a*x + (b*x^2)/2 + (c + a*x + (b*x^2)/2)^(1 + n)/(1 + n)
```

Rubi [A] time = 0.0200527, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043

$$\frac{\left(ax+\frac{bx^2}{2}+c\right)^{n+1}}{n+1}+ax+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x)*(1 + (c + a*x + (b*x^2)/2)^n),x]
```

```
[Out] a*x + (b*x^2)/2 + (c + a*x + (b*x^2)/2)^(1 + n)/(1 + n)
```

Rubi in Sympy [A] time = 2.38454, size = 29, normalized size = 0.83

$$ax+\frac{bx^2}{2}+c+\frac{\left(ax+\frac{bx^2}{2}+c\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x+a)*(1+(c+a*x+1/2*b*x**2)**n),x)
```

```
[Out] a*x + b*x**2/2 + c + (a*x + b*x**2/2 + c)**(n + 1)/(n + 1)
```

Mathematica [B] time = 0.066281, size = 73, normalized size = 2.09

$$\frac{2c\left(ax+\frac{bx^2}{2}+c\right)^n+bx^2\left(\left(ax+\frac{bx^2}{2}+c\right)^n+n+1\right)+2ax\left(\left(ax+\frac{bx^2}{2}+c\right)^n+n+1\right)}{2(n+1)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x)*(1 + (c + a*x + (b*x^2)/2)^n),x]
```

```
[Out] (2*c*(c + a*x + (b*x^2)/2)^n + 2*a*x*(1 + n + (c + a*x + (b*x^2)/
2)^n) + b*x^2*(1 + n + (c + a*x + (b*x^2)/2)^n))/(2*(1 + n))
```

Maple [A] time = 0.003, size = 33, normalized size = 0.9

$$c+ax+\frac{bx^2}{2}+\frac{1}{1+n}\left(c+ax+\frac{bx^2}{2}\right)^{1+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x+a)*(1+(c+a*x+1/2*b*x^2)^n),x)
```

```
[Out] c+a*x+1/2*b*x^2+(c+a*x+1/2*b*x^2)^(1+n)/(1+n)
```

Maxima [A] time = 0.96775, size = 73, normalized size = 2.09

$$\frac{1}{2}\,bx^2 + ax + \frac{\left(bx^2 + 2\,ax + 2\,c\right)\left(bx^2 + 2\,ax + 2\,c\right)^n}{2^{n+1}n + 2^{n+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x + a)*((1/2*b*x^2 + a*x + c)^n + 1),x, algorithm="maxima")
```

```
[Out] 1/2*b*x^2 + a*x + (b*x^2 + 2*a*x + 2*c)*(b*x^2 + 2*a*x + 2*c)^n/(
2^(n + 1)*n + 2^(n + 1))
```

Fricas [A] time = 0.272704, size = 70, normalized size = 2.

$$\frac{(bn + b)x^2 + \left(bx^2 + 2\,ax + 2\,c\right)\left(\frac{1}{2}\,bx^2 + ax + c\right)^n + 2\,(an + a)x}{2\,(n + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x + a)*((1/2*b*x^2 + a*x + c)^n + 1),x, algorithm="fricas")
```

```
[Out] 1/2*((b*n + b)*x^2 + (b*x^2 + 2*a*x + 2*c)*(1/2*b*x^2 + a*x + c)^
n + 2*(a*n + a)*x)/(n + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x+a)*(1+(c+a*x+1/2*b*x**2)**n),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.26976, size = 115, normalized size = 3.29

$$\frac{bnx^2 + bx^2e^{\left(n\ln\left(\frac{1}{2}\,bx^2+ax+c\right)\right)} + 2\,anx + bx^2 + 2\,axe^{\left(n\ln\left(\frac{1}{2}\,bx^2+ax+c\right)\right)} + 2\,ax + 2\,ce^{\left(n\ln\left(\frac{1}{2}\,bx^2+ax+c\right)\right)}}{2\,(n + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x + a)*((1/2*b*x^2 + a*x + c)^n + 1),x, algorithm="giac")
```

```
[Out] 1/2*(b*n*x^2 + b*x^2*e^(n*ln(1/2*b*x^2 + a*x + c)) + 2*a*n*x + b*
x^2 + 2*a*x*e^(n*ln(1/2*b*x^2 + a*x + c)) + 2*a*x + 2*c*e^(n*ln(1
/2*b*x^2 + a*x + c)))/(n + 1)
```

3.210 $\int \left(a+cx^2\right)\left(1+\left(ax+\frac{cx^3}{3}\right)^5\right)\,dx$

Optimal. Leaf size=30

$$\frac{1}{6}\left(ax+\frac{cx^3}{3}\right)^6+ax+\frac{cx^3}{3}$$

```
[Out] a*x + (c*x^3)/3 + (a*x + (c*x^3)/3)^6/6
```

Rubi [A] time = 0.0295571, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042

$$\frac{\left(3ax+cx^3\right)^6}{4374}+ax+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + c*x^2)*(1 + (a*x + (c*x^3)/3)^5),x]

[Out] a*x + (c*x^3)/3 + (3*a*x + c*x^3)^6/4374
```

Rubi in Sympy [A] time = 4.07227, size = 22, normalized size = 0.73

$$ax+\frac{cx^3}{3}+\frac{\left(ax+\frac{cx^3}{3}\right)^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**2+a)*(1+(a*x+1/3*c*x**3)**5),x)

[Out] a*x + c*x**3/3 + (a*x + c*x**3/3)**6/6
```

Mathematica [B] time = 0.00919983, size = 93, normalized size = 3.1

$$\frac{a^6x^6}{6}+\frac{1}{3}a^5cx^8+\frac{5}{18}a^4c^2x^{10}+\frac{10}{81}a^3c^3x^{12}+\frac{5}{162}a^2c^4x^{14}+\frac{1}{243}ac^5x^{16}+ax+\frac{c^6x^{18}}{4374}+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + c*x^2)*(1 + (a*x + (c*x^3)/3)^5),x]

[Out] a*x + (c*x^3)/3 + (a^6*x^6)/6 + (a^5*c*x^8)/3 + (5*a^4*c^2*x^10)/
18 + (10*a^3*c^3*x^12)/81 + (5*a^2*c^4*x^14)/162 + (a*c^5*x^16)/2
43 + (c^6*x^18)/4374
```

Maple [B] time = 0.005, size = 78, normalized size = 2.6

$$\frac{c^6x^{18}}{4374}+\frac{ac^5x^{16}}{243}+\frac{5\,a^2c^4x^{14}}{162}+\frac{10\,a^3c^3x^{12}}{81}+\frac{5\,a^4c^2x^{10}}{18}+\frac{a^5cx^8}{3}+\frac{a^6x^6}{6}+\frac{cx^3}{3}+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^2+a)*(1+(a*x+1/3*c*x^3)^5),x)
```

```
[Out] 1/4374*c^6*x^18+1/243*a*c^5*x^16+5/162*a^2*c^4*x^14+10/81*a^3*c^3
*x^12+5/18*a^4*c^2*x^10+1/3*a^5*c*x^8+1/6*a^6*x^6+1/3*c*x^3+a*x
```

Maxima [A] time = 0.80384, size = 104, normalized size = 3.47

$$\frac{1}{4374}c^6x^{18} + \frac{1}{243}ac^5x^{16} + \frac{5}{162}a^2c^4x^{14} + \frac{10}{81}a^3c^3x^{12} + \frac{5}{18}a^4c^2x^{10} + \frac{1}{3}a^5cx^8 + \frac{1}{6}a^6x^6 + \frac{1}{3}cx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/243*((c*x^3 + 3*a*x)^5 + 243)*(c*x^2 + a),x, algorithm="maxima")
```

```
[Out] 1/4374*c^6*x^18 + 1/243*a*c^5*x^16 + 5/162*a^2*c^4*x^14 + 10/81*a
^3*c^3*x^12 + 5/18*a^4*c^2*x^10 + 1/3*a^5*c*x^8 + 1/6*a^6*x^6 + 1
/3*c*x^3 + a*x
```

Fricas [A] time = 0.256895, size = 104, normalized size = 3.47

$$\frac{1}{4374}c^6x^{18} + \frac{1}{243}ac^5x^{16} + \frac{5}{162}a^2c^4x^{14} + \frac{10}{81}a^3c^3x^{12} + \frac{5}{18}a^4c^2x^{10} + \frac{1}{3}a^5cx^8 + \frac{1}{6}a^6x^6 + \frac{1}{3}cx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/243*((c*x^3 + 3*a*x)^5 + 243)*(c*x^2 + a),x, algorithm="fricas")
```

```
[Out] 1/4374*c^6*x^18 + 1/243*a*c^5*x^16 + 5/162*a^2*c^4*x^14 + 10/81*a
^3*c^3*x^12 + 5/18*a^4*c^2*x^10 + 1/3*a^5*c*x^8 + 1/6*a^6*x^6 + 1
/3*c*x^3 + a*x
```

Sympy [A] time = 0.174432, size = 87, normalized size = 2.9

$$\frac{a^6x^6}{6} + \frac{a^5cx^8}{3} + \frac{5a^4c^2x^{10}}{18} + \frac{10a^3c^3x^{12}}{81} + \frac{5a^2c^4x^{14}}{162} + \frac{ac^5x^{16}}{243} + ax + \frac{c^6x^{18}}{4374} + \frac{cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**2+a)*(1+(a*x+1/3*c*x**3)**5),x)
```

```
[Out] a**6*x**6/6 + a**5*c*x**8/3 + 5*a**4*c**2*x**10/18 + 10*a**3*c**3
*x**12/81 + 5*a**2*c**4*x**14/162 + a*c**5*x**16/243 + a*x + c**6
*x**18/4374 + c*x**3/3
```

GIAC/XCAS [A] time = 0.258867, size = 104, normalized size = 3.47

$$\frac{1}{4374}c^6x^{18} + \frac{1}{243}ac^5x^{16} + \frac{5}{162}a^2c^4x^{14} + \frac{10}{81}a^3c^3x^{12} + \frac{5}{18}a^4c^2x^{10} + \frac{1}{3}a^5cx^8 + \frac{1}{6}a^6x^6 + \frac{1}{3}cx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/243*((c*x^3 + 3*a*x)^5 + 243)*(c*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/4374*c^6*x^18 + 1/243*a*c^5*x^16 + 5/162*a^2*c^4*x^14 + 10/81*a
^3*c^3*x^12 + 5/18*a^4*c^2*x^10 + 1/3*a^5*c*x^8 + 1/6*a^6*x^6 + 1
/3*c*x^3 + a*x
```

3.211 $\int (a + cx^2) \left(1 + \left(d + ax + \frac{cx^3}{3}\right)^5\right) dx$

Optimal. Leaf size=31

$$\frac{1}{6}\left(ax + \frac{cx^3}{3} + d\right)^6 + ax + \frac{cx^3}{3}$$

```
[Out] a*x + (c*x^3)/3 + (d + a*x + (c*x^3)/3)^6/6
```

Rubi [A] time = 0.048952, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.04$

$$\frac{(3ax + cx^3 + 3d)^6}{4374} + ax + \frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + c*x^2)*(1 + (d + a*x + (c*x^3)/3)^5),x]

[Out] a*x + (c*x^3)/3 + (3*d + 3*a*x + c*x^3)^6/4374
```

Rubi in Sympy [A] time = 6.82399, size = 26, normalized size = 0.84

$$ax + \frac{cx^3}{3} + d + \frac{\left(ax + \frac{cx^3}{3} + d\right)^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**2+a)*(1+(d+a*x+1/3*c*x**3)**5),x)

[Out] a*x + c*x**3/3 + d + (a*x + c*x**3/3 + d)**6/6
```

Mathematica [B] time = 0.0750482, size = 140, normalized size = 4.52

$$\frac{x (3a + cx^2) \left(243a^5x^5 + 405a^4cx^7 + 270a^3c^2x^9 + 90a^2c^3x^{11} + 15ac^4x^{13} + 1215d^4 (3ax + cx^3) + 540d^3 (3ax + cx^3)^2 + 135d^2\right.}{4374}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + c*x^2)*(1 + (d + a*x + (c*x^3)/3)^5),x]

[Out] (x*(3*a + c*x^2)*(1458 + 1458*d^5 + 243*a^5*x^5 + 405*a^4*c*x^7 +
 270*a^3*c^2*x^9 + 90*a^2*c^3*x^11 + 15*a*c^4*x^13 + c^5*x^15 + 1
215*d^4*(3*a*x + c*x^3) + 540*d^3*(3*a*x + c*x^3)^2 + 135*d^2*(3*
a*x + c*x^3)^3 + 18*d*(3*a*x + c*x^3)^4))/4374
```

Maple [B] time = 0.004, size = 618, normalized size = 19.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^2+a)*(1+(d+a*x+1/3*c*x^3)^5),x)
```

```
[Out] 1/4374*c^6*x^18+1/243*a*c^5*x^16+1/243*c^5*d*x^15+5/162*a^2*c^4*x
^14+5/81*a*c^4*d*x^13+1/12*(10/27*a^3*c^3+c*(4/27*d^2*c^3+2/3*c^2
*a^3+1/3*c*(2/3*c^2*d^2+4/3*a^3*c)))*x^12+10/27*a^2*c^3*d*x^11+1/
10*(a*(4/27*d^2*c^3+2/3*c^2*a^3+1/3*c*(2/3*c^2*d^2+4/3*a^3*c))+c*
(4/3*a*c^2*d^2+a*(2/3*c^2*d^2+4/3*a^3*c)+1/3*c*(a^4+4*a*c*d^2)))*
x^10+1/9*(10/3*a^3*c^2*d+c*(d*(2/3*c^2*d^2+4/3*a^3*c)+4*a^3*c*d+1
/3*c*(4/3*c*d^3+4*a^3*d)))*x^9+1/8*(a*(4/3*a*c^2*d^2+a*(2/3*c^2*d
^2+4/3*a^3*c)+1/3*c*(a^4+4*a*c*d^2))+c*(6*d^2*a^2*c+a*(a^4+4*a*c*
d^2)))*x^8+1/7*(a*(d*(2/3*c^2*d^2+4/3*a^3*c)+4*a^3*c*d+1/3*c*(4/3
*c*d^3+4*a^3*d))+c*(d*(a^4+4*a*c*d^2)+a*(4/3*c*d^3+4*a^3*d)+4/3*a
*c*d^3))*x^7+1/6*(a*(6*d^2*a^2*c+a*(a^4+4*a*c*d^2))+c*(d*(4/3*c*d
^3+4*a^3*d)+6*a^3*d^2+1/3*c*d^4))*x^6+1/5*(a*(d*(a^4+4*a*c*d^2)+a
*(4/3*c*d^3+4*a^3*d)+4/3*a*c*d^3)+10*a^2*c*d^3)*x^5+1/4*(a*(d*(4/
3*c*d^3+4*a^3*d)+6*a^3*d^2+1/3*c*d^4)+5*a*c*d^4)*x^4+1/3*(10*a^3*
d^3+c*(d^5+1))*x^3+5/2*a^2*d^4*x^2+a*(d^5+1)*x
```

Maxima [A] time = 0.81392, size = 378, normalized size = 12.19

$$\frac{1}{4374}c^6x^{18}+\frac{1}{243}ac^5x^{16}+\frac{1}{243}c^5dx^{15}+\frac{5}{162}a^2c^4x^{14}+\frac{5}{81}ac^4dx^{13}+\frac{10}{27}a^2c^3dx^{11}+\frac{5}{162}\left(4a^3c^3+c^4d^2\right)x^{12}+\frac{5}{54}\left(3a^4c^2+4ac^3d^2\right)x^{10}+\frac{10}{81}\left(9a^3c^2d+c^3d^3\right)x^9+\frac{1}{3}\left(a^5c+5a^2c^2d^2\right)x^8+\frac{5}{2}a^2d^4x^2+\frac{5}{9}\left(3a^4cd+2ac^2d^3\right)x^7+\frac{1}{18}\left(3a^6+60a^3cd^2+5c^2d^4\right)x^6+\frac{1}{3}\left(3a^5d+10a^2cd^3\right)x^5+\frac{5}{6}\left(3a^4d^2+2acd^4\right)x^4+\frac{1}{3}\left(10a^3d^3+cd^5+c\right)x^3+\left(ad^5+a\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/243*((c*x^3 + 3*a*x + 3*d)^5 + 243)*(c*x^2 + a),x, algorithm="maxima"
```

```
[Out] 1/4374*c^6*x^18 + 1/243*a*c^5*x^16 + 1/243*c^5*d*x^15 + 5/162*a^2
*c^4*x^14 + 5/81*a*c^4*d*x^13 + 10/27*a^2*c^3*d*x^11 + 5/162*(4*a
^3*c^3 + c^4*d^2)*x^12 + 5/54*(3*a^4*c^2 + 4*a*c^3*d^2)*x^10 + 10
/81*(9*a^3*c^2*d + c^3*d^3)*x^9 + 1/3*(a^5*c + 5*a^2*c^2*d^2)*x^8
 + 5/2*a^2*d^4*x^2 + 5/9*(3*a^4*c*d + 2*a*c^2*d^3)*x^7 + 1/18*(3*
a^6 + 60*a^3*c*d^2 + 5*c^2*d^4)*x^6 + 1/3*(3*a^5*d + 10*a^2*c*d^3
)*x^5 + 5/6*(3*a^4*d^2 + 2*a*c*d^4)*x^4 + 1/3*(10*a^3*d^3 + c*d^5
 + c)*x^3 + (a*d^5 + a)*x
```

Fricas [A] time = 0.268085, size = 378, normalized size = 12.19

$$\frac{1}{4374}c^6x^{18}+\frac{1}{243}ac^5x^{16}+\frac{1}{243}c^5dx^{15}+\frac{5}{162}a^2c^4x^{14}+\frac{5}{81}ac^4dx^{13}+\frac{10}{27}a^2c^3dx^{11}+\frac{5}{162}\left(4a^3c^3+c^4d^2\right)x^{12}+\frac{5}{54}\left(3a^4c^2+4ac^3d^2\right)x^{10}+\frac{10}{81}\left(9a^3c^2d+c^3d^3\right)x^9+\frac{1}{3}\left(a^5c+5a^2c^2d^2\right)x^8+\frac{5}{2}a^2d^4x^2+\frac{5}{9}\left(3a^4cd+2ac^2d^3\right)x^7+\frac{1}{18}\left(3a^6+60a^3cd^2+5c^2d^4\right)x^6+\frac{1}{3}\left(3a^5d+10a^2cd^3\right)x^5+\frac{5}{6}\left(3a^4d^2+2acd^4\right)x^4+\frac{1}{3}\left(10a^3d^3+cd^5+c\right)x^3+\left(ad^5+a\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/243*((c*x^3 + 3*a*x + 3*d)^5 + 243)*(c*x^2 + a),x, algorithm="fricas"
```

```
[Out] 1/4374*c^6*x^18 + 1/243*a*c^5*x^16 + 1/243*c^5*d*x^15 + 5/162*a^2
*c^4*x^14 + 5/81*a*c^4*d*x^13 + 10/27*a^2*c^3*d*x^11 + 5/162*(4*a
^3*c^3 + c^4*d^2)*x^12 + 5/54*(3*a^4*c^2 + 4*a*c^3*d^2)*x^10 + 10
```

```
/81*(9*a^3*c^2*d + c^3*d^3)*x^9 + 1/3*(a^5*c + 5*a^2*c^2*d^2)*x^8
 + 5/2*a^2*d^4*x^2 + 5/9*(3*a^4*c*d + 2*a*c^2*d^3)*x^7 + 1/18*(3*
a^6 + 60*a^3*c*d^2 + 5*c^2*d^4)*x^6 + 1/3*(3*a^5*d + 10*a^2*c*d^3
)*x^5 + 5/6*(3*a^4*d^2 + 2*a*c*d^4)*x^4 + 1/3*(10*a^3*d^3 + c*d^5
 + c)*x^3 + (a*d^5 + a)*x
```

Sympy [A] time = 0.308761, size = 314, normalized size = 10.13

$$\frac{5a^2c^4x^{14}}{162}+\frac{10a^2c^3dx^{11}}{27}+\frac{5a^2d^4x^2}{2}+\frac{ac^5x^{16}}{243}+\frac{5ac^4dx^{13}}{81}+\frac{c^6x^{18}}{4374}+\frac{c^5dx^{15}}{243}+x^{12}\left(\frac{10a^3c^3}{81}+\frac{5c^4d^2}{162}\right)+x^{10}\left(\frac{5a^4c^2}{18}+\frac{10ac^3d^2}{27}\right)+x^9\left(\frac{10a^3c^2d}{9}+\frac{10c^3d^3}{81}\right)+x^8\left(\frac{a^5c}{3}+\frac{5a^2c^2d^2}{3}\right)+x^7\left(\frac{5a^4cd}{3}+\frac{10ac^2d^3}{9}\right)+x^6\left(\frac{a^6}{6}+\frac{10a^3cd^2}{3}+\frac{5c^2d^4}{18}\right)+x^5\left(a^5d+\frac{10a^2cd^3}{3}\right)+x^4\left(\frac{5a^4d^2}{2}+\frac{5acd^4}{3}\right)+x^3\left(\frac{10a^3d^3}{3}+\frac{cd^5}{3}+\frac{c}{3}\right)+x\left(ad^5+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**2+a)*(1+(d+a*x+1/3*c*x**3)**5),x)
```

```
[Out] 5*a**2*c**4*x**14/162 + 10*a**2*c**3*d*x**11/27 + 5*a**2*d**4*x**
2/2 + a*c**5*x**16/243 + 5*a*c**4*d*x**13/81 + c**6*x**18/4374 +
c**5*d*x**15/243 + x**12*(10*a**3*c**3/81 + 5*c**4*d**2/162) + x*
*10*(5*a**4*c**2/18 + 10*a*c**3*d**2/27) + x**9*(10*a**3*c**2*d/9
 + 10*c**3*d**3/81) + x**8*(a**5*c/3 + 5*a**2*c**2*d**2/3) + x**7
*(5*a**4*c*d/3 + 10*a*c**2*d**3/9) + x**6*(a**6/6 + 10*a**3*c*d**
2/3 + 5*c**2*d**4/18) + x**5*(a**5*d + 10*a**2*c*d**3/3) + x**4*(
5*a**4*d**2/2 + 5*a*c*d**4/3) + x**3*(10*a**3*d**3/3 + c*d**5/3 +
 c/3) + x*(a*d**5 + a)
```

GIAC/XCAS [A] time = 0.259174, size = 393, normalized size = 12.68

$$\frac{1}{4374}c^6x^{18}+\frac{1}{243}ac^5x^{16}+\frac{1}{243}c^5dx^{15}+\frac{5}{162}a^2c^4x^{14}+\frac{5}{81}ac^4dx^{13}+\frac{10}{81}a^3c^3x^{12}+\frac{5}{162}c^4d^2x^{12}+\frac{10}{27}a^2c^3dx^{11}+\frac{5}{18}a^4c^2x^{10}+\frac{10}{27}ac^3d^2x^{10}+\frac{10}{9}a^3c^2dx^9+\frac{10}{81}c^3d^3x^9+\frac{1}{3}a^5cx^8+\frac{5}{3}a^2c^2d^2x^8+\frac{5}{3}a^4cdx^7+\frac{10}{9}ac^2d^3x^7+\frac{1}{6}a^6x^6+\frac{10}{3}a^3cd^2x^6+\frac{5}{18}c^2d^4x^6+a^5dx^5+\frac{10}{3}a^2cd^3x^5+\frac{5}{2}a^4d^2x^4+\frac{5}{3}acd^4x^4+\frac{10}{3}a^3d^3x^3+\frac{1}{3}cd^5x^3+\frac{5}{2}a^2d^4x^2+ad^5x+\frac{1}{3}cx^3+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/243*((c*x^3 + 3*a*x + 3*d)^5 + 243)*(c*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/4374*c^6*x^18 + 1/243*a*c^5*x^16 + 1/243*c^5*d*x^15 + 5/162*a^2
*c^4*x^14 + 5/81*a*c^4*d*x^13 + 10/81*a^3*c^3*x^12 + 5/162*c^4*d^
2*x^12 + 10/27*a^2*c^3*d*x^11 + 5/18*a^4*c^2*x^10 + 10/27*a*c^3*d
^2*x^10 + 10/9*a^3*c^2*d*x^9 + 10/81*c^3*d^3*x^9 + 1/3*a^5*c*x^8
+ 5/3*a^2*c^2*d^2*x^8 + 5/3*a^4*c*d*x^7 + 10/9*a*c^2*d^3*x^7 + 1/
6*a^6*x^6 + 10/3*a^3*c*d^2*x^6 + 5/18*c^2*d^4*x^6 + a^5*d*x^5 + 1
0/3*a^2*c*d^3*x^5 + 5/2*a^4*d^2*x^4 + 5/3*a*c*d^4*x^4 + 10/3*a^3*
d^3*x^3 + 1/3*c*d^5*x^3 + 5/2*a^2*d^4*x^2 + a*d^5*x + 1/3*c*x^3 +
 a*x
```

3.212 $\int \left(bx+cx^2\right)\left(1+\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx$

Optimal. Leaf size=34

$$\frac{x^{12}(3b+2cx)^6}{279936}+\frac{bx^2}{2}+\frac{cx^3}{3}$$

```
[Out] (b*x^2)/2 + (c*x^3)/3 + (x^12*(3*b + 2*c*x)^6)/279936
```

Rubi [A] time = 0.0307772, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032

$$\frac{x^{12}(3b+2cx)^6}{279936}+\frac{bx^2}{2}+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)*(1 + ((b*x^2)/2 + (c*x^3)/3)^5),x]
```

```
[Out] (b*x^2)/2 + (c*x^3)/3 + (x^12*(3*b + 2*c*x)^6)/279936
```

Rubi in Sympy [A] time = 4.68213, size = 29, normalized size = 0.85

$$\frac{bx^2}{2}+\frac{cx^3}{3}+\frac{\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((c*x**2+b*x)*(1+(1/2*b*x**2+1/3*c*x**3)**5),x)
```

```
[Out] b*x**2/2 + c*x**3/3 + (b*x**2/2 + c*x**3/3)**6/6
```

Mathematica [B] time = 0.0121498, size = 98, normalized size = 2.88

$$\frac{b^6x^{12}}{384}+\frac{1}{96}b^5cx^{13}+\frac{5}{288}b^4c^2x^{14}+\frac{5}{324}b^3c^3x^{15}+\frac{5}{648}b^2c^4x^{16}+\frac{1}{486}bc^5x^{17}+\frac{bx^2}{2}+\frac{c^6x^{18}}{4374}+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)*(1 + ((b*x^2)/2 + (c*x^3)/3)^5),x]
```

```
[Out] (b*x^2)/2 + (c*x^3)/3 + (b^6*x^12)/384 + (b^5*c*x^13)/96 + (5*b^4
*c^2*x^14)/288 + (5*b^3*c^3*x^15)/324 + (5*b^2*c^4*x^16)/648 + (b
*c^5*x^17)/486 + (c^6*x^18)/4374
```

Maple [B] time = 0.003, size = 81, normalized size = 2.4

$$\frac{c^6x^{18}}{4374}+\frac{bc^5x^{17}}{486}+\frac{5\,b^2c^4x^{16}}{648}+\frac{5\,b^3c^3x^{15}}{324}+\frac{5\,b^4c^2x^{14}}{288}+\frac{b^5cx^{13}}{96}+\frac{b^6x^{12}}{384}+\frac{cx^3}{3}+\frac{bx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^5),x)
```

```
[Out] 1/4374*c^6*x^18+1/486*b*c^5*x^17+5/648*b^2*c^4*x^16+5/324*b^3*c^3
*x^15+5/288*b^4*c^2*x^14+1/96*b^5*c*x^13+1/384*b^6*x^12+1/3*c*x^3
+1/2*b*x^2
```

Maxima [A] time = 0.791433, size = 108, normalized size = 3.18

$$\frac{1}{4374}c^6x^{18} + \frac{1}{486}bc^5x^{17} + \frac{5}{648}b^2c^4x^{16} + \frac{5}{324}b^3c^3x^{15} + \frac{5}{288}b^4c^2x^{14} + \frac{1}{96}b^5cx^{13} + \frac{1}{384}b^6x^{12} + \frac{1}{3}cx^3 + \frac{1}{2}bx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/7776*((2*c*x^3 + 3*b*x^2)^5 + 7776)*(c*x^2 + b*x),x, algorithm="maxi
```

```
[Out] 1/4374*c^6*x^18 + 1/486*b*c^5*x^17 + 5/648*b^2*c^4*x^16 + 5/324*b
^3*c^3*x^15 + 5/288*b^4*c^2*x^14 + 1/96*b^5*c*x^13 + 1/384*b^6*x^
12 + 1/3*c*x^3 + 1/2*b*x^2
```

Fricas [A] time = 0.248413, size = 108, normalized size = 3.18

$$\frac{1}{4374}c^6x^{18} + \frac{1}{486}bc^5x^{17} + \frac{5}{648}b^2c^4x^{16} + \frac{5}{324}b^3c^3x^{15} + \frac{5}{288}b^4c^2x^{14} + \frac{1}{96}b^5cx^{13} + \frac{1}{384}b^6x^{12} + \frac{1}{3}cx^3 + \frac{1}{2}bx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/7776*((2*c*x^3 + 3*b*x^2)^5 + 7776)*(c*x^2 + b*x),x, algorithm="fric
```

```
[Out] 1/4374*c^6*x^18 + 1/486*b*c^5*x^17 + 5/648*b^2*c^4*x^16 + 5/324*b
^3*c^3*x^15 + 5/288*b^4*c^2*x^14 + 1/96*b^5*c*x^13 + 1/384*b^6*x^
12 + 1/3*c*x^3 + 1/2*b*x^2
```

Sympy [A] time = 0.18019, size = 90, normalized size = 2.65

$$\frac{b^6x^{12}}{384} + \frac{b^5cx^{13}}{96} + \frac{5b^4c^2x^{14}}{288} + \frac{5b^3c^3x^{15}}{324} + \frac{5b^2c^4x^{16}}{648} + \frac{bc^5x^{17}}{486} + \frac{bx^2}{2} + \frac{c^6x^{18}}{4374} + \frac{cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**2+b*x)*(1+(1/2*b*x**2+1/3*c*x**3)**5),x)
```

```
[Out] b**6*x**12/384 + b**5*c*x**13/96 + 5*b**4*c**2*x**14/288 + 5*b**3
*c**3*x**15/324 + 5*b**2*c**4*x**16/648 + b*c**5*x**17/486 + b*x*
*2/2 + c**6*x**18/4374 + c*x**3/3
```

GIAC/XCAS [A] time = 0.260631, size = 108, normalized size = 3.18

$$\frac{1}{4374}c^6x^{18} + \frac{1}{486}bc^5x^{17} + \frac{5}{648}b^2c^4x^{16} + \frac{5}{324}b^3c^3x^{15} + \frac{5}{288}b^4c^2x^{14} + \frac{1}{96}b^5cx^{13} + \frac{1}{384}b^6x^{12} + \frac{1}{3}cx^3 + \frac{1}{2}bx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/7776*((2*c*x^3 + 3*b*x^2)^5 + 7776)*(c*x^2 + b*x),x, algorithm="giac
```

```
[Out] 1/4374*c^6*x^18 + 1/486*b*c^5*x^17 + 5/648*b^2*c^4*x^16 + 5/324*b
^3*c^3*x^15 + 5/288*b^4*c^2*x^14 + 1/96*b^5*c*x^13 + 1/384*b^6*x^
12 + 1/3*c*x^3 + 1/2*b*x^2
```

3.213 $\int \left(bx+cx^2\right)\left(1+\left(d+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx$

Optimal. Leaf size=41

$$\frac{1}{6}\left(\frac{bx^2}{2}+\frac{cx^3}{3}+d\right)^6+\frac{bx^2}{2}+\frac{cx^3}{3}$$

```
[Out] (b*x^2)/2 + (c*x^3)/3 + (d + (b*x^2)/2 + (c*x^3)/3)^6/6
```

Rubi [A] time = 0.0517969, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.031

$$\frac{\left(3bx^2+2cx^3+6d\right)^6}{279936}+\frac{bx^2}{2}+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x + c*x^2)*(1 + (d + (b*x^2)/2 + (c*x^3)/3)^5),x]
```

```
[Out] (b*x^2)/2 + (c*x^3)/3 + (6*d + 3*b*x^2 + 2*c*x^3)^6/279936
```

Rubi in Sympy [A] time = 6.77656, size = 32, normalized size = 0.78

$$\frac{bx^2}{2}+\frac{cx^3}{3}+d+\frac{\left(\frac{bx^2}{2}+\frac{cx^3}{3}+d\right)^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**2+b*x)*(1+(d+1/2*b*x**2+1/3*c*x**3)**5),x)
```

```
[Out] b*x**2/2 + c*x**3/3 + d + (b*x**2/2 + c*x**3/3 + d)**6/6
```

Mathematica [B] time = 0.0868715, size = 146, normalized size = 3.56

$$\frac{x^2(3b+2cx)\left(243b^5x^{10}+810b^4cx^{11}+1080b^3c^2x^{12}+720b^2c^3x^{13}+240bc^4x^{14}+19440d^4x^2(3b+2cx)+4320d^3x^4(3b+2cx)^2\right.}{279936}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x + c*x^2)*(1 + (d + (b*x^2)/2 + (c*x^3)/3)^5),x]
```

```
[Out] (x^2*(3*b + 2*c*x)*(46656 + 46656*d^5 + 243*b^5*x^10 + 810*b^4*c*
x^11 + 1080*b^3*c^2*x^12 + 720*b^2*c^3*x^13 + 240*b*c^4*x^14 + 32
*c^5*x^15 + 19440*d^4*x^2*(3*b + 2*c*x) + 4320*d^3*x^4*(3*b + 2*c
*x)^2 + 540*d^2*x^6*(3*b + 2*c*x)^3 + 36*d*x^8*(3*b + 2*c*x)^4))/
279936
```

Maple [B] time = 0.003, size = 646, normalized size = 15.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((c*x^2+b*x)*(1+(d+1/2*b*x^2+1/3*c*x^3)^5),x)
```

```
[Out] 1/4374*c^6*x^18+1/486*b*c^5*x^17+5/648*b^2*c^4*x^16+1/15*(5/54*b^
3*c^3+c*(1/81*c^4*d+1/12*b^3*c^2+1/3*c*(4/27*c^3*d+1/6*b^3*c)))*x
^15+1/14*(b*(1/81*c^4*d+1/12*b^3*c^2+1/3*c*(4/27*c^3*d+1/6*b^3*c)
)+c*(2/27*d*b*c^3+1/2*b*(4/27*c^3*d+1/6*b^3*c)+1/3*c*(2/3*c^2*d*b
+1/16*b^4)))*x^14+1/13*(b*(2/27*d*b*c^3+1/2*b*(4/27*c^3*d+1/6*b^3
*c)+1/3*c*(2/3*c^2*d*b+1/16*b^4))+c*(1/2*b^2*c^2*d+1/2*b*(2/3*c^2
*d*b+1/16*b^4)))*x^13+1/12*(b*(1/2*b^2*c^2*d+1/2*b*(2/3*c^2*d*b+1
/16*b^4))+c*(d*(4/27*c^3*d+1/6*b^3*c)+1/2*b^3*c*d+1/3*c*(2/3*c^2*
d^2+1/2*b^3*d)))*x^12+1/11*(b*(d*(4/27*c^3*d+1/6*b^3*c)+1/2*b^3*c
*d+1/3*c*(2/3*c^2*d^2+1/2*b^3*d))+c*(d*(2/3*c^2*d*b+1/16*b^4)+1/2
*b*(2/3*c^2*d^2+1/2*b^3*d)+2/3*c^2*b*d^2))*x^11+1/10*(b*(d*(2/3*c
^2*d*b+1/16*b^4)+1/2*b*(2/3*c^2*d^2+1/2*b^3*d)+2/3*c^2*b*d^2)+5/2
*b^2*c^2*d^2)*x^10+1/9*(5/2*b^3*c*d^2+c*(d*(2/3*c^2*d^2+1/2*b^3*d
)+3/4*b^3*d^2+4/9*c^2*d^3))*x^9+1/8*(b*(d*(2/3*c^2*d^2+1/2*b^3*d)
+3/4*b^3*d^2+4/9*c^2*d^3)+10/3*b*c^2*d^3)*x^8+5/6*b^2*c*d^3*x^7+1
/6*(5/2*b^3*d^3+5/3*c^2*d^4)*x^6+5/6*c*d^4*b*x^5+5/8*b^2*d^4*x^4+
1/3*c*(d^5+1)*x^3+1/2*b*(d^5+1)*x^2
```

Maxima [A] time = 0.809356, size = 390, normalized size = 9.51

$$\frac{1}{4374}c^6x^{18}+\frac{1}{486}bc^5x^{17}+\frac{5}{648}b^2c^4x^{16}+\frac{1}{972}\left(15\,b^3c^3+4\,c^5d\right)x^{15}+\frac{5}{2592}\left(9\,b^4c^2+16\,bc^4d\right)x^{14}+\frac{1}{864}\left(9\,b^5c+80\,b^2c^3d\right)x^{13}+\frac{5}{6}b^2cd^3x^7+\frac{1}{10368}\left(27\,b^6+1440\,b^3c^2d+320\,c^4d^2\right)x^{12}+\frac{5}{432}\left(9\,b^4cd+16\,bc^3d^2\right)x^{11}+\frac{5}{6}bcd^4x^5+\frac{1}{96}\left(3\,b^5d+40\,b^2c^2d^2\right)x^{10}+\frac{5}{8}b^2d^4x^4+\frac{5}{324}\left(27\,b^3cd^2+8\,c^3d^3\right)x^9+\frac{5}{288}\left(9\,b^4d^2+32\,bc^2d^3\right)x^8+\frac{5}{36}\left(3\,b^3d^3+2\,c^2d^4\right)x^6+\frac{1}{3}\left(cd^5+c\right)x^3+\frac{1}{2}\left(bd^5+b\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/7776*((2*c*x^3 + 3*b*x^2 + 6*d)^5 + 7776)*(c*x^2 + b*x),x, algorithm=
```

```
[Out] 1/4374*c^6*x^18 + 1/486*b*c^5*x^17 + 5/648*b^2*c^4*x^16 + 1/972*(
15*b^3*c^3 + 4*c^5*d)*x^15 + 5/2592*(9*b^4*c^2 + 16*b*c^4*d)*x^14
 + 1/864*(9*b^5*c + 80*b^2*c^3*d)*x^13 + 5/6*b^2*c*d^3*x^7 + 1/10
368*(27*b^6 + 1440*b^3*c^2*d + 320*c^4*d^2)*x^12 + 5/432*(9*b^4*c
*d + 16*b*c^3*d^2)*x^11 + 5/6*b*c*d^4*x^5 + 1/96*(3*b^5*d + 40*b^
2*c^2*d^2)*x^10 + 5/8*b^2*d^4*x^4 + 5/324*(27*b^3*c*d^2 + 8*c^3*d
^3)*x^9 + 5/288*(9*b^4*d^2 + 32*b*c^2*d^3)*x^8 + 5/36*(3*b^3*d^3
+ 2*c^2*d^4)*x^6 + 1/3*(c*d^5 + c)*x^3 + 1/2*(b*d^5 + b)*x^2
```

Fricas [A] time = 0.248399, size = 390, normalized size = 9.51

$$\frac{1}{4374}c^6x^{18}+\frac{1}{486}bc^5x^{17}+\frac{5}{648}b^2c^4x^{16}+\frac{1}{972}\left(15\,b^3c^3+4\,c^5d\right)x^{15}+\frac{5}{2592}\left(9\,b^4c^2+16\,bc^4d\right)x^{14}+\frac{1}{864}\left(9\,b^5c+80\,b^2c^3d\right)x^{13}+\frac{5}{6}b^2cd^3x^7+\frac{1}{10368}\left(27\,b^6+1440\,b^3c^2d+320\,c^4d^2\right)x^{12}+\frac{5}{432}\left(9\,b^4cd+16\,bc^3d^2\right)x^{11}+\frac{5}{6}bcd^4x^5+\frac{1}{96}\left(3\,b^5d+40\,b^2c^2d^2\right)x^{10}+\frac{5}{8}b^2d^4x^4+\frac{5}{324}\left(27\,b^3cd^2+8\,c^3d^3\right)x^9+\frac{5}{288}\left(9\,b^4d^2+32\,bc^2d^3\right)x^8+\frac{5}{36}\left(3\,b^3d^3+2\,c^2d^4\right)x^6+\frac{1}{3}\left(cd^5+c\right)x^3+\frac{1}{2}\left(bd^5+b\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/7776*((2*c*x^3 + 3*b*x^2 + 6*d)^5 + 7776)*(c*x^2 + b*x),x, algorithm=
```

```
[Out] 1/4374*c^6*x^18 + 1/486*b*c^5*x^17 + 5/648*b^2*c^4*x^16 + 1/972*(
15*b^3*c^3 + 4*c^5*d)*x^15 + 5/2592*(9*b^4*c^2 + 16*b*c^4*d)*x^14
 + 1/864*(9*b^5*c + 80*b^2*c^3*d)*x^13 + 5/6*b^2*c*d^3*x^7 + 1/10
368*(27*b^6 + 1440*b^3*c^2*d + 320*c^4*d^2)*x^12 + 5/432*(9*b^4*c
*d + 16*b*c^3*d^2)*x^11 + 5/6*b*c*d^4*x^5 + 1/96*(3*b^5*d + 40*b^
2*c^2*d^2)*x^10 + 5/8*b^2*d^4*x^4 + 5/324*(27*b^3*c*d^2 + 8*c^3*d
^3)*x^9 + 5/288*(9*b^4*d^2 + 32*b*c^2*d^3)*x^8 + 5/36*(3*b^3*d^3
+ 2*c^2*d^4)*x^6 + 1/3*(c*d^5 + c)*x^3 + 1/2*(b*d^5 + b)*x^2
```

Sympy [A] time = 0.327803, size = 321, normalized size = 7.83

$$\frac{5b^2c^4x^{16}}{648}+\frac{5b^2cd^3x^7}{6}+\frac{5b^2d^4x^4}{8}+\frac{bc^5x^{17}}{486}+\frac{5bcd^4x^5}{6}+\frac{c^6x^{18}}{4374}+x^{15}\left(\frac{5b^3c^3}{324}+\frac{c^5d}{243}\right)+x^{14}\left(\frac{5b^4c^2}{288}+\frac{5bc^4d}{162}\right)+x^{13}\left(\frac{b^5c}{96}+\frac{5b^2c^3d}{54}\right)+x^{12}\left(\frac{b^6}{384}+\frac{5b^3c^2d}{36}+\frac{5c^4d^2}{162}\right)+x^{11}\left(\frac{5b^4cd}{48}+\frac{5bc^3d^2}{27}\right)+x^{10}\left(\frac{b^5d}{32}+\frac{5b^2c^2d^2}{12}\right)+x^9\left(\frac{5b^3cd^2}{12}+\frac{10c^3d^3}{81}\right)+x^8\left(\frac{5b^4d^2}{32}+\frac{5bc^2d^3}{9}\right)+x^6\left(\frac{5b^3d^3}{12}+\frac{5c^2d^4}{18}\right)+x^3\left(\frac{cd^5}{3}+\frac{c}{3}\right)+x^2\left(\frac{bd^5}{2}+\frac{b}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)*(1+(d+1/2*b*x**2+1/3*c*x**3)**5),x)
```

```
[Out] 5*b**2*c**4*x**16/648 + 5*b**2*c*d**3*x**7/6 + 5*b**2*d**4*x**4/8
 + b*c**5*x**17/486 + 5*b*c*d**4*x**5/6 + c**6*x**18/4374 + x**15
*(5*b**3*c**3/324 + c**5*d/243) + x**14*(5*b**4*c**2/288 + 5*b*c*
*4*d/162) + x**13*(b**5*c/96 + 5*b**2*c**3*d/54) + x**12*(b**6/38
4 + 5*b**3*c**2*d/36 + 5*c**4*d**2/162) + x**11*(5*b**4*c*d/48 +
5*b*c**3*d**2/27) + x**10*(b**5*d/32 + 5*b**2*c**2*d**2/12) + x**
9*(5*b**3*c*d**2/12 + 10*c**3*d**3/81) + x**8*(5*b**4*d**2/32 + 5
*b*c**2*d**3/9) + x**6*(5*b**3*d**3/12 + 5*c**2*d**4/18) + x**3*(
c*d**5/3 + c/3) + x**2*(b*d**5/2 + b/2)
```

GIAC/XCAS [A] time = 0.262493, size = 402, normalized size = 9.8

$$\frac{1}{4374}c^6x^{18}+\frac{1}{486}bc^5x^{17}+\frac{5}{648}b^2c^4x^{16}+\frac{5}{324}b^3c^3x^{15}+\frac{1}{243}c^5dx^{15}+\frac{5}{288}b^4c^2x^{14}+\frac{5}{162}bc^4dx^{14}+\frac{1}{96}b^5cx^{13}+\frac{5}{54}b^2c^3dx^{13}+\frac{1}{384}b^6x^{12}+\frac{5}{36}b^3c^2dx^{12}+\frac{5}{162}c^4d^2x^{12}+\frac{5}{48}b^4cdx^{11}+\frac{5}{27}bc^3d^2x^{11}+\frac{1}{32}b^5dx^{10}+\frac{5}{12}b^2c^2d^2x^{10}+\frac{5}{12}b^3cd^2x^9+\frac{10}{81}c^3d^3x^9+\frac{5}{32}b^4d^2x^8+\frac{5}{9}bc^2d^3x^8+\frac{5}{6}b^2cd^3x^7+\frac{5}{12}b^3d^3x^6+\frac{5}{18}c^2d^4x^6+\frac{5}{6}bcd^4x^5+\frac{5}{8}b^2d^4x^4+\frac{1}{3}cd^5x^3+\frac{1}{2}bd^5x^2+\frac{1}{3}cx^3+\frac{1}{2}bx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/7776*((2*c*x^3 + 3*b*x^2 + 6*d)^5 + 7776)*(c*x^2 + b*x),x, algorithm=
```

```
[Out] 1/4374*c^6*x^18 + 1/486*b*c^5*x^17 + 5/648*b^2*c^4*x^16 + 5/324*b
^3*c^3*x^15 + 1/243*c^5*d*x^15 + 5/288*b^4*c^2*x^14 + 5/162*b*c^4
*d*x^14 + 1/96*b^5*c*x^13 + 5/54*b^2*c^3*d*x^13 + 1/384*b^6*x^12
+ 5/36*b^3*c^2*d*x^12 + 5/162*c^4*d^2*x^12 + 5/48*b^4*c*d*x^11 +
5/27*b*c^3*d^2*x^11 + 1/32*b^5*d*x^10 + 5/12*b^2*c^2*d^2*x^10 + 5
/12*b^3*c*d^2*x^9 + 10/81*c^3*d^3*x^9 + 5/32*b^4*d^2*x^8 + 5/9*b*
c^2*d^3*x^8 + 5/6*b^2*c*d^3*x^7 + 5/12*b^3*d^3*x^6 + 5/18*c^2*d^4
*x^6 + 5/6*b*c*d^4*x^5 + 5/8*b^2*d^4*x^4 + 1/3*c*d^5*x^3 + 1/2*b*
d^5*x^2 + 1/3*c*x^3 + 1/2*b*x^2
```

3.214 $\int (a+bx+cx^2)\left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx$

Optimal. Leaf size=46

$$\frac{1}{6}\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^6+ax+\frac{bx^2}{2}+\frac{cx^3}{3}$$

```
[Out] a*x + (b*x^2)/2 + (c*x^3)/3 + (a*x + (b*x^2)/2 + (c*x^3)/3)^6/6
```

Rubi [A] time = 0.050086, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$

$$\frac{\left(6ax+3bx^2+2cx^3\right)^6}{279936}+ax+\frac{bx^2}{2}+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x + c*x^2)*(1 + (a*x + (b*x^2)/2 + (c*x^3)/3)^5),x]
```

```
[Out] a*x + (b*x^2)/2 + (c*x^3)/3 + (6*a*x + 3*b*x^2 + 2*c*x^3)^6/27993
6
```

Rubi in Sympy [A] time = 9.87422, size = 36, normalized size = 0.78

$$ax+\frac{bx^2}{2}+\frac{cx^3}{3}+\frac{\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**2+b*x+a)*(1+(a*x+1/2*b*x**2+1/3*c*x**3)**5),x)
```

```
[Out] a*x + b*x**2/2 + c*x**3/3 + (a*x + b*x**2/2 + c*x**3/3)**6/6
```

Mathematica [B] time = 0.135737, size = 244, normalized size = 5.3

$$\frac{a^6x^6}{6}+\frac{1}{6}a^5x^7(3b+2cx)+\frac{5}{72}a^4x^8(3b+2cx)^2+\frac{5}{324}a^3x^9(3b+2cx)^3+\frac{5a^2x^{10}(3b+2cx)^4}{2592}$$
$$+a\left(\frac{b^5x^{11}}{32}+\frac{5}{48}b^4cx^{12}+\frac{5}{36}b^3c^2x^{13}+\frac{5}{54}b^2c^3x^{14}+\frac{5}{162}bc^4x^{15}+\frac{c^5x^{16}}{243}+x\right)$$
$$+\frac{x^2\left(729b^6x^{10}+2916b^5cx^{11}+4860b^4c^2x^{12}+4320b^3c^3x^{13}+2160b^2c^4x^{14}+576b\left(c^5x^{15}+243\right)+64cx\left(c^5x^{15}+1458\right)\right)}{279936}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x + c*x^2)*(1 + (a*x + (b*x^2)/2 + (c*x^3)/3)^5),x]
```

```
[Out] (a^6*x^6)/6 + (a^5*x^7*(3*b + 2*c*x))/6 + (5*a^4*x^8*(3*b + 2*c*x
)^2)/72 + (5*a^3*x^9*(3*b + 2*c*x)^3)/324 + (5*a^2*x^10*(3*b + 2*
c*x)^4)/2592 + a*(x + (b^5*x^11)/32 + (5*b^4*c*x^12)/48 + (5*b^3*
c^2*x^13)/36 + (5*b^2*c^3*x^14)/54 + (5*b*c^4*x^15)/162 + (c^5*x^
16)/243) + (x^2*(729*b^6*x^10 + 2916*b^5*c*x^11 + 4860*b^4*c^2*x^
12 + 4320*b^3*c^3*x^13 + 2160*b^2*c^4*x^14 + 576*b*(243 + c^5*x^1
```

```
5) + 64*c*x*(1458 + c^5*x^15)))/279936
```

Maple [B] time = 0.004, size = 1523, normalized size = 33.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^5),x)
```

```
[Out] 1/4374*c^6*x^18+1/486*b*c^5*x^17+1/16*(1/243*a*c^5+5/162*b^2*c^4+
c*(1/81*a*c^4+1/27*b^2*c^3+1/3*c*(2/9*(2/3*a*c+1/4*b^2)*c^2+1/9*b
^2*c^2)))*x^16+1/15*(5/162*a*b*c^4+b*(1/81*a*c^4+1/27*b^2*c^3+1/3
*c*(2/9*(2/3*a*c+1/4*b^2)*c^2+1/9*b^2*c^2))+c*(2/27*a*b*c^3+1/2*b
*(2/9*(2/3*a*c+1/4*b^2)*c^2+1/9*b^2*c^2)+1/3*c*(2/9*a*b*c^2+2/3*(
2/3*a*c+1/4*b^2)*b*c)))*x^15+1/14*(a*(1/81*a*c^4+1/27*b^2*c^3+1/3
*c*(2/9*(2/3*a*c+1/4*b^2)*c^2+1/9*b^2*c^2))+b*(2/27*a*b*c^3+1/2*b
*(2/9*(2/3*a*c+1/4*b^2)*c^2+1/9*b^2*c^2)+1/3*c*(2/9*a*b*c^2+2/3*(
2/3*a*c+1/4*b^2)*b*c))+c*(a*(2/9*(2/3*a*c+1/4*b^2)*c^2+1/9*b^2*c^
2)+1/2*b*(2/9*a*b*c^2+2/3*(2/3*a*c+1/4*b^2)*b*c)+1/3*c*(2/9*a^2*c
^2+2/3*a*b^2*c+(2/3*a*c+1/4*b^2)^2)))*x^14+1/13*(a*(2/27*a*b*c^3+
1/2*b*(2/9*(2/3*a*c+1/4*b^2)*c^2+1/9*b^2*c^2)+1/3*c*(2/9*a*b*c^2+
2/3*(2/3*a*c+1/4*b^2)*b*c))+b*(a*(2/9*(2/3*a*c+1/4*b^2)*c^2+1/9*b
^2*c^2)+1/2*b*(2/9*a*b*c^2+2/3*(2/3*a*c+1/4*b^2)*b*c)+1/3*c*(2/9*
a^2*c^2+2/3*a*b^2*c+(2/3*a*c+1/4*b^2)^2))+c*(a*(2/9*a*b*c^2+2/3*(
2/3*a*c+1/4*b^2)*b*c)+1/2*b*(2/9*a^2*c^2+2/3*a*b^2*c+(2/3*a*c+1/4
*b^2)^2)+1/3*c*(2/3*a^2*b*c+2*a*b*(2/3*a*c+1/4*b^2))))*x^13+1/12*
(a*(a*(2/9*(2/3*a*c+1/4*b^2)*c^2+1/9*b^2*c^2)+1/2*b*(2/9*a*b*c^2+
2/3*(2/3*a*c+1/4*b^2)*b*c)+1/3*c*(2/9*a^2*c^2+2/3*a*b^2*c+(2/3*a*
c+1/4*b^2)^2))+b*(a*(2/9*a*b*c^2+2/3*(2/3*a*c+1/4*b^2)*b*c)+1/2*b
*(2/9*a^2*c^2+2/3*a*b^2*c+(2/3*a*c+1/4*b^2)^2)+1/3*c*(2/3*a^2*b*c
+2*a*b*(2/3*a*c+1/4*b^2)))+c*(a*(2/9*a^2*c^2+2/3*a*b^2*c+(2/3*a*c
+1/4*b^2)^2)+1/2*b*(2/3*a^2*b*c+2*a*b*(2/3*a*c+1/4*b^2))+1/3*c*(2
*a^2*(2/3*a*c+1/4*b^2)+a^2*b^2)))*x^12+1/11*(a*(a*(2/9*a*b*c^2+2/
3*(2/3*a*c+1/4*b^2)*b*c)+1/2*b*(2/9*a^2*c^2+2/3*a*b^2*c+(2/3*a*c+
1/4*b^2)^2)+1/3*c*(2/3*a^2*b*c+2*a*b*(2/3*a*c+1/4*b^2)))+b*(a*(2/
9*a^2*c^2+2/3*a*b^2*c+(2/3*a*c+1/4*b^2)^2)+1/2*b*(2/3*a^2*b*c+2*a
*b*(2/3*a*c+1/4*b^2))+1/3*c*(2*a^2*(2/3*a*c+1/4*b^2)+a^2*b^2))+c*
(a*(2/3*a^2*b*c+2*a*b*(2/3*a*c+1/4*b^2))+1/2*b*(2*a^2*(2/3*a*c+1/
4*b^2)+a^2*b^2)+2/3*c*a^3*b))*x^11+1/10*(a*(a*(2/9*a^2*c^2+2/3*a*
b^2*c+(2/3*a*c+1/4*b^2)^2)+1/2*b*(2/3*a^2*b*c+2*a*b*(2/3*a*c+1/4*
b^2))+1/3*c*(2*a^2*(2/3*a*c+1/4*b^2)+a^2*b^2))+b*(a*(2/3*a^2*b*c+
2*a*b*(2/3*a*c+1/4*b^2))+1/2*b*(2*a^2*(2/3*a*c+1/4*b^2)+a^2*b^2)+
2/3*c*a^3*b)+c*(a*(2*a^2*(2/3*a*c+1/4*b^2)+a^2*b^2)+a^3*b^2+1/3*c
*a^4))*x^10+1/9*(a*(a*(2/3*a^2*b*c+2*a*b*(2/3*a*c+1/4*b^2))+1/2*b
*(2*a^2*(2/3*a*c+1/4*b^2)+a^2*b^2)+2/3*c*a^3*b)+b*(a*(2*a^2*(2/3*
a*c+1/4*b^2)+a^2*b^2)+a^3*b^2+1/3*c*a^4)+5/2*c*a^4*b)*x^9+1/8*(a*
(a*(2*a^2*(2/3*a*c+1/4*b^2)+a^2*b^2)+a^3*b^2+1/3*c*a^4)+5/2*a^4*b
^2+a^5*c)*x^8+1/2*a^5*b*x^7+1/6*a^6*x^6+1/3*c*x^3+1/2*b*x^2+a*x
```

Maxima [A] time = 0.809744, size = 390, normalized size = 8.48

$$\frac{1}{4374}c^6x^{18} + \frac{1}{486}bc^5x^{17} + \frac{1}{1944}\left(15\,b^2c^4 + 8\,ac^5\right)x^{16} + \frac{5}{324}\left(b^3c^3 + 2\,abc^4\right)x^{15} + \frac{5}{2592}\left(9\,b^4c^2 + 48\,ab^2c^3 + 16\,a^2c^4\right)x^{14} + \frac{1}{864}\left(9\,b^5c + 120\,ab^3c^2 + 160\,a^2bc^3\right)x^{13} + \frac{1}{2}a^5bx^7 + \frac{1}{10368}\left(27\,b^6 + 1080\,ab^4c + 4320\,a^2b^2c^2 + 1280\,a^3c^3\right)x^{12} + \frac{1}{6}a^6x^6 + \frac{1}{288}\left(9\,ab^5 + 120\,a^2b^3c + 160\,a^3bc^2\right)x^{11} + \frac{5}{288}\left(9\,a^2b^4 + 48\,a^3b^2c + 16\,a^4c^2\right)x^{10} + \frac{5}{12}\left(a^3b^3 + 2\,a^4bc\right)x^9 + \frac{1}{24}\left(15\,a^4b^2 + 8\,a^5c\right)x^8 + \frac{1}{3}cx^3 + \frac{1}{2}bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/7776*((2*c*x^3 + 3*b*x^2 + 6*a*x)^5 + 7776)*(c*x^2 + b*x + a),x, algor
```

```
[Out] 1/4374*c^6*x^18 + 1/486*b*c^5*x^17 + 1/1944*(15*b^2*c^4 + 8*a*c^5
)*x^16 + 5/324*(b^3*c^3 + 2*a*b*c^4)*x^15 + 5/2592*(9*b^4*c^2 + 4
8*a*b^2*c^3 + 16*a^2*c^4)*x^14 + 1/864*(9*b^5*c + 120*a*b^3*c^2 +
 160*a^2*b*c^3)*x^13 + 1/2*a^5*b*x^7 + 1/10368*(27*b^6 + 1080*a*b
^4*c + 4320*a^2*b^2*c^2 + 1280*a^3*c^3)*x^12 + 1/6*a^6*x^6 + 1/28
8*(9*a*b^5 + 120*a^2*b^3*c + 160*a^3*b*c^2)*x^11 + 5/288*(9*a^2*b
^4 + 48*a^3*b^2*c + 16*a^4*c^2)*x^10 + 5/12*(a^3*b^3 + 2*a^4*b*c)
*x^9 + 1/24*(15*a^4*b^2 + 8*a^5*c)*x^8 + 1/3*c*x^3 + 1/2*b*x^2 +
a*x
```

Fricas [A] time = 0.251339, size = 390, normalized size = 8.48

$$\frac{1}{4374}\,c^{6}x^{18} + \frac{1}{486}\,bc^{5}x^{17} + \frac{1}{1944}\left(15\,b^{2}c^{4} + 8\,ac^{5}\right)x^{16} + \frac{5}{324}\left(b^{3}c^{3} + 2\,abc^{4}\right)x^{15} + \frac{5}{2592}\left(9\,b^{4}c^{2} + 48\,ab^{2}c^{3} + 16\,a^{2}c^{4}\right)x^{14} + \frac{1}{864}\left(9\,b^{5}c + 120\,ab^{3}c^{2} + 160\,a^{2}bc^{3}\right)x^{13} + \frac{1}{2}\,a^{5}bx^{7} + \frac{1}{10368}\left(27\,b^{6} + 1080\,ab^{4}c + 4320\,a^{2}b^{2}c^{2} + 1280\,a^{3}c^{3}\right)x^{12} + \frac{1}{6}\,a^{6}x^{6} + \frac{1}{288}\left(9\,ab^{5} + 120\,a^{2}b^{3}c + 160\,a^{3}bc^{2}\right)x^{11} + \frac{5}{288}\left(9\,a^{2}b^{4} + 48\,a^{3}b^{2}c + 16\,a^{4}c^{2}\right)x^{10} + \frac{5}{12}\left(a^{3}b^{3} + 2\,a^{4}bc\right)x^{9} + \frac{1}{24}\left(15\,a^{4}b^{2} + 8\,a^{5}c\right)x^{8} + \frac{1}{3}\,cx^{3} + \frac{1}{2}\,bx^{2} + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/7776*((2*c*x^3 + 3*b*x^2 + 6*a*x)^5 + 7776)*(c*x^2 + b*x + a),x, algor
```

```
[Out] 1/4374*c^6*x^18 + 1/486*b*c^5*x^17 + 1/1944*(15*b^2*c^4 + 8*a*c^5
)*x^16 + 5/324*(b^3*c^3 + 2*a*b*c^4)*x^15 + 5/2592*(9*b^4*c^2 + 4
8*a*b^2*c^3 + 16*a^2*c^4)*x^14 + 1/864*(9*b^5*c + 120*a*b^3*c^2 +
 160*a^2*b*c^3)*x^13 + 1/2*a^5*b*x^7 + 1/10368*(27*b^6 + 1080*a*b
^4*c + 4320*a^2*b^2*c^2 + 1280*a^3*c^3)*x^12 + 1/6*a^6*x^6 + 1/28
8*(9*a*b^5 + 120*a^2*b^3*c + 160*a^3*b*c^2)*x^11 + 5/288*(9*a^2*b
^4 + 48*a^3*b^2*c + 16*a^4*c^2)*x^10 + 5/12*(a^3*b^3 + 2*a^4*b*c)
*x^9 + 1/24*(15*a^4*b^2 + 8*a^5*c)*x^8 + 1/3*c*x^3 + 1/2*b*x^2 +
a*x
```

Sympy [A] time = 0.347034, size = 323, normalized size = 7.02

$$\frac{a^{6}x^{6}}{6} + \frac{a^{5}bx^{7}}{2} + ax + \frac{bc^{5}x^{17}}{486} + \frac{bx^{2}}{2} + \frac{c^{6}x^{18}}{4374} + \frac{cx^{3}}{3} + x^{16}\left(\frac{ac^{5}}{243} + \frac{5b^{2}c^{4}}{648}\right) + x^{15}\left(\frac{5abc^{4}}{162} + \frac{5b^{3}c^{3}}{324}\right) + x^{14}\left(\frac{5a^{2}c^{4}}{162} + \frac{5ab^{2}c^{3}}{54} + \frac{5b^{4}c^{2}}{288}\right) + x^{13}\left(\frac{5a^{2}bc^{3}}{27} + \frac{5ab^{3}c^{2}}{36} + \frac{b^{5}c}{96}\right) + x^{12}\left(\frac{10a^{3}c^{3}}{81} + \frac{5a^{2}b^{2}c^{2}}{12} + \frac{5ab^{4}c}{48} + \frac{b^{6}}{384}\right) + x^{11}\left(\frac{5a^{3}bc^{2}}{9} + \frac{5a^{2}b^{3}c}{12} + \frac{ab^{5}}{32}\right) + x^{10}\left(\frac{5a^{4}c^{2}}{18} + \frac{5a^{3}b^{2}c}{6} + \frac{5a^{2}b^{4}}{32}\right) + x^{9}\left(\frac{5a^{4}bc}{6} + \frac{5a^{3}b^{3}}{12}\right) + x^{8}\left(\frac{a^{5}c}{3} + \frac{5a^{4}b^{2}}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**2+b*x+a)*(1+(a*x+1/2*b*x**2+1/3*c*x**3)**5),x)
```

```
[Out] a**6*x**6/6 + a**5*b*x**7/2 + a*x + b*c**5*x**17/486 + b*x**2/2 +
 c**6*x**18/4374 + c*x**3/3 + x**16*(a*c**5/243 + 5*b**2*c**4/648
) + x**15*(5*a*b*c**4/162 + 5*b**3*c**3/324) + x**14*(5*a**2*c**4
/162 + 5*a*b**2*c**3/54 + 5*b**4*c**2/288) + x**13*(5*a**2*b*c**3
/27 + 5*a*b**3*c**2/36 + b**5*c/96) + x**12*(10*a**3*c**3/81 + 5*
a**2*b**2*c**2/12 + 5*a*b**4*c/48 + b**6/384) + x**11*(5*a**3*b*c
```

```
**2/9 + 5*a**2*b**3*c/12 + a*b**5/32) + x**10*(5*a**4*c**2/18 + 5
*a**3*b**2*c/6 + 5*a**2*b**4/32) + x**9*(5*a**4*b*c/6 + 5*a**3*b*
*3/12) + x**8*(a**5*c/3 + 5*a**4*b**2/8)
```

GIAC/XCAS [A] time = 0.261076, size = 417, normalized size = 9.07

$$\frac{1}{4374}\,c^{6}x^{18} + \frac{1}{486}\,bc^{5}x^{17} + \frac{5}{648}\,b^{2}c^{4}x^{16} + \frac{1}{243}\,ac^{5}x^{16} + \frac{5}{324}\,b^{3}c^{3}x^{15} + \frac{5}{162}\,abc^{4}x^{15} + \frac{5}{288}\,b^{4}c^{2}x^{14} + \frac{5}{54}\,ab^{2}c^{3}x^{14} + \frac{5}{162}\,a^{2}c^{4}x^{14} + \frac{1}{96}\,b^{5}cx^{13} + \frac{5}{36}\,ab^{3}c^{2}x^{13} + \frac{5}{27}\,a^{2}bc^{3}x^{13} + \frac{1}{384}\,b^{6}x^{12} + \frac{5}{48}\,ab^{4}cx^{12} + \frac{5}{12}\,a^{2}b^{2}c^{2}x^{12} + \frac{10}{81}\,a^{3}c^{3}x^{12} + \frac{1}{32}\,ab^{5}x^{11} + \frac{5}{12}\,a^{2}b^{3}cx^{11} + \frac{5}{9}\,a^{3}bc^{2}x^{11} + \frac{5}{32}\,a^{2}b^{4}x^{10} + \frac{5}{6}\,a^{3}b^{2}cx^{10} + \frac{5}{18}\,a^{4}c^{2}x^{10} + \frac{5}{12}\,a^{3}b^{3}x^{9} + \frac{5}{6}\,a^{4}bcx^{9} + \frac{5}{8}\,a^{4}b^{2}x^{8} + \frac{1}{3}\,a^{5}cx^{8} + \frac{1}{2}\,a^{5}bx^{7} + \frac{1}{6}\,a^{6}x^{6} + \frac{1}{3}\,cx^{3} + \frac{1}{2}\,bx^{2} + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/7776*((2*c*x^3 + 3*b*x^2 + 6*a*x)^5 + 7776)*(c*x^2 + b*x + a),x, algor
```

```
[Out] 1/4374*c^6*x^18 + 1/486*b*c^5*x^17 + 5/648*b^2*c^4*x^16 + 1/243*a
*c^5*x^16 + 5/324*b^3*c^3*x^15 + 5/162*a*b*c^4*x^15 + 5/288*b^4*c
^2*x^14 + 5/54*a*b^2*c^3*x^14 + 5/162*a^2*c^4*x^14 + 1/96*b^5*c*x
^13 + 5/36*a*b^3*c^2*x^13 + 5/27*a^2*b*c^3*x^13 + 1/384*b^6*x^12
+ 5/48*a*b^4*c*x^12 + 5/12*a^2*b^2*c^2*x^12 + 10/81*a^3*c^3*x^12
+ 1/32*a*b^5*x^11 + 5/12*a^2*b^3*c*x^11 + 5/9*a^3*b*c^2*x^11 + 5/
32*a^2*b^4*x^10 + 5/6*a^3*b^2*c*x^10 + 5/18*a^4*c^2*x^10 + 5/12*a
^3*b^3*x^9 + 5/6*a^4*b*c*x^9 + 5/8*a^4*b^2*x^8 + 1/3*a^5*c*x^8 +
1/2*a^5*b*x^7 + 1/6*a^6*x^6 + 1/3*c*x^3 + 1/2*b*x^2 + a*x
```

3.215 $\int \left(a+bx+cx^2\right)\left(1+\left(d+ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)dx$

Optimal. Leaf size=47

$$\frac{1}{6}\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}+d\right)^6+ax+\frac{bx^2}{2}+\frac{cx^3}{3}$$

```
[Out] a*x + (b*x^2)/2 + (c*x^3)/3 + (d + a*x + (b*x^2)/2 + (c*x^3)/3)^6
/6
```

Rubi [A] time = 0.101271, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028

$$\frac{\left(6ax+3bx^2+2cx^3+6d\right)^6}{279936}+ax+\frac{bx^2}{2}+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)*(1 + (d + a*x + (b*x^2)/2 + (c*x^3)/3)^5),x]
```

```
[Out] a*x + (b*x^2)/2 + (c*x^3)/3 + (6*d + 6*a*x + 3*b*x^2 + 2*c*x^3)^6
/279936
```

Rubi in Sympy [A] time = 21.7639, size = 39, normalized size = 0.83

$$ax+\frac{bx^2}{2}+\frac{cx^3}{3}+d+\frac{\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}+d\right)^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((c*x**2+b*x+a)*(1+(d+a*x+1/2*b*x**2+1/3*c*x**3)**5),x)
```

```
[Out] a*x + b*x**2/2 + c*x**3/3 + d + (a*x + b*x**2/2 + c*x**3/3 + d)**
6/6
```

Mathematica [B] time = 0.22098, size = 248, normalized size = 5.28

$$x(6a+x(3b+2cx))\left(7776a^5x^5+6480a^4x^6(3b+2cx)+2160a^3x^7(3b+2cx)^2+360a^2x^8(3b+2cx)^3+19440d^4x(6a+x(3b+2$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)*(1 + (d + a*x + (b*x^2)/2 + (c*x^3)/3)^5),x]
```

```
[Out] (x*(6*a + x*(3*b + 2*c*x))*(46656 + 46656*d^5 + 7776*a^5*x^5 + 24
3*b^5*x^10 + 810*b^4*c*x^11 + 1080*b^3*c^2*x^12 + 720*b^2*c^3*x^1
3 + 240*b*c^4*x^14 + 32*c^5*x^15 + 6480*a^4*x^6*(3*b + 2*c*x) + 2
160*a^3*x^7*(3*b + 2*c*x)^2 + 360*a^2*x^8*(3*b + 2*c*x)^3 + 30*a*
x^9*(3*b + 2*c*x)^4 + 19440*d^4*x*(6*a + x*(3*b + 2*c*x)) + 4320*
d^3*x^2*(6*a + x*(3*b + 2*c*x))^2 + 540*d^2*x^3*(6*a + x*(3*b + 2
*c*x))^3 + 36*d*x^4*(6*a + x*(3*b + 2*c*x))^4))/279936
```

Maple [B] time = 0.004, size = 4284, normalized size = 91.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^2+b*x+a)*(1+(d+a*x+1/2*b*x^2+1/3*c*x^3)^5),x)
```

```
[Out] 1/4374*c^6*x^18+1/486*b*c^5*x^17+1/16*(1/243*a*c^5+5/162*b^2*c^4+
c*(1/81*a*c^4+1/27*b^2*c^3+1/3*c*(2/9*(2/3*a*c+1/4*b^2)*c^2+1/9*b
^2*c^2)))*x^16+1/15*(5/162*a*b*c^4+b*(1/81*a*c^4+1/27*b^2*c^3+1/3
*c*(2/9*(2/3*a*c+1/4*b^2)*c^2+1/9*b^2*c^2))+c*(1/81*c^4*d+2/27*a*
b*c^3+1/2*b*(2/9*(2/3*a*c+1/4*b^2)*c^2+1/9*b^2*c^2)+1/3*c*(2/9*(2
/3*c*d+a*b)*c^2+2/3*(2/3*a*c+1/4*b^2)*b*c)))*x^15+1/14*(a*(1/81*a
*c^4+1/27*b^2*c^3+1/3*c*(2/9*(2/3*a*c+1/4*b^2)*c^2+1/9*b^2*c^2))+
b*(1/81*c^4*d+2/27*a*b*c^3+1/2*b*(2/9*(2/3*a*c+1/4*b^2)*c^2+1/9*b
^2*c^2)+1/3*c*(2/9*(2/3*c*d+a*b)*c^2+2/3*(2/3*a*c+1/4*b^2)*b*c))+
c*(2/27*d*b*c^3+a*(2/9*(2/3*a*c+1/4*b^2)*c^2+1/9*b^2*c^2)+1/2*b*(
2/9*(2/3*c*d+a*b)*c^2+2/3*(2/3*a*c+1/4*b^2)*b*c)+1/3*c*(2/9*(a^2+
b*d)*c^2+2/3*(2/3*c*d+a*b)*b*c+(2/3*a*c+1/4*b^2)^2)))*x^14+1/13*(
a*(1/81*c^4*d+2/27*a*b*c^3+1/2*b*(2/9*(2/3*a*c+1/4*b^2)*c^2+1/9*b
^2*c^2)+1/3*c*(2/9*(2/3*c*d+a*b)*c^2+2/3*(2/3*a*c+1/4*b^2)*b*c))+
b*(2/27*d*b*c^3+a*(2/9*(2/3*a*c+1/4*b^2)*c^2+1/9*b^2*c^2)+1/2*b*(
2/9*(2/3*c*d+a*b)*c^2+2/3*(2/3*a*c+1/4*b^2)*b*c)+1/3*c*(2/9*(a^2+
b*d)*c^2+2/3*(2/3*c*d+a*b)*b*c+(2/3*a*c+1/4*b^2)^2))+c*(d*(2/9*(2
/3*a*c+1/4*b^2)*c^2+1/9*b^2*c^2)+a*(2/9*(2/3*c*d+a*b)*c^2+2/3*(2/
3*a*c+1/4*b^2)*b*c)+1/2*b*(2/9*(a^2+b*d)*c^2+2/3*(2/3*c*d+a*b)*b*
c+(2/3*a*c+1/4*b^2)^2)+1/3*c*(4/9*a*c^2*d+2/3*(a^2+b*d)*b*c+2*(2/
3*c*d+a*b)*(2/3*a*c+1/4*b^2))))*x^13+1/12*(a*(2/27*d*b*c^3+a*(2/9
*(2/3*a*c+1/4*b^2)*c^2+1/9*b^2*c^2)+1/2*b*(2/9*(2/3*c*d+a*b)*c^2+
2/3*(2/3*a*c+1/4*b^2)*b*c)+1/3*c*(2/9*(a^2+b*d)*c^2+2/3*(2/3*c*d+
a*b)*b*c+(2/3*a*c+1/4*b^2)^2))+b*(d*(2/9*(2/3*a*c+1/4*b^2)*c^2+1/
9*b^2*c^2)+a*(2/9*(2/3*c*d+a*b)*c^2+2/3*(2/3*a*c+1/4*b^2)*b*c)+1/
2*b*(2/9*(a^2+b*d)*c^2+2/3*(2/3*c*d+a*b)*b*c+(2/3*a*c+1/4*b^2)^2)
+1/3*c*(4/9*a*c^2*d+2/3*(a^2+b*d)*b*c+2*(2/3*c*d+a*b)*(2/3*a*c+1/
4*b^2)))+c*(d*(2/9*(2/3*c*d+a*b)*c^2+2/3*(2/3*a*c+1/4*b^2)*b*c)+a
*(2/9*(a^2+b*d)*c^2+2/3*(2/3*c*d+a*b)*b*c+(2/3*a*c+1/4*b^2)^2)+1/
2*b*(4/9*a*c^2*d+2/3*(a^2+b*d)*b*c+2*(2/3*c*d+a*b)*(2/3*a*c+1/4*b
^2))+1/3*c*(2/9*c^2*d^2+4/3*a*b*c*d+2*(a^2+b*d)*(2/3*a*c+1/4*b^2)
+(2/3*c*d+a*b)^2)))*x^12+1/11*(a*(d*(2/9*(2/3*a*c+1/4*b^2)*c^2+1/
9*b^2*c^2)+a*(2/9*(2/3*c*d+a*b)*c^2+2/3*(2/3*a*c+1/4*b^2)*b*c)+1/
2*b*(2/9*(a^2+b*d)*c^2+2/3*(2/3*c*d+a*b)*b*c+(2/3*a*c+1/4*b^2)^2)
+1/3*c*(4/9*a*c^2*d+2/3*(a^2+b*d)*b*c+2*(2/3*c*d+a*b)*(2/3*a*c+1/
4*b^2)))+b*(d*(2/9*(2/3*c*d+a*b)*c^2+2/3*(2/3*a*c+1/4*b^2)*b*c)+a
*(2/9*(a^2+b*d)*c^2+2/3*(2/3*c*d+a*b)*b*c+(2/3*a*c+1/4*b^2)^2)+1/
2*b*(4/9*a*c^2*d+2/3*(a^2+b*d)*b*c+2*(2/3*c*d+a*b)*(2/3*a*c+1/4*b
^2))+1/3*c*(2/9*c^2*d^2+4/3*a*b*c*d+2*(a^2+b*d)*(2/3*a*c+1/4*b^2)
+(2/3*c*d+a*b)^2))+c*(d*(2/9*(a^2+b*d)*c^2+2/3*(2/3*c*d+a*b)*b*c+
(2/3*a*c+1/4*b^2)^2)+a*(4/9*a*c^2*d+2/3*(a^2+b*d)*b*c+2*(2/3*c*d+
a*b)*(2/3*a*c+1/4*b^2))+1/2*b*(2/9*c^2*d^2+4/3*a*b*c*d+2*(a^2+b*d
)*(2/3*a*c+1/4*b^2)+(2/3*c*d+a*b)^2)+1/3*c*(2/3*b*c*d^2+4*a*d*(2/
3*a*c+1/4*b^2)+2*(a^2+b*d)*(2/3*c*d+a*b))))*x^11+1/10*(a*(d*(2/9*
(2/3*c*d+a*b)*c^2+2/3*(2/3*a*c+1/4*b^2)*b*c)+a*(2/9*(a^2+b*d)*c^2
+2/3*(2/3*c*d+a*b)*b*c+(2/3*a*c+1/4*b^2)^2)+1/2*b*(4/9*a*c^2*d+2/
3*(a^2+b*d)*b*c+2*(2/3*c*d+a*b)*(2/3*a*c+1/4*b^2))+1/3*c*(2/9*c^2
*d^2+4/3*a*b*c*d+2*(a^2+b*d)*(2/3*a*c+1/4*b^2)+(2/3*c*d+a*b)^2))+
b*(d*(2/9*(a^2+b*d)*c^2+2/3*(2/3*c*d+a*b)*b*c+(2/3*a*c+1/4*b^2)^2
)+a*(4/9*a*c^2*d+2/3*(a^2+b*d)*b*c+2*(2/3*c*d+a*b)*(2/3*a*c+1/4*b
^2))+1/2*b*(2/9*c^2*d^2+4/3*a*b*c*d+2*(a^2+b*d)*(2/3*a*c+1/4*b^2)
+(2/3*c*d+a*b)^2)+1/3*c*(2/3*b*c*d^2+4*a*d*(2/3*a*c+1/4*b^2)+2*(a
^2+b*d)*(2/3*c*d+a*b)))+c*(d*(4/9*a*c^2*d+2/3*(a^2+b*d)*b*c+2*(2/
3*c*d+a*b)*(2/3*a*c+1/4*b^2))+a*(2/9*c^2*d^2+4/3*a*b*c*d+2*(a^2+b
*d)*(2/3*a*c+1/4*b^2)+(2/3*c*d+a*b)^2)+1/2*b*(2/3*b*c*d^2+4*a*d*(
2/3*a*c+1/4*b^2)+2*(a^2+b*d)*(2/3*c*d+a*b))+1/3*c*(2*d^2*(2/3*a*c
+1/4*b^2)+4*a*d*(2/3*c*d+a*b)+(a^2+b*d)^2)))*x^10+1/9*(a*(d*(2/9*
(a^2+b*d)*c^2+2/3*(2/3*c*d+a*b)*b*c+(2/3*a*c+1/4*b^2)^2)+a*(4/9*a
*c^2*d+2/3*(a^2+b*d)*b*c+2*(2/3*c*d+a*b)*(2/3*a*c+1/4*b^2))+1/2*b
*(2/9*c^2*d^2+4/3*a*b*c*d+2*(a^2+b*d)*(2/3*a*c+1/4*b^2)+(2/3*c*d+
a*b)^2)+1/3*c*(2/3*b*c*d^2+4*a*d*(2/3*a*c+1/4*b^2)+2*(a^2+b*d)*(2
/3*c*d+a*b)))+b*(d*(4/9*a*c^2*d+2/3*(a^2+b*d)*b*c+2*(2/3*c*d+a*b)
```

```
*(2/3*a*c+1/4*b^2))+a*(2/9*c^2*d^2+4/3*a*b*c*d+2*(a^2+b*d)*(2/3*a
*c+1/4*b^2)+(2/3*c*d+a*b)^2)+1/2*b*(2/3*b*c*d^2+4*a*d*(2/3*a*c+1/
4*b^2)+2*(a^2+b*d)*(2/3*c*d+a*b))+1/3*c*(2*d^2*(2/3*a*c+1/4*b^2)+
4*a*d*(2/3*c*d+a*b)+(a^2+b*d)^2))+c*(d*(2/9*c^2*d^2+4/3*a*b*c*d+2
*(a^2+b*d)*(2/3*a*c+1/4*b^2)+(2/3*c*d+a*b)^2)+a*(2/3*b*c*d^2+4*a*
d*(2/3*a*c+1/4*b^2)+2*(a^2+b*d)*(2/3*c*d+a*b))+1/2*b*(2*d^2*(2/3*
a*c+1/4*b^2)+4*a*d*(2/3*c*d+a*b)+(a^2+b*d)^2)+1/3*c*(2*d^2*(2/3*c
*d+a*b)+4*a*d*(a^2+b*d))))*x^9+1/8*(a*(d*(4/9*a*c^2*d+2/3*(a^2+b*
d)*b*c+2*(2/3*c*d+a*b)*(2/3*a*c+1/4*b^2))+a*(2/9*c^2*d^2+4/3*a*b*
c*d+2*(a^2+b*d)*(2/3*a*c+1/4*b^2)+(2/3*c*d+a*b)^2)+1/2*b*(2/3*b*c
*d^2+4*a*d*(2/3*a*c+1/4*b^2)+2*(a^2+b*d)*(2/3*c*d+a*b))+1/3*c*(2*
d^2*(2/3*a*c+1/4*b^2)+4*a*d*(2/3*c*d+a*b)+(a^2+b*d)^2))+b*(d*(2/9
*c^2*d^2+4/3*a*b*c*d+2*(a^2+b*d)*(2/3*a*c+1/4*b^2)+(2/3*c*d+a*b)^
2)+a*(2/3*b*c*d^2+4*a*d*(2/3*a*c+1/4*b^2)+2*(a^2+b*d)*(2/3*c*d+a*
b))+1/2*b*(2*d^2*(2/3*a*c+1/4*b^2)+4*a*d*(2/3*c*d+a*b)+(a^2+b*d)^
2)+1/3*c*(2*d^2*(2/3*c*d+a*b)+4*a*d*(a^2+b*d)))+c*(d*(2/3*b*c*d^2
+4*a*d*(2/3*a*c+1/4*b^2)+2*(a^2+b*d)*(2/3*c*d+a*b))+a*(2*d^2*(2/3
*a*c+1/4*b^2)+4*a*d*(2/3*c*d+a*b)+(a^2+b*d)^2)+1/2*b*(2*d^2*(2/3*
c*d+a*b)+4*a*d*(a^2+b*d))+1/3*c*(2*d^2*(a^2+b*d)+4*a^2*d^2)))*x^8
+1/7*(a*(d*(2/9*c^2*d^2+4/3*a*b*c*d+2*(a^2+b*d)*(2/3*a*c+1/4*b^2)
+(2/3*c*d+a*b)^2)+a*(2/3*b*c*d^2+4*a*d*(2/3*a*c+1/4*b^2)+2*(a^2+b
*d)*(2/3*c*d+a*b))+1/2*b*(2*d^2*(2/3*a*c+1/4*b^2)+4*a*d*(2/3*c*d+
a*b)+(a^2+b*d)^2)+1/3*c*(2*d^2*(2/3*c*d+a*b)+4*a*d*(a^2+b*d)))+b*
(d*(2/3*b*c*d^2+4*a*d*(2/3*a*c+1/4*b^2)+2*(a^2+b*d)*(2/3*c*d+a*b)
)+a*(2*d^2*(2/3*a*c+1/4*b^2)+4*a*d*(2/3*c*d+a*b)+(a^2+b*d)^2)+1/2
*b*(2*d^2*(2/3*c*d+a*b)+4*a*d*(a^2+b*d))+1/3*c*(2*d^2*(a^2+b*d)+4
*a^2*d^2))+c*(d*(2*d^2*(2/3*a*c+1/4*b^2)+4*a*d*(2/3*c*d+a*b)+(a^2
+b*d)^2)+a*(2*d^2*(2/3*c*d+a*b)+4*a*d*(a^2+b*d))+1/2*b*(2*d^2*(a^
2+b*d)+4*a^2*d^2)+4/3*a*c*d^3))*x^7+1/6*(a*(d*(2/3*b*c*d^2+4*a*d*
(2/3*a*c+1/4*b^2)+2*(a^2+b*d)*(2/3*c*d+a*b))+a*(2*d^2*(2/3*a*c+1/
4*b^2)+4*a*d*(2/3*c*d+a*b)+(a^2+b*d)^2)+1/2*b*(2*d^2*(2/3*c*d+a*b
)+4*a*d*(a^2+b*d))+1/3*c*(2*d^2*(a^2+b*d)+4*a^2*d^2))+b*(d*(2*d^2
*(2/3*a*c+1/4*b^2)+4*a*d*(2/3*c*d+a*b)+(a^2+b*d)^2)+a*(2*d^2*(2/3
*c*d+a*b)+4*a*d*(a^2+b*d))+1/2*b*(2*d^2*(a^2+b*d)+4*a^2*d^2)+4/3*
a*c*d^3)+c*(d*(2*d^2*(2/3*c*d+a*b)+4*a*d*(a^2+b*d))+a*(2*d^2*(a^2
+b*d)+4*a^2*d^2)+2*b*a*d^3+1/3*c*d^4))*x^6+1/5*(a*(d*(2*d^2*(2/3*
a*c+1/4*b^2)+4*a*d*(2/3*c*d+a*b)+(a^2+b*d)^2)+a*(2*d^2*(2/3*c*d+a
*b)+4*a*d*(a^2+b*d))+1/2*b*(2*d^2*(a^2+b*d)+4*a^2*d^2)+4/3*a*c*d^
3)+b*(d*(2*d^2*(2/3*c*d+a*b)+4*a*d*(a^2+b*d))+a*(2*d^2*(a^2+b*d)+
4*a^2*d^2)+2*b*a*d^3+1/3*c*d^4)+c*(d*(2*d^2*(a^2+b*d)+4*a^2*d^2)+
4*a^2*d^3+1/2*b*d^4))*x^5+1/4*(a*(d*(2*d^2*(2/3*c*d+a*b)+4*a*d*(a
^2+b*d))+a*(2*d^2*(a^2+b*d)+4*a^2*d^2)+2*b*a*d^3+1/3*c*d^4)+b*(d*
(2*d^2*(a^2+b*d)+4*a^2*d^2)+4*a^2*d^3+1/2*b*d^4)+5*a*c*d^4)*x^4+1
/3*(a*(d*(2*d^2*(a^2+b*d)+4*a^2*d^2)+4*a^2*d^3+1/2*b*d^4)+5*b*d^4
*a+c*(d^5+1))*x^3+1/2*(5*a^2*d^4+b*(d^5+1))*x^2+a*(d^5+1)*x
```

Maxima [A] time = 0.81788, size = 1044, normalized size = 22.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/7776*((2*c*x^3 + 3*b*x^2 + 6*a*x + 6*d)^5 + 7776)*(c*x^2 + b*x + a),x,
```

```
[Out] 1/4374*c^6*x^18 + 1/486*b*c^5*x^17 + 1/1944*(15*b^2*c^4 + 8*a*c^5
)*x^16 + 1/972*(15*b^3*c^3 + 30*a*b*c^4 + 4*c^5*d)*x^15 + 5/2592*
(9*b^4*c^2 + 48*a*b^2*c^3 + 16*a^2*c^4 + 16*b*c^4*d)*x^14 + 1/259
2*(27*b^5*c + 360*a*b^3*c^2 + 480*a^2*b*c^3 + 80*(3*b^2*c^3 + 2*a
*c^4)*d)*x^13 + 1/10368*(27*b^6 + 1080*a*b^4*c + 4320*a^2*b^2*c^2
 + 1280*a^3*c^3 + 320*c^4*d^2 + 480*(3*b^3*c^2 + 8*a*b*c^3)*d)*x^
12 + 1/864*(27*a*b^5 + 360*a^2*b^3*c + 480*a^3*b*c^2 + 160*b*c^3*
d^2 + 10*(9*b^4*c + 72*a*b^2*c^2 + 32*a^2*c^3)*d)*x^11 + 1/864*(1
35*a^2*b^4 + 720*a^3*b^2*c + 240*a^4*c^2 + 40*(9*b^2*c^2 + 8*a*c^
3)*d^2 + 9*(3*b^5 + 80*a*b^3*c + 160*a^2*b*c^2)*d)*x^10 + 5/1296*
(108*a^3*b^3 + 216*a^4*b*c + 32*c^3*d^3 + 108*(b^3*c + 4*a*b*c^2)
*d^2 + 9*(9*a*b^4 + 72*a^2*b^2*c + 32*a^3*c^2)*d)*x^9 + 1/288*(18
0*a^4*b^2 + 96*a^5*c + 160*b*c^2*d^3 + 15*(3*b^4 + 48*a*b^2*c + 3
2*a^2*c^2)*d^2 + 120*(3*a^2*b^3 + 8*a^3*b*c)*d)*x^8 + 1/36*(18*a^
```

```
5*b + 10*(3*b^2*c + 4*a*c^2)*d^3 + 45*(a*b^3 + 4*a^2*b*c)*d^2 + 3
0*(3*a^3*b^2 + 2*a^4*c)*d)*x^7 + 1/36*(6*a^6 + 90*a^4*b*d + 10*c^
2*d^4 + 15*(b^3 + 8*a*b*c)*d^3 + 15*(9*a^2*b^2 + 8*a^3*c)*d^2)*x^
6 + 1/6*(6*a^5*d + 30*a^3*b*d^2 + 5*b*c*d^4 + 5*(3*a*b^2 + 4*a^2*
c)*d^3)*x^5 + 5/24*(12*a^4*d^2 + 24*a^2*b*d^3 + (3*b^2 + 8*a*c)*d
^4)*x^4 + 1/6*(20*a^3*d^3 + 15*a*b*d^4 + 2*c*d^5 + 2*c)*x^3 + 1/2
*(5*a^2*d^4 + b*d^5 + b)*x^2 + (a*d^5 + a)*x
```

Fricas [A] time = 0.256481, size = 1044, normalized size = 22.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/7776*((2*c*x^3 + 3*b*x^2 + 6*a*x + 6*d)^5 + 7776)*(c*x^2 + b*x + a),x,
```

```
[Out] 1/4374*c^6*x^18 + 1/486*b*c^5*x^17 + 1/1944*(15*b^2*c^4 + 8*a*c^5
)*x^16 + 1/972*(15*b^3*c^3 + 30*a*b*c^4 + 4*c^5*d)*x^15 + 5/2592*
(9*b^4*c^2 + 48*a*b^2*c^3 + 16*a^2*c^4 + 16*b*c^4*d)*x^14 + 1/259
2*(27*b^5*c + 360*a*b^3*c^2 + 480*a^2*b*c^3 + 80*(3*b^2*c^3 + 2*a
*c^4)*d)*x^13 + 1/10368*(27*b^6 + 1080*a*b^4*c + 4320*a^2*b^2*c^2
 + 1280*a^3*c^3 + 320*c^4*d^2 + 480*(3*b^3*c^2 + 8*a*b*c^3)*d)*x^
12 + 1/864*(27*a*b^5 + 360*a^2*b^3*c + 480*a^3*b*c^2 + 160*b*c^3*
d^2 + 10*(9*b^4*c + 72*a*b^2*c^2 + 32*a^2*c^3)*d)*x^11 + 1/864*(1
35*a^2*b^4 + 720*a^3*b^2*c + 240*a^4*c^2 + 40*(9*b^2*c^2 + 8*a*c^
3)*d^2 + 9*(3*b^5 + 80*a*b^3*c + 160*a^2*b*c^2)*d)*x^10 + 5/1296*
(108*a^3*b^3 + 216*a^4*b*c + 32*c^3*d^3 + 108*(b^3*c + 4*a*b*c^2)
*d^2 + 9*(9*a*b^4 + 72*a^2*b^2*c + 32*a^3*c^2)*d)*x^9 + 1/288*(18
0*a^4*b^2 + 96*a^5*c + 160*b*c^2*d^3 + 15*(3*b^4 + 48*a*b^2*c + 3
2*a^2*c^2)*d^2 + 120*(3*a^2*b^3 + 8*a^3*b*c)*d)*x^8 + 1/36*(18*a^
5*b + 10*(3*b^2*c + 4*a*c^2)*d^3 + 45*(a*b^3 + 4*a^2*b*c)*d^2 + 3
0*(3*a^3*b^2 + 2*a^4*c)*d)*x^7 + 1/36*(6*a^6 + 90*a^4*b*d + 10*c^
2*d^4 + 15*(b^3 + 8*a*b*c)*d^3 + 15*(9*a^2*b^2 + 8*a^3*c)*d^2)*x^
6 + 1/6*(6*a^5*d + 30*a^3*b*d^2 + 5*b*c*d^4 + 5*(3*a*b^2 + 4*a^2*
c)*d^3)*x^5 + 5/24*(12*a^4*d^2 + 24*a^2*b*d^3 + (3*b^2 + 8*a*c)*d
^4)*x^4 + 1/6*(20*a^3*d^3 + 15*a*b*d^4 + 2*c*d^5 + 2*c)*x^3 + 1/2
*(5*a^2*d^4 + b*d^5 + b)*x^2 + (a*d^5 + a)*x
```

Sympy [A] time = 0.676009, size = 930, normalized size = 19.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)*(1+(d+a*x+1/2*b*x**2+1/3*c*x**3)**5),x)
```

```
[Out] b*c**5*x**17/486 + c**6*x**18/4374 + x**16*(a*c**5/243 + 5*b**2*c
**4/648) + x**15*(5*a*b*c**4/162 + 5*b**3*c**3/324 + c**5*d/243)
+ x**14*(5*a**2*c**4/162 + 5*a*b**2*c**3/54 + 5*b**4*c**2/288 + 5
*b*c**4*d/162) + x**13*(5*a**2*b*c**3/27 + 5*a*b**3*c**2/36 + 5*a
*c**4*d/81 + b**5*c/96 + 5*b**2*c**3*d/54) + x**12*(10*a**3*c**3/
81 + 5*a**2*b**2*c**2/12 + 5*a*b**4*c/48 + 10*a*b*c**3*d/27 + b**
6/384 + 5*b**3*c**2*d/36 + 5*c**4*d**2/162) + x**11*(5*a**3*b*c**
2/9 + 5*a**2*b**3*c/12 + 10*a**2*c**3*d/27 + a*b**5/32 + 5*a*b**2
*c**2*d/6 + 5*b**4*c*d/48 + 5*b*c**3*d**2/27) + x**10*(5*a**4*c**
2/18 + 5*a**3*b**2*c/6 + 5*a**2*b**4/32 + 5*a**2*b*c**2*d/3 + 5*a
*b**3*c*d/6 + 10*a*c**3*d**2/27 + b**5*d/32 + 5*b**2*c**2*d**2/12
) + x**9*(5*a**4*b*c/6 + 5*a**3*b**3/12 + 10*a**3*c**2*d/9 + 5*a*
*2*b**2*c*d/2 + 5*a*b**4*d/16 + 5*a*b*c**2*d**2/3 + 5*b**3*c*d**2
/12 + 10*c**3*d**3/81) + x**8*(a**5*c/3 + 5*a**4*b**2/8 + 10*a**3
*b*c*d/3 + 5*a**2*b**3*d/4 + 5*a**2*c**2*d**2/3 + 5*a*b**2*c*d**2
/2 + 5*b**4*d**2/32 + 5*b*c**2*d**3/9) + x**7*(a**5*b/2 + 5*a**4*
c*d/3 + 5*a**3*b**2*d/2 + 5*a**2*b*c*d**2 + 5*a*b**3*d**2/4 + 10*
```

```
a*c**2*d**3/9 + 5*b**2*c*d**3/6) + x**6*(a**6/6 + 5*a**4*b*d/2 +
10*a**3*c*d**2/3 + 15*a**2*b**2*d**2/4 + 10*a*b*c*d**3/3 + 5*b**3
*d**3/12 + 5*c**2*d**4/18) + x**5*(a**5*d + 5*a**3*b*d**2 + 10*a*
*2*c*d**3/3 + 5*a*b**2*d**3/2 + 5*b*c*d**4/6) + x**4*(5*a**4*d**2
/2 + 5*a**2*b*d**3 + 5*a*c*d**4/3 + 5*b**2*d**4/8) + x**3*(10*a**
3*d**3/3 + 5*a*b*d**4/2 + c*d**5/3 + c/3) + x**2*(5*a**2*d**4/2 +
 b*d**5/2 + b/2) + x*(a*d**5 + a)
```

GIAC/XCAS [A] time = 0.265059, size = 1253, normalized size = 26.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/7776*((2*c*x^3 + 3*b*x^2 + 6*a*x + 6*d)^5 + 7776)*(c*x^2 + b*x + a),x,
```

```
[Out] 1/4374*c^6*x^18 + 1/486*b*c^5*x^17 + 5/648*b^2*c^4*x^16 + 1/243*a
*c^5*x^16 + 5/324*b^3*c^3*x^15 + 5/162*a*b*c^4*x^15 + 1/243*c^5*d
*x^15 + 5/288*b^4*c^2*x^14 + 5/54*a*b^2*c^3*x^14 + 5/162*a^2*c^4*
x^14 + 5/162*b*c^4*d*x^14 + 1/96*b^5*c*x^13 + 5/36*a*b^3*c^2*x^13
 + 5/27*a^2*b*c^3*x^13 + 5/54*b^2*c^3*d*x^13 + 5/81*a*c^4*d*x^13
+ 1/384*b^6*x^12 + 5/48*a*b^4*c*x^12 + 5/12*a^2*b^2*c^2*x^12 + 10
/81*a^3*c^3*x^12 + 5/36*b^3*c^2*d*x^12 + 10/27*a*b*c^3*d*x^12 + 5
/162*c^4*d^2*x^12 + 1/32*a*b^5*x^11 + 5/12*a^2*b^3*c*x^11 + 5/9*a
^3*b*c^2*x^11 + 5/48*b^4*c*d*x^11 + 5/6*a*b^2*c^2*d*x^11 + 10/27*
a^2*c^3*d*x^11 + 5/27*b*c^3*d^2*x^11 + 5/32*a^2*b^4*x^10 + 5/6*a^
3*b^2*c*x^10 + 5/18*a^4*c^2*x^10 + 1/32*b^5*d*x^10 + 5/6*a*b^3*c*
d*x^10 + 5/3*a^2*b*c^2*d*x^10 + 5/12*b^2*c^2*d^2*x^10 + 10/27*a*c
^3*d^2*x^10 + 5/12*a^3*b^3*x^9 + 5/6*a^4*b*c*x^9 + 5/16*a*b^4*d*x
^9 + 5/2*a^2*b^2*c*d*x^9 + 10/9*a^3*c^2*d*x^9 + 5/12*b^3*c*d^2*x^
9 + 5/3*a*b*c^2*d^2*x^9 + 10/81*c^3*d^3*x^9 + 5/8*a^4*b^2*x^8 + 1
/3*a^5*c*x^8 + 5/4*a^2*b^3*d*x^8 + 10/3*a^3*b*c*d*x^8 + 5/32*b^4*
d^2*x^8 + 5/2*a*b^2*c*d^2*x^8 + 5/3*a^2*c^2*d^2*x^8 + 5/9*b*c^2*d
^3*x^8 + 1/2*a^5*b*x^7 + 5/2*a^3*b^2*d*x^7 + 5/3*a^4*c*d*x^7 + 5/
4*a*b^3*d^2*x^7 + 5*a^2*b*c*d^2*x^7 + 5/6*b^2*c*d^3*x^7 + 10/9*a*
c^2*d^3*x^7 + 1/6*a^6*x^6 + 5/2*a^4*b*d*x^6 + 15/4*a^2*b^2*d^2*x^
6 + 10/3*a^3*c*d^2*x^6 + 5/12*b^3*d^3*x^6 + 10/3*a*b*c*d^3*x^6 +
5/18*c^2*d^4*x^6 + a^5*d*x^5 + 5*a^3*b*d^2*x^5 + 5/2*a*b^2*d^3*x^
5 + 10/3*a^2*c*d^3*x^5 + 5/6*b*c*d^4*x^5 + 5/2*a^4*d^2*x^4 + 5*a^
2*b*d^3*x^4 + 5/8*b^2*d^4*x^4 + 5/3*a*c*d^4*x^4 + 10/3*a^3*d^3*x^
3 + 5/2*a*b*d^4*x^3 + 1/3*c*d^5*x^3 + 5/2*a^2*d^4*x^2 + 1/2*b*d^5
*x^2 + a*d^5*x + 1/3*c*x^3 + 1/2*b*x^2 + a*x
```

3.216 $\int \left(a+cx^2\right)\left(1+\left(ax+\frac{cx^3}{3}\right)^n\right)\,dx$

Optimal. Leaf size=34

$$\frac{\left(ax+\frac{cx^3}{3}\right)^{n+1}}{n+1}+ax+\frac{cx^3}{3}$$

```
[Out] a*x + (c*x^3)/3 + (a*x + (c*x^3)/3)^(1 + n)/(1 + n)
```

Rubi [A] time = 0.0195225, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.042$

$$\frac{\left(ax+\frac{cx^3}{3}\right)^{n+1}}{n+1}+ax+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + c*x^2)*(1 + (a*x + (c*x^3)/3)^n),x]

[Out] a*x + (c*x^3)/3 + (a*x + (c*x^3)/3)^(1 + n)/(1 + n)
```

Rubi in Sympy [A] time = 2.50489, size = 26, normalized size = 0.76

$$ax+\frac{cx^3}{3}+\frac{\left(ax+\frac{cx^3}{3}\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**2+a)*(1+(a*x+1/3*c*x**3)**n),x)

[Out] a*x + c*x**3/3 + (a*x + c*x**3/3)**(n + 1)/(n + 1)
```

Mathematica [A] time = 0.033954, size = 36, normalized size = 1.06

$$\frac{x\left(3a+cx^2\right)\left(\left(ax+\frac{cx^3}{3}\right)^n+n+1\right)}{3(n+1)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + c*x^2)*(1 + (a*x + (c*x^3)/3)^n),x]

[Out] (x*(3*a + c*x^2)*(1 + n + (a*x + (c*x^3)/3)^n))/(3*(1 + n))
```

Maple [A] time = 0.003, size = 31, normalized size = 0.9

$$ax+\frac{cx^3}{3}+\frac{1}{1+n}\left(ax+\frac{cx^3}{3}\right)^{1+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^2+a)*(1+(a*x+1/3*c*x^3)^n),x)
```

```
[Out] a*x+1/3*c*x^3+(a*x+1/3*c*x^3)^(1+n)/(1+n)
```

Maxima [A] time = 0.998622, size = 73, normalized size = 2.15

$$\frac{1}{3}cx^3 + ax + \frac{\left(cx^3 + 3\,ax\right)e^{\left(n\log\left(cx^2+3\,a\right)+n\log(x)\right)}}{3^{n+1}n + 3^{n+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + a)*((1/3*c*x^3 + a*x)^n + 1),x, algorithm="maxima")
```

```
[Out] 1/3*c*x^3 + a*x + (c*x^3 + 3*a*x)*e^(n*log(c*x^2 + 3*a) + n*log(x
))/(3^(n + 1)*n + 3^(n + 1))
```

Fricas [A] time = 0.290244, size = 65, normalized size = 1.91

$$\frac{(cn+c)x^3 + \left(cx^3 + 3\,ax\right)\left(\frac{1}{3}\,cx^3 + ax\right)^n + 3\,(an+a)x}{3\,(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + a)*((1/3*c*x^3 + a*x)^n + 1),x, algorithm="fricas")
```

```
[Out] 1/3*((c*n + c)*x^3 + (c*x^3 + 3*a*x)*(1/3*c*x^3 + a*x)^n + 3*(a*n
 + a)*x)/(n + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**2+a)*(1+(a*x+1/3*c*x**3)**n),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.258519, size = 41, normalized size = 1.21

$$\frac{1}{3}cx^3 + ax + \frac{\left(\frac{1}{3}\,cx^3 + ax\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + a)*((1/3*c*x^3 + a*x)^n + 1),x, algorithm="giac")
```

```
[Out] 1/3*c*x^3 + a*x + (1/3*c*x^3 + a*x)^(n + 1)/(n + 1)
```

3.217 $\int (bx + cx^2) \left(1 + \left(\frac{bx^2}{2} + \frac{cx^3}{3}\right)^n\right) dx$

Optimal. Leaf size=44

$$\frac{\left(\frac{bx^2}{2} + \frac{cx^3}{3}\right)^{n+1}}{n+1} + \frac{bx^2}{2} + \frac{cx^3}{3}$$

```
[Out] (b*x^2)/2 + (c*x^3)/3 + ((b*x^2)/2 + (c*x^3)/3)^(1 + n)/(1 + n)
```

Rubi [A] time = 0.0206251, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.032$

$$\frac{\left(\frac{bx^2}{2} + \frac{cx^3}{3}\right)^{n+1}}{n+1} + \frac{bx^2}{2} + \frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x + c*x^2)*(1 + ((b*x^2)/2 + (c*x^3)/3)^n),x]
```

```
[Out] (b*x^2)/2 + (c*x^3)/3 + ((b*x^2)/2 + (c*x^3)/3)^(1 + n)/(1 + n)
```

Rubi in Sympy [A] time = 3.32383, size = 32, normalized size = 0.73

$$\frac{bx^2}{2} + \frac{cx^3}{3} + \frac{\left(\frac{bx^2}{2} + \frac{cx^3}{3}\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**2+b*x)*(1+(1/2*b*x**2+1/3*c*x**3)**n),x)
```

```
[Out] b*x**2/2 + c*x**3/3 + (b*x**2/2 + c*x**3/3)**(n + 1)/(n + 1)
```

Mathematica [A] time = 0.0371577, size = 42, normalized size = 0.95

$$\frac{x^2(3b + 2cx)\left(\left(\frac{bx^2}{2} + \frac{cx^3}{3}\right)^n + n + 1\right)}{6(n+1)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x + c*x^2)*(1 + ((b*x^2)/2 + (c*x^3)/3)^n),x]
```

```
[Out] (x^2*(3*b + 2*c*x)*(1 + n + ((b*x^2)/2 + (c*x^3)/3)^n))/(6*(1 + n
))
```

Maple [A] time = 0.004, size = 37, normalized size = 0.8

$$\frac{bx^2}{2} + \frac{cx^3}{3} + \frac{1}{1+n}\left(\frac{bx^2}{2} + \frac{cx^3}{3}\right)^{1+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^n),x)
```

```
[Out] 1/2*b*x^2+1/3*c*x^3+(1/2*b*x^2+1/3*c*x^3)^(1+n)/(1+n)
```

Maxima [A] time = 0.992911, size = 96, normalized size = 2.18

$$\frac{1}{3}\,cx^3 + \frac{1}{2}\,bx^2 + \frac{\left(2\,cx^3 + 3\,bx^2\right)e^{\left(n\log\left(2\,cx+3\,b\right)+2\,n\log\left(x\right)\right)}}{3^{n+1}2^{n+1}n + 3^{n+1}2^{n+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2 + b*x)*((1/3*c*x^3 + 1/2*b*x^2)^n + 1),x, algorithm="maxima")
```

```
[Out] 1/3*c*x^3 + 1/2*b*x^2 + (2*c*x^3 + 3*b*x^2)*e^(n*log(2*c*x + 3*b)
 + 2*n*log(x))/(3^(n + 1)*2^(n + 1)*n + 3^(n + 1)*2^(n + 1))
```

Fricas [A] time = 0.279799, size = 77, normalized size = 1.75

$$\frac{2\left(cn + c\right)x^3 + 3\left(bn + b\right)x^2 + \left(2\,cx^3 + 3\,bx^2\right)\left(\frac{1}{3}\,cx^3 + \frac{1}{2}\,bx^2\right)^n}{6\left(n + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2 + b*x)*((1/3*c*x^3 + 1/2*b*x^2)^n + 1),x, algorithm="fricas")
```

```
[Out] 1/6*(2*(c*n + c)*x^3 + 3*(b*n + b)*x^2 + (2*c*x^3 + 3*b*x^2)*(1/3
*c*x^3 + 1/2*b*x^2)^n)/(n + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**2+b*x)*(1+(1/2*b*x**2+1/3*c*x**3)**n),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.260366, size = 49, normalized size = 1.11

$$\frac{1}{3}\,cx^3 + \frac{1}{2}\,bx^2 + \frac{\left(\frac{1}{3}\,cx^3 + \frac{1}{2}\,bx^2\right)^{n+1}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2 + b*x)*((1/3*c*x^3 + 1/2*b*x^2)^n + 1),x, algorithm="giac")
```

```
[Out] 1/3*c*x^3 + 1/2*b*x^2 + (1/3*c*x^3 + 1/2*b*x^2)^(n + 1)/(n + 1)
```

3.218 $\int (a+bx+cx^2)\left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right)\,dx$

Optimal. Leaf size=50

$$\frac{\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^{n+1}}{n+1}+ax+\frac{bx^2}{2}+\frac{cx^3}{3}$$

```
[Out] a*x + (b*x^2)/2 + (c*x^3)/3 + (a*x + (b*x^2)/2 + (c*x^3)/3)^(1 +
n)/(1 + n)
```

Rubi [A] time = 0.021181, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029

$$\frac{\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^{n+1}}{n+1}+ax+\frac{bx^2}{2}+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x + c*x^2)*(1 + (a*x + (b*x^2)/2 + (c*x^3)/3)^n),x]
```

```
[Out] a*x + (b*x^2)/2 + (c*x^3)/3 + (a*x + (b*x^2)/2 + (c*x^3)/3)^(1 +
n)/(1 + n)
```

Rubi in Sympy [A] time = 3.1566, size = 39, normalized size = 0.78

$$ax+\frac{bx^2}{2}+\frac{cx^3}{3}+\frac{\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**2+b*x+a)*(1+(a*x+1/2*b*x**2+1/3*c*x**3)**n),x)
```

```
[Out] a*x + b*x**2/2 + c*x**3/3 + (a*x + b*x**2/2 + c*x**3/3)**(n + 1)/
(n + 1)
```

Mathematica [A] time = 0.057532, size = 49, normalized size = 0.98

$$\frac{x(6a+x(3b+2cx))\left(\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n+n+1\right)}{6(n+1)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x + c*x^2)*(1 + (a*x + (b*x^2)/2 + (c*x^3)/3)^n),x]
```

```
[Out] (x*(6*a + x*(3*b + 2*c*x))*(1 + n + (a*x + (b*x^2)/2 + (c*x^3)/3)
^n))/(6*(1 + n))
```

Maple [A] time = 0.003, size = 43, normalized size = 0.9

$$ax+\frac{bx^2}{2}+\frac{cx^3}{3}+\frac{1}{1+n}\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^{1+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^n),x)
```

```
[Out] a*x+1/2*b*x^2+1/3*c*x^3+(a*x+1/2*b*x^2+1/3*c*x^3)^(1+n)/(1+n)
```

Maxima [A] time = 1.01791, size = 112, normalized size = 2.24

$$\frac{1}{3}cx^3 + \frac{1}{2}bx^2 + ax + \frac{\left(2\,cx^3 + 3\,bx^2 + 6\,ax\right)e^{\left(n\log\left(2\,cx^2+3\,bx+6\,a\right)+n\log\left(x\right)\right)}}{3^{n+1}2^{n+1}n + 3^{n+1}2^{n+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2 + b*x + a)*((1/3*c*x^3 + 1/2*b*x^2 + a*x)^n + 1),x, algorithm="ma
```

```
[Out] 1/3*c*x^3 + 1/2*b*x^2 + a*x + (2*c*x^3 + 3*b*x^2 + 6*a*x)*e^(n*lo
g(2*c*x^2 + 3*b*x + 6*a) + n*log(x))/(3^(n + 1)*2^(n + 1)*n + 3^(
n + 1)*2^(n + 1))
```

Fricas [A] time = 0.275281, size = 97, normalized size = 1.94

$$\frac{2\,(cn+c)x^3 + 3\,(bn+b)x^2 + \left(2\,cx^3 + 3\,bx^2 + 6\,ax\right)\left(\frac{1}{3}\,cx^3 + \frac{1}{2}\,bx^2 + ax\right)^n + 6\,(an+a)x}{6\,(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2 + b*x + a)*((1/3*c*x^3 + 1/2*b*x^2 + a*x)^n + 1),x, algorithm="fr
```

```
[Out] 1/6*(2*(c*n + c)*x^3 + 3*(b*n + b)*x^2 + (2*c*x^3 + 3*b*x^2 + 6*a
*x)*(1/3*c*x^3 + 1/2*b*x^2 + a*x)^n + 6*(a*n + a)*x)/(n + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**2+b*x+a)*(1+(a*x+1/2*b*x**2+1/3*c*x**3)**n),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.259917, size = 57, normalized size = 1.14

$$\frac{1}{3}cx^3 + \frac{1}{2}bx^2 + ax + \frac{\left(\frac{1}{3}\,cx^3 + \frac{1}{2}\,bx^2 + ax\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2 + b*x + a)*((1/3*c*x^3 + 1/2*b*x^2 + a*x)^n + 1),x, algorithm="gi
```

```
[Out] 1/3*c*x^3 + 1/2*b*x^2 + a*x + (1/3*c*x^3 + 1/2*b*x^2 + a*x)^(n +
1)/(n + 1)
```

3.219 $\int (-4+4x+x^2)(5-12x+6x^2+x^3)\,dx$

Optimal. Leaf size=19

$$\frac{1}{6}\left(x^3+6x^2-12x+5\right)^2$$

```
[Out] (5 - 12*x + 6*x^2 + x^3)^2/6
```

Rubi [A] time = 0.0115114, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045

$$\frac{1}{6}\left(x^3+6x^2-12x+5\right)^2$$

Antiderivative was successfully verified.

```
[In] Int[(-4 + 4*x + x^2)*(5 - 12*x + 6*x^2 + x^3),x]
```

```
[Out] (5 - 12*x + 6*x^2 + x^3)^2/6
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int^{x^3+6x^2-12x+5} x\,dx}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((x**2+4*x-4)*(x**3+6*x**2-12*x+5),x)
```

```
[Out] Integral(x, (x, x**3 + 6*x**2 - 12*x + 5))/3
```

Mathematica [A] time = 0.0023042, size = 33, normalized size = 1.74

$$\frac{x^6}{6}+2x^5+2x^4-\frac{67x^3}{3}+34x^2-20x$$

Antiderivative was successfully verified.

```
[In] Integrate[(-4 + 4*x + x^2)*(5 - 12*x + 6*x^2 + x^3),x]
```

```
[Out] -20*x + 34*x^2 - (67*x^3)/3 + 2*x^4 + 2*x^5 + x^6/6
```

Maple [A] time = 0.001, size = 30, normalized size = 1.6

$$\frac{x^6}{6}+2\,x^5+2\,x^4-\frac{67\,x^3}{3}+34\,x^2-20\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+4*x-4)*(x^3+6*x^2-12*x+5),x)
```

```
[Out] 1/6*x^6+2*x^5+2*x^4-67/3*x^3+34*x^2-20*x
```

Maxima [A] time = 0.784678, size = 23, normalized size = 1.21

$$\frac{1}{6}\left(x^3 + 6\,x^2 - 12\,x + 5\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3 + 6*x^2 - 12*x + 5)*(x^2 + 4*x - 4),x, algorithm="maxima")
```

```
[Out] 1/6*(x^3 + 6*x^2 - 12*x + 5)^2
```

Fricas [A] time = 0.231034, size = 1, normalized size = 0.05

$$\frac{1}{6}x^6 + 2x^5 + 2x^4 - \frac{67}{3}x^3 + 34x^2 - 20x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3 + 6*x^2 - 12*x + 5)*(x^2 + 4*x - 4),x, algorithm="fricas")
```

```
[Out] 1/6*x^6 + 2*x^5 + 2*x^4 - 67/3*x^3 + 34*x^2 - 20*x
```

Sympy [A] time = 0.072797, size = 29, normalized size = 1.53

$$\frac{x^6}{6} + 2x^5 + 2x^4 - \frac{67x^3}{3} + 34x^2 - 20x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+4*x-4)*(x**3+6*x**2-12*x+5),x)
```

```
[Out] x**6/6 + 2*x**5 + 2*x**4 - 67*x**3/3 + 34*x**2 - 20*x
```

GIAC/XCAS [A] time = 0.260408, size = 39, normalized size = 2.05

$$\frac{1}{6}\,x^6 + 2\,x^5 + 2\,x^4 - \frac{67}{3}\,x^3 + 34\,x^2 - 20\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3 + 6*x^2 - 12*x + 5)*(x^2 + 4*x - 4),x, algorithm="giac")
```

```
[Out] 1/6*x^6 + 2*x^5 + 2*x^4 - 67/3*x^3 + 34*x^2 - 20*x
```

3.220 $\int (2x+x^3) (1+4x^2+x^4) \, dx$

Optimal. Leaf size=16

$$\frac{1}{8}\left(x^4+4x^2+1\right)^2$$

```
[Out] (1 + 4*x^2 + x^4)^2/8
```

Rubi [A] time = 0.0102545, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056

$$\frac{1}{8}\left(x^4+4x^2+1\right)^2$$

Antiderivative was successfully verified.

```
[In]  Int[(2*x + x^3)*(1 + 4*x^2 + x^4),x]

[Out] (1 + 4*x^2 + x^4)^2/8
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int^{x^4+4x^2+1} x\,dx}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**3+2*x)*(x**4+4*x**2+1),x)

[Out] Integral(x, (x, x**4 + 4*x**2 + 1))/4
```

Mathematica [A] time = 0.00227188, size = 21, normalized size = 1.31

$$\frac{x^8}{8}+x^6+\frac{9x^4}{4}+x^2$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2*x + x^3)*(1 + 4*x^2 + x^4),x]

[Out] x^2 + (9*x^4)/4 + x^6 + x^8/8
```

Maple [A] time = 0.001, size = 18, normalized size = 1.1

$$\frac{x^8}{8}+x^6+\frac{9\,x^4}{4}+x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^3+2*x)*(x^4+4*x^2+1),x)
```

```
[Out] 1/8*x^8+x^6+9/4*x^4+x^2
```

Maxima [A] time = 0.841396, size = 19, normalized size = 1.19

$$\frac{1}{8}\left(x^4 + 4x^2 + 1\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 4*x^2 + 1)*(x^3 + 2*x),x, algorithm="maxima")
```

```
[Out] 1/8*(x^4 + 4*x^2 + 1)^2
```

Fricas [A] time = 0.236894, size = 1, normalized size = 0.06

$$\frac{1}{8}x^8 + x^6 + \frac{9}{4}x^4 + x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 4*x^2 + 1)*(x^3 + 2*x),x, algorithm="fricas")
```

```
[Out] 1/8*x^8 + x^6 + 9/4*x^4 + x^2
```

Sympy [A] time = 0.064193, size = 17, normalized size = 1.06

$$\frac{x^8}{8} + x^6 + \frac{9x^4}{4} + x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3+2*x)*(x**4+4*x**2+1),x)
```

```
[Out] x**8/8 + x**6 + 9*x**4/4 + x**2
```

GIAC/XCAS [A] time = 0.258055, size = 23, normalized size = 1.44

$$\frac{1}{8}x^8 + x^6 + \frac{9}{4}x^4 + x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 4*x^2 + 1)*(x^3 + 2*x),x, algorithm="giac")
```

```
[Out] 1/8*x^8 + x^6 + 9/4*x^4 + x^2
```

3.221 $\int (1+2x)\left(x+x^2\right)^3\left(-18+7\left(x+x^2\right)^3\right)^2\,dx$

Optimal. Leaf size=33

$$\frac{49}{10}x^{10}(x+1)^{10}-36x^7(x+1)^7+81x^4(x+1)^4$$

```
[Out] 81*x^4*(1 + x)^4 - 36*x^7*(1 + x)^7 + (49*x^10*(1 + x)^10)/10
```

Rubi [B] time = 0.317868, antiderivative size = 96, normalized size of antiderivative = 2.91, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{49x^{20}}{10}+49x^{19}+\frac{441x^{18}}{2}+588x^{17}+1029x^{16}+\frac{6174x^{15}}{5}+993x^{14}+336x^{13}-\frac{1071x^{12}}{2}-1211x^{11}-\frac{12551x^{10}}{10}-756x^9-171x^8+288x^7+486x^6+324x^5+81x^4$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + 2*x)*(x + x^2)^3*(-18 + 7*(x + x^2)^3)^2,x]
```

```
[Out] 81*x^4 + 324*x^5 + 486*x^6 + 288*x^7 - 171*x^8 - 756*x^9 - (12551
*x^10)/10 - 1211*x^11 - (1071*x^12)/2 + 336*x^13 + 993*x^14 + (61
74*x^15)/5 + 1029*x^16 + 588*x^17 + (441*x^18)/2 + 49*x^19 + (49*
x^20)/10
```

Rubi in Sympy [A] time = 27.5124, size = 94, normalized size = 2.85

$$\frac{49x^{20}}{10}+49x^{19}+\frac{441x^{18}}{2}+588x^{17}+1029x^{16}+\frac{6174x^{15}}{5}+993x^{14}+336x^{13}-\frac{1071x^{12}}{2}-1211x^{11}-\frac{12551x^{10}}{10}-756x^9-171x^8+288x^7+486x^6+324x^5+81x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+2*x)*(x**2+x)**3*(-18+7*(x**2+x)**3)**2,x)
```

```
[Out] 49*x**20/10 + 49*x**19 + 441*x**18/2 + 588*x**17 + 1029*x**16 + 6
174*x**15/5 + 993*x**14 + 336*x**13 - 1071*x**12/2 - 1211*x**11 -
 12551*x**10/10 - 756*x**9 - 171*x**8 + 288*x**7 + 486*x**6 + 324
*x**5 + 81*x**4
```

Mathematica [B] time = 0.00910928, size = 96, normalized size = 2.91

$$\frac{49x^{20}}{10}+49x^{19}+\frac{441x^{18}}{2}+588x^{17}+1029x^{16}+\frac{6174x^{15}}{5}+993x^{14}+336x^{13}-\frac{1071x^{12}}{2}-1211x^{11}-\frac{12551x^{10}}{10}-756x^9-171x^8+288x^7+486x^6+324x^5+81x^4$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + 2*x)*(x + x^2)^3*(-18 + 7*(x + x^2)^3)^2,x]
```

```
[Out] 81*x^4 + 324*x^5 + 486*x^6 + 288*x^7 - 171*x^8 - 756*x^9 - (12551
*x^10)/10 - 1211*x^11 - (1071*x^12)/2 + 336*x^13 + 993*x^14 + (61
```

```
74*x^15)/5 + 1029*x^16 + 588*x^17 + (441*x^18)/2 + 49*x^19 + (49*
x^20)/10
```

Maple [B] time = 0.002, size = 87, normalized size = 2.6

$$\frac{49\,x^{20}}{10}+49\,x^{19}+\frac{441\,x^{18}}{2}+588\,x^{17}+1029\,x^{16}+\frac{6174\,x^{15}}{5}+993\,x^{14}+336\,x^{13}-\frac{1071\,x^{12}}{2}-1211\,x^{11}-\frac{12551\,x^{10}}{10}-756\,x^{9}-171\,x^{8}+288\,x^{7}+486\,x^{6}+324\,x^{5}+81\,x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+2*x)*(x^2+x)^3*(-18+7*(x^2+x)^3)^2,x)
```

```
[Out] 49/10*x^20+49*x^19+441/2*x^18+588*x^17+1029*x^16+6174/5*x^15+993*
x^14+336*x^13-1071/2*x^12-1211*x^11-12551/10*x^10-756*x^9-171*x^8
+288*x^7+486*x^6+324*x^5+81*x^4
```

Maxima [A] time = 0.830847, size = 116, normalized size = 3.52

$$\frac{49}{10}\,x^{20}+49\,x^{19}+\frac{441}{2}\,x^{18}+588\,x^{17}+1029\,x^{16}+\frac{6174}{5}\,x^{15}+993\,x^{14}+336\,x^{13}-\frac{1071}{2}\,x^{12}-1211\,x^{11}-\frac{12551}{10}\,x^{10}-756\,x^{9}-171\,x^{8}+288\,x^{7}+486\,x^{6}+324\,x^{5}+81\,x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((7*(x^2 + x)^3 - 18)^2*(x^2 + x)^3*(2*x + 1),x, algorithm="maxima")
```

```
[Out] 49/10*x^20 + 49*x^19 + 441/2*x^18 + 588*x^17 + 1029*x^16 + 6174/5
*x^15 + 993*x^14 + 336*x^13 - 1071/2*x^12 - 1211*x^11 - 12551/10*
x^10 - 756*x^9 - 171*x^8 + 288*x^7 + 486*x^6 + 324*x^5 + 81*x^4
```

Fricas [A] time = 0.232719, size = 1, normalized size = 0.03

$$\frac{49}{10}x^{20}+49x^{19}+\frac{441}{2}x^{18}+588x^{17}+1029x^{16}+\frac{6174}{5}x^{15}+993x^{14}+336x^{13}-\frac{1071}{2}x^{12}-1211x^{11}-\frac{12551}{10}x^{10}-756x^{9}-171x^{8}+288x^{7}+486x^{6}+324x^{5}+81x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((7*(x^2 + x)^3 - 18)^2*(x^2 + x)^3*(2*x + 1),x, algorithm="fricas")
```

```
[Out] 49/10*x^20 + 49*x^19 + 441/2*x^18 + 588*x^17 + 1029*x^16 + 6174/5
*x^15 + 993*x^14 + 336*x^13 - 1071/2*x^12 - 1211*x^11 - 12551/10*
x^10 - 756*x^9 - 171*x^8 + 288*x^7 + 486*x^6 + 324*x^5 + 81*x^4
```

Sympy [A] time = 0.132587, size = 94, normalized size = 2.85

$$\frac{49x^{20}}{10}+49x^{19}+\frac{441x^{18}}{2}+588x^{17}+1029x^{16}+\frac{6174x^{15}}{5}+993x^{14}+336x^{13}-\frac{1071x^{12}}{2}-1211x^{11}-\frac{12551x^{10}}{10}-756x^{9}-171x^{8}+288x^{7}+486x^{6}+324x^{5}+81x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+2*x)*(x**2+x)**3*(-18+7*(x**2+x)**3)**2,x)
```

```
[Out] 49*x**20/10 + 49*x**19 + 441*x**18/2 + 588*x**17 + 1029*x**16 + 6
174*x**15/5 + 993*x**14 + 336*x**13 - 1071*x**12/2 - 1211*x**11 -
 12551*x**10/10 - 756*x**9 - 171*x**8 + 288*x**7 + 486*x**6 + 324
*x**5 + 81*x**4
```

GIAC/XCAS [A] time = 0.257468, size = 116, normalized size = 3.52

$$\frac{49}{10}\,x^{20} + 49\,x^{19} + \frac{441}{2}\,x^{18} + 588\,x^{17} + 1029\,x^{16} + \frac{6174}{5}\,x^{15} + 993\,x^{14} + 336\,x^{13} - \frac{1071}{2}\,x^{12} - 1211\,x^{11} - \frac{12551}{10}\,x^{10} - 756\,x^{9} - 171\,x^{8} + 288\,x^{7} + 486\,x^{6} + 324\,x^{5} + 81\,x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((7*(x^2 + x)^3 - 18)^2*(x^2 + x)^3*(2*x + 1),x, algorithm="giac")
```

```
[Out] 49/10*x^20 + 49*x^19 + 441/2*x^18 + 588*x^17 + 1029*x^16 + 6174/5
*x^15 + 993*x^14 + 336*x^13 - 1071/2*x^12 - 1211*x^11 - 12551/10*
x^10 - 756*x^9 - 171*x^8 + 288*x^7 + 486*x^6 + 324*x^5 + 81*x^4
```

3.222 $\int x^3(1+x)^3(1+2x)\left(-18+7x^3(1+x)^3\right)^2\,dx$

Optimal. Leaf size=33

$$\frac{49}{10}x^{10}(x+1)^{10}-36x^7(x+1)^7+81x^4(x+1)^4$$

```
[Out] 81*x^4*(1 + x)^4 - 36*x^7*(1 + x)^7 + (49*x^10*(1 + x)^10)/10
```

Rubi [B] time = 0.266328, antiderivative size = 96, normalized size of antiderivative = 2.91, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}=0.036$

$$\frac{49x^{20}}{10}+49x^{19}+\frac{441x^{18}}{2}+588x^{17}+1029x^{16}+\frac{6174x^{15}}{5}+993x^{14}+336x^{13}-\frac{1071x^{12}}{2}-1211x^{11}-\frac{12551x^{10}}{10}-756x^9-171x^8+288x^7+486x^6+324x^5+81x^4$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*(1 + x)^3*(1 + 2*x)*(-18 + 7*x^3*(1 + x)^3)^2,x]
```

```
[Out] 81*x^4 + 324*x^5 + 486*x^6 + 288*x^7 - 171*x^8 - 756*x^9 - (12551
*x^10)/10 - 1211*x^11 - (1071*x^12)/2 + 336*x^13 + 993*x^14 + (61
74*x^15)/5 + 1029*x^16 + 588*x^17 + (441*x^18)/2 + 49*x^19 + (49*
x^20)/10
```

Rubi in Sympy [A] time = 27.9094, size = 94, normalized size = 2.85

$$\frac{49x^{20}}{10}+49x^{19}+\frac{441x^{18}}{2}+588x^{17}+1029x^{16}+\frac{6174x^{15}}{5}+993x^{14}+336x^{13}-\frac{1071x^{12}}{2}-1211x^{11}-\frac{12551x^{10}}{10}-756x^9-171x^8+288x^7+486x^6+324x^5+81x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(1+x)**3*(1+2*x)*(-18+7*x**3*(1+x)**3)**2,x)
```

```
[Out] 49*x**20/10 + 49*x**19 + 441*x**18/2 + 588*x**17 + 1029*x**16 + 6
174*x**15/5 + 993*x**14 + 336*x**13 - 1071*x**12/2 - 1211*x**11 -
 12551*x**10/10 - 756*x**9 - 171*x**8 + 288*x**7 + 486*x**6 + 324
*x**5 + 81*x**4
```

Mathematica [B] time = 0.0078719, size = 96, normalized size = 2.91

$$\frac{49x^{20}}{10}+49x^{19}+\frac{441x^{18}}{2}+588x^{17}+1029x^{16}+\frac{6174x^{15}}{5}+993x^{14}+336x^{13}-\frac{1071x^{12}}{2}-1211x^{11}-\frac{12551x^{10}}{10}-756x^9-171x^8+288x^7+486x^6+324x^5+81x^4$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*(1 + x)^3*(1 + 2*x)*(-18 + 7*x^3*(1 + x)^3)^2,x]
```

```
[Out] 81*x^4 + 324*x^5 + 486*x^6 + 288*x^7 - 171*x^8 - 756*x^9 - (12551
*x^10)/10 - 1211*x^11 - (1071*x^12)/2 + 336*x^13 + 993*x^14 + (61
74*x^15)/5 + 1029*x^16 + 588*x^17 + (441*x^18)/2 + 49*x^19 + (49*
```

```
x^20)/10
```

Maple [B] time = 0.002, size = 87, normalized size = 2.6

$$\frac{49 x^{20}}{10}+49 x^{19}+\frac{441 x^{18}}{2}+588 x^{17}+1029 x^{16}+\frac{6174 x^{15}}{5}+993 x^{14}+336 x^{13}-\frac{1071 x^{12}}{2}-1211 x^{11}-\frac{12551 x^{10}}{10}-756 x^{9}-171 x^{8}+288 x^{7}+486 x^{6}+324 x^{5}+81 x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(1+x)^3*(1+2*x)*(-18+7*x^3*(1+x)^3)^2,x)
```

```
[Out] 49/10*x^20+49*x^19+441/2*x^18+588*x^17+1029*x^16+6174/5*x^15+993*
x^14+336*x^13-1071/2*x^12-1211*x^11-12551/10*x^10-756*x^9-171*x^8
+288*x^7+486*x^6+324*x^5+81*x^4
```

Maxima [A] time = 0.896517, size = 116, normalized size = 3.52

$$\frac{49}{10} x^{20}+49 x^{19}+\frac{441}{2} x^{18}+588 x^{17}+1029 x^{16}+\frac{6174}{5} x^{15}+993 x^{14}+336 x^{13}-\frac{1071}{2} x^{12}-1211 x^{11}-\frac{12551}{10} x^{10}-756 x^{9}-171 x^{8}+288 x^{7}+486 x^{6}+324 x^{5}+81 x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((7*(x + 1)^3*x^3 - 18)^2*(2*x + 1)*(x + 1)^3*x^3,x, algorithm="maxima")
```

```
[Out] 49/10*x^20 + 49*x^19 + 441/2*x^18 + 588*x^17 + 1029*x^16 + 6174/5
*x^15 + 993*x^14 + 336*x^13 - 1071/2*x^12 - 1211*x^11 - 12551/10*
x^10 - 756*x^9 - 171*x^8 + 288*x^7 + 486*x^6 + 324*x^5 + 81*x^4
```

Fricas [A] time = 0.230773, size = 1, normalized size = 0.03

$$\frac{49}{10}x^{20}+49x^{19}+\frac{441}{2}x^{18}+588x^{17}+1029x^{16}+\frac{6174}{5}x^{15}+993x^{14}+336x^{13}-\frac{1071}{2}x^{12}-1211x^{11}-\frac{12551}{10}x^{10}-756x^{9}-171x^{8}+288x^{7}+486x^{6}+324x^{5}+81x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((7*(x + 1)^3*x^3 - 18)^2*(2*x + 1)*(x + 1)^3*x^3,x, algorithm="fricas")
```

```
[Out] 49/10*x^20 + 49*x^19 + 441/2*x^18 + 588*x^17 + 1029*x^16 + 6174/5
*x^15 + 993*x^14 + 336*x^13 - 1071/2*x^12 - 1211*x^11 - 12551/10*
x^10 - 756*x^9 - 171*x^8 + 288*x^7 + 486*x^6 + 324*x^5 + 81*x^4
```

Sympy [A] time = 0.158426, size = 94, normalized size = 2.85

$$\frac{49x^{20}}{10}+49x^{19}+\frac{441x^{18}}{2}+588x^{17}+1029x^{16}+\frac{6174x^{15}}{5}+993x^{14}+336x^{13}-\frac{1071x^{12}}{2}-1211x^{11}-\frac{12551x^{10}}{10}-756x^{9}-171x^{8}+288x^{7}+486x^{6}+324x^{5}+81x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(1+x)**3*(1+2*x)*(-18+7*x**3*(1+x)**3)**2,x)
```

```
[Out] 49*x**20/10 + 49*x**19 + 441*x**18/2 + 588*x**17 + 1029*x**16 + 6
174*x**15/5 + 993*x**14 + 336*x**13 - 1071*x**12/2 - 1211*x**11 -
 12551*x**10/10 - 756*x**9 - 171*x**8 + 288*x**7 + 486*x**6 + 324
*x**5 + 81*x**4
```

GIAC/XCAS [A] time = 0.259429, size = 116, normalized size = 3.52

$$\frac{49}{10}\,x^{20} + 49\,x^{19} + \frac{441}{2}\,x^{18} + 588\,x^{17} + 1029\,x^{16} + \frac{6174}{5}\,x^{15} + 993\,x^{14} + 336\,x^{13} - \frac{1071}{2}\,x^{12} - 1211\,x^{11} - \frac{12551}{10}\,x^{10} - 756\,x^{9} - 171\,x^{8} + 288\,x^{7} + 486\,x^{6} + 324\,x^{5} + 81\,x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((7*(x + 1)^3*x^3 - 18)^2*(2*x + 1)*(x + 1)^3*x^3,x, algorithm="giac")
```

```
[Out] 49/10*x^20 + 49*x^19 + 441/2*x^18 + 588*x^17 + 1029*x^16 + 6174/5
*x^15 + 993*x^14 + 336*x^13 - 1071/2*x^12 - 1211*x^11 - 12551/10*
x^10 - 756*x^9 - 171*x^8 + 288*x^7 + 486*x^6 + 324*x^5 + 81*x^4
```

3.223 $\int \frac{2-x^2}{(1-6x+x^3)^5}\,dx$

Optimal. Leaf size=14

$$\frac{1}{12(x^3-6x+1)^4}$$

```
[Out] 1/(12*(1 - 6*x + x^3)^4)
```

Rubi [A] time = 0.00863122, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$

$$\frac{1}{12(x^3-6x+1)^4}$$

Antiderivative was successfully verified.

```
[In]  Int[(2 - x^2)/(1 - 6*x + x^3)^5,x]

[Out] 1/(12*(1 - 6*x + x^3)^4)
```

Rubi in Sympy [A] time = 2.93853, size = 12, normalized size = 0.86

$$\frac{1}{12(x^3-6x+1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2+2)/(x**3-6*x+1)**5,x)

[Out] 1/(12*(x**3 - 6*x + 1)**4)
```

Mathematica [A] time = 0.00842163, size = 14, normalized size = 1.

$$\frac{1}{12(x^3-6x+1)^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 - x^2)/(1 - 6*x + x^3)^5,x]

[Out] 1/(12*(1 - 6*x + x^3)^4)
```

Maple [A] time = 0.002, size = 13, normalized size = 0.9

$$\frac{1}{12(x^3-6x+1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2+2)/(x^3-6*x+1)^5,x)
```

```
[Out] 1/12/(x^3-6*x+1)^4
```

Maxima [A] time = 0.810874, size = 16, normalized size = 1.14

$$\frac{1}{12\,(x^3-6\,x+1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 2)/(x^3 - 6*x + 1)^5,x, algorithm="maxima")
```

```
[Out] 1/12/(x^3 - 6*x + 1)^4
```

Fricas [A] time = 0.244749, size = 77, normalized size = 5.5

$$\frac{1}{12\,(x^{12}-24\,x^{10}+4\,x^9+216\,x^8-72\,x^7-858\,x^6+432\,x^5+1224\,x^4-860\,x^3+216\,x^2-24\,x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 2)/(x^3 - 6*x + 1)^5,x, algorithm="fricas")
```

```
[Out] 1/12/(x^12 - 24*x^10 + 4*x^9 + 216*x^8 - 72*x^7 - 858*x^6 + 432*x
^5 + 1224*x^4 - 860*x^3 + 216*x^2 - 24*x + 1)
```

Sympy [A] time = 0.666687, size = 56, normalized size = 4.

$$\frac{1}{12x^{12}-288x^{10}+48x^9+2592x^8-864x^7-10296x^6+5184x^5+14688x^4-10320x^3+2592x^2-288x+12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+2)/(x**3-6*x+1)**5,x)
```

```
[Out] 1/(12*x**12 - 288*x**10 + 48*x**9 + 2592*x**8 - 864*x**7 - 10296*
x**6 + 5184*x**5 + 14688*x**4 - 10320*x**3 + 2592*x**2 - 288*x +
12)
```

GIAC/XCAS [A] time = 0.259587, size = 16, normalized size = 1.14

$$\frac{1}{12\,(x^3-6\,x+1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 2)/(x^3 - 6*x + 1)^5,x, algorithm="giac")
```

```
[Out] 1/12/(x^3 - 6*x + 1)^4
```

3.224 $\int \frac{2x+x^2}{4+3x^2+x^3}\,dx$

Optimal. Leaf size=15

$$\frac{1}{3}\log\left(x^3+3x^2+4\right)$$

```
[Out] Log[4 + 3*x^2 + x^3]/3
```

Rubi [A] time = 0.00958381, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05

$$\frac{1}{3}\log\left(x^3+3x^2+4\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(2*x + x^2)/(4 + 3*x^2 + x^3),x]

[Out] Log[4 + 3*x^2 + x^3]/3
```

Rubi in Sympy [A] time = 3.95154, size = 12, normalized size = 0.8

$$\frac{\log\left(x^3+3x^2+4\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+2*x)/(x**3+3*x**2+4),x)

[Out] log(x**3 + 3*x**2 + 4)/3
```

Mathematica [A] time = 0.00710682, size = 15, normalized size = 1.

$$\frac{1}{3}\log\left(x^3+3x^2+4\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2*x + x^2)/(4 + 3*x^2 + x^3),x]

[Out] Log[4 + 3*x^2 + x^3]/3
```

Maple [A] time = 0.002, size = 14, normalized size = 0.9

$$\frac{\ln\left(x^3+3\,x^2+4\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+2*x)/(x^3+3*x^2+4),x)
```

```
[Out] 1/3*ln(x^3+3*x^2+4)
```

Maxima [A] time = 0.848944, size = 18, normalized size = 1.2

$$\frac{1}{3}\log\left(x^3+3x^2+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 2*x)/(x^3 + 3*x^2 + 4),x, algorithm="maxima")
```

```
[Out] 1/3*log(x^3 + 3*x^2 + 4)
```

Fricas [A] time = 0.250565, size = 18, normalized size = 1.2

$$\frac{1}{3}\log\left(x^3+3x^2+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 2*x)/(x^3 + 3*x^2 + 4),x, algorithm="fricas")
```

```
[Out] 1/3*log(x^3 + 3*x^2 + 4)
```

Sympy [A] time = 0.157732, size = 12, normalized size = 0.8

$$\frac{\log\left(x^3+3x^2+4\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+2*x)/(x**3+3*x**2+4),x)
```

```
[Out] log(x**3 + 3*x**2 + 4)/3
```

GIAC/XCAS [A] time = 0.2608, size = 19, normalized size = 1.27

$$\frac{1}{3}\ln\left(\left|x^3+3x^2+4\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 2*x)/(x^3 + 3*x^2 + 4),x, algorithm="giac")
```

```
[Out] 1/3*ln(abs(x^3 + 3*x^2 + 4))
```

3.225 $\int \frac{1+x+x^3}{4x+2x^2+x^4}\,dx$

Optimal. Leaf size=17

$$\frac{1}{4}\log\left(x^4+2x^2+4x\right)$$

```
[Out] Log[4*x + 2*x^2 + x^4]/4
```

Rubi [A] time = 0.0101633, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048

$$\frac{1}{4}\log\left(x^4+2x^2+4x\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x + x^3)/(4*x + 2*x^2 + x^4),x]

[Out] Log[4*x + 2*x^2 + x^4]/4
```

Rubi in Sympy [A] time = 14.9665, size = 12, normalized size = 0.71

$$\frac{\log\left(x\left(x^3+2x+4\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**3+x+1)/(x**4+2*x**2+4*x),x)

[Out] log(x*(x**3 + 2*x + 4))/4
```

Mathematica [A] time = 0.0086725, size = 20, normalized size = 1.18

$$\frac{1}{4}\log\left(x^3+2x+4\right)+\frac{\log(x)}{4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x + x^3)/(4*x + 2*x^2 + x^4),x]

[Out] Log[x]/4 + Log[4 + 2*x + x^3]/4
```

Maple [A] time = 0.002, size = 14, normalized size = 0.8

$$\frac{\ln\left(x\left(x^3+2\,x+4\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^3+x+1)/(x^4+2*x^2+4*x),x)
```

```
[Out] 1/4*ln(x*(x^3+2*x+4))
```

Maxima [A] time = 0.822876, size = 20, normalized size = 1.18

$$\frac{1}{4}\log\left(x^4 + 2x^2 + 4x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + x + 1)/(x^4 + 2*x^2 + 4*x),x, algorithm="maxima")
```

```
[Out] 1/4*log(x^4 + 2*x^2 + 4*x)
```

Fricas [A] time = 0.246695, size = 20, normalized size = 1.18

$$\frac{1}{4}\log\left(x^4 + 2x^2 + 4x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + x + 1)/(x^4 + 2*x^2 + 4*x),x, algorithm="fricas")
```

```
[Out] 1/4*log(x^4 + 2*x^2 + 4*x)
```

Sympy [A] time = 0.201967, size = 14, normalized size = 0.82

$$\frac{\log\left(x^4 + 2x^2 + 4x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3+x+1)/(x**4+2*x**2+4*x),x)
```

```
[Out] log(x**4 + 2*x**2 + 4*x)/4
```

GIAC/XCAS [A] time = 0.263046, size = 24, normalized size = 1.41

$$\frac{1}{4}\ln\left(\left|x^3 + 2x + 4\right|\right) + \frac{1}{4}\ln\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + x + 1)/(x^4 + 2*x^2 + 4*x),x, algorithm="giac")
```

```
[Out] 1/4*ln(abs(x^3 + 2*x + 4)) + 1/4*ln(abs(x))
```

3.226 $\int \frac{bc-ad-2aex-bex^2-3afx^2-2bfx^3}{(c+dx+ex^2+fx^3)^2}\,dx$

Optimal. Leaf size=40

$$\frac{a}{c+dx+ex^2+fx^3}+\frac{bx}{c+dx+ex^2+fx^3}$$

```
[Out] a/(c + d*x + e*x^2 + f*x^3) + (b*x)/(c + d*x + e*x^2 + f*x^3)
```

Rubi [A] time = 0.142924, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 52, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.058

$$\frac{a}{c+dx+ex^2+fx^3}+\frac{bx}{c+dx+ex^2+fx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*c - a*d - 2*a*e*x - b*e*x^2 - 3*a*f*x^2 - 2*b*f*x^3)/(c + d*x + e*x^2 + f*x^
```

```
[Out] a/(c + d*x + e*x^2 + f*x^3) + (b*x)/(c + d*x + e*x^2 + f*x^3)
```

Rubi in Sympy [A] time = 64.6483, size = 34, normalized size = 0.85

$$\frac{a}{c+dx+ex^2+fx^3}+\frac{bx}{c+dx+ex^2+fx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-2*b*f*x**3-3*a*f*x**2-b*e*x**2-2*a*e*x-a*d+b*c)/(f*x**3+e*x**
```

```
[Out] a/(c + d*x + e*x**2 + f*x**3) + b*x/(c + d*x + e*x**2 + f*x**3)
```

Mathematica [A] time = 0.100928, size = 23, normalized size = 0.57

$$\frac{a+bx}{c+dx+ex^2+fx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*c - a*d - 2*a*e*x - b*e*x^2 - 3*a*f*x^2 - 2*b*f*x^3)/(c + d*x + e*x^2
```

```
[Out] (a + b*x)/(c + d*x + e*x^2 + f*x^3)
```

Maple [A] time = 0.014, size = 28, normalized size = 0.7

$$-\frac{-bx-a}{fx^3+ex^2+dx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*b*f*x^3-3*a*f*x^2-b*e*x^2-2*a*e*x-a*d+b*c)/(f*x^3+e*x^2+d*x+c)^2,x)
```

```
[Out] -(-b*x-a)/(f*x^3+e*x^2+d*x+c)
```

Maxima [A] time = 0.842004, size = 31, normalized size = 0.78

$$\frac{bx + a}{fx^3 + ex^2 + dx + c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(2*b*f*x^3 + b*e*x^2 + 3*a*f*x^2 + 2*a*e*x - b*c + a*d)/(f*x^3 + e*x^2 +
```

```
[Out] (b*x + a)/(f*x^3 + e*x^2 + d*x + c)
```

Fricas [A] time = 0.253686, size = 31, normalized size = 0.78

$$\frac{bx + a}{fx^3 + ex^2 + dx + c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(2*b*f*x^3 + b*e*x^2 + 3*a*f*x^2 + 2*a*e*x - b*c + a*d)/(f*x^3 + e*x^2 +
```

```
[Out] (b*x + a)/(f*x^3 + e*x^2 + d*x + c)
```

Sympy [A] time = 111.918, size = 19, normalized size = 0.48

$$\frac{a + bx}{c + dx + ex^2 + fx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*b*f*x**3-3*a*f*x**2-b*e*x**2-2*a*e*x-a*d+b*c)/(f*x**3+e*x**2+d*x
```

```
[Out] (a + b*x)/(c + d*x + e*x**2 + f*x**3)
```

GIAC/XCAS [A] time = 0.262876, size = 32, normalized size = 0.8

$$\frac{bx + a}{fx^3 + x^2e + dx + c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(2*b*f*x^3 + b*e*x^2 + 3*a*f*x^2 + 2*a*e*x - b*c + a*d)/(f*x^3 + e*x^2 +
```

```
[Out] (b*x + a)/(f*x^3 + x^2*e + d*x + c)
```

3.227 $\int \frac{A+Bx+Cx^2+Dx^3}{a+bx+cx^2+bx^3+ax^4} \, dx$

Optimal. Leaf size=605

$$\frac{\tan^{-1}\left(\frac{-\sqrt{8a^2-4ac+b^2}+4ax+b}{\sqrt{2}\sqrt{-b\left(b-\sqrt{8a^2-4ac+b^2}\right)+4a^2+2ac}}\right)\left(-a\left(A\left(b-\sqrt{8a^2-4ac+b^2}\right)-C\sqrt{8a^2-4ac+b^2}+bC+2cD\right)+bD\left(b-\sqrt{8a^2-4\cdots}\right)\right.}{\sqrt{2}a\sqrt{8a^2-4ac+b^2}\sqrt{-b\left(b-\sqrt{8a^2-4ac+b^2}\right)+4a^2+2ac}}$$

$$-\frac{\tan^{-1}\left(\frac{\sqrt{8a^2-4ac+b^2}+4ax+b}{\sqrt{2}\sqrt{-b\left(\sqrt{8a^2-4ac+b^2}+b\right)+4a^2+2ac}}\right)\left(-a\left(A\left(\sqrt{8a^2-4ac+b^2}+b\right)+C\sqrt{8a^2-4ac+b^2}+bC+2cD\right)+bD\left(\sqrt{8a^2-4ac\cdots}\right)\right.}{\sqrt{2}a\sqrt{8a^2-4ac+b^2}\sqrt{-b\left(\sqrt{8a^2-4ac+b^2}+b\right)+4a^2+2ac}}$$

$$-\frac{\log\left(x\left(b-\sqrt{8a^2-4ac+b^2}\right)+2ax^2+2a\right)\left(D\left(b-\sqrt{8a^2-4ac+b^2}\right)+2a(A-C)\right)}{4a\sqrt{8a^2-4ac+b^2}}$$

$$+\frac{\log\left(x\left(\sqrt{8a^2-4ac+b^2}+b\right)+2ax^2+2a\right)\left(D\left(\sqrt{8a^2-4ac+b^2}+b\right)+2a(A-C)\right)}{4a\sqrt{8a^2-4ac+b^2}}$$

```
[Out] ((4*a^2*B + b*(b - Sqrt[8*a^2 + b^2 - 4*a*c])*D - a*(A*(b - Sqrt[
8*a^2 + b^2 - 4*a*c]) + b*C - Sqrt[8*a^2 + b^2 - 4*a*c]*C + 2*c*D
))*ArcTan[(b - Sqrt[8*a^2 + b^2 - 4*a*c] + 4*a*x)/(Sqrt[2]*Sqrt[4
*a^2 + 2*a*c - b*(b - Sqrt[8*a^2 + b^2 - 4*a*c])])])/(Sqrt[2]*a*S
qrt[8*a^2 + b^2 - 4*a*c]*Sqrt[4*a^2 + 2*a*c - b*(b - Sqrt[8*a^2 +
 b^2 - 4*a*c])]) - ((4*a^2*B + b*(b + Sqrt[8*a^2 + b^2 - 4*a*c])*
D - a*(A*(b + Sqrt[8*a^2 + b^2 - 4*a*c]) + b*C + Sqrt[8*a^2 + b^2
 - 4*a*c]*C + 2*c*D))*ArcTan[(b + Sqrt[8*a^2 + b^2 - 4*a*c] + 4*a
*x)/(Sqrt[2]*Sqrt[4*a^2 + 2*a*c - b*(b + Sqrt[8*a^2 + b^2 - 4*a*c
])])])/(Sqrt[2]*a*Sqrt[8*a^2 + b^2 - 4*a*c]*Sqrt[4*a^2 + 2*a*c -
b*(b + Sqrt[8*a^2 + b^2 - 4*a*c])]) - ((2*a*(A - C) + (b - Sqrt[8
*a^2 + b^2 - 4*a*c])*D)*Log[2*a + (b - Sqrt[8*a^2 + b^2 - 4*a*c])
*x + 2*a*x^2])/(4*a*Sqrt[8*a^2 + b^2 - 4*a*c]) + ((2*a*(A - C) +
(b + Sqrt[8*a^2 + b^2 - 4*a*c])*D)*Log[2*a + (b + Sqrt[8*a^2 + b^
2 - 4*a*c])*x + 2*a*x^2])/(4*a*Sqrt[8*a^2 + b^2 - 4*a*c])
```

Rubi [A] time = 10.6993, antiderivative size = 605, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$

$$\frac{\tan^{-1}\left(\frac{-\sqrt{8a^2-4ac+b^2}+4ax+b}{\sqrt{2}\sqrt{-b\left(b-\sqrt{8a^2-4ac+b^2}\right)+4a^2+2ac}}\right)\left(-a\left(A\left(b-\sqrt{8a^2-4ac+b^2}\right)-C\sqrt{8a^2-4ac+b^2}+bC+2cD\right)+bD\left(b-\sqrt{8a^2-4\cdots}\right)\right.}{\sqrt{2}a\sqrt{8a^2-4ac+b^2}\sqrt{-b\left(b-\sqrt{8a^2-4ac+b^2}\right)+4a^2+2ac}}$$

$$-\frac{\tan^{-1}\left(\frac{\sqrt{8a^2-4ac+b^2}+4ax+b}{\sqrt{2}\sqrt{-b\left(\sqrt{8a^2-4ac+b^2}+b\right)+4a^2+2ac}}\right)\left(-a\left(A\left(\sqrt{8a^2-4ac+b^2}+b\right)+C\sqrt{8a^2-4ac+b^2}+bC+2cD\right)+bD\left(\sqrt{8a^2-4ac\cdots}\right)\right.}{\sqrt{2}a\sqrt{8a^2-4ac+b^2}\sqrt{-b\left(\sqrt{8a^2-4ac+b^2}+b\right)+4a^2+2ac}}$$

$$-\frac{\log\left(x\left(b-\sqrt{8a^2-4ac+b^2}\right)+2ax^2+2a\right)\left(D\left(b-\sqrt{8a^2-4ac+b^2}\right)+2a(A-C)\right)}{4a\sqrt{8a^2-4ac+b^2}}$$

$$+\frac{\log\left(x\left(\sqrt{8a^2-4ac+b^2}+b\right)+2ax^2+2a\right)\left(D\left(\sqrt{8a^2-4ac+b^2}+b\right)+2a(A-C)\right)}{4a\sqrt{8a^2-4ac+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2 + D*x^3)/(a + b*x + c*x^2 + b*x^3 + a*x^4),x]
```

```
[Out] ((4*a^2*B + b*(b - Sqrt[8*a^2 + b^2 - 4*a*c])*D - a*(A*(b - Sqrt[
8*a^2 + b^2 - 4*a*c]) + b*C - Sqrt[8*a^2 + b^2 - 4*a*c]*C + 2*c*D
))*ArcTan[(b - Sqrt[8*a^2 + b^2 - 4*a*c] + 4*a*x)/(Sqrt[2]*Sqrt[4
*a^2 + 2*a*c - b*(b - Sqrt[8*a^2 + b^2 - 4*a*c])])])/(Sqrt[2]*a*S
qrt[8*a^2 + b^2 - 4*a*c]*Sqrt[4*a^2 + 2*a*c - b*(b - Sqrt[8*a^2 +
 b^2 - 4*a*c])]) - ((4*a^2*B + b*(b + Sqrt[8*a^2 + b^2 - 4*a*c])*
D - a*(A*(b + Sqrt[8*a^2 + b^2 - 4*a*c]) + b*C + Sqrt[8*a^2 + b^2
 - 4*a*c]*C + 2*c*D))*ArcTan[(b + Sqrt[8*a^2 + b^2 - 4*a*c] + 4*a
*x)/(Sqrt[2]*Sqrt[4*a^2 + 2*a*c - b*(b + Sqrt[8*a^2 + b^2 - 4*a*c
])])])/(Sqrt[2]*a*Sqrt[8*a^2 + b^2 - 4*a*c]*Sqrt[4*a^2 + 2*a*c -
b*(b + Sqrt[8*a^2 + b^2 - 4*a*c])]) - ((2*a*(A - C) + (b - Sqrt[8
*a^2 + b^2 - 4*a*c])*D)*Log[2*a + (b - Sqrt[8*a^2 + b^2 - 4*a*c])
*x + 2*a*x^2])/(4*a*Sqrt[8*a^2 + b^2 - 4*a*c]) + ((2*a*(A - C) +
(b + Sqrt[8*a^2 + b^2 - 4*a*c])*D)*Log[2*a + (b + Sqrt[8*a^2 + b^
2 - 4*a*c])*x + 2*a*x^2])/(4*a*Sqrt[8*a^2 + b^2 - 4*a*c])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((D*x**3+C*x**2+B*x+A)/(a*x**4+b*x**3+c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.117919, size = 98, normalized size = 0.16

$$\text{RootSum}\left[\#1^4 a + \#1^3 b + \#1^2 c + \#1 b + a\&, \frac{\#1^3 D\log(x-\#1) + \#1^2 C\log(x-\#1) + A\log(x-\#1) + \#1 B\log(x-\#1)}{4\#1^3 a + 3\#1^2 b + 2\#1 c + b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2 + D*x^3)/(a + b*x + c*x^2 + b*x^3 + a*x^4),x]
```

```
[Out] RootSum[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , (A*Log[x - #1] +
B*Log[x - #1]*#1 + C*Log[x - #1]*#1^2 + D*Log[x - #1]*#1^3)/(b +
2*c*#1 + 3*b*#1^2 + 4*a*#1^3) & ]
```

Maple [B] time = 0.071, size = 2105, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^3+C*x^2+B*x+A)/(a*x^4+b*x^3+c*x^2+b*x+a),x)
```

```
[Out] 1/2/(8*a^2-4*a*c+b^2)^(1/2)*ln(2*a*x^2+(8*a^2-4*a*c+b^2)^(1/2)*x+
b*x+2*a)*A-1/2/(8*a^2-4*a*c+b^2)^(1/2)*ln(2*a*x^2+(8*a^2-4*a*c+b^
2)^(1/2)*x+b*x+2*a)*C+1/4/a*ln(2*a*x^2+(8*a^2-4*a*c+b^2)^(1/2)*x+
b*x+2*a)*D+1/4/a/(8*a^2-4*a*c+b^2)^(1/2)*ln(2*a*x^2+(8*a^2-4*a*c+
b^2)^(1/2)*x+b*x+2*a)*D*b+1/(8*a^2+4*a*c-2*b^2-2*b*(8*a^2-4*a*c+b
^2)^(1/2))^(1/2)*arctan((b+4*a*x+(8*a^2-4*a*c+b^2)^(1/2))/(8*a^2+
4*a*c-2*b^2-2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2))*A+1/(8*a^2-4*a*c+
b^2)^(1/2)/(8*a^2+4*a*c-2*b^2-2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2)*
```

```
arctan((b+4*a*x+(8*a^2-4*a*c+b^2)^(1/2))/(8*a^2+4*a*c-2*b^2-2*b*(
8*a^2-4*a*c+b^2)^(1/2))^(1/2))*A*b+1/(8*a^2+4*a*c-2*b^2-2*b*(8*a^
2-4*a*c+b^2)^(1/2))^(1/2)*arctan((b+4*a*x+(8*a^2-4*a*c+b^2)^(1/2)
)/(8*a^2+4*a*c-2*b^2-2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2))*C+1/(8*a
^2-4*a*c+b^2)^(1/2)/(8*a^2+4*a*c-2*b^2-2*b*(8*a^2-4*a*c+b^2)^(1/2
))^(1/2)*arctan((b+4*a*x+(8*a^2-4*a*c+b^2)^(1/2))/(8*a^2+4*a*c-2*
b^2-2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2))*b*C+2/(8*a^2-4*a*c+b^2)^(
1/2)/(8*a^2+4*a*c-2*b^2-2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2)*arctan
((b+4*a*x+(8*a^2-4*a*c+b^2)^(1/2))/(8*a^2+4*a*c-2*b^2-2*b*(8*a^2-
4*a*c+b^2)^(1/2))^(1/2))*D*c-1/a/(8*a^2-4*a*c+b^2)^(1/2)/(8*a^2+4
*a*c-2*b^2-2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2)*arctan((b+4*a*x+(8*
a^2-4*a*c+b^2)^(1/2))/(8*a^2+4*a*c-2*b^2-2*b*(8*a^2-4*a*c+b^2)^(1
/2))^(1/2))*D*b^2-1/a/(8*a^2+4*a*c-2*b^2-2*b*(8*a^2-4*a*c+b^2)^(1
/2))^(1/2)*arctan((b+4*a*x+(8*a^2-4*a*c+b^2)^(1/2))/(8*a^2+4*a*c-
2*b^2-2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2))*D*b-4*a/(8*a^2-4*a*c+b^
2)^(1/2)/(8*a^2+4*a*c-2*b^2-2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2)*ar
ctan((b+4*a*x+(8*a^2-4*a*c+b^2)^(1/2))/(8*a^2+4*a*c-2*b^2-2*b*(8*
a^2-4*a*c+b^2)^(1/2))^(1/2))*B-1/2/(8*a^2-4*a*c+b^2)^(1/2)*ln(-2*
a*x^2+(8*a^2-4*a*c+b^2)^(1/2)*x-b*x-2*a)*A+1/2/(8*a^2-4*a*c+b^2)^
(1/2)*ln(-2*a*x^2+(8*a^2-4*a*c+b^2)^(1/2)*x-b*x-2*a)*C+1/4/a*ln(-
2*a*x^2+(8*a^2-4*a*c+b^2)^(1/2)*x-b*x-2*a)*D-1/4/a/(8*a^2-4*a*c+b
^2)^(1/2)*ln(-2*a*x^2+(8*a^2-4*a*c+b^2)^(1/2)*x-b*x-2*a)*D*b-1/(8
*a^2+4*a*c-2*b^2+2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2)*arctan((-4*a*
x+(8*a^2-4*a*c+b^2)^(1/2)-b)/(8*a^2+4*a*c-2*b^2+2*b*(8*a^2-4*a*c+
b^2)^(1/2))^(1/2))*A+1/(8*a^2-4*a*c+b^2)^(1/2)/(8*a^2+4*a*c-2*b^2
+2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2)*arctan((-4*a*x+(8*a^2-4*a*c+b
^2)^(1/2)-b)/(8*a^2+4*a*c-2*b^2+2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2
))*A*b-1/(8*a^2+4*a*c-2*b^2+2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2)*ar
ctan((-4*a*x+(8*a^2-4*a*c+b^2)^(1/2)-b)/(8*a^2+4*a*c-2*b^2+2*b*(8
*a^2-4*a*c+b^2)^(1/2))^(1/2))*C+1/(8*a^2-4*a*c+b^2)^(1/2)/(8*a^2+
4*a*c-2*b^2+2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2)*arctan((-4*a*x+(8*
a^2-4*a*c+b^2)^(1/2)-b)/(8*a^2+4*a*c-2*b^2+2*b*(8*a^2-4*a*c+b^2)^
(1/2))^(1/2))*b*C+2/(8*a^2-4*a*c+b^2)^(1/2)/(8*a^2+4*a*c-2*b^2+2*
b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2)*arctan((-4*a*x+(8*a^2-4*a*c+b^2)
^(1/2)-b)/(8*a^2+4*a*c-2*b^2+2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2))*
D*c-1/a/(8*a^2-4*a*c+b^2)^(1/2)/(8*a^2+4*a*c-2*b^2+2*b*(8*a^2-4*a
*c+b^2)^(1/2))^(1/2)*arctan((-4*a*x+(8*a^2-4*a*c+b^2)^(1/2)-b)/(8
*a^2+4*a*c-2*b^2+2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2))*D*b^2+1/a/(8
*a^2+4*a*c-2*b^2+2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2)*arctan((-4*a*
x+(8*a^2-4*a*c+b^2)^(1/2)-b)/(8*a^2+4*a*c-2*b^2+2*b*(8*a^2-4*a*c+
b^2)^(1/2))^(1/2))*D*b-4*a/(8*a^2-4*a*c+b^2)^(1/2)/(8*a^2+4*a*c-2
*b^2+2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2)*arctan((-4*a*x+(8*a^2-4*a
*c+b^2)^(1/2)-b)/(8*a^2+4*a*c-2*b^2+2*b*(8*a^2-4*a*c+b^2)^(1/2))^
(1/2))*B
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Dx^3 + Cx^2 + Bx + A}{ax^4 + bx^3 + cx^2 + bx + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3 + C*x^2 + B*x + A)/(a*x^4 + b*x^3 + c*x^2 + b*x + a),x, algorithm=
```

```
[Out] integrate((D*x^3 + C*x^2 + B*x + A)/(a*x^4 + b*x^3 + c*x^2 + b*x
+ a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3 + C*x^2 + B*x + A)/(a*x^4 + b*x^3 + c*x^2 + b*x + a),x, algorithm=
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((D*x**3+C*x**2+B*x+A)/(a*x**4+b*x**3+c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Dx^3 + Cx^2 + Bx + A}{ax^4 + bx^3 + cx^2 + bx + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3 + C*x^2 + B*x + A)/(a*x^4 + b*x^3 + c*x^2 + b*x + a),x, algorithm=
```

```
[Out] integrate((D*x^3 + C*x^2 + B*x + A)/(a*x^4 + b*x^3 + c*x^2 + b*x
+ a), x)
```

3.228 $\int \frac{2+x-4x^2+2x^3}{1-x+x^2-x^3+x^4}\,dx$

Optimal. Leaf size=63

$$\frac{1}{2}\left(1+\sqrt{5}\right)\log\left(2x^2-\left(1-\sqrt{5}\right)x+2\right)+\frac{1}{2}\left(1-\sqrt{5}\right)\log\left(2x^2-\left(1+\sqrt{5}\right)x+2\right)$$

```
[Out] ((1 + Sqrt[5])*Log[2 - (1 - Sqrt[5])*x + 2*x^2])/2 + ((1 - Sqrt[5
])*Log[2 - (1 + Sqrt[5])*x + 2*x^2])/2
```

Rubi [A] time = 0.131887, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062

$$\frac{1}{2}\left(1+\sqrt{5}\right)\log\left(2x^2-\left(1-\sqrt{5}\right)x+2\right)+\frac{1}{2}\left(1-\sqrt{5}\right)\log\left(2x^2-\left(1+\sqrt{5}\right)x+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x - 4*x^2 + 2*x^3)/(1 - x + x^2 - x^3 + x^4),x]
```

```
[Out] ((1 + Sqrt[5])*Log[2 - (1 - Sqrt[5])*x + 2*x^2])/2 + ((1 - Sqrt[5
])*Log[2 - (1 + Sqrt[5])*x + 2*x^2])/2
```

Rubi in Sympy [A] time = 37.2821, size = 53, normalized size = 0.84

$$-\frac{2\log\left(2x^2+x\left(-1+\sqrt{5}\right)+2\right)}{-\sqrt{5}+1}-\frac{2\log\left(2x^2+x\left(-\sqrt{5}-1\right)+2\right)}{1+\sqrt{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((2*x**3-4*x**2+x+2)/(x**4-x**3+x**2-x+1),x)
```

```
[Out] -2*log(2*x**2 + x*(-1 + sqrt(5)) + 2)/(-sqrt(5) + 1) - 2*log(2*x*
*2 + x*(-sqrt(5) - 1) + 2)/(1 + sqrt(5))
```

Mathematica [A] time = 0.0402801, size = 55, normalized size = 0.87

$$\frac{1}{2}\left(\left(1+\sqrt{5}\right)\log\left(2x^2+\left(\sqrt{5}-1\right)x+2\right)-\left(\sqrt{5}-1\right)\log\left(-2x^2+\sqrt{5}x+x-2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x - 4*x^2 + 2*x^3)/(1 - x + x^2 - x^3 + x^4),x]
```

```
[Out] (-((-1 + Sqrt[5])*Log[-2 + x + Sqrt[5]*x - 2*x^2]) + (1 + Sqrt[5]
)*Log[2 + (-1 + Sqrt[5])*x + 2*x^2])/2
```

Maple [A] time = 0.036, size = 82, normalized size = 1.3

$$\frac{\ln\left(-x\sqrt{5}+2\,x^2-x+2\right)}{2}-\frac{\ln\left(-x\sqrt{5}+2\,x^2-x+2\right)\sqrt{5}}{2}+\frac{\ln\left(x\sqrt{5}+2\,x^2-x+2\right)}{2}+\frac{\ln\left(x\sqrt{5}+2\,x^2-x+2\right)\sqrt{5}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((2*x^3-4*x^2+x+2)/(x^4-x^3+x^2-x+1),x)
```

```
[Out] 1/2*ln(-x*5^(1/2)+2*x^2-x+2)-1/2*ln(-x*5^(1/2)+2*x^2-x+2)*5^(1/2)
+1/2*ln(x*5^(1/2)+2*x^2-x+2)+1/2*ln(x*5^(1/2)+2*x^2-x+2)*5^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2\,x^3 - 4\,x^2 + x + 2}{x^4 - x^3 + x^2 - x + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3 - 4*x^2 + x + 2)/(x^4 - x^3 + x^2 - x + 1),x, algorithm="maxima")
```

```
[Out] integrate((2*x^3 - 4*x^2 + x + 2)/(x^4 - x^3 + x^2 - x + 1), x)
```

Fricas [A] time = 0.264406, size = 112, normalized size = 1.78

$$\frac{1}{2}\sqrt{5}\log\left(\frac{2\,x^4 - 2\,x^3 + 7\,x^2 + \sqrt{5}\left(2\,x^3 - x^2 + 2\,x\right) - 2\,x + 2}{x^4 - x^3 + x^2 - x + 1}\right) + \frac{1}{2}\log\left(x^4 - x^3 + x^2 - x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3 - 4*x^2 + x + 2)/(x^4 - x^3 + x^2 - x + 1),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(5)*log((2*x^4 - 2*x^3 + 7*x^2 + sqrt(5)*(2*x^3 - x^2 + 2
*x) - 2*x + 2)/(x^4 - x^3 + x^2 - x + 1)) + 1/2*log(x^4 - x^3 + x
^2 - x + 1)
```

Sympy [A] time = 0.254638, size = 58, normalized size = 0.92

$$\left(\frac{1}{2} + \frac{\sqrt{5}}{2}\right)\log\left(x^2 + x\left(-\frac{1}{2} + \frac{\sqrt{5}}{2}\right) + 1\right) + \left(-\frac{\sqrt{5}}{2} + \frac{1}{2}\right)\log\left(x^2 + x\left(-\frac{\sqrt{5}}{2} - \frac{1}{2}\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((2*x**3-4*x**2+x+2)/(x**4-x**3+x**2-x+1),x)
```

```
[Out] (1/2 + sqrt(5)/2)*log(x**2 + x*(-1/2 + sqrt(5)/2) + 1) + (-sqrt(5
)/2 + 1/2)*log(x**2 + x*(-sqrt(5)/2 - 1/2) + 1)
```

GIAC/XCAS [A] time = 0.275274, size = 78, normalized size = 1.24

$$-\frac{1}{2}\sqrt{5}\ln\left(x^2 - \frac{1}{2}x\left(\sqrt{5} + 1\right) + 1\right) + \frac{1}{2}\sqrt{5}\ln\left(x^2 + \frac{1}{2}x\left(\sqrt{5} - 1\right) + 1\right) + \frac{1}{2}\ln\left(x^4 - x^3 + x^2 - x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3 - 4*x^2 + x + 2)/(x^4 - x^3 + x^2 - x + 1),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(5)*ln(x^2 - 1/2*x*(sqrt(5) + 1) + 1) + 1/2*sqrt(5)*ln(x
^2 + 1/2*x*(sqrt(5) - 1) + 1) + 1/2*ln(x^4 - x^3 + x^2 - x + 1)
```

3.229 $\int \frac{3x+3x^2+x^3}{1+4x+6x^2+4x^3+x^4} \, dx$

Optimal. Leaf size=14

$$\frac{1}{3(x+1)^3} + \log(x+1)$$

```
[Out] 1/(3*(1 + x)^3) + Log[1 + x]
```

Rubi [A] time = 0.0605462, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{1}{3(x+1)^3} + \log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[(3*x + 3*x^2 + x^3)/(1 + 4*x + 6*x^2 + 4*x^3 + x^4),x]

[Out] 1/(3*(1 + x)^3) + Log[1 + x]
```

Rubi in Sympy [A] time = 34.3685, size = 12, normalized size = 0.86

$$\log(x+1) + \frac{1}{3(x+1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**3+3*x**2+3*x)/(x**4+4*x**3+6*x**2+4*x+1),x)

[Out] log(x + 1) + 1/(3*(x + 1)**3)
```

Mathematica [A] time = 0.0112096, size = 14, normalized size = 1.

$$\frac{1}{3(x+1)^3} + \log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3*x + 3*x^2 + x^3)/(1 + 4*x + 6*x^2 + 4*x^3 + x^4),x]

[Out] 1/(3*(1 + x)^3) + Log[1 + x]
```

Maple [A] time = 0.009, size = 13, normalized size = 0.9

$$\frac{1}{3(1+x)^3} + \ln(1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^3+3*x^2+3*x)/(x^4+4*x^3+6*x^2+4*x+1),x)
```

```
[Out] 1/3/(1+x)^3+ln(1+x)
```

Maxima [A] time = 0.837061, size = 30, normalized size = 2.14

$$\frac{1}{3\,(x^3+3\,x^2+3\,x+1)}+\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3 + 3*x^2 + 3*x)/(x^4 + 4*x^3 + 6*x^2 + 4*x + 1),x, algorithm="maxima
```

```
[Out] 1/3/(x^3 + 3*x^2 + 3*x + 1) + log(x + 1)
```

Fricas [A] time = 0.26641, size = 51, normalized size = 3.64

$$\frac{3\left(x^3+3\,x^2+3\,x+1\right)\log\left(x+1\right)+1}{3\,(x^3+3\,x^2+3\,x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3 + 3*x^2 + 3*x)/(x^4 + 4*x^3 + 6*x^2 + 4*x + 1),x, algorithm="fricas
```

```
[Out] 1/3*(3*(x^3 + 3*x^2 + 3*x + 1)*log(x + 1) + 1)/(x^3 + 3*x^2 + 3*x
 + 1)
```

Sympy [A] time = 0.195147, size = 20, normalized size = 1.43

$$\log\left(x+1\right)+\frac{1}{3x^3+9x^2+9x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3+3*x**2+3*x)/(x**4+4*x**3+6*x**2+4*x+1),x)
```

```
[Out] log(x + 1) + 1/(3*x**3 + 9*x**2 + 9*x + 3)
```

GIAC/XCAS [A] time = 0.259003, size = 18, normalized size = 1.29

$$\frac{1}{3\,(x+1)^3}+\ln\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3 + 3*x^2 + 3*x)/(x^4 + 4*x^3 + 6*x^2 + 4*x + 1),x, algorithm="giac")
```

```
[Out] 1/3/(x + 1)^3 + ln(abs(x + 1))
```

3.230 $\int \frac{-1+3x-3x^2+x^3}{1+4x+6x^2+4x^3+x^4}\,dx$

Optimal. Leaf size=28

$$\frac{6}{x+1}-\frac{6}{(x+1)^2}+\frac{8}{3(x+1)^3}+\log(x+1)$$

```
[Out] 8/(3*(1 + x)^3) - 6/(1 + x)^2 + 6/(1 + x) + Log[1 + x]
```

Rubi [A] time = 0.0508693, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059

$$\frac{6}{x+1}-\frac{6}{(x+1)^2}+\frac{8}{3(x+1)^3}+\log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + 3*x - 3*x^2 + x^3)/(1 + 4*x + 6*x^2 + 4*x^3 + x^4),x]
```

```
[Out] 8/(3*(1 + x)^3) - 6/(1 + x)^2 + 6/(1 + x) + Log[1 + x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int^{x+1} \frac{x^3-6x^2+12x-8}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**3-3*x**2+3*x-1)/(x**4+4*x**3+6*x**2+4*x+1),x)
```

```
[Out] Integral((x**3 - 6*x**2 + 12*x - 8)/x**4, (x, x + 1))
```

Mathematica [A] time = 0.0197026, size = 24, normalized size = 0.86

$$\frac{2\left(9x^2+9x+4\right)}{3(x+1)^3}+\log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + 3*x - 3*x^2 + x^3)/(1 + 4*x + 6*x^2 + 4*x^3 + x^4),x]
```

```
[Out] (2*(4 + 9*x + 9*x^2))/(3*(1 + x)^3) + Log[1 + x]
```

Maple [A] time = 0.01, size = 27, normalized size = 1.

$$\frac{8}{3\,(1+x)^3}-6\,(1+x)^{-2}+6\,(1+x)^{-1}+\ln(1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^3-3*x^2+3*x-1)/(x^4+4*x^3+6*x^2+4*x+1),x)
```

```
[Out] 8/3/(1+x)^3-6/(1+x)^2+6/(1+x)+ln(1+x)
```

Maxima [A] time = 0.830648, size = 43, normalized size = 1.54

$$\frac{2\left(9\,x^2+9\,x+4\right)}{3\,(x^3+3\,x^2+3\,x+1)}+\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3 - 3*x^2 + 3*x - 1)/(x^4 + 4*x^3 + 6*x^2 + 4*x + 1),x, algorithm="max
```

```
[Out] 2/3*(9*x^2 + 9*x + 4)/(x^3 + 3*x^2 + 3*x + 1) + log(x + 1)
```

Fricas [A] time = 0.264645, size = 62, normalized size = 2.21

$$\frac{18\,x^2+3\left(x^3+3\,x^2+3\,x+1\right)\log\left(x+1\right)+18\,x+8}{3\,(x^3+3\,x^2+3\,x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3 - 3*x^2 + 3*x - 1)/(x^4 + 4*x^3 + 6*x^2 + 4*x + 1),x, algorithm="fri
```

```
[Out] 1/3*(18*x^2 + 3*(x^3 + 3*x^2 + 3*x + 1)*log(x + 1) + 18*x + 8)/(x
^3 + 3*x^2 + 3*x + 1)
```

Sympy [A] time = 0.231938, size = 29, normalized size = 1.04

$$\frac{18x^2+18x+8}{3x^3+9x^2+9x+3}+\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3-3*x**2+3*x-1)/(x**4+4*x**3+6*x**2+4*x+1),x)
```

```
[Out] (18*x**2 + 18*x + 8)/(3*x**3 + 9*x**2 + 9*x + 3) + log(x + 1)
```

GIAC/XCAS [A] time = 0.260213, size = 31, normalized size = 1.11

$$\frac{2\left(9\,x^2+9\,x+4\right)}{3\,(x+1)^3}+\ln\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3 - 3*x^2 + 3*x - 1)/(x^4 + 4*x^3 + 6*x^2 + 4*x + 1),x, algorithm="gia
```

```
[Out] 2/3*(9*x^2 + 9*x + 4)/(x + 1)^3 + ln(abs(x + 1))
```

3.231 $\int \frac{9-40x-18x^2+174x^4+24x^5+26x^6-39x^8}{(3+2x^2+x^4)^3}\,dx$

Optimal. Leaf size=59

$$\frac{13x}{x^4+2x^2+3}-\frac{2\left(13x^2+18\right)x}{\left(x^4+2x^2+3\right)^2}+\frac{2\left(1-2x^2\right)}{\left(x^4+2x^2+3\right)^2}$$

```
[Out] (2*(1 - 2*x^2))/(3 + 2*x^2 + x^4)^2 - (2*x*(18 + 13*x^2))/(3 + 2*
x^2 + x^4)^2 + (13*x)/(3 + 2*x^2 + x^4)
```

Rubi [A] time = 0.130675, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.14$

$$\frac{13x}{x^4+2x^2+3}-\frac{2\left(13x^2+18\right)x}{\left(x^4+2x^2+3\right)^2}+\frac{2\left(1-2x^2\right)}{\left(x^4+2x^2+3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(9 - 40*x - 18*x^2 + 174*x^4 + 24*x^5 + 26*x^6 - 39*x^8)/(3 + 2*x^2 + x^4)^3,x]
```

```
[Out] (2*(1 - 2*x^2))/(3 + 2*x^2 + x^4)^2 - (2*x*(18 + 13*x^2))/(3 + 2*
x^2 + x^4)^2 + (13*x)/(3 + 2*x^2 + x^4)
```

Rubi in Sympy [A] time = 35.4857, size = 53, normalized size = 0.9

$$\frac{x\left(-65536x^3-131072x+3833856\right)}{294912\left(x^4+2x^2+3\right)}-\frac{x\left(1024x^3+39936x^2+8192x+55296\right)}{1536\left(x^4+2x^2+3\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-39*x**8+26*x**6+24*x**5+174*x**4-18*x**2-40*x+9)/(x**4+2*x**2
```

```
[Out] x*(-65536*x**3 - 131072*x + 3833856)/(294912*(x**4 + 2*x**2 + 3))
 - x*(1024*x**3 + 39936*x**2 + 8192*x + 55296)/(1536*(x**4 + 2*x*
*2 + 3)**2)
```

Mathematica [A] time = 0.0187123, size = 28, normalized size = 0.47

$$\frac{13x^5-4x^2+3x+2}{\left(x^4+2x^2+3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(9 - 40*x - 18*x^2 + 174*x^4 + 24*x^5 + 26*x^6 - 39*x^8)/(3 + 2*x^2 + x^4
```

```
[Out] (2 + 3*x - 4*x^2 + 13*x^5)/(3 + 2*x^2 + x^4)^2
```

Maple [A] time = 0.01, size = 30, normalized size = 0.5

$$-\frac{-13\,x^5+4\,x^2-3\,x-2}{\left(x^4+2\,x^2+3\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-39*x^8+26*x^6+24*x^5+174*x^4-18*x^2-40*x+9)/(x^4+2*x^2+3)^3,x)
```

```
[Out] -(-13*x^5+4*x^2-3*x-2)/(x^4+2*x^2+3)^2
```

Maxima [A] time = 0.854581, size = 51, normalized size = 0.86

$$\frac{13\,x^5-4\,x^2+3\,x+2}{x^8+4\,x^6+10\,x^4+12\,x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(39*x^8 - 26*x^6 - 24*x^5 - 174*x^4 + 18*x^2 + 40*x - 9)/(x^4 + 2*x^2 +
```

```
[Out] (13*x^5 - 4*x^2 + 3*x + 2)/(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)
```

Fricas [A] time = 0.258463, size = 51, normalized size = 0.86

$$\frac{13\,x^5-4\,x^2+3\,x+2}{x^8+4\,x^6+10\,x^4+12\,x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(39*x^8 - 26*x^6 - 24*x^5 - 174*x^4 + 18*x^2 + 40*x - 9)/(x^4 + 2*x^2 +
```

```
[Out] (13*x^5 - 4*x^2 + 3*x + 2)/(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)
```

Sympy [A] time = 0.548461, size = 34, normalized size = 0.58

$$\frac{13x^5-4x^2+3x+2}{x^8+4x^6+10x^4+12x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-39*x**8+26*x**6+24*x**5+174*x**4-18*x**2-40*x+9)/(x**4+2*x**2+3)**
```

```
[Out] (13*x**5 - 4*x**2 + 3*x + 2)/(x**8 + 4*x**6 + 10*x**4 + 12*x**2 +
 9)
```

GIAC/XCAS [A] time = 0.262823, size = 38, normalized size = 0.64

$$\frac{13\,x^5-4\,x^2+3\,x+2}{\left(x^4+2\,x^2+3\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(39*x^8 - 26*x^6 - 24*x^5 - 174*x^4 + 18*x^2 + 40*x - 9)/(x^4 + 2*x^2 +
```

```
[Out] (13*x^5 - 4*x^2 + 3*x + 2)/(x^4 + 2*x^2 + 3)^2
```

3.232 $\int \frac{-1+4x^5}{(1+x+x^5)^2} \, dx$

Optimal. Leaf size=11

$$-\frac{x}{x^5 + x + 1}$$

```
[Out] -(x/(1 + x + x^5))
```

Rubi [A] time = 0.00803861, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$

$$-\frac{x}{x^5 + x + 1}$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + 4*x^5)/(1 + x + x^5)^2,x]

[Out] -(x/(1 + x + x^5))
```

Rubi in Sympy [A] time = 6.7951, size = 8, normalized size = 0.73

$$-\frac{x}{x^5 + x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((4*x**5-1)/(x**5+x+1)**2,x)

[Out] -x/(x**5 + x + 1)
```

Mathematica [A] time = 0.0106846, size = 11, normalized size = 1.

$$-\frac{x}{x^5 + x + 1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + 4*x^5)/(1 + x + x^5)^2,x]

[Out] -(x/(1 + x + x^5))
```

Maple [B] time = 0.013, size = 41, normalized size = 3.7

$$\frac{-1 - 3\,x}{7\,x^2 + 7\,x + 7} - \frac{-3\,x^2 + 5\,x - 1}{7\,x^3 - 7\,x^2 + 7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((4*x^5-1)/(x^5+x+1)^2,x)

[Out] 1/7*(-1-3*x)/(x^2+x+1)-1/7*(-3*x^2+5*x-1)/(x^3-x^2+1)
```

Maxima [A] time = 0.857491, size = 15, normalized size = 1.36

$$-\frac{x}{x^5 + x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^5 - 1)/(x^5 + x + 1)^2,x, algorithm="maxima")
```

```
[Out] -x/(x^5 + x + 1)
```

Fricas [A] time = 0.259103, size = 15, normalized size = 1.36

$$-\frac{x}{x^5 + x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^5 - 1)/(x^5 + x + 1)^2,x, algorithm="fricas")
```

```
[Out] -x/(x^5 + x + 1)
```

Sympy [A] time = 0.332222, size = 8, normalized size = 0.73

$$-\frac{x}{x^5 + x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x**5-1)/(x**5+x+1)**2,x)
```

```
[Out] -x/(x**5 + x + 1)
```

GIAC/XCAS [A] time = 0.259282, size = 15, normalized size = 1.36

$$-\frac{x}{x^5 + x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^5 - 1)/(x^5 + x + 1)^2,x, algorithm="giac")
```

```
[Out] -x/(x^5 + x + 1)
```

3.233 $\int \frac{1+x^2}{(1-7x^2+7x^4-x^6)^2}\,dx$

Optimal. Leaf size=91

$$\frac{x}{16(1-x^2)}+\frac{(29-5x^2)\,x}{32(x^4-6x^2+1)}+\frac{1}{4}\tanh^{-1}(x)+\frac{1}{64}\left(\left(3-2\sqrt{2}\right)\tanh^{-1}\left(\left(\sqrt{2}-1\right)x\right)-\left(3+2\sqrt{2}\right)\tanh^{-1}\left(\left(1+\sqrt{2}\right)x\right)\right)$$

```
[Out] x/(16*(1 - x^2)) + (x*(29 - 5*x^2))/(32*(1 - 6*x^2 + x^4)) + ArcT
anh[x]/4 + ((3 - 2*Sqrt[2])*ArcTanh[(-1 + Sqrt[2])*x] - (3 + 2*Sq
rt[2])*ArcTanh[(1 + Sqrt[2])*x])/64
```

Rubi [B] time = 0.34491, antiderivative size = 205, normalized size of antiderivative = 2.25, number of steps used = 15, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28

$$-\frac{12-5x}{64(-x^2+2x+1)}+\frac{5x+12}{64(-x^2-2x+1)}+\frac{1}{32(1-x)}-\frac{1}{32(x+1)}-\frac{3}{256}\left(2+3\sqrt{2}\right)\log\left(-x-\sqrt{2}+1\right)-\frac{3}{256}\left(2-3\sqrt{2}\right)\log\left(-x+\sqrt{2}+1\right)+\frac{3}{256}\left(2+3\sqrt{2}\right)\log\left(x-\sqrt{2}+1\right)+\frac{3}{256}\left(2-3\sqrt{2}\right)\log\left(x+\sqrt{2}+1\right)-\frac{5\tanh^{-1}\left(\frac{1-x}{\sqrt{2}}\right)}{64\sqrt{2}}+\frac{1}{4}\tanh^{-1}(x)+\frac{5\tanh^{-1}\left(\frac{x+1}{\sqrt{2}}\right)}{64\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^2)/(1 - 7*x^2 + 7*x^4 - x^6)^2,x]
```

```
[Out] 1/(32*(1 - x)) - 1/(32*(1 + x)) + (12 + 5*x)/(64*(1 - 2*x - x^2))
 - (12 - 5*x)/(64*(1 + 2*x - x^2)) - (5*ArcTanh[(1 - x)/Sqrt[2]])
/(64*Sqrt[2]) + ArcTanh[x]/4 + (5*ArcTanh[(1 + x)/Sqrt[2]])/(64*S
qrt[2]) - (3*(2 + 3*Sqrt[2])*Log[1 - Sqrt[2] - x])/256 - (3*(2 -
3*Sqrt[2])*Log[1 + Sqrt[2] - x])/256 + (3*(2 + 3*Sqrt[2])*Log[1 -
 Sqrt[2] + x])/256 + (3*(2 - 3*Sqrt[2])*Log[1 + Sqrt[2] + x])/256
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+1)/(-x**6+7*x**4-7*x**2+1)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.153365, size = 132, normalized size = 1.45

$$\frac{1}{128}\left(-\frac{4x\left(7x^4-46x^2+31\right)}{x^6-7x^4+7x^2-1}-16\log(1-x)+\left(3+2\sqrt{2}\right)\log\left(-x+\sqrt{2}-1\right)+\left(2\sqrt{2}-3\right)\log\left(-x+\sqrt{2}+1\right)+16\log(x+1)-\left(3+2\sqrt{2}\right)\log\left(x+\sqrt{2}-1\right)+\left(3-2\sqrt{2}\right)\log\left(x+\sqrt{2}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^2)/(1 - 7*x^2 + 7*x^4 - x^6)^2,x]
```

```
[Out] ((-4*x*(31 - 46*x^2 + 7*x^4))/(-1 + 7*x^2 - 7*x^4 + x^6) - 16*Log
[1 - x] + (3 + 2*Sqrt[2])*Log[-1 + Sqrt[2] - x] + (-3 + 2*Sqrt[2]
)*Log[1 + Sqrt[2] - x] + 16*Log[1 + x] - (3 + 2*Sqrt[2])*Log[-1 +
 Sqrt[2] + x] + (3 - 2*Sqrt[2])*Log[1 + Sqrt[2] + x])/128
```

Maple [A] time = 0.026, size = 116, normalized size = 1.3

$$-\frac{-12+5x}{64x^2-128x-64}-\frac{3\ln\left(x^2-2x-1\right)}{128}-\frac{\sqrt{2}}{32}Artanh\left(\frac{(2x-2)\sqrt{2}}{4}\right)-\frac{1}{-32+32x}-\frac{\ln(-1+x)}{8}-\frac{1}{32+32x}+\frac{\ln(1+x)}{8}+\frac{-5x-12}{64x^2+128x-64}+\frac{3\ln\left(x^2+2x-1\right)}{128}-\frac{\sqrt{2}}{32}Artanh\left(\frac{(2+2x)\sqrt{2}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+1)/(-x^6+7*x^4-7*x^2+1)^2,x)
```

```
[Out] -1/64*(-12+5*x)/(x^2-2*x-1)-3/128*ln(x^2-2*x-1)-1/32*2^(1/2)*arct
anh(1/4*(2*x-2)*2^(1/2))-1/32/(-1+x)-1/8*ln(-1+x)-1/32/(1+x)+1/8*
ln(1+x)+1/64*(-5*x-12)/(x^2+2*x-1)+3/128*ln(x^2+2*x-1)-1/32*2^(1/
2)*arctanh(1/4*(2+2*x)*2^(1/2))
```

Maxima [A] time = 0.89445, size = 162, normalized size = 1.78

$$\frac{1}{64}\sqrt{2}\log\left(\frac{2\left(x-\sqrt{2}+1\right)}{2x+2\sqrt{2}+2}\right)+\frac{1}{64}\sqrt{2}\log\left(\frac{2\left(x-\sqrt{2}-1\right)}{2x+2\sqrt{2}-2}\right)-\frac{7x^5-46x^3+31x}{32\left(x^6-7x^4+7x^2-1\right)}+\frac{3}{128}\log\left(x^2+2x-1\right)-\frac{3}{128}\log\left(x^2-2x-1\right)+\frac{1}{8}\log(x+1)-\frac{1}{8}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/(x^6 - 7*x^4 + 7*x^2 - 1)^2,x, algorithm="maxima")
```

```
[Out] 1/64*sqrt(2)*log(2*(x - sqrt(2) + 1)/((2*sqrt(2)) + 2*x + 2)) + 1
/64*sqrt(2)*log(2*(x - sqrt(2) - 1)/((2*sqrt(2)) + 2*x - 2)) - 1/
32*(7*x^5 - 46*x^3 + 31*x)/(x^6 - 7*x^4 + 7*x^2 - 1) + 3/128*log(
x^2 + 2*x - 1) - 3/128*log(x^2 - 2*x - 1) + 1/8*log(x + 1) - 1/8*
log(x - 1)
```

Fricas [A] time = 0.275451, size = 324, normalized size = 3.56

$$\sqrt{2}\left(3\sqrt{2}\left(x^6-7x^4+7x^2-1\right)\log\left(x^2+2x-1\right)-3\sqrt{2}\left(x^6-7x^4+7x^2-1\right)\log\left(x^2-2x-1\right)+16\sqrt{2}\left(x^6-7x^4+7x^2-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/(x^6 - 7*x^4 + 7*x^2 - 1)^2,x, algorithm="fricas")
```

```
[Out] 1/256*sqrt(2)*(3*sqrt(2)*(x^6 - 7*x^4 + 7*x^2 - 1)*log(x^2 + 2*x
- 1) - 3*sqrt(2)*(x^6 - 7*x^4 + 7*x^2 - 1)*log(x^2 - 2*x - 1) + 1
6*sqrt(2)*(x^6 - 7*x^4 + 7*x^2 - 1)*log(x + 1) - 16*sqrt(2)*(x^6
- 7*x^4 + 7*x^2 - 1)*log(x - 1) + 4*(x^6 - 7*x^4 + 7*x^2 - 1)*log
((sqrt(2)*(x^2 + 2*x + 3) - 4*x - 4)/(x^2 + 2*x - 1)) + 4*(x^6 -
7*x^4 + 7*x^2 - 1)*log((sqrt(2)*(x^2 - 2*x + 3) - 4*x + 4)/(x^2 -
 2*x - 1)) - 4*sqrt(2)*(7*x^5 - 46*x^3 + 31*x))/(x^6 - 7*x^4 + 7*
```

```
x^2 - 1)
```

Sympy [A] time = 3.89062, size = 272, normalized size = 2.99

$$-\frac{7x^5-46x^3+31x}{32x^6-224x^4+224x^2-32}-\frac{\log(x-1)}{8}+\frac{\log(x+1)}{8}+\left(-\frac{3}{128}-\frac{\sqrt{2}}{64}\right)\log\left(x-\frac{38423555}{909328}-\frac{38423555\sqrt{2}}{1363992}+\frac{9549859782656\left(-\frac{3}{128}-\frac{\sqrt{2}}{64}\right)^5}{170499}-\frac{56267374592\left(-\frac{3}{128}-\frac{\sqrt{2}}{64}\right)^3}{56833}\right)+\left(-\frac{3}{128}+\frac{\sqrt{2}}{64}\right)\log\left(x-\frac{38423555}{909328}+\frac{9549859782656\left(-\frac{3}{128}+\frac{\sqrt{2}}{64}\right)^5}{170499}-\frac{56267374592\left(-\frac{3}{128}+\frac{\sqrt{2}}{64}\right)^3}{56833}+\frac{38423555\sqrt{2}}{1363992}\right)+\left(-\frac{\sqrt{2}}{64}+\frac{3}{128}\right)\log\left(x-\frac{38423555\sqrt{2}}{1363992}-\frac{56267374592\left(-\frac{\sqrt{2}}{64}+\frac{3}{128}\right)^3}{56833}+\frac{9549859782656\left(-\frac{\sqrt{2}}{64}+\frac{3}{128}\right)^5}{170499}+\frac{38423555}{909328}\right)+\left(\frac{\sqrt{2}}{64}+\frac{3}{128}\right)\log\left(x-\frac{56267374592\left(\frac{\sqrt{2}}{64}+\frac{3}{128}\right)^3}{56833}+\frac{9549859782656\left(\frac{\sqrt{2}}{64}+\frac{3}{128}\right)^5}{170499}+\frac{38423555\sqrt{2}}{1363992}+\frac{38423555}{909328}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+1)/(-x**6+7*x**4-7*x**2+1)**2,x)
```

```
[Out] -(7*x**5 - 46*x**3 + 31*x)/(32*x**6 - 224*x**4 + 224*x**2 - 32) -
 log(x - 1)/8 + log(x + 1)/8 + (-3/128 - sqrt(2)/64)*log(x - 3842
3555/909328 - 38423555*sqrt(2)/1363992 + 9549859782656*(-3/128 -
sqrt(2)/64)**5/170499 - 56267374592*(-3/128 - sqrt(2)/64)**3/5683
3) + (-3/128 + sqrt(2)/64)*log(x - 38423555/909328 + 954985978265
6*(-3/128 + sqrt(2)/64)**5/170499 - 56267374592*(-3/128 + sqrt(2)
/64)**3/56833 + 38423555*sqrt(2)/1363992) + (-sqrt(2)/64 + 3/128)
*log(x - 38423555*sqrt(2)/1363992 - 56267374592*(-sqrt(2)/64 + 3/
128)**3/56833 + 9549859782656*(-sqrt(2)/64 + 3/128)**5/170499 + 3
8423555/909328) + (sqrt(2)/64 + 3/128)*log(x - 56267374592*(sqrt(
2)/64 + 3/128)**3/56833 + 9549859782656*(sqrt(2)/64 + 3/128)**5/1
70499 + 38423555*sqrt(2)/1363992 + 38423555/909328)
```

GIAC/XCAS [A] time = 0.266122, size = 181, normalized size = 1.99

$$\frac{1}{64}\sqrt{2}\ln\left(\frac{\left|2x-2\sqrt{2}+2\right|}{\left|2x+2\sqrt{2}+2\right|}\right)+\frac{1}{64}\sqrt{2}\ln\left(\frac{\left|2x-2\sqrt{2}-2\right|}{\left|2x+2\sqrt{2}-2\right|}\right)-\frac{7x^5-46x^3+31x}{32(x^6-7x^4+7x^2-1)}+\frac{3}{128}\ln\left(\left|x^2+2x-1\right|\right)-\frac{3}{128}\ln\left(\left|x^2-2x-1\right|\right)+\frac{1}{8}\ln\left(\left|x+1\right|\right)-\frac{1}{8}\ln\left(\left|x-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/(x^6 - 7*x^4 + 7*x^2 - 1)^2,x, algorithm="giac")
```

```
[Out] 1/64*sqrt(2)*ln(abs(2*x - 2*sqrt(2) + 2)/abs(2*x + 2*sqrt(2) + 2)
) + 1/64*sqrt(2)*ln(abs(2*x - 2*sqrt(2) - 2)/abs(2*x + 2*sqrt(2)
- 2)) - 1/32*(7*x^5 - 46*x^3 + 31*x)/(x^6 - 7*x^4 + 7*x^2 - 1) +
3/128*ln(abs(x^2 + 2*x - 1)) - 3/128*ln(abs(x^2 - 2*x - 1)) + 1/8
*ln(abs(x + 1)) - 1/8*ln(abs(x - 1))
```

3.234 $\int x^m \left(a+bx+cx^2+dx^3\right)^p (a(1+m)+x(b(2+m+p)+x(c(3+m+2p)+d(4+m+3p)x)))\,dx$

Optimal. Leaf size=25

$$x^{m+1}\left(a+bx+cx^2+dx^3\right)^{p+1}$$

```
[Out] x^(1 + m)*(a + b*x + c*x^2 + d*x^3)^(1 + p)
```

Rubi [A] time = 0.0304605, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 56, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.018

$$x^{m+1}\left(a+bx+cx^2+dx^3\right)^{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x + c*x^2 + d*x^3)^p*(a*(1 + m) + x*(b*(2 + m + p) + x*(c*(3 + m + 2*p
```

```
[Out] x^(1 + m)*(a + b*x + c*x^2 + d*x^3)^(1 + p)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**m*(d*x**3+c*x**2+b*x+a)**p*(a*(1+m)+x*(b*(2+m+p)+x*(c*(3+m+2
```

```
[Out] Timed out
```

Mathematica [A] time = 0.137723, size = 23, normalized size = 0.92

$$x^{m+1}(a+x(b+x(c+dx)))^{p+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x + c*x^2 + d*x^3)^p*(a*(1 + m) + x*(b*(2 + m + p) + x*(c*(3 +
```

```
[Out] x^(1 + m)*(a + x*(b + x*(c + d*x)))^(1 + p)
```

Maple [A] time = 0.011, size = 26, normalized size = 1.

$$x^{1+m}\left(dx^3+cx^2+bx+a\right)^{1+p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(d*x^3+c*x^2+b*x+a)^p*(a*(1+m)+x*(b*(2+m+p)+x*(c*(3+m+2*p)+d*(4+m+3*p)
```

```
[Out] x^(1+m)*(d*x^3+c*x^2+b*x+a)^(1+p)
```

Maxima [A] time = 0.993762, size = 59, normalized size = 2.36

$$\left(dx^4 + cx^3 + bx^2 + ax\right)e^{\left(p\log\left(dx^3+cx^2+bx+a\right)+m\log(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(m + 1) + (b*(m + p + 2) + (d*(m + 3*p + 4)*x + c*(m + 2*p + 3))*x)*x)*(
```

```
[Out] (d*x^4 + c*x^3 + b*x^2 + a*x)*e^(p*log(d*x^3 + c*x^2 + b*x + a) +
 m*log(x))
```

Fricas [A] time = 0.506426, size = 54, normalized size = 2.16

$$\left(dx^4 + cx^3 + bx^2 + ax\right)\left(dx^3 + cx^2 + bx + a\right)^p x^m$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(m + 1) + (b*(m + p + 2) + (d*(m + 3*p + 4)*x + c*(m + 2*p + 3))*x)*x)*(
```

```
[Out] (d*x^4 + c*x^3 + b*x^2 + a*x)*(d*x^3 + c*x^2 + b*x + a)^p*x^m
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(d*x**3+c*x**2+b*x+a)**p*(a*(1+m)+x*(b*(2+m+p)+x*(c*(3+m+2*p)+d
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.414685, size = 155, normalized size = 6.2

$$dx^4e^{\left(p\ln\left(dx^3+cx^2+bx+a\right)+m\ln(x)\right)} + cx^3e^{\left(p\ln\left(dx^3+cx^2+bx+a\right)+m\ln(x)\right)} + bx^2e^{\left(p\ln\left(dx^3+cx^2+bx+a\right)+m\ln(x)\right)} + axe^{\left(p\ln\left(dx^3+cx^2+bx+a\right)+m\ln(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(m + 1) + (b*(m + p + 2) + (d*(m + 3*p + 4)*x + c*(m + 2*p + 3))*x)*x)*(
```

```
[Out] d*x^4*e^(p*ln(d*x^3 + c*x^2 + b*x + a) + m*ln(x)) + c*x^3*e^(p*ln
(d*x^3 + c*x^2 + b*x + a) + m*ln(x)) + b*x^2*e^(p*ln(d*x^3 + c*x^
2 + b*x + a) + m*ln(x)) + a*x*e^(p*ln(d*x^3 + c*x^2 + b*x + a) +
m*ln(x))
```

3.235 $\int x^2 (a + bx + cx^2 + dx^3)^p (3a + b(4 + p)x + c(5 + 2p)x^2 + d(6 + 3p)$

Optimal. Leaf size=23

$$x^3 (a + bx + cx^2 + dx^3)^{p+1}$$

```
[Out] x^3*(a + b*x + c*x^2 + d*x^3)^(1 + p)
```

Rubi [A] time = 0.0232666, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}} = 0.02$

$$x^3 (a + bx + cx^2 + dx^3)^{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x + c*x^2 + d*x^3)^p*(3*a + b*(4 + p)*x + c*(5 + 2*p)*x^2 + d*(6 + 3*
```

```
[Out] x^3*(a + b*x + c*x^2 + d*x^3)^(1 + p)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**2*(d*x**3+c*x**2+b*x+a)**p*(3*a+b*(4+p)*x+c*(5+2*p)*x**2+d*(
```

```
[Out] Timed out
```

Mathematica [A] time = 0.063995, size = 21, normalized size = 0.91

$$x^3(a + x(b + x(c + dx)))^{p+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x + c*x^2 + d*x^3)^p*(3*a + b*(4 + p)*x + c*(5 + 2*p)*x^2 + d*
```

```
[Out] x^3*(a + x*(b + x*(c + d*x)))^(1 + p)
```

Maple [A] time = 0.011, size = 24, normalized size = 1.

$$x^3 (dx^3 + cx^2 + bx + a)^{1+p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^3+c*x^2+b*x+a)^p*(3*a+b*(4+p)*x+c*(5+2*p)*x^2+d*(6+3*p)*x^3),x)
```

```
[Out] x^3*(d*x^3+c*x^2+b*x+a)^(1+p)
```

Maxima [A] time = 0.939416, size = 53, normalized size = 2.3

$$\left(dx^6 + cx^5 + bx^4 + ax^3\right)\left(dx^3 + cx^2 + bx + a\right)^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*(p + 2)*x^3 + c*(2*p + 5)*x^2 + b*(p + 4)*x + 3*a)*(d*x^3 + c*x^2 + 
```

```
[Out] (d*x^6 + c*x^5 + b*x^4 + a*x^3)*(d*x^3 + c*x^2 + b*x + a)^p
```

Fricas [A] time = 0.293899, size = 53, normalized size = 2.3

$$\left(dx^6 + cx^5 + bx^4 + ax^3\right)\left(dx^3 + cx^2 + bx + a\right)^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*(p + 2)*x^3 + c*(2*p + 5)*x^2 + b*(p + 4)*x + 3*a)*(d*x^3 + c*x^2 + 
```

```
[Out] (d*x^6 + c*x^5 + b*x^4 + a*x^3)*(d*x^3 + c*x^2 + b*x + a)^p
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**3+c*x**2+b*x+a)**p*(3*a+b*(4+p)*x+c*(5+2*p)*x**2+d*(6+3*p
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.469378, size = 131, normalized size = 5.7

$$dx^6e^{\left(p\ln\left(dx^3+cx^2+bx+a\right)\right)} + cx^5e^{\left(p\ln\left(dx^3+cx^2+bx+a\right)\right)} + bx^4e^{\left(p\ln\left(dx^3+cx^2+bx+a\right)\right)} + ax^3e^{\left(p\ln\left(dx^3+cx^2+bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*(p + 2)*x^3 + c*(2*p + 5)*x^2 + b*(p + 4)*x + 3*a)*(d*x^3 + c*x^2 + 
```

```
[Out] d*x^6*e^(p*ln(d*x^3 + c*x^2 + b*x + a)) + c*x^5*e^(p*ln(d*x^3 + c
*x^2 + b*x + a)) + b*x^4*e^(p*ln(d*x^3 + c*x^2 + b*x + a)) + a*x^
3*e^(p*ln(d*x^3 + c*x^2 + b*x + a))
```

3.236 $\int x\left(a+bx+cx^2+dx^3\right)^p\left(2a+b(3+p)x+c(4+2p)x^2+d(5+3p)x\right.$

Optimal. Leaf size=23

$$x^2\left(a+bx+cx^2+dx^3\right)^{p+1}$$

```
[Out] x^2*(a + b*x + c*x^2 + d*x^3)^(1 + p)
```

Rubi [A] time = 0.0210664, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.02

$$x^2\left(a+bx+cx^2+dx^3\right)^{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x + c*x^2 + d*x^3)^p*(2*a + b*(3 + p)*x + c*(4 + 2*p)*x^2 + d*(5 + 3*p)
```

```
[Out] x^2*(a + b*x + c*x^2 + d*x^3)^(1 + p)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x*(d*x**3+c*x**2+b*x+a)**p*(2*a+b*(3+p)*x+c*(4+2*p)*x**2+d*(5+3
```

```
[Out] Timed out
```

Mathematica [A] time = 0.0590705, size = 21, normalized size = 0.91

$$x^2(a+x(b+x(c+dx)))^{p+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x + c*x^2 + d*x^3)^p*(2*a + b*(3 + p)*x + c*(4 + 2*p)*x^2 + d*(5
```

```
[Out] x^2*(a + x*(b + x*(c + d*x)))^(1 + p)
```

Maple [A] time = 0.009, size = 24, normalized size = 1.

$$x^2\left(dx^3+cx^2+bx+a\right)^{1+p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^3+c*x^2+b*x+a)^p*(2*a+b*(3+p)*x+c*(4+2*p)*x^2+d*(5+3*p)*x^3),x)
```

```
[Out] x^2*(d*x^3+c*x^2+b*x+a)^(1+p)
```

Maxima [A] time = 0.90099, size = 53, normalized size = 2.3

$$\left(dx^5 + cx^4 + bx^3 + ax^2\right)\left(dx^3 + cx^2 + bx + a\right)^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*(3*p + 5)*x^3 + 2*c*(p + 2)*x^2 + b*(p + 3)*x + 2*a)*(d*x^3 + c*x^2 + 
```

```
[Out] (d*x^5 + c*x^4 + b*x^3 + a*x^2)*(d*x^3 + c*x^2 + b*x + a)^p
```

Fricas [A] time = 0.290735, size = 53, normalized size = 2.3

$$\left(dx^5 + cx^4 + bx^3 + ax^2\right)\left(dx^3 + cx^2 + bx + a\right)^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*(3*p + 5)*x^3 + 2*c*(p + 2)*x^2 + b*(p + 3)*x + 2*a)*(d*x^3 + c*x^2 + 
```

```
[Out] (d*x^5 + c*x^4 + b*x^3 + a*x^2)*(d*x^3 + c*x^2 + b*x + a)^p
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**3+c*x**2+b*x+a)**p*(2*a+b*(3+p)*x+c*(4+2*p)*x**2+d*(5+3*p)*x
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.328122, size = 131, normalized size = 5.7

$$dx^5e^{\left(p\ln\left(dx^3+cx^2+bx+a\right)\right)} + cx^4e^{\left(p\ln\left(dx^3+cx^2+bx+a\right)\right)} + bx^3e^{\left(p\ln\left(dx^3+cx^2+bx+a\right)\right)} + ax^2e^{\left(p\ln\left(dx^3+cx^2+bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*(3*p + 5)*x^3 + 2*c*(p + 2)*x^2 + b*(p + 3)*x + 2*a)*(d*x^3 + c*x^2 + 
```

```
[Out] d*x^5*e^(p*ln(d*x^3 + c*x^2 + b*x + a)) + c*x^4*e^(p*ln(d*x^3 + c
*x^2 + b*x + a)) + b*x^3*e^(p*ln(d*x^3 + c*x^2 + b*x + a)) + a*x^
2*e^(p*ln(d*x^3 + c*x^2 + b*x + a))
```

3.237 $\int \left(a+bx+cx^2+dx^3\right)^p \left(a+b(2+p)x+c(3+2p)x^2+d(4+3p)x^3\right)$

Optimal. Leaf size=21

$$x\left(a+bx+cx^2+dx^3\right)^{p+1}$$

```
[Out] x*(a + b*x + c*x^2 + d*x^3)^(1 + p)
```

Rubi [A] time = 0.0163447, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.022$

$$x\left(a+bx+cx^2+dx^3\right)^{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2 + d*x^3)^p*(a + b*(2 + p)*x + c*(3 + 2*p)*x^2 + d*(4 + 3*p)*x^3
```

```
[Out] x*(a + b*x + c*x^2 + d*x^3)^(1 + p)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((d*x**3+c*x**2+b*x+a)**p*(a+b*(2+p)*x+c*(3+2*p)*x**2+d*(4+3*p)*
```

```
[Out] Timed out
```

Mathematica [A] time = 0.0527342, size = 19, normalized size = 0.9

$$x(a+x(b+x(c+dx)))^{p+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2 + d*x^3)^p*(a + b*(2 + p)*x + c*(3 + 2*p)*x^2 + d*(4 + 3*
```

```
[Out] x*(a + x*(b + x*(c + d*x)))^(1 + p)
```

Maple [A] time = 0.009, size = 22, normalized size = 1.1

$$x\left(dx^3+cx^2+bx+a\right)^{1+p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c*x^2+b*x+a)^p*(a+b*(2+p)*x+c*(3+2*p)*x^2+d*(4+3*p)*x^3),x)
```

```
[Out] x*(d*x^3+c*x^2+b*x+a)^(1+p)
```

Maxima [A] time = 0.89127, size = 50, normalized size = 2.38

$$\left(dx^4 + cx^3 + bx^2 + ax\right)\left(dx^3 + cx^2 + bx + a\right)^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*(3*p + 4)*x^3 + c*(2*p + 3)*x^2 + b*(p + 2)*x + a)*(d*x^3 + c*x^2 + b*
```

```
[Out] (d*x^4 + c*x^3 + b*x^2 + a*x)*(d*x^3 + c*x^2 + b*x + a)^p
```

Fricas [A] time = 0.290751, size = 50, normalized size = 2.38

$$\left(dx^4 + cx^3 + bx^2 + ax\right)\left(dx^3 + cx^2 + bx + a\right)^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*(3*p + 4)*x^3 + c*(2*p + 3)*x^2 + b*(p + 2)*x + a)*(d*x^3 + c*x^2 + b*
```

```
[Out] (d*x^4 + c*x^3 + b*x^2 + a*x)*(d*x^3 + c*x^2 + b*x + a)^p
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c*x**2+b*x+a)**p*(a+b*(2+p)*x+c*(3+2*p)*x**2+d*(4+3*p)*x**3)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.320907, size = 128, normalized size = 6.1

$$dx^4e^{\left(p\ln\left(dx^3+cx^2+bx+a\right)\right)} + cx^3e^{\left(p\ln\left(dx^3+cx^2+bx+a\right)\right)} + bx^2e^{\left(p\ln\left(dx^3+cx^2+bx+a\right)\right)} + axe^{\left(p\ln\left(dx^3+cx^2+bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*(3*p + 4)*x^3 + c*(2*p + 3)*x^2 + b*(p + 2)*x + a)*(d*x^3 + c*x^2 + b*
```

```
[Out] d*x^4*e^(p*ln(d*x^3 + c*x^2 + b*x + a)) + c*x^3*e^(p*ln(d*x^3 + c
*x^2 + b*x + a)) + b*x^2*e^(p*ln(d*x^3 + c*x^2 + b*x + a)) + a*x*
e^(p*ln(d*x^3 + c*x^2 + b*x + a))
```

3.238 $\int \frac{(a+bx+cx^2+dx^3)^p (b(1+p)x+c(2+2p)x^2+d(3+3p)x^3)}{x} \, dx$

Optimal. Leaf size=19

$$\left(a+bx+cx^2+dx^3\right)^{p+1}$$

```
[Out] (a + b*x + c*x^2 + d*x^3)^(1 + p)
```

Rubi [A] time = 0.0220254, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.021

$$\left(a+bx+cx^2+dx^3\right)^{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x + c*x^2 + d*x^3)^p*(b*(1 + p)*x + c*(2 + 2*p)*x^2 + d*(3 + 3*p)*x^3))

[Out] (a + b*x + c*x^2 + d*x^3)^(1 + p)
```

Rubi in Sympy [A] time = 17.5511, size = 17, normalized size = 0.89

$$\left(a+bx+cx^2+dx^3\right)^{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((d*x**3+c*x**2+b*x+a)**p*(b*(1+p)*x+c*(2+2*p)*x**2+d*(3+3*p)*x*

[Out] (a + b*x + c*x**2 + d*x**3)**(p + 1)
```

Mathematica [A] time = 0.0399336, size = 17, normalized size = 0.89

$$(a+x(b+x(c+dx)))^{p+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2 + d*x^3)^p*(b*(1 + p)*x + c*(2 + 2*p)*x^2 + d*(3 + 3*p)

[Out] (a + x*(b + x*(c + d*x)))^(1 + p)
```

Maple [A] time = 0.007, size = 20, normalized size = 1.1

$$\left(dx^3+cx^2+bx+a\right)^{1+p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c*x^2+b*x+a)^p*(b*(1+p)*x+c*(2+2*p)*x^2+d*(3+3*p)*x^3)/x,x)

[Out] (d*x^3+c*x^2+b*x+a)^(1+p)
```

Maxima [A] time = 0.923027, size = 45, normalized size = 2.37

$$\left(dx^3 + cx^2 + bx + a\right)\left(dx^3 + cx^2 + bx + a\right)^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*(p + 1)*x^3 + 2*c*(p + 1)*x^2 + b*(p + 1)*x)*(d*x^3 + c*x^2 + b*x +
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p
```

Fricas [A] time = 0.289565, size = 45, normalized size = 2.37

$$\left(dx^3 + cx^2 + bx + a\right)\left(dx^3 + cx^2 + bx + a\right)^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*(p + 1)*x^3 + 2*c*(p + 1)*x^2 + b*(p + 1)*x)*(d*x^3 + c*x^2 + b*x +
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c*x**2+b*x+a)**p*(b*(1+p)*x+c*(2+2*p)*x**2+d*(3+3*p)*x**3)/x
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(3\,d(p+1)x^3 + 2\,c(p+1)x^2 + b(p+1)x\right)\left(dx^3 + cx^2 + bx + a\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*(p + 1)*x^3 + 2*c*(p + 1)*x^2 + b*(p + 1)*x)*(d*x^3 + c*x^2 + b*x +
```

```
[Out] integrate((3*d*(p + 1)*x^3 + 2*c*(p + 1)*x^2 + b*(p + 1)*x)*(d*x^
3 + c*x^2 + b*x + a)^p/x, x)
```

3.239 $\int \frac{(a+bx+cx^2+dx^3)^p(-a+bpx+c(1+2p)x^2+d(2+3p)x^3)}{x^2}\,dx$

Optimal. Leaf size=23

$$\frac{(a+bx+cx^2+dx^3)^{p+1}}{x}$$

```
[Out] (a + b*x + c*x^2 + d*x^3)^(1 + p)/x
```

Rubi [A] time = 0.0211701, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}} = 0.02$

$$\frac{(a+bx+cx^2+dx^3)^{p+1}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x + c*x^2 + d*x^3)^p*(-a + b*p*x + c*(1 + 2*p)*x^2 + d*(2 + 3*p)*x^3))/
```

```
[Out] (a + b*x + c*x^2 + d*x^3)^(1 + p)/x
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((d*x**3+c*x**2+b*x+a)**p*(-a+b*p*x+c*(1+2*p)*x**2+d*(2+3*p)*x**
```

```
[Out] Timed out
```

Mathematica [A] time = 0.0715709, size = 21, normalized size = 0.91

$$\frac{(a+x(b+x(c+dx)))^{p+1}}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2 + d*x^3)^p*(-a + b*p*x + c*(1 + 2*p)*x^2 + d*(2 + 3*p)*
```

```
[Out] (a + x*(b + x*(c + d*x)))^(1 + p)/x
```

Maple [A] time = 0.009, size = 24, normalized size = 1.

$$\frac{(dx^3+cx^2+bx+a)^{1+p}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c*x^2+b*x+a)^p*(-a+b*p*x+c*(1+2*p)*x^2+d*(2+3*p)*x^3)/x^2,x)
```

```
[Out] (d*x^3+c*x^2+b*x+a)^(1+p)/x
```

Maxima [A] time = 0.901203, size = 49, normalized size = 2.13

$$\frac{\left(dx^3 + cx^2 + bx + a\right)\left(dx^3 + cx^2 + bx + a\right)^p}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*(3*p + 2)*x^3 + c*(2*p + 1)*x^2 + b*p*x - a)*(d*x^3 + c*x^2 + b*x + a)
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p/x
```

Fricas [A] time = 0.320247, size = 49, normalized size = 2.13

$$\frac{\left(dx^3 + cx^2 + bx + a\right)\left(dx^3 + cx^2 + bx + a\right)^p}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*(3*p + 2)*x^3 + c*(2*p + 1)*x^2 + b*p*x - a)*(d*x^3 + c*x^2 + b*x + a)
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p/x
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c*x**2+b*x+a)**p*(-a+b*p*x+c*(1+2*p)*x**2+d*(2+3*p)*x**3)/x*
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d(3p+2)x^3 + c(2p+1)x^2 + bpx - a\right)\left(dx^3 + cx^2 + bx + a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*(3*p + 2)*x^3 + c*(2*p + 1)*x^2 + b*p*x - a)*(d*x^3 + c*x^2 + b*x + a)
```

```
[Out] integrate((d*(3*p + 2)*x^3 + c*(2*p + 1)*x^2 + b*p*x - a)*(d*x^3
+ c*x^2 + b*x + a)^p/x^2, x)
```

3.240 $\int \frac{(a+bx+cx^2+dx^3)^p(-2a+b(-1+p)x+2cpx^2+d(1+3p)x^3)}{x^3}\,dx$

Optimal. Leaf size=23

$$\frac{(a+bx+cx^2+dx^3)^{p+1}}{x^2}$$

```
[Out] (a + b*x + c*x^2 + d*x^3)^(1 + p)/x^2
```

Rubi [A] time = 0.0214587, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.021$

$$\frac{(a+bx+cx^2+dx^3)^{p+1}}{x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x + c*x^2 + d*x^3)^p*(-2*a + b*(-1 + p)*x + 2*c*p*x^2 + d*(1 + 3*p)*x^3
```

```
[Out] (a + b*x + c*x^2 + d*x^3)^(1 + p)/x^2
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((d*x**3+c*x**2+b*x+a)**p*(-2*a+b*(-1+p)*x+2*c*p*x**2+d*(1+3*p)*
```

```
[Out] Timed out
```

Mathematica [A] time = 0.0783587, size = 21, normalized size = 0.91

$$\frac{(a+x(b+x(c+dx)))^{p+1}}{x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2 + d*x^3)^p*(-2*a + b*(-1 + p)*x + 2*c*p*x^2 + d*(1 + 3*
```

```
[Out] (a + x*(b + x*(c + d*x)))^(1 + p)/x^2
```

Maple [A] time = 0.008, size = 24, normalized size = 1.

$$\frac{(dx^3+cx^2+bx+a)^{1+p}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c*x^2+b*x+a)^p*(-2*a+b*(-1+p)*x+2*c*p*x^2+d*(1+3*p)*x^3)/x^3,x)
```

```
[Out] (d*x^3+c*x^2+b*x+a)^(1+p)/x^2
```

Maxima [A] time = 0.90696, size = 49, normalized size = 2.13

$$\frac{\left(dx^3+cx^2+bx+a\right)\left(dx^3+cx^2+bx+a\right)^p}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*(3*p + 1)*x^3 + 2*c*p*x^2 + b*(p - 1)*x - 2*a)*(d*x^3 + c*x^2 + b*x + 
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p/x^2
```

Fricas [A] time = 0.357415, size = 49, normalized size = 2.13

$$\frac{\left(dx^3+cx^2+bx+a\right)\left(dx^3+cx^2+bx+a\right)^p}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*(3*p + 1)*x^3 + 2*c*p*x^2 + b*(p - 1)*x - 2*a)*(d*x^3 + c*x^2 + b*x + 
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p/x^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c*x**2+b*x+a)**p*(-2*a+b*(-1+p)*x+2*c*p*x**2+d*(1+3*p)*x**3)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d(3\,p+1)x^3+2\,cpx^2+b(p-1)x-2\,a\right)\left(dx^3+cx^2+bx+a\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*(3*p + 1)*x^3 + 2*c*p*x^2 + b*(p - 1)*x - 2*a)*(d*x^3 + c*x^2 + b*x + 
```

```
[Out] integrate((d*(3*p + 1)*x^3 + 2*c*p*x^2 + b*(p - 1)*x - 2*a)*(d*x^
3 + c*x^2 + b*x + a)^p/x^3, x)
```

3.241 $\int \frac{(a+bx+cx^2+dx^3)^p(-3a+b(-2+p)x+c(-1+2p)x^2+3dpx^3)}{x^4}\,dx$

Optimal. Leaf size=23

$$\frac{(a+bx+cx^2+dx^3)^{p+1}}{x^3}$$

```
[Out] (a + b*x + c*x^2 + d*x^3)^(1 + p)/x^3
```

Rubi [A] time = 0.0209346, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.021$

$$\frac{(a+bx+cx^2+dx^3)^{p+1}}{x^3}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x + c*x^2 + d*x^3)^p*(-3*a + b*(-2 + p)*x + c*(-
1 + 2*p)*x^2 + 3*d*p*x^3))/x^4,x]
```

```
[Out] (a + b*x + c*x^2 + d*x^3)^(1 + p)/x^3
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((d*x**3+c*x**2+b*x+a)**p*(-3*a+b*(-2+p)*x+c*(-1+2*p)*x**2+3*d*p
```

```
[Out] Timed out
```

Mathematica [A] time = 0.0958093, size = 21, normalized size = 0.91

$$\frac{(a+x(b+x(c+dx)))^{p+1}}{x^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x + c*x^2 + d*x^3)^p*(-3*a + b*(-2 + p)*x + c*(-
1 + 2*p)*x^2 + 3*d*p*x^3))/x^4,x]
```

```
[Out] (a + x*(b + x*(c + d*x)))^(1 + p)/x^3
```

Maple [A] time = 0.009, size = 24, normalized size = 1.

$$\frac{(dx^3+cx^2+bx+a)^{1+p}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c*x^2+b*x+a)^p*(-3*a+b*(-2+p)*x+c*(-1+2*p)*x^2+3*d*p*x^3)/x^4,x)
```

```
[Out] (d*x^3+c*x^2+b*x+a)^(1+p)/x^3
```

Maxima [A] time = 0.941787, size = 49, normalized size = 2.13

$$\frac{\left(dx^3+cx^2+bx+a\right)\left(dx^3+cx^2+bx+a\right)^p}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*p*x^3 + c*(2*p - 1)*x^2 + b*(p - 2)*x - 3*a)*(d*x^3 + c*x^2 + b*x +
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p/x^3
```

Fricas [A] time = 0.352541, size = 49, normalized size = 2.13

$$\frac{\left(dx^3+cx^2+bx+a\right)\left(dx^3+cx^2+bx+a\right)^p}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*p*x^3 + c*(2*p - 1)*x^2 + b*(p - 2)*x - 3*a)*(d*x^3 + c*x^2 + b*x +
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p/x^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c*x**2+b*x+a)**p*(-3*a+b*(-2+p)*x+c*(-1+2*p)*x**2+3*d*p*x**3
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(3\,dpx^3+c(2\,p-1)x^2+b(p-2)x-3\,a\right)\left(dx^3+cx^2+bx+a\right)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*p*x^3 + c*(2*p - 1)*x^2 + b*(p - 2)*x - 3*a)*(d*x^3 + c*x^2 + b*x +
```

```
[Out] integrate((3*d*p*x^3 + c*(2*p - 1)*x^2 + b*(p - 2)*x - 3*a)*(d*x^
3 + c*x^2 + b*x + a)^p/x^4, x)
```

3.242 $\int \frac{x^4(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx$

Optimal. Leaf size=97

$$\frac{x^4}{4}+\frac{x^3}{3}-\frac{3x^2}{4}+\frac{1}{3}\log\left(x^2+x+1\right)-\frac{13}{48}\log\left(2x^2-x+2\right)+\frac{5x}{4}+\frac{1}{24}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)-\frac{10\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] (5*x)/4 - (3*x^2)/4 + x^3/3 + x^4/4 + (Sqrt[5/3]*ArcTan[(1 - 4*x)
/Sqrt[15]])/24 - (10*ArcTan[(1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]) + Log
[1 + x + x^2]/3 - (13*Log[2 - x + 2*x^2])/48
```

Rubi [A] time = 0.265033, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{x^4}{4}+\frac{x^3}{3}-\frac{3x^2}{4}+\frac{1}{3}\log\left(x^2+x+1\right)-\frac{13}{48}\log\left(2x^2-x+2\right)+\frac{5x}{4}+\frac{1}{24}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)-\frac{10\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

```
[Out] (5*x)/4 - (3*x^2)/4 + x^3/3 + x^4/4 + (Sqrt[5/3]*ArcTan[(1 - 4*x)
/Sqrt[15]])/24 - (10*ArcTan[(1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]) + Log
[1 + x + x^2]/3 - (13*Log[2 - x + 2*x^2])/48
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+3*x**2+x+2),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.0566248, size = 83, normalized size = 0.86

$$\frac{1}{144}\left(36x^4+48x^3-108x^2+48\log\left(x^2+x+1\right)-39\log\left(2x^2-x+2\right)+180x-160\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)-2\sqrt{15}\tan^{-1}\left(\frac{4x-1}{\sqrt{15}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^4*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

```
[Out] (180*x - 108*x^2 + 48*x^3 + 36*x^4 - 160*Sqrt[3]*ArcTan[(1 + 2*x)
/Sqrt[3]] - 2*Sqrt[15]*ArcTan[(-1 + 4*x)/Sqrt[15]] + 48*Log[1 + x
 + x^2] - 39*Log[2 - x + 2*x^2])/144
```

Maple [A] time = 0.012, size = 74, normalized size = 0.8

$$\frac{x^4}{4}+\frac{x^3}{3}-\frac{3\,x^2}{4}+\frac{5\,x}{4}+\frac{\ln\left(x^2+x+1\right)}{3}-\frac{10\,\sqrt{3}}{9}\arctan\left(\frac{(1+2\,x)\,\sqrt{3}}{3}\right)-\frac{13\,\ln\left(2\,x^2-x+2\right)}{48}-\frac{\sqrt{15}}{72}\arctan\left(\frac{(-1+4\,x)\,\sqrt{15}}{15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^4*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x)
```

```
[Out] 1/4*x^4+1/3*x^3-3/4*x^2+5/4*x+1/3*ln(x^2+x+1)-10/9*arctan(1/3*(1+
2*x)*3^(1/2))*3^(1/2)-13/48*ln(2*x^2-x+2)-1/72*15^(1/2)*arctan(1/
15*(-1+4*x)*15^(1/2))
```

Maxima [A] time = 0.904407, size = 99, normalized size = 1.02

$$\frac{1}{4}\,x^4+\frac{1}{3}\,x^3-\frac{3}{4}\,x^2-\frac{1}{72}\,\sqrt{15}\arctan\left(\frac{1}{15}\,\sqrt{15}(4\,x-1)\right)-\frac{10}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)+\frac{5}{4}\,x-\frac{13}{48}\,\log\left(2\,x^2-x+2\right)+\frac{1}{3}\,\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^3 + 3*x^2 + x + 5)*x^4/(2*x^4 + x^3 + 3*x^2 + x + 2),x, algorithm="m
```

```
[Out] 1/4*x^4 + 1/3*x^3 - 3/4*x^2 - 1/72*sqrt(15)*arctan(1/15*sqrt(15)*
(4*x - 1)) - 10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 5/4*x -
 13/48*log(2*x^2 - x + 2) + 1/3*log(x^2 + x + 1)
```

Fricas [A] time = 0.273661, size = 122, normalized size = 1.26

$$\frac{1}{144}\,\sqrt{3}\left(4\,\sqrt{3}\left(3\,x^4+4\,x^3-9\,x^2+15\,x\right)-2\,\sqrt{5}\arctan\left(\frac{1}{15}\,\sqrt{5}\sqrt{3}(4\,x-1)\right)-13\,\sqrt{3}\log\left(2\,x^2-x+2\right)+16\,\sqrt{3}\log\left(x^2+x+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^3 + 3*x^2 + x + 5)*x^4/(2*x^4 + x^3 + 3*x^2 + x + 2),x, algorithm="f
```

```
[Out] 1/144*sqrt(3)*(4*sqrt(3)*(3*x^4 + 4*x^3 - 9*x^2 + 15*x) - 2*sqrt(
5)*arctan(1/15*sqrt(5)*sqrt(3)*(4*x - 1)) - 13*sqrt(3)*log(2*x^2
- x + 2) + 16*sqrt(3)*log(x^2 + x + 1) - 160*arctan(1/3*sqrt(3)*(
2*x + 1)))
```

Sympy [A] time = 0.722517, size = 97, normalized size = 1.

$$\frac{x^4}{4}+\frac{x^3}{3}-\frac{3x^2}{4}+\frac{5x}{4}-\frac{13\log\left(x^2-\frac{x}{2}+1\right)}{48}+\frac{\log\left(x^2+x+1\right)}{3}-\frac{\sqrt{15}\,\mathrm{atan}\left(\frac{4\sqrt{15}x}{15}-\frac{\sqrt{15}}{15}\right)}{72}-\frac{10\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x**4*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+3*x**2+x+2),x)
```

```
[Out] x**4/4 + x**3/3 - 3*x**2/4 + 5*x/4 - 13*log(x**2 - x/2 + 1)/48 +
log(x**2 + x + 1)/3 - sqrt(15)*atan(4*sqrt(15)*x/15 - sqrt(15)/15
)/72 - 10*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/9
```

GIAC/XCAS [A] time = 0.263833, size = 99, normalized size = 1.02

$$\frac{1}{4}x^4 + \frac{1}{3}x^3 - \frac{3}{4}x^2 - \frac{1}{72}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right) - \frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{5}{4}x - \frac{13}{48}\ln\left(2x^2 - x + 2\right) + \frac{1}{3}\ln\left(x^2 + x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3 + 3*x^2 + x + 5)*x^4/(2*x^4 + x^3 + 3*x^2 + x + 2),x, algorithm="g
```

```
[Out] 1/4*x^4 + 1/3*x^3 - 3/4*x^2 - 1/72*sqrt(15)*arctan(1/15*sqrt(15)*
(4*x - 1)) - 10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 5/4*x -
 13/48*ln(2*x^2 - x + 2) + 1/3*ln(x^2 + x + 1)
```

3.243 $\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4} dx$

Optimal. Leaf size=90

$$\frac{x^3}{3}+\frac{x^2}{2}+\frac{2}{3}\log\left(x^2+x+1\right)-\frac{1}{24}\log\left(2x^2-x+2\right)-\frac{3x}{2}+\frac{5}{12}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{8\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] (-3*x)/2 + x^2/2 + x^3/3 + (5*Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]
])/12 + (8*ArcTan[(1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]) + (2*Log[1 + x
+ x^2])/3 - Log[2 - x + 2*x^2]/24
```

Rubi [A] time = 0.236407, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{x^3}{3}+\frac{x^2}{2}+\frac{2}{3}\log\left(x^2+x+1\right)-\frac{1}{24}\log\left(2x^2-x+2\right)-\frac{3x}{2}+\frac{5}{12}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{8\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

```
[Out] (-3*x)/2 + x^2/2 + x^3/3 + (5*Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]
])/12 + (8*ArcTan[(1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]) + (2*Log[1 + x
+ x^2])/3 - Log[2 - x + 2*x^2]/24
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**3*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+3*x**2+x+2),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.0406212, size = 78, normalized size = 0.87

$$\frac{1}{72}\left(24x^3+36x^2+48\log\left(x^2+x+1\right)-3\log\left(2x^2-x+2\right)-108x+64\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)-10\sqrt{15}\tan^{-1}\left(\frac{4x-1}{\sqrt{15}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

```
[Out] (-108*x + 36*x^2 + 24*x^3 + 64*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]]
- 10*Sqrt[15]*ArcTan[(-1 + 4*x)/Sqrt[15]] + 48*Log[1 + x + x^2] -
 3*Log[2 - x + 2*x^2])/72
```

Maple [A] time = 0.009, size = 69, normalized size = 0.8

$$\frac{x^3}{3}+\frac{x^2}{2}-\frac{3\,x}{2}+\frac{2\,\ln\left(x^2+x+1\right)}{3}+\frac{8\sqrt{3}}{9}\arctan\left(\frac{(1+2\,x)\sqrt{3}}{3}\right)-\frac{\ln\left(2\,x^2-x+2\right)}{24}-\frac{5\sqrt{15}}{36}\arctan\left(\frac{(-1+4\,x)\sqrt{15}}{15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x)
```

```
[Out] 1/3*x^3+1/2*x^2-3/2*x+2/3*ln(x^2+x+1)+8/9*arctan(1/3*(1+2*x)*3^(1
/2))*3^(1/2)-1/24*ln(2*x^2-x+2)-5/36*15^(1/2)*arctan(1/15*(-1+4*x
)*15^(1/2))
```

Maxima [A] time = 0.914183, size = 92, normalized size = 1.02

$$\frac{1}{3}\,x^3+\frac{1}{2}\,x^2-\frac{5}{36}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4\,x-1)\right)+\frac{8}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x+1)\right)-\frac{3}{2}\,x-\frac{1}{24}\log\left(2\,x^2-x+2\right)+\frac{2}{3}\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^3 + 3*x^2 + x + 5)*x^3/(2*x^4 + x^3 + 3*x^2 + x + 2),x, algorithm="m
```

```
[Out] 1/3*x^3 + 1/2*x^2 - 5/36*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1))
 + 8/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 3/2*x - 1/24*log(2
*x^2 - x + 2) + 2/3*log(x^2 + x + 1)
```

Fricas [A] time = 0.274183, size = 115, normalized size = 1.28

$$\frac{1}{72}\sqrt{3}\left(4\sqrt{3}\left(2\,x^3+3\,x^2-9\,x\right)-10\sqrt{5}\arctan\left(\frac{1}{15}\sqrt{5}\sqrt{3}(4\,x-1)\right)-\sqrt{3}\log\left(2\,x^2-x+2\right)+16\sqrt{3}\log\left(x^2+x+1\right)+64\text{ ar}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^3 + 3*x^2 + x + 5)*x^3/(2*x^4 + x^3 + 3*x^2 + x + 2),x, algorithm="f
```

```
[Out] 1/72*sqrt(3)*(4*sqrt(3)*(2*x^3 + 3*x^2 - 9*x) - 10*sqrt(5)*arctan
(1/15*sqrt(5)*sqrt(3)*(4*x - 1)) - sqrt(3)*log(2*x^2 - x + 2) + 1
6*sqrt(3)*log(x^2 + x + 1) + 64*arctan(1/3*sqrt(3)*(2*x + 1)))
```

Sympy [A] time = 0.684268, size = 92, normalized size = 1.02

$$\frac{x^3}{3}+\frac{x^2}{2}-\frac{3x}{2}-\frac{\log\left(x^2-\frac{x}{2}+1\right)}{24}+\frac{2\log\left(x^2+x+1\right)}{3}-\frac{5\sqrt{15}\operatorname{atan}\left(\frac{4\sqrt{15}x}{15}-\frac{\sqrt{15}}{15}\right)}{36}+\frac{8\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+3*x**2+x+2),x)
```

```
[Out] x**3/3 + x**2/2 - 3*x/2 - log(x**2 - x/2 + 1)/24 + 2*log(x**2 + x
 + 1)/3 - 5*sqrt(15)*atan(4*sqrt(15)*x/15 - sqrt(15)/15)/36 + 8*s
```

```
qrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/9
```

GIAC/XCAS [A] time = 0.261816, size = 92, normalized size = 1.02

$$\frac{1}{3}\,x^3 + \frac{1}{2}\,x^2 - \frac{5}{36}\,\sqrt{15}\arctan\left(\frac{1}{15}\,\sqrt{15}(4\,x - 1)\right) + \frac{8}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x + 1)\right) - \frac{3}{2}\,x - \frac{1}{24}\ln\left(2\,x^2 - x + 2\right) + \frac{2}{3}\ln\left(x^2 + x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3 + 3*x^2 + x + 5)*x^3/(2*x^4 + x^3 + 3*x^2 + x + 2),x, algorithm="g
```

```
[Out] 1/3*x^3 + 1/2*x^2 - 5/36*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1))
 + 8/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 3/2*x - 1/24*ln(2*
x^2 - x + 2) + 2/3*ln(x^2 + x + 1)
```

3.244 $\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4} \, dx$

Optimal. Leaf size=77

$$\frac{x^2}{2}-\log\left(x^2+x+1\right)+\frac{1}{4}\log\left(2x^2-x+2\right)+x+\frac{1}{6}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{2\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] x + x^2/2 + (Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/6 + (2*ArcTan[
(1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]) - Log[1 + x + x^2] + Log[2 - x +
2*x^2]/4
```

Rubi [A] time = 0.236505, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{x^2}{2}-\log\left(x^2+x+1\right)+\frac{1}{4}\log\left(2x^2-x+2\right)+x+\frac{1}{6}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{2\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

```
[Out] x + x^2/2 + (Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/6 + (2*ArcTan[
(1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]) - Log[1 + x + x^2] + Log[2 - x +
2*x^2]/4
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+3*x**2+x+2),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.049007, size = 72, normalized size = 0.94

$$\frac{1}{36}\left(9\left(-4\log\left(x^2+x+1\right)+\log\left(2x^2-x+2\right)+2x(x+2)\right)+8\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)-2\sqrt{15}\tan^{-1}\left(\frac{4x-1}{\sqrt{15}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

```
[Out] (8*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 2*Sqrt[15]*ArcTan[(-1 + 4*
x)/Sqrt[15]] + 9*(2*x*(2 + x) - 4*Log[1 + x + x^2] + Log[2 - x +
2*x^2]))/36
```

Maple [A] time = 0.008, size = 62, normalized size = 0.8

$$x+\frac{x^2}{2}-\ln\left(x^2+x+1\right)+\frac{2\sqrt{3}}{9}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)+\frac{\ln\left(2x^2-x+2\right)}{4}-\frac{\sqrt{15}}{18}\arctan\left(\frac{(-1+4x)\sqrt{15}}{15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x)
```

```
[Out] x+1/2*x^2-ln(x^2+x+1)+2/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/4
*ln(2*x^2-x+2)-1/18*15^(1/2)*arctan(1/15*(-1+4*x)*15^(1/2))
```

Maxima [A] time = 0.914693, size = 82, normalized size = 1.06

$$\frac{1}{2}x^2 - \frac{1}{18}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right) + \frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + x + \frac{1}{4}\log\left(2x^2 - x + 2\right) - \log\left(x^2 + x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3 + 3*x^2 + x + 5)*x^2/(2*x^4 + x^3 + 3*x^2 + x + 2),x, algorithm="m
```

```
[Out] 1/2*x^2 - 1/18*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 2/9*sqr
t(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + x + 1/4*log(2*x^2 - x + 2) -
 log(x^2 + x + 1)
```

Fricas [A] time = 0.275754, size = 105, normalized size = 1.36

$$\frac{1}{36}\sqrt{3}\left(6\sqrt{3}\left(x^2 + 2x\right) - 2\sqrt{5}\arctan\left(\frac{1}{15}\sqrt{5}\sqrt{3}(4x-1)\right) + 3\sqrt{3}\log\left(2x^2 - x + 2\right) - 12\sqrt{3}\log\left(x^2 + x + 1\right) + 8\arctan\left(\frac{1}{3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3 + 3*x^2 + x + 5)*x^2/(2*x^4 + x^3 + 3*x^2 + x + 2),x, algorithm="f
```

```
[Out] 1/36*sqrt(3)*(6*sqrt(3)*(x^2 + 2*x) - 2*sqrt(5)*arctan(1/15*sqrt(
5)*sqrt(3)*(4*x - 1)) + 3*sqrt(3)*log(2*x^2 - x + 2) - 12*sqrt(3)
*log(x^2 + x + 1) + 8*arctan(1/3*sqrt(3)*(2*x + 1)))
```

Sympy [A] time = 0.806577, size = 78, normalized size = 1.01

$$\frac{x^2}{2} + x + \frac{\log\left(x^2 - \frac{x}{2} + 1\right)}{4} - \log\left(x^2 + x + 1\right) - \frac{\sqrt{15}\operatorname{atan}\left(\frac{4\sqrt{15}x}{15} - \frac{\sqrt{15}}{15}\right)}{18} + \frac{2\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x**2*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+3*x**2+x+2),x)
```

```
[Out] x**2/2 + x + log(x**2 - x/2 + 1)/4 - log(x**2 + x + 1) - sqrt(15)
*atan(4*sqrt(15)*x/15 - sqrt(15)/15)/18 + 2*sqrt(3)*atan(2*sqrt(3
)*x/3 + sqrt(3)/3)/9
```

GIAC/XCAS [A] time = 0.260861, size = 82, normalized size = 1.06

$$\frac{1}{2}x^2 - \frac{1}{18}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right) + \frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + x + \frac{1}{4}\ln\left(2x^2 - x + 2\right) - \ln\left(x^2 + x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^3 + 3*x^2 + x + 5)*x^2/(2*x^4 + x^3 + 3*x^2 + x + 2),x, algorithm="g
```

```
[Out] 1/2*x^2 - 1/18*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 2/9*sqr
t(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + x + 1/4*ln(2*x^2 - x + 2) -
ln(x^2 + x + 1)
```

3.245 $\int \frac{x(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4} dx$

Optimal. Leaf size=72

$$\frac{1}{3}\log\left(x^2+x+1\right)+\frac{1}{6}\log\left(2x^2-x+2\right)+x-\frac{1}{3}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)-\frac{10\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] x - (Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/3 - (10*ArcTan[(1 + 2*
x)/Sqrt[3]])/(3*Sqrt[3]) + Log[1 + x + x^2]/3 + Log[2 - x + 2*x^2
]/6
```

Rubi [A] time = 0.186873, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152

$$\frac{1}{3}\log\left(x^2+x+1\right)+\frac{1}{6}\log\left(2x^2-x+2\right)+x-\frac{1}{3}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)-\frac{10\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

```
[Out] x - (Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/3 - (10*ArcTan[(1 + 2*
x)/Sqrt[3]])/(3*Sqrt[3]) + Log[1 + x + x^2]/3 + Log[2 - x + 2*x^2
]/6
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+3*x**2+x+2),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.036799, size = 69, normalized size = 0.96

$$\frac{1}{18}\left(3\left(2\log\left(x^2+x+1\right)+\log\left(2x^2-x+2\right)+6x\right)-20\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+2\sqrt{15}\tan^{-1}\left(\frac{4x-1}{\sqrt{15}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

```
[Out] (-20*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 2*Sqrt[15]*ArcTan[(-1 +
4*x)/Sqrt[15]] + 3*(6*x + 2*Log[1 + x + x^2] + Log[2 - x + 2*x^2]
))/18
```

Maple [A] time = 0.005, size = 57, normalized size = 0.8

$$x+\frac{\ln\left(x^2+x+1\right)}{3}-\frac{10\sqrt{3}}{9}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)+\frac{\ln\left(2x^2-x+2\right)}{6}+\frac{\sqrt{15}}{9}\arctan\left(\frac{(-1+4x)\sqrt{15}}{15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x)
```

```
[Out] x+1/3*ln(x^2+x+1)-10/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/6*ln
(2*x^2-x+2)+1/9*15^(1/2)*arctan(1/15*(-1+4*x)*15^(1/2))
```

Maxima [A] time = 0.914368, size = 76, normalized size = 1.06

$$\frac{1}{9}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right)-\frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+x+\frac{1}{6}\log\left(2x^2-x+2\right)+\frac{1}{3}\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^3 + 3*x^2 + x + 5)*x/(2*x^4 + x^3 + 3*x^2 + x + 2),x, algorithm="max
```

```
[Out] 1/9*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) - 10/9*sqrt(3)*arcta
n(1/3*sqrt(3)*(2*x + 1)) + x + 1/6*log(2*x^2 - x + 2) + 1/3*log(x
^2 + x + 1)
```

Fricas [A] time = 0.272233, size = 96, normalized size = 1.33

$$\frac{1}{18}\sqrt{3}\left(6\sqrt{3}x+2\sqrt{5}\arctan\left(\frac{1}{15}\sqrt{5}\sqrt{3}(4x-1)\right)+\sqrt{3}\log\left(2x^2-x+2\right)+2\sqrt{3}\log\left(x^2+x+1\right)-20\arctan\left(\frac{1}{3}\sqrt{3}(2x+1\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^3 + 3*x^2 + x + 5)*x/(2*x^4 + x^3 + 3*x^2 + x + 2),x, algorithm="fri
```

```
[Out] 1/18*sqrt(3)*(6*sqrt(3)*x + 2*sqrt(5)*arctan(1/15*sqrt(5)*sqrt(3)
*(4*x - 1)) + sqrt(3)*log(2*x^2 - x + 2) + 2*sqrt(3)*log(x^2 + x
+ 1) - 20*arctan(1/3*sqrt(3)*(2*x + 1)))
```

Sympy [A] time = 0.730982, size = 75, normalized size = 1.04

$$x+\frac{\log\left(x^2-\frac{x}{2}+1\right)}{6}+\frac{\log\left(x^2+x+1\right)}{3}+\frac{\sqrt{15}\operatorname{atan}\left(\frac{4\sqrt{15}x}{15}-\frac{\sqrt{15}}{15}\right)}{9}-\frac{10\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+3*x**2+x+2),x)
```

```
[Out] x + log(x**2 - x/2 + 1)/6 + log(x**2 + x + 1)/3 + sqrt(15)*atan(4
*sqrt(15)*x/15 - sqrt(15)/15)/9 - 10*sqrt(3)*atan(2*sqrt(3)*x/3 +
 sqrt(3)/3)/9
```

GIAC/XCAS [A] time = 0.263231, size = 76, normalized size = 1.06

$$\frac{1}{9}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right)-\frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+x+\frac{1}{6}\ln\left(2x^2-x+2\right)+\frac{1}{3}\ln\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^3 + 3*x^2 + x + 5)*x/(2*x^4 + x^3 + 3*x^2 + x + 2),x, algorithm="gia
```

```
[Out] 1/9*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) - 10/9*sqrt(3)*arcta
n(1/3*sqrt(3)*(2*x + 1)) + x + 1/6*ln(2*x^2 - x + 2) + 1/3*ln(x^2
 + x + 1)
```

3.246 $\int \frac{5+x+3x^2+2x^3}{2+x+3x^2+x^3+2x^4}\,dx$

Optimal. Leaf size=71

$$\frac{2}{3}\log\left(x^2+x+1\right)-\frac{1}{6}\log\left(2x^2-x+2\right)-\frac{1}{3}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{8\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] -(Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/3 + (8*ArcTan[(1 + 2*x)/S
qrt[3]])/(3*Sqrt[3]) + (2*Log[1 + x + x^2])/3 - Log[2 - x + 2*x^2
]/6
```

Rubi [A] time = 0.166109, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156

$$\frac{2}{3}\log\left(x^2+x+1\right)-\frac{1}{6}\log\left(2x^2-x+2\right)-\frac{1}{3}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{8\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(5 + x + 3*x^2 + 2*x^3)/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

```
[Out] -(Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/3 + (8*ArcTan[(1 + 2*x)/S
qrt[3]])/(3*Sqrt[3]) + (2*Log[1 + x + x^2])/3 - Log[2 - x + 2*x^2
]/6
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*x**3+3*x**2+x+5)/(2*x**4+x**3+3*x**2+x+2),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.0284967, size = 65, normalized size = 0.92

$$\frac{1}{18}\left(12\log\left(x^2+x+1\right)-3\log\left(2x^2-x+2\right)+16\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+2\sqrt{15}\tan^{-1}\left(\frac{4x-1}{\sqrt{15}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(5 + x + 3*x^2 + 2*x^3)/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

```
[Out] (16*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 2*Sqrt[15]*ArcTan[(-1 + 4
*x)/Sqrt[15]] + 12*Log[1 + x + x^2] - 3*Log[2 - x + 2*x^2])/18
```

Maple [A] time = 0.006, size = 56, normalized size = 0.8

$$\frac{2\ln\left(x^2+x+1\right)}{3}+\frac{8\sqrt{3}}{9}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)-\frac{\ln\left(2x^2-x+2\right)}{6}+\frac{\sqrt{15}}{9}\arctan\left(\frac{(-1+4x)\sqrt{15}}{15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x)
```

```
[Out] 2/3*ln(x^2+x+1)+8/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-1/6*ln(2*
x^2-x+2)+1/9*15^(1/2)*arctan(1/15*(-1+4*x)*15^(1/2))
```

Maxima [A] time = 0.895591, size = 74, normalized size = 1.04

$$\frac{1}{9}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right)+\frac{8}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{6}\log\left(2x^2-x+2\right)+\frac{2}{3}\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3 + 3*x^2 + x + 5)/(2*x^4 + x^3 + 3*x^2 + x + 2),x, algorithm="maxim
```

```
[Out] 1/9*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 8/9*sqrt(3)*arctan
(1/3*sqrt(3)*(2*x + 1)) - 1/6*log(2*x^2 - x + 2) + 2/3*log(x^2 +
x + 1)
```

Fricas [A] time = 0.275955, size = 89, normalized size = 1.25

$$\frac{1}{18}\sqrt{3}\left(2\sqrt{5}\arctan\left(\frac{1}{15}\sqrt{5}\sqrt{3}(4x-1)\right)-\sqrt{3}\log\left(2x^2-x+2\right)+4\sqrt{3}\log\left(x^2+x+1\right)+16\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3 + 3*x^2 + x + 5)/(2*x^4 + x^3 + 3*x^2 + x + 2),x, algorithm="frica
```

```
[Out] 1/18*sqrt(3)*(2*sqrt(5)*arctan(1/15*sqrt(5)*sqrt(3)*(4*x - 1)) -
sqrt(3)*log(2*x^2 - x + 2) + 4*sqrt(3)*log(x^2 + x + 1) + 16*arct
an(1/3*sqrt(3)*(2*x + 1)))
```

Sympy [A] time = 0.663386, size = 75, normalized size = 1.06

$$-\frac{\log\left(x^2-\frac{x}{2}+1\right)}{6}+\frac{2\log\left(x^2+x+1\right)}{3}+\frac{\sqrt{15}\,\mathrm{atan}\left(\frac{4\sqrt{15}x}{15}-\frac{\sqrt{15}}{15}\right)}{9}+\frac{8\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x**3+3*x**2+x+5)/(2*x**4+x**3+3*x**2+x+2),x)
```

```
[Out] -log(x**2 - x/2 + 1)/6 + 2*log(x**2 + x + 1)/3 + sqrt(15)*atan(4*
sqrt(15)*x/15 - sqrt(15)/15)/9 + 8*sqrt(3)*atan(2*sqrt(3)*x/3 + s
qrt(3)/3)/9
```

GIAC/XCAS [A] time = 0.263081, size = 74, normalized size = 1.04

$$\frac{1}{9}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right)+\frac{8}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{6}\ln\left(2x^2-x+2\right)+\frac{2}{3}\ln\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3 + 3*x^2 + x + 5)/(2*x^4 + x^3 + 3*x^2 + x + 2),x, algorithm="giac"

[Out] 1/9*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 8/9*sqrt(3)*arctan
(1/3*sqrt(3)*(2*x + 1)) - 1/6*ln(2*x^2 - x + 2) + 2/3*ln(x^2 + x
+ 1)
```

3.247 $\int \frac{5+x+3x^2+2x^3}{x(2+x+3x^2+x^3+2x^4)}\,dx$

Optimal. Leaf size=75

$$-\log\left(x^2+x+1\right)-\frac{1}{4}\log\left(2x^2-x+2\right)+\frac{5\log(x)}{2}+\frac{1}{6}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{2\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] (Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/6 + (2*ArcTan[(1 + 2*x)/Sq
rt[3]])/(3*Sqrt[3]) + (5*Log[x])/2 - Log[1 + x + x^2] - Log[2 - x
 + 2*x^2]/4
```

Rubi [A] time = 0.291587, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171

$$-\log\left(x^2+x+1\right)-\frac{1}{4}\log\left(2x^2-x+2\right)+\frac{5\log(x)}{2}+\frac{1}{6}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{2\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(5 + x + 3*x^2 + 2*x^3)/(x*(2 + x + 3*x^2 + x^3 + 2*x^4)),x]
```

```
[Out] (Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/6 + (2*ArcTan[(1 + 2*x)/Sq
rt[3]])/(3*Sqrt[3]) + (5*Log[x])/2 - Log[1 + x + x^2] - Log[2 - x
 + 2*x^2]/4
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*x**3+3*x**2+x+5)/x/(2*x**4+x**3+3*x**2+x+2),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.0321589, size = 69, normalized size = 0.92

$$\frac{1}{36}\left(-36\log\left(x^2+x+1\right)-9\log\left(2x^2-x+2\right)+90\log(x)+8\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)-2\sqrt{15}\tan^{-1}\left(\frac{4x-1}{\sqrt{15}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(5 + x + 3*x^2 + 2*x^3)/(x*(2 + x + 3*x^2 + x^3 + 2*x^4)),x]
```

```
[Out] (8*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 2*Sqrt[15]*ArcTan[(-1 + 4*
x)/Sqrt[15]] + 90*Log[x] - 36*Log[1 + x + x^2] - 9*Log[2 - x + 2*
x^2])/36
```

Maple [A] time = 0.01, size = 60, normalized size = 0.8

$$-\ln\left(x^2+x+1\right)+\frac{2\sqrt{3}}{9}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)-\frac{\ln\left(2x^2-x+2\right)}{4}-\frac{\sqrt{15}}{18}\arctan\left(\frac{(-1+4x)\sqrt{15}}{15}\right)+\frac{5\ln(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+3*x^2+x+2),x)
```

```
[Out] -ln(x^2+x+1)+2/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-1/4*ln(2*x^2
-x+2)-1/18*15^(1/2)*arctan(1/15*(-1+4*x)*15^(1/2))+5/2*ln(x)
```

Maxima [A] time = 0.919276, size = 80, normalized size = 1.07

$$-\frac{1}{18}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right)+\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{4}\log\left(2x^2-x+2\right)-\log\left(x^2+x+1\right)+\frac{5}{2}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^3 + 3*x^2 + x + 5)/((2*x^4 + x^3 + 3*x^2 + x + 2)*x),x, algorithm="m
```

```
[Out] -1/18*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 2/9*sqrt(3)*arct
an(1/3*sqrt(3)*(2*x + 1)) - 1/4*log(2*x^2 - x + 2) - log(x^2 + x
+ 1) + 5/2*log(x)
```

Fricas [A] time = 0.274499, size = 99, normalized size = 1.32

$$-\frac{1}{36}\sqrt{3}\left(2\sqrt{5}\arctan\left(\frac{1}{15}\sqrt{5}\sqrt{3}(4x-1)\right)+3\sqrt{3}\log\left(2x^2-x+2\right)+12\sqrt{3}\log\left(x^2+x+1\right)-30\sqrt{3}\log\left(x\right)-8\arctan\left(\frac{1}{3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^3 + 3*x^2 + x + 5)/((2*x^4 + x^3 + 3*x^2 + x + 2)*x),x, algorithm="f
```

```
[Out] -1/36*sqrt(3)*(2*sqrt(5)*arctan(1/15*sqrt(5)*sqrt(3)*(4*x - 1)) +
 3*sqrt(3)*log(2*x^2 - x + 2) + 12*sqrt(3)*log(x^2 + x + 1) - 30*
sqrt(3)*log(x) - 8*arctan(1/3*sqrt(3)*(2*x + 1)))
```

Sympy [A] time = 0.900786, size = 78, normalized size = 1.04

$$\frac{5\log\left(x\right)}{2}-\frac{\log\left(x^2-\frac{x}{2}+1\right)}{4}-\log\left(x^2+x+1\right)-\frac{\sqrt{15}\,\mathrm{atan}\left(\frac{4\sqrt{15}x}{15}-\frac{\sqrt{15}}{15}\right)}{18}+\frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((2*x**3+3*x**2+x+5)/x/(2*x**4+x**3+3*x**2+x+2),x)
```

```
[Out] 5*log(x)/2 - log(x**2 - x/2 + 1)/4 - log(x**2 + x + 1) - sqrt(15)
*atan(4*sqrt(15)*x/15 - sqrt(15)/15)/18 + 2*sqrt(3)*atan(2*sqrt(3
)*x/3 + sqrt(3)/3)/9
```

GIAC/XCAS [A] time = 0.262842, size = 81, normalized size = 1.08

$$-\frac{1}{18}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right)+\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{4}\ln\left(2x^2-x+2\right)-\ln\left(x^2+x+1\right)+\frac{5}{2}\ln\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^3 + 3*x^2 + x + 5)/((2*x^4 + x^3 + 3*x^2 + x + 2)*x),x, algorithm="g
```

```
[Out] -1/18*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 2/9*sqrt(3)*arct
an(1/3*sqrt(3)*(2*x + 1)) - 1/4*ln(2*x^2 - x + 2) - ln(x^2 + x +
1) + 5/2*ln(abs(x))
```

3.248 $\int \frac{5+x+3x^2+2x^3}{x^2(2+x+3x^2+x^3+2x^4)}\,dx$

Optimal. Leaf size=84

$$\frac{1}{3}\log\left(x^2+x+1\right)+\frac{1}{24}\log\left(2x^2-x+2\right)-\frac{5}{2x}-\frac{3\log(x)}{4}+\frac{5}{12}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)-\frac{10\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] -5/(2*x) + (5*Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/12 - (10*ArcT
an[(1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]) - (3*Log[x])/4 + Log[1 + x + x
^2]/3 + Log[2 - x + 2*x^2]/24
```

Rubi [A] time = 0.320216, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171

$$\frac{1}{3}\log\left(x^2+x+1\right)+\frac{1}{24}\log\left(2x^2-x+2\right)-\frac{5}{2x}-\frac{3\log(x)}{4}+\frac{5}{12}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)-\frac{10\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(5 + x + 3*x^2 + 2*x^3)/(x^2*(2 + x + 3*x^2 + x^3 + 2*x^4)),x]
```

```
[Out] -5/(2*x) + (5*Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/12 - (10*ArcT
an[(1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]) - (3*Log[x])/4 + Log[1 + x + x
^2]/3 + Log[2 - x + 2*x^2]/24
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*x**3+3*x**2+x+5)/x**2/(2*x**4+x**3+3*x**2+x+2),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.0565308, size = 78, normalized size = 0.93

$$-\frac{-24x\log\left(x^2+x+1\right)-3x\log\left(2x^2-x+2\right)+54x\log(x)+80\sqrt{3}x\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+10\sqrt{15}x\tan^{-1}\left(\frac{4x-1}{\sqrt{15}}\right)+180}{72x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(5 + x + 3*x^2 + 2*x^3)/(x^2*(2 + x + 3*x^2 + x^3 + 2*x^4)),x]
```

```
[Out] -(180 + 80*Sqrt[3]*x*ArcTan[(1 + 2*x)/Sqrt[3]] + 10*Sqrt[15]*x*Ar
cTan[(-1 + 4*x)/Sqrt[15]] + 54*x*Log[x] - 24*x*Log[1 + x + x^2] -
 3*x*Log[2 - x + 2*x^2])/(72*x)
```

Maple [A] time = 0.012, size = 65, normalized size = 0.8

$$\frac{\ln\left(x^2+x+1\right)}{3}-\frac{10\sqrt{3}}{9}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)+\frac{\ln\left(2x^2-x+2\right)}{24}-\frac{5\sqrt{15}}{36}\arctan\left(\frac{(-1+4x)\sqrt{15}}{15}\right)-\frac{5}{2x}-\frac{3\ln(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+3*x^2+x+2),x)
```

```
[Out] 1/3*ln(x^2+x+1)-10/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/24*ln(
2*x^2-x+2)-5/36*15^(1/2)*arctan(1/15*(-1+4*x)*15^(1/2))-5/2/x-3/4
*ln(x)
```

Maxima [A] time = 0.893414, size = 86, normalized size = 1.02

$$-\frac{5}{36}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right)-\frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{5}{2x}+\frac{1}{24}\log\left(2x^2-x+2\right)+\frac{1}{3}\log\left(x^2+x+1\right)-\frac{3}{4}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3 + 3*x^2 + x + 5)/((2*x^4 + x^3 + 3*x^2 + x + 2)*x^2),x, algorithm=
```

```
[Out] -5/36*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) - 10/9*sqrt(3)*arc
tan(1/3*sqrt(3)*(2*x + 1)) - 5/2/x + 1/24*log(2*x^2 - x + 2) + 1/
3*log(x^2 + x + 1) - 3/4*log(x)
```

Fricas [A] time = 0.275648, size = 116, normalized size = 1.38

$$-\frac{\sqrt{3}\left(10\sqrt{5}x\arctan\left(\frac{1}{15}\sqrt{5}\sqrt{3}(4x-1)\right)-\sqrt{3}x\log\left(2x^2-x+2\right)-8\sqrt{3}x\log\left(x^2+x+1\right)+18\sqrt{3}x\log(x)+80x\arctan\left(\right.\right.}{72x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3 + 3*x^2 + x + 5)/((2*x^4 + x^3 + 3*x^2 + x + 2)*x^2),x, algorithm=
```

```
[Out] -1/72*sqrt(3)*(10*sqrt(5)*x*arctan(1/15*sqrt(5)*sqrt(3)*(4*x - 1)
) - sqrt(3)*x*log(2*x^2 - x + 2) - 8*sqrt(3)*x*log(x^2 + x + 1) +
 18*sqrt(3)*x*log(x) + 80*x*arctan(1/3*sqrt(3)*(2*x + 1)) + 60*sq
rt(3))/x
```

Sympy [A] time = 0.942979, size = 87, normalized size = 1.04

$$-\frac{3\log(x)}{4}+\frac{\log\left(x^2-\frac{x}{2}+1\right)}{24}+\frac{\log\left(x^2+x+1\right)}{3}-\frac{5\sqrt{15}\operatorname{atan}\left(\frac{4\sqrt{15}x}{15}-\frac{\sqrt{15}}{15}\right)}{36}-\frac{10\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{9}-\frac{5}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x**3+3*x**2+x+5)/x**2/(2*x**4+x**3+3*x**2+x+2),x)
```

```
[Out] -3*log(x)/4 + log(x**2 - x/2 + 1)/24 + log(x**2 + x + 1)/3 - 5*sq
rt(15)*atan(4*sqrt(15)*x/15 - sqrt(15)/15)/36 - 10*sqrt(3)*atan(2
*sqrt(3)*x/3 + sqrt(3)/3)/9 - 5/(2*x)
```

GIAC/XCAS [A] time = 0.262388, size = 88, normalized size = 1.05

$$-\frac{5}{36}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right)-\frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{5}{2x}+\frac{1}{24}\ln\left(2x^2-x+2\right)+\frac{1}{3}\ln\left(x^2+x+1\right)-\frac{3}{4}\ln\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3 + 3*x^2 + x + 5)/((2*x^4 + x^3 + 3*x^2 + x + 2)*x^2),x, algorithm=
```

```
[Out] -5/36*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) - 10/9*sqrt(3)*arc
tan(1/3*sqrt(3)*(2*x + 1)) - 5/2/x + 1/24*ln(2*x^2 - x + 2) + 1/3
*ln(x^2 + x + 1) - 3/4*ln(abs(x))
```

3.249 $\int \frac{5+x+3x^2+2x^3}{x^3(2+x+3x^2+x^3+2x^4)}\,dx$

Optimal. Leaf size=91

$$-\frac{5}{4x^2}+\frac{2}{3}\log\left(x^2+x+1\right)+\frac{13}{48}\log\left(2x^2-x+2\right)+\frac{3}{4x}-\frac{15\log(x)}{8}+\frac{1}{24}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{8\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] -5/(4*x^2) + 3/(4*x) + (Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/24
+ (8*ArcTan[(1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]) - (15*Log[x])/8 + (2*
Log[1 + x + x^2])/3 + (13*Log[2 - x + 2*x^2])/48
```

Rubi [A] time = 0.330969, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$

$$-\frac{5}{4x^2}+\frac{2}{3}\log\left(x^2+x+1\right)+\frac{13}{48}\log\left(2x^2-x+2\right)+\frac{3}{4x}-\frac{15\log(x)}{8}+\frac{1}{24}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{8\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(5 + x + 3*x^2 + 2*x^3)/(x^3*(2 + x + 3*x^2 + x^3 + 2*x^4)),x]
```

```
[Out] -5/(4*x^2) + 3/(4*x) + (Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/24
+ (8*ArcTan[(1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]) - (15*Log[x])/8 + (2*
Log[1 + x + x^2])/3 + (13*Log[2 - x + 2*x^2])/48
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*x**3+3*x**2+x+5)/x**3/(2*x**4+x**3+3*x**2+x+2),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.102335, size = 82, normalized size = 0.9

$$\frac{1}{144}\left(3\left(-\frac{60}{x^2}+32\log\left(x^2+x+1\right)+13\log\left(2x^2-x+2\right)+\frac{36}{x}-90\log(x)\right)+128\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)-2\sqrt{15}\tan^{-1}\left(\frac{4x-1}{\sqrt{15}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(5 + x + 3*x^2 + 2*x^3)/(x^3*(2 + x + 3*x^2 + x^3 + 2*x^4)),x]
```

```
[Out] (128*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 2*Sqrt[15]*ArcTan[(-1 +
4*x)/Sqrt[15]] + 3*(-60/x^2 + 36/x - 90*Log[x] + 32*Log[1 + x + x
^2] + 13*Log[2 - x + 2*x^2]))/144
```

Maple [A] time = 0.012, size = 70, normalized size = 0.8

$$\frac{2\,\ln\left(x^{2}+x+1\right)}{3}+\frac{8\,\sqrt{3}}{9}\arctan\left(\frac{\left(1+2\,x\right)\sqrt{3}}{3}\right)+\frac{13\,\ln\left(2\,x^{2}-x+2\right)}{48}-\frac{\sqrt{15}}{72}\arctan\left(\frac{\left(-1+4\,x\right)\sqrt{15}}{15}\right)-\frac{5}{4\,x^{2}}+\frac{3}{4\,x}-\frac{15\,\ln\left(x\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+3*x^2+x+2),x)
```

```
[Out] 2/3*ln(x^2+x+1)+8/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+13/48*ln(
2*x^2-x+2)-1/72*15^(1/2)*arctan(1/15*(-1+4*x)*15^(1/2))-5/4/x^2+3
/4/x-15/8*ln(x)
```

Maxima [A] time = 0.918459, size = 93, normalized size = 1.02

$$-\frac{1}{72}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4\,x-1)\right)+\frac{8}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x+1)\right)+\frac{3\,x-5}{4\,x^{2}}+\frac{13}{48}\log\left(2\,x^{2}-x+2\right)+\frac{2}{3}\log\left(x^{2}+x+1\right)-\frac{15}{8}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^3 + 3*x^2 + x + 5)/((2*x^4 + x^3 + 3*x^2 + x + 2)*x^3),x, algorithm=
```

```
[Out] -1/72*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 8/9*sqrt(3)*arct
an(1/3*sqrt(3)*(2*x + 1)) + 1/4*(3*x - 5)/x^2 + 13/48*log(2*x^2 -
 x + 2) + 2/3*log(x^2 + x + 1) - 15/8*log(x)
```

Fricas [A] time = 0.276062, size = 136, normalized size = 1.49

$$-\frac{\sqrt{3}\left(2\,\sqrt{5}x^{2}\arctan\left(\frac{1}{15}\,\sqrt{5}\sqrt{3}(4\,x-1)\right)-13\,\sqrt{3}x^{2}\log\left(2\,x^{2}-x+2\right)-32\,\sqrt{3}x^{2}\log\left(x^{2}+x+1\right)+90\,\sqrt{3}x^{2}\log\left(x\right)-128\,x^{2}\right.}{144\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^3 + 3*x^2 + x + 5)/((2*x^4 + x^3 + 3*x^2 + x + 2)*x^3),x, algorithm=
```

```
[Out] -1/144*sqrt(3)*(2*sqrt(5)*x^2*arctan(1/15*sqrt(5)*sqrt(3)*(4*x -
1)) - 13*sqrt(3)*x^2*log(2*x^2 - x + 2) - 32*sqrt(3)*x^2*log(x^2
+ x + 1) + 90*sqrt(3)*x^2*log(x) - 128*x^2*arctan(1/3*sqrt(3)*(2*
x + 1)) - 12*sqrt(3)*(3*x - 5))/x^2
```

Sympy [A] time = 1.01566, size = 94, normalized size = 1.03

$$-\frac{15\log\left(x\right)}{8}+\frac{13\log\left(x^{2}-\frac{x}{2}+1\right)}{48}+\frac{2\log\left(x^{2}+x+1\right)}{3}-\frac{\sqrt{15}\operatorname{atan}\left(\frac{4\sqrt{15}x}{15}-\frac{\sqrt{15}}{15}\right)}{72}+\frac{8\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{9}+\frac{3x-5}{4x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((2*x**3+3*x**2+x+5)/x**3/(2*x**4+x**3+3*x**2+x+2),x)
```

```
[Out] -15*log(x)/8 + 13*log(x**2 - x/2 + 1)/48 + 2*log(x**2 + x + 1)/3
- sqrt(15)*atan(4*sqrt(15)*x/15 - sqrt(15)/15)/72 + 8*sqrt(3)*ata
n(2*sqrt(3)*x/3 + sqrt(3)/3)/9 + (3*x - 5)/(4*x**2)
```

GIAC/XCAS [A] time = 0.263831, size = 95, normalized size = 1.04

$$-\frac{1}{72}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right)+\frac{8}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{3x-5}{4x^2}+\frac{13}{48}\ln\left(2x^2-x+2\right)+\frac{2}{3}\ln\left(x^2+x+1\right)-\frac{15}{8}\ln\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3 + 3*x^2 + x + 5)/((2*x^4 + x^3 + 3*x^2 + x + 2)*x^3),x, algorithm=
```

```
[Out] -1/72*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 8/9*sqrt(3)*arct
an(1/3*sqrt(3)*(2*x + 1)) + 1/4*(3*x - 5)/x^2 + 13/48*ln(2*x^2 -
x + 2) + 2/3*ln(x^2 + x + 1) - 15/8*ln(abs(x))
```

3.250 $\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx$

Optimal. Leaf size=307

$$\frac{1}{42}\left(7+5i\sqrt{7}\right)x^3+\frac{1}{42}\left(7-5i\sqrt{7}\right)x^3+\frac{1}{28}\left(7+5i\sqrt{7}\right)x^2+\frac{1}{28}\left(7-5i\sqrt{7}\right)x^2+\frac{3}{112}\left(7-11i\sqrt{7}\right)\log\left(4x^2+\left(1-i\sqrt{7}\right)x+4\right)+\frac{3}{112}\left(7+11i\sqrt{7}\right)\log\left(4x^2+\left(1+i\sqrt{7}\right)x+4\right)-\frac{1}{28}\left(35+9i\sqrt{7}\right)x-\frac{1}{28}\left(35-9i\sqrt{7}\right)x+\frac{11\left(5\sqrt{7}+9i\right)\tan^{-1}\left(\frac{8x-i\sqrt{7}+1}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{4\sqrt{14\left(35+i\sqrt{7}\right)}}-\frac{11\left(-5\sqrt{7}+9i\right)\tan^{-1}\left(\frac{8x+i\sqrt{7}+1}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{4\sqrt{14\left(35-i\sqrt{7}\right)}}$$

```
[Out] -((35 - (9*I)*Sqrt[7])*x)/28 - ((35 + (9*I)*Sqrt[7])*x)/28 + ((7
- (5*I)*Sqrt[7])*x^2)/28 + ((7 + (5*I)*Sqrt[7])*x^2)/28 + ((7 - (
5*I)*Sqrt[7])*x^3)/42 + ((7 + (5*I)*Sqrt[7])*x^3)/42 + (11*(9*I +
 5*Sqrt[7])*ArcTan[(1 - I*Sqrt[7] + 8*x)/Sqrt[2*(35 + I*Sqrt[7])]
])/(4*Sqrt[14*(35 + I*Sqrt[7])]) - (11*(9*I - 5*Sqrt[7])*ArcTan[(
1 + I*Sqrt[7] + 8*x)/Sqrt[2*(35 - I*Sqrt[7])]])/(4*Sqrt[14*(35 -
I*Sqrt[7])]) + (3*(7 - (11*I)*Sqrt[7])*Log[4 + (1 - I*Sqrt[7])*x
+ 4*x^2])/112 + (3*(7 + (11*I)*Sqrt[7])*Log[4 + (1 + I*Sqrt[7])*x
 + 4*x^2])/112
```

Rubi [A] time = 1.30302, antiderivative size = 307, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171

$$\frac{1}{42}\left(7+5i\sqrt{7}\right)x^3+\frac{1}{42}\left(7-5i\sqrt{7}\right)x^3+\frac{1}{28}\left(7+5i\sqrt{7}\right)x^2+\frac{1}{28}\left(7-5i\sqrt{7}\right)x^2+\frac{3}{112}\left(7-11i\sqrt{7}\right)\log\left(4x^2+\left(1-i\sqrt{7}\right)x+4\right)+\frac{3}{112}\left(7+11i\sqrt{7}\right)\log\left(4x^2+\left(1+i\sqrt{7}\right)x+4\right)-\frac{1}{28}\left(35+9i\sqrt{7}\right)x-\frac{1}{28}\left(35-9i\sqrt{7}\right)x+\frac{11\left(5\sqrt{7}+9i\right)\tan^{-1}\left(\frac{8x-i\sqrt{7}+1}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{4\sqrt{14\left(35+i\sqrt{7}\right)}}-\frac{11\left(-5\sqrt{7}+9i\right)\tan^{-1}\left(\frac{8x+i\sqrt{7}+1}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{4\sqrt{14\left(35-i\sqrt{7}\right)}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x^4),x]
```


```
[Out] -((35 - (9*I)*Sqrt[7])*x)/28 - ((35 + (9*I)*Sqrt[7])*x)/28 + ((7
- (5*I)*Sqrt[7])*x^2)/28 + ((7 + (5*I)*Sqrt[7])*x^2)/28 + ((7 - (
5*I)*Sqrt[7])*x^3)/42 + ((7 + (5*I)*Sqrt[7])*x^3)/42 + (11*(9*I +
 5*Sqrt[7])*ArcTan[(1 - I*Sqrt[7] + 8*x)/Sqrt[2*(35 + I*Sqrt[7])]
])/(4*Sqrt[14*(35 + I*Sqrt[7])]) - (11*(9*I - 5*Sqrt[7])*ArcTan[(
1 + I*Sqrt[7] + 8*x)/Sqrt[2*(35 - I*Sqrt[7])]])/(4*Sqrt[14*(35 -
I*Sqrt[7])]) + (3*(7 - (11*I)*Sqrt[7])*Log[4 + (1 - I*Sqrt[7])*x
+ 4*x^2])/112 + (3*(7 + (11*I)*Sqrt[7])*Log[4 + (1 + I*Sqrt[7])*x
 + 4*x^2])/112
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$x^3\left(\frac{1}{6}-\frac{5\sqrt{7}i}{42}\right)+x^3\left(\frac{1}{6}+\frac{5\sqrt{7}i}{42}\right)+\left(\frac{3}{16}-\frac{33\sqrt{7}i}{112}\right)\log\left(4x^2+x\left(1-\sqrt{7}i\right)+4\right)$$
$$+\left(\frac{3}{16}+\frac{33\sqrt{7}i}{112}\right)\log\left(4x^2+x\left(1+\sqrt{7}i\right)+4\right)-\frac{\left(\frac{55}{4}-\frac{99\sqrt{7}i}{28}\right)\operatorname{atan}\left(\frac{8x+1+\sqrt{7}i}{\sqrt{35+4\sqrt{77}}-i\sqrt{-35+4\sqrt{77}}}\right)}{-\sqrt{35+4\sqrt{77}}+i\sqrt{-35+4\sqrt{77}}}$$
$$+\frac{\left(\frac{55}{4}+\frac{99\sqrt{7}i}{28}\right)\operatorname{atan}\left(\frac{8x+1-\sqrt{7}i}{\sqrt{35+4\sqrt{77}}+i\sqrt{-35+4\sqrt{77}}}\right)}{\sqrt{35+4\sqrt{77}}+i\sqrt{-35+4\sqrt{77}}}+\int\left(-\frac{5}{4}\right)dx$$
$$-\int\frac{5}{4}\,dx+\left(\frac{1}{2}-\frac{5\sqrt{7}i}{14}\right)\int x\,dx+\left(\frac{1}{2}+\frac{5\sqrt{7}i}{14}\right)\int x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**3*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+5*x**2+x+2),x)
```

```
[Out] x**3*(1/6 - 5*sqrt(7)*I/42) + x**3*(1/6 + 5*sqrt(7)*I/42) + (3/16
 - 33*sqrt(7)*I/112)*log(4*x**2 + x*(1 - sqrt(7)*I) + 4) + (3/16
+ 33*sqrt(7)*I/112)*log(4*x**2 + x*(1 + sqrt(7)*I) + 4) - (55/4 -
 99*sqrt(7)*I/28)*atan((8*x + 1 + sqrt(7)*I)/(sqrt(35 + 4*sqrt(77
)) - I*sqrt(-35 + 4*sqrt(77))))/(-sqrt(35 + 4*sqrt(77)) + I*sqrt(
-35 + 4*sqrt(77))) + (55/4 + 99*sqrt(7)*I/28)*atan((8*x + 1 - sqr
t(7)*I)/(sqrt(35 + 4*sqrt(77)) + I*sqrt(-35 + 4*sqrt(77))))/(sqrt
(35 + 4*sqrt(77)) + I*sqrt(-35 + 4*sqrt(77))) + Integral(-5/4, x)
 - Integral(5/4, x) + (1/2 - 5*sqrt(7)*I/14)*Integral(x, x) + (1/
2 + 5*sqrt(7)*I/14)*Integral(x, x)
```

Mathematica [C] time = 0.032629, size = 109, normalized size = 0.36

$$\frac{1}{6}\left(3\text{RootSum}\left[2\#1^4+\#1^3+5\#1^2+\#1+2\&,\frac{3\#1^3\log(x-\#1)+19\#1^2\log(x-\#1)+\#1\log(x-\#1)+10\log(x-\#1)}{8\#1^3+3\#1^2+10\#1+1}\&\right]+x\left(2x^2+3x-15\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x^4),x]
```

```
[Out] (x*(-15 + 3*x + 2*x^2) + 3*RootSum[2 + #1 + 5*#1^2 + #1^3 + 2*#1^
4 & , (10*Log[x - #1] + Log[x - #1]*#1 + 19*Log[x - #1]*#1^2 + 3*
Log[x - #1]*#1^3)/(1 + 10*#1 + 3*#1^2 + 8*#1^3) & ])/6
```

Maple [C] time = 0.01, size = 74, normalized size = 0.2

$$\frac{x^3}{3}+\frac{x^2}{2}-\frac{5x}{2}+\frac{1}{2}\sum_{\_R=RootOf\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)}\frac{\left(3\_R^3+19\_R^2+\_R+10\right)\ln\left(x-\_R\right)}{8\_R^3+3\_R^2+10\_R+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x)
```

```
[Out] 1/3*x^3+1/2*x^2-5/2*x+1/2*sum((3*_R^3+19*_R^2+_R+10)/(8*_R^3+3*_R
^2+10*_R+1)*ln(x-_R),_R=RootOf(2*_Z^4+_Z^3+5*_Z^2+_Z+2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{3}x^3 + \frac{1}{2}x^2 - \frac{5}{2}x + \frac{1}{2}\int \frac{3x^3 + 19x^2 + x + 10}{2x^4 + x^3 + 5x^2 + x + 2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3 + 3*x^2 + x + 5)*x^3/(2*x^4 + x^3 + 5*x^2 + x + 2),x, algorithm="m
```

```
[Out] 1/3*x^3 + 1/2*x^2 - 5/2*x + 1/2*integrate((3*x^3 + 19*x^2 + x + 1
0)/(2*x^4 + x^3 + 5*x^2 + x + 2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3 + 3*x^2 + x + 5)*x^3/(2*x^4 + x^3 + 5*x^2 + x + 2),x, algorithm="f
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 3.4075, size = 61, normalized size = 0.2

$$\frac{x^3}{3} + \frac{x^2}{2} - \frac{5x}{2} + \text{RootSum}\left(1372t^4 - 1029t^3 + 3136t^2 + 2688t + 512, \left(t \mapsto t\log\left(\frac{5831t^3}{1936} - \frac{23765t^2}{7744} + \frac{2065t}{242} + x + \frac{415}{121}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+5*x**2+x+2),x)
```

```
[Out] x**3/3 + x**2/2 - 5*x/2 + RootSum(1372*_t**4 - 1029*_t**3 + 3136*
_t**2 + 2688*_t + 512, Lambda(_t, _t*log(5831*_t**3/1936 - 23765*
_t**2/7744 + 2065*_t/242 + x + 415/121)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2x^3 + 3x^2 + x + 5)x^3}{2x^4 + x^3 + 5x^2 + x + 2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3 + 3*x^2 + x + 5)*x^3/(2*x^4 + x^3 + 5*x^2 + x + 2),x, algorithm="g
```

```
[Out] integrate((2*x^3 + 3*x^2 + x + 5)*x^3/(2*x^4 + x^3 + 5*x^2 + x +
2), x)
```

3.251 $\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx$

Optimal. Leaf size=269

$$\frac{1}{28}\left(7+5i\sqrt{7}\right)x^2+\frac{1}{28}\left(7-5i\sqrt{7}\right)x^2-\frac{1}{56}\left(35+9i\sqrt{7}\right)\log\left(4x^2+\left(1-i\sqrt{7}\right)x+4\right)-\frac{1}{56}\left(35-9i\sqrt{7}\right)\log\left(4x^2+\left(1+i\sqrt{7}\right)x+4\right)+\frac{1}{14}\left(7+5i\sqrt{7}\right)x+\frac{1}{14}\left(7-5i\sqrt{7}\right)x-\frac{\left(\sqrt{7}+53i\right)\tan^{-1}\left(\frac{8x-i\sqrt{7}+1}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{2\sqrt{14\left(35+i\sqrt{7}\right)}}+\frac{\left(-\sqrt{7}+53i\right)\tan^{-1}\left(\frac{8x+i\sqrt{7}+1}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{2\sqrt{14\left(35-i\sqrt{7}\right)}}$$

```
[Out] ((7 - (5*I)*Sqrt[7])*x)/14 + ((7 + (5*I)*Sqrt[7])*x)/14 + ((7 - (
5*I)*Sqrt[7])*x^2)/28 + ((7 + (5*I)*Sqrt[7])*x^2)/28 - ((53*I + S
qrt[7])*ArcTan[(1 - I*Sqrt[7] + 8*x)/Sqrt[2*(35 + I*Sqrt[7])]])/(
2*Sqrt[14*(35 + I*Sqrt[7])]) + ((53*I - Sqrt[7])*ArcTan[(1 + I*Sq
rt[7] + 8*x)/Sqrt[2*(35 - I*Sqrt[7])]])/(2*Sqrt[14*(35 - I*Sqrt[7
])]) - ((35 + (9*I)*Sqrt[7])*Log[4 + (1 - I*Sqrt[7])*x + 4*x^2])/
56 - ((35 - (9*I)*Sqrt[7])*Log[4 + (1 + I*Sqrt[7])*x + 4*x^2])/56
```

Rubi [A] time = 1.04639, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171

$$\frac{1}{28}\left(7+5i\sqrt{7}\right)x^2+\frac{1}{28}\left(7-5i\sqrt{7}\right)x^2-\frac{1}{56}\left(35+9i\sqrt{7}\right)\log\left(4x^2+\left(1-i\sqrt{7}\right)x+4\right)-\frac{1}{56}\left(35-9i\sqrt{7}\right)\log\left(4x^2+\left(1+i\sqrt{7}\right)x+4\right)+\frac{1}{14}\left(7+5i\sqrt{7}\right)x+\frac{1}{14}\left(7-5i\sqrt{7}\right)x-\frac{\left(\sqrt{7}+53i\right)\tan^{-1}\left(\frac{8x-i\sqrt{7}+1}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{2\sqrt{14\left(35+i\sqrt{7}\right)}}+\frac{\left(-\sqrt{7}+53i\right)\tan^{-1}\left(\frac{8x+i\sqrt{7}+1}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{2\sqrt{14\left(35-i\sqrt{7}\right)}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x^4),x]
```

```
[Out] ((7 - (5*I)*Sqrt[7])*x)/14 + ((7 + (5*I)*Sqrt[7])*x)/14 + ((7 - (
5*I)*Sqrt[7])*x^2)/28 + ((7 + (5*I)*Sqrt[7])*x^2)/28 - ((53*I + S
qrt[7])*ArcTan[(1 - I*Sqrt[7] + 8*x)/Sqrt[2*(35 + I*Sqrt[7])]])/(
2*Sqrt[14*(35 + I*Sqrt[7])]) + ((53*I - Sqrt[7])*ArcTan[(1 + I*Sq
rt[7] + 8*x)/Sqrt[2*(35 - I*Sqrt[7])]])/(2*Sqrt[14*(35 - I*Sqrt[7
])]) - ((35 + (9*I)*Sqrt[7])*Log[4 + (1 - I*Sqrt[7])*x + 4*x^2])/
56 - ((35 - (9*I)*Sqrt[7])*Log[4 + (1 + I*Sqrt[7])*x + 4*x^2])/56
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\left(\frac{5}{8}+\frac{9\sqrt{7}i}{56}\right)\log\left(4x^2+x\left(1-\sqrt{7}i\right)+4\right)-\left(\frac{5}{8}-\frac{9\sqrt{7}i}{56}\right)\log\left(4x^2+x\left(1+\sqrt{7}i\right)+4\right)+\frac{\left(7-53\sqrt{7}i\right)\operatorname{atan}\left(\frac{8x+1+\sqrt{7}i}{\sqrt{35+4\sqrt{77}}-i\sqrt{-35+4\sqrt{77}}}\right)}{14\left(-\sqrt{35+4\sqrt{77}}+i\sqrt{-35+4\sqrt{77}}\right)}-\frac{\left(7+53\sqrt{7}i\right)\operatorname{atan}\left(\frac{8x+1-\sqrt{7}i}{\sqrt{35+4\sqrt{77}}+i\sqrt{-35+4\sqrt{77}}}\right)}{14\left(\sqrt{35+4\sqrt{77}}+i\sqrt{-35+4\sqrt{77}}\right)}-\int\left(-\frac{1}{2}\right)dx+\int\frac{1}{2}\,dx+\left(\frac{1}{2}-\frac{5\sqrt{7}i}{14}\right)\int x\,dx+\left(\frac{1}{2}+\frac{5\sqrt{7}i}{14}\right)\int x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**2*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+5*x**2+x+2),x)
```

```
[Out] -(5/8 + 9*sqrt(7)*I/56)*log(4*x**2 + x*(1 - sqrt(7)*I) + 4) - (5/
8 - 9*sqrt(7)*I/56)*log(4*x**2 + x*(1 + sqrt(7)*I) + 4) + (7 - 53
*sqrt(7)*I)*atan((8*x + 1 + sqrt(7)*I)/(sqrt(35 + 4*sqrt(77)) - I
*sqrt(-35 + 4*sqrt(77))))/(14*(-sqrt(35 + 4*sqrt(77)) + I*sqrt(-3
5 + 4*sqrt(77)))) - (7 + 53*sqrt(7)*I)*atan((8*x + 1 - sqrt(7)*I)
/(sqrt(35 + 4*sqrt(77)) + I*sqrt(-35 + 4*sqrt(77))))/(14*(sqrt(35
 + 4*sqrt(77)) + I*sqrt(-35 + 4*sqrt(77)))) - Integral(-1/2, x) +
 Integral(1/2, x) + (1/2 - 5*sqrt(7)*I/14)*Integral(x, x) + (1/2
+ 5*sqrt(7)*I/14)*Integral(x, x)
```

Mathematica [C] time = 0.0269464, size = 101, normalized size = 0.38

$$-\text{RootSum}\left[2\#1^4+\#1^3+5\#1^2+\#1+2\&,\frac{5\#1^3\log(x-\#1)+\#1^2\log(x-\#1)+3\#1\log(x-\#1)+2\log(x-\#1)}{8\#1^3+3\#1^2+10\#1+1}\&\right]+\frac{x^2}{2}+x$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x^4),x]
```

```
[Out] x + x^2/2 - RootSum[2 + #1 + 5*#1^2 + #1^3 + 2*#1^4 & , (2*Log[x
- #1] + 3*Log[x - #1]*#1 + Log[x - #1]*#1^2 + 5*Log[x - #1]*#1^3)
/(1 + 10*#1 + 3*#1^2 + 8*#1^3) & ]
```

Maple [C] time = 0.009, size = 67, normalized size = 0.3

$$x+\frac{x^2}{2}+\sum_{\_R=RootOf\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)}\frac{\left(-5\_R^3-\_R^2-3\_R-2\right)\ln\left(x-\_R\right)}{8\_R^3+3\_R^2+10\_R+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x)
```

```
[Out] x+1/2*x^2+sum((-5*_R^3-_R^2-3*_R-2)/(8*_R^3+3*_R^2+10*_R+1)*ln(x-
_R),_R=RootOf(2*_Z^4+_Z^3+5*_Z^2+_Z+2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}x^2+x-\int\frac{5x^3+x^2+3x+2}{2x^4+x^3+5x^2+x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3 + 3*x^2 + x + 5)*x^2/(2*x^4 + x^3 + 5*x^2 + x + 2),x, algorithm="m
```

```
[Out] 1/2*x^2 + x - integrate((5*x^3 + x^2 + 3*x + 2)/(2*x^4 + x^3 + 5*
x^2 + x + 2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3 + 3*x^2 + x + 5)*x^2/(2*x^4 + x^3 + 5*x^2 + x + 2),x, algorithm="f
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 2.97885, size = 53, normalized size = 0.2

$$\frac{x^2}{2} + x + \operatorname{RootSum}\left(686t^4 + 1715t^3 + 1372t^2 + 448t + 256, \left(t \mapsto t\log\left(\frac{5145t^3}{4192} + \frac{1421t^2}{8384} - \frac{2541t}{2096} + x + \frac{17}{262}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+5*x**2+x+2),x)
```

```
[Out] x**2/2 + x + RootSum(686*_t**4 + 1715*_t**3 + 1372*_t**2 + 448*_t
 + 256, Lambda(_t, _t*log(5145*_t**3/4192 + 1421*_t**2/8384 - 254
1*_t/2096 + x + 17/262)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(2\,x^3 + 3\,x^2 + x + 5\right)x^2}{2\,x^4 + x^3 + 5\,x^2 + x + 2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3 + 3*x^2 + x + 5)*x^2/(2*x^4 + x^3 + 5*x^2 + x + 2),x, algorithm="g
```

```
[Out] integrate((2*x^3 + 3*x^2 + x + 5)*x^2/(2*x^4 + x^3 + 5*x^2 + x +
2), x)
```

3.252 $\int \frac{x(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx$

Optimal. Leaf size=230

$$\frac{1}{28}\left(7+5i\sqrt{7}\right)\log\left(4x^2+\left(1-i\sqrt{7}\right)x+4\right)+\frac{1}{28}\left(7-5i\sqrt{7}\right)\log\left(4x^2+\left(1+i\sqrt{7}\right)x+4\right)+\frac{1}{14}\left(7+5i\sqrt{7}\right)x+\frac{1}{14}\left(7-5i\sqrt{7}\right)x-\frac{\left(7\sqrt{7}+19i\right)\tan^{-1}\left(\frac{8x-i\sqrt{7}+1}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{\sqrt{14\left(35+i\sqrt{7}\right)}}+\frac{\left(-7\sqrt{7}+19i\right)\tan^{-1}\left(\frac{8x+i\sqrt{7}+1}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{\sqrt{14\left(35-i\sqrt{7}\right)}}$$

```
[Out] ((7 - (5*I)*Sqrt[7])*x)/14 + ((7 + (5*I)*Sqrt[7])*x)/14 - ((19*I
+ 7*Sqrt[7])*ArcTan[(1 - I*Sqrt[7] + 8*x)/Sqrt[2*(35 + I*Sqrt[7])
]])/Sqrt[14*(35 + I*Sqrt[7])] + ((19*I - 7*Sqrt[7])*ArcTan[(1 + I
*Sqrt[7] + 8*x)/Sqrt[2*(35 - I*Sqrt[7])]])/Sqrt[14*(35 - I*Sqrt[7
])] + ((7 + (5*I)*Sqrt[7])*Log[4 + (1 - I*Sqrt[7])*x + 4*x^2])/28
 + ((7 - (5*I)*Sqrt[7])*Log[4 + (1 + I*Sqrt[7])*x + 4*x^2])/28
```

Rubi [A] time = 0.950496, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{1}{28}\left(7+5i\sqrt{7}\right)\log\left(4x^2+\left(1-i\sqrt{7}\right)x+4\right)+\frac{1}{28}\left(7-5i\sqrt{7}\right)\log\left(4x^2+\left(1+i\sqrt{7}\right)x+4\right)+\frac{1}{14}\left(7+5i\sqrt{7}\right)x+\frac{1}{14}\left(7-5i\sqrt{7}\right)x-\frac{\left(7\sqrt{7}+19i\right)\tan^{-1}\left(\frac{8x-i\sqrt{7}+1}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{\sqrt{14\left(35+i\sqrt{7}\right)}}+\frac{\left(-7\sqrt{7}+19i\right)\tan^{-1}\left(\frac{8x+i\sqrt{7}+1}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{\sqrt{14\left(35-i\sqrt{7}\right)}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x^4),x]
```

```
[Out] ((7 - (5*I)*Sqrt[7])*x)/14 + ((7 + (5*I)*Sqrt[7])*x)/14 - ((19*I
+ 7*Sqrt[7])*ArcTan[(1 - I*Sqrt[7] + 8*x)/Sqrt[2*(35 + I*Sqrt[7])
]])/Sqrt[14*(35 + I*Sqrt[7])] + ((19*I - 7*Sqrt[7])*ArcTan[(1 + I
*Sqrt[7] + 8*x)/Sqrt[2*(35 - I*Sqrt[7])]])/Sqrt[14*(35 - I*Sqrt[7
])] + ((7 + (5*I)*Sqrt[7])*Log[4 + (1 - I*Sqrt[7])*x + 4*x^2])/28
 + ((7 - (5*I)*Sqrt[7])*Log[4 + (1 + I*Sqrt[7])*x + 4*x^2])/28
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\left(\frac{1}{4}+\frac{5\sqrt{7}i}{28}\right)\log\left(4x^2+x\left(1-\sqrt{7}i\right)+4\right)+\left(\frac{1}{4}-\frac{5\sqrt{7}i}{28}\right)\log\left(4x^2+x\left(1+\sqrt{7}i\right)+4\right)+\frac{\left(7-\frac{19\sqrt{7}i}{7}\right)\operatorname{atan}\left(\frac{8x+1+\sqrt{7}i}{\sqrt{35+4\sqrt{77}}-i\sqrt{-35+4\sqrt{77}}}\right)}{-\sqrt{35+4\sqrt{77}}+i\sqrt{-35+4\sqrt{77}}}-\frac{\left(7+\frac{19\sqrt{7}i}{7}\right)\operatorname{atan}\left(\frac{8x+1-\sqrt{7}i}{\sqrt{35+4\sqrt{77}}+i\sqrt{-35+4\sqrt{77}}}\right)}{\sqrt{35+4\sqrt{77}}+i\sqrt{-35+4\sqrt{77}}}-\int\left(-\frac{1}{2}\right)dx+\int\frac{1}{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+5*x**2+x+2),x)
```

```
[Out] (1/4 + 5*sqrt(7)*I/28)*log(4*x**2 + x*(1 - sqrt(7)*I) + 4) + (1/4
 - 5*sqrt(7)*I/28)*log(4*x**2 + x*(1 + sqrt(7)*I) + 4) + (7 - 19*
sqrt(7)*I/7)*atan((8*x + 1 + sqrt(7)*I)/(sqrt(35 + 4*sqrt(77)) -
I*sqrt(-35 + 4*sqrt(77))))/(-sqrt(35 + 4*sqrt(77)) + I*sqrt(-35 +
 4*sqrt(77))) - (7 + 19*sqrt(7)*I/7)*atan((8*x + 1 - sqrt(7)*I)/(
sqrt(35 + 4*sqrt(77)) + I*sqrt(-35 + 4*sqrt(77))))/(sqrt(35 + 4*s
qrt(77)) + I*sqrt(-35 + 4*sqrt(77))) - Integral(-1/2, x) + Integr
al(1/2, x)
```

Mathematica [C] time = 0.0249216, size = 94, normalized size = 0.41

$$2\text{RootSum}\left[2\#1^4+\#1^3+5\#1^2+\#1+2\&,\frac{\#1^3\log(x-\#1)-2\#1^2\log(x-\#1)+2\#1\log(x-\#1)-\log(x-\#1)}{8\#1^3+3\#1^2+10\#1+1}\&\right]+x$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x^4),x]
```

```
[Out] x + 2*RootSum[2 + #1 + 5*#1^2 + #1^3 + 2*#1^4 & , (-Log[x - #1] +
 2*Log[x - #1]*#1 - 2*Log[x - #1]*#1^2 + Log[x - #1]*#1^3)/(1 + 1
0*#1 + 3*#1^2 + 8*#1^3) & ]
```

Maple [C] time = 0.009, size = 62, normalized size = 0.3

$$x+2\sum_{\_R=RootOf\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)}\frac{\left(\_R^3-2\_R^2+2\_R-1\right)\ln\left(x-\_R\right)}{8\_R^3+3\_R^2+10\_R+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x)
```

```
[Out] x+2*sum((_R^3-2*_R^2+2*_R-1)/(8*_R^3+3*_R^2+10*_R+1)*ln(x-_R),_R=
RootOf(2*_Z^4+_Z^3+5*_Z^2+_Z+2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$x+2\int\frac{x^3-2x^2+2x-1}{2x^4+x^3+5x^2+x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^3 + 3*x^2 + x + 5)*x/(2*x^4 + x^3 + 5*x^2 + x + 2),x, algorithm="max
```

```
[Out] x + 2*integrate((x^3 - 2*x^2 + 2*x - 1)/(2*x^4 + x^3 + 5*x^2 + x
+ 2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3 + 3*x^2 + x + 5)*x/(2*x^4 + x^3 + 5*x^2 + x + 2),x, algorithm="fri
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 2.83781, size = 48, normalized size = 0.21

$$x + \operatorname{RootSum}\left(343t^4 - 343t^3 + 294t^2 - 336t + 128, \left(t \mapsto t\log\left(\frac{3773t^3}{304} - \frac{1029t^2}{304} + \frac{1001t}{152} + x - \frac{121}{19}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+5*x**2+x+2),x)
```

```
[Out] x + RootSum(343*_t**4 - 343*_t**3 + 294*_t**2 - 336*_t + 128, Lam
bda(_t, _t*log(3773*_t**3/304 - 1029*_t**2/304 + 1001*_t/152 + x
- 121/19)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(2\,x^3 + 3\,x^2 + x + 5\right)x}{2\,x^4 + x^3 + 5\,x^2 + x + 2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3 + 3*x^2 + x + 5)*x/(2*x^4 + x^3 + 5*x^2 + x + 2),x, algorithm="gia
```

```
[Out] integrate((2*x^3 + 3*x^2 + x + 5)*x/(2*x^4 + x^3 + 5*x^2 + x + 2)
, x)
```

3.253 $\int \frac{5+x+3x^2+2x^3}{2+x+5x^2+x^3+2x^4} \, dx$

Optimal. Leaf size=198

$$\frac{1}{28}\left(7+5i\sqrt{7}\right)\log\left(4x^2+\left(1-i\sqrt{7}\right)x+4\right)+\frac{1}{28}\left(7-5i\sqrt{7}\right)\log\left(4x^2+\left(1+i\sqrt{7}\right)x+4\right)+\frac{\left(7\sqrt{7}+19i\right)\tan^{-1}\left(\frac{8x-i\sqrt{7}+1}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{\sqrt{14\left(35+i\sqrt{7}\right)}}-\frac{\left(-7\sqrt{7}+19i\right)\tan^{-1}\left(\frac{8x+i\sqrt{7}+1}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{\sqrt{14\left(35-i\sqrt{7}\right)}}$$

```
[Out] ((19*I + 7*Sqrt[7])*ArcTan[(1 - I*Sqrt[7] + 8*x)/Sqrt[2*(35 + I*S
qrt[7])]])/Sqrt[14*(35 + I*Sqrt[7])] - ((19*I - 7*Sqrt[7])*ArcTan
[(1 + I*Sqrt[7] + 8*x)/Sqrt[2*(35 - I*Sqrt[7])]])/Sqrt[14*(35 - I
*Sqrt[7])] + ((7 + (5*I)*Sqrt[7])*Log[4 + (1 - I*Sqrt[7])*x + 4*x
^2])/28 + ((7 - (5*I)*Sqrt[7])*Log[4 + (1 + I*Sqrt[7])*x + 4*x^2]
)/28
```

Rubi [A] time = 0.560258, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156

$$\frac{1}{28}\left(7+5i\sqrt{7}\right)\log\left(4x^2+\left(1-i\sqrt{7}\right)x+4\right)+\frac{1}{28}\left(7-5i\sqrt{7}\right)\log\left(4x^2+\left(1+i\sqrt{7}\right)x+4\right)+\frac{\left(7\sqrt{7}+19i\right)\tan^{-1}\left(\frac{8x-i\sqrt{7}+1}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{\sqrt{14\left(35+i\sqrt{7}\right)}}-\frac{\left(-7\sqrt{7}+19i\right)\tan^{-1}\left(\frac{8x+i\sqrt{7}+1}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{\sqrt{14\left(35-i\sqrt{7}\right)}}$$

Antiderivative was successfully verified.

```
[In]  Int[(5 + x + 3*x^2 + 2*x^3)/(2 + x + 5*x^2 + x^3 + 2*x^4),x]
```

```
[Out] ((19*I + 7*Sqrt[7])*ArcTan[(1 - I*Sqrt[7] + 8*x)/Sqrt[2*(35 + I*S
qrt[7])]])/Sqrt[14*(35 + I*Sqrt[7])] - ((19*I - 7*Sqrt[7])*ArcTan
[(1 + I*Sqrt[7] + 8*x)/Sqrt[2*(35 - I*Sqrt[7])]])/Sqrt[14*(35 - I
*Sqrt[7])] + ((7 + (5*I)*Sqrt[7])*Log[4 + (1 - I*Sqrt[7])*x + 4*x
^2])/28 + ((7 - (5*I)*Sqrt[7])*Log[4 + (1 + I*Sqrt[7])*x + 4*x^2]
)/28
```

Rubi in Sympy [A] time = 84.7509, size = 216, normalized size = 1.09

$$\left(\frac{1}{4}+\frac{5\sqrt{7}i}{28}\right)\log\left(4x^2+x\left(1-\sqrt{7}i\right)+4\right)+\left(\frac{1}{4}-\frac{5\sqrt{7}i}{28}\right)\log\left(4x^2+x\left(1+\sqrt{7}i\right)+4\right)-\frac{\left(7-\frac{19\sqrt{7}i}{7}\right)\operatorname{atan}\left(\frac{8x+1+\sqrt{7}i}{\sqrt{35+4\sqrt{77}}-i\sqrt{-35+4\sqrt{77}}}\right)}{-\sqrt{35+4\sqrt{77}}+i\sqrt{-35+4\sqrt{77}}}+\frac{\left(7+\frac{19\sqrt{7}i}{7}\right)\operatorname{atan}\left(\frac{8x+1-\sqrt{7}i}{\sqrt{35+4\sqrt{77}}+i\sqrt{-35+4\sqrt{77}}}\right)}{\sqrt{35+4\sqrt{77}}+i\sqrt{-35+4\sqrt{77}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*x**3+3*x**2+x+5)/(2*x**4+x**3+5*x**2+x+2),x)
```

```
[Out] (1/4 + 5*sqrt(7)*I/28)*log(4*x**2 + x*(1 - sqrt(7)*I) + 4) + (1/4
 - 5*sqrt(7)*I/28)*log(4*x**2 + x*(1 + sqrt(7)*I) + 4) - (7 - 19*
sqrt(7)*I/7)*atan((8*x + 1 + sqrt(7)*I)/(sqrt(35 + 4*sqrt(77)) -
I*sqrt(-35 + 4*sqrt(77))))/(-sqrt(35 + 4*sqrt(77)) + I*sqrt(-35 +
```

```
 4*sqrt(77))) + (7 + 19*sqrt(7)*I/7)*atan((8*x + 1 - sqrt(7)*I)/(
sqrt(35 + 4*sqrt(77)) + I*sqrt(-35 + 4*sqrt(77))))/(sqrt(35 + 4*s
qrt(77)) + I*sqrt(-35 + 4*sqrt(77)))
```

Mathematica [C] time = 0.0216715, size = 90, normalized size = 0.45

$$\text{RootSum}\left[2\#1^4+\#1^3+5\#1^2+\#1+2\&,\frac{2\#1^3\log(x-\#1)+3\#1^2\log(x-\#1)+\#1\log(x-\#1)+5\log(x-\#1)}{8\#1^3+3\#1^2+10\#1+1}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[(5 + x + 3*x^2 + 2*x^3)/(2 + x + 5*x^2 + x^3 + 2*x^4),x]
```

```
[Out] RootSum[2 + #1 + 5*#1^2 + #1^3 + 2*#1^4 & , (5*Log[x - #1] + Log[
x - #1]*#1 + 3*Log[x - #1]*#1^2 + 2*Log[x - #1]*#1^3)/(1 + 10*#1
+ 3*#1^2 + 8*#1^3) & ]
```

Maple [C] time = 0.007, size = 58, normalized size = 0.3

$$\sum_{\_R=RootOf\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)}\frac{\left(2\_R^3+3\_R^2+\_R+5\right)\ln\left(x-\_R\right)}{8\_R^3+3\_R^2+10\_R+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x)
```

```
[Out] sum((2*_R^3+3*_R^2+_R+5)/(8*_R^3+3*_R^2+10*_R+1)*ln(x-_R),_R=Root
Of(2*_Z^4+_Z^3+5*_Z^2+_Z+2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2\,x^3+3\,x^2+x+5}{2\,x^4+x^3+5\,x^2+x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^3 + 3*x^2 + x + 5)/(2*x^4 + x^3 + 5*x^2 + x + 2),x, algorithm="maxim
```

```
[Out] integrate((2*x^3 + 3*x^2 + x + 5)/(2*x^4 + x^3 + 5*x^2 + x + 2),
x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^3 + 3*x^2 + x + 5)/(2*x^4 + x^3 + 5*x^2 + x + 2),x, algorithm="frica
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 2.85058, size = 46, normalized size = 0.23

$$\operatorname{RootSum}\left(343t^4 - 343t^3 + 294t^2 - 336t + 128, \left(t \mapsto t\log\left(-\frac{7203t^3}{304} + \frac{2303t^2}{304} - \frac{2177t}{152} + x + \frac{250}{19}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x**3+3*x**2+x+5)/(2*x**4+x**3+5*x**2+x+2),x)
```

```
[Out] RootSum(343*_t**4 - 343*_t**3 + 294*_t**2 - 336*_t + 128, Lambda(
_t, _t*log(-7203*_t**3/304 + 2303*_t**2/304 - 2177*_t/152 + x + 2
50/19)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2\,x^3 + 3\,x^2 + x + 5}{2\,x^4 + x^3 + 5\,x^2 + x + 2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3 + 3*x^2 + x + 5)/(2*x^4 + x^3 + 5*x^2 + x + 2),x, algorithm="giac"
```

```
[Out] integrate((2*x^3 + 3*x^2 + x + 5)/(2*x^4 + x^3 + 5*x^2 + x + 2),
x)
```

3.254 $\int \frac{5+x+3x^2+2x^3}{x(2+x+5x^2+x^3+2x^4)}\,dx$

Optimal. Leaf size=245

$$-\frac{1}{56}\left(35-9i\sqrt{7}\right)\log\left(4ix^2+\left(-\sqrt{7}+i\right)x+4i\right)-\frac{1}{56}\left(35+9i\sqrt{7}\right)\log\left(4ix^2+\left(\sqrt{7}+i\right)x+4i\right)+\frac{1}{28}\left(35+9i\sqrt{7}\right)\log(x)+\frac{1}{28}\left(35-9i\sqrt{7}\right)\log(x)-\frac{\left(53+i\sqrt{7}\right)\tanh^{-1}\left(\frac{8ix-\sqrt{7}+i}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{2\sqrt{14\left(35-i\sqrt{7}\right)}}+\frac{\left(53-i\sqrt{7}\right)\tanh^{-1}\left(\frac{8ix+\sqrt{7}+i}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{2\sqrt{14\left(35+i\sqrt{7}\right)}}$$

```
[Out] -((53 + I*Sqrt[7])*ArcTanh[(I - Sqrt[7] + (8*I)*x)/Sqrt[2*(35 - I
*Sqrt[7])]])/(2*Sqrt[14*(35 - I*Sqrt[7])]) + ((53 - I*Sqrt[7])*Ar
cTanh[(I + Sqrt[7] + (8*I)*x)/Sqrt[2*(35 + I*Sqrt[7])]])/(2*Sqrt[
14*(35 + I*Sqrt[7])]) + ((35 - (9*I)*Sqrt[7])*Log[x])/28 + ((35 +
 (9*I)*Sqrt[7])*Log[x])/28 - ((35 - (9*I)*Sqrt[7])*Log[4*I + (I -
 Sqrt[7])*x + (4*I)*x^2])/56 - ((35 + (9*I)*Sqrt[7])*Log[4*I + (I
 + Sqrt[7])*x + (4*I)*x^2])/56
```

Rubi [A] time = 1.19434, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171

$$-\frac{1}{56}\left(35-9i\sqrt{7}\right)\log\left(4ix^2+\left(-\sqrt{7}+i\right)x+4i\right)-\frac{1}{56}\left(35+9i\sqrt{7}\right)\log\left(4ix^2+\left(\sqrt{7}+i\right)x+4i\right)+\frac{1}{28}\left(35+9i\sqrt{7}\right)\log(x)+\frac{1}{28}\left(35-9i\sqrt{7}\right)\log(x)-\frac{\left(53+i\sqrt{7}\right)\tanh^{-1}\left(\frac{8ix-\sqrt{7}+i}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{2\sqrt{14\left(35-i\sqrt{7}\right)}}+\frac{\left(53-i\sqrt{7}\right)\tanh^{-1}\left(\frac{8ix+\sqrt{7}+i}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{2\sqrt{14\left(35+i\sqrt{7}\right)}}$$

Antiderivative was successfully verified.

```
[In]  Int[(5 + x + 3*x^2 + 2*x^3)/(x*(2 + x + 5*x^2 + x^3 + 2*x^4)),x]
```

```
[Out] -((53 + I*Sqrt[7])*ArcTanh[(I - Sqrt[7] + (8*I)*x)/Sqrt[2*(35 - I
*Sqrt[7])]])/(2*Sqrt[14*(35 - I*Sqrt[7])]) + ((53 - I*Sqrt[7])*Ar
cTanh[(I + Sqrt[7] + (8*I)*x)/Sqrt[2*(35 + I*Sqrt[7])]])/(2*Sqrt[
14*(35 + I*Sqrt[7])]) + ((35 - (9*I)*Sqrt[7])*Log[x])/28 + ((35 +
 (9*I)*Sqrt[7])*Log[x])/28 - ((35 - (9*I)*Sqrt[7])*Log[4*I + (I -
 Sqrt[7])*x + (4*I)*x^2])/56 - ((35 + (9*I)*Sqrt[7])*Log[4*I + (I
 + Sqrt[7])*x + (4*I)*x^2])/56
```

Rubi in Sympy [A] time = 115.025, size = 253, normalized size = 1.03

$$\left(\frac{5}{4}-\frac{9\sqrt{7}i}{28}\right)\log(x)+\left(\frac{5}{4}+\frac{9\sqrt{7}i}{28}\right)\log(x)-\left(\frac{5}{8}+\frac{9\sqrt{7}i}{56}\right)\log\left(4x^2+x\left(1-\sqrt{7}i\right)+4\right)-\left(\frac{5}{8}-\frac{9\sqrt{7}i}{56}\right)\log\left(4x^2+x\left(1+\sqrt{7}i\right)+4\right)-\frac{\left(\frac{1}{2}-\frac{53\sqrt{7}i}{14}\right)\operatorname{atan}\left(\frac{8x+1+\sqrt{7}i}{\sqrt{35+4\sqrt{77}}-i\sqrt{-35+4\sqrt{77}}}\right)}{-\sqrt{35+4\sqrt{77}}+i\sqrt{-35+4\sqrt{77}}}+\frac{\left(7+53\sqrt{7}i\right)\operatorname{atan}\left(\frac{8x+1-\sqrt{7}i}{\sqrt{35+4\sqrt{77}}+i\sqrt{-35+4\sqrt{77}}}\right)}{14\left(\sqrt{35+4\sqrt{77}}+i\sqrt{-35+4\sqrt{77}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*x**3+3*x**2+x+5)/x/(2*x**4+x**3+5*x**2+x+2),x)
```

```
[Out] (5/4 - 9*sqrt(7)*I/28)*log(x) + (5/4 + 9*sqrt(7)*I/28)*log(x) - (
5/8 + 9*sqrt(7)*I/56)*log(4*x**2 + x*(1 - sqrt(7)*I) + 4) - (5/8
- 9*sqrt(7)*I/56)*log(4*x**2 + x*(1 + sqrt(7)*I) + 4) - (1/2 - 53
*sqrt(7)*I/14)*atan((8*x + 1 + sqrt(7)*I)/(sqrt(35 + 4*sqrt(77))
- I*sqrt(-35 + 4*sqrt(77))))/(-sqrt(35 + 4*sqrt(77)) + I*sqrt(-35
 + 4*sqrt(77))) + (7 + 53*sqrt(7)*I)*atan((8*x + 1 - sqrt(7)*I)/(
sqrt(35 + 4*sqrt(77)) + I*sqrt(-35 + 4*sqrt(77))))/(14*(sqrt(35 +
 4*sqrt(77)) + I*sqrt(-35 + 4*sqrt(77))))
```

Mathematica [C] time = 0.0282212, size = 101, normalized size = 0.41

$$\frac{5\log(x)}{2}-\frac{1}{2}\text{RootSum}\left[2\#1^4+\#1^3+5\#1^2+\#1+2\&,\frac{10\#1^3\log(x-\#1)+\#1^2\log(x-\#1)+19\#1\log(x-\#1)+3\log(x-\#1)}{8\#1^3+3\#1^2+10\#1+1}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[(5 + x + 3*x^2 + 2*x^3)/(x*(2 + x + 5*x^2 + x^3 + 2*x^4)),x]
```

```
[Out] (5*Log[x])/2 - RootSum[2 + #1 + 5*#1^2 + #1^3 + 2*#1^4 & , (3*Log
[x - #1] + 19*Log[x - #1]*#1 + Log[x - #1]*#1^2 + 10*Log[x - #1]*
#1^3)/(1 + 10*#1 + 3*#1^2 + 8*#1^3) & ]/2
```

Maple [C] time = 0.012, size = 67, normalized size = 0.3

$$\frac{5\ln(x)}{2}+\frac{1}{2}\sum_{\_R=RootOf(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2)}\frac{(-10\_R^3-\_R^2-19\_R-3)\ln(x-\_R)}{8\_R^3+3\_R^2+10\_R+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+5*x^2+x+2),x)
```

```
[Out] 5/2*ln(x)+1/2*sum((-10*_R^3-_R^2-19*_R-3)/(8*_R^3+3*_R^2+10*_R+1)
*ln(x-_R),_R=RootOf(2*_Z^4+_Z^3+5*_Z^2+_Z+2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}\int\frac{10x^3+x^2+19x+3}{2x^4+x^3+5x^2+x+2}\,dx+\frac{5}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^3 + 3*x^2 + x + 5)/((2*x^4 + x^3 + 5*x^2 + x + 2)*x),x, algorithm="m
```

```
[Out] -1/2*integrate((10*x^3 + x^2 + 19*x + 3)/(2*x^4 + x^3 + 5*x^2 + x
 + 2), x) + 5/2*log(x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3 + 3*x^2 + x + 5)/((2*x^4 + x^3 + 5*x^2 + x + 2)*x),x, algorithm="f
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 30.5423, size = 60, normalized size = 0.24

$$\frac{5\log(x)}{2} + \text{RootSum}\left(686t^4 + 1715t^3 + 1372t^2 + 448t + 256, \left(t \mapsto t\log\left(-\frac{160344611t^4}{532759184} - \frac{16880402t^3}{33297449} + \frac{4010520787t^2}{2131036736} + \frac{153753567}{53275918}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x**3+3*x**2+x+5)/x/(2*x**4+x**3+5*x**2+x+2),x)
```

```
[Out] 5*log(x)/2 + RootSum(686*_t**4 + 1715*_t**3 + 1372*_t**2 + 448*_t
 + 256, Lambda(_t, _t*log(-160344611*_t**4/532759184 - 16880402*_
t**3/33297449 + 4010520787*_t**2/2131036736 + 1537535671*_t/53275
9184 + x + 46660495/66594898)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2\,x^3 + 3\,x^2 + x + 5}{(2\,x^4 + x^3 + 5\,x^2 + x + 2)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3 + 3*x^2 + x + 5)/((2*x^4 + x^3 + 5*x^2 + x + 2)*x),x, algorithm="g
```

```
[Out] integrate((2*x^3 + 3*x^2 + x + 5)/((2*x^4 + x^3 + 5*x^2 + x + 2)*
x), x)
```

3.255 $\int \frac{5+x+3x^2+2x^3}{x^2(2+x+5x^2+x^3+2x^4)}\,dx$

Optimal. Leaf size=281

$$\frac{3}{112}\left(7+11i\sqrt{7}\right)\log\left(4ix^2+\left(-\sqrt{7}+i\right)x+4i\right)+\frac{3}{112}\left(7-11i\sqrt{7}\right)\log\left(4ix^2+\left(\sqrt{7}+i\right)x+4i\right)-\frac{35+9i\sqrt{7}}{28x}-\frac{35-9i\sqrt{7}}{28x}-\frac{3}{56}\left(7+11i\sqrt{7}\right)\log(x)-\frac{3}{56}\left(7-11i\sqrt{7}\right)\log(x)+\frac{11\left(9+5i\sqrt{7}\right)\tanh^{-1}\left(\frac{8ix-\sqrt{7}+i}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{4\sqrt{14\left(35-i\sqrt{7}\right)}}-\frac{11\left(9-5i\sqrt{7}\right)\tanh^{-1}\left(\frac{8ix+\sqrt{7}+i}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{4\sqrt{14\left(35+i\sqrt{7}\right)}}$$

```
[Out] -(35 - (9*I)*Sqrt[7])/(28*x) - (35 + (9*I)*Sqrt[7])/(28*x) + (11*
(9 + (5*I)*Sqrt[7])*ArcTanh[(I - Sqrt[7] + (8*I)*x)/Sqrt[2*(35 -
I*Sqrt[7])]])/(4*Sqrt[14*(35 - I*Sqrt[7])]) - (11*(9 - (5*I)*Sqrt
[7])*ArcTanh[(I + Sqrt[7] + (8*I)*x)/Sqrt[2*(35 + I*Sqrt[7])]])/(
4*Sqrt[14*(35 + I*Sqrt[7])]) - (3*(7 - (11*I)*Sqrt[7])*Log[x])/56
 - (3*(7 + (11*I)*Sqrt[7])*Log[x])/56 + (3*(7 + (11*I)*Sqrt[7])*L
og[4*I + (I - Sqrt[7])*x + (4*I)*x^2])/112 + (3*(7 - (11*I)*Sqrt[
7])*Log[4*I + (I + Sqrt[7])*x + (4*I)*x^2])/112
```

Rubi [A] time = 1.30279, antiderivative size = 281, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$

$$\frac{3}{112}\left(7+11i\sqrt{7}\right)\log\left(4ix^2+\left(-\sqrt{7}+i\right)x+4i\right)+\frac{3}{112}\left(7-11i\sqrt{7}\right)\log\left(4ix^2+\left(\sqrt{7}+i\right)x+4i\right)-\frac{35+9i\sqrt{7}}{28x}-\frac{35-9i\sqrt{7}}{28x}-\frac{3}{56}\left(7+11i\sqrt{7}\right)\log(x)-\frac{3}{56}\left(7-11i\sqrt{7}\right)\log(x)+\frac{11\left(9+5i\sqrt{7}\right)\tanh^{-1}\left(\frac{8ix-\sqrt{7}+i}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{4\sqrt{14\left(35-i\sqrt{7}\right)}}-\frac{11\left(9-5i\sqrt{7}\right)\tanh^{-1}\left(\frac{8ix+\sqrt{7}+i}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{4\sqrt{14\left(35+i\sqrt{7}\right)}}$$

Antiderivative was successfully verified.

```
[In]  Int[(5 + x + 3*x^2 + 2*x^3)/(x^2*(2 + x + 5*x^2 + x^3 + 2*x^4)),x]
```

```
[Out] -(35 - (9*I)*Sqrt[7])/(28*x) - (35 + (9*I)*Sqrt[7])/(28*x) + (11*
(9 + (5*I)*Sqrt[7])*ArcTanh[(I - Sqrt[7] + (8*I)*x)/Sqrt[2*(35 -
I*Sqrt[7])]])/(4*Sqrt[14*(35 - I*Sqrt[7])]) - (11*(9 - (5*I)*Sqrt
[7])*ArcTanh[(I + Sqrt[7] + (8*I)*x)/Sqrt[2*(35 + I*Sqrt[7])]])/(
4*Sqrt[14*(35 + I*Sqrt[7])]) - (3*(7 - (11*I)*Sqrt[7])*Log[x])/56
 - (3*(7 + (11*I)*Sqrt[7])*Log[x])/56 + (3*(7 + (11*I)*Sqrt[7])*L
og[4*I + (I - Sqrt[7])*x + (4*I)*x^2])/112 + (3*(7 - (11*I)*Sqrt[
7])*Log[4*I + (I + Sqrt[7])*x + (4*I)*x^2])/112
```

Rubi in Sympy [A] time = 133.822, size = 284, normalized size = 1.01

$$-\left(\frac{3}{8}+\frac{33\sqrt{7}i}{56}\right)\log(x)-\left(\frac{3}{8}-\frac{33\sqrt{7}i}{56}\right)\log(x)+\left(\frac{3}{16}-\frac{33\sqrt{7}i}{112}\right)\log\left(4x^2+x\left(1-\sqrt{7}i\right)+4\right)+\left(\frac{3}{16}+\frac{33\sqrt{7}i}{112}\right)\log\left(4x^2+x\left(1+\sqrt{7}i\right)+4\right)+\frac{\left(\frac{55}{4}-\frac{99\sqrt{7}i}{28}\right)\operatorname{atan}\left(\frac{8x+1+\sqrt{7}i}{\sqrt{35+4\sqrt{77}}-i\sqrt{-35+4\sqrt{77}}}\right)}{-\sqrt{35+4\sqrt{77}}+i\sqrt{-35+4\sqrt{77}}}-\frac{\left(\frac{55}{4}+\frac{99\sqrt{7}i}{28}\right)\operatorname{atan}\left(\frac{8x+1-\sqrt{7}i}{\sqrt{35+4\sqrt{77}}+i\sqrt{-35+4\sqrt{77}}}\right)}{\sqrt{35+4\sqrt{77}}+i\sqrt{-35+4\sqrt{77}}}-\frac{\frac{5}{4}+\frac{9\sqrt{7}i}{28}}{x}-\frac{\frac{5}{4}-\frac{9\sqrt{7}i}{28}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((2*x**3+3*x**2+x+5)/x**2/(2*x**4+x**3+5*x**2+x+2),x)
```

```
[Out] -(3/8 + 33*sqrt(7)*I/56)*log(x) - (3/8 - 33*sqrt(7)*I/56)*log(x)
+ (3/16 - 33*sqrt(7)*I/112)*log(4*x**2 + x*(1 - sqrt(7)*I) + 4) +
 (3/16 + 33*sqrt(7)*I/112)*log(4*x**2 + x*(1 + sqrt(7)*I) + 4) +
(55/4 - 99*sqrt(7)*I/28)*atan((8*x + 1 + sqrt(7)*I)/(sqrt(35 + 4*
sqrt(77)) - I*sqrt(-35 + 4*sqrt(77))))/(-sqrt(35 + 4*sqrt(77)) +
I*sqrt(-35 + 4*sqrt(77))) - (55/4 + 99*sqrt(7)*I/28)*atan((8*x +
1 - sqrt(7)*I)/(sqrt(35 + 4*sqrt(77)) + I*sqrt(-35 + 4*sqrt(77)))
)/(sqrt(35 + 4*sqrt(77)) + I*sqrt(-35 + 4*sqrt(77))) - (5/4 + 9*s
qrt(7)*I/28)/x - (5/4 - 9*sqrt(7)*I/28)/x
```

Mathematica [C] time = 0.0285086, size = 109, normalized size = 0.39

$$\frac{1}{4}\text{RootSum}\left[2\#1^4+\#1^3+5\#1^2+\#1+2\&,\frac{6\#1^3\log(x-\#1)-17\#1^2\log(x-\#1)+13\#1\log(x-\#1)-35\log(x-\#1)}{8\#1^3+3\#1^2+10\#1+1}\&\right]-\frac{5}{2x}-\frac{3\log(x)}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + x + 3*x^2 + 2*x^3)/(x^2*(2 + x + 5*x^2 + x^3 + 2*x^4)),x]
```

```
[Out] -5/(2*x) - (3*Log[x])/4 + RootSum[2 + #1 + 5*#1^2 + #1^3 + 2*#1^4
 & , (-35*Log[x - #1] + 13*Log[x - #1]*#1 - 17*Log[x - #1]*#1^2 +
 6*Log[x - #1]*#1^3)/(1 + 10*#1 + 3*#1^2 + 8*#1^3) & ]/4
```

Maple [C] time = 0.012, size = 72, normalized size = 0.3

$$-\frac{5}{2x}-\frac{3\ln(x)}{4}+\frac{1}{4}\sum_{\_R=RootOf(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2)}\frac{(6\_R^3-17\_R^2+13\_R-35)\ln(x-\_R)}{8\_R^3+3\_R^2+10\_R+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+5*x^2+x+2),x)
```

```
[Out] -5/2/x-3/4*ln(x)+1/4*sum((6*_R^3-17*_R^2+13*_R-35)/(8*_R^3+3*_R^2
+10*_R+1)*ln(x-_R),_R=RootOf(2*_Z^4+_Z^3+5*_Z^2+_Z+2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{5}{2x}+\frac{1}{4}\int\frac{6x^3-17x^2+13x-35}{2x^4+x^3+5x^2+x+2}\,dx-\frac{3}{4}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3 + 3*x^2 + x + 5)/((2*x^4 + x^3 + 5*x^2 + x + 2)*x^2),x, algorithm=
```

```
[Out] -5/2/x + 1/4*integrate((6*x^3 - 17*x^2 + 13*x - 35)/(2*x^4 + x^3
+ 5*x^2 + x + 2), x) - 3/4*log(x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3 + 3*x^2 + x + 5)/((2*x^4 + x^3 + 5*x^2 + x + 2)*x^2),x, algorithm=
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 9.64667, size = 65, normalized size = 0.23

$$-\frac{3\log(x)}{4} + \mathrm{RootSum}\left(1372t^4 - 1029t^3 + 3136t^2 + 2688t + 512, \left(t \mapsto t\log\left(-\frac{506797249t^4}{34947704} + \frac{21584647t^3}{4368463} - \frac{14969669687t^2}{559163264} - \frac{282513}{6354}\right.\right.\right. - \frac{5}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x**3+3*x**2+x+5)/x**2/(2*x**4+x**3+5*x**2+x+2),x)
```

```
[Out] -3*log(x)/4 + RootSum(1372*_t**4 - 1029*_t**3 + 3136*_t**2 + 2688
*_t + 512, Lambda(_t, _t*log(-506797249*_t**4/34947704 + 21584647
*_t**3/4368463 - 14969669687*_t**2/559163264 - 282513301*_t/63541
28 + x - 101471979/8736926))) - 5/(2*x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2\,x^3 + 3\,x^2 + x + 5}{(2\,x^4 + x^3 + 5\,x^2 + x + 2)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3 + 3*x^2 + x + 5)/((2*x^4 + x^3 + 5*x^2 + x + 2)*x^2),x, algorithm=
```

```
[Out] integrate((2*x^3 + 3*x^2 + x + 5)/((2*x^4 + x^3 + 5*x^2 + x + 2)*
x^2), x)
```

3.256 $\int \frac{5+x+3x^2+2x^3}{x^3(2+x+5x^2+x^3+2x^4)}\,dx$

Optimal. Leaf size=317

$$-\frac{35+9i\sqrt{7}}{56x^2}-\frac{35-9i\sqrt{7}}{56x^2}+\frac{1}{32}\left(35-9i\sqrt{7}\right)\log\left(4ix^2+\left(-\sqrt{7}+i\right)x+4i\right)+\frac{1}{32}\left(35+9i\sqrt{7}\right)\log\left(4ix^2+\left(\sqrt{7}+i\right)x+4i\right)+\frac{3\left(7+11i\sqrt{7}\right)}{56x}+\frac{3\left(7-11i\sqrt{7}\right)}{56x}-\frac{1}{16}\left(35+9i\sqrt{7}\right)\log(x)-\frac{1}{16}\left(35-9i\sqrt{7}\right)\log(x)+\frac{\left(355-73i\sqrt{7}\right)\tanh^{-1}\left(\frac{8ix-\sqrt{7}+i}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{8\sqrt{14\left(35-i\sqrt{7}\right)}}-\frac{\left(355+73i\sqrt{7}\right)\tanh^{-1}\left(\frac{8ix+\sqrt{7}+i}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{8\sqrt{14\left(35+i\sqrt{7}\right)}}$$

```
[Out] -(35 - (9*I)*Sqrt[7])/(56*x^2) - (35 + (9*I)*Sqrt[7])/(56*x^2) +
(3*(7 - (11*I)*Sqrt[7]))/(56*x) + (3*(7 + (11*I)*Sqrt[7]))/(56*x)
 + ((355 - (73*I)*Sqrt[7])*ArcTanh[(I - Sqrt[7] + (8*I)*x)/Sqrt[2
*(35 - I*Sqrt[7])]])/(8*Sqrt[14*(35 - I*Sqrt[7])]) - ((355 + (73*
I)*Sqrt[7])*ArcTanh[(I + Sqrt[7] + (8*I)*x)/Sqrt[2*(35 + I*Sqrt[7
])]])/(8*Sqrt[14*(35 + I*Sqrt[7])]) - ((35 - (9*I)*Sqrt[7])*Log[x
])/16 - ((35 + (9*I)*Sqrt[7])*Log[x])/16 + ((35 - (9*I)*Sqrt[7])*
Log[4*I + (I - Sqrt[7])*x + (4*I)*x^2])/32 + ((35 + (9*I)*Sqrt[7]
)*Log[4*I + (I + Sqrt[7])*x + (4*I)*x^2])/32
```

Rubi [A] time = 1.46743, antiderivative size = 317, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171

$$-\frac{35+9i\sqrt{7}}{56x^2}-\frac{35-9i\sqrt{7}}{56x^2}+\frac{1}{32}\left(35-9i\sqrt{7}\right)\log\left(4ix^2+\left(-\sqrt{7}+i\right)x+4i\right)+\frac{1}{32}\left(35+9i\sqrt{7}\right)\log\left(4ix^2+\left(\sqrt{7}+i\right)x+4i\right)+\frac{3\left(7+11i\sqrt{7}\right)}{56x}+\frac{3\left(7-11i\sqrt{7}\right)}{56x}-\frac{1}{16}\left(35+9i\sqrt{7}\right)\log(x)-\frac{1}{16}\left(35-9i\sqrt{7}\right)\log(x)+\frac{\left(355-73i\sqrt{7}\right)\tanh^{-1}\left(\frac{8ix-\sqrt{7}+i}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{8\sqrt{14\left(35-i\sqrt{7}\right)}}-\frac{\left(355+73i\sqrt{7}\right)\tanh^{-1}\left(\frac{8ix+\sqrt{7}+i}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{8\sqrt{14\left(35+i\sqrt{7}\right)}}$$

Antiderivative was successfully verified.

```
[In]  Int[(5 + x + 3*x^2 + 2*x^3)/(x^3*(2 + x + 5*x^2 + x^3 + 2*x^4)),x]
```


```
[Out] -(35 - (9*I)*Sqrt[7])/(56*x^2) - (35 + (9*I)*Sqrt[7])/(56*x^2) +
(3*(7 - (11*I)*Sqrt[7]))/(56*x) + (3*(7 + (11*I)*Sqrt[7]))/(56*x)
 + ((355 - (73*I)*Sqrt[7])*ArcTanh[(I - Sqrt[7] + (8*I)*x)/Sqrt[2
*(35 - I*Sqrt[7])]])/(8*Sqrt[14*(35 - I*Sqrt[7])]) - ((355 + (73*
I)*Sqrt[7])*ArcTanh[(I + Sqrt[7] + (8*I)*x)/Sqrt[2*(35 + I*Sqrt[7
])]])/(8*Sqrt[14*(35 + I*Sqrt[7])]) - ((35 - (9*I)*Sqrt[7])*Log[x
])/16 - ((35 + (9*I)*Sqrt[7])*Log[x])/16 + ((35 - (9*I)*Sqrt[7])*
Log[4*I + (I - Sqrt[7])*x + (4*I)*x^2])/32 + ((35 + (9*I)*Sqrt[7]
)*Log[4*I + (I + Sqrt[7])*x + (4*I)*x^2])/32
```

Rubi in Sympy [A] time = 137.984, size = 318, normalized size = 1.

$$-\left(\frac{35}{16}+\frac{9\sqrt{7}i}{16}\right)\log(x)-\left(\frac{35}{16}-\frac{9\sqrt{7}i}{16}\right)\log(x)+\left(\frac{35}{32}+\frac{9\sqrt{7}i}{32}\right)\log\left(4x^2+x\left(1-\sqrt{7}i\right)+4\right)+\left(\frac{35}{32}-\frac{9\sqrt{7}i}{32}\right)\log\left(4x^2+x\left(1+\sqrt{7}i\right)+4\right)-\frac{\left(511+355\sqrt{7}i\right)\operatorname{atan}\left(\frac{8x+1+\sqrt{7}i}{\sqrt{35+4\sqrt{77}}-i\sqrt{-35+4\sqrt{77}}}\right)}{56\left(-\sqrt{35+4\sqrt{77}}+i\sqrt{-35+4\sqrt{77}}\right)}+\frac{\left(\frac{73}{8}-\frac{355\sqrt{7}i}{56}\right)\operatorname{atan}\left(\frac{8x+1-\sqrt{7}i}{\sqrt{35+4\sqrt{77}}+i\sqrt{-35+4\sqrt{77}}}\right)}{\sqrt{35+4\sqrt{77}}+i\sqrt{-35+4\sqrt{77}}}+\frac{\frac{3}{8}-\frac{33\sqrt{7}i}{56}}{x}+\frac{\frac{3}{8}+\frac{33\sqrt{7}i}{56}}{x}-\frac{\frac{5}{8}+\frac{9\sqrt{7}i}{56}}{x^2}-\frac{\frac{5}{8}-\frac{9\sqrt{7}i}{56}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((2*x**3+3*x**2+x+5)/x**3/(2*x**4+x**3+5*x**2+x+2),x)
```

```
[Out] -(35/16 + 9*sqrt(7)*I/16)*log(x) - (35/16 - 9*sqrt(7)*I/16)*log(x
) + (35/32 + 9*sqrt(7)*I/32)*log(4*x**2 + x*(1 - sqrt(7)*I) + 4)
+ (35/32 - 9*sqrt(7)*I/32)*log(4*x**2 + x*(1 + sqrt(7)*I) + 4) -
(511 + 355*sqrt(7)*I)*atan((8*x + 1 + sqrt(7)*I)/(sqrt(35 + 4*sqr
t(77)) - I*sqrt(-35 + 4*sqrt(77))))/(56*(-sqrt(35 + 4*sqrt(77)) +
 I*sqrt(-35 + 4*sqrt(77)))) + (73/8 - 355*sqrt(7)*I/56)*atan((8*x
 + 1 - sqrt(7)*I)/(sqrt(35 + 4*sqrt(77)) + I*sqrt(-35 + 4*sqrt(77
))))/(sqrt(35 + 4*sqrt(77)) + I*sqrt(-35 + 4*sqrt(77))) + (3/8 -
33*sqrt(7)*I/56)/x + (3/8 + 33*sqrt(7)*I/56)/x - (5/8 + 9*sqrt(7)
*I/56)/x**2 - (5/8 - 9*sqrt(7)*I/56)/x**2
```

Mathematica [C] time = 0.0300998, size = 116, normalized size = 0.37

$$\frac{1}{8}\text{RootSum}\left[2\#1^4+\#1^3+5\#1^2+\#1+2\&,\frac{70\#1^3\log(x-\#1)+47\#1^2\log(x-\#1)+141\#1\log(x-\#1)+61\log(x-\#1)}{8\#1^3+3\#1^2+10\#1+1}\&\right]-\frac{5}{4x^2}+\frac{3}{4x}-\frac{35\log(x)}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + x + 3*x^2 + 2*x^3)/(x^3*(2 + x + 5*x^2 + x^3 + 2*x^4)),x]
```

```
[Out] -5/(4*x^2) + 3/(4*x) - (35*Log[x])/8 + RootSum[2 + #1 + 5*#1^2 +
#1^3 + 2*#1^4 & , (61*Log[x - #1] + 141*Log[x - #1]*#1 + 47*Log[x
 - #1]*#1^2 + 70*Log[x - #1]*#1^3)/(1 + 10*#1 + 3*#1^2 + 8*#1^3)
& ]/8
```

Maple [C] time = 0.015, size = 77, normalized size = 0.2

$$-\frac{5}{4x^2}+\frac{3}{4x}-\frac{35\ln(x)}{8}+\frac{1}{8}\sum_{\_R=RootOf(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2)}\frac{\left(70\_R^3+47\_R^2+141\_R+61\right)\ln(x-\_R)}{8\_R^3+3\_R^2+10\_R+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+5*x^2+x+2),x)
```

```
[Out] -5/4/x^2+3/4/x-35/8*ln(x)+1/8*sum((70*_R^3+47*_R^2+141*_R+61)/(8*
_R^3+3*_R^2+10*_R+1)*ln(x-_R),_R=RootOf(2*_Z^4+_Z^3+5*_Z^2+_Z+2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{3x-5}{4x^2}+\frac{1}{8}\int \frac{70x^3+47x^2+141x+61}{2x^4+x^3+5x^2+x+2}\,dx-\frac{35}{8}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3 + 3*x^2 + x + 5)/((2*x^4 + x^3 + 5*x^2 + x + 2)*x^3),x, algorithm=
```

```
[Out] 1/4*(3*x - 5)/x^2 + 1/8*integrate((70*x^3 + 47*x^2 + 141*x + 61)/
(2*x^4 + x^3 + 5*x^2 + x + 2), x) - 35/8*log(x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3 + 3*x^2 + x + 5)/((2*x^4 + x^3 + 5*x^2 + x + 2)*x^3),x, algorithm=
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 6.67984, size = 70, normalized size = 0.22

$$-\frac{35\log(x)}{8}$$
$$+\text{RootSum}\left(2744t^4-12005t^3+18424t^2-3136t+1024,\left(t\mapsto t\log\left(-\frac{20101387287723t^4}{91907904361586}+\frac{944515214496t^3}{45953952180793}+\frac{165723270}{58821058}\right.\right.\right.$$
$$+\frac{3x-5}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x**3+3*x**2+x+5)/x**3/(2*x**4+x**3+5*x**2+x+2),x)
```

```
[Out] -35*log(x)/8 + RootSum(2744*_t**4 - 12005*_t**3 + 18424*_t**2 - 3
136*_t + 1024, Lambda(_t, _t*log(-20101387287723*_t**4/9190790436
1586 + 944515214496*_t**3/45953952180793 + 16572327093911939*_t**
2/5882105879141504 - 4564471749800865*_t/735263234892688 + x + 70
084064010625/91907904361586))) + (3*x - 5)/(4*x**2)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x^3+3x^2+x+5}{(2x^4+x^3+5x^2+x+2)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3 + 3*x^2 + x + 5)/((2*x^4 + x^3 + 5*x^2 + x + 2)*x^3),x, algorithm=
```

```
[Out] integrate((2*x^3 + 3*x^2 + x + 5)/((2*x^4 + x^3 + 5*x^2 + x + 2)*
x^3), x)
```

3.257 $\int \frac{x^2(3a+bx^2)}{a^2+2abx^2+b^2x^4+c^2x^6} dx$

Optimal. Leaf size=19

$$\frac{\tan^{-1}\left(\frac{cx^3}{a+bx^2}\right)}{c}$$

```
[Out] ArcTan[(c*x^3)/(a + b*x^2)]/c
```

Rubi [A] time = 0.174633, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.05$

$$\frac{\tan^{-1}\left(\frac{cx^3}{a+bx^2}\right)}{c}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(3*a + b*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4 + c^2*x^6),x]
```

```
[Out] ArcTan[(c*x^3)/(a + b*x^2)]/c
```

Rubi in Sympy [A] time = 53.0187, size = 14, normalized size = 0.74

$$\frac{\operatorname{atan}\left(\frac{cx^3}{a+bx^2}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**2*(b*x**2+3*a)/(c**2*x**6+b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] atan(c*x**3/(a + b*x**2))/c
```

Mathematica [C] time = 0.071569, size = 87, normalized size = 4.58

$$\frac{1}{2}\text{RootSum}\left[\#1^6c^2 + \#1^4b^2 + 2\#1^2ab + a^2\&, \frac{\#1^3b\log(x-\#1) + 3\#1a\log(x-\#1)}{3\#1^4c^2 + 2\#1^2b^2 + 2ab}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(3*a + b*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4 + c^2*x^6),x]
```

```
[Out] RootSum[a^2 + 2*a*b*#1^2 + b^2*#1^4 + c^2*#1^6 & , (3*a*Log[x - #
1]*#1 + b*Log[x - #1]*#1^3)/(2*a*b + 2*b^2*#1^2 + 3*c^2*#1^4) & ]
/2
```

Maple [C] time = 0.428, size = 75, normalized size = 4.

$$\frac{1}{2}\sum_{\_R=RootOf(c^2\_Z^6+b^2\_Z^4+2ab\_Z^2+a^2)} \frac{(\_R^4b + 3\_R^2a)\ln(x-\_R)}{3\_R^5c^2 + 2\_R^3b^2 + 2\_Rab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(b*x^2+3*a)/(c^2*x^6+b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] 1/2*sum((_R^4*b+3*_R^2*a)/(3*_R^5*c^2+2*_R^3*b^2+2*_R*a*b)*ln(x-_
R),_R=RootOf(_Z^6*c^2+_Z^4*b^2+2*_Z^2*a*b+a^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + 3a)x^2}{c^2x^6 + b^2x^4 + 2abx^2 + a^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + 3*a)*x^2/(c^2*x^6 + b^2*x^4 + 2*a*b*x^2 + a^2),x, algorithm="m
```

```
[Out] integrate((b*x^2 + 3*a)*x^2/(c^2*x^6 + b^2*x^4 + 2*a*b*x^2 + a^2)
, x)
```

Fricas [A] time = 0.264245, size = 112, normalized size = 5.89

$$\frac{\arctan\left(\frac{cx}{b}\right) - \arctan\left(\frac{bc^2x^5+ab^2x+(b^3-ac^2)x^3}{a^2c}\right) + \arctan\left(\frac{bc^2x^3+(b^3-ac^2)x}{abc}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + 3*a)*x^2/(c^2*x^6 + b^2*x^4 + 2*a*b*x^2 + a^2),x, algorithm="f
```

```
[Out] (arctan(c*x/b) - arctan((b*c^2*x^5 + a*b^2*x + (b^3 - a*c^2)*x^3)
/(a^2*c)) + arctan((b*c^2*x^3 + (b^3 - a*c^2)*x)/(a*b*c)))/c
```

Sympy [A] time = 4.11852, size = 44, normalized size = 2.32

$$\frac{-\frac{i\log\left(-\frac{ia}{c}-\frac{ibx^2}{c}+x^3\right)}{2}+\frac{i\log\left(\frac{ia}{c}+\frac{ibx^2}{c}+x^3\right)}{2}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(b*x**2+3*a)/(c**2*x**6+b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] (-I*log(-I*a/c - I*b*x**2/c + x**3)/2 + I*log(I*a/c + I*b*x**2/c
+ x**3)/2)/c
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + 3a)x^2}{c^2x^6 + b^2x^4 + 2abx^2 + a^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + 3*a)*x^2/(c^2*x^6 + b^2*x^4 + 2*a*b*x^2 + a^2),x, algorithm="g
```

```
[Out] integrate((b*x^2 + 3*a)*x^2/(c^2*x^6 + b^2*x^4 + 2*a*b*x^2 + a^2)
, x)
```

3.258 $\int \frac{1-3x^4}{(-2+x)(1+x^2)^2}\,dx$

Optimal. Leaf size=43

$$-\frac{1-2x}{5(x^2+1)}-\frac{14}{25}\log\left(x^2+1\right)-\frac{47}{25}\log(2-x)-\frac{46}{25}\tan^{-1}(x)$$

```
[Out] -(1 - 2*x)/(5*(1 + x^2)) - (46*ArcTan[x])/25 - (47*Log[2 - x])/25
 - (14*Log[1 + x^2])/25
```

Rubi [A] time = 0.114839, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$-\frac{1-2x}{5(x^2+1)}-\frac{14}{25}\log\left(x^2+1\right)-\frac{47}{25}\log(2-x)-\frac{46}{25}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - 3*x^4)/((-2 + x)*(1 + x^2)^2),x]
```

```
[Out] -(1 - 2*x)/(5*(1 + x^2)) - (46*ArcTan[x])/25 - (47*Log[2 - x])/25
 - (14*Log[1 + x^2])/25
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-3*x**4+1)/(-2+x)/(x**2+1)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.036632, size = 57, normalized size = 1.33

$$\frac{2(x-2)+3}{5\left((x-2)^2+4(x-2)+5\right)}-\frac{14}{25}\log\left((x-2)^2+4(x-2)+5\right)-\frac{47}{25}\log(x-2)-\frac{46}{25}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - 3*x^4)/((-2 + x)*(1 + x^2)^2),x]
```

```
[Out] (3 + 2*(-2 + x))/(5*(5 + 4*(-2 + x) + (-2 + x)^2)) - (46*ArcTan[x
])/25 - (14*Log[5 + 4*(-2 + x) + (-2 + x)^2])/25 - (47*Log[-2 + x
])/25
```

Maple [A] time = 0.013, size = 34, normalized size = 0.8

$$-\frac{2}{25x^2+25}\left(-5x+\frac{5}{2}\right)-\frac{14\ln\left(x^2+1\right)}{25}-\frac{46\arctan(x)}{25}-\frac{47\ln(x-2)}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-3*x^4+1)/(x-2)/(x^2+1)^2,x)
```

```
[Out] -2/25*(-5*x+5/2)/(x^2+1)-14/25*ln(x^2+1)-46/25*arctan(x)-47/25*ln
(x-2)
```

Maxima [A] time = 0.90292, size = 45, normalized size = 1.05

$$\frac{2\,x-1}{5\,(x^2+1)}-\frac{46}{25}\,\arctan\left(x\right)-\frac{14}{25}\,\log\left(x^2+1\right)-\frac{47}{25}\,\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(3*x^4 - 1)/((x^2 + 1)^2*(x - 2)),x, algorithm="maxima")
```

```
[Out] 1/5*(2*x - 1)/(x^2 + 1) - 46/25*arctan(x) - 14/25*log(x^2 + 1) -
47/25*log(x - 2)
```

Fricas [A] time = 0.274598, size = 63, normalized size = 1.47

$$-\frac{46\,\left(x^2+1\right)\,\arctan\left(x\right)+14\,\left(x^2+1\right)\,\log\left(x^2+1\right)+47\,\left(x^2+1\right)\,\log\left(x-2\right)-10\,x+5}{25\,(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(3*x^4 - 1)/((x^2 + 1)^2*(x - 2)),x, algorithm="fricas")
```

```
[Out] -1/25*(46*(x^2 + 1)*arctan(x) + 14*(x^2 + 1)*log(x^2 + 1) + 47*(x
^2 + 1)*log(x - 2) - 10*x + 5)/(x^2 + 1)
```

Sympy [A] time = 0.421405, size = 36, normalized size = 0.84

$$\frac{2x-1}{5x^2+5}-\frac{47\log\left(x-2\right)}{25}-\frac{14\log\left(x^2+1\right)}{25}-\frac{46\,\mathrm{atan}\left(x\right)}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-3*x**4+1)/(-2+x)/(x**2+1)**2,x)
```

```
[Out] (2*x - 1)/(5*x**2 + 5) - 47*log(x - 2)/25 - 14*log(x**2 + 1)/25 -
 46*atan(x)/25
```

GIAC/XCAS [A] time = 0.261812, size = 46, normalized size = 1.07

$$\frac{2\,x-1}{5\,(x^2+1)}-\frac{46}{25}\,\arctan\left(x\right)-\frac{14}{25}\ln\left(x^2+1\right)-\frac{47}{25}\ln\left(|x-2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(3*x^4 - 1)/((x^2 + 1)^2*(x - 2)),x, algorithm="giac")
```

```
[Out] 1/5*(2*x - 1)/(x^2 + 1) - 46/25*arctan(x) - 14/25*ln(x^2 + 1) - 4
7/25*ln(abs(x - 2))
```

3.259 $\int \frac{-9-9x+2x^2}{-9x+x^3}\,dx$

Optimal. Leaf size=17

$$-\log(3-x)+\log(x)+2\log(x+3)$$

```
[Out] -Log[3 - x] + Log[x] + 2*Log[3 + x]
```

Rubi [A] time = 0.0526702, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$-\log(3-x)+\log(x)+2\log(x+3)$$

Antiderivative was successfully verified.

```
[In]  Int[(-9 - 9*x + 2*x^2)/(-9*x + x^3),x]

[Out] -Log[3 - x] + Log[x] + 2*Log[3 + x]
```

Rubi in Sympy [A] time = 9.1575, size = 14, normalized size = 0.82

$$\log(x)-\log(-x+3)+2\log(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*x**2-9*x-9)/(x**3-9*x),x)

[Out] log(x) - log(-x + 3) + 2*log(x + 3)
```

Mathematica [A] time = 0.00906288, size = 17, normalized size = 1.

$$-\log(3-x)+\log(x)+2\log(x+3)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-9 - 9*x + 2*x^2)/(-9*x + x^3),x]

[Out] -Log[3 - x] + Log[x] + 2*Log[3 + x]
```

Maple [A] time = 0.011, size = 16, normalized size = 0.9

$$-\ln(-3+x)+\ln(x)+2\ln(3+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*x^2-9*x-9)/(x^3-9*x),x)

[Out] -ln(-3+x)+ln(x)+2*ln(3+x)
```

Maxima [A] time = 0.996735, size = 20, normalized size = 1.18

$$2\log(x+3) - \log(x-3) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 - 9*x - 9)/(x^3 - 9*x),x, algorithm="maxima")
```

```
[Out] 2*log(x + 3) - log(x - 3) + log(x)
```

Fricas [A] time = 0.258079, size = 20, normalized size = 1.18

$$2\log(x+3) - \log(x-3) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 - 9*x - 9)/(x^3 - 9*x),x, algorithm="fricas")
```

```
[Out] 2*log(x + 3) - log(x - 3) + log(x)
```

Sympy [A] time = 0.270863, size = 14, normalized size = 0.82

$$\log(x) - \log(x-3) + 2\log(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x**2-9*x-9)/(x**3-9*x),x)
```

```
[Out] log(x) - log(x - 3) + 2*log(x + 3)
```

GIAC/XCAS [A] time = 0.262644, size = 24, normalized size = 1.41

$$2\ln(|x+3|) - \ln(|x-3|) + \ln(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 - 9*x - 9)/(x^3 - 9*x),x, algorithm="giac")
```

```
[Out] 2*ln(abs(x + 3)) - ln(abs(x - 3)) + ln(abs(x))
```

3.260 $\int \frac{1+2x^2+x^5}{-x+x^3}\,dx$

Optimal. Leaf size=25

$$\frac{x^3}{3}+x+2\log(1-x)-\log(x)+\log(x+1)$$

```
[Out] x + x^3/3 + 2*Log[1 - x] - Log[x] + Log[1 + x]
```

Rubi [A] time = 0.0520808, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{x^3}{3}+x+2\log(1-x)-\log(x)+\log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + 2*x^2 + x^5)/(-x + x^3),x]

[Out] x + x^3/3 + 2*Log[1 - x] - Log[x] + Log[1 + x]
```

Rubi in Sympy [A] time = 12.6099, size = 20, normalized size = 0.8

$$\frac{x^3}{3}+x-\log\left(x\right)+2\log\left(-x+1\right)+\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**5+2*x**2+1)/(x**3-x),x)

[Out] x**3/3 + x - log(x) + 2*log(-x + 1) + log(x + 1)
```

Mathematica [A] time = 0.00868306, size = 25, normalized size = 1.

$$\frac{x^3}{3}+x+2\log(1-x)-\log(x)+\log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + 2*x^2 + x^5)/(-x + x^3),x]

[Out] x + x^3/3 + 2*Log[1 - x] - Log[x] + Log[1 + x]
```

Maple [A] time = 0.01, size = 22, normalized size = 0.9

$$\frac{x^3}{3}+x+2\,\ln\left(-1+x\right)+\ln\left(1+x\right)-\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^5+2*x^2+1)/(x^3-x),x)
```

```
[Out] 1/3*x^3+x+2*ln(-1+x)+ln(1+x)-ln(x)
```

Maxima [A] time = 0.823995, size = 28, normalized size = 1.12

$$\frac{1}{3}x^3 + x + \log(x+1) + 2\log(x-1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^5 + 2*x^2 + 1)/(x^3 - x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3 + x + log(x + 1) + 2*log(x - 1) - log(x)
```

Fricas [A] time = 0.258774, size = 28, normalized size = 1.12

$$\frac{1}{3}x^3 + x + \log(x+1) + 2\log(x-1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^5 + 2*x^2 + 1)/(x^3 - x),x, algorithm="fricas")
```

```
[Out] 1/3*x^3 + x + log(x + 1) + 2*log(x - 1) - log(x)
```

Sympy [A] time = 0.277487, size = 20, normalized size = 0.8

$$\frac{x^3}{3} + x - \log(x) + 2\log(x-1) + \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**5+2*x**2+1)/(x**3-x),x)
```

```
[Out] x**3/3 + x - log(x) + 2*log(x - 1) + log(x + 1)
```

GIAC/XCAS [A] time = 0.260412, size = 32, normalized size = 1.28

$$\frac{1}{3}x^3 + x + \ln(|x+1|) + 2\ln(|x-1|) - \ln(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^5 + 2*x^2 + 1)/(x^3 - x),x, algorithm="giac")
```

```
[Out] 1/3*x^3 + x + ln(abs(x + 1)) + 2*ln(abs(x - 1)) - ln(abs(x))
```

3.261 $\int \frac{3+2x^2}{(-1+x)^2x} \, dx$

Optimal. Leaf size=22

$$\frac{5}{1-x} - \log(1-x) + 3\log(x)$$

```
[Out] 5/(1 - x) - Log[1 - x] + 3*Log[x]
```

Rubi [A] time = 0.0420829, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$

$$\frac{5}{1-x} - \log(1-x) + 3\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 2*x^2)/((-1 + x)^2*x),x]

[Out] 5/(1 - x) - Log[1 - x] + 3*Log[x]
```

Rubi in Sympy [A] time = 4.40424, size = 14, normalized size = 0.64

$$3\log(x) - \log(-x+1) + \frac{5}{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((2*x**2+3)/(-1+x)**2/x,x)

[Out] 3*log(x) - log(-x + 1) + 5/(-x + 1)
```

Mathematica [A] time = 0.0132863, size = 20, normalized size = 0.91

$$-\frac{5}{x-1} - \log(1-x) + 3\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 2*x^2)/((-1 + x)^2*x),x]

[Out] -5/(-1 + x) - Log[1 - x] + 3*Log[x]
```

Maple [A] time = 0.011, size = 19, normalized size = 0.9

$$-5(-1+x)^{-1} - \ln(-1+x) + 3\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+3)/(-1+x)^2/x,x)

[Out] -5/(-1+x)-ln(-1+x)+3*ln(x)
```

Maxima [A] time = 0.944092, size = 24, normalized size = 1.09

$$-\frac{5}{x-1}-\log(x-1)+3\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + 3)/((x - 1)^2*x),x, algorithm="maxima")
```

```
[Out] -5/(x - 1) - log(x - 1) + 3*log(x)
```

Fricas [A] time = 0.244985, size = 32, normalized size = 1.45

$$-\frac{(x-1)\log(x-1)-3(x-1)\log(x)+5}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + 3)/((x - 1)^2*x),x, algorithm="fricas")
```

```
[Out] -((x - 1)*log(x - 1) - 3*(x - 1)*log(x) + 5)/(x - 1)
```

Sympy [A] time = 0.225205, size = 14, normalized size = 0.64

$$3\log(x)-\log(x-1)-\frac{5}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x**2+3)/(-1+x)**2/x,x)
```

```
[Out] 3*log(x) - log(x - 1) - 5/(x - 1)
```

GIAC/XCAS [A] time = 0.259594, size = 38, normalized size = 1.73

$$-\frac{5}{x-1}+2\ln(|x-1|)+3\ln\left(\left|-\frac{1}{x-1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + 3)/((x - 1)^2*x),x, algorithm="giac")
```

```
[Out] -5/(x - 1) + 2*ln(abs(x - 1)) + 3*ln(abs(-1/(x - 1) - 1))
```

3.262 $\int \frac{-1+2x^2}{(-1+4x)(1+x^2)}\,dx$

Optimal. Leaf size=27

$$\frac{6}{17}\log\left(x^2+1\right)-\frac{7}{34}\log(1-4x)+\frac{3}{17}\tan^{-1}(x)$$

```
[Out] (3*ArcTan[x])/17 - (7*Log[1 - 4*x])/34 + (6*Log[1 + x^2])/17
```

Rubi [A] time = 0.0737282, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{6}{17}\log\left(x^2+1\right)-\frac{7}{34}\log(1-4x)+\frac{3}{17}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + 2*x^2)/((-1 + 4*x)*(1 + x^2)),x]

[Out] (3*ArcTan[x])/17 - (7*Log[1 - 4*x])/34 + (6*Log[1 + x^2])/17
```

Rubi in Sympy [A] time = 8.09436, size = 26, normalized size = 0.96

$$-\frac{7\log\left(-4x+1\right)}{34}+\frac{6\log\left(x^2+1\right)}{17}+\frac{3\,\mathrm{atan}\left(x\right)}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*x**2-1)/(-1+4*x)/(x**2+1),x)

[Out] -7*log(-4*x + 1)/34 + 6*log(x**2 + 1)/17 + 3*atan(x)/17
```

Mathematica [A] time = 0.0165882, size = 38, normalized size = 1.41

$$-\frac{7}{34}\log(4x-1)+\frac{6}{17}\log\left((4x-1)^2+2(4x-1)+17\right)+\frac{3}{17}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + 2*x^2)/((-1 + 4*x)*(1 + x^2)),x]

[Out] (3*ArcTan[x])/17 - (7*Log[-1 + 4*x])/34 + (6*Log[17 + 2*(-1 + 4*x
) + (-1 + 4*x)^2])/17
```

Maple [A] time = 0.008, size = 22, normalized size = 0.8

$$\frac{6\,\ln\left(x^2+1\right)}{17}+\frac{3\,\arctan\left(x\right)}{17}-\frac{7\,\ln\left(-1+4\,x\right)}{34}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*x^2-1)/(-1+4*x)/(x^2+1),x)
```

```
[Out] 6/17*ln(x^2+1)+3/17*arctan(x)-7/34*ln(-1+4*x)
```

Maxima [A] time = 0.982327, size = 28, normalized size = 1.04

$$\frac{3}{17} \arctan(x) + \frac{6}{17} \log\left(x^2+1\right) - \frac{7}{34} \log(4x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 - 1)/((x^2 + 1)*(4*x - 1)),x, algorithm="maxima")
```

```
[Out] 3/17*arctan(x) + 6/17*log(x^2 + 1) - 7/34*log(4*x - 1)
```

Fricas [A] time = 0.256568, size = 28, normalized size = 1.04

$$\frac{3}{17} \arctan(x) + \frac{6}{17} \log\left(x^2+1\right) - \frac{7}{34} \log(4x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 - 1)/((x^2 + 1)*(4*x - 1)),x, algorithm="fricas")
```

```
[Out] 3/17*arctan(x) + 6/17*log(x^2 + 1) - 7/34*log(4*x - 1)
```

Sympy [A] time = 0.313382, size = 26, normalized size = 0.96

$$-\frac{7\log\left(x-\frac{1}{4}\right)}{34} + \frac{6\log\left(x^2+1\right)}{17} + \frac{3\,\mathrm{atan}(x)}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x**2-1)/(-1+4*x)/(x**2+1),x)
```

```
[Out] -7*log(x - 1/4)/34 + 6*log(x**2 + 1)/17 + 3*atan(x)/17
```

GIAC/XCAS [A] time = 0.264672, size = 30, normalized size = 1.11

$$\frac{3}{17} \arctan(x) + \frac{6}{17} \ln\left(x^2+1\right) - \frac{7}{34} \ln(|4x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 - 1)/((x^2 + 1)*(4*x - 1)),x, algorithm="giac")
```

```
[Out] 3/17*arctan(x) + 6/17*ln(x^2 + 1) - 7/34*ln(abs(4*x - 1))
```

3.263 $\int \frac{-3+2x-3x^2+x^3}{1+x^2}\,dx$

Optimal. Leaf size=21

$$\frac{x^2}{2}+\frac{1}{2}\log\left(x^2+1\right)-3x$$

```
[Out] -3*x + x^2/2 + Log[1 + x^2]/2
```

Rubi [A] time = 0.0252335, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095

$$\frac{x^2}{2}+\frac{1}{2}\log\left(x^2+1\right)-3x$$

Antiderivative was successfully verified.

```
[In]  Int[(-3 + 2*x - 3*x^2 + x^3)/(1 + x^2),x]

[Out] -3*x + x^2/2 + Log[1 + x^2]/2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-3x+\frac{\log\left(x^2+1\right)}{2}+\int x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**3-3*x**2+2*x-3)/(x**2+1),x)

[Out] -3*x + log(x**2 + 1)/2 + Integral(x, x)
```

Mathematica [A] time = 0.00677724, size = 21, normalized size = 1.

$$\frac{x^2}{2}+\frac{1}{2}\log\left(x^2+1\right)-3x$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-3 + 2*x - 3*x^2 + x^3)/(1 + x^2),x]

[Out] -3*x + x^2/2 + Log[1 + x^2]/2
```

Maple [A] time = 0.004, size = 18, normalized size = 0.9

$$-3\,x+\frac{x^2}{2}+\frac{\ln\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^3-3*x^2+2*x-3)/(x^2+1),x)
```

```
[Out] -3*x+1/2*x^2+1/2*ln(x^2+1)
```

Maxima [A] time = 0.89792, size = 23, normalized size = 1.1

$$\frac{1}{2}x^2 - 3x + \frac{1}{2}\log\left(x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - 3*x^2 + 2*x - 3)/(x^2 + 1),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 3*x + 1/2*log(x^2 + 1)
```

Fricas [A] time = 0.257006, size = 23, normalized size = 1.1

$$\frac{1}{2}x^2 - 3x + \frac{1}{2}\log\left(x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - 3*x^2 + 2*x - 3)/(x^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 - 3*x + 1/2*log(x^2 + 1)
```

Sympy [A] time = 0.135727, size = 15, normalized size = 0.71

$$\frac{x^2}{2} - 3x + \frac{\log\left(x^2 + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3-3*x**2+2*x-3)/(x**2+1),x)
```

```
[Out] x**2/2 - 3*x + log(x**2 + 1)/2
```

GIAC/XCAS [A] time = 0.260172, size = 23, normalized size = 1.1

$$\frac{1}{2}x^2 - 3x + \frac{1}{2}\ln\left(x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - 3*x^2 + 2*x - 3)/(x^2 + 1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 3*x + 1/2*ln(x^2 + 1)
```

3.264 $\int \frac{x+10x^2+6x^3+x^4}{10+6x+x^2}\,dx$

Optimal. Leaf size=27

$$\frac{x^3}{3}+\frac{1}{2}\log\left(x^2+6x+10\right)-3\tan^{-1}(x+3)$$

```
[Out] x^3/3 - 3*ArcTan[3 + x] + Log[10 + 6*x + x^2]/2
```

Rubi [A] time = 0.0486547, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192

$$\frac{x^3}{3}+\frac{1}{2}\log\left(x^2+6x+10\right)-3\tan^{-1}(x+3)$$

Antiderivative was successfully verified.

```
[In]  Int[(x + 10*x^2 + 6*x^3 + x^4)/(10 + 6*x + x^2),x]

[Out] x^3/3 - 3*ArcTan[3 + x] + Log[10 + 6*x + x^2]/2
```

Rubi in Sympy [A] time = 36.6503, size = 22, normalized size = 0.81

$$\frac{x^3}{3}+\frac{\log\left(x^2+6x+10\right)}{2}-3\operatorname{atan}(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**4+6*x**3+10*x**2+x)/(x**2+6*x+10),x)

[Out] x**3/3 + log(x**2 + 6*x + 10)/2 - 3*atan(x + 3)
```

Mathematica [A] time = 0.0115037, size = 27, normalized size = 1.

$$\frac{x^3}{3}+\frac{1}{2}\log\left(x^2+6x+10\right)-3\tan^{-1}(x+3)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x + 10*x^2 + 6*x^3 + x^4)/(10 + 6*x + x^2),x]

[Out] x^3/3 - 3*ArcTan[3 + x] + Log[10 + 6*x + x^2]/2
```

Maple [A] time = 0.006, size = 24, normalized size = 0.9

$$\frac{x^3}{3}-3\arctan(3+x)+\frac{\ln\left(x^2+6x+10\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^4+6*x^3+10*x^2+x)/(x^2+6*x+10),x)
```

```
[Out] 1/3*x^3-3*arctan(3+x)+1/2*ln(x^2+6*x+10)
```

Maxima [A] time = 0.865046, size = 31, normalized size = 1.15

$$\frac{1}{3}x^3 - 3\arctan(x+3) + \frac{1}{2}\log\left(x^2 + 6x + 10\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 + 6*x^3 + 10*x^2 + x)/(x^2 + 6*x + 10),x, algorithm="maxima")
```

```
[Out] 1/3*x^3 - 3*arctan(x + 3) + 1/2*log(x^2 + 6*x + 10)
```

Fricas [A] time = 0.273257, size = 31, normalized size = 1.15

$$\frac{1}{3}x^3 - 3\arctan(x+3) + \frac{1}{2}\log\left(x^2 + 6x + 10\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 + 6*x^3 + 10*x^2 + x)/(x^2 + 6*x + 10),x, algorithm="fricas")
```

```
[Out] 1/3*x^3 - 3*arctan(x + 3) + 1/2*log(x^2 + 6*x + 10)
```

Sympy [A] time = 0.203127, size = 22, normalized size = 0.81

$$\frac{x^3}{3} + \frac{\log\left(x^2 + 6x + 10\right)}{2} - 3\operatorname{atan}(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+6*x**3+10*x**2+x)/(x**2+6*x+10),x)
```

```
[Out] x**3/3 + log(x**2 + 6*x + 10)/2 - 3*atan(x + 3)
```

GIAC/XCAS [A] time = 0.260273, size = 31, normalized size = 1.15

$$\frac{1}{3}x^3 - 3\arctan(x+3) + \frac{1}{2}\ln\left(x^2 + 6x + 10\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 + 6*x^3 + 10*x^2 + x)/(x^2 + 6*x + 10),x, algorithm="giac")
```

```
[Out] 1/3*x^3 - 3*arctan(x + 3) + 1/2*ln(x^2 + 6*x + 10)
```

3.265 $\int \frac{1}{-18+27x-7x^2-3x^3+x^4} \, dx$

Optimal. Leaf size=39

$$\frac{1}{8}\log(1-x)-\frac{1}{5}\log(2-x)+\frac{1}{12}\log(3-x)-\frac{1}{120}\log(x+3)$$

```
[Out] Log[1 - x]/8 - Log[2 - x]/5 + Log[3 - x]/12 - Log[3 + x]/120
```

Rubi [A] time = 0.044821, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05

$$\frac{1}{8}\log(1-x)-\frac{1}{5}\log(2-x)+\frac{1}{12}\log(3-x)-\frac{1}{120}\log(x+3)$$

Antiderivative was successfully verified.

```
[In]  Int[(-18 + 27*x - 7*x^2 - 3*x^3 + x^4)^(-1),x]
```

```
[Out] Log[1 - x]/8 - Log[2 - x]/5 + Log[3 - x]/12 - Log[3 + x]/120
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**4-3*x**3-7*x**2+27*x-18),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.00952781, size = 39, normalized size = 1.

$$\frac{1}{8}\log(1-x)-\frac{1}{5}\log(2-x)+\frac{1}{12}\log(3-x)-\frac{1}{120}\log(x+3)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-18 + 27*x - 7*x^2 - 3*x^3 + x^4)^(-1),x]
```

```
[Out] Log[1 - x]/8 - Log[2 - x]/5 + Log[3 - x]/12 - Log[3 + x]/120
```

Maple [A] time = 0.012, size = 26, normalized size = 0.7

$$\frac{\ln(-1+x)}{8}+\frac{\ln(-3+x)}{12}-\frac{\ln(x-2)}{5}-\frac{\ln(3+x)}{120}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^4-3*x^3-7*x^2+27*x-18),x)
```

```
[Out] 1/8*ln(-1+x)+1/12*ln(-3+x)-1/5*ln(x-2)-1/120*ln(3+x)
```

Maxima [A] time = 0.800236, size = 34, normalized size = 0.87

$$-\frac{1}{120}\log(x+3)+\frac{1}{8}\log(x-1)-\frac{1}{5}\log(x-2)+\frac{1}{12}\log(x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 - 3*x^3 - 7*x^2 + 27*x - 18),x, algorithm="maxima")
```

```
[Out] -1/120*log(x + 3) + 1/8*log(x - 1) - 1/5*log(x - 2) + 1/12*log(x
- 3)
```

Fricas [A] time = 0.262999, size = 34, normalized size = 0.87

$$-\frac{1}{120}\log(x+3)+\frac{1}{8}\log(x-1)-\frac{1}{5}\log(x-2)+\frac{1}{12}\log(x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 - 3*x^3 - 7*x^2 + 27*x - 18),x, algorithm="fricas")
```

```
[Out] -1/120*log(x + 3) + 1/8*log(x - 1) - 1/5*log(x - 2) + 1/12*log(x
- 3)
```

Sympy [A] time = 0.654566, size = 26, normalized size = 0.67

$$\frac{\log(x-3)}{12}-\frac{\log(x-2)}{5}+\frac{\log(x-1)}{8}-\frac{\log(x+3)}{120}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**4-3*x**3-7*x**2+27*x-18),x)
```

```
[Out] log(x - 3)/12 - log(x - 2)/5 + log(x - 1)/8 - log(x + 3)/120
```

GIAC/XCAS [A] time = 0.265846, size = 39, normalized size = 1.

$$-\frac{1}{120}\ln(|x+3|)+\frac{1}{8}\ln(|x-1|)-\frac{1}{5}\ln(|x-2|)+\frac{1}{12}\ln(|x-3|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 - 3*x^3 - 7*x^2 + 27*x - 18),x, algorithm="giac")
```

```
[Out] -1/120*ln(abs(x + 3)) + 1/8*ln(abs(x - 1)) - 1/5*ln(abs(x - 2)) +
 1/12*ln(abs(x - 3))
```

3.266 $\int \frac{1+x^3}{-2+x}\,dx$

Optimal. Leaf size=22

$$\frac{x^3}{3}+x^2+4x+9\log(2-x)$$

```
[Out] 4*x + x^2 + x^3/3 + 9*Log[2 - x]
```

Rubi [A] time = 0.0316454, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{x^3}{3}+x^2+4x+9\log(2-x)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^3)/(-2 + x),x]

[Out] 4*x + x^2 + x^3/3 + 9*Log[2 - x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{x^3}{3}+4x+9\log(-x+2)+2\int x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**3+1)/(-2+x),x)

[Out] x**3/3 + 4*x + 9*log(-x + 2) + 2*Integral(x, x)
```

Mathematica [A] time = 0.0063875, size = 23, normalized size = 1.05

$$\frac{x^3}{3}+x^2+4x+9\log(x-2)-\frac{44}{3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^3)/(-2 + x),x]

[Out] -44/3 + 4*x + x^2 + x^3/3 + 9*Log[-2 + x]
```

Maple [A] time = 0.003, size = 19, normalized size = 0.9

$$\frac{x^3}{3}+x^2+4\,x+9\,\ln(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^3+1)/(x-2),x)
```

```
[Out] 1/3*x^3+x^2+4*x+9*ln(x-2)
```

Maxima [A] time = 0.813851, size = 24, normalized size = 1.09

$$\frac{1}{3}x^3 + x^2 + 4x + 9 \log(x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + 1)/(x - 2),x, algorithm="maxima")
```

```
[Out] 1/3*x^3 + x^2 + 4*x + 9*log(x - 2)
```

Fricas [A] time = 0.262207, size = 24, normalized size = 1.09

$$\frac{1}{3}x^3 + x^2 + 4x + 9 \log(x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + 1)/(x - 2),x, algorithm="fricas")
```

```
[Out] 1/3*x^3 + x^2 + 4*x + 9*log(x - 2)
```

Sympy [A] time = 0.114196, size = 17, normalized size = 0.77

$$\frac{x^3}{3} + x^2 + 4x + 9\log(x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3+1)/(-2+x),x)
```

```
[Out] x**3/3 + x**2 + 4*x + 9*log(x - 2)
```

GIAC/XCAS [A] time = 0.259357, size = 26, normalized size = 1.18

$$\frac{1}{3}x^3 + x^2 + 4x + 9\ln(|x - 2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + 1)/(x - 2),x, algorithm="giac")
```

```
[Out] 1/3*x^3 + x^2 + 4*x + 9*ln(abs(x - 2))
```

3.267 $\int \frac{3x-4x^2+3x^3}{1+x^2}\,dx$

Optimal. Leaf size=15

$$\frac{3x^2}{2} - 4x + 4\tan^{-1}(x)$$

```
[Out] -4*x + (3*x^2)/2 + 4*ArcTan[x]
```

Rubi [A] time = 0.0401547, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$\frac{3x^2}{2} - 4x + 4\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(3*x - 4*x^2 + 3*x^3)/(1 + x^2),x]

[Out] -4*x + (3*x^2)/2 + 4*ArcTan[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-4x + 4\,\mathrm{atan}\,(x) + 3\int x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**3-4*x**2+3*x)/(x**2+1),x)

[Out] -4*x + 4*atan(x) + 3*Integral(x, x)
```

Mathematica [A] time = 0.00745848, size = 15, normalized size = 1.

$$\frac{3x^2}{2} - 4x + 4\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3*x - 4*x^2 + 3*x^3)/(1 + x^2),x]

[Out] -4*x + (3*x^2)/2 + 4*ArcTan[x]
```

Maple [A] time = 0.004, size = 14, normalized size = 0.9

$$-4\,x + \frac{3\,x^2}{2} + 4\,\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^3-4*x^2+3*x)/(x^2+1),x)
```

```
[Out] -4*x+3/2*x^2+4*arctan(x)
```

Maxima [A] time = 0.904657, size = 18, normalized size = 1.2

$$\frac{3}{2}x^2 - 4x + 4\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3 - 4*x^2 + 3*x)/(x^2 + 1),x, algorithm="maxima")
```

```
[Out] 3/2*x^2 - 4*x + 4*arctan(x)
```

Fricas [A] time = 0.253062, size = 18, normalized size = 1.2

$$\frac{3}{2}x^2 - 4x + 4\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3 - 4*x^2 + 3*x)/(x^2 + 1),x, algorithm="fricas")
```

```
[Out] 3/2*x^2 - 4*x + 4*arctan(x)
```

Sympy [A] time = 0.169562, size = 14, normalized size = 0.93

$$\frac{3x^2}{2} - 4x + 4\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**3-4*x**2+3*x)/(x**2+1),x)
```

```
[Out] 3*x**2/2 - 4*x + 4*atan(x)
```

GIAC/XCAS [A] time = 0.258449, size = 18, normalized size = 1.2

$$\frac{3}{2}x^2 - 4x + 4\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3 - 4*x^2 + 3*x)/(x^2 + 1),x, algorithm="giac")
```

```
[Out] 3/2*x^2 - 4*x + 4*arctan(x)
```

3.268 $\int \frac{5+3x}{1-x-x^2+x^3} \, dx$

Optimal. Leaf size=12

$$\frac{4}{1-x}+\tanh^{-1}(x)$$

```
[Out] 4/(1 - x) + ArcTanh[x]
```

Rubi [A] time = 0.0402657, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095

$$\frac{4}{1-x}+\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(5 + 3*x)/(1 - x - x^2 + x^3),x]

[Out] 4/(1 - x) + ArcTanh[x]
```

Rubi in Sympy [A] time = 15.4377, size = 7, normalized size = 0.58

$$\operatorname{atanh}(x)+\frac{4}{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5+3*x)/(x**3-x**2-x+1),x)

[Out] atanh(x) + 4/(-x + 1)
```

Mathematica [A] time = 0.0158097, size = 24, normalized size = 2.

$$-\frac{4}{x-1}-\frac{1}{2}\log(x-1)+\frac{1}{2}\log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(5 + 3*x)/(1 - x - x^2 + x^3),x]

[Out] -4/(-1 + x) - Log[-1 + x]/2 + Log[1 + x]/2
```

Maple [A] time = 0.011, size = 21, normalized size = 1.8

$$-4(-1+x)^{-1}-\frac{\ln(-1+x)}{2}+\frac{\ln(1+x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5+3*x)/(x^3-x^2-x+1),x)

[Out] -4/(-1+x)-1/2*ln(-1+x)+1/2*ln(1+x)
```

Maxima [A] time = 0.789688, size = 27, normalized size = 2.25

$$-\frac{4}{x-1} + \frac{1}{2}\,\log\left(x+1\right) - \frac{1}{2}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x + 5)/(x^3 - x^2 - x + 1),x, algorithm="maxima")
```

```
[Out] -4/(x - 1) + 1/2*log(x + 1) - 1/2*log(x - 1)
```

Fricas [A] time = 0.248448, size = 35, normalized size = 2.92

$$\frac{(x-1)\log\left(x+1\right) - (x-1)\log\left(x-1\right) - 8}{2\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x + 5)/(x^3 - x^2 - x + 1),x, algorithm="fricas")
```

```
[Out] 1/2*((x - 1)*log(x + 1) - (x - 1)*log(x - 1) - 8)/(x - 1)
```

Sympy [A] time = 0.186911, size = 17, normalized size = 1.42

$$-\frac{\log\left(x-1\right)}{2} + \frac{\log\left(x+1\right)}{2} - \frac{4}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5+3*x)/(x**3-x**2-x+1),x)
```

```
[Out] -log(x - 1)/2 + log(x + 1)/2 - 4/(x - 1)
```

GIAC/XCAS [A] time = 0.264683, size = 30, normalized size = 2.5

$$-\frac{4}{x-1} + \frac{1}{2}\ln\left(|x+1|\right) - \frac{1}{2}\ln\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x + 5)/(x^3 - x^2 - x + 1),x, algorithm="giac")
```

```
[Out] -4/(x - 1) + 1/2*ln(abs(x + 1)) - 1/2*ln(abs(x - 1))
```

3.269 $\int \frac{-1-x-x^3+x^4}{-x^2+x^3} dx$

Optimal. Leaf size=25

$$\frac{x^2}{2} - \frac{1}{x} - 2\log(1-x) + 2\log(x)$$

```
[Out] -x^(-1) + x^2/2 - 2*Log[1 - x] + 2*Log[x]
```

Rubi [A] time = 0.0442415, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$

$$\frac{x^2}{2} - \frac{1}{x} - 2\log(1-x) + 2\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 - x - x^3 + x^4)/(-x^2 + x^3),x]

[Out] -x^(-1) + x^2/2 - 2*Log[1 - x] + 2*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$2\log(x) - 2\log(-x+1) + \int x\,dx - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**4-x**3-x-1)/(x**3-x**2),x)

[Out] 2*log(x) - 2*log(-x + 1) + Integral(x, x) - 1/x
```

Mathematica [A] time = 0.00956973, size = 25, normalized size = 1.

$$\frac{x^2}{2} - \frac{1}{x} - 2\log(1-x) + 2\log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 - x - x^3 + x^4)/(-x^2 + x^3),x]

[Out] -x^(-1) + x^2/2 - 2*Log[1 - x] + 2*Log[x]
```

Maple [A] time = 0.011, size = 22, normalized size = 0.9

$$\frac{x^2}{2} - 2\ln(-1+x) - x^{-1} + 2\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^4-x^3-x-1)/(x^3-x^2),x)
```

```
[Out] 1/2*x^2-2*ln(-1+x)-1/x+2*ln(x)
```

Maxima [A] time = 0.824758, size = 28, normalized size = 1.12

$$\frac{1}{2}x^2 - \frac{1}{x} - 2\log(x-1) + 2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 - x^3 - x - 1)/(x^3 - x^2),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 1/x - 2*log(x - 1) + 2*log(x)
```

Fricas [A] time = 0.252043, size = 30, normalized size = 1.2

$$\frac{x^3 - 4x\log(x-1) + 4x\log(x) - 2}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 - x^3 - x - 1)/(x^3 - x^2),x, algorithm="fricas")
```

```
[Out] 1/2*(x^3 - 4*x*log(x - 1) + 4*x*log(x) - 2)/x
```

Sympy [A] time = 0.200612, size = 19, normalized size = 0.76

$$\frac{x^2}{2} + 2\log(x) - 2\log(x-1) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**4-x**3-x-1)/(x**3-x**2),x)
```

```
[Out] x**2/2 + 2*log(x) - 2*log(x - 1) - 1/x
```

GIAC/XCAS [A] time = 0.261422, size = 31, normalized size = 1.24

$$\frac{1}{2}x^2 - \frac{1}{x} - 2\ln(|x-1|) + 2\ln(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 - x^3 - x - 1)/(x^3 - x^2),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 1/x - 2*ln(abs(x - 1)) + 2*ln(abs(x))
```

3.270 $\int \frac{2+x+x^2+x^3}{2+3x^2+x^4}\,dx$

Optimal. Leaf size=13

$$\frac{1}{2}\log\left(x^2+2\right)+\tan^{-1}(x)$$

```
[Out] ArcTan[x] + Log[2 + x^2]/2
```

Rubi [A] time = 0.0407511, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{1}{2}\log\left(x^2+2\right)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + x + x^2 + x^3)/(2 + 3*x^2 + x^4),x]

[Out] ArcTan[x] + Log[2 + x^2]/2
```

Rubi in Sympy [A] time = 17.3369, size = 10, normalized size = 0.77

$$\frac{\log\left(x^2+2\right)}{2}+\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**3+x**2+x+2)/(x**4+3*x**2+2),x)

[Out] log(x**2 + 2)/2 + atan(x)
```

Mathematica [A] time = 0.01097, size = 13, normalized size = 1.

$$\frac{1}{2}\log\left(x^2+2\right)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + x + x^2 + x^3)/(2 + 3*x^2 + x^4),x]

[Out] ArcTan[x] + Log[2 + x^2]/2
```

Maple [A] time = 0.007, size = 12, normalized size = 0.9

$$\arctan(x)+\frac{\ln\left(x^2+2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^3+x^2+x+2)/(x^4+3*x^2+2),x)
```

```
[Out] arctan(x)+1/2*ln(x^2+2)
```

Maxima [A] time = 0.865083, size = 15, normalized size = 1.15

$$\arctan(x) + \frac{1}{2}\log(x^2 + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3 + x^2 + x + 2)/(x^4 + 3*x^2 + 2),x, algorithm="maxima")
```

```
[Out] arctan(x) + 1/2*log(x^2 + 2)
```

Fricas [A] time = 0.248314, size = 15, normalized size = 1.15

$$\arctan(x) + \frac{1}{2}\log(x^2 + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3 + x^2 + x + 2)/(x^4 + 3*x^2 + 2),x, algorithm="fricas")
```

```
[Out] arctan(x) + 1/2*log(x^2 + 2)
```

Sympy [A] time = 0.243299, size = 10, normalized size = 0.77

$$\frac{\log(x^2 + 2)}{2} + \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3+x**2+x+2)/(x**4+3*x**2+2),x)
```

```
[Out] log(x**2 + 2)/2 + atan(x)
```

GIAC/XCAS [A] time = 0.261356, size = 15, normalized size = 1.15

$$\arctan(x) + \frac{1}{2}\ln(x^2 + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + x^2 + x + 2)/(x^4 + 3*x^2 + 2),x, algorithm="giac")
```

```
[Out] arctan(x) + 1/2*ln(x^2 + 2)
```

3.271 $\int \frac{-4+8x-4x^2+4x^3-x^4+x^5}{(2+x^2)^3}\,dx$

Optimal. Leaf size=35

$$-\frac{1}{(x^2+2)^2}+\frac{1}{2}\log\left(x^2+2\right)-\frac{\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] -(2 + x^2)^(-2) - ArcTan[x/Sqrt[2]]/Sqrt[2] + Log[2 + x^2]/2
```

Rubi [A] time = 0.0566319, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161

$$-\frac{1}{(x^2+2)^2}+\frac{1}{2}\log\left(x^2+2\right)-\frac{\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(-4 + 8*x - 4*x^2 + 4*x^3 - x^4 + x^5)/(2 + x^2)^3,x]

[Out] -(2 + x^2)^(-2) - ArcTan[x/Sqrt[2]]/Sqrt[2] + Log[2 + x^2]/2
```

Rubi in Sympy [A] time = 23.9828, size = 46, normalized size = 1.31

$$\frac{x^2}{4(x^2+2)}+\frac{x^2}{2(x^2+2)^2}+\frac{\log\left(x^2+2\right)}{2}-\frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**5-x**4+4*x**3-4*x**2+8*x-4)/(x**2+2)**3,x)

[Out] x**2/(4*(x**2 + 2)) + x**2/(2*(x**2 + 2)**2) + log(x**2 + 2)/2 -
sqrt(2)*atan(sqrt(2)*x/2)/2
```

Mathematica [A] time = 0.0259836, size = 35, normalized size = 1.

$$-\frac{1}{(x^2+2)^2}+\frac{1}{2}\log\left(x^2+2\right)-\frac{\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-4 + 8*x - 4*x^2 + 4*x^3 - x^4 + x^5)/(2 + x^2)^3,x]

[Out] -(2 + x^2)^(-2) - ArcTan[x/Sqrt[2]]/Sqrt[2] + Log[2 + x^2]/2
```

Maple [A] time = 0.008, size = 31, normalized size = 0.9

$$-\left(x^2+2\right)^{-2}+\frac{\ln\left(x^2+2\right)}{2}-\frac{\sqrt{2}}{2}\arctan\left(\frac{\sqrt{2}x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^5-x^4+4*x^3-4*x^2+8*x-4)/(x^2+2)^3,x)
```

```
[Out] -1/(x^2+2)^2+1/2*ln(x^2+2)-1/2*arctan(1/2*2^(1/2)*x)*2^(1/2)
```

Maxima [A] time = 0.886028, size = 47, normalized size = 1.34

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - \frac{1}{x^4 + 4x^2 + 4} + \frac{1}{2}\log\left(x^2 + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5 - x^4 + 4*x^3 - 4*x^2 + 8*x - 4)/(x^2 + 2)^3,x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/(x^4 + 4*x^2 + 4) + 1/2*lo
g(x^2 + 2)
```

Fricas [A] time = 0.249461, size = 84, normalized size = 2.4

$$\frac{\sqrt{2}\left(\sqrt{2}\left(x^4 + 4x^2 + 4\right)\log\left(x^2 + 2\right) - 2\left(x^4 + 4x^2 + 4\right)\arctan\left(\frac{1}{2}\sqrt{2}x\right) - 2\sqrt{2}\right)}{4\left(x^4 + 4x^2 + 4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5 - x^4 + 4*x^3 - 4*x^2 + 8*x - 4)/(x^2 + 2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*(sqrt(2)*(x^4 + 4*x^2 + 4)*log(x^2 + 2) - 2*(x^4 + 4*
x^2 + 4)*arctan(1/2*sqrt(2)*x) - 2*sqrt(2))/(x^4 + 4*x^2 + 4)
```

Sympy [A] time = 0.363944, size = 36, normalized size = 1.03

$$\frac{\log\left(x^2 + 2\right)}{2} - \frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{2} - \frac{1}{x^4 + 4x^2 + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**5-x**4+4*x**3-4*x**2+8*x-4)/(x**2+2)**3,x)
```

```
[Out] log(x**2 + 2)/2 - sqrt(2)*atan(sqrt(2)*x/2)/2 - 1/(x**4 + 4*x**2
+ 4)
```

GIAC/XCAS [A] time = 0.262179, size = 41, normalized size = 1.17

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - \frac{1}{\left(x^2 + 2\right)^2} + \frac{1}{2}\ln\left(x^2 + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5 - x^4 + 4*x^3 - 4*x^2 + 8*x - 4)/(x^2 + 2)^3,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/(x^2 + 2)^2 + 1/2*ln(x^2 +
 2)
```

3.272 $\int \frac{-1-3x+x^2}{-2x+x^2+x^3}\,dx$

Optimal. Leaf size=23

$$-\log(1-x)+\frac{\log(x)}{2}+\frac{3}{2}\log(x+2)$$

```
[Out] -Log[1 - x] + Log[x]/2 + (3*Log[2 + x])/2
```

Rubi [A] time = 0.0552012, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095

$$-\log(1-x)+\frac{\log(x)}{2}+\frac{3}{2}\log(x+2)$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 - 3*x + x^2)/(-2*x + x^2 + x^3),x]

[Out] -Log[1 - x] + Log[x]/2 + (3*Log[2 + x])/2
```

Rubi in Sympy [A] time = 11.4245, size = 17, normalized size = 0.74

$$\frac{\log(x)}{2}-\log(-x+1)+\frac{3\log(x+2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2-3*x-1)/(x**3+x**2-2*x),x)

[Out] log(x)/2 - log(-x + 1) + 3*log(x + 2)/2
```

Mathematica [A] time = 0.00895568, size = 23, normalized size = 1.

$$-\log(1-x)+\frac{\log(x)}{2}+\frac{3}{2}\log(x+2)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 - 3*x + x^2)/(-2*x + x^2 + x^3),x]

[Out] -Log[1 - x] + Log[x]/2 + (3*Log[2 + x])/2
```

Maple [A] time = 0.011, size = 18, normalized size = 0.8

$$\frac{3\ln(2+x)}{2}-\ln(-1+x)+\frac{\ln(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2-3*x-1)/(x^3+x^2-2*x),x)
```

```
[Out] 3/2*ln(2+x)-ln(-1+x)+1/2*ln(x)
```

Maxima [A] time = 0.806418, size = 23, normalized size = 1.

$$\frac{3}{2}\log(x+2)-\log(x-1)+\frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 3*x - 1)/(x^3 + x^2 - 2*x),x, algorithm="maxima")
```

```
[Out] 3/2*log(x + 2) - log(x - 1) + 1/2*log(x)
```

Fricas [A] time = 0.253737, size = 23, normalized size = 1.

$$\frac{3}{2}\log(x+2)-\log(x-1)+\frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 3*x - 1)/(x^3 + x^2 - 2*x),x, algorithm="fricas")
```

```
[Out] 3/2*log(x + 2) - log(x - 1) + 1/2*log(x)
```

Sympy [A] time = 0.302288, size = 17, normalized size = 0.74

$$\frac{\log(x)}{2}-\log(x-1)+\frac{3\log(x+2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2-3*x-1)/(x**3+x**2-2*x),x)
```

```
[Out] log(x)/2 - log(x - 1) + 3*log(x + 2)/2
```

GIAC/XCAS [A] time = 0.261631, size = 27, normalized size = 1.17

$$\frac{3}{2}\ln(|x+2|)-\ln(|x-1|)+\frac{1}{2}\ln(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 3*x - 1)/(x^3 + x^2 - 2*x),x, algorithm="giac")
```

```
[Out] 3/2*ln(abs(x + 2)) - ln(abs(x - 1)) + 1/2*ln(abs(x))
```

3.273 $\int \frac{3-x+3x^2-2x^3+x^4}{3x-2x^2+x^3}\,dx$

Optimal. Leaf size=23

$$\frac{x^2}{2}-\frac{1}{2}\log\left(x^2-2x+3\right)+\log(x)$$

```
[Out] x^2/2 + Log[x] - Log[3 - 2*x + x^2]/2
```

Rubi [A] time = 0.0636577, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{x^2}{2}-\frac{1}{2}\log\left(x^2-2x+3\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(3 - x + 3*x^2 - 2*x^3 + x^4)/(3*x - 2*x^2 + x^3),x]

[Out] x^2/2 + Log[x] - Log[3 - 2*x + x^2]/2
```

Rubi in Sympy [A] time = 30.1732, size = 19, normalized size = 0.83

$$\frac{x^2}{2}+\log(x)-\frac{\log\left(x^2-2x+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**4-2*x**3+3*x**2-x+3)/(x**3-2*x**2+3*x),x)

[Out] x**2/2 + log(x) - log(x**2 - 2*x + 3)/2
```

Mathematica [A] time = 0.0131068, size = 23, normalized size = 1.

$$\frac{x^2}{2}-\frac{1}{2}\log\left(x^2-2x+3\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3 - x + 3*x^2 - 2*x^3 + x^4)/(3*x - 2*x^2 + x^3),x]

[Out] x^2/2 + Log[x] - Log[3 - 2*x + x^2]/2
```

Maple [A] time = 0.007, size = 20, normalized size = 0.9

$$\frac{x^2}{2}+\ln(x)-\frac{\ln\left(x^2-2\,x+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^4-2*x^3+3*x^2-x+3)/(x^3-2*x^2+3*x),x)
```

```
[Out] 1/2*x^2+ln(x)-1/2*ln(x^2-2*x+3)
```

Maxima [A] time = 0.801188, size = 26, normalized size = 1.13

$$\frac{1}{2}x^2 - \frac{1}{2}\log\left(x^2 - 2x + 3\right) + \log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 - 2*x^3 + 3*x^2 - x + 3)/(x^3 - 2*x^2 + 3*x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 1/2*log(x^2 - 2*x + 3) + log(x)
```

Fricas [A] time = 0.24787, size = 26, normalized size = 1.13

$$\frac{1}{2}x^2 - \frac{1}{2}\log\left(x^2 - 2x + 3\right) + \log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 - 2*x^3 + 3*x^2 - x + 3)/(x^3 - 2*x^2 + 3*x),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 - 1/2*log(x^2 - 2*x + 3) + log(x)
```

Sympy [A] time = 0.21191, size = 19, normalized size = 0.83

$$\frac{x^2}{2} + \log\left(x\right) - \frac{\log\left(x^2 - 2x + 3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**4-2*x**3+3*x**2-x+3)/(x**3-2*x**2+3*x),x)
```

```
[Out] x**2/2 + log(x) - log(x**2 - 2*x + 3)/2
```

GIAC/XCAS [A] time = 0.261477, size = 27, normalized size = 1.17

$$\frac{1}{2}x^2 - \frac{1}{2}\ln\left(x^2 - 2x + 3\right) + \ln\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 - 2*x^3 + 3*x^2 - x + 3)/(x^3 - 2*x^2 + 3*x),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 1/2*ln(x^2 - 2*x + 3) + ln(abs(x))
```

3.274 $\int \frac{-1+x+x^3}{(1+x^2)^2}\,dx$

Optimal. Leaf size=29

$$-\frac{x}{2(x^2+1)}+\frac{1}{2}\log\left(x^2+1\right)-\frac{1}{2}\tan^{-1}(x)$$

```
[Out] -x/(2*(1 + x^2)) - ArcTan[x]/2 + Log[1 + x^2]/2
```

Rubi [A] time = 0.032797, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286

$$-\frac{x}{2(x^2+1)}+\frac{1}{2}\log\left(x^2+1\right)-\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + x + x^3)/(1 + x^2)^2,x]
```

```
[Out] -x/(2*(1 + x^2)) - ArcTan[x]/2 + Log[1 + x^2]/2
```

Rubi in Sympy [A] time = 7.91506, size = 20, normalized size = 0.69

$$-\frac{x}{2(x^2+1)}+\frac{\log\left(x^2+1\right)}{2}-\frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**3+x-1)/(x**2+1)**2,x)
```

```
[Out] -x/(2*(x**2 + 1)) + log(x**2 + 1)/2 - atan(x)/2
```

Mathematica [A] time = 0.0153806, size = 25, normalized size = 0.86

$$\frac{1}{2}\left(-\frac{x}{x^2+1}+\log\left(x^2+1\right)-\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + x + x^3)/(1 + x^2)^2,x]
```

```
[Out] (-(x/(1 + x^2)) - ArcTan[x] + Log[1 + x^2])/2
```

Maple [A] time = 0.007, size = 24, normalized size = 0.8

$$-\frac{x}{2x^2+2}-\frac{\arctan(x)}{2}+\frac{\ln\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^3+x-1)/(x^2+1)^2,x)
```

```
[Out] -1/2*x/(x^2+1)-1/2*arctan(x)+1/2*ln(x^2+1)
```

Maxima [A] time = 0.898812, size = 31, normalized size = 1.07

$$-\frac{x}{2\left(x^2+1\right)}-\frac{1}{2}\arctan\left(x\right)+\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + x - 1)/(x^2 + 1)^2,x, algorithm="maxima")

[Out] -1/2*x/(x^2 + 1) - 1/2*arctan(x) + 1/2*log(x^2 + 1)
```

Fricas [A] time = 0.245212, size = 43, normalized size = 1.48

$$-\frac{\left(x^2+1\right)\arctan\left(x\right)-\left(x^2+1\right)\log\left(x^2+1\right)+x}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + x - 1)/(x^2 + 1)^2,x, algorithm="fricas")

[Out] -1/2*((x^2 + 1)*arctan(x) - (x^2 + 1)*log(x^2 + 1) + x)/(x^2 + 1)
```

Sympy [A] time = 0.263982, size = 20, normalized size = 0.69

$$-\frac{x}{2x^2+2}+\frac{\log\left(x^2+1\right)}{2}-\frac{\operatorname{atan}\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3+x-1)/(x**2+1)**2,x)

[Out] -x/(2*x**2 + 2) + log(x**2 + 1)/2 - atan(x)/2
```

GIAC/XCAS [A] time = 0.260965, size = 31, normalized size = 1.07

$$-\frac{x}{2\left(x^2+1\right)}-\frac{1}{2}\arctan\left(x\right)+\frac{1}{2}\ln\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + x - 1)/(x^2 + 1)^2,x, algorithm="giac")

[Out] -1/2*x/(x^2 + 1) - 1/2*arctan(x) + 1/2*ln(x^2 + 1)
```

3.275 $\int \frac{1+2x-x^2+8x^3+x^4}{(x+x^2)(1+x^3)}\,dx$

Optimal. Leaf size=44

$$\log\left(x^2-x+1\right)-\frac{3}{x+1}+\log(x)-2\log(x+1)-\frac{2\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -3/(1 + x) - (2*ArcTan[(1 - 2*x)/Sqrt[3]])/Sqrt[3] + Log[x] - 2*L
og[1 + x] + Log[1 - x + x^2]
```

Rubi [A] time = 0.422485, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\log\left(x^2-x+1\right)-\frac{3}{x+1}+\log(x)-2\log(x+1)-\frac{2\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + 2*x - x^2 + 8*x^3 + x^4)/((x + x^2)*(1 + x^3)),x]
```

```
[Out] -3/(1 + x) - (2*ArcTan[(1 - 2*x)/Sqrt[3]])/Sqrt[3] + Log[x] - 2*L
og[1 + x] + Log[1 - x + x^2]
```

Rubi in Sympy [A] time = 148.204, size = 46, normalized size = 1.05

$$\log(x)-2\log(x+1)+\log\left(x^2-x+1\right)+\frac{2\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}-\frac{1}{3}\right)\right)}{3}-\frac{3}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**4+8*x**3-x**2+2*x+1)/(x**2+x)/(x**3+1),x)
```

```
[Out] log(x) - 2*log(x + 1) + log(x**2 - x + 1) + 2*sqrt(3)*atan(sqrt(3
)*(2*x/3 - 1/3))/3 - 3/(x + 1)
```

Mathematica [A] time = 0.0376188, size = 44, normalized size = 1.

$$\log\left(x^2-x+1\right)-\frac{3}{x+1}+\log(x)-2\log(x+1)+\frac{2\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + 2*x - x^2 + 8*x^3 + x^4)/((x + x^2)*(1 + x^3)),x]
```

```
[Out] -3/(1 + x) + (2*ArcTan[(-1 + 2*x)/Sqrt[3]])/Sqrt[3] + Log[x] - 2*
Log[1 + x] + Log[1 - x + x^2]
```

Maple [A] time = 0.013, size = 42, normalized size = 1.

$$-3\,(1+x)^{-1}-2\,\ln(1+x)+\ln(x)+\ln\left(x^2-x+1\right)+\frac{2\sqrt{3}}{3}\arctan\left(\frac{(2\,x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((x^4+8*x^3-x^2+2*x+1)/(x^2+x)/(x^3+1),x)
```

```
[Out] -3/(1+x)-2*ln(1+x)+ln(x)+ln(x^2-x+1)+2/3*3^(1/2)*arctan(1/3*(2*x-
1)*3^(1/2))
```

Maxima [A] time = 0.888519, size = 55, normalized size = 1.25

$$\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{3}{x+1}+\log\left(x^2-x+1\right)-2\log\left(x+1\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 + 8*x^3 - x^2 + 2*x + 1)/((x^3 + 1)*(x^2 + x)),x, algorithm="maxima
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 3/(x + 1) + log(x^2 -
 x + 1) - 2*log(x + 1) + log(x)
```

Fricas [A] time = 0.253884, size = 93, normalized size = 2.11

$$\frac{\sqrt{3}\left(\sqrt{3}(x+1)\log\left(x^2-x+1\right)-2\sqrt{3}(x+1)\log\left(x+1\right)+\sqrt{3}(x+1)\log\left(x\right)+2\left(x+1\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-3\sqrt{3}\right)}{3\left(x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 + 8*x^3 - x^2 + 2*x + 1)/((x^3 + 1)*(x^2 + x)),x, algorithm="fricas
```

```
[Out] 1/3*sqrt(3)*(sqrt(3)*(x + 1)*log(x^2 - x + 1) - 2*sqrt(3)*(x + 1)
*log(x + 1) + sqrt(3)*(x + 1)*log(x) + 2*(x + 1)*arctan(1/3*sqrt(
3)*(2*x - 1)) - 3*sqrt(3))/(x + 1)
```

Sympy [A] time = 0.606367, size = 49, normalized size = 1.11

$$\log\left(x\right)-2\log\left(x+1\right)+\log\left(x^2-x+1\right)+\frac{2\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}-\frac{3}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((x**4+8*x**3-x**2+2*x+1)/(x**2+x)/(x**3+1),x)
```

```
[Out] log(x) - 2*log(x + 1) + log(x**2 - x + 1) + 2*sqrt(3)*atan(2*sqrt
(3)*x/3 - sqrt(3)/3)/3 - 3/(x + 1)
```

GIAC/XCAS [A] time = 0.262553, size = 58, normalized size = 1.32

$$\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{3}{x+1}+\ln\left(x^2-x+1\right)-2\ln\left(|x+1|\right)+\ln\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 + 8*x^3 - x^2 + 2*x + 1)/((x^3 + 1)*(x^2 + x)),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 3/(x + 1) + ln(x^2 -
x + 1) - 2*ln(abs(x + 1)) + ln(abs(x))
```

3.276 $\int \frac{15-5x+x^2+x^3}{(5+x^2)(3+2x+x^2)}\,dx$

Optimal. Leaf size=46

$$\frac{1}{2}\log\left(x^2+2x+3\right)-\sqrt{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+\frac{5\tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] -(Sqrt[5]*ArcTan[x/Sqrt[5]]) + (5*ArcTan[(1 + x)/Sqrt[2]])/Sqrt[2
] + Log[3 + 2*x + x^2]/2
```

Rubi [A] time = 0.256851, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207

$$\frac{1}{2}\log\left(x^2+2x+3\right)-\sqrt{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+\frac{5\tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(15 - 5*x + x^2 + x^3)/((5 + x^2)*(3 + 2*x + x^2)),x]
```

```
[Out] -(Sqrt[5]*ArcTan[x/Sqrt[5]]) + (5*ArcTan[(1 + x)/Sqrt[2]])/Sqrt[2
] + Log[3 + 2*x + x^2]/2
```

Rubi in Sympy [A] time = 47.2279, size = 48, normalized size = 1.04

$$\frac{\log\left(x^2+2x+3\right)}{2}+\frac{5\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{x}{2}+\frac{1}{2}\right)\right)}{2}-\sqrt{5}\,\mathrm{atan}\left(\frac{\sqrt{5}x}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**3+x**2-5*x+15)/(x**2+5)/(x**2+2*x+3),x)
```

```
[Out] log(x**2 + 2*x + 3)/2 + 5*sqrt(2)*atan(sqrt(2)*(x/2 + 1/2))/2 - s
qrt(5)*atan(sqrt(5)*x/5)
```

Mathematica [A] time = 0.0350858, size = 46, normalized size = 1.

$$\frac{1}{2}\log\left(x^2+2x+3\right)-\sqrt{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+\frac{5\tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(15 - 5*x + x^2 + x^3)/((5 + x^2)*(3 + 2*x + x^2)),x]
```

```
[Out] -(Sqrt[5]*ArcTan[x/Sqrt[5]]) + (5*ArcTan[(1 + x)/Sqrt[2]])/Sqrt[2
] + Log[3 + 2*x + x^2]/2
```

Maple [A] time = 0.009, size = 41, normalized size = 0.9

$$\frac{\ln\left(x^2+2\,x+3\right)}{2}+\frac{5\,\sqrt{2}}{2}\arctan\left(\frac{(2+2\,x)\sqrt{2}}{4}\right)-\arctan\left(\frac{x\sqrt{5}}{5}\right)\sqrt{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x)
```

```
[Out] 1/2*ln(x^2+2*x+3)+5/2*2^(1/2)*arctan(1/4*(2+2*x)*2^(1/2))-arctan(
1/5*x*5^(1/2))*5^(1/2)
```

Maxima [A] time = 0.875632, size = 51, normalized size = 1.11

$$\frac{5}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x+1)\right)-\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right)+\frac{1}{2}\log\left(x^2+2x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3 + x^2 - 5*x + 15)/((x^2 + 2*x + 3)*(x^2 + 5)),x, algorithm="maxima"
```

```
[Out] 5/2*sqrt(2)*arctan(1/2*sqrt(2)*(x + 1)) - sqrt(5)*arctan(1/5*sqrt
(5)*x) + 1/2*log(x^2 + 2*x + 3)
```

Fricas [A] time = 0.261166, size = 62, normalized size = 1.35

$$-\frac{1}{4}\sqrt{2}\left(2\sqrt{5}\sqrt{2}\arctan\left(\frac{1}{5}\sqrt{5}x\right)-\sqrt{2}\log\left(x^2+2x+3\right)-10\arctan\left(\frac{1}{2}\sqrt{2}(x+1)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3 + x^2 - 5*x + 15)/((x^2 + 2*x + 3)*(x^2 + 5)),x, algorithm="fricas"
```

```
[Out] -1/4*sqrt(2)*(2*sqrt(5)*sqrt(2)*arctan(1/5*sqrt(5)*x) - sqrt(2)*l
og(x^2 + 2*x + 3) - 10*arctan(1/2*sqrt(2)*(x + 1)))
```

Sympy [A] time = 0.588429, size = 51, normalized size = 1.11

$$\frac{\log\left(x^2+2x+3\right)}{2}-\sqrt{5}\operatorname{atan}\left(\frac{\sqrt{5}x}{5}\right)+\frac{5\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}+\frac{\sqrt{2}}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3+x**2-5*x+15)/(x**2+5)/(x**2+2*x+3),x)
```

```
[Out] log(x**2 + 2*x + 3)/2 - sqrt(5)*atan(sqrt(5)*x/5) + 5*sqrt(2)*ata
n(sqrt(2)*x/2 + sqrt(2)/2)/2
```

GIAC/XCAS [A] time = 0.260872, size = 51, normalized size = 1.11

$$\frac{5}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x+1)\right)-\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right)+\frac{1}{2}\ln\left(x^2+2x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3 + x^2 - 5*x + 15)/((x^2 + 2*x + 3)*(x^2 + 5)),x, algorithm="giac")
```

```
[Out] 5/2*sqrt(2)*arctan(1/2*sqrt(2)*(x + 1)) - sqrt(5)*arctan(1/5*sqrt
(5)*x) + 1/2*ln(x^2 + 2*x + 3)
```

3.277 $\int \frac{-3+25x+23x^2+32x^3+15x^4+7x^5+x^6}{(1+x^2)^2(2+x+x^2)^2}\,dx$

Optimal. Leaf size=33

$$-\frac{3}{x^2+1}+\frac{1}{x^2+x+2}+\log\left(x^2+1\right)-\log\left(x^2+x+2\right)$$

```
[Out] -3/(1 + x^2) + (2 + x + x^2)^(-1) + Log[1 + x^2] - Log[2 + x + x^
2]
```

Rubi [A] time = 0.30324, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.068

$$-\frac{3}{x^2+1}+\frac{1}{x^2+x+2}+\log\left(x^2+1\right)-\log\left(x^2+x+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + 25*x + 23*x^2 + 32*x^3 + 15*x^4 + 7*x^5 + x^6)/((1 + x^2)^2*(2 + x + x^2)^
```

```
[Out] -3/(1 + x^2) + (2 + x + x^2)^(-1) + Log[1 + x^2] - Log[2 + x + x^
2]
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((x**6+7*x**5+15*x**4+32*x**3+23*x**2+25*x-3)/(x**2+1)**2/(x**2+
```

```
[Out] Timed out
```

Mathematica [A] time = 0.0319205, size = 33, normalized size = 1.

$$-\frac{3}{x^2+1}+\frac{1}{x^2+x+2}+\log\left(x^2+1\right)-\log\left(x^2+x+2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + 25*x + 23*x^2 + 32*x^3 + 15*x^4 + 7*x^5 + x^6)/((1 + x^2)^2*(2 + x +
```

```
[Out] -3/(1 + x^2) + (2 + x + x^2)^(-1) + Log[1 + x^2] - Log[2 + x + x^
2]
```

Maple [A] time = 0.015, size = 34, normalized size = 1.

$$-3\left(x^2+1\right)^{-1}+\left(x^2+x+2\right)^{-1}+\ln\left(x^2+1\right)-\ln\left(x^2+x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6+7*x^5+15*x^4+32*x^3+23*x^2+25*x-3)/(x^2+1)^2/(x^2+x+2)^2,x)
```

```
[Out] -3/(x^2+1)+1/(x^2+x+2)+ln(x^2+1)-ln(x^2+x+2)
```

Maxima [A] time = 0.793608, size = 59, normalized size = 1.79

$$-\frac{2\,x^2+3\,x+5}{x^4+x^3+3\,x^2+x+2}-\log\left(x^2+x+2\right)+\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6 + 7*x^5 + 15*x^4 + 32*x^3 + 23*x^2 + 25*x - 3)/((x^2 + x + 2)^2*(x^2
```

```
[Out] -(2*x^2 + 3*x + 5)/(x^4 + x^3 + 3*x^2 + x + 2) - log(x^2 + x + 2)
 + log(x^2 + 1)
```

Fricas [A] time = 0.264881, size = 97, normalized size = 2.94

$$-\frac{2\,x^2+\left(x^4+x^3+3\,x^2+x+2\right)\log\left(x^2+x+2\right)-\left(x^4+x^3+3\,x^2+x+2\right)\log\left(x^2+1\right)+3\,x+5}{x^4+x^3+3\,x^2+x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6 + 7*x^5 + 15*x^4 + 32*x^3 + 23*x^2 + 25*x - 3)/((x^2 + x + 2)^2*(x^2
```

```
[Out] -(2*x^2 + (x^4 + x^3 + 3*x^2 + x + 2)*log(x^2 + x + 2) - (x^4 + x
^3 + 3*x^2 + x + 2)*log(x^2 + 1) + 3*x + 5)/(x^4 + x^3 + 3*x^2 +
x + 2)
```

Sympy [A] time = 0.514569, size = 39, normalized size = 1.18

$$-\frac{2x^2+3x+5}{x^4+x^3+3x^2+x+2}+\log\left(x^2+1\right)-\log\left(x^2+x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+7*x**5+15*x**4+32*x**3+23*x**2+25*x-3)/(x**2+1)**2/(x**2+x+2)*
```

```
[Out] -(2*x**2 + 3*x + 5)/(x**4 + x**3 + 3*x**2 + x + 2) + log(x**2 + 1
) - log(x**2 + x + 2)
```

GIAC/XCAS [A] time = 0.262739, size = 59, normalized size = 1.79

$$-\frac{2\,x^2+3\,x+5}{x^4+x^3+3\,x^2+x+2}-\ln\left(x^2+x+2\right)+\ln\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6 + 7*x^5 + 15*x^4 + 32*x^3 + 23*x^2 + 25*x - 3)/((x^2 + x + 2)^2*(x^2
```

```
[Out] -(2*x^2 + 3*x + 5)/(x^4 + x^3 + 3*x^2 + x + 2) - ln(x^2 + x + 2)
+ ln(x^2 + 1)
```

3.278 $\int \frac{1}{(1+x^2)(4+x^2)} \, dx$

Optimal. Leaf size=17

$$\frac{1}{3}\tan^{-1}(x) - \frac{1}{6}\tan^{-1}\left(\frac{x}{2}\right)$$

```
[Out] -ArcTan[x/2]/6 + ArcTan[x]/3
```

Rubi [A] time = 0.022618, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{1}{3}\tan^{-1}(x) - \frac{1}{6}\tan^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/((1 + x^2)*(4 + x^2)),x]

[Out] -ArcTan[x/2]/6 + ArcTan[x]/3
```

Rubi in Sympy [A] time = 5.12088, size = 10, normalized size = 0.59

$$-\frac{\operatorname{atan}\left(\frac{x}{2}\right)}{6} + \frac{\operatorname{atan}(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**2+1)/(x**2+4),x)

[Out] -atan(x/2)/6 + atan(x)/3
```

Mathematica [A] time = 0.00993099, size = 17, normalized size = 1.

$$\frac{1}{6}\tan^{-1}\left(\frac{2}{x}\right) + \frac{1}{3}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((1 + x^2)*(4 + x^2)),x]

[Out] ArcTan[2/x]/6 + ArcTan[x]/3
```

Maple [A] time = 0.01, size = 12, normalized size = 0.7

$$-\frac{1}{6}\arctan\left(\frac{x}{2}\right) + \frac{\arctan(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^2+1)/(x^2+4),x)
```

```
[Out] -1/6*arctan(1/2*x)+1/3*arctan(x)
```

Maxima [A] time = 0.89781, size = 15, normalized size = 0.88

$$-\frac{1}{6}\arctan\left(\frac{1}{2}x\right)+\frac{1}{3}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 4)*(x^2 + 1)),x, algorithm="maxima")

[Out] -1/6*arctan(1/2*x) + 1/3*arctan(x)
```

Fricas [A] time = 0.262774, size = 15, normalized size = 0.88

$$-\frac{1}{6}\arctan\left(\frac{1}{2}x\right)+\frac{1}{3}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 4)*(x^2 + 1)),x, algorithm="fricas")

[Out] -1/6*arctan(1/2*x) + 1/3*arctan(x)
```

Sympy [A] time = 0.40096, size = 10, normalized size = 0.59

$$-\frac{\operatorname{atan}\left(\frac{x}{2}\right)}{6}+\frac{\operatorname{atan}\left(x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**2+1)/(x**2+4),x)

[Out] -atan(x/2)/6 + atan(x)/3
```

GIAC/XCAS [A] time = 0.262438, size = 15, normalized size = 0.88

$$-\frac{1}{6}\arctan\left(\frac{1}{2}x\right)+\frac{1}{3}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 4)*(x^2 + 1)),x, algorithm="giac")

[Out] -1/6*arctan(1/2*x) + 1/3*arctan(x)
```

3.279 $\int \frac{a+bx^3}{1+x^2} dx$

Optimal. Leaf size=24

$$a\tan^{-1}(x)+\frac{bx^2}{2}-\frac{1}{2}b\log\left(x^2+1\right)$$

```
[Out] (b*x^2)/2 + a*ArcTan[x] - (b*Log[1 + x^2])/2
```

Rubi [A] time = 0.0419264, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267

$$a\tan^{-1}(x)+\frac{bx^2}{2}-\frac{1}{2}b\log\left(x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)/(1 + x^2),x]
```

```
[Out] (b*x^2)/2 + a*ArcTan[x] - (b*Log[1 + x^2])/2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\operatorname{atan}(x)-\frac{b\log\left(x^2+1\right)}{2}+b\int x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((b*x**3+a)/(x**2+1),x)
```

```
[Out] a*atan(x) - b*log(x**2 + 1)/2 + b*Integral(x, x)
```

Mathematica [A] time = 0.0148715, size = 22, normalized size = 0.92

$$a\tan^{-1}(x)+\frac{1}{2}b\left(x^2-\log\left(x^2+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)/(1 + x^2),x]
```

```
[Out] a*ArcTan[x] + (b*(x^2 - Log[1 + x^2]))/2
```

Maple [A] time = 0.003, size = 21, normalized size = 0.9

$$\frac{bx^2}{2}+a\arctan(x)-\frac{b\ln\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)/(x^2+1),x)
```

```
[Out] 1/2*b*x^2+a*arctan(x)-1/2*b*ln(x^2+1)
```

Maxima [A] time = 0.87947, size = 27, normalized size = 1.12

$$\frac{1}{2}bx^2 + a\arctan(x) - \frac{1}{2}b\log(x^2+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)/(x^2 + 1),x, algorithm="maxima")
```

```
[Out] 1/2*b*x^2 + a*arctan(x) - 1/2*b*log(x^2 + 1)
```

Fricas [A] time = 0.257545, size = 27, normalized size = 1.12

$$\frac{1}{2}bx^2 + a\arctan(x) - \frac{1}{2}b\log(x^2+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)/(x^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/2*b*x^2 + a*arctan(x) - 1/2*b*log(x^2 + 1)
```

Sympy [A] time = 1.29482, size = 34, normalized size = 1.42

$$\frac{bx^2}{2} + \left(-\frac{ia}{2} - \frac{b}{2}\right)\log(x-i) + \left(\frac{ia}{2} - \frac{b}{2}\right)\log(x+i)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**3+a)/(x**2+1),x)
```

```
[Out] b*x**2/2 + (-I*a/2 - b/2)*log(x - I) + (I*a/2 - b/2)*log(x + I)
```

GIAC/XCAS [A] time = 0.259862, size = 27, normalized size = 1.12

$$\frac{1}{2}bx^2 + a\arctan(x) - \frac{1}{2}b\ln(x^2+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)/(x^2 + 1),x, algorithm="giac")
```

```
[Out] 1/2*b*x^2 + a*arctan(x) - 1/2*b*ln(x^2 + 1)
```

3.280 $\int \frac{x+x^2}{(4+x)(-4+x^2)} \, dx$

Optimal. Leaf size=15

$$\log(x+4)-\frac{1}{2}\tanh^{-1}\left(\frac{x}{2}\right)$$

```
[Out] -ArcTanh[x/2]/2 + Log[4 + x]
```

Rubi [A] time = 0.0849651, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\log(x+4)-\frac{1}{2}\tanh^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(x + x^2)/((4 + x)*(-4 + x^2)),x]

[Out] -ArcTanh[x/2]/2 + Log[4 + x]
```

Rubi in Sympy [A] time = 34.0988, size = 10, normalized size = 0.67

$$\log(x+4)-\frac{\operatorname{atanh}\left(\frac{x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+x)/(4+x)/(x**2-4),x)

[Out] log(x + 4) - atanh(x/2)/2
```

Mathematica [A] time = 0.00925871, size = 23, normalized size = 1.53

$$\frac{1}{4}\log(2-x)-\frac{1}{4}\log(x+2)+\log(x+4)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x + x^2)/((4 + x)*(-4 + x^2)),x]

[Out] Log[2 - x]/4 - Log[2 + x]/4 + Log[4 + x]
```

Maple [A] time = 0.012, size = 18, normalized size = 1.2

$$-\frac{\ln(2+x)}{4}+\ln(4+x)+\frac{\ln(x-2)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+x)/(4+x)/(x^2-4),x)
```

```
[Out] -1/4*ln(2+x)+ln(4+x)+1/4*ln(x-2)
```

Maxima [A] time = 0.879987, size = 23, normalized size = 1.53

$$\log(x+4) - \frac{1}{4}\log(x+2) + \frac{1}{4}\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + x)/((x^2 - 4)*(x + 4)),x, algorithm="maxima")

[Out] log(x + 4) - 1/4*log(x + 2) + 1/4*log(x - 2)
```

Fricas [A] time = 0.275746, size = 23, normalized size = 1.53

$$\log(x+4) - \frac{1}{4}\log(x+2) + \frac{1}{4}\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + x)/((x^2 - 4)*(x + 4)),x, algorithm="fricas")

[Out] log(x + 4) - 1/4*log(x + 2) + 1/4*log(x - 2)
```

Sympy [A] time = 0.291078, size = 17, normalized size = 1.13

$$\frac{\log(x-2)}{4} - \frac{\log(x+2)}{4} + \log(x+4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+x)/(4+x)/(x**2-4),x)

[Out] log(x - 2)/4 - log(x + 2)/4 + log(x + 4)
```

GIAC/XCAS [A] time = 0.261533, size = 27, normalized size = 1.8

$$\ln(|x+4|) - \frac{1}{4}\ln(|x+2|) + \frac{1}{4}\ln(|x-2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + x)/((x^2 - 4)*(x + 4)),x, algorithm="giac")

[Out] ln(abs(x + 4)) - 1/4*ln(abs(x + 2)) + 1/4*ln(abs(x - 2))
```

3.281 $\int \frac{4+x^2}{(1+x^2)(2+x^2)}\,dx$

Optimal. Leaf size=20

$$3\tan^{-1}(x)-\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

```
[Out] 3*ArcTan[x] - Sqrt[2]*ArcTan[x/Sqrt[2]]
```

Rubi [A] time = 0.0411773, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$3\tan^{-1}(x)-\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(4 + x^2)/((1 + x^2)*(2 + x^2)),x]

[Out] 3*ArcTan[x] - Sqrt[2]*ArcTan[x/Sqrt[2]]
```

Rubi in Sympy [A] time = 8.12611, size = 19, normalized size = 0.95

$$3\operatorname{atan}(x)-\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+4)/(x**2+1)/(x**2+2),x)

[Out] 3*atan(x) - sqrt(2)*atan(sqrt(2)*x/2)
```

Mathematica [A] time = 0.0152242, size = 20, normalized size = 1.

$$3\tan^{-1}(x)-\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(4 + x^2)/((1 + x^2)*(2 + x^2)),x]

[Out] 3*ArcTan[x] - Sqrt[2]*ArcTan[x/Sqrt[2]]
```

Maple [A] time = 0.009, size = 18, normalized size = 0.9

$$3\,\arctan(x)-\arctan\left(\frac{\sqrt{2}x}{2}\right)\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+4)/(x^2+1)/(x^2+2),x)

[Out] 3*arctan(x)-arctan(1/2*2^(1/2)*x)*2^(1/2)
```

Maxima [A] time = 0.889019, size = 23, normalized size = 1.15

$$-\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) + 3\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 4)/((x^2 + 2)*(x^2 + 1)),x, algorithm="maxima")

[Out] -sqrt(2)*arctan(1/2*sqrt(2)*x) + 3*arctan(x)
```

Fricas [A] time = 0.260523, size = 23, normalized size = 1.15

$$-\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) + 3\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 4)/((x^2 + 2)*(x^2 + 1)),x, algorithm="fricas")

[Out] -sqrt(2)*arctan(1/2*sqrt(2)*x) + 3*arctan(x)
```

Sympy [A] time = 0.404922, size = 19, normalized size = 0.95

$$3\operatorname{atan}{\left(x\right)} - \sqrt{2}\operatorname{atan}{\left(\frac{\sqrt{2}x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+4)/(x**2+1)/(x**2+2),x)

[Out] 3*atan(x) - sqrt(2)*atan(sqrt(2)*x/2)
```

GIAC/XCAS [A] time = 0.262265, size = 23, normalized size = 1.15

$$-\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) + 3\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 4)/((x^2 + 2)*(x^2 + 1)),x, algorithm="giac")

[Out] -sqrt(2)*arctan(1/2*sqrt(2)*x) + 3*arctan(x)
```

3.282 $\int \frac{5-4x+3x^2+x^4}{(-1+x)^2(1+x^2)}\,dx$

Optimal. Leaf size=37

$$\frac{3}{4}\log\left(x^2+1\right)+x+\frac{5}{2(1-x)}+\frac{1}{2}\log(1-x)+2\tan^{-1}(x)$$

```
[Out] 5/(2*(1 - x)) + x + 2*ArcTan[x] + Log[1 - x]/2 + (3*Log[1 + x^2])
/4
```

Rubi [A] time = 0.0781491, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$\frac{3}{4}\log\left(x^2+1\right)+x+\frac{5}{2(1-x)}+\frac{1}{2}\log(1-x)+2\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(5 - 4*x + 3*x^2 + x^4)/((-1 + x)^2*(1 + x^2)),x]

[Out] 5/(2*(1 - x)) + x + 2*ArcTan[x] + Log[1 - x]/2 + (3*Log[1 + x^2])
/4
```

Rubi in Sympy [A] time = 34.322, size = 29, normalized size = 0.78

$$x+\frac{\log(-x+1)}{2}+\frac{3\log\left(x^2+1\right)}{4}+2\,\text{atan}\,(x)+\frac{5}{2\,(-x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**4+3*x**2-4*x+5)/(-1+x)**2/(x**2+1),x)

[Out] x + log(-x + 1)/2 + 3*log(x**2 + 1)/4 + 2*atan(x) + 5/(2*(-x + 1)
)
```

Mathematica [A] time = 0.0385096, size = 33, normalized size = 0.89

$$\frac{3}{4}\log\left(x^2+1\right)+x+\frac{5}{2-2x}+\frac{1}{2}\log(x-1)+2\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(5 - 4*x + 3*x^2 + x^4)/((-1 + x)^2*(1 + x^2)),x]

[Out] 5/(2 - 2*x) + x + 2*ArcTan[x] + Log[-1 + x]/2 + (3*Log[1 + x^2])/
4
```

Maple [A] time = 0.011, size = 28, normalized size = 0.8

$$x-\frac{5}{2\,x-2}+\frac{\ln(-1+x)}{2}+\frac{3\,\ln\left(x^2+1\right)}{4}+2\,\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^4+3*x^2-4*x+5)/(-1+x)^2/(x^2+1),x)
```

```
[Out] x-5/2/(-1+x)+1/2*ln(-1+x)+3/4*ln(x^2+1)+2*arctan(x)
```

Maxima [A] time = 0.888388, size = 36, normalized size = 0.97

$$x - \frac{5}{2(x-1)} + 2\arctan(x) + \frac{3}{4}\log\left(x^2+1\right) + \frac{1}{2}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 + 3*x^2 - 4*x + 5)/((x^2 + 1)*(x - 1)^2),x, algorithm="maxima")
```

```
[Out] x - 5/2/(x - 1) + 2*arctan(x) + 3/4*log(x^2 + 1) + 1/2*log(x - 1)
```

Fricas [A] time = 0.256245, size = 59, normalized size = 1.59

$$\frac{4x^2 + 8(x-1)\arctan(x) + 3(x-1)\log\left(x^2+1\right) + 2(x-1)\log(x-1) - 4x - 10}{4(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 + 3*x^2 - 4*x + 5)/((x^2 + 1)*(x - 1)^2),x, algorithm="fricas")
```

```
[Out] 1/4*(4*x^2 + 8*(x - 1)*arctan(x) + 3*(x - 1)*log(x^2 + 1) + 2*(x
- 1)*log(x - 1) - 4*x - 10)/(x - 1)
```

Sympy [A] time = 0.382463, size = 29, normalized size = 0.78

$$x + \frac{\log(x-1)}{2} + \frac{3\log\left(x^2+1\right)}{4} + 2\operatorname{atan}(x) - \frac{5}{2x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**4+3*x**2-4*x+5)/(-1+x)**2/(x**2+1),x)
```

```
[Out] x + log(x - 1)/2 + 3*log(x**2 + 1)/4 + 2*atan(x) - 5/(2*x - 2)
```

GIAC/XCAS [A] time = 0.26266, size = 81, normalized size = 2.19

$$\frac{1}{2}\pi - 2\pi\left\lfloor\frac{\pi + 4\arctan(x)}{4\pi} + \frac{1}{2}\right\rfloor + x - \frac{5}{2(x-1)} + 2\arctan(x) + \frac{3}{4}\ln\left(\frac{2}{x-1} + \frac{2}{(x-1)^2} + 1\right) + 2\ln(|x-1|) - 1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 + 3*x^2 - 4*x + 5)/((x^2 + 1)*(x - 1)^2),x, algorithm="giac")
```

```
[Out] 1/2*pi - 2*pi*floor(1/4*(pi + 4*arctan(x))/pi + 1/2) + x - 5/2/(x
 - 1) + 2*arctan(x) + 3/4*ln(2/(x - 1) + 2/(x - 1)^2 + 1) + 2*ln(
abs(x - 1)) - 1
```

3.283 $\int \frac{1+x^4}{2+x^2} \, dx$

Optimal. Leaf size=26

$$\frac{x^3}{3} - 2x + \frac{5\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] -2*x + x^3/3 + (5*ArcTan[x/Sqrt[2]])/Sqrt[2]
```

Rubi [A] time = 0.0312438, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$\frac{x^3}{3} - 2x + \frac{5\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^4)/(2 + x^2),x]

[Out] -2*x + x^3/3 + (5*ArcTan[x/Sqrt[2]])/Sqrt[2]
```

Rubi in Sympy [A] time = 7.05666, size = 26, normalized size = 1.

$$\frac{x^3}{3} - 2x + \frac{5\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**4+1)/(x**2+2),x)

[Out] x**3/3 - 2*x + 5*sqrt(2)*atan(sqrt(2)*x/2)/2
```

Mathematica [A] time = 0.0146098, size = 26, normalized size = 1.

$$\frac{x^3}{3} - 2x + \frac{5\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^4)/(2 + x^2),x]

[Out] -2*x + x^3/3 + (5*ArcTan[x/Sqrt[2]])/Sqrt[2]
```

Maple [A] time = 0.003, size = 22, normalized size = 0.9

$$-2\,x + \frac{x^3}{3} + \frac{5\,\sqrt{2}}{2}\arctan\left(\frac{\sqrt{2}x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^4+1)/(x^2+2),x)

[Out] -2*x+1/3*x^3+5/2*arctan(1/2*2^(1/2)*x)*2^(1/2)
```

Maxima [A] time = 0.891567, size = 28, normalized size = 1.08

$$\frac{1}{3}x^3 + \frac{5}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 1)/(x^2 + 2),x, algorithm="maxima")

[Out] 1/3*x^3 + 5/2*sqrt(2)*arctan(1/2*sqrt(2)*x) - 2*x
```

Fricas [A] time = 0.26264, size = 35, normalized size = 1.35

$$\frac{1}{6}\sqrt{2}\left(\sqrt{2}\left(x^3 - 6x\right) + 15\arctan\left(\frac{1}{2}\sqrt{2}x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 1)/(x^2 + 2),x, algorithm="fricas")

[Out] 1/6*sqrt(2)*(sqrt(2)*(x^3 - 6*x) + 15*arctan(1/2*sqrt(2)*x))
```

Sympy [A] time = 0.171308, size = 26, normalized size = 1.

$$\frac{x^3}{3} - 2x + \frac{5\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**4+1)/(x**2+2),x)

[Out] x**3/3 - 2*x + 5*sqrt(2)*atan(sqrt(2)*x/2)/2
```

GIAC/XCAS [A] time = 0.259129, size = 28, normalized size = 1.08

$$\frac{1}{3}x^3 + \frac{5}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 1)/(x^2 + 2),x, algorithm="giac")

[Out] 1/3*x^3 + 5/2*sqrt(2)*arctan(1/2*sqrt(2)*x) - 2*x
```

3.284 $\int \frac{2+2x+x^4}{x^4+x^5} \, dx$

Optimal. Leaf size=12

$$\log(x+1) - \frac{2}{3x^3}$$

```
[Out] -2/(3*x^3) + Log[1 + x]
```

Rubi [A] time = 0.035673, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$\log(x+1) - \frac{2}{3x^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + 2*x + x^4)/(x^4 + x^5),x]

[Out] -2/(3*x^3) + Log[1 + x]
```

Rubi in Sympy [A] time = 8.43478, size = 10, normalized size = 0.83

$$\log(x+1) - \frac{2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**4+2*x+2)/(x**5+x**4),x)

[Out] log(x + 1) - 2/(3*x**3)
```

Mathematica [A] time = 0.00655485, size = 12, normalized size = 1.

$$\log(x+1) - \frac{2}{3x^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + 2*x + x^4)/(x^4 + x^5),x]

[Out] -2/(3*x^3) + Log[1 + x]
```

Maple [A] time = 0.007, size = 11, normalized size = 0.9

$$-\frac{2}{3\,x^3} + \ln(1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^4+2*x+2)/(x^5+x^4),x)
```

```
[Out] -2/3/x^3+ln(1+x)
```

Maxima [A] time = 0.786608, size = 14, normalized size = 1.17

$$-\frac{2}{3\,x^3}+\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 2*x + 2)/(x^5 + x^4),x, algorithm="maxima")
```

```
[Out] -2/3/x^3 + log(x + 1)
```

Fricas [A] time = 0.245954, size = 22, normalized size = 1.83

$$\frac{3\,x^3\log\left(x+1\right)-2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 2*x + 2)/(x^5 + x^4),x, algorithm="fricas")
```

```
[Out] 1/3*(3*x^3*log(x + 1) - 2)/x^3
```

Sympy [A] time = 0.190422, size = 10, normalized size = 0.83

$$\log\left(x+1\right)-\frac{2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**4+2*x+2)/(x**5+x**4),x)
```

```
[Out] log(x + 1) - 2/(3*x**3)
```

GIAC/XCAS [A] time = 0.262198, size = 15, normalized size = 1.25

$$-\frac{2}{3\,x^3}+\ln\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 2*x + 2)/(x^5 + x^4),x, algorithm="giac")
```

```
[Out] -2/3/x^3 + ln(abs(x + 1))
```

3.285 $\int \frac{-1-5x+2x^2}{2-x-2x^2+x^3} \, dx$

Optimal. Leaf size=21

$$2\log(1-x)-\log(2-x)+\log(x+1)$$

```
[Out] 2*Log[1 - x] - Log[2 - x] + Log[1 + x]
```

Rubi [A] time = 0.0551408, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$

$$2\log(1-x)-\log(2-x)+\log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 - 5*x + 2*x^2)/(2 - x - 2*x^2 + x^3),x]

[Out] 2*Log[1 - x] - Log[2 - x] + Log[1 + x]
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*x**2-5*x-1)/(x**3-2*x**2-x+2),x)

[Out] Timed out
```

Mathematica [A] time = 0.0111232, size = 21, normalized size = 1.

$$2\log(1-x)-\log(2-x)+\log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 - 5*x + 2*x^2)/(2 - x - 2*x^2 + x^3),x]

[Out] 2*Log[1 - x] - Log[2 - x] + Log[1 + x]
```

Maple [A] time = 0.011, size = 18, normalized size = 0.9

$$2\ln(-1+x)+\ln(1+x)-\ln(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*x^2-5*x-1)/(x^3-2*x^2-x+2),x)

[Out] 2*ln(-1+x)+ln(1+x)-ln(x-2)
```

Maxima [A] time = 0.796338, size = 23, normalized size = 1.1

$$\log(x+1) + 2\log(x-1) - \log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2 - 5*x - 1)/(x^3 - 2*x^2 - x + 2),x, algorithm="maxima")
```

```
[Out] log(x + 1) + 2*log(x - 1) - log(x - 2)
```

Fricas [A] time = 0.249862, size = 23, normalized size = 1.1

$$\log(x+1) + 2\log(x-1) - \log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2 - 5*x - 1)/(x^3 - 2*x^2 - x + 2),x, algorithm="fricas")
```

```
[Out] log(x + 1) + 2*log(x - 1) - log(x - 2)
```

Sympy [A] time = 0.284621, size = 15, normalized size = 0.71

$$-\log(x-2) + 2\log(x-1) + \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x**2-5*x-1)/(x**3-2*x**2-x+2),x)
```

```
[Out] -log(x - 2) + 2*log(x - 1) + log(x + 1)
```

GIAC/XCAS [A] time = 0.261692, size = 27, normalized size = 1.29

$$\ln(|x+1|) + 2\ln(|x-1|) - \ln(|x-2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2 - 5*x - 1)/(x^3 - 2*x^2 - x + 2),x, algorithm="giac")
```

```
[Out] ln(abs(x + 1)) + 2*ln(abs(x - 1)) - ln(abs(x - 2))
```

3.286 $\int \frac{2+x+x^3}{1+2x^2+x^4} \, dx$

Optimal. Leaf size=22

$$\frac{x}{x^2+1}+\frac{1}{2}\log\left(x^2+1\right)+\tan^{-1}(x)$$

```
[Out] x/(1 + x^2) + ArcTan[x] + Log[1 + x^2]/2
```

Rubi [A] time = 0.0329247, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263

$$\frac{x}{x^2+1}+\frac{1}{2}\log\left(x^2+1\right)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + x + x^3)/(1 + 2*x^2 + x^4),x]
```

```
[Out] x/(1 + x^2) + ArcTan[x] + Log[1 + x^2]/2
```

Rubi in Sympy [A] time = 9.50715, size = 17, normalized size = 0.77

$$\frac{x}{x^2+1}+\frac{\log\left(x^2+1\right)}{2}+\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**3+x+2)/(x**4+2*x**2+1),x)
```

```
[Out] x/(x**2 + 1) + log(x**2 + 1)/2 + atan(x)
```

Mathematica [A] time = 0.0142639, size = 22, normalized size = 1.

$$\frac{x}{x^2+1}+\frac{1}{2}\log\left(x^2+1\right)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + x + x^3)/(1 + 2*x^2 + x^4),x]
```

```
[Out] x/(1 + x^2) + ArcTan[x] + Log[1 + x^2]/2
```

Maple [A] time = 0.007, size = 21, normalized size = 1.

$$\frac{x}{x^2+1}+\arctan(x)+\frac{\ln\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^3+x+2)/(x^4+2*x^2+1),x)
```

```
[Out] x/(x^2+1)+arctan(x)+1/2*ln(x^2+1)
```

Maxima [A] time = 0.873432, size = 27, normalized size = 1.23

$$\frac{x}{x^2+1} + \arctan(x) + \frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + x + 2)/(x^4 + 2*x^2 + 1),x, algorithm="maxima")
```

```
[Out] x/(x^2 + 1) + arctan(x) + 1/2*log(x^2 + 1)
```

Fricas [A] time = 0.249065, size = 46, normalized size = 2.09

$$\frac{2\left(x^2+1\right)\arctan(x) + \left(x^2+1\right)\log\left(x^2+1\right) + 2x}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + x + 2)/(x^4 + 2*x^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(x^2 + 1)*arctan(x) + (x^2 + 1)*log(x^2 + 1) + 2*x)/(x^2 +
 1)
```

Sympy [A] time = 0.235448, size = 17, normalized size = 0.77

$$\frac{x}{x^2+1} + \frac{\log\left(x^2+1\right)}{2} + \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3+x+2)/(x**4+2*x**2+1),x)
```

```
[Out] x/(x**2 + 1) + log(x**2 + 1)/2 + atan(x)
```

GIAC/XCAS [A] time = 0.260583, size = 27, normalized size = 1.23

$$\frac{x}{x^2+1} + \arctan(x) + \frac{1}{2}\ln\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + x + 2)/(x^4 + 2*x^2 + 1),x, algorithm="giac")
```

```
[Out] x/(x^2 + 1) + arctan(x) + 1/2*ln(x^2 + 1)
```

3.287 $\int \frac{1+2x+x^2+x^3}{1+2x^2+x^4} \, dx$

Optimal. Leaf size=24

$$-\frac{1}{2(x^2+1)} + \frac{1}{2}\log\left(x^2+1\right) + \tan^{-1}(x)$$

```
[Out] -1/(2*(1 + x^2)) + ArcTan[x] + Log[1 + x^2]/2
```

Rubi [A] time = 0.0354224, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$

$$-\frac{1}{2(x^2+1)} + \frac{1}{2}\log\left(x^2+1\right) + \tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x + x^2 + x^3)/(1 + 2*x^2 + x^4),x]

[Out] -1/(2*(1 + x^2)) + ArcTan[x] + Log[1 + x^2]/2
```

Rubi in Sympy [A] time = 12.894, size = 20, normalized size = 0.83

$$\frac{x^2}{2(x^2+1)} + \frac{\log\left(x^2+1\right)}{2} + \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((x**3+x**2+2*x+1)/(x**4+2*x**2+1),x)

[Out] x**2/(2*(x**2 + 1)) + log(x**2 + 1)/2 + atan(x)
```

Mathematica [A] time = 0.0172592, size = 24, normalized size = 1.

$$-\frac{1}{2(x^2+1)} + \frac{1}{2}\log\left(x^2+1\right) + \tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x + x^2 + x^3)/(1 + 2*x^2 + x^4),x]

[Out] -1/(2*(1 + x^2)) + ArcTan[x] + Log[1 + x^2]/2
```

Maple [A] time = 0.007, size = 21, normalized size = 0.9

$$-\frac{1}{2x^2+2} + \arctan(x) + \frac{\ln\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2+2*x+1)/(x^4+2*x^2+1),x)
```

```
[Out] -1/2/(x^2+1)+arctan(x)+1/2*ln(x^2+1)
```

Maxima [A] time = 0.88082, size = 27, normalized size = 1.12

$$-\frac{1}{2\left(x^{2}+1\right)}+\arctan\left(x\right)+\frac{1}{2}\log\left(x^{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3 + x^2 + 2*x + 1)/(x^4 + 2*x^2 + 1),x, algorithm="maxima")
```

```
[Out] -1/2/(x^2 + 1) + arctan(x) + 1/2*log(x^2 + 1)
```

Fricas [A] time = 0.246784, size = 43, normalized size = 1.79

$$\frac{2\left(x^{2}+1\right)\arctan\left(x\right)+\left(x^{2}+1\right)\log\left(x^{2}+1\right)-1}{2\left(x^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3 + x^2 + 2*x + 1)/(x^4 + 2*x^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(x^2 + 1)*arctan(x) + (x^2 + 1)*log(x^2 + 1) - 1)/(x^2 + 1
)
```

Sympy [A] time = 0.241453, size = 19, normalized size = 0.79

$$\frac{\log\left(x^{2}+1\right)}{2}+\operatorname{atan}\left(x\right)-\frac{1}{2x^{2}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3+x**2+2*x+1)/(x**4+2*x**2+1),x)
```

```
[Out] log(x**2 + 1)/2 + atan(x) - 1/(2*x**2 + 2)
```

GIAC/XCAS [A] time = 0.261496, size = 27, normalized size = 1.12

$$-\frac{1}{2\left(x^{2}+1\right)}+\arctan\left(x\right)+\frac{1}{2}\ln\left(x^{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3 + x^2 + 2*x + 1)/(x^4 + 2*x^2 + 1),x, algorithm="giac")
```

```
[Out] -1/2/(x^2 + 1) + arctan(x) + 1/2*ln(x^2 + 1)
```

3.288 $\int \frac{3+4x}{(1+x^2)(2+x^2)}\,dx$

Optimal. Leaf size=36

$$2\log\left(x^2+1\right)-2\log\left(x^2+2\right)+3\tan^{-1}(x)-\frac{3\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] 3*ArcTan[x] - (3*ArcTan[x/Sqrt[2]])/Sqrt[2] + 2*Log[1 + x^2] - 2*
Log[2 + x^2]
```

Rubi [A] time = 0.085023, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$2\log\left(x^2+1\right)-2\log\left(x^2+2\right)+3\tan^{-1}(x)-\frac{3\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(3 + 4*x)/((1 + x^2)*(2 + x^2)),x]
```

```
[Out] 3*ArcTan[x] - (3*ArcTan[x/Sqrt[2]])/Sqrt[2] + 2*Log[1 + x^2] - 2*
Log[2 + x^2]
```

Rubi in Sympy [A] time = 17.3543, size = 39, normalized size = 1.08

$$2\log\left(x^2+1\right)-2\log\left(x^2+2\right)+3\,\mathrm{atan}\,(x)-\frac{3\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3+4*x)/(x**2+1)/(x**2+2),x)
```

```
[Out] 2*log(x**2 + 1) - 2*log(x**2 + 2) + 3*atan(x) - 3*sqrt(2)*atan(sq
rt(2)*x/2)/2
```

Mathematica [A] time = 0.0253715, size = 36, normalized size = 1.

$$2\log\left(x^2+1\right)-2\log\left(x^2+2\right)+3\tan^{-1}(x)-\frac{3\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3 + 4*x)/((1 + x^2)*(2 + x^2)),x]
```

```
[Out] 3*ArcTan[x] - (3*ArcTan[x/Sqrt[2]])/Sqrt[2] + 2*Log[1 + x^2] - 2*
Log[2 + x^2]
```

Maple [A] time = 0.005, size = 34, normalized size = 0.9

$$3\arctan(x)+2\ln\left(x^2+1\right)-2\ln\left(x^2+2\right)-\frac{3\sqrt{2}}{2}\arctan\left(\frac{\sqrt{2}x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3+4*x)/(x^2+1)/(x^2+2),x)
```

```
[Out] 3*arctan(x)+2*ln(x^2+1)-2*ln(x^2+2)-3/2*arctan(1/2*2^(1/2)*x)*2^(
1/2)
```

Maxima [A] time = 0.867264, size = 45, normalized size = 1.25

$$-\frac{3}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + 3\arctan(x) - 2\log\left(x^2+2\right) + 2\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x + 3)/((x^2 + 2)*(x^2 + 1)),x, algorithm="maxima")
```

```
[Out] -3/2*sqrt(2)*arctan(1/2*sqrt(2)*x) + 3*arctan(x) - 2*log(x^2 + 2)
 + 2*log(x^2 + 1)
```

Fricas [A] time = 0.252258, size = 59, normalized size = 1.64

$$\frac{1}{2}\sqrt{2}\left(3\sqrt{2}\arctan(x) - 2\sqrt{2}\log\left(x^2+2\right) + 2\sqrt{2}\log\left(x^2+1\right) - 3\arctan\left(\frac{1}{2}\sqrt{2}x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x + 3)/((x^2 + 2)*(x^2 + 1)),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*(3*sqrt(2)*arctan(x) - 2*sqrt(2)*log(x^2 + 2) + 2*sqr
t(2)*log(x^2 + 1) - 3*arctan(1/2*sqrt(2)*x))
```

Sympy [A] time = 0.514732, size = 39, normalized size = 1.08

$$2\log\left(x^2+1\right) - 2\log\left(x^2+2\right) + 3\operatorname{atan}(x) - \frac{3\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3+4*x)/(x**2+1)/(x**2+2),x)
```

```
[Out] 2*log(x**2 + 1) - 2*log(x**2 + 2) + 3*atan(x) - 3*sqrt(2)*atan(sq
rt(2)*x/2)/2
```

GIAC/XCAS [A] time = 0.260829, size = 45, normalized size = 1.25

$$-\frac{3}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + 3\arctan(x) - 2\ln\left(x^2+2\right) + 2\ln\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x + 3)/((x^2 + 2)*(x^2 + 1)),x, algorithm="giac")
```

```
[Out] -3/2*sqrt(2)*arctan(1/2*sqrt(2)*x) + 3*arctan(x) - 2*ln(x^2 + 2)
+ 2*ln(x^2 + 1)
```

3.289 $\int \frac{2+x}{(1+x^2)(4+x^2)}\,dx$

Optimal. Leaf size=37

$$\frac{1}{6}\log\left(x^2+1\right)-\frac{1}{6}\log\left(x^2+4\right)-\frac{1}{3}\tan^{-1}\left(\frac{x}{2}\right)+\frac{2}{3}\tan^{-1}(x)$$

```
[Out] -ArcTan[x/2]/3 + (2*ArcTan[x])/3 + Log[1 + x^2]/6 - Log[4 + x^2]/
6
```

Rubi [A] time = 0.0796073, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{1}{6}\log\left(x^2+1\right)-\frac{1}{6}\log\left(x^2+4\right)-\frac{1}{3}\tan^{-1}\left(\frac{x}{2}\right)+\frac{2}{3}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x)/((1 + x^2)*(4 + x^2)),x]
```

```
[Out] -ArcTan[x/2]/3 + (2*ArcTan[x])/3 + Log[1 + x^2]/6 - Log[4 + x^2]/
6
```

Rubi in Sympy [A] time = 17.1073, size = 29, normalized size = 0.78

$$\frac{\log\left(x^2+1\right)}{6}-\frac{\log\left(x^2+4\right)}{6}-\frac{\operatorname{atan}\left(\frac{x}{2}\right)}{3}+\frac{2\operatorname{atan}(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((2+x)/(x**2+1)/(x**2+4),x)
```

```
[Out] log(x**2 + 1)/6 - log(x**2 + 4)/6 - atan(x/2)/3 + 2*atan(x)/3
```

Mathematica [A] time = 0.0124736, size = 37, normalized size = 1.

$$\frac{1}{6}\log\left(x^2+1\right)-\frac{1}{6}\log\left(x^2+4\right)-\frac{1}{3}\tan^{-1}\left(\frac{x}{2}\right)+\frac{2}{3}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x)/((1 + x^2)*(4 + x^2)),x]
```

```
[Out] -ArcTan[x/2]/3 + (2*ArcTan[x])/3 + Log[1 + x^2]/6 - Log[4 + x^2]/
6
```

Maple [A] time = 0.005, size = 28, normalized size = 0.8

$$-\frac{1}{3}\arctan\left(\frac{x}{2}\right)+\frac{2\arctan(x)}{3}+\frac{\ln\left(x^2+1\right)}{6}-\frac{\ln\left(x^2+4\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2+x)/(x^2+1)/(x^2+4),x)
```

```
[Out] -1/3*arctan(1/2*x)+2/3*arctan(x)+1/6*ln(x^2+1)-1/6*ln(x^2+4)
```

Maxima [A]   time = 0.894388, size = 36, normalized size = 0.97

$$-\frac{1}{3}\arctan\left(\frac{1}{2}x\right)+\frac{2}{3}\arctan\left(x\right)-\frac{1}{6}\log\left(x^2+4\right)+\frac{1}{6}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 2)/((x^2 + 4)*(x^2 + 1)),x, algorithm="maxima")
```

```
[Out] -1/3*arctan(1/2*x) + 2/3*arctan(x) - 1/6*log(x^2 + 4) + 1/6*log(x
^2 + 1)
```

Fricas [A]   time = 0.251885, size = 36, normalized size = 0.97

$$-\frac{1}{3}\arctan\left(\frac{1}{2}x\right)+\frac{2}{3}\arctan\left(x\right)-\frac{1}{6}\log\left(x^2+4\right)+\frac{1}{6}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 2)/((x^2 + 4)*(x^2 + 1)),x, algorithm="fricas")
```

```
[Out] -1/3*arctan(1/2*x) + 2/3*arctan(x) - 1/6*log(x^2 + 4) + 1/6*log(x
^2 + 1)
```

Sympy [A]   time = 0.480808, size = 29, normalized size = 0.78

$$\frac{\log\left(x^2+1\right)}{6}-\frac{\log\left(x^2+4\right)}{6}-\frac{\operatorname{atan}\left(\frac{x}{2}\right)}{3}+\frac{2\operatorname{atan}\left(x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2+x)/(x**2+1)/(x**2+4),x)
```

```
[Out] log(x**2 + 1)/6 - log(x**2 + 4)/6 - atan(x/2)/3 + 2*atan(x)/3
```

GIAC/XCAS [A]   time = 0.262499, size = 36, normalized size = 0.97

$$-\frac{1}{3}\arctan\left(\frac{1}{2}x\right)+\frac{2}{3}\arctan\left(x\right)-\frac{1}{6}\ln\left(x^2+4\right)+\frac{1}{6}\ln\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 2)/((x^2 + 4)*(x^2 + 1)),x, algorithm="giac")
```

```
[Out] -1/3*arctan(1/2*x) + 2/3*arctan(x) - 1/6*ln(x^2 + 4) + 1/6*ln(x^2
 + 1)
```

3.290 $\int \frac{2-x+x^3}{-7-6x+x^2} \, dx$

Optimal. Leaf size=29

$$\frac{x^2}{2}+6x+\frac{169}{4}\log(7-x)-\frac{1}{4}\log(x+1)$$

```
[Out] 6*x + x^2/2 + (169*Log[7 - x])/4 - Log[1 + x]/4
```

Rubi [A] time = 0.0392552, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$\frac{x^2}{2}+6x+\frac{169}{4}\log(7-x)-\frac{1}{4}\log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[(2 - x + x^3)/(-7 - 6*x + x^2),x]

[Out] 6*x + x^2/2 + (169*Log[7 - x])/4 - Log[1 + x]/4
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$6x+\frac{169\log(-x+7)}{4}-\frac{\log(x+1)}{4}+\int x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**3-x+2)/(x**2-6*x-7),x)

[Out] 6*x + 169*log(-x + 7)/4 - log(x + 1)/4 + Integral(x, x)
```

Mathematica [A] time = 0.00925839, size = 29, normalized size = 1.

$$\frac{x^2}{2}+6x+\frac{169}{4}\log(7-x)-\frac{1}{4}\log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 - x + x^3)/(-7 - 6*x + x^2),x]

[Out] 6*x + x^2/2 + (169*Log[7 - x])/4 - Log[1 + x]/4
```

Maple [A] time = 0.009, size = 22, normalized size = 0.8

$$\frac{x^2}{2}+6\,x-\frac{\ln(1+x)}{4}+\frac{169\,\ln(x-7)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^3-x+2)/(x^2-6*x-7),x)
```

```
[Out] 1/2*x^2+6*x-1/4*ln(1+x)+169/4*ln(x-7)
```

Maxima [A] time = 0.802924, size = 28, normalized size = 0.97

$$\frac{1}{2}x^2 + 6x - \frac{1}{4}\log(x+1) + \frac{169}{4}\log(x-7)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - x + 2)/(x^2 - 6*x - 7),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + 6*x - 1/4*log(x + 1) + 169/4*log(x - 7)
```

Fricas [A] time = 0.249926, size = 28, normalized size = 0.97

$$\frac{1}{2}x^2 + 6x - \frac{1}{4}\log(x+1) + \frac{169}{4}\log(x-7)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - x + 2)/(x^2 - 6*x - 7),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 + 6*x - 1/4*log(x + 1) + 169/4*log(x - 7)
```

Sympy [A] time = 0.213654, size = 22, normalized size = 0.76

$$\frac{x^2}{2} + 6x + \frac{169\log(x-7)}{4} - \frac{\log(x+1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3-x+2)/(x**2-6*x-7),x)
```

```
[Out] x**2/2 + 6*x + 169*log(x - 7)/4 - log(x + 1)/4
```

GIAC/XCAS [A] time = 0.264865, size = 31, normalized size = 1.07

$$\frac{1}{2}x^2 + 6x - \frac{1}{4}\ln(|x+1|) + \frac{169}{4}\ln(|x-7|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - x + 2)/(x^2 - 6*x - 7),x, algorithm="giac")
```

```
[Out] 1/2*x^2 + 6*x - 1/4*ln(abs(x + 1)) + 169/4*ln(abs(x - 7))
```

3.291 $\int \frac{-1+x^5}{-1+x^2}\,dx$

Optimal. Leaf size=19

$$\frac{x^4}{4}+\frac{x^2}{2}+\log(x+1)$$

```
[Out] x^2/2 + x^4/4 + Log[1 + x]
```

Rubi [A] time = 0.0357949, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$\frac{x^4}{4}+\frac{x^2}{2}+\log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + x^5)/(-1 + x^2),x]

[Out] x^2/2 + x^4/4 + Log[1 + x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{x^4}{4}+\log(x+1)+\int x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**5-1)/(x**2-1),x)

[Out] x**4/4 + log(x + 1) + Integral(x, x)
```

Mathematica [A] time = 0.00634878, size = 19, normalized size = 1.

$$\frac{x^4}{4}+\frac{x^2}{2}+\log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + x^5)/(-1 + x^2),x]

[Out] x^2/2 + x^4/4 + Log[1 + x]
```

Maple [A] time = 0.003, size = 16, normalized size = 0.8

$$\frac{x^2}{2}+\frac{x^4}{4}+\ln(1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^5-1)/(x^2-1),x)
```

```
[Out] 1/2*x^2+1/4*x^4+ln(1+x)
```

Maxima [A] time = 0.80068, size = 20, normalized size = 1.05

$$\frac{1}{4}x^4 + \frac{1}{2}x^2 + \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^5 - 1)/(x^2 - 1),x, algorithm="maxima")
```

```
[Out] 1/4*x^4 + 1/2*x^2 + log(x + 1)
```

Fricas [A] time = 0.244984, size = 20, normalized size = 1.05

$$\frac{1}{4}x^4 + \frac{1}{2}x^2 + \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^5 - 1)/(x^2 - 1),x, algorithm="fricas")
```

```
[Out] 1/4*x^4 + 1/2*x^2 + log(x + 1)
```

Sympy [A] time = 0.129648, size = 14, normalized size = 0.74

$$\frac{x^4}{4} + \frac{x^2}{2} + \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**5-1)/(x**2-1),x)
```

```
[Out] x**4/4 + x**2/2 + log(x + 1)
```

GIAC/XCAS [A] time = 0.260975, size = 22, normalized size = 1.16

$$\frac{1}{4}x^4 + \frac{1}{2}x^2 + \ln(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^5 - 1)/(x^2 - 1),x, algorithm="giac")
```

```
[Out] 1/4*x^4 + 1/2*x^2 + ln(abs(x + 1))
```

3.292 $\int \frac{5+2x-x^2+x^3}{1+x+x^2} \, dx$

Optimal. Leaf size=41

$$\frac{x^2}{2}+\frac{3}{2}\log\left(x^2+x+1\right)-2x+\frac{11\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -2*x + x^2/2 + (11*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] + (3*Log[1
+ x + x^2])/2
```

Rubi [A] time = 0.0629781, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227

$$\frac{x^2}{2}+\frac{3}{2}\log\left(x^2+x+1\right)-2x+\frac{11\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(5 + 2*x - x^2 + x^3)/(1 + x + x^2),x]
```

```
[Out] -2*x + x^2/2 + (11*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] + (3*Log[1
+ x + x^2])/2
```

Rubi in Sympy [A] time = 22.9296, size = 42, normalized size = 1.02

$$\frac{x^2}{2}-2x+\frac{3\log\left(x^2+x+1\right)}{2}+\frac{11\sqrt{3}\,\operatorname{atan}\left(\sqrt{3}\left(\frac{2x}{3}+\frac{1}{3}\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**3-x**2+2*x+5)/(x**2+x+1),x)
```

```
[Out] x**2/2 - 2*x + 3*log(x**2 + x + 1)/2 + 11*sqrt(3)*atan(sqrt(3)*(2
*x/3 + 1/3))/3
```

Mathematica [A] time = 0.0247558, size = 41, normalized size = 1.

$$\frac{x^2}{2}+\frac{3}{2}\log\left(x^2+x+1\right)-2x+\frac{11\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(5 + 2*x - x^2 + x^3)/(1 + x + x^2),x]
```

```
[Out] -2*x + x^2/2 + (11*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] + (3*Log[1
+ x + x^2])/2
```

Maple [A] time = 0.004, size = 35, normalized size = 0.9

$$-2\,x+\frac{x^2}{2}+\frac{3\,\ln\left(x^2+x+1\right)}{2}+\frac{11\,\sqrt{3}}{3}\arctan\left(\frac{(1+2\,x)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^3-x^2+2*x+5)/(x^2+x+1),x)
```

```
[Out] -2*x+1/2*x^2+3/2*ln(x^2+x+1)+11/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(
1/2)
```

Maxima [A] time = 0.881089, size = 46, normalized size = 1.12

$$\frac{1}{2}x^2 + \frac{11}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - 2x + \frac{3}{2}\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - x^2 + 2*x + 5)/(x^2 + x + 1),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + 11/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*x + 3/2*
log(x^2 + x + 1)
```

Fricas [A] time = 0.249227, size = 57, normalized size = 1.39

$$\frac{1}{6}\sqrt{3}\left(\sqrt{3}\left(x^2-4x\right) + 3\sqrt{3}\log\left(x^2+x+1\right) + 22\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - x^2 + 2*x + 5)/(x^2 + x + 1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*(sqrt(3)*(x^2 - 4*x) + 3*sqrt(3)*log(x^2 + x + 1) + 2
2*arctan(1/3*sqrt(3)*(2*x + 1)))
```

Sympy [A] time = 0.230536, size = 46, normalized size = 1.12

$$\frac{x^2}{2} - 2x + \frac{3\log\left(x^2+x+1\right)}{2} + \frac{11\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3-x**2+2*x+5)/(x**2+x+1),x)
```

```
[Out] x**2/2 - 2*x + 3*log(x**2 + x + 1)/2 + 11*sqrt(3)*atan(2*sqrt(3)*
x/3 + sqrt(3)/3)/3
```

GIAC/XCAS [A] time = 0.262447, size = 46, normalized size = 1.12

$$\frac{1}{2}x^2 + \frac{11}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - 2x + \frac{3}{2}\ln\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - x^2 + 2*x + 5)/(x^2 + x + 1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 + 11/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*x + 3/2*
ln(x^2 + x + 1)
```

3.293 $\int \frac{-3+x-2x^3+x^4}{10-8x+2x^2}\,dx$

Optimal. Leaf size=41

$$\frac{x^3}{6}+\frac{x^2}{2}+\frac{3}{4}\log\left(x^2-4x+5\right)+\frac{3x}{2}+6\tan^{-1}(2-x)$$

```
[Out] (3*x)/2 + x^2/2 + x^3/6 + 6*ArcTan[2 - x] + (3*Log[5 - 4*x + x^2]
)/4
```

Rubi [A] time = 0.0580116, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$\frac{x^3}{6}+\frac{x^2}{2}+\frac{3}{4}\log\left(x^2-4x+5\right)+\frac{3x}{2}+6\tan^{-1}(2-x)$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + x - 2*x^3 + x^4)/(10 - 8*x + 2*x^2),x]
```

```
[Out] (3*x)/2 + x^2/2 + x^3/6 + 6*ArcTan[2 - x] + (3*Log[5 - 4*x + x^2]
)/4
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{x^3}{6}+\frac{x^2}{2}+\frac{3\log\left(2x^2-8x+10\right)}{4}-6\,\mathrm{atan}\,(x-2)+\int\frac{3}{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((x**4-2*x**3+x-3)/(2*x**2-8*x+10),x)
```

```
[Out] x**3/6 + x**2/2 + 3*log(2*x**2 - 8*x + 10)/4 - 6*atan(x - 2) + In
tegral(3/2, x)
```

Mathematica [A] time = 0.0113821, size = 39, normalized size = 0.95

$$\frac{1}{2}\left(\frac{x^3}{3}+x^2+\frac{3}{2}\log\left(x^2-4x+5\right)+3x+12\tan^{-1}(2-x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + x - 2*x^3 + x^4)/(10 - 8*x + 2*x^2),x]
```

```
[Out] (3*x + x^2 + x^3/3 + 12*ArcTan[2 - x] + (3*Log[5 - 4*x + x^2])/2)
/2
```

Maple [A] time = 0.006, size = 32, normalized size = 0.8

$$\frac{3\,x}{2}+\frac{x^2}{2}+\frac{x^3}{6}-6\ \arctan\left(x-2\right)+\frac{3\ \ln\left(x^2-4\,x+5\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^4-2*x^3+x-3)/(2*x^2-8*x+10),x)
```

```
[Out] 3/2*x+1/2*x^2+1/6*x^3-6*arctan(x-2)+3/4*ln(x^2-4*x+5)
```

Maxima [A] time = 0.873511, size = 42, normalized size = 1.02

$$\frac{1}{6}x^3 + \frac{1}{2}x^2 + \frac{3}{2}x - 6\arctan(x-2) + \frac{3}{4}\log\left(x^2 - 4x + 5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/2*(x^4 - 2*x^3 + x - 3)/(x^2 - 4*x + 5),x, algorithm="maxima")
```

```
[Out] 1/6*x^3 + 1/2*x^2 + 3/2*x - 6*arctan(x - 2) + 3/4*log(x^2 - 4*x +
 5)
```

Fricas [A] time = 0.255419, size = 42, normalized size = 1.02

$$\frac{1}{6}x^3 + \frac{1}{2}x^2 + \frac{3}{2}x - 6\arctan(x-2) + \frac{3}{4}\log\left(x^2 - 4x + 5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/2*(x^4 - 2*x^3 + x - 3)/(x^2 - 4*x + 5),x, algorithm="fricas")
```

```
[Out] 1/6*x^3 + 1/2*x^2 + 3/2*x - 6*arctan(x - 2) + 3/4*log(x^2 - 4*x +
 5)
```

Sympy [A] time = 0.236565, size = 34, normalized size = 0.83

$$\frac{x^3}{6} + \frac{x^2}{2} + \frac{3x}{2} + \frac{3\log\left(x^2 - 4x + 5\right)}{4} - 6\,\mathrm{atan}\,(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**4-2*x**3+x-3)/(2*x**2-8*x+10),x)
```

```
[Out] x**3/6 + x**2/2 + 3*x/2 + 3*log(x**2 - 4*x + 5)/4 - 6*atan(x - 2)
```

GIAC/XCAS [A] time = 0.261423, size = 42, normalized size = 1.02

$$\frac{1}{6}x^3 + \frac{1}{2}x^2 + \frac{3}{2}x - 6\arctan(x-2) + \frac{3}{4}\ln\left(x^2 - 4x + 5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/2*(x^4 - 2*x^3 + x - 3)/(x^2 - 4*x + 5),x, algorithm="giac")
```

```
[Out] 1/6*x^3 + 1/2*x^2 + 3/2*x - 6*arctan(x - 2) + 3/4*ln(x^2 - 4*x +
5)
```

3.294 $\int \frac{1+2x+3x^2+x^3}{(-3+x)(-2+x)(-1+x)}\,dx$

Optimal. Leaf size=30

$$x+\frac{7}{2}\log(1-x)-25\log(2-x)+\frac{61}{2}\log(3-x)$$

```
[Out] x + (7*Log[1 - x])/2 - 25*Log[2 - x] + (61*Log[3 - x])/2
```

Rubi [A] time = 0.101971, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034

$$x+\frac{7}{2}\log(1-x)-25\log(2-x)+\frac{61}{2}\log(3-x)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + 2*x + 3*x^2 + x^3)/((-3 + x)*(-2 + x)*(-1 + x)),x]

[Out] x + (7*Log[1 - x])/2 - 25*Log[2 - x] + (61*Log[3 - x])/2
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**3+3*x**2+2*x+1)/(-3+x)/(-2+x)/(-1+x),x)

[Out] Timed out
```

Mathematica [A] time = 0.0183478, size = 24, normalized size = 0.8

$$x+\frac{61}{2}\log(x-3)-25\log(x-2)+\frac{7}{2}\log(x-1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + 2*x + 3*x^2 + x^3)/((-3 + x)*(-2 + x)*(-1 + x)),x]

[Out] x + (61*Log[-3 + x])/2 - 25*Log[-2 + x] + (7*Log[-1 + x])/2
```

Maple [A] time = 0.011, size = 21, normalized size = 0.7

$$x+\frac{7\ln(-1+x)}{2}+\frac{61\ln(-3+x)}{2}-25\ln(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^3+3*x^2+2*x+1)/(-3+x)/(x-2)/(-1+x),x)

[Out] x+7/2*ln(-1+x)+61/2*ln(-3+x)-25*ln(x-2)
```

Maxima [A] time = 0.801119, size = 27, normalized size = 0.9

$$x + \frac{7}{2}\log(x-1) - 25\log(x-2) + \frac{61}{2}\log(x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3 + 3*x^2 + 2*x + 1)/((x - 1)*(x - 2)*(x - 3)),x, algorithm="maxima")
```

```
[Out] x + 7/2*log(x - 1) - 25*log(x - 2) + 61/2*log(x - 3)
```

Fricas [A] time = 0.253226, size = 27, normalized size = 0.9

$$x + \frac{7}{2}\log(x-1) - 25\log(x-2) + \frac{61}{2}\log(x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3 + 3*x^2 + 2*x + 1)/((x - 1)*(x - 2)*(x - 3)),x, algorithm="fricas")
```

```
[Out] x + 7/2*log(x - 1) - 25*log(x - 2) + 61/2*log(x - 3)
```

Sympy [A] time = 0.328071, size = 24, normalized size = 0.8

$$x + \frac{61\log(x-3)}{2} - 25\log(x-2) + \frac{7\log(x-1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3+3*x**2+2*x+1)/(-3+x)/(-2+x)/(-1+x),x)
```

```
[Out] x + 61*log(x - 3)/2 - 25*log(x - 2) + 7*log(x - 1)/2
```

GIAC/XCAS [A] time = 0.262492, size = 31, normalized size = 1.03

$$x + \frac{7}{2}\ln(|x-1|) - 25\ln(|x-2|) + \frac{61}{2}\ln(|x-3|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3 + 3*x^2 + 2*x + 1)/((x - 1)*(x - 2)*(x - 3)),x, algorithm="giac")
```

```
[Out] x + 7/2*ln(abs(x - 1)) - 25*ln(abs(x - 2)) + 61/2*ln(abs(x - 3))
```

3.295 $\int \frac{2-7x+x^2-x^3+x^4}{-24-14x+x^2+x^3}\,dx$

Optimal. Leaf size=35

$$\frac{x^2}{2}-2x+\frac{13}{3}\log(4-x)-\frac{22}{3}\log(x+2)+20\log(x+3)$$

```
[Out] -2*x + x^2/2 + (13*Log[4 - x])/3 - (22*Log[2 + x])/3 + 20*Log[3 +
 x]
```

Rubi [A] time = 0.0735875, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033

$$\frac{x^2}{2}-2x+\frac{13}{3}\log(4-x)-\frac{22}{3}\log(x+2)+20\log(x+3)$$

Antiderivative was successfully verified.

```
[In]  Int[(2 - 7*x + x^2 - x^3 + x^4)/(-24 - 14*x + x^2 + x^3),x]
```

```
[Out] -2*x + x^2/2 + (13*Log[4 - x])/3 - (22*Log[2 + x])/3 + 20*Log[3 +
 x]
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**4-x**3+x**2-7*x+2)/(x**3+x**2-14*x-24),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.0128668, size = 35, normalized size = 1.

$$\frac{x^2}{2}-2x+\frac{13}{3}\log(4-x)-\frac{22}{3}\log(x+2)+20\log(x+3)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 - 7*x + x^2 - x^3 + x^4)/(-24 - 14*x + x^2 + x^3),x]
```

```
[Out] -2*x + x^2/2 + (13*Log[4 - x])/3 - (22*Log[2 + x])/3 + 20*Log[3 +
 x]
```

Maple [A] time = 0.013, size = 28, normalized size = 0.8

$$\frac{x^2}{2}-2\,x-\frac{22\,\ln(2+x)}{3}+\frac{13\,\ln(x-4)}{3}+20\,\ln(3+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^4-x^3+x^2-7*x+2)/(x^3+x^2-14*x-24),x)

[Out] 1/2*x^2-2*x-22/3*ln(2+x)+13/3*ln(x-4)+20*ln(3+x)
```

Maxima [A] time = 0.79717, size = 36, normalized size = 1.03

$$\frac{1}{2}x^2 - 2x + 20\log(x+3) - \frac{22}{3}\log(x+2) + \frac{13}{3}\log(x-4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 - x^3 + x^2 - 7*x + 2)/(x^3 + x^2 - 14*x - 24),x, algorithm="maxima"

[Out] 1/2*x^2 - 2*x + 20*log(x + 3) - 22/3*log(x + 2) + 13/3*log(x - 4)
```

Fricas [A] time = 0.255236, size = 36, normalized size = 1.03

$$\frac{1}{2}x^2 - 2x + 20\log(x+3) - \frac{22}{3}\log(x+2) + \frac{13}{3}\log(x-4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 - x^3 + x^2 - 7*x + 2)/(x^3 + x^2 - 14*x - 24),x, algorithm="fricas"

[Out] 1/2*x^2 - 2*x + 20*log(x + 3) - 22/3*log(x + 2) + 13/3*log(x - 4)
```

Sympy [A] time = 0.323575, size = 31, normalized size = 0.89

$$\frac{x^2}{2} - 2x + \frac{13\log(x-4)}{3} - \frac{22\log(x+2)}{3} + 20\log(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**4-x**3+x**2-7*x+2)/(x**3+x**2-14*x-24),x)

[Out] x**2/2 - 2*x + 13*log(x - 4)/3 - 22*log(x + 2)/3 + 20*log(x + 3)
```

GIAC/XCAS [A] time = 0.261888, size = 41, normalized size = 1.17

$$\frac{1}{2}x^2 - 2x + 20\ln(|x+3|) - \frac{22}{3}\ln(|x+2|) + \frac{13}{3}\ln(|x-4|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 - x^3 + x^2 - 7*x + 2)/(x^3 + x^2 - 14*x - 24),x, algorithm="giac")

[Out] 1/2*x^2 - 2*x + 20*ln(abs(x + 3)) - 22/3*ln(abs(x + 2)) + 13/3*ln
(abs(x - 4))
```

3.296 $\int \frac{2+x^2}{(-1+x)^2x(1+x)}\,dx$

Optimal. Leaf size=34

$$\frac{3}{2(1-x)}-\frac{5}{4}\log(1-x)+2\log(x)-\frac{3}{4}\log(x+1)$$

```
[Out] 3/(2*(1 - x)) - (5*Log[1 - x])/4 + 2*Log[x] - (3*Log[1 + x])/4
```

Rubi [A] time = 0.0894976, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053

$$\frac{3}{2(1-x)}-\frac{5}{4}\log(1-x)+2\log(x)-\frac{3}{4}\log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + x^2)/((-1 + x)^2*x*(1 + x)),x]

[Out] 3/(2*(1 - x)) - (5*Log[1 - x])/4 + 2*Log[x] - (3*Log[1 + x])/4
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+2)/(-1+x)**2/x/(1+x),x)

[Out] Timed out
```

Mathematica [A] time = 0.0253209, size = 32, normalized size = 0.94

$$-\frac{3}{2(x-1)}-\frac{5}{4}\log(1-x)+2\log(x)-\frac{3}{4}\log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + x^2)/((-1 + x)^2*x*(1 + x)),x]

[Out] -3/(2*(-1 + x)) - (5*Log[1 - x])/4 + 2*Log[x] - (3*Log[1 + x])/4
```

Maple [A] time = 0.013, size = 25, normalized size = 0.7

$$-\frac{3}{2\,x-2}-\frac{5\,\ln(-1+x)}{4}-\frac{3\,\ln(1+x)}{4}+2\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+2)/(-1+x)^2/x/(1+x),x)
```

```
[Out] -3/2/(-1+x)-5/4*ln(-1+x)-3/4*ln(1+x)+2*ln(x)
```

Maxima [A] time = 0.804361, size = 32, normalized size = 0.94

$$-\frac{3}{2\,(x-1)} - \frac{3}{4}\,\log\left(x+1\right) - \frac{5}{4}\,\log\left(x-1\right) + 2\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 2)/((x + 1)*(x - 1)^2*x),x, algorithm="maxima")
```

```
[Out] -3/2/(x - 1) - 3/4*log(x + 1) - 5/4*log(x - 1) + 2*log(x)
```

Fricas [A] time = 0.253541, size = 46, normalized size = 1.35

$$-\frac{3\,(x-1)\log\left(x+1\right) + 5\,(x-1)\log\left(x-1\right) - 8\,(x-1)\log\left(x\right) + 6}{4\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 2)/((x + 1)*(x - 1)^2*x),x, algorithm="fricas")
```

```
[Out] -1/4*(3*(x - 1)*log(x + 1) + 5*(x - 1)*log(x - 1) - 8*(x - 1)*log
(x) + 6)/(x - 1)
```

Sympy [A] time = 0.336332, size = 27, normalized size = 0.79

$$2\log\left(x\right) - \frac{5\log\left(x-1\right)}{4} - \frac{3\log\left(x+1\right)}{4} - \frac{3}{2x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+2)/(-1+x)**2/x/(1+x),x)
```

```
[Out] 2*log(x) - 5*log(x - 1)/4 - 3*log(x + 1)/4 - 3/(2*x - 2)
```

GIAC/XCAS [A] time = 0.261878, size = 46, normalized size = 1.35

$$-\frac{3}{2\,(x-1)} + 2\ln\left(\left|-\frac{1}{x-1} - 1\right|\right) - \frac{3}{4}\ln\left(\left|-\frac{2}{x-1} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 2)/((x + 1)*(x - 1)^2*x),x, algorithm="giac")
```

```
[Out] -3/2/(x - 1) + 2*ln(abs(-1/(x - 1) - 1)) - 3/4*ln(abs(-2/(x - 1)
- 1))
```

3.297 $\int \frac{3+x^2+x^3}{(2+x^2)^2}\,dx$

Optimal. Leaf size=42

$$\frac{x+4}{4(x^2+2)}+\frac{1}{2}\log\left(x^2+2\right)+\frac{5\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}}$$

```
[Out] (4 + x)/(4*(2 + x^2)) + (5*ArcTan[x/Sqrt[2]])/(4*Sqrt[2]) + Log[2
 + x^2]/2
```

Rubi [A] time = 0.0449307, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{x+4}{4(x^2+2)}+\frac{1}{2}\log\left(x^2+2\right)+\frac{5\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(3 + x^2 + x^3)/(2 + x^2)^2,x]
```

```
[Out] (4 + x)/(4*(2 + x^2)) + (5*ArcTan[x/Sqrt[2]])/(4*Sqrt[2]) + Log[2
 + x^2]/2
```

Rubi in Sympy [A] time = 8.94224, size = 39, normalized size = 0.93

$$\frac{x(-2x+1)}{4(x^2+2)}+\frac{\log\left(x^2+2\right)}{2}+\frac{5\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**3+x**2+3)/(x**2+2)**2,x)
```

```
[Out] x*(-2*x + 1)/(4*(x**2 + 2)) + log(x**2 + 2)/2 + 5*sqrt(2)*atan(sq
rt(2)*x/2)/8
```

Mathematica [A] time = 0.0313027, size = 42, normalized size = 1.

$$\frac{x+4}{4(x^2+2)}+\frac{1}{2}\log\left(x^2+2\right)+\frac{5\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3 + x^2 + x^3)/(2 + x^2)^2,x]
```

```
[Out] (4 + x)/(4*(2 + x^2)) + (5*ArcTan[x/Sqrt[2]])/(4*Sqrt[2]) + Log[2
 + x^2]/2
```

Maple [A] time = 0.007, size = 35, normalized size = 0.8

$$\frac{1}{x^2+2}\left(\frac{x}{4}+1\right)+\frac{\ln\left(x^2+2\right)}{2}+\frac{5\sqrt{2}}{8}\arctan\left(\frac{\sqrt{2}x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((x^3+x^2+3)/(x^2+2)^2,x)
```

```
[Out] (1/4*x+1)/(x^2+2)+1/2*ln(x^2+2)+5/8*arctan(1/2*2^(1/2)*x)*2^(1/2)
```

Maxima [A] time = 0.871203, size = 45, normalized size = 1.07

$$\frac{5}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+\frac{x+4}{4\left(x^2+2\right)}+\frac{1}{2}\log\left(x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((x^3 + x^2 + 3)/(x^2 + 2)^2,x, algorithm="maxima")
```

```
[Out] 5/8*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/4*(x + 4)/(x^2 + 2) + 1/2*l
og(x^2 + 2)
```

Fricas [A] time = 0.253075, size = 68, normalized size = 1.62

$$\frac{\sqrt{2}\left(2\sqrt{2}\left(x^2+2\right)\log\left(x^2+2\right)+5\left(x^2+2\right)\arctan\left(\frac{1}{2}\sqrt{2}x\right)+\sqrt{2}(x+4)\right)}{8\left(x^2+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((x^3 + x^2 + 3)/(x^2 + 2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2)*(2*sqrt(2)*(x^2 + 2)*log(x^2 + 2) + 5*(x^2 + 2)*arcta
n(1/2*sqrt(2)*x) + sqrt(2)*(x + 4))/(x^2 + 2)
```

Sympy [A] time = 0.291692, size = 36, normalized size = 0.86

$$\frac{x+4}{4x^2+8}+\frac{\log\left(x^2+2\right)}{2}+\frac{5\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((x**3+x**2+3)/(x**2+2)**2,x)
```

```
[Out] (x + 4)/(4*x**2 + 8) + log(x**2 + 2)/2 + 5*sqrt(2)*atan(sqrt(2)*x
/2)/8
```

GIAC/XCAS [A] time = 0.262728, size = 45, normalized size = 1.07

$$\frac{5}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+\frac{x+4}{4\left(x^2+2\right)}+\frac{1}{2}\ln\left(x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((x^3 + x^2 + 3)/(x^2 + 2)^2,x, algorithm="giac")
```

```
[Out] 5/8*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/4*(x + 4)/(x^2 + 2) + 1/2*l
n(x^2 + 2)
```

3.298 $\int \frac{-35+70x-4x^2+2x^3}{(26-10x+x^2)(17-2x+x^2)}\,dx$

Optimal. Leaf size=49

$$\frac{1003\log\left(x^2-10x+26\right)}{1025}+\frac{22\log\left(x^2-2x+17\right)}{1025}-\frac{15033\tan^{-1}(5-x)}{1025}-\frac{4607\tan^{-1}\left(\frac{x-1}{4}\right)}{4100}$$

```
[Out] (-15033*ArcTan[5 - x])/1025 - (4607*ArcTan[(-1 + x)/4])/4100 + (1
003*Log[26 - 10*x + x^2])/1025 + (22*Log[17 - 2*x + x^2])/1025
```

Rubi [A] time = 0.290272, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$

$$\frac{1003\log\left(x^2-10x+26\right)}{1025}+\frac{22\log\left(x^2-2x+17\right)}{1025}-\frac{15033\tan^{-1}(5-x)}{1025}-\frac{4607\tan^{-1}\left(\frac{x-1}{4}\right)}{4100}$$

Antiderivative was successfully verified.

```
[In] Int[(-35 + 70*x - 4*x^2 + 2*x^3)/((26 - 10*x + x^2)*(17 - 2*x + x^2)),x]
```

```
[Out] (-15033*ArcTan[5 - x])/1025 - (4607*ArcTan[(-1 + x)/4])/4100 + (1
003*Log[26 - 10*x + x^2])/1025 + (22*Log[17 - 2*x + x^2])/1025
```

Rubi in Sympy [A] time = 82.3177, size = 46, normalized size = 0.94

$$\frac{1003\log\left(x^2-10x+26\right)}{1025}+\frac{22\log\left(x^2-2x+17\right)}{1025}-\frac{4607\,\mathrm{atan}\left(\frac{x}{4}-\frac{1}{4}\right)}{4100}+\frac{15033\,\mathrm{atan}\,(x-5)}{1025}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((2*x**3-4*x**2+70*x-35)/(x**2-10*x+26)/(x**2-2*x+17),x)
```

```
[Out] 1003*log(x**2 - 10*x + 26)/1025 + 22*log(x**2 - 2*x + 17)/1025 -
4607*atan(x/4 - 1/4)/4100 + 15033*atan(x - 5)/1025
```

Mathematica [A] time = 0.0229319, size = 49, normalized size = 1.

$$\frac{1003\log\left(x^2-10x+26\right)}{1025}+\frac{22\log\left(x^2-2x+17\right)}{1025}-\frac{15033\tan^{-1}(5-x)}{1025}-\frac{4607\tan^{-1}\left(\frac{x-1}{4}\right)}{4100}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-35 + 70*x - 4*x^2 + 2*x^3)/((26 - 10*x + x^2)*(17 - 2*x + x^2)),x]
```

```
[Out] (-15033*ArcTan[5 - x])/1025 - (4607*ArcTan[(-1 + x)/4])/4100 + (1
003*Log[26 - 10*x + x^2])/1025 + (22*Log[17 - 2*x + x^2])/1025
```

Maple [A] time = 0.009, size = 38, normalized size = 0.8

$$\frac{15033\,\arctan(-5+x)}{1025}-\frac{4607}{4100}\arctan\left(-\frac{1}{4}+\frac{x}{4}\right)+\frac{1003\,\ln\left(x^2-10\,x+26\right)}{1025}+\frac{22\,\ln\left(x^2-2\,x+17\right)}{1025}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*x^3-4*x^2+70*x-35)/(x^2-10*x+26)/(x^2-2*x+17),x)
```

```
[Out] 15033/1025*arctan(-5+x)-4607/4100*arctan(-1/4+1/4*x)+1003/1025*ln
(x^2-10*x+26)+22/1025*ln(x^2-2*x+17)
```

Maxima [A] time = 0.887453, size = 50, normalized size = 1.02

$$\frac{15033}{1025}\arctan\left(x-5\right)-\frac{4607}{4100}\arctan\left(\frac{1}{4}x-\frac{1}{4}\right)+\frac{22}{1025}\log\left(x^2-2x+17\right)+\frac{1003}{1025}\log\left(x^2-10x+26\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3 - 4*x^2 + 70*x - 35)/((x^2 - 2*x + 17)*(x^2 - 10*x + 26)),x, algor
```

```
[Out] 15033/1025*arctan(x - 5) - 4607/4100*arctan(1/4*x - 1/4) + 22/102
5*log(x^2 - 2*x + 17) + 1003/1025*log(x^2 - 10*x + 26)
```

Fricas [A] time = 0.255186, size = 50, normalized size = 1.02

$$\frac{15033}{1025}\arctan\left(x-5\right)-\frac{4607}{4100}\arctan\left(\frac{1}{4}x-\frac{1}{4}\right)+\frac{22}{1025}\log\left(x^2-2x+17\right)+\frac{1003}{1025}\log\left(x^2-10x+26\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3 - 4*x^2 + 70*x - 35)/((x^2 - 2*x + 17)*(x^2 - 10*x + 26)),x, algor
```

```
[Out] 15033/1025*arctan(x - 5) - 4607/4100*arctan(1/4*x - 1/4) + 22/102
5*log(x^2 - 2*x + 17) + 1003/1025*log(x^2 - 10*x + 26)
```

Sympy [A] time = 0.627449, size = 46, normalized size = 0.94

$$\frac{1003\log\left(x^2-10x+26\right)}{1025}+\frac{22\log\left(x^2-2x+17\right)}{1025}-\frac{4607\operatorname{atan}\left(\frac{x}{4}-\frac{1}{4}\right)}{4100}+\frac{15033\operatorname{atan}\left(x-5\right)}{1025}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x**3-4*x**2+70*x-35)/(x**2-10*x+26)/(x**2-2*x+17),x)
```

```
[Out] 1003*log(x**2 - 10*x + 26)/1025 + 22*log(x**2 - 2*x + 17)/1025 -
4607*atan(x/4 - 1/4)/4100 + 15033*atan(x - 5)/1025
```

GIAC/XCAS [A] time = 0.261597, size = 50, normalized size = 1.02

$$\frac{15033}{1025}\arctan\left(x-5\right)-\frac{4607}{4100}\arctan\left(\frac{1}{4}x-\frac{1}{4}\right)+\frac{22}{1025}\ln\left(x^2-2x+17\right)+\frac{1003}{1025}\ln\left(x^2-10x+26\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3 - 4*x^2 + 70*x - 35)/((x^2 - 2*x + 17)*(x^2 - 10*x + 26)),x, algor
```

```
[Out] 15033/1025*arctan(x - 5) - 4607/4100*arctan(1/4*x - 1/4) + 22/102
5*ln(x^2 - 2*x + 17) + 1003/1025*ln(x^2 - 10*x + 26)
```

3.299 $\int \frac{2+x^2}{(-5+x)(-3+x)(4+x)}\,dx$

Optimal. Leaf size=29

$$-\frac{11}{14}\log(3-x)+\frac{3}{2}\log(5-x)+\frac{2}{7}\log(x+4)$$

```
[Out] (-11*Log[3 - x])/14 + (3*Log[5 - x])/2 + (2*Log[4 + x])/7
```

Rubi [A] time = 0.0939585, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048

$$-\frac{11}{14}\log(3-x)+\frac{3}{2}\log(5-x)+\frac{2}{7}\log(x+4)$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + x^2)/((-5 + x)*(-3 + x)*(4 + x)),x]

[Out] (-11*Log[3 - x])/14 + (3*Log[5 - x])/2 + (2*Log[4 + x])/7
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+2)/(-5+x)/(-3+x)/(4+x),x)

[Out] Timed out
```

Mathematica [A] time = 0.0118387, size = 29, normalized size = 1.

$$-\frac{11}{14}\log(3-x)+\frac{3}{2}\log(5-x)+\frac{2}{7}\log(x+4)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + x^2)/((-5 + x)*(-3 + x)*(4 + x)),x]

[Out] (-11*Log[3 - x])/14 + (3*Log[5 - x])/2 + (2*Log[4 + x])/7
```

Maple [A] time = 0.011, size = 20, normalized size = 0.7

$$-\frac{11\ln(-3+x)}{14}+\frac{2\ln(4+x)}{7}+\frac{3\ln(-5+x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+2)/(-5+x)/(-3+x)/(4+x),x)

[Out] -11/14*ln(-3+x)+2/7*ln(4+x)+3/2*ln(-5+x)
```

Maxima [A] time = 0.815033, size = 26, normalized size = 0.9

$$\frac{2}{7}\log\left(x+4\right)-\frac{11}{14}\log\left(x-3\right)+\frac{3}{2}\log\left(x-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 2)/((x + 4)*(x - 3)*(x - 5)),x, algorithm="maxima")
```

```
[Out] 2/7*log(x + 4) - 11/14*log(x - 3) + 3/2*log(x - 5)
```

Fricas [A] time = 0.256919, size = 26, normalized size = 0.9

$$\frac{2}{7}\log\left(x+4\right)-\frac{11}{14}\log\left(x-3\right)+\frac{3}{2}\log\left(x-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 2)/((x + 4)*(x - 3)*(x - 5)),x, algorithm="fricas")
```

```
[Out] 2/7*log(x + 4) - 11/14*log(x - 3) + 3/2*log(x - 5)
```

Sympy [A] time = 0.309087, size = 24, normalized size = 0.83

$$\frac{3\log\left(x-5\right)}{2}-\frac{11\log\left(x-3\right)}{14}+\frac{2\log\left(x+4\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+2)/(-5+x)/(-3+x)/(4+x),x)
```

```
[Out] 3*log(x - 5)/2 - 11*log(x - 3)/14 + 2*log(x + 4)/7
```

GIAC/XCAS [A] time = 0.258399, size = 30, normalized size = 1.03

$$\frac{2}{7}\ln\left(|x+4|\right)-\frac{11}{14}\ln\left(|x-3|\right)+\frac{3}{2}\ln\left(|x-5|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 2)/((x + 4)*(x - 3)*(x - 5)),x, algorithm="giac")
```

```
[Out] 2/7*ln(abs(x + 4)) - 11/14*ln(abs(x - 3)) + 3/2*ln(abs(x - 5))
```

3.300 $\int \frac{x^4}{(-1+x)(2+x^2)} dx$

Optimal. Leaf size=46

$$\frac{x^2}{2}-\frac{2}{3}\log\left(x^2+2\right)+x+\frac{1}{3}\log(1-x)-\frac{2}{3}\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

```
[Out] x + x^2/2 - (2*Sqrt[2]*ArcTan[x/Sqrt[2]])/3 + Log[1 - x]/3 - (2*L
og[2 + x^2])/3
```

Rubi [A] time = 0.0830231, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{x^2}{2}-\frac{2}{3}\log\left(x^2+2\right)+x+\frac{1}{3}\log(1-x)-\frac{2}{3}\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/((-1 + x)*(2 + x^2)),x]
```

```
[Out] x + x^2/2 - (2*Sqrt[2]*ArcTan[x/Sqrt[2]])/3 + Log[1 - x]/3 - (2*L
og[2 + x^2])/3
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$x+\frac{\log(-x+1)}{3}-\frac{2\log\left(x^2+2\right)}{3}-\frac{2\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{3}+\int x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(-1+x)/(x**2+2),x)
```

```
[Out] x + log(-x + 1)/3 - 2*log(x**2 + 2)/3 - 2*sqrt(2)*atan(sqrt(2)*x/
2)/3 + Integral(x, x)
```

Mathematica [A] time = 0.0260565, size = 43, normalized size = 0.93

$$\frac{1}{6}\left(3x^2-4\log\left(x^2+2\right)+6x+2\log(x-1)-4\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)-9\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/((-1 + x)*(2 + x^2)),x]
```

```
[Out] (-9 + 6*x + 3*x^2 - 4*Sqrt[2]*ArcTan[x/Sqrt[2]] + 2*Log[-1 + x] -
 4*Log[2 + x^2])/6
```

Maple [A] time = 0.008, size = 34, normalized size = 0.7

$$x+\frac{x^2}{2}+\frac{\ln(-1+x)}{3}-\frac{2\ln\left(x^2+2\right)}{3}-\frac{2\sqrt{2}}{3}\arctan\left(\frac{\sqrt{2}x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^4/(-1+x)/(x^2+2),x)
```

```
[Out] x+1/2*x^2+1/3*ln(-1+x)-2/3*ln(x^2+2)-2/3*arctan(1/2*2^(1/2)*x)*2^
(1/2)
```

Maxima [A] time = 0.890108, size = 45, normalized size = 0.98

$$\frac{1}{2}x^2 - \frac{2}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + x - \frac{2}{3}\log\left(x^2+2\right) + \frac{1}{3}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x^4/((x^2 + 2)*(x - 1)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 2/3*sqrt(2)*arctan(1/2*sqrt(2)*x) + x - 2/3*log(x^2 + 2
) + 1/3*log(x - 1)
```

Fricas [A] time = 0.254892, size = 45, normalized size = 0.98

$$\frac{1}{2}x^2 - \frac{2}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + x - \frac{2}{3}\log\left(x^2+2\right) + \frac{1}{3}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x^4/((x^2 + 2)*(x - 1)),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 - 2/3*sqrt(2)*arctan(1/2*sqrt(2)*x) + x - 2/3*log(x^2 + 2
) + 1/3*log(x - 1)
```

Sympy [A] time = 0.315118, size = 41, normalized size = 0.89

$$\frac{x^2}{2} + x + \frac{\log\left(x-1\right)}{3} - \frac{2\log\left(x^2+2\right)}{3} - \frac{2\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x**4/(-1+x)/(x**2+2),x)
```

```
[Out] x**2/2 + x + log(x - 1)/3 - 2*log(x**2 + 2)/3 - 2*sqrt(2)*atan(sq
rt(2)*x/2)/3
```

GIAC/XCAS [A] time = 0.261926, size = 46, normalized size = 1.

$$\frac{1}{2}x^2 - \frac{2}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + x - \frac{2}{3}\ln\left(x^2+2\right) + \frac{1}{3}\ln\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x^4/((x^2 + 2)*(x - 1)),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 2/3*sqrt(2)*arctan(1/2*sqrt(2)*x) + x - 2/3*ln(x^2 + 2)
 + 1/3*ln(abs(x - 1))
```

3.301 $\int \frac{-1+7x+2x^2}{-1-x+x^2+x^3}\,dx$

Optimal. Leaf size=16

$$2\log(1-x)-\frac{3}{x+1}$$

```
[Out] -3/(1 + x) + 2*Log[1 - x]
```

Rubi [A] time = 0.0495433, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042

$$2\log(1-x)-\frac{3}{x+1}$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + 7*x + 2*x^2)/(-1 - x + x^2 + x^3),x]

[Out] -3/(1 + x) + 2*Log[1 - x]
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*x**2+7*x-1)/(x**3+x**2-x-1),x)

[Out] Timed out
```

Mathematica [A] time = 0.0121834, size = 14, normalized size = 0.88

$$2\log(x-1)-\frac{3}{x+1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + 7*x + 2*x^2)/(-1 - x + x^2 + x^3),x]

[Out] -3/(1 + x) + 2*Log[-1 + x]
```

Maple [A] time = 0.008, size = 15, normalized size = 0.9

$$2\ln(-1+x)-3(1+x)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*x^2+7*x-1)/(x^3+x^2-x-1),x)

[Out] 2*ln(-1+x)-3/(1+x)
```

Maxima [A] time = 0.786818, size = 19, normalized size = 1.19

$$-\frac{3}{x+1} + 2 \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2 + 7*x - 1)/(x^3 + x^2 - x - 1),x, algorithm="maxima")
```

```
[Out] -3/(x + 1) + 2*log(x - 1)
```

Fricas [A] time = 0.251811, size = 23, normalized size = 1.44

$$\frac{2(x+1)\log(x-1)-3}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2 + 7*x - 1)/(x^3 + x^2 - x - 1),x, algorithm="fricas")
```

```
[Out] (2*(x + 1)*log(x - 1) - 3)/(x + 1)
```

Sympy [A] time = 0.185855, size = 10, normalized size = 0.62

$$2\log(x-1) - \frac{3}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x**2+7*x-1)/(x**3+x**2-x-1),x)
```

```
[Out] 2*log(x - 1) - 3/(x + 1)
```

GIAC/XCAS [A] time = 0.262542, size = 20, normalized size = 1.25

$$-\frac{3}{x+1} + 2\ln(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + 7*x - 1)/(x^3 + x^2 - x - 1),x, algorithm="giac")
```

```
[Out] -3/(x + 1) + 2*ln(abs(x - 1))
```

3.302 $\int \frac{1+2x}{-1+3x-3x^2+x^3}\,dx$

Optimal. Leaf size=21

$$\frac{2}{1-x}-\frac{3}{2(1-x)^2}$$

```
[Out] -3/(2*(1 - x)^2) + 2/(1 - x)
```

Rubi [A] time = 0.0404679, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048

$$\frac{2}{1-x}-\frac{3}{2(1-x)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + 2*x)/(-1 + 3*x - 3*x^2 + x^3),x]

[Out] -3/(2*(1 - x)^2) + 2/(1 - x)
```

Rubi in Sympy [A] time = 13.1956, size = 17, normalized size = 0.81

$$-\frac{3\left(2x+1\right)^2}{2\left(-3x+3\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+2*x)/(x**3-3*x**2+3*x-1),x)

[Out] -3*(2*x + 1)**2/(2*(-3*x + 3)**2)
```

Mathematica [A] time = 0.00436105, size = 14, normalized size = 0.67

$$\frac{1-4x}{2(x-1)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + 2*x)/(-1 + 3*x - 3*x^2 + x^3),x]

[Out] (1 - 4*x)/(2*(-1 + x)^2)
```

Maple [A] time = 0.007, size = 16, normalized size = 0.8

$$-2\left(-1+x\right)^{-1}-\frac{3}{2\left(-1+x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+2*x)/(x^3-3*x^2+3*x-1),x)
```

```
[Out] -2/(-1+x)-3/2/(-1+x)^2
```

Maxima [A] time = 0.807282, size = 23, normalized size = 1.1

$$-\frac{4x-1}{2(x^2-2x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x + 1)/(x^3 - 3*x^2 + 3*x - 1),x, algorithm="maxima")
```

```
[Out] -1/2*(4*x - 1)/(x^2 - 2*x + 1)
```

Fricas [A] time = 0.239378, size = 23, normalized size = 1.1

$$-\frac{4x-1}{2(x^2-2x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x + 1)/(x^3 - 3*x^2 + 3*x - 1),x, algorithm="fricas")
```

```
[Out] -1/2*(4*x - 1)/(x^2 - 2*x + 1)
```

Sympy [A] time = 0.165503, size = 15, normalized size = 0.71

$$-\frac{4x-1}{2x^2-4x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+2*x)/(x**3-3*x**2+3*x-1),x)
```

```
[Out] -(4*x - 1)/(2*x**2 - 4*x + 2)
```

GIAC/XCAS [A] time = 0.259027, size = 16, normalized size = 0.76

$$-\frac{4x-1}{2(x-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x + 1)/(x^3 - 3*x^2 + 3*x - 1),x, algorithm="giac")
```

```
[Out] -1/2*(4*x - 1)/(x - 1)^2
```

3.303 $\int \frac{5-5x+7x^2+x^3}{(-1+x)^2(1+x)^3}\,dx$

Optimal. Leaf size=15

$$\frac{1}{1-x}-\frac{2}{(x+1)^2}$$

```
[Out] (1 - x)^(-1) - 2/(1 + x)^2
```

Rubi [A] time = 0.0370854, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042

$$\frac{1}{1-x}-\frac{2}{(x+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(5 - 5*x + 7*x^2 + x^3)/((-1 + x)^2*(1 + x)^3),x]

[Out] (1 - x)^(-1) - 2/(1 + x)^2
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((x**3+7*x**2-5*x+5)/(-1+x)**2/(1+x)**3,x)

[Out] Timed out
```

Mathematica [A] time = 0.0191993, size = 15, normalized size = 1.

$$-\frac{2}{(x+1)^2}-\frac{1}{x-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 - 5*x + 7*x^2 + x^3)/((-1 + x)^2*(1 + x)^3),x]

[Out] -(-1 + x)^(-1) - 2/(1 + x)^2
```

Maple [A] time = 0.008, size = 16, normalized size = 1.1

$$-(-1+x)^{-1}-2\,(1+x)^{-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+7*x^2-5*x+5)/(-1+x)^2/(1+x)^3,x)

[Out] -1/(-1+x)-2/(1+x)^2
```

Maxima [A] time = 0.797168, size = 31, normalized size = 2.07

$$-\frac{x^2+4x-1}{x^3+x^2-x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3 + 7*x^2 - 5*x + 5)/((x + 1)^3*(x - 1)^2),x, algorithm="maxima")
```

```
[Out] -(x^2 + 4*x - 1)/(x^3 + x^2 - x - 1)
```

Fricas [A] time = 0.242329, size = 31, normalized size = 2.07

$$-\frac{x^2+4x-1}{x^3+x^2-x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3 + 7*x^2 - 5*x + 5)/((x + 1)^3*(x - 1)^2),x, algorithm="fricas")
```

```
[Out] -(x^2 + 4*x - 1)/(x^3 + x^2 - x - 1)
```

Sympy [A] time = 0.274454, size = 19, normalized size = 1.27

$$-\frac{x^2+4x-1}{x^3+x^2-x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3+7*x**2-5*x+5)/(-1+x)**2/(1+x)**3,x)
```

```
[Out] -(x**2 + 4*x - 1)/(x**3 + x**2 - x - 1)
```

GIAC/XCAS [A] time = 0.262833, size = 41, normalized size = 2.73

$$-\frac{1}{x-1}+\frac{\frac{4}{x-1}+1}{2\left(\frac{2}{x-1}+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3 + 7*x^2 - 5*x + 5)/((x + 1)^3*(x - 1)^2),x, algorithm="giac")
```

```
[Out] -1/(x - 1) + 1/2*(4/(x - 1) + 1)/(2/(x - 1) + 1)^2
```

3.304 $\int \frac{1+3x+3x^2}{1+2x+2x^2+x^3}\,dx$

Optimal. Leaf size=31

$$\log\left(x^2+x+1\right)+\log(x+1)-\frac{2\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] (-2*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] + Log[1 + x] + Log[1 + x +
 x^2]
```

Rubi [A] time = 0.0885131, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192

$$\log\left(x^2+x+1\right)+\log(x+1)-\frac{2\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + 3*x + 3*x^2)/(1 + 2*x + 2*x^2 + x^3),x]
```

```
[Out] (-2*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] + Log[1 + x] + Log[1 + x +
 x^2]
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+3*x+1)/(x**3+2*x**2+2*x+1),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.0216145, size = 31, normalized size = 1.

$$\log\left(x^2+x+1\right)+\log(x+1)-\frac{2\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + 3*x + 3*x^2)/(1 + 2*x + 2*x^2 + x^3),x]
```

```
[Out] (-2*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] + Log[1 + x] + Log[1 + x +
 x^2]
```

Maple [A] time = 0.008, size = 29, normalized size = 0.9

$$\ln(1+x)+\ln\left(x^2+x+1\right)-\frac{2\sqrt{3}}{3}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((3*x^2+3*x+1)/(x^3+2*x^2+2*x+1),x)
```

```
[Out] ln(1+x)+ln(x^2+x+1)-2/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Maxima [A] time = 0.882593, size = 38, normalized size = 1.23

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\log\left(x^2+x+1\right)+\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2 + 3*x + 1)/(x^3 + 2*x^2 + 2*x + 1),x, algorithm="maxima")
```

```
[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + log(x^2 + x + 1) + l
og(x + 1)
```

Fricas [A] time = 0.258849, size = 51, normalized size = 1.65

$$\frac{1}{3}\sqrt{3}\left(\sqrt{3}\log\left(x^2+x+1\right)+\sqrt{3}\log\left(x+1\right)-2\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2 + 3*x + 1)/(x^3 + 2*x^2 + 2*x + 1),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*(sqrt(3)*log(x^2 + x + 1) + sqrt(3)*log(x + 1) - 2*ar
ctan(1/3*sqrt(3)*(2*x + 1)))
```

Sympy [A] time = 0.270119, size = 3, normalized size = 0.1

$$\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((3*x**2+3*x+1)/(x**3+2*x**2+2*x+1),x)
```

```
[Out] log(x + 1)
```

GIAC/XCAS [A] time = 0.261863, size = 39, normalized size = 1.26

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\ln\left(x^2+x+1\right)+\ln\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2 + 3*x + 1)/(x^3 + 2*x^2 + 2*x + 1),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + ln(x^2 + x + 1) + ln
(abs(x + 1))
```

3.305 $\int \frac{-1+2x+x^2}{-2x+3x^2+2x^3} \, dx$

Optimal. Leaf size=25

$$\frac{1}{10}\log(1-2x)+\frac{\log(x)}{2}-\frac{1}{10}\log(x+2)$$

```
[Out] Log[1 - 2*x]/10 + Log[x]/2 - Log[2 + x]/10
```

Rubi [A] time = 0.0619174, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08

$$\frac{1}{10}\log(1-2x)+\frac{\log(x)}{2}-\frac{1}{10}\log(x+2)$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + 2*x + x^2)/(-2*x + 3*x^2 + 2*x^3),x]

[Out] Log[1 - 2*x]/10 + Log[x]/2 - Log[2 + x]/10
```

Rubi in Sympy [A] time = 13.7888, size = 19, normalized size = 0.76

$$\frac{\log(x)}{2}+\frac{\log(-2x+1)}{10}-\frac{\log(x+2)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+2*x-1)/(2*x**3+3*x**2-2*x),x)

[Out] log(x)/2 + log(-2*x + 1)/10 - log(x + 2)/10
```

Mathematica [A] time = 0.00957453, size = 25, normalized size = 1.

$$\frac{1}{10}\log(1-2x)+\frac{\log(x)}{2}-\frac{1}{10}\log(x+2)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + 2*x + x^2)/(-2*x + 3*x^2 + 2*x^3),x]

[Out] Log[1 - 2*x]/10 + Log[x]/2 - Log[2 + x]/10
```

Maple [A] time = 0.01, size = 20, normalized size = 0.8

$$-\frac{\ln(2+x)}{10}+\frac{\ln(x)}{2}+\frac{\ln(2x-1)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+2*x-1)/(2*x^3+3*x^2-2*x),x)
```

```
[Out] -1/10*ln(2+x)+1/2*ln(x)+1/10*ln(2*x-1)
```

Maxima [A] time = 0.815332, size = 26, normalized size = 1.04

$$\frac{1}{10}\log(2x-1)-\frac{1}{10}\log(x+2)+\frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 + 2*x - 1)/(2*x^3 + 3*x^2 - 2*x),x, algorithm="maxima")
```

```
[Out] 1/10*log(2*x - 1) - 1/10*log(x + 2) + 1/2*log(x)
```

Fricas [A] time = 0.275075, size = 26, normalized size = 1.04

$$\frac{1}{10}\log(2x-1)-\frac{1}{10}\log(x+2)+\frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 + 2*x - 1)/(2*x^3 + 3*x^2 - 2*x),x, algorithm="fricas")
```

```
[Out] 1/10*log(2*x - 1) - 1/10*log(x + 2) + 1/2*log(x)
```

Sympy [A] time = 0.296881, size = 19, normalized size = 0.76

$$\frac{\log(x)}{2}+\frac{\log\left(x-\frac{1}{2}\right)}{10}-\frac{\log(x+2)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2*x-1)/(2*x**3+3*x**2-2*x),x)
```

```
[Out] log(x)/2 + log(x - 1/2)/10 - log(x + 2)/10
```

GIAC/XCAS [A] time = 0.25954, size = 30, normalized size = 1.2

$$\frac{1}{10}\ln(|2x-1|)-\frac{1}{10}\ln(|x+2|)+\frac{1}{2}\ln(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 + 2*x - 1)/(2*x^3 + 3*x^2 - 2*x),x, algorithm="giac")
```

```
[Out] 1/10*ln(abs(2*x - 1)) - 1/10*ln(abs(x + 2)) + 1/2*ln(abs(x))
```

3.306 $\int \frac{1+4x-2x^2+x^4}{1-x-x^2+x^3}\,dx$

Optimal. Leaf size=30

$$\frac{x^2}{2}+x+\frac{2}{1-x}+\log(1-x)-\log(x+1)$$

```
[Out] 2/(1 - x) + x + x^2/2 + Log[1 - x] - Log[1 + x]
```

Rubi [A] time = 0.0609606, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034

$$\frac{x^2}{2}+x+\frac{2}{1-x}+\log(1-x)-\log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + 4*x - 2*x^2 + x^4)/(1 - x - x^2 + x^3),x]

[Out] 2/(1 - x) + x + x^2/2 + Log[1 - x] - Log[1 + x]
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**4-2*x**2+4*x+1)/(x**3-x**2-x+1),x)

[Out] Timed out
```

Mathematica [A] time = 0.0261004, size = 29, normalized size = 0.97

$$\frac{1}{2}(x+1)^2-\frac{2}{x-1}+\log(1-x)-\log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + 4*x - 2*x^2 + x^4)/(1 - x - x^2 + x^3),x]

[Out] -2/(-1 + x) + (1 + x)^2/2 + Log[1 - x] - Log[1 + x]
```

Maple [A] time = 0.01, size = 25, normalized size = 0.8

$$x+\frac{x^2}{2}+\ln\left(-1+x\right)-2\left(-1+x\right)^{-1}-\ln\left(1+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^4-2*x^2+4*x+1)/(x^3-x^2-x+1),x)

[Out] x+1/2*x^2+ln(-1+x)-2/(-1+x)-ln(1+x)
```

Maxima [A] time = 0.797626, size = 32, normalized size = 1.07

$$\frac{1}{2}x^2 + x - \frac{2}{x-1} - \log(x+1) + \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 - 2*x^2 + 4*x + 1)/(x^3 - x^2 - x + 1),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + x - 2/(x - 1) - log(x + 1) + log(x - 1)
```

Fricas [A] time = 0.253166, size = 49, normalized size = 1.63

$$\frac{x^3 + x^2 - 2(x-1)\log(x+1) + 2(x-1)\log(x-1) - 2x - 4}{2(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 - 2*x^2 + 4*x + 1)/(x^3 - x^2 - x + 1),x, algorithm="fricas")
```

```
[Out] 1/2*(x^3 + x^2 - 2*(x - 1)*log(x + 1) + 2*(x - 1)*log(x - 1) - 2*
x - 4)/(x - 1)
```

Sympy [A] time = 0.194386, size = 20, normalized size = 0.67

$$\frac{x^2}{2} + x + \log(x-1) - \log(x+1) - \frac{2}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**4-2*x**2+4*x+1)/(x**3-x**2-x+1),x)
```

```
[Out] x**2/2 + x + log(x - 1) - log(x + 1) - 2/(x - 1)
```

GIAC/XCAS [A] time = 0.260329, size = 35, normalized size = 1.17

$$\frac{1}{2}x^2 + x - \frac{2}{x-1} - \ln(|x+1|) + \ln(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 - 2*x^2 + 4*x + 1)/(x^3 - x^2 - x + 1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 + x - 2/(x - 1) - ln(abs(x + 1)) + ln(abs(x - 1))
```

3.307 $\int \frac{4-x+2x^2}{4x+x^3} \, dx$

Optimal. Leaf size=23

$$\frac{1}{2}\log\left(x^2+4\right)+\log(x)-\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)$$

```
[Out] -ArcTan[x/2]/2 + Log[x] + Log[4 + x^2]/2
```

Rubi [A] time = 0.0589412, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{1}{2}\log\left(x^2+4\right)+\log(x)-\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(4 - x + 2*x^2)/(4*x + x^3),x]

[Out] -ArcTan[x/2]/2 + Log[x] + Log[4 + x^2]/2
```

Rubi in Sympy [A] time = 8.87468, size = 17, normalized size = 0.74

$$\log(x)+\frac{\log\left(x^2+4\right)}{2}-\frac{\operatorname{atan}\left(\frac{x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*x**2-x+4)/(x**3+4*x),x)

[Out] log(x) + log(x**2 + 4)/2 - atan(x/2)/2
```

Mathematica [A] time = 0.00855635, size = 23, normalized size = 1.

$$\frac{1}{2}\log\left(x^2+4\right)+\log(x)-\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(4 - x + 2*x^2)/(4*x + x^3),x]

[Out] -ArcTan[x/2]/2 + Log[x] + Log[4 + x^2]/2
```

Maple [A] time = 0.007, size = 18, normalized size = 0.8

$$-\frac{1}{2}\arctan\left(\frac{x}{2}\right)+\ln(x)+\frac{\ln\left(x^2+4\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*x^2-x+4)/(x^3+4*x),x)
```

```
[Out] -1/2*arctan(1/2*x)+ln(x)+1/2*ln(x^2+4)
```

Maxima [A] time = 0.870545, size = 23, normalized size = 1.

$$-\frac{1}{2}\,\arctan\left(\frac{1}{2}\,x\right)+\frac{1}{2}\,\log\left(x^2+4\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 - x + 4)/(x^3 + 4*x),x, algorithm="maxima")
```

```
[Out] -1/2*arctan(1/2*x) + 1/2*log(x^2 + 4) + log(x)
```

Fricas [A] time = 0.25604, size = 23, normalized size = 1.

$$-\frac{1}{2}\,\arctan\left(\frac{1}{2}\,x\right)+\frac{1}{2}\,\log\left(x^2+4\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 - x + 4)/(x^3 + 4*x),x, algorithm="fricas")
```

```
[Out] -1/2*arctan(1/2*x) + 1/2*log(x^2 + 4) + log(x)
```

Sympy [A] time = 0.303118, size = 17, normalized size = 0.74

$$\log\left(x\right)+\frac{\log\left(x^2+4\right)}{2}-\frac{\operatorname{atan}\left(\frac{x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x**2-x+4)/(x**3+4*x),x)
```

```
[Out] log(x) + log(x**2 + 4)/2 - atan(x/2)/2
```

GIAC/XCAS [A] time = 0.262433, size = 24, normalized size = 1.04

$$-\frac{1}{2}\,\arctan\left(\frac{1}{2}\,x\right)+\frac{1}{2}\,\ln\left(x^2+4\right)+\ln\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 - x + 4)/(x^3 + 4*x),x, algorithm="giac")
```

```
[Out] -1/2*arctan(1/2*x) + 1/2*ln(x^2 + 4) + ln(abs(x))
```

3.308 $\int \frac{1+x^2+x^3}{(-1+x)x(1+x^2)^3(1+x+x^2)}\,dx$

Optimal. Leaf size=103

$$-\frac{3(1-x)}{8(x^2+1)}+\frac{3x}{16(x^2+1)}+\frac{x+1}{8(x^2+1)^2}+\frac{15}{16}\log\left(x^2+1\right)-\frac{1}{2}\log\left(x^2+x+1\right)+\frac{1}{8}\log(1-x)-\log(x)+\frac{7}{16}\tan^{-1}(x)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] (1 + x)/(8*(1 + x^2)^2) - (3*(1 - x))/(8*(1 + x^2)) + (3*x)/(16*(
1 + x^2)) + (7*ArcTan[x])/16 - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3]
+ Log[1 - x]/8 - Log[x] + (15*Log[1 + x^2])/16 - Log[1 + x + x^2]
/2
```

Rubi [A] time = 0.94226, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.281

$$-\frac{3(1-x)}{8(x^2+1)}+\frac{3x}{16(x^2+1)}+\frac{x+1}{8(x^2+1)^2}+\frac{15}{16}\log\left(x^2+1\right)-\frac{1}{2}\log\left(x^2+x+1\right)+\frac{1}{8}\log(1-x)-\log(x)+\frac{7}{16}\tan^{-1}(x)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^2 + x^3)/((-1 + x)*x*(1 + x^2)^3*(1 + x + x^2)),x]
```


```
[Out] (1 + x)/(8*(1 + x^2)^2) - (3*(1 - x))/(8*(1 + x^2)) + (3*x)/(16*(
1 + x^2)) + (7*ArcTan[x])/16 - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3]
+ Log[1 - x]/8 - Log[x] + (15*Log[1 + x^2])/16 - Log[1 + x + x^2]
/2
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**3+x**2+1)/(-1+x)/x/(x**2+1)**3/(x**2+x+1),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.07196, size = 93, normalized size = 0.9

$$\frac{1}{48}\left(-14\log\left(1-x^3\right)+\frac{6(x+1)}{(x^2+1)^2}+\frac{9(3x-2)}{x^2+1}+45\log\left(x^2+1\right)-10\log\left(x^2+x+1\right)+20\log(1-x)-48\log(x)+21\tan^{-1}(x)-16\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^2 + x^3)/((-1 + x)*x*(1 + x^2)^3*(1 + x + x^2)),x]
```

```
[Out] ((6*(1 + x))/(1 + x^2)^2 + (9*(-2 + 3*x))/(1 + x^2) + 21*ArcTan[x
] - 16*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 20*Log[1 - x] - 48*Log
[x] + 45*Log[1 + x^2] - 10*Log[1 + x + x^2] - 14*Log[1 - x^3])/48
```

Maple [A] time = 0.017, size = 73, normalized size = 0.7

$$-\frac{\ln\left(x^{2}+x+1\right)}{2}-\frac{\sqrt{3}}{3}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)+\frac{\ln(-1+x)}{8}-\ln(x)+\frac{1}{8\left(x^{2}+1\right)^{2}}\left(\frac{9x^{3}}{2}-3x^{2}+\frac{11x}{2}-2\right)+\frac{15\ln\left(x^{2}+1\right)}{16}+\frac{7\arctan(x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2+1)/(-1+x)/x/(x^2+1)^3/(x^2+x+1),x)
```

```
[Out] -1/2*ln(x^2+x+1)-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/8*ln(-
1+x)-ln(x)+1/8*(9/2*x^3-3*x^2+11/2*x-2)/(x^2+1)^2+15/16*ln(x^2+1)
+7/16*arctan(x)
```

Maxima [A] time = 0.886691, size = 104, normalized size = 1.01

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{9x^{3}-6x^{2}+11x-4}{16\left(x^{4}+2x^{2}+1\right)}+\frac{7}{16}\arctan(x)-\frac{1}{2}\log\left(x^{2}+x+1\right)+\frac{15}{16}\log\left(x^{2}+1\right)+\frac{1}{8}\log(x-1)-\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3 + x^2 + 1)/((x^2 + x + 1)*(x^2 + 1)^3*(x - 1)*x),x, algorithm="maxi
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/16*(9*x^3 - 6*x^2
+ 11*x - 4)/(x^4 + 2*x^2 + 1) + 7/16*arctan(x) - 1/2*log(x^2 + x
+ 1) + 15/16*log(x^2 + 1) + 1/8*log(x - 1) - log(x)
```

Fricas [A] time = 0.26447, size = 211, normalized size = 2.05

$$\frac{\sqrt{3}\left(7\sqrt{3}\left(x^{4}+2x^{2}+1\right)\arctan(x)-8\sqrt{3}\left(x^{4}+2x^{2}+1\right)\log\left(x^{2}+x+1\right)+15\sqrt{3}\left(x^{4}+2x^{2}+1\right)\log\left(x^{2}+1\right)+2\sqrt{3}\left(x^{4}+2\right.}{48\left(x^{4}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3 + x^2 + 1)/((x^2 + x + 1)*(x^2 + 1)^3*(x - 1)*x),x, algorithm="fric
```

```
[Out] 1/48*sqrt(3)*(7*sqrt(3)*(x^4 + 2*x^2 + 1)*arctan(x) - 8*sqrt(3)*(
x^4 + 2*x^2 + 1)*log(x^2 + x + 1) + 15*sqrt(3)*(x^4 + 2*x^2 + 1)*
log(x^2 + 1) + 2*sqrt(3)*(x^4 + 2*x^2 + 1)*log(x - 1) - 16*sqrt(3
)*(x^4 + 2*x^2 + 1)*log(x) - 16*(x^4 + 2*x^2 + 1)*arctan(1/3*sqrt
(3)*(2*x + 1)) + sqrt(3)*(9*x^3 - 6*x^2 + 11*x - 4))/(x^4 + 2*x^2
 + 1)
```

Sympy [A] time = 1.66314, size = 88, normalized size = 0.85

$$-\log(x)+\frac{\log(x-1)}{8}+\frac{15\log(x^2+1)}{16}-\frac{\log(x^2+x+1)}{2}+\frac{7\operatorname{atan}(x)}{16}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}+\frac{9x^3-6x^2+11x-4}{16x^4+32x^2+16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3+x**2+1)/(-1+x)/x/(x**2+1)**3/(x**2+x+1),x)
```

```
[Out] -log(x) + log(x - 1)/8 + 15*log(x**2 + 1)/16 - log(x**2 + x + 1)/
2 + 7*atan(x)/16 - sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/3 + (9
*x**3 - 6*x**2 + 11*x - 4)/(16*x**4 + 32*x**2 + 16)
```

GIAC/XCAS [A] time = 0.263779, size = 100, normalized size = 0.97

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{9x^3-6x^2+11x-4}{16(x^2+1)^2}+\frac{7}{16}\arctan(x)-\frac{1}{2}\ln\left(x^2+x+1\right)+\frac{15}{16}\ln\left(x^2+1\right)+\frac{1}{8}\ln(|x-1|)-\ln(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + x^2 + 1)/((x^2 + x + 1)*(x^2 + 1)^3*(x - 1)*x),x, algorithm="giac
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/16*(9*x^3 - 6*x^2
+ 11*x - 4)/(x^2 + 1)^2 + 7/16*arctan(x) - 1/2*ln(x^2 + x + 1) +
15/16*ln(x^2 + 1) + 1/8*ln(abs(x - 1)) - ln(abs(x))
```

3.309 $\int \frac{1-3x+2x^2-x^3}{(1+x^2)^2} \, dx$

Optimal. Leaf size=33

$$\frac{2-x}{2(x^2+1)} - \frac{1}{2}\log(x^2+1) + \frac{3}{2}\tan^{-1}(x)$$

```
[Out] (2 - x)/(2*(1 + x^2)) + (3*ArcTan[x])/2 - Log[1 + x^2]/2
```

Rubi [A] time = 0.0369622, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$

$$\frac{2-x}{2(x^2+1)} - \frac{1}{2}\log(x^2+1) + \frac{3}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - 3*x + 2*x^2 - x^3)/(1 + x^2)^2,x]

[Out] (2 - x)/(2*(1 + x^2)) + (3*ArcTan[x])/2 - Log[1 + x^2]/2
```

Rubi in Sympy [A] time = 11.7374, size = 27, normalized size = 0.82

$$-\frac{x(2x+1)}{2(x^2+1)} - \frac{\log(x^2+1)}{2} + \frac{3\,\mathrm{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**3+2*x**2-3*x+1)/(x**2+1)**2,x)

[Out] -x*(2*x + 1)/(2*(x**2 + 1)) - log(x**2 + 1)/2 + 3*atan(x)/2
```

Mathematica [A] time = 0.0178586, size = 30, normalized size = 0.91

$$\frac{1}{2}\left(\frac{2-x}{x^2+1} - \log(x^2+1) + 3\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - 3*x + 2*x^2 - x^3)/(1 + x^2)^2,x]

[Out] ((2 - x)/(1 + x^2) + 3*ArcTan[x] - Log[1 + x^2])/2
```

Maple [A] time = 0.007, size = 28, normalized size = 0.9

$$-\frac{1}{x^2+1}\left(\frac{x}{2}-1\right) - \frac{\ln(x^2+1)}{2} + \frac{3\arctan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^3+2*x^2-3*x+1)/(x^2+1)^2,x)
```

```
[Out] -(1/2*x-1)/(x^2+1)-1/2*ln(x^2+1)+3/2*arctan(x)
```

Maxima [A] time = 0.883924, size = 34, normalized size = 1.03

$$-\frac{x-2}{2\left(x^2+1\right)}+\frac{3}{2}\arctan\left(x\right)-\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x^3 - 2*x^2 + 3*x - 1)/(x^2 + 1)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(x - 2)/(x^2 + 1) + 3/2*arctan(x) - 1/2*log(x^2 + 1)
```

Fricas [A] time = 0.255712, size = 49, normalized size = 1.48

$$\frac{3\left(x^2+1\right)\arctan\left(x\right)-\left(x^2+1\right)\log\left(x^2+1\right)-x+2}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x^3 - 2*x^2 + 3*x - 1)/(x^2 + 1)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(3*(x^2 + 1)*arctan(x) - (x^2 + 1)*log(x^2 + 1) - x + 2)/(x^2
 + 1)
```

Sympy [A] time = 0.280811, size = 24, normalized size = 0.73

$$-\frac{x-2}{2x^2+2}-\frac{\log\left(x^2+1\right)}{2}+\frac{3\operatorname{atan}\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**3+2*x**2-3*x+1)/(x**2+1)**2,x)
```

```
[Out] -(x - 2)/(2*x**2 + 2) - log(x**2 + 1)/2 + 3*atan(x)/2
```

GIAC/XCAS [A] time = 0.26147, size = 34, normalized size = 1.03

$$-\frac{x-2}{2\left(x^2+1\right)}+\frac{3}{2}\arctan\left(x\right)-\frac{1}{2}\ln\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^3 - 2*x^2 + 3*x - 1)/(x^2 + 1)^2,x, algorithm="giac")
```

```
[Out] -1/2*(x - 2)/(x^2 + 1) + 3/2*arctan(x) - 1/2*ln(x^2 + 1)
```

3.310 $\int \frac{1-3x+2x^2-x^3}{x(1+x^2)^2} dx$

Optimal. Leaf size=33

$$-\frac{2x+1}{2(x^2+1)} - \frac{1}{2}\log\left(x^2+1\right) + \log(x) - 2\tan^{-1}(x)$$

```
[Out] -(1 + 2*x)/(2*(1 + x^2)) - 2*ArcTan[x] + Log[x] - Log[1 + x^2]/2
```

Rubi [A] time = 0.0833581, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192

$$-\frac{2x+1}{2(x^2+1)} - \frac{1}{2}\log\left(x^2+1\right) + \log(x) - 2\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - 3*x + 2*x^2 - x^3)/(x*(1 + x^2)^2),x]
```

```
[Out] -(1 + 2*x)/(2*(1 + x^2)) - 2*ArcTan[x] + Log[x] - Log[1 + x^2]/2
```

Rubi in Sympy [A] time = 16.7973, size = 29, normalized size = 0.88

$$-\frac{x\left(2+\frac{1}{x}\right)}{2(x^2+1)} + 2\log(x) - \log\left(x^2+1\right) - 2\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**3+2*x**2-3*x+1)/x/(x**2+1)**2,x)
```

```
[Out] -x*(2 + 1/x)/(2*(x**2 + 1)) + 2*log(x) - log(x**2 + 1) - 2*atan(x
)
```

Mathematica [A] time = 0.0374092, size = 33, normalized size = 1.

$$\frac{-2x-1}{2(x^2+1)} - \frac{1}{2}\log\left(x^2+1\right) + \log(x) - 2\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - 3*x + 2*x^2 - x^3)/(x*(1 + x^2)^2),x]
```

```
[Out] (-1 - 2*x)/(2*(1 + x^2)) - 2*ArcTan[x] + Log[x] - Log[1 + x^2]/2
```

Maple [A] time = 0.011, size = 28, normalized size = 0.9

$$\ln(x) - \frac{1}{x^2+1}\left(x+\frac{1}{2}\right) - \frac{\ln\left(x^2+1\right)}{2} - 2\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^3+2*x^2-3*x+1)/x/(x^2+1)^2,x)
```

```
[Out] ln(x)-(x+1/2)/(x^2+1)-1/2*ln(x^2+1)-2*arctan(x)
```

Maxima [A] time = 0.884581, size = 39, normalized size = 1.18

$$-\frac{2x+1}{2(x^2+1)} - 2\arctan(x) - \frac{1}{2}\log\left(x^2+1\right) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x^3 - 2*x^2 + 3*x - 1)/((x^2 + 1)^2*x),x, algorithm="maxima")
```

```
[Out] -1/2*(2*x + 1)/(x^2 + 1) - 2*arctan(x) - 1/2*log(x^2 + 1) + log(x
)
```

Fricas [A] time = 0.253904, size = 59, normalized size = 1.79

$$-\frac{4\left(x^2+1\right)\arctan(x) + \left(x^2+1\right)\log\left(x^2+1\right) - 2\left(x^2+1\right)\log(x) + 2x + 1}{2(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x^3 - 2*x^2 + 3*x - 1)/((x^2 + 1)^2*x),x, algorithm="fricas")
```

```
[Out] -1/2*(4*(x^2 + 1)*arctan(x) + (x^2 + 1)*log(x^2 + 1) - 2*(x^2 + 1
)*log(x) + 2*x + 1)/(x^2 + 1)
```

Sympy [A] time = 0.359238, size = 27, normalized size = 0.82

$$-\frac{2x+1}{2x^2+2} + \log(x) - \frac{\log\left(x^2+1\right)}{2} - 2\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**3+2*x**2-3*x+1)/x/(x**2+1)**2,x)
```

```
[Out] -(2*x + 1)/(2*x**2 + 2) + log(x) - log(x**2 + 1)/2 - 2*atan(x)
```

GIAC/XCAS [A] time = 0.260669, size = 41, normalized size = 1.24

$$-\frac{2x+1}{2(x^2+1)} - 2\arctan(x) - \frac{1}{2}\ln\left(x^2+1\right) + \ln(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x^3 - 2*x^2 + 3*x - 1)/((x^2 + 1)^2*x),x, algorithm="giac")
```

```
[Out] -1/2*(2*x + 1)/(x^2 + 1) - 2*arctan(x) - 1/2*ln(x^2 + 1) + ln(abs
(x))
```

3.311 $\int \frac{1-x-x^2+x^3+x^4}{-x+x^3} dx$

Optimal. Leaf size=25

$$\frac{x^2}{2} + \frac{1}{2}\log\left(1-x^2\right) + x - \log(x)$$

```
[Out] x + x^2/2 - Log[x] + Log[1 - x^2]/2
```

Rubi [A] time = 0.0540746, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{x^2}{2} + \frac{1}{2}\log\left(1-x^2\right) + x - \log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - x - x^2 + x^3 + x^4)/(-x + x^3),x]

[Out] x + x^2/2 - Log[x] + Log[1 - x^2]/2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$x - \log(x) + \frac{\log\left(-x^2+1\right)}{2} + \int x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**4+x**3-x**2-x+1)/(x**3-x),x)

[Out] x - log(x) + log(-x**2 + 1)/2 + Integral(x, x)
```

Mathematica [A] time = 0.00918447, size = 25, normalized size = 1.

$$\frac{x^2}{2} + \frac{1}{2}\log\left(1-x^2\right) + x - \log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - x - x^2 + x^3 + x^4)/(-x + x^3),x]

[Out] x + x^2/2 - Log[x] + Log[1 - x^2]/2
```

Maple [A] time = 0.01, size = 24, normalized size = 1.

$$x + \frac{x^2}{2} + \frac{\ln(-1+x)}{2} + \frac{\ln(1+x)}{2} - \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^4+x^3-x^2-x+1)/(x^3-x),x)
```

```
[Out] x+1/2*x^2+1/2*ln(-1+x)+1/2*ln(1+x)-ln(x)
```

Maxima [A] time = 0.800667, size = 31, normalized size = 1.24

$$\frac{1}{2}x^2 + x + \frac{1}{2}\log(x+1) + \frac{1}{2}\log(x-1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + x^3 - x^2 - x + 1)/(x^3 - x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + x + 1/2*log(x + 1) + 1/2*log(x - 1) - log(x)
```

Fricas [A] time = 0.252461, size = 26, normalized size = 1.04

$$\frac{1}{2}x^2 + x + \frac{1}{2}\log\left(x^2-1\right) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + x^3 - x^2 - x + 1)/(x^3 - x),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 + x + 1/2*log(x^2 - 1) - log(x)
```

Sympy [A] time = 0.177694, size = 17, normalized size = 0.68

$$\frac{x^2}{2} + x - \log(x) + \frac{\log\left(x^2-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**4+x**3-x**2-x+1)/(x**3-x),x)
```

```
[Out] x**2/2 + x - log(x) + log(x**2 - 1)/2
```

GIAC/XCAS [A] time = 0.260299, size = 35, normalized size = 1.4

$$\frac{1}{2}x^2 + x + \frac{1}{2}\ln(|x+1|) + \frac{1}{2}\ln(|x-1|) - \ln(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + x^3 - x^2 - x + 1)/(x^3 - x),x, algorithm="giac")
```

```
[Out] 1/2*x^2 + x + 1/2*ln(abs(x + 1)) + 1/2*ln(abs(x - 1)) - ln(abs(x)
)
```

3.312 $\int \frac{2-4x^2+x^3}{(1+x^2)(2+x^2)}\,dx$

Optimal. Leaf size=36

$$-\frac{1}{2}\log\left(x^2+1\right)+\log\left(x^2+2\right)+6\tan^{-1}(x)-5\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

```
[Out] 6*ArcTan[x] - 5*Sqrt[2]*ArcTan[x/Sqrt[2]] - Log[1 + x^2]/2 + Log[
2 + x^2]
```

Rubi [A] time = 0.217059, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$

$$-\frac{1}{2}\log\left(x^2+1\right)+\log\left(x^2+2\right)+6\tan^{-1}(x)-5\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(2 - 4*x^2 + x^3)/((1 + x^2)*(2 + x^2)),x]
```

```
[Out] 6*ArcTan[x] - 5*Sqrt[2]*ArcTan[x/Sqrt[2]] - Log[1 + x^2]/2 + Log[
2 + x^2]
```

Rubi in Sympy [A] time = 35.1153, size = 36, normalized size = 1.

$$-\frac{\log\left(x^2+1\right)}{2}+\log\left(x^2+2\right)+6\,\mathrm{atan}\,(x)-5\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**3-4*x**2+2)/(x**2+1)/(x**2+2),x)
```

```
[Out] -log(x**2 + 1)/2 + log(x**2 + 2) + 6*atan(x) - 5*sqrt(2)*atan(sqr
t(2)*x/2)
```

Mathematica [A] time = 0.0259529, size = 36, normalized size = 1.

$$-\frac{1}{2}\log\left(x^2+1\right)+\log\left(x^2+2\right)+6\tan^{-1}(x)-5\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 - 4*x^2 + x^3)/((1 + x^2)*(2 + x^2)),x]
```

```
[Out] 6*ArcTan[x] - 5*Sqrt[2]*ArcTan[x/Sqrt[2]] - Log[1 + x^2]/2 + Log[
2 + x^2]
```

Maple [A] time = 0.005, size = 32, normalized size = 0.9

$$6\ \arctan(x)-\frac{\ln\left(x^2+1\right)}{2}+\ln\left(x^2+2\right)-5\ \arctan\left(1/2\,\sqrt{2}x\right)\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^3-4*x^2+2)/(x^2+1)/(x^2+2),x)
```

```
[Out] 6*arctan(x)-1/2*ln(x^2+1)+ln(x^2+2)-5*arctan(1/2*2^(1/2)*x)*2^(1/
2)
```

Maxima [A] time = 0.896717, size = 42, normalized size = 1.17

$$-5\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+6\arctan\left(x\right)+\log\left(x^2+2\right)-\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3 - 4*x^2 + 2)/((x^2 + 2)*(x^2 + 1)),x, algorithm="maxima")
```

```
[Out] -5*sqrt(2)*arctan(1/2*sqrt(2)*x) + 6*arctan(x) + log(x^2 + 2) - 1
/2*log(x^2 + 1)
```

Fricas [A] time = 0.253366, size = 42, normalized size = 1.17

$$-5\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+6\arctan\left(x\right)+\log\left(x^2+2\right)-\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3 - 4*x^2 + 2)/((x^2 + 2)*(x^2 + 1)),x, algorithm="fricas")
```

```
[Out] -5*sqrt(2)*arctan(1/2*sqrt(2)*x) + 6*arctan(x) + log(x^2 + 2) - 1
/2*log(x^2 + 1)
```

Sympy [A] time = 0.547452, size = 36, normalized size = 1.

$$-\frac{\log\left(x^2+1\right)}{2}+\log\left(x^2+2\right)+6\operatorname{atan}\left(x\right)-5\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3-4*x**2+2)/(x**2+1)/(x**2+2),x)
```

```
[Out] -log(x**2 + 1)/2 + log(x**2 + 2) + 6*atan(x) - 5*sqrt(2)*atan(sqr
t(2)*x/2)
```

GIAC/XCAS [A] time = 0.264769, size = 42, normalized size = 1.17

$$-5\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+6\arctan\left(x\right)+\ln\left(x^2+2\right)-\frac{1}{2}\ln\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3 - 4*x^2 + 2)/((x^2 + 2)*(x^2 + 1)),x, algorithm="giac")
```

```
[Out] -5*sqrt(2)*arctan(1/2*sqrt(2)*x) + 6*arctan(x) + ln(x^2 + 2) - 1/
2*ln(x^2 + 1)
```

3.313 $\int \frac{1+x^2+x^4}{(1+x^2)(4+x^2)^2} \, dx$

Optimal. Leaf size=29

$$-\frac{13x}{24\left(x^2+4\right)}+\frac{25}{144}\tan^{-1}\left(\frac{x}{2}\right)+\frac{1}{9}\tan^{-1}(x)$$

```
[Out] (-13*x)/(24*(4 + x^2)) + (25*ArcTan[x/2])/144 + ArcTan[x]/9
```

Rubi [A] time = 0.193392, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$

$$-\frac{13x}{24\left(x^2+4\right)}+\frac{25}{144}\tan^{-1}\left(\frac{x}{2}\right)+\frac{1}{9}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^2 + x^4)/((1 + x^2)*(4 + x^2)^2),x]

[Out] (-13*x)/(24*(4 + x^2)) + (25*ArcTan[x/2])/144 + ArcTan[x]/9
```

Rubi in Sympy [A] time = 30.8389, size = 22, normalized size = 0.76

$$-\frac{13x}{24\left(x^2+4\right)}+\frac{25\,\mathrm{atan}\left(\frac{x}{2}\right)}{144}+\frac{\mathrm{atan}(x)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**4+x**2+1)/(x**2+1)/(x**2+4)**2,x)

[Out] -13*x/(24*(x**2 + 4)) + 25*atan(x/2)/144 + atan(x)/9
```

Mathematica [A] time = 0.0272968, size = 29, normalized size = 1.

$$-\frac{13x}{24\left(x^2+4\right)}+\frac{25}{144}\tan^{-1}\left(\frac{x}{2}\right)+\frac{1}{9}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^2 + x^4)/((1 + x^2)*(4 + x^2)^2),x]

[Out] (-13*x)/(24*(4 + x^2)) + (25*ArcTan[x/2])/144 + ArcTan[x]/9
```

Maple [A] time = 0.013, size = 22, normalized size = 0.8

$$-\frac{13\,x}{24\,x^2+96}+\frac{25}{144}\arctan\left(\frac{x}{2}\right)+\frac{\arctan(x)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^4+x^2+1)/(x^2+1)/(x^2+4)^2,x)
```

```
[Out] -13/24*x/(x^2+4)+25/144*arctan(1/2*x)+1/9*arctan(x)
```

Maxima [A] time = 0.883595, size = 28, normalized size = 0.97

$$-\frac{13\,x}{24\,(x^2+4)}+\frac{25}{144}\,\arctan\left(\frac{1}{2}\,x\right)+\frac{1}{9}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 + x^2 + 1)/((x^2 + 4)^2*(x^2 + 1)),x, algorithm="maxima")
```

```
[Out] -13/24*x/(x^2 + 4) + 25/144*arctan(1/2*x) + 1/9*arctan(x)
```

Fricas [A] time = 0.25219, size = 45, normalized size = 1.55

$$\frac{25\,\left(x^2+4\right)\,\arctan\left(\frac{1}{2}\,x\right)+16\,\left(x^2+4\right)\,\arctan\left(x\right)-78\,x}{144\,(x^2+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 + x^2 + 1)/((x^2 + 4)^2*(x^2 + 1)),x, algorithm="fricas")
```

```
[Out] 1/144*(25*(x^2 + 4)*arctan(1/2*x) + 16*(x^2 + 4)*arctan(x) - 78*x
)/(x^2 + 4)
```

Sympy [A] time = 0.510357, size = 22, normalized size = 0.76

$$-\frac{13x}{24x^2+96}+\frac{25\,\operatorname{atan}\left(\frac{x}{2}\right)}{144}+\frac{\operatorname{atan}\left(x\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**4+x**2+1)/(x**2+1)/(x**2+4)**2,x)
```

```
[Out] -13*x/(24*x**2 + 96) + 25*atan(x/2)/144 + atan(x)/9
```

GIAC/XCAS [A] time = 0.260678, size = 28, normalized size = 0.97

$$-\frac{13\,x}{24\,(x^2+4)}+\frac{25}{144}\,\arctan\left(\frac{1}{2}\,x\right)+\frac{1}{9}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 + x^2 + 1)/((x^2 + 4)^2*(x^2 + 1)),x, algorithm="giac")
```

```
[Out] -13/24*x/(x^2 + 4) + 25/144*arctan(1/2*x) + 1/9*arctan(x)
```

3.314 $\int \frac{1+x^2+x^3}{2x^2+x^3+x^4} \, dx$

Optimal. Leaf size=46

$$\frac{5}{8}\log\left(x^2+x+2\right)-\frac{1}{2x}-\frac{\log(x)}{4}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{7}}\right)}{4\sqrt{7}}$$

```
[Out] -1/(2*x) + ArcTan[(1 + 2*x)/Sqrt[7]]/(4*Sqrt[7]) - Log[x]/4 + (5*
Log[2 + x + x^2])/8
```

Rubi [A] time = 0.0999931, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261

$$\frac{5}{8}\log\left(x^2+x+2\right)-\frac{1}{2x}-\frac{\log(x)}{4}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{7}}\right)}{4\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^2 + x^3)/(2*x^2 + x^3 + x^4),x]
```

```
[Out] -1/(2*x) + ArcTan[(1 + 2*x)/Sqrt[7]]/(4*Sqrt[7]) - Log[x]/4 + (5*
Log[2 + x + x^2])/8
```

Rubi in Sympy [A] time = 21.5263, size = 42, normalized size = 0.91

$$-\frac{\log(x)}{4}+\frac{5\log\left(x^2+x+2\right)}{8}+\frac{\sqrt{7}\operatorname{atan}\left(\sqrt{7}\left(\frac{2x}{7}+\frac{1}{7}\right)\right)}{28}-\frac{1}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**3+x**2+1)/(x**4+x**3+2*x**2),x)
```

```
[Out] -log(x)/4 + 5*log(x**2 + x + 2)/8 + sqrt(7)*atan(sqrt(7)*(2*x/7 +
 1/7))/28 - 1/(2*x)
```

Mathematica [A] time = 0.0459745, size = 46, normalized size = 1.

$$\frac{5}{8}\log\left(x^2+x+2\right)-\frac{1}{2x}-\frac{\log(x)}{4}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{7}}\right)}{4\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^2 + x^3)/(2*x^2 + x^3 + x^4),x]
```

```
[Out] -1/(2*x) + ArcTan[(1 + 2*x)/Sqrt[7]]/(4*Sqrt[7]) - Log[x]/4 + (5*
Log[2 + x + x^2])/8
```

Maple [A] time = 0.01, size = 36, normalized size = 0.8

$$-\frac{1}{2\,x}-\frac{\ln(x)}{4}+\frac{5\,\ln\left(x^2+x+2\right)}{8}+\frac{\sqrt{7}}{28}\arctan\left(\frac{(1+2\,x)\sqrt{7}}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((x^3+x^2+1)/(x^4+x^3+2*x^2),x)
```

```
[Out] -1/2/x-1/4*ln(x)+5/8*ln(x^2+x+2)+1/28*arctan(1/7*(1+2*x)*7^(1/2))
*7^(1/2)
```

Maxima [A] time = 0.903441, size = 47, normalized size = 1.02

$$\frac{1}{28}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2x+1)\right)-\frac{1}{2x}+\frac{5}{8}\log\left(x^2+x+2\right)-\frac{1}{4}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + x^2 + 1)/(x^4 + x^3 + 2*x^2),x, algorithm="maxima")
```

```
[Out] 1/28*sqrt(7)*arctan(1/7*sqrt(7)*(2*x + 1)) - 1/2/x + 5/8*log(x^2
+ x + 2) - 1/4*log(x)
```

Fricas [A] time = 0.255271, size = 66, normalized size = 1.43

$$\frac{\sqrt{7}\left(5\sqrt{7}x\log\left(x^2+x+2\right)-2\sqrt{7}x\log\left(x\right)+2x\arctan\left(\frac{1}{7}\sqrt{7}(2x+1)\right)-4\sqrt{7}\right)}{56x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + x^2 + 1)/(x^4 + x^3 + 2*x^2),x, algorithm="fricas")
```

```
[Out] 1/56*sqrt(7)*(5*sqrt(7)*x*log(x^2 + x + 2) - 2*sqrt(7)*x*log(x) +
 2*x*arctan(1/7*sqrt(7)*(2*x + 1)) - 4*sqrt(7))/x
```

Sympy [A] time = 0.378752, size = 46, normalized size = 1.

$$-\frac{\log\left(x\right)}{4}+\frac{5\log\left(x^2+x+2\right)}{8}+\frac{\sqrt{7}\operatorname{atan}\left(\frac{2\sqrt{7}x}{7}+\frac{\sqrt{7}}{7}\right)}{28}-\frac{1}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((x**3+x**2+1)/(x**4+x**3+2*x**2),x)
```

```
[Out] -log(x)/4 + 5*log(x**2 + x + 2)/8 + sqrt(7)*atan(2*sqrt(7)*x/7 +
sqrt(7)/7)/28 - 1/(2*x)
```

GIAC/XCAS [A] time = 0.262228, size = 49, normalized size = 1.07

$$\frac{1}{28}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2x+1)\right)-\frac{1}{2x}+\frac{5}{8}\ln\left(x^2+x+2\right)-\frac{1}{4}\ln\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + x^2 + 1)/(x^4 + x^3 + 2*x^2),x, algorithm="giac")
```

```
[Out] 1/28*sqrt(7)*arctan(1/7*sqrt(7)*(2*x + 1)) - 1/2/x + 5/8*ln(x^2 +
 x + 2) - 1/4*ln(abs(x))
```

3.315 $\int \frac{1-12x+x^2+x^3}{-12+x+x^2} \, dx$

Optimal. Leaf size=22

$$\frac{x^2}{2}-\frac{2}{7}\tanh^{-1}\left(\frac{1}{7}(2x+1)\right)$$

```
[Out] x^2/2 - (2*ArcTanh[(1 + 2*x)/7])/7
```

Rubi [A] time = 0.0304985, antiderivative size = 26, normalized size of antiderivative = 1.18, number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15

$$\frac{x^2}{2}+\frac{1}{7}\log(3-x)-\frac{1}{7}\log(x+4)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 12*x + x^2 + x^3)/(-12 + x + x^2),x]

[Out] x^2/2 + Log[3 - x]/7 - Log[4 + x]/7
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\log(-x+3)}{7}-\frac{\log(x+4)}{7}+\int x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((x**3+x**2-12*x+1)/(x**2+x-12),x)

[Out] log(-x + 3)/7 - log(x + 4)/7 + Integral(x, x)
```

Mathematica [A] time = 0.00839475, size = 26, normalized size = 1.18

$$\frac{x^2}{2}+\frac{1}{7}\log(3-x)-\frac{1}{7}\log(x+4)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 12*x + x^2 + x^3)/(-12 + x + x^2),x]

[Out] x^2/2 + Log[3 - x]/7 - Log[4 + x]/7
```

Maple [A] time = 0.008, size = 19, normalized size = 0.9

$$\frac{x^2}{2}+\frac{\ln(-3+x)}{7}-\frac{\ln(4+x)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2-12*x+1)/(x^2+x-12),x)
```

```
[Out] 1/2*x^2+1/7*ln(-3+x)-1/7*ln(4+x)
```

Maxima [A] time = 0.79174, size = 24, normalized size = 1.09

$$\frac{1}{2}x^2 - \frac{1}{7}\log(x+4) + \frac{1}{7}\log(x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3 + x^2 - 12*x + 1)/(x^2 + x - 12),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 1/7*log(x + 4) + 1/7*log(x - 3)
```

Fricas [A] time = 0.251206, size = 24, normalized size = 1.09

$$\frac{1}{2}x^2 - \frac{1}{7}\log(x+4) + \frac{1}{7}\log(x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3 + x^2 - 12*x + 1)/(x^2 + x - 12),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 - 1/7*log(x + 4) + 1/7*log(x - 3)
```

Sympy [A] time = 0.172967, size = 17, normalized size = 0.77

$$\frac{x^2}{2} + \frac{\log(x-3)}{7} - \frac{\log(x+4)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3+x**2-12*x+1)/(x**2+x-12),x)
```

```
[Out] x**2/2 + log(x - 3)/7 - log(x + 4)/7
```

GIAC/XCAS [A] time = 0.260745, size = 27, normalized size = 1.23

$$\frac{1}{2}x^2 - \frac{1}{7}\ln(|x+4|) + \frac{1}{7}\ln(|x-3|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + x^2 - 12*x + 1)/(x^2 + x - 12),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 1/7*ln(abs(x + 4)) + 1/7*ln(abs(x - 3))
```

3.316 $\int \frac{-3+5x+6x^2}{-3x+2x^2+x^3}\,dx$

Optimal. Leaf size=17

$$2\log(1-x)+\log(x)+3\log(x+3)$$

```
[Out] 2*Log[1 - x] + Log[x] + 3*Log[3 + x]
```

Rubi [A] time = 0.0561465, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$

$$2\log(1-x)+\log(x)+3\log(x+3)$$

Antiderivative was successfully verified.

```
[In]  Int[(-3 + 5*x + 6*x^2)/(-3*x + 2*x^2 + x^3),x]

[Out] 2*Log[1 - x] + Log[x] + 3*Log[3 + x]
```

Rubi in Sympy [A] time = 13.5897, size = 15, normalized size = 0.88

$$\log(x)+2\log(-x+1)+3\log(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((6*x**2+5*x-3)/(x**3+2*x**2-3*x),x)

[Out] log(x) + 2*log(-x + 1) + 3*log(x + 3)
```

Mathematica [A] time = 0.00983628, size = 17, normalized size = 1.

$$2\log(1-x)+\log(x)+3\log(x+3)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-3 + 5*x + 6*x^2)/(-3*x + 2*x^2 + x^3),x]

[Out] 2*Log[1 - x] + Log[x] + 3*Log[3 + x]
```

Maple [A] time = 0.01, size = 16, normalized size = 0.9

$$2\ln(-1+x)+\ln(x)+3\ln(3+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((6*x^2+5*x-3)/(x^3+2*x^2-3*x),x)

[Out] 2*ln(-1+x)+ln(x)+3*ln(3+x)
```

Maxima [A] time = 0.79942, size = 20, normalized size = 1.18

$$3 \log(x + 3) + 2 \log(x - 1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6*x^2 + 5*x - 3)/(x^3 + 2*x^2 - 3*x),x, algorithm="maxima")
```

```
[Out] 3*log(x + 3) + 2*log(x - 1) + log(x)
```

Fricas [A] time = 0.252112, size = 20, normalized size = 1.18

$$3 \log(x + 3) + 2 \log(x - 1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6*x^2 + 5*x - 3)/(x^3 + 2*x^2 - 3*x),x, algorithm="fricas")
```

```
[Out] 3*log(x + 3) + 2*log(x - 1) + log(x)
```

Sympy [A] time = 0.286836, size = 15, normalized size = 0.88

$$\log(x) + 2\log(x - 1) + 3\log(x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((6*x**2+5*x-3)/(x**3+2*x**2-3*x),x)
```

```
[Out] log(x) + 2*log(x - 1) + 3*log(x + 3)
```

GIAC/XCAS [A] time = 0.258676, size = 24, normalized size = 1.41

$$3\ln(|x + 3|) + 2\ln(|x - 1|) + \ln(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((6*x^2 + 5*x - 3)/(x^3 + 2*x^2 - 3*x),x, algorithm="giac")
```

```
[Out] 3*ln(abs(x + 3)) + 2*ln(abs(x - 1)) + ln(abs(x))
```

3.317 $\int \frac{-2+3x+5x^2}{2x^2+x^3}\,dx$

Optimal. Leaf size=14

$$\frac{1}{x}+2\log(x)+3\log(x+2)$$

```
[Out] x^(-1) + 2*Log[x] + 3*Log[2 + x]
```

Rubi [A] time = 0.0478355, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{1}{x}+2\log(x)+3\log(x+2)$$

Antiderivative was successfully verified.

```
[In]  Int[(-2 + 3*x + 5*x^2)/(2*x^2 + x^3),x]
```

```
[Out] x^(-1) + 2*Log[x] + 3*Log[2 + x]
```

Rubi in Sympy [A] time = 7.06518, size = 14, normalized size = 1.

$$2\log(x)+3\log(x+2)+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**2+3*x-2)/(x**3+2*x**2),x)
```

```
[Out] 2*log(x) + 3*log(x + 2) + 1/x
```

Mathematica [A] time = 0.00591745, size = 14, normalized size = 1.

$$\frac{1}{x}+2\log(x)+3\log(x+2)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-2 + 3*x + 5*x^2)/(2*x^2 + x^3),x]
```

```
[Out] x^(-1) + 2*Log[x] + 3*Log[2 + x]
```

Maple [A] time = 0.01, size = 15, normalized size = 1.1

$$x^{-1}+2\ln(x)+3\ln(2+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5*x^2+3*x-2)/(x^3+2*x^2),x)
```

```
[Out] 1/x+2*ln(x)+3*ln(2+x)
```

Maxima [A] time = 0.792024, size = 19, normalized size = 1.36

$$\frac{1}{x} + 3\,\log\left(x + 2\right) + 2\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^2 + 3*x - 2)/(x^3 + 2*x^2),x, algorithm="maxima")
```

```
[Out] 1/x + 3*log(x + 2) + 2*log(x)
```

Fricas [A] time = 0.24723, size = 24, normalized size = 1.71

$$\frac{3\,x \log\left(x + 2\right) + 2\,x \log\left(x\right) + 1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^2 + 3*x - 2)/(x^3 + 2*x^2),x, algorithm="fricas")
```

```
[Out] (3*x*log(x + 2) + 2*x*log(x) + 1)/x
```

Sympy [A] time = 0.236777, size = 14, normalized size = 1.

$$2 \log\left(x\right) + 3 \log\left(x + 2\right) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x**2+3*x-2)/(x**3+2*x**2),x)
```

```
[Out] 2*log(x) + 3*log(x + 2) + 1/x
```

GIAC/XCAS [A] time = 0.259838, size = 22, normalized size = 1.57

$$\frac{1}{x} + 3 \ln\left(|x + 2|\right) + 2 \ln\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^2 + 3*x - 2)/(x^3 + 2*x^2),x, algorithm="giac")
```

```
[Out] 1/x + 3*ln(abs(x + 2)) + 2*ln(abs(x))
```

3.318 $\int \frac{18-2x-4x^2}{-6+x+4x^2+x^3}\,dx$

Optimal. Leaf size=19

$$\log(1-x)-2\log(x+2)-3\log(x+3)$$

```
[Out] Log[1 - x] - 2*Log[2 + x] - 3*Log[3 + x]
```

Rubi [A] time = 0.054352, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042

$$\log(1-x)-2\log(x+2)-3\log(x+3)$$

Antiderivative was successfully verified.

```
[In]  Int[(18 - 2*x - 4*x^2)/(-6 + x + 4*x^2 + x^3),x]

[Out] Log[1 - x] - 2*Log[2 + x] - 3*Log[3 + x]
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-4*x**2-2*x+18)/(x**3+4*x**2+x-6),x)

[Out] Timed out
```

Mathematica [A] time = 0.0113373, size = 25, normalized size = 1.32

$$-2\left(-\frac{1}{2}\log(1-x)+\log(x+2)+\frac{3}{2}\log(x+3)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(18 - 2*x - 4*x^2)/(-6 + x + 4*x^2 + x^3),x]

[Out] -2*(-Log[1 - x]/2 + Log[2 + x] + (3*Log[3 + x])/2)
```

Maple [A] time = 0.011, size = 18, normalized size = 1.

$$-2\ln(2+x)+\ln(-1+x)-3\ln(3+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-4*x^2-2*x+18)/(x^3+4*x^2+x-6),x)

[Out] -2*ln(2+x)+ln(-1+x)-3*ln(3+x)
```

Maxima [A] time = 0.78928, size = 23, normalized size = 1.21

$$-3\log\left(x+3\right)-2\log\left(x+2\right)+\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2*(2*x^2 + x - 9)/(x^3 + 4*x^2 + x - 6),x, algorithm="maxima")
```

```
[Out] -3*log(x + 3) - 2*log(x + 2) + log(x - 1)
```

Fricas [A] time = 0.254814, size = 23, normalized size = 1.21

$$-3\log\left(x+3\right)-2\log\left(x+2\right)+\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2*(2*x^2 + x - 9)/(x^3 + 4*x^2 + x - 6),x, algorithm="fricas")
```

```
[Out] -3*log(x + 3) - 2*log(x + 2) + log(x - 1)
```

Sympy [A] time = 0.273726, size = 17, normalized size = 0.89

$$\log\left(x-1\right)-2\log\left(x+2\right)-3\log\left(x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-4*x**2-2*x+18)/(x**3+4*x**2+x-6),x)
```

```
[Out] log(x - 1) - 2*log(x + 2) - 3*log(x + 3)
```

GIAC/XCAS [A] time = 0.261884, size = 27, normalized size = 1.42

$$-3\ln\left(|x+3|\right)-2\ln\left(|x+2|\right)+\ln\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2*(2*x^2 + x - 9)/(x^3 + 4*x^2 + x - 6),x, algorithm="giac")
```

```
[Out] -3*ln(abs(x + 3)) - 2*ln(abs(x + 2)) + ln(abs(x - 1))
```

3.319 $\int \frac{1+x-2x^2+x^3}{4+5x^2+x^4} \, dx$

Optimal. Leaf size=23

$$\frac{1}{2}\log\left(x^2+4\right)-\frac{3}{2}\tan^{-1}\left(\frac{x}{2}\right)+\tan^{-1}(x)$$

```
[Out] (-3*ArcTan[x/2])/2 + ArcTan[x] + Log[4 + x^2]/2
```

Rubi [A] time = 0.0555356, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$\frac{1}{2}\log\left(x^2+4\right)-\frac{3}{2}\tan^{-1}\left(\frac{x}{2}\right)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x - 2*x^2 + x^3)/(4 + 5*x^2 + x^4),x]

[Out] (-3*ArcTan[x/2])/2 + ArcTan[x] + Log[4 + x^2]/2
```

Rubi in Sympy [A] time = 22.5066, size = 19, normalized size = 0.83

$$\frac{\log\left(x^2+4\right)}{2}-\frac{3\operatorname{atan}\left(\frac{x}{2}\right)}{2}+\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((x**3-2*x**2+x+1)/(x**4+5*x**2+4),x)

[Out] log(x**2 + 4)/2 - 3*atan(x/2)/2 + atan(x)
```

Mathematica [A] time = 0.0143496, size = 23, normalized size = 1.

$$\frac{1}{2}\log\left(x^2+4\right)-\frac{3}{2}\tan^{-1}\left(\frac{x}{2}\right)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x - 2*x^2 + x^3)/(4 + 5*x^2 + x^4),x]

[Out] (-3*ArcTan[x/2])/2 + ArcTan[x] + Log[4 + x^2]/2
```

Maple [A] time = 0.006, size = 18, normalized size = 0.8

$$-\frac{3}{2}\arctan\left(\frac{x}{2}\right)+\arctan(x)+\frac{\ln\left(x^2+4\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-2*x^2+x+1)/(x^4+5*x^2+4),x)
```

```
[Out] -3/2*arctan(1/2*x)+arctan(x)+1/2*ln(x^2+4)
```

Maxima [A] time = 0.892481, size = 23, normalized size = 1.

$$-\frac{3}{2}\arctan\left(\frac{1}{2}x\right)+\arctan\left(x\right)+\frac{1}{2}\log\left(x^2+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3 - 2*x^2 + x + 1)/(x^4 + 5*x^2 + 4),x, algorithm="maxima")
```

```
[Out] -3/2*arctan(1/2*x) + arctan(x) + 1/2*log(x^2 + 4)
```

Fricas [A] time = 0.255242, size = 23, normalized size = 1.

$$-\frac{3}{2}\arctan\left(\frac{1}{2}x\right)+\arctan\left(x\right)+\frac{1}{2}\log\left(x^2+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3 - 2*x^2 + x + 1)/(x^4 + 5*x^2 + 4),x, algorithm="fricas")
```

```
[Out] -3/2*arctan(1/2*x) + arctan(x) + 1/2*log(x^2 + 4)
```

Sympy [A] time = 0.493768, size = 19, normalized size = 0.83

$$\frac{\log\left(x^2+4\right)}{2}-\frac{3\operatorname{atan}\left(\frac{x}{2}\right)}{2}+\operatorname{atan}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3-2*x**2+x+1)/(x**4+5*x**2+4),x)
```

```
[Out] log(x**2 + 4)/2 - 3*atan(x/2)/2 + atan(x)
```

GIAC/XCAS [A] time = 0.261755, size = 23, normalized size = 1.

$$-\frac{3}{2}\arctan\left(\frac{1}{2}x\right)+\arctan\left(x\right)+\frac{1}{2}\ln\left(x^2+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3 - 2*x^2 + x + 1)/(x^4 + 5*x^2 + 4),x, algorithm="giac")
```

```
[Out] -3/2*arctan(1/2*x) + arctan(x) + 1/2*ln(x^2 + 4)
```

3.320 $\int \frac{-32+5x-27x^2+4x^3}{-70-299x-286x^2+50x^3-13x^4+30x^5}\,dx$

Optimal. Leaf size=63

$$\frac{11049\log\left(x^2+x+5\right)}{260015}-\frac{3146\log(7-3x)}{80155}-\frac{334}{323}\log(2x+1)+\frac{4822\log(5x+2)}{4879}+\frac{3988\tan^{-1}\left(\frac{2x+1}{\sqrt{19}}\right)}{13685\sqrt{19}}$$

```
[Out] (3988*ArcTan[(1 + 2*x)/Sqrt[19]])/(13685*Sqrt[19]) - (3146*Log[7
- 3*x])/80155 - (334*Log[1 + 2*x])/323 + (4822*Log[2 + 5*x])/4879
 + (11049*Log[5 + x + x^2])/260015
```

Rubi [A] time = 0.166806, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116

$$\frac{11049\log\left(x^2+x+5\right)}{260015}-\frac{3146\log(7-3x)}{80155}-\frac{334}{323}\log(2x+1)+\frac{4822\log(5x+2)}{4879}+\frac{3988\tan^{-1}\left(\frac{2x+1}{\sqrt{19}}\right)}{13685\sqrt{19}}$$

Antiderivative was successfully verified.

```
[In] Int[(-32 + 5*x - 27*x^2 + 4*x^3)/(-70 - 299*x - 286*x^2 + 50*x^3 - 13*x^4 + 30*x^5)
```

```
[Out] (3988*ArcTan[(1 + 2*x)/Sqrt[19]])/(13685*Sqrt[19]) - (3146*Log[7
- 3*x])/80155 - (334*Log[1 + 2*x])/323 + (4822*Log[2 + 5*x])/4879
 + (11049*Log[5 + x + x^2])/260015
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((4*x**3-27*x**2+5*x-32)/(30*x**5-13*x**4+50*x**3-
286*x**2-299*x-70),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.0407911, size = 57, normalized size = 0.9

$$\frac{453009\log\left(x^2+x+5\right)-418418\log(7-3x)-11023670\log(2x+1)+10536070\log(5x+2)+163508\sqrt{19}\tan^{-1}\left(\frac{2x+1}{\sqrt{19}}\right)}{10660615}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-32 + 5*x - 27*x^2 + 4*x^3)/(-70 - 299*x - 286*x^2 + 50*x^3 - 13*x^4 + 3
```

```
[Out] (163508*Sqrt[19]*ArcTan[(1 + 2*x)/Sqrt[19]] - 418418*Log[7 - 3*x]
 - 11023670*Log[1 + 2*x] + 10536070*Log[2 + 5*x] + 453009*Log[5 +
 x + x^2])/10660615
```

Maple [A] time = 0.016, size = 51, normalized size = 0.8

$$\frac{4822\ \ln(2+5x)}{4879}-\frac{334\ \ln(1+2x)}{323}+\frac{11049\ \ln(x^2+x+5)}{260015}+\frac{3988\sqrt{19}}{260015}\arctan\left(\frac{(1+2x)\sqrt{19}}{19}\right)-\frac{3146\ \ln(3x-7)}{80155}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^3-27*x^2+5*x-32)/(30*x^5-13*x^4+50*x^3-286*x^2-299*x-70),x)
```

```
[Out] 4822/4879*ln(2+5*x)-334/323*ln(1+2*x)+11049/260015*ln(x^2+x+5)+39
88/260015*arctan(1/19*(1+2*x)*19^(1/2))*19^(1/2)-3146/80155*ln(3*
x-7)
```

Maxima [A] time = 0.893196, size = 68, normalized size = 1.08

$$\frac{3988}{260015}\sqrt{19}\arctan\left(\frac{1}{19}\sqrt{19}(2x+1)\right)+\frac{11049}{260015}\log\left(x^2+x+5\right)+\frac{4822}{4879}\log(5x+2)-\frac{3146}{80155}\log(3x-7)-\frac{334}{323}\log(2x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3 - 27*x^2 + 5*x - 32)/(30*x^5 - 13*x^4 + 50*x^3 - 286*x^2 - 299*x -
```

```
[Out] 3988/260015*sqrt(19)*arctan(1/19*sqrt(19)*(2*x + 1)) + 11049/2600
15*log(x^2 + x + 5) + 4822/4879*log(5*x + 2) - 3146/80155*log(3*x
 - 7) - 334/323*log(2*x + 1)
```

Fricas [A] time = 0.259556, size = 86, normalized size = 1.37

$$\frac{1}{202551685}\sqrt{19}\Big(453009\sqrt{19}\log\left(x^2+x+5\right)+10536070\sqrt{19}\log(5x+2)-418418\sqrt{19}\log(3x-7)-11023670\sqrt{19}\log($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3 - 27*x^2 + 5*x - 32)/(30*x^5 - 13*x^4 + 50*x^3 - 286*x^2 - 299*x -
```

```
[Out] 1/202551685*sqrt(19)*(453009*sqrt(19)*log(x^2 + x + 5) + 10536070
*sqrt(19)*log(5*x + 2) - 418418*sqrt(19)*log(3*x - 7) - 11023670*
sqrt(19)*log(2*x + 1) + 3106652*arctan(1/19*sqrt(19)*(2*x + 1)))
```

Sympy [A] time = 1.08499, size = 68, normalized size = 1.08

$$-\frac{3146\log\left(x-\frac{7}{3}\right)}{80155}+\frac{4822\log\left(x+\frac{2}{5}\right)}{4879}-\frac{334\log\left(x+\frac{1}{2}\right)}{323}+\frac{11049\log\left(x^2+x+5\right)}{260015}+\frac{3988\sqrt{19}\operatorname{atan}\left(\frac{2\sqrt{19}x}{19}+\frac{\sqrt{19}}{19}\right)}{260015}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**3-27*x**2+5*x-32)/(30*x**5-13*x**4+50*x**3-286*x**2-
299*x-70),x)
```

```
[Out] -3146*log(x - 7/3)/80155 + 4822*log(x + 2/5)/4879 - 334*log(x + 1
/2)/323 + 11049*log(x**2 + x + 5)/260015 + 3988*sqrt(19)*atan(2*s
qrt(19)*x/19 + sqrt(19)/19)/260015
```

GIAC/XCAS [A] time = 0.262111, size = 72, normalized size = 1.14

$$\frac{3988}{260015}\sqrt{19}\arctan\left(\frac{1}{19}\sqrt{19}(2x+1)\right)+\frac{11049}{260015}\ln\left(x^2+x+5\right)+\frac{4822}{4879}\ln\left(|5x+2|\right)-\frac{3146}{80155}\ln\left(|3x-7|\right)-\frac{334}{323}\ln\left(|2x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3 - 27*x^2 + 5*x - 32)/(30*x^5 - 13*x^4 + 50*x^3 - 286*x^2 - 299*x -
```

```
[Out] 3988/260015*sqrt(19)*arctan(1/19*sqrt(19)*(2*x + 1)) + 11049/2600
15*ln(x^2 + x + 5) + 4822/4879*ln(abs(5*x + 2)) - 3146/80155*ln(a
bs(3*x - 7)) - 334/323*ln(abs(2*x + 1))
```

3.321 $\int \frac{8-13x^2-7x^3+12x^5}{4-20x+41x^2-80x^3+116x^4-80x^5+100x^6}\,dx$

Optimal. Leaf size=69

$$-\frac{502x+313}{1452\left(2x^2+1\right)}+\frac{2843\log\left(2x^2+1\right)}{7986}+\frac{5828}{9075(2-5x)}-\frac{59096\log(2-5x)}{99825}+\frac{503\tan^{-1}\left(\sqrt{2}x\right)}{7986\sqrt{2}}$$

```
[Out] 5828/(9075*(2 - 5*x)) - (313 + 502*x)/(1452*(1 + 2*x^2)) + (503*A
rcTan[Sqrt[2]*x])/(7986*Sqrt[2]) - (59096*Log[2 - 5*x])/99825 + (
2843*Log[1 + 2*x^2])/7986
```

Rubi [A] time = 0.199109, antiderivative size = 86, normalized size of antiderivative = 1.25, number of steps used = 7, number of rules used = 5, integrand size = 50, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$-\frac{502x+313}{1452\left(2x^2+1\right)}+\frac{2843\log\left(2x^2+1\right)}{7986}+\frac{5828}{9075(2-5x)}-\frac{59096\log(2-5x)}{99825}+\frac{272\sqrt{2}\tan^{-1}\left(\sqrt{2}x\right)}{1331}-\frac{251\tan^{-1}\left(\sqrt{2}x\right)}{726\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(8 - 13*x^2 - 7*x^3 + 12*x^5)/(4 - 20*x + 41*x^2 - 80*x^3 + 116*x^4 - 80*x^5 + 1
```

```
[Out] 5828/(9075*(2 - 5*x)) - (313 + 502*x)/(1452*(1 + 2*x^2)) - (251*A
rcTan[Sqrt[2]*x])/(726*Sqrt[2]) + (272*Sqrt[2]*ArcTan[Sqrt[2]*x])
/1331 - (59096*Log[2 - 5*x])/99825 + (2843*Log[1 + 2*x^2])/7986
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((12*x**5-7*x**3-13*x**2+8)/(100*x**6-80*x**5+116*x**4-
80*x**3+41*x**2-20*x+4),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.0749282, size = 67, normalized size = 0.97

$$\frac{142150\log\left(2x^2+1\right)-\frac{33\left(36458x^2+4675x+2554\right)}{10x^3-4x^2+5x-2}-236384\log(2-5x)+12575\sqrt{2}\tan^{-1}\left(\sqrt{2}x\right)}{399300}$$

Antiderivative was successfully verified.

```
[In] Integrate[(8 - 13*x^2 - 7*x^3 + 12*x^5)/(4 - 20*x + 41*x^2 - 80*x^3 + 116*x^4 - 80*
```

```
[Out] ((-33*(2554 + 4675*x + 36458*x^2))/(-2 + 5*x - 4*x^2 + 10*x^3) +
12575*Sqrt[2]*ArcTan[Sqrt[2]*x] - 236384*Log[2 - 5*x] + 142150*Lo
g[1 + 2*x^2])/399300
```

Maple [A] time = 0.02, size = 54, normalized size = 0.8

$$\frac{1}{3993}\left(-\frac{2761\,x}{4}-\frac{3443}{8}\right)\left(x^{2}+\frac{1}{2}\right)^{-1}+\frac{2843\,\ln\left(4\,x^{2}+2\right)}{7986}+\frac{503\,\arctan\left(\sqrt{2}x\right)\sqrt{2}}{15972}-\frac{5828}{45375\,x-18150}-\frac{59096\,\ln\left(5\,x-2\right)}{99825}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((12*x^5-7*x^3-13*x^2+8)/(100*x^6-80*x^5+116*x^4-80*x^3+41*x^2-
20*x+4),x)
```

```
[Out] 1/3993*(-2761/4*x-3443/8)/(x^2+1/2)+2843/7986*ln(4*x^2+2)+503/159
72*arctan(2^(1/2)*x)*2^(1/2)-5828/9075/(5*x-2)-59096/99825*ln(5*x
-2)
```

Maxima [A] time = 0.875479, size = 80, normalized size = 1.16

$$\frac{503}{15972}\sqrt{2}\arctan\left(\sqrt{2}x\right)-\frac{36458\,x^{2}+4675\,x+2554}{12100\,(10\,x^{3}-4\,x^{2}+5\,x-2)}+\frac{2843}{7986}\log\left(2\,x^{2}+1\right)-\frac{59096}{99825}\log\left(5\,x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x^5 - 7*x^3 - 13*x^2 + 8)/(100*x^6 - 80*x^5 + 116*x^4 - 80*x^3 + 41*x
```

```
[Out] 503/15972*sqrt(2)*arctan(sqrt(2)*x) - 1/12100*(36458*x^2 + 4675*x
 + 2554)/(10*x^3 - 4*x^2 + 5*x - 2) + 2843/7986*log(2*x^2 + 1) -
59096/99825*log(5*x - 2)
```

Fricas [A] time = 0.266174, size = 155, normalized size = 2.25

$$\frac{\sqrt{2}\left(142150\,\sqrt{2}\left(10\,x^{3}-4\,x^{2}+5\,x-2\right)\log\left(2\,x^{2}+1\right)-236384\,\sqrt{2}\left(10\,x^{3}-4\,x^{2}+5\,x-2\right)\log\left(5\,x-2\right)+25150\,\left(10\,x^{3}-4\,x\right.\right.}{798600\,(10\,x^{3}-4\,x^{2}+5\,x-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x^5 - 7*x^3 - 13*x^2 + 8)/(100*x^6 - 80*x^5 + 116*x^4 - 80*x^3 + 41*x
```

```
[Out] 1/798600*sqrt(2)*(142150*sqrt(2)*(10*x^3 - 4*x^2 + 5*x - 2)*log(2
*x^2 + 1) - 236384*sqrt(2)*(10*x^3 - 4*x^2 + 5*x - 2)*log(5*x - 2
) + 25150*(10*x^3 - 4*x^2 + 5*x - 2)*arctan(sqrt(2)*x) - 33*sqrt(
2)*(36458*x^2 + 4675*x + 2554))/(10*x^3 - 4*x^2 + 5*x - 2)
```

Sympy [A] time = 0.565926, size = 63, normalized size = 0.91

$$-\frac{36458x^{2}+4675x+2554}{121000x^{3}-48400x^{2}+60500x-24200}-\frac{59096\log\left(x-\frac{2}{5}\right)}{99825}+\frac{2843\log\left(x^{2}+\frac{1}{2}\right)}{7986}+\frac{503\sqrt{2}\,\operatorname{atan}\left(\sqrt{2}x\right)}{15972}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((12*x**5-7*x**3-13*x**2+8)/(100*x**6-80*x**5+116*x**4-
80*x**3+41*x**2-20*x+4),x)
```

```
[Out] -(36458*x**2 + 4675*x + 2554)/(121000*x**3 - 48400*x**2 + 60500*x
 - 24200) - 59096*log(x - 2/5)/99825 + 2843*log(x**2 + 1/2)/7986
+ 503*sqrt(2)*atan(sqrt(2)*x)/15972
```

GIAC/XCAS [A] time = 0.26206, size = 80, normalized size = 1.16

$$\frac{503}{15972}\sqrt{2}\arctan\left(\sqrt{2}x\right) - \frac{36458\,x^2 + 4675\,x + 2554}{12100\,(2\,x^2 + 1)(5\,x - 2)} + \frac{2843}{7986}\ln\left(2\,x^2 + 1\right) - \frac{59096}{99825}\ln\left(|5\,x - 2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x^5 - 7*x^3 - 13*x^2 + 8)/(100*x^6 - 80*x^5 + 116*x^4 - 80*x^3 + 41*x
```

```
[Out] 503/15972*sqrt(2)*arctan(sqrt(2)*x) - 1/12100*(36458*x^2 + 4675*x
 + 2554)/((2*x^2 + 1)*(5*x - 2)) + 2843/7986*ln(2*x^2 + 1) - 5909
6/99825*ln(abs(5*x - 2))
```

3.322 $\int \frac{9+x^4}{x^2(9+x^2)} \, dx$

Optimal. Leaf size=17

$$x - \frac{1}{x} - \frac{10}{3}\tan^{-1}\left(\frac{x}{3}\right)$$

```
[Out] -x^(-1) + x - (10*ArcTan[x/3])/3
```

Rubi [A] time = 0.0415431, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$x - \frac{1}{x} - \frac{10}{3}\tan^{-1}\left(\frac{x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(9 + x^4)/(x^2*(9 + x^2)),x]
```

```
[Out] -x^(-1) + x - (10*ArcTan[x/3])/3
```

Rubi in Sympy [A] time = 6.98288, size = 12, normalized size = 0.71

$$x - \frac{10\,\mathrm{atan}\left(\frac{x}{3}\right)}{3} - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((x**4+9)/x**2/(x**2+9),x)
```

```
[Out] x - 10*atan(x/3)/3 - 1/x
```

Mathematica [A] time = 0.00869106, size = 17, normalized size = 1.

$$x - \frac{1}{x} - \frac{10}{3}\tan^{-1}\left(\frac{x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(9 + x^4)/(x^2*(9 + x^2)),x]
```

```
[Out] -x^(-1) + x - (10*ArcTan[x/3])/3
```

Maple [A] time = 0.009, size = 14, normalized size = 0.8

$$-x^{-1} + x - \frac{10}{3}\arctan\left(\frac{x}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+9)/x^2/(x^2+9),x)
```

```
[Out] -1/x+x-10/3*arctan(1/3*x)
```

Maxima [A] time = 0.88416, size = 18, normalized size = 1.06

$$x - \frac{1}{x} - \frac{10}{3} \arctan\left(\frac{1}{3} x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 9)/((x^2 + 9)*x^2),x, algorithm="maxima")
```

```
[Out] x - 1/x - 10/3*arctan(1/3*x)
```

Fricas [A] time = 0.252626, size = 26, normalized size = 1.53

$$\frac{3 x^2 - 10 x \arctan\left(\frac{1}{3} x\right) - 3}{3 x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 9)/((x^2 + 9)*x^2),x, algorithm="fricas")
```

```
[Out] 1/3*(3*x^2 - 10*x*arctan(1/3*x) - 3)/x
```

Sympy [A] time = 0.208994, size = 12, normalized size = 0.71

$$x - \frac{10 \operatorname{atan}\left(\frac{x}{3}\right)}{3} - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**4+9)/x**2/(x**2+9),x)
```

```
[Out] x - 10*atan(x/3)/3 - 1/x
```

GIAC/XCAS [A] time = 0.259991, size = 18, normalized size = 1.06

$$x - \frac{1}{x} - \frac{10}{3} \arctan\left(\frac{1}{3} x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 9)/((x^2 + 9)*x^2),x, algorithm="giac")
```

```
[Out] x - 1/x - 10/3*arctan(1/3*x)
```

3.323 $\int \frac{2x+x^4}{1+x^2} \, dx$

Optimal. Leaf size=19

$$\frac{x^3}{3} + \log\left(x^2+1\right) - x + \tan^{-1}(x)$$

```
[Out] -x + x^3/3 + ArcTan[x] + Log[1 + x^2]
```

Rubi [A] time = 0.0421123, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{x^3}{3} + \log\left(x^2+1\right) - x + \tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(2*x + x^4)/(1 + x^2),x]

[Out] -x + x^3/3 + ArcTan[x] + Log[1 + x^2]
```

Rubi in Sympy [A] time = 9.7387, size = 15, normalized size = 0.79

$$\frac{x^3}{3} - x + \log\left(x^2+1\right) + \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**4+2*x)/(x**2+1),x)

[Out] x**3/3 - x + log(x**2 + 1) + atan(x)
```

Mathematica [A] time = 0.00757208, size = 19, normalized size = 1.

$$\frac{x^3}{3} + \log\left(x^2+1\right) - x + \tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2*x + x^4)/(1 + x^2),x]

[Out] -x + x^3/3 + ArcTan[x] + Log[1 + x^2]
```

Maple [A] time = 0.003, size = 18, normalized size = 1.

$$-x + \frac{x^3}{3} + \arctan(x) + \ln\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^4+2*x)/(x^2+1),x)
```

```
[Out] -x+1/3*x^3+arctan(x)+ln(x^2+1)
```

Maxima [A] time = 0.922846, size = 23, normalized size = 1.21

$$\frac{1}{3}x^3 - x + \arctan(x) + \log\left(x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 + 2*x)/(x^2 + 1),x, algorithm="maxima")
```

```
[Out] 1/3*x^3 - x + arctan(x) + log(x^2 + 1)
```

Fricas [A] time = 0.246352, size = 23, normalized size = 1.21

$$\frac{1}{3}x^3 - x + \arctan(x) + \log\left(x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 + 2*x)/(x^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/3*x^3 - x + arctan(x) + log(x^2 + 1)
```

Sympy [A] time = 0.183933, size = 15, normalized size = 0.79

$$\frac{x^3}{3} - x + \log\left(x^2 + 1\right) + \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+2*x)/(x**2+1),x)
```

```
[Out] x**3/3 - x + log(x**2 + 1) + atan(x)
```

GIAC/XCAS [A] time = 0.259785, size = 23, normalized size = 1.21

$$\frac{1}{3}x^3 - x + \arctan(x) + \ln\left(x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 + 2*x)/(x^2 + 1),x, algorithm="giac")
```

```
[Out] 1/3*x^3 - x + arctan(x) + ln(x^2 + 1)
```

3.324 $\int \frac{-x+x^3}{(-1+x)^2(1+x^2)}\,dx$

Optimal. Leaf size=9

$$\log(1-x)+\tan^{-1}(x)$$

```
[Out] ArcTan[x] + Log[1 - x]
```

Rubi [A] time = 0.104009, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\log(1-x)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(-x + x^3)/((-1 + x)^2*(1 + x^2)),x]

[Out] ArcTan[x] + Log[1 - x]
```

Rubi in Sympy [A] time = 34.7981, size = 7, normalized size = 0.78

$$\log(-x+1)+\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**3-x)/(-1+x)**2/(x**2+1),x)

[Out] log(-x + 1) + atan(x)
```

Mathematica [A] time = 0.00955501, size = 9, normalized size = 1.

$$\log(1-x)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-x + x^3)/((-1 + x)^2*(1 + x^2)),x]

[Out] ArcTan[x] + Log[1 - x]
```

Maple [A] time = 0.006, size = 8, normalized size = 0.9

$$\ln(-1+x)+\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^3-x)/(-1+x)^2/(x^2+1),x)

[Out] ln(-1+x)+arctan(x)
```

Maxima [A] time = 0.90075, size = 9, normalized size = 1.

$$\arctan(x) + \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - x)/((x^2 + 1)*(x - 1)^2),x, algorithm="maxima")
```

```
[Out] arctan(x) + log(x - 1)
```

Fricas [A] time = 0.255742, size = 9, normalized size = 1.

$$\arctan(x) + \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - x)/((x^2 + 1)*(x - 1)^2),x, algorithm="fricas")
```

```
[Out] arctan(x) + log(x - 1)
```

Sympy [A] time = 0.369989, size = 7, normalized size = 0.78

$$\log(x - 1) + \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3-x)/(-1+x)**2/(x**2+1),x)
```

```
[Out] log(x - 1) + atan(x)
```

GIAC/XCAS [A] time = 0.264345, size = 38, normalized size = 4.22

$$\frac{1}{4}\pi - \pi\left\lfloor\frac{\pi + 4\arctan(x)}{4\pi} + \frac{1}{2}\right\rfloor + \arctan(x) + \ln(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - x)/((x^2 + 1)*(x - 1)^2),x, algorithm="giac")
```

```
[Out] 1/4*pi - pi*floor(1/4*(pi + 4*arctan(x))/pi + 1/2) + arctan(x) +
ln(abs(x - 1))
```

3.325 $\int \frac{2+5x+3x^2+2x^3}{1+x+x^2}\,dx$

Optimal. Leaf size=12

$$x^2 + \log\left(x^2 + x + 1\right) + x$$

```
[Out] x + x^2 + Log[1 + x + x^2]
```

Rubi [A] time = 0.0236842, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$x^2 + \log\left(x^2 + x + 1\right) + x$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + 5*x + 3*x^2 + 2*x^3)/(1 + x + x^2),x]
```

```
[Out] x + x^2 + Log[1 + x + x^2]
```

Rubi in Sympy [A] time = 21.6663, size = 12, normalized size = 1.

$$x^2 + x + \log\left(x^2 + x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*x**3+3*x**2+5*x+2)/(x**2+x+1),x)
```

```
[Out] x**2 + x + log(x**2 + x + 1)
```

Mathematica [A] time = 0.00860242, size = 12, normalized size = 1.

$$x^2 + \log\left(x^2 + x + 1\right) + x$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + 5*x + 3*x^2 + 2*x^3)/(1 + x + x^2),x]
```

```
[Out] x + x^2 + Log[1 + x + x^2]
```

Maple [A] time = 0.003, size = 13, normalized size = 1.1

$$x + x^2 + \ln\left(x^2 + x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*x^3+3*x^2+5*x+2)/(x^2+x+1),x)
```

```
[Out] x+x^2+ln(x^2+x+1)
```

Maxima [A] time = 0.809502, size = 16, normalized size = 1.33

$$x^2 + x + \log\left(x^2 + x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3 + 3*x^2 + 5*x + 2)/(x^2 + x + 1),x, algorithm="maxima")
```

```
[Out] x^2 + x + log(x^2 + x + 1)
```

Fricas [A] time = 0.246551, size = 16, normalized size = 1.33

$$x^2 + x + \log\left(x^2 + x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3 + 3*x^2 + 5*x + 2)/(x^2 + x + 1),x, algorithm="fricas")
```

```
[Out] x^2 + x + log(x^2 + x + 1)
```

Sympy [A] time = 0.148496, size = 12, normalized size = 1.

$$x^2 + x + \log\left(x^2 + x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x**3+3*x**2+5*x+2)/(x**2+x+1),x)
```

```
[Out] x**2 + x + log(x**2 + x + 1)
```

GIAC/XCAS [A] time = 0.26417, size = 16, normalized size = 1.33

$$x^2 + x + \ln\left(x^2 + x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3 + 3*x^2 + 5*x + 2)/(x^2 + x + 1),x, algorithm="giac")
```

```
[Out] x^2 + x + ln(x^2 + x + 1)
```

3.326 $\int \frac{3-4x-5x^2+3x^3}{x^3(-1+x+x^2)}\,dx$

Optimal. Leaf size=65

$$\frac{3}{2x^2}-\frac{1}{x}+3\log(x)-\frac{1}{10}\left(15-\sqrt{5}\right)\log\left(2x-\sqrt{5}+1\right)-\frac{1}{10}\left(15+\sqrt{5}\right)\log\left(2x+\sqrt{5}+1\right)$$

```
[Out] 3/(2*x^2) - x^(-1) + 3*Log[x] - ((15 - Sqrt[5])*Log[1 - Sqrt[5] +
 2*x])/10 - ((15 + Sqrt[5])*Log[1 + Sqrt[5] + 2*x])/10
```

Rubi [A] time = 0.123368, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\frac{3}{2x^2}-\frac{1}{x}+3\log(x)-\frac{1}{10}\left(15-\sqrt{5}\right)\log\left(2x-\sqrt{5}+1\right)-\frac{1}{10}\left(15+\sqrt{5}\right)\log\left(2x+\sqrt{5}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 4*x - 5*x^2 + 3*x^3)/(x^3*(-1 + x + x^2)),x]
```

```
[Out] 3/(2*x^2) - x^(-1) + 3*Log[x] - ((15 - Sqrt[5])*Log[1 - Sqrt[5] +
 2*x])/10 - ((15 + Sqrt[5])*Log[1 + Sqrt[5] + 2*x])/10
```

Rubi in Sympy [A] time = 24.1092, size = 75, normalized size = 1.15

$$3\log(x)-\frac{\sqrt{5}\left(\frac{1}{2}+\frac{3\sqrt{5}}{2}\right)\log\left(2x+1+\sqrt{5}\right)}{5}+\frac{\sqrt{5}\left(-\frac{3\sqrt{5}}{2}+\frac{1}{2}\right)\log\left(2x-\sqrt{5}+1\right)}{5}-\frac{1}{x}+\frac{3}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((3*x**3-5*x**2-4*x+3)/x**3/(x**2+x-1),x)
```

```
[Out] 3*log(x) - sqrt(5)*(1/2 + 3*sqrt(5)/2)*log(2*x + 1 + sqrt(5))/5 +
 sqrt(5)*(-3*sqrt(5)/2 + 1/2)*log(2*x - sqrt(5) + 1)/5 - 1/x + 3/
(2*x**2)
```

Mathematica [A] time = 0.0588385, size = 58, normalized size = 0.89

$$\frac{1}{10}\left(\frac{15}{x^2}-\frac{10}{x}+\left(\sqrt{5}-15\right)\log\left(-2x+\sqrt{5}-1\right)+30\log(x)-\left(15+\sqrt{5}\right)\log\left(2x+\sqrt{5}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 4*x - 5*x^2 + 3*x^3)/(x^3*(-1 + x + x^2)),x]
```

```
[Out] (15/x^2 - 10/x + (-15 + Sqrt[5])*Log[-1 + Sqrt[5] - 2*x] + 30*Log
[x] - (15 + Sqrt[5])*Log[1 + Sqrt[5] + 2*x])/10
```

Maple [A] time = 0.01, size = 41, normalized size = 0.6

$$-x^{-1}+\frac{3}{2\,x^2}+3\,\ln(x)-\frac{3\,\ln\left(x^2+x-1\right)}{2}-\frac{\sqrt{5}}{5}\mathit{Artanh}\left(\frac{(1+2\,x)\sqrt{5}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^3-5*x^2-4*x+3)/x^3/(x^2+x-1),x)
```

```
[Out] -1/x+3/2/x^2+3*ln(x)-3/2*ln(x^2+x-1)-1/5*5^(1/2)*arctanh(1/5*(1+2
*x)*5^(1/2))
```

Maxima [A] time = 0.892482, size = 69, normalized size = 1.06

$$\frac{1}{10}\sqrt{5}\log\left(\frac{2x-\sqrt{5}+1}{2x+\sqrt{5}+1}\right)-\frac{2x-3}{2x^2}-\frac{3}{2}\log\left(x^2+x-1\right)+3\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^3 - 5*x^2 - 4*x + 3)/((x^2 + x - 1)*x^3),x, algorithm="maxima")
```

```
[Out] 1/10*sqrt(5)*log((2*x - sqrt(5) + 1)/(2*x + sqrt(5) + 1)) - 1/2*(
2*x - 3)/x^2 - 3/2*log(x^2 + x - 1) + 3*log(x)
```

Fricas [A] time = 0.255881, size = 104, normalized size = 1.6

$$-\frac{\sqrt{5}\left(3\sqrt{5}x^2\log\left(x^2+x-1\right)-6\sqrt{5}x^2\log\left(x\right)-x^2\log\left(\frac{\sqrt{5}(2x^2+2x+3)-10x-5}{x^2+x-1}\right)+\sqrt{5}(2x-3)\right)}{10x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^3 - 5*x^2 - 4*x + 3)/((x^2 + x - 1)*x^3),x, algorithm="fricas")
```

```
[Out] -1/10*sqrt(5)*(3*sqrt(5)*x^2*log(x^2 + x - 1) - 6*sqrt(5)*x^2*log
(x) - x^2*log((sqrt(5)*(2*x^2 + 2*x + 3) - 10*x - 5)/(x^2 + x - 1
)) + sqrt(5)*(2*x - 3))/x^2
```

Sympy [A] time = 2.00271, size = 99, normalized size = 1.52

$$3\log\left(x\right)+\left(-\frac{3}{2}+\frac{\sqrt{5}}{10}\right)\log\left(x-\frac{405}{202}-\frac{35\sqrt{5}}{202}+\frac{110\left(-\frac{3}{2}+\frac{\sqrt{5}}{10}\right)^2}{101}\right)+\left(-\frac{3}{2}-\frac{\sqrt{5}}{10}\right)\log\left(x-\frac{405}{202}+\frac{35\sqrt{5}}{202}+\frac{110\left(-\frac{3}{2}-\frac{\sqrt{5}}{10}\right)^2}{101}\right)-\frac{2x-3}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**3-5*x**2-4*x+3)/x**3/(x**2+x-1),x)
```

```
[Out] 3*log(x) + (-3/2 + sqrt(5)/10)*log(x - 405/202 - 35*sqrt(5)/202 +
 110*(-3/2 + sqrt(5)/10)**2/101) + (-3/2 - sqrt(5)/10)*log(x - 40
5/202 + 35*sqrt(5)/202 + 110*(-3/2 - sqrt(5)/10)**2/101) - (2*x -
 3)/(2*x**2)
```

GIAC/XCAS [A] time = 0.260298, size = 74, normalized size = 1.14

$$\frac{1}{10}\sqrt{5}\ln\left(\frac{\left|2x-\sqrt{5}+1\right|}{\left|2x+\sqrt{5}+1\right|}\right)-\frac{2x-3}{2x^2}-\frac{3}{2}\ln\left(\left|x^2+x-1\right|\right)+3\ln\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3 - 5*x^2 - 4*x + 3)/((x^2 + x - 1)*x^3),x, algorithm="giac")
```

```
[Out] 1/10*sqrt(5)*ln(abs(2*x - sqrt(5) + 1)/abs(2*x + sqrt(5) + 1)) -
1/2*(2*x - 3)/x^2 - 3/2*ln(abs(x^2 + x - 1)) + 3*ln(abs(x))
```

3.327 $\int \frac{4+8x+5x^2+2x^3}{(2+2x+x^2)^2}\,dx$

Optimal. Leaf size=28

$$-\frac{1}{x^2+2x+2}+\log\left(x^2+2x+2\right)-\tan^{-1}(x+1)$$

```
[Out] -(2 + 2*x + x^2)^(-1) - ArcTan[1 + x] + Log[2 + 2*x + x^2]
```

Rubi [A] time = 0.0462366, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192

$$-\frac{1}{x^2+2x+2}+\log\left(x^2+2x+2\right)-\tan^{-1}(x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[(4 + 8*x + 5*x^2 + 2*x^3)/(2 + 2*x + x^2)^2,x]

[Out] -(2 + 2*x + x^2)^(-1) - ArcTan[1 + x] + Log[2 + 2*x + x^2]
```

Rubi in Sympy [A] time = 18.2682, size = 32, normalized size = 1.14

$$\frac{0.25x(2x+4)}{x^2+2x+2}+\log\left(x^2+2x+2\right)-1.0\,\mathrm{atan}\,(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*x**3+5*x**2+8*x+4)/(x**2+2*x+2)**2,x)

[Out] 0.25*x*(2*x + 4)/(x**2 + 2*x + 2) + log(x**2 + 2*x + 2) - 1.0*ata
n(x + 1)
```

Mathematica [A] time = 0.0164666, size = 28, normalized size = 1.

$$-\frac{1}{x^2+2x+2}+\log\left(x^2+2x+2\right)-\tan^{-1}(x+1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(4 + 8*x + 5*x^2 + 2*x^3)/(2 + 2*x + x^2)^2,x]

[Out] -(2 + 2*x + x^2)^(-1) - ArcTan[1 + x] + Log[2 + 2*x + x^2]
```

Maple [A] time = 0.009, size = 29, normalized size = 1.

$$-\left(x^2+2\,x+2\right)^{-1}-\arctan\left(1+x\right)+\ln\left(x^2+2\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*x^3+5*x^2+8*x+4)/(x^2+2*x+2)^2,x)
```

```
[Out] -1/(x^2+2*x+2)-arctan(1+x)+ln(x^2+2*x+2)
```

Maxima [A] time = 0.900423, size = 38, normalized size = 1.36

$$-\frac{1}{x^2 + 2x + 2} - \arctan(x + 1) + \log(x^2 + 2x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3 + 5*x^2 + 8*x + 4)/(x^2 + 2*x + 2)^2,x, algorithm="maxima")
```

```
[Out] -1/(x^2 + 2*x + 2) - arctan(x + 1) + log(x^2 + 2*x + 2)
```

Fricas [A] time = 0.252745, size = 62, normalized size = 2.21

$$-\frac{(x^2 + 2x + 2)\arctan(x + 1) - (x^2 + 2x + 2)\log(x^2 + 2x + 2) + 1}{x^2 + 2x + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3 + 5*x^2 + 8*x + 4)/(x^2 + 2*x + 2)^2,x, algorithm="fricas")
```

```
[Out] -((x^2 + 2*x + 2)*arctan(x + 1) - (x^2 + 2*x + 2)*log(x^2 + 2*x +
 2) + 1)/(x^2 + 2*x + 2)
```

Sympy [A] time = 0.300702, size = 24, normalized size = 0.86

$$\log(x^2 + 2x + 2) - \operatorname{atan}(x + 1) - \frac{1}{x^2 + 2x + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x**3+5*x**2+8*x+4)/(x**2+2*x+2)**2,x)
```

```
[Out] log(x**2 + 2*x + 2) - atan(x + 1) - 1/(x**2 + 2*x + 2)
```

GIAC/XCAS [A] time = 0.260316, size = 38, normalized size = 1.36

$$-\frac{1}{x^2 + 2x + 2} - \arctan(x + 1) + \ln(x^2 + 2x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3 + 5*x^2 + 8*x + 4)/(x^2 + 2*x + 2)^2,x, algorithm="giac")
```

```
[Out] -1/(x^2 + 2*x + 2) - arctan(x + 1) + ln(x^2 + 2*x + 2)
```

3.328 $\int \frac{(-1+x)^4 x^4}{1+x^2} \, dx$

Optimal. Leaf size=32

$$\frac{x^7}{7} - \frac{2x^6}{3} + x^5 - \frac{4x^3}{3} + 4x - 4\tan^{-1}(x)$$

```
[Out] 4*x - (4*x^3)/3 + x^5 - (2*x^6)/3 + x^7/7 - 4*ArcTan[x]
```

Rubi [A] time = 0.0634226, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{x^7}{7} - \frac{2x^6}{3} + x^5 - \frac{4x^3}{3} + 4x - 4\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[((-1 + x)^4*x^4)/(1 + x^2),x]
```

```
[Out] 4*x - (4*x^3)/3 + x^5 - (2*x^6)/3 + x^7/7 - 4*ArcTan[x]
```

Rubi in Sympy [A] time = 5.00429, size = 29, normalized size = 0.91

$$\frac{x^7}{7} - \frac{2x^6}{3} + x^5 - \frac{4x^3}{3} + 4x - 4\,\mathrm{atan}\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-1+x)**4*x**4/(x**2+1),x)
```

```
[Out] x**7/7 - 2*x**6/3 + x**5 - 4*x**3/3 + 4*x - 4*atan(x)
```

Mathematica [A] time = 0.0365245, size = 32, normalized size = 1.

$$\frac{x^7}{7} - \frac{2x^6}{3} + x^5 - \frac{4x^3}{3} + 4x - 4\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((-1 + x)^4*x^4)/(1 + x^2),x]
```

```
[Out] 4*x - (4*x^3)/3 + x^5 - (2*x^6)/3 + x^7/7 - 4*ArcTan[x]
```

Maple [A] time = 0.004, size = 27, normalized size = 0.8

$$4\,x - \frac{4\,x^3}{3} + x^5 - \frac{2\,x^6}{3} + \frac{x^7}{7} - 4\,\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-1+x)^4*x^4/(x^2+1),x)
```

```
[Out] 4*x-4/3*x^3+x^5-2/3*x^6+1/7*x^7-4*arctan(x)
```

Maxima [A] time = 0.901996, size = 35, normalized size = 1.09

$$\frac{1}{7}x^7 - \frac{2}{3}x^6 + x^5 - \frac{4}{3}x^3 + 4x - 4\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - 1)^4*x^4/(x^2 + 1),x, algorithm="maxima")
```

```
[Out] 1/7*x^7 - 2/3*x^6 + x^5 - 4/3*x^3 + 4*x - 4*arctan(x)
```

Fricas [A] time = 0.282765, size = 35, normalized size = 1.09

$$\frac{1}{7}x^7 - \frac{2}{3}x^6 + x^5 - \frac{4}{3}x^3 + 4x - 4\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - 1)^4*x^4/(x^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/7*x^7 - 2/3*x^6 + x^5 - 4/3*x^3 + 4*x - 4*arctan(x)
```

Sympy [A] time = 0.180476, size = 29, normalized size = 0.91

$$\frac{x^7}{7} - \frac{2x^6}{3} + x^5 - \frac{4x^3}{3} + 4x - 4\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-1+x)**4*x**4/(x**2+1),x)
```

```
[Out] x**7/7 - 2*x**6/3 + x**5 - 4*x**3/3 + 4*x - 4*atan(x)
```

GIAC/XCAS [A] time = 0.258654, size = 35, normalized size = 1.09

$$\frac{1}{7}x^7 - \frac{2}{3}x^6 + x^5 - \frac{4}{3}x^3 + 4x - 4\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - 1)^4*x^4/(x^2 + 1),x, algorithm="giac")
```

```
[Out] 1/7*x^7 - 2/3*x^6 + x^5 - 4/3*x^3 + 4*x - 4*arctan(x)
```

3.329 $\int \frac{-20x+4x^2}{9-10x^2+x^4}\,dx$

Optimal. Leaf size=31

$$\log(1-x)-\frac{1}{2}\log(3-x)+\frac{3}{2}\log(x+1)-2\log(x+3)$$

```
[Out] Log[1 - x] - Log[3 - x]/2 + (3*Log[1 + x])/2 - 2*Log[3 + x]
```

Rubi [A] time = 0.087748, antiderivative size = 41, normalized size of antiderivative = 1.32, number of steps used = 11, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364

$$\frac{5}{4}\log\left(1-x^2\right)-\frac{5}{4}\log\left(9-x^2\right)-\frac{3}{2}\tanh^{-1}\left(\frac{x}{3}\right)+\frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(-20*x + 4*x^2)/(9 - 10*x^2 + x^4),x]

[Out] (-3*ArcTanh[x/3])/2 + ArcTanh[x]/2 + (5*Log[1 - x^2])/4 - (5*Log[
9 - x^2])/4
```

Rubi in Sympy [A] time = 21.913, size = 32, normalized size = 1.03

$$\frac{5\log\left(-x^2+1\right)}{4}-\frac{5\log\left(-x^2+9\right)}{4}-\frac{3\operatorname{atanh}\left(\frac{x}{3}\right)}{2}+\frac{\operatorname{atanh}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((4*x**2-20*x)/(x**4-10*x**2+9),x)

[Out] 5*log(-x**2 + 1)/4 - 5*log(-x**2 + 9)/4 - 3*atanh(x/3)/2 + atanh(
x)/2
```

Mathematica [A] time = 0.0109546, size = 39, normalized size = 1.26

$$4\left(\frac{1}{4}\log(1-x)-\frac{1}{8}\log(3-x)+\frac{3}{8}\log(x+1)-\frac{1}{2}\log(x+3)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-20*x + 4*x^2)/(9 - 10*x^2 + x^4),x]

[Out] 4*(Log[1 - x]/4 - Log[3 - x]/8 + (3*Log[1 + x])/8 - Log[3 + x]/2)
```

Maple [A] time = 0.013, size = 24, normalized size = 0.8

$$\ln(-1+x)-\frac{\ln(-3+x)}{2}+\frac{3\ln(1+x)}{2}-2\ln(3+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((4*x^2-20*x)/(x^4-10*x^2+9),x)

[Out] ln(-1+x)-1/2*ln(-3+x)+3/2*ln(1+x)-2*ln(3+x)
```

Maxima [A] time = 0.817465, size = 31, normalized size = 1.

$$-2\log(x+3)+\frac{3}{2}\log(x+1)+\log(x-1)-\frac{1}{2}\log(x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(4*(x^2 - 5*x)/(x^4 - 10*x^2 + 9),x, algorithm="maxima")

[Out] -2*log(x + 3) + 3/2*log(x + 1) + log(x - 1) - 1/2*log(x - 3)
```

Fricas [A] time = 0.257986, size = 31, normalized size = 1.

$$-2\log(x+3)+\frac{3}{2}\log(x+1)+\log(x-1)-\frac{1}{2}\log(x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(4*(x^2 - 5*x)/(x^4 - 10*x^2 + 9),x, algorithm="fricas")

[Out] -2*log(x + 3) + 3/2*log(x + 1) + log(x - 1) - 1/2*log(x - 3)
```

Sympy [A] time = 0.54298, size = 26, normalized size = 0.84

$$-\frac{\log(x-3)}{2}+\log(x-1)+\frac{3\log(x+1)}{2}-2\log(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x**2-20*x)/(x**4-10*x**2+9),x)

[Out] -log(x - 3)/2 + log(x - 1) + 3*log(x + 1)/2 - 2*log(x + 3)
```

GIAC/XCAS [A] time = 0.260978, size = 36, normalized size = 1.16

$$-2\ln(|x+3|)+\frac{3}{2}\ln(|x+1|)+\ln(|x-1|)-\frac{1}{2}\ln(|x-3|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(4*(x^2 - 5*x)/(x^4 - 10*x^2 + 9),x, algorithm="giac")

[Out] -2*ln(abs(x + 3)) + 3/2*ln(abs(x + 1)) + ln(abs(x - 1)) - 1/2*ln(
abs(x - 3))
```

3.330 $\int \frac{-1+x+4x^3}{(-1+x)x^2(1+x^2)}\,dx$

Optimal. Leaf size=24

$$-\log\left(x^2+1\right)-\frac{1}{x}+2\log(1-x)+\tan^{-1}(x)$$

```
[Out] -x^(-1) + ArcTan[x] + 2*Log[1 - x] - Log[1 + x^2]
```

Rubi [A] time = 0.31502, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$-\log\left(x^2+1\right)-\frac{1}{x}+2\log(1-x)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + x + 4*x^3)/((-1 + x)*x^2*(1 + x^2)),x]

[Out] -x^(-1) + ArcTan[x] + 2*Log[1 - x] - Log[1 + x^2]
```

Rubi in Sympy [A] time = 85.723, size = 19, normalized size = 0.79

$$2\log(-x+1)-\log\left(x^2+1\right)+\operatorname{atan}(x)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((4*x**3+x-1)/(-1+x)/x**2/(x**2+1),x)

[Out] 2*log(-x + 1) - log(x**2 + 1) + atan(x) - 1/x
```

Mathematica [A] time = 0.0143346, size = 24, normalized size = 1.

$$-\log\left(x^2+1\right)-\frac{1}{x}+2\log(1-x)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + x + 4*x^3)/((-1 + x)*x^2*(1 + x^2)),x]

[Out] -x^(-1) + ArcTan[x] + 2*Log[1 - x] - Log[1 + x^2]
```

Maple [A] time = 0.01, size = 23, normalized size = 1.

$$2\ln(-1+x)-x^{-1}-\ln\left(x^2+1\right)+\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((4*x^3+x-1)/(-1+x)/x^2/(x^2+1),x)

[Out] 2*ln(-1+x)-1/x-ln(x^2+1)+arctan(x)
```

Maxima [A] time = 0.893454, size = 30, normalized size = 1.25

$$-\frac{1}{x} + \arctan(x) - \log\left(x^2 + 1\right) + 2\log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3 + x - 1)/((x^2 + 1)*(x - 1)*x^2),x, algorithm="maxima")
```

```
[Out] -1/x + arctan(x) - log(x^2 + 1) + 2*log(x - 1)
```

Fricas [A] time = 0.274551, size = 35, normalized size = 1.46

$$\frac{x\arctan(x) - x\log\left(x^2 + 1\right) + 2x\log(x - 1) - 1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3 + x - 1)/((x^2 + 1)*(x - 1)*x^2),x, algorithm="fricas")
```

```
[Out] (x*arctan(x) - x*log(x^2 + 1) + 2*x*log(x - 1) - 1)/x
```

Sympy [A] time = 0.355072, size = 19, normalized size = 0.79

$$2\log(x - 1) - \log\left(x^2 + 1\right) + \operatorname{atan}(x) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x**3+x-1)/(-1+x)/x**2/(x**2+1),x)
```

```
[Out] 2*log(x - 1) - log(x**2 + 1) + atan(x) - 1/x
```

GIAC/XCAS [A] time = 0.26134, size = 31, normalized size = 1.29

$$-\frac{1}{x} + \arctan(x) - \ln\left(x^2 + 1\right) + 2\ln(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3 + x - 1)/((x^2 + 1)*(x - 1)*x^2),x, algorithm="giac")
```

```
[Out] -1/x + arctan(x) - ln(x^2 + 1) + 2*ln(abs(x - 1))
```

3.331 $\int \frac{1-3x+2x^2-4x^3+x^4}{(1+x^2)^3} \, dx$

Optimal. Leaf size=23

$$\frac{2}{x^2+1} - \frac{1}{4(x^2+1)^2} + \tan^{-1}(x)$$

```
[Out] -1/(4*(1 + x^2)^2) + 2/(1 + x^2) + ArcTan[x]
```

Rubi [A] time = 0.0407249, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$

$$\frac{2}{x^2+1} - \frac{1}{4(x^2+1)^2} + \tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 3*x + 2*x^2 - 4*x^3 + x^4)/(1 + x^2)^3,x]

[Out] -1/(4*(1 + x^2)^2) + 2/(1 + x^2) + ArcTan[x]
```

Rubi in Sympy [A] time = 15.5979, size = 22, normalized size = 0.96

$$\frac{x^2}{4(x^2+1)^2} + \operatorname{atan}(x) + \frac{7}{4(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((x**4-4*x**3+2*x**2-3*x+1)/(x**2+1)**3,x)

[Out] x**2/(4*(x**2 + 1)**2) + atan(x) + 7/(4*(x**2 + 1))
```

Mathematica [A] time = 0.0189779, size = 23, normalized size = 1.

$$\frac{2}{x^2+1} - \frac{1}{4(x^2+1)^2} + \tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 3*x + 2*x^2 - 4*x^3 + x^4)/(1 + x^2)^3,x]

[Out] -1/(4*(1 + x^2)^2) + 2/(1 + x^2) + ArcTan[x]
```

Maple [A] time = 0.007, size = 19, normalized size = 0.8

$$\frac{1}{(x^2+1)^2}\left(2x^2+\frac{7}{4}\right) + \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-4*x^3+2*x^2-3*x+1)/(x^2+1)^3,x)
```

```
[Out] (2*x^2+7/4)/(x^2+1)^2+arctan(x)
```

Maxima [A] time = 0.893775, size = 32, normalized size = 1.39

$$\frac{8\,x^2+7}{4\,(x^4+2\,x^2+1)}+\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 - 4*x^3 + 2*x^2 - 3*x + 1)/(x^2 + 1)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(8*x^2 + 7)/(x^4 + 2*x^2 + 1) + arctan(x)
```

Fricas [A] time = 0.253124, size = 47, normalized size = 2.04

$$\frac{8\,x^2+4\,\left(x^4+2\,x^2+1\right)\,\arctan(x)+7}{4\,(x^4+2\,x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 - 4*x^3 + 2*x^2 - 3*x + 1)/(x^2 + 1)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(8*x^2 + 4*(x^4 + 2*x^2 + 1)*arctan(x) + 7)/(x^4 + 2*x^2 + 1)
```

Sympy [A] time = 0.32248, size = 20, normalized size = 0.87

$$\frac{8x^2+7}{4x^4+8x^2+4}+\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**4-4*x**3+2*x**2-3*x+1)/(x**2+1)**3,x)
```

```
[Out] (8*x**2 + 7)/(4*x**4 + 8*x**2 + 4) + atan(x)
```

GIAC/XCAS [A] time = 0.261317, size = 26, normalized size = 1.13

$$\frac{8\,x^2+7}{4\,(x^2+1)^2}+\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 - 4*x^3 + 2*x^2 - 3*x + 1)/(x^2 + 1)^3,x, algorithm="giac")
```

```
[Out] 1/4*(8*x^2 + 7)/(x^2 + 1)^2 + arctan(x)
```

3.332 $\int \frac{1-3x+2x^2-4x^3+x^4}{1+3x^2+3x^4+x^6}\,dx$

Optimal. Leaf size=23

$$\frac{2}{x^2+1}-\frac{1}{4(x^2+1)^2}+\tan^{-1}(x)$$

```
[Out] -1/(4*(1 + x^2)^2) + 2/(1 + x^2) + ArcTan[x]
```

Rubi [A] time = 0.0720995, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$\frac{2}{x^2+1}-\frac{1}{4(x^2+1)^2}+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - 3*x + 2*x^2 - 4*x^3 + x^4)/(1 + 3*x^2 + 3*x^4 + x^6),x]
```

```
[Out] -1/(4*(1 + x^2)^2) + 2/(1 + x^2) + ArcTan[x]
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**4-4*x**3+2*x**2-3*x+1)/(x**6+3*x**4+3*x**2+1),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.00887441, size = 23, normalized size = 1.

$$\frac{2}{x^2+1}-\frac{1}{4(x^2+1)^2}+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - 3*x + 2*x^2 - 4*x^3 + x^4)/(1 + 3*x^2 + 3*x^4 + x^6),x]
```

```
[Out] -1/(4*(1 + x^2)^2) + 2/(1 + x^2) + ArcTan[x]
```

Maple [A] time = 0.004, size = 19, normalized size = 0.8

$$\frac{1}{(x^2+1)^2}\left(2x^2+\frac{7}{4}\right)+\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^4-4*x^3+2*x^2-3*x+1)/(x^6+3*x^4+3*x^2+1),x)
```

```
[Out] (2*x^2+7/4)/(x^2+1)^2+arctan(x)
```

Maxima [A] time = 0.902664, size = 32, normalized size = 1.39

$$\frac{8\,x^2+7}{4\,(x^4+2\,x^2+1)}+\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 - 4*x^3 + 2*x^2 - 3*x + 1)/(x^6 + 3*x^4 + 3*x^2 + 1),x, algorithm="m
```

```
[Out] 1/4*(8*x^2 + 7)/(x^4 + 2*x^2 + 1) + arctan(x)
```

Fricas [A] time = 0.245796, size = 47, normalized size = 2.04

$$\frac{8\,x^2+4\,\left(x^4+2\,x^2+1\right)\arctan(x)+7}{4\,(x^4+2\,x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 - 4*x^3 + 2*x^2 - 3*x + 1)/(x^6 + 3*x^4 + 3*x^2 + 1),x, algorithm="f
```

```
[Out] 1/4*(8*x^2 + 4*(x^4 + 2*x^2 + 1)*arctan(x) + 7)/(x^4 + 2*x^2 + 1)
```

Sympy [A] time = 0.306455, size = 20, normalized size = 0.87

$$\frac{8x^2+7}{4x^4+8x^2+4}+\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**4-4*x**3+2*x**2-3*x+1)/(x**6+3*x**4+3*x**2+1),x)
```

```
[Out] (8*x**2 + 7)/(4*x**4 + 8*x**2 + 4) + atan(x)
```

GIAC/XCAS [A] time = 0.258109, size = 26, normalized size = 1.13

$$\frac{8\,x^2+7}{4\,(x^2+1)^2}+\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 - 4*x^3 + 2*x^2 - 3*x + 1)/(x^6 + 3*x^4 + 3*x^2 + 1),x, algorithm="g
```

```
[Out] 1/4*(8*x^2 + 7)/(x^2 + 1)^2 + arctan(x)
```

3.333 $\int \frac{1+x+2x^2+2x^3}{x^2+x^3+x^4} dx$

Optimal. Leaf size=13

$$\log \left(x^2+x+1\right)-\frac{1}{x}$$

```
[Out] -x^(-1) + Log[1 + x + x^2]
```

Rubi [A] time = 0.0551603, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\log \left(x^2+x+1\right)-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + 2*x^2 + 2*x^3)/(x^2 + x^3 + x^4),x]
```

```
[Out] -x^(-1) + Log[1 + x + x^2]
```

Rubi in Sympy [A] time = 22.1478, size = 10, normalized size = 0.77

$$\log \left(x^2+x+1\right)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((2*x**3+2*x**2+x+1)/(x**4+x**3+x**2),x)
```

```
[Out] log(x**2 + x + 1) - 1/x
```

Mathematica [A] time = 0.00960045, size = 13, normalized size = 1.

$$\log \left(x^2+x+1\right)-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + 2*x^2 + 2*x^3)/(x^2 + x^3 + x^4),x]
```

```
[Out] -x^(-1) + Log[1 + x + x^2]
```

Maple [A] time = 0.006, size = 14, normalized size = 1.1

$$-x^{-1}+\ln \left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+2*x^2+x+1)/(x^4+x^3+x^2),x)
```

```
[Out] -1/x+ln(x^2+x+1)
```

Maxima [A] time = 0.798441, size = 18, normalized size = 1.38

$$-\frac{1}{x} + \log\left(x^2 + x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3 + 2*x^2 + x + 1)/(x^4 + x^3 + x^2),x, algorithm="maxima")
```

```
[Out] -1/x + log(x^2 + x + 1)
```

Fricas [A] time = 0.247324, size = 20, normalized size = 1.54

$$\frac{x\log\left(x^2 + x + 1\right) - 1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3 + 2*x^2 + x + 1)/(x^4 + x^3 + x^2),x, algorithm="fricas")
```

```
[Out] (x*log(x^2 + x + 1) - 1)/x
```

Sympy [A] time = 0.208887, size = 10, normalized size = 0.77

$$\log\left(x^2 + x + 1\right) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x**3+2*x**2+x+1)/(x**4+x**3+x**2),x)
```

```
[Out] log(x**2 + x + 1) - 1/x
```

GIAC/XCAS [A] time = 0.258066, size = 18, normalized size = 1.38

$$-\frac{1}{x} + \ln\left(x^2 + x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3 + 2*x^2 + x + 1)/(x^4 + x^3 + x^2),x, algorithm="giac")
```

```
[Out] -1/x + ln(x^2 + x + 1)
```

3.334 $\int \frac{x^2(c+dx)^2}{a+bx^3}\,dx$

Optimal. Leaf size=206

$$\frac{\sqrt[3]{a}d\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{5/3}}-\frac{\sqrt[3]{a}d\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}}+\frac{\sqrt[3]{a}d\left(\sqrt[3]{a}d+2\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{5/3}}+\frac{c^2\log\left(a+bx^3\right)}{3b}+\frac{2cdx}{b}+\frac{d^2x^2}{2b}$$

```
[Out] (2*c*d*x)/b + (d^2*x^2)/(2*b) + (a^(1/3)*d*(2*b^(1/3)*c + a^(1/3)
*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b
^(5/3)) - (a^(1/3)*d*(2*b^(1/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1
/3)*x])/(3*b^(5/3)) + (a^(1/3)*d*(2*b^(1/3)*c - a^(1/3)*d)*Log[a^
(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(5/3)) + (c^2*Log[
a + b*x^3])/(3*b)
```

Rubi [A] time = 0.560343, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4

$$\frac{\sqrt[3]{a}d\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{5/3}}-\frac{\sqrt[3]{a}d\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}}+\frac{\sqrt[3]{a}d\left(\sqrt[3]{a}d+2\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{5/3}}+\frac{c^2\log\left(a+bx^3\right)}{3b}+\frac{2cdx}{b}+\frac{d^2x^2}{2b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(c + d*x)^2)/(a + b*x^3),x]
```

```
[Out] (2*c*d*x)/b + (d^2*x^2)/(2*b) + (a^(1/3)*d*(2*b^(1/3)*c + a^(1/3)
*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b
^(5/3)) - (a^(1/3)*d*(2*b^(1/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1
/3)*x])/(3*b^(5/3)) + (a^(1/3)*d*(2*b^(1/3)*c - a^(1/3)*d)*Log[a^
(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(5/3)) + (c^2*Log[
a + b*x^3])/(3*b)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt[3]{a}d\left(\sqrt[3]{a}d-2\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{\frac{5}{3}}}-\frac{\sqrt[3]{a}d\left(\sqrt[3]{a}d-2\sqrt[3]{b}c\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{6b^{\frac{5}{3}}}+\frac{\sqrt{3}\sqrt[3]{a}d\left(\sqrt[3]{a}d+2\sqrt[3]{b}c\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{3b^{\frac{5}{3}}}+\frac{c^2\log\left(a+bx^3\right)}{3b}+\frac{2cdx}{b}+\frac{d^2\int x\,dx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(d*x+c)**2/(b*x**3+a),x)
```

```
[Out] a**(1/3)*d*(a**(1/3)*d - 2*b**(1/3)*c)*log(a**(1/3) + b**(1/3)*x)
/(3*b**(5/3)) - a**(1/3)*d*(a**(1/3)*d - 2*b**(1/3)*c)*log(a**(2/
3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(6*b**(5/3)) + sqrt(3)*
a**(1/3)*d*(a**(1/3)*d + 2*b**(1/3)*c)*atan(sqrt(3)*(a**(1/3)/3 -
```

```
 2*b**(1/3)*x/3)/a**(1/3))/(3*b**(5/3)) + c**2*log(a + b*x**3)/(3
*b) + 2*c*d*x/b + d**2*Integral(x, x)/b
```

Mathematica [A] time = 0.185539, size = 193, normalized size = 0.94

$$\frac{-\sqrt[3]{a}d\left(\sqrt[3]{a}d-2\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+2b^{2/3}c^2\log\left(a+bx^3\right)+2\sqrt[3]{a}d\left(\sqrt[3]{a}d-2\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+2\sqrt{3}\sqrt[3]{}}{6b^{5/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(c + d*x)^2)/(a + b*x^3),x]
```

```
[Out] (12*b^(2/3)*c*d*x + 3*b^(2/3)*d^2*x^2 + 2*Sqrt[3]*a^(1/3)*d*(2*b^
(1/3)*c + a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]]
+ 2*a^(1/3)*d*(-2*b^(1/3)*c + a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x]
 - a^(1/3)*d*(-2*b^(1/3)*c + a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2] + 2*b^(2/3)*c^2*Log[a + b*x^3])/(6*b^(5/3))
```

Maple [A] time = 0.009, size = 236, normalized size = 1.2

$$\frac{d^2x^2}{2\,b}+2\,\frac{cdx}{b}-\frac{2\,acd}{3\,b^2}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{acd}{3\,b^2}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{2\,acd\sqrt{3}}{3\,b^2}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{ad^2}{3\,b^2}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{ad^2}{6\,b^2}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{ad^2\sqrt{3}}{3\,b^2}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{c^2\ln\left(bx^3+a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(d*x+c)^2/(b*x^3+a),x)
```

```
[Out] 1/2*d^2*x^2/b+2*c*d*x/b-2/3/b^2*a*c*d/(a/b)^(2/3)*ln(x+(a/b)^(1/3
))+1/3/b^2*a*c*d/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))-2/
3/b^2*a*c*d/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)
*x-1))+1/3/b^2*a*d^2/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-1/6/b^2*a*d^2/
(a/b)^(1/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))-1/3/b^2*a*d^2*3^(1/
2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/3*c^2*ln
(b*x^3+a)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)^2*x^2/(b*x^3 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)^2*x^2/(b*x^3 + a),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 4.95384, size = 138, normalized size = 0.67

$$\operatorname{RootSum}\left(27t^3b^5 - 27t^2b^4c^2 + t\left(18ab^2cd^3 + 9b^3c^4\right) - a^2d^6 + 2abc^3d^3 - b^2c^6, \left(t \mapsto t\log\left(x + \frac{9t^2b^3 - 18tb^2c^2 + 4acd^3 + 5b}{ad^4 + 8bc^3d}\right.\right.\right.$$
$$+ \frac{2cdx}{b} + \frac{d^2x^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(d*x+c)**2/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*b**5 - 27*_t**2*b**4*c**2 + _t*(18*a*b**2*c*d**3
 + 9*b**3*c**4) - a**2*d**6 + 2*a*b*c**3*d**3 - b**2*c**6, Lambda
(_t, _t*log(x + (9*_t**2*b**3 - 18*_t*b**2*c**2 + 4*a*c*d**3 + 5*
b*c**4)/(a*d**4 + 8*b*c**3*d)))) + 2*c*d*x/b + d**2*x**2/(2*b)
```

GIAC/XCAS [A] time = 0.267007, size = 302, normalized size = 1.47

$$\frac{c^2\ln\left(\left|bx^3 + a\right|\right)}{3\,b} + \frac{bd^2x^2 + 4\,bcdx}{2\,b^2} - \frac{\sqrt{3}\left(2\,\left(-ab^2\right)^{\frac{1}{3}}ab^2cd - \left(-ab^2\right)^{\frac{2}{3}}abd^2\right)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^4}$$
$$- \frac{\left(2\,\left(-ab^2\right)^{\frac{1}{3}}ab^2cd + \left(-ab^2\right)^{\frac{2}{3}}abd^2\right)\ln\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,ab^4}$$
$$+ \frac{\left(ab^4d^2\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 2\,ab^4cd\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x + c)^2*x^2/(b*x^3 + a),x, algorithm="giac")
```

```
[Out] 1/3*c^2*ln(abs(b*x^3 + a))/b + 1/2*(b*d^2*x^2 + 4*b*c*d*x)/b^2 -
1/3*sqrt(3)*(2*(-a*b^2)^(1/3)*a*b^2*c*d - (-a*b^2)^(2/3)*a*b*d^2)
*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^4) -
1/6*(2*(-a*b^2)^(1/3)*a*b^2*c*d + (-a*b^2)^(2/3)*a*b*d^2)*ln(x^2
+ x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^4) + 1/3*(a*b^4*d^2*(-a/b)^
(1/3) + 2*a*b^4*c*d)*(-a/b)^(1/3)*ln(abs(x - (-a/b)^(1/3)))/(a*b^
5)
```

3.335 $\int \frac{-x+2x^3+4x^5}{(3+2x^2+x^4)^2} dx$

Optimal. Leaf size=45

$$\frac{9\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}+\frac{5-7x^2}{8(x^4+2x^2+3)}$$

```
[Out] (5 - 7*x^2)/(8*(3 + 2*x^2 + x^4)) + (9*ArcTan[(1 + x^2)/Sqrt[2]])
/(8*Sqrt[2])
```

Rubi [A] time = 0.111402, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$\frac{9\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}+\frac{5-7x^2}{8(x^4+2x^2+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(-x + 2*x^3 + 4*x^5)/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] (5 - 7*x^2)/(8*(3 + 2*x^2 + x^4)) + (9*ArcTan[(1 + x^2)/Sqrt[2]])
/(8*Sqrt[2])
```

Rubi in Sympy [A] time = 23.4117, size = 41, normalized size = 0.91

$$\frac{-14x^2+10}{16(x^4+2x^2+3)}+\frac{9\sqrt{2}\operatorname{atan}\left(\sqrt{2}\left(\frac{x^2}{2}+\frac{1}{2}\right)\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((4*x**5+2*x**3-x)/(x**4+2*x**2+3)**2,x)
```

```
[Out] (-14*x**2 + 10)/(16*(x**4 + 2*x**2 + 3)) + 9*sqrt(2)*atan(sqrt(2)
*(x**2/2 + 1/2))/16
```

Mathematica [A] time = 0.0421578, size = 45, normalized size = 1.

$$\frac{9\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}+\frac{5-7x^2}{8(x^4+2x^2+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x + 2*x^3 + 4*x^5)/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] (5 - 7*x^2)/(8*(3 + 2*x^2 + x^4)) + (9*ArcTan[(1 + x^2)/Sqrt[2]])
/(8*Sqrt[2])
```

Maple [A] time = 0.013, size = 41, normalized size = 0.9

$$\frac{1}{2x^4+4x^2+6}\left(-\frac{7x^2}{4}+\frac{5}{4}\right)+\frac{9\sqrt{2}}{16}\arctan\left(\frac{(2x^2+2)\sqrt{2}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((4*x^5+2*x^3-x)/(x^4+2*x^2+3)^2,x)
```

```
[Out] 1/2*(-7/4*x^2+5/4)/(x^4+2*x^2+3)+9/16*2^(1/2)*arctan(1/4*(2*x^2+2
)*2^(1/2))
```

Maxima [A] time = 0.92412, size = 51, normalized size = 1.13

$$\frac{9}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x^2+1)\right)-\frac{7x^2-5}{8(x^4+2x^2+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^5 + 2*x^3 - x)/(x^4 + 2*x^2 + 3)^2,x, algorithm="maxima")
```

```
[Out] 9/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 1/8*(7*x^2 - 5)/(x^4
 + 2*x^2 + 3)
```

Fricas [A] time = 0.267367, size = 72, normalized size = 1.6

$$\frac{\sqrt{2}\left(9\left(x^4+2x^2+3\right)\arctan\left(\frac{1}{2}\sqrt{2}(x^2+1)\right)-\sqrt{2}(7x^2-5)\right)}{16(x^4+2x^2+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^5 + 2*x^3 - x)/(x^4 + 2*x^2 + 3)^2,x, algorithm="fricas")
```

```
[Out] 1/16*sqrt(2)*(9*(x^4 + 2*x^2 + 3)*arctan(1/2*sqrt(2)*(x^2 + 1)) -
 sqrt(2)*(7*x^2 - 5))/(x^4 + 2*x^2 + 3)
```

Sympy [A] time = 0.397817, size = 44, normalized size = 0.98

$$-\frac{7x^2-5}{8x^4+16x^2+24}+\frac{9\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x**5+2*x**3-x)/(x**4+2*x**2+3)**2,x)
```

```
[Out] -(7*x**2 - 5)/(8*x**4 + 16*x**2 + 24) + 9*sqrt(2)*atan(sqrt(2)*x*
*2/2 + sqrt(2)/2)/16
```

GIAC/XCAS [A] time = 0.262093, size = 51, normalized size = 1.13

$$\frac{9}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x^2+1)\right)-\frac{7x^2-5}{8(x^4+2x^2+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^5 + 2*x^3 - x)/(x^4 + 2*x^2 + 3)^2,x, algorithm="giac")
```

```
[Out] 9/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 1/8*(7*x^2 - 5)/(x^4
 + 2*x^2 + 3)
```

3.336 $\int \frac{x+x^5}{(1+2x^2+2x^4)^3}\,dx$

Optimal. Leaf size=59

$$\tan^{-1}\left(2x^2+1\right)+\frac{2x^2+1}{2\left(2x^4+2x^2+1\right)}+\frac{4x^2+3}{16\left(2x^4+2x^2+1\right)^2}$$

```
[Out] (3 + 4*x^2)/(16*(1 + 2*x^2 + 2*x^4)^2) + (1 + 2*x^2)/(2*(1 + 2*x^
2 + 2*x^4)) + ArcTan[1 + 2*x^2]
```

Rubi [A] time = 0.110603, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35

$$\tan^{-1}\left(2x^2+1\right)+\frac{2x^2+1}{2\left(2x^4+2x^2+1\right)}+\frac{4x^2+3}{16\left(2x^4+2x^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x + x^5)/(1 + 2*x^2 + 2*x^4)^3,x]
```

```
[Out] (3 + 4*x^2)/(16*(1 + 2*x^2 + 2*x^4)^2) + (1 + 2*x^2)/(2*(1 + 2*x^
2 + 2*x^4)) + ArcTan[1 + 2*x^2]
```

Rubi in Sympy [A] time = 16.4578, size = 48, normalized size = 0.81

$$\frac{4x^2+2}{4\left(2x^4+2x^2+1\right)}+\frac{8x^2+6}{32\left(2x^4+2x^2+1\right)^2}+\operatorname{atan}\left(2x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**5+x)/(2*x**4+2*x**2+1)**3,x)
```

```
[Out] (4*x**2 + 2)/(4*(2*x**4 + 2*x**2 + 1)) + (8*x**2 + 6)/(32*(2*x**4
 + 2*x**2 + 1)**2) + atan(2*x**2 + 1)
```

Mathematica [A] time = 0.0348983, size = 44, normalized size = 0.75

$$\tan^{-1}\left(2x^2+1\right)+\frac{32x^6+48x^4+36x^2+11}{16\left(2x^4+2x^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x + x^5)/(1 + 2*x^2 + 2*x^4)^3,x]
```

```
[Out] (11 + 36*x^2 + 48*x^4 + 32*x^6)/(16*(1 + 2*x^2 + 2*x^4)^2) + ArcT
an[1 + 2*x^2]
```

Maple [A] time = 0.019, size = 41, normalized size = 0.7

$$2\,\frac{1}{\left(2\,x^4+2\,x^2+1\right)^2}\left(x^6+3/2\,x^4+\frac{9\,x^2}{8}+\frac{11}{32}\right)+\arctan\left(2\,x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^5+x)/(2*x^4+2*x^2+1)^3,x)
```

```
[Out] 2*(x^6+3/2*x^4+9/8*x^2+11/32)/(2*x^4+2*x^2+1)^2+arctan(2*x^2+1)
```

Maxima [A] time = 0.87856, size = 70, normalized size = 1.19

$$\frac{32\,x^6+48\,x^4+36\,x^2+11}{16\,(4\,x^8+8\,x^6+8\,x^4+4\,x^2+1)}+\arctan\left(2\,x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^5 + x)/(2*x^4 + 2*x^2 + 1)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(32*x^6 + 48*x^4 + 36*x^2 + 11)/(4*x^8 + 8*x^6 + 8*x^4 + 4*x
^2 + 1) + arctan(2*x^2 + 1)
```

Fricas [A] time = 0.261244, size = 101, normalized size = 1.71

$$\frac{32\,x^6+48\,x^4+36\,x^2+16\left(4\,x^8+8\,x^6+8\,x^4+4\,x^2+1\right)\arctan\left(2\,x^2+1\right)+11}{16\,(4\,x^8+8\,x^6+8\,x^4+4\,x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^5 + x)/(2*x^4 + 2*x^2 + 1)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(32*x^6 + 48*x^4 + 36*x^2 + 16*(4*x^8 + 8*x^6 + 8*x^4 + 4*x^
2 + 1)*arctan(2*x^2 + 1) + 11)/(4*x^8 + 8*x^6 + 8*x^4 + 4*x^2 + 1
)
```

Sympy [A] time = 0.558776, size = 46, normalized size = 0.78

$$\frac{32x^6+48x^4+36x^2+11}{64x^8+128x^6+128x^4+64x^2+16}+\operatorname{atan}\left(2x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**5+x)/(2*x**4+2*x**2+1)**3,x)
```

```
[Out] (32*x**6 + 48*x**4 + 36*x**2 + 11)/(64*x**8 + 128*x**6 + 128*x**4
 + 64*x**2 + 16) + atan(2*x**2 + 1)
```

GIAC/XCAS [A] time = 0.259366, size = 57, normalized size = 0.97

$$\frac{32\,x^6+48\,x^4+36\,x^2+11}{16\,(2\,x^4+2\,x^2+1)^2}+\arctan\left(2\,x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^5 + x)/(2*x^4 + 2*x^2 + 1)^3,x, algorithm="giac")
```

```
[Out] 1/16*(32*x^6 + 48*x^4 + 36*x^2 + 11)/(2*x^4 + 2*x^2 + 1)^2 + arct
an(2*x^2 + 1)
```

3.337 $\int \frac{a+bx+cx^2}{d+ex^2+fx^4} dx$

Optimal. Leaf size=209

$$\frac{\left(c-\frac{ce-2af}{\sqrt{e^2-4df}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{e-\sqrt{e^2-4df}}}\right)}{\sqrt{2}\sqrt{f}\sqrt{e-\sqrt{e^2-4df}}}+\frac{\left(\frac{ce-2af}{\sqrt{e^2-4df}}+c\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{\sqrt{e^2-4df}+e}}\right)}{\sqrt{2}\sqrt{f}\sqrt{\sqrt{e^2-4df}+e}}-\frac{b\tanh^{-1}\left(\frac{e+2fx^2}{\sqrt{e^2-4df}}\right)}{\sqrt{e^2-4df}}$$

```
[Out] ((c - (c*e - 2*a*f)/Sqrt[e^2 - 4*d*f])*ArcTan[(Sqrt[2]*Sqrt[f]*x)
/Sqrt[e - Sqrt[e^2 - 4*d*f]]])/(Sqrt[2]*Sqrt[f]*Sqrt[e - Sqrt[e^2
 - 4*d*f]]) + ((c + (c*e - 2*a*f)/Sqrt[e^2 - 4*d*f])*ArcTan[(Sqrt
[2]*Sqrt[f]*x)/Sqrt[e + Sqrt[e^2 - 4*d*f]]])/(Sqrt[2]*Sqrt[f]*Sqr
t[e + Sqrt[e^2 - 4*d*f]]) - (b*ArcTanh[(e + 2*f*x^2)/Sqrt[e^2 - 4
*d*f]])/Sqrt[e^2 - 4*d*f]
```

Rubi [A] time = 0.869576, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28

$$\frac{\left(c-\frac{ce-2af}{\sqrt{e^2-4df}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{e-\sqrt{e^2-4df}}}\right)}{\sqrt{2}\sqrt{f}\sqrt{e-\sqrt{e^2-4df}}}+\frac{\left(\frac{ce-2af}{\sqrt{e^2-4df}}+c\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{\sqrt{e^2-4df}+e}}\right)}{\sqrt{2}\sqrt{f}\sqrt{\sqrt{e^2-4df}+e}}-\frac{b\tanh^{-1}\left(\frac{e+2fx^2}{\sqrt{e^2-4df}}\right)}{\sqrt{e^2-4df}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x + c*x^2)/(d + e*x^2 + f*x^4),x]
```

```
[Out] ((c - (c*e - 2*a*f)/Sqrt[e^2 - 4*d*f])*ArcTan[(Sqrt[2]*Sqrt[f]*x)
/Sqrt[e - Sqrt[e^2 - 4*d*f]]])/(Sqrt[2]*Sqrt[f]*Sqrt[e - Sqrt[e^2
 - 4*d*f]]) + ((c + (c*e - 2*a*f)/Sqrt[e^2 - 4*d*f])*ArcTan[(Sqrt
[2]*Sqrt[f]*x)/Sqrt[e + Sqrt[e^2 - 4*d*f]]])/(Sqrt[2]*Sqrt[f]*Sqr
t[e + Sqrt[e^2 - 4*d*f]]) - (b*ArcTanh[(e + 2*f*x^2)/Sqrt[e^2 - 4
*d*f]])/Sqrt[e^2 - 4*d*f]
```

Rubi in Sympy [A] time = 60.4013, size = 221, normalized size = 1.06

$$-\frac{b\,\text{atanh}\left(\frac{e+2fx^2}{\sqrt{-4df+e^2}}\right)}{\sqrt{-4df+e^2}}-\frac{\sqrt{2}\left(2af-ce-c\sqrt{-4df+e^2}\right)\text{atan}\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{e+\sqrt{-4df+e^2}}}\right)}{2\sqrt{f}\sqrt{e+\sqrt{-4df+e^2}}\sqrt{-4df+e^2}}+\frac{\sqrt{2}\left(2af-ce+c\sqrt{-4df+e^2}\right)\text{atan}\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{e-\sqrt{-4df+e^2}}}\right)}{2\sqrt{f}\sqrt{e-\sqrt{-4df+e^2}}\sqrt{-4df+e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**2+b*x+a)/(f*x**4+e*x**2+d),x)
```

```
[Out] -b*atanh((e + 2*f*x**2)/sqrt(-4*d*f + e**2))/sqrt(-4*d*f + e**2)
- sqrt(2)*(2*a*f - c*e - c*sqrt(-4*d*f + e**2))*atan(sqrt(2)*sqrt
(f)*x/sqrt(e + sqrt(-4*d*f + e**2)))/(2*sqrt(f)*sqrt(e + sqrt(-4*
d*f + e**2))*sqrt(-4*d*f + e**2)) + sqrt(2)*(2*a*f - c*e + c*sqrt
(-4*d*f + e**2))*atan(sqrt(2)*sqrt(f)*x/sqrt(e - sqrt(-4*d*f + e*
*2)))/(2*sqrt(f)*sqrt(e - sqrt(-4*d*f + e**2))*sqrt(-4*d*f + e**2
```

```
))
```

Mathematica [A] time = 0.425308, size = 234, normalized size = 1.12

$$\frac{\frac{\sqrt{2}\left(2af+c\left(\sqrt{e^2-4df}-e\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{e-\sqrt{e^2-4df}}}\right)}{\sqrt{f}\sqrt{e-\sqrt{e^2-4df}}}+\frac{\sqrt{2}\left(c\left(\sqrt{e^2-4df}+e\right)-2af\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{\sqrt{e^2-4df}+e}}\right)}{\sqrt{f}\sqrt{\sqrt{e^2-4df}+e}}+b\log\left(\sqrt{e^2-4df}-e-2fx^2\right)-b\log\left(\sqrt{e^2-4df}+e+2fx^2\right)}{2\sqrt{e^2-4df}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x + c*x^2)/(d + e*x^2 + f*x^4),x]
```

```
[Out] ((Sqrt[2]*(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f]))*ArcTan[(Sqrt[2]*Sq
rt[f]*x)/Sqrt[e - Sqrt[e^2 - 4*d*f]]])/(Sqrt[f]*Sqrt[e - Sqrt[e^2
 - 4*d*f]]) + (Sqrt[2]*(-2*a*f + c*(e + Sqrt[e^2 - 4*d*f]))*ArcTa
n[(Sqrt[2]*Sqrt[f]*x)/Sqrt[e + Sqrt[e^2 - 4*d*f]]])/(Sqrt[f]*Sqrt
[e + Sqrt[e^2 - 4*d*f]]) + b*Log[-e + Sqrt[e^2 - 4*d*f] - 2*f*x^2
] - b*Log[e + Sqrt[e^2 - 4*d*f] + 2*f*x^2])/(2*Sqrt[e^2 - 4*d*f])
```

Maple [B] time = 0.073, size = 616, normalized size = 3.

$$
\begin{aligned}
&-\frac{b}{8\,df-2\,e^{2}}\sqrt{-4\,df+e^{2}}\ln\left(-2\,fx^{2}+\sqrt{-4\,df+e^{2}}-e\right)\\
&-2\,\frac{f\sqrt{2}cd}{\left(4\,df-e^{2}\right)\sqrt{\left(-e+\sqrt{-4\,df+e^{2}}\right)f}}\mathit{Artanh}\left(\frac{fx\sqrt{2}}{\sqrt{\left(-e+\sqrt{-4\,df+e^{2}}\right)f}}\right)\\
&+\frac{c\sqrt{2}e^{2}}{8\,df-2\,e^{2}}\mathit{Artanh}\left(fx\sqrt{2}\frac{1}{\sqrt{\left(-e+\sqrt{-4\,df+e^{2}}\right)f}}\right)\frac{1}{\sqrt{\left(-e+\sqrt{-4\,df+e^{2}}\right)f}}\\
&+\frac{f\sqrt{2}a}{4\,df-e^{2}}\sqrt{-4\,df+e^{2}}\mathit{Artanh}\left(fx\sqrt{2}\frac{1}{\sqrt{\left(-e+\sqrt{-4\,df+e^{2}}\right)f}}\right)\frac{1}{\sqrt{\left(-e+\sqrt{-4\,df+e^{2}}\right)f}}\\
&-\frac{c\sqrt{2}e}{8\,df-2\,e^{2}}\sqrt{-4\,df+e^{2}}\mathit{Artanh}\left(fx\sqrt{2}\frac{1}{\sqrt{\left(-e+\sqrt{-4\,df+e^{2}}\right)f}}\right)\frac{1}{\sqrt{\left(-e+\sqrt{-4\,df+e^{2}}\right)f}}\\
&+\frac{b}{8\,df-2\,e^{2}}\sqrt{-4\,df+e^{2}}\ln\left(2\,fx^{2}+\sqrt{-4\,df+e^{2}}+e\right)\\
&+2\,\frac{f\sqrt{2}cd}{\left(4\,df-e^{2}\right)\sqrt{\left(e+\sqrt{-4\,df+e^{2}}\right)f}}\arctan\left(\frac{fx\sqrt{2}}{\sqrt{\left(e+\sqrt{-4\,df+e^{2}}\right)f}}\right)\\
&-\frac{c\sqrt{2}e^{2}}{8\,df-2\,e^{2}}\arctan\left(fx\sqrt{2}\frac{1}{\sqrt{\left(e+\sqrt{-4\,df+e^{2}}\right)f}}\right)\frac{1}{\sqrt{\left(e+\sqrt{-4\,df+e^{2}}\right)f}}\\
&+\frac{f\sqrt{2}a}{4\,df-e^{2}}\sqrt{-4\,df+e^{2}}\arctan\left(fx\sqrt{2}\frac{1}{\sqrt{\left(e+\sqrt{-4\,df+e^{2}}\right)f}}\right)\frac{1}{\sqrt{\left(e+\sqrt{-4\,df+e^{2}}\right)f}}\\
&-\frac{c\sqrt{2}e}{8\,df-2\,e^{2}}\sqrt{-4\,df+e^{2}}\arctan\left(fx\sqrt{2}\frac{1}{\sqrt{\left(e+\sqrt{-4\,df+e^{2}}\right)f}}\right)\frac{1}{\sqrt{\left(e+\sqrt{-4\,df+e^{2}}\right)f}}
\end{aligned}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((c*x^2+b*x+a)/(f*x^4+e*x^2+d),x)
```

```
[Out] -1/2*(-4*d*f+e^2)^(1/2)/(4*d*f-e^2)*b*ln(-2*f*x^2+(-4*d*f+e^2)^(1
/2)-e)-2*f/(4*d*f-e^2)*2^(1/2)/((-e+(-4*d*f+e^2)^(1/2))*f)^(1/2)*
arctanh(x*f*2^(1/2)/((-e+(-4*d*f+e^2)^(1/2))*f)^(1/2))*c*d+1/2/(4
*d*f-e^2)*2^(1/2)/((-e+(-4*d*f+e^2)^(1/2))*f)^(1/2)*arctanh(x*f*2
^(1/2)/((-e+(-4*d*f+e^2)^(1/2))*f)^(1/2))*c*e^2+f*(-4*d*f+e^2)^(1
/2)/(4*d*f-e^2)*2^(1/2)/((-e+(-4*d*f+e^2)^(1/2))*f)^(1/2)*arctanh
(x*f*2^(1/2)/((-e+(-4*d*f+e^2)^(1/2))*f)^(1/2))*a-1/2*(-4*d*f+e^2
)^(1/2)/(4*d*f-e^2)*2^(1/2)/((-e+(-4*d*f+e^2)^(1/2))*f)^(1/2)*arc
tanh(x*f*2^(1/2)/((-e+(-4*d*f+e^2)^(1/2))*f)^(1/2))*c*e+1/2*(-4*d
*f+e^2)^(1/2)/(4*d*f-e^2)*b*ln(2*f*x^2+(-4*d*f+e^2)^(1/2)+e)+2*f/
(4*d*f-e^2)*2^(1/2)/((e+(-4*d*f+e^2)^(1/2))*f)^(1/2)*arctan(x*f*2
^(1/2)/((e+(-4*d*f+e^2)^(1/2))*f)^(1/2))*c*d-1/2/(4*d*f-e^2)*2^(1
/2)/((e+(-4*d*f+e^2)^(1/2))*f)^(1/2)*arctan(x*f*2^(1/2)/((e+(-4*d
*f+e^2)^(1/2))*f)^(1/2))*c*e^2+f*(-4*d*f+e^2)^(1/2)/(4*d*f-e^2)*2
^(1/2)/((e+(-4*d*f+e^2)^(1/2))*f)^(1/2)*arctan(x*f*2^(1/2)/((e+(-
4*d*f+e^2)^(1/2))*f)^(1/2))*a-1/2*(-4*d*f+e^2)^(1/2)/(4*d*f-e^2)*
2^(1/2)/((e+(-4*d*f+e^2)^(1/2))*f)^(1/2)*arctan(x*f*2^(1/2)/((e+(
```

```
-4*d*f+e^2)^(1/2))*f)^(1/2))*c*e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{cx^2 + bx + a}{fx^4 + ex^2 + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2 + b*x + a)/(f*x^4 + e*x^2 + d),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)/(f*x^4 + e*x^2 + d), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2 + b*x + a)/(f*x^4 + e*x^2 + d),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**2+b*x+a)/(f*x**4+e*x**2+d),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 1.0255, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x + a)/(f*x^4 + e*x^2 + d),x, algorithm="giac")
```

```
[Out] Done
```

3.338 $\int \frac{(d+ex)^2}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=224

$$\frac{\left(\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}+e^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(e^2-\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{2de\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

```
[Out] ((e^2 + (2*c*d^2 - b*e^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt
[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sq
rt[b^2 - 4*a*c]]) + ((e^2 - (2*c*d^2 - b*e^2)/Sqrt[b^2 - 4*a*c])*
ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]
*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (2*d*e*ArcTanh[(b + 2*c*x
^2)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]
```

Rubi [A] time = 0.66834, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318

$$\frac{\left(\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}+e^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(e^2-\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{2de\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x)^2/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((e^2 + (2*c*d^2 - b*e^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt
[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sq
rt[b^2 - 4*a*c]]) + ((e^2 - (2*c*d^2 - b*e^2)/Sqrt[b^2 - 4*a*c])*
ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]
*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (2*d*e*ArcTanh[(b + 2*c*x
^2)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]
```

Rubi in Sympy [A] time = 61.612, size = 235, normalized size = 1.05

$$-\frac{2de\,\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{\sqrt{-4ac+b^2}}+\frac{\sqrt{2}\left(be^2-2cd^2+e^2\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{2\sqrt{c}\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}-\frac{\sqrt{2}\left(be^2-2cd^2-e^2\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{2\sqrt{c}\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x+d)**2/(c*x**4+b*x**2+a),x)
```

```
[Out] -2*d*e*atanh((b + 2*c*x**2)/sqrt(-4*a*c + b**2))/sqrt(-4*a*c + b*
*2) + sqrt(2)*(b*e**2 - 2*c*d**2 + e**2*sqrt(-4*a*c + b**2))*atan
(sqrt(2)*sqrt(c)*x/sqrt(b + sqrt(-4*a*c + b**2)))/(2*sqrt(c)*sqrt
(b + sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2)) - sqrt(2)*(b*e**2
- 2*c*d**2 - e**2*sqrt(-4*a*c + b**2))*atan(sqrt(2)*sqrt(c)*x/sqr
t(b - sqrt(-4*a*c + b**2)))/(2*sqrt(c)*sqrt(b - sqrt(-4*a*c + b**
2))*sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 0.453746, size = 245, normalized size = 1.09

$$\frac{\frac{\sqrt{2}\left(e^2\left(\sqrt{b^2-4ac}-b\right)+2cd^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(e^2\left(\sqrt{b^2-4ac}+b\right)-2cd^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}+2de\log\left(\sqrt{b^2-4ac}-b-2cx^2\right)-2de\,\mathrm{lo}}{2\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x)^2/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((Sqrt[2]*(2*c*d^2 + (-b + Sqrt[b^2 - 4*a*c])*e^2)*ArcTan[(Sqrt[2
]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*Sqrt[b - Sqrt
[b^2 - 4*a*c]]) + (Sqrt[2]*(-2*c*d^2 + (b + Sqrt[b^2 - 4*a*c])*e^
2)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt
[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + 2*d*e*Log[-b + Sqrt[b^2 - 4*a*
c] - 2*c*x^2] - 2*d*e*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(2*Sq
rt[b^2 - 4*a*c])
```

Maple [B] time = 0.039, size = 633, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x+d)^2/(c*x^4+b*x^2+a),x)
```

```
[Out] (-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*d*e*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+
b)+2*c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
n(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*e^2*a-1/2/(4*a*c-
b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*e^2*b^2-1/2*(-4*a*c+b^2)^(1/2)/
(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*e^2+c*(-4*a*c+b^2)^(1/
2)/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*
x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*d^2-(-4*a*c+b^2)^(1/2
)/(4*a*c-b^2)*d*e*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)-2*c/(4*a*c-b^
2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/
((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*e^2*a+1/2/(4*a*c-b^2)*2^(1/2)/
((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*
c+b^2)^(1/2))*c)^(1/2))*e^2*b^2-1/2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2
)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/(
(-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*e^2+c*(-4*a*c+b^2)^(1/2)/(4*a
*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(
1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*d^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^2}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)^2/(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^2/(c*x^4 + b*x^2 + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)^2/(c*x^4 + b*x^2 + a),x, algorithm="fricas")

[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x+d)**2/(c*x**4+b*x**2+a),x)

[Out] Timed out
```

GIAC/XCAS [A] time = 1.04054, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)^2/(c*x^4 + b*x^2 + a),x, algorithm="giac")

[Out] Done
```

3.339 $\int \frac{x^2}{(a+bx)(c+dx)}\,dx$

Optimal. Leaf size=56

$$\frac{a^2\log(a+bx)}{b^2(bc-ad)}-\frac{c^2\log(c+dx)}{d^2(bc-ad)}+\frac{x}{bd}$$

```
[Out] x/(b*d) + (a^2*Log[a + b*x])/(b^2*(b*c - a*d)) - (c^2*Log[c + d*x
])/(d^2*(b*c - a*d))
```

Rubi [A] time = 0.10977, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056

$$\frac{a^2\log(a+bx)}{b^2(bc-ad)}-\frac{c^2\log(c+dx)}{d^2(bc-ad)}+\frac{x}{bd}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/((a + b*x)*(c + d*x)),x]
```

```
[Out] x/(b*d) + (a^2*Log[a + b*x])/(b^2*(b*c - a*d)) - (c^2*Log[c + d*x
])/(d^2*(b*c - a*d))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^2\log\left(a+bx\right)}{b^2\left(ad-bc\right)}+\frac{c^2\log\left(c+dx\right)}{d^2\left(ad-bc\right)}+\frac{\int\frac{1}{b}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(b*x+a)/(d*x+c),x)
```

```
[Out] -a**2*log(a + b*x)/(b**2*(a*d - b*c)) + c**2*log(c + d*x)/(d**2*(
a*d - b*c)) + Integral(1/b, x)/d
```

Mathematica [A] time = 0.0427302, size = 56, normalized size = 1.

$$\frac{a^2\log(a+bx)}{b^2(bc-ad)}-\frac{c^2\log(c+dx)}{d^2(bc-ad)}+\frac{x}{bd}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/((a + b*x)*(c + d*x)),x]
```

```
[Out] x/(b*d) + (a^2*Log[a + b*x])/(b^2*(b*c - a*d)) - (c^2*Log[c + d*x
])/(d^2*(b*c - a*d))
```

Maple [A] time = 0.009, size = 57, normalized size = 1.

$$\frac{x}{bd}+\frac{c^2\ln\left(dx+c\right)}{d^2\left(ad-bc\right)}-\frac{a^2\ln\left(bx+a\right)}{b^2\left(ad-bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(b*x+a)/(d*x+c),x)
```

```
[Out] x/b/d+1/d^2*c^2/(a*d-b*c)*ln(d*x+c)-1/b^2*a^2/(a*d-b*c)*ln(b*x+a)
```

Maxima [A] time = 0.818043, size = 81, normalized size = 1.45

$$\frac{a^2 \log(bx + a)}{b^3c - ab^2d} - \frac{c^2 \log(dx + c)}{bcd^2 - ad^3} + \frac{x}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((b*x + a)*(d*x + c)),x, algorithm="maxima")
```

```
[Out] a^2*log(b*x + a)/(b^3*c - a*b^2*d) - c^2*log(d*x + c)/(b*c*d^2 -
a*d^3) + x/(b*d)
```

Fricas [A] time = 0.296374, size = 88, normalized size = 1.57

$$\frac{a^2d^2 \log(bx + a) - b^2c^2 \log(dx + c) + \left(b^2cd - abd^2\right)x}{b^3cd^2 - ab^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((b*x + a)*(d*x + c)),x, algorithm="fricas")
```

```
[Out] (a^2*d^2*log(b*x + a) - b^2*c^2*log(d*x + c) + (b^2*c*d - a*b*d^2
)*x)/(b^3*c*d^2 - a*b^2*d^3)
```

Sympy [A] time = 4.04331, size = 190, normalized size = 3.39

$$-\frac{a^2 \log\left(x + \frac{\frac{a^4d^3}{b(ad-bc)} - \frac{2a^3cd^2}{ad-bc} + \frac{a^2bc^2d}{ad-bc} + a^2cd + abc^2}{a^2d^2+b^2c^2}\right)}{b^2(ad - bc)} + \frac{c^2 \log\left(x + \frac{-\frac{a^2bc^2d}{ad-bc} + a^2cd + \frac{2ab^2c^3}{ad-bc} + abc^2 - \frac{b^3c^4}{d(ad-bc)}}{a^2d^2+b^2c^2}\right)}{d^2(ad - bc)} + \frac{x}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(b*x+a)/(d*x+c),x)
```

```
[Out] -a**2*log(x + (a**4*d**3/(b*(a*d - b*c)) - 2*a**3*c*d**2/(a*d - b
*c) + a**2*b*c**2*d/(a*d - b*c) + a**2*c*d + a*b*c**2)/(a**2*d**2
 + b**2*c**2))/(b**2*(a*d - b*c)) + c**2*log(x + (-a**2*b*c**2*d/
(a*d - b*c) + a**2*c*d + 2*a*b**2*c**3/(a*d - b*c) + a*b*c**2 - b
**3*c**4/(d*(a*d - b*c)))/(a**2*d**2 + b**2*c**2))/(d**2*(a*d - b
*c)) + x/(b*d)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((b*x + a)*(d*x + c)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.340 $\int \frac{x^2}{(c+dx)(a+bx^2)}\,dx$

Optimal. Leaf size=96

$$\frac{ad\log\left(a+bx^2\right)}{2b\left(ad^2+bc^2\right)}+\frac{c^2\log(c+dx)}{d\left(ad^2+bc^2\right)}-\frac{\sqrt{a}c\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}\left(ad^2+bc^2\right)}$$

```
[Out] -((Sqrt[a]*c*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[b]*(b*c^2 + a*d^2
))) + (c^2*Log[c + d*x])/(d*(b*c^2 + a*d^2)) + (a*d*Log[a + b*x^2
])/(2*b*(b*c^2 + a*d^2))
```

Rubi [A] time = 0.203124, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{ad\log\left(a+bx^2\right)}{2b\left(ad^2+bc^2\right)}+\frac{c^2\log(c+dx)}{d\left(ad^2+bc^2\right)}-\frac{\sqrt{a}c\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}\left(ad^2+bc^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((c + d*x)*(a + b*x^2)),x]
```

```
[Out] -((Sqrt[a]*c*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[b]*(b*c^2 + a*d^2
))) + (c^2*Log[c + d*x])/(d*(b*c^2 + a*d^2)) + (a*d*Log[a + b*x^2
])/(2*b*(b*c^2 + a*d^2))
```

Rubi in Sympy [A] time = 23.6938, size = 82, normalized size = 0.85

$$-\frac{\sqrt{a}c\,\mathrm{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}\left(ad^2+bc^2\right)}+\frac{ad\log\left(a+bx^2\right)}{2b\left(ad^2+bc^2\right)}+\frac{c^2\log\left(c+dx\right)}{d\left(ad^2+bc^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**2/(d*x+c)/(b*x**2+a),x)
```

```
[Out] -sqrt(a)*c*atan(sqrt(b)*x/sqrt(a))/(sqrt(b)*(a*d**2 + b*c**2)) +
a*d*log(a + b*x**2)/(2*b*(a*d**2 + b*c**2)) + c**2*log(c + d*x)/(
d*(a*d**2 + b*c**2))
```

Mathematica [A] time = 0.0560853, size = 73, normalized size = 0.76

$$\frac{-2\sqrt{a}\sqrt{b}cd\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)+ad^2\log\left(a+bx^2\right)+2bc^2\log(c+dx)}{2abd^3+2b^2c^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((c + d*x)*(a + b*x^2)),x]
```

```
[Out] (-2*Sqrt[a]*Sqrt[b]*c*d*ArcTan[(Sqrt[b]*x)/Sqrt[a]] + 2*b*c^2*Log
[c + d*x] + a*d^2*Log[a + b*x^2])/(2*b^2*c^2*d + 2*a*b*d^3)
```

Maple [A] time = 0.009, size = 87, normalized size = 0.9

$$\frac{ad\ln\left(bx^2+a\right)}{2b\left(ad^2+c^2b\right)}-\frac{ac}{ad^2+c^2b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{c^2\ln\left(dx+c\right)}{d\left(ad^2+c^2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(d*x+c)/(b*x^2+a),x)
```

```
[Out] 1/2*a*d*ln(b*x^2+a)/b/(a*d^2+b*c^2)-a/(a*d^2+b*c^2)*c/(a*b)^(1/2)
*arctan(x*b/(a*b)^(1/2))+c^2*ln(d*x+c)/d/(a*d^2+b*c^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((b*x^2 + a)*(d*x + c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.292032, size = 1, normalized size = 0.01

$$\left[\frac{bcd\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+ad^2\log\left(bx^2+a\right)+2\,bc^2\log\left(dx+c\right)}{2\left(b^2c^2d+abd^3\right)},\right.$$
$$\left.-\frac{2\,bcd\sqrt{\frac{a}{b}}\arctan\left(\frac{x}{\sqrt{\frac{a}{b}}}\right)-ad^2\log\left(bx^2+a\right)-2\,bc^2\log\left(dx+c\right)}{2\left(b^2c^2d+abd^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((b*x^2 + a)*(d*x + c)),x, algorithm="fricas")
```

```
[Out] [1/2*(b*c*d*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2
+ a)) + a*d^2*log(b*x^2 + a) + 2*b*c^2*log(d*x + c))/(b^2*c^2*d +
 a*b*d^3), -1/2*(2*b*c*d*sqrt(a/b)*arctan(x/sqrt(a/b)) - a*d^2*lo
g(b*x^2 + a) - 2*b*c^2*log(d*x + c))/(b^2*c^2*d + a*b*d^3)]
```

Sympy [A] time = 22.4507, size = 1355, normalized size = 14.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(d*x+c)/(b*x**2+a),x)
```

```
[Out] c**2*log(x + (-4*a**3*b*c**4*d**5/(a*d**2 + b*c**2)**2 + 2*a**3*c
**2*d**5/(a*d**2 + b*c**2) + 4*a**2*b**2*c**6*d**3/(a*d**2 + b*c*
```

```
*2)**2 - 4*a**2*b*c**4*d**3/(a*d**2 + b*c**2) - a**2*c**2*d**3 +
20*a*b**3*c**8*d/(a*d**2 + b*c**2)**2 - 14*a*b**2*c**6*d/(a*d**2
+ b*c**2) + 7*a*b*c**4*d + 12*b**4*c**10/(d*(a*d**2 + b*c**2)**2)
 - 8*b**3*c**8/(d*(a*d**2 + b*c**2)))/(a**2*c*d**4 - 3*a*b*c**3*d
**2 + 4*b**2*c**5))/(d*(a*d**2 + b*c**2)) + (a*d/(2*b*(a*d**2 + b
*c**2)) - c*sqrt(-a*b**3)/(2*b**2*(a*d**2 + b*c**2)))*log(x + (-4
*a**3*b*d**7*(a*d/(2*b*(a*d**2 + b*c**2)) - c*sqrt(-a*b**3)/(2*b*
*2*(a*d**2 + b*c**2)))**2 + 2*a**3*d**6*(a*d/(2*b*(a*d**2 + b*c**
2)) - c*sqrt(-a*b**3)/(2*b**2*(a*d**2 + b*c**2))) + 4*a**2*b**2*c
**2*d**5*(a*d/(2*b*(a*d**2 + b*c**2)) - c*sqrt(-a*b**3)/(2*b**2*(
a*d**2 + b*c**2)))**2 - 4*a**2*b*c**2*d**4*(a*d/(2*b*(a*d**2 + b*
c**2)) - c*sqrt(-a*b**3)/(2*b**2*(a*d**2 + b*c**2))) - a**2*c**2*
d**3 + 20*a*b**3*c**4*d**3*(a*d/(2*b*(a*d**2 + b*c**2)) - c*sqrt(
-a*b**3)/(2*b**2*(a*d**2 + b*c**2)))**2 - 14*a*b**2*c**4*d**2*(a*
d/(2*b*(a*d**2 + b*c**2)) - c*sqrt(-a*b**3)/(2*b**2*(a*d**2 + b*c
**2))) + 7*a*b*c**4*d + 12*b**4*c**6*d*(a*d/(2*b*(a*d**2 + b*c**2
)) - c*sqrt(-a*b**3)/(2*b**2*(a*d**2 + b*c**2)))**2 - 8*b**3*c**6
*(a*d/(2*b*(a*d**2 + b*c**2)) - c*sqrt(-a*b**3)/(2*b**2*(a*d**2 +
 b*c**2))))/(a**2*c*d**4 - 3*a*b*c**3*d**2 + 4*b**2*c**5)) + (a*d
/(2*b*(a*d**2 + b*c**2)) + c*sqrt(-a*b**3)/(2*b**2*(a*d**2 + b*c*
*2)))*log(x + (-4*a**3*b*d**7*(a*d/(2*b*(a*d**2 + b*c**2)) + c*sq
rt(-a*b**3)/(2*b**2*(a*d**2 + b*c**2)))**2 + 2*a**3*d**6*(a*d/(2*
b*(a*d**2 + b*c**2)) + c*sqrt(-a*b**3)/(2*b**2*(a*d**2 + b*c**2))
) + 4*a**2*b**2*c**2*d**5*(a*d/(2*b*(a*d**2 + b*c**2)) + c*sqrt(-
a*b**3)/(2*b**2*(a*d**2 + b*c**2)))**2 - 4*a**2*b*c**2*d**4*(a*d/
(2*b*(a*d**2 + b*c**2)) + c*sqrt(-a*b**3)/(2*b**2*(a*d**2 + b*c**
2))) - a**2*c**2*d**3 + 20*a*b**3*c**4*d**3*(a*d/(2*b*(a*d**2 + b
*c**2)) + c*sqrt(-a*b**3)/(2*b**2*(a*d**2 + b*c**2)))**2 - 14*a*b
**2*c**4*d**2*(a*d/(2*b*(a*d**2 + b*c**2)) + c*sqrt(-a*b**3)/(2*b
**2*(a*d**2 + b*c**2))) + 7*a*b*c**4*d + 12*b**4*c**6*d*(a*d/(2*b
*(a*d**2 + b*c**2)) + c*sqrt(-a*b**3)/(2*b**2*(a*d**2 + b*c**2)))
**2 - 8*b**3*c**6*(a*d/(2*b*(a*d**2 + b*c**2)) + c*sqrt(-a*b**3)/
(2*b**2*(a*d**2 + b*c**2))))/(a**2*c*d**4 - 3*a*b*c**3*d**2 + 4*b
**2*c**5))
```

GIAC/XCAS [A] time = 0.264403, size = 115, normalized size = 1.2

$$\frac{ad\ln\left(bx^2+a\right)}{2\left(b^2c^2+abd^2\right)}+\frac{c^2\ln\left(|dx+c|\right)}{bc^2d+ad^3}-\frac{ac\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(bc^2+ad^2)\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((b*x^2 + a)*(d*x + c)),x, algorithm="giac")
```

```
[Out] 1/2*a*d*ln(b*x^2 + a)/(b^2*c^2 + a*b*d^2) + c^2*ln(abs(d*x + c))/
(b*c^2*d + a*d^3) - a*c*arctan(b*x/sqrt(a*b))/((b*c^2 + a*d^2)*sq
rt(a*b))
```

3.341 $\int \frac{x^2}{(c+dx)(a+bx^3)}\,dx$

Optimal. Leaf size=264

$$-\frac{\sqrt[3]{a}d\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{2/3}\left(bc^3-ad^3\right)}-\frac{\sqrt[3]{a}d\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{2/3}\left(a^{2/3}d^2+\sqrt[3]{a}\sqrt[3]{b}cd+b^{2/3}c^2\right)}+\frac{\sqrt[3]{a}d\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{2/3}\left(bc^3-ad^3\right)}+\frac{c^2\log\left(a+bx^3\right)}{3\left(bc^3-ad^3\right)}-\frac{c^2\log(c+dx)}{bc^3-ad^3}$$

```
[Out] -((a^(1/3)*d*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(
Sqrt[3]*b^(2/3)*(b^(2/3)*c^2 + a^(1/3)*b^(1/3)*c*d + a^(2/3)*d^2)
)) + (a^(1/3)*d*(b^(1/3)*c + a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])
/(3*b^(2/3)*(b*c^3 - a*d^3)) - (c^2*Log[c + d*x])/(b*c^3 - a*d^3)
 - (a^(1/3)*d*(b^(1/3)*c + a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2])/(6*b^(2/3)*(b*c^3 - a*d^3)) + (c^2*Log[a + b
*x^3])/(3*(b*c^3 - a*d^3))
```

Rubi [A] time = 0.960649, antiderivative size = 264, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4

$$-\frac{\sqrt[3]{a}d\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{2/3}\left(bc^3-ad^3\right)}-\frac{\sqrt[3]{a}d\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{2/3}\left(a^{2/3}d^2+\sqrt[3]{a}\sqrt[3]{b}cd+b^{2/3}c^2\right)}+\frac{\sqrt[3]{a}d\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{2/3}\left(bc^3-ad^3\right)}+\frac{c^2\log\left(a+bx^3\right)}{3\left(bc^3-ad^3\right)}-\frac{c^2\log(c+dx)}{bc^3-ad^3}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/((c + d*x)*(a + b*x^3)),x]
```

```
[Out] -((a^(1/3)*d*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(
Sqrt[3]*b^(2/3)*(b^(2/3)*c^2 + a^(1/3)*b^(1/3)*c*d + a^(2/3)*d^2)
)) + (a^(1/3)*d*(b^(1/3)*c + a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])
/(3*b^(2/3)*(b*c^3 - a*d^3)) - (c^2*Log[c + d*x])/(b*c^3 - a*d^3)
 - (a^(1/3)*d*(b^(1/3)*c + a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2])/(6*b^(2/3)*(b*c^3 - a*d^3)) + (c^2*Log[a + b
*x^3])/(3*(b*c^3 - a*d^3))
```

Rubi in Sympy [A] time = 113.591, size = 231, normalized size = 0.88

$$-\frac{\sqrt{3}\sqrt[3]{a}d\left(\sqrt[3]{a}d-\sqrt[3]{b}c\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{3b^{\frac{2}{3}}\left(ad^3-bc^3\right)}-\frac{\sqrt[3]{a}d\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{\frac{2}{3}}\left(ad^3-bc^3\right)}+\frac{\sqrt[3]{a}d\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{6b^{\frac{2}{3}}\left(ad^3-bc^3\right)}-\frac{c^2\log\left(a+bx^3\right)}{3\left(ad^3-bc^3\right)}+\frac{c^2\log\left(c+dx\right)}{ad^3-bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(d*x+c)/(b*x**3+a),x)
```

```
[Out] -sqrt(3)*a**(1/3)*d*(a**(1/3)*d - b**(1/3)*c)*atan(sqrt(3)*(a**(1
/3)/3 - 2*b**(1/3)*x/3)/a**(1/3))/(3*b**(2/3)*(a*d**3 - b*c**3))
- a**(1/3)*d*(a**(1/3)*d + b**(1/3)*c)*log(a**(1/3) + b**(1/3)*x)
/(3*b**(2/3)*(a*d**3 - b*c**3)) + a**(1/3)*d*(a**(1/3)*d + b**(1/
3)*c)*log(a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(6*b**(
2/3)*(a*d**3 - b*c**3)) - c**2*log(a + b*x**3)/(3*(a*d**3 - b*c**
3)) + c**2*log(c + d*x)/(a*d**3 - b*c**3)
```

Mathematica [A] time = 0.169607, size = 228, normalized size = 0.86

$$\frac{-\sqrt[3]{a}\sqrt[3]{b}cd\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-a^{2/3}d^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+2b^{2/3}c^2\log\left(a+bx^3\right)+2\sqrt[3]{a}d\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right.}{6b^{2/3}\left(bc^3-ad^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/((c + d*x)*(a + b*x^3)),x]
```

```
[Out] (2*Sqrt[3]*a^(1/3)*d*(-(b^(1/3)*c) + a^(1/3)*d)*ArcTan[(1 - (2*b^
(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*a^(1/3)*d*(b^(1/3)*c + a^(1/3)*d)*
Log[a^(1/3) + b^(1/3)*x] - 6*b^(2/3)*c^2*Log[c + d*x] - a^(1/3)*b
^(1/3)*c*d*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - a^(2/
3)*d^2*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 2*b^(2/3)
*c^2*Log[a + b*x^3])/(6*b^(2/3)*(b*c^3 - a*d^3))
```

Maple [A] time = 0.008, size = 336, normalized size = 1.3

$$-\frac{acd}{(3\,ad^3-3\,bc^3)\,b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{acd}{(6\,ad^3-6\,bc^3)\,b}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}$$
$$-\frac{acd\sqrt{3}}{(3\,ad^3-3\,bc^3)\,b}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}$$
$$-\frac{ad^2}{(3\,ad^3-3\,bc^3)\,b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{ad^2}{(6\,ad^3-6\,bc^3)\,b}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}$$
$$+\frac{ad^2\sqrt{3}}{(3\,ad^3-3\,bc^3)\,b}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{c^2\ln\left(bx^3+a\right)}{3\,ad^3-3\,bc^3}+\frac{c^2\ln\left(dx+c\right)}{ad^3-bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(d*x+c)/(b*x^3+a),x)
```

```
[Out] -1/3/(a*d^3-b*c^3)*a*c*d/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+1/6/(a*d
^3-b*c^3)*a*c*d/b/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))-1
/3/(a*d^3-b*c^3)*a*c*d/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(
2/(a/b)^(1/3)*x-1))-1/3/(a*d^3-b*c^3)*a*d^2/b/(a/b)^(1/3)*ln(x+(a
/b)^(1/3))+1/6/(a*d^3-b*c^3)*a*d^2/b/(a/b)^(1/3)*ln(x^2-x*(a/b)^(
1/3)+(a/b)^(2/3))+1/3/(a*d^3-b*c^3)*a*d^2*3^(1/2)/b/(a/b)^(1/3)*a
rctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/3/(a*d^3-b*c^3)*c^2*ln(b
*x^3+a)+c^2/(a*d^3-b*c^3)*ln(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((b*x^3 + a)*(d*x + c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((b*x^3 + a)*(d*x + c)),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(d*x+c)/(b*x**3+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.270792, size = 432, normalized size = 1.64

$$-\frac{c^2d\ln\left(|dx+c|\right)}{bc^3d-ad^4}+\frac{c^2\ln\left(|bx^3+a|\right)}{3\,(bc^3-ad^3)}+\frac{\left(-ab^2\right)^{\frac{1}{3}}d\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^2c^2-\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}bcd+\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}}d^2}+\frac{\left(ab^2c^3d^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^2bd^5\left(-\frac{a}{b}\right)^{\frac{1}{3}}-ab^2c^4d+a^2bcd^4\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,(ab^3c^6-2\,a^2b^2c^3d^3+a^3bd^6)}+\frac{\left(\left(-ab^2\right)^{\frac{1}{3}}bcd-\left(-ab^2\right)^{\frac{2}{3}}d^2\right)\ln\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,(b^3c^3-ab^2d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((b*x^3 + a)*(d*x + c)),x, algorithm="giac")
```

```
[Out] -c^2*d*ln(abs(d*x + c))/(b*c^3*d - a*d^4) + 1/3*c^2*ln(abs(b*x^3
+ a))/(b*c^3 - a*d^3) + (-a*b^2)^(1/3)*d*arctan(1/3*sqrt(3)*(2*x
+ (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*b^2*c^2 - sqrt(3)*(-a*b^2)
^(1/3)*b*c*d + sqrt(3)*(-a*b^2)^(2/3)*d^2) + 1/3*(a*b^2*c^3*d^2*(
-a/b)^(1/3) - a^2*b*d^5*(-a/b)^(1/3) - a*b^2*c^4*d + a^2*b*c*d^4)
*(-a/b)^(1/3)*ln(abs(x - (-a/b)^(1/3)))/(a*b^3*c^6 - 2*a^2*b^2*c^
3*d^3 + a^3*b*d^6) + 1/6*((-a*b^2)^(1/3)*b*c*d - (-a*b^2)^(2/3)*d
^2)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(b^3*c^3 - a*b^2*d^3)
```

3.342 $\int \frac{x^2}{(c+dx)(a+bx^4)}\,dx$

Optimal. Leaf size=417

$$\frac{\sqrt{a}d^3\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}\left(ad^4+bc^4\right)}-\frac{c^2d\log\left(a+bx^4\right)}{4\left(ad^4+bc^4\right)}+\frac{c^2d\log(c+dx)}{ad^4+bc^4}+\frac{c\left(\sqrt{a}d^2+\sqrt{b}c^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\left(ad^4+bc^4\right)}-\frac{c\left(\sqrt{a}d^2+\sqrt{b}c^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\left(ad^4+bc^4\right)}-\frac{c\left(\sqrt{b}c^2-\sqrt{a}d^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\left(ad^4+bc^4\right)}+\frac{c\left(\sqrt{b}c^2-\sqrt{a}d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\left(ad^4+bc^4\right)}$$

```
[Out] (Sqrt[a]*d^3*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[b]*(b*c^4 + a
*d^4)) - (c*(Sqrt[b]*c^2 - Sqrt[a]*d^2)*ArcTan[1 - (Sqrt[2]*b^(1/
4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(1/4)*b^(1/4)*(b*c^4 + a*d^4)) + (c*
(Sqrt[b]*c^2 - Sqrt[a]*d^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4
)])/(2*Sqrt[2]*a^(1/4)*b^(1/4)*(b*c^4 + a*d^4)) + (c^2*d*Log[c +
d*x])/(b*c^4 + a*d^4) + (c*(Sqrt[b]*c^2 + Sqrt[a]*d^2)*Log[Sqrt[a
] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(1/4)*
b^(1/4)*(b*c^4 + a*d^4)) - (c*(Sqrt[b]*c^2 + Sqrt[a]*d^2)*Log[Sqr
t[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(1/
4)*b^(1/4)*(b*c^4 + a*d^4)) - (c^2*d*Log[a + b*x^4])/(4*(b*c^4 +
a*d^4))
```

Rubi [A] time = 1.17695, antiderivative size = 417, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.55

$$\frac{\sqrt{a}d^3\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}\left(ad^4+bc^4\right)}-\frac{c^2d\log\left(a+bx^4\right)}{4\left(ad^4+bc^4\right)}+\frac{c^2d\log(c+dx)}{ad^4+bc^4}+\frac{c\left(\sqrt{a}d^2+\sqrt{b}c^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\left(ad^4+bc^4\right)}-\frac{c\left(\sqrt{a}d^2+\sqrt{b}c^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\left(ad^4+bc^4\right)}-\frac{c\left(\sqrt{b}c^2-\sqrt{a}d^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\left(ad^4+bc^4\right)}+\frac{c\left(\sqrt{b}c^2-\sqrt{a}d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\left(ad^4+bc^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/((c + d*x)*(a + b*x^4)),x]
```

```
[Out] (Sqrt[a]*d^3*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[b]*(b*c^4 + a
*d^4)) - (c*(Sqrt[b]*c^2 - Sqrt[a]*d^2)*ArcTan[1 - (Sqrt[2]*b^(1/
4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(1/4)*b^(1/4)*(b*c^4 + a*d^4)) + (c*
(Sqrt[b]*c^2 - Sqrt[a]*d^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4
)])/(2*Sqrt[2]*a^(1/4)*b^(1/4)*(b*c^4 + a*d^4)) + (c^2*d*Log[c +
d*x])/(b*c^4 + a*d^4) + (c*(Sqrt[b]*c^2 + Sqrt[a]*d^2)*Log[Sqrt[a
] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(1/4)*
b^(1/4)*(b*c^4 + a*d^4)) - (c*(Sqrt[b]*c^2 + Sqrt[a]*d^2)*Log[Sqr
t[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(1/
4)*b^(1/4)*(b*c^4 + a*d^4)) - (c^2*d*Log[a + b*x^4])/(4*(b*c^4 +
a*d^4))
```

Rubi in Sympy [A] time = 148.362, size = 381, normalized size = 0.91

$$\frac{\sqrt{a}d^{3}\operatorname{atan}\left(\frac{\sqrt{b}x^{2}}{\sqrt{a}}\right)}{2\sqrt{b}\left(ad^{4}+bc^{4}\right)}-\frac{c^{2}d\log\left(a+bx^{4}\right)}{4\left(ad^{4}+bc^{4}\right)}+\frac{c^{2}d\log\left(c+dx\right)}{ad^{4}+bc^{4}}+\frac{\sqrt{2}c\left(\sqrt{a}d^{2}-\sqrt{b}c^{2}\right)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{4\sqrt[4]{a}\sqrt[4]{b}\left(ad^{4}+bc^{4}\right)}$$
$$-\frac{\sqrt{2}c\left(\sqrt{a}d^{2}-\sqrt{b}c^{2}\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{4\sqrt[4]{a}\sqrt[4]{b}\left(ad^{4}+bc^{4}\right)}+\frac{\sqrt{2}c\left(\sqrt{a}d^{2}+\sqrt{b}c^{2}\right)\log\left(-\sqrt{2}\sqrt[4]{a}b^{\frac{3}{4}}x+\sqrt{a}\sqrt{b}+bx^{2}\right)}{8\sqrt[4]{a}\sqrt[4]{b}\left(ad^{4}+bc^{4}\right)}$$
$$-\frac{\sqrt{2}c\left(\sqrt{a}d^{2}+\sqrt{b}c^{2}\right)\log\left(\sqrt{2}\sqrt[4]{a}b^{\frac{3}{4}}x+\sqrt{a}\sqrt{b}+bx^{2}\right)}{8\sqrt[4]{a}\sqrt[4]{b}\left(ad^{4}+bc^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(d*x+c)/(b*x**4+a),x)
```

```
[Out] sqrt(a)*d**3*atan(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)*(a*d**4 + b*c*
*4)) - c**2*d*log(a + b*x**4)/(4*(a*d**4 + b*c**4)) + c**2*d*log(
c + d*x)/(a*d**4 + b*c**4) + sqrt(2)*c*(sqrt(a)*d**2 - sqrt(b)*c*
*2)*atan(1 - sqrt(2)*b**(1/4)*x/a**(1/4))/(4*a**(1/4)*b**(1/4)*(a
*d**4 + b*c**4)) - sqrt(2)*c*(sqrt(a)*d**2 - sqrt(b)*c**2)*atan(1
 + sqrt(2)*b**(1/4)*x/a**(1/4))/(4*a**(1/4)*b**(1/4)*(a*d**4 + b*
c**4)) + sqrt(2)*c*(sqrt(a)*d**2 + sqrt(b)*c**2)*log(-sqrt(2)*a**
(1/4)*b**(3/4)*x + sqrt(a)*sqrt(b) + b*x**2)/(8*a**(1/4)*b**(1/4)
*(a*d**4 + b*c**4)) - sqrt(2)*c*(sqrt(a)*d**2 + sqrt(b)*c**2)*log
(sqrt(2)*a**(1/4)*b**(3/4)*x + sqrt(a)*sqrt(b) + b*x**2)/(8*a**(1
/4)*b**(1/4)*(a*d**4 + b*c**4))
```

Mathematica [A] time = 0.486246, size = 370, normalized size = 0.89

$$\frac{-2\left(2a^{3/4}d^{3}-\sqrt{2}\sqrt{a}\sqrt[4]{b}cd^{2}+\sqrt{2}b^{3/4}c^{3}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+2\left(-2a^{3/4}d^{3}-\sqrt{2}\sqrt{a}\sqrt[4]{b}cd^{2}+\sqrt{2}b^{3/4}c^{3}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/((c + d*x)*(a + b*x^4)),x]
```

```
[Out] (-2*(Sqrt[2]*b^(3/4)*c^3 - Sqrt[2]*Sqrt[a]*b^(1/4)*c*d^2 + 2*a^(3
/4)*d^3)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(Sqrt[2]*b^(
3/4)*c^3 - Sqrt[2]*Sqrt[a]*b^(1/4)*c*d^2 - 2*a^(3/4)*d^3)*ArcTan[
1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + b^(1/4)*c*(8*a^(1/4)*b^(1/4)*c
*d*Log[c + d*x] + Sqrt[2]*(Sqrt[b]*c^2 + Sqrt[a]*d^2)*Log[Sqrt[a]
 - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] - Sqrt[2]*Sqrt[b]*c^2
*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] - Sqrt[2]
*Sqrt[a]*d^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^
2] - 2*a^(1/4)*b^(1/4)*c*d*Log[a + b*x^4]))/(8*a^(1/4)*Sqrt[b]*(b
*c^4 + a*d^4))
```

Maple [A] time = 0.019, size = 422, normalized size = 1.

$$-\frac{cd^2\sqrt{2}}{4\,ad^4+4\,bc^4}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)-\frac{cd^2\sqrt{2}}{8\,ad^4+8\,bc^4}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)-\frac{cd^2\sqrt{2}}{4\,ad^4+4\,bc^4}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)+\frac{ad^3}{2\,ad^4+2\,bc^4}\arctan\left(x^2\sqrt{\frac{b}{a}}\right)\frac{1}{\sqrt{ab}}+\frac{c^3\sqrt{2}}{8\,ad^4+8\,bc^4}\ln\left(1\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{c^3\sqrt{2}}{4\,ad^4+4\,bc^4}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{c^3\sqrt{2}}{4\,ad^4+4\,bc^4}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}-\frac{c^2d\ln\left(bx^4+a\right)}{4\,ad^4+4\,bc^4}+\frac{c^2d\ln\left(dx+c\right)}{ad^4+bc^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(d*x+c)/(b*x^4+a),x)
```

```
[Out] -1/4/(a*d^4+b*c^4)*c*d^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)
^(1/4)*x-1)-1/8/(a*d^4+b*c^4)*c*d^2*(a/b)^(1/4)*2^(1/2)*ln((x^2+(
a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b
)^(1/2)))-1/4/(a*d^4+b*c^4)*c*d^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1
/2)/(a/b)^(1/4)*x+1)+1/2/(a*d^4+b*c^4)*a*d^3/(a*b)^(1/2)*arctan(x
^2*(b/a)^(1/2))+1/8/(a*d^4+b*c^4)*c^3/(a/b)^(1/4)*2^(1/2)*ln((x^2
-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a
/b)^(1/2)))+1/4/(a*d^4+b*c^4)*c^3/(a/b)^(1/4)*2^(1/2)*arctan(2^(1
/2)/(a/b)^(1/4)*x+1)+1/4/(a*d^4+b*c^4)*c^3/(a/b)^(1/4)*2^(1/2)*ar
ctan(2^(1/2)/(a/b)^(1/4)*x-1)-1/4*c^2*d*ln(b*x^4+a)/(a*d^4+b*c^4)
+c^2*d*ln(d*x+c)/(a*d^4+b*c^4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((b*x^4 + a)*(d*x + c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((b*x^4 + a)*(d*x + c)),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(d*x+c)/(b*x**4+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.294233, size = 575, normalized size = 1.38

$$\frac{c^2d^2\ln\left(|dx+c|\right)}{bc^4d+ad^5}-\frac{c^2d\ln\left(|bx^4+a|\right)}{4\,(bc^4+ad^4)}-\frac{\left(\sqrt{2}a^2b^3d-\left(ab^3\right)^{\frac{3}{4}}abc\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}a^2b^4c^2+\sqrt{2}\sqrt{ab}a^2b^3d^2-2\,(ab^3)^{\frac{1}{4}}a^2b^3cd\right)}+\frac{\left(\sqrt{2}a^2b^3d+\left(ab^3\right)^{\frac{3}{4}}abc\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}a^2b^4c^2+\sqrt{2}\sqrt{ab}a^2b^3d^2+2\,(ab^3)^{\frac{1}{4}}a^2b^3cd\right)}-\frac{\left(\left(ab^3\right)^{\frac{1}{4}}abcd^2+\left(ab^3\right)^{\frac{3}{4}}c^3\right)\ln\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}ab^3c^4+\sqrt{2}a^2b^2d^4\right)}+\frac{\left(\left(ab^3\right)^{\frac{1}{4}}abcd^2+\left(ab^3\right)^{\frac{3}{4}}c^3\right)\ln\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}ab^3c^4+\sqrt{2}a^2b^2d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((b*x^4 + a)*(d*x + c)),x, algorithm="giac")
```

```
[Out] c^2*d^2*ln(abs(d*x + c))/(b*c^4*d + a*d^5) - 1/4*c^2*d*ln(abs(b*x
^4 + a))/(b*c^4 + a*d^4) - 1/2*(sqrt(2)*a^2*b^3*d - (a*b^3)^(3/4)
*a*b*c)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4
))/(sqrt(2)*a^2*b^4*c^2 + sqrt(2)*sqrt(a*b)*a^2*b^3*d^2 - 2*(a*b^
3)^(1/4)*a^2*b^3*c*d) + 1/2*(sqrt(2)*a^2*b^3*d + (a*b^3)^(3/4)*a*
b*c)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/
(sqrt(2)*a^2*b^4*c^2 + sqrt(2)*sqrt(a*b)*a^2*b^3*d^2 + 2*(a*b^3)^
(1/4)*a^2*b^3*c*d) - 1/4*((a*b^3)^(1/4)*a*b*c*d^2 + (a*b^3)^(3/4)
*c^3)*ln(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a*b^3*
c^4 + sqrt(2)*a^2*b^2*d^4) + 1/4*((a*b^3)^(1/4)*a*b*c*d^2 + (a*b^
3)^(3/4)*c^3)*ln(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2
)*a*b^3*c^4 + sqrt(2)*a^2*b^2*d^4)
```

3.343 $\int \frac{x}{(1-x)(1+x)^2} \, dx$

Optimal. Leaf size=16

$$\frac{1}{2(x+1)} + \frac{1}{2}\tanh^{-1}(x)$$

```
[Out] 1/(2*(1 + x)) + ArcTanh[x]/2
```

Rubi [A] time = 0.0292138, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$\frac{1}{2(x+1)} + \frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[x/((1 - x)*(1 + x)^2),x]

[Out] 1/(2*(1 + x)) + ArcTanh[x]/2
```

Rubi in Sympy [A] time = 4.71427, size = 10, normalized size = 0.62

$$\frac{\operatorname{atanh}(x)}{2} + \frac{1}{2\,(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(1-x)/(1+x)**2,x)

[Out] atanh(x)/2 + 1/(2*(x + 1))
```

Mathematica [A] time = 0.013501, size = 24, normalized size = 1.5

$$\frac{1}{4}\left(\frac{2}{x+1} - \log(1-x) + \log(x+1)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/((1 - x)*(1 + x)^2),x]

[Out] (2/(1 + x) - Log[1 - x] + Log[1 + x])/4
```

Maple [A] time = 0.01, size = 21, normalized size = 1.3

$$-\frac{\ln(-1+x)}{4} + \frac{1}{2+2\,x} + \frac{\ln(1+x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(1-x)/(1+x)^2,x)
```

```
[Out] -1/4*ln(-1+x)+1/2/(1+x)+1/4*ln(1+x)
```

Maxima [A] time = 0.830957, size = 27, normalized size = 1.69

$$\frac{1}{2(x+1)} + \frac{1}{4}\log(x+1) - \frac{1}{4}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x/((x + 1)^2*(x - 1)),x, algorithm="maxima")
```

```
[Out] 1/2/(x + 1) + 1/4*log(x + 1) - 1/4*log(x - 1)
```

Fricas [A] time = 0.24334, size = 35, normalized size = 2.19

$$\frac{(x+1)\log(x+1) - (x+1)\log(x-1) + 2}{4(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x/((x + 1)^2*(x - 1)),x, algorithm="fricas")
```

```
[Out] 1/4*((x + 1)*log(x + 1) - (x + 1)*log(x - 1) + 2)/(x + 1)
```

Sympy [A] time = 0.223792, size = 19, normalized size = 1.19

$$-\frac{\log(x-1)}{4} + \frac{\log(x+1)}{4} + \frac{1}{2x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1-x)/(1+x)**2,x)
```

```
[Out] -log(x - 1)/4 + log(x + 1)/4 + 1/(2*x + 2)
```

GIAC/XCAS [A] time = 0.25892, size = 28, normalized size = 1.75

$$\frac{1}{2(x+1)} - \frac{1}{4}\ln\left(\left|-\frac{2}{x+1} + 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x/((x + 1)^2*(x - 1)),x, algorithm="giac")
```

```
[Out] 1/2/(x + 1) - 1/4*ln(abs(-2/(x + 1) + 1))
```

3.344 $\int \frac{x^2}{(1-x^2)(1+x^2)^2} \, dx$

Optimal. Leaf size=19

$$\frac{1}{4}\tanh^{-1}(x) - \frac{x}{4\,(x^2+1)}$$

```
[Out] -x/(4*(1 + x^2)) + ArcTanh[x]/4
```

Rubi [A] time = 0.0440524, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$\frac{1}{4}\tanh^{-1}(x) - \frac{x}{4\,(x^2+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/((1 - x^2)*(1 + x^2)^2),x]
```

```
[Out] -x/(4*(1 + x^2)) + ArcTanh[x]/4
```

Rubi in Sympy [A] time = 8.29075, size = 12, normalized size = 0.63

$$-\frac{x}{4\,(x^2+1)} + \frac{\operatorname{atanh}(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(-x**2+1)/(x**2+1)**2,x)
```

```
[Out] -x/(4*(x**2 + 1)) + atanh(x)/4
```

Mathematica [A] time = 0.0178874, size = 27, normalized size = 1.42

$$\frac{1}{8}\left(-\frac{2x}{x^2+1} - \log(1-x) + \log(x+1)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/((1 - x^2)*(1 + x^2)^2),x]
```

```
[Out] ((-2*x)/(1 + x^2) - Log[1 - x] + Log[1 + x])/8
```

Maple [A] time = 0.014, size = 24, normalized size = 1.3

$$-\frac{\ln(-1+x)}{8} + \frac{\ln(1+x)}{8} - \frac{x}{4\,x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(-x^2+1)/(x^2+1)^2,x)
```

```
[Out] -1/8*ln(-1+x)+1/8*ln(1+x)-1/4*x/(x^2+1)
```

Maxima [A] time = 0.804053, size = 31, normalized size = 1.63

$$-\frac{x}{4\left(x^{2}+1\right)}+\frac{1}{8}\log\left(x+1\right)-\frac{1}{8}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x^2/((x^2 + 1)^2*(x^2 - 1)),x, algorithm="maxima")

[Out] -1/4*x/(x^2 + 1) + 1/8*log(x + 1) - 1/8*log(x - 1)
```

Fricas [A] time = 0.254631, size = 46, normalized size = 2.42

$$\frac{\left(x^{2}+1\right)\log\left(x+1\right)-\left(x^{2}+1\right)\log\left(x-1\right)-2\,x}{8\left(x^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x^2/((x^2 + 1)^2*(x^2 - 1)),x, algorithm="fricas")

[Out] 1/8*((x^2 + 1)*log(x + 1) - (x^2 + 1)*log(x - 1) - 2*x)/(x^2 + 1)
```

Sympy [A] time = 0.279657, size = 20, normalized size = 1.05

$$-\frac{x}{4x^{2}+4}-\frac{\log\left(x-1\right)}{8}+\frac{\log\left(x+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(-x**2+1)/(x**2+1)**2,x)

[Out] -x/(4*x**2 + 4) - log(x - 1)/8 + log(x + 1)/8
```

GIAC/XCAS [A] time = 0.260747, size = 41, normalized size = 2.16

$$-\frac{1}{4\left(x+\frac{1}{x}\right)}+\frac{1}{16}\ln\left(\left|x+\frac{1}{x}+2\right|\right)-\frac{1}{16}\ln\left(\left|x+\frac{1}{x}-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x^2/((x^2 + 1)^2*(x^2 - 1)),x, algorithm="giac")

[Out] -1/4/(x + 1/x) + 1/16*ln(abs(x + 1/x + 2)) - 1/16*ln(abs(x + 1/x
- 2))
```

3.345 $\int \frac{x^3}{(1-x^3)(1+x^3)^2} \, dx$

Optimal. Leaf size=97

$$-\frac{x}{6\left(x^3+1\right)}+\frac{1}{72}\log\left(x^2-x+1\right)+\frac{1}{24}\log\left(x^2+x+1\right)-\frac{1}{12}\log(1-x)-\frac{1}{36}\log(x+1)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{12\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

```
[Out] -x/(6*(1 + x^3)) + ArcTan[(1 - 2*x)/Sqrt[3]]/(12*Sqrt[3]) + ArcTa
n[(1 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) - Log[1 - x]/12 - Log[1 + x]/36
+ Log[1 - x + x^2]/72 + Log[1 + x + x^2]/24
```

Rubi [A] time = 0.184403, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$

$$-\frac{x}{6\left(x^3+1\right)}+\frac{1}{72}\log\left(x^2-x+1\right)+\frac{1}{24}\log\left(x^2+x+1\right)-\frac{1}{12}\log(1-x)-\frac{1}{36}\log(x+1)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{12\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/((1 - x^3)*(1 + x^3)^2),x]
```


```
[Out] -x/(6*(1 + x^3)) + ArcTan[(1 - 2*x)/Sqrt[3]]/(12*Sqrt[3]) + ArcTa
n[(1 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) - Log[1 - x]/12 - Log[1 + x]/36
+ Log[1 - x + x^2]/72 + Log[1 + x + x^2]/24
```

Rubi in Sympy [A] time = 25.7174, size = 85, normalized size = 0.88

$$-\frac{x}{6\left(x^3+1\right)}-\frac{\log\left(-x+1\right)}{12}-\frac{\log\left(x+1\right)}{36}+\frac{\log\left(x^2-x+1\right)}{72}+\frac{\log\left(x^2+x+1\right)}{24}-\frac{\sqrt{3}\operatorname{atan}\left(\sqrt{3}\left(\frac{2x}{3}-\frac{1}{3}\right)\right)}{36}+\frac{\sqrt{3}\operatorname{atan}\left(\sqrt{3}\left(\frac{2x}{3}+\frac{1}{3}\right)\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(-x**3+1)/(x**3+1)**2,x)
```

```
[Out] -x/(6*(x**3 + 1)) - log(-x + 1)/12 - log(x + 1)/36 + log(x**2 - x
 + 1)/72 + log(x**2 + x + 1)/24 - sqrt(3)*atan(sqrt(3)*(2*x/3 - 1
/3))/36 + sqrt(3)*atan(sqrt(3)*(2*x/3 + 1/3))/12
```

Mathematica [A] time = 0.0825719, size = 85, normalized size = 0.88

$$\frac{1}{72}\left(-\frac{12x}{x^3+1}+\log\left(x^2-x+1\right)+3\log\left(x^2+x+1\right)-6\log(1-x)-2\log(x+1)-2\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)+6\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/((1 - x^3)*(1 + x^3)^2),x]
```

```
[Out] ((-12*x)/(1 + x^3) - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 6*Sqr
t[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 6*Log[1 - x] - 2*Log[1 + x] + Lo
g[1 - x + x^2] + 3*Log[1 + x + x^2])/72
```

Maple [A] time = 0.019, size = 90, normalized size = 0.9

$$\frac{\ln\left(x^2+x+1\right)}{24}+\frac{\sqrt{3}}{12}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)-\frac{\ln(-1+x)}{12}+\frac{1}{18+18x}-\frac{\ln(1+x)}{36}+\frac{-2x-2}{36x^2-36x+36}+\frac{\ln\left(x^2-x+1\right)}{72}-\frac{\sqrt{3}}{36}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(-x^3+1)/(x^3+1)^2,x)
```

```
[Out] 1/24*ln(x^2+x+1)+1/12*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-1/12*ln
(-1+x)+1/18/(1+x)-1/36*ln(1+x)+1/36*(-2*x-2)/(x^2-x+1)+1/72*ln(x^
2-x+1)-1/36*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A] time = 0.905521, size = 101, normalized size = 1.04

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{36}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{x}{6\left(x^3+1\right)}+\frac{1}{24}\log\left(x^2+x+1\right)+\frac{1}{72}\log\left(x^2-x+1\right)-\frac{1}{36}\log(x+1)-\frac{1}{12}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x^3/((x^3 + 1)^2*(x^3 - 1)),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/36*sqrt(3)*arctan(
1/3*sqrt(3)*(2*x - 1)) - 1/6*x/(x^3 + 1) + 1/24*log(x^2 + x + 1)
+ 1/72*log(x^2 - x + 1) - 1/36*log(x + 1) - 1/12*log(x - 1)
```

Fricas [A] time = 0.260667, size = 159, normalized size = 1.64

$$\frac{\sqrt{3}\left(3\sqrt{3}\left(x^3+1\right)\log\left(x^2+x+1\right)+\sqrt{3}\left(x^3+1\right)\log\left(x^2-x+1\right)-2\sqrt{3}\left(x^3+1\right)\log(x+1)-6\sqrt{3}\left(x^3+1\right)\log(x-1)+18\,(\right.}{216\left(x^3+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x^3/((x^3 + 1)^2*(x^3 - 1)),x, algorithm="fricas")
```

```
[Out] 1/216*sqrt(3)*(3*sqrt(3)*(x^3 + 1)*log(x^2 + x + 1) + sqrt(3)*(x^
3 + 1)*log(x^2 - x + 1) - 2*sqrt(3)*(x^3 + 1)*log(x + 1) - 6*sqrt
(3)*(x^3 + 1)*log(x - 1) + 18*(x^3 + 1)*arctan(1/3*sqrt(3)*(2*x +
 1)) - 6*(x^3 + 1)*arctan(1/3*sqrt(3)*(2*x - 1)) - 12*sqrt(3)*x)/
(x^3 + 1)
```

Sympy [A] time = 1.15848, size = 92, normalized size = 0.95

$$-\frac{x}{6x^3+6}-\frac{\log(x-1)}{12}-\frac{\log(x+1)}{36}+\frac{\log(x^2-x+1)}{72}+\frac{\log(x^2+x+1)}{24}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{36}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(-x**3+1)/(x**3+1)**2,x)
```

```
[Out] -x/(6*x**3 + 6) - log(x - 1)/12 - log(x + 1)/36 + log(x**2 - x +
1)/72 + log(x**2 + x + 1)/24 - sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(
3)/3)/36 + sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/12
```

GIAC/XCAS [A] time = 0.264014, size = 104, normalized size = 1.07

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{36}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{x}{6(x^3+1)}+\frac{1}{24}\ln\left(x^2+x+1\right)+\frac{1}{72}\ln\left(x^2-x+1\right)-\frac{1}{36}\ln\left(|x+1|\right)-\frac{1}{12}\ln\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x^3/((x^3 + 1)^2*(x^3 - 1)),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/36*sqrt(3)*arctan(
1/3*sqrt(3)*(2*x - 1)) - 1/6*x/(x^3 + 1) + 1/24*ln(x^2 + x + 1) +
 1/72*ln(x^2 - x + 1) - 1/36*ln(abs(x + 1)) - 1/12*ln(abs(x - 1))
```

3.346 $\int \frac{9+x+3x^2+x^3}{(1+x^2)(3+x^2)}\,dx$

Optimal. Leaf size=15

$$\frac{1}{2}\log\left(x^2+3\right)+3\tan^{-1}(x)$$

```
[Out] 3*ArcTan[x] + Log[3 + x^2]/2
```

Rubi [A] time = 0.190183, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{1}{2}\log\left(x^2+3\right)+3\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(9 + x + 3*x^2 + x^3)/((1 + x^2)*(3 + x^2)),x]

[Out] 3*ArcTan[x] + Log[3 + x^2]/2
```

Rubi in Sympy [A] time = 32.7411, size = 12, normalized size = 0.8

$$\frac{\log\left(x^2+3\right)}{2}+3\,\mathrm{atan}\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**3+3*x**2+x+9)/(x**2+1)/(x**2+3),x)

[Out] log(x**2 + 3)/2 + 3*atan(x)
```

Mathematica [A] time = 0.0122429, size = 15, normalized size = 1.

$$\frac{1}{2}\log\left(x^2+3\right)+3\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(9 + x + 3*x^2 + x^3)/((1 + x^2)*(3 + x^2)),x]

[Out] 3*ArcTan[x] + Log[3 + x^2]/2
```

Maple [A] time = 0.006, size = 14, normalized size = 0.9

$$3\arctan(x)+\frac{\ln\left(x^2+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^3+3*x^2+x+9)/(x^2+1)/(x^2+3),x)
```

```
[Out] 3*arctan(x)+1/2*ln(x^2+3)
```

Maxima [A] time = 0.901664, size = 18, normalized size = 1.2

$$3 \arctan(x) + \frac{1}{2} \log(x^2 + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3 + 3*x^2 + x + 9)/((x^2 + 3)*(x^2 + 1)),x, algorithm="maxima")
```

```
[Out] 3*arctan(x) + 1/2*log(x^2 + 3)
```

Fricas [A] time = 0.262736, size = 18, normalized size = 1.2

$$3 \arctan(x) + \frac{1}{2} \log(x^2 + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3 + 3*x^2 + x + 9)/((x^2 + 3)*(x^2 + 1)),x, algorithm="fricas")
```

```
[Out] 3*arctan(x) + 1/2*log(x^2 + 3)
```

Sympy [A] time = 0.263852, size = 12, normalized size = 0.8

$$\frac{\log(x^2 + 3)}{2} + 3 \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+3*x**2+x+9)/(x**2+1)/(x**2+3),x)
```

```
[Out] log(x**2 + 3)/2 + 3*atan(x)
```

GIAC/XCAS [A] time = 0.263762, size = 18, normalized size = 1.2

$$3 \arctan(x) + \frac{1}{2} \ln(x^2 + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3 + 3*x^2 + x + 9)/((x^2 + 3)*(x^2 + 1)),x, algorithm="giac")
```

```
[Out] 3*arctan(x) + 1/2*ln(x^2 + 3)
```

3.347 $\int \frac{3+x+x^2+x^3}{(1+x^2)(3+x^2)} dx$

Optimal. Leaf size=13

$$\frac{1}{2}\log(x^2+3)+\tan^{-1}(x)$$

```
[Out] ArcTan[x] + Log[3 + x^2]/2
```

Rubi [A] time = 0.166696, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{1}{2}\log(x^2+3)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + x + x^2 + x^3)/((1 + x^2)*(3 + x^2)),x]
```

```
[Out] ArcTan[x] + Log[3 + x^2]/2
```

Rubi in Sympy [A] time = 28.9777, size = 10, normalized size = 0.77

$$\frac{\log(x^2+3)}{2}+\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((x**3+x**2+x+3)/(x**2+1)/(x**2+3),x)
```

```
[Out] log(x**2 + 3)/2 + atan(x)
```

Mathematica [A] time = 0.0104689, size = 13, normalized size = 1.

$$\frac{1}{2}\log(x^2+3)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + x + x^2 + x^3)/((1 + x^2)*(3 + x^2)),x]
```

```
[Out] ArcTan[x] + Log[3 + x^2]/2
```

Maple [A] time = 0.005, size = 12, normalized size = 0.9

$$\arctan(x)+\frac{\ln(x^2+3)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2+x+3)/(x^2+1)/(x^2+3),x)
```

```
[Out] arctan(x)+1/2*ln(x^2+3)
```

Maxima [A] time = 0.89021, size = 15, normalized size = 1.15

$$\arctan(x) + \frac{1}{2}\log(x^2 + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3 + x^2 + x + 3)/((x^2 + 3)*(x^2 + 1)),x, algorithm="maxima")
```

```
[Out] arctan(x) + 1/2*log(x^2 + 3)
```

Fricas [A] time = 0.254576, size = 15, normalized size = 1.15

$$\arctan(x) + \frac{1}{2}\log(x^2 + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3 + x^2 + x + 3)/((x^2 + 3)*(x^2 + 1)),x, algorithm="fricas")
```

```
[Out] arctan(x) + 1/2*log(x^2 + 3)
```

Sympy [A] time = 0.274042, size = 10, normalized size = 0.77

$$\frac{\log(x^2 + 3)}{2} + \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3+x**2+x+3)/(x**2+1)/(x**2+3),x)
```

```
[Out] log(x**2 + 3)/2 + atan(x)
```

GIAC/XCAS [A] time = 0.260913, size = 15, normalized size = 1.15

$$\arctan(x) + \frac{1}{2}\ln(x^2 + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3 + x^2 + x + 3)/((x^2 + 3)*(x^2 + 1)),x, algorithm="giac")
```

```
[Out] arctan(x) + 1/2*ln(x^2 + 3)
```

3.348 $\int \frac{-4+6x-x^2+3x^3}{(1+x^2)(2+x^2)}\,dx$

Optimal. Leaf size=29

$$\frac{3}{2}\log\left(x^2+1\right)-3\tan^{-1}(x)+\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

```
[Out] -3*ArcTan[x] + Sqrt[2]*ArcTan[x/Sqrt[2]] + (3*Log[1 + x^2])/2
```

Rubi [A] time = 0.217307, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$\frac{3}{2}\log\left(x^2+1\right)-3\tan^{-1}(x)+\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(-4 + 6*x - x^2 + 3*x^3)/((1 + x^2)*(2 + x^2)),x]

[Out] -3*ArcTan[x] + Sqrt[2]*ArcTan[x/Sqrt[2]] + (3*Log[1 + x^2])/2
```

Rubi in Sympy [A] time = 40.5898, size = 29, normalized size = 1.

$$\frac{3\log\left(x^2+1\right)}{2}-3\,\mathrm{atan}\,(x)+\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**3-x**2+6*x-4)/(x**2+1)/(x**2+2),x)

[Out] 3*log(x**2 + 1)/2 - 3*atan(x) + sqrt(2)*atan(sqrt(2)*x/2)
```

Mathematica [A] time = 0.0252063, size = 29, normalized size = 1.

$$\frac{3}{2}\log\left(x^2+1\right)-3\tan^{-1}(x)+\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-4 + 6*x - x^2 + 3*x^3)/((1 + x^2)*(2 + x^2)),x]

[Out] -3*ArcTan[x] + Sqrt[2]*ArcTan[x/Sqrt[2]] + (3*Log[1 + x^2])/2
```

Maple [A] time = 0.006, size = 25, normalized size = 0.9

$$-3\arctan(x)+\frac{3\ln\left(x^2+1\right)}{2}+\arctan\left(\frac{\sqrt{2}x}{2}\right)\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^3-x^2+6*x-4)/(x^2+1)/(x^2+2),x)

[Out] -3*arctan(x)+3/2*ln(x^2+1)+arctan(1/2*2^(1/2)*x)*2^(1/2)
```

Maxima [A] time = 0.883609, size = 32, normalized size = 1.1

$$\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - 3\arctan\left(x\right) + \frac{3}{2}\log\left(x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3 - x^2 + 6*x - 4)/((x^2 + 2)*(x^2 + 1)),x, algorithm="maxima")

[Out] sqrt(2)*arctan(1/2*sqrt(2)*x) - 3*arctan(x) + 3/2*log(x^2 + 1)
```

Fricas [A] time = 0.255936, size = 32, normalized size = 1.1

$$\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - 3\arctan\left(x\right) + \frac{3}{2}\log\left(x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3 - x^2 + 6*x - 4)/((x^2 + 2)*(x^2 + 1)),x, algorithm="fricas")

[Out] sqrt(2)*arctan(1/2*sqrt(2)*x) - 3*arctan(x) + 3/2*log(x^2 + 1)
```

Sympy [A] time = 0.525867, size = 29, normalized size = 1.

$$\frac{3\log\left(x^2 + 1\right)}{2} - 3\,\mathrm{atan}\left(x\right) + \sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**3-x**2+6*x-4)/(x**2+1)/(x**2+2),x)

[Out] 3*log(x**2 + 1)/2 - 3*atan(x) + sqrt(2)*atan(sqrt(2)*x/2)
```

GIAC/XCAS [A] time = 0.260815, size = 32, normalized size = 1.1

$$\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - 3\arctan\left(x\right) + \frac{3}{2}\ln\left(x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3 - x^2 + 6*x - 4)/((x^2 + 2)*(x^2 + 1)),x, algorithm="giac")

[Out] sqrt(2)*arctan(1/2*sqrt(2)*x) - 3*arctan(x) + 3/2*ln(x^2 + 1)
```

3.349 $\int \frac{1}{(4-4x+x^2)(5-4x+x^2)}\,dx$

Optimal. Leaf size=14

$$\frac{1}{2-x}+\tan^{-1}(2-x)$$

```
[Out] (2 - x)^(-1) + ArcTan[2 - x]
```

Rubi [A] time = 0.0302973, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19

$$\frac{1}{2-x}+\tan^{-1}(2-x)$$

Antiderivative was successfully verified.

```
[In] Int[1/((4 - 4*x + x^2)*(5 - 4*x + x^2)),x]

[Out] (2 - x)^(-1) + ArcTan[2 - x]
```

Rubi in Sympy [A] time = 9.02525, size = 10, normalized size = 0.71

$$-\operatorname{atan}(x-2)+\frac{2}{-2x+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(x**2-4*x+4)/(x**2-4*x+5),x)

[Out] -atan(x - 2) + 2/(-2*x + 4)
```

Mathematica [A] time = 0.0129667, size = 14, normalized size = 1.

$$\tan^{-1}(2-x)-\frac{1}{x-2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((4 - 4*x + x^2)*(5 - 4*x + x^2)),x]

[Out] -(-2 + x)^(-1) + ArcTan[2 - x]
```

Maple [A] time = 0.007, size = 15, normalized size = 1.1

$$-(x-2)^{-1}-\arctan(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-4*x+4)/(x^2-4*x+5),x)

[Out] -1/(x-2)-arctan(x-2)
```

Maxima [A] time = 0.883689, size = 19, normalized size = 1.36

$$-\frac{1}{x-2} - \arctan(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 - 4*x + 5)*(x^2 - 4*x + 4)),x, algorithm="maxima")
```

```
[Out] -1/(x - 2) - arctan(x - 2)
```

Fricas [A] time = 0.24803, size = 23, normalized size = 1.64

$$-\frac{(x-2)\arctan(x-2)+1}{x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 - 4*x + 5)*(x^2 - 4*x + 4)),x, algorithm="fricas")
```

```
[Out] -((x - 2)*arctan(x - 2) + 1)/(x - 2)
```

Sympy [A] time = 0.303711, size = 10, normalized size = 0.71

$$-\operatorname{atan}(x-2) - \frac{1}{x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**2-4*x+4)/(x**2-4*x+5),x)
```

```
[Out] -atan(x - 2) - 1/(x - 2)
```

GIAC/XCAS [A] time = 0.259219, size = 19, normalized size = 1.36

$$-\frac{1}{x-2} - \arctan(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 - 4*x + 5)*(x^2 - 4*x + 4)),x, algorithm="giac")
```

```
[Out] -1/(x - 2) - arctan(x - 2)
```

3.350 $\int \frac{-3+x+x^2}{(-3+x)x^2} \, dx$

Optimal. Leaf size=12

$$\log(3-x)-\frac{1}{x}$$

```
[Out] -x^(-1) + Log[3 - x]
```

Rubi [A] time = 0.034114, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$\log(3-x)-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + x + x^2)/((-3 + x)*x^2),x]
```

```
[Out] -x^(-1) + Log[3 - x]
```

Rubi in Sympy [A] time = 4.45373, size = 7, normalized size = 0.58

$$\log(-x+3)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((x**2+x-3)/(-3+x)/x**2,x)
```

```
[Out] log(-x + 3) - 1/x
```

Mathematica [A] time = 0.00500165, size = 12, normalized size = 1.

$$\log(3-x)-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + x + x^2)/((-3 + x)*x^2),x]
```

```
[Out] -x^(-1) + Log[3 - x]
```

Maple [A] time = 0.008, size = 11, normalized size = 0.9

$$\ln(-3+x)-x^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x-3)/(-3+x)/x^2,x)
```

```
[Out] ln(-3+x)-1/x
```

Maxima [A] time = 0.812887, size = 14, normalized size = 1.17

$$-\frac{1}{x} + \log(x - 3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + x - 3)/((x - 3)*x^2),x, algorithm="maxima")
```

```
[Out] -1/x + log(x - 3)
```

Fricas [A] time = 0.2497, size = 16, normalized size = 1.33

$$\frac{x\log(x - 3) - 1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + x - 3)/((x - 3)*x^2),x, algorithm="fricas")
```

```
[Out] (x*log(x - 3) - 1)/x
```

Sympy [A] time = 0.174778, size = 7, normalized size = 0.58

$$\log(x - 3) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+x-3)/(-3+x)/x**2,x)
```

```
[Out] log(x - 3) - 1/x
```

GIAC/XCAS [A] time = 0.261107, size = 15, normalized size = 1.25

$$-\frac{1}{x} + \ln(|x - 3|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + x - 3)/((x - 3)*x^2),x, algorithm="giac")
```

```
[Out] -1/x + ln(abs(x - 3))
```

3.351 $\int \frac{1+x+4x^2}{x+4x^3} \, dx$

Optimal. Leaf size=11

$$\log(x) + \frac{1}{2}\tan^{-1}(2x)$$

```
[Out] ArcTan[2*x]/2 + Log[x]
```

Rubi [A] time = 0.0482499, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$\log(x) + \frac{1}{2}\tan^{-1}(2x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + 4*x^2)/(x + 4*x^3),x]

[Out] ArcTan[2*x]/2 + Log[x]
```

Rubi in Sympy [A] time = 7.83613, size = 8, normalized size = 0.73

$$\log(x) + \frac{\operatorname{atan}(2x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((4*x**2+x+1)/(4*x**3+x),x)

[Out] log(x) + atan(2*x)/2
```

Mathematica [A] time = 0.00733817, size = 11, normalized size = 1.

$$\log(x) + \frac{1}{2}\tan^{-1}(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + 4*x^2)/(x + 4*x^3),x]

[Out] ArcTan[2*x]/2 + Log[x]
```

Maple [A] time = 0.007, size = 10, normalized size = 0.9

$$\frac{\arctan(2x)}{2} + \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+x+1)/(4*x^3+x),x)
```

```
[Out] 1/2*arctan(2*x)+ln(x)
```

Maxima [A] time = 0.877128, size = 12, normalized size = 1.09

$$\frac{1}{2}\arctan(2x) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^2 + x + 1)/(4*x^3 + x),x, algorithm="maxima")
```

```
[Out] 1/2*arctan(2*x) + log(x)
```

Fricas [A] time = 0.290213, size = 12, normalized size = 1.09

$$\frac{1}{2}\arctan(2x) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^2 + x + 1)/(4*x^3 + x),x, algorithm="fricas")
```

```
[Out] 1/2*arctan(2*x) + log(x)
```

Sympy [A] time = 0.39055, size = 8, normalized size = 0.73

$$\log(x) + \frac{\operatorname{atan}(2x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x**2+x+1)/(4*x**3+x),x)
```

```
[Out] log(x) + atan(2*x)/2
```

GIAC/XCAS [A] time = 0.263086, size = 14, normalized size = 1.27

$$\frac{1}{2}\arctan(2x) + \ln(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^2 + x + 1)/(4*x^3 + x),x, algorithm="giac")
```

```
[Out] 1/2*arctan(2*x) + ln(abs(x))
```

3.352 $\int \frac{1-x+3x^2}{-x^2+x^3}\,dx$

Optimal. Leaf size=12

$$\frac{1}{x}+3\log(1-x)$$

```
[Out] x^(-1) + 3*Log[1 - x]
```

Rubi [A] time = 0.0433017, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{1}{x}+3\log(1-x)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - x + 3*x^2)/(-x^2 + x^3),x]

[Out] x^(-1) + 3*Log[1 - x]
```

Rubi in Sympy [A] time = 7.00016, size = 8, normalized size = 0.67

$$3\log(-x+1)+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2-x+1)/(x**3-x**2),x)

[Out] 3*log(-x + 1) + 1/x
```

Mathematica [A] time = 0.00578145, size = 12, normalized size = 1.

$$\frac{1}{x}+3\log(1-x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - x + 3*x^2)/(-x^2 + x^3),x]

[Out] x^(-1) + 3*Log[1 - x]
```

Maple [A] time = 0.007, size = 11, normalized size = 0.9

$$3\,\ln(-1+x)+x^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2-x+1)/(x^3-x^2),x)

[Out] 3*ln(-1+x)+1/x
```

Maxima [A] time = 0.812893, size = 14, normalized size = 1.17

$$\frac{1}{x} + 3 \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 - x + 1)/(x^3 - x^2),x, algorithm="maxima")
```

```
[Out] 1/x + 3*log(x - 1)
```

Fricas [A] time = 0.249535, size = 18, normalized size = 1.5

$$\frac{3x\log(x - 1) + 1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 - x + 1)/(x^3 - x^2),x, algorithm="fricas")
```

```
[Out] (3*x*log(x - 1) + 1)/x
```

Sympy [A] time = 0.169184, size = 8, normalized size = 0.67

$$3\log(x - 1) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**2-x+1)/(x**3-x**2),x)
```

```
[Out] 3*log(x - 1) + 1/x
```

GIAC/XCAS [A] time = 0.262483, size = 15, normalized size = 1.25

$$\frac{1}{x} + 3\ln(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 - x + 1)/(x^3 - x^2),x, algorithm="giac")
```

```
[Out] 1/x + 3*ln(abs(x - 1))
```

3.353 $\int \frac{4+3x+x^2}{x+x^2} \, dx$

Optimal. Leaf size=12

$$x + 4\log(x) - 2\log(x + 1)$$

```
[Out] x + 4*Log[x] - 2*Log[1 + x]
```

Rubi [A] time = 0.0379996, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$x + 4\log(x) - 2\log(x + 1)$$

Antiderivative was successfully verified.

```
[In]  Int[(4 + 3*x + x^2)/(x + x^2),x]

[Out] x + 4*Log[x] - 2*Log[1 + x]
```

Rubi in Sympy [A] time = 6.11268, size = 12, normalized size = 1.

$$x + 4\log(x) - 2\log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+3*x+4)/(x**2+x),x)

[Out] x + 4*log(x) - 2*log(x + 1)
```

Mathematica [A] time = 0.00510085, size = 12, normalized size = 1.

$$x + 4\log(x) - 2\log(x + 1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(4 + 3*x + x^2)/(x + x^2),x]

[Out] x + 4*Log[x] - 2*Log[1 + x]
```

Maple [A] time = 0.008, size = 13, normalized size = 1.1

$$x + 4\ln(x) - 2\ln(1 + x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+3*x+4)/(x^2+x),x)

[Out] x+4*ln(x)-2*ln(1+x)
```

Maxima [A] time = 0.817265, size = 16, normalized size = 1.33

$$x - 2\,\log\left(x + 1\right) + 4\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 3*x + 4)/(x^2 + x),x, algorithm="maxima")

[Out] x - 2*log(x + 1) + 4*log(x)
```

Fricas [A] time = 0.251381, size = 16, normalized size = 1.33

$$x - 2\,\log\left(x + 1\right) + 4\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 3*x + 4)/(x^2 + x),x, algorithm="fricas")

[Out] x - 2*log(x + 1) + 4*log(x)
```

Sympy [A] time = 0.198646, size = 12, normalized size = 1.

$$x + 4\log\left(x\right) - 2\log\left(x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+3*x+4)/(x**2+x),x)

[Out] x + 4*log(x) - 2*log(x + 1)
```

GIAC/XCAS [A] time = 0.259427, size = 19, normalized size = 1.58

$$x - 2\ln\left(|x + 1|\right) + 4\ln\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 3*x + 4)/(x^2 + x),x, algorithm="giac")

[Out] x - 2*ln(abs(x + 1)) + 4*ln(abs(x))
```

3.354 $\int \frac{4+x+3x^2}{x+x^3}\,dx$

Optimal. Leaf size=17

$$-\frac{1}{2}\log\left(x^2+1\right)+4\log(x)+\tan^{-1}(x)$$

```
[Out] ArcTan[x] + 4*Log[x] - Log[1 + x^2]/2
```

Rubi [A] time = 0.0560719, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$

$$-\frac{1}{2}\log\left(x^2+1\right)+4\log(x)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(4 + x + 3*x^2)/(x + x^3),x]

[Out] ArcTan[x] + 4*Log[x] - Log[1 + x^2]/2
```

Rubi in Sympy [A] time = 8.40866, size = 15, normalized size = 0.88

$$4\log(x)-\frac{\log\left(x^2+1\right)}{2}+\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+x+4)/(x**3+x),x)

[Out] 4*log(x) - log(x**2 + 1)/2 + atan(x)
```

Mathematica [A] time = 0.00711386, size = 17, normalized size = 1.

$$-\frac{1}{2}\log\left(x^2+1\right)+4\log(x)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(4 + x + 3*x^2)/(x + x^3),x]

[Out] ArcTan[x] + 4*Log[x] - Log[1 + x^2]/2
```

Maple [A] time = 0.007, size = 16, normalized size = 0.9

$$\arctan(x)+4\ln(x)-\frac{\ln\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2+x+4)/(x^3+x),x)
```

```
[Out] arctan(x)+4*ln(x)-1/2*ln(x^2+1)
```

Maxima [A] time = 0.891133, size = 20, normalized size = 1.18

$$\arctan(x) - \frac{1}{2}\log\left(x^2+1\right) + 4\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + x + 4)/(x^3 + x),x, algorithm="maxima")
```

```
[Out] arctan(x) - 1/2*log(x^2 + 1) + 4*log(x)
```

Fricas [A] time = 0.25447, size = 20, normalized size = 1.18

$$\arctan(x) - \frac{1}{2}\log\left(x^2+1\right) + 4\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + x + 4)/(x^3 + x),x, algorithm="fricas")
```

```
[Out] arctan(x) - 1/2*log(x^2 + 1) + 4*log(x)
```

Sympy [A] time = 0.294637, size = 15, normalized size = 0.88

$$4\log(x) - \frac{\log\left(x^2+1\right)}{2} + \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**2+x+4)/(x**3+x),x)
```

```
[Out] 4*log(x) - log(x**2 + 1)/2 + atan(x)
```

GIAC/XCAS [A] time = 0.259018, size = 22, normalized size = 1.29

$$\arctan(x) - \frac{1}{2}\ln\left(x^2+1\right) + 4\ln(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + x + 4)/(x^3 + x),x, algorithm="giac")
```

```
[Out] arctan(x) - 1/2*ln(x^2 + 1) + 4*ln(abs(x))
```

3.355 $\int \frac{7-4x+8x^2}{(1+4x)(1+x^2)}\,dx$

Optimal. Leaf size=13

$$2\log(4x+1)-\tan^{-1}(x)$$

```
[Out] -ArcTan[x] + 2*Log[1 + 4*x]
```

Rubi [A] time = 0.056637, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$

$$2\log(4x+1)-\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(7 - 4*x + 8*x^2)/((1 + 4*x)*(1 + x^2)),x]

[Out] -ArcTan[x] + 2*Log[1 + 4*x]
```

Rubi in Sympy [A] time = 8.37743, size = 10, normalized size = 0.77

$$2\log(4x+1)-\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((8*x**2-4*x+7)/(1+4*x)/(x**2+1),x)

[Out] 2*log(4*x + 1) - atan(x)
```

Mathematica [A] time = 0.013479, size = 13, normalized size = 1.

$$2\log(4x+1)-\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 - 4*x + 8*x^2)/((1 + 4*x)*(1 + x^2)),x]

[Out] -ArcTan[x] + 2*Log[1 + 4*x]
```

Maple [A] time = 0.008, size = 14, normalized size = 1.1

$$-\arctan(x)+2\ln(1+4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x^2-4*x+7)/(1+4*x)/(x^2+1),x)

[Out] -arctan(x)+2*ln(1+4*x)
```

Maxima [A] time = 0.902382, size = 18, normalized size = 1.38

$$-\arctan(x) + 2\log(4x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^2 - 4*x + 7)/((x^2 + 1)*(4*x + 1)),x, algorithm="maxima")
```

```
[Out] -arctan(x) + 2*log(4*x + 1)
```

Fricas [A] time = 0.254999, size = 18, normalized size = 1.38

$$-\arctan(x) + 2\log(4x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^2 - 4*x + 7)/((x^2 + 1)*(4*x + 1)),x, algorithm="fricas")
```

```
[Out] -arctan(x) + 2*log(4*x + 1)
```

Sympy [A] time = 0.365506, size = 10, normalized size = 0.77

$$2\log\left(x + \frac{1}{4}\right) - \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((8*x**2-4*x+7)/(1+4*x)/(x**2+1),x)
```

```
[Out] 2*log(x + 1/4) - atan(x)
```

GIAC/XCAS [A] time = 0.263136, size = 19, normalized size = 1.46

$$-\arctan(x) + 2\ln(|4x + 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^2 - 4*x + 7)/((x^2 + 1)*(4*x + 1)),x, algorithm="giac")
```

```
[Out] -arctan(x) + 2*ln(abs(4*x + 1))
```

3.356 $\int \frac{x^2}{(-1+x)(1+2x+x^2)}\,dx$

Optimal. Leaf size=28

$$\frac{1}{2(x+1)}+\frac{1}{4}\log(1-x)+\frac{3}{4}\log(x+1)$$

```
[Out] 1/(2*(1 + x)) + Log[1 - x]/4 + (3*Log[1 + x])/4
```

Rubi [A] time = 0.0363008, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$\frac{1}{2(x+1)}+\frac{1}{4}\log(1-x)+\frac{3}{4}\log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/((-1 + x)*(1 + 2*x + x^2)),x]

[Out] 1/(2*(1 + x)) + Log[1 - x]/4 + (3*Log[1 + x])/4
```

Rubi in Sympy [A] time = 6.78452, size = 20, normalized size = 0.71

$$\frac{\log(-x+1)}{4}+\frac{3\log(x+1)}{4}+\frac{1}{2(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(-1+x)/(x**2+2*x+1),x)

[Out] log(-x + 1)/4 + 3*log(x + 1)/4 + 1/(2*(x + 1))
```

Mathematica [A] time = 0.0212229, size = 22, normalized size = 0.79

$$\frac{1}{4}\left(\frac{2}{x+1}+\log(x-1)+3\log(x+1)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/((-1 + x)*(1 + 2*x + x^2)),x]

[Out] (2/(1 + x) + Log[-1 + x] + 3*Log[1 + x])/4
```

Maple [A] time = 0.011, size = 21, normalized size = 0.8

$$\frac{\ln(-1+x)}{4}+\frac{1}{2+2x}+\frac{3\ln(1+x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(-1+x)/(x^2+2*x+1),x)
```

```
[Out] 1/4*ln(-1+x)+1/2/(1+x)+3/4*ln(1+x)
```

Maxima [A] time = 0.802438, size = 27, normalized size = 0.96

$$\frac{1}{2\,(x+1)} + \frac{3}{4}\,\log\left(x+1\right) + \frac{1}{4}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((x^2 + 2*x + 1)*(x - 1)),x, algorithm="maxima")

[Out] 1/2/(x + 1) + 3/4*log(x + 1) + 1/4*log(x - 1)
```

Fricas [A] time = 0.253225, size = 35, normalized size = 1.25

$$\frac{3\,(x+1)\log\left(x+1\right) + (x+1)\log\left(x-1\right) + 2}{4\,(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((x^2 + 2*x + 1)*(x - 1)),x, algorithm="fricas")

[Out] 1/4*(3*(x + 1)*log(x + 1) + (x + 1)*log(x - 1) + 2)/(x + 1)
```

Sympy [A] time = 0.23563, size = 20, normalized size = 0.71

$$\frac{\log\left(x-1\right)}{4} + \frac{3\log\left(x+1\right)}{4} + \frac{1}{2x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(-1+x)/(x**2+2*x+1),x)

[Out] log(x - 1)/4 + 3*log(x + 1)/4 + 1/(2*x + 2)
```

GIAC/XCAS [A] time = 0.260459, size = 30, normalized size = 1.07

$$\frac{1}{2\,(x+1)} + \frac{3}{4}\ln\left(|x+1|\right) + \frac{1}{4}\ln\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((x^2 + 2*x + 1)*(x - 1)),x, algorithm="giac")

[Out] 1/2/(x + 1) + 3/4*ln(abs(x + 1)) + 1/4*ln(abs(x - 1))
```

3.357 $\int \frac{-4+3x+x^2}{(-1+2x)^2(3+2x)}\,dx$

Optimal. Leaf size=32

$$-\frac{9}{32(1-2x)}+\frac{41}{128}\log(1-2x)-\frac{25}{128}\log(2x+3)$$

```
[Out] -9/(32*(1 - 2*x)) + (41*Log[1 - 2*x])/128 - (25*Log[3 + 2*x])/128
```

Rubi [A] time = 0.0641163, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$

$$-\frac{9}{32(1-2x)}+\frac{41}{128}\log(1-2x)-\frac{25}{128}\log(2x+3)$$

Antiderivative was successfully verified.

```
[In]  Int[(-4 + 3*x + x^2)/((-1 + 2*x)^2*(3 + 2*x)),x]

[Out] -9/(32*(1 - 2*x)) + (41*Log[1 - 2*x])/128 - (25*Log[3 + 2*x])/128
```

Rubi in Sympy [A] time = 8.42037, size = 26, normalized size = 0.81

$$\frac{41\log(-2x+1)}{128}-\frac{25\log(2x+3)}{128}-\frac{9}{32(-2x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+3*x-4)/(-1+2*x)**2/(3+2*x),x)

[Out] 41*log(-2*x + 1)/128 - 25*log(2*x + 3)/128 - 9/(32*(-2*x + 1))
```

Mathematica [A] time = 0.0246441, size = 32, normalized size = 1.

$$\frac{9}{32(2x-1)}+\frac{41}{128}\log(1-2x)-\frac{25}{128}\log(2x+3)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-4 + 3*x + x^2)/((-1 + 2*x)^2*(3 + 2*x)),x]

[Out] 9/(32*(-1 + 2*x)) + (41*Log[1 - 2*x])/128 - (25*Log[3 + 2*x])/128
```

Maple [A] time = 0.011, size = 27, normalized size = 0.8

$$-\frac{25\ln(3+2x)}{128}+\frac{9}{64x-32}+\frac{41\ln(2x-1)}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+3*x-4)/(2*x-1)^2/(3+2*x),x)
```

```
[Out] -25/128*ln(3+2*x)+9/32/(2*x-1)+41/128*ln(2*x-1)
```

Maxima [A] time = 0.811985, size = 35, normalized size = 1.09

$$\frac{9}{32\,(2\,x-1)}-\frac{25}{128}\,\log\left(2\,x+3\right)+\frac{41}{128}\,\log\left(2\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 + 3*x - 4)/((2*x + 3)*(2*x - 1)^2),x, algorithm="maxima")
```

```
[Out] 9/32/(2*x - 1) - 25/128*log(2*x + 3) + 41/128*log(2*x - 1)
```

Fricas [A] time = 0.248939, size = 50, normalized size = 1.56

$$-\frac{25\,(2\,x-1)\log\left(2\,x+3\right)-41\,(2\,x-1)\log\left(2\,x-1\right)-36}{128\,(2\,x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 + 3*x - 4)/((2*x + 3)*(2*x - 1)^2),x, algorithm="fricas")
```

```
[Out] -1/128*(25*(2*x - 1)*log(2*x + 3) - 41*(2*x - 1)*log(2*x - 1) - 3
6)/(2*x - 1)
```

Sympy [A] time = 0.317317, size = 26, normalized size = 0.81

$$\frac{41\log\left(x-\frac{1}{2}\right)}{128}-\frac{25\log\left(x+\frac{3}{2}\right)}{128}+\frac{9}{64x-32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+3*x-4)/(-1+2*x)**2/(3+2*x),x)
```

```
[Out] 41*log(x - 1/2)/128 - 25*log(x + 3/2)/128 + 9/(64*x - 32)
```

GIAC/XCAS [A] time = 0.261922, size = 58, normalized size = 1.81

$$\frac{9}{32\,(2\,x-1)}-\frac{1}{8}\ln\left(\frac{|2\,x-1|}{2\,(2\,x-1)^{2}}\right)-\frac{25}{128}\ln\left(\left|-\frac{4}{2\,x-1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 + 3*x - 4)/((2*x + 3)*(2*x - 1)^2),x, algorithm="giac")
```

```
[Out] 9/32/(2*x - 1) - 1/8*ln(1/2*abs(2*x - 1)/(2*x - 1)^2) - 25/128*ln
(abs(-4/(2*x - 1) - 1))
```

3.358 $\int \frac{5-4x+3x^2}{(-1+x)(1+x^2)}\,dx$

Optimal. Leaf size=23

$$\frac{1}{2}\log\left(x^2+1\right)+2\log(1-x)-3\tan^{-1}(x)$$

```
[Out] -3*ArcTan[x] + 2*Log[1 - x] + Log[1 + x^2]/2
```

Rubi [A] time = 0.0635736, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174

$$\frac{1}{2}\log\left(x^2+1\right)+2\log(1-x)-3\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(5 - 4*x + 3*x^2)/((-1 + x)*(1 + x^2)),x]

[Out] -3*ArcTan[x] + 2*Log[1 - x] + Log[1 + x^2]/2
```

Rubi in Sympy [A] time = 9.04946, size = 19, normalized size = 0.83

$$2\log(-x+1)+\frac{\log\left(x^2+1\right)}{2}-3\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2-4*x+5)/(-1+x)/(x**2+1),x)

[Out] 2*log(-x + 1) + log(x**2 + 1)/2 - 3*atan(x)
```

Mathematica [A] time = 0.0116205, size = 28, normalized size = 1.22

$$\frac{1}{2}\log\left((x-1)^2+2(x-1)+2\right)+2\log(x-1)-3\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(5 - 4*x + 3*x^2)/((-1 + x)*(1 + x^2)),x]

[Out] -3*ArcTan[x] + Log[2 + 2*(-1 + x) + (-1 + x)^2]/2 + 2*Log[-1 + x]
```

Maple [A] time = 0.007, size = 20, normalized size = 0.9

$$2\ln(-1+x)+\frac{\ln\left(x^2+1\right)}{2}-3\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2-4*x+5)/(-1+x)/(x^2+1),x)
```

```
[Out] 2*ln(-1+x)+1/2*ln(x^2+1)-3*arctan(x)
```

Maxima [A] time = 0.885305, size = 26, normalized size = 1.13

$$-3\arctan(x) + \frac{1}{2}\log\left(x^2+1\right) + 2\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2 - 4*x + 5)/((x^2 + 1)*(x - 1)),x, algorithm="maxima")
```

```
[Out] -3*arctan(x) + 1/2*log(x^2 + 1) + 2*log(x - 1)
```

Fricas [A] time = 0.254199, size = 26, normalized size = 1.13

$$-3\arctan(x) + \frac{1}{2}\log\left(x^2+1\right) + 2\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2 - 4*x + 5)/((x^2 + 1)*(x - 1)),x, algorithm="fricas")
```

```
[Out] -3*arctan(x) + 1/2*log(x^2 + 1) + 2*log(x - 1)
```

Sympy [A] time = 0.309548, size = 19, normalized size = 0.83

$$2\log(x-1) + \frac{\log\left(x^2+1\right)}{2} - 3\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**2-4*x+5)/(-1+x)/(x**2+1),x)
```

```
[Out] 2*log(x - 1) + log(x**2 + 1)/2 - 3*atan(x)
```

GIAC/XCAS [A] time = 0.263705, size = 27, normalized size = 1.17

$$-3\arctan(x) + \frac{1}{2}\ln\left(x^2+1\right) + 2\ln(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 - 4*x + 5)/((x^2 + 1)*(x - 1)),x, algorithm="giac")
```

```
[Out] -3*arctan(x) + 1/2*ln(x^2 + 1) + 2*ln(abs(x - 1))
```

3.359 $\int \frac{-1-2x+x^2}{(-1+x)^2(1+x^2)}\,dx$

Optimal. Leaf size=24

$$-\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{x-1}+\log(1-x)+\tan^{-1}(x)$$

```
[Out] (-1 + x)^(-1) + ArcTan[x] + Log[1 - x] - Log[1 + x^2]/2
```

Rubi [A] time = 0.0611676, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$

$$-\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{x-1}+\log(1-x)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 - 2*x + x^2)/((-1 + x)^2*(1 + x^2)),x]

[Out] (-1 + x)^(-1) + ArcTan[x] + Log[1 - x] - Log[1 + x^2]/2
```

Rubi in Sympy [A] time = 9.07149, size = 20, normalized size = 0.83

$$\log(-x+1)-\frac{\log\left(x^2+1\right)}{2}+\operatorname{atan}(x)-\frac{1}{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2-2*x-1)/(-1+x)**2/(x**2+1),x)

[Out] log(-x + 1) - log(x**2 + 1)/2 + atan(x) - 1/(-x + 1)
```

Mathematica [A] time = 0.0224606, size = 22, normalized size = 0.92

$$-\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{x-1}+\log(x-1)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 - 2*x + x^2)/((-1 + x)^2*(1 + x^2)),x]

[Out] (-1 + x)^(-1) + ArcTan[x] + Log[-1 + x] - Log[1 + x^2]/2
```

Maple [A] time = 0.009, size = 21, normalized size = 0.9

$$\ln(-1+x)+(-1+x)^{-1}-\frac{\ln\left(x^2+1\right)}{2}+\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2-2*x-1)/(-1+x)^2/(x^2+1),x)
```

```
[Out] ln(-1+x)+1/(-1+x)-1/2*ln(x^2+1)+arctan(x)
```

Maxima [A] time = 0.907467, size = 27, normalized size = 1.12

$$\frac{1}{x-1}+\arctan(x)-\frac{1}{2}\log\left(x^{2}+1\right)+\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 - 2*x - 1)/((x^2 + 1)*(x - 1)^2),x, algorithm="maxima")
```

```
[Out] 1/(x - 1) + arctan(x) - 1/2*log(x^2 + 1) + log(x - 1)
```

Fricas [A] time = 0.258675, size = 49, normalized size = 2.04

$$\frac{2(x-1)\arctan(x)-(x-1)\log\left(x^{2}+1\right)+2(x-1)\log(x-1)+2}{2(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 - 2*x - 1)/((x^2 + 1)*(x - 1)^2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(x - 1)*arctan(x) - (x - 1)*log(x^2 + 1) + 2*(x - 1)*log(x
 - 1) + 2)/(x - 1)
```

Sympy [A] time = 0.335632, size = 20, normalized size = 0.83

$$\log(x-1)-\frac{\log\left(x^{2}+1\right)}{2}+\operatorname{atan}(x)+\frac{1}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2-2*x-1)/(-1+x)**2/(x**2+1),x)
```

```
[Out] log(x - 1) - log(x**2 + 1)/2 + atan(x) + 1/(x - 1)
```

GIAC/XCAS [A] time = 0.263389, size = 63, normalized size = 2.62

$$\frac{1}{4}\pi-\pi\left\lfloor\frac{\pi+4\arctan(x)}{4\pi}+\frac{1}{2}\right\rfloor+\frac{1}{x-1}+\arctan(x)-\frac{1}{2}\ln\left(\frac{2}{x-1}+\frac{2}{(x-1)^{2}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 2*x - 1)/((x^2 + 1)*(x - 1)^2),x, algorithm="giac")
```

```
[Out] 1/4*pi - pi*floor(1/4*(pi + 4*arctan(x))/pi + 1/2) + 1/(x - 1) +
arctan(x) - 1/2*ln(2/(x - 1) + 2/(x - 1)^2 + 1)
```

3.360 $\int \frac{5+x^3}{(10-6x+x^2)\left(\frac{1}{2}-x+x^2\right)} \, dx$

Optimal. Leaf size=49

$$\frac{56}{221}\log\left(x^2-6x+10\right)+\frac{109}{442}\log\left(2x^2-2x+1\right)-\frac{261}{221}\tan^{-1}(1-2x)-\frac{1026}{221}\tan^{-1}(3-x)$$

```
[Out] (-261*ArcTan[1 - 2*x])/221 - (1026*ArcTan[3 - x])/221 + (56*Log[1
0 - 6*x + x^2])/221 + (109*Log[1 - 2*x + 2*x^2])/442
```

Rubi [A] time = 0.257859, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214

$$\frac{56}{221}\log\left(x^2-6x+10\right)+\frac{109}{442}\log\left(2x^2-2x+1\right)-\frac{261}{221}\tan^{-1}(1-2x)-\frac{1026}{221}\tan^{-1}(3-x)$$

Antiderivative was successfully verified.

```
[In] Int[(5 + x^3)/((10 - 6*x + x^2)*(1/2 - x + x^2)),x]
```

```
[Out] (-261*ArcTan[1 - 2*x])/221 - (1026*ArcTan[3 - x])/221 + (56*Log[1
0 - 6*x + x^2])/221 + (109*Log[1 - 2*x + 2*x^2])/442
```

Rubi in Sympy [A] time = 54.7232, size = 46, normalized size = 0.94

$$\frac{56\log\left(x^2-6x+10\right)}{221}+\frac{109\log\left(2x^2-2x+1\right)}{442}+\frac{1026\,\text{atan}\,(x-3)}{221}+\frac{261\,\text{atan}\,(2x-1)}{221}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((x**3+5)/(x**2-6*x+10)/(1/2-x+x**2),x)
```

```
[Out] 56*log(x**2 - 6*x + 10)/221 + 109*log(2*x**2 - 2*x + 1)/442 + 102
6*atan(x - 3)/221 + 261*atan(2*x - 1)/221
```

Mathematica [A] time = 0.0229265, size = 49, normalized size = 1.

$$\frac{56}{221}\log\left(x^2-6x+10\right)+\frac{109}{442}\log\left(2x^2-2x+1\right)-\frac{261}{221}\tan^{-1}(1-2x)-\frac{1026}{221}\tan^{-1}(3-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + x^3)/((10 - 6*x + x^2)*(1/2 - x + x^2)),x]
```

```
[Out] (-261*ArcTan[1 - 2*x])/221 - (1026*ArcTan[3 - x])/221 + (56*Log[1
0 - 6*x + x^2])/221 + (109*Log[1 - 2*x + 2*x^2])/442
```

Maple [A] time = 0.01, size = 40, normalized size = 0.8

$$\frac{261\,\arctan\left(2\,x-1\right)}{221}+\frac{1026\,\arctan\left(-3+x\right)}{221}+\frac{56\,\ln\left(x^2-6\,x+10\right)}{221}+\frac{109\,\ln\left(2\,x^2-2\,x+1\right)}{442}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^3+5)/(x^2-6*x+10)/(1/2-x+x^2),x)
```

```
[Out] 261/221*arctan(2*x-1)+1026/221*arctan(-3+x)+56/221*ln(x^2-6*x+10)
+109/442*ln(2*x^2-2*x+1)
```

Maxima [A] time = 0.883293, size = 53, normalized size = 1.08

$$\frac{261}{221}\arctan\left(2x-1\right)+\frac{1026}{221}\arctan\left(x-3\right)+\frac{109}{442}\log\left(2x^{2}-2x+1\right)+\frac{56}{221}\log\left(x^{2}-6x+10\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*(x^3 + 5)/((2*x^2 - 2*x + 1)*(x^2 - 6*x + 10)),x, algorithm="maxima")
```

```
[Out] 261/221*arctan(2*x - 1) + 1026/221*arctan(x - 3) + 109/442*log(2*
x^2 - 2*x + 1) + 56/221*log(x^2 - 6*x + 10)
```

Fricas [A] time = 0.277207, size = 50, normalized size = 1.02

$$\frac{261}{221}\arctan\left(2x-1\right)+\frac{1026}{221}\arctan\left(x-3\right)+\frac{109}{442}\log\left(x^{2}-x+\frac{1}{2}\right)+\frac{56}{221}\log\left(x^{2}-6x+10\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*(x^3 + 5)/((2*x^2 - 2*x + 1)*(x^2 - 6*x + 10)),x, algorithm="fricas")
```

```
[Out] 261/221*arctan(2*x - 1) + 1026/221*arctan(x - 3) + 109/442*log(x^
2 - x + 1/2) + 56/221*log(x^2 - 6*x + 10)
```

Sympy [A] time = 0.63495, size = 44, normalized size = 0.9

$$\frac{56\log\left(x^{2}-6x+10\right)}{221}+\frac{109\log\left(x^{2}-x+\frac{1}{2}\right)}{442}+\frac{1026\operatorname{atan}\left(x-3\right)}{221}+\frac{261\operatorname{atan}\left(2x-1\right)}{221}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3+5)/(x**2-6*x+10)/(1/2-x+x**2),x)
```

```
[Out] 56*log(x**2 - 6*x + 10)/221 + 109*log(x**2 - x + 1/2)/442 + 1026*
atan(x - 3)/221 + 261*atan(2*x - 1)/221
```

GIAC/XCAS [A] time = 0.25985, size = 53, normalized size = 1.08

$$\frac{261}{221}\arctan\left(2x-1\right)+\frac{1026}{221}\arctan\left(x-3\right)+\frac{109}{442}\ln\left(2x^{2}-2x+1\right)+\frac{56}{221}\ln\left(x^{2}-6x+10\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*(x^3 + 5)/((2*x^2 - 2*x + 1)*(x^2 - 6*x + 10)),x, algorithm="giac")
```

```
[Out] 261/221*arctan(2*x - 1) + 1026/221*arctan(x - 3) + 109/442*ln(2*x
^2 - 2*x + 1) + 56/221*ln(x^2 - 6*x + 10)
```

3.361 $\int \frac{4+3x+x^2}{(-3+x)(-2+x)(-1+x)}\,dx$

Optimal. Leaf size=25

$$4\log(1-x)-14\log(2-x)+11\log(3-x)$$

```
[Out] 4*Log[1 - x] - 14*Log[2 - x] + 11*Log[3 - x]
```

Rubi [A] time = 0.0958125, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$

$$4\log(1-x)-14\log(2-x)+11\log(3-x)$$

Antiderivative was successfully verified.

```
[In]  Int[(4 + 3*x + x^2)/((-3 + x)*(-2 + x)*(-1 + x)),x]

[Out] 4*Log[1 - x] - 14*Log[2 - x] + 11*Log[3 - x]
```

Rubi in Sympy [A] time = 10.398, size = 19, normalized size = 0.76

$$4\log(-x+1)-14\log(-x+2)+11\log(-x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+3*x+4)/(-3+x)/(-2+x)/(-1+x),x)

[Out] 4*log(-x + 1) - 14*log(-x + 2) + 11*log(-x + 3)
```

Mathematica [A] time = 0.0127385, size = 19, normalized size = 0.76

$$11\log(x-3)-14\log(x-2)+4\log(x-1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(4 + 3*x + x^2)/((-3 + x)*(-2 + x)*(-1 + x)),x]

[Out] 11*Log[-3 + x] - 14*Log[-2 + x] + 4*Log[-1 + x]
```

Maple [A] time = 0.01, size = 20, normalized size = 0.8

$$4\ln(-1+x)+11\ln(-3+x)-14\ln(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+3*x+4)/(-3+x)/(x-2)/(-1+x),x)

[Out] 4*ln(-1+x)+11*ln(-3+x)-14*ln(x-2)
```

Maxima [A] time = 0.804719, size = 26, normalized size = 1.04

$$4 \log(x-1) - 14 \log(x-2) + 11 \log(x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 + 3*x + 4)/((x - 1)*(x - 2)*(x - 3)),x, algorithm="maxima")
```

```
[Out] 4*log(x - 1) - 14*log(x - 2) + 11*log(x - 3)
```

Fricas [A] time = 0.262204, size = 26, normalized size = 1.04

$$4 \log(x-1) - 14 \log(x-2) + 11 \log(x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 + 3*x + 4)/((x - 1)*(x - 2)*(x - 3)),x, algorithm="fricas")
```

```
[Out] 4*log(x - 1) - 14*log(x - 2) + 11*log(x - 3)
```

Sympy [A] time = 0.309479, size = 19, normalized size = 0.76

$$11 \log(x-3) - 14 \log(x-2) + 4 \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+3*x+4)/(-3+x)/(-2+x)/(-1+x),x)
```

```
[Out] 11*log(x - 3) - 14*log(x - 2) + 4*log(x - 1)
```

GIAC/XCAS [A] time = 0.261981, size = 30, normalized size = 1.2

$$4 \ln(|x-1|) - 14 \ln(|x-2|) + 11 \ln(|x-3|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 + 3*x + 4)/((x - 1)*(x - 2)*(x - 3)),x, algorithm="giac")
```

```
[Out] 4*ln(abs(x - 1)) - 14*ln(abs(x - 2)) + 11*ln(abs(x - 3))
```

3.362 $\int \frac{1+16x}{(5+x)^2(-3+2x)(1+x+x^2)}\,dx$

Optimal. Leaf size=60

$$-\frac{481\log\left(x^2+x+1\right)}{5586}-\frac{79}{273(x+5)}+\frac{200\log(3-2x)}{3211}+\frac{2731\log(x+5)}{24843}+\frac{451\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2793\sqrt{3}}$$

```
[Out] -79/(273*(5 + x)) + (451*ArcTan[(1 + 2*x)/Sqrt[3]])/(2793*Sqrt[3]
) + (200*Log[3 - 2*x])/3211 + (2731*Log[5 + x])/24843 - (481*Log[
1 + x + x^2])/5586
```

Rubi [A] time = 0.456796, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192

$$-\frac{481\log\left(x^2+x+1\right)}{5586}-\frac{79}{273(x+5)}+\frac{200\log(3-2x)}{3211}+\frac{2731\log(x+5)}{24843}+\frac{451\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2793\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + 16*x)/((5 + x)^2*(-3 + 2*x)*(1 + x + x^2)),x]
```

```
[Out] -79/(273*(5 + x)) + (451*ArcTan[(1 + 2*x)/Sqrt[3]])/(2793*Sqrt[3]
) + (200*Log[3 - 2*x])/3211 + (2731*Log[5 + x])/24843 - (481*Log[
1 + x + x^2])/5586
```

Rubi in Sympy [A] time = 164.427, size = 60, normalized size = 1.

$$\frac{200\log\left(-2x+3\right)}{3211}+\frac{2731\log\left(x+5\right)}{24843}-\frac{481\log\left(x^2+x+1\right)}{5586}+\frac{451\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}+\frac{1}{3}\right)\right)}{8379}-\frac{79}{273\left(x+5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+16*x)/(5+x)**2/(-3+2*x)/(x**2+x+1),x)
```

```
[Out] 200*log(-2*x + 3)/3211 + 2731*log(x + 5)/24843 - 481*log(x**2 + x
 + 1)/5586 + 451*sqrt(3)*atan(sqrt(3)*(2*x/3 + 1/3))/8379 - 79/(2
73*(x + 5))
```

Mathematica [A] time = 0.0896317, size = 54, normalized size = 0.9

$$\frac{-243867\log\left(x^2+x+1\right)-\frac{819546}{x+5}+176400\log(3-2x)+311334\log(x+5)+152438\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2832102}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + 16*x)/((5 + x)^2*(-3 + 2*x)*(1 + x + x^2)),x]
```

```
[Out] (-819546/(5 + x) + 152438*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 176
400*Log[3 - 2*x] + 311334*Log[5 + x] - 243867*Log[1 + x + x^2])/2
832102
```

Maple [A] time = 0.014, size = 48, normalized size = 0.8

$$-\frac{481\ \ln\left(x^{2}+x+1\right)}{5586}+\frac{451\sqrt{3}}{8379}\arctan\left(\frac{(1+2\,x)\sqrt{3}}{3}\right)-\frac{79}{1365+273\,x}+\frac{2731\ \ln\left(5+x\right)}{24843}+\frac{200\ \ln\left(-3+2\,x\right)}{3211}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+16*x)/(5+x)^2/(-3+2*x)/(x^2+x+1),x)
```

```
[Out] -481/5586*ln(x^2+x+1)+451/8379*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2
)-79/273/(5+x)+2731/24843*ln(5+x)+200/3211*ln(-3+2*x)
```

Maxima [A] time = 0.900518, size = 63, normalized size = 1.05

$$\frac{451}{8379}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x+1)\right)-\frac{79}{273\,(x+5)}-\frac{481}{5586}\log\left(x^{2}+x+1\right)+\frac{200}{3211}\log\left(2\,x-3\right)+\frac{2731}{24843}\log\left(x+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((16*x + 1)/((x^2 + x + 1)*(2*x - 3)*(x + 5)^2),x, algorithm="maxima")
```

```
[Out] 451/8379*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 79/273/(x + 5) -
 481/5586*log(x^2 + x + 1) + 200/3211*log(2*x - 3) + 2731/24843*l
og(x + 5)
```

Fricas [A] time = 0.295344, size = 99, normalized size = 1.65

$$-\frac{\sqrt{3}\left(81289\sqrt{3}(x+5)\log\left(x^{2}+x+1\right)-58800\sqrt{3}(x+5)\log\left(2\,x-3\right)-103778\sqrt{3}(x+5)\log\left(x+5\right)-152438\,(x+5)\,\text{arcta}\right.}{2832102\,(x+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((16*x + 1)/((x^2 + x + 1)*(2*x - 3)*(x + 5)^2),x, algorithm="fricas")
```

```
[Out] -1/2832102*sqrt(3)*(81289*sqrt(3)*(x + 5)*log(x^2 + x + 1) - 5880
0*sqrt(3)*(x + 5)*log(2*x - 3) - 103778*sqrt(3)*(x + 5)*log(x + 5
) - 152438*(x + 5)*arctan(1/3*sqrt(3)*(2*x + 1)) + 273182*sqrt(3)
)/(x + 5)
```

Sympy [A] time = 0.762493, size = 63, normalized size = 1.05

$$\frac{200\log\left(x-\frac{3}{2}\right)}{3211}+\frac{2731\log\left(x+5\right)}{24843}-\frac{481\log\left(x^{2}+x+1\right)}{5586}+\frac{451\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{8379}-\frac{79}{273x+1365}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+16*x)/(5+x)**2/(-3+2*x)/(x**2+x+1),x)
```

```
[Out] 200*log(x - 3/2)/3211 + 2731*log(x + 5)/24843 - 481*log(x**2 + x
+ 1)/5586 + 451*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/8379 - 79
/(273*x + 1365)
```

GIAC/XCAS [A] time = 0.263523, size = 81, normalized size = 1.35

$$\frac{451}{8379}\sqrt{3}\arctan\left(-\sqrt{3}\left(\frac{14}{x+5}-3\right)\right)-\frac{79}{273(x+5)}-\frac{481}{5586}\ln\left(-\frac{9}{x+5}+\frac{21}{(x+5)^2}+1\right)+\frac{200}{3211}\ln\left(\left|-\frac{13}{x+5}+2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((16*x + 1)/((x^2 + x + 1)*(2*x - 3)*(x + 5)^2),x, algorithm="giac")
```

```
[Out] 451/8379*sqrt(3)*arctan(-sqrt(3)*(14/(x + 5) - 3)) - 79/273/(x +
5) - 481/5586*ln(-9/(x + 5) + 21/(x + 5)^2 + 1) + 200/3211*ln(abs
(-13/(x + 5) + 2))
```

3.363 $\int \frac{-1+x^3}{1+x+x^2} \, dx$

Optimal. Leaf size=11

$$\frac{x^2}{2}-x$$

```
[Out] -x + x^2/2
```

Rubi [A] time = 0.0118247, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071

$$\frac{x^2}{2}-x$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + x^3)/(1 + x + x^2),x]

[Out] -x + x^2/2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-x+\int x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**3-1)/(x**2+x+1),x)

[Out] -x + Integral(x, x)
```

Mathematica [A] time = 0.000599968, size = 11, normalized size = 1.

$$\frac{x^2}{2}-x$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + x^3)/(1 + x + x^2),x]

[Out] -x + x^2/2
```

Maple [A] time = 0.001, size = 10, normalized size = 0.9

$$-x+\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^3-1)/(x^2+x+1),x)
```

```
[Out] -x+1/2*x^2
```

Maxima [A] time = 0.831675, size = 12, normalized size = 1.09

$$\frac{1}{2}x^2 - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - 1)/(x^2 + x + 1),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - x
```

Fricas [A] time = 0.247404, size = 12, normalized size = 1.09

$$\frac{1}{2}x^2 - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - 1)/(x^2 + x + 1),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 - x
```

Sympy [A] time = 0.072308, size = 5, normalized size = 0.45

$$\frac{x^2}{2} - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3-1)/(x**2+x+1),x)
```

```
[Out] x**2/2 - x
```

GIAC/XCAS [A] time = 0.259574, size = 12, normalized size = 1.09

$$\frac{1}{2}x^2 - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - 1)/(x^2 + x + 1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - x
```

3.364 $\int \frac{-3+x^3}{-7-6x+x^2} \, dx$

Optimal. Leaf size=29

$$\frac{x^2}{2} + 6x + \frac{85}{2}\log(7-x) + \frac{1}{2}\log(x+1)$$

```
[Out] 6*x + x^2/2 + (85*Log[7 - x])/2 + Log[1 + x]/2
```

Rubi [A] time = 0.0359319, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$

$$\frac{x^2}{2} + 6x + \frac{85}{2}\log(7-x) + \frac{1}{2}\log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[(-3 + x^3)/(-7 - 6*x + x^2),x]

[Out] 6*x + x^2/2 + (85*Log[7 - x])/2 + Log[1 + x]/2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$6x + \frac{85\log(-x+7)}{2} + \frac{\log(x+1)}{2} + \int x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**3-3)/(x**2-6*x-7),x)

[Out] 6*x + 85*log(-x + 7)/2 + log(x + 1)/2 + Integral(x, x)
```

Mathematica [A] time = 0.00906544, size = 29, normalized size = 1.

$$\frac{x^2}{2} + 6x + \frac{85}{2}\log(7-x) + \frac{1}{2}\log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-3 + x^3)/(-7 - 6*x + x^2),x]

[Out] 6*x + x^2/2 + (85*Log[7 - x])/2 + Log[1 + x]/2
```

Maple [A] time = 0.008, size = 22, normalized size = 0.8

$$\frac{x^2}{2} + 6\,x + \frac{\ln(1+x)}{2} + \frac{85\,\ln(x-7)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^3-3)/(x^2-6*x-7),x)
```

[Out] 1/2*x^2+6*x+1/2*ln(1+x)+85/2*ln(x-7)

Maxima [A] time = 0.825232, size = 28, normalized size = 0.97

$$\frac{1}{2}x^2 + 6x + \frac{1}{2}\log(x+1) + \frac{85}{2}\log(x-7)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - 3)/(x^2 - 6*x - 7),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + 6*x + 1/2*log(x + 1) + 85/2*log(x - 7)
```

Fricas [A] time = 0.253094, size = 28, normalized size = 0.97

$$\frac{1}{2}x^2 + 6x + \frac{1}{2}\log(x+1) + \frac{85}{2}\log(x-7)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - 3)/(x^2 - 6*x - 7),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 + 6*x + 1/2*log(x + 1) + 85/2*log(x - 7)
```

Sympy [A] time = 0.212784, size = 22, normalized size = 0.76

$$\frac{x^2}{2} + 6x + \frac{85\log(x-7)}{2} + \frac{\log(x+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3-3)/(x**2-6*x-7),x)
```

```
[Out] x**2/2 + 6*x + 85*log(x - 7)/2 + log(x + 1)/2
```

GIAC/XCAS [A] time = 0.261119, size = 31, normalized size = 1.07

$$\frac{1}{2}x^2 + 6x + \frac{1}{2}\ln(|x+1|) + \frac{85}{2}\ln(|x-7|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - 3)/(x^2 - 6*x - 7),x, algorithm="giac")
```

```
[Out] 1/2*x^2 + 6*x + 1/2*ln(abs(x + 1)) + 85/2*ln(abs(x - 7))
```

3.365 $\int \frac{1+x^3}{(13+4x+x^2)^2} dx$

Optimal. Leaf size=45

$$\frac{47x + 67}{18(x^2 + 4x + 13)} + \frac{1}{2}\log(x^2 + 4x + 13) - \frac{61}{54}\tan^{-1}\left(\frac{x+2}{3}\right)$$

```
[Out] (67 + 47*x)/(18*(13 + 4*x + x^2)) - (61*ArcTan[(2 + x)/3])/54 + L
og[13 + 4*x + x^2]/2
```

Rubi [A] time = 0.0528657, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$

$$\frac{47x + 67}{18(x^2 + 4x + 13)} + \frac{1}{2}\log(x^2 + 4x + 13) - \frac{61}{54}\tan^{-1}\left(\frac{x+2}{3}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^3)/(13 + 4*x + x^2)^2,x]
```

```
[Out] (67 + 47*x)/(18*(13 + 4*x + x^2)) - (61*ArcTan[(2 + x)/3])/54 + L
og[13 + 4*x + x^2]/2
```

Rubi in Sympy [A] time = 13.9818, size = 34, normalized size = 0.76

$$\frac{0.0138888888888889(188x + 268)}{x^2 + 4x + 13} + \frac{\log(x^2 + 4x + 13)}{2} - 1.12962962962963\,\text{atan}\left(\frac{x}{3} + \frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**3+1)/(x**2+4*x+13)**2,x)
```

```
[Out] 0.0138888888888889*(188*x + 268)/(x**2 + 4*x + 13) + log(x**2 + 4
*x + 13)/2 - 1.12962962962963*atan(x/3 + 2/3)
```

Mathematica [A] time = 0.0195404, size = 45, normalized size = 1.

$$\frac{47x + 67}{18(x^2 + 4x + 13)} + \frac{1}{2}\log(x^2 + 4x + 13) - \frac{61}{54}\tan^{-1}\left(\frac{x+2}{3}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^3)/(13 + 4*x + x^2)^2,x]
```

```
[Out] (67 + 47*x)/(18*(13 + 4*x + x^2)) - (61*ArcTan[(2 + x)/3])/54 + L
og[13 + 4*x + x^2]/2
```

Maple [A] time = 0.009, size = 37, normalized size = 0.8

$$\frac{1}{x^2 + 4x + 13}\left(\frac{47x}{18} + \frac{67}{18}\right) + \frac{\ln(x^2 + 4x + 13)}{2} - \frac{61}{54}\arctan\left(\frac{2}{3} + \frac{x}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^3+1)/(x^2+4*x+13)^2,x)
```

```
[Out] (47/18*x+67/18)/(x^2+4*x+13)+1/2*ln(x^2+4*x+13)-61/54*arctan(2/3+
1/3*x)
```

Maxima [A] time = 0.889976, size = 50, normalized size = 1.11

$$\frac{47\,x+67}{18\,(x^2+4\,x+13)}-\frac{61}{54}\,\arctan\left(\frac{1}{3}\,x+\frac{2}{3}\right)+\frac{1}{2}\,\log\left(x^2+4\,x+13\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + 1)/(x^2 + 4*x + 13)^2,x, algorithm="maxima")
```

```
[Out] 1/18*(47*x + 67)/(x^2 + 4*x + 13) - 61/54*arctan(1/3*x + 2/3) + 1
/2*log(x^2 + 4*x + 13)
```

Fricas [A] time = 0.258079, size = 70, normalized size = 1.56

$$-\frac{61\,\left(x^2+4\,x+13\right)\arctan\left(\frac{1}{3}\,x+\frac{2}{3}\right)-27\,\left(x^2+4\,x+13\right)\log\left(x^2+4\,x+13\right)-141\,x-201}{54\,(x^2+4\,x+13)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + 1)/(x^2 + 4*x + 13)^2,x, algorithm="fricas")
```

```
[Out] -1/54*(61*(x^2 + 4*x + 13)*arctan(1/3*x + 2/3) - 27*(x^2 + 4*x +
13)*log(x^2 + 4*x + 13) - 141*x - 201)/(x^2 + 4*x + 13)
```

Sympy [A] time = 0.324539, size = 37, normalized size = 0.82

$$\frac{47x+67}{18x^2+72x+234}+\frac{\log\left(x^2+4x+13\right)}{2}-\frac{61\operatorname{atan}\left(\frac{x}{3}+\frac{2}{3}\right)}{54}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3+1)/(x**2+4*x+13)**2,x)
```

```
[Out] (47*x + 67)/(18*x**2 + 72*x + 234) + log(x**2 + 4*x + 13)/2 - 61*
atan(x/3 + 2/3)/54
```

GIAC/XCAS [A] time = 0.260441, size = 50, normalized size = 1.11

$$\frac{47\,x+67}{18\,(x^2+4\,x+13)}-\frac{61}{54}\,\arctan\left(\frac{1}{3}\,x+\frac{2}{3}\right)+\frac{1}{2}\,\ln\left(x^2+4\,x+13\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + 1)/(x^2 + 4*x + 13)^2,x, algorithm="giac")
```

```
[Out] 1/18*(47*x + 67)/(x^2 + 4*x + 13) - 61/54*arctan(1/3*x + 2/3) + 1
/2*ln(x^2 + 4*x + 13)
```

3.366 $\int \frac{-32+36x-42x^2+21x^3-10x^4+3x^5}{x(1+x^2)(4+x^2)^2} \, dx$

Optimal. Leaf size=32

$$\frac{1}{x^2+4} + \log\left(x^2+4\right) - 2\log(x) + \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + 2\tan^{-1}(x)$$

```
[Out] (4 + x^2)^(-1) + ArcTan[x/2]/2 + 2*ArcTan[x] - 2*Log[x] + Log[4 +
 x^2]
```

Rubi [A] time = 0.464191, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116

$$\frac{1}{x^2+4} + \log\left(x^2+4\right) - 2\log(x) + \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + 2\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-32 + 36*x - 42*x^2 + 21*x^3 - 10*x^4 + 3*x^5)/(x*(1 + x^2)*(4 + x^2)^2),x]
```

```
[Out] (4 + x^2)^(-1) + ArcTan[x/2]/2 + 2*ArcTan[x] - 2*Log[x] + Log[4 +
 x^2]
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((3*x**5-10*x**4+21*x**3-42*x**2+36*x-32)/x/(x**2+1)/(x**2+4)**2
```

```
[Out] Timed out
```

Mathematica [A] time = 0.0290071, size = 32, normalized size = 1.

$$\frac{1}{x^2+4} + \log\left(x^2+4\right) - 2\log(x) + \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + 2\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-32 + 36*x - 42*x^2 + 21*x^3 - 10*x^4 + 3*x^5)/(x*(1 + x^2)*(4 + x^2)^2
```

```
[Out] (4 + x^2)^(-1) + ArcTan[x/2]/2 + 2*ArcTan[x] - 2*Log[x] + Log[4 +
 x^2]
```

Maple [A] time = 0.013, size = 29, normalized size = 0.9

$$\left(x^2+4\right)^{-1} + \frac{1}{2}\arctan\left(\frac{x}{2}\right) + 2\arctan(x) - 2\ln(x) + \ln\left(x^2+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^5-10*x^4+21*x^3-42*x^2+36*x-32)/x/(x^2+1)/(x^2+4)^2,x)
```

```
[Out] 1/(x^2+4)+1/2*arctan(1/2*x)+2*arctan(x)-2*ln(x)+ln(x^2+4)
```

Maxima [A] time = 0.898172, size = 38, normalized size = 1.19

$$\frac{1}{x^2+4}+\frac{1}{2}\arctan\left(\frac{1}{2}x\right)+2\arctan(x)+\log\left(x^2+4\right)-2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^5 - 10*x^4 + 21*x^3 - 42*x^2 + 36*x - 32)/((x^2 + 4)^2*(x^2 + 1)*x),
```

```
[Out] 1/(x^2 + 4) + 1/2*arctan(1/2*x) + 2*arctan(x) + log(x^2 + 4) - 2*
log(x)
```

Fricas [A] time = 0.327498, size = 70, normalized size = 2.19

$$\frac{\left(x^2+4\right)\arctan\left(\frac{1}{2}x\right)+4\left(x^2+4\right)\arctan(x)+2\left(x^2+4\right)\log\left(x^2+4\right)-4\left(x^2+4\right)\log(x)+2}{2\left(x^2+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^5 - 10*x^4 + 21*x^3 - 42*x^2 + 36*x - 32)/((x^2 + 4)^2*(x^2 + 1)*x),
```

```
[Out] 1/2*((x^2 + 4)*arctan(1/2*x) + 4*(x^2 + 4)*arctan(x) + 2*(x^2 + 4
)*log(x^2 + 4) - 4*(x^2 + 4)*log(x) + 2)/(x^2 + 4)
```

Sympy [A] time = 0.783684, size = 29, normalized size = 0.91

$$-2\log(x)+\log\left(x^2+4\right)+\frac{\operatorname{atan}\left(\frac{x}{2}\right)}{2}+2\operatorname{atan}(x)+\frac{1}{x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**5-10*x**4+21*x**3-42*x**2+36*x-32)/x/(x**2+1)/(x**2+4)**2,x)
```

```
[Out] -2*log(x) + log(x**2 + 4) + atan(x/2)/2 + 2*atan(x) + 1/(x**2 + 4
)
```

GIAC/XCAS [A] time = 0.264355, size = 39, normalized size = 1.22

$$\frac{1}{x^2+4}+\frac{1}{2}\arctan\left(\frac{1}{2}x\right)+2\arctan(x)+\ln\left(x^2+4\right)-2\ln(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^5 - 10*x^4 + 21*x^3 - 42*x^2 + 36*x - 32)/((x^2 + 4)^2*(x^2 + 1)*x),
```

```
[Out] 1/(x^2 + 4) + 1/2*arctan(1/2*x) + 2*arctan(x) + ln(x^2 + 4) - 2*l
n(abs(x))
```

3.367 $\int \frac{-1+x^4+7x^5+x^9}{-7+6x^4+x^8}\,dx$

Optimal. Leaf size=148

$$\frac{x^2}{2}-\frac{\log\left(x^2-\sqrt{2}\sqrt[4]{7}x+\sqrt{7}\right)}{4\sqrt{2}7^{3/4}}+\frac{\log\left(x^2+\sqrt{2}\sqrt[4]{7}x+\sqrt{7}\right)}{4\sqrt{2}7^{3/4}}-\frac{1}{2}\tanh^{-1}\left(x^2\right)-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}x}{\sqrt[4]{7}}\right)}{2\sqrt{2}7^{3/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{7}}+1\right)}{2\sqrt{2}7^{3/4}}$$

```
[Out] x^2/2 - ArcTan[1 - (Sqrt[2]*x)/7^(1/4)]/(2*Sqrt[2]*7^(3/4)) + Arc
Tan[1 + (Sqrt[2]*x)/7^(1/4)]/(2*Sqrt[2]*7^(3/4)) - ArcTanh[x^2]/2
 - Log[Sqrt[7] - Sqrt[2]*7^(1/4)*x + x^2]/(4*Sqrt[2]*7^(3/4)) + L
og[Sqrt[7] + Sqrt[2]*7^(1/4)*x + x^2]/(4*Sqrt[2]*7^(3/4))
```

Rubi [A] time = 0.271577, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 12, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462

$$\frac{x^2}{2}-\frac{\log\left(x^2-\sqrt{2}\sqrt[4]{7}x+\sqrt{7}\right)}{4\sqrt{2}7^{3/4}}+\frac{\log\left(x^2+\sqrt{2}\sqrt[4]{7}x+\sqrt{7}\right)}{4\sqrt{2}7^{3/4}}-\frac{1}{2}\tanh^{-1}\left(x^2\right)-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}x}{\sqrt[4]{7}}\right)}{2\sqrt{2}7^{3/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{7}}+1\right)}{2\sqrt{2}7^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + x^4 + 7*x^5 + x^9)/(-7 + 6*x^4 + x^8),x]
```

```
[Out] x^2/2 - ArcTan[1 - (Sqrt[2]*x)/7^(1/4)]/(2*Sqrt[2]*7^(3/4)) + Arc
Tan[1 + (Sqrt[2]*x)/7^(1/4)]/(2*Sqrt[2]*7^(3/4)) - ArcTanh[x^2]/2
 - Log[Sqrt[7] - Sqrt[2]*7^(1/4)*x + x^2]/(4*Sqrt[2]*7^(3/4)) + L
og[Sqrt[7] + Sqrt[2]*7^(1/4)*x + x^2]/(4*Sqrt[2]*7^(3/4))
```

Rubi in Sympy [A] time = 40.9431, size = 136, normalized size = 0.92

$$\frac{x^2}{2}-\frac{\sqrt{2}\sqrt[4]{7}\log\left(x^2-\sqrt{2}\sqrt[4]{7}x+\sqrt{7}\right)}{56}+\frac{\sqrt{2}\sqrt[4]{7}\log\left(x^2+\sqrt{2}\sqrt[4]{7}x+\sqrt{7}\right)}{56}+\frac{\sqrt{2}\sqrt[4]{7}\,\mathrm{atan}\left(\frac{\sqrt{2}\cdot 7^{\frac{3}{4}}x}{7}-1\right)}{28}+\frac{\sqrt{2}\sqrt[4]{7}\,\mathrm{atan}\left(\frac{\sqrt{2}\cdot 7^{\frac{3}{4}}x}{7}+1\right)}{28}-\frac{\mathrm{atanh}\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**9+7*x**5+x**4-1)/(x**8+6*x**4-7),x)
```

```
[Out] x**2/2 - sqrt(2)*7**(1/4)*log(x**2 - sqrt(2)*7**(1/4)*x + sqrt(7)
)/56 + sqrt(2)*7**(1/4)*log(x**2 + sqrt(2)*7**(1/4)*x + sqrt(7))/
56 + sqrt(2)*7**(1/4)*atan(sqrt(2)*7**(3/4)*x/7 - 1)/28 + sqrt(2)
*7**(1/4)*atan(sqrt(2)*7**(3/4)*x/7 + 1)/28 - atanh(x**2)/2
```

Mathematica [A] time = 0.10222, size = 159, normalized size = 1.07

$$\frac{1}{56}\left(28x^2-14\log\left(x^2+1\right)-\sqrt{2}\sqrt[4]{7}\log\left(\sqrt{7}x^2-\sqrt{2}7^{3/4}x+7\right)+\sqrt{2}\sqrt[4]{7}\log\left(\sqrt{7}x^2+\sqrt{2}7^{3/4}x+7\right)+14\log(1-x)+14\log(x+1)-2\sqrt{2}\sqrt[4]{7}\tan^{-1}\left(1-\frac{\sqrt{2}x}{\sqrt[4]{7}}\right)+2\sqrt{2}\sqrt[4]{7}\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{7}}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + x^4 + 7*x^5 + x^9)/(-7 + 6*x^4 + x^8),x]
```

```
[Out] (28*x^2 - 2*Sqrt[2]*7^(1/4)*ArcTan[1 - (Sqrt[2]*x)/7^(1/4)] + 2*S
qrt[2]*7^(1/4)*ArcTan[1 + (Sqrt[2]*x)/7^(1/4)] + 14*Log[1 - x] +
14*Log[1 + x] - 14*Log[1 + x^2] - Sqrt[2]*7^(1/4)*Log[7 - Sqrt[2]
*7^(3/4)*x + Sqrt[7]*x^2] + Sqrt[2]*7^(1/4)*Log[7 + Sqrt[2]*7^(3/
4)*x + Sqrt[7]*x^2])/56
```

Maple [A] time = 0.017, size = 110, normalized size = 0.7

$$\frac{x^2}{2}+\frac{\ln(-1+x)}{4}+\frac{\sqrt[4]{7}\sqrt{2}}{28}\arctan\left(-1+\frac{x\sqrt{2}7^{\frac{3}{4}}}{7}\right)+\frac{\sqrt[4]{7}\sqrt{2}}{56}\ln\left(\frac{x^2+\sqrt[4]{7}x\sqrt{2}+\sqrt{7}}{x^2-\sqrt[4]{7}x\sqrt{2}+\sqrt{7}}\right)+\frac{\sqrt[4]{7}\sqrt{2}}{28}\arctan\left(1+\frac{x\sqrt{2}7^{\frac{3}{4}}}{7}\right)+\frac{\ln(1+x)}{4}-\frac{\ln\left(x^2+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^9+7*x^5+x^4-1)/(x^8+6*x^4-7),x)
```

```
[Out] 1/2*x^2+1/4*ln(-1+x)+1/28*arctan(-1+1/7*x*2^(1/2)*7^(3/4))*7^(1/4
)*2^(1/2)+1/56*7^(1/4)*2^(1/2)*ln((x^2+7^(1/4)*x*2^(1/2)+7^(1/2))
/(x^2-7^(1/4)*x*2^(1/2)+7^(1/2)))+1/28*arctan(1+1/7*x*2^(1/2)*7^(
3/4))*7^(1/4)*2^(1/2)+1/4*ln(1+x)-1/4*ln(x^2+1)
```

Maxima [A] time = 0.87486, size = 178, normalized size = 1.2

$$\frac{1}{2}x^2+\frac{1}{28}\cdot 7^{\frac{1}{4}}\sqrt{2}\arctan\left(\frac{1}{14}\cdot 7^{\frac{3}{4}}\sqrt{2}\left(2x+7^{\frac{1}{4}}\sqrt{2}\right)\right)+\frac{1}{28}\cdot 7^{\frac{1}{4}}\sqrt{2}\arctan\left(\frac{1}{14}\cdot 7^{\frac{3}{4}}\sqrt{2}\left(2x-7^{\frac{1}{4}}\sqrt{2}\right)\right)+\frac{1}{56}\cdot 7^{\frac{1}{4}}\sqrt{2}\log\left(x^2+7^{\frac{1}{4}}\sqrt{2}x+\sqrt{7}\right)-\frac{1}{56}\cdot 7^{\frac{1}{4}}\sqrt{2}\log\left(x^2-7^{\frac{1}{4}}\sqrt{2}x+\sqrt{7}\right)-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{4}\log(x+1)+\frac{1}{4}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^9 + 7*x^5 + x^4 - 1)/(x^8 + 6*x^4 - 7),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + 1/28*7^(1/4)*sqrt(2)*arctan(1/14*7^(3/4)*sqrt(2)*(2*x +
 7^(1/4)*sqrt(2))) + 1/28*7^(1/4)*sqrt(2)*arctan(1/14*7^(3/4)*sqr
t(2)*(2*x - 7^(1/4)*sqrt(2))) + 1/56*7^(1/4)*sqrt(2)*log(x^2 + 7^
(1/4)*sqrt(2)*x + sqrt(7)) - 1/56*7^(1/4)*sqrt(2)*log(x^2 - 7^(1/
4)*sqrt(2)*x + sqrt(7)) - 1/4*log(x^2 + 1) + 1/4*log(x + 1) + 1/4
*log(x - 1)
```

Fricas [A] time = 0.295521, size = 257, normalized size = 1.74

$$\frac{1}{2744} \cdot 343^{\frac{3}{4}}\sqrt{2}\left(2\cdot 343^{\frac{1}{4}}\sqrt{2}x^2 - 343^{\frac{1}{4}}\sqrt{2}\log\left(x^2+1\right) + 343^{\frac{1}{4}}\sqrt{2}\log\left(x^2-1\right) - 4\arctan\left(\frac{7}{343^{\frac{1}{4}}\sqrt{2}\sqrt{\frac{1}{7}}\sqrt{\sqrt{7}\left(\sqrt{7}x^2+343^{\frac{1}{4}}\sqrt{2}x+7\right)}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^9 + 7*x^5 + x^4 - 1)/(x^8 + 6*x^4 - 7),x, algorithm="fricas")
```

```
[Out] 1/2744*343^(3/4)*sqrt(2)*(2*343^(1/4)*sqrt(2)*x^2 - 343^(1/4)*sqr
t(2)*log(x^2 + 1) + 343^(1/4)*sqrt(2)*log(x^2 - 1) - 4*arctan(7/(
343^(1/4)*sqrt(2)*sqrt(1/7)*sqrt(sqrt(7)*(sqrt(7)*x^2 + 343^(1/4)
*sqrt(2)*x + 7)) + 343^(1/4)*sqrt(2)*x + 7)) - 4*arctan(7/(343^(1
/4)*sqrt(2)*sqrt(1/7)*sqrt(sqrt(7)*(sqrt(7)*x^2 - 343^(1/4)*sqrt(
2)*x + 7)) + 343^(1/4)*sqrt(2)*x - 7)) + log(7*sqrt(7)*x^2 + 7*34
3^(1/4)*sqrt(2)*x + 49) - log(7*sqrt(7)*x^2 - 7*343^(1/4)*sqrt(2)
*x + 49))
```

Sympy [A] time = 1.7953, size = 146, normalized size = 0.99

$$\frac{x^2}{2} + \frac{\log\left(x^2-1\right)}{4} - \frac{\log\left(x^2+1\right)}{4} - \frac{\sqrt{2}\sqrt[4]{7}\log\left(x^2-\sqrt{2}\sqrt[4]{7}x+\sqrt{7}\right)}{56} + \frac{\sqrt{2}\sqrt[4]{7}\log\left(x^2+\sqrt{2}\sqrt[4]{7}x+\sqrt{7}\right)}{56} + \frac{\sqrt{2}\sqrt[4]{7}\operatorname{atan}\left(\frac{\sqrt{2}\cdot 7^{\frac{3}{4}}x}{7}-1\right)}{28} + \frac{\sqrt{2}\sqrt[4]{7}\operatorname{atan}\left(\frac{\sqrt{2}\cdot 7^{\frac{3}{4}}x}{7}+1\right)}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**9+7*x**5+x**4-1)/(x**8+6*x**4-7),x)
```

```
[Out] x**2/2 + log(x**2 - 1)/4 - log(x**2 + 1)/4 - sqrt(2)*7**(1/4)*log
(x**2 - sqrt(2)*7**(1/4)*x + sqrt(7))/56 + sqrt(2)*7**(1/4)*log(x
**2 + sqrt(2)*7**(1/4)*x + sqrt(7))/56 + sqrt(2)*7**(1/4)*atan(sq
rt(2)*7**(3/4)*x/7 - 1)/28 + sqrt(2)*7**(1/4)*atan(sqrt(2)*7**(3/
4)*x/7 + 1)/28
```

GIAC/XCAS [A] time = 0.273128, size = 165, normalized size = 1.11

$$\frac{1}{2}x^2 + \frac{1}{28}\cdot 28^{\frac{1}{4}}\arctan\left(\frac{1}{14}\cdot 7^{\frac{3}{4}}\sqrt{2}\left(2x+7^{\frac{1}{4}}\sqrt{2}\right)\right) + \frac{1}{28}\cdot 28^{\frac{1}{4}}\arctan\left(\frac{1}{14}\cdot 7^{\frac{3}{4}}\sqrt{2}\left(2x-7^{\frac{1}{4}}\sqrt{2}\right)\right) + \frac{1}{56}\cdot 28^{\frac{1}{4}}\ln\left(x^2+7^{\frac{1}{4}}\sqrt{2}x+\sqrt{7}\right) - \frac{1}{56}\cdot 28^{\frac{1}{4}}\ln\left(x^2-7^{\frac{1}{4}}\sqrt{2}x+\sqrt{7}\right) - \frac{1}{4}\ln\left(x^2+1\right) + \frac{1}{4}\ln\left(|x+1|\right) + \frac{1}{4}\ln\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^9 + 7*x^5 + x^4 - 1)/(x^8 + 6*x^4 - 7),x, algorithm="giac")
```

```
[Out] 1/2*x^2 + 1/28*28^(1/4)*arctan(1/14*7^(3/4)*sqrt(2)*(2*x + 7^(1/4
)*sqrt(2))) + 1/28*28^(1/4)*arctan(1/14*7^(3/4)*sqrt(2)*(2*x - 7^
(1/4)*sqrt(2))) + 1/56*28^(1/4)*ln(x^2 + 7^(1/4)*sqrt(2)*x + sqrt
(7)) - 1/56*28^(1/4)*ln(x^2 - 7^(1/4)*sqrt(2)*x + sqrt(7)) - 1/4*
ln(x^2 + 1) + 1/4*ln(abs(x + 1)) + 1/4*ln(abs(x - 1))
```

3.368 $\int \frac{1+x^3+x^6}{x+x^5} \, dx$

Optimal. Leaf size=112

$$-\frac{1}{4}\log\left(x^4+1\right)+\frac{x^2}{2}+\frac{\log\left(x^2-\sqrt{2}x+1\right)}{4\sqrt{2}}-\frac{\log\left(x^2+\sqrt{2}x+1\right)}{4\sqrt{2}}-\frac{1}{2}\tan^{-1}\left(x^2\right)+\log(x)-\frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}x+1\right)}{2\sqrt{2}}$$

```
[Out] x^2/2 - ArcTan[x^2]/2 - ArcTan[1 - Sqrt[2]*x]/(2*Sqrt[2]) + ArcTa
n[1 + Sqrt[2]*x]/(2*Sqrt[2]) + Log[x] + Log[1 - Sqrt[2]*x + x^2]/
(4*Sqrt[2]) - Log[1 + Sqrt[2]*x + x^2]/(4*Sqrt[2]) - Log[1 + x^4]
/4
```

Rubi [A] time = 0.232911, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 12, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75

$$-\frac{1}{4}\log\left(x^4+1\right)+\frac{x^2}{2}+\frac{\log\left(x^2-\sqrt{2}x+1\right)}{4\sqrt{2}}-\frac{\log\left(x^2+\sqrt{2}x+1\right)}{4\sqrt{2}}-\frac{1}{2}\tan^{-1}\left(x^2\right)+\log(x)-\frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}x+1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^3 + x^6)/(x + x^5),x]
```

```
[Out] x^2/2 - ArcTan[x^2]/2 - ArcTan[1 - Sqrt[2]*x]/(2*Sqrt[2]) + ArcTa
n[1 + Sqrt[2]*x]/(2*Sqrt[2]) + Log[x] + Log[1 - Sqrt[2]*x + x^2]/
(4*Sqrt[2]) - Log[1 + Sqrt[2]*x + x^2]/(4*Sqrt[2]) - Log[1 + x^4]
/4
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\log(x)-\frac{\log\left(x^4+1\right)}{4}+\frac{\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{8}-\frac{\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{8}-\frac{\operatorname{atan}\left(x^2\right)}{2}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}x-1\right)}{4}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}x+1\right)}{4}+\int x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**6+x**3+1)/(x**5+x),x)
```

```
[Out] log(x) - log(x**4 + 1)/4 + sqrt(2)*log(x**2 - sqrt(2)*x + 1)/8 -
sqrt(2)*log(x**2 + sqrt(2)*x + 1)/8 - atan(x**2)/2 + sqrt(2)*atan
(sqrt(2)*x - 1)/4 + sqrt(2)*atan(sqrt(2)*x + 1)/4 + Integral(x, x
)
```

Mathematica [A] time = 0.0780835, size = 101, normalized size = 0.9

$$\frac{1}{8}\left(-2\log\left(x^4+1\right)+4x^2+\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)-\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)+8\log(x)-2\left(\sqrt{2}-2\right)\tan^{-1}\left(1-\sqrt{2}x\right)+2\left(2+\sqrt{2}\right)\tan^{-1}\left(\sqrt{2}x+1\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^3 + x^6)/(x + x^5),x]
```

```
[Out] (4*x^2 - 2*(-2 + Sqrt[2])*ArcTan[1 - Sqrt[2]*x] + 2*(2 + Sqrt[2])
*ArcTan[1 + Sqrt[2]*x] + 8*Log[x] + Sqrt[2]*Log[1 - Sqrt[2]*x + x
^2] - Sqrt[2]*Log[1 + Sqrt[2]*x + x^2] - 2*Log[1 + x^4])/8
```

Maple [A] time = 0.01, size = 79, normalized size = 0.7

$$\frac{x^2}{2} - \frac{\arctan(x^2)}{2} + \frac{\arctan\left(\sqrt{2}x - 1\right)\sqrt{2}}{4} + \frac{\sqrt{2}}{8}\ln\left(\frac{1 + x^2 - \sqrt{2}x}{1 + x^2 + \sqrt{2}x}\right) + \frac{\arctan\left(1 + \sqrt{2}x\right)\sqrt{2}}{4} - \frac{\ln(x^4 + 1)}{4} + \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^6+x^3+1)/(x^5+x),x)
```

```
[Out] 1/2*x^2-1/2*arctan(x^2)+1/4*arctan(2^(1/2)*x-1)*2^(1/2)+1/8*2^(1/
2)*ln((1+x^2-2^(1/2)*x)/(1+x^2+2^(1/2)*x))+1/4*arctan(1+2^(1/2)*x
)*2^(1/2)-1/4*ln(x^4+1)+ln(x)
```

Maxima [A] time = 0.877967, size = 134, normalized size = 1.2

$$\frac{1}{4}\sqrt{2}\left(\sqrt{2} + 1\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2x + \sqrt{2}\right)\right) - \frac{1}{4}\sqrt{2}\left(\sqrt{2} - 1\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2x - \sqrt{2}\right)\right) - \frac{1}{8}\sqrt{2}\left(\sqrt{2} + 1\right)\log\left(x^2 + \sqrt{2}x + 1\right) - \frac{1}{8}\sqrt{2}\left(\sqrt{2} - 1\right)\log\left(x^2 - \sqrt{2}x + 1\right) + \frac{1}{2}x^2 + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^6 + x^3 + 1)/(x^5 + x),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*(sqrt(2) + 1)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 1
/4*sqrt(2)*(sqrt(2) - 1)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 1/
8*sqrt(2)*(sqrt(2) + 1)*log(x^2 + sqrt(2)*x + 1) - 1/8*sqrt(2)*(s
qrt(2) - 1)*log(x^2 - sqrt(2)*x + 1) + 1/2*x^2 + log(x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^6 + x^3 + 1)/(x^5 + x),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 3.12875, size = 61, normalized size = 0.54

$$\frac{x^2}{2} + \log(x) + \text{RootSum}\left(256t^4 + 256t^3 + 128t^2 + 16t + 1, \left(t \mapsto t\log\left(\frac{1792t^4}{73} + \frac{704t^3}{219} - \frac{3152t^2}{219} - \frac{2584t}{219} + x - \frac{344}{219}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**6+x**3+1)/(x**5+x),x)
```

```
[Out] x**2/2 + log(x) + RootSum(256*_t**4 + 256*_t**3 + 128*_t**2 + 16*
_t + 1, Lambda(_t, _t*log(1792*_t**4/73 + 704*_t**3/219 - 3152*_t
**2/219 - 2584*_t/219 + x - 344/219)))
```

GIAC/XCAS [A] time = 0.261579, size = 124, normalized size = 1.11

$$\frac{1}{2}x^2 + \frac{1}{4}\left(\sqrt{2}+2\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right) + \frac{1}{4}\left(\sqrt{2}-2\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right) - \frac{1}{8}\sqrt{2}\ln\left(x^2+\sqrt{2}x+1\right) + \frac{1}{8}\sqrt{2}\ln\left(x^2-\sqrt{2}x+1\right) - \frac{1}{4}\ln\left(x^4+1\right) + \ln\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^6 + x^3 + 1)/(x^5 + x),x, algorithm="giac")
```

```
[Out] 1/2*x^2 + 1/4*(sqrt(2) + 2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) +
 1/4*(sqrt(2) - 2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 1/8*sqrt
(2)*ln(x^2 + sqrt(2)*x + 1) + 1/8*sqrt(2)*ln(x^2 - sqrt(2)*x + 1)
 - 1/4*ln(x^4 + 1) + ln(abs(x))
```

3.369 $\int \frac{1+x^2}{-x+x^2} dx$

Optimal. Leaf size=14

$$x + 2\log(1 - x) - \log(x)$$

```
[Out] x + 2*Log[1 - x] - Log[x]
```

Rubi [A] time = 0.0380729, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$x + 2\log(1 - x) - \log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^2)/(-x + x^2),x]

[Out] x + 2*Log[1 - x] - Log[x]
```

Rubi in Sympy [A] time = 4.66953, size = 10, normalized size = 0.71

$$x - \log(x) + 2\log(-x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+1)/(x**2-x),x)

[Out] x - log(x) + 2*log(-x + 1)
```

Mathematica [A] time = 0.00509605, size = 14, normalized size = 1.

$$x + 2\log(1 - x) - \log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^2)/(-x + x^2),x]

[Out] x + 2*Log[1 - x] - Log[x]
```

Maple [A] time = 0.007, size = 13, normalized size = 0.9

$$x + 2\ln(-1 + x) - \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+1)/(x^2-x),x)

[Out] x+2*ln(-1+x)-ln(x)
```

Maxima [A] time = 0.795736, size = 16, normalized size = 1.14

$$x + 2 \log(x - 1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/(x^2 - x),x, algorithm="maxima")
```

```
[Out] x + 2*log(x - 1) - log(x)
```

Fricas [A] time = 0.269769, size = 16, normalized size = 1.14

$$x + 2 \log(x - 1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/(x^2 - x),x, algorithm="fricas")
```

```
[Out] x + 2*log(x - 1) - log(x)
```

Sympy [A] time = 0.18792, size = 10, normalized size = 0.71

$$x - \log(x) + 2\log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+1)/(x**2-x),x)
```

```
[Out] x - log(x) + 2*log(x - 1)
```

GIAC/XCAS [A] time = 0.258812, size = 19, normalized size = 1.36

$$x + 2\ln(|x - 1|) - \ln(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/(x^2 - x),x, algorithm="giac")
```

```
[Out] x + 2*ln(abs(x - 1)) - ln(abs(x))
```

3.370 $\int \frac{1+x^3}{-x+x^3} \, dx$

Optimal. Leaf size=12

$$x + \log(1 - x) - \log(x)$$

```
[Out] x + Log[1 - x] - Log[x]
```

Rubi [A] time = 0.0463124, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$x + \log(1 - x) - \log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^3)/(-x + x^3),x]

[Out] x + Log[1 - x] - Log[x]
```

Rubi in Sympy [A] time = 7.05429, size = 8, normalized size = 0.67

$$x - \log(x) + \log(-x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**3+1)/(x**3-x),x)

[Out] x - log(x) + log(-x + 1)
```

Mathematica [A] time = 0.00727673, size = 12, normalized size = 1.

$$x + \log(1 - x) - \log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^3)/(-x + x^3),x]

[Out] x + Log[1 - x] - Log[x]
```

Maple [A] time = 0.008, size = 11, normalized size = 0.9

$$x + \ln(-1 + x) - \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^3+1)/(x^3-x),x)

[Out] x+ln(-1+x)-ln(x)
```

Maxima [A] time = 0.805075, size = 14, normalized size = 1.17

$$x + \log(x-1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + 1)/(x^3 - x),x, algorithm="maxima")
```

```
[Out] x + log(x - 1) - log(x)
```

Fricas [A] time = 0.258767, size = 14, normalized size = 1.17

$$x + \log(x-1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + 1)/(x^3 - x),x, algorithm="fricas")
```

```
[Out] x + log(x - 1) - log(x)
```

Sympy [A] time = 0.186061, size = 8, normalized size = 0.67

$$x - \log(x) + \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3+1)/(x**3-x),x)
```

```
[Out] x - log(x) + log(x - 1)
```

GIAC/XCAS [A] time = 0.259719, size = 16, normalized size = 1.33

$$x + \ln(|x-1|) - \ln(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + 1)/(x^3 - x),x, algorithm="giac")
```

```
[Out] x + ln(abs(x - 1)) - ln(abs(x))
```

3.371 $\int \frac{1+x^3}{-x^2+x^3} \, dx$

Optimal. Leaf size=17

$$x + \frac{1}{x} + 2\log(1-x) - \log(x)$$

```
[Out] x^(-1) + x + 2*Log[1 - x] - Log[x]
```

Rubi [A] time = 0.0358791, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118

$$x + \frac{1}{x} + 2\log(1-x) - \log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^3)/(-x^2 + x^3),x]
```

```
[Out] x^(-1) + x + 2*Log[1 - x] - Log[x]
```

Rubi in Sympy [A] time = 7.94137, size = 14, normalized size = 0.82

$$x - \log(x) + 2\log(-x+1) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**3+1)/(x**3-x**2),x)
```

```
[Out] x - log(x) + 2*log(-x + 1) + 1/x
```

Mathematica [A] time = 0.0055549, size = 17, normalized size = 1.

$$x + \frac{1}{x} + 2\log(1-x) - \log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^3)/(-x^2 + x^3),x]
```

```
[Out] x^(-1) + x + 2*Log[1 - x] - Log[x]
```

Maple [A] time = 0.01, size = 16, normalized size = 0.9

$$x + 2\ln(-1+x) + x^{-1} - \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^3+1)/(x^3-x^2),x)
```

```
[Out] x+2*ln(-1+x)+1/x-ln(x)
```

Maxima [A] time = 0.825409, size = 20, normalized size = 1.18

$$x + \frac{1}{x} + 2 \log(x-1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + 1)/(x^3 - x^2),x, algorithm="maxima")
```

```
[Out] x + 1/x + 2*log(x - 1) - log(x)
```

Fricas [A] time = 0.269518, size = 28, normalized size = 1.65

$$\frac{x^2 + 2x\log(x-1) - x\log(x) + 1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + 1)/(x^3 - x^2),x, algorithm="fricas")
```

```
[Out] (x^2 + 2*x*log(x - 1) - x*log(x) + 1)/x
```

Sympy [A] time = 0.213749, size = 14, normalized size = 0.82

$$x - \log(x) + 2\log(x-1) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3+1)/(x**3-x**2),x)
```

```
[Out] x - log(x) + 2*log(x - 1) + 1/x
```

GIAC/XCAS [A] time = 0.259033, size = 23, normalized size = 1.35

$$x + \frac{1}{x} + 2\ln(|x-1|) - \ln(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + 1)/(x^3 - x^2),x, algorithm="giac")
```

```
[Out] x + 1/x + 2*ln(abs(x - 1)) - ln(abs(x))
```

3.372 $\int \frac{-1+x^5}{-x+x^3} \, dx$

Optimal. Leaf size=17

$$\frac{x^3}{3} + x + \log(x) - \log(x+1)$$

```
[Out] x + x^3/3 + Log[x] - Log[1 + x]
```

Rubi [A] time = 0.0447349, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{x^3}{3} + x + \log(x) - \log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + x^5)/(-x + x^3),x]

[Out] x + x^3/3 + Log[x] - Log[1 + x]
```

Rubi in Sympy [A] time = 12.5824, size = 14, normalized size = 0.82

$$\frac{x^3}{3} + x + \log(x) - \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**5-1)/(x**3-x),x)

[Out] x**3/3 + x + log(x) - log(x + 1)
```

Mathematica [A] time = 0.00688539, size = 17, normalized size = 1.

$$\frac{x^3}{3} + x + \log(x) - \log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + x^5)/(-x + x^3),x]

[Out] x + x^3/3 + Log[x] - Log[1 + x]
```

Maple [A] time = 0.008, size = 16, normalized size = 0.9

$$x + \frac{x^3}{3} + \ln(x) - \ln(1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^5-1)/(x^3-x),x)
```

```
[Out] x+1/3*x^3+ln(x)-ln(1+x)
```

Maxima [A] time = 0.791284, size = 20, normalized size = 1.18

$$\frac{1}{3}x^3 + x - \log(x+1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^5 - 1)/(x^3 - x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3 + x - log(x + 1) + log(x)
```

Fricas [A] time = 0.262422, size = 20, normalized size = 1.18

$$\frac{1}{3}x^3 + x - \log(x+1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^5 - 1)/(x^3 - x),x, algorithm="fricas")
```

```
[Out] 1/3*x^3 + x - log(x + 1) + log(x)
```

Sympy [A] time = 0.184863, size = 14, normalized size = 0.82

$$\frac{x^3}{3} + x + \log(x) - \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**5-1)/(x**3-x),x)
```

```
[Out] x**3/3 + x + log(x) - log(x + 1)
```

GIAC/XCAS [A] time = 0.258978, size = 23, normalized size = 1.35

$$\frac{1}{3}x^3 + x - \ln(|x+1|) + \ln(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^5 - 1)/(x^3 - x),x, algorithm="giac")
```

```
[Out] 1/3*x^3 + x - ln(abs(x + 1)) + ln(abs(x))
```

3.373 $\int \frac{1+x^4}{x^3+x^5} \, dx$

Optimal. Leaf size=18

$$-\frac{1}{2x^2} + \log\left(x^2+1\right) - \log(x)$$

```
[Out] -1/(2*x^2) - Log[x] + Log[1 + x^2]
```

Rubi [A] time = 0.0718099, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\frac{1}{2x^2} + \log\left(x^2+1\right) - \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)/(x^3 + x^5),x]
```

```
[Out] -1/(2*x^2) - Log[x] + Log[1 + x^2]
```

Rubi in Sympy [A] time = 9.38737, size = 19, normalized size = 1.06

$$-\frac{\log\left(x^2\right)}{2} + \log\left(x^2+1\right) - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((x**4+1)/(x**5+x**3),x)
```

```
[Out] -log(x**2)/2 + log(x**2 + 1) - 1/(2*x**2)
```

Mathematica [A] time = 0.00701179, size = 18, normalized size = 1.

$$-\frac{1}{2x^2} + \log\left(x^2+1\right) - \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)/(x^3 + x^5),x]
```

```
[Out] -1/(2*x^2) - Log[x] + Log[1 + x^2]
```

Maple [A] time = 0.009, size = 17, normalized size = 0.9

$$-\frac{1}{2\,x^2} - \ln(x) + \ln\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)/(x^5+x^3),x)
```

```
[Out] -1/2/x^2-ln(x)+ln(x^2+1)
```

Maxima [A] time = 0.887873, size = 22, normalized size = 1.22

$$-\frac{1}{2\,x^2}+\log\left(x^2+1\right)-\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 1)/(x^5 + x^3),x, algorithm="maxima")
```

```
[Out] -1/2/x^2 + log(x^2 + 1) - log(x)
```

Fricas [A] time = 0.257971, size = 34, normalized size = 1.89

$$\frac{2\,x^2\log\left(x^2+1\right)-2\,x^2\log\left(x\right)-1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 1)/(x^5 + x^3),x, algorithm="fricas")
```

```
[Out] 1/2*(2*x^2*log(x^2 + 1) - 2*x^2*log(x) - 1)/x^2
```

Sympy [A] time = 0.229886, size = 15, normalized size = 0.83

$$-\log\left(x\right)+\log\left(x^2+1\right)-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**4+1)/(x**5+x**3),x)
```

```
[Out] -log(x) + log(x**2 + 1) - 1/(2*x**2)
```

GIAC/XCAS [A] time = 0.259642, size = 31, normalized size = 1.72

$$\frac{x^2-1}{2\,x^2}+\ln\left(x^2+1\right)-\frac{1}{2}\ln\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 1)/(x^5 + x^3),x, algorithm="giac")
```

```
[Out] 1/2*(x^2 - 1)/x^2 + ln(x^2 + 1) - 1/2*ln(x^2)
```

3.374 $\int \frac{1+x^2}{x+2x^2+x^3} dx$

Optimal. Leaf size=10

$$\frac{2}{x+1} + \log(x)$$

```
[Out] 2/(1 + x) + Log[x]
```

Rubi [A] time = 0.0378543, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$\frac{2}{x+1} + \log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^2)/(x + 2*x^2 + x^3),x]

[Out] 2/(1 + x) + Log[x]
```

Rubi in Sympy [A] time = 7.47058, size = 7, normalized size = 0.7

$$\log(x) + \frac{2}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+1)/(x**3+2*x**2+x),x)

[Out] log(x) + 2/(x + 1)
```

Mathematica [A] time = 0.00793174, size = 10, normalized size = 1.

$$\frac{2}{x+1} + \log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^2)/(x + 2*x^2 + x^3),x]

[Out] 2/(1 + x) + Log[x]
```

Maple [A] time = 0.008, size = 11, normalized size = 1.1

$$2 (1+x)^{-1} + \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+1)/(x^3+2*x^2+x),x)

[Out] 2/(1+x)+ln(x)
```

Maxima [A] time = 0.801148, size = 14, normalized size = 1.4

$$\frac{2}{x+1} + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/(x^3 + 2*x^2 + x),x, algorithm="maxima")
```

```
[Out] 2/(x + 1) + log(x)
```

Fricas [A] time = 0.287341, size = 19, normalized size = 1.9

$$\frac{(x+1)\log(x)+2}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/(x^3 + 2*x^2 + x),x, algorithm="fricas")
```

```
[Out] ((x + 1)*log(x) + 2)/(x + 1)
```

Sympy [A] time = 0.164596, size = 7, normalized size = 0.7

$$\log(x) + \frac{2}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+1)/(x**3+2*x**2+x),x)
```

```
[Out] log(x) + 2/(x + 1)
```

GIAC/XCAS [A] time = 0.260025, size = 15, normalized size = 1.5

$$\frac{2}{x+1} + \ln(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/(x^3 + 2*x^2 + x),x, algorithm="giac")
```

```
[Out] 2/(x + 1) + ln(abs(x))
```

3.375 $\int \frac{1+x^5}{-10x-3x^2+x^3}\,dx$

Optimal. Leaf size=42

$$\frac{x^3}{3}+\frac{3x^2}{2}+19x+\frac{3126}{35}\log(5-x)-\frac{\log(x)}{10}-\frac{31}{14}\log(x+2)$$

```
[Out] 19*x + (3*x^2)/2 + x^3/3 + (3126*Log[5 - x])/35 - Log[x]/10 - (31
*Log[2 + x])/14
```

Rubi [A] time = 0.0612502, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{x^3}{3}+\frac{3x^2}{2}+19x+\frac{3126}{35}\log(5-x)-\frac{\log(x)}{10}-\frac{31}{14}\log(x+2)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^5)/(-10*x - 3*x^2 + x^3),x]
```

```
[Out] 19*x + (3*x^2)/2 + x^3/3 + (3126*Log[5 - x])/35 - Log[x]/10 - (31
*Log[2 + x])/14
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{x^3}{3}+19x-\frac{\log(x)}{10}+\frac{3126\log(-x+5)}{35}-\frac{31\log(x+2)}{14}+3\int x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**5+1)/(x**3-3*x**2-10*x),x)
```

```
[Out] x**3/3 + 19*x - log(x)/10 + 3126*log(-x + 5)/35 - 31*log(x + 2)/1
4 + 3*Integral(x, x)
```

Mathematica [A] time = 0.0103377, size = 42, normalized size = 1.

$$\frac{x^3}{3}+\frac{3x^2}{2}+19x+\frac{3126}{35}\log(5-x)-\frac{\log(x)}{10}-\frac{31}{14}\log(x+2)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^5)/(-10*x - 3*x^2 + x^3),x]
```

```
[Out] 19*x + (3*x^2)/2 + x^3/3 + (3126*Log[5 - x])/35 - Log[x]/10 - (31
*Log[2 + x])/14
```

Maple [A] time = 0.011, size = 31, normalized size = 0.7

$$\frac{x^3}{3}+\frac{3\,x^2}{2}+19\,x-\frac{31\,\ln(2+x)}{14}+\frac{3126\,\ln(-5+x)}{35}-\frac{\ln(x)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^5+1)/(x^3-3*x^2-10*x),x)
```

```
[Out] 1/3*x^3+3/2*x^2+19*x-31/14*ln(2+x)+3126/35*ln(-5+x)-1/10*ln(x)
```

Maxima [A] time = 0.815855, size = 41, normalized size = 0.98

$$\frac{1}{3}x^3 + \frac{3}{2}x^2 + 19x - \frac{31}{14}\log(x+2) + \frac{3126}{35}\log(x-5) - \frac{1}{10}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^5 + 1)/(x^3 - 3*x^2 - 10*x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3 + 3/2*x^2 + 19*x - 31/14*log(x + 2) + 3126/35*log(x - 5)
 - 1/10*log(x)
```

Fricas [A] time = 0.255831, size = 41, normalized size = 0.98

$$\frac{1}{3}x^3 + \frac{3}{2}x^2 + 19x - \frac{31}{14}\log(x+2) + \frac{3126}{35}\log(x-5) - \frac{1}{10}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^5 + 1)/(x^3 - 3*x^2 - 10*x),x, algorithm="fricas")
```

```
[Out] 1/3*x^3 + 3/2*x^2 + 19*x - 31/14*log(x + 2) + 3126/35*log(x - 5)
 - 1/10*log(x)
```

Sympy [A] time = 0.339296, size = 36, normalized size = 0.86

$$\frac{x^3}{3} + \frac{3x^2}{2} + 19x - \frac{\log(x)}{10} + \frac{3126\log(x-5)}{35} - \frac{31\log(x+2)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**5+1)/(x**3-3*x**2-10*x),x)
```

```
[Out] x**3/3 + 3*x**2/2 + 19*x - log(x)/10 + 3126*log(x - 5)/35 - 31*lo
g(x + 2)/14
```

GIAC/XCAS [A] time = 0.260089, size = 45, normalized size = 1.07

$$\frac{1}{3}x^3 + \frac{3}{2}x^2 + 19x - \frac{31}{14}\ln(|x+2|) + \frac{3126}{35}\ln(|x-5|) - \frac{1}{10}\ln(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^5 + 1)/(x^3 - 3*x^2 - 10*x),x, algorithm="giac")
```

```
[Out] 1/3*x^3 + 3/2*x^2 + 19*x - 31/14*ln(abs(x + 2)) + 3126/35*ln(abs(
x - 5)) - 1/10*ln(abs(x))
```

3.376 $\int \frac{15-5x+x^2+x^3}{(5+x^2)(3+2x+x^2)}\,dx$

Optimal. Leaf size=46

$$\frac{1}{2}\log\left(x^2+2x+3\right)-\sqrt{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+\frac{5\tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] -(Sqrt[5]*ArcTan[x/Sqrt[5]]) + (5*ArcTan[(1 + x)/Sqrt[2]])/Sqrt[2
] + Log[3 + 2*x + x^2]/2
```

Rubi [A] time = 0.246034, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207

$$\frac{1}{2}\log\left(x^2+2x+3\right)-\sqrt{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+\frac{5\tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(15 - 5*x + x^2 + x^3)/((5 + x^2)*(3 + 2*x + x^2)),x]
```

```
[Out] -(Sqrt[5]*ArcTan[x/Sqrt[5]]) + (5*ArcTan[(1 + x)/Sqrt[2]])/Sqrt[2
] + Log[3 + 2*x + x^2]/2
```

Rubi in Sympy [A] time = 47.8555, size = 48, normalized size = 1.04

$$\frac{\log\left(x^2+2x+3\right)}{2}+\frac{5\sqrt{2}\operatorname{atan}\left(\sqrt{2}\left(\frac{x}{2}+\frac{1}{2}\right)\right)}{2}-\sqrt{5}\operatorname{atan}\left(\frac{\sqrt{5}x}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**3+x**2-5*x+15)/(x**2+5)/(x**2+2*x+3),x)
```

```
[Out] log(x**2 + 2*x + 3)/2 + 5*sqrt(2)*atan(sqrt(2)*(x/2 + 1/2))/2 - s
qrt(5)*atan(sqrt(5)*x/5)
```

Mathematica [A] time = 0.0337915, size = 46, normalized size = 1.

$$\frac{1}{2}\log\left(x^2+2x+3\right)-\sqrt{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+\frac{5\tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(15 - 5*x + x^2 + x^3)/((5 + x^2)*(3 + 2*x + x^2)),x]
```

```
[Out] -(Sqrt[5]*ArcTan[x/Sqrt[5]]) + (5*ArcTan[(1 + x)/Sqrt[2]])/Sqrt[2
] + Log[3 + 2*x + x^2]/2
```

Maple [A] time = 0.001, size = 41, normalized size = 0.9

$$\frac{\ln\left(x^2+2x+3\right)}{2}+\frac{5\sqrt{2}}{2}\arctan\left(\frac{(2+2x)\sqrt{2}}{4}\right)-\arctan\left(\frac{x\sqrt{5}}{5}\right)\sqrt{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x)
```

```
[Out] 1/2*ln(x^2+2*x+3)+5/2*2^(1/2)*arctan(1/4*(2+2*x)*2^(1/2))-arctan(
1/5*x*5^(1/2))*5^(1/2)
```

Maxima [A] time = 0.882939, size = 51, normalized size = 1.11

$$\frac{5}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x+1)\right)-\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right)+\frac{1}{2}\log\left(x^2+2x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + x^2 - 5*x + 15)/((x^2 + 2*x + 3)*(x^2 + 5)),x, algorithm="maxima"
```

```
[Out] 5/2*sqrt(2)*arctan(1/2*sqrt(2)*(x + 1)) - sqrt(5)*arctan(1/5*sqrt
(5)*x) + 1/2*log(x^2 + 2*x + 3)
```

Fricas [A] time = 0.259253, size = 62, normalized size = 1.35

$$-\frac{1}{4}\sqrt{2}\left(2\sqrt{5}\sqrt{2}\arctan\left(\frac{1}{5}\sqrt{5}x\right)-\sqrt{2}\log\left(x^2+2x+3\right)-10\arctan\left(\frac{1}{2}\sqrt{2}(x+1)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + x^2 - 5*x + 15)/((x^2 + 2*x + 3)*(x^2 + 5)),x, algorithm="fricas"
```

```
[Out] -1/4*sqrt(2)*(2*sqrt(5)*sqrt(2)*arctan(1/5*sqrt(5)*x) - sqrt(2)*l
og(x^2 + 2*x + 3) - 10*arctan(1/2*sqrt(2)*(x + 1)))
```

Sympy [A] time = 0.607985, size = 51, normalized size = 1.11

$$\frac{\log\left(x^2+2x+3\right)}{2}-\sqrt{5}\,\mathrm{atan}\left(\frac{\sqrt{5}x}{5}\right)+\frac{5\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}+\frac{\sqrt{2}}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3+x**2-5*x+15)/(x**2+5)/(x**2+2*x+3),x)
```

```
[Out] log(x**2 + 2*x + 3)/2 - sqrt(5)*atan(sqrt(5)*x/5) + 5*sqrt(2)*ata
n(sqrt(2)*x/2 + sqrt(2)/2)/2
```

GIAC/XCAS [A] time = 0.260258, size = 51, normalized size = 1.11

$$\frac{5}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x+1)\right)-\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right)+\frac{1}{2}\ln\left(x^2+2x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + x^2 - 5*x + 15)/((x^2 + 2*x + 3)*(x^2 + 5)),x, algorithm="giac")
```

```
[Out] 5/2*sqrt(2)*arctan(1/2*sqrt(2)*(x + 1)) - sqrt(5)*arctan(1/5*sqrt
(5)*x) + 1/2*ln(x^2 + 2*x + 3)
```

3.377 $\int \frac{1}{(1+x^2)\left(3+\frac{10x}{1+x^2}\right)} \, dx$

Optimal. Leaf size=19

$$\frac{1}{8}\log(3x+1) - \frac{1}{8}\log(x+3)$$

```
[Out] -Log[3 + x]/8 + Log[1 + 3*x]/8
```

Rubi [A] time = 0.109778, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$

$$\frac{1}{8}\log(3x+1) - \frac{1}{8}\log(x+3)$$

Antiderivative was successfully verified.

```
[In]  Int[1/((1 + x^2)*(3 + (10*x)/(1 + x^2))),x]

[Out] -Log[3 + x]/8 + Log[1 + 3*x]/8
```

Rubi in Sympy [A] time = 14.7092, size = 14, normalized size = 0.74

$$-\frac{\log(x+3)}{8} + \frac{\log(3x+1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**2+1)/(3+10*x/(x**2+1)),x)

[Out] -log(x + 3)/8 + log(3*x + 1)/8
```

Mathematica [A] time = 0.00511877, size = 19, normalized size = 1.

$$\frac{1}{8}\log(3x+1) - \frac{1}{8}\log(x+3)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((1 + x^2)*(3 + (10*x)/(1 + x^2))),x]

[Out] -Log[3 + x]/8 + Log[1 + 3*x]/8
```

Maple [A] time = 0.008, size = 16, normalized size = 0.8

$$-\frac{\ln(3+x)}{8} + \frac{\ln(1+3x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^2+1)/(3+10*x/(x^2+1)),x)
```

```
[Out] -1/8*ln(3+x)+1/8*ln(1+3*x)
```

Maxima [A] time = 0.804295, size = 20, normalized size = 1.05

$$\frac{1}{8}\log(3x+1) - \frac{1}{8}\log(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 1)*(10*x/(x^2 + 1) + 3)),x, algorithm="maxima")
```

```
[Out] 1/8*log(3*x + 1) - 1/8*log(x + 3)
```

Fricas [A] time = 0.275956, size = 20, normalized size = 1.05

$$\frac{1}{8}\log(3x+1) - \frac{1}{8}\log(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 1)*(10*x/(x^2 + 1) + 3)),x, algorithm="fricas")
```

```
[Out] 1/8*log(3*x + 1) - 1/8*log(x + 3)
```

Sympy [A] time = 0.209658, size = 14, normalized size = 0.74

$$\frac{\log\left(x+\frac{1}{3}\right)}{8} - \frac{\log(x+3)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**2+1)/(3+10*x/(x**2+1)),x)
```

```
[Out] log(x + 1/3)/8 - log(x + 3)/8
```

GIAC/XCAS [A] time = 0.261683, size = 23, normalized size = 1.21

$$\frac{1}{8}\ln(|3x+1|) - \frac{1}{8}\ln(|x+3|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 1)*(10*x/(x^2 + 1) + 3)),x, algorithm="giac")
```

```
[Out] 1/8*ln(abs(3*x + 1)) - 1/8*ln(abs(x + 3))
```

3.378 $\int \frac{x^3}{13+\frac{2}{x}+15x} \, dx$

Optimal. Leaf size=40

$$\frac{x^3}{45} - \frac{13x^2}{450} + \frac{139x}{3375} - \frac{16}{567} \log(3x+2) + \frac{\log(5x+1)}{4375}$$

```
[Out] (139*x)/3375 - (13*x^2)/450 + x^3/45 - (16*Log[2 + 3*x])/567 + Lo
g[1 + 5*x]/4375
```

Rubi [A] time = 0.0615334, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{x^3}{45} - \frac{13x^2}{450} + \frac{139x}{3375} - \frac{16}{567} \log(3x+2) + \frac{\log(5x+1)}{4375}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(13 + 2/x + 15*x),x]
```

```
[Out] (139*x)/3375 - (13*x^2)/450 + x^3/45 - (16*Log[2 + 3*x])/567 + Lo
g[1 + 5*x]/4375
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{x^3}{45} - \frac{16 \log(3x+2)}{567} + \frac{\log(5x+1)}{4375} + \int \frac{139}{3375} \, dx - \frac{13 \int x \, dx}{225}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**3/(13+2/x+15*x),x)
```

```
[Out] x**3/45 - 16*log(3*x + 2)/567 + log(5*x + 1)/4375 + Integral(139/
3375, x) - 13*Integral(x, x)/225
```

Mathematica [A] time = 0.00869042, size = 40, normalized size = 1.

$$\frac{x^3}{45} - \frac{13x^2}{450} + \frac{139x}{3375} - \frac{16}{567} \log(3x+2) + \frac{\log(5x+1)}{4375}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(13 + 2/x + 15*x),x]
```

```
[Out] (139*x)/3375 - (13*x^2)/450 + x^3/45 - (16*Log[2 + 3*x])/567 + Lo
g[1 + 5*x]/4375
```

Maple [A] time = 0.009, size = 31, normalized size = 0.8

$$\frac{139\,x}{3375} - \frac{13\,x^2}{450} + \frac{x^3}{45} - \frac{16\,\ln(2+3\,x)}{567} + \frac{\ln(1+5\,x)}{4375}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(13+2/x+15*x),x)
```

```
[Out] 139/3375*x-13/450*x^2+1/45*x^3-16/567*ln(2+3*x)+1/4375*ln(1+5*x)
```

Maxima [A] time = 0.800001, size = 41, normalized size = 1.02

$$\frac{1}{45}\,x^{3} - \frac{13}{450}\,x^{2} + \frac{139}{3375}\,x + \frac{1}{4375}\,\log\left(5\,x + 1\right) - \frac{16}{567}\,\log\left(3\,x + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(15*x + 2/x + 13),x, algorithm="maxima")
```

```
[Out] 1/45*x^3 - 13/450*x^2 + 139/3375*x + 1/4375*log(5*x + 1) - 16/567
*log(3*x + 2)
```

Fricas [A] time = 0.267442, size = 41, normalized size = 1.02

$$\frac{1}{45}\,x^{3} - \frac{13}{450}\,x^{2} + \frac{139}{3375}\,x + \frac{1}{4375}\,\log\left(5\,x + 1\right) - \frac{16}{567}\,\log\left(3\,x + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(15*x + 2/x + 13),x, algorithm="fricas")
```

```
[Out] 1/45*x^3 - 13/450*x^2 + 139/3375*x + 1/4375*log(5*x + 1) - 16/567
*log(3*x + 2)
```

Sympy [A] time = 0.261761, size = 34, normalized size = 0.85

$$\frac{x^{3}}{45} - \frac{13x^{2}}{450} + \frac{139x}{3375} + \frac{\log\left(x + \frac{1}{5}\right)}{4375} - \frac{16\log\left(x + \frac{2}{3}\right)}{567}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(13+2/x+15*x),x)
```

```
[Out] x**3/45 - 13*x**2/450 + 139*x/3375 + log(x + 1/5)/4375 - 16*log(x
 + 2/3)/567
```

GIAC/XCAS [A] time = 0.260944, size = 43, normalized size = 1.08

$$\frac{1}{45}\,x^{3} - \frac{13}{450}\,x^{2} + \frac{139}{3375}\,x + \frac{1}{4375}\,\ln\left(|5\,x + 1|\right) - \frac{16}{567}\,\ln\left(|3\,x + 2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(15*x + 2/x + 13),x, algorithm="giac")
```

```
[Out] 1/45*x^3 - 13/450*x^2 + 139/3375*x + 1/4375*ln(abs(5*x + 1)) - 16
/567*ln(abs(3*x + 2))
```

3.379 $\int \frac{x^2}{13+\frac{2}{x}+15x}\,dx$

Optimal. Leaf size=33

$$\frac{x^2}{30}-\frac{13x}{225}+\frac{8}{189}\log(3x+2)-\frac{1}{875}\log(5x+1)$$

```
[Out] (-13*x)/225 + x^2/30 + (8*Log[2 + 3*x])/189 - Log[1 + 5*x]/875
```

Rubi [A] time = 0.057556, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{x^2}{30}-\frac{13x}{225}+\frac{8}{189}\log(3x+2)-\frac{1}{875}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(13 + 2/x + 15*x),x]
```

```
[Out] (-13*x)/225 + x^2/30 + (8*Log[2 + 3*x])/189 - Log[1 + 5*x]/875
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{8\log(3x+2)}{189}-\frac{\log(5x+1)}{875}+\int\left(-\frac{13}{225}\right)dx+\frac{\int x\,dx}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(13+2/x+15*x),x)
```

```
[Out] 8*log(3*x + 2)/189 - log(5*x + 1)/875 + Integral(-13/225, x) + In
tegral(x, x)/15
```

Mathematica [A] time = 0.00672796, size = 33, normalized size = 1.

$$\frac{x^2}{30}-\frac{13x}{225}+\frac{8}{189}\log(3x+2)-\frac{1}{875}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(13 + 2/x + 15*x),x]
```

```
[Out] (-13*x)/225 + x^2/30 + (8*Log[2 + 3*x])/189 - Log[1 + 5*x]/875
```

Maple [A] time = 0.008, size = 26, normalized size = 0.8

$$-\frac{13\,x}{225}+\frac{x^2}{30}+\frac{8\,\ln(2+3\,x)}{189}-\frac{\ln(1+5\,x)}{875}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(13+2/x+15*x),x)
```

```
[Out] -13/225*x+1/30*x^2+8/189*ln(2+3*x)-1/875*ln(1+5*x)
```

Maxima [A] time = 0.829853, size = 34, normalized size = 1.03

$$\frac{1}{30}\,x^2-\frac{13}{225}\,x-\frac{1}{875}\,\log\left(5\,x+1\right)+\frac{8}{189}\,\log\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(15*x + 2/x + 13),x, algorithm="maxima")
```

```
[Out] 1/30*x^2 - 13/225*x - 1/875*log(5*x + 1) + 8/189*log(3*x + 2)
```

Fricas [A] time = 0.267203, size = 34, normalized size = 1.03

$$\frac{1}{30}\,x^2-\frac{13}{225}\,x-\frac{1}{875}\,\log\left(5\,x+1\right)+\frac{8}{189}\,\log\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(15*x + 2/x + 13),x, algorithm="fricas")
```

```
[Out] 1/30*x^2 - 13/225*x - 1/875*log(5*x + 1) + 8/189*log(3*x + 2)
```

Sympy [A] time = 0.241234, size = 27, normalized size = 0.82

$$\frac{x^2}{30}-\frac{13x}{225}-\frac{\log\left(x+\frac{1}{5}\right)}{875}+\frac{8\log\left(x+\frac{2}{3}\right)}{189}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(13+2/x+15*x),x)
```

```
[Out] x**2/30 - 13*x/225 - log(x + 1/5)/875 + 8*log(x + 2/3)/189
```

GIAC/XCAS [A] time = 0.261983, size = 36, normalized size = 1.09

$$\frac{1}{30}\,x^2-\frac{13}{225}\,x-\frac{1}{875}\,\ln\left(|5\,x+1|\right)+\frac{8}{189}\,\ln\left(|3\,x+2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(15*x + 2/x + 13),x, algorithm="giac")
```

```
[Out] 1/30*x^2 - 13/225*x - 1/875*ln(abs(5*x + 1)) + 8/189*ln(abs(3*x +
 2))
```

3.380 $\int \frac{x}{13+\frac{2}{x}+15x}\,dx$

Optimal. Leaf size=26

$$\frac{x}{15}-\frac{4}{63}\log(3x+2)+\frac{1}{175}\log(5x+1)$$

```
[Out] x/15 - (4*Log[2 + 3*x])/63 + Log[1 + 5*x]/175
```

Rubi [A] time = 0.0444354, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286

$$\frac{x}{15}-\frac{4}{63}\log(3x+2)+\frac{1}{175}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[x/(13 + 2/x + 15*x),x]

[Out] x/15 - (4*Log[2 + 3*x])/63 + Log[1 + 5*x]/175
```

Rubi in Sympy [A] time = 10.9559, size = 20, normalized size = 0.77

$$\frac{x}{15}-\frac{4\log(3x+2)}{63}+\frac{\log(5x+1)}{175}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(13+2/x+15*x),x)

[Out] x/15 - 4*log(3*x + 2)/63 + log(5*x + 1)/175
```

Mathematica [A] time = 0.00706266, size = 26, normalized size = 1.

$$\frac{x}{15}-\frac{4}{63}\log(3x+2)+\frac{1}{175}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(13 + 2/x + 15*x),x]

[Out] x/15 - (4*Log[2 + 3*x])/63 + Log[1 + 5*x]/175
```

Maple [A] time = 0.008, size = 21, normalized size = 0.8

$$\frac{x}{15}-\frac{4\ln(2+3x)}{63}+\frac{\ln(1+5x)}{175}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(13+2/x+15*x),x)
```

```
[Out] 1/15*x-4/63*ln(2+3*x)+1/175*ln(1+5*x)
```

Maxima [A] time = 0.814758, size = 27, normalized size = 1.04

$$\frac{1}{15}\,x+\frac{1}{175}\,\log\left(5\,x+1\right)-\frac{4}{63}\,\log\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(15*x + 2/x + 13),x, algorithm="maxima")
```

```
[Out] 1/15*x + 1/175*log(5*x + 1) - 4/63*log(3*x + 2)
```

Fricas [A] time = 0.262954, size = 27, normalized size = 1.04

$$\frac{1}{15}\,x+\frac{1}{175}\,\log\left(5\,x+1\right)-\frac{4}{63}\,\log\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(15*x + 2/x + 13),x, algorithm="fricas")
```

```
[Out] 1/15*x + 1/175*log(5*x + 1) - 4/63*log(3*x + 2)
```

Sympy [A] time = 0.230857, size = 20, normalized size = 0.77

$$\frac{x}{15}+\frac{\log\left(x+\frac{1}{5}\right)}{175}-\frac{4\log\left(x+\frac{2}{3}\right)}{63}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(13+2/x+15*x),x)
```

```
[Out] x/15 + log(x + 1/5)/175 - 4*log(x + 2/3)/63
```

GIAC/XCAS [A] time = 0.262967, size = 30, normalized size = 1.15

$$\frac{1}{15}\,x+\frac{1}{175}\,\ln\left(|5\,x+1|\right)-\frac{4}{63}\,\ln\left(|3\,x+2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(15*x + 2/x + 13),x, algorithm="giac")
```

```
[Out] 1/15*x + 1/175*ln(abs(5*x + 1)) - 4/63*ln(abs(3*x + 2))
```

3.381 $\int \frac{1}{13+\frac{2}{x}+15x}\,dx$

Optimal. Leaf size=21

$$\frac{2}{21}\log(3x+2)-\frac{1}{35}\log(5x+1)$$

```
[Out] (2*Log[2 + 3*x])/21 - Log[1 + 5*x]/35
```

Rubi [A] time = 0.0234647, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{2}{21}\log(3x+2)-\frac{1}{35}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(13 + 2/x + 15*x)^(-1),x]

[Out] (2*Log[2 + 3*x])/21 - Log[1 + 5*x]/35
```

Rubi in Sympy [A] time = 4.01834, size = 17, normalized size = 0.81

$$\frac{2\log(3x+2)}{21}-\frac{\log(5x+1)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(13+2/x+15*x),x)

[Out] 2*log(3*x + 2)/21 - log(5*x + 1)/35
```

Mathematica [A] time = 0.00456296, size = 21, normalized size = 1.

$$\frac{2}{21}\log(3x+2)-\frac{1}{35}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(13 + 2/x + 15*x)^(-1),x]

[Out] (2*Log[2 + 3*x])/21 - Log[1 + 5*x]/35
```

Maple [A] time = 0.007, size = 18, normalized size = 0.9

$$\frac{2\ln(2+3x)}{21}-\frac{\ln(1+5x)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(13+2/x+15*x),x)
```

```
[Out] 2/21*ln(2+3*x)-1/35*ln(1+5*x)
```

Maxima [A] time = 0.815769, size = 23, normalized size = 1.1

$$-\frac{1}{35}\log(5x+1)+\frac{2}{21}\log(3x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(15*x + 2/x + 13),x, algorithm="maxima")
```

```
[Out] -1/35*log(5*x + 1) + 2/21*log(3*x + 2)
```

Fricas [A] time = 0.263147, size = 23, normalized size = 1.1

$$-\frac{1}{35}\log(5x+1)+\frac{2}{21}\log(3x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(15*x + 2/x + 13),x, algorithm="fricas")
```

```
[Out] -1/35*log(5*x + 1) + 2/21*log(3*x + 2)
```

Sympy [A] time = 0.211935, size = 17, normalized size = 0.81

$$-\frac{\log\left(x+\frac{1}{5}\right)}{35}+\frac{2\log\left(x+\frac{2}{3}\right)}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(13+2/x+15*x),x)
```

```
[Out] -log(x + 1/5)/35 + 2*log(x + 2/3)/21
```

GIAC/XCAS [A] time = 0.259582, size = 26, normalized size = 1.24

$$-\frac{1}{35}\ln(|5x+1|)+\frac{2}{21}\ln(|3x+2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(15*x + 2/x + 13),x, algorithm="giac")
```

```
[Out] -1/35*ln(abs(5*x + 1)) + 2/21*ln(abs(3*x + 2))
```

3.382 $\int \frac{1}{x\left(13+\frac{2}{x}+15x\right)}\,dx$

Optimal. Leaf size=21

$$\frac{1}{7}\log(5x+1)-\frac{1}{7}\log(3x+2)$$

```
[Out] -Log[2 + 3*x]/7 + Log[1 + 5*x]/7
```

Rubi [A] time = 0.0286702, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188

$$\frac{1}{7}\log(5x+1)-\frac{1}{7}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(13 + 2/x + 15*x)),x]
```

```
[Out] -Log[2 + 3*x]/7 + Log[1 + 5*x]/7
```

Rubi in Sympy [A] time = 6.87726, size = 15, normalized size = 0.71

$$-\frac{\log(3x+2)}{7}+\frac{\log(5x+1)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(13+2/x+15*x),x)
```

```
[Out] -log(3*x + 2)/7 + log(5*x + 1)/7
```

Mathematica [A] time = 0.00406794, size = 21, normalized size = 1.

$$\frac{1}{7}\log(5x+1)-\frac{1}{7}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(13 + 2/x + 15*x)),x]
```

```
[Out] -Log[2 + 3*x]/7 + Log[1 + 5*x]/7
```

Maple [A] time = 0.007, size = 18, normalized size = 0.9

$$\frac{\ln(1+5x)}{7}-\frac{\ln(2+3x)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(13+2/x+15*x),x)
```

```
[Out] 1/7*ln(1+5*x)-1/7*ln(2+3*x)
```

Maxima [A] time = 0.804716, size = 23, normalized size = 1.1

$$\frac{1}{7}\log(5x+1) - \frac{1}{7}\log(3x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((15*x + 2/x + 13)*x),x, algorithm="maxima")
```

```
[Out] 1/7*log(5*x + 1) - 1/7*log(3*x + 2)
```

Fricas [A] time = 0.280503, size = 23, normalized size = 1.1

$$\frac{1}{7}\log(5x+1) - \frac{1}{7}\log(3x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((15*x + 2/x + 13)*x),x, algorithm="fricas")
```

```
[Out] 1/7*log(5*x + 1) - 1/7*log(3*x + 2)
```

Sympy [A] time = 0.201988, size = 15, normalized size = 0.71

$$\frac{\log\left(x+\frac{1}{5}\right)}{7} - \frac{\log\left(x+\frac{2}{3}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(13+2/x+15*x),x)
```

```
[Out] log(x + 1/5)/7 - log(x + 2/3)/7
```

GIAC/XCAS [A] time = 0.260332, size = 26, normalized size = 1.24

$$\frac{1}{7}\ln(|5x+1|) - \frac{1}{7}\ln(|3x+2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((15*x + 2/x + 13)*x),x, algorithm="giac")
```

```
[Out] 1/7*ln(abs(5*x + 1)) - 1/7*ln(abs(3*x + 2))
```

3.383 $\int \frac{1}{x^2\left(13+\frac{2}{x}+15x\right)}\,dx$

Optimal. Leaf size=27

$$\frac{\log(x)}{2}+\frac{3}{14}\log(3x+2)-\frac{5}{7}\log(5x+1)$$

```
[Out] Log[x]/2 + (3*Log[2 + 3*x])/14 - (5*Log[1 + 5*x])/7
```

Rubi [A] time = 0.0463537, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312

$$\frac{\log(x)}{2}+\frac{3}{14}\log(3x+2)-\frac{5}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(13 + 2/x + 15*x)),x]

[Out] Log[x]/2 + (3*Log[2 + 3*x])/14 - (5*Log[1 + 5*x])/7
```

Rubi in Sympy [A] time = 12.1012, size = 24, normalized size = 0.89

$$\frac{\log\left(x\right)}{2}+\frac{3\log\left(3x+2\right)}{14}-\frac{5\log\left(5x+1\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(13+2/x+15*x),x)

[Out] log(x)/2 + 3*log(3*x + 2)/14 - 5*log(5*x + 1)/7
```

Mathematica [A] time = 0.00698715, size = 27, normalized size = 1.

$$\frac{\log(x)}{2}+\frac{3}{14}\log(3x+2)-\frac{5}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(13 + 2/x + 15*x)),x]

[Out] Log[x]/2 + (3*Log[2 + 3*x])/14 - (5*Log[1 + 5*x])/7
```

Maple [A] time = 0.009, size = 22, normalized size = 0.8

$$\frac{\ln\left(x\right)}{2}+\frac{3\ \ln\left(2+3\,x\right)}{14}-\frac{5\ \ln\left(1+5\,x\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(13+2/x+15*x),x)
```

```
[Out] 1/2*ln(x)+3/14*ln(2+3*x)-5/7*ln(1+5*x)
```

Maxima [A] time = 0.820349, size = 28, normalized size = 1.04

$$-\frac{5}{7}\log(5x+1)+\frac{3}{14}\log(3x+2)+\frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((15*x + 2/x + 13)*x^2),x, algorithm="maxima")
```

```
[Out] -5/7*log(5*x + 1) + 3/14*log(3*x + 2) + 1/2*log(x)
```

Fricas [A] time = 0.270517, size = 28, normalized size = 1.04

$$-\frac{5}{7}\log(5x+1)+\frac{3}{14}\log(3x+2)+\frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((15*x + 2/x + 13)*x^2),x, algorithm="fricas")
```

```
[Out] -5/7*log(5*x + 1) + 3/14*log(3*x + 2) + 1/2*log(x)
```

Sympy [A] time = 0.310984, size = 24, normalized size = 0.89

$$\frac{\log(x)}{2}-\frac{5\log\left(x+\frac{1}{5}\right)}{7}+\frac{3\log\left(x+\frac{2}{3}\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(13+2/x+15*x),x)
```

```
[Out] log(x)/2 - 5*log(x + 1/5)/7 + 3*log(x + 2/3)/14
```

GIAC/XCAS [A] time = 0.264026, size = 32, normalized size = 1.19

$$-\frac{5}{7}\ln(|5x+1|)+\frac{3}{14}\ln(|3x+2|)+\frac{1}{2}\ln(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((15*x + 2/x + 13)*x^2),x, algorithm="giac")
```

```
[Out] -5/7*ln(abs(5*x + 1)) + 3/14*ln(abs(3*x + 2)) + 1/2*ln(abs(x))
```

3.384 $\int \frac{1}{x^3\left(13+\frac{2}{x}+15x\right)}\,dx$

Optimal. Leaf size=34

$$-\frac{1}{2x}-\frac{13\log(x)}{4}-\frac{9}{28}\log(3x+2)+\frac{25}{7}\log(5x+1)$$

```
[Out] -1/(2*x) - (13*Log[x])/4 - (9*Log[2 + 3*x])/28 + (25*Log[1 + 5*x]
)/7
```

Rubi [A] time = 0.0779943, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188

$$-\frac{1}{2x}-\frac{13\log(x)}{4}-\frac{9}{28}\log(3x+2)+\frac{25}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*(13 + 2/x + 15*x)),x]
```

```
[Out] -1/(2*x) - (13*Log[x])/4 - (9*Log[2 + 3*x])/28 + (25*Log[1 + 5*x]
)/7
```

Rubi in Sympy [A] time = 16.2613, size = 31, normalized size = 0.91

$$-\frac{13\log\left(x\right)}{4}-\frac{9\log\left(3x+2\right)}{28}+\frac{25\log\left(5x+1\right)}{7}-\frac{1}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(13+2/x+15*x),x)
```

```
[Out] -13*log(x)/4 - 9*log(3*x + 2)/28 + 25*log(5*x + 1)/7 - 1/(2*x)
```

Mathematica [A] time = 0.00602208, size = 34, normalized size = 1.

$$-\frac{1}{2x}-\frac{13\log(x)}{4}-\frac{9}{28}\log(3x+2)+\frac{25}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*(13 + 2/x + 15*x)),x]
```

```
[Out] -1/(2*x) - (13*Log[x])/4 - (9*Log[2 + 3*x])/28 + (25*Log[1 + 5*x]
)/7
```

Maple [A] time = 0.012, size = 27, normalized size = 0.8

$$-\frac{1}{2\,x}-\frac{13\,\ln\left(x\right)}{4}-\frac{9\,\ln\left(2+3\,x\right)}{28}+\frac{25\,\ln\left(1+5\,x\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(13+2/x+15*x),x)
```

```
[Out] -1/2/x-13/4*ln(x)-9/28*ln(2+3*x)+25/7*ln(1+5*x)
```

Maxima [A] time = 0.812505, size = 35, normalized size = 1.03

$$-\frac{1}{2\,x}+\frac{25}{7}\,\log\left(5\,x+1\right)-\frac{9}{28}\,\log\left(3\,x+2\right)-\frac{13}{4}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((15*x + 2/x + 13)*x^3),x, algorithm="maxima")
```

```
[Out] -1/2/x + 25/7*log(5*x + 1) - 9/28*log(3*x + 2) - 13/4*log(x)
```

Fricas [A] time = 0.274376, size = 41, normalized size = 1.21

$$\frac{100\,x\log\left(5\,x+1\right)-9\,x\log\left(3\,x+2\right)-91\,x\log\left(x\right)-14}{28\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((15*x + 2/x + 13)*x^3),x, algorithm="fricas")
```

```
[Out] 1/28*(100*x*log(5*x + 1) - 9*x*log(3*x + 2) - 91*x*log(x) - 14)/x
```

Sympy [A] time = 0.354642, size = 31, normalized size = 0.91

$$-\frac{13\log\left(x\right)}{4}+\frac{25\log\left(x+\frac{1}{5}\right)}{7}-\frac{9\log\left(x+\frac{2}{3}\right)}{28}-\frac{1}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(13+2/x+15*x),x)
```

```
[Out] -13*log(x)/4 + 25*log(x + 1/5)/7 - 9*log(x + 2/3)/28 - 1/(2*x)
```

GIAC/XCAS [A] time = 0.263741, size = 39, normalized size = 1.15

$$-\frac{1}{2\,x}+\frac{25}{7}\ln\left(|5\,x+1|\right)-\frac{9}{28}\ln\left(|3\,x+2|\right)-\frac{13}{4}\ln\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((15*x + 2/x + 13)*x^3),x, algorithm="giac")
```

```
[Out] -1/2/x + 25/7*ln(abs(5*x + 1)) - 9/28*ln(abs(3*x + 2)) - 13/4*ln(
abs(x))
```

3.385 $\int \frac{1}{x^4\left(13+\frac{2}{x}+15x\right)}\,dx$

Optimal. Leaf size=41

$$-\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\log(x)}{8}+\frac{27}{56}\log(3x+2)-\frac{125}{7}\log(5x+1)$$

```
[Out] -1/(4*x^2) + 13/(4*x) + (139*Log[x])/8 + (27*Log[2 + 3*x])/56 - (
125*Log[1 + 5*x])/7
```

Rubi [A] time = 0.0846669, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188

$$-\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\log(x)}{8}+\frac{27}{56}\log(3x+2)-\frac{125}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^4*(13 + 2/x + 15*x)),x]
```

```
[Out] -1/(4*x^2) + 13/(4*x) + (139*Log[x])/8 + (27*Log[2 + 3*x])/56 - (
125*Log[1 + 5*x])/7
```

Rubi in Sympy [A] time = 16.9341, size = 37, normalized size = 0.9

$$\frac{139\log(x)}{8}+\frac{27\log(3x+2)}{56}-\frac{125\log(5x+1)}{7}+\frac{13}{4x}-\frac{1}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**4/(13+2/x+15*x),x)
```

```
[Out] 139*log(x)/8 + 27*log(3*x + 2)/56 - 125*log(5*x + 1)/7 + 13/(4*x)
 - 1/(4*x**2)
```

Mathematica [A] time = 0.00749304, size = 41, normalized size = 1.

$$-\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\log(x)}{8}+\frac{27}{56}\log(3x+2)-\frac{125}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^4*(13 + 2/x + 15*x)),x]
```

```
[Out] -1/(4*x^2) + 13/(4*x) + (139*Log[x])/8 + (27*Log[2 + 3*x])/56 - (
125*Log[1 + 5*x])/7
```

Maple [A] time = 0.013, size = 32, normalized size = 0.8

$$-\frac{1}{4\,x^2}+\frac{13}{4\,x}+\frac{139\,\ln(x)}{8}+\frac{27\,\ln(2+3\,x)}{56}-\frac{125\,\ln(1+5\,x)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^4/(13+2/x+15*x),x)
```

```
[Out] -1/4/x^2+13/4/x+139/8*ln(x)+27/56*ln(2+3*x)-125/7*ln(1+5*x)
```

Maxima [A] time = 0.808357, size = 42, normalized size = 1.02

$$\frac{13\,x-1}{4\,x^2}-\frac{125}{7}\,\log\left(5\,x+1\right)+\frac{27}{56}\,\log\left(3\,x+2\right)+\frac{139}{8}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((15*x + 2/x + 13)*x^4),x, algorithm="maxima")
```

```
[Out] 1/4*(13*x - 1)/x^2 - 125/7*log(5*x + 1) + 27/56*log(3*x + 2) + 13
9/8*log(x)
```

Fricas [A] time = 0.259382, size = 53, normalized size = 1.29

$$-\frac{1000\,x^2\log\left(5\,x+1\right)-27\,x^2\log\left(3\,x+2\right)-973\,x^2\log\left(x\right)-182\,x+14}{56\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((15*x + 2/x + 13)*x^4),x, algorithm="fricas")
```

```
[Out] -1/56*(1000*x^2*log(5*x + 1) - 27*x^2*log(3*x + 2) - 973*x^2*log(
x) - 182*x + 14)/x^2
```

Sympy [A] time = 0.399477, size = 36, normalized size = 0.88

$$\frac{139\log\left(x\right)}{8}-\frac{125\log\left(x+\frac{1}{5}\right)}{7}+\frac{27\log\left(x+\frac{2}{3}\right)}{56}+\frac{13x-1}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**4/(13+2/x+15*x),x)
```

```
[Out] 139*log(x)/8 - 125*log(x + 1/5)/7 + 27*log(x + 2/3)/56 + (13*x -
1)/(4*x**2)
```

GIAC/XCAS [A] time = 0.262306, size = 46, normalized size = 1.12

$$\frac{13\,x-1}{4\,x^2}-\frac{125}{7}\,\ln\left(|5\,x+1|\right)+\frac{27}{56}\,\ln\left(|3\,x+2|\right)+\frac{139}{8}\,\ln\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((15*x + 2/x + 13)*x^4),x, algorithm="giac")
```

```
[Out] 1/4*(13*x - 1)/x^2 - 125/7*ln(abs(5*x + 1)) + 27/56*ln(abs(3*x +
2)) + 139/8*ln(abs(x))
```

3.386 $\int \frac{1}{x^5\left(13+\frac{2}{x}+15x\right)}\,dx$

Optimal. Leaf size=48

$$-\frac{1}{6x^3}+\frac{13}{8x^2}-\frac{139}{8x}-\frac{1417\log(x)}{16}-\frac{81}{112}\log(3x+2)+\frac{625}{7}\log(5x+1)$$

```
[Out] -1/(6*x^3) + 13/(8*x^2) - 139/(8*x) - (1417*Log[x])/16 - (81*Log[
2 + 3*x])/112 + (625*Log[1 + 5*x])/7
```

Rubi [A] time = 0.0993714, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188

$$-\frac{1}{6x^3}+\frac{13}{8x^2}-\frac{139}{8x}-\frac{1417\log(x)}{16}-\frac{81}{112}\log(3x+2)+\frac{625}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^5*(13 + 2/x + 15*x)),x]
```

```
[Out] -1/(6*x^3) + 13/(8*x^2) - 139/(8*x) - (1417*Log[x])/16 - (81*Log[
2 + 3*x])/112 + (625*Log[1 + 5*x])/7
```

Rubi in Sympy [A] time = 17.4917, size = 44, normalized size = 0.92

$$-\frac{1417\log(x)}{16}-\frac{81\log(3x+2)}{112}+\frac{625\log(5x+1)}{7}-\frac{139}{8x}+\frac{13}{8x^2}-\frac{1}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**5/(13+2/x+15*x),x)
```

```
[Out] -1417*log(x)/16 - 81*log(3*x + 2)/112 + 625*log(5*x + 1)/7 - 139/
(8*x) + 13/(8*x**2) - 1/(6*x**3)
```

Mathematica [A] time = 0.00771351, size = 48, normalized size = 1.

$$-\frac{1}{6x^3}+\frac{13}{8x^2}-\frac{139}{8x}-\frac{1417\log(x)}{16}-\frac{81}{112}\log(3x+2)+\frac{625}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^5*(13 + 2/x + 15*x)),x]
```

```
[Out] -1/(6*x^3) + 13/(8*x^2) - 139/(8*x) - (1417*Log[x])/16 - (81*Log[
2 + 3*x])/112 + (625*Log[1 + 5*x])/7
```

Maple [A] time = 0.013, size = 37, normalized size = 0.8

$$-\frac{1}{6\,x^3}+\frac{13}{8\,x^2}-\frac{139}{8\,x}-\frac{1417\,\ln(x)}{16}-\frac{81\,\ln(2+3\,x)}{112}+\frac{625\,\ln(1+5\,x)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^5/(13+2/x+15*x),x)
```

```
[Out] -1/6/x^3+13/8/x^2-139/8/x-1417/16*ln(x)-81/112*ln(2+3*x)+625/7*ln
(1+5*x)
```

Maxima [A] time = 0.788821, size = 49, normalized size = 1.02

$$-\frac{417\,x^2-39\,x+4}{24\,x^3}+\frac{625}{7}\,\log\left(5\,x+1\right)-\frac{81}{112}\,\log\left(3\,x+2\right)-\frac{1417}{16}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((15*x + 2/x + 13)*x^5),x, algorithm="maxima")
```

```
[Out] -1/24*(417*x^2 - 39*x + 4)/x^3 + 625/7*log(5*x + 1) - 81/112*log(
3*x + 2) - 1417/16*log(x)
```

Fricas [A] time = 0.271926, size = 59, normalized size = 1.23

$$\frac{30000\,x^3\log\left(5\,x+1\right)-243\,x^3\log\left(3\,x+2\right)-29757\,x^3\log\left(x\right)-5838\,x^2+546\,x-56}{336\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((15*x + 2/x + 13)*x^5),x, algorithm="fricas")
```

```
[Out] 1/336*(30000*x^3*log(5*x + 1) - 243*x^3*log(3*x + 2) - 29757*x^3*
log(x) - 5838*x^2 + 546*x - 56)/x^3
```

Sympy [A] time = 0.440754, size = 41, normalized size = 0.85

$$-\frac{1417\log\left(x\right)}{16}+\frac{625\log\left(x+\frac{1}{5}\right)}{7}-\frac{81\log\left(x+\frac{2}{3}\right)}{112}-\frac{417x^2-39x+4}{24x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**5/(13+2/x+15*x),x)
```

```
[Out] -1417*log(x)/16 + 625*log(x + 1/5)/7 - 81*log(x + 2/3)/112 - (417
*x**2 - 39*x + 4)/(24*x**3)
```

GIAC/XCAS [A] time = 0.262599, size = 53, normalized size = 1.1

$$-\frac{417\,x^2-39\,x+4}{24\,x^3}+\frac{625}{7}\ln\left(\left|5\,x+1\right|\right)-\frac{81}{112}\ln\left(\left|3\,x+2\right|\right)-\frac{1417}{16}\ln\left(\left|x\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((15*x + 2/x + 13)*x^5),x, algorithm="giac")
```

```
[Out] -1/24*(417*x^2 - 39*x + 4)/x^3 + 625/7*ln(abs(5*x + 1)) - 81/112*
ln(abs(3*x + 2)) - 1417/16*ln(abs(x))
```

3.387 $\int \frac{x^2}{2-(1+x^2)^4}\,dx$

Optimal. Leaf size=157

$$\frac{i\sqrt{1-i\sqrt[4]{2}}\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}-\frac{i\sqrt{1+i\sqrt[4]{2}}\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}-\frac{\sqrt{1+\sqrt[4]{2}}\tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}+\frac{\sqrt{\sqrt[4]{2}-1}\tanh^{-1}\left(\frac{x}{\sqrt{\sqrt[4]{2}-1}}\right)}{4\ 2^{3/4}}$$

```
[Out] ((I/4)*Sqrt[1 - I*2^(1/4)]*ArcTan[x/Sqrt[1 - I*2^(1/4)]])/2^(3/4)
 - ((I/4)*Sqrt[1 + I*2^(1/4)]*ArcTan[x/Sqrt[1 + I*2^(1/4)]])/2^(3
/4) - (Sqrt[1 + 2^(1/4)]*ArcTan[x/Sqrt[1 + 2^(1/4)]])/(4*2^(3/4))
 + (Sqrt[-1 + 2^(1/4)]*ArcTanh[x/Sqrt[-1 + 2^(1/4)]])/(4*2^(3/4))
```

Rubi [A] time = 0.3435, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294

$$\frac{i\sqrt{1-i\sqrt[4]{2}}\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}-\frac{i\sqrt{1+i\sqrt[4]{2}}\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}-\frac{\sqrt{1+\sqrt[4]{2}}\tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}+\frac{\sqrt{\sqrt[4]{2}-1}\tanh^{-1}\left(\frac{x}{\sqrt{\sqrt[4]{2}-1}}\right)}{4\ 2^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(2 - (1 + x^2)^4),x]
```

```
[Out] ((I/4)*Sqrt[1 - I*2^(1/4)]*ArcTan[x/Sqrt[1 - I*2^(1/4)]])/2^(3/4)
 - ((I/4)*Sqrt[1 + I*2^(1/4)]*ArcTan[x/Sqrt[1 + I*2^(1/4)]])/2^(3
/4) - (Sqrt[1 + 2^(1/4)]*ArcTan[x/Sqrt[1 + 2^(1/4)]])/(4*2^(3/4))
 + (Sqrt[-1 + 2^(1/4)]*ArcTanh[x/Sqrt[-1 + 2^(1/4)]])/(4*2^(3/4))
```

Rubi in Sympy [A] time = 38.6925, size = 158, normalized size = 1.01

$$-\frac{\left(\sqrt[4]{2}+\sqrt{2}\right)\operatorname{atan}\left(\frac{x}{\sqrt{1+\sqrt[4]{2}}}\right)}{8\sqrt{1+\sqrt[4]{2}}}+\frac{\sqrt[4]{2}i\sqrt{1-\sqrt[4]{2}i}\operatorname{atan}\left(\frac{x}{\sqrt{1-\sqrt[4]{2}i}}\right)}{8}-\frac{\left(-2^{\frac{3}{4}}+2-2i-2^{\frac{3}{4}}i\right)\operatorname{atan}\left(\frac{\sqrt{2}x(-1+i)}{2\sqrt{\sqrt[4]{2}-i}}\right)}{16\sqrt{\sqrt[4]{2}-i}}-\frac{\left(-\sqrt{2}+\sqrt[4]{2}\right)\operatorname{atanh}\left(\frac{x}{\sqrt{-1+\sqrt[4]{2}}}\right)}{8\sqrt{-1+\sqrt[4]{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(2-(x**2+1)**4),x)
```

```
[Out] -(2**(1/4) + sqrt(2))*atan(x/sqrt(1 + 2**(1/4)))/(8*sqrt(1 + 2**(
1/4))) + 2**(1/4)*I*sqrt(1 - 2**(1/4)*I)*atan(x/sqrt(1 - 2**(1/4)
*I))/8 - (-2**(3/4) + 2 - 2*I - 2**(3/4)*I)*atan(sqrt(2)*x*(-1 +
I)/(2*sqrt(2**(1/4) - I)))/(16*sqrt(2**(1/4) - I)) - (-sqrt(2) +
2**(1/4))*atanh(x/sqrt(-1 + 2**(1/4)))/(8*sqrt(-1 + 2**(1/4)))
```

Mathematica [C] time = 0.0260018, size = 61, normalized size = 0.39

$$-\frac{1}{8}\text{RootSum}\left[\#1^8+4\#1^6+6\#1^4+4\#1^2-1\&,\frac{\#1\log(x-\#1)}{\#1^6+3\#1^4+3\#1^2+1}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(2 - (1 + x^2)^4),x]
```

```
[Out] -RootSum[-1 + 4*#1^2 + 6*#1^4 + 4*#1^6 + #1^8 & , (Log[x - #1]*#1
)/(1 + 3*#1^2 + 3*#1^4 + #1^6) & ]/8
```

Maple [C] time = 0.013, size = 54, normalized size = 0.3

$$-\frac{1}{8}\sum_{\_R=RootOf\left(\_Z^8+4\_Z^6+6\_Z^4+4\_Z^2-1\right)}\frac{\_R^2\ln\left(x-\_R\right)}{\_R^7+3\_R^5+3\_R^3+\_R}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(2-(x^2+1)^4),x)
```

```
[Out] -1/8*sum(_R^2/(_R^7+3*_R^5+3*_R^3+_R)*ln(x-_R),_R=RootOf(_Z^8+4*_
Z^6+6*_Z^4+4*_Z^2-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x^2}{(x^2+1)^4-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x^2/((x^2 + 1)^4 - 2),x, algorithm="maxima")
```

```
[Out] -integrate(x^2/((x^2 + 1)^4 - 2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x^2/((x^2 + 1)^4 - 2),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 0.666037, size = 41, normalized size = 0.26

$$-\operatorname{RootSum}\left(1073741824t^8-65536t^4+1024t^2-1,\left(t\mapsto t\log\left(-\frac{67108864t^7}{3}-\frac{262144t^5}{3}-\frac{40t}{3}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(2-(x**2+1)**4),x)
```

```
[Out] -RootSum(1073741824*_t**8 - 65536*_t**4 + 1024*_t**2 - 1, Lambda(
_t, _t*log(-67108864*_t**7/3 - 262144*_t**5/3 - 40*_t/3 + x)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2}{(x^2+1)^4-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x^2/((x^2 + 1)^4 - 2),x, algorithm="giac")
```

```
[Out] integrate(-x^2/((x^2 + 1)^4 - 2), x)
```

3.388 $\int \frac{x^2}{2-(1-x^2)^4}\,dx$

Optimal. Leaf size=157

$$-\frac{\sqrt{\sqrt[4]{2}-1}\tan^{-1}\left(\frac{x}{\sqrt{\sqrt[4]{2}-1}}\right)}{4\ 2^{3/4}}-\frac{i\sqrt{1-i\sqrt[4]{2}}\tanh^{-1}\left(\frac{x}{\sqrt{1-i\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}+\frac{i\sqrt{1+i\sqrt[4]{2}}\tanh^{-1}\left(\frac{x}{\sqrt{1+i\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}+\frac{\sqrt{1+\sqrt[4]{2}}\tanh^{-1}\left(\frac{x}{\sqrt{1+\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}$$

```
[Out] -(Sqrt[-1 + 2^(1/4)]*ArcTan[x/Sqrt[-1 + 2^(1/4)]])/(4*2^(3/4)) -
((I/4)*Sqrt[1 - I*2^(1/4)]*ArcTanh[x/Sqrt[1 - I*2^(1/4)]])/2^(3/4
) + ((I/4)*Sqrt[1 + I*2^(1/4)]*ArcTanh[x/Sqrt[1 + I*2^(1/4)]])/2^
(3/4) + (Sqrt[1 + 2^(1/4)]*ArcTanh[x/Sqrt[1 + 2^(1/4)]])/(4*2^(3/
4))
```

Rubi [A] time = 0.267073, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263

$$-\frac{\sqrt{\sqrt[4]{2}-1}\tan^{-1}\left(\frac{x}{\sqrt{\sqrt[4]{2}-1}}\right)}{4\ 2^{3/4}}-\frac{i\sqrt{1-i\sqrt[4]{2}}\tanh^{-1}\left(\frac{x}{\sqrt{1-i\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}+\frac{i\sqrt{1+i\sqrt[4]{2}}\tanh^{-1}\left(\frac{x}{\sqrt{1+i\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}+\frac{\sqrt{1+\sqrt[4]{2}}\tanh^{-1}\left(\frac{x}{\sqrt{1+\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(2 - (1 - x^2)^4),x]
```

```
[Out] -(Sqrt[-1 + 2^(1/4)]*ArcTan[x/Sqrt[-1 + 2^(1/4)]])/(4*2^(3/4)) -
((I/4)*Sqrt[1 - I*2^(1/4)]*ArcTanh[x/Sqrt[1 - I*2^(1/4)]])/2^(3/4
) + ((I/4)*Sqrt[1 + I*2^(1/4)]*ArcTanh[x/Sqrt[1 + I*2^(1/4)]])/2^
(3/4) + (Sqrt[1 + 2^(1/4)]*ArcTanh[x/Sqrt[1 + 2^(1/4)]])/(4*2^(3/
4))
```

Rubi in Sympy [A] time = 39.0292, size = 163, normalized size = 1.04

$$\frac{\left(\sqrt[4]{2}+\sqrt{2}\right)\operatorname{atan}\left(\frac{x}{\sqrt{-1+\sqrt[4]{2}}}\right)}{8\sqrt{-1+\sqrt[4]{2}}}+\frac{\left(-\sqrt{2}+\sqrt[4]{2}\right)\operatorname{atanh}\left(\frac{x}{\sqrt{1+\sqrt[4]{2}}}\right)}{8\sqrt{1+\sqrt[4]{2}}}+\frac{\left(\sqrt{2}-\sqrt[4]{2}i\right)\operatorname{atanh}\left(\frac{x}{\sqrt{1-\sqrt[4]{2}i}}\right)}{8\sqrt{1-\sqrt[4]{2}i}}-\frac{\left(2^{\frac{3}{4}}+2-2i+2^{\frac{3}{4}}i\right)\operatorname{atanh}\left(\frac{\sqrt{2}x(-1+i)}{2\sqrt{\sqrt[4]{2}-i}}\right)}{16\sqrt{\sqrt[4]{2}-i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(2-(-x**2+1)**4),x)
```

```
[Out] (2**(1/4) + sqrt(2))*atan(x/sqrt(-1 + 2**(1/4)))/(8*sqrt(-1 + 2**
(1/4))) + (-sqrt(2) + 2**(1/4))*atanh(x/sqrt(1 + 2**(1/4)))/(8*sq
rt(1 + 2**(1/4))) + (sqrt(2) - 2**(1/4)*I)*atanh(x/sqrt(1 - 2**(1
```

```
/4)*I))/(8*sqrt(1 - 2**(1/4)*I)) - (2**(3/4) + 2 - 2*I + 2**(3/4)
*I)*atanh(sqrt(2)*x*(-1 + I)/(2*sqrt(2**(1/4) - I)))/(16*sqrt(2**
(1/4) - I))
```

Mathematica [C] time = 0.0249484, size = 61, normalized size = 0.39

$$-\frac{1}{8}\text{RootSum}\left[\#1^8-4\#1^6+6\#1^4-4\#1^2-1\&,\frac{\#1\log(x-\#1)}{\#1^6-3\#1^4+3\#1^2-1}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(2 - (1 - x^2)^4),x]
```

```
[Out] -RootSum[-1 - 4*#1^2 + 6*#1^4 - 4*#1^6 + #1^8 & , (Log[x - #1]*#1
)/(-1 + 3*#1^2 - 3*#1^4 + #1^6) & ]/8
```

Maple [C] time = 0.013, size = 56, normalized size = 0.4

$$-\frac{1}{8}\sum_{\_R=RootOf(\_Z^8-4\_Z^6+6\_Z^4-4\_Z^2-1)}\frac{\_R^2\ln(x-\_R)}{\_R^7-3\_R^5+3\_R^3-\_R}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(2-(-x^2+1)^4),x)
```

```
[Out] -1/8*sum(_R^2/(_R^7-3*_R^5+3*_R^3-_R)*ln(x-_R),_R=RootOf(_Z^8-4*_
Z^6+6*_Z^4-4*_Z^2-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x^2}{(x^2-1)^4-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x^2/((x^2 - 1)^4 - 2),x, algorithm="maxima")
```

```
[Out] -integrate(x^2/((x^2 - 1)^4 - 2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x^2/((x^2 - 1)^4 - 2),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 0.666073, size = 41, normalized size = 0.26

$$-\operatorname{RootSum}\left(1073741824t^8-65536t^4-1024t^2-1,\left(t\mapsto t\log\left(-\frac{67108864t^7}{3}+\frac{262144t^5}{3}+\frac{40t}{3}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(2-(-x**2+1)**4),x)
```

```
[Out] -RootSum(1073741824*_t**8 - 65536*_t**4 - 1024*_t**2 - 1, Lambda(
_t, _t*log(-67108864*_t**7/3 + 262144*_t**5/3 + 40*_t/3 + x)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2}{(x^2-1)^4-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x^2/((x^2 - 1)^4 - 2),x, algorithm="giac")
```

```
[Out] integrate(-x^2/((x^2 - 1)^4 - 2), x)
```

3.389 $\int \frac{x^2}{2+(1+x^2)^4} \, dx$

Optimal. Leaf size=188

$$\frac{\sqrt[4]{-1}\sqrt{1-\sqrt[4]{-2}}\tan^{-1}\left(\frac{x}{\sqrt{1-\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}}-\frac{(-1)^{3/4}\sqrt{1+i\sqrt[4]{-2}}\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}}-\frac{\sqrt[4]{-1}\sqrt{1+\sqrt[4]{-2}}\tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}}+\frac{1}{8}i\left(\sqrt[4]{-2}+\sqrt{2}\right)\sqrt{\frac{1+i}{2^{3/4}+(1+i)}}\tan^{-1}\left(\sqrt{\frac{1+i}{2^{3/4}+(1+i)}}x\right)$$

```
[Out] ((-1)^(1/4)*Sqrt[1 - (-2)^(1/4)]*ArcTan[x/Sqrt[1 - (-2)^(1/4)]])/
(4*2^(3/4)) - ((-1)^(3/4)*Sqrt[1 + I*(-2)^(1/4)]*ArcTan[x/Sqrt[1
+ I*(-2)^(1/4)]])/(4*2^(3/4)) - ((-1)^(1/4)*Sqrt[1 + (-2)^(1/4)]*
ArcTan[x/Sqrt[1 + (-2)^(1/4)]])/(4*2^(3/4)) + (I/8)*((-2)^(1/4) +
 Sqrt[2])*Sqrt[(1 + I)/((1 + I) + 2^(3/4))]*ArcTan[Sqrt[(1 + I)/(
(1 + I) + 2^(3/4))]*x]
```

Rubi [A] time = 0.574583, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{\sqrt[4]{-1}\sqrt{1-\sqrt[4]{-2}}\tan^{-1}\left(\frac{x}{\sqrt{1-\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}}-\frac{(-1)^{3/4}\sqrt{1+i\sqrt[4]{-2}}\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}}-\frac{\sqrt[4]{-1}\sqrt{1+\sqrt[4]{-2}}\tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}}+\frac{1}{8}i\left(\sqrt[4]{-2}+\sqrt{2}\right)\sqrt{\frac{1+i}{2^{3/4}+(1+i)}}\tan^{-1}\left(\sqrt{\frac{1+i}{2^{3/4}+(1+i)}}x\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(2 + (1 + x^2)^4),x]
```

```
[Out] ((-1)^(1/4)*Sqrt[1 - (-2)^(1/4)]*ArcTan[x/Sqrt[1 - (-2)^(1/4)]])/
(4*2^(3/4)) - ((-1)^(3/4)*Sqrt[1 + I*(-2)^(1/4)]*ArcTan[x/Sqrt[1
+ I*(-2)^(1/4)]])/(4*2^(3/4)) - ((-1)^(1/4)*Sqrt[1 + (-2)^(1/4)]*
ArcTan[x/Sqrt[1 + (-2)^(1/4)]])/(4*2^(3/4)) + (I/8)*((-2)^(1/4) +
 Sqrt[2])*Sqrt[(1 + I)/((1 + I) + 2^(3/4))]*ArcTan[Sqrt[(1 + I)/(
(1 + I) + 2^(3/4))]*x]
```

Rubi in Sympy [A] time = 83.5536, size = 236, normalized size = 1.26

$$\frac{\left(\sqrt[4]{2}-2i+\sqrt[4]{2}i\right)\operatorname{atan}\left(\frac{\sqrt{2}x}{\sqrt{-2^{\frac{3}{4}}+2-2^{\frac{3}{4}}i}}\right)}{8\sqrt{-2^{\frac{3}{4}}+2-2^{\frac{3}{4}}i}}+\frac{\left(\sqrt[4]{2}-\sqrt[4]{2}i+2i\right)\operatorname{atan}\left(\frac{\sqrt{2}x}{\sqrt{-2^{\frac{3}{4}}+2+2^{\frac{3}{4}}i}}\right)}{8\sqrt{-2^{\frac{3}{4}}+2+2^{\frac{3}{4}}i}}-\frac{\left(\sqrt[4]{2}-2i-\sqrt[4]{2}i\right)\operatorname{atan}\left(\frac{\sqrt{2}x}{\sqrt{2^{\frac{3}{4}}+2-2^{\frac{3}{4}}i}}\right)}{8\sqrt{2^{\frac{3}{4}}+2-2^{\frac{3}{4}}i}}-\frac{\left(\sqrt[4]{2}+\sqrt[4]{2}i+2i\right)\operatorname{atan}\left(\frac{\sqrt{2}x}{\sqrt{2^{\frac{3}{4}}+2+2^{\frac{3}{4}}i}}\right)}{8\sqrt{2^{\frac{3}{4}}+2+2^{\frac{3}{4}}i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(2+(x**2+1)**4),x)
```

```
[Out] (2**(1/4) - 2*I + 2**(1/4)*I)*atan(sqrt(2)*x/sqrt(-2**(3/4) + 2 -
 2**(3/4)*I))/(8*sqrt(-2**(3/4) + 2 - 2**(3/4)*I)) + (2**(1/4) -
2**(1/4)*I + 2*I)*atan(sqrt(2)*x/sqrt(-2**(3/4) + 2 + 2**(3/4)*I)
)/(8*sqrt(-2**(3/4) + 2 + 2**(3/4)*I)) - (2**(1/4) - 2*I - 2**(1/
4)*I)*atan(sqrt(2)*x/sqrt(2**(3/4) + 2 - 2**(3/4)*I))/(8*sqrt(2**
(3/4) + 2 - 2**(3/4)*I)) - (2**(1/4) + 2**(1/4)*I + 2*I)*atan(sqr
t(2)*x/sqrt(2**(3/4) + 2 + 2**(3/4)*I))/(8*sqrt(2**(3/4) + 2 + 2*
*(3/4)*I))
```

Mathematica [C] time = 0.0188428, size = 61, normalized size = 0.32

$$\frac{1}{8}\text{RootSum}\left[\#1^8+4\#1^6+6\#1^4+4\#1^2+3\&,\frac{\#1\log(x-\#1)}{\#1^6+3\#1^4+3\#1^2+1}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(2 + (1 + x^2)^4),x]
```

```
[Out] RootSum[3 + 4*#1^2 + 6*#1^4 + 4*#1^6 + #1^8 & , (Log[x - #1]*#1)/
(1 + 3*#1^2 + 3*#1^4 + #1^6) & ]/8
```

Maple [C] time = 0.009, size = 54, normalized size = 0.3

$$\frac{1}{8}\sum_{\_R=RootOf\left(\_Z^8+4\_Z^6+6\_Z^4+4\_Z^2+3\right)}\frac{\_R^2\ln\left(x-\_R\right)}{\_R^7+3\_R^5+3\_R^3+\_R}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(2+(x^2+1)^4),x)
```

```
[Out] 1/8*sum(_R^2/(_R^7+3*_R^5+3*_R^3+_R)*ln(x-_R),_R=RootOf(_Z^8+4*_Z
^6+6*_Z^4+4*_Z^2+3))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2}{\left(x^2+1\right)^4+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((x^2 + 1)^4 + 2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((x^2 + 1)^4 + 2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((x^2 + 1)^4 + 2),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 0.591718, size = 39, normalized size = 0.21

$$\text{RootSum}\left(1073741824t^8 + 65536t^4 + 1024t^2 + 3, \left(t \mapsto t\log\left(67108864t^7 - 262144t^5 + 4096t^3 + 40t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(2+(x**2+1)**4),x)
```

```
[Out] RootSum(1073741824*_t**8 + 65536*_t**4 + 1024*_t**2 + 3, Lambda(_
t, _t*log(67108864*_t**7 - 262144*_t**5 + 4096*_t**3 + 40*_t + x)
))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(x^2+1)^4+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((x^2 + 1)^4 + 2),x, algorithm="giac")
```

```
[Out] integrate(x^2/((x^2 + 1)^4 + 2), x)
```

3.390 $\int \frac{x^2}{2+(1-x^2)^4}\,dx$

Optimal. Leaf size=188

$$-\frac{\sqrt[4]{-1}\sqrt{1-\sqrt[4]{-2}}\tanh^{-1}\left(\frac{x}{\sqrt{1-\sqrt[4]{-2}}}\right)}{4\,2^{3/4}}+\frac{(-1)^{3/4}\sqrt{1+i\sqrt[4]{-2}}\tanh^{-1}\left(\frac{x}{\sqrt{1+i\sqrt[4]{-2}}}\right)}{4\,2^{3/4}}+\frac{\sqrt[4]{-1}\sqrt{1+\sqrt[4]{-2}}\tanh^{-1}\left(\frac{x}{\sqrt{1+\sqrt[4]{-2}}}\right)}{4\,2^{3/4}}-\frac{1}{8}i\left(\sqrt[4]{-2}+\sqrt{2}\right)\sqrt{\frac{1+i}{2^{3/4}+(1+i)}}\tanh^{-1}\left(\sqrt{\frac{1+i}{2^{3/4}+(1+i)}}x\right)$$

```
[Out] -((-1)^(1/4)*Sqrt[1 - (-2)^(1/4)]*ArcTanh[x/Sqrt[1 - (-2)^(1/4)]]
)/(4*2^(3/4)) + ((-1)^(3/4)*Sqrt[1 + I*(-2)^(1/4)]*ArcTanh[x/Sqrt
[1 + I*(-2)^(1/4)]])/(4*2^(3/4)) + ((-1)^(1/4)*Sqrt[1 + (-2)^(1/4
)]*ArcTanh[x/Sqrt[1 + (-2)^(1/4)]])/(4*2^(3/4)) - (I/8)*((-2)^(1/
4) + Sqrt[2])*Sqrt[(1 + I)/((1 + I) + 2^(3/4))]*ArcTanh[Sqrt[(1 +
 I)/((1 + I) + 2^(3/4))]*x]
```

Rubi [A] time = 0.379113, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294

$$-\frac{\sqrt[4]{-1}\sqrt{1-\sqrt[4]{-2}}\tanh^{-1}\left(\frac{x}{\sqrt{1-\sqrt[4]{-2}}}\right)}{4\,2^{3/4}}+\frac{(-1)^{3/4}\sqrt{1+i\sqrt[4]{-2}}\tanh^{-1}\left(\frac{x}{\sqrt{1+i\sqrt[4]{-2}}}\right)}{4\,2^{3/4}}+\frac{\sqrt[4]{-1}\sqrt{1+\sqrt[4]{-2}}\tanh^{-1}\left(\frac{x}{\sqrt{1+\sqrt[4]{-2}}}\right)}{4\,2^{3/4}}-\frac{1}{8}i\left(\sqrt[4]{-2}+\sqrt{2}\right)\sqrt{\frac{1+i}{2^{3/4}+(1+i)}}\tanh^{-1}\left(\sqrt{\frac{1+i}{2^{3/4}+(1+i)}}x\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(2 + (1 - x^2)^4),x]
```



```
[Out] -((-1)^(1/4)*Sqrt[1 - (-2)^(1/4)]*ArcTanh[x/Sqrt[1 - (-2)^(1/4)]]
)/(4*2^(3/4)) + ((-1)^(3/4)*Sqrt[1 + I*(-2)^(1/4)]*ArcTanh[x/Sqrt
[1 + I*(-2)^(1/4)]])/(4*2^(3/4)) + ((-1)^(1/4)*Sqrt[1 + (-2)^(1/4
)]*ArcTanh[x/Sqrt[1 + (-2)^(1/4)]])/(4*2^(3/4)) - (I/8)*((-2)^(1/
4) + Sqrt[2])*Sqrt[(1 + I)/((1 + I) + 2^(3/4))]*ArcTanh[Sqrt[(1 +
 I)/((1 + I) + 2^(3/4))]*x]
```

Rubi in Sympy [A] time = 86.2258, size = 236, normalized size = 1.26

$$-\frac{\left(\sqrt[4]{2}+\sqrt[4]{2}i+2i\right)\operatorname{atanh}\left(\frac{\sqrt{2}x}{\sqrt{-2^{\frac{3}{4}}+2-2^{\frac{3}{4}}i}}\right)}{8\sqrt{-2^{\frac{3}{4}}+2-2^{\frac{3}{4}}i}}-\frac{\left(\sqrt[4]{2}-2i-\sqrt[4]{2}i\right)\operatorname{atanh}\left(\frac{\sqrt{2}x}{\sqrt{-2^{\frac{3}{4}}+2+2^{\frac{3}{4}}i}}\right)}{8\sqrt{-2^{\frac{3}{4}}+2+2^{\frac{3}{4}}i}}+\frac{\left(\sqrt[4]{2}-\sqrt[4]{2}i+2i\right)\operatorname{atanh}\left(\frac{\sqrt{2}x}{\sqrt{2^{\frac{3}{4}}+2-2^{\frac{3}{4}}i}}\right)}{8\sqrt{2^{\frac{3}{4}}+2-2^{\frac{3}{4}}i}}+\frac{\left(\sqrt[4]{2}-2i+\sqrt[4]{2}i\right)\operatorname{atanh}\left(\frac{\sqrt{2}x}{\sqrt{2^{\frac{3}{4}}+2+2^{\frac{3}{4}}i}}\right)}{8\sqrt{2^{\frac{3}{4}}+2+2^{\frac{3}{4}}i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(2+(-x**2+1)**4),x)
```

```
[Out] -(2**(1/4) + 2**(1/4)*I + 2*I)*atanh(sqrt(2)*x/sqrt(-2**(3/4) + 2
 - 2**(3/4)*I))/(8*sqrt(-2**(3/4) + 2 - 2**(3/4)*I)) - (2**(1/4)
- 2*I - 2**(1/4)*I)*atanh(sqrt(2)*x/sqrt(-2**(3/4) + 2 + 2**(3/4)
*I))/(8*sqrt(-2**(3/4) + 2 + 2**(3/4)*I)) + (2**(1/4) - 2**(1/4)*
I + 2*I)*atanh(sqrt(2)*x/sqrt(2**(3/4) + 2 - 2**(3/4)*I))/(8*sqrt
(2**(3/4) + 2 - 2**(3/4)*I)) + (2**(1/4) - 2*I + 2**(1/4)*I)*atan
h(sqrt(2)*x/sqrt(2**(3/4) + 2 + 2**(3/4)*I))/(8*sqrt(2**(3/4) + 2
 + 2**(3/4)*I))
```

Mathematica [C] time = 0.0192255, size = 61, normalized size = 0.32

$$\frac{1}{8}\text{RootSum}\left[\#1^8-4\#1^6+6\#1^4-4\#1^2+3\&,\frac{\#1\log(x-\#1)}{\#1^6-3\#1^4+3\#1^2-1}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(2 + (1 - x^2)^4),x]
```

```
[Out] RootSum[3 - 4*#1^2 + 6*#1^4 - 4*#1^6 + #1^8 & , (Log[x - #1]*#1)/
(-1 + 3*#1^2 - 3*#1^4 + #1^6) & ]/8
```

Maple [C] time = 0.009, size = 56, normalized size = 0.3

$$\frac{1}{8}\sum_{\_R=RootOf(\_Z^8-4\_Z^6+6\_Z^4-4\_Z^2+3)}\frac{\_R^2\ln(x-\_R)}{\_R^7-3\_R^5+3\_R^3-\_R}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(2+(-x^2+1)^4),x)
```

```
[Out] 1/8*sum(_R^2/(_R^7-3*_R^5+3*_R^3-_R)*ln(x-_R),_R=RootOf(_Z^8-4*_Z
^6+6*_Z^4-4*_Z^2+3))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2}{(x^2-1)^4+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((x^2 - 1)^4 + 2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((x^2 - 1)^4 + 2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((x^2 - 1)^4 + 2),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 0.601099, size = 39, normalized size = 0.21

$$\text{RootSum}\left(1073741824t^8 + 65536t^4 - 1024t^2 + 3, \left(t \mapsto t\log\left(67108864t^7 + 262144t^5 + 4096t^3 - 40t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(2+(-x**2+1)**4),x)
```

```
[Out] RootSum(1073741824*_t**8 + 65536*_t**4 - 1024*_t**2 + 3, Lambda(_
t, _t*log(67108864*_t**7 + 262144*_t**5 + 4096*_t**3 - 40*_t + x)
))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(x^2-1)^4+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((x^2 - 1)^4 + 2),x, algorithm="giac")
```

```
[Out] integrate(x^2/((x^2 - 1)^4 + 2), x)
```

3.391 $\int \frac{1-x^2}{a+b(1-x^2)^4}\,dx$

Optimal. Leaf size=663

$$\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\log\left(-\sqrt{2}\sqrt[8]{b}x\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}+\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}x^2\right)}{8\sqrt{2}\sqrt{-a}b^{3/8}\sqrt{\sqrt{-a}+\sqrt{b}}}-\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\log\left(\sqrt{2}\sqrt[8]{b}x\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}+\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}x^2\right)}{8\sqrt{2}\sqrt{-a}b^{3/8}\sqrt{\sqrt{-a}+\sqrt{b}}}-\frac{\tan^{-1}\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}\right)}{4\sqrt{-a}b^{3/8}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}-\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}\tan^{-1}\left(\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}-\sqrt{2}\sqrt[8]{b}x}{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}}\right)}{4\sqrt{2}\sqrt{-a}b^{3/8}\sqrt{\sqrt{-a}+\sqrt{b}}}+\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}\tan^{-1}\left(\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}+\sqrt{2}\sqrt[8]{b}x}{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}}\right)}{4\sqrt{2}\sqrt{-a}b^{3/8}\sqrt{\sqrt{-a}+\sqrt{b}}}+\frac{\tanh^{-1}\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}\right)}{4\sqrt{-a}b^{3/8}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}$$

```
[Out] -ArcTan[(b^(1/8)*x)/Sqrt[(-a)^(1/4) - b^(1/4)]]/(4*Sqrt[-a]*Sqrt[
(-a)^(1/4) - b^(1/4)]*b^(3/8)) - (Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] -
 b^(1/4)]*ArcTan[(Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)] - Sqrt
[2]*b^(1/8)*x)/Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] - b^(1/4)]])/(4*Sqrt
[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^(3/8)) + (Sqrt[Sqrt[Sqrt[
-a] + Sqrt[b]] - b^(1/4)]*ArcTan[(Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] +
 b^(1/4)] + Sqrt[2]*b^(1/8)*x)/Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] - b^
(1/4)]])/(4*Sqrt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^(3/8)) +
ArcTanh[(b^(1/8)*x)/Sqrt[(-a)^(1/4) + b^(1/4)]]/(4*Sqrt[-a]*Sqrt[
(-a)^(1/4) + b^(1/4)]*b^(3/8)) + (Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] +
 b^(1/4)]*Log[Sqrt[Sqrt[-a] + Sqrt[b]] - Sqrt[2]*Sqrt[Sqrt[Sqrt[-
a] + Sqrt[b]] + b^(1/4)]*b^(1/8)*x + b^(1/4)*x^2])/(8*Sqrt[2]*Sqr
t[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^(3/8)) - (Sqrt[Sqrt[Sqrt[-a] + S
qrt[b]] + b^(1/4)]*Log[Sqrt[Sqrt[-a] + Sqrt[b]] + Sqrt[2]*Sqrt[Sq
rt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*b^(1/8)*x + b^(1/4)*x^2])/(8*Sq
rt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^(3/8))
```

Rubi [A] time = 3.07506, antiderivative size = 663, normalized size of antiderivative = 1., number of

steps used = 16, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$

$$\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\log\left(-\sqrt{2}\sqrt[8]{b}x\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}+\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}x^2\right)}{8\sqrt{2}\sqrt{-a}b^{3/8}\sqrt{\sqrt{-a}+\sqrt{b}}} - \frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\log\left(\sqrt{2}\sqrt[8]{b}x\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}+\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}x^2\right)}{8\sqrt{2}\sqrt{-a}b^{3/8}\sqrt{\sqrt{-a}+\sqrt{b}}} - \frac{\tan^{-1}\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}\right)}{4\sqrt{-a}b^{3/8}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}} - \frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}\tan^{-1}\left(\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}-\sqrt{2}\sqrt[8]{b}x}{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}}\right)}{4\sqrt{2}\sqrt{-a}b^{3/8}\sqrt{\sqrt{-a}+\sqrt{b}}} + \frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}\tan^{-1}\left(\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}+\sqrt{2}\sqrt[8]{b}x}{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}}\right)}{4\sqrt{2}\sqrt{-a}b^{3/8}\sqrt{\sqrt{-a}+\sqrt{b}}} + \frac{\tanh^{-1}\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}\right)}{4\sqrt{-a}b^{3/8}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - x^2)/(a + b*(1 - x^2)^4),x]
```

```
[Out] -ArcTan[(b^(1/8)*x)/Sqrt[(-a)^(1/4) - b^(1/4)]]/(4*Sqrt[-a]*Sqrt[
(-a)^(1/4) - b^(1/4)]*b^(3/8)) - (Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] -
 b^(1/4)]*ArcTan[(Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)] - Sqrt
[2]*b^(1/8)*x)/Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] - b^(1/4)]])/(4*Sqrt
[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^(3/8)) + (Sqrt[Sqrt[Sqrt[
-a] + Sqrt[b]] - b^(1/4)]*ArcTan[(Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] +
 b^(1/4)] + Sqrt[2]*b^(1/8)*x)/Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] - b^
(1/4)]])/(4*Sqrt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^(3/8)) +
ArcTanh[(b^(1/8)*x)/Sqrt[(-a)^(1/4) + b^(1/4)]]/(4*Sqrt[-a]*Sqrt[
(-a)^(1/4) + b^(1/4)]*b^(3/8)) + (Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] +
 b^(1/4)]*Log[Sqrt[Sqrt[-a] + Sqrt[b]] - Sqrt[2]*Sqrt[Sqrt[Sqrt[-
a] + Sqrt[b]] + b^(1/4)]*b^(1/8)*x + b^(1/4)*x^2])/(8*Sqrt[2]*Sqr
t[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^(3/8)) - (Sqrt[Sqrt[Sqrt[-a] + S
qrt[b]] + b^(1/4)]*Log[Sqrt[Sqrt[-a] + Sqrt[b]] + Sqrt[2]*Sqrt[Sq
rt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*b^(1/8)*x + b^(1/4)*x^2])/(8*Sq
rt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^(3/8))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(-\frac{-x^2+1}{a+b\left(-x^2+1\right)^4}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2+1)/(a+b*(-x**2+1)**4),x)
```

```
[Out] Integral(-(-x**2 + 1)/(a + b*(-x**2 + 1)**4), x)
```

Mathematica [C] time = 0.0530493, size = 63, normalized size = 0.1

$$-\frac{\text{RootSum}\left[\#1^8b-4\#1^6b+6\#1^4b-4\#1^2b+a+b\&,\frac{\log(x-\#1)}{\#1^5-2\#1^3+\#1}\&\right]}{8b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - x^2)/(a + b*(1 - x^2)^4),x]
```

```
[Out] -RootSum[a + b - 4*b*#1^2 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , Log[
x - #1]/(#1 - 2*#1^3 + #1^5) & ]/(8*b)
```

Maple [C] time = 0.095, size = 69, normalized size = 0.1

$$\frac{1}{8b}\sum_{\_R=RootOf\left(b\_Z^8-4b\_Z^6+6b\_Z^4-4b\_Z^2+a+b\right)}\frac{\left(-\_R^2+1\right)\ln\left(x-\_R\right)}{\_R^7-3\_R^5+3\_R^3-\_R}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2+1)/(a+b*(-x^2+1)^4),x)
```

```
[Out] 1/8/b*sum((-_R^2+1)/(_R^7-3*_R^5+3*_R^3-_R)*ln(x-_R),_R=RootOf(_Z
^8*b-4*_Z^6*b+6*_Z^4*b-4*_Z^2*b+a+b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2-1}{(x^2-1)^4 b + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 1)/((x^2 - 1)^4*b + a),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 1)/((x^2 - 1)^4*b + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 1)/((x^2 - 1)^4*b + a),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 11.2101, size = 133, normalized size = 0.2

$$-\operatorname{RootSum}\left(t^8\left(16777216a^5b^3+16777216a^4b^4\right)+1048576t^6a^3b^3+24576t^4a^2b^2+256t^2ab+1,\left(t\mapsto t\log\left(-6291456t^7a^4b\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+1)/(a+b*(-x**2+1)**4),x)
```

```
[Out] -RootSum(_t**8*(16777216*a**5*b**3 + 16777216*a**4*b**4) + 104857
6*_t**6*a**3*b**3 + 24576*_t**4*a**2*b**2 + 256*_t**2*a*b + 1, La
mbda(_t, _t*log(-6291456*_t**7*a**4*b**3 - 6291456*_t**7*a**3*b**
```

```
4 + 65536*_t**5*a**3*b**2 - 327680*_t**5*a**2*b**3 - 512*_t**3*a*
*2*b - 5632*_t**3*a*b**2 - 32*_t*b + x)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2-1}{(x^2-1)^4 b + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 1)/((x^2 - 1)^4*b + a),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 1)/((x^2 - 1)^4*b + a), x)
```

3.392 $\int \frac{1-x^2}{a+b(-1+x^2)^4}\,dx$

Optimal. Leaf size=663

$$\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\log\left(-\sqrt{2}\sqrt[8]{b}x\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}+\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}x^2\right)}{8\sqrt{2}\sqrt{-a}b^{3/8}\sqrt{\sqrt{-a}+\sqrt{b}}} - \frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\log\left(\sqrt{2}\sqrt[8]{b}x\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}+\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}x^2\right)}{8\sqrt{2}\sqrt{-a}b^{3/8}\sqrt{\sqrt{-a}+\sqrt{b}}} - \frac{\tan^{-1}\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}\right)}{4\sqrt{-a}b^{3/8}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}} - \frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}\tan^{-1}\left(\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}-\sqrt{2}\sqrt[8]{b}x}{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}}\right)}{4\sqrt{2}\sqrt{-a}b^{3/8}\sqrt{\sqrt{-a}+\sqrt{b}}} + \frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}\tan^{-1}\left(\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}+\sqrt{2}\sqrt[8]{b}x}{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}}\right)}{4\sqrt{2}\sqrt{-a}b^{3/8}\sqrt{\sqrt{-a}+\sqrt{b}}} + \frac{\tanh^{-1}\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}\right)}{4\sqrt{-a}b^{3/8}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}$$

```
[Out] -ArcTan[(b^(1/8)*x)/Sqrt[(-a)^(1/4) - b^(1/4)]]/(4*Sqrt[-a]*Sqrt[
(-a)^(1/4) - b^(1/4)]*b^(3/8)) - (Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] -
 b^(1/4)]*ArcTan[(Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)] - Sqrt
[2]*b^(1/8)*x)/Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] - b^(1/4)]])/(4*Sqrt
[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^(3/8)) + (Sqrt[Sqrt[Sqrt[
-a] + Sqrt[b]] - b^(1/4)]*ArcTan[(Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] +
 b^(1/4)] + Sqrt[2]*b^(1/8)*x)/Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] - b^
(1/4)]])/(4*Sqrt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^(3/8)) +
ArcTanh[(b^(1/8)*x)/Sqrt[(-a)^(1/4) + b^(1/4)]]/(4*Sqrt[-a]*Sqrt[
(-a)^(1/4) + b^(1/4)]*b^(3/8)) + (Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] +
 b^(1/4)]*Log[Sqrt[Sqrt[-a] + Sqrt[b]] - Sqrt[2]*Sqrt[Sqrt[Sqrt[-
a] + Sqrt[b]] + b^(1/4)]*b^(1/8)*x + b^(1/4)*x^2])/(8*Sqrt[2]*Sqr
t[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^(3/8)) - (Sqrt[Sqrt[Sqrt[-a] + S
qrt[b]] + b^(1/4)]*Log[Sqrt[Sqrt[-a] + Sqrt[b]] + Sqrt[2]*Sqrt[Sq
rt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*b^(1/8)*x + b^(1/4)*x^2])/(8*Sq
rt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^(3/8))
```

Rubi [A] time = 2.69441, antiderivative size = 663, normalized size of antiderivative = 1., number of

steps used = 17, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$

$$\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\log\left(-\sqrt{2}\sqrt[8]{b}x\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}+\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}x^2\right)}{8\sqrt{2}\sqrt{-a}b^{3/8}\sqrt{\sqrt{-a}+\sqrt{b}}}-\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\log\left(\sqrt{2}\sqrt[8]{b}x\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}+\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}x^2\right)}{8\sqrt{2}\sqrt{-a}b^{3/8}\sqrt{\sqrt{-a}+\sqrt{b}}}-\frac{\tan^{-1}\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}\right)}{4\sqrt{-a}b^{3/8}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}-\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}\tan^{-1}\left(\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}-\sqrt{2}\sqrt[8]{b}x}{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}}\right)}{4\sqrt{2}\sqrt{-a}b^{3/8}\sqrt{\sqrt{-a}+\sqrt{b}}}+\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}\tan^{-1}\left(\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}+\sqrt{2}\sqrt[8]{b}x}{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}}\right)}{4\sqrt{2}\sqrt{-a}b^{3/8}\sqrt{\sqrt{-a}+\sqrt{b}}}+\frac{\tanh^{-1}\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}\right)}{4\sqrt{-a}b^{3/8}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - x^2)/(a + b*(-1 + x^2)^4),x]
```

```
[Out] -ArcTan[(b^(1/8)*x)/Sqrt[(-a)^(1/4) - b^(1/4)]]/(4*Sqrt[-a]*Sqrt[
(-a)^(1/4) - b^(1/4)]*b^(3/8)) - (Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] -
 b^(1/4)]*ArcTan[(Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)] - Sqrt
[2]*b^(1/8)*x)/Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] - b^(1/4)]])/(4*Sqrt
[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^(3/8)) + (Sqrt[Sqrt[Sqrt[
-a] + Sqrt[b]] - b^(1/4)]*ArcTan[(Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] +
 b^(1/4)] + Sqrt[2]*b^(1/8)*x)/Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] - b^
(1/4)]])/(4*Sqrt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^(3/8)) +
ArcTanh[(b^(1/8)*x)/Sqrt[(-a)^(1/4) + b^(1/4)]]/(4*Sqrt[-a]*Sqrt[
(-a)^(1/4) + b^(1/4)]*b^(3/8)) + (Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] +
 b^(1/4)]*Log[Sqrt[Sqrt[-a] + Sqrt[b]] - Sqrt[2]*Sqrt[Sqrt[Sqrt[-
a] + Sqrt[b]] + b^(1/4)]*b^(1/8)*x + b^(1/4)*x^2])/(8*Sqrt[2]*Sqr
t[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^(3/8)) - (Sqrt[Sqrt[Sqrt[-a] + S
qrt[b]] + b^(1/4)]*Log[Sqrt[Sqrt[-a] + Sqrt[b]] + Sqrt[2]*Sqrt[Sq
rt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*b^(1/8)*x + b^(1/4)*x^2])/(8*Sq
rt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^(3/8))
```

Rubi in Sympy [A] time = 117.049, size = 216, normalized size = 0.33

$$-\frac{\operatorname{atanh}\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{b}+i\sqrt[4]{-a}}}\right)}{4b^{\frac{3}{8}}\sqrt{-a}\sqrt{\sqrt[4]{b}+i\sqrt[4]{-a}}}-\frac{\operatorname{atanh}\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{b}-i\sqrt[4]{-a}}}\right)}{4b^{\frac{3}{8}}\sqrt{-a}\sqrt{\sqrt[4]{b}-i\sqrt[4]{-a}}}+\frac{\operatorname{atanh}\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{b}+\sqrt[4]{-a}}}\right)}{4b^{\frac{3}{8}}\sqrt{-a}\sqrt{\sqrt[4]{b}+\sqrt[4]{-a}}}+\frac{\operatorname{atanh}\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{b}-\sqrt[4]{-a}}}\right)}{4b^{\frac{3}{8}}\sqrt{-a}\sqrt{\sqrt[4]{b}-\sqrt[4]{-a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2+1)/(a+b*(x**2-1)**4),x)
```

```
[Out] -atanh(b**(1/8)*x/sqrt(b**(1/4) + I*(-a)**(1/4)))/(4*b**(3/8)*sqr
t(-a)*sqrt(b**(1/4) + I*(-a)**(1/4))) - atanh(b**(1/8)*x/sqrt(b**
(1/4) - I*(-a)**(1/4)))/(4*b**(3/8)*sqrt(-a)*sqrt(b**(1/4) - I*(-
a)**(1/4))) + atanh(b**(1/8)*x/sqrt(b**(1/4) + (-a)**(1/4)))/(4*b
**(3/8)*sqrt(-a)*sqrt(b**(1/4) + (-a)**(1/4))) + atanh(b**(1/8)*x
/sqrt(b**(1/4) - (-a)**(1/4)))/(4*b**(3/8)*sqrt(-a)*sqrt(b**(1/4)
 - (-a)**(1/4)))
```

Mathematica [C] time = 0.0433942, size = 63, normalized size = 0.1

$$-\frac{\text{RootSum}\left[\#1^8b-4\#1^6b+6\#1^4b-4\#1^2b+a+b\&,\frac{\log(x-\#1)}{\#1^5-2\#1^3+\#1}\&\right]}{8b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - x^2)/(a + b*(-1 + x^2)^4),x]
```

```
[Out] -RootSum[a + b - 4*b*#1^2 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , Log[
x - #1]/(#1 - 2*#1^3 + #1^5) & ]/(8*b)
```

Maple [C] time = 0.002, size = 69, normalized size = 0.1

$$\frac{1}{8b}\sum_{\_R=RootOf(\_Z^8b-4\_Z^6b+6\_Z^4b-4\_Z^2b+a+b)}\frac{(-\_R^2+1)\ln(x-\_R)}{\_R^7-3\_R^5+3\_R^3-\_R}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2+1)/(a+b*(x^2-1)^4),x)
```

```
[Out] 1/8/b*sum((-_R^2+1)/(_R^7-3*_R^5+3*_R^3-_R)*ln(x-_R),_R=RootOf(_Z
^8*b-4*_Z^6*b+6*_Z^4*b-4*_Z^2*b+a+b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x^2-1}{(x^2-1)^4b+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 1)/((x^2 - 1)^4*b + a),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 1)/((x^2 - 1)^4*b + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 1)/((x^2 - 1)^4*b + a),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 11.2901, size = 133, normalized size = 0.2

$$-\text{RootSum}\left(t^8\left(16777216a^5b^3+16777216a^4b^4\right)+1048576t^6a^3b^3+24576t^4a^2b^2+256t^2ab+1,\left(t\mapsto t\log\left(-6291456t^7a^4b\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+1)/(a+b*(x**2-1)**4),x)
```

```
[Out] -RootSum(_t**8*(16777216*a**5*b**3 + 16777216*a**4*b**4) + 104857
6*_t**6*a**3*b**3 + 24576*_t**4*a**2*b**2 + 256*_t**2*a*b + 1, La
mbda(_t, _t*log(-6291456*_t**7*a**4*b**3 - 6291456*_t**7*a**3*b**
4 + 65536*_t**5*a**3*b**2 - 327680*_t**5*a**2*b**3 - 512*_t**3*a*
*2*b - 5632*_t**3*a*b**2 - 32*_t*b + x)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2-1}{(x^2-1)^4 b + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 1)/((x^2 - 1)^4*b + a),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 1)/((x^2 - 1)^4*b + a), x)
```

3.393 $\int \frac{(1+x^2)^2}{ax^6+b(1+x^2)^3}\,dx$

Optimal. Leaf size=168

$$\frac{\tan^{-1}\left(\frac{x\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}}{\sqrt[6]{b}}\right)}{3b^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}}+\frac{\tan^{-1}\left(\frac{x\sqrt{\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}}}{\sqrt[6]{b}}\right)}{3b^{5/6}\sqrt{\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}}}+\frac{\tan^{-1}\left(\frac{x\sqrt{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}}}{\sqrt[6]{b}}\right)}{3b^{5/6}\sqrt{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}}}$$

```
[Out] ArcTan[(Sqrt[a^(1/3) + b^(1/3)]*x)/b^(1/6)]/(3*Sqrt[a^(1/3) + b^(
1/3)]*b^(5/6)) + ArcTan[(Sqrt[-((-1)^(1/3)*a^(1/3)) + b^(1/3)]*x)
/b^(1/6)]/(3*Sqrt[-((-1)^(1/3)*a^(1/3)) + b^(1/3)]*b^(5/6)) + Arc
Tan[(Sqrt[(-1)^(2/3)*a^(1/3) + b^(1/3)]*x)/b^(1/6)]/(3*Sqrt[(-1)^
(2/3)*a^(1/3) + b^(1/3)]*b^(5/6))
```

Rubi [F] time = 0.675554, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$

$$\text{Int}\left(\frac{(1+x^2)^2}{ax^6+b(1+x^2)^3},x\right)$$

Verification is Not applicable to the result.

```
[In]  Int[(1 + x^2)^2/(a*x^6 + b*(1 + x^2)^3),x]
```

```
[Out] Defer[Int][(a*x^6 + b*(1 + x^2)^3)^(-1), x] + 2*Defer[Int][x^2/(a
*x^6 + b*(1 + x^2)^3), x] + Defer[Int][x^4/(a*x^6 + b*(1 + x^2)^3
), x]
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+1)**2/(a*x**6+b*(x**2+1)**3),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.0963126, size = 95, normalized size = 0.57

$$\frac{1}{6}\text{RootSum}\left[\#1^6a+\#1^6b+3\#1^4b+3\#1^2b+b\&,\frac{\#1^4\log(x-\#1)+2\#1^2\log(x-\#1)+\log(x-\#1)}{\#1^5a+\#1^5b+2\#1^3b+\#1b}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^2)^2/(a*x^6 + b*(1 + x^2)^3),x]
```

```
[Out] RootSum[b + 3*b*#1^2 + 3*b*#1^4 + a*#1^6 + b*#1^6 & , (Log[x - #1
] + 2*Log[x - #1]*#1^2 + Log[x - #1]*#1^4)/(b*#1 + 2*b*#1^3 + a*#
1^5 + b*#1^5) & ]/6
```

Maple [C] time = 0.235, size = 67, normalized size = 0.4

$$\frac{1}{6}\sum_{\_R=RootOf\left((a+b)\_Z^6+3b\_Z^4+3b\_Z^2+b\right)}\frac{\left(\_R^4+2\_R^2+1\right)\ln\left(x-\_R\right)}{\_R^5a+\_R^5b+2\_R^3b+\_Rb}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+1)^2/(a*x^6+b*(x^2+1)^3),x)
```

```
[Out] 1/6*sum((_R^4+2*_R^2+1)/(_R^5*a+_R^5*b+2*_R^3*b+_R*b)*ln(x-_R),_R
=RootOf((a+b)*_Z^6+3*b*_Z^4+3*b*_Z^2+b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(x^2+1)^2}{ax^6+(x^2+1)^3b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)^2/(a*x^6 + (x^2 + 1)^3*b),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 1)^2/(a*x^6 + (x^2 + 1)^3*b), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)^2/(a*x^6 + (x^2 + 1)^3*b),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 6.13289, size = 42, normalized size = 0.25

$$\operatorname{RootSum}\left(t^6\left(46656ab^5+46656b^6\right)+3888t^4b^4+108t^2b^2+1,\left(t\mapsto t\log\left(6tb+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+1)**2/(a*x**6+b*(x**2+1)**3),x)
```

```
[Out] RootSum(_t**6*(46656*a*b**5 + 46656*b**6) + 3888*_t**4*b**4 + 108
*_t**2*b**2 + 1, Lambda(_t, _t*log(6*_t*b + x)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(x^2+1)^2}{ax^6+(x^2+1)^3b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)^2/(a*x^6 + (x^2 + 1)^3*b),x, algorithm="giac")

[Out] integrate((x^2 + 1)^2/(a*x^6 + (x^2 + 1)^3*b), x)
```

3.394 $\int \frac{(d+ex)^3}{a+cx^4} dx$

Optimal. Leaf size=320

$$-\frac{d\left(\sqrt{c}d^2-3\sqrt{a}e^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}}+\frac{d\left(\sqrt{c}d^2-3\sqrt{a}e^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}}-\frac{d\left(3\sqrt{a}e^2+\sqrt{c}d^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{3/4}}+\frac{d\left(3\sqrt{a}e^2+\sqrt{c}d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}c^{3/4}}+\frac{3d^2e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{c}}+\frac{e^3\log\left(a+cx^4\right)}{4c}$$

```
[Out] (3*d^2*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[c]) - (d*
(Sqrt[c]*d^2 + 3*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1
/4)])/(2*Sqrt[2]*a^(3/4)*c^(3/4)) + (d*(Sqrt[c]*d^2 + 3*Sqrt[a]*e
^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c
^(3/4)) - (d*(Sqrt[c]*d^2 - 3*Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4)) + (
d*(Sqrt[c]*d^2 - 3*Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(
1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4)) + (e^3*Log[a +
 c*x^4])/(4*c)
```

Rubi [A] time = 0.583225, antiderivative size = 320, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.588

$$-\frac{d\left(\sqrt{c}d^2-3\sqrt{a}e^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}}+\frac{d\left(\sqrt{c}d^2-3\sqrt{a}e^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}}-\frac{d\left(3\sqrt{a}e^2+\sqrt{c}d^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{3/4}}+\frac{d\left(3\sqrt{a}e^2+\sqrt{c}d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}c^{3/4}}+\frac{3d^2e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{c}}+\frac{e^3\log\left(a+cx^4\right)}{4c}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x)^3/(a + c*x^4),x]
```

```
[Out] (3*d^2*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[c]) - (d*
(Sqrt[c]*d^2 + 3*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1
/4)])/(2*Sqrt[2]*a^(3/4)*c^(3/4)) + (d*(Sqrt[c]*d^2 + 3*Sqrt[a]*e
^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c
^(3/4)) - (d*(Sqrt[c]*d^2 - 3*Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4)) + (
d*(Sqrt[c]*d^2 - 3*Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(
1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4)) + (e^3*Log[a +
 c*x^4])/(4*c)
```

Rubi in Sympy [A] time = 85.5033, size = 306, normalized size = 0.96

$$\frac{e^3\log\left(a+cx^4\right)}{4c}+\frac{3d^2e\operatorname{atan}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{c}}+\frac{\sqrt{2}d\left(3\sqrt{a}e^2-\sqrt{c}d^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}c^{\frac{3}{4}}x+\sqrt{a}\sqrt{c}+cx^2\right)}{8a^{\frac{3}{4}}c^{\frac{3}{4}}}
-\frac{\sqrt{2}d\left(3\sqrt{a}e^2-\sqrt{c}d^2\right)\log\left(\sqrt{2}\sqrt[4]{a}c^{\frac{3}{4}}x+\sqrt{a}\sqrt{c}+cx^2\right)}{8a^{\frac{3}{4}}c^{\frac{3}{4}}}
-\frac{\sqrt{2}d\left(3\sqrt{a}e^2+\sqrt{c}d^2\right)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}d\left(3\sqrt{a}e^2+\sqrt{c}d^2\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}c^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x+d)**3/(c*x**4+a),x)
```

```
[Out] e**3*log(a + c*x**4)/(4*c) + 3*d**2*e*atan(sqrt(c)*x**2/sqrt(a))/
(2*sqrt(a)*sqrt(c)) + sqrt(2)*d*(3*sqrt(a)*e**2 - sqrt(c)*d**2)*l
og(-sqrt(2)*a**(1/4)*c**(3/4)*x + sqrt(a)*sqrt(c) + c*x**2)/(8*a*
*(3/4)*c**(3/4)) - sqrt(2)*d*(3*sqrt(a)*e**2 - sqrt(c)*d**2)*log(
sqrt(2)*a**(1/4)*c**(3/4)*x + sqrt(a)*sqrt(c) + c*x**2)/(8*a**(3/
4)*c**(3/4)) - sqrt(2)*d*(3*sqrt(a)*e**2 + sqrt(c)*d**2)*atan(1 -
 sqrt(2)*c**(1/4)*x/a**(1/4))/(4*a**(3/4)*c**(3/4)) + sqrt(2)*d*(
3*sqrt(a)*e**2 + sqrt(c)*d**2)*atan(1 + sqrt(2)*c**(1/4)*x/a**(1/
4))/(4*a**(3/4)*c**(3/4))
```

Mathematica [A] time = 0.541466, size = 322, normalized size = 1.01

$$\frac{-\sqrt{2}\sqrt[4]{c}\left(\sqrt[4]{a}\sqrt{c}d^3-3a^{3/4}de^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)+\sqrt{2}\sqrt[4]{c}\left(\sqrt[4]{a}\sqrt{c}d^3-3a^{3/4}de^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x)^3/(a + c*x^4),x]
```

```
[Out] (-2*a^(1/4)*c^(1/4)*d*(Sqrt[2]*Sqrt[c]*d^2 + 6*a^(1/4)*c^(1/4)*d*
e + 3*Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)
] + 2*a^(1/4)*c^(1/4)*d*(Sqrt[2]*Sqrt[c]*d^2 - 6*a^(1/4)*c^(1/4)*
d*e + 3*Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/
4)] - Sqrt[2]*c^(1/4)*(a^(1/4)*Sqrt[c]*d^3 - 3*a^(3/4)*d*e^2)*Log
[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + Sqrt[2]*c^(
1/4)*(a^(1/4)*Sqrt[c]*d^3 - 3*a^(3/4)*d*e^2)*Log[Sqrt[a] + Sqrt[2
]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + 2*a*e^3*Log[a + c*x^4])/(8*a
*c)
```

Maple [A] time = 0.011, size = 314, normalized size = 1.

$$\frac{d^3\sqrt{2}}{8\,a}\sqrt[4]{\frac{a}{c}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^2-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)$$
$$+\frac{d^3\sqrt{2}}{4\,a}\sqrt[4]{\frac{a}{c}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}+1\right)+\frac{d^3\sqrt{2}}{4\,a}\sqrt[4]{\frac{a}{c}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}-1\right)+\frac{3\,d^2e}{2}\arctan\left(x^2\sqrt{\frac{c}{a}}\right)\frac{1}{\sqrt{ac}}$$
$$+\frac{3\,e^2d\sqrt{2}}{8\,c}\ln\left(1\left(x^2-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{c}}}$$
$$+\frac{3\,e^2d\sqrt{2}}{4\,c}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{c}}}+\frac{3\,e^2d\sqrt{2}}{4\,c}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{c}}}+\frac{e^3\ln\left(cx^4+a\right)}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x+d)^3/(c*x^4+a),x)
```

```
[Out] 1/8*d^3*(1/c*a)^(1/4)/a*2^(1/2)*ln((x^2+(1/c*a)^(1/4)*x*2^(1/2)+(
1/c*a)^(1/2))/(x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))+1/4*d^
3*(1/c*a)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)+1/4*d
^3*(1/c*a)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)+3/2*
d^2*e/(a*c)^(1/2)*arctan(x^2*(c/a)^(1/2))+3/8*e^2*d/c/(1/c*a)^(1/
4)*2^(1/2)*ln((x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2+(1
/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))+3/4*e^2*d/c/(1/c*a)^(1/4)*2
^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)+3/4*e^2*d/c/(1/c*a)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)+1/4*e^3*ln(c*x^4+a)/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)^3/(c*x^4 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)^3/(c*x^4 + a),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 13.5301, size = 384, normalized size = 1.2

$$\operatorname{RootSum}\left(256t^4a^3c^4-256t^3a^3c^3e^3+t^2\left(96a^3c^2e^6+480a^2c^3d^4e^2\right)+t\left(-16a^3ce^9+192a^2c^2d^4e^5-48ac^3d^8e\right)+a^3e^{12}+3a^2c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x+d)**3/(c*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*c**4 - 256*_t**3*a**3*c**3*e**3 + _t**2*(9
6*a**3*c**2*e**6 + 480*a**2*c**3*d**4*e**2) + _t*(-16*a**3*c*e**9
 + 192*a**2*c**2*d**4*e**5 - 48*a*c**3*d**8*e) + a**3*e**12 + 3*a
**2*c*d**4*e**8 + 3*a*c**2*d**8*e**4 + c**3*d**12, Lambda(_t, _t*
log(x + (1728*_t**3*a**4*c**3*e**6 + 960*_t**3*a**3*c**4*d**4*e**
2 - 1296*_t**2*a**4*c**2*e**9 - 2016*_t**2*a**3*c**3*d**4*e**5 +
48*_t**2*a**2*c**4*d**8*e + 324*_t*a**4*c*e**12 + 4716*_t*a**3*c*
*2*d**4*e**8 + 1452*_t*a**2*c**3*d**8*e**4 + 4*_t*a*c**4*d**12 -
27*a**4*e**15 + 1119*a**3*c*d**4*e**11 - 609*a**2*c**2*d**8*e**7
- 91*a*c**3*d**12*e**3)/(729*a**3*c*d**3*e**12 - 1053*a**2*c**2*d
**7*e**8 - 117*a*c**3*d**11*e**4 + c**4*d**15))))
```

GIAC/XCAS [A] time = 0.273609, size = 420, normalized size = 1.31

$$\frac{e^3\ln\left(\left|cx^4+a\right|\right)}{4\,c}+\frac{\sqrt{2}\left(3\,\sqrt{2}\sqrt{ac}c^2d^2e+\left(ac^3\right)^{\frac{1}{4}}c^2d^3+3\,\left(ac^3\right)^{\frac{3}{4}}de^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^3}+\frac{\sqrt{2}\left(3\,\sqrt{2}\sqrt{ac}c^2d^2e+\left(ac^3\right)^{\frac{1}{4}}c^2d^3+3\,\left(ac^3\right)^{\frac{3}{4}}de^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^3}+\frac{\sqrt{2}\left(\left(ac^3\right)^{\frac{1}{4}}c^2d^3-3\,\left(ac^3\right)^{\frac{3}{4}}de^2\right)\ln\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,ac^3}-\frac{\sqrt{2}\left(\left(ac^3\right)^{\frac{1}{4}}c^2d^3-3\,\left(ac^3\right)^{\frac{3}{4}}de^2\right)\ln\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)^3/(c*x^4 + a),x, algorithm="giac")
```

```
[Out] 1/4*e^3*ln(abs(c*x^4 + a))/c + 1/4*sqrt(2)*(3*sqrt(2)*sqrt(a*c)*c
^2*d^2*e + (a*c^3)^(1/4)*c^2*d^3 + 3*(a*c^3)^(3/4)*d*e^2)*arctan(
1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^3) + 1/
4*sqrt(2)*(3*sqrt(2)*sqrt(a*c)*c^2*d^2*e + (a*c^3)^(1/4)*c^2*d^3
+ 3*(a*c^3)^(3/4)*d*e^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^
(1/4))/(a/c)^(1/4))/(a*c^3) + 1/8*sqrt(2)*((a*c^3)^(1/4)*c^2*d^3
- 3*(a*c^3)^(3/4)*d*e^2)*ln(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/
c))/(a*c^3) - 1/8*sqrt(2)*((a*c^3)^(1/4)*c^2*d^3 - 3*(a*c^3)^(3/4
)*d*e^2)*ln(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^3)
```

3.395 $\int \frac{(d+ex)^2}{a+cx^4}\,dx$

Optimal. Leaf size=291

$$-\frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}}+\frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}}-\frac{\left(\sqrt{a}e^2+\sqrt{c}d^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{3/4}}+\frac{\left(\sqrt{a}e^2+\sqrt{c}d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}c^{3/4}}+\frac{de\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}}$$

```
[Out] (d*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(Sqrt[a]*Sqrt[c]) - ((Sqrt[c]
*d^2 + Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*S
qrt[2]*a^(3/4)*c^(3/4)) + ((Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 +
 (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(3/4)) - ((Sq
rt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x
+ Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4)) + ((Sqrt[c]*d^2 - Sqr
t[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])
/(4*Sqrt[2]*a^(3/4)*c^(3/4))
```

Rubi [A] time = 0.46707, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529

$$-\frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}}+\frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}}-\frac{\left(\sqrt{a}e^2+\sqrt{c}d^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{3/4}}+\frac{\left(\sqrt{a}e^2+\sqrt{c}d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}c^{3/4}}+\frac{de\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x)^2/(a + c*x^4),x]
```

```
[Out] (d*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(Sqrt[a]*Sqrt[c]) - ((Sqrt[c]
*d^2 + Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*S
qrt[2]*a^(3/4)*c^(3/4)) + ((Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 +
 (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(3/4)) - ((Sq
rt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x
+ Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4)) + ((Sqrt[c]*d^2 - Sqr
t[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])
/(4*Sqrt[2]*a^(3/4)*c^(3/4))
```

Rubi in Sympy [A] time = 79.4004, size = 272, normalized size = 0.93

$$\frac{de\,\mathrm{atan}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}}+\frac{\sqrt{2}\left(\sqrt{a}e^2-\sqrt{c}d^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}c^{\frac{3}{4}}x+\sqrt{a}\sqrt{c}+cx^2\right)}{8a^{\frac{3}{4}}c^{\frac{3}{4}}}-\frac{\sqrt{2}\left(\sqrt{a}e^2-\sqrt{c}d^2\right)\log\left(\sqrt{2}\sqrt[4]{a}c^{\frac{3}{4}}x+\sqrt{a}\sqrt{c}+cx^2\right)}{8a^{\frac{3}{4}}c^{\frac{3}{4}}}-\frac{\sqrt{2}\left(\sqrt{a}e^2+\sqrt{c}d^2\right)\mathrm{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\left(\sqrt{a}e^2+\sqrt{c}d^2\right)\mathrm{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}c^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x+d)**2/(c*x**4+a),x)
```

```
[Out] d*e*atan(sqrt(c)*x**2/sqrt(a))/(sqrt(a)*sqrt(c)) + sqrt(2)*(sqrt(
a)*e**2 - sqrt(c)*d**2)*log(-sqrt(2)*a**(1/4)*c**(3/4)*x + sqrt(a
)*sqrt(c) + c*x**2)/(8*a**(3/4)*c**(3/4)) - sqrt(2)*(sqrt(a)*e**2
 - sqrt(c)*d**2)*log(sqrt(2)*a**(1/4)*c**(3/4)*x + sqrt(a)*sqrt(c
) + c*x**2)/(8*a**(3/4)*c**(3/4)) - sqrt(2)*(sqrt(a)*e**2 + sqrt(
c)*d**2)*atan(1 - sqrt(2)*c**(1/4)*x/a**(1/4))/(4*a**(3/4)*c**(3/
4)) + sqrt(2)*(sqrt(a)*e**2 + sqrt(c)*d**2)*atan(1 + sqrt(2)*c**(
1/4)*x/a**(1/4))/(4*a**(3/4)*c**(3/4))
```

Mathematica [A] time = 0.198703, size = 243, normalized size = 0.84

$$\frac{-\sqrt{2}\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)-\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)\right)-2\left(4\sqrt[4]{a}\sqrt[4]{c}de+\sqrt{2}\sqrt{a}e^2+\sqrt{2}\sqrt{c}d^2\right.}{8a^{3/4}c^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x)^2/(a + c*x^4),x]
```

```
[Out] (-2*(Sqrt[2]*Sqrt[c]*d^2 + 4*a^(1/4)*c^(1/4)*d*e + Sqrt[2]*Sqrt[a
]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*(Sqrt[2]*Sqrt[
c]*d^2 - 4*a^(1/4)*c^(1/4)*d*e + Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 +
(Sqrt[2]*c^(1/4)*x)/a^(1/4)] - Sqrt[2]*(Sqrt[c]*d^2 - Sqrt[a]*e^2
)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - Log[S
qrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]))/(8*a^(3/4)*c^
(3/4))
```

Maple [A] time = 0.005, size = 292, normalized size = 1.

$$\frac{d^2\sqrt{2}}{8\,a}\sqrt[4]{\frac{a}{c}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^2-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)$$
$$+\frac{d^2\sqrt{2}}{4\,a}\sqrt[4]{\frac{a}{c}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}+1\right)+\frac{d^2\sqrt{2}}{4\,a}\sqrt[4]{\frac{a}{c}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}-1\right)$$
$$+de\arctan\left(x^2\sqrt{\frac{c}{a}}\right)\frac{1}{\sqrt{ac}}+\frac{e^2\sqrt{2}}{8\,c}\ln\left(1\left(x^2-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{c}}}$$
$$+\frac{e^2\sqrt{2}}{4\,c}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{c}}}+\frac{e^2\sqrt{2}}{4\,c}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x+d)^2/(c*x^4+a),x)
```

```
[Out] 1/8*d^2*(1/c*a)^(1/4)/a*2^(1/2)*ln((x^2+(1/c*a)^(1/4)*x*2^(1/2)+(
1/c*a)^(1/2))/(x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))+1/4*d^
2*(1/c*a)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)+1/4*d
^2*(1/c*a)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)+d*e/
(a*c)^(1/2)*arctan(x^2*(c/a)^(1/2))+1/8*e^2/c/(1/c*a)^(1/4)*2^(1/
2)*ln((x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2+(1/c*a)^(1
/4)*x*2^(1/2)+(1/c*a)^(1/2)))+1/4*e^2/c/(1/c*a)^(1/4)*2^(1/2)*arc
tan(2^(1/2)/(1/c*a)^(1/4)*x+1)+1/4*e^2/c/(1/c*a)^(1/4)*2^(1/2)*ar
ctan(2^(1/2)/(1/c*a)^(1/4)*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)^2/(c*x^4 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)^2/(c*x^4 + a),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 10.0858, size = 277, normalized size = 0.95

$$\operatorname{RootSum}\left(256t^4a^3c^3+192t^2a^2c^2d^2e^2+t\left(32a^2cde^5-32ac^2d^5e\right)+a^2e^8+2acd^4e^4+c^2d^8,\left(t\mapsto t\log\left(x+\frac{64t^3a^4c^2e^6+448\cdots}{\cdots}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x+d)**2/(c*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*c**3 + 192*_t**2*a**2*c**2*d**2*e**2 + _t*
(32*a**2*c*d*e**5 - 32*a*c**2*d**5*e) + a**2*e**8 + 2*a*c*d**4*e*
*4 + c**2*d**8, Lambda(_t, _t*log(x + (64*_t**3*a**4*c**2*e**6 +
448*_t**3*a**3*c**3*d**4*e**2 - 160*_t**2*a**3*c**2*d**3*e**5 + 3
2*_t**2*a**2*c**3*d**7*e + 60*_t*a**3*c*d**2*e**8 + 256*_t*a**2*c
**2*d**6*e**4 + 4*_t*a*c**3*d**10 + 6*a**3*d*e**11 - 24*a**2*c*d*
*5*e**7 - 30*a*c**2*d**9*e**3)/(a**3*e**12 - 33*a**2*c*d**4*e**8
- 33*a*c**2*d**8*e**4 + c**3*d**12))))
```

GIAC/XCAS [A] time = 0.271084, size = 385, normalized size = 1.32

$$\frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ac}c^2de+\left(ac^3\right)^{\frac14}c^2d^2+\left(ac^3\right)^{\frac34}e^2\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac14}\right)}{2\left(\frac{a}{c}\right)^{\frac14}}\right)}{4ac^3}+\frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ac}c^2de+\left(ac^3\right)^{\frac14}c^2d^2+\left(ac^3\right)^{\frac34}e^2\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac14}\right)}{2\left(\frac{a}{c}\right)^{\frac14}}\right)}{4ac^3}+\frac{\sqrt{2}\left(\left(ac^3\right)^{\frac14}c^2d^2-\left(ac^3\right)^{\frac34}e^2\right)\ln\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac14}+\sqrt{\frac{a}{c}}\right)}{8ac^3}-\frac{\sqrt{2}\left(\left(ac^3\right)^{\frac14}c^2d^2-\left(ac^3\right)^{\frac34}e^2\right)\ln\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac14}+\sqrt{\frac{a}{c}}\right)}{8ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)^2/(c*x^4 + a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*(2*sqrt(2)*sqrt(a*c)*c^2*d*e + (a*c^3)^(1/4)*c^2*d^2
+ (a*c^3)^(3/4)*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4
))/(a/c)^(1/4))/(a*c^3) + 1/4*sqrt(2)*(2*sqrt(2)*sqrt(a*c)*c^2*d*
e + (a*c^3)^(1/4)*c^2*d^2 + (a*c^3)^(3/4)*e^2)*arctan(1/2*sqrt(2)
*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^3) + 1/8*sqrt(2)*(
(a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(3/4)*e^2)*ln(x^2 + sqrt(2)*x*(a/
c)^(1/4) + sqrt(a/c))/(a*c^3) - 1/8*sqrt(2)*((a*c^3)^(1/4)*c^2*d^
2 - (a*c^3)^(3/4)*e^2)*ln(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c)
)/(a*c^3)
```

3.396 $\int \frac{d+ex}{a+cx^4} \, dx$

Optimal. Leaf size=219

$$-\frac{d \log \left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{d \log \left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}-\frac{d \tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{d \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{e \tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{c}}$$

```
[Out] (e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[c]) - (d*ArcTan
[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(1/4)) +
(d*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^
(1/4)) - (d*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2
])/(4*Sqrt[2]*a^(3/4)*c^(1/4)) + (d*Log[Sqrt[a] + Sqrt[2]*a^(1/4)
*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(1/4))
```

Rubi [A] time = 0.35954, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.6$

$$-\frac{d \log \left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{d \log \left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}-\frac{d \tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{d \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{e \tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x)/(a + c*x^4),x]
```

```
[Out] (e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[c]) - (d*ArcTan
[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(1/4)) +
(d*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^
(1/4)) - (d*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2
])/(4*Sqrt[2]*a^(3/4)*c^(1/4)) + (d*Log[Sqrt[a] + Sqrt[2]*a^(1/4)
*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(1/4))
```

Rubi in Sympy [A] time = 60.1606, size = 207, normalized size = 0.95

$$\frac{e \operatorname{atan}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{c}}-\frac{\sqrt{2}d \log \left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{8a^{\frac{3}{4}}\sqrt[4]{c}}+\frac{\sqrt{2}d \log \left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{8a^{\frac{3}{4}}\sqrt[4]{c}}-\frac{\sqrt{2}d \operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}\sqrt[4]{c}}+\frac{\sqrt{2}d \operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}\sqrt[4]{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x+d)/(c*x**4+a),x)
```

```
[Out] e*atan(sqrt(c)*x**2/sqrt(a))/(2*sqrt(a)*sqrt(c)) - sqrt(2)*d*log(
-sqrt(2)*a**(1/4)*c**(1/4)*x + sqrt(a) + sqrt(c)*x**2)/(8*a**(3/4
)*c**(1/4)) + sqrt(2)*d*log(sqrt(2)*a**(1/4)*c**(1/4)*x + sqrt(a)
 + sqrt(c)*x**2)/(8*a**(3/4)*c**(1/4)) - sqrt(2)*d*atan(1 - sqrt(
2)*c**(1/4)*x/a**(1/4))/(4*a**(3/4)*c**(1/4)) + sqrt(2)*d*atan(1
```

```
+ sqrt(2)*c**(1/4)*x/a**(1/4))/(4*a**(3/4)*c**(1/4))
```

Mathematica [A] time = 0.101186, size = 184, normalized size = 0.84

$$\frac{-2\left(2\sqrt[4]{a}e+\sqrt{2}\sqrt[4]{c}d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+2\left(\sqrt{2}\sqrt[4]{c}d-2\sqrt[4]{a}e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)+\sqrt{2}\sqrt[4]{c}d\left(\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)\right.}{8a^{3/4}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x)/(a + c*x^4),x]
```

```
[Out] (-2*(Sqrt[2]*c^(1/4)*d + 2*a^(1/4)*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)
*x)/a^(1/4)] + 2*(Sqrt[2]*c^(1/4)*d - 2*a^(1/4)*e)*ArcTan[1 + (Sq
rt[2]*c^(1/4)*x)/a^(1/4)] + Sqrt[2]*c^(1/4)*d*(-Log[Sqrt[a] - Sqr
t[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + Log[Sqrt[a] + Sqrt[2]*a^(
1/4)*c^(1/4)*x + Sqrt[c]*x^2]))/(8*a^(3/4)*Sqrt[c])
```

Maple [A] time = 0.005, size = 151, normalized size = 0.7

$$\frac{d\sqrt{2}}{8a}\sqrt[4]{\frac{a}{c}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^2-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)$$
$$+\frac{d\sqrt{2}}{4a}\sqrt[4]{\frac{a}{c}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}+1\right)+\frac{d\sqrt{2}}{4a}\sqrt[4]{\frac{a}{c}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}-1\right)+\frac{e}{2}\arctan\left(x^2\sqrt{\frac{c}{a}}\right)\frac{1}{\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x+d)/(c*x^4+a),x)
```

```
[Out] 1/8*d*(1/c*a)^(1/4)/a*2^(1/2)*ln((x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/
c*a)^(1/2))/(x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))+1/4*d*(1
/c*a)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)+1/4*d*(1/
c*a)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)+1/2*e/(a*c
)^(1/2)*arctan(x^2*(c/a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)/(c*x^4 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)/(c*x^4 + a),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 2.91482, size = 124, normalized size = 0.57

$$\operatorname{RootSum}\left(256t^4a^3c^2 + 32t^2a^2ce^2 - 16tacd^2e + ae^4 + cd^4, \left(t \mapsto t\log\left(x + \frac{-128t^3a^3ce^2 - 16t^2a^2cd^2e - 8ta^2e^4 - 4tacd^4 + 5}{4ade^4 - cd^5}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x+d)/(c*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*c**2 + 32*_t**2*a**2*c*e**2 - 16*_t*a*c*d*
*2*e + a*e**4 + c*d**4, Lambda(_t, _t*log(x + (-128*_t**3*a**3*c*
e**2 - 16*_t**2*a**2*c*d**2*e - 8*_t*a**2*e**4 - 4*_t*a*c*d**4 +
5*a*d**2*e**3)/(4*a*d*e**4 - c*d**5))))
```

GIAC/XCAS [A] time = 0.268959, size = 290, normalized size = 1.32

$$\frac{\sqrt{2}\left(ac^3\right)^{\frac{1}{4}} d\ln\left(x^2 + \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{8\,ac} - \frac{\sqrt{2}\left(ac^3\right)^{\frac{1}{4}} d\ln\left(x^2 - \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{8\,ac} - \frac{\sqrt{2}\left(\sqrt{2}\sqrt{ac}ce - \left(ac^3\right)^{\frac{1}{4}} cd\right)\arctan\left(\frac{\sqrt{2}\left(2\,x + \sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^2} - \frac{\sqrt{2}\left(\sqrt{2}\sqrt{ac}ce - \left(ac^3\right)^{\frac{1}{4}} cd\right)\arctan\left(\frac{\sqrt{2}\left(2\,x - \sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)/(c*x^4 + a),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(a*c^3)^(1/4)*d*ln(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt
(a/c))/(a*c) - 1/8*sqrt(2)*(a*c^3)^(1/4)*d*ln(x^2 - sqrt(2)*x*(a/
c)^(1/4) + sqrt(a/c))/(a*c) - 1/4*sqrt(2)*(sqrt(2)*sqrt(a*c)*c*e
- (a*c^3)^(1/4)*c*d)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4
))/(a/c)^(1/4))/(a*c^2) - 1/4*sqrt(2)*(sqrt(2)*sqrt(a*c)*c*e - (a
*c^3)^(1/4)*c*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(
a/c)^(1/4))/(a*c^2)
```

3.397 $\int \frac{1}{a+cx^4} dx$

Optimal. Leaf size=185

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}$$

```
[Out] -ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*c^(1/
4)) + ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*
c^(1/4)) - Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]
/(4*Sqrt[2]*a^(3/4)*c^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1
/4)*x + Sqrt[c]*x^2]/(4*Sqrt[2]*a^(3/4)*c^(1/4))
```

Rubi [A] time = 0.226112, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + c*x^4)^(-1),x]
```

```
[Out] -ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*c^(1/
4)) + ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*
c^(1/4)) - Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]
/(4*Sqrt[2]*a^(3/4)*c^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1
/4)*x + Sqrt[c]*x^2]/(4*Sqrt[2]*a^(3/4)*c^(1/4))
```

Rubi in Sympy [A] time = 47.6741, size = 172, normalized size = 0.93

$$-\frac{\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{8a^{\frac{3}{4}}\sqrt[4]{c}}+\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{8a^{\frac{3}{4}}\sqrt[4]{c}}-\frac{\sqrt{2}\,\mathrm{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}\sqrt[4]{c}}+\frac{\sqrt{2}\,\mathrm{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}\sqrt[4]{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c*x**4+a),x)
```

```
[Out] -sqrt(2)*log(-sqrt(2)*a**(1/4)*c**(1/4)*x + sqrt(a) + sqrt(c)*x**
2)/(8*a**(3/4)*c**(1/4)) + sqrt(2)*log(sqrt(2)*a**(1/4)*c**(1/4)*
x + sqrt(a) + sqrt(c)*x**2)/(8*a**(3/4)*c**(1/4)) - sqrt(2)*atan(
1 - sqrt(2)*c**(1/4)*x/a**(1/4))/(4*a**(3/4)*c**(1/4)) + sqrt(2)*
atan(1 + sqrt(2)*c**(1/4)*x/a**(1/4))/(4*a**(3/4)*c**(1/4))
```

Mathematica [A] time = 0.0333429, size = 134, normalized size = 0.72

$$\frac{-\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)+\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)-2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + c*x^4)^(-1),x]
```

```
[Out] (-2*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*ArcTan[1 + (Sqrt[
2]*c^(1/4)*x)/a^(1/4)] - Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x
+ Sqrt[c]*x^2] + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c
]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(1/4))
```

Maple [A] time = 0.002, size = 128, normalized size = 0.7

$$\frac{\sqrt{2}}{8\,a}\sqrt[4]{\frac{a}{c}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^2-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)$$
$$+\frac{\sqrt{2}}{4\,a}\sqrt[4]{\frac{a}{c}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}+1\right)+\frac{\sqrt{2}}{4\,a}\sqrt[4]{\frac{a}{c}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c*x^4+a),x)
```

```
[Out] 1/8*(1/c*a)^(1/4)/a*2^(1/2)*ln((x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*
a)^(1/2))/(x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))+1/4*(1/c*a
)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)+1/4*(1/c*a)^(
1/4)/a*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x^4 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.259772, size = 142, normalized size = 0.77

$$-\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}\arctan\left(\frac{a\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}}{x+\sqrt{a^2\sqrt{-\frac{1}{a^3c}}+x^2}}\right)$$
$$+\frac{1}{4}\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}\log\left(a\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}+x\right)-\frac{1}{4}\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}\log\left(-a\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x^4 + a),x, algorithm="fricas")
```

```
[Out] -(-1/(a^3*c))^(1/4)*arctan(a*(-1/(a^3*c))^(1/4)/(x + sqrt(a^2*sqr
t(-1/(a^3*c)) + x^2))) + 1/4*(-1/(a^3*c))^(1/4)*log(a*(-1/(a^3*c)
)^(1/4) + x) - 1/4*(-1/(a^3*c))^(1/4)*log(-a*(-1/(a^3*c))^(1/4) +
 x)
```

Sympy [A] time = 0.3852, size = 20, normalized size = 0.11

$$\text{RootSum}\left(256t^4a^3c + 1, \left(t \mapsto t\log\left(4ta + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*c + 1, Lambda(_t, _t*log(4*_t*a + x)))
```

GIAC/XCAS [A] time = 0.263749, size = 242, normalized size = 1.31

$$\frac{\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac} + \frac{\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac} + \frac{\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}\ln\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,ac} - \frac{\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}\ln\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x^4 + a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)
^(1/4))/(a/c)^(1/4))/(a*c) + 1/4*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2
*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c) + 1/8*sqr
t(2)*(a*c^3)^(1/4)*ln(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a
*c) - 1/8*sqrt(2)*(a*c^3)^(1/4)*ln(x^2 - sqrt(2)*x*(a/c)^(1/4) +
sqrt(a/c))/(a*c)
```

3.398 $\int \frac{1}{(d+ex)(a+cx^4)}\,dx$

Optimal. Leaf size=416

$$-\frac{\sqrt[4]{c}d\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^4+cd^4\right)}+\frac{\sqrt[4]{c}d\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^4+cd^4\right)}-\frac{\sqrt[4]{c}d\left(\sqrt{a}e^2+\sqrt{c}d^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(ae^4+cd^4\right)}+\frac{\sqrt[4]{c}d\left(\sqrt{a}e^2+\sqrt{c}d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\left(ae^4+cd^4\right)}-\frac{e^3\log\left(a+cx^4\right)}{4\left(ae^4+cd^4\right)}+\frac{e^3\log(d+ex)}{ae^4+cd^4}-\frac{\sqrt{c}d^2e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(ae^4+cd^4\right)}$$

```
[Out] -(Sqrt[c]*d^2*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*(c*d^4
+ a*e^4)) - (c^(1/4)*d*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 - (Sq
rt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)) +
(c^(1/4)*d*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4
)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)) + (e^3*Log[d +
 e*x])/(c*d^4 + a*e^4) - (c^(1/4)*d*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*L
og[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]
*a^(3/4)*(c*d^4 + a*e^4)) + (c^(1/4)*d*(Sqrt[c]*d^2 - Sqrt[a]*e^2
)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt
[2]*a^(3/4)*(c*d^4 + a*e^4)) - (e^3*Log[a + c*x^4])/(4*(c*d^4 + a
*e^4))
```

Rubi [A] time = 0.945997, antiderivative size = 416, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 11, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.647

$$-\frac{\sqrt[4]{c}d\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^4+cd^4\right)}+\frac{\sqrt[4]{c}d\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^4+cd^4\right)}-\frac{\sqrt[4]{c}d\left(\sqrt{a}e^2+\sqrt{c}d^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(ae^4+cd^4\right)}+\frac{\sqrt[4]{c}d\left(\sqrt{a}e^2+\sqrt{c}d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\left(ae^4+cd^4\right)}-\frac{e^3\log\left(a+cx^4\right)}{4\left(ae^4+cd^4\right)}+\frac{e^3\log(d+ex)}{ae^4+cd^4}-\frac{\sqrt{c}d^2e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(ae^4+cd^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d + e*x)*(a + c*x^4)),x]
```


```
[Out] -(Sqrt[c]*d^2*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*(c*d^4
+ a*e^4)) - (c^(1/4)*d*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 - (Sq
rt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)) +
(c^(1/4)*d*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4
)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)) + (e^3*Log[d +
 e*x])/(c*d^4 + a*e^4) - (c^(1/4)*d*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*L
og[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]
*a^(3/4)*(c*d^4 + a*e^4)) + (c^(1/4)*d*(Sqrt[c]*d^2 - Sqrt[a]*e^2
)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt
[2]*a^(3/4)*(c*d^4 + a*e^4)) - (e^3*Log[a + c*x^4])/(4*(c*d^4 + a
*e^4))
```

Rubi in Sympy [A] time = 133.876, size = 379, normalized size = 0.91

$$-\frac{e^3\log\left(a+cx^4\right)}{4\left(ae^4+cd^4\right)}+\frac{e^3\log\left(d+ex\right)}{ae^4+cd^4}-\frac{\sqrt{c}d^2e\operatorname{atan}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(ae^4+cd^4\right)}+\frac{\sqrt{2}\sqrt[4]{c}d\left(\sqrt{a}e^2-\sqrt{c}d^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}c^{\frac{3}{4}}x+\sqrt{a}\sqrt{c}+cx^2\right)}{8a^{\frac{3}{4}}\left(ae^4+cd^4\right)}-\frac{\sqrt{2}\sqrt[4]{c}d\left(\sqrt{a}e^2-\sqrt{c}d^2\right)\log\left(\sqrt{2}\sqrt[4]{a}c^{\frac{3}{4}}x+\sqrt{a}\sqrt{c}+cx^2\right)}{8a^{\frac{3}{4}}\left(ae^4+cd^4\right)}-\frac{\sqrt{2}\sqrt[4]{c}d\left(\sqrt{a}e^2+\sqrt{c}d^2\right)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}\left(ae^4+cd^4\right)}+\frac{\sqrt{2}\sqrt[4]{c}d\left(\sqrt{a}e^2+\sqrt{c}d^2\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}\left(ae^4+cd^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x+d)/(c*x**4+a),x)
```

```
[Out] -e**3*log(a + c*x**4)/(4*(a*e**4 + c*d**4)) + e**3*log(d + e*x)/(
a*e**4 + c*d**4) - sqrt(c)*d**2*e*atan(sqrt(c)*x**2/sqrt(a))/(2*s
qrt(a)*(a*e**4 + c*d**4)) + sqrt(2)*c**(1/4)*d*(sqrt(a)*e**2 - sq
rt(c)*d**2)*log(-sqrt(2)*a**(1/4)*c**(3/4)*x + sqrt(a)*sqrt(c) +
c*x**2)/(8*a**(3/4)*(a*e**4 + c*d**4)) - sqrt(2)*c**(1/4)*d*(sqrt
(a)*e**2 - sqrt(c)*d**2)*log(sqrt(2)*a**(1/4)*c**(3/4)*x + sqrt(a
)*sqrt(c) + c*x**2)/(8*a**(3/4)*(a*e**4 + c*d**4)) - sqrt(2)*c**(
1/4)*d*(sqrt(a)*e**2 + sqrt(c)*d**2)*atan(1 - sqrt(2)*c**(1/4)*x/
a**(1/4))/(4*a**(3/4)*(a*e**4 + c*d**4)) + sqrt(2)*c**(1/4)*d*(sq
rt(a)*e**2 + sqrt(c)*d**2)*atan(1 + sqrt(2)*c**(1/4)*x/a**(1/4))/
(4*a**(3/4)*(a*e**4 + c*d**4))
```

Mathematica [A] time = 0.285521, size = 404, normalized size = 0.97

$$-2a^{3/4}e^3\log\left(a+cx^4\right)+8a^{3/4}e^3\log(d+ex)-\sqrt{2}c^{3/4}d^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)+\sqrt{2}c^{3/4}d^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d + e*x)*(a + c*x^4)),x]
```

```
[Out] (-2*c^(1/4)*d*(Sqrt[2]*Sqrt[c]*d^2 - 2*a^(1/4)*c^(1/4)*d*e + Sqrt
[2]*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*c^(1
/4)*d*(Sqrt[2]*Sqrt[c]*d^2 + 2*a^(1/4)*c^(1/4)*d*e + Sqrt[2]*Sqrt
[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 8*a^(3/4)*e^3*
Log[d + e*x] - Sqrt[2]*c^(3/4)*d^3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*
c^(1/4)*x + Sqrt[c]*x^2] + Sqrt[2]*Sqrt[a]*c^(1/4)*d*e^2*Log[Sqrt
[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + Sqrt[2]*c^(3/4)*
d^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - Sqrt
[2]*Sqrt[a]*c^(1/4)*d*e^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x
 + Sqrt[c]*x^2] - 2*a^(3/4)*e^3*Log[a + c*x^4])/(8*a^(3/4)*(c*d^4
 + a*e^4))
```

Maple [A] time = 0.012, size = 433, normalized size = 1.

$$\frac{cd^3\sqrt{2}}{(8\,ae^4+8\,cd^4)\,a}\sqrt[4]{\frac{a}{c}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^2-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)$$
$$+\frac{cd^3\sqrt{2}}{(4\,ae^4+4\,cd^4)\,a}\sqrt[4]{\frac{a}{c}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}+1\right)$$
$$+\frac{cd^3\sqrt{2}}{(4\,ae^4+4\,cd^4)\,a}\sqrt[4]{\frac{a}{c}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}-1\right)-\frac{d^2ec}{2\,ae^4+2\,cd^4}\arctan\left(x^2\sqrt{\frac{c}{a}}\right)\frac{1}{\sqrt{ac}}$$
$$+\frac{e^2d\sqrt{2}}{8\,ae^4+8\,cd^4}\ln\left(1\left(x^2-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{c}}}$$
$$+\frac{e^2d\sqrt{2}}{4\,ae^4+4\,cd^4}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{c}}}$$
$$+\frac{e^2d\sqrt{2}}{4\,ae^4+4\,cd^4}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{c}}}-\frac{e^3\ln\left(cx^4+a\right)}{4\,ae^4+4\,cd^4}+\frac{e^3\ln\left(ex+d\right)}{ae^4+cd^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*x+d)/(c*x^4+a),x)
```

```
[Out] 1/8/(a*e^4+c*d^4)*c*d^3*(1/c*a)^(1/4)/a*2^(1/2)*ln((x^2+(1/c*a)^(
1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a
)^(1/2)))+1/4/(a*e^4+c*d^4)*c*d^3*(1/c*a)^(1/4)/a*2^(1/2)*arctan(
2^(1/2)/(1/c*a)^(1/4)*x+1)+1/4/(a*e^4+c*d^4)*c*d^3*(1/c*a)^(1/4)/
a*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)-1/2/(a*e^4+c*d^4)*c*d
^2*e/(a*c)^(1/2)*arctan(x^2*(c/a)^(1/2))+1/8/(a*e^4+c*d^4)*e^2*d/
(1/c*a)^(1/4)*2^(1/2)*ln((x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/
2))/(x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))+1/4/(a*e^4+c*d^4
)*e^2*d/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)+1
/4/(a*e^4+c*d^4)*e^2*d/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*
a)^(1/4)*x-1)-1/4*e^3*ln(c*x^4+a)/(a*e^4+c*d^4)+e^3*ln(e*x+d)/(a*
e^4+c*d^4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)*(e*x + d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)*(e*x + d)),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*x+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.292374, size = 520, normalized size = 1.25

$$\frac{\left(ac^3\right)^{\frac{1}{4}}c^2d\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ac^3d^2-2\left(ac^3\right)^{\frac{1}{4}}ac^2de-\sqrt{2}\sqrt{ac}ac^2e^2\right)}+\frac{\left(ac^3\right)^{\frac{1}{4}}c^2d\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ac^3d^2+2\left(ac^3\right)^{\frac{1}{4}}ac^2de-\sqrt{2}\sqrt{ac}ac^2e^2\right)}$$
$$+\frac{\left(\left(ac^3\right)^{\frac{1}{4}}c^2d^3-\left(ac^3\right)^{\frac{3}{4}}de^2\right)\ln\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}ac^3d^4+\sqrt{2}a^2c^2e^4\right)}$$
$$-\frac{\left(\left(ac^3\right)^{\frac{1}{4}}c^2d^3-\left(ac^3\right)^{\frac{3}{4}}de^2\right)\ln\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}ac^3d^4+\sqrt{2}a^2c^2e^4\right)}-\frac{e^3\ln\left(\left|cx^4+a\right|\right)}{4\left(cd^4+ae^4\right)}+\frac{e^4\ln\left(\left|xe+d\right|\right)}{cd^4e+ae^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)*(e*x + d)),x, algorithm="giac")
```

```
[Out] 1/2*(a*c^3)^(1/4)*c^2*d*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(
1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^3*d^2 - 2*(a*c^3)^(1/4)*a*c^2*d*e
 - sqrt(2)*sqrt(a*c)*a*c^2*e^2) + 1/2*(a*c^3)^(1/4)*c^2*d*arctan(
1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c
^3*d^2 + 2*(a*c^3)^(1/4)*a*c^2*d*e - sqrt(2)*sqrt(a*c)*a*c^2*e^2)
 + 1/4*((a*c^3)^(1/4)*c^2*d^3 - (a*c^3)^(3/4)*d*e^2)*ln(x^2 + sqr
t(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a*c^3*d^4 + sqrt(2)*a^2*
c^2*e^4) - 1/4*((a*c^3)^(1/4)*c^2*d^3 - (a*c^3)^(3/4)*d*e^2)*ln(x
^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a*c^3*d^4 + sqrt
(2)*a^2*c^2*e^4) - 1/4*e^3*ln(abs(c*x^4 + a))/(c*d^4 + a*e^4) + e
^4*ln(abs(x*e + d))/(c*d^4*e + a*e^5)
```

3.399 $\int \frac{1}{(d+ex)^2(a+cx^4)}\,dx$

Optimal. Leaf size=552

$$-\frac{\sqrt[4]{c}\left(\sqrt{c}d^2\left(cd^4-3ae^4\right)-\sqrt{a}e^2\left(3cd^4-ae^4\right)\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^4+cd^4\right)^2}+\frac{\sqrt[4]{c}\left(\sqrt{c}d^2\left(cd^4-3ae^4\right)-\sqrt{a}e^2\left(3cd^4-ae^4\right)\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^4+cd^4\right)^2}-\frac{\sqrt[4]{c}\left(\sqrt{a}e^2\left(3cd^4-ae^4\right)+\sqrt{c}d^2\left(cd^4-3ae^4\right)\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(ae^4+cd^4\right)^2}+\frac{\sqrt[4]{c}\left(\sqrt{a}e^2\left(3cd^4-ae^4\right)+\sqrt{c}d^2\left(cd^4-3ae^4\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\left(ae^4+cd^4\right)^2}-\frac{\sqrt{c}de\left(cd^4-ae^4\right)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{\sqrt{a}\left(ae^4+cd^4\right)^2}-\frac{e^3}{(d+ex)\left(ae^4+cd^4\right)}-\frac{cd^3e^3\log\left(a+cx^4\right)}{\left(ae^4+cd^4\right)^2}+\frac{4cd^3e^3\log(d+ex)}{\left(ae^4+cd^4\right)^2}$$

```
[Out] -(e^3/((c*d^4 + a*e^4)*(d + e*x))) - (Sqrt[c]*d*e*(c*d^4 - a*e^4)
*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(Sqrt[a]*(c*d^4 + a*e^4)^2) - (c^
(1/4)*(Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) + Sqrt[a]*e^2*(3*c*d^4 - a*e
^4))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*
(c*d^4 + a*e^4)^2) + (c^(1/4)*(Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) + Sq
rt[a]*e^2*(3*c*d^4 - a*e^4))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/
4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2) + (4*c*d^3*e^3*Log[d +
 e*x])/(c*d^4 + a*e^4)^2 - (c^(1/4)*(Sqrt[c]*d^2*(c*d^4 - 3*a*e^4
) - Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*
c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2) +
 (c^(1/4)*(Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) - Sqrt[a]*e^2*(3*c*d^4 -
 a*e^4))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/
(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2) - (c*d^3*e^3*Log[a + c*x^4]
)/(c*d^4 + a*e^4)^2
```

Rubi [A] time = 1.82237, antiderivative size = 552, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 11, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.647

$$-\frac{\sqrt[4]{c}\left(\sqrt{c}d^2\left(cd^4-3ae^4\right)-\sqrt{a}e^2\left(3cd^4-ae^4\right)\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^4+cd^4\right)^2}+\frac{\sqrt[4]{c}\left(\sqrt{c}d^2\left(cd^4-3ae^4\right)-\sqrt{a}e^2\left(3cd^4-ae^4\right)\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^4+cd^4\right)^2}-\frac{\sqrt[4]{c}\left(\sqrt{a}e^2\left(3cd^4-ae^4\right)+\sqrt{c}d^2\left(cd^4-3ae^4\right)\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(ae^4+cd^4\right)^2}+\frac{\sqrt[4]{c}\left(\sqrt{a}e^2\left(3cd^4-ae^4\right)+\sqrt{c}d^2\left(cd^4-3ae^4\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\left(ae^4+cd^4\right)^2}-\frac{\sqrt{c}de\left(cd^4-ae^4\right)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{\sqrt{a}\left(ae^4+cd^4\right)^2}-\frac{e^3}{(d+ex)\left(ae^4+cd^4\right)}-\frac{cd^3e^3\log\left(a+cx^4\right)}{\left(ae^4+cd^4\right)^2}+\frac{4cd^3e^3\log(d+ex)}{\left(ae^4+cd^4\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d + e*x)^2*(a + c*x^4)),x]
```

```
[Out] -(e^3/((c*d^4 + a*e^4)*(d + e*x))) - (Sqrt[c]*d*e*(c*d^4 - a*e^4)
*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(Sqrt[a]*(c*d^4 + a*e^4)^2) - (c^
(1/4)*(Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) + Sqrt[a]*e^2*(3*c*d^4 - a*e
```

```
^4))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*
(c*d^4 + a*e^4)^2) + (c^(1/4)*(Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) + Sq
rt[a]*e^2*(3*c*d^4 - a*e^4))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/
4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2) + (4*c*d^3*e^3*Log[d +
 e*x])/(c*d^4 + a*e^4)^2 - (c^(1/4)*(Sqrt[c]*d^2*(c*d^4 - 3*a*e^4
) - Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*
c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2) +
 (c^(1/4)*(Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) - Sqrt[a]*e^2*(3*c*d^4 -
 a*e^4))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/
(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2) - (c*d^3*e^3*Log[a + c*x^4]
)/(c*d^4 + a*e^4)^2
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x+d)**2/(c*x**4+a),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 1.37711, size = 524, normalized size = 0.95

$$-\frac{\frac{\sqrt{2}\sqrt[4]{c}\left(a^{3/2}e^6-3\sqrt{a}cd^4e^2-3a\sqrt{c}d^2e^4+c^{3/2}d^6\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{a^{3/4}}+\frac{\sqrt{2}\sqrt[4]{c}\left(a^{3/2}e^6-3\sqrt{a}cd^4e^2-3a\sqrt{c}d^2e^4+c^{3/2}d^6\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x\right.}{a^{3/4}}}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d + e*x)^2*(a + c*x^4)),x]
```

```
[Out] ((-8*e^3*(c*d^4 + a*e^4))/(d + e*x) + (2*c^(1/4)*(-(Sqrt[c]*d^2)
+ Sqrt[a]*e^2)*(Sqrt[2]*c*d^4 - 4*a^(1/4)*c^(3/4)*d^3*e + 4*Sqrt[
2]*Sqrt[a]*Sqrt[c]*d^2*e^2 - 4*a^(3/4)*c^(1/4)*d*e^3 + Sqrt[2]*a*
e^4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(3/4) + (2*c^(1/4
)*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*(Sqrt[2]*c*d^4 + 4*a^(1/4)*c^(3/4)*
d^3*e + 4*Sqrt[2]*Sqrt[a]*Sqrt[c]*d^2*e^2 + 4*a^(3/4)*c^(1/4)*d*e
^3 + Sqrt[2]*a*e^4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(3
/4) + 32*c*d^3*e^3*Log[d + e*x] - (Sqrt[2]*c^(1/4)*(c^(3/2)*d^6 -
 3*Sqrt[a]*c*d^4*e^2 - 3*a*Sqrt[c]*d^2*e^4 + a^(3/2)*e^6)*Log[Sqr
t[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(3/4) + (Sqrt[
2]*c^(1/4)*(c^(3/2)*d^6 - 3*Sqrt[a]*c*d^4*e^2 - 3*a*Sqrt[c]*d^2*e
^4 + a^(3/2)*e^6)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[
c]*x^2])/a^(3/4) - 8*c*d^3*e^3*Log[a + c*x^4])/(8*(c*d^4 + a*e^4)
^2)
```

Maple [A] time = 0.016, size = 866, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*x+d)^2/(c*x^4+a),x)
```

```
[Out] -3/4/(a*e^4+c*d^4)^2*c*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*
a)^(1/4)*x+1)*d^2*e^4+1/4/(a*e^4+c*d^4)^2*c^2*(1/c*a)^(1/4)/a*2^(
```

```
1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*d^6-3/4/(a*e^4+c*d^4)^2*c*
(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*d^2*e^4+1
/4/(a*e^4+c*d^4)^2*c^2*(1/c*a)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(1/
c*a)^(1/4)*x-1)*d^6-3/8/(a*e^4+c*d^4)^2*c*(1/c*a)^(1/4)*2^(1/2)*l
n((x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2-(1/c*a)^(1/4)*
x*2^(1/2)+(1/c*a)^(1/2)))*d^2*e^4+1/8/(a*e^4+c*d^4)^2*c^2*(1/c*a)
^(1/4)/a*2^(1/2)*ln((x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(
x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))*d^6+1/(a*e^4+c*d^4)^2
*c/(a*c)^(1/2)*arctan(x^2*(c/a)^(1/2))*a*d*e^5-1/(a*e^4+c*d^4)^2*
c^2/(a*c)^(1/2)*arctan(x^2*(c/a)^(1/2))*d^5*e-1/8/(a*e^4+c*d^4)^2
/(1/c*a)^(1/4)*2^(1/2)*ln((x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1
/2))/(x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))*a*e^6+3/8/(a*e^
4+c*d^4)^2*c/(1/c*a)^(1/4)*2^(1/2)*ln((x^2-(1/c*a)^(1/4)*x*2^(1/2
)+(1/c*a)^(1/2))/(x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))*d^4
*e^2-1/4/(a*e^4+c*d^4)^2/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/
c*a)^(1/4)*x+1)*a*e^6+3/4/(a*e^4+c*d^4)^2*c/(1/c*a)^(1/4)*2^(1/2)
*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*d^4*e^2-1/4/(a*e^4+c*d^4)^2/(1
/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*a*e^6+3/4/(
a*e^4+c*d^4)^2*c/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/
4)*x-1)*d^4*e^2-c*d^3*e^3*ln(c*x^4+a)/(a*e^4+c*d^4)^2-e^3/(a*e^4+
c*d^4)/(e*x+d)+4*c*d^3*e^3*ln(e*x+d)/(a*e^4+c*d^4)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)*(e*x + d)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)*(e*x + d)^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*x+d)**2/(c*x**4+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^4 + a)(ex + d)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)*(e*x + d)^2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + a)*(e*x + d)^2), x)
```

3.400 $\int \frac{1}{(d+ex)^3(a+cx^4)}\,dx$

Optimal. Leaf size=680

$$-\frac{\sqrt{c}e\left(a^2e^8-12acd^4e^4+3c^2d^8\right)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(ae^4+cd^4\right)^3}-\frac{c^{3/4}d\left(3a^2e^8-12acd^4e^4-2\sqrt{a}\sqrt{c}d^2e^2\left(3cd^4-5ae^4\right)+c^2d^8\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^4+cd^4\right)^3}+\frac{c^{3/4}d\left(3a^2e^8-12acd^4e^4-2\sqrt{a}\sqrt{c}d^2e^2\left(3cd^4-5ae^4\right)+c^2d^8\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^4+cd^4\right)^3}-\frac{c^{3/4}d\left(3a^2e^8-12acd^4e^4+2\sqrt{a}\sqrt{c}d^2e^2\left(3cd^4-5ae^4\right)+c^2d^8\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(ae^4+cd^4\right)^3}+\frac{c^{3/4}d\left(3a^2e^8-12acd^4e^4+2\sqrt{a}\sqrt{c}d^2e^2\left(3cd^4-5ae^4\right)+c^2d^8\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\left(ae^4+cd^4\right)^3}-\frac{e^3}{2(d+ex)^2\left(ae^4+cd^4\right)}-\frac{4cd^3e^3}{(d+ex)\left(ae^4+cd^4\right)^2}-\frac{cd^2e^3\left(5cd^4-3ae^4\right)\log\left(a+cx^4\right)}{2\left(ae^4+cd^4\right)^3}+\frac{2cd^2e^3\left(5cd^4-3ae^4\right)\log(d+ex)}{\left(ae^4+cd^4\right)^3}$$

```
[Out] -e^3/(2*(c*d^4 + a*e^4)*(d + e*x)^2) - (4*c*d^3*e^3)/((c*d^4 + a*
e^4)^2*(d + e*x)) - (Sqrt[c]*e*(3*c^2*d^8 - 12*a*c*d^4*e^4 + a^2*
e^8)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*(c*d^4 + a*e^4)^3)
 - (c^(3/4)*d*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8 + 2*Sqrt[a]*S
qrt[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x
)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^3) + (c^(3/4)*d*(c
^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8 + 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(
3*c*d^4 - 5*a*e^4))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*S
qrt[2]*a^(3/4)*(c*d^4 + a*e^4)^3) + (2*c*d^2*e^3*(5*c*d^4 - 3*a*e
^4)*Log[d + e*x])/(c*d^4 + a*e^4)^3 - (c^(3/4)*d*(c^2*d^8 - 12*a*
c*d^4*e^4 + 3*a^2*e^8 - 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d^4 - 5*a*
e^4))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*
Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^3) + (c^(3/4)*d*(c^2*d^8 - 12*a*c
*d^4*e^4 + 3*a^2*e^8 - 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d^4 - 5*a*e
^4))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*S
qrt[2]*a^(3/4)*(c*d^4 + a*e^4)^3) - (c*d^2*e^3*(5*c*d^4 - 3*a*e^4
)*Log[a + c*x^4])/(2*(c*d^4 + a*e^4)^3)
```

Rubi [A] time = 2.13323, antiderivative size = 680, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 11, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.647$

$$-\frac{\sqrt{c}e\left(a^2e^8-12acd^4e^4+3c^2d^8\right)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(ae^4+cd^4\right)^3}-\frac{c^{3/4}d\left(3a^2e^8-12acd^4e^4-2\sqrt{a}\sqrt{c}d^2e^2\left(3cd^4-5ae^4\right)+c^2d^8\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^4+cd^4\right)^3}+\frac{c^{3/4}d\left(3a^2e^8-12acd^4e^4-2\sqrt{a}\sqrt{c}d^2e^2\left(3cd^4-5ae^4\right)+c^2d^8\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^4+cd^4\right)^3}-\frac{c^{3/4}d\left(3a^2e^8-12acd^4e^4+2\sqrt{a}\sqrt{c}d^2e^2\left(3cd^4-5ae^4\right)+c^2d^8\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(ae^4+cd^4\right)^3}+\frac{c^{3/4}d\left(3a^2e^8-12acd^4e^4+2\sqrt{a}\sqrt{c}d^2e^2\left(3cd^4-5ae^4\right)+c^2d^8\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\left(ae^4+cd^4\right)^3}-\frac{e^3}{2(d+ex)^2\left(ae^4+cd^4\right)}-\frac{4cd^3e^3}{(d+ex)\left(ae^4+cd^4\right)^2}-\frac{cd^2e^3\left(5cd^4-3ae^4\right)\log\left(a+cx^4\right)}{2\left(ae^4+cd^4\right)^3}+\frac{2cd^2e^3\left(5cd^4-3ae^4\right)\log(d+ex)}{\left(ae^4+cd^4\right)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d + e*x)^3*(a + c*x^4)),x]
```

```
[Out] -e^3/(2*(c*d^4 + a*e^4)*(d + e*x)^2) - (4*c*d^3*e^3)/((c*d^4 + a*
e^4)^2*(d + e*x)) - (Sqrt[c]*e*(3*c^2*d^8 - 12*a*c*d^4*e^4 + a^2*
e^8)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*(c*d^4 + a*e^4)^3)
 - (c^(3/4)*d*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8 + 2*Sqrt[a]*S
qrt[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x
)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^3) + (c^(3/4)*d*(c
^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8 + 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(
3*c*d^4 - 5*a*e^4))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*S
qrt[2]*a^(3/4)*(c*d^4 + a*e^4)^3) + (2*c*d^2*e^3*(5*c*d^4 - 3*a*e
^4)*Log[d + e*x])/(c*d^4 + a*e^4)^3 - (c^(3/4)*d*(c^2*d^8 - 12*a*
c*d^4*e^4 + 3*a^2*e^8 - 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d^4 - 5*a*
e^4))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*
Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^3) + (c^(3/4)*d*(c^2*d^8 - 12*a*c
*d^4*e^4 + 3*a^2*e^8 - 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d^4 - 5*a*e
^4))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*S
qrt[2]*a^(3/4)*(c*d^4 + a*e^4)^3) - (c*d^2*e^3*(5*c*d^4 - 3*a*e^4
)*Log[a + c*x^4])/(2*(c*d^4 + a*e^4)^3)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x+d)**3/(c*x**4+a),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 1.9669, size = 738, normalized size = 1.09

$$\frac{-4a^{3/4}e^3\left(ae^4+cd^4\right)^2-32a^{3/4}cd^3e^3(d+ex)\left(ae^4+cd^4\right)+4a^{3/4}cd^2e^3(d+ex)^2\left(3ae^4-5cd^4\right)\log\left(a+cx^4\right)+16a^{3/4}cd^2e^3(d}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d + e*x)^3*(a + c*x^4)),x]
```

```
[Out] (-4*a^(3/4)*e^3*(c*d^4 + a*e^4)^2 - 32*a^(3/4)*c*d^3*e^3*(c*d^4 +
 a*e^4)*(d + e*x) - 2*Sqrt[c]*(Sqrt[2]*c^(9/4)*d^9 - 6*a^(1/4)*c^
2*d^8*e + 6*Sqrt[2]*Sqrt[a]*c^(7/4)*d^7*e^2 - 12*Sqrt[2]*a*c^(5/4
)*d^5*e^4 + 24*a^(5/4)*c*d^4*e^5 - 10*Sqrt[2]*a^(3/2)*c^(3/4)*d^3
*e^6 + 3*Sqrt[2]*a^2*c^(1/4)*d*e^8 - 2*a^(9/4)*e^9)*(d + e*x)^2*A
rcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*Sqrt[c]*(Sqrt[2]*c^(9/
4)*d^9 + 6*a^(1/4)*c^2*d^8*e + 6*Sqrt[2]*Sqrt[a]*c^(7/4)*d^7*e^2
- 12*Sqrt[2]*a*c^(5/4)*d^5*e^4 - 24*a^(5/4)*c*d^4*e^5 - 10*Sqrt[2
]*a^(3/2)*c^(3/4)*d^3*e^6 + 3*Sqrt[2]*a^2*c^(1/4)*d*e^8 + 2*a^(9/
4)*e^9)*(d + e*x)^2*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 16*
a^(3/4)*c*d^2*e^3*(5*c*d^4 - 3*a*e^4)*(d + e*x)^2*Log[d + e*x] -
Sqrt[2]*c^(3/4)*d*(c^2*d^8 - 6*Sqrt[a]*c^(3/2)*d^6*e^2 - 12*a*c*d
^4*e^4 + 10*a^(3/2)*Sqrt[c]*d^2*e^6 + 3*a^2*e^8)*(d + e*x)^2*Log[
Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + Sqrt[2]*c^(3
/4)*d*(c^2*d^8 - 6*Sqrt[a]*c^(3/2)*d^6*e^2 - 12*a*c*d^4*e^4 + 10*
a^(3/2)*Sqrt[c]*d^2*e^6 + 3*a^2*e^8)*(d + e*x)^2*Log[Sqrt[a] + Sq
rt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + 4*a^(3/4)*c*d^2*e^3*(-5*
c*d^4 + 3*a*e^4)*(d + e*x)^2*Log[a + c*x^4])/(8*a^(3/4)*(c*d^4 +
a*e^4)^3*(d + e*x)^2)
```

Maple [B] time = 0.019, size = 1201, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*x+d)^3/(c*x^4+a),x)
```

```
[Out] 3/4/(a*e^4+c*d^4)^3*c*(1/c*a)^(1/4)*a*2^(1/2)*arctan(2^(1/2)/(1/c
*a)^(1/4)*x+1)*d*e^8-3/(a*e^4+c*d^4)^3*c^2*(1/c*a)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*d^5*e^4+1/4/(a*e^4+c*d^4)^3*c^3
*(1/c*a)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*d^9+3/
4/(a*e^4+c*d^4)^3*c*(1/c*a)^(1/4)*a*2^(1/2)*arctan(2^(1/2)/(1/c*a
)^(1/4)*x-1)*d*e^8-3/(a*e^4+c*d^4)^3*c^2*(1/c*a)^(1/4)*2^(1/2)*ar
ctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*d^5*e^4+1/4/(a*e^4+c*d^4)^3*c^3*(
1/c*a)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*d^9+3/8/
(a*e^4+c*d^4)^3*c*(1/c*a)^(1/4)*a*2^(1/2)*ln((x^2+(1/c*a)^(1/4)*x
*2^(1/2)+(1/c*a)^(1/2))/(x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2
)))*d*e^8-3/2/(a*e^4+c*d^4)^3*c^2*(1/c*a)^(1/4)*2^(1/2)*ln((x^2+(
1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2-(1/c*a)^(1/4)*x*2^(1/2
)+(1/c*a)^(1/2)))*d^5*e^4+1/8/(a*e^4+c*d^4)^3*c^3*(1/c*a)^(1/4)/a
*2^(1/2)*ln((x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2-(1/c
*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))*d^9-1/2/(a*e^4+c*d^4)^3*c/(a*
c)^(1/2)*arctan(x^2*(c/a)^(1/2))*a^2*e^9+6/(a*e^4+c*d^4)^3*c^2/(a
*c)^(1/2)*arctan(x^2*(c/a)^(1/2))*a*d^4*e^5-3/2/(a*e^4+c*d^4)^3*c
^3/(a*c)^(1/2)*arctan(x^2*(c/a)^(1/2))*d^8*e-5/4/(a*e^4+c*d^4)^3*
c/(1/c*a)^(1/4)*2^(1/2)*ln((x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(
1/2))/(x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))*a*d^3*e^6+3/4/
(a*e^4+c*d^4)^3*c^2/(1/c*a)^(1/4)*2^(1/2)*ln((x^2-(1/c*a)^(1/4)*x
*2^(1/2)+(1/c*a)^(1/2))/(x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2
)))*d^7*e^2-5/2/(a*e^4+c*d^4)^3*c/(1/c*a)^(1/4)*2^(1/2)*arctan(2^
(1/2)/(1/c*a)^(1/4)*x+1)*a*d^3*e^6+3/2/(a*e^4+c*d^4)^3*c^2/(1/c*a
)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*d^7*e^2-5/2/(a*
e^4+c*d^4)^3*c/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)
*x-1)*a*d^3*e^6+3/2/(a*e^4+c*d^4)^3*c^2/(1/c*a)^(1/4)*2^(1/2)*arc
tan(2^(1/2)/(1/c*a)^(1/4)*x-1)*d^7*e^2+3/2/(a*e^4+c*d^4)^3*c*ln(c
*x^4+a)*a*d^2*e^7-5/2/(a*e^4+c*d^4)^3*c^2*ln(c*x^4+a)*d^6*e^3-1/2
*e^3/(a*e^4+c*d^4)/(e*x+d)^2-4*c*d^3*e^3/(a*e^4+c*d^4)^2/(e*x+d)-
6*e^7*c*d^2/(a*e^4+c*d^4)^3*ln(e*x+d)*a+10*e^3*c^2*d^6/(a*e^4+c*d
^4)^3*ln(e*x+d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)*(e*x + d)^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)*(e*x + d)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(c*x**4+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.365284, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^4 + a)*(e*x + d)^3),x, algorithm="giac")
```

```
[Out] Done
```

3.401 $\int \frac{(d+ex)^3}{(a+cx^4)^2}\,dx$

Optimal. Leaf size=349

$$-\frac{3d\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{3/4}}+\frac{3d\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{3/4}}-\frac{3d\left(\sqrt{a}e^2+\sqrt{c}d^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{3/4}}+\frac{3d\left(\sqrt{a}e^2+\sqrt{c}d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}c^{3/4}}+\frac{3d^2e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{c}}-\frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{4ac\left(a+cx^4\right)}$$

```
[Out] -(a*e^3 - c*x*(d^3 + 3*d^2*e*x + 3*d*e^2*x^2))/(4*a*c*(a + c*x^4)
) + (3*d^2*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(3/2)*Sqrt[c]) -
 (3*d*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/
a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(3/4)) + (3*d*(Sqrt[c]*d^2 + Sqrt[
a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/
4)*c^(3/4)) - (3*d*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(3/4)
) + (3*d*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4
)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(3/4))
```

Rubi [A] time = 0.712593, antiderivative size = 349, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 12, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.706$

$$-\frac{3d\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{3/4}}+\frac{3d\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{3/4}}-\frac{3d\left(\sqrt{a}e^2+\sqrt{c}d^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{3/4}}+\frac{3d\left(\sqrt{a}e^2+\sqrt{c}d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}c^{3/4}}+\frac{3d^2e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{c}}-\frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{4ac\left(a+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x)^3/(a + c*x^4)^2,x]
```

```
[Out] -(a*e^3 - c*x*(d^3 + 3*d^2*e*x + 3*d*e^2*x^2))/(4*a*c*(a + c*x^4)
) + (3*d^2*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(3/2)*Sqrt[c]) -
 (3*d*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/
a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(3/4)) + (3*d*(Sqrt[c]*d^2 + Sqrt[
a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/
4)*c^(3/4)) - (3*d*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(3/4)
) + (3*d*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4
)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(3/4))
```

Rubi in Sympy [A] time = 145.379, size = 333, normalized size = 0.95

$$-\frac{ae^3 - cx\left(d^3 + 3d^2ex + 3de^2x^2\right)}{4ac\left(a + cx^4\right)} + \frac{3d^2e\operatorname{atan}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{\frac{3}{2}}\sqrt{c}} + \frac{3\sqrt{2}d\left(\sqrt{a}e^2 - \sqrt{c}d^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}c^{\frac{3}{4}}x + \sqrt{a}\sqrt{c} + cx^2\right)}{32a^{\frac{7}{4}}c^{\frac{3}{4}}} - \frac{3\sqrt{2}d\left(\sqrt{a}e^2 - \sqrt{c}d^2\right)\log\left(\sqrt{2}\sqrt[4]{a}c^{\frac{3}{4}}x + \sqrt{a}\sqrt{c} + cx^2\right)}{32a^{\frac{7}{4}}c^{\frac{3}{4}}} - \frac{3\sqrt{2}d\left(\sqrt{a}e^2 + \sqrt{c}d^2\right)\operatorname{atan}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{16a^{\frac{7}{4}}c^{\frac{3}{4}}} + \frac{3\sqrt{2}d\left(\sqrt{a}e^2 + \sqrt{c}d^2\right)\operatorname{atan}\left(1 + \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{16a^{\frac{7}{4}}c^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x+d)**3/(c*x**4+a)**2,x)
```

```
[Out] -(a*e**3 - c*x*(d**3 + 3*d**2*e*x + 3*d*e**2*x**2))/(4*a*c*(a + c
*x**4)) + 3*d**2*e*atan(sqrt(c)*x**2/sqrt(a))/(4*a**(3/2)*sqrt(c)
) + 3*sqrt(2)*d*(sqrt(a)*e**2 - sqrt(c)*d**2)*log(-sqrt(2)*a**(1/
4)*c**(3/4)*x + sqrt(a)*sqrt(c) + c*x**2)/(32*a**(7/4)*c**(3/4))
- 3*sqrt(2)*d*(sqrt(a)*e**2 - sqrt(c)*d**2)*log(sqrt(2)*a**(1/4)*
c**(3/4)*x + sqrt(a)*sqrt(c) + c*x**2)/(32*a**(7/4)*c**(3/4)) - 3
*sqrt(2)*d*(sqrt(a)*e**2 + sqrt(c)*d**2)*atan(1 - sqrt(2)*c**(1/4
)*x/a**(1/4))/(16*a**(7/4)*c**(3/4)) + 3*sqrt(2)*d*(sqrt(a)*e**2
+ sqrt(c)*d**2)*atan(1 + sqrt(2)*c**(1/4)*x/a**(1/4))/(16*a**(7/4
)*c**(3/4))
```

Mathematica [A] time = 0.726295, size = 347, normalized size = 0.99

$$3\sqrt{2}\sqrt[4]{c}\left(a^{3/4}de^2 - \sqrt[4]{a}\sqrt{c}d^3\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2\right) + 3\sqrt{2}\sqrt[4]{c}\left(\sqrt[4]{a}\sqrt{c}d^3 - a^{3/4}de^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2\right) -$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x)^3/(a + c*x^4)^2,x]
```

```
[Out] ((-8*a*(a*e^3 - c*d*x*(d^2 + 3*d*e*x + 3*e^2*x^2)))/(a + c*x^4) -
 6*a^(1/4)*c^(1/4)*d*(Sqrt[2]*Sqrt[c]*d^2 + 4*a^(1/4)*c^(1/4)*d*e
 + Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] +
 6*a^(1/4)*c^(1/4)*d*(Sqrt[2]*Sqrt[c]*d^2 - 4*a^(1/4)*c^(1/4)*d*e
 + Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] +
 3*Sqrt[2]*c^(1/4)*(-(a^(1/4)*Sqrt[c]*d^3) + a^(3/4)*d*e^2)*Log[S
qrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + 3*Sqrt[2]*c^(
1/4)*(a^(1/4)*Sqrt[c]*d^3 - a^(3/4)*d*e^2)*Log[Sqrt[a] + Sqrt[2]*
a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(32*a^2*c)
```

Maple [A] time = 0.008, size = 390, normalized size = 1.1

$$\frac{d^3x}{4a(cx^4+a)}+\frac{3d^3\sqrt{2}}{32a^2}\sqrt[4]{\frac{a}{c}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^2-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)$$
$$+\frac{3d^3\sqrt{2}}{16a^2}\sqrt[4]{\frac{a}{c}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}+1\right)+\frac{3d^3\sqrt{2}}{16a^2}\sqrt[4]{\frac{a}{c}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}-1\right)$$
$$+\frac{3d^2ex^2}{4a(cx^4+a)}+\frac{3d^2e}{4a}\arctan\left(x^2\sqrt{\frac{c}{a}}\right)\frac{1}{\sqrt{ac}}+\frac{3de^2x^3}{4a(cx^4+a)}$$
$$+\frac{3e^2d\sqrt{2}}{32ac}\ln\left(1\left(x^2-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{c}}}$$
$$+\frac{3e^2d\sqrt{2}}{16ac}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{c}}}+\frac{3e^2d\sqrt{2}}{16ac}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{c}}}+\frac{e^3x^4}{4a(cx^4+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x+d)^3/(c*x^4+a)^2,x)
```

```
[Out] 1/4*d^3*x/a/(c*x^4+a)+3/32*d^3/a^2*(1/c*a)^(1/4)*2^(1/2)*ln((x^2+
(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2-(1/c*a)^(1/4)*x*2^(1/
2)+(1/c*a)^(1/2)))+3/16*d^3/a^2*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1
/2)/(1/c*a)^(1/4)*x+1)+3/16*d^3/a^2*(1/c*a)^(1/4)*2^(1/2)*arctan(
2^(1/2)/(1/c*a)^(1/4)*x-1)+3/4*d^2*e*x^2/a/(c*x^4+a)+3/4*d^2*e/a/
(a*c)^(1/2)*arctan(x^2*(c/a)^(1/2))+3/4*e^2*d*x^3/a/(c*x^4+a)+3/3
2*e^2*d/a/c/(1/c*a)^(1/4)*2^(1/2)*ln((x^2-(1/c*a)^(1/4)*x*2^(1/2)
+(1/c*a)^(1/2))/(x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))+3/16
*e^2*d/a/c/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1
)+3/16*e^2*d/a/c/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/
4)*x-1)+1/4*e^3*x^4/a/(c*x^4+a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)^3/(c*x^4 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)^3/(c*x^4 + a)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 22.8319, size = 350, normalized size = 1.

$$\operatorname{RootSum}\left(65536t^4a^7c^3 + 27648t^2a^4c^2d^4e^2 + t\left(3456a^3cd^4e^5 - 3456a^2c^2d^8e\right) + 81a^2d^4e^8 + 162acd^8e^4 + 81c^2d^{12}, \left(t \mapsto t\log\right.\right.$$
$$+ \frac{-ae^3 + cd^3x + 3cd^2ex^2 + 3cde^2x^3}{4a^2c + 4ac^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x+d)**3/(c*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**7*c**3 + 27648*_t**2*a**4*c**2*d**4*e**2 +
 _t*(3456*a**3*c*d**4*e**5 - 3456*a**2*c**2*d**8*e) + 81*a**2*d**
4*e**8 + 162*a*c*d**8*e**4 + 81*c**2*d**12, Lambda(_t, _t*log(x +
 (4096*_t**3*a**7*c**2*e**6 + 28672*_t**3*a**6*c**3*d**4*e**2 - 7
680*_t**2*a**5*c**2*d**4*e**5 + 1536*_t**2*a**4*c**3*d**8*e + 216
0*_t*a**4*c*d**4*e**8 + 9216*_t*a**3*c**2*d**8*e**4 + 144*_t*a**2
*c**3*d**12 + 162*a**3*d**4*e**11 - 648*a**2*c*d**8*e**7 - 810*a*
c**2*d**12*e**3)/(27*a**3*d**3*e**12 - 891*a**2*c*d**7*e**8 - 891
*a*c**2*d**11*e**4 + 27*c**3*d**15)))) + (-a*e**3 + c*d**3*x + 3*
c*d**2*e*x**2 + 3*c*d*e**2*x**3)/(4*a**2*c + 4*a*c**2*x**4)
```

GIAC/XCAS [A] time = 0.270418, size = 462, normalized size = 1.32

$$\frac{3\,cdx^3e^2 + 3\,cd^2x^2e + cd^3x - ae^3}{4\,(cx^4+a)ac} + \frac{3\,\sqrt{2}\left(2\,\sqrt{2}\sqrt{ac}c^2d^2e + \left(ac^3\right)^{\frac{1}{4}}c^2d^3 + \left(ac^3\right)^{\frac{3}{4}}de^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c^3} + \frac{3\,\sqrt{2}\left(2\,\sqrt{2}\sqrt{ac}c^2d^2e + \left(ac^3\right)^{\frac{1}{4}}c^2d^3 + \left(ac^3\right)^{\frac{3}{4}}de^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c^3} + \frac{3\,\sqrt{2}\left(\left(ac^3\right)^{\frac{1}{4}}c^2d^3 - \left(ac^3\right)^{\frac{3}{4}}de^2\right)\ln\left(x^2 + \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{32\,a^2c^3} - \frac{3\,\sqrt{2}\left(\left(ac^3\right)^{\frac{1}{4}}c^2d^3 - \left(ac^3\right)^{\frac{3}{4}}de^2\right)\ln\left(x^2 - \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{32\,a^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)^3/(c*x^4 + a)^2,x, algorithm="giac")
```

```
[Out] 1/4*(3*c*d*x^3*e^2 + 3*c*d^2*x^2*e + c*d^3*x - a*e^3)/((c*x^4 + a
)*a*c) + 3/16*sqrt(2)*(2*sqrt(2)*sqrt(a*c)*c^2*d^2*e + (a*c^3)^(1
/4)*c^2*d^3 + (a*c^3)^(3/4)*d*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt
(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^3) + 3/16*sqrt(2)*(2*sqrt(2)
*sqrt(a*c)*c^2*d^2*e + (a*c^3)^(1/4)*c^2*d^3 + (a*c^3)^(3/4)*d*e^
2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a
^2*c^3) + 3/32*sqrt(2)*((a*c^3)^(1/4)*c^2*d^3 - (a*c^3)^(3/4)*d*e
^2)*ln(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^3) - 3/32*
sqrt(2)*((a*c^3)^(1/4)*c^2*d^3 - (a*c^3)^(3/4)*d*e^2)*ln(x^2 - sq
rt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^3)
```

3.402 $\int \frac{(d+ex)^2}{(a+cx^4)^2}\,dx$

Optimal. Leaf size=322

$$-\frac{\left(3\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{3/4}}+\frac{\left(3\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{3/4}}-\frac{\left(\sqrt{a}e^2+3\sqrt{c}d^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{3/4}}+\frac{\left(\sqrt{a}e^2+3\sqrt{c}d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}c^{3/4}}+\frac{de\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}}+\frac{x(d+ex)^2}{4a\left(a+cx^4\right)}$$

```
[Out] (x*(d + e*x)^2)/(4*a*(a + c*x^4)) + (d*e*ArcTan[(Sqrt[c]*x^2)/Sqr
t[a]])/(2*a^(3/2)*Sqrt[c]) - ((3*Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTa
n[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(3/4)) +
 ((3*Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^
(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(3/4)) - ((3*Sqrt[c]*d^2 - Sqrt[a]*e
^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*S
qrt[2]*a^(7/4)*c^(3/4)) + ((3*Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt
[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/
4)*c^(3/4))
```

Rubi [A] time = 0.579997, antiderivative size = 322, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.588

$$-\frac{\left(3\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{3/4}}+\frac{\left(3\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{3/4}}-\frac{\left(\sqrt{a}e^2+3\sqrt{c}d^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{3/4}}+\frac{\left(\sqrt{a}e^2+3\sqrt{c}d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}c^{3/4}}+\frac{de\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}}+\frac{x(d+ex)^2}{4a\left(a+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x)^2/(a + c*x^4)^2,x]
```

```
[Out] (x*(d + e*x)^2)/(4*a*(a + c*x^4)) + (d*e*ArcTan[(Sqrt[c]*x^2)/Sqr
t[a]])/(2*a^(3/2)*Sqrt[c]) - ((3*Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTa
n[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(3/4)) +
 ((3*Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^
(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(3/4)) - ((3*Sqrt[c]*d^2 - Sqrt[a]*e
^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*S
qrt[2]*a^(7/4)*c^(3/4)) + ((3*Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt
[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/
4)*c^(3/4))
```

Rubi in Sympy [A] time = 98.9791, size = 299, normalized size = 0.93

$$\frac{x(d+ex)^2}{4a(a+cx^4)}+\frac{de\,\mathrm{atan}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}\sqrt{c}}+\frac{\sqrt{2}\left(\sqrt{a}e^2-3\sqrt{c}d^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}c^{\frac{3}{4}}x+\sqrt{a}\sqrt{c}+cx^2\right)}{32a^{\frac{7}{4}}c^{\frac{3}{4}}}$$
$$-\frac{\sqrt{2}\left(\sqrt{a}e^2-3\sqrt{c}d^2\right)\log\left(\sqrt{2}\sqrt[4]{a}c^{\frac{3}{4}}x+\sqrt{a}\sqrt{c}+cx^2\right)}{32a^{\frac{7}{4}}c^{\frac{3}{4}}}$$
$$-\frac{\sqrt{2}\left(\sqrt{a}e^2+3\sqrt{c}d^2\right)\mathrm{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{16a^{\frac{7}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\left(\sqrt{a}e^2+3\sqrt{c}d^2\right)\mathrm{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{16a^{\frac{7}{4}}c^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x+d)**2/(c*x**4+a)**2,x)
```

```
[Out] x*(d + e*x)**2/(4*a*(a + c*x**4)) + d*e*atan(sqrt(c)*x**2/sqrt(a)
)/(2*a**(3/2)*sqrt(c)) + sqrt(2)*(sqrt(a)*e**2 - 3*sqrt(c)*d**2)*
log(-sqrt(2)*a**(1/4)*c**(3/4)*x + sqrt(a)*sqrt(c) + c*x**2)/(32*
a**(7/4)*c**(3/4)) - sqrt(2)*(sqrt(a)*e**2 - 3*sqrt(c)*d**2)*log(
sqrt(2)*a**(1/4)*c**(3/4)*x + sqrt(a)*sqrt(c) + c*x**2)/(32*a**(7
/4)*c**(3/4)) - sqrt(2)*(sqrt(a)*e**2 + 3*sqrt(c)*d**2)*atan(1 -
sqrt(2)*c**(1/4)*x/a**(1/4))/(16*a**(7/4)*c**(3/4)) + sqrt(2)*(sq
rt(a)*e**2 + 3*sqrt(c)*d**2)*atan(1 + sqrt(2)*c**(1/4)*x/a**(1/4)
)/(16*a**(7/4)*c**(3/4))
```

Mathematica [A] time = 0.653972, size = 321, normalized size = 1.

$$\frac{\frac{\sqrt{2}\left(a^{3/4}e^2-3\sqrt[4]{a}\sqrt{c}d^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{c^{3/4}}+\frac{\sqrt{2}\left(3\sqrt[4]{a}\sqrt{c}d^2-a^{3/4}e^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{c^{3/4}}-\frac{2\sqrt[4]{a}\left(8\sqrt[4]{a}\sqrt[4]{c}de+\sqrt{2}\sqrt{a}e^2+3\sqrt{2}\sqrt{c}d^2\right)\mathrm{ta}}{c^{3/4}}}{32a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x)^2/(a + c*x^4)^2,x]
```

```
[Out] ((8*a*x*(d + e*x)^2)/(a + c*x^4) - (2*a^(1/4)*(3*Sqrt[2]*Sqrt[c]*
d^2 + 8*a^(1/4)*c^(1/4)*d*e + Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 - (Sq
rt[2]*c^(1/4)*x)/a^(1/4)])/c^(3/4) + (2*a^(1/4)*(3*Sqrt[2]*Sqrt[c
]*d^2 - 8*a^(1/4)*c^(1/4)*d*e + Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 + (
Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(3/4) + (Sqrt[2]*(-3*a^(1/4)*Sqrt[
c]*d^2 + a^(3/4)*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + S
qrt[c]*x^2])/c^(3/4) + (Sqrt[2]*(3*a^(1/4)*Sqrt[c]*d^2 - a^(3/4)*
e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(3
/4))/(32*a^2)
```

Maple [A] time = 0.007, size = 362, normalized size = 1.1

$$\frac{d^2x}{4a(cx^4+a)} + \frac{3d^2\sqrt{2}}{32a^2}\sqrt[4]{\frac{a}{c}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^2-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)$$
$$+\frac{3d^2\sqrt{2}}{16a^2}\sqrt[4]{\frac{a}{c}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}+1\right)+\frac{3d^2\sqrt{2}}{16a^2}\sqrt[4]{\frac{a}{c}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}-1\right)$$
$$+\frac{dex^2}{2a(cx^4+a)}+\frac{de}{2a}\arctan\left(x^2\sqrt{\frac{c}{a}}\right)\frac{1}{\sqrt{ac}}+\frac{e^2x^3}{4a(cx^4+a)}$$
$$+\frac{e^2\sqrt{2}}{32ac}\ln\left(1\left(x^2-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{c}}}$$
$$+\frac{e^2\sqrt{2}}{16ac}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{c}}}+\frac{e^2\sqrt{2}}{16ac}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x+d)^2/(c*x^4+a)^2,x)
```

```
[Out] 1/4*d^2*x/a/(c*x^4+a)+3/32*d^2/a^2*(1/c*a)^(1/4)*2^(1/2)*ln((x^2+
(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2-(1/c*a)^(1/4)*x*2^(1/
2)+(1/c*a)^(1/2)))+3/16*d^2/a^2*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1
/2)/(1/c*a)^(1/4)*x+1)+3/16*d^2/a^2*(1/c*a)^(1/4)*2^(1/2)*arctan(
2^(1/2)/(1/c*a)^(1/4)*x-1)+1/2*d*e*x^2/a/(c*x^4+a)+1/2*d*e/a/(a*c
)^(1/2)*arctan(x^2*(c/a)^(1/2))+1/4*e^2*x^3/a/(c*x^4+a)+1/32*e^2/
a/c/(1/c*a)^(1/4)*2^(1/2)*ln((x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)
^(1/2))/(x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))+1/16*e^2/a/c
/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)+1/16*e^2
/a/c/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)^2/(c*x^4 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)^2/(c*x^4 + a)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 13.061, size = 318, normalized size = 0.99

$$\operatorname{RootSum}\left(65536t^4a^7c^3 + 11264t^2a^4c^2d^2e^2 + t\left(256a^3cde^5 - 2304a^2c^2d^5e\right) + a^2e^8 + 82acd^4e^4 + 81c^2d^8, \left(t \mapsto t\log\left(x + \frac{409}{}\right.\right.\right.$$
$$+ \frac{d^2x + 2dex^2 + e^2x^3}{4a^2 + 4acx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x+d)**2/(c*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**7*c**3 + 11264*_t**2*a**4*c**2*d**2*e**2 +
 _t*(256*a**3*c*d*e**5 - 2304*a**2*c**2*d**5*e) + a**2*e**8 + 82*
a*c*d**4*e**4 + 81*c**2*d**8, Lambda(_t, _t*log(x + (4096*_t**3*a
**7*c**2*e**6 + 356352*_t**3*a**6*c**3*d**4*e**2 - 23552*_t**2*a*
*5*c**2*d**3*e**5 + 27648*_t**2*a**4*c**3*d**7*e + 912*_t*a**4*c*
d**2*e**8 + 43584*_t*a**3*c**2*d**6*e**4 + 3888*_t*a**2*c**3*d**1
0 + 12*a**3*d*e**11 - 1088*a**2*c*d**5*e**7 - 7020*a*c**2*d**9*e*
*3)/(a**3*e**12 - 649*a**2*c*d**4*e**8 - 5841*a*c**2*d**8*e**4 +
729*c**3*d**12)))) + (d**2*x + 2*d*e*x**2 + e**2*x**3)/(4*a**2 +
4*a*c*x**4)
```

GIAC/XCAS [A] time = 0.270812, size = 436, normalized size = 1.35

$$\frac{x^3e^2 + 2\,dx^2e + d^2x}{4\,(cx^4 + a)a} + \frac{\sqrt{2}\left(4\,\sqrt{2}\sqrt{ac}c^2de + 3\,\left(ac^3\right)^{\frac{1}{4}}c^2d^2 + \left(ac^3\right)^{\frac{3}{4}}e^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x + \sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c^3}$$
$$+ \frac{\sqrt{2}\left(4\,\sqrt{2}\sqrt{ac}c^2de + 3\,\left(ac^3\right)^{\frac{1}{4}}c^2d^2 + \left(ac^3\right)^{\frac{3}{4}}e^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x - \sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c^3}$$
$$+ \frac{\sqrt{2}\left(3\,\left(ac^3\right)^{\frac{1}{4}}c^2d^2 - \left(ac^3\right)^{\frac{3}{4}}e^2\right)\ln\left(x^2 + \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{32\,a^2c^3}$$
$$- \frac{\sqrt{2}\left(3\,\left(ac^3\right)^{\frac{1}{4}}c^2d^2 - \left(ac^3\right)^{\frac{3}{4}}e^2\right)\ln\left(x^2 - \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{32\,a^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)^2/(c*x^4 + a)^2,x, algorithm="giac")
```

```
[Out] 1/4*(x^3*e^2 + 2*d*x^2*e + d^2*x)/((c*x^4 + a)*a) + 1/16*sqrt(2)*
(4*sqrt(2)*sqrt(a*c)*c^2*d*e + 3*(a*c^3)^(1/4)*c^2*d^2 + (a*c^3)^
(3/4)*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(
1/4))/(a^2*c^3) + 1/16*sqrt(2)*(4*sqrt(2)*sqrt(a*c)*c^2*d*e + 3*(
a*c^3)^(1/4)*c^2*d^2 + (a*c^3)^(3/4)*e^2)*arctan(1/2*sqrt(2)*(2*x
 - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^3) + 1/32*sqrt(2)*(3*
(a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(3/4)*e^2)*ln(x^2 + sqrt(2)*x*(a/
c)^(1/4) + sqrt(a/c))/(a^2*c^3) - 1/32*sqrt(2)*(3*(a*c^3)^(1/4)*c
^2*d^2 - (a*c^3)^(3/4)*e^2)*ln(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt
(a/c))/(a^2*c^3)
```

3.403 $\int \frac{d+ex}{(a+cx^4)^2}\,dx$

Optimal. Leaf size=241

$$-\frac{3d\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}}+\frac{3d\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}}-\frac{3d\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}}+\frac{3d\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}}+\frac{e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{c}}+\frac{x(d+ex)}{4a\left(a+cx^4\right)}$$

```
[Out] (x*(d + e*x))/(4*a*(a + c*x^4)) + (e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]
])/(4*a^(3/2)*Sqrt[c]) - (3*d*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1
/4)])/(8*Sqrt[2]*a^(7/4)*c^(1/4)) + (3*d*ArcTan[1 + (Sqrt[2]*c^(1
/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(1/4)) - (3*d*Log[Sqrt[a] -
 Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^
(1/4)) + (3*d*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x
^2])/(16*Sqrt[2]*a^(7/4)*c^(1/4))
```

Rubi [A] time = 0.424374, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667

$$-\frac{3d\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}}+\frac{3d\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}}-\frac{3d\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}}+\frac{3d\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}}+\frac{e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{c}}+\frac{x(d+ex)}{4a\left(a+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x)/(a + c*x^4)^2,x]
```

```
[Out] (x*(d + e*x))/(4*a*(a + c*x^4)) + (e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]
])/(4*a^(3/2)*Sqrt[c]) - (3*d*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1
/4)])/(8*Sqrt[2]*a^(7/4)*c^(1/4)) + (3*d*ArcTan[1 + (Sqrt[2]*c^(1
/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(1/4)) - (3*d*Log[Sqrt[a] -
 Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^
(1/4)) + (3*d*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x
^2])/(16*Sqrt[2]*a^(7/4)*c^(1/4))
```

Rubi in Sympy [A] time = 82.947, size = 231, normalized size = 0.96

$$\frac{x\left(d+ex\right)}{4a\left(a+cx^4\right)}+\frac{e\,\mathrm{atan}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{\frac{3}{2}}\sqrt{c}}-\frac{3\sqrt{2}d\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{32a^{\frac{7}{4}}\sqrt[4]{c}}+\frac{3\sqrt{2}d\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{32a^{\frac{7}{4}}\sqrt[4]{c}}-\frac{3\sqrt{2}d\,\mathrm{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{16a^{\frac{7}{4}}\sqrt[4]{c}}+\frac{3\sqrt{2}d\,\mathrm{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{16a^{\frac{7}{4}}\sqrt[4]{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x+d)/(c*x**4+a)**2,x)
```

```
[Out] x*(d + e*x)/(4*a*(a + c*x**4)) + e*atan(sqrt(c)*x**2/sqrt(a))/(4*
a**(3/2)*sqrt(c)) - 3*sqrt(2)*d*log(-sqrt(2)*a**(1/4)*c**(1/4)*x
+ sqrt(a) + sqrt(c)*x**2)/(32*a**(7/4)*c**(1/4)) + 3*sqrt(2)*d*lo
```

```
g(sqrt(2)*a**(1/4)*c**(1/4)*x + sqrt(a) + sqrt(c)*x**2)/(32*a**(7
/4)*c**(1/4)) - 3*sqrt(2)*d*atan(1 - sqrt(2)*c**(1/4)*x/a**(1/4))
/(16*a**(7/4)*c**(1/4)) + 3*sqrt(2)*d*atan(1 + sqrt(2)*c**(1/4)*x
/a**(1/4))/(16*a**(7/4)*c**(1/4))
```

Mathematica [A] time = 0.35193, size = 224, normalized size = 0.93

$$\frac{\frac{8a^{3/4}x(d+ex)}{a+cx^4}-\frac{2\left(4\sqrt[4]{a}e+3\sqrt{2}\sqrt[4]{c}d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{c}}+\frac{2\left(3\sqrt{2}\sqrt[4]{c}d-4\sqrt[4]{a}e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{c}}-\frac{3\sqrt{2}d\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{\sqrt[4]{c}}+\frac{3\sqrt{2}d\log\left(\sqrt{\right.}}{}}{32a^{7/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x)/(a + c*x^4)^2,x]
```

```
[Out] ((8*a^(3/4)*x*(d + e*x))/(a + c*x^4) - (2*(3*Sqrt[2]*c^(1/4)*d +
4*a^(1/4)*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/Sqrt[c] + (
2*(3*Sqrt[2]*c^(1/4)*d - 4*a^(1/4)*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)
*x)/a^(1/4)])/Sqrt[c] - (3*Sqrt[2]*d*Log[Sqrt[a] - Sqrt[2]*a^(1/4
)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/4) + (3*Sqrt[2]*d*Log[Sqrt[a] +
Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/4))/(32*a^(7/4))
```

Maple [A] time = 0.006, size = 188, normalized size = 0.8

$$\frac{dx}{4\,a\,(cx^4+a)}+\frac{3\,d\sqrt{2}}{32\,a^2}\sqrt[4]{\frac{a}{c}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^2-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)$$
$$+\frac{3\,d\sqrt{2}}{16\,a^2}\sqrt[4]{\frac{a}{c}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}+1\right)+\frac{3\,d\sqrt{2}}{16\,a^2}\sqrt[4]{\frac{a}{c}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}-1\right)$$
$$+\frac{ex^2}{4\,a\,(cx^4+a)}+\frac{e}{4\,a}\arctan\left(x^2\sqrt{\frac{c}{a}}\right)\frac{1}{\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x+d)/(c*x^4+a)^2,x)
```

```
[Out] 1/4*d*x/a/(c*x^4+a)+3/32*d/a^2*(1/c*a)^(1/4)*2^(1/2)*ln((x^2+(1/c
*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2-(1/c*a)^(1/4)*x*2^(1/2)+(
1/c*a)^(1/2)))+3/16*d/a^2*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1
/c*a)^(1/4)*x+1)+3/16*d/a^2*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/
(1/c*a)^(1/4)*x-1)+1/4*e*x^2/a/(c*x^4+a)+1/4*e/a/(a*c)^(1/2)*arct
an(x^2*(c/a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)/(c*x^4 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)/(c*x^4 + a)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 4.99889, size = 155, normalized size = 0.64

$$\operatorname{RootSum}\left(65536t^4a^7c^2 + 2048t^2a^4ce^2 - 1152ta^2cd^2e + 16ae^4 + 81cd^4, \left(t \mapsto t\log\left(x + \frac{-32768t^3a^6ce^2 - 4608t^2a^4cd^2e - 51}{192ade^4 -}\right.\right.\right.$$
$$+ \frac{dx + ex^2}{4a^2 + 4acx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x+d)/(c*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**7*c**2 + 2048*_t**2*a**4*c*e**2 - 1152*_t*
a**2*c*d**2*e + 16*a*e**4 + 81*c*d**4, Lambda(_t, _t*log(x + (-32
768*_t**3*a**6*c*e**2 - 4608*_t**2*a**4*c*d**2*e - 512*_t*a**3*e*
*4 - 1296*_t*a**2*c*d**4 + 360*a*d**2*e**3)/(192*a*d*e**4 - 243*c
*d**5)))) + (d*x + e*x**2)/(4*a**2 + 4*a*c*x**4)
```

GIAC/XCAS [A] time = 0.270282, size = 325, normalized size = 1.35

$$\frac{3\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}d\ln\left(x^2 + \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{32\,a^2c} - \frac{3\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}d\ln\left(x^2 - \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{32\,a^2c} + \frac{x^2e + dx}{4\left(cx^4 + a\right)a} + \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ac}ce + 3\left(ac^3\right)^{\frac{1}{4}}cd\right)\arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c^2} + \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ac}ce + 3\left(ac^3\right)^{\frac{1}{4}}cd\right)\arctan\left(\frac{\sqrt{2}\left(2x - \sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)/(c*x^4 + a)^2,x, algorithm="giac")
```

```
[Out] 3/32*sqrt(2)*(a*c^3)^(1/4)*d*ln(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqr
t(a/c))/(a^2*c) - 3/32*sqrt(2)*(a*c^3)^(1/4)*d*ln(x^2 - sqrt(2)*x
*(a/c)^(1/4) + sqrt(a/c))/(a^2*c) + 1/4*(x^2*e + d*x)/((c*x^4 + a
)*a) + 1/16*sqrt(2)*(2*sqrt(2)*sqrt(a*c)*c*e + 3*(a*c^3)^(1/4)*c*
d)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a
^2*c^2) + 1/16*sqrt(2)*(2*sqrt(2)*sqrt(a*c)*c*e + 3*(a*c^3)^(1/4)
*c*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))
/(a^2*c^2)
```

3.404 $\int \frac{1}{(a+cx^4)^2} dx$

Optimal. Leaf size=202

$$-\frac{3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}}+\frac{3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}}+\frac{x}{4a\left(a+cx^4\right)}$$

```
[Out] x/(4*a*(a + c*x^4)) - (3*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])
/(8*Sqrt[2]*a^(7/4)*c^(1/4)) + (3*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/
a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(1/4)) - (3*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(1/4)) +
(3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sq
rt[2]*a^(7/4)*c^(1/4))
```

Rubi [A] time = 0.255305, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778

$$-\frac{3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}}+\frac{3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}}+\frac{x}{4a\left(a+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + c*x^4)^(-2),x]
```

```
[Out] x/(4*a*(a + c*x^4)) - (3*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])
/(8*Sqrt[2]*a^(7/4)*c^(1/4)) + (3*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/
a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(1/4)) - (3*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(1/4)) +
(3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sq
rt[2]*a^(7/4)*c^(1/4))
```

Rubi in Sympy [A] time = 55.0951, size = 190, normalized size = 0.94

$$\frac{x}{4a\left(a+cx^4\right)}-\frac{3\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{32a^{\frac{7}{4}}\sqrt[4]{c}}+\frac{3\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{32a^{\frac{7}{4}}\sqrt[4]{c}}-\frac{3\sqrt{2}\,\mathrm{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{16a^{\frac{7}{4}}\sqrt[4]{c}}+\frac{3\sqrt{2}\,\mathrm{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{16a^{\frac{7}{4}}\sqrt[4]{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c*x**4+a)**2,x)
```

```
[Out] x/(4*a*(a + c*x**4)) - 3*sqrt(2)*log(-sqrt(2)*a**(1/4)*c**(1/4)*x
 + sqrt(a) + sqrt(c)*x**2)/(32*a**(7/4)*c**(1/4)) + 3*sqrt(2)*log
(sqrt(2)*a**(1/4)*c**(1/4)*x + sqrt(a) + sqrt(c)*x**2)/(32*a**(7/
4)*c**(1/4)) - 3*sqrt(2)*atan(1 - sqrt(2)*c**(1/4)*x/a**(1/4))/(1
6*a**(7/4)*c**(1/4)) + 3*sqrt(2)*atan(1 + sqrt(2)*c**(1/4)*x/a**(
```

```
1/4))/(16*a**(7/4)*c**(1/4))
```

Mathematica [A] time = 0.242037, size = 183, normalized size = 0.91

$$\frac{\frac{8a^{3/4}x}{a+cx^4}-\frac{3\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{\sqrt[4]{c}}+\frac{3\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{\sqrt[4]{c}}-\frac{6\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt[4]{c}}+\frac{6\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt[4]{c}}}{32a^{7/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + c*x^4)^(-2),x]
```

```
[Out] ((8*a^(3/4)*x)/(a + c*x^4) - (6*Sqrt[2]*ArcTan[1 - (Sqrt[2]*c^(1/
4)*x)/a^(1/4)])/c^(1/4) + (6*Sqrt[2]*ArcTan[1 + (Sqrt[2]*c^(1/4)*
x)/a^(1/4)])/c^(1/4) - (3*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c
^(1/4)*x + Sqrt[c]*x^2])/c^(1/4) + (3*Sqrt[2]*Log[Sqrt[a] + Sqrt[
2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/4))/(32*a^(7/4))
```

Maple [A] time = 0.002, size = 143, normalized size = 0.7

$$\frac{x}{4\,a\,(cx^4+a)}+\frac{3\,\sqrt{2}}{32\,a^2}\sqrt[4]{\frac{a}{c}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^2-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)+\frac{3\,\sqrt{2}}{16\,a^2}\sqrt[4]{\frac{a}{c}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}+1\right)+\frac{3\,\sqrt{2}}{16\,a^2}\sqrt[4]{\frac{a}{c}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c*x^4+a)^2,x)
```

```
[Out] 1/4*x/a/(c*x^4+a)+3/32/a^2*(1/c*a)^(1/4)*2^(1/2)*ln((x^2+(1/c*a)^
(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*
a)^(1/2)))+3/16/a^2*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^
(1/4)*x+1)+3/16/a^2*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^
(1/4)*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)^(-2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.281534, size = 213, normalized size = 1.05

$$-\frac{12\left(acx^4+a^2\right)\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}}\arctan\left(\frac{a^2\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}}}{x+\sqrt{a^4\sqrt{-\frac{1}{a^7c}}+x^2}}\right)-3\left(acx^4+a^2\right)\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}}\log\left(a^2\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}}+x\right)+3\left(acx^4+a^2\right)\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}}}{16\left(acx^4+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)^(-2),x, algorithm="fricas")
```

```
[Out] -1/16*(12*(a*c*x^4 + a^2)*(-1/(a^7*c))^(1/4)*arctan(a^2*(-1/(a^7*
c))^(1/4)/(x + sqrt(a^4*sqrt(-1/(a^7*c)) + x^2))) - 3*(a*c*x^4 +
a^2)*(-1/(a^7*c))^(1/4)*log(a^2*(-1/(a^7*c))^(1/4) + x) + 3*(a*c*
x^4 + a^2)*(-1/(a^7*c))^(1/4)*log(-a^2*(-1/(a^7*c))^(1/4) + x) -
4*x)/(a*c*x^4 + a^2)
```

Sympy [A] time = 2.01766, size = 39, normalized size = 0.19

$$\frac{x}{4a^2 + 4acx^4} + \operatorname{RootSum}\left(65536t^4a^7c + 81, \left(t \mapsto t\log\left(\frac{16ta^2}{3} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x**4+a)**2,x)
```

```
[Out] x/(4*a**2 + 4*a*c*x**4) + RootSum(65536*_t**4*a**7*c + 81, Lambda
(_t, _t*log(16*_t*a**2/3 + x)))
```

GIAC/XCAS [A] time = 0.261857, size = 262, normalized size = 1.3

$$\frac{x}{4\,(cx^4 + a)a} + \frac{3\,\sqrt{2}\,\left(ac^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x + \sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c} + \frac{3\,\sqrt{2}\,\left(ac^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x - \sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c} + \frac{3\,\sqrt{2}\,\left(ac^3\right)^{\frac{1}{4}}\ln\left(x^2 + \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{32\,a^2c} - \frac{3\,\sqrt{2}\,\left(ac^3\right)^{\frac{1}{4}}\ln\left(x^2 - \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{32\,a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)^(-2),x, algorithm="giac")
```

```
[Out] 1/4*x/((c*x^4 + a)*a) + 3/16*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqr
t(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c) + 3/16*sqrt
(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/
(a/c)^(1/4))/(a^2*c) + 3/32*sqrt(2)*(a*c^3)^(1/4)*ln(x^2 + sqrt(2
)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c) - 3/32*sqrt(2)*(a*c^3)^(1/4)
*ln(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c)
```

3.405 $\int \frac{1}{(d+ex)(a+cx^4)^2}\,dx$

Optimal. Leaf size=855

$$\frac{\log(d+ex)e^7}{(cd^4+ae^4)^2}-\frac{\log\left(cx^4+a\right)e^7}{4\left(cd^4+ae^4\right)^2}-\frac{\sqrt{c}d^2\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)e^5}{2\sqrt{a}\left(cd^4+ae^4\right)^2}$$
$$-\frac{\sqrt[4]{c}d\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)e^4}{2\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^2}+\frac{\sqrt[4]{c}d\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)e^4}{2\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^2}$$
$$-\frac{\sqrt[4]{c}d\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{c}x^2-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)e^4}{4\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^2}$$
$$+\frac{\sqrt[4]{c}d\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{c}x^2+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)e^4}{4\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^2}$$
$$-\frac{\sqrt{c}d^2\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)e}{4a^{3/2}\left(cd^4+ae^4\right)}+\frac{ae^3+cx\left(d^3-exd^2+e^2x^2d\right)}{4a\left(cd^4+ae^4\right)\left(cx^4+a\right)}$$
$$-\frac{\sqrt[4]{c}d\left(3\sqrt{c}d^2+\sqrt{a}e^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\left(cd^4+ae^4\right)}+\frac{\sqrt[4]{c}d\left(3\sqrt{c}d^2+\sqrt{a}e^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}\left(cd^4+ae^4\right)}$$
$$-\frac{\sqrt[4]{c}d\left(3\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{c}x^2-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)}{16\sqrt{2}a^{7/4}\left(cd^4+ae^4\right)}$$
$$+\frac{\sqrt[4]{c}d\left(3\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{c}x^2+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)}{16\sqrt{2}a^{7/4}\left(cd^4+ae^4\right)}$$

```
[Out] (a*e^3 + c*x*(d^3 - d^2*e*x + d*e^2*x^2))/(4*a*(c*d^4 + a*e^4)*(a
 + c*x^4)) - (Sqrt[c]*d^2*e^5*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*S
qrt[a]*(c*d^4 + a*e^4)^2) - (Sqrt[c]*d^2*e*ArcTan[(Sqrt[c]*x^2)/S
qrt[a]])/(4*a^(3/2)*(c*d^4 + a*e^4)) - (c^(1/4)*d*e^4*(Sqrt[c]*d^
2 + Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt
[2]*a^(3/4)*(c*d^4 + a*e^4)^2) - (c^(1/4)*d*(3*Sqrt[c]*d^2 + Sqrt
[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7
/4)*(c*d^4 + a*e^4)) + (c^(1/4)*d*e^4*(Sqrt[c]*d^2 + Sqrt[a]*e^2)
*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d
^4 + a*e^4)^2) + (c^(1/4)*d*(3*Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[
1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*e
^4)) + (e^7*Log[d + e*x])/(c*d^4 + a*e^4)^2 - (c^(1/4)*d*e^4*(Sqr
t[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x +
 Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2) - (c^(1/4)*d
*(3*Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1
/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)) + (c^(
1/4)*d*e^4*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1
/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^
2) + (c^(1/4)*d*(3*Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[
2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^4 +
 a*e^4)) - (e^7*Log[a + c*x^4])/(4*(c*d^4 + a*e^4)^2)
```

Rubi [A] time = 1.99194, antiderivative size = 855, normalized size of antiderivative = 1., number of

steps used = 31, number of rules used = 13, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.765$

$$\frac{\log(d+ex)e^7}{(cd^4+ae^4)^2}-\frac{\log(cx^4+a)\,e^7}{4(cd^4+ae^4)^2}-\frac{\sqrt{c}d^2\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)e^5}{2\sqrt{a}(cd^4+ae^4)^2}-\frac{\sqrt[4]{c}d\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)e^4}{2\sqrt{2}a^{3/4}(cd^4+ae^4)^2}+\frac{\sqrt[4]{c}d\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)e^4}{2\sqrt{2}a^{3/4}(cd^4+ae^4)^2}-\frac{\sqrt[4]{c}d\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{c}x^2-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)e^4}{4\sqrt{2}a^{3/4}(cd^4+ae^4)^2}+\frac{\sqrt[4]{c}d\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{c}x^2+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)e^4}{4\sqrt{2}a^{3/4}(cd^4+ae^4)^2}-\frac{\sqrt{c}d^2\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)e}{4a^{3/2}(cd^4+ae^4)}+\frac{ae^3+cx\left(d^3-exd^2+e^2x^2d\right)}{4a(cd^4+ae^4)(cx^4+a)}-\frac{\sqrt[4]{c}d\left(3\sqrt{c}d^2+\sqrt{a}e^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}(cd^4+ae^4)}+\frac{\sqrt[4]{c}d\left(3\sqrt{c}d^2+\sqrt{a}e^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}(cd^4+ae^4)}-\frac{\sqrt[4]{c}d\left(3\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{c}x^2-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)}{16\sqrt{2}a^{7/4}(cd^4+ae^4)}+\frac{\sqrt[4]{c}d\left(3\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{c}x^2+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)}{16\sqrt{2}a^{7/4}(cd^4+ae^4)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d + e*x)*(a + c*x^4)^2),x]
```

```
[Out] (a*e^3 + c*x*(d^3 - d^2*e*x + d*e^2*x^2))/(4*a*(c*d^4 + a*e^4)*(a
 + c*x^4)) - (Sqrt[c]*d^2*e^5*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*S
qrt[a]*(c*d^4 + a*e^4)^2) - (Sqrt[c]*d^2*e*ArcTan[(Sqrt[c]*x^2)/S
qrt[a]])/(4*a^(3/2)*(c*d^4 + a*e^4)) - (c^(1/4)*d*e^4*(Sqrt[c]*d^
2 + Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt
[2]*a^(3/4)*(c*d^4 + a*e^4)^2) - (c^(1/4)*d*(3*Sqrt[c]*d^2 + Sqrt
[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7
/4)*(c*d^4 + a*e^4)) + (c^(1/4)*d*e^4*(Sqrt[c]*d^2 + Sqrt[a]*e^2)
*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d
^4 + a*e^4)^2) + (c^(1/4)*d*(3*Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[
1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*e
^4)) + (e^7*Log[d + e*x])/(c*d^4 + a*e^4)^2 - (c^(1/4)*d*e^4*(Sqr
t[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x +
 Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2) - (c^(1/4)*d
*(3*Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1
/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)) + (c^(
1/4)*d*e^4*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1
/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^
2) + (c^(1/4)*d*(3*Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[
2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^4 +
 a*e^4)) - (e^7*Log[a + c*x^4])/(4*(c*d^4 + a*e^4)^2)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(a+cx^4)^2\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x+d)/(c*x**4+a)**2,x)
```

```
[Out] Integral(1/((a + c*x**4)**2*(d + e*x)), x)
```

Mathematica [A] time = 0.722225, size = 558, normalized size = 0.65

$$\frac{\frac{\sqrt{2}\sqrt[4]{c}\left(5a^{3/2}de^6+\sqrt{a}cd^5e^2-7a\sqrt{c}d^3e^4-3c^{3/2}d^7\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{a^{7/4}}+\frac{\sqrt{2}\sqrt[4]{c}\left(-5a^{3/2}de^6-\sqrt{a}cd^5e^2+7a\sqrt{c}d^3e^4+3c^{3/2}d^7\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\cdots\right.}{a^{7/4}}}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d + e*x)*(a + c*x^4)^2),x]
```

```
[Out] ((8*(c*d^4 + a*e^4)*(a*e^3 + c*d*x*(d^2 - d*e*x + e^2*x^2)))/(a*(
a + c*x^4)) - (2*c^(1/4)*d*(3*Sqrt[2]*c^(3/2)*d^6 - 4*a^(1/4)*c^(
5/4)*d^5*e + Sqrt[2]*Sqrt[a]*c*d^4*e^2 + 7*Sqrt[2]*a*Sqrt[c]*d^2*
e^4 - 12*a^(5/4)*c^(1/4)*d*e^5 + 5*Sqrt[2]*a^(3/2)*e^6)*ArcTan[1
- (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(7/4) + (2*c^(1/4)*d*(3*Sqrt[2]
*c^(3/2)*d^6 + 4*a^(1/4)*c^(5/4)*d^5*e + Sqrt[2]*Sqrt[a]*c*d^4*e^
2 + 7*Sqrt[2]*a*Sqrt[c]*d^2*e^4 + 12*a^(5/4)*c^(1/4)*d*e^5 + 5*Sq
rt[2]*a^(3/2)*e^6)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(7/
4) + 32*e^7*Log[d + e*x] + (Sqrt[2]*c^(1/4)*(-3*c^(3/2)*d^7 + Sqr
t[a]*c*d^5*e^2 - 7*a*Sqrt[c]*d^3*e^4 + 5*a^(3/2)*d*e^6)*Log[Sqrt[
a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(7/4) + (Sqrt[2]
*c^(1/4)*(3*c^(3/2)*d^7 - Sqrt[a]*c*d^5*e^2 + 7*a*Sqrt[c]*d^3*e^4
 - 5*a^(3/2)*d*e^6)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqr
t[c]*x^2])/a^(7/4) - 8*e^7*Log[a + c*x^4])/(32*(c*d^4 + a*e^4)^2)
```

Maple [A] time = 0.038, size = 1126, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*x+d)/(c*x^4+a)^2,x)
```

```
[Out] 1/4/(a*e^4+c*d^4)^2*c/(c*x^4+a)*x^3*d*e^6+1/4/(a*e^4+c*d^4)^2*c^2
/(c*x^4+a)*e^2*d^5/a*x^3-1/4/(a*e^4+c*d^4)^2*c/(c*x^4+a)*x^2*d^2*
e^5-1/4/(a*e^4+c*d^4)^2*c^2/(c*x^4+a)*d^6*e/a*x^2+1/4/(a*e^4+c*d^
4)^2*c/(c*x^4+a)*x*d^3*e^4+1/4/(a*e^4+c*d^4)^2*c^2/(c*x^4+a)*d^7/
a*x+1/4/(a*e^4+c*d^4)^2/(c*x^4+a)*e^7*a+1/4/(a*e^4+c*d^4)^2*c/(c*
x^4+a)*e^3*d^4+7/16/(a*e^4+c*d^4)^2*c/a*(1/c*a)^(1/4)*2^(1/2)*arc
tan(2^(1/2)/(1/c*a)^(1/4)*x+1)*d^3*e^4+3/16/(a*e^4+c*d^4)^2*c^2/a
^2*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*d^7+7/
16/(a*e^4+c*d^4)^2*c/a*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*
a)^(1/4)*x-1)*d^3*e^4+3/16/(a*e^4+c*d^4)^2*c^2/a^2*(1/c*a)^(1/4)*
2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*d^7+7/32/(a*e^4+c*d^4)^
2*c/a*(1/c*a)^(1/4)*2^(1/2)*ln((x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*
a)^(1/2))/(x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))*d^3*e^4+3/
32/(a*e^4+c*d^4)^2*c^2/a^2*(1/c*a)^(1/4)*2^(1/2)*ln((x^2+(1/c*a)^
(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*
a)^(1/2)))*d^7-3/4/(a*e^4+c*d^4)^2*c/(a^3*c)^(1/2)*arctan(x^2*(c/
a)^(1/2))*a*d^2*e^5-1/4/(a*e^4+c*d^4)^2*c^2/(a^3*c)^(1/2)*arctan(
x^2*(c/a)^(1/2))*d^6*e+5/32/(a*e^4+c*d^4)^2/(1/c*a)^(1/4)*2^(1/2)
*ln((x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2+(1/c*a)^(1/4
)*x*2^(1/2)+(1/c*a)^(1/2)))*d*e^6+1/32/(a*e^4+c*d^4)^2*c/a/(1/c*a
)^(1/4)*2^(1/2)*ln((x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x
^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))*d^5*e^2+5/16/(a*e^4+c*
d^4)^2/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*d*
e^6+1/16/(a*e^4+c*d^4)^2*c/a/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)
/(1/c*a)^(1/4)*x+1)*d^5*e^2+5/16/(a*e^4+c*d^4)^2/(1/c*a)^(1/4)*2^
(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*d*e^6+1/16/(a*e^4+c*d^4)^
2*c/a/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*d^5
```

```
*e^2-1/4/(a*e^4+c*d^4)^2*e^7*ln(a*(c*x^4+a))+e^7*ln(e*x+d)/(a*e^4
+c*d^4)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)^2*(e*x + d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)^2*(e*x + d)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*x+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.322321, size = 1038, normalized size = 1.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)^2*(e*x + d)),x, algorithm="giac")
```

```
[Out] 1/8*(4*sqrt(2)*sqrt(a*c)*c^3*d^2*e + 3*(a*c^3)^(1/4)*c^3*d^3 + 5*
(a*c^3)^(3/4)*c*d*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1
/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^4*d^4 - 4*(a*c^3)^(1/4)*a^2*c^3*
d^3*e + 4*sqrt(2)*sqrt(a*c)*a^2*c^3*d^2*e^2 + sqrt(2)*a^3*c^3*e^4
 - 4*(a*c^3)^(3/4)*a^2*c*d*e^3) + 1/8*(4*sqrt(2)*sqrt(a*c)*c^3*d^
2*e + 3*(a*c^3)^(1/4)*c^3*d^3 + 5*(a*c^3)^(3/4)*c*d*e^2)*arctan(1
/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*
c^4*d^4 + 4*(a*c^3)^(1/4)*a^2*c^3*d^3*e + 4*sqrt(2)*sqrt(a*c)*a^2
*c^3*d^2*e^2 + sqrt(2)*a^3*c^3*e^4 + 4*(a*c^3)^(3/4)*a^2*c*d*e^3)
 + 1/16*(3*(a*c^3)^(1/4)*c^3*d^7 - (a*c^3)^(3/4)*c*d^5*e^2 + 7*(a
*c^3)^(1/4)*a*c^2*d^3*e^4 - 5*(a*c^3)^(3/4)*a*d*e^6)*ln(x^2 + sqr
t(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^2*c^4*d^8 + 2*sqrt(2)*
```

```
a^3*c^3*d^4*e^4 + sqrt(2)*a^4*c^2*e^8) - 1/16*(3*(a*c^3)^(1/4)*c^
3*d^7 - (a*c^3)^(3/4)*c*d^5*e^2 + 7*(a*c^3)^(1/4)*a*c^2*d^3*e^4 -
 5*(a*c^3)^(3/4)*a*d*e^6)*ln(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a
/c))/(sqrt(2)*a^2*c^4*d^8 + 2*sqrt(2)*a^3*c^3*d^4*e^4 + sqrt(2)*a
^4*c^2*e^8) - 1/4*e^7*ln(abs(c*x^4 + a))/(c^2*d^8 + 2*a*c*d^4*e^4
 + a^2*e^8) + e^8*ln(abs(x*e + d))/(c^2*d^8*e + 2*a*c*d^4*e^5 + a
^2*e^9) + 1/4*(a*c*d^4*e^3 + (c^2*d^5*e^2 + a*c*d*e^6)*x^3 - (c^2
*d^6*e + a*c*d^2*e^5)*x^2 + a^2*e^7 + (c^2*d^7 + a*c*d^3*e^4)*x)/
((c*d^4 + a*e^4)^2*(c*x^4 + a)*a)
```

3.406 $\int \frac{1}{(d+ex)^2(a+cx^4)^2}\,dx$

Optimal. Leaf size=1141

result too large to display

```
[Out] -(e^7/((c*d^4 + a*e^4)^2*(d + e*x))) + (c*(4*a*d^3*e^3 + x*(d^2*(
c*d^4 - 3*a*e^4) - 2*d*e*(c*d^4 - a*e^4)*x + e^2*(3*c*d^4 - a*e^4
)*x^2)))/(4*a*(c*d^4 + a*e^4)^2*(a + c*x^4)) - (Sqrt[c]*d*e^5*(3*
c*d^4 - a*e^4)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(Sqrt[a]*(c*d^4 + a
*e^4)^3) - (Sqrt[c]*d*e*(c*d^4 - a*e^4)*ArcTan[(Sqrt[c]*x^2)/Sqrt
[a]])/(2*a^(3/2)*(c*d^4 + a*e^4)^2) - (c^(1/4)*(3*Sqrt[c]*d^2*(c*
d^4 - 3*a*e^4) + Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*ArcTan[1 - (Sqrt[
2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^2) - (
c^(1/4)*e^4*(Sqrt[c]*d^2*(5*c*d^4 - 3*a*e^4) + Sqrt[a]*e^2*(7*c*d
^4 - a*e^4))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*
a^(3/4)*(c*d^4 + a*e^4)^3) + (c^(1/4)*(3*Sqrt[c]*d^2*(c*d^4 - 3*a
*e^4) + Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*ArcTan[1 + (Sqrt[2]*c^(1/4
)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)*e
^4*(Sqrt[c]*d^2*(5*c*d^4 - 3*a*e^4) + Sqrt[a]*e^2*(7*c*d^4 - a*e^
4))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(
c*d^4 + a*e^4)^3) + (8*c*d^3*e^7*Log[d + e*x])/(c*d^4 + a*e^4)^3
- (c^(1/4)*(3*Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) - Sqrt[a]*e^2*(3*c*d^
4 - a*e^4))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2
])/(16*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^2) - (c^(1/4)*e^4*(Sqrt[c]
*d^2*(5*c*d^4 - 3*a*e^4) - Sqrt[a]*e^2*(7*c*d^4 - a*e^4))*Log[Sqr
t[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/
4)*(c*d^4 + a*e^4)^3) + (c^(1/4)*(3*Sqrt[c]*d^2*(c*d^4 - 3*a*e^4)
 - Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c
^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^2) +
 (c^(1/4)*e^4*(Sqrt[c]*d^2*(5*c*d^4 - 3*a*e^4) - Sqrt[a]*e^2*(7*c
*d^4 - a*e^4))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*
x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^3) - (2*c*d^3*e^7*Log[a
+ c*x^4])/(c*d^4 + a*e^4)^3
```

Rubi [A] time = 3.95327, antiderivative size = 1141, normalized size of antiderivative = 1., number

of steps used = 31, number of rules used = 13, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.765

$$\frac{8cd^3\log(d+ex)e^7}{(cd^4+ae^4)^3}-\frac{2cd^3\log\left(cx^4+a\right)e^7}{(cd^4+ae^4)^3}-\frac{e^7}{(cd^4+ae^4)^2(d+ex)}-\frac{\sqrt{c}d\left(3cd^4-ae^4\right)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)e^5}{\sqrt{a}(cd^4+ae^4)^3}-\frac{\sqrt[4]{c}\left(\sqrt{c}\left(5cd^4-3ae^4\right)d^2+\sqrt{a}e^2\left(7cd^4-ae^4\right)\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)e^4}{2\sqrt{2}a^{3/4}(cd^4+ae^4)^3}+\frac{\sqrt[4]{c}\left(\sqrt{c}\left(5cd^4-3ae^4\right)d^2+\sqrt{a}e^2\left(7cd^4-ae^4\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)e^4}{2\sqrt{2}a^{3/4}(cd^4+ae^4)^3}-\frac{\sqrt[4]{c}\left(\sqrt{c}d^2\left(5cd^4-3ae^4\right)-\sqrt{a}e^2\left(7cd^4-ae^4\right)\right)\log\left(\sqrt{c}x^2-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)e^4}{4\sqrt{2}a^{3/4}(cd^4+ae^4)^3}+\frac{\sqrt[4]{c}\left(\sqrt{c}d^2\left(5cd^4-3ae^4\right)-\sqrt{a}e^2\left(7cd^4-ae^4\right)\right)\log\left(\sqrt{c}x^2+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)e^4}{4\sqrt{2}a^{3/4}(cd^4+ae^4)^3}-\frac{\sqrt{c}d\left(cd^4-ae^4\right)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)e}{2a^{3/2}(cd^4+ae^4)^2}+\frac{c\left(4ad^3e^3+x\left(\left(cd^4-3ae^4\right)d^2-2e\left(cd^4-ae^4\right)xd+e^2\left(3cd^4-ae^4\right)x^2\right)\right)}{4a(cd^4+ae^4)^2(cx^4+a)}-\frac{\sqrt[4]{c}\left(3\sqrt{c}\left(cd^4-3ae^4\right)d^2+\sqrt{a}e^2\left(3cd^4-ae^4\right)\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}(cd^4+ae^4)^2}+\frac{\sqrt[4]{c}\left(3\sqrt{c}\left(cd^4-3ae^4\right)d^2+\sqrt{a}e^2\left(3cd^4-ae^4\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}(cd^4+ae^4)^2}-\frac{\sqrt[4]{c}\left(3\sqrt{c}d^2\left(cd^4-3ae^4\right)-\sqrt{a}e^2\left(3cd^4-ae^4\right)\right)\log\left(\sqrt{c}x^2-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)}{16\sqrt{2}a^{7/4}(cd^4+ae^4)^2}+\frac{\sqrt[4]{c}\left(3\sqrt{c}d^2\left(cd^4-3ae^4\right)-\sqrt{a}e^2\left(3cd^4-ae^4\right)\right)\log\left(\sqrt{c}x^2+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)}{16\sqrt{2}a^{7/4}(cd^4+ae^4)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d + e*x)^2*(a + c*x^4)^2),x]
```

```
[Out] -(e^7/((c*d^4 + a*e^4)^2*(d + e*x))) + (c*(4*a*d^3*e^3 + x*(d^2*(
c*d^4 - 3*a*e^4) - 2*d*e*(c*d^4 - a*e^4)*x + e^2*(3*c*d^4 - a*e^4
)*x^2)))/(4*a*(c*d^4 + a*e^4)^2*(a + c*x^4)) - (Sqrt[c]*d*e^5*(3*
c*d^4 - a*e^4)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(Sqrt[a]*(c*d^4 + a
*e^4)^3) - (Sqrt[c]*d*e*(c*d^4 - a*e^4)*ArcTan[(Sqrt[c]*x^2)/Sqrt
[a]])/(2*a^(3/2)*(c*d^4 + a*e^4)^2) - (c^(1/4)*(3*Sqrt[c]*d^2*(c*
d^4 - 3*a*e^4) + Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*ArcTan[1 - (Sqrt[
2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^2) - (
c^(1/4)*e^4*(Sqrt[c]*d^2*(5*c*d^4 - 3*a*e^4) + Sqrt[a]*e^2*(7*c*d
^4 - a*e^4))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*
a^(3/4)*(c*d^4 + a*e^4)^3) + (c^(1/4)*(3*Sqrt[c]*d^2*(c*d^4 - 3*a
*e^4) + Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*ArcTan[1 + (Sqrt[2]*c^(1/4
)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)*e
^4*(Sqrt[c]*d^2*(5*c*d^4 - 3*a*e^4) + Sqrt[a]*e^2*(7*c*d^4 - a*e^
4))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(
c*d^4 + a*e^4)^3) + (8*c*d^3*e^7*Log[d + e*x])/(c*d^4 + a*e^4)^3
 - (c^(1/4)*(3*Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) - Sqrt[a]*e^2*(3*c*d^
4 - a*e^4))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2
])/(16*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^2) - (c^(1/4)*e^4*(Sqrt[c]
*d^2*(5*c*d^4 - 3*a*e^4) - Sqrt[a]*e^2*(7*c*d^4 - a*e^4))*Log[Sqr
t[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/
4)*(c*d^4 + a*e^4)^3) + (c^(1/4)*(3*Sqrt[c]*d^2*(c*d^4 - 3*a*e^4)
 - Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c
```

```
^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^2) +
 (c^(1/4)*e^4*(Sqrt[c]*d^2*(5*c*d^4 - 3*a*e^4) - Sqrt[a]*e^2*(7*c
*d^4 - a*e^4))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*
x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^3) - (2*c*d^3*e^7*Log[a
+ c*x^4])/(c*d^4 + a*e^4)^3
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x+d)**2/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 1.73309, size = 807, normalized size = 0.71

$$\frac{256cd^3\log(d+ex)e^7 - 64cd^3\log\left(cx^4+a\right)e^7 - \frac{32\left(cd^4+ae^4\right)e^7}{d+ex} + \frac{8c\left(cd^4+ae^4\right)\left(cx\left(d^2-2exd+3e^2x^2\right)d^4+ae^3\left(4d^3-3exd^2+2e^2x^2d-e^3x^3\right)\right)}{a\left(cx^4+a\right)} + \cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d + e*x)^2*(a + c*x^4)^2),x]
```

```
[Out] ((-32*e^7*(c*d^4 + a*e^4))/(d + e*x) + (8*c*(c*d^4 + a*e^4)*(c*d^
4*x*(d^2 - 2*d*e*x + 3*e^2*x^2) + a*e^3*(4*d^3 - 3*d^2*e*x + 2*d*
e^2*x^2 - e^3*x^3)))/(a*(a + c*x^4)) + (2*c^(1/4)*(-3*Sqrt[2]*c^(
5/2)*d^10 + 8*a^(1/4)*c^(9/4)*d^9*e - 3*Sqrt[2]*Sqrt[a]*c^2*d^8*e
^2 - 14*Sqrt[2]*a*c^(3/2)*d^6*e^4 + 48*a^(5/4)*c^(5/4)*d^5*e^5 -
30*Sqrt[2]*a^(3/2)*c*d^4*e^6 + 21*Sqrt[2]*a^2*Sqrt[c]*d^2*e^8 - 2
4*a^(9/4)*c^(1/4)*d*e^9 + 5*Sqrt[2]*a^(5/2)*e^10)*ArcTan[1 - (Sqr
t[2]*c^(1/4)*x)/a^(1/4)])/a^(7/4) + (2*c^(1/4)*(3*Sqrt[2]*c^(5/2)
*d^10 + 8*a^(1/4)*c^(9/4)*d^9*e + 3*Sqrt[2]*Sqrt[a]*c^2*d^8*e^2 +
 14*Sqrt[2]*a*c^(3/2)*d^6*e^4 + 48*a^(5/4)*c^(5/4)*d^5*e^5 + 30*S
qrt[2]*a^(3/2)*c*d^4*e^6 - 21*Sqrt[2]*a^2*Sqrt[c]*d^2*e^8 - 24*a^
(9/4)*c^(1/4)*d*e^9 - 5*Sqrt[2]*a^(5/2)*e^10)*ArcTan[1 + (Sqrt[2]
*c^(1/4)*x)/a^(1/4)])/a^(7/4) + 256*c*d^3*e^7*Log[d + e*x] - (Sqr
t[2]*c^(1/4)*(3*c^(5/2)*d^10 - 3*Sqrt[a]*c^2*d^8*e^2 + 14*a*c^(3/
2)*d^6*e^4 - 30*a^(3/2)*c*d^4*e^6 - 21*a^2*Sqrt[c]*d^2*e^8 + 5*a^
(5/2)*e^10)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2
])/a^(7/4) + (Sqrt[2]*c^(1/4)*(3*c^(5/2)*d^10 - 3*Sqrt[a]*c^2*d^8
*e^2 + 14*a*c^(3/2)*d^6*e^4 - 30*a^(3/2)*c*d^4*e^6 - 21*a^2*Sqrt[
c]*d^2*e^8 + 5*a^(5/2)*e^10)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4
)*x + Sqrt[c]*x^2])/a^(7/4) - 64*c*d^3*e^7*Log[a + c*x^4])/(32*(c
*d^4 + a*e^4)^3)
```

Maple [A] time = 0.032, size = 1644, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*x+d)^2/(c*x^4+a)^2,x)
```

```
[Out] 7/8*c^2/(a*e^4+c*d^4)^3/a*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1
/c*a)^(1/4)*x+1)*d^6*e^4+3/2*c/(a*e^4+c*d^4)^3/(a^3*c)^(1/2)*arct
```

```
an(x^2*(c/a)^(1/2))*a^2*e^9*d+15/8*c/(a*e^4+c*d^4)^3/(1/c*a)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*d^4*e^6+15/16*c/(a*e^
4+c*d^4)^3/(1/c*a)^(1/4)*2^(1/2)*ln((x^2-(1/c*a)^(1/4)*x*2^(1/2)+
(1/c*a)^(1/2))/(x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))*d^4*e
^6+15/8*c/(a*e^4+c*d^4)^3/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1
/c*a)^(1/4)*x+1)*d^4*e^6+c^2/(a*e^4+c*d^4)^3/(c*x^4+a)*d^7*e^3+8*
c*d^3*e^7*ln(e*x+d)/(a*e^4+c*d^4)^3+7/16*c^2/(a*e^4+c*d^4)^3/a*(1
/c*a)^(1/4)*2^(1/2)*ln((x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)
)/(x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))*d^6*e^4+3/16*c^2/(
a*e^4+c*d^4)^3/a/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/
4)*x+1)*d^8*e^2+3/16*c^2/(a*e^4+c*d^4)^3/a/(1/c*a)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*d^8*e^2+3/32*c^2/(a*e^4+c*d^4)^
3/a/(1/c*a)^(1/4)*2^(1/2)*ln((x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)
^(1/2))/(x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))*d^8*e^2+7/8*
c^2/(a*e^4+c*d^4)^3/a*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a
)^(1/4)*x-1)*d^6*e^4-3/4*c/(a*e^4+c*d^4)^3/(c*x^4+a)*d^2*a*x*e^8+
3/4*c^3/(a*e^4+c*d^4)^3/(c*x^4+a)*e^2/a*x^3*d^8-21/32*c/(a*e^4+c*
d^4)^3*(1/c*a)^(1/4)*2^(1/2)*ln((x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c
*a)^(1/2))/(x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))*d^2*e^8+3
/32*c^3/(a*e^4+c*d^4)^3/a^2*(1/c*a)^(1/4)*2^(1/2)*ln((x^2+(1/c*a)
^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c
*a)^(1/2)))*d^10-3*c^2/(a*e^4+c*d^4)^3/(a^3*c)^(1/2)*arctan(x^2*(
c/a)^(1/2))*a*d^5*e^5-21/16*c/(a*e^4+c*d^4)^3*(1/c*a)^(1/4)*2^(1/
2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*d^2*e^8+3/16*c^3/(a*e^4+c*d^
4)^3/a^2*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*
d^10-21/16*c/(a*e^4+c*d^4)^3*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)
/(1/c*a)^(1/4)*x-1)*d^2*e^8+3/16*c^3/(a*e^4+c*d^4)^3/a^2*(1/c*a)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*d^10-2*c/(a*e^4+c
*d^4)^3*d^3*e^7*ln(a*(c*x^4+a))+c/(a*e^4+c*d^4)^3/(c*x^4+a)*a*d^3
*e^7+1/4*c^3/(a*e^4+c*d^4)^3/(c*x^4+a)*d^10/a*x-1/4*c/(a*e^4+c*d^
4)^3/(c*x^4+a)*e^10*a*x^3+1/2*c^2/(a*e^4+c*d^4)^3/(c*x^4+a)*e^6*x
^3*d^4-1/2*c^3/(a*e^4+c*d^4)^3/(a^3*c)^(1/2)*arctan(x^2*(c/a)^(1/
2))*d^9*e-1/2*c^3/(a*e^4+c*d^4)^3/(c*x^4+a)*d^9*e/a*x^2+1/2*c/(a*
e^4+c*d^4)^3/(c*x^4+a)*d*e^9*a*x^2-e^7/(a*e^4+c*d^4)^2/(e*x+d)-5/
32/(a*e^4+c*d^4)^3*a/(1/c*a)^(1/4)*2^(1/2)*ln((x^2-(1/c*a)^(1/4)*
x*2^(1/2)+(1/c*a)^(1/2))/(x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/
2)))*e^10-5/16/(a*e^4+c*d^4)^3*a/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(
1/2)/(1/c*a)^(1/4)*x+1)*e^10-5/16/(a*e^4+c*d^4)^3*a/(1/c*a)^(1/4)
*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*e^10-1/2*c^2/(a*e^4+c*
d^4)^3/(c*x^4+a)*d^6*x*e^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)^2*(e*x + d)^2),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)^2*(e*x + d)^2),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*x+d)**2/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.361535, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)^2*(e*x + d)^2),x, algorithm="giac")
```

```
[Out] Done
```

3.407 $\int \frac{1}{(d+ex)^3(a+cx^4)^2}\,dx$

Optimal. Leaf size=1384

result too large to display

```
[Out] -e^7/(2*(c*d^4 + a*e^4)^2*(d + e*x)^2) - (8*c*d^3*e^7)/((c*d^4 +
a*e^4)^3*(d + e*x)) + (c*(2*a*d^2*e^3*(5*c*d^4 - 3*a*e^4) + x*(d*
(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8) - e*(3*c^2*d^8 - 12*a*c*d^
4*e^4 + a^2*e^8)*x + 2*c*d^3*e^2*(3*c*d^4 - 5*a*e^4)*x^2)))/(4*a*
(c*d^4 + a*e^4)^3*(a + c*x^4)) - (Sqrt[c]*e^5*(21*c^2*d^8 - 26*a*
c*d^4*e^4 + a^2*e^8)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*(c
*d^4 + a*e^4)^4) - (Sqrt[c]*e*(3*c^2*d^8 - 12*a*c*d^4*e^4 + a^2*e
^8)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(3/2)*(c*d^4 + a*e^4)^3)
- (c^(3/4)*d*(3*c^2*d^8 - 36*a*c*d^4*e^4 + 9*a^2*e^8 + 2*Sqrt[a]*
Sqrt[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4))*ArcTan[1 - (Sqrt[2]*c^(1/4)*
x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^3) - (c^(3/4)*d*e
^4*(4*Sqrt[a]*Sqrt[c]*d^2*e^2*(7*c*d^4 - 5*a*e^4) + 3*(5*c^2*d^8
- 10*a*c*d^4*e^4 + a^2*e^8))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/
4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^4) + (c^(3/4)*d*(3*c^2*d^
8 - 36*a*c*d^4*e^4 + 9*a^2*e^8 + 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d
^4 - 5*a*e^4))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2
]*a^(7/4)*(c*d^4 + a*e^4)^3) + (c^(3/4)*d*e^4*(4*Sqrt[a]*Sqrt[c]*
d^2*e^2*(7*c*d^4 - 5*a*e^4) + 3*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2
*e^8))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4
)*(c*d^4 + a*e^4)^4) + (12*c*d^2*e^7*(3*c*d^4 - a*e^4)*Log[d + e*
x])/(c*d^4 + a*e^4)^4 - (c^(3/4)*d*(3*c^2*d^8 - 36*a*c*d^4*e^4 +
9*a^2*e^8 - 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4))*Log[Sq
rt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(
7/4)*(c*d^4 + a*e^4)^3) + (c^(3/4)*d*e^4*(4*Sqrt[a]*Sqrt[c]*d^2*e
^2*(7*c*d^4 - 5*a*e^4) - 3*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8)
)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt
[2]*a^(3/4)*(c*d^4 + a*e^4)^4) + (c^(3/4)*d*(3*c^2*d^8 - 36*a*c*d
^4*e^4 + 9*a^2*e^8 - 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4
))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sq
rt[2]*a^(7/4)*(c*d^4 + a*e^4)^3) - (c^(3/4)*d*e^4*(4*Sqrt[a]*Sqrt
[c]*d^2*e^2*(7*c*d^4 - 5*a*e^4) - 3*(5*c^2*d^8 - 10*a*c*d^4*e^4 +
 a^2*e^8))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]
)/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^4) - (3*c*d^2*e^7*(3*c*d^4 -
 a*e^4)*Log[a + c*x^4])/(c*d^4 + a*e^4)^4
```

Rubi [A] time = 4.66989, antiderivative size = 1384, normalized size of antiderivative = 1., number

of steps used = 31, number of rules used = 13, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.765$

$$\frac{12cd^2(3cd^4 - ae^4)\log(d+ex)e^7}{(cd^4+ae^4)^4} - \frac{3cd^2(3cd^4-ae^4)\log(cx^4+a)e^7}{(cd^4+ae^4)^4} - \frac{8cd^3e^7}{(cd^4+ae^4)^3(d+ex)} - \frac{e^7}{2(cd^4+ae^4)^2(d+ex)^2} - \frac{\sqrt{c}(21c^2d^8 - 26ace^4d^4 + a^2e^8)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)e^5}{2\sqrt{a}(cd^4+ae^4)^4} - \frac{c^{3/4}d\left(4\sqrt{a}\sqrt{c}d^2(7cd^4-5ae^4)e^2 + 3(5c^2d^8 - 10ace^4d^4 + a^2e^8)\right)\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)e^4}{2\sqrt{2}a^{3/4}(cd^4+ae^4)^4} + \frac{c^{3/4}d\left(4\sqrt{a}\sqrt{c}d^2(7cd^4-5ae^4)e^2 + 3(5c^2d^8 - 10ace^4d^4 + a^2e^8)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}} + 1\right)e^4}{2\sqrt{2}a^{3/4}(cd^4+ae^4)^4} + \frac{c^{3/4}d\left(4\sqrt{a}\sqrt{c}d^2e^2(7cd^4-5ae^4) - 3(5c^2d^8 - 10ace^4d^4 + a^2e^8)\right)\log\left(\sqrt{c}x^2 - \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a}\right)e^4}{4\sqrt{2}a^{3/4}(cd^4+ae^4)^4} - \frac{c^{3/4}d\left(4\sqrt{a}\sqrt{c}d^2e^2(7cd^4-5ae^4) - 3(5c^2d^8 - 10ace^4d^4 + a^2e^8)\right)\log\left(\sqrt{c}x^2 + \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a}\right)e^4}{4\sqrt{2}a^{3/4}(cd^4+ae^4)^4} - \frac{\sqrt{c}(3c^2d^8 - 12ace^4d^4 + a^2e^8)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)e}{4a^{3/2}(cd^4+ae^4)^3} + \frac{c\left(2ad^2(5cd^4-3ae^4)e^3 + x\left(2ce^2(3cd^4-5ae^4)x^2d^3 + (c^2d^8 - 12ace^4d^4 + 3a^2e^8)d - e(3c^2d^8 - 12ace^4d^4 + a^2e^8)x\right)\right)}{4a(cd^4+ae^4)^3(cx^4+a)} - \frac{c^{3/4}d\left(3c^2d^8 - 36ace^4d^4 + 2\sqrt{a}\sqrt{c}e^2(3cd^4-5ae^4)d^2 + 9a^2e^8\right)\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}(cd^4+ae^4)^3} + \frac{c^{3/4}d\left(3c^2d^8 - 36ace^4d^4 + 2\sqrt{a}\sqrt{c}e^2(3cd^4-5ae^4)d^2 + 9a^2e^8\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}} + 1\right)}{8\sqrt{2}a^{7/4}(cd^4+ae^4)^3} - \frac{c^{3/4}d\left(3c^2d^8 - 36ace^4d^4 - 2\sqrt{a}\sqrt{c}e^2(3cd^4-5ae^4)d^2 + 9a^2e^8\right)\log\left(\sqrt{c}x^2 - \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a}\right)}{16\sqrt{2}a^{7/4}(cd^4+ae^4)^3} + \frac{c^{3/4}d\left(3c^2d^8 - 36ace^4d^4 - 2\sqrt{a}\sqrt{c}e^2(3cd^4-5ae^4)d^2 + 9a^2e^8\right)\log\left(\sqrt{c}x^2 + \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a}\right)}{16\sqrt{2}a^{7/4}(cd^4+ae^4)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d + e*x)^3*(a + c*x^4)^2),x]
```

```
[Out] -e^7/(2*(c*d^4 + a*e^4)^2*(d + e*x)^2) - (8*c*d^3*e^7)/((c*d^4 +
a*e^4)^3*(d + e*x)) + (c*(2*a*d^2*e^3*(5*c*d^4 - 3*a*e^4) + x*(d*
(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8) - e*(3*c^2*d^8 - 12*a*c*d^
4*e^4 + a^2*e^8)*x + 2*c*d^3*e^2*(3*c*d^4 - 5*a*e^4)*x^2)))/(4*a*
(c*d^4 + a*e^4)^3*(a + c*x^4)) - (Sqrt[c]*e^5*(21*c^2*d^8 - 26*a*
c*d^4*e^4 + a^2*e^8)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*(c
*d^4 + a*e^4)^4) - (Sqrt[c]*e*(3*c^2*d^8 - 12*a*c*d^4*e^4 + a^2*e
^8)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(3/2)*(c*d^4 + a*e^4)^3)
- (c^(3/4)*d*(3*c^2*d^8 - 36*a*c*d^4*e^4 + 9*a^2*e^8 + 2*Sqrt[a]*
Sqrt[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4))*ArcTan[1 - (Sqrt[2]*c^(1/4)*
x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^3) - (c^(3/4)*d*e
^4*(4*Sqrt[a]*Sqrt[c]*d^2*e^2*(7*c*d^4 - 5*a*e^4) + 3*(5*c^2*d^8
- 10*a*c*d^4*e^4 + a^2*e^8))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/
4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^4) + (c^(3/4)*d*(3*c^2*d^
8 - 36*a*c*d^4*e^4 + 9*a^2*e^8 + 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d
^4 - 5*a*e^4))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2
]*a^(7/4)*(c*d^4 + a*e^4)^3) + (c^(3/4)*d*e^4*(4*Sqrt[a]*Sqrt[c]*
d^2*e^2*(7*c*d^4 - 5*a*e^4) + 3*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2
*e^8))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4
)*(c*d^4 + a*e^4)^4) + (12*c*d^2*e^7*(3*c*d^4 - a*e^4)*Log[d + e*
x])/(c*d^4 + a*e^4)^4 - (c^(3/4)*d*(3*c^2*d^8 - 36*a*c*d^4*e^4 +
9*a^2*e^8 - 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4))*Log[Sq
rt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(
```

```
7/4)*(c*d^4 + a*e^4)^3) + (c^(3/4)*d*e^4*(4*Sqrt[a]*Sqrt[c]*d^2*e
^2*(7*c*d^4 - 5*a*e^4) - 3*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8)
)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt
[2]*a^(3/4)*(c*d^4 + a*e^4)^4) + (c^(3/4)*d*(3*c^2*d^8 - 36*a*c*d
^4*e^4 + 9*a^2*e^8 - 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4
))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sq
rt[2]*a^(7/4)*(c*d^4 + a*e^4)^3) - (c^(3/4)*d*e^4*(4*Sqrt[a]*Sqrt
[c]*d^2*e^2*(7*c*d^4 - 5*a*e^4) - 3*(5*c^2*d^8 - 10*a*c*d^4*e^4 +
 a^2*e^8))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]
)/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^4) - (3*c*d^2*e^7*(3*c*d^4 -
 a*e^4)*Log[a + c*x^4])/(c*d^4 + a*e^4)^4
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x+d)**3/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 2.43567, size = 996, normalized size = 0.72

$$\frac{384cd^2\left(3cd^4-ae^4\right)\log(d+ex)e^7-96cd^2\left(3cd^4-ae^4\right)\log\left(cx^4+a\right)e^7-\frac{256cd^3\left(cd^4+ae^4\right)e^7}{d+ex}-\frac{16\left(cd^4+ae^4\right)^2e^7}{(d+ex)^2}+\frac{8c\left(cd^4+ae^4\right)\left(c^2\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d + e*x)^3*(a + c*x^4)^2),x]
```

```
[Out] ((-16*e^7*(c*d^4 + a*e^4)^2)/(d + e*x)^2 - (256*c*d^3*e^7*(c*d^4
+ a*e^4))/(d + e*x) + (8*c*(c*d^4 + a*e^4)*(-(a^2*e^7*(6*d^2 - 3*
d*e*x + e^2*x^2)) + c^2*d^7*x*(d^2 - 3*d*e*x + 6*e^2*x^2) + 2*a*c
*d^3*e^3*(5*d^3 - 6*d^2*e*x + 6*d*e^2*x^2 - 5*e^3*x^3)))/(a*(a +
c*x^4)) - (6*Sqrt[c]*(Sqrt[2]*c^(13/4)*d^13 - 4*a^(1/4)*c^3*d^12*
e + 2*Sqrt[2]*Sqrt[a]*c^(11/4)*d^11*e^2 + 9*Sqrt[2]*a*c^(9/4)*d^9
*e^4 - 44*a^(5/4)*c^2*d^8*e^5 + 36*Sqrt[2]*a^(3/2)*c^(7/4)*d^7*e^
6 - 49*Sqrt[2]*a^2*c^(5/4)*d^5*e^8 + 84*a^(9/4)*c*d^4*e^9 - 30*Sq
rt[2]*a^(5/2)*c^(3/4)*d^3*e^10 + 7*Sqrt[2]*a^3*c^(1/4)*d*e^12 - 4
*a^(13/4)*e^13)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(7/4)
+ (6*Sqrt[c]*(Sqrt[2]*c^(13/4)*d^13 + 4*a^(1/4)*c^3*d^12*e + 2*Sq
rt[2]*Sqrt[a]*c^(11/4)*d^11*e^2 + 9*Sqrt[2]*a*c^(9/4)*d^9*e^4 + 4
4*a^(5/4)*c^2*d^8*e^5 + 36*Sqrt[2]*a^(3/2)*c^(7/4)*d^7*e^6 - 49*S
qrt[2]*a^2*c^(5/4)*d^5*e^8 - 84*a^(9/4)*c*d^4*e^9 - 30*Sqrt[2]*a^
(5/2)*c^(3/4)*d^3*e^10 + 7*Sqrt[2]*a^3*c^(1/4)*d*e^12 + 4*a^(13/4
)*e^13)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(7/4) + 384*c*
d^2*e^7*(3*c*d^4 - a*e^4)*Log[d + e*x] - (3*Sqrt[2]*c^(3/4)*(c^3*
d^13 - 2*Sqrt[a]*c^(5/2)*d^11*e^2 + 9*a*c^2*d^9*e^4 - 36*a^(3/2)*
c^(3/2)*d^7*e^6 - 49*a^2*c*d^5*e^8 + 30*a^(5/2)*Sqrt[c]*d^3*e^10
+ 7*a^3*d*e^12)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]
*x^2])/a^(7/4) + (3*Sqrt[2]*c^(3/4)*(c^3*d^13 - 2*Sqrt[a]*c^(5/2)
*d^11*e^2 + 9*a*c^2*d^9*e^4 - 36*a^(3/2)*c^(3/2)*d^7*e^6 - 49*a^2
*c*d^5*e^8 + 30*a^(5/2)*Sqrt[c]*d^3*e^10 + 7*a^3*d*e^12)*Log[Sqrt
[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(7/4) - 96*c*d^
2*e^7*(3*c*d^4 - a*e^4)*Log[a + c*x^4])/(32*(c*d^4 + a*e^4)^4)
```

Maple [A] time = 0.037, size = 2133, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^3/(c*x^4+a)^2,x)
```

```
[Out] 27/16*c^3/(a*e^4+c*d^4)^4/a*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/
(1/c*a)^(1/4)*x-1)*d^9*e^4-1/2*e^7/(a*e^4+c*d^4)^2/(e*x+d)^2-45/1
6*c/(a*e^4+c*d^4)^4*a/(1/c*a)^(1/4)*2^(1/2)*ln((x^2-(1/c*a)^(1/4)
*x*2^(1/2)+(1/c*a)^(1/2))/(x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1
/2)))*d^3*e^10+21/16*c/(a*e^4+c*d^4)^4*a*(1/c*a)^(1/4)*2^(1/2)*ar
ctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*d*e^12-45/8*c/(a*e^4+c*d^4)^4*a/(
1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*d^3*e^10+2
1/16*c/(a*e^4+c*d^4)^4*a*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/
c*a)^(1/4)*x-1)*d*e^12+27/32*c^3/(a*e^4+c*d^4)^4/a*(1/c*a)^(1/4)*
2^(1/2)*ln((x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2-(1/c*
a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))*d^9*e^4+21/32*c/(a*e^4+c*d^4)^
4*a*(1/c*a)^(1/4)*2^(1/2)*ln((x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)
^(1/2))/(x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))*d*e^12+3/8*c
^3/(a*e^4+c*d^4)^4/a/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)
^(1/4)*x-1)*d^11*e^2+3/8*c^3/(a*e^4+c*d^4)^4/a/(1/c*a)^(1/4)*2^(1
/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*d^11*e^2-45/8*c/(a*e^4+c*d^
4)^4*a/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*d^
3*e^10+3/16*c^3/(a*e^4+c*d^4)^4/a/(1/c*a)^(1/4)*2^(1/2)*ln((x^2-(
1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2+(1/c*a)^(1/4)*x*2^(1/2
)+(1/c*a)^(1/2)))*d^11*e^2+3/2*c^4/(a*e^4+c*d^4)^4/(c*x^4+a)*d^11
*e^2/a*x^3+3/32*c^4/(a*e^4+c*d^4)^4/a^2*(1/c*a)^(1/4)*2^(1/2)*ln(
(x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2-(1/c*a)^(1/4)*x*
2^(1/2)+(1/c*a)^(1/2)))*d^13-147/16*c^2/(a*e^4+c*d^4)^4*(1/c*a)^(
1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*d^5*e^8-147/16*c^2
/(a*e^4+c*d^4)^4*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/
4)*x-1)*d^5*e^8-147/32*c^2/(a*e^4+c*d^4)^4*(1/c*a)^(1/4)*2^(1/2)*
ln((x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2-(1/c*a)^(1/4)
*x*2^(1/2)+(1/c*a)^(1/2)))*d^5*e^8+27/4*c^2/(a*e^4+c*d^4)^4/(1/c*
a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*d^7*e^6+1/4*c^
4/(a*e^4+c*d^4)^4/(c*x^4+a)*d^13/a*x+c^2/(a*e^4+c*d^4)^4/(c*x^4+a
)*a*d^6*e^7+3*c/(a*e^4+c*d^4)^4*a*ln(a*(c*x^4+a))*d^2*e^11-3/4*c/
(a*e^4+c*d^4)^4/(a^3*c)^(1/2)*arctan(x^2*(c/a)^(1/2))*a^3*e^13-3/
4*c^4/(a*e^4+c*d^4)^4/(a^3*c)^(1/2)*arctan(x^2*(c/a)^(1/2))*d^12*
e-12*e^11*c*d^2/(a*e^4+c*d^4)^4*ln(e*x+d)*a+36*e^7*c^2*d^6/(a*e^4
+c*d^4)^4*ln(e*x+d)+5/2*c^3/(a*e^4+c*d^4)^4/(c*x^4+a)*d^10*e^3-9*
c^2/(a*e^4+c*d^4)^4*ln(a*(c*x^4+a))*d^6*e^7+27/16*c^3/(a*e^4+c*d^
4)^4/a*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*d^
9*e^4+27/4*c^2/(a*e^4+c*d^4)^4/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/
2)/(1/c*a)^(1/4)*x+1)*d^7*e^6+3/16*c^4/(a*e^4+c*d^4)^4/a^2*(1/c*a
)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*d^13+3/16*c^4/(
a*e^4+c*d^4)^4/a^2*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(
1/4)*x-1)*d^13+11/4*c^2/(a*e^4+c*d^4)^4/(c*x^4+a)*e^9*a*x^2*d^4-3
/4*c^4/(a*e^4+c*d^4)^4/(c*x^4+a)*e/a*x^2*d^12-9/4*c^2/(a*e^4+c*d^
4)^4/(c*x^4+a)*d^5*a*x*e^8-5/2*c^2/(a*e^4+c*d^4)^4/(c*x^4+a)*d^3*
e^10*a*x^3-1/4*c/(a*e^4+c*d^4)^4/(c*x^4+a)*e^13*a^2*x^2-3/2*c/(a*
e^4+c*d^4)^4/(c*x^4+a)*a^2*d^2*e^11-c^3/(a*e^4+c*d^4)^4/(c*x^4+a)
*d^7*e^6*x^3+9/4*c^3/(a*e^4+c*d^4)^4/(c*x^4+a)*e^5*x^2*d^8-11/4*c
^3/(a*e^4+c*d^4)^4/(c*x^4+a)*d^9*x*e^4-8*c*d^3*e^7/(a*e^4+c*d^4)^
3/(e*x+d)-33/4*c^3/(a*e^4+c*d^4)^4/(a^3*c)^(1/2)*arctan(x^2*(c/a)
^(1/2))*a*d^8*e^5+27/8*c^2/(a*e^4+c*d^4)^4/(1/c*a)^(1/4)*2^(1/2)*
ln((x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2+(1/c*a)^(1/4)
*x*2^(1/2)+(1/c*a)^(1/2)))*d^7*e^6+3/4*c/(a*e^4+c*d^4)^4/(c*x^4+a
)*d*a^2*x*e^12+63/4*c^2/(a*e^4+c*d^4)^4/(a^3*c)^(1/2)*arctan(x^2*
(c/a)^(1/2))*a^2*d^4*e^9
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)^2*(e*x + d)^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)^2*(e*x + d)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*x+d)**3/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.429836, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)^2*(e*x + d)^3),x, algorithm="giac")
```

```
[Out] Done
```

3.408 $\int \frac{(d+ex)^3}{(a+cx^4)^3} dx$

Optimal. Leaf size=394

$$-\frac{3d\left(7\sqrt{c}d^2-5\sqrt{a}e^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}c^{3/4}}+\frac{3d\left(7\sqrt{c}d^2-5\sqrt{a}e^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}c^{3/4}}-\frac{3d\left(5\sqrt{a}e^2+7\sqrt{c}d^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}c^{3/4}}+\frac{3d\left(5\sqrt{a}e^2+7\sqrt{c}d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{64\sqrt{2}a^{11/4}c^{3/4}}+\frac{9d^2e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{c}}+\frac{x\left(7d^3+18d^2ex+15de^2x^2\right)}{32a^2\left(a+cx^4\right)}-\frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{8ac\left(a+cx^4\right)^2}$$

```
[Out] (x*(7*d^3 + 18*d^2*e*x + 15*d*e^2*x^2))/(32*a^2*(a + c*x^4)) - (a
*e^3 - c*x*(d^3 + 3*d^2*e*x + 3*d*e^2*x^2))/(8*a*c*(a + c*x^4)^2)
 + (9*d^2*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(16*a^(5/2)*Sqrt[c]) -
 (3*d*(7*Sqrt[c]*d^2 + 5*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)
*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*c^(3/4)) + (3*d*(7*Sqrt[c]*d^2
 + 5*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sq
rt[2]*a^(11/4)*c^(3/4)) - (3*d*(7*Sqrt[c]*d^2 - 5*Sqrt[a]*e^2)*Lo
g[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2
]*a^(11/4)*c^(3/4)) + (3*d*(7*Sqrt[c]*d^2 - 5*Sqrt[a]*e^2)*Log[Sq
rt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^
(11/4)*c^(3/4))
```

Rubi [A] time = 0.780661, antiderivative size = 394, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.647

$$-\frac{3d\left(7\sqrt{c}d^2-5\sqrt{a}e^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}c^{3/4}}+\frac{3d\left(7\sqrt{c}d^2-5\sqrt{a}e^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}c^{3/4}}-\frac{3d\left(5\sqrt{a}e^2+7\sqrt{c}d^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}c^{3/4}}+\frac{3d\left(5\sqrt{a}e^2+7\sqrt{c}d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{64\sqrt{2}a^{11/4}c^{3/4}}+\frac{9d^2e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{c}}+\frac{x\left(7d^3+18d^2ex+15de^2x^2\right)}{32a^2\left(a+cx^4\right)}-\frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{8ac\left(a+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x)^3/(a + c*x^4)^3,x]
```


```
[Out] (x*(7*d^3 + 18*d^2*e*x + 15*d*e^2*x^2))/(32*a^2*(a + c*x^4)) - (a
*e^3 - c*x*(d^3 + 3*d^2*e*x + 3*d*e^2*x^2))/(8*a*c*(a + c*x^4)^2)
 + (9*d^2*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(16*a^(5/2)*Sqrt[c]) -
 (3*d*(7*Sqrt[c]*d^2 + 5*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)
*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*c^(3/4)) + (3*d*(7*Sqrt[c]*d^2
 + 5*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sq
rt[2]*a^(11/4)*c^(3/4)) - (3*d*(7*Sqrt[c]*d^2 - 5*Sqrt[a]*e^2)*Lo
g[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2
]*a^(11/4)*c^(3/4)) + (3*d*(7*Sqrt[c]*d^2 - 5*Sqrt[a]*e^2)*Log[Sq
rt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^
(11/4)*c^(3/4))
```

Rubi in Sympy [A] time = 121.822, size = 386, normalized size = 0.98

$$-\frac{ae^3 - cx\left(d^3 + 3d^2ex + 3de^2x^2\right)}{8ac\left(a + cx^4\right)^2} + \frac{x\left(7d^3 + 18d^2ex + 15de^2x^2\right)}{32a^2\left(a + cx^4\right)} + \frac{9d^2e\,\mathrm{atan}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{16a^{\frac{5}{2}}\sqrt{c}} + \frac{3\sqrt{2}d\left(5\sqrt{a}e^2 - 7\sqrt{c}d^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}c^{\frac{3}{4}}x + \sqrt{a}\sqrt{c} + cx^2\right)}{256a^{\frac{11}{4}}c^{\frac{3}{4}}} - \frac{3\sqrt{2}d\left(5\sqrt{a}e^2 - 7\sqrt{c}d^2\right)\log\left(\sqrt{2}\sqrt[4]{a}c^{\frac{3}{4}}x + \sqrt{a}\sqrt{c} + cx^2\right)}{256a^{\frac{11}{4}}c^{\frac{3}{4}}} - \frac{3\sqrt{2}d\left(5\sqrt{a}e^2 + 7\sqrt{c}d^2\right)\mathrm{atan}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{128a^{\frac{11}{4}}c^{\frac{3}{4}}} + \frac{3\sqrt{2}d\left(5\sqrt{a}e^2 + 7\sqrt{c}d^2\right)\mathrm{atan}\left(1 + \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{128a^{\frac{11}{4}}c^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x+d)**3/(c*x**4+a)**3,x)
```

```
[Out] -(a*e**3 - c*x*(d**3 + 3*d**2*e*x + 3*d*e**2*x**2))/(8*a*c*(a + c
*x**4)**2) + x*(7*d**3 + 18*d**2*e*x + 15*d*e**2*x**2)/(32*a**2*(
a + c*x**4)) + 9*d**2*e*atan(sqrt(c)*x**2/sqrt(a))/(16*a**(5/2)*s
qrt(c)) + 3*sqrt(2)*d*(5*sqrt(a)*e**2 - 7*sqrt(c)*d**2)*log(-sqrt
(2)*a**(1/4)*c**(3/4)*x + sqrt(a)*sqrt(c) + c*x**2)/(256*a**(11/4
)*c**(3/4)) - 3*sqrt(2)*d*(5*sqrt(a)*e**2 - 7*sqrt(c)*d**2)*log(s
qrt(2)*a**(1/4)*c**(3/4)*x + sqrt(a)*sqrt(c) + c*x**2)/(256*a**(1
1/4)*c**(3/4)) - 3*sqrt(2)*d*(5*sqrt(a)*e**2 + 7*sqrt(c)*d**2)*at
an(1 - sqrt(2)*c**(1/4)*x/a**(1/4))/(128*a**(11/4)*c**(3/4)) + 3*
sqrt(2)*d*(5*sqrt(a)*e**2 + 7*sqrt(c)*d**2)*atan(1 + sqrt(2)*c**(
1/4)*x/a**(1/4))/(128*a**(11/4)*c**(3/4))
```

Mathematica [A] time = 0.835254, size = 388, normalized size = 0.98

$$\frac{\frac{3\sqrt{2}\left(5a^{3/4}de^2-7\sqrt[4]{a}\sqrt{c}d^3\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{c^{3/4}}+\frac{3\sqrt{2}\left(7\sqrt[4]{a}\sqrt{c}d^3-5a^{3/4}de^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{c^{3/4}}-\frac{32a^2\left(ae^3-cdx\left(d^2+3dex+3e^2x^2\right)\right)}{c\left(a+cx^4\right)^2}}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x)^3/(a + c*x^4)^3,x]
```

```
[Out] ((8*a*d*x*(7*d^2 + 18*d*e*x + 15*e^2*x^2))/(a + c*x^4) - (32*a^2*
(a*e^3 - c*d*x*(d^2 + 3*d*e*x + 3*e^2*x^2)))/(c*(a + c*x^4)^2) -
(6*a^(1/4)*d*(7*Sqrt[2]*Sqrt[c]*d^2 + 24*a^(1/4)*c^(1/4)*d*e + 5*
Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(
3/4) + (6*a^(1/4)*d*(7*Sqrt[2]*Sqrt[c]*d^2 - 24*a^(1/4)*c^(1/4)*d
*e + 5*Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4
)])/c^(3/4) + (3*Sqrt[2]*(-7*a^(1/4)*Sqrt[c]*d^3 + 5*a^(3/4)*d*e^
2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(3/4
) + (3*Sqrt[2]*(7*a^(1/4)*Sqrt[c]*d^3 - 5*a^(3/4)*d*e^2)*Log[Sqrt
[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(3/4))/(256*a^3
)
```

Maple [A] time = 0.01, size = 470, normalized size = 1.2

$$\frac{d^3x}{8a(cx^4+a)^2}+\frac{7d^3x}{32a^2(cx^4+a)}+\frac{21d^3\sqrt{2}}{256a^3}\sqrt[4]{\frac{a}{c}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^2-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)+\frac{21d^3\sqrt{2}}{128a^3}\sqrt[4]{\frac{a}{c}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}+1\right)+\frac{21d^3\sqrt{2}}{128a^3}\sqrt[4]{\frac{a}{c}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}-1\right)+\frac{3d^2ex^2}{8a(cx^4+a)^2}+\frac{9d^2ex^2}{16a^2(cx^4+a)}+\frac{9d^2e}{16a^2}\arctan\left(x^2\sqrt{\frac{c}{a}}\right)\frac{1}{\sqrt{ac}}+\frac{3de^2x^3}{8a(cx^4+a)^2}+\frac{15de^2x^3}{32a^2(cx^4+a)}+\frac{15e^2d\sqrt{2}}{256a^2c}\ln\left(1\left(x^2-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{c}}}+\frac{15e^2d\sqrt{2}}{128a^2c}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{c}}}+\frac{15e^2d\sqrt{2}}{128a^2c}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{c}}}+\frac{e^3x^4}{8a(cx^4+a)^2}+\frac{e^3x^4}{8a^2(cx^4+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x+d)^3/(c*x^4+a)^3,x)
```

```
[Out] 1/8*d^3*x/a/(c*x^4+a)^2+7/32*d^3/a^2*x/(c*x^4+a)+21/256*d^3/a^3*(
1/c*a)^(1/4)*2^(1/2)*ln((x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2
))/(x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))+21/128*d^3/a^3*(1
/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)+21/128*d^3/
a^3*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)+3/8*d
^2*e*x^2/a/(c*x^4+a)^2+9/16*d^2*e/a^2*x^2/(c*x^4+a)+9/16*d^2*e/a^
2/(a*c)^(1/2)*arctan(x^2*(c/a)^(1/2))+3/8*e^2*d*x^3/a/(c*x^4+a)^2
+15/32*e^2*d/a^2*x^3/(c*x^4+a)+15/256*e^2*d/a^2/c/(1/c*a)^(1/4)*2
^(1/2)*ln((x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2+(1/c*a
)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))+15/128*e^2*d/a^2/c/(1/c*a)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)+15/128*e^2*d/a^2/c/(1
/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)+1/8*e^3*x^4
/a/(c*x^4+a)^2+1/8*e^3/a^2*x^4/(c*x^4+a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)^3/(c*x^4 + a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)^3/(c*x^4 + a)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 30.204, size = 413, normalized size = 1.05

$$\mathrm{RootSum}\left(268435456t^4a^{11}c^3 + 63111168t^2a^6c^2d^4e^2 + t\left(4147200a^4cd^4e^5 - 8128512a^3c^2d^8e\right) + 50625a^2d^4e^8 + 245106acd^8e\right.$$

$$+\frac{-4a^2e^3 + 11acd^3x + 30acd^2ex^2 + 27acde^2x^3 + 7c^2d^3x^5 + 18c^2d^2ex^6 + 15c^2de^2x^7}{32a^4c + 64a^3c^2x^4 + 32a^2c^3x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x+d)**3/(c*x**4+a)**3,x)
```

```
[Out] RootSum(268435456*_t**4*a**11*c**3 + 63111168*_t**2*a**6*c**2*d**
4*e**2 + _t*(4147200*a**4*c*d**4*e**5 - 8128512*a**3*c**2*d**8*e)
 + 50625*a**2*d**4*e**8 + 245106*a*c*d**8*e**4 + 194481*c**2*d**1
2, Lambda(_t, _t*log(x + (262144000*_t**3*a**10*c**2*e**6 + 37140
56192*_t**3*a**9*c**3*d**4*e**2 - 539688960*_t**2*a**7*c**2*d**4*
e**5 + 202309632*_t**2*a**6*c**3*d**8*e + 77328000*_t*a**5*c*d**4
*e**8 + 660699648*_t*a**4*c**2*d**8*e**4 + 19361664*_t*a**3*c**3*
d**12 + 3037500*a**3*d**4*e**11 - 26360640*a**2*c*d**8*e**7 - 605
66940*a*c**2*d**12*e**3)/(421875*a**3*d**3*e**12 - 29598075*a**2*
c*d**7*e**8 - 58012227*a*c**2*d**11*e**4 + 3176523*c**3*d**15))))
 + (-4*a**2*e**3 + 11*a*c*d**3*x + 30*a*c*d**2*e*x**2 + 27*a*c*d*
e**2*x**3 + 7*c**2*d**3*x**5 + 18*c**2*d**2*e*x**6 + 15*c**2*d*e*
*2*x**7)/(32*a**4*c + 64*a**3*c**2*x**4 + 32*a**2*c**3*x**8)
```

GIAC/XCAS [A] time = 0.276112, size = 525, normalized size = 1.33

$$\frac{3\sqrt{2}\left(12\sqrt{2}\sqrt{ac}c^2d^2e + 7\left(ac^3\right)^{\frac{1}{4}}c^2d^3 + 5\left(ac^3\right)^{\frac{3}{4}}de^2\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128a^3c^3} + \frac{3\sqrt{2}\left(12\sqrt{2}\sqrt{ac}c^2d^2e + 7\left(ac^3\right)^{\frac{1}{4}}c^2d^3 + 5\left(ac^3\right)^{\frac{3}{4}}de^2\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128a^3c^3} + \frac{3\sqrt{2}\left(7\left(ac^3\right)^{\frac{1}{4}}c^2d^3 - 5\left(ac^3\right)^{\frac{3}{4}}de^2\right)\ln\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{256a^3c^3} - \frac{3\sqrt{2}\left(7\left(ac^3\right)^{\frac{1}{4}}c^2d^3 - 5\left(ac^3\right)^{\frac{3}{4}}de^2\right)\ln\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{256a^3c^3} + \frac{15c^2dx^7e^2 + 18c^2d^2x^6e + 7c^2d^3x^5 + 27acdx^3e^2 + 30acd^2x^2e + 11acd^3x - 4a^2e^3}{32\left(cx^4+a\right)^2a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)^3/(c*x^4 + a)^3,x, algorithm="giac")
```

```
[Out] 3/128*sqrt(2)*(12*sqrt(2)*sqrt(a*c)*c^2*d^2*e + 7*(a*c^3)^(1/4)*c
^2*d^3 + 5*(a*c^3)^(3/4)*d*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)
*(a/c)^(1/4))/(a/c)^(1/4))/(a^3*c^3) + 3/128*sqrt(2)*(12*sqrt(2)*
sqrt(a*c)*c^2*d^2*e + 7*(a*c^3)^(1/4)*c^2*d^3 + 5*(a*c^3)^(3/4)*d
*e^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))
/(a^3*c^3) + 3/256*sqrt(2)*(7*(a*c^3)^(1/4)*c^2*d^3 - 5*(a*c^3)^(
```

```
3/4)*d*e^2)*ln(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^3*c^3)
 - 3/256*sqrt(2)*(7*(a*c^3)^(1/4)*c^2*d^3 - 5*(a*c^3)^(3/4)*d*e^2
)*ln(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^3*c^3) + 1/32*(1
5*c^2*d*x^7*e^2 + 18*c^2*d^2*x^6*e + 7*c^2*d^3*x^5 + 27*a*c*d*x^3
*e^2 + 30*a*c*d^2*x^2*e + 11*a*c*d^3*x - 4*a^2*e^3)/((c*x^4 + a)^
2*a^2*c)
```

3.409 $\int \frac{(d+ex)^2}{(a+cx^4)^3} dx$

Optimal. Leaf size=360

$$-\frac{\left(21\sqrt{c}d^2-5\sqrt{a}e^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}c^{3/4}}+\frac{\left(21\sqrt{c}d^2-5\sqrt{a}e^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}c^{3/4}}-\frac{\left(5\sqrt{a}e^2+21\sqrt{c}d^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}c^{3/4}}+\frac{\left(5\sqrt{a}e^2+21\sqrt{c}d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{64\sqrt{2}a^{11/4}c^{3/4}}+\frac{3de\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{c}}+\frac{x\left(7d^2+12dex+5e^2x^2\right)}{32a^2\left(a+cx^4\right)}+\frac{x(d+ex)^2}{8a\left(a+cx^4\right)^2}$$

```
[Out] (x*(d + e*x)^2)/(8*a*(a + c*x^4)^2) + (x*(7*d^2 + 12*d*e*x + 5*e^
2*x^2))/(32*a^2*(a + c*x^4)) + (3*d*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a
]])/(8*a^(5/2)*Sqrt[c]) - ((21*Sqrt[c]*d^2 + 5*Sqrt[a]*e^2)*ArcTa
n[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*c^(3/4))
 + ((21*Sqrt[c]*d^2 + 5*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*
x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*c^(3/4)) - ((21*Sqrt[c]*d^2 - 5
*Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x
^2])/(128*Sqrt[2]*a^(11/4)*c^(3/4)) + ((21*Sqrt[c]*d^2 - 5*Sqrt[a
]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(1
28*Sqrt[2]*a^(11/4)*c^(3/4))
```

Rubi [A] time = 0.697319, antiderivative size = 360, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.588

$$-\frac{\left(21\sqrt{c}d^2-5\sqrt{a}e^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}c^{3/4}}+\frac{\left(21\sqrt{c}d^2-5\sqrt{a}e^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}c^{3/4}}-\frac{\left(5\sqrt{a}e^2+21\sqrt{c}d^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}c^{3/4}}+\frac{\left(5\sqrt{a}e^2+21\sqrt{c}d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{64\sqrt{2}a^{11/4}c^{3/4}}+\frac{3de\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{c}}+\frac{x\left(7d^2+12dex+5e^2x^2\right)}{32a^2\left(a+cx^4\right)}+\frac{x(d+ex)^2}{8a\left(a+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x)^2/(a + c*x^4)^3,x]
```


```
[Out] (x*(d + e*x)^2)/(8*a*(a + c*x^4)^2) + (x*(7*d^2 + 12*d*e*x + 5*e^
2*x^2))/(32*a^2*(a + c*x^4)) + (3*d*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a
]])/(8*a^(5/2)*Sqrt[c]) - ((21*Sqrt[c]*d^2 + 5*Sqrt[a]*e^2)*ArcTa
n[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*c^(3/4))
 + ((21*Sqrt[c]*d^2 + 5*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*
x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*c^(3/4)) - ((21*Sqrt[c]*d^2 - 5
*Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x
^2])/(128*Sqrt[2]*a^(11/4)*c^(3/4)) + ((21*Sqrt[c]*d^2 - 5*Sqrt[a
]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(1
28*Sqrt[2]*a^(11/4)*c^(3/4))
```

Rubi in Sympy [A] time = 119.628, size = 343, normalized size = 0.95

$$\frac{x(d+ex)^2}{8a(a+cx^4)^2}+\frac{x(7d^2+12dex+5e^2x^2)}{32a^2(a+cx^4)}+\frac{3de\operatorname{atan}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{8a^{\frac{5}{2}}\sqrt{c}}$$
$$+\frac{\sqrt{2}\left(5\sqrt{a}e^2-21\sqrt{c}d^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}c^{\frac{3}{4}}x+\sqrt{a}\sqrt{c}+cx^2\right)}{256a^{\frac{11}{4}}c^{\frac{3}{4}}}$$
$$-\frac{\sqrt{2}\left(5\sqrt{a}e^2-21\sqrt{c}d^2\right)\log\left(\sqrt{2}\sqrt[4]{a}c^{\frac{3}{4}}x+\sqrt{a}\sqrt{c}+cx^2\right)}{256a^{\frac{11}{4}}c^{\frac{3}{4}}}$$
$$-\frac{\sqrt{2}\left(5\sqrt{a}e^2+21\sqrt{c}d^2\right)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{128a^{\frac{11}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\left(5\sqrt{a}e^2+21\sqrt{c}d^2\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{128a^{\frac{11}{4}}c^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x+d)**2/(c*x**4+a)**3,x)
```

```
[Out] x*(d + e*x)**2/(8*a*(a + c*x**4)**2) + x*(7*d**2 + 12*d*e*x + 5*e
**2*x**2)/(32*a**2*(a + c*x**4)) + 3*d*e*atan(sqrt(c)*x**2/sqrt(a
))/(8*a**(5/2)*sqrt(c)) + sqrt(2)*(5*sqrt(a)*e**2 - 21*sqrt(c)*d*
*2)*log(-sqrt(2)*a**(1/4)*c**(3/4)*x + sqrt(a)*sqrt(c) + c*x**2)/
(256*a**(11/4)*c**(3/4)) - sqrt(2)*(5*sqrt(a)*e**2 - 21*sqrt(c)*d
**2)*log(sqrt(2)*a**(1/4)*c**(3/4)*x + sqrt(a)*sqrt(c) + c*x**2)/
(256*a**(11/4)*c**(3/4)) - sqrt(2)*(5*sqrt(a)*e**2 + 21*sqrt(c)*d
**2)*atan(1 - sqrt(2)*c**(1/4)*x/a**(1/4))/(128*a**(11/4)*c**(3/4
)) + sqrt(2)*(5*sqrt(a)*e**2 + 21*sqrt(c)*d**2)*atan(1 + sqrt(2)*
c**(1/4)*x/a**(1/4))/(128*a**(11/4)*c**(3/4))
```

Mathematica [A] time = 0.601113, size = 358, normalized size = 0.99

$$\frac{\frac{\sqrt{2}\left(5a^{3/4}e^2-21\sqrt[4]{a}\sqrt{c}d^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{c^{3/4}}+\frac{\sqrt{2}\left(21\sqrt[4]{a}\sqrt{c}d^2-5a^{3/4}e^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{c^{3/4}}+\frac{32a^2x(d+ex)^2}{(a+cx^4)^2}-\frac{2\sqrt[4]{a}\left(48\sqrt[4]{a}\sqrt[4]{c}de\cdots\right.}{\cdots}}{256a^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x)^2/(a + c*x^4)^3,x]
```

```
[Out] ((32*a^2*x*(d + e*x)^2)/(a + c*x^4)^2 + (8*a*x*(7*d^2 + 12*d*e*x
+ 5*e^2*x^2))/(a + c*x^4) - (2*a^(1/4)*(21*Sqrt[2]*Sqrt[c]*d^2 +
48*a^(1/4)*c^(1/4)*d*e + 5*Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[
2]*c^(1/4)*x)/a^(1/4)])/c^(3/4) + (2*a^(1/4)*(21*Sqrt[2]*Sqrt[c]*
d^2 - 48*a^(1/4)*c^(1/4)*d*e + 5*Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 +
(Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(3/4) + (Sqrt[2]*(-21*a^(1/4)*Sqr
t[c]*d^2 + 5*a^(3/4)*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x
 + Sqrt[c]*x^2])/c^(3/4) + (Sqrt[2]*(21*a^(1/4)*Sqrt[c]*d^2 - 5*a
^(3/4)*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2
])/c^(3/4))/(256*a^3)
```

Maple [A] time = 0.007, size = 419, normalized size = 1.2

$$\frac{d^2x}{8a(cx^4+a)^2}+\frac{7d^2x}{32a^2(cx^4+a)}+\frac{21d^2\sqrt{2}}{256a^3}\sqrt[4]{\frac{a}{c}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^2-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)+\frac{21d^2\sqrt{2}}{128a^3}\sqrt[4]{\frac{a}{c}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}+1\right)+\frac{21d^2\sqrt{2}}{128a^3}\sqrt[4]{\frac{a}{c}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}-1\right)+\frac{dex^2}{4a(cx^4+a)^2}+\frac{3dex^2}{8a^2(cx^4+a)}+\frac{3de}{8a^2}\arctan\left(x^2\sqrt{\frac{c}{a}}\right)\frac{1}{\sqrt{ac}}+\frac{e^2x^3}{8a(cx^4+a)^2}+\frac{5e^2x^3}{32a^2(cx^4+a)}+\frac{5e^2\sqrt{2}}{256a^2c}\ln\left(1\left(x^2-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{c}}}+\frac{5e^2\sqrt{2}}{128a^2c}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{c}}}+\frac{5e^2\sqrt{2}}{128a^2c}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x+d)^2/(c*x^4+a)^3,x)
```

```
[Out] 1/8*d^2*x/a/(c*x^4+a)^2+7/32*d^2/a^2*x/(c*x^4+a)+21/256*d^2/a^3*(
1/c*a)^(1/4)*2^(1/2)*ln((x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2
))/(x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))+21/128*d^2/a^3*(1
/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)+21/128*d^2/
a^3*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)+1/4*d
*e*x^2/a/(c*x^4+a)^2+3/8*d*e/a^2*x^2/(c*x^4+a)+3/8*d*e/a^2/(a*c)^
(1/2)*arctan(x^2*(c/a)^(1/2))+1/8*e^2*x^3/a/(c*x^4+a)^2+5/32*e^2/
a^2*x^3/(c*x^4+a)+5/256*e^2/a^2/c/(1/c*a)^(1/4)*2^(1/2)*ln((x^2-(
1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2+(1/c*a)^(1/4)*x*2^(1/2
)+(1/c*a)^(1/2)))+5/128*e^2/a^2/c/(1/c*a)^(1/4)*2^(1/2)*arctan(2^
(1/2)/(1/c*a)^(1/4)*x+1)+5/128*e^2/a^2/c/(1/c*a)^(1/4)*2^(1/2)*ar
ctan(2^(1/2)/(1/c*a)^(1/4)*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)^2/(c*x^4 + a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)^2/(c*x^4 + a)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 16.8138, size = 374, normalized size = 1.04

$$\operatorname{RootSum}\left(268435456t^4a^{11}c^3 + 25755648t^2a^6c^2d^2e^2 + t\left(307200a^4cde^5 - 5419008a^3c^2d^5e\right) + 625a^2e^8 + 111906acd^4e^4 + 194\right.$$
$$+\frac{11ad^2x + 20adex^2 + 9ae^2x^3 + 7cd^2x^5 + 12cdex^6 + 5ce^2x^7}{32a^4 + 64a^3cx^4 + 32a^2c^2x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x+d)**2/(c*x**4+a)**3,x)
```

```
[Out] RootSum(268435456*_t**4*a**11*c**3 + 25755648*_t**2*a**6*c**2*d**
2*e**2 + _t*(307200*a**4*c*d*e**5 - 5419008*a**3*c**2*d**5*e) + 6
25*a**2*e**8 + 111906*a*c*d**4*e**4 + 194481*c**2*d**8, Lambda(_t
, _t*log(x + (262144000*_t**3*a**10*c**2*e**6 + 46110081024*_t**3
*a**9*c**3*d**4*e**2 - 1645608960*_t**2*a**7*c**2*d**3*e**5 + 364
1573376*_t**2*a**6*c**3*d**7*e + 32688000*_t*a**5*c*d**2*e**8 + 3
128219136*_t*a**4*c**2*d**6*e**4 + 522764928*_t*a**3*c**3*d**10 +
 225000*a**3*d*e**11 - 43338240*a**2*c*d**5*e**7 - 523431720*a*c*
*2*d**9*e**3)/(15625*a**3*e**12 - 21357225*a**2*c*d**4*e**8 - 376
741449*a*c**2*d**8*e**4 + 85766121*c**3*d**12)))) + (11*a*d**2*x
+ 20*a*d*e*x**2 + 9*a*e**2*x**3 + 7*c*d**2*x**5 + 12*c*d*e*x**6 +
 5*c*e**2*x**7)/(32*a**4 + 64*a**3*c*x**4 + 32*a**2*c**2*x**8)
```

GIAC/XCAS [A] time = 0.274006, size = 481, normalized size = 1.34

$$\frac{5\,cx^7e^2 + 12\,cdx^6e + 7\,cd^2x^5 + 9\,ax^3e^2 + 20\,adx^2e + 11\,ad^2x}{32\,(cx^4+a)^2a^2} + \frac{\sqrt{2}\left(24\,\sqrt{2}\sqrt{ac}c^2de + 21\,\left(ac^3\right)^{\frac{1}{4}}c^2d^2 + 5\,\left(ac^3\right)^{\frac{3}{4}}e^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\,a^3c^3} + \frac{\sqrt{2}\left(24\,\sqrt{2}\sqrt{ac}c^2de + 21\,\left(ac^3\right)^{\frac{1}{4}}c^2d^2 + 5\,\left(ac^3\right)^{\frac{3}{4}}e^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\,a^3c^3} + \frac{\sqrt{2}\left(21\,\left(ac^3\right)^{\frac{1}{4}}c^2d^2 - 5\,\left(ac^3\right)^{\frac{3}{4}}e^2\right)\ln\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{256\,a^3c^3} - \frac{\sqrt{2}\left(21\,\left(ac^3\right)^{\frac{1}{4}}c^2d^2 - 5\,\left(ac^3\right)^{\frac{3}{4}}e^2\right)\ln\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{256\,a^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)^2/(c*x^4 + a)^3,x, algorithm="giac")
```

```
[Out] 1/32*(5*c*x^7*e^2 + 12*c*d*x^6*e + 7*c*d^2*x^5 + 9*a*x^3*e^2 + 20
*a*d*x^2*e + 11*a*d^2*x)/((c*x^4 + a)^2*a^2) + 1/128*sqrt(2)*(24*
sqrt(2)*sqrt(a*c)*c^2*d*e + 21*(a*c^3)^(1/4)*c^2*d^2 + 5*(a*c^3)^
(3/4)*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(
1/4))/(a^3*c^3) + 1/128*sqrt(2)*(24*sqrt(2)*sqrt(a*c)*c^2*d*e + 2
1*(a*c^3)^(1/4)*c^2*d^2 + 5*(a*c^3)^(3/4)*e^2)*arctan(1/2*sqrt(2)
*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^3*c^3) + 1/256*sqrt(
2)*(21*(a*c^3)^(1/4)*c^2*d^2 - 5*(a*c^3)^(3/4)*e^2)*ln(x^2 + sqrt
(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^3*c^3) - 1/256*sqrt(2)*(21*(a*c
^3)^(1/4)*c^2*d^2 - 5*(a*c^3)^(3/4)*e^2)*ln(x^2 - sqrt(2)*x*(a/c)
^(1/4) + sqrt(a/c))/(a^3*c^3)
```

3.410 $\int \frac{d+ex}{(a+cx^4)^3} \, dx$

Optimal. Leaf size=266

$$-\frac{21d\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}\sqrt[4]{c}}+\frac{21d\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}\sqrt[4]{c}}-\frac{21d\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{c}}+\frac{21d\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{c}}+\frac{3e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{c}}+\frac{x(7d+6ex)}{32a^2(a+cx^4)}+\frac{x(d+ex)}{8a(a+cx^4)^2}$$

```
[Out] (x*(d + e*x))/(8*a*(a + c*x^4)^2) + (x*(7*d + 6*e*x))/(32*a^2*(a
+ c*x^4)) + (3*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(16*a^(5/2)*Sqrt[
c]) - (21*d*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*
a^(11/4)*c^(1/4)) + (21*d*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]
)/(64*Sqrt[2]*a^(11/4)*c^(1/4)) - (21*d*Log[Sqrt[a] - Sqrt[2]*a^(
1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(11/4)*c^(1/4)) + (
21*d*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128
*Sqrt[2]*a^(11/4)*c^(1/4))
```

Rubi [A] time = 0.520925, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$

$$-\frac{21d\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}\sqrt[4]{c}}+\frac{21d\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}\sqrt[4]{c}}-\frac{21d\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{c}}+\frac{21d\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{c}}+\frac{3e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{c}}+\frac{x(7d+6ex)}{32a^2(a+cx^4)}+\frac{x(d+ex)}{8a(a+cx^4)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x)/(a + c*x^4)^3,x]
```


```
[Out] (x*(d + e*x))/(8*a*(a + c*x^4)^2) + (x*(7*d + 6*e*x))/(32*a^2*(a
+ c*x^4)) + (3*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(16*a^(5/2)*Sqrt[
c]) - (21*d*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*
a^(11/4)*c^(1/4)) + (21*d*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]
)/(64*Sqrt[2]*a^(11/4)*c^(1/4)) - (21*d*Log[Sqrt[a] - Sqrt[2]*a^(
1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(11/4)*c^(1/4)) + (
21*d*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128
*Sqrt[2]*a^(11/4)*c^(1/4))
```

Rubi in Sympy [A] time = 90.5236, size = 257, normalized size = 0.97

$$\frac{x(d+ex)}{8a(a+cx^4)^2}+\frac{x(7d+6ex)}{32a^2(a+cx^4)}+\frac{3e\,\text{atan}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{16a^{\frac{5}{2}}\sqrt{c}}-\frac{21\sqrt{2}d\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{256a^{\frac{11}{4}}\sqrt[4]{c}}+\frac{21\sqrt{2}d\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{256a^{\frac{11}{4}}\sqrt[4]{c}}-\frac{21\sqrt{2}d\,\text{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{128a^{\frac{11}{4}}\sqrt[4]{c}}+\frac{21\sqrt{2}d\,\text{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{128a^{\frac{11}{4}}\sqrt[4]{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x+d)/(c*x**4+a)**3,x)
```

```
[Out] x*(d + e*x)/(8*a*(a + c*x**4)**2) + x*(7*d + 6*e*x)/(32*a**2*(a +
 c*x**4)) + 3*e*atan(sqrt(c)*x**2/sqrt(a))/(16*a**(5/2)*sqrt(c))
- 21*sqrt(2)*d*log(-sqrt(2)*a**(1/4)*c**(1/4)*x + sqrt(a) + sqrt(
c)*x**2)/(256*a**(11/4)*c**(1/4)) + 21*sqrt(2)*d*log(sqrt(2)*a**(
1/4)*c**(1/4)*x + sqrt(a) + sqrt(c)*x**2)/(256*a**(11/4)*c**(1/4)
) - 21*sqrt(2)*d*atan(1 - sqrt(2)*c**(1/4)*x/a**(1/4))/(128*a**(1
1/4)*c**(1/4)) + 21*sqrt(2)*d*atan(1 + sqrt(2)*c**(1/4)*x/a**(1/4
))/(128*a**(11/4)*c**(1/4))
```

Mathematica [A] time = 0.352981, size = 249, normalized size = 0.94

$$\frac{\frac{32a^{7/4}x(d+ex)}{(a+cx^4)^2}+\frac{8a^{3/4}x(7d+6ex)}{a+cx^4}-\frac{6\left(8\sqrt[4]{a}e+7\sqrt{2}\sqrt[4]{c}d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{c}}+\frac{6\left(7\sqrt{2}\sqrt[4]{c}d-8\sqrt[4]{a}e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{c}}-\frac{21\sqrt{2}d\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\right.}{\sqrt[4]{c}}}{256a^{11/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x)/(a + c*x^4)^3,x]
```

```
[Out] ((32*a^(7/4)*x*(d + e*x))/(a + c*x^4)^2 + (8*a^(3/4)*x*(7*d + 6*e
*x))/(a + c*x^4) - (6*(7*Sqrt[2]*c^(1/4)*d + 8*a^(1/4)*e)*ArcTan[
1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/Sqrt[c] + (6*(7*Sqrt[2]*c^(1/4)
*d - 8*a^(1/4)*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/Sqrt[c
] - (21*Sqrt[2]*d*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[
c]*x^2])/c^(1/4) + (21*Sqrt[2]*d*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^
(1/4)*x + Sqrt[c]*x^2])/c^(1/4))/(256*a^(11/4))
```

Maple [A] time = 0.007, size = 222, normalized size = 0.8

$$\frac{dx}{8\,a\,(cx^4+a)^2}+\frac{7\,dx}{32\,a^2\,(cx^4+a)}+\frac{21\,d\sqrt{2}}{256\,a^3}\sqrt[4]{\frac{a}{c}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^2-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)+\frac{21\,d\sqrt{2}}{128\,a^3}\sqrt[4]{\frac{a}{c}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}+1\right)+\frac{21\,d\sqrt{2}}{128\,a^3}\sqrt[4]{\frac{a}{c}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}-1\right)+\frac{ex^2}{8\,a\,(cx^4+a)^2}+\frac{3\,ex^2}{16\,a^2\,(cx^4+a)}+\frac{3\,e}{16\,a^2}\arctan\left(x^2\sqrt{\frac{c}{a}}\right)\frac{1}{\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x+d)/(c*x^4+a)^3,x)
```

```
[Out] 1/8*d*x/a/(c*x^4+a)^2+7/32*d/a^2*x/(c*x^4+a)+21/256*d/a^3*(1/c*a)
^(1/4)*2^(1/2)*ln((x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^
2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))+21/128*d/a^3*(1/c*a)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)+21/128*d/a^3*(1/c*a
)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)+1/8*e*x^2/a/(c*
x^4+a)^2+3/16*e/a^2*x^2/(c*x^4+a)+3/16*e/a^2/(a*c)^(1/2)*arctan(x
^2*(c/a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)/(c*x^4 + a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)/(c*x^4 + a)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 8.86643, size = 192, normalized size = 0.72

$$\operatorname{RootSum}\left(268435456t^4a^{11}c^2 + 4718592t^2a^6ce^2 - 2709504ta^3cd^2e + 20736ae^4 + 194481cd^4, \left(t \mapsto t\log\left(x + \frac{-67108864t^3a^9\ldots}{\ldots}\right)\right)\right) + \frac{11adx + 10aex^2 + 7cdx^5 + 6cex^6}{32a^4 + 64a^3cx^4 + 32a^2c^2x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x+d)/(c*x**4+a)**3,x)
```

```
[Out] RootSum(268435456*_t**4*a**11*c**2 + 4718592*_t**2*a**6*c*e**2 -
2709504*_t*a**3*c*d**2*e + 20736*a*e**4 + 194481*c*d**4, Lambda(_
t, _t*log(x + (-67108864*_t**3*a**9*c*e**2 - 9633792*_t**2*a**6*c
*d**2*e - 589824*_t*a**4*e**4 - 2765952*_t*a**3*c*d**4 + 423360*a
*d**2*e**3)/(193536*a*d*e**4 - 453789*c*d**5)))) + (11*a*d*x + 10
*a*e*x**2 + 7*c*d*x**5 + 6*c*e*x**6)/(32*a**4 + 64*a**3*c*x**4 +
32*a**2*c**2*x**8)
```

GIAC/XCAS [A] time = 0.271612, size = 351, normalized size = 1.32

$$\frac{21\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}d\ln\left(x^2 + \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{256\,a^3c} - \frac{21\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}d\ln\left(x^2 - \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{256\,a^3c} + \frac{3\sqrt{2}\left(4\sqrt{2}\sqrt{ac}ce + 7\left(ac^3\right)^{\frac{1}{4}}cd\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\,a^3c^2} + \frac{3\sqrt{2}\left(4\sqrt{2}\sqrt{ac}ce + 7\left(ac^3\right)^{\frac{1}{4}}cd\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\,a^3c^2} + \frac{6\,cx^6e + 7\,cdx^5 + 10\,ax^2e + 11\,adx}{32\left(cx^4 + a\right)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x + d)/(c*x^4 + a)^3,x, algorithm="giac")
```

```
[Out] 21/256*sqrt(2)*(a*c^3)^(1/4)*d*ln(x^2 + sqrt(2)*x*(a/c)^(1/4) + s
qrt(a/c))/(a^3*c) - 21/256*sqrt(2)*(a*c^3)^(1/4)*d*ln(x^2 - sqrt(
```

```
2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^3*c) + 3/128*sqrt(2)*(4*sqrt(2)*
sqrt(a*c)*c*e + 7*(a*c^3)^(1/4)*c*d)*arctan(1/2*sqrt(2)*(2*x + sq
rt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^3*c^2) + 3/128*sqrt(2)*(4*sqrt
(2)*sqrt(a*c)*c*e + 7*(a*c^3)^(1/4)*c*d)*arctan(1/2*sqrt(2)*(2*x
- sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^3*c^2) + 1/32*(6*c*x^6*e +
 7*c*d*x^5 + 10*a*x^2*e + 11*a*d*x)/((c*x^4 + a)^2*a^2)
```

3.411 $\int \frac{1}{(a+cx^4)^3}\,dx$

Optimal. Leaf size=219

$$-\frac{21\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}\sqrt[4]{c}}+\frac{21\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}\sqrt[4]{c}}-\frac{21\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{c}}+\frac{21\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{c}}+\frac{7x}{32a^2\left(a+cx^4\right)}+\frac{x}{8a\left(a+cx^4\right)^2}$$

```
[Out] x/(8*a*(a + c*x^4)^2) + (7*x)/(32*a^2*(a + c*x^4)) - (21*ArcTan[1
 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*c^(1/4)) +
(21*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)
*c^(1/4)) - (21*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]
*x^2])/(128*Sqrt[2]*a^(11/4)*c^(1/4)) + (21*Log[Sqrt[a] + Sqrt[2]
*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(11/4)*c^(1/4))
```

Rubi [A] time = 0.314379, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.778$

$$-\frac{21\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}\sqrt[4]{c}}+\frac{21\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}\sqrt[4]{c}}-\frac{21\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{c}}+\frac{21\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{c}}+\frac{7x}{32a^2\left(a+cx^4\right)}+\frac{x}{8a\left(a+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + c*x^4)^(-3),x]
```

```
[Out] x/(8*a*(a + c*x^4)^2) + (7*x)/(32*a^2*(a + c*x^4)) - (21*ArcTan[1
 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*c^(1/4)) +
(21*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)
*c^(1/4)) - (21*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]
*x^2])/(128*Sqrt[2]*a^(11/4)*c^(1/4)) + (21*Log[Sqrt[a] + Sqrt[2]
*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(11/4)*c^(1/4))
```

Rubi in Sympy [A] time = 59.9886, size = 207, normalized size = 0.95

$$\frac{x}{8a\left(a+cx^4\right)^2}+\frac{7x}{32a^2\left(a+cx^4\right)}-\frac{21\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{256a^{\frac{11}{4}}\sqrt[4]{c}}+\frac{21\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{256a^{\frac{11}{4}}\sqrt[4]{c}}-\frac{21\sqrt{2}\,\mathrm{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{128a^{\frac{11}{4}}\sqrt[4]{c}}+\frac{21\sqrt{2}\,\mathrm{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{128a^{\frac{11}{4}}\sqrt[4]{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c*x**4+a)**3,x)
```

```
[Out] x/(8*a*(a + c*x**4)**2) + 7*x/(32*a**2*(a + c*x**4)) - 21*sqrt(2)
*log(-sqrt(2)*a**(1/4)*c**(1/4)*x + sqrt(a) + sqrt(c)*x**2)/(256*
a**(11/4)*c**(1/4)) + 21*sqrt(2)*log(sqrt(2)*a**(1/4)*c**(1/4)*x
+ sqrt(a) + sqrt(c)*x**2)/(256*a**(11/4)*c**(1/4)) - 21*sqrt(2)*a
tan(1 - sqrt(2)*c**(1/4)*x/a**(1/4))/(128*a**(11/4)*c**(1/4)) + 2
```

```
1*sqrt(2)*atan(1 + sqrt(2)*c**(1/4)*x/a**(1/4))/(128*a**(11/4)*c*
*(1/4))
```

Mathematica [A] time = 0.181054, size = 200, normalized size = 0.91

$$\frac{\frac{32a^{7/4}x}{(a+cx^4)^2}+\frac{56a^{3/4}x}{a+cx^4}-\frac{21\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{\sqrt[4]{c}}+\frac{21\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{\sqrt[4]{c}}-\frac{42\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt[4]{c}}+\frac{42\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt[4]{c}}}{256a^{11/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + c*x^4)^(-3),x]
```

```
[Out] ((32*a^(7/4)*x)/(a + c*x^4)^2 + (56*a^(3/4)*x)/(a + c*x^4) - (42*
Sqrt[2]*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(1/4) + (42*Sq
rt[2]*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(1/4) - (21*Sqrt
[2]*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/
4) + (21*Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c
]*x^2])/c^(1/4))/(256*a^(11/4))
```

Maple [A] time = 0.006, size = 158, normalized size = 0.7

$$\frac{x}{8\,a\,(cx^4+a)^2}+\frac{7\,x}{32\,a^2\,(cx^4+a)}+\frac{21\,\sqrt{2}}{256\,a^3}\sqrt[4]{\frac{a}{c}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^2-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)$$
$$+\frac{21\,\sqrt{2}}{128\,a^3}\sqrt[4]{\frac{a}{c}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}+1\right)+\frac{21\,\sqrt{2}}{128\,a^3}\sqrt[4]{\frac{a}{c}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c*x^4+a)^3,x)
```

```
[Out] 1/8*x/a/(c*x^4+a)^2+7/32*x/a^2/(c*x^4+a)+21/256/a^3*(1/c*a)^(1/4)
*2^(1/2)*ln((x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2-(1/c
*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))+21/128/a^3*(1/c*a)^(1/4)*2^(1
/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)+21/128/a^3*(1/c*a)^(1/4)*2^
(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)^(-3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.300502, size = 293, normalized size = 1.34

$$\frac{28\,cx^5 - 84\left(a^2c^2x^8 + 2\,a^3cx^4 + a^4\right)\left(-\frac{1}{a^{11}c}\right)^{\frac{1}{4}}\arctan\left(\frac{a^3\left(-\frac{1}{a^{11}c}\right)^{\frac{1}{4}}}{x+\sqrt{a^6\sqrt{-\frac{1}{a^{11}c}}+x^2}}\right) + 21\left(a^2c^2x^8 + 2\,a^3cx^4 + a^4\right)\left(-\frac{1}{a^{11}c}\right)^{\frac{1}{4}}\log\left(a^3\left(-\frac{1}{a^{11}c}\right.\right.}{128\left(a^2c^2x^8 + 2\,a^3cx^4 + a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)^(-3),x, algorithm="fricas")
```

```
[Out] 1/128*(28*c*x^5 - 84*(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4)*(-1/(a^11*
c))^(1/4)*arctan(a^3*(-1/(a^11*c))^(1/4)/(x + sqrt(a^6*sqrt(-1/(a
^11*c)) + x^2))) + 21*(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4)*(-1/(a^11
*c))^(1/4)*log(a^3*(-1/(a^11*c))^(1/4) + x) - 21*(a^2*c^2*x^8 + 2
*a^3*c*x^4 + a^4)*(-1/(a^11*c))^(1/4)*log(-a^3*(-1/(a^11*c))^(1/4
) + x) + 44*a*x)/(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4)
```

Sympy [A] time = 5.04828, size = 63, normalized size = 0.29

$$\frac{11ax + 7cx^5}{32a^4 + 64a^3cx^4 + 32a^2c^2x^8} + \operatorname{RootSum}\left(268435456t^4a^{11}c + 194481, \left(t \mapsto t\log\left(\frac{128ta^3}{21} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x**4+a)**3,x)
```

```
[Out] (11*a*x + 7*c*x**5)/(32*a**4 + 64*a**3*c*x**4 + 32*a**2*c**2*x**8
) + RootSum(268435456*_t**4*a**11*c + 194481, Lambda(_t, _t*log(1
28*_t*a**3/21 + x)))
```

GIAC/XCAS [A] time = 0.266752, size = 275, normalized size = 1.26

$$\frac{21\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\,a^3c} + \frac{21\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\,a^3c} + \frac{21\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}\ln\left(x^2 + \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{256\,a^3c} - \frac{21\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}\ln\left(x^2 - \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{256\,a^3c} + \frac{7\,cx^5 + 11\,ax}{32\left(cx^4 + a\right)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)^(-3),x, algorithm="giac")
```

```
[Out] 21/128*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a
/c)^(1/4))/(a/c)^(1/4))/(a^3*c) + 21/128*sqrt(2)*(a*c^3)^(1/4)*ar
ctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^3*c)
 + 21/256*sqrt(2)*(a*c^3)^(1/4)*ln(x^2 + sqrt(2)*x*(a/c)^(1/4) +
sqrt(a/c))/(a^3*c) - 21/256*sqrt(2)*(a*c^3)^(1/4)*ln(x^2 - sqrt(2
)*x*(a/c)^(1/4) + sqrt(a/c))/(a^3*c) + 1/32*(7*c*x^5 + 11*a*x)/((
c*x^4 + a)^2*a^2)
```

3.412 $\int \frac{1}{(d+ex)(a+cx^4)^3}\,dx$

Optimal. Leaf size=1352

result too large to display

```
[Out] (c*x*(7*d^3 - 6*d^2*e*x + 5*d*e^2*x^2))/(32*a^2*(c*d^4 + a*e^4)*(
a + c*x^4)) + (a*e^3 + c*x*(d^3 - d^2*e*x + d*e^2*x^2))/(8*a*(c*d
^4 + a*e^4)*(a + c*x^4)^2) + (e^4*(a*e^3 + c*x*(d^3 - d^2*e*x + d
*e^2*x^2)))/(4*a*(c*d^4 + a*e^4)^2*(a + c*x^4)) - (Sqrt[c]*d^2*e^
9*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*(c*d^4 + a*e^4)^3) -
(Sqrt[c]*d^2*e^5*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(3/2)*(c*d^4
 + a*e^4)^2) - (3*Sqrt[c]*d^2*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(1
6*a^(5/2)*(c*d^4 + a*e^4)) - (c^(1/4)*d*e^8*(Sqrt[c]*d^2 + Sqrt[a
]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4
)*(c*d^4 + a*e^4)^3) - (c^(1/4)*d*e^4*(3*Sqrt[c]*d^2 + Sqrt[a]*e^
2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c
*d^4 + a*e^4)^2) - (c^(1/4)*d*(21*Sqrt[c]*d^2 + 5*Sqrt[a]*e^2)*Ar
cTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*(c*d^
4 + a*e^4)) + (c^(1/4)*d*e^8*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1
 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^
4)^3) + (c^(1/4)*d*e^4*(3*Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 + (
Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^2
) + (c^(1/4)*d*(21*Sqrt[c]*d^2 + 5*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[
2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*(c*d^4 + a*e^4)) + (
e^11*Log[d + e*x])/(c*d^4 + a*e^4)^3 - (c^(1/4)*d*e^8*(Sqrt[c]*d^
2 - Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c
]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^3) - (c^(1/4)*d*e^4*(3
*Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)
*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^2) - (c^(1
/4)*d*(21*Sqrt[c]*d^2 - 5*Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1
/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(11/4)*(c*d^4 + a*e^
4)) + (c^(1/4)*d*e^8*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] + Sq
rt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4
 + a*e^4)^3) + (c^(1/4)*d*e^4*(3*Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[S
qrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^
(7/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)*d*(21*Sqrt[c]*d^2 - 5*Sqrt[a]
*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(12
8*Sqrt[2]*a^(11/4)*(c*d^4 + a*e^4)) - (e^11*Log[a + c*x^4])/(4*(c
*d^4 + a*e^4)^3)
```

Rubi [A] time = 3.20965, antiderivative size = 1352, normalized size of antiderivative = 1., number of steps used = 46, number of rules used = 14, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.824$

result too large to display

Antiderivative was successfully verified.

```
[In]  Int[1/((d + e*x)*(a + c*x^4)^3),x]
```

```
[Out] (c*x*(7*d^3 - 6*d^2*e*x + 5*d*e^2*x^2))/(32*a^2*(c*d^4 + a*e^4)*(
a + c*x^4)) + (a*e^3 + c*x*(d^3 - d^2*e*x + d*e^2*x^2))/(8*a*(c*d
^4 + a*e^4)*(a + c*x^4)^2) + (e^4*(a*e^3 + c*x*(d^3 - d^2*e*x + d
*e^2*x^2)))/(4*a*(c*d^4 + a*e^4)^2*(a + c*x^4)) - (Sqrt[c]*d^2*e^
9*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*(c*d^4 + a*e^4)^3) -
(Sqrt[c]*d^2*e^5*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(3/2)*(c*d^4
 + a*e^4)^2) - (3*Sqrt[c]*d^2*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(1
6*a^(5/2)*(c*d^4 + a*e^4)) - (c^(1/4)*d*e^8*(Sqrt[c]*d^2 + Sqrt[a
]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4
)*(c*d^4 + a*e^4)^3) - (c^(1/4)*d*e^4*(3*Sqrt[c]*d^2 + Sqrt[a]*e^
2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c
*d^4 + a*e^4)^2) - (c^(1/4)*d*(21*Sqrt[c]*d^2 + 5*Sqrt[a]*e^2)*Ar
cTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*(c*d^
4 + a*e^4)) + (c^(1/4)*d*e^8*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1
 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^
4)^3) + (c^(1/4)*d*e^4*(3*Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 + (
```

```
Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^2
) + (c^(1/4)*d*(21*Sqrt[c]*d^2 + 5*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[
2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*(c*d^4 + a*e^4)) + (
e^11*Log[d + e*x])/(c*d^4 + a*e^4)^3 - (c^(1/4)*d*e^8*(Sqrt[c]*d^
2 - Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c
]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^3) - (c^(1/4)*d*e^4*(3
*Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)
*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^2) - (c^(1
/4)*d*(21*Sqrt[c]*d^2 - 5*Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1
/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(11/4)*(c*d^4 + a*e^
4)) + (c^(1/4)*d*e^8*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] + Sq
rt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4
 + a*e^4)^3) + (c^(1/4)*d*e^4*(3*Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[S
qrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^
(7/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)*d*(21*Sqrt[c]*d^2 - 5*Sqrt[a]
*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(12
8*Sqrt[2]*a^(11/4)*(c*d^4 + a*e^4)) - (e^11*Log[a + c*x^4])/(4*(c
*d^4 + a*e^4)^3)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x+d)/(c*x**4+a)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 1.52247, size = 835, normalized size = 0.62

$$256\log(d+ex)e^{11} - 64\log\left(cx^4+a\right)e^{11} + \frac{32\left(cd^4+ae^4\right)^2\left(ae^3+cdx\left(d^2-exd+e^2x^2\right)\right)}{a\left(cx^4+a\right)^2} + \frac{8\left(cd^4+ae^4\right)\left(8a^2e^7+acdx\left(15d^2-14exd+13e^2x^2\right)e^4+c^2d^5\right.}{a^2\left(cx^4+a\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d + e*x)*(a + c*x^4)^3),x]
```

```
[Out] ((32*(c*d^4 + a*e^4)^2*(a*e^3 + c*d*x*(d^2 - d*e*x + e^2*x^2)))/(
a*(a + c*x^4)^2) + (8*(c*d^4 + a*e^4)*(8*a^2*e^7 + c^2*d^5*x*(7*d
^2 - 6*d*e*x + 5*e^2*x^2) + a*c*d*e^4*x*(15*d^2 - 14*d*e*x + 13*e
^2*x^2)))/(a^2*(a + c*x^4)) - (2*c^(1/4)*d*(21*Sqrt[2]*c^(5/2)*d^
10 - 24*a^(1/4)*c^(9/4)*d^9*e + 5*Sqrt[2]*Sqrt[a]*c^2*d^8*e^2 + 6
6*Sqrt[2]*a*c^(3/2)*d^6*e^4 - 80*a^(5/4)*c^(5/4)*d^5*e^5 + 18*Sqr
t[2]*a^(3/2)*c*d^4*e^6 + 77*Sqrt[2]*a^2*Sqrt[c]*d^2*e^8 - 120*a^(
9/4)*c^(1/4)*d*e^9 + 45*Sqrt[2]*a^(5/2)*e^10)*ArcTan[1 - (Sqrt[2]
*c^(1/4)*x)/a^(1/4)])/a^(11/4) + (2*c^(1/4)*d*(21*Sqrt[2]*c^(5/2)
*d^10 + 24*a^(1/4)*c^(9/4)*d^9*e + 5*Sqrt[2]*Sqrt[a]*c^2*d^8*e^2
+ 66*Sqrt[2]*a*c^(3/2)*d^6*e^4 + 80*a^(5/4)*c^(5/4)*d^5*e^5 + 18*
Sqrt[2]*a^(3/2)*c*d^4*e^6 + 77*Sqrt[2]*a^2*Sqrt[c]*d^2*e^8 + 120*
a^(9/4)*c^(1/4)*d*e^9 + 45*Sqrt[2]*a^(5/2)*e^10)*ArcTan[1 + (Sqrt
[2]*c^(1/4)*x)/a^(1/4)])/a^(11/4) + 256*e^11*Log[d + e*x] + (Sqrt
[2]*c^(1/4)*(-21*c^(5/2)*d^11 + 5*Sqrt[a]*c^2*d^9*e^2 - 66*a*c^(3
/2)*d^7*e^4 + 18*a^(3/2)*c*d^5*e^6 - 77*a^2*Sqrt[c]*d^3*e^8 + 45*
a^(5/2)*d*e^10)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]
*x^2])/a^(11/4) + (Sqrt[2]*c^(1/4)*(21*c^(5/2)*d^11 - 5*Sqrt[a]*c
^2*d^9*e^2 + 66*a*c^(3/2)*d^7*e^4 - 18*a^(3/2)*c*d^5*e^6 + 77*a^2
*Sqrt[c]*d^3*e^8 - 45*a^(5/2)*d*e^10)*Log[Sqrt[a] + Sqrt[2]*a^(1/
4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(11/4) - 64*e^11*Log[a + c*x^4])/(
256*(c*d^4 + a*e^4)^3)
```

Maple [A] time = 0.035, size = 2106, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*x+d)/(c*x^4+a)^3,x)
```

```
[Out] 9/64/(a*e^4+c*d^4)^3*c/a/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/
c*a)^(1/4)*x-1)*d^5*e^6+5/128/(a*e^4+c*d^4)^3*c^2/a^2/(1/c*a)^(1/
4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*d^9*e^2+9/128/(a*e^4
+c*d^4)^3*c/a/(1/c*a)^(1/4)*2^(1/2)*ln((x^2-(1/c*a)^(1/4)*x*2^(1/
2)+(1/c*a)^(1/2))/(x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))*d^
5*e^6+5/256/(a*e^4+c*d^4)^3*c^2/a^2/(1/c*a)^(1/4)*2^(1/2)*ln((x^2
-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2+(1/c*a)^(1/4)*x*2^(1
/2)+(1/c*a)^(1/2)))*d^9*e^2+77/128/(a*e^4+c*d^4)^3*c*(1/c*a)^(1/4
)/a*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*d^3*e^8+77/128/(a*e
^4+c*d^4)^3*c*(1/c*a)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4
)*x-1)*d^3*e^8+21/256/(a*e^4+c*d^4)^3*c^3*(1/c*a)^(1/4)/a^3*2^(1/
2)*ln((x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2-(1/c*a)^(1
/4)*x*2^(1/2)+(1/c*a)^(1/2)))*d^11-5/8/(a*e^4+c*d^4)^3*c^2/(a^5*c
)^(1/2)*arctan(x^2*(c/a)^(1/2))*a*d^6*e^5+5/32/(a*e^4+c*d^4)^3*c^
4/(c*x^4+a)^2*d^9*e^2/a^2*x^7-5/8/(a*e^4+c*d^4)^3*c^3/(c*x^4+a)^2
*d^6*e^5/a*x^6+15/32/(a*e^4+c*d^4)^3*c^2/(c*x^4+a)^2*d^3*x^5*e^8+
7/32/(a*e^4+c*d^4)^3*c^4/(c*x^4+a)^2*d^11/a^2*x^5+1/4/(a*e^4+c*d^
4)^3*c^2/(c*x^4+a)^2*x^4*d^4*e^7+13/16/(a*e^4+c*d^4)^3*c^2/(c*x^4
+a)^2*d^5*e^6*x^3+1/8/(a*e^4+c*d^4)^3*c^2/(c*x^4+a)^2*e^3*d^8-3/1
6/(a*e^4+c*d^4)^3*c^4/(c*x^4+a)^2*d^10*e/a^2*x^6+9/16/(a*e^4+c*d^
4)^3*c^3/(c*x^4+a)^2*d^5*e^6/a*x^7-15/16/(a*e^4+c*d^4)^3*c/(a^5*c
)^(1/2)*arctan(x^2*(c/a)^(1/2))*a^2*d^2*e^9+11/16/(a*e^4+c*d^4)^3
*c^3/(c*x^4+a)^2*d^7/a*x^5*e^4+3/8/(a*e^4+c*d^4)^3/(c*x^4+a)^2*e^
11*a^2-1/4/(a*e^4+c*d^4)^3*e^11*ln(a^2*(c*x^4+a))+9/64/(a*e^4+c*d
^4)^3*c/a/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)
*d^5*e^6+33/64/(a*e^4+c*d^4)^3*c^2*(1/c*a)^(1/4)/a^2*2^(1/2)*arct
an(2^(1/2)/(1/c*a)^(1/4)*x-1)*d^7*e^4+9/32/(a*e^4+c*d^4)^3*c^3/(c
*x^4+a)^2*d^9*e^2/a*x^3-5/16/(a*e^4+c*d^4)^3*c^3/(c*x^4+a)^2*d^10
*e/a*x^2+21/128/(a*e^4+c*d^4)^3*c^3*(1/c*a)^(1/4)/a^3*2^(1/2)*arc
tan(2^(1/2)/(1/c*a)^(1/4)*x+1)*d^11+19/32/(a*e^4+c*d^4)^3*c/(c*x^
4+a)^2*d^3*a*x*e^8+17/32/(a*e^4+c*d^4)^3*c/(c*x^4+a)^2*d*e^10*a*x
^3-9/16/(a*e^4+c*d^4)^3*c/(c*x^4+a)^2*d^2*e^9*a*x^2+21/128/(a*e^4
+c*d^4)^3*c^3*(1/c*a)^(1/4)/a^3*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1
/4)*x-1)*d^11+77/256/(a*e^4+c*d^4)^3*c*(1/c*a)^(1/4)/a*2^(1/2)*ln
((x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2-(1/c*a)^(1/4)*x
*2^(1/2)+(1/c*a)^(1/2)))*d^3*e^8+e^11*ln(e*x+d)/(a*e^4+c*d^4)^3+5
/128/(a*e^4+c*d^4)^3*c^2/a^2/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)
/(1/c*a)^(1/4)*x+1)*d^9*e^2+33/128/(a*e^4+c*d^4)^3*c^2*(1/c*a)^(1
/4)/a^2*2^(1/2)*ln((x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x
^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))*d^7*e^4+33/64/(a*e^4+c
*d^4)^3*c^2*(1/c*a)^(1/4)/a^2*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4
)*x+1)*d^7*e^4-7/8/(a*e^4+c*d^4)^3*c^2/(c*x^4+a)^2*d^6*e^5*x^2+15
/16/(a*e^4+c*d^4)^3*c^2/(c*x^4+a)^2*d^7*x*e^4+11/32/(a*e^4+c*d^4)
^3*c^3/(c*x^4+a)^2*d^11/a*x-3/16/(a*e^4+c*d^4)^3*c^3/(a^5*c)^(1/2
)*arctan(x^2*(c/a)^(1/2))*d^10*e+45/128/(a*e^4+c*d^4)^3/(1/c*a)^(
1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*d*e^10+45/256/(a*e
^4+c*d^4)^3/(1/c*a)^(1/4)*2^(1/2)*ln((x^2-(1/c*a)^(1/4)*x*2^(1/2)
+(1/c*a)^(1/2))/(x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))*d*e^
10+45/128/(a*e^4+c*d^4)^3/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1
/c*a)^(1/4)*x+1)*d*e^10+1/4/(a*e^4+c*d^4)^3*c/(c*x^4+a)^2*x^4*a*e
^11+1/2/(a*e^4+c*d^4)^3*c/(c*x^4+a)^2*a*d^4*e^7+13/32/(a*e^4+c*d^
4)^3*c^2/(c*x^4+a)^2*d*e^10*x^7-7/16/(a*e^4+c*d^4)^3*c^2/(c*x^4+a
)^2*d^2*e^9*x^6
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)^3*(e*x + d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)^3*(e*x + d)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*x+d)/(c*x**4+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.388769, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)^3*(e*x + d)),x, algorithm="giac")
```

```
[Out] Done
```

3.413 $\int \frac{1}{(d+ex)^2(a+cx^4)^3}\,dx$

Optimal. Leaf size=1830

result too large to display

```
[Out] -(e^11/((c*d^4 + a*e^4)^3*(d + e*x))) + (c*x*(7*d^2*(c*d^4 - 3*a*
e^4) - 12*d*e*(c*d^4 - a*e^4)*x + 5*e^2*(3*c*d^4 - a*e^4)*x^2))/(
32*a^2*(c*d^4 + a*e^4)^2*(a + c*x^4)) + (c*(4*a*d^3*e^3 + x*(d^2*
(c*d^4 - 3*a*e^4) - 2*d*e*(c*d^4 - a*e^4)*x + e^2*(3*c*d^4 - a*e^
4)*x^2)))/(8*a*(c*d^4 + a*e^4)^2*(a + c*x^4)^2) + (c*e^4*(8*a*d^3
*e^3 + x*(d^2*(5*c*d^4 - 3*a*e^4) - 2*d*e*(3*c*d^4 - a*e^4)*x + e
^2*(7*c*d^4 - a*e^4)*x^2)))/(4*a*(c*d^4 + a*e^4)^3*(a + c*x^4)) -
 (Sqrt[c]*d*e^9*(5*c*d^4 - a*e^4)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/
(Sqrt[a]*(c*d^4 + a*e^4)^4) - (Sqrt[c]*d*e^5*(3*c*d^4 - a*e^4)*Ar
cTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*a^(3/2)*(c*d^4 + a*e^4)^3) - (3*S
qrt[c]*d*e*(c*d^4 - a*e^4)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(8*a^(5
/2)*(c*d^4 + a*e^4)^2) - (c^(1/4)*(21*Sqrt[c]*d^2*(c*d^4 - 3*a*e^
4) + 5*Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*ArcTan[1 - (Sqrt[2]*c^(1/4)
*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*(c*d^4 + a*e^4)^2) - (c^(1/4)*
e^4*(3*Sqrt[c]*d^2*(5*c*d^4 - 3*a*e^4) + Sqrt[a]*e^2*(7*c*d^4 - a
*e^4))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4
)*(c*d^4 + a*e^4)^3) - (c^(1/4)*e^8*(3*Sqrt[c]*d^2*(3*c*d^4 - a*e
^4) + Sqrt[a]*e^2*(11*c*d^4 - a*e^4))*ArcTan[1 - (Sqrt[2]*c^(1/4)
*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^4) + (c^(1/4)*(2
1*Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) + 5*Sqrt[a]*e^2*(3*c*d^4 - a*e^4)
)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*(
c*d^4 + a*e^4)^2) + (c^(1/4)*e^4*(3*Sqrt[c]*d^2*(5*c*d^4 - 3*a*e^
4) + Sqrt[a]*e^2*(7*c*d^4 - a*e^4))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x
)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^3) + (c^(1/4)*e^8*
(3*Sqrt[c]*d^2*(3*c*d^4 - a*e^4) + Sqrt[a]*e^2*(11*c*d^4 - a*e^4)
)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*
d^4 + a*e^4)^4) + (12*c*d^3*e^11*Log[d + e*x])/(c*d^4 + a*e^4)^4
- (c^(1/4)*e^8*(9*c^(3/2)*d^6 - 11*Sqrt[a]*c*d^4*e^2 - 3*a*Sqrt[c
]*d^2*e^4 + a^(3/2)*e^6)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x
+ Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^4) - (c^(1/4)*
(21*Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) - 5*Sqrt[a]*e^2*(3*c*d^4 - a*e^
4))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*
Sqrt[2]*a^(11/4)*(c*d^4 + a*e^4)^2) - (c^(1/4)*e^4*(3*Sqrt[c]*d^2
*(5*c*d^4 - 3*a*e^4) - Sqrt[a]*e^2*(7*c*d^4 - a*e^4))*Log[Sqrt[a]
 - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*
(c*d^4 + a*e^4)^3) + (c^(1/4)*e^8*(9*c^(3/2)*d^6 - 11*Sqrt[a]*c*d
^4*e^2 - 3*a*Sqrt[c]*d^2*e^4 + a^(3/2)*e^6)*Log[Sqrt[a] + Sqrt[2]
*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*
e^4)^4) + (c^(1/4)*(21*Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) - 5*Sqrt[a]*
e^2*(3*c*d^4 - a*e^4))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x +
Sqrt[c]*x^2])/(128*Sqrt[2]*a^(11/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)
*e^4*(3*Sqrt[c]*d^2*(5*c*d^4 - 3*a*e^4) - Sqrt[a]*e^2*(7*c*d^4 -
a*e^4))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(
16*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^3) - (3*c*d^3*e^11*Log[a + c*x
^4])/(c*d^4 + a*e^4)^4
```

Rubi [A] time = 6.58406, antiderivative size = 1830, normalized size of antiderivative = 1., number of steps used = 46, number of rules used = 14, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.824$

result too large to display

Antiderivative was successfully verified.

```
[In]  Int[1/((d + e*x)^2*(a + c*x^4)^3),x]
```

```
[Out] -(e^11/((c*d^4 + a*e^4)^3*(d + e*x))) + (c*x*(7*d^2*(c*d^4 - 3*a*
e^4) - 12*d*e*(c*d^4 - a*e^4)*x + 5*e^2*(3*c*d^4 - a*e^4)*x^2))/(
32*a^2*(c*d^4 + a*e^4)^2*(a + c*x^4)) + (c*(4*a*d^3*e^3 + x*(d^2*
(c*d^4 - 3*a*e^4) - 2*d*e*(c*d^4 - a*e^4)*x + e^2*(3*c*d^4 - a*e^
4)*x^2)))/(8*a*(c*d^4 + a*e^4)^2*(a + c*x^4)^2) + (c*e^4*(8*a*d^3
```

```
*e^3 + x*(d^2*(5*c*d^4 - 3*a*e^4) - 2*d*e*(3*c*d^4 - a*e^4)*x + e
^2*(7*c*d^4 - a*e^4)*x^2)))/(4*a*(c*d^4 + a*e^4)^3*(a + c*x^4)) -
 (Sqrt[c]*d*e^9*(5*c*d^4 - a*e^4)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/
(Sqrt[a]*(c*d^4 + a*e^4)^4) - (Sqrt[c]*d*e^5*(3*c*d^4 - a*e^4)*Ar
cTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*a^(3/2)*(c*d^4 + a*e^4)^3) - (3*S
qrt[c]*d*e*(c*d^4 - a*e^4)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(8*a^(5
/2)*(c*d^4 + a*e^4)^2) - (c^(1/4)*(21*Sqrt[c]*d^2*(c*d^4 - 3*a*e^
4) + 5*Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*ArcTan[1 - (Sqrt[2]*c^(1/4)
*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*(c*d^4 + a*e^4)^2) - (c^(1/4)*
e^4*(3*Sqrt[c]*d^2*(5*c*d^4 - 3*a*e^4) + Sqrt[a]*e^2*(7*c*d^4 - a
*e^4))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4
)*(c*d^4 + a*e^4)^3) - (c^(1/4)*e^8*(3*Sqrt[c]*d^2*(3*c*d^4 - a*e
^4) + Sqrt[a]*e^2*(11*c*d^4 - a*e^4))*ArcTan[1 - (Sqrt[2]*c^(1/4)
*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^4) + (c^(1/4)*(2
1*Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) + 5*Sqrt[a]*e^2*(3*c*d^4 - a*e^4)
)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*(
c*d^4 + a*e^4)^2) + (c^(1/4)*e^4*(3*Sqrt[c]*d^2*(5*c*d^4 - 3*a*e^
4) + Sqrt[a]*e^2*(7*c*d^4 - a*e^4))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x
)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^3) + (c^(1/4)*e^8*
(3*Sqrt[c]*d^2*(3*c*d^4 - a*e^4) + Sqrt[a]*e^2*(11*c*d^4 - a*e^4)
)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*
d^4 + a*e^4)^4) + (12*c*d^3*e^11*Log[d + e*x])/(c*d^4 + a*e^4)^4
- (c^(1/4)*e^8*(9*c^(3/2)*d^6 - 11*Sqrt[a]*c*d^4*e^2 - 3*a*Sqrt[c
]*d^2*e^4 + a^(3/2)*e^6)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x
+ Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^4) - (c^(1/4)*
(21*Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) - 5*Sqrt[a]*e^2*(3*c*d^4 - a*e^
4))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*
Sqrt[2]*a^(11/4)*(c*d^4 + a*e^4)^2) - (c^(1/4)*e^4*(3*Sqrt[c]*d^2
*(5*c*d^4 - 3*a*e^4) - Sqrt[a]*e^2*(7*c*d^4 - a*e^4))*Log[Sqrt[a]
 - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*
(c*d^4 + a*e^4)^3) + (c^(1/4)*e^8*(9*c^(3/2)*d^6 - 11*Sqrt[a]*c*d
^4*e^2 - 3*a*Sqrt[c]*d^2*e^4 + a^(3/2)*e^6)*Log[Sqrt[a] + Sqrt[2]
*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*
e^4)^4) + (c^(1/4)*(21*Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) - 5*Sqrt[a]*
e^2*(3*c*d^4 - a*e^4))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x +
Sqrt[c]*x^2])/(128*Sqrt[2]*a^(11/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)
*e^4*(3*Sqrt[c]*d^2*(5*c*d^4 - 3*a*e^4) - Sqrt[a]*e^2*(7*c*d^4 -
a*e^4))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(
16*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^3) - (3*c*d^3*e^11*Log[a + c*x
^4])/(c*d^4 + a*e^4)^4
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x+d)**2/(c*x**4+a)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 3.03633, size = 1115, normalized size = 0.61

$$3072cd^3\log(d+ex)e^{11}-768cd^3\log\left(cx^4+a\right)e^{11}-\frac{256\left(cd^4+ae^4\right)e^{11}}{d+ex}+\frac{8c\left(cd^4+ae^4\right)\left(c^2x\left(7d^2-12exd+15e^2x^2\right)d^8+2ace^4x\left(13d^2-24exd+33e\right.\right.}{a^2\left(cx^4+a\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d + e*x)^2*(a + c*x^4)^3),x]
```

```
[Out] ((-256*e^11*(c*d^4 + a*e^4))/(d + e*x) + (8*c*(c*d^4 + a*e^4)*(c^
2*d^8*x*(7*d^2 - 12*d*e*x + 15*e^2*x^2) + 2*a*c*d^4*e^4*x*(13*d^2
```

```
 - 24*d*e*x + 33*e^2*x^2) + a^2*e^7*(64*d^3 - 45*d^2*e*x + 28*d*e
^2*x^2 - 13*e^3*x^3)))/(a^2*(a + c*x^4)) + (32*c*(c*d^4 + a*e^4)^
2*(c*d^4*x*(d^2 - 2*d*e*x + 3*e^2*x^2) + a*e^3*(4*d^3 - 3*d^2*e*x
 + 2*d*e^2*x^2 - e^3*x^3)))/(a*(a + c*x^4)^2) - (6*c^(1/4)*(7*Sqr
t[2]*c^(7/2)*d^14 - 16*a^(1/4)*c^(13/4)*d^13*e + 5*Sqrt[2]*Sqrt[a
]*c^3*d^12*e^2 + 33*Sqrt[2]*a*c^(5/2)*d^10*e^4 - 80*a^(5/4)*c^(9/
4)*d^9*e^5 + 27*Sqrt[2]*a^(3/2)*c^2*d^8*e^6 + 77*Sqrt[2]*a^2*c^(3
/2)*d^6*e^8 - 240*a^(9/4)*c^(5/4)*d^5*e^9 + 135*Sqrt[2]*a^(5/2)*c
*d^4*e^10 - 77*Sqrt[2]*a^3*Sqrt[c]*d^2*e^12 + 80*a^(13/4)*c^(1/4)
*d*e^13 - 15*Sqrt[2]*a^(7/2)*e^14)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)
/a^(1/4)])/a^(11/4) + (6*c^(1/4)*(7*Sqrt[2]*c^(7/2)*d^14 + 16*a^(
1/4)*c^(13/4)*d^13*e + 5*Sqrt[2]*Sqrt[a]*c^3*d^12*e^2 + 33*Sqrt[2
]*a*c^(5/2)*d^10*e^4 + 80*a^(5/4)*c^(9/4)*d^9*e^5 + 27*Sqrt[2]*a^
(3/2)*c^2*d^8*e^6 + 77*Sqrt[2]*a^2*c^(3/2)*d^6*e^8 + 240*a^(9/4)*
c^(5/4)*d^5*e^9 + 135*Sqrt[2]*a^(5/2)*c*d^4*e^10 - 77*Sqrt[2]*a^3
*Sqrt[c]*d^2*e^12 - 80*a^(13/4)*c^(1/4)*d*e^13 - 15*Sqrt[2]*a^(7/
2)*e^14)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(11/4) + 3072
*c*d^3*e^11*Log[d + e*x] - (3*Sqrt[2]*c^(1/4)*(7*c^(7/2)*d^14 - 5
*Sqrt[a]*c^3*d^12*e^2 + 33*a*c^(5/2)*d^10*e^4 - 27*a^(3/2)*c^2*d^
8*e^6 + 77*a^2*c^(3/2)*d^6*e^8 - 135*a^(5/2)*c*d^4*e^10 - 77*a^3*
Sqrt[c]*d^2*e^12 + 15*a^(7/2)*e^14)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)
*c^(1/4)*x + Sqrt[c]*x^2])/a^(11/4) + (3*Sqrt[2]*c^(1/4)*(7*c^(7/
2)*d^14 - 5*Sqrt[a]*c^3*d^12*e^2 + 33*a*c^(5/2)*d^10*e^4 - 27*a^(
3/2)*c^2*d^8*e^6 + 77*a^2*c^(3/2)*d^6*e^8 - 135*a^(5/2)*c*d^4*e^1
0 - 77*a^3*Sqrt[c]*d^2*e^12 + 15*a^(7/2)*e^14)*Log[Sqrt[a] + Sqrt
[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(11/4) - 768*c*d^3*e^11*L
og[a + c*x^4])/(256*(c*d^4 + a*e^4)^4)
```

Maple [A] time = 0.042, size = 2781, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*x+d)^2/(c*x^4+a)^3,x)
```

```
[Out] 231/128/(a*e^4+c*d^4)^4*c^2*(1/c*a)^(1/4)/a*2^(1/2)*arctan(2^(1/2
)/(1/c*a)^(1/4)*x-1)*d^6*e^8+9/8/(a*e^4+c*d^4)^4*c/(c*x^4+a)^2*d*
e^13*a^2*x^2-57/32/(a*e^4+c*d^4)^4*c/(c*x^4+a)^2*d^2*a^2*x*e^12+2
7/32/(a*e^4+c*d^4)^4*c^4/(c*x^4+a)^2*e^2/a*x^3*d^12-3/8/(a*e^4+c*
d^4)^4*c^2/(c*x^4+a)^2*d^5*e^9*a*x^2-5/8/(a*e^4+c*d^4)^4*c^4/(c*x
^4+a)^2*d^13*e/a*x^2-39/32/(a*e^4+c*d^4)^4*c^2/(c*x^4+a)^2*d^6*a*
x*e^8+81/256/(a*e^4+c*d^4)^4*c^2/a/(1/c*a)^(1/4)*2^(1/2)*ln((x^2-
(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2+(1/c*a)^(1/4)*x*2^(1/
2)+(1/c*a)^(1/2)))*d^8*e^6+99/128/(a*e^4+c*d^4)^4*c^3*(1/c*a)^(1/
4)/a^2*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*d^10*e^4-5/8/(a*
e^4+c*d^4)^4*c^3/(c*x^4+a)^2*d^5*e^9*x^6-19/32/(a*e^4+c*d^4)^4*c^
3/(c*x^4+a)^2*d^6*x^5*e^8+7/32/(a*e^4+c*d^4)^4*c^5/(c*x^4+a)^2*d^
14/a^2*x^5-45/256/(a*e^4+c*d^4)^4*a/(1/c*a)^(1/4)*2^(1/2)*ln((x^2
-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2+(1/c*a)^(1/4)*x*2^(1
/2)+(1/c*a)^(1/2)))*e^14-45/128/(a*e^4+c*d^4)^4*a/(1/c*a)^(1/4)*2
^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*e^14-45/128/(a*e^4+c*d^4
)^4*a/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*e^1
4-3/8/(a*e^4+c*d^4)^4*c^4/(a^5*c)^(1/2)*arctan(x^2*(c/a)^(1/2))*d
^13*e+21/128/(a*e^4+c*d^4)^4*c^4*(1/c*a)^(1/4)/a^3*2^(1/2)*arctan
(2^(1/2)/(1/c*a)^(1/4)*x+1)*d^14-45/8/(a*e^4+c*d^4)^4*c^2/(a^5*c)
^(1/2)*arctan(x^2*(c/a)^(1/2))*a^2*d^5*e^9-15/8/(a*e^4+c*d^4)^4*c
^3/(a^5*c)^(1/2)*arctan(x^2*(c/a)^(1/2))*a*d^9*e^5+21/128/(a*e^4+
c*d^4)^4*c^4*(1/c*a)^(1/4)/a^3*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/
4)*x-1)*d^14-231/256/(a*e^4+c*d^4)^4*c*(1/c*a)^(1/4)*2^(1/2)*ln((
x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2-(1/c*a)^(1/4)*x*2
^(1/2)+(1/c*a)^(1/2)))*d^2*e^12+21/256/(a*e^4+c*d^4)^4*c^4*(1/c*a
)^(1/4)/a^3*2^(1/2)*ln((x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)
)/(x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))*d^14+405/256/(a*e^
4+c*d^4)^4*c/(1/c*a)^(1/4)*2^(1/2)*ln((x^2-(1/c*a)^(1/4)*x*2^(1/2
)+(1/c*a)^(1/2))/(x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))*d^4
*e^10+405/128/(a*e^4+c*d^4)^4*c/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1
```

```
/2)/(1/c*a)^(1/4)*x-1)*d^4*e^10+405/128/(a*e^4+c*d^4)^4*c/(1/c*a)
^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*d^4*e^10+15/8/(a
*e^4+c*d^4)^4*c/(a^5*c)^(1/2)*arctan(x^2*(c/a)^(1/2))*a^3*d*e^13-
231/128/(a*e^4+c*d^4)^4*c*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1
/c*a)^(1/4)*x-1)*d^2*e^12-3/8/(a*e^4+c*d^4)^4*c^5/(c*x^4+a)^2*d^1
3*e/a^2*x^6-45/32/(a*e^4+c*d^4)^4*c^2/(c*x^4+a)^2*d^2*a*x^5*e^12+
33/32/(a*e^4+c*d^4)^4*c^4/(c*x^4+a)^2*d^10/a*x^5*e^4+57/32/(a*e^4
+c*d^4)^4*c^2/(c*x^4+a)^2*e^10*a*x^3*d^4+81/128/(a*e^4+c*d^4)^4*c
^2/a/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*d^8*
e^6+15/256/(a*e^4+c*d^4)^4*c^3/a^2/(1/c*a)^(1/4)*2^(1/2)*ln((x^2-
(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2+(1/c*a)^(1/4)*x*2^(1/
2)+(1/c*a)^(1/2)))*d^12*e^2+15/128/(a*e^4+c*d^4)^4*c^3/a^2/(1/c*a
)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*d^12*e^2-e^11/(
a*e^4+c*d^4)^3/(e*x+d)+1/2/(a*e^4+c*d^4)^4*c^3/(c*x^4+a)^2*d^11*e
^3-3/(a*e^4+c*d^4)^4*c*d^3*e^11*ln(a^2*(c*x^4+a))+81/128/(a*e^4+c
*d^4)^4*c^2/a/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*
x+1)*d^8*e^6+231/256/(a*e^4+c*d^4)^4*c^2*(1/c*a)^(1/4)/a*2^(1/2)*
ln((x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2-(1/c*a)^(1/4)
*x*2^(1/2)+(1/c*a)^(1/2)))*d^6*e^8+99/256/(a*e^4+c*d^4)^4*c^3*(1/
c*a)^(1/4)/a^2*2^(1/2)*ln((x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1
/2))/(x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))*d^10*e^4+231/12
8/(a*e^4+c*d^4)^4*c^2*(1/c*a)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(1/c
*a)^(1/4)*x+1)*d^6*e^8+99/128/(a*e^4+c*d^4)^4*c^3*(1/c*a)^(1/4)/a
^2*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*d^10*e^4+12*c*d^3*e^
11*ln(e*x+d)/(a*e^4+c*d^4)^4+3/(a*e^4+c*d^4)^4*c^2/(c*x^4+a)^2*a*
d^7*e^7+11/32/(a*e^4+c*d^4)^4*c^4/(c*x^4+a)^2*d^14/a*x+2/(a*e^4+c
*d^4)^4*c^3/(c*x^4+a)^2*x^4*d^7*e^7-13/32/(a*e^4+c*d^4)^4*c^2/(c*
x^4+a)^2*e^14*a*x^7-17/32/(a*e^4+c*d^4)^4*c/(c*x^4+a)^2*e^14*a^2*
x^3+5/2/(a*e^4+c*d^4)^4*c/(c*x^4+a)^2*a^2*d^3*e^11+101/32/(a*e^4+
c*d^4)^4*c^3/(c*x^4+a)^2*e^6*x^3*d^8-17/8/(a*e^4+c*d^4)^4*c^3/(c*
x^4+a)^2*d^9*e^5*x^2+29/32/(a*e^4+c*d^4)^4*c^3/(c*x^4+a)^2*d^10*x
*e^4+53/32/(a*e^4+c*d^4)^4*c^3/(c*x^4+a)^2*e^10*x^7*d^4+2/(a*e^4+
c*d^4)^4*c^2/(c*x^4+a)^2*x^4*a*d^3*e^11+81/32/(a*e^4+c*d^4)^4*c^4
/(c*x^4+a)^2*e^6/a*x^7*d^8+15/32/(a*e^4+c*d^4)^4*c^5/(c*x^4+a)^2*
e^2/a^2*x^7*d^12+7/8/(a*e^4+c*d^4)^4*c^2/(c*x^4+a)^2*d*e^13*a*x^6
-15/8/(a*e^4+c*d^4)^4*c^4/(c*x^4+a)^2*d^9*e^5/a*x^6-231/128/(a*e^
4+c*d^4)^4*c*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x
+1)*d^2*e^12+15/128/(a*e^4+c*d^4)^4*c^3/a^2/(1/c*a)^(1/4)*2^(1/2)
*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*d^12*e^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)^3*(e*x + d)^2),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)^3*(e*x + d)^2),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*x+d)**2/(c*x**4+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.410421, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)^3*(e*x + d)^2),x, algorithm="giac")
```

```
[Out] Done
```

3.414 $\int \frac{1}{(d+ex)^3(a+cx^4)^3} dx$

Optimal. Leaf size=2204

result too large to display

```
[Out] -e^11/(2*(c*d^4 + a*e^4)^3*(d + e*x)^2) - (12*c*d^3*e^11)/((c*d^4
 + a*e^4)^4*(d + e*x)) + (c*x*(7*d*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*
a^2*e^8) - 6*e*(3*c^2*d^8 - 12*a*c*d^4*e^4 + a^2*e^8)*x + 10*c*d^
3*e^2*(3*c*d^4 - 5*a*e^4)*x^2))/(32*a^2*(c*d^4 + a*e^4)^3*(a + c*
x^4)) + (c*(2*a*d^2*e^3*(5*c*d^4 - 3*a*e^4) + x*(d*(c^2*d^8 - 12*
a*c*d^4*e^4 + 3*a^2*e^8) - e*(3*c^2*d^8 - 12*a*c*d^4*e^4 + a^2*e^
8)*x + 2*c*d^3*e^2*(3*c*d^4 - 5*a*e^4)*x^2)))/(8*a*(c*d^4 + a*e^4
)^3*(a + c*x^4)^2) + (c*e^4*(12*a*d^2*e^3*(3*c*d^4 - a*e^4) + x*(
3*d*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8) - e*(21*c^2*d^8 - 26*a
*c*d^4*e^4 + a^2*e^8)*x + 4*c*d^3*e^2*(7*c*d^4 - 5*a*e^4)*x^2)))/
(4*a*(c*d^4 + a*e^4)^4*(a + c*x^4)) - (Sqrt[c]*e^9*(55*c^2*d^8 -
40*a*c*d^4*e^4 + a^2*e^8)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[
a]*(c*d^4 + a*e^4)^5) - (Sqrt[c]*e^5*(21*c^2*d^8 - 26*a*c*d^4*e^4
 + a^2*e^8)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(3/2)*(c*d^4 + a*
e^4)^4) - (3*Sqrt[c]*e*(3*c^2*d^8 - 12*a*c*d^4*e^4 + a^2*e^8)*Arc
Tan[(Sqrt[c]*x^2)/Sqrt[a]])/(16*a^(5/2)*(c*d^4 + a*e^4)^3) - (3*c
^(3/4)*d*e^8*(15*c^2*d^8 - 16*a*c*d^4*e^4 + a^2*e^8 + 2*Sqrt[a]*S
qrt[c]*d^2*e^2*(11*c*d^4 - 5*a*e^4))*ArcTan[1 - (Sqrt[2]*c^(1/4)*
x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^5) - (c^(3/4)*d*e
^4*(4*Sqrt[a]*Sqrt[c]*d^2*e^2*(7*c*d^4 - 5*a*e^4) + 9*(5*c^2*d^8
- 10*a*c*d^4*e^4 + a^2*e^8))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/
4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^4) - (c^(3/4)*d*(10*Sqrt[
a]*Sqrt[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4) + 21*(c^2*d^8 - 12*a*c*d^4
*e^4 + 3*a^2*e^8))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*S
qrt[2]*a^(11/4)*(c*d^4 + a*e^4)^3) + (3*c^(3/4)*d*e^8*(15*c^2*d^8
 - 16*a*c*d^4*e^4 + a^2*e^8 + 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(11*c*d^4
 - 5*a*e^4))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*
a^(3/4)*(c*d^4 + a*e^4)^5) + (c^(3/4)*d*e^4*(4*Sqrt[a]*Sqrt[c]*d^
2*e^2*(7*c*d^4 - 5*a*e^4) + 9*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e
^8))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*
(c*d^4 + a*e^4)^4) + (c^(3/4)*d*(10*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*
d^4 - 5*a*e^4) + 21*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8))*ArcTa
n[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*(c*d^4 +
 a*e^4)^3) + (6*c*d^2*e^11*(13*c*d^4 - 3*a*e^4)*Log[d + e*x])/(c*
d^4 + a*e^4)^5 - (3*c^(3/4)*d*e^8*(15*c^2*d^8 - 16*a*c*d^4*e^4 +
a^2*e^8 - 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(11*c*d^4 - 5*a*e^4))*Log[Sqr
t[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/
4)*(c*d^4 + a*e^4)^5) + (c^(3/4)*d*e^4*(4*Sqrt[a]*Sqrt[c]*d^2*e^2
*(7*c*d^4 - 5*a*e^4) - 9*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8))*
Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[
2]*a^(7/4)*(c*d^4 + a*e^4)^4) + (c^(3/4)*d*(10*Sqrt[a]*Sqrt[c]*d^
2*e^2*(3*c*d^4 - 5*a*e^4) - 21*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*
e^8))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(12
8*Sqrt[2]*a^(11/4)*(c*d^4 + a*e^4)^3) + (3*c^(3/4)*d*e^8*(15*c^2*
d^8 - 16*a*c*d^4*e^4 + a^2*e^8 - 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(11*c*
d^4 - 5*a*e^4))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]
*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^5) - (c^(3/4)*d*e^4*(4*
Sqrt[a]*Sqrt[c]*d^2*e^2*(7*c*d^4 - 5*a*e^4) - 9*(5*c^2*d^8 - 10*a
*c*d^4*e^4 + a^2*e^8))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x +
Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^4) - (c^(3/4)*d
*(10*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4) - 21*(c^2*d^8 -
12*a*c*d^4*e^4 + 3*a^2*e^8))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4
)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(11/4)*(c*d^4 + a*e^4)^3) - (3
*c*d^2*e^11*(13*c*d^4 - 3*a*e^4)*Log[a + c*x^4])/(2*(c*d^4 + a*e^
4)^5)
```

Rubi [A] time = 7.56783, antiderivative size = 2204, normalized size of antiderivative = 1., number of steps used = 46, number of rules used = 14, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.824

result too large to display

Antiderivative was successfully verified.

```
[In]  Int[1/((d + e*x)^3*(a + c*x^4)^3),x]
```

```
[Out] -e^11/(2*(c*d^4 + a*e^4)^3*(d + e*x)^2) - (12*c*d^3*e^11)/((c*d^4
 + a*e^4)^4*(d + e*x)) + (c*x*(7*d*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*
a^2*e^8) - 6*e*(3*c^2*d^8 - 12*a*c*d^4*e^4 + a^2*e^8)*x + 10*c*d^
3*e^2*(3*c*d^4 - 5*a*e^4)*x^2))/(32*a^2*(c*d^4 + a*e^4)^3*(a + c*
x^4)) + (c*(2*a*d^2*e^3*(5*c*d^4 - 3*a*e^4) + x*(d*(c^2*d^8 - 12*
a*c*d^4*e^4 + 3*a^2*e^8) - e*(3*c^2*d^8 - 12*a*c*d^4*e^4 + a^2*e^
8)*x + 2*c*d^3*e^2*(3*c*d^4 - 5*a*e^4)*x^2)))/(8*a*(c*d^4 + a*e^4
)^3*(a + c*x^4)^2) + (c*e^4*(12*a*d^2*e^3*(3*c*d^4 - a*e^4) + x*(
3*d*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8) - e*(21*c^2*d^8 - 26*a
*c*d^4*e^4 + a^2*e^8)*x + 4*c*d^3*e^2*(7*c*d^4 - 5*a*e^4)*x^2)))/
(4*a*(c*d^4 + a*e^4)^4*(a + c*x^4)) - (Sqrt[c]*e^9*(55*c^2*d^8 -
40*a*c*d^4*e^4 + a^2*e^8)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[
a]*(c*d^4 + a*e^4)^5) - (Sqrt[c]*e^5*(21*c^2*d^8 - 26*a*c*d^4*e^4
 + a^2*e^8)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(3/2)*(c*d^4 + a*
e^4)^4) - (3*Sqrt[c]*e*(3*c^2*d^8 - 12*a*c*d^4*e^4 + a^2*e^8)*Arc
Tan[(Sqrt[c]*x^2)/Sqrt[a]])/(16*a^(5/2)*(c*d^4 + a*e^4)^3) - (3*c
^(3/4)*d*e^8*(15*c^2*d^8 - 16*a*c*d^4*e^4 + a^2*e^8 + 2*Sqrt[a]*S
qrt[c]*d^2*e^2*(11*c*d^4 - 5*a*e^4))*ArcTan[1 - (Sqrt[2]*c^(1/4)*
x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^5) - (c^(3/4)*d*e
^4*(4*Sqrt[a]*Sqrt[c]*d^2*e^2*(7*c*d^4 - 5*a*e^4) + 9*(5*c^2*d^8
- 10*a*c*d^4*e^4 + a^2*e^8))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/
4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^4) - (c^(3/4)*d*(10*Sqrt[
a]*Sqrt[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4) + 21*(c^2*d^8 - 12*a*c*d^4
*e^4 + 3*a^2*e^8))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*S
qrt[2]*a^(11/4)*(c*d^4 + a*e^4)^3) + (3*c^(3/4)*d*e^8*(15*c^2*d^8
 - 16*a*c*d^4*e^4 + a^2*e^8 + 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(11*c*d^4
 - 5*a*e^4))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*
a^(3/4)*(c*d^4 + a*e^4)^5) + (c^(3/4)*d*e^4*(4*Sqrt[a]*Sqrt[c]*d^
2*e^2*(7*c*d^4 - 5*a*e^4) + 9*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e
^8))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*
(c*d^4 + a*e^4)^4) + (c^(3/4)*d*(10*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*
d^4 - 5*a*e^4) + 21*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8))*ArcTa
n[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*(c*d^4 +
 a*e^4)^3) + (6*c*d^2*e^11*(13*c*d^4 - 3*a*e^4)*Log[d + e*x])/(c*
d^4 + a*e^4)^5 - (3*c^(3/4)*d*e^8*(15*c^2*d^8 - 16*a*c*d^4*e^4 +
a^2*e^8 - 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(11*c*d^4 - 5*a*e^4))*Log[Sqr
t[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/
4)*(c*d^4 + a*e^4)^5) + (c^(3/4)*d*e^4*(4*Sqrt[a]*Sqrt[c]*d^2*e^2
*(7*c*d^4 - 5*a*e^4) - 9*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8))*
Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[
2]*a^(7/4)*(c*d^4 + a*e^4)^4) + (c^(3/4)*d*(10*Sqrt[a]*Sqrt[c]*d^
2*e^2*(3*c*d^4 - 5*a*e^4) - 21*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*
e^8))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(12
8*Sqrt[2]*a^(11/4)*(c*d^4 + a*e^4)^3) + (3*c^(3/4)*d*e^8*(15*c^2*
d^8 - 16*a*c*d^4*e^4 + a^2*e^8 - 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(11*c*
d^4 - 5*a*e^4))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]
*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^5) - (c^(3/4)*d*e^4*(4*
Sqrt[a]*Sqrt[c]*d^2*e^2*(7*c*d^4 - 5*a*e^4) - 9*(5*c^2*d^8 - 10*a
*c*d^4*e^4 + a^2*e^8))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x +
Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^4) - (c^(3/4)*d
*(10*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4) - 21*(c^2*d^8 -
12*a*c*d^4*e^4 + 3*a^2*e^8))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4
)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(11/4)*(c*d^4 + a*e^4)^3) - (3
*c*d^2*e^11*(13*c*d^4 - 3*a*e^4)*Log[a + c*x^4])/(2*(c*d^4 + a*e^
4)^5)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x+d)**3/(c*x**4+a)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 5.7129, size = 1338, normalized size = 0.61

$$\frac{1536cd^2\left(13cd^4-3ae^4\right)\log(d+ex)e^{11}-384cd^2\left(13cd^4-3ae^4\right)\log\left(cx^4+a\right)e^{11}-\frac{3072cd^3\left(cd^4+ae^4\right)e^{11}}{d+ex}-\frac{128\left(cd^4+ae^4\right)^2e^{11}}{(d+ex)^2}+8c}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d + e*x)^3*(a + c*x^4)^3),x]
```

```
[Out] ((-128*e^11*(c*d^4 + a*e^4)^2)/(d + e*x)^2 - (3072*c*d^3*e^11*(c*
d^4 + a*e^4))/(d + e*x) + (8*c*(c*d^4 + a*e^4)*(a^3*e^11*(-96*d^2
 + 45*d*e*x - 14*e^2*x^2) + c^3*d^11*x*(7*d^2 - 18*d*e*x + 30*e^2
*x^2) + a*c^2*d^7*e^4*x*(43*d^2 - 114*d*e*x + 204*e^2*x^2) + a^2*
c*d^3*e^7*(288*d^3 - 303*d^2*e*x + 274*d*e^2*x^2 - 210*e^3*x^3)))
/(a^2*(a + c*x^4)) + (32*c*(c*d^4 + a*e^4)^2*(-(a^2*e^7*(6*d^2 -
3*d*e*x + e^2*x^2)) + c^2*d^7*x*(d^2 - 3*d*e*x + 6*e^2*x^2) + 2*a
*c*d^3*e^3*(5*d^3 - 6*d^2*e*x + 6*d*e^2*x^2 - 5*e^3*x^3)))/(a*(a
+ c*x^4)^2) - (6*Sqrt[c]*(7*Sqrt[2]*c^(17/4)*d^17 - 24*a^(1/4)*c^
4*d^16*e + 10*Sqrt[2]*Sqrt[a]*c^(15/4)*d^15*e^2 + 50*Sqrt[2]*a*c^
(13/4)*d^13*e^4 - 176*a^(5/4)*c^3*d^12*e^5 + 78*Sqrt[2]*a^(3/2)*c
^(11/4)*d^11*e^6 + 220*Sqrt[2]*a^2*c^(9/4)*d^9*e^8 - 960*a^(9/4)*
c^2*d^8*e^9 + 702*Sqrt[2]*a^(5/2)*c^(7/4)*d^7*e^10 - 770*Sqrt[2]*
a^3*c^(5/4)*d^5*e^12 + 1200*a^(13/4)*c*d^4*e^13 - 390*Sqrt[2]*a^(
7/2)*c^(3/4)*d^3*e^14 + 77*Sqrt[2]*a^4*c^(1/4)*d*e^16 - 40*a^(17/
4)*e^17)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(11/4) + (6*S
qrt[c]*(7*Sqrt[2]*c^(17/4)*d^17 + 24*a^(1/4)*c^4*d^16*e + 10*Sqrt
[2]*Sqrt[a]*c^(15/4)*d^15*e^2 + 50*Sqrt[2]*a*c^(13/4)*d^13*e^4 +
176*a^(5/4)*c^3*d^12*e^5 + 78*Sqrt[2]*a^(3/2)*c^(11/4)*d^11*e^6 +
 220*Sqrt[2]*a^2*c^(9/4)*d^9*e^8 + 960*a^(9/4)*c^2*d^8*e^9 + 702*
Sqrt[2]*a^(5/2)*c^(7/4)*d^7*e^10 - 770*Sqrt[2]*a^3*c^(5/4)*d^5*e^
12 - 1200*a^(13/4)*c*d^4*e^13 - 390*Sqrt[2]*a^(7/2)*c^(3/4)*d^3*e
^14 + 77*Sqrt[2]*a^4*c^(1/4)*d*e^16 + 40*a^(17/4)*e^17)*ArcTan[1
+ (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(11/4) + 1536*c*d^2*e^11*(13*c*
d^4 - 3*a*e^4)*Log[d + e*x] - (3*Sqrt[2]*c^(3/4)*(7*c^4*d^17 - 10
*Sqrt[a]*c^(7/2)*d^15*e^2 + 50*a*c^3*d^13*e^4 - 78*a^(3/2)*c^(5/2
)*d^11*e^6 + 220*a^2*c^2*d^9*e^8 - 702*a^(5/2)*c^(3/2)*d^7*e^10 -
 770*a^3*c*d^5*e^12 + 390*a^(7/2)*Sqrt[c]*d^3*e^14 + 77*a^4*d*e^1
6)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(11/
4) + (3*Sqrt[2]*c^(3/4)*(7*c^4*d^17 - 10*Sqrt[a]*c^(7/2)*d^15*e^2
 + 50*a*c^3*d^13*e^4 - 78*a^(3/2)*c^(5/2)*d^11*e^6 + 220*a^2*c^2*
d^9*e^8 - 702*a^(5/2)*c^(3/2)*d^7*e^10 - 770*a^3*c*d^5*e^12 + 390
*a^(7/2)*Sqrt[c]*d^3*e^14 + 77*a^4*d*e^16)*Log[Sqrt[a] + Sqrt[2]*
a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(11/4) - 384*c*d^2*e^11*(13*c
*d^4 - 3*a*e^4)*Log[a + c*x^4])/(256*(c*d^4 + a*e^4)^5)
```

Maple [A] time = 0.048, size = 3352, normalized size = 1.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*x+d)^3/(c*x^4+a)^3,x)
```

```
[Out] 21/128/(a*e^4+c*d^4)^5*c^5*(1/c*a)^(1/4)/a^3*2^(1/2)*arctan(2^(1/
2)/(1/c*a)^(1/4)*x-1)*d^17+65/8/(a*e^4+c*d^4)^5*c^3/(c*x^4+a)^2*e
^13*a*x^6*d^4-33/8/(a*e^4+c*d^4)^5*c^5/(c*x^4+a)^2*e^5/a*x^6*d^12
+225/8/(a*e^4+c*d^4)^5*c^2/(a^5*c)^(1/2)*arctan(x^2*(c/a)^(1/2))*
a^3*d^4*e^13-45/2/(a*e^4+c*d^4)^5*c^3/(a^5*c)^(1/2)*arctan(x^2*(c
```

```
/a)^(1/2))*a^2*d^8*e^9-33/8/(a*e^4+c*d^4)^5*c^4/(a^5*c)^(1/2)*arc
tan(x^2*(c/a)^(1/2))*a*d^12*e^5+21/128/(a*e^4+c*d^4)^5*c^5*(1/c*a
)^(1/4)/a^3*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*d^17+25/16/
(a*e^4+c*d^4)^5*c^5/(c*x^4+a)^2*d^13/a*x^5*e^4-125/16/(a*e^4+c*d^
4)^5*c^2/(c*x^4+a)^2*d^3*e^14*a^2*x^3-31/16/(a*e^4+c*d^4)^5*c^3/(
c*x^4+a)^2*d^7*e^10*a*x^3+27/16/(a*e^4+c*d^4)^5*c^5/(c*x^4+a)^2*d
^15*e^2/a*x^3+75/8/(a*e^4+c*d^4)^5*c^2/(c*x^4+a)^2*e^13*a^2*x^2*d
^4+15/2/(a*e^4+c*d^4)^5*c^3/(c*x^4+a)^2*e^9*a*x^2*d^8-15/16/(a*e^
4+c*d^4)^5*c^5/(c*x^4+a)^2*e/a*x^2*d^16-141/16/(a*e^4+c*d^4)^5*c^
2/(c*x^4+a)^2*d^5*a^2*x*e^12-85/8/(a*e^4+c*d^4)^5*c^3/(c*x^4+a)^2
*d^9*a*x*e^8-3/(a*e^4+c*d^4)^5*c^2/(c*x^4+a)^2*x^4*a^2*d^2*e^15-1
05/16/(a*e^4+c*d^4)^5*c^3/(c*x^4+a)^2*d^3*e^14*a*x^7+117/16/(a*e^
4+c*d^4)^5*c^5/(c*x^4+a)^2*d^11*e^6/a*x^7+15/16/(a*e^4+c*d^4)^5*c
^6/(c*x^4+a)^2*d^15*e^2/a^2*x^7-1155/128/(a*e^4+c*d^4)^5*c^2*(1/c
*a)^(1/4)*2^(1/2)*ln((x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/
(x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))*d^5*e^12+21/256/(a*e
^4+c*d^4)^5*c^5*(1/c*a)^(1/4)/a^3*2^(1/2)*ln((x^2+(1/c*a)^(1/4)*x
*2^(1/2)+(1/c*a)^(1/2))/(x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2
)))*d^17+15/128/(a*e^4+c*d^4)^5*c^4/a^2/(1/c*a)^(1/4)*2^(1/2)*ln(
(x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2+(1/c*a)^(1/4)*x*
2^(1/2)+(1/c*a)^(1/2)))*d^15*e^2+15/64/(a*e^4+c*d^4)^5*c^4/a^2/(1
/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*d^15*e^2-58
5/64/(a*e^4+c*d^4)^5*c*a/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/
c*a)^(1/4)*x-1)*d^3*e^14+231/128/(a*e^4+c*d^4)^5*c*(1/c*a)^(1/4)*
a*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*d*e^16-585/128/(a*e^4
+c*d^4)^5*c*a/(1/c*a)^(1/4)*2^(1/2)*ln((x^2-(1/c*a)^(1/4)*x*2^(1/
2)+(1/c*a)^(1/2))/(x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))*d^
3*e^14+117/128/(a*e^4+c*d^4)^5*c^3/a/(1/c*a)^(1/4)*2^(1/2)*ln((x^
2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2+(1/c*a)^(1/4)*x*2^(
1/2)+(1/c*a)^(1/2)))*d^11*e^6+75/64/(a*e^4+c*d^4)^5*c^4*(1/c*a)^(
1/4)/a^2*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*d^13*e^4+165/6
4/(a*e^4+c*d^4)^5*c^3*(1/c*a)^(1/4)/a*2^(1/2)*ln((x^2+(1/c*a)^(1/
4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^
(1/2)))*d^9*e^8+75/128/(a*e^4+c*d^4)^5*c^4*(1/c*a)^(1/4)/a^2*2^(1
/2)*ln((x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2-(1/c*a)^(
1/4)*x*2^(1/2)+(1/c*a)^(1/2)))*d^13*e^4+231/256/(a*e^4+c*d^4)^5*c
*(1/c*a)^(1/4)*a*2^(1/2)*ln((x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^
(1/2))/(x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))*d*e^16-1/2*e^
11/(a*e^4+c*d^4)^3/(e*x+d)^2-9/16/(a*e^4+c*d^4)^5*c^6/(c*x^4+a)^2
*e/a^2*x^6*d^16+45/32/(a*e^4+c*d^4)^5*c^2/(c*x^4+a)^2*d*a^2*x^5*e
^16-129/16/(a*e^4+c*d^4)^5*c^3/(c*x^4+a)^2*d^5*a*x^5*e^12+6/(a*e^
4+c*d^4)^5*c^3/(c*x^4+a)^2*x^4*a*d^6*e^11+57/32/(a*e^4+c*d^4)^5*c
/(c*x^4+a)^2*d*a^3*x*e^16-65/8/(a*e^4+c*d^4)^5*c^4/(c*x^4+a)^2*d^
9*x^5*e^8+121/16/(a*e^4+c*d^4)^5*c^4/(c*x^4+a)^2*d^11*e^6*x^3-27/
8/(a*e^4+c*d^4)^5*c^4/(c*x^4+a)^2*e^5*x^2*d^12+5/16/(a*e^4+c*d^4)
^5*c^4/(c*x^4+a)^2*d^13*x*e^4+43/4/(a*e^4+c*d^4)^5*c^3/(c*x^4+a)^
2*a*d^10*e^7+9/(a*e^4+c*d^4)^5*c^4/(c*x^4+a)^2*x^4*d^10*e^7-7/16/
(a*e^4+c*d^4)^5*c^2/(c*x^4+a)^2*e^17*a^2*x^6+7/32/(a*e^4+c*d^4)^5
*c^6/(c*x^4+a)^2*d^17/a^2*x^5+11/32/(a*e^4+c*d^4)^5*c^5/(c*x^4+a)
^2*d^17/a*x-3/16/(a*e^4+c*d^4)^5*c^4/(c*x^4+a)^2*d^7*e^10*x^7-18*
e^15*c*d^2/(a*e^4+c*d^4)^5*ln(e*x+d)*a+5/4/(a*e^4+c*d^4)^5*c^4/(c
*x^4+a)^2*d^14*e^3-15/4/(a*e^4+c*d^4)^5*c/(c*x^4+a)^2*a^3*d^2*e^1
5-15/16/(a*e^4+c*d^4)^5*c/(a^5*c)^(1/2)*arctan(x^2*(c/a)^(1/2))*a
^4*e^17-9/16/(a*e^4+c*d^4)^5*c^5/(a^5*c)^(1/2)*arctan(x^2*(c/a)^(
1/2))*d^16*e+9/2/(a*e^4+c*d^4)^5*c*a*ln(a^2*(c*x^4+a))*d^2*e^15-9
/16/(a*e^4+c*d^4)^5*c/(c*x^4+a)^2*e^17*a^3*x^2+23/4/(a*e^4+c*d^4)
^5*c^2/(c*x^4+a)^2*a^2*d^6*e^11+5/(a*e^4+c*d^4)^5*c^4/(c*x^4+a)^2
*e^9*x^6*d^8+1053/128/(a*e^4+c*d^4)^5*c^2/(1/c*a)^(1/4)*2^(1/2)*l
n((x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2+(1/c*a)^(1/4)*
x*2^(1/2)+(1/c*a)^(1/2)))*d^7*e^10+1053/64/(a*e^4+c*d^4)^5*c^2/(1
/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*d^7*e^10+10
53/64/(a*e^4+c*d^4)^5*c^2/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1
/c*a)^(1/4)*x-1)*d^7*e^10-1155/64/(a*e^4+c*d^4)^5*c^2*(1/c*a)^(1/
4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*d^5*e^12-1155/64/(a*
e^4+c*d^4)^5*c^2*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/
4)*x-1)*d^5*e^12+117/64/(a*e^4+c*d^4)^5*c^3/a/(1/c*a)^(1/4)*2^(1/
2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*d^11*e^6+15/64/(a*e^4+c*d^4)
^5*c^4/a^2/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1
)*d^15*e^2+165/32/(a*e^4+c*d^4)^5*c^3*(1/c*a)^(1/4)/a*2^(1/2)*arc
tan(2^(1/2)/(1/c*a)^(1/4)*x+1)*d^9*e^8+231/128/(a*e^4+c*d^4)^5*c*
(1/c*a)^(1/4)*a*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*d*e^16+
75/64/(a*e^4+c*d^4)^5*c^4*(1/c*a)^(1/4)/a^2*2^(1/2)*arctan(2^(1/2
```

```
)/(1/c*a)^(1/4)*x+1)*d^13*e^4+165/32/(a*e^4+c*d^4)^5*c^3*(1/c*a)^
(1/4)/a*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*d^9*e^8-585/64/
(a*e^4+c*d^4)^5*c*a/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^
(1/4)*x+1)*d^3*e^14+117/64/(a*e^4+c*d^4)^5*c^3/a/(1/c*a)^(1/4)*2^
(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*d^11*e^6-39/2/(a*e^4+c*d^
4)^5*c^2*ln(a^2*(c*x^4+a))*d^6*e^11+78*e^11*c^2*d^6/(a*e^4+c*d^4)
^5*ln(e*x+d)-12*c*d^3*e^11/(a*e^4+c*d^4)^4/(e*x+d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)^3*(e*x + d)^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)^3*(e*x + d)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*x+d)**3/(c*x**4+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.531328, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)^3*(e*x + d)^3),x, algorithm="giac")
```

```
[Out] Done
```

3.415 $\int \frac{-1+x}{1-x+x^2}\,dx$

Optimal. Leaf size=32

$$\frac{1}{2}\log\left(x^2-x+1\right)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x + x^2]/2
```

Rubi [A] time = 0.046202, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286

$$\frac{1}{2}\log\left(x^2-x+1\right)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + x)/(1 - x + x^2),x]

[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x + x^2]/2
```

Rubi in Sympy [A] time = 5.62089, size = 31, normalized size = 0.97

$$\frac{\log\left(x^2-x+1\right)}{2}-\frac{\sqrt{3}\,\text{atan}\left(\sqrt{3}\left(\frac{2x}{3}-\frac{1}{3}\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-1+x)/(x**2-x+1),x)

[Out] log(x**2 - x + 1)/2 - sqrt(3)*atan(sqrt(3)*(2*x/3 - 1/3))/3
```

Mathematica [A] time = 0.0158734, size = 33, normalized size = 1.03

$$\frac{1}{2}\log\left(x^2-x+1\right)-\frac{\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + x)/(1 - x + x^2),x]

[Out] -(ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3]) + Log[1 - x + x^2]/2
```

Maple [A] time = 0.002, size = 29, normalized size = 0.9

$$\frac{\ln\left(x^2-x+1\right)}{2}-\frac{\sqrt{3}}{3}\arctan\left(\frac{(2\,x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-1+x)/(x^2-x+1),x)

[Out] 1/2*ln(x^2-x+1)-1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A] time = 0.889137, size = 38, normalized size = 1.19

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{2}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - 1)/(x^2 - x + 1),x, algorithm="maxima")

[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)
```

Fricas [A] time = 0.272673, size = 43, normalized size = 1.34

$$\frac{1}{6}\sqrt{3}\left(\sqrt{3}\log\left(x^2-x+1\right)-2\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - 1)/(x^2 - x + 1),x, algorithm="fricas")

[Out] 1/6*sqrt(3)*(sqrt(3)*log(x^2 - x + 1) - 2*arctan(1/3*sqrt(3)*(2*x
 - 1)))
```

Sympy [A] time = 0.208901, size = 34, normalized size = 1.06

$$\frac{\log\left(x^2-x+1\right)}{2}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-1+x)/(x**2-x+1),x)

[Out] log(x**2 - x + 1)/2 - sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/3
```

GIAC/XCAS [A] time = 0.259034, size = 38, normalized size = 1.19

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{2}\ln\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - 1)/(x^2 - x + 1),x, algorithm="giac")

[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*ln(x^2 - x + 1)
```

3.416 $\int \frac{-1+x^2}{1+x^3} \, dx$

Optimal. Leaf size=32

$$\frac{1}{2}\log\left(x^2-x+1\right)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x + x^2]/2
```

Rubi [A] time = 0.0449211, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385

$$\frac{1}{2}\log\left(x^2-x+1\right)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + x^2)/(1 + x^3),x]

[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x + x^2]/2
```

Rubi in Sympy [A] time = 6.69553, size = 31, normalized size = 0.97

$$\frac{\log\left(x^2-x+1\right)}{2}-\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}-\frac{1}{3}\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2-1)/(x**3+1),x)

[Out] log(x**2 - x + 1)/2 - sqrt(3)*atan(sqrt(3)*(2*x/3 - 1/3))/3
```

Mathematica [A] time = 0.00842515, size = 33, normalized size = 1.03

$$\frac{1}{2}\log\left(x^2-x+1\right)-\frac{\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + x^2)/(1 + x^3),x]

[Out] -(ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3]) + Log[1 - x + x^2]/2
```

Maple [A] time = 0.002, size = 29, normalized size = 0.9

$$\frac{\ln\left(x^2-x+1\right)}{2}-\frac{\sqrt{3}}{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2-1)/(x^3+1),x)

[Out] 1/2*ln(x^2-x+1)-1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A] time = 0.882456, size = 38, normalized size = 1.19

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{2}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 1)/(x^3 + 1),x, algorithm="maxima")

[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)
```

Fricas [A] time = 0.269043, size = 43, normalized size = 1.34

$$\frac{1}{6}\sqrt{3}\left(\sqrt{3}\log\left(x^2-x+1\right)-2\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 1)/(x^3 + 1),x, algorithm="fricas")

[Out] 1/6*sqrt(3)*(sqrt(3)*log(x^2 - x + 1) - 2*arctan(1/3*sqrt(3)*(2*x
 - 1)))
```

Sympy [A] time = 0.225929, size = 34, normalized size = 1.06

$$\frac{\log\left(x^2-x+1\right)}{2}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2-1)/(x**3+1),x)

[Out] log(x**2 - x + 1)/2 - sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/3
```

GIAC/XCAS [A] time = 0.261187, size = 38, normalized size = 1.19

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{2}\ln\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 1)/(x^3 + 1),x, algorithm="giac")

[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*ln(x^2 - x + 1)
```

3.417 $\int \frac{-4+3x}{4-2x+x^2}\,dx$

Optimal. Leaf size=32

$$\frac{3}{2}\log\left(x^2-2x+4\right)+\frac{\tan^{-1}\left(\frac{1-x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] ArcTan[(1 - x)/Sqrt[3]]/Sqrt[3] + (3*Log[4 - 2*x + x^2])/2
```

Rubi [A] time = 0.0406884, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{3}{2}\log\left(x^2-2x+4\right)+\frac{\tan^{-1}\left(\frac{1-x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(-4 + 3*x)/(4 - 2*x + x^2),x]

[Out] ArcTan[(1 - x)/Sqrt[3]]/Sqrt[3] + (3*Log[4 - 2*x + x^2])/2
```

Rubi in Sympy [A] time = 5.83509, size = 32, normalized size = 1.

$$\frac{3\log\left(x^2-2x+4\right)}{2}-\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{x}{3}-\frac{1}{3}\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-4+3*x)/(x**2-2*x+4),x)

[Out] 3*log(x**2 - 2*x + 4)/2 - sqrt(3)*atan(sqrt(3)*(x/3 - 1/3))/3
```

Mathematica [A] time = 0.0154952, size = 31, normalized size = 0.97

$$\frac{3}{2}\log\left(x^2-2x+4\right)-\frac{\tan^{-1}\left(\frac{x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-4 + 3*x)/(4 - 2*x + x^2),x]

[Out] -(ArcTan[(-1 + x)/Sqrt[3]]/Sqrt[3]) + (3*Log[4 - 2*x + x^2])/2
```

Maple [A] time = 0.007, size = 29, normalized size = 0.9

$$\frac{3\ln\left(x^2-2x+4\right)}{2}-\frac{\sqrt{3}}{3}\arctan\left(\frac{(2x-2)\sqrt{3}}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-4+3*x)/(x^2-2*x+4),x)
```

```
[Out] 3/2*ln(x^2-2*x+4)-1/3*3^(1/2)*arctan(1/6*(2*x-2)*3^(1/2))
```

Maxima [A] time = 0.874097, size = 35, normalized size = 1.09

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(x-1)\right)+\frac{3}{2}\log\left(x^2-2x+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x - 4)/(x^2 - 2*x + 4),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x - 1)) + 3/2*log(x^2 - 2*x + 4)
```

Fricas [A] time = 0.274774, size = 42, normalized size = 1.31

$$\frac{1}{6}\sqrt{3}\left(3\sqrt{3}\log\left(x^2-2x+4\right)-2\arctan\left(\frac{1}{3}\sqrt{3}(x-1)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x - 4)/(x^2 - 2*x + 4),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*(3*sqrt(3)*log(x^2 - 2*x + 4) - 2*arctan(1/3*sqrt(3)*
(x - 1)))
```

Sympy [A] time = 0.220161, size = 36, normalized size = 1.12

$$\frac{3\log\left(x^2-2x+4\right)}{2}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-4+3*x)/(x**2-2*x+4),x)
```

```
[Out] 3*log(x**2 - 2*x + 4)/2 - sqrt(3)*atan(sqrt(3)*x/3 - sqrt(3)/3)/3
```

GIAC/XCAS [A] time = 0.26077, size = 35, normalized size = 1.09

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(x-1)\right)+\frac{3}{2}\ln\left(x^2-2x+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x - 4)/(x^2 - 2*x + 4),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x - 1)) + 3/2*ln(x^2 - 2*x + 4)
```

3.418 $\int \frac{-8+2x+3x^2}{8+x^3} \, dx$

Optimal. Leaf size=32

$$\frac{3}{2}\log\left(x^2-2x+4\right)+\frac{\tan^{-1}\left(\frac{1-x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] ArcTan[(1 - x)/Sqrt[3]]/Sqrt[3] + (3*Log[4 - 2*x + x^2])/2
```

Rubi [A] time = 0.0435554, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$

$$\frac{3}{2}\log\left(x^2-2x+4\right)+\frac{\tan^{-1}\left(\frac{1-x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(-8 + 2*x + 3*x^2)/(8 + x^3),x]

[Out] ArcTan[(1 - x)/Sqrt[3]]/Sqrt[3] + (3*Log[4 - 2*x + x^2])/2
```

Rubi in Sympy [A] time = 7.99735, size = 32, normalized size = 1.

$$\frac{3\log\left(x^2-2x+4\right)}{2}-\frac{\sqrt{3}\,\text{atan}\left(\sqrt{3}\left(\frac{x}{3}-\frac{1}{3}\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+2*x-8)/(x**3+8),x)

[Out] 3*log(x**2 - 2*x + 4)/2 - sqrt(3)*atan(sqrt(3)*(x/3 - 1/3))/3
```

Mathematica [A] time = 0.00737497, size = 31, normalized size = 0.97

$$\frac{3}{2}\log\left(x^2-2x+4\right)-\frac{\tan^{-1}\left(\frac{x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-8 + 2*x + 3*x^2)/(8 + x^3),x]

[Out] -(ArcTan[(-1 + x)/Sqrt[3]]/Sqrt[3]) + (3*Log[4 - 2*x + x^2])/2
```

Maple [A] time = 0.002, size = 29, normalized size = 0.9

$$\frac{3\ln\left(x^2-2x+4\right)}{2}-\frac{\sqrt{3}}{3}\arctan\left(\frac{(2x-2)\sqrt{3}}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2+2*x-8)/(x^3+8),x)
```

```
[Out] 3/2*ln(x^2-2*x+4)-1/3*3^(1/2)*arctan(1/6*(2*x-2)*3^(1/2))
```

Maxima [A] time = 0.910806, size = 35, normalized size = 1.09

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(x-1)\right)+\frac{3}{2}\log\left(x^2-2x+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2*x - 8)/(x^3 + 8),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x - 1)) + 3/2*log(x^2 - 2*x + 4)
```

Fricas [A] time = 0.269858, size = 42, normalized size = 1.31

$$\frac{1}{6}\sqrt{3}\left(3\sqrt{3}\log\left(x^2-2x+4\right)-2\arctan\left(\frac{1}{3}\sqrt{3}(x-1)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2*x - 8)/(x^3 + 8),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*(3*sqrt(3)*log(x^2 - 2*x + 4) - 2*arctan(1/3*sqrt(3)*
(x - 1)))
```

Sympy [A] time = 0.234807, size = 36, normalized size = 1.12

$$\frac{3\log\left(x^2-2x+4\right)}{2}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**2+2*x-8)/(x**3+8),x)
```

```
[Out] 3*log(x**2 - 2*x + 4)/2 - sqrt(3)*atan(sqrt(3)*x/3 - sqrt(3)/3)/3
```

GIAC/XCAS [A] time = 0.26192, size = 35, normalized size = 1.09

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(x-1)\right)+\frac{3}{2}\ln\left(x^2-2x+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2*x - 8)/(x^3 + 8),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x - 1)) + 3/2*ln(x^2 - 2*x + 4)
```

3.419 $\int \frac{2+x}{-1+2x+x^2}\,dx$

Optimal. Leaf size=45

$$\frac{1}{4}\left(2+\sqrt{2}\right)\log\left(x-\sqrt{2}+1\right)+\frac{1}{4}\left(2-\sqrt{2}\right)\log\left(x+\sqrt{2}+1\right)$$

```
[Out] ((2 + Sqrt[2])*Log[1 - Sqrt[2] + x])/4 + ((2 - Sqrt[2])*Log[1 + S
qrt[2] + x])/4
```

Rubi [A] time = 0.0446536, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{1}{4}\left(2+\sqrt{2}\right)\log\left(x-\sqrt{2}+1\right)+\frac{1}{4}\left(2-\sqrt{2}\right)\log\left(x+\sqrt{2}+1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + x)/(-1 + 2*x + x^2),x]
```

```
[Out] ((2 + Sqrt[2])*Log[1 - Sqrt[2] + x])/4 + ((2 - Sqrt[2])*Log[1 + S
qrt[2] + x])/4
```

Rubi in Sympy [A] time = 4.65621, size = 46, normalized size = 1.02

$$-\frac{\sqrt{2}\left(-\sqrt{2}+1\right)\log\left(x+1+\sqrt{2}\right)}{4}+\frac{\sqrt{2}\left(1+\sqrt{2}\right)\log\left(x-\sqrt{2}+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2+x)/(x**2+2*x-1),x)
```

```
[Out] -sqrt(2)*(-sqrt(2) + 1)*log(x + 1 + sqrt(2))/4 + sqrt(2)*(1 + sqr
t(2))*log(x - sqrt(2) + 1)/4
```

Mathematica [A] time = 0.0348445, size = 42, normalized size = 0.93

$$\frac{1}{4}\left(\left(2+\sqrt{2}\right)\log\left(-x+\sqrt{2}-1\right)-\left(\sqrt{2}-2\right)\log\left(x+\sqrt{2}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + x)/(-1 + 2*x + x^2),x]
```

```
[Out] ((2 + Sqrt[2])*Log[-1 + Sqrt[2] - x] - (-2 + Sqrt[2])*Log[1 + Sqr
t[2] + x])/4
```

Maple [A] time = 0.003, size = 29, normalized size = 0.6

$$\frac{\ln\left(x^2+2\,x-1\right)}{2}-\frac{\sqrt{2}}{2}\mathit{Artanh}\left(\frac{(2+2\,x)\sqrt{2}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2+x)/(x^2+2*x-1),x)
```

```
[Out] 1/2*ln(x^2+2*x-1)-1/2*2^(1/2)*arctanh(1/4*(2+2*x)*2^(1/2))
```

Maxima [A] time = 0.895592, size = 51, normalized size = 1.13

$$\frac{1}{4}\,\sqrt{2}\log\left(\frac{2\left(x-\sqrt{2}+1\right)}{2\,x+2\,\sqrt{2}+2}\right)+\frac{1}{2}\,\log\left(x^2+2\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 2)/(x^2 + 2*x - 1),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*log(2*(x - sqrt(2) + 1)/((2*sqrt(2)) + 2*x + 2)) + 1/
2*log(x^2 + 2*x - 1)
```

Fricas [A] time = 0.271927, size = 65, normalized size = 1.44

$$\frac{1}{4}\,\sqrt{2}\left(\sqrt{2}\log\left(x^2+2\,x-1\right)+\log\left(\frac{\sqrt{2}\left(x^2+2\,x+3\right)-4\,x-4}{x^2+2\,x-1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 2)/(x^2 + 2*x - 1),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*(sqrt(2)*log(x^2 + 2*x - 1) + log((sqrt(2)*(x^2 + 2*x
 + 3) - 4*x - 4)/(x^2 + 2*x - 1)))
```

Sympy [A] time = 0.206105, size = 39, normalized size = 0.87

$$\left(-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)\log\left(x+1+\sqrt{2}\right)+\left(\frac{\sqrt{2}}{4}+\frac{1}{2}\right)\log\left(x-\sqrt{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2+x)/(x**2+2*x-1),x)
```

```
[Out] (-sqrt(2)/4 + 1/2)*log(x + 1 + sqrt(2)) + (sqrt(2)/4 + 1/2)*log(x
 - sqrt(2) + 1)
```

GIAC/XCAS [A] time = 0.262357, size = 59, normalized size = 1.31

$$\frac{1}{4}\,\sqrt{2}\ln\left(\frac{\left|2\,x-2\,\sqrt{2}+2\right|}{\left|2\,x+2\,\sqrt{2}+2\right|}\right)+\frac{1}{2}\,\ln\left(\left|x^2+2\,x-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 2)/(x^2 + 2*x - 1),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*ln(abs(2*x - 2*sqrt(2) + 2)/abs(2*x + 2*sqrt(2) + 2))
 + 1/2*ln(abs(x^2 + 2*x - 1))
```

3.420 $\int \frac{-4+x^2}{2-5x+x^3}\,dx$

Optimal. Leaf size=45

$$\frac{1}{4}\left(2+\sqrt{2}\right)\log\left(x-\sqrt{2}+1\right)+\frac{1}{4}\left(2-\sqrt{2}\right)\log\left(x+\sqrt{2}+1\right)$$

```
[Out] ((2 + Sqrt[2])*Log[1 - Sqrt[2] + x])/4 + ((2 - Sqrt[2])*Log[1 + S
qrt[2] + x])/4
```

Rubi [A] time = 0.0327935, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188

$$\frac{1}{4}\left(2+\sqrt{2}\right)\log\left(x-\sqrt{2}+1\right)+\frac{1}{4}\left(2-\sqrt{2}\right)\log\left(x+\sqrt{2}+1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(-4 + x^2)/(2 - 5*x + x^3),x]
```

```
[Out] ((2 + Sqrt[2])*Log[1 - Sqrt[2] + x])/4 + ((2 - Sqrt[2])*Log[1 + S
qrt[2] + x])/4
```

Rubi in Sympy [A] time = 6.58991, size = 46, normalized size = 1.02

$$-\frac{\sqrt{2}\left(-\sqrt{2}+1\right)\log\left(x+1+\sqrt{2}\right)}{4}+\frac{\sqrt{2}\left(1+\sqrt{2}\right)\log\left(x-\sqrt{2}+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2-4)/(x**3-5*x+2),x)
```

```
[Out] -sqrt(2)*(-sqrt(2) + 1)*log(x + 1 + sqrt(2))/4 + sqrt(2)*(1 + sqr
t(2))*log(x - sqrt(2) + 1)/4
```

Mathematica [A] time = 0.00756792, size = 42, normalized size = 0.93

$$\frac{1}{4}\left(\left(2+\sqrt{2}\right)\log\left(-x+\sqrt{2}-1\right)-\left(\sqrt{2}-2\right)\log\left(x+\sqrt{2}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-4 + x^2)/(2 - 5*x + x^3),x]
```

```
[Out] ((2 + Sqrt[2])*Log[-1 + Sqrt[2] - x] - (-2 + Sqrt[2])*Log[1 + Sqr
t[2] + x])/4
```

Maple [A] time = 0.003, size = 29, normalized size = 0.6

$$\frac{\ln\left(x^2+2\,x-1\right)}{2}-\frac{\sqrt{2}}{2}Artanh\left(\frac{(2+2\,x)\sqrt{2}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((x^2-4)/(x^3-5*x+2),x)
```

```
[Out] 1/2*ln(x^2+2*x-1)-1/2*2^(1/2)*arctanh(1/4*(2+2*x)*2^(1/2))
```

Maxima [A] time = 0.918364, size = 51, normalized size = 1.13

$$\frac{1}{4}\sqrt{2}\log\left(\frac{2\left(x-\sqrt{2}+1\right)}{2x+2\sqrt{2}+2}\right)+\frac{1}{2}\log\left(x^2+2x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((x^2 - 4)/(x^3 - 5*x + 2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*log(2*(x - sqrt(2) + 1)/((2*sqrt(2)) + 2*x + 2)) + 1/
2*log(x^2 + 2*x - 1)
```

Fricas [A] time = 0.276496, size = 65, normalized size = 1.44

$$\frac{1}{4}\sqrt{2}\left(\sqrt{2}\log\left(x^2+2x-1\right)+\log\left(\frac{\sqrt{2}\left(x^2+2x+3\right)-4x-4}{x^2+2x-1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((x^2 - 4)/(x^3 - 5*x + 2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*(sqrt(2)*log(x^2 + 2*x - 1) + log((sqrt(2)*(x^2 + 2*x
 + 3) - 4*x - 4)/(x^2 + 2*x - 1)))
```

Sympy [A] time = 0.228284, size = 39, normalized size = 0.87

$$\left(-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)\log\left(x+1+\sqrt{2}\right)+\left(\frac{\sqrt{2}}{4}+\frac{1}{2}\right)\log\left(x-\sqrt{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((x**2-4)/(x**3-5*x+2),x)
```

```
[Out] (-sqrt(2)/4 + 1/2)*log(x + 1 + sqrt(2)) + (sqrt(2)/4 + 1/2)*log(x
 - sqrt(2) + 1)
```

GIAC/XCAS [A] time = 0.259936, size = 59, normalized size = 1.31

$$\frac{1}{4}\sqrt{2}\ln\left(\frac{\left|2x-2\sqrt{2}+2\right|}{\left|2x+2\sqrt{2}+2\right|}\right)+\frac{1}{2}\ln\left(\left|x^2+2x-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 4)/(x^3 - 5*x + 2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*ln(abs(2*x - 2*sqrt(2) + 2)/abs(2*x + 2*sqrt(2) + 2))
 + 1/2*ln(abs(x^2 + 2*x - 1))
```

3.421 $\int \frac{2}{-1+4x^2}\,dx$

Optimal. Leaf size=6

$$-\tanh^{-1}(2x)$$

[Out] -ArcTanh[2*x]

Rubi [A] time = 0.00846515, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$-\tanh^{-1}(2x)$$

Antiderivative was successfully verified.

[In] Int[2/(-1 + 4*x^2),x]

[Out] -ArcTanh[2*x]

Rubi in Sympy [A] time = 2.06193, size = 5, normalized size = 0.83

$$-\operatorname{atanh}(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate(2/(4*x**2-1),x)

[Out] -atanh(2*x)

Mathematica [B] time = 0.00452808, size = 23, normalized size = 3.83

$$2\left(\frac{1}{4}\log(1-2x)-\frac{1}{4}\log(2x+1)\right)$$

Antiderivative was successfully verified.

[In] Integrate[2/(-1 + 4*x^2),x]

[Out] 2*(Log[1 - 2*x]/4 - Log[1 + 2*x]/4)

Maple [B] time = 0.008, size = 18, normalized size = 3.

$$\frac{\ln(2x-1)}{2}-\frac{\ln(1+2x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(2/(4*x^2-1),x)

[Out] 1/2*ln(2*x-1)-1/2*ln(1+2*x)

Maxima [A] time = 0.789849, size = 23, normalized size = 3.83

$$-\frac{1}{2}\log(2x+1)+\frac{1}{2}\log(2x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(2/(4*x^2 - 1),x, algorithm="maxima")

[Out] -1/2*log(2*x + 1) + 1/2*log(2*x - 1)
```

Fricas [A] time = 0.266751, size = 23, normalized size = 3.83

$$-\frac{1}{2}\log(2x+1)+\frac{1}{2}\log(2x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(2/(4*x^2 - 1),x, algorithm="fricas")

[Out] -1/2*log(2*x + 1) + 1/2*log(2*x - 1)
```

Sympy [A] time = 0.160301, size = 15, normalized size = 2.5

$$\frac{\log\left(x-\frac{1}{2}\right)}{2}-\frac{\log\left(x+\frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(2/(4*x**2-1),x)

[Out] log(x - 1/2)/2 - log(x + 1/2)/2
```

GIAC/XCAS [A] time = 0.258425, size = 20, normalized size = 3.33

$$-\frac{1}{2}\ln\left(\left|x+\frac{1}{2}\right|\right)+\frac{1}{2}\ln\left(\left|x-\frac{1}{2}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(2/(4*x^2 - 1),x, algorithm="giac")

[Out] -1/2*ln(abs(x + 1/2)) + 1/2*ln(abs(x - 1/2))
```

3.422 $\int \left(\frac{1}{-1+2x}-\frac{1}{1+2x}\right)\,dx$

Optimal. Leaf size=21

$$\frac{1}{2}\log(1-2x)-\frac{1}{2}\log(2x+1)$$

```
[Out] Log[1 - 2*x]/2 - Log[1 + 2*x]/2
```

Rubi [A] time = 0.012799, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.

$$\frac{1}{2}\log(1-2x)-\frac{1}{2}\log(2x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + 2*x)^(-1) - (1 + 2*x)^(-1),x]

[Out] Log[1 - 2*x]/2 - Log[1 + 2*x]/2
```

Rubi in Sympy [A] time = 1.93035, size = 15, normalized size = 0.71

$$\frac{\log(-2x+1)}{2}-\frac{\log(2x+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-1+2*x)-1/(1+2*x),x)

[Out] log(-2*x + 1)/2 - log(2*x + 1)/2
```

Mathematica [A] time = 0.00370188, size = 23, normalized size = 1.1

$$2\left(\frac{1}{4}\log(1-2x)-\frac{1}{4}\log(2x+1)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + 2*x)^(-1) - (1 + 2*x)^(-1),x]

[Out] 2*(Log[1 - 2*x]/4 - Log[1 + 2*x]/4)
```

Maple [A] time = 0.002, size = 18, normalized size = 0.9

$$\frac{\ln(2x-1)}{2}-\frac{\ln(1+2x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(2*x-1)-1/(1+2*x),x)
```

```
[Out] 1/2*ln(2*x-1)-1/2*ln(1+2*x)
```

Maxima [A] time = 0.84113, size = 23, normalized size = 1.1

$$-\frac{1}{2}\log(2x+1)+\frac{1}{2}\log(2x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(2*x + 1) + 1/(2*x - 1),x, algorithm="maxima")
```

```
[Out] -1/2*log(2*x + 1) + 1/2*log(2*x - 1)
```

Fricas [A] time = 0.269485, size = 23, normalized size = 1.1

$$-\frac{1}{2}\log(2x+1)+\frac{1}{2}\log(2x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(2*x + 1) + 1/(2*x - 1),x, algorithm="fricas")
```

```
[Out] -1/2*log(2*x + 1) + 1/2*log(2*x - 1)
```

Sympy [A] time = 0.177171, size = 15, normalized size = 0.71

$$\frac{\log\left(x-\frac{1}{2}\right)}{2}-\frac{\log\left(x+\frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-1+2*x)-1/(1+2*x),x)
```

```
[Out] log(x - 1/2)/2 - log(x + 1/2)/2
```

GIAC/XCAS [A] time = 0.263533, size = 26, normalized size = 1.24

$$-\frac{1}{2}\ln(|2x+1|)+\frac{1}{2}\ln(|2x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(2*x + 1) + 1/(2*x - 1),x, algorithm="giac")
```

```
[Out] -1/2*ln(abs(2*x + 1)) + 1/2*ln(abs(2*x - 1))
```

3.423 $\int \frac{x}{(1-x^2)^5} \, dx$

Optimal. Leaf size=13

$$\frac{1}{8(1-x^2)^4}$$

```
[Out] 1/(8*(1 - x^2)^4)
```

Rubi [A] time = 0.0078271, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{1}{8(1-x^2)^4}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(1 - x^2)^5,x]

[Out] 1/(8*(1 - x^2)^4)
```

Rubi in Sympy [A] time = 1.92051, size = 8, normalized size = 0.62

$$\frac{1}{8(-x^2+1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(-x**2+1)**5,x)

[Out] 1/(8*(-x**2 + 1)**4)
```

Mathematica [A] time = 0.0037918, size = 11, normalized size = 0.85

$$\frac{1}{8(x^2-1)^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(1 - x^2)^5,x]

[Out] 1/(8*(-1 + x^2)^4)
```

Maple [A] time = 0.001, size = 10, normalized size = 0.8

$$\frac{1}{8 (x^2-1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(-x^2+1)^5,x)
```

```
[Out] 1/8/(x^2-1)^4
```

Maxima [A] time = 0.782708, size = 12, normalized size = 0.92

$$\frac{1}{8\,(x^2-1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x/(x^2 - 1)^5,x, algorithm="maxima")
```

```
[Out] 1/8/(x^2 - 1)^4
```

Fricas [A] time = 0.263157, size = 32, normalized size = 2.46

$$\frac{1}{8\,(x^8-4\,x^6+6\,x^4-4\,x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x/(x^2 - 1)^5,x, algorithm="fricas")
```

```
[Out] 1/8/(x^8 - 4*x^6 + 6*x^4 - 4*x^2 + 1)
```

Sympy [A] time = 0.320626, size = 22, normalized size = 1.69

$$\frac{1}{8x^8-32x^6+48x^4-32x^2+8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(-x**2+1)**5,x)
```

```
[Out] 1/(8*x**8 - 32*x**6 + 48*x**4 - 32*x**2 + 8)
```

GIAC/XCAS [A] time = 0.25912, size = 12, normalized size = 0.92

$$\frac{1}{8\,(x^2-1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x/(x^2 - 1)^5,x, algorithm="giac")
```

```
[Out] 1/8/(x^2 - 1)^4
```

3.424 $\int \left(-\frac{1}{32(-1+x)^5}+\frac{3}{64(-1+x)^4}-\frac{5}{128(-1+x)^3}+\frac{5}{256(-1+x)^2}-\frac{1}{32(1+x)^5}-\frac{3}{64(1+x)^4}-\right.$

Optimal. Leaf size=13

$$\frac{1}{8\left(1-x^2\right)^4}$$

```
[Out] 1/(8*(1 - x^2)^4)
```

Rubi [B] time = 0.031487, antiderivative size = 81, normalized size of antiderivative = 6.23, number of steps used = 1, number of rules used = 0, integrand size = 73, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.

$$\frac{5}{256(x+1)}+\frac{5}{256(x+1)^2}+\frac{1}{64(x+1)^3}+\frac{1}{128(x+1)^4}+\frac{5}{256(1-x)}+\frac{5}{256(1-x)^2}+\frac{1}{64(1-x)^3}+\frac{1}{128(1-x)^4}$$

Antiderivative was successfully verified.

```
[In] Int[-1/(32*(-1 + x)^5) + 3/(64*(-1 + x)^4) - 5/(128*(-1 + x)^3) + 5/(256*(-
1 + x)^2) - 1/(32*(1 + x)^5) - 3/(64*(1 + x)^4) - 5/(128*(1 + x)^3) - 5/(256*(1 + x)^2),x
```

```
[Out] 1/(128*(1 - x)^4) + 1/(64*(1 - x)^3) + 5/(256*(1 - x)^2) + 5/(256
*(1 - x)) + 1/(128*(1 + x)^4) + 1/(64*(1 + x)^3) + 5/(256*(1 + x)
^2) + 5/(256*(1 + x))
```

Rubi in Sympy [A] time = 5.52815, size = 63, normalized size = 4.85

$$\frac{5}{256(x+1)}+\frac{5}{256(x+1)^2}+\frac{1}{64(x+1)^3}+\frac{1}{128(x+1)^4}+\frac{5}{256(-x+1)}+\frac{5}{256(-x+1)^2}+\frac{1}{64(-x+1)^3}+\frac{1}{128(-x+1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(-1/32/(-1+x)**5+3/64/(-1+x)**4-5/128/(-1+x)**3+5/256/(-
1+x)**2-1/32/(1+x)**5-3/64/(1+x)**4-5/128/(1+x)**3-5/256/(1+x)**2,x)
```

```
[Out] 5/(256*(x + 1)) + 5/(256*(x + 1)**2) + 1/(64*(x + 1)**3) + 1/(128
*(x + 1)**4) + 5/(256*(-x + 1)) + 5/(256*(-x + 1)**2) + 1/(64*(-x
 + 1)**3) + 1/(128*(-x + 1)**4)
```

Mathematica [A] time = 0.00230004, size = 11, normalized size = 0.85

$$\frac{1}{8\left(x^2-1\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[-1/(32*(-1 + x)^5) + 3/(64*(-1 + x)^4) - 5/(128*(-
1 + x)^3) + 5/(256*(-1 + x)^2) - 1/(32*(1 + x)^5) - 3/(64*(1 + x)^4) - 5/(128*(1 + x)^3) -
```

```
[Out] 1/(8*(-1 + x^2)^4)
```

Maple [B] time = 0.004, size = 58, normalized size = 4.5

$$\frac{1}{128\,(1+x)^4}+\frac{1}{64\,(1+x)^3}+\frac{5}{256\,(1+x)^2}+\frac{5}{256+256\,x}+\frac{1}{128\,(-1+x)^4}-\frac{1}{64\,(-1+x)^3}+\frac{5}{256\,(-1+x)^2}-\frac{5}{-256+256\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/32/(-1+x)^5+3/64/(-1+x)^4-5/128/(-1+x)^3+5/256/(-1+x)^2-1/32/(1+x)^5-
3/64/(1+x)^4-5/128/(1+x)^3-5/256/(1+x)^2,x)
```

```
[Out] 1/128/(1+x)^4+1/64/(1+x)^3+5/256/(1+x)^2+5/256/(1+x)+1/128/(-1+x)
^4-1/64/(-1+x)^3+5/256/(-1+x)^2-5/256/(-1+x)
```

Maxima [A] time = 0.796665, size = 77, normalized size = 5.92

$$\frac{5}{256\,(x+1)}-\frac{5}{256\,(x-1)}+\frac{5}{256\,(x+1)^2}+\frac{5}{256\,(x-1)^2}+\frac{1}{64\,(x+1)^3}-\frac{1}{64\,(x-1)^3}+\frac{1}{128\,(x+1)^4}+\frac{1}{128\,(x-1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-5/256/(x + 1)^2 + 5/256/(x - 1)^2 - 5/128/(x + 1)^3 - 5/128/(x - 1)^3 -
```

```
[Out] 5/256/(x + 1) - 5/256/(x - 1) + 5/256/(x + 1)^2 + 5/256/(x - 1)^2
 + 1/64/(x + 1)^3 - 1/64/(x - 1)^3 + 1/128/(x + 1)^4 + 1/128/(x -
 1)^4
```

Fricas [A] time = 0.265836, size = 32, normalized size = 2.46

$$\frac{1}{8\,(x^8-4\,x^6+6\,x^4-4\,x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-5/256/(x + 1)^2 + 5/256/(x - 1)^2 - 5/128/(x + 1)^3 - 5/128/(x - 1)^3 -
```

```
[Out] 1/8/(x^8 - 4*x^6 + 6*x^4 - 4*x^2 + 1)
```

Sympy [A] time = 0.833841, size = 22, normalized size = 1.69

$$\frac{1}{8x^8-32x^6+48x^4-32x^2+8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/32/(-1+x)**5+3/64/(-1+x)**4-5/128/(-1+x)**3+5/256/(-
1+x)**2-1/32/(1+x)**5-3/64/(1+x)**4-5/128/(1+x)**3-5/256/(1+x)**2,x)
```

```
[Out] 1/(8*x**8 - 32*x**6 + 48*x**4 - 32*x**2 + 8)
```

GIAC/XCAS [A] time = 0.259185, size = 77, normalized size = 5.92

$$\frac{5}{256\,(x+1)} - \frac{5}{256\,(x-1)} + \frac{5}{256\,(x+1)^2} + \frac{5}{256\,(x-1)^2} + \frac{1}{64\,(x+1)^3} - \frac{1}{64\,(x-1)^3} + \frac{1}{128\,(x+1)^4} + \frac{1}{128\,(x-1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-5/256/(x + 1)^2 + 5/256/(x - 1)^2 - 5/128/(x + 1)^3 - 5/128/(x - 1)^3 -

[Out] 5/256/(x + 1) - 5/256/(x - 1) + 5/256/(x + 1)^2 + 5/256/(x - 1)^2
 + 1/64/(x + 1)^3 - 1/64/(x - 1)^3 + 1/128/(x + 1)^4 + 1/128/(x -
 1)^4
```

3.425 $\int \frac{1+x^6}{-1+x^6} \, dx$

Optimal. Leaf size=69

$$\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{6}\log\left(x^2+x+1\right)+x+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2}{3}\tanh^{-1}(x)$$

```
[Out] x + ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] - ArcTan[(1 + 2*x)/Sqrt[3]]
/Sqrt[3] - (2*ArcTanh[x])/3 + Log[1 - x + x^2]/6 - Log[1 + x + x^
2]/6
```

Rubi [A] time = 0.232518, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538

$$\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{6}\log\left(x^2+x+1\right)+x+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2}{3}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^6)/(-1 + x^6),x]
```

```
[Out] x + ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] - ArcTan[(1 + 2*x)/Sqrt[3]]
/Sqrt[3] - (2*ArcTanh[x])/3 + Log[1 - x + x^2]/6 - Log[1 + x + x^
2]/6
```

Rubi in Sympy [A] time = 45.5037, size = 71, normalized size = 1.03

$$x+\frac{\log\left(x^2-x+1\right)}{6}-\frac{\log\left(x^2+x+1\right)}{6}-\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}-\frac{1}{3}\right)\right)}{3}-\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}+\frac{1}{3}\right)\right)}{3}-\frac{2\,\mathrm{atanh}(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**6+1)/(x**6-1),x)
```

```
[Out] x + log(x**2 - x + 1)/6 - log(x**2 + x + 1)/6 - sqrt(3)*atan(sqrt
(3)*(2*x/3 - 1/3))/3 - sqrt(3)*atan(sqrt(3)*(2*x/3 + 1/3))/3 - 2*
atanh(x)/3
```

Mathematica [A] time = 0.0244281, size = 78, normalized size = 1.13

$$\frac{1}{6}\left(\log\left(x^2-x+1\right)-\log\left(x^2+x+1\right)+6x+2\log(1-x)-2\log(x+1)-2\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)-2\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^6)/(-1 + x^6),x]
```

```
[Out] (6*x - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1
 + 2*x)/Sqrt[3]] + 2*Log[1 - x] - 2*Log[1 + x] + Log[1 - x + x^2]
 - Log[1 + x + x^2])/6
```

Maple [A] time = 0.015, size = 67, normalized size = 1.

$$x-\frac{\ln\left(x^{2}+x+1\right)}{6}-\frac{\sqrt{3}}{3}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)+\frac{\ln(-1+x)}{3}-\frac{\ln(1+x)}{3}+\frac{\ln\left(x^{2}-x+1\right)}{6}-\frac{\sqrt{3}}{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^6+1)/(x^6-1),x)
```

```
[Out] x-1/6*ln(x^2+x+1)-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/3*ln(
-1+x)-1/3*ln(1+x)+1/6*ln(x^2-x+1)-1/3*3^(1/2)*arctan(1/3*(2*x-1)*
3^(1/2))
```

Maxima [A] time = 0.860649, size = 89, normalized size = 1.29

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+x-\frac{1}{6}\log\left(x^{2}+x+1\right)+\frac{1}{6}\log\left(x^{2}-x+1\right)-\frac{1}{3}\log(x+1)+\frac{1}{3}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^6 + 1)/(x^6 - 1),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*sqrt(3)*arctan(1
/3*sqrt(3)*(2*x - 1)) + x - 1/6*log(x^2 + x + 1) + 1/6*log(x^2 -
x + 1) - 1/3*log(x + 1) + 1/3*log(x - 1)
```

Fricas [A] time = 0.285753, size = 109, normalized size = 1.58

$$\frac{1}{18}\sqrt{3}\left(6\sqrt{3}x-\sqrt{3}\log\left(x^{2}+x+1\right)+\sqrt{3}\log\left(x^{2}-x+1\right)-2\sqrt{3}\log(x+1)+2\sqrt{3}\log(x-1)-6\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^6 + 1)/(x^6 - 1),x, algorithm="fricas")
```

```
[Out] 1/18*sqrt(3)*(6*sqrt(3)*x - sqrt(3)*log(x^2 + x + 1) + sqrt(3)*lo
g(x^2 - x + 1) - 2*sqrt(3)*log(x + 1) + 2*sqrt(3)*log(x - 1) - 6*
arctan(1/3*sqrt(3)*(2*x + 1)) - 6*arctan(1/3*sqrt(3)*(2*x - 1)))
```

Sympy [A] time = 0.720682, size = 85, normalized size = 1.23

$$x+\frac{\log(x-1)}{3}-\frac{\log(x+1)}{3}+\frac{\log\left(x^{2}-x+1\right)}{6}-\frac{\log\left(x^{2}+x+1\right)}{6}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**6+1)/(x**6-1),x)
```

```
[Out] x + log(x - 1)/3 - log(x + 1)/3 + log(x**2 - x + 1)/6 - log(x**2
+ x + 1)/6 - sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/3 - sqrt(3)*
atan(2*sqrt(3)*x/3 + sqrt(3)/3)/3
```

GIAC/XCAS [A] time = 0.262784, size = 92, normalized size = 1.33

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+x-\frac{1}{6}\ln\left(x^2+x+1\right)+\frac{1}{6}\ln\left(x^2-x+1\right)-\frac{1}{3}\ln\left(|x+1|\right)+\frac{1}{3}\ln\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^6 + 1)/(x^6 - 1),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*sqrt(3)*arctan(1
/3*sqrt(3)*(2*x - 1)) + x - 1/6*ln(x^2 + x + 1) + 1/6*ln(x^2 - x
+ 1) - 1/3*ln(abs(x + 1)) + 1/3*ln(abs(x - 1))
```

3.426 $\int \frac{\frac{1}{x^3}+x^3}{-\frac{1}{x^3}+x^3}\,dx$

Optimal. Leaf size=69

$$\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{6}\log\left(x^2+x+1\right)+x+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2}{3}\tanh^{-1}(x)$$

```
[Out] x + ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] - ArcTan[(1 + 2*x)/Sqrt[3]]
/Sqrt[3] - (2*ArcTanh[x])/3 + Log[1 - x + x^2]/6 - Log[1 + x + x^
2]/6
```

Rubi [A] time = 0.243243, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421

$$\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{6}\log\left(x^2+x+1\right)+x+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2}{3}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(x^(-3) + x^3)/(-x^(-3) + x^3),x]
```

```
[Out] x + ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] - ArcTan[(1 + 2*x)/Sqrt[3]]
/Sqrt[3] - (2*ArcTanh[x])/3 + Log[1 - x + x^2]/6 - Log[1 + x + x^
2]/6
```

Rubi in Sympy [A] time = 47.4503, size = 71, normalized size = 1.03

$$x+\frac{\log\left(x^2-x+1\right)}{6}-\frac{\log\left(x^2+x+1\right)}{6}-\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}-\frac{1}{3}\right)\right)}{3}-\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}+\frac{1}{3}\right)\right)}{3}-\frac{2\,\mathrm{atanh}\,(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1/x**3+x**3)/(-1/x**3+x**3),x)
```

```
[Out] x + log(x**2 - x + 1)/6 - log(x**2 + x + 1)/6 - sqrt(3)*atan(sqrt
(3)*(2*x/3 - 1/3))/3 - sqrt(3)*atan(sqrt(3)*(2*x/3 + 1/3))/3 - 2*
atanh(x)/3
```

Mathematica [A] time = 0.00817237, size = 78, normalized size = 1.13

$$\frac{1}{6}\left(\log\left(x^2-x+1\right)-\log\left(x^2+x+1\right)+6x+2\log(1-x)-2\log(x+1)-2\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)-2\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^(-3) + x^3)/(-x^(-3) + x^3),x]
```

```
[Out] (6*x - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1
 + 2*x)/Sqrt[3]] + 2*Log[1 - x] - 2*Log[1 + x] + Log[1 - x + x^2]
```

```
 - Log[1 + x + x^2])/6
```

Maple [A] time = 0.003, size = 67, normalized size = 1.

$$x-\frac{\ln\left(x^2+x+1\right)}{6}-\frac{\sqrt{3}}{3}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)+\frac{\ln(-1+x)}{3}-\frac{\ln(1+x)}{3}+\frac{\ln\left(x^2-x+1\right)}{6}-\frac{\sqrt{3}}{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1/x^3+x^3)/(-1/x^3+x^3),x)
```

```
[Out] x-1/6*ln(x^2+x+1)-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/3*ln(
-1+x)-1/3*ln(1+x)+1/6*ln(x^2-x+1)-1/3*3^(1/2)*arctan(1/3*(2*x-1)*
3^(1/2))
```

Maxima [A] time = 0.86105, size = 89, normalized size = 1.29

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+x-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log(x+1)+\frac{1}{3}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + 1/x^3)/(x^3 - 1/x^3),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*sqrt(3)*arctan(1
/3*sqrt(3)*(2*x - 1)) + x - 1/6*log(x^2 + x + 1) + 1/6*log(x^2 -
x + 1) - 1/3*log(x + 1) + 1/3*log(x - 1)
```

Fricas [A] time = 0.284883, size = 109, normalized size = 1.58

$$\frac{1}{18}\sqrt{3}\left(6\sqrt{3}x-\sqrt{3}\log\left(x^2+x+1\right)+\sqrt{3}\log\left(x^2-x+1\right)-2\sqrt{3}\log(x+1)+2\sqrt{3}\log(x-1)-6\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + 1/x^3)/(x^3 - 1/x^3),x, algorithm="fricas")
```

```
[Out] 1/18*sqrt(3)*(6*sqrt(3)*x - sqrt(3)*log(x^2 + x + 1) + sqrt(3)*lo
g(x^2 - x + 1) - 2*sqrt(3)*log(x + 1) + 2*sqrt(3)*log(x - 1) - 6*
arctan(1/3*sqrt(3)*(2*x + 1)) - 6*arctan(1/3*sqrt(3)*(2*x - 1)))
```

Sympy [A] time = 0.736112, size = 85, normalized size = 1.23

$$x+\frac{\log(x-1)}{3}-\frac{\log(x+1)}{3}+\frac{\log\left(x^2-x+1\right)}{6}-\frac{\log\left(x^2+x+1\right)}{6}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1/x**3+x**3)/(-1/x**3+x**3),x)
```

```
[Out] x + log(x - 1)/3 - log(x + 1)/3 + log(x**2 - x + 1)/6 - log(x**2
+ x + 1)/6 - sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/3 - sqrt(3)*
atan(2*sqrt(3)*x/3 + sqrt(3)/3)/3
```

GIAC/XCAS [A] time = 0.26149, size = 92, normalized size = 1.33

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+x-\frac{1}{6}\ln\left(x^2+x+1\right)+\frac{1}{6}\ln\left(x^2-x+1\right)-\frac{1}{3}\ln\left(|x+1|\right)+\frac{1}{3}\ln\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + 1/x^3)/(x^3 - 1/x^3),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*sqrt(3)*arctan(1
/3*sqrt(3)*(2*x - 1)) + x - 1/6*ln(x^2 + x + 1) + 1/6*ln(x^2 - x
+ 1) - 1/3*ln(abs(x + 1)) + 1/3*ln(abs(x - 1))
```

3.427 $\int \frac{-x+x^3}{6+2x} \, dx$

Optimal. Leaf size=24

$$\frac{x^3}{6} - \frac{3x^2}{4} + 4x - 12\log(x+3)$$

```
[Out] 4*x - (3*x^2)/4 + x^3/6 - 12*Log[3 + x]
```

Rubi [A] time = 0.0411261, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{x^3}{6} - \frac{3x^2}{4} + 4x - 12\log(x+3)$$

Antiderivative was successfully verified.

```
[In]  Int[(-x + x^3)/(6 + 2*x),x]

[Out] 4*x - (3*x^2)/4 + x^3/6 - 12*Log[3 + x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{x^3}{6} + 4x - 12\log(x+3) - \frac{3\int x \, dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**3-x)/(6+2*x),x)

[Out] x**3/6 + 4*x - 12*log(x + 3) - 3*Integral(x, x)/2
```

Mathematica [A] time = 0.00901904, size = 31, normalized size = 1.29

$$\frac{1}{2}\left(\frac{x^3}{3} - \frac{3x^2}{2} + 8x - 24\log(x+3) + \frac{93}{2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-x + x^3)/(6 + 2*x),x]

[Out] (93/2 + 8*x - (3*x^2)/2 + x^3/3 - 24*Log[3 + x])/2
```

Maple [A] time = 0.004, size = 21, normalized size = 0.9

$$4x - \frac{3x^2}{4} + \frac{x^3}{6} - 12\ln(3+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^3-x)/(6+2*x),x)
```

```
[Out] 4*x-3/4*x^2+1/6*x^3-12*ln(3+x)
```

Maxima [A] time = 0.79585, size = 27, normalized size = 1.12

$$\frac{1}{6}x^3 - \frac{3}{4}x^2 + 4x - 12\log(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/2*(x^3 - x)/(x + 3),x, algorithm="maxima")

[Out] 1/6*x^3 - 3/4*x^2 + 4*x - 12*log(x + 3)
```

Fricas [A] time = 0.272784, size = 27, normalized size = 1.12

$$\frac{1}{6}x^3 - \frac{3}{4}x^2 + 4x - 12\log(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/2*(x^3 - x)/(x + 3),x, algorithm="fricas")

[Out] 1/6*x^3 - 3/4*x^2 + 4*x - 12*log(x + 3)
```

Sympy [A] time = 0.12358, size = 20, normalized size = 0.83

$$\frac{x^3}{6} - \frac{3x^2}{4} + 4x - 12\log(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3-x)/(6+2*x),x)

[Out] x**3/6 - 3*x**2/4 + 4*x - 12*log(x + 3)
```

GIAC/XCAS [A] time = 0.260812, size = 28, normalized size = 1.17

$$\frac{1}{6}x^3 - \frac{3}{4}x^2 + 4x - 12\ln(|x+3|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/2*(x^3 - x)/(x + 3),x, algorithm="giac")

[Out] 1/6*x^3 - 3/4*x^2 + 4*x - 12*ln(abs(x + 3))
```

3.428 $\int \frac{x+x^3}{-1+x}\,dx$

Optimal. Leaf size=26

$$\frac{x^3}{3}+\frac{x^2}{2}+2x+2\log(1-x)$$

```
[Out] 2*x + x^2/2 + x^3/3 + 2*Log[1 - x]
```

Rubi [A] time = 0.0351514, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{x^3}{3}+\frac{x^2}{2}+2x+2\log(1-x)$$

Antiderivative was successfully verified.

```
[In]  Int[(x + x^3)/(-1 + x),x]

[Out] 2*x + x^2/2 + x^3/3 + 2*Log[1 - x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{x^3}{3}+2x+2\log(-x+1)+\int x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**3+x)/(-1+x),x)

[Out] x**3/3 + 2*x + 2*log(-x + 1) + Integral(x, x)
```

Mathematica [A] time = 0.00657661, size = 25, normalized size = 0.96

$$\frac{1}{6}\left(2x^3+3x^2+12x+12\log(x-1)-17\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x + x^3)/(-1 + x),x]

[Out] (-17 + 12*x + 3*x^2 + 2*x^3 + 12*Log[-1 + x])/6
```

Maple [A] time = 0.003, size = 21, normalized size = 0.8

$$\frac{x^3}{3}+\frac{x^2}{2}+2x+2\ln(-1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^3+x)/(-1+x),x)
```

```
[Out] 1/3*x^3+1/2*x^2+2*x+2*ln(-1+x)
```

Maxima [A] time = 0.793256, size = 27, normalized size = 1.04

$$\frac{1}{3}x^3 + \frac{1}{2}x^2 + 2x + 2\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + x)/(x - 1),x, algorithm="maxima")
```

```
[Out] 1/3*x^3 + 1/2*x^2 + 2*x + 2*log(x - 1)
```

Fricas [A] time = 0.269372, size = 27, normalized size = 1.04

$$\frac{1}{3}x^3 + \frac{1}{2}x^2 + 2x + 2\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + x)/(x - 1),x, algorithm="fricas")
```

```
[Out] 1/3*x^3 + 1/2*x^2 + 2*x + 2*log(x - 1)
```

Sympy [A] time = 0.115514, size = 19, normalized size = 0.73

$$\frac{x^3}{3} + \frac{x^2}{2} + 2x + 2\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3+x)/(-1+x),x)
```

```
[Out] x**3/3 + x**2/2 + 2*x + 2*log(x - 1)
```

GIAC/XCAS [A] time = 0.260184, size = 28, normalized size = 1.08

$$\frac{1}{3}x^3 + \frac{1}{2}x^2 + 2x + 2\ln(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + x)/(x - 1),x, algorithm="giac")
```

```
[Out] 1/3*x^3 + 1/2*x^2 + 2*x + 2*ln(abs(x - 1))
```

3.429 $\int (ac + (bc + d)x)\,dx$

Optimal. Leaf size=17

$$acx + \frac{1}{2}x^2(bc + d)$$

```
[Out] a*c*x + ((b*c + d)*x^2)/2
```

Rubi [A] time = 0.0192166, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$

$$acx + \frac{1}{2}x^2(bc + d)$$

Antiderivative was successfully verified.

```
[In]  Int[a*c + (b*c + d)*x,x]

[Out] a*c*x + ((b*c + d)*x^2)/2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$c\int a\,dx + (bc + d)\int x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(a*c+(b*c+d)*x,x)

[Out] c*Integral(a, x) + (b*c + d)*Integral(x, x)
```

Mathematica [A] time = 0.0000479975, size = 22, normalized size = 1.29

$$acx + \frac{1}{2}bcx^2 + \frac{dx^2}{2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[a*c + (b*c + d)*x,x]

[Out] a*c*x + (b*c*x^2)/2 + (d*x^2)/2
```

Maple [A] time = 0.001, size = 16, normalized size = 0.9

$$acx + \frac{(bc + d)\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(a*c+(b*c+d)*x,x)
```

```
[Out] a*c*x+1/2*(b*c+d)*x^2
```

Maxima [A] time = 0.78335, size = 20, normalized size = 1.18

$$acx + \frac{1}{2}(bc + d)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(a*c + (b*c + d)*x,x, algorithm="maxima")
```

```
[Out] a*c*x + 1/2*(b*c + d)*x^2
```

Fricas [A] time = 0.247731, size = 1, normalized size = 0.06

$$\frac{1}{2}x^2cb + \frac{1}{2}x^2d + xca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(a*c + (b*c + d)*x,x, algorithm="fricas")
```

```
[Out] 1/2*x^2*c*b + 1/2*x^2*d + x*c*a
```

Sympy [A] time = 0.069167, size = 15, normalized size = 0.88

$$acx + x^2\left(\frac{bc}{2} + \frac{d}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(a*c+(b*c+d)*x,x)
```

```
[Out] a*c*x + x**2*(b*c/2 + d/2)
```

GIAC/XCAS [A] time = 0.257955, size = 20, normalized size = 1.18

$$acx + \frac{1}{2}(bc + d)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(a*c + (b*c + d)*x,x, algorithm="giac")
```

```
[Out] a*c*x + 1/2*(b*c + d)*x^2
```

3.430 $\int (dx + c(a + bx))\,dx$

Optimal. Leaf size=24

$$\frac{c(a+bx)^2}{2b}+\frac{dx^2}{2}$$

```
[Out] (d*x^2)/2 + (c*(a + b*x)^2)/(2*b)
```

Rubi [A] time = 0.0166772, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.

$$\frac{c(a+bx)^2}{2b}+\frac{dx^2}{2}$$

Antiderivative was successfully verified.

```
[In]  Int[d*x + c*(a + b*x),x]

[Out] (d*x^2)/2 + (c*(a + b*x)^2)/(2*b)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$d\int x\,dx+\frac{c\,(a+bx)^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(d*x+(b*x+a)*c,x)

[Out] d*Integral(x, x) + c*(a + b*x)**2/(2*b)
```

Mathematica [A] time = 0.00166935, size = 22, normalized size = 0.92

$$acx+\frac{1}{2}bcx^2+\frac{dx^2}{2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[d*x + c*(a + b*x),x]

[Out] a*c*x + (b*c*x^2)/2 + (d*x^2)/2
```

Maple [A] time = 0.001, size = 20, normalized size = 0.8

$$\frac{dx^2}{2}+c\left(ax+\frac{bx^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(d*x+(b*x+a)*c,x)
```

```
[Out] 1/2*d*x^2+c*(a*x+1/2*b*x^2)
```

Maxima [A] time = 0.791181, size = 27, normalized size = 1.12

$$\frac{1}{2}\,dx^2 + \frac{1}{2}\,\left(bx^2 + 2\,ax\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x + a)*c + d*x,x, algorithm="maxima")
```

```
[Out] 1/2*d*x^2 + 1/2*(b*x^2 + 2*a*x)*c
```

Fricas [A] time = 0.248856, size = 1, normalized size = 0.04

$$\frac{1}{2}x^2cb + \frac{1}{2}x^2d + xca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x + a)*c + d*x,x, algorithm="fricas")
```

```
[Out] 1/2*x^2*c*b + 1/2*x^2*d + x*c*a
```

Sympy [A] time = 0.074944, size = 15, normalized size = 0.62

$$acx + x^2\left(\frac{bc}{2} + \frac{d}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x+(b*x+a)*c,x)
```

```
[Out] a*c*x + x**2*(b*c/2 + d/2)
```

GIAC/XCAS [A] time = 0.256977, size = 27, normalized size = 1.12

$$\frac{1}{2}\,dx^2 + \frac{1}{2}\,\left(bx^2 + 2\,ax\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x + a)*c + d*x,x, algorithm="giac")
```

```
[Out] 1/2*d*x^2 + 1/2*(b*x^2 + 2*a*x)*c
```

3.431 $\int \frac{4+4x}{x^2(1+x^2)}\,dx$

Optimal. Leaf size=22

$$-2\log\left(x^2+1\right)-\frac{4}{x}+4\log(x)-4\tan^{-1}(x)$$

```
[Out] -4/x - 4*ArcTan[x] + 4*Log[x] - 2*Log[1 + x^2]
```

Rubi [A] time = 0.0545094, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$-2\log\left(x^2+1\right)-\frac{4}{x}+4\log(x)-4\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(4 + 4*x)/(x^2*(1 + x^2)),x]

[Out] -4/x - 4*ArcTan[x] + 4*Log[x] - 2*Log[1 + x^2]
```

Rubi in Sympy [A] time = 7.06004, size = 20, normalized size = 0.91

$$4\log(x)-2\log\left(x^2+1\right)-4\,\mathrm{atan}\,(x)-\frac{4}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((4+4*x)/x**2/(x**2+1),x)

[Out] 4*log(x) - 2*log(x**2 + 1) - 4*atan(x) - 4/x
```

Mathematica [A] time = 0.00812693, size = 24, normalized size = 1.09

$$4\left(-\frac{1}{2}\log\left(x^2+1\right)-\frac{1}{x}+\log(x)-\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(4 + 4*x)/(x^2*(1 + x^2)),x]

[Out] 4*(-x^(-1) - ArcTan[x] + Log[x] - Log[1 + x^2]/2)
```

Maple [A] time = 0.01, size = 23, normalized size = 1.1

$$-4x^{-1}-4\arctan(x)+4\ln(x)-2\ln\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((4+4*x)/x^2/(x^2+1),x)

[Out] -4/x-4*arctan(x)+4*ln(x)-2*ln(x^2+1)
```

Maxima [A] time = 0.858785, size = 30, normalized size = 1.36

$$-\frac{4}{x} - 4 \arctan(x) - 2 \log\left(x^2 + 1\right) + 4 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(4*(x + 1)/((x^2 + 1)*x^2),x, algorithm="maxima")

[Out] -4/x - 4*arctan(x) - 2*log(x^2 + 1) + 4*log(x)
```

Fricas [A] time = 0.28056, size = 34, normalized size = 1.55

$$-\frac{2\left(2\,x\arctan(x) + x\log\left(x^2+1\right) - 2\,x\log(x) + 2\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(4*(x + 1)/((x^2 + 1)*x^2),x, algorithm="fricas")

[Out] -2*(2*x*arctan(x) + x*log(x^2 + 1) - 2*x*log(x) + 2)/x
```

Sympy [A] time = 0.280935, size = 20, normalized size = 0.91

$$4\log(x) - 2\log\left(x^2+1\right) - 4\operatorname{atan}(x) - \frac{4}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4+4*x)/x**2/(x**2+1),x)

[Out] 4*log(x) - 2*log(x**2 + 1) - 4*atan(x) - 4/x
```

GIAC/XCAS [A] time = 0.260692, size = 31, normalized size = 1.41

$$-\frac{4}{x} - 4 \arctan(x) - 2\ln\left(x^2+1\right) + 4\ln\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(4*(x + 1)/((x^2 + 1)*x^2),x, algorithm="giac")

[Out] -4/x - 4*arctan(x) - 2*ln(x^2 + 1) + 4*ln(abs(x))
```

3.432 $\int \frac{24+8x}{x(-4+x^2)}\,dx$

Optimal. Leaf size=17

$$5\log(2-x)-6\log(x)+\log(x+2)$$

```
[Out] 5*Log[2 - x] - 6*Log[x] + Log[2 + x]
```

Rubi [A] time = 0.041434, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062

$$5\log(2-x)-6\log(x)+\log(x+2)$$

Antiderivative was successfully verified.

```
[In]  Int[(24 + 8*x)/(x*(-4 + x^2)),x]

[Out] 5*Log[2 - x] - 6*Log[x] + Log[2 + x]
```

Rubi in Sympy [A] time = 5.6143, size = 15, normalized size = 0.88

$$-6\log(x)+5\log(-x+2)+\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((24+8*x)/x/(x**2-4),x)

[Out] -6*log(x) + 5*log(-x + 2) + log(x + 2)
```

Mathematica [A] time = 0.00966221, size = 27, normalized size = 1.59

$$8\left(\frac{5}{8}\log(2-x)-\frac{3\log(x)}{4}+\frac{1}{8}\log(x+2)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(24 + 8*x)/(x*(-4 + x^2)),x]

[Out] 8*((5*Log[2 - x])/8 - (3*Log[x])/4 + Log[2 + x]/8)
```

Maple [A] time = 0.01, size = 16, normalized size = 0.9

$$\ln(2+x)-6\ln(x)+5\ln(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((24+8*x)/x/(x^2-4),x)

[Out] ln(2+x)-6*ln(x)+5*ln(x-2)
```

Maxima [A] time = 0.796886, size = 20, normalized size = 1.18

$$\log(x+2) + 5 \log(x-2) - 6 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(8*(x + 3)/((x^2 - 4)*x),x, algorithm="maxima")

[Out] log(x + 2) + 5*log(x - 2) - 6*log(x)
```

Fricas [A] time = 0.27961, size = 20, normalized size = 1.18

$$\log(x+2) + 5 \log(x-2) - 6 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(8*(x + 3)/((x^2 - 4)*x),x, algorithm="fricas")

[Out] log(x + 2) + 5*log(x - 2) - 6*log(x)
```

Sympy [A] time = 0.264976, size = 15, normalized size = 0.88

$$-6\log(x) + 5\log(x-2) + \log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((24+8*x)/x/(x**2-4),x)

[Out] -6*log(x) + 5*log(x - 2) + log(x + 2)
```

GIAC/XCAS [A] time = 0.259095, size = 24, normalized size = 1.41

$$\ln(|x+2|) + 5\ln(|x-2|) - 6\ln(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(8*(x + 3)/((x^2 - 4)*x),x, algorithm="giac")

[Out] ln(abs(x + 2)) + 5*ln(abs(x - 2)) - 6*ln(abs(x))
```

3.433 $\int \frac{-1+x^2}{-2x+x^3}\,dx$

Optimal. Leaf size=19

$$\frac{1}{4}\log\left(2-x^2\right)+\frac{\log(x)}{2}$$

```
[Out] Log[x]/2 + Log[2 - x^2]/4
```

Rubi [A] time = 0.0496399, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{1}{4}\log\left(2-x^2\right)+\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + x^2)/(-2*x + x^3),x]
```

```
[Out] Log[x]/2 + Log[2 - x^2]/4
```

Rubi in Sympy [A] time = 8.20481, size = 14, normalized size = 0.74

$$\frac{\log\left(x^2\right)}{4}+\frac{\log\left(-x^2+2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2-1)/(x**3-2*x),x)
```

```
[Out] log(x**2)/4 + log(-x**2 + 2)/4
```

Mathematica [A] time = 0.00559458, size = 19, normalized size = 1.

$$\frac{1}{4}\log\left(2-x^2\right)+\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + x^2)/(-2*x + x^3),x]
```

```
[Out] Log[x]/2 + Log[2 - x^2]/4
```

Maple [A] time = 0.007, size = 14, normalized size = 0.7

$$\frac{\ln(x)}{2}+\frac{\ln\left(x^2-2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2-1)/(x^3-2*x),x)
```

```
[Out] 1/2*ln(x)+1/4*ln(x^2-2)
```

Maxima [A] time = 0.78842, size = 18, normalized size = 0.95

$$\frac{1}{4}\log\left(x^2-2\right)+\frac{1}{2}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 1)/(x^3 - 2*x),x, algorithm="maxima")

[Out] 1/4*log(x^2 - 2) + 1/2*log(x)
```

Fricas [A] time = 0.258814, size = 18, normalized size = 0.95

$$\frac{1}{4}\log\left(x^2-2\right)+\frac{1}{2}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 1)/(x^3 - 2*x),x, algorithm="fricas")

[Out] 1/4*log(x^2 - 2) + 1/2*log(x)
```

Sympy [A] time = 0.178408, size = 12, normalized size = 0.63

$$\frac{\log\left(x\right)}{2}+\frac{\log\left(x^2-2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2-1)/(x**3-2*x),x)

[Out] log(x)/2 + log(x**2 - 2)/4
```

GIAC/XCAS [A] time = 0.259422, size = 22, normalized size = 1.16

$$\frac{1}{4}\ln\left(x^2\right)+\frac{1}{4}\ln\left(\left|x^2-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 1)/(x^3 - 2*x),x, algorithm="giac")

[Out] 1/4*ln(x^2) + 1/4*ln(abs(x^2 - 2))
```

3.434 $\int \frac{1+x^2}{3x+x^3} \, dx$

Optimal. Leaf size=12

$$\frac{1}{3}\log\left(x^3+3x\right)$$

```
[Out] Log[3*x + x^3]/3
```

Rubi [A] time = 0.00871026, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$\frac{1}{3}\log\left(x^3+3x\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^2)/(3*x + x^3),x]

[Out] Log[3*x + x^3]/3
```

Rubi in Sympy [A] time = 8.12128, size = 14, normalized size = 1.17

$$\frac{\log\left(x^2\right)}{6}+\frac{\log\left(x^2+3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+1)/(x**3+3*x),x)

[Out] log(x**2)/6 + log(x**2 + 3)/3
```

Mathematica [A] time = 0.00546691, size = 17, normalized size = 1.42

$$\frac{1}{3}\log\left(x^2+3\right)+\frac{\log(x)}{3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^2)/(3*x + x^3),x]

[Out] Log[x]/3 + Log[3 + x^2]/3
```

Maple [A] time = 0.002, size = 11, normalized size = 0.9

$$\frac{\ln\left(x\left(x^2+3\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+1)/(x^3+3*x),x)
```

```
[Out] 1/3*ln(x*(x^2+3))
```

Maxima [A] time = 0.800529, size = 14, normalized size = 1.17

$$\frac{1}{3}\log\left(x^3 + 3x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/(x^3 + 3*x),x, algorithm="maxima")

[Out] 1/3*log(x^3 + 3*x)
```

Fricas [A] time = 0.265983, size = 14, normalized size = 1.17

$$\frac{1}{3}\log\left(x^3 + 3x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/(x^3 + 3*x),x, algorithm="fricas")

[Out] 1/3*log(x^3 + 3*x)
```

Sympy [A] time = 0.159458, size = 8, normalized size = 0.67

$$\frac{\log\left(x^3 + 3x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+1)/(x**3+3*x),x)

[Out] log(x**3 + 3*x)/3
```

GIAC/XCAS [A] time = 0.261002, size = 20, normalized size = 1.67

$$\frac{1}{3}\ln\left(x^2 + 3\right) + \frac{1}{6}\ln\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/(x^3 + 3*x),x, algorithm="giac")

[Out] 1/3*ln(x^2 + 3) + 1/6*ln(x^2)
```

3.435 $\int \frac{a+3bx^2}{ax+bx^3}\,dx$

Optimal. Leaf size=10

$$\log\left(ax+bx^3\right)$$

```
[Out] Log[a*x + b*x^3]
```

Rubi [A] time = 0.0103937, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05

$$\log\left(ax+bx^3\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + 3*b*x^2)/(a*x + b*x^3),x]

[Out] Log[a*x + b*x^3]
```

Rubi in Sympy [A] time = 12.4693, size = 14, normalized size = 1.4

$$\frac{\log\left(x^2\right)}{2}+\log\left(a+bx^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((3*b*x**2+a)/(b*x**3+a*x),x)

[Out] log(x**2)/2 + log(a + b*x**2)
```

Mathematica [A] time = 0.00922223, size = 11, normalized size = 1.1

$$\log\left(a+bx^2\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + 3*b*x^2)/(a*x + b*x^3),x]

[Out] Log[x] + Log[a + b*x^2]
```

Maple [A] time = 0.002, size = 11, normalized size = 1.1

$$\ln\left(x\left(bx^2+a\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*b*x^2+a)/(b*x^3+a*x),x)

[Out] ln(x*(b*x^2+a))
```

Maxima [A] time = 0.789497, size = 14, normalized size = 1.4

$$\log(bx^3 + ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*b*x^2 + a)/(b*x^3 + a*x),x, algorithm="maxima")
```

```
[Out] log(b*x^3 + a*x)
```

Fricas [A] time = 0.259774, size = 14, normalized size = 1.4

$$\log(bx^3 + ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*b*x^2 + a)/(b*x^3 + a*x),x, algorithm="fricas")
```

```
[Out] log(b*x^3 + a*x)
```

Sympy [A] time = 1.11232, size = 8, normalized size = 0.8

$$\log(ax + bx^3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*b*x**2+a)/(b*x**3+a*x),x)
```

```
[Out] log(a*x + b*x**3)
```

GIAC/XCAS [A] time = 0.261366, size = 22, normalized size = 2.2

$$\frac{1}{2}\ln(x^2) + \ln(|bx^2 + a|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*b*x^2 + a)/(b*x^3 + a*x),x, algorithm="giac")
```

```
[Out] 1/2*ln(x^2) + ln(abs(b*x^2 + a))
```

3.436 $\int \frac{-2+4x}{-x+x^3}\,dx$

Optimal. Leaf size=17

$$\log(1-x)+2\log(x)-3\log(x+1)$$

```
[Out] Log[1 - x] + 2*Log[x] - 3*Log[1 + x]
```

Rubi [A] time = 0.0436409, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$\log(1-x)+2\log(x)-3\log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[(-2 + 4*x)/(-x + x^3),x]

[Out] Log[1 - x] + 2*Log[x] - 3*Log[1 + x]
```

Rubi in Sympy [A] time = 6.5731, size = 15, normalized size = 0.88

$$2\log(x)+\log(-x+1)-3\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-2+4*x)/(x**3-x),x)

[Out] 2*log(x) + log(-x + 1) - 3*log(x + 1)
```

Mathematica [A] time = 0.00896208, size = 17, normalized size = 1.

$$\log(1-x)+2\log(x)-3\log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-2 + 4*x)/(-x + x^3),x]

[Out] Log[1 - x] + 2*Log[x] - 3*Log[1 + x]
```

Maple [A] time = 0.01, size = 16, normalized size = 0.9

$$\ln(-1+x)-3\ln(1+x)+2\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-2+4*x)/(x^3-x),x)

[Out] ln(-1+x)-3*ln(1+x)+2*ln(x)
```

Maxima [A] time = 0.794937, size = 20, normalized size = 1.18

$$-3\ \log\left(x+1\right)+\log\left(x-1\right)+2\ \log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(2*(2*x - 1)/(x^3 - x),x, algorithm="maxima")
```

```
[Out] -3*log(x + 1) + log(x - 1) + 2*log(x)
```

Fricas [A] time = 0.264144, size = 20, normalized size = 1.18

$$-3\ \log\left(x+1\right)+\log\left(x-1\right)+2\ \log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(2*(2*x - 1)/(x^3 - x),x, algorithm="fricas")
```

```
[Out] -3*log(x + 1) + log(x - 1) + 2*log(x)
```

Sympy [A] time = 0.252031, size = 15, normalized size = 0.88

$$2\log\left(x\right)+\log\left(x-1\right)-3\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-2+4*x)/(x**3-x),x)
```

```
[Out] 2*log(x) + log(x - 1) - 3*log(x + 1)
```

GIAC/XCAS [A] time = 0.260208, size = 24, normalized size = 1.41

$$-3\ln\left(|x+1|\right)+\ln\left(|x-1|\right)+2\ln\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(2*(2*x - 1)/(x^3 - x),x, algorithm="giac")
```

```
[Out] -3*ln(abs(x + 1)) + ln(abs(x - 1)) + 2*ln(abs(x))
```

3.437 $\int \frac{4+x}{4x+x^3}\,dx$

Optimal. Leaf size=23

$$-\frac{1}{2}\log\left(x^2+4\right)+\log(x)+\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)$$

```
[Out] ArcTan[x/2]/2 + Log[x] - Log[4 + x^2]/2
```

Rubi [A] time = 0.0480656, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385

$$-\frac{1}{2}\log\left(x^2+4\right)+\log(x)+\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(4 + x)/(4*x + x^3),x]

[Out] ArcTan[x/2]/2 + Log[x] - Log[4 + x^2]/2
```

Rubi in Sympy [A] time = 6.97383, size = 17, normalized size = 0.74

$$\log(x)-\frac{\log\left(x^2+4\right)}{2}+\frac{\operatorname{atan}\left(\frac{x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((4+x)/(x**3+4*x),x)

[Out] log(x) - log(x**2 + 4)/2 + atan(x/2)/2
```

Mathematica [A] time = 0.00664285, size = 23, normalized size = 1.

$$-\frac{1}{2}\log\left(x^2+4\right)+\log(x)+\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(4 + x)/(4*x + x^3),x]

[Out] ArcTan[x/2]/2 + Log[x] - Log[4 + x^2]/2
```

Maple [A] time = 0.007, size = 18, normalized size = 0.8

$$\frac{1}{2}\arctan\left(\frac{x}{2}\right)+\ln(x)-\frac{\ln\left(x^2+4\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((4+x)/(x^3+4*x),x)
```

```
[Out] 1/2*arctan(1/2*x)+ln(x)-1/2*ln(x^2+4)
```

Maxima [A] time = 0.872465, size = 23, normalized size = 1.

$$\frac{1}{2}\arctan\left(\frac{1}{2}x\right)-\frac{1}{2}\log\left(x^2+4\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 4)/(x^3 + 4*x),x, algorithm="maxima")
```

```
[Out] 1/2*arctan(1/2*x) - 1/2*log(x^2 + 4) + log(x)
```

Fricas [A] time = 0.263439, size = 23, normalized size = 1.

$$\frac{1}{2}\arctan\left(\frac{1}{2}x\right)-\frac{1}{2}\log\left(x^2+4\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 4)/(x^3 + 4*x),x, algorithm="fricas")
```

```
[Out] 1/2*arctan(1/2*x) - 1/2*log(x^2 + 4) + log(x)
```

Sympy [A] time = 0.272453, size = 17, normalized size = 0.74

$$\log\left(x\right)-\frac{\log\left(x^2+4\right)}{2}+\frac{\operatorname{atan}\left(\frac{x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4+x)/(x**3+4*x),x)
```

```
[Out] log(x) - log(x**2 + 4)/2 + atan(x/2)/2
```

GIAC/XCAS [A] time = 0.260941, size = 24, normalized size = 1.04

$$\frac{1}{2}\arctan\left(\frac{1}{2}x\right)-\frac{1}{2}\ln\left(x^2+4\right)+\ln\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 4)/(x^3 + 4*x),x, algorithm="giac")
```

```
[Out] 1/2*arctan(1/2*x) - 1/2*ln(x^2 + 4) + ln(abs(x))
```

3.438 $\int \frac{-x+2x^3}{1-x^2+x^4} \, dx$

Optimal. Leaf size=15

$$\frac{1}{2}\log\left(x^4-x^2+1\right)$$

```
[Out] Log[1 - x^2 + x^4]/2
```

Rubi [A] time = 0.00969548, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045

$$\frac{1}{2}\log\left(x^4-x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-x + 2*x^3)/(1 - x^2 + x^4),x]

[Out] Log[1 - x^2 + x^4]/2
```

Rubi in Sympy [A] time = 5.06028, size = 10, normalized size = 0.67

$$\frac{\log\left(x^4-x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((2*x**3-x)/(x**4-x**2+1),x)

[Out] log(x**4 - x**2 + 1)/2
```

Mathematica [A] time = 0.00788278, size = 15, normalized size = 1.

$$\frac{1}{2}\log\left(x^4-x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x + 2*x^3)/(1 - x^2 + x^4),x]

[Out] Log[1 - x^2 + x^4]/2
```

Maple [A] time = 0.002, size = 14, normalized size = 0.9

$$\frac{\ln\left(x^4-x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3-x)/(x^4-x^2+1),x)
```

```
[Out] 1/2*ln(x^4-x^2+1)
```

Maxima [A] time = 0.800427, size = 18, normalized size = 1.2

$$\frac{1}{2}\log\left(x^4-x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^3 - x)/(x^4 - x^2 + 1),x, algorithm="maxima")
```

```
[Out] 1/2*log(x^4 - x^2 + 1)
```

Fricas [A] time = 0.26653, size = 18, normalized size = 1.2

$$\frac{1}{2}\log\left(x^4-x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^3 - x)/(x^4 - x^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/2*log(x^4 - x^2 + 1)
```

Sympy [A] time = 0.194188, size = 10, normalized size = 0.67

$$\frac{\log\left(x^4-x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x**3-x)/(x**4-x**2+1),x)
```

```
[Out] log(x**4 - x**2 + 1)/2
```

GIAC/XCAS [A] time = 0.260562, size = 18, normalized size = 1.2

$$\frac{1}{2}\ln\left(x^4-x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^3 - x)/(x^4 - x^2 + 1),x, algorithm="giac")
```

```
[Out] 1/2*ln(x^4 - x^2 + 1)
```

3.439 $\int \frac{-3+x}{2x+3x^2+x^3}\,dx$

Optimal. Leaf size=21

$$-\frac{3\log(x)}{2}+4\log(x+1)-\frac{5}{2}\log(x+2)$$

```
[Out] (-3*Log[x])/2 + 4*Log[1 + x] - (5*Log[2 + x])/2
```

Rubi [A] time = 0.0476858, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$-\frac{3\log(x)}{2}+4\log(x+1)-\frac{5}{2}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + x)/(2*x + 3*x^2 + x^3),x]

[Out] (-3*Log[x])/2 + 4*Log[1 + x] - (5*Log[2 + x])/2
```

Rubi in Sympy [A] time = 12.6167, size = 20, normalized size = 0.95

$$-\frac{3\log(x)}{2}+4\log(x+1)-\frac{5\log(x+2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-3+x)/(x**3+3*x**2+2*x),x)

[Out] -3*log(x)/2 + 4*log(x + 1) - 5*log(x + 2)/2
```

Mathematica [A] time = 0.00860018, size = 21, normalized size = 1.

$$-\frac{3\log(x)}{2}+4\log(x+1)-\frac{5}{2}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + x)/(2*x + 3*x^2 + x^3),x]

[Out] (-3*Log[x])/2 + 4*Log[1 + x] - (5*Log[2 + x])/2
```

Maple [A] time = 0.01, size = 18, normalized size = 0.9

$$-\frac{3\ln(x)}{2}+4\ln(1+x)-\frac{5\ln(2+x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3+x)/(x^3+3*x^2+2*x),x)
```

```
[Out] -3/2*ln(x)+4*ln(1+x)-5/2*ln(2+x)
```

Maxima [A] time = 0.785245, size = 23, normalized size = 1.1

$$-\frac{5}{2}\log(x+2)+4\log(x+1)-\frac{3}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - 3)/(x^3 + 3*x^2 + 2*x),x, algorithm="maxima")

[Out] -5/2*log(x + 2) + 4*log(x + 1) - 3/2*log(x)
```

Fricas [A] time = 0.271498, size = 23, normalized size = 1.1

$$-\frac{5}{2}\log(x+2)+4\log(x+1)-\frac{3}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - 3)/(x^3 + 3*x^2 + 2*x),x, algorithm="fricas")

[Out] -5/2*log(x + 2) + 4*log(x + 1) - 3/2*log(x)
```

Sympy [A] time = 0.275557, size = 20, normalized size = 0.95

$$-\frac{3\log(x)}{2}+4\log(x+1)-\frac{5\log(x+2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-3+x)/(x**3+3*x**2+2*x),x)

[Out] -3*log(x)/2 + 4*log(x + 1) - 5*log(x + 2)/2
```

GIAC/XCAS [A] time = 0.259774, size = 27, normalized size = 1.29

$$-\frac{5}{2}\ln(|x+2|)+4\ln(|x+1|)-\frac{3}{2}\ln(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - 3)/(x^3 + 3*x^2 + 2*x),x, algorithm="giac")

[Out] -5/2*ln(abs(x + 2)) + 4*ln(abs(x + 1)) - 3/2*ln(abs(x))
```

3.440 $\int \frac{2+4x}{x^2+2x^3+x^4}\,dx$

Optimal. Leaf size=10

$$-\frac{2}{x(x+1)}$$

```
[Out] -2/(x*(1 + x))
```

Rubi [A] time = 0.0158958, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15

$$-\frac{2}{x(x+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 4*x)/(x^2 + 2*x^3 + x^4),x]

[Out] -2/(x*(1 + x))
```

Rubi in Sympy [A] time = 6.90936, size = 7, normalized size = 0.7

$$-\frac{2}{x\,(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((2+4*x)/(x**4+2*x**3+x**2),x)

[Out] -2/(x*(x + 1))
```

Mathematica [A] time = 0.00890193, size = 9, normalized size = 0.9

$$-\frac{2}{x^2+x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 4*x)/(x^2 + 2*x^3 + x^4),x]

[Out] -2/(x + x^2)
```

Maple [A] time = 0.007, size = 14, normalized size = 1.4

$$2\,(1+x)^{-1}-2\,x^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+4*x)/(x^4+2*x^3+x^2),x)

[Out] 2/(1+x)-2/x
```

Maxima [A] time = 0.791325, size = 12, normalized size = 1.2

$$-\frac{2}{x^2+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(2*(2*x + 1)/(x^4 + 2*x^3 + x^2),x, algorithm="maxima")

[Out] -2/(x^2 + x)
```

Fricas [A] time = 0.25147, size = 12, normalized size = 1.2

$$-\frac{2}{x^2+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(2*(2*x + 1)/(x^4 + 2*x^3 + x^2),x, algorithm="fricas")

[Out] -2/(x^2 + x)
```

Sympy [A] time = 0.161714, size = 7, normalized size = 0.7

$$-\frac{2}{x^2+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2+4*x)/(x**4+2*x**3+x**2),x)

[Out] -2/(x**2 + x)
```

GIAC/XCAS [A] time = 0.258634, size = 12, normalized size = 1.2

$$-\frac{2}{x^2+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(2*(2*x + 1)/(x^4 + 2*x^3 + x^2),x, algorithm="giac")

[Out] -2/(x^2 + x)
```

3.441 $\int \frac{1+x}{-6x+x^2+x^3}\,dx$

Optimal. Leaf size=25

$$\frac{3}{10}\log(2-x)-\frac{\log(x)}{6}-\frac{2}{15}\log(x+3)$$

```
[Out] (3*Log[2 - x])/10 - Log[x]/6 - (2*Log[3 + x])/15
```

Rubi [A] time = 0.0466046, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{3}{10}\log(2-x)-\frac{\log(x)}{6}-\frac{2}{15}\log(x+3)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x)/(-6*x + x^2 + x^3),x]

[Out] (3*Log[2 - x])/10 - Log[x]/6 - (2*Log[3 + x])/15
```

Rubi in Sympy [A] time = 9.82311, size = 20, normalized size = 0.8

$$-\frac{\log(x)}{6}+\frac{3\log(-x+2)}{10}-\frac{2\log(x+3)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+x)/(x**3+x**2-6*x),x)

[Out] -log(x)/6 + 3*log(-x + 2)/10 - 2*log(x + 3)/15
```

Mathematica [A] time = 0.00781111, size = 25, normalized size = 1.

$$\frac{3}{10}\log(2-x)-\frac{\log(x)}{6}-\frac{2}{15}\log(x+3)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x)/(-6*x + x^2 + x^3),x]

[Out] (3*Log[2 - x])/10 - Log[x]/6 - (2*Log[3 + x])/15
```

Maple [A] time = 0.01, size = 18, normalized size = 0.7

$$-\frac{\ln(x)}{6}+\frac{3\ln(x-2)}{10}-\frac{2\ln(3+x)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+x)/(x^3+x^2-6*x),x)
```

```
[Out] -1/6*ln(x)+3/10*ln(x-2)-2/15*ln(3+x)
```

Maxima [A] time = 0.793163, size = 23, normalized size = 0.92

$$-\frac{2}{15} \log(x+3) + \frac{3}{10} \log(x-2) - \frac{1}{6} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 1)/(x^3 + x^2 - 6*x),x, algorithm="maxima")
```

```
[Out] -2/15*log(x + 3) + 3/10*log(x - 2) - 1/6*log(x)
```

Fricas [A] time = 0.25752, size = 23, normalized size = 0.92

$$-\frac{2}{15} \log(x+3) + \frac{3}{10} \log(x-2) - \frac{1}{6} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 1)/(x^3 + x^2 - 6*x),x, algorithm="fricas")
```

```
[Out] -2/15*log(x + 3) + 3/10*log(x - 2) - 1/6*log(x)
```

Sympy [A] time = 0.298799, size = 20, normalized size = 0.8

$$-\frac{\log(x)}{6} + \frac{3\log(x-2)}{10} - \frac{2\log(x+3)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+x)/(x**3+x**2-6*x),x)
```

```
[Out] -log(x)/6 + 3*log(x - 2)/10 - 2*log(x + 3)/15
```

GIAC/XCAS [A] time = 0.263813, size = 27, normalized size = 1.08

$$-\frac{2}{15} \ln(|x+3|) + \frac{3}{10} \ln(|x-2|) - \frac{1}{6} \ln(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 1)/(x^3 + x^2 - 6*x),x, algorithm="giac")
```

```
[Out] -2/15*ln(abs(x + 3)) + 3/10*ln(abs(x - 2)) - 1/6*ln(abs(x))
```

3.442 $\int \frac{4x^2+x^3}{x+x^3} \, dx$

Optimal. Leaf size=14

$$2\log\left(x^2+1\right)+x-\tan^{-1}(x)$$

```
[Out] x - ArcTan[x] + 2*Log[1 + x^2]
```

Rubi [A] time = 0.0444789, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294

$$2\log\left(x^2+1\right)+x-\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(4*x^2 + x^3)/(x + x^3),x]

[Out] x - ArcTan[x] + 2*Log[1 + x^2]
```

Rubi in Sympy [A] time = 7.14112, size = 12, normalized size = 0.86

$$x+2\log\left(x^2+1\right)-\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**3+4*x**2)/(x**3+x),x)

[Out] x + 2*log(x**2 + 1) - atan(x)
```

Mathematica [A] time = 0.00588897, size = 14, normalized size = 1.

$$2\log\left(x^2+1\right)+x-\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(4*x^2 + x^3)/(x + x^3),x]

[Out] x - ArcTan[x] + 2*Log[1 + x^2]
```

Maple [A] time = 0.003, size = 15, normalized size = 1.1

$$x-\arctan(x)+2\ln\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^3+4*x^2)/(x^3+x),x)

[Out] x-arctan(x)+2*ln(x^2+1)
```

Maxima [A] time = 0.87066, size = 19, normalized size = 1.36

$$x - \arctan(x) + 2 \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + 4*x^2)/(x^3 + x),x, algorithm="maxima")

[Out] x - arctan(x) + 2*log(x^2 + 1)
```

Fricas [A] time = 0.265371, size = 19, normalized size = 1.36

$$x - \arctan(x) + 2 \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + 4*x^2)/(x^3 + x),x, algorithm="fricas")

[Out] x - arctan(x) + 2*log(x^2 + 1)
```

Sympy [A] time = 0.197064, size = 12, normalized size = 0.86

$$x + 2\log(x^2 + 1) - \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3+4*x**2)/(x**3+x),x)

[Out] x + 2*log(x**2 + 1) - atan(x)
```

GIAC/XCAS [A] time = 0.261053, size = 19, normalized size = 1.36

$$x - \arctan(x) + 2\ln(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + 4*x^2)/(x^3 + x),x, algorithm="giac")

[Out] x - arctan(x) + 2*ln(x^2 + 1)
```

3.443 $\int \frac{x+2x^3}{(x^2+x^4)^3}\,dx$

Optimal. Leaf size=13

$$-\frac{1}{4\left(x^4+x^2\right)^2}$$

```
[Out] -1/(4*(x^2 + x^4)^2)
```

Rubi [A] time = 0.0083106, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$

$$-\frac{1}{4\left(x^4+x^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x + 2*x^3)/(x^2 + x^4)^3,x]

[Out] -1/(4*(x^2 + x^4)^2)
```

Rubi in Sympy [A] time = 8.47684, size = 14, normalized size = 1.08

$$-\frac{1}{4x^4\left(x^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((2*x**3+x)/(x**4+x**2)**3,x)

[Out] -1/(4*x**4*(x**2 + 1)**2)
```

Mathematica [A] time = 0.0102078, size = 14, normalized size = 1.08

$$-\frac{1}{4x^4\left(x^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + 2*x^3)/(x^2 + x^4)^3,x]

[Out] -1/(4*x^4*(1 + x^2)^2)
```

Maple [B] time = 0.017, size = 30, normalized size = 2.3

$$-\frac{1}{4\,x^4}+\frac{1}{2\,x^2}-\frac{1}{4\,\left(x^2+1\right)^2}-\frac{1}{2\,x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+x)/(x^4+x^2)^3,x)
```

```
[Out] -1/4/x^4+1/2/x^2-1/4/(x^2+1)^2-1/2/(x^2+1)
```

Maxima [A] time = 0.788247, size = 15, normalized size = 1.15

$$-\frac{1}{4\,(x^4+x^2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^3 + x)/(x^4 + x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4/(x^4 + x^2)^2
```

Fricas [A] time = 0.244984, size = 22, normalized size = 1.69

$$-\frac{1}{4\,(x^8+2\,x^6+x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^3 + x)/(x^4 + x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/4/(x^8 + 2*x^6 + x^4)
```

Sympy [A] time = 0.367543, size = 17, normalized size = 1.31

$$-\frac{1}{4x^8+8x^6+4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x**3+x)/(x**4+x**2)**3,x)
```

```
[Out] -1/(4*x**8 + 8*x**6 + 4*x**4)
```

GIAC/XCAS [A] time = 0.259861, size = 15, normalized size = 1.15

$$-\frac{1}{4\,(x^4+x^2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^3 + x)/(x^4 + x^2)^3,x, algorithm="giac")
```

```
[Out] -1/4/(x^4 + x^2)^2
```

3.444 $\int \frac{ax^2+bx^3}{cx^2+dx^3}\,dx$

Optimal. Leaf size=26

$$\frac{bx}{d}-\frac{(bc-ad)\log(c+dx)}{d^2}$$

```
[Out] (b*x)/d - ((b*c - a*d)*Log[c + d*x])/d^2
```

Rubi [A] time = 0.0695624, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08

$$\frac{bx}{d}-\frac{(bc-ad)\log(c+dx)}{d^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a*x^2 + b*x^3)/(c*x^2 + d*x^3),x]

[Out] (b*x)/d - ((b*c - a*d)*Log[c + d*x])/d^2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int b\,dx}{d}+\frac{(ad-bc)\log(c+dx)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**3+a*x**2)/(d*x**3+c*x**2),x)

[Out] Integral(b, x)/d + (a*d - b*c)*log(c + d*x)/d**2
```

Mathematica [A] time = 0.0114378, size = 25, normalized size = 0.96

$$\frac{(ad-bc)\log(c+dx)}{d^2}+\frac{bx}{d}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a*x^2 + b*x^3)/(c*x^2 + d*x^3),x]

[Out] (b*x)/d + ((-(b*c) + a*d)*Log[c + d*x])/d^2
```

Maple [A] time = 0.004, size = 32, normalized size = 1.2

$$\frac{bx}{d}+\frac{\ln(dx+c)\,a}{d}-\frac{\ln(dx+c)\,bc}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^3+a*x^2)/(d*x^3+c*x^2),x)
```

```
[Out] b*x/d+1/d*ln(d*x+c)*a-1/d^2*ln(d*x+c)*b*c
```

Maxima [A] time = 0.789119, size = 35, normalized size = 1.35

$$\frac{bx}{d} - \frac{(bc - ad)\log(dx + c)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a*x^2)/(d*x^3 + c*x^2),x, algorithm="maxima")
```

```
[Out] b*x/d - (b*c - a*d)*log(d*x + c)/d^2
```

Fricas [A] time = 0.272052, size = 34, normalized size = 1.31

$$\frac{bdx - (bc - ad)\log(dx + c)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a*x^2)/(d*x^3 + c*x^2),x, algorithm="fricas")
```

```
[Out] (b*d*x - (b*c - a*d)*log(d*x + c))/d^2
```

Sympy [A] time = 1.24133, size = 20, normalized size = 0.77

$$\frac{bx}{d} + \frac{(ad - bc)\log(c + dx)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**3+a*x**2)/(d*x**3+c*x**2),x)
```

```
[Out] b*x/d + (a*d - b*c)*log(c + d*x)/d**2
```

GIAC/XCAS [A] time = 0.26258, size = 36, normalized size = 1.38

$$\frac{bx}{d} - \frac{(bc - ad)\ln(|dx + c|)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a*x^2)/(d*x^3 + c*x^2),x, algorithm="giac")
```

```
[Out] b*x/d - (b*c - a*d)*ln(abs(d*x + c))/d^2
```

3.445 $\int \frac{x+x^2}{-2x-x^2+x^3} \, dx$

Optimal. Leaf size=6

$$\log(2-x)$$

```
[Out] Log[2 - x]
```

Rubi [A] time = 0.0249443, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\log(2-x)$$

Antiderivative was successfully verified.

```
[In]  Int[(x + x^2)/(-2*x - x^2 + x^3),x]
```

```
[Out] Log[2 - x]
```

Rubi in Sympy [A] time = 6.11693, size = 3, normalized size = 0.5

$$\log(-x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+x)/(x**3-x**2-2*x),x)
```

```
[Out] log(-x + 2)
```

Mathematica [A] time = 0.00123161, size = 4, normalized size = 0.67

$$\log(x-2)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x + x^2)/(-2*x - x^2 + x^3),x]
```

```
[Out] Log[-2 + x]
```

Maple [A] time = 0.001, size = 5, normalized size = 0.8

$$\ln(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+x)/(x^3-x^2-2*x),x)
```

```
[Out] ln(x-2)
```

Maxima [A] time = 0.781833, size = 5, normalized size = 0.83

$$\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + x)/(x^3 - x^2 - 2*x),x, algorithm="maxima")
```

```
[Out] log(x - 2)
```

Fricas [A] time = 0.252075, size = 5, normalized size = 0.83

$$\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + x)/(x^3 - x^2 - 2*x),x, algorithm="fricas")
```

```
[Out] log(x - 2)
```

Sympy [A] time = 0.084186, size = 3, normalized size = 0.5

$$\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+x)/(x**3-x**2-2*x),x)
```

```
[Out] log(x - 2)
```

GIAC/XCAS [A] time = 0.260901, size = 7, normalized size = 1.17

$$\ln(|x-2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + x)/(x^3 - x^2 - 2*x),x, algorithm="giac")
```

```
[Out] ln(abs(x - 2))
```

3.446 $\int \frac{1-5x^2}{x^3(1+x^2)}\,dx$

Optimal. Leaf size=20

$$-\frac{1}{2x^2}+3\log\left(x^2+1\right)-6\log(x)$$

```
[Out] -1/(2*x^2) - 6*Log[x] + 3*Log[1 + x^2]
```

Rubi [A] time = 0.0516574, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.111$

$$-\frac{1}{2x^2}+3\log\left(x^2+1\right)-6\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - 5*x^2)/(x^3*(1 + x^2)),x]

[Out] -1/(2*x^2) - 6*Log[x] + 3*Log[1 + x^2]
```

Rubi in Sympy [A] time = 8.0897, size = 20, normalized size = 1.

$$-3\log\left(x^2\right)+3\log\left(x^2+1\right)-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-5*x**2+1)/x**3/(x**2+1),x)

[Out] -3*log(x**2) + 3*log(x**2 + 1) - 1/(2*x**2)
```

Mathematica [A] time = 0.007486, size = 20, normalized size = 1.

$$-\frac{1}{2x^2}+3\log\left(x^2+1\right)-6\log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - 5*x^2)/(x^3*(1 + x^2)),x]

[Out] -1/(2*x^2) - 6*Log[x] + 3*Log[1 + x^2]
```

Maple [A] time = 0.009, size = 19, normalized size = 1.

$$-\frac{1}{2\,x^2}-6\,\ln(x)+3\,\ln\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-5*x^2+1)/x^3/(x^2+1),x)
```

```
[Out] -1/2/x^2-6*ln(x)+3*ln(x^2+1)
```

Maxima [A] time = 0.781725, size = 27, normalized size = 1.35

$$-\frac{1}{2\,x^2} + 3\,\log\left(x^2+1\right) - 3\,\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(5*x^2 - 1)/((x^2 + 1)*x^3),x, algorithm="maxima")
```

```
[Out] -1/2/x^2 + 3*log(x^2 + 1) - 3*log(x^2)
```

Fricas [A] time = 0.254087, size = 34, normalized size = 1.7

$$\frac{6\,x^2\log\left(x^2+1\right) - 12\,x^2\log\left(x\right) - 1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(5*x^2 - 1)/((x^2 + 1)*x^3),x, algorithm="fricas")
```

```
[Out] 1/2*(6*x^2*log(x^2 + 1) - 12*x^2*log(x) - 1)/x^2
```

Sympy [A] time = 0.229946, size = 19, normalized size = 0.95

$$-6\log\left(x\right) + 3\log\left(x^2+1\right) - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-5*x**2+1)/x**3/(x**2+1),x)
```

```
[Out] -6*log(x) + 3*log(x**2 + 1) - 1/(2*x**2)
```

GIAC/XCAS [A] time = 0.259261, size = 36, normalized size = 1.8

$$\frac{6\,x^2-1}{2\,x^2} + 3\ln\left(x^2+1\right) - 3\ln\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(5*x^2 - 1)/((x^2 + 1)*x^3),x, algorithm="giac")
```

```
[Out] 1/2*(6*x^2 - 1)/x^2 + 3*ln(x^2 + 1) - 3*ln(x^2)
```

3.447 $\int \frac{2x}{(-1+x)(5+x^2)}\,dx$

Optimal. Leaf size=38

$$-\frac{1}{6}\log\left(x^2+5\right)+\frac{1}{3}\log(1-x)+\frac{1}{3}\sqrt{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)$$

```
[Out] (Sqrt[5]*ArcTan[x/Sqrt[5]])/3 + Log[1 - x]/3 - Log[5 + x^2]/6
```

Rubi [A] time = 0.0696901, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$-\frac{1}{6}\log\left(x^2+5\right)+\frac{1}{3}\log(1-x)+\frac{1}{3}\sqrt{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(2*x)/((-1 + x)*(5 + x^2)),x]

[Out] (Sqrt[5]*ArcTan[x/Sqrt[5]])/3 + Log[1 - x]/3 - Log[5 + x^2]/6
```

Rubi in Sympy [A] time = 6.35133, size = 31, normalized size = 0.82

$$\frac{\log(-x+1)}{3}-\frac{\log\left(x^2+5\right)}{6}+\frac{\sqrt{5}\operatorname{atan}\left(\frac{\sqrt{5}x}{5}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(2*x/(-1+x)/(x**2+5),x)

[Out] log(-x + 1)/3 - log(x**2 + 5)/6 + sqrt(5)*atan(sqrt(5)*x/5)/3
```

Mathematica [A] time = 0.01681, size = 40, normalized size = 1.05

$$2\left(-\frac{1}{12}\log\left(x^2+5\right)+\frac{1}{6}\log(1-x)+\frac{1}{6}\sqrt{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2*x)/((-1 + x)*(5 + x^2)),x]

[Out] 2*((Sqrt[5]*ArcTan[x/Sqrt[5]])/6 + Log[1 - x]/6 - Log[5 + x^2]/12
)
```

Maple [A] time = 0.008, size = 28, normalized size = 0.7

$$\frac{\ln(-1+x)}{3}-\frac{\ln\left(x^2+5\right)}{6}+\frac{\sqrt{5}}{3}\arctan\left(\frac{x\sqrt{5}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(2*x/(-1+x)/(x^2+5),x)
```

```
[Out] 1/3*ln(-1+x)-1/6*ln(x^2+5)+1/3*arctan(1/5*x*5^(1/2))*5^(1/2)
```

Maxima [A] time = 0.880584, size = 36, normalized size = 0.95

$$\frac{1}{3}\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right) - \frac{1}{6}\log\left(x^2+5\right) + \frac{1}{3}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(2*x/((x^2 + 5)*(x - 1)),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(5)*arctan(1/5*sqrt(5)*x) - 1/6*log(x^2 + 5) + 1/3*log(x
- 1)
```

Fricas [A] time = 0.252537, size = 36, normalized size = 0.95

$$\frac{1}{3}\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right) - \frac{1}{6}\log\left(x^2+5\right) + \frac{1}{3}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(2*x/((x^2 + 5)*(x - 1)),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(5)*arctan(1/5*sqrt(5)*x) - 1/6*log(x^2 + 5) + 1/3*log(x
- 1)
```

Sympy [A] time = 0.307288, size = 31, normalized size = 0.82

$$\frac{\log\left(x-1\right)}{3} - \frac{\log\left(x^2+5\right)}{6} + \frac{\sqrt{5}\,\mathrm{atan}\left(\frac{\sqrt{5}x}{5}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(2*x/(-1+x)/(x**2+5),x)
```

```
[Out] log(x - 1)/3 - log(x**2 + 5)/6 + sqrt(5)*atan(sqrt(5)*x/5)/3
```

GIAC/XCAS [A] time = 0.265648, size = 38, normalized size = 1.

$$\frac{1}{3}\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right) - \frac{1}{6}\ln\left(x^2+5\right) + \frac{1}{3}\ln\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(2*x/((x^2 + 5)*(x - 1)),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(5)*arctan(1/5*sqrt(5)*x) - 1/6*ln(x^2 + 5) + 1/3*ln(abs(
x - 1))
```

3.448 $\int \frac{2+x^2}{2+x}\,dx$

Optimal. Leaf size=17

$$\frac{x^2}{2} - 2x + 6\log(x+2)$$

```
[Out] -2*x + x^2/2 + 6*Log[2 + x]
```

Rubi [A] time = 0.0244233, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{x^2}{2} - 2x + 6\log(x+2)$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + x^2)/(2 + x),x]
```

```
[Out] -2*x + x^2/2 + 6*Log[2 + x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-2x + 6\log(x+2) + \int x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+2)/(2+x),x)
```

```
[Out] -2*x + 6*log(x + 2) + Integral(x, x)
```

Mathematica [A] time = 0.00535812, size = 18, normalized size = 1.06

$$\frac{x^2}{2} - 2x + 6\log(x+2) - 6$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + x^2)/(2 + x),x]
```

```
[Out] -6 - 2*x + x^2/2 + 6*Log[2 + x]
```

Maple [A] time = 0.003, size = 16, normalized size = 0.9

$$-2\,x + \frac{x^2}{2} + 6\,\ln(2+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+2)/(2+x),x)
```

```
[Out] -2*x+1/2*x^2+6*ln(2+x)
```

Maxima [A] time = 0.7841, size = 20, normalized size = 1.18

$$\frac{1}{2}x^2 - 2x + 6 \log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 2)/(x + 2),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 2*x + 6*log(x + 2)
```

Fricas [A] time = 0.251039, size = 20, normalized size = 1.18

$$\frac{1}{2}x^2 - 2x + 6 \log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 2)/(x + 2),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 - 2*x + 6*log(x + 2)
```

Sympy [A] time = 0.118431, size = 14, normalized size = 0.82

$$\frac{x^2}{2} - 2x + 6\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+2)/(2+x),x)
```

```
[Out] x**2/2 - 2*x + 6*log(x + 2)
```

GIAC/XCAS [A] time = 0.258381, size = 22, normalized size = 1.29

$$\frac{1}{2}x^2 - 2x + 6\ln(|x+2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 2)/(x + 2),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 2*x + 6*ln(abs(x + 2))
```

3.449 $\int \frac{1}{(-3+x)(4+x^2)}\, dx$

Optimal. Leaf size=31

$$-\frac{1}{26}\log\left(x^2+4\right)+\frac{1}{13}\log(3-x)-\frac{3}{26}\tan^{-1}\left(\frac{x}{2}\right)$$

```
[Out] (-3*ArcTan[x/2])/26 + Log[3 - x]/13 - Log[4 + x^2]/26
```

Rubi [A] time = 0.0359802, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$

$$-\frac{1}{26}\log\left(x^2+4\right)+\frac{1}{13}\log(3-x)-\frac{3}{26}\tan^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/((-3 + x)*(4 + x^2)),x]
```

```
[Out] (-3*ArcTan[x/2])/26 + Log[3 - x]/13 - Log[4 + x^2]/26
```

Rubi in Sympy [A] time = 5.00589, size = 22, normalized size = 0.71

$$\frac{\log(-x+3)}{13}-\frac{\log\left(x^2+4\right)}{26}-\frac{3\,\text{atan}\left(\frac{x}{2}\right)}{26}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-3+x)/(x**2+4),x)
```

```
[Out] log(-x + 3)/13 - log(x**2 + 4)/26 - 3*atan(x/2)/26
```

Mathematica [A] time = 0.00788278, size = 36, normalized size = 1.16

$$-\frac{1}{26}\log\left((x-3)^2+6(x-3)+13\right)+\frac{1}{13}\log(x-3)-\frac{3}{26}\tan^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((-3 + x)*(4 + x^2)),x]
```

```
[Out] (-3*ArcTan[x/2])/26 - Log[13 + 6*(-3 + x) + (-3 + x)^2]/26 + Log[
-3 + x]/13
```

Maple [A] time = 0.007, size = 22, normalized size = 0.7

$$-\frac{\ln\left(x^2+4\right)}{26}-\frac{3}{26}\arctan\left(\frac{x}{2}\right)+\frac{\ln(-3+x)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-3+x)/(x^2+4),x)
```

```
[Out] -1/26*ln(x^2+4)-3/26*arctan(1/2*x)+1/13*ln(-3+x)
```

Maxima [A] time = 0.871102, size = 28, normalized size = 0.9

$$-\frac{3}{26}\arctan\left(\frac{1}{2}x\right)-\frac{1}{26}\log\left(x^2+4\right)+\frac{1}{13}\log\left(x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((x^2 + 4)*(x - 3)),x, algorithm="maxima")
```

```
[Out] -3/26*arctan(1/2*x) - 1/26*log(x^2 + 4) + 1/13*log(x - 3)
```

Fricas [A] time = 0.253326, size = 28, normalized size = 0.9

$$-\frac{3}{26}\arctan\left(\frac{1}{2}x\right)-\frac{1}{26}\log\left(x^2+4\right)+\frac{1}{13}\log\left(x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((x^2 + 4)*(x - 3)),x, algorithm="fricas")
```

```
[Out] -3/26*arctan(1/2*x) - 1/26*log(x^2 + 4) + 1/13*log(x - 3)
```

Sympy [A] time = 0.318353, size = 22, normalized size = 0.71

$$\frac{\log\left(x-3\right)}{13}-\frac{\log\left(x^2+4\right)}{26}-\frac{3\operatorname{atan}\left(\frac{x}{2}\right)}{26}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3+x)/(x**2+4),x)
```

```
[Out] log(x - 3)/13 - log(x**2 + 4)/26 - 3*atan(x/2)/26
```

GIAC/XCAS [A] time = 0.260597, size = 30, normalized size = 0.97

$$-\frac{3}{26}\arctan\left(\frac{1}{2}x\right)-\frac{1}{26}\ln\left(x^2+4\right)+\frac{1}{13}\ln\left(|x-3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((x^2 + 4)*(x - 3)),x, algorithm="giac")
```

```
[Out] -3/26*arctan(1/2*x) - 1/26*ln(x^2 + 4) + 1/13*ln(abs(x - 3))
```

3.450 $\int \frac{-2+3x^6}{x(5+2x^6)}\,dx$

Optimal. Leaf size=19

$$\frac{19}{60}\log\left(2x^6+5\right)-\frac{2\log(x)}{5}$$

```
[Out] (-2*Log[x])/5 + (19*Log[5 + 2*x^6])/60
```

Rubi [A] time = 0.0529364, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{19}{60}\log\left(2x^6+5\right)-\frac{2\log(x)}{5}$$

Antiderivative was successfully verified.

```
[In]  Int[(-2 + 3*x^6)/(x*(5 + 2*x^6)),x]

[Out] (-2*Log[x])/5 + (19*Log[5 + 2*x^6])/60
```

Rubi in Sympy [A] time = 8.06544, size = 17, normalized size = 0.89

$$-\frac{\log\left(x^6\right)}{15}+\frac{19\log\left(2x^6+5\right)}{60}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**6-2)/x/(2*x**6+5),x)

[Out] -log(x**6)/15 + 19*log(2*x**6 + 5)/60
```

Mathematica [A] time = 0.0083234, size = 19, normalized size = 1.

$$\frac{19}{60}\log\left(2x^6+5\right)-\frac{2\log(x)}{5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-2 + 3*x^6)/(x*(5 + 2*x^6)),x]

[Out] (-2*Log[x])/5 + (19*Log[5 + 2*x^6])/60
```

Maple [A] time = 0.008, size = 16, normalized size = 0.8

$$-\frac{2\ln(x)}{5}+\frac{19\ln\left(2x^6+5\right)}{60}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^6-2)/x/(2*x^6+5),x)
```

```
[Out] -2/5*ln(x)+19/60*ln(2*x^6+5)
```

Maxima [A] time = 0.78885, size = 23, normalized size = 1.21

$$\frac{19}{60}\log\left(2x^6+5\right)-\frac{1}{15}\log\left(x^6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^6 - 2)/((2*x^6 + 5)*x),x, algorithm="maxima")
```

```
[Out] 19/60*log(2*x^6 + 5) - 1/15*log(x^6)
```

Fricas [A] time = 0.254399, size = 20, normalized size = 1.05

$$\frac{19}{60}\log\left(2x^6+5\right)-\frac{2}{5}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^6 - 2)/((2*x^6 + 5)*x),x, algorithm="fricas")
```

```
[Out] 19/60*log(2*x^6 + 5) - 2/5*log(x)
```

Sympy [A] time = 0.257758, size = 17, normalized size = 0.89

$$-\frac{2\log\left(x\right)}{5}+\frac{19\log\left(2x^6+5\right)}{60}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**6-2)/x/(2*x**6+5),x)
```

```
[Out] -2*log(x)/5 + 19*log(2*x**6 + 5)/60
```

GIAC/XCAS [A] time = 0.266006, size = 23, normalized size = 1.21

$$\frac{19}{60}\ln\left(2x^6+5\right)-\frac{1}{15}\ln\left(x^6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^6 - 2)/((2*x^6 + 5)*x),x, algorithm="giac")
```

```
[Out] 19/60*ln(2*x^6 + 5) - 1/15*ln(x^6)
```

3.451 $\int \frac{3+2x}{(-2+x)(5+x)} \, dx$

Optimal. Leaf size=11

$$\log(2-x)+\log(x+5)$$

```
[Out] Log[2 - x] + Log[5 + x]
```

Rubi [A] time = 0.0224881, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$

$$\log(2-x)+\log(x+5)$$

Antiderivative was successfully verified.

```
[In]  Int[(3 + 2*x)/((-2 + x)*(5 + x)),x]

[Out] Log[2 - x] + Log[5 + x]
```

Rubi in Sympy [A] time = 3.47357, size = 8, normalized size = 0.73

$$\log(-x+2)+\log(x+5)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3+2*x)/(-2+x)/(5+x),x)

[Out] log(-x + 2) + log(x + 5)
```

Mathematica [A] time = 0.00542211, size = 9, normalized size = 0.82

$$\log(x-2)+\log(x+5)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3 + 2*x)/((-2 + x)*(5 + x)),x]

[Out] Log[-2 + x] + Log[5 + x]
```

Maple [A] time = 0.002, size = 9, normalized size = 0.8

$$\ln((x-2)(5+x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3+2*x)/(x-2)/(5+x),x)

[Out] ln((x-2)*(5+x))
```

Maxima [A] time = 0.80073, size = 12, normalized size = 1.09

$$\log(x+5) + \log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x + 3)/((x + 5)*(x - 2)),x, algorithm="maxima")
```

```
[Out] log(x + 5) + log(x - 2)
```

Fricas [A] time = 0.249137, size = 12, normalized size = 1.09

$$\log\left(x^2 + 3x - 10\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x + 3)/((x + 5)*(x - 2)),x, algorithm="fricas")
```

```
[Out] log(x^2 + 3*x - 10)
```

Sympy [A] time = 0.164508, size = 8, normalized size = 0.73

$$\log\left(x^2 + 3x - 10\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3+2*x)/(-2+x)/(5+x),x)
```

```
[Out] log(x**2 + 3*x - 10)
```

GIAC/XCAS [A] time = 0.260732, size = 15, normalized size = 1.36

$$\ln(|x+5|) + \ln(|x-2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x + 3)/((x + 5)*(x - 2)),x, algorithm="giac")
```

```
[Out] ln(abs(x + 5)) + ln(abs(x - 2))
```

3.452 $\int \frac{x^4}{4+5x^2+x^4}\,dx$

Optimal. Leaf size=18

$$x-\frac{8}{3}\tan^{-1}\left(\frac{x}{2}\right)+\frac{1}{3}\tan^{-1}(x)$$

```
[Out] x - (8*ArcTan[x/2])/3 + ArcTan[x]/3
```

Rubi [A] time = 0.0404273, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188

$$x-\frac{8}{3}\tan^{-1}\left(\frac{x}{2}\right)+\frac{1}{3}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/(4 + 5*x^2 + x^4),x]

[Out] x - (8*ArcTan[x/2])/3 + ArcTan[x]/3
```

Rubi in Sympy [A] time = 15.1725, size = 14, normalized size = 0.78

$$x-\frac{8\,\mathrm{atan}\left(\frac{x}{2}\right)}{3}+\frac{\mathrm{atan}(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(x**4+5*x**2+4),x)

[Out] x - 8*atan(x/2)/3 + atan(x)/3
```

Mathematica [A] time = 0.0114762, size = 18, normalized size = 1.

$$x+\frac{8}{3}\tan^{-1}\left(\frac{2}{x}\right)+\frac{1}{3}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/(4 + 5*x^2 + x^4),x]

[Out] x + (8*ArcTan[2/x])/3 + ArcTan[x]/3
```

Maple [A] time = 0.009, size = 13, normalized size = 0.7

$$x-\frac{8}{3}\arctan\left(\frac{x}{2}\right)+\frac{\arctan(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(x^4+5*x^2+4),x)
```

```
[Out] x-8/3*arctan(1/2*x)+1/3*arctan(x)
```

Maxima [A] time = 0.865284, size = 16, normalized size = 0.89

$$x - \frac{8}{3}\arctan\left(\frac{1}{2}x\right) + \frac{1}{3}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(x^4 + 5*x^2 + 4),x, algorithm="maxima")
```

```
[Out] x - 8/3*arctan(1/2*x) + 1/3*arctan(x)
```

Fricas [A] time = 0.258066, size = 16, normalized size = 0.89

$$x - \frac{8}{3}\arctan\left(\frac{1}{2}x\right) + \frac{1}{3}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(x^4 + 5*x^2 + 4),x, algorithm="fricas")
```

```
[Out] x - 8/3*arctan(1/2*x) + 1/3*arctan(x)
```

Sympy [A] time = 0.426887, size = 14, normalized size = 0.78

$$x - \frac{8\operatorname{atan}\left(\frac{x}{2}\right)}{3} + \frac{\operatorname{atan}(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(x**4+5*x**2+4),x)
```

```
[Out] x - 8*atan(x/2)/3 + atan(x)/3
```

GIAC/XCAS [A] time = 0.259284, size = 16, normalized size = 0.89

$$x - \frac{8}{3}\arctan\left(\frac{1}{2}x\right) + \frac{1}{3}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(x^4 + 5*x^2 + 4),x, algorithm="giac")
```

```
[Out] x - 8/3*arctan(1/2*x) + 1/3*arctan(x)
```

3.453 $\int \frac{1}{(1+x)(2+x)^2(3+x)^3} \, dx$

Optimal. Leaf size=46

$$\frac{1}{x+2}+\frac{5}{4(x+3)}+\frac{1}{4(x+3)^2}+\frac{1}{8}\log(x+1)+2\log(x+2)-\frac{17}{8}\log(x+3)$$

```
[Out] (2 + x)^(-1) + 1/(4*(3 + x)^2) + 5/(4*(3 + x)) + Log[1 + x]/8 + 2
*Log[2 + x] - (17*Log[3 + x])/8
```

Rubi [A] time = 0.0595168, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062

$$\frac{1}{x+2}+\frac{5}{4(x+3)}+\frac{1}{4(x+3)^2}+\frac{1}{8}\log(x+1)+2\log(x+2)-\frac{17}{8}\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x)*(2 + x)^2*(3 + x)^3),x]
```

```
[Out] (2 + x)^(-1) + 1/(4*(3 + x)^2) + 5/(4*(3 + x)) + Log[1 + x]/8 + 2
*Log[2 + x] - (17*Log[3 + x])/8
```

Rubi in Sympy [A] time = 7.23463, size = 41, normalized size = 0.89

$$\frac{\log(x+1)}{8}+2\log(x+2)-\frac{17\log(x+3)}{8}+\frac{5}{4(x+3)}+\frac{1}{4(x+3)^2}+\frac{1}{x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(1+x)/(2+x)**2/(3+x)**3,x)
```

```
[Out] log(x + 1)/8 + 2*log(x + 2) - 17*log(x + 3)/8 + 5/(4*(x + 3)) + 1
/(4*(x + 3)**2) + 1/(x + 2)
```

Mathematica [A] time = 0.0266584, size = 44, normalized size = 0.96

$$\frac{1}{8}\left(\frac{8}{x+2}+\frac{10}{x+3}+\frac{2}{(x+3)^2}+\log(-x-1)+16\log(x+2)-17\log(x+3)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x)*(2 + x)^2*(3 + x)^3),x]
```

```
[Out] (8/(2 + x) + 2/(3 + x)^2 + 10/(3 + x) + Log[-1 - x] + 16*Log[2 +
x] - 17*Log[3 + x])/8
```

Maple [A] time = 0.016, size = 39, normalized size = 0.9

$$(2+x)^{-1}+\frac{1}{4(3+x)^2}+\frac{5}{12+4x}+\frac{\ln(1+x)}{8}+2\ln(2+x)-\frac{17\ln(3+x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1+x)/(2+x)^2/(3+x)^3,x)
```

```
[Out] 1/(2+x)+1/4/(3+x)^2+5/4/(3+x)+1/8*ln(1+x)+2*ln(2+x)-17/8*ln(3+x)
```

Maxima [A] time = 0.780402, size = 62, normalized size = 1.35

$$\frac{9\,x^2+50\,x+68}{4\,(x^3+8\,x^2+21\,x+18)}-\frac{17}{8}\,\log\left(x+3\right)+2\,\log\left(x+2\right)+\frac{1}{8}\,\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x + 3)^3*(x + 2)^2*(x + 1)),x, algorithm="maxima")
```

```
[Out] 1/4*(9*x^2 + 50*x + 68)/(x^3 + 8*x^2 + 21*x + 18) - 17/8*log(x +
3) + 2*log(x + 2) + 1/8*log(x + 1)
```

Fricas [A] time = 0.275284, size = 112, normalized size = 2.43

$$\frac{18\,x^2-17\,\left(x^3+8\,x^2+21\,x+18\right)\log\left(x+3\right)+16\,\left(x^3+8\,x^2+21\,x+18\right)\log\left(x+2\right)+\left(x^3+8\,x^2+21\,x+18\right)\log\left(x+1\right)+1}{8\,(x^3+8\,x^2+21\,x+18)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x + 3)^3*(x + 2)^2*(x + 1)),x, algorithm="fricas")
```

```
[Out] 1/8*(18*x^2 - 17*(x^3 + 8*x^2 + 21*x + 18)*log(x + 3) + 16*(x^3 +
 8*x^2 + 21*x + 18)*log(x + 2) + (x^3 + 8*x^2 + 21*x + 18)*log(x
+ 1) + 100*x + 136)/(x^3 + 8*x^2 + 21*x + 18)
```

Sympy [A] time = 0.508482, size = 46, normalized size = 1.

$$\frac{9x^2+50x+68}{4x^3+32x^2+84x+72}+\frac{\log\left(x+1\right)}{8}+2\log\left(x+2\right)-\frac{17\log\left(x+3\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1+x)/(2+x)**2/(3+x)**3,x)
```

```
[Out] (9*x**2 + 50*x + 68)/(4*x**3 + 32*x**2 + 84*x + 72) + log(x + 1)/
8 + 2*log(x + 2) - 17*log(x + 3)/8
```

GIAC/XCAS [A] time = 0.260756, size = 70, normalized size = 1.52

$$\frac{1}{x+2}-\frac{\frac{7}{x+2}+6}{4\left(\frac{1}{x+2}+1\right)^2}+\frac{1}{8}\ln\left(\left|-\frac{1}{x+2}+1\right|\right)-\frac{17}{8}\ln\left(\left|-\frac{1}{x+2}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x + 3)^3*(x + 2)^2*(x + 1)),x, algorithm="giac")
```

```
[Out] 1/(x + 2) - 1/4*(7/(x + 2) + 6)/(1/(x + 2) + 1)^2 + 1/8*ln(abs(-1
/(x + 2) + 1)) - 17/8*ln(abs(-1/(x + 2) - 1))
```

3.454 $\int \frac{x}{-1+x^2} \, dx$

Optimal. Leaf size=12

$$\frac{1}{2} \log \left(1-x^2\right)$$

```
[Out] Log[1 - x^2]/2
```

Rubi [A] time = 0.00737657, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\frac{1}{2} \log \left(1-x^2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x/(-1 + x^2),x]

[Out] Log[1 - x^2]/2
```

Rubi in Sympy [A] time = 1.70047, size = 7, normalized size = 0.58

$$\frac{\log \left(-x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(x**2-1),x)

[Out] log(-x**2 + 1)/2
```

Mathematica [A] time = 0.00214421, size = 10, normalized size = 0.83

$$\frac{1}{2} \log \left(x^2-1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(-1 + x^2),x]

[Out] Log[-1 + x^2]/2
```

Maple [A] time = 0.003, size = 14, normalized size = 1.2

$$\frac{\ln(-1+x)}{2}+\frac{\ln(1+x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(x^2-1),x)
```

```
[Out] 1/2*ln(-1+x)+1/2*ln(1+x)
```

Maxima [A] time = 0.775098, size = 11, normalized size = 0.92

$$\frac{1}{2}\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^2 - 1),x, algorithm="maxima")
```

```
[Out] 1/2*log(x^2 - 1)
```

Fricas [A] time = 0.25895, size = 11, normalized size = 0.92

$$\frac{1}{2}\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^2 - 1),x, algorithm="fricas")
```

```
[Out] 1/2*log(x^2 - 1)
```

Sympy [A] time = 0.122413, size = 7, normalized size = 0.58

$$\frac{\log\left(x^2-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x**2-1),x)
```

```
[Out] log(x**2 - 1)/2
```

GIAC/XCAS [A] time = 0.259873, size = 12, normalized size = 1.

$$\frac{1}{2}\ln\left(\left|x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^2 - 1),x, algorithm="giac")
```

```
[Out] 1/2*ln(abs(x^2 - 1))
```

3.455 $\int \frac{1}{(-1+x^2)^2}\,dx$

Optimal. Leaf size=21

$$\frac{x}{2\,(1-x^2)}+\frac{1}{2}\tanh^{-1}(x)$$

```
[Out] x/(2*(1 - x^2)) + ArcTanh[x]/2
```

Rubi [A] time = 0.0106945, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286

$$\frac{x}{2\,(1-x^2)}+\frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + x^2)^(-2),x]
```

```
[Out] x/(2*(1 - x^2)) + ArcTanh[x]/2
```

Rubi in Sympy [A] time = 1.18394, size = 12, normalized size = 0.57

$$\frac{x}{2\,(-x^2+1)}+\frac{\operatorname{atanh}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**2-1)**2,x)
```

```
[Out] x/(2*(-x**2 + 1)) + atanh(x)/2
```

Mathematica [A] time = 0.0122346, size = 27, normalized size = 1.29

$$\frac{1}{4}\left(-\frac{2x}{x^2-1}-\log(1-x)+\log(x+1)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + x^2)^(-2),x]
```

```
[Out] ((-2*x)/(-1 + x^2) - Log[1 - x] + Log[1 + x])/4
```

Maple [A] time = 0.013, size = 28, normalized size = 1.3

$$-\frac{1}{-4+4\,x}-\frac{\ln(-1+x)}{4}-\frac{1}{4+4\,x}+\frac{\ln(1+x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^2-1)^2,x)
```

```
[Out] -1/4/(-1+x)-1/4*ln(-1+x)-1/4/(1+x)+1/4*ln(1+x)
```

Maxima [A] time = 0.804754, size = 31, normalized size = 1.48

$$-\frac{x}{2\left(x^2-1\right)}+\frac{1}{4}\log\left(x+1\right)-\frac{1}{4}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 1)^(-2),x, algorithm="maxima")
```

```
[Out] -1/2*x/(x^2 - 1) + 1/4*log(x + 1) - 1/4*log(x - 1)
```

Fricas [A] time = 0.265184, size = 46, normalized size = 2.19

$$\frac{\left(x^2-1\right)\log\left(x+1\right)-\left(x^2-1\right)\log\left(x-1\right)-2\,x}{4\left(x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 1)^(-2),x, algorithm="fricas")
```

```
[Out] 1/4*((x^2 - 1)*log(x + 1) - (x^2 - 1)*log(x - 1) - 2*x)/(x^2 - 1)
```

Sympy [A] time = 0.218957, size = 20, normalized size = 0.95

$$-\frac{x}{2x^2-2}-\frac{\log\left(x-1\right)}{4}+\frac{\log\left(x+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**2-1)**2,x)
```

```
[Out] -x/(2*x**2 - 2) - log(x - 1)/4 + log(x + 1)/4
```

GIAC/XCAS [A] time = 0.259816, size = 34, normalized size = 1.62

$$-\frac{x}{2\left(x^2-1\right)}+\frac{1}{4}\ln\left(|x+1|\right)-\frac{1}{4}\ln\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 1)^(-2),x, algorithm="giac")
```

```
[Out] -1/2*x/(x^2 - 1) + 1/4*ln(abs(x + 1)) - 1/4*ln(abs(x - 1))
```

3.456 $\int \frac{x^2}{(1+x^2)^2} dx$

Optimal. Leaf size=19

$$\frac{1}{2}\tan^{-1}(x) - \frac{x}{2(x^2+1)}$$

```
[Out] -x/(2*(1 + x^2)) + ArcTan[x]/2
```

Rubi [A] time = 0.0160692, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{1}{2}\tan^{-1}(x) - \frac{x}{2(x^2+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(1 + x^2)^2,x]

[Out] -x/(2*(1 + x^2)) + ArcTan[x]/2
```

Rubi in Sympy [A] time = 3.28705, size = 12, normalized size = 0.63

$$-\frac{x}{2(x^2+1)} + \frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(x**2+1)**2,x)

[Out] -x/(2*(x**2 + 1)) + atan(x)/2
```

Mathematica [A] time = 0.0121731, size = 19, normalized size = 1.

$$\frac{1}{2}\tan^{-1}(x) - \frac{x}{2(x^2+1)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(1 + x^2)^2,x]

[Out] -x/(2*(1 + x^2)) + ArcTan[x]/2
```

Maple [A] time = 0.009, size = 16, normalized size = 0.8

$$-\frac{x}{2x^2+2} + \frac{\arctan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(x^2+1)^2,x)
```

```
[Out] -1/2*x/(x^2+1)+1/2*arctan(x)
```

Maxima [A] time = 0.869146, size = 20, normalized size = 1.05

$$-\frac{x}{2\,(x^2+1)}+\frac{1}{2}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2 + 1)^2,x, algorithm="maxima")
```

```
[Out] -1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

Fricas [A] time = 0.260039, size = 28, normalized size = 1.47

$$\frac{\left(x^2+1\right)\arctan\left(x\right)-x}{2\,(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2 + 1)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((x^2 + 1)*arctan(x) - x)/(x^2 + 1)
```

Sympy [A] time = 0.203589, size = 12, normalized size = 0.63

$$-\frac{x}{2x^2+2}+\frac{\operatorname{atan}\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**2+1)**2,x)
```

```
[Out] -x/(2*x**2 + 2) + atan(x)/2
```

GIAC/XCAS [A] time = 0.259511, size = 20, normalized size = 1.05

$$-\frac{x}{2\,(x^2+1)}+\frac{1}{2}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2 + 1)^2,x, algorithm="giac")
```

```
[Out] -1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

3.457 $\int \frac{1}{2+3x} \, dx$

Optimal. Leaf size=10

$$\frac{1}{3} \log(3x+2)$$

```
[Out] Log[2 + 3*x]/3
```

Rubi [A] time = 0.0064259, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{1}{3} \log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x)^(-1),x]
```

```
[Out] Log[2 + 3*x]/3
```

Rubi in Sympy [A] time = 1.07953, size = 7, normalized size = 0.7

$$\frac{\log(3x+2)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(2+3*x),x)
```

```
[Out] log(3*x + 2)/3
```

Mathematica [A] time = 0.00116986, size = 10, normalized size = 1.

$$\frac{1}{3} \log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x)^(-1),x]
```

```
[Out] Log[2 + 3*x]/3
```

Maple [A] time = 0.001, size = 9, normalized size = 0.9

$$\frac{\ln(2+3x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2+3*x),x)
```

```
[Out] 1/3*ln(2+3*x)
```

Maxima [A] time = 0.782142, size = 11, normalized size = 1.1

$$\frac{1}{3}\log(3x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3*x + 2),x, algorithm="maxima")
```

```
[Out] 1/3*log(3*x + 2)
```

Fricas [A] time = 0.267878, size = 11, normalized size = 1.1

$$\frac{1}{3}\log(3x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3*x + 2),x, algorithm="fricas")
```

```
[Out] 1/3*log(3*x + 2)
```

Sympy [A] time = 0.061809, size = 7, normalized size = 0.7

$$\frac{\log(3x+2)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(2+3*x),x)
```

```
[Out] log(3*x + 2)/3
```

GIAC/XCAS [A] time = 0.262542, size = 12, normalized size = 1.2

$$\frac{1}{3}\ln(|3x+2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3*x + 2),x, algorithm="giac")
```

```
[Out] 1/3*ln(abs(3*x + 2))
```

3.458 $\int \frac{1}{a^2+x^2}\,dx$

Optimal. Leaf size=10

$$\frac{\tan^{-1}\left(\frac{x}{a}\right)}{a}$$

```
[Out] ArcTan[x/a]/a
```

Rubi [A] time = 0.0089864, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\frac{\tan^{-1}\left(\frac{x}{a}\right)}{a}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + x^2)^(-1),x]
```

```
[Out] ArcTan[x/a]/a
```

Rubi in Sympy [A] time = 1.5478, size = 5, normalized size = 0.5

$$\frac{\operatorname{atan}\left(\frac{x}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a**2+x**2),x)
```

```
[Out] atan(x/a)/a
```

Mathematica [A] time = 0.00324175, size = 10, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{x}{a}\right)}{a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + x^2)^(-1),x]
```

```
[Out] ArcTan[x/a]/a
```

Maple [A] time = 0.005, size = 11, normalized size = 1.1

$$\frac{1}{a}\arctan\left(\frac{x}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a^2+x^2),x)
```

```
[Out] arctan(x/a)/a
```

Maxima [A] time = 0.871347, size = 14, normalized size = 1.4

$$\frac{\arctan\left(\frac{x}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a^2 + x^2),x, algorithm="maxima")
```

```
[Out] arctan(x/a)/a
```

Fricas [A] time = 0.265683, size = 14, normalized size = 1.4

$$\frac{\arctan\left(\frac{x}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a^2 + x^2),x, algorithm="fricas")
```

```
[Out] arctan(x/a)/a
```

Sympy [A] time = 0.244491, size = 20, normalized size = 2.

$$\frac{-\frac{i\log(-ia+x)}{2}+\frac{i\log(ia+x)}{2}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a**2+x**2),x)
```

```
[Out] (-I*log(-I*a + x)/2 + I*log(I*a + x)/2)/a
```

GIAC/XCAS [A] time = 0.260493, size = 14, normalized size = 1.4

$$\frac{\arctan\left(\frac{x}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a^2 + x^2),x, algorithm="giac")
```

```
[Out] arctan(x/a)/a
```

3.459 $\int \frac{1}{a+bx^2}\,dx$

Optimal. Leaf size=24

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

```
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

Rubi [A] time = 0.0192345, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

Rubi in Sympy [A] time = 2.47579, size = 22, normalized size = 0.92

$$\frac{\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a),x)
```

```
[Out] atan(sqrt(b)*x/sqrt(a))/(sqrt(a)*sqrt(b))
```

Mathematica [A] time = 0.00730585, size = 24, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

Maple [A] time = 0.001, size = 16, normalized size = 0.7

$$1 \arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a),x)
```

```
[Out] 1/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.258547, size = 1, normalized size = 0.04

$$\left[\frac{\log\left(\frac{2\,abx+\left(bx^2-a\right)\sqrt{-ab}}{bx^2+a}\right)}{2\,\sqrt{-ab}},\ \frac{\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{\sqrt{ab}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/2*log((2*a*b*x + (b*x^2 - a)*sqrt(-a*b))/(b*x^2 + a))/sqrt(-a*
b), arctan(sqrt(a*b)*x/a)/sqrt(a*b)]
```

Sympy [A] time = 0.299633, size = 53, normalized size = 2.21

$$-\frac{\sqrt{-\frac{1}{ab}}\log\left(-a\sqrt{-\frac{1}{ab}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{ab}}\log\left(a\sqrt{-\frac{1}{ab}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a),x)
```

```
[Out] -sqrt(-1/(a*b))*log(-a*sqrt(-1/(a*b)) + x)/2 + sqrt(-1/(a*b))*log
(a*sqrt(-1/(a*b)) + x)/2
```

GIAC/XCAS [A] time = 0.258685, size = 20, normalized size = 0.83

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] arctan(b*x/sqrt(a*b))/sqrt(a*b)
```

3.460 $\int \frac{1}{2-x+x^2}\,dx$

Optimal. Leaf size=19

$$-\frac{2\tan^{-1}\left(\frac{1-2x}{\sqrt{7}}\right)}{\sqrt{7}}$$

```
[Out] (-2*ArcTan[(1 - 2*x)/Sqrt[7]])/Sqrt[7]
```

Rubi [A] time = 0.0315247, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$-\frac{2\tan^{-1}\left(\frac{1-2x}{\sqrt{7}}\right)}{\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In]  Int[(2 - x + x^2)^(-1),x]

[Out] (-2*ArcTan[(1 - 2*x)/Sqrt[7]])/Sqrt[7]
```

Rubi in Sympy [A] time = 1.40504, size = 22, normalized size = 1.16

$$\frac{2\sqrt{7}\,\operatorname{atan}\left(\sqrt{7}\left(\frac{2x}{7}-\frac{1}{7}\right)\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**2-x+2),x)

[Out] 2*sqrt(7)*atan(sqrt(7)*(2*x/7 - 1/7))/7
```

Mathematica [A] time = 0.00943982, size = 19, normalized size = 1.

$$\frac{2\tan^{-1}\left(\frac{2x-1}{\sqrt{7}}\right)}{\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 - x + x^2)^(-1),x]

[Out] (2*ArcTan[(-1 + 2*x)/Sqrt[7]])/Sqrt[7]
```

Maple [A] time = 0.004, size = 17, normalized size = 0.9

$$\frac{2\sqrt{7}}{7}\arctan\left(\frac{(2x-1)\sqrt{7}}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^2-x+2),x)

[Out] 2/7*7^(1/2)*arctan(1/7*(2*x-1)*7^(1/2))
```

Maxima [A] time = 0.871936, size = 22, normalized size = 1.16

$$\frac{2}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^2 - x + 2),x, algorithm="maxima")

[Out] 2/7*sqrt(7)*arctan(1/7*sqrt(7)*(2*x - 1))
```

Fricas [A] time = 0.270579, size = 22, normalized size = 1.16

$$\frac{2}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^2 - x + 2),x, algorithm="fricas")

[Out] 2/7*sqrt(7)*arctan(1/7*sqrt(7)*(2*x - 1))
```

Sympy [A] time = 0.212476, size = 26, normalized size = 1.37

$$\frac{2\sqrt{7}\operatorname{atan}\left(\frac{2\sqrt{7}x}{7}-\frac{\sqrt{7}}{7}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**2-x+2),x)

[Out] 2*sqrt(7)*atan(2*sqrt(7)*x/7 - sqrt(7)/7)/7
```

GIAC/XCAS [A] time = 0.25931, size = 22, normalized size = 1.16

$$\frac{2}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^2 - x + 2),x, algorithm="giac")

[Out] 2/7*sqrt(7)*arctan(1/7*sqrt(7)*(2*x - 1))
```

3.461 $\int x^2 (4-x^2)^2 \, dx$

Optimal. Leaf size=22

$$\frac{x^7}{7}-\frac{8x^5}{5}+\frac{16x^3}{3}$$

```
[Out] (16*x^3)/3 - (8*x^5)/5 + x^7/7
```

Rubi [A] time = 0.0237059, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{x^7}{7}-\frac{8x^5}{5}+\frac{16x^3}{3}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(4 - x^2)^2,x]

[Out] (16*x^3)/3 - (8*x^5)/5 + x^7/7
```

Rubi in Sympy [A] time = 4.29155, size = 17, normalized size = 0.77

$$\frac{x^7}{7}-\frac{8x^5}{5}+\frac{16x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(-x**2+4)**2,x)

[Out] x**7/7 - 8*x**5/5 + 16*x**3/3
```

Mathematica [A] time = 0.00120954, size = 22, normalized size = 1.

$$\frac{x^7}{7}-\frac{8x^5}{5}+\frac{16x^3}{3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(4 - x^2)^2,x]

[Out] (16*x^3)/3 - (8*x^5)/5 + x^7/7
```

Maple [A] time = 0.001, size = 17, normalized size = 0.8

$$\frac{16\,x^3}{3}-\frac{8\,x^5}{5}+\frac{x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(-x^2+4)^2,x)
```

```
[Out] 16/3*x^3-8/5*x^5+1/7*x^7
```

Maxima [A] time = 0.817242, size = 22, normalized size = 1.

$$\frac{1}{7}\,x^{7} - \frac{8}{5}\,x^{5} + \frac{16}{3}\,x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 4)^2*x^2,x, algorithm="maxima")

[Out] 1/7*x^7 - 8/5*x^5 + 16/3*x^3
```

Fricas [A] time = 0.236009, size = 1, normalized size = 0.05

$$\frac{1}{7}x^{7} - \frac{8}{5}x^{5} + \frac{16}{3}x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 4)^2*x^2,x, algorithm="fricas")

[Out] 1/7*x^7 - 8/5*x^5 + 16/3*x^3
```

Sympy [A] time = 0.067429, size = 17, normalized size = 0.77

$$\frac{x^{7}}{7} - \frac{8x^{5}}{5} + \frac{16x^{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(-x**2+4)**2,x)

[Out] x**7/7 - 8*x**5/5 + 16*x**3/3
```

GIAC/XCAS [A] time = 0.259366, size = 22, normalized size = 1.

$$\frac{1}{7}\,x^{7} - \frac{8}{5}\,x^{5} + \frac{16}{3}\,x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 4)^2*x^2,x, algorithm="giac")

[Out] 1/7*x^7 - 8/5*x^5 + 16/3*x^3
```

3.462 $\int x (1-x^3)^2 \, dx$

Optimal. Leaf size=22

$$\frac{x^8}{8} - \frac{2x^5}{5} + \frac{x^2}{2}$$

```
[Out] x^2/2 - (2*x^5)/5 + x^8/8
```

Rubi [A] time = 0.0204988, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{x^8}{8} - \frac{2x^5}{5} + \frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(1 - x^3)^2,x]

[Out] x^2/2 - (2*x^5)/5 + x^8/8
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{x^8}{8} - \frac{2x^5}{5} + \int x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(-x**3+1)**2,x)

[Out] x**8/8 - 2*x**5/5 + Integral(x, x)
```

Mathematica [A] time = 0.00106874, size = 22, normalized size = 1.

$$\frac{x^8}{8} - \frac{2x^5}{5} + \frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(1 - x^3)^2,x]

[Out] x^2/2 - (2*x^5)/5 + x^8/8
```

Maple [A] time = 0.001, size = 17, normalized size = 0.8

$$\frac{x^2}{2} - \frac{2x^5}{5} + \frac{x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(-x^3+1)^2,x)
```

```
[Out] 1/2*x^2-2/5*x^5+1/8*x^8
```

Maxima [A] time = 0.810013, size = 22, normalized size = 1.

$$\frac{1}{8}x^8 - \frac{2}{5}x^5 + \frac{1}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - 1)^2*x,x, algorithm="maxima")

[Out] 1/8*x^8 - 2/5*x^5 + 1/2*x^2
```

Fricas [A] time = 0.229898, size = 1, normalized size = 0.05

$$\frac{1}{8}x^8 - \frac{2}{5}x^5 + \frac{1}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - 1)^2*x,x, algorithm="fricas")

[Out] 1/8*x^8 - 2/5*x^5 + 1/2*x^2
```

Sympy [A] time = 0.064682, size = 15, normalized size = 0.68

$$\frac{x^8}{8} - \frac{2x^5}{5} + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(-x**3+1)**2,x)

[Out] x**8/8 - 2*x**5/5 + x**2/2
```

GIAC/XCAS [A] time = 0.258153, size = 22, normalized size = 1.

$$\frac{1}{8}x^8 - \frac{2}{5}x^5 + \frac{1}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - 1)^2*x,x, algorithm="giac")

[Out] 1/8*x^8 - 2/5*x^5 + 1/2*x^2
```

3.463 $\int \frac{-4+5x^2+x^3}{x^2}\,dx$

Optimal. Leaf size=16

$$\frac{x^2}{2}+5x+\frac{4}{x}$$

```
[Out] 4/x + 5*x + x^2/2
```

Rubi [A] time = 0.0138338, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071

$$\frac{x^2}{2}+5x+\frac{4}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(-4 + 5*x^2 + x^3)/x^2,x]

[Out] 4/x + 5*x + x^2/2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$5x+\int x\,dx+\frac{4}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((x**3+5*x**2-4)/x**2,x)

[Out] 5*x + Integral(x, x) + 4/x
```

Mathematica [A] time = 0.00175415, size = 16, normalized size = 1.

$$\frac{x^2}{2}+5x+\frac{4}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-4 + 5*x^2 + x^3)/x^2,x]

[Out] 4/x + 5*x + x^2/2
```

Maple [A] time = 0.005, size = 15, normalized size = 0.9

$$4\,x^{-1}+5\,x+\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+5*x^2-4)/x^2,x)
```

```
[Out] 4/x+5*x+1/2*x^2
```

Maxima [A] time = 0.819913, size = 19, normalized size = 1.19

$$\frac{1}{2}x^2 + 5x + \frac{4}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3 + 5*x^2 - 4)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + 5*x + 4/x
```

Fricas [A] time = 0.253396, size = 20, normalized size = 1.25

$$\frac{x^3 + 10x^2 + 8}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3 + 5*x^2 - 4)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(x^3 + 10*x^2 + 8)/x
```

Sympy [A] time = 0.10831, size = 10, normalized size = 0.62

$$\frac{x^2}{2} + 5x + \frac{4}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+5*x**2-4)/x**2,x)
```

```
[Out] x**2/2 + 5*x + 4/x
```

GIAC/XCAS [A] time = 0.261158, size = 19, normalized size = 1.19

$$\frac{1}{2}x^2 + 5x + \frac{4}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3 + 5*x^2 - 4)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*x^2 + 5*x + 4/x
```

3.464 $\int \frac{-1+x}{3-4x+3x^2}\,dx$

Optimal. Leaf size=37

$$\frac{1}{6}\log\left(3x^2-4x+3\right)+\frac{\tan^{-1}\left(\frac{2-3x}{\sqrt{5}}\right)}{3\sqrt{5}}$$

```
[Out] ArcTan[(2 - 3*x)/Sqrt[5]]/(3*Sqrt[5]) + Log[3 - 4*x + 3*x^2]/6
```

Rubi [A] time = 0.0497561, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{1}{6}\log\left(3x^2-4x+3\right)+\frac{\tan^{-1}\left(\frac{2-3x}{\sqrt{5}}\right)}{3\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + x)/(3 - 4*x + 3*x^2),x]

[Out] ArcTan[(2 - 3*x)/Sqrt[5]]/(3*Sqrt[5]) + Log[3 - 4*x + 3*x^2]/6
```

Rubi in Sympy [A] time = 6.37102, size = 34, normalized size = 0.92

$$\frac{\log\left(3x^2-4x+3\right)}{6}-\frac{\sqrt{5}\,\mathrm{atan}\left(\sqrt{5}\left(\frac{3x}{5}-\frac{2}{5}\right)\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-1+x)/(3*x**2-4*x+3),x)

[Out] log(3*x**2 - 4*x + 3)/6 - sqrt(5)*atan(sqrt(5)*(3*x/5 - 2/5))/15
```

Mathematica [A] time = 0.0194873, size = 37, normalized size = 1.

$$\frac{1}{6}\log\left(3x^2-4x+3\right)-\frac{\tan^{-1}\left(\frac{3x-2}{\sqrt{5}}\right)}{3\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + x)/(3 - 4*x + 3*x^2),x]

[Out] -ArcTan[(-2 + 3*x)/Sqrt[5]]/(3*Sqrt[5]) + Log[3 - 4*x + 3*x^2]/6
```

Maple [A] time = 0.006, size = 31, normalized size = 0.8

$$\frac{\ln\left(3\,x^2-4\,x+3\right)}{6}-\frac{\sqrt{5}}{15}\arctan\left(\frac{(6\,x-4)\sqrt{5}}{10}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-1+x)/(3*x^2-4*x+3),x)
```

```
[Out] 1/6*ln(3*x^2-4*x+3)-1/15*5^(1/2)*arctan(1/10*(6*x-4)*5^(1/2))
```

Maxima [A] time = 0.888227, size = 41, normalized size = 1.11

$$-\frac{1}{15}\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}(3x-2)\right)+\frac{1}{6}\log\left(3x^2-4x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - 1)/(3*x^2 - 4*x + 3),x, algorithm="maxima")
```

```
[Out] -1/15*sqrt(5)*arctan(1/5*sqrt(5)*(3*x - 2)) + 1/6*log(3*x^2 - 4*x
 + 3)
```

Fricas [A] time = 0.253113, size = 46, normalized size = 1.24

$$\frac{1}{30}\sqrt{5}\left(\sqrt{5}\log\left(3x^2-4x+3\right)-2\arctan\left(\frac{1}{5}\sqrt{5}(3x-2)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - 1)/(3*x^2 - 4*x + 3),x, algorithm="fricas")
```

```
[Out] 1/30*sqrt(5)*(sqrt(5)*log(3*x^2 - 4*x + 3) - 2*arctan(1/5*sqrt(5)
*(3*x - 2)))
```

Sympy [A] time = 0.232691, size = 39, normalized size = 1.05

$$\frac{\log\left(x^2-\frac{4x}{3}+1\right)}{6}-\frac{\sqrt{5}\operatorname{atan}\left(\frac{3\sqrt{5}x}{5}-\frac{2\sqrt{5}}{5}\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-1+x)/(3*x**2-4*x+3),x)
```

```
[Out] log(x**2 - 4*x/3 + 1)/6 - sqrt(5)*atan(3*sqrt(5)*x/5 - 2*sqrt(5)/
5)/15
```

GIAC/XCAS [A] time = 0.259884, size = 41, normalized size = 1.11

$$-\frac{1}{15}\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}(3x-2)\right)+\frac{1}{6}\ln\left(3x^2-4x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - 1)/(3*x^2 - 4*x + 3),x, algorithm="giac")
```

```
[Out] -1/15*sqrt(5)*arctan(1/5*sqrt(5)*(3*x - 2)) + 1/6*ln(3*x^2 - 4*x
+ 3)
```

3.465 $\int \left(2+x^3\right)^2 \, dx$

Optimal. Leaf size=14

$$\frac{x^7}{7}+x^4+4x$$

```
[Out] 4*x + x^4 + x^7/7
```

Rubi [A] time = 0.0117251, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{x^7}{7}+x^4+4x$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + x^3)^2,x]

[Out] 4*x + x^4 + x^7/7
```

Rubi in Sympy [A] time = 1.56586, size = 10, normalized size = 0.71

$$\frac{x^7}{7}+x^4+4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**3+2)**2,x)

[Out] x**7/7 + x**4 + 4*x
```

Mathematica [A] time = 0.000632926, size = 14, normalized size = 1.

$$\frac{x^7}{7}+x^4+4x$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + x^3)^2,x]

[Out] 4*x + x^4 + x^7/7
```

Maple [A] time = 0.001, size = 13, normalized size = 0.9

$$4\,x+x^4+\frac{x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^3+2)^2,x)
```

```
[Out] 4*x+x^4+1/7*x^7
```

Maxima [A] time = 0.805882, size = 16, normalized size = 1.14

$$\frac{1}{7}x^7 + x^4 + 4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3 + 2)^2,x, algorithm="maxima")
```

```
[Out] 1/7*x^7 + x^4 + 4*x
```

Fricas [A] time = 0.223933, size = 1, normalized size = 0.07

$$\frac{1}{7}x^7 + x^4 + 4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3 + 2)^2,x, algorithm="fricas")
```

```
[Out] 1/7*x^7 + x^4 + 4*x
```

Sympy [A] time = 0.059704, size = 10, normalized size = 0.71

$$\frac{x^7}{7} + x^4 + 4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+2)**2,x)
```

```
[Out] x**7/7 + x**4 + 4*x
```

GIAC/XCAS [A] time = 0.257433, size = 16, normalized size = 1.14

$$\frac{1}{7}x^7 + x^4 + 4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3 + 2)^2,x, algorithm="giac")
```

```
[Out] 1/7*x^7 + x^4 + 4*x
```

3.466 $\int \frac{-4+x^2}{2+x} \, dx$

Optimal. Leaf size=11

$$\frac{x^2}{2} - 2x$$

```
[Out] -2*x + x^2/2
```

Rubi [A] time = 0.0107793, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{x^2}{2} - 2x$$

Antiderivative was successfully verified.

```
[In]  Int[(-4 + x^2)/(2 + x),x]

[Out] -2*x + x^2/2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-2x + \int x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2-4)/(2+x),x)

[Out] -2*x + Integral(x, x)
```

Mathematica [A] time = 0.00056637, size = 11, normalized size = 1.

$$\frac{x^2}{2} - 2x$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-4 + x^2)/(2 + x),x]

[Out] -2*x + x^2/2
```

Maple [A] time = 0.001, size = 10, normalized size = 0.9

$$-2x + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2-4)/(2+x),x)
```

```
[Out] -2*x+1/2*x^2
```

Maxima [A] time = 0.807066, size = 12, normalized size = 1.09

$$\frac{1}{2}x^2 - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 - 4)/(x + 2),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 2*x
```

Fricas [A] time = 0.25402, size = 12, normalized size = 1.09

$$\frac{1}{2}x^2 - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 - 4)/(x + 2),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 - 2*x
```

Sympy [A] time = 0.062842, size = 7, normalized size = 0.64

$$\frac{x^2}{2} - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-4)/(2+x),x)
```

```
[Out] x**2/2 - 2*x
```

GIAC/XCAS [A] time = 0.261172, size = 12, normalized size = 1.09

$$\frac{1}{2}x^2 - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 - 4)/(x + 2),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 2*x
```

3.467 $\int \frac{1}{(2+x)(1+x^2)} \, dx$

Optimal. Leaf size=25

$$-\frac{1}{10}\log\left(x^2+1\right)+\frac{1}{5}\log(x+2)+\frac{2}{5}\tan^{-1}(x)$$

```
[Out] (2*ArcTan[x])/5 + Log[2 + x]/5 - Log[1 + x^2]/10
```

Rubi [A] time = 0.0320511, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$

$$-\frac{1}{10}\log\left(x^2+1\right)+\frac{1}{5}\log(x+2)+\frac{2}{5}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[1/((2 + x)*(1 + x^2)),x]
```

```
[Out] (2*ArcTan[x])/5 + Log[2 + x]/5 - Log[1 + x^2]/10
```

Rubi in Sympy [A] time = 4.8145, size = 20, normalized size = 0.8

$$\frac{\log(x+2)}{5}-\frac{\log\left(x^2+1\right)}{10}+\frac{2\,\mathrm{atan}\,(x)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(2+x)/(x**2+1),x)
```

```
[Out] log(x + 2)/5 - log(x**2 + 1)/10 + 2*atan(x)/5
```

Mathematica [A] time = 0.00912464, size = 25, normalized size = 1.

$$-\frac{1}{10}\log\left(x^2+1\right)+\frac{1}{5}\log(x+2)+\frac{2}{5}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((2 + x)*(1 + x^2)),x]
```

```
[Out] (2*ArcTan[x])/5 + Log[2 + x]/5 - Log[1 + x^2]/10
```

Maple [A] time = 0.008, size = 20, normalized size = 0.8

$$\frac{2\arctan(x)}{5}+\frac{\ln(2+x)}{5}-\frac{\ln\left(x^2+1\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(2+x)/(x^2+1),x)
```

```
[Out] 2/5*arctan(x)+1/5*ln(2+x)-1/10*ln(x^2+1)
```

Maxima [A] time = 0.889416, size = 26, normalized size = 1.04

$$\frac{2}{5}\arctan(x) - \frac{1}{10}\log\left(x^2+1\right) + \frac{1}{5}\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 1)*(x + 2)),x, algorithm="maxima")
```

```
[Out] 2/5*arctan(x) - 1/10*log(x^2 + 1) + 1/5*log(x + 2)
```

Fricas [A] time = 0.260785, size = 26, normalized size = 1.04

$$\frac{2}{5}\arctan(x) - \frac{1}{10}\log\left(x^2+1\right) + \frac{1}{5}\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 1)*(x + 2)),x, algorithm="fricas")
```

```
[Out] 2/5*arctan(x) - 1/10*log(x^2 + 1) + 1/5*log(x + 2)
```

Sympy [A] time = 0.287354, size = 20, normalized size = 0.8

$$\frac{\log(x+2)}{5} - \frac{\log\left(x^2+1\right)}{10} + \frac{2\,\mathrm{atan}(x)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(2+x)/(x**2+1),x)
```

```
[Out] log(x + 2)/5 - log(x**2 + 1)/10 + 2*atan(x)/5
```

GIAC/XCAS [A] time = 0.261556, size = 27, normalized size = 1.08

$$\frac{2}{5}\arctan(x) - \frac{1}{10}\ln\left(x^2+1\right) + \frac{1}{5}\ln(|x+2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 1)*(x + 2)),x, algorithm="giac")
```

```
[Out] 2/5*arctan(x) - 1/10*ln(x^2 + 1) + 1/5*ln(abs(x + 2))
```

3.468 $\int \frac{1}{(1+x)(1+x^2)}\,dx$

Optimal. Leaf size=25

$$-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2}\log(x+1)+\frac{1}{2}\tan^{-1}(x)$$

```
[Out] ArcTan[x]/2 + Log[1 + x]/2 - Log[1 + x^2]/4
```

Rubi [A] time = 0.0310307, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385

$$-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2}\log(x+1)+\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[1/((1 + x)*(1 + x^2)),x]

[Out] ArcTan[x]/2 + Log[1 + x]/2 - Log[1 + x^2]/4
```

Rubi in Sympy [A] time = 5.12394, size = 19, normalized size = 0.76

$$\frac{\log(x+1)}{2}-\frac{\log\left(x^2+1\right)}{4}+\frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1+x)/(x**2+1),x)

[Out] log(x + 1)/2 - log(x**2 + 1)/4 + atan(x)/2
```

Mathematica [A] time = 0.00840627, size = 25, normalized size = 1.

$$-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2}\log(x+1)+\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((1 + x)*(1 + x^2)),x]

[Out] ArcTan[x]/2 + Log[1 + x]/2 - Log[1 + x^2]/4
```

Maple [A] time = 0.006, size = 20, normalized size = 0.8

$$\frac{\arctan(x)}{2}+\frac{\ln(1+x)}{2}-\frac{\ln\left(x^2+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1+x)/(x^2+1),x)
```

```
[Out] 1/2*arctan(x)+1/2*ln(1+x)-1/4*ln(x^2+1)
```

Maxima [A] time = 0.883932, size = 26, normalized size = 1.04

$$\frac{1}{2}\arctan(x) - \frac{1}{4}\log\left(x^2+1\right) + \frac{1}{2}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 1)*(x + 1)),x, algorithm="maxima")
```

```
[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(x + 1)
```

Fricas [A] time = 0.266685, size = 26, normalized size = 1.04

$$\frac{1}{2}\arctan(x) - \frac{1}{4}\log\left(x^2+1\right) + \frac{1}{2}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 1)*(x + 1)),x, algorithm="fricas")
```

```
[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(x + 1)
```

Sympy [A] time = 0.261416, size = 19, normalized size = 0.76

$$\frac{\log(x+1)}{2} - \frac{\log\left(x^2+1\right)}{4} + \frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1+x)/(x**2+1),x)
```

```
[Out] log(x + 1)/2 - log(x**2 + 1)/4 + atan(x)/2
```

GIAC/XCAS [A] time = 0.259575, size = 27, normalized size = 1.08

$$\frac{1}{2}\arctan(x) - \frac{1}{4}\ln\left(x^2+1\right) + \frac{1}{2}\ln(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 1)*(x + 1)),x, algorithm="giac")
```

```
[Out] 1/2*arctan(x) - 1/4*ln(x^2 + 1) + 1/2*ln(abs(x + 1))
```

3.469 $\int \frac{x}{(1+x)(1+x^2)} \, dx$

Optimal. Leaf size=25

$$\frac{1}{4}\log\left(x^2+1\right)-\frac{1}{2}\log(x+1)+\frac{1}{2}\tan^{-1}(x)$$

```
[Out] ArcTan[x]/2 - Log[1 + x]/2 + Log[1 + x^2]/4
```

Rubi [A] time = 0.0526596, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286

$$\frac{1}{4}\log\left(x^2+1\right)-\frac{1}{2}\log(x+1)+\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[x/((1 + x)*(1 + x^2)),x]

[Out] ArcTan[x]/2 - Log[1 + x]/2 + Log[1 + x^2]/4
```

Rubi in Sympy [A] time = 5.40097, size = 19, normalized size = 0.76

$$-\frac{\log(x+1)}{2}+\frac{\log\left(x^2+1\right)}{4}+\frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(1+x)/(x**2+1),x)

[Out] -log(x + 1)/2 + log(x**2 + 1)/4 + atan(x)/2
```

Mathematica [A] time = 0.00928431, size = 25, normalized size = 1.

$$\frac{1}{4}\log\left(x^2+1\right)-\frac{1}{2}\log(x+1)+\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/((1 + x)*(1 + x^2)),x]

[Out] ArcTan[x]/2 - Log[1 + x]/2 + Log[1 + x^2]/4
```

Maple [A] time = 0.007, size = 20, normalized size = 0.8

$$\frac{\arctan(x)}{2}-\frac{\ln(1+x)}{2}+\frac{\ln\left(x^2+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(1+x)/(x^2+1),x)
```

```
[Out] 1/2*arctan(x)-1/2*ln(1+x)+1/4*ln(x^2+1)
```

Maxima [A] time = 0.881122, size = 26, normalized size = 1.04

$$\frac{1}{2}\arctan(x) + \frac{1}{4}\log\left(x^2+1\right) - \frac{1}{2}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((x^2 + 1)*(x + 1)),x, algorithm="maxima")

[Out] 1/2*arctan(x) + 1/4*log(x^2 + 1) - 1/2*log(x + 1)
```

Fricas [A] time = 0.281666, size = 26, normalized size = 1.04

$$\frac{1}{2}\arctan(x) + \frac{1}{4}\log\left(x^2+1\right) - \frac{1}{2}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((x^2 + 1)*(x + 1)),x, algorithm="fricas")

[Out] 1/2*arctan(x) + 1/4*log(x^2 + 1) - 1/2*log(x + 1)
```

Sympy [A] time = 0.264509, size = 19, normalized size = 0.76

$$-\frac{\log(x+1)}{2} + \frac{\log\left(x^2+1\right)}{4} + \frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(1+x)/(x**2+1),x)

[Out] -log(x + 1)/2 + log(x**2 + 1)/4 + atan(x)/2
```

GIAC/XCAS [A] time = 0.260959, size = 27, normalized size = 1.08

$$\frac{1}{2}\arctan(x) + \frac{1}{4}\ln\left(x^2+1\right) - \frac{1}{2}\ln(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((x^2 + 1)*(x + 1)),x, algorithm="giac")

[Out] 1/2*arctan(x) + 1/4*ln(x^2 + 1) - 1/2*ln(abs(x + 1))
```

3.470 $\int \frac{2x+x^2}{(1+x)^2} dx$

Optimal. Leaf size=9

$$\frac{x^2}{x+1}$$

```
[Out] x^2/(1 + x)
```

Rubi [A] time = 0.00739929, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{x^2}{x+1}$$

Antiderivative was successfully verified.

```
[In]  Int[(2*x + x^2)/(1 + x)^2,x]

[Out] x^2/(1 + x)
```

Rubi in Sympy [A] time = 3.01009, size = 5, normalized size = 0.56

$$\frac{x^2}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+2*x)/(1+x)**2,x)

[Out] x**2/(x + 1)
```

Mathematica [A] time = 0.00467687, size = 7, normalized size = 0.78

$$x + \frac{1}{x+1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2*x + x^2)/(1 + x)^2,x]

[Out] x + (1 + x)^(-1)
```

Maple [A] time = 0.005, size = 8, normalized size = 0.9

$$x + (1+x)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+2*x)/(1+x)^2,x)
```

```
[Out] x+1/(1+x)
```

Maxima [A] time = 0.775601, size = 9, normalized size = 1.

$$x + \frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 2*x)/(x + 1)^2,x, algorithm="maxima")
```

```
[Out] x + 1/(x + 1)
```

Fricas [A] time = 0.254757, size = 16, normalized size = 1.78

$$\frac{x^2+x+1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 2*x)/(x + 1)^2,x, algorithm="fricas")
```

```
[Out] (x^2 + x + 1)/(x + 1)
```

Sympy [A] time = 0.123863, size = 5, normalized size = 0.56

$$x + \frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+2*x)/(1+x)**2,x)
```

```
[Out] x + 1/(x + 1)
```

GIAC/XCAS [A] time = 0.25882, size = 11, normalized size = 1.22

$$x + \frac{1}{x+1} + 1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 2*x)/(x + 1)^2,x, algorithm="giac")
```

```
[Out] x + 1/(x + 1) + 1
```

3.471 $\int \frac{-10+x^2}{4+9x^2+2x^4}\,dx$

Optimal. Leaf size=22

$$\tan^{-1}\left(\frac{x}{2}\right)-\frac{3\tan^{-1}\left(\sqrt{2}x\right)}{\sqrt{2}}$$

```
[Out] ArcTan[x/2] - (3*ArcTan[Sqrt[2]*x])/Sqrt[2]
```

Rubi [A] time = 0.031989, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\tan^{-1}\left(\frac{x}{2}\right)-\frac{3\tan^{-1}\left(\sqrt{2}x\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-10 + x^2)/(4 + 9*x^2 + 2*x^4),x]

[Out] ArcTan[x/2] - (3*ArcTan[Sqrt[2]*x])/Sqrt[2]
```

Rubi in Sympy [A] time = 8.63619, size = 20, normalized size = 0.91

$$\operatorname{atan}\left(\frac{x}{2}\right)-\frac{3\sqrt{2}\operatorname{atan}\left(\sqrt{2}x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((x**2-10)/(2*x**4+9*x**2+4),x)

[Out] atan(x/2) - 3*sqrt(2)*atan(sqrt(2)*x)/2
```

Mathematica [A] time = 0.0196729, size = 22, normalized size = 1.

$$\tan^{-1}\left(\frac{x}{2}\right)-\frac{3\tan^{-1}\left(\sqrt{2}x\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-10 + x^2)/(4 + 9*x^2 + 2*x^4),x]

[Out] ArcTan[x/2] - (3*ArcTan[Sqrt[2]*x])/Sqrt[2]
```

Maple [A] time = 0.01, size = 17, normalized size = 0.8

$$\arctan\left(\frac{x}{2}\right)-\frac{3\arctan\left(\sqrt{2}x\right)\sqrt{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2-10)/(2*x^4+9*x^2+4),x)

[Out] arctan(1/2*x)-3/2*arctan(2^(1/2)*x)*2^(1/2)
```

Maxima [A] time = 0.857709, size = 22, normalized size = 1.

$$-\frac{3}{2}\sqrt{2}\arctan\left(\sqrt{2}x\right)+\arctan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 10)/(2*x^4 + 9*x^2 + 4),x, algorithm="maxima")

[Out] -3/2*sqrt(2)*arctan(sqrt(2)*x) + arctan(1/2*x)
```

Fricas [A] time = 0.265483, size = 30, normalized size = 1.36

$$\frac{1}{2}\sqrt{2}\left(\sqrt{2}\arctan\left(\frac{1}{2}x\right)-3\arctan\left(\sqrt{2}x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 10)/(2*x^4 + 9*x^2 + 4),x, algorithm="fricas")

[Out] 1/2*sqrt(2)*(sqrt(2)*arctan(1/2*x) - 3*arctan(sqrt(2)*x))
```

Sympy [A] time = 0.411319, size = 20, normalized size = 0.91

$$\operatorname{atan}\left(\frac{x}{2}\right)-\frac{3\sqrt{2}\operatorname{atan}\left(\sqrt{2}x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2-10)/(2*x**4+9*x**2+4),x)

[Out] atan(x/2) - 3*sqrt(2)*atan(sqrt(2)*x)/2
```

GIAC/XCAS [A] time = 0.259908, size = 22, normalized size = 1.

$$-\frac{3}{2}\sqrt{2}\arctan\left(\sqrt{2}x\right)+\arctan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 10)/(2*x^4 + 9*x^2 + 4),x, algorithm="giac")

[Out] -3/2*sqrt(2)*arctan(sqrt(2)*x) + arctan(1/2*x)
```

3.472 $\int \frac{31+5x}{11-4x+3x^2}\,dx$

Optimal. Leaf size=37

$$\frac{5}{6}\log\left(3x^2-4x+11\right)-\frac{103\tan^{-1}\left(\frac{2-3x}{\sqrt{29}}\right)}{3\sqrt{29}}$$

```
[Out] (-103*ArcTan[(2 - 3*x)/Sqrt[29]])/(3*Sqrt[29]) + (5*Log[11 - 4*x
+ 3*x^2])/6
```

Rubi [A] time = 0.0526654, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$\frac{5}{6}\log\left(3x^2-4x+11\right)-\frac{103\tan^{-1}\left(\frac{2-3x}{\sqrt{29}}\right)}{3\sqrt{29}}$$

Antiderivative was successfully verified.

```
[In]  Int[(31 + 5*x)/(11 - 4*x + 3*x^2),x]
```

```
[Out] (-103*ArcTan[(2 - 3*x)/Sqrt[29]])/(3*Sqrt[29]) + (5*Log[11 - 4*x
+ 3*x^2])/6
```

Rubi in Sympy [A] time = 6.5666, size = 37, normalized size = 1.

$$\frac{5\log\left(3x^2-4x+11\right)}{6}+\frac{103\sqrt{29}\,\mathrm{atan}\left(\sqrt{29}\left(\frac{3x}{29}-\frac{2}{29}\right)\right)}{87}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((31+5*x)/(3*x**2-4*x+11),x)
```

```
[Out] 5*log(3*x**2 - 4*x + 11)/6 + 103*sqrt(29)*atan(sqrt(29)*(3*x/29 -
 2/29))/87
```

Mathematica [A] time = 0.0211698, size = 37, normalized size = 1.

$$\frac{5}{6}\log\left(3x^2-4x+11\right)+\frac{103\tan^{-1}\left(\frac{3x-2}{\sqrt{29}}\right)}{3\sqrt{29}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(31 + 5*x)/(11 - 4*x + 3*x^2),x]
```

```
[Out] (103*ArcTan[(-2 + 3*x)/Sqrt[29]])/(3*Sqrt[29]) + (5*Log[11 - 4*x
+ 3*x^2])/6
```

Maple [A] time = 0.006, size = 31, normalized size = 0.8

$$\frac{5\ln\left(3x^2-4x+11\right)}{6}+\frac{103\sqrt{29}}{87}\arctan\left(\frac{(6x-4)\sqrt{29}}{58}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((31+5*x)/(3*x^2-4*x+11),x)
```

```
[Out] 5/6*ln(3*x^2-4*x+11)+103/87*29^(1/2)*arctan(1/58*(6*x-4)*29^(1/2)
)
```

Maxima [A] time = 0.855363, size = 41, normalized size = 1.11

$$\frac{103}{87}\sqrt{29}\arctan\left(\frac{1}{29}\sqrt{29}(3x-2)\right)+\frac{5}{6}\log\left(3x^2-4x+11\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((5*x + 31)/(3*x^2 - 4*x + 11),x, algorithm="maxima")
```

```
[Out] 103/87*sqrt(29)*arctan(1/29*sqrt(29)*(3*x - 2)) + 5/6*log(3*x^2 -
 4*x + 11)
```

Fricas [A] time = 0.264668, size = 47, normalized size = 1.27

$$\frac{1}{174}\sqrt{29}\left(5\sqrt{29}\log\left(3x^2-4x+11\right)+206\arctan\left(\frac{1}{29}\sqrt{29}(3x-2)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((5*x + 31)/(3*x^2 - 4*x + 11),x, algorithm="fricas")
```

```
[Out] 1/174*sqrt(29)*(5*sqrt(29)*log(3*x^2 - 4*x + 11) + 206*arctan(1/2
9*sqrt(29)*(3*x - 2)))
```

Sympy [A] time = 0.252585, size = 44, normalized size = 1.19

$$\frac{5\log\left(x^2-\frac{4x}{3}+\frac{11}{3}\right)}{6}+\frac{103\sqrt{29}\operatorname{atan}\left(\frac{3\sqrt{29}x}{29}-\frac{2\sqrt{29}}{29}\right)}{87}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((31+5*x)/(3*x**2-4*x+11),x)
```

```
[Out] 5*log(x**2 - 4*x/3 + 11/3)/6 + 103*sqrt(29)*atan(3*sqrt(29)*x/29
- 2*sqrt(29)/29)/87
```

GIAC/XCAS [A] time = 0.261712, size = 41, normalized size = 1.11

$$\frac{103}{87}\sqrt{29}\arctan\left(\frac{1}{29}\sqrt{29}(3x-2)\right)+\frac{5}{6}\ln\left(3x^2-4x+11\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((5*x + 31)/(3*x^2 - 4*x + 11),x, algorithm="giac")
```

```
[Out] 103/87*sqrt(29)*arctan(1/29*sqrt(29)*(3*x - 2)) + 5/6*ln(3*x^2 -
4*x + 11)
```

3.473 $\int \frac{-2+x^2+x^3}{x^4} \, dx$

Optimal. Leaf size=15

$$\frac{2}{3x^3} - \frac{1}{x} + \log(x)$$

```
[Out] 2/(3*x^3) - x^(-1) + Log[x]
```

Rubi [A] time = 0.0125453, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$\frac{2}{3x^3} - \frac{1}{x} + \log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(-2 + x^2 + x^3)/x^4,x]

[Out] 2/(3*x^3) - x^(-1) + Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 + x^2 - 2}{x^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**3+x**2-2)/x**4,x)

[Out] Integral((x**3 + x**2 - 2)/x**4, x)
```

Mathematica [A] time = 0.00273745, size = 15, normalized size = 1.

$$\frac{2}{3x^3} - \frac{1}{x} + \log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-2 + x^2 + x^3)/x^4,x]

[Out] 2/(3*x^3) - x^(-1) + Log[x]
```

Maple [A] time = 0.008, size = 14, normalized size = 0.9

$$\frac{2}{3\,x^3} - x^{-1} + \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^3+x^2-2)/x^4,x)
```

```
[Out] 2/3/x^3-1/x+ln(x)
```

Maxima [A] time = 0.78114, size = 20, normalized size = 1.33

$$-\frac{3x^2-2}{3x^3}+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + x^2 - 2)/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*(3*x^2 - 2)/x^3 + log(x)
```

Fricas [A] time = 0.263128, size = 26, normalized size = 1.73

$$\frac{3x^3\log(x)-3x^2+2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + x^2 - 2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*x^3*log(x) - 3*x^2 + 2)/x^3
```

Sympy [A] time = 0.169219, size = 14, normalized size = 0.93

$$\log(x)-\frac{3x^2-2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3+x**2-2)/x**4,x)
```

```
[Out] log(x) - (3*x**2 - 2)/(3*x**3)
```

GIAC/XCAS [A] time = 0.260572, size = 22, normalized size = 1.47

$$-\frac{3x^2-2}{3x^3}+\ln(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + x^2 - 2)/x^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*x^2 - 2)/x^3 + ln(abs(x))
```

3.474 $\int \frac{1+x+x^3}{x^2} \, dx$

Optimal. Leaf size=15

$$\frac{x^2}{2} - \frac{1}{x} + \log(x)$$

```
[Out] -x^(-1) + x^2/2 + Log[x]
```

Rubi [A] time = 0.0113741, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{x^2}{2} - \frac{1}{x} + \log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x + x^3)/x^2,x]

[Out] -x^(-1) + x^2/2 + Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 + x + 1}{x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**3+x+1)/x**2,x)

[Out] Integral((x**3 + x + 1)/x**2, x)
```

Mathematica [A] time = 0.00126265, size = 15, normalized size = 1.

$$\frac{x^2}{2} - \frac{1}{x} + \log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x + x^3)/x^2,x]

[Out] -x^(-1) + x^2/2 + Log[x]
```

Maple [A] time = 0.005, size = 14, normalized size = 0.9

$$-x^{-1} + \frac{x^2}{2} + \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^3+x+1)/x^2,x)
```

```
[Out] -1/x+1/2*x^2+ln(x)
```

Maxima [A] time = 0.802605, size = 18, normalized size = 1.2

$$\frac{1}{2}x^2 - \frac{1}{x} + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + x + 1)/x^2,x, algorithm="maxima")

[Out] 1/2*x^2 - 1/x + log(x)
```

Fricas [A] time = 0.264821, size = 20, normalized size = 1.33

$$\frac{x^3 + 2x\log(x) - 2}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + x + 1)/x^2,x, algorithm="fricas")

[Out] 1/2*(x^3 + 2*x*log(x) - 2)/x
```

Sympy [A] time = 0.124718, size = 10, normalized size = 0.67

$$\frac{x^2}{2} + \log(x) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3+x+1)/x**2,x)

[Out] x**2/2 + log(x) - 1/x
```

GIAC/XCAS [A] time = 0.259521, size = 19, normalized size = 1.27

$$\frac{1}{2}x^2 - \frac{1}{x} + \ln(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + x + 1)/x^2,x, algorithm="giac")

[Out] 1/2*x^2 - 1/x + ln(abs(x))
```

3.475 $\int \frac{-2+x^2}{x(2+x^2)}\,dx$

Optimal. Leaf size=11

$$\log\left(x^2+2\right)-\log(x)$$

```
[Out] -Log[x] + Log[2 + x^2]
```

Rubi [A] time = 0.0411748, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\log\left(x^2+2\right)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + x^2)/(x*(2 + x^2)),x]

[Out] -Log[x] + Log[2 + x^2]
```

Rubi in Sympy [A] time = 6.69438, size = 12, normalized size = 1.09

$$-\frac{\log\left(x^2\right)}{2}+\log\left(x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((x**2-2)/x/(x**2+2),x)

[Out] -log(x**2)/2 + log(x**2 + 2)
```

Mathematica [A] time = 0.00503397, size = 11, normalized size = 1.

$$\log\left(x^2+2\right)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + x^2)/(x*(2 + x^2)),x]

[Out] -Log[x] + Log[2 + x^2]
```

Maple [A] time = 0.006, size = 12, normalized size = 1.1

$$-\ln(x)+\ln\left(x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-2)/x/(x^2+2),x)

[Out] -ln(x)+ln(x^2+2)
```

Maxima [A] time = 0.792677, size = 18, normalized size = 1.64

$$\log\left(x^2+2\right) - \frac{1}{2}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 2)/((x^2 + 2)*x),x, algorithm="maxima")
```

```
[Out] log(x^2 + 2) - 1/2*log(x^2)
```

Fricas [A] time = 0.257375, size = 15, normalized size = 1.36

$$\log\left(x^2+2\right) - \log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 2)/((x^2 + 2)*x),x, algorithm="fricas")
```

```
[Out] log(x^2 + 2) - log(x)
```

Sympy [A] time = 0.189321, size = 8, normalized size = 0.73

$$-\log\left(x\right) + \log\left(x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2-2)/x/(x**2+2),x)
```

```
[Out] -log(x) + log(x**2 + 2)
```

GIAC/XCAS [A] time = 0.260607, size = 18, normalized size = 1.64

$$\ln\left(x^2+2\right) - \frac{1}{2}\ln\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 2)/((x^2 + 2)*x),x, algorithm="giac")
```

```
[Out] ln(x^2 + 2) - 1/2*ln(x^2)
```

3.476 $\int (-3+x)\left(-7+4x^2\right)\,dx$

Optimal. Leaf size=22

$$-4x^3+\frac{1}{16}\left(7-4x^2\right)^2+21x$$

```
[Out] 21*x - 4*x^3 + (7 - 4*x^2)^2/16
```

Rubi [A] time = 0.0181222, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$-4x^3+\frac{1}{16}\left(7-4x^2\right)^2+21x$$

Antiderivative was successfully verified.

```
[In]  Int[(-3 + x)*(-7 + 4*x^2),x]

[Out] 21*x - 4*x^3 + (7 - 4*x^2)^2/16
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$x^4-4x^3+21x-7\int x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-3+x)*(4*x**2-7),x)

[Out] x**4 - 4*x**3 + 21*x - 7*Integral(x, x)
```

Mathematica [A] time = 0.00169975, size = 19, normalized size = 0.86

$$x^4-4x^3-\frac{7x^2}{2}+21x$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-3 + x)*(-7 + 4*x^2),x]

[Out] 21*x - (7*x^2)/2 - 4*x^3 + x^4
```

Maple [A] time = 0.001, size = 18, normalized size = 0.8

$$x^4-4\,x^3-\frac{7\,x^2}{2}+21\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-3+x)*(4*x^2-7),x)
```

```
[Out] x^4-4*x^3-7/2*x^2+21*x
```

Maxima [A] time = 0.781584, size = 23, normalized size = 1.05

$$x^4 - 4\,x^3 - \frac{7}{2}\,x^2 + 21\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^2 - 7)*(x - 3),x, algorithm="maxima")
```

```
[Out] x^4 - 4*x^3 - 7/2*x^2 + 21*x
```

Fricas [A] time = 0.246489, size = 1, normalized size = 0.05

$$x^4 - 4x^3 - \frac{7}{2}x^2 + 21x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^2 - 7)*(x - 3),x, algorithm="fricas")
```

```
[Out] x^4 - 4*x^3 - 7/2*x^2 + 21*x
```

Sympy [A] time = 0.058897, size = 17, normalized size = 0.77

$$x^4 - 4x^3 - \frac{7x^2}{2} + 21x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-3+x)*(4*x**2-7),x)
```

```
[Out] x**4 - 4*x**3 - 7*x**2/2 + 21*x
```

GIAC/XCAS [A] time = 0.261148, size = 23, normalized size = 1.05

$$x^4 - 4\,x^3 - \frac{7}{2}\,x^2 + 21\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^2 - 7)*(x - 3),x, algorithm="giac")
```

```
[Out] x^4 - 4*x^3 - 7/2*x^2 + 21*x
```

3.477 $\int (-2+7x)^3\,dx$

Optimal. Leaf size=11

$$\frac{1}{28}(2-7x)^4$$

```
[Out] (2 - 7*x)^4/28
```

Rubi [A] time = 0.00656477, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{1}{28}(2-7x)^4$$

Antiderivative was successfully verified.

```
[In]  Int[(-2 + 7*x)^3,x]
```

```
[Out] (2 - 7*x)^4/28
```

Rubi in Sympy [A] time = 1.05605, size = 7, normalized size = 0.64

$$\frac{(-7x+2)^4}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-2+7*x)**3,x)
```

```
[Out] (-7*x + 2)**4/28
```

Mathematica [A] time = 0.00230804, size = 11, normalized size = 1.

$$\frac{1}{28}(7x-2)^4$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-2 + 7*x)^3,x]
```

```
[Out] (-2 + 7*x)^4/28
```

Maple [A] time = 0.001, size = 10, normalized size = 0.9

$$\frac{(-2+7x)^4}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-2+7*x)^3,x)
```

```
[Out] 1/28*(-2+7*x)^4
```

Maxima [A] time = 0.777655, size = 26, normalized size = 2.36

$$\frac{343}{4}\,x^4 - 98\,x^3 + 42\,x^2 - 8\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((7*x - 2)^3,x, algorithm="maxima")
```

```
[Out] 343/4*x^4 - 98*x^3 + 42*x^2 - 8*x
```

Fricas [A] time = 0.224479, size = 1, normalized size = 0.09

$$\frac{343}{4}x^4 - 98x^3 + 42x^2 - 8x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((7*x - 2)^3,x, algorithm="fricas")
```

```
[Out] 343/4*x^4 - 98*x^3 + 42*x^2 - 8*x
```

Sympy [A] time = 0.065656, size = 19, normalized size = 1.73

$$\frac{343x^4}{4} - 98x^3 + 42x^2 - 8x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-2+7*x)**3,x)
```

```
[Out] 343*x**4/4 - 98*x**3 + 42*x**2 - 8*x
```

GIAC/XCAS [A] time = 0.258236, size = 12, normalized size = 1.09

$$\frac{1}{28}\,(7\,x - 2)^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((7*x - 2)^3,x, algorithm="giac")
```

```
[Out] 1/28*(7*x - 2)^4
```

3.478 $\int \frac{-7+4x^2}{3+2x}\,dx$

Optimal. Leaf size=13

$$x^2 - 3x + \log(2x + 3)$$

```
[Out] -3*x + x^2 + Log[3 + 2*x]
```

Rubi [A] time = 0.026308, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$x^2 - 3x + \log(2x + 3)$$

Antiderivative was successfully verified.

```
[In]  Int[(-7 + 4*x^2)/(3 + 2*x),x]

[Out] -3*x + x^2 + Log[3 + 2*x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-3x + \log(2x + 3) + 2\int x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((4*x**2-7)/(3+2*x),x)

[Out] -3*x + log(2*x + 3) + 2*Integral(x, x)
```

Mathematica [A] time = 0.00589985, size = 16, normalized size = 1.23

$$x^2 - 3x + \log(2x + 3) - \frac{27}{4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-7 + 4*x^2)/(3 + 2*x),x]

[Out] -27/4 - 3*x + x^2 + Log[3 + 2*x]
```

Maple [A] time = 0.003, size = 14, normalized size = 1.1

$$-3x + x^2 + \ln(3 + 2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((4*x^2-7)/(3+2*x),x)

[Out] -3*x+x^2+ln(3+2*x)
```

Maxima [A] time = 0.782027, size = 18, normalized size = 1.38

$$x^2 - 3x + \log(2x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^2 - 7)/(2*x + 3),x, algorithm="maxima")

[Out] x^2 - 3*x + log(2*x + 3)
```

Fricas [A] time = 0.261004, size = 18, normalized size = 1.38

$$x^2 - 3x + \log(2x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^2 - 7)/(2*x + 3),x, algorithm="fricas")

[Out] x^2 - 3*x + log(2*x + 3)
```

Sympy [A] time = 0.123443, size = 12, normalized size = 0.92

$$x^2 - 3x + \log(2x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x**2-7)/(3+2*x),x)

[Out] x**2 - 3*x + log(2*x + 3)
```

GIAC/XCAS [A] time = 0.26133, size = 19, normalized size = 1.46

$$x^2 - 3x + \ln(|2x + 3|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^2 - 7)/(2*x + 3),x, algorithm="giac")

[Out] x^2 - 3*x + ln(abs(2*x + 3))
```

3.479 $\int \frac{1+x}{(-1+x)x^2} \, dx$

Optimal. Leaf size=16

$$\frac{1}{x} + 2\log(1-x) - 2\log(x)$$

```
[Out] x^(-1) + 2*Log[1 - x] - 2*Log[x]
```

Rubi [A] time = 0.025342, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$

$$\frac{1}{x} + 2\log(1-x) - 2\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x)/((-1 + x)*x^2),x]

[Out] x^(-1) + 2*Log[1 - x] - 2*Log[x]
```

Rubi in Sympy [A] time = 3.26899, size = 14, normalized size = 0.88

$$-2\log(x) + 2\log(-x+1) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+x)/(-1+x)/x**2,x)

[Out] -2*log(x) + 2*log(-x + 1) + 1/x
```

Mathematica [A] time = 0.00437705, size = 16, normalized size = 1.

$$\frac{1}{x} + 2\log(1-x) - 2\log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x)/((-1 + x)*x^2),x]

[Out] x^(-1) + 2*Log[1 - x] - 2*Log[x]
```

Maple [A] time = 0.009, size = 15, normalized size = 0.9

$$2\ln(-1+x) + x^{-1} - 2\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+x)/(-1+x)/x^2,x)

[Out] 2*ln(-1+x)+1/x-2*ln(x)
```

Maxima [A] time = 0.771998, size = 19, normalized size = 1.19

$$\frac{1}{x} + 2\log(x-1) - 2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 1)/((x - 1)*x^2),x, algorithm="maxima")

[Out] 1/x + 2*log(x - 1) - 2*log(x)
```

Fricas [A] time = 0.260379, size = 24, normalized size = 1.5

$$\frac{2x\log(x-1) - 2x\log(x) + 1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 1)/((x - 1)*x^2),x, algorithm="fricas")

[Out] (2*x*log(x - 1) - 2*x*log(x) + 1)/x
```

Sympy [A] time = 0.194442, size = 14, normalized size = 0.88

$$-2\log(x) + 2\log(x-1) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+x)/(-1+x)/x**2,x)

[Out] -2*log(x) + 2*log(x - 1) + 1/x
```

GIAC/XCAS [A] time = 0.26267, size = 22, normalized size = 1.38

$$\frac{1}{x} + 2\ln(|x-1|) - 2\ln(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 1)/((x - 1)*x^2),x, algorithm="giac")

[Out] 1/x + 2*ln(abs(x - 1)) - 2*ln(abs(x))
```

3.480 $\int \frac{1}{4x^2+4x^3+x^4}\,dx$

Optimal. Leaf size=25

$$-\frac{1}{4x}-\frac{1}{4(x+2)}+\frac{1}{2}\tanh^{-1}(x+1)$$

```
[Out] -1/(4*x) - 1/(4*(2 + x)) + ArcTanh[1 + x]/2
```

Rubi [A] time = 0.0274699, antiderivative size = 31, normalized size of antiderivative = 1.24, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188

$$-\frac{1}{4x}-\frac{1}{4(x+2)}-\frac{\log(x)}{4}+\frac{1}{4}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(4*x^2 + 4*x^3 + x^4)^(-1),x]

[Out] -1/(4*x) - 1/(4*(2 + x)) - Log[x]/4 + Log[2 + x]/4
```

Rubi in Sympy [A] time = 15.5941, size = 22, normalized size = 0.88

$$-\frac{\log(x)}{4}+\frac{\log(x+2)}{4}-\frac{1}{4(x+2)}-\frac{1}{4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(x**4+4*x**3+4*x**2),x)

[Out] -log(x)/4 + log(x + 2)/4 - 1/(4*(x + 2)) - 1/(4*x)
```

Mathematica [A] time = 0.03099, size = 26, normalized size = 1.04

$$\frac{1}{4}\left(-\frac{2(x+1)}{x(x+2)}-\log(x)+\log(x+2)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4*x^2 + 4*x^3 + x^4)^(-1),x]

[Out] ((-2*(1 + x))/(x*(2 + x)) - Log[x] + Log[2 + x])/4
```

Maple [A] time = 0.013, size = 24, normalized size = 1.

$$-\frac{1}{4x}-\frac{1}{8+4x}-\frac{\ln(x)}{4}+\frac{\ln(2+x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+4*x^3+4*x^2),x)
```

```
[Out] -1/4/x-1/4/(2+x)-1/4*ln(x)+1/4*ln(2+x)
```

Maxima [A] time = 0.780537, size = 34, normalized size = 1.36

$$-\frac{x+1}{2\,(x^2+2\,x)}+\frac{1}{4}\,\log\left(x+2\right)-\frac{1}{4}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 + 4*x^3 + 4*x^2),x, algorithm="maxima")
```

```
[Out] -1/2*(x + 1)/(x^2 + 2*x) + 1/4*log(x + 2) - 1/4*log(x)
```

Fricas [A] time = 0.276176, size = 53, normalized size = 2.12

$$\frac{\left(x^2+2\,x\right)\,\log\left(x+2\right)-\left(x^2+2\,x\right)\,\log\left(x\right)-2\,x-2}{4\,(x^2+2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 + 4*x^3 + 4*x^2),x, algorithm="fricas")
```

```
[Out] 1/4*((x^2 + 2*x)*log(x + 2) - (x^2 + 2*x)*log(x) - 2*x - 2)/(x^2
+ 2*x)
```

Sympy [A] time = 0.235672, size = 22, normalized size = 0.88

$$-\frac{x+1}{2x^2+4x}-\frac{\log\left(x\right)}{4}+\frac{\log\left(x+2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**4+4*x**3+4*x**2),x)
```

```
[Out] -(x + 1)/(2*x**2 + 4*x) - log(x)/4 + log(x + 2)/4
```

GIAC/XCAS [A] time = 0.260364, size = 36, normalized size = 1.44

$$-\frac{x+1}{2\,(x^2+2\,x)}+\frac{1}{4}\ln\left(|x+2|\right)-\frac{1}{4}\ln\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 + 4*x^3 + 4*x^2),x, algorithm="giac")
```

```
[Out] -1/2*(x + 1)/(x^2 + 2*x) + 1/4*ln(abs(x + 2)) - 1/4*ln(abs(x))
```

3.481 $\int \frac{1+x^2}{1+x} \, dx$

Optimal. Leaf size=17

$$\frac{x^2}{2} - x + 2\log(x+1)$$

```
[Out] -x + x^2/2 + 2*Log[1 + x]
```

Rubi [A] time = 0.0235443, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{x^2}{2} - x + 2\log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^2)/(1 + x),x]

[Out] -x + x^2/2 + 2*Log[1 + x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-x + 2\log(x+1) + \int x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+1)/(1+x),x)

[Out] -x + 2*log(x + 1) + Integral(x, x)
```

Mathematica [A] time = 0.00470055, size = 18, normalized size = 1.06

$$\frac{1}{2}\left(x^2 - 2x + 4\log(x+1) - 3\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^2)/(1 + x),x]

[Out] (-3 - 2*x + x^2 + 4*Log[1 + x])/2
```

Maple [A] time = 0.003, size = 16, normalized size = 0.9

$$-x + \frac{x^2}{2} + 2\ln(1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+1)/(1+x),x)
```

```
[Out] -x+1/2*x^2+2*ln(1+x)
```

Maxima [A] time = 0.78098, size = 20, normalized size = 1.18

$$\frac{1}{2}x^2 - x + 2\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/(x + 1),x, algorithm="maxima")

[Out] 1/2*x^2 - x + 2*log(x + 1)
```

Fricas [A] time = 0.252692, size = 20, normalized size = 1.18

$$\frac{1}{2}x^2 - x + 2\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/(x + 1),x, algorithm="fricas")

[Out] 1/2*x^2 - x + 2*log(x + 1)
```

Sympy [A] time = 0.108367, size = 12, normalized size = 0.71

$$\frac{x^2}{2} - x + 2\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+1)/(1+x),x)

[Out] x**2/2 - x + 2*log(x + 1)
```

GIAC/XCAS [A] time = 0.260125, size = 22, normalized size = 1.29

$$\frac{1}{2}x^2 - x + 2\ln(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/(x + 1),x, algorithm="giac")

[Out] 1/2*x^2 - x + 2*ln(abs(x + 1))
```

3.482 $\int \frac{-1+3x-3x^2+x^3}{x^2}\,dx$

Optimal. Leaf size=18

$$\frac{x^2}{2}-3x+\frac{1}{x}+3\log(x)$$

```
[Out] x^(-1) - 3*x + x^2/2 + 3*Log[x]
```

Rubi [A] time = 0.016003, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059

$$\frac{x^2}{2}-3x+\frac{1}{x}+3\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + 3*x - 3*x^2 + x^3)/x^2,x]

[Out] x^(-1) - 3*x + x^2/2 + 3*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3-3x^2+3x-1}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**3-3*x**2+3*x-1)/x**2,x)

[Out] Integral((x**3 - 3*x**2 + 3*x - 1)/x**2, x)
```

Mathematica [A] time = 0.00166039, size = 18, normalized size = 1.

$$\frac{x^2}{2}-3x+\frac{1}{x}+3\log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + 3*x - 3*x^2 + x^3)/x^2,x]

[Out] x^(-1) - 3*x + x^2/2 + 3*Log[x]
```

Maple [A] time = 0.008, size = 17, normalized size = 0.9

$$x^{-1}-3\,x+\frac{x^2}{2}+3\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^3-3*x^2+3*x-1)/x^2,x)
```

```
[Out] 1/x-3*x+1/2*x^2+3*ln(x)
```

Maxima [A] time = 0.781365, size = 22, normalized size = 1.22

$$\frac{1}{2}x^2 - 3x + \frac{1}{x} + 3\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - 3*x^2 + 3*x - 1)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 3*x + 1/x + 3*log(x)
```

Fricas [A] time = 0.249545, size = 27, normalized size = 1.5

$$\frac{x^3 - 6x^2 + 6x\log(x) + 2}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - 3*x^2 + 3*x - 1)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(x^3 - 6*x^2 + 6*x*log(x) + 2)/x
```

Sympy [A] time = 0.129684, size = 15, normalized size = 0.83

$$\frac{x^2}{2} - 3x + 3\log(x) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3-3*x**2+3*x-1)/x**2,x)
```

```
[Out] x**2/2 - 3*x + 3*log(x) + 1/x
```

GIAC/XCAS [A] time = 0.259882, size = 23, normalized size = 1.28

$$\frac{1}{2}x^2 - 3x + \frac{1}{x} + 3\ln(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - 3*x^2 + 3*x - 1)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 3*x + 1/x + 3*ln(abs(x))
```

3.483 $\int \left(\frac{1}{2}\left(3-\sqrt{37}\right)+x\right)\left(\frac{1}{2}\left(3+\sqrt{37}\right)+x\right)\,dx$

Optimal. Leaf size=18

$$\frac{x^3}{3}+\frac{3x^2}{2}-7x$$

```
[Out] -7*x + (3*x^2)/2 + x^3/3
```

Rubi [A] time = 0.0308022, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034

$$\frac{x^3}{3}+\frac{3x^2}{2}-7x$$

Antiderivative was successfully verified.

```
[In]  Int[((3 - Sqrt[37])/2 + x)*((3 + Sqrt[37])/2 + x),x]

[Out] -7*x + (3*x^2)/2 + x^3/3
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{x^3}{3}-7x+3\int x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x+3/2-1/2*37**(1/2))*(x+3/2+1/2*37**(1/2)),x)

[Out] x**3/3 - 7*x + 3*Integral(x, x)
```

Mathematica [A] time = 0.000988747, size = 18, normalized size = 1.

$$\frac{x^3}{3}+\frac{3x^2}{2}-7x$$

Antiderivative was successfully verified.

```
[In]  Integrate[((3 - Sqrt[37])/2 + x)*((3 + Sqrt[37])/2 + x),x]

[Out] -7*x + (3*x^2)/2 + x^3/3
```

Maple [A] time = 0.001, size = 28, normalized size = 1.6

$$\frac{x^3}{3}+\frac{3\,x^2}{2}+\left(\frac{3}{2}-\frac{\sqrt{37}}{2}\right)\left(\frac{3}{2}+\frac{\sqrt{37}}{2}\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x+3/2-1/2*37^(1/2))*(x+3/2+1/2*37^(1/2)),x)
```

```
[Out] 1/3*x^3+3/2*x^2+(3/2-1/2*37^(1/2))*(3/2+1/2*37^(1/2))*x
```

Maxima [A] time = 0.869247, size = 19, normalized size = 1.06

$$\frac{1}{3}x^3 + \frac{3}{2}x^2 - 7x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/4*(2*x + sqrt(37) + 3)*(2*x - sqrt(37) + 3),x, algorithm="maxima")
```

```
[Out] 1/3*x^3 + 3/2*x^2 - 7*x
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: SyntaxError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/4*(2*x + sqrt(37) + 3)*(2*x - sqrt(37) + 3),x, algorithm="fricas")
```

```
[Out] Exception raised: SyntaxError
```

Sympy [A] time = 0.066213, size = 14, normalized size = 0.78

$$\frac{x^3}{3} + \frac{3x^2}{2} - 7x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x+3/2-1/2*37**(1/2))*(x+3/2+1/2*37**(1/2)),x)
```

```
[Out] x**3/3 + 3*x**2/2 - 7*x
```

GIAC/XCAS [A] time = 0.25815, size = 19, normalized size = 1.06

$$\frac{1}{3}x^3 + \frac{3}{2}x^2 - 7x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/4*(2*x + sqrt(37) + 3)*(2*x - sqrt(37) + 3),x, algorithm="giac")
```

```
[Out] 1/3*x^3 + 3/2*x^2 - 7*x
```

3.484 $\int \frac{4+3x^2+2x^3}{(1+x)^4} \, dx$

Optimal. Leaf size=23

$$\frac{3}{x+1} - \frac{5}{3(x+1)^3} + 2\log(x+1)$$

```
[Out] -5/(3*(1 + x)^3) + 3/(1 + x) + 2*Log[1 + x]
```

Rubi [A] time = 0.0374137, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056

$$\frac{3}{x+1} - \frac{5}{3(x+1)^3} + 2\log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[(4 + 3*x^2 + 2*x^3)/(1 + x)^4,x]

[Out] -5/(3*(1 + x)^3) + 3/(1 + x) + 2*Log[1 + x]
```

Rubi in Sympy [A] time = 7.27385, size = 19, normalized size = 0.83

$$2\log(x+1) + \frac{3}{x+1} - \frac{5}{3(x+1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*x**3+3*x**2+4)/(1+x)**4,x)

[Out] 2*log(x + 1) + 3/(x + 1) - 5/(3*(x + 1)**3)
```

Mathematica [A] time = 0.0176103, size = 23, normalized size = 1.

$$\frac{3}{x+1} - \frac{5}{3(x+1)^3} + 2\log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(4 + 3*x^2 + 2*x^3)/(1 + x)^4,x]

[Out] -5/(3*(1 + x)^3) + 3/(1 + x) + 2*Log[1 + x]
```

Maple [A] time = 0.009, size = 22, normalized size = 1.

$$-\frac{5}{3(1+x)^3} + 3(1+x)^{-1} + 2\ln(1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*x^3+3*x^2+4)/(1+x)^4,x)
```

```
[Out] -5/3/(1+x)^3+3/(1+x)+2*ln(1+x)
```

Maxima [A] time = 0.794135, size = 46, normalized size = 2.

$$\frac{9\,x^2+18\,x+4}{3\,(x^3+3\,x^2+3\,x+1)}+2\,\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^3 + 3*x^2 + 4)/(x + 1)^4,x, algorithm="maxima")
```

```
[Out] 1/3*(9*x^2 + 18*x + 4)/(x^3 + 3*x^2 + 3*x + 1) + 2*log(x + 1)
```

Fricas [A] time = 0.24939, size = 62, normalized size = 2.7

$$\frac{9\,x^2+6\,\left(x^3+3\,x^2+3\,x+1\right)\,\log\left(x+1\right)+18\,x+4}{3\,(x^3+3\,x^2+3\,x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^3 + 3*x^2 + 4)/(x + 1)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(9*x^2 + 6*(x^3 + 3*x^2 + 3*x + 1)*log(x + 1) + 18*x + 4)/(x^
3 + 3*x^2 + 3*x + 1)
```

Sympy [A] time = 0.249484, size = 31, normalized size = 1.35

$$\frac{9x^2+18x+4}{3x^3+9x^2+9x+3}+2\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x**3+3*x**2+4)/(1+x)**4,x)
```

```
[Out] (9*x**2 + 18*x + 4)/(3*x**3 + 9*x**2 + 9*x + 3) + 2*log(x + 1)
```

GIAC/XCAS [A] time = 0.260678, size = 34, normalized size = 1.48

$$\frac{9\,x^2+18\,x+4}{3\,(x+1)^3}+2\ln\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^3 + 3*x^2 + 4)/(x + 1)^4,x, algorithm="giac")
```

```
[Out] 1/3*(9*x^2 + 18*x + 4)/(x + 1)^3 + 2*ln(abs(x + 1))
```

3.485 $\int \frac{x}{(1+x)^2(1+x^2)}\,dx$

Optimal. Leaf size=16

$$\frac{1}{2(x+1)}+\frac{1}{2}\tan^{-1}(x)$$

```
[Out] 1/(2*(1 + x)) + ArcTan[x]/2
```

Rubi [A] time = 0.0374706, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{1}{2(x+1)}+\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[x/((1 + x)^2*(1 + x^2)),x]

[Out] 1/(2*(1 + x)) + ArcTan[x]/2
```

Rubi in Sympy [A] time = 4.70513, size = 10, normalized size = 0.62

$$\frac{\operatorname{atan}(x)}{2}+\frac{1}{2(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(1+x)**2/(x**2+1),x)

[Out] atan(x)/2 + 1/(2*(x + 1))
```

Mathematica [A] time = 0.0105242, size = 12, normalized size = 0.75

$$\frac{1}{2}\left(\frac{1}{x+1}+\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/((1 + x)^2*(1 + x^2)),x]

[Out] ((1 + x)^(-1) + ArcTan[x])/2
```

Maple [A] time = 0.006, size = 13, normalized size = 0.8

$$\frac{1}{2+2x}+\frac{\arctan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(1+x)^2/(x^2+1),x)
```

```
[Out] 1/2/(1+x)+1/2*arctan(x)
```

Maxima [A] time = 0.876112, size = 16, normalized size = 1.

$$\frac{1}{2(x+1)} + \frac{1}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((x^2 + 1)*(x + 1)^2),x, algorithm="maxima")
```

```
[Out] 1/2/(x + 1) + 1/2*arctan(x)
```

Fricas [A] time = 0.248934, size = 20, normalized size = 1.25

$$\frac{(x+1)\arctan(x)+1}{2(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((x^2 + 1)*(x + 1)^2),x, algorithm="fricas")
```

```
[Out] 1/2*((x + 1)*arctan(x) + 1)/(x + 1)
```

Sympy [A] time = 0.242806, size = 10, normalized size = 0.62

$$\frac{\operatorname{atan}(x)}{2} + \frac{1}{2x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(1+x)**2/(x**2+1),x)
```

```
[Out] atan(x)/2 + 1/(2*x + 2)
```

GIAC/XCAS [A] time = 0.264845, size = 43, normalized size = 2.69

$$-\frac{1}{8}\pi - \frac{1}{2}\pi\left\lfloor -\frac{\pi - 4\arctan(x)}{4\pi} + \frac{1}{2}\right\rfloor + \frac{1}{2(x+1)} + \frac{1}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((x^2 + 1)*(x + 1)^2),x, algorithm="giac")
```

```
[Out] -1/8*pi - 1/2*pi*floor(-1/4*(pi - 4*arctan(x))/pi + 1/2) + 1/2/(x
 + 1) + 1/2*arctan(x)
```

3.486 $\int \frac{7-2x+3x^2-x^3+x^4}{2+x} dx$

Optimal. Leaf size=29

$$\frac{x^4}{4} - x^3 + \frac{9x^2}{2} - 20x + 47\log(x+2)$$

```
[Out] -20*x + (9*x^2)/2 - x^3 + x^4/4 + 47*Log[2 + x]
```

Rubi [A] time = 0.0425133, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$

$$\frac{x^4}{4} - x^3 + \frac{9x^2}{2} - 20x + 47\log(x+2)$$

Antiderivative was successfully verified.

```
[In]  Int[(7 - 2*x + 3*x^2 - x^3 + x^4)/(2 + x),x]

[Out] -20*x + (9*x^2)/2 - x^3 + x^4/4 + 47*Log[2 + x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{x^4}{4} - x^3 - 20x + 47\log(x+2) + 9\int x\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**4-x**3+3*x**2-2*x+7)/(2+x),x)

[Out] x**4/4 - x**3 - 20*x + 47*log(x + 2) + 9*Integral(x, x)
```

Mathematica [A] time = 0.0130473, size = 30, normalized size = 1.03

$$\frac{x^4}{4} - x^3 + \frac{9x^2}{2} - 20x + 47\log(x+2) - 70$$

Antiderivative was successfully verified.

```
[In]  Integrate[(7 - 2*x + 3*x^2 - x^3 + x^4)/(2 + x),x]

[Out] -70 - 20*x + (9*x^2)/2 - x^3 + x^4/4 + 47*Log[2 + x]
```

Maple [A] time = 0.003, size = 26, normalized size = 0.9

$$-20\,x + \frac{9\,x^2}{2} - x^3 + \frac{x^4}{4} + 47\,\ln(2+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^4-x^3+3*x^2-2*x+7)/(2+x),x)
```

```
[Out] -20*x+9/2*x^2-x^3+1/4*x^4+47*ln(2+x)
```

Maxima [A] time = 0.795033, size = 34, normalized size = 1.17

$$\frac{1}{4}x^4 - x^3 + \frac{9}{2}x^2 - 20x + 47\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 - x^3 + 3*x^2 - 2*x + 7)/(x + 2),x, algorithm="maxima")
```

```
[Out] 1/4*x^4 - x^3 + 9/2*x^2 - 20*x + 47*log(x + 2)
```

Fricas [A] time = 0.25018, size = 34, normalized size = 1.17

$$\frac{1}{4}x^4 - x^3 + \frac{9}{2}x^2 - 20x + 47\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 - x^3 + 3*x^2 - 2*x + 7)/(x + 2),x, algorithm="fricas")
```

```
[Out] 1/4*x^4 - x^3 + 9/2*x^2 - 20*x + 47*log(x + 2)
```

Sympy [A] time = 0.13882, size = 24, normalized size = 0.83

$$\frac{x^4}{4} - x^3 + \frac{9x^2}{2} - 20x + 47\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**4-x**3+3*x**2-2*x+7)/(2+x),x)
```

```
[Out] x**4/4 - x**3 + 9*x**2/2 - 20*x + 47*log(x + 2)
```

GIAC/XCAS [A] time = 0.259807, size = 35, normalized size = 1.21

$$\frac{1}{4}x^4 - x^3 + \frac{9}{2}x^2 - 20x + 47\ln(|x+2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 - x^3 + 3*x^2 - 2*x + 7)/(x + 2),x, algorithm="giac")
```

```
[Out] 1/4*x^4 - x^3 + 9/2*x^2 - 20*x + 47*ln(abs(x + 2))
```

3.487 $\int \frac{-1+x^3}{-1+x} \, dx$

Optimal. Leaf size=16

$$\frac{x^3}{3}+\frac{x^2}{2}+x$$

```
[Out] x + x^2/2 + x^3/3
```

Rubi [A] time = 0.0118253, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{x^3}{3}+\frac{x^2}{2}+x$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^3)/(-1 + x),x]

[Out] x + x^2/2 + x^3/3
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{x^3}{3}+x+\int x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((x**3-1)/(-1+x),x)

[Out] x**3/3 + x + Integral(x, x)
```

Mathematica [A] time = 0.000656925, size = 16, normalized size = 1.

$$\frac{x^3}{3}+\frac{x^2}{2}+x$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^3)/(-1 + x),x]

[Out] x + x^2/2 + x^3/3
```

Maple [A] time = 0.001, size = 13, normalized size = 0.8

$$x+\frac{x^2}{2}+\frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)/(-1+x),x)
```

```
[Out] x+1/2*x^2+1/3*x^3
```

Maxima [A] time = 0.785423, size = 16, normalized size = 1.

$$\frac{1}{3}x^3 + \frac{1}{2}x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - 1)/(x - 1),x, algorithm="maxima")
```

```
[Out] 1/3*x^3 + 1/2*x^2 + x
```

Fricas [A] time = 0.24503, size = 16, normalized size = 1.

$$\frac{1}{3}x^3 + \frac{1}{2}x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - 1)/(x - 1),x, algorithm="fricas")
```

```
[Out] 1/3*x^3 + 1/2*x^2 + x
```

Sympy [A] time = 0.07334, size = 10, normalized size = 0.62

$$\frac{x^3}{3} + \frac{x^2}{2} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3-1)/(-1+x),x)
```

```
[Out] x**3/3 + x**2/2 + x
```

GIAC/XCAS [A] time = 0.259049, size = 16, normalized size = 1.

$$\frac{1}{3}x^3 + \frac{1}{2}x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - 1)/(x - 1),x, algorithm="giac")
```

```
[Out] 1/3*x^3 + 1/2*x^2 + x
```

3.488 $\int \frac{2+2x}{(-1+x)^3(1+x^2)}\,dx$

Optimal. Leaf size=17

$$\frac{1}{x-1}-\frac{1}{(1-x)^2}+\tan^{-1}(x)$$

```
[Out] -(1 - x)^(-2) + (-1 + x)^(-1) + ArcTan[x]
```

Rubi [A] time = 0.0448863, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\frac{1}{x-1}-\frac{1}{(1-x)^2}+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + 2*x)/((-1 + x)^3*(1 + x^2)),x]

[Out] -(1 - x)^(-2) + (-1 + x)^(-1) + ArcTan[x]
```

Rubi in Sympy [A] time = 5.93668, size = 14, normalized size = 0.82

$$\operatorname{atan}(x)-\frac{1}{-x+1}-\frac{1}{(-x+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2+2*x)/(-1+x)**3/(x**2+1),x)

[Out] atan(x) - 1/(-x + 1) - 1/(-x + 1)**2
```

Mathematica [A] time = 0.0194569, size = 17, normalized size = 1.

$$\frac{x+(x-1)^2\tan^{-1}(x)-2}{(x-1)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + 2*x)/((-1 + x)^3*(1 + x^2)),x]

[Out] (-2 + x + (-1 + x)^2*ArcTan[x])/(-1 + x)^2
```

Maple [A] time = 0.009, size = 16, normalized size = 0.9

$$-(-1+x)^{-2}+(-1+x)^{-1}+\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2+2*x)/(-1+x)^3/(x^2+1),x)
```

```
[Out] -1/(-1+x)^2+1/(-1+x)+arctan(x)
```

Maxima [A] time = 0.855404, size = 23, normalized size = 1.35

$$\frac{x-2}{x^2-2\,x+1}+\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*(x + 1)/((x^2 + 1)*(x - 1)^3),x, algorithm="maxima")
```

```
[Out] (x - 2)/(x^2 - 2*x + 1) + arctan(x)
```

Fricas [A] time = 0.263458, size = 34, normalized size = 2.

$$\frac{\left(x^2-2\,x+1\right)\arctan\left(x\right)+x-2}{x^2-2\,x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*(x + 1)/((x^2 + 1)*(x - 1)^3),x, algorithm="fricas")
```

```
[Out] ((x^2 - 2*x + 1)*arctan(x) + x - 2)/(x^2 - 2*x + 1)
```

Sympy [A] time = 0.308353, size = 14, normalized size = 0.82

$$\frac{x-2}{x^2-2x+1}+\operatorname{atan}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+2*x)/(-1+x)**3/(x**2+1),x)
```

```
[Out] (x - 2)/(x**2 - 2*x + 1) + atan(x)
```

GIAC/XCAS [A] time = 0.261325, size = 16, normalized size = 0.94

$$\frac{x-2}{\left(x-1\right)^2}+\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*(x + 1)/((x^2 + 1)*(x - 1)^3),x, algorithm="giac")
```

```
[Out] (x - 2)/(x - 1)^2 + arctan(x)
```

3.489 $\int \frac{1}{bx+c(d+ex)^2} dx$

Optimal. Leaf size=47

$$-\frac{2\tanh^{-1}\left(\frac{b+2ce(d+ex)}{\sqrt{b}\sqrt{b+4cde}}\right)}{\sqrt{b}\sqrt{b+4cde}}$$

```
[Out] (-2*ArcTanh[(b + 2*c*e*(d + e*x))/(Sqrt[b]*Sqrt[b + 4*c*d*e])])/(
Sqrt[b]*Sqrt[b + 4*c*d*e])
```

Rubi [A] time = 0.128169, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$-\frac{2\tanh^{-1}\left(\frac{b+2ce(d+ex)}{\sqrt{b}\sqrt{b+4cde}}\right)}{\sqrt{b}\sqrt{b+4cde}}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x + c*(d + e*x)^2)^(-1),x]
```

```
[Out] (-2*ArcTanh[(b + 2*c*e*(d + e*x))/(Sqrt[b]*Sqrt[b + 4*c*d*e])])/(
Sqrt[b]*Sqrt[b + 4*c*d*e])
```

Rubi in Sympy [A] time = 8.43813, size = 54, normalized size = 1.15

$$-\frac{2\,\mathrm{atanh}\left(\frac{b+2cde+2ce^2x}{\sqrt{b}\sqrt{b+4cde}}\right)}{\sqrt{b}\sqrt{b+4cde}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x+c*(e*x+d)**2),x)
```

```
[Out] -2*atanh((b + 2*c*d*e + 2*c*e**2*x)/(sqrt(b)*sqrt(b + 4*c*d*e)))/
(sqrt(b)*sqrt(b + 4*c*d*e))
```

Mathematica [A] time = 0.0403079, size = 47, normalized size = 1.

$$-\frac{2\tanh^{-1}\left(\frac{b+2ce(d+ex)}{\sqrt{b}\sqrt{b+4cde}}\right)}{\sqrt{b}\sqrt{b+4cde}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x + c*(d + e*x)^2)^(-1),x]
```

```
[Out] (-2*ArcTanh[(b + 2*c*e*(d + e*x))/(Sqrt[b]*Sqrt[b + 4*c*d*e])])/(
Sqrt[b]*Sqrt[b + 4*c*d*e])
```

Maple [A] time = 0.009, size = 43, normalized size = 0.9

$$-2\,\frac{1}{\sqrt{4\,bcde+b^2}}\mathit{Artanh}\left(\frac{2\,ce^2x+2\,cde+b}{\sqrt{4\,bcde+b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x+c*(e*x+d)^2),x)
```

```
[Out] -2/(4*b*c*d*e+b^2)^(1/2)*arctanh((2*c*e^2*x+2*c*d*e+b)/(4*b*c*d*e
+b^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((e*x + d)^2*c + b*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.283352, size = 1, normalized size = 0.02

$$\left[\frac{\log\left(-\frac{8\,bc^{2}d^{2}e^{2}+6\,b^{2}cde+b^{3}+2\,\left(4\,bc^{2}de^{3}+b^{2}ce^{2}\right)x-\left(2\,c^{2}e^{4}x^{2}+2\,c^{2}d^{2}e^{2}+4\,bcde+b^{2}+2\,\left(2\,c^{2}de^{3}+bce^{2}\right)x\right)\sqrt{4\,bcde+b^{2}}}{ce^{2}x^{2}+cd^{2}+(2\,cde+b)x}\right)}{\sqrt{4\,bcde+b^{2}}},\ -\frac{2\,\arctan\left(\frac{\sqrt{-4\,bcde-b^{2}}\left(2\,ce^{2}x+2\,cde+b\right)}{4\,bcde+b^{2}}\right)}{\sqrt{-4\,bcde-b^{2}}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((e*x + d)^2*c + b*x),x, algorithm="fricas")
```

```
[Out] [log(-(8*b*c^2*d^2*e^2 + 6*b^2*c*d*e + b^3 + 2*(4*b*c^2*d*e^3 + b
^2*c*e^2)*x - (2*c^2*e^4*x^2 + 2*c^2*d^2*e^2 + 4*b*c*d*e + b^2 +
2*(2*c^2*d*e^3 + b*c*e^2)*x)*sqrt(4*b*c*d*e + b^2))/(c*e^2*x^2 +
c*d^2 + (2*c*d*e + b)*x))/sqrt(4*b*c*d*e + b^2), -2*arctan(sqrt(-
4*b*c*d*e - b^2)*(2*c*e^2*x + 2*c*d*e + b)/(4*b*c*d*e + b^2))/sqr
t(-4*b*c*d*e - b^2)]
```

Sympy [A] time = 0.830143, size = 151, normalized size = 3.21

$$\sqrt{\frac{1}{b\,(b+4cde)}}\log\left(x+\frac{-b^{2}\sqrt{\frac{1}{b(b+4cde)}}-4bcde\sqrt{\frac{1}{b(b+4cde)}}+b+2cde}{2ce^{2}}\right)-\sqrt{\frac{1}{b\,(b+4cde)}}\log\left(x+\frac{b^{2}\sqrt{\frac{1}{b(b+4cde)}}+4bcde\sqrt{\frac{1}{b(b+4cde)}}+b+2cde}{2ce^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x+c*(e*x+d)**2),x)
```

```
[Out] sqrt(1/(b*(b + 4*c*d*e)))*log(x + (-b**2*sqrt(1/(b*(b + 4*c*d*e))
) - 4*b*c*d*e*sqrt(1/(b*(b + 4*c*d*e))) + b + 2*c*d*e)/(2*c*e**2)
```

```
) - sqrt(1/(b*(b + 4*c*d*e)))*log(x + (b**2*sqrt(1/(b*(b + 4*c*d*
e))) + 4*b*c*d*e*sqrt(1/(b*(b + 4*c*d*e))) + b + 2*c*d*e)/(2*c*e*
*2))
```

GIAC/XCAS [A] time = 0.261169, size = 65, normalized size = 1.38

$$\frac{2\,\arctan\left(\frac{2\,cxe^2+2\,cde+b}{\sqrt{-4\,bcde-b^2}}\right)}{\sqrt{-4\,bcde-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((e*x + d)^2*c + b*x),x, algorithm="giac")
```

```
[Out] 2*arctan((2*c*x*e^2 + 2*c*d*e + b)/sqrt(-4*b*c*d*e - b^2))/sqrt(-
4*b*c*d*e - b^2)
```

3.490 $\int \frac{1}{a+bx+c(d+ex)^2} dx$

Optimal. Leaf size=57

$$-\frac{2\tanh^{-1}\left(\frac{b+2ce(d+ex)}{\sqrt{-4ace^2+b^2+4bcde}}\right)}{\sqrt{-4ace^2+b^2+4bcde}}$$

```
[Out] (-2*ArcTanh[(b + 2*c*e*(d + e*x))/Sqrt[b^2 + 4*b*c*d*e - 4*a*c*e^
2]])/Sqrt[b^2 + 4*b*c*d*e - 4*a*c*e^2]
```

Rubi [A] time = 0.158596, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$

$$-\frac{2\tanh^{-1}\left(\frac{b+2ce(d+ex)}{\sqrt{-4ace^2+b^2+4bcde}}\right)}{\sqrt{-4ace^2+b^2+4bcde}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x + c*(d + e*x)^2)^(-1),x]
```

```
[Out] (-2*ArcTanh[(b + 2*c*e*(d + e*x))/Sqrt[b^2 + 4*b*c*d*e - 4*a*c*e^
2]])/Sqrt[b^2 + 4*b*c*d*e - 4*a*c*e^2]
```

Rubi in Sympy [A] time = 11.8583, size = 68, normalized size = 1.19

$$-\frac{2\,\mathrm{atanh}\left(\frac{b+2cde+2ce^2x}{\sqrt{-4ace^2+b^2+4bcde}}\right)}{\sqrt{-4ace^2+b^2+4bcde}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a+b*x+c*(e*x+d)**2),x)
```

```
[Out] -2*atanh((b + 2*c*d*e + 2*c*e**2*x)/sqrt(-4*a*c*e**2 + b**2 + 4*b
*c*d*e))/sqrt(-4*a*c*e**2 + b**2 + 4*b*c*d*e)
```

Mathematica [A] time = 0.0476144, size = 61, normalized size = 1.07

$$\frac{2\tan^{-1}\left(\frac{b+2ce(d+ex)}{\sqrt{4ace^2-b^2-4bcde}}\right)}{\sqrt{4ace^2-b^2-4bcde}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x + c*(d + e*x)^2)^(-1),x]
```

```
[Out] (2*ArcTan[(b + 2*c*e*(d + e*x))/Sqrt[-b^2 - 4*b*c*d*e + 4*a*c*e^2
]])/Sqrt[-b^2 - 4*b*c*d*e + 4*a*c*e^2]
```

Maple [A] time = 0.009, size = 61, normalized size = 1.1

$$2\frac{1}{\sqrt{4ace^2-4bcde-b^2}}\arctan\left(\frac{2ce^2x+2cde+b}{\sqrt{4ace^2-4bcde-b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a+b*x+c*(e*x+d)^2),x)
```

```
[Out] 2/(4*a*c*e^2-4*b*c*d*e-b^2)^(1/2)*arctan((2*c*e^2*x+2*c*d*e+b)/(4
*a*c*e^2-4*b*c*d*e-b^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((e*x + d)^2*c + b*x + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.276118, size = 1, normalized size = 0.02

$$\left[\frac{\log\left(\frac{8\,ac^2de^3-6\,b^2cde-b^3-4\,(2\,bc^2d^2-abc)\,e^2-2\,(4\,bc^2de^3-4\,ac^2e^4+b^2ce^2)\,x+(2\,c^2e^4x^2+4\,bcde+2\,(c^2d^2-ac)\,e^2+b^2+2\,(2\,c^2de^3+bce^2)\,x)\,\sqrt{4\,bcde-4\,ace}}{ce^2x^2+cd^2+(2\,cde+b)x+a}\right.}{\sqrt{4\,bcde-4\,ace^2+b^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((e*x + d)^2*c + b*x + a),x, algorithm="fricas")
```

```
[Out] [log((8*a*c^2*d*e^3 - 6*b^2*c*d*e - b^3 - 4*(2*b*c^2*d^2 - a*b*c)
*e^2 - 2*(4*b*c^2*d*e^3 - 4*a*c^2*e^4 + b^2*c*e^2)*x + (2*c^2*e^4
*x^2 + 4*b*c*d*e + 2*(c^2*d^2 - a*c)*e^2 + b^2 + 2*(2*c^2*d*e^3 +
 b*c*e^2)*x)*sqrt(4*b*c*d*e - 4*a*c*e^2 + b^2))/(c*e^2*x^2 + c*d^
2 + (2*c*d*e + b)*x + a))/sqrt(4*b*c*d*e - 4*a*c*e^2 + b^2), 2*ar
ctan(-sqrt(-4*b*c*d*e + 4*a*c*e^2 - b^2)*(2*c*e^2*x + 2*c*d*e + b
)/(4*b*c*d*e - 4*a*c*e^2 + b^2))/sqrt(-4*b*c*d*e + 4*a*c*e^2 - b^
2)]
```

Sympy [A] time = 1.0462, size = 294, normalized size = 5.16

$$-\sqrt{-\frac{1}{4ace^2-b^2-4bcde}}\log\left(x+\frac{-4ace^2\sqrt{-\frac{1}{4ace^2-b^2-4bcde}}+b^2\sqrt{-\frac{1}{4ace^2-b^2-4bcde}}+4bcde\sqrt{-\frac{1}{4ace^2-b^2-4bcde}}+b+2cde}{2ce^2}\right)$$
$$+\sqrt{-\frac{1}{4ace^2-b^2-4bcde}}\log\left(x+\frac{4ace^2\sqrt{-\frac{1}{4ace^2-b^2-4bcde}}-b^2\sqrt{-\frac{1}{4ace^2-b^2-4bcde}}-4bcde\sqrt{-\frac{1}{4ace^2-b^2-4bcde}}+b+2cde}{2ce^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a+b*x+c*(e*x+d)**2),x)
```

```
[Out] -sqrt(-1/(4*a*c*e**2 - b**2 - 4*b*c*d*e))*log(x + (-4*a*c*e**2*sq
rt(-1/(4*a*c*e**2 - b**2 - 4*b*c*d*e)) + b**2*sqrt(-1/(4*a*c*e**2
 - b**2 - 4*b*c*d*e)) + 4*b*c*d*e*sqrt(-1/(4*a*c*e**2 - b**2 - 4*
b*c*d*e)) + b + 2*c*d*e)/(2*c*e**2)) + sqrt(-1/(4*a*c*e**2 - b**2
 - 4*b*c*d*e))*log(x + (4*a*c*e**2*sqrt(-1/(4*a*c*e**2 - b**2 - 4
```

```
*b*c*d*e)) - b**2*sqrt(-1/(4*a*c*e**2 - b**2 - 4*b*c*d*e)) - 4*b*
c*d*e*sqrt(-1/(4*a*c*e**2 - b**2 - 4*b*c*d*e)) + b + 2*c*d*e)/(2*
c*e**2))
```

GIAC/XCAS [A] time = 0.260004, size = 81, normalized size = 1.42

$$\frac{2\,\arctan\left(\frac{2\,cxe^{2}+2\,cde+b}{\sqrt{-4\,bcde+4\,ace^{2}-b^{2}}}\right)}{\sqrt{-4\,bcde+4\,ace^{2}-b^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((e*x + d)^2*c + b*x + a),x, algorithm="giac")
```

```
[Out] 2*arctan((2*c*x*e^2 + 2*c*d*e + b)/sqrt(-4*b*c*d*e + 4*a*c*e^2 -
b^2))/sqrt(-4*b*c*d*e + 4*a*c*e^2 - b^2)
```

3.491 $\int \frac{x^2}{1+(-1+x^2)^2}\,dx$

Optimal. Leaf size=188

$$\frac{\log\left(x^2-\sqrt{2\left(1+\sqrt{2}\right)}x+\sqrt{2}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{\log\left(x^2+\sqrt{2\left(1+\sqrt{2}\right)}x+\sqrt{2}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2x}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)+\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\tan^{-1}\left(\frac{2x+\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)$$

```
[Out] -(Sqrt[(1 + Sqrt[2])/2]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] - 2*x)/Sqrt
[2*(-1 + Sqrt[2])]])/2 + (Sqrt[(1 + Sqrt[2])/2]*ArcTan[(Sqrt[2*(1
 + Sqrt[2])] + 2*x)/Sqrt[2*(-1 + Sqrt[2])]])/2 + Log[Sqrt[2] - Sq
rt[2*(1 + Sqrt[2])]*x + x^2]/(4*Sqrt[2*(1 + Sqrt[2])]) - Log[Sqrt
[2] + Sqrt[2*(1 + Sqrt[2])]*x + x^2]/(4*Sqrt[2*(1 + Sqrt[2])])
```

Rubi [A] time = 0.397477, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$

$$\frac{\log\left(x^2-\sqrt{2\left(1+\sqrt{2}\right)}x+\sqrt{2}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{\log\left(x^2+\sqrt{2\left(1+\sqrt{2}\right)}x+\sqrt{2}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2x}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)+\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\tan^{-1}\left(\frac{2x+\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(1 + (-1 + x^2)^2),x]
```

```
[Out] -(Sqrt[(1 + Sqrt[2])/2]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] - 2*x)/Sqrt
[2*(-1 + Sqrt[2])]])/2 + (Sqrt[(1 + Sqrt[2])/2]*ArcTan[(Sqrt[2*(1
 + Sqrt[2])] + 2*x)/Sqrt[2*(-1 + Sqrt[2])]])/2 + Log[Sqrt[2] - Sq
rt[2*(1 + Sqrt[2])]*x + x^2]/(4*Sqrt[2*(1 + Sqrt[2])]) - Log[Sqrt
[2] + Sqrt[2*(1 + Sqrt[2])]*x + x^2]/(4*Sqrt[2*(1 + Sqrt[2])])
```

Rubi in Sympy [A] time = 36.3409, size = 185, normalized size = 0.98

$$\frac{\sqrt{2}\log\left(x^2-\sqrt{2}x\sqrt{1+\sqrt{2}}+\sqrt{2}\right)}{8\sqrt{1+\sqrt{2}}}-\frac{\sqrt{2}\log\left(x^2+\sqrt{2}x\sqrt{1+\sqrt{2}}+\sqrt{2}\right)}{8\sqrt{1+\sqrt{2}}}+\frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\left(x-\frac{\sqrt{2+2\sqrt{2}}}{2}\right)}{\sqrt{-1+\sqrt{2}}}\right)}{4\sqrt{-1+\sqrt{2}}}+\frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\left(x+\frac{\sqrt{2+2\sqrt{2}}}{2}\right)}{\sqrt{-1+\sqrt{2}}}\right)}{4\sqrt{-1+\sqrt{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(1+(x**2-1)**2),x)
```

```
[Out] sqrt(2)*log(x**2 - sqrt(2)*x*sqrt(1 + sqrt(2)) + sqrt(2))/(8*sqrt
(1 + sqrt(2))) - sqrt(2)*log(x**2 + sqrt(2)*x*sqrt(1 + sqrt(2)) +
 sqrt(2))/(8*sqrt(1 + sqrt(2))) + sqrt(2)*atan(sqrt(2)*(x - sqrt(
2 + 2*sqrt(2))/2)/sqrt(-1 + sqrt(2)))/(4*sqrt(-1 + sqrt(2))) + sq
rt(2)*atan(sqrt(2)*(x + sqrt(2 + 2*sqrt(2))/2)/sqrt(-1 + sqrt(2))
)/(4*sqrt(-1 + sqrt(2)))
```

Mathematica [C] time = 0.0456302, size = 39, normalized size = 0.21

$$-\frac{\tan^{-1}\left(\frac{x}{\sqrt{-1-i}}\right)}{(-1-i)^{3/2}}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{-1+i}}\right)}{(-1+i)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(1 + (-1 + x^2)^2),x]
```

```
[Out] -(ArcTan[x/Sqrt[-1 - I]]/(-1 - I)^(3/2)) - ArcTan[x/Sqrt[-1 + I]]
/(-1 + I)^(3/2)
```

Maple [B] time = 0.047, size = 308, normalized size = 1.6

$$\frac{\sqrt{2+2\sqrt{2}}\sqrt{2}\ln\left(x^2+\sqrt{2}-x\sqrt{2+2\sqrt{2}}\right)}{8}+\frac{\sqrt{2}\left(2+2\sqrt{2}\right)}{4\sqrt{-2+2\sqrt{2}}}\arctan\left(\frac{2x-\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)-\frac{\sqrt{2+2\sqrt{2}}\ln\left(x^2+\sqrt{2}-x\sqrt{2+2\sqrt{2}}\right)}{8}-\frac{2+2\sqrt{2}}{4\sqrt{-2+2\sqrt{2}}}\arctan\left(\frac{2x-\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)-\frac{\sqrt{2+2\sqrt{2}}\sqrt{2}\ln\left(x^2+\sqrt{2}+x\sqrt{2+2\sqrt{2}}\right)}{8}+\frac{\sqrt{2}\left(2+2\sqrt{2}\right)}{4\sqrt{-2+2\sqrt{2}}}\arctan\left(\frac{2x+\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)+\frac{\sqrt{2+2\sqrt{2}}\ln\left(x^2+\sqrt{2}+x\sqrt{2+2\sqrt{2}}\right)}{8}-\frac{2+2\sqrt{2}}{4\sqrt{-2+2\sqrt{2}}}\arctan\left(\frac{2x+\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(1+(x^2-1)^2),x)
```

```
[Out] 1/8*(2+2*2^(1/2))^(1/2)*2^(1/2)*ln(x^2+2^(1/2)-x*(2+2*2^(1/2))^(1
/2))+1/4*2^(1/2)*(2+2*2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*x-(
2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))-1/8*(2+2*2^(1/2))^(1/2)
*ln(x^2+2^(1/2)-x*(2+2*2^(1/2))^(1/2))-1/4*(2+2*2^(1/2))/(-2+2*2^
(1/2))^(1/2)*arctan((2*x-(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2
))-1/8*(2+2*2^(1/2))^(1/2)*2^(1/2)*ln(x^2+2^(1/2)+x*(2+2*2^(1/2))
^(1/2))+1/4*2^(1/2)*(2+2*2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*
x+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))+1/8*(2+2*2^(1/2))^(1
/2)*ln(x^2+2^(1/2)+x*(2+2*2^(1/2))^(1/2))-1/4*(2+2*2^(1/2))/(-2+2
*2^(1/2))^(1/2)*arctan((2*x+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(
1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2}{(x^2-1)^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((x^2 - 1)^2 + 1),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((x^2 - 1)^2 + 1), x)
```

Fricas [A] time = 0.28938, size = 630, normalized size = 3.35

$$\sqrt{2}\left(2^{\frac{1}{4}}\left(\sqrt{2}-1\right)\log\left(24\sqrt{2}x^2+2^{\frac{3}{4}}\left(17\sqrt{2}x-24x\right)\sqrt{\frac{\sqrt{2}-2}{2\sqrt{2}-3}}-34x^2+2\sqrt{2}\left(12\sqrt{2}-17\right)\right)-2^{\frac{1}{4}}\left(\sqrt{2}-1\right)\log\left(24\sqrt{2}x^2-2^{\frac{3}{4}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((x^2 - 1)^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2)*(2^(1/4)*(sqrt(2) - 1)*log(24*sqrt(2)*x^2 + 2^(3/4)*(
17*sqrt(2)*x - 24*x)*sqrt((sqrt(2) - 2)/(2*sqrt(2) - 3)) - 34*x^2
 + 2*sqrt(2)*(12*sqrt(2) - 17)) - 2^(1/4)*(sqrt(2) - 1)*log(24*sq
rt(2)*x^2 - 2^(3/4)*(17*sqrt(2)*x - 24*x)*sqrt((sqrt(2) - 2)/(2*s
qrt(2) - 3)) - 34*x^2 + 2*sqrt(2)*(12*sqrt(2) - 17)) - 4*2^(1/4)*
arctan(2^(1/4)*(sqrt(2) - 1)/(sqrt(2)*sqrt(1/2)*(sqrt(2) - 1)*sqr
t((24*sqrt(2)*x^2 + 2^(3/4)*(17*sqrt(2)*x - 24*x)*sqrt((sqrt(2) -
 2)/(2*sqrt(2) - 3)) - 34*x^2 + 2*sqrt(2)*(12*sqrt(2) - 17))/(12*
sqrt(2) - 17))*sqrt((sqrt(2) - 2)/(2*sqrt(2) - 3)) + sqrt(2)*(sqr
t(2)*x - x)*sqrt((sqrt(2) - 2)/(2*sqrt(2) - 3)) - 2^(1/4))) - 4*2
^(1/4)*arctan(2^(1/4)*(sqrt(2) - 1)/(sqrt(2)*sqrt(1/2)*(sqrt(2) -
 1)*sqrt((24*sqrt(2)*x^2 - 2^(3/4)*(17*sqrt(2)*x - 24*x)*sqrt((sq
rt(2) - 2)/(2*sqrt(2) - 3)) - 34*x^2 + 2*sqrt(2)*(12*sqrt(2) - 17
))/(12*sqrt(2) - 17))*sqrt((sqrt(2) - 2)/(2*sqrt(2) - 3)) + sqrt(
2)*(sqrt(2)*x - x)*sqrt((sqrt(2) - 2)/(2*sqrt(2) - 3)) + 2^(1/4))
))/((sqrt(2) - 1)*sqrt((sqrt(2) - 2)/(2*sqrt(2) - 3)))
```

Sympy [A] time = 1.76616, size = 24, normalized size = 0.13

$$\operatorname{RootSum}\left(128t^4+16t^2+1,\left(t\mapsto t\log\left(64t^3+4t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(1+(x**2-1)**2),x)
```

```
[Out] RootSum(128*_t**4 + 16*_t**2 + 1, Lambda(_t, _t*log(64*_t**3 + 4*
_t + x)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2}{\left(x^2-1\right)^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((x^2 - 1)^2 + 1),x, algorithm="giac")
```

```
[Out] integrate(x^2/((x^2 - 1)^2 + 1), x)
```

3.492 $\int -\frac{15-36x+5x^2+12x^3-34x^4+140x^5+15x^6+8x^7-30x^9}{(3+x+x^4)^4}\,dx$

Optimal. Leaf size=60

$$\frac{x^4}{(x^4+x+3)^3}-\frac{3x}{(x^4+x+3)^3}+\frac{2}{(x^4+x+3)^3}-\frac{5x^6}{(x^4+x+3)^3}+\frac{5x^2}{(x^4+x+3)^3}$$

```
[Out] 2/(3 + x + x^4)^3 - (3*x)/(3 + x + x^4)^3 + (5*x^2)/(3 + x + x^4)
^3 + x^4/(3 + x + x^4)^3 - (5*x^6)/(3 + x + x^4)^3
```

Rubi [A] time = 0.231907, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 50, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04

$$\frac{x^4}{(x^4+x+3)^3}-\frac{3x}{(x^4+x+3)^3}+\frac{2}{(x^4+x+3)^3}-\frac{5x^6}{(x^4+x+3)^3}+\frac{5x^2}{(x^4+x+3)^3}$$

Antiderivative was successfully verified.

```
[In] Int[-((15 - 36*x + 5*x^2 + 12*x^3 - 34*x^4 + 140*x^5 + 15*x^6 + 8*x^7 - 30*x^9)/(3 +
```

```
[Out] 2/(3 + x + x^4)^3 - (3*x)/(3 + x + x^4)^3 + (5*x^2)/(3 + x + x^4)
^3 + x^4/(3 + x + x^4)^3 - (5*x^6)/(3 + x + x^4)^3
```

Rubi in Sympy [A] time = 57.8957, size = 60, normalized size = 1.

$$-\frac{5x^6}{(x^4+x+3)^3}+\frac{x^4}{(x^4+x+3)^3}+\frac{5x^2}{(x^4+x+3)^3}-\frac{3x}{(x^4+x+3)^3}+\frac{2}{(x^4+x+3)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((30*x**9-8*x**7-15*x**6-140*x**5+34*x**4-12*x**3-
5*x**2+36*x-15)/(x**4+x+3)**4,x)
```

```
[Out] -5*x**6/(x**4 + x + 3)**3 + x**4/(x**4 + x + 3)**3 + 5*x**2/(x**4
 + x + 3)**3 - 3*x/(x**4 + x + 3)**3 + 2/(x**4 + x + 3)**3
```

Mathematica [A] time = 0.0232864, size = 27, normalized size = 0.45

$$\frac{-5x^6+x^4+5x^2-3x+2}{(x^4+x+3)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[-((15 - 36*x + 5*x^2 + 12*x^3 - 34*x^4 + 140*x^5 + 15*x^6 + 8*x^7 - 30*x^
```

```
[Out] (2 - 3*x + 5*x^2 + x^4 - 5*x^6)/(3 + x + x^4)^3
```

Maple [A] time = 0.012, size = 28, normalized size = 0.5

$$\frac{-5\,x^6+x^4+5\,x^2-3\,x+2}{(x^4+x+3)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((30*x^9-8*x^7-15*x^6-140*x^5+34*x^4-12*x^3-5*x^2+36*x-15)/(x^4+x+3)^4,x)
```

```
[Out] (-5*x^6+x^4+5*x^2-3*x+2)/(x^4+x+3)^3
```

Maxima [A] time = 0.798956, size = 88, normalized size = 1.47

$$-\frac{5\,x^{6}-x^{4}-5\,x^{2}+3\,x-2}{x^{12}+3\,x^{9}+9\,x^{8}+3\,x^{6}+18\,x^{5}+27\,x^{4}+x^{3}+9\,x^{2}+27\,x+27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((30*x^9 - 8*x^7 - 15*x^6 - 140*x^5 + 34*x^4 - 12*x^3 - 5*x^2 + 36*x - 15)
```

```
[Out] -(5*x^6 - x^4 - 5*x^2 + 3*x - 2)/(x^12 + 3*x^9 + 9*x^8 + 3*x^6 +
18*x^5 + 27*x^4 + x^3 + 9*x^2 + 27*x + 27)
```

Fricas [A] time = 0.255867, size = 88, normalized size = 1.47

$$-\frac{5\,x^{6}-x^{4}-5\,x^{2}+3\,x-2}{x^{12}+3\,x^{9}+9\,x^{8}+3\,x^{6}+18\,x^{5}+27\,x^{4}+x^{3}+9\,x^{2}+27\,x+27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((30*x^9 - 8*x^7 - 15*x^6 - 140*x^5 + 34*x^4 - 12*x^3 - 5*x^2 + 36*x - 15)
```

```
[Out] -(5*x^6 - x^4 - 5*x^2 + 3*x - 2)/(x^12 + 3*x^9 + 9*x^8 + 3*x^6 +
18*x^5 + 27*x^4 + x^3 + 9*x^2 + 27*x + 27)
```

Sympy [A] time = 0.823337, size = 61, normalized size = 1.02

$$-\frac{5x^{6}-x^{4}-5x^{2}+3x-2}{x^{12}+3x^{9}+9x^{8}+3x^{6}+18x^{5}+27x^{4}+x^{3}+9x^{2}+27x+27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((30*x**9-8*x**7-15*x**6-140*x**5+34*x**4-12*x**3-5*x**2+36*x-
15)/(x**4+x+3)**4,x)
```

```
[Out] -(5*x**6 - x**4 - 5*x**2 + 3*x - 2)/(x**12 + 3*x**9 + 9*x**8 + 3*
x**6 + 18*x**5 + 27*x**4 + x**3 + 9*x**2 + 27*x + 27)
```

GIAC/XCAS [A] time = 0.259847, size = 41, normalized size = 0.68

$$-\frac{5\,x^{6}-x^{4}-5\,x^{2}+3\,x-2}{\left(x^{4}+x+3\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((30*x^9 - 8*x^7 - 15*x^6 - 140*x^5 + 34*x^4 - 12*x^3 - 5*x^2 + 36*x - 15)
```

```
[Out] -(5*x^6 - x^4 - 5*x^2 + 3*x - 2)/(x^4 + x + 3)^3
```

3.493 $\int \left(\frac{3\left(-47+228x+120x^2+19x^3\right)}{\left(3+x+x^4\right)^4}+\frac{42-320x-75x^2-8x^3}{\left(3+x+x^4\right)^3}+\frac{30x}{\left(3+x+x^4\right)^2}\right)\,dx$

Optimal. Leaf size=27

$$\frac{-5x^6+x^4+5x^2-3x+2}{\left(x^4+x+3\right)^3}$$

```
[Out] (2 - 3*x + 5*x^2 + x^4 - 5*x^6)/(3 + x + x^4)^3
```

Rubi [F] time = 0.54944, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.

$$\text{Int}\left(\frac{3\left(-47+228x+120x^2+19x^3\right)}{\left(3+x+x^4\right)^4}+\frac{42-320x-75x^2-8x^3}{\left(3+x+x^4\right)^3}+\frac{30x}{\left(3+x+x^4\right)^2},x\right)$$

Verification is Not applicable to the result.

```
[In] Int[(3*(-47 + 228*x + 120*x^2 + 19*x^3))/(3 + x + x^4)^4 + (42 - 320*x - 75*x^2 - 8*
```

```
[Out] -19/(4*(3 + x + x^4)^3) + (3 + x + x^4)^(-2) - (621*Defer[Int][(3
 + x + x^4)^(-4), x])/4 + 684*Defer[Int][x/(3 + x + x^4)^4, x] +
360*Defer[Int][x^2/(3 + x + x^4)^4, x] + 44*Defer[Int][(3 + x + x
^4)^(-3), x] - 320*Defer[Int][x/(3 + x + x^4)^3, x] - 75*Defer[In
t][x^2/(3 + x + x^4)^3, x] + 30*Defer[Int][x/(3 + x + x^4)^2, x]
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(3*(19*x**3+120*x**2+228*x-47)/(x**4+x+3)**4+(-8*x**3-
75*x**2-320*x+42)/(x**4+x+3)**3+30*x/(x**4+x+3)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.0144376, size = 27, normalized size = 1.

$$\frac{-5x^6+x^4+5x^2-3x+2}{\left(x^4+x+3\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*(-47 + 228*x + 120*x^2 + 19*x^3))/(3 + x + x^4)^4 + (42 - 320*x - 75*x
```

```
[Out] (2 - 3*x + 5*x^2 + x^4 - 5*x^6)/(3 + x + x^4)^3
```

Maple [C] time = 0.04, size = 250, normalized size = 9.3

$$\frac{1}{(x^4+x+3)^2}\left(\frac{377432\,x^7}{195075}-\frac{1404328\,x^6}{195075}+\frac{234517\,x^5}{195075}+\frac{660506\,x^4}{195075}-\frac{208792\,x^3}{195075}-\frac{13339729\,x^2}{390150}+\frac{89881\,x}{13005}+\frac{121303}{21675}\right)$$
$$+\frac{1}{195075}\sum_{\_R=RootOf(\_Z^4+\_Z+3)}\frac{(377432\,\_R^2-2808656\,\_R+703551)\ln(x-\_R)}{4\,\_R^3+1}$$
$$+30\frac{1}{x^4+x+3}\left(-\frac{16\,x^3}{765}+\frac{64\,x^2}{765}-\frac{x}{765}-\frac{4}{255}\right)$$
$$+\frac{2}{51}\sum_{\_R=RootOf(\_Z^4+\_Z+3)}\frac{(-16\,\_R^2+128\,\_R-3)\ln(x-\_R)}{4\,\_R^3+1}$$
$$+3\frac{1}{(x^4+x+3)^3}\left(-\frac{255032\,x^{11}}{585225}+\frac{914728\,x^{10}}{585225}-\frac{226867\,x^9}{585225}-\frac{701338\,x^8}{585225}+\frac{236024\,x^7}{585225}+\frac{13501313\,x^6}{1170450}-\frac{2360372\,x^5}{585225}-\frac{187}{5}\right.$$
$$+\frac{1}{195075}\sum_{\_R=RootOf(\_Z^4+\_Z+3)}\frac{(-255032\,\_R^2+1829456\,\_R-680601)\ln(x-\_R)}{4\,\_R^3+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(3*(19*x^3+120*x^2+228*x-47)/(x^4+x+3)^4+(-8*x^3-75*x^2-320*x+42)/(x^4+x+3)
```

```
[Out] (377432/195075*x^7-1404328/195075*x^6+234517/195075*x^5+660506/19
5075*x^4-208792/195075*x^3-13339729/390150*x^2+89881/13005*x+1213
03/21675)/(x^4+x+3)^2+1/195075*sum((377432*_R^2-2808656*_R+703551
)/(4*_R^3+1)*ln(x-_R),_R=RootOf(_Z^4+_Z+3))+30*(-16/765*x^3+64/76
5*x^2-1/765*x-4/255)/(x^4+x+3)+2/51*sum((-16*_R^2+128*_R-3)/(4*_R
^3+1)*ln(x-_R),_R=RootOf(_Z^4+_Z+3))+3*(-255032/585225*x^11+91472
8/585225*x^10-226867/585225*x^9-701338/585225*x^8+236024/585225*x
^7+13501313/1170450*x^6-2360372/585225*x^5-1873778/585225*x^4+109
35781/1170450*x^3+3415123/130050*x^2-62987/7225*x-76253/21675)/(x
^4+x+3)^3+1/195075*sum((-255032*_R^2+1829456*_R-680601)/(4*_R^3+1
)*ln(x-_R),_R=RootOf(_Z^4+_Z+3))
```

Maxima [A] time = 0.81328, size = 88, normalized size = 3.26

$$-\frac{5\,x^6-x^4-5\,x^2+3\,x-2}{x^{12}+3\,x^9+9\,x^8+3\,x^6+18\,x^5+27\,x^4+x^3+9\,x^2+27\,x+27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(30*x/(x^4 + x + 3)^2 - (8*x^3 + 75*x^2 + 320*x - 42)/(x^4 + x + 3)^3 + 3*(
```

```
[Out] -(5*x^6 - x^4 - 5*x^2 + 3*x - 2)/(x^12 + 3*x^9 + 9*x^8 + 3*x^6 +
18*x^5 + 27*x^4 + x^3 + 9*x^2 + 27*x + 27)
```

Fricas [A] time = 0.25761, size = 88, normalized size = 3.26

$$-\frac{5\,x^6-x^4-5\,x^2+3\,x-2}{x^{12}+3\,x^9+9\,x^8+3\,x^6+18\,x^5+27\,x^4+x^3+9\,x^2+27\,x+27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(30*x/(x^4 + x + 3)^2 - (8*x^3 + 75*x^2 + 320*x - 42)/(x^4 + x + 3)^3 + 3*(
```

```
[Out] -(5*x^6 - x^4 - 5*x^2 + 3*x - 2)/(x^12 + 3*x^9 + 9*x^8 + 3*x^6 +
18*x^5 + 27*x^4 + x^3 + 9*x^2 + 27*x + 27)
```

Sympy [A] time = 1.06688, size = 61, normalized size = 2.26

$$-\frac{5x^6 - x^4 - 5x^2 + 3x - 2}{x^{12} + 3x^9 + 9x^8 + 3x^6 + 18x^5 + 27x^4 + x^3 + 9x^2 + 27x + 27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3*(19*x**3+120*x**2+228*x-47)/(x**4+x+3)**4+(-8*x**3-75*x**2-
320*x+42)/(x**4+x+3)**3+30*x/(x**4+x+3)**2,x)
```

```
[Out] -(5*x**6 - x**4 - 5*x**2 + 3*x - 2)/(x**12 + 3*x**9 + 9*x**8 + 3*
x**6 + 18*x**5 + 27*x**4 + x**3 + 9*x**2 + 27*x + 27)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{30\,x}{\left(x^4 + x + 3\right)^2} - \frac{8\,x^3 + 75\,x^2 + 320\,x - 42}{\left(x^4 + x + 3\right)^3} + \frac{3\,\left(19\,x^3 + 120\,x^2 + 228\,x - 47\right)}{\left(x^4 + x + 3\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(30*x/(x^4 + x + 3)^2 - (8*x^3 + 75*x^2 + 320*x - 42)/(x^4 + x + 3)^3 + 3*(
```

```
[Out] integrate(30*x/(x^4 + x + 3)^2 - (8*x^3 + 75*x^2 + 320*x - 42)/(x
^4 + x + 3)^3 + 3*(19*x^3 + 120*x^2 + 228*x - 47)/(x^4 + x + 3)^4
, x)
```

3.494 $\int \left(\frac{-3+10x+4x^3-30x^5}{(3+x+x^4)^3}-\frac{3(1+4x^3)(2-3x+5x^2+x^4-5x^6)}{(3+x+x^4)^4}\right)\,dx$

Optimal. Leaf size=27

$$\frac{-5x^6+x^4+5x^2-3x+2}{(x^4+x+3)^3}$$

```
[Out] (2 - 3*x + 5*x^2 + x^4 - 5*x^6)/(3 + x + x^4)^3
```

Rubi [F] time = 0.786527, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.

$$\text{Int}\left(\frac{-3+10x+4x^3-30x^5}{(3+x+x^4)^3}-\frac{3(1+4x^3)(2-3x+5x^2+x^4-5x^6)}{(3+x+x^4)^4},x\right)$$

Verification is Not applicable to the result.

```
[In] Int[(-3 + 10*x + 4*x^3 - 30*x^5)/(3 + x + x^4)^3 - (3*(1 + 4*x^3)*(2 - 3*x + 5*x^2 +
```

```
[Out] 7/(2*(3 + x + x^4)^3) - (63*x)/(22*(3 + x + x^4)^3) - (12*x^2)/(3
 + x + x^4)^3 - (5*x^3)/(3 + x + x^4)^3 + (3*x^4)/(2*(3 + x + x^4
)^3) - (10*x^6)/(3 + x + x^4)^3 - 1/(2*(3 + x + x^4)^2) + (5*x^2)
/(3 + x + x^4)^2 + (144*Defer[Int][(3 + x + x^4)^(-4), x])/11 + (
828*Defer[Int][x/(3 + x + x^4)^4, x])/11 + 18*Defer[Int][x^2/(3 +
 x + x^4)^4, x] - 4*Defer[Int][(3 + x + x^4)^(-3), x] - 20*Defer[
Int][x/(3 + x + x^4)^3, x]
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-30*x**5+4*x**3+10*x-3)/(x**4+x+3)**3-3*(4*x**3+1)*(-
5*x**6+x**4+5*x**2-3*x+2)/(x**4+x+3)**4,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.0138268, size = 27, normalized size = 1.

$$\frac{-5x^6+x^4+5x^2-3x+2}{(x^4+x+3)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + 10*x + 4*x^3 - 30*x^5)/(3 + x + x^4)^3 - (3*(1 + 4*x^3)*(2 - 3*x + 5
```

```
[Out] (2 - 3*x + 5*x^2 + x^4 - 5*x^6)/(3 + x + x^4)^3
```

Maple [B] time = 0.024, size = 112, normalized size = 4.2

$$-\frac{1}{(x^4+x+3)^2}\left(-\frac{34568\,x^7}{195075}+\frac{73672\,x^6}{195075}+\frac{15392\,x^5}{195075}-\frac{60494\,x^4}{195075}-\frac{68792\,x^3}{195075}-\frac{583927\,x^2}{195075}+\frac{3356\,x}{13005}-\frac{2069}{43350}\right)$$
$$+3\,\frac{1}{(x^4+x+3)^3}\left(-\frac{34568\,x^{11}}{585225}+\frac{73672\,x^{10}}{585225}+\frac{15392\,x^9}{585225}-\frac{95062\,x^8}{585225}-\frac{98824\,x^7}{585225}-\frac{1322894\,x^6}{585225}+\frac{36022\,x^5}{585225}-\frac{129019\,x^4}{1170450}-\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-30*x^5+4*x^3+10*x-3)/(x^4+x+3)^3-3*(4*x^3+1)*(-5*x^6+x^4+5*x^2-
3*x+2)/(x^4+x+3)^4,x)
```

```
[Out] -(-34568/195075*x^7+73672/195075*x^6+15392/195075*x^5-60494/19507
5*x^4-68792/195075*x^3-583927/195075*x^2+3356/13005*x-2069/43350)
/(x^4+x+3)^2+3*(-34568/585225*x^11+73672/585225*x^10+15392/585225
*x^9-95062/585225*x^8-98824/585225*x^7-1322894/585225*x^6+36022/5
85225*x^5-129019/1170450*x^4-790303/585225*x^3-80674/65025*x^2-10
951/14450*x+26831/43350)/(x^4+x+3)^3
```

Maxima [A] time = 0.810754, size = 88, normalized size = 3.26

$$-\frac{5\,x^6-x^4-5\,x^2+3\,x-2}{x^{12}+3\,x^9+9\,x^8+3\,x^6+18\,x^5+27\,x^4+x^3+9\,x^2+27\,x+27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(30*x^5 - 4*x^3 - 10*x + 3)/(x^4 + x + 3)^3 + 3*(5*x^6 - x^4 - 5*x^2 + 3*
```

```
[Out] -(5*x^6 - x^4 - 5*x^2 + 3*x - 2)/(x^12 + 3*x^9 + 9*x^8 + 3*x^6 +
18*x^5 + 27*x^4 + x^3 + 9*x^2 + 27*x + 27)
```

Fricas [A] time = 0.277168, size = 88, normalized size = 3.26

$$-\frac{5\,x^6-x^4-5\,x^2+3\,x-2}{x^{12}+3\,x^9+9\,x^8+3\,x^6+18\,x^5+27\,x^4+x^3+9\,x^2+27\,x+27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(30*x^5 - 4*x^3 - 10*x + 3)/(x^4 + x + 3)^3 + 3*(5*x^6 - x^4 - 5*x^2 + 3*
```

```
[Out] -(5*x^6 - x^4 - 5*x^2 + 3*x - 2)/(x^12 + 3*x^9 + 9*x^8 + 3*x^6 +
18*x^5 + 27*x^4 + x^3 + 9*x^2 + 27*x + 27)
```

Sympy [A] time = 0.952438, size = 61, normalized size = 2.26

$$-\frac{5x^6-x^4-5x^2+3x-2}{x^{12}+3x^9+9x^8+3x^6+18x^5+27x^4+x^3+9x^2+27x+27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-30*x**5+4*x**3+10*x-3)/(x**4+x+3)**3-3*(4*x**3+1)*(-
5*x**6+x**4+5*x**2-3*x+2)/(x**4+x+3)**4,x)
```

```
[Out] -(5*x**6 - x**4 - 5*x**2 + 3*x - 2)/(x**12 + 3*x**9 + 9*x**8 + 3*
x**6 + 18*x**5 + 27*x**4 + x**3 + 9*x**2 + 27*x + 27)
```

GIAC/XCAS [A] time = 0.26246, size = 150, normalized size = 5.56

$$\frac{69136\,x^{7}-147344\,x^{6}-30784\,x^{5}+120988\,x^{4}+137584\,x^{3}+1167854\,x^{2}-100680\,x+18621}{390150\,(x^{4}+x+3)^{2}}-\frac{69136\,x^{11}-147344\,x^{10}-30784\,x^{9}+190124\,x^{8}+197648\,x^{7}+2645788\,x^{6}-72044\,x^{5}+129019\,x^{4}+1580606\,x^{3}+1452132}{390150\,(x^{4}+x+3)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(30*x^5 - 4*x^3 - 10*x + 3)/(x^4 + x + 3)^3 + 3*(5*x^6 - x^4 - 5*x^2 + 3*
```

```
[Out] 1/390150*(69136*x^7 - 147344*x^6 - 30784*x^5 + 120988*x^4 + 13758
4*x^3 + 1167854*x^2 - 100680*x + 18621)/(x^4 + x + 3)^2 - 1/39015
0*(69136*x^11 - 147344*x^10 - 30784*x^9 + 190124*x^8 + 197648*x^7
 + 2645788*x^6 - 72044*x^5 + 129019*x^4 + 1580606*x^3 + 1452132*x
^2 + 887031*x - 724437)/(x^4 + x + 3)^3
```

4 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Mathematica/Rubi followed by one for Maple. The following are links to the source code.

The following are the listing of the above functions.

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)
(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)
```

```
ExpnType[expn_] :=
  If[AtomQ[expn], 1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]

SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]
HypergeometricFunctionQ[func_] := MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
AppellFunctionQ[func_] := MemberQ[{AppellF1},func]
```

```
# File: GradeAntiderivative.mpl Original version thanks to Albert Rich emailed on 03/21/2017
#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);
  #This check below actually is not needed, since I only call this grading only for
  #passed integrals. i.e. I check for "F" before calling this.

  if not type(result,freeof('int')) then
     return "F";
  end if;

  if ExpnType_result<=ExpnType_optimal then
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
         if leaf_count_result<=2*leaf_count_optimal then
            return "A";
         else
            return "B";
         end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      return "C";
  end if
end proc:
```

```
# is_contains_complex(result) takes expressions and returns true if it contains "I"
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1 else
    max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1 else
      max(2,ExpnType(op(1,expn))) end if else
    max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' or op(0,expn)='integrate' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:
```

```
ElementaryFunctionQ := proc(func)
  member(func,[exp,log,ln, sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[erf,erfc,erfi,FresnelS,FresnelC,Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u) else
  apply(op(0,u),op(2..nops(u),u))
  end if
end proc:
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```