Computer algebra independent integration tests
0_Independent_test_suites/Welz_Problems

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November 25, 2018 Compiled on November 25, 2018 at 10:43pm

Contents

1 Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from Albert Rich Rubi web site.

1.1 Listing of CAS systems tested

The following systems were tested at this time.

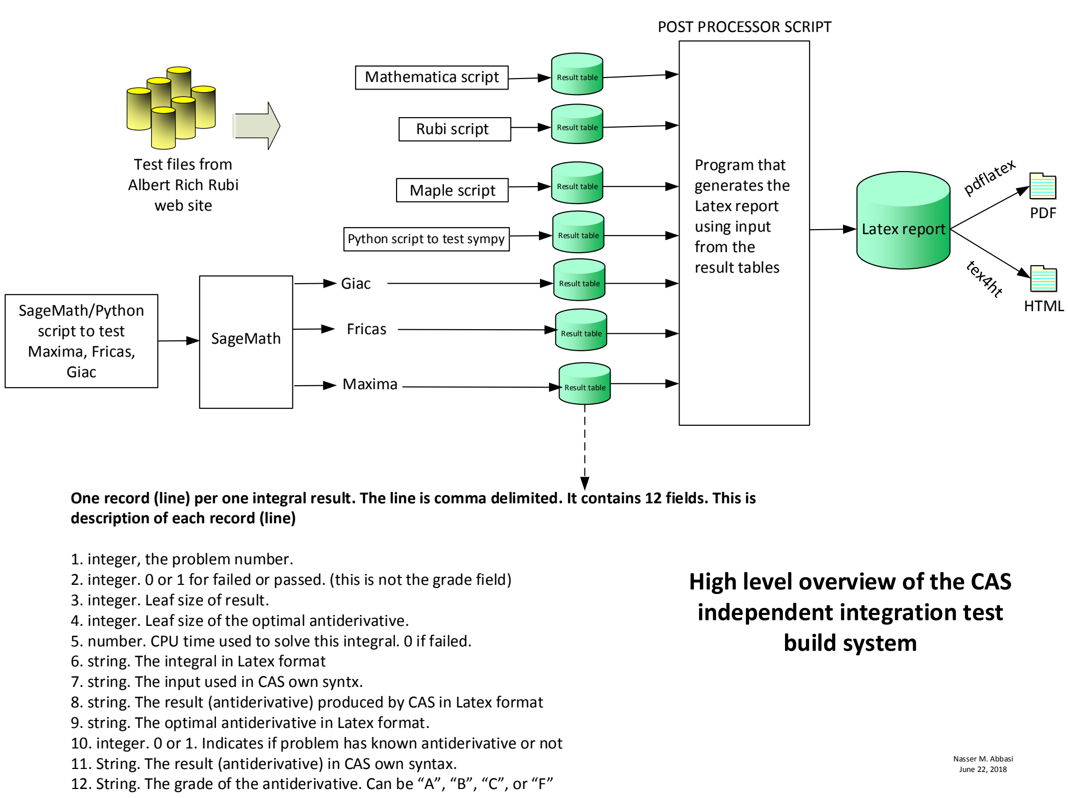
1. Mathematica 11.3 (64 bit).
2. Rubi 4.15.2 in Mathematica 11.3.
3. Rubi in Sympy (Version 1.3) under Python 3.7.0 using Anaconda distribution.
4. Maple 2018.1 (64 bit).
5. Maxima 5.41 Using Lisp ECL 16.1.2.
6. Fricas 1.3.4.
7. Sympy 1.3 under Python 3.7.0 using Anaconda distribution.
8. Giac/Xcas 1.4.9.

Maxima, Fricas and Giac/Xcas were called from inside SageMath version 8.3. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python. Rubi in Sympy was also called directly using sympy 1.3 in python.

1.2 Design of the test system

The following diagram gives a high level view of the current test build system.



1.3 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.4 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.5 Important notes about some of the results

Important note about Maxima results Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS and Giac/XCAS results There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

Important note about finding leaf size of antiderivative For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-express

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems implement a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.6 Grading of results

The table below summarizes the grading of each CAS system.

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 94.51 (86)	% 5.49 (5)
Rubi in Sympy	% 70.33 (64)	% 29.67 (27)
Mathematica	% 84.62 (77)	% 15.38 (14)
Maple	% 60.44 (55)	% 39.56 (36)
Maxima	% 21.98 (20)	% 78.02 (71)
Fricas	% 59.34 (54)	% 40.66 (37)
Sympy	% 29.67 (27)	% 70.33 (64)
Giac	% 18.68 (17)	% 81.32 (74)

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

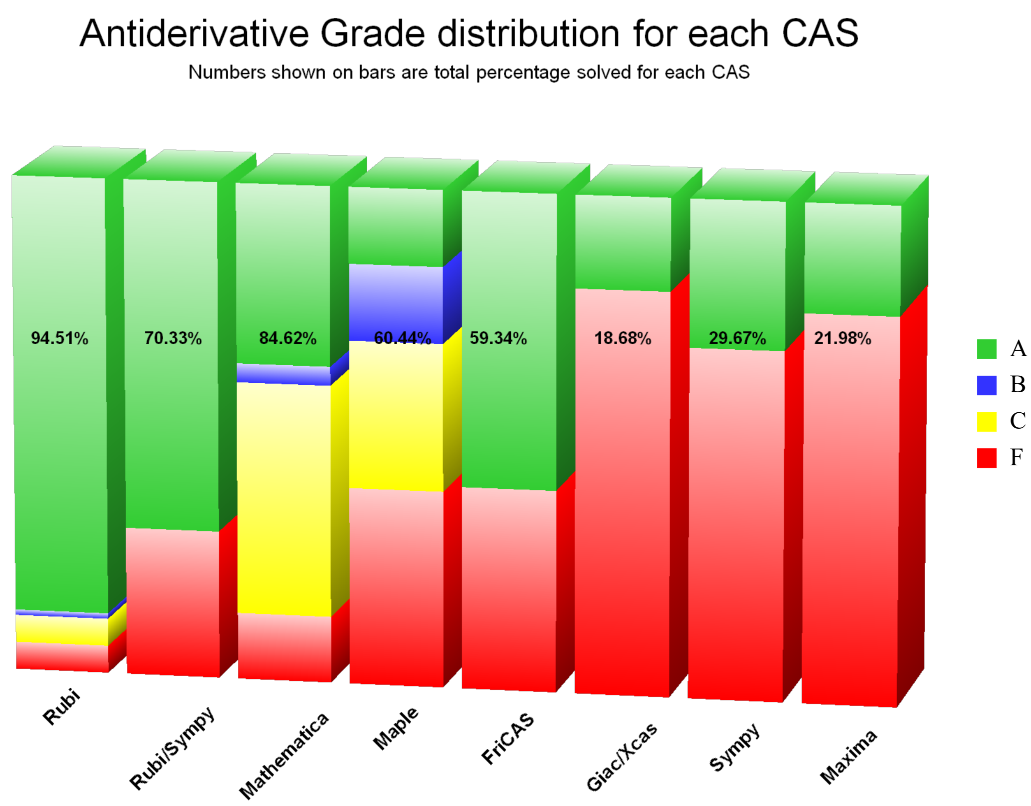
grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

Grading is currently implemented only for for Mathematica, Rubi and Maple results. For all other CAS systems (Maxima, Fricas, Sympy, Giac, Rubi in sympy), the grading function is not yet implemented. For these systems, a grade of A is assigned if the integrate command completes successfully and a grade of F otherwise.

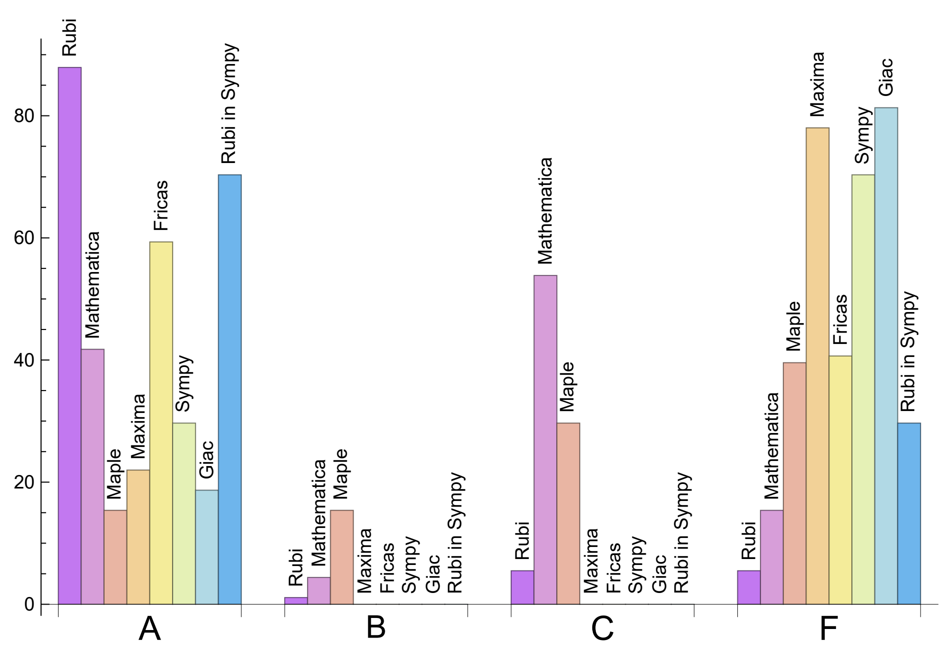
Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	87.91	1.1	5.49	5.49
Rubi in Sympy	70.33	0.	0.	29.67
Mathematica	41.76	4.4	53.85	15.38
Maple	15.38	15.38	29.67	39.56
Maxima	21.98	0.	0.	78.02
Fricas	59.34	0.	0.	40.66
Sympy	29.67	0.	0.	70.33
Giac	18.68	0.	0.	81.32

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.7 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.52	145.79	7.87	81.	1.
Rubi in Sympy	11.56	106.11	1.5	55.	0.89
Mathematica	0.85	205.52	3.43	118.	1.
Maple	0.19	2243.62	12.27	206.	1.64
Maxima	1.46	93.65	1.47	67.	1.26
Fricas	0.26	487.78	5.07	82.5	1.65
Sympy	4.3	214.41	5.29	37.	0.86
Giac	0.22	70.	1.58	55.	1.45

1.8 list of integrals that has no closed form antiderivative

{}

1.9 list of integrals not solved by each system

Not solved by Rubi {43, 44, 45, 58, 59}

Not solved by Rubi in Sympy {3, 5, 9, 10, 17, 18, 29, 31, 37, 38, 43, 44, 45, 46, 48, 49, 51, 52, 58, 59, 79, 80, 81, 88, 89, 90, 91}

Not solved by Mathematica {9, 10, 12, 13, 37, 38, 44, 45, 46, 58, 59, 65, 90, 91}

Not solved by Maple {12, 13, 14, 18, 19, 20, 25, 26, 27, 29, 30, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 53, 54, 55, 58, 59, 67, 68, 69, 70, 75, 76, 77, 78, 90, 91}

Not solved by Maxima {3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 17, 18, 24, 25, 26, 27, 28, 31, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 58, 59, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91}

Not solved by Fricas {12, 13, 29, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 49, 51, 53, 54, 55, 58, 59, 67, 68, 69, 70, 75, 76, 77, 78, 82, 84, 85, 86, 87, 88, 89, 90, 91}

Not solved by Sympy {3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 18, 24, 29, 32, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 58, 59, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91}

Not solved by Giac {5, 6, 9, 10, 11, 12, 13, 14, 15, 17, 18, 19, 20, 24, 25, 26, 27, 28, 29, 30, 31, 32, 35, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 50, 52, 53, 54, 55, 56, 58, 59, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91}

1.10 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Rubi in Sympy {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {9, 10, 49, 80}

Mathematica {39, 40, 53, 54, 55, 63, 64, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 82, 83, 84, 85, 86, 87, 88, 89}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Rubi in Sympy Verification phase not implemented yet.

2 detailed summary tables of results

2.1 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	18	12	18	12
normalized size	1	1.	1.	0.93	1.2	1.2	0.8	1.2	0.8
time (sec)	N/A	0.009	0.005	0.004	1.365	0.206	0.038	0.199	0.669

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	C	C	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	52	37	34	55	1	42	55	42
normalized size	1	3.47	2.47	2.27	3.67	0.07	2.8	3.67	2.8
time (sec)	N/A	0.097	0.02	0.005	1.375	0.21	52.66	0.212	8.535

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	119	370	0	363	0	239	0
normalized size	1	1.	1.45	4.51	0.	4.43	0.	2.91	0.
time (sec)	N/A	0.144	0.159	0.062	0.	0.22	0.	0.206	0.

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	172	0	201	0	127	34
normalized size	1	1.	1.	4.	0.	4.67	0.	2.95	0.79
time (sec)	N/A	0.038	0.042	0.069	0.	0.216	0.	0.233	4.029

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	65	115	0	274	0	0	0
normalized size	1	1.	0.88	1.55	0.	3.7	0.	0.	0.
time (sec)	N/A	0.146	0.146	0.023	0.	0.225	0.	0.	0.

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	165	125	72	230	0	0	53
normalized size	1	1.	2.58	1.95	1.12	3.59	0.	0.	0.83
time (sec)	N/A	0.076	0.179	0.038	1.546	0.231	0.	0.	5.039

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	90	45	0	216	0	136	41
normalized size	1	1.	1.88	0.94	0.	4.5	0.	2.83	0.85
time (sec)	N/A	0.039	0.071	0.026	0.	0.222	0.	0.203	3.715

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	29	21	0	24	53	27	26
normalized size	1	1.	0.97	0.7	0.	0.8	1.77	0.9	0.87
time (sec)	N/A	0.128	0.023	0.004	0.	0.208	1.502	0.206	4.971

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	F	F
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	220	365	0	902	0	1185	0	0	0
normalized size	1	1.66	0.	4.1	0.	5.39	0.	0.	0.
time (sec)	N/A	0.997	4.931	0.177	0.	0.253	0.	0.	0.

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	B	A	B	F	A	F(-1)	F	F(-1)
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	220	541	0	1542	0	1185	0	0	0
normalized size	1	2.46	0.	7.01	0.	5.39	0.	0.	0.
time (sec)	N/A	1.561	1.66	0.033	0.	0.255	0.	0.	0.

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	F(-2)	A	F(-2)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	403	278	0	680	0	0	184
normalized size	1	1.	2.92	2.01	0.	4.93	0.	0.	1.33
time (sec)	N/A	0.162	0.413	0.032	0.	0.287	0.	0.	18.456

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F(-1)	F	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	0	0	0	0	0	0	202
normalized size	1	1.	0.	0.	0.	0.	0.	0.	1.62
time (sec)	N/A	0.328	0.154	0.036	0.	0.	0.	0.	21.574

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F(-1)	F	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	0	0	0	0	0	0	155
normalized size	1	1.	0.	0.	0.	0.	0.	0.	1.91
time (sec)	N/A	0.278	0.119	0.037	0.	0.	0.	0.	17.113

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	145	0	0	81	15	0	29
normalized size	1	1.	4.68	0.	0.	2.61	0.48	0.	0.94
time (sec)	N/A	0.089	2.357	0.024	0.	0.39	2.212	0.	3.712

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	C	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	162	22	0	39	15	0	29
normalized size	1	1.	4.91	0.67	0.	1.18	0.45	0.	0.88
time (sec)	N/A	0.091	1.435	0.055	0.	0.4	1.164	0.	3.951

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	20	31	56	20	17
normalized size	1	1.	1.	0.84	1.05	1.63	2.95	1.05	0.89
time (sec)	N/A	0.434	0.025	0.004	1.341	0.203	12.491	0.214	11.273

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	36	120	0	43	2147	0	0
normalized size	1	1.	0.69	2.31	0.	0.83	41.29	0.	0.
time (sec)	N/A	0.047	0.04	0.045	0.	0.229	4.98	0.	0.

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	39	0	0	45	0	0	0
normalized size	1	1.	0.7	0.	0.	0.8	0.	0.	0.
time (sec)	N/A	0.046	0.04	0.045	0.	0.23	0.	0.	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	0	20	20	311	0	12
normalized size	1	1.	1.	0.	1.18	1.18	18.29	0.	0.71
time (sec)	N/A	0.087	0.027	0.029	1.396	0.23	5.359	0.	4.054

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	0	24	24	36	0	14
normalized size	1	1.	1.	0.	1.2	1.2	1.8	0.	0.7
time (sec)	N/A	0.092	0.028	0.045	1.397	0.232	3.559	0.	4.364

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	37	48	54	70	36	55	39
normalized size	1	1.	0.88	1.14	1.29	1.67	0.86	1.31	0.93
time (sec)	N/A	0.063	0.063	0.017	1.352	0.224	0.176	0.198	5.391

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	27	26	22	26	15
normalized size	1	1.	1.	0.95	1.23	1.18	1.	1.18	0.68
time (sec)	N/A	0.041	0.021	0.004	1.342	0.216	0.116	0.201	4.678

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	49	51	69	78	49	100	54
normalized size	1	1.	0.79	0.82	1.11	1.26	0.79	1.61	0.87
time (sec)	N/A	0.154	0.052	0.019	1.368	0.231	0.216	0.199	10.102

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	961	455	0	720	0	0	87
normalized size	1	1.	11.17	5.29	0.	8.37	0.	0.	1.01
time (sec)	N/A	0.226	5.47	0.057	0.	0.252	0.	0.	18.81

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	0	0	20	15	0	15
normalized size	1	1.	1.	0.	0.	1.05	0.79	0.	0.79
time (sec)	N/A	0.106	0.026	0.039	0.	0.218	0.282	0.	3.644

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	0	0	30	27	0	20
normalized size	1	1.	1.	0.	0.	1.15	1.04	0.	0.77
time (sec)	N/A	0.161	0.041	0.05	0.	0.223	1.344	0.	4.681

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	85	0	0	1	46	0	58
normalized size	1	1.	1.35	0.	0.	0.02	0.73	0.	0.92
time (sec)	N/A	0.365	0.251	0.028	0.	0.243	2.87	0.	9.559

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	161	25	0	1	51	0	73
normalized size	1	1.	1.96	0.3	0.	0.01	0.62	0.	0.89
time (sec)	N/A	0.14	0.081	0.008	0.	0.241	2.482	0.	8.607

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	679	679	412	0	699	0	0	0	0
normalized size	1	1.	0.61	0.	1.03	0.	0.	0.	0.
time (sec)	N/A	7.496	0.267	0.063	1.399	0.	0.	0.	0.

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	0	20	20	311	0	12
normalized size	1	1.	1.	0.	1.18	1.18	18.29	0.	0.71
time (sec)	N/A	0.088	0.04	0.037	1.402	0.241	7.301	0.	3.919

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	36	120	0	43	2147	0	0
normalized size	1	1.	0.69	2.31	0.	0.83	41.29	0.	0.
time (sec)	N/A	0.046	0.023	0.033	0.	0.244	6.697	0.	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	33	44	65	65	0	0	27
normalized size	1	1.	0.97	1.29	1.91	1.91	0.	0.	0.79
time (sec)	N/A	0.06	0.056	0.066	1.766	0.224	0.	0.	9.298

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	41	65	84	84	36	86	49
normalized size	1	1.	0.71	1.12	1.45	1.45	0.62	1.48	0.84
time (sec)	N/A	0.078	0.021	0.056	1.497	0.214	1.607	0.207	2.456

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	41	48	84	84	37	86	49
normalized size	1	1.	0.71	0.83	1.45	1.45	0.64	1.48	0.84
time (sec)	N/A	0.069	0.02	0.063	1.5	0.216	1.72	0.204	2.453

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	86	12	105	122	29	0	71
normalized size	1	1.	1.76	0.24	2.14	2.49	0.59	0.	1.45
time (sec)	N/A	0.016	0.071	0.034	1.502	0.216	1.61	0.	3.587

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	39	65	84	96	32	85	48
normalized size	1	1.	0.71	1.18	1.53	1.75	0.58	1.55	0.87
time (sec)	N/A	0.07	0.019	0.057	1.474	0.215	1.605	0.209	2.299

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F(-1)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	121	97	0	0	0	0	0	0	0
normalized size	1	0.8	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.083	0.073	0.049	0.	0.	0.	0.	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	0	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.194	0.089	0.059	0.	0.	0.	0.	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	110	176	109	0	0	0	0	0	155
normalized size	1	1.6	0.99	0.	0.	0.	0.	0.	1.41
time (sec)	N/A	0.099	0.19	0.098	0.	0.	0.	0.	4.678

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	81	131	85	0	0	0	0	0	204
normalized size	1	1.62	1.05	0.	0.	0.	0.	0.	2.52
time (sec)	N/A	0.139	0.022	0.019	0.	0.	0.	0.	7.393

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	A	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	117	49	0	104	0	0	0	155
normalized size	1	1.77	0.74	0.	1.58	0.	0.	0.	2.35
time (sec)	N/A	0.113	0.035	0.064	1.524	0.	0.	0.	5.457

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	145	61	0	0	0	0	0	216
normalized size	1	1.84	0.77	0.	0.	0.	0.	0.	2.73
time (sec)	N/A	0.188	0.058	0.02	0.	0.	0.	0.	9.593

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	F	C	F	F	F(-1)	F(-1)	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	0	72	0	0	0	0	0	0
normalized size	1	0.	0.61	0.	0.	0.	0.	0.	0.
time (sec)	N/A	27.68	0.228	0.028	0.	0.	0.	0.	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	F	A	F	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	111	0	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.338	1.235	0.062	0.	0.	0.	0.	0.

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	F	A	F	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	176	0	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.82	0.415	0.073	0.	0.	0.	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	C	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	326	576	0	0	0	0	0	0	0
normalized size	1	1.77	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.525	0.165	0.183	0.	0.	0.	0.	0.

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	407	407	206	584	0	1827	0	0	478
normalized size	1	1.	0.51	1.43	0.	4.49	0.	0.	1.17
time (sec)	N/A	1.477	2.259	0.132	0.	0.281	0.	0.	48.171

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	648	648	662	719	0	2839	0	0	0
normalized size	1	1.	1.02	1.11	0.	4.38	0.	0.	0.
time (sec)	N/A	2.686	6.077	0.072	0.	0.34	0.	0.	0.

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F(-1)	F(-1)	A	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1058	1058	1242	989	0	0	0	1	0
normalized size	1	1.	1.17	0.93	0.	0.	0.	0.	0.
time (sec)	N/A	5.375	6.165	0.138	0.	0.	0.	0.227	0.

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	378	378	1236	21028	0	6251	0	0	379
normalized size	1	1.	3.27	55.63	0.	16.54	0.	0.	1.
time (sec)	N/A	1.586	6.467	0.765	0.	0.363	0.	0.	79.056

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	638	638	1431	86793	0	0	0	1	0
normalized size	1	1.	2.24	136.04	0.	0.	0.	0.	0.
time (sec)	N/A	2.835	6.717	4.895	0.	0.	0.	0.351	0.

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	C	C	C	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	204	213	1275	0	1	0	0	0
normalized size	1	3.09	3.23	19.32	0.	0.02	0.	0.	0.
time (sec)	N/A	2.372	0.563	0.168	0.	0.617	0.	0.	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-2)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	205	0	0	0	0	0	231
normalized size	1	1.	1.04	0.	0.	0.	0.	0.	1.17
time (sec)	N/A	0.241	0.468	0.052	0.	0.	0.	0.	13.653

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-2)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	220	0	0	0	0	0	282
normalized size	1	1.	1.11	0.	0.	0.	0.	0.	1.42
time (sec)	N/A	0.212	0.442	0.073	0.	0.	0.	0.	13.12

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	88	97	273	0	0	0	0	0	94
normalized size	1	1.1	3.1	0.	0.	0.	0.	0.	1.07
time (sec)	N/A	0.047	0.81	0.199	0.	0.	0.	0.	2.42

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	34	69	142	143	32	0	85
normalized size	1	1.	0.32	0.64	1.33	1.34	0.3	0.	0.79
time (sec)	N/A	0.102	0.018	0.069	1.666	0.21	1.783	0.	4.775

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	48	49	96	115	37	97	56
normalized size	1	1.	0.72	0.73	1.43	1.72	0.55	1.45	0.84
time (sec)	N/A	0.084	0.025	0.073	1.534	0.208	1.678	0.24	2.768

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	F	A	F	F	F(-2)	F	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1	0	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.099	0.372	0.072	0.	0.	0.	0.	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	F	A	F	F	F(-1)	F	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	280	0	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.385	0.042	0.112	0.	0.	0.	0.	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	31	31	22	31	22
normalized size	1	1.	1.	0.96	1.24	1.24	0.88	1.24	0.88
time (sec)	N/A	0.012	0.012	0.001	1.39	0.194	0.107	0.245	3.601

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	86	52	0	51	73	0	56
normalized size	1	1.	1.46	0.88	0.	0.86	1.24	0.	0.95
time (sec)	N/A	0.06	0.027	0.043	0.	0.195	0.213	0.	7.633

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	99	75	0	89	100	0	80
normalized size	1	1.	1.27	0.96	0.	1.14	1.28	0.	1.03
time (sec)	N/A	0.124	0.029	0.033	0.	0.198	0.218	0.	14.771

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	110	100	0	105	0	0	36
normalized size	1	1.	2.24	2.04	0.	2.14	0.	0.	0.73
time (sec)	N/A	0.026	0.144	0.022	0.	0.257	0.	0.	2.408

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	108	365	0	81	0	0	46
normalized size	1	1.	2.04	6.89	0.	1.53	0.	0.	0.87
time (sec)	N/A	0.037	0.134	0.04	0.	0.254	0.	0.	2.585

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	F	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	0	1421	0	1	0	0	63
normalized size	1	1.	0.	18.95	0.	0.01	0.	0.	0.84
time (sec)	N/A	0.161	0.329	0.098	0.	0.27	0.	0.	9.109

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	322	456	0	2419	0	0	144
normalized size	1	1.	1.88	2.67	0.	14.15	0.	0.	0.84
time (sec)	N/A	0.142	0.427	0.101	0.	0.339	0.	0.	6.94

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	203	0	0	0	0	0	178
normalized size	1	1.	2.54	0.	0.	0.	0.	0.	2.22
time (sec)	N/A	0.118	0.56	0.088	0.	0.	0.	0.	28.634

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	221	0	0	0	0	0	199
normalized size	1	1.	2.51	0.	0.	0.	0.	0.	2.26
time (sec)	N/A	0.131	0.555	0.043	0.	0.	0.	0.	29.508

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	205	0	0	0	0	0	170
normalized size	1	1.	1.38	0.	0.	0.	0.	0.	1.14
time (sec)	N/A	0.136	0.446	0.07	0.	0.	0.	0.	39.319

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	219	0	0	0	0	0	173
normalized size	1	1.	1.62	0.	0.	0.	0.	0.	1.28
time (sec)	N/A	0.109	0.425	0.045	0.	0.	0.	0.	36.093

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	120	164	0	1451	0	0	155
normalized size	1	1.	0.94	1.29	0.	11.43	0.	0.	1.22
time (sec)	N/A	0.068	0.161	0.289	0.	0.358	0.	0.	3.059

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	135	240	0	2641	0	0	223
normalized size	1	1.	0.86	1.53	0.	16.82	0.	0.	1.42
time (sec)	N/A	0.101	0.249	0.153	0.	0.461	0.	0.	4.538

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	118	286	0	950	0	0	37
normalized size	1	1.	1.59	3.86	0.	12.84	0.	0.	0.5
time (sec)	N/A	0.288	0.191	0.169	0.	0.342	0.	0.	4.325

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	139	383	0	914	0	0	22
normalized size	1	1.	1.35	3.72	0.	8.87	0.	0.	0.21
time (sec)	N/A	0.545	0.181	0.197	0.	0.328	0.	0.	3.314

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	126	0	0	0	0	0	17
normalized size	1	1.	1.56	0.	0.	0.	0.	0.	0.21
time (sec)	N/A	0.04	0.152	0.07	0.	0.	0.	0.	2.603

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	126	0	0	0	0	0	20
normalized size	1	1.	1.56	0.	0.	0.	0.	0.	0.25
time (sec)	N/A	0.037	0.14	0.04	0.	0.	0.	0.	3.072

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	118	0	0	0	0	0	144
normalized size	1	1.	1.04	0.	0.	0.	0.	0.	1.27
time (sec)	N/A	0.046	0.131	0.05	0.	0.	0.	0.	5.869

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	124	0	0	0	0	0	192
normalized size	1	1.	1.14	0.	0.	0.	0.	0.	1.76
time (sec)	N/A	0.044	0.102	0.056	0.	0.	0.	0.	5.238

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	159	206	0	72	0	0	0
normalized size	1	1.	1.83	2.37	0.	0.83	0.	0.	0.
time (sec)	N/A	1.479	0.553	0.053	0.	0.266	0.	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	C	C	C	F	A	F	F	F(-1)
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	579	100	317	0	95	0	0	0
normalized size	1	579.	100.	317.	0.	95.	0.	0.	0.
time (sec)	N/A	3.072	0.464	0.063	0.	0.264	0.	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	C	C	C	F	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	180	133	536	0	85	0	0	0
normalized size	1	3.91	2.89	11.65	0.	1.85	0.	0.	0.
time (sec)	N/A	2.933	0.59	0.079	0.	0.259	0.	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-2)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	323	258	0	0	0	0	469
normalized size	1	1.	10.09	8.06	0.	0.	0.	0.	14.66
time (sec)	N/A	0.145	0.516	0.093	0.	0.	0.	0.	63.878

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	262	240	0	59	0	0	371
normalized size	1	1.	11.39	10.43	0.	2.57	0.	0.	16.13
time (sec)	N/A	0.091	0.283	0.031	0.	0.258	0.	0.	41.593

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	206	353	0	0	0	0	34
normalized size	1	1.	0.94	1.62	0.	0.	0.	0.	0.16
time (sec)	N/A	0.116	0.43	0.283	0.	0.	0.	0.	3.449

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	207	350	0	0	0	0	34
normalized size	1	1.	0.99	1.67	0.	0.	0.	0.	0.16
time (sec)	N/A	0.105	0.656	0.266	0.	0.	0.	0.	7.693

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	196	349	0	0	0	0	53
normalized size	1	1.	0.88	1.57	0.	0.	0.	0.	0.24
time (sec)	N/A	0.1	0.623	0.257	0.	0.	0.	0.	11.817

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	198	350	0	0	0	0	51
normalized size	1	1.	0.93	1.64	0.	0.	0.	0.	0.24
time (sec)	N/A	0.096	0.539	0.256	0.	0.	0.	0.	6.47

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	685	327	0	0	0	0	0
normalized size	1	1.	10.54	5.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.211	4.031	0.24	0.	0.	0.	0.	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	1137	311	0	0	0	0	0
normalized size	1	1.	18.05	4.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.213	10.714	0.235	0.	0.	0.	0.	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	0	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.096	0.144	0.045	0.	0.	0.	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F(-1)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	0	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.16	0.047	0.039	0.	0.	0.	0.	0.

2.2 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [29] had the largest ratio of [2.143]

Table 1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.	10	0.1
2	C	5	2	3.47	37	0.054
3	A	9	6	1.	15	0.4
4	A	3	3	1.	19	0.158
5	A	8	7	1.	17	0.412
6	A	6	6	1.	17	0.353
7	A	3	3	1.	17	0.176
8	A	4	2	1.	23	0.087
9	A	18	12	1.66	27	0.444
10	B	25	13	2.46	39	0.333
11	A	7	3	1.	45	0.067
12	A	7	4	1.	32	0.125
13	A	5	3	1.	32	0.094
14	A	2	2	1.	27	0.074
15	A	2	2	1.	29	0.069
16	A	2	1	1.	30	0.033
17	A	3	2	1.	13	0.154
18	A	3	2	1.	15	0.133
19	A	2	2	1.	23	0.087
20	A	2	2	1.	25	0.08
21	A	3	2	1.	11	0.182
22	A	2	2	1.	18	0.111
23	A	6	6	1.	20	0.3
24	A	6	5	1.	31	0.161
25	A	2	2	1.	29	0.069
26	A	2	2	1.	35	0.057
27	A	5	5	1.	32	0.156
28	A	6	6	1.	21	0.286
29	A	359	30	1.	14	2.143
30	A	2	2	1.	23	0.087
31	A	3	2	1.	13	0.154
32	A	2	2	1.	33	0.061

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
33	A	5	5	1.	15	0.333
34	A	5	5	1.	15	0.333
35	A	1	1	1.	11	0.091
36	A	5	5	1.	15	0.333
37	A	1	1	0.8	17	0.059
38	A	3	3	1.	18	0.167
39	A	2	2	1.6	16	0.125
40	A	5	5	1.62	17	0.294
41	A	5	5	1.77	13	0.385
42	A	5	5	1.84	16	0.312
43	F	0	0	N/A	0	N/A
44	F	0	0	N/A	0	N/A
45	F	0	0	N/A	0	N/A
46	C	7	3	1.77	32	0.094
47	A	19	9	1.	20	0.45
48	A	29	9	1.	20	0.45
49	A	49	9	1.	20	0.45
50	A	14	6	1.	23	0.261
51	A	24	6	1.	23	0.261
52	C	9	8	3.09	48	0.167
53	A	7	7	1.	24	0.292
54	A	7	7	1.	24	0.292
55	A	1	1	1.1	18	0.056
56	A	8	8	1.	13	0.615
57	A	6	6	1.	15	0.4
58	F	0	0	N/A	0	N/A
59	F	0	0	N/A	0	N/A
60	A	1	1	1.	38	0.026
61	A	1	1	1.	33	0.03
62	A	2	2	1.	39	0.051
63	A	1	1	1.	19	0.053
64	A	4	4	1.	19	0.21
65	A	4	4	1.	24	0.167
66	A	1	1	1.	24	0.042
67	A	7	7	1.	24	0.292
68	A	7	7	1.	24	0.292
69	A	3	3	1.	26	0.115
70	A	3	3	1.	22	0.136
71	A	1	1	1.	22	0.045
72	A	1	1	1.	23	0.043
73	A	8	8	1.	18	0.444
74	A	8	8	1.	23	0.348
75	A	1	1	1.	21	0.048
76	A	1	1	1.	19	0.053
77	A	1	1	1.	19	0.053
78	A	1	1	1.	19	0.053
79	A	4	4	1.	34	0.118
80	C	5	5	579.	40	0.125
81	C	7	7	3.91	51	0.137

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
82	A	2	2	1.	29	0.069
83	A	2	2	1.	18	0.111
84	A	1	1	1.	25	0.04
85	A	1	1	1.	25	0.04
86	A	1	1	1.	25	0.04
87	A	1	1	1.	25	0.04
88	A	2	2	1.	40	0.05
89	A	2	2	1.	40	0.05
90	A	1	1	1.	18	0.056
91	A	3	3	1.	15	0.2

3 Listing of integrals

3.1 $\int \frac{1}{\sqrt{1-ax}}\,dx$

Optimal. Leaf size=15

$$-\frac{2\sqrt{1-ax}}{a}$$

```
[Out] (-2*Sqrt[1 - a*x])/a
```

Rubi [A] time = 0.00895376, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$-\frac{2\sqrt{1-ax}}{a}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[1 - a*x],x]

[Out] (-2*Sqrt[1 - a*x])/a
```

Rubi in Sympy [A] time = 0.668796, size = 12, normalized size = 0.8

$$-\frac{2\sqrt{-ax+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-a*x+1)**(1/2),x)

[Out] -2*sqrt(-a*x + 1)/a
```

Mathematica [A] time = 0.00485158, size = 15, normalized size = 1.

$$-\frac{2\sqrt{1-ax}}{a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[1 - a*x],x]

[Out] (-2*Sqrt[1 - a*x])/a
```

Maple [A] time = 0.004, size = 14, normalized size = 0.9

$$-2\,\frac{\sqrt{-ax+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-a*x+1)^(1/2),x)
```

```
[Out] -2*(-a*x+1)^(1/2)/a
```

Maxima [A] time = 1.36485, size = 18, normalized size = 1.2

$$-\frac{2\sqrt{-ax+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-a*x + 1),x, algorithm="maxima")

[Out] -2*sqrt(-a*x + 1)/a
```

Fricas [A] time = 0.205924, size = 18, normalized size = 1.2

$$-\frac{2\sqrt{-ax+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-a*x + 1),x, algorithm="fricas")

[Out] -2*sqrt(-a*x + 1)/a
```

Sympy [A] time = 0.038173, size = 12, normalized size = 0.8

$$-\frac{2\sqrt{-ax+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-a*x+1)**(1/2),x)

[Out] -2*sqrt(-a*x + 1)/a
```

GIAC/XCAS [A] time = 0.199346, size = 18, normalized size = 1.2

$$-\frac{2\sqrt{-ax+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-a*x + 1),x, algorithm="giac")

[Out] -2*sqrt(-a*x + 1)/a
```

3.2 $\int \frac{-2\log\left(-\sqrt{-1+ax}\right)+\log(-1+ax)}{2\pi\sqrt{-1+ax}}\,dx$

Optimal. Leaf size=15

$$-\frac{2\sqrt{1-ax}}{a}$$

```
[Out] (-2*Sqrt[1 - a*x])/a
```

Rubi [C] time = 0.0971257, antiderivative size = 52, normalized size of antiderivative = 3.47, number of steps used = 5, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054

$$\frac{\sqrt{ax-1}\log(ax-1)}{\pi a}-\frac{2\sqrt{ax-1}\log\left(-\sqrt{ax-1}\right)}{\pi a}$$

Antiderivative was successfully verified.

```
[In] Int[(-2*Log[-Sqrt[-1 + a*x]] + Log[-1 + a*x])/(2*Pi*Sqrt[-1 + a*x]),x]
```

```
[Out] (-2*Sqrt[-1 + a*x]*Log[-Sqrt[-1 + a*x]])/(a*Pi) + (Sqrt[-1 + a*x]
*Log[-1 + a*x])/(a*Pi)
```

Rubi in Sympy [A] time = 8.53478, size = 42, normalized size = 2.8

$$-\frac{2\sqrt{ax-1}\log\left(-\sqrt{ax-1}\right)}{\pi a}+\frac{\sqrt{ax-1}\log\left(ax-1\right)}{\pi a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/2*(ln(a*x-1)-2*ln(-(a*x-1)**(1/2)))/pi/(a*x-1)**(1/2),x)
```

```
[Out] -2*sqrt(a*x - 1)*log(-sqrt(a*x - 1))/(pi*a) + sqrt(a*x - 1)*log(a
*x - 1)/(pi*a)
```

Mathematica [C] time = 0.0202415, size = 37, normalized size = 2.47

$$\frac{\sqrt{ax-1}\left(\log(ax-1)-2\log\left(-\sqrt{ax-1}\right)\right)}{\pi a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-2*Log[-Sqrt[-1 + a*x]] + Log[-1 + a*x])/(2*Pi*Sqrt[-
1 + a*x]),x]
```

```
[Out] (Sqrt[-1 + a*x]*(-2*Log[-Sqrt[-1 + a*x]] + Log[-1 + a*x]))/(a*Pi)
```

Maple [C] time = 0.005, size = 34, normalized size = 2.3

$$\frac{1}{a\pi}\sqrt{ax-1}\left(\ln\left(ax-1\right)-2\,\ln\left(-\sqrt{ax-1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/2*(ln(a*x-1)-2*ln(-(a*x-1)^(1/2)))/Pi/(a*x-1)^(1/2),x)
```

```
[Out] (a*x-1)^(1/2)*(ln(a*x-1)-2*ln(-(a*x-1)^(1/2)))/a/Pi
```

Maxima [A] time = 1.37476, size = 55, normalized size = 3.67

$$\frac{\sqrt{ax-1}\log(ax-1) - 2\sqrt{ax-1}\log\left(-\sqrt{ax-1}\right)}{\pi a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*(log(a*x - 1) - 2*log(-sqrt(a*x - 1)))/(pi*sqrt(a*x - 1)),x, algori
```

```
[Out] (sqrt(a*x - 1)*log(a*x - 1) - 2*sqrt(a*x - 1)*log(-sqrt(a*x - 1))
)/(pi*a)
```

Fricas [A] time = 0.210294, size = 1, normalized size = 0.07

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*(log(a*x - 1) - 2*log(-sqrt(a*x - 1)))/(pi*sqrt(a*x - 1)),x, algori
```

```
[Out] 0
```

Sympy [A] time = 52.66, size = 42, normalized size = 2.8

$$\frac{\begin{cases} \frac{-2\sqrt{ax-1}\log\left(-\sqrt{ax-1}\right)+\sqrt{ax-1}\log(ax-1)}{a} & \text{for } a \neq 0 \\ \pi x & \text{otherwise} \end{cases}}{\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*(ln(a*x-1)-2*ln(-(a*x-1)**(1/2)))/pi/(a*x-1)**(1/2),x)
```

```
[Out] Piecewise(((-2*sqrt(a*x - 1)*log(-sqrt(a*x - 1)) + sqrt(a*x - 1)*
log(a*x - 1))/a, Ne(a, 0)), (pi*x, True))/pi
```

GIAC/XCAS [A] time = 0.212235, size = 55, normalized size = 3.67

$$\frac{\sqrt{ax-1}\ln(ax-1) - 2\sqrt{ax-1}\ln\left(-\sqrt{ax-1}\right)}{\pi a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*(log(a*x - 1) - 2*log(-sqrt(a*x - 1)))/(pi*sqrt(a*x - 1)),x, algori
```

```
[Out] (sqrt(a*x - 1)*ln(a*x - 1) - 2*sqrt(a*x - 1)*ln(-sqrt(a*x - 1)))/
(pi*a)
```

3.3 $\int \frac{1}{\left(2x+\sqrt{1+x^2}\right)^2}\,dx$

Optimal. Leaf size=82

$$\frac{4x}{3(1-3x^2)}-\frac{2\sqrt{x^2+1}}{3(1-3x^2)}+\frac{\tanh^{-1}\left(\frac{1}{2}\sqrt{3}\sqrt{x^2+1}\right)}{3\sqrt{3}}-\frac{\tanh^{-1}\left(\sqrt{3}x\right)}{3\sqrt{3}}$$

```
[Out] (4*x)/(3*(1 - 3*x^2)) - (2*Sqrt[1 + x^2])/(3*(1 - 3*x^2)) - ArcTa
nh[Sqrt[3]*x]/(3*Sqrt[3]) + ArcTanh[(Sqrt[3]*Sqrt[1 + x^2])/2]/(3
*Sqrt[3])
```

Rubi [A] time = 0.143501, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4

$$\frac{4x}{3(1-3x^2)}-\frac{2\sqrt{x^2+1}}{3(1-3x^2)}+\frac{\tanh^{-1}\left(\frac{1}{2}\sqrt{3}\sqrt{x^2+1}\right)}{3\sqrt{3}}-\frac{\tanh^{-1}\left(\sqrt{3}x\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(2*x + Sqrt[1 + x^2])^(-2),x]
```

```
[Out] (4*x)/(3*(1 - 3*x^2)) - (2*Sqrt[1 + x^2])/(3*(1 - 3*x^2)) - ArcTa
nh[Sqrt[3]*x]/(3*Sqrt[3]) + ArcTanh[(Sqrt[3]*Sqrt[1 + x^2])/2]/(3
*Sqrt[3])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(2x+\sqrt{x^2+1}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(2*x+(x**2+1)**(1/2))**2,x)
```

```
[Out] Integral((2*x + sqrt(x**2 + 1))**(-2), x)
```

Mathematica [A] time = 0.158967, size = 119, normalized size = 1.45

$$\frac{1}{18}\left(\sqrt{3}\log\left(2\sqrt{3}\sqrt{x^2+1}-\sqrt{3}x+3\right)+\frac{-12\sqrt{x^2+1}+\sqrt{3}\left(1-3x^2\right)\log\left(2\sqrt{3}\sqrt{x^2+1}+\sqrt{3}x+3\right)+24x}{1-3x^2}-2\sqrt{3}\log\left(3x+\sqrt{3}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2*x + Sqrt[1 + x^2])^(-2),x]
```

```
[Out] (-2*Sqrt[3]*Log[Sqrt[3] + 3*x] + Sqrt[3]*Log[3 - Sqrt[3]*x + 2*Sq
rt[3]*Sqrt[1 + x^2]] + (24*x - 12*Sqrt[1 + x^2] + Sqrt[3]*(1 - 3*
x^2)*Log[3 + Sqrt[3]*x + 2*Sqrt[3]*Sqrt[1 + x^2]])/(1 - 3*x^2))/1
```

8

Maple [B] time = 0.062, size = 370, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(1/(2*x+(x^2+1)^(1/2))^2,x)
```

```
[Out] -1/2*x/(3*x^2-1)-1/9*arctanh(x*3^(1/2))*3^(1/2)-5/18*x/(x^2-1/3)-
3^(1/2)*(-1/12/(x-1/3*3^(1/2))*((x-1/3*3^(1/2))^2+2/3*3^(1/2)*(x-
1/3*3^(1/2))+4/3)^(3/2)+1/36*3^(1/2)*(1/3*(9*(x-1/3*3^(1/2))^2+6*
3^(1/2)*(x-1/3*3^(1/2))+12)^(1/2)+1/3*3^(1/2)*arcsinh(x)-2/3*3^(1
/2)*arctanh(3/4*(8/3+2/3*3^(1/2)*(x-1/3*3^(1/2)))*3^(1/2)/(9*(x-1
/3*3^(1/2))^2+6*3^(1/2)*(x-1/3*3^(1/2))+12)^(1/2)))+1/12*x*((x-1/
3*3^(1/2))^2+2/3*3^(1/2)*(x-1/3*3^(1/2))+4/3)^(1/2)+1/12*arcsinh(
x))+3^(1/2)*(-1/12/(x+1/3*3^(1/2))*((x+1/3*3^(1/2))^2-2/3*3^(1/2)
*(x+1/3*3^(1/2))+4/3)^(3/2)-1/36*3^(1/2)*(1/3*(9*(x+1/3*3^(1/2))^
2-6*3^(1/2)*(x+1/3*3^(1/2))+12)^(1/2)-1/3*3^(1/2)*arcsinh(x)-2/3*
3^(1/2)*arctanh(3/4*(8/3-2/3*3^(1/2)*(x+1/3*3^(1/2)))*3^(1/2)/(9*
(x+1/3*3^(1/2))^2-6*3^(1/2)*(x+1/3*3^(1/2))+12)^(1/2)))+1/12*x*((
x+1/3*3^(1/2))^2-2/3*3^(1/2)*(x+1/3*3^(1/2))+4/3)^(1/2)+1/12*arcs
inh(x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(2\,x + \sqrt{x^2 + 1}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((2*x + sqrt(x^2 + 1))^(-2),x, algorithm="maxima")
```

```
[Out] integrate((2*x + sqrt(x^2 + 1))^(-2), x)
```

Fricas [A] time = 0.220191, size = 363, normalized size = 4.43

$$-\frac{\left(6\,x^4 + x^2 - 2\left(3\,x^3 - x\right)\sqrt{x^2 + 1} - 1\right)\log\left(-\frac{24\,x^3 - \sqrt{3}\left(6\,x^4 + 17\,x^2 + 7\right) - 2\left(12\,x^2 - \sqrt{3}\left(3\,x^3 + 7\,x\right) + 6\right)\sqrt{x^2+1} + 24\,x}{6\,x^4 + x^2 - 2\left(3\,x^3 - x\right)\sqrt{x^2+1} - 1}\right) + \left(6\,x^4 + x^2 - 1\right)\log\left(\cdots\right.}{6\left(2\,\sqrt{3}\left(3\,x^3 - x\right)\sqrt{x^2 + 1} - \sqrt{3}\left(6\,x^4 \cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((2*x + sqrt(x^2 + 1))^(-2),x, algorithm="fricas")
```

```
[Out] -1/6*((6*x^4 + x^2 - 2*(3*x^3 - x)*sqrt(x^2 + 1) - 1)*log(-(24*x^
3 - sqrt(3)*(6*x^4 + 17*x^2 + 7) - 2*(12*x^2 - sqrt(3)*(3*x^3 + 7
*x) + 6)*sqrt(x^2 + 1) + 24*x)/(6*x^4 + x^2 - 2*(3*x^3 - x)*sqrt(
x^2 + 1) - 1)) + (6*x^4 + x^2 - 1)*log((sqrt(3)*(3*x^2 + 1) - 6*x
)/(3*x^2 - 1)) - 8*sqrt(3)*(3*x^3 + 2*x) - 2*sqrt(x^2 + 1)*((3*x^
3 - x)*log((sqrt(3)*(3*x^2 + 1) - 6*x)/(3*x^2 - 1)) - 2*sqrt(3)*(
6*x^2 + 1)))/(2*sqrt(3)*(3*x^3 - x)*sqrt(x^2 + 1) - sqrt(3)*(6*x^
4 + x^2 - 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(2x + \sqrt{x^2 + 1}\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(2*x+(x**2+1)**(1/2))**2,x)
```

```
[Out] Integral((2*x + sqrt(x**2 + 1))**(-2), x)
```

GIAC/XCAS [A] time = 0.206009, size = 239, normalized size = 2.91

$$\frac{1}{18}\,\sqrt{3}\ln\left(\frac{\left|6\,x - 2\,\sqrt{3}\right|}{\left|6\,x + 2\,\sqrt{3}\right|}\right) - \frac{1}{18}\,\sqrt{3}\ln\left(-\frac{\left|-6\,x - 8\,\sqrt{3} + 6\,\sqrt{x^2 + 1} - \frac{6}{x - \sqrt{x^2+1}}\right|}{2\left(3\,x - 4\,\sqrt{3} - 3\,\sqrt{x^2 + 1} + \frac{3}{x - \sqrt{x^2+1}}\right)}\right) - \frac{4\left(x - \sqrt{x^2 + 1} + \frac{1}{x - \sqrt{x^2+1}}\right)}{3\left(3\left(x - \sqrt{x^2 + 1} + \frac{1}{x - \sqrt{x^2+1}}\right)^2 - 16\right)} - \frac{4\,x}{3\,(3\,x^2 - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x + sqrt(x^2 + 1))^(-2),x, algorithm="giac")
```

```
[Out] 1/18*sqrt(3)*ln(abs(6*x - 2*sqrt(3))/abs(6*x + 2*sqrt(3))) - 1/18
*sqrt(3)*ln(-1/2*abs(-6*x - 8*sqrt(3) + 6*sqrt(x^2 + 1) - 6/(x -
sqrt(x^2 + 1)))/(3*x - 4*sqrt(3) - 3*sqrt(x^2 + 1) + 3/(x - sqrt(
x^2 + 1)))) - 4/3*(x - sqrt(x^2 + 1) + 1/(x - sqrt(x^2 + 1)))/(3*
(x - sqrt(x^2 + 1) + 1/(x - sqrt(x^2 + 1)))^2 - 16) - 4/3*x/(3*x^
2 - 1)
```

3.4 $\int \frac{1}{\sqrt{-1+x^2}(-4+3x^2)^2}\,dx$

Optimal. Leaf size=43

$$\frac{3\sqrt{x^2-1}x}{8(4-3x^2)}+\frac{5}{16}\tanh^{-1}\left(\frac{x}{2\sqrt{x^2-1}}\right)$$

```
[Out] (3*x*Sqrt[-1 + x^2])/(8*(4 - 3*x^2)) + (5*ArcTanh[x/(2*Sqrt[-1 +
x^2])])/16
```

Rubi [A] time = 0.0379042, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$\frac{3\sqrt{x^2-1}x}{8(4-3x^2)}+\frac{5}{16}\tanh^{-1}\left(\frac{x}{2\sqrt{x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[-1 + x^2]*(-4 + 3*x^2)^2),x]
```

```
[Out] (3*x*Sqrt[-1 + x^2])/(8*(4 - 3*x^2)) + (5*ArcTanh[x/(2*Sqrt[-1 +
x^2])])/16
```

Rubi in Sympy [A] time = 4.02935, size = 34, normalized size = 0.79

$$\frac{3x\sqrt{x^2-1}}{8(-3x^2+4)}+\frac{5\,\mathrm{atanh}\left(\frac{x}{2\sqrt{x^2-1}}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(3*x**2-4)**2/(x**2-1)**(1/2),x)
```

```
[Out] 3*x*sqrt(x**2 - 1)/(8*(-3*x**2 + 4)) + 5*atanh(x/(2*sqrt(x**2 - 1
)))/16
```

Mathematica [A] time = 0.0422125, size = 43, normalized size = 1.

$$\frac{5}{16}\tanh^{-1}\left(\frac{x}{2\sqrt{x^2-1}}\right)-\frac{3x\sqrt{x^2-1}}{8(3x^2-4)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[-1 + x^2]*(-4 + 3*x^2)^2),x]
```

```
[Out] (-3*x*Sqrt[-1 + x^2])/(8*(-4 + 3*x^2)) + (5*ArcTanh[x/(2*Sqrt[-1
+ x^2])])/16
```

Maple [B] time = 0.069, size = 172, normalized size = 4.

$$-\frac{1}{16}\sqrt{\left(x-\frac{2\sqrt{3}}{3}\right)^2+\frac{4\sqrt{3}}{3}\left(x-\frac{2\sqrt{3}}{3}\right)+\frac{1}{3}}\left(x-\frac{2\sqrt{3}}{3}\right)^{-1}$$
$$+\frac{5}{32}\mathit{Artanh}\left(\frac{3\sqrt{3}}{2}\left(\frac{2}{3}+\frac{4\sqrt{3}}{3}\left(x-\frac{2\sqrt{3}}{3}\right)\right)\frac{1}{\sqrt{9\left(x-2/3\sqrt{3}\right)^2+12\sqrt{3}\left(x-2/3\sqrt{3}\right)+3}}\right)$$
$$-\frac{1}{16}\sqrt{\left(x+\frac{2\sqrt{3}}{3}\right)^2-\frac{4\sqrt{3}}{3}\left(x+\frac{2\sqrt{3}}{3}\right)+\frac{1}{3}}\left(x+\frac{2\sqrt{3}}{3}\right)^{-1}$$
$$-\frac{5}{32}\mathit{Artanh}\left(\frac{3\sqrt{3}}{2}\left(\frac{2}{3}-\frac{4\sqrt{3}}{3}\left(x+\frac{2\sqrt{3}}{3}\right)\right)\frac{1}{\sqrt{9\left(x+2/3\sqrt{3}\right)^2-12\sqrt{3}\left(x+2/3\sqrt{3}\right)+3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(1/(3*x^2-4)^2/(x^2-1)^(1/2),x)
```

```
[Out] -1/16/(x-2/3*3^(1/2))*((x-2/3*3^(1/2))^2+4/3*3^(1/2)*(x-2/3*3^(1/
2))+1/3)^(1/2)+5/32*arctanh(3/2*(2/3+4/3*3^(1/2)*(x-2/3*3^(1/2)))
*3^(1/2)/(9*(x-2/3*3^(1/2))^2+12*3^(1/2)*(x-2/3*3^(1/2))+3)^(1/2)
)-1/16/(x+2/3*3^(1/2))*((x+2/3*3^(1/2))^2-4/3*3^(1/2)*(x+2/3*3^(1
/2))+1/3)^(1/2)-5/32*arctanh(3/2*(2/3-4/3*3^(1/2)*(x+2/3*3^(1/2))
)*3^(1/2)/(9*(x+2/3*3^(1/2))^2-12*3^(1/2)*(x+2/3*3^(1/2))+3)^(1/2
))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3x^2-4)^2\sqrt{x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(1/((3*x^2 - 4)^2*sqrt(x^2 - 1)),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 - 4)^2*sqrt(x^2 - 1)), x)
```

Fricas [A] time = 0.21575, size = 201, normalized size = 4.67

$$\frac{20x^2-5\left(6x^4-11x^2-2\left(3x^3-4x\right)\sqrt{x^2-1}+4\right)\log\left(3x^2-3\sqrt{x^2-1}x-2\right)+5\left(6x^4-11x^2-2\left(3x^3-4x\right)\sqrt{x^2-1}\right.}{32\left(6x^4-11x^2-2(3x^3-4x)\sqrt{x^2-1}+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(1/((3*x^2 - 4)^2*sqrt(x^2 - 1)),x, algorithm="fricas")
```

```
[Out] 1/32*(20*x^2 - 5*(6*x^4 - 11*x^2 - 2*(3*x^3 - 4*x)*sqrt(x^2 - 1)
+ 4)*log(3*x^2 - 3*sqrt(x^2 - 1)*x - 2) + 5*(6*x^4 - 11*x^2 - 2*(
3*x^3 - 4*x)*sqrt(x^2 - 1) + 4)*log(x^2 - sqrt(x^2 - 1)*x - 2) -
20*sqrt(x^2 - 1)*x - 16)/(6*x^4 - 11*x^2 - 2*(3*x^3 - 4*x)*sqrt(x
^2 - 1) + 4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(x-1)(x+1)}\,(3x^2-4)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(1/(3*x**2-4)**2/(x**2-1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x - 1)*(x + 1))*(3*x**2 - 4)**2), x)
```

GIAC/XCAS [A] time = 0.23251, size = 127, normalized size = 2.95

$$\frac{5\left(x-\sqrt{x^2-1}\right)^2-3}{4\left(3\left(x-\sqrt{x^2-1}\right)^4-10\left(x-\sqrt{x^2-1}\right)^2+3\right)} - \frac{5}{32}\ln\left(\left|3\left(x-\sqrt{x^2-1}\right)^2-1\right|\right) + \frac{5}{32}\ln\left(\left|\left(x-\sqrt{x^2-1}\right)^2-3\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(1/((3*x^2 - 4)^2*sqrt(x^2 - 1)),x, algorithm="giac")
```

```
[Out] 1/4*(5*(x - sqrt(x^2 - 1))^2 - 3)/(3*(x - sqrt(x^2 - 1))^4 - 10*(
x - sqrt(x^2 - 1))^2 + 3) - 5/32*ln(abs(3*(x - sqrt(x^2 - 1))^2 -
 1)) + 5/32*ln(abs((x - sqrt(x^2 - 1))^2 - 3))
```

3.5 $\int \frac{1}{\left(2\sqrt{x}+\sqrt{1+x}\right)^2}\,dx$

Optimal. Leaf size=74

$$-\frac{4\sqrt{x}\sqrt{x+1}}{3(1-3x)}+\frac{8}{9(1-3x)}+\frac{5}{9}\log(1-3x)-\frac{8}{9}\sinh^{-1}\left(\sqrt{x}\right)+\frac{10}{9}\tanh^{-1}\left(\frac{2\sqrt{x}}{\sqrt{x+1}}\right)$$

```
[Out] 8/(9*(1 - 3*x)) - (4*Sqrt[x]*Sqrt[1 + x])/(3*(1 - 3*x)) - (8*ArcS
inh[Sqrt[x]])/9 + (10*ArcTanh[(2*Sqrt[x])/Sqrt[1 + x]])/9 + (5*Lo
g[1 - 3*x])/9
```

Rubi [A] time = 0.145885, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$

$$-\frac{4\sqrt{x}\sqrt{x+1}}{3(1-3x)}+\frac{8}{9(1-3x)}+\frac{5}{9}\log(1-3x)-\frac{8}{9}\sinh^{-1}\left(\sqrt{x}\right)+\frac{10}{9}\tanh^{-1}\left(\frac{2\sqrt{x}}{\sqrt{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(2*Sqrt[x] + Sqrt[1 + x])^(-2),x]
```

```
[Out] 8/(9*(1 - 3*x)) - (4*Sqrt[x]*Sqrt[1 + x])/(3*(1 - 3*x)) - (8*ArcS
inh[Sqrt[x]])/9 + (10*ArcTanh[(2*Sqrt[x])/Sqrt[1 + x]])/9 + (5*Lo
g[1 - 3*x])/9
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$2\int^{\sqrt{x+1}}\frac{x}{\left(x+2\sqrt{x^2-1}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(2*x**(1/2)+(1+x)**(1/2))**2,x)
```

```
[Out] 2*Integral(x/(x + 2*sqrt(x**2 - 1))**2, (x, sqrt(x + 1)))
```

Mathematica [A] time = 0.145594, size = 65, normalized size = 0.88

$$\frac{1}{9}\left(\frac{-12\sqrt{x}\sqrt{x+1}+(5-15x)\log(1-3x)+8}{1-3x}-8\sinh^{-1}\left(\sqrt{x}\right)+10\tanh^{-1}\left(2\sqrt{\frac{x}{x+1}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2*Sqrt[x] + Sqrt[1 + x])^(-2),x]
```

```
[Out] (-8*ArcSinh[Sqrt[x]] + 10*ArcTanh[2*Sqrt[x/(1 + x)]] + (8 - 12*Sq
rt[x]*Sqrt[1 + x] + (5 - 15*x)*Log[1 - 3*x])/(1 - 3*x))/9
```

Maple [B] time = 0.023, size = 115, normalized size = 1.6

$$-\frac{8}{27\,x-9}+\frac{5\,\ln(3\,x-1)}{9}$$
$$-\frac{1}{27\,x-9}\sqrt{x}\sqrt{1+x}\left(12\,\ln\left(1/2+x+\sqrt{(1+x)\,x}\right)x-15\,Artanh\left(1/4\,\frac{1+5\,x}{\sqrt{(1+x)\,x}}\right)x-4\,\ln\left(1/2+x+\sqrt{(1+x)\,x}\right)+5\,Arta\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(1/(2*x^(1/2)+(1+x)^(1/2))^2,x)
```

```
[Out] -8/9/(3*x-1)+5/9*ln(3*x-1)-1/9*x^(1/2)*(1+x)^(1/2)*(12*ln(1/2+x+(
(1+x)*x)^(1/2))*x-15*arctanh(1/4*(1+5*x)/((1+x)*x)^(1/2))*x-4*ln(
1/2+x+((1+x)*x)^(1/2))+5*arctanh(1/4*(1+5*x)/((1+x)*x)^(1/2))-12*
((1+x)*x)^(1/2))/((1+x)*x)^(1/2)/(3*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\sqrt{x+1}+2\sqrt{x}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((sqrt(x + 1) + 2*sqrt(x))^(-2),x, algorithm="maxima")
```

```
[Out] integrate((sqrt(x + 1) + 2*sqrt(x))^(-2), x)
```

Fricas [A] time = 0.224661, size = 274, normalized size = 3.7

$$\frac{2\left(5\left(3x-1\right)\log\left(3x-1\right)-18\right)\sqrt{x+1}\sqrt{x}-5\left(2\left(3x-1\right)\sqrt{x+1}\sqrt{x}-6x^{2}-x+1\right)\log\left(3\sqrt{x+1}\sqrt{x}-3x-1\right)+4\left(2(3x-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((sqrt(x + 1) + 2*sqrt(x))^(-2),x, algorithm="fricas")
```

```
[Out] 1/9*(2*(5*(3*x - 1)*log(3*x - 1) - 18)*sqrt(x + 1)*sqrt(x) - 5*(2
*(3*x - 1)*sqrt(x + 1)*sqrt(x) - 6*x^2 - x + 1)*log(3*sqrt(x + 1)
*sqrt(x) - 3*x - 1) + 4*(2*(3*x - 1)*sqrt(x + 1)*sqrt(x) - 6*x^2
- x + 1)*log(2*sqrt(x + 1)*sqrt(x) - 2*x - 1) + 5*(2*(3*x - 1)*sq
rt(x + 1)*sqrt(x) - 6*x^2 - x + 1)*log(sqrt(x + 1)*sqrt(x) - x +
1) - 5*(6*x^2 + x - 1)*log(3*x - 1) + 36*x + 12)/(2*(3*x - 1)*sqr
t(x + 1)*sqrt(x) - 6*x^2 - x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(2\sqrt{x}+\sqrt{x+1}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(1/(2*x**(1/2)+(1+x)**(1/2))**2,x)
```

```
[Out] Integral((2*sqrt(x) + sqrt(x + 1))**(-2), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(x + 1) + 2*sqrt(x))^(-2),x, algorithm="giac")

[Out] Exception raised: NotImplementedError
```

3.6 $\int \frac{\sqrt{-1+x^2}}{(-i+x)^2}\,dx$

Optimal. Leaf size=64

$$\frac{\sqrt{x^2-1}}{-x+i}-\frac{i\tan^{-1}\left(\frac{1-ix}{\sqrt{2}\sqrt{x^2-1}}\right)}{\sqrt{2}}+\tanh^{-1}\left(\frac{x}{\sqrt{x^2-1}}\right)$$

```
[Out] Sqrt[-1 + x^2]/(I - x) - (I*ArcTan[(1 - I*x)/(Sqrt[2]*Sqrt[-1 + x
^2])])/Sqrt[2] + ArcTanh[x/Sqrt[-1 + x^2]]
```

Rubi [A] time = 0.0759454, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353

$$\frac{\sqrt{x^2-1}}{-x+i}-\frac{i\tan^{-1}\left(\frac{1-ix}{\sqrt{2}\sqrt{x^2-1}}\right)}{\sqrt{2}}+\tanh^{-1}\left(\frac{x}{\sqrt{x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[-1 + x^2]/(-I + x)^2,x]
```

```
[Out] Sqrt[-1 + x^2]/(I - x) - (I*ArcTan[(1 - I*x)/(Sqrt[2]*Sqrt[-1 + x
^2])])/Sqrt[2] + ArcTanh[x/Sqrt[-1 + x^2]]
```

Rubi in Sympy [A] time = 5.03943, size = 53, normalized size = 0.83

$$\frac{\sqrt{2}i\,\mathrm{atan}\left(\frac{\sqrt{2}(ix-1)}{2\sqrt{x^2-1}}\right)}{2}+\mathrm{atanh}\left(\frac{x}{\sqrt{x^2-1}}\right)+\frac{\sqrt{x^2-1}}{-x+i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2-1)**(1/2)/(-I+x)**2,x)
```

```
[Out] sqrt(2)*I*atan(sqrt(2)*(I*x - 1)/(2*sqrt(x**2 - 1)))/2 + atanh(x/
sqrt(x**2 - 1)) + sqrt(x**2 - 1)/(-x + I)
```

Mathematica [B] time = 0.178963, size = 165, normalized size = 2.58

$$\frac{1}{4}\left(-\frac{4\sqrt{x^2-1}}{x-i}+\sqrt{2}\log\left(2\sqrt{2}\sqrt{x^2-1}-3x-i\right)+2\log\left(-2x^2-2\sqrt{x^2-1}x+2i\sqrt{x^2-1}+2ix+1\right)-2i\sqrt{2}\tan^{-1}\left(\frac{1}{2}\left(-\sqrt{2}\sqrt{x^2-1}+x-i\right)\right)+4\tanh^{-1}\left(\frac{2x}{\sqrt{x^2-1}-x+i}\right)-\sqrt{2}\log(x-i)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[-1 + x^2]/(-I + x)^2,x]
```

```
[Out] ((-4*Sqrt[-1 + x^2])/(-I + x) - (2*I)*Sqrt[2]*ArcTan[(-I + x - Sq
rt[2]*Sqrt[-1 + x^2])/2] + 4*ArcTanh[(2*x)/(I - x + Sqrt[-1 + x^2
])] - Sqrt[2]*Log[-I + x] + Sqrt[2]*Log[-I - 3*x + 2*Sqrt[2]*Sqrt
[-1 + x^2]] + 2*Log[1 + (2*I)*x - 2*x^2 + (2*I)*Sqrt[-1 + x^2] -
2*x*Sqrt[-1 + x^2]])/4
```

Maple [B] time = 0.038, size = 125, normalized size = 2.

$$\frac{1}{2x-2i}\left((x-i)^2-2+2i(x-i)\right)^{\frac{3}{2}}+\ln\left(x+\sqrt{(x-i)^2-2+2i(x-i)}\right)+\frac{i}{2}\sqrt{2}\arctan\left(\frac{(-4+2i(x-i))\sqrt{2}}{4}\frac{1}{\sqrt{(x-i)^2-2+2i(x-i)}}\right)-\frac{i}{2}\sqrt{(x-i)^2-2+2i(x-i)}-\frac{x}{2}\sqrt{(x-i)^2-2+2i(x-i)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((x^2-1)^(1/2)/(x-I)^2,x)
```

```
[Out] 1/2/(x-I)*((x-I)^2-2+2*I*(x-I))^(3/2)+ln(x+((x-I)^2-2+2*I*(x-I))^
(1/2))+1/2*I*2^(1/2)*arctan(1/4*(-4+2*I*(x-I))*2^(1/2)/((x-I)^2-2
+2*I*(x-I))^(1/2))-1/2*I*((x-I)^2-2+2*I*(x-I))^(1/2)-1/2*x*((x-I)
^2-2+2*I*(x-I))^(1/2)
```

Maxima [A] time = 1.54556, size = 72, normalized size = 1.12

$$\frac{1}{2}i\sqrt{2}\arcsin\left(\frac{ix}{|x-i|}-\frac{1}{|x-i|}\right)-\frac{\sqrt{x^2-1}}{x-i}+\log\left(2x+2\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(sqrt(x^2 - 1)/(x - I)^2,x, algorithm="maxima")
```

```
[Out] 1/2*I*sqrt(2)*arcsin(I*x/abs(x - I) - 1/abs(x - I)) - sqrt(x^2 -
1)/(x - I) + log(2*x + 2*sqrt(x^2 - 1))
```

Fricas [A] time = 0.231246, size = 230, normalized size = 3.59

$$\frac{\left(\sqrt{2}\sqrt{x^2-1}(x-i)-\sqrt{2}(x^2-ix)\right)\log\left(-x+i\sqrt{2}+\sqrt{x^2-1}+i\right)-\left(\sqrt{2}\sqrt{x^2-1}(x-i)-\sqrt{2}(x^2-ix)\right)\log\left(-x-i\sqrt{2}+\right.}{2x^2-\sqrt{x^2-1}(2x-2i)-2ix}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(sqrt(x^2 - 1)/(x - I)^2,x, algorithm="fricas")
```

```
[Out] ((sqrt(2)*sqrt(x^2 - 1)*(x - I) - sqrt(2)*(x^2 - I*x))*log(-x + I
*sqrt(2) + sqrt(x^2 - 1) + I) - (sqrt(2)*sqrt(x^2 - 1)*(x - I) -
sqrt(2)*(x^2 - I*x))*log(-x - I*sqrt(2) + sqrt(x^2 - 1) + I) - (2
*x^2 - sqrt(x^2 - 1)*(2*x - 2*I) - 2*I*x)*log(-x + sqrt(x^2 - 1))
 + 2*I*x - 2*I*sqrt(x^2 - 1) - 2)/(2*x^2 - sqrt(x^2 - 1)*(2*x - 2
*I) - 2*I*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{(x-1)(x+1)}}{(x-i)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((x**2-1)**(1/2)/(-I+x)**2,x)
```

```
[Out] Integral(sqrt((x - 1)*(x + 1))/(x - I)**2, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2-1}}{(x-i)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 - 1)/(x - I)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 - 1)/(x - I)^2, x)
```

3.7 $\int \frac{1}{\sqrt{-1+x^2}(1+x^2)^2} dx$

Optimal. Leaf size=48

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right)}{4\sqrt{2}} - \frac{x\sqrt{x^2-1}}{4(x^2+1)}$$

```
[Out] -(x*Sqrt[-1 + x^2])/(4*(1 + x^2)) + (3*ArcTanh[(Sqrt[2]*x)/Sqrt[-
1 + x^2]])/(4*Sqrt[2])
```

Rubi [A] time = 0.0388008, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right)}{4\sqrt{2}} - \frac{x\sqrt{x^2-1}}{4(x^2+1)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 + x^2]*(1 + x^2)^2),x]
```

```
[Out] -(x*Sqrt[-1 + x^2])/(4*(1 + x^2)) + (3*ArcTanh[(Sqrt[2]*x)/Sqrt[-
1 + x^2]])/(4*Sqrt[2])
```

Rubi in Sympy [A] time = 3.71454, size = 41, normalized size = 0.85

$$-\frac{x\sqrt{x^2-1}}{4(x^2+1)} + \frac{3\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(x**2+1)**2/(x**2-1)**(1/2),x)
```

```
[Out] -x*sqrt(x**2 - 1)/(4*(x**2 + 1)) + 3*sqrt(2)*atanh(sqrt(2)*x/sqrt
(x**2 - 1))/8
```

Mathematica [A] time = 0.0706282, size = 90, normalized size = 1.88

$$-\frac{\sqrt{x^2-1}x}{4(x^2+1)} + \frac{3\log\left(-3x^2-2\sqrt{2}\sqrt{x^2-1}x+1\right)}{16\sqrt{2}} - \frac{3\log\left(-3x^2+2\sqrt{2}\sqrt{x^2-1}x+1\right)}{16\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 + x^2]*(1 + x^2)^2),x]
```

```
[Out] -(x*Sqrt[-1 + x^2])/(4*(1 + x^2)) + (3*Log[1 - 3*x^2 - 2*Sqrt[2]*
x*Sqrt[-1 + x^2]])/(16*Sqrt[2]) - (3*Log[1 - 3*x^2 + 2*Sqrt[2]*x*
Sqrt[-1 + x^2]])/(16*Sqrt[2])
```

Maple [A] time = 0.026, size = 45, normalized size = 0.9

$$-\frac{x}{8}\frac{1}{\sqrt{x^2-1}}\left(\frac{x^2}{x^2-1}-\frac{1}{2}\right)^{-1} + \frac{3\sqrt{2}}{8}\mathit{Artanh}\left(x\sqrt{2}\frac{1}{\sqrt{x^2-1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(1/(x^2+1)^2/(x^2-1)^(1/2),x)
```

```
[Out] -1/8/(x^2-1)^(1/2)*x/(x^2/(x^2-1)-1/2)+3/8*arctanh(x*2^(1/2)/(x^2
-1)^(1/2))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2+1)^2\sqrt{x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(1/((x^2 + 1)^2*sqrt(x^2 - 1)),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 1)^2*sqrt(x^2 - 1)), x)
```

Fricas [A] time = 0.222009, size = 216, normalized size = 4.5

$$-\frac{6\sqrt{2}\sqrt{x^2-1}x+3\left(2x^4+x^2-2\left(x^3+x\right)\sqrt{x^2-1}-1\right)\log\left(\frac{4x^2+\sqrt{2}\left(2x^4+x^2+3\right)-2\sqrt{x^2-1}\left(\sqrt{2}\left(x^3+x\right)+2x\right)+4}{2x^4+x^2-2\left(x^3+x\right)\sqrt{x^2-1}-1}\right)-2\sqrt{2}\left(3x^2-1\right)}{8\left(2\sqrt{2}\left(x^3+x\right)\sqrt{x^2-1}-\sqrt{2}\left(2x^4+x^2-1\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(1/((x^2 + 1)^2*sqrt(x^2 - 1)),x, algorithm="fricas")
```

```
[Out] -1/8*(6*sqrt(2)*sqrt(x^2 - 1)*x + 3*(2*x^4 + x^2 - 2*(x^3 + x)*sq
rt(x^2 - 1) - 1)*log((4*x^2 + sqrt(2)*(2*x^4 + x^2 + 3) - 2*sqrt(
x^2 - 1)*(sqrt(2)*(x^3 + x) + 2*x) + 4)/(2*x^4 + x^2 - 2*(x^3 + x
)*sqrt(x^2 - 1) - 1)) - 2*sqrt(2)*(3*x^2 - 1))/(2*sqrt(2)*(x^3 +
x)*sqrt(x^2 - 1) - sqrt(2)*(2*x^4 + x^2 - 1))
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(1/(x**2+1)**2/(x**2-1)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

GIAC/XCAS [A] time = 0.202751, size = 136, normalized size = 2.83

$$-\frac{3}{16}\sqrt{2}\ln\left(\frac{\left(x-\sqrt{x^2-1}\right)^2-2\sqrt{2}+3}{\left(x-\sqrt{x^2-1}\right)^2+2\sqrt{2}+3}\right)-\frac{3\left(x-\sqrt{x^2-1}\right)^2+1}{2\left(\left(x-\sqrt{x^2-1}\right)^4+6\left(x-\sqrt{x^2-1}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 1)^2*sqrt(x^2 - 1)),x, algorithm="giac")
```

```
[Out] -3/16*sqrt(2)*ln(((x - sqrt(x^2 - 1))^2 - 2*sqrt(2) + 3)/((x - sq
rt(x^2 - 1))^2 + 2*sqrt(2) + 3)) - 1/2*(3*(x - sqrt(x^2 - 1))^2 +
 1)/((x - sqrt(x^2 - 1))^4 + 6*(x - sqrt(x^2 - 1))^2 + 1)
```

3.8 $\int \frac{1}{\left(\sqrt{-1+x}+\sqrt{x}\right)^2 \sqrt{-1+x}} \, dx$

Optimal. Leaf size=30

$$-\frac{4x^{3/2}}{3} + \frac{4}{3}(x-1)^{3/2} + 2\sqrt{x-1}$$

```
[Out] 2*Sqrt[-1 + x] + (4*(-1 + x)^(3/2))/3 - (4*x^(3/2))/3
```

Rubi [A] time = 0.127849, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$

$$-\frac{4x^{3/2}}{3} + \frac{4}{3}(x-1)^{3/2} + 2\sqrt{x-1}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((Sqrt[-1 + x] + Sqrt[x])^2*Sqrt[-1 + x]),x]

[Out] 2*Sqrt[-1 + x] + (4*(-1 + x)^(3/2))/3 - (4*x^(3/2))/3
```

Rubi in Sympy [A] time = 4.97118, size = 26, normalized size = 0.87

$$-\frac{4x^{\frac{3}{2}}}{3} + \frac{4(x-1)^{\frac{3}{2}}}{3} + 2\sqrt{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-1+x)**(1/2)/((-1+x)**(1/2)+x**(1/2))**2,x)

[Out] -4*x**(3/2)/3 + 4*(x - 1)**(3/2)/3 + 2*sqrt(x - 1)
```

Mathematica [A] time = 0.0229325, size = 29, normalized size = 0.97

$$\frac{2}{3}\left(-2x^{3/2} + 2\sqrt{x-1}x + \sqrt{x-1}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((Sqrt[-1 + x] + Sqrt[x])^2*Sqrt[-1 + x]),x]

[Out] (2*(Sqrt[-1 + x] + 2*Sqrt[-1 + x]*x - 2*x^(3/2)))/3
```

Maple [A] time = 0.004, size = 21, normalized size = 0.7

$$\frac{4}{3}(-1+x)^{\frac{3}{2}} - \frac{4}{3}x^{\frac{3}{2}} + 2\sqrt{-1+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-1+x)^(1/2)/((-1+x)^(1/2)+x^(1/2))^2,x)

[Out] 4/3*(-1+x)^(3/2)-4/3*x^(3/2)+2*(-1+x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x-1}\left(\sqrt{x-1}+\sqrt{x}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(x - 1)*(sqrt(x - 1) + sqrt(x))^2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x - 1)*(sqrt(x - 1) + sqrt(x))^2), x)
```

Fricas [A] time = 0.207534, size = 24, normalized size = 0.8

$$\frac{2}{3}\,(2\,x+1)\sqrt{x-1}-\frac{4}{3}\,x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(x - 1)*(sqrt(x - 1) + sqrt(x))^2),x, algorithm="fricas")
```

```
[Out] 2/3*(2*x + 1)*sqrt(x - 1) - 4/3*x^(3/2)
```

Sympy [A] time = 1.50173, size = 53, normalized size = 1.77

$$-\frac{4\sqrt{x}}{6\sqrt{x}\sqrt{x-1}+6x-3}-\frac{2\sqrt{x-1}}{6\sqrt{x}\sqrt{x-1}+6x-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-1+x)**(1/2)/((-1+x)**(1/2)+x**(1/2))**2,x)
```

```
[Out] -4*sqrt(x)/(6*sqrt(x)*sqrt(x - 1) + 6*x - 3) - 2*sqrt(x - 1)/(6*s
qrt(x)*sqrt(x - 1) + 6*x - 3)
```

GIAC/XCAS [A] time = 0.205963, size = 27, normalized size = 0.9

$$\frac{4}{3}\,(x-1)^{\frac{3}{2}}-\frac{4}{3}\,x^{\frac{3}{2}}+2\,\sqrt{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(x - 1)*(sqrt(x - 1) + sqrt(x))^2),x, algorithm="giac")
```

```
[Out] 4/3*(x - 1)^(3/2) - 4/3*x^(3/2) + 2*sqrt(x - 1)
```

3.9 $\int \frac{1}{\sqrt{-1+x^2}\left(\sqrt{x}+\sqrt{-1+x^2}\right)^2}\,dx$

Optimal. Leaf size=220

$$\frac{2-4x}{5\left(\sqrt{x^2-1}+\sqrt{x}\right)}-\frac{1}{50}\sqrt{50\sqrt{5}-110}\tan^{-1}\left(\frac{\sqrt{2\sqrt{5}-2}\sqrt{x^2-1}}{2-\left(1-\sqrt{5}\right)x}\right)-\frac{1}{50}\sqrt{110+50\sqrt{5}}\tanh^{-1}\left(\frac{\sqrt{2+2\sqrt{5}}\sqrt{x^2-1}}{-\sqrt{5}x-x+2}\right)+\frac{1}{25}\sqrt{50\sqrt{5}-110}\tan^{-1}\left(\frac{1}{2}\sqrt{2+2\sqrt{5}}\sqrt{x}\right)-\frac{1}{25}\sqrt{110+50\sqrt{5}}\tanh^{-1}\left(\frac{1}{2}\sqrt{2\sqrt{5}-2}\sqrt{x}\right)$$

```
[Out] (2 - 4*x)/(5*(Sqrt[x] + Sqrt[-1 + x^2])) + (Sqrt[-110 + 50*Sqrt[5
]]*ArcTan[(Sqrt[2 + 2*Sqrt[5]]*Sqrt[x])/2])/25 - (Sqrt[-110 + 50*
Sqrt[5]]*ArcTan[(Sqrt[-2 + 2*Sqrt[5]]*Sqrt[-1 + x^2])/(2 - (1 - S
qrt[5])*x)])/50 - (Sqrt[110 + 50*Sqrt[5]]*ArcTanh[(Sqrt[-2 + 2*Sq
rt[5]]*Sqrt[x])/2])/25 - (Sqrt[110 + 50*Sqrt[5]]*ArcTanh[(Sqrt[2
+ 2*Sqrt[5]]*Sqrt[-1 + x^2])/(2 - x - Sqrt[5]*x)])/50
```

Rubi [A] time = 0.997216, antiderivative size = 365, normalized size of antiderivative = 1.66, number of steps used = 18, number of rules used = 12, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444

$$-\frac{2\sqrt{x^2-1}(1-2x)}{5\left(-x^2+x+1\right)}+\frac{2\sqrt{x}(1-2x)}{5\left(-x^2+x+1\right)}-\frac{2}{5}\sqrt{\frac{1}{5}\left(5\sqrt{5}-2\right)}\tan^{-1}\left(\frac{2-\left(1-\sqrt{5}\right)x}{\sqrt{2\left(\sqrt{5}-1\right)}\sqrt{x^2-1}}\right)+\sqrt{\frac{2}{5\left(\sqrt{5}-1\right)}}\tan^{-1}\left(\frac{2-\left(1-\sqrt{5}\right)x}{\sqrt{2\left(\sqrt{5}-1\right)}\sqrt{x^2-1}}\right)-\frac{2}{5}\sqrt{\frac{1}{5}\left(2+5\sqrt{5}\right)}\tanh^{-1}\left(\frac{2-\left(1+\sqrt{5}\right)x}{\sqrt{2\left(1+\sqrt{5}\right)}\sqrt{x^2-1}}\right)+\sqrt{\frac{2}{5\left(1+\sqrt{5}\right)}}\tanh^{-1}\left(\frac{2-\left(1+\sqrt{5}\right)x}{\sqrt{2\left(1+\sqrt{5}\right)}\sqrt{x^2-1}}\right)+\frac{1}{5}\sqrt{\frac{2}{5}\left(5\sqrt{5}-11\right)}\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\sqrt{x}\right)-\frac{1}{5}\sqrt{\frac{2}{5}\left(11+5\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{x}\right)$$

Warning: Unable to verify antiderivative.

```
[In]  Int[1/(Sqrt[-1 + x^2]*(Sqrt[x] + Sqrt[-1 + x^2])^2),x]
```

```
[Out] (2*(1 - 2*x)*Sqrt[x])/(5*(1 + x - x^2)) - (2*(1 - 2*x)*Sqrt[-1 +
x^2])/(5*(1 + x - x^2)) + (Sqrt[(2*(-11 + 5*Sqrt[5]))/5]*ArcTan[S
qrt[2/(-1 + Sqrt[5])]*Sqrt[x]])/5 + Sqrt[2/(5*(-1 + Sqrt[5]))]*Ar
cTan[(2 - (1 - Sqrt[5])*x)/(Sqrt[2*(-1 + Sqrt[5])]*Sqrt[-1 + x^2]
)] - (2*Sqrt[(-2 + 5*Sqrt[5])/5]*ArcTan[(2 - (1 - Sqrt[5])*x)/(Sq
rt[2*(-1 + Sqrt[5])]*Sqrt[-1 + x^2])])/5 - (Sqrt[(2*(11 + 5*Sqrt[
5]))/5]*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*Sqrt[x]])/5 + Sqrt[2/(5*(1
+ Sqrt[5]))]*ArcTanh[(2 - (1 + Sqrt[5])*x)/(Sqrt[2*(1 + Sqrt[5])]
*Sqrt[-1 + x^2])] - (2*Sqrt[(2 + 5*Sqrt[5])/5]*ArcTanh[(2 - (1 +
Sqrt[5])*x)/(Sqrt[2*(1 + Sqrt[5])]*Sqrt[-1 + x^2])])/5
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$2\int^{\sqrt{x}}\frac{x}{\left(x+\sqrt{x^4-1}\right)^2\sqrt{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**2-1)**(1/2)/(x**(1/2)+(x**2-1)**(1/2))**2,x)
```

```
[Out] 2*Integral(x/((x + sqrt(x**4 - 1))**2*sqrt(x**4 - 1)), (x, sqrt(x
)))
```

Mathematica [A] time = 4.93058, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-1+x^2}\left(\sqrt{x}+\sqrt{-1+x^2}\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[1/(Sqrt[-1 + x^2]*(Sqrt[x] + Sqrt[-1 + x^2])^2),x]
```

```
[Out] Integrate[1/(Sqrt[-1 + x^2]*(Sqrt[x] + Sqrt[-1 + x^2])^2), x]
```

Maple [B] time = 0.177, size = 902, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^2-1)^(1/2)/(x^(1/2)+(x^2-1)^(1/2))^2,x)
```

```
[Out] -6/25*5^(1/2)/(2+2*5^(1/2))^(1/2)*arctanh(2*(1+5^(1/2)+(5^(1/2)+1
)*(x-1/2*5^(1/2)-1/2))/(2+2*5^(1/2))^(1/2)/(4*(x-1/2*5^(1/2)-1/2)
^2+4*(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+2+2*5^(1/2))^(1/2))-6/25*5^(
1/2)/(-2+2*5^(1/2))^(1/2)*arctan(2*(1-5^(1/2)+(-5^(1/2)+1)*(x+1/2
*5^(1/2)-1/2))/(-2+2*5^(1/2))^(1/2)/(4*(x+1/2*5^(1/2)-1/2)^2+4*(-
5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+2-2*5^(1/2))^(1/2))-1/5/(1/2+1/2*5
^(1/2))/(x-1/2*5^(1/2)-1/2)*((x-1/2*5^(1/2)-1/2)^2+(5^(1/2)+1)*(x
-1/2*5^(1/2)-1/2)+1/2+1/2*5^(1/2))^(1/2)+6/5/(1/2+1/2*5^(1/2))/(2
+2*5^(1/2))^(1/2)*arctanh(2*(1+5^(1/2)+(5^(1/2)+1)*(x-1/2*5^(1/2)
-1/2))/(2+2*5^(1/2))^(1/2)/(4*(x-1/2*5^(1/2)-1/2)^2+4*(5^(1/2)+1)
*(x-1/2*5^(1/2)-1/2)+2+2*5^(1/2))^(1/2))+2/5/(1/2+1/2*5^(1/2))/(2
+2*5^(1/2))^(1/2)*arctanh(2*(1+5^(1/2)+(5^(1/2)+1)*(x-1/2*5^(1/2)
-1/2))/(2+2*5^(1/2))^(1/2)/(4*(x-1/2*5^(1/2)-1/2)^2+4*(5^(1/2)+1)
*(x-1/2*5^(1/2)-1/2)+2+2*5^(1/2))^(1/2))*5^(1/2)-1/5*5^(1/2)/(1/2
+1/2*5^(1/2))/(x-1/2*5^(1/2)-1/2)*((x-1/2*5^(1/2)-1/2)^2+(5^(1/2)
+1)*(x-1/2*5^(1/2)-1/2)+1/2+1/2*5^(1/2))^(1/2)-1/5/(1/2-1/2*5^(1/
2))/(x+1/2*5^(1/2)-1/2)*((x+1/2*5^(1/2)-1/2)^2+(-5^(1/2)+1)*(x+1/
2*5^(1/2)-1/2)+1/2-1/2*5^(1/2))^(1/2)+2/5/(1/2-1/2*5^(1/2))/(-2+2
*5^(1/2))^(1/2)*arctan(2*(1-5^(1/2)+(-5^(1/2)+1)*(x+1/2*5^(1/2)-1
/2))/(-2+2*5^(1/2))^(1/2)/(4*(x+1/2*5^(1/2)-1/2)^2+4*(-5^(1/2)+1)
*(x+1/2*5^(1/2)-1/2)+2-2*5^(1/2))^(1/2))*5^(1/2)-6/5/(1/2-1/2*5^(
1/2))/(-2+2*5^(1/2))^(1/2)*arctan(2*(1-5^(1/2)+(-5^(1/2)+1)*(x+1/
2*5^(1/2)-1/2))/(-2+2*5^(1/2))^(1/2)/(4*(x+1/2*5^(1/2)-1/2)^2+4*(
-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+2-2*5^(1/2))^(1/2))+1/5*5^(1/2)/(
1/2-1/2*5^(1/2))/(x+1/2*5^(1/2)-1/2)*((x+1/2*5^(1/2)-1/2)^2+(-5^(
1/2)+1)*(x+1/2*5^(1/2)-1/2)+1/2-1/2*5^(1/2))^(1/2)+2/5*x^(1/2)/(x
+1/2*5^(1/2)-1/2)+4/5/(-2+2*5^(1/2))^(1/2)*arctan(2*x^(1/2)/(-2+2
*5^(1/2))^(1/2))-8/25/(-2+2*5^(1/2))^(1/2)*arctan(2*x^(1/2)/(-2+2
*5^(1/2))^(1/2))*5^(1/2)+2/5*x^(1/2)/(x-1/2*5^(1/2)-1/2)-4/5/(2+2
*5^(1/2))^(1/2)*arctanh(2*x^(1/2)/(2+2*5^(1/2))^(1/2))-8/25/(2+2*
5^(1/2))^(1/2)*arctanh(2*x^(1/2)/(2+2*5^(1/2))^(1/2))*5^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2-1}\left(\sqrt{x^2-1}+\sqrt{x}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(x^2 - 1)*(sqrt(x^2 - 1) + sqrt(x))^2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2 - 1)*(sqrt(x^2 - 1) + sqrt(x))^2), x)
```

Fricas [A] time = 0.252677, size = 1185, normalized size = 5.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(x^2 - 1)*(sqrt(x^2 - 1) + sqrt(x))^2),x, algorithm="fricas")
```

```
[Out] 1/50*(40*x^3 + 40*x^2 - 4*(2*sqrt(2)*(x^3 - x^2 - x)*sqrt(x^2 - 1
)*sqrt(-sqrt(5)*(11*sqrt(5) - 25)) - sqrt(2)*(2*x^4 - 2*x^3 - 3*x
^2 + x + 1)*sqrt(-sqrt(5)*(11*sqrt(5) - 25)))*arctan(-1/2*sqrt(2)
*sqrt(-sqrt(5)*(11*sqrt(5) - 25))*(sqrt(5) + 3)/(sqrt(5)*(2*x - 1
) - 2*sqrt(5)*sqrt(x^2 - 1) - 2*sqrt(sqrt(5)*(sqrt(5)*(2*x^2 - x)
 - sqrt(x^2 - 1)*(sqrt(5)*(2*x - 1) + 5) + 5*x)) + 5)) + 4*(2*sqr
t(2)*(x^3 - x^2 - x)*sqrt(x^2 - 1)*sqrt(-sqrt(5)*(11*sqrt(5) - 25
)) - sqrt(2)*(2*x^4 - 2*x^3 - 3*x^2 + x + 1)*sqrt(-sqrt(5)*(11*sq
rt(5) - 25)))*arctan(1/2*sqrt(2)*sqrt(-sqrt(5)*(11*sqrt(5) - 25))
*(sqrt(5) + 3)/(sqrt(2)*sqrt(sqrt(5)*(sqrt(5)*(2*x - 1) + 5)) + 2
*sqrt(5)*sqrt(x))) + (2*sqrt(2)*(x^3 - x^2 - x)*sqrt(x^2 - 1)*sqr
t(sqrt(5)*(11*sqrt(5) + 25)) - sqrt(2)*(2*x^4 - 2*x^3 - 3*x^2 + x
 + 1)*sqrt(sqrt(5)*(11*sqrt(5) + 25)))*log(sqrt(2)*sqrt(sqrt(5)*(
11*sqrt(5) + 25))*(sqrt(5) - 3) - 2*sqrt(5)*(2*x - 1) + 4*sqrt(5)
*sqrt(x^2 - 1) + 10) - (2*sqrt(2)*(x^3 - x^2 - x)*sqrt(x^2 - 1)*s
qrt(sqrt(5)*(11*sqrt(5) + 25)) - sqrt(2)*(2*x^4 - 2*x^3 - 3*x^2 +
 x + 1)*sqrt(sqrt(5)*(11*sqrt(5) + 25)))*log(sqrt(2)*sqrt(sqrt(5)
*(11*sqrt(5) + 25))*(sqrt(5) - 3) + 4*sqrt(5)*sqrt(x)) - (2*sqrt(
2)*(x^3 - x^2 - x)*sqrt(x^2 - 1)*sqrt(sqrt(5)*(11*sqrt(5) + 25))
- sqrt(2)*(2*x^4 - 2*x^3 - 3*x^2 + x + 1)*sqrt(sqrt(5)*(11*sqrt(5
) + 25)))*log(-sqrt(2)*sqrt(sqrt(5)*(11*sqrt(5) + 25))*(sqrt(5) -
 3) - 2*sqrt(5)*(2*x - 1) + 4*sqrt(5)*sqrt(x^2 - 1) + 10) + (2*sq
rt(2)*(x^3 - x^2 - x)*sqrt(x^2 - 1)*sqrt(sqrt(5)*(11*sqrt(5) + 25
)) - sqrt(2)*(2*x^4 - 2*x^3 - 3*x^2 + x + 1)*sqrt(sqrt(5)*(11*sqr
t(5) + 25)))*log(-sqrt(2)*sqrt(sqrt(5)*(11*sqrt(5) + 25))*(sqrt(5
) - 3) + 4*sqrt(5)*sqrt(x)) - 20*(2*x^2 + 2*(2*x^2 - x)*sqrt(x) +
 2*x + 1)*sqrt(x^2 - 1) + 20*(4*x^3 - 2*x^2 - 2*x + 1)*sqrt(x) -
40)/(2*x^4 - 2*x^3 - 3*x^2 - 2*(x^3 - x^2 - x)*sqrt(x^2 - 1) + x
+ 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(x-1)(x+1)}\left(\sqrt{x}+\sqrt{x^2-1}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-1)**(1/2)/(x**(1/2)+(x**2-1)**(1/2))**2,x)
```

```
[Out] Integral(1/(sqrt((x - 1)*(x + 1))*(sqrt(x) + sqrt(x**2 - 1))**2),
 x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2-1}\left(\sqrt{x^2-1}+\sqrt{x}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(x^2 - 1)*(sqrt(x^2 - 1) + sqrt(x))^2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^2 - 1)*(sqrt(x^2 - 1) + sqrt(x))^2), x)
```

3.10 $\int \frac{\left(\sqrt{x}-\sqrt{-1+x^2}\right)^2}{(1+x-x^2)^2\sqrt{-1+x^2}}\,dx$

Optimal. Leaf size=220

$$\frac{2-4x}{5\left(\sqrt{x^2-1}+\sqrt{x}\right)}-\frac{1}{50}\sqrt{50\sqrt{5}-110}\tan^{-1}\left(\frac{\sqrt{2\sqrt{5}-2}\sqrt{x^2-1}}{2-\left(1-\sqrt{5}\right)x}\right)$$
$$-\frac{1}{50}\sqrt{110+50\sqrt{5}}\tanh^{-1}\left(\frac{\sqrt{2+2\sqrt{5}}\sqrt{x^2-1}}{-\sqrt{5}x-x+2}\right)$$
$$+\frac{1}{25}\sqrt{50\sqrt{5}-110}\tan^{-1}\left(\frac{1}{2}\sqrt{2+2\sqrt{5}}\sqrt{x}\right)-\frac{1}{25}\sqrt{110+50\sqrt{5}}\tanh^{-1}\left(\frac{1}{2}\sqrt{2\sqrt{5}-2}\sqrt{x}\right)$$

```
[Out] (2 - 4*x)/(5*(Sqrt[x] + Sqrt[-1 + x^2])) + (Sqrt[-110 + 50*Sqrt[5
]]*ArcTan[(Sqrt[2 + 2*Sqrt[5]]*Sqrt[x])/2])/25 - (Sqrt[-110 + 50*
Sqrt[5]]*ArcTan[(Sqrt[-2 + 2*Sqrt[5]]*Sqrt[-1 + x^2])/(2 - (1 - S
qrt[5])*x)])/50 - (Sqrt[110 + 50*Sqrt[5]]*ArcTanh[(Sqrt[-2 + 2*Sq
rt[5]]*Sqrt[x])/2])/25 - (Sqrt[110 + 50*Sqrt[5]]*ArcTanh[(Sqrt[2
+ 2*Sqrt[5]]*Sqrt[-1 + x^2])/(2 - x - Sqrt[5]*x)])/50
```

Rubi [B] time = 1.56062, antiderivative size = 541, normalized size of antiderivative = 2.46, number of steps used = 25, number of rules used = 13, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$-\frac{\sqrt{x^2-1}(1-2x)}{5(-x^2+x+1)}+\frac{2\sqrt{x}(1-2x)}{5(-x^2+x+1)}-\frac{(3-x)\sqrt{x^2-1}}{5(-x^2+x+1)}$$
$$+\frac{(x+2)\sqrt{x^2-1}}{5(-x^2+x+1)}+\frac{1}{5}\sqrt{\frac{1}{5}\left(2+5\sqrt{5}\right)}\tan^{-1}\left(\frac{2-\left(1-\sqrt{5}\right)x}{\sqrt{2\left(\sqrt{5}-1\right)}\sqrt{x^2-1}}\right)$$
$$-\frac{1}{5}\sqrt{\frac{1}{5}\left(5\sqrt{5}-2\right)}\tan^{-1}\left(\frac{2-\left(1-\sqrt{5}\right)x}{\sqrt{2\left(\sqrt{5}-1\right)}\sqrt{x^2-1}}\right)$$
$$-\frac{1}{5}\sqrt{\frac{1}{10}\left(5\sqrt{5}-11\right)}\tan^{-1}\left(\frac{2-\left(1-\sqrt{5}\right)x}{\sqrt{2\left(\sqrt{5}-1\right)}\sqrt{x^2-1}}\right)$$
$$+\frac{1}{5}\sqrt{\frac{1}{10}\left(11+5\sqrt{5}\right)}\tanh^{-1}\left(\frac{2-\left(1+\sqrt{5}\right)x}{\sqrt{2\left(1+\sqrt{5}\right)}\sqrt{x^2-1}}\right)$$
$$-\frac{1}{5}\sqrt{\frac{1}{5}\left(2+5\sqrt{5}\right)}\tanh^{-1}\left(\frac{2-\left(1+\sqrt{5}\right)x}{\sqrt{2\left(1+\sqrt{5}\right)}\sqrt{x^2-1}}\right)$$
$$-\frac{1}{5}\sqrt{\frac{1}{5}\left(5\sqrt{5}-2\right)}\tanh^{-1}\left(\frac{2-\left(1+\sqrt{5}\right)x}{\sqrt{2\left(1+\sqrt{5}\right)}\sqrt{x^2-1}}\right)$$
$$+\frac{1}{5}\sqrt{\frac{2}{5}\left(5\sqrt{5}-11\right)}\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\sqrt{x}\right)-\frac{1}{5}\sqrt{\frac{2}{5}\left(11+5\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{x}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[(Sqrt[x] - Sqrt[-1 + x^2])^2/((1 + x - x^2)^2*Sqrt[-1 + x^2]),x]
```

```
[Out] (2*(1 - 2*x)*Sqrt[x])/(5*(1 + x - x^2)) - ((1 - 2*x)*Sqrt[-1 + x^
2])/(5*(1 + x - x^2)) - ((3 - x)*Sqrt[-1 + x^2])/(5*(1 + x - x^2)
) + ((2 + x)*Sqrt[-1 + x^2])/(5*(1 + x - x^2)) + (Sqrt[(2*(-11 +
5*Sqrt[5]))/5]*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*Sqrt[x]])/5 - (Sqrt[
(-11 + 5*Sqrt[5])/10]*ArcTan[(2 - (1 - Sqrt[5])*x)/(Sqrt[2*(-1 +
Sqrt[5])]*Sqrt[-1 + x^2])])/5 - (Sqrt[(-2 + 5*Sqrt[5])/5]*ArcTan[
(2 - (1 - Sqrt[5])*x)/(Sqrt[2*(-1 + Sqrt[5])]*Sqrt[-1 + x^2])])/5
 + (Sqrt[(2 + 5*Sqrt[5])/5]*ArcTan[(2 - (1 - Sqrt[5])*x)/(Sqrt[2*
(-1 + Sqrt[5])]*Sqrt[-1 + x^2])])/5 - (Sqrt[(2*(11 + 5*Sqrt[5]))/
5]*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*Sqrt[x]])/5 - (Sqrt[(-2 + 5*Sqrt
[5])/5]*ArcTanh[(2 - (1 + Sqrt[5])*x)/(Sqrt[2*(1 + Sqrt[5])]*Sqrt
[-1 + x^2])])/5 - (Sqrt[(2 + 5*Sqrt[5])/5]*ArcTanh[(2 - (1 + Sqrt
[5])*x)/(Sqrt[2*(1 + Sqrt[5])]*Sqrt[-1 + x^2])])/5 + (Sqrt[(11 +
5*Sqrt[5])/10]*ArcTanh[(2 - (1 + Sqrt[5])*x)/(Sqrt[2*(1 + Sqrt[5]
)]*Sqrt[-1 + x^2])])/5
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((x**(1/2)-(x**2-1)**(1/2))**2/(-x**2+x+1)**2/(x**2-
1)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 1.66001, size = 0, normalized size = 0.

$$\int \frac{\left(\sqrt{x}-\sqrt{-1+x^2}\right)^2}{(1+x-x^2)^2\sqrt{-1+x^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sqrt[x] - Sqrt[-1 + x^2])^2/((1 + x - x^2)^2*Sqrt[-
1 + x^2]),x]
```

```
[Out] Integrate[(Sqrt[x] - Sqrt[-1 + x^2])^2/((1 + x - x^2)^2*Sqrt[-1 +
 x^2]), x]
```

Maple [B] time = 0.033, size = 1542, normalized size = 7.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)-(x^2-1)^(1/2))^2/(-x^2+x+1)^2/(x^2-1)^(1/2),x)
```

```
[Out] 4/5/(-2+2*5^(1/2))^(1/2)*arctan(2*x^(1/2)/(-2+2*5^(1/2))^(1/2))+2
/5*x^(1/2)/(x-1/2*5^(1/2)-1/2)-4/5/(2+2*5^(1/2))^(1/2)*arctanh(2*
x^(1/2)/(2+2*5^(1/2))^(1/2))+4/25*5^(1/2)/(-2+2*5^(1/2))^(1/2)*ar
ctan(2*(1-5^(1/2)+(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2))/(-2+2*5^(1/2)
)^(1/2)/(4*(x+1/2*5^(1/2)-1/2)^2+4*(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/
2)+2-2*5^(1/2))^(1/2))-1/10/(1/2+1/2*5^(1/2))/(x-1/2*5^(1/2)-1/2)
*((x-1/2*5^(1/2)-1/2)^2+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+1/2+1/2*5
^(1/2))^(1/2)-1/10/(1/2-1/2*5^(1/2))/(x+1/2*5^(1/2)-1/2)*((x+1/2*
5^(1/2)-1/2)^2+(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+1/2-1/2*5^(1/2))^
(1/2)-8/25/(-2+2*5^(1/2))^(1/2)*arctan(2*x^(1/2)/(-2+2*5^(1/2))^(
1/2))*5^(1/2)-8/25/(2+2*5^(1/2))^(1/2)*arctanh(2*x^(1/2)/(2+2*5^(
```

```
1/2))^(1/2))*5^(1/2)+2/5/(2+2*5^(1/2))^(1/2)*arctanh(2*(1+5^(1/2)
+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2))/(2+2*5^(1/2))^(1/2)/(4*(x-1/2*5
^(1/2)-1/2)^2+4*(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+2+2*5^(1/2))^(1/2
))+1/25*5^(1/2)*(4*(x+1/2*5^(1/2)-1/2)^2+4*(-5^(1/2)+1)*(x+1/2*5^
(1/2)-1/2)+2-2*5^(1/2))^(1/2)+1/10/(1/2+1/2*5^(1/2))*ln(x+((x-1/2
*5^(1/2)-1/2)^2+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+1/2+1/2*5^(1/2))^
(1/2))+1/20/(1/2+1/2*5^(1/2))*(4*(x-1/2*5^(1/2)-1/2)^2+4*(5^(1/2)
+1)*(x-1/2*5^(1/2)-1/2)+2+2*5^(1/2))^(1/2)-1/25*ln(x+((x-1/2*5^(1
/2)-1/2)^2+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+1/2+1/2*5^(1/2))^(1/2)
)*5^(1/2)+4/25*5^(1/2)/(2+2*5^(1/2))^(1/2)*arctanh(2*(1+5^(1/2)+(
5^(1/2)+1)*(x-1/2*5^(1/2)-1/2))/(2+2*5^(1/2))^(1/2)/(4*(x-1/2*5^(
1/2)-1/2)^2+4*(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+2+2*5^(1/2))^(1/2))
-1/25*5^(1/2)*(4*(x-1/2*5^(1/2)-1/2)^2+4*(5^(1/2)+1)*(x-1/2*5^(1/
2)-1/2)+2+2*5^(1/2))^(1/2)-2/5/(-2+2*5^(1/2))^(1/2)*arctan(2*(1-5
^(1/2)+(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2))/(-2+2*5^(1/2))^(1/2)/(4*
(x+1/2*5^(1/2)-1/2)^2+4*(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+2-2*5^(1
/2))^(1/2))+1/10/(1/2-1/2*5^(1/2))*ln(x+((x+1/2*5^(1/2)-1/2)^2+(-
5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+1/2-1/2*5^(1/2))^(1/2))+1/20/(1/2-
1/2*5^(1/2))*(4*(x+1/2*5^(1/2)-1/2)^2+4*(-5^(1/2)+1)*(x+1/2*5^(1/
2)-1/2)+2-2*5^(1/2))^(1/2)+1/25*ln(x+((x+1/2*5^(1/2)-1/2)^2+(-5^(
1/2)+1)*(x+1/2*5^(1/2)-1/2)+1/2-1/2*5^(1/2))^(1/2))*5^(1/2)+2/5*x
^(1/2)/(x+1/2*5^(1/2)-1/2)-1/10*5^(1/2)/(1/2+1/2*5^(1/2))/(x-1/2*
5^(1/2)-1/2)*((x-1/2*5^(1/2)-1/2)^2+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/
2)+1/2+1/2*5^(1/2))^(1/2)+1/10*5^(1/2)/(1/2-1/2*5^(1/2))/(x+1/2*5
^(1/2)-1/2)*((x+1/2*5^(1/2)-1/2)^2+(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/
2)+1/2-1/2*5^(1/2))^(1/2)-1/5/(1/2-1/2*5^(1/2))/(x+1/2*5^(1/2)-1/
2)*((x+1/2*5^(1/2)-1/2)^2+(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+1/2-1/
2*5^(1/2))^(3/2)+1/5/(1/2-1/2*5^(1/2))*x*((x+1/2*5^(1/2)-1/2)^2+(
-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+1/2-1/2*5^(1/2))^(1/2)-1/10/(1/2-
1/2*5^(1/2))*ln(x+((x+1/2*5^(1/2)-1/2)^2+(-5^(1/2)+1)*(x+1/2*5^(1
/2)-1/2)+1/2-1/2*5^(1/2))^(1/2))*5^(1/2)-1/20/(1/2-1/2*5^(1/2))*5
^(1/2)*(4*(x+1/2*5^(1/2)-1/2)^2+4*(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2
)+2-2*5^(1/2))^(1/2)+1/5/(1/2+1/2*5^(1/2))*x*((x-1/2*5^(1/2)-1/2)
^2+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+1/2+1/2*5^(1/2))^(1/2)+1/20/(1
/2+1/2*5^(1/2))*5^(1/2)*(4*(x-1/2*5^(1/2)-1/2)^2+4*(5^(1/2)+1)*(x
-1/2*5^(1/2)-1/2)+2+2*5^(1/2))^(1/2)+1/10/(1/2+1/2*5^(1/2))*ln(x+
((x-1/2*5^(1/2)-1/2)^2+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+1/2+1/2*5^
(1/2))^(1/2))*5^(1/2)-1/5/(1/2+1/2*5^(1/2))/(x-1/2*5^(1/2)-1/2)*(
(x-1/2*5^(1/2)-1/2)^2+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+1/2+1/2*5^(
1/2))^(3/2)-1/5*ln(x+((x+1/2*5^(1/2)-1/2)^2+(-5^(1/2)+1)*(x+1/2*5
^(1/2)-1/2)+1/2-1/2*5^(1/2))^(1/2))-1/5*ln(x+((x-1/2*5^(1/2)-1/2)
^2+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+1/2+1/2*5^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2\left(x^{\frac{5}{2}}-3x^{\frac{3}{2}}\right)}{5\left(x^{2}-x-1\right)}+\int \frac{x^{\frac{3}{2}}+\sqrt{x}}{5\left(x^{2}-x-1\right)}\,dx+\int \frac{x^{2}+x-1}{\left(x^{4}-2x^{3}-x^{2}+2x+1\right)\sqrt{x+1}\sqrt{x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sqrt(x^2 - 1) - sqrt(x))^2/((x^2 - x - 1)^2*sqrt(x^2 - 1)),x, algorithm
```

```
[Out] -2/5*(x^(5/2) - 3*x^(3/2))/(x^2 - x - 1) + integrate(1/5*(x^(3/2)
 + sqrt(x))/(x^2 - x - 1), x) + integrate((x^2 + x - 1)/((x^4 - 2
*x^3 - x^2 + 2*x + 1)*sqrt(x + 1)*sqrt(x - 1)), x)
```

Fricas [A] time = 0.255298, size = 1185, normalized size = 5.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sqrt(x^2 - 1) - sqrt(x))^2/((x^2 - x - 1)^2*sqrt(x^2 - 1)),x, algorithm
```

```
[Out] 1/50*(40*x^3 + 40*x^2 - 4*(2*sqrt(2)*(x^3 - x^2 - x)*sqrt(x^2 - 1
)*sqrt(-sqrt(5)*(11*sqrt(5) - 25)) - sqrt(2)*(2*x^4 - 2*x^3 - 3*x
^2 + x + 1)*sqrt(-sqrt(5)*(11*sqrt(5) - 25)))*arctan(-1/2*sqrt(2)
*sqrt(-sqrt(5)*(11*sqrt(5) - 25))*(sqrt(5) + 3)/(sqrt(5)*(2*x - 1
) - 2*sqrt(5)*sqrt(x^2 - 1) - 2*sqrt(sqrt(5)*(sqrt(5)*(2*x^2 - x)
 - sqrt(x^2 - 1)*(sqrt(5)*(2*x - 1) + 5) + 5*x)) + 5)) + 4*(2*sqr
t(2)*(x^3 - x^2 - x)*sqrt(x^2 - 1)*sqrt(-sqrt(5)*(11*sqrt(5) - 25
)) - sqrt(2)*(2*x^4 - 2*x^3 - 3*x^2 + x + 1)*sqrt(-sqrt(5)*(11*sq
rt(5) - 25)))*arctan(1/2*sqrt(2)*sqrt(-sqrt(5)*(11*sqrt(5) - 25))
*(sqrt(5) + 3)/(sqrt(2)*sqrt(sqrt(5)*(sqrt(5)*(2*x - 1) + 5)) + 2
*sqrt(5)*sqrt(x))) + (2*sqrt(2)*(x^3 - x^2 - x)*sqrt(x^2 - 1)*sqr
t(sqrt(5)*(11*sqrt(5) + 25)) - sqrt(2)*(2*x^4 - 2*x^3 - 3*x^2 + x
 + 1)*sqrt(sqrt(5)*(11*sqrt(5) + 25)))*log(sqrt(2)*sqrt(sqrt(5)*(
11*sqrt(5) + 25))*(sqrt(5) - 3) - 2*sqrt(5)*(2*x - 1) + 4*sqrt(5)
*sqrt(x^2 - 1) + 10) - (2*sqrt(2)*(x^3 - x^2 - x)*sqrt(x^2 - 1)*s
qrt(sqrt(5)*(11*sqrt(5) + 25)) - sqrt(2)*(2*x^4 - 2*x^3 - 3*x^2 +
 x + 1)*sqrt(sqrt(5)*(11*sqrt(5) + 25)))*log(sqrt(2)*sqrt(sqrt(5)
*(11*sqrt(5) + 25))*(sqrt(5) - 3) + 4*sqrt(5)*sqrt(x)) - (2*sqrt(
2)*(x^3 - x^2 - x)*sqrt(x^2 - 1)*sqrt(sqrt(5)*(11*sqrt(5) + 25))
- sqrt(2)*(2*x^4 - 2*x^3 - 3*x^2 + x + 1)*sqrt(sqrt(5)*(11*sqrt(5
) + 25)))*log(-sqrt(2)*sqrt(sqrt(5)*(11*sqrt(5) + 25))*(sqrt(5) -
 3) - 2*sqrt(5)*(2*x - 1) + 4*sqrt(5)*sqrt(x^2 - 1) + 10) + (2*sq
rt(2)*(x^3 - x^2 - x)*sqrt(x^2 - 1)*sqrt(sqrt(5)*(11*sqrt(5) + 25
)) - sqrt(2)*(2*x^4 - 2*x^3 - 3*x^2 + x + 1)*sqrt(sqrt(5)*(11*sqr
t(5) + 25)))*log(-sqrt(2)*sqrt(sqrt(5)*(11*sqrt(5) + 25))*(sqrt(5
) - 3) + 4*sqrt(5)*sqrt(x)) - 20*(2*x^2 + 2*(2*x^2 - x)*sqrt(x) +
 2*x + 1)*sqrt(x^2 - 1) + 20*(4*x^3 - 2*x^2 - 2*x + 1)*sqrt(x) -
40)/(2*x^4 - 2*x^3 - 3*x^2 - 2*(x^3 - x^2 - x)*sqrt(x^2 - 1) + x
+ 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**(1/2)-(x**2-1)**(1/2))**2/(-x**2+x+1)**2/(x**2-1)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\sqrt{x^2-1}-\sqrt{x}\right)^2}{\left(x^2-x-1\right)^2\sqrt{x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sqrt(x^2 - 1) - sqrt(x))^2/((x^2 - x - 1)^2*sqrt(x^2 - 1)),x, algorithm
```

```
[Out] integrate((sqrt(x^2 - 1) - sqrt(x))^2/((x^2 - x - 1)^2*sqrt(x^2 -
 1)), x)
```

3.11 $\int \left(\frac{1}{\sqrt{2}(1+x)^2\sqrt{-i+x^2}}+\frac{1}{\sqrt{2}(1+x)^2\sqrt{i+x^2}}\right)\,dx$

Optimal. Leaf size=138

$$-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{x^2-i}}{\sqrt{2}(x+1)}-\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{x^2+i}}{\sqrt{2}(x+1)}+\frac{\tanh^{-1}\left(\frac{x+i}{\sqrt{1-i}\sqrt{x^2-i}}\right)}{(1-i)^{3/2}\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{-x+i}{\sqrt{1+i}\sqrt{x^2+i}}\right)}{(1+i)^{3/2}\sqrt{2}}$$

```
[Out] ((-1/2 - I/2)*Sqrt[-I + x^2])/(Sqrt[2]*(1 + x)) - ((1/2 - I/2)*Sq
rt[I + x^2])/(Sqrt[2]*(1 + x)) + ArcTanh[(I + x)/(Sqrt[1 - I]*Sqr
t[-I + x^2])]/((1 - I)^(3/2)*Sqrt[2]) - ArcTanh[(I - x)/(Sqrt[1 +
 I]*Sqrt[I + x^2])]/((1 + I)^(3/2)*Sqrt[2])
```

Rubi [A] time = 0.161835, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{x^2-i}}{\sqrt{2}(x+1)}-\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{x^2+i}}{\sqrt{2}(x+1)}+\frac{\tanh^{-1}\left(\frac{x+i}{\sqrt{1-i}\sqrt{x^2-i}}\right)}{(1-i)^{3/2}\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{-x+i}{\sqrt{1+i}\sqrt{x^2+i}}\right)}{(1+i)^{3/2}\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2]*(1 + x)^2*Sqrt[-I + x^2]) + 1/(Sqrt[2]*(1 + x)^2*Sqrt[I + x^2]),x]
```

```
[Out] ((-1/2 - I/2)*Sqrt[-I + x^2])/(Sqrt[2]*(1 + x)) - ((1/2 - I/2)*Sq
rt[I + x^2])/(Sqrt[2]*(1 + x)) + ArcTanh[(I + x)/(Sqrt[1 - I]*Sqr
t[-I + x^2])]/((1 - I)^(3/2)*Sqrt[2]) - ArcTanh[(I - x)/(Sqrt[1 +
 I]*Sqrt[I + x^2])]/((1 + I)^(3/2)*Sqrt[2])
```

Rubi in Sympy [A] time = 18.4555, size = 184, normalized size = 1.33

$$\frac{(1+i)\,\text{atanh}\left(\frac{\sqrt{2}(-x-i)}{\sqrt{x^2-i}\left(\sqrt{1+\sqrt{2}}-i\sqrt{-1+\sqrt{2}}\right)}\right)}{2\left(-\sqrt{1+\sqrt{2}}+i\sqrt{-1+\sqrt{2}}\right)}-\frac{(1-i)\,\text{atanh}\left(\frac{\sqrt{2}(-x+i)}{\sqrt{x^2+i}\left(\sqrt{1+\sqrt{2}}+i\sqrt{-1+\sqrt{2}}\right)}\right)}{2\left(\sqrt{1+\sqrt{2}}+i\sqrt{-1+\sqrt{2}}\right)}-\frac{\sqrt{2}\,(1+i)\sqrt{x^2-i}}{4\,(x+1)}-\frac{\sqrt{2}\,(1-i)\sqrt{x^2+i}}{4\,(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/2/(1+x)**2*2**(1/2)/(-I+x**2)**(1/2)+1/2/(1+x)**2*2**(1/2)/(I
```

```
[Out] (1 + I)*atanh(sqrt(2)*(-x - I)/(sqrt(x**2 - I)*(sqrt(1 + sqrt(2))
 - I*sqrt(-1 + sqrt(2)))))/(2*(-sqrt(1 + sqrt(2)) + I*sqrt(-1 + s
qrt(2)))) - (1 - I)*atanh(sqrt(2)*(-x + I)/(sqrt(x**2 + I)*(sqrt(
1 + sqrt(2)) + I*sqrt(-1 + sqrt(2)))))/(2*(sqrt(1 + sqrt(2)) + I*
sqrt(-1 + sqrt(2)))) - sqrt(2)*(1 + I)*sqrt(x**2 - I)/(4*(x + 1))
 - sqrt(2)*(1 - I)*sqrt(x**2 + I)/(4*(x + 1))
```

Mathematica [B] time = 0.4127, size = 403, normalized size = 2.92

$$-\frac{(2+2i)\sqrt{x^2-i}+(2-2i)\sqrt{x^2+i}+i\sqrt{1-i}x\log\left((-2+i)x^2+2\sqrt{1-i}\sqrt{x^2-i}x+i\right)+i\sqrt{1-i}\log\left((-2+i)x^2+2\sqrt{1-i}\sqrt{x\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2]*(1 + x)^2*Sqrt[-I + x^2]) + 1/(Sqrt[2]*(1 + x)^2*Sqrt[I + x^
```

```
[Out] -((2 + 2*I)*Sqrt[-I + x^2] + (2 - 2*I)*Sqrt[I + x^2] + 2*Sqrt[1 -
 I]*(1 + x)*ArcTan[(1 + x^2 + (2*I)*Sqrt[1 - I]*Sqrt[-I + x^2])/(
(1 - 2*I) - (2*I)*x + x^2)] + 2*Sqrt[1 + I]*(1 + x)*ArcTan[(1 + x
^2 - (2*I)*Sqrt[1 + I]*Sqrt[I + x^2])/((1 + 2*I) + (2*I)*x + x^2)
] - I*Sqrt[1 - I]*Log[(1 + x)^2] + I*Sqrt[1 + I]*Log[(1 + x)^2] -
 I*Sqrt[1 - I]*x*Log[(1 + x)^2] + I*Sqrt[1 + I]*x*Log[(1 + x)^2]
+ I*Sqrt[1 - I]*Log[I - (2 - I)*x^2 + 2*Sqrt[1 - I]*x*Sqrt[-I + x
^2]] + I*Sqrt[1 - I]*x*Log[I - (2 - I)*x^2 + 2*Sqrt[1 - I]*x*Sqrt
[-I + x^2]] - I*Sqrt[1 + I]*Log[-I - (2 + I)*x^2 + 2*Sqrt[1 + I]*
x*Sqrt[I + x^2]] - I*Sqrt[1 + I]*x*Log[-I - (2 + I)*x^2 + 2*Sqrt[
1 + I]*x*Sqrt[I + x^2]])/(4*Sqrt[2]*(1 + x))
```

Maple [B] time = 0.032, size = 278, normalized size = 2.

$$
\begin{aligned}
&-\frac{\sqrt{2}}{4+4x}\sqrt{(1+x)^2-1-i-2x}-\frac{\frac{i}{4}\sqrt{2}}{1+x}\sqrt{(1+x)^2-1-i-2x}\\
&-\frac{\sqrt{2}}{4\sqrt{1-i}}\ln\left(\frac{1}{1+x}\left(-2i-2x+2\sqrt{1-i}\sqrt{(1+x)^2-1-i-2x}\right)\right)\\
&-\frac{\frac{i}{4}\sqrt{2}}{\sqrt{1-i}}\ln\left(\frac{1}{1+x}\left(-2i-2x+2\sqrt{1-i}\sqrt{(1+x)^2-1-i-2x}\right)\right)\\
&-\frac{\sqrt{2}}{4+4x}\sqrt{(1+x)^2-1+i-2x}+\frac{\frac{i}{4}\sqrt{2}}{1+x}\sqrt{(1+x)^2-1+i-2x}\\
&-\frac{\sqrt{2}}{4\sqrt{1+i}}\ln\left(\frac{1}{1+x}\left(2i-2x+2\sqrt{1+i}\sqrt{(1+x)^2-1+i-2x}\right)\right)\\
&+\frac{\frac{i}{4}\sqrt{2}}{\sqrt{1+i}}\ln\left(\frac{1}{1+x}\left(2i-2x+2\sqrt{1+i}\sqrt{(1+x)^2-1+i-2x}\right)\right)
\end{aligned}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2/(1+x)^2*2^(1/2)/(-I+x^2)^(1/2)+1/2/(1+x)^2*2^(1/2)/(I+x^2)^(1/2),x)
```

```
[Out] -1/4*2^(1/2)/(1+x)*((1+x)^2-1-I-2*x)^(1/2)-1/4*I*2^(1/2)/(1+x)*((
1+x)^2-1-I-2*x)^(1/2)-1/4*2^(1/2)/(1-I)^(1/2)*ln((-2*I-2*x+2*(1-I
)^(1/2)*((1+x)^2-1-I-2*x)^(1/2))/(1+x))-1/4*I*2^(1/2)/(1-I)^(1/2)
*ln((-2*I-2*x+2*(1-I)^(1/2)*((1+x)^2-1-I-2*x)^(1/2))/(1+x))-1/4*2
^(1/2)/(1+x)*((1+x)^2-1+I-2*x)^(1/2)+1/4*I*2^(1/2)/(1+x)*((1+x)^2
-1+I-2*x)^(1/2)-1/4*2^(1/2)/(1+I)^(1/2)*ln((2*I-2*x+2*(1+I)^(1/2)
*((1+x)^2-1+I-2*x)^(1/2))/(1+x))+1/4*I*2^(1/2)/(1+I)^(1/2)*ln((2*
I-2*x+2*(1+I)^(1/2)*((1+x)^2-1+I-2*x)^(1/2))/(1+x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*sqrt(2)/(sqrt(x^2 + I)*(x + 1)^2) + 1/2*sqrt(2)/(sqrt(x^2 - I)*(x +
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0.286757, size = 680, normalized size = 4.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*sqrt(2)/(sqrt(x^2 + I)*(x + 1)^2) + 1/2*sqrt(2)/(sqrt(x^2 - I)*(x +
```

```
[Out] (sqrt(2)*sqrt(x^2 - I)*((I + 1)*x - I) + (sqrt(-1/2*I + 1/2)*sqrt
(x^2 - I)*((I - 1)*x^2 + (I - 1)*x) + sqrt(-1/2*I + 1/2)*(-(I - 1
)*x^3 - (I - 1)*x^2) + sqrt(x^2 + I)*(sqrt(-1/2*I + 1/2)*sqrt(x^2
 - I)*(-(I - 1)*x - I + 1) + sqrt(-1/2*I + 1/2)*((I - 1)*x^2 + (I
 - 1)*x)))*log(sqrt(2)*sqrt(-1/2*I + 1/2) - x + sqrt(x^2 - I) - 1
) + (sqrt(-1/2*I + 1/2)*sqrt(x^2 - I)*(-(I - 1)*x^2 - (I - 1)*x)
+ sqrt(-1/2*I + 1/2)*((I - 1)*x^3 + (I - 1)*x^2) + sqrt(x^2 + I)*
(sqrt(-1/2*I + 1/2)*sqrt(x^2 - I)*((I - 1)*x + I - 1) + sqrt(-1/2
*I + 1/2)*(-(I - 1)*x^2 - (I - 1)*x)))*log(-sqrt(2)*sqrt(-1/2*I +
 1/2) - x + sqrt(x^2 - I) - 1) + (sqrt(-1/2*I - 1/2)*((I + 1)*x^2
 + (I + 1)*x)*sqrt(x^2 - I) + sqrt(-1/2*I - 1/2)*(-(I + 1)*x^3 -
(I + 1)*x^2) + sqrt(x^2 + I)*(sqrt(-1/2*I - 1/2)*sqrt(x^2 - I)*(-
(I + 1)*x - I - 1) + sqrt(-1/2*I - 1/2)*((I + 1)*x^2 + (I + 1)*x)
))*log(I*sqrt(2)*sqrt(-1/2*I - 1/2) - x + sqrt(x^2 + I) - 1) + (s
qrt(-1/2*I - 1/2)*sqrt(x^2 - I)*(-(I + 1)*x^2 - (I + 1)*x) + sqrt
(-1/2*I - 1/2)*((I + 1)*x^3 + (I + 1)*x^2) + (sqrt(-1/2*I - 1/2)*
sqrt(x^2 - I)*((I + 1)*x + I + 1) + sqrt(-1/2*I - 1/2)*(-(I + 1)*
x^2 - (I + 1)*x))*sqrt(x^2 + I))*log(-I*sqrt(2)*sqrt(-1/2*I - 1/2
) - x + sqrt(x^2 + I) - 1) + sqrt(2)*(-(I + 1)*x^2 + (I + 1)*x) +
 sqrt(x^2 + I)*(sqrt(2)*((I + 1)*x - 1) - (I + 1)*sqrt(2)*sqrt(x^
2 - I)))/((2*I + 2)*x^3 + (2*I + 2)*x^2 + sqrt(x^2 + I)*(-(2*I +
2)*x^2 + sqrt(x^2 - I)*((2*I + 2)*x + 2*I + 2) - (2*I + 2)*x) + s
qrt(x^2 - I)*(-(2*I + 2)*x^2 - (2*I + 2)*x))
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/(1+x)**2*2**(1/2)/(-I+x**2)**(1/2)+1/2/(1+x)**2*2**(1/2)/(I+x**2
```

```
[Out] Exception raised: TypeError
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*sqrt(2)/(sqrt(x^2 + I)*(x + 1)^2) + 1/2*sqrt(2)/(sqrt(x^2 - I)*(x +
```

```
[Out] Exception raised: TypeError
```

3.12 $\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)^2\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=125

$$-\frac{\sqrt{1-ix^2}}{2(x+1)}-\frac{\sqrt{1+ix^2}}{2(x+1)}-\frac{1}{4}(1-i)^{3/2}\tanh^{-1}\left(\frac{1+ix}{\sqrt{1-i}\sqrt{1-ix^2}}\right)-\frac{1}{4}(1+i)^{3/2}\tanh^{-1}\left(\frac{1-ix}{\sqrt{1+i}\sqrt{1+ix^2}}\right)$$

```
[Out] -Sqrt[1 - I*x^2]/(2*(1 + x)) - Sqrt[1 + I*x^2]/(2*(1 + x)) - ((1
- I)^(3/2)*ArcTanh[(1 + I*x)/(Sqrt[1 - I]*Sqrt[1 - I*x^2])])/4 -
((1 + I)^(3/2)*ArcTanh[(1 - I*x)/(Sqrt[1 + I]*Sqrt[1 + I*x^2])])/
4
```

Rubi [A] time = 0.328194, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$-\frac{\sqrt{1-ix^2}}{2(x+1)}-\frac{\sqrt{1+ix^2}}{2(x+1)}-\frac{1}{4}(1-i)^{3/2}\tanh^{-1}\left(\frac{1+ix}{\sqrt{1-i}\sqrt{1-ix^2}}\right)-\frac{1}{4}(1+i)^{3/2}\tanh^{-1}\left(\frac{1-ix}{\sqrt{1+i}\sqrt{1+ix^2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x)^2*Sqrt[1 + x^4]),x]
```

```
[Out] -Sqrt[1 - I*x^2]/(2*(1 + x)) - Sqrt[1 + I*x^2]/(2*(1 + x)) - ((1
- I)^(3/2)*ArcTanh[(1 + I*x)/(Sqrt[1 - I]*Sqrt[1 - I*x^2])])/4 -
((1 + I)^(3/2)*ArcTanh[(1 - I*x)/(Sqrt[1 + I]*Sqrt[1 + I*x^2])])/
4
```

Rubi in Sympy [A] time = 21.5739, size = 202, normalized size = 1.62

$$\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}(-ix+1)}{\sqrt{ix^2+1}\left(\sqrt{1+\sqrt{2}}+i\sqrt{-1+\sqrt{2}}\right)}\right)}{2\left(-\sqrt{-1+\sqrt{2}}+i\sqrt{1+\sqrt{2}}\right)}-\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}(ix+1)}{\sqrt{-ix^2+1}\left(\sqrt{1+\sqrt{2}}-i\sqrt{-1+\sqrt{2}}\right)}\right)}{2\left(\sqrt{-1+\sqrt{2}}+i\sqrt{1+\sqrt{2}}\right)}-\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{-ix^2+1}}{x+1}-\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{ix^2+1}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+(x**4+1)**(1/2))**(1/2)/(1+x)**2/(x**4+1)**(1/2),x)
```

```
[Out] sqrt(2)*atanh(sqrt(2)*(-I*x + 1)/(sqrt(I*x**2 + 1)*(sqrt(1 + sqrt
(2)) + I*sqrt(-1 + sqrt(2)))))/(2*(-sqrt(-1 + sqrt(2)) + I*sqrt(1
 + sqrt(2)))) - sqrt(2)*atanh(sqrt(2)*(I*x + 1)/(sqrt(-I*x**2 + 1
)*(sqrt(1 + sqrt(2)) - I*sqrt(-1 + sqrt(2)))))/(2*(sqrt(-1 + sqrt
(2)) + I*sqrt(1 + sqrt(2)))) - (1/2 - I/2)*(1/2 + I/2)*sqrt(-I*x*
*2 + 1)/(x + 1) - (1/2 - I/2)*(1/2 + I/2)*sqrt(I*x**2 + 1)/(x + 1
)
```

Mathematica [A] time = 0.154463, size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)^2\sqrt{1+x^4}}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x)^2*Sqrt[1 + x^4]),x]
```

```
[Out] Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x)^2*Sqrt[1 + x^4]), x]
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int \frac{1}{(1+x)^2}\sqrt{x^2+\sqrt{x^4+1}}\frac{1}{\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)^2/(x^4+1)^(1/2),x)
```

```
[Out] int((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)^2/(x^4+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}(x+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*(x + 1)^2),x, algorithm="maxim
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*(x + 1)^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*(x + 1)^2),x, algorithm="frica
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{(x+1)^2\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+(x**4+1)**(1/2))**(1/2)/(1+x)**2/(x**4+1)**(1/2),x)
```

```
[Out] Integral(sqrt(x**2 + sqrt(x**4 + 1))/((x + 1)**2*sqrt(x**4 + 1)),
 x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}(x+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*(x + 1)^2),x, algorithm="giac"
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*(x + 1)^2), x)
```

3.13 $\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=81

$$-\frac{1}{2}\sqrt{1-i}\tanh^{-1}\left(\frac{1+ix}{\sqrt{1-i}\sqrt{1-ix^2}}\right)-\frac{1}{2}\sqrt{1+i}\tanh^{-1}\left(\frac{1-ix}{\sqrt{1+i}\sqrt{1+ix^2}}\right)$$

```
[Out] -(Sqrt[1 - I]*ArcTanh[(1 + I*x)/(Sqrt[1 - I]*Sqrt[1 - I*x^2])])/2
 - (Sqrt[1 + I]*ArcTanh[(1 - I*x)/(Sqrt[1 + I]*Sqrt[1 + I*x^2])])
/2
```

Rubi [A] time = 0.278117, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094

$$-\frac{1}{2}\sqrt{1-i}\tanh^{-1}\left(\frac{1+ix}{\sqrt{1-i}\sqrt{1-ix^2}}\right)-\frac{1}{2}\sqrt{1+i}\tanh^{-1}\left(\frac{1-ix}{\sqrt{1+i}\sqrt{1+ix^2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x)*Sqrt[1 + x^4]),x]
```

```
[Out] -(Sqrt[1 - I]*ArcTanh[(1 + I*x)/(Sqrt[1 - I]*Sqrt[1 - I*x^2])])/2
 - (Sqrt[1 + I]*ArcTanh[(1 - I*x)/(Sqrt[1 + I]*Sqrt[1 + I*x^2])])
/2
```

Rubi in Sympy [A] time = 17.1132, size = 155, normalized size = 1.91

$$-\frac{\sqrt{2}\,(1+i)\,\text{atanh}\left(\frac{\sqrt{2}(-ix+1)}{\sqrt{ix^2+1}\left(\sqrt{1+\sqrt{2}}+i\sqrt{-1+\sqrt{2}}\right)}\right)}{2\left(\sqrt{1+\sqrt{2}}+i\sqrt{-1+\sqrt{2}}\right)}+\frac{\sqrt{2}\,(1-i)\,\text{atanh}\left(\frac{\sqrt{2}(ix+1)}{\sqrt{-ix^2+1}\left(\sqrt{1+\sqrt{2}}-i\sqrt{-1+\sqrt{2}}\right)}\right)}{2\left(-\sqrt{1+\sqrt{2}}+i\sqrt{-1+\sqrt{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((x**2+(x**4+1)**(1/2))**(1/2)/(1+x)/(x**4+1)**(1/2),x)
```

```
[Out] -sqrt(2)*(1 + I)*atanh(sqrt(2)*(-I*x + 1)/(sqrt(I*x**2 + 1)*(sqrt
(1 + sqrt(2)) + I*sqrt(-1 + sqrt(2)))))/(2*(sqrt(1 + sqrt(2)) + I
*sqrt(-1 + sqrt(2)))) + sqrt(2)*(1 - I)*atanh(sqrt(2)*(I*x + 1)/(
sqrt(-I*x**2 + 1)*(sqrt(1 + sqrt(2)) - I*sqrt(-1 + sqrt(2)))))/(2
*(-sqrt(1 + sqrt(2)) + I*sqrt(-1 + sqrt(2))))
```

Mathematica [A] time = 0.118799, size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)\sqrt{1+x^4}}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x)*Sqrt[1 + x^4]),x]
```

```
[Out] Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x)*Sqrt[1 + x^4]), x]
```

Maple [F] time = 0.037, size = 0, normalized size = 0.

$$\int \frac{1}{1+x}\sqrt{x^2+\sqrt{x^4+1}}\frac{1}{\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)/(x^4+1)^(1/2),x)

[Out] int((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)/(x^4+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*(x + 1)),x, algorithm="maxima"

[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*(x + 1)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*(x + 1)),x, algorithm="fricas"

[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{(x+1)\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+(x**4+1)**(1/2))**(1/2)/(1+x)/(x**4+1)**(1/2),x)

[Out] Integral(sqrt(x**2 + sqrt(x**4 + 1))/((x + 1)*sqrt(x**4 + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*(x + 1)),x, algorithm="giac")

[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*(x + 1)), x)
```

3.14 $\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=31

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{\sqrt{x^4+1}+x^2}}\right)}{\sqrt{2}}$$

```
[Out] ArcTanh[(Sqrt[2]*x)/Sqrt[x^2 + Sqrt[1 + x^4]]]/Sqrt[2]
```

Rubi [A] time = 0.088642, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{\sqrt{x^4+1}+x^2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x^2 + Sqrt[1 + x^4]]/Sqrt[1 + x^4],x]

[Out] ArcTanh[(Sqrt[2]*x)/Sqrt[x^2 + Sqrt[1 + x^4]]]/Sqrt[2]
```

Rubi in Sympy [A] time = 3.71235, size = 29, normalized size = 0.94

$$\frac{\sqrt{2}\,\operatorname{atanh}\left(\frac{\sqrt{2}x}{\sqrt{x^2+\sqrt{x^4+1}}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+(x**4+1)**(1/2))**(1/2)/(x**4+1)**(1/2),x)

[Out] sqrt(2)*atanh(sqrt(2)*x/sqrt(x**2 + sqrt(x**4 + 1)))/2
```

Mathematica [B] time = 2.35662, size = 145, normalized size = 4.68

$$-\frac{x\left(x^4+\sqrt{x^4+1}x^2+1\right)\left(\log\left(1-\frac{\sqrt{x^2\left(\sqrt{x^4+1}+x^2\right)}}{\sqrt{2}x^2}\right)-\log\left(\frac{\sqrt{x^2\left(\sqrt{x^4+1}+x^2\right)}}{\sqrt{2}x^2}+1\right)\right)}{2\sqrt{2}\sqrt{x^4+1}\sqrt{\sqrt{x^4+1}+x^2}\sqrt{x^2\left(\sqrt{x^4+1}+x^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/Sqrt[1 + x^4],x]

[Out] -(x*(1 + x^4 + x^2*Sqrt[1 + x^4])*(Log[1 - Sqrt[x^2*(x^2 + Sqrt[1
 + x^4])]/(Sqrt[2]*x^2)] - Log[1 + Sqrt[x^2*(x^2 + Sqrt[1 + x^4])
]/(Sqrt[2]*x^2)]))/(2*Sqrt[2]*Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x
^4]]*Sqrt[x^2*(x^2 + Sqrt[1 + x^4])])
```

Maple [F] time = 0.024, size = 0, normalized size = 0.

$$\int 1\sqrt{x^2+\sqrt{x^4+1}}\frac{1}{\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x)
```

```
[Out] int((x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2 + \sqrt{x^4 + 1}}}{\sqrt{x^4 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x^2 + sqrt(x^4 + 1))/sqrt(x^4 + 1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/sqrt(x^4 + 1), x)
```

Fricas [A] time = 0.389951, size = 81, normalized size = 2.61

$$\frac{1}{4}\,\sqrt{2}\log\left(4\,x^4 + 4\,\sqrt{x^4 + 1}x^2 + 2\,\left(\sqrt{2}x^3 + \sqrt{2}\sqrt{x^4 + 1}x\right)\sqrt{x^2 + \sqrt{x^4 + 1}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x^2 + sqrt(x^4 + 1))/sqrt(x^4 + 1),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log(4*x^4 + 4*sqrt(x^4 + 1)*x^2 + 2*(sqrt(2)*x^3 + sq
rt(2)*sqrt(x^4 + 1)*x)*sqrt(x^2 + sqrt(x^4 + 1)) + 1)
```

Sympy [A] time = 2.21216, size = 15, normalized size = 0.48

$$\frac{G_{3,3}^{2,2}\left(\begin{matrix} 1, 1 & \frac{1}{2} \\ \frac{1}{4}, \frac{3}{4} & 0 \end{matrix} \middle| {x^{4}} \right)}{4\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+(x**4+1)**(1/2))**(1/2)/(x**4+1)**(1/2),x)
```

```
[Out] meijerg(((1, 1), (1/2,)), ((1/4, 3/4), (0,)), x**4)/(4*sqrt(pi))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2 + \sqrt{x^4 + 1}}}{\sqrt{x^4 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x^2 + sqrt(x^4 + 1))/sqrt(x^4 + 1),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/sqrt(x^4 + 1), x)
```

3.15 $\int \frac{\sqrt{-x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=33

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{\sqrt{x^4+1}-x^2}}\right)}{\sqrt{2}}$$

```
[Out] ArcTan[(Sqrt[2]*x)/Sqrt[-x^2 + Sqrt[1 + x^4]]]/Sqrt[2]
```

Rubi [A] time = 0.0914918, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{\sqrt{x^4+1}-x^2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[-x^2 + Sqrt[1 + x^4]]/Sqrt[1 + x^4],x]
```

```
[Out] ArcTan[(Sqrt[2]*x)/Sqrt[-x^2 + Sqrt[1 + x^4]]]/Sqrt[2]
```

Rubi in Sympy [A] time = 3.95058, size = 29, normalized size = 0.88

$$\frac{\sqrt{2}\,\text{atan}\left(\frac{\sqrt{2}x}{\sqrt{-x^2+\sqrt{x^4+1}}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2+(x**4+1)**(1/2))**(1/2)/(x**4+1)**(1/2),x)
```

```
[Out] sqrt(2)*atan(sqrt(2)*x/sqrt(-x**2 + sqrt(x**4 + 1)))/2
```

Mathematica [B] time = 1.43463, size = 162, normalized size = 4.91

$$\frac{x\left(2x^4-2\sqrt{x^4+1}x^2+1\right)^2\left(x^4-\sqrt{x^4+1}x^2+1\right)\sin^{-1}\left(x^2-\sqrt{x^4+1}\right)}{\sqrt{2}\sqrt{\sqrt{x^4+1}-x^2}\sqrt{x^2\left(\sqrt{x^4+1}-x^2\right)}\left(-8x^{10}-12x^6+8\sqrt{x^4+1}x^4+\sqrt{x^4+1}-4x^2+8\sqrt{x^4+1}x^8\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[-x^2 + Sqrt[1 + x^4]]/Sqrt[1 + x^4],x]
```

```
[Out] (x*(1 + 2*x^4 - 2*x^2*Sqrt[1 + x^4])^2*(1 + x^4 - x^2*Sqrt[1 + x^
4])*ArcSin[x^2 - Sqrt[1 + x^4]])/(Sqrt[2]*Sqrt[-x^2 + Sqrt[1 + x^
4]]*Sqrt[x^2*(-x^2 + Sqrt[1 + x^4])]*(-4*x^2 - 12*x^6 - 8*x^10 +
Sqrt[1 + x^4] + 8*x^4*Sqrt[1 + x^4] + 8*x^8*Sqrt[1 + x^4]))
```

Maple [C] time = 0.055, size = 22, normalized size = 0.7

$$-\frac{\sqrt{2}}{4\,x^2}\,{}_3F_2\left(\frac{1}{2},\frac{3}{4},\frac{5}{4};\frac{3}{2},\frac{3}{2};-x^{-4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x)
```

```
[Out] -1/4*2^(1/2)/x^2*hypergeom([1/2,3/4,5/4],[3/2,3/2],-1/x^4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-x^2 + sqrt(x^4 + 1))/sqrt(x^4 + 1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + sqrt(x^4 + 1))/sqrt(x^4 + 1), x)
```

Fricas [A] time = 0.400327, size = 39, normalized size = 1.18

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-x^2+\sqrt{x^4+1}}}{2\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-x^2 + sqrt(x^4 + 1))/sqrt(x^4 + 1),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-x^2 + sqrt(x^4 + 1))/x)
```

Sympy [A] time = 1.16438, size = 15, normalized size = 0.45

$$\frac{G_{3,3}^{2,2}\left(\begin{matrix}\frac{1}{2},1 & 1\\ \frac{1}{4},\frac{3}{4} & 0\end{matrix}\middle|x^4\right)}{4\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+(x**4+1)**(1/2))**(1/2)/(x**4+1)**(1/2),x)
```

```
[Out] meijerg(((1/2, 1), (1,)), ((1/4, 3/4), (0,)), x**4)/(4*sqrt(pi))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-x^2 + sqrt(x^4 + 1))/sqrt(x^4 + 1),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^2 + sqrt(x^4 + 1))/sqrt(x^4 + 1), x)
```

3.16 $\int \frac{(-1+x)^{3/2}+(1+x)^{3/2}}{(-1+x)^{3/2}(1+x)^{3/2}}\,dx$

Optimal. Leaf size=19

$$-\frac{2}{\sqrt{x+1}}-\frac{2}{\sqrt{x-1}}$$

```
[Out] -2/Sqrt[-1 + x] - 2/Sqrt[1 + x]
```

Rubi [A] time = 0.434125, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033

$$-\frac{2}{\sqrt{x+1}}-\frac{2}{\sqrt{x-1}}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x)^(3/2) + (1 + x)^(3/2))/((-1 + x)^(3/2)*(1 + x)^(3/2)),x]

[Out] -2/Sqrt[-1 + x] - 2/Sqrt[1 + x]
```

Rubi in Sympy [A] time = 11.2729, size = 17, normalized size = 0.89

$$-\frac{2}{\sqrt{x+1}}-\frac{2}{\sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(((-1+x)**(3/2)+(1+x)**(3/2))/(-1+x)**(3/2)/(1+x)**(3/2),x)

[Out] -2/sqrt(x + 1) - 2/sqrt(x - 1)
```

Mathematica [A] time = 0.0247948, size = 19, normalized size = 1.

$$-\frac{2}{\sqrt{x+1}}-\frac{2}{\sqrt{x-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x)^(3/2) + (1 + x)^(3/2))/((-1 + x)^(3/2)*(1 + x)^(3/2)),x]

[Out] -2/Sqrt[-1 + x] - 2/Sqrt[1 + x]
```

Maple [A] time = 0.004, size = 16, normalized size = 0.8

$$-2\,\frac{1}{\sqrt{-1+x}}-2\,\frac{1}{\sqrt{1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((-1+x)^(3/2)+(1+x)^(3/2))/(-1+x)^(3/2)/(1+x)^(3/2),x)

[Out] -2/(-1+x)^(1/2)-2/(1+x)^(1/2)
```

Maxima [A] time = 1.34085, size = 20, normalized size = 1.05

$$-\frac{2}{\sqrt{x+1}} - \frac{2}{\sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x + 1)^(3/2) + (x - 1)^(3/2))/((x + 1)^(3/2)*(x - 1)^(3/2)),x, algorit
```

```
[Out] -2/sqrt(x + 1) - 2/sqrt(x - 1)
```

Fricas [A] time = 0.203075, size = 31, normalized size = 1.63

$$-\frac{2\left(\sqrt{x+1} + \sqrt{x-1}\right)}{\sqrt{x+1}\sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x + 1)^(3/2) + (x - 1)^(3/2))/((x + 1)^(3/2)*(x - 1)^(3/2)),x, algorit
```

```
[Out] -2*(sqrt(x + 1) + sqrt(x - 1))/(sqrt(x + 1)*sqrt(x - 1))
```

Sympy [A] time = 12.4909, size = 56, normalized size = 2.95

$$-\frac{2x\sqrt{x-1}}{x^2-1} - \frac{2x\sqrt{x+1}}{x^2-1} - \frac{2\sqrt{x-1}}{x^2-1} + \frac{2\sqrt{x+1}}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)**(3/2)+(1+x)**(3/2))/(-1+x)**(3/2)/(1+x)**(3/2),x)
```

```
[Out] -2*x*sqrt(x - 1)/(x**2 - 1) - 2*x*sqrt(x + 1)/(x**2 - 1) - 2*sqrt
(x - 1)/(x**2 - 1) + 2*sqrt(x + 1)/(x**2 - 1)
```

GIAC/XCAS [A] time = 0.213949, size = 20, normalized size = 1.05

$$-\frac{2}{\sqrt{x+1}} - \frac{2}{\sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x + 1)^(3/2) + (x - 1)^(3/2))/((x + 1)^(3/2)*(x - 1)^(3/2)),x, algorit
```

```
[Out] -2/sqrt(x + 1) - 2/sqrt(x - 1)
```

3.17 $\int \left(x+\sqrt{a+x^2}\right)^b \, dx$

Optimal. Leaf size=52

$$\frac{\left(\sqrt{a+x^2}+x\right)^{b+1}}{2(b+1)}-\frac{a\left(\sqrt{a+x^2}+x\right)^{b-1}}{2(1-b)}$$

```
[Out] -(a*(x + Sqrt[a + x^2])^(-1 + b))/(2*(1 - b)) + (x + Sqrt[a + x^2
])^(1 + b)/(2*(1 + b))
```

Rubi [A] time = 0.0466202, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$\frac{\left(\sqrt{a+x^2}+x\right)^{b+1}}{2(b+1)}-\frac{a\left(\sqrt{a+x^2}+x\right)^{b-1}}{2(1-b)}$$

Antiderivative was successfully verified.

```
[In]  Int[(x + Sqrt[a + x^2])^b,x]
```

```
[Out] -(a*(x + Sqrt[a + x^2])^(-1 + b))/(2*(1 - b)) + (x + Sqrt[a + x^2
])^(1 + b)/(2*(1 + b))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x+\sqrt{a+x^2}\right)^b \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x+(x**2+a)**(1/2))**b,x)
```

```
[Out] Integral((x + sqrt(a + x**2))**b, x)
```

Mathematica [A] time = 0.040084, size = 36, normalized size = 0.69

$$\frac{\left(\sqrt{a+x^2}+x\right)^b\left(b\sqrt{a+x^2}-x\right)}{b^2-1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x + Sqrt[a + x^2])^b,x]
```

```
[Out] ((x + Sqrt[a + x^2])^b*(-x + b*Sqrt[a + x^2]))/(-1 + b^2)
```

Maple [B] time = 0.045, size = 120, normalized size = 2.3

$$\frac{b}{4\sqrt{\pi}}a^{\frac{b}{2}+\frac{1}{2}}\left(8\,\frac{\sqrt{\pi}x^{1+b}a^{-b/2-1/2}}{(1+b)\,b\,(-2+2\,b)}\left(\frac{ab}{x^2}+b-1\right)\left(\sqrt{\frac{a}{x^2}+1}+1\right)^{-1+b}+4\,\frac{\sqrt{\pi}x^{1+b}a^{-b/2-1/2}}{(1+b)\,b}\sqrt{\frac{a}{x^2}+1}\left(\sqrt{\frac{a}{x^2}+1}+1\right)^{-1+b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x+(x^2+a)^(1/2))^b,x)
```

```
[Out] 1/4*a^(1/2*b+1/2)/Pi^(1/2)*b*(8*Pi^(1/2)/(1+b)/b*x^(1+b)*a^(-1/2*
b-1/2)*(1/x^2*a*b+b-1)/(-2+2*b)*((1/x^2*a+1)^(1/2)+1)^(-1+b)+4*Pi
^(1/2)/(1+b)/b*x^(1+b)*a^(-1/2*b-1/2)*(1/x^2*a+1)^(1/2)*((1/x^2*a
+1)^(1/2)+1)^(-1+b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x+\sqrt{x^2+a}\right)^b\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sqrt(x^2 + a))^b,x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(x^2 + a))^b, x)
```

Fricas [A] time = 0.229006, size = 43, normalized size = 0.83

$$\frac{\left(\sqrt{x^2+a}b-x\right)\left(x+\sqrt{x^2+a}\right)^b}{b^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sqrt(x^2 + a))^b,x, algorithm="fricas")
```

```
[Out] (sqrt(x^2 + a)*b - x)*(x + sqrt(x^2 + a))^b/(b^2 - 1)
```

Sympy [A] time = 4.97977, size = 2147, normalized size = 41.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x+(x**2+a)**(1/2))**b,x)
```

```
[Out] Piecewise((-a**(9/2)*a**(b/2)*b**2*x*sqrt(a/x**2 + 1)*sinh(b*asin
h(x/sqrt(a)))*gamma(-b/2)/(2*a**(9/2)*b**2*gamma(-b/2 + 1) - 2*a*
*(9/2)*gamma(-b/2 + 1) + 2*a**(7/2)*b**2*x**2*gamma(-b/2 + 1) - 2
*a**(7/2)*x**2*gamma(-b/2 + 1)) + a**(9/2)*a**(b/2)*b*x*cosh(b*as
inh(x/sqrt(a)))*gamma(-b/2)/(2*a**(9/2)*b**2*gamma(-b/2 + 1) - 2*
a**(9/2)*gamma(-b/2 + 1) + 2*a**(7/2)*b**2*x**2*gamma(-b/2 + 1) -
 2*a**(7/2)*x**2*gamma(-b/2 + 1)) - a**(7/2)*a**(b/2)*b**2*x**3*s
qrt(a/x**2 + 1)*sinh(b*asinh(x/sqrt(a)))*gamma(-b/2)/(2*a**(9/2)*
b**2*gamma(-b/2 + 1) - 2*a**(9/2)*gamma(-b/2 + 1) + 2*a**(7/2)*b*
*2*x**2*gamma(-b/2 + 1) - 2*a**(7/2)*x**2*gamma(-b/2 + 1)) + a**(
7/2)*a**(b/2)*b*x**3*cosh(b*asinh(x/sqrt(a)))*gamma(-b/2)/(2*a**(
9/2)*b**2*gamma(-b/2 + 1) - 2*a**(9/2)*gamma(-b/2 + 1) + 2*a**(7/
2)*b**2*x**2*gamma(-b/2 + 1) - 2*a**(7/2)*x**2*gamma(-b/2 + 1)) +
 2*a**5*a**(b/2)*b*cosh(b*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*ga
mma(-b/2 + 1)/(2*a**(9/2)*b**2*gamma(-b/2 + 1) - 2*a**(9/2)*gamma
(-b/2 + 1) + 2*a**(7/2)*b**2*x**2*gamma(-b/2 + 1) - 2*a**(7/2)*x*
*2*gamma(-b/2 + 1)) - 2*a**5*a**(b/2)*b*gamma(-b/2 + 1)/(2*a**(9/
2)*b**2*gamma(-b/2 + 1) - 2*a**(9/2)*gamma(-b/2 + 1) + 2*a**(7/2)
*b**2*x**2*gamma(-b/2 + 1) - 2*a**(7/2)*x**2*gamma(-b/2 + 1)) - 2
*a**4*a**(b/2)*b*x**2*sqrt(a/x**2 + 1)*sinh(b*asinh(x/sqrt(a)) +
asinh(x/sqrt(a)))*gamma(-b/2 + 1)/(2*a**(9/2)*b**2*gamma(-b/2 + 1
) - 2*a**(9/2)*gamma(-b/2 + 1) + 2*a**(7/2)*b**2*x**2*gamma(-b/2
```

```
+ 1) - 2*a**(7/2)*x**2*gamma(-b/2 + 1)) + 4*a**4*a**(b/2)*b*x**2*
cosh(b*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(-b/2 + 1)/(2*a*
*(9/2)*b**2*gamma(-b/2 + 1) - 2*a**(9/2)*gamma(-b/2 + 1) + 2*a**(
7/2)*b**2*x**2*gamma(-b/2 + 1) - 2*a**(7/2)*x**2*gamma(-b/2 + 1))
 - 2*a**4*a**(b/2)*b*x**2*gamma(-b/2 + 1)/(2*a**(9/2)*b**2*gamma(
-b/2 + 1) - 2*a**(9/2)*gamma(-b/2 + 1) + 2*a**(7/2)*b**2*x**2*gam
ma(-b/2 + 1) - 2*a**(7/2)*x**2*gamma(-b/2 + 1)) - 2*a**4*a**(b/2)
*x**2*sqrt(a/x**2 + 1)*sinh(b*asinh(x/sqrt(a)) + asinh(x/sqrt(a))
)*gamma(-b/2 + 1)/(2*a**(9/2)*b**2*gamma(-b/2 + 1) - 2*a**(9/2)*g
amma(-b/2 + 1) + 2*a**(7/2)*b**2*x**2*gamma(-b/2 + 1) - 2*a**(7/2
)*x**2*gamma(-b/2 + 1)) + 2*a**4*a**(b/2)*x**2*cosh(b*asinh(x/sqr
t(a)) + asinh(x/sqrt(a)))*gamma(-b/2 + 1)/(2*a**(9/2)*b**2*gamma(
-b/2 + 1) - 2*a**(9/2)*gamma(-b/2 + 1) + 2*a**(7/2)*b**2*x**2*gam
ma(-b/2 + 1) - 2*a**(7/2)*x**2*gamma(-b/2 + 1)) - 2*a**3*a**(b/2)
*b*x**4*sqrt(a/x**2 + 1)*sinh(b*asinh(x/sqrt(a)) + asinh(x/sqrt(a
)))*gamma(-b/2 + 1)/(2*a**(9/2)*b**2*gamma(-b/2 + 1) - 2*a**(9/2)
*gamma(-b/2 + 1) + 2*a**(7/2)*b**2*x**2*gamma(-b/2 + 1) - 2*a**(7
/2)*x**2*gamma(-b/2 + 1)) + 2*a**3*a**(b/2)*b*x**4*cosh(b*asinh(x
/sqrt(a)) + asinh(x/sqrt(a)))*gamma(-b/2 + 1)/(2*a**(9/2)*b**2*ga
mma(-b/2 + 1) - 2*a**(9/2)*gamma(-b/2 + 1) + 2*a**(7/2)*b**2*x**2
*gamma(-b/2 + 1) - 2*a**(7/2)*x**2*gamma(-b/2 + 1)) - 2*a**3*a**(
b/2)*x**4*sqrt(a/x**2 + 1)*sinh(b*asinh(x/sqrt(a)) + asinh(x/sqrt
(a)))*gamma(-b/2 + 1)/(2*a**(9/2)*b**2*gamma(-b/2 + 1) - 2*a**(9/
2)*gamma(-b/2 + 1) + 2*a**(7/2)*b**2*x**2*gamma(-b/2 + 1) - 2*a**
(7/2)*x**2*gamma(-b/2 + 1)) + 2*a**3*a**(b/2)*x**4*cosh(b*asinh(x
/sqrt(a)) + asinh(x/sqrt(a)))*gamma(-b/2 + 1)/(2*a**(9/2)*b**2*ga
mma(-b/2 + 1) - 2*a**(9/2)*gamma(-b/2 + 1) + 2*a**(7/2)*b**2*x**2
*gamma(-b/2 + 1) - 2*a**(7/2)*x**2*gamma(-b/2 + 1)), Abs(x**2/a)
> 1), (-2*a**(5/2)*a**(b/2)*b*x*sqrt(1 + x**2/a)*sinh(b*asinh(x/s
qrt(a)) + asinh(x/sqrt(a)))*gamma(-b/2 + 1)/(2*a**(5/2)*b**2*gamm
a(-b/2 + 1) - 2*a**(5/2)*gamma(-b/2 + 1)) + a**(5/2)*a**(b/2)*b*x
*cosh(b*asinh(x/sqrt(a)))*gamma(-b/2)/(2*a**(5/2)*b**2*gamma(-b/2
 + 1) - 2*a**(5/2)*gamma(-b/2 + 1)) - 2*a**(5/2)*a**(b/2)*x*sqrt(
1 + x**2/a)*sinh(b*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(-b/
2 + 1)/(2*a**(5/2)*b**2*gamma(-b/2 + 1) - 2*a**(5/2)*gamma(-b/2 +
 1)) - a**3*a**(b/2)*b**2*sqrt(1 + x**2/a)*sinh(b*asinh(x/sqrt(a)
))*gamma(-b/2)/(2*a**(5/2)*b**2*gamma(-b/2 + 1) - 2*a**(5/2)*gamm
a(-b/2 + 1)) + 2*a**3*a**(b/2)*b*cosh(b*asinh(x/sqrt(a)) + asinh(
x/sqrt(a)))*gamma(-b/2 + 1)/(2*a**(5/2)*b**2*gamma(-b/2 + 1) - 2*
a**(5/2)*gamma(-b/2 + 1)) + 2*a**2*a**(b/2)*b*x**2*cosh(b*asinh(x
/sqrt(a)) + asinh(x/sqrt(a)))*gamma(-b/2 + 1)/(2*a**(5/2)*b**2*ga
mma(-b/2 + 1) - 2*a**(5/2)*gamma(-b/2 + 1)) + 2*a**2*a**(b/2)*x**
2*cosh(b*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(-b/2 + 1)/(2*
a**(5/2)*b**2*gamma(-b/2 + 1) - 2*a**(5/2)*gamma(-b/2 + 1)), True
))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x+\sqrt{x^2+a}\right)^b \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sqrt(x^2 + a))^b,x, algorithm="giac")
```

```
[Out] integrate((x + sqrt(x^2 + a))^b, x)
```

3.18 $\int \left(x-\sqrt{a+x^2}\right)^b\,dx$

Optimal. Leaf size=56

$$\frac{\left(x-\sqrt{a+x^2}\right)^{b+1}}{2(b+1)}-\frac{a\left(x-\sqrt{a+x^2}\right)^{b-1}}{2(1-b)}$$

```
[Out] -(a*(x - Sqrt[a + x^2])^(-1 + b))/(2*(1 - b)) + (x - Sqrt[a + x^2
])^(1 + b)/(2*(1 + b))
```

Rubi [A] time = 0.0458865, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$\frac{\left(x-\sqrt{a+x^2}\right)^{b+1}}{2(b+1)}-\frac{a\left(x-\sqrt{a+x^2}\right)^{b-1}}{2(1-b)}$$

Antiderivative was successfully verified.

```
[In]  Int[(x - Sqrt[a + x^2])^b,x]
```

```
[Out] -(a*(x - Sqrt[a + x^2])^(-1 + b))/(2*(1 - b)) + (x - Sqrt[a + x^2
])^(1 + b)/(2*(1 + b))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x-\sqrt{a+x^2}\right)^b\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x-(x**2+a)**(1/2))**b,x)
```

```
[Out] Integral((x - sqrt(a + x**2))**b, x)
```

Mathematica [A] time = 0.03978, size = 39, normalized size = 0.7

$$\frac{\left(x-\sqrt{a+x^2}\right)^b\left(b\left(-\sqrt{a+x^2}\right)-x\right)}{b^2-1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x - Sqrt[a + x^2])^b,x]
```

```
[Out] ((x - Sqrt[a + x^2])^b*(-x - b*Sqrt[a + x^2]))/(-1 + b^2)
```

Maple [F] time = 0.045, size = 0, normalized size = 0.

$$\int \left(x-\sqrt{x^2+a}\right)^b\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x-(x^2+a)^(1/2))^b,x)

[Out] int((x-(x^2+a)^(1/2))^b,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x - \sqrt{x^2 + a}\right)^b dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - sqrt(x^2 + a))^b,x, algorithm="maxima")

[Out] integrate((x - sqrt(x^2 + a))^b, x)
```

Fricas [A] time = 0.229939, size = 45, normalized size = 0.8

$$-\frac{\left(\sqrt{x^2 + a}b + x\right)\left(x - \sqrt{x^2 + a}\right)^b}{b^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - sqrt(x^2 + a))^b,x, algorithm="fricas")

[Out] -(sqrt(x^2 + a)*b + x)*(x - sqrt(x^2 + a))^b/(b^2 - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x - \sqrt{a + x^2}\right)^b dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x-(x**2+a)**(1/2))**b,x)

[Out] Integral((x - sqrt(a + x**2))**b, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x - \sqrt{x^2 + a}\right)^b dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - sqrt(x^2 + a))^b,x, algorithm="giac")

[Out] integrate((x - sqrt(x^2 + a))^b, x)
```

3.19 $\int \frac{\left(x+\sqrt{a+x^2}\right)^b}{\sqrt{a+x^2}}\,dx$

Optimal. Leaf size=17

$$\frac{\left(\sqrt{a+x^2}+x\right)^b}{b}$$

```
[Out] (x + Sqrt[a + x^2])^b/b
```

Rubi [A] time = 0.0872216, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087

$$\frac{\left(\sqrt{a+x^2}+x\right)^b}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[a + x^2])^b/Sqrt[a + x^2],x]

[Out] (x + Sqrt[a + x^2])^b/b
```

Rubi in Sympy [A] time = 4.05421, size = 12, normalized size = 0.71

$$\frac{\left(x+\sqrt{a+x^2}\right)^b}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((x+(x**2+a)**(1/2))**b/(x**2+a)**(1/2),x)

[Out] (x + sqrt(a + x**2))**b/b
```

Mathematica [A] time = 0.027324, size = 17, normalized size = 1.

$$\frac{\left(\sqrt{a+x^2}+x\right)^b}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[a + x^2])^b/Sqrt[a + x^2],x]

[Out] (x + Sqrt[a + x^2])^b/b
```

Maple [F] time = 0.029, size = 0, normalized size = 0.

$$\int 1\left(x+\sqrt{x^2+a}\right)^b\frac{1}{\sqrt{x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x+(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x)
```

```
[Out] int((x+(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x)
```

Maxima [A] time = 1.39616, size = 20, normalized size = 1.18

$$\frac{\left(x+\sqrt{x^2+a}\right)^b}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sqrt(x^2 + a))^b/sqrt(x^2 + a),x, algorithm="maxima")
```

```
[Out] (x + sqrt(x^2 + a))^b/b
```

Fricas [A] time = 0.230442, size = 20, normalized size = 1.18

$$\frac{\left(x+\sqrt{x^2+a}\right)^b}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sqrt(x^2 + a))^b/sqrt(x^2 + a),x, algorithm="fricas")
```

```
[Out] (x + sqrt(x^2 + a))^b/b
```

Sympy [A] time = 5.35941, size = 311, normalized size = 18.29

$$\begin{cases} -\frac{\sqrt{a}a^{\frac{b}{2}}\sinh\left(-b\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)+\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{bx\sqrt{\frac{a}{x^2}+1}}-\frac{2a^{\frac{b}{2}}\cosh\left(b\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)\left(-\frac{b}{2}+1\right)}{b^2\left(-\frac{b}{2}\right)}+\frac{a^{\frac{b}{2}}x\cosh\left(-b\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)+\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{\sqrt{a}b}-\frac{a^{\frac{b}{2}}x\sinh\left(-b\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)+\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{\sqrt{a}b\sqrt{\frac{a}{x^2}+1}} & \text{for}\ \left|\frac{x^2}{a}\right|>1 \\ -\frac{a^{\frac{b}{2}}\sinh\left(-b\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)+\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{b\sqrt{1+\frac{x^2}{a}}}-\frac{2a^{\frac{b}{2}}\cosh\left(b\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)\left(-\frac{b}{2}+1\right)}{b^2\left(-\frac{b}{2}\right)}-\frac{a^{\frac{b}{2}}x^2\sinh\left(-b\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)+\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{ab\sqrt{1+\frac{x^2}{a}}}+\frac{a^{\frac{b}{2}}x\cosh\left(-b\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)+\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{\sqrt{a}b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x+(x**2+a)**(1/2))**b/(x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-sqrt(a)*a**(b/2)*sinh(-b*asinh(x/sqrt(a)) + asinh(x/s
qrt(a)))/(b*x*sqrt(a/x**2 + 1)) - 2*a**(b/2)*cosh(b*asinh(x/sqrt(
a)))*gamma(-b/2 + 1)/(b**2*gamma(-b/2)) + a**(b/2)*x*cosh(-b*asin
h(x/sqrt(a)) + asinh(x/sqrt(a)))/(sqrt(a)*b) - a**(b/2)*x*sinh(-b
*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))/(sqrt(a)*b*sqrt(a/x**2 + 1)
), Abs(x**2/a) > 1), (-a**(b/2)*sinh(-b*asinh(x/sqrt(a)) + asinh(
x/sqrt(a)))/(b*sqrt(1 + x**2/a)) - 2*a**(b/2)*cosh(b*asinh(x/sqrt
(a)))*gamma(-b/2 + 1)/(b**2*gamma(-b/2)) - a**(b/2)*x**2*sinh(-b*
asinh(x/sqrt(a)) + asinh(x/sqrt(a)))/(a*b*sqrt(1 + x**2/a)) + a**
(b/2)*x*cosh(-b*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))/(sqrt(a)*b),
 True))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x+\sqrt{x^2+a}\right)^b}{\sqrt{x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sqrt(x^2 + a))^b/sqrt(x^2 + a),x, algorithm="giac")

[Out] integrate((x + sqrt(x^2 + a))^b/sqrt(x^2 + a), x)
```

3.20 $\int \frac{\left(x-\sqrt{a+x^2}\right)^b}{\sqrt{a+x^2}} \, dx$

Optimal. Leaf size=20

$$-\frac{\left(x-\sqrt{a+x^2}\right)^b}{b}$$

```
[Out] -((x - Sqrt[a + x^2])^b/b)
```

Rubi [A] time = 0.0922341, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$

$$-\frac{\left(x-\sqrt{a+x^2}\right)^b}{b}$$

Antiderivative was successfully verified.

```
[In]  Int[(x - Sqrt[a + x^2])^b/Sqrt[a + x^2],x]

[Out] -((x - Sqrt[a + x^2])^b/b)
```

Rubi in Sympy [A] time = 4.36387, size = 14, normalized size = 0.7

$$-\frac{\left(x-\sqrt{a+x^2}\right)^b}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x-(x**2+a)**(1/2))**b/(x**2+a)**(1/2),x)

[Out] -(x - sqrt(a + x**2))**b/b
```

Mathematica [A] time = 0.0279361, size = 20, normalized size = 1.

$$-\frac{\left(x-\sqrt{a+x^2}\right)^b}{b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x - Sqrt[a + x^2])^b/Sqrt[a + x^2],x]

[Out] -((x - Sqrt[a + x^2])^b/b)
```

Maple [F] time = 0.045, size = 0, normalized size = 0.

$$\int 1 \left(x-\sqrt{x^2+a}\right)^b \frac{1}{\sqrt{x^2+a}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x-(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x)

[Out] int((x-(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x)
```

Maxima [A] time = 1.39686, size = 24, normalized size = 1.2

$$-\frac{\left(x-\sqrt{x^2+a}\right)^b}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - sqrt(x^2 + a))^b/sqrt(x^2 + a),x, algorithm="maxima")

[Out] -(x - sqrt(x^2 + a))^b/b
```

Fricas [A] time = 0.232349, size = 24, normalized size = 1.2

$$-\frac{\left(x-\sqrt{x^2+a}\right)^b}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - sqrt(x^2 + a))^b/sqrt(x^2 + a),x, algorithm="fricas")

[Out] -(x - sqrt(x^2 + a))^b/b
```

Sympy [A] time = 3.55928, size = 36, normalized size = 1.8

$$\begin{cases} -\frac{\left(x-\sqrt{a+x^2}\right)^b}{b} & \text{for } b \neq 0 \\ \begin{cases} \operatorname{asinh}\left(x\sqrt{\frac{1}{a}}\right) & \text{for } a > 0 \\ \operatorname{acosh}\left(x\sqrt{-\frac{1}{a}}\right) & \text{for } a < 0 \end{cases} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x-(x**2+a)**(1/2))**b/(x**2+a)**(1/2),x)

[Out] Piecewise((-(x - sqrt(a + x**2))**b/b, Ne(b, 0)), (Piecewise((asi
nh(x*sqrt(1/a)), a > 0), (acosh(x*sqrt(-1/a)), a < 0)), True))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x-\sqrt{x^2+a}\right)^b}{\sqrt{x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - sqrt(x^2 + a))^b/sqrt(x^2 + a),x, algorithm="giac")

[Out] integrate((x - sqrt(x^2 + a))^b/sqrt(x^2 + a), x)
```

3.21 $\int \frac{1}{(a+be^{px})^2}\,dx$

Optimal. Leaf size=42

$$-\frac{\log\left(a+be^{px}\right)}{a^2p}+\frac{x}{a^2}+\frac{1}{ap\left(a+be^{px}\right)}$$

```
[Out] 1/(a*(a + b*E^(p*x))*p) + x/a^2 - Log[a + b*E^(p*x)]/(a^2*p)
```

Rubi [A] time = 0.0631128, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$-\frac{\log\left(a+be^{px}\right)}{a^2p}+\frac{x}{a^2}+\frac{1}{ap\left(a+be^{px}\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*E^(p*x))^(-2),x]
```

```
[Out] 1/(a*(a + b*E^(p*x))*p) + x/a^2 - Log[a + b*E^(p*x)]/(a^2*p)
```

Rubi in Sympy [A] time = 5.39135, size = 39, normalized size = 0.93

$$\frac{1}{ap\left(a+be^{px}\right)}-\frac{\log\left(a+be^{px}\right)}{a^2p}+\frac{\log\left(e^{px}\right)}{a^2p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a+b*exp(p*x))**2,x)
```

```
[Out] 1/(a*p*(a + b*exp(p*x))) - log(a + b*exp(p*x))/(a**2*p) + log(exp
(p*x))/(a**2*p)
```

Mathematica [A] time = 0.0630322, size = 37, normalized size = 0.88

$$\frac{\frac{a}{ap+bpe^{px}}-\frac{\log(a+be^{px})}{p}+x}{a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*E^(p*x))^(-2),x]
```

```
[Out] (a/(a*p + b*E^(p*x)*p) + x - Log[a + b*E^(p*x)]/p)/a^2
```

Maple [A] time = 0.017, size = 48, normalized size = 1.1

$$-\frac{\ln\left(a+b\mathrm{e}^{px}\right)}{a^2p}+\frac{1}{a\left(a+b\mathrm{e}^{px}\right)p}+\frac{\ln\left(\mathrm{e}^{px}\right)}{a^2p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a+b*exp(p*x))^2,x)
```

```
[Out] -ln(a+b*exp(p*x))/a^2/p+1/a/(a+b*exp(p*x))/p+1/p/a^2*ln(exp(p*x))
```

Maxima [A] time = 1.35229, size = 54, normalized size = 1.29

$$\frac{x}{a^2} + \frac{1}{\left(abe^{(px)} + a^2\right)p} - \frac{\log\left(be^{(px)} + a\right)}{a^2p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*e^(p*x) + a)^(-2),x, algorithm="maxima")
```

```
[Out] x/a^2 + 1/((a*b*e^(p*x) + a^2)*p) - log(b*e^(p*x) + a)/(a^2*p)
```

Fricas [A] time = 0.224159, size = 70, normalized size = 1.67

$$\frac{bpxe^{(px)} + apx - \left(be^{(px)} + a\right)\log\left(be^{(px)} + a\right) + a}{a^2bpe^{(px)} + a^3p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*e^(p*x) + a)^(-2),x, algorithm="fricas")
```

```
[Out] (b*p*x*e^(p*x) + a*p*x - (b*e^(p*x) + a)*log(b*e^(p*x) + a) + a)/
(a^2*b*p*e^(p*x) + a^3*p)
```

Sympy [A] time = 0.17583, size = 36, normalized size = 0.86

$$\frac{1}{a^2p + abpe^{px}} + \frac{x}{a^2} - \frac{\log\left(\frac{a}{b} + e^{px}\right)}{a^2p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a+b*exp(p*x))**2,x)
```

```
[Out] 1/(a**2*p + a*b*p*exp(p*x)) + x/a**2 - log(a/b + exp(p*x))/(a**2*
p)
```

GIAC/XCAS [A] time = 0.197701, size = 55, normalized size = 1.31

$$\frac{x}{a^2} - \frac{\ln\left(\left|be^{(px)} + a\right|\right)}{a^2p} + \frac{1}{\left(be^{(px)} + a\right)ap}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*e^(p*x) + a)^(-2),x, algorithm="giac")
```

```
[Out] x/a^2 - ln(abs(b*e^(p*x) + a))/(a^2*p) + 1/((b*e^(p*x) + a)*a*p)
```

3.22 $\int \frac{1}{(be^{-px}+ae^{px})^2}\,dx$

Optimal. Leaf size=22

$$-\frac{1}{2ap\left(ae^{2px}+b\right)}$$

```
[Out] -1/(2*a*(b + a*E^(2*p*x))*p)
```

Rubi [A] time = 0.0413316, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$-\frac{1}{2ap\left(ae^{2px}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(b/E^(p*x) + a*E^(p*x))^(-2),x]

[Out] -1/(2*a*(b + a*E^(2*p*x))*p)
```

Rubi in Sympy [A] time = 4.67805, size = 15, normalized size = 0.68

$$\frac{1}{2bp\left(a+be^{-2px}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(b/exp(p*x)+a*exp(p*x))**2,x)

[Out] 1/(2*b*p*(a + b*exp(-2*p*x)))
```

Mathematica [A] time = 0.0211906, size = 22, normalized size = 1.

$$-\frac{1}{2ap\left(ae^{2px}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b/E^(p*x) + a*E^(p*x))^(-2),x]

[Out] -1/(2*a*(b + a*E^(2*p*x))*p)
```

Maple [A] time = 0.004, size = 21, normalized size = 1.

$$-\frac{1}{2\,pa\left(a\left(e^{px}\right)^{2}+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/exp(p*x)+a*exp(p*x))^2,x)

[Out] -1/2/p/a/(a*exp(p*x)^2+b)
```

Maxima [A] time = 1.34244, size = 27, normalized size = 1.23

$$\frac{1}{2\left(b^2e^{(-2\,px)} + ab\right)p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a*e^(p*x) + b*e^(-p*x))^(-2),x, algorithm="maxima")
```

```
[Out] 1/2/((b^2*e^(-2*p*x) + a*b)*p)
```

Fricas [A] time = 0.215691, size = 26, normalized size = 1.18

$$-\frac{1}{2\left(a^2pe^{(2\,px)} + abp\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a*e^(p*x) + b*e^(-p*x))^(-2),x, algorithm="fricas")
```

```
[Out] -1/2/(a^2*p*e^(2*p*x) + a*b*p)
```

Sympy [A] time = 0.116086, size = 22, normalized size = 1.

$$-\frac{1}{2a^2pe^{2px} + 2abp}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b/exp(p*x)+a*exp(p*x))**2,x)
```

```
[Out] -1/(2*a**2*p*exp(2*p*x) + 2*a*b*p)
```

GIAC/XCAS [A] time = 0.200777, size = 26, normalized size = 1.18

$$-\frac{1}{2\left(ae^{(2\,px)} + b\right)ap}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a*e^(p*x) + b*e^(-p*x))^(-2),x, algorithm="giac")
```

```
[Out] -1/2/((a*e^(2*p*x) + b)*a*p)
```

3.23 $\int \frac{x}{(be^{-px}+ae^{px})^2}\,dx$

Optimal. Leaf size=62

$$-\frac{\log\left(ae^{2px}+b\right)}{4abp^2}+\frac{x}{2abp}-\frac{x}{2ap\left(ae^{2px}+b\right)}$$

```
[Out] x/(2*a*b*p) - x/(2*a*(b + a*E^(2*p*x))*p) - Log[b + a*E^(2*p*x)]/
(4*a*b*p^2)
```

Rubi [A] time = 0.154055, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$-\frac{\log\left(ae^{2px}+b\right)}{4abp^2}+\frac{x}{2abp}-\frac{x}{2ap\left(ae^{2px}+b\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(b/E^(p*x) + a*E^(p*x))^2,x]
```

```
[Out] x/(2*a*b*p) - x/(2*a*(b + a*E^(2*p*x))*p) - Log[b + a*E^(2*p*x)]/
(4*a*b*p^2)
```

Rubi in Sympy [A] time = 10.1021, size = 54, normalized size = 0.87

$$\frac{x}{2bp\left(a+be^{-2px}\right)}-\frac{\log\left(a+be^{-2px}\right)}{4abp^2}+\frac{\log\left(e^{-2px}\right)}{4abp^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(b/exp(p*x)+a*exp(p*x))**2,x)
```

```
[Out] x/(2*b*p*(a + b*exp(-2*p*x))) - log(a + b*exp(-2*p*x))/(4*a*b*p**
2) + log(exp(-2*p*x))/(4*a*b*p**2)
```

Mathematica [A] time = 0.0517093, size = 49, normalized size = 0.79

$$\frac{\frac{2pxe^{2px}}{ae^{2px}+b}-\frac{\log\left(ae^{2px}+b\right)}{a}}{4bp^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(b/E^(p*x) + a*E^(p*x))^2,x]
```

```
[Out] ((2*E^(2*p*x)*p*x)/(b + a*E^(2*p*x)) - Log[b + a*E^(2*p*x)]/a)/(4
*b*p^2)
```

Maple [A] time = 0.019, size = 51, normalized size = 0.8

$$-\frac{\ln\left(a\left(e^{px}\right)^2+b\right)}{4\,p^2ba}+\frac{x\left(e^{px}\right)^2}{2\,bp\left(a\left(e^{px}\right)^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(b/exp(p*x)+a*exp(p*x))^2,x)

[Out] -1/4/p^2/b/a*ln(a*exp(p*x)^2+b)+1/2/p*x*exp(p*x)^2/b/(a*exp(p*x)^
2+b)
```

Maxima [A] time = 1.36774, size = 69, normalized size = 1.11

$$\frac{xe^{(2\,px)}}{2\,\left(abpe^{(2\,px)}+b^2p\right)}-\frac{\log\left(\frac{ae^{(2\,px)}+b}{a}\right)}{4\,abp^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(a*e^(p*x) + b*e^(-p*x))^2,x, algorithm="maxima")

[Out] 1/2*x*e^(2*p*x)/(a*b*p*e^(2*p*x) + b^2*p) - 1/4*log((a*e^(2*p*x)
+ b)/a)/(a*b*p^2)
```

Fricas [A] time = 0.230856, size = 78, normalized size = 1.26

$$\frac{2\,apxe^{(2\,px)}-\left(ae^{(2\,px)}+b\right)\log\left(ae^{(2\,px)}+b\right)}{4\,\left(a^2bp^2e^{(2\,px)}+ab^2p^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(a*e^(p*x) + b*e^(-p*x))^2,x, algorithm="fricas")

[Out] 1/4*(2*a*p*x*e^(2*p*x) - (a*e^(2*p*x) + b)*log(a*e^(2*p*x) + b))/
(a^2*b*p^2*e^(2*p*x) + a*b^2*p^2)
```

Sympy [A] time = 0.215637, size = 49, normalized size = 0.79

$$-\frac{x}{2a^2pe^{2px}+2abp}+\frac{x}{2abp}-\frac{\log\left(e^{2px}+\frac{b}{a}\right)}{4abp^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(b/exp(p*x)+a*exp(p*x))**2,x)

[Out] -x/(2*a**2*p*exp(2*p*x) + 2*a*b*p) + x/(2*a*b*p) - log(exp(2*p*x)
 + b/a)/(4*a*b*p**2)
```

GIAC/XCAS [A] time = 0.198747, size = 100, normalized size = 1.61

$$\frac{2\,apxe^{(2\,px)}-ae^{(2\,px)}\ln\left(-ae^{(2\,px)}-b\right)-b\ln\left(-ae^{(2\,px)}-b\right)}{4\,\left(a^2bp^2e^{(2\,px)}+ab^2p^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(a*e^(p*x) + b*e^(-p*x))^2,x, algorithm="giac")

[Out] 1/4*(2*a*p*x*e^(2*p*x) - a*e^(2*p*x)*ln(-a*e^(2*p*x) - b) - b*ln(
-a*e^(2*p*x) - b))/(a^2*b*p^2*e^(2*p*x) + a*b^2*p^2)
```

3.24 $\int \frac{1-x+3x^2}{\sqrt{1-x+x^2}(1+x+x^2)^2} \, dx$

Optimal. Leaf size=86

$$\frac{\sqrt{x^2-x+1}(x+1)}{x^2+x+1}+\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}(x+1)}{\sqrt{x^2-x+1}}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{2}{3}}(1-x)}{\sqrt{x^2-x+1}}\right)}{\sqrt{6}}$$

```
[Out] ((1 + x)*Sqrt[1 - x + x^2])/(1 + x + x^2) + Sqrt[2]*ArcTan[(Sqrt[
2]*(1 + x))/Sqrt[1 - x + x^2]] - ArcTanh[(Sqrt[2/3]*(1 - x))/Sqrt
[1 - x + x^2]]/Sqrt[6]
```

Rubi [A] time = 0.225655, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$

$$\frac{\sqrt{x^2-x+1}(x+1)}{x^2+x+1}+\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}(x+1)}{\sqrt{x^2-x+1}}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{2}{3}}(1-x)}{\sqrt{x^2-x+1}}\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - x + 3*x^2)/(Sqrt[1 - x + x^2]*(1 + x + x^2)^2),x]
```

```
[Out] ((1 + x)*Sqrt[1 - x + x^2])/(1 + x + x^2) + Sqrt[2]*ArcTan[(Sqrt[
2]*(1 + x))/Sqrt[1 - x + x^2]] - ArcTanh[(Sqrt[2/3]*(1 - x))/Sqrt
[1 - x + x^2]]/Sqrt[6]
```

Rubi in Sympy [A] time = 18.81, size = 87, normalized size = 1.01

$$\frac{(12x+12)\sqrt{x^2-x+1}}{12(x^2+x+1)}+\sqrt{2}\,\mathrm{atan}\left(-\frac{\sqrt{2}\,(-144x-144)}{144\sqrt{x^2-x+1}}\right)+\frac{\sqrt{6}\,\mathrm{atanh}\left(\frac{\sqrt{6}(24x-24)}{72\sqrt{x^2-x+1}}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2-x+1)/(x**2+x+1)**2/(x**2-x+1)**(1/2),x)
```

```
[Out] (12*x + 12)*sqrt(x**2 - x + 1)/(12*(x**2 + x + 1)) + sqrt(2)*atan
(-sqrt(2)*(-144*x - 144)/(144*sqrt(x**2 - x + 1))) + sqrt(6)*atan
h(sqrt(6)*(24*x - 24)/(72*sqrt(x**2 - x + 1)))/6
```

Mathematica [C] time = 5.47027, size = 961, normalized size = 11.17

$$\frac{\sqrt{x^2-x+1}(x+1)}{x^2+x+1} + \frac{\left(7-i\sqrt{3}\right)\tan^{-1}\left(\frac{3\left(\left(-21-4i\sqrt{3}\right)x^4+14\left(7-2i\sqrt{3}\right)x^3+\left(-103-36i\sqrt{3}\right)x^2+\left(94+32i\sqrt{3}\right)x-64i\sqrt{3}-17\right)}{\left(84i-113\sqrt{3}\right)x^4+2\left(52\sqrt{3-3i\sqrt{3}}\sqrt{x^2-x+1}+21\sqrt{3}+138i\right)x^3+\left(52\sqrt{3-3i\sqrt{3}}\sqrt{x^2-x+1}-59\sqrt{3}-180i\right)x^2+2\left(26\sqrt{3-3i\sqrt{3}}\sqrt{x^2-x+1}-69\sqrt{3}+132i\right.}\right.}{4\sqrt{3-3i\sqrt{3}}} - \frac{i\left(-7i+\sqrt{3}\right)\tan^{-1}\left(\frac{3\left(\left(-21+4i\sqrt{3}\right)x^4+14\left(7+2i\sqrt{3}\right)x^3+\left(-103+36i\sqrt{3}\right)x^2+\left(94-32i\sqrt{3}\right)x+64i\sqrt{3}-17\right)}{\left(84i+113\sqrt{3}\right)x^4-2\left(52\sqrt{3+3i\sqrt{3}}\sqrt{x^2-x+1}+21\sqrt{3}-138i\right)x^3+\left(-52\sqrt{3+3i\sqrt{3}}\sqrt{x^2-x+1}+59\sqrt{3}-180i\right)x^2+\left(-52\sqrt{3+3i\sqrt{3}}\sqrt{x^2-x+1}+138\sqrt{3}\right.}\right.}{4\sqrt{3+3i\sqrt{3}}} - \frac{\left(7i+\sqrt{3}\right)\log\left(16\left(x^2+x+1\right)^2\right)}{8\sqrt{3-3i\sqrt{3}}} - \frac{\left(-7i+\sqrt{3}\right)\log\left(16\left(x^2+x+1\right)^2\right)}{8\sqrt{3+3i\sqrt{3}}} + \frac{\left(7i+\sqrt{3}\right)\log\left(\left(x^2+x+1\right)\left(\left(11i+4\sqrt{3}\right)x^2-\left(8i\sqrt{1-i\sqrt{3}}\sqrt{x^2-x+1}+4\sqrt{3}+17i\right)x+10i\sqrt{1-i\sqrt{3}}\sqrt{x^2-x+1}+4\sqrt{3}\right.\right.}{8\sqrt{3-3i\sqrt{3}}} + \frac{\left(-7i+\sqrt{3}\right)\log\left(\left(x^2+x+1\right)\left(\left(-11i+4\sqrt{3}\right)x^2+\left(8i\sqrt{1+i\sqrt{3}}\sqrt{x^2-x+1}-4\sqrt{3}+17i\right)x-10i\sqrt{1+i\sqrt{3}}\sqrt{x^2-x+1}+4\right.\right.}{8\sqrt{3+3i\sqrt{3}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - x + 3*x^2)/(Sqrt[1 - x + x^2]*(1 + x + x^2)^2),x]
```

```
[Out] ((1 + x)*Sqrt[1 - x + x^2])/(1 + x + x^2) + ((7 - I*Sqrt[3])*ArcT
an[(3*(-17 - (64*I)*Sqrt[3] + (94 + (32*I)*Sqrt[3])*x + (-103 - (
36*I)*Sqrt[3])*x^2 + 14*(7 - (2*I)*Sqrt[3])*x^3 + (-21 - (4*I)*Sq
rt[3])*x^4))/(96*I + 67*Sqrt[3] + (84*I - 113*Sqrt[3])*x^4 - 52*S
qrt[3 - (3*I)*Sqrt[3]]*Sqrt[1 - x + x^2] + 2*x*(132*I - 69*Sqrt[3
] + 26*Sqrt[3 - (3*I)*Sqrt[3]]*Sqrt[1 - x + x^2]) + x^2*(-180*I -
 59*Sqrt[3] + 52*Sqrt[3 - (3*I)*Sqrt[3]]*Sqrt[1 - x + x^2]) + 2*x
^3*(138*I + 21*Sqrt[3] + 52*Sqrt[3 - (3*I)*Sqrt[3]]*Sqrt[1 - x +
x^2]))])/(4*Sqrt[3 - (3*I)*Sqrt[3]]) - ((I/4)*(-7*I + Sqrt[3])*Ar
cTan[(3*(-17 + (64*I)*Sqrt[3] + (94 - (32*I)*Sqrt[3])*x + (-103 +
 (36*I)*Sqrt[3])*x^2 + 14*(7 + (2*I)*Sqrt[3])*x^3 + (-21 + (4*I)*
Sqrt[3])*x^4))/(96*I - 67*Sqrt[3] + (84*I + 113*Sqrt[3])*x^4 + 52
*Sqrt[3 + (3*I)*Sqrt[3]]*Sqrt[1 - x + x^2] + x^2*(-180*I + 59*Sqr
t[3] - 52*Sqrt[3 + (3*I)*Sqrt[3]]*Sqrt[1 - x + x^2]) + x*(264*I +
 138*Sqrt[3] - 52*Sqrt[3 + (3*I)*Sqrt[3]]*Sqrt[1 - x + x^2]) - 2*
x^3*(-138*I + 21*Sqrt[3] + 52*Sqrt[3 + (3*I)*Sqrt[3]]*Sqrt[1 - x
+ x^2]))])/Sqrt[3 + (3*I)*Sqrt[3]] - ((-7*I + Sqrt[3])*Log[16*(1
+ x + x^2)^2])/(8*Sqrt[3 + (3*I)*Sqrt[3]]) - ((7*I + Sqrt[3])*Log
[16*(1 + x + x^2)^2])/(8*Sqrt[3 - (3*I)*Sqrt[3]]) + ((7*I + Sqrt[
3])*Log[(1 + x + x^2)*(11*I + 4*Sqrt[3] + (11*I + 4*Sqrt[3])*x^2
+ (10*I)*Sqrt[1 - I*Sqrt[3]]*Sqrt[1 - x + x^2] - x*(17*I + 4*Sqrt
[3] + (8*I)*Sqrt[1 - I*Sqrt[3]]*Sqrt[1 - x + x^2]))])/(8*Sqrt[3 -
 (3*I)*Sqrt[3]]) + ((-7*I + Sqrt[3])*Log[(1 + x + x^2)*(-11*I + 4
*Sqrt[3] + (-11*I + 4*Sqrt[3])*x^2 - (10*I)*Sqrt[1 + I*Sqrt[3]]*S
qrt[1 - x + x^2] + x*(17*I - 4*Sqrt[3] + (8*I)*Sqrt[1 + I*Sqrt[3]
]*Sqrt[1 - x + x^2]))])/(8*Sqrt[3 + (3*I)*Sqrt[3]])
```

Maple [B] time = 0.057, size = 455, normalized size = 5.3

$$-\frac{1}{6}\left(9\,\frac{\sqrt{2}\,(1+x)^2}{(1-x)^2}\arctan\left(2\,\frac{(1+x)\sqrt{2}}{1-x}\,\frac{1}{\sqrt{\frac{(1+x)^2}{(1-x)^2}+3}}\right)\sqrt{\frac{(1+x)^2}{(1-x)^2}+3}-6\,\frac{\sqrt{6}\,(1+x)^2}{(1-x)^2}\,Artanh\left(1/4\sqrt{\frac{(1+x)^2}{(1-x)^2}+3}\sqrt{6}\right)\sqrt{\frac{(1+x}{(1-x}}\right. + \frac{1}{2}\sqrt{\frac{(1+x)^2}{(1-x)^2}+3}\left(3\sqrt{2}\arctan\left(2\,\frac{(1+x)\sqrt{2}}{1-x}\,\frac{1}{\sqrt{\frac{(1+x)^2}{(1-x)^2}+3}}\right)-\sqrt{6}\,Artanh\left(\frac{\sqrt{6}}{4}\sqrt{\frac{(1+x)^2}{(1-x)^2}+3}\right)\right)\frac{1}{\sqrt{1\left(\frac{(1+x)^2}{(1-x)^2}+3\right)\left(1+\frac{1+x}{1-x}\right)^{-2}}}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2-x+1)/(x^2+x+1)^2/(x^2-x+1)^(1/2),x)
```

```
[Out] -1/6*(9*2^(1/2)*arctan(2*2^(1/2)/((1+x)^2/(1-x)^2+3)^(1/2)*(1+x)/
(1-x))*((1+x)^2/(1-x)^2+3)^(1/2)*(1+x)^2/(1-x)^2-6*6^(1/2)*arctan
h(1/4*((1+x)^2/(1-x)^2+3)^(1/2)*6^(1/2))*((1+x)^2/(1-x)^2+3)^(1/2
)*(1+x)^2/(1-x)^2+3*2^(1/2)*arctan(2*2^(1/2)/((1+x)^2/(1-x)^2+3)^
(1/2)*(1+x)/(1-x))*((1+x)^2/(1-x)^2+3)^(1/2)-2*6^(1/2)*arctanh(1/
4*((1+x)^2/(1-x)^2+3)^(1/2)*6^(1/2))*((1+x)^2/(1-x)^2+3)^(1/2)-12
*(1+x)^3/(1-x)^3-36*(1+x)/(1-x))/(((1+x)^2/(1-x)^2+3)/(1+(1+x)/(1
-x))^2)^(1/2)/(1+(1+x)/(1-x))/(3*(1+x)^2/(1-x)^2+1)+1/2*((1+x)^2/
(1-x)^2+3)^(1/2)*(3*2^(1/2)*arctan(2*2^(1/2)/((1+x)^2/(1-x)^2+3)^
(1/2)*(1+x)/(1-x))-6^(1/2)*arctanh(1/4*((1+x)^2/(1-x)^2+3)^(1/2)*
6^(1/2)))/(1+(1+x)/(1-x))/(((1+x)^2/(1-x)^2+3)/(1+(1+x)/(1-x))^2)
^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3x^2 - x + 1}{(x^2 + x + 1)^2 \sqrt{x^2 - x + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 - x + 1)/((x^2 + x + 1)^2*sqrt(x^2 - x + 1)),x, algorithm="maxima
```

```
[Out] integrate((3*x^2 - x + 1)/((x^2 + x + 1)^2*sqrt(x^2 - x + 1)), x)
```

Fricas [A] time = 0.25221, size = 720, normalized size = 8.37

$$2\sqrt{6}\sqrt{3}\left(4x^2+7x-9\right)\sqrt{x^2-x+1}-2\sqrt{6}\sqrt{3}\left(4x^3+5x^2-11x+9\right)-24\left(8x^4+5x^2-4\left(2x^3+x^2+x-1\right)\sqrt{x^2-x+1}\right. -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 - x + 1)/((x^2 + x + 1)^2*sqrt(x^2 - x + 1)),x, algorithm="fricas
```

```
[Out] 1/2*(2*sqrt(6)*sqrt(3)*(4*x^2 + 7*x - 9)*sqrt(x^2 - x + 1) - 2*sq
rt(6)*sqrt(3)*(4*x^3 + 5*x^2 - 11*x + 9) - 24*(8*x^4 + 5*x^2 - 4*
(2*x^3 + x^2 + x - 1)*sqrt(x^2 - x + 1) - 3*x + 5)*arctan(-(sqrt(
6)*sqrt(3) + 2*sqrt(3))/(sqrt(6)*(2*x + 1) - 2*sqrt(6)*sqrt(2*x^2
 - sqrt(x^2 - x + 1)*(2*x + sqrt(6) + 1) + sqrt(6)*(x + 1) + 4) -
 2*sqrt(6)*sqrt(x^2 - x + 1) + 6)) + 24*(8*x^4 + 5*x^2 - 4*(2*x^3
 + x^2 + x - 1)*sqrt(x^2 - x + 1) - 3*x + 5)*arctan(-(sqrt(6)*sqr
t(3) - 2*sqrt(3))/(sqrt(6)*(2*x + 1) - 2*sqrt(6)*sqrt(2*x^2 - sqr
t(x^2 - x + 1)*(2*x - sqrt(6) + 1) - sqrt(6)*(x + 1) + 4) - 2*sqr
t(6)*sqrt(x^2 - x + 1) - 6)) + (4*sqrt(3)*(2*x^3 + x^2 + x - 1)*s
qrt(x^2 - x + 1) - sqrt(3)*(8*x^4 + 5*x^2 - 3*x + 5))*log(4056*x^
2 - 2028*sqrt(x^2 - x + 1)*(2*x + sqrt(6) + 1) + 2028*sqrt(6)*(x
+ 1) + 8112) - (4*sqrt(3)*(2*x^3 + x^2 + x - 1)*sqrt(x^2 - x + 1)
 - sqrt(3)*(8*x^4 + 5*x^2 - 3*x + 5))*log(4056*x^2 - 2028*sqrt(x^
2 - x + 1)*(2*x - sqrt(6) + 1) - 2028*sqrt(6)*(x + 1) + 8112))/(4
*sqrt(6)*sqrt(3)*(2*x^3 + x^2 + x - 1)*sqrt(x^2 - x + 1) - sqrt(6
)*sqrt(3)*(8*x^4 + 5*x^2 - 3*x + 5))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3x^2 - x + 1}{\sqrt{x^2 - x + 1}\,(x^2 + x + 1)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**2-x+1)/(x**2+x+1)**2/(x**2-x+1)**(1/2),x)
```

```
[Out] Integral((3*x**2 - x + 1)/(sqrt(x**2 - x + 1)*(x**2 + x + 1)**2),
 x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3\,x^2 - x + 1}{(x^2 + x + 1)^2 \sqrt{x^2 - x + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 - x + 1)/((x^2 + x + 1)^2*sqrt(x^2 - x + 1)),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 - x + 1)/((x^2 + x + 1)^2*sqrt(x^2 - x + 1)), x)
```

3.25 $\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a^2+x^2}}\,dx$

Optimal. Leaf size=19

$$2\sqrt{\sqrt{a^2+x^2}+x}$$

```
[Out] 2*Sqrt[x + Sqrt[a^2 + x^2]]
```

Rubi [A] time = 0.10589, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$

$$2\sqrt{\sqrt{a^2+x^2}+x}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a^2 + x^2],x]

[Out] 2*Sqrt[x + Sqrt[a^2 + x^2]]
```

Rubi in Sympy [A] time = 3.64378, size = 15, normalized size = 0.79

$$2\sqrt{x+\sqrt{a^2+x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x+(a**2+x**2)**(1/2))**(1/2)/(a**2+x**2)**(1/2),x)

[Out] 2*sqrt(x + sqrt(a**2 + x**2))
```

Mathematica [A] time = 0.0264594, size = 19, normalized size = 1.

$$2\sqrt{\sqrt{a^2+x^2}+x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a^2 + x^2],x]

[Out] 2*Sqrt[x + Sqrt[a^2 + x^2]]
```

Maple [F] time = 0.039, size = 0, normalized size = 0.

$$\int 1\sqrt{x+\sqrt{a^2+x^2}}\frac{1}{\sqrt{a^2+x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x+(a^2+x^2)^(1/2))^(1/2)/(a^2+x^2)^(1/2),x)

[Out] int((x+(a^2+x^2)^(1/2))^(1/2)/(a^2+x^2)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x + \sqrt{a^2 + x^2}}}{\sqrt{a^2 + x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x + sqrt(a^2 + x^2))/sqrt(a^2 + x^2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x + sqrt(a^2 + x^2))/sqrt(a^2 + x^2), x)
```

Fricas [A] time = 0.218391, size = 20, normalized size = 1.05

$$2\sqrt{x + \sqrt{a^2 + x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x + sqrt(a^2 + x^2))/sqrt(a^2 + x^2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x + sqrt(a^2 + x^2))
```

Sympy [A] time = 0.282316, size = 15, normalized size = 0.79

$$2\sqrt{x + \sqrt{a^2 + x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(a**2+x**2)**(1/2))**(1/2)/(a**2+x**2)**(1/2),x)
```

```
[Out] 2*sqrt(x + sqrt(a**2 + x**2))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x + \sqrt{a^2 + x^2}}}{\sqrt{a^2 + x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x + sqrt(a^2 + x^2))/sqrt(a^2 + x^2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x + sqrt(a^2 + x^2))/sqrt(a^2 + x^2), x)
```

3.26 $\int \frac{\sqrt{bx+\sqrt{a+b^2x^2}}}{\sqrt{a+b^2x^2}}\,dx$

Optimal. Leaf size=26

$$\frac{2\sqrt{\sqrt{a+b^2x^2}+bx}}{b}$$

```
[Out] (2*Sqrt[b*x + Sqrt[a + b^2*x^2]])/b
```

Rubi [A] time = 0.160506, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$

$$\frac{2\sqrt{\sqrt{a+b^2x^2}+bx}}{b}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[b*x + Sqrt[a + b^2*x^2]]/Sqrt[a + b^2*x^2],x]
```

```
[Out] (2*Sqrt[b*x + Sqrt[a + b^2*x^2]])/b
```

Rubi in Sympy [A] time = 4.68085, size = 20, normalized size = 0.77

$$\frac{2\sqrt{bx+\sqrt{a+b^2x^2}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x+(b**2*x**2+a)**(1/2))**(1/2)/(b**2*x**2+a)**(1/2),x)
```

```
[Out] 2*sqrt(b*x + sqrt(a + b**2*x**2))/b
```

Mathematica [A] time = 0.0409988, size = 26, normalized size = 1.

$$\frac{2\sqrt{\sqrt{a+b^2x^2}+bx}}{b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[b*x + Sqrt[a + b^2*x^2]]/Sqrt[a + b^2*x^2],x]
```

```
[Out] (2*Sqrt[b*x + Sqrt[a + b^2*x^2]])/b
```

Maple [F] time = 0.05, size = 0, normalized size = 0.

$$\int 1\sqrt{bx+\sqrt{b^2x^2+a}}\frac{1}{\sqrt{b^2x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x+(b^2*x^2+a)^(1/2))^(1/2)/(b^2*x^2+a)^(1/2),x)
```

```
[Out] int((b*x+(b^2*x^2+a)^(1/2))^(1/2)/(b^2*x^2+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx + \sqrt{b^2x^2 + a}}}{\sqrt{b^2x^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b*x + sqrt(b^2*x^2 + a))/sqrt(b^2*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x + sqrt(b^2*x^2 + a))/sqrt(b^2*x^2 + a), x)
```

Fricas [A] time = 0.222811, size = 30, normalized size = 1.15

$$\frac{2\sqrt{bx + \sqrt{b^2x^2 + a}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b*x + sqrt(b^2*x^2 + a))/sqrt(b^2*x^2 + a),x, algorithm="fricas")
```

```
[Out] 2*sqrt(b*x + sqrt(b^2*x^2 + a))/b
```

Sympy [A] time = 1.34415, size = 27, normalized size = 1.04

$$\begin{cases} \frac{2\sqrt{bx+\sqrt{a+b^2x^2}}}{b} & \text{for}\ b \neq 0 \\ \frac{x}{\sqrt[4]{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+(b**2*x**2+a)**(1/2))**(1/2)/(b**2*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((2*sqrt(b*x + sqrt(a + b**2*x**2))/b, Ne(b, 0)), (x/a**
(1/4), True))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx + \sqrt{b^2x^2 + a}}}{\sqrt{b^2x^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b*x + sqrt(b^2*x^2 + a))/sqrt(b^2*x^2 + a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x + sqrt(b^2*x^2 + a))/sqrt(b^2*x^2 + a), x)
```

3.27 $\int \frac{1}{x\sqrt{a^2+x^2}\sqrt{x+\sqrt{a^2+x^2}}}\,dx$

Optimal. Leaf size=63

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)}{a^{3/2}}$$

```
[Out] (-2*ArcTan[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a]])/a^(3/2) - (2*ArcTa
nh[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a]])/a^(3/2)
```

Rubi [A] time = 0.365434, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*Sqrt[a^2 + x^2]*Sqrt[x + Sqrt[a^2 + x^2]]),x]
```

```
[Out] (-2*ArcTan[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a]])/a^(3/2) - (2*ArcTa
nh[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a]])/a^(3/2)
```

Rubi in Sympy [A] time = 9.55944, size = 58, normalized size = 0.92

$$-\frac{2\,\mathrm{atan}\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}-\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(a**2+x**2)**(1/2)/(x+(a**2+x**2)**(1/2))**(1/2),x)
```

```
[Out] -2*atan(sqrt(x + sqrt(a**2 + x**2))/sqrt(a))/a**(3/2) - 2*atanh(s
qrt(x + sqrt(a**2 + x**2))/sqrt(a))/a**(3/2)
```

Mathematica [A] time = 0.250593, size = 85, normalized size = 1.35

$$\frac{\log\left(\sqrt{a}-\sqrt{\sqrt{a^2+x^2}+x}\right)-\log\left(\sqrt{\sqrt{a^2+x^2}+x}+\sqrt{a}\right)-2\tan^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*Sqrt[a^2 + x^2]*Sqrt[x + Sqrt[a^2 + x^2]]),x]
```

```
[Out] (-2*ArcTan[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a]] + Log[Sqrt[a] - Sqr
t[x + Sqrt[a^2 + x^2]]] - Log[Sqrt[a] + Sqrt[x + Sqrt[a^2 + x^2]]
])/a^(3/2)
```

Maple [F] time = 0.028, size = 0, normalized size = 0.

$$\int \frac{1}{x}\frac{1}{\sqrt{a^2+x^2}}\frac{1}{\sqrt{x+\sqrt{a^2+x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(a^2+x^2)^(1/2)/(x+(a^2+x^2)^(1/2))^(1/2),x)
```

```
[Out] int(1/x/(a^2+x^2)^(1/2)/(x+(a^2+x^2)^(1/2))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a^2 + x^2}\sqrt{x + \sqrt{a^2 + x^2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(a^2 + x^2)*sqrt(x + sqrt(a^2 + x^2))*x),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a^2 + x^2)*sqrt(x + sqrt(a^2 + x^2))*x), x)
```

Fricas [A] time = 0.243175, size = 1, normalized size = 0.02

$$\left[\frac{2\arctan\left(\frac{\sqrt{a}}{\sqrt{x+\sqrt{a^2+x^2}}}\right)+\log\left(-\frac{(a+x)\sqrt{a}-2a\sqrt{x+\sqrt{a^2+x^2}}+\sqrt{a^2+x^2}\sqrt{a}}{a-x-\sqrt{a^2+x^2}}\right)}{a^{\frac{3}{2}}}, \frac{2\arctan\left(\frac{a}{\sqrt{-a}\sqrt{x+\sqrt{a^2+x^2}}}\right)+\log\left(-\frac{\sqrt{-a}(a-x)+2a\sqrt{x+\sqrt{a^2+x^2}}-}{a+x+\sqrt{a^2+x^2}}\right.}{\sqrt{-a}a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(a^2 + x^2)*sqrt(x + sqrt(a^2 + x^2))*x),x, algorithm="fricas")
```

```
[Out] [(2*arctan(sqrt(a)/sqrt(x + sqrt(a^2 + x^2))) + log(-((a + x)*sqr
t(a) - 2*a*sqrt(x + sqrt(a^2 + x^2)) + sqrt(a^2 + x^2)*sqrt(a))/(
a - x - sqrt(a^2 + x^2))))/a^(3/2), (2*arctan(a/(sqrt(-a)*sqrt(x
+ sqrt(a^2 + x^2)))) + log(-(sqrt(-a)*(a - x) + 2*a*sqrt(x + sqrt
(a^2 + x^2)) - sqrt(a^2 + x^2)*sqrt(-a))/(a + x + sqrt(a^2 + x^2)
)))/(sqrt(-a)*a)]
```

Sympy [A] time = 2.87045, size = 46, normalized size = 0.73

$$-\frac{2\,\Gamma^2\left(\frac{3}{4}\right)\Gamma\left(\frac{5}{4}\right){}_3F_2\left(\begin{matrix}\frac{3}{4},\frac{3}{4},\frac{5}{4}\\ \frac{3}{2},\frac{7}{4}\end{matrix}\middle|\frac{a^2e^{i\pi}}{x^2}\right)}{\pi x^{\frac{3}{2}}\,\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(a**2+x**2)**(1/2)/(x+(a**2+x**2)**(1/2))**(1/2),x)
```

```
[Out] -gamma(3/4)**2*gamma(5/4)*hyper((3/4, 3/4, 5/4), (3/2, 7/4), a**2
*exp_polar(I*pi)/x**2)/(pi*x**(3/2)*gamma(7/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a^2 + x^2}\sqrt{x + \sqrt{a^2 + x^2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(a^2 + x^2)*sqrt(x + sqrt(a^2 + x^2))*x),x, algorithm="giac")

[Out] integrate(1/(sqrt(a^2 + x^2)*sqrt(x + sqrt(a^2 + x^2))*x), x)
```

3.28 $\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x}\,dx$

Optimal. Leaf size=82

$$2\sqrt{\sqrt{a^2+x^2}+x}-2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)$$

```
[Out] 2*Sqrt[x + Sqrt[a^2 + x^2]] - 2*Sqrt[a]*ArcTan[Sqrt[x + Sqrt[a^2
+ x^2]]/Sqrt[a]] - 2*Sqrt[a]*ArcTanh[Sqrt[x + Sqrt[a^2 + x^2]]/Sq
rt[a]]
```

Rubi [A] time = 0.140034, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$2\sqrt{\sqrt{a^2+x^2}+x}-2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x + Sqrt[a^2 + x^2]]/x,x]
```

```
[Out] 2*Sqrt[x + Sqrt[a^2 + x^2]] - 2*Sqrt[a]*ArcTan[Sqrt[x + Sqrt[a^2
+ x^2]]/Sqrt[a]] - 2*Sqrt[a]*ArcTanh[Sqrt[x + Sqrt[a^2 + x^2]]/Sq
rt[a]]
```

Rubi in Sympy [A] time = 8.60686, size = 73, normalized size = 0.89

$$-2\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)-2\sqrt{a}\,\mathrm{atanh}\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)+2\sqrt{x+\sqrt{a^2+x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x+(a**2+x**2)**(1/2))**(1/2)/x,x)
```

```
[Out] -2*sqrt(a)*atan(sqrt(x + sqrt(a**2 + x**2))/sqrt(a)) - 2*sqrt(a)*
atanh(sqrt(x + sqrt(a**2 + x**2))/sqrt(a)) + 2*sqrt(x + sqrt(a**2
 + x**2))
```

Mathematica [A] time = 0.081156, size = 161, normalized size = 1.96

$$-\frac{\sqrt{a^2+x^2}\left(\sqrt{a^2+x^2}+x\right)\left(-2\sqrt{\sqrt{a^2+x^2}+x}-\sqrt{a}\log\left(\sqrt{a}-\sqrt{\sqrt{a^2+x^2}+x}\right)+\sqrt{a}\log\left(\sqrt{\sqrt{a^2+x^2}+x}+\sqrt{a}\right)+2\sqrt{a}\tan^{-}\right.}{x\left(\sqrt{a^2+x^2}+x\right)+a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[x + Sqrt[a^2 + x^2]]/x,x]
```

```
[Out] -((Sqrt[a^2 + x^2]*(x + Sqrt[a^2 + x^2])*(-2*Sqrt[x + Sqrt[a^2 +
x^2]] + 2*Sqrt[a]*ArcTan[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a]] - Sqr
t[a]*Log[Sqrt[a] - Sqrt[x + Sqrt[a^2 + x^2]]] + Sqrt[a]*Log[Sqrt[
a] + Sqrt[x + Sqrt[a^2 + x^2]]]))/(a^2 + x*(x + Sqrt[a^2 + x^2]))
)
```

Maple [C] time = 0.008, size = 25, normalized size = 0.3

$$2\sqrt{2}\sqrt{x}{}_3\mathrm{F}_2(-1/4,-1/4,1/4;\ 1/2,3/4;\ -\frac{a^2}{x^2})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x+(a^2+x^2)^(1/2))^(1/2)/x,x)
```

```
[Out] 2*2^(1/2)*x^(1/2)*hypergeom([-1/4,-1/4,1/4],[1/2,3/4],-1/x^2*a^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x + sqrt(a^2 + x^2))/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x + sqrt(a^2 + x^2))/x, x)
```

Fricas [A] time = 0.240564, size = 1, normalized size = 0.01

$$\left[-2\sqrt{a}\arctan\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)+\sqrt{a}\log\left(-\frac{a-2\sqrt{a}\sqrt{x+\sqrt{a^2+x^2}}+x+\sqrt{a^2+x^2}}{a-x-\sqrt{a^2+x^2}}\right)+2\sqrt{x+\sqrt{a^2+x^2}},\ -2\sqrt{-a}\arctan\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{-a}}\right)+\sqrt{-a}\log\left(-\frac{a+2\sqrt{-a}\sqrt{x+\sqrt{a^2+x^2}}-x-\sqrt{a^2+x^2}}{a+x+\sqrt{a^2+x^2}}\right)+2\sqrt{x+\sqrt{a^2+x^2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x + sqrt(a^2 + x^2))/x,x, algorithm="fricas")
```

```
[Out] [-2*sqrt(a)*arctan(sqrt(x + sqrt(a^2 + x^2))/sqrt(a)) + sqrt(a)*l
og(-(a - 2*sqrt(a)*sqrt(x + sqrt(a^2 + x^2)) + x + sqrt(a^2 + x^2
))/(a - x - sqrt(a^2 + x^2))) + 2*sqrt(x + sqrt(a^2 + x^2)), -2*s
qrt(-a)*arctan(sqrt(x + sqrt(a^2 + x^2))/sqrt(-a)) + sqrt(-a)*log
(-(a + 2*sqrt(-a)*sqrt(x + sqrt(a^2 + x^2)) - x - sqrt(a^2 + x^2)
)/(a + x + sqrt(a^2 + x^2))) + 2*sqrt(x + sqrt(a^2 + x^2))]
```

Sympy [A] time = 2.48222, size = 51, normalized size = 0.62

$$\frac{\sqrt{x^2}\,\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right){}_3F_2\left(\begin{matrix}-\frac{1}{4},-\frac{1}{4},\frac{1}{4}\\ \frac{1}{2},\frac{3}{4}\end{matrix}\middle|\frac{a^2e^{i\pi}}{x^2}\right)}{8\pi\,\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x+(a**2+x**2)**(1/2))**(1/2)/x,x)
```

```
[Out] sqrt(x)*gamma(-1/4)**2*gamma(1/4)*hyper((-1/4, -1/4, 1/4), (1/2,
3/4), a**2*exp_polar(I*pi)/x**2)/(8*pi*gamma(3/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x + sqrt(a^2 + x^2))/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x + sqrt(a^2 + x^2))/x, x)
```

3.29 $\int x^3 \log^3(2+x)\log(3+x)\,dx$

Optimal. Leaf size=679

$$
\begin{aligned}
&-\frac{5609}{96}\text{PolyLog}(2,-x-2)-\frac{563}{8}\text{PolyLog}(3,-x-2)-\frac{195}{2}\text{PolyLog}(4,-x-2)\\
&-\frac{195}{4}\log^2(x+2)\text{PolyLog}(2,-x-2)+\frac{563}{8}\log(x+2)\text{PolyLog}(2,-x-2)\\
&+\frac{195}{2}\log(x+2)\text{PolyLog}(3,-x-2)+\frac{3x^4}{256}+\frac{1}{4}x^4\log^3(x+2)\log(x+3)\\
&+\frac{3}{64}x^4\log^2(x+2)-\frac{3}{16}x^4\log^2(x+2)\log(x+3)-\frac{3}{128}x^4\log(x+2)\\
&+\frac{3}{32}x^4\log(x+2)\log(x+3)-\frac{3}{128}x^4\log(x+3)-\frac{763x^3}{3456}-\frac{17}{48}x^3\log^2(x+2)\\
&+\frac{1}{2}x^3\log^2(x+2)\log(x+3)+\frac{83}{288}x^3\log(x+2)-\frac{7}{12}x^3\log(x+2)\log(x+3)\\
&+\frac{37}{144}x^3\log(x+3)+\frac{8029x^2}{2304}-\frac{3}{2}x^2\log^2(x+2)\log(x+3)-\frac{187}{64}x^2\log(x+2)\\
&+\frac{13}{4}x^2\log(x+2)\log(x+3)-\frac{115}{48}x^2\log(x+3)-\frac{302177x}{1152}+\frac{3}{256}(x+2)^4\\
&-\frac{71}{216}(x+2)^3+\frac{377}{64}(x+2)^2-\frac{1}{16}(x+2)^4\log^3(x+2)+\frac{3}{4}(x+2)^3\log^3(x+2)\\
&-\frac{33}{8}(x+2)^2\log^3(x+2)+\frac{65}{4}(x+2)\log^3(x+2)-\frac{81}{4}\log^3(x+2)\log(x+3)\\
&+6x\log^2(x+2)\log(x+3)+\frac{3}{64}(x+2)^4\log^2(x+2)-\frac{3}{4}(x+2)^3\log^2(x+2)\\
&+\frac{273}{32}(x+2)^2\log^2(x+2)-\frac{1251}{16}(x+2)\log^2(x+2)+\frac{43}{12}\log^2(x+2)\\
&+\frac{963}{16}\log^2(x+2)\log(x+3)-25x\log(x+2)\log(x+3)-\frac{3}{128}(x+2)^4\log(x+2)\\
&+\frac{1}{2}(x+2)^3\log(x+2)-\frac{273}{32}(x+2)^2\log(x+2)+\frac{6365}{32}(x+2)\log(x+2)\\
&+\frac{1}{128}\left(-3(x+2)^4+32(x+2)^3-144(x+2)^2+384(x+2)-192\log(x+2)\right)\log(x+2)\\
&+\frac{17}{72}\left((x+2)^3-9(x+2)^2+36(x+2)-24\log(x+2)\right)\log(x+2)+\frac{2069}{144}\log(x+2)\\
&+\frac{415}{12}(x+3)\log(x+3)-\frac{4083}{32}\log(x+2)\log(x+3)+\frac{3891}{128}\log(x+3)
\end{aligned}
$$

```
[Out] (-302177*x)/1152 + (8029*x^2)/2304 - (763*x^3)/3456 + (3*x^4)/256
 + (377*(2 + x)^2)/64 - (71*(2 + x)^3)/216 + (3*(2 + x)^4)/256 +
(2069*Log[2 + x])/144 - (187*x^2*Log[2 + x])/64 + (83*x^3*Log[2 +
 x])/288 - (3*x^4*Log[2 + x])/128 + (6365*(2 + x)*Log[2 + x])/32
- (273*(2 + x)^2*Log[2 + x])/32 + ((2 + x)^3*Log[2 + x])/2 - (3*(
2 + x)^4*Log[2 + x])/128 + ((384*(2 + x) - 144*(2 + x)^2 + 32*(2
+ x)^3 - 3*(2 + x)^4 - 192*Log[2 + x])*Log[2 + x])/128 + (17*(36*
(2 + x) - 9*(2 + x)^2 + (2 + x)^3 - 24*Log[2 + x])*Log[2 + x])/72
 + (43*Log[2 + x]^2)/12 - (17*x^3*Log[2 + x]^2)/48 + (3*x^4*Log[2
 + x]^2)/64 - (1251*(2 + x)*Log[2 + x]^2)/16 + (273*(2 + x)^2*Log
[2 + x]^2)/32 - (3*(2 + x)^3*Log[2 + x]^2)/4 + (3*(2 + x)^4*Log[2
 + x]^2)/64 + (65*(2 + x)*Log[2 + x]^3)/4 - (33*(2 + x)^2*Log[2 +
 x]^3)/8 + (3*(2 + x)^3*Log[2 + x]^3)/4 - ((2 + x)^4*Log[2 + x]^3
)/16 + (3891*Log[3 + x])/128 - (115*x^2*Log[3 + x])/48 + (37*x^3*
Log[3 + x])/144 - (3*x^4*Log[3 + x])/128 + (415*(3 + x)*Log[3 + x
])/12 - (4083*Log[2 + x]*Log[3 + x])/32 - 25*x*Log[2 + x]*Log[3 +
 x] + (13*x^2*Log[2 + x]*Log[3 + x])/4 - (7*x^3*Log[2 + x]*Log[3
+ x])/12 + (3*x^4*Log[2 + x]*Log[3 + x])/32 + (963*Log[2 + x]^2*L
og[3 + x])/16 + 6*x*Log[2 + x]^2*Log[3 + x] - (3*x^2*Log[2 + x]^2
*Log[3 + x])/2 + (x^3*Log[2 + x]^2*Log[3 + x])/2 - (3*x^4*Log[2 +
 x]^2*Log[3 + x])/16 - (81*Log[2 + x]^3*Log[3 + x])/4 + (x^4*Log[
2 + x]^3*Log[3 + x])/4 - (5609*PolyLog[2, -2 - x])/96 + (563*Log[
2 + x]*PolyLog[2, -2 - x])/8 - (195*Log[2 + x]^2*PolyLog[2, -2 -
x])/4 - (563*PolyLog[3, -2 - x])/8 + (195*Log[2 + x]*PolyLog[3, -
2 - x])/2 - (195*PolyLog[4, -2 - x])/2
```

Rubi [A] time = 7.49551, antiderivative size = 679, normalized size of antiderivative = 1., number of

steps used = 359, number of rules used = 30, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 2.143$

$$
\begin{aligned}
&-\frac{5609}{96}\text{PolyLog}(2,-x-2)-\frac{563}{8}\text{PolyLog}(3,-x-2)-\frac{195}{2}\text{PolyLog}(4,-x-2)\\
&-\frac{195}{4}\log^2(x+2)\text{PolyLog}(2,-x-2)+\frac{563}{8}\log(x+2)\text{PolyLog}(2,-x-2)\\
&+\frac{195}{2}\log(x+2)\text{PolyLog}(3,-x-2)+\frac{3x^4}{256}+\frac{1}{4}x^4\log^3(x+2)\log(x+3)\\
&+\frac{3}{64}x^4\log^2(x+2)-\frac{3}{16}x^4\log^2(x+2)\log(x+3)-\frac{3}{128}x^4\log(x+2)\\
&+\frac{3}{32}x^4\log(x+2)\log(x+3)-\frac{3}{128}x^4\log(x+3)-\frac{763x^3}{3456}-\frac{17}{48}x^3\log^2(x+2)\\
&+\frac{1}{2}x^3\log^2(x+2)\log(x+3)+\frac{83}{288}x^3\log(x+2)-\frac{7}{12}x^3\log(x+2)\log(x+3)\\
&+\frac{37}{144}x^3\log(x+3)+\frac{8029x^2}{2304}-\frac{3}{2}x^2\log^2(x+2)\log(x+3)-\frac{187}{64}x^2\log(x+2)\\
&+\frac{13}{4}x^2\log(x+2)\log(x+3)-\frac{115}{48}x^2\log(x+3)-\frac{302177x}{1152}+\frac{3}{256}(x+2)^4\\
&-\frac{71}{216}(x+2)^3+\frac{377}{64}(x+2)^2-\frac{1}{16}(x+2)^4\log^3(x+2)+\frac{3}{4}(x+2)^3\log^3(x+2)\\
&-\frac{33}{8}(x+2)^2\log^3(x+2)+\frac{65}{4}(x+2)\log^3(x+2)-\frac{81}{4}\log^3(x+2)\log(x+3)\\
&+6x\log^2(x+2)\log(x+3)+\frac{3}{64}(x+2)^4\log^2(x+2)-\frac{3}{4}(x+2)^3\log^2(x+2)\\
&+\frac{273}{32}(x+2)^2\log^2(x+2)-\frac{1251}{16}(x+2)\log^2(x+2)+\frac{43}{12}\log^2(x+2)\\
&+\frac{963}{16}\log^2(x+2)\log(x+3)-25x\log(x+2)\log(x+3)-\frac{3}{128}(x+2)^4\log(x+2)\\
&+\frac{1}{2}(x+2)^3\log(x+2)-\frac{273}{32}(x+2)^2\log(x+2)+\frac{6365}{32}(x+2)\log(x+2)\\
&+\frac{1}{128}\left(-3(x+2)^4+32(x+2)^3-144(x+2)^2+384(x+2)-192\log(x+2)\right)\log(x+2)\\
&+\frac{17}{72}\left((x+2)^3-9(x+2)^2+36(x+2)-24\log(x+2)\right)\log(x+2)+\frac{2069}{144}\log(x+2)\\
&+\frac{415}{12}(x+3)\log(x+3)-\frac{4083}{32}\log(x+2)\log(x+3)+\frac{3891}{128}\log(x+3)
\end{aligned}
$$

Antiderivative was successfully verified.

```
[In]   Int[x^3*Log[2 + x]^3*Log[3 + x],x]
```

```
[Out] (-302177*x)/1152 + (8029*x^2)/2304 - (763*x^3)/3456 + (3*x^4)/256
 + (377*(2 + x)^2)/64 - (71*(2 + x)^3)/216 + (3*(2 + x)^4)/256 +
(2069*Log[2 + x])/144 - (187*x^2*Log[2 + x])/64 + (83*x^3*Log[2 +
 x])/288 - (3*x^4*Log[2 + x])/128 + (6365*(2 + x)*Log[2 + x])/32
- (273*(2 + x)^2*Log[2 + x])/32 + ((2 + x)^3*Log[2 + x])/2 - (3*(
2 + x)^4*Log[2 + x])/128 + ((384*(2 + x) - 144*(2 + x)^2 + 32*(2
+ x)^3 - 3*(2 + x)^4 - 192*Log[2 + x])*Log[2 + x])/128 + (17*(36*
(2 + x) - 9*(2 + x)^2 + (2 + x)^3 - 24*Log[2 + x])*Log[2 + x])/72
 + (43*Log[2 + x]^2)/12 - (17*x^3*Log[2 + x]^2)/48 + (3*x^4*Log[2
 + x]^2)/64 - (1251*(2 + x)*Log[2 + x]^2)/16 + (273*(2 + x)^2*Log
[2 + x]^2)/32 - (3*(2 + x)^3*Log[2 + x]^2)/4 + (3*(2 + x)^4*Log[2
 + x]^2)/64 + (65*(2 + x)*Log[2 + x]^3)/4 - (33*(2 + x)^2*Log[2 +
 x]^3)/8 + (3*(2 + x)^3*Log[2 + x]^3)/4 - ((2 + x)^4*Log[2 + x]^3
)/16 + (3891*Log[3 + x])/128 - (115*x^2*Log[3 + x])/48 + (37*x^3*
Log[3 + x])/144 - (3*x^4*Log[3 + x])/128 + (415*(3 + x)*Log[3 + x
])/12 - (4083*Log[2 + x]*Log[3 + x])/32 - 25*x*Log[2 + x]*Log[3 +
 x] + (13*x^2*Log[2 + x]*Log[3 + x])/4 - (7*x^3*Log[2 + x]*Log[3
+ x])/12 + (3*x^4*Log[2 + x]*Log[3 + x])/32 + (963*Log[2 + x]^2*L
og[3 + x])/16 + 6*x*Log[2 + x]^2*Log[3 + x] - (3*x^2*Log[2 + x]^2
*Log[3 + x])/2 + (x^3*Log[2 + x]^2*Log[3 + x])/2 - (3*x^4*Log[2 +
 x]^2*Log[3 + x])/16 - (81*Log[2 + x]^3*Log[3 + x])/4 + (x^4*Log[
2 + x]^3*Log[3 + x])/4 - (5609*PolyLog[2, -2 - x])/96 + (563*Log[
2 + x]*PolyLog[2, -2 - x])/8 - (195*Log[2 + x]^2*PolyLog[2, -2 -
x])/4 - (563*PolyLog[3, -2 - x])/8 + (195*Log[2 + x]*PolyLog[3, -
2 - x])/2 - (195*PolyLog[4, -2 - x])/2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \log(x+2)^3 \log(x+3)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*ln(2+x)**3*ln(3+x),x)

[Out] Integral(x**3*log(x + 2)**3*log(x + 3), x)
```

Mathematica [A] time = 0.266705, size = 412, normalized size = 0.61

$$\frac{-224640\text{PolyLog}(4,-x-2)-24\left(4680\log^2(x+2)-6756\log(x+2)+5609\right)\text{PolyLog}(2,-x-2)+288(780\log(x+2)-56}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*Log[2 + x]^3*Log[3 + x],x]

[Out] (-759744 - 558290*x + 17705*x^2 - 1050*x^3 + 54*x^4 + 910528*Log[
2 + x] + 400008*x*Log[2 + x] - 22836*x^2*Log[2 + x] + 2072*x^3*Lo
g[2 + x] - 162*x^4*Log[2 + x] - 302016*Log[2 + x]^2 - 118800*x*Lo
g[2 + x]^2 + 11880*x^2*Log[2 + x]^2 - 1680*x^3*Log[2 + x]^2 + 216
*x^4*Log[2 + x]^2 + 48384*Log[2 + x]^3 + 15552*x*Log[2 + x]^3 - 2
592*x^2*Log[2 + x]^3 + 576*x^3*Log[2 + x]^3 - 144*x^4*Log[2 + x]^
3 + 309078*Log[3 + x] + 79680*x*Log[3 + x] - 5520*x^2*Log[3 + x]
+ 592*x^3*Log[3 + x] - 54*x^4*Log[3 + x] - 293976*Log[2 + x]*Log[
3 + x] - 57600*x*Log[2 + x]*Log[3 + x] + 7488*x^2*Log[2 + x]*Log[
3 + x] - 1344*x^3*Log[2 + x]*Log[3 + x] + 216*x^4*Log[2 + x]*Log[
3 + x] + 138672*Log[2 + x]^2*Log[3 + x] + 13824*x*Log[2 + x]^2*Lo
g[3 + x] - 3456*x^2*Log[2 + x]^2*Log[3 + x] + 1152*x^3*Log[2 + x]
^2*Log[3 + x] - 432*x^4*Log[2 + x]^2*Log[3 + x] - 46656*Log[2 + x
]^3*Log[3 + x] + 576*x^4*Log[2 + x]^3*Log[3 + x] - 24*(5609 - 675
6*Log[2 + x] + 4680*Log[2 + x]^2)*PolyLog[2, -2 - x] + 288*(-563
+ 780*Log[2 + x])*PolyLog[3, -2 - x] - 224640*PolyLog[4, -2 - x])
/2304
```

Maple [F] time = 0.063, size = 0, normalized size = 0.

$$\int x^3 (\ln(2+x))^3 \ln(3+x)\; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*ln(2+x)^3*ln(3+x),x)

[Out] int(x^3*ln(2+x)^3*ln(3+x),x)
```

Maxima [A] time = 1.39935, size = 699, normalized size = 1.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*log(x + 3)*log(x + 2)^3,x, algorithm="maxima")

[Out] 3/128*x^4 + 1/16*(4*x^4*log(x + 3) - x^4 + 4*x^3 - 18*x^2 + 108*x
 - 324*log(x + 3))*log(x + 2)^3 - 65/4*log(x + 3)*log(x + 2)^3 +
```

```
195/4*log(x + 3)*log(x + 2)^2*log(-x - 2) - 175/384*x^3 + 1/96*(9
*x^4 - 70*x^3 + 495*x^2 - 6*(3*x^4 - 8*x^3 + 24*x^2 - 96*x)*log(x
 + 3) + 4680*log(x + 3)*log(-x - 2) - 4950*x + 4680*dilog(x + 3)
+ 5778*log(x + 3) + 6048*log(x + 2))*log(x + 2)^2 + 195/4*dilog(x
 + 3)*log(x + 2)^2 - 195/4*dilog(-x - 2)*log(x + 2)^2 + 563/16*lo
g(x + 3)*log(x + 2)^2 + 21*log(x + 2)^3 + 17705/2304*x^2 + 1/8*(7
80*log(x + 2)^2 - 563*log(x + 2))*dilog(-x - 2) - 1/1152*(27*x^4
- 296*x^3 - 18720*log(x + 2)^3 + 2760*x^2 + 40536*log(x + 2)^2 -
39840*x - 67308*log(x + 2))*log(x + 3) - 1/1152*(81*x^4 - 1036*x^
3 + 56160*log(x + 3)*log(x + 2)^2 + 112320*log(x + 3)*log(x + 2)*
log(-x - 2) + 11418*x^2 - 12*(9*x^4 - 56*x^3 + 312*x^2 + 4680*log
(x + 2)^2 - 2400*x - 6756*log(x + 2))*log(x + 3) + 112320*dilog(x
 + 3)*log(x + 2) + 112320*dilog(-x - 2)*log(x + 2) - 81072*log(x
+ 3)*log(x + 2) + 72576*log(x + 2)^2 - 200004*x - 81072*dilog(-x
- 2) + 146988*log(x + 3) + 302016*log(x + 2) - 112320*polylog(3,
-x - 2))*log(x + 2) + 563/8*dilog(-x - 2)*log(x + 2) - 5609/96*lo
g(x + 3)*log(x + 2) + 1573/12*log(x + 2)^2 - 279145/1152*x - 5609
/96*dilog(-x - 2) + 17171/128*log(x + 3) + 14227/36*log(x + 2) -
195/2*polylog(4, -x - 2) - 563/8*polylog(3, -x - 2)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^3 \log\left(x+3\right) \log\left(x+2\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*log(x + 3)*log(x + 2)^3,x, algorithm="fricas")
```

```
[Out] integral(x^3*log(x + 3)*log(x + 2)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*ln(2+x)**3*ln(3+x),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \log\left(x+3\right) \log\left(x+2\right)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*log(x + 3)*log(x + 2)^3,x, algorithm="giac")
```

```
[Out] integrate(x^3*log(x + 3)*log(x + 2)^3, x)
```

3.30 $\int \frac{\left(x+\sqrt{b+x^2}\right)^a}{\sqrt{b+x^2}}\,dx$

Optimal. Leaf size=17

$$\frac{\left(\sqrt{b+x^2}+x\right)^a}{a}$$

```
[Out] (x + Sqrt[b + x^2])^a/a
```

Rubi [A] time = 0.0877249, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$

$$\frac{\left(\sqrt{b+x^2}+x\right)^a}{a}$$

Antiderivative was successfully verified.

```
[In]  Int[(x + Sqrt[b + x^2])^a/Sqrt[b + x^2],x]

[Out] (x + Sqrt[b + x^2])^a/a
```

Rubi in Sympy [A] time = 3.91902, size = 12, normalized size = 0.71

$$\frac{\left(x+\sqrt{b+x^2}\right)^a}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x+(x**2+b)**(1/2))**a/(x**2+b)**(1/2),x)

[Out] (x + sqrt(b + x**2))**a/a
```

Mathematica [A] time = 0.040226, size = 17, normalized size = 1.

$$\frac{\left(\sqrt{b+x^2}+x\right)^a}{a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x + Sqrt[b + x^2])^a/Sqrt[b + x^2],x]

[Out] (x + Sqrt[b + x^2])^a/a
```

Maple [F] time = 0.037, size = 0, normalized size = 0.

$$\int 1\left(x+\sqrt{x^2+b}\right)^a\frac{1}{\sqrt{x^2+b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x+(x^2+b)^(1/2))^a/(x^2+b)^(1/2),x)
```

```
[Out] int((x+(x^2+b)^(1/2))^a/(x^2+b)^(1/2),x)
```

Maxima [A] time = 1.40199, size = 20, normalized size = 1.18

$$\frac{\left(x + \sqrt{x^2 + b}\right)^a}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x + sqrt(x^2 + b))^a/sqrt(x^2 + b),x, algorithm="maxima")
```

```
[Out] (x + sqrt(x^2 + b))^a/a
```

Fricas [A] time = 0.241325, size = 20, normalized size = 1.18

$$\frac{\left(x + \sqrt{x^2 + b}\right)^a}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x + sqrt(x^2 + b))^a/sqrt(x^2 + b),x, algorithm="fricas")
```

```
[Out] (x + sqrt(x^2 + b))^a/a
```

Sympy [A] time = 7.30149, size = 311, normalized size = 18.29

$$\begin{cases} -\frac{\sqrt{b}b^{\frac{a}{2}}\sinh\left(-a\operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)+\operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)\right)}{ax\sqrt{\frac{b}{x^2}+1}} + \frac{b^{\frac{a}{2}}x\cosh\left(-a\operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)+\operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)\right)}{a\sqrt{b}} - \frac{b^{\frac{a}{2}}x\sinh\left(-a\operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)+\operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)\right)}{a\sqrt{b}\sqrt{\frac{b}{x^2}+1}} - \frac{2b^{\frac{a}{2}}\cosh\left(a\operatorname{asinh}\left(\right.\right.}{a^2\left(-\frac{a}{2}\right.} \\ -\frac{b^{\frac{a}{2}}\sinh\left(-a\operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)+\operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)\right)}{a\sqrt{1+\frac{x^2}{b}}} - \frac{b^{\frac{a}{2}}x^2\sinh\left(-a\operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)+\operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)\right)}{ab\sqrt{1+\frac{x^2}{b}}} + \frac{b^{\frac{a}{2}}x\cosh\left(-a\operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)+\operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)\right)}{a\sqrt{b}} - \frac{2b^{\frac{a}{2}}\cosh\left(a\operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right.\right.}{a^2\left(-\frac{a}{2}\right)} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x+(x**2+b)**(1/2))**a/(x**2+b)**(1/2),x)
```

```
[Out] Piecewise((-sqrt(b)*b**(a/2)*sinh(-a*asinh(x/sqrt(b)) + asinh(x/s
qrt(b)))/(a*x*sqrt(b/x**2 + 1)) + b**(a/2)*x*cosh(-a*asinh(x/sqrt
(b)) + asinh(x/sqrt(b)))/(a*sqrt(b)) - b**(a/2)*x*sinh(-a*asinh(x
/sqrt(b)) + asinh(x/sqrt(b)))/(a*sqrt(b)*sqrt(b/x**2 + 1)) - 2*b*
*(a/2)*cosh(a*asinh(x/sqrt(b)))*gamma(-a/2 + 1)/(a**2*gamma(-a/2)
), Abs(x**2/b) > 1), (-b**(a/2)*sinh(-a*asinh(x/sqrt(b)) + asinh(
x/sqrt(b)))/(a*sqrt(1 + x**2/b)) - b**(a/2)*x**2*sinh(-a*asinh(x/
sqrt(b)) + asinh(x/sqrt(b)))/(a*b*sqrt(1 + x**2/b)) + b**(a/2)*x*
cosh(-a*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))/(a*sqrt(b)) - 2*b**(
a/2)*cosh(a*asinh(x/sqrt(b)))*gamma(-a/2 + 1)/(a**2*gamma(-a/2)),
 True))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x + \sqrt{x^2 + b}\right)^a}{\sqrt{x^2 + b}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sqrt(x^2 + b))^a/sqrt(x^2 + b),x, algorithm="giac")

[Out] integrate((x + sqrt(x^2 + b))^a/sqrt(x^2 + b), x)
```

3.31 $\int \left(x+\sqrt{b+x^2}\right)^a \, dx$

Optimal. Leaf size=52

$$\frac{\left(\sqrt{b+x^2}+x\right)^{a+1}}{2(a+1)}-\frac{b\left(\sqrt{b+x^2}+x\right)^{a-1}}{2(1-a)}$$

```
[Out] -(b*(x + Sqrt[b + x^2])^(-1 + a))/(2*(1 - a)) + (x + Sqrt[b + x^2
])^(1 + a)/(2*(1 + a))
```

Rubi [A] time = 0.0464369, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$\frac{\left(\sqrt{b+x^2}+x\right)^{a+1}}{2(a+1)}-\frac{b\left(\sqrt{b+x^2}+x\right)^{a-1}}{2(1-a)}$$

Antiderivative was successfully verified.

```
[In]  Int[(x + Sqrt[b + x^2])^a,x]

[Out] -(b*(x + Sqrt[b + x^2])^(-1 + a))/(2*(1 - a)) + (x + Sqrt[b + x^2
])^(1 + a)/(2*(1 + a))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x+\sqrt{b+x^2}\right)^a \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x+(x**2+b)**(1/2))**a,x)

[Out] Integral((x + sqrt(b + x**2))**a, x)
```

Mathematica [A] time = 0.0233956, size = 36, normalized size = 0.69

$$\frac{\left(\sqrt{b+x^2}+x\right)^a\left(a\sqrt{b+x^2}-x\right)}{a^2-1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x + Sqrt[b + x^2])^a,x]

[Out] ((x + Sqrt[b + x^2])^a*(-x + a*Sqrt[b + x^2]))/(-1 + a^2)
```

Maple [B] time = 0.033, size = 120, normalized size = 2.3

$$\frac{a}{4\sqrt{\pi}}b^{\frac{a}{2}+\frac{1}{2}}\left(8\,\frac{\sqrt{\pi}x^{1+a}b^{-a/2-1/2}}{(1+a)\,a\,(2\,a-2)}\left(\frac{ab}{x^2}+a-1\right)\left(\sqrt{\frac{b}{x^2}+1}+1\right)^{a-1}+4\,\frac{\sqrt{\pi}x^{1+a}b^{-a/2-1/2}}{(1+a)\,a}\sqrt{\frac{b}{x^2}+1}\left(\sqrt{\frac{b}{x^2}+1}+1\right)^{a-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x+(x^2+b)^(1/2))^a,x)
```

```
[Out] 1/4*b^(1/2*a+1/2)/Pi^(1/2)*a*(8*Pi^(1/2)/(1+a)/a*x^(1+a)*b^(-1/2*
a-1/2)*(1/x^2*a*b+a-1)/(2*a-2)*((1/x^2*b+1)^(1/2)+1)^(a-1)+4*Pi^(
1/2)/(1+a)/a*x^(1+a)*b^(-1/2*a-1/2)*(1/x^2*b+1)^(1/2)*((1/x^2*b+1
)^(1/2)+1)^(a-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x + \sqrt{x^2 + b}\right)^a dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sqrt(x^2 + b))^a,x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(x^2 + b))^a, x)
```

Fricas [A] time = 0.244311, size = 43, normalized size = 0.83

$$\frac{\left(\sqrt{x^2 + b}a - x\right)\left(x + \sqrt{x^2 + b}\right)^a}{a^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sqrt(x^2 + b))^a,x, algorithm="fricas")
```

```
[Out] (sqrt(x^2 + b)*a - x)*(x + sqrt(x^2 + b))^a/(a^2 - 1)
```

Sympy [A] time = 6.69675, size = 2147, normalized size = 41.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x+(x**2+b)**(1/2))**a,x)
```

```
[Out] Piecewise((-a**2*b**(9/2)*b**(a/2)*x*sqrt(b/x**2 + 1)*sinh(a*asin
h(x/sqrt(b)))*gamma(-a/2)/(2*a**2*b**(9/2)*gamma(-a/2 + 1) + 2*a*
*2*b**(7/2)*x**2*gamma(-a/2 + 1) - 2*b**(9/2)*gamma(-a/2 + 1) - 2
*b**(7/2)*x**2*gamma(-a/2 + 1)) - a**2*b**(7/2)*b**(a/2)*x**3*sqr
t(b/x**2 + 1)*sinh(a*asinh(x/sqrt(b)))*gamma(-a/2)/(2*a**2*b**(9/
2)*gamma(-a/2 + 1) + 2*a**2*b**(7/2)*x**2*gamma(-a/2 + 1) - 2*b**
(9/2)*gamma(-a/2 + 1) - 2*b**(7/2)*x**2*gamma(-a/2 + 1)) + a*b**(
9/2)*b**(a/2)*x*cosh(a*asinh(x/sqrt(b)))*gamma(-a/2)/(2*a**2*b**(
9/2)*gamma(-a/2 + 1) + 2*a**2*b**(7/2)*x**2*gamma(-a/2 + 1) - 2*b
**(9/2)*gamma(-a/2 + 1) - 2*b**(7/2)*x**2*gamma(-a/2 + 1)) + a*b*
*(7/2)*b**(a/2)*x**3*cosh(a*asinh(x/sqrt(b)))*gamma(-a/2)/(2*a**2
*b**(9/2)*gamma(-a/2 + 1) + 2*a**2*b**(7/2)*x**2*gamma(-a/2 + 1)
- 2*b**(9/2)*gamma(-a/2 + 1) - 2*b**(7/2)*x**2*gamma(-a/2 + 1)) +
 2*a*b**5*b**(a/2)*cosh(a*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))*ga
mma(-a/2 + 1)/(2*a**2*b**(9/2)*gamma(-a/2 + 1) + 2*a**2*b**(7/2)*
x**2*gamma(-a/2 + 1) - 2*b**(9/2)*gamma(-a/2 + 1) - 2*b**(7/2)*x*
*2*gamma(-a/2 + 1)) - 2*a*b**5*b**(a/2)*gamma(-a/2 + 1)/(2*a**2*b
**(9/2)*gamma(-a/2 + 1) + 2*a**2*b**(7/2)*x**2*gamma(-a/2 + 1) -
2*b**(9/2)*gamma(-a/2 + 1) - 2*b**(7/2)*x**2*gamma(-a/2 + 1)) - 2
*a*b**4*b**(a/2)*x**2*sqrt(b/x**2 + 1)*sinh(a*asinh(x/sqrt(b)) +
asinh(x/sqrt(b)))*gamma(-a/2 + 1)/(2*a**2*b**(9/2)*gamma(-a/2 + 1
) + 2*a**2*b**(7/2)*x**2*gamma(-a/2 + 1) - 2*b**(9/2)*gamma(-a/2
```

```
+ 1) - 2*b**(7/2)*x**2*gamma(-a/2 + 1)) + 4*a*b**4*b**(a/2)*x**2*
cosh(a*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))*gamma(-a/2 + 1)/(2*a*
*2*b**(9/2)*gamma(-a/2 + 1) + 2*a**2*b**(7/2)*x**2*gamma(-a/2 + 1
) - 2*b**(9/2)*gamma(-a/2 + 1) - 2*b**(7/2)*x**2*gamma(-a/2 + 1))
 - 2*a*b**4*b**(a/2)*x**2*gamma(-a/2 + 1)/(2*a**2*b**(9/2)*gamma(
-a/2 + 1) + 2*a**2*b**(7/2)*x**2*gamma(-a/2 + 1) - 2*b**(9/2)*gam
ma(-a/2 + 1) - 2*b**(7/2)*x**2*gamma(-a/2 + 1)) - 2*a*b**3*b**(a/
2)*x**4*sqrt(b/x**2 + 1)*sinh(a*asinh(x/sqrt(b)) + asinh(x/sqrt(b
)))*gamma(-a/2 + 1)/(2*a**2*b**(9/2)*gamma(-a/2 + 1) + 2*a**2*b**
(7/2)*x**2*gamma(-a/2 + 1) - 2*b**(9/2)*gamma(-a/2 + 1) - 2*b**(7
/2)*x**2*gamma(-a/2 + 1)) + 2*a*b**3*b**(a/2)*x**4*cosh(a*asinh(x
/sqrt(b)) + asinh(x/sqrt(b)))*gamma(-a/2 + 1)/(2*a**2*b**(9/2)*ga
mma(-a/2 + 1) + 2*a**2*b**(7/2)*x**2*gamma(-a/2 + 1) - 2*b**(9/2)
*gamma(-a/2 + 1) - 2*b**(7/2)*x**2*gamma(-a/2 + 1)) - 2*b**4*b**(
a/2)*x**2*sqrt(b/x**2 + 1)*sinh(a*asinh(x/sqrt(b)) + asinh(x/sqrt
(b)))*gamma(-a/2 + 1)/(2*a**2*b**(9/2)*gamma(-a/2 + 1) + 2*a**2*b
**(7/2)*x**2*gamma(-a/2 + 1) - 2*b**(9/2)*gamma(-a/2 + 1) - 2*b**
(7/2)*x**2*gamma(-a/2 + 1)) + 2*b**4*b**(a/2)*x**2*cosh(a*asinh(x
/sqrt(b)) + asinh(x/sqrt(b)))*gamma(-a/2 + 1)/(2*a**2*b**(9/2)*ga
mma(-a/2 + 1) + 2*a**2*b**(7/2)*x**2*gamma(-a/2 + 1) - 2*b**(9/2)
*gamma(-a/2 + 1) - 2*b**(7/2)*x**2*gamma(-a/2 + 1)) - 2*b**3*b**(
a/2)*x**4*sqrt(b/x**2 + 1)*sinh(a*asinh(x/sqrt(b)) + asinh(x/sqrt
(b)))*gamma(-a/2 + 1)/(2*a**2*b**(9/2)*gamma(-a/2 + 1) + 2*a**2*b
**(7/2)*x**2*gamma(-a/2 + 1) - 2*b**(9/2)*gamma(-a/2 + 1) - 2*b**
(7/2)*x**2*gamma(-a/2 + 1)) + 2*b**3*b**(a/2)*x**4*cosh(a*asinh(x
/sqrt(b)) + asinh(x/sqrt(b)))*gamma(-a/2 + 1)/(2*a**2*b**(9/2)*ga
mma(-a/2 + 1) + 2*a**2*b**(7/2)*x**2*gamma(-a/2 + 1) - 2*b**(9/2)
*gamma(-a/2 + 1) - 2*b**(7/2)*x**2*gamma(-a/2 + 1)), Abs(x**2/b)
> 1), (-a**2*b**3*b**(a/2)*sqrt(1 + x**2/b)*sinh(a*asinh(x/sqrt(b
)))*gamma(-a/2)/(2*a**2*b**(5/2)*gamma(-a/2 + 1) - 2*b**(5/2)*gam
ma(-a/2 + 1)) - 2*a*b**(5/2)*b**(a/2)*x*sqrt(1 + x**2/b)*sinh(a*a
sinh(x/sqrt(b)) + asinh(x/sqrt(b)))*gamma(-a/2 + 1)/(2*a**2*b**(5
/2)*gamma(-a/2 + 1) - 2*b**(5/2)*gamma(-a/2 + 1)) + a*b**(5/2)*b*
*(a/2)*x*cosh(a*asinh(x/sqrt(b)))*gamma(-a/2)/(2*a**2*b**(5/2)*ga
mma(-a/2 + 1) - 2*b**(5/2)*gamma(-a/2 + 1)) + 2*a*b**3*b**(a/2)*c
osh(a*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))*gamma(-a/2 + 1)/(2*a**
2*b**(5/2)*gamma(-a/2 + 1) - 2*b**(5/2)*gamma(-a/2 + 1)) + 2*a*b*
*2*b**(a/2)*x**2*cosh(a*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))*gamm
a(-a/2 + 1)/(2*a**2*b**(5/2)*gamma(-a/2 + 1) - 2*b**(5/2)*gamma(-
a/2 + 1)) - 2*b**(5/2)*b**(a/2)*x*sqrt(1 + x**2/b)*sinh(a*asinh(x
/sqrt(b)) + asinh(x/sqrt(b)))*gamma(-a/2 + 1)/(2*a**2*b**(5/2)*ga
mma(-a/2 + 1) - 2*b**(5/2)*gamma(-a/2 + 1)) + 2*b**2*b**(a/2)*x**
2*cosh(a*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))*gamma(-a/2 + 1)/(2*
a**2*b**(5/2)*gamma(-a/2 + 1) - 2*b**(5/2)*gamma(-a/2 + 1)), True
))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x+\sqrt{x^2+b}\right)^a dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sqrt(x^2 + b))^a,x, algorithm="giac")
```

```
[Out] integrate((x + sqrt(x^2 + b))^a, x)
```

3.32 $\int \left(6+3x^a+2x^{2a}\right)^{\frac{1}{a}} \left(x^a+x^{2a}+x^{3a}\right) \, dx$

Optimal. Leaf size=34

$$\frac{x^{a+1}\left(2x^{2a}+3x^a+6\right)^{\frac{1}{a}+1}}{6(a+1)}$$

```
[Out] (x^(1 + a)*(6 + 3*x^a + 2*x^(2*a))^(1 + a^(-1)))/(6*(1 + a))
```

Rubi [A] time = 0.0596816, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061

$$\frac{x^{a+1}\left(2x^{2a}+3x^a+6\right)^{\frac{1}{a}+1}}{6(a+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(6 + 3*x^a + 2*x^(2*a))^a^(-1)*(x^a + x^(2*a) + x^(3*a)),x]

[Out] (x^(1 + a)*(6 + 3*x^a + 2*x^(2*a))^(1 + a^(-1)))/(6*(1 + a))
```

Rubi in Sympy [A] time = 9.29847, size = 27, normalized size = 0.79

$$\frac{x^{a+1}\left(2x^{2a}+3x^a+6\right)^{1+\frac{1}{a}}}{6\left(a+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((6+3*x**a+2*x**(2*a))**(1/a)*(x**a+x**(2*a)+x**(3*a)),x)

[Out] x**(a + 1)*(2*x**(2*a) + 3*x**a + 6)**(1 + 1/a)/(6*(a + 1))
```

Mathematica [A] time = 0.0564303, size = 33, normalized size = 0.97

$$\frac{x^{a+1}\left(2x^{2a}+3x^a+6\right)^{\frac{1}{a}+1}}{6a+6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(6 + 3*x^a + 2*x^(2*a))^a^(-1)*(x^a + x^(2*a) + x^(3*a)),x]

[Out] (x^(1 + a)*(6 + 3*x^a + 2*x^(2*a))^(1 + a^(-1)))/(6 + 6*a)
```

Maple [A] time = 0.066, size = 44, normalized size = 1.3

$$\frac{x{x}^{a}\left(6+3\,{x}^{a}+2\,\left({x}^{a}\right)^{2}\right)\sqrt[a]{6+3\,{x}^{a}+2\,\left({x}^{a}\right)^{2}}}{6+6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((6+3*x^a+2*x^(2*a))^(1/a)*(x^a+x^(2*a)+x^(3*a)),x)
```

```
[Out] 1/6*x*x^a*(6+3*x^a+2*(x^a)^2)/(1+a)*(6+3*x^a+2*(x^a)^2)^(1/a)
```

Maxima [A] time = 1.76613, size = 65, normalized size = 1.91

$$\frac{\left(2\,xx^{3\,a} + 3\,xx^{2\,a} + 6\,xx^{a}\right)\left(2\,x^{2\,a} + 3\,x^{a} + 6\right)^{\left(\frac{1}{a}\right)}}{6\,(a+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^(2*a) + 3*x^a + 6)^(1/a)*(x^(3*a) + x^(2*a) + x^a),x, algorithm="m
```

```
[Out] 1/6*(2*x*x^(3*a) + 3*x*x^(2*a) + 6*x*x^a)*(2*x^(2*a) + 3*x^a + 6)
^(1/a)/(a + 1)
```

Fricas [A] time = 0.223727, size = 65, normalized size = 1.91

$$\frac{\left(2\,xx^{3\,a} + 3\,xx^{2\,a} + 6\,xx^{a}\right)\left(2\,x^{2\,a} + 3\,x^{a} + 6\right)^{\left(\frac{1}{a}\right)}}{6\,(a+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^(2*a) + 3*x^a + 6)^(1/a)*(x^(3*a) + x^(2*a) + x^a),x, algorithm="f
```

```
[Out] 1/6*(2*x*x^(3*a) + 3*x*x^(2*a) + 6*x*x^a)*(2*x^(2*a) + 3*x^a + 6)
^(1/a)/(a + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6+3*x**a+2*x**(2*a))**(1/a)*(x**a+x**(2*a)+x**(3*a)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(2\,x^{2\,a} + 3\,x^{a} + 6\right)^{\left(\frac{1}{a}\right)}\left(x^{3\,a} + x^{2\,a} + x^{a}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^(2*a) + 3*x^a + 6)^(1/a)*(x^(3*a) + x^(2*a) + x^a),x, algorithm="g
```

```
[Out] integrate((2*x^(2*a) + 3*x^a + 6)^(1/a)*(x^(3*a) + x^(2*a) + x^a)
, x)
```

3.33 $\int \frac{1}{x\sqrt[3]{1-x^2}}\,dx$

Optimal. Leaf size=58

$$\frac{3}{4}\log\left(1-\sqrt[3]{1-x^2}\right)+\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)-\frac{\log(x)}{2}$$

```
[Out] (Sqrt[3]*ArcTan[(1 + 2*(1 - x^2)^(1/3))/Sqrt[3]])/2 - Log[x]/2 +
(3*Log[1 - (1 - x^2)^(1/3)])/4
```

Rubi [A] time = 0.0778211, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{3}{4}\log\left(1-\sqrt[3]{1-x^2}\right)+\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 - x^2)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 + 2*(1 - x^2)^(1/3))/Sqrt[3]])/2 - Log[x]/2 +
(3*Log[1 - (1 - x^2)^(1/3)])/4
```

Rubi in Sympy [A] time = 2.45576, size = 49, normalized size = 0.84

$$-\frac{\log\left(x^2\right)}{4}+\frac{3\log\left(-\sqrt[3]{-x^2+1}+1\right)}{4}+\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2\sqrt[3]{-x^2+1}}{3}+\frac{1}{3}\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/x/(-x**2+1)**(1/3),x)
```

```
[Out] -log(x**2)/4 + 3*log(-(-x**2 + 1)**(1/3) + 1)/4 + sqrt(3)*atan(sq
rt(3)*(2*(-x**2 + 1)**(1/3)/3 + 1/3))/2
```

Mathematica [C] time = 0.0209582, size = 41, normalized size = 0.71

$$-\frac{3\sqrt[3]{\frac{x^2-1}{x^2}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};\frac{1}{x^2}\right)}{2\sqrt[3]{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 - x^2)^(1/3)),x]
```

```
[Out] (-3*((-1 + x^2)/x^2)^(1/3)*Hypergeometric2F1[1/3, 1/3, 4/3, x^(-2
)])/(2*(1 - x^2)^(1/3))
```

Maple [C] time = 0.056, size = 65, normalized size = 1.1

$$\frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)}{4\pi}\left(\frac{2\pi\sqrt{3}}{3\,\Gamma(2/3)}\left(-\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+2\ln(x)+i\pi\right)+\frac{2\pi\sqrt{3}x^2}{9\,\Gamma(2/3)}{}_3F_2(1,1,\frac{4}{3};2,2;x^2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(1/x/(-x^2+1)^(1/3),x)
```

```
[Out] 1/4/Pi*3^(1/2)*GAMMA(2/3)*(2/3*(-1/6*Pi*3^(1/2)-3/2*ln(3)+2*ln(x)
+I*Pi)*Pi*3^(1/2)/GAMMA(2/3)+2/9*Pi*3^(1/2)/GAMMA(2/3)*x^2*hyperg
eom([1,1,4/3],[2,2],x^2))
```

Maxima [A] time = 1.49653, size = 84, normalized size = 1.45

$$\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^2+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{1}{4}\log\left(\left(-x^2+1\right)^{\frac{2}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{2}\log\left(\left(-x^2+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(1/((-x^2 + 1)^(1/3)*x),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^2 + 1)^(1/3) + 1)) - 1/4*lo
g((-x^2 + 1)^(2/3) + (-x^2 + 1)^(1/3) + 1) + 1/2*log((-x^2 + 1)^(
1/3) - 1)
```

Fricas [A] time = 0.21369, size = 84, normalized size = 1.45

$$\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^2+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{1}{4}\log\left(\left(-x^2+1\right)^{\frac{2}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{2}\log\left(\left(-x^2+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(1/((-x^2 + 1)^(1/3)*x),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^2 + 1)^(1/3) + 1)) - 1/4*lo
g((-x^2 + 1)^(2/3) + (-x^2 + 1)^(1/3) + 1) + 1/2*log((-x^2 + 1)^(
1/3) - 1)
```

Sympy [A] time = 1.60745, size = 36, normalized size = 0.62

$$-\frac{e^{-\frac{i\pi}{3}}\,\Gamma\left(\frac{1}{3}\right)\,{}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\right|\frac{1}{x^2}\right)}{2x^{\frac{2}{3}}\,\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(1/x/(-x**2+1)**(1/3),x)
```

```
[Out] -exp(-I*pi/3)*gamma(1/3)*hyper((1/3, 1/3), (4/3,), x**(-2))/(2*x*
*(2/3)*gamma(4/3))
```

GIAC/XCAS [A] time = 0.20711, size = 86, normalized size = 1.48

$$\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^2+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{1}{4}\ln\left(\left(-x^2+1\right)^{\frac{2}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{2}\ln\left(-\left(-x^2+1\right)^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((-x^2 + 1)^(1/3)*x),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^2 + 1)^(1/3) + 1)) - 1/4*ln
((-x^2 + 1)^(2/3) + (-x^2 + 1)^(1/3) + 1) + 1/2*ln(-(-x^2 + 1)^(1
/3) + 1)
```

3.34 $\int \frac{1}{x(1-x^2)^{2/3}}\,dx$

Optimal. Leaf size=58

$$\frac{3}{4}\log\left(1-\sqrt[3]{1-x^2}\right)-\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)-\frac{\log(x)}{2}$$

```
[Out] -(Sqrt[3]*ArcTan[(1 + 2*(1 - x^2)^(1/3))/Sqrt[3]])/2 - Log[x]/2 +
 (3*Log[1 - (1 - x^2)^(1/3)])/4
```

Rubi [A] time = 0.069181, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{3}{4}\log\left(1-\sqrt[3]{1-x^2}\right)-\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 - x^2)^(2/3)),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(1 + 2*(1 - x^2)^(1/3))/Sqrt[3]])/2 - Log[x]/2 +
 (3*Log[1 - (1 - x^2)^(1/3)])/4
```

Rubi in Sympy [A] time = 2.45289, size = 49, normalized size = 0.84

$$-\frac{\log\left(x^2\right)}{4}+\frac{3\log\left(-\sqrt[3]{-x^2+1}+1\right)}{4}-\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2\sqrt[3]{-x^2+1}}{3}+\frac{1}{3}\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/x/(-x**2+1)**(2/3),x)
```

```
[Out] -log(x**2)/4 + 3*log(-(-x**2 + 1)**(1/3) + 1)/4 - sqrt(3)*atan(sq
rt(3)*(2*(-x**2 + 1)**(1/3)/3 + 1/3))/2
```

Mathematica [C] time = 0.0202607, size = 41, normalized size = 0.71

$$-\frac{3\left(\frac{x^2-1}{x^2}\right)^{2/3}{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};\frac{1}{x^2}\right)}{4\left(1-x^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 - x^2)^(2/3)),x]
```

```
[Out] (-3*((-1 + x^2)/x^2)^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, x^(-2
)])/(4*(1 - x^2)^(2/3))
```

Maple [C] time = 0.063, size = 48, normalized size = 0.8

$$\frac{1}{2\,(2/3)}\left(\left(\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+2\ln(x)+i\pi\right)\left(\frac{2}{3}\right)+\frac{2\,(2/3)\,x^2}{3}{}_3\mathrm{F}_2(1,1,\frac{5}{3};\,2,2;\,x^2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(1/x/(-x^2+1)^(2/3),x)
```

```
[Out] 1/2/GAMMA(2/3)*((1/6*Pi*3^(1/2)-3/2*ln(3)+2*ln(x)+I*Pi)*GAMMA(2/3
)+2/3*GAMMA(2/3)*x^2*hypergeom([1,1,5/3],[2,2],x^2))
```

Maxima [A]   time = 1.49964, size = 84, normalized size = 1.45

$$-\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^2+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{1}{4}\log\left(\left(-x^2+1\right)^{\frac{2}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{2}\log\left(\left(-x^2+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(1/((-x^2 + 1)^(2/3)*x),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^2 + 1)^(1/3) + 1)) - 1/4*l
og((-x^2 + 1)^(2/3) + (-x^2 + 1)^(1/3) + 1) + 1/2*log((-x^2 + 1)^
(1/3) - 1)
```

Fricas [A]   time = 0.21626, size = 84, normalized size = 1.45

$$-\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^2+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{1}{4}\log\left(\left(-x^2+1\right)^{\frac{2}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{2}\log\left(\left(-x^2+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(1/((-x^2 + 1)^(2/3)*x),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^2 + 1)^(1/3) + 1)) - 1/4*l
og((-x^2 + 1)^(2/3) + (-x^2 + 1)^(1/3) + 1) + 1/2*log((-x^2 + 1)^
(1/3) - 1)
```

Sympy [A]   time = 1.71959, size = 37, normalized size = 0.64

$$-\frac{e^{-\frac{2i\pi}{3}}\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|\frac{1}{x^2}\right)}{2x^{\frac{4}{3}}\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(1/x/(-x**2+1)**(2/3),x)
```

```
[Out] -exp(-2*I*pi/3)*gamma(2/3)*hyper((2/3, 2/3), (5/3,), x**(-2))/(2*
x**(4/3)*gamma(5/3))
```

GIAC/XCAS [A]   time = 0.204186, size = 86, normalized size = 1.48

$$-\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^2+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{1}{4}\ln\left(\left(-x^2+1\right)^{\frac{2}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{2}\ln\left(-\left(-x^2+1\right)^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(1/((-x^2 + 1)^(2/3)*x),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^2 + 1)^(1/3) + 1)) - 1/4*l
n((-x^2 + 1)^(2/3) + (-x^2 + 1)^(1/3) + 1) + 1/2*ln(-(-x^2 + 1)^(
1/3) + 1)
```

3.35 $\int \frac{1}{\sqrt[3]{1-x^3}}\,dx$

Optimal. Leaf size=49

$$\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)-\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -(ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3]) + Log[x +
(1 - x^3)^(1/3)]/2
```

Rubi [A] time = 0.0162811, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)-\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^3)^(-1/3),x]
```

```
[Out] -(ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3]) + Log[x +
(1 - x^3)^(1/3)]/2
```

Rubi in Sympy [A] time = 3.58718, size = 71, normalized size = 1.45

$$\frac{\log\left(\frac{x}{\sqrt[3]{-x^3+1}}+1\right)}{3}-\frac{\log\left(\frac{x^2}{(-x^3+1)^{\frac{2}{3}}}-\frac{x}{\sqrt[3]{-x^3+1}}+1\right)}{6}+\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3\sqrt[3]{-x^3+1}}-\frac{1}{3}\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(-x**3+1)**(1/3),x)
```

```
[Out] log(x/(-x**3 + 1)**(1/3) + 1)/3 - log(x**2/(-x**3 + 1)**(2/3) - x
/(-x**3 + 1)**(1/3) + 1)/6 + sqrt(3)*atan(sqrt(3)*(2*x/(3*(-x**3
+ 1)**(1/3)) - 1/3))/3
```

Mathematica [A] time = 0.0709057, size = 86, normalized size = 1.76

$$\frac{1}{3}\log\left(\frac{x}{\sqrt[3]{1-x^3}}+1\right)+\frac{\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{1-x^3}}-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{6}\log\left(-\frac{x}{\sqrt[3]{1-x^3}}+\frac{x^2}{(1-x^3)^{2/3}}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^3)^(-1/3),x]
```

```
[Out] ArcTan[(-1 + (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[1 + x^
2/(1 - x^3)^(2/3) - x/(1 - x^3)^(1/3)]/6 + Log[1 + x/(1 - x^3)^(1
/3)]/3
```

Maple [C] time = 0.034, size = 12, normalized size = 0.2

$$x\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{3};\ \frac{4}{3};\ x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(1/(-x^3+1)^(1/3),x)
```

```
[Out] x*hypergeom([1/3,1/3],[4/3],x^3)
```

Maxima [A] time = 1.50195, size = 105, normalized size = 2.14

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(-x^3+1\right)^{\frac{1}{3}}}{x}-1\right)\right)+\frac{1}{3}\log\left(\frac{\left(-x^3+1\right)^{\frac{1}{3}}}{x}+1\right)$$
$$-\frac{1}{6}\log\left(-\frac{\left(-x^3+1\right)^{\frac{1}{3}}}{x}+\frac{\left(-x^3+1\right)^{\frac{2}{3}}}{x^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((-x^3 + 1)^(-1/3),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3)/x - 1)) + 1/3
*log((-x^3 + 1)^(1/3)/x + 1) - 1/6*log(-(-x^3 + 1)^(1/3)/x + (-x^
3 + 1)^(2/3)/x^2 + 1)
```

Fricas [A] time = 0.216066, size = 122, normalized size = 2.49

$$\frac{1}{18}\sqrt{3}\left(2\sqrt{3}\log\left(\frac{x+\left(-x^3+1\right)^{\frac{1}{3}}}{x}\right)-\sqrt{3}\log\left(\frac{x^2-\left(-x^3+1\right)^{\frac{1}{3}}x+\left(-x^3+1\right)^{\frac{2}{3}}}{x^2}\right)-6\arctan\left(-\frac{\sqrt{3}x-2\sqrt{3}\left(-x^3+1\right)^{\frac{1}{3}}}{3x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((-x^3 + 1)^(-1/3),x, algorithm="fricas")
```

```
[Out] 1/18*sqrt(3)*(2*sqrt(3)*log((x + (-x^3 + 1)^(1/3))/x) - sqrt(3)*l
og((x^2 - (-x^3 + 1)^(1/3)*x + (-x^3 + 1)^(2/3))/x^2) - 6*arctan(
-1/3*(sqrt(3)*x - 2*sqrt(3)*(-x^3 + 1)^(1/3))/x))
```

Sympy [A] time = 1.60956, size = 29, normalized size = 0.59

$$\frac{x\,\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\,x^3e^{2i\pi}\right)}{3\,\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(1/(-x**3+1)**(1/3),x)
```

```
[Out] x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), x**3*exp_polar(2*I*pi))/(3
*gamma(4/3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-x^3+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^3 + 1)^(-1/3),x, algorithm="giac")
```

```
[Out] integrate((-x^3 + 1)^(-1/3), x)
```

3.36 $\int \frac{1}{x\sqrt[3]{1-x^3}}\,dx$

Optimal. Leaf size=55

$$\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\log(x)}{2}$$

```
[Out] ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[x]/2 + Log[
1 - (1 - x^3)^(1/3)]/2
```

Rubi [A] time = 0.0699828, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(1 - x^3)^(1/3)),x]
```

```
[Out] ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[x]/2 + Log[
1 - (1 - x^3)^(1/3)]/2
```

Rubi in Sympy [A] time = 2.29865, size = 48, normalized size = 0.87

$$-\frac{\log\left(x^3\right)}{6}+\frac{\log\left(-\sqrt[3]{-x^3+1}+1\right)}{2}+\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2\sqrt[3]{-x^3+1}}{3}+\frac{1}{3}\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(-x**3+1)**(1/3),x)
```

```
[Out] -log(x**3)/6 + log(-(-x**3 + 1)**(1/3) + 1)/2 + sqrt(3)*atan(sqrt
(3)*(2*(-x**3 + 1)**(1/3)/3 + 1/3))/3
```

Mathematica [C] time = 0.0189008, size = 39, normalized size = 0.71

$$-\frac{\sqrt[3]{\frac{x^3-1}{x^3}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};\frac{1}{x^3}\right)}{\sqrt[3]{1-x^3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(1 - x^3)^(1/3)),x]
```

```
[Out] -((((-1 + x^3)/x^3)^(1/3)*Hypergeometric2F1[1/3, 1/3, 4/3, x^(-3)
])/(1 - x^3)^(1/3))
```

Maple [C] time = 0.057, size = 65, normalized size = 1.2

$$\frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)}{6\,\pi}\left(\frac{2\,\pi\,\sqrt{3}}{3\,\Gamma(2/3)}\left(-\frac{\pi\,\sqrt{3}}{6}-\frac{3\,\ln(3)}{2}+3\,\ln(x)+i\pi\right)+\frac{2\,\pi\,\sqrt{3}x^3}{9\,\Gamma(2/3)}{}_3F_2(1,1,\frac{4}{3};\,2,2;\,x^3)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(1/x/(-x^3+1)^(1/3),x)
```

```
[Out] 1/6/Pi*3^(1/2)*GAMMA(2/3)*(2/3*(-1/6*Pi*3^(1/2)-3/2*ln(3)+3*ln(x)
+I*Pi)*Pi*3^(1/2)/GAMMA(2/3)+2/9*Pi*3^(1/2)/GAMMA(2/3)*x^3*hyperg
eom([1,1,4/3],[2,2],x^3))
```

Maxima [A] time = 1.47398, size = 84, normalized size = 1.53

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^3+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{1}{6}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left(-x^3+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(1/((-x^3 + 1)^(1/3)*x),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) - 1/6*lo
g((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log((-x^3 + 1)^(
1/3) - 1)
```

Fricas [A] time = 0.214686, size = 96, normalized size = 1.75

$$-\frac{1}{18}\sqrt{3}\left(\sqrt{3}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)-2\sqrt{3}\log\left(\left(-x^3+1\right)^{\frac{1}{3}}-1\right)-6\arctan\left(\frac{2}{3}\sqrt{3}\left(-x^3+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(1/((-x^3 + 1)^(1/3)*x),x, algorithm="fricas")
```

```
[Out] -1/18*sqrt(3)*(sqrt(3)*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) +
1) - 2*sqrt(3)*log((-x^3 + 1)^(1/3) - 1) - 6*arctan(2/3*sqrt(3)*(
-x^3 + 1)^(1/3) + 1/3*sqrt(3)))
```

Sympy [A] time = 1.60516, size = 32, normalized size = 0.58

$$-\frac{e^{-\frac{i\pi}{3}}\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{1}{x^3}\right)}{3x\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(1/x/(-x**3+1)**(1/3),x)
```

```
[Out] -exp(-I*pi/3)*gamma(1/3)*hyper((1/3, 1/3), (4/3,), x**(-3))/(3*x*
gamma(4/3))
```

GIAC/XCAS [A] time = 0.20865, size = 85, normalized size = 1.55

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^3+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{1}{6}\ln\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\ln\left(\left|\left(-x^3+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((-x^3 + 1)^(1/3)*x),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) - 1/6*ln
((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*ln(abs((-x^3 + 1)
^(1/3) - 1))
```

3.37 $\int \frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx$

Optimal. Leaf size=121

$$\frac{3\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{4\sqrt[3]{2}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+2(1-x)}{2^{2/3}\sqrt{3}\sqrt[3]{1-x^3}}\right)}{2\sqrt[3]{2}}-\frac{\log(1-x)}{4\sqrt[3]{2}}-\frac{\log(x+1)}{2\sqrt[3]{2}}$$

```
[Out] -(Sqrt[3]*ArcTan[(2*(1 - x) + 2^(2/3)*(1 - x^3)^(1/3))/(2^(2/3)*S
qrt[3]*(1 - x^3)^(1/3))])/(2*2^(1/3)) - Log[1 - x]/(4*2^(1/3)) -
Log[1 + x]/(2*2^(1/3)) + (3*Log[-1 + x + 2^(2/3)*(1 - x^3)^(1/3)]
)/(4*2^(1/3))
```

Rubi [A] time = 0.0830356, antiderivative size = 97, normalized size of antiderivative = 0.8, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059

$$\frac{3\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{4\sqrt[3]{2}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}}-\frac{\log\left((1-x)(x+1)^2\right)}{4\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((1 + x)*(1 - x^3)^(1/3)),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]
)/(2*2^(1/3)) - Log[(1 - x)*(1 + x)^2]/(4*2^(1/3)) + (3*Log[-1 +
x + 2^(2/3)*(1 - x^3)^(1/3)])/(4*2^(1/3))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x+1)\sqrt[3]{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1+x)/(-x**3+1)**(1/3),x)
```

```
[Out] Integral(1/((x + 1)*(-x**3 + 1)**(1/3)), x)
```

Mathematica [A] time = 0.0732422, size = 0, normalized size = 0.

$$\int \frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[1/((1 + x)*(1 - x^3)^(1/3)),x]
```

```
[Out] Integrate[1/((1 + x)*(1 - x^3)^(1/3)), x]
```

Maple [F] time = 0.049, size = 0, normalized size = 0.

$$\int \frac{1}{1+x}\frac{1}{\sqrt[3]{-x^{3}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1+x)/(-x^3+1)^(1/3),x)

[Out] int(1/(1+x)/(-x^3+1)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-x^{3}+1)^{\frac{1}{3}}(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((-x^3 + 1)^(1/3)*(x + 1)),x, algorithm="maxima")

[Out] integrate(1/((-x^3 + 1)^(1/3)*(x + 1)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((-x^3 + 1)^(1/3)*(x + 1)),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{-(x-1)(x^{2}+x+1)}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1+x)/(-x**3+1)**(1/3),x)

[Out] Integral(1/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-x^{3}+1)^{\frac{1}{3}}(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((-x^3 + 1)^(1/3)*(x + 1)),x, algorithm="giac")

[Out] integrate(1/((-x^3 + 1)^(1/3)*(x + 1)), x)
```

3.38 $\int \frac{x}{(1+x)\sqrt[3]{1-x^3}}\,dx$

Optimal. Leaf size=145

$$\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)-\frac{3\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{4\sqrt[3]{2}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\log\left((1-x)(x+1)^2\right)}{4\sqrt[3]{2}}$$

```
[Out] (Sqrt[3]*ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])
/(2*2^(1/3)) - ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3
] + Log[(1 - x)*(1 + x)^2]/(4*2^(1/3)) + Log[x + (1 - x^3)^(1/3)]
/2 - (3*Log[-1 + x + 2^(2/3)*(1 - x^3)^(1/3)])/(4*2^(1/3))
```

Rubi [A] time = 0.193547, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)-\frac{3\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{4\sqrt[3]{2}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\log\left((1-x)(x+1)^2\right)}{4\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/((1 + x)*(1 - x^3)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])
/(2*2^(1/3)) - ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3
] + Log[(1 - x)*(1 + x)^2]/(4*2^(1/3)) + Log[x + (1 - x^3)^(1/3)]
/2 - (3*Log[-1 + x + 2^(2/3)*(1 - x^3)^(1/3)])/(4*2^(1/3))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(x+1)\sqrt[3]{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(1+x)/(-x**3+1)**(1/3),x)
```

```
[Out] Integral(x/((x + 1)*(-x**3 + 1)**(1/3)), x)
```

Mathematica [A] time = 0.0889764, size = 0, normalized size = 0.

$$\int \frac{x}{(1+x)\sqrt[3]{1-x^3}}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[x/((1 + x)*(1 - x^3)^(1/3)),x]

[Out] Integrate[x/((1 + x)*(1 - x^3)^(1/3)), x]
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int \frac{x}{1+x} \frac{1}{\sqrt[3]{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(1+x)/(-x^3+1)^(1/3),x)

[Out] int(x/(1+x)/(-x^3+1)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(-x^3+1)^{\frac{1}{3}}(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((-x^3 + 1)^(1/3)*(x + 1)),x, algorithm="maxima")

[Out] integrate(x/((-x^3 + 1)^(1/3)*(x + 1)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((-x^3 + 1)^(1/3)*(x + 1)),x, algorithm="fricas")

[Out] Exception raised: TypeError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt[3]{-(x-1)(x^2+x+1)}(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(1+x)/(-x**3+1)**(1/3),x)

[Out] Integral(x/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(-x^3+1)^{\frac{1}{3}}(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((-x^3 + 1)^(1/3)*(x + 1)),x, algorithm="giac")

[Out] integrate(x/((-x^3 + 1)^(1/3)*(x + 1)), x)
```

3.39 $\int \frac{1}{x\sqrt[3]{2-3x+x^2}}\,dx$

Optimal. Leaf size=110

$$\frac{3\log\left(-2^{2/3}\sqrt[3]{x^2-3x+2}-x+2\right)}{4\sqrt[3]{2}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{2}(2-x)}{\sqrt{3}\sqrt[3]{x^2-3x+2}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt[3]{2}}-\frac{\log(2-x)}{4\sqrt[3]{2}}-\frac{\log(x)}{2\sqrt[3]{2}}$$

```
[Out] -(Sqrt[3]*ArcTan[1/Sqrt[3] + (2^(1/3)*(2 - x))/(Sqrt[3]*(2 - 3*x
+ x^2)^(1/3))])/(2*2^(1/3)) - Log[2 - x]/(4*2^(1/3)) - Log[x]/(2*
2^(1/3)) + (3*Log[2 - x - 2^(2/3)*(2 - 3*x + x^2)^(1/3)])/(4*2^(1
/3))
```

Rubi [A] time = 0.0992248, antiderivative size = 176, normalized size of antiderivative = 1.6, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{3\sqrt[3]{x-2}\sqrt[3]{x-1}\log\left(-\frac{(x-2)^{2/3}}{\sqrt[3]{2}}-\sqrt[3]{2}\sqrt[3]{x-1}\right)}{4\sqrt[3]{2}\sqrt[3]{x^2-3x+2}}-\frac{\sqrt[3]{x-2}\sqrt[3]{x-1}\log(x)}{2\sqrt[3]{2}\sqrt[3]{x^2-3x+2}}-\frac{\sqrt{3}\sqrt[3]{x-2}\sqrt[3]{x-1}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{\sqrt[3]{2}(x-2)^{2/3}}{\sqrt{3}\sqrt[3]{x-1}}\right)}{2\sqrt[3]{2}\sqrt[3]{x^2-3x+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(2 - 3*x + x^2)^(1/3)),x]
```

```
[Out] -(Sqrt[3]*(-2 + x)^(1/3)*(-1 + x)^(1/3)*ArcTan[1/Sqrt[3] - (2^(1/
3)*(-2 + x)^(2/3))/(Sqrt[3]*(-1 + x)^(1/3))])/(2*2^(1/3)*(2 - 3*x
 + x^2)^(1/3)) + (3*(-2 + x)^(1/3)*(-1 + x)^(1/3)*Log[-((-2 + x)^
(2/3)/2^(1/3)) - 2^(1/3)*(-1 + x)^(1/3)])/(4*2^(1/3)*(2 - 3*x + x
^2)^(1/3)) - ((-2 + x)^(1/3)*(-1 + x)^(1/3)*Log[x])/(2*2^(1/3)*(2
 - 3*x + x^2)^(1/3))
```

Rubi in Sympy [A] time = 4.67779, size = 155, normalized size = 1.41

$$-\frac{\sqrt[3]{2x-4}\sqrt[3]{2x-2}\log{(x)}}{4\sqrt[3]{x^2-3x+2}}+\frac{3\sqrt[3]{2x-4}\sqrt[3]{2x-2}\log{\left(-\frac{(2x-4)^{\frac{2}{3}}}{2}-\sqrt[3]{2x-2}\right)}}{8\sqrt[3]{x^2-3x+2}}+\frac{\sqrt{3}\sqrt[3]{2x-4}\sqrt[3]{2x-2}\operatorname{atan}{\left(\frac{\sqrt{3}(2x-4)^{\frac{2}{3}}}{3\sqrt[3]{2x-2}}-\frac{\sqrt{3}}{3}\right)}}{4\sqrt[3]{x^2-3x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/x/(x**2-3*x+2)**(1/3),x)
```

```
[Out] -(2*x - 4)**(1/3)*(2*x - 2)**(1/3)*log(x)/(4*(x**2 - 3*x + 2)**(1
/3)) + 3*(2*x - 4)**(1/3)*(2*x - 2)**(1/3)*log(-(2*x - 4)**(2/3)/
2 - (2*x - 2)**(1/3))/(8*(x**2 - 3*x + 2)**(1/3)) + sqrt(3)*(2*x
- 4)**(1/3)*(2*x - 2)**(1/3)*atan(sqrt(3)*(2*x - 4)**(2/3)/(3*(2*
x - 2)**(1/3)) - sqrt(3)/3)/(4*(x**2 - 3*x + 2)**(1/3))
```

Mathematica [C] time = 0.19044, size = 109, normalized size = 0.99

$$-\frac{15xF_1\left(\frac{2}{3};\frac{1}{3},\frac{1}{3};\frac{5}{3};\frac{1}{x},\frac{2}{x}\right)}{2\sqrt[3]{x^2-3x+2}\left(5xF_1\left(\frac{2}{3};\frac{1}{3},\frac{1}{3};\frac{5}{3};\frac{1}{x},\frac{2}{x}\right)+2F_1\left(\frac{5}{3};\frac{1}{3},\frac{4}{3};\frac{8}{3};\frac{1}{x},\frac{2}{x}\right)+F_1\left(\frac{5}{3};\frac{4}{3},\frac{1}{3};\frac{8}{3};\frac{1}{x},\frac{2}{x}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]   Integrate[1/(x*(2 - 3*x + x^2)^(1/3)),x]
```

```
[Out] (-15*x*AppellF1[2/3, 1/3, 1/3, 5/3, x^(-1), 2/x])/(2*(2 - 3*x + x
^2)^(1/3)*(5*x*AppellF1[2/3, 1/3, 1/3, 5/3, x^(-1), 2/x] + 2*Appe
llF1[5/3, 1/3, 4/3, 8/3, x^(-1), 2/x] + AppellF1[5/3, 4/3, 1/3, 8
/3, x^(-1), 2/x]))
```

Maple [F] time = 0.098, size = 0, normalized size = 0.

$$\int \frac{1}{x}\frac{1}{\sqrt[3]{x^{2}-3\,x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(1/x/(x^2-3*x+2)^(1/3),x)
```

```
[Out] int(1/x/(x^2-3*x+2)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{{\left(x^{2}-3\,x+2\right)}^{\frac{1}{3}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(1/((x^2 - 3*x + 2)^(1/3)*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 - 3*x + 2)^(1/3)*x), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(1/((x^2 - 3*x + 2)^(1/3)*x),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt[3]{\left(x-2\right)\left(x-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(1/x/(x**2-3*x+2)**(1/3),x)
```

```
[Out] Integral(1/(x*((x - 2)*(x - 1))**(1/3)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2 - 3x + 2)^{\frac{1}{3}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 - 3*x + 2)^(1/3)*x),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 - 3*x + 2)^(1/3)*x), x)
```

3.40 $\int \frac{1}{\sqrt[3]{-5+7x-3x^2+x^3}}\,dx$

Optimal. Leaf size=81

$$-\frac{3}{4}\log\left(\sqrt[3]{x^3-3x^2+7x-5}-x+1\right)+\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2(x-1)}{\sqrt{3}\sqrt[3]{x^3-3x^2+7x-5}}+\frac{1}{\sqrt{3}}\right)+\frac{1}{4}\log(1-x)$$

```
[Out] (Sqrt[3]*ArcTan[1/Sqrt[3] + (2*(-1 + x))/(Sqrt[3]*(-5 + 7*x - 3*x
^2 + x^3)^(1/3))])/2 + Log[1 - x]/4 - (3*Log[1 - x + (-5 + 7*x -
3*x^2 + x^3)^(1/3)])/4
```

Rubi [A] time = 0.139111, antiderivative size = 131, normalized size of antiderivative = 1.62, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$

$$\frac{\sqrt{3}\sqrt[3]{(x-1)^2+4}\sqrt[3]{x-1}\tan^{-1}\left(\frac{\frac{2(x-1)^{2/3}}{\sqrt[3]{(x-1)^2+4}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{(x-1)^3-4(1-x)}}-\frac{3\sqrt[3]{(x-1)^2+4}\sqrt[3]{x-1}\log\left((x-1)^{2/3}-\sqrt[3]{(x-1)^2+4}\right)}{4\sqrt[3]{(x-1)^3-4(1-x)}}$$

Antiderivative was successfully verified.

```
[In]  Int[(-5 + 7*x - 3*x^2 + x^3)^(-1/3),x]
```

```
[Out] (Sqrt[3]*(4 + (-1 + x)^2)^(1/3)*(-1 + x)^(1/3)*ArcTan[(1 + (2*(-1
 + x)^(2/3))/(4 + (-1 + x)^2)^(1/3))/Sqrt[3]])/(2*(-4*(1 - x) + (
-1 + x)^3)^(1/3)) - (3*(4 + (-1 + x)^2)^(1/3)*(-1 + x)^(1/3)*Log[
-(4 + (-1 + x)^2)^(1/3) + (-1 + x)^(2/3)])/(4*(-4*(1 - x) + (-1 +
 x)^3)^(1/3))
```

Rubi in Sympy [A] time = 7.39252, size = 204, normalized size = 2.52

$$-\frac{\sqrt[3]{x-1}\sqrt[3]{x^2-2x+5}\log\left(-\frac{(x-1)^{\frac{2}{3}}}{\sqrt[3]{(x-1)^2+4}}+1\right)}{2\sqrt[3]{x^3-3x^2+7x-5}}+\frac{\sqrt[3]{x-1}\sqrt[3]{x^2-2x+5}\log\left(\frac{(x-1)^{\frac{4}{3}}}{\left((x-1)^2+4\right)^{\frac{2}{3}}}+\frac{(x-1)^{\frac{2}{3}}}{\sqrt[3]{(x-1)^2+4}}+1\right)}{4\sqrt[3]{x^3-3x^2+7x-5}}+\frac{\sqrt{3}\sqrt[3]{x-1}\sqrt[3]{x^2-2x+5}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2(x-1)^{\frac{2}{3}}}{3\sqrt[3]{(x-1)^2+4}}+\frac{1}{3}\right)\right)}{2\sqrt[3]{x^3-3x^2+7x-5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**3-3*x**2+7*x-5)**(1/3),x)
```

```
[Out] -(x - 1)**(1/3)*(x**2 - 2*x + 5)**(1/3)*log(-(x - 1)**(2/3)/((x -
 1)**2 + 4)**(1/3) + 1)/(2*(x**3 - 3*x**2 + 7*x - 5)**(1/3)) + (x
 - 1)**(1/3)*(x**2 - 2*x + 5)**(1/3)*log((x - 1)**(4/3)/((x - 1)*
*2 + 4)**(2/3) + (x - 1)**(2/3)/((x - 1)**2 + 4)**(1/3) + 1)/(4*(
x**3 - 3*x**2 + 7*x - 5)**(1/3)) + sqrt(3)*(x - 1)**(1/3)*(x**2 -
 2*x + 5)**(1/3)*atan(sqrt(3)*(2*(x - 1)**(2/3)/(3*((x - 1)**2 +
4)**(1/3)) + 1/3))/(2*(x**3 - 3*x**2 + 7*x - 5)**(1/3))
```

Mathematica [C] time = 0.0224462, size = 85, normalized size = 1.05

$$\frac{3\sqrt[3]{ix+(2-i)}\sqrt[3]{i(x-1)}(x-(1-2i))F_1\left(\frac{2}{3};\frac{1}{3},\frac{1}{3};\frac{5}{3};-\frac{1}{4}i(x-(1-2i)),-\frac{1}{2}i(x-(1-2i))\right)}{4\sqrt[3]{x^3-3x^2+7x-5}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(-5 + 7*x - 3*x^2 + x^3)^(-1/3),x]
```

```
[Out] (3*((2 - I) + I*x)^(1/3)*(I*(-1 + x))^(1/3)*((-1 + 2*I) + x)*Appe
llF1[2/3, 1/3, 1/3, 5/3, (-I/4)*((-1 + 2*I) + x), (-I/2)*((-1 + 2
*I) + x)])/(4*(-5 + 7*x - 3*x^2 + x^3)^(1/3))
```

Maple [F] time = 0.019, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{x^3-3\,x^2+7\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^3-3*x^2+7*x-5)^(1/3),x)
```

```
[Out] int(1/(x^3-3*x^2+7*x-5)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^3-3\,x^2+7\,x-5)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - 3*x^2 + 7*x - 5)^(-1/3),x, algorithm="maxima")
```

```
[Out] integrate((x^3 - 3*x^2 + 7*x - 5)^(-1/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - 3*x^2 + 7*x - 5)^(-1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{x^3-3x^2+7x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**3-3*x**2+7*x-5)**(1/3),x)

[Out] Integral((x**3 - 3*x**2 + 7*x - 5)**(-1/3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^3 - 3\,x^2 + 7\,x - 5\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - 3*x^2 + 7*x - 5)^(-1/3),x, algorithm="giac")

[Out] integrate((x^3 - 3*x^2 + 7*x - 5)^(-1/3), x)
```

3.41 $\int \frac{1}{\sqrt[3]{x(-q+x^2)}}\,dx$

Optimal. Leaf size=66

$$-\frac{3}{4}\log\left(\sqrt[3]{x(x^2-q)}-x\right)+\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2x}{\sqrt{3}\sqrt[3]{x(x^2-q)}}+\frac{1}{\sqrt{3}}\right)+\frac{\log(x)}{4}$$

```
[Out] (Sqrt[3]*ArcTan[1/Sqrt[3] + (2*x)/(Sqrt[3]*(x*(-q + x^2))^(1/3))]
)/2 + Log[x]/4 - (3*Log[-x + (x*(-q + x^2))^(1/3)])/4
```

Rubi [A] time = 0.112684, antiderivative size = 117, normalized size of antiderivative = 1.77, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$

$$\frac{\sqrt{3}\sqrt[3]{x}\sqrt[3]{x^2-q}\tan^{-1}\left(\frac{\frac{2x^{2/3}}{\sqrt[3]{x^2-q}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{x^3-qx}}-\frac{3\sqrt[3]{x}\sqrt[3]{x^2-q}\log\left(x^{2/3}-\sqrt[3]{x^2-q}\right)}{4\sqrt[3]{x^3-qx}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(-q + x^2))^(-1/3),x]
```

```
[Out] (Sqrt[3]*x^(1/3)*(-q + x^2)^(1/3)*ArcTan[(1 + (2*x^(2/3))/(-q + x
^2)^(1/3))/Sqrt[3]])/(2*(-(q*x) + x^3)^(1/3)) - (3*x^(1/3)*(-q +
x^2)^(1/3)*Log[x^(2/3) - (-q + x^2)^(1/3)])/(4*(-(q*x) + x^3)^(1/
3))
```

Rubi in Sympy [A] time = 5.45697, size = 155, normalized size = 2.35

$$-\frac{\left(-qx+x^3\right)^{\frac{2}{3}}\log\left(-\frac{x^{\frac{2}{3}}}{\sqrt[3]{-q+x^2}}+1\right)}{2x^{\frac{2}{3}}\left(-q+x^2\right)^{\frac{2}{3}}}+\frac{\left(-qx+x^3\right)^{\frac{2}{3}}\log\left(\frac{x^{\frac{4}{3}}}{(-q+x^2)^{\frac{2}{3}}}+\frac{x^{\frac{2}{3}}}{\sqrt[3]{-q+x^2}}+1\right)}{4x^{\frac{2}{3}}\left(-q+x^2\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\left(-qx+x^3\right)^{\frac{2}{3}}\operatorname{atan}\left(\sqrt{3}\left(\frac{2x^{\frac{2}{3}}}{3\sqrt[3]{-q+x^2}}+\frac{1}{3}\right)\right)}{2x^{\frac{2}{3}}\left(-q+x^2\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x*(x**2-q))**(1/3),x)
```

```
[Out] -(-q*x + x**3)**(2/3)*log(-x**(2/3)/(-q + x**2)**(1/3) + 1)/(2*x*
*(2/3)*(-q + x**2)**(2/3)) + (-q*x + x**3)**(2/3)*log(x**(4/3)/(-
q + x**2)**(2/3) + x**(2/3)/(-q + x**2)**(1/3) + 1)/(4*x**(2/3)*(
-q + x**2)**(2/3)) + sqrt(3)*(-q*x + x**3)**(2/3)*atan(sqrt(3)*(2
*x**(2/3)/(3*(-q + x**2)**(1/3)) + 1/3))/(2*x**(2/3)*(-q + x**2)*
*(2/3))
```

Mathematica [C] time = 0.0347719, size = 49, normalized size = 0.74

$$\frac{3x\sqrt[3]{\frac{q-x^2}{q}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};\frac{x^2}{q}\right)}{2\sqrt[3]{x^3-qx}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(-q + x^2))^(-1/3),x]
```

```
[Out] (3*x*((q - x^2)/q)^(1/3)*Hypergeometric2F1[1/3, 1/3, 4/3, x^2/q])
/(2*(-(q*x) + x^3)^(1/3))
```

Maple [F] time = 0.064, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{x\,(x^2-q)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x*(x^2-q))^(1/3),x)
```

```
[Out] int(1/(x*(x^2-q))^(1/3),x)
```

Maxima [A] time = 1.52352, size = 104, normalized size = 1.58

$$-\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(x^2-q\right)^{\frac{1}{3}}}{x^{\frac{2}{3}}}+1\right)\right)+\frac{1}{4}\log\left(\frac{\left(x^2-q\right)^{\frac{1}{3}}}{x^{\frac{2}{3}}}+\frac{\left(x^2-q\right)^{\frac{2}{3}}}{x^{\frac{4}{3}}}+1\right)-\frac{1}{2}\log\left(\frac{\left(x^2-q\right)^{\frac{1}{3}}}{x^{\frac{2}{3}}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((x^2 - q)*x)^(-1/3),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^2 - q)^(1/3)/x^(2/3) + 1))
+ 1/4*log((x^2 - q)^(1/3)/x^(2/3) + (x^2 - q)^(2/3)/x^(4/3) + 1)
- 1/2*log((x^2 - q)^(1/3)/x^(2/3) - 1)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((x^2 - q)*x)^(-1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{x\,(-q+x^2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x*(x**2-q))**(1/3),x)
```

```
[Out] Integral((x*(-q + x**2))**(-1/3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{((x^2-q)x)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((x^2 - q)*x)^(-1/3),x, algorithm="giac")
```

```
[Out] integrate(((x^2 - q)*x)^(-1/3), x)
```

3.42 $\int \frac{1}{\sqrt[3]{(-1+x)(q-2x+x^2)}}\,dx$

Optimal. Leaf size=79

$$-\frac{3}{4}\log\left(\sqrt[3]{(x-1)(q+x^2-2x)}-x+1\right)+\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2(x-1)}{\sqrt{3}\sqrt[3]{(x-1)(q+x^2-2x)}}+\frac{1}{\sqrt{3}}\right)+\frac{1}{4}\log(1-x)$$

```
[Out] (Sqrt[3]*ArcTan[1/Sqrt[3] + (2*(-1 + x))/(Sqrt[3]*((-1 + x)*(q -
2*x + x^2))^(1/3))])/2 + Log[1 - x]/4 - (3*Log[1 - x + ((-1 + x)*
(q - 2*x + x^2))^(1/3)])/4
```

Rubi [A] time = 0.187644, antiderivative size = 145, normalized size of antiderivative = 1.84, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$

$$\frac{\sqrt{3}\sqrt[3]{x-1}\sqrt[3]{q+(x-1)^2-1}\tan^{-1}\left(\frac{\frac{2(x-1)^{2/3}}{\sqrt[3]{q+(x-1)^2-1}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{(1-q)(1-x)+(x-1)^3}}-\frac{3\sqrt[3]{x-1}\sqrt[3]{q+(x-1)^2-1}\log\left((x-1)^{2/3}-\sqrt[3]{q+(x-1)^2-1}\right)}{4\sqrt[3]{(1-q)(1-x)+(x-1)^3}}$$

Antiderivative was successfully verified.

```
[In]  Int[((-1 + x)*(q - 2*x + x^2))^(-1/3),x]
```

```
[Out] (Sqrt[3]*(-1 + q + (-1 + x)^2)^(1/3)*(-1 + x)^(1/3)*ArcTan[(1 + (
2*(-1 + x)^(2/3))/(-1 + q + (-1 + x)^2)^(1/3))/Sqrt[3]])/(2*((1 -
 q)*(1 - x) + (-1 + x)^3)^(1/3)) - (3*(-1 + q + (-1 + x)^2)^(1/3)
*(-1 + x)^(1/3)*Log[-(-1 + q + (-1 + x)^2)^(1/3) + (-1 + x)^(2/3)
])/(4*((1 - q)*(1 - x) + (-1 + x)^3)^(1/3))
```

Rubi in Sympy [A] time = 9.5927, size = 216, normalized size = 2.73

$$-\frac{\sqrt[3]{x-1}\sqrt[3]{q+x^2-2x}\log\left(-\frac{(x-1)^{\frac{2}{3}}}{\sqrt[3]{q+(x-1)^2-1}}+1\right)}{2\sqrt[3]{-q+x^3-3x^2+x(q+2)}}+\frac{\sqrt[3]{x-1}\sqrt[3]{q+x^2-2x}\log\left(\frac{(x-1)^{\frac{4}{3}}}{\left(q+(x-1)^2-1\right)^{\frac{2}{3}}}+\frac{(x-1)^{\frac{2}{3}}}{\sqrt[3]{q+(x-1)^2-1}}+1\right)}{4\sqrt[3]{-q+x^3-3x^2+x(q+2)}}+\frac{\sqrt{3}\sqrt[3]{x-1}\sqrt[3]{q+x^2-2x}\operatorname{atan}\left(\sqrt{3}\left(\frac{2(x-1)^{\frac{2}{3}}}{3\sqrt[3]{q+(x-1)^2-1}}+\frac{1}{3}\right)\right)}{2\sqrt[3]{-q+x^3-3x^2+x(q+2)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/((-1+x)*(x**2+q-2*x))**(1/3),x)
```

```
[Out] -(x - 1)**(1/3)*(q + x**2 - 2*x)**(1/3)*log(-(x - 1)**(2/3)/(q +
(x - 1)**2 - 1)**(1/3) + 1)/(2*(-q + x**3 - 3*x**2 + x*(q + 2))**
(1/3)) + (x - 1)**(1/3)*(q + x**2 - 2*x)**(1/3)*log((x - 1)**(4/3
)/(q + (x - 1)**2 - 1)**(2/3) + (x - 1)**(2/3)/(q + (x - 1)**2 -
```

```
1)**(1/3) + 1)/(4*(-q + x**3 - 3*x**2 + x*(q + 2))**(1/3)) + sqrt
(3)*(x - 1)**(1/3)*(q + x**2 - 2*x)**(1/3)*atan(sqrt(3)*(2*(x - 1
)**(2/3)/(3*(q + (x - 1)**2 - 1)**(1/3)) + 1/3))/(2*(-q + x**3 -
3*x**2 + x*(q + 2))**(1/3))
```

Mathematica [C] time = 0.0579134, size = 61, normalized size = 0.77

$$\frac{3(x-1)\sqrt[3]{\frac{q+(x-2)x}{q-1}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{(x-1)^2}{q-1}\right)}{2\sqrt[3]{(x-1)(q+(x-2)x)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((-1 + x)*(q - 2*x + x^2))^(-1/3),x]
```

```
[Out] (3*(-1 + x)*((q + (-2 + x)*x)/(-1 + q))^(1/3)*Hypergeometric2F1[1
/3, 1/3, 4/3, -((-1 + x)^2/(-1 + q))])/(2*((-1 + x)*(q + (-2 + x)
*x))^(1/3))
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{(-1+x)\,(x^2+q-2\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/((-1+x)*(x^2+q-2*x))^(1/3),x)
```

```
[Out] int(1/((-1+x)*(x^2+q-2*x))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left((x^2+q-2\,x)(x-1)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((x^2 + q - 2*x)*(x - 1))^(-1/3),x, algorithm="maxima")
```

```
[Out] integrate(((x^2 + q - 2*x)*(x - 1))^(-1/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((x^2 + q - 2*x)*(x - 1))^(-1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((-1+x)*(x**2+q-2*x))**(1/3),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left((x^2 + q - 2\,x)(x - 1)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((x^2 + q - 2*x)*(x - 1))^(-1/3),x, algorithm="giac")
```

```
[Out] integrate(((x^2 + q - 2*x)*(x - 1))^(-1/3), x)
```

3.43 $\int \frac{1}{x\sqrt[3]{(-1+x)\left(q-2qx+x^2\right)}}\,dx$

Optimal. Leaf size=118

$$-\frac{3\log\left(\sqrt[3]{(x-1)\left(-2qx+q+x^2\right)}-\sqrt[3]{q}(x-1)\right)}{4\sqrt[3]{q}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{q}(x-1)}{\sqrt{3}\sqrt[3]{(x-1)\left(-2qx+q+x^2\right)}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt[3]{q}}+\frac{\log(1-x)}{4\sqrt[3]{q}}+\frac{\log(x)}{2\sqrt[3]{q}}$$

```
[Out] (Sqrt[3]*ArcTan[1/Sqrt[3] + (2*q^(1/3)*(-1 + x))/(Sqrt[3]*((-1 +
x)*(q - 2*q*x + x^2))^(1/3))])/(2*q^(1/3)) + Log[1 - x]/(4*q^(1/3
)) + Log[x]/(2*q^(1/3)) - (3*Log[-(q^(1/3)*(-1 + x)) + ((-1 + x)*
(q - 2*q*x + x^2))^(1/3)])/(4*q^(1/3))
```

Rubi [F] time = 27.6796, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.

$$\text{Int}\left(\frac{1}{x\sqrt[3]{(-1+x)\left(q-2qx+x^2\right)}},x\right)$$

Verification is Not applicable to the result.

```
[In]  Int[1/(x*((-1 + x)*(q - 2*q*x + x^2))^(1/3)),x]
```

```
[Out] ((-1 - 2*q - (1 - 5*q + 4*q^2 + (1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqr
t[3]*Sqrt[(1 - q)^3*q])^(2/3))/(1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt
[3]*Sqrt[(1 - q)^3*q])^(1/3) + 3*x)^(1/3)*(-1 + 5*q - 4*q^2 + ((1
 - 4*q)^2*(1 - q)^2)/(1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(
1 - q)^3*q])^(2/3) + (1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(
1 - q)^3*q])^(2/3) - ((1 - 5*q + 4*q^2 + (1 + 6*q - 15*q^2 + 8*q^
3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(2/3))*(1 + 2*q - 3*x))/(1 + 6*q
 - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(1/3) + (-1 - 2*
q + 3*x)^2)^(1/3)*Defer[Subst][Defer[Int][1/(((1 + 2*q)/3 + x)*(-
(1 - 5*q + 4*q^2 + (1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1
- q)^3*q])^(2/3))/(3*(1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(
1 - q)^3*q])^(1/3)) + x)^(1/3)*((-1 + 5*q - 4*q^2 + ((1 - 4*q)^2*
(1 - q)^2)/(1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q
])^(2/3) + (1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q
])^(2/3))/9 + ((1 - 5*q + 4*q^2 + (1 + 6*q - 15*q^2 + 8*q^3 + 3*S
qrt[3]*Sqrt[(1 - q)^3*q])^(2/3))*x)/(3*(1 + 6*q - 15*q^2 + 8*q^3
+ 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(1/3)) + x^2)^(1/3)), x], x, (-1 -
 2*q)/3 + x])/(3*(-q + 3*q*x - (1 + 2*q)*x^2 + x^3)^(1/3))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt[3]{x-1}\sqrt[3]{-2qx+q+x^2}\int\frac{1}{x\sqrt[3]{x-1}\sqrt[3]{-2qx+q+x^2}}\,dx}{\sqrt[3]{3qx-q+x^3+x^2(-2q-1)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/((-1+x)*(-2*q*x+x**2+q))**(1/3),x)
```

```
[Out] (x - 1)**(1/3)*(-2*q*x + q + x**2)**(1/3)*Integral(1/(x*(x - 1)**
(1/3)*(-2*q*x + q + x**2)**(1/3)), x)/(3*q*x - q + x**3 + x**2*(-
2*q - 1))**(1/3)
```

Mathematica [C] time = 0.22792, size = 72, normalized size = 0.61

$$\frac{3(x-1)\sqrt[3]{-\frac{-2qx+q+x^2}{(q-1)x^2}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};\frac{q(x-1)^2}{(q-1)x^2}\right)}{2\sqrt[3]{(x-1)\left(-2qx+q+x^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*((-1 + x)*(q - 2*q*x + x^2))^(1/3)),x]
```

```
[Out] (3*(-1 + x)*(-((q - 2*q*x + x^2)/((-1 + q)*x^2)))^(1/3)*Hypergeom
etric2F1[1/3, 1/3, 4/3, (q*(-1 + x)^2)/((-1 + q)*x^2)])/(2*((-1 +
 x)*(q - 2*q*x + x^2))^(1/3))
```

Maple [F] time = 0.028, size = 0, normalized size = 0.

$$\int \frac{1}{x}\frac{1}{\sqrt[3]{(-1+x)\left(-2\,qx+x^2+q\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/((-1+x)*(-2*q*x+x^2+q))^(1/3),x)
```

```
[Out] int(1/x/((-1+x)*(-2*q*x+x^2+q))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-\left(2\,qx-x^2-q\right)(x-1)\right)^{\frac{1}{3}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-(2*q*x - x^2 - q)*(x - 1))^(1/3)*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((-(2*q*x - x^2 - q)*(x - 1))^(1/3)*x), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-(2*q*x - x^2 - q)*(x - 1))^(1/3)*x),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/((-1+x)*(-2*q*x+x**2+q))**(1/3),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-(2\,qx - x^2 - q)(x - 1)\right)^{\frac{1}{3}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-(2*q*x - x^2 - q)*(x - 1))^(1/3)*x),x, algorithm="giac")
```

```
[Out] integrate(1/((-(2*q*x - x^2 - q)*(x - 1))^(1/3)*x), x)
```

3.44 $\int \frac{2-(1+k)x}{\sqrt[3]{(1-x)x(1-kx)}(1-(1+k)x)}\,dx$

Optimal. Leaf size=111

$$\frac{\log(x)}{2\sqrt[3]{k}}+\frac{\log(1-(k+1)x)}{2\sqrt[3]{k}}-\frac{3\log\left(\sqrt[3]{(1-x)x(1-kx)}-\sqrt[3]{k}x\right)}{2\sqrt[3]{k}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{k}x}{\sqrt[3]{(1-x)x(1-kx)}}+1}{\sqrt{3}}\right)}{\sqrt[3]{k}}$$

```
[Out] (Sqrt[3]*ArcTan[(1 + (2*k^(1/3)*x)/((1 - x)*x*(1 - k*x))^(1/3))/S
qrt[3]])/k^(1/3) + Log[x]/(2*k^(1/3)) + Log[1 - (1 + k)*x]/(2*k^(
1/3)) - (3*Log[-(k^(1/3)*x) + ((1 - x)*x*(1 - k*x))^(1/3)])/(2*k^
(1/3))
```

Rubi [F] time = 1.33793, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.

$$\text{Int}\left(\frac{2-(1+k)x}{\sqrt[3]{(1-x)x(1-kx)}(1-(1+k)x)},x\right)$$

Verification is Not applicable to the result.

```
[In] Int[(2 - (1 + k)*x)/(((1 - x)*x*(1 - k*x))^(1/3)*(1 - (1 + k)*x)),x]
```

```
[Out] (3*(1 - x)^(1/3)*x*(1 - k*x)^(1/3)*AppellF1[2/3, 1/3, 1/3, 5/3, x
, k*x])/(2*((1 - x)*x*(1 - k*x))^(1/3)) + ((1 - x)^(1/3)*x^(1/3)*
(1 - k*x)^(1/3)*Defer[Int][1/((1 - x)^(1/3)*x^(1/3)*(1 + (-1 - k)
*x)*(1 - k*x)^(1/3)), x])/((1 - x)*x*(1 - k*x))^(1/3)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((2-(1+k)*x)/((1-x)*x*(-k*x+1))**(1/3)/(1-(1+k)*x),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 1.23486, size = 0, normalized size = 0.

$$\int \frac{2-(1+k)x}{\sqrt[3]{(1-x)x(1-kx)}(1-(1+k)x)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(2 - (1 + k)*x)/(((1 - x)*x*(1 - k*x))^(1/3)*(1 - (1 + k)*x)),x]
```

```
[Out] Integrate[(2 - (1 + k)*x)/(((1 - x)*x*(1 - k*x))^(1/3)*(1 - (1 +
k)*x)), x]
```

Maple [F] time = 0.062, size = 0, normalized size = 0.

$$\int \frac{2-(1+k)\,x}{1-(1+k)\,x}\,\frac{1}{\sqrt[3]{(1-x)\,x\,(-kx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2-(1+k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(1-(1+k)*x),x)
```

```
[Out] int((2-(1+k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(1-(1+k)*x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(k+1)x-2}{((kx-1)(x-1)x)^{\frac{1}{3}}\,((k+1)x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((k + 1)*x - 2)/(((k*x - 1)*(x - 1)*x)^(1/3)*((k + 1)*x - 1)),x, algorit
```

```
[Out] integrate(((k + 1)*x - 2)/(((k*x - 1)*(x - 1)*x)^(1/3)*((k + 1)*x
 - 1)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((k + 1)*x - 2)/(((k*x - 1)*(x - 1)*x)^(1/3)*((k + 1)*x - 1)),x, algorit
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2-(1+k)*x)/((1-x)*x*(-k*x+1))**(1/3)/(1-(1+k)*x),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(k+1)x-2}{((kx-1)(x-1)x)^{\frac{1}{3}}\,((k+1)x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((k + 1)*x - 2)/(((k*x - 1)*(x - 1)*x)^(1/3)*((k + 1)*x - 1)),x, algorit
```

```
[Out] integrate(((k + 1)*x - 2)/(((k*x - 1)*(x - 1)*x)^(1/3)*((k + 1)*x
 - 1)), x)
```

3.45 $\int \frac{1-kx}{(1+(-2+k)x)((1-x)x(1-kx))^{2/3}}\,dx$

Optimal. Leaf size=176

$$\frac{\log(1-(2-k)x)}{2^{2/3}\sqrt[3]{1-k}}+\frac{\log(1-kx)}{2\,2^{2/3}\sqrt[3]{1-k}}-\frac{3\log\left(kx+2^{2/3}\sqrt[3]{1-k}\sqrt[3]{(1-x)x(1-kx)}-1\right)}{2\,2^{2/3}\sqrt[3]{1-k}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{\sqrt[3]{2}(1-kx)}{\sqrt[3]{1-k}\sqrt[3]{(1-x)x(1-kx)}}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt[3]{1-k}}$$

```
[Out] -((Sqrt[3]*ArcTan[(1 + (2^(1/3)*(1 - k*x))/((1 - k)^(1/3)*((1 - x
)*x*(1 - k*x))^(1/3)))/Sqrt[3]])/(2^(2/3)*(1 - k)^(1/3))) + Log[1
 - (2 - k)*x]/(2^(2/3)*(1 - k)^(1/3)) + Log[1 - k*x]/(2*2^(2/3)*(
1 - k)^(1/3)) - (3*Log[-1 + k*x + 2^(2/3)*(1 - k)^(1/3)*((1 - x)*
x*(1 - k*x))^(1/3)])/(2*2^(2/3)*(1 - k)^(1/3))
```

Rubi [F] time = 0.819896, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$

$$\text{Int}\left(\frac{1-kx}{(1+(-2+k)x)((1-x)x(1-kx))^{2/3}},x\right)$$

Verification is Not applicable to the result.

```
[In]  Int[(1 - k*x)/((1 + (-2 + k)*x)*((1 - x)*x*(1 - k*x))^(2/3)),x]
```

```
[Out] ((1 - x)^(2/3)*x^(2/3)*(1 - k*x)^(2/3)*Defer[Int][(1 - k*x)^(1/3)
/((1 - x)^(2/3)*x^(2/3)*(1 + (-2 + k)*x)), x])/((1 - x)*x*(1 - k*
x))^(2/3)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-k*x+1)/(1+(-2+k)*x)/((1-x)*x*(-k*x+1))**(2/3),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.41476, size = 0, normalized size = 0.

$$\int \frac{1-kx}{(1+(-2+k)x)((1-x)x(1-kx))^{2/3}}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(1 - k*x)/((1 + (-2 + k)*x)*((1 - x)*x*(1 - k*x))^(2/3)),x]
```

```
[Out] Integrate[(1 - k*x)/((1 + (-2 + k)*x)*((1 - x)*x*(1 - k*x))^(2/3)
), x]
```

Maple [F] time = 0.073, size = 0, normalized size = 0.

$$\int \frac{-kx+1}{1+(-2+k)\,x}\left((1-x)\,x\,(-kx+1)\right)^{-\frac{2}{3}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-k*x+1)/(1+(-2+k)*x)/((1-x)*x*(-k*x+1))^(2/3),x)
```

```
[Out] int((-k*x+1)/(1+(-2+k)*x)/((1-x)*x*(-k*x+1))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{kx-1}{\left((kx-1)(x-1)x\right)^{\frac{2}{3}}\left((k-2)x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(k*x - 1)/(((k*x - 1)*(x - 1)*x)^(2/3)*((k - 2)*x + 1)),x, algorithm="m
```

```
[Out] -integrate((k*x - 1)/(((k*x - 1)*(x - 1)*x)^(2/3)*((k - 2)*x + 1)
), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(k*x - 1)/(((k*x - 1)*(x - 1)*x)^(2/3)*((k - 2)*x + 1)),x, algorithm="f
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-k*x+1)/(1+(-2+k)*x)/((1-x)*x*(-k*x+1))**(2/3),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{kx-1}{\left((kx-1)(x-1)x\right)^{\frac{2}{3}}\left((k-2)x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(k*x - 1)/(((k*x - 1)*(x - 1)*x)^(2/3)*((k - 2)*x + 1)),x, algorithm="g
```

```
[Out] integrate(-(k*x - 1)/(((k*x - 1)*(x - 1)*x)^(2/3)*((k - 2)*x + 1)
), x)
```

3.46 $\int \frac{a+bx+cx^2}{(1-x+x^2)\sqrt[3]{1-x^3}}\,dx$

Optimal. Leaf size=326

$$\frac{(a-b-2c)\left(-3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)-2\sqrt{3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)\right)}{12\sqrt[3]{2}}-\frac{(a-b-2c)\left(2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)-3\log\left(\sqrt[3]{1-x^3}+\sqrt[3]{2}x\right)\right)}{12\sqrt[3]{2}}+\frac{(a+b)\left(-3\log\left(\sqrt[3]{1-x^3}-\sqrt[3]{2}(x-1)\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}(x-1)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)+\log\left(-3x^3+6x^2-6x+3\right)\right)}{4\sqrt[3]{2}}-\frac{1}{6}c\left(-2\log\left(\frac{x}{\sqrt[3]{1-x^3}}+1\right)+2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)+\log\left(-\frac{x}{\sqrt[3]{1-x^3}}+\frac{x^2}{(1-x^3)^{2/3}}+1\right)\right)$$

```
[Out] -(c*(2*Sqrt[3]*ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]] + Log[
1 + x^2/(1 - x^3)^(2/3) - x/(1 - x^3)^(1/3)] - 2*Log[1 + x/(1 - x
^3)^(1/3)]))/6 + ((a - b - 2*c)*(-2*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(
1 - x^3)^(1/3))/Sqrt[3]] - 3*Log[2^(1/3) - (1 - x^3)^(1/3)]))/(12
*2^(1/3)) + ((a + b)*(2*Sqrt[3]*ArcTan[(1 + (2*2^(1/3)*(-1 + x))/
(1 - x^3)^(1/3))/Sqrt[3]] + Log[3 - 6*x + 6*x^2 - 3*x^3] - 3*Log[
-(2^(1/3)*(-1 + x)) + (1 - x^3)^(1/3)]))/(4*2^(1/3)) - ((a - b -
2*c)*(2*Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3
]] - 3*Log[2^(1/3)*x + (1 - x^3)^(1/3)]))/(12*2^(1/3))
```

Rubi [C] time = 1.52529, antiderivative size = 576, normalized size of antiderivative = 1.77, number of steps used = 7, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094

$$-\frac{\log\left(2\,2^{2/3}\sqrt[3]{1-x^3}+2x-i\sqrt{3}+1\right)\left(3ib-\sqrt{3}\left(2a+b-i\sqrt{3}c-c\right)\right)}{4\sqrt[3]{2}\left(\sqrt{3}+i\right)}-\frac{\log\left(2\,2^{2/3}\sqrt[3]{1-x^3}+2x+i\sqrt{3}+1\right)\left(\sqrt{3}\left(2a+b+i\sqrt{3}c-c\right)+3ib\right)}{4\sqrt[3]{2}\left(-\sqrt{3}+i\right)}-\frac{\tan^{-1}\left(\frac{2-\frac{\sqrt[3]{2}\left(2x-i\sqrt{3}+1\right)}{\sqrt[3]{1-x^3}}}{2\sqrt{3}}\right)\left(2a-i\sqrt{3}b+b-\left(1+i\sqrt{3}\right)c\right)}{2\sqrt[3]{2}\left(\sqrt{3}+i\right)}+\frac{\tan^{-1}\left(\frac{2-\frac{\sqrt[3]{2}\left(2x+i\sqrt{3}+1\right)}{\sqrt[3]{1-x^3}}}{2\sqrt{3}}\right)\left(2a+i\sqrt{3}b+b+i\sqrt{3}c-c\right)}{2\sqrt[3]{2}\left(-\sqrt{3}+i\right)}+\frac{\log\left(-\left(-2x-i\sqrt{3}+1\right)^2\left(2x-i\sqrt{3}+1\right)\right)\left(3ib-\sqrt{3}\left(2a+b-i\sqrt{3}c-c\right)\right)}{12\sqrt[3]{2}\left(\sqrt{3}+i\right)}+\frac{\log\left(-\left(-2x+i\sqrt{3}+1\right)^2\left(2x+i\sqrt{3}+1\right)\right)\left(\sqrt{3}\left(2a+b+i\sqrt{3}c-c\right)+3ib\right)}{12\sqrt[3]{2}\left(-\sqrt{3}+i\right)}+\frac{1}{2}c\log\left(\sqrt[3]{1-x^3}+x\right)-\frac{c\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x + c*x^2)/((1 - x + x^2)*(1 - x^3)^(1/3)),x]
```

```
[Out] -((c*ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3]) - ((2*
a + b - I*Sqrt[3]*b - (1 + I*Sqrt[3])*c)*ArcTan[(2 - (2^(1/3)*(1
- I*Sqrt[3] + 2*x))/(1 - x^3)^(1/3))/(2*Sqrt[3])])/(2*2^(1/3)*(I
+ Sqrt[3])) + ((2*a + b + I*Sqrt[3]*b - c + I*Sqrt[3]*c)*ArcTan[(
2 - (2^(1/3)*(1 + I*Sqrt[3] + 2*x))/(1 - x^3)^(1/3))/(2*Sqrt[3])]
)/(2*2^(1/3)*(I - Sqrt[3])) + (((3*I)*b - Sqrt[3]*(2*a + b - c -
I*Sqrt[3]*c))*Log[-((1 - I*Sqrt[3] - 2*x)^2*(1 - I*Sqrt[3] + 2*x)
)])/(12*2^(1/3)*(I + Sqrt[3])) + (((3*I)*b + Sqrt[3]*(2*a + b - c
 + I*Sqrt[3]*c))*Log[-((1 + I*Sqrt[3] - 2*x)^2*(1 + I*Sqrt[3] + 2
*x))])/(12*2^(1/3)*(I - Sqrt[3])) + (c*Log[x + (1 - x^3)^(1/3)])/
2 - (((3*I)*b - Sqrt[3]*(2*a + b - c - I*Sqrt[3]*c))*Log[1 - I*Sq
rt[3] + 2*x + 2*2^(2/3)*(1 - x^3)^(1/3)])/(4*2^(1/3)*(I + Sqrt[3]
)) - (((3*I)*b + Sqrt[3]*(2*a + b - c + I*Sqrt[3]*c))*Log[1 + I*S
qrt[3] + 2*x + 2*2^(2/3)*(1 - x^3)^(1/3)])/(4*2^(1/3)*(I - Sqrt[3
]))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**2+b*x+a)/(x**2-x+1)/(-x**3+1)**(1/3),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.165259, size = 0, normalized size = 0.

$$\int \frac{a+bx+cx^2}{(1-x+x^2)\sqrt[3]{1-x^3}}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(a + b*x + c*x^2)/((1 - x + x^2)*(1 - x^3)^(1/3)),x]

[Out] Integrate[(a + b*x + c*x^2)/((1 - x + x^2)*(1 - x^3)^(1/3)), x]
```

Maple [F] time = 0.183, size = 0, normalized size = 0.

$$\int \frac{cx^2+bx+a}{x^2-x+1}\frac{1}{\sqrt[3]{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^2+b*x+a)/(x^2-x+1)/(-x^3+1)^(1/3),x)

[Out] int((c*x^2+b*x+a)/(x^2-x+1)/(-x^3+1)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{cx^2+bx+a}{(-x^3+1)^{\frac{1}{3}}(x^2-x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2 + b*x + a)/((-x^3 + 1)^(1/3)*(x^2 - x + 1)),x, algorithm="maxima'

[Out] integrate((c*x^2 + b*x + a)/((-x^3 + 1)^(1/3)*(x^2 - x + 1)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2 + b*x + a)/((-x^3 + 1)^(1/3)*(x^2 - x + 1)),x, algorithm="fricas'

[Out] Exception raised: TypeError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+bx+cx^2}{\sqrt[3]{-(x-1)(x^2+x+1)}(x^2-x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**2+b*x+a)/(x**2-x+1)/(-x**3+1)**(1/3),x)
```

```
[Out] Integral((a + b*x + c*x**2)/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x*
*2 - x + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{cx^2 + bx + a}{(-x^3 + 1)^{\frac{1}{3}}(x^2 - x + 1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2 + b*x + a)/((-x^3 + 1)^(1/3)*(x^2 - x + 1)),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)/((-x^3 + 1)^(1/3)*(x^2 - x + 1)), x)
```

3.47 $\int \frac{1}{(3-2x)^{11/2}(1+x+2x^2)^5}\,dx$

Optimal. Leaf size=407

$$\frac{x}{28(3-2x)^{9/2}\left(2x^2+x+1\right)^4}+\frac{5(4377x+3049)}{153664(3-2x)^{9/2}\left(2x^2+x+1\right)}+\frac{3049x+1387}{32928(3-2x)^{9/2}\left(2x^2+x+1\right)^2}+\frac{73x+23}{1176(3-2x)^{9/2}\left(2x^2+x+1\right)^3}-\frac{38225}{240945152\sqrt{3-2x}}-\frac{141045}{120472576(3-2x)^{3/2}}-\frac{38491}{8605184(3-2x)^{5/2}}-\frac{462025}{30118144(3-2x)^{7/2}}-\frac{19255}{395136(3-2x)^{9/2}}+\frac{5\sqrt{\frac{1}{2}\left(40815066112\sqrt{14}-149046503977\right)}\log\left(-2x-\sqrt{7+2\sqrt{14}}\sqrt{3-2x}+\sqrt{14}+3\right)}{6746464256}-\frac{5\sqrt{\frac{1}{2}\left(40815066112\sqrt{14}-149046503977\right)}\log\left(-2x+\sqrt{7+2\sqrt{14}}\sqrt{3-2x}+\sqrt{14}+3\right)}{6746464256}+\frac{5\sqrt{\frac{1}{2}\left(149046503977+40815066112\sqrt{14}\right)}\tan^{-1}\left(\frac{\sqrt{7+2\sqrt{14}}-2\sqrt{3-2x}}{\sqrt{2\sqrt{14}-7}}\right)}{3373232128}-\frac{5\sqrt{\frac{1}{2}\left(149046503977+40815066112\sqrt{14}\right)}\tan^{-1}\left(\frac{2\sqrt{3-2x}+\sqrt{7+2\sqrt{14}}}{\sqrt{2\sqrt{14}-7}}\right)}{3373232128}$$

```
[Out] -19255/(395136*(3 - 2*x)^(9/2)) - 462025/(30118144*(3 - 2*x)^(7/2
)) - 38491/(8605184*(3 - 2*x)^(5/2)) - 141045/(120472576*(3 - 2*x
)^(3/2)) - 38225/(240945152*Sqrt[3 - 2*x]) + x/(28*(3 - 2*x)^(9/2
)*(1 + x + 2*x^2)^4) + (23 + 73*x)/(1176*(3 - 2*x)^(9/2)*(1 + x +
 2*x^2)^3) + (1387 + 3049*x)/(32928*(3 - 2*x)^(9/2)*(1 + x + 2*x^
2)^2) + (5*(3049 + 4377*x))/(153664*(3 - 2*x)^(9/2)*(1 + x + 2*x^
2)) + (5*Sqrt[(149046503977 + 40815066112*Sqrt[14])/2]*ArcTan[(Sq
rt[7 + 2*Sqrt[14]] - 2*Sqrt[3 - 2*x])/Sqrt[-7 + 2*Sqrt[14]]])/337
3232128 - (5*Sqrt[(149046503977 + 40815066112*Sqrt[14])/2]*ArcTan
[(Sqrt[7 + 2*Sqrt[14]] + 2*Sqrt[3 - 2*x])/Sqrt[-7 + 2*Sqrt[14]]])
/3373232128 + (5*Sqrt[(-149046503977 + 40815066112*Sqrt[14])/2]*L
og[3 + Sqrt[14] - Sqrt[7 + 2*Sqrt[14]]*Sqrt[3 - 2*x] - 2*x])/6746
464256 - (5*Sqrt[(-149046503977 + 40815066112*Sqrt[14])/2]*Log[3
+ Sqrt[14] + Sqrt[7 + 2*Sqrt[14]]*Sqrt[3 - 2*x] - 2*x])/674646425
6
```

Rubi [A] time = 1.47675, antiderivative size = 407, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45

$$\frac{x}{28(3-2x)^{9/2}\left(2x^2+x+1\right)^4}+\frac{5(4377x+3049)}{153664(3-2x)^{9/2}\left(2x^2+x+1\right)}+\frac{3049x+1387}{32928(3-2x)^{9/2}\left(2x^2+x+1\right)^2}+\frac{73x+23}{1176(3-2x)^{9/2}\left(2x^2+x+1\right)^3}-\frac{38225}{240945152\sqrt{3-2x}}-\frac{141045}{120472576(3-2x)^{3/2}}-\frac{38491}{8605184(3-2x)^{5/2}}-\frac{462025}{30118144(3-2x)^{7/2}}-\frac{19255}{395136(3-2x)^{9/2}}+\frac{5\sqrt{\frac{1}{2}\left(40815066112\sqrt{14}-149046503977\right)}\log\left(-2x-\sqrt{7+2\sqrt{14}}\sqrt{3-2x}+\sqrt{14}+3\right)}{6746464256}-\frac{5\sqrt{\frac{1}{2}\left(40815066112\sqrt{14}-149046503977\right)}\log\left(-2x+\sqrt{7+2\sqrt{14}}\sqrt{3-2x}+\sqrt{14}+3\right)}{6746464256}+\frac{5\sqrt{\frac{1}{2}\left(149046503977+40815066112\sqrt{14}\right)}\tan^{-1}\left(\frac{\sqrt{7+2\sqrt{14}}-2\sqrt{3-2x}}{\sqrt{2\sqrt{14}-7}}\right)}{3373232128}-\frac{5\sqrt{\frac{1}{2}\left(149046503977+40815066112\sqrt{14}\right)}\tan^{-1}\left(\frac{2\sqrt{3-2x}+\sqrt{7+2\sqrt{14}}}{\sqrt{2\sqrt{14}-7}}\right)}{3373232128}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((3 - 2*x)^(11/2)*(1 + x + 2*x^2)^5),x]
```

```
[Out] -19255/(395136*(3 - 2*x)^(9/2)) - 462025/(30118144*(3 - 2*x)^(7/2
)) - 38491/(8605184*(3 - 2*x)^(5/2)) - 141045/(120472576*(3 - 2*x
)^(3/2)) - 38225/(240945152*Sqrt[3 - 2*x]) + x/(28*(3 - 2*x)^(9/2
)*(1 + x + 2*x^2)^4) + (23 + 73*x)/(1176*(3 - 2*x)^(9/2)*(1 + x +
 2*x^2)^3) + (1387 + 3049*x)/(32928*(3 - 2*x)^(9/2)*(1 + x + 2*x^
2)^2) + (5*(3049 + 4377*x))/(153664*(3 - 2*x)^(9/2)*(1 + x + 2*x^
2)) + (5*Sqrt[(149046503977 + 40815066112*Sqrt[14])/2]*ArcTan[(Sq
rt[7 + 2*Sqrt[14]] - 2*Sqrt[3 - 2*x])/Sqrt[-7 + 2*Sqrt[14]]])/337
3232128 - (5*Sqrt[(149046503977 + 40815066112*Sqrt[14])/2]*ArcTan
[(Sqrt[7 + 2*Sqrt[14]] + 2*Sqrt[3 - 2*x])/Sqrt[-7 + 2*Sqrt[14]]])
/3373232128 + (5*Sqrt[(-149046503977 + 40815066112*Sqrt[14])/2]*L
og[3 + Sqrt[14] - Sqrt[7 + 2*Sqrt[14]]*Sqrt[3 - 2*x] - 2*x])/6746
464256 - (5*Sqrt[(-149046503977 + 40815066112*Sqrt[14])/2]*Log[3
+ Sqrt[14] + Sqrt[7 + 2*Sqrt[14]]*Sqrt[3 - 2*x] - 2*x])/674646425
6
```

Rubi in Sympy [A] time = 48.1711, size = 478, normalized size = 1.17

$$\frac{x}{28\left(-2x+3\right)^{\frac{9}{2}}\left(2x^{2}+x+1\right)^{4}}+\frac{\sqrt{14}\left(-48353174284800\sqrt{14}+732027212367360\right)\log\left(-2x-\sqrt{7+2\sqrt{14}}\sqrt{-2x+3}+3+\sqrt{14}\right)}{17068042484057112576\sqrt{7+2\sqrt{14}}}-\frac{\sqrt{14}\left(-48353174284800\sqrt{14}+732027212367360\right)\log\left(-2x+\sqrt{7+2\sqrt{14}}\sqrt{-2x+3}+3+\sqrt{14}\right)}{17068042484057112576\sqrt{7+2\sqrt{14}}}+\frac{\sqrt{14}\left(-1464054424734720\sqrt{7+2\sqrt{14}}+\frac{\sqrt{7+2\sqrt{14}}\left(-96706348569600\sqrt{14}+1464054424734720\right)}{2}\right)\operatorname{atan}\left(\frac{2\sqrt{-2x+3}-\sqrt{7+2\sqrt{14}}}{\sqrt{-7+2\sqrt{14}}}\right)}{8534021242028556288\sqrt{-7+2\sqrt{14}}\sqrt{7+2\sqrt{14}}}+\frac{\sqrt{14}\left(-1464054424734720\sqrt{7+2\sqrt{14}}+\frac{\sqrt{7+2\sqrt{14}}\left(-96706348569600\sqrt{14}+1464054424734720\right)}{2}\right)\operatorname{atan}\left(\frac{2\sqrt{-2x+3}+\sqrt{7+2\sqrt{14}}}{\sqrt{-7+2\sqrt{14}}}\right)}{8534021242028556288\sqrt{-7+2\sqrt{14}}\sqrt{7+2\sqrt{14}}}-\frac{38225}{240945152\sqrt{-2x+3}}-\frac{141045}{120472576\left(-2x+3\right)^{\frac{3}{2}}}-\frac{38491}{8605184\left(-2x+3\right)^{\frac{5}{2}}}-\frac{462025}{30118144\left(-2x+3\right)^{\frac{7}{2}}}+\frac{28616x+9016}{460992\left(-2x+3\right)^{\frac{9}{2}}\left(2x^{2}+x+1\right)^{3}}+\frac{16732912x+7611856}{180708864\left(-2x+3\right)^{\frac{9}{2}}\left(2x^{2}+x+1\right)^{2}}+\frac{5044404960x+3513911520}{35418937344\left(-2x+3\right)^{\frac{9}{2}}\left(2x^{2}+x+1\right)}-\frac{19255}{395136\left(-2x+3\right)^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(3-2*x)**(11/2)/(2*x**2+x+1)**5,x)
```

```
[Out] x/(28*(-2*x + 3)**(9/2)*(2*x**2 + x + 1)**4) + sqrt(14)*(-4835317
4284800*sqrt(14) + 732027212367360)*log(-2*x - sqrt(7 + 2*sqrt(14
))*sqrt(-2*x + 3) + 3 + sqrt(14))/(17068042484057112576*sqrt(7 +
2*sqrt(14))) - sqrt(14)*(-48353174284800*sqrt(14) + 7320272123673
60)*log(-2*x + sqrt(7 + 2*sqrt(14))*sqrt(-2*x + 3) + 3 + sqrt(14)
)/(17068042484057112576*sqrt(7 + 2*sqrt(14))) + sqrt(14)*(-146405
4424734720*sqrt(7 + 2*sqrt(14)) + sqrt(7 + 2*sqrt(14))*(-96706348
569600*sqrt(14) + 1464054424734720)/2)*atan((2*sqrt(-2*x + 3) - s
qrt(7 + 2*sqrt(14)))/sqrt(-7 + 2*sqrt(14)))/(8534021242028556288*
sqrt(-7 + 2*sqrt(14))*sqrt(7 + 2*sqrt(14))) + sqrt(14)*(-14640544
24734720*sqrt(7 + 2*sqrt(14)) + sqrt(7 + 2*sqrt(14))*(-9670634856
9600*sqrt(14) + 1464054424734720)/2)*atan((2*sqrt(-2*x + 3) + sqr
t(7 + 2*sqrt(14)))/sqrt(-7 + 2*sqrt(14)))/(8534021242028556288*sq
rt(-7 + 2*sqrt(14))*sqrt(7 + 2*sqrt(14))) - 38225/(240945152*sqrt
(-2*x + 3)) - 141045/(120472576*(-2*x + 3)**(3/2)) - 38491/(86051
84*(-2*x + 3)**(5/2)) - 462025/(30118144*(-2*x + 3)**(7/2)) + (28
616*x + 9016)/(460992*(-2*x + 3)**(9/2)*(2*x**2 + x + 1)**3) + (1
6732912*x + 7611856)/(180708864*(-2*x + 3)**(9/2)*(2*x**2 + x + 1
```

```
)**2) + (5044404960*x + 3513911520)/(35418937344*(-2*x + 3)**(9/2
)*(2*x**2 + x + 1)) - 19255/(395136*(-2*x + 3)**(9/2))
```

Mathematica [C] time = 2.25894, size = 206, normalized size = 0.51

$$\frac{\frac{45i\left(284993\sqrt{7}+53515i\right)\tan^{-1}\left(\frac{\sqrt{6-4x}}{\sqrt{-7-i\sqrt{7}}}\right)}{\sqrt{-\frac{1}{2}i\left(\sqrt{7}-7i\right)}}-\frac{45i\left(284993\sqrt{7}-53515i\right)\tan^{-1}\left(\frac{\sqrt{6-4x}}{\sqrt{-7+i\sqrt{7}}}\right)}{\sqrt{\frac{1}{2}i\left(\sqrt{7}+7i\right)}}-\frac{14\left(88070400x^{12}-677249280x^{11}+1873554048x^{10}-2443779648x^{9}+2343370\right.}{}}{30359089152}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((3 - 2*x)^(11/2)*(1 + x + 2*x^2)^5),x]
```

```
[Out] ((-14*(40289347 - 429812744*x + 135202154*x^2 - 1073855156*x^3 +
1627773523*x^4 - 1470758860*x^5 + 2888625656*x^6 - 3106712560*x^7
 + 2343370048*x^8 - 2443779648*x^9 + 1873554048*x^10 - 677249280*
x^11 + 88070400*x^12))/((3 - 2*x)^(9/2)*(1 + x + 2*x^2)^4) + ((45
*I)*(53515*I + 284993*Sqrt[7])*ArcTan[Sqrt[6 - 4*x]/Sqrt[-7 - I*S
qrt[7]]])/Sqrt[(-I/2)*(-7*I + Sqrt[7])] - ((45*I)*(-53515*I + 284
993*Sqrt[7])*ArcTan[Sqrt[6 - 4*x]/Sqrt[-7 + I*Sqrt[7]]])/Sqrt[(I/
2)*(7*I + Sqrt[7])])/30359089152
```

Maple [A] time = 0.132, size = 584, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(3-2*x)^(11/2)/(2*x^2+x+1)^5,x)
```

```
[Out] 1/6588344*(567651623/32*(3-2*x)^(1/2)-6194606411/192*(3-2*x)^(3/2
)+9801432515/384*(3-2*x)^(5/2)-8763772549/768*(3-2*x)^(7/2)+14963
0663/48*(3-2*x)^(9/2)-200063633/384*(3-2*x)^(11/2)+18969965/384*(
3-2*x)^(13/2)-526135/256*(3-2*x)^(15/2))/((3-2*x)^2-7+14*x)^4-731
595/13492928512*ln(3-2*x+14^(1/2)-(3-2*x)^(1/2)*(7+2*14^(1/2))^(1
/2))*(7+2*14^(1/2))^(1/2)*14^(1/2)+1424965/6746464256*ln(3-2*x+14
^(1/2)-(3-2*x)^(1/2)*(7+2*14^(1/2))^(1/2))*(7+2*14^(1/2))^(1/2)-7
31595/6746464256/(-7+2*14^(1/2))^(1/2)*arctan((2*(3-2*x)^(1/2)-(7
+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))*(7+2*14^(1/2))*14^(1/2
)+1424965/3373232128/(-7+2*14^(1/2))^(1/2)*arctan((2*(3-2*x)^(1/2
)-(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))*(7+2*14^(1/2))-578
695/3373232128/(-7+2*14^(1/2))^(1/2)*arctan((2*(3-2*x)^(1/2)-(7+2
*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))*14^(1/2)+731595/13492928
512*ln(3-2*x+14^(1/2)+(3-2*x)^(1/2)*(7+2*14^(1/2))^(1/2))*(7+2*14
^(1/2))^(1/2)*14^(1/2)-1424965/6746464256*ln(3-2*x+14^(1/2)+(3-2*
x)^(1/2)*(7+2*14^(1/2))^(1/2))*(7+2*14^(1/2))^(1/2)-731595/674646
4256/(-7+2*14^(1/2))^(1/2)*arctan((2*(3-2*x)^(1/2)+(7+2*14^(1/2))
^(1/2))/(-7+2*14^(1/2))^(1/2))*(7+2*14^(1/2))*14^(1/2)+1424965/33
73232128/(-7+2*14^(1/2))^(1/2)*arctan((2*(3-2*x)^(1/2)+(7+2*14^(1
/2))^(1/2))/(-7+2*14^(1/2))^(1/2))*(7+2*14^(1/2))-578695/33732321
28/(-7+2*14^(1/2))^(1/2)*arctan((2*(3-2*x)^(1/2)+(7+2*14^(1/2))^(
1/2))/(-7+2*14^(1/2))^(1/2))*14^(1/2)+1/151263/(3-2*x)^(9/2)+5/23
5298/(3-2*x)^(7/2)+19/470596/(3-2*x)^(5/2)+185/2823576/(3-2*x)^(3
/2)+505/3294172/(3-2*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(2\,x^{2}+x+1\right)^{5}\left(-2\,x+3\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2*x^2 + x + 1)^5*(-2*x + 3)^(11/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/((2*x^2 + x + 1)^5*(-2*x + 3)^(11/2)), x)
```

Fricas [A] time = 0.280812, size = 1827, normalized size = 4.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2*x^2 + x + 1)^5*(-2*x + 3)^(11/2)),x, algorithm="fricas")
```

```
[Out] 1/948692239236095606784*sqrt(79716926)*2744^(3/4)*(2263908918780*
sqrt(39858463)*sqrt(14)*(256*x^12 - 1024*x^11 + 1280*x^10 - 1024*
x^9 + 1888*x^8 - 1664*x^7 + 560*x^6 - 1280*x^5 + 577*x^4 + 12*x^3
 + 486*x^2 + 108*x + 81)*sqrt(-2*x + 3)*arctan(1116036964*sqrt(39
858463)*(284993*sqrt(14)*sqrt(7) - 1024233*sqrt(7))/(sqrt(7971692
6)*sqrt(39858463)*2744^(1/4)*(21292357711*sqrt(14)*sqrt(7) - 8163
0132224*sqrt(7))*sqrt(-sqrt(14)*(sqrt(79716926)*sqrt(39858463)*27
44^(1/4)*sqrt(-2*x + 3)*(40473233043683538146117038154862650739*s
qrt(14) - 151437300139796837407365155111661919174)*sqrt((21292357
711*sqrt(14) - 81630132224)/(1738097975309635979264*sqrt(14) - 65
05290721706129709735)) + 79716926*sqrt(14)*(280394144603408538329
670261219721*sqrt(14)*(2*x - 3) - 2098285190694476737313101842132
992*x + 3147427786041715105969652763199488) - 3129302298665650491
69122829452744361767044*sqrt(14) + 117088191691441243473976984865
8455837958144)/(280394144603408538329670261219721*sqrt(14) - 1049
142595347238368656550921066496))*sqrt((21292357711*sqrt(14) - 816
30132224)/(1738097975309635979264*sqrt(14) - 65052907217061297097
35)) + 558018482*sqrt(79716926)*2744^(1/4)*sqrt(-2*x + 3)*(212923
57711*sqrt(14) - 81630132224)*sqrt((21292357711*sqrt(14) - 816301
32224)/(1738097975309635979264*sqrt(14) - 6505290721706129709735)
) - 7812258748*sqrt(39858463)*(7645*sqrt(14) - 115739))) + 226390
8918780*sqrt(39858463)*sqrt(14)*(256*x^12 - 1024*x^11 + 1280*x^10
 - 1024*x^9 + 1888*x^8 - 1664*x^7 + 560*x^6 - 1280*x^5 + 577*x^4
+ 12*x^3 + 486*x^2 + 108*x + 81)*sqrt(-2*x + 3)*arctan(1116036964
*sqrt(39858463)*(284993*sqrt(14)*sqrt(7) - 1024233*sqrt(7))/(sqrt
(79716926)*sqrt(39858463)*2744^(1/4)*(21292357711*sqrt(14)*sqrt(7
) - 81630132224*sqrt(7))*sqrt(sqrt(14)*(sqrt(79716926)*sqrt(39858
463)*2744^(1/4)*sqrt(-2*x + 3)*(404732330436835381461170381548626
50739*sqrt(14) - 151437300139796837407365155111661919174)*sqrt((2
1292357711*sqrt(14) - 81630132224)/(1738097975309635979264*sqrt(1
4) - 6505290721706129709735)) - 79716926*sqrt(14)*(28039414460340
8538329670261219721*sqrt(14)*(2*x - 3) - 209828519069447673731310
1842132992*x + 3147427786041715105969652763199488) + 312930229866
565049169122829452744361767044*sqrt(14) - 11708819169144124347397
69848658455837958144)/(280394144603408538329670261219721*sqrt(14)
 - 1049142595347238368656550921066496))*sqrt((21292357711*sqrt(14
) - 81630132224)/(1738097975309635979264*sqrt(14) - 6505290721706
129709735)) + 558018482*sqrt(79716926)*2744^(1/4)*sqrt(-2*x + 3)*
(21292357711*sqrt(14) - 81630132224)*sqrt((21292357711*sqrt(14) -
 81630132224)/(1738097975309635979264*sqrt(14) - 6505290721706129
709735)) + 7812258748*sqrt(39858463)*(7645*sqrt(14) - 115739))) +
 45*sqrt(39858463)*(21292357711*sqrt(14)*sqrt(7)*(256*x^12 - 1024
*x^11 + 1280*x^10 - 1024*x^9 + 1888*x^8 - 1664*x^7 + 560*x^6 - 12
80*x^5 + 577*x^4 + 12*x^3 + 486*x^2 + 108*x + 81) - 81630132224*s
qrt(7)*(256*x^12 - 1024*x^11 + 1280*x^10 - 1024*x^9 + 1888*x^8 -
1664*x^7 + 560*x^6 - 1280*x^5 + 577*x^4 + 12*x^3 + 486*x^2 + 108*
x + 81))*sqrt(-2*x + 3)*log(-996461575/4*sqrt(14)*(sqrt(79716926)
*sqrt(39858463)*2744^(1/4)*sqrt(-2*x + 3)*(4047323304368353814611
7038154862650739*sqrt(14) - 1514373001397968374073651551116619191
74)*sqrt((21292357711*sqrt(14) - 81630132224)/(173809797530963597
9264*sqrt(14) - 6505290721706129709735)) + 79716926*sqrt(14)*(280
394144603408538329670261219721*sqrt(14)*(2*x - 3) - 2098285190694
476737313101842132992*x + 3147427786041715105969652763199488) - 3
12930229866565049169122829452744361767044*sqrt(14) + 117088191691
4412434739769848658455837958144)/(2803941446034085383296702612197
21*sqrt(14) - 1049142595347238368656550921066496)) - 45*sqrt(3985
8463)*(21292357711*sqrt(14)*sqrt(7)*(256*x^12 - 1024*x^11 + 1280*
```

```
x^10 - 1024*x^9 + 1888*x^8 - 1664*x^7 + 560*x^6 - 1280*x^5 + 577*
x^4 + 12*x^3 + 486*x^2 + 108*x + 81) - 81630132224*sqrt(7)*(256*x
^12 - 1024*x^11 + 1280*x^10 - 1024*x^9 + 1888*x^8 - 1664*x^7 + 56
0*x^6 - 1280*x^5 + 577*x^4 + 12*x^3 + 486*x^2 + 108*x + 81))*sqrt
(-2*x + 3)*log(996461575/4*sqrt(14)*(sqrt(79716926)*sqrt(39858463
)*2744^(1/4)*sqrt(-2*x + 3)*(404732330436835381461170381548626507
39*sqrt(14) - 151437300139796837407365155111661919174)*sqrt((2129
2357711*sqrt(14) - 81630132224)/(173809797530963597926 4*sqrt(14)
- 6505290721706129709735)) - 79716926*sqrt(14)*(28039414460340853
8329670261219721*sqrt(14)*(2*x - 3) - 209828519069447673731310184
2132992*x + 314742778604171510596965276319948 8) + 312930229866565
049169122829452744361767044*sqrt(14) - 117088191691441243473976 98
48658455837958144)/(2803941446034085383296702612197 21*sqrt(14) -
10491425953472383686565509210664 96)) - 2*sqrt(79716926)*2744^(1/4
)*(21292357711*sqrt(14)*sqrt(7)*(88070400*x^12 - 677249280*x^11 +
 1873554048*x^10 - 2443779648*x^9 + 2343370048*x^8 - 3106712560*x
^7 + 2888625656*x^6 - 1470758860*x^5 + 1627773523*x^4 - 107385515
6*x^3 + 135202154*x^2 - 429812744*x + 40289347) - 81630132224*sqr
t(7)*(88070400*x^12 - 677249280*x^11 + 1873554048*x^10 - 24437796
48*x^9 + 2343370048*x^8 - 3106712560*x^7 + 2888625656*x^6 - 14707
58860*x^5 + 1627773523*x^4 - 1073855156*x^3 + 135202154*x^2 - 429
812744*x + 40289347))*sqrt((21292357711*sqrt(14) - 81630132224)/(
173809797530963597926 4*sqrt(14) - 6505290721706129709735)))/((212
92357711*sqrt(14)*sqrt(7)*(256*x^12 - 1024*x^11 + 1280*x^10 - 102
4*x^9 + 1888*x^8 - 1664*x^7 + 560*x^6 - 1280*x^5 + 577*x^4 + 12*x
^3 + 486*x^2 + 108*x + 81) - 81630132224*sqrt(7)*(256*x^12 - 1024
*x^11 + 1280*x^10 - 1024*x^9 + 1888*x^8 - 1664*x^7 + 560*x^6 - 12
80*x^5 + 577*x^4 + 12*x^3 + 486*x^2 + 108*x + 81))*sqrt(-2*x + 3)
*sqrt((21292357711*sqrt(14) - 81630132224)/(173809797530963597926
4*sqrt(14) - 6505290721706129709735)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3-2*x)**(11/2)/(2*x**2+x+1)**5,x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2x^2 + x + 1)^5(-2x + 3)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2*x^2 + x + 1)^5*(-2*x + 3)^(11/2)),x, algorithm="giac")

[Out] integrate(1/((2*x^2 + x + 1)^5*(-2*x + 3)^(11/2)), x)
```

3.48 $\int \frac{1}{(3-2x)^{21/2}(1+x+2x^2)^{10}}\,dx$

Optimal. Leaf size=648

result too large to display

```
[Out] 4718120139975/(351733660450816*(3 - 2*x)^(19/2)) - 815900548375/(
629418129227776*(3 - 2*x)^(17/2)) - 3029508823715/(15550330251509
76*(3 - 2*x)^(15/2)) - 13515743021825/(13476952884641792*(3 - 2*x
)^(13/2)) - 5846828446875/(14513641568075776*(3 - 2*x)^(11/2)) -
37283626871975/(261245548225363968*(3 - 2*x)^(9/2)) - 13235516227
2575/(2844673747342852096*(3 - 2*x)^(7/2)) - 11557581705725/(8127
63927812243456*(3 - 2*x)^(5/2)) - 46601678385075/(113786949893714
08384*(3 - 2*x)^(3/2)) - 24229218097975/(22757389978742816768*Sqr
t[3 - 2*x]) + x/(63*(3 - 2*x)^(19/2)*(1 + x + 2*x^2)^9) + (53 + 1
73*x)/(7056*(3 - 2*x)^(19/2)*(1 + x + 2*x^2)^8) + (8477 + 21409*x
)/(691488*(3 - 2*x)^(19/2)*(1 + x + 2*x^2)^7) + (5*(21409 + 47471
*x))/(6453888*(3 - 2*x)^(19/2)*(1 + x + 2*x^2)^6) + (41*(47471 +
92875*x))/(90354432*(3 - 2*x)^(19/2)*(1 + x + 2*x^2)^5) + (41*(34
36375 + 5677637*x))/(5059848192*(3 - 2*x)^(19/2)*(1 + x + 2*x^2)^
4) + (451*(811091 + 998691*x))/(10119696384*(3 - 2*x)^(19/2)*(1 +
 x + 2*x^2)^3) + (451*(28962039 + 14627273*x))/(283351498752*(3 -
 2*x)^(19/2)*(1 + x + 2*x^2)^2) + (11275*(14627273 - 35058731*x))
/(3966920982528*(3 - 2*x)^(19/2)*(1 + x + 2*x^2)) + (11275*Sqrt[(
7 + 2*Sqrt[14])/2]*(9756589235 + 2148932869*Sqrt[14])*ArcTan[(Sqr
t[7 + 2*Sqrt[14]] - 2*Sqrt[3 - 2*x])/Sqrt[-7 + 2*Sqrt[14]]])/3186
03459702399434752 - (11275*Sqrt[(7 + 2*Sqrt[14])/2]*(9756589235 +
 2148932869*Sqrt[14])*ArcTan[(Sqrt[7 + 2*Sqrt[14]] + 2*Sqrt[3 - 2
*x])/Sqrt[-7 + 2*Sqrt[14]]])/318603459702399434752 + (11275*(9756
589235 - 2148932869*Sqrt[14])*Sqrt[(-7 + 2*Sqrt[14])/2]*Log[3 + S
qrt[14] - Sqrt[7 + 2*Sqrt[14]]*Sqrt[3 - 2*x] - 2*x])/637206919404
798869504 - (11275*(9756589235 - 2148932869*Sqrt[14])*Sqrt[(-7 +
2*Sqrt[14])/2]*Log[3 + Sqrt[14] + Sqrt[7 + 2*Sqrt[14]]*Sqrt[3 - 2
*x] - 2*x])/637206919404798869504
```

Rubi [A] time = 2.68597, antiderivative size = 648, normalized size of antiderivative = 1., number of

steps used = 29, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.45$

$$\frac{11275(14627273-35058731x)}{3966920982528(3-2x)^{19/2}(2x^2+x+1)}+\frac{451(14627273x+28962039)}{283351498752(3-2x)^{19/2}(2x^2+x+1)^2}+\frac{451(998691x+811091)}{10119696384(3-2x)^{19/2}(2x^2+x+1)^3}+\frac{41(5677637x+3436375)}{5059848192(3-2x)^{19/2}(2x^2+x+1)^4}+\frac{41(92875x+47471)}{90354432(3-2x)^{19/2}(2x^2+x+1)^5}+\frac{5(47471x+21409)}{6453888(3-2x)^{19/2}(2x^2+x+1)^6}+\frac{21409x+8477}{691488(3-2x)^{19/2}(2x^2+x+1)^7}+\frac{173x+53}{7056(3-2x)^{19/2}(2x^2+x+1)^8}+\frac{x}{63(3-2x)^{19/2}(2x^2+x+1)^9}-\frac{24229218097975}{22757389978742816768\sqrt{3-2x}}-\frac{46601678385075}{11378694989371408384(3-2x)^{3/2}}-\frac{11557581705725}{812763927812243456(3-2x)^{5/2}}-\frac{132355162272575}{2844673747342852096(3-2x)^{7/2}}-\frac{37283626871975}{261245548225363968(3-2x)^{9/2}}-\frac{5846828446875}{14513641568075776(3-2x)^{11/2}}-\frac{13515743021825}{13476952884641792(3-2x)^{13/2}}-\frac{3029508823715}{1555033025150976(3-2x)^{15/2}}-\frac{815900548375}{629418129227776(3-2x)^{17/2}}+\frac{4718120139975}{351733660450816(3-2x)^{19/2}}+\frac{11275\left(9756589235-2148932869\sqrt{14}\right)\sqrt{\frac{1}{2}\left(2\sqrt{14}-7\right)}\log\left(-2x-\sqrt{7+2\sqrt{14}}\sqrt{3-2x}+\sqrt{14}+3\right)}{637206919404798869504}-\frac{11275\left(9756589235-2148932869\sqrt{14}\right)\sqrt{\frac{1}{2}\left(2\sqrt{14}-7\right)}\log\left(-2x+\sqrt{7+2\sqrt{14}}\sqrt{3-2x}+\sqrt{14}+3\right)}{637206919404798869504}+\frac{11275\sqrt{\frac{1}{2}\left(7+2\sqrt{14}\right)}\left(9756589235+2148932869\sqrt{14}\right)\tan^{-1}\left(\frac{\sqrt{7+2\sqrt{14}}-2\sqrt{3-2x}}{\sqrt{2\sqrt{14}-7}}\right)}{318603459702399434752}-\frac{11275\sqrt{\frac{1}{2}\left(7+2\sqrt{14}\right)}\left(9756589235+2148932869\sqrt{14}\right)\tan^{-1}\left(\frac{2\sqrt{3-2x}+\sqrt{7+2\sqrt{14}}}{\sqrt{2\sqrt{14}-7}}\right)}{318603459702399434752}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((3 - 2*x)^(21/2)*(1 + x + 2*x^2)^10),x]
```

```
[Out] 4718120139975/(351733660450816*(3 - 2*x)^(19/2)) - 815900548375/(
629418129227776*(3 - 2*x)^(17/2)) - 3029508823715/(15550330251509
76*(3 - 2*x)^(15/2)) - 13515743021825/(13476952884641792*(3 - 2*x
)^(13/2)) - 5846828446875/(14513641568075776*(3 - 2*x)^(11/2)) -
37283626871975/(261245548225363968*(3 - 2*x)^(9/2)) - 13235516227
2575/(2844673747342852096*(3 - 2*x)^(7/2)) - 11557581705725/(8127
63927812243456*(3 - 2*x)^(5/2)) - 46601678385075/(113786949893714
08384*(3 - 2*x)^(3/2)) - 24229218097975/(22757389978742816768*Sqr
t[3 - 2*x]) + x/(63*(3 - 2*x)^(19/2)*(1 + x + 2*x^2)^9) + (53 + 1
73*x)/(7056*(3 - 2*x)^(19/2)*(1 + x + 2*x^2)^8) + (8477 + 21409*x
)/(691488*(3 - 2*x)^(19/2)*(1 + x + 2*x^2)^7) + (5*(21409 + 47471
*x))/(6453888*(3 - 2*x)^(19/2)*(1 + x + 2*x^2)^6) + (41*(47471 +
92875*x))/(90354432*(3 - 2*x)^(19/2)*(1 + x + 2*x^2)^5) + (41*(34
36375 + 5677637*x))/(5059848192*(3 - 2*x)^(19/2)*(1 + x + 2*x^2)^
4) + (451*(811091 + 998691*x))/(10119696384*(3 - 2*x)^(19/2)*(1 +
 x + 2*x^2)^3) + (451*(28962039 + 14627273*x))/(283351498752*(3 -
 2*x)^(19/2)*(1 + x + 2*x^2)^2) + (11275*(14627273 - 35058731*x))
/(3966920982528*(3 - 2*x)^(19/2)*(1 + x + 2*x^2)) + (11275*Sqrt[(
7 + 2*Sqrt[14])/2]*(9756589235 + 2148932869*Sqrt[14])*ArcTan[(Sqr
t[7 + 2*Sqrt[14]] - 2*Sqrt[3 - 2*x])/Sqrt[-7 + 2*Sqrt[14]]])/3186
03459702399434752 - (11275*Sqrt[(7 + 2*Sqrt[14])/2]*(9756589235 +
 2148932869*Sqrt[14])*ArcTan[(Sqrt[7 + 2*Sqrt[14]] + 2*Sqrt[3 - 2
*x])/Sqrt[-7 + 2*Sqrt[14]]])/318603459702399434752 + (11275*(9756
589235 - 2148932869*Sqrt[14])*Sqrt[(-7 + 2*Sqrt[14])/2]*Log[3 + S
qrt[14] - Sqrt[7 + 2*Sqrt[14]]*Sqrt[3 - 2*x] - 2*x])/637206919404
798869504 - (11275*(9756589235 - 2148932869*Sqrt[14])*Sqrt[(-7 +
2*Sqrt[14])/2]*Log[3 + Sqrt[14] + Sqrt[7 + 2*Sqrt[14]]*Sqrt[3 - 2
*x] - 2*x])/637206919404798869504
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{x}{63\,(-2x+3)^{\frac{19}{2}}\,(2x^2+x+1)^9}+\frac{-154357191466591365584640000x+6440121232839552246912000}{154905798615955861175009280\,(-2x+3)^{\frac{19}{2}}\,(2x^2+x+1)}+\frac{\int\frac{-1181393221838810620537497600000x-62490796859310559680227696640000}{(-2x+3)^{\frac{11}{2}}\,(2x^2+x+1)}\,dx}{266598579951449104257882211221504 0}-\frac{5846828446875}{14513641568075776\,(-2x+3)^{\frac{11}{2}}}-\frac{13515743021825}{13476952884641792\,(-2x+3)^{\frac{13}{2}}}-\frac{3029508823715}{1555033025150976\,(-2x+3)^{\frac{15}{2}}}-\frac{815900548375}{629418129227776\,(-2x+3)^{\frac{17}{2}}}+\frac{67816x+20776}{2765952\,(-2x+3)^{\frac{19}{2}}\,(2x^2+x+1)^8}+\frac{117492592x+46521776}{3794886144\,(-2x+3)^{\frac{19}{2}}\,(2x^2+x+1)^7}+\frac{164128134240x+74020332960}{4462786105344\,(-2x+3)^{\frac{19}{2}}\,(2x^2+x+1)^6}+\frac{184316990760000x+94209549053760}{4373530383237120\,(-2x+3)^{\frac{19}{2}}\,(2x^2+x+1)^5}+\frac{157747397367934080x+95476201213680000}{3428847820457902080\,(-2x+3)^{\frac{19}{2}}\,(2x^2+x+1)^4}+\frac{89735798552133000960x+72879297583985544960}{2016162518429246423040\,(-2x+3)^{\frac{19}{2}}\,(2x^2+x+1)^3}+\frac{18400346379541577848320x+36432734212165998389760}{790335707224264597831680\,(-2x+3)^{\frac{19}{2}}\,(2x^2+x+1)^2}+\frac{4718120139975}{351733660450816\,(-2x+3)^{\frac{19}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(3-2*x)**(21/2)/(2*x**2+x+1)**10,x)
```

```
[Out] x/(63*(-2*x + 3)**(19/2)*(2*x**2 + x + 1)**9) + (-154357191466591
36558464000*x + 6440121232839552246912000)/(154905798615955861175
009280*(-2*x + 3)**(19/2)*(2*x**2 + x + 1)) + Integral((-11813932
2183881062053749760000000*x - 6249079685931055968022769664000)/((-
2*x + 3)**(11/2)*(2*x**2 + x + 1)), x)/26659857995144910425788221
12215040 - 5846828446875/(14513641568075776*(-2*x + 3)**(11/2)) -
 13515743021825/(13476952884641792*(-2*x + 3)**(13/2)) - 30295088
23715/(1555033025150976*(-2*x + 3)**(15/2)) - 815900548375/(62941
8129227776*(-2*x + 3)**(17/2)) + (67816*x + 20776)/(2765952*(-2*x
 + 3)**(19/2)*(2*x**2 + x + 1)**8) + (117492592*x + 46521776)/(37
94886144*(-2*x + 3)**(19/2)*(2*x**2 + x + 1)**7) + (164128134240*
x + 74020332960)/(4462786105344*(-2*x + 3)**(19/2)*(2*x**2 + x +
1)**6) + (184316990760000*x + 94209549053760)/(4373530383237120*(
-2*x + 3)**(19/2)*(2*x**2 + x + 1)**5) + (157747397367934080*x +
95476201213680000)/(3428847820457902080*(-2*x + 3)**(19/2)*(2*x**
2 + x + 1)**4) + (89735798552133000960*x + 72879297583985544960)/
(2016162518429246423040*(-2*x + 3)**(19/2)*(2*x**2 + x + 1)**3) +
 (18400346379541577848320*x + 36432734212165998389760)/(790335707
224264597831680*(-2*x + 3)**(19/2)*(2*x**2 + x + 1)**2) + 4718120
139975/(351733660450816*(-2*x + 3)**(19/2))
```

Mathematica [C] time = 6.07724, size = 662, normalized size = 1.02

$$-\frac{44193\sqrt{3-2x}-11993(3-2x)^{3/2}}{948721536\left((3-2x)^2-7(3-2x)+14\right)^8}+\frac{891605}{12401793332096\sqrt{3-2x}}-\frac{55\left(1410835658499(3-2x)^{3/2}-4751425354423\sqrt{3-2x}\right)}{68272169936228450304\left((3-2x)^2-7(3-2x)+14\right)}+\frac{8519225}{260437659974016(3-2x)^{3/2}}-\frac{11\left(1953387138017(3-2x)^{3/2}-6489356793153\sqrt{3-2x}\right)}{1706804248405711257 6\left((3-2x)^2-7(3-2x)+14\right)^2}+\frac{75933}{3100448333024(3-2x)^{5/2}}-\frac{1406968826615(3-2x)^{3/2}-4402987778403\sqrt{3-2x}}{914359418788773888\left((3-2x)^2-7(3-2x)+14\right)^3}+\frac{854095}{43406276662336(3-2x)^{7/2}}-\frac{52802422641(3-2x)^{3/2}-132204145097\sqrt{3-2x}}{32655693528170496\left((3-2x)^2-7(3-2x)+14\right)^4}+\frac{30349}{1993145356944(3-2x)^{9/2}}-\frac{5\left(38010319(3-2x)^{3/2}+107643741\sqrt{3-2x}\right)}{291568692215808\left((3-2x)^2-7(3-2x)+14\right)^5}+\frac{2365}{221460595216(3-2x)^{11/2}}+\frac{5\left(5912661(3-2x)^{3/2}-37938085\sqrt{3-2x}\right)}{10413167579136\left((3-2x)^2-7(3-2x)+14\right)^6}+\frac{165}{25705247659(3-2x)^{13/2}}+\frac{5\left(340449(3-2x)^{3/2}-1574149\sqrt{3-2x}\right)}{185949421056\left((3-2x)^2-7(3-2x)+14\right)^7}+\frac{73}{23727920916(3-2x)^{15/2}}+\frac{5}{4802079233(3-2x)^{17/2}}-\frac{47\sqrt{3-2x}-23(3-2x)^{3/2}}{4235364\left((3-2x)^2-7(3-2x)+14\right)^9}+\frac{1}{5367029731(3-2x)^{19/2}}-\frac{11275\left(2148932869\sqrt{7}-34555708553i\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{3-2x}}{\sqrt{-7-i\sqrt{7}}}\right)}{22757389978742816768\sqrt{14\left(-7-i\sqrt{7}\right)}}-\frac{11275\left(2148932869\sqrt{7}+34555708553i\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{3-2x}}{\sqrt{-7+i\sqrt{7}}}\right)}{22757389978742816768\sqrt{14\left(-7+i\sqrt{7}\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((3 - 2*x)^(21/2)*(1 + x + 2*x^2)^10),x]
```

```
[Out] -(47*Sqrt[3 - 2*x] - 23*(3 - 2*x)^(3/2))/(4235364*(14 - 7*(3 - 2*
x) + (3 - 2*x)^2)^9) - (44193*Sqrt[3 - 2*x] - 11993*(3 - 2*x)^(3/
2))/(948721536*(14 - 7*(3 - 2*x) + (3 - 2*x)^2)^8) + (5*(-1574149
*Sqrt[3 - 2*x] + 340449*(3 - 2*x)^(3/2)))/(185949421056*(14 - 7*(
3 - 2*x) + (3 - 2*x)^2)^7) + (5*(-37938085*Sqrt[3 - 2*x] + 591266
1*(3 - 2*x)^(3/2)))/(10413167579136*(14 - 7*(3 - 2*x) + (3 - 2*x)
^2)^6) - (5*(107643741*Sqrt[3 - 2*x] + 38010319*(3 - 2*x)^(3/2)))
/(291568692215808*(14 - 7*(3 - 2*x) + (3 - 2*x)^2)^5) - (-1322041
45097*Sqrt[3 - 2*x] + 52802422641*(3 - 2*x)^(3/2))/(3265569352817
0496*(14 - 7*(3 - 2*x) + (3 - 2*x)^2)^4) - (-4402987778403*Sqrt[3
 - 2*x] + 1406968826615*(3 - 2*x)^(3/2))/(914359418788773888*(14
- 7*(3 - 2*x) + (3 - 2*x)^2)^3) - (11*(-6489356793153*Sqrt[3 - 2*
x] + 1953387138017*(3 - 2*x)^(3/2)))/(17068042484057112576*(14 -
7*(3 - 2*x) + (3 - 2*x)^2)^2) - (55*(-4751425354423*Sqrt[3 - 2*x]
 + 1410835658499*(3 - 2*x)^(3/2)))/(68272169936228450304*(14 - 7*
(3 - 2*x) + (3 - 2*x)^2)) + 1/(5367029731*(3 - 2*x)^(19/2)) + 5/(
4802079233*(3 - 2*x)^(17/2)) + 73/(23727920916*(3 - 2*x)^(15/2))
+ 165/(25705247659*(3 - 2*x)^(13/2)) + 2365/(221460595216*(3 - 2*
x)^(11/2)) + 30349/(1993145356944*(3 - 2*x)^(9/2)) + 854095/(4340
6276662336*(3 - 2*x)^(7/2)) + 75933/(3100448333024*(3 - 2*x)^(5/2
)) + 8519225/(260437659974016*(3 - 2*x)^(3/2)) + 891605/(12401793
332096*Sqrt[3 - 2*x]) - (11275*(-34555708553*I + 2148932869*Sqrt[
7])*ArcTan[(Sqrt[2]*Sqrt[3 - 2*x])/Sqrt[-7 - I*Sqrt[7]]])/(227573
89978742816768*Sqrt[14*(-7 - I*Sqrt[7])]) - (11275*(34555708553*I
```

```
 + 2148932869*Sqrt[7])*ArcTan[(Sqrt[2]*Sqrt[3 - 2*x])/Sqrt[-7 + I
*Sqrt[7]]])/(22757389978742816768*Sqrt[14*(-7 + I*Sqrt[7])])
```

Maple [A] time = 0.072, size = 719, normalized size = 1.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(3-2*x)^(21/2)/(2*x^2+x+1)^10,x)
```

```
[Out] 1/86812553324672*(-16557498921138789 4481/65536*(3-2*x)^(23/2)+454
06001689183688581/131072*(3-2*x)^(25/2)-43462358811134257841/1179
648*(3-2*x)^(27/2)+192384852501874197/65536*(3-2*x)^(29/2)-135284
1099712333/8192*(3-2*x)^(31/2)+4606702222670185/786432*(3-2*x)^(3
3/2)-25865320405815/262144*(3-2*x)^(35/2)+544765170330150812273/1
024*(3-2*x)^(1/2)-347698778390586025 8979/1536*(3-2*x)^(3/2)+93649
99706478908741137/2048*(3-2*x)^(5/2)-2385190577290327905 4347/4096
*(3-2*x)^(7/2)+192983613795383541041317/36864*(3-2*x)^(9/2)-57758
42147534844975 0643/16384*(3-2*x)^(11/2)+603330358695846954 11551/3
2768*(3-2*x)^(13/2)-1497708850834939780 40723/196608*(3-2*x)^(15/2
)+6625689994458215569 6811/262144*(3-2*x)^(17/2)-1772997884154363 0
405471/262144*(3-2*x)^(19/2)+2869878271121283060373/196608*(3-2*x
)^(21/2))/((3-2*x)^2-7+14*x)^9-206922416016525/127441383880959773
9008*ln(3-2*x+14^(1/2)-(3-2*x)^(1/2)*(7+2*14^(1/2))^(1/2))*(7+2*1
4^(1/2))^(1/2)*14^(1/2)+389615613935075/63720691940479886 9504*ln(
3-2*x+14^(1/2)-(3-2*x)^(1/2)*(7+2*14^(1/2))^(1/2))*(7+2*14^(1/2))
^(1/2)-206922416016525/637206919404798869504/(-7+2*14^(1/2))^(1/2
)*arctan((2*(3-2*x)^(1/2)-(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(
1/2))*(7+2*14^(1/2))*14^(1/2)+389615613935075/318603459702399434 7
52/(-7+2*14^(1/2))^(1/2)*arctan((2*(3-2*x)^(1/2)-(7+2*14^(1/2))^(
1/2))/(-7+2*14^(1/2))^(1/2))*(7+2*14^(1/2))-110005543624625/31860
3459702399434752/(-7+2*14^(1/2))^(1/2)*arctan((2*(3-2*x)^(1/2)-(7
+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))*14^(1/2)+2069224160165
25/1274413838809597739008*ln(3-2*x+14^(1/2)+(3-2*x)^(1/2)*(7+2*14
^(1/2))^(1/2))*(7+2*14^(1/2))^(1/2)*14^(1/2)-389615613935075/6372
06919404798869504*ln(3-2*x+14^(1/2)+(3-2*x)^(1/2)*(7+2*14^(1/2))^
(1/2))*(7+2*14^(1/2))^(1/2)-206922416016525/637206919404798869504
/(-7+2*14^(1/2))^(1/2)*arctan((2*(3-2*x)^(1/2)+(7+2*14^(1/2))^(1/
2))/(-7+2*14^(1/2))^(1/2))*(7+2*14^(1/2))*14^(1/2)+38961561393507
5/318603459702399434752/(-7+2*14^(1/2))^(1/2)*arctan((2*(3-2*x)^(
1/2)+(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))*(7+2*14^(1/2))-
110005543624625/318603459702399434752/(-7+2*14^(1/2))^(1/2)*arcta
n((2*(3-2*x)^(1/2)+(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))*1
4^(1/2)+1/5367029731/(3-2*x)^(19/2)+5/4802079233/(3-2*x)^(17/2)+7
3/23727920916/(3-2*x)^(15/2)+165/25705247659/(3-2*x)^(13/2)+2365/
221460595216/(3-2*x)^(11/2)+30349/1993145356944/(3-2*x)^(9/2)+854
095/43406276662336/(3-2*x)^(7/2)+75933/3100448333024/(3-2*x)^(5/2
)+8519225/260437659974016/(3-2*x)^(3/2)+891605/12401793332096/(3-
2*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2x^2 + x + 1)^{10}(-2x + 3)^{\frac{21}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((2*x^2 + x + 1)^10*(-2*x + 3)^(21/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/((2*x^2 + x + 1)^10*(-2*x + 3)^(21/2)), x)
```

Fricas [A] time = 0.340481, size = 2839, normalized size = 4.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2*x^2 + x + 1)^10*(-2*x + 3)^(21/2)),x, algorithm="fricas")
```

```
[Out] 1/5520330666373161526201587121296845762985984*sqrt(11696075257569
86)*2744^(3/4)*(52052026180937767452848051100*sqrt(58480376287849
3)*sqrt(14)*(262144*x^27 - 2359296*x^26 + 8847360*x^25 - 19070976
*x^24 + 31260672*x^23 - 50135040*x^22 + 72400896*x^21 - 84787200*
x^20 + 97449984*x^19 - 111622144*x^18 + 102818304*x^17 - 94063104
*x^16 + 92761344*x^15 - 66772224*x^14 + 51609024*x^13 - 46803648*
x^12 + 23042340*x^11 - 18046404*x^10 + 14741135*x^9 - 2785131*x^8
 + 5374836*x^7 - 1955988*x^6 - 185166*x^5 - 1395306*x^4 - 454896*
x^3 - 314928*x^2 - 59049*x - 19683)*sqrt(-2*x + 3)*arctan(1637450
5360597804*sqrt(584803762878493)*(34555708553*sqrt(14)*sqrt(7) -
128466244977*sqrt(7))/(sqrt(1169607525756986)*sqrt(58480376287849
3)*2744^(1/4)*(327571850528462403199*sqrt(14)*sqrt(7) - 122642238
0928157351936*sqrt(7))*sqrt(-sqrt(14)*(sqrt(1169607525756986)*sqr
t(584803762878493)*2744^(1/4)*sqrt(-2*x + 3)*(1753205114611442492
1588415510503669733478055782584434677759736860464224411*sqrt(14)
- 655989286765353399282614782661181821311507917509232773908588405
51081105174)*sqrt((327571850528462403199*sqrt(14) - 1226422380928
157351936)/(401741448850159339627110059970918575243264*sqrt(14) -
 1503179149031234349960285217920460999509255)) + 1169607525756986
*sqrt(14)*(985103879740300769348116449646663111706904845321946338
203465849*sqrt(14)*(2*x - 3) - 7371842417180200746144727139314482
572723741509811671017998778368*x + 110577636257703011192170907089
71723859085612264717506526998167552) - 16130588759553249395427127
271762836634823565933565015784697865123790835338395596*sqrt(14) +
 6035513658879973504399141330264538203774112504578512162215455452
0755897091751936)/(9851038797403007693481164496466631117069048453
21946338203465849*sqrt(14) - 368592120859010037307236356965724128
6361870754905835508999389184))*sqrt((327571850528462403199*sqrt(1
4) - 1226422380928157351936)/(40174144885015933962711005997091857
5243264*sqrt(14) - 1503179149031234349960285217920460999509255))
+ 8187252680298902*sqrt(1169607525756986)*2744^(1/4)*sqrt(-2*x +
3)*(327571850528462403199*sqrt(14) - 1226422380928157351936)*sqrt
((327571850528462403199*sqrt(14) - 1226422380928157351936)/(40174
1448850159339627110059970918575243264*sqrt(14) - 1503179149031234
349960285217920460999509255)) - 114621537524184628*sqrt(584803762
878493)*(2148932869*sqrt(14) - 9756589235))) + 520520261809377674
52848051100*sqrt(584803762878493)*sqrt(14)*(262144*x^27 - 2359296
*x^26 + 8847360*x^25 - 19070976*x^24 + 31260672*x^23 - 50135040*x
^22 + 72400896*x^21 - 84787200*x^20 + 97449984*x^19 - 111622144*x
^18 + 102818304*x^17 - 94063104*x^16 + 92761344*x^15 - 66772224*x
^14 + 51609024*x^13 - 46803648*x^12 + 23042340*x^11 - 18046404*x^
10 + 14741135*x^9 - 2785131*x^8 + 5374836*x^7 - 1955988*x^6 - 185
166*x^5 - 1395306*x^4 - 454896*x^3 - 314928*x^2 - 59049*x - 19683
)*sqrt(-2*x + 3)*arctan(16374505360597804*sqrt(584803762878493)*(
34555708553*sqrt(14)*sqrt(7) - 128466244977*sqrt(7))/(sqrt(116960
7525756986)*sqrt(584803762878493)*2744^(1/4)*(3275718505284624031
99*sqrt(14)*sqrt(7) - 1226422380928157351936*sqrt(7))*sqrt(sqrt(1
4)*(sqrt(1169607525756986)*sqrt(584803762878493)*2744^(1/4)*sqrt(
-2*x + 3)*(175320511461144249215884155105036697334780557825844346
77759736860464224411*sqrt(14) - 655989286765353399282614782661181
82131150791750923277390858840551081105174)*sqrt((3275718505284624
03199*sqrt(14) - 1226422380928157351936)/(40174144885015933962711
0059970918575243264*sqrt(14) - 1503179149031234349960285217920460
999509255)) - 1169607525756986*sqrt(14)*(985103879740300769348116
449646663111706904845321946338203465849*sqrt(14)*(2*x - 3) - 7371
842417180200746144727139314482572723741509811671017998778368*x +
11057763625770301119217090708971723859085612264717506526998167552
) + 1613058875955324939542712727176283663482356593356501578469786
5123790835338395596*sqrt(14) - 6035513658879973504399141330264538
2037741125045785121622154554520755897091751936)/(9851038797403007
69348116449646663111706904845321946338203465849*sqrt(14) - 368592
1208590100373072363569657241286361870754905835508999389184))*sqrt
((327571850528462403199*sqrt(14) - 1226422380928157351936)/(40174
1448850159339627110059970918575243264*sqrt(14) - 1503179149031234
349960285217920460999509255)) + 8187252680298902*sqrt(11696075257
```

```
56986)*2744^(1/4)*sqrt(-2*x + 3)*(327571850528462403199*sqrt(14)
- 122642238092815735193 6)*sqrt((327571850528462403199*sqrt(14) -
122642238092815735193 6)/(401741448850159339627110059970918575243 2
64*sqrt(14) - 15031791490312343499602852179204609995092 55)) + 114
62153752418462 8*sqrt(584803762878493)*(2148932869*sqrt(14) - 9756
589235))) + 426093525*sqrt(584803762878493)*(3275718505284624031 9
9*sqrt(14)*sqrt(7)*(262144*x^27 - 2359296*x^26 + 8847360*x^25 - 1
9070976*x^24 + 31260672*x^23 - 50135040*x^22 + 72400896*x^21 - 84
787200*x^20 + 97449984*x^19 - 111622144*x^18 + 102818304*x^17 - 9
4063104*x^16 + 92761344*x^15 - 66772224*x^14 + 51609024*x^13 - 46
803648*x^12 + 23042340*x^11 - 18046404*x^10 + 14741135*x^9 - 2785
131*x^8 + 5374836*x^7 - 1955988*x^6 - 185166*x^5 - 1395306*x^4 -
454896*x^3 - 314928*x^2 - 59049*x - 19683) - 1226422380928157351 9
36*sqrt(7)*(262144*x^27 - 2359296*x^26 + 8847360*x^25 - 19070976*
x^24 + 31260672*x^23 - 50135040*x^22 + 72400896*x^21 - 84787200*x
^20 + 97449984*x^19 - 111622144*x^18 + 102818304*x^17 - 94063104*
x^16 + 92761344*x^15 - 66772224*x^14 + 51609024*x^13 - 46803648*x
^12 + 23042340*x^11 - 18046404*x^10 + 14741135*x^9 - 2785131*x^8
+ 5374836*x^7 - 1955988*x^6 - 185166*x^5 - 1395306*x^4 - 454896*x
^3 - 314928*x^2 - 59049*x - 19683))*sqrt(-2*x + 3)*log(-743435438
5828022168312 5/4*sqrt(14)*(sqrt(1169607525756986)*sqrt(5848037628
78493)*2744^(1/4)*sqrt(-2*x + 3)*(1753205114611442492158841551050
366973347805578258443467775973686046422441 1*sqrt(14) - 6559892867
65353399282614782661181821311507917509232773908588405510811051 74)
*sqrt((327571850528462403199*sqrt(14) - 122642238092815735193 6)/(
401741448850159339627110059970918575243264*sqrt(14) - 15031791490
31234349960285217920460999509255)) + 1169607525756986*sqrt(14)*(9
85103879740300769348116449646663111706904845321946338203465849*sq
rt(14)*(2*x - 3) - 737184241718020074614472713931448257272374150 9
8116710179987783 68*x + 1105776362577030111921709070897172385908 56
1226471750652699816755 2) - 1613058875955324939542712727176283663 4
82356593356501578469786512379083533839559 6*sqrt(14) + 60355136588
79973504399141330264538203774112504578512162215455452075589709175
1936)/(985103879740300769348116449646663111706904845321946338203 4
65849*sqrt(14) - 368592120859010037307236356965724128636187075490
5835508999389184)) - 426093525*sqrt(584803762878493)*(32757185052
8462403199*sqrt(14)*sqrt(7)*(262144*x^27 - 2359296*x^26 + 8847360
*x^25 - 19070976*x^24 + 31260672*x^23 - 50135040*x^22 + 72400896*
x^21 - 84787200*x^20 + 97449984*x^19 - 111622144*x^18 + 102818304
*x^17 - 94063104*x^16 + 92761344*x^15 - 66772224*x^14 + 51609024*
x^13 - 46803648*x^12 + 23042340*x^11 - 18046404*x^10 + 14741135*x
^9 - 2785131*x^8 + 5374836*x^7 - 1955988*x^6 - 185166*x^5 - 13953
06*x^4 - 454896*x^3 - 314928*x^2 - 59049*x - 19683) - 12264223809
28157351936*sqrt(7)*(262144*x^27 - 2359296*x^26 + 8847360*x^25 -
19070976*x^24 + 31260672*x^23 - 50135040*x^22 + 72400896*x^21 - 8
4787200*x^20 + 97449984*x^19 - 111622144*x^18 + 102818304*x^17 -
94063104*x^16 + 92761344*x^15 - 66772224*x^14 + 51609024*x^13 - 4
6803648*x^12 + 23042340*x^11 - 18046404*x^10 + 14741135*x^9 - 278
5131*x^8 + 5374836*x^7 - 1955988*x^6 - 185166*x^5 - 1395306*x^4 -
 454896*x^3 - 314928*x^2 - 59049*x - 19683))*sqrt(-2*x + 3)*log(7
434354385828022168312 5/4*sqrt(14)*(sqrt(1169607525756986)*sqrt(58
4803762878493)*2744^(1/4)*sqrt(-2*x + 3)*(175320511461144249215 88
415510503669733478055782584434677759736860464224411*sqrt(14) - 65
598928676535339928261478266118182131150791750923277390858840551 08
1105174)*sqrt((327571850528462403199*sqrt(14) - 1226422380928157 3
51936)/(401741448850159339627110059970918575243264*sqrt(14) - 150
31791490312343499602852179204609995092 55)) - 1169607525756986*sqr
t(14)*(985103879740300769348116449646663111706904845321946338203 4
65849*sqrt(14)*(2*x - 3) - 73718424171802007461447271393144825727
237415098116710179987783 68*x + 110577636257703011192170907089717 2
385908561226471750652699816755 2) + 161305887595532493954271272717
628366348235659335650157846978651237908353383955 96*sqrt(14) - 603
551365887997350439914133026453820377411250457851216221545545207 55
897091751936)/(985103879740300769348116449646663111706904845321 94
6338203465849*sqrt(14) - 3685921208590100373072363569657241286361
87075490583550899938918 4)) - 2*sqrt(1169607525756986)*2744^(1/4)*
(327571850528462403199*sqrt(14)*sqrt(7)*(24003120493771442749440 0
*x^27 - 262194894159623706378240 0*x^26 + 123650450558968111054848
00*x^25 - 3396989006438128411115520 0*x^24 + 65360120291258796757 8
11200*x^23 - 106701725825102321939251200*x^22 + 16229030722324950
2039654400*x^21 - 2166342283264706095475097 60*x^20 + 253788172995
391086570485760*x^19 - 287279159180291305208156160*x^18 + 3040105
910109668111559552 00*x^17 - 282644664539994827031006720*x^16 + 25
881925681516324984544793 6*x^15 - 229408132984166521977166336*x^14
```

```
 + 172649692294614969274168896*x^13 - 133312541377246386115890240
*x^12 + 102031573634317834547976132*x^11 - 5979110268149411757214
9176*x^10 + 41613884937255303086792337*x^9 - 27246604251076689552
043953*x^8 + 10718131725916893151555068*x^7 - 8685973988079840377
705700*x^6 + 3673303058277822225386926*x^5 - 80999036209504421005
4958*x^4 + 1362587089603925431664856*x^3 + 1119267686976029998061
16*x^2 + 205702452014540322797289*x - 4884417100172357749737) - 1
226422380928157351936*sqrt(7)*(240031204937714427494400*x^27 - 26
21948941596237063782400*x^26 + 12365045055896811105484800*x^25 -
33969890064381284111155200*x^24 + 65360120291258796757811200*x^23
 - 106701725825102321939251200*x^22 + 162290307223249502039654400
*x^21 - 216634228326470609547509760*x^20 + 2537881729953910865704
85760*x^19 - 287279159180291305208156160*x^18 + 30401059101096681
1155955200*x^17 - 282644664539994827031006720*x^16 + 258819256815
163249845447936*x^15 - 229408132984166521977166336*x^14 + 1726496
92294614969274168896*x^13 - 133312541377246386115890240*x^12 + 10
2031573634317834547976132*x^11 - 59791102681494117572149176*x^10
+ 41613884937255303086792337*x^9 - 27246604251076689552043953*x^8
 + 10718131725916893151555068*x^7 - 8685973988079840377705700*x^6
 + 3673303058277822225386926*x^5 - 809990362095044210054958*x^4 +
 1362587089603925431664856*x^3 + 111926768697602999806116*x^2 + 2
05702452014540322797289*x - 4884417100172357749737))*sqrt((327571
850528462403199*sqrt(14) - 1226422380928157351936)/(4017414488501
59339627110059970918575243264*sqrt(14) - 150317914903123434996028
5217920460999509255)))/((327571850528462403199*sqrt(14)*sqrt(7)*(
262144*x^27 - 2359296*x^26 + 8847360*x^25 - 19070976*x^24 + 31260
672*x^23 - 50135040*x^22 + 72400896*x^21 - 84787200*x^20 + 974499
84*x^19 - 111622144*x^18 + 102818304*x^17 - 94063104*x^16 + 92761
344*x^15 - 66772224*x^14 + 51609024*x^13 - 46803648*x^12 + 230423
40*x^11 - 18046404*x^10 + 14741135*x^9 - 2785131*x^8 + 5374836*x^
7 - 1955988*x^6 - 185166*x^5 - 1395306*x^4 - 454896*x^3 - 314928*
x^2 - 59049*x - 19683) - 1226422380928157351936*sqrt(7)*(262144*x
^27 - 2359296*x^26 + 8847360*x^25 - 19070976*x^24 + 31260672*x^23
 - 50135040*x^22 + 72400896*x^21 - 84787200*x^20 + 97449984*x^19
- 111622144*x^18 + 102818304*x^17 - 94063104*x^16 + 92761344*x^15
 - 66772224*x^14 + 51609024*x^13 - 46803648*x^12 + 23042340*x^11
- 18046404*x^10 + 14741135*x^9 - 2785131*x^8 + 5374836*x^7 - 1955
988*x^6 - 185166*x^5 - 1395306*x^4 - 454896*x^3 - 314928*x^2 - 59
049*x - 19683))*sqrt(-2*x + 3)*sqrt((327571850528462403199*sqrt(1
4) - 1226422380928157351936)/(40174144885015933962711005997091857
5243264*sqrt(14) - 1503179149031234349960285217920460999509255)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3-2*x)**(21/2)/(2*x**2+x+1)**10,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2x^2 + x + 1)^{10}(-2x + 3)^{\frac{21}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2*x^2 + x + 1)^10*(-2*x + 3)^(21/2)),x, algorithm="giac")
```

```
[Out] integrate(1/((2*x^2 + x + 1)^10*(-2*x + 3)^(21/2)), x)
```

3.49 $\int \frac{1}{(3-2x)^{41/2}(1+x+2x^2)^{20}}\,dx$

Optimal. Leaf size=1058

result too large to display

```
[Out] -13056959628363355534285785425/(106924014357253562723941220352*(3
 - 2*x)^(39/2)) - 3948194343291401740321996415/(20288146313940419
5937734623232*(3 - 2*x)^(37/2)) - 3046882292626202227364808l1/(53
736171318004354599724305612 8*(3 - 2*x)^(35/2)) + 2124315846756567
455653862925/(168885109856585114456276389068 8*(3 - 2*x)^(33/2)) +
 476575150745141187960959295 35/(66632852434325399703658138959872*
(3 - 2*x)^(31/2)) + 349116199939747140621727519 85/(12466791745777
0102671360389021696*(3 - 2*x)^(29/2)) + 14906630980879476084301 74
04825/(16249818206564516830956630017310 72*(3 - 2*x)^(27/2)) + 158
4861396416906654373438017 1/(60184511876164877151691222286 3360*(3
 - 2*x)^(25/2)) + 11155168222970774232376891145/(16851663325326165
60247354224017408*(3 - 2*x)^(23/2)) + 140118184980910202724749563
75/(1011099799519569936148412534410 4448*(3 - 2*x)^(21/2)) + 17344
1368149804378661935869705/(8965084889073520100515924471772610 56*(
3 - 2*x)^(19/2)) - 2272409082346990515271351954 5/(160427834857105
0965355481221264572416*(3 - 2*x)^(17/2)) - 101190274412779618678 5
73275245/(396351121411671414970177713488894361 6*(3 - 2*x)^(15/2))
 - 460503190416958283087439337135/(343504305223448559640820685023
70844672*(3 - 2*x)^(13/2)) - 2211619588790911794826342607495/(406
920484649315986036049119181931544576*(3 - 2*x)^(11/2)) - 14340146
755077724762794043702 5/(739855426635119974610998398512602808 32*(3
 - 2*x)^(9/2)) - 4611053278117143010907562317585/(725058318102417
575118778430542350752153 6*(3 - 2*x)^(7/2)) - 40596537244063051072
0926890227/(207159519457833592891079551583528786329 6*(3 - 2*x)^(5
/2)) - 498668147918778185341731652277 5/(870069981722901090142534 1
1665082090258432*(3 - 2*x)^(3/2)) - 9270277547814767462080476205 0
5/(580046654481934060095022744433880601722 88*Sqrt[3 - 2*x]) + x/(
133*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^19) + (113 + 373*x)/(33516*(
3 - 2*x)^(39/2)*(1 + x + 2*x^2)^18) + (40657 + 107329*x)/(7976808
*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^17) + (5*(751303 + 1831285*x))/
(595601664*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^16) + (184959785 + 42
9411497*x)/(25015269888*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^15) + (4
1652915209 + 92630823167*x)/(4902992898048*(3 - 2*x)^(39/2)*(1 +
x + 2*x^2)^14) + (2871555518177 + 6100156355517*x)/(2974482358149
12*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^13) + (77559130805859 + 15627
4047129113*x)/(7138757659557888*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^
12) + (5*(2656658801194921 + 5020880176134289*x))/(10993686795719
14752*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^11) + (45187921585208601 +
 78752911037377255*x)/(3420258114223734784*(3 - 2*x)^(39/2)*(1 +
x + 2*x^2)^10) + (6063974149878048635 + 9477172618423641847*x)/(4
309525223921905827 84*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^9) + (69183
3601144925854831 + 919498192874055581221*x)/(48266682507925345271
808*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^8) + (23*(919498192874055581
221 + 908287136092467468517*x))/(1576711628592227945545728*(3 - 2
*x)^(39/2)*(1 + x + 2*x^2)^7) + (115*(908287136092467468517 + 298
28188494452222 5747*x))/(1018798283090362672506470 4*(3 - 2*x)^(39/
2)*(1 + x + 2*x^2)^6) + (23*(2599313568802265110081 - 10426142448
623187379187*x))/(203759656618072534501294 08*(3 - 2*x)^(39/2)*(1
 + x + 2*x^2)^5) - (23*(10426142448623187379187 + 2751372346319426
2383705*x))/(200184925800211612843376 64*(3 - 2*x)^(39/2)*(1 + x +
 2*x^2)^4) - (115*(265132244281690164788 43 + 3067341540655378934 2
019*x))/(764342443964444339947438 08*(3 - 2*x)^(39/2)*(1 + x + 2*x
^2)^3) - (115*(884116091130079810446 43 - 5712269536245152162963*x
))/(125891696652967303050166272*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^
2) + (115*(2856134768122576081481 5 + 965934812839019490346107*x))
/(195831528126838026966925312*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)) +
 (115*Sqrt[(7 + 2*Sqrt[14])/2]*(302971189122193607250286930 61 + 8
061110911143276053983022787*Sqrt[14])*ArcTan[(Sqrt[7 + 2*Sqrt[14]
] - 2*Sqrt[3 - 2*x])/Sqrt[-7 + 2*Sqrt[14]]])/8120653162747076841 3
303184220743284241203 2 - (115*Sqrt[(7 + 2*Sqrt[14])/2]*(302971189
1221936072502869306 1 + 8061110911143276053983022787*Sqrt[14])*Arc
Tan[(Sqrt[7 + 2*Sqrt[14]] + 2*Sqrt[3 - 2*x])/Sqrt[-7 + 2*Sqrt[14]
]])/812065316274707684133031842207432842412032 + (115*(3029711891
221936072502869306 1 - 8061110911143276053983022787*Sqrt[14])*Sqrt
[(-7 + 2*Sqrt[14])/2]*Log[3 + Sqrt[14] - Sqrt[7 + 2*Sqrt[14]]*Sqr
t[3 - 2*x] - 2*x])/1624130632549415368266063684414865684824064 -
(115*(302971189122193607250286930 61 - 806111091114327605398302278
```

```
7*Sqrt[14])*Sqrt[(-7 + 2*Sqrt[14])/2]*Log[3 + Sqrt[14] + Sqrt[7 +
 2*Sqrt[14]]*Sqrt[3 - 2*x] - 2*x])/162413063254941536826606368441
4865684824064
```

Rubi [A] time = 5.37521, antiderivative size = 1058, normalized size of antiderivative = 1., number of steps used = 49, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.45$

result too large to display

Warning: Unable to verify antiderivative.

```
[In]  Int[1/((3 - 2*x)^(41/2)*(1 + x + 2*x^2)^20),x]
```

```
[Out] -1305695962836335553428578542 5/(106924014357253562723941220352*(3
 - 2*x)^(39/2)) - 394819434329140174032199641 5/(20288146313940419
5937734623232*(3 - 2*x)^(37/2)) - 3046882292626202227364808 11/(53
7361713180043545997243056128*(3 - 2*x)^(35/2)) + 212431584675656 7
455653862925/(168885109856585114456276389068 8*(3 - 2*x)^(33/2)) +
 476575150745141187960959295 35/(666328524343253997036581389598 72*
(3 - 2*x)^(31/2)) + 349116199939747140621727519 85/(12466791745777
010267136038902169 6*(3 - 2*x)^(29/2)) + 149066309808794760843017 4
04825/(162498182065645168309566300173107 2*(3 - 2*x)^(27/2)) + 158
486139641690665437343801 71/(601845118761648771516912228633 60*(3
 - 2*x)^(25/2)) + 111551682229707742323768911 45/(168516633253261 65
602473542240174 08*(3 - 2*x)^(23/2)) + 140118184980910202724749563
75/(101109979951956993614841253441044 48*(3 - 2*x)^(21/2)) + 17344
136814980437866193586970 5/(896508488907352010051592447177261056*(
3 - 2*x)^(19/2)) - 227240908234699051527135195 45/(160427834857105
09653554812212645724 16*(3 - 2*x)^(17/2)) - 101190274412779618678 5
73275245/(396351121411671414970177713488894361 6*(3 - 2*x)^(15/2))
 - 460503190416958283087439337135/(343504305223448559640820685023
70844672*(3 - 2*x)^(13/2)) - 221161958879091179482634260749 5/(406
920484649315986036049119181931544576*(3 - 2*x)^(11/2)) - 14340146
755077724762794043702 5/(739855426635119974610998398512602808 32*(3
 - 2*x)^(9/2)) - 461105327811714301090756231758 5/(725058318102417
57511877843054235075215 36*(3 - 2*x)^(7/2)) - 405965372440630510 72
0926890227/(207159519457833592891079551583528786329 6*(3 - 2*x)^(5
/2)) - 498668147918778185341731652277 5/(870069981722901090142534 1
166508209025843 2*(3 - 2*x)^(3/2)) - 927027754781476746208047620 50
5/(5800466544819340600950227444338806017228 8*Sqrt[3 - 2*x]) + x/(
133*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^19) + (113 + 373*x)/(33516*(
3 - 2*x)^(39/2)*(1 + x + 2*x^2)^18) + (40657 + 107329*x)/(7976808
*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^17) + (5*(751303 + 1831285*x))/
(595601664*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^16) + (184959785 + 42
9411497*x)/(25015269888*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^15) + (4
1652915209 + 92630823167*x)/(4902992898048*(3 - 2*x)^(39/2)*(1 +
x + 2*x^2)^14) + (2871555518177 + 6100156355517*x)/(297448235814 9
12*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^13) + (77559130805859 + 15627
4047129113*x)/(7138757659557888*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^
12) + (5*(2656658801194921 + 5020880176134289*x))/(10993686795719
14752*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^11) + (45187921585208601 +
 78752911037377255*x)/(342025811422373478 4*(3 - 2*x)^(39/2)*(1 +
x + 2*x^2)^10) + (606397414987804863 5 + 947717261842364184 7*x)/(4
309525223921905827 84*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^9) + (69183
360114492585483 1 + 919498192874055581221*x)/(482666825079253452 71
808*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^8) + (23*(919498192874055581
221 + 908287136092467468517*x))/(1576711628592227945545728*(3 - 2
*x)^(39/2)*(1 + x + 2*x^2)^7) + (115*(908287136092467468517 + 298
281884944522225747*x))/(101879828309036267250647 04*(3 - 2*x)^(39/
2)*(1 + x + 2*x^2)^6) + (23*(25993135688022651100 81 - 10426142448
623187379187*x))/(203759656618072534501294 08*(3 - 2*x)^(39/2)*(1
+ x + 2*x^2)^5) - (23*(104261424486231873791 87 + 275137234631942 6
2383705*x))/(200184925800211612843376 64*(3 - 2*x)^(39/2)*(1 + x +
 2*x^2)^4) - (115*(265132244281690164788 43 + 306734154065537893 42
019*x))/(764342443964444339947438 08*(3 - 2*x)^(39/2)*(1 + x + 2*x
^2)^3) - (115*(884116091130079810446 43 - 571226953624515216296 3*x
))/(125891696652967303050166272*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^
2) + (115*(285613476812257608148 15 + 965934812839019490346107*x))
/(195831528126838026966925312*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)) +
 (115*Sqrt[(7 + 2*Sqrt[14])/2]*(302971189122193607250286930 61 + 8
```

```
061110911143276053983022787*Sqrt[14])*ArcTan[(Sqrt[7 + 2*Sqrt[14]
] - 2*Sqrt[3 - 2*x])/Sqrt[-7 + 2*Sqrt[14]]])/81206531627470768413
3031842207432842412032 - (115*Sqrt[(7 + 2*Sqrt[14])/2]*(302971189
12219360725028693061 + 8061110911143276053983022787*Sqrt[14])*Arc
Tan[(Sqrt[7 + 2*Sqrt[14]] + 2*Sqrt[3 - 2*x])/Sqrt[-7 + 2*Sqrt[14]
]])/812065316274707684133031842207432842412032 + (115*(3029711891
2219360725028693061 - 8061110911143276053983022787*Sqrt[14])*Sqrt
[(-7 + 2*Sqrt[14])/2]*Log[3 + Sqrt[14] - Sqrt[7 + 2*Sqrt[14]]*Sqr
t[3 - 2*x] - 2*x])/1624130632549415368266063684414865684824064 -
(115*(30297118912219360725028693061 - 806111091114327605398302278
7*Sqrt[14])*Sqrt[(-7 + 2*Sqrt[14])/2]*Log[3 + Sqrt[14] + Sqrt[7 +
 2*Sqrt[14]]*Sqrt[3 - 2*x] - 2*x])/162413063254941536826606368441
4865684824064
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$
\begin{aligned}
&\frac{x}{133\left(-2x+3\right)^{\frac{39}{2}}\left(2x^{2}+x+1\right)^{19}} \\
&+\frac{\int\frac{-79426075962231715392089154535476326400x+13068536369629429401640635068702208000}{\left(-2x+3\right)^{\frac{41}{2}}\left(2x^{2}+x+1\right)^{9}}\,dx}{49475578698391671468350845128120729600} \\
&+\frac{146216x+44296}{13138272\left(-2x+3\right)^{\frac{39}{2}}\left(2x^{2}+x+1\right)^{18}}+\frac{589021552x+223125616}{43776722304\left(-2x+3\right)^{\frac{39}{2}}\left(2x^{2}+x+1\right)^{17}} \\
&+\frac{2110519336800x+865861681440}{137283801145344\left(-2x+3\right)^{\frac{39}{2}}\left(2x^{2}+x+1\right)^{16}}+\frac{6928434268875840x+2984274342235200}{403614375367311360\left(-2x+3\right)^{\frac{39}{2}}\left(2x^{2}+x+1\right)^{15}} \\
&+\frac{20924013532366815360x+9408813737133390720}{1107517846007902371840\left(-2x+3\right)^{\frac{39}{2}}\left(2x^{2}+x+1\right)^{14}} \\
&+\frac{57873497074462503141120x+27243065619141593598720}{2821955471628135243448320\left(-2x+3\right)^{\frac{39}{2}}\left(2x^{2}+x+1\right)^{13}} \\
&+\frac{145295342948683106164016640x+72110377354780278913835520}{6637239269269374092590448640\left(-2x+3\right)^{\frac{39}{2}}\left(2x^{2}+x+1\right)^{12}} \\
&+\frac{326770416680301421681066214400x+172901458108932896335179801600}{14309887864544770543625007267840\left(-2x+3\right)^{\frac{39}{2}}\left(2x^{2}+x+1\right)^{11}} \\
&+\frac{645802967231886306826540424448000x+370557652515461812186329087129600}{28047380214507750265505014244966400\left(-2x+3\right)^{\frac{39}{2}}\left(2x^{2}+x+1\right)^{10}} \\
&+\frac{1088028437838790621809440473088716800x+696175598675973438759010577554944000}{49475578698391671468350845128120729600\left(-2x+3\right)^{\frac{39}{2}}\left(2x^{2}+x+1\right)^{9}}
\end{aligned}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   rubi_integrate(1/(3-2*x)**(41/2)/(2*x**2+x+1)**20,x)
```

```
[Out] x/(133*(-2*x + 3)**(39/2)*(2*x**2 + x + 1)**19) + Integral((-7942
6075962231715392089154535476326400*x + 13068536369629429401640635
068702208000)/((-2*x + 3)**(41/2)*(2*x**2 + x + 1)**9), x)/494755
78698391671468350845128120729600 + (146216*x + 44296)/(13138272*(
-2*x + 3)**(39/2)*(2*x**2 + x + 1)**18) + (589021552*x + 22312561
6)/(43776722304*(-2*x + 3)**(39/2)*(2*x**2 + x + 1)**17) + (21105
19336800*x + 865861681440)/(137283801145344*(-2*x + 3)**(39/2)*(2
*x**2 + x + 1)**16) + (6928434268875840*x + 2984274342235200)/(40
3614375367311360*(-2*x + 3)**(39/2)*(2*x**2 + x + 1)**15) + (2092
4013532366815360*x + 9408813737133390720)/(1107517846007902371840
*(-2*x + 3)**(39/2)*(2*x**2 + x + 1)**14) + (57873497074462503141
120*x + 27243065619141593598720)/(2821955471628135243448320*(-2*x
 + 3)**(39/2)*(2*x**2 + x + 1)**13) + (14529534294868310616401664
0*x + 72110377354780278913835520)/(6637239269269374092590448640*(
-2*x + 3)**(39/2)*(2*x**2 + x + 1)**12) + (3267704166803014216810
66214400*x + 172901458108932896335179801600)/(1430988786454477054
3625007267840*(-2*x + 3)**(39/2)*(2*x**2 + x + 1)**11) + (6458029
67231886306826540424448000*x + 370557652515461812186329087129600)
/(28047380214507750265505014244966400*(-2*x + 3)**(39/2)*(2*x**2
+ x + 1)**10) + (1088028437838790621809440473088716800*x + 696175
598675973438759010577554944000)/(49475578698391671468350845128120
729600*(-2*x + 3)**(39/2)*(2*x**2 + x + 1)**9)
```

Mathematica [C] time = 6.16477, size = 1242, normalized size = 1.17

result too large to display

Antiderivative was successfully verified.

```
[In]  Integrate[1/((3 - 2*x)^(41/2)*(1 + x + 2*x^2)^20),x]
```

```
[Out] -(393*Sqrt[3 - 2*x] + 287*(3 - 2*x)^(3/2))/(150276832468*(14 - 7*
(3 - 2*x) + (3 - 2*x)^2)^19) - (-4226921*Sqrt[3 - 2*x] + 1313129*
(3 - 2*x)^(3/2))/(75739523563872*(14 - 7*(3 - 2*x) + (3 - 2*x)^2)
^18) - (-3401932701*Sqrt[3 - 2*x] + 760755809*(3 - 2*x)^(3/2))/(3
6052013216403072*(14 - 7*(3 - 2*x) + (3 - 2*x)^2)^17) - (5*(-1464
90500023*Sqrt[3 - 2*x] + 16144709919*(3 - 2*x)^(3/2)))/(161513019
20948576256*(14 - 7*(3 - 2*x) + (3 - 2*x)^2)^16) - (9745709632283
*Sqrt[3 - 2*x] - 4557912048927*(3 - 2*x)^(3/2))/(4522364537865601
35168*(14 - 7*(3 - 2*x) + (3 - 2*x)^2)^15) - (435856117815771*Sqr
t[3 - 2*x] - 123609208162571*(3 - 2*x)^(3/2))/(933035209917534594
6624*(14 - 7*(3 - 2*x) + (3 - 2*x)^2)^14) - (12743552265699763l*S
qrt[3 - 2*x] - 31270302414674811*(3 - 2*x)^(3/2))/(33962481640998
25924571136*(14 - 7*(3 - 2*x) + (3 - 2*x)^2)^13) + (5*(-154035916
7602841319*Sqrt[3 - 2*x] + 342026557757088031*(3 - 2*x)^(3/2)))/(
3803797943791805035519672 32*(14 - 7*(3 - 2*x) + (3 - 2*x)^2)^12)
+ (5*(-2108462813948119068 7*Sqrt[3 - 2*x] + 4158669924550257827*(
3 - 2*x)^(3/2)))/(130174418520875105660006563 84*(14 - 7*(3 - 2*x)
 + (3 - 2*x)^2)^11) - (163329397359734271258 1*Sqrt[3 - 2*x] - 237
080744154193384005*(3 - 2*x)^(3/2))/(7289767437169005916960367575
04*(14 - 7*(3 - 2*x) + (3 - 2*x)^2)^10) - (735043251343102201715 5
*Sqrt[3 - 2*x] + 5131564318471376538977*(3 - 2*x)^(3/2))/(6123404
6472219649702467087630336*(14 - 7*(3 - 2*x) + (3 - 2*x)^2)^9) - (
-113207386492327172550771*Sqrt[3 - 2*x] + 434211603673429008953 87
*(3 - 2*x)^(3/2))/(279927069587289827211278114881536*(14 - 7*(3 -
 2*x) + (3 - 2*x)^2)^8) - (-224637967205021836248421 07*Sqrt[3 - 2
*x] + 709497819442478643117366 3*(3 - 2*x)^(3/2))/(548657056391088
06133410510516781056*(14 - 7*(3 - 2*x) + (3 - 2*x)^2)^7) - (5*(-1
86257412289925530757362143*Sqrt[3 - 2*x] + 555401785887220466671 1
3711*(3 - 2*x)^(3/2)))/(3072479515790093143470988588939739136*(14
 - 7*(3 - 2*x) + (3 - 2*x)^2)^6) - (23*(-255056047077847659080618
951*Sqrt[3 - 2*x] + 7444398847327232818931635 5*(3 - 2*x)^(3/2)))/
(28676475480707536005729226830104231936*(14 - 7*(3 - 2*x) + (3 -
2*x)^2)^5) - (23*(-11100577882868065896562605 77*Sqrt[3 - 2*x] + 3
21533953909984640923113289*(3 - 2*x)^(3/2)))/(1889273678728967078
02451376763039645696*(14 - 7*(3 - 2*x) + (3 - 2*x)^2)^4) - (23*(-
4820387670797872511726954245*Sqrt[3 - 2*x] + 13943044905313772031
11252689*(3 - 2*x)^(3/2)))/(1220761453947947958108147357545794633
728*(14 - 7*(3 - 2*x) + (3 - 2*x)^2)^3) - (23*(-17490402570151108
581128226213*Sqrt[3 - 2*x] + 5072167085782230110284731077*(3 - 2*
x)^(3/2)))/(6214785583735007786732386547505863589888*(14 - 7*(3 -
 2*x) + (3 - 2*x)^2)^2) - (115*(-82782386138609724168863115877*Sq
rt[3 - 2*x] + 24217623575858523510208130121*(3 - 2*x)^(3/2)))/(17
40139963445802180285068233301641805168 64*(14 - 7*(3 - 2*x) + (3 -
 2*x)^2)) + 1/(3111898385606868039*(3 - 2*x)^(39/2)) + 10/(295231
3853011644037*(3 - 2*x)^(37/2)) + 143/(7819642097165976098*(3 - 2
*x)^(35/2)) + 355/(5266289575642392066*(3 - 2*x)^(33/2)) + 52865/
(27703874858530886747 2*(3 - 2*x)^(31/2)) + 14333/(323956601168304
72406*(3 - 2*x)^(29/2)) + 1478345/(1689042692987850837168*(3 - 2*
x)^(27/2)) + 475387/(312785683886639043920*(3 - 2*x)^(25/2)) + 16
575515/(7006399319060714583808*(3 - 2*x)^(23/2)) + 246866015/(735
67192850137503129984*(3 - 2*x)^(21/2)) + 8192823353/(186370221887
0150079292928*(3 - 2*x)^(19/2)) + 8972680075/(1667523037936450070
946304*(3 - 2*x)^(17/2)) + 102495360575/(164790511984308007011164
16*(3 - 2*x)^(15/2)) + 122484655975/(178523054649667007595427 84*(
3 - 2*x)^(13/2)) + 10815878546425/(1480368099325700262983624704*(
3 - 2*x)^(11/2)) + 769045155125/(100934188590388654294338048*(3 -
 2*x)^(9/2)) + 838467657280275/(10550987180648627328901470617 6*(3
 - 2*x)^(7/2)) + 9270470094105/(1076631344964145645806272512*(3 -
 2*x)^(5/2)) + 320421783064625/(30145677658996078082575630336*(3
- 2*x)^(3/2)) + 683151246370725/(30145677658996078082575630336*Sq
rt[3 - 2*x]) - (115*(-117022014202441653827938545631*I + 80611109
11143276053983022787*Sqrt[7])*ArcTan[(Sqrt[2]*Sqrt[3 - 2*x])/Sqrt
[-7 - I*Sqrt[7]]])/(58004665448193406009502274443388060172288*Sqr
```

```
t[14*(-7 - I*Sqrt[7])]) - (115*(117022014202441653827938545631*I
+ 80611109111432760539830227877*Sqrt[7])*ArcTan[(Sqrt[2]*Sqrt[3 -
2*x])/Sqrt[-7 + I*Sqrt[7]]])/(58004665448193406009502274443388060
172288*Sqrt[14*(-7 + I*Sqrt[7])])
```

Maple [A] time = 0.138, size = 989, normalized size = 0.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(3-2*x)^(41/2)/(2*x^2+x+1)^20,x)
```

```
[Out] -7192279694031133468210490184035/32482612650988307365321273688297
31369648128*ln(3-2*x+14^(1/2)-(3-2*x)^(1/2)*(7+2*14^(1/2))^(1/2))
*(7+2*14^(1/2))^(1/2)*14^(1/2)+13457531633280790190212932747565/8
12065316274707684133031842207432842412032/(-7+2*14^(1/2))^(1/2)*a
rctan((2*(3-2*x)^(1/2)-(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2
))*(7+2*14^(1/2))-3484168674905226483378299702015/812065316274707
684133031842207432842412032/(-7+2*14^(1/2))^(1/2)*arctan((2*(3-2*
x)^(1/2)-(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))*14^(1/2)+71
92279694031133468210490184035/32482612650988307365321273688297313
69648128*ln(3-2*x+14^(1/2)+(3-2*x)^(1/2)*(7+2*14^(1/2))^(1/2))*(7
+2*14^(1/2))^(1/2)*14^(1/2)+13457531633280790190212932747565/8120
65316274707684133031842207432842412032/(-7+2*14^(1/2))^(1/2)*arct
an((2*(3-2*x)^(1/2)+(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))*
(7+2*14^(1/2))-3484168674905226483378299702015/812065316274707684
133031842207432842412032/(-7+2*14^(1/2))^(1/2)*arctan((2*(3-2*x)^
(1/2)+(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))*14^(1/2)+13457
531633280790190212932747565/1624130632549415368266063684414865684
824064*ln(3-2*x+14^(1/2)-(3-2*x)^(1/2)*(7+2*14^(1/2))^(1/2))*(7+2
*14^(1/2))^(1/2)-13457531633280790190212932747565/162413063254941
5368266063684414865684824064*ln(3-2*x+14^(1/2)+(3-2*x)^(1/2)*(7+2
*14^(1/2))^(1/2))*(7+2*14^(1/2))^(1/2)+683151246370725/3014567765
8996078082575630336/(3-2*x)^(1/2)+1/30145677658996078082575630336
*(-5059022664167725408892162874688680417923742003781/294806555197
44*(3-2*x)^(47/2)-44796329357069082297154473725670903546220392558
695/9070970929152*(3-2*x)^(43/2)-19392429209015348214540269031324
33081580221023737/501171143835648*(3-2*x)^(51/2)-1006304725834560
333245233940167063186576585913370455/10720238370816*(3-2*x)^(39/2
)+13805722741822612586258592099428566280191230197271405/393075406
92992*(3-2*x)^(37/2)-928342237074576734557978321305/1924145348608
*(3-2*x)^(75/2)+339556544641293541759958988614814460549873/982688
5173248*(3-2*x)^(61/2)+133883313322119397348791732981953297/82463
3720832*(3-2*x)^(69/2)+490738543064879423955077165987434152441563
270473/1002342287671296*(3-2*x)^(53/2)-75593011640468285657019519
0192032441294632160945523631/3915399561216*(3-2*x)^(23/2)+8535085
02207214511987093866021124087908041634697244059/7830799122432*(3-
2*x)^(25/2)-6886173809894005543994516442461871486007042005189775/
125627793408*(3-2*x)^(27/2)+1363299879672453951418482537651472082
79814148352958009/5527622909952*(3-2*x)^(29/2)-550660914208175901
67865401986871791412011888132876913/5527622909952*(3-2*x)^(31/2)+
2737487528928439357869138774910126923363791747141675/755914244096
*(3-2*x)^(33/2)-1166457217021587688420366823074349521448831011337
1105/9826885173248*(3-2*x)^(35/2)+1264662933338272271690443076373
2665179119615389552413/25098715136*(3-2*x)^(17/2)-259367320368504
4441695042001860835122939346700333136537/6199382638592*(3-2*x)^(1
9/2)+807597736492641378942268937217995835353849465/1048576*(3-2*x
)^(1/2)-503502693505289734438057515605193725/103079215104*(3-2*x)
^(67/2)-3254850748003483429666738850178379/824633720832*(3-2*x)^(
71/2)+360433340020130123942335063779145/5772436045824*(3-2*x)^(73
/2)+1808668971148992206490172102870787954874541181/33411409589043
2*(3-2*x)^(57/2)+129886852748727110357425618672922324659/11338713
66144*(3-2*x)^(65/2)+28607223317693223698395672584150593863016075
796143/29480655519744*(3-2*x)^(45/2)-1196897725308288065129289211
1395530933265219/25701084299264*(3-2*x)^(59/2)+730124764525775715
33836489036461787385135079265/2680059592704*(3-2*x)^(49/2)-642433
96719140374998473027009027485263697/29480655519744*(3-2*x)^(63/2)
+2672239984790337844292019294315182385216573077301785/11792262207
8976*(3-2*x)^(41/2)+118632384645382623721251719631219381945276176
```

```
4018822545/3915399561216*(3-2*x)^(21/2)-17650942358963262675871173
16622980931674493927114 3/51904512*(3-2*x)^(11/2)-223975463212094
86953062074374795737299957063565/3145728*(3-2*x)^(3/2)+4045315666
89883337048499233527781983599187634017/12582912*(3-2*x)^(5/2)-118
8598027552254830082683218064697188605612952419/12582912*(3-2*x)^(
7/2)+38315833791662940918235729539899936257724714453 45/18874368*(
3-2*x)^(9/2)+99778501261680101871691304247745683309731234125 51261
/21592276992*(3-2*x)^(13/2)-1255696718499588580979726331572072320
357969297077745/2399141888*(3-2*x)^(15/2)-55011835288361289002011
693179378316699033102675/1002342287671296*(3-2*x)^(55/2))/((3-2*x
)^2-7+14*x)^19-719227969403113346821049018403 5/16241306325494153 6
82660636844148656848240 64/(-7+2*14^(1/2))^(1/2)*arctan((2*(3-2*x)
^(1/2)-(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))*(7+2*14^(1/2)
)*14^(1/2)-7192279694031133468210490184035/1624130632549415368266
06368441486568482 4064/(-7+2*14^(1/2))^(1/2)*arctan((2*(3-2*x)^(1/
2)+(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))*(7+2*14^(1/2))*14
^(1/2)+52865/2770387485853088674 72/(3-2*x)^(31/2)+14333/323956601
16830472406/(3-2*x)^(29/2)+1478345/16890426929878508371 68/(3-2*x)
^(27/2)+475387/312785683886639043920/(3-2*x)^(25/2)+16575515/7006
39931906071458380 8/(3-2*x)^(23/2)+246866015/7356719285013750312 99
84/(3-2*x)^(21/2)+355/5266289575642392066/(3-2*x)^(33/2)+1/311189
838560686803 9/(3-2*x)^(39/2)+10/29523138530116440 37/(3-2*x)^(37/2
)+143/7819642097165976098/(3-2*x)^(35/2)+8192823353/186370221887 0
150079292928/(3-2*x)^(19/2)+8972680075/16675230379364500709463 04/
(3-2*x)^(17/2)+102495360575/1647905119843080070111641 6/(3-2*x)^(1
5/2)+122484655975/178523054649667007595427 84/(3-2*x)^(13/2)+10815
878546425/1480368099325700262983624704/(3-2*x)^(11/2)+92704700941
05/10766313449641456458062725 12/(3-2*x)^(5/2)+320421783064625/301
45677658996078082575630336/(3-2*x)^(3/2)+769045155125/10093418859
0388654294338048/(3-2*x)^(9/2)+838467657280275/105509871806486273
289014706176/(3-2*x)^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(2\,x^2+x+1\right)^{20}\left(-2\,x+3\right)^{\frac{41}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2*x^2 + x + 1)^20*(-2*x + 3)^(41/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/((2*x^2 + x + 1)^20*(-2*x + 3)^(41/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2*x^2 + x + 1)^20*(-2*x + 3)^(41/2)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)**(41/2)/(2*x**2+x+1)**20,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.227316, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2*x^2 + x + 1)^20*(-2*x + 3)^(41/2)),x, algorithm="giac")

[Out] Done
```

3.50 $\int \frac{1}{(3-2x+x^2)^{11/2}(1+x+2x^2)^5}\,dx$

Optimal. Leaf size=378

$$-\frac{63043297-29625922x}{41160000000\,(x^2-2x+3)^{3/2}}-\frac{31(7434109-3088870x)}{411600000000\sqrt{x^2-2x+3}}+\frac{3(8233x+8822)}{343000\,(x^2-2x+3)^{9/2}\,(2x^2+x+1)}+\frac{8878x+5485}{117600\,(x^2-2x+3)^{9/2}\,(2x^2+x+1)^2}-\frac{30316369-15043110x}{6860000000\,(x^2-2x+3)^{5/2}}+\frac{67x+28}{1050\,(x^2-2x+3)^{9/2}\,(2x^2+x+1)^3}-\frac{4878869-2578034x}{411600000\,(x^2-2x+3)^{7/2}}-\frac{1-10x}{280\,(x^2-2x+3)^{9/2}\,(2x^2+x+1)^4}-\frac{3450497-2004270x}{123480000\,(x^2-2x+3)^{9/2}}+\frac{\sqrt{\frac{1}{70}\left(151363871237318045+110320475741093888\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{5}{7\left(151363871237318045+110320475741093888\sqrt{2}\right)}}\left(\left(932587773+620347970\sqrt{2}\right)x+3122\right.}{\sqrt{x^2-2x+3}}\right.}{137200000000}-\frac{\sqrt{\frac{1}{70}\left(110320475741093888\sqrt{2}-151363871237318045\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{5}{7\left(110320475741093888\sqrt{2}-151363871237318045\right)}}\left(\left(932587773-620347970\sqrt{2}\right)x-3\right.}{\sqrt{x^2-2x+3}}\right.}{137200000000}$$

```
[Out] -(3450497 - 2004270*x)/(123480000*(3 - 2*x + x^2)^(9/2)) - (48788
69 - 2578034*x)/(411600000*(3 - 2*x + x^2)^(7/2)) - (30316369 - 1
5043110*x)/(6860000000*(3 - 2*x + x^2)^(5/2)) - (63043297 - 29625
922*x)/(41160000000*(3 - 2*x + x^2)^(3/2)) - (31*(7434109 - 30888
70*x))/(411600000000*Sqrt[3 - 2*x + x^2]) - (1 - 10*x)/(280*(3 -
2*x + x^2)^(9/2)*(1 + x + 2*x^2)^4) + (28 + 67*x)/(1050*(3 - 2*x
+ x^2)^(9/2)*(1 + x + 2*x^2)^3) + (5485 + 8878*x)/(117600*(3 - 2*
x + x^2)^(9/2)*(1 + x + 2*x^2)^2) + (3*(8822 + 8233*x))/(343000*(
3 - 2*x + x^2)^(9/2)*(1 + x + 2*x^2)) + (Sqrt[(151363871237318045
 + 110320475741093888*Sqrt[2])/70]*ArcTan[(Sqrt[5/(7*(15136387123
7318045 + 110320475741093888*Sqrt[2]))]*(308108167 + 312239803*Sq
rt[2] + (932587773 + 620347970*Sqrt[2])*x))/Sqrt[3 - 2*x + x^2]])
/137200000000 - (Sqrt[(-151363871237318045 + 110320475741093888*S
qrt[2])/70]*ArcTanh[(Sqrt[5/(7*(-151363871237318045 + 11032047574
1093888*Sqrt[2]))]*(308108167 - 312239803*Sqrt[2] + (932587773 -
620347970*Sqrt[2])*x))/Sqrt[3 - 2*x + x^2]])/137200000000
```

Rubi [A] time = 1.58625, antiderivative size = 378, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$

$$-\frac{63043297-29625922x}{41160000000\,(x^2-2x+3)^{3/2}}-\frac{31(7434109-3088870x)}{411600000000\sqrt{x^2-2x+3}}+\frac{3(8233x+8822)}{343000\,(x^2-2x+3)^{9/2}\,(2x^2+x+1)}+\frac{8878x+5485}{117600\,(x^2-2x+3)^{9/2}\,(2x^2+x+1)^2}-\frac{30316369-15043110x}{6860000000\,(x^2-2x+3)^{5/2}}+\frac{67x+28}{1050\,(x^2-2x+3)^{9/2}\,(2x^2+x+1)^3}-\frac{4878869-2578034x}{411600000\,(x^2-2x+3)^{7/2}}-\frac{1-10x}{280\,(x^2-2x+3)^{9/2}\,(2x^2+x+1)^4}-\frac{3450497-2004270x}{123480000\,(x^2-2x+3)^{9/2}}+\frac{\sqrt{\frac{1}{70}\left(151363871237318045+110320475741093888\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{5}{7\left(151363871237318045+110320475741093888\sqrt{2}\right)}}\left(\left(932587773+620347970\sqrt{2}\right)x+3122\right.}{\sqrt{x^2-2x+3}}\right.}{137200000000}-\frac{\sqrt{\frac{1}{70}\left(110320475741093888\sqrt{2}-151363871237318045\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{5}{7\left(110320475741093888\sqrt{2}-151363871237318045\right)}}\left(\left(932587773-620347970\sqrt{2}\right)x-3\right.}{\sqrt{x^2-2x+3}}\right.}{137200000000}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - 2*x + x^2)^(11/2)*(1 + x + 2*x^2)^5),x]
```

```
[Out] -(3450497 - 2004270*x)/(123480000*(3 - 2*x + x^2)^(9/2)) - (48788
69 - 2578034*x)/(411600000*(3 - 2*x + x^2)^(7/2)) - (30316369 - 1
5043110*x)/(6860000000*(3 - 2*x + x^2)^(5/2)) - (63043297 - 29625
922*x)/(41160000000*(3 - 2*x + x^2)^(3/2)) - (31*(7434109 - 30888
70*x))/(411600000000*Sqrt[3 - 2*x + x^2]) - (1 - 10*x)/(280*(3 -
2*x + x^2)^(9/2)*(1 + x + 2*x^2)^4) + (28 + 67*x)/(1050*(3 - 2*x
+ x^2)^(9/2)*(1 + x + 2*x^2)^3) + (5485 + 8878*x)/(117600*(3 - 2*
x + x^2)^(9/2)*(1 + x + 2*x^2)^2) + (3*(8822 + 8233*x))/(343000*(
3 - 2*x + x^2)^(9/2)*(1 + x + 2*x^2)) + (Sqrt[(151363871237318045
 + 110320475741093888*Sqrt[2])/70]*ArcTan[(Sqrt[5/(7*(15136387123
7318045 + 110320475741093888*Sqrt[2]))]*(308108167 + 312239803*Sq
rt[2] + (932587773 + 620347970*Sqrt[2])*x))/Sqrt[3 - 2*x + x^2]])
/137200000000 - (Sqrt[(-151363871237318045 + 110320475741093888*S
qrt[2])/70]*ArcTanh[(Sqrt[5/(7*(-151363871237318045 + 11032047574
1093888*Sqrt[2]))]*(308108167 - 312239803*Sqrt[2] + (932587773 -
620347970*Sqrt[2])*x))/Sqrt[3 - 2*x + x^2]])/137200000000
```

Rubi in Sympy [A] time = 79.0556, size = 379, normalized size = 1.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**2-2*x+3)**(11/2)/(2*x**2+x+1)**5,x)
```

```
[Out] -(-25336765062000000000000*x + 60979022483400000000000)/(10890936
0000000000000000000*sqrt(x**2 - 2*x + 3)) - (-3919509480600000000
00*x + 834062819310000000000)/(5445468000000000000000000*(x**2 - 2
*x + 3)**(3/2)) - (-1990203453000000000*x + 4010855618700000000)/
(9075780000000000000000*(x**2 - 2*x + 3)**(5/2)) - (-5684564970000
000*x + 10757906145000000)/(90757800000000000*(x**2 - 2*x + 3)**
(7/2)) - (-10522417500000*x + 18115109250000)/(648270000000000*(x
**2 - 2*x + 3)**(9/2)) - (-50*x + 5)/(1400*(x**2 - 2*x + 3)**(9/2
)*(2*x**2 + x + 1)**4) + (93800*x + 39200)/(1470000*(x**2 - 2*x +
 3)**(9/2)*(2*x**2 + x + 1)**3) + (77682500*x + 47993750)/(102900
0000*(x**2 - 2*x + 3)**(9/2)*(2*x**2 + x + 1)**2) + (25933950000*
x + 27789300000)/(360150000000*(x**2 - 2*x + 3)**(9/2)*(2*x**2 +
x + 1)) + sqrt(70)*(12228813148230000000000 + 12392797781070000
00000000*sqrt(2))*(10528692593580000000000*sqrt(2) + 1765714407
786000000000000)*atan(sqrt(35)*(x*(2462161092930000000000*sqrt
(2) + 3701440871037000000000) + 12228813148230000000000 + 12
392797781070000000000*sqrt(2))/(27783000000000000*sqrt(15136387
1237318045 + 110320475741093888*sqrt(2))*sqrt(x**2 - 2*x + 3)))/(
30258287488800000000000000000000000000*sqrt(15136387123731
8045 + 110320475741093888*sqrt(2))) + sqrt(70)*(-1239279778107000
000000000*sqrt(2) + 12228813148230000000000)*(-1052869259358000
000000000*sqrt(2) + 1765714407786000000000000)*atanh(sqrt(35)*(x*
(-2462161092930000000000*sqrt(2) + 3701440871037000000000)
- 1239279778107000000000*sqrt(2) + 12228813148230000000000)/
(27783000000000000*sqrt(-151363871237318045 + 110320475741093888*
sqrt(2))*sqrt(x**2 - 2*x + 3)))/(302582874888000000000000000000
0000000000000*sqrt(-151363871237318045 + 110320475741093888*sqrt(
2)))
```

Mathematica [C] time = 6.46651, size = 1236, normalized size = 3.27

result too large to display

Antiderivative was successfully verified.

```
[In]  Integrate[1/((3 - 2*x + x^2)^(11/2)*(1 + x + 2*x^2)^5),x]
```

```
[Out] Sqrt[3 - 2*x + x^2]*(1/(225000*(3 - 2*x + x^2)^5) + (1 + 2*x)/(35
0000*(3 - 2*x + x^2)^4) + (3*(-38 + 45*x))/(8750000*(3 - 2*x + x^
2)^3) + (-2003 + 1198*x)/(52500000*(3 - 2*x + x^2)^2) + (-97229 +
```

```
 29420*x)/(1050000000*(3 - 2*x + x^2)) + (-797 - 1998*x)/(2800000
0*(1 + x + 2*x^2)^4) + (-14087 - 5995*x)/(105000000*(1 + x + 2*x^
2)^3) + (-795589 + 1892994*x)/(11760000000*(1 + x + 2*x^2)^2) + (
3035369 + 14037055*x)/(34300000000*(1 + x + 2*x^2))) + ((31017398
5*I + 44900803*Sqrt[7])*ArcTan[(9627448535205165 + (3579775365292
28045*I)*Sqrt[7] - 2892591314086740000*x + (36106220736881480*I)*
Sqrt[7]*x + 464983088285203040*x^2 - (103856972562252438O*I)*Sqrt
[7]*x^2 + 12836598046940220*x^3 + (32874806474606454O*I)*Sqrt[7]*
x^3 - 487447134867348425*x^4 - (428071291440525685*I)*Sqrt[7]*x^4
 + (358541546158555136*I)*Sqrt[10*(-5 + I*Sqrt[7])]*Sqrt[3 - 2*x
+ x^2] + (220640951482187776*I)*Sqrt[10*(-5 + I*Sqrt[7])]*x*Sqrt[
3 - 2*x + x^2] + (579182497640742912*I)*Sqrt[10*(-5 + I*Sqrt[7])]
*x^2*Sqrt[3 - 2*x + x^2] - (275801189352734720*I)*Sqrt[10*(-5 + I
*Sqrt[7])]*x^3*Sqrt[3 - 2*x + x^2])/(4321741285513437647*I + 8273
87564543169945*Sqrt[7] + (3694994885631086104*I)*x + 285423303382
928480*Sqrt[7]*x + (5471192788852131980*I)*x^2 - 7052553231648848
0*Sqrt[7]*x^2 - (6268363351511187532*I)*x^3 + 137879256656321740*
Sqrt[7]*x^3 + (2092254277956040633*I)*x^4 + 70562873851568315*Sqr
t[7]*x^4)])/(68600000000*Sqrt[70*(-5 + I*Sqrt[7])]) - ((I/6860000
0000)*(-310173985*I + 44900803*Sqrt[7])*ArcTan[(35*(1521027563127
6955*I + 23639644701233427*Sqrt[7] - (80355173705781000*I)*x + 81
54951525226528*Sqrt[7]*x + (32801021588957180*I)*x^2 - 2015015209
042528*Sqrt[7]*x^2 - (22632774169109180*I)*x^3 + 3939407333037764
*Sqrt[7]*x^3 - (9346476174243955*I)*x^4 + 2016082110044809*Sqrt[7
]*x^4))/(-9627448535205165 + (357977536529228045*I)*Sqrt[7] + 289
2591314086740000*x + (36106220736881480*I)*Sqrt[7]*x - 4649830882
85203040*x^2 - (1038569725622524380*I)*Sqrt[7]*x^2 - 128365980469
40220*x^3 + (328748064746064540*I)*Sqrt[7]*x^3 + 4874471348673484
25*x^4 - (428071291440525685*I)*Sqrt[7]*x^4 - (27580118935273472*
I)*Sqrt[70*(5 + I*Sqrt[7])]*Sqrt[3 - 2*x + x^2] - (27580118935273
472*I)*Sqrt[70*(5 + I*Sqrt[7])]*x^2*Sqrt[3 - 2*x + x^2] + (551602
37870546944*I)*Sqrt[70*(5 + I*Sqrt[7])]*x^3*Sqrt[3 - 2*x + x^2])]
)/Sqrt[70*(5 + I*Sqrt[7])] - ((-310173985*I + 44900803*Sqrt[7])*L
og[(-I + Sqrt[7] - (4*I)*x)^2*(I + Sqrt[7] + (4*I)*x)^2])/(137200
000000*Sqrt[70*(5 + I*Sqrt[7])]) + ((I/137200000000)*(310173985*I
 + 44900803*Sqrt[7])*Log[(-I + Sqrt[7] - (4*I)*x)^2*(I + Sqrt[7]
+ (4*I)*x)^2])/Sqrt[70*(-5 + I*Sqrt[7])] - ((I/137200000000)*(310
173985*I + 44900803*Sqrt[7])*Log[(1 + x + 2*x^2)*(-13*I + 15*Sqrt
[7] + (22*I)*x - 10*Sqrt[7]*x + (9*I)*x^2 + 5*Sqrt[7]*x^2 + I*Sqr
t[70*(-5 + I*Sqrt[7])]*Sqrt[3 - 2*x + x^2] - I*Sqrt[70*(-5 + I*Sq
rt[7])]*x*Sqrt[3 - 2*x + x^2])])/Sqrt[70*(-5 + I*Sqrt[7])] + ((-3
10173985*I + 44900803*Sqrt[7])*Log[(1 + x + 2*x^2)*(-163*I + 15*S
qrt[7] + (122*I)*x - 10*Sqrt[7]*x - (41*I)*x^2 + 5*Sqrt[7]*x^2 -
(13*I)*Sqrt[10*(5 + I*Sqrt[7])]*Sqrt[3 - 2*x + x^2] + (5*I)*Sqrt[
10*(5 + I*Sqrt[7])]*x*Sqrt[3 - 2*x + x^2])])/(137200000000*Sqrt[7
0*(5 + I*Sqrt[7])])
```

Maple [B] time = 0.765, size = 21028, normalized size = 55.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^2-2*x+3)^(11/2)/(2*x^2+x+1)^5,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2x^2 + x + 1)^5 (x^2 - 2x + 3)^{\frac{11}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((2*x^2 + x + 1)^5*(x^2 - 2*x + 3)^(11/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/((2*x^2 + x + 1)^5*(x^2 - 2*x + 3)^(11/2)), x)
```

Fricas [A] time = 0.362639, size = 6251, normalized size = 16.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2*x^2 + x + 1)^5*(x^2 - 2*x + 3)^(11/2)),x, algorithm="fricas")
```

```
[Out] 1/4939200000000*(4*sqrt(205487899)*sqrt(35)*sqrt(7)*(250540991585
0655911199179366640*x^32 - 58848315492759694117962566860800*x^31 +
70336698901347132902202470830080*x^30 - 5640114722627557282137279
38611200*x^29 + 33877828431307192657415395726886640*x^28 - 1616441
69981873538644976209121561600*x^27 + 63515930350911189944750096784
965760*x^26 - 210557817277442257216056696290868480*x^25 + 5990799
856246088078898940680822064000*x^24 - 14817882401959265024195004744
927552000*x^23 + 32181812056071451825852615591863465660*x^22 - 618
7184772955802395440882682138814720*x^21 + 10603803272759527543249
407132512637520*x^20 - 16301932856715307988417262259057211200*x^1
9 + 22613684765066685707216182128734364240*x^18 - 284629023420049
20706947119363091816160*x^17 + 32676135176862669211706329839458900
0360*x^16 - 34375259168148954959453196199877528480*x^15 + 3326409
52547642703451041322531359964000*x^14 - 29690304639514302322193623
022642067720*x^13 + 24484631887122483709778594663137626775*x^12 -
 18669195666477658318850179379968512720*x^11 + 131586975136960148
49995752136862496980*x^10 - 8560917034642778106789153295046959300
*x^9 + 5123941961010059680373987398344568415*x^8 - 28042263206249
50569135022727301669360*x^7 + 13885358123968654961822588100641973
60*x^6 - 610465831798347956368523388448548480*x^5 + 2302162989556
27150529634520155221125*x^4 - 69964383404373320595002167294609600
*x^3 + 15304284300591900638465660638051420*x^2 - 1103204757410938
88*sqrt(2)*(1655223201792*x^32 - 38878706663424*x^31 + 4646861785
86624*x^30 - 3726196136847360*x^29 + 22381713783067392*x^28 - 106
791778421442048*x^27 + 419624113942796928*x^26 - 1391070508148646
144*x^25 + 3957879649400169920*x^24 - 9789576786607705600*x^23 +
21261224222796687168*x^22 - 40876232368919362816*x^21 + 700550480
51289600656*x^20 - 107700290191151927360*x^19 + 14939948734273247
7072*x^18 - 188042906866321790048*x^17 + 215878035555994176808*x^
16 - 227103461923573594144*x^15 + 219762450463563231920*x^14 - 19
6151858411205186216*x^13 + 161760079779763942595*x^12 - 123339840
041530707216*x^11 + 86934203030952873444*x^10 - 56558523276802427
540*x^9 + 33851816283004633387*x^8 - 18526391388525625008*x^7 + 9
173495637012543408*x^6 - 4033101339230550144*x^5 + 15209461615492
06025*x^4 - 462226440381394880*x^3 + 101109228876664076*x^2 - 125
95090393724612*x + 382982001236017) - 190644164057811360913005718
8223540*x + 57969638321298857545326138026765)*sqrt(x^2 - 2*x + 3)
*sqrt((151363871237318045*sqrt(2) - 220640951482187776)/(33397068
569829208576933551323217920*sqrt(2) - 472522362514299670284917461
44535113)) - 4*sqrt(205487899)*sqrt(35)*sqrt(7)*(2505409915850655
91119917936640*x^33 - 6135372540861035002916174622720*x^32 + 7647
2071442208167905118645452800*x^31 - 63998246172179376493660674819
0720*x^30 + 4016371453241338690241233684412160*x^29 - 20048942300
949912243052144039937280*x^28 + 825418536737044412995458488180524
80*x^27 - 287159104433155596834389825111898240*x^26 + 85886862956
4315376337541934649544640*x^25 - 22369469991084197838786150484501
48800*x^24 + 5124102521174037211790204381357664960*x^23 - 1040593
3975960152123149730201711967680*x^22 + 18860766060525068433557020
025072619600*x^21 - 30691274851422616119553452520028489680*x^20 +
 45081528653462797168790944458441243760*x^19 - 600801563940866631
40994853987619185680*x^18 + 7299981596253669161782603304452647908
0*x^17 - 81231743782608564292636240096715327240*x^16 + 8310994559
3953060776505494062289747840*x^15 - 78427494994130809902434683204
783737480*x^14 + 68411527510419695509612014370957102335*x^13 - 55
231057943303058323856167435958954875*x^12 + 412850518651089793032
10529619078972420*x^11 - 28556677261894993605897587670410952880*x
^10 + 18243967900859125271164543196209642035*x^9 - 10724669541859
622266496198594992688955*x^8 + 5762245664163959895719267159339201
680*x^7 - 2797454333407067178417649149855271440*x^6 + 12027767505
15386982146365413070444165*x^5 - 44196391443110601682853133346133
```

```
1825*x^4 + 130380975576388033973331946562853820*x^3 - 27619638829
65256229043925522388836O*x^2 - 110320475741093888*sqrt(2)*(165522
3201792*x^33 - 40533929865216*x^32 + 505220108451840*x^31 - 42281
05798895616*x^30 + 26534545003438848*x^29 - 132455269127702784*x^
28 + 545320709618281344*x^27 - 1897144292662342272*x^26 + 5674198
357530944192*x^25 - 14778605890709480640*x^24 + 33852877039198730
688*x^23 - 68747805476282099904*x^22 + 124605468308576372880*x^21
 - 202764864564694271504*x^20 + 297835462881238465328*x^19 - 3969
25342242926100304*x^18 + 482280318056102498824*x^17 - 53666534238
7075938472*x^16 + 549073863627919009152*x^15 - 518138802562515874
344*x^14 + 451967348292510885963*x^13 - 364889306092787766775*x^1
2 + 272753673169336497876*x^11 - 188662439910260964464*x^10 + 120
530531835136900623*x^9 - 70853562704172604599*x^8 + 3806883120166
4625104*x^7 - 18481651602455633232*x^6 + 7946260495872201737*x^5
- 2919877186136224485*x^4 + 861374478008482796*x^3 - 182471805219
283208*x^2 + 22036451262907845*x - 663344282706293) + 33355225714
86217667919065790563025*x - 10040635859356643287398776395718 5)*sq
rt((151363871237318045*sqrt(2) - 220640951482187776)/(33397068569
82920857693355132321792O*sqrt(2) - 47252236251429967028491746144 5
35113)) - 360204287698287824 4*(337802213083473608^(1/4)*(4096*x^3
3 - 94208*x^32 + 1105920*x^31 - 8726528*x^30 + 51652864*x^29 - 24
3142912*x^28 + 943504512*x^27 - 3091758976*x^26 + 8703838016*x^25
 - 21323296320*x^24 + 45922747584*x^23 - 87669031872*x^22 + 14942
9442480*x^21 - 228892406512*x^20 + 317020649584*x^19 - 3993174399
52*x^18 + 459889940952*x^17 - 486569477336*x^16 + 474741729456*x^
15 - 428370113832*x^14 + 358128604129*x^13 - 277690283125*x^12 +
199741426598*x^11 - 133169510462*x^10 + 82124768139*x^9 - 4668881
9967*x^8 + 24352777172*x^7 - 11572941916*x^6 + 4956109991*x^5 - 1
881604195*x^4 + 618995238*x^3 - 169803054*x^2 + 35318565*x - 4164
129)*sqrt(x^2 - 2*x + 3) - 337802213083473608^(1/4)*(4096*x^34 -
98304*x^33 + 1204224*x^32 - 9922560*x^31 + 61393152*x^30 - 302465
536*x^29 + 1230167680*x^28 - 4231534592*x^27 + 12524632256*x^26 -
 32311376256*x^25 + 73387152384*x^24 - 147941398656*x^23 + 266538
692976*x^22 - 431807810720*x^21 + 632607571360*x^20 - 84257939129
6*x^19 + 1025439024680*x^18 - 1145619691952*x^17 + 1179623583016*
x^16 - 1123076849304*x^15 + 990931422841*x^14 - 811490311782*x^13
 + 617205254013*x^12 - 435943782908*x^11 + 285629584397*x^10 - 17
3202055986*x^9 + 96858530705*x^8 - 49702089632*x^7 + 23230667691*
x^6 - 9779204266*x^5 + 3646868583*x^4 - 1176696972*x^3 + 31530008
7*x^2 - 63577710*x + 7212483))*arctan(1438415293*(2*sqrt(20548789
9)*sqrt(35)*sqrt(7)*(110320475741093888*sqrt(2) - 151363871237318
045)*sqrt((151363871237318045*sqrt(2) - 220640951482187776)/(3339
706856982920857693355132321792O*sqrt(2) - 472522362514299670284 91
746144535113)) + 5*337802213083473608^(1/4)*sqrt(7)*(932587773*sq
rt(2) - 1240695940))/(4*sqrt(1438415293)*sqrt(205487899)*sqrt(35)
*sqrt(7)*(110320475741093888*sqrt(2) - 151363871237318045)*sqrt(-
(337802213083473608^(1/4)*sqrt(205487899)*sqrt(35)*(sqrt(2)*(2466
174192810249028691853897589514172583272025134182276224653*x - 595
385919413896708238305351741300870299462875468675571296839 3) - 348
768500132871805369119961982349453041135672955257343674374 0*x + 84
200333869492161110749074150025228755779007798209379891930 46)*sqrt
((151363871237318045*sqrt(2) - 220640951482187776)/(3339706856982
92085769335513232179 20*sqrt(2) - 472522362514299670284917461445 35
113)) + 337591919062388654431586980530568515903559031612841934150
69696*x^2 - 596779698385758914219233068840496250857369013003383 14
64558990*sqrt(2)*(4*x^2 - 3*x + 7) - (337802213083473608^(1/4)*sq
rt(205487899)*sqrt(35)*(246617419281024902869185389758951417258 32
7202513418227622465 3*sqrt(2) - 348768500132871805369119961982349 4
530411356729552573436743740)*sqrt((151363871237318045*sqrt(2) - 2
20640951482187776)/(33397068569829208576933551323217920*sqrt(2) -
 47252236251429967028491746144535113)) - 596779698385758914219233
06884049625085736901300338314645589 90*sqrt(2)*(4*x + 1) + 3375919
19062388654431586980530568515903559031612841934150 69696*x + 84397
97976559716360789674513264212897588975790321048353767424)*sqrt(x^
2 - 2*x + 3) - 2054878990*sqrt(2)*(145210423895997621304997884289
16104954126009459160005*sqrt(2) - 205359975395916534257564103890 7
16525784055434581114 88) - 253193939296791490823690235397926386927
66927370963145061302272*x + 590785858359180145255277215928494902 8
31228305322473384763719 68)/(145210423895997621304997884289161049 5
4126009459160005*sqrt(2) - 205359975395916534257564103890716525 78
405543458111488))*sqrt((151363871237318045*sqrt(2) - 220640951482
187776)/(33397068569829208576933551323217920*sqrt(2) - 4725223625
14299670284917461445 35113)) + 11507322344*sqrt(205487899)*sqrt(35
)*sqrt(x^2 - 2*x + 3)*(110320475741093888*sqrt(2) - 1513638712373
```

```
18045)*sqrt((151363871237318045*sqrt(2) - 220640951482187776)/(33
3970685698292085769335513232179 20*sqrt(2) - 472522362514299670284
91746144535113)) - 2876830586*sqrt(205487899)*sqrt(35)*(110320475
741093888*sqrt(2)*(4*x + 1) - 605455484949272180*x - 151363871237
318045)*sqrt((151363871237318045*sqrt(2) - 220640951482187776)/(3
33970685698292085769335513232179 20*sqrt(2) - 47252236251429967028
491746144535113)) + 50344535255*337802213083473608^(1/4)*(4283498
5*sqrt(2) - 179603212))) + 3602042876982878244*(33780221308347360
8^(1/4)*(4096*x^33 - 94208*x^32 + 1105920*x^31 - 8726528*x^30 + 5
1652864*x^29 - 243142912*x^28 + 943504512*x^27 - 3091758976*x^26
+ 8703838016*x^25 - 21323296320*x^24 + 45922747584*x^23 - 8766903
1872*x^22 + 149429442480*x^21 - 228892406512*x^20 + 317020649584*
x^19 - 399317439952*x^18 + 459889940952*x^17 - 486569477336*x^16
+ 474741729456*x^15 - 428370113832*x^14 + 358128604129*x^13 - 277
690283125*x^12 + 199741426598*x^11 - 133169510462*x^10 + 82124768
139*x^9 - 46688819967*x^8 + 24352777172*x^7 - 11572941916*x^6 + 4
956109991*x^5 - 1881604195*x^4 + 618995238*x^3 - 169803054*x^2 +
35318565*x - 4164129)*sqrt(x^2 - 2*x + 3) - 337802213083473608^(1
/4)*(4096*x^34 - 98304*x^33 + 1204224*x^32 - 9922560*x^31 + 61393
152*x^30 - 302465536*x^29 + 1230167680*x^28 - 4231534592*x^27 + 1
2524632256*x^26 - 32311376256*x^25 + 73387152384*x^24 - 147941398
656*x^23 + 266538692976*x^22 - 431807810720*x^21 + 632607571360*x
^20 - 842579391296*x^19 + 1025439024680*x^18 - 1145619691952*x^17
 + 1179623583016*x^16 - 1123076849304*x^15 + 990931422841*x^14 -
811490311782*x^13 + 617205254013*x^12 - 435943782908*x^11 + 28562
9584397*x^10 - 173202055986*x^9 + 96858530705*x^8 - 49702089632*x
^7 + 23230667691*x^6 - 9779204266*x^5 + 3646868583*x^4 - 11766969
72*x^3 + 315300087*x^2 - 63577710*x + 7212483))*arctan(1438415293
*(2*sqrt(205487899)*sqrt(35)*sqrt(7)*(110320475741093888*sqrt(2)
- 151363871237318045)*sqrt((151363871237318045*sqrt(2) - 22064095
1482187776)/(333970685698292085769335513232179 20*sqrt(2) - 472522
36251429967028491746144535113)) - 5*337802213083473608^(1/4)*sqrt
(7)*(932587773*sqrt(2) - 1240695940))/(4*sqrt(1438415293)*sqrt(20
5487899)*sqrt(35)*sqrt(7)*(110320475741093888*sqrt(2) - 151363871
237318045)*sqrt((337802213083473608^(1/4)*sqrt(205487899)*sqrt(35
)*(sqrt(2)*(24661741928102490286918538975895141725832720251341822
76224653*x - 59538591941389670823830535174130087029946287546867 55
712968393) - 34876850013287180536911996198234945304113567295525 73
436743740*x + 84200333869492161110749074150025228755779007798209 3
7989193046)*sqrt((151363871237318045*sqrt(2) - 220640951482187776
)/(333970685698292085769335513232179 20*sqrt(2) - 4725223625142996
7028491746144535113)) - 33759191906238865443158698053056851590355
90316128419341506969 6*x^2 + 59677969838575891421923306884049625 08
57369013003383146455899 0*sqrt(2)*(4*x^2 - 3*x + 7) - (33780221308
3473608^(1/4)*sqrt(205487899)*sqrt(35)*(2466174192810249028691853
89758951417258327202513418227622465 3*sqrt(2) - 348768500132871805
36911996198234945304113567295525734367437 40)*sqrt((15136387123731
8045*sqrt(2) - 220640951482187776)/(3339706856982920857693355 1323
217920*sqrt(2) - 47252236251429967028491746144535113)) + 59677969
83857589142192330688404962508573690130033831464558990*sqrt(2)*(4*
x + 1) - 33759191906238865443158698053056851590355903161284193415
069696*x - 84397979765597163607896745132642128975889757903210483 5
3767424)*sqrt(x^2 - 2*x + 3) + 2054878990*sqrt(2)*(14521042389599
762130499788428916104954126009459160005*sqrt(2) - 205359975395916
53425756410389071652578405543458111488) + 25319393929679149082369
02353979263869276692737096314506130227 2*x - 590785858359180145255
27721592849490283122830532247338476371968)/(14521042389599762130 4
99788428916104954126009459160005*sqrt(2) - 2053599753959165342575
64103890716525784055434581114 88))*sqrt((151363871237318045*sqrt(2
) - 220640951482187776)/(333970685698292085769335513232179 20*sqrt
(2) - 47252236251429967028491746144535113)) + 11507322344*sqrt(20
5487899)*sqrt(35)*sqrt(x^2 - 2*x + 3)*(110320475741093888*sqrt(2)
 - 151363871237318045)*sqrt((151363871237318045*sqrt(2) - 2206409
51482187776)/(333970685698292085769335513232179 20*sqrt(2) - 47252
236251429967028491746144535113)) - 2876830586*sqrt(205487899)*sqr
t(35)*(110320475741093888*sqrt(2)*(4*x + 1) - 605455484949272180*
x - 151363871237318045)*sqrt((151363871237318045*sqrt(2) - 220640
951482187776)/(333970685698292085769335513232179 20*sqrt(2) - 4725
22362514299670284917461445351 13)) - 50344535255*33780221308347360
8^(1/4)*(42834985*sqrt(2) - 179603212))) - 9*(337802213083473608^
(1/4)*sqrt(7)*(619986416588054712320*x^33 - 142596875815252583833
60*x^32 + 167396332478774772326400*x^31 - 132088106054085056459 77
60*x^30 + 7818377455534700703130880*x^29 - 36803052424234552531 44
7040*x^28 + 142812495466196598236519040*x^27 - 467980607540086291
```

```
795521920*x^26 + 1317446616724297757953798720*x^25 - 322757667853
5657715618094400*x^24 + 6951044852168434341663353280*x^23 - 13269
92405177373976290573024 0*x^22 + 2261821889060694323479355160 0*x^2
1 - 3464604074648222638077310904 0*x^20 + 4798547278320350044790 49
43280*x^19 - 6044223356371000837581353384 0*x^18 + 696107218055963
26893894078840*x^17 - 7364903971549544477405132812 0*x^16 + 718587
460083596632904168335 20*x^15 - 6483975875198212162303769844 0*x^14
 + 5420773192178240354947320780 5*x^13 - 4203227625878689198172149
0625*x^12 + 3023363557633788572374836091 0*x^11 - 2015705263430684
6278448886790*x^10 + 1243072282998619548982576825 5*x^9 - 70670005
3370731173492540451 5*x^8 + 3686130628133706280779668740*x^7 - 175
1725290010384786403674220*x^6 + 750175994515709494869087595*x^5 -
 284806895091577474021198775*x^4 + 93693515501145037746469710*x^3
 - 25702047601359362810309430*x^2 - 11032047574109388 8*sqrt(2)*(4
096*x^33 - 94208*x^32 + 1105920*x^31 - 8726528*x^30 + 51652864*x^
29 - 243142912*x^28 + 943504512*x^27 - 3091758976*x^26 + 87038380
16*x^25 - 21323296320*x^24 + 45922747584*x^23 - 87669031872*x^22
+ 149429442480*x^21 - 228892406512*x^20 + 317020649584*x^19 - 399
317439952*x^18 + 459889940952*x^17 - 486569477336*x^16 + 47474172
9456*x^15 - 428370113832*x^14 + 358128604129*x^13 - 277690283125*
x^12 + 199741426598*x^11 - 133169510462*x^10 + 82124768139*x^9 -
46688819967*x^8 + 24352777172*x^7 - 11572941916*x^6 + 4956109991*
x^5 - 1881604195*x^4 + 618995238*x^3 - 169803054*x^2 + 35318565*x
 - 4164129) + 534595472494684779800542 5*x - 630298685771581953407
805)*sqrt(x^2 - 2*x + 3) - 337802213083473608^(1/4)*sqrt(7)*(6199
86416588054712320*x^34 - 14879673998113313095680*x^33 + 182276006
476888085422080*x^32 - 1501917094184562540595200*x^31 + 929270515
4181094809027840*x^30 - 45782354444830385683397120*x^29 + 1862029
42315830268839785600*x^28 - 640501457119745148723312640*x^27 + 18
95776824091944217337859520*x^26 - 4890774995113719620333339520*x^
25 + 11108163483925213995887969280*x^24 - 22392982816835520879607
547520*x^23 + 40344328403382311629919551920*x^22 - 65360101861090
282575800442400*x^21 + 95753930975087526647353191200*x^20 - 12753
6078491345560715656736320*x^19 + 155214420493384520883764350600*x
^18 - 173405431539558491799550873840*x^17 + 178552392128137577493
252323720*x^16 - 169993259607663498045584890680*x^15 + 1499912162
91917485508694465845*x^14 - 122830315062901722450544706190*x^13 +
 93422576595419908569093564585*x^12 - 659861386227958431576309748
60*x^11 + 43233999634236175350258543865*x^10 - 262165337003036551
22578067370*x^9 + 14660882169867436203483071725*x^8 - 75231006952
83688215881009440*x^7 + 3516283793137448501472784095*x^6 - 148021
8215322255324062779970*x^5 + 552004146616632515489480235*x^4 - 17
8109408955150036952459740*x^3 + 47725041769783177235169915*x^2 -
11032047574109388 8*sqrt(2)*(4096*x^34 - 98304*x^33 + 1204224*x^32
 - 9922560*x^31 + 61393152*x^30 - 302465536*x^29 + 1230167680*x^2
8 - 4231534592*x^27 + 12524632256*x^26 - 32311376256*x^25 + 73387
152384*x^24 - 147941398656*x^23 + 266538692976*x^22 - 43180781072
0*x^21 + 632607571360*x^20 - 842579391296*x^19 + 1025439024680*x^
18 - 1145619691952*x^17 + 1179623583016*x^16 - 1123076849304*x^15
 + 990931422841*x^14 - 811490311782*x^13 + 617205254013*x^12 - 43
5943782908*x^11 + 285629584397*x^10 - 173202055986*x^9 + 96858530
705*x^8 - 49702089632*x^7 + 23230667691*x^6 - 9779204266*x^5 + 36
46868583*x^4 - 1176696972*x^3 + 315300087*x^2 - 63577710*x + 7212
483) - 962336831000354784277695 0*x + 1091709348113345365155735))*
log(-1581554340224321978368*(337802213083473608^(1/4)*sqrt(205487
899)*sqrt(35)*(sqrt(2)*(246617419281024902869185389758951417258 32
720251341822762246 53*x - 595385919413896708238305351741300870299 4
6287546867557129683 93) - 3487685001328718053691199619823494530411
35672955257343674374 0*x + 842003338694921611107490741500252287557
7900779820937989193046)*sqrt((15136387123731804 5*sqrt(2) - 220640
951482187776)/(33397068569829208576933551323217920*sqrt(2) - 4725
22362514299670284917461445351 13)) + 337591919062388654431586980 53
056851590355903161284193415069696*x^2 - 596779698385758914219233 0
688404962508573690130033831464558990*sqrt(2)*(4*x^2 - 3*x + 7) -
(337802213083473608^(1/4)*sqrt(205487899)*sqrt(35)*(2466174192810
24902869185389758951417258327202513418227622465 3*sqrt(2) - 348768
50013287180536911996198234945304113567295525734367437 40)*sqrt((15
13638712373180 45*sqrt(2) - 220640951482187776)/(33397068569829208
576933551323217920*sqrt(2) - 47252236251429967028491746144535113)
) - 5967796983857589142192330688404962508573690130033831464558990
*sqrt(2)*(4*x + 1) + 3375919190623886544315869805305685159035590 3
161284193415069696*x + 843979797655971636078967451326421289758897
579032104835376742 4)*sqrt(x^2 - 2*x + 3) - 2054878990*sqrt(2)*(14
52104238959976213049978842891610495412600945916000 5*sqrt(2) - 205
```

```
35997539591653425756410389071652578405543458111488) - 25319393929
67914908236902353979263869276692737096314506130227 2*x + 590785858
35918014525527721592849490283122830532247338476371968)/(145210423
8959976213049978842891610495412600945916 0005*sqrt(2) - 2053599753
95916534257564103890716525784055434581114 88)) + 9*(33780221308347
3608^(1/4)*sqrt(7)*(619986416588054712320*x^33 - 1425968758152525
8383360*x^32 + 16739633247877477232640 0*x^31 - 13208810605408505 6
4597760*x^30 + 78183774555347007031308 80*x^29 - 36803052424234552
531447040*x^28 + 142812495466196598236519040*x^27 - 4679806075400
86291795521920*x^26 + 1317446616724297757953798720*x^25 - 3227576
67853565771561809440 0*x^24 + 6951044852168434341663353280*x^23 -
13269924051773739762905730240*x^22 + 226182188906069432347935516 0
0*x^21 - 34646040746482226380773109040*x^20 + 479854727832035004 4
7904943280*x^19 - 60442233563710008375813533840*x^18 + 6961072180
5596326893894078840*x^17 - 73649039715495444774051328120*x^16 + 7
18587460083596632904168335 20*x^15 - 64839758751982121623037698440
*x^14 + 54207731921782403549473207805*x^13 - 42032276258786891981
721490625*x^12 + 30233635576337885723748360910*x^11 - 20157052634
306846278448886790*x^10 + 12430722829986195489825768255*x^9 - 706
70005337073117349254045 15*x^8 + 368613062813370628077966 8740*x^7
- 17517252900103847864036742 20*x^6 + 7501759945157094948690875 95*
x^5 - 2848068950915774740211987 75*x^4 + 936935155011450377464697 1
0*x^3 - 257020476013593628103094 30*x^2 - 110320475741093888*sqrt(
2)*(4096*x^33 - 94208*x^32 + 1105920*x^31 - 8726528*x^30 + 516528
64*x^29 - 243142912*x^28 + 943504512*x^27 - 3091758976*x^26 + 870
3838016*x^25 - 21323296320*x^24 + 45922747584*x^23 - 87669031872*
x^22 + 149429442480*x^21 - 228892406512*x^20 + 317020649584*x^19
- 399317439952*x^18 + 459889940952*x^17 - 486569477336*x^16 + 474
741729456*x^15 - 428370113832*x^14 + 358128604129*x^13 - 27769028
3125*x^12 + 199741426598*x^11 - 133169510462*x^10 + 82124768139*x
^9 - 46688819967*x^8 + 24352777172*x^7 - 11572941916*x^6 + 495610
9991*x^5 - 1881604195*x^4 + 618995238*x^3 - 169803054*x^2 + 35318
565*x - 4164129) + 53459547249468477980054 25*x - 6302986857715819
53407805)*sqrt(x^2 - 2*x + 3) - 337802213083473608^(1/4)*sqrt(7)*
(619986416588054712320*x^34 - 14879673998113313095680*x^33 + 1822
76006476888085422080*x^32 - 1501917094184562540595200*x^31 + 9292
70515418109480902784 0*x^30 - 45782354444830385683397120*x^29 + 18
6202942315830268839785600*x^28 - 640501457119745148723312640*x^27
 + 1895776824091944217337859520*x^26 - 489077499511371962033339 5
20*x^25 + 11108163483925213995887969280*x^24 - 22392982816835520 8
79607547520*x^23 + 40344328403382311629919551920*x^22 - 653601018
61090282575800442400*x^21 + 95753930975087526647353191200*x^20 -
127536078491345560715656736320*x^19 + 155214420493384520883764350
600*x^18 - 173405431539558491799550873840*x^17 + 178552392128137 5
77493252323720*x^16 - 169993259607663498045584890680*x^15 + 14999
1216291917485508694465845*x^14 - 122830315062901722450544706190*x
^13 + 93422576595419908569093564585*x^12 - 65986138622795843157 63
0974860*x^11 + 43233999634236175350258543865*x^10 - 2621653370030
3655122578067370*x^9 + 14660882169867436203483071725*x^8 - 752310
06952836882158810094 40*x^7 + 3516283793137448501472784095*x^6 - 1
48021821532225532406277997 0*x^5 + 552004146616632515489480235*x^4
 - 178109408955150036952459740*x^3 + 47725041769783177235169915*x
^2 - 110320475741093888*sqrt(2)*(4096*x^34 - 98304*x^33 + 1204224
*x^32 - 9922560*x^31 + 61393152*x^30 - 302465536*x^29 + 123016768
0*x^28 - 4231534592*x^27 + 12524632256*x^26 - 32311376256*x^25 +
73387152384*x^24 - 147941398656*x^23 + 266538692976*x^22 - 431807
810720*x^21 + 632607571360*x^20 - 842579391296*x^19 + 10254390246
80*x^18 - 1145619691952*x^17 + 1179623583016*x^16 - 1123076849304
*x^15 + 990931422841*x^14 - 811490311782*x^13 + 617205254013*x^12
 - 435943782908*x^11 + 285629584397*x^10 - 173202055986*x^9 + 968
58530705*x^8 - 49702089632*x^7 + 23230667691*x^6 - 9779204266*x^5
 + 3646868583*x^4 - 1176696972*x^3 + 315300087*x^2 - 63577710*x +
 7212483) - 9623368310003547842776950*x + 10917093481133453651557
35))*log(158155434022432197836 8*(337802213083473608^(1/4)*sqrt(20
5487899)*sqrt(35)*(sqrt(2)*(2466174192810249028691853897589514172
58327202513418227622465 3*x - 59538591941389670823830535174130087 0
29946287546867557129683 93) - 34876850013287180536911996198234945 3
04113567295525734367437 40*x + 84200333869492161110749074150025228
75557790077982093798919 3046)*sqrt((15136387123731804 5*sqrt(2) - 22
064095148218777 6)/(3339706856982920857693355132321792 0*sqrt(2) -
4725223625142996702849174614453511 3)) - 33759191906238865443158 69
80530568515903559031612841934150696 96*x^2 + 59677969838575891421 9
23306884049625085736901300338314645589 90*sqrt(2)*(4*x^2 - 3*x + 7
) - (337802213083473608^(1/4)*sqrt(205487899)*sqrt(35)*(246617419
```

```
2810249028691853897589514172583272025134182276224653*sqrt(2) - 34
87685001328718053691199619823494530411356729552573436743740)*sqrt
((151363871237318045*sqrt(2) - 220640951482187776)/(3339706856982
9208576933551323217920*sqrt(2) - 47252236251429967028491746144535
113)) + 596779698385758914219233068840496250857369013003383146455
8990*sqrt(2)*(4*x + 1) - 3375919190623886544315869805305685159035
5903161284193415069696*x - 84397979765597163607896745132642128975
88975790321048353767424)*sqrt(x^2 - 2*x + 3) + 2054878990*sqrt(2)
*(14521042389599762130499788428916104954126009459160005*sqrt(2) -
 20535997539591653425756410389071652578405543458111488) + 2531939
3929679149082369023539792638692766927370963145061302272*x - 59078
585835918014525527721592849490283122830532247338476371968)/(14521
042389599762130499788428916104954126009459160005*sqrt(2) - 205359
97539591653425756410389071652578405543458111488)))/(sqrt(20548789
9)*sqrt(35)*sqrt(7)*(619986416588054712320*x^33 - 142596875815252
58383360*x^32 + 167396332478774772326400*x^31 - 13208810605408505
64597760*x^30 + 7818377455534700703130880*x^29 - 3680305242423455
2531447040*x^28 + 142812495466196598236519040*x^27 - 467980607540
086291795521920*x^26 + 1317446616724297757953798720*x^25 - 322757
6678535657715618094400*x^24 + 6951044852168434341663353280*x^23 -
 13269924051773739762905730240*x^22 + 226182188906069432347935516
00*x^21 - 34646040746482226380773109040*x^20 + 479854727832035004
47904943280*x^19 - 60442233563710008375813533840*x^18 + 696107218
05596326893894078840*x^17 - 73649039715495444774051328120*x^16 +
71858746008359663290416833520*x^15 - 6483975875198212162303769844
0*x^14 + 54207731921782403549473207805*x^13 - 4203227625878689198
1721490625*x^12 + 30233635576337885723748360910*x^11 - 2015705263
4306846278448886790*x^10 + 12430722829986195489825768255*x^9 - 70
67000533707311734925404515*x^8 + 3686130628133706280779668740*x^7
 - 1751725290010384786403674220*x^6 + 750175994515709494869087595
*x^5 - 284806895091577474021198775*x^4 + 936935155011450377464697
10*x^3 - 25702047601359362810309430*x^2 - 110320475741093888*sqrt
(2)*(4096*x^33 - 94208*x^32 + 1105920*x^31 - 8726528*x^30 + 51652
864*x^29 - 243142912*x^28 + 943504512*x^27 - 3091758976*x^26 + 87
03838016*x^25 - 21323296320*x^24 + 45922747584*x^23 - 87669031872
*x^22 + 149429442480*x^21 - 228892406512*x^20 + 317020649584*x^19
 - 399317439952*x^18 + 459889940952*x^17 - 486569477336*x^16 + 47
4741729456*x^15 - 428370113832*x^14 + 358128604129*x^13 - 2776902
83125*x^12 + 199741426598*x^11 - 133169510462*x^10 + 82124768139*
x^9 - 46688819967*x^8 + 24352777172*x^7 - 11572941916*x^6 + 49561
09991*x^5 - 1881604195*x^4 + 618995238*x^3 - 169803054*x^2 + 3531
8565*x - 4164129) + 5345954724946847798005425*x - 630298685771581
953407805)*sqrt(x^2 - 2*x + 3)*sqrt((151363871237318045*sqrt(2) -
 220640951482187776)/(33397068569829208576933551323217920*sqrt(2)
 - 47252236251429967028491746144535113)) - sqrt(205487899)*sqrt(3
5)*sqrt(7)*(619986416588054712320*x^34 - 14879673998113313095680*
x^33 + 182276006476888085422080*x^32 - 1501917094184562540595200*
x^31 + 9292705154181094809027840*x^30 - 4578235444483038568339712
0*x^29 + 186202942315830268839785600*x^28 - 640501457119745148723
312640*x^27 + 1895776824091944217337859520*x^26 - 489077499511371
9620333339520*x^25 + 11108163483925213995887969280*x^24 - 2239298
2816835520879607547520*x^23 + 40344328403382311629919551920*x^22
- 65360101861090282575800442400*x^21 + 95753930975087526647353191
200*x^20 - 127536078491345560715656736320*x^19 + 1552144204933845
20883764350600*x^18 - 173405431539558491799550873840*x^17 + 17855
2392128137577493252323720*x^16 - 169993259607663498045584890680*x
^15 + 149991216291917485508694465845*x^14 - 122830315062901722450
544706190*x^13 + 93422576595419908569093564585*x^12 - 65986138622
795843157630974860*x^11 + 43233999634236175350258543865*x^10 - 26
216533700303655122578067370*x^9 + 14660882169867436203483071725*x
^8 - 7523100695283688215881009440*x^7 + 3516283793137448501472784
095*x^6 - 1480218215322255324062779970*x^5 + 55200414661663251548
9480235*x^4 - 178109408955150036952459740*x^3 + 47725041769783177
235169915*x^2 - 110320475741093888*sqrt(2)*(4096*x^34 - 98304*x^3
3 + 1204224*x^32 - 9922560*x^31 + 61393152*x^30 - 302465536*x^29
+ 1230167680*x^28 - 4231534592*x^27 + 12524632256*x^26 - 32311376
256*x^25 + 73387152384*x^24 - 147941398656*x^23 + 266538692976*x^
22 - 431807810720*x^21 + 632607571360*x^20 - 842579391296*x^19 +
1025439024680*x^18 - 1145619691952*x^17 + 1179623583016*x^16 - 11
23076849304*x^15 + 990931422841*x^14 - 811490311782*x^13 + 617205
254013*x^12 - 435943782908*x^11 + 285629584397*x^10 - 17320205598
6*x^9 + 96858530705*x^8 - 49702089632*x^7 + 23230667691*x^6 - 977
9204266*x^5 + 3646868583*x^4 - 1176696972*x^3 + 315300087*x^2 - 6
3577710*x + 7212483) - 9623368310003547842776950*x + 109170934811
```

```
3345365155735)*sqrt((151363871237318045*sqrt(2) - 220640951482187
776)/(333970685698292085769335513232179920*sqrt(2) - 4725223625142
99670284917461445351113)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-2*x+3)**(11/2)/(2*x**2+x+1)**5,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2x^2+x+1)^5(x^2-2x+3)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2*x^2 + x + 1)^5*(x^2 - 2*x + 3)^(11/2)),x, algorithm="giac")
```

```
[Out] integrate(1/((2*x^2 + x + 1)^5*(x^2 - 2*x + 3)^(11/2)), x)
```

3.51 $\int \frac{1}{(3-2x+x^2)^{21/2}(1+x+2x^2)^{10}}\,dx$

Optimal. Leaf size=638

result too large to display

```
[Out] (37358055634422583 - 14024622879097678*x)/(18401244792000000*(3
 - 2*x + x^2)^(19/2)) + (476849951294984711 - 125181871472148210*
x)/(1042737204880000000*(3 - 2*x + x^2)^(17/2)) + (785175837548
3333511 + 1942164996204584234*x)/(15641058073200000000*(3 - 2*
x + x^2)^(15/2)) - (11*(7502325106308201089 - 7813986379726516886
*x))/(406667509903200000000*(3 - 2*x + x^2)^(13/2)) - (3*(6905
3268515296359011 - 44840736195018286006*x))/(11470109253680000000
00000*(3 - 2*x + x^2)^(11/2)) - (838519439380295335657 - 46618939
0555853643870*x)/(938463484392000000000000*(3 - 2*x + x^2)^(9/2)
) - (1117646664729238460189 - 568839749685437871554*x)/(312821161
4640000000000000*(3 - 2*x + x^2)^(7/2)) - (655140551156544930168
9 - 312729855998330930191 0*x)/(52136860244000000000000000*(3 - 2
*x + x^2)^(5/2)) - (417903978239845985081 9 - 1886993445589652402 6
94*x)/(104273720488000000000000000*(3 - 2*x + x^2)^(3/2)) - (121
05495874518671061833 - 51176564350436793381 90*x)/(104273720488000
0000000000000*Sqrt[3 - 2*x + x^2]) - (1 - 10*x)/(630*(3 - 2*x +
x^2)^(19/2)*(1 + x + 2*x^2)^9) + (887 + 2218*x)/(88200*(3 - 2*x +
 x^2)^(19/2)*(1 + x + 2*x^2)^8) + (14453 + 29371*x)/(1080450*(3 -
 2*x + x^2)^(19/2)*(1 + x + 2*x^2)^7) + (8837931 + 17459234*x)/(6
05052000*(3 - 2*x + x^2)^(19/2)*(1 + x + 2*x^2)^6) + (447940041 +
 813432205*x)/(26471025000*(3 - 2*x + x^2)^(19/2)*(1 + x + 2*x^2)
^5) + (592729157441 + 911061463974*x)/(29647548000000*(3 - 2*x +
x^2)^(19/2)*(1 + x + 2*x^2)^4) + (277010166219 + 310705340015*x)/
(12353145000000*(3 - 2*x + x^2)^(19/2)*(1 + x + 2*x^2)^3) + (5488
221294349 + 1384103301166*x)/(276710448000000*(3 - 2*x + x^2)^(19
/2)*(1 + x + 2*x^2)^2) - (37857197792117 + 146548895467025*x)/(24
2121642000000*(3 - 2*x + x^2)^(19/2)*(1 + x + 2*x^2)) + (Sqrt[(8
104222592127468960547894479780085484640 5 + 573059225230017071260 2
6363878666500308992*Sqrt[2])/70]*ArcTan[(Sqrt[5/(7*(810422259212 7
4689605478944797800854846405 + 573059225230017071260263638786665 0
0308992*Sqrt[2]))]*(27294458952324838174 9 + 191941026386645109841
*Sqrt[2] + (656826642296538601431 + 464885615909893491590*Sqrt[2]
)*x))/Sqrt[3 - 2*x + x^2]])/322828856000000000000000 - (Sqrt[(-
8104222592127468960547894479780085484640 5 + 573059225230017071260
26363878666500308992*Sqrt[2])/70]*ArcTanh[(Sqrt[5/(7*(-8104222592
12746896054789447978008548464 05 + 573059225230017071260263638786 6
6500308992*Sqrt[2]))]*(27294458952324838174 9 - 191941026386645109
841*Sqrt[2] + (656826642296538601431 - 464885615909893491590*Sqrt
[2])*x))/Sqrt[3 - 2*x + x^2]])/322828856000000000000000
```

Rubi [A] time = 2.83471, antiderivative size = 638, normalized size of antiderivative = 1., number of

steps used = 24, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261

$$-\frac{1210549587451867106 1833-5117656435043679338190x}{104273720488000000000000000\sqrt{x^2-2x+3}}-\frac{146548895467025x+37857197792117}{2421216420000000\left(x^2-2x+3\right)^{19/2}\left(2x^2+x+1\right)}-\frac{417903978239845985 0819-1886993445589652402694x}{104273720488000000000000000\left(x^2-2x+3\right)^{3/2}}+\frac{1384103301166x+5488221294349}{276710448000000\left(x^2-2x+3\right)^{19/2}\left(2x^2+x+1\right)^2}-\frac{655140551156544930168 9-3127298559983309301910x}{52136860244000000000000000\left(x^2-2x+3\right)^{5/2}}+\frac{310705340015x+277010166219}{12353145000000\left(x^2-2x+3\right)^{19/2}\left(2x^2+x+1\right)^3}-\frac{1117646664729238460189-568839749685437871554x}{3128211614640000000000000\left(x^2-2x+3\right)^{7/2}}+\frac{911061463974x+592729157441}{29647548000000\left(x^2-2x+3\right)^{19/2}\left(2x^2+x+1\right)^4}-\frac{838519439380295335657-466189390555853643870x}{938463484392000000000000\left(x^2-2x+3\right)^{9/2}}+\frac{813432205x+447940041}{26471025000\left(x^2-2x+3\right)^{19/2}\left(2x^2+x+1\right)^5}-\frac{3(69053268515296359011-44840736195018286006x)}{114701092536800000000000\left(x^2-2x+3\right)^{11/2}}+\frac{17459234x+8837931}{605052000\left(x^2-2x+3\right)^{19/2}\left(2x^2+x+1\right)^6}-\frac{11(7502325106308201089-7813986379726516886x)}{406667509903200000000000\left(x^2-2x+3\right)^{13/2}}+\frac{29371x+14453}{1080450\left(x^2-2x+3\right)^{19/2}\left(2x^2+x+1\right)^7}+\frac{194216499620458 4234x+7851758375483333511}{15641058073200000000000\left(x^2-2x+3\right)^{15/2}}+\frac{2218x+887}{88200\left(x^2-2x+3\right)^{19/2}\left(2x^2+x+1\right)^8}+\frac{476849951294984711-125181871472148210x}{1042737204880000000000\left(x^2-2x+3\right)^{17/2}}-\frac{1-10x}{630\left(x^2-2x+3\right)^{19/2}\left(2x^2+x+1\right)^9}+\frac{37358055634422583-14024622879097678x}{1840124479200000000\left(x^2-2x+3\right)^{19/2}}+\frac{\sqrt{\frac{1}{70}\left(8104222592127468960547894479780085484640 5+57305922523001707126026363878666500308992\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{7(}\cdots}{\cdots}\right)}{3228\cdots}-\frac{\sqrt{\frac{1}{70}\left(57305922523001707126026363878666500308992\sqrt{2}-81042225921274689605478944797800854846405\right)}\tanh^{-1}\left(\frac{\sqrt{\cdots}}{\cdots}\right)}{32\cdots}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((3 - 2*x + x^2)^(21/2)*(1 + x + 2*x^2)^10),x]
```

```
[Out] (37358055634422583 - 14024622879097678*x)/(1840124479200000000*(3
 - 2*x + x^2)^(19/2)) + (476849951294984711 - 125181871472148210*
x)/(1042737204880000000000*(3 - 2*x + x^2)^(17/2)) + (785175837548
3333511 + 1942164996204584234*x)/(15641058073200000000000*(3 - 2*
x + x^2)^(15/2)) - (11*(7502325106308201089 - 7813986379726516886
*x))/(406667509903200000000000*(3 - 2*x + x^2)^(13/2)) - (3*(6905
3268515296359011 - 44840736195018286006*x))/(11470109253680000000
00000*(3 - 2*x + x^2)^(11/2)) - (838519439380295335657 - 46618939
0555853643870*x)/(938463484392000000000000*(3 - 2*x + x^2)^(9/2)
) - (1117646664729238460189 - 5688397496854378715 54*x)/(312821161
46400000000000000*(3 - 2*x + x^2)^(7/2)) - (655140551156544930168
9 - 3127298559983309301910*x)/(52136860244000000000000000*(3 - 2
*x + x^2)^(5/2)) - (417903978239845985 0819 - 1886993445589652402 6
94*x)/(104273720488000000000000000*(3 - 2*x + x^2)^(3/2)) - (121
0549587451867106 1833 - 5117656435043679338190*x)/(104273720488000
000000000000000*Sqrt[3 - 2*x + x^2]) - (1 - 10*x)/(630*(3 - 2*x +
x^2)^(19/2)*(1 + x + 2*x^2)^9) + (887 + 2218*x)/(88200*(3 - 2*x +
 x^2)^(19/2)*(1 + x + 2*x^2)^8) + (14453 + 29371*x)/(1080450*(3 -
 2*x + x^2)^(19/2)*(1 + x + 2*x^2)^7) + (8837931 + 17459234*x)/(6
05052000*(3 - 2*x + x^2)^(19/2)*(1 + x + 2*x^2)^6) + (447940041 +
 813432205*x)/(26471025000*(3 - 2*x + x^2)^(19/2)*(1 + x + 2*x^2)
^5) + (592729157441 + 911061463974*x)/(29647548000000*(3 - 2*x +
x^2)^(19/2)*(1 + x + 2*x^2)^4) + (277010166219 + 310705340015*x)/
(12353145000000*(3 - 2*x + x^2)^(19/2)*(1 + x + 2*x^2)^3) + (5488
221294349 + 1384103301166*x)/(276710448000000*(3 - 2*x + x^2)^(19
```

```
/2)*(1 + x + 2*x^2)^2) - (37857197792117 + 146548895467025*x)/(24
21216420000000*(3 - 2*x + x^2)^(19/2)*(1 + x + 2*x^2)) + (Sqrt[(8
104222592127468960547894479780085484640‌5 + 5730592252300170712602
6363878666500308992*Sqrt[2])/70]*ArcTan[(Sqrt[5/(7*(8104222592127
4689605478944797800854846405 + 5730592252300170712602636387866650
0308992*Sqrt[2]))]*(272944589523248381749 + 191941026386645109841
*Sqrt[2] + (656826642296538601431 + 464885615909893491590*Sqrt[2]
)*x))/Sqrt[3 - 2*x + x^2]])/322828856000000000000000000 - (Sqrt[(-
81042225921274689605478944797800854846405 + 573059225230017071260
26363878666500308992*Sqrt[2])/70]*ArcTanh[(Sqrt[5/(7*(-8104222592
1274689605478944797800854846405 + 5730592252300170712602636387866
6500308992*Sqrt[2]))]*(272944589523248381749 - 191941026386645109
841*Sqrt[2] + (656826642296538601431 - 464885615909893491590*Sqrt
[2])*x))/Sqrt[3 - 2*x + x^2]])/322828856000000000000000000
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{-443613257029425219187500000000000x+16898370149016020696062500000000000}{369519996979350000000000000000000000\left(x^{2}-2x+3\right)^{\frac{17}{2}}}+\frac{-82832928879670660687500000000x+220646016090808380843750000000}{1086823520527500000000000000000000\left(x^{2}-2x+3\right)^{\frac{19}{2}}}-\frac{-50x+5}{3150\left(x^{2}-2x+3\right)^{\frac{19}{2}}\left(2x^{2}+x+1\right)^{9}}+\frac{221800x+88700}{8820000\left(x^{2}-2x+3\right)^{\frac{19}{2}}\left(2x^{2}+x+1\right)^{8}}+\frac{587420000x+289060000}{21609000000\left(x^{2}-2x+3\right)^{\frac{19}{2}}\left(2x^{2}+x+1\right)^{7}}+\frac{1309442550000x+662844825000}{45378900000000\left(x^{2}-2x+3\right)^{\frac{19}{2}}\left(2x^{2}+x+1\right)^{6}}+\frac{2440296615000000x+1343820123000000}{79413075000000000\left(x^{2}-2x+3\right)^{\frac{19}{2}}\left(2x^{2}+x+1\right)^{5}}+\frac{3416480489902500000x+2222734340403750000}{111178305000000000000\left(x^{2}-2x+3\right)^{\frac{19}{2}}\left(2x^{2}+x+1\right)^{4}}+\frac{2936165463141750000000x+2617746070769550000000}{116737220250000000000000\left(x^{2}-2x+3\right)^{\frac{19}{2}}\left(2x^{2}+x+1\right)^{3}}+\frac{4087430061255843750000000x+16207403509874390625000000}{817160541750000000000000000\left(x^{2}-2x+3\right)^{\frac{19}{2}}\left(2x^{2}+x+1\right)^{2}}-\frac{17311088277042328125000000000x+44718814891938206250000000000}{2860061896125000000000000000000\left(x^{2}-2x+3\right)^{\frac{19}{2}}\left(2x^{2}+x+1\right)}-\frac{\int\frac{1419562422494160701400000000000000000x^{2}-352737956551832094072375000000000000x+1476220393170575791239375000000000000}{\left(x^{2}-2x+3\right)^{\frac{17}{2}}\left(2x^{2}+x+1\right)}\,dx}{369519996979350000000000000000000000}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**2-2*x+3)**(21/2)/(2*x**2+x+1)**10,x)
```

```
[Out] (-443613257029425219187500000000000*x + 1689837014901602069606250
000000000)/(369519996979350000000000000000000000*(x**2 - 2*x + 3)
**(17/2)) + (-82832928879670660687500000000*x + 2206460160908083
80843750000000)/(1086823520527500000000000000000000*(x**2 - 2*x +
 3)**(19/2)) - (-50*x + 5)/(3150*(x**2 - 2*x + 3)**(19/2)*(2*x**2
 + x + 1)**9) + (221800*x + 88700)/(8820000*(x**2 - 2*x + 3)**(19
/2)*(2*x**2 + x + 1)**8) + (587420000*x + 289060000)/(21609000000
*(x**2 - 2*x + 3)**(19/2)*(2*x**2 + x + 1)**7) + (1309442550000*x
 + 662844825000)/(45378900000000*(x**2 - 2*x + 3)**(19/2)*(2*x**2
 + x + 1)**6) + (2440296615000000*x + 1343820123000000)/(79413075
000000000*(x**2 - 2*x + 3)**(19/2)*(2*x**2 + x + 1)**5) + (341648
0489902500000*x + 2222734340403750000)/(111178305000000000000*(x*
*2 - 2*x + 3)**(19/2)*(2*x**2 + x + 1)**4) + (2936165463141750000
000*x + 2617746070769550000000)/(116737220250000000000000*(x**2 -
 2*x + 3)**(19/2)*(2*x**2 + x + 1)**3) + (40874300612558437500000
0*x + 16207403509874390625000000)/(817160541750000000000000000*(x**
2 - 2*x + 3)**(19/2)*(2*x**2 + x + 1)**2) - (17311088277042328125
000000000*x + 44718814891938206250000000000)/(2860061896125000000000
0000000*(x**2 - 2*x + 3)**(19/2)*(2*x**2 + x + 1)) - Integral((14
19562422494160701400000000000000000*x**2 - 352737956551832094072375
000000000000*x + 1476220393170575791239375000000000000)/((x**2 - 2*x
 + 3)**(17/2)*(2*x**2 + x + 1)), x)/369519996979350000000000000000
```

0000000

Mathematica [C] time = 6.71662, size = 1431, normalized size = 2.24

result too large to display

Antiderivative was successfully verified.

```
[In]  Integrate[1/((3 - 2*x + x^2)^(21/2)*(1 + x + 2*x^2)^10),x]
```

```
[Out] Sqrt[3 - 2*x + x^2]*((1 - x)/(11875000000*(3 - 2*x + x^2)^10) + (
265 - 113*x)/(403750000000*(3 - 2*x + x^2)^9) + (82361 - 4841*x)/
(60562500000000*(3 - 2*x + x^2)^8) + (1062937 + 1642511*x)/(15746
25000000000*(3 - 2*x + x^2)^7) + (7*(-678331 + 833371*x))/(222062
5000000000*(3 - 2*x + x^2)^6) + (7*(-73161291 + 43964675*x))/(908
43750000000000*(3 - 2*x + x^2)^5) + (-1340879383 + 430593031*x)/(
181687500000000000*(3 - 2*x + x^2)^4) - (11*(1626125723 + 1129502
05*x))/(3028125000000000000*(3 - 2*x + x^2)^3) - (11*(3311570647
+ 15286717673*x))/(36337500000000000000*(3 - 2*x + x^2)^2) - (11*
(-411521923277 + 484788625685*x))/(363375000000000000000*(3 - 2*x
 + x^2)) + (251943 + 221770*x)/(6300000000000*(1 + x + 2*x^2)^9)
- (73*(-888423 + 1604678*x))/(882000000000000*(1 + x + 2*x^2)^8)
+ (-2596903794 - 4965311863*x)/(108045000000000000*(1 + x + 2*x^2)
^7) + (-539608494637 - 334647150510*x)/(12101040000000000000*(1 +
x + 2*x^2)^6) + (-40800462989458 + 56711874696335*x)/(26471025000
0000000000*(1 + x + 2*x^2)^5) + (42018358198215561 + 129196597088
670934*x)/(2964754800000000000000000*(1 + x + 2*x^2)^4) + (6281955
9864314747 + 169630389653846945*x)/(370594350000000000000000*(1 +
 x + 2*x^2)^3) + (1082422109196374795 + 4797048907791526114*x)/(8
301313440000000000000000*(1 + x + 2*x^2)^2) + (655712031444299227
47 + 367152793968978953465*x)/(3631824630000000000000000000*(1 + x
 + 2*x^2))) + ((232442807954946745795*I + 21634177831191924841*Sq
rt[7])*ArcTan[(-1350637388604350168995865589487332591135 15 + (188
6308946264666902168552859950458893964 05*I)*Sqrt[7] - 150624136187
2688008559268776761430483700000*x - (105711500937472192718115651 3
50352447938680*I)*Sqrt[7]*x + 491153540508443587025809789813541 98
5707360*x^2 - (4607640641771399933999751008726633103994 20*I)*Sqrt
[7]*x^2 - 180084985147246689199448745264977678818020*x^3 + (19786
829637791387086383768095344600939686 0*I)*Sqrt[7]*x^3 - 1760048165
007618809267744855998310477758 25*x^4 - (2073428332284595771635570
430355582648351 65*I)*Sqrt[7]*x^4 + (186244248199755548159585682 60
5666126004224*I)*Sqrt[10*(-5 + I*Sqrt[7])]*Sqrt[3 - 2*x + x^2] +
(11461184504600341425205272775733300061798 4*I)*Sqrt[10*(-5 + I*Sq
rt[7])]*x*Sqrt[3 - 2*x + x^2] + (3008560932457589624116384103629 9
9126622208*I)*Sqrt[10*(-5 + I*Sqrt[7])]*x^2*Sqrt[3 - 2*x + x^2] -
 (143264806307504267815065909696662507724 80*I)*Sqrt[10*(-5 + I*S
qrt[7])]*x^3*Sqrt[3 - 2*x + x^2])/(236877329083883697986467849302
3884746594823*I + 4236429402592387354739426631800259567295 05*Sqrt
[7] + (1890613486065620301760074218556745311646936*I)*x + 6150574
559311228258394328777942059796320*Sqrt[7]*x + (25113002598558229 6
2340893027852239157667820*I)*x^2 - 202786755080110618986776343109
4227596320*Sqrt[7]*x^2 - (31342177462307603571283187974993808123 0
3788*I)*x^3 + 63430431602720043279192866968369397935660*Sqrt[7]*x
^3 + (9447490648866264673283853691904607036696 97*I)*x^4 + 1638131
7765107264789462917221030750634835*Sqrt[7]*x^4)])/(16141442800000
000000000000*Sqrt[70*(-5 + I*Sqrt[7])]) - ((I/1614144280000000000
0000000)*(-232442807954946745795*I + 21634177831191924841*Sqrt[7]
)*ArcTan[(35*(43624942906639466765851862182126076285 95*I + 121040
8400740682101354121894800074162 0843*Sqrt[7] - (409190315966173327
07196094500783237405000*I)*x + 17573070169460652166840939365548 74
22752*Sqrt[7]*x + (2648728832926512757773396585336431031062 0*I)*x
^2 - 5793907288003160542479324088840650275 2*Sqrt[7]*x^2 - (152388
9414975282568392481402100786307062 0*I)*x^3 + 1812298045792001236 5
48367627667697083876*Sqrt[7]*x^3 - (795837271959975808913244203 76
5619963595*I)*x^4 + 46803765043163613684179763488659287528 1*Sqrt[
7]*x^4))/(135063738860435016899586558948733259113515 + (188630894
6264666902168552859950458893964 05*I)*Sqrt[7] + 150624136187268800
855926877676143048370000 0*x - (105711500937472192718115651350352 4
47938680*I)*Sqrt[7]*x - 4911535405084435870258097898135419857073 6
0*x^2 - (4607640641771399933999751008726633103994 20*I)*Sqrt[7]*x^
```

```
2 + 180084985147246689199448745264977678818020*x^3 + (19786829637
791387086383768095344600939686O*I)*Sqrt[7]*x^3 + 1760048165007618
809267744855998310477758825*x^4 - (20734283322845957716355704303555
58264835165*I)*Sqrt[7]*x^4 - (143264806307504267815065909696666625
077248*I)*Sqrt[70*(5 + I*Sqrt[7])]*Sqrt[3 - 2*x + x^2] - (1432648
06307504267815065909696666625077248*I)*Sqrt[70*(5 + I*Sqrt[7])]*x^
2*Sqrt[3 - 2*x + x^2] + (286529612615008535630131819393332501544 9
6*I)*Sqrt[70*(5 + I*Sqrt[7])]*x^3*Sqrt[3 - 2*x + x^2])])/Sqrt[70*
(5 + I*Sqrt[7])] - ((-232442807954946745795*I + 21634177831191924
841*Sqrt[7])*Log[(-I + Sqrt[7] - (4*I)*x)^2*(I + Sqrt[7] + (4*I)*
x)^2])/(322828856000000000000000000*Sqrt[70*(5 + I*Sqrt[7])]) + ((
I/322828856000000000000000000)*(232442807954946745795*I + 21634177
831191924841*Sqrt[7])*Log[(-I + Sqrt[7] - (4*I)*x)^2*(I + Sqrt[7]
 + (4*I)*x)^2])/Sqrt[70*(-5 + I*Sqrt[7])] - ((I/3228288560000000
00000000000)*(232442807954946745795*I + 21634177831191924841*Sqrt[7
])*Log[(1 + x + 2*x^2)*(-13*I + 15*Sqrt[7] + (22*I)*x - 10*Sqrt[7
]*x + (9*I)*x^2 + 5*Sqrt[7]*x^2 + I*Sqrt[70*(-5 + I*Sqrt[7])]*Sqr
t[3 - 2*x + x^2] - I*Sqrt[70*(-5 + I*Sqrt[7])]*x*Sqrt[3 - 2*x + x
^2])])/Sqrt[70*(-5 + I*Sqrt[7])] + ((-232442807954946745795*I + 2
1634177831191924841*Sqrt[7])*Log[(1 + x + 2*x^2)*(-163*I + 15*Sqr
t[7] + (122*I)*x - 10*Sqrt[7]*x - (41*I)*x^2 + 5*Sqrt[7]*x^2 - (1
3*I)*Sqrt[10*(5 + I*Sqrt[7])]*Sqrt[3 - 2*x + x^2] + (5*I)*Sqrt[10
*(5 + I*Sqrt[7])]*x*Sqrt[3 - 2*x + x^2])])/(322828856000000000000
00000*Sqrt[70*(5 + I*Sqrt[7])])
```

Maple [B] time = 4.895, size = 86793, normalized size = 136.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^2-2*x+3)^(21/2)/(2*x^2+x+1)^10,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2x^2 + x + 1)^{10} (x^2 - 2x + 3)^{\frac{21}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((2*x^2 + x + 1)^10*(x^2 - 2*x + 3)^(21/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/((2*x^2 + x + 1)^10*(x^2 - 2*x + 3)^(21/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((2*x^2 + x + 1)^10*(x^2 - 2*x + 3)^(21/2)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**2-2*x+3)**(21/2)/(2*x**2+x+1)**10,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.350931, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2*x^2 + x + 1)^10*(x^2 - 2*x + 3)^(21/2)),x, algorithm="giac")
```

```
[Out] Done
```

3.52 $\int \frac{-a-\sqrt{1+a^2}+x}{\left(-a+\sqrt{1+a^2}+x\right)\sqrt{(-a+x)(1+x^2)}}\,dx$

Optimal. Leaf size=66

$$-\sqrt{2}\sqrt{\sqrt{a^2+1}+a}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sqrt{a^2+1}-a}(x-a)}{\sqrt{(x^2+1)(x-a)}}\right)$$

```
[Out] -(Sqrt[2]*Sqrt[a + Sqrt[1 + a^2]]*ArcTan[(Sqrt[2]*Sqrt[-a + Sqrt[
1 + a^2]]*(-a + x))/Sqrt[(-a + x)*(1 + x^2)]])
```

Rubi [C] time = 2.37246, antiderivative size = 204, normalized size of antiderivative = 3.09, number of steps used = 9, number of rules used = 8, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$\frac{4\sqrt{a^2+1}\sqrt{x^2+1}\sqrt{\frac{a-x}{a+i}}\,\Pi\left(\frac{2}{1-i\left(a-\sqrt{a^2+1}\right)};\sin^{-1}\left(\frac{\sqrt{1-ix}}{\sqrt{2}}\right)|\frac{2}{1-ia}\right)}{\left(1-i\left(a-\sqrt{a^2+1}\right)\right)\sqrt{(x^2+1)(-(a-x))}}+\frac{2i\sqrt{x^2+1}\sqrt{\frac{a-x}{a+i}}F\left(\sin^{-1}\left(\frac{\sqrt{1-ix}}{\sqrt{2}}\right)|\frac{2}{1-ia}\right)}{\sqrt{(x^2+1)(-(a-x))}}$$

Antiderivative was successfully verified.

```
[In]  Int[(-a - Sqrt[1 + a^2] + x)/((-a + Sqrt[1 + a^2] + x)*Sqrt[(-
a + x)*(1 + x^2)]),x]
```

```
[Out] ((2*I)*Sqrt[(a - x)/(I + a)]*Sqrt[1 + x^2]*EllipticF[ArcSin[Sqrt[
1 - I*x]/Sqrt[2]], 2/(1 - I*a)])/Sqrt[-((a - x)*(1 + x^2))] + (4*
Sqrt[1 + a^2]*Sqrt[(a - x)/(I + a)]*Sqrt[1 + x^2]*EllipticPi[2/(1
 - I*(a - Sqrt[1 + a^2])), ArcSin[Sqrt[1 - I*x]/Sqrt[2]], 2/(1 -
I*a)])/((1 - I*(a - Sqrt[1 + a^2]))*Sqrt[-((a - x)*(1 + x^2))])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-a+x-(a**2+1)**(1/2))/(-a+x+(a**2+1)**(1/2))/((-
a+x)*(x**2+1))**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.56329, size = 213, normalized size = 3.23

$$\frac{2\sqrt{\frac{a-x}{a+i}}\left(2i\sqrt{a^2+1}\sqrt{1-ix}\sqrt{x^2+1}\,\Pi\left(\frac{2i}{a-\sqrt{a^2+1}+i};\sin^{-1}\left(\frac{\sqrt{1-ix}}{\sqrt{2}}\right)|\frac{2i}{a+i}\right)-\left(\sqrt{a^2+1}-a-i\right)\sqrt{1+ix}(x+i)F\left(\sin^{-1}\left(\frac{\sqrt{1-ix}}{\sqrt{2}}\right)|\frac{2i}{a+i}\right.\right.}{\left(-\sqrt{a^2+1}+a+i\right)\sqrt{1-ix}\sqrt{(x^2+1)(x-a)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-a - Sqrt[1 + a^2] + x)/((-a + Sqrt[1 + a^2] + x)*Sqrt[(-
a + x)*(1 + x^2)]),x]
```

```
[Out] (2*Sqrt[(a - x)/(I + a)]*(-((-I - a + Sqrt[1 + a^2])*Sqrt[1 + I*x
]*(I + x)*EllipticF[ArcSin[Sqrt[1 - I*x]/Sqrt[2]], (2*I)/(I + a)]
) + (2*I)*Sqrt[1 + a^2]*Sqrt[1 - I*x]*Sqrt[1 + x^2]*EllipticPi[(2
```

```
*I)/(I + a - Sqrt[1 + a^2]), ArcSin[Sqrt[1 - I*x]/Sqrt[2]], (2*I)
/(I + a)]))/((I + a - Sqrt[1 + a^2])*Sqrt[1 - I*x]*Sqrt[(-a + x)*
(1 + x^2)])
```

Maple [C] time = 0.168, size = 1275, normalized size = 19.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a+x-(a^2+1)^(1/2))/(-a+x+(a^2+1)^(1/2))/((-a+x)*(x^2+1))^(1/2),x)
```

```
[Out] 2*(-a-I)*((-a+x)/(-a-I))^(1/2)*((x-I)/(a-I))^(1/2)*((x+I)/(a+I))^
(1/2)/(-a*x^2+x^3-a+x)^(1/2)*EllipticF(((-a+x)/(-a-I))^(1/2),((a+
I)/(a-I))^(1/2))-2*(a^2+1)^(1/2)*(-I/(a^2+1)^(1/2)*(1-I*x)^(1/2)*
(-1/(-a-I)*a+1/(-a-I)*x)^(1/2)*(1+I*x)^(1/2)/(-a^3*x^2+a^2*x^3-a^
3+a^2*x-a*x^2+x^3-a+x)^(1/2)/(-I-a-(a^2+1)^(1/2))*EllipticPi(1/2*
2^(1/2)*(-I*(x+I))^(1/2),-2*I/(-I-a-(a^2+1)^(1/2)),2^(1/2)*(-I/(-
a-I))^(1/2))*a^2-I/(a^2+1)^(1/2)*(1-I*x)^(1/2)*(-1/(-a-I)*a+1/(-a
-I)*x)^(1/2)*(1+I*x)^(1/2)/(-a^3*x^2+a^2*x^3-a^3+a^2*x-a*x^2+x^3-
a+x)^(1/2)/(-I-a-(a^2+1)^(1/2))*EllipticPi(1/2*2^(1/2)*(-I*(x+I))
^(1/2),-2*I/(-I-a-(a^2+1)^(1/2)),2^(1/2)*(-I/(-a-I))^(1/2))+I/(a^
2+1)^(1/2)*(1-I*x)^(1/2)*(-1/(-a-I)*a+1/(-a-I)*x)^(1/2)*(1+I*x)^(
1/2)/(-a^3*x^2+a^2*x^3-a^3+a^2*x-a*x^2+x^3-a+x)^(1/2)/(-I-a+(a^2+
1)^(1/2))*EllipticPi(1/2*2^(1/2)*(-I*(x+I))^(1/2),-2*I/(-I-a+(a^2
+1)^(1/2)),2^(1/2)*(-I/(-a-I))^(1/2))*a^2+I/(a^2+1)^(1/2)*(1-I*x)
^(1/2)*(-1/(-a-I)*a+1/(-a-I)*x)^(1/2)*(1+I*x)^(1/2)/(-a^3*x^2+a^2
*x^3-a^3+a^2*x-a*x^2+x^3-a+x)^(1/2)/(-I-a+(a^2+1)^(1/2))*Elliptic
Pi(1/2*2^(1/2)*(-I*(x+I))^(1/2),-2*I/(-I-a+(a^2+1)^(1/2)),2^(1/2)
*(-I/(-a-I))^(1/2))+(-1/(-a-I)*a+1/(-a-I)*x)^(1/2)*(1/(a-I)*x-I/(
a-I))^(1/2)*(1/(a+I)*x+I/(a+I))^(1/2)/(-a*x^2+x^3-a+x)^(1/2)/(a^2
+1)^(1/2)*EllipticPi(((-a+x)/(-a-I))^(1/2),-(a+I)/(a^2+1)^(1/2),(
(a+I)/(a-I))^(1/2))*a+I*(-1/(-a-I)*a+1/(-a-I)*x)^(1/2)*(1/(a-I)*x
-I/(a-I))^(1/2)*(1/(a+I)*x+I/(a+I))^(1/2)/(-a*x^2+x^3-a+x)^(1/2)/
(a^2+1)^(1/2)*EllipticPi(((-a+x)/(-a-I))^(1/2),-(a+I)/(a^2+1)^(1/
2),((a+I)/(a-I))^(1/2))-(-1/(-a-I)*a+1/(-a-I)*x)^(1/2)*(1/(a-I)*x
-I/(a-I))^(1/2)*(1/(a+I)*x+I/(a+I))^(1/2)/(-a*x^2+x^3-a+x)^(1/2)/
(a^2+1)^(1/2)*EllipticPi(((-a+x)/(-a-I))^(1/2),(a+I)/(a^2+1)^(1/2
),((a+I)/(a-I))^(1/2))*a-I*(-1/(-a-I)*a+1/(-a-I)*x)^(1/2)*(1/(a-I
)*x-I/(a-I))^(1/2)*(1/(a+I)*x+I/(a+I))^(1/2)/(-a*x^2+x^3-a+x)^(1/
2)/(a^2+1)^(1/2)*EllipticPi(((-a+x)/(-a-I))^(1/2),(a+I)/(a^2+1)^(
1/2),((a+I)/(a-I))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a - x + \sqrt{a^2 + 1}}{\sqrt{-(x^2 + 1)(a - x)}\left(a - x - \sqrt{a^2 + 1}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a - x + sqrt(a^2 + 1))/(sqrt(-(x^2 + 1)*(a - x))*(a - x - sqrt(a^2 + 1))
```

```
[Out] integrate((a - x + sqrt(a^2 + 1))/(sqrt(-(x^2 + 1)*(a - x))*(a -
x - sqrt(a^2 + 1))), x)
```

Fricas [A] time = 0.616919, size = 1, normalized size = 0.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a - x + sqrt(a^2 + 1))/(sqrt(-(x^2 + 1)*(a - x))*(a - x - sqrt(a^2 + 1))
```

```
[Out] [1/4*sqrt(-2*a - 2*sqrt(a^2 + 1))*log((256*a^6 + (32*a^6 + 48*a^4
 + 18*a^2 + 1)*x^4 + 320*a^4 - 12*(16*a^5 + 20*a^3 + 5*a)*x^3 + 2
*(128*a^6 + 184*a^4 + 64*a^2 + 3)*x^2 + 82*a^2 + 4*(32*a^5 + 36*a
^3 + (16*a^5 + 20*a^3 + 5*a)*x^2 - 2*(8*a^4 + 8*a^2 + 1)*x + (32*
a^4 + (16*a^4 + 12*a^2 + 1)*x^2 + 20*a^2 - 8*(2*a^3 + a)*x + 1)*s
qrt(a^2 + 1) + 7*a)*sqrt(-a*x^2 + x^3 - a + x)*sqrt(-2*a - 2*sqrt
(a^2 + 1)) - 4*(64*a^5 + 76*a^3 + 17*a)*x + 2*(128*a^5 + (16*a^5
+ 16*a^3 + 3*a)*x^4 - 6*(16*a^4 + 12*a^2 + 1)*x^3 + 96*a^3 + 4*(3
2*a^5 + 30*a^3 + 5*a)*x^2 - 2*(64*a^4 + 44*a^2 + 3)*x + 9*a)*sqrt
(a^2 + 1) + 1)/((32*a^6 + 48*a^4 + 18*a^2 + 1)*x^4 + 4*(16*a^5 +
20*a^3 + 5*a)*x^3 + 6*(8*a^4 + 8*a^2 + 1)*x^2 + 2*a^2 + 4*(4*a^3
+ 3*a)*x + 2*((16*a^5 + 16*a^3 + 3*a)*x^4 + 2*(16*a^4 + 12*a^2 +
1)*x^3 + 12*(2*a^3 + a)*x^2 + 2*(4*a^2 + 1)*x + a)*sqrt(a^2 + 1)
+ 1)), 1/2*sqrt(2*a + 2*sqrt(a^2 + 1))*arctan(1/2*(8*a^3 + (4*a^3
 + 3*a)*x^2 - 2*(2*a^2 + 1)*x + ((4*a^2 + 1)*x^2 + 8*a^2 - 4*a*x
+ 1)*sqrt(a^2 + 1) + 5*a)/(sqrt(-a*x^2 + x^3 - a + x)*(2*a^2 + 2*
sqrt(a^2 + 1)*a + 1)*sqrt(2*a + 2*sqrt(a^2 + 1))))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x-(a**2+1)**(1/2))/(-a+x+(a**2+1)**(1/2))/((-a+x)*(x**2+1))**(1/
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a - x + \sqrt{a^2 + 1}}{\sqrt{-(x^2 + 1)(a - x)}\left(a - x - \sqrt{a^2 + 1}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a - x + sqrt(a^2 + 1))/(sqrt(-(x^2 + 1)*(a - x))*(a - x - sqrt(a^2 + 1))
```

```
[Out] integrate((a - x + sqrt(a^2 + 1))/(sqrt(-(x^2 + 1)*(a - x))*(a -
x - sqrt(a^2 + 1))), x)
```

3.53 $\int \frac{a+bx}{\sqrt[3]{1-x^2}(3+x^2)}\,dx$

Optimal. Leaf size=198

$$\frac{a\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\,2^{2/3}\sqrt{3}}+\frac{a\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{2\,2^{2/3}}+\frac{a\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}\sqrt{3}}-\frac{a\tanh^{-1}(x)}{6\,2^{2/3}}-\frac{b\log\left(x^2+3\right)}{4\,2^{2/3}}+\frac{3b\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{4\,2^{2/3}}+\frac{\sqrt{3}b\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{2\,2^{2/3}}$$

```
[Out] (a*ArcTan[Sqrt[3]/x])/(2*2^(2/3)*Sqrt[3]) + (Sqrt[3]*b*ArcTan[(1
+ (2 - 2*x^2)^(1/3))/Sqrt[3]])/(2*2^(2/3)) + (a*ArcTan[(Sqrt[3]*(
1 - 2^(1/3)*(1 - x^2)^(1/3)))/x])/(2*2^(2/3)*Sqrt[3]) - (a*ArcTan
h[x])/(6*2^(2/3)) + (a*ArcTanh[x/(1 + 2^(1/3)*(1 - x^2)^(1/3))])/
(2*2^(2/3)) - (b*Log[3 + x^2])/(4*2^(2/3)) + (3*b*Log[2^(2/3) - (
1 - x^2)^(1/3)])/(4*2^(2/3))
```

Rubi [A] time = 0.241473, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292

$$\frac{a\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\,2^{2/3}\sqrt{3}}+\frac{a\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{2\,2^{2/3}}+\frac{a\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}\sqrt{3}}-\frac{a\tanh^{-1}(x)}{6\,2^{2/3}}-\frac{b\log\left(x^2+3\right)}{4\,2^{2/3}}+\frac{3b\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{4\,2^{2/3}}+\frac{\sqrt{3}b\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{2\,2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (a*ArcTan[Sqrt[3]/x])/(2*2^(2/3)*Sqrt[3]) + (Sqrt[3]*b*ArcTan[(1
+ (2 - 2*x^2)^(1/3))/Sqrt[3]])/(2*2^(2/3)) + (a*ArcTan[(Sqrt[3]*(
1 - 2^(1/3)*(1 - x^2)^(1/3)))/x])/(2*2^(2/3)*Sqrt[3]) - (a*ArcTan
h[x])/(6*2^(2/3)) + (a*ArcTanh[x/(1 + 2^(1/3)*(1 - x^2)^(1/3))])/
(2*2^(2/3)) - (b*Log[3 + x^2])/(4*2^(2/3)) + (3*b*Log[2^(2/3) - (
1 - x^2)^(1/3)])/(4*2^(2/3))
```

Rubi in Sympy [A] time = 13.6533, size = 231, normalized size = 1.17

$$\frac{\sqrt[3]{2}a\log\left(\sqrt[3]{2}\sqrt[3]{-x+1}+(x+1)^{\frac{2}{3}}\right)}{8}-\frac{\sqrt[3]{2}a\log\left((-x+1)^{\frac{2}{3}}+\sqrt[3]{2}\sqrt[3]{x+1}\right)}{8}-\frac{\sqrt[3]{2}\sqrt{3}a\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2^{\frac{2}{3}}\sqrt{3}(x+1)^{\frac{2}{3}}}{3\sqrt[3]{-x+1}}\right)}{12}-\frac{\sqrt[3]{2}\sqrt{3}a\,\mathrm{atan}\left(\frac{2^{\frac{2}{3}}\sqrt{3}(-x+1)^{\frac{2}{3}}}{3\sqrt[3]{x+1}}-\frac{\sqrt{3}}{3}\right)}{12}-\frac{\sqrt[3]{2}b\log\left(x^2+3\right)}{8}+\frac{3\sqrt[3]{2}b\log\left(-\sqrt[3]{-x^2+1}+2^{\frac{2}{3}}\right)}{8}+\frac{\sqrt[3]{2}\sqrt{3}b\,\mathrm{atan}\left(\sqrt{3}\left(\frac{\sqrt[3]{2}\sqrt[3]{-x^2+1}}{3}+\frac{1}{3}\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((b*x+a)/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] 2**(1/3)*a*log(2**(1/3)*(-x + 1)**(1/3) + (x + 1)**(2/3))/8 - 2**
(1/3)*a*log((-x + 1)**(2/3) + 2**(1/3)*(x + 1)**(1/3))/8 - 2**(1/
```

```
3)*sqrt(3)*a*atan(sqrt(3)/3 - 2**(2/3)*sqrt(3)*(x + 1)**(2/3)/(3*
(-x + 1)**(1/3)))/12 - 2**(1/3)*sqrt(3)*a*atan(2**(2/3)*sqrt(3)*(
-x + 1)**(2/3)/(3*(x + 1)**(1/3)) - sqrt(3)/3)/12 - 2**(1/3)*b*lo
g(x**2 + 3)/8 + 3*2**(1/3)*b*log(-(-x**2 + 1)**(1/3) + 2**(2/3))/
8 + 2**(1/3)*sqrt(3)*b*atan(sqrt(3)*(2**(1/3)*(-x**2 + 1)**(1/3)/
3 + 1/3))/4
```

Mathematica [C] time = 0.468093, size = 205, normalized size = 1.04

$$\frac{3x\left(\frac{3aF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)}{2x^2\left(F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)-F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};x^2,-\frac{x^2}{3}\right)\right)+9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)}+\frac{bxF_1\left(1;\frac{1}{3},1;2;x^2,-\frac{x^2}{3}\right)}{x^2\left(F_1\left(2;\frac{4}{3},1;3;x^2,-\frac{x^2}{3}\right)-F_1\left(2;\frac{1}{3},2;3;x^2,-\frac{x^2}{3}\right)\right)+6F_1\left(1;\frac{1}{3},1;2;x^2,-\frac{x^2}{3}\right)}\right)}{\sqrt[3]{1-x^2}\,(x^2+3)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x)/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (3*x*((3*a*AppellF1[1/2, 1/3, 1, 3/2, x^2, -x^2/3])/(9*AppellF1[1
/2, 1/3, 1, 3/2, x^2, -x^2/3] + 2*x^2*(-AppellF1[3/2, 1/3, 2, 5/2
, x^2, -x^2/3] + AppellF1[3/2, 4/3, 1, 5/2, x^2, -x^2/3])) + (b*x
*AppellF1[1, 1/3, 1, 2, x^2, -x^2/3])/(6*AppellF1[1, 1/3, 1, 2, x
^2, -x^2/3] + x^2*(-AppellF1[2, 1/3, 2, 3, x^2, -x^2/3] + AppellF
1[2, 4/3, 1, 3, x^2, -x^2/3]))))/((1 - x^2)^(1/3)*(3 + x^2))
```

Maple [F] time = 0.052, size = 0, normalized size = 0.

$$\int \frac{bx+a}{x^2+3}\frac{1}{\sqrt[3]{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x+a)/(-x^2+1)^(1/3)/(x^2+3),x)
```

```
[Out] int((b*x+a)/(-x^2+1)^(1/3)/(x^2+3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx+a}{(x^2+3)(-x^2+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x + a)/((x^2 + 3)*(-x^2 + 1)^(1/3)),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x + a)/((x^2 + 3)*(-x^2 + 1)^(1/3)),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+bx}{\sqrt[3]{-\left(x-1\right)\left(x+1\right)}\left(x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x+a)/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] Integral((a + b*x)/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx+a}{(x^2+3)(-x^2+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x + a)/((x^2 + 3)*(-x^2 + 1)^(1/3)),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

3.54 $\int \frac{a+bx}{(3-x^2)\sqrt[3]{1+x^2}}\,dx$

Optimal. Leaf size=198

$$\frac{a\tan^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{x^2+1}+1}\right)}{2\,2^{2/3}}-\frac{a\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{x^2+1}\right)}{x}\right)}{2\,2^{2/3}\sqrt{3}}-\frac{a\tan^{-1}(x)}{6\,2^{2/3}}-\frac{a\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}\sqrt{3}}+\frac{b\log\left(3-x^2\right)}{4\,2^{2/3}}-\frac{3b\log\left(2^{2/3}-\sqrt[3]{x^2+1}\right)}{4\,2^{2/3}}-\frac{\sqrt{3}b\tan^{-1}\left(\frac{\sqrt[3]{2}\sqrt[3]{x^2+1}+1}{\sqrt{3}}\right)}{2\,2^{2/3}}$$

```
[Out] -(a*ArcTan[x])/(6*2^(2/3)) + (a*ArcTan[x/(1 + 2^(1/3)*(1 + x^2)^(
1/3))])/(2*2^(2/3)) - (Sqrt[3]*b*ArcTan[(1 + 2^(1/3)*(1 + x^2)^(1
/3))/Sqrt[3]])/(2*2^(2/3)) - (a*ArcTanh[Sqrt[3]/x])/(2*2^(2/3)*Sq
rt[3]) - (a*ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*(1 + x^2)^(1/3)))/x])/(
2*2^(2/3)*Sqrt[3]) + (b*Log[3 - x^2])/(4*2^(2/3)) - (3*b*Log[2^(2
/3) - (1 + x^2)^(1/3)])/(4*2^(2/3))
```

Rubi [A] time = 0.21155, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292

$$\frac{a\tan^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{x^2+1}+1}\right)}{2\,2^{2/3}}-\frac{a\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{x^2+1}\right)}{x}\right)}{2\,2^{2/3}\sqrt{3}}-\frac{a\tan^{-1}(x)}{6\,2^{2/3}}-\frac{a\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}\sqrt{3}}+\frac{b\log\left(3-x^2\right)}{4\,2^{2/3}}-\frac{3b\log\left(2^{2/3}-\sqrt[3]{x^2+1}\right)}{4\,2^{2/3}}-\frac{\sqrt{3}b\tan^{-1}\left(\frac{\sqrt[3]{2}\sqrt[3]{x^2+1}+1}{\sqrt{3}}\right)}{2\,2^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x)/((3 - x^2)*(1 + x^2)^(1/3)),x]
```


```
[Out] -(a*ArcTan[x])/(6*2^(2/3)) + (a*ArcTan[x/(1 + 2^(1/3)*(1 + x^2)^(
1/3))])/(2*2^(2/3)) - (Sqrt[3]*b*ArcTan[(1 + 2^(1/3)*(1 + x^2)^(1
/3))/Sqrt[3]])/(2*2^(2/3)) - (a*ArcTanh[Sqrt[3]/x])/(2*2^(2/3)*Sq
rt[3]) - (a*ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*(1 + x^2)^(1/3)))/x])/(
2*2^(2/3)*Sqrt[3]) + (b*Log[3 - x^2])/(4*2^(2/3)) - (3*b*Log[2^(2
/3) - (1 + x^2)^(1/3)])/(4*2^(2/3))
```

Rubi in Sympy [A] time = 13.1202, size = 282, normalized size = 1.42

$$\frac{\sqrt[3]{2}\sqrt{3}a\log\left(-x+\sqrt{3}\right)}{24}-\frac{\sqrt[3]{2}\sqrt{3}a\log\left(x+\sqrt{3}\right)}{24}+\frac{\sqrt[3]{2}\sqrt{3}a\log\left(-x-\sqrt[3]{2}\sqrt{3}\sqrt[3]{x^2+1}+\sqrt{3}\right)}{24}-\frac{\sqrt[3]{2}\sqrt{3}a\log\left(x-\sqrt[3]{2}\sqrt{3}\sqrt[3]{x^2+1}+\sqrt{3}\right)}{24}-\frac{\sqrt[3]{2}a\,\mathrm{atan}\left(\frac{2^{\frac{2}{3}}\left(-x+\sqrt{3}\right)}{3\sqrt[3]{x^2+1}}+\frac{\sqrt{3}}{3}\right)}{12}+\frac{\sqrt[3]{2}a\,\mathrm{atan}\left(\frac{2^{\frac{2}{3}}\left(x+\sqrt{3}\right)}{3\sqrt[3]{x^2+1}}+\frac{\sqrt{3}}{3}\right)}{12}+\frac{\sqrt[3]{2}b\log\left(-x^2+3\right)}{8}-\frac{3\sqrt[3]{2}b\log\left(-\sqrt[3]{x^2+1}+2^{\frac{2}{3}}\right)}{8}-\frac{\sqrt[3]{2}\sqrt{3}b\,\mathrm{atan}\left(\sqrt{3}\left(\frac{\sqrt[3]{2}\sqrt[3]{x^2+1}}{3}+\frac{1}{3}\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x+a)/(-x**2+3)/(x**2+1)**(1/3),x)
```

```
[Out] 2**(1/3)*sqrt(3)*a*log(-x + sqrt(3))/24 - 2**(1/3)*sqrt(3)*a*log(
x + sqrt(3))/24 + 2**(1/3)*sqrt(3)*a*log(-x - 2**(1/3)*sqrt(3)*(x
**2 + 1)**(1/3) + sqrt(3))/24 - 2**(1/3)*sqrt(3)*a*log(x - 2**(1/
3)*sqrt(3)*(x**2 + 1)**(1/3) + sqrt(3))/24 - 2**(1/3)*a*atan(2**(
2/3)*(-x + sqrt(3))/(3*(x**2 + 1)**(1/3)) + sqrt(3)/3)/12 + 2**(1
/3)*a*atan(2**(2/3)*(x + sqrt(3))/(3*(x**2 + 1)**(1/3)) + sqrt(3)
/3)/12 + 2**(1/3)*b*log(-x**2 + 3)/8 - 3*2**(1/3)*b*log(-(x**2 +
1)**(1/3) + 2**(2/3))/8 - 2**(1/3)*sqrt(3)*b*atan(sqrt(3)*(2**(1/
3)*(x**2 + 1)**(1/3)/3 + 1/3))/4
```

Mathematica [C] time = 0.441686, size = 220, normalized size = 1.11

$$\frac{3x\left(-\frac{3aF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-x^2,\frac{x^2}{3}\right)}{2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};-x^2,\frac{x^2}{3}\right)-F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};-x^2,\frac{x^2}{3}\right)\right)+9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-x^2,\frac{x^2}{3}\right)}-\frac{bxF_1\left(1;\frac{1}{3},1;2;-x^2,\frac{x^2}{3}\right)}{x^2\left(F_1\left(2;\frac{1}{3},2;3;-x^2,\frac{x^2}{3}\right)-F_1\left(2;\frac{4}{3},1;3;-x^2,\frac{x^2}{3}\right)\right)+6F_1\left(1;\frac{1}{3},1;2;-x^2,\frac{x^2}{3}\right)}\right)}{(x^2-3)\sqrt[3]{x^2+1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x)/((3 - x^2)*(1 + x^2)^(1/3)),x]
```

```
[Out] (3*x*((-3*a*AppellF1[1/2, 1/3, 1, 3/2, -x^2, x^2/3])/(9*AppellF1[
1/2, 1/3, 1, 3/2, -x^2, x^2/3] + 2*x^2*(AppellF1[3/2, 1/3, 2, 5/2
, -x^2, x^2/3] - AppellF1[3/2, 4/3, 1, 5/2, -x^2, x^2/3])) - (b*x
*AppellF1[1, 1/3, 1, 2, -x^2, x^2/3])/(6*AppellF1[1, 1/3, 1, 2, -
x^2, x^2/3] + x^2*(AppellF1[2, 1/3, 2, 3, -x^2, x^2/3] - AppellF1
[2, 4/3, 1, 3, -x^2, x^2/3]))))/((-3 + x^2)*(1 + x^2)^(1/3))
```

Maple [F] time = 0.073, size = 0, normalized size = 0.

$$\int \frac{bx+a}{-x^2+3}\frac{1}{\sqrt[3]{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x+a)/(-x^2+3)/(x^2+1)^(1/3),x)
```

```
[Out] int((b*x+a)/(-x^2+3)/(x^2+1)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{bx+a}{(x^2+1)^{\frac{1}{3}}(x^2-3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(b*x + a)/((x^2 + 1)^(1/3)*(x^2 - 3)),x, algorithm="maxima")
```

```
[Out] -integrate((b*x + a)/((x^2 + 1)^(1/3)*(x^2 - 3)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(b*x + a)/((x^2 + 1)^(1/3)*(x^2 - 3)),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{a}{x^2\sqrt[3]{x^2+1}-3\sqrt[3]{x^2+1}}\,dx-\int \frac{bx}{x^2\sqrt[3]{x^2+1}-3\sqrt[3]{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x**2+3)/(x**2+1)**(1/3),x)
```

```
[Out] -Integral(a/(x**2*(x**2 + 1)**(1/3) - 3*(x**2 + 1)**(1/3)), x) -
Integral(b*x/(x**2*(x**2 + 1)**(1/3) - 3*(x**2 + 1)**(1/3)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{bx+a}{(x^2+1)^{\frac{1}{3}}(x^2-3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(b*x + a)/((x^2 + 1)^(1/3)*(x^2 - 3)),x, algorithm="giac")
```

```
[Out] integrate(-(b*x + a)/((x^2 + 1)^(1/3)*(x^2 - 3)), x)
```

3.55 $\int \frac{1}{x\sqrt[3]{4-6x+3x^2}}\,dx$

Optimal. Leaf size=88

$$\frac{\log\left(\frac{2\sqrt[3]{2}\sqrt[3]{3x^2-6x+4}+2x-4}{x}\right)}{2\,2^{2/3}}+\frac{\tan^{-1}\left(\frac{-2\sqrt[3]{2}\sqrt[3]{3x^2-6x+4}+x-2}{\sqrt{3}(x-2)}\right)}{2^{2/3}\sqrt{3}}$$

```
[Out] ArcTan[(-2 + x - 2*2^(1/3)*(4 - 6*x + 3*x^2)^(1/3))/(Sqrt[3]*(-2
+ x))]/(2^(2/3)*Sqrt[3]) + Log[(-4 + 2*x + 2*2^(1/3)*(4 - 6*x + 3
*x^2)^(1/3))/x]/(2*2^(2/3))
```

Rubi [A] time = 0.0472327, antiderivative size = 97, normalized size of antiderivative = 1.1, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056

$$\frac{\log\left(-3\sqrt[3]{2}\sqrt[3]{3x^2-6x+4}-3x+6\right)}{2\,2^{2/3}}-\frac{\tan^{-1}\left(\frac{2^{2/3}(2-x)}{\sqrt{3}\sqrt[3]{3x^2-6x+4}}+\frac{1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\log(x)}{2\,2^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(4 - 6*x + 3*x^2)^(1/3)),x]
```

```
[Out] -(ArcTan[1/Sqrt[3] + (2^(2/3)*(2 - x))/(Sqrt[3]*(4 - 6*x + 3*x^2)
^(1/3))]/(2^(2/3)*Sqrt[3])) - Log[x]/(2*2^(2/3)) + Log[6 - 3*x -
3*2^(1/3)*(4 - 6*x + 3*x^2)^(1/3)]/(2*2^(2/3))
```

Rubi in Sympy [A] time = 2.41973, size = 94, normalized size = 1.07

$$-\frac{\sqrt[3]{2}\log(x)}{4}+\frac{\sqrt[3]{2}\log\left(-3x-3\sqrt[3]{2}\sqrt[3]{3x^2-6x+4}+6\right)}{4}-\frac{\sqrt[3]{2}\sqrt{3}\,\mathrm{atan}\left(\frac{2^{\frac{2}{3}}\sqrt{3}(-3x+6)}{9\sqrt[3]{3x^2-6x+4}}+\frac{\sqrt{3}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(3*x**2-6*x+4)**(1/3),x)
```

```
[Out] -2**(1/3)*log(x)/4 + 2**(1/3)*log(-3*x - 3*2**(1/3)*(3*x**2 - 6*x
 + 4)**(1/3) + 6)/4 - 2**(1/3)*sqrt(3)*atan(2**(2/3)*sqrt(3)*(-3*
x + 6)/(9*(3*x**2 - 6*x + 4)**(1/3)) + sqrt(3)/3)/6
```

Mathematica [C] time = 0.810162, size = 273, normalized size = 3.1

$$-\frac{15x\left(3x-i\sqrt{3}-3\right)\left(3x+i\sqrt{3}-3\right)F_1\left(\frac{2}{3};\frac{1}{3},\frac{1}{3};\frac{5}{3};\frac{3-i\sqrt{3}}{3x},\frac{3+i\sqrt{3}}{3x}\right)}{2\left(3x^2-6x+4\right)^{4/3}\left(15xF_1\left(\frac{2}{3};\frac{1}{3},\frac{1}{3};\frac{5}{3};\frac{3-i\sqrt{3}}{3x},\frac{3+i\sqrt{3}}{3x}\right)+\left(3+i\sqrt{3}\right)F_1\left(\frac{5}{3};\frac{1}{3},\frac{4}{3};\frac{8}{3};\frac{3-i\sqrt{3}}{3x},\frac{3+i\sqrt{3}}{3x}\right)+\left(3-i\sqrt{3}\right)F_1\left(\frac{5}{3};\frac{4}{3},\frac{1}{3};\frac{8}{3};\frac{3-i}{3x}\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/(x*(4 - 6*x + 3*x^2)^(1/3)),x]
```

```
[Out] (-15*x*(-3 - I*Sqrt[3] + 3*x)*(-3 + I*Sqrt[3] + 3*x)*AppellF1[2/3
, 1/3, 1/3, 5/3, (3 - I*Sqrt[3])/(3*x), (3 + I*Sqrt[3])/(3*x)])/(
2*(4 - 6*x + 3*x^2)^(4/3)*(15*x*AppellF1[2/3, 1/3, 1/3, 5/3, (3 -
 I*Sqrt[3])/(3*x), (3 + I*Sqrt[3])/(3*x)] + (3 + I*Sqrt[3])*Appel
```

```
lF1[5/3, 1/3, 4/3, 8/3, (3 - I*Sqrt[3])/(3*x), (3 + I*Sqrt[3])/(3
*x)] + (3 - I*Sqrt[3])*AppellF1[5/3, 4/3, 1/3, 8/3, (3 - I*Sqrt[3
])/(3*x), (3 + I*Sqrt[3])/(3*x)]))
```

Maple [F] time = 0.199, size = 0, normalized size = 0.

$$\int \frac{1}{x}\frac{1}{\sqrt[3]{3\,x^{2}-6\,x+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(3*x^2-6*x+4)^(1/3),x)

[Out] int(1/x/(3*x^2-6*x+4)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3\,x^{2}-6\,x+4)^{\frac{1}{3}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((3*x^2 - 6*x + 4)^(1/3)*x),x, algorithm="maxima")

[Out] integrate(1/((3*x^2 - 6*x + 4)^(1/3)*x), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((3*x^2 - 6*x + 4)^(1/3)*x),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt[3]{3x^{2}-6x+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(3*x**2-6*x+4)**(1/3),x)

[Out] Integral(1/(x*(3*x**2 - 6*x + 4)**(1/3)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3\,x^{2}-6\,x+4)^{\frac{1}{3}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((3*x^2 - 6*x + 4)^(1/3)*x),x, algorithm="giac")

[Out] integrate(1/((3*x^2 - 6*x + 4)^(1/3)*x), x)
```

3.56 $\int x\sqrt[3]{1-x^3}\,dx$

Optimal. Leaf size=107

$$-\frac{1}{9}\log\left(\frac{x}{\sqrt[3]{1-x^3}}+1\right)-\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{3}\sqrt[3]{1-x^3}x^2+\frac{1}{18}\log\left(-\frac{x}{\sqrt[3]{1-x^3}}+\frac{x^2}{(1-x^3)^{2/3}}+1\right)$$

```
[Out] (x^2*(1 - x^3)^(1/3))/3 - ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt
[3]]/(3*Sqrt[3]) + Log[1 + x^2/(1 - x^3)^(2/3) - x/(1 - x^3)^(1/3
)]/18 - Log[1 + x/(1 - x^3)^(1/3)]/9
```

Rubi [A] time = 0.10215, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615

$$-\frac{1}{9}\log\left(\frac{x}{\sqrt[3]{1-x^3}}+1\right)-\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{3}\sqrt[3]{1-x^3}x^2+\frac{1}{18}\log\left(-\frac{x}{\sqrt[3]{1-x^3}}+\frac{x^2}{(1-x^3)^{2/3}}+1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x*(1 - x^3)^(1/3),x]
```

```
[Out] (x^2*(1 - x^3)^(1/3))/3 - ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt
[3]]/(3*Sqrt[3]) + Log[1 + x^2/(1 - x^3)^(2/3) - x/(1 - x^3)^(1/3
)]/18 - Log[1 + x/(1 - x^3)^(1/3)]/9
```

Rubi in Sympy [A] time = 4.77537, size = 85, normalized size = 0.79

$$\frac{x^2\sqrt[3]{-x^3+1}}{3}-\frac{\log\left(\frac{x}{\sqrt[3]{-x^3+1}}+1\right)}{9}+\frac{\log\left(\frac{x^2}{(-x^3+1)^{\frac{2}{3}}}-\frac{x}{\sqrt[3]{-x^3+1}}+1\right)}{18}+\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3\sqrt[3]{-x^3+1}}-\frac{1}{3}\right)\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(-x**3+1)**(1/3),x)
```

```
[Out] x**2*(-x**3 + 1)**(1/3)/3 - log(x/(-x**3 + 1)**(1/3) + 1)/9 + log
(x**2/(-x**3 + 1)**(2/3) - x/(-x**3 + 1)**(1/3) + 1)/18 + sqrt(3)
*atan(sqrt(3)*(2*x/(3*(-x**3 + 1)**(1/3)) - 1/3))/9
```

Mathematica [C] time = 0.0183334, size = 34, normalized size = 0.32

$$\frac{1}{6}x^2\left({}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};x^3\right)+2\sqrt[3]{1-x^3}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(1 - x^3)^(1/3),x]
```

```
[Out] (x^2*(2*(1 - x^3)^(1/3) + Hypergeometric2F1[2/3, 2/3, 5/3, x^3]))
/6
```

Maple [C] time = 0.069, size = 69, normalized size = 0.6

$$-\frac{x^2(x^3-1)}{3}(-x^3+1)^{-\frac{2}{3}} + \frac{x^2}{6}(x^3-1)^{\frac{2}{3}}(-signum(x^3-1))^{\frac{2}{3}}\,{}_2\mathrm{F}_1(\frac{2}{3},\frac{2}{3};\frac{5}{3};x^3)(signum(x^3-1))^{-\frac{2}{3}}(-x^3+1)^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(-x^3+1)^(1/3),x)
```

```
[Out] -1/3*x^2*(x^3-1)/(-x^3+1)^(2/3)+1/6*(x^3-1)^(2/3)/signum(x^3-1)^(
2/3)*(-signum(x^3-1))^(2/3)*x^2*hypergeom([2/3,2/3],[5/3],x^3)/(-
x^3+1)^(2/3)
```

Maxima [A] time = 1.66624, size = 142, normalized size = 1.33

$$-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2(-x^3+1)^{\frac{1}{3}}}{x}-1\right)\right)-\frac{(-x^3+1)^{\frac{1}{3}}}{3x\left(\frac{x^3-1}{x^3}-1\right)} - \frac{1}{9}\log\left(\frac{(-x^3+1)^{\frac{1}{3}}}{x}+1\right)+\frac{1}{18}\log\left(-\frac{(-x^3+1)^{\frac{1}{3}}}{x}+\frac{(-x^3+1)^{\frac{2}{3}}}{x^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^3 + 1)^(1/3)*x,x, algorithm="maxima")
```

```
[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3)/x - 1)) - 1/3
*(-x^3 + 1)^(1/3)/(x*((x^3 - 1)/x^3 - 1)) - 1/9*log((-x^3 + 1)^(1
/3)/x + 1) + 1/18*log(-(-x^3 + 1)^(1/3)/x + (-x^3 + 1)^(2/3)/x^2
+ 1)
```

Fricas [A] time = 0.209585, size = 143, normalized size = 1.34

$$\frac{1}{54}\sqrt{3}\left(6\sqrt{3}(-x^3+1)^{\frac{1}{3}}x^2-2\sqrt{3}\log\left(\frac{x+(-x^3+1)^{\frac{1}{3}}}{x}\right)+\sqrt{3}\log\left(\frac{x^2-(-x^3+1)^{\frac{1}{3}}x+(-x^3+1)^{\frac{2}{3}}}{x^2}\right)-6\arctan\left(-\frac{\sqrt{3}x-2}{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^3 + 1)^(1/3)*x,x, algorithm="fricas")
```

```
[Out] 1/54*sqrt(3)*(6*sqrt(3)*(-x^3 + 1)^(1/3)*x^2 - 2*sqrt(3)*log((x +
 (-x^3 + 1)^(1/3))/x) + sqrt(3)*log((x^2 - (-x^3 + 1)^(1/3)*x + (
-x^3 + 1)^(2/3))/x^2) - 6*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(-x^
3 + 1)^(1/3))/x))
```

Sympy [A] time = 1.78304, size = 32, normalized size = 0.3

$$\frac{x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle| x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(-x**3+1)**(1/3),x)
```

```
[Out] x**2*gamma(2/3)*hyper((-1/3, 2/3), (5/3,), x**3*exp_polar(2*I*pi)
)/(3*gamma(5/3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-x^3+1\right)^{\frac{1}{3}} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^3 + 1)^(1/3)*x,x, algorithm="giac")
```

```
[Out] integrate((-x^3 + 1)^(1/3)*x, x)
```

3.57 $\int \frac{\sqrt[3]{1-x^3}}{x}\,dx$

Optimal. Leaf size=67

$$\sqrt[3]{1-x^3}+\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)-\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\log(x)}{2}$$

```
[Out] (1 - x^3)^(1/3) - ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3]
 - Log[x]/2 + Log[1 - (1 - x^3)^(1/3)]/2
```

Rubi [A] time = 0.0840064, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4

$$\sqrt[3]{1-x^3}+\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)-\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - x^3)^(1/3)/x,x]
```

```
[Out] (1 - x^3)^(1/3) - ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3]
 - Log[x]/2 + Log[1 - (1 - x^3)^(1/3)]/2
```

Rubi in Sympy [A] time = 2.76762, size = 56, normalized size = 0.84

$$\sqrt[3]{-x^3+1}-\frac{\log\left(x^3\right)}{6}+\frac{\log\left(-\sqrt[3]{-x^3+1}+1\right)}{2}-\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2\sqrt[3]{-x^3+1}}{3}+\frac{1}{3}\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**3+1)**(1/3)/x,x)
```

```
[Out] (-x**3 + 1)**(1/3) - log(x**3)/6 + log(-(-x**3 + 1)**(1/3) + 1)/2
 - sqrt(3)*atan(sqrt(3)*(2*(-x**3 + 1)**(1/3)/3 + 1/3))/3
```

Mathematica [C] time = 0.0246976, size = 48, normalized size = 0.72

$$\frac{-\left(1-\frac{1}{x^3}\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};\frac{1}{x^3}\right)-2x^3+2}{2\left(1-x^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - x^3)^(1/3)/x,x]
```

```
[Out] (2 - 2*x^3 - (1 - x^(-3))^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3,
x^(-3)])/(2*(1 - x^3)^(2/3))
```

Maple [C] time = 0.073, size = 49, normalized size = 0.7

$$-\frac{1}{9\,(2/3)}\left(-3\left(3+1/6\,\pi\sqrt{3}-3/2\,\ln(3)+3\,\ln(x)+i\pi\right)(2/3)+\left(\frac{2}{3}\right)x^3{}_3F_2\left(\frac{2}{3},1,1;\,2,2;\,x^3\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((-x^3+1)^(1/3)/x,x)
```

```
[Out] -1/9/GAMMA(2/3)*(-3*(3+1/6*Pi*3^(1/2)-3/2*ln(3)+3*ln(x)+I*Pi)*GAM
MA(2/3)+GAMMA(2/3)*x^3*hypergeom([2/3,1,1],[2,2],x^3))
```

Maxima [A] time = 1.53367, size = 96, normalized size = 1.43

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^3+1\right)^{\frac{1}{3}}+1\right)\right)+\left(-x^3+1\right)^{\frac{1}{3}}-\frac{1}{6}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left(-x^3+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((-x^3 + 1)^(1/3)/x,x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) + (-x^3
 + 1)^(1/3) - 1/6*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) +
1/3*log((-x^3 + 1)^(1/3) - 1)
```

Fricas [A] time = 0.208472, size = 115, normalized size = 1.72

$$-\frac{1}{18}\sqrt{3}\left(\sqrt{3}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)-2\sqrt{3}\log\left(\left(-x^3+1\right)^{\frac{1}{3}}-1\right)-6\sqrt{3}\left(-x^3+1\right)^{\frac{1}{3}}+6\arctan\left(\frac{2}{3}\sqrt{3}\left(-x^3+1\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((-x^3 + 1)^(1/3)/x,x, algorithm="fricas")
```

```
[Out] -1/18*sqrt(3)*(sqrt(3)*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) +
1) - 2*sqrt(3)*log((-x^3 + 1)^(1/3) - 1) - 6*sqrt(3)*(-x^3 + 1)^(
1/3) + 6*arctan(2/3*sqrt(3)*(-x^3 + 1)^(1/3) + 1/3*sqrt(3)))
```

Sympy [A] time = 1.67805, size = 37, normalized size = 0.55

$$-\frac{xe^{\frac{i\pi}{3}}\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{1}{x^3}\right)}{3\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((-x**3+1)**(1/3)/x,x)
```

```
[Out] -x*exp(I*pi/3)*gamma(-1/3)*hyper((-1/3, -1/3), (2/3,), x**(-3))/(
3*gamma(2/3))
```

GIAC/XCAS [A] time = 0.24, size = 97, normalized size = 1.45

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^3+1\right)^{\frac{1}{3}}+1\right)\right)+\left(-x^3+1\right)^{\frac{1}{3}}-\frac{1}{6}\ln\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\ln\left(\left|\left(-x^3+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^3 + 1)^(1/3)/x,x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) + (-x^3
 + 1)^(1/3) - 1/6*ln((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1
/3*ln(abs((-x^3 + 1)^(1/3) - 1))
```

3.58 $\int \frac{\sqrt[3]{1-x^3}}{1+x}\,dx$

Optimal. Leaf size=1

$$0$$

```
[Out] 0
```

Rubi [F] time = 0.0989775, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$

$$\text{Int}\left(\frac{\sqrt[3]{1-x^3}}{1+x}, x\right)$$

Verification is Not applicable to the result.

```
[In] Int[(1 - x^3)^(1/3)/(1 + x),x]

[Out] Defer[Int][(1 - x^3)^(1/3)/(1 + x), x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{-x^3+1}}{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-x**3+1)**(1/3)/(1+x),x)

[Out] Integral((-x**3 + 1)**(1/3)/(x + 1), x)
```

Mathematica [A] time = 0.371783, size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{1-x^3}}{1+x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - x^3)^(1/3)/(1 + x),x]

[Out] Integrate[(1 - x^3)^(1/3)/(1 + x), x]
```

Maple [F] time = 0.072, size = 0, normalized size = 0.

$$\int \frac{1}{1+x}\sqrt[3]{-x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)^(1/3)/(1+x),x)

[Out] int((-x^3+1)^(1/3)/(1+x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-x^3+1\right)^{\frac{1}{3}}}{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^3 + 1)^(1/3)/(x + 1),x, algorithm="maxima")

[Out] integrate((-x^3 + 1)^(1/3)/(x + 1), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^3 + 1)^(1/3)/(x + 1),x, algorithm="fricas")

[Out] Exception raised: TypeError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{-(x-1)(x^2+x+1)}}{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**3+1)**(1/3)/(1+x),x)

[Out] Integral((-(x - 1)*(x**2 + x + 1))**(1/3)/(x + 1), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-x^3+1\right)^{\frac{1}{3}}}{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^3 + 1)^(1/3)/(x + 1),x, algorithm="giac")

[Out] integrate((-x^3 + 1)^(1/3)/(x + 1), x)
```

3.59 $\int \frac{\sqrt[3]{1-x^3}}{1-x+x^2}\,dx$

Optimal. Leaf size=280

$$\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}+\frac{3\log\left(\sqrt[3]{1-x^3}-\sqrt[3]{2}(x-1)\right)}{2\ 2^{2/3}}+\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)-\frac{\log\left(\sqrt[3]{1-x^3}+\sqrt[3]{2}x\right)}{2\ 2^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}(x-1)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2^{2/3}}+\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\log\left(-3(x-1)\left(x^2-x+1\right)\right)}{2\ 2^{2/3}}$$

```
[Out] (Sqrt[3]*ArcTan[(1 + (2*2^(1/3)*(-1 + x))/(1 - x^3)^(1/3))/Sqrt[3
]])/2^(2/3) + ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3]
 - ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*S
qrt[3]) - ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*
Sqrt[3]) - Log[-3*(-1 + x)*(1 - x + x^2)]/(2*2^(2/3)) + Log[2^(1/
3) - (1 - x^3)^(1/3)]/(2*2^(2/3)) + (3*Log[-(2^(1/3)*(-1 + x)) +
(1 - x^3)^(1/3)])/(2*2^(2/3)) + Log[x + (1 - x^3)^(1/3)]/2 - Log[
2^(1/3)*x + (1 - x^3)^(1/3)]/(2*2^(2/3))
```

Rubi [F] time = 0.385211, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$

$$\text{Int}\left(\frac{\sqrt[3]{1-x^3}}{1-x+x^2},x\right)$$

Verification is Not applicable to the result.

```
[In] Int[(1 - x^3)^(1/3)/(1 - x + x^2),x]
```

```
[Out] ((2*I)*Defer[Int][(1 - x^3)^(1/3)/(1 + I*Sqrt[3] - 2*x), x])/Sqrt
[3] + ((2*I)*Defer[Int][(1 - x^3)^(1/3)/(-1 + I*Sqrt[3] + 2*x), x
])/Sqrt[3]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2\sqrt{3}i\int\frac{\sqrt[3]{-x^3+1}}{2x-1-\sqrt{3}i}\,dx}{3}+\frac{2\sqrt{3}i\int\frac{\sqrt[3]{-x^3+1}}{2x-1+\sqrt{3}i}\,dx}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-x**3+1)**(1/3)/(x**2-x+1),x)
```

```
[Out] -2*sqrt(3)*I*Integral((-x**3 + 1)**(1/3)/(2*x - 1 - sqrt(3)*I), x
)/3 + 2*sqrt(3)*I*Integral((-x**3 + 1)**(1/3)/(2*x - 1 + sqrt(3)*
I), x)/3
```

Mathematica [A] time = 0.0416973, size = 0, normalized size = 0.

$$\int\frac{\sqrt[3]{1-x^3}}{1-x+x^2}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(1 - x^3)^(1/3)/(1 - x + x^2),x]

[Out] Integrate[(1 - x^3)^(1/3)/(1 - x + x^2), x]
```

Maple [F] time = 0.112, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 - x + 1}\sqrt[3]{-x^3 + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^3+1)^(1/3)/(x^2-x+1),x)

[Out] int((-x^3+1)^(1/3)/(x^2-x+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-x^3 + 1\right)^{\frac{1}{3}}}{x^2 - x + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^3 + 1)^(1/3)/(x^2 - x + 1),x, algorithm="maxima")

[Out] integrate((-x^3 + 1)^(1/3)/(x^2 - x + 1), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^3 + 1)^(1/3)/(x^2 - x + 1),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{-\left(x - 1\right)\left(x^2 + x + 1\right)}}{x^2 - x + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**3+1)**(1/3)/(x**2-x+1),x)

[Out] Integral((-(x - 1)*(x**2 + x + 1))**(1/3)/(x**2 - x + 1), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-x^3 + 1\right)^{\frac{1}{3}}}{x^2 - x + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^3 + 1)^(1/3)/(x^2 - x + 1),x, algorithm="giac")

[Out] integrate((-x^3 + 1)^(1/3)/(x^2 - x + 1), x)
```

3.60 $\int \frac{3-3x+30x^2+160x^3}{9+24x-12x^2+80x^3+320x^4}\,dx$

Optimal. Leaf size=25

$$\frac{1}{8}\log\left(320x^4+80x^3-12x^2+24x+9\right)$$

```
[Out] Log[9 + 24*x - 12*x^2 + 80*x^3 + 320*x^4]/8
```

Rubi [A] time = 0.0121434, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026

$$\frac{1}{8}\log\left(320x^4+80x^3-12x^2+24x+9\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 3*x + 30*x^2 + 160*x^3)/(9 + 24*x - 12*x^2 + 80*x^3 + 320*x^4),x]
```

```
[Out] Log[9 + 24*x - 12*x^2 + 80*x^3 + 320*x^4]/8
```

Rubi in Sympy [A] time = 3.60071, size = 22, normalized size = 0.88

$$\frac{\log\left(320x^4+80x^3-12x^2+24x+9\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((160*x**3+30*x**2-3*x+3)/(320*x**4+80*x**3-12*x**2+24*x+9),x)
```

```
[Out] log(320*x**4 + 80*x**3 - 12*x**2 + 24*x + 9)/8
```

Mathematica [A] time = 0.0117437, size = 25, normalized size = 1.

$$\frac{1}{8}\log\left(320x^4+80x^3-12x^2+24x+9\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 3*x + 30*x^2 + 160*x^3)/(9 + 24*x - 12*x^2 + 80*x^3 + 320*x^4),x]
```

```
[Out] Log[9 + 24*x - 12*x^2 + 80*x^3 + 320*x^4]/8
```

Maple [A] time = 0.001, size = 24, normalized size = 1.

$$\frac{\ln\left(320\,x^4+80\,x^3-12\,x^2+24\,x+9\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((160*x^3+30*x^2-3*x+3)/(320*x^4+80*x^3-12*x^2+24*x+9),x)
```

```
[Out] 1/8*ln(320*x^4+80*x^3-12*x^2+24*x+9)
```

Maxima [A] time = 1.38999, size = 31, normalized size = 1.24

$$\frac{1}{8}\log\left(320\,x^4 + 80\,x^3 - 12\,x^2 + 24\,x + 9\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((160*x^3 + 30*x^2 - 3*x + 3)/(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9),x, al
```

```
[Out] 1/8*log(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9)
```

Fricas [A] time = 0.194498, size = 31, normalized size = 1.24

$$\frac{1}{8}\log\left(320\,x^4 + 80\,x^3 - 12\,x^2 + 24\,x + 9\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((160*x^3 + 30*x^2 - 3*x + 3)/(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9),x, al
```

```
[Out] 1/8*log(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9)
```

Sympy [A] time = 0.107169, size = 22, normalized size = 0.88

$$\frac{\log\left(320x^4 + 80x^3 - 12x^2 + 24x + 9\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((160*x**3+30*x**2-3*x+3)/(320*x**4+80*x**3-12*x**2+24*x+9),x)
```

```
[Out] log(320*x**4 + 80*x**3 - 12*x**2 + 24*x + 9)/8
```

GIAC/XCAS [A] time = 0.244751, size = 31, normalized size = 1.24

$$\frac{1}{8}\ln\left(320\,x^4 + 80\,x^3 - 12\,x^2 + 24\,x + 9\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((160*x^3 + 30*x^2 - 3*x + 3)/(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9),x, al
```

```
[Out] 1/8*ln(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9)
```

3.61 $\int \frac{3+12x+20x^2}{9+24x-12x^2+80x^3+320x^4} dx$

Optimal. Leaf size=59

$$\frac{\tan^{-1}\left(\frac{800x^3-40x^2+30x+57}{6\sqrt{11}}\right)}{2\sqrt{11}}-\frac{\tan^{-1}\left(\frac{7-40x}{5\sqrt{11}}\right)}{2\sqrt{11}}$$

```
[Out] -ArcTan[(7 - 40*x)/(5*Sqrt[11])]/(2*Sqrt[11]) + ArcTan[(57 + 30*x
 - 40*x^2 + 800*x^3)/(6*Sqrt[11])]/(2*Sqrt[11])
```

Rubi [A] time = 0.0595764, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.03

$$\frac{\tan^{-1}\left(\frac{800x^3-40x^2+30x+57}{6\sqrt{11}}\right)}{2\sqrt{11}}-\frac{\tan^{-1}\left(\frac{7-40x}{5\sqrt{11}}\right)}{2\sqrt{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 12*x + 20*x^2)/(9 + 24*x - 12*x^2 + 80*x^3 + 320*x^4),x]
```

```
[Out] -ArcTan[(7 - 40*x)/(5*Sqrt[11])]/(2*Sqrt[11]) + ArcTan[(57 + 30*x
 - 40*x^2 + 800*x^3)/(6*Sqrt[11])]/(2*Sqrt[11])
```

Rubi in Sympy [A] time = 7.63274, size = 56, normalized size = 0.95

$$\frac{\sqrt{11}\,\text{atan}\left(\sqrt{11}\left(\frac{8x}{11}-\frac{7}{55}\right)\right)}{22}+\frac{\sqrt{11}\,\text{atan}\left(\sqrt{11}\left(\frac{400x^3}{33}-\frac{20x^2}{33}+\frac{5x}{11}+\frac{19}{22}\right)\right)}{22}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((20*x**2+12*x+3)/(320*x**4+80*x**3-12*x**2+24*x+9),x)
```

```
[Out] sqrt(11)*atan(sqrt(11)*(8*x/11 - 7/55))/22 + sqrt(11)*atan(sqrt(1
1)*(400*x**3/33 - 20*x**2/33 + 5*x/11 + 19/22))/22
```

Mathematica [C] time = 0.0273115, size = 86, normalized size = 1.46

$$\frac{1}{8}\text{RootSum}\left[320\#1^4+80\#1^3-12\#1^2+24\#1+9\&,\frac{20\#1^2\log(x-\#1)+12\#1\log(x-\#1)+3\log(x-\#1)}{160\#1^3+30\#1^2-3\#1+3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 12*x + 20*x^2)/(9 + 24*x - 12*x^2 + 80*x^3 + 320*x^4),x]
```

```
[Out] RootSum[9 + 24*#1 - 12*#1^2 + 80*#1^3 + 320*#1^4 & , (3*Log[x - #
1] + 12*Log[x - #1]*#1 + 20*Log[x - #1]*#1^2)/(3 - 3*#1 + 30*#1^2
 + 160*#1^3) & ]/8
```

Maple [A] time = 0.043, size = 52, normalized size = 0.9

$$\frac{\sqrt{11}}{22}\arctan\left(\frac{(40\,x-7)\sqrt{11}}{55}\right)+\frac{\sqrt{11}}{22}\arctan\left(-\frac{20\sqrt{11}x^2}{33}+\frac{5\sqrt{11}x}{11}+\frac{19\sqrt{11}}{22}+\frac{400\sqrt{11}x^3}{33}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((20*x^2+12*x+3)/(320*x^4+80*x^3-12*x^2+24*x+9),x)
```

```
[Out] 1/22*11^(1/2)*arctan(1/55*(40*x-7)*11^(1/2))+1/22*11^(1/2)*arctan
(-20/33*11^(1/2)*x^2+5/11*11^(1/2)*x+19/22*11^(1/2)+400/33*11^(1/
2)*x^3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{20\,x^2+12\,x+3}{320\,x^4+80\,x^3-12\,x^2+24\,x+9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((20*x^2 + 12*x + 3)/(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9),x, algorithm=
```

```
[Out] integrate((20*x^2 + 12*x + 3)/(320*x^4 + 80*x^3 - 12*x^2 + 24*x +
 9), x)
```

Fricas [A] time = 0.195492, size = 51, normalized size = 0.86

$$\frac{1}{22}\,\sqrt{11}\left(\arctan\left(\frac{1}{66}\,\sqrt{11}\left(800\,x^3-40\,x^2+30\,x+57\right)\right)+\arctan\left(\frac{1}{55}\,\sqrt{11}(40\,x-7)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((20*x^2 + 12*x + 3)/(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9),x, algorithm=
```

```
[Out] 1/22*sqrt(11)*(arctan(1/66*sqrt(11)*(800*x^3 - 40*x^2 + 30*x + 57
)) + arctan(1/55*sqrt(11)*(40*x - 7)))
```

Sympy [A] time = 0.212998, size = 73, normalized size = 1.24

$$\frac{\sqrt{11}\left(2\operatorname{atan}\left(\frac{8\sqrt{11}x}{11}-\frac{7\sqrt{11}}{55}\right)+2\operatorname{atan}\left(\frac{400\sqrt{11}x^3}{33}-\frac{20\sqrt{11}x^2}{33}+\frac{5\sqrt{11}x}{11}+\frac{19\sqrt{11}}{22}\right)\right)}{44}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((20*x**2+12*x+3)/(320*x**4+80*x**3-12*x**2+24*x+9),x)
```

```
[Out] sqrt(11)*(2*atan(8*sqrt(11)*x/11 - 7*sqrt(11)/55) + 2*atan(400*sq
rt(11)*x**3/33 - 20*sqrt(11)*x**2/33 + 5*sqrt(11)*x/11 + 19*sqrt(
11)/22))/44
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{20\,x^2+12\,x+3}{320\,x^4+80\,x^3-12\,x^2+24\,x+9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((20*x^2 + 12*x + 3)/(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9),x, algorithm=
```

```
[Out] integrate((20*x^2 + 12*x + 3)/(320*x^4 + 80*x^3 - 12*x^2 + 24*x +
 9), x)
```

3.62 $\int -\frac{84+576x+400x^2-2560x^3}{9+24x-12x^2+80x^3+320x^4}\,dx$

Optimal. Leaf size=78

$$-2\sqrt{11}\tan^{-1}\left(\frac{800x^3-40x^2+30x+57}{6\sqrt{11}}\right)+2\log\left(320x^4+80x^3-12x^2+24x+9\right)+2\sqrt{11}\tan^{-1}\left(\frac{7-40x}{5\sqrt{11}}\right)$$

```
[Out] 2*Sqrt[11]*ArcTan[(7 - 40*x)/(5*Sqrt[11])] - 2*Sqrt[11]*ArcTan[(5
7 + 30*x - 40*x^2 + 800*x^3)/(6*Sqrt[11])] + 2*Log[9 + 24*x - 12*
x^2 + 80*x^3 + 320*x^4]
```

Rubi [A] time = 0.124319, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.051$

$$-2\sqrt{11}\tan^{-1}\left(\frac{800x^3-40x^2+30x+57}{6\sqrt{11}}\right)+2\log\left(320x^4+80x^3-12x^2+24x+9\right)+2\sqrt{11}\tan^{-1}\left(\frac{7-40x}{5\sqrt{11}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[-((84 + 576*x + 400*x^2 - 2560*x^3)/(9 + 24*x - 12*x^2 + 80*x^3 + 320*x^4)),x]
```

```
[Out] 2*Sqrt[11]*ArcTan[(7 - 40*x)/(5*Sqrt[11])] - 2*Sqrt[11]*ArcTan[(5
7 + 30*x - 40*x^2 + 800*x^3)/(6*Sqrt[11])] + 2*Log[9 + 24*x - 12*
x^2 + 80*x^3 + 320*x^4]
```

Rubi in Sympy [A] time = 14.7712, size = 80, normalized size = 1.03

$$2\log\left(320x^4+80x^3-12x^2+24x+9\right)-2\sqrt{11}\,\mathrm{atan}\left(\sqrt{11}\left(\frac{8x}{11}-\frac{7}{55}\right)\right)$$
$$-2\sqrt{11}\,\mathrm{atan}\left(\sqrt{11}\left(\frac{400x^3}{33}-\frac{20x^2}{33}+\frac{5x}{11}+\frac{19}{22}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((2560*x**3-400*x**2-576*x-84)/(320*x**4+80*x**3-12*x**2+24*x+9)
```

```
[Out] 2*log(320*x**4 + 80*x**3 - 12*x**2 + 24*x + 9) - 2*sqrt(11)*atan(
sqrt(11)*(8*x/11 - 7/55)) - 2*sqrt(11)*atan(sqrt(11)*(400*x**3/33
 - 20*x**2/33 + 5*x/11 + 19/22))
```

Mathematica [C] time = 0.0294842, size = 99, normalized size = 1.27

$$\frac{1}{2}\mathrm{RootSum}\left[320\#1^4+80\#1^3-12\#1^2+24\#1+9\&,\frac{640\#1^3\log(x-\#1)-100\#1^2\log(x-\#1)-144\#1\log(x-\#1)-21\log(x-\#1)}{160\#1^3+30\#1^2-3\#1+3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[-((84 + 576*x + 400*x^2 - 2560*x^3)/(9 + 24*x - 12*x^2 + 80*x^3 + 320*x^4
```

```
[Out] RootSum[9 + 24*#1 - 12*#1^2 + 80*#1^3 + 320*#1^4 & , (-21*Log[x -
 #1] - 144*Log[x - #1]*#1 - 100*Log[x - #1]*#1^2 + 640*Log[x - #1
]*#1^3)/(3 - 3*#1 + 30*#1^2 + 160*#1^3) & ]/2
```

Maple [A] time = 0.033, size = 75, normalized size = 1.

$$2\ln\left(6400x^4+1600x^3-240x^2+480x+180\right)-2\sqrt{11}\arctan\left(-\frac{20\sqrt{11}x^2}{33}+\frac{5\sqrt{11}x}{11}+\frac{19\sqrt{11}}{22}+\frac{400\sqrt{11}x^3}{33}\right)-2\sqrt{11}\arctan\left(\frac{(40x-7)\sqrt{11}}{55}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2560*x^3-400*x^2-576*x-84)/(320*x^4+80*x^3-12*x^2+24*x+9),x)
```

```
[Out] 2*ln(6400*x^4+1600*x^3-240*x^2+480*x+180)-2*11^(1/2)*arctan(-20/3
3*11^(1/2)*x^2+5/11*11^(1/2)*x+19/22*11^(1/2)+400/33*11^(1/2)*x^3
)-2*11^(1/2)*arctan(1/55*(40*x-7)*11^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$4\int\frac{640x^3-100x^2-144x-21}{320x^4+80x^3-12x^2+24x+9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*(640*x^3 - 100*x^2 - 144*x - 21)/(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9)
```

```
[Out] 4*integrate((640*x^3 - 100*x^2 - 144*x - 21)/(320*x^4 + 80*x^3 -
12*x^2 + 24*x + 9), x)
```

Fricas [A] time = 0.198484, size = 89, normalized size = 1.14

$$-2\sqrt{11}\arctan\left(\frac{1}{66}\sqrt{11}\left(800x^3-40x^2+30x+57\right)\right)-2\sqrt{11}\arctan\left(\frac{1}{55}\sqrt{11}(40x-7)\right)+2\log\left(320x^4+80x^3-12x^2+24x+9\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*(640*x^3 - 100*x^2 - 144*x - 21)/(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9)
```

```
[Out] -2*sqrt(11)*arctan(1/66*sqrt(11)*(800*x^3 - 40*x^2 + 30*x + 57))
- 2*sqrt(11)*arctan(1/55*sqrt(11)*(40*x - 7)) + 2*log(320*x^4 + 8
0*x^3 - 12*x^2 + 24*x + 9)
```

Sympy [A] time = 0.218207, size = 100, normalized size = 1.28

$$\sqrt{11}\left(-2\operatorname{atan}\left(\frac{8\sqrt{11}x}{11}-\frac{7\sqrt{11}}{55}\right)-2\operatorname{atan}\left(\frac{400\sqrt{11}x^3}{33}-\frac{20\sqrt{11}x^2}{33}+\frac{5\sqrt{11}x}{11}+\frac{19\sqrt{11}}{22}\right)\right)+2\log\left(x^4+\frac{x^3}{4}-\frac{3x^2}{80}+\frac{3x}{40}+\frac{9}{320}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2560*x**3-400*x**2-576*x-84)/(320*x**4+80*x**3-12*x**2+24*x+9),x)
```

```
[Out] sqrt(11)*(-2*atan(8*sqrt(11)*x/11 - 7*sqrt(11)/55) - 2*atan(400*s
qrt(11)*x**3/33 - 20*sqrt(11)*x**2/33 + 5*sqrt(11)*x/11 + 19*sqrt
(11)/22)) + 2*log(x**4 + x**3/4 - 3*x**2/80 + 3*x/40 + 9/320)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{4\left(640\,x^3-100\,x^2-144\,x-21\right)}{320\,x^4+80\,x^3-12\,x^2+24\,x+9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*(640*x^3 - 100*x^2 - 144*x - 21)/(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9)
```

```
[Out] integrate(4*(640*x^3 - 100*x^2 - 144*x - 21)/(320*x^4 + 80*x^3 -
12*x^2 + 24*x + 9), x)
```

3.63 $\int \frac{\sqrt{1-x^4}}{1+x^4}\,dx$

Optimal. Leaf size=49

$$\frac{1}{2}\tan^{-1}\left(\frac{x\left(x^2+1\right)}{\sqrt{1-x^4}}\right)+\frac{1}{2}\tanh^{-1}\left(\frac{x\left(1-x^2\right)}{\sqrt{1-x^4}}\right)$$

```
[Out] ArcTan[(x*(1 + x^2))/Sqrt[1 - x^4]]/2 + ArcTanh[(x*(1 - x^2))/Sqr
t[1 - x^4]]/2
```

Rubi [A] time = 0.0261983, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053

$$\frac{1}{2}\tan^{-1}\left(\frac{x\left(x^2+1\right)}{\sqrt{1-x^4}}\right)+\frac{1}{2}\tanh^{-1}\left(\frac{x\left(1-x^2\right)}{\sqrt{1-x^4}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 - x^4]/(1 + x^4),x]
```

```
[Out] ArcTan[(x*(1 + x^2))/Sqrt[1 - x^4]]/2 + ArcTanh[(x*(1 - x^2))/Sqr
t[1 - x^4]]/2
```

Rubi in Sympy [A] time = 2.40816, size = 36, normalized size = 0.73

$$\frac{\operatorname{atan}\left(\frac{x\left(x^2+1\right)}{\sqrt{-x^4+1}}\right)}{2}+\frac{\operatorname{atanh}\left(\frac{x\left(-x^2+1\right)}{\sqrt{-x^4+1}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**4+1)**(1/2)/(x**4+1),x)
```

```
[Out] atan(x*(x**2 + 1)/sqrt(-x**4 + 1))/2 + atanh(x*(-x**2 + 1)/sqrt(-
x**4 + 1))/2
```

Mathematica [C] time = 0.144058, size = 110, normalized size = 2.24

$$-\frac{5x\sqrt{1-x^4}F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};x^4,-x^4\right)}{\left(x^4+1\right)\left(2x^4\left(2F_1\left(\frac{5}{4};-\frac{1}{2},2;\frac{9}{4};x^4,-x^4\right)+F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};x^4,-x^4\right)\right)-5F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};x^4,-x^4\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[Sqrt[1 - x^4]/(1 + x^4),x]
```

```
[Out] (-5*x*Sqrt[1 - x^4]*AppellF1[1/4, -1/2, 1, 5/4, x^4, -x^4])/((1 +
 x^4)*(-5*AppellF1[1/4, -1/2, 1, 5/4, x^4, -x^4] + 2*x^4*(2*Appel
lF1[5/4, -1/2, 2, 9/4, x^4, -x^4] + AppellF1[5/4, 1/2, 1, 9/4, x^
4, -x^4])))
```

Maple [B] time = 0.022, size = 100, normalized size = 2.

$$-\frac{1}{4}\arctan\left(\frac{1}{x}\sqrt{-x^4+1}+1\right)+\frac{1}{4}\arctan\left(-\frac{1}{x}\sqrt{-x^4+1}+1\right)$$
$$-\frac{1}{8}\ln\left(1\left(\frac{-x^4+1}{2\,x^2}-\frac{1}{x}\sqrt{-x^4+1}+1\right)\left(\frac{-x^4+1}{2\,x^2}+\frac{1}{x}\sqrt{-x^4+1}+1\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^4+1)^(1/2)/(x^4+1),x)
```

```
[Out] -1/4*arctan((-x^4+1)^(1/2)/x+1)+1/4*arctan(-(-x^4+1)^(1/2)/x+1)-1
/8*ln((1/2*(-x^4+1)/x^2-(-x^4+1)^(1/2)/x+1)/(1/2*(-x^4+1)/x^2+(-x
^4+1)^(1/2)/x+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^4+1}}{x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^4 + 1)/(x^4 + 1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 + 1)/(x^4 + 1), x)
```

Fricas [A] time = 0.257111, size = 105, normalized size = 2.14

$$\frac{1}{2}\arctan\left(-\frac{x^3+\sqrt{-x^4+1}x^2+x}{x^3-x-\sqrt{-x^4+1}}\right)+\frac{1}{4}\log\left(-\frac{x^4-2\,x^2-2\,\sqrt{-x^4+1}x-1}{x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^4 + 1)/(x^4 + 1),x, algorithm="fricas")
```

```
[Out] 1/2*arctan(-(x^3 + sqrt(-x^4 + 1)*x^2 + x)/(x^3 - x - sqrt(-x^4 +
 1))) + 1/4*log(-(x^4 - 2*x^2 - 2*sqrt(-x^4 + 1)*x - 1)/(x^4 + 1)
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(x-1)(x+1)(x^2+1)}}{x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**4+1)**(1/2)/(x**4+1),x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 1)*(x**2 + 1))/(x**4 + 1), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^4+1}}{x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^4 + 1)/(x^4 + 1),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^4 + 1)/(x^4 + 1), x)
```

3.64 $\int \frac{\sqrt{1+x^4}}{1-x^4}\,dx$

Optimal. Leaf size=53

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{2\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{2\sqrt{2}}$$

```
[Out] ArcTan[(Sqrt[2]*x)/Sqrt[1 + x^4]]/(2*Sqrt[2]) + ArcTanh[(Sqrt[2]*
x)/Sqrt[1 + x^4]]/(2*Sqrt[2])
```

Rubi [A] time = 0.0367356, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{2\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 + x^4]/(1 - x^4),x]
```

```
[Out] ArcTan[(Sqrt[2]*x)/Sqrt[1 + x^4]]/(2*Sqrt[2]) + ArcTanh[(Sqrt[2]*
x)/Sqrt[1 + x^4]]/(2*Sqrt[2])
```

Rubi in Sympy [A] time = 2.58519, size = 46, normalized size = 0.87

$$\frac{\sqrt{2}\,\operatorname{atan}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{4}+\frac{\sqrt{2}\,\operatorname{atanh}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**4+1)**(1/2)/(-x**4+1),x)
```

```
[Out] sqrt(2)*atan(sqrt(2)*x/sqrt(x**4 + 1))/4 + sqrt(2)*atanh(sqrt(2)*
x/sqrt(x**4 + 1))/4
```

Mathematica [C] time = 0.134014, size = 108, normalized size = 2.04

$$-\frac{5x\sqrt{x^4+1}F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};-x^4,x^4\right)}{\left(x^4-1\right)\left(2x^4\left(2F_1\left(\frac{5}{4};-\frac{1}{2},2;\frac{9}{4};-x^4,x^4\right)+F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};-x^4,x^4\right)\right)+5F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};-x^4,x^4\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[Sqrt[1 + x^4]/(1 - x^4),x]
```

```
[Out] (-5*x*Sqrt[1 + x^4]*AppellF1[1/4, -1/2, 1, 5/4, -x^4, x^4])/((-1
+ x^4)*(5*AppellF1[1/4, -1/2, 1, 5/4, -x^4, x^4] + 2*x^4*(2*Appel
lF1[5/4, -1/2, 2, 9/4, -x^4, x^4] + AppellF1[5/4, 1/2, 1, 9/4, -x
^4, x^4])))
```

Maple [C] time = 0.04, size = 365, normalized size = 6.9

$$-\frac{\mathit{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right),i\right)}{\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}}\sqrt{1-ix^2}\sqrt{1+ix^2}\frac{1}{\sqrt{x^4+1}}$$
$$-\frac{\frac{i}{2}\left(\mathit{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right),i\right)-\mathit{EllipticE}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right),i\right)\right)}{\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}}\sqrt{1-ix^2}\sqrt{1+ix^2}\frac{1}{\sqrt{x^4+1}}$$
$$-(-1)^{\frac{3}{4}}\mathit{EllipticPi}\left(\sqrt[4]{-1}x,-i,\sqrt{-i}-(-1)^{\frac{3}{4}}\right)\sqrt{1-ix^2}\sqrt{1+ix^2}\frac{1}{\sqrt{x^4+1}}$$
$$+\frac{\frac{i}{2}\mathit{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right),i\right)}{\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}}\sqrt{1-ix^2}\sqrt{1+ix^2}\frac{1}{\sqrt{x^4+1}}$$
$$-\frac{\frac{i}{2}\mathit{EllipticE}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right),i\right)}{\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}}\sqrt{1-ix^2}\sqrt{1+ix^2}\frac{1}{\sqrt{x^4+1}}$$
$$-(-1)^{\frac{3}{4}}\mathit{EllipticPi}\left(\sqrt[4]{-1}x,i,\sqrt{-i}-(-1)^{\frac{3}{4}}\right)\sqrt{1-ix^2}\sqrt{1+ix^2}\frac{1}{\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((x^4+1)^(1/2)/(-x^4+1),x)
```

```
[Out] -1/(1/2*2^(1/2)+1/2*I*2^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x
^4+1)^(1/2)*EllipticF(x*(1/2*2^(1/2)+1/2*I*2^(1/2)),I)-1/2*I/(1/2
*2^(1/2)+1/2*I*2^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(
1/2)*(EllipticF(x*(1/2*2^(1/2)+1/2*I*2^(1/2)),I)-EllipticE(x*(1/2
*2^(1/2)+1/2*I*2^(1/2)),I))-(-1)^(3/4)*(1-I*x^2)^(1/2)*(1+I*x^2)^
(1/2)/(x^4+1)^(1/2)*EllipticPi((-1)^(1/4)*x,-I,(-I)^(1/2)/(-1)^(1
/4))+1/2*I/(1/2*2^(1/2)+1/2*I*2^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^
(1/2)/(x^4+1)^(1/2)*EllipticF(x*(1/2*2^(1/2)+1/2*I*2^(1/2)),I)-1/
2*I/(1/2*2^(1/2)+1/2*I*2^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(
x^4+1)^(1/2)*EllipticE(x*(1/2*2^(1/2)+1/2*I*2^(1/2)),I)-(-1)^(3/4
)*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticPi((-1)^(
1/4)*x,I,(-I)^(1/2)/(-1)^(1/4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{x^4+1}}{x^4-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(-sqrt(x^4 + 1)/(x^4 - 1),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(x^4 + 1)/(x^4 - 1), x)
```

Fricas [A] time = 0.25386, size = 81, normalized size = 1.53

$$\frac{1}{8}\sqrt{2}\left(2\arctan\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)+\log\left(\frac{\sqrt{2}\left(x^4+2x^2+1\right)+4\sqrt{x^4+1}x}{x^4-2x^2+1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(-sqrt(x^4 + 1)/(x^4 - 1),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2)*(2*arctan(sqrt(2)*x/sqrt(x^4 + 1)) + log((sqrt(2)*(x^
4 + 2*x^2 + 1) + 4*sqrt(x^4 + 1)*x)/(x^4 - 2*x^2 + 1)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{x^4+1}}{x^4-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**4+1)**(1/2)/(-x**4+1),x)
```

```
[Out] -Integral(sqrt(x**4 + 1)/(x**4 - 1), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\sqrt{x^4+1}}{x^4-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sqrt(x^4 + 1)/(x^4 - 1),x, algorithm="giac")
```

```
[Out] integrate(-sqrt(x^4 + 1)/(x^4 - 1), x)
```

3.65 $\int \frac{\sqrt{1+px^2+x^4}}{1-x^4} \, dx$

Optimal. Leaf size=75

$$\frac{1}{4}\sqrt{2-p}\tan^{-1}\left(\frac{\sqrt{2-p}x}{\sqrt{px^2+x^4+1}}\right)+\frac{1}{4}\sqrt{p+2}\tanh^{-1}\left(\frac{\sqrt{p+2}x}{\sqrt{px^2+x^4+1}}\right)$$

```
[Out] (Sqrt[2 - p]*ArcTan[(Sqrt[2 - p]*x)/Sqrt[1 + p*x^2 + x^4]])/4 + (
Sqrt[2 + p]*ArcTanh[(Sqrt[2 + p]*x)/Sqrt[1 + p*x^2 + x^4]])/4
```

Rubi [A] time = 0.161053, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$\frac{1}{4}\sqrt{2-p}\tan^{-1}\left(\frac{\sqrt{2-p}x}{\sqrt{px^2+x^4+1}}\right)+\frac{1}{4}\sqrt{p+2}\tanh^{-1}\left(\frac{\sqrt{p+2}x}{\sqrt{px^2+x^4+1}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 + p*x^2 + x^4]/(1 - x^4),x]
```

```
[Out] (Sqrt[2 - p]*ArcTan[(Sqrt[2 - p]*x)/Sqrt[1 + p*x^2 + x^4]])/4 + (
Sqrt[2 + p]*ArcTanh[(Sqrt[2 + p]*x)/Sqrt[1 + p*x^2 + x^4]])/4
```

Rubi in Sympy [A] time = 9.10875, size = 63, normalized size = 0.84

$$\frac{\sqrt{-p+2}\,\text{atan}\left(\frac{x\sqrt{-p+2}}{\sqrt{px^2+x^4+1}}\right)}{4}+\frac{\sqrt{p+2}\,\text{atanh}\left(\frac{x\sqrt{p+2}}{\sqrt{px^2+x^4+1}}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**4+p*x**2+1)**(1/2)/(-x**4+1),x)
```

```
[Out] sqrt(-p + 2)*atan(x*sqrt(-p + 2)/sqrt(p*x**2 + x**4 + 1))/4 + sqr
t(p + 2)*atanh(x*sqrt(p + 2)/sqrt(p*x**2 + x**4 + 1))/4
```

Mathematica [A] time = 0.329071, size = 0, normalized size = 0.

$$\int \frac{\sqrt{1+px^2+x^4}}{1-x^4} \, dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[Sqrt[1 + p*x^2 + x^4]/(1 - x^4),x]
```

```
[Out] Integrate[Sqrt[1 + p*x^2 + x^4]/(1 - x^4), x]
```

Maple [C] time = 0.098, size = 1421, normalized size = 19.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^4+p*x^2+1)^(1/2)/(-x^4+1),x)
```

```
[Out] -1/2*(1+p)/(-2*p+2*(p^2-4)^(1/2))^(1/2)*(1-(-1/2*p+1/2*(p^2-4)^(1
/2))*x^2)^(1/2)*(1-(-1/2*p-1/2*(p^2-4)^(1/2))*x^2)^(1/2)/(x^4+p*x
^2+1)^(1/2)*EllipticF(1/2*x*(-2*p+2*(p^2-4)^(1/2))^(1/2),(-1-p*(-
1/2*p-1/2*(p^2-4)^(1/2)))^(1/2))+2/(-2*p+2*(p^2-4)^(1/2))^(1/2)*(
1-(-1/2*p+1/2*(p^2-4)^(1/2))*x^2)^(1/2)*(1-(-1/2*p-1/2*(p^2-4)^(1
/2))*x^2)^(1/2)/(x^4+p*x^2+1)^(1/2)/(p+(p^2-4)^(1/2))*(EllipticF(
1/2*x*(-2*p+2*(p^2-4)^(1/2))^(1/2),(-1-p*(-1/2*p-1/2*(p^2-4)^(1/2
)))^(1/2))-EllipticE(1/2*x*(-2*p+2*(p^2-4)^(1/2))^(1/2),(-1-p*(-1
/2*p-1/2*(p^2-4)^(1/2)))^(1/2)))+1/(-1/2*p+1/2*(p^2-4)^(1/2))^(1/
2)*(1-(-1/2*p+1/2*(p^2-4)^(1/2))*x^2)^(1/2)*(1-(-1/2*p-1/2*(p^2-4
)^(1/2))*x^2)^(1/2)/(x^4+p*x^2+1)^(1/2)*EllipticPi((-1/2*p+1/2*(p
^2-4)^(1/2))^(1/2)*x,1/(-1/2*p+1/2*(p^2-4)^(1/2)),(-1/2*p-1/2*(p^
2-4)^(1/2))^(1/2)/(-1/2*p+1/2*(p^2-4)^(1/2))^(1/2))+1/2*p/(-1/2*p
+1/2*(p^2-4)^(1/2))^(1/2)*(1-(-1/2*p+1/2*(p^2-4)^(1/2))*x^2)^(1/2
)*(1-(-1/2*p-1/2*(p^2-4)^(1/2))*x^2)^(1/2)/(x^4+p*x^2+1)^(1/2)*El
lipticPi((-1/2*p+1/2*(p^2-4)^(1/2))^(1/2)*x,1/(-1/2*p+1/2*(p^2-4)
^(1/2)),(-1/2*p-1/2*(p^2-4)^(1/2))^(1/2)/(-1/2*p+1/2*(p^2-4)^(1/2
))^(1/2))+1/2*(-1-p)/(-2*p+2*(p^2-4)^(1/2))^(1/2)*(1-(-1/2*p+1/2*
(p^2-4)^(1/2))*x^2)^(1/2)*(1-(-1/2*p-1/2*(p^2-4)^(1/2))*x^2)^(1/2
)/(x^4+p*x^2+1)^(1/2)*EllipticF(1/2*x*(-2*p+2*(p^2-4)^(1/2))^(1/2
),(-1-p*(-1/2*p-1/2*(p^2-4)^(1/2)))^(1/2))+1/(-2*p+2*(p^2-4)^(1/2
))^(1/2)*(1+1/2*p*x^2-1/2*x^2*(p^2-4)^(1/2))^(1/2)*(1+1/2*p*x^2+1
/2*x^2*(p^2-4)^(1/2))^(1/2)/(x^4+p*x^2+1)^(1/2)*EllipticF(1/2*x*(
-2*p+2*(p^2-4)^(1/2))^(1/2),(-1-p*(-1/2*p-1/2*(p^2-4)^(1/2)))^(1/
2))*p-1/(-2*p+2*(p^2-4)^(1/2))^(1/2)*(1+1/2*p*x^2-1/2*x^2*(p^2-4)
^(1/2))^(1/2)*(1+1/2*p*x^2+1/2*x^2*(p^2-4)^(1/2))^(1/2)/(x^4+p*x^
2+1)^(1/2)*EllipticF(1/2*x*(-2*p+2*(p^2-4)^(1/2))^(1/2),(-1-p*(-1
/2*p-1/2*(p^2-4)^(1/2)))^(1/2))-2/(-2*p+2*(p^2-4)^(1/2))^(1/2)*(1
+1/2*p*x^2-1/2*x^2*(p^2-4)^(1/2))^(1/2)*(1+1/2*p*x^2+1/2*x^2*(p^2
-4)^(1/2))^(1/2)/(x^4+p*x^2+1)^(1/2)/(p+(p^2-4)^(1/2))*EllipticF(
1/2*x*(-2*p+2*(p^2-4)^(1/2))^(1/2),(-1-p*(-1/2*p-1/2*(p^2-4)^(1/2
)))^(1/2))+2/(-2*p+2*(p^2-4)^(1/2))^(1/2)*(1+1/2*p*x^2-1/2*x^2*(p
^2-4)^(1/2))^(1/2)*(1+1/2*p*x^2+1/2*x^2*(p^2-4)^(1/2))^(1/2)/(x^4
+p*x^2+1)^(1/2)/(p+(p^2-4)^(1/2))*EllipticE(1/2*x*(-2*p+2*(p^2-4)
^(1/2))^(1/2),(-1-p*(-1/2*p-1/2*(p^2-4)^(1/2)))^(1/2))+1/(-1/2*p+
1/2*(p^2-4)^(1/2))^(1/2)*(1+1/2*p*x^2-1/2*x^2*(p^2-4)^(1/2))^(1/2
)*(1+1/2*p*x^2+1/2*x^2*(p^2-4)^(1/2))^(1/2)/(x^4+p*x^2+1)^(1/2)*E
llipticPi((-1/2*p+1/2*(p^2-4)^(1/2))^(1/2)*x,-1/(-1/2*p+1/2*(p^2-
4)^(1/2)),(-1/2*p-1/2*(p^2-4)^(1/2))^(1/2)/(-1/2*p+1/2*(p^2-4)^(1
/2))^(1/2))-1/2*p/(-1/2*p+1/2*(p^2-4)^(1/2))^(1/2)*(1+1/2*p*x^2-1
/2*x^2*(p^2-4)^(1/2))^(1/2)*(1+1/2*p*x^2+1/2*x^2*(p^2-4)^(1/2))^(
1/2)/(x^4+p*x^2+1)^(1/2)*EllipticPi((-1/2*p+1/2*(p^2-4)^(1/2))^(1
/2)*x,-1/(-1/2*p+1/2*(p^2-4)^(1/2)),(-1/2*p-1/2*(p^2-4)^(1/2))^(1
/2)/(-1/2*p+1/2*(p^2-4)^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{x^4 + px^2 + 1}}{x^4 - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sqrt(x^4 + p*x^2 + 1)/(x^4 - 1),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(x^4 + p*x^2 + 1)/(x^4 - 1), x)
```

Fricas [A] time = 0.27045, size = 1, normalized size = 0.01

$$\left[\frac{1}{8}\sqrt{p-2}\log\left(\frac{x^4+2(p-1)x^2-2\sqrt{x^4+px^2+1}\sqrt{p-2}x+1}{x^4+2x^2+1}\right) + \frac{1}{8}\sqrt{p+2}\log\left(\frac{x^4+2(p+1)x^2+2\sqrt{x^4+px^2+1}\sqrt{p+2}x+1}{x^4-2x^2+1}\right), \frac{1}{4}\sqrt{-p+2}\arctan\left(\frac{\sqrt{-p+2}x}{\sqrt{x^4+px^2+1}}\right) + \frac{1}{8}\sqrt{p+2}\log\left(\frac{x^4+2(p+1)x^2+2\sqrt{x^4+px^2+1}\sqrt{p+2}x+1}{x^4-2x^2+1}\right), \frac{1}{4}\sqrt{-p-2}\arctan\left(\frac{\sqrt{x^4+px^2+1}}{\sqrt{-p-2}x}\right) + \frac{1}{8}\sqrt{p-2}\log\left(\frac{x^4+2(p-1)x^2-2\sqrt{x^4+px^2+1}\sqrt{p-2}x+1}{x^4+2x^2+1}\right), \frac{1}{4}\sqrt{-p+2}\arctan\left(\frac{\sqrt{-p+2}x}{\sqrt{x^4+px^2+1}}\right) + \frac{1}{4}\sqrt{-p-2}\arctan\left(\frac{\sqrt{x^4+px^2+1}}{\sqrt{-p-2}x}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sqrt(x^4 + p*x^2 + 1)/(x^4 - 1),x, algorithm="fricas")
```

```
[Out] [1/8*sqrt(p - 2)*log((x^4 + 2*(p - 1)*x^2 - 2*sqrt(x^4 + p*x^2 +
1)*sqrt(p - 2)*x + 1)/(x^4 + 2*x^2 + 1)) + 1/8*sqrt(p + 2)*log((x
^4 + 2*(p + 1)*x^2 + 2*sqrt(x^4 + p*x^2 + 1)*sqrt(p + 2)*x + 1)/(
x^4 - 2*x^2 + 1)), 1/4*sqrt(-p + 2)*arctan(sqrt(-p + 2)*x/sqrt(x^
4 + p*x^2 + 1)) + 1/8*sqrt(p + 2)*log((x^4 + 2*(p + 1)*x^2 + 2*sq
rt(x^4 + p*x^2 + 1)*sqrt(p + 2)*x + 1)/(x^4 - 2*x^2 + 1)), 1/4*sq
rt(-p - 2)*arctan(sqrt(x^4 + p*x^2 + 1)/(sqrt(-p - 2)*x)) + 1/8*s
qrt(p - 2)*log((x^4 + 2*(p - 1)*x^2 - 2*sqrt(x^4 + p*x^2 + 1)*sqr
t(p - 2)*x + 1)/(x^4 + 2*x^2 + 1)), 1/4*sqrt(-p + 2)*arctan(sqrt(
-p + 2)*x/sqrt(x^4 + p*x^2 + 1)) + 1/4*sqrt(-p - 2)*arctan(sqrt(x
^4 + p*x^2 + 1)/(sqrt(-p - 2)*x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{px^2+x^4+1}}{x^4-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**4+p*x**2+1)**(1/2)/(-x**4+1),x)
```

```
[Out] -Integral(sqrt(p*x**2 + x**4 + 1)/(x**4 - 1), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\sqrt{x^4+px^2+1}}{x^4-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sqrt(x^4 + p*x^2 + 1)/(x^4 - 1),x, algorithm="giac")
```

```
[Out] integrate(-sqrt(x^4 + p*x^2 + 1)/(x^4 - 1), x)
```

3.66 $\int \frac{\sqrt{1+px^2-x^4}}{1+x^4} \, dx$

Optimal. Leaf size=171

$$\frac{\sqrt{\sqrt{p^2+4}-p}\tanh^{-1}\left(\frac{\sqrt{\sqrt{p^2+4}-p}x\left(\sqrt{p^2+4}+p-2x^2\right)}{2\sqrt{2}\sqrt{px^2-x^4+1}}\right)}{2\sqrt{2}}-\frac{\sqrt{\sqrt{p^2+4}+p}\tan^{-1}\left(\frac{\sqrt{\sqrt{p^2+4}+p}x\left(-\sqrt{p^2+4}+p-2x^2\right)}{2\sqrt{2}\sqrt{px^2-x^4+1}}\right)}{2\sqrt{2}}$$

```
[Out] -(Sqrt[p + Sqrt[4 + p^2]]*ArcTan[(Sqrt[p + Sqrt[4 + p^2]]*x*(p -
Sqrt[4 + p^2] - 2*x^2))/(2*Sqrt[2]*Sqrt[1 + p*x^2 - x^4])])/(2*Sq
rt[2]) + (Sqrt[-p + Sqrt[4 + p^2]]*ArcTanh[(Sqrt[-p + Sqrt[4 + p^
2]]*x*(p + Sqrt[4 + p^2] - 2*x^2))/(2*Sqrt[2]*Sqrt[1 + p*x^2 - x^
4])])/(2*Sqrt[2])
```

Rubi [A] time = 0.141518, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$

$$\frac{\sqrt{\sqrt{p^2+4}-p}\tanh^{-1}\left(\frac{\sqrt{\sqrt{p^2+4}-p}x\left(\sqrt{p^2+4}+p-2x^2\right)}{2\sqrt{2}\sqrt{px^2-x^4+1}}\right)}{2\sqrt{2}}-\frac{\sqrt{\sqrt{p^2+4}+p}\tan^{-1}\left(\frac{\sqrt{\sqrt{p^2+4}+p}x\left(-\sqrt{p^2+4}+p-2x^2\right)}{2\sqrt{2}\sqrt{px^2-x^4+1}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 + p*x^2 - x^4]/(1 + x^4),x]
```

```
[Out] -(Sqrt[p + Sqrt[4 + p^2]]*ArcTan[(Sqrt[p + Sqrt[4 + p^2]]*x*(p -
Sqrt[4 + p^2] - 2*x^2))/(2*Sqrt[2]*Sqrt[1 + p*x^2 - x^4])])/(2*Sq
rt[2]) + (Sqrt[-p + Sqrt[4 + p^2]]*ArcTanh[(Sqrt[-p + Sqrt[4 + p^
2]]*x*(p + Sqrt[4 + p^2] - 2*x^2))/(2*Sqrt[2]*Sqrt[1 + p*x^2 - x^
4])])/(2*Sqrt[2])
```

Rubi in Sympy [A] time = 6.9404, size = 144, normalized size = 0.84

$$\frac{\sqrt{2}\sqrt{-p+\sqrt{p^2+4}}\operatorname{atanh}\left(\frac{\sqrt{2}x\sqrt{-p+\sqrt{p^2+4}}\left(p-2x^2+\sqrt{p^2+4}\right)}{4\sqrt{px^2-x^4+1}}\right)}{4}-\frac{\sqrt{2}\sqrt{p+\sqrt{p^2+4}}\operatorname{atan}\left(\frac{\sqrt{2}x\sqrt{p+\sqrt{p^2+4}}\left(p-2x^2-\sqrt{p^2+4}\right)}{4\sqrt{px^2-x^4+1}}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**4+p*x**2+1)**(1/2)/(x**4+1),x)
```

```
[Out] sqrt(2)*sqrt(-p + sqrt(p**2 + 4))*atanh(sqrt(2)*x*sqrt(-p + sqrt(
p**2 + 4))*(p - 2*x**2 + sqrt(p**2 + 4))/(4*sqrt(p*x**2 - x**4 +
1)))/4 - sqrt(2)*sqrt(p + sqrt(p**2 + 4))*atan(sqrt(2)*x*sqrt(p +
 sqrt(p**2 + 4))*(p - 2*x**2 - sqrt(p**2 + 4))/(4*sqrt(p*x**2 - x
**4 + 1)))/4
```

Mathematica [C] time = 0.426933, size = 322, normalized size = 1.88

$$\frac{\sqrt{\frac{4x^2}{\sqrt{p^2+4}-p}+2}\sqrt{1-\frac{2x^2}{\sqrt{p^2+4}+p}}\left(2iF\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{1}{\sqrt{p^2+4}-p}}x\right)|\frac{p-\sqrt{p^2+4}}{p+\sqrt{p^2+4}}\right)-(p+2i)\left(\frac{1}{2}i\left(p-\sqrt{p^2+4}\right);i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{1}{\sqrt{p^2+4}-p}}\cdots\right.\right.\right.}{4\sqrt{\frac{1}{\sqrt{p^2+4}-p}}\sqrt{px^2-x^4+1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 + p*x^2 - x^4]/(1 + x^4),x]
```

```
[Out] (Sqrt[2 + (4*x^2)/(-p + Sqrt[4 + p^2])]*Sqrt[1 - (2*x^2)/(p + Sqr
t[4 + p^2])]*((2*I)*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[(-p + Sqrt[4
 + p^2])^(-1)]*x], (p - Sqrt[4 + p^2])/(p + Sqrt[4 + p^2])] - (2*
I + p)*EllipticPi[(I/2)*(p - Sqrt[4 + p^2]), I*ArcSinh[Sqrt[2]*Sq
rt[(-p + Sqrt[4 + p^2])^(-1)]*x], (p - Sqrt[4 + p^2])/(p + Sqrt[4
 + p^2])] + (-2*I + p)*EllipticPi[(I/2)*(-p + Sqrt[4 + p^2]), I*A
rcSinh[Sqrt[2]*Sqrt[(-p + Sqrt[4 + p^2])^(-1)]*x], (p - Sqrt[4 +
p^2])/(p + Sqrt[4 + p^2])]))/(4*Sqrt[(-p + Sqrt[4 + p^2])^(-1)]*S
qrt[1 + p*x^2 - x^4])
```

Maple [B] time = 0.101, size = 456, normalized size = 2.7

$$
\begin{aligned}
&-\frac{\sqrt{2}}{32}\sqrt{p+\sqrt{p^2+4}}\sqrt{p^2+4}\ln\left(\frac{\sqrt{2}}{x}\sqrt{p+\sqrt{p^2+4}}\sqrt{-x^4+px^2+1}-\frac{-x^4+px^2+1}{x^2}-\sqrt{p^2+4}\right)\\
&+\frac{\sqrt{2}}{4}\arctan\left(\frac{1}{2}\left(2\sqrt{p+\sqrt{p^2+4}}-2\frac{\sqrt{-x^4+px^2+1}\sqrt{2}}{x}\right)\frac{1}{\sqrt{-p+\sqrt{p^2+4}}}\right)\frac{1}{\sqrt{-p+\sqrt{p^2+4}}}\\
&+\frac{\sqrt{2}p}{32}\sqrt{p+\sqrt{p^2+4}}\ln\left(\frac{\sqrt{2}}{x}\sqrt{p+\sqrt{p^2+4}}\sqrt{-x^4+px^2+1}-\frac{-x^4+px^2+1}{x^2}-\sqrt{p^2+4}\right)\\
&+\frac{\sqrt{2}}{32}\sqrt{p+\sqrt{p^2+4}}\sqrt{p^2+4}\ln\left(\frac{-x^4+px^2+1}{x^2}+\frac{\sqrt{2}}{x}\sqrt{p+\sqrt{p^2+4}}\sqrt{-x^4+px^2+1}+\sqrt{p^2+4}\right)\\
&-\frac{\sqrt{2}}{4}\arctan\left(\frac{1}{2}\left(2\frac{\sqrt{-x^4+px^2+1}\sqrt{2}}{x}+2\sqrt{p+\sqrt{p^2+4}}\right)\frac{1}{\sqrt{-p+\sqrt{p^2+4}}}\right)\frac{1}{\sqrt{-p+\sqrt{p^2+4}}}\\
&-\frac{\sqrt{2}p}{32}\sqrt{p+\sqrt{p^2+4}}\ln\left(\frac{-x^4+px^2+1}{x^2}+\frac{\sqrt{2}}{x}\sqrt{p+\sqrt{p^2+4}}\sqrt{-x^4+px^2+1}+\sqrt{p^2+4}\right)
\end{aligned}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^4+p*x^2+1)^(1/2)/(x^4+1),x)
```

```
[Out] -1/32*2^(1/2)*(p+(p^2+4)^(1/2))^(1/2)*(p^2+4)^(1/2)*ln((p+(p^2+4)
^(1/2))^(1/2)*(-x^4+p*x^2+1)^(1/2)*2^(1/2)/x-(-x^4+p*x^2+1)/x^2-(
p^2+4)^(1/2))+1/4*2^(1/2)/(-p+(p^2+4)^(1/2))^(1/2)*arctan(1/2*(2*
(p+(p^2+4)^(1/2))^(1/2)-2*(-x^4+p*x^2+1)^(1/2)*2^(1/2)/x)/(-p+(p^
2+4)^(1/2))^(1/2))+1/32*2^(1/2)*(p+(p^2+4)^(1/2))^(1/2)*p*ln((p+(
p^2+4)^(1/2))^(1/2)*(-x^4+p*x^2+1)^(1/2)*2^(1/2)/x-(-x^4+p*x^2+1)
/x^2-(p^2+4)^(1/2))+1/32*2^(1/2)*(p+(p^2+4)^(1/2))^(1/2)*(p^2+4)^
(1/2)*ln((-x^4+p*x^2+1)/x^2+(p+(p^2+4)^(1/2))^(1/2)*(-x^4+p*x^2+1
)^(1/2)*2^(1/2)/x+(p^2+4)^(1/2))-1/4*2^(1/2)/(-p+(p^2+4)^(1/2))^(
1/2)*arctan(1/2*(2*(-x^4+p*x^2+1)^(1/2)*2^(1/2)/x+2*(p+(p^2+4)^(1
/2))^(1/2))/(-p+(p^2+4)^(1/2))^(1/2))-1/32*2^(1/2)*(p+(p^2+4)^(1/
2))^(1/2)*p*ln((-x^4+p*x^2+1)/x^2+(p+(p^2+4)^(1/2))^(1/2)*(-x^4+p
*x^2+1)^(1/2)*2^(1/2)/x+(p^2+4)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^4+px^2+1}}{x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^4 + p*x^2 + 1)/(x^4 + 1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 + p*x^2 + 1)/(x^4 + 1), x)
```

Fricas [A] time = 0.339132, size = 2419, normalized size = 14.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^4 + p*x^2 + 1)/(x^4 + 1),x, algorithm="fricas")
```

```
[Out] -1/8*((p^2 + 4)^(1/4)*(p - sqrt(p^2 + 4))*log(-((p^4 + 5*p^2 + 4)
*x^4 - p^4 - (p^5 + 5*p^3 + 4*p)*x^2 - 5*p^2 + 2*(sqrt(-x^4 + p*x
^2 + 1)*sqrt(p^2 + 4)*(p^2 + 1)*x - sqrt(-x^4 + p*x^2 + 1)*(p^3 +
 3*p)*x)*(p^2 + 4)^(1/4)*sqrt((p^2 - sqrt(p^2 + 4)*p + 4)/(p^2 -
sqrt(p^2 + 4)*p + 2)) - ((p^3 + 3*p)*x^4 - p^3 - (p^4 + 3*p^2)*x^
2 - 3*p)*sqrt(p^2 + 4) + ((p^3 + 3*p)*sqrt(p^2 + 4)*x^2 - (p^4 +
5*p^2 + 4)*x^2)*sqrt(p^2 + 4) - 4)/((p^4 + 5*p^2 + 4)*x^4 + p^4 +
 5*p^2 - ((p^3 + 3*p)*x^4 + p^3 + 3*p)*sqrt(p^2 + 4) + 4)) - (p^2
 + 4)^(1/4)*(p - sqrt(p^2 + 4))*log(-((p^4 + 5*p^2 + 4)*x^4 - p^4
 - (p^5 + 5*p^3 + 4*p)*x^2 - 5*p^2 - 2*(sqrt(-x^4 + p*x^2 + 1)*sq
rt(p^2 + 4)*(p^2 + 1)*x - sqrt(-x^4 + p*x^2 + 1)*(p^3 + 3*p)*x)*(
p^2 + 4)^(1/4)*sqrt((p^2 - sqrt(p^2 + 4)*p + 4)/(p^2 - sqrt(p^2 +
 4)*p + 2)) - ((p^3 + 3*p)*x^4 - p^3 - (p^4 + 3*p^2)*x^2 - 3*p)*s
qrt(p^2 + 4) + ((p^3 + 3*p)*sqrt(p^2 + 4)*x^2 - (p^4 + 5*p^2 + 4)
*x^2)*sqrt(p^2 + 4) - 4)/((p^4 + 5*p^2 + 4)*x^4 + p^4 + 5*p^2 - (
(p^3 + 3*p)*x^4 + p^3 + 3*p)*sqrt(p^2 + 4) + 4)) - 8*(p^2 + 4)^(1
/4)*arctan(((sqrt(-x^4 + p*x^2 + 1)*p - sqrt(-x^4 + p*x^2 + 1)*sq
rt(p^2 + 4))*sqrt((p^2 - sqrt(p^2 + 4)*p + 4)/(p^2 - sqrt(p^2 + 4
)*p + 2)) + (p*x^3 - sqrt(p^2 + 4)*x^3 + 2*x)*(p^2 + 4)^(1/4))/((
p*x^4 - (x^4 + 1)*sqrt(p^2 + 4) + p)*sqrt(-((p^4 + 5*p^2 + 4)*x^4
 - p^4 - (p^5 + 5*p^3 + 4*p)*x^2 - 5*p^2 + 2*(sqrt(-x^4 + p*x^2 +
 1)*sqrt(p^2 + 4)*(p^2 + 1)*x - sqrt(-x^4 + p*x^2 + 1)*(p^3 + 3*p
)*x)*(p^2 + 4)^(1/4)*sqrt((p^2 - sqrt(p^2 + 4)*p + 4)/(p^2 - sqrt
(p^2 + 4)*p + 2)) - ((p^3 + 3*p)*x^4 - p^3 - (p^4 + 3*p^2)*x^2 -
3*p)*sqrt(p^2 + 4) + ((p^3 + 3*p)*sqrt(p^2 + 4)*x^2 - (p^4 + 5*p^
2 + 4)*x^2)*sqrt(p^2 + 4) - 4)/((p^4 + 5*p^2 + 4)*x^4 + p^4 + 5*p
^2 - ((p^3 + 3*p)*x^4 + p^3 + 3*p)*sqrt(p^2 + 4) + 4))*sqrt((p^2
- sqrt(p^2 + 4)*p + 4)/(p^2 - sqrt(p^2 + 4)*p + 2)) - (sqrt(-x^4
+ p*x^2 + 1)*p*x^2 - sqrt(-x^4 + p*x^2 + 1)*sqrt(p^2 + 4)*x^2)*sq
rt((p^2 - sqrt(p^2 + 4)*p + 4)/(p^2 - sqrt(p^2 + 4)*p + 2)) - (2*
x^3 - p*x + sqrt(p^2 + 4)*x)*(p^2 + 4)^(1/4))) + 8*(p^2 + 4)^(1/4
)*arctan(((sqrt(-x^4 + p*x^2 + 1)*p - sqrt(-x^4 + p*x^2 + 1)*sqrt
(p^2 + 4))*sqrt((p^2 - sqrt(p^2 + 4)*p + 4)/(p^2 - sqrt(p^2 + 4)*
p + 2)) - (p*x^3 - sqrt(p^2 + 4)*x^3 + 2*x)*(p^2 + 4)^(1/4))/((p*
x^4 - (x^4 + 1)*sqrt(p^2 + 4) + p)*sqrt(-((p^4 + 5*p^2 + 4)*x^4 -
 p^4 - (p^5 + 5*p^3 + 4*p)*x^2 - 5*p^2 - 2*(sqrt(-x^4 + p*x^2 + 1
)*sqrt(p^2 + 4)*(p^2 + 1)*x - sqrt(-x^4 + p*x^2 + 1)*(p^3 + 3*p)*
x)*(p^2 + 4)^(1/4)*sqrt((p^2 - sqrt(p^2 + 4)*p + 4)/(p^2 - sqrt(p
^2 + 4)*p + 2)) - ((p^3 + 3*p)*x^4 - p^3 - (p^4 + 3*p^2)*x^2 - 3*
p)*sqrt(p^2 + 4) + ((p^3 + 3*p)*sqrt(p^2 + 4)*x^2 - (p^4 + 5*p^2
+ 4)*x^2)*sqrt(p^2 + 4) - 4)/((p^4 + 5*p^2 + 4)*x^4 + p^4 + 5*p^2
 - ((p^3 + 3*p)*x^4 + p^3 + 3*p)*sqrt(p^2 + 4) + 4))*sqrt((p^2 -
sqrt(p^2 + 4)*p + 4)/(p^2 - sqrt(p^2 + 4)*p + 2)) - (sqrt(-x^4 +
p*x^2 + 1)*p*x^2 - sqrt(-x^4 + p*x^2 + 1)*sqrt(p^2 + 4)*x^2)*sqrt
((p^2 - sqrt(p^2 + 4)*p + 4)/(p^2 - sqrt(p^2 + 4)*p + 2)) + (2*x^
3 - p*x + sqrt(p^2 + 4)*x)*(p^2 + 4)^(1/4))))/((p - sqrt(p^2 + 4)
)*sqrt((p^2 - sqrt(p^2 + 4)*p + 4)/(p^2 - sqrt(p^2 + 4)*p + 2)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{px^2 - x^4 + 1}}{x^4 + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**4+p*x**2+1)**(1/2)/(x**4+1),x)
```

```
[Out] Integral(sqrt(p*x**2 - x**4 + 1)/(x**4 + 1), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^4 + px^2 + 1}}{x^4 + 1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^4 + p*x^2 + 1)/(x^4 + 1),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^4 + p*x^2 + 1)/(x^4 + 1), x)
```

3.67 $\int \frac{a+bx}{(2-x^2)\sqrt[4]{-1+x^2}}\,dx$

Optimal. Leaf size=80

$$\frac{a\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{2\sqrt{2}}+\frac{a\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{2\sqrt{2}}-b\tan^{-1}\left(\sqrt[4]{x^2-1}\right)+b\tanh^{-1}\left(\sqrt[4]{x^2-1}\right)$$

```
[Out] (a*ArcTan[x/(Sqrt[2]*(-1 + x^2)^(1/4))])/(2*Sqrt[2]) - b*ArcTan[(
-1 + x^2)^(1/4)] + (a*ArcTanh[x/(Sqrt[2]*(-1 + x^2)^(1/4))])/(2*S
qrt[2]) + b*ArcTanh[(-1 + x^2)^(1/4)]
```

Rubi [A] time = 0.118325, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292

$$\frac{a\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{2\sqrt{2}}+\frac{a\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{2\sqrt{2}}-b\tan^{-1}\left(\sqrt[4]{x^2-1}\right)+b\tanh^{-1}\left(\sqrt[4]{x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x)/((2 - x^2)*(-1 + x^2)^(1/4)),x]
```

```
[Out] (a*ArcTan[x/(Sqrt[2]*(-1 + x^2)^(1/4))])/(2*Sqrt[2]) - b*ArcTan[(
-1 + x^2)^(1/4)] + (a*ArcTanh[x/(Sqrt[2]*(-1 + x^2)^(1/4))])/(2*S
qrt[2]) + b*ArcTanh[(-1 + x^2)^(1/4)]
```

Rubi in Sympy [A] time = 28.6337, size = 178, normalized size = 2.22

$$-\frac{\sqrt{2}ax(1-i)\left(i;\operatorname{asin}\left(\frac{\sqrt{2}(1+i)\sqrt[4]{x^2-1}}{2}\right)\middle|-1\right)}{2\sqrt{-i\sqrt{x^2-1}+1}\sqrt{i\sqrt{x^2-1}+1}}+\frac{a\sqrt{\frac{x^2}{\left(\sqrt{x^2-1}+1\right)^2}}\left(\sqrt{x^2-1}+1\right)F\left(2\operatorname{atan}\left(\sqrt[4]{x^2-1}\right)\middle|\frac{1}{2}\right)}{4x}$$
$$+\frac{\sqrt{2}a\sqrt{x^2}\operatorname{atanh}\left(\frac{\sqrt{2}\sqrt[4]{x^2-1}}{\sqrt{x^2}}\right)}{4x}-b\operatorname{atan}\left(\sqrt[4]{x^2-1}\right)+b\operatorname{atanh}\left(\sqrt[4]{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x+a)/(-x**2+2)/(x**2-1)**(1/4),x)
```

```
[Out] -sqrt(2)*a*x*(1 - I)*elliptic_pi(I, asin(sqrt(2)*(1 + I)*(x**2 -
1)**(1/4)/2), -1)/(2*sqrt(-I*sqrt(x**2 - 1) + 1)*sqrt(I*sqrt(x**2
 - 1) + 1)) + a*sqrt(x**2/(sqrt(x**2 - 1) + 1)**2)*(sqrt(x**2 - 1
) + 1)*elliptic_f(2*atan((x**2 - 1)**(1/4)), 1/2)/(4*x) + sqrt(2)
*a*sqrt(x**2)*atanh(sqrt(2)*(x**2 - 1)**(1/4)/sqrt(x**2))/(4*x) -
 b*atan((x**2 - 1)**(1/4)) + b*atanh((x**2 - 1)**(1/4))
```

Mathematica [C] time = 0.560316, size = 203, normalized size = 2.54

$$\frac{2x\left(-\frac{3aF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};x^2,\frac{x^2}{2}\right)}{x^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};x^2,\frac{x^2}{2}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};x^2,\frac{x^2}{2}\right)\right)+6F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};x^2,\frac{x^2}{2}\right)}-\frac{2bxF_1\left(1;\frac{1}{4},1;2;x^2,\frac{x^2}{2}\right)}{x^2\left(2F_1\left(2;\frac{1}{4},2;3;x^2,\frac{x^2}{2}\right)+F_1\left(2;\frac{5}{4},1;3;x^2,\frac{x^2}{2}\right)\right)+8F_1\left(1;\frac{1}{4},1;2;x^2,\frac{x^2}{2}\right)}\right)}{(x^2-2)\sqrt[4]{x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x)/((2 - x^2)*(-1 + x^2)^(1/4)),x]
```

```
[Out] (2*x*((-3*a*AppellF1[1/2, 1/4, 1, 3/2, x^2, x^2/2])/(6*AppellF1[1
/2, 1/4, 1, 3/2, x^2, x^2/2] + x^2*(2*AppellF1[3/2, 1/4, 2, 5/2,
x^2, x^2/2] + AppellF1[3/2, 5/4, 1, 5/2, x^2, x^2/2])) - (2*b*x*A
ppellF1[1, 1/4, 1, 2, x^2, x^2/2])/(8*AppellF1[1, 1/4, 1, 2, x^2,
 x^2/2] + x^2*(2*AppellF1[2, 1/4, 2, 3, x^2, x^2/2] + AppellF1[2,
 5/4, 1, 3, x^2, x^2/2]))))/((-2 + x^2)*(-1 + x^2)^(1/4))
```

Maple [F] time = 0.088, size = 0, normalized size = 0.

$$\int \frac{bx + a}{-x^2 + 2} \frac{1}{\sqrt[4]{x^2 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x+a)/(-x^2+2)/(x^2-1)^(1/4),x)
```

```
[Out] int((b*x+a)/(-x^2+2)/(x^2-1)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{bx + a}{(x^2 - 1)^{\frac{1}{4}}(x^2 - 2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(b*x + a)/((x^2 - 1)^(1/4)*(x^2 - 2)),x, algorithm="maxima")
```

```
[Out] -integrate((b*x + a)/((x^2 - 1)^(1/4)*(x^2 - 2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(b*x + a)/((x^2 - 1)^(1/4)*(x^2 - 2)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{a}{x^2\sqrt[4]{x^2 - 1} - 2\sqrt[4]{x^2 - 1}} dx - \int \frac{bx}{x^2\sqrt[4]{x^2 - 1} - 2\sqrt[4]{x^2 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x+a)/(-x**2+2)/(x**2-1)**(1/4),x)
```

```
[Out] -Integral(a/(x**2*(x**2 - 1)**(1/4) - 2*(x**2 - 1)**(1/4)), x) -
Integral(b*x/(x**2*(x**2 - 1)**(1/4) - 2*(x**2 - 1)**(1/4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{bx + a}{\left(x^2 - 1\right)^{\frac{1}{4}}\left(x^2 - 2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(b*x + a)/((x^2 - 1)^(1/4)*(x^2 - 2)),x, algorithm="giac")
```

```
[Out] integrate(-(b*x + a)/((x^2 - 1)^(1/4)*(x^2 - 2)), x)
```

3.68 $\int \frac{a+bx}{\sqrt[4]{-1-x^2}(2+x^2)}\,dx$

Optimal. Leaf size=88

$$\frac{a\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-x^2-1}}\right)}{2\sqrt{2}}+\frac{a\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-x^2-1}}\right)}{2\sqrt{2}}+b\tan^{-1}\left(\sqrt[4]{-x^2-1}\right)-b\tanh^{-1}\left(\sqrt[4]{-x^2-1}\right)$$

```
[Out] (a*ArcTan[x/(Sqrt[2]*(-1 - x^2)^(1/4))])/(2*Sqrt[2]) + b*ArcTan[(
-1 - x^2)^(1/4)] + (a*ArcTanh[x/(Sqrt[2]*(-1 - x^2)^(1/4))])/(2*S
qrt[2]) - b*ArcTanh[(-1 - x^2)^(1/4)]
```

Rubi [A] time = 0.130639, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292

$$\frac{a\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-x^2-1}}\right)}{2\sqrt{2}}+\frac{a\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-x^2-1}}\right)}{2\sqrt{2}}+b\tan^{-1}\left(\sqrt[4]{-x^2-1}\right)-b\tanh^{-1}\left(\sqrt[4]{-x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x)/((-1 - x^2)^(1/4)*(2 + x^2)),x]
```

```
[Out] (a*ArcTan[x/(Sqrt[2]*(-1 - x^2)^(1/4))])/(2*Sqrt[2]) + b*ArcTan[(
-1 - x^2)^(1/4)] + (a*ArcTanh[x/(Sqrt[2]*(-1 - x^2)^(1/4))])/(2*S
qrt[2]) - b*ArcTanh[(-1 - x^2)^(1/4)]
```

Rubi in Sympy [A] time = 29.508, size = 199, normalized size = 2.26

$$-\frac{\sqrt{2}ax(1-i)\left(i;\operatorname{asin}\left(\frac{\sqrt{2}(1+i)\sqrt[4]{-x^2-1}}{2}\right)\middle|-1\right)}{2\sqrt{-i\sqrt{-x^2-1}+1}\sqrt{i\sqrt{-x^2-1}+1}}-\frac{\sqrt{2}a\sqrt{-x^2}\operatorname{atanh}\left(\frac{\sqrt{2}\sqrt[4]{-x^2-1}}{\sqrt{-x^2}}\right)}{4x}$$
$$-\frac{a\sqrt{-\frac{x^2}{\left(\sqrt{-x^2-1}+1\right)^2}}\left(\sqrt{-x^2-1}+1\right)F\left(2\operatorname{atan}\left(\sqrt[4]{-x^2-1}\right)\middle|\frac{1}{2}\right)}{4x}$$
$$+b\operatorname{atan}\left(\sqrt[4]{-x^2-1}\right)-b\operatorname{atanh}\left(\sqrt[4]{-x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x+a)/(-x**2-1)**(1/4)/(x**2+2),x)
```

```
[Out] -sqrt(2)*a*x*(1 - I)*elliptic_pi(I, asin(sqrt(2)*(1 + I)*(-x**2 -
 1)**(1/4)/2), -1)/(2*sqrt(-I*sqrt(-x**2 - 1) + 1)*sqrt(I*sqrt(-x
**2 - 1) + 1)) - sqrt(2)*a*sqrt(-x**2)*atanh(sqrt(2)*(-x**2 - 1)*
*(1/4)/sqrt(-x**2))/(4*x) - a*sqrt(-x**2/(sqrt(-x**2 - 1) + 1)**2
)*(sqrt(-x**2 - 1) + 1)*elliptic_f(2*atan((-x**2 - 1)**(1/4)), 1/
2)/(4*x) + b*atan((-x**2 - 1)**(1/4)) - b*atanh((-x**2 - 1)**(1/4
))
```

Mathematica [C] time = 0.554591, size = 221, normalized size = 2.51

$$\frac{2x\left(-\frac{3aF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-x^2,-\frac{x^2}{2}\right)}{x^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-x^2,-\frac{x^2}{2}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-x^2,-\frac{x^2}{2}\right)\right)-6F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-x^2,-\frac{x^2}{2}\right)}-\frac{2bxF_1\left(1;\frac{1}{4},1;2;-x^2,-\frac{x^2}{2}\right)}{x^2\left(2F_1\left(2;\frac{1}{4},2;3;-x^2,-\frac{x^2}{2}\right)+F_1\left(2;\frac{5}{4},1;3;-x^2,-\frac{x^2}{2}\right)\right)-8F_1\left(1;\frac{1}{4},1;2;-x^2\right.}\right.}{\sqrt[4]{-x^2-1}\left(x^2+2\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x)/((-1 - x^2)^(1/4)*(2 + x^2)),x]
```

```
[Out] (2*x*((-3*a*AppellF1[1/2, 1/4, 1, 3/2, -x^2, -x^2/2])/(-6*AppellF
1[1/2, 1/4, 1, 3/2, -x^2, -x^2/2] + x^2*(2*AppellF1[3/2, 1/4, 2,
5/2, -x^2, -x^2/2] + AppellF1[3/2, 5/4, 1, 5/2, -x^2, -x^2/2])) -
 (2*b*x*AppellF1[1, 1/4, 1, 2, -x^2, -x^2/2])/(-8*AppellF1[1, 1/4
, 1, 2, -x^2, -x^2/2] + x^2*(2*AppellF1[2, 1/4, 2, 3, -x^2, -x^2/
2] + AppellF1[2, 5/4, 1, 3, -x^2, -x^2/2]))))/((-1 - x^2)^(1/4)*(
2 + x^2))
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int \frac{bx+a}{x^2+2}\frac{1}{\sqrt[4]{-x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x+a)/(-x^2-1)^(1/4)/(x^2+2),x)
```

```
[Out] int((b*x+a)/(-x^2-1)^(1/4)/(x^2+2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx+a}{(x^2+2)(-x^2-1)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x + a)/((x^2 + 2)*(-x^2 - 1)^(1/4)),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)/((x^2 + 2)*(-x^2 - 1)^(1/4)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x + a)/((x^2 + 2)*(-x^2 - 1)^(1/4)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+bx}{\sqrt[4]{-x^2-1}\,(x^2+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x+a)/(-x**2-1)**(1/4)/(x**2+2),x)
```

```
[Out] Integral((a + b*x)/((-x**2 - 1)**(1/4)*(x**2 + 2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx + a}{(x^2 + 2)(-x^2 - 1)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x + a)/((x^2 + 2)*(-x^2 - 1)^(1/4)),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)/((x^2 + 2)*(-x^2 - 1)^(1/4)), x)
```

3.69 $\int \frac{a+bx}{\sqrt[4]{1-x^2}(2-x^2)}\,dx$

Optimal. Leaf size=149

$$\frac{1}{2}a\tan^{-1}\left(\frac{1-\sqrt{1-x^2}}{x\sqrt[4]{1-x^2}}\right)+\frac{1}{2}a\tanh^{-1}\left(\frac{\sqrt{1-x^2}+1}{x\sqrt[4]{1-x^2}}\right)+\frac{b\tan^{-1}\left(\frac{1-\sqrt{1-x^2}}{\sqrt{2}\sqrt[4]{1-x^2}}\right)}{\sqrt{2}}+\frac{b\tanh^{-1}\left(\frac{\sqrt{1-x^2}+1}{\sqrt{2}\sqrt[4]{1-x^2}}\right)}{\sqrt{2}}$$

```
[Out] (b*ArcTan[(1 - Sqrt[1 - x^2])/(Sqrt[2]*(1 - x^2)^(1/4))])/Sqrt[2]
 + (a*ArcTan[(1 - Sqrt[1 - x^2])/(x*(1 - x^2)^(1/4))])/2 + (b*Arc
Tanh[(1 + Sqrt[1 - x^2])/(Sqrt[2]*(1 - x^2)^(1/4))])/Sqrt[2] + (a
*ArcTanh[(1 + Sqrt[1 - x^2])/(x*(1 - x^2)^(1/4))])/2
```

Rubi [A] time = 0.136095, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{1}{2}a\tan^{-1}\left(\frac{1-\sqrt{1-x^2}}{x\sqrt[4]{1-x^2}}\right)+\frac{1}{2}a\tanh^{-1}\left(\frac{\sqrt{1-x^2}+1}{x\sqrt[4]{1-x^2}}\right)+\frac{b\tan^{-1}\left(\frac{1-\sqrt{1-x^2}}{\sqrt{2}\sqrt[4]{1-x^2}}\right)}{\sqrt{2}}+\frac{b\tanh^{-1}\left(\frac{\sqrt{1-x^2}+1}{\sqrt{2}\sqrt[4]{1-x^2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x)/((1 - x^2)^(1/4)*(2 - x^2)),x]
```

```
[Out] (b*ArcTan[(1 - Sqrt[1 - x^2])/(Sqrt[2]*(1 - x^2)^(1/4))])/Sqrt[2]
 + (a*ArcTan[(1 - Sqrt[1 - x^2])/(x*(1 - x^2)^(1/4))])/2 + (b*Arc
Tanh[(1 + Sqrt[1 - x^2])/(Sqrt[2]*(1 - x^2)^(1/4))])/Sqrt[2] + (a
*ArcTanh[(1 + Sqrt[1 - x^2])/(x*(1 - x^2)^(1/4))])/2
```

Rubi in Sympy [A] time = 39.3188, size = 170, normalized size = 1.14

$$-\frac{ia\sqrt{x^2}\left(-i;\operatorname{asin}\left(\sqrt[4]{-x^2+1}\right)\middle|-1\right)}{x}+\frac{ia\sqrt{x^2}\left(i;\operatorname{asin}\left(\sqrt[4]{-x^2+1}\right)\middle|-1\right)}{x}-\frac{\sqrt{2}b\log\left(-\sqrt{2}\sqrt[4]{-x^2+1}+\sqrt{-x^2+1}+1\right)}{4}+\frac{\sqrt{2}b\log\left(\sqrt{2}\sqrt[4]{-x^2+1}+\sqrt{-x^2+1}+1\right)}{4}-\frac{\sqrt{2}b\operatorname{atan}\left(\sqrt{2}\sqrt[4]{-x^2+1}-1\right)}{2}-\frac{\sqrt{2}b\operatorname{atan}\left(\sqrt{2}\sqrt[4]{-x^2+1}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x+a)/(-x**2+1)**(1/4)/(-x**2+2),x)
```

```
[Out] -I*a*sqrt(x**2)*elliptic_pi(-I, asin((-x**2 + 1)**(1/4)), -1)/x +
 I*a*sqrt(x**2)*elliptic_pi(I, asin((-x**2 + 1)**(1/4)), -1)/x -
sqrt(2)*b*log(-sqrt(2)*(-x**2 + 1)**(1/4) + sqrt(-x**2 + 1) + 1)/
4 + sqrt(2)*b*log(sqrt(2)*(-x**2 + 1)**(1/4) + sqrt(-x**2 + 1) +
1)/4 - sqrt(2)*b*atan(sqrt(2)*(-x**2 + 1)**(1/4) - 1)/2 - sqrt(2)
*b*atan(sqrt(2)*(-x**2 + 1)**(1/4) + 1)/2
```

Mathematica [C] time = 0.445522, size = 205, normalized size = 1.38

$$\frac{2x\left(-\frac{3aF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};x^2,\frac{x^2}{2}\right)}{x^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};x^2,\frac{x^2}{2}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};x^2,\frac{x^2}{2}\right)\right)+6F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};x^2,\frac{x^2}{2}\right)}-\frac{2bxF_1\left(1;\frac{1}{4},1;2;x^2,\frac{x^2}{2}\right)}{x^2\left(2F_1\left(2;\frac{1}{4},2;3;x^2,\frac{x^2}{2}\right)+F_1\left(2;\frac{5}{4},1;3;x^2,\frac{x^2}{2}\right)\right)+8F_1\left(1;\frac{1}{4},1;2;x^2,\frac{x^2}{2}\right)}\right)}{\sqrt[4]{1-x^2}\,(x^2-2)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x)/((1 - x^2)^(1/4)*(2 - x^2)),x]
```

```
[Out] (2*x*((-3*a*AppellF1[1/2, 1/4, 1, 3/2, x^2, x^2/2])/(6*AppellF1[1
/2, 1/4, 1, 3/2, x^2, x^2/2] + x^2*(2*AppellF1[3/2, 1/4, 2, 5/2,
x^2, x^2/2] + AppellF1[3/2, 5/4, 1, 5/2, x^2, x^2/2])) - (2*b*x*A
ppellF1[1, 1/4, 1, 2, x^2, x^2/2])/(8*AppellF1[1, 1/4, 1, 2, x^2,
 x^2/2] + x^2*(2*AppellF1[2, 1/4, 2, 3, x^2, x^2/2] + AppellF1[2,
 5/4, 1, 3, x^2, x^2/2]))))/((1 - x^2)^(1/4)*(-2 + x^2))
```

Maple [F] time = 0.07, size = 0, normalized size = 0.

$$\int \frac{bx+a}{-x^2+2}\frac{1}{\sqrt[4]{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x+a)/(-x^2+1)^(1/4)/(-x^2+2),x)
```

```
[Out] int((b*x+a)/(-x^2+1)^(1/4)/(-x^2+2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{bx+a}{(x^2-2)(-x^2+1)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(b*x + a)/((x^2 - 2)*(-x^2 + 1)^(1/4)),x, algorithm="maxima")
```

```
[Out] -integrate((b*x + a)/((x^2 - 2)*(-x^2 + 1)^(1/4)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(b*x + a)/((x^2 - 2)*(-x^2 + 1)^(1/4)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{a}{x^2\sqrt[4]{-x^2+1}-2\sqrt[4]{-x^2+1}}\,dx-\int \frac{bx}{x^2\sqrt[4]{-x^2+1}-2\sqrt[4]{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x+a)/(-x**2+1)**(1/4)/(-x**2+2),x)
```

```
[Out] -Integral(a/(x**2*(-x**2 + 1)**(1/4) - 2*(-x**2 + 1)**(1/4)), x)
- Integral(b*x/(x**2*(-x**2 + 1)**(1/4) - 2*(-x**2 + 1)**(1/4)),
```

```
x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{bx+a}{(x^2-2)(-x^2+1)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(b*x + a)/((x^2 - 2)*(-x^2 + 1)^(1/4)),x, algorithm="giac")
```

```
[Out] integrate(-(b*x + a)/((x^2 - 2)*(-x^2 + 1)^(1/4)), x)
```

3.70 $\int \frac{a+bx}{\sqrt[4]{1+x^2}(2+x^2)}\,dx$

Optimal. Leaf size=135

$$-\frac{1}{2}a\tan^{-1}\left(\frac{\sqrt{x^2+1}+1}{x\sqrt[4]{x^2+1}}\right)-\frac{1}{2}a\tanh^{-1}\left(\frac{1-\sqrt{x^2+1}}{x\sqrt[4]{x^2+1}}\right)-\frac{b\tan^{-1}\left(\frac{1-\sqrt{x^2+1}}{\sqrt{2}\sqrt[4]{x^2+1}}\right)}{\sqrt{2}}-\frac{b\tanh^{-1}\left(\frac{\sqrt{x^2+1}+1}{\sqrt{2}\sqrt[4]{x^2+1}}\right)}{\sqrt{2}}$$

```
[Out] -((b*ArcTan[(1 - Sqrt[1 + x^2])/(Sqrt[2]*(1 + x^2)^(1/4))])/Sqrt[
2]) - (a*ArcTan[(1 + Sqrt[1 + x^2])/(x*(1 + x^2)^(1/4))])/2 - (a*
ArcTanh[(1 - Sqrt[1 + x^2])/(x*(1 + x^2)^(1/4))])/2 - (b*ArcTanh[
(1 + Sqrt[1 + x^2])/(Sqrt[2]*(1 + x^2)^(1/4))])/Sqrt[2]
```

Rubi [A] time = 0.108503, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$-\frac{1}{2}a\tan^{-1}\left(\frac{\sqrt{x^2+1}+1}{x\sqrt[4]{x^2+1}}\right)-\frac{1}{2}a\tanh^{-1}\left(\frac{1-\sqrt{x^2+1}}{x\sqrt[4]{x^2+1}}\right)-\frac{b\tan^{-1}\left(\frac{1-\sqrt{x^2+1}}{\sqrt{2}\sqrt[4]{x^2+1}}\right)}{\sqrt{2}}-\frac{b\tanh^{-1}\left(\frac{\sqrt{x^2+1}+1}{\sqrt{2}\sqrt[4]{x^2+1}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x)/((1 + x^2)^(1/4)*(2 + x^2)),x]
```

```
[Out] -((b*ArcTan[(1 - Sqrt[1 + x^2])/(Sqrt[2]*(1 + x^2)^(1/4))])/Sqrt[
2]) - (a*ArcTan[(1 + Sqrt[1 + x^2])/(x*(1 + x^2)^(1/4))])/2 - (a*
ArcTanh[(1 - Sqrt[1 + x^2])/(x*(1 + x^2)^(1/4))])/2 - (b*ArcTanh[
(1 + Sqrt[1 + x^2])/(Sqrt[2]*(1 + x^2)^(1/4))])/Sqrt[2]
```

Rubi in Sympy [A] time = 36.0933, size = 173, normalized size = 1.28

$$\frac{ia\sqrt{-x^2}\left(-i;\operatorname{asin}\left(\sqrt[4]{x^2+1}\right)\middle|-1\right)}{x}-\frac{ia\sqrt{-x^2}\left(i;\operatorname{asin}\left(\sqrt[4]{x^2+1}\right)\middle|-1\right)}{x}+\frac{\sqrt{2}b\log\left(-\sqrt{2}\sqrt[4]{x^2+1}+\sqrt{x^2+1}+1\right)}{4}-\frac{\sqrt{2}b\log\left(\sqrt{2}\sqrt[4]{x^2+1}+\sqrt{x^2+1}+1\right)}{4}+\frac{\sqrt{2}b\operatorname{atan}\left(\sqrt{2}\sqrt[4]{x^2+1}-1\right)}{2}+\frac{\sqrt{2}b\operatorname{atan}\left(\sqrt{2}\sqrt[4]{x^2+1}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x+a)/(x**2+1)**(1/4)/(x**2+2),x)
```

```
[Out] I*a*sqrt(-x**2)*elliptic_pi(-I, asin((x**2 + 1)**(1/4)), -1)/x -
I*a*sqrt(-x**2)*elliptic_pi(I, asin((x**2 + 1)**(1/4)), -1)/x + s
qrt(2)*b*log(-sqrt(2)*(x**2 + 1)**(1/4) + sqrt(x**2 + 1) + 1)/4 -
 sqrt(2)*b*log(sqrt(2)*(x**2 + 1)**(1/4) + sqrt(x**2 + 1) + 1)/4
+ sqrt(2)*b*atan(sqrt(2)*(x**2 + 1)**(1/4) - 1)/2 + sqrt(2)*b*ata
n(sqrt(2)*(x**2 + 1)**(1/4) + 1)/2
```

Mathematica [C] time = 0.425114, size = 219, normalized size = 1.62

$$\frac{2x\left(-\frac{3aF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-x^2,-\frac{x^2}{2}\right)}{x^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-x^2,-\frac{x^2}{2}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-x^2,-\frac{x^2}{2}\right)\right)-6F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-x^2,-\frac{x^2}{2}\right)}-\frac{2bxF_1\left(1;\frac{1}{4},1;2;-x^2,-\frac{x^2}{2}\right)}{x^2\left(2F_1\left(2;\frac{1}{4},2;3;-x^2,-\frac{x^2}{2}\right)+F_1\left(2;\frac{5}{4},1;3;-x^2,-\frac{x^2}{2}\right)\right)-8F_1\left(1;\frac{1}{4},1;2;-x^2\right.}\right.}{\sqrt[4]{x^2+1}\,(x^2+2)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x)/((1 + x^2)^(1/4)*(2 + x^2)),x]
```

```
[Out] (2*x*((-3*a*AppellF1[1/2, 1/4, 1, 3/2, -x^2, -x^2/2])/(-6*AppellF
1[1/2, 1/4, 1, 3/2, -x^2, -x^2/2] + x^2*(2*AppellF1[3/2, 1/4, 2,
5/2, -x^2, -x^2/2] + AppellF1[3/2, 5/4, 1, 5/2, -x^2, -x^2/2])) -
 (2*b*x*AppellF1[1, 1/4, 1, 2, -x^2, -x^2/2])/(-8*AppellF1[1, 1/4
, 1, 2, -x^2, -x^2/2] + x^2*(2*AppellF1[2, 1/4, 2, 3, -x^2, -x^2/
2] + AppellF1[2, 5/4, 1, 3, -x^2, -x^2/2]))))/((1 + x^2)^(1/4)*(2
 + x^2))
```

Maple [F] time = 0.045, size = 0, normalized size = 0.

$$\int \frac{bx + a}{x^2 + 2} \frac{1}{\sqrt[4]{x^2 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x+a)/(x^2+1)^(1/4)/(x^2+2),x)
```

```
[Out] int((b*x+a)/(x^2+1)^(1/4)/(x^2+2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx + a}{(x^2 + 2)(x^2 + 1)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x + a)/((x^2 + 2)*(x^2 + 1)^(1/4)),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)/((x^2 + 2)*(x^2 + 1)^(1/4)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x + a)/((x^2 + 2)*(x^2 + 1)^(1/4)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + bx}{\sqrt[4]{x^2 + 1}\,(x^2 + 2)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x+a)/(x**2+1)**(1/4)/(x**2+2),x)
```

```
[Out] Integral((a + b*x)/((x**2 + 1)**(1/4)*(x**2 + 2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx + a}{(x^2 + 2)(x^2 + 1)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x + a)/((x^2 + 2)*(x^2 + 1)^(1/4)),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)/((x^2 + 2)*(x^2 + 1)^(1/4)), x)
```

3.71 $\int \frac{x}{\sqrt{1-x^3}(4-x^3)}\,dx$

Optimal. Leaf size=127

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt{1-x^3}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{\sqrt[3]{2}x+1}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}}+\frac{\tanh^{-1}\left(\sqrt{1-x^3}\right)}{9\ 2^{2/3}}$$

```
[Out] -ArcTan[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[1 - x^3]]/(3*2^(2/3)*Sqrt[
3]) + ArcTan[Sqrt[1 - x^3]/Sqrt[3]]/(3*2^(2/3)*Sqrt[3]) - ArcTanh
[(1 + 2^(1/3)*x)/Sqrt[1 - x^3]]/(3*2^(2/3)) + ArcTanh[Sqrt[1 - x^
3]]/(9*2^(2/3))
```

Rubi [A] time = 0.0675644, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt{1-x^3}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{\sqrt[3]{2}x+1}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}}+\frac{\tanh^{-1}\left(\sqrt{1-x^3}\right)}{9\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(Sqrt[1 - x^3]*(4 - x^3)),x]
```

```
[Out] -ArcTan[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[1 - x^3]]/(3*2^(2/3)*Sqrt[
3]) + ArcTan[Sqrt[1 - x^3]/Sqrt[3]]/(3*2^(2/3)*Sqrt[3]) - ArcTanh
[(1 + 2^(1/3)*x)/Sqrt[1 - x^3]]/(3*2^(2/3)) + ArcTanh[Sqrt[1 - x^
3]]/(9*2^(2/3))
```

Rubi in Sympy [A] time = 3.05896, size = 155, normalized size = 1.22

$$\frac{\sqrt[3]{2}\log\left(\sqrt[3]{2}x-\sqrt{-x^3+1}+1\right)}{12}-\frac{\sqrt[3]{2}\log\left(\sqrt[3]{2}x+\sqrt{-x^3+1}+1\right)}{12}-\frac{\sqrt[3]{2}\sqrt{3}\,\operatorname{atan}\left(\frac{\sqrt{3}}{3}-\frac{2^{\frac{2}{3}}\sqrt{3}\left(-\sqrt{-x^3+1}+1\right)}{3x}\right)}{18}+\frac{\sqrt[3]{2}\sqrt{3}\,\operatorname{atan}\left(\frac{\sqrt{3}}{3}-\frac{2^{\frac{2}{3}}\sqrt{3}\left(\sqrt{-x^3+1}+1\right)}{3x}\right)}{18}+\frac{\sqrt[3]{2}\,\operatorname{atanh}\left(\sqrt{-x^3+1}\right)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(-x**3+4)/(-x**3+1)**(1/2),x)
```

```
[Out] 2**(1/3)*log(2**(1/3)*x - sqrt(-x**3 + 1) + 1)/12 - 2**(1/3)*log(
2**(1/3)*x + sqrt(-x**3 + 1) + 1)/12 - 2**(1/3)*sqrt(3)*atan(sqrt
(3)/3 - 2**(2/3)*sqrt(3)*(-sqrt(-x**3 + 1) + 1)/(3*x))/18 + 2**(1
/3)*sqrt(3)*atan(sqrt(3)/3 - 2**(2/3)*sqrt(3)*(sqrt(-x**3 + 1) +
1)/(3*x))/18 + 2**(1/3)*atanh(sqrt(-x**3 + 1))/18
```

Mathematica [C] time = 0.160891, size = 120, normalized size = 0.94

$$-\frac{10x^2F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};x^3,\frac{x^3}{4}\right)}{\sqrt{1-x^3}\left(x^3-4\right)\left(3x^3\left(F_1\left(\frac{5}{3};\frac{1}{2},2;\frac{8}{3};x^3,\frac{x^3}{4}\right)+2F_1\left(\frac{5}{3};\frac{3}{2},1;\frac{8}{3};x^3,\frac{x^3}{4}\right)\right)+20F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};x^3,\frac{x^3}{4}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x/(Sqrt[1 - x^3]*(4 - x^3)),x]
```

```
[Out] (-10*x^2*AppellF1[2/3, 1/2, 1, 5/3, x^3, x^3/4])/(Sqrt[1 - x^3]*(
-4 + x^3)*(20*AppellF1[2/3, 1/2, 1, 5/3, x^3, x^3/4] + 3*x^3*(App
ellF1[5/3, 1/2, 2, 8/3, x^3, x^3/4] + 2*AppellF1[5/3, 3/2, 1, 8/3
, x^3, x^3/4])))
```

Maple [C] time = 0.289, size = 164, normalized size = 1.3

$$\frac{i}{36}\sqrt{2}\sum_{\_alpha=RootOf(\_Z^3-4)}\_alpha^2\left(-2\,\_alpha^2+\_alpha+1+i\sqrt{3}\,(1-\_alpha)\right)\sqrt{\frac{i}{2}\left(2\,x+1-i\sqrt{3}\right)}\sqrt{\frac{-1+x}{i\sqrt{3}-3}}\sqrt{-\frac{i}{2}\left(2\,x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(-x^3+4)/(-x^3+1)^(1/2),x)
```

```
[Out] 1/36*I*2^(1/2)*sum(_alpha^2*(1/2*I*(2*x+1-I*3^(1/2)))^(1/2)*((-1+
x)/(I*3^(1/2)-3))^(1/2)*(-1/2*I*(2*x+1+I*3^(1/2)))^(1/2)/(-x^3+1)
^(1/2)*(-2*_alpha^2+_alpha+1+I*3^(1/2)*(1-_alpha))*EllipticPi(1/3
*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),1/2*_alpha-1/3*I
*_alpha^2*3^(1/2)-1/2+1/6*I*_alpha*3^(1/2)+1/6*I*3^(1/2),(I*3^(1/
2)/(-3/2+1/2*I*3^(1/2)))^(1/2)),_alpha=RootOf(_Z^3-4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{(x^3-4)\sqrt{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x/((x^3 - 4)*sqrt(-x^3 + 1)),x, algorithm="maxima")
```

```
[Out] -integrate(x/((x^3 - 4)*sqrt(-x^3 + 1)), x)
```

Fricas [A] time = 0.358083, size = 1451, normalized size = 11.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x/((x^3 - 4)*sqrt(-x^3 + 1)),x, algorithm="fricas")
```

```
[Out] 1/15552*432^(5/6)*(sqrt(3)*log(2592*(1296*x^7 - 1296*x^4 + 6*2^(2
/3)*(x^9 - 228*x^6 + 264*x^3 - 64) + (72*x^7 - 1872*x^4 + 432^(5/
6)*sqrt(3)*(7*x^5 - 4*x^2) - 144*432^(1/6)*sqrt(3)*(x^6 - x^3) +
1152*x)*sqrt(-x^3 + 1) - 216*2^(1/3)*(x^8 - 5*x^5 + 4*x^2))/(x^9
- 12*x^6 + 48*x^3 - 64)) - sqrt(3)*log(2592*(1296*x^7 - 1296*x^4
+ 6*2^(2/3)*(x^9 - 228*x^6 + 264*x^3 - 64) - (72*x^7 - 1872*x^4 +
 432^(5/6)*sqrt(3)*(7*x^5 - 4*x^2) - 144*432^(1/6)*sqrt(3)*(x^6 -
 x^3) + 1152*x)*sqrt(-x^3 + 1) - 216*2^(1/3)*(x^8 - 5*x^5 + 4*x^2
))/(x^9 - 12*x^6 + 48*x^3 - 64)) + 8*arctan(-432*(18*x^5 + 2^(2/3
)*(x^7 + 16*x^4 - 8*x) + 2^(1/3)*(5*x^6 + 20*x^3 - 16))*sqrt(-x^3
 + 1)/(432^(5/6)*(x^9 + 66*x^6 - 72*x^3 + 32) - 72*sqrt(3)*2^(1/3
)*(x^9 - 12*x^6 + 48*x^3 - 64) + 864*sqrt(3)*(x^8 + 7*x^5 - 8*x^2
) + 1728*432^(1/6)*(x^7 + x^4 - 2*x))) + 4*arctan(-216*(6*x^8 + 4
2*x^5 - 48*x^2 - 4*432^(1/6)*sqrt(3)*(x^7 + x^4 - 2*x) - (18*x^5
+ 2^(2/3)*(x^7 + 16*x^4 - 8*x) - 2*2^(1/3)*(5*x^6 + 20*x^3 - 16))
*sqrt(-x^3 + 1))/(36*sqrt(2)*(x^9 - 12*x^6 + 48*x^3 - 64)*sqrt((1
```

```
296*x^7 - 1296*x^4 + 6*2^(2/3)*(x^9 - 228*x^6 + 264*x^3 - 64) + (
72*x^7 - 1872*x^4 + 432^(5/6)*sqrt(3)*(7*x^5 - 4*x^2) - 144*432^(
1/6)*sqrt(3)*(x^6 - x^3) + 1152*x)*sqrt(-x^3 + 1) - 216*2^(1/3)*(
x^8 - 5*x^5 + 4*x^2))/(x^9 - 12*x^6 + 48*x^3 - 64)) - 432^(5/6)*(
x^9 + 66*x^6 - 72*x^3 + 32) + 432*sqrt(3)*(x^8 + 7*x^5 - 8*x^2) +
 216*(18*sqrt(3)*x^5 - sqrt(3)*2^(2/3)*(x^7 + 16*x^4 - 8*x))*sqrt
(-x^3 + 1) + 864*432^(1/6)*(x^7 + x^4 - 2*x))) + 4*arctan(216*(6*
x^8 + 42*x^5 - 48*x^2 - 4*432^(1/6)*sqrt(3)*(x^7 + x^4 - 2*x) + (
18*x^5 + 2^(2/3)*(x^7 + 16*x^4 - 8*x) - 2*2^(1/3)*(5*x^6 + 20*x^3
 - 16))*sqrt(-x^3 + 1))/(36*sqrt(2)*(x^9 - 12*x^6 + 48*x^3 - 64)*
sqrt((1296*x^7 - 1296*x^4 + 6*2^(2/3)*(x^9 - 228*x^6 + 264*x^3 -
64) - (72*x^7 - 1872*x^4 + 432^(5/6)*sqrt(3)*(7*x^5 - 4*x^2) - 14
4*432^(1/6)*sqrt(3)*(x^6 - x^3) + 1152*x)*sqrt(-x^3 + 1) - 216*2^
(1/3)*(x^8 - 5*x^5 + 4*x^2))/(x^9 - 12*x^6 + 48*x^3 - 64)) - 432^
(5/6)*(x^9 + 66*x^6 - 72*x^3 + 32) + 432*sqrt(3)*(x^8 + 7*x^5 - 8
*x^2) - 216*(18*sqrt(3)*x^5 - sqrt(3)*2^(2/3)*(x^7 + 16*x^4 - 8*x
))*sqrt(-x^3 + 1) + 864*432^(1/6)*(x^7 + x^4 - 2*x))))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{x^3\sqrt{-x^3+1}-4\sqrt{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(-x**3+4)/(-x**3+1)**(1/2),x)

[Out] -Integral(x/(x**3*sqrt(-x**3 + 1) - 4*sqrt(-x**3 + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x}{(x^3-4)\sqrt{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x/((x^3 - 4)*sqrt(-x^3 + 1)),x, algorithm="giac")

[Out] integrate(-x/((x^3 - 4)*sqrt(-x^3 + 1)), x)
```

3.72 $\int \frac{x}{(4-dx^3)\sqrt{-1+dx^3}}\,dx$

Optimal. Leaf size=157

$$-\frac{\tan^{-1}\left(\frac{\sqrt[3]{2}\sqrt[3]{d}x+1}{\sqrt{dx^3-1}}\right)}{3\ 2^{2/3}d^{2/3}}-\frac{\tan^{-1}\left(\sqrt{dx^3-1}\right)}{9\ 2^{2/3}d^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{dx^3-1}}\right)}{3\ 2^{2/3}\sqrt{3}d^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{dx^3-1}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt{3}d^{2/3}}$$

```
[Out] -ArcTan[(1 + 2^(1/3)*d^(1/3)*x)/Sqrt[-1 + d*x^3]]/(3*2^(2/3)*d^(2
/3)) - ArcTan[Sqrt[-1 + d*x^3]]/(9*2^(2/3)*d^(2/3)) - ArcTanh[(Sq
rt[3]*(1 - 2^(1/3)*d^(1/3)*x))/Sqrt[-1 + d*x^3]]/(3*2^(2/3)*Sqrt[
3]*d^(2/3)) - ArcTanh[Sqrt[-1 + d*x^3]/Sqrt[3]]/(3*2^(2/3)*Sqrt[3
]*d^(2/3))
```

Rubi [A] time = 0.100935, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043

$$-\frac{\tan^{-1}\left(\frac{\sqrt[3]{2}\sqrt[3]{d}x+1}{\sqrt{dx^3-1}}\right)}{3\ 2^{2/3}d^{2/3}}-\frac{\tan^{-1}\left(\sqrt{dx^3-1}\right)}{9\ 2^{2/3}d^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{dx^3-1}}\right)}{3\ 2^{2/3}\sqrt{3}d^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{dx^3-1}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt{3}d^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/((4 - d*x^3)*Sqrt[-1 + d*x^3]),x]
```

```
[Out] -ArcTan[(1 + 2^(1/3)*d^(1/3)*x)/Sqrt[-1 + d*x^3]]/(3*2^(2/3)*d^(2
/3)) - ArcTan[Sqrt[-1 + d*x^3]]/(9*2^(2/3)*d^(2/3)) - ArcTanh[(Sq
rt[3]*(1 - 2^(1/3)*d^(1/3)*x))/Sqrt[-1 + d*x^3]]/(3*2^(2/3)*Sqrt[
3]*d^(2/3)) - ArcTanh[Sqrt[-1 + d*x^3]/Sqrt[3]]/(3*2^(2/3)*Sqrt[3
]*d^(2/3))
```

Rubi in Sympy [A] time = 4.53822, size = 223, normalized size = 1.42

$$\frac{\sqrt[3]{2}i\log\left(\sqrt[3]{2}\sqrt[3]{d}x-i\sqrt{dx^3-1}+1\right)}{12d^{\frac{2}{3}}}-\frac{\sqrt[3]{2}i\log\left(\sqrt[3]{2}\sqrt[3]{d}x+i\sqrt{dx^3-1}+1\right)}{12d^{\frac{2}{3}}}+\frac{\sqrt[3]{2}\sqrt{3}i\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}+\frac{2^{\frac{2}{3}}\sqrt{3}i\left(-\sqrt{dx^3-1}+i\right)}{3\sqrt[3]{d}x}\right)}{18d^{\frac{2}{3}}}-\frac{\sqrt[3]{2}\sqrt{3}i\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}+\frac{2^{\frac{2}{3}}\sqrt{3}i\left(\sqrt{dx^3-1}+i\right)}{3\sqrt[3]{d}x}\right)}{18d^{\frac{2}{3}}}-\frac{\sqrt[3]{2}\,\mathrm{atan}\left(\sqrt{dx^3-1}\right)}{18d^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(-d*x**3+4)/(d*x**3-1)**(1/2),x)
```

```
[Out] 2**(1/3)*I*log(2**(1/3)*d**(1/3)*x - I*sqrt(d*x**3 - 1) + 1)/(12*
d**(2/3)) - 2**(1/3)*I*log(2**(1/3)*d**(1/3)*x + I*sqrt(d*x**3 -
1) + 1)/(12*d**(2/3)) + 2**(1/3)*sqrt(3)*I*atan(sqrt(3)/3 + 2**(2
/3)*sqrt(3)*I*(-sqrt(d*x**3 - 1) + I)/(3*d**(1/3)*x))/(18*d**(2/3
)) - 2**(1/3)*sqrt(3)*I*atan(sqrt(3)/3 + 2**(2/3)*sqrt(3)*I*(sqrt
(d*x**3 - 1) + I)/(3*d**(1/3)*x))/(18*d**(2/3)) - 2**(1/3)*atan(s
qrt(d*x**3 - 1))/(18*d**(2/3))
```

Mathematica [C] time = 0.249286, size = 135, normalized size = 0.86

$$-\frac{10x^2F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};dx^3,\frac{dx^3}{4}\right)}{\left(dx^3-4\right)\sqrt{dx^3-1}\left(3dx^3\left(F_1\left(\frac{5}{3};\frac{1}{2},2;\frac{8}{3};dx^3,\frac{dx^3}{4}\right)+2F_1\left(\frac{5}{3};\frac{3}{2},1;\frac{8}{3};dx^3,\frac{dx^3}{4}\right)\right)+20F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};dx^3,\frac{dx^3}{4}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x/((4 - d*x^3)*Sqrt[-1 + d*x^3]),x]
```

```
[Out] (-10*x^2*AppellF1[2/3, 1/2, 1, 5/3, d*x^3, (d*x^3)/4])/((-4 + d*x
^3)*Sqrt[-1 + d*x^3]*(20*AppellF1[2/3, 1/2, 1, 5/3, d*x^3, (d*x^3
)/4] + 3*d*x^3*(AppellF1[5/3, 1/2, 2, 8/3, d*x^3, (d*x^3)/4] + 2*
AppellF1[5/3, 3/2, 1, 8/3, d*x^3, (d*x^3)/4])))
```

Maple [C] time = 0.153, size = 240, normalized size = 1.5

$$-\frac{i}{9}\sqrt{2}\sum_{\_alpha=RootOf(d\_Z^3-4)}\frac{1}{\_alpha}\sqrt{-\frac{i}{2}\left(2x+\frac{1}{\sqrt[3]{d}}+i\sqrt{3}\frac{1}{\sqrt[3]{d}}\right)\sqrt[3]{d}}\sqrt{1\left(x-\frac{1}{\sqrt[3]{d}}\right)\left(-3\frac{1}{\sqrt[3]{d}}-i\sqrt{3}\frac{1}{\sqrt[3]{d}}\right)^{-1}}\sqrt{\frac{i}{2}\left(2x+\frac{1}{\sqrt[3]{d}}-i\sqrt{3}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(-d*x^3+4)/(d*x^3-1)^(1/2),x)
```

```
[Out] -1/9*I*2^(1/2)*sum(1/_alpha/d^(4/3)*(-1/2*I*(2*x+1/d^(1/3)+I*3^(1
/2)/d^(1/3))*d^(1/3))^(1/2)*((x-1/d^(1/3))/(-3/d^(1/3)-I*3^(1/2)/
d^(1/3)))^(1/2)*(1/2*I*(2*x+1/d^(1/3)-I*3^(1/2)/d^(1/3))*d^(1/3))
^(1/2)/(d*x^3-1)^(1/2)*(-2*_alpha^2*d+I*3^(1/2)*_alpha*d^(2/3)+_a
lpha*d^(2/3)-I*3^(1/2)*d^(1/3)+d^(1/3))*EllipticPi(1/3*3^(1/2)*(-
I*(x+1/2/d^(1/3)+1/2*I*3^(1/2)/d^(1/3))*3^(1/2)*d^(1/3))^(1/2),1/
3*I*_alpha^2*3^(1/2)*d^(2/3)-1/6*I*_alpha*3^(1/2)*d^(1/3)+1/2*_al
pha*d^(1/3)-1/6*I*3^(1/2)-1/2,(-I*3^(1/2)/d^(1/3)/(-3/2/d^(1/3)-1
/2*I*3^(1/2)/d^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{\sqrt{dx^3-1}(dx^3-4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x/(sqrt(d*x^3 - 1)*(d*x^3 - 4)),x, algorithm="maxima")
```

```
[Out] -integrate(x/(sqrt(d*x^3 - 1)*(d*x^3 - 4)), x)
```

Fricas [A] time = 0.460739, size = 2641, normalized size = 16.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x/(sqrt(d*x^3 - 1)*(d*x^3 - 4)),x, algorithm="fricas")
```

```
[Out] 1/9*sqrt(3)*(1/432)^(1/6)*(d^(-4))^(1/6)*arctan(6*(4*sqrt(3)*(1/2
)^(2/3)*(d^5*x^7 + d^4*x^4 - 2*d^3*x)*(d^(-4))^(2/3) - sqrt(3)*(1
/2)^(1/3)*(d^4*x^8 + 7*d^3*x^5 - 8*d^2*x^2)*(d^(-4))^(1/3) + (648
*sqrt(3)*(1/432)^(5/6)*d^5*(d^(-4))^(5/6)*x^5 - sqrt(3)*(1/432)^(
1/6)*(d^3*x^7 + 16*d^2*x^4 - 8*d*x)*(d^(-4))^(1/6))*sqrt(d*x^3 -
1))/(d^3*x^9 + 66*d^2*x^6 - 72*d*x^3 - 24*(1/2)^(2/3)*(d^5*x^7 +
d^4*x^4 - 2*d^3*x)*(d^(-4))^(2/3) - 6*(1/2)^(1/3)*(d^4*x^8 + 7*d^
3*x^5 - 8*d^2*x^2)*(d^(-4))^(1/3) + 6*(648*(1/432)^(5/6)*d^5*(d^(
-4))^(5/6)*x^5 - sqrt(1/3)*(5*d^4*x^6 + 20*d^3*x^3 - 16*d^2)*sqrt
(d^(-4)) + (1/432)^(1/6)*(d^3*x^7 + 16*d^2*x^4 - 8*d*x)*(d^(-4))^
```

```
(1/6))*sqrt(d*x^3 - 1) + (d^3*x^9 - 12*d^2*x^6 + 48*d*x^3 - 64)*s
qrt((d^3*x^9 - 60*d^2*x^6 - 24*(1/2)^(2/3)*(d^5*x^7 - 5*d^4*x^4 +
 4*d^3*x)*(d^(-4))^(2/3) + 12*(1/2)^(1/3)*(d^4*x^8 + 7*d^3*x^5 -
8*d^2*x^2)*(d^(-4))^(1/3) - 12*(648*(1/432)^(5/6)*d^5*(d^(-4))^(5
/6)*x^5 - sqrt(1/3)*(d^4*x^6 + 16*d^3*x^3 - 8*d^2)*sqrt(d^(-4)) -
 (1/432)^(1/6)*(d^3*x^7 - 2*d^2*x^4 - 8*d*x)*(d^(-4))^(1/6))*sqrt
(d*x^3 - 1) + 32)/(d^3*x^9 - 12*d^2*x^6 + 48*d*x^3 - 64)) + 32))
+ 1/9*sqrt(3)*(1/432)^(1/6)*(d^(-4))^(1/6)*arctan(-6*(4*sqrt(3)*(
1/2)^(2/3)*(d^5*x^7 + d^4*x^4 - 2*d^3*x)*(d^(-4))^(2/3) - sqrt(3)
*(1/2)^(1/3)*(d^4*x^8 + 7*d^3*x^5 - 8*d^2*x^2)*(d^(-4))^(1/3) - (
648*sqrt(3)*(1/432)^(5/6)*d^5*(d^(-4))^(5/6)*x^5 - sqrt(3)*(1/432
)^(1/6)*(d^3*x^7 + 16*d^2*x^4 - 8*d*x)*(d^(-4))^(1/6))*sqrt(d*x^3
 - 1))/(d^3*x^9 + 66*d^2*x^6 - 72*d*x^3 - 24*(1/2)^(2/3)*(d^5*x^7
 + d^4*x^4 - 2*d^3*x)*(d^(-4))^(2/3) - 6*(1/2)^(1/3)*(d^4*x^8 + 7
*d^3*x^5 - 8*d^2*x^2)*(d^(-4))^(1/3) - 6*(648*(1/432)^(5/6)*d^5*(
d^(-4))^(5/6)*x^5 - sqrt(1/3)*(5*d^4*x^6 + 20*d^3*x^3 - 16*d^2)*s
qrt(d^(-4)) + (1/432)^(1/6)*(d^3*x^7 + 16*d^2*x^4 - 8*d*x)*(d^(-4
))^(1/6))*sqrt(d*x^3 - 1) + (d^3*x^9 - 12*d^2*x^6 + 48*d*x^3 - 64
)*sqrt((d^3*x^9 - 60*d^2*x^6 - 24*(1/2)^(2/3)*(d^5*x^7 - 5*d^4*x^
4 + 4*d^3*x)*(d^(-4))^(2/3) + 12*(1/2)^(1/3)*(d^4*x^8 + 7*d^3*x^5
 - 8*d^2*x^2)*(d^(-4))^(1/3) + 12*(648*(1/432)^(5/6)*d^5*(d^(-4))
^(5/6)*x^5 - sqrt(1/3)*(d^4*x^6 + 16*d^3*x^3 - 8*d^2)*sqrt(d^(-4)
) - (1/432)^(1/6)*(d^3*x^7 - 2*d^2*x^4 - 8*d*x)*(d^(-4))^(1/6))*s
qrt(d*x^3 - 1) + 32)/(d^3*x^9 - 12*d^2*x^6 + 48*d*x^3 - 64)) + 32
)) + 1/18*(1/432)^(1/6)*(d^(-4))^(1/6)*log((d^3*x^9 + 66*d^2*x^6
- 72*d*x^3 + 48*(1/2)^(2/3)*(d^5*x^7 + d^4*x^4 - 2*d^3*x)*(d^(-4)
)^(2/3) + 12*(1/2)^(1/3)*(d^4*x^8 + 7*d^3*x^5 - 8*d^2*x^2)*(d^(-4
))^(1/3) + 6*(1296*(1/432)^(5/6)*d^5*(d^(-4))^(5/6)*x^5 + sqrt(1/
3)*(5*d^4*x^6 + 20*d^3*x^3 - 16*d^2)*sqrt(d^(-4)) + 2*(1/432)^(1/
6)*(d^3*x^7 + 16*d^2*x^4 - 8*d*x)*(d^(-4))^(1/6))*sqrt(d*x^3 - 1)
 + 32)/(d^3*x^9 - 12*d^2*x^6 + 48*d*x^3 - 64)) - 1/18*(1/432)^(1/
6)*(d^(-4))^(1/6)*log((d^3*x^9 + 66*d^2*x^6 - 72*d*x^3 + 48*(1/2)
^(2/3)*(d^5*x^7 + d^4*x^4 - 2*d^3*x)*(d^(-4))^(2/3) + 12*(1/2)^(1
/3)*(d^4*x^8 + 7*d^3*x^5 - 8*d^2*x^2)*(d^(-4))^(1/3) - 6*(1296*(1
/432)^(5/6)*d^5*(d^(-4))^(5/6)*x^5 + sqrt(1/3)*(5*d^4*x^6 + 20*d^
3*x^3 - 16*d^2)*sqrt(d^(-4)) + 2*(1/432)^(1/6)*(d^3*x^7 + 16*d^2*
x^4 - 8*d*x)*(d^(-4))^(1/6))*sqrt(d*x^3 - 1) + 32)/(d^3*x^9 - 12*
d^2*x^6 + 48*d*x^3 - 64)) - 1/36*(1/432)^(1/6)*(d^(-4))^(1/6)*log
((d^3*x^9 - 60*d^2*x^6 - 24*(1/2)^(2/3)*(d^5*x^7 - 5*d^4*x^4 + 4*
d^3*x)*(d^(-4))^(2/3) + 12*(1/2)^(1/3)*(d^4*x^8 + 7*d^3*x^5 - 8*d
^2*x^2)*(d^(-4))^(1/3) + 12*(648*(1/432)^(5/6)*d^5*(d^(-4))^(5/6)
*x^5 - sqrt(1/3)*(d^4*x^6 + 16*d^3*x^3 - 8*d^2)*sqrt(d^(-4)) - (1
/432)^(1/6)*(d^3*x^7 - 2*d^2*x^4 - 8*d*x)*(d^(-4))^(1/6))*sqrt(d*
x^3 - 1) + 32)/(d^3*x^9 - 12*d^2*x^6 + 48*d*x^3 - 64)) + 1/36*(1/
432)^(1/6)*(d^(-4))^(1/6)*log((d^3*x^9 - 60*d^2*x^6 - 24*(1/2)^(2
/3)*(d^5*x^7 - 5*d^4*x^4 + 4*d^3*x)*(d^(-4))^(2/3) + 12*(1/2)^(1/
3)*(d^4*x^8 + 7*d^3*x^5 - 8*d^2*x^2)*(d^(-4))^(1/3) - 12*(648*(1/
432)^(5/6)*d^5*(d^(-4))^(5/6)*x^5 - sqrt(1/3)*(d^4*x^6 + 16*d^3*x
^3 - 8*d^2)*sqrt(d^(-4)) - (1/432)^(1/6)*(d^3*x^7 - 2*d^2*x^4 - 8
*d*x)*(d^(-4))^(1/6))*sqrt(d*x^3 - 1) + 32)/(d^3*x^9 - 12*d^2*x^6
 + 48*d*x^3 - 64))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{dx^3\sqrt{dx^3-1}-4\sqrt{dx^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(-d*x**3+4)/(d*x**3-1)**(1/2),x)

[Out] -Integral(x/(d*x**3*sqrt(d*x**3 - 1) - 4*sqrt(d*x**3 - 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x}{\sqrt{dx^3-1}(dx^3-4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x/(sqrt(d*x^3 - 1)*(d*x^3 - 4)),x, algorithm="giac")
```

```
[Out] integrate(-x/(sqrt(d*x^3 - 1)*(d*x^3 - 4)), x)
```

3.73 $\int \frac{x}{\sqrt{-1+x^3}(8+x^3)}\,dx$

Optimal. Leaf size=74

$$\frac{1}{18}\tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{x^3-1}}\right)+\frac{1}{18}\tan^{-1}\left(\frac{\sqrt{x^3-1}}{3}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{3}(1-x)}{\sqrt{x^3-1}}\right)}{6\sqrt{3}}$$

```
[Out] ArcTan[(1 - x)^2/(3*Sqrt[-1 + x^3])]/18 + ArcTan[Sqrt[-1 + x^3]/3
]/18 - ArcTanh[(Sqrt[3]*(1 - x))/Sqrt[-1 + x^3]]/(6*Sqrt[3])
```

Rubi [A] time = 0.288252, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444

$$\frac{1}{18}\tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{x^3-1}}\right)+\frac{1}{18}\tan^{-1}\left(\frac{\sqrt{x^3-1}}{3}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{3}(1-x)}{\sqrt{x^3-1}}\right)}{6\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(Sqrt[-1 + x^3]*(8 + x^3)),x]
```

```
[Out] ArcTan[(1 - x)^2/(3*Sqrt[-1 + x^3])]/18 + ArcTan[Sqrt[-1 + x^3]/3
]/18 - ArcTanh[(Sqrt[3]*(1 - x))/Sqrt[-1 + x^3]]/(6*Sqrt[3])
```

Rubi in Sympy [A] time = 4.32494, size = 37, normalized size = 0.5

$$-\frac{x^2\sqrt{x^3-1}\,\text{appellf}_1\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},x^3,-\frac{x^3}{8}\right)}{16\sqrt{-x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(x**3+8)/(x**3-1)**(1/2),x)
```

```
[Out] -x**2*sqrt(x**3 - 1)*appellf1(2/3, 1/2, 1, 5/3, x**3, -x**3/8)/(1
6*sqrt(-x**3 + 1))
```

Mathematica [C] time = 0.191349, size = 118, normalized size = 1.59

$$-\frac{20x^2F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};x^3,-\frac{x^3}{8}\right)}{\sqrt{x^3-1}\,(x^3+8)\left(3x^3\left(F_1\left(\frac{5}{3};\frac{1}{2},2;\frac{8}{3};x^3,-\frac{x^3}{8}\right)-4F_1\left(\frac{5}{3};\frac{3}{2},1;\frac{8}{3};x^3,-\frac{x^3}{8}\right)\right)-40F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};x^3,-\frac{x^3}{8}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x/(Sqrt[-1 + x^3]*(8 + x^3)),x]
```

```
[Out] (-20*x^2*AppellF1[2/3, 1/2, 1, 5/3, x^3, -x^3/8])/(Sqrt[-1 + x^3]
*(8 + x^3)*(-40*AppellF1[2/3, 1/2, 1, 5/3, x^3, -x^3/8] + 3*x^3*(
AppellF1[5/3, 1/2, 2, 8/3, x^3, -x^3/8] - 4*AppellF1[5/3, 3/2, 1,
 8/3, x^3, -x^3/8])))
```

Maple [C] time = 0.169, size = 286, normalized size = 3.9

$$-\frac{-\frac{3}{2}-\frac{i}{2}\sqrt{3}}{9}\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i}{2}\sqrt{3}}}\sqrt{\frac{1}{\frac{3}{2}-\frac{i}{2}\sqrt{3}}\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)}\sqrt{\frac{1}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)}\,EllipticPi\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i}{2}\sqrt{3}}},\frac{i}{6}\sqrt{3}+\frac{1}{2},\sqrt{\frac{\frac{3}{2}+\frac{i}{2}}{\frac{3}{2}-\frac{i}{2}}}\right.$$
$$+\frac{\sqrt{2}}{36}\sum_{\_alpha=RootOf(\_Z^2-2\_Z+4)}(2-\_alpha)(-1+\_alpha)\left(-i\sqrt{3}-3\right)\sqrt{\frac{-1+x}{-i\sqrt{3}-3}}\sqrt{\frac{2x+1-i\sqrt{3}}{-i\sqrt{3}+3}}\sqrt{\frac{2x+1+i\sqrt{3}}{i\sqrt{3}+3}}\,EllipticPi\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(x^3+8)/(x^3-1)^(1/2),x)
```

```
[Out] -1/9*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x
+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1
/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticPi(((-1+x)/
(-3/2-1/2*I*3^(1/2)))^(1/2),1/6*I*3^(1/2)+1/2,((3/2+1/2*I*3^(1/2)
)/(3/2-1/2*I*3^(1/2)))^(1/2))+1/36*2^(1/2)*sum((2-_alpha)*(-1+_al
pha)*(-I*3^(1/2)-3)*((-1+x)/(-I*3^(1/2)-3))^(1/2)*((2*x+1-I*3^(1/
2))/(-I*3^(1/2)+3))^(1/2)*((2*x+1+I*3^(1/2))/(I*3^(1/2)+3))^(1/2)
/(x^3-1)^(1/2)*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),1/6
*I*_alpha*3^(1/2)+1/2*_alpha-1/6*I*3^(1/2)-1/2,((3/2+1/2*I*3^(1/2
))/(3/2-1/2*I*3^(1/2)))^(1/2)),_alpha=RootOf(_Z^2-2*_Z+4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(x^3+8)\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((x^3 + 8)*sqrt(x^3 - 1)),x, algorithm="maxima")
```

```
[Out] integrate(x/((x^3 + 8)*sqrt(x^3 - 1)), x)
```

Fricas [A] time = 0.342107, size = 950, normalized size = 12.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((x^3 + 8)*sqrt(x^3 - 1)),x, algorithm="fricas")
```

```
[Out] 1/216*sqrt(3)*log(3*(x^6 + 48*x^5 + 186*x^4 - 56*x^3 + 6*sqrt(3)*
(x^4 + 12*x^3 + 12*x^2 - 16*x)*sqrt(x^3 - 1) - 120*x^2 - 96*x + 6
4)/(x^6 - 6*x^5 + 24*x^4 - 56*x^3 + 96*x^2 - 96*x + 64)) - 1/216*
sqrt(3)*log(3*(x^6 + 48*x^5 + 186*x^4 - 56*x^3 - 6*sqrt(3)*(x^4 +
 12*x^3 + 12*x^2 - 16*x)*sqrt(x^3 - 1) - 120*x^2 - 96*x + 64)/(x^
6 - 6*x^5 + 24*x^4 - 56*x^3 + 96*x^2 - 96*x + 64)) - 1/54*arctan(
3*(9*x^5 - 9*x^4 - 54*x^3 + sqrt(3)*(x^4 + 8*x^3 - 30*x^2 - 4*x +
 16)*sqrt(x^3 - 1) + 18*x^2 + 36*x)/(sqrt(3)*(x^6 - 6*x^5 + 24*x^
4 - 56*x^3 + 96*x^2 - 96*x + 64)*sqrt((x^6 + 48*x^5 + 186*x^4 - 5
6*x^3 + 6*sqrt(3)*(x^4 + 12*x^3 + 12*x^2 - 16*x)*sqrt(x^3 - 1) -
120*x^2 - 96*x + 64)/(x^6 - 6*x^5 + 24*x^4 - 56*x^3 + 96*x^2 - 96
*x + 64)) + sqrt(3)*(x^6 + 3*x^5 - 75*x^4 + 88*x^3 + 6*x^2 + 84*x
 - 80) + 9*(x^4 - 6*x^3 - 6*x^2 + 20*x)*sqrt(x^3 - 1))) - 1/54*ar
ctan(-3*(9*x^5 - 9*x^4 - 54*x^3 - sqrt(3)*(x^4 + 8*x^3 - 30*x^2 -
 4*x + 16)*sqrt(x^3 - 1) + 18*x^2 + 36*x)/(sqrt(3)*(x^6 - 6*x^5 +
 24*x^4 - 56*x^3 + 96*x^2 - 96*x + 64)*sqrt((x^6 + 48*x^5 + 186*x
^4 - 56*x^3 - 6*sqrt(3)*(x^4 + 12*x^3 + 12*x^2 - 16*x)*sqrt(x^3 -
 1) - 120*x^2 - 96*x + 64)/(x^6 - 6*x^5 + 24*x^4 - 56*x^3 + 96*x^
```

```
2 - 96*x + 64)) + sqrt(3)*(x^6 + 3*x^5 - 75*x^4 + 88*x^3 + 6*x^2
+ 84*x - 80) - 9*(x^4 - 6*x^3 - 6*x^2 + 20*x)*sqrt(x^3 - 1))) + 1
/54*arctan(1/6*(x^3 - 12*x^2 - 6*x - 10)/(sqrt(x^3 - 1)*(x - 1)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{(x-1)(x^2+x+1)}(x+2)(x^2-2x+4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x**3+8)/(x**3-1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt((x - 1)*(x**2 + x + 1))*(x + 2)*(x**2 - 2*x + 4)
), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(x^3+8)\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((x^3 + 8)*sqrt(x^3 - 1)),x, algorithm="giac")
```

```
[Out] integrate(x/((x^3 + 8)*sqrt(x^3 - 1)), x)
```

3.74 $\int \frac{x}{(8-dx^3)\sqrt{1+dx^3}}\,dx$

Optimal. Leaf size=103

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{d}x+1\right)}{\sqrt{dx^3+1}}\right)}{6\sqrt{3}d^{2/3}}+\frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{d}x+1\right)^2}{3\sqrt{dx^3+1}}\right)}{18d^{2/3}}-\frac{\tanh^{-1}\left(\frac{1}{3}\sqrt{dx^3+1}\right)}{18d^{2/3}}$$

```
[Out] -ArcTan[(Sqrt[3]*(1 + d^(1/3)*x))/Sqrt[1 + d*x^3]]/(6*Sqrt[3]*d^(
2/3)) + ArcTanh[(1 + d^(1/3)*x)^2/(3*Sqrt[1 + d*x^3])]/(18*d^(2/3
)) - ArcTanh[Sqrt[1 + d*x^3]/3]/(18*d^(2/3))
```

Rubi [A] time = 0.544631, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{d}x+1\right)}{\sqrt{dx^3+1}}\right)}{6\sqrt{3}d^{2/3}}+\frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{d}x+1\right)^2}{3\sqrt{dx^3+1}}\right)}{18d^{2/3}}-\frac{\tanh^{-1}\left(\frac{1}{3}\sqrt{dx^3+1}\right)}{18d^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/((8 - d*x^3)*Sqrt[1 + d*x^3]),x]
```

```
[Out] -ArcTan[(Sqrt[3]*(1 + d^(1/3)*x))/Sqrt[1 + d*x^3]]/(6*Sqrt[3]*d^(
2/3)) + ArcTanh[(1 + d^(1/3)*x)^2/(3*Sqrt[1 + d*x^3])]/(18*d^(2/3
)) - ArcTanh[Sqrt[1 + d*x^3]/3]/(18*d^(2/3))
```

Rubi in Sympy [A] time = 3.31384, size = 22, normalized size = 0.21

$$\frac{x^2\,\text{appellf}_1\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-dx^3,\frac{dx^3}{8}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(-d*x**3+8)/(d*x**3+1)**(1/2),x)
```

```
[Out] x**2*appellf1(2/3, 1/2, 1, 5/3, -d*x**3, d*x**3/8)/16
```

Mathematica [C] time = 0.18094, size = 139, normalized size = 1.35

$$-\frac{20x^2F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-dx^3,\frac{dx^3}{8}\right)}{(dx^3-8)\sqrt{dx^3+1}\left(3dx^3\left(F_1\left(\frac{5}{3};\frac{1}{2},2;\frac{8}{3};-dx^3,\frac{dx^3}{8}\right)-4F_1\left(\frac{5}{3};\frac{3}{2},1;\frac{8}{3};-dx^3,\frac{dx^3}{8}\right)\right)+40F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-dx^3,\frac{dx^3}{8}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x/((8 - d*x^3)*Sqrt[1 + d*x^3]),x]
```

```
[Out] (-20*x^2*AppellF1[2/3, 1/2, 1, 5/3, -(d*x^3), (d*x^3)/8])/((-8 +
d*x^3)*Sqrt[1 + d*x^3]*(40*AppellF1[2/3, 1/2, 1, 5/3, -(d*x^3), (
d*x^3)/8] + 3*d*x^3*(AppellF1[5/3, 1/2, 2, 8/3, -(d*x^3), (d*x^3)
/8] - 4*AppellF1[5/3, 3/2, 1, 8/3, -(d*x^3), (d*x^3)/8])))
```

Maple [C] time = 0.197, size = 383, normalized size = 3.7

$$\frac{-\frac{i}{27}\sqrt{2}}{d^3}\sum_{\_alpha=RootOf(d\_Z^3-8)}\frac{1}{\_alpha}\sqrt[3]{-d^2}\sqrt{\frac{i}{2}d\left(2x+\frac{1}{d}\left(-i\sqrt{3}\sqrt[3]{-d^2}+\sqrt[3]{-d^2}\right)\right)\frac{1}{\sqrt[3]{-d^2}}}\sqrt{d\left(x-\frac{1}{d}\sqrt[3]{-d^2}\right)\left(-3\sqrt[3]{-d^2}+i\sqrt{3}\sqrt[3]{\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(-d*x^3+8)/(d*x^3+1)^(1/2),x)
```

```
[Out] -1/27*I/d^3*2^(1/2)*sum(1/_alpha*(-d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(
-I*3^(1/2)*(-d^2)^(1/3)+(-d^2)^(1/3)))/(-d^2)^(1/3))^(1/2)*(d*(x-
1/d*(-d^2)^(1/3))/(-3*(-d^2)^(1/3)+I*3^(1/2)*(-d^2)^(1/3)))^(1/2)
*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2)^(1/3)+(-d^2)^(1/3)))/(-d^2)
^(1/3))^(1/2)/(d*x^3+1)^(1/2)*(I*(-d^2)^(1/3)*_alpha*3^(1/2)*d-I*
3^(1/2)*(-d^2)^(2/3)+2*_alpha^2*d^2-(-d^2)^(1/3)*_alpha*d-(-d^2)^
(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2)^(1/3)-1/2*I*3^(1
/2)/d*(-d^2)^(1/3))*3^(1/2)*d/(-d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-
d^2)^(1/3)*_alpha^2*3^(1/2)*d-I*(-d^2)^(2/3)*_alpha*3^(1/2)-3*(-d
^2)^(2/3)*_alpha+I*3^(1/2)*d-3*d),(I*3^(1/2)/d*(-d^2)^(1/3)/(-3/2
/d*(-d^2)^(1/3)+1/2*I*3^(1/2)/d*(-d^2)^(1/3)))^(1/2)),_alpha=Root
Of(_Z^3*d-8))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x}{\sqrt{dx^3+1}(dx^3-8)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x/(sqrt(d*x^3 + 1)*(d*x^3 - 8)),x, algorithm="maxima")
```

```
[Out] -integrate(x/(sqrt(d*x^3 + 1)*(d*x^3 - 8)), x)
```

Fricas [A] time = 0.328065, size = 914, normalized size = 8.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x/(sqrt(d*x^3 + 1)*(d*x^3 - 8)),x, algorithm="fricas")
```

```
[Out] 1/324*sqrt(3)*(2*sqrt(3)*log((18*d^4*x^8 + 684*d^3*x^5 + 1152*d^2
*x^2 + (d^3*x^9 + 318*d^2*x^6 + 1200*d*x^3 + 640)*(d^2)^(2/3) + 6
*(30*d^3*x^5 + 192*d^2*x^2 + (7*d^2*x^6 + 152*d*x^3 + 64)*(d^2)^(
2/3) + (d^3*x^7 + 80*d^2*x^4 + 160*d*x)*(d^2)^(1/3))*sqrt(d*x^3 +
 1) + 18*(5*d^3*x^7 + 64*d^2*x^4 + 32*d*x)*(d^2)^(1/3))/(d^3*x^9
- 24*d^2*x^6 + 192*d*x^3 - 512)) - sqrt(3)*log(-(18*d^3*x^7 - 936
*d^2*x^4 - 1440*d*x - 18*(d^2*x^8 + 20*d*x^5 - 8*x^2)*(d^2)^(2/3)
 + 6*(d^3*x^7 - 28*d^2*x^4 - 272*d*x - 24*(d*x^5 + x^2)*(d^2)^(2/
3) + 4*(d^2*x^6 + 41*d*x^3 + 40)*(d^2)^(1/3))*sqrt(d*x^3 + 1) - (
d^3*x^9 - 276*d^2*x^6 - 1608*d*x^3 - 1088)*(d^2)^(1/3))/(d^3*x^9
- 24*d^2*x^6 + 192*d*x^3 - 512)) + 6*arctan(-1/9*(sqrt(3)*(d^3*x^
9 + 318*d^2*x^6 + 1200*d*x^3 + 640)*(d^2)^(2/3) - 9*sqrt(3)*(5*d^
3*x^7 + 64*d^2*x^4 + 32*d*x)*(d^2)^(1/3) - 9*sqrt(3)*(d^4*x^8 + 3
8*d^3*x^5 + 64*d^2*x^2) + 3*sqrt(d*x^3 + 1)*(2*sqrt(3)*(7*d^2*x^6
 + 152*d*x^3 + 64)*(d^2)^(2/3) - sqrt(3)*(d^3*x^7 + 80*d^2*x^4 +
160*d*x)*(d^2)^(1/3) - 6*sqrt(3)*(5*d^3*x^5 + 32*d^2*x^2)))/(3*d^
4*x^8 + 114*d^3*x^5 + 192*d^2*x^2 + (30*d^3*x^5 + 192*d^2*x^2 - (
d^3*x^7 + 80*d^2*x^4 + 160*d*x)*(d^2)^(1/3))*sqrt(d*x^3 + 1) - 3*
```

```
(5*d^3*x^7 + 64*d^2*x^4 + 32*d*x)*(d^2)^(1/3)))))/(d^2)^(1/3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{dx^3\sqrt{dx^3+1}-8\sqrt{dx^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(-d*x**3+8)/(d*x**3+1)**(1/2),x)

[Out] -Integral(x/(d*x**3*sqrt(d*x**3 + 1) - 8*sqrt(d*x**3 + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x}{\sqrt{dx^3+1}(dx^3-8)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x/(sqrt(d*x^3 + 1)*(d*x^3 - 8)),x, algorithm="giac")

[Out] integrate(-x/(sqrt(d*x^3 + 1)*(d*x^3 - 8)), x)
```

3.75 $\int \frac{1}{\sqrt[3]{1-3x^2}\,(3-x^2)}\,dx$

Optimal. Leaf size=81

$$\frac{1}{4}\tan^{-1}\left(\frac{1-\sqrt[3]{1-3x^2}}{x}\right)-\frac{\tanh^{-1}\left(\frac{\left(1-\sqrt[3]{1-3x^2}\right)^2}{3\sqrt{3}x}\right)}{4\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt{3}}\right)}{4\sqrt{3}}$$

```
[Out] ArcTan[(1 - (1 - 3*x^2)^(1/3))/x]/4 + ArcTanh[x/Sqrt[3]]/(4*Sqrt[
3]) - ArcTanh[(1 - (1 - 3*x^2)^(1/3))^2/(3*Sqrt[3]*x)]/(4*Sqrt[3]
)
```

Rubi [A] time = 0.0404513, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048

$$\frac{1}{4}\tan^{-1}\left(\frac{1-\sqrt[3]{1-3x^2}}{x}\right)-\frac{\tanh^{-1}\left(\frac{\left(1-\sqrt[3]{1-3x^2}\right)^2}{3\sqrt{3}x}\right)}{4\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((1 - 3*x^2)^(1/3)*(3 - x^2)),x]
```

```
[Out] ArcTan[(1 - (1 - 3*x^2)^(1/3))/x]/4 + ArcTanh[x/Sqrt[3]]/(4*Sqrt[
3]) - ArcTanh[(1 - (1 - 3*x^2)^(1/3))^2/(3*Sqrt[3]*x)]/(4*Sqrt[3]
)
```

Rubi in Sympy [A] time = 2.60271, size = 17, normalized size = 0.21

$$\frac{x\,\mathrm{appellf}_1\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},3x^2,\frac{x^2}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-3*x**2+1)**(1/3)/(-x**2+3),x)
```

```
[Out] x*appellf1(1/2, 1/3, 1, 3/2, 3*x**2, x**2/3)/3
```

Mathematica [C] time = 0.151867, size = 126, normalized size = 1.56

$$-\frac{9xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};3x^2,\frac{x^2}{3}\right)}{\sqrt[3]{1-3x^2}\,(x^2-3)\left(2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};3x^2,\frac{x^2}{3}\right)+3F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};3x^2,\frac{x^2}{3}\right)\right)+9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};3x^2,\frac{x^2}{3}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((1 - 3*x^2)^(1/3)*(3 - x^2)),x]
```

```
[Out] (-9*x*AppellF1[1/2, 1/3, 1, 3/2, 3*x^2, x^2/3])/((1 - 3*x^2)^(1/3
)*(-3 + x^2)*(9*AppellF1[1/2, 1/3, 1, 3/2, 3*x^2, x^2/3] + 2*x^2*
(AppellF1[3/2, 1/3, 2, 5/2, 3*x^2, x^2/3] + 3*AppellF1[3/2, 4/3,
```

```
1, 5/2, 3*x^2, x^2/3])))
```

Maple [F] time = 0.07, size = 0, normalized size = 0.

$$\int \frac{1}{-x^2+3}\frac{1}{\sqrt[3]{-3\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-3*x^2+1)^(1/3)/(-x^2+3),x)
```

```
[Out] int(1/(-3*x^2+1)^(1/3)/(-x^2+3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{(x^2-3)(-3\,x^2+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((x^2 - 3)*(-3*x^2 + 1)^(1/3)),x, algorithm="maxima")
```

```
[Out] -integrate(1/((x^2 - 3)*(-3*x^2 + 1)^(1/3)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((x^2 - 3)*(-3*x^2 + 1)^(1/3)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{x^2\sqrt[3]{-3x^2+1}-3\sqrt[3]{-3x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-3*x**2+1)**(1/3)/(-x**2+3),x)
```

```
[Out] -Integral(1/(x**2*(-3*x**2 + 1)**(1/3) - 3*(-3*x**2 + 1)**(1/3)),
 x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{(x^2-3)(-3\,x^2+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((x^2 - 3)*(-3*x^2 + 1)^(1/3)),x, algorithm="giac")

[Out] integrate(-1/((x^2 - 3)*(-3*x^2 + 1)^(1/3)), x)
```

3.76 $\int \frac{1}{(3+x^2)\sqrt[3]{1+3x^2}}\,dx$

Optimal. Leaf size=81

$$\frac{\tan^{-1}\left(\frac{\left(1-\sqrt[3]{3x^2+1}\right)^2}{3\sqrt{3}x}\right)}{4\sqrt{3}}-\frac{1}{4}\tanh^{-1}\left(\frac{1-\sqrt[3]{3x^2+1}}{x}\right)+\frac{\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{4\sqrt{3}}$$

```
[Out] ArcTan[x/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[(1 - (1 + 3*x^2)^(1/3))^2/
(3*Sqrt[3]*x)]/(4*Sqrt[3]) - ArcTanh[(1 - (1 + 3*x^2)^(1/3))/x]/4
```

Rubi [A] time = 0.0369219, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053

$$\frac{\tan^{-1}\left(\frac{\left(1-\sqrt[3]{3x^2+1}\right)^2}{3\sqrt{3}x}\right)}{4\sqrt{3}}-\frac{1}{4}\tanh^{-1}\left(\frac{1-\sqrt[3]{3x^2+1}}{x}\right)+\frac{\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((3 + x^2)*(1 + 3*x^2)^(1/3)),x]
```

```
[Out] ArcTan[x/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[(1 - (1 + 3*x^2)^(1/3))^2/
(3*Sqrt[3]*x)]/(4*Sqrt[3]) - ArcTanh[(1 - (1 + 3*x^2)^(1/3))/x]/4
```

Rubi in Sympy [A] time = 3.07163, size = 20, normalized size = 0.25

$$\frac{x\,\mathrm{appellf}_1\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-3x^2,-\frac{x^2}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**2+3)/(3*x**2+1)**(1/3),x)
```

```
[Out] x*appellf1(1/2, 1/3, 1, 3/2, -3*x**2, -x**2/3)/3
```

Mathematica [C] time = 0.140462, size = 126, normalized size = 1.56

$$-\frac{9xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-3x^2,-\frac{x^2}{3}\right)}{(x^2+3)\sqrt[3]{3x^2+1}\left(2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};-3x^2,-\frac{x^2}{3}\right)+3F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};-3x^2,-\frac{x^2}{3}\right)\right)-9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-3x^2,-\frac{x^2}{3}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((3 + x^2)*(1 + 3*x^2)^(1/3)),x]
```

```
[Out] (-9*x*AppellF1[1/2, 1/3, 1, 3/2, -3*x^2, -x^2/3])/((3 + x^2)*(1 +
 3*x^2)^(1/3)*(-9*AppellF1[1/2, 1/3, 1, 3/2, -3*x^2, -x^2/3] + 2*
x^2*(AppellF1[3/2, 1/3, 2, 5/2, -3*x^2, -x^2/3] + 3*AppellF1[3/2,
 4/3, 1, 5/2, -3*x^2, -x^2/3])))
```

Maple [F] time = 0.04, size = 0, normalized size = 0.

$$\int \frac{1}{x^2+3}\frac{1}{\sqrt[3]{3\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^2+3)/(3*x^2+1)^(1/3),x)
```

```
[Out] int(1/(x^2+3)/(3*x^2+1)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3\,x^2+1)^{\frac{1}{3}}(x^2+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((3*x^2 + 1)^(1/3)*(x^2 + 3)),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 + 1)^(1/3)*(x^2 + 3)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((3*x^2 + 1)^(1/3)*(x^2 + 3)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2+3)\sqrt[3]{3x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**2+3)/(3*x**2+1)**(1/3),x)
```

```
[Out] Integral(1/((x**2 + 3)*(3*x**2 + 1)**(1/3)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3\,x^2+1)^{\frac{1}{3}}(x^2+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((3*x^2 + 1)^(1/3)*(x^2 + 3)),x, algorithm="giac")
```

```
[Out] integrate(1/((3*x^2 + 1)^(1/3)*(x^2 + 3)), x)
```

3.77 $\int \frac{1}{\sqrt[3]{1-x^2}(3+x^2)}\,dx$

Optimal. Leaf size=113

$$\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\,2^{2/3}\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{2\,2^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}(x)}{6\,2^{2/3}}$$

```
[Out] ArcTan[Sqrt[3]/x]/(2*2^(2/3)*Sqrt[3]) + ArcTan[(Sqrt[3]*(1 - 2^(1
/3)*(1 - x^2)^(1/3)))/x]/(2*2^(2/3)*Sqrt[3]) - ArcTanh[x]/(6*2^(2
/3)) + ArcTanh[x/(1 + 2^(1/3)*(1 - x^2)^(1/3))]/(2*2^(2/3))
```

Rubi [A] time = 0.0461671, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053

$$\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\,2^{2/3}\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{2\,2^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}(x)}{6\,2^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] ArcTan[Sqrt[3]/x]/(2*2^(2/3)*Sqrt[3]) + ArcTan[(Sqrt[3]*(1 - 2^(1
/3)*(1 - x^2)^(1/3)))/x]/(2*2^(2/3)*Sqrt[3]) - ArcTanh[x]/(6*2^(2
/3)) + ArcTanh[x/(1 + 2^(1/3)*(1 - x^2)^(1/3))]/(2*2^(2/3))
```

Rubi in Sympy [A] time = 5.86888, size = 144, normalized size = 1.27

$$\frac{\sqrt[3]{2}\log\left(\sqrt[3]{2}\sqrt[3]{-x+1}+(x+1)^{\frac{2}{3}}\right)}{8}-\frac{\sqrt[3]{2}\log\left((-x+1)^{\frac{2}{3}}+\sqrt[3]{2}\sqrt[3]{x+1}\right)}{8}-\frac{\sqrt[3]{2}\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}}{3}-\frac{2^{\frac{2}{3}}\sqrt{3}(x+1)^{\frac{2}{3}}}{3\sqrt[3]{-x+1}}\right)}{12}-\frac{\sqrt[3]{2}\sqrt{3}\operatorname{atan}\left(\frac{2^{\frac{2}{3}}\sqrt{3}(-x+1)^{\frac{2}{3}}}{3\sqrt[3]{x+1}}-\frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] 2**(1/3)*log(2**(1/3)*(-x + 1)**(1/3) + (x + 1)**(2/3))/8 - 2**(1
/3)*log((-x + 1)**(2/3) + 2**(1/3)*(x + 1)**(1/3))/8 - 2**(1/3)*s
qrt(3)*atan(sqrt(3)/3 - 2**(2/3)*sqrt(3)*(x + 1)**(2/3)/(3*(-x +
1)**(1/3)))/12 - 2**(1/3)*sqrt(3)*atan(2**(2/3)*sqrt(3)*(-x + 1)*
*(2/3)/(3*(x + 1)**(1/3)) - sqrt(3)/3)/12
```

Mathematica [C] time = 0.130754, size = 118, normalized size = 1.04

$$-\frac{9xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)}{\sqrt[3]{1-x^2}\left(x^2+3\right)\left(2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};x^2,-\frac{x^2}{3}\right)-F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)\right)-9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (-9*x*AppellF1[1/2, 1/3, 1, 3/2, x^2, -x^2/3])/((1 - x^2)^(1/3)*(
3 + x^2)*(-9*AppellF1[1/2, 1/3, 1, 3/2, x^2, -x^2/3] + 2*x^2*(App
ellF1[3/2, 1/3, 2, 5/2, x^2, -x^2/3] - AppellF1[3/2, 4/3, 1, 5/2,
 x^2, -x^2/3])))
```

Maple [F] time = 0.05, size = 0, normalized size = 0.

$$\int \frac{1}{x^2+3}\frac{1}{\sqrt[3]{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-x^2+1)^(1/3)/(x^2+3),x)
```

```
[Out] int(1/(-x^2+1)^(1/3)/(x^2+3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2+3)(-x^2+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{-(x-1)(x+1)}\,(x^2+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] Integral(1/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2+3)(-x^2+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)),x, algorithm="giac")

[Out] integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

3.78 $\int \frac{1}{(3-x^2)\sqrt[3]{1+x^2}}\,dx$

Optimal. Leaf size=109

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{x^2+1}+1}\right)}{2\,2^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{x^2+1}\right)}{x}\right)}{2\,2^{2/3}\sqrt{3}}-\frac{\tan^{-1}(x)}{6\,2^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}\sqrt{3}}$$

```
[Out] -ArcTan[x]/(6*2^(2/3)) + ArcTan[x/(1 + 2^(1/3)*(1 + x^2)^(1/3))]/
(2*2^(2/3)) - ArcTanh[Sqrt[3]/x]/(2*2^(2/3)*Sqrt[3]) - ArcTanh[(S
qrt[3]*(1 - 2^(1/3)*(1 + x^2)^(1/3)))/x]/(2*2^(2/3)*Sqrt[3])
```

Rubi [A] time = 0.0437177, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{x^2+1}+1}\right)}{2\,2^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{x^2+1}\right)}{x}\right)}{2\,2^{2/3}\sqrt{3}}-\frac{\tan^{-1}(x)}{6\,2^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((3 - x^2)*(1 + x^2)^(1/3)),x]
```

```
[Out] -ArcTan[x]/(6*2^(2/3)) + ArcTan[x/(1 + 2^(1/3)*(1 + x^2)^(1/3))]/
(2*2^(2/3)) - ArcTanh[Sqrt[3]/x]/(2*2^(2/3)*Sqrt[3]) - ArcTanh[(S
qrt[3]*(1 - 2^(1/3)*(1 + x^2)^(1/3)))/x]/(2*2^(2/3)*Sqrt[3])
```

Rubi in Sympy [A] time = 5.23769, size = 192, normalized size = 1.76

$$\frac{\sqrt[3]{2}\sqrt{3}\log\left(-x+\sqrt{3}\right)}{24}-\frac{\sqrt[3]{2}\sqrt{3}\log\left(x+\sqrt{3}\right)}{24}+\frac{\sqrt[3]{2}\sqrt{3}\log\left(-x-\sqrt[3]{2}\sqrt{3}\sqrt[3]{x^2+1}+\sqrt{3}\right)}{24}-\frac{\sqrt[3]{2}\sqrt{3}\log\left(x-\sqrt[3]{2}\sqrt{3}\sqrt[3]{x^2+1}+\sqrt{3}\right)}{24}-\frac{\sqrt[3]{2}\,\mathrm{atan}\left(\frac{2^{\frac{2}{3}}\left(-x+\sqrt{3}\right)}{3\sqrt[3]{x^2+1}}+\frac{\sqrt{3}}{3}\right)}{12}+\frac{\sqrt[3]{2}\,\mathrm{atan}\left(\frac{2^{\frac{2}{3}}\left(x+\sqrt{3}\right)}{3\sqrt[3]{x^2+1}}+\frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-x**2+3)/(x**2+1)**(1/3),x)
```

```
[Out] 2**(1/3)*sqrt(3)*log(-x + sqrt(3))/24 - 2**(1/3)*sqrt(3)*log(x +
sqrt(3))/24 + 2**(1/3)*sqrt(3)*log(-x - 2**(1/3)*sqrt(3)*(x**2 +
1)**(1/3) + sqrt(3))/24 - 2**(1/3)*sqrt(3)*log(x - 2**(1/3)*sqrt(
3)*(x**2 + 1)**(1/3) + sqrt(3))/24 - 2**(1/3)*atan(2**(2/3)*(-x +
 sqrt(3))/(3*(x**2 + 1)**(1/3)) + sqrt(3)/3)/12 + 2**(1/3)*atan(2
**(2/3)*(x + sqrt(3))/(3*(x**2 + 1)**(1/3)) + sqrt(3)/3)/12
```

Mathematica [C] time = 0.10238, size = 124, normalized size = 1.14

$$-\frac{9xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-x^2,\frac{x^2}{3}\right)}{(x^2-3)\sqrt[3]{x^2+1}\left(2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};-x^2,\frac{x^2}{3}\right)-F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};-x^2,\frac{x^2}{3}\right)\right)+9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-x^2,\frac{x^2}{3}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((3 - x^2)*(1 + x^2)^(1/3)),x]
```

```
[Out] (-9*x*AppellF1[1/2, 1/3, 1, 3/2, -x^2, x^2/3])/((-3 + x^2)*(1 + x
^2)^(1/3)*(9*AppellF1[1/2, 1/3, 1, 3/2, -x^2, x^2/3] + 2*x^2*(App
ellF1[3/2, 1/3, 2, 5/2, -x^2, x^2/3] - AppellF1[3/2, 4/3, 1, 5/2,
 -x^2, x^2/3])))
```

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int \frac{1}{-x^2+3}\frac{1}{\sqrt[3]{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-x^2+3)/(x^2+1)^(1/3),x)
```

```
[Out] int(1/(-x^2+3)/(x^2+1)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{(x^2+1)^{\frac{1}{3}}(x^2-3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((x^2 + 1)^(1/3)*(x^2 - 3)),x, algorithm="maxima")
```

```
[Out] -integrate(1/((x^2 + 1)^(1/3)*(x^2 - 3)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((x^2 + 1)^(1/3)*(x^2 - 3)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{x^2\sqrt[3]{x^2+1}-3\sqrt[3]{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-x**2+3)/(x**2+1)**(1/3),x)
```

```
[Out] -Integral(1/(x**2*(x**2 + 1)**(1/3) - 3*(x**2 + 1)**(1/3)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{(x^2+1)^{\frac{1}{3}}(x^2-3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((x^2 + 1)^(1/3)*(x^2 - 3)),x, algorithm="giac")
```

```
[Out] integrate(-1/((x^2 + 1)^(1/3)*(x^2 - 3)), x)
```

3.79 $\int \frac{a+x}{(-a+x)\sqrt{a^2x-(1+a^2)x^2+x^3}}\,dx$

Optimal. Leaf size=87

$$-\frac{2\sqrt{x}\sqrt{-(a^2+1)x+a^2+x^2}\tan^{-1}\left(\frac{(1-a)\sqrt{x}}{\sqrt{-(a^2+1)x+a^2+x^2}}\right)}{(1-a)\sqrt{-(a^2+1)x^2+a^2x+x^3}}$$

```
[Out] (-2*Sqrt[x]*Sqrt[a^2 - (1 + a^2)*x + x^2]*ArcTan[((1 - a)*Sqrt[x]
)/Sqrt[a^2 - (1 + a^2)*x + x^2]])/((1 - a)*Sqrt[a^2*x - (1 + a^2)
*x^2 + x^3])
```

Rubi [A] time = 1.47925, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118

$$-\frac{2\sqrt{x}\sqrt{-(a^2+1)x+a^2+x^2}\tan^{-1}\left(\frac{(1-a)\sqrt{x}}{\sqrt{-(a^2+1)x+a^2+x^2}}\right)}{(1-a)\sqrt{-(a^2+1)x^2+a^2x+x^3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + x)/((-a + x)*Sqrt[a^2*x - (1 + a^2)*x^2 + x^3]),x]
```

```
[Out] (-2*Sqrt[x]*Sqrt[a^2 - (1 + a^2)*x + x^2]*ArcTan[((1 - a)*Sqrt[x]
)/Sqrt[a^2 - (1 + a^2)*x + x^2]])/((1 - a)*Sqrt[a^2*x - (1 + a^2)
*x^2 + x^3])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+x)/(-a+x)/(a**2*x-(a**2+1)*x**2+x**3)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.553469, size = 159, normalized size = 1.83

$$-\frac{2i\left(a^2-x\right)^{3/2}\sqrt{\frac{x-1}{x-a^2}}\sqrt{\frac{x}{x-a^2}}\left((a+1)F\left(i\sinh^{-1}\left(\frac{\sqrt{-a^2}}{\sqrt{a^2-x}}\right)|1-\frac{1}{a^2}\right)-2\left(\frac{a-1}{a};i\sinh^{-1}\left(\frac{\sqrt{-a^2}}{\sqrt{a^2-x}}\right)|1-\frac{1}{a^2}\right)\right)}{(a-1)\sqrt{-a^2}\sqrt{(x-1)x\left(x-a^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + x)/((-a + x)*Sqrt[a^2*x - (1 + a^2)*x^2 + x^3]),x]
```

```
[Out] ((-2*I)*(a^2 - x)^(3/2)*Sqrt[(-1 + x)/(-a^2 + x)]*Sqrt[x/(-a^2 +
x)]*((1 + a)*EllipticF[I*ArcSinh[Sqrt[-a^2]/Sqrt[a^2 - x]], 1 - a
^(-2)] - 2*EllipticPi[(-1 + a)/a, I*ArcSinh[Sqrt[-a^2]/Sqrt[a^2 -
 x]], 1 - a^(-2)]))/((-1 + a)*Sqrt[-a^2]*Sqrt[(-1 + x)*x*(-a^2 +
x)])
```

Maple [C] time = 0.053, size = 206, normalized size = 2.4

$$-2\frac{a^{2}}{\sqrt{-a^{2}x^{2}+a^{2}x+x^{3}-x^{2}}}\sqrt{-\frac{-a^{2}+x}{a^{2}}}\sqrt{\frac{-1+x}{a^{2}-1}}\sqrt{\frac{x}{a^{2}}}EllipticF\left(\sqrt{-\frac{-a^{2}+x}{a^{2}}},\sqrt{\frac{a^{2}}{a^{2}-1}}\right)-4\frac{a^{3}}{\sqrt{-a^{2}x^{2}+a^{2}x+x^{3}-x^{2}}\,(a^{2}-a)}\sqrt{-\frac{-a^{2}+x}{a^{2}}}\sqrt{\frac{-1+x}{a^{2}-1}}\sqrt{\frac{x}{a^{2}}}EllipticPi\left(\sqrt{-\frac{-a^{2}+x}{a^{2}}},\frac{a^{2}}{a^{2}-a},\sqrt{\frac{a^{2}}{a^{2}-1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+x)/(-a+x)/(a^2*x-(a^2+1)*x^2+x^3)^(1/2),x)
```

```
[Out] -2*a^2*(-(-a^2+x)/a^2)^(1/2)*((-1+x)/(a^2-1))^(1/2)*(x/a^2)^(1/2)
/(-a^2*x^2+a^2*x+x^3-x^2)^(1/2)*EllipticF((-(-a^2+x)/a^2)^(1/2),(
a^2/(a^2-1))^(1/2))-4*a^3*(-(-a^2+x)/a^2)^(1/2)*((-1+x)/(a^2-1))^
(1/2)*(x/a^2)^(1/2)/(-a^2*x^2+a^2*x+x^3-x^2)^(1/2)/(a^2-a)*Ellipt
icPi((-(-a^2+x)/a^2)^(1/2),a^2/(a^2-a),(a^2/(a^2-1))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{a+x}{\sqrt{a^{2}x-(a^{2}+1)x^{2}+x^{3}}(a-x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(a + x)/(sqrt(a^2*x - (a^2 + 1)*x^2 + x^3)*(a - x)),x, algorithm="maxim
```

```
[Out] -integrate((a + x)/(sqrt(a^2*x - (a^2 + 1)*x^2 + x^3)*(a - x)), x
)
```

Fricas [A] time = 0.265928, size = 72, normalized size = 0.83

$$\frac{\arctan\left(\frac{a^{2}-2\left(a^{2}-a+1\right)x+x^{2}}{2\sqrt{a^{2}x-(a^{2}+1)x^{2}+x^{3}}(a-1)}\right)}{a-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(a + x)/(sqrt(a^2*x - (a^2 + 1)*x^2 + x^3)*(a - x)),x, algorithm="frica
```

```
[Out] arctan(1/2*(a^2 - 2*(a^2 - a + 1)*x + x^2)/(sqrt(a^2*x - (a^2 + 1
)*x^2 + x^3)*(a - 1)))/(a - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{a+x}{\sqrt{x\left(-a^{2}+x\right)\left(x-1\right)}\left(-a+x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+x)/(-a+x)/(a**2*x-(a**2+1)*x**2+x**3)**(1/2),x)
```

```
[Out] Integral((a + x)/(sqrt(x*(-a**2 + x)*(x - 1))*(-a + x)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{a+x}{\sqrt{a^2x-(a^2+1)x^2+x^3}(a-x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(a + x)/(sqrt(a^2*x - (a^2 + 1)*x^2 + x^3)*(a - x)),x, algorithm="giac"
```

```
[Out] integrate(-(a + x)/(sqrt(a^2*x - (a^2 + 1)*x^2 + x^3)*(a - x)), x
)
```

3.80 $\int \frac{-2+a+x}{(-a+x)\sqrt{(2-a)ax+(-1-2a+a^2)x^2+x^3}}\,dx$

Optimal. Leaf size=1

$$0$$

```
[Out] 0
```

Rubi [C] time = 3.07177, antiderivative size = 579, normalized size of antiderivative = 579., number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$\frac{2(1-a)\sqrt{x}\sqrt{-(-a^2+2a+1)x+(2-a)a+x^2}\tan^{-1}\left(\frac{\sqrt{-a^2+2a-1}\sqrt{x}}{\sqrt{-(-a^2+2a+1)x+(2-a)a+x^2}}\right)}{a\sqrt{-a^2+2a-1}\sqrt{-(-a^2+2a+1)x^2+(2-a)ax+x^3}}+\frac{\sqrt[4]{(2-a)a}\left(-a+\sqrt{(2-a)a}+2\right)\sqrt{x}\left(\frac{x}{\sqrt{(2-a)a}}+1\right)\sqrt{\frac{-(-a^2+2a+1)x+(2-a)a+x^2}{(2-a)a\left(\frac{x}{\sqrt{(2-a)a}}+1\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{(2-a)a}}\right)|\frac{1}{4}\left(\frac{-a^2+2a+1}{\sqrt{(2-a)a}}+2\right)\right)}{\left(a+\sqrt{(2-a)a}\right)\sqrt{-(-a^2+2a+1)x^2+(2-a)ax+x^3}}+\frac{\left(\sqrt{2-a}-\sqrt{a}\right)(1-a)\sqrt[4]{(2-a)a}\sqrt{x}\left(\frac{x}{\sqrt{(2-a)a}}+1\right)\sqrt{\frac{-(-a^2+2a+1)x+(2-a)a+x^2}{(2-a)a\left(\frac{x}{\sqrt{(2-a)a}}+1\right)^2}}\left(\frac{\left(\sqrt{2-a}+\sqrt{a}\right)^2}{4\sqrt{(2-a)a}};2\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{(2-a)a}}\right)|\frac{1}{4}\left(\frac{-a^2+2a+1}{\sqrt{(2-a)a}}\right.\right.}{\left(\sqrt{2-a}+\sqrt{a}\right)a\sqrt{-(-a^2+2a+1)x^2+(2-a)ax+x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(-2 + a + x)/((-a + x)*Sqrt[(2 - a)*a*x + (-1 - 2*a + a^2)*x^2 + x^3]),x]
```

```
[Out] (2*(1 - a)*Sqrt[x]*Sqrt[(2 - a)*a - (1 + 2*a - a^2)*x + x^2]*ArcT
an[(Sqrt[-1 + 2*a - a^2]*Sqrt[x])/Sqrt[(2 - a)*a - (1 + 2*a - a^2
)*x + x^2]])/(a*Sqrt[-1 + 2*a - a^2]*Sqrt[(2 - a)*a*x - (1 + 2*a
- a^2)*x^2 + x^3]) + (((2 - a)*a)^(1/4)*(2 - a + Sqrt[(2 - a)*a])
*Sqrt[x]*(1 + x/Sqrt[(2 - a)*a])*Sqrt[((2 - a)*a - (1 + 2*a - a^2
)*x + x^2)/((2 - a)*a*(1 + x/Sqrt[(2 - a)*a])^2)]*EllipticF[2*Arc
Tan[Sqrt[x]/((2 - a)*a)^(1/4)], (2 + (1 + 2*a - a^2)/Sqrt[(2 - a)
*a])/4])/((a + Sqrt[(2 - a)*a])*Sqrt[(2 - a)*a*x - (1 + 2*a - a^2
)*x^2 + x^3]) + ((Sqrt[2 - a] - Sqrt[a])*(1 - a)*((2 - a)*a)^(1/4
)*Sqrt[x]*(1 + x/Sqrt[(2 - a)*a])*Sqrt[((2 - a)*a - (1 + 2*a - a^
2)*x + x^2)/((2 - a)*a*(1 + x/Sqrt[(2 - a)*a])^2)]*EllipticPi[(Sq
rt[2 - a] + Sqrt[a])^2/(4*Sqrt[(2 - a)*a]), 2*ArcTan[Sqrt[x]/((2
- a)*a)^(1/4)], (2 + (1 + 2*a - a^2)/Sqrt[(2 - a)*a])/4])/((Sqrt[
2 - a] + Sqrt[a])*a*Sqrt[(2 - a)*a*x - (1 + 2*a - a^2)*x^2 + x^3]
)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-2+a+x)/(-a+x)/((-a+2)*a*x+(a**2-2*a-1)*x**2+x**3)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.464059, size = 100, normalized size = 100.

$$-\frac{2i\sqrt{\frac{1}{x-1}+1}(x-1)^{3/2}\sqrt{\frac{(a-1)^2}{x-1}+1}\left(F\left(i\sinh^{-1}\left(\frac{1}{\sqrt{x-1}}\right)|(a-1)^2\right)-2\left(1-a;i\sinh^{-1}\left(\frac{1}{\sqrt{x-1}}\right)|(a-1)^2\right)\right)}{\sqrt{(x-1)x\left(a^2-2a+x\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + a + x)/((-a + x)*Sqrt[(2 - a)*a*x + (-1 - 2*a + a^2)*x^2 + x^3]),x]
```

```
[Out] ((-2*I)*Sqrt[1 + (-1 + x)^(-1)]*Sqrt[1 + (-1 + a)^2/(-1 + x)]*(-1
 + x)^(3/2)*(EllipticF[I*ArcSinh[1/Sqrt[-1 + x]], (-1 + a)^2] - 2
*EllipticPi[1 - a, I*ArcSinh[1/Sqrt[-1 + x]], (-1 + a)^2]))/Sqrt[
(-1 + x)*x*(-2*a + a^2 + x)]
```

Maple [C] time = 0.063, size = 317, normalized size = 317.

$$2\,{\frac {{a}^{2}-2\,a}{\sqrt {{a}^{2}{x}^{2}-{a}^{2}x-2\,a{x}^{2}+{x}^{3}+2\,ax-{x}^{2}}}\sqrt {{\frac {{a}^{2}-2\,a+x}{{a}^{2}-2\,a}}}\sqrt {{\frac {-1+x}{-{a}^{2}+2\,a-1}}}\sqrt {{\frac {x}{-{a}^{2}+2\,a}}}{\it EllipticF} \left(\sqrt {{\frac {{a}^{2}-2\,a+x}{{a}^{2}-2\,a}}},\sqrt {{\frac {-{a}^{2}+2\,a}{-{a}^{2}+2\,a-}}} \right.}$$
$$+2\,{\frac { \left(2\,a-2 \right) \left({a}^{2}-2\,a \right) }{\sqrt {{a}^{2}{x}^{2}-{a}^{2}x-2\,a{x}^{2}+{x}^{3}+2\,ax-{x}^{2}} \left(-{a}^{2}+a \right) }\sqrt {{\frac {{a}^{2}-2\,a+x}{{a}^{2}-2\,a}}}\sqrt {{\frac {-1+x}{-{a}^{2}+2\,a-1}}}\sqrt {{\frac {x}{-{a}^{2}+2\,a}}}{\it EllipticPi} \left(\sqrt {{\frac {{a}^{2}-2\,a+x}{{a}^{2}-2\,a}}},{\frac{-a}{-}} \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2+a+x)/(-a+x)/((2-a)*a*x+(a^2-2*a-1)*x^2+x^3)^(1/2),x)
```

```
[Out] 2*(a^2-2*a)*((a^2-2*a+x)/(a^2-2*a))^(1/2)*((-1+x)/(-a^2+2*a-1))^(
1/2)*(x/(-a^2+2*a))^(1/2)/(a^2*x^2-a^2*x-2*a*x^2+x^3+2*a*x-x^2)^(
1/2)*EllipticF(((a^2-2*a+x)/(a^2-2*a))^(1/2),((-a^2+2*a)/(-a^2+2*
a-1))^(1/2))+2*(2*a-2)*(a^2-2*a)*((a^2-2*a+x)/(a^2-2*a))^(1/2)*((
-1+x)/(-a^2+2*a-1))^(1/2)*(x/(-a^2+2*a))^(1/2)/(a^2*x^2-a^2*x-2*a
*x^2+x^3+2*a*x-x^2)^(1/2)/(-a^2+a)*EllipticPi(((a^2-2*a+x)/(a^2-2
*a))^(1/2),(-a^2+2*a)/(-a^2+a),((-a^2+2*a)/(-a^2+2*a-1))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{a + x - 2}{\sqrt{-(a - 2)ax + (a^2 - 2\,a - 1)x^2 + x^3}(a - x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(a + x - 2)/(sqrt(-(a - 2)*a*x + (a^2 - 2*a - 1)*x^2 + x^3)*(a - x)),x, a
```

```
[Out] -integrate((a + x - 2)/(sqrt(-(a - 2)*a*x + (a^2 - 2*a - 1)*x^2 +
 x^3)*(a - x)), x)
```

Fricas [A] time = 0.263826, size = 95, normalized size = 95.

$$\frac{\log\left(-\frac{a^2 - 2\,(a^2 - a)\,x - x^2 + 2\,\sqrt{(a^2 - 2\,a - 1)x^2 + x^3 - (a^2 - 2\,a)x}\,a}{a^2 - 2\,ax + x^2}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(a + x - 2)/(sqrt(-(a - 2)*a*x + (a^2 - 2*a - 1)*x^2 + x^3)*(a - x)),x, a
```

```
[Out] log(-(a^2 - 2*(a^2 - a)*x - x^2 + 2*sqrt((a^2 - 2*a - 1)*x^2 + x^
3 - (a^2 - 2*a)*x)*a)/(a^2 - 2*a*x + x^2))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + x - 2}{\sqrt{x(x - 1)(a^2 - 2a + x)}(-a + x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+a+x)/(-a+x)/((-a+2)*a*x+(a**2-2*a-1)*x**2+x**3)**(1/2),x)
```

```
[Out] Integral((a + x - 2)/(sqrt(x*(x - 1)*(a**2 - 2*a + x))*(-a + x)),
 x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{a + x - 2}{\sqrt{-(a - 2)ax + (a^2 - 2\,a - 1)x^2 + x^3}(a - x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(a + x - 2)/(sqrt(-(a - 2)*a*x + (a^2 - 2*a - 1)*x^2 + x^3)*(a - x)),x, a
```

```
[Out] integrate(-(a + x - 2)/(sqrt(-(a - 2)*a*x + (a^2 - 2*a - 1)*x^2 +
 x^3)*(a - x)), x)
```

3.81 $\int \frac{-a+(-1+2a)x}{(-a+x)\sqrt{a^2x-(-1+2a+a^2)x^2+(-1+2a)x^3}}\,dx$

Optimal. Leaf size=46

$$\log\left(\frac{-2\left(\sqrt{(1-x)x\left(a^2-2ax+x\right)}+x\right)-a^2+2ax+x^2}{(a-x)^2}\right)$$

```
[Out] Log[(-a^2 + 2*a*x + x^2 - 2*(x + Sqrt[(1 - x)*x*(a^2 + x - 2*a*x)
]))/(a - x)^2]
```

Rubi [C] time = 2.93293, antiderivative size = 180, normalized size of antiderivative = 3.91, number of steps used = 7, number of rules used = 7, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}} = 0.137$

$$\frac{4(1-a)\sqrt{1-x}\sqrt{x}\sqrt{\frac{(1-2a)x}{a^2}+1}\,\Pi\left(\frac{1}{a};\sin^{-1}\left(\sqrt{x}\right)|-\frac{1-2a}{a^2}\right)}{\sqrt{\left(-a^2-2a+1\right)x^2+a^2x-(1-2a)x^3}}-\frac{2(1-2a)\sqrt{1-x}\sqrt{x}\sqrt{\frac{(1-2a)x}{a^2}+1}F\left(\sin^{-1}\left(\sqrt{x}\right)|-\frac{1-2a}{a^2}\right)}{\sqrt{\left(-a^2-2a+1\right)x^2+a^2x-(1-2a)x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-a + (-1 + 2*a)*x)/((-a + x)*Sqrt[a^2*x - (-1 + 2*a + a^2)*x^2 + (-
1 + 2*a)*x^3]),x]
```

```
[Out] (-2*(1 - 2*a)*Sqrt[1 - x]*Sqrt[x]*Sqrt[1 + ((1 - 2*a)*x)/a^2]*Ell
ipticF[ArcSin[Sqrt[x]], -((1 - 2*a)/a^2)])/Sqrt[a^2*x + (1 - 2*a
- a^2)*x^2 - (1 - 2*a)*x^3] + (4*(1 - a)*Sqrt[1 - x]*Sqrt[x]*Sqrt
[1 + ((1 - 2*a)*x)/a^2]*EllipticPi[a^(-1), ArcSin[Sqrt[x]], -((1
- 2*a)/a^2)])/Sqrt[a^2*x + (1 - 2*a - a^2)*x^2 - (1 - 2*a)*x^3]
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-a+(-1+2*a)*x)/(-a+x)/(a**2*x-(a**2+2*a-1)*x**2+(-
1+2*a)*x**3)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.590397, size = 133, normalized size = 2.89

$$\frac{2i(x-1)^{3/2}\sqrt{\frac{x}{x-1}}\sqrt{-\frac{a^2-2ax+x}{(2a-1)(x-1)}}\left(2a\,\Pi\left(1-a;i\sinh^{-1}\left(\frac{1}{\sqrt{x-1}}\right)|-\frac{(a-1)^2}{2a-1}\right)-F\left(i\sinh^{-1}\left(\frac{1}{\sqrt{x-1}}\right)|-\frac{(a-1)^2}{2a-1}\right)\right)}{\sqrt{-(x-1)x\left(a^2-2ax+x\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a + (-1 + 2*a)*x)/((-a + x)*Sqrt[a^2*x - (-1 + 2*a + a^2)*x^2 + (-
1 + 2*a)*x^3]),x]
```

```
[Out] ((2*I)*(-1 + x)^(3/2)*Sqrt[x/(-1 + x)]*Sqrt[-((a^2 + x - 2*a*x)/(
(-1 + 2*a)*(-1 + x)))]*(-EllipticF[I*ArcSinh[1/Sqrt[-1 + x]], -((
```

```
-1 + a)^2/(-1 + 2*a))] + 2*a*EllipticPi[1 - a, I*ArcSinh[1/Sqrt[-
1 + x]], -((-1 + a)^2/(-1 + 2*a))]))/Sqrt[-((-1 + x)*x*(a^2 + x -
 2*a*x))]
```

Maple [C] time = 0.079, size = 536, normalized size = 11.7

$$2\frac{a^2}{(-1+2a)\sqrt{-a^2x^2+2ax^3+a^2x-2ax^2-x^3+x^2}}\sqrt{-\frac{-1+2a}{a^2}\left(x-\frac{a^2}{-1+2a}\right)}\sqrt{(-1+x)\left(\frac{a^2}{-1+2a}-1\right)^{-1}}\sqrt{\frac{(-1+2a)x}{a^2}}$$
$$-4\frac{a^3}{(-1+2a)\sqrt{-a^2x^2+2ax^3+a^2x-2ax^2-x^3+x^2}}\sqrt{-\frac{-1+2a}{a^2}\left(x-\frac{a^2}{-1+2a}\right)}\sqrt{(-1+x)\left(\frac{a^2}{-1+2a}-1\right)^{-1}}\sqrt{\frac{(-1+2a)}{a^2}}$$
$$-4\frac{a^3(a-1)}{(-1+2a)\sqrt{-a^2x^2+2ax^3+a^2x-2ax^2-x^3+x^2}}\sqrt{-\frac{-1+2a}{a^2}\left(x-\frac{a^2}{-1+2a}\right)}\sqrt{(-1+x)\left(\frac{a^2}{-1+2a}-1\right)^{-1}}\sqrt{\frac{(-1+2a)}{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a+(-1+2*a)*x)/(-a+x)/(a^2*x-(a^2+2*a-1)*x^2+(-1+2*a)*x^3)^(1/2),x)
```

```
[Out] 2*a^2/(-1+2*a)*(-(x-a^2/(-1+2*a))/a^2*(-1+2*a))^(1/2)*((-1+x)/(a^
2/(-1+2*a)-1))^(1/2)*(x/a^2*(-1+2*a))^(1/2)/(-a^2*x^2+2*a*x^3+a^2
*x-2*a*x^2-x^3+x^2)^(1/2)*EllipticF((-(x-a^2/(-1+2*a))/a^2*(-1+2*
a))^(1/2),(a^2/(-1+2*a)/(a^2/(-1+2*a)-1))^(1/2))-4*a^3/(-1+2*a)*(
-(x-a^2/(-1+2*a))/a^2*(-1+2*a))^(1/2)*((-1+x)/(a^2/(-1+2*a)-1))^(
1/2)*(x/a^2*(-1+2*a))^(1/2)/(-a^2*x^2+2*a*x^3+a^2*x-2*a*x^2-x^3+x
^2)^(1/2)*EllipticF((-(x-a^2/(-1+2*a))/a^2*(-1+2*a))^(1/2),(a^2/(
-1+2*a)/(a^2/(-1+2*a)-1))^(1/2))-4*a^3*(a-1)/(-1+2*a)*(-(x-a^2/(-
1+2*a))/a^2*(-1+2*a))^(1/2)*((-1+x)/(a^2/(-1+2*a)-1))^(1/2)*(x/a^
2*(-1+2*a))^(1/2)/(-a^2*x^2+2*a*x^3+a^2*x-2*a*x^2-x^3+x^2)^(1/2)/
(a^2/(-1+2*a)-a)*EllipticPi((-(x-a^2/(-1+2*a))/a^2*(-1+2*a))^(1/2
),a^2/(-1+2*a)/(a^2/(-1+2*a)-a),(a^2/(-1+2*a)/(a^2/(-1+2*a)-1))^(
1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(2a-1)x-a}{\sqrt{(2a-1)x^3+a^2x-(a^2+2a-1)x^2}(a-x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-((2*a - 1)*x - a)/(sqrt((2*a - 1)*x^3 + a^2*x - (a^2 + 2*a - 1)*x^2)*(a
```

```
[Out] -integrate(((2*a - 1)*x - a)/(sqrt((2*a - 1)*x^3 + a^2*x - (a^2 +
 2*a - 1)*x^2)*(a - x)), x)
```

Fricas [A] time = 0.25922, size = 85, normalized size = 1.85

$$\log\left(-\frac{a^2-2(a-1)x-x^2+2\sqrt{(2a-1)x^3+a^2x-(a^2+2a-1)x^2}}{a^2-2ax+x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-((2*a - 1)*x - a)/(sqrt((2*a - 1)*x^3 + a^2*x - (a^2 + 2*a - 1)*x^2)*(a
```

```
[Out] log(-(a^2 - 2*(a - 1)*x - x^2 + 2*sqrt((2*a - 1)*x^3 + a^2*x - (a
^2 + 2*a - 1)*x^2))/(a^2 - 2*a*x + x^2))
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+(-1+2*a)*x)/(-a+x)/(a**2*x-(a**2+2*a-1)*x**2+(-1+2*a)*x**3)**(1/
```

```
[Out] Exception raised: RecursionError
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(2a-1)x-a}{\sqrt{(2a-1)x^3+a^2x-(a^2+2a-1)x^2}(a-x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-((2*a - 1)*x - a)/(sqrt((2*a - 1)*x^3 + a^2*x - (a^2 + 2*a - 1)*x^2)*(a
```

```
[Out] integrate(-((2*a - 1)*x - a)/(sqrt((2*a - 1)*x^3 + a^2*x - (a^2 +
 2*a - 1)*x^2)*(a - x)), x)
```

3.82 $\int \frac{1-\sqrt[3]{2}x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=32

$$\frac{2\tan^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{x^3+1}}\right)}{\sqrt{3}}$$

```
[Out] (2*ArcTan[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[1 + x^3]])/Sqrt[3]
```

Rubi [A] time = 0.144692, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069

$$\frac{2\tan^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{x^3+1}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - 2^(1/3)*x)/((2^(2/3) + x)*Sqrt[1 + x^3]),x]

[Out] (2*ArcTan[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[1 + x^3]])/Sqrt[3]
```

Rubi in Sympy [A] time = 63.8776, size = 469, normalized size = 14.66

$$\frac{6\sqrt{\frac{x^2-x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{-\sqrt{3}+2}\,(x+1)\,\text{atan}\left(\frac{3^{\frac{3}{4}}\sqrt{1+\sqrt[3]{2}}\sqrt{-4\sqrt{3}+8}\sqrt{-\frac{\left(-x-1+\sqrt{3}\right)^2}{\left(x+1+\sqrt{3}\right)^2}+1}}{6\sqrt{-1+\sqrt[3]{2}}\sqrt{\frac{\left(-x-1+\sqrt{3}\right)^2}{\left(x+1+\sqrt{3}\right)^2}-4\sqrt{3}+7}}\right)}{\sqrt{\frac{x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{-1+\sqrt[3]{2}}\left(1+\sqrt[3]{2}\right)^{\frac{3}{2}}\sqrt{-4\sqrt{3}+8}\sqrt{x^3+1}} - \frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{x^2-x+1}{\left(x+1+\sqrt{3}\right)^2}}\left(1+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)\sqrt{\sqrt{3}+2}\,(x+1)\,F\left(\text{asin}\left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{3\sqrt{\frac{x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{x^3+1}\left(-2^{\frac{2}{3}}+1+\sqrt{3}\right)} + \frac{12\sqrt[4]{3}\sqrt{\frac{x^2-x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{-\sqrt{3}+2}\,(x+1)\,\Pi\left(\frac{\left(-2^{\frac{2}{3}}+1+\sqrt{3}\right)^2}{\left(-1+2^{\frac{2}{3}}+\sqrt{3}\right)^2};\text{asin}\left(\frac{-x-1+\sqrt{3}}{x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{-4\sqrt{3}+7}\sqrt{x^3+1}\left(-2^{\frac{2}{3}}+1+\sqrt{3}\right)\left(-\sqrt{3}-2^{\frac{2}{3}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1-2**(1/3)*x)/(2**(2/3)+x)/(x**3+1)**(1/2),x)

[Out] 6*sqrt((x**2 - x + 1)/(x + 1 + sqrt(3))**2)*sqrt(-sqrt(3) + 2)*(x
 + 1)*atan(3**(3/4)*sqrt(1 + 2**(1/3))*sqrt(-4*sqrt(3) + 8)*sqrt(
-(-x - 1 + sqrt(3))**2/(x + 1 + sqrt(3))**2 + 1)/(6*sqrt(-1 + 2**
(1/3))*sqrt((-x - 1 + sqrt(3))**2/(x + 1 + sqrt(3))**2 - 4*sqrt(3
) + 7)))/(sqrt((x + 1)/(x + 1 + sqrt(3))**2)*sqrt(-1 + 2**(1/3))*
(1 + 2**(1/3))**(3/2)*sqrt(-4*sqrt(3) + 8)*sqrt(x**3 + 1)) - 2*3*
*(3/4)*sqrt((x**2 - x + 1)/(x + 1 + sqrt(3))**2)*(1 + 2**(1/3)*(1
 + sqrt(3)))*sqrt(sqrt(3) + 2)*(x + 1)*elliptic_f(asin((x - sqrt(
3) + 1)/(x + 1 + sqrt(3))), -7 - 4*sqrt(3))/(3*sqrt((x + 1)/(x +
1 + sqrt(3))**2)*sqrt(x**3 + 1)*(-2**(2/3) + 1 + sqrt(3))) + 12*3
```

```
**(1/4)*sqrt((x**2 - x + 1)/(x + 1 + sqrt(3))**2)*sqrt(-sqrt(3) +
 2)*(x + 1)*elliptic_pi((-2**(2/3) + 1 + sqrt(3))**2/(-1 + 2**(2/
3) + sqrt(3))**2, asin((-x - 1 + sqrt(3))/(x + 1 + sqrt(3))), -7
- 4*sqrt(3))/(sqrt((x + 1)/(x + 1 + sqrt(3))**2)*sqrt(-4*sqrt(3)
+ 7)*sqrt(x**3 + 1)*(-2**(2/3) + 1 + sqrt(3))*(-sqrt(3) - 2**(2/3
) + 1))
```

Mathematica [C] time = 0.516328, size = 323, normalized size = 10.09

$$-\frac{2\sqrt{\frac{2}{3}}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\left(\sqrt{2ix+\sqrt{3}-i}\left(\left(-3i\sqrt[3]{2}+4\sqrt{3}+\sqrt[3]{2}\sqrt{3}\right)x+\sqrt[3]{2}\sqrt{3}-2\sqrt{3}+3i\sqrt[3]{2}+6i\right)F\left(\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)-6i\sqrt{}}{\left(1+2\,2^{2/3}-i\sqrt{3}\right)\sqrt{-2ix+\sqrt{3}+i}\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(1 - 2^(1/3)*x)/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*Sqrt[2/3]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(Sqrt[-I + Sqrt[3
] + (2*I)*x]*(6*I + (3*I)*2^(1/3) - 2*Sqrt[3] + 2^(1/3)*Sqrt[3] +
 ((-3*I)*2^(1/3) + 4*Sqrt[3] + 2^(1/3)*Sqrt[3])*x)*EllipticF[ArcS
in[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3
*I + Sqrt[3])] - (6*I)*Sqrt[3]*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1
 - x + x^2]*EllipticPi[(2*Sqrt[3])/(I + (2*I)*2^(2/3) + Sqrt[3]),
 ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3
])/(3*I + Sqrt[3])]))/((1 + 2*2^(2/3) - I*Sqrt[3])*Sqrt[I + Sqrt[
3] - (2*I)*x]*Sqrt[1 + x^3])
```

Maple [C] time = 0.093, size = 258, normalized size = 8.1

$$-2\,\frac{\sqrt[3]{2}\left(3/2-i/2\sqrt{3}\right)}{\sqrt{x^3+1}}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}\,EllipticF\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},\sqrt{\frac{-3/2+i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\right)$$
$$+6\,\frac{3/2-i/2\sqrt{3}}{\sqrt{x^3+1}\left(2^{2/3}-1\right)}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}\,EllipticPi\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},\frac{-3/2+i/2\sqrt{3}}{2^{2/3}-1},\sqrt{\frac{-3}{-3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1-2^(1/3)*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x)
```

```
[Out] -2*2^(1/3)*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*
((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*
3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+
x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3
^(1/2)))^(1/2))+6*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))
^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2
+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)/(2^(2/3
)-1)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(
1/2))/(2^(2/3)-1),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/
2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{2^{\frac{1}{3}}x-1}{\sqrt{x^3+1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(2^(1/3)*x - 1)/(sqrt(x^3 + 1)*(x + 2^(2/3))),x, algorithm="maxima")
```

```
[Out] -integrate((2^(1/3)*x - 1)/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(2^(1/3)*x - 1)/(sqrt(x^3 + 1)*(x + 2^(2/3))),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt[3]{2}x}{x\sqrt{x^3+1}+2^{\frac{2}{3}}\sqrt{x^3+1}}\,dx-\int \left(-\frac{1}{x\sqrt{x^3+1}+2^{\frac{2}{3}}\sqrt{x^3+1}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1-2**(1/3)*x)/(2**(2/3)+x)/(x**3+1)**(1/2),x)
```

```
[Out] -Integral(2**(1/3)*x/(x*sqrt(x**3 + 1) + 2**(2/3)*sqrt(x**3 + 1))
, x) - Integral(-1/(x*sqrt(x**3 + 1) + 2**(2/3)*sqrt(x**3 + 1)),
x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{2^{\frac{1}{3}}x-1}{\sqrt{x^3+1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(2^(1/3)*x - 1)/(sqrt(x^3 + 1)*(x + 2^(2/3))),x, algorithm="giac")
```

```
[Out] integrate(-(2^(1/3)*x - 1)/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

3.83 $\int \frac{1+x}{(-2+x)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=23

$$-\frac{2}{3}\tanh^{-1}\left(\frac{(x+1)^2}{3\sqrt{x^3+1}}\right)$$

```
[Out] (-2*ArcTanh[(1 + x)^2/(3*Sqrt[1 + x^3])])/3
```

Rubi [A] time = 0.0910752, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$-\frac{2}{3}\tanh^{-1}\left(\frac{(x+1)^2}{3\sqrt{x^3+1}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x)/((-2 + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*ArcTanh[(1 + x)^2/(3*Sqrt[1 + x^3])])/3
```

Rubi in Sympy [A] time = 41.593, size = 371, normalized size = 16.13

$$-\frac{3\sqrt{\frac{x^2-x+1}{\left(x+1+\sqrt{3}\right)^2}}\left(\frac{\sqrt{3}}{3}+1\right)(x+1)\operatorname{atanh}\left(\frac{3^{\frac{3}{4}}\sqrt{-\sqrt{3}+2}\sqrt{-\frac{\left(-x-1+\sqrt{3}\right)^2}{\left(x+1+\sqrt{3}\right)^2}+1}}{3\sqrt{\frac{\left(-x-1+\sqrt{3}\right)^2}{\left(x+1+\sqrt{3}\right)^2}-4\sqrt{3}+7}}\right)}{\sqrt{\frac{x+1}{\left(x+1+\sqrt{3}\right)^2}}\left(\sqrt{3}+3\right)\sqrt{x^3+1}}+\frac{2\sqrt[4]{3}\sqrt{\frac{x^2-x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{\sqrt{3}+2}\,(x+1)\,F\left(\operatorname{asin}\left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+1+\sqrt{3}\right)^2}}\left(\sqrt{3}+3\right)\sqrt{x^3+1}}+\frac{12\sqrt[4]{3}\sqrt{\frac{x^2-x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{-\sqrt{3}+2}\,(x+1)\left(4\sqrt{3}+7;\operatorname{asin}\left(\frac{-x-1+\sqrt{3}}{x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+1+\sqrt{3}\right)^2}}\sqrt{-4\sqrt{3}+7}\left(-\sqrt{3}+3\right)\left(\sqrt{3}+3\right)\sqrt{x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+x)/(-2+x)/(x**3+1)**(1/2),x)
```

```
[Out] -3*sqrt((x**2 - x + 1)/(x + 1 + sqrt(3))**2)*(sqrt(3)/3 + 1)*(x +
 1)*atanh(3**(3/4)*sqrt(-sqrt(3) + 2)*sqrt(-(-x - 1 + sqrt(3))**2
/(x + 1 + sqrt(3))**2 + 1)/(3*sqrt((-x - 1 + sqrt(3))**2/(x + 1 +
 sqrt(3))**2 - 4*sqrt(3) + 7)))/(sqrt((x + 1)/(x + 1 + sqrt(3))**
2)*(sqrt(3) + 3)*sqrt(x**3 + 1)) + 2*3**(1/4)*sqrt((x**2 - x + 1)
/(x + 1 + sqrt(3))**2)*sqrt(sqrt(3) + 2)*(x + 1)*elliptic_f(asin(
(x - sqrt(3) + 1)/(x + 1 + sqrt(3))), -7 - 4*sqrt(3))/(sqrt((x +
1)/(x + 1 + sqrt(3))**2)*(sqrt(3) + 3)*sqrt(x**3 + 1)) + 12*3**(1
/4)*sqrt((x**2 - x + 1)/(x + 1 + sqrt(3))**2)*sqrt(-sqrt(3) + 2)*
(x + 1)*elliptic_pi(4*sqrt(3) + 7, asin((-x - 1 + sqrt(3))/(x + 1
 + sqrt(3))), -7 - 4*sqrt(3))/(sqrt((x + 1)/(x + 1 + sqrt(3))**2)
*sqrt(-4*sqrt(3) + 7)*(-sqrt(3) + 3)*(sqrt(3) + 3)*sqrt(x**3 + 1)
)
```

Mathematica [C] time = 0.282679, size = 262, normalized size = 11.39

$$\frac{2\sqrt{6}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\left(\sqrt{2ix+\sqrt{3}-i}\left(-i\sqrt{3}x+x+i\sqrt{3}+1\right)F\left(\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)-2\sqrt{3}\sqrt{-2ix+\sqrt{3}+i}\sqrt{x^2-x+1}\left(\frac{2\sqrt{3}}{-3i+\cdots}\right.\right.}{\left(\sqrt{3}-3i\right)\sqrt{-2ix+\sqrt{3}+i}\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(1 + x)/((-2 + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[6]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(Sqrt[-I + Sqrt[3] +
 (2*I)*x]*(1 + I*Sqrt[3] + x - I*Sqrt[3]*x)*EllipticF[ArcSin[Sqrt
[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sq
rt[3])] - 2*Sqrt[3]*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 - x + x^2]
*EllipticPi[(2*Sqrt[3])/(-3*I + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3]
 - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/((
-3*I + Sqrt[3])*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 + x^3])
```

Maple [C] time = 0.031, size = 240, normalized size = 10.4

$$2\,\frac{3/2-i/2\sqrt{3}}{\sqrt{x^3+1}}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}\,EllipticF\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},\sqrt{\frac{-3/2+i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\right)$$
$$-2\,\frac{3/2-i/2\sqrt{3}}{\sqrt{x^3+1}}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}\,EllipticPi\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},-i/6\sqrt{3}+1/2,\sqrt{\frac{-3/2+i/\cdots}{-3/2-i/\cdots}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+x)/(-2+x)/(x^3+1)^(1/2),x)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1
/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/
(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1
/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^
(1/2))-2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((
x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^
(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticPi(((1+x
)/(3/2-1/2*I*3^(1/2)))^(1/2),-1/6*I*3^(1/2)+1/2,((-3/2+1/2*I*3^(1
/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1}{\sqrt{x^3+1}(x-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 1)/(sqrt(x^3 + 1)*(x - 2)),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)/(sqrt(x^3 + 1)*(x - 2)), x)
```

Fricas [A] time = 0.257884, size = 59, normalized size = 2.57

$$\frac{1}{3}\log\left(\frac{x^3+12\,x^2-6\,\sqrt{x^3+1}(x+1)-6\,x+10}{x^3-6\,x^2+12\,x-8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 1)/(sqrt(x^3 + 1)*(x - 2)),x, algorithm="fricas")
```

```
[Out] 1/3*log((x^3 + 12*x^2 - 6*sqrt(x^3 + 1)*(x + 1) - 6*x + 10)/(x^3
- 6*x^2 + 12*x - 8))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1}{\sqrt{(x+1)(x^2-x+1)}(x-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+x)/(-2+x)/(x**3+1)**(1/2),x)
```

```
[Out] Integral((x + 1)/(sqrt((x + 1)*(x**2 - x + 1))*(x - 2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1}{\sqrt{x^3+1}(x-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 1)/(sqrt(x^3 + 1)*(x - 2)),x, algorithm="giac")
```

```
[Out] integrate((x + 1)/(sqrt(x^3 + 1)*(x - 2)), x)
```

3.84 $\int \frac{x}{\sqrt{1+x^3}\left(10+6\sqrt{3}+x^3\right)}\,dx$

Optimal. Leaf size=218

$$-\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)(x+1)}{\sqrt{2}\sqrt{x^3+1}}\right)}{2\sqrt{2}3^{3/4}}-\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{x^3+1}}{\sqrt{2}3^{3/4}}\right)}{3\sqrt{2}3^{3/4}}-\frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(-2x+\sqrt{3}+1\right)}{\sqrt{2}\sqrt{x^3+1}}\right)}{3\sqrt{2}\sqrt[4]{3}}-\frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)(x+1)}{\sqrt{2}\sqrt{x^3+1}}\right)}{6\sqrt{2}\sqrt[4]{3}}$$

```
[Out] -((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3])*(1 + x))/(Sqrt[2]*S
qrt[1 + x^3])])/(2*Sqrt[2]*3^(3/4)) - ((2 - Sqrt[3])*ArcTan[((1 -
 Sqrt[3])*Sqrt[1 + x^3])/(Sqrt[2]*3^(3/4))])/(3*Sqrt[2]*3^(3/4))
- ((2 - Sqrt[3])*ArcTanh[(3^(1/4)*(1 + Sqrt[3] - 2*x))/(Sqrt[2]*S
qrt[1 + x^3])])/(3*Sqrt[2]*3^(1/4)) - ((2 - Sqrt[3])*ArcTanh[(3^(
1/4)*(1 - Sqrt[3])*(1 + x))/(Sqrt[2]*Sqrt[1 + x^3])])/(6*Sqrt[2]*
3^(1/4))
```

Rubi [A] time = 0.115921, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.04$

$$-\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)(x+1)}{\sqrt{2}\sqrt{x^3+1}}\right)}{2\sqrt{2}3^{3/4}}-\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{x^3+1}}{\sqrt{2}3^{3/4}}\right)}{3\sqrt{2}3^{3/4}}-\frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(-2x+\sqrt{3}+1\right)}{\sqrt{2}\sqrt{x^3+1}}\right)}{3\sqrt{2}\sqrt[4]{3}}-\frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)(x+1)}{\sqrt{2}\sqrt{x^3+1}}\right)}{6\sqrt{2}\sqrt[4]{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(Sqrt[1 + x^3]*(10 + 6*Sqrt[3] + x^3)),x]
```

```
[Out] -((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3])*(1 + x))/(Sqrt[2]*S
qrt[1 + x^3])])/(2*Sqrt[2]*3^(3/4)) - ((2 - Sqrt[3])*ArcTan[((1 -
 Sqrt[3])*Sqrt[1 + x^3])/(Sqrt[2]*3^(3/4))])/(3*Sqrt[2]*3^(3/4))
- ((2 - Sqrt[3])*ArcTanh[(3^(1/4)*(1 + Sqrt[3] - 2*x))/(Sqrt[2]*S
qrt[1 + x^3])])/(3*Sqrt[2]*3^(1/4)) - ((2 - Sqrt[3])*ArcTanh[(3^(
1/4)*(1 - Sqrt[3])*(1 + x))/(Sqrt[2]*Sqrt[1 + x^3])])/(6*Sqrt[2]*
3^(1/4))
```

Rubi in Sympy [A] time = 3.44886, size = 34, normalized size = 0.16

$$\frac{x^2\,\text{appellf}_1\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-x^3,-\frac{x^3}{10+6\sqrt{3}}\right)}{2\left(10+6\sqrt{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(10+x**3+6*3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] x**2*appellf1(2/3, 1/2, 1, 5/3, -x**3, -x**3/(10 + 6*sqrt(3)))/(2
*(10 + 6*sqrt(3)))
```

Mathematica [C] time = 0.429673, size = 206, normalized size = 0.94

$$-\frac{10\left(26+15\sqrt{3}\right)x^2F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-x^3,-\frac{x^3}{10+6\sqrt{3}}\right)}{\left(5+3\sqrt{3}\right)\sqrt{x^3+1}\left(x^3+6\sqrt{3}+10\right)\left(3x^3\left(F_1\left(\frac{5}{3};\frac{1}{2},2;\frac{8}{3};-x^3,-\frac{x^3}{10+6\sqrt{3}}\right)+\left(5+3\sqrt{3}\right)F_1\left(\frac{5}{3};\frac{3}{2},1;\frac{8}{3};-x^3,-\frac{x^3}{10+6\sqrt{3}}\right)\right)-10\left(5\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x/(Sqrt[1 + x^3]*(10 + 6*Sqrt[3] + x^3)),x]
```

```
[Out] (-10*(26 + 15*Sqrt[3])*x^2*AppellF1[2/3, 1/2, 1, 5/3, -x^3, -(x^3
/(10 + 6*Sqrt[3]))])/((5 + 3*Sqrt[3])*Sqrt[1 + x^3]*(10 + 6*Sqrt[
3] + x^3)*(-10*(5 + 3*Sqrt[3])*AppellF1[2/3, 1/2, 1, 5/3, -x^3, -
(x^3/(10 + 6*Sqrt[3]))] + 3*x^3*(AppellF1[5/3, 1/2, 2, 8/3, -x^3,
 -(x^3/(10 + 6*Sqrt[3]))] + (5 + 3*Sqrt[3])*AppellF1[5/3, 3/2, 1,
 8/3, -x^3, -(x^3/(10 + 6*Sqrt[3]))])))
```

Maple [C] time = 0.283, size = 353, normalized size = 1.6

$$\frac{\left(-1-\sqrt{3}\right)\left(\frac{3}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}{18+9\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i}{2}\sqrt{3}}}\sqrt{\frac{1}{-\frac{3}{2}-\frac{i}{2}\sqrt{3}}\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)}\sqrt{\frac{1}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)}EllipticPi\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i}{2}\sqrt{3}}},\left(-\right.\right.$$

$$-\frac{\sqrt{2}}{18}\sum_{\_alpha=RootOf\left(\_Z^2+\left(-1-\sqrt{3}\right)\_Z+2\sqrt{3}+4\right)}\frac{\left(-\_alpha\sqrt{3}+\_alpha-2\right)\left(-i\sqrt{3}+3\right)\left(-1+2\_alpha-\_alpha\sqrt{3}\right)}{-\sqrt{3}+2\_alpha-1}\sqrt{\frac{1+x}{-i\sqrt{3}+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(10+x^3+6*3^(1/2))/(x^3+1)^(1/2),x)
```

```
[Out] 1/9*(-1-3^(1/2))/(2+3^(1/2))*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*
I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1
/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1
/2)*3^(1/2)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),1/3*(-3/
2+1/2*I*3^(1/2))*3^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2
)))^(1/2))-1/18*2^(1/2)*sum((-_alpha*3^(1/2)+_alpha-2)/(-3^(1/2)+
2*_alpha-1)*(-I*3^(1/2)+3)*((1+x)/(-I*3^(1/2)+3))^(1/2)*((2*x-1-I
*3^(1/2))/(-I*3^(1/2)-3))^(1/2)*((2*x-1+I*3^(1/2))/(I*3^(1/2)-3))
^(1/2)/(x^3+1)^(1/2)*(-1+2*_alpha-_alpha*3^(1/2))*EllipticPi(((1+
x)/(3/2-1/2*I*3^(1/2)))^(1/2),-1/2*I*_alpha+1/3*I*_alpha*3^(1/2)+
1/2*_alpha*3^(1/2)-_alpha-1/6*I*3^(1/2)+1/2,((-3/2+1/2*I*3^(1/2))
/(-3/2-1/2*I*3^(1/2)))^(1/2)),_alpha=RootOf(_Z^2+(-1-3^(1/2))*_Z+
2*3^(1/2)+4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{\left(x^3+6\sqrt{3}+10\right)\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((x^3 + 6*sqrt(3) + 10)*sqrt(x^3 + 1)),x, algorithm="maxima")
```

```
[Out] integrate(x/((x^3 + 6*sqrt(3) + 10)*sqrt(x^3 + 1)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((x^3 + 6*sqrt(3) + 10)*sqrt(x^3 + 1)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{(x+1)(x^2-x+1)}\left(x^3+10+6\sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(10+x**3+6*3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt((x + 1)*(x**2 - x + 1))*(x**3 + 10 + 6*sqrt(3)))
, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(x^3+6\sqrt{3}+10\right)\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((x^3 + 6*sqrt(3) + 10)*sqrt(x^3 + 1)),x, algorithm="giac")
```

```
[Out] integrate(x/((x^3 + 6*sqrt(3) + 10)*sqrt(x^3 + 1)), x)
```

3.85 $\int \frac{x}{\sqrt{1+x^3}\left(10-6\sqrt{3}+x^3\right)}\,dx$

Optimal. Leaf size=210

$$-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(-2x-\sqrt{3}+1\right)}{\sqrt{2}\sqrt{x^3+1}}\right)}{3\sqrt{2}\sqrt[4]{3}}-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)(x+1)}{\sqrt{2}\sqrt{x^3+1}}\right)}{6\sqrt{2}\sqrt[4]{3}}+\frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)(x+1)}{\sqrt{2}\sqrt{x^3+1}}\right)}{2\sqrt{2}3^{3/4}}+\frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{x^3+1}}{\sqrt{2}3^{3/4}}\right)}{3\sqrt{2}3^{3/4}}$$

```
[Out] -((2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3] - 2*x))/(Sqrt[2]*Sqr
t[1 + x^3])])/(3*Sqrt[2]*3^(1/4)) - ((2 + Sqrt[3])*ArcTan[(3^(1/4
)*(1 + Sqrt[3])*(1 + x))/(Sqrt[2]*Sqrt[1 + x^3])])/(6*Sqrt[2]*3^(
1/4)) + ((2 + Sqrt[3])*ArcTanh[(3^(1/4)*(1 - Sqrt[3])*(1 + x))/(S
qrt[2]*Sqrt[1 + x^3])])/(2*Sqrt[2]*3^(3/4)) + ((2 + Sqrt[3])*ArcT
anh[((1 + Sqrt[3])*Sqrt[1 + x^3])/(Sqrt[2]*3^(3/4))])/(3*Sqrt[2]*
3^(3/4))
```

Rubi [A] time = 0.104841, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.04$

$$-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(-2x-\sqrt{3}+1\right)}{\sqrt{2}\sqrt{x^3+1}}\right)}{3\sqrt{2}\sqrt[4]{3}}-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)(x+1)}{\sqrt{2}\sqrt{x^3+1}}\right)}{6\sqrt{2}\sqrt[4]{3}}+\frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)(x+1)}{\sqrt{2}\sqrt{x^3+1}}\right)}{2\sqrt{2}3^{3/4}}+\frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{x^3+1}}{\sqrt{2}3^{3/4}}\right)}{3\sqrt{2}3^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(Sqrt[1 + x^3]*(10 - 6*Sqrt[3] + x^3)),x]
```


```
[Out] -((2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3] - 2*x))/(Sqrt[2]*Sqr
t[1 + x^3])])/(3*Sqrt[2]*3^(1/4)) - ((2 + Sqrt[3])*ArcTan[(3^(1/4
)*(1 + Sqrt[3])*(1 + x))/(Sqrt[2]*Sqrt[1 + x^3])])/(6*Sqrt[2]*3^(
1/4)) + ((2 + Sqrt[3])*ArcTanh[(3^(1/4)*(1 - Sqrt[3])*(1 + x))/(S
qrt[2]*Sqrt[1 + x^3])])/(2*Sqrt[2]*3^(3/4)) + ((2 + Sqrt[3])*ArcT
anh[((1 + Sqrt[3])*Sqrt[1 + x^3])/(Sqrt[2]*3^(3/4))])/(3*Sqrt[2]*
3^(3/4))
```

Rubi in Sympy [A] time = 7.69253, size = 34, normalized size = 0.16

$$\frac{x^2\,\text{appellf}_1\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-x^3,-\frac{x^3}{-6\sqrt{3}+10}\right)}{2\left(-6\sqrt{3}+10\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(10+x**3-6*3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] x**2*appellf1(2/3, 1/2, 1, 5/3, -x**3, -x**3/(-6*sqrt(3) + 10))/(
2*(-6*sqrt(3) + 10))
```

Mathematica [C] time = 0.656094, size = 207, normalized size = 0.99

$$\frac{10\left(26-15\sqrt{3}\right)x^2F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-x^3,\frac{1}{4}\left(5+3\sqrt{3}\right)x^3\right)}{\left(3\sqrt{3}-5\right)\left(-x^3+6\sqrt{3}-10\right)\sqrt{x^3+1}\left(\left(50-30\sqrt{3}\right)F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-x^3,\frac{1}{4}\left(5+3\sqrt{3}\right)x^3\right)-3x^3\left(F_1\left(\frac{5}{3};\frac{1}{2},2;\frac{8}{3};-x^3,\frac{1}{4}\left(5+3\right.\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x/(Sqrt[1 + x^3]*(10 - 6*Sqrt[3] + x^3)),x]
```

```
[Out] (10*(26 - 15*Sqrt[3])*x^2*AppellF1[2/3, 1/2, 1, 5/3, -x^3, ((5 +
3*Sqrt[3])*x^3)/4])/((-5 + 3*Sqrt[3])*(-10 + 6*Sqrt[3] - x^3)*Sqr
t[1 + x^3]*((50 - 30*Sqrt[3])*AppellF1[2/3, 1/2, 1, 5/3, -x^3, ((
5 + 3*Sqrt[3])*x^3)/4] - 3*x^3*(AppellF1[5/3, 1/2, 2, 8/3, -x^3,
((5 + 3*Sqrt[3])*x^3)/4] + (5 - 3*Sqrt[3])*AppellF1[5/3, 3/2, 1,
8/3, -x^3, ((5 + 3*Sqrt[3])*x^3)/4])))
```

Maple [C] time = 0.266, size = 350, normalized size = 1.7

$$\frac{\left(\sqrt{3}-1\right)\left(\frac{3}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}{-18+9\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i}{2}\sqrt{3}}}\sqrt{\frac{1}{-\frac{3}{2}-\frac{i}{2}\sqrt{3}}\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)}\sqrt{\frac{1}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)}EllipticPi\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i}{2}\sqrt{3}}},-\frac{\left(-\right.}{}\right.$$

$$-\frac{\sqrt{2}}{18}\sum_{\_alpha=RootOf\left(\_Z^2+\left(\sqrt{3}-1\right)\_Z-2\sqrt{3}+4\right)}\frac{\left(-\_alpha\sqrt{3}-\_alpha+2\right)\left(-i\sqrt{3}+3\right)\left(-1+2\_alpha+\_alpha\sqrt{3}\right)}{-\sqrt{3}-2\_alpha+1}\sqrt{\frac{1+x}{-i\sqrt{3}+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(10+x^3-6*3^(1/2))/(x^3+1)^(1/2),x)
```

```
[Out] 1/9*(3^(1/2)-1)/(-2+3^(1/2))*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*
I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1
/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1
/2)*3^(1/2)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(-3
/2+1/2*I*3^(1/2))*3^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/
2)))^(1/2))-1/18*2^(1/2)*sum((-_alpha*3^(1/2)-_alpha+2)/(-3^(1/2)
-2*_alpha+1)*(-I*3^(1/2)+3)*((1+x)/(-I*3^(1/2)+3))^(1/2)*((2*x-1-
I*3^(1/2))/(-I*3^(1/2)-3))^(1/2)*((2*x-1+I*3^(1/2))/(I*3^(1/2)-3)
)^(1/2)/(x^3+1)^(1/2)*(-1+2*_alpha+_alpha*3^(1/2))*EllipticPi(((1
+x)/(3/2-1/2*I*3^(1/2)))^(1/2),1/2*I*_alpha+1/3*I*_alpha*3^(1/2)-
1/2*_alpha*3^(1/2)-_alpha-1/6*I*3^(1/2)+1/2,((-3/2+1/2*I*3^(1/2))
/(-3/2-1/2*I*3^(1/2)))^(1/2)),_alpha=RootOf(_Z^2+(3^(1/2)-1)*_Z-2
*3^(1/2)+4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(x^3-6\sqrt{3}+10\right)\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((x^3 - 6*sqrt(3) + 10)*sqrt(x^3 + 1)),x, algorithm="maxima")
```

```
[Out] integrate(x/((x^3 - 6*sqrt(3) + 10)*sqrt(x^3 + 1)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((x^3 - 6*sqrt(3) + 10)*sqrt(x^3 + 1)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{(x+1)(x^2-x+1)}\left(x^3-6\sqrt{3}+10\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(10+x**3-6*3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt((x + 1)*(x**2 - x + 1))*(x**3 - 6*sqrt(3) + 10))
, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(x^3-6\sqrt{3}+10\right)\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((x^3 - 6*sqrt(3) + 10)*sqrt(x^3 + 1)),x, algorithm="giac")
```

```
[Out] integrate(x/((x^3 - 6*sqrt(3) + 10)*sqrt(x^3 + 1)), x)
```

3.86 $\int \frac{x}{\sqrt{-1+x^3}\left(-10-6\sqrt{3}+x^3\right)}\,dx$

Optimal. Leaf size=222

$$\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{x^3-1}}\right)}{6\sqrt{2}\sqrt[4]{3}}+\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(2x+\sqrt{3}+1\right)}{\sqrt{2}\sqrt{x^3-1}}\right)}{3\sqrt{2}\sqrt[4]{3}}+\frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{x^3-1}}\right)}{2\sqrt{2}3^{3/4}}-\frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{x^3-1}}{\sqrt{2}3^{3/4}}\right)}{3\sqrt{2}3^{3/4}}$$

```
[Out] ((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3])*(1 - x))/(Sqrt[2]*Sq
rt[-1 + x^3])])/(6*Sqrt[2]*3^(1/4)) + ((2 - Sqrt[3])*ArcTan[(3^(1
/4)*(1 + Sqrt[3] + 2*x))/(Sqrt[2]*Sqrt[-1 + x^3])])/(3*Sqrt[2]*3^
(1/4)) + ((2 - Sqrt[3])*ArcTanh[(3^(1/4)*(1 + Sqrt[3])*(1 - x))/(
Sqrt[2]*Sqrt[-1 + x^3])])/(2*Sqrt[2]*3^(3/4)) - ((2 - Sqrt[3])*Ar
cTanh[((1 - Sqrt[3])*Sqrt[-1 + x^3])/(Sqrt[2]*3^(3/4))])/(3*Sqrt[
2]*3^(3/4))
```

Rubi [A] time = 0.100172, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.04$

$$\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{x^3-1}}\right)}{6\sqrt{2}\sqrt[4]{3}}+\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(2x+\sqrt{3}+1\right)}{\sqrt{2}\sqrt{x^3-1}}\right)}{3\sqrt{2}\sqrt[4]{3}}+\frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{x^3-1}}\right)}{2\sqrt{2}3^{3/4}}-\frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{x^3-1}}{\sqrt{2}3^{3/4}}\right)}{3\sqrt{2}3^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(Sqrt[-1 + x^3]*(-10 - 6*Sqrt[3] + x^3)),x]
```

```
[Out] ((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3])*(1 - x))/(Sqrt[2]*Sq
rt[-1 + x^3])])/(6*Sqrt[2]*3^(1/4)) + ((2 - Sqrt[3])*ArcTan[(3^(1
/4)*(1 + Sqrt[3] + 2*x))/(Sqrt[2]*Sqrt[-1 + x^3])])/(3*Sqrt[2]*3^
(1/4)) + ((2 - Sqrt[3])*ArcTanh[(3^(1/4)*(1 + Sqrt[3])*(1 - x))/(
Sqrt[2]*Sqrt[-1 + x^3])])/(2*Sqrt[2]*3^(3/4)) - ((2 - Sqrt[3])*Ar
cTanh[((1 - Sqrt[3])*Sqrt[-1 + x^3])/(Sqrt[2]*3^(3/4))])/(3*Sqrt[
2]*3^(3/4))
```

Rubi in Sympy [A] time = 11.817, size = 53, normalized size = 0.24

$$\frac{x^2\sqrt{x^3-1}\,\text{appellf}_1\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},x^3,-\frac{x^3}{-6\sqrt{3}-10}\right)}{2\left(10+6\sqrt{3}\right)\sqrt{-x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(-10+x**3-6*3**(1/2))/(x**3-1)**(1/2),x)
```

```
[Out] x**2*sqrt(x**3 - 1)*appellf1(2/3, 1/2, 1, 5/3, x**3, -x**3/(-6*sq
rt(3) - 10))/(2*(10 + 6*sqrt(3))*sqrt(-x**3 + 1))
```

Mathematica [C] time = 0.623126, size = 196, normalized size = 0.88

$$-\frac{10\left(26+15\sqrt{3}\right)x^2F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};x^3,\frac{x^3}{10+6\sqrt{3}}\right)}{\left(5+3\sqrt{3}\right)\left(-x^3+6\sqrt{3}+10\right)\sqrt{x^3-1}\left(3x^3\left(F_1\left(\frac{5}{3};\frac{1}{2},2;\frac{8}{3};x^3,\frac{x^3}{10+6\sqrt{3}}\right)+\left(5+3\sqrt{3}\right)F_1\left(\frac{5}{3};\frac{3}{2},1;\frac{8}{3};x^3,\frac{x^3}{10+6\sqrt{3}}\right)\right)+10\left(5+3\sqrt{}\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x/(Sqrt[-1 + x^3]*(-10 - 6*Sqrt[3] + x^3)),x]
```

```
[Out] (-10*(26 + 15*Sqrt[3])*x^2*AppellF1[2/3, 1/2, 1, 5/3, x^3, x^3/(1
0 + 6*Sqrt[3])])/((5 + 3*Sqrt[3])*(10 + 6*Sqrt[3] - x^3)*Sqrt[-1
+ x^3]*(10*(5 + 3*Sqrt[3])*AppellF1[2/3, 1/2, 1, 5/3, x^3, x^3/(1
0 + 6*Sqrt[3])] + 3*x^3*(AppellF1[5/3, 1/2, 2, 8/3, x^3, x^3/(10
+ 6*Sqrt[3])] + (5 + 3*Sqrt[3])*AppellF1[5/3, 3/2, 1, 8/3, x^3, x
^3/(10 + 6*Sqrt[3])])))
```

Maple [C] time = 0.257, size = 349, normalized size = 1.6

$$\frac{\left(-1-\sqrt{3}\right)\left(-\frac{3}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}{18+9\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i}{2}\sqrt{3}}}\sqrt{\frac{1}{\frac{3}{2}-\frac{i}{2}\sqrt{3}}\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)}\sqrt{\frac{1}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)}\mathit{EllipticPi}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i}{2}\sqrt{3}}},-\right.$$

$$-\frac{\sqrt{2}}{18}\sum_{\_alpha=RootOf\left(\_Z^2+\left(1+\sqrt{3}\right)\_Z+2\sqrt{3}+4\right)}\frac{\left(-\_alpha\sqrt{3}+\_alpha+2\right)\left(-i\sqrt{3}-3\right)\left(1+2\_alpha-\_alpha\sqrt{3}\right)}{-\sqrt{3}-2\_alpha-1}\sqrt{\frac{-1+x}{-i\sqrt{3}-3}}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(-10+x^3-6*3^(1/2))/(x^3-1)^(1/2),x)
```

```
[Out] 1/9*(-1-3^(1/2))/(2+3^(1/2))*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1
/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^
(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(
1/2)*3^(1/2)*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),-1/3*
(3/2+1/2*I*3^(1/2))*3^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/
2)))^(1/2))-1/18*2^(1/2)*sum((-_alpha*3^(1/2)+_alpha+2)/(-3^(1/2)
-2*_alpha-1)*(-I*3^(1/2)-3)*((-1+x)/(-I*3^(1/2)-3))^(1/2)*((2*x+1
-I*3^(1/2))/(-I*3^(1/2)+3))^(1/2)*((2*x+1+I*3^(1/2))/(I*3^(1/2)+3
))^(1/2)/(x^3-1)^(1/2)*(1+2*_alpha-_alpha*3^(1/2))*EllipticPi(((-
1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),-1/2*I*_alpha+1/3*I*_alpha*3^(1/
2)-1/2*_alpha*3^(1/2)+_alpha+1/6*I*3^(1/2)+1/2,((3/2+1/2*I*3^(1/2
))/(3/2-1/2*I*3^(1/2)))^(1/2)),_alpha=RootOf(_Z^2+(1+3^(1/2))*_Z+
2*3^(1/2)+4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(x^3-6\sqrt{3}-10\right)\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((x^3 - 6*sqrt(3) - 10)*sqrt(x^3 - 1)),x, algorithm="maxima")
```

```
[Out] integrate(x/((x^3 - 6*sqrt(3) - 10)*sqrt(x^3 - 1)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((x^3 - 6*sqrt(3) - 10)*sqrt(x^3 - 1)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{(x-1)(x^2+x+1)}\left(x^3-6\sqrt{3}-10\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-10+x**3-6*3**(1/2))/(x**3-1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt((x - 1)*(x**2 + x + 1))*(x**3 - 6*sqrt(3) - 10))
, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(x^3-6\sqrt{3}-10\right)\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((x^3 - 6*sqrt(3) - 10)*sqrt(x^3 - 1)),x, algorithm="giac")
```

```
[Out] integrate(x/((x^3 - 6*sqrt(3) - 10)*sqrt(x^3 - 1)), x)
```

3.87 $\int \frac{x}{\sqrt{-1+x^3}\left(-10+6\sqrt{3}+x^3\right)}\,dx$

Optimal. Leaf size=214

$$-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{x^3-1}}\right)}{2\sqrt{2}3^{3/4}}+\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{x^3-1}}{\sqrt{2}3^{3/4}}\right)}{3\sqrt{2}3^{3/4}}+\frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{x^3-1}}\right)}{6\sqrt{2}\sqrt[4]{3}}+\frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(2x-\sqrt{3}+1\right)}{\sqrt{2}\sqrt{x^3-1}}\right)}{3\sqrt{2}\sqrt[4]{3}}$$

```
[Out] -((2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3])*(1 - x))/(Sqrt[2]*S
qrt[-1 + x^3])])/(2*Sqrt[2]*3^(3/4)) + ((2 + Sqrt[3])*ArcTan[((1
+ Sqrt[3])*Sqrt[-1 + x^3])/(Sqrt[2]*3^(3/4))])/(3*Sqrt[2]*3^(3/4)
) + ((2 + Sqrt[3])*ArcTanh[(3^(1/4)*(1 + Sqrt[3])*(1 - x))/(Sqrt[
2]*Sqrt[-1 + x^3])])/(6*Sqrt[2]*3^(1/4)) + ((2 + Sqrt[3])*ArcTanh
[(3^(1/4)*(1 - Sqrt[3] + 2*x))/(Sqrt[2]*Sqrt[-1 + x^3])])/(3*Sqrt
[2]*3^(1/4))
```

Rubi [A] time = 0.0962502, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.04$

$$-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{x^3-1}}\right)}{2\sqrt{2}3^{3/4}}+\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{x^3-1}}{\sqrt{2}3^{3/4}}\right)}{3\sqrt{2}3^{3/4}}+\frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{x^3-1}}\right)}{6\sqrt{2}\sqrt[4]{3}}+\frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(2x-\sqrt{3}+1\right)}{\sqrt{2}\sqrt{x^3-1}}\right)}{3\sqrt{2}\sqrt[4]{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(Sqrt[-1 + x^3]*(-10 + 6*Sqrt[3] + x^3)),x]
```

```
[Out] -((2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3])*(1 - x))/(Sqrt[2]*S
qrt[-1 + x^3])])/(2*Sqrt[2]*3^(3/4)) + ((2 + Sqrt[3])*ArcTan[((1
+ Sqrt[3])*Sqrt[-1 + x^3])/(Sqrt[2]*3^(3/4))])/(3*Sqrt[2]*3^(3/4)
) + ((2 + Sqrt[3])*ArcTanh[(3^(1/4)*(1 + Sqrt[3])*(1 - x))/(Sqrt[
2]*Sqrt[-1 + x^3])])/(6*Sqrt[2]*3^(1/4)) + ((2 + Sqrt[3])*ArcTanh
[(3^(1/4)*(1 - Sqrt[3] + 2*x))/(Sqrt[2]*Sqrt[-1 + x^3])])/(3*Sqrt
[2]*3^(1/4))
```

Rubi in Sympy [A] time = 6.47048, size = 51, normalized size = 0.24

$$\frac{x^2\sqrt{x^3-1}\,\mathrm{appellf}_1\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},x^3,-\frac{x^3}{-10+6\sqrt{3}}\right)}{2\left(-6\sqrt{3}+10\right)\sqrt{-x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(-10+x**3+6*3**(1/2))/(x**3-1)**(1/2),x)
```

```
[Out] x**2*sqrt(x**3 - 1)*appellf1(2/3, 1/2, 1, 5/3, x**3, -x**3/(-10 +
 6*sqrt(3)))/(2*(-6*sqrt(3) + 10)*sqrt(-x**3 + 1))
```

Mathematica [C] time = 0.53932, size = 198, normalized size = 0.93

$$\frac{10\left(26-15\sqrt{3}\right)x^2F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};x^3,-\frac{1}{4}\left(5+3\sqrt{3}\right)x^3\right)}{\left(3\sqrt{3}-5\right)\sqrt{x^3-1}\left(x^3+6\sqrt{3}-10\right)\left(10\left(3\sqrt{3}-5\right)F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};x^3,-\frac{1}{4}\left(5+3\sqrt{3}\right)x^3\right)-3x^3\left(F_1\left(\frac{5}{3};\frac{1}{2},2;\frac{8}{3};x^3,-\frac{1}{4}\left(5+3\sqrt{\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x/(Sqrt[-1 + x^3]*(-10 + 6*Sqrt[3] + x^3)),x]
```

```
[Out] (10*(26 - 15*Sqrt[3])*x^2*AppellF1[2/3, 1/2, 1, 5/3, x^3, -((5 +
3*Sqrt[3])*x^3)/4])/((-5 + 3*Sqrt[3])*Sqrt[-1 + x^3]*(-10 + 6*Sqr
t[3] + x^3)*(10*(-5 + 3*Sqrt[3])*AppellF1[2/3, 1/2, 1, 5/3, x^3,
-((5 + 3*Sqrt[3])*x^3)/4] - 3*x^3*(AppellF1[5/3, 1/2, 2, 8/3, x^3
, -((5 + 3*Sqrt[3])*x^3)/4] + (5 - 3*Sqrt[3])*AppellF1[5/3, 3/2,
1, 8/3, x^3, -((5 + 3*Sqrt[3])*x^3)/4])))
```

Maple [C] time = 0.256, size = 350, normalized size = 1.6

$$\frac{\left(\sqrt{3}-1\right)\left(-\frac{3}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}{-18+9\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i}{2}\sqrt{3}}}\sqrt{\frac{1}{\frac{3}{2}-\frac{i}{2}\sqrt{3}}\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)}\sqrt{\frac{1}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)}EllipticPi\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i}{2}\sqrt{3}}},\frac{\left(\frac{3}{2}\right.}{}\right.$$

$$-\frac{\sqrt{2}}{18}\sum_{\_alpha=RootOf\left(\_Z^2+\left(1-\sqrt{3}\right)\_Z-2\sqrt{3}+4\right)}\frac{\left(-\_alpha\sqrt{3}-\_alpha-2\right)\left(-i\sqrt{3}-3\right)\left(1+2\_alpha+\_alpha\sqrt{3}\right)}{-\sqrt{3}+2\_alpha+1}\sqrt{\frac{-1+x}{-i\sqrt{3}-3}}\sqrt{\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(-10+x^3+6*3^(1/2))/(x^3-1)^(1/2),x)
```

```
[Out] 1/9*(3^(1/2)-1)/(-2+3^(1/2))*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1
/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^
(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(
1/2)*3^(1/2)*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),1/3*(
3/2+1/2*I*3^(1/2))*3^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2
)))^(1/2))-1/18*2^(1/2)*sum((-_alpha*3^(1/2)-_alpha-2)/(-3^(1/2)+
2*_alpha+1)*(-I*3^(1/2)-3)*((-1+x)/(-I*3^(1/2)-3))^(1/2)*((2*x+1-
I*3^(1/2))/(-I*3^(1/2)+3))^(1/2)*((2*x+1+I*3^(1/2))/(I*3^(1/2)+3)
)^(1/2)/(x^3-1)^(1/2)*(1+2*_alpha+_alpha*3^(1/2))*EllipticPi(((-1
+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),1/2*I*_alpha+1/3*I*_alpha*3^(1/2)
+1/2*_alpha*3^(1/2)+_alpha+1/6*I*3^(1/2)+1/2,((3/2+1/2*I*3^(1/2))
/(3/2-1/2*I*3^(1/2)))^(1/2)),_alpha=RootOf(_Z^2+(1-3^(1/2))*_Z-2*
3^(1/2)+4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(x^3+6\sqrt{3}-10\right)\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((x^3 + 6*sqrt(3) - 10)*sqrt(x^3 - 1)),x, algorithm="maxima")
```

```
[Out] integrate(x/((x^3 + 6*sqrt(3) - 10)*sqrt(x^3 - 1)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((x^3 + 6*sqrt(3) - 10)*sqrt(x^3 - 1)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{(x-1)(x^2+x+1)}\left(x^3-10+6\sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(-10+x**3+6*3**(1/2))/(x**3-1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt((x - 1)*(x**2 + x + 1))*(x**3 - 10 + 6*sqrt(3)))
, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(x^3+6\sqrt{3}-10\right)\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((x^3 + 6*sqrt(3) - 10)*sqrt(x^3 - 1)),x, algorithm="giac")
```

```
[Out] integrate(x/((x^3 + 6*sqrt(3) - 10)*sqrt(x^3 - 1)), x)
```

3.88 $\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-4+4\sqrt{3}x^2+x^4}}\,dx$

Optimal. Leaf size=65

$$\frac{1}{3}\sqrt{2\sqrt{3}-3}\tanh^{-1}\left(\frac{\left(x-\sqrt{3}+1\right)^2}{\sqrt{3\left(2\sqrt{3}-3\right)}\sqrt{x^4+4\sqrt{3}x^2-4}}\right)$$

```
[Out] (Sqrt[-3 + 2*Sqrt[3]]*ArcTanh[(1 - Sqrt[3] + x)^2/(Sqrt[3*(-3 + 2
*Sqrt[3])]*Sqrt[-4 + 4*Sqrt[3]*x^2 + x^4])])/3
```

Rubi [A] time = 0.211471, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05

$$\frac{1}{3}\sqrt{2\sqrt{3}-3}\tanh^{-1}\left(\frac{\left(x-\sqrt{3}+1\right)^2}{\sqrt{3\left(2\sqrt{3}-3\right)}\sqrt{x^4+4\sqrt{3}x^2-4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[-4 + 4*Sqrt[3]*x^2 + x^4]),x]
```

```
[Out] (Sqrt[-3 + 2*Sqrt[3]]*ArcTanh[(1 - Sqrt[3] + x)^2/(Sqrt[3*(-3 + 2
*Sqrt[3])]*Sqrt[-4 + 4*Sqrt[3]*x^2 + x^4])])/3
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((1+x-3**(1/2))/(1+x+3**(1/2))/(-4+x**4+4*3**(1/2)*x**2)**(1/2),
```

```
[Out] Timed out
```

Mathematica [C] time = 4.03084, size = 685, normalized size = 10.54

$$\frac{\left(x+\sqrt{3}-1\right)^2\sqrt{-x^3+\left(\sqrt{3}-1\right)x^2-2\left(2+\sqrt{3}\right)x+2\left(1+\sqrt{3}\right)}\sqrt{\frac{-\frac{4}{x+\sqrt{3}-1}+\sqrt{3}+1}{3+\sqrt{3}+i\sqrt{2\left(2+\sqrt{3}\right)}}}\left(2\sqrt{6}\sqrt{\frac{x^2+2\sqrt{3}+4}{\left(x+\sqrt{3}-1\right)^2}}\sqrt{\sqrt{2\left(2+\sqrt{3}\right)}-i\left(\frac{\ldots}{x+\ldots}\right.}\right.}{\left(\sqrt{2\ldots}\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[-4 + 4*Sqrt[3]*x^2 + x^4]),x]
```

```
[Out] ((-1 + Sqrt[3] + x)^2*Sqrt[2*(1 + Sqrt[3]) - 2*(2 + Sqrt[3])*x +
(-1 + Sqrt[3])*x^2 - x^3]*Sqrt[(1 + Sqrt[3] - 4/(-1 + Sqrt[3] + x
))/(3 + Sqrt[3] + I*Sqrt[2*(2 + Sqrt[3])])]*((I*(-1 + Sqrt[3] + I
*Sqrt[2*(2 + Sqrt[3])]) + (2*((2*I)*Sqrt[3] - Sqrt[2*(2 + Sqrt[3]
```

```
)] + Sqrt[6*(2 + Sqrt[3])]))/(-1 + Sqrt[3] + x))*Sqrt[Sqrt[2*(2 +
 Sqrt[3])] + I*(1 - Sqrt[3] + 8/(-1 + Sqrt[3] + x))]*EllipticF[Ar
cSin[Sqrt[Sqrt[2*(2 + Sqrt[3])] - I*(1 - Sqrt[3] + 8/(-1 + Sqrt[3
] + x))]/(2^(3/4)*(2 + Sqrt[3])^(1/4))], ((2*I)*Sqrt[2*(2 + Sqrt[
3])])/(3 + Sqrt[3] + I*Sqrt[2*(2 + Sqrt[3])])] + 2*Sqrt[6]*Sqrt[(
4 + 2*Sqrt[3] + x^2)/(-1 + Sqrt[3] + x)^2]*Sqrt[Sqrt[2*(2 + Sqrt[
3])] - I*(1 - Sqrt[3] + 8/(-1 + Sqrt[3] + x))]*EllipticPi[(2*Sqrt
[2*(2 + Sqrt[3])])/(Sqrt[2*(2 + Sqrt[3])] + I*(3 + Sqrt[3])), Arc
Sin[Sqrt[Sqrt[2*(2 + Sqrt[3])] - I*(1 - Sqrt[3] + 8/(-1 + Sqrt[3]
 + x))]/(2^(3/4)*(2 + Sqrt[3])^(1/4))], ((2*I)*Sqrt[2*(2 + Sqrt[3
])])/(3 + Sqrt[3] + I*Sqrt[2*(2 + Sqrt[3])])]))/((Sqrt[2*(2 + Sqr
t[3])] + I*(3 + Sqrt[3]))*Sqrt[1 + Sqrt[3] - (2 + Sqrt[3])*x + ((
-1 + Sqrt[3])*x^2)/2 - x^3/2]*Sqrt[-4 + 4*Sqrt[3]*x^2 + x^4]*Sqrt
[Sqrt[2*(2 + Sqrt[3])] - I*(1 - Sqrt[3] + 8/(-1 + Sqrt[3] + x))])
```

Maple [C] time = 0.24, size = 327, normalized size = 5.

$$\frac{EllipticF\left(x\left(\frac{i}{2}\sqrt{3}-\frac{i}{2}\right), i\sqrt{1+4\sqrt{3}\left(1+1/2\sqrt{3}\right)}\right)}{\frac{i}{2}\sqrt{3}-\frac{i}{2}}\sqrt{1-\left(-1+\frac{\sqrt{3}}{2}\right)x^2}\sqrt{1-\left(1+\frac{\sqrt{3}}{2}\right)x^2}\frac{1}{\sqrt{-4+x^4+4\sqrt{3}x^2}}$$
$$-2\sqrt{3}\left(-1/2\frac{1}{\sqrt{\left(-1-\sqrt{3}\right)^4+4\sqrt{3}\left(-1-\sqrt{3}\right)^2-4}}Artanh\left(1/2\frac{4\sqrt{3}\left(-1-\sqrt{3}\right)^2-8+4\sqrt{3}x^2+2x^2\left(-1-\sqrt{3}\right)^2}{\sqrt{\left(-1-\sqrt{3}\right)^4+4\sqrt{3}\left(-1-\sqrt{3}\right)^2-4}\sqrt{-4+x^4+4\sqrt{3}x^2}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((1+x-3^(1/2))/(1+x+3^(1/2))/(-4+x^4+4*3^(1/2)*x^2)^(1/2),x)
```

```
[Out] 1/(1/2*I*3^(1/2)-1/2*I)*(1-(-1+1/2*3^(1/2))*x^2)^(1/2)*(1-(1+1/2*
3^(1/2))*x^2)^(1/2)/(-4+x^4+4*3^(1/2)*x^2)^(1/2)*EllipticF(x*(1/2
*I*3^(1/2)-1/2*I),I*(1+4*3^(1/2)*(1+1/2*3^(1/2)))^(1/2))-2*3^(1/2
)*(-1/2/((-1-3^(1/2))^4+4*3^(1/2)*(-1-3^(1/2))^2-4)^(1/2)*arctanh
(1/2*(4*3^(1/2)*(-1-3^(1/2))^2-8+4*3^(1/2)*x^2+2*x^2*(-1-3^(1/2))
^2)/((-1-3^(1/2))^4+4*3^(1/2)*(-1-3^(1/2))^2-4)^(1/2)/(-4+x^4+4*3
^(1/2)*x^2)^(1/2))-1/(-1+1/2*3^(1/2))^(1/2)/(-1-3^(1/2))*(1-(-1+1
/2*3^(1/2))*x^2)^(1/2)*(1-(1+1/2*3^(1/2))*x^2)^(1/2)/(-4+x^4+4*3^
(1/2)*x^2)^(1/2)*EllipticPi((-1+1/2*3^(1/2))^(1/2)*x,1/(-1+1/2*3^
(1/2))/(-1-3^(1/2))^2,(1+1/2*3^(1/2))^(1/2)/(-1+1/2*3^(1/2))^(1/2
)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-\sqrt{3}+1}{\sqrt{x^4+4\sqrt{3}x^2-4}\left(x+\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x - sqrt(3) + 1)/(sqrt(x^4 + 4*sqrt(3)*x^2 - 4)*(x + sqrt(3) + 1)),x, a
```

```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(x^4 + 4*sqrt(3)*x^2 - 4)*(x + s
qrt(3) + 1)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x - sqrt(3) + 1)/(sqrt(x^4 + 4*sqrt(3)*x^2 - 4)*(x + sqrt(3) + 1)),x, a
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x - \sqrt{3} + 1}{\left(x + 1 + \sqrt{3}\right)\sqrt{x^4 + 4\sqrt{3}x^2 - 4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3**(1/2))/(1+x+3**(1/2))/(-4+x**4+4*3**(1/2)*x**2)**(1/2),x)
```

```
[Out] Integral((x - sqrt(3) + 1)/((x + 1 + sqrt(3))*sqrt(x**4 + 4*sqrt(
3)*x**2 - 4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x - \sqrt{3} + 1}{\sqrt{x^4 + 4\sqrt{3}x^2 - 4}\left(x + \sqrt{3} + 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x - sqrt(3) + 1)/(sqrt(x^4 + 4*sqrt(3)*x^2 - 4)*(x + sqrt(3) + 1)),x, a
```

```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(x^4 + 4*sqrt(3)*x^2 - 4)*(x + s
qrt(3) + 1)), x)
```

3.89 $\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-4-4\sqrt{3}x^2+x^4}}\,dx$

Optimal. Leaf size=63

$$-\frac{1}{3}\sqrt{3+2\sqrt{3}}\tan^{-1}\left(\frac{\left(x+\sqrt{3}+1\right)^2}{\sqrt{3\left(3+2\sqrt{3}\right)}\sqrt{x^4-4\sqrt{3}x^2-4}}\right)$$

```
[Out] -(Sqrt[3 + 2*Sqrt[3]]*ArcTan[(1 + Sqrt[3] + x)^2/(Sqrt[3*(3 + 2*S
qrt[3])]*Sqrt[-4 - 4*Sqrt[3]*x^2 + x^4])])/3
```

Rubi [A] time = 0.21298, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05

$$-\frac{1}{3}\sqrt{3+2\sqrt{3}}\tan^{-1}\left(\frac{\left(x+\sqrt{3}+1\right)^2}{\sqrt{3\left(3+2\sqrt{3}\right)}\sqrt{x^4-4\sqrt{3}x^2-4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[-4 - 4*Sqrt[3]*x^2 + x^4]),x]
```

```
[Out] -(Sqrt[3 + 2*Sqrt[3]]*ArcTan[(1 + Sqrt[3] + x)^2/(Sqrt[3*(3 + 2*S
qrt[3])]*Sqrt[-4 - 4*Sqrt[3]*x^2 + x^4])])/3
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((1+x+3**(1/2))/(1+x-3**(1/2))/(-4+x**4-4*3**(1/2)*x**2)**(1/2),
```

```
[Out] Timed out
```

Mathematica [C] time = 10.7136, size = 1137, normalized size = 18.05

$$-\frac{\left(x-\sqrt{3}-1\right)^2\sqrt{\frac{\sqrt{3}-1+\frac{4}{x-\sqrt{3}-1}}{-3+\sqrt{3}-i\sqrt{4-2\sqrt{3}}}}\sqrt{\left(x-\sqrt{3}+1\right)^3+\left(-2+4\sqrt{3}\right)\left(x-\sqrt{3}+1\right)^2+\left(20-8\sqrt{3}\right)\left(x-\sqrt{3}+1\right)+16\sqrt{3}-24}\left(\cdots\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[-4 - 4*Sqrt[3]*x^2 + x^4]),x]
```

```
[Out] -(((-1 - Sqrt[3] + x)^2*Sqrt[(-1 + Sqrt[3] + 4/(-1 - Sqrt[3] + x)
)/(-3 + Sqrt[3] - I*Sqrt[4 - 2*Sqrt[3]])]*Sqrt[-24 + 16*Sqrt[3] +
 (20 - 8*Sqrt[3])*(1 - Sqrt[3] + x) + (-2 + 4*Sqrt[3])*(1 - Sqrt[
```

```
3] + x)^2 + (1 - Sqrt[3] + x)^3]*((I*Sqrt[Sqrt[4 - 2*Sqrt[3]] + I
*(1 + Sqrt[3]) + (8*I)/(-1 - Sqrt[3] + x)] + I*Sqrt[3]*Sqrt[Sqrt[
4 - 2*Sqrt[3]] + I*(1 + Sqrt[3]) + (8*I)/(-1 - Sqrt[3] + x)] + Sq
rt[-2*I + (2*I)*Sqrt[3] - 2*Sqrt[12 - 6*Sqrt[3]] + 4*Sqrt[4 - 2*S
qrt[3]] - ((16*I)*(-2 + Sqrt[3]))/(-1 - Sqrt[3] + x)] + (2*((2*I)
*Sqrt[3]*Sqrt[Sqrt[4 - 2*Sqrt[3]] + I*(1 + Sqrt[3]) + (8*I)/(-1 -
 Sqrt[3] + x)] + Sqrt[6]*Sqrt[-I + I*Sqrt[3] - Sqrt[12 - 6*Sqrt[3
]] + 2*Sqrt[4 - 2*Sqrt[3]] - ((8*I)*(-2 + Sqrt[3]))/(-1 - Sqrt[3]
 + x)] + Sqrt[-2*I + (2*I)*Sqrt[3] - 2*Sqrt[12 - 6*Sqrt[3]] + 4*S
qrt[4 - 2*Sqrt[3]] - ((16*I)*(-2 + Sqrt[3]))/(-1 - Sqrt[3] + x)])
)/(-1 - Sqrt[3] + x))*EllipticF[ArcSin[Sqrt[Sqrt[4 - 2*Sqrt[3]] -
 I*(1 + Sqrt[3]) - (8*I)/(-1 - Sqrt[3] + x)]/(2^(3/4)*(2 - Sqrt[3
])^(1/4))], (2*Sqrt[4 - 2*Sqrt[3]])/(Sqrt[4 - 2*Sqrt[3]] + I*(-3
+ Sqrt[3]))] + 2*Sqrt[6]*Sqrt[Sqrt[4 - 2*Sqrt[3]] - I*(1 + Sqrt[3
]) - (8*I)/(-1 - Sqrt[3] + x)]*Sqrt[1 + 8/(-1 - Sqrt[3] + x)^2 +
(2*(1 + Sqrt[3]))/(-1 - Sqrt[3] + x)]*EllipticPi[(2*Sqrt[4 - 2*Sq
rt[3]])/(Sqrt[4 - 2*Sqrt[3]] - I*(-3 + Sqrt[3])), ArcSin[Sqrt[Sqr
t[4 - 2*Sqrt[3]] - I*(1 + Sqrt[3]) - (8*I)/(-1 - Sqrt[3] + x)]/(2
^(3/4)*(2 - Sqrt[3])^(1/4))], (2*Sqrt[4 - 2*Sqrt[3]])/(Sqrt[4 - 2
*Sqrt[3]] + I*(-3 + Sqrt[3]))]))/((Sqrt[4 - 2*Sqrt[3]] - I*(-3 +
Sqrt[3]))*Sqrt[Sqrt[4 - 2*Sqrt[3]] - I*(1 + Sqrt[3]) - (8*I)/(-1
- Sqrt[3] + x)]*Sqrt[8*(1 + Sqrt[3]) + 4*(3 + Sqrt[3])*(-1 - Sqrt
[3] + x) + 2*(1 + Sqrt[3])*(-1 - Sqrt[3] + x)^2 + (-1 - Sqrt[3] +
 x)^3/2]*Sqrt[48 - 32*Sqrt[3] - 64*(1 - Sqrt[3] + x) + 32*Sqrt[3]
*(1 - Sqrt[3] + x) + 24*(1 - Sqrt[3] + x)^2 - 16*Sqrt[3]*(1 - Sqr
t[3] + x)^2 - 4*(1 - Sqrt[3] + x)^3 + 4*Sqrt[3]*(1 - Sqrt[3] + x)
^3 + (1 - Sqrt[3] + x)^4]))
```

Maple [C] time = 0.235, size = 311, normalized size = 4.9

$$\frac{EllipticF\left(x\left(\frac{i}{2}+\frac{i}{2}\sqrt{3}\right), i\sqrt{1-4\sqrt{3}\left(1-1/2\sqrt{3}\right)}\right)}{\frac{i}{2}+\frac{i}{2}\sqrt{3}}\sqrt{1-\left(-1-\frac{\sqrt{3}}{2}\right)x^2}\sqrt{1-\left(1-\frac{\sqrt{3}}{2}\right)x^2}\frac{1}{\sqrt{-4+x^4-4\sqrt{3}x^2}} +2\sqrt{3}\left(-1/2\frac{1}{\sqrt{\left(\sqrt{3}-1\right)^4-4\sqrt{3}\left(\sqrt{3}-1\right)^2-4}}Artanh\left(1/2\frac{-4\sqrt{3}\left(\sqrt{3}-1\right)^2-8-4\sqrt{3}x^2+2x^2\left(\sqrt{3}-1\right)^2}{\sqrt{\left(\sqrt{3}-1\right)^4-4\sqrt{3}\left(\sqrt{3}-1\right)^2-4}\sqrt{-4+x^4-4\sqrt{3}x^2}}\right)-\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+x+3^(1/2))/(1+x-3^(1/2))/(-4+x^4-4*3^(1/2)*x^2)^(1/2),x)
```

```
[Out] 1/(1/2*I+1/2*I*3^(1/2))*(1-(-1-1/2*3^(1/2))*x^2)^(1/2)*(1-(1-1/2*
3^(1/2))*x^2)^(1/2)/(-4+x^4-4*3^(1/2)*x^2)^(1/2)*EllipticF(x*(1/2
*I+1/2*I*3^(1/2)),I*(1-4*3^(1/2)*(1-1/2*3^(1/2)))^(1/2))+2*3^(1/2
)*(-1/2/((3^(1/2)-1)^4-4*3^(1/2)*(3^(1/2)-1)^2-4)^(1/2)*arctanh(1
/2*(-4*3^(1/2)*(3^(1/2)-1)^2-8-4*3^(1/2)*x^2+2*x^2*(3^(1/2)-1)^2)
/((3^(1/2)-1)^4-4*3^(1/2)*(3^(1/2)-1)^2-4)^(1/2)/(-4+x^4-4*3^(1/2
)*x^2)^(1/2))-1/(-1-1/2*3^(1/2))^(1/2)/(3^(1/2)-1)*(1-(-1-1/2*3^(
1/2))*x^2)^(1/2)*(1-(1-1/2*3^(1/2))*x^2)^(1/2)/(-4+x^4-4*3^(1/2)*
x^2)^(1/2)*EllipticPi((-1-1/2*3^(1/2))^(1/2)*x,1/(-1-1/2*3^(1/2))
/(3^(1/2)-1)^2,(1-1/2*3^(1/2))^(1/2)/(-1-1/2*3^(1/2))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+\sqrt{3}+1}{\sqrt{x^4-4\sqrt{3}x^2-4}\left(x-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sqrt(3) + 1)/(sqrt(x^4 - 4*sqrt(3)*x^2 - 4)*(x - sqrt(3) + 1)),x, a
```

```
[Out] integrate((x + sqrt(3) + 1)/(sqrt(x^4 - 4*sqrt(3)*x^2 - 4)*(x - s
qrt(3) + 1)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x + sqrt(3) + 1)/(sqrt(x^4 - 4*sqrt(3)*x^2 - 4)*(x - sqrt(3) + 1)),x, a
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x + 1 + \sqrt{3}}{\left(x - \sqrt{3} + 1\right)\sqrt{x^4 - 4\sqrt{3}x^2 - 4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3**(1/2))/(1+x-3**(1/2))/(-4+x**4-4*3**(1/2)*x**2)**(1/2),x)
```

```
[Out] Integral((x + 1 + sqrt(3))/((x - sqrt(3) + 1)*sqrt(x**4 - 4*sqrt(
3)*x**2 - 4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x + \sqrt{3} + 1}{\sqrt{x^4 - 4\sqrt{3}x^2 - 4}\left(x - \sqrt{3} + 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x + sqrt(3) + 1)/(sqrt(x^4 - 4*sqrt(3)*x^2 - 4)*(x - sqrt(3) + 1)),x, a
```

```
[Out] integrate((x + sqrt(3) + 1)/(sqrt(x^4 - 4*sqrt(3)*x^2 - 4)*(x - s
qrt(3) + 1)), x)
```

3.90 $\int \frac{-1+x}{(1+x)\sqrt[3]{2+x^3}}\,dx$

Optimal. Leaf size=53

$$-\frac{3}{2}\log\left(-\sqrt[3]{x^3+2}+x+2\right)+\sqrt{3}\tan^{-1}\left(\frac{\frac{2(x+2)}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)+\log(x+1)$$

```
[Out] Sqrt[3]*ArcTan[(1 + (2*(2 + x))/(2 + x^3)^(1/3))/Sqrt[3]] + Log[1
 + x] - (3*Log[2 + x - (2 + x^3)^(1/3)])/2
```

Rubi [A] time = 0.0963491, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056

$$-\frac{3}{2}\log\left(-\sqrt[3]{x^3+2}+x+2\right)+\sqrt{3}\tan^{-1}\left(\frac{\frac{2(x+2)}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)+\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x)/((1 + x)*(2 + x^3)^(1/3)),x]
```

```
[Out] Sqrt[3]*ArcTan[(1 + (2*(2 + x))/(2 + x^3)^(1/3))/Sqrt[3]] + Log[1
 + x] - (3*Log[2 + x - (2 + x^3)^(1/3)])/2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-1}{(x+1)\sqrt[3]{x^3+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-1+x)/(1+x)/(x**3+2)**(1/3),x)
```

```
[Out] Integral((x - 1)/((x + 1)*(x**3 + 2)**(1/3)), x)
```

Mathematica [A] time = 0.143547, size = 0, normalized size = 0.

$$\int \frac{-1+x}{(1+x)\sqrt[3]{2+x^3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(-1 + x)/((1 + x)*(2 + x^3)^(1/3)),x]
```

```
[Out] Integrate[(-1 + x)/((1 + x)*(2 + x^3)^(1/3)), x]
```

Maple [F] time = 0.045, size = 0, normalized size = 0.

$$\int \frac{-1+x}{1+x}\frac{1}{\sqrt[3]{x^3+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-1+x)/(1+x)/(x^3+2)^(1/3),x)
```

```
[Out] int((-1+x)/(1+x)/(x^3+2)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-1}{(x^3+2)^{\frac{1}{3}}(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - 1)/((x^3 + 2)^(1/3)*(x + 1)),x, algorithm="maxima")
```

```
[Out] integrate((x - 1)/((x^3 + 2)^(1/3)*(x + 1)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - 1)/((x^3 + 2)^(1/3)*(x + 1)),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-1}{(x+1)\sqrt[3]{x^3+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-1+x)/(1+x)/(x**3+2)**(1/3),x)
```

```
[Out] Integral((x - 1)/((x + 1)*(x**3 + 2)**(1/3)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-1}{(x^3+2)^{\frac{1}{3}}(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - 1)/((x^3 + 2)^(1/3)*(x + 1)),x, algorithm="giac")
```

```
[Out] integrate((x - 1)/((x^3 + 2)^(1/3)*(x + 1)), x)
```

3.91 $\int \frac{1}{(1+x)\sqrt[3]{2+x^3}}\,dx$

Optimal. Leaf size=108

$$\frac{3}{4}\log\left(-\sqrt[3]{x^3+2}+x+2\right)-\frac{1}{4}\log\left(\sqrt[3]{x^3+2}-x\right)+\frac{\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{\frac{2(x+2)}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)-\frac{1}{2}\log(x+1)$$

```
[Out] ArcTan[(1 + (2*x)/(2 + x^3)^(1/3))/Sqrt[3]]/(2*Sqrt[3]) - (Sqrt[3
]*ArcTan[(1 + (2*(2 + x))/(2 + x^3)^(1/3))/Sqrt[3]])/2 - Log[1 +
x]/2 + (3*Log[2 + x - (2 + x^3)^(1/3)])/4 - Log[-x + (2 + x^3)^(1
/3)]/4
```

Rubi [A] time = 0.160376, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{3}{4}\log\left(-\sqrt[3]{x^3+2}+x+2\right)-\frac{1}{4}\log\left(\sqrt[3]{x^3+2}-x\right)+\frac{\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{\frac{2(x+2)}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)-\frac{1}{2}\log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[1/((1 + x)*(2 + x^3)^(1/3)),x]
```

```
[Out] ArcTan[(1 + (2*x)/(2 + x^3)^(1/3))/Sqrt[3]]/(2*Sqrt[3]) - (Sqrt[3
]*ArcTan[(1 + (2*(2 + x))/(2 + x^3)^(1/3))/Sqrt[3]])/2 - Log[1 +
x]/2 + (3*Log[2 + x - (2 + x^3)^(1/3)])/4 - Log[-x + (2 + x^3)^(1
/3)]/4
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x+1)\sqrt[3]{x^3+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1+x)/(x**3+2)**(1/3),x)
```

```
[Out] Integral(1/((x + 1)*(x**3 + 2)**(1/3)), x)
```

Mathematica [A] time = 0.0469073, size = 0, normalized size = 0.

$$\int \frac{1}{(1+x)\sqrt[3]{2+x^3}}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[1/((1 + x)*(2 + x^3)^(1/3)),x]
```

```
[Out] Integrate[1/((1 + x)*(2 + x^3)^(1/3)), x]
```

Maple [F] time = 0.039, size = 0, normalized size = 0.

$$\int \frac{1}{1+x} \frac{1}{\sqrt[3]{x^3+2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1+x)/(x^3+2)^(1/3),x)
```

```
[Out] int(1/(1+x)/(x^3+2)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^3+2)^{\frac{1}{3}}(x+1)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^3 + 2)^(1/3)*(x + 1)),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 2)^(1/3)*(x + 1)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^3 + 2)^(1/3)*(x + 1)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x+1)\sqrt[3]{x^3+2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1+x)/(x**3+2)**(1/3),x)
```

```
[Out] Integral(1/((x + 1)*(x**3 + 2)**(1/3)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^3+2)^{\frac{1}{3}}(x+1)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^3 + 2)^(1/3)*(x + 1)),x, algorithm="giac")

[Out] integrate(1/((x^3 + 2)^(1/3)*(x + 1)), x)
```

4 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Mathematica/Rubi followed by one for Maple. The following are links to the source code.

The following are the listing of the above functions.

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)
(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)
```

```
ExpnType[expn_] :=
  If[AtomQ[expn], 1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]

SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]
HypergeometricFunctionQ[func_] := MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
AppellFunctionQ[func_] := MemberQ[{AppellF1},func]
```

```
# File: GradeAntiderivative.mpl Original version thanks to Albert Rich emailed on 03/21/2017
#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);
  #This check below actually is not needed, since I only call this grading only for
  #passed integrals. i.e. I check for "F" before calling this.

  if not type(result,freeof('int')) then
     return "F";
  end if;

  if ExpnType_result<=ExpnType_optimal then
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
         if leaf_count_result<=2*leaf_count_optimal then
            return "A";
         else
            return "B";
         end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      return "C";
  end if
end proc:
```

```
# is_contains_complex(result) takes expressions and returns true if it contains "I"
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1 else
    max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1 else
      max(2,ExpnType(op(1,expn))) end if else
    max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' or op(0,expn)='integrate' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:
```

```
ElementaryFunctionQ := proc(func)
  member(func,[exp,log,ln, sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[erf,erfc,erfi,FresnelS,FresnelC,Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u) else
  apply(op(0,u),op(2..nops(u),u))
  end if
end proc:
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```