

Computer algebra independent integration tests

0_Independent_test_suites/Welz_Problems

Nasser M. Abbasi

November 25, 2018

Compiled on November 25, 2018 at 10:44pm

Contents

1	Introduction	2
2	detailed summary tables of results	11
3	Listing of integrals	30
4	Listing of Grading functions	376

1 Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from Albert Rich Rubi web site.

1.1 Listing of CAS systems tested

The following systems were tested at this time.

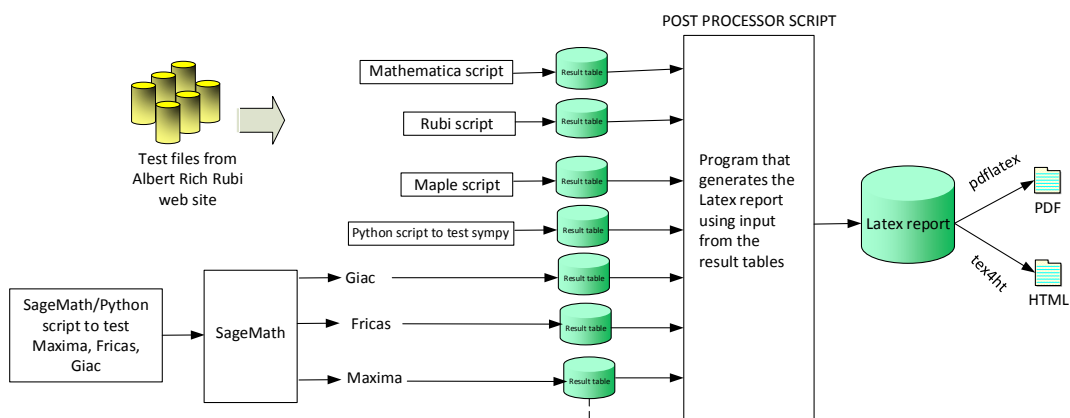
1. Mathematica 11.3 (64 bit).
2. Rubi 4.15.2 in Mathematica 11.3.
3. Rubi in Sympy (Version 1.3) under Python 3.7.0 using Anaconda distribution.
4. Maple 2018.1 (64 bit).
5. Maxima 5.41 Using Lisp ECL 16.1.2.
6. Fricas 1.3.4.
7. Sympy 1.3 under Python 3.7.0 using Anaconda distribution.
8. Giac/Xcas 1.4.9.

Maxima, Fricas and Giac/Xcas were called from inside SageMath version 8.3. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python. Rubi in Sympy was also called directly using sympy 1.3 in python.

1.2 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is comma delimited. It contains 12 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"

High level overview of the CAS independent integration test build system

Nasser M. Abbasi
June 22, 2018

1.3 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int', int(expr, x)), output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.4 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked

in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.5 Important notes about some of the results

Important note about Maxima results Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

Important note about FriCAS and Giac/XCAS results There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

Important note about finding leaf size of antiderivative For Mathematica, Rubi and Maple, the builtin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-express
def tree(expr):
    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

    except Exception as ee:
        leafCount =1
```

When these cas systems implement a builtin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.6 Grading of results

The table below summarizes the grading of each CAS system.

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in the table below reflect the above.

System	solved	Failed
Rubi	% 94.51 (86)	% 5.49 (5)
Rubi in Sympy	% 70.33 (64)	% 29.67 (27)
Mathematica	% 84.62 (77)	% 15.38 (14)
Maple	% 60.44 (55)	% 39.56 (36)
Maxima	% 21.98 (20)	% 78.02 (71)
Fricas	% 59.34 (54)	% 40.66 (37)
Sympy	% 29.67 (27)	% 70.33 (64)
Giac	% 18.68 (17)	% 81.32 (74)

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

Grading is currently implemented only for for Mathematica, Rubi and Maple results. For all other CAS systems (Maxima, Fricas, Sympy, Giac, Rubi in sympy), the grading function is not yet implemented. For these systems, a grade of A is assigned if the integrate command completes successfully and a grade of F otherwise.

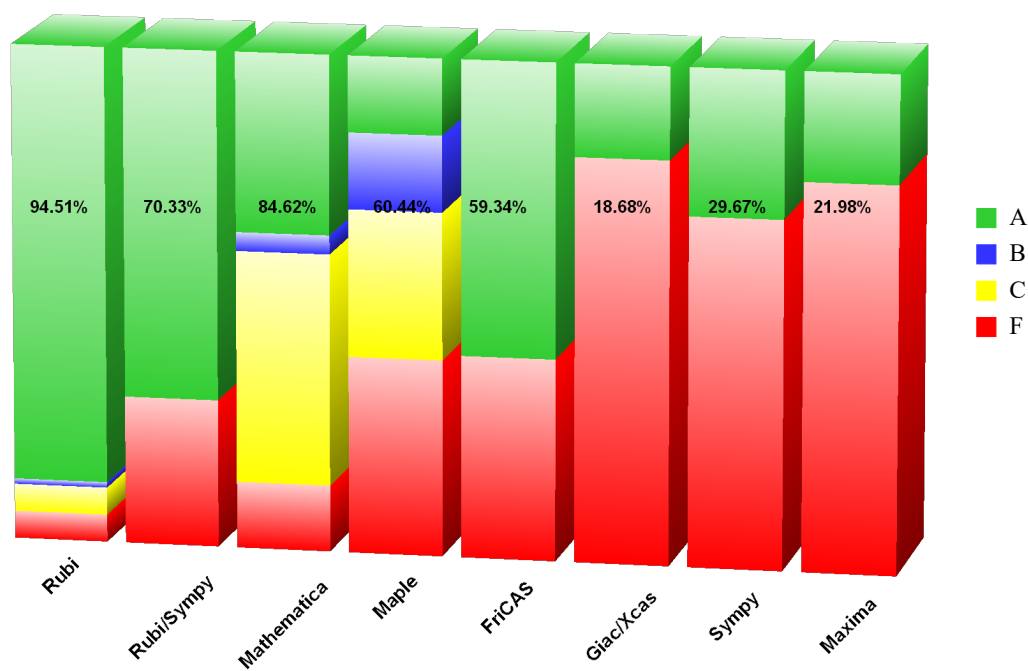
Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	87.91	1.1	5.49	5.49
Rubi in Sympy	70.33	0.	0.	29.67
Mathematica	41.76	4.4	53.85	15.38
Maple	15.38	15.38	29.67	39.56
Maxima	21.98	0.	0.	78.02
Fricas	59.34	0.	0.	40.66
Sympy	29.67	0.	0.	70.33
Giac	18.68	0.	0.	81.32

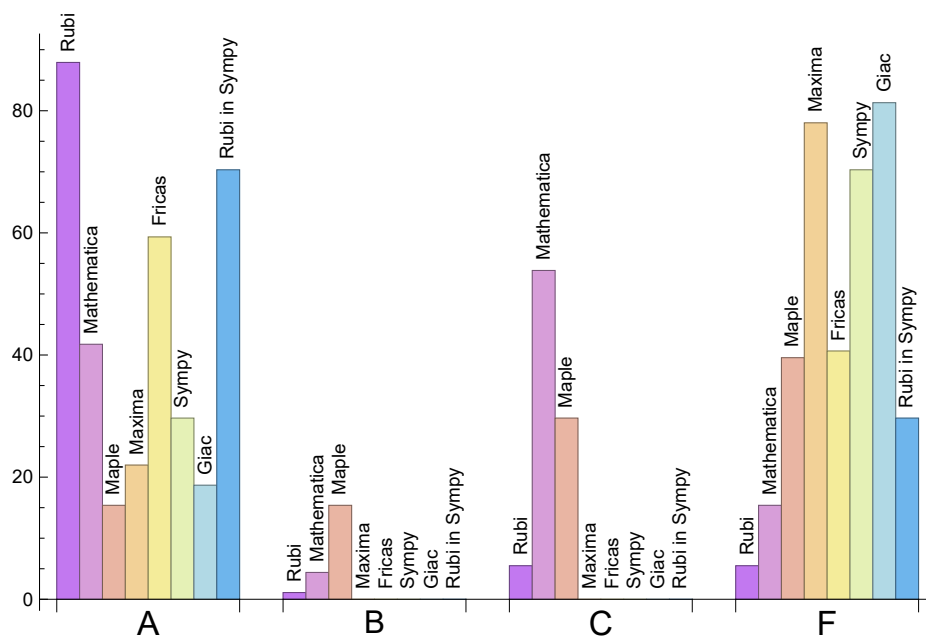
The following is a Bar chart illustration of the data in the above table.

Antiderivative Grade distribution for each CAS

Numbers shown on bars are total percentage solved for each CAS



The figure below compares the CAS systems for each grade level.



1.7 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.52	145.79	7.87	81.	1.
Rubi in Sympy	11.56	106.11	1.5	55.	0.89
Mathematica	0.85	205.52	3.43	118.	1.
Maple	0.19	2243.62	12.27	206.	1.64
Maxima	1.46	93.65	1.47	67.	1.26
Fricas	0.26	487.78	5.07	82.5	1.65
Sympy	4.3	214.41	5.29	37.	0.86
Giac	0.22	70.	1.58	55.	1.45

1.8 list of integrals that has no closed form antiderivative

{

1.9 list of integrals not solved by each system

Not solved by Rubi {43, 44, 45, 58, 59}

Not solved by Rubi in Sympy {3, 5, 9, 10, 17, 18, 29, 31, 37, 38, 43, 44, 45, 46, 48, 49, 51, 52, 58, 59, 79, 80, 81, 88, 89, 90, 91}

Not solved by Mathematica {9, 10, 12, 13, 37, 38, 44, 45, 46, 58, 59, 65, 90, 91}

Not solved by Maple {12, 13, 14, 18, 19, 20, 25, 26, 27, 29, 30, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 53, 54, 55, 58, 59, 67, 68, 69, 70, 75, 76, 77, 78, 90, 91}

Not solved by Maxima {3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 17, 18, 24, 25, 26, 27, 28, 31, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 58, 59, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91}

Not solved by Fricas {12, 13, 29, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 49, 51, 53, 54, 55, 58, 59, 67, 68, 69, 70, 75, 76, 77, 78, 82, 84, 85, 86, 87, 88, 89, 90, 91}

Not solved by Sympy {3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 18, 24, 29, 32, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 58, 59, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91}

Not solved by Giac {5, 6, 9, 10, 11, 12, 13, 14, 15, 17, 18, 19, 20, 24, 25, 26, 27, 28, 29, 30, 31, 32, 35, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 50, 52, 53, 54, 55, 56, 58, 59, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91}

1.10 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Rubi in Sympy {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional

methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {9, 10, 49, 80}

Mathematica {39, 40, 53, 54, 55, 63, 64, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 82, 83, 84, 85, 86, 87, 88, 89}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Rubi in Sympy Verification phase not implemented yet.

2 detailed summary tables of results

2.1 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	18	12	18	12
normalized size	1	1.	1.	0.93	1.2	1.2	0.8	1.2	0.8
time (sec)	N/A	0.009	0.005	0.004	1.365	0.206	0.038	0.199	0.669

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	C	C	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	52	37	34	55	1	42	55	42
normalized size	1	3.47	2.47	2.27	3.67	0.07	2.8	3.67	2.8
time (sec)	N/A	0.097	0.02	0.005	1.375	0.21	52.66	0.212	8.535

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	119	370	0	363	0	239	0
normalized size	1	1.	1.45	4.51	0.	4.43	0.	2.91	0.
time (sec)	N/A	0.144	0.159	0.062	0.	0.22	0.	0.206	0.

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	172	0	201	0	127	34
normalized size	1	1.	1.	4.	0.	4.67	0.	2.95	0.79
time (sec)	N/A	0.038	0.042	0.069	0.	0.216	0.	0.233	4.029

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	65	115	0	274	0	0	0
normalized size	1	1.	0.88	1.55	0.	3.7	0.	0.	0.
time (sec)	N/A	0.146	0.146	0.023	0.	0.225	0.	0.	0.

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	165	125	72	230	0	0	53
normalized size	1	1.	2.58	1.95	1.12	3.59	0.	0.	0.83
time (sec)	N/A	0.076	0.179	0.038	1.546	0.231	0.	0.	5.039

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	90	45	0	216	0	136	41
normalized size	1	1.	1.88	0.94	0.	4.5	0.	2.83	0.85
time (sec)	N/A	0.039	0.071	0.026	0.	0.222	0.	0.203	3.715

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	29	21	0	24	53	27	26
normalized size	1	1.	0.97	0.7	0.	0.8	1.77	0.9	0.87
time (sec)	N/A	0.128	0.023	0.004	0.	0.208	1.502	0.206	4.971

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	F	F
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	220	365	0	902	0	1185	0	0	0
normalized size	1	1.66	0.	4.1	0.	5.39	0.	0.	0.
time (sec)	N/A	0.997	4.931	0.177	0.	0.253	0.	0.	0.

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	B	A	B	F	A	F(-1)	F	F(-1)
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	220	541	0	1542	0	1185	0	0	0
normalized size	1	2.46	0.	7.01	0.	5.39	0.	0.	0.
time (sec)	N/A	1.561	1.66	0.033	0.	0.255	0.	0.	0.

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	F(-2)	A	F(-2)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	403	278	0	680	0	0	184
normalized size	1	1.	2.92	2.01	0.	4.93	0.	0.	1.33
time (sec)	N/A	0.162	0.413	0.032	0.	0.287	0.	0.	18.456

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F(-1)	F	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	0	0	0	0	0	0	202
normalized size	1	1.	0.	0.	0.	0.	0.	0.	1.62
time (sec)	N/A	0.328	0.154	0.036	0.	0.	0.	0.	21.574

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F(-1)	F	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	0	0	0	0	0	0	155
normalized size	1	1.	0.	0.	0.	0.	0.	0.	1.91
time (sec)	N/A	0.278	0.119	0.037	0.	0.	0.	0.	17.113

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	145	0	0	81	15	0	29
normalized size	1	1.	4.68	0.	0.	2.61	0.48	0.	0.94
time (sec)	N/A	0.089	2.357	0.024	0.	0.39	2.212	0.	3.712

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	C	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	162	22	0	39	15	0	29
normalized size	1	1.	4.91	0.67	0.	1.18	0.45	0.	0.88
time (sec)	N/A	0.091	1.435	0.055	0.	0.4	1.164	0.	3.951

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	20	31	56	20	17
normalized size	1	1.	1.	0.84	1.05	1.63	2.95	1.05	0.89
time (sec)	N/A	0.434	0.025	0.004	1.341	0.203	12.491	0.214	11.273

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	36	120	0	43	2147	0	0
normalized size	1	1.	0.69	2.31	0.	0.83	41.29	0.	0.
time (sec)	N/A	0.047	0.04	0.045	0.	0.229	4.98	0.	0.

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	39	0	0	45	0	0	0
normalized size	1	1.	0.7	0.	0.	0.8	0.	0.	0.
time (sec)	N/A	0.046	0.04	0.045	0.	0.23	0.	0.	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	0	20	20	311	0	12
normalized size	1	1.	1.	0.	1.18	1.18	18.29	0.	0.71
time (sec)	N/A	0.087	0.027	0.029	1.396	0.23	5.359	0.	4.054

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	0	24	24	36	0	14
normalized size	1	1.	1.	0.	1.2	1.2	1.8	0.	0.7
time (sec)	N/A	0.092	0.028	0.045	1.397	0.232	3.559	0.	4.364

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	37	48	54	70	36	55	39
normalized size	1	1.	0.88	1.14	1.29	1.67	0.86	1.31	0.93
time (sec)	N/A	0.063	0.063	0.017	1.352	0.224	0.176	0.198	5.391

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	27	26	22	26	15
normalized size	1	1.	1.	0.95	1.23	1.18	1.	1.18	0.68
time (sec)	N/A	0.041	0.021	0.004	1.342	0.216	0.116	0.201	4.678

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	49	51	69	78	49	100	54
normalized size	1	1.	0.79	0.82	1.11	1.26	0.79	1.61	0.87
time (sec)	N/A	0.154	0.052	0.019	1.368	0.231	0.216	0.199	10.102

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	961	455	0	720	0	0	87
normalized size	1	1.	11.17	5.29	0.	8.37	0.	0.	1.01
time (sec)	N/A	0.226	5.47	0.057	0.	0.252	0.	0.	18.81

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	0	0	20	15	0	15
normalized size	1	1.	1.	0.	0.	1.05	0.79	0.	0.79
time (sec)	N/A	0.106	0.026	0.039	0.	0.218	0.282	0.	3.644

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	0	0	30	27	0	20
normalized size	1	1.	1.	0.	0.	1.15	1.04	0.	0.77
time (sec)	N/A	0.161	0.041	0.05	0.	0.223	1.344	0.	4.681

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	85	0	0	1	46	0	58
normalized size	1	1.	1.35	0.	0.	0.02	0.73	0.	0.92
time (sec)	N/A	0.365	0.251	0.028	0.	0.243	2.87	0.	9.559

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	161	25	0	1	51	0	73
normalized size	1	1.	1.96	0.3	0.	0.01	0.62	0.	0.89
time (sec)	N/A	0.14	0.081	0.008	0.	0.241	2.482	0.	8.607

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	679	679	412	0	699	0	0	0	0
normalized size	1	1.	0.61	0.	1.03	0.	0.	0.	0.
time (sec)	N/A	7.496	0.267	0.063	1.399	0.	0.	0.	0.

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	0	20	20	311	0	12
normalized size	1	1.	1.	0.	1.18	1.18	18.29	0.	0.71
time (sec)	N/A	0.088	0.04	0.037	1.402	0.241	7.301	0.	3.919

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	36	120	0	43	2147	0	0
normalized size	1	1.	0.69	2.31	0.	0.83	41.29	0.	0.
time (sec)	N/A	0.046	0.023	0.033	0.	0.244	6.697	0.	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	33	44	65	65	0	0	27
normalized size	1	1.	0.97	1.29	1.91	1.91	0.	0.	0.79
time (sec)	N/A	0.06	0.056	0.066	1.766	0.224	0.	0.	9.298

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	41	65	84	84	36	86	49
normalized size	1	1.	0.71	1.12	1.45	1.45	0.62	1.48	0.84
time (sec)	N/A	0.078	0.021	0.056	1.497	0.214	1.607	0.207	2.456

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	41	48	84	84	37	86	49
normalized size	1	1.	0.71	0.83	1.45	1.45	0.64	1.48	0.84
time (sec)	N/A	0.069	0.02	0.063	1.5	0.216	1.72	0.204	2.453

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	86	12	105	122	29	0	71
normalized size	1	1.	1.76	0.24	2.14	2.49	0.59	0.	1.45
time (sec)	N/A	0.016	0.071	0.034	1.502	0.216	1.61	0.	3.587

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	39	65	84	96	32	85	48
normalized size	1	1.	0.71	1.18	1.53	1.75	0.58	1.55	0.87
time (sec)	N/A	0.07	0.019	0.057	1.474	0.215	1.605	0.209	2.299

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F(-1)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	121	97	0	0	0	0	0	0	0
normalized size	1	0.8	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.083	0.073	0.049	0.	0.	0.	0.	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	0	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.194	0.089	0.059	0.	0.	0.	0.	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	110	176	109	0	0	0	0	0	155
normalized size	1	1.6	0.99	0.	0.	0.	0.	0.	1.41
time (sec)	N/A	0.099	0.19	0.098	0.	0.	0.	0.	4.678

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	81	131	85	0	0	0	0	0	204
normalized size	1	1.62	1.05	0.	0.	0.	0.	0.	2.52
time (sec)	N/A	0.139	0.022	0.019	0.	0.	0.	0.	7.393

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	A	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	117	49	0	104	0	0	0	155
normalized size	1	1.77	0.74	0.	1.58	0.	0.	0.	2.35
time (sec)	N/A	0.113	0.035	0.064	1.524	0.	0.	0.	5.457

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	145	61	0	0	0	0	0	216
normalized size	1	1.84	0.77	0.	0.	0.	0.	0.	2.73
time (sec)	N/A	0.188	0.058	0.02	0.	0.	0.	0.	9.593

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	F	C	F	F	F(-1)	F(-1)	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	0	72	0	0	0	0	0	0
normalized size	1	0.	0.61	0.	0.	0.	0.	0.	0.
time (sec)	N/A	27.68	0.228	0.028	0.	0.	0.	0.	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	F	A	F	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	111	0	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.338	1.235	0.062	0.	0.	0.	0.	0.

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	F	A	F	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	176	0	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.82	0.415	0.073	0.	0.	0.	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	C	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	326	576	0	0	0	0	0	0	0
normalized size	1	1.77	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.525	0.165	0.183	0.	0.	0.	0.	0.

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	407	407	206	584	0	1827	0	0	478
normalized size	1	1.	0.51	1.43	0.	4.49	0.	0.	1.17
time (sec)	N/A	1.477	2.259	0.132	0.	0.281	0.	0.	48.171

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	648	648	662	719	0	2839	0	0	0
normalized size	1	1.	1.02	1.11	0.	4.38	0.	0.	0.
time (sec)	N/A	2.686	6.077	0.072	0.	0.34	0.	0.	0.

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F(-1)	F(-1)	A	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1058	1058	1242	989	0	0	0	1	0
normalized size	1	1.	1.17	0.93	0.	0.	0.	0.	0.
time (sec)	N/A	5.375	6.165	0.138	0.	0.	0.	0.227	0.

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	378	378	1236	21028	0	6251	0	0	379
normalized size	1	1.	3.27	55.63	0.	16.54	0.	0.	1.
time (sec)	N/A	1.586	6.467	0.765	0.	0.363	0.	0.	79.056

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	638	638	1431	86793	0	0	0	1	0
normalized size	1	1.	2.24	136.04	0.	0.	0.	0.	0.
time (sec)	N/A	2.835	6.717	4.895	0.	0.	0.	0.351	0.

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	C	C	C	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	204	213	1275	0	1	0	0	0
normalized size	1	3.09	3.23	19.32	0.	0.02	0.	0.	0.
time (sec)	N/A	2.372	0.563	0.168	0.	0.617	0.	0.	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-2)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	205	0	0	0	0	0	231
normalized size	1	1.	1.04	0.	0.	0.	0.	0.	1.17
time (sec)	N/A	0.241	0.468	0.052	0.	0.	0.	0.	13.653

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-2)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	220	0	0	0	0	0	282
normalized size	1	1.	1.11	0.	0.	0.	0.	0.	1.42
time (sec)	N/A	0.212	0.442	0.073	0.	0.	0.	0.	13.12

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	88	97	273	0	0	0	0	0	94
normalized size	1	1.1	3.1	0.	0.	0.	0.	0.	1.07
time (sec)	N/A	0.047	0.81	0.199	0.	0.	0.	0.	2.42

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	34	69	142	143	32	0	85
normalized size	1	1.	0.32	0.64	1.33	1.34	0.3	0.	0.79
time (sec)	N/A	0.102	0.018	0.069	1.666	0.21	1.783	0.	4.775

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	48	49	96	115	37	97	56
normalized size	1	1.	0.72	0.73	1.43	1.72	0.55	1.45	0.84
time (sec)	N/A	0.084	0.025	0.073	1.534	0.208	1.678	0.24	2.768

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	F	A	F	F	F(-2)	F	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1	0	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.099	0.372	0.072	0.	0.	0.	0.	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	F	A	F	F	F(-1)	F	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	280	0	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.385	0.042	0.112	0.	0.	0.	0.	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	31	31	22	31	22
normalized size	1	1.	1.	0.96	1.24	1.24	0.88	1.24	0.88
time (sec)	N/A	0.012	0.012	0.001	1.39	0.194	0.107	0.245	3.601

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	86	52	0	51	73	0	56
normalized size	1	1.	1.46	0.88	0.	0.86	1.24	0.	0.95
time (sec)	N/A	0.06	0.027	0.043	0.	0.195	0.213	0.	7.633

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	99	75	0	89	100	0	80
normalized size	1	1.	1.27	0.96	0.	1.14	1.28	0.	1.03
time (sec)	N/A	0.124	0.029	0.033	0.	0.198	0.218	0.	14.771

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	110	100	0	105	0	0	36
normalized size	1	1.	2.24	2.04	0.	2.14	0.	0.	0.73
time (sec)	N/A	0.026	0.144	0.022	0.	0.257	0.	0.	2.408

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	108	365	0	81	0	0	46
normalized size	1	1.	2.04	6.89	0.	1.53	0.	0.	0.87
time (sec)	N/A	0.037	0.134	0.04	0.	0.254	0.	0.	2.585

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	F	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	0	1421	0	1	0	0	63
normalized size	1	1.	0.	18.95	0.	0.01	0.	0.	0.84
time (sec)	N/A	0.161	0.329	0.098	0.	0.27	0.	0.	9.109

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	322	456	0	2419	0	0	144
normalized size	1	1.	1.88	2.67	0.	14.15	0.	0.	0.84
time (sec)	N/A	0.142	0.427	0.101	0.	0.339	0.	0.	6.94

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	203	0	0	0	0	0	178
normalized size	1	1.	2.54	0.	0.	0.	0.	0.	2.22
time (sec)	N/A	0.118	0.56	0.088	0.	0.	0.	0.	28.634

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	221	0	0	0	0	0	199
normalized size	1	1.	2.51	0.	0.	0.	0.	0.	2.26
time (sec)	N/A	0.131	0.555	0.043	0.	0.	0.	0.	29.508

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	205	0	0	0	0	0	170
normalized size	1	1.	1.38	0.	0.	0.	0.	0.	1.14
time (sec)	N/A	0.136	0.446	0.07	0.	0.	0.	0.	39.319

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	219	0	0	0	0	0	173
normalized size	1	1.	1.62	0.	0.	0.	0.	0.	1.28
time (sec)	N/A	0.109	0.425	0.045	0.	0.	0.	0.	36.093

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	120	164	0	1451	0	0	155
normalized size	1	1.	0.94	1.29	0.	11.43	0.	0.	1.22
time (sec)	N/A	0.068	0.161	0.289	0.	0.358	0.	0.	3.059

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	135	240	0	2641	0	0	223
normalized size	1	1.	0.86	1.53	0.	16.82	0.	0.	1.42
time (sec)	N/A	0.101	0.249	0.153	0.	0.461	0.	0.	4.538

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	118	286	0	950	0	0	37
normalized size	1	1.	1.59	3.86	0.	12.84	0.	0.	0.5
time (sec)	N/A	0.288	0.191	0.169	0.	0.342	0.	0.	4.325

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	139	383	0	914	0	0	22
normalized size	1	1.	1.35	3.72	0.	8.87	0.	0.	0.21
time (sec)	N/A	0.545	0.181	0.197	0.	0.328	0.	0.	3.314

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	126	0	0	0	0	0	17
normalized size	1	1.	1.56	0.	0.	0.	0.	0.	0.21
time (sec)	N/A	0.04	0.152	0.07	0.	0.	0.	0.	2.603

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	126	0	0	0	0	0	20
normalized size	1	1.	1.56	0.	0.	0.	0.	0.	0.25
time (sec)	N/A	0.037	0.14	0.04	0.	0.	0.	0.	3.072

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	118	0	0	0	0	0	144
normalized size	1	1.	1.04	0.	0.	0.	0.	0.	1.27
time (sec)	N/A	0.046	0.131	0.05	0.	0.	0.	0.	5.869

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	124	0	0	0	0	0	192
normalized size	1	1.	1.14	0.	0.	0.	0.	0.	1.76
time (sec)	N/A	0.044	0.102	0.056	0.	0.	0.	0.	5.238

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	159	206	0	72	0	0	0
normalized size	1	1.	1.83	2.37	0.	0.83	0.	0.	0.
time (sec)	N/A	1.479	0.553	0.053	0.	0.266	0.	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	C	C	C	F	A	F	F	F(-1)
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	579	100	317	0	95	0	0	0
normalized size	1	579.	100.	317.	0.	95.	0.	0.	0.
time (sec)	N/A	3.072	0.464	0.063	0.	0.264	0.	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	C	C	C	F	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	180	133	536	0	85	0	0	0
normalized size	1	3.91	2.89	11.65	0.	1.85	0.	0.	0.
time (sec)	N/A	2.933	0.59	0.079	0.	0.259	0.	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-2)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	323	258	0	0	0	0	469
normalized size	1	1.	10.09	8.06	0.	0.	0.	0.	14.66
time (sec)	N/A	0.145	0.516	0.093	0.	0.	0.	0.	63.878

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	262	240	0	59	0	0	371
normalized size	1	1.	11.39	10.43	0.	2.57	0.	0.	16.13
time (sec)	N/A	0.091	0.283	0.031	0.	0.258	0.	0.	41.593

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	206	353	0	0	0	0	34
normalized size	1	1.	0.94	1.62	0.	0.	0.	0.	0.16
time (sec)	N/A	0.116	0.43	0.283	0.	0.	0.	0.	3.449

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	207	350	0	0	0	0	34
normalized size	1	1.	0.99	1.67	0.	0.	0.	0.	0.16
time (sec)	N/A	0.105	0.656	0.266	0.	0.	0.	0.	7.693

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	196	349	0	0	0	0	53
normalized size	1	1.	0.88	1.57	0.	0.	0.	0.	0.24
time (sec)	N/A	0.1	0.623	0.257	0.	0.	0.	0.	11.817

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	198	350	0	0	0	0	51
normalized size	1	1.	0.93	1.64	0.	0.	0.	0.	0.24
time (sec)	N/A	0.096	0.539	0.256	0.	0.	0.	0.	6.47

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	685	327	0	0	0	0	0
normalized size	1	1.	10.54	5.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.211	4.031	0.24	0.	0.	0.	0.	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	1137	311	0	0	0	0	0
normalized size	1	1.	18.05	4.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.213	10.714	0.235	0.	0.	0.	0.	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	0	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.096	0.144	0.045	0.	0.	0.	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F(-1)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	0	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.16	0.047	0.039	0.	0.	0.	0.	0.

2.2 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [29] had the largest ratio of [2.143]

Table 1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.	10	0.1
2	C	5	2	3.47	37	0.054
3	A	9	6	1.	15	0.4
4	A	3	3	1.	19	0.158
5	A	8	7	1.	17	0.412
6	A	6	6	1.	17	0.353

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
7	A	3	3	1.	17	0.176
8	A	4	2	1.	23	0.087
9	A	18	12	1.66	27	0.444
10	B	25	13	2.46	39	0.333
11	A	7	3	1.	45	0.067
12	A	7	4	1.	32	0.125
13	A	5	3	1.	32	0.094
14	A	2	2	1.	27	0.074
15	A	2	2	1.	29	0.069
16	A	2	1	1.	30	0.033
17	A	3	2	1.	13	0.154
18	A	3	2	1.	15	0.133
19	A	2	2	1.	23	0.087
20	A	2	2	1.	25	0.08
21	A	3	2	1.	11	0.182
22	A	2	2	1.	18	0.111
23	A	6	6	1.	20	0.3
24	A	6	5	1.	31	0.161
25	A	2	2	1.	29	0.069
26	A	2	2	1.	35	0.057
27	A	5	5	1.	32	0.156
28	A	6	6	1.	21	0.286
29	A	359	30	1.	14	2.143
30	A	2	2	1.	23	0.087
31	A	3	2	1.	13	0.154
32	A	2	2	1.	33	0.061
33	A	5	5	1.	15	0.333
34	A	5	5	1.	15	0.333
35	A	1	1	1.	11	0.091
36	A	5	5	1.	15	0.333
37	A	1	1	0.8	17	0.059
38	A	3	3	1.	18	0.167
39	A	2	2	1.6	16	0.125
40	A	5	5	1.62	17	0.294
41	A	5	5	1.77	13	0.385
42	A	5	5	1.84	16	0.312
Continued on next page						

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
43	F	0	0	N/A	0	N/A
44	F	0	0	N/A	0	N/A
45	F	0	0	N/A	0	N/A
46	C	7	3	1.77	32	0.094
47	A	19	9	1.	20	0.45
48	A	29	9	1.	20	0.45
49	A	49	9	1.	20	0.45
50	A	14	6	1.	23	0.261
51	A	24	6	1.	23	0.261
52	C	9	8	3.09	48	0.167
53	A	7	7	1.	24	0.292
54	A	7	7	1.	24	0.292
55	A	1	1	1.1	18	0.056
56	A	8	8	1.	13	0.615
57	A	6	6	1.	15	0.4
58	F	0	0	N/A	0	N/A
59	F	0	0	N/A	0	N/A
60	A	1	1	1.	38	0.026
61	A	1	1	1.	33	0.03
62	A	2	2	1.	39	0.051
63	A	1	1	1.	19	0.053
64	A	4	4	1.	19	0.21
65	A	4	4	1.	24	0.167
66	A	1	1	1.	24	0.042
67	A	7	7	1.	24	0.292
68	A	7	7	1.	24	0.292
69	A	3	3	1.	26	0.115
70	A	3	3	1.	22	0.136
71	A	1	1	1.	22	0.045
72	A	1	1	1.	23	0.043
73	A	8	8	1.	18	0.444
74	A	8	8	1.	23	0.348
75	A	1	1	1.	21	0.048
76	A	1	1	1.	19	0.053
77	A	1	1	1.	19	0.053
78	A	1	1	1.	19	0.053

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
79	A	4	4	1.	34	0.118
80	C	5	5	579.	40	0.125
81	C	7	7	3.91	51	0.137
82	A	2	2	1.	29	0.069
83	A	2	2	1.	18	0.111
84	A	1	1	1.	25	0.04
85	A	1	1	1.	25	0.04
86	A	1	1	1.	25	0.04
87	A	1	1	1.	25	0.04
88	A	2	2	1.	40	0.05
89	A	2	2	1.	40	0.05
90	A	1	1	1.	18	0.056
91	A	3	3	1.	15	0.2

3 Listing of integrals

$$3.1 \quad \int \frac{1}{\sqrt{1-ax}} dx$$

Optimal. Leaf size=15

$$-\frac{2\sqrt{1-ax}}{a}$$

[Out] (-2*Sqrt[1 - a*x])/a

Rubi [A] time = 0.00895376, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$-\frac{2\sqrt{1-ax}}{a}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[1 - a*x], x]

[Out] (-2*Sqrt[1 - a*x])/a

Rubi in Sympy [A] time = 0.668796, size = 12, normalized size = 0.8

$$-\frac{2\sqrt{-ax+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate(1/(-a*x+1)**(1/2), x)

[Out] -2*sqrt(-a*x + 1)/a

Mathematica [A] time = 0.00485158, size = 15, normalized size = 1.

$$-\frac{2\sqrt{1-ax}}{a}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[1 - a*x], x]

[Out] (-2*Sqrt[1 - a*x])/a

Maple [A] time = 0.004, size = 14, normalized size = 0.9

$$-2 \frac{\sqrt{-ax + 1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-a*x+1)^(1/2), x)

[Out] -2*(-a*x+1)^(1/2)/a

Maxima [A] time = 1.36485, size = 18, normalized size = 1.2

$$-\frac{2\sqrt{-ax + 1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sqrt(-a*x + 1), x, algorithm="maxima")

[Out] -2*sqrt(-a*x + 1)/a

Fricas [A] time = 0.205924, size = 18, normalized size = 1.2

$$-\frac{2\sqrt{-ax + 1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sqrt(-a*x + 1), x, algorithm="fricas")

[Out] -2*sqrt(-a*x + 1)/a

Sympy [A] time = 0.038173, size = 12, normalized size = 0.8

$$-\frac{2\sqrt{-ax + 1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x+1)**(1/2),x)
```

```
[Out] -2*sqrt(-a*x + 1)/a
```

GIAC/XCAS [A] time = 0.199346, size = 18, normalized size = 1.2

$$-\frac{2\sqrt{-ax+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sqrt(-a*x + 1),x, algorithm="giac")
```

```
[Out] -2*sqrt(-a*x + 1)/a
```


$$3.2 \quad \int \frac{-2 \log\left(-\sqrt{-1+ax}\right) + \log(-1+ax)}{2\pi\sqrt{-1+ax}} dx$$

Optimal. Leaf size=15

$$-\frac{2\sqrt{1-ax}}{a}$$

[Out] $(-2*\text{Sqrt}[1 - a*x])/a$

Rubi [C] time = 0.0971257, antiderivative size = 52, normalized size of antiderivative = 3.47, number of steps used = 5, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$

$$\frac{\sqrt{ax-1} \log(ax-1)}{\pi a} - \frac{2\sqrt{ax-1} \log(-\sqrt{ax-1})}{\pi a}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(-2*\text{Log}[-\text{Sqrt}[-1 + a*x]] + \text{Log}[-1 + a*x])/(2*\text{Pi}*\text{Sqrt}[-1 + a*x]), x]$

[Out] $(-2*\text{Sqrt}[-1 + a*x]*\text{Log}[-\text{Sqrt}[-1 + a*x]])/(a*\text{Pi}) + (\text{Sqrt}[-1 + a*x]*\text{Log}[-1 + a*x])/(a*\text{Pi})$

Rubi in Sympy [A] time = 8.53478, size = 42, normalized size = 2.8

$$-\frac{2\sqrt{ax-1} \log(-\sqrt{ax-1})}{\pi a} + \frac{\sqrt{ax-1} \log(ax-1)}{\pi a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{rubi_integrate}(1/2*(\ln(a*x-1)-2*\ln(-(a*x-1)**(1/2)))/\text{pi}/(a*x-1)**(1/2), x)$

[Out] $-2*\text{sqrt}(a*x - 1)*\log(-\text{sqrt}(a*x - 1))/(\text{pi}*a) + \text{sqrt}(a*x - 1)*\log(a*x - 1)/(\text{pi}*a)$

Mathematica [C] time = 0.0202415, size = 37, normalized size = 2.47

$$\frac{\sqrt{ax-1} \left(\log(ax-1) - 2 \log(-\sqrt{ax-1}) \right)}{\pi a}$$

Antiderivative was successfully verified.

[In] Integrate[(-2*Log[-Sqrt[-1 + a*x]] + Log[-1 + a*x])/(2*Pi*Sqrt[-1 + a*x]),x]

[Out] (Sqrt[-1 + a*x]*(-2*Log[-Sqrt[-1 + a*x]] + Log[-1 + a*x]))/(a*Pi)

Maple [C] time = 0.005, size = 34, normalized size = 2.3

$$\frac{1}{a\pi} \sqrt{ax-1} \left(\ln(ax-1) - 2 \ln(-\sqrt{ax-1}) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/2*(ln(a*x-1)-2*ln(-(a*x-1)^(1/2)))/Pi/(a*x-1)^(1/2),x)

[Out] (a*x-1)^(1/2)*(ln(a*x-1)-2*ln(-(a*x-1)^(1/2)))/a/Pi

Maxima [A] time = 1.37476, size = 55, normalized size = 3.67

$$\frac{\sqrt{ax-1} \log(ax-1) - 2\sqrt{ax-1} \log(-\sqrt{ax-1})}{\pi a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/2*(log(a*x - 1) - 2*log(-sqrt(a*x - 1)))/(pi*sqrt(a*x - 1)),x, algorithm="maxima")

[Out] (sqrt(a*x - 1)*log(a*x - 1) - 2*sqrt(a*x - 1)*log(-sqrt(a*x - 1)))/(pi*a)

Fricas [A] time = 0.210294, size = 1, normalized size = 0.07

0

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/2*(log(a*x - 1) - 2*log(-sqrt(a*x - 1)))/(pi*sqrt(a*x - 1)),x, algorithm="fricas")

[Out] 0

Sympy [A] time = 52.66, size = 42, normalized size = 2.8

$$\frac{\begin{cases} \frac{-2\sqrt{ax-1}\log(-\sqrt{ax-1})+\sqrt{ax-1}\log(ax-1)}{a} & \text{for } a \neq 0 \\ \pi x & \text{otherwise} \end{cases}}{\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/2*(ln(a*x-1)-2*ln(-(a*x-1)**(1/2)))/pi/(a*x-1)**(1/2),x)

[Out] Piecewise((((-2*sqrt(a*x - 1)*log(-sqrt(a*x - 1)) + sqrt(a*x - 1)*log(a*x - 1))/a, Ne(a, 0)), (pi*x, True))/pi

GIAC/XCAS [A] time = 0.212235, size = 55, normalized size = 3.67

$$\frac{\sqrt{ax-1}\ln(ax-1)-2\sqrt{ax-1}\ln(-\sqrt{ax-1})}{\pi a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/2*(log(a*x - 1) - 2*log(-sqrt(a*x - 1)))/(pi*sqrt(a*x - 1)),x, algorithm=

[Out] (sqrt(a*x - 1)*ln(a*x - 1) - 2*sqrt(a*x - 1)*ln(-sqrt(a*x - 1)))/(pi*a)

$$3.3 \quad \int \frac{1}{(2x + \sqrt{1+x^2})^2} dx$$

Optimal. Leaf size=82

$$\frac{4x}{3(1-3x^2)} - \frac{2\sqrt{x^2+1}}{3(1-3x^2)} + \frac{\tanh^{-1}\left(\frac{1}{2}\sqrt{3}\sqrt{x^2+1}\right)}{3\sqrt{3}} - \frac{\tanh^{-1}(\sqrt{3}x)}{3\sqrt{3}}$$

[Out] (4*x)/(3*(1 - 3*x^2)) - (2*Sqrt[1 + x^2])/(3*(1 - 3*x^2)) - ArcTanh[Sqrt[3]*x]/(3*Sqrt[3]) + ArcTanh[(Sqrt[3]*Sqrt[1 + x^2])/2]/(3*Sqrt[3])

Rubi [A] time = 0.143501, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$

$$\frac{4x}{3(1-3x^2)} - \frac{2\sqrt{x^2+1}}{3(1-3x^2)} + \frac{\tanh^{-1}\left(\frac{1}{2}\sqrt{3}\sqrt{x^2+1}\right)}{3\sqrt{3}} - \frac{\tanh^{-1}(\sqrt{3}x)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(2*x + Sqrt[1 + x^2])^(-2), x]

[Out] (4*x)/(3*(1 - 3*x^2)) - (2*Sqrt[1 + x^2])/(3*(1 - 3*x^2)) - ArcTanh[Sqrt[3]*x]/(3*Sqrt[3]) + ArcTanh[(Sqrt[3]*Sqrt[1 + x^2])/2]/(3*Sqrt[3])

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2x + \sqrt{x^2 + 1})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate(1/(2*x+(x**2+1)**(1/2))**2, x)

[Out] Integral((2*x + sqrt(x**2 + 1))**(-2), x)

Mathematica [A] time = 0.158967, size = 119, normalized size = 1.45

$$\frac{1}{18} \left(\sqrt{3} \log \left(2\sqrt{3}\sqrt{x^2+1} - \sqrt{3}x + 3 \right) \right. \\ \left. + \frac{-12\sqrt{x^2+1} + \sqrt{3}(1-3x^2) \log \left(2\sqrt{3}\sqrt{x^2+1} + \sqrt{3}x + 3 \right) + 24x}{1-3x^2} - 2\sqrt{3} \log \left(3x + \sqrt{3} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(2*x + Sqrt[1 + x^2])^(-2), x]

[Out] (-2*Sqrt[3]*Log[Sqrt[3] + 3*x] + Sqrt[3]*Log[3 - Sqrt[3]*x + 2*Sqrt[3]*Sqrt[1 + x^2]] + (24*x - 12*Sqrt[1 + x^2] + Sqrt[3]*(1 - 3*x^2))*Log[3 + Sqrt[3]*x + 2*Sqrt[3]*Sqrt[1 + x^2]])/(1 - 3*x^2)/18

Maple [B] time = 0.062, size = 370, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*x+(x^2+1)^(1/2))^2, x)

[Out] -1/2*x/(3*x^2-1)-1/9*arctanh(x*3^(1/2))*3^(1/2)-5/18*x/(x^2-1/3)-3^(1/2)*(-1/12/(x-1/3*3^(1/2))*((x-1/3*3^(1/2))^2+2/3*3^(1/2)*(x-1/3*3^(1/2))+4/3)^(3/2)+1/36*3^(1/2)*(1/3*(9*(x-1/3*3^(1/2))^2+6*3^(1/2)*(x-1/3*3^(1/2))+12)^(1/2)+1/3*3^(1/2)*arcsinh(x)-2/3*3^(1/2)*arctanh(3/4*(8/3+2/3*3^(1/2)*(x-1/3*3^(1/2))))*3^(1/2)/(9*(x-1/3*3^(1/2))^2+6*3^(1/2)*(x-1/3*3^(1/2))+12)^(1/2))+1/12*x*((x-1/3*3^(1/2))^2+2/3*3^(1/2)*(x-1/3*3^(1/2))+4/3)^(1/2)+1/12*arcsinh(x))+3^(1/2)*(-1/12/(x+1/3*3^(1/2))*((x+1/3*3^(1/2))^2-2/3*3^(1/2)*(x+1/3*3^(1/2))+4/3)^(3/2)-1/36*3^(1/2)*(1/3*(9*(x+1/3*3^(1/2))^2-6*3^(1/2)*(x+1/3*3^(1/2))+12)^(1/2)-1/3*3^(1/2)*arcsinh(x)-2/3*3^(1/2)*arctanh(3/4*(8/3-2/3*3^(1/2)*(x+1/3*3^(1/2))))*3^(1/2)/(9*(x+1/3*3^(1/2))^2-6*3^(1/2)*(x+1/3*3^(1/2))+12)^(1/2))+1/12*x*((x+1/3*3^(1/2))^2-2/3*3^(1/2)*(x+1/3*3^(1/2))+4/3)^(1/2)+1/12*arcsinh(x))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2x + \sqrt{x^2 + 1})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x + sqrt(x^2 + 1))^-2),x, algorithm="maxima")`

[Out] `integrate((2*x + sqrt(x^2 + 1))^-2), x)`

Fricas [A] time = 0.220191, size = 363, normalized size = 4.43

$$\frac{\left(6x^4 + x^2 - 2(3x^3 - x)\sqrt{x^2 + 1} - 1\right) \log\left(-\frac{24x^3 - \sqrt{3}(6x^4 + 17x^2 + 7) - 2(12x^2 - \sqrt{3}(3x^3 + 7x) + 6)\sqrt{x^2 + 1} + 24x}{6x^4 + x^2 - 2(3x^3 - x)\sqrt{x^2 + 1} - 1}\right) + (6x^4 + x^2 - 1) \log\left(\frac{24x^3 - \sqrt{3}(6x^4 + 17x^2 + 7) - 2(12x^2 - \sqrt{3}(3x^3 + 7x) + 6)\sqrt{x^2 + 1} + 24x}{6x^4 + x^2 - 2(3x^3 - x)\sqrt{x^2 + 1} - 1}\right)}{6\left(2\sqrt{3}(3x^3 - x)\sqrt{x^2 + 1} - \sqrt{3}(6x^4 + x^2 - 1)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x + sqrt(x^2 + 1))^-2),x, algorithm="fricas")`

[Out] `-1/6*((6*x^4 + x^2 - 2*(3*x^3 - x)*sqrt(x^2 + 1) - 1)*log(-(24*x^3 - sqrt(3)*(6*x^4 + 17*x^2 + 7) - 2*(12*x^2 - sqrt(3)*(3*x^3 + 7*x) + 6)*sqrt(x^2 + 1) + 24*x)/(6*x^4 + x^2 - 2*(3*x^3 - x)*sqrt(x^2 + 1) - 1)) + (6*x^4 + x^2 - 1)*log((sqrt(3)*(3*x^2 + 1) - 6*x)/(3*x^2 - 1)) - 8*sqrt(3)*(3*x^3 + 2*x) - 2*sqrt(x^2 + 1)*((3*x^3 - x)*log((sqrt(3)*(3*x^2 + 1) - 6*x)/(3*x^2 - 1)) - 2*sqrt(3)*(6*x^2 + 1)))/(2*sqrt(3)*(3*x^3 - x)*sqrt(x^2 + 1) - sqrt(3)*(6*x^4 + x^2 - 1))`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2x + \sqrt{x^2 + 1})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(2*x+(x**2+1)**(1/2))**2,x)`

[Out] `Integral((2*x + sqrt(x**2 + 1))**(-2), x)`

GIAC/XCAS [A] time = 0.206009, size = 239, normalized size = 2.91

$$\frac{1}{18} \sqrt{3} \ln \left(\frac{|6x - 2\sqrt{3}|}{|6x + 2\sqrt{3}|} \right) - \frac{1}{18} \sqrt{3} \ln \left(-\frac{\left| -6x - 8\sqrt{3} + 6\sqrt{x^2 + 1} - \frac{6}{x - \sqrt{x^2 + 1}} \right|}{2 \left(3x - 4\sqrt{3} - 3\sqrt{x^2 + 1} + \frac{3}{x - \sqrt{x^2 + 1}} \right)} \right) - \frac{4 \left(x - \sqrt{x^2 + 1} + \frac{1}{x - \sqrt{x^2 + 1}} \right)}{3 \left(3 \left(x - \sqrt{x^2 + 1} + \frac{1}{x - \sqrt{x^2 + 1}} \right)^2 - 16 \right)} - \frac{4x}{3(3x^2 - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x + sqrt(x^2 + 1))^(-2), x, algorithm="giac")

[Out] 1/18*sqrt(3)*ln(abs(6*x - 2*sqrt(3))/abs(6*x + 2*sqrt(3))) - 1/18*sqrt(3)*ln(-1/2*abs(-6*x - 8*sqrt(3) + 6*sqrt(x^2 + 1) - 6/(x - sqrt(x^2 + 1)))/(3*x - 4*sqrt(3) - 3*sqrt(x^2 + 1) + 3/(x - sqrt(x^2 + 1)))) - 4/3*(x - sqrt(x^2 + 1) + 1/(x - sqrt(x^2 + 1)))/(3*(x - sqrt(x^2 + 1) + 1/(x - sqrt(x^2 + 1)))^2 - 16) - 4/3*x/(3*x^2 - 1)

$$3.4 \quad \int \frac{1}{\sqrt{-1+x^2}(-4+3x^2)^2} dx$$

Optimal. Leaf size=43

$$\frac{3\sqrt{x^2-1}x}{8(4-3x^2)} + \frac{5}{16} \tanh^{-1}\left(\frac{x}{2\sqrt{x^2-1}}\right)$$

[Out] (3*x*Sqrt[-1 + x^2])/(8*(4 - 3*x^2)) + (5*ArcTanh[x/(2*Sqrt[-1 + x^2])])/16

Rubi [A] time = 0.0379042, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$

$$\frac{3\sqrt{x^2-1}x}{8(4-3x^2)} + \frac{5}{16} \tanh^{-1}\left(\frac{x}{2\sqrt{x^2-1}}\right)$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[-1 + x^2]*(-4 + 3*x^2)^2), x]

[Out] (3*x*Sqrt[-1 + x^2])/(8*(4 - 3*x^2)) + (5*ArcTanh[x/(2*Sqrt[-1 + x^2])])/16

Rubi in Sympy [A] time = 4.02935, size = 34, normalized size = 0.79

$$\frac{3x\sqrt{x^2-1}}{8(-3x^2+4)} + \frac{5 \operatorname{atanh}\left(\frac{x}{2\sqrt{x^2-1}}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate(1/(3*x**2-4)**2/(x**2-1)**(1/2), x)

[Out] 3*x*sqrt(x**2 - 1)/(8*(-3*x**2 + 4)) + 5*atanh(x/(2*sqrt(x**2 - 1)))/16

Mathematica [A] time = 0.0422125, size = 43, normalized size = 1.

$$\frac{5}{16} \tanh^{-1}\left(\frac{x}{2\sqrt{x^2-1}}\right) - \frac{3x\sqrt{x^2-1}}{8(3x^2-4)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[-1 + x^2]*(-4 + 3*x^2)^2),x]

[Out] (-3*x*Sqrt[-1 + x^2])/(8*(-4 + 3*x^2)) + (5*ArcTanh[x/(2*Sqrt[-1 + x^2])])/16

Maple [B] time = 0.069, size = 172, normalized size = 4.

$$\begin{aligned}
 & -\frac{1}{16}\sqrt{\left(x - \frac{2\sqrt{3}}{3}\right)^2 + \frac{4\sqrt{3}}{3}\left(x - \frac{2\sqrt{3}}{3}\right) + \frac{1}{3}\left(x - \frac{2\sqrt{3}}{3}\right)^{-1}} \\
 & + \frac{5}{32}\operatorname{Artanh}\left(\frac{3\sqrt{3}}{2}\left(\frac{2}{3} + \frac{4\sqrt{3}}{3}\left(x - \frac{2\sqrt{3}}{3}\right)\right)\right) \frac{1}{\sqrt{9\left(x - \frac{2}{3}\sqrt{3}\right)^2 + 12\sqrt{3}\left(x - \frac{2}{3}\sqrt{3}\right) + 3}} \\
 & - \frac{1}{16}\sqrt{\left(x + \frac{2\sqrt{3}}{3}\right)^2 - \frac{4\sqrt{3}}{3}\left(x + \frac{2\sqrt{3}}{3}\right) + \frac{1}{3}\left(x + \frac{2\sqrt{3}}{3}\right)^{-1}} \\
 & - \frac{5}{32}\operatorname{Artanh}\left(\frac{3\sqrt{3}}{2}\left(\frac{2}{3} - \frac{4\sqrt{3}}{3}\left(x + \frac{2\sqrt{3}}{3}\right)\right)\right) \frac{1}{\sqrt{9\left(x + \frac{2}{3}\sqrt{3}\right)^2 - 12\sqrt{3}\left(x + \frac{2}{3}\sqrt{3}\right) + 3}}
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(3*x^2-4)^2/(x^2-1)^(1/2),x)

[Out] -1/16/(x-2/3*3^(1/2))*((x-2/3*3^(1/2))^2+4/3*3^(1/2)*(x-2/3*3^(1/2))+1/3)^(1/2)+5/32*arctanh(3/2*(2/3+4/3*3^(1/2)*(x-2/3*3^(1/2)))*3^(1/2)/(9*(x-2/3*3^(1/2))^2+12*3^(1/2)*(x-2/3*3^(1/2))+3)^(1/2))-1/16/(x+2/3*3^(1/2))*((x+2/3*3^(1/2))^2-4/3*3^(1/2)*(x+2/3*3^(1/2))+1/3)^(1/2)-5/32*arctanh(3/2*(2/3-4/3*3^(1/2)*(x+2/3*3^(1/2)))*3^(1/2)/(9*(x+2/3*3^(1/2))^2-12*3^(1/2)*(x+2/3*3^(1/2))+3)^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3x^2 - 4)^2 \sqrt{x^2 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((3*x^2 - 4)^2*sqrt(x^2 - 1)),x, algorithm="maxima")

[Out] integrate(1/((3*x^2 - 4)^2*sqrt(x^2 - 1)), x)

Fricas [A] time = 0.21575, size = 201, normalized size = 4.67

$$\frac{20x^2 - 5 \left(6x^4 - 11x^2 - 2(3x^3 - 4x)\sqrt{x^2 - 1} + 4 \right) \log \left(3x^2 - 3\sqrt{x^2 - 1}x - 2 \right) + 5 \left(6x^4 - 11x^2 - 2(3x^3 - 4x)\sqrt{x^2 - 1} \right)}{32 \left(6x^4 - 11x^2 - 2(3x^3 - 4x)\sqrt{x^2 - 1} + 4 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((3*x^2 - 4)^2*sqrt(x^2 - 1)),x, algorithm="fricas")

[Out] 1/32*(20*x^2 - 5*(6*x^4 - 11*x^2 - 2*(3*x^3 - 4*x)*sqrt(x^2 - 1) + 4)*log(3*x^2 - 3*sqrt(x^2 - 1)*x - 2) + 5*(6*x^4 - 11*x^2 - 2*(3*x^3 - 4*x)*sqrt(x^2 - 1) + 4)*log(x^2 - sqrt(x^2 - 1)*x - 2) - 20*sqrt(x^2 - 1)*x - 16)/(6*x^4 - 11*x^2 - 2*(3*x^3 - 4*x)*sqrt(x^2 - 1) + 4)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(x-1)(x+1)}(3x^2-4)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3*x**2-4)**2/(x**2-1)**(1/2),x)

[Out] Integral(1/(sqrt((x - 1)*(x + 1))*(3*x**2 - 4)**2), x)

GIAC/XCAS [A] time = 0.23251, size = 127, normalized size = 2.95

$$\frac{5 \left(x - \sqrt{x^2 - 1} \right)^2 - 3}{4 \left(3 \left(x - \sqrt{x^2 - 1} \right)^4 - 10 \left(x - \sqrt{x^2 - 1} \right)^2 + 3 \right)} - \frac{5}{32} \ln \left(\left| 3 \left(x - \sqrt{x^2 - 1} \right)^2 - 1 \right| \right) + \frac{5}{32} \ln \left(\left| \left(x - \sqrt{x^2 - 1} \right)^2 - 3 \right| \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((3*x^2 - 4)^2*sqrt(x^2 - 1)),x, algorithm="giac")

[Out] 1/4*(5*(x - sqrt(x^2 - 1))^2 - 3)/(3*(x - sqrt(x^2 - 1))^4 - 10*(x - sqrt(x^2 - 1))^2 + 3) - 5/32*ln(abs(3*(x - sqrt(x^2 - 1))^2 - 1)) + 5/32*ln(abs((x - sqrt(x^2 - 1))^2 - 3))

$$3.5 \quad \int \frac{1}{(2\sqrt{x} + \sqrt{1+x})^2} dx$$

Optimal. Leaf size=74

$$-\frac{4\sqrt{x}\sqrt{x+1}}{3(1-3x)} + \frac{8}{9(1-3x)} + \frac{5}{9}\log(1-3x) - \frac{8}{9}\sinh^{-1}(\sqrt{x}) + \frac{10}{9}\tanh^{-1}\left(\frac{2\sqrt{x}}{\sqrt{x+1}}\right)$$

[Out] 8/(9*(1 - 3*x)) - (4*Sqrt[x]*Sqrt[1 + x])/(3*(1 - 3*x)) - (8*ArcSinh[Sqrt[x]])/9 + (10*ArcTanh[(2*Sqrt[x])/Sqrt[1 + x]])/9 + (5*Log[1 - 3*x])/9

Rubi [A] time = 0.145885, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$

$$-\frac{4\sqrt{x}\sqrt{x+1}}{3(1-3x)} + \frac{8}{9(1-3x)} + \frac{5}{9}\log(1-3x) - \frac{8}{9}\sinh^{-1}(\sqrt{x}) + \frac{10}{9}\tanh^{-1}\left(\frac{2\sqrt{x}}{\sqrt{x+1}}\right)$$

Antiderivative was successfully verified.

[In] Int[(2*Sqrt[x] + Sqrt[1 + x])^(-2), x]

[Out] 8/(9*(1 - 3*x)) - (4*Sqrt[x]*Sqrt[1 + x])/(3*(1 - 3*x)) - (8*ArcSinh[Sqrt[x]])/9 + (10*ArcTanh[(2*Sqrt[x])/Sqrt[1 + x]])/9 + (5*Log[1 - 3*x])/9

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$2 \int^{\sqrt{x+1}} \frac{x}{(x + 2\sqrt{x^2 - 1})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate(1/(2*x**(1/2)+(1+x)**(1/2))**2, x)

[Out] 2*Integral(x/(x + 2*sqrt(x**2 - 1))**2, (x, sqrt(x + 1)))

Mathematica [A] time = 0.145594, size = 65, normalized size = 0.88

$$\frac{1}{9} \left(\frac{-12\sqrt{x}\sqrt{x+1} + (5-15x)\log(1-3x) + 8}{1-3x} - 8\sinh^{-1}(\sqrt{x}) + 10\tanh^{-1}\left(2\sqrt{\frac{x}{x+1}}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(2*Sqrt[x] + Sqrt[1 + x])^(-2), x]

[Out] (-8*ArcSinh[Sqrt[x]] + 10*ArcTanh[2*Sqrt[x/(1 + x)]]) + (8 - 12*Sqrt[x]*Sqrt[1 + x] + (5 - 15*x)*Log[1 - 3*x])/(1 - 3*x)/9

Maple [B] time = 0.023, size = 115, normalized size = 1.6

$$-\frac{8}{27x-9} + \frac{5 \ln(3x-1)}{9} - \frac{1}{27x-9} \sqrt{x} \sqrt{1+x} \left(12 \ln \left(\frac{1}{2} + x + \sqrt{(1+x)x} \right) x - 15 \operatorname{Artanh} \left(\frac{1}{4} \frac{1+5x}{\sqrt{(1+x)x}} \right) x - 4 \ln \left(\frac{1}{2} + x + \sqrt{(1+x)x} \right) + 5 \operatorname{Artanh} \left(\frac{1}{4} \frac{1+5x}{\sqrt{(1+x)x}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*x^(1/2)+(1+x)^(1/2))^2, x)

[Out] -8/9/(3*x-1)+5/9*ln(3*x-1)-1/9*x^(1/2)*(1+x)^(1/2)*(12*ln(1/2+x+(1+x)*x)^(1/2))*x-15*arctanh(1/4*(1+5*x)/((1+x)*x)^(1/2))*x-4*ln(1/2+x+((1+x)*x)^(1/2))+5*arctanh(1/4*(1+5*x)/((1+x)*x)^(1/2))-12*((1+x)*x)^(1/2)/((1+x)*x)^(1/2)/(3*x-1)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(\sqrt{x+1} + 2\sqrt{x})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((sqrt(x + 1) + 2*sqrt(x))^(-2), x, algorithm="maxima")

[Out] integrate((sqrt(x + 1) + 2*sqrt(x))^(-2), x)

Fricas [A] time = 0.224661, size = 274, normalized size = 3.7

$$2(5(3x-1)\log(3x-1)-18)\sqrt{x+1}\sqrt{x}-5\left(2(3x-1)\sqrt{x+1}\sqrt{x}-6x^2-x+1\right)\log\left(3\sqrt{x+1}\sqrt{x}-3x-1\right)+4\left(2(3x-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sqrt(x + 1) + 2*sqrt(x))^-2),x, algorithm="fricas")
```

```
[Out] 1/9*(2*(5*(3*x - 1)*log(3*x - 1) - 18)*sqrt(x + 1)*sqrt(x) - 5*(2
*(3*x - 1)*sqrt(x + 1)*sqrt(x) - 6*x^2 - x + 1)*log(3*sqrt(x + 1)
*sqrt(x) - 3*x - 1) + 4*(2*(3*x - 1)*sqrt(x + 1)*sqrt(x) - 6*x^2
- x + 1)*log(2*sqrt(x + 1)*sqrt(x) - 2*x - 1) + 5*(2*(3*x - 1)*sq
rt(x + 1)*sqrt(x) - 6*x^2 - x + 1)*log(sqrt(x + 1)*sqrt(x) - x +
1) - 5*(6*x^2 + x - 1)*log(3*x - 1) + 36*x + 12)/(2*(3*x - 1)*sqr
t(x + 1)*sqrt(x) - 6*x^2 - x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2\sqrt{x} + \sqrt{x+1})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**(1/2)+(1+x)**(1/2))**2,x)
```

```
[Out] Integral((2*sqrt(x) + sqrt(x + 1))**(-2), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sqrt(x + 1) + 2*sqrt(x))^-2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

$$3.6 \quad \int \frac{\sqrt{-1+x^2}}{(-i+x)^2} dx$$

Optimal. Leaf size=64

$$\frac{\sqrt{x^2-1}}{-x+i} - \frac{i \tan^{-1}\left(\frac{1-ix}{\sqrt{2}\sqrt{x^2-1}}\right)}{\sqrt{2}} + \tanh^{-1}\left(\frac{x}{\sqrt{x^2-1}}\right)$$

[Out] Sqrt[-1 + x^2]/(I - x) - (I * ArcTan[(1 - I * x)/(Sqrt[2] * Sqrt[-1 + x^2])])/Sqrt[2] + ArcTanh[x/Sqrt[-1 + x^2]]

Rubi [A] time = 0.0759454, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$

$$\frac{\sqrt{x^2-1}}{-x+i} - \frac{i \tan^{-1}\left(\frac{1-ix}{\sqrt{2}\sqrt{x^2-1}}\right)}{\sqrt{2}} + \tanh^{-1}\left(\frac{x}{\sqrt{x^2-1}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[-1 + x^2]/(-I + x)^2, x]

[Out] Sqrt[-1 + x^2]/(I - x) - (I * ArcTan[(1 - I * x)/(Sqrt[2] * Sqrt[-1 + x^2])])/Sqrt[2] + ArcTanh[x/Sqrt[-1 + x^2]]

Rubi in Sympy [A] time = 5.03943, size = 53, normalized size = 0.83

$$\frac{\sqrt{2}i \operatorname{atan}\left(\frac{\sqrt{2}(ix-1)}{2\sqrt{x^2-1}}\right)}{2} + \operatorname{atanh}\left(\frac{x}{\sqrt{x^2-1}}\right) + \frac{\sqrt{x^2-1}}{-x+i}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate((x**2-1)**(1/2)/(-I+x)**2, x)

[Out] sqrt(2)*I*atan(sqrt(2)*(I*x - 1)/(2*sqrt(x**2 - 1)))/2 + atanh(x/sqrt(x**2 - 1)) + sqrt(x**2 - 1)/(-x + I)

Mathematica [B] time = 0.178963, size = 165, normalized size = 2.58

$$\frac{1}{4} \left(-\frac{4\sqrt{x^2-1}}{x-i} + \sqrt{2} \log \left(2\sqrt{2}\sqrt{x^2-1} - 3x - i \right) + 2 \log \left(-2x^2 - 2\sqrt{x^2-1}x + 2i\sqrt{x^2-1} + 2ix + 1 \right) \right. \\ \left. - 2i\sqrt{2} \tan^{-1} \left(\frac{1}{2} \left(-\sqrt{2}\sqrt{x^2-1} + x - i \right) \right) + 4 \tanh^{-1} \left(\frac{2x}{\sqrt{x^2-1} - x + i} \right) - \sqrt{2} \log(x-i) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[-1 + x^2]/(-I + x)^2, x]

[Out] $((-4*\text{Sqrt}[-1 + x^2])/(-I + x) - (2*I)*\text{Sqrt}[2]*\text{ArcTan}[(-I + x - \text{Sqrt}[2]*\text{Sqrt}[-1 + x^2])/2] + 4*\text{ArcTanh}[(2*x)/(I - x + \text{Sqrt}[-1 + x^2])] - \text{Sqrt}[2]*\text{Log}[-I + x] + \text{Sqrt}[2]*\text{Log}[-I - 3*x + 2*\text{Sqrt}[2]*\text{Sqrt}[-1 + x^2]] + 2*\text{Log}[1 + (2*I)*x - 2*x^2 + (2*I)*\text{Sqrt}[-1 + x^2] - 2*x*\text{Sqrt}[-1 + x^2]])/4$

Maple [B] time = 0.038, size = 125, normalized size = 2.

$$\frac{1}{2x-2i} \left((x-i)^2 - 2 + 2i(x-i) \right)^{\frac{3}{2}} + \ln \left(x + \sqrt{(x-i)^2 - 2 + 2i(x-i)} \right) \\ + \frac{i}{2} \sqrt{2} \arctan \left(\frac{(-4 + 2i(x-i))\sqrt{2}}{4} \frac{1}{\sqrt{(x-i)^2 - 2 + 2i(x-i)}} \right) \\ - \frac{i}{2} \sqrt{(x-i)^2 - 2 + 2i(x-i)} - \frac{x}{2} \sqrt{(x-i)^2 - 2 + 2i(x-i)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2-1)^(1/2)/(x-I)^2, x)

[Out] $1/2/(x-I) * ((x-I)^2 - 2 + 2*I*(x-I))^{3/2} + \ln(x + ((x-I)^2 - 2 + 2*I*(x-I))^{1/2}) + 1/2*I^2 * ((x-I)^2 - 2 + 2*I*(x-I))^{1/2} * \arctan(1/4 * (-4 + 2*I*(x-I)) * ((x-I)^2 - 2 + 2*I*(x-I))^{1/2}) - 1/2*I * ((x-I)^2 - 2 + 2*I*(x-I))^{1/2} - 1/2*x * ((x-I)^2 - 2 + 2*I*(x-I))^{1/2}$

Maxima [A] time = 1.54556, size = 72, normalized size = 1.12

$$\frac{1}{2} i \sqrt{2} \arcsin \left(\frac{ix}{|x-i|} - \frac{1}{|x-i|} \right) - \frac{\sqrt{x^2-1}}{x-i} + \log \left(2x + 2\sqrt{x^2-1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(x^2 - 1)/(x - I)^2,x, algorithm="maxima")

[Out] 1/2*I*sqrt(2)*arcsin(I*x/abs(x - I) - 1/abs(x - I)) - sqrt(x^2 - 1)/(x - I) + log(2*x + 2*sqrt(x^2 - 1))

Fricas [A] time = 0.231246, size = 230, normalized size = 3.59

$$\frac{\left(\sqrt{2}\sqrt{x^2-1}(x-i)-\sqrt{2}(x^2-ix)\right)\log\left(-x+i\sqrt{2}+\sqrt{x^2-1}+i\right)-\left(\sqrt{2}\sqrt{x^2-1}(x-i)-\sqrt{2}(x^2-ix)\right)\log\left(-x-i\sqrt{2}+\sqrt{x^2-1}-i\right)}{2x^2-\sqrt{x^2-1}(2x-2i)-2ix}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(x^2 - 1)/(x - I)^2,x, algorithm="fricas")

[Out] ((sqrt(2)*sqrt(x^2 - 1)*(x - I) - sqrt(2)*(x^2 - I*x))*log(-x + I*sqrt(2) + sqrt(x^2 - 1) + I) - (sqrt(2)*sqrt(x^2 - 1)*(x - I) - sqrt(2)*(x^2 - I*x))*log(-x - I*sqrt(2) + sqrt(x^2 - 1) + I) - (2*x^2 - sqrt(x^2 - 1)*(2*x - 2*I) - 2*I*x)*log(-x + sqrt(x^2 - 1)) + 2*I*x - 2*I*sqrt(x^2 - 1) - 2)/(2*x^2 - sqrt(x^2 - 1)*(2*x - 2*I) - 2*I*x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(x-1)(x+1)}}{(x-i)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2-1)**(1/2)/(-I+x)**2,x)

[Out] Integral(sqrt((x - 1)*(x + 1))/(x - I)**2, x)

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2-1}}{(x-i)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(x^2 - 1)/(x - I)^2,x, algorithm="giac")


```
[Out] integrate(sqrt(x^2 - 1)/(x - 1)^2, x)
```

$$3.7 \quad \int \frac{1}{\sqrt{-1+x^2}(1+x^2)^2} dx$$

Optimal. Leaf size=48

$$\frac{3 \tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right)}{4\sqrt{2}} - \frac{x\sqrt{x^2-1}}{4(x^2+1)}$$

[Out] $-(x*\text{Sqrt}[-1 + x^2])/(4*(1 + x^2)) + (3*\text{ArcTanh}[(\text{Sqrt}[2]*x)/\text{Sqrt}[-1 + x^2]])/(4*\text{Sqrt}[2])$

Rubi [A] time = 0.0388008, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$

$$\frac{3 \tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right)}{4\sqrt{2}} - \frac{x\sqrt{x^2-1}}{4(x^2+1)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[1/(\text{Sqrt}[-1 + x^2]*(1 + x^2)^2), x]$

[Out] $-(x*\text{Sqrt}[-1 + x^2])/(4*(1 + x^2)) + (3*\text{ArcTanh}[(\text{Sqrt}[2]*x)/\text{Sqrt}[-1 + x^2]])/(4*\text{Sqrt}[2])$

Rubi in Sympy [A] time = 3.71454, size = 41, normalized size = 0.85

$$-\frac{x\sqrt{x^2-1}}{4(x^2+1)} + \frac{3\sqrt{2} \operatorname{atanh}\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{rubi_integrate}(1/(x^{**2}+1)**2/(x^{**2}-1)**(1/2), x)$

[Out] $-x*\text{sqrt}(x^{**2} - 1)/(4*(x^{**2} + 1)) + 3*\text{sqrt}(2)*\text{atanh}(\text{sqrt}(2)*x/\text{sqrt}(x^{**2} - 1))/8$

Mathematica [A] time = 0.0706282, size = 90, normalized size = 1.88

$$-\frac{\sqrt{x^2-1}x}{4(x^2+1)} + \frac{3 \log\left(-3x^2 - 2\sqrt{2}\sqrt{x^2-1}x + 1\right)}{16\sqrt{2}} - \frac{3 \log\left(-3x^2 + 2\sqrt{2}\sqrt{x^2-1}x + 1\right)}{16\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[-1 + x^2] * (1 + x^2)^2), x]

[Out] $-(x \sqrt{-1 + x^2}) / (4 (1 + x^2)) + (3 \operatorname{Log}[1 - 3 x^2 - 2 \sqrt{2} x \sqrt{-1 + x^2}]) / (16 \sqrt{2}) - (3 \operatorname{Log}[1 - 3 x^2 + 2 \sqrt{2} x \sqrt{-1 + x^2}]) / (16 \sqrt{2})$

Maple [A] time = 0.026, size = 45, normalized size = 0.9

$$-\frac{x}{8} \frac{1}{\sqrt{x^2-1}} \left(\frac{x^2}{x^2-1} - \frac{1}{2} \right)^{-1} + \frac{3\sqrt{2}}{8} \operatorname{Artanh} \left(x \sqrt{2} \frac{1}{\sqrt{x^2-1}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2+1)^2/(x^2-1)^(1/2), x)

[Out] $-1/8/(x^2-1)^{(1/2)} * x/(x^2/(x^2-1)-1/2) + 3/8 * \operatorname{arctanh}(x \sqrt{2} / (x^2-1)^{(1/2)}) * 2^{(1/2)}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2 + 1)^2 \sqrt{x^2 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((x^2 + 1)^2 * sqrt(x^2 - 1)), x, algorithm="maxima")

[Out] integrate(1/((x^2 + 1)^2 * sqrt(x^2 - 1)), x)

Fricas [A] time = 0.222009, size = 216, normalized size = 4.5

$$\frac{6 \sqrt{2} \sqrt{x^2 - 1} x + 3 \left(2 x^4 + x^2 - 2 (x^3 + x) \sqrt{x^2 - 1} - 1 \right) \log \left(\frac{4 x^2 + \sqrt{2} (2 x^4 + x^2 + 3) - 2 \sqrt{x^2 - 1} (\sqrt{2} (x^3 + x) + 2 x) + 4}{2 x^4 + x^2 - 2 (x^3 + x) \sqrt{x^2 - 1} - 1} \right) - 2 \sqrt{2} (3 x^2 - 1)}{8 \left(2 \sqrt{2} (x^3 + x) \sqrt{x^2 - 1} - \sqrt{2} (2 x^4 + x^2 - 1) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((x^2 + 1)^2 * sqrt(x^2 - 1)), x, algorithm="fricas")

```
[Out] -1/8*(6*sqrt(2)*sqrt(x^2 - 1)*x + 3*(2*x^4 + x^2 - 2*(x^3 + x)*sqrt(x^2 - 1) - 1)*log((4*x^2 + sqrt(2)*(2*x^4 + x^2 + 3) - 2*sqrt(x^2 - 1)*(sqrt(2)*(x^3 + x) + 2*x) + 4)/(2*x^4 + x^2 - 2*(x^3 + x)*sqrt(x^2 - 1) - 1)) - 2*sqrt(2)*(3*x^2 - 1))/(2*sqrt(2)*(x^3 + x)*sqrt(x^2 - 1) - sqrt(2)*(2*x^4 + x^2 - 1))
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)**2/(x**2-1)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

GIAC/XCAS [A] time = 0.202751, size = 136, normalized size = 2.83

$$-\frac{3}{16}\sqrt{2}\ln\left(\frac{(x-\sqrt{x^2-1})^2-2\sqrt{2}+3}{(x-\sqrt{x^2-1})^2+2\sqrt{2}+3}\right)-\frac{3(x-\sqrt{x^2-1})^2+1}{2\left((x-\sqrt{x^2-1})^4+6(x-\sqrt{x^2-1})^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((x^2 + 1)^2*sqrt(x^2 - 1)),x, algorithm="giac")
```

```
[Out] -3/16*sqrt(2)*ln(((x - sqrt(x^2 - 1))^2 - 2*sqrt(2) + 3)/((x - sqrt(x^2 - 1))^2 + 2*sqrt(2) + 3)) - 1/2*(3*(x - sqrt(x^2 - 1))^2 + 1)/((x - sqrt(x^2 - 1))^4 + 6*(x - sqrt(x^2 - 1))^2 + 1)
```

$$3.8 \quad \int \frac{1}{\left(\sqrt{-1+x}+\sqrt{x}\right)^2 \sqrt{-1+x}} dx$$

Optimal. Leaf size=30

$$-\frac{4x^{3/2}}{3} + \frac{4}{3}(x-1)^{3/2} + 2\sqrt{x-1}$$

[Out] 2*Sqrt[-1 + x] + (4*(-1 + x)^(3/2))/3 - (4*x^(3/2))/3

Rubi [A] time = 0.127849, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$

$$-\frac{4x^{3/2}}{3} + \frac{4}{3}(x-1)^{3/2} + 2\sqrt{x-1}$$

Antiderivative was successfully verified.

[In] Int[1/((Sqrt[-1 + x] + Sqrt[x])^2*Sqrt[-1 + x]), x]

[Out] 2*Sqrt[-1 + x] + (4*(-1 + x)^(3/2))/3 - (4*x^(3/2))/3

Rubi in Sympy [A] time = 4.97118, size = 26, normalized size = 0.87

$$-\frac{4x^{3/2}}{3} + \frac{4(x-1)^{3/2}}{3} + 2\sqrt{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate(1/(-1+x)**(1/2)/((-1+x)**(1/2)+x**(1/2))**2, x)

[Out] -4*x**(3/2)/3 + 4*(x - 1)**(3/2)/3 + 2*sqrt(x - 1)

Mathematica [A] time = 0.0229325, size = 29, normalized size = 0.97

$$\frac{2}{3} \left(-2x^{3/2} + 2\sqrt{x-1}x + \sqrt{x-1} \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/((Sqrt[-1 + x] + Sqrt[x])^2*Sqrt[-1 + x]), x]

[Out] $(2 * (\text{Sqrt}[-1 + x] + 2 * \text{Sqrt}[-1 + x] * x - 2 * x^{(3/2)})) / 3$

Maple [A] time = 0.004, size = 21, normalized size = 0.7

$$\frac{4}{3}(-1+x)^{\frac{3}{2}} - \frac{4}{3}x^{\frac{3}{2}} + 2\sqrt{-1+x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(-1+x)^(1/2)/((-1+x)^(1/2)+x^(1/2))^2,x)`

[Out] $4/3 * (-1+x)^{(3/2)} - 4/3 * x^{(3/2)} + 2 * (-1+x)^{(1/2)}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x-1}(\sqrt{x-1} + \sqrt{x})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(x - 1)*(sqrt(x - 1) + sqrt(x))^2),x, algorithm="maxima")`

[Out] `integrate(1/(sqrt(x - 1)*(sqrt(x - 1) + sqrt(x))^2), x)`

Fricas [A] time = 0.207534, size = 24, normalized size = 0.8

$$\frac{2}{3}(2x+1)\sqrt{x-1} - \frac{4}{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(x - 1)*(sqrt(x - 1) + sqrt(x))^2),x, algorithm="fricas")`

[Out] $2/3 * (2 * x + 1) * \text{sqrt}(x - 1) - 4/3 * x^{(3/2)}$

Sympy [A] time = 1.50173, size = 53, normalized size = 1.77

$$-\frac{4\sqrt{x}}{6\sqrt{x}\sqrt{x-1}+6x-3} - \frac{2\sqrt{x-1}}{6\sqrt{x}\sqrt{x-1}+6x-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)**(1/2)/((-1+x)**(1/2)+x**(1/2))**2,x)
```

```
[Out] -4*sqrt(x)/(6*sqrt(x)*sqrt(x - 1) + 6*x - 3) - 2*sqrt(x - 1)/(6*sqrt(x)*sqrt(x - 1) + 6*x - 3)
```

GIAC/XCAS [A] time = 0.205963, size = 27, normalized size = 0.9

$$\frac{4}{3}(x-1)^{\frac{3}{2}} - \frac{4}{3}x^{\frac{3}{2}} + 2\sqrt{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(x - 1)*(sqrt(x - 1) + sqrt(x))^2),x, algorithm="giac")
```

```
[Out] 4/3*(x - 1)^(3/2) - 4/3*x^(3/2) + 2*sqrt(x - 1)
```

$$3.9 \quad \int \frac{1}{\sqrt{-1+x^2}(\sqrt{x}+\sqrt{-1+x^2})^2} dx$$

Optimal. Leaf size=220

$$\begin{aligned} & \frac{2-4x}{5(\sqrt{x^2-1}+\sqrt{x})} - \frac{1}{50}\sqrt{50\sqrt{5}-110}\tan^{-1}\left(\frac{\sqrt{2\sqrt{5}-2\sqrt{x^2-1}}}{2-(1-\sqrt{5})x}\right) \\ & - \frac{1}{50}\sqrt{110+50\sqrt{5}}\tanh^{-1}\left(\frac{\sqrt{2+2\sqrt{5}\sqrt{x^2-1}}}{-\sqrt{5}x-x+2}\right) \\ & + \frac{1}{25}\sqrt{50\sqrt{5}-110}\tan^{-1}\left(\frac{1}{2}\sqrt{2+2\sqrt{5}\sqrt{x}}\right) - \frac{1}{25}\sqrt{110+50\sqrt{5}}\tanh^{-1}\left(\frac{1}{2}\sqrt{2\sqrt{5}-2\sqrt{x}}\right) \end{aligned}$$

[Out] (2 - 4*x)/(5*(Sqrt[x] + Sqrt[-1 + x^2])) + (Sqrt[-110 + 50*Sqrt[5]]*ArcTan[(Sqrt[2 + 2*Sqrt[5]]*Sqrt[x])/2])/25 - (Sqrt[-110 + 50*Sqrt[5]]*ArcTan[(Sqrt[-2 + 2*Sqrt[5]]*Sqrt[-1 + x^2])/(2 - (1 - Sqrt[5])*x))]/50 - (Sqrt[110 + 50*Sqrt[5]]*ArcTanh[(Sqrt[-2 + 2*Sqrt[5]]*Sqrt[x])/2])/25 - (Sqrt[110 + 50*Sqrt[5]]*ArcTanh[(Sqrt[2 + 2*Sqrt[5]]*Sqrt[-1 + x^2])/(2 - x - Sqrt[5]*x))]/50

Rubi [A] time = 0.997216, antiderivative size = 365, normalized size of antiderivative = 1.66, number of steps used = 18, number of rules used = 12, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$

$$\begin{aligned} & -\frac{2\sqrt{x^2-1}(1-2x)}{5(-x^2+x+1)} + \frac{2\sqrt{x}(1-2x)}{5(-x^2+x+1)} - \frac{2}{5}\sqrt{\frac{1}{5}(5\sqrt{5}-2)}\tan^{-1}\left(\frac{2-(1-\sqrt{5})x}{\sqrt{2(\sqrt{5}-1)\sqrt{x^2-1}}}\right) \\ & + \sqrt{\frac{2}{5(\sqrt{5}-1)}}\tan^{-1}\left(\frac{2-(1-\sqrt{5})x}{\sqrt{2(\sqrt{5}-1)\sqrt{x^2-1}}}\right) - \frac{2}{5}\sqrt{\frac{1}{5}(2+5\sqrt{5})}\tanh^{-1}\left(\frac{2-(1+\sqrt{5})x}{\sqrt{2(1+\sqrt{5})\sqrt{x^2-1}}}\right) \\ & + \sqrt{\frac{2}{5(1+\sqrt{5})}}\tanh^{-1}\left(\frac{2-(1+\sqrt{5})x}{\sqrt{2(1+\sqrt{5})\sqrt{x^2-1}}}\right) \\ & + \frac{1}{5}\sqrt{\frac{2}{5}(5\sqrt{5}-11)}\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\sqrt{x}\right) - \frac{1}{5}\sqrt{\frac{2}{5}(11+5\sqrt{5})}\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{x}\right) \end{aligned}$$

Warning: Unable to verify antiderivative.

[In] Int[1/(Sqrt[-1 + x^2]*(Sqrt[x] + Sqrt[-1 + x^2])^2), x]

[Out] (2*(1 - 2*x)*Sqrt[x])/(5*(1 + x - x^2)) - (2*(1 - 2*x)*Sqrt[-1 + x^2])/(5*(1 + x - x^2)) + (Sqrt[(2*(-11 + 5*Sqrt[5]))]/5)*ArcTan[S

$$\frac{\sqrt{2/(-1 + \sqrt{5})} \sqrt{x}}{5} + \sqrt{2/(5(-1 + \sqrt{5}))} \operatorname{ArcTan}\left[\frac{2 - (1 - \sqrt{5})x}{\sqrt{2(-1 + \sqrt{5})} \sqrt{-1 + x^2}}\right] - \frac{2\sqrt{(-2 + 5\sqrt{5})/5} \operatorname{ArcTan}\left[\frac{2 - (1 - \sqrt{5})x}{\sqrt{2(-1 + \sqrt{5})} \sqrt{-1 + x^2}}\right]}{5} - \frac{\sqrt{(2(11 + 5\sqrt{5}))/5} \operatorname{ArcTanh}\left[\frac{\sqrt{2/(1 + \sqrt{5})} \sqrt{x}}{5} + \sqrt{2/(5(1 + \sqrt{5}))} \operatorname{ArcTanh}\left[\frac{2 - (1 + \sqrt{5})x}{\sqrt{2(1 + \sqrt{5})} \sqrt{-1 + x^2}}\right] - \frac{2\sqrt{(2 + 5\sqrt{5})/5} \operatorname{ArcTanh}\left[\frac{2 - (1 + \sqrt{5})x}{\sqrt{2(1 + \sqrt{5})} \sqrt{-1 + x^2}}\right]}{5}$$

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$2 \int^{\sqrt{x}} \frac{x}{(x + \sqrt{x^4 - 1})^2 \sqrt{x^4 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/(x**2-1)**(1/2)/(x**(1/2)+(x**2-1)**(1/2))**2,x)`

[Out] `2*Integral(x/((x + sqrt(x**4 - 1))**2*sqrt(x**4 - 1)), (x, sqrt(x)))`

Mathematica [A] time = 4.93058, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-1 + x^2} (\sqrt{x} + \sqrt{-1 + x^2})^2} dx$$

Verification is Not applicable to the result.

[In] `Integrate[1/(Sqrt[-1 + x^2] * (Sqrt[x] + Sqrt[-1 + x^2])^2), x]`

[Out] `Integrate[1/(Sqrt[-1 + x^2] * (Sqrt[x] + Sqrt[-1 + x^2])^2), x]`

Maple [B] time = 0.177, size = 902, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^2-1)^(1/2)/(x^(1/2)+(x^2-1)^(1/2))^2,x)`

```
[Out] -6/25*5^(1/2)/(2+2*5^(1/2))^(1/2)*arctanh(2*(1+5^(1/2)+(5^(1/2)+1)
)* (x-1/2*5^(1/2)-1/2))/(2+2*5^(1/2))^(1/2)/(4*(x-1/2*5^(1/2)-1/2)
^2+4*(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+2+2*5^(1/2))^(1/2))-6/25*5^(
1/2)/(-2+2*5^(1/2))^(1/2)*arctan(2*(1-5^(1/2)+(-5^(1/2)+1)*(x+1/2
*5^(1/2)-1/2)))/(-2+2*5^(1/2))^(1/2)/(4*(x+1/2*5^(1/2)-1/2)^2+4*(-
5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+2-2*5^(1/2))^(1/2))-1/5/(1/2+1/2*5
^(1/2))/(x-1/2*5^(1/2)-1/2)*((x-1/2*5^(1/2)-1/2)^2+(5^(1/2)+1)*(x
-1/2*5^(1/2)-1/2)+1/2+1/2*5^(1/2))^(1/2)+6/5/(1/2+1/2*5^(1/2)))/(2
+2*5^(1/2))^(1/2)*arctanh(2*(1+5^(1/2)+(5^(1/2)+1)*(x-1/2*5^(1/2)
-1/2))/(2+2*5^(1/2))^(1/2)/(4*(x-1/2*5^(1/2)-1/2)^2+4*(5^(1/2)+1)
*(x-1/2*5^(1/2)-1/2)+2+2*5^(1/2))^(1/2))+2/5/(1/2+1/2*5^(1/2)))/(2
+2*5^(1/2))^(1/2)*arctanh(2*(1+5^(1/2)+(5^(1/2)+1)*(x-1/2*5^(1/2)
-1/2))/(2+2*5^(1/2))^(1/2)/(4*(x-1/2*5^(1/2)-1/2)^2+4*(5^(1/2)+1)
*(x-1/2*5^(1/2)-1/2)+2+2*5^(1/2))^(1/2))*5^(1/2)-1/5*5^(1/2)/(1/2
+1/2*5^(1/2))/(x-1/2*5^(1/2)-1/2)*((x-1/2*5^(1/2)-1/2)^2+(5^(1/2)
+1)*(x-1/2*5^(1/2)-1/2)+1/2+1/2*5^(1/2))^(1/2)-1/5/(1/2-1/2*5^(1/
2)))/(x+1/2*5^(1/2)-1/2)*((x+1/2*5^(1/2)-1/2)^2+(-5^(1/2)+1)*(x+1/
2*5^(1/2)-1/2)+1/2-1/2*5^(1/2))^(1/2)+2/5/(1/2-1/2*5^(1/2)))/(-2+2
*5^(1/2))^(1/2)*arctan(2*(1-5^(1/2)+(-5^(1/2)+1)*(x+1/2*5^(1/2)-1
/2)))/(-2+2*5^(1/2))^(1/2)/(4*(x+1/2*5^(1/2)-1/2)^2+4*(-5^(1/2)+1)
*(x+1/2*5^(1/2)-1/2)+2-2*5^(1/2))^(1/2))*5^(1/2)-6/5/(1/2-1/2*5^(
1/2)))/(-2+2*5^(1/2))^(1/2)*arctan(2*(1-5^(1/2)+(-5^(1/2)+1)*(x+1/
2*5^(1/2)-1/2)))/(-2+2*5^(1/2))^(1/2)/(4*(x+1/2*5^(1/2)-1/2)^2+4*(
-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+2-2*5^(1/2))^(1/2))+1/5*5^(1/2)/(
1/2-1/2*5^(1/2))/(x+1/2*5^(1/2)-1/2)*((x+1/2*5^(1/2)-1/2)^2+(-5^(
1/2)+1)*(x+1/2*5^(1/2)-1/2)+1/2-1/2*5^(1/2))^(1/2)+2/5*x^(1/2)/(x
+1/2*5^(1/2)-1/2)+4/5/(-2+2*5^(1/2))^(1/2)*arctan(2*x^(1/2)/(-2+2
*5^(1/2))^(1/2))-8/25/(-2+2*5^(1/2))^(1/2)*arctan(2*x^(1/2)/(-2+2
*5^(1/2))^(1/2))*5^(1/2)+2/5*x^(1/2)/(x-1/2*5^(1/2)-1/2)-4/5/(2+2
*5^(1/2))^(1/2)*arctanh(2*x^(1/2)/(2+2*5^(1/2))^(1/2))-8/25/(2+2*
5^(1/2))^(1/2)*arctanh(2*x^(1/2)/(2+2*5^(1/2))^(1/2))*5^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2-1}(\sqrt{x^2-1}+\sqrt{x})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(x^2 - 1)*(sqrt(x^2 - 1) + sqrt(x))^2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2 - 1)*(sqrt(x^2 - 1) + sqrt(x))^2), x)
```

Fricas [A] time = 0.252677, size = 1185, normalized size = 5.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(x^2 - 1)*(sqrt(x^2 - 1) + sqrt(x))^2),x, algorithm="fricas")

[Out] $\frac{1}{50} (40x^3 + 40x^2 - 4(2\sqrt{2})(x^3 - x^2 - x)\sqrt{x^2 - 1})\sqrt{-\sqrt{5}(11\sqrt{5} - 25)} - \sqrt{2}(2x^4 - 2x^3 - 3x^2 + x + 1)\sqrt{-\sqrt{5}(11\sqrt{5} - 25))} \arctan\left(\frac{-1/2\sqrt{2}\sqrt{-\sqrt{5}(11\sqrt{5} - 25)}(\sqrt{5} + 3)}{(\sqrt{5})(2x - 1)}\right) - 2\sqrt{5}\sqrt{x^2 - 1} - 2\sqrt{\sqrt{5}(\sqrt{5}(2x^2 - x) - \sqrt{x^2 - 1}(\sqrt{5}(2x - 1) + 5) + 5x)) + 5} + 4(2\sqrt{2})(x^3 - x^2 - x)\sqrt{x^2 - 1}\sqrt{-\sqrt{5}(11\sqrt{5} - 25)} - \sqrt{2}(2x^4 - 2x^3 - 3x^2 + x + 1)\sqrt{-\sqrt{5}(11\sqrt{5} - 25))} \arctan\left(\frac{1/2\sqrt{2}\sqrt{-\sqrt{5}(11\sqrt{5} - 25)}(\sqrt{5} + 3)}{(\sqrt{2})\sqrt{\sqrt{5}(\sqrt{5}(2x - 1) + 5))} + 2\sqrt{5}\sqrt{x}}\right) + (2\sqrt{2})(x^3 - x^2 - x)\sqrt{x^2 - 1}\sqrt{\sqrt{5}(11\sqrt{5} + 25)} - \sqrt{2}(2x^4 - 2x^3 - 3x^2 + x + 1)\sqrt{\sqrt{5}(11\sqrt{5} + 25))} \log(\sqrt{2}\sqrt{\sqrt{5}(11\sqrt{5} + 25)}(\sqrt{5} - 3) - 2\sqrt{5}(2x - 1) + 4\sqrt{5}\sqrt{x^2 - 1} + 10) - (2\sqrt{2})(x^3 - x^2 - x)\sqrt{x^2 - 1}\sqrt{\sqrt{5}(11\sqrt{5} + 25)} - \sqrt{2}(2x^4 - 2x^3 - 3x^2 + x + 1)\sqrt{\sqrt{5}(11\sqrt{5} + 25))} \log(\sqrt{2}\sqrt{\sqrt{5}(11\sqrt{5} + 25)}(\sqrt{5} - 3) + 4\sqrt{5}\sqrt{x}) - (2\sqrt{2})(x^3 - x^2 - x)\sqrt{x^2 - 1}\sqrt{\sqrt{5}(11\sqrt{5} + 25)} - \sqrt{2}(2x^4 - 2x^3 - 3x^2 + x + 1)\sqrt{\sqrt{5}(11\sqrt{5} + 25))} \log(-\sqrt{2}\sqrt{\sqrt{5}(11\sqrt{5} + 25)}(\sqrt{5} - 3) - 2\sqrt{5}(2x - 1) + 4\sqrt{5}\sqrt{x^2 - 1} + 10) + (2\sqrt{2})(x^3 - x^2 - x)\sqrt{x^2 - 1}\sqrt{\sqrt{5}(11\sqrt{5} + 25)} - \sqrt{2}(2x^4 - 2x^3 - 3x^2 + x + 1)\sqrt{\sqrt{5}(11\sqrt{5} + 25))} \log(-\sqrt{2}\sqrt{\sqrt{5}(11\sqrt{5} + 25)}(\sqrt{5} - 3) + 4\sqrt{5}\sqrt{x}) - 20(2x^2 + 2(2x^2 - x)\sqrt{x} + 2x + 1)\sqrt{x^2 - 1} + 20(4x^3 - 2x^2 - 2x + 1)\sqrt{x} - 40)/(2x^4 - 2x^3 - 3x^2 - 2(x^3 - x^2 - x)\sqrt{x^2 - 1} + x + 1)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(x-1)(x+1)}(\sqrt{x} + \sqrt{x^2-1})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**2-1)**(1/2)/(x**(1/2)+(x**2-1)**(1/2))**2,x)

[Out] Integral(1/(sqrt((x - 1)*(x + 1))*(sqrt(x) + sqrt(x**2 - 1))**2), x)

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2-1}(\sqrt{x^2-1} + \sqrt{x})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(x^2 - 1)*(sqrt(x^2 - 1) + sqrt(x))^2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^2 - 1)*(sqrt(x^2 - 1) + sqrt(x))^2), x)
```

$$3.10 \quad \int \frac{(\sqrt{x} - \sqrt{-1+x^2})^2}{(1+x-x^2)^2 \sqrt{-1+x^2}} dx$$

Optimal. Leaf size=220

$$\begin{aligned} & \frac{2-4x}{5(\sqrt{x^2-1} + \sqrt{x})} - \frac{1}{50} \sqrt{50\sqrt{5}-110} \tan^{-1} \left(\frac{\sqrt{2\sqrt{5}-2\sqrt{x^2-1}}}{2-(1-\sqrt{5})x} \right) \\ & - \frac{1}{50} \sqrt{110+50\sqrt{5}} \tanh^{-1} \left(\frac{\sqrt{2+2\sqrt{5}\sqrt{x^2-1}}}{-\sqrt{5}x-x+2} \right) \\ & + \frac{1}{25} \sqrt{50\sqrt{5}-110} \tan^{-1} \left(\frac{1}{2} \sqrt{2+2\sqrt{5}\sqrt{x}} \right) - \frac{1}{25} \sqrt{110+50\sqrt{5}} \tanh^{-1} \left(\frac{1}{2} \sqrt{2\sqrt{5}-2\sqrt{x}} \right) \end{aligned}$$

[Out] (2 - 4*x)/(5*(Sqrt[x] + Sqrt[-1 + x^2])) + (Sqrt[-110 + 50*Sqrt[5]]*ArcTan[(Sqrt[2 + 2*Sqrt[5]]*Sqrt[x])/2])/25 - (Sqrt[-110 + 50*Sqrt[5]]*ArcTan[(Sqrt[-2 + 2*Sqrt[5]]*Sqrt[-1 + x^2])/(2 - (1 - Sqrt[5])*x))]/50 - (Sqrt[110 + 50*Sqrt[5]]*ArcTanh[(Sqrt[-2 + 2*Sqrt[5]]*Sqrt[x])/2])/25 - (Sqrt[110 + 50*Sqrt[5]]*ArcTanh[(Sqrt[2 + 2*Sqrt[5]]*Sqrt[-1 + x^2])/(2 - x - Sqrt[5]*x))]/50

Rubi [B] time = 1.56062, antiderivative size = 541, normalized size of antiderivative = 2.46, number

of steps used = 25, number of rules used = 13, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\begin{aligned}
& -\frac{\sqrt{x^2-1}(1-2x)}{5(-x^2+x+1)} + \frac{2\sqrt{x}(1-2x)}{5(-x^2+x+1)} - \frac{(3-x)\sqrt{x^2-1}}{5(-x^2+x+1)} \\
& + \frac{(x+2)\sqrt{x^2-1}}{5(-x^2+x+1)} + \frac{1}{5}\sqrt{\frac{1}{5}(2+5\sqrt{5})} \tan^{-1}\left(\frac{2-(1-\sqrt{5})x}{\sqrt{2(\sqrt{5}-1)}\sqrt{x^2-1}}\right) \\
& - \frac{1}{5}\sqrt{\frac{1}{5}(5\sqrt{5}-2)} \tan^{-1}\left(\frac{2-(1-\sqrt{5})x}{\sqrt{2(\sqrt{5}-1)}\sqrt{x^2-1}}\right) \\
& - \frac{1}{5}\sqrt{\frac{1}{10}(5\sqrt{5}-11)} \tan^{-1}\left(\frac{2-(1-\sqrt{5})x}{\sqrt{2(\sqrt{5}-1)}\sqrt{x^2-1}}\right) \\
& + \frac{1}{5}\sqrt{\frac{1}{10}(11+5\sqrt{5})} \tanh^{-1}\left(\frac{2-(1+\sqrt{5})x}{\sqrt{2(1+\sqrt{5})}\sqrt{x^2-1}}\right) \\
& - \frac{1}{5}\sqrt{\frac{1}{5}(2+5\sqrt{5})} \tanh^{-1}\left(\frac{2-(1+\sqrt{5})x}{\sqrt{2(1+\sqrt{5})}\sqrt{x^2-1}}\right) \\
& - \frac{1}{5}\sqrt{\frac{1}{5}(5\sqrt{5}-2)} \tanh^{-1}\left(\frac{2-(1+\sqrt{5})x}{\sqrt{2(1+\sqrt{5})}\sqrt{x^2-1}}\right) \\
& + \frac{1}{5}\sqrt{\frac{2}{5}(5\sqrt{5}-11)} \tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\sqrt{x}\right) - \frac{1}{5}\sqrt{\frac{2}{5}(11+5\sqrt{5})} \tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{x}\right)
\end{aligned}$$

Warning: Unable to verify antiderivative.

[In] Int[(Sqrt[x] - Sqrt[-1 + x^2])^2/((1 + x - x^2)^2*Sqrt[-1 + x^2]), x]

[Out] (2*(1 - 2*x)*Sqrt[x])/(5*(1 + x - x^2)) - ((1 - 2*x)*Sqrt[-1 + x^2])/(5*(1 + x - x^2)) - ((3 - x)*Sqrt[-1 + x^2])/(5*(1 + x - x^2)) + ((2 + x)*Sqrt[-1 + x^2])/(5*(1 + x - x^2)) + (Sqrt[(2*(-11 + 5*Sqrt[5]))/5]*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*Sqrt[x]])/5 - (Sqrt[(-11 + 5*Sqrt[5])/10]*ArcTan[(2 - (1 - Sqrt[5])*x)/(Sqrt[2*(-1 + Sqrt[5])]*Sqrt[-1 + x^2])])/5 - (Sqrt[(-2 + 5*Sqrt[5])/5]*ArcTan[(2 - (1 - Sqrt[5])*x)/(Sqrt[2*(-1 + Sqrt[5])]*Sqrt[-1 + x^2])])/5 + (Sqrt[(2 + 5*Sqrt[5])/5]*ArcTan[(2 - (1 - Sqrt[5])*x)/(Sqrt[2*(-1 + Sqrt[5])]*Sqrt[-1 + x^2])])/5 - (Sqrt[(2*(11 + 5*Sqrt[5]))/5]*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*Sqrt[x]])/5 - (Sqrt[(-2 + 5*Sqrt[5])/5]*ArcTanh[(2 - (1 + Sqrt[5])*x)/(Sqrt[2*(1 + Sqrt[5])]*Sqrt

$$\frac{[-1 + x^2])]/5 - (\text{Sqrt}[(2 + 5*\text{Sqrt}[5])/5]*\text{ArcTanh}[(2 - (1 + \text{Sqrt}[5])*x)/(\text{Sqrt}[2*(1 + \text{Sqrt}[5])]*\text{Sqrt}[-1 + x^2])]/5 + (\text{Sqrt}[(11 + 5*\text{Sqrt}[5])/10]*\text{ArcTanh}[(2 - (1 + \text{Sqrt}[5])*x)/(\text{Sqrt}[2*(1 + \text{Sqrt}[5])]*\text{Sqrt}[-1 + x^2])]/5$$

Rubi in Sympy [F-1] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((x**(1/2)-(x**2-1)**(1/2))**2/(-x**2+x+1)**2/(x**2-1)**(1/2),x)`

[Out] Timed out

Mathematica [A] time = 1.66001, size = 0, normalized size = 0.

$$\int \frac{(\sqrt{x} - \sqrt{-1 + x^2})^2}{(1 + x - x^2)^2 \sqrt{-1 + x^2}} dx$$

Verification is Not applicable to the result.

[In] `Integrate[(Sqrt[x] - Sqrt[-1 + x^2])^2/((1 + x - x^2)^2*Sqrt[-1 + x^2]),x]`

[Out] `Integrate[(Sqrt[x] - Sqrt[-1 + x^2])^2/((1 + x - x^2)^2*Sqrt[-1 + x^2]), x]`

Maple [B] time = 0.033, size = 1542, normalized size = 7.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^(1/2)-(x^2-1)^(1/2))^2/(-x^2+x+1)^2/(x^2-1)^(1/2),x)`

[Out] `4/5/(-2+2*5^(1/2))^(1/2)*arctan(2*x^(1/2)/(-2+2*5^(1/2))^(1/2))+2/5*x^(1/2)/(x-1/2*5^(1/2)-1/2)-4/5/(2+2*5^(1/2))^(1/2)*arctanh(2*x^(1/2)/(2+2*5^(1/2))^(1/2))+4/25*5^(1/2)/(-2+2*5^(1/2))^(1/2)*arctan(2*(1-5^(1/2))+(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2))/(-2+2*5^(1/2))`

$$\begin{aligned}
&)^{(1/2)} / (4 * (x + 1/2 * 5^{(1/2)} - 1/2)^{2+4} * (-5^{(1/2)} + 1) * (x + 1/2 * 5^{(1/2)} - 1/2) \\
& + 2 - 2 * 5^{(1/2)})^{(1/2)} - 1/10 / (1/2 + 1/2 * 5^{(1/2)}) / (x - 1/2 * 5^{(1/2)} - 1/2) \\
& * ((x - 1/2 * 5^{(1/2)} - 1/2)^{2+(5^{(1/2)}+1)} * (x - 1/2 * 5^{(1/2)} - 1/2) + 1/2 + 1/2 * 5 \\
& ^{(1/2)})^{(1/2)} - 1/10 / (1/2 - 1/2 * 5^{(1/2)}) / (x + 1/2 * 5^{(1/2)} - 1/2) * ((x + 1/2 * \\
& 5^{(1/2)} - 1/2)^{2+(-5^{(1/2)}+1)} * (x + 1/2 * 5^{(1/2)} - 1/2) + 1/2 - 1/2 * 5^{(1/2)})^{(1/2)} \\
& - 8/25 / (-2 + 2 * 5^{(1/2)})^{(1/2)} * \arctan(2 * x^{(1/2)} / (-2 + 2 * 5^{(1/2)}))^{(1/2)} \\
& * 5^{(1/2)} - 8/25 / (2 + 2 * 5^{(1/2)})^{(1/2)} * \operatorname{arctanh}(2 * x^{(1/2)} / (2 + 2 * 5^{(1/2)}))^{(1/2)} \\
& * 5^{(1/2)} + 2/5 / (2 + 2 * 5^{(1/2)})^{(1/2)} * \operatorname{arctanh}(2 * (1 + 5^{(1/2)} \\
& + (5^{(1/2)} + 1) * (x - 1/2 * 5^{(1/2)} - 1/2)) / (2 + 2 * 5^{(1/2)}))^{(1/2)} / (4 * (x - 1/2 * 5 \\
& ^{(1/2)} - 1/2)^{2+4} * (5^{(1/2)} + 1) * (x - 1/2 * 5^{(1/2)} - 1/2) + 2 + 2 * 5^{(1/2)})^{(1/2)} \\
&) + 1/25 * 5^{(1/2)} * (4 * (x + 1/2 * 5^{(1/2)} - 1/2)^{2+4} * (-5^{(1/2)} + 1) * (x + 1/2 * 5^{(1/2)} \\
& ^{(1/2)} - 1/2) + 2 - 2 * 5^{(1/2)})^{(1/2)} + 1/10 / (1/2 + 1/2 * 5^{(1/2)}) * \ln(x + ((x - 1/2 \\
& * 5^{(1/2)} - 1/2)^{2+(5^{(1/2)}+1)} * (x - 1/2 * 5^{(1/2)} - 1/2) + 1/2 + 1/2 * 5^{(1/2)})^{(1/2)} \\
& + 1/20 / (1/2 + 1/2 * 5^{(1/2)}) * (4 * (x - 1/2 * 5^{(1/2)} - 1/2)^{2+4} * (5^{(1/2)} \\
& + 1) * (x - 1/2 * 5^{(1/2)} - 1/2) + 2 + 2 * 5^{(1/2)})^{(1/2)} - 1/25 * \ln(x + ((x - 1/2 * 5^{(1/2)} \\
& ^{(1/2)} - 1/2)^{2+(5^{(1/2)}+1)} * (x - 1/2 * 5^{(1/2)} - 1/2) + 1/2 + 1/2 * 5^{(1/2)})^{(1/2)} \\
&) * 5^{(1/2)} + 4/25 * 5^{(1/2)} / (2 + 2 * 5^{(1/2)})^{(1/2)} * \operatorname{arctanh}(2 * (1 + 5^{(1/2)} + (\\
& 5^{(1/2)} + 1) * (x - 1/2 * 5^{(1/2)} - 1/2)) / (2 + 2 * 5^{(1/2)}))^{(1/2)} / (4 * (x - 1/2 * 5^{(1/2)} \\
& ^{(1/2)} - 1/2)^{2+4} * (5^{(1/2)} + 1) * (x - 1/2 * 5^{(1/2)} - 1/2) + 2 + 2 * 5^{(1/2)})^{(1/2)} \\
& - 1/25 * 5^{(1/2)} * (4 * (x - 1/2 * 5^{(1/2)} - 1/2)^{2+4} * (5^{(1/2)} + 1) * (x - 1/2 * 5^{(1/2)} \\
& ^{(1/2)} - 1/2) + 2 + 2 * 5^{(1/2)})^{(1/2)} - 2/5 / (-2 + 2 * 5^{(1/2)})^{(1/2)} * \arctan(2 * (1 - 5 \\
& ^{(1/2)} + (-5^{(1/2)} + 1) * (x + 1/2 * 5^{(1/2)} - 1/2)) / (-2 + 2 * 5^{(1/2)})^{(1/2)} / (4 * \\
& (x + 1/2 * 5^{(1/2)} - 1/2)^{2+4} * (-5^{(1/2)} + 1) * (x + 1/2 * 5^{(1/2)} - 1/2) + 2 - 2 * 5^{(1/2)} \\
& ^{(1/2)})^{(1/2)} + 1/10 / (1/2 - 1/2 * 5^{(1/2)}) * \ln(x + ((x + 1/2 * 5^{(1/2)} - 1/2)^{2+(-5^{(1/2)}+1)} * (x + 1/2 * 5^{(1/2)} - 1/2) + 1/2 - 1/2 * 5^{(1/2)})^{(1/2)} + 1/20 / (1/2 - \\
& 1/2 * 5^{(1/2)}) * (4 * (x + 1/2 * 5^{(1/2)} - 1/2)^{2+4} * (-5^{(1/2)} + 1) * (x + 1/2 * 5^{(1/2)} \\
& ^{(1/2)} - 1/2) + 2 - 2 * 5^{(1/2)})^{(1/2)} + 1/25 * \ln(x + ((x + 1/2 * 5^{(1/2)} - 1/2)^{2+(-5^{(1/2)}+1)} * (x + 1/2 * 5^{(1/2)} - 1/2) + 1/2 - 1/2 * 5^{(1/2)})^{(1/2)} * 5^{(1/2)} + 2/5 * x \\
& ^{(1/2)} / (x + 1/2 * 5^{(1/2)} - 1/2) - 1/10 * 5^{(1/2)} / (1/2 + 1/2 * 5^{(1/2)}) / (x - 1/2 * 5^{(1/2)} \\
& ^{(1/2)} - 1/2) * ((x - 1/2 * 5^{(1/2)} - 1/2)^{2+(5^{(1/2)}+1)} * (x - 1/2 * 5^{(1/2)} - 1/2) \\
& + 1/2 + 1/2 * 5^{(1/2)})^{(1/2)} + 1/10 * 5^{(1/2)} / (1/2 - 1/2 * 5^{(1/2)}) / (x + 1/2 * 5^{(1/2)} \\
& ^{(1/2)} - 1/2) * ((x + 1/2 * 5^{(1/2)} - 1/2)^{2+(-5^{(1/2)}+1)} * (x + 1/2 * 5^{(1/2)} - 1/2) \\
& + 1/2 - 1/2 * 5^{(1/2)})^{(1/2)} - 1/5 / (1/2 - 1/2 * 5^{(1/2)}) / (x + 1/2 * 5^{(1/2)} - 1/2) * \\
& ((x + 1/2 * 5^{(1/2)} - 1/2)^{2+(-5^{(1/2)}+1)} * (x + 1/2 * 5^{(1/2)} - 1/2) + 1/2 - 1/2 * 5^{(1/2)} \\
& ^{(1/2)})^{(3/2)} + 1/5 / (1/2 - 1/2 * 5^{(1/2)}) * x * ((x + 1/2 * 5^{(1/2)} - 1/2)^{2+(-5^{(1/2)}+1)} * (x + 1/2 * 5^{(1/2)} - 1/2) + 1/2 - 1/2 * 5^{(1/2)})^{(1/2)} - 1/10 / (1/2 - \\
& 1/2 * 5^{(1/2)}) * \ln(x + ((x + 1/2 * 5^{(1/2)} - 1/2)^{2+(-5^{(1/2)}+1)} * (x + 1/2 * 5^{(1/2)} \\
& ^{(1/2)} - 1/2) + 1/2 - 1/2 * 5^{(1/2)})^{(1/2)} * 5^{(1/2)} - 1/20 / (1/2 - 1/2 * 5^{(1/2)}) * 5 \\
& ^{(1/2)} * (4 * (x + 1/2 * 5^{(1/2)} - 1/2)^{2+4} * (-5^{(1/2)} + 1) * (x + 1/2 * 5^{(1/2)} - 1/2) \\
& + 2 - 2 * 5^{(1/2)})^{(1/2)} + 1/5 / (1/2 + 1/2 * 5^{(1/2)}) * x * ((x - 1/2 * 5^{(1/2)} - 1/2) \\
& ^{2+(5^{(1/2)}+1)} * (x - 1/2 * 5^{(1/2)} - 1/2) + 1/2 + 1/2 * 5^{(1/2)})^{(1/2)} + 1/20 / (1 \\
& /2 + 1/2 * 5^{(1/2)}) * 5^{(1/2)} * (4 * (x - 1/2 * 5^{(1/2)} - 1/2)^{2+4} * (5^{(1/2)} + 1) * (x \\
& - 1/2 * 5^{(1/2)} - 1/2) + 2 + 2 * 5^{(1/2)})^{(1/2)} + 1/10 / (1/2 + 1/2 * 5^{(1/2)}) * \ln(x + \\
& ((x - 1/2 * 5^{(1/2)} - 1/2)^{2+(5^{(1/2)}+1)} * (x - 1/2 * 5^{(1/2)} - 1/2) + 1/2 + 1/2 * 5^{(1/2)} \\
& ^{(1/2)})^{(1/2)} * 5^{(1/2)} - 1/5 / (1/2 + 1/2 * 5^{(1/2)}) / (x - 1/2 * 5^{(1/2)} - 1/2) * (\\
& (x - 1/2 * 5^{(1/2)} - 1/2)^{2+(5^{(1/2)}+1)} * (x - 1/2 * 5^{(1/2)} - 1/2) + 1/2 + 1/2 * 5^{(1/2)} \\
& ^{(1/2)})^{(3/2)} - 1/5 * \ln(x + ((x + 1/2 * 5^{(1/2)} - 1/2)^{2+(-5^{(1/2)}+1)} * (x + 1/2 * 5^{(1/2)} \\
& ^{(1/2)} - 1/2) + 1/2 - 1/2 * 5^{(1/2)})^{(1/2)} - 1/5 * \ln(x + ((x - 1/2 * 5^{(1/2)} - 1/2) \\
& ^{2+(5^{(1/2)}+1)} * (x - 1/2 * 5^{(1/2)} - 1/2) + 1/2 + 1/2 * 5^{(1/2)})^{(1/2)})
\end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2\left(x^{\frac{5}{2}} - 3x^{\frac{3}{2}}\right)}{5(x^2 - x - 1)} + \int \frac{x^{\frac{3}{2}} + \sqrt{x}}{5(x^2 - x - 1)} dx + \int \frac{x^2 + x - 1}{(x^4 - 2x^3 - x^2 + 2x + 1)\sqrt{x+1}\sqrt{x-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((sqrt(x^2 - 1) - sqrt(x))^2/((x^2 - x - 1)^2*sqrt(x^2 - 1)),x, algorithm

[Out] -2/5*(x^(5/2) - 3*x^(3/2))/(x^2 - x - 1) + integrate(1/5*(x^(3/2) + sqrt(x))/(x^2 - x - 1), x) + integrate((x^2 + x - 1)/((x^4 - 2*x^3 - x^2 + 2*x + 1)*sqrt(x + 1)*sqrt(x - 1)), x)

Fricas [A] time = 0.255298, size = 1185, normalized size = 5.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((sqrt(x^2 - 1) - sqrt(x))^2/((x^2 - x - 1)^2*sqrt(x^2 - 1)),x, algorithm

[Out] 1/50*(40*x^3 + 40*x^2 - 4*(2*sqrt(2)*(x^3 - x^2 - x)*sqrt(x^2 - 1)*sqrt(-sqrt(5)*(11*sqrt(5) - 25)) - sqrt(2)*(2*x^4 - 2*x^3 - 3*x^2 + x + 1)*sqrt(-sqrt(5)*(11*sqrt(5) - 25))))*arctan(-1/2*sqrt(2)*sqrt(-sqrt(5)*(11*sqrt(5) - 25))*(sqrt(5) + 3)/(sqrt(5)*(2*x - 1) - 2*sqrt(5)*sqrt(x^2 - 1) - 2*sqrt(sqrt(5)*(sqrt(5)*(2*x^2 - x) - sqrt(x^2 - 1)*(sqrt(5)*(2*x - 1) + 5) + 5*x)) + 5)) + 4*(2*sqrt(2)*(x^3 - x^2 - x)*sqrt(x^2 - 1)*sqrt(-sqrt(5)*(11*sqrt(5) - 25)) - sqrt(2)*(2*x^4 - 2*x^3 - 3*x^2 + x + 1)*sqrt(-sqrt(5)*(11*sqrt(5) - 25))))*arctan(1/2*sqrt(2)*sqrt(-sqrt(5)*(11*sqrt(5) - 25))*(sqrt(5) + 3)/(sqrt(2)*sqrt(sqrt(5)*(sqrt(5)*(2*x - 1) + 5)) + 2*sqrt(5)*sqrt(x))) + (2*sqrt(2)*(x^3 - x^2 - x)*sqrt(x^2 - 1)*sqrt(sqrt(5)*(11*sqrt(5) + 25)) - sqrt(2)*(2*x^4 - 2*x^3 - 3*x^2 + x + 1)*sqrt(sqrt(5)*(11*sqrt(5) + 25))))*log(sqrt(2)*sqrt(sqrt(5)*(11*sqrt(5) + 25))*(sqrt(5) - 3) - 2*sqrt(5)*(2*x - 1) + 4*sqrt(5)*sqrt(x^2 - 1) + 10) - (2*sqrt(2)*(x^3 - x^2 - x)*sqrt(x^2 - 1)*sqrt(sqrt(5)*(11*sqrt(5) + 25)) - sqrt(2)*(2*x^4 - 2*x^3 - 3*x^2 + x + 1)*sqrt(sqrt(5)*(11*sqrt(5) + 25))))*log(sqrt(2)*sqrt(sqrt(5)*(11*sqrt(5) + 25))*(sqrt(5) - 3) + 4*sqrt(5)*sqrt(x)) - (2*sqrt(2)*(x^3 - x^2 - x)*sqrt(x^2 - 1)*sqrt(sqrt(5)*(11*sqrt(5) + 25)) - sqrt(2)*(2*x^4 - 2*x^3 - 3*x^2 + x + 1)*sqrt(sqrt(5)*(11*sqrt(5) + 25))))*log(-sqrt(2)*sqrt(sqrt(5)*(11*sqrt(5) + 25))*(sqrt(5) - 3) - 2*sqrt(5)*(2*x - 1) + 4*sqrt(5)*sqrt(x^2 - 1) + 10) + (2*sqrt(2)*(x^3 - x^2 - x)*sqrt(x^2 - 1)*sqrt(sqrt(5)*(11*sqrt(5) + 25)) - sqrt(2)*(2*x^4 - 2*x^3 - 3*x^2 + x + 1)*sqrt(sqrt(5)*(11*sqrt(5) + 25))))*log(-sqrt(2)*sqrt(sqrt(5)*(11*sqrt(5) + 25))*(sqrt(5) - 3) + 4*sqrt(5)*sqrt(x)) - 20*(2*x^2 + 2*(2*x^2 - x)*sqrt(x) + 2*x + 1)*sqrt(x^2 - 1) + 20*(4*x^3 - 2*x^2 - 2*x + 1)*sqrt(x) - 40)/(2*x^4 - 2*x^3 - 3*x^2 - 2*(x^3 - x^2 - x)*sqrt(x^2 - 1) + x

+ 1)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**(1/2)-(x**2-1)**(1/2))**2/(-x**2+x+1)**2/(x**2-1)**(1/2), x)

[Out] Timed out

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(\sqrt{x^2 - 1} - \sqrt{x})^2}{(x^2 - x - 1)^2 \sqrt{x^2 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((sqrt(x^2 - 1) - sqrt(x))^2/((x^2 - x - 1)^2*sqrt(x^2 - 1)), x, algorithm="giac")

[Out] integrate((sqrt(x^2 - 1) - sqrt(x))^2/((x^2 - x - 1)^2*sqrt(x^2 - 1)), x)

$$3.11 \quad \int \left(\frac{1}{\sqrt{2}(1+x)^2\sqrt{-i+x^2}} + \frac{1}{\sqrt{2}(1+x)^2\sqrt{i+x^2}} \right) dx$$

Optimal. Leaf size=138

$$-\frac{\left(\frac{1}{2} + \frac{i}{2}\right)\sqrt{x^2 - i}}{\sqrt{2}(x+1)} - \frac{\left(\frac{1}{2} - \frac{i}{2}\right)\sqrt{x^2 + i}}{\sqrt{2}(x+1)} + \frac{\tanh^{-1}\left(\frac{x+i}{\sqrt{1-i}\sqrt{x^2-i}}\right)}{(1-i)^{3/2}\sqrt{2}} - \frac{\tanh^{-1}\left(\frac{-x+i}{\sqrt{1+i}\sqrt{x^2+i}}\right)}{(1+i)^{3/2}\sqrt{2}}$$

[Out] $((-1/2 - I/2)*\text{Sqrt}[-I + x^2])/(\text{Sqrt}[2]*(1 + x)) - ((1/2 - I/2)*\text{Sqrt}[I + x^2])/(\text{Sqrt}[2]*(1 + x)) + \text{ArcTanh}[(I + x)/(\text{Sqrt}[1 - I]*\text{Sqrt}[-I + x^2])]/((1 - I)^{(3/2)}*\text{Sqrt}[2]) - \text{ArcTanh}[(I - x)/(\text{Sqrt}[1 + I]*\text{Sqrt}[I + x^2])]/((1 + I)^{(3/2)}*\text{Sqrt}[2])$

Rubi [A] time = 0.161835, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$-\frac{\left(\frac{1}{2} + \frac{i}{2}\right)\sqrt{x^2 - i}}{\sqrt{2}(x+1)} - \frac{\left(\frac{1}{2} - \frac{i}{2}\right)\sqrt{x^2 + i}}{\sqrt{2}(x+1)} + \frac{\tanh^{-1}\left(\frac{x+i}{\sqrt{1-i}\sqrt{x^2-i}}\right)}{(1-i)^{3/2}\sqrt{2}} - \frac{\tanh^{-1}\left(\frac{-x+i}{\sqrt{1+i}\sqrt{x^2+i}}\right)}{(1+i)^{3/2}\sqrt{2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[1/(\text{Sqrt}[2]*(1 + x)^2*\text{Sqrt}[-I + x^2]) + 1/(\text{Sqrt}[2]*(1 + x)^2*\text{Sqrt}[I + x^2]), x]$

[Out] $((-1/2 - I/2)*\text{Sqrt}[-I + x^2])/(\text{Sqrt}[2]*(1 + x)) - ((1/2 - I/2)*\text{Sqrt}[I + x^2])/(\text{Sqrt}[2]*(1 + x)) + \text{ArcTanh}[(I + x)/(\text{Sqrt}[1 - I]*\text{Sqrt}[-I + x^2])]/((1 - I)^{(3/2)}*\text{Sqrt}[2]) - \text{ArcTanh}[(I - x)/(\text{Sqrt}[1 + I]*\text{Sqrt}[I + x^2])]/((1 + I)^{(3/2)}*\text{Sqrt}[2])$

Rubi in Sympy [A] time = 18.4555, size = 184, normalized size = 1.33

$$\frac{(1+i)\operatorname{atanh}\left(\frac{\sqrt{2}(-x-i)}{\sqrt{x^2-i}(\sqrt{1+\sqrt{2}-i}\sqrt{-1+\sqrt{2}})}\right)}{2\left(-\sqrt{1+\sqrt{2}}+i\sqrt{-1+\sqrt{2}}\right)} - \frac{(1-i)\operatorname{atanh}\left(\frac{\sqrt{2}(-x+i)}{\sqrt{x^2+i}(\sqrt{1+\sqrt{2}+i}\sqrt{-1+\sqrt{2}})}\right)}{2\left(\sqrt{1+\sqrt{2}}+i\sqrt{-1+\sqrt{2}}\right)} - \frac{\sqrt{2}(1+i)\sqrt{x^2-i}}{4(x+1)} - \frac{\sqrt{2}(1-i)\sqrt{x^2+i}}{4(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{rubi_integrate}(1/2/(1+x)**2*2**(1/2)/(-I+x**2)**(1/2)+1/2/(1+x)**2*2**(1/2)/(I$

[Out] $(1 + I)*\operatorname{atanh}(\operatorname{sqrt}(2)*(-x - I)/(\operatorname{sqrt}(x**2 - I)*(\operatorname{sqrt}(1 + \operatorname{sqrt}(2)) - I*\operatorname{sqrt}(-1 + \operatorname{sqrt}(2)))))/(2*(-\operatorname{sqrt}(1 + \operatorname{sqrt}(2)) + I*\operatorname{sqrt}(-1 + s$

$$\begin{aligned} & \text{qrt}(2))) - (1 - I) \cdot \text{atanh}(\text{sqrt}(2) \cdot (-x + I) / (\text{sqrt}(x^{**2} + I) \cdot (\text{sqrt}(1 + \text{sqrt}(2)) + I \cdot \text{sqrt}(-1 + \text{sqrt}(2)))))) / (2 \cdot (\text{sqrt}(1 + \text{sqrt}(2)) + I \cdot \text{sqrt}(-1 + \text{sqrt}(2)))) - \text{sqrt}(2) \cdot (1 + I) \cdot \text{sqrt}(x^{**2} - I) / (4 \cdot (x + 1)) \\ & - \text{sqrt}(2) \cdot (1 - I) \cdot \text{sqrt}(x^{**2} + I) / (4 \cdot (x + 1)) \end{aligned}$$

Mathematica [B] time = 0.4127, size = 403, normalized size = 2.92

$$(2 + 2i)\sqrt{x^2 - i} + (2 - 2i)\sqrt{x^2 + i} + i\sqrt{1 - ix} \log\left((-2 + i)x^2 + 2\sqrt{1 - i}\sqrt{x^2 - ix} + i\right) + i\sqrt{1 - i} \log\left((-2 + i)x^2 + 2\sqrt{1 - i}\sqrt{x^2 + ix} + i\right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[2]*(1+x)^2*Sqrt[-I+x^2]) + 1/(Sqrt[2]*(1+x)^2*Sqrt[I+x^2])]

[Out] -((2 + 2*I)*Sqrt[-I + x^2] + (2 - 2*I)*Sqrt[I + x^2] + 2*Sqrt[1 - I]*(1 + x)*ArcTan[(1 + x^2 + (2*I)*Sqrt[1 - I]*Sqrt[-I + x^2])]/((1 - 2*I) - (2*I)*x + x^2)) + 2*Sqrt[1 + I]*(1 + x)*ArcTan[(1 + x^2 - (2*I)*Sqrt[1 + I]*Sqrt[I + x^2])]/((1 + 2*I) + (2*I)*x + x^2) - I*Sqrt[1 - I]*Log[(1 + x)^2] + I*Sqrt[1 + I]*Log[(1 + x)^2] - I*Sqrt[1 - I]*x*Log[(1 + x)^2] + I*Sqrt[1 + I]*x*Log[(1 + x)^2] + I*Sqrt[1 - I]*Log[I - (2 - I)*x^2 + 2*Sqrt[1 - I]*x*Sqrt[-I + x^2]] + I*Sqrt[1 - I]*x*Log[I - (2 - I)*x^2 + 2*Sqrt[1 - I]*x*Sqrt[-I + x^2]] - I*Sqrt[1 + I]*Log[-I - (2 + I)*x^2 + 2*Sqrt[1 + I]*x*Sqrt[I + x^2]] - I*Sqrt[1 + I]*x*Log[-I - (2 + I)*x^2 + 2*Sqrt[1 + I]*x*Sqrt[I + x^2]]/(4*Sqrt[2]*(1 + x))

Maple [B] time = 0.032, size = 278, normalized size = 2.

$$\begin{aligned} & -\frac{\sqrt{2}}{4+4x}\sqrt{(1+x)^2-1-i-2x}-\frac{i\sqrt{2}}{1+x}\sqrt{(1+x)^2-1-i-2x} \\ & -\frac{\sqrt{2}}{4\sqrt{1-i}}\ln\left(\frac{1}{1+x}\left(-2i-2x+2\sqrt{1-i}\sqrt{(1+x)^2-1-i-2x}\right)\right) \\ & -\frac{i\sqrt{2}}{\sqrt{1-i}}\ln\left(\frac{1}{1+x}\left(-2i-2x+2\sqrt{1-i}\sqrt{(1+x)^2-1-i-2x}\right)\right) \\ & -\frac{\sqrt{2}}{4+4x}\sqrt{(1+x)^2-1+i-2x}+\frac{i\sqrt{2}}{1+x}\sqrt{(1+x)^2-1+i-2x} \\ & -\frac{\sqrt{2}}{4\sqrt{1+i}}\ln\left(\frac{1}{1+x}\left(2i-2x+2\sqrt{1+i}\sqrt{(1+x)^2-1+i-2x}\right)\right) \\ & +\frac{i\sqrt{2}}{\sqrt{1+i}}\ln\left(\frac{1}{1+x}\left(2i-2x+2\sqrt{1+i}\sqrt{(1+x)^2-1+i-2x}\right)\right) \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/2/(1+x)^2 \cdot 2^{1/2}/(-I+x^2)^{1/2} + 1/2/(1+x)^2 \cdot 2^{1/2}/(I+x^2)^{1/2}, x)$

[Out] $-1/4 \cdot 2^{1/2}/(1+x) \cdot ((1+x)^2 - 1 - I - 2 \cdot x)^{1/2} - 1/4 \cdot I \cdot 2^{1/2}/(1+x) \cdot ((1+x)^2 - 1 - I - 2 \cdot x)^{1/2} - 1/4 \cdot 2^{1/2}/(1-I)^{1/2} \cdot \ln((-2 \cdot I - 2 \cdot x + 2 \cdot (1-I))^{1/2} \cdot ((1+x)^2 - 1 - I - 2 \cdot x)^{1/2})/(1+x) - 1/4 \cdot I \cdot 2^{1/2}/(1-I)^{1/2} \cdot \ln((-2 \cdot I - 2 \cdot x + 2 \cdot (1-I))^{1/2} \cdot ((1+x)^2 - 1 - I - 2 \cdot x)^{1/2})/(1+x) - 1/4 \cdot 2^{1/2}/(1+x) \cdot ((1+x)^2 - 1 + I - 2 \cdot x)^{1/2} + 1/4 \cdot I \cdot 2^{1/2}/(1+x) \cdot ((1+x)^2 - 1 + I - 2 \cdot x)^{1/2} - 1/4 \cdot 2^{1/2}/(1+I)^{1/2} \cdot \ln((2 \cdot I - 2 \cdot x + 2 \cdot (1+I))^{1/2} \cdot ((1+x)^2 - 1 + I - 2 \cdot x)^{1/2})/(1+x) + 1/4 \cdot I \cdot 2^{1/2}/(1+I)^{1/2} \cdot \ln((2 \cdot I - 2 \cdot x + 2 \cdot (1+I))^{1/2} \cdot ((1+x)^2 - 1 + I - 2 \cdot x)^{1/2})/(1+x)$

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(1/2 \cdot \sqrt{2}/(\sqrt{x^2 + I}) \cdot (x + 1)^2 + 1/2 \cdot \sqrt{2}/(\sqrt{x^2 - I}) \cdot (x + 1)^2, x)$

[Out] Exception raised: RuntimeError

Fricas [A] time = 0.286757, size = 680, normalized size = 4.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(1/2 \cdot \sqrt{2}/(\sqrt{x^2 + I}) \cdot (x + 1)^2 + 1/2 \cdot \sqrt{2}/(\sqrt{x^2 - I}) \cdot (x + 1)^2, x)$

[Out] $(\sqrt{2}) \cdot \sqrt{x^2 - I} \cdot ((I + 1) \cdot x - I) + (\sqrt{-1/2 \cdot I + 1/2}) \cdot \sqrt{x^2 - I} \cdot ((I - 1) \cdot x^2 + (I - 1) \cdot x) + \sqrt{-1/2 \cdot I + 1/2} \cdot (-(I - 1) \cdot x^3 - (I - 1) \cdot x^2) + \sqrt{x^2 + I} \cdot (\sqrt{-1/2 \cdot I + 1/2}) \cdot \sqrt{x^2 - I} \cdot (-(I - 1) \cdot x - I + 1) + \sqrt{-1/2 \cdot I + 1/2} \cdot ((I - 1) \cdot x^2 + (I - 1) \cdot x) \cdot \log(\sqrt{2}) \cdot \sqrt{-1/2 \cdot I + 1/2} - x + \sqrt{x^2 - I} - 1) + (\sqrt{-1/2 \cdot I + 1/2}) \cdot \sqrt{x^2 - I} \cdot (-(I - 1) \cdot x^2 - (I - 1) \cdot x) + \sqrt{-1/2 \cdot I + 1/2} \cdot ((I - 1) \cdot x^3 + (I - 1) \cdot x^2) + \sqrt{x^2 + I} \cdot (\sqrt{-1/2 \cdot I + 1/2}) \cdot \sqrt{x^2 - I} \cdot ((I - 1) \cdot x + I - 1) + \sqrt{-1/2 \cdot I + 1/2} \cdot (-(I - 1) \cdot x^2 - (I - 1) \cdot x) \cdot \log(-\sqrt{2}) \cdot \sqrt{-1/2 \cdot I + 1/2} - x + \sqrt{x^2 - I} - 1) + (\sqrt{-1/2 \cdot I - 1/2}) \cdot ((I + 1) \cdot x^2 + (I + 1) \cdot x) \cdot \sqrt{x^2 - I} + \sqrt{-1/2 \cdot I - 1/2} \cdot (-(I + 1) \cdot x^3 - (I + 1) \cdot x^2) + \sqrt{x^2 + I} \cdot (\sqrt{-1/2 \cdot I - 1/2}) \cdot \sqrt{x^2 - I} \cdot (-(I + 1) \cdot x - I - 1) + \sqrt{-1/2 \cdot I - 1/2} \cdot ((I + 1) \cdot x^2 + (I + 1) \cdot x) \cdot \log(I \cdot \sqrt{2}) \cdot \sqrt{-1/2 \cdot I - 1/2} - x + \sqrt{x^2 + I} - 1) + (\sqrt{-1/2 \cdot I - 1/2}) \cdot \sqrt{x^2 - I} \cdot (-(I + 1) \cdot x^2 - (I + 1) \cdot x) + \sqrt{-1/2 \cdot I - 1/2} \cdot ((I + 1) \cdot x^3 + (I + 1) \cdot x^2) + (\sqrt{-1/2 \cdot I - 1/2}) \cdot \sqrt{x^2 - I} \cdot ((I + 1) \cdot x + I + 1) + \sqrt{-1/2 \cdot I - 1/2} \cdot (-(I + 1) \cdot x^3 - (I + 1) \cdot x^2) + \sqrt{x^2 + I} \cdot (\sqrt{-1/2 \cdot I - 1/2}) \cdot \sqrt{x^2 - I} \cdot (-(I + 1) \cdot x - I - 1) + \sqrt{-1/2 \cdot I - 1/2} \cdot ((I + 1) \cdot x^2 + (I + 1) \cdot x) \cdot \log(-\sqrt{2}) \cdot \sqrt{-1/2 \cdot I - 1/2} - x + \sqrt{x^2 - I} - 1)$

$$\begin{aligned} & x^2 - (I + 1)x) \sqrt{x^2 + I} \log(-I \sqrt{2} \sqrt{-1/2 I - 1/2} \\ &) - x + \sqrt{x^2 + I} - 1) + \sqrt{2} \sqrt{-(I + 1)x^2 + (I + 1)x} + \\ & \sqrt{x^2 + I} (\sqrt{2} \sqrt{(I + 1)x - 1} - (I + 1) \sqrt{2} \sqrt{x^2 \\ & 2 - I})) / ((2I + 2)x^3 + (2I + 2)x^2 + \sqrt{x^2 + I} \sqrt{-(2I + \\ & 2)x^2 + \sqrt{x^2 - I} \sqrt{(2I + 2)x + 2I + 2}} - (2I + 2)x) + \sqrt{x^2 - I} \sqrt{-(2I + 2)x^2 - (2I + 2)x}) \end{aligned}$$

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/2/(1+x)**2*2**(1/2)/(-I+x**2)**(1/2)+1/2/(1+x)**2*2**(1/2)/(I+x**2

[Out] Exception raised: TypeError

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/2*sqrt(2)/(sqrt(x^2 + I)*(x + 1)^2) + 1/2*sqrt(2)/(sqrt(x^2 - I)*(x +

[Out] Exception raised: TypeError

$$3.12 \quad \int \frac{\sqrt{x^2 + \sqrt{1+x^4}}}{(1+x)^2 \sqrt{1+x^4}} dx$$

Optimal. Leaf size=125

$$-\frac{\sqrt{1-ix^2}}{2(x+1)} - \frac{\sqrt{1+ix^2}}{2(x+1)} - \frac{1}{4}(1-i)^{3/2} \tanh^{-1}\left(\frac{1+ix}{\sqrt{1-i}\sqrt{1-ix^2}}\right) - \frac{1}{4}(1+i)^{3/2} \tanh^{-1}\left(\frac{1-ix}{\sqrt{1+i}\sqrt{1+ix^2}}\right)$$

[Out] $-\text{Sqrt}[1 - I*x^2]/(2*(1 + x)) - \text{Sqrt}[1 + I*x^2]/(2*(1 + x)) - ((1 - I)^{(3/2)}*\text{ArcTanh}[(1 + I*x)/(\text{Sqrt}[1 - I]*\text{Sqrt}[1 - I*x^2])])/4 - ((1 + I)^{(3/2)}*\text{ArcTanh}[(1 - I*x)/(\text{Sqrt}[1 + I]*\text{Sqrt}[1 + I*x^2])])/4$

Rubi [A] time = 0.328194, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$-\frac{\sqrt{1-ix^2}}{2(x+1)} - \frac{\sqrt{1+ix^2}}{2(x+1)} - \frac{1}{4}(1-i)^{3/2} \tanh^{-1}\left(\frac{1+ix}{\sqrt{1-i}\sqrt{1-ix^2}}\right) - \frac{1}{4}(1+i)^{3/2} \tanh^{-1}\left(\frac{1-ix}{\sqrt{1+i}\sqrt{1+ix^2}}\right)$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sqrt}[x^2 + \text{Sqrt}[1 + x^4]]/((1 + x)^2*\text{Sqrt}[1 + x^4]), x]$

[Out] $-\text{Sqrt}[1 - I*x^2]/(2*(1 + x)) - \text{Sqrt}[1 + I*x^2]/(2*(1 + x)) - ((1 - I)^{(3/2)}*\text{ArcTanh}[(1 + I*x)/(\text{Sqrt}[1 - I]*\text{Sqrt}[1 - I*x^2])])/4 - ((1 + I)^{(3/2)}*\text{ArcTanh}[(1 - I*x)/(\text{Sqrt}[1 + I]*\text{Sqrt}[1 + I*x^2])])/4$

Rubi in Sympy [A] time = 21.5739, size = 202, normalized size = 1.62

$$\frac{\sqrt{2} \operatorname{atanh}\left(\frac{\sqrt{2}(-ix+1)}{\sqrt{ix^2+1}(\sqrt{1+\sqrt{2}+i}\sqrt{-1+\sqrt{2}})}\right)}{2\left(-\sqrt{-1+\sqrt{2}}+i\sqrt{1+\sqrt{2}}\right)} - \frac{\sqrt{2} \operatorname{atanh}\left(\frac{\sqrt{2}(ix+1)}{\sqrt{-ix^2+1}(\sqrt{1+\sqrt{2}-i}\sqrt{-1+\sqrt{2}})}\right)}{2\left(\sqrt{-1+\sqrt{2}}+i\sqrt{1+\sqrt{2}}\right)}$$

$$- \frac{\left(\frac{1}{2} - \frac{i}{2}\right)\left(\frac{1}{2} + \frac{i}{2}\right)\sqrt{-ix^2+1}}{x+1} - \frac{\left(\frac{1}{2} - \frac{i}{2}\right)\left(\frac{1}{2} + \frac{i}{2}\right)\sqrt{ix^2+1}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{rubi_integrate}((x^{**2}+(x^{**4}+1)^{(1/2)})^{(1/2)}/((1+x)^{**2}/(x^{**4}+1)^{(1/2})), x)$

[Out] $\text{sqrt}(2)*\text{atanh}(\text{sqrt}(2)*(-I*x + 1)/(\text{sqrt}(I*x^{**2} + 1)*(\text{sqrt}(1 + \text{sqrt}(2)) + I*\text{sqrt}(-1 + \text{sqrt}(2))))) / (2*(-\text{sqrt}(-1 + \text{sqrt}(2)) + I*\text{sqrt}(1$

+ sqrt(2)))) - sqrt(2)*atanh(sqrt(2)*(I*x + 1)/(sqrt(-I*x**2 + 1)
)*(sqrt(1 + sqrt(2)) - I*sqrt(-1 + sqrt(2)))))/(2*(sqrt(-1 + sqrt
 (2)) + I*sqrt(1 + sqrt(2)))) - (1/2 - I/2)*(1/2 + I/2)*sqrt(-I*x*
 2 + 1)/(x + 1) - (1/2 - I/2)(1/2 + I/2)*sqrt(I*x**2 + 1)/(x + 1
)

Mathematica [A] time = 0.154463, size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2 + \sqrt{1 + x^4}}}{(1 + x)^2 \sqrt{1 + x^4}} dx$$

Verification is Not applicable to the result.

[In] Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x)^2*Sqrt[1 + x^4]),x]

[Out] Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x)^2*Sqrt[1 + x^4]), x]

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int \frac{1}{(1+x)^2} \sqrt{x^2 + \sqrt{x^4 + 1}} \frac{1}{\sqrt{x^4 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)^2/(x^4+1)^(1/2),x)

[Out] int((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)^2/(x^4+1)^(1/2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2 + \sqrt{x^4 + 1}}}{\sqrt{x^4 + 1}(x + 1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*(x + 1)^2),x, algorithm="maxim

[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*(x + 1)^2), x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*(x + 1)^2), x, algorithm="fricas")

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2 + \sqrt{x^4 + 1}}}{(x + 1)^2 \sqrt{x^4 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+(x**4+1)**(1/2))**(1/2)/(1+x)**2/(x**4+1)**(1/2), x)

[Out] Integral(sqrt(x**2 + sqrt(x**4 + 1))/((x + 1)**2*sqrt(x**4 + 1)), x)

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2 + \sqrt{x^4 + 1}}}{\sqrt{x^4 + 1}(x + 1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*(x + 1)^2), x, algorithm="giac")

[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*(x + 1)^2), x)

$$3.13 \quad \int \frac{\sqrt{x^2 + \sqrt{1+x^4}}}{(1+x)\sqrt{1+x^4}} dx$$

Optimal. Leaf size=81

$$-\frac{1}{2}\sqrt{1-i}\tanh^{-1}\left(\frac{1+ix}{\sqrt{1-i}\sqrt{1-ix^2}}\right) - \frac{1}{2}\sqrt{1+i}\tanh^{-1}\left(\frac{1-ix}{\sqrt{1+i}\sqrt{1+ix^2}}\right)$$

[Out] $-(\text{Sqrt}[1 - I] * \text{ArcTanh}[(1 + I * x) / (\text{Sqrt}[1 - I] * \text{Sqrt}[1 - I * x^2])]) / 2$
 $- (\text{Sqrt}[1 + I] * \text{ArcTanh}[(1 - I * x) / (\text{Sqrt}[1 + I] * \text{Sqrt}[1 + I * x^2])]) / 2$

Rubi [A] time = 0.278117, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$

$$-\frac{1}{2}\sqrt{1-i}\tanh^{-1}\left(\frac{1+ix}{\sqrt{1-i}\sqrt{1-ix^2}}\right) - \frac{1}{2}\sqrt{1+i}\tanh^{-1}\left(\frac{1-ix}{\sqrt{1+i}\sqrt{1+ix^2}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x)*Sqrt[1 + x^4]),x]

[Out] $-(\text{Sqrt}[1 - I] * \text{ArcTanh}[(1 + I * x) / (\text{Sqrt}[1 - I] * \text{Sqrt}[1 - I * x^2])]) / 2$
 $- (\text{Sqrt}[1 + I] * \text{ArcTanh}[(1 - I * x) / (\text{Sqrt}[1 + I] * \text{Sqrt}[1 + I * x^2])]) / 2$

Rubi in Sympy [A] time = 17.1132, size = 155, normalized size = 1.91

$$\frac{\sqrt{2}(1+i)\operatorname{atanh}\left(\frac{\sqrt{2}(-ix+1)}{\sqrt{ix^2+1}(\sqrt{1+\sqrt{2}+i}\sqrt{-1+\sqrt{2}})}\right)}{2(\sqrt{1+\sqrt{2}+i}\sqrt{-1+\sqrt{2}})} + \frac{\sqrt{2}(1-i)\operatorname{atanh}\left(\frac{\sqrt{2}(ix+1)}{\sqrt{-ix^2+1}(\sqrt{1+\sqrt{2}-i}\sqrt{-1+\sqrt{2}})}\right)}{2(-\sqrt{1+\sqrt{2}+i}\sqrt{-1+\sqrt{2}})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate((x**2+(x**4+1)**(1/2))**(1/2)/(1+x)/(x**4+1)**(1/2),x)

[Out] $-\text{sqrt}(2) * (1 + I) * \operatorname{atanh}(\text{sqrt}(2) * (-I * x + 1) / (\text{sqrt}(I * x^2 + 1) * (\text{sqrt}(1 + \text{sqrt}(2)) + I * \text{sqrt}(-1 + \text{sqrt}(2))))) / (2 * (\text{sqrt}(1 + \text{sqrt}(2)) + I * \text{sqrt}(-1 + \text{sqrt}(2)))) + \text{sqrt}(2) * (1 - I) * \operatorname{atanh}(\text{sqrt}(2) * (I * x + 1) / (\text{sqrt}(-I * x^2 + 1) * (\text{sqrt}(1 + \text{sqrt}(2)) - I * \text{sqrt}(-1 + \text{sqrt}(2))))) / (2 * (-\text{sqrt}(1 + \text{sqrt}(2)) + I * \text{sqrt}(-1 + \text{sqrt}(2))))$

Mathematica [A] time = 0.118799, size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2 + \sqrt{1 + x^4}}}{(1 + x)\sqrt{1 + x^4}} dx$$

Verification is Not applicable to the result.

[In] Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x)*Sqrt[1 + x^4]),x]

[Out] Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x)*Sqrt[1 + x^4]), x]

Maple [F] time = 0.037, size = 0, normalized size = 0.

$$\int \frac{1}{1+x} \sqrt{x^2 + \sqrt{x^4 + 1}} \frac{1}{\sqrt{x^4 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)/(x^4+1)^(1/2),x)

[Out] int((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)/(x^4+1)^(1/2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2 + \sqrt{x^4 + 1}}}{\sqrt{x^4 + 1}(x + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*(x + 1)),x, algorithm="maxima"

[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*(x + 1)), x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*(x + 1)),x, algorithm="fricas"`

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2 + \sqrt{x^4 + 1}}}{(x + 1)\sqrt{x^4 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**2+(x**4+1)**(1/2))**(1/2)/(1+x)/(x**4+1)**(1/2),x)`

[Out] `Integral(sqrt(x**2 + sqrt(x**4 + 1))/((x + 1)*sqrt(x**4 + 1)), x)`

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2 + \sqrt{x^4 + 1}}}{\sqrt{x^4 + 1}(x + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*(x + 1)),x, algorithm="giac")`

[Out] `integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*(x + 1)), x)`

$$3.14 \quad \int \frac{\sqrt{x^2 + \sqrt{1+x^4}}}{\sqrt{1+x^4}} dx$$

Optimal. Leaf size=31

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{\sqrt{x^4+1}+x^2}}\right)}{\sqrt{2}}$$

[Out] ArcTanh[(Sqrt[2]*x)/Sqrt[x^2 + Sqrt[1 + x^4]]]/Sqrt[2]

Rubi [A] time = 0.088642, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{\sqrt{x^4+1}+x^2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x^2 + Sqrt[1 + x^4]]/Sqrt[1 + x^4], x]

[Out] ArcTanh[(Sqrt[2]*x)/Sqrt[x^2 + Sqrt[1 + x^4]]]/Sqrt[2]

Rubi in Sympy [A] time = 3.71235, size = 29, normalized size = 0.94

$$\frac{\sqrt{2} \operatorname{atanh}\left(\frac{\sqrt{2}x}{\sqrt{x^2 + \sqrt{x^4+1}}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate((x**2+(x**4+1)**(1/2))**(1/2)/(x**4+1)**(1/2), x)

[Out] sqrt(2)*atanh(sqrt(2)*x/sqrt(x**2 + sqrt(x**4 + 1)))/2

Mathematica [B] time = 2.35662, size = 145, normalized size = 4.68

$$\frac{x \left(x^4 + \sqrt{x^4 + 1} x^2 + 1 \right) \left(\log \left(1 - \frac{\sqrt{x^2 (\sqrt{x^4 + 1} + x^2)}}{\sqrt{2} x^2} \right) - \log \left(\frac{\sqrt{x^2 (\sqrt{x^4 + 1} + x^2)}}{\sqrt{2} x^2} + 1 \right) \right)}{2\sqrt{2}\sqrt{x^4 + 1}\sqrt{\sqrt{x^4 + 1} + x^2}\sqrt{x^2 (\sqrt{x^4 + 1} + x^2)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/Sqrt[1 + x^4],x]

[Out] $-(x*(1 + x^4 + x^2*\text{Sqrt}[1 + x^4])*(\text{Log}[1 - \text{Sqrt}[x^2*(x^2 + \text{Sqrt}[1 + x^4])]]/(\text{Sqrt}[2]*x^2)) - \text{Log}[1 + \text{Sqrt}[x^2*(x^2 + \text{Sqrt}[1 + x^4])]]/(\text{Sqrt}[2]*x^2)))/(2*\text{Sqrt}[2]*\text{Sqrt}[1 + x^4]*\text{Sqrt}[x^2 + \text{Sqrt}[1 + x^4]]*\text{Sqrt}[x^2*(x^2 + \text{Sqrt}[1 + x^4])])$

Maple [F] time = 0.024, size = 0, normalized size = 0.

$$\int 1\sqrt{x^2 + \sqrt{x^4 + 1}} \frac{1}{\sqrt{x^4 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x)

[Out] int((x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2 + \sqrt{x^4 + 1}}}{\sqrt{x^4 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(x^2 + sqrt(x^4 + 1))/sqrt(x^4 + 1),x, algorithm="maxima")

[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/sqrt(x^4 + 1), x)

Fricas [A] time = 0.389951, size = 81, normalized size = 2.61

$$\frac{1}{4} \sqrt{2} \log \left(4x^4 + 4\sqrt{x^4 + 1}x^2 + 2 \left(\sqrt{2}x^3 + \sqrt{2}\sqrt{x^4 + 1}x \right) \sqrt{x^2 + \sqrt{x^4 + 1} + 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(x^2 + sqrt(x^4 + 1))/sqrt(x^4 + 1),x, algorithm="fricas")

[Out] $\frac{1}{4}\sqrt{2}\log(4x^4 + 4\sqrt{x^4 + 1})x^2 + 2(\sqrt{2}x^3 + \sqrt{2}\sqrt{x^4 + 1}x)\sqrt{x^2 + \sqrt{x^4 + 1}} + 1$

Sympy [A] time = 2.21216, size = 15, normalized size = 0.48

$$\frac{G_{3,3}^{2,2}\left(\begin{matrix} 1, 1 & \frac{1}{2} \\ \frac{1}{4}, \frac{3}{4} & 0 \end{matrix} \middle| x^4\right)}{4\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**2+(x**4+1)**(1/2))**(1/2)/(x**4+1)**(1/2),x)`

[Out] `meijerg(((1, 1), (1/2,)), ((1/4, 3/4), (0,)), x**4)/(4*sqrt(pi))`

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2 + \sqrt{x^4 + 1}}}{\sqrt{x^4 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(x^2 + sqrt(x^4 + 1))/sqrt(x^4 + 1),x, algorithm="giac")`

[Out] `integrate(sqrt(x^2 + sqrt(x^4 + 1))/sqrt(x^4 + 1), x)`

$$3.15 \quad \int \frac{\sqrt{-x^2 + \sqrt{1+x^4}}}{\sqrt{1+x^4}} dx$$

Optimal. Leaf size=33

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{\sqrt{x^4+1}-x^2}}\right)}{\sqrt{2}}$$

[Out] ArcTan[(Sqrt[2]*x)/Sqrt[-x^2 + Sqrt[1 + x^4]]]/Sqrt[2]

Rubi [A] time = 0.0914918, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{\sqrt{x^4+1}-x^2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[-x^2 + Sqrt[1 + x^4]]/Sqrt[1 + x^4], x]

[Out] ArcTan[(Sqrt[2]*x)/Sqrt[-x^2 + Sqrt[1 + x^4]]]/Sqrt[2]

Rubi in Sympy [A] time = 3.95058, size = 29, normalized size = 0.88

$$\frac{\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}x}{\sqrt{-x^2 + \sqrt{x^4+1}}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate((-x**2+(x**4+1)**(1/2))**(1/2)/(x**4+1)**(1/2), x)

[Out] sqrt(2)*atan(sqrt(2)*x/sqrt(-x**2 + sqrt(x**4 + 1)))/2

Mathematica [B] time = 1.43463, size = 162, normalized size = 4.91

$$\frac{x \left(2x^4 - 2\sqrt{x^4 + 1}x^2 + 1\right)^2 \left(x^4 - \sqrt{x^4 + 1}x^2 + 1\right) \sin^{-1}\left(x^2 - \sqrt{x^4 + 1}\right)}{\sqrt{2}\sqrt{\sqrt{x^4 + 1} - x^2}\sqrt{x^2\left(\sqrt{x^4 + 1} - x^2\right)}\left(-8x^{10} - 12x^6 + 8\sqrt{x^4 + 1}x^4 + \sqrt{x^4 + 1} - 4x^2 + 8\sqrt{x^4 + 1}x^8\right)}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[-x^2 + Sqrt[1 + x^4]]/Sqrt[1 + x^4],x]

[Out] (x*(1 + 2*x^4 - 2*x^2*Sqrt[1 + x^4])^2*(1 + x^4 - x^2*Sqrt[1 + x^4])*ArcSin[x^2 - Sqrt[1 + x^4]])/(Sqrt[2]*Sqrt[-x^2 + Sqrt[1 + x^4]]*Sqrt[x^2*(-x^2 + Sqrt[1 + x^4])]*(-4*x^2 - 12*x^6 - 8*x^10 + Sqrt[1 + x^4] + 8*x^4*Sqrt[1 + x^4] + 8*x^8*Sqrt[1 + x^4]))

Maple [C] time = 0.055, size = 22, normalized size = 0.7

$$-\frac{\sqrt{2}}{4x^2} {}_3F_2\left(\frac{1}{2}, \frac{3}{4}, \frac{5}{4}; \frac{3}{2}, \frac{3}{2}; -x^{-4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x)

[Out] -1/4*2^(1/2)/x^2*hypergeom([1/2, 3/4, 5/4], [3/2, 3/2], -1/x^4)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^2 + \sqrt{x^4 + 1}}}{\sqrt{x^4 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-x^2 + sqrt(x^4 + 1))/sqrt(x^4 + 1),x, algorithm="maxima")

[Out] integrate(sqrt(-x^2 + sqrt(x^4 + 1))/sqrt(x^4 + 1), x)

Fricas [A] time = 0.400327, size = 39, normalized size = 1.18

$$-\frac{1}{2} \sqrt{2} \arctan\left(\frac{\sqrt{2}\sqrt{-x^2 + \sqrt{x^4 + 1}}}{2x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-x^2 + sqrt(x^4 + 1))/sqrt(x^4 + 1),x, algorithm="fricas")

[Out] $-1/2 \cdot \sqrt{2} \cdot \arctan(1/2 \cdot \sqrt{2} \cdot \sqrt{-x^2 + \sqrt{x^4 + 1}}) / x$

Sympy [A] time = 1.16438, size = 15, normalized size = 0.45

$$\frac{G_{3,3}^{2,2} \left(\begin{matrix} \frac{1}{2}, 1 & 1 \\ \frac{1}{4}, \frac{3}{4} & 0 \end{matrix} \middle| x^4 \right)}{4\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x**2+(x**4+1)**(1/2))**(1/2)/(x**4+1)**(1/2),x)`

[Out] `meijerg(((1/2, 1), (1,)), ((1/4, 3/4), (0,)), x**4)/(4*sqrt(pi))`

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^2 + \sqrt{x^4 + 1}}}{\sqrt{x^4 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-x^2 + sqrt(x^4 + 1))/sqrt(x^4 + 1),x, algorithm="giac")`

[Out] `integrate(sqrt(-x^2 + sqrt(x^4 + 1))/sqrt(x^4 + 1), x)`

$$3.16 \quad \int \frac{(-1+x)^{3/2} + (1+x)^{3/2}}{(-1+x)^{3/2}(1+x)^{3/2}} dx$$

Optimal. Leaf size=19

$$-\frac{2}{\sqrt{x+1}} - \frac{2}{\sqrt{x-1}}$$

[Out] -2/Sqrt[-1 + x] - 2/Sqrt[1 + x]

Rubi [A] time = 0.434125, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$

$$-\frac{2}{\sqrt{x+1}} - \frac{2}{\sqrt{x-1}}$$

Antiderivative was successfully verified.

[In] Int[((-1 + x)^(3/2) + (1 + x)^(3/2))/((-1 + x)^(3/2)*(1 + x)^(3/2)), x]

[Out] -2/Sqrt[-1 + x] - 2/Sqrt[1 + x]

Rubi in Sympy [A] time = 11.2729, size = 17, normalized size = 0.89

$$-\frac{2}{\sqrt{x+1}} - \frac{2}{\sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate(((1-x)**(3/2)+(1+x)**(3/2))/((1-x)**(3/2)/(1+x)**(3/2)), x)

[Out] -2/sqrt(x + 1) - 2/sqrt(x - 1)

Mathematica [A] time = 0.0247948, size = 19, normalized size = 1.

$$-\frac{2}{\sqrt{x+1}} - \frac{2}{\sqrt{x-1}}$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + x)^(3/2) + (1 + x)^(3/2)/((-1 + x)^(3/2)*(1 + x)^(3/2)), x]

[Out] $-2/\text{sqrt}[-1 + x] - 2/\text{sqrt}[1 + x]$

Maple [A] time = 0.004, size = 16, normalized size = 0.8

$$-2 \frac{1}{\sqrt{-1+x}} - 2 \frac{1}{\sqrt{1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((−1+x)^(3/2)+(1+x)^(3/2))/(−1+x)^(3/2)/(1+x)^(3/2),x)`

[Out] $-2/(-1+x)^{(1/2)}-2/(1+x)^{(1/2)}$

Maxima [A] time = 1.34085, size = 20, normalized size = 1.05

$$-\frac{2}{\sqrt{x+1}} - \frac{2}{\sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(((x+1)^(3/2)+(x-1)^(3/2))/((x+1)^(3/2)*(x-1)^(3/2)),x,algorithm="maxima")`

[Out] $-2/\text{sqrt}(x + 1) - 2/\text{sqrt}(x - 1)$

Fricas [A] time = 0.203075, size = 31, normalized size = 1.63

$$-\frac{2\left(\sqrt{x+1} + \sqrt{x-1}\right)}{\sqrt{x+1}\sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(((x+1)^(3/2)+(x-1)^(3/2))/((x+1)^(3/2)*(x-1)^(3/2)),x,algorithm="fricas")`

[Out] $-2*(\text{sqrt}(x + 1) + \text{sqrt}(x - 1))/(\text{sqrt}(x + 1)*\text{sqrt}(x - 1))$

Sympy [A] time = 12.4909, size = 56, normalized size = 2.95

$$-\frac{2x\sqrt{x-1}}{x^2-1} - \frac{2x\sqrt{x+1}}{x^2-1} - \frac{2\sqrt{x-1}}{x^2-1} + \frac{2\sqrt{x+1}}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((( -1+x)**(3/2)+(1+x)**(3/2))/(-1+x)**(3/2)/(1+x)**(3/2),x)
```

```
[Out] -2*x*sqrt(x - 1)/(x**2 - 1) - 2*x*sqrt(x + 1)/(x**2 - 1) - 2*sqrt
(x - 1)/(x**2 - 1) + 2*sqrt(x + 1)/(x**2 - 1)
```

GIAC/XCAS [A] time = 0.213949, size = 20, normalized size = 1.05

$$-\frac{2}{\sqrt{x+1}} - \frac{2}{\sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x + 1)^(3/2) + (x - 1)^(3/2))/((x + 1)^(3/2)*(x - 1)^(3/2)),x, algorithm=
```

```
[Out] -2/sqrt(x + 1) - 2/sqrt(x - 1)
```

$$3.17 \quad \int \left(x + \sqrt{a + x^2} \right)^b dx$$

Optimal. Leaf size=52

$$\frac{\left(\sqrt{a+x^2}+x\right)^{b+1}}{2(b+1)} - \frac{a\left(\sqrt{a+x^2}+x\right)^{b-1}}{2(1-b)}$$

[Out] $-(a*(x + \text{Sqrt}[a + x^2])^{(-1 + b)})/(2*(1 - b)) + (x + \text{Sqrt}[a + x^2])^{(1 + b)}/(2*(1 + b))$

Rubi [A] time = 0.0466202, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{\left(\sqrt{a+x^2}+x\right)^{b+1}}{2(b+1)} - \frac{a\left(\sqrt{a+x^2}+x\right)^{b-1}}{2(1-b)}$$

Antiderivative was successfully verified.

[In] Int[(x + Sqrt[a + x^2])^b, x]

[Out] $-(a*(x + \text{Sqrt}[a + x^2])^{(-1 + b)})/(2*(1 - b)) + (x + \text{Sqrt}[a + x^2])^{(1 + b)}/(2*(1 + b))$

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x + \sqrt{a + x^2} \right)^b dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate((x+(x**2+a)**(1/2))**b, x)

[Out] Integral((x + sqrt(a + x**2))**b, x)

Mathematica [A] time = 0.040084, size = 36, normalized size = 0.69

$$\frac{\left(\sqrt{a+x^2}+x\right)^b \left(b\sqrt{a+x^2}-x\right)}{b^2-1}$$

Antiderivative was successfully verified.

[In] Integrate[(x + Sqrt[a + x^2])^b, x]

[Out] ((x + Sqrt[a + x^2])^b*(-x + b*Sqrt[a + x^2]))/(-1 + b^2)

Maple [B] time = 0.045, size = 120, normalized size = 2.3

$$\frac{b}{4\sqrt{\pi}} a^{\frac{b}{2} + \frac{1}{2}} \left(8 \frac{\sqrt{\pi} x^{1+b} a^{-b/2-1/2}}{(1+b)b(-2+2b)} \left(\frac{ab}{x^2} + b - 1 \right) \left(\sqrt{\frac{a}{x^2} + 1} + 1 \right)^{-1+b} + 4 \frac{\sqrt{\pi} x^{1+b} a^{-b/2-1/2}}{(1+b)b} \sqrt{\frac{a}{x^2} + 1} \left(\sqrt{\frac{a}{x^2} + 1} + 1 \right)^{-1+b} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x+(x^2+a)^(1/2))^b, x)

[Out] 1/4*a^(1/2*b+1/2)/Pi^(1/2)*b*(8*Pi^(1/2)/(1+b)/b*x^(1+b)*a^(-1/2*b-1/2)*(1/x^2*a*b+b-1)/(-2+2*b)*((1/x^2*a+1)^(1/2)+1)^(-1+b)+4*Pi^(1/2)/(1+b)/b*x^(1+b)*a^(-1/2*b-1/2)*(1/x^2*a+1)^(1/2)*((1/x^2*a+1)^(1/2)+1)^(-1+b))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (x + \sqrt{x^2 + a})^b dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x + sqrt(x^2 + a))^b, x, algorithm="maxima")

[Out] integrate((x + sqrt(x^2 + a))^b, x)

Fricas [A] time = 0.229006, size = 43, normalized size = 0.83

$$\frac{(\sqrt{x^2 + ab} - x)(x + \sqrt{x^2 + a})^b}{b^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x + sqrt(x^2 + a))^b, x, algorithm="fricas")

[Out] $(\sqrt{x^2 + a} \cdot b - x) \cdot (x + \sqrt{x^2 + a})^{b/(b^2 - 1)}$

Sympy [A] time = 4.97977, size = 2147, normalized size = 41.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x+(x**2+a)**(1/2))**b,x)`

[Out] $\text{Piecewise}\left(\left(-a^{(9/2)} \cdot a^{(b/2)} \cdot b^{*2} \cdot x \cdot \sqrt{a/x^{*2} + 1} \cdot \sinh(b \cdot \text{asinh}(x/\sqrt{a})) \cdot \frac{\Gamma(-b/2)}{(2 \cdot a^{(9/2)}) \cdot b^{*2} \cdot \Gamma(-b/2 + 1)} - 2 \cdot a^{(9/2)} \cdot \Gamma(-b/2 + 1) + 2 \cdot a^{(7/2)} \cdot b^{*2} \cdot x^{*2} \cdot \Gamma(-b/2 + 1) - 2 \cdot a^{(7/2)} \cdot x^{*2} \cdot \Gamma(-b/2 + 1)\right) + a^{(9/2)} \cdot a^{(b/2)} \cdot b \cdot x \cdot \cosh(b \cdot \text{asinh}(x/\sqrt{a})) \cdot \frac{\Gamma(-b/2)}{(2 \cdot a^{(9/2)}) \cdot b^{*2} \cdot \Gamma(-b/2 + 1)} - 2 \cdot a^{(9/2)} \cdot \Gamma(-b/2 + 1) + 2 \cdot a^{(7/2)} \cdot b^{*2} \cdot x^{*2} \cdot \Gamma(-b/2 + 1) - 2 \cdot a^{(7/2)} \cdot x^{*2} \cdot \Gamma(-b/2 + 1)\right) - a^{(7/2)} \cdot a^{(b/2)} \cdot b^{*2} \cdot x^{*3} \cdot \sqrt{a/x^{*2} + 1} \cdot \sinh(b \cdot \text{asinh}(x/\sqrt{a})) \cdot \frac{\Gamma(-b/2)}{(2 \cdot a^{(9/2)}) \cdot b^{*2} \cdot \Gamma(-b/2 + 1)} - 2 \cdot a^{(9/2)} \cdot \Gamma(-b/2 + 1) + 2 \cdot a^{(7/2)} \cdot b^{*2} \cdot x^{*2} \cdot \Gamma(-b/2 + 1) - 2 \cdot a^{(7/2)} \cdot x^{*2} \cdot \Gamma(-b/2 + 1)\right) + a^{(7/2)} \cdot a^{(b/2)} \cdot b \cdot x^{*3} \cdot \cosh(b \cdot \text{asinh}(x/\sqrt{a})) \cdot \frac{\Gamma(-b/2)}{(2 \cdot a^{(9/2)}) \cdot b^{*2} \cdot \Gamma(-b/2 + 1)} - 2 \cdot a^{(9/2)} \cdot \Gamma(-b/2 + 1) + 2 \cdot a^{(7/2)} \cdot b^{*2} \cdot x^{*2} \cdot \Gamma(-b/2 + 1) - 2 \cdot a^{(7/2)} \cdot x^{*2} \cdot \Gamma(-b/2 + 1)\right) + 2 \cdot a^{*5} \cdot a^{(b/2)} \cdot b \cdot \cosh(b \cdot \text{asinh}(x/\sqrt{a}) + \text{asinh}(x/\sqrt{a})) \cdot \frac{\Gamma(-b/2 + 1)}{(2 \cdot a^{(9/2)}) \cdot b^{*2} \cdot \Gamma(-b/2 + 1)} - 2 \cdot a^{(9/2)} \cdot \Gamma(-b/2 + 1) + 2 \cdot a^{(7/2)} \cdot b^{*2} \cdot x^{*2} \cdot \Gamma(-b/2 + 1) - 2 \cdot a^{(7/2)} \cdot x^{*2} \cdot \Gamma(-b/2 + 1)\right) - 2 \cdot a^{*5} \cdot a^{(b/2)} \cdot b \cdot \Gamma(-b/2 + 1) / (2 \cdot a^{(9/2)}) \cdot b^{*2} \cdot \Gamma(-b/2 + 1) - 2 \cdot a^{(9/2)} \cdot \Gamma(-b/2 + 1) + 2 \cdot a^{(7/2)} \cdot b^{*2} \cdot x^{*2} \cdot \Gamma(-b/2 + 1) - 2 \cdot a^{(7/2)} \cdot x^{*2} \cdot \Gamma(-b/2 + 1)\right) - 2 \cdot a^{*4} \cdot a^{(b/2)} \cdot b \cdot x^{*2} \cdot \sqrt{a/x^{*2} + 1} \cdot \sinh(b \cdot \text{asinh}(x/\sqrt{a})) + \text{asinh}(x/\sqrt{a})) \cdot \frac{\Gamma(-b/2 + 1)}{(2 \cdot a^{(9/2)}) \cdot b^{*2} \cdot \Gamma(-b/2 + 1)} - 2 \cdot a^{(9/2)} \cdot \Gamma(-b/2 + 1) + 2 \cdot a^{(7/2)} \cdot b^{*2} \cdot x^{*2} \cdot \Gamma(-b/2 + 1) - 2 \cdot a^{(7/2)} \cdot x^{*2} \cdot \Gamma(-b/2 + 1)\right) + 4 \cdot a^{*4} \cdot a^{(b/2)} \cdot b \cdot x^{*2} \cdot \cosh(b \cdot \text{asinh}(x/\sqrt{a}) + \text{asinh}(x/\sqrt{a})) \cdot \frac{\Gamma(-b/2 + 1)}{(2 \cdot a^{(9/2)}) \cdot b^{*2} \cdot \Gamma(-b/2 + 1)} - 2 \cdot a^{(9/2)} \cdot \Gamma(-b/2 + 1) + 2 \cdot a^{(7/2)} \cdot b^{*2} \cdot x^{*2} \cdot \Gamma(-b/2 + 1) - 2 \cdot a^{(7/2)} \cdot x^{*2} \cdot \Gamma(-b/2 + 1)\right) - 2 \cdot a^{*4} \cdot a^{(b/2)} \cdot b \cdot x^{*2} \cdot \Gamma(-b/2 + 1) / (2 \cdot a^{(9/2)}) \cdot b^{*2} \cdot \Gamma(-b/2 + 1) - 2 \cdot a^{(9/2)} \cdot \Gamma(-b/2 + 1) + 2 \cdot a^{(7/2)} \cdot b^{*2} \cdot x^{*2} \cdot \Gamma(-b/2 + 1) - 2 \cdot a^{(7/2)} \cdot x^{*2} \cdot \Gamma(-b/2 + 1)\right) - 2 \cdot a^{*4} \cdot a^{(b/2)} \cdot x^{*2} \cdot \sqrt{a/x^{*2} + 1} \cdot \sinh(b \cdot \text{asinh}(x/\sqrt{a}) + \text{asinh}(x/\sqrt{a})) \cdot \frac{\Gamma(-b/2 + 1)}{(2 \cdot a^{(9/2)}) \cdot b^{*2} \cdot \Gamma(-b/2 + 1)} - 2 \cdot a^{(9/2)} \cdot \Gamma(-b/2 + 1) + 2 \cdot a^{(7/2)} \cdot b^{*2} \cdot x^{*2} \cdot \Gamma(-b/2 + 1) - 2 \cdot a^{(7/2)} \cdot x^{*2} \cdot \Gamma(-b/2 + 1)\right) + 2 \cdot a^{*4} \cdot a^{(b/2)} \cdot x^{*2} \cdot \cosh(b \cdot \text{asinh}(x/\sqrt{a}) + \text{asinh}(x/\sqrt{a})) \cdot \frac{\Gamma(-b/2 + 1)}{(2 \cdot a^{(9/2)}) \cdot b^{*2} \cdot \Gamma(-b/2 + 1)} - 2 \cdot a^{(9/2)} \cdot \Gamma(-b/2 + 1) + 2 \cdot a^{(7/2)} \cdot b^{*2} \cdot x^{*2} \cdot \Gamma(-b/2 + 1) - 2 \cdot a^{(7/2)} \cdot x^{*2} \cdot \Gamma(-b/2 + 1)\right) + 2 \cdot a^{*3} \cdot a^{(b/2)} \cdot b \cdot x^{*4} \cdot \sqrt{a/x^{*2} + 1} \cdot \sinh(b \cdot \text{asinh}(x/\sqrt{a}) + \text{asinh}(x/\sqrt{a})) \cdot \frac{\Gamma(-b/2 + 1)}{(2 \cdot a^{(9/2)}) \cdot b^{*2} \cdot \Gamma(-b/2 + 1)} - 2 \cdot a^{(9/2)} \cdot \Gamma(-b/2 + 1) + 2 \cdot a^{(7/2)} \cdot b^{*2} \cdot x^{*2} \cdot \Gamma(-b/2 + 1) - 2 \cdot a^{(7/2)} \cdot x^{*2} \cdot \Gamma(-b/2 + 1)\right) + 2 \cdot a^{*3} \cdot a^{(b/2)} \cdot b \cdot x^{*4} \cdot \cosh(b \cdot \text{asinh}(x/\sqrt{a}) + \text{asinh}(x/\sqrt{a})) \cdot \frac{\Gamma(-b/2 + 1)}{(2 \cdot a^{(9/2)}) \cdot b^{*2} \cdot \Gamma(-b/2 + 1)} - 2 \cdot a^{(9/2)} \cdot \Gamma(-b/2 + 1) + 2 \cdot a^{(7/2)} \cdot b^{*2} \cdot x^{*2} \cdot \Gamma(-b/2 + 1) - 2 \cdot a^{(7/2)} \cdot x^{*2} \cdot \Gamma(-b/2 + 1)\right) + 2 \cdot a^{(9/2)} \cdot \Gamma(-b/2 + 1) + 2 \cdot a^{(7/2)} \cdot b^{*2} \cdot x^{*2} \cdot \Gamma(-b/2 + 1) - 2 \cdot a^{(7/2)} \cdot x^{*2} \cdot \Gamma(-b/2 + 1)$


```

*gamma(-b/2 + 1) - 2*a**(7/2)*x**2*gamma(-b/2 + 1)) - 2*a**3*a**
b/2)*x**4*sqrt(a/x**2 + 1)*sinh(b*asinh(x/sqrt(a)) + asinh(x/sqrt
(a))) *gamma(-b/2 + 1)/(2*a**(9/2)*b**2*gamma(-b/2 + 1) - 2*a**(9/
2)*gamma(-b/2 + 1) + 2*a**(7/2)*b**2*x**2*gamma(-b/2 + 1) - 2*a**
(7/2)*x**2*gamma(-b/2 + 1)) + 2*a**3*a**(b/2)*x**4*cosh(b*asinh(x
/sqrt(a)) + asinh(x/sqrt(a))) *gamma(-b/2 + 1)/(2*a**(9/2)*b**2*ga
mma(-b/2 + 1) - 2*a**(9/2)*gamma(-b/2 + 1) + 2*a**(7/2)*b**2*x**2
*gamma(-b/2 + 1) - 2*a**(7/2)*x**2*gamma(-b/2 + 1)), Abs(x**2/a)
> 1), (-2*a**(5/2)*a**(b/2)*b*x*sqrt(1 + x**2/a)*sinh(b*asinh(x/s
qrt(a)) + asinh(x/sqrt(a))) *gamma(-b/2 + 1)/(2*a**(5/2)*b**2*gamma
a(-b/2 + 1) - 2*a**(5/2)*gamma(-b/2 + 1)) + a**(5/2)*a**(b/2)*b*x
*cosh(b*asinh(x/sqrt(a))) *gamma(-b/2)/(2*a**(5/2)*b**2*gamma(-b/2
+ 1) - 2*a**(5/2)*gamma(-b/2 + 1)) - 2*a**(5/2)*a**(b/2)*x*sqrt(
1 + x**2/a)*sinh(b*asinh(x/sqrt(a)) + asinh(x/sqrt(a))) *gamma(-b/
2 + 1)/(2*a**(5/2)*b**2*gamma(-b/2 + 1) - 2*a**(5/2)*gamma(-b/2 +
1)) - a**3*a**(b/2)*b**2*sqrt(1 + x**2/a)*sinh(b*asinh(x/sqrt(a)
)) *gamma(-b/2)/(2*a**(5/2)*b**2*gamma(-b/2 + 1) - 2*a**(5/2)*gamma
a(-b/2 + 1)) + 2*a**3*a**(b/2)*b*cosh(b*asinh(x/sqrt(a)) + asinh(
x/sqrt(a))) *gamma(-b/2 + 1)/(2*a**(5/2)*b**2*gamma(-b/2 + 1) - 2*
a**(5/2)*gamma(-b/2 + 1)) + 2*a**2*a**(b/2)*b*x**2*cosh(b*asinh(x
/sqrt(a)) + asinh(x/sqrt(a))) *gamma(-b/2 + 1)/(2*a**(5/2)*b**2*ga
mma(-b/2 + 1) - 2*a**(5/2)*gamma(-b/2 + 1)) + 2*a**2*a**(b/2)*x**
2*cosh(b*asinh(x/sqrt(a)) + asinh(x/sqrt(a))) *gamma(-b/2 + 1)/(2*
a**(5/2)*b**2*gamma(-b/2 + 1) - 2*a**(5/2)*gamma(-b/2 + 1)), True
))

```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x + \sqrt{x^2 + a} \right)^b dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x + sqrt(x^2 + a))^b,x, algorithm="giac")

[Out] integrate((x + sqrt(x^2 + a))^b, x)

$$3.18 \quad \int \left(x - \sqrt{a + x^2} \right)^b dx$$

Optimal. Leaf size=56

$$\frac{\left(x - \sqrt{a + x^2} \right)^{b+1}}{2(b+1)} - \frac{a \left(x - \sqrt{a + x^2} \right)^{b-1}}{2(1-b)}$$

[Out] $-(a*(x - \text{Sqrt}[a + x^2])^{(-1 + b)})/(2*(1 - b)) + (x - \text{Sqrt}[a + x^2])^{(1 + b)}/(2*(1 + b))$

Rubi [A] time = 0.0458865, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{\left(x - \sqrt{a + x^2} \right)^{b+1}}{2(b+1)} - \frac{a \left(x - \sqrt{a + x^2} \right)^{b-1}}{2(1-b)}$$

Antiderivative was successfully verified.

[In] Int[(x - Sqrt[a + x^2])^b, x]

[Out] $-(a*(x - \text{Sqrt}[a + x^2])^{(-1 + b)})/(2*(1 - b)) + (x - \text{Sqrt}[a + x^2])^{(1 + b)}/(2*(1 + b))$

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x - \sqrt{a + x^2} \right)^b dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate((x-(x**2+a)**(1/2))**b, x)

[Out] Integral((x - sqrt(a + x**2))**b, x)

Mathematica [A] time = 0.03978, size = 39, normalized size = 0.7

$$\frac{\left(x - \sqrt{a + x^2} \right)^b \left(b \left(-\sqrt{a + x^2} \right) - x \right)}{b^2 - 1}$$

Antiderivative was successfully verified.

[In] Integrate[(x - Sqrt[a + x^2])^b, x]

[Out] ((x - Sqrt[a + x^2])^b*(-x - b*Sqrt[a + x^2]))/(-1 + b^2)

Maple [F] time = 0.045, size = 0, normalized size = 0.

$$\int (x - \sqrt{x^2 + a})^b dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x-(x^2+a)^(1/2))^b, x)

[Out] int((x-(x^2+a)^(1/2))^b, x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (x - \sqrt{x^2 + a})^b dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x - sqrt(x^2 + a))^b, x, algorithm="maxima")

[Out] integrate((x - sqrt(x^2 + a))^b, x)

Fricas [A] time = 0.229939, size = 45, normalized size = 0.8

$$\frac{(\sqrt{x^2 + a} + x)(x - \sqrt{x^2 + a})^b}{b^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x - sqrt(x^2 + a))^b, x, algorithm="fricas")

[Out] -(sqrt(x^2 + a)*b + x)*(x - sqrt(x^2 + a))^b/(b^2 - 1)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (x - \sqrt{a + x^2})^b dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x-(x**2+a)**(1/2))**b,x)

[Out] Integral((x - sqrt(a + x**2))**b, x)

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int (x - \sqrt{x^2 + a})^b dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x - sqrt(x^2 + a))^b,x, algorithm="giac")

[Out] integrate((x - sqrt(x^2 + a))^b, x)

$$3.19 \quad \int \frac{(x + \sqrt{a+x^2})^b}{\sqrt{a+x^2}} dx$$

Optimal. Leaf size=17

$$\frac{(\sqrt{a+x^2} + x)^b}{b}$$

[Out] (x + Sqrt[a + x^2])^b/b

Rubi [A] time = 0.0872216, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$

$$\frac{(\sqrt{a+x^2} + x)^b}{b}$$

Antiderivative was successfully verified.

[In] Int[(x + Sqrt[a + x^2])^b/Sqrt[a + x^2], x]

[Out] (x + Sqrt[a + x^2])^b/b

Rubi in Sympy [A] time = 4.05421, size = 12, normalized size = 0.71

$$\frac{(x + \sqrt{a+x^2})^b}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate((x+(x**2+a)**(1/2))**b/(x**2+a)**(1/2), x)

[Out] (x + sqrt(a + x**2))**b/b

Mathematica [A] time = 0.027324, size = 17, normalized size = 1.

$$\frac{(\sqrt{a+x^2} + x)^b}{b}$$

Antiderivative was successfully verified.

[In] Integrate[(x + Sqrt[a + x^2])^b/Sqrt[a + x^2],x]

[Out] (x + Sqrt[a + x^2])^b/b

Maple [F] time = 0.029, size = 0, normalized size = 0.

$$\int 1 \left(x + \sqrt{x^2 + a} \right)^b \frac{1}{\sqrt{x^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x+(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x)

[Out] int((x+(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x)

Maxima [A] time = 1.39616, size = 20, normalized size = 1.18

$$\frac{\left(x + \sqrt{x^2 + a} \right)^b}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x + sqrt(x^2 + a))^b/sqrt(x^2 + a),x, algorithm="maxima")

[Out] (x + sqrt(x^2 + a))^b/b

Fricas [A] time = 0.230442, size = 20, normalized size = 1.18

$$\frac{\left(x + \sqrt{x^2 + a} \right)^b}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x + sqrt(x^2 + a))^b/sqrt(x^2 + a),x, algorithm="fricas")

[Out] (x + sqrt(x^2 + a))^b/b

Sympy [A] time = 5.35941, size = 311, normalized size = 18.29

$$\left\{ \begin{array}{l} -\frac{\sqrt{a} a^{\frac{b}{2}} \sinh\left(-b \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right) + \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{b x \sqrt{\frac{a}{x^2} + 1}} - \frac{2 a^{\frac{b}{2}} \cosh\left(b \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right) \left(-\frac{b}{2} + 1\right)}{b^2 \left(-\frac{b}{2}\right)} + \frac{a^{\frac{b}{2}} x \cosh\left(-b \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right) + \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{\sqrt{a} b} - \frac{a^{\frac{b}{2}} x \sinh\left(-b \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right) + \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{\sqrt{a} b \sqrt{\frac{a}{x^2} + 1}} \\ -\frac{a^{\frac{b}{2}} \sinh\left(-b \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right) + \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{b \sqrt{1 + \frac{x^2}{a}}} - \frac{2 a^{\frac{b}{2}} \cosh\left(b \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right) \left(-\frac{b}{2} + 1\right)}{b^2 \left(-\frac{b}{2}\right)} - \frac{a^{\frac{b}{2}} x^2 \sinh\left(-b \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right) + \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{a b \sqrt{1 + \frac{x^2}{a}}} + \frac{a^{\frac{b}{2}} x \cosh\left(-b \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right) + \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{\sqrt{a} b} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+(x**2+a)**(1/2))**b/(x**2+a)**(1/2), x)

[Out] Piecewise((-sqrt(a)*a**(b/2)*sinh(-b*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))/(b*x*sqrt(a/x**2 + 1)) - 2*a**(b/2)*cosh(b*asinh(x/sqrt(a)))*gamma(-b/2 + 1)/(b**2*gamma(-b/2)) + a**(b/2)*x*cosh(-b*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))/(sqrt(a)*b) - a**(b/2)*x*sinh(-b*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))/(sqrt(a)*b*sqrt(a/x**2 + 1)), Abs(x**2/a) > 1), (-a**(b/2)*sinh(-b*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))/(b*sqrt(1 + x**2/a)) - 2*a**(b/2)*cosh(b*asinh(x/sqrt(a)))*gamma(-b/2 + 1)/(b**2*gamma(-b/2)) - a**(b/2)*x**2*sinh(-b*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))/(a*b*sqrt(1 + x**2/a)) + a**(b/2)*x*cosh(-b*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))/(sqrt(a)*b), True))

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x + \sqrt{x^2 + a})^b}{\sqrt{x^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x + sqrt(x^2 + a))^b/sqrt(x^2 + a), x, algorithm="giac")

[Out] integrate((x + sqrt(x^2 + a))^b/sqrt(x^2 + a), x)

$$3.20 \quad \int \frac{(x - \sqrt{a+x^2})^b}{\sqrt{a+x^2}} dx$$

Optimal. Leaf size=20

$$-\frac{(x - \sqrt{a+x^2})^b}{b}$$

[Out] -((x - Sqrt[a + x^2])^b/b)

Rubi [A] time = 0.0922341, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$

$$-\frac{(x - \sqrt{a+x^2})^b}{b}$$

Antiderivative was successfully verified.

[In] Int[(x - Sqrt[a + x^2])^b/Sqrt[a + x^2], x]

[Out] -((x - Sqrt[a + x^2])^b/b)

Rubi in Sympy [A] time = 4.36387, size = 14, normalized size = 0.7

$$-\frac{(x - \sqrt{a+x^2})^b}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate((x-(x**2+a)**(1/2))**b/(x**2+a)**(1/2), x)

[Out] -(x - sqrt(a + x**2))**b/b

Mathematica [A] time = 0.0279361, size = 20, normalized size = 1.

$$-\frac{(x - \sqrt{a+x^2})^b}{b}$$

Antiderivative was successfully verified.

[In] Integrate[(x - Sqrt[a + x^2])^b/Sqrt[a + x^2],x]

[Out] -((x - Sqrt[a + x^2])^b/b)

Maple [F] time = 0.045, size = 0, normalized size = 0.

$$\int 1 \left(x - \sqrt{x^2 + a} \right)^b \frac{1}{\sqrt{x^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x-(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x)

[Out] int((x-(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x)

Maxima [A] time = 1.39686, size = 24, normalized size = 1.2

$$-\frac{\left(x - \sqrt{x^2 + a} \right)^b}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x - sqrt(x^2 + a))^b/sqrt(x^2 + a),x, algorithm="maxima")

[Out] -(x - sqrt(x^2 + a))^b/b

Fricas [A] time = 0.232349, size = 24, normalized size = 1.2

$$-\frac{\left(x - \sqrt{x^2 + a} \right)^b}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x - sqrt(x^2 + a))^b/sqrt(x^2 + a),x, algorithm="fricas")

[Out] -(x - sqrt(x^2 + a))^b/b

Sympy [A] time = 3.55928, size = 36, normalized size = 1.8

$$\begin{cases} \frac{(x-\sqrt{a+x^2})^b}{b} & \text{for } b \neq 0 \\ \operatorname{asinh}\left(x\sqrt{\frac{1}{a}}\right) & \text{for } a > 0 \\ \operatorname{acosh}\left(x\sqrt{-\frac{1}{a}}\right) & \text{for } a < 0 \end{cases} \quad \text{otherwise}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x-(x**2+a)**(1/2))**b/(x**2+a)**(1/2), x)

[Out] Piecewise((- (x - sqrt(a + x**2))**b/b, Ne(b, 0)), (Piecewise((asinh(x*sqrt(1/a)), a > 0), (acosh(x*sqrt(-1/a)), a < 0)), True))

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x - \sqrt{x^2 + a})^b}{\sqrt{x^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x - sqrt(x^2 + a))^b/sqrt(x^2 + a), x, algorithm="giac")

[Out] integrate((x - sqrt(x^2 + a))^b/sqrt(x^2 + a), x)

$$3.21 \quad \int \frac{1}{(a+be^{px})^2} dx$$

Optimal. Leaf size=42

$$-\frac{\log(a+be^{px})}{a^2p} + \frac{x}{a^2} + \frac{1}{ap(a+be^{px})}$$

[Out] $1/(a*(a + b*E^{(p*x)})^p) + x/a^2 - \text{Log}[a + b*E^{(p*x)}]/(a^2*p)$

Rubi [A] time = 0.0631128, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$-\frac{\log(a+be^{px})}{a^2p} + \frac{x}{a^2} + \frac{1}{ap(a+be^{px})}$$

Antiderivative was successfully verified.

[In] Int[(a + b*E^(p*x))^(-2), x]

[Out] $1/(a*(a + b*E^{(p*x)})^p) + x/a^2 - \text{Log}[a + b*E^{(p*x)}]/(a^2*p)$

Rubi in Sympy [A] time = 5.39135, size = 39, normalized size = 0.93

$$\frac{1}{ap(a+be^{px})} - \frac{\log(a+be^{px})}{a^2p} + \frac{\log(e^{px})}{a^2p}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate(1/(a+b*exp(p*x))**2, x)

[Out] $1/(a*p*(a + b*\exp(p*x))) - \log(a + b*\exp(p*x))/(a**2*p) + \log(\exp(p*x))/(a**2*p)$

Mathematica [A] time = 0.0630322, size = 37, normalized size = 0.88

$$\frac{\frac{a}{ap+bp e^{px}} - \frac{\log(a+be^{px})}{p} + x}{a^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*E^(p*x))^(-2), x]

[Out] $(a/(a^*p + b^*E^(p^*x)^*p) + x - \text{Log}[a + b^*E^(p^*x)]/p)/a^2$

Maple [A] time = 0.017, size = 48, normalized size = 1.1

$$-\frac{\ln(a + be^{px})}{a^2p} + \frac{1}{a(a + be^{px})p} + \frac{\ln(e^{px})}{a^2p}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a+b*exp(p*x))^2,x)`

[Out] $-\ln(a+b^*\exp(p^*x))/a^2/p+1/a/(a+b^*\exp(p^*x))/p+1/p/a^2*\ln(\exp(p^*x))$

Maxima [A] time = 1.35229, size = 54, normalized size = 1.29

$$\frac{x}{a^2} + \frac{1}{(abe^{px} + a^2)p} - \frac{\log\left(be^{px} + a\right)}{a^2p}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*e^(p*x) + a)^(-2),x, algorithm="maxima")`

[Out] $x/a^2 + 1/((a*b*e^(p*x) + a^2)*p) - \log(b*e^(p*x) + a)/(a^2*p)$

Fricas [A] time = 0.224159, size = 70, normalized size = 1.67

$$\frac{bpXe^{(px)} + apx - (be^{(px)} + a) \log\left(be^{(px)} + a\right) + a}{a^2bpe^{(px)} + a^3p}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*e^(p*x) + a)^(-2),x, algorithm="fricas")`

[Out] $(b^*p^*x^*e^(p^*x) + a^*p^*x - (b^*e^(p^*x) + a)^*\log(b^*e^(p^*x) + a) + a)/(a^2*b^*p^*e^(p^*x) + a^3*p)$

Sympy [A] time = 0.17583, size = 36, normalized size = 0.86

$$\frac{1}{a^2p + abpe^{px}} + \frac{x}{a^2} - \frac{\log\left(\frac{a}{b} + e^{px}\right)}{a^2p}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*exp(p*x))**2,x)`

[Out] $1/(a^{**2}p + a*b*p*exp(p*x)) + x/a^{**2} - \log(a/b + exp(p*x))/(a^{**2}p)$

GIAC/XCAS [A] time = 0.197701, size = 55, normalized size = 1.31

$$\frac{x}{a^2} - \frac{\ln(|be^{px} + a|)}{a^2p} + \frac{1}{(be^{px} + a)ap}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*e^(p*x) + a)^(-2),x, algorithm="giac")`

[Out] $x/a^2 - \ln(\text{abs}(b*e^{p*x} + a))/(a^2*p) + 1/((b*e^{p*x} + a)*a*p)$

$$3.22 \quad \int \frac{1}{(be^{-px} + ae^{px})^2} dx$$

Optimal. Leaf size=22

$$-\frac{1}{2ap(ae^{2px} + b)}$$

[Out] $-1/(2*a*(b + a*E^{(2*p*x)})^p)$

Rubi [A] time = 0.0413316, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$-\frac{1}{2ap(ae^{2px} + b)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(b/E^{(p*x)} + a*E^{(p*x)})^{(-2)}, x]$

[Out] $-1/(2*a*(b + a*E^{(2*p*x)})^p)$

Rubi in Sympy [A] time = 4.67805, size = 15, normalized size = 0.68

$$\frac{1}{2bp(a + be^{-2px})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{rubi_integrate}(1/(b/\exp(p*x)+a*\exp(p*x))^{*2}, x)$

[Out] $1/(2*b*p*(a + b*\exp(-2*p*x)))$

Mathematica [A] time = 0.0211906, size = 22, normalized size = 1.

$$-\frac{1}{2ap(ae^{2px} + b)}$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[(b/E^{(p*x)} + a*E^{(p*x)})^{(-2)}, x]$

[Out] $-1/(2*a*(b + a*E^(2*p*x))^*p)$

Maple [A] time = 0.004, size = 21, normalized size = 1.

$$-\frac{1}{2pa(a(e^{px})^2 + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b/exp(p*x)+a*exp(p*x))^2, x)`

[Out] $-1/2/p/a/(a*\exp(p*x)^2+b)$

Maxima [A] time = 1.34244, size = 27, normalized size = 1.23

$$\frac{1}{2(b^2e^{(-2px)} + ab)p}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*e^(p*x) + b*e^(-p*x))^(-2), x, algorithm="maxima")`

[Out] $1/2/((b^2*e^{(-2*p*x)} + a*b)*p)$

Fricas [A] time = 0.215691, size = 26, normalized size = 1.18

$$-\frac{1}{2(a^2pe^{(2px)} + abp)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*e^(p*x) + b*e^(-p*x))^(-2), x, algorithm="fricas")`

[Out] $-1/2/(a^2*p*e^{(2*p*x)} + a*b*p)$

Sympy [A] time = 0.116086, size = 22, normalized size = 1.

$$-\frac{1}{2a^2pe^{2px} + 2abp}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b/exp(p*x)+a*exp(p*x))**2,x)`

[Out] $-1/(2*a**2*p*exp(2*p*x) + 2*a*b*p)$

GIAC/XCAS [A] time = 0.200777, size = 26, normalized size = 1.18

$$\frac{1}{2(ae^{2px} + b)ap}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*e^(p*x) + b*e^(-p*x))^-2,x, algorithm="giac")`

[Out] $-1/2/((a*e^{2*p*x} + b)*a*p)$

$$3.23 \quad \int \frac{x}{(be^{-px} + ae^{px})^2} dx$$

Optimal. Leaf size=62

$$-\frac{\log(ae^{2px} + b)}{4abp^2} + \frac{x}{2abp} - \frac{x}{2ap(ae^{2px} + b)}$$

[Out] $x/(2*a*b*p) - x/(2*a*(b + a*E^(2*p*x))*p) - \text{Log}[b + a*E^(2*p*x)]/(4*a*b*p^2)$

Rubi [A] time = 0.154055, antiderivative size = 62, normalized size of antiderivative = 1., number of rules used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$-\frac{\log(ae^{2px} + b)}{4abp^2} + \frac{x}{2abp} - \frac{x}{2ap(ae^{2px} + b)}$$

Antiderivative was successfully verified.

[In] Int[x/(b/E^(p*x) + a*E^(p*x))^2, x]

[Out] $x/(2*a*b*p) - x/(2*a*(b + a*E^(2*p*x))*p) - \text{Log}[b + a*E^(2*p*x)]/(4*a*b*p^2)$

Rubi in Sympy [A] time = 10.1021, size = 54, normalized size = 0.87

$$\frac{x}{2bp(a + be^{-2px})} - \frac{\log(a + be^{-2px})}{4abp^2} + \frac{\log(e^{-2px})}{4abp^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate(x/(b/exp(p*x)+a*exp(p*x))**2, x)

[Out] $x/(2*b*p*(a + b*\exp(-2*p*x))) - \log(a + b*\exp(-2*p*x))/(4*a*b*p**2) + \log(\exp(-2*p*x))/(4*a*b*p**2)$

Mathematica [A] time = 0.0517093, size = 49, normalized size = 0.79

$$\frac{2pxe^{2px}}{ae^{2px}+b} - \frac{\log(ae^{2px}+b)}{a}$$

$$4bp^2$$

Antiderivative was successfully verified.

[In] Integrate[x/(b/E^(p*x) + a*E^(p*x))^2, x]

[Out] ((2*E^(2*p*x)*p*x)/(b + a*E^(2*p*x)) - Log[b + a*E^(2*p*x)]/a)/(4*b*p^2)

Maple [A] time = 0.019, size = 51, normalized size = 0.8

$$-\frac{\ln\left(a(e^{px})^2 + b\right)}{4p^2ba} + \frac{x(e^{px})^2}{2bp\left(a(e^{px})^2 + b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(b/exp(p*x)+a*exp(p*x))^2, x)

[Out] -1/4/p^2/b/a*ln(a*exp(p*x)^2+b)+1/2/p*x*exp(p*x)^2/b/(a*exp(p*x)^2+b)

Maxima [A] time = 1.36774, size = 69, normalized size = 1.11

$$\frac{xe^{(2px)}}{2(abpe^{(2px)} + b^2p)} - \frac{\log\left(\frac{ae^{(2px)}+b}{a}\right)}{4abp^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a*e^(p*x) + b*e^(-p*x))^2, x, algorithm="maxima")

[Out] 1/2*x*e^(2*p*x)/(a*b*p*e^(2*p*x) + b^2*p) - 1/4*log((a*e^(2*p*x) + b)/a)/(a*b*p^2)

Fricas [A] time = 0.230856, size = 78, normalized size = 1.26

$$\frac{2apxe^{(2px)} - \left(ae^{(2px)} + b\right) \log\left(ae^{(2px)} + b\right)}{4\left(a^2bp^2e^{(2px)} + ab^2p^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a*e^(p*x) + b*e^(-p*x))^2, x, algorithm="fricas")

[Out] $\frac{1}{4} \cdot (2 \cdot a \cdot p \cdot x \cdot e^{(2 \cdot p \cdot x)} - (a \cdot e^{(2 \cdot p \cdot x)} + b) \cdot \log(a \cdot e^{(2 \cdot p \cdot x)} + b)) / (a^2 \cdot b \cdot p^2 \cdot e^{(2 \cdot p \cdot x)} + a \cdot b^2 \cdot p^2)$

Sympy [A] time = 0.215637, size = 49, normalized size = 0.79

$$-\frac{x}{2a^2pe^{2px} + 2abp} + \frac{x}{2abp} - \frac{\log\left(e^{2px} + \frac{b}{a}\right)}{4abp^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(b/exp(p*x)+a*exp(p*x))**2,x)`

[Out] $-\frac{x}{(2 \cdot a \cdot p \cdot \exp(2 \cdot p \cdot x) + 2 \cdot a \cdot b \cdot p)} + \frac{x}{(2 \cdot a \cdot b \cdot p)} - \frac{\log(\exp(2 \cdot p \cdot x) + b/a)}{(4 \cdot a \cdot b \cdot p^2)}$

GIAC/XCAS [A] time = 0.198747, size = 100, normalized size = 1.61

$$\frac{2apxe^{(2px)} - ae^{(2px)}\ln(-ae^{(2px)} - b) - b\ln(-ae^{(2px)} - b)}{4(a^2bp^2e^{(2px)} + ab^2p^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(a*e^(p*x) + b*e^(-p*x))^2,x, algorithm="giac")`

[Out] $\frac{1}{4} \cdot (2 \cdot a \cdot p \cdot x \cdot e^{(2 \cdot p \cdot x)} - a \cdot e^{(2 \cdot p \cdot x)} \cdot \ln(-a \cdot e^{(2 \cdot p \cdot x)} - b) - b \cdot \ln(-a \cdot e^{(2 \cdot p \cdot x)} - b)) / (a^2 \cdot b \cdot p^2 \cdot e^{(2 \cdot p \cdot x)} + a \cdot b^2 \cdot p^2)$

$$3.24 \quad \int \frac{1-x+3x^2}{\sqrt{1-x+x^2}(1+x+x^2)^2} dx$$

Optimal. Leaf size=86

$$\frac{\sqrt{x^2-x+1}(x+1)}{x^2+x+1} + \sqrt{2} \tan^{-1} \left(\frac{\sqrt{2}(x+1)}{\sqrt{x^2-x+1}} \right) - \frac{\tanh^{-1} \left(\frac{\sqrt{\frac{2}{3}}(1-x)}{\sqrt{x^2-x+1}} \right)}{\sqrt{6}}$$

[Out] ((1 + x)*Sqrt[1 - x + x^2])/(1 + x + x^2) + Sqrt[2]*ArcTan[(Sqrt[2]*(1 + x))/Sqrt[1 - x + x^2]] - ArcTanh[(Sqrt[2/3]*(1 - x))/Sqrt[1 - x + x^2]]/Sqrt[6]

Rubi [A] time = 0.225655, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$

$$\frac{\sqrt{x^2-x+1}(x+1)}{x^2+x+1} + \sqrt{2} \tan^{-1} \left(\frac{\sqrt{2}(x+1)}{\sqrt{x^2-x+1}} \right) - \frac{\tanh^{-1} \left(\frac{\sqrt{\frac{2}{3}}(1-x)}{\sqrt{x^2-x+1}} \right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

[In] Int[(1 - x + 3*x^2)/(Sqrt[1 - x + x^2]*(1 + x + x^2)^2), x]

[Out] ((1 + x)*Sqrt[1 - x + x^2])/(1 + x + x^2) + Sqrt[2]*ArcTan[(Sqrt[2]*(1 + x))/Sqrt[1 - x + x^2]] - ArcTanh[(Sqrt[2/3]*(1 - x))/Sqrt[1 - x + x^2]]/Sqrt[6]

Rubi in Sympy [A] time = 18.81, size = 87, normalized size = 1.01

$$\frac{(12x+12)\sqrt{x^2-x+1}}{12(x^2+x+1)} + \sqrt{2} \operatorname{atan} \left(-\frac{\sqrt{2}(-144x-144)}{144\sqrt{x^2-x+1}} \right) + \frac{\sqrt{6} \operatorname{atanh} \left(\frac{\sqrt{6}(24x-24)}{72\sqrt{x^2-x+1}} \right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate((3*x**2-x+1)/(x**2+x+1)**2/(x**2-x+1)**(1/2), x)

[Out] (12*x + 12)*sqrt(x**2 - x + 1)/(12*(x**2 + x + 1)) + sqrt(2)*atan(-sqrt(2)*(-144*x - 144)/(144*sqrt(x**2 - x + 1))) + sqrt(6)*atan(h(sqrt(6)*(24*x - 24)/(72*sqrt(x**2 - x + 1))))/6

Mathematica [C] time = 5.47027, size = 961, normalized size = 11.17

$$\frac{\sqrt{x^2 - x + 1}(x + 1)}{x^2 + x + 1} + \frac{(7 - i\sqrt{3}) \tan^{-1} \left(\frac{3((-21 - 4i\sqrt{3})x^4 + 14(7 - 2i\sqrt{3})x^3 + (-103 - 36i\sqrt{3})x^2 + (94 + 32i\sqrt{3})x - 64i\sqrt{3} - 17)}{(84i - 113\sqrt{3})x^4 + 2(52\sqrt{3} - 3i\sqrt{3}\sqrt{x^2 - x + 1} + 21\sqrt{3} + 138i)x^3 + (52\sqrt{3} - 3i\sqrt{3}\sqrt{x^2 - x + 1} - 59\sqrt{3} - 180i)x^2 + 2(26\sqrt{3} - 3i\sqrt{3}\sqrt{x^2 - x + 1} - 69\sqrt{3} + 132i)} \right)}{4\sqrt{3 - 3i\sqrt{3}}} + \frac{i(-7i + \sqrt{3}) \tan^{-1} \left(\frac{3((-21 + 4i\sqrt{3})x^4 + 14(7 + 2i\sqrt{3})x^3 + (-103 + 36i\sqrt{3})x^2 + (94 - 32i\sqrt{3})x + 64i\sqrt{3} - 17)}{(84i + 113\sqrt{3})x^4 - 2(52\sqrt{3} + 3i\sqrt{3}\sqrt{x^2 - x + 1} + 21\sqrt{3} - 138i)x^3 + (-52\sqrt{3} + 3i\sqrt{3}\sqrt{x^2 - x + 1} + 59\sqrt{3} - 180i)x^2 + (-52\sqrt{3} + 3i\sqrt{3}\sqrt{x^2 - x + 1} + 138\sqrt{3})} \right)}{4\sqrt{3 + 3i\sqrt{3}}} - \frac{(7i + \sqrt{3}) \log(16(x^2 + x + 1)^2)}{8\sqrt{3 - 3i\sqrt{3}}} - \frac{(-7i + \sqrt{3}) \log(16(x^2 + x + 1)^2)}{8\sqrt{3 + 3i\sqrt{3}}} + \frac{(7i + \sqrt{3}) \log((x^2 + x + 1) \left((11i + 4\sqrt{3})x^2 - (8i\sqrt{1 - i\sqrt{3}}\sqrt{x^2 - x + 1} + 4\sqrt{3} + 17i)x + 10i\sqrt{1 - i\sqrt{3}}\sqrt{x^2 - x + 1} + 4\sqrt{3} \right))}{8\sqrt{3 - 3i\sqrt{3}}} + \frac{(-7i + \sqrt{3}) \log((x^2 + x + 1) \left((-11i + 4\sqrt{3})x^2 + (8i\sqrt{1 + i\sqrt{3}}\sqrt{x^2 - x + 1} - 4\sqrt{3} + 17i)x - 10i\sqrt{1 + i\sqrt{3}}\sqrt{x^2 - x + 1} + 4\sqrt{3} \right))}{8\sqrt{3 + 3i\sqrt{3}}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x + 3*x^2)/(Sqrt[1 - x + x^2]*(1 + x + x^2)^2), x]

[Out] ((1 + x)*Sqrt[1 - x + x^2])/(1 + x + x^2) + ((7 - I*Sqrt[3])*ArcTan[(3*(-17 - (64*I)*Sqrt[3] + (94 + (32*I)*Sqrt[3])*x + (-103 - (36*I)*Sqrt[3])*x^2 + 14*(7 - (2*I)*Sqrt[3])*x^3 + (-21 - (4*I)*Sqrt[3])*x^4)]/(96*I + 67*Sqrt[3] + (84*I - 113*Sqrt[3])*x^4 - 52*Sqrt[3 - (3*I)*Sqrt[3]]*Sqrt[1 - x + x^2] + 2*x*(132*I - 69*Sqrt[3] + 26*Sqrt[3 - (3*I)*Sqrt[3]]*Sqrt[1 - x + x^2]) + x^2*(-180*I - 59*Sqrt[3] + 52*Sqrt[3 - (3*I)*Sqrt[3]]*Sqrt[1 - x + x^2]) + 2*x^3*(138*I + 21*Sqrt[3] + 52*Sqrt[3 - (3*I)*Sqrt[3]]*Sqrt[1 - x + x^2])))/(4*Sqrt[3 - (3*I)*Sqrt[3]]) - ((I/4)*(-7*I + Sqrt[3])*ArcTan[(3*(-17 + (64*I)*Sqrt[3] + (94 - (32*I)*Sqrt[3])*x + (-103 + (36*I)*Sqrt[3])*x^2 + 14*(7 + (2*I)*Sqrt[3])*x^3 + (-21 + (4*I)*Sqrt[3])*x^4)]/(96*I - 67*Sqrt[3] + (84*I + 113*Sqrt[3])*x^4 + 52*Sqrt[3 + (3*I)*Sqrt[3]]*Sqrt[1 - x + x^2] + x^2*(-180*I + 59*Sqrt[3] - 52*Sqrt[3 + (3*I)*Sqrt[3]]*Sqrt[1 - x + x^2]) + x*(264*I + 138*Sqrt[3] - 52*Sqrt[3 + (3*I)*Sqrt[3]]*Sqrt[1 - x + x^2]) - 2*x^3*(-138*I + 21*Sqrt[3] + 52*Sqrt[3 + (3*I)*Sqrt[3]]*Sqrt[1 - x + x^2])))/Sqrt[3 + (3*I)*Sqrt[3]]) - ((-7*I + Sqrt[3])*Log[16*(1 + x + x^2)^2])/(8*Sqrt[3 + (3*I)*Sqrt[3]]) - ((7*I + Sqrt[3])*Log[16*(1 + x + x^2)^2])/(8*Sqrt[3 - (3*I)*Sqrt[3]]) + ((7*I + Sqrt[3])*Log[(1 + x + x^2)*(11*I + 4*Sqrt[3] + (11*I + 4*Sqrt[3])*x^2 + (10*I)*Sqrt[1 - I*Sqrt[3]]*Sqrt[1 - x + x^2] - x*(17*I + 4*Sqrt[3] + (8*I)*Sqrt[1 - I*Sqrt[3]]*Sqrt[1 - x + x^2])))/(8*Sqrt[3 - (3*I)*Sqrt[3]]) + ((-7*I + Sqrt[3])*Log[(1 + x + x^2)*(-11*I + 4*Sqrt[3] + (-11*I + 4*Sqrt[3])*x^2 - (10*I)*Sqrt[1 + I*Sqrt[3]]*S

$\text{qrt}[1 - x + x^2] + x(17\sqrt{3} - 4\sqrt{3} + (8\sqrt{3})\sqrt{1 + \sqrt{3}}) \sqrt{1 - x + x^2} / (8\sqrt{3} + (3\sqrt{3})\sqrt{3})$

Maple [B] time = 0.057, size = 455, normalized size = 5.3

$$-\frac{1}{6} \left(9 \frac{\sqrt{2}(1+x)^2}{(1-x)^2} \arctan \left(2 \frac{(1+x)\sqrt{2}}{1-x} \frac{1}{\sqrt{\frac{(1+x)^2}{(1-x)^2} + 3}} \right) \sqrt{\frac{(1+x)^2}{(1-x)^2} + 3} - 6 \frac{\sqrt{6}(1+x)^2}{(1-x)^2} \operatorname{Artanh} \left(\frac{1}{4} \sqrt{\frac{(1+x)^2}{(1-x)^2} + 3\sqrt{6}} \right) \sqrt{\frac{(1+x)^2}{(1-x)^2} + 3} \right) \\ + \frac{1}{2} \sqrt{\frac{(1+x)^2}{(1-x)^2} + 3} \left(3\sqrt{2} \arctan \left(2 \frac{(1+x)\sqrt{2}}{1-x} \frac{1}{\sqrt{\frac{(1+x)^2}{(1-x)^2} + 3}} \right) - \sqrt{6} \operatorname{Artanh} \left(\frac{\sqrt{6}}{4} \sqrt{\frac{(1+x)^2}{(1-x)^2} + 3} \right) \right) \frac{1}{\sqrt{1 \left(\frac{(1+x)^2}{(1-x)^2} + 3 \right) \left(1 + \frac{1+x}{1-x} \right)^{-2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*x^2-x+1)/(x^2+x+1)^2/(x^2-x+1)^(1/2),x)`

[Out] $-1/6*(9*2^{(1/2)}*\arctan(2*2^{(1/2)}/((1+x)^2/(1-x)^2+3)^{(1/2)}*(1+x)/(1-x))*((1+x)^2/(1-x)^2+3)^{(1/2)}*(1+x)^2/(1-x)^2-6*6^{(1/2)}*\arctan h(1/4*((1+x)^2/(1-x)^2+3)^{(1/2)}*6^{(1/2)})*((1+x)^2/(1-x)^2+3)^{(1/2)}*(1+x)^2/(1-x)^2+3*2^{(1/2)}*\arctan(2*2^{(1/2)}/((1+x)^2/(1-x)^2+3)^{(1/2)}*(1+x)/(1-x))*((1+x)^2/(1-x)^2+3)^{(1/2)}-2*6^{(1/2)}*\operatorname{artanh}(1/4*((1+x)^2/(1-x)^2+3)^{(1/2)}*6^{(1/2)})*((1+x)^2/(1-x)^2+3)^{(1/2)}-12*(1+x)^3/(1-x)^3-36*(1+x)/(1-x))/(((1+x)^2/(1-x)^2+3)/(1+(1+x)/(1-x))^2)^{(1/2)}/(1+(1+x)/(1-x))/(3*(1+x)^2/(1-x)^2+1)+1/2*((1+x)^2/(1-x)^2+3)^{(1/2)}*(3*2^{(1/2)}*\arctan(2*2^{(1/2)}/((1+x)^2/(1-x)^2+3)^{(1/2)}*(1+x)/(1-x))-6^{(1/2)}*\operatorname{artanh}(1/4*((1+x)^2/(1-x)^2+3)^{(1/2)}*6^{(1/2)}))/((1+(1+x)/(1-x))/(((1+x)^2/(1-x)^2+3)/(1+(1+x)/(1-x))^2)^{(1/2)}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3x^2 - x + 1}{(x^2 + x + 1)^2 \sqrt{x^2 - x + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x^2 - x + 1)/((x^2 + x + 1)^2*sqrt(x^2 - x + 1)),x, algorithm="maxima)`

[Out] `integrate((3*x^2 - x + 1)/((x^2 + x + 1)^2*sqrt(x^2 - x + 1)), x)`

Fricas [A] time = 0.25221, size = 720, normalized size = 8.37

$$2\sqrt{6}\sqrt{3}(4x^2 + 7x - 9)\sqrt{x^2 - x + 1} - 2\sqrt{6}\sqrt{3}(4x^3 + 5x^2 - 11x + 9) - 24\left(8x^4 + 5x^2 - 4(2x^3 + x^2 + x - 1)\sqrt{x^2 - x + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2 - x + 1)/((x^2 + x + 1)^2*sqrt(x^2 - x + 1)),x, algorithm="fricas

[Out] 1/2*(2*sqrt(6)*sqrt(3)*(4*x^2 + 7*x - 9)*sqrt(x^2 - x + 1) - 2*sqrt(6)*sqrt(3)*(4*x^3 + 5*x^2 - 11*x + 9) - 24*(8*x^4 + 5*x^2 - 4*(2*x^3 + x^2 + x - 1)*sqrt(x^2 - x + 1) - 3*x + 5)*arctan(-(sqrt(6)*sqrt(3) + 2*sqrt(3))/(sqrt(6)*(2*x + 1) - 2*sqrt(6)*sqrt(2*x^2 - sqrt(x^2 - x + 1)*(2*x + sqrt(6) + 1) + sqrt(6)*(x + 1) + 4) - 2*sqrt(6)*sqrt(x^2 - x + 1) + 6)) + 24*(8*x^4 + 5*x^2 - 4*(2*x^3 + x^2 + x - 1)*sqrt(x^2 - x + 1) - 3*x + 5)*arctan(-(sqrt(6)*sqrt(3) - 2*sqrt(3))/(sqrt(6)*(2*x + 1) - 2*sqrt(6)*sqrt(2*x^2 - sqrt(x^2 - x + 1)*(2*x - sqrt(6) + 1) - sqrt(6)*(x + 1) + 4) - 2*sqrt(6)*sqrt(x^2 - x + 1) - 6)) + (4*sqrt(3)*(2*x^3 + x^2 + x - 1)*sqrt(x^2 - x + 1) - sqrt(3)*(8*x^4 + 5*x^2 - 3*x + 5))*log(4056*x^2 - 2028*sqrt(x^2 - x + 1)*(2*x + sqrt(6) + 1) + 2028*sqrt(6)*(x + 1) + 8112) - (4*sqrt(3)*(2*x^3 + x^2 + x - 1)*sqrt(x^2 - x + 1) - sqrt(3)*(8*x^4 + 5*x^2 - 3*x + 5))*log(4056*x^2 - 2028*sqrt(x^2 - x + 1)*(2*x - sqrt(6) + 1) - 2028*sqrt(6)*(x + 1) + 8112))/(4*sqrt(6)*sqrt(3)*(2*x^3 + x^2 + x - 1)*sqrt(x^2 - x + 1) - sqrt(6)*sqrt(3)*(8*x^4 + 5*x^2 - 3*x + 5))

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3x^2 - x + 1}{\sqrt{x^2 - x + 1}(x^2 + x + 1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x**2-x+1)/(x**2+x+1)**2/(x**2-x+1)**(1/2),x)

[Out] Integral((3*x**2 - x + 1)/(sqrt(x**2 - x + 1)*(x**2 + x + 1)**2), x)

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3x^2 - x + 1}{(x^2 + x + 1)^2\sqrt{x^2 - x + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2 - x + 1)/((x^2 + x + 1)^2*sqrt(x^2 - x + 1)),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 - x + 1)/((x^2 + x + 1)^2*sqrt(x^2 - x + 1)), x)
```


$$3.25 \quad \int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a^2+x^2}} dx$$

Optimal. Leaf size=19

$$2\sqrt{\sqrt{a^2+x^2}+x}$$

[Out] 2*Sqrt[x + Sqrt[a^2 + x^2]]

Rubi [A] time = 0.10589, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$

$$2\sqrt{\sqrt{a^2+x^2}+x}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a^2 + x^2], x]

[Out] 2*Sqrt[x + Sqrt[a^2 + x^2]]

Rubi in Sympy [A] time = 3.64378, size = 15, normalized size = 0.79

$$2\sqrt{x + \sqrt{a^2 + x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate((x+(a**2+x**2)**(1/2))**(1/2)/(a**2+x**2)**(1/2), x)

[Out] 2*sqrt(x + sqrt(a**2 + x**2))

Mathematica [A] time = 0.0264594, size = 19, normalized size = 1.

$$2\sqrt{\sqrt{a^2+x^2}+x}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a^2 + x^2], x]

[Out] $2\sqrt{x + \sqrt{a^2 + x^2}}$

Maple [F] time = 0.039, size = 0, normalized size = 0.

$$\int 1\sqrt{x + \sqrt{a^2 + x^2}} \frac{1}{\sqrt{a^2 + x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x+(a^2+x^2)^(1/2))^(1/2)/(a^2+x^2)^(1/2), x)`

[Out] `int((x+(a^2+x^2)^(1/2))^(1/2)/(a^2+x^2)^(1/2), x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x + \sqrt{a^2 + x^2}}}{\sqrt{a^2 + x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(x + sqrt(a^2 + x^2))/sqrt(a^2 + x^2), x, algorithm="maxima")`

[Out] `integrate(sqrt(x + sqrt(a^2 + x^2))/sqrt(a^2 + x^2), x)`

Fricas [A] time = 0.218391, size = 20, normalized size = 1.05

$$2\sqrt{x + \sqrt{a^2 + x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(x + sqrt(a^2 + x^2))/sqrt(a^2 + x^2), x, algorithm="fricas")`

[Out] `2*sqrt(x + sqrt(a^2 + x^2))`

Sympy [A] time = 0.282316, size = 15, normalized size = 0.79

$$2\sqrt{x + \sqrt{a^2 + x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(a**2+x**2)**(1/2))**(1/2)/(a**2+x**2)**(1/2),x)
```

```
[Out] 2*sqrt(x + sqrt(a**2 + x**2))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x + \sqrt{a^2 + x^2}}}{\sqrt{a^2 + x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x + sqrt(a^2 + x^2))/sqrt(a^2 + x^2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x + sqrt(a^2 + x^2))/sqrt(a^2 + x^2), x)
```

$$3.26 \quad \int \frac{\sqrt{bx + \sqrt{a + b^2x^2}}}{\sqrt{a + b^2x^2}} dx$$

Optimal. Leaf size=26

$$\frac{2\sqrt{\sqrt{a + b^2x^2} + bx}}{b}$$

[Out] (2*Sqrt[b*x + Sqrt[a + b^2*x^2]])/b

Rubi [A] time = 0.160506, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$

$$\frac{2\sqrt{\sqrt{a + b^2x^2} + bx}}{b}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[b*x + Sqrt[a + b^2*x^2]]/Sqrt[a + b^2*x^2], x]

[Out] (2*Sqrt[b*x + Sqrt[a + b^2*x^2]])/b

Rubi in Sympy [A] time = 4.68085, size = 20, normalized size = 0.77

$$\frac{2\sqrt{bx + \sqrt{a + b^2x^2}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate((b*x+(b**2*x**2+a)**(1/2))**(1/2)/(b**2*x**2+a)**(1/2), x)

[Out] 2*sqrt(b*x + sqrt(a + b**2*x**2))/b

Mathematica [A] time = 0.0409988, size = 26, normalized size = 1.

$$\frac{2\sqrt{\sqrt{a + b^2x^2} + bx}}{b}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[b*x + Sqrt[a + b^2*x^2]]/Sqrt[a + b^2*x^2], x]

[Out] $(2*\text{Sqrt}[b*x + \text{Sqrt}[a + b^2*x^2]])/b$

Maple [F] time = 0.05, size = 0, normalized size = 0.

$$\int 1\sqrt{bx + \sqrt{b^2x^2 + a}} \frac{1}{\sqrt{b^2x^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x+(b^2*x^2+a)^(1/2))^(1/2)/(b^2*x^2+a)^(1/2),x)`

[Out] `int((b*x+(b^2*x^2+a)^(1/2))^(1/2)/(b^2*x^2+a)^(1/2),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx + \sqrt{b^2x^2 + a}}}{\sqrt{b^2x^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(b*x + sqrt(b^2*x^2 + a))/sqrt(b^2*x^2 + a),x, algorithm="maxima")`

[Out] `integrate(sqrt(b*x + sqrt(b^2*x^2 + a))/sqrt(b^2*x^2 + a), x)`

Fricas [A] time = 0.222811, size = 30, normalized size = 1.15

$$\frac{2\sqrt{bx + \sqrt{b^2x^2 + a}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(b*x + sqrt(b^2*x^2 + a))/sqrt(b^2*x^2 + a),x, algorithm="fricas")`

[Out] `2*sqrt(b*x + sqrt(b^2*x^2 + a))/b`

Sympy [A] time = 1.34415, size = 27, normalized size = 1.04

$$\begin{cases} \frac{2\sqrt{bx + \sqrt{a + b^2x^2}}}{b} & \text{for } b \neq 0 \\ \frac{x}{\sqrt[4]{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+(b**2*x**2+a)**(1/2))**(1/2)/(b**2*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((2*sqrt(b*x + sqrt(a + b**2*x**2))/b, Ne(b, 0)), (x/a**
(1/4), True))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx + \sqrt{b^2x^2 + a}}}{\sqrt{b^2x^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b*x + sqrt(b^2*x^2 + a))/sqrt(b^2*x^2 + a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x + sqrt(b^2*x^2 + a))/sqrt(b^2*x^2 + a), x)
```

$$3.27 \quad \int \frac{1}{x\sqrt{a^2+x^2}\sqrt{x+\sqrt{a^2+x^2}}} dx$$

Optimal. Leaf size=63

$$-\frac{2 \tan^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{2 \tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)}{a^{3/2}}$$

[Out] $(-2*\text{ArcTan}[\text{Sqrt}[x + \text{Sqrt}[a^2 + x^2]]/\text{Sqrt}[a]])/a^{(3/2)} - (2*\text{ArcTanh}[\text{Sqrt}[x + \text{Sqrt}[a^2 + x^2]]/\text{Sqrt}[a]])/a^{(3/2)}$

Rubi [A] time = 0.365434, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$

$$-\frac{2 \tan^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{2 \tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[1/(x*\text{Sqrt}[a^2 + x^2]*\text{Sqrt}[x + \text{Sqrt}[a^2 + x^2]]),x]$

[Out] $(-2*\text{ArcTan}[\text{Sqrt}[x + \text{Sqrt}[a^2 + x^2]]/\text{Sqrt}[a]])/a^{(3/2)} - (2*\text{ArcTanh}[\text{Sqrt}[x + \text{Sqrt}[a^2 + x^2]]/\text{Sqrt}[a]])/a^{(3/2)}$

Rubi in Sympy [A] time = 9.55944, size = 58, normalized size = 0.92

$$-\frac{2 \operatorname{atan}\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}} - \frac{2 \operatorname{atanh}\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{rubi_integrate}(1/x/(a^{**2}+x^{**2})^{**}(1/2)/(x+(a^{**2}+x^{**2})^{**}(1/2))^{**}(1/2),x)$

[Out] $-2*\operatorname{atan}(\text{sqrt}(x + \text{sqrt}(a^{**2} + x^{**2}))/\text{sqrt}(a))/a^{**}(3/2) - 2*\operatorname{atanh}(\text{sqrt}(x + \text{sqrt}(a^{**2} + x^{**2}))/\text{sqrt}(a))/a^{**}(3/2)$

Mathematica [A] time = 0.250593, size = 85, normalized size = 1.35

$$\frac{\log\left(\sqrt{a} - \sqrt{\sqrt{a^2+x^2}+x}\right) - \log\left(\sqrt{\sqrt{a^2+x^2}+x} + \sqrt{a}\right) - 2 \tan^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*Sqrt[a^2 + x^2]*Sqrt[x + Sqrt[a^2 + x^2]]),x]

[Out] (-2*ArcTan[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a]] + Log[Sqrt[a] - Sqrt[x + Sqrt[a^2 + x^2]]] - Log[Sqrt[a] + Sqrt[x + Sqrt[a^2 + x^2]]])/a^(3/2)

Maple [F] time = 0.028, size = 0, normalized size = 0.

$$\int \frac{1}{x} \frac{1}{\sqrt{a^2 + x^2}} \frac{1}{\sqrt{x + \sqrt{a^2 + x^2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(a^2+x^2)^(1/2)/(x+(a^2+x^2)^(1/2))^(1/2),x)

[Out] int(1/x/(a^2+x^2)^(1/2)/(x+(a^2+x^2)^(1/2))^(1/2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a^2 + x^2} \sqrt{x + \sqrt{a^2 + x^2}} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(a^2 + x^2)*sqrt(x + sqrt(a^2 + x^2))*x),x, algorithm="maxima")

[Out] integrate(1/(sqrt(a^2 + x^2)*sqrt(x + sqrt(a^2 + x^2))*x), x)

Fricas [A] time = 0.243175, size = 1, normalized size = 0.02

$$\left[\frac{2 \arctan\left(\frac{\sqrt{a}}{\sqrt{x + \sqrt{a^2 + x^2}}}\right) + \log\left(-\frac{(a+x)\sqrt{a-2a\sqrt{x + \sqrt{a^2 + x^2}} + \sqrt{a^2 + x^2}\sqrt{a}}{a-x-\sqrt{a^2 + x^2}}\right)}{a^{\frac{3}{2}}}, \frac{2 \arctan\left(\frac{a}{\sqrt{-a}\sqrt{x + \sqrt{a^2 + x^2}}}\right) + \log\left(-\frac{\sqrt{-a}(a-x) + 2a\sqrt{x + \sqrt{a^2 + x^2}}}{a+x + \sqrt{a^2 + x^2}}\right)}{\sqrt{-aa}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(a^2 + x^2)*sqrt(x + sqrt(a^2 + x^2))*x),x, algorithm="fricas")


```
[Out] [(2*arctan(sqrt(a)/sqrt(x + sqrt(a^2 + x^2))) + log(-((a + x)*sqrt(a) - 2*a*sqrt(x + sqrt(a^2 + x^2))) + sqrt(a^2 + x^2)*sqrt(a))/(a - x - sqrt(a^2 + x^2))))/a^(3/2), (2*arctan(a/(sqrt(-a)*sqrt(x + sqrt(a^2 + x^2)))) + log(-(sqrt(-a)*(a - x) + 2*a*sqrt(x + sqrt(a^2 + x^2))) - sqrt(a^2 + x^2)*sqrt(-a))/(a + x + sqrt(a^2 + x^2)))/(sqrt(-a)*a)]
```

Sympy [A] time = 2.87045, size = 46, normalized size = 0.73

$$\frac{{}_2F_2\left(\begin{matrix} \frac{3}{4}, \frac{3}{4}, \frac{5}{4} \\ \frac{3}{2}, \frac{7}{4} \end{matrix} \middle| \frac{a^2 e^{i\pi}}{x^2}\right)}{\pi x^{\frac{3}{2}} \Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2+x**2)**(1/2)/(x+(a**2+x**2)**(1/2))**(1/2),x)
```

```
[Out] -gamma(3/4)**2*gamma(5/4)*hyper((3/4, 3/4, 5/4), (3/2, 7/4), a**2*exp_polar(I*pi)/x**2)/(pi*x**(3/2)*gamma(7/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a^2 + x^2} \sqrt{x + \sqrt{a^2 + x^2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(a^2 + x^2)*sqrt(x + sqrt(a^2 + x^2)))*x),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a^2 + x^2)*sqrt(x + sqrt(a^2 + x^2)))*x), x)
```

$$3.28 \quad \int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x} dx$$

Optimal. Leaf size=82

$$2\sqrt{\sqrt{a^2+x^2}+x} - 2\sqrt{a} \tan^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right) - 2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)$$

[Out] 2*Sqrt[x + Sqrt[a^2 + x^2]] - 2*Sqrt[a]*ArcTan[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a]] - 2*Sqrt[a]*ArcTanh[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a]]

Rubi [A] time = 0.140034, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$2\sqrt{\sqrt{a^2+x^2}+x} - 2\sqrt{a} \tan^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right) - 2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x + Sqrt[a^2 + x^2]]/x, x]

[Out] 2*Sqrt[x + Sqrt[a^2 + x^2]] - 2*Sqrt[a]*ArcTan[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a]] - 2*Sqrt[a]*ArcTanh[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a]]

Rubi in Sympy [A] time = 8.60686, size = 73, normalized size = 0.89

$$-2\sqrt{a} \operatorname{atan}\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right) - 2\sqrt{a} \operatorname{atanh}\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right) + 2\sqrt{x+\sqrt{a^2+x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate((x+(a**2+x**2)**(1/2))**(1/2)/x, x)

[Out] -2*sqrt(a)*atan(sqrt(x + sqrt(a**2 + x**2))/sqrt(a)) - 2*sqrt(a)*atanh(sqrt(x + sqrt(a**2 + x**2))/sqrt(a)) + 2*sqrt(x + sqrt(a**2 + x**2))

Mathematica [A] time = 0.081156, size = 161, normalized size = 1.96

$$\frac{\sqrt{a^2 + x^2} \left(\sqrt{a^2 + x^2} + x \right) \left(-2\sqrt{\sqrt{a^2 + x^2} + x} - \sqrt{a} \log \left(\sqrt{a} - \sqrt{\sqrt{a^2 + x^2} + x} \right) + \sqrt{a} \log \left(\sqrt{\sqrt{a^2 + x^2} + x} + \sqrt{a} \right) + 2\sqrt{a} \tan^{-1} \left(\frac{\sqrt{a^2 + x^2} + x}{\sqrt{a}} \right) \right)}{x \left(\sqrt{a^2 + x^2} + x \right) + a^2}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x + Sqrt[a^2 + x^2]]/x,x]

[Out] -((Sqrt[a^2 + x^2]*(x + Sqrt[a^2 + x^2])*(-2*Sqrt[x + Sqrt[a^2 + x^2]] + 2*Sqrt[a]*ArcTan[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a]] - Sqrt[a]*Log[Sqrt[a] - Sqrt[x + Sqrt[a^2 + x^2]]] + Sqrt[a]*Log[Sqrt[a] + Sqrt[x + Sqrt[a^2 + x^2]]]))/(a^2 + x*(x + Sqrt[a^2 + x^2]))

Maple [C] time = 0.008, size = 25, normalized size = 0.3

$$2\sqrt{2}\sqrt{x}{}_3F_2\left(-\frac{1}{4}, -\frac{1}{4}, \frac{1}{4}; \frac{1}{2}, \frac{3}{4}; -\frac{a^2}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x+(a^2+x^2)^(1/2))^(1/2)/x,x)

[Out] 2*2^(1/2)*x^(1/2)*hypergeom([-1/4, -1/4, 1/4], [1/2, 3/4], -1/x^2*a^2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x + \sqrt{a^2 + x^2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(x + sqrt(a^2 + x^2))/x,x, algorithm="maxima")

[Out] integrate(sqrt(x + sqrt(a^2 + x^2))/x, x)

Fricas [A] time = 0.240564, size = 1, normalized size = 0.01

$$\left[-2\sqrt{a} \arctan\left(\frac{\sqrt{x + \sqrt{a^2 + x^2}}}{\sqrt{a}}\right) + \sqrt{a} \log\left(-\frac{a - 2\sqrt{a}\sqrt{x + \sqrt{a^2 + x^2}} + x + \sqrt{a^2 + x^2}}{a - x - \sqrt{a^2 + x^2}}\right) \right. \\ \left. + 2\sqrt{x + \sqrt{a^2 + x^2}}, -2\sqrt{-a} \arctan\left(\frac{\sqrt{x + \sqrt{a^2 + x^2}}}{\sqrt{-a}}\right) \right. \\ \left. + \sqrt{-a} \log\left(-\frac{a + 2\sqrt{-a}\sqrt{x + \sqrt{a^2 + x^2}} - x - \sqrt{a^2 + x^2}}{a + x + \sqrt{a^2 + x^2}}\right) + 2\sqrt{x + \sqrt{a^2 + x^2}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(x + sqrt(a^2 + x^2))/x,x, algorithm="fricas")

[Out] [-2*sqrt(a)*arctan(sqrt(x + sqrt(a^2 + x^2))/sqrt(a)) + sqrt(a)*log(-(a - 2*sqrt(a)*sqrt(x + sqrt(a^2 + x^2)) + x + sqrt(a^2 + x^2))/(a - x - sqrt(a^2 + x^2))) + 2*sqrt(x + sqrt(a^2 + x^2)), -2*sqrt(-a)*arctan(sqrt(x + sqrt(a^2 + x^2))/sqrt(-a)) + sqrt(-a)*log(-(a + 2*sqrt(-a)*sqrt(x + sqrt(a^2 + x^2)) - x - sqrt(a^2 + x^2))/(a + x + sqrt(a^2 + x^2))) + 2*sqrt(x + sqrt(a^2 + x^2))]

Sympy [A] time = 2.48222, size = 51, normalized size = 0.62

$$\frac{\sqrt{x^2} \left(-\frac{1}{4}\right) \left(\frac{1}{4}\right) {}_3F_2\left(\begin{matrix} -\frac{1}{4}, -\frac{1}{4}, \frac{1}{4} \\ \frac{1}{2}, \frac{3}{4} \end{matrix} \middle| \frac{a^2 e^{i\pi}}{x^2}\right)}{8\pi \left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+(a**2+x**2)**(1/2))**(1/2)/x,x)

[Out] sqrt(x)*gamma(-1/4)**2*gamma(1/4)*hyper((-1/4, -1/4, 1/4), (1/2, 3/4), a**2*exp_polar(I*pi)/x**2)/(8*pi*gamma(3/4))

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x + \sqrt{a^2 + x^2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(x + sqrt(a^2 + x^2))/x,x, algorithm="giac")

```
[Out] integrate(sqrt(x + sqrt(a^2 + x^2))/x, x)
```

3.29 $\int x^3 \log^3(2+x) \log(3+x) dx$

Optimal. Leaf size=679

$$\begin{aligned}
& -\frac{5609}{96}\text{PolyLog}(2, -x-2) - \frac{563}{8}\text{PolyLog}(3, -x-2) - \frac{195}{2}\text{PolyLog}(4, -x-2) \\
& - \frac{195}{4}\log^2(x+2)\text{PolyLog}(2, -x-2) + \frac{563}{8}\log(x+2)\text{PolyLog}(2, -x-2) \\
& + \frac{195}{2}\log(x+2)\text{PolyLog}(3, -x-2) + \frac{3x^4}{256} + \frac{1}{4}x^4\log^3(x+2)\log(x+3) \\
& + \frac{3}{64}x^4\log^2(x+2) - \frac{3}{16}x^4\log^2(x+2)\log(x+3) - \frac{3}{128}x^4\log(x+2) \\
& + \frac{3}{32}x^4\log(x+2)\log(x+3) - \frac{3}{128}x^4\log(x+3) - \frac{763x^3}{3456} - \frac{17}{48}x^3\log^2(x+2) \\
& + \frac{1}{2}x^3\log^2(x+2)\log(x+3) + \frac{83}{288}x^3\log(x+2) - \frac{7}{12}x^3\log(x+2)\log(x+3) \\
& + \frac{37}{144}x^3\log(x+3) + \frac{8029x^2}{2304} - \frac{3}{2}x^2\log^2(x+2)\log(x+3) - \frac{187}{64}x^2\log(x+2) \\
& + \frac{13}{4}x^2\log(x+2)\log(x+3) - \frac{115}{48}x^2\log(x+3) - \frac{302177x}{1152} + \frac{3}{256}(x+2)^4 \\
& - \frac{71}{216}(x+2)^3 + \frac{377}{64}(x+2)^2 - \frac{1}{16}(x+2)^4\log^3(x+2) + \frac{3}{4}(x+2)^3\log^3(x+2) \\
& - \frac{33}{8}(x+2)^2\log^3(x+2) + \frac{65}{4}(x+2)\log^3(x+2) - \frac{81}{4}\log^3(x+2)\log(x+3) \\
& + 6x\log^2(x+2)\log(x+3) + \frac{3}{64}(x+2)^4\log^2(x+2) - \frac{3}{4}(x+2)^3\log^2(x+2) \\
& + \frac{273}{32}(x+2)^2\log^2(x+2) - \frac{1251}{16}(x+2)\log^2(x+2) + \frac{43}{12}\log^2(x+2) \\
& + \frac{963}{16}\log^2(x+2)\log(x+3) - 25x\log(x+2)\log(x+3) - \frac{3}{128}(x+2)^4\log(x+2) \\
& + \frac{1}{2}(x+2)^3\log(x+2) - \frac{273}{32}(x+2)^2\log(x+2) + \frac{6365}{32}(x+2)\log(x+2) \\
& + \frac{1}{128}(-3(x+2)^4 + 32(x+2)^3 - 144(x+2)^2 + 384(x+2) - 192\log(x+2))\log(x+2) \\
& + \frac{17}{72}((x+2)^3 - 9(x+2)^2 + 36(x+2) - 24\log(x+2))\log(x+2) + \frac{2069}{144}\log(x+2) \\
& + \frac{415}{12}(x+3)\log(x+3) - \frac{4083}{32}\log(x+2)\log(x+3) + \frac{3891}{128}\log(x+3)
\end{aligned}$$

[Out] $(-302177*x)/1152 + (8029*x^2)/2304 - (763*x^3)/3456 + (3*x^4)/256 + (377*(2+x)^2)/64 - (71*(2+x)^3)/216 + (3*(2+x)^4)/256 + (2069*\text{Log}[2+x])/144 - (187*x^2*\text{Log}[2+x])/64 + (83*x^3*\text{Log}[2+x])/288 - (3*x^4*\text{Log}[2+x])/128 + (6365*(2+x)*\text{Log}[2+x])/32 - (273*(2+x)^2*\text{Log}[2+x])/32 + ((2+x)^3*\text{Log}[2+x])/2 - (3*(2+x)^4*\text{Log}[2+x])/128 + ((384*(2+x) - 144*(2+x)^2 + 32*(2+x)^3 - 3*(2+x)^4 - 192*\text{Log}[2+x])* \text{Log}[2+x])/128 + (17*(36*(2+x) - 9*(2+x)^2 + (2+x)^3 - 24*\text{Log}[2+x])* \text{Log}[2+x])/72 + (43*\text{Log}[2+x]^2)/12 - (17*x^3*\text{Log}[2+x]^2)/48 + (3*x^4*\text{Log}[2+x]^2)/64 - (1251*(2+x)*\text{Log}[2+x]^2)/16 + (273*(2+x)^2*\text{Log}[2+x]^2)/32 - (3*(2+x)^3*\text{Log}[2+x]^2)/4 + (3*(2+x)^4*\text{Log}[2+x]^2)/64 + (65*(2+x)*\text{Log}[2+x]^3)/4 - (33*(2+x)^2*\text{Log}[2+x]^3)/8 + (3*(2+x)^3*\text{Log}[2+x]^3)/4 - ((2+x)^4*\text{Log}[2+x]^3)/16 + (3891*\text{Log}[3+x])/128 - (115*x^2*\text{Log}[3+x])/48 + (37*x^3*$

$$\begin{aligned} & \text{Log}[3 + x])/144 - (3*x^4*\text{Log}[3 + x])/128 + (415*(3 + x)*\text{Log}[3 + x])/12 \\ & - (4083*\text{Log}[2 + x]*\text{Log}[3 + x])/32 - 25*x*\text{Log}[2 + x]*\text{Log}[3 + x] \\ & + (13*x^2*\text{Log}[2 + x]*\text{Log}[3 + x])/4 - (7*x^3*\text{Log}[2 + x]*\text{Log}[3 + x])/12 \\ & + (3*x^4*\text{Log}[2 + x]*\text{Log}[3 + x])/32 + (963*\text{Log}[2 + x]^2*\text{Log}[3 + x])/16 \\ & + 6*x*\text{Log}[2 + x]^2*\text{Log}[3 + x] - (3*x^2*\text{Log}[2 + x]^2*\text{Log}[3 + x])/2 \\ & + (x^3*\text{Log}[2 + x]^2*\text{Log}[3 + x])/2 - (3*x^4*\text{Log}[2 + x]^2*\text{Log}[3 + x])/16 \\ & - (81*\text{Log}[2 + x]^3*\text{Log}[3 + x])/4 + (x^4*\text{Log}[2 + x]^3*\text{Log}[3 + x])/4 \\ & - (5609*\text{PolyLog}[2, -2 - x])/96 + (563*\text{Log}[2 + x]*\text{PolyLog}[2, -2 - x])/8 \\ & - (195*\text{Log}[2 + x]^2*\text{PolyLog}[2, -2 - x])/4 - (563*\text{PolyLog}[3, -2 - x])/8 \\ & + (195*\text{Log}[2 + x]*\text{PolyLog}[3, -2 - x])/2 - (195*\text{PolyLog}[4, -2 - x])/2 \end{aligned}$$

Rubi [A] time = 7.49551, antiderivative size = 679, normalized size of antiderivative = 1., number of steps used = 359, number of rules used = 30, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 2.143$

$$\begin{aligned} & -\frac{5609}{96}\text{PolyLog}(2, -x - 2) - \frac{563}{8}\text{PolyLog}(3, -x - 2) - \frac{195}{2}\text{PolyLog}(4, -x - 2) \\ & - \frac{195}{4}\log^2(x + 2)\text{PolyLog}(2, -x - 2) + \frac{563}{8}\log(x + 2)\text{PolyLog}(2, -x - 2) \\ & + \frac{195}{2}\log(x + 2)\text{PolyLog}(3, -x - 2) + \frac{3x^4}{256} + \frac{1}{4}x^4\log^3(x + 2)\log(x + 3) \\ & + \frac{3}{64}x^4\log^2(x + 2) - \frac{3}{16}x^4\log^2(x + 2)\log(x + 3) - \frac{3}{128}x^4\log(x + 2) \\ & + \frac{3}{32}x^4\log(x + 2)\log(x + 3) - \frac{3}{128}x^4\log(x + 3) - \frac{763x^3}{3456} - \frac{17}{48}x^3\log^2(x + 2) \\ & + \frac{1}{2}x^3\log^2(x + 2)\log(x + 3) + \frac{83}{288}x^3\log(x + 2) - \frac{7}{12}x^3\log(x + 2)\log(x + 3) \\ & + \frac{37}{144}x^3\log(x + 3) + \frac{8029x^2}{2304} - \frac{3}{2}x^2\log^2(x + 2)\log(x + 3) - \frac{187}{64}x^2\log(x + 2) \\ & + \frac{13}{4}x^2\log(x + 2)\log(x + 3) - \frac{115}{48}x^2\log(x + 3) - \frac{302177x}{1152} + \frac{3}{256}(x + 2)^4 \\ & - \frac{71}{216}(x + 2)^3 + \frac{377}{64}(x + 2)^2 - \frac{1}{16}(x + 2)^4\log^3(x + 2) + \frac{3}{4}(x + 2)^3\log^3(x + 2) \\ & - \frac{33}{8}(x + 2)^2\log^3(x + 2) + \frac{65}{4}(x + 2)\log^3(x + 2) - \frac{81}{4}\log^3(x + 2)\log(x + 3) \\ & + 6x\log^2(x + 2)\log(x + 3) + \frac{3}{64}(x + 2)^4\log^2(x + 2) - \frac{3}{4}(x + 2)^3\log^2(x + 2) \\ & + \frac{273}{32}(x + 2)^2\log^2(x + 2) - \frac{1251}{16}(x + 2)\log^2(x + 2) + \frac{43}{12}\log^2(x + 2) \\ & + \frac{963}{16}\log^2(x + 2)\log(x + 3) - 25x\log(x + 2)\log(x + 3) - \frac{3}{128}(x + 2)^4\log(x + 2) \\ & + \frac{1}{2}(x + 2)^3\log(x + 2) - \frac{273}{32}(x + 2)^2\log(x + 2) + \frac{6365}{32}(x + 2)\log(x + 2) \\ & + \frac{1}{128}(-3(x + 2)^4 + 32(x + 2)^3 - 144(x + 2)^2 + 384(x + 2) - 192\log(x + 2))\log(x + 2) \\ & + \frac{17}{72}((x + 2)^3 - 9(x + 2)^2 + 36(x + 2) - 24\log(x + 2))\log(x + 2) + \frac{2069}{144}\log(x + 2) \\ & + \frac{415}{12}(x + 3)\log(x + 3) - \frac{4083}{32}\log(x + 2)\log(x + 3) + \frac{3891}{128}\log(x + 3) \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[x^3*Log[2 + x]^3*Log[3 + x],x]

[Out]
$$\begin{aligned} & (-302177*x)/1152 + (8029*x^2)/2304 - (763*x^3)/3456 + (3*x^4)/256 \\ & + (377*(2 + x)^2)/64 - (71*(2 + x)^3)/216 + (3*(2 + x)^4)/256 + \\ & (2069*\text{Log}[2 + x])/144 - (187*x^2*\text{Log}[2 + x])/64 + (83*x^3*\text{Log}[2 + \\ & x])/288 - (3*x^4*\text{Log}[2 + x])/128 + (6365*(2 + x)*\text{Log}[2 + x])/32 \\ & - (273*(2 + x)^2*\text{Log}[2 + x])/32 + ((2 + x)^3*\text{Log}[2 + x])/2 - (3*(\\ & 2 + x)^4*\text{Log}[2 + x])/128 + ((384*(2 + x) - 144*(2 + x)^2 + 32*(2 \\ & + x)^3 - 3*(2 + x)^4 - 192*\text{Log}[2 + x])*\text{Log}[2 + x])/128 + (17*(36* \\ & (2 + x) - 9*(2 + x)^2 + (2 + x)^3 - 24*\text{Log}[2 + x])*\text{Log}[2 + x])/72 \\ & + (43*\text{Log}[2 + x]^2)/12 - (17*x^3*\text{Log}[2 + x]^2)/48 + (3*x^4*\text{Log}[2 \\ & + x]^2)/64 - (1251*(2 + x)*\text{Log}[2 + x]^2)/16 + (273*(2 + x)^2*\text{Log} \\ & [2 + x]^2)/32 - (3*(2 + x)^3*\text{Log}[2 + x]^2)/4 + (3*(2 + x)^4*\text{Log}[2 \\ & + x]^2)/64 + (65*(2 + x)*\text{Log}[2 + x]^3)/4 - (33*(2 + x)^2*\text{Log}[2 + \\ & x]^3)/8 + (3*(2 + x)^3*\text{Log}[2 + x]^3)/4 - ((2 + x)^4*\text{Log}[2 + x]^3 \\ &)/16 + (3891*\text{Log}[3 + x])/128 - (115*x^2*\text{Log}[3 + x])/48 + (37*x^3* \\ & \text{Log}[3 + x])/144 - (3*x^4*\text{Log}[3 + x])/128 + (415*(3 + x)*\text{Log}[3 + x \\ &])/12 - (4083*\text{Log}[2 + x]*\text{Log}[3 + x])/32 - 25*x*\text{Log}[2 + x]*\text{Log}[3 + \\ & x] + (13*x^2*\text{Log}[2 + x]*\text{Log}[3 + x])/4 - (7*x^3*\text{Log}[2 + x]*\text{Log}[3 \\ & + x])/12 + (3*x^4*\text{Log}[2 + x]*\text{Log}[3 + x])/32 + (963*\text{Log}[2 + x]^2*\text{L} \\ & \text{og}[3 + x])/16 + 6*x*\text{Log}[2 + x]^2*\text{Log}[3 + x] - (3*x^2*\text{Log}[2 + x]^2 \\ & *\text{Log}[3 + x])/2 + (x^3*\text{Log}[2 + x]^2*\text{Log}[3 + x])/2 - (3*x^4*\text{Log}[2 + \\ & x]^2*\text{Log}[3 + x])/16 - (81*\text{Log}[2 + x]^3*\text{Log}[3 + x])/4 + (x^4*\text{Log}[\\ & 2 + x]^3*\text{Log}[3 + x])/4 - (5609*\text{PolyLog}[2, -2 - x])/96 + (563*\text{Log}[\\ & 2 + x]*\text{PolyLog}[2, -2 - x])/8 - (195*\text{Log}[2 + x]^2*\text{PolyLog}[2, -2 - \\ & x])/4 - (563*\text{PolyLog}[3, -2 - x])/8 + (195*\text{Log}[2 + x]*\text{PolyLog}[3, - \\ & 2 - x])/2 - (195*\text{PolyLog}[4, -2 - x])/2 \end{aligned}$$

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \log(x+2)^3 \log(x+3) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate(x**3*ln(2+x)**3*ln(3+x),x)

[Out] Integral(x**3*log(x + 2)**3*log(x + 3), x)

Mathematica [A] time = 0.266705, size = 412, normalized size = 0.61

$$-224640\text{PolyLog}(4, -x - 2) - 24(4680 \log^2(x + 2) - 6756 \log(x + 2) + 5609) \text{PolyLog}(2, -x - 2) + 288(780 \log(x + 2) - 563)$$

Antiderivative was successfully verified.

[In] Integrate[x^3*Log[2 + x]^3*Log[3 + x],x]


```
[Out] (-759744 - 558290*x + 17705*x^2 - 1050*x^3 + 54*x^4 + 910528*Log[
2 + x] + 400008*x*Log[2 + x] - 22836*x^2*Log[2 + x] + 2072*x^3*Lo
g[2 + x] - 162*x^4*Log[2 + x] - 302016*Log[2 + x]^2 - 118800*x*Lo
g[2 + x]^2 + 11880*x^2*Log[2 + x]^2 - 1680*x^3*Log[2 + x]^2 + 216
*x^4*Log[2 + x]^2 + 48384*Log[2 + x]^3 + 15552*x*Log[2 + x]^3 - 2
592*x^2*Log[2 + x]^3 + 576*x^3*Log[2 + x]^3 - 144*x^4*Log[2 + x]^
3 + 309078*Log[3 + x] + 79680*x*Log[3 + x] - 5520*x^2*Log[3 + x]
+ 592*x^3*Log[3 + x] - 54*x^4*Log[3 + x] - 293976*Log[2 + x]*Log[
3 + x] - 57600*x*Log[2 + x]*Log[3 + x] + 7488*x^2*Log[2 + x]*Log[
3 + x] - 1344*x^3*Log[2 + x]*Log[3 + x] + 216*x^4*Log[2 + x]*Log[
3 + x] + 138672*Log[2 + x]^2*Log[3 + x] + 13824*x*Log[2 + x]^2*Lo
g[3 + x] - 3456*x^2*Log[2 + x]^2*Log[3 + x] + 1152*x^3*Log[2 + x]
^2*Log[3 + x] - 432*x^4*Log[2 + x]^2*Log[3 + x] - 46656*Log[2 + x
]^3*Log[3 + x] + 576*x^4*Log[2 + x]^3*Log[3 + x] - 24*(5609 - 675
6*Log[2 + x] + 4680*Log[2 + x]^2)*PolyLog[2, -2 - x] + 288*(-563
+ 780*Log[2 + x])*PolyLog[3, -2 - x] - 224640*PolyLog[4, -2 - x])
/2304
```

Maple [F] time = 0.063, size = 0, normalized size = 0.

$$\int x^3 (\ln(2+x))^3 \ln(3+x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(2+x)^3*ln(3+x), x)
```

```
[Out] int(x^3*ln(2+x)^3*ln(3+x), x)
```

Maxima [A] time = 1.39935, size = 699, normalized size = 1.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(x + 3)*log(x + 2)^3,x, algorithm="maxima")
```

```
[Out] 3/128*x^4 + 1/16*(4*x^4*log(x + 3) - x^4 + 4*x^3 - 18*x^2 + 108*x
- 324*log(x + 3))*log(x + 2)^3 - 65/4*log(x + 3)*log(x + 2)^3 +
195/4*log(x + 3)*log(x + 2)^2*log(-x - 2) - 175/384*x^3 + 1/96*(9
*x^4 - 70*x^3 + 495*x^2 - 6*(3*x^4 - 8*x^3 + 24*x^2 - 96*x)*log(x
+ 3) + 4680*log(x + 3)*log(-x - 2) - 4950*x + 4680*dilog(x + 3)
+ 5778*log(x + 3) + 6048*log(x + 2))*log(x + 2)^2 + 195/4*dilog(x
+ 3)*log(x + 2)^2 - 195/4*dilog(-x - 2)*log(x + 2)^2 + 563/16*lo
g(x + 3)*log(x + 2)^2 + 21*log(x + 2)^3 + 17705/2304*x^2 + 1/8*(7
80*log(x + 2)^2 - 563*log(x + 2))*dilog(-x - 2) - 1/1152*(27*x^4
- 296*x^3 - 18720*log(x + 2)^3 + 2760*x^2 + 40536*log(x + 2)^2 -
```

$$\begin{aligned}
& 39840*x - 67308*\log(x + 2))*\log(x + 3) - 1/1152*(81*x^4 - 1036*x^3 \\
& + 56160*\log(x + 3)*\log(x + 2)^2 + 112320*\log(x + 3)*\log(x + 2)* \\
& \log(-x - 2) + 11418*x^2 - 12*(9*x^4 - 56*x^3 + 312*x^2 + 4680*\log \\
& (x + 2)^2 - 2400*x - 6756*\log(x + 2))*\log(x + 3) + 112320*\operatorname{dilog}(x \\
& + 3)*\log(x + 2) + 112320*\operatorname{dilog}(-x - 2)*\log(x + 2) - 81072*\log(x \\
& + 3)*\log(x + 2) + 72576*\log(x + 2)^2 - 200004*x - 81072*\operatorname{dilog}(-x \\
& - 2) + 146988*\log(x + 3) + 302016*\log(x + 2) - 112320*\operatorname{polylog}(3, \\
& -x - 2))*\log(x + 2) + 563/8*\operatorname{dilog}(-x - 2)*\log(x + 2) - 5609/96*\log \\
& (x + 3)*\log(x + 2) + 1573/12*\log(x + 2)^2 - 279145/1152*x - 5609 \\
& /96*\operatorname{dilog}(-x - 2) + 17171/128*\log(x + 3) + 14227/36*\log(x + 2) - \\
& 195/2*\operatorname{polylog}(4, -x - 2) - 563/8*\operatorname{polylog}(3, -x - 2)
\end{aligned}$$

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}(x^3 \log(x + 3) \log(x + 2)^3, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*log(x + 3)*log(x + 2)^3,x, algorithm="fricas")`

[Out] `integral(x^3*log(x + 3)*log(x + 2)^3, x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*ln(2+x)**3*ln(3+x),x)`

[Out] Timed out

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \log(x + 3) \log(x + 2)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*log(x + 3)*log(x + 2)^3,x, algorithm="giac")`

[Out] `integrate(x^3*log(x + 3)*log(x + 2)^3, x)`

$$3.30 \quad \int \frac{(x + \sqrt{b+x^2})^a}{\sqrt{b+x^2}} dx$$

Optimal. Leaf size=17

$$\frac{(\sqrt{b+x^2} + x)^a}{a}$$

[Out] (x + Sqrt[b + x^2])^a/a

Rubi [A] time = 0.0877249, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$

$$\frac{(\sqrt{b+x^2} + x)^a}{a}$$

Antiderivative was successfully verified.

[In] Int[(x + Sqrt[b + x^2])^a/Sqrt[b + x^2], x]

[Out] (x + Sqrt[b + x^2])^a/a

Rubi in Sympy [A] time = 3.91902, size = 12, normalized size = 0.71

$$\frac{(x + \sqrt{b+x^2})^a}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate((x+(x**2+b)**(1/2))**a/(x**2+b)**(1/2), x)

[Out] (x + sqrt(b + x**2))**a/a

Mathematica [A] time = 0.040226, size = 17, normalized size = 1.

$$\frac{(\sqrt{b+x^2} + x)^a}{a}$$

Antiderivative was successfully verified.

[In] Integrate[(x + Sqrt[b + x^2])^a/Sqrt[b + x^2],x]

[Out] (x + Sqrt[b + x^2])^a/a

Maple [F] time = 0.037, size = 0, normalized size = 0.

$$\int 1 \left(x + \sqrt{x^2 + b} \right)^a \frac{1}{\sqrt{x^2 + b}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x+(x^2+b)^(1/2))^a/(x^2+b)^(1/2),x)

[Out] int((x+(x^2+b)^(1/2))^a/(x^2+b)^(1/2),x)

Maxima [A] time = 1.40199, size = 20, normalized size = 1.18

$$\frac{\left(x + \sqrt{x^2 + b} \right)^a}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x + sqrt(x^2 + b))^a/sqrt(x^2 + b),x, algorithm="maxima")

[Out] (x + sqrt(x^2 + b))^a/a

Fricas [A] time = 0.241325, size = 20, normalized size = 1.18

$$\frac{\left(x + \sqrt{x^2 + b} \right)^a}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x + sqrt(x^2 + b))^a/sqrt(x^2 + b),x, algorithm="fricas")

[Out] (x + sqrt(x^2 + b))^a/a

Sympy [A] time = 7.30149, size = 311, normalized size = 18.29

$$\left\{ \begin{array}{l} -\frac{\sqrt{b} b^{\frac{a}{2}} \sinh\left(-a \operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right) + \operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)\right)}{ax\sqrt{\frac{b}{x^2}+1}} + \frac{b^{\frac{a}{2}} x \cosh\left(-a \operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right) + \operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)\right)}{a\sqrt{b}} - \frac{b^{\frac{a}{2}} x \sinh\left(-a \operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right) + \operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)\right)}{a\sqrt{b}\sqrt{\frac{b}{x^2}+1}} - \frac{2b^{\frac{a}{2}} \cosh\left(a \operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)\right)}{a^2\left(-\frac{a}{2}\right)} \\ -\frac{b^{\frac{a}{2}} \sinh\left(-a \operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right) + \operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)\right)}{a\sqrt{1+\frac{x^2}{b}}} - \frac{b^{\frac{a}{2}} x^2 \sinh\left(-a \operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right) + \operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)\right)}{ab\sqrt{1+\frac{x^2}{b}}} + \frac{b^{\frac{a}{2}} x \cosh\left(-a \operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right) + \operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)\right)}{a\sqrt{b}} - \frac{2b^{\frac{a}{2}} \cosh\left(a \operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)\right)}{a^2\left(-\frac{a}{2}\right)} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+(x**2+b)**(1/2))**a/(x**2+b)**(1/2), x)

[Out] Piecewise((-sqrt(b)*b**(a/2)*sinh(-a*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))/(a*x*sqrt(b/x**2 + 1)) + b**(a/2)*x*cosh(-a*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))/(a*sqrt(b)) - b**(a/2)*x*sinh(-a*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))/(a*sqrt(b)*sqrt(b/x**2 + 1)) - 2*b**(a/2)*cosh(a*asinh(x/sqrt(b)))*gamma(-a/2 + 1)/(a**2*gamma(-a/2)), Abs(x**2/b) > 1), (-b**(a/2)*sinh(-a*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))/(a*sqrt(1 + x**2/b)) - b**(a/2)*x**2*sinh(-a*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))/(a*b*sqrt(1 + x**2/b)) + b**(a/2)*x*cosh(-a*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))/(a*sqrt(b)) - 2*b**(a/2)*cosh(a*asinh(x/sqrt(b)))*gamma(-a/2 + 1)/(a**2*gamma(-a/2)), True))

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x + \sqrt{x^2 + b})^a}{\sqrt{x^2 + b}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x + sqrt(x^2 + b))^a/sqrt(x^2 + b), x, algorithm="giac")

[Out] integrate((x + sqrt(x^2 + b))^a/sqrt(x^2 + b), x)

$$3.31 \quad \int \left(x + \sqrt{b + x^2} \right)^a dx$$

Optimal. Leaf size=52

$$\frac{\left(\sqrt{b+x^2}+x\right)^{a+1}}{2(a+1)} - \frac{b\left(\sqrt{b+x^2}+x\right)^{a-1}}{2(1-a)}$$

[Out] $-(b*(x + \text{Sqrt}[b + x^2])^{(-1 + a)})/(2*(1 - a)) + (x + \text{Sqrt}[b + x^2])^{(1 + a)}/(2*(1 + a))$

Rubi [A] time = 0.0464369, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{\left(\sqrt{b+x^2}+x\right)^{a+1}}{2(a+1)} - \frac{b\left(\sqrt{b+x^2}+x\right)^{a-1}}{2(1-a)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(x + \text{Sqrt}[b + x^2])^a, x]$

[Out] $-(b*(x + \text{Sqrt}[b + x^2])^{(-1 + a)})/(2*(1 - a)) + (x + \text{Sqrt}[b + x^2])^{(1 + a)}/(2*(1 + a))$

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x + \sqrt{b + x^2} \right)^a dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{rubi_integrate}((x+(x**2+b)**(1/2))**a,x)$

[Out] $\text{Integral}((x + \text{sqrt}(b + x**2))**a, x)$

Mathematica [A] time = 0.0233956, size = 36, normalized size = 0.69

$$\frac{\left(\sqrt{b+x^2}+x\right)^a \left(a\sqrt{b+x^2}-x\right)}{a^2-1}$$

Antiderivative was successfully verified.

[In] Integrate[(x + Sqrt[b + x^2])^a, x]

[Out] ((x + Sqrt[b + x^2])^a*(-x + a*Sqrt[b + x^2]))/(-1 + a^2)

Maple [B] time = 0.033, size = 120, normalized size = 2.3

$$\frac{a}{4\sqrt{\pi}} b^{\frac{a}{2} + \frac{1}{2}} \left(8 \frac{\sqrt{\pi} x^{1+a} b^{-a/2-1/2}}{(1+a)a(2a-2)} \left(\frac{ab}{x^2} + a - 1 \right) \left(\sqrt{\frac{b}{x^2} + 1} + 1 \right)^{a-1} + 4 \frac{\sqrt{\pi} x^{1+a} b^{-a/2-1/2}}{(1+a)a} \sqrt{\frac{b}{x^2} + 1} \left(\sqrt{\frac{b}{x^2} + 1} + 1 \right)^{a-1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x+(x^2+b)^(1/2))^a, x)

[Out] 1/4*b^(1/2*a+1/2)/Pi^(1/2)*a*(8*Pi^(1/2)/(1+a)/a*x^(1+a)*b^(-1/2*a-1/2)*(1/x^2*a*b+a-1)/(2*a-2)*((1/x^2*b+1)^(1/2)+1)^(a-1)+4*Pi^(1/2)/(1+a)/a*x^(1+a)*b^(-1/2*a-1/2)*(1/x^2*b+1)^(1/2)*((1/x^2*b+1)^(1/2)+1)^(a-1))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (x + \sqrt{x^2 + b})^a dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x + sqrt(x^2 + b))^a, x, algorithm="maxima")

[Out] integrate((x + sqrt(x^2 + b))^a, x)

Fricas [A] time = 0.244311, size = 43, normalized size = 0.83

$$\frac{(\sqrt{x^2 + b}a - x)(x + \sqrt{x^2 + b})^a}{a^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x + sqrt(x^2 + b))^a, x, algorithm="fricas")

[Out] (sqrt(x^2 + b)*a - x)*(x + sqrt(x^2 + b))^a/(a^2 - 1)

Sympy [A] time = 6.69675, size = 2147, normalized size = 41.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+(x**2+b)**(1/2))**a,x)

[Out] Piecewise((-a**2*b**(9/2)*b**(a/2)*x*sqrt(b/x**2 + 1)*sinh(a*asin
h(x/sqrt(b))) * gamma(-a/2)/(2*a**2*b**(9/2)*gamma(-a/2 + 1) + 2*a*
*2*b**(7/2)*x**2*gamma(-a/2 + 1) - 2*b**(9/2)*gamma(-a/2 + 1) - 2
*b**(7/2)*x**2*gamma(-a/2 + 1)) - a**2*b**(7/2)*b**(a/2)*x**3*sq
r t(b/x**2 + 1)*sinh(a*asinh(x/sqrt(b))) * gamma(-a/2)/(2*a**2*b**(9/
2)*gamma(-a/2 + 1) + 2*a**2*b**(7/2)*x**2*gamma(-a/2 + 1) - 2*b**
(9/2)*gamma(-a/2 + 1) - 2*b**(7/2)*x**2*gamma(-a/2 + 1)) + a*b**(
9/2)*b**(a/2)*x*cosh(a*asinh(x/sqrt(b))) * gamma(-a/2)/(2*a**2*b**(
9/2)*gamma(-a/2 + 1) + 2*a**2*b**(7/2)*x**2*gamma(-a/2 + 1) - 2*b
** (9/2)*gamma(-a/2 + 1) - 2*b**(7/2)*x**2*gamma(-a/2 + 1)) + a*b*
*(7/2)*b**(a/2)*x**3*cosh(a*asinh(x/sqrt(b))) * gamma(-a/2)/(2*a**2
*b**(9/2)*gamma(-a/2 + 1) + 2*a**2*b**(7/2)*x**2*gamma(-a/2 + 1)
- 2*b**(9/2)*gamma(-a/2 + 1) - 2*b**(7/2)*x**2*gamma(-a/2 + 1)) +
2*a*b**5*b**(a/2)*cosh(a*asinh(x/sqrt(b)) + asinh(x/sqrt(b))) * ga
mma(-a/2 + 1)/(2*a**2*b**(9/2)*gamma(-a/2 + 1) + 2*a**2*b**(7/2)*
x**2*gamma(-a/2 + 1) - 2*b**(9/2)*gamma(-a/2 + 1) - 2*b**(7/2)*x**
*2*gamma(-a/2 + 1)) - 2*a*b**5*b**(a/2)*gamma(-a/2 + 1)/(2*a**2*b
** (9/2)*gamma(-a/2 + 1) + 2*a**2*b**(7/2)*x**2*gamma(-a/2 + 1) -
2*b**(9/2)*gamma(-a/2 + 1) - 2*b**(7/2)*x**2*gamma(-a/2 + 1)) - 2
*a*b**4*b**(a/2)*x**2*sqrt(b/x**2 + 1)*sinh(a*asinh(x/sqrt(b)) +
asinh(x/sqrt(b))) * gamma(-a/2 + 1)/(2*a**2*b**(9/2)*gamma(-a/2 + 1
) + 2*a**2*b**(7/2)*x**2*gamma(-a/2 + 1) - 2*b**(9/2)*gamma(-a/2
+ 1) - 2*b**(7/2)*x**2*gamma(-a/2 + 1)) + 4*a*b**4*b**(a/2)*x**2*
cosh(a*asinh(x/sqrt(b)) + asinh(x/sqrt(b))) * gamma(-a/2 + 1)/(2*a*
*2*b**(9/2)*gamma(-a/2 + 1) + 2*a**2*b**(7/2)*x**2*gamma(-a/2 + 1
) - 2*b**(9/2)*gamma(-a/2 + 1) - 2*b**(7/2)*x**2*gamma(-a/2 + 1))
- 2*a*b**4*b**(a/2)*x**2*gamma(-a/2 + 1)/(2*a**2*b**(9/2)*gamma(
-a/2 + 1) + 2*a**2*b**(7/2)*x**2*gamma(-a/2 + 1) - 2*b**(9/2)*gam
ma(-a/2 + 1) - 2*b**(7/2)*x**2*gamma(-a/2 + 1)) - 2*a*b**3*b**(
a/2)*x**4*sqrt(b/x**2 + 1)*sinh(a*asinh(x/sqrt(b)) + asinh(x/sqrt(b
))) * gamma(-a/2 + 1)/(2*a**2*b**(9/2)*gamma(-a/2 + 1) + 2*a**2*b**
(7/2)*x**2*gamma(-a/2 + 1) - 2*b**(9/2)*gamma(-a/2 + 1) - 2*b**(7
/2)*x**2*gamma(-a/2 + 1)) + 2*a*b**3*b**(a/2)*x**4*cosh(a*asinh(x
/sqrt(b)) + asinh(x/sqrt(b))) * gamma(-a/2 + 1)/(2*a**2*b**(9/2)*ga
mma(-a/2 + 1) + 2*a**2*b**(7/2)*x**2*gamma(-a/2 + 1) - 2*b**(9/2)
*gamma(-a/2 + 1) - 2*b**(7/2)*x**2*gamma(-a/2 + 1)) - 2*b**4*b**(
a/2)*x**2*sqrt(b/x**2 + 1)*sinh(a*asinh(x/sqrt(b)) + asinh(x/sqrt
(b))) * gamma(-a/2 + 1)/(2*a**2*b**(9/2)*gamma(-a/2 + 1) + 2*a**2*b
** (7/2)*x**2*gamma(-a/2 + 1) - 2*b**(9/2)*gamma(-a/2 + 1) - 2*b**
(7/2)*x**2*gamma(-a/2 + 1)) + 2*b**4*b**(a/2)*x**2*cosh(a*asinh(x
/sqrt(b)) + asinh(x/sqrt(b))) * gamma(-a/2 + 1)/(2*a**2*b**(9/2)*ga
mma(-a/2 + 1) + 2*a**2*b**(7/2)*x**2*gamma(-a/2 + 1) - 2*b**(9/2)
*gamma(-a/2 + 1) - 2*b**(7/2)*x**2*gamma(-a/2 + 1)) - 2*b**3*b**(
a/2)*x**4*sqrt(b/x**2 + 1)*sinh(a*asinh(x/sqrt(b)) + asinh(x/sqrt
(b))) * gamma(-a/2 + 1)/(2*a**2*b**(9/2)*gamma(-a/2 + 1) + 2*a**2*b


```

** (7/2)*x**2*gamma(-a/2 + 1) - 2*b**(9/2)*gamma(-a/2 + 1) - 2*b**
(7/2)*x**2*gamma(-a/2 + 1)) + 2*b**3*b**(a/2)*x**4*cosh(a*asinh(x
/sqrt(b)) + asinh(x/sqrt(b))) * gamma(-a/2 + 1)/(2*a**2*b**(9/2)*ga
mma(-a/2 + 1) + 2*a**2*b**(7/2)*x**2*gamma(-a/2 + 1) - 2*b**(9/2)
* gamma(-a/2 + 1) - 2*b**(7/2)*x**2*gamma(-a/2 + 1)), Abs(x**2/b)
> 1), (-a**2*b**3*b**(a/2)*sqrt(1 + x**2/b)*sinh(a*asinh(x/sqrt(b)
))) * gamma(-a/2)/(2*a**2*b**(5/2)*gamma(-a/2 + 1) - 2*b**(5/2)*gam
ma(-a/2 + 1)) - 2*a*b**(5/2)*b**(a/2)*x*sqrt(1 + x**2/b)*sinh(a*a
sinh(x/sqrt(b)) + asinh(x/sqrt(b))) * gamma(-a/2 + 1)/(2*a**2*b**(5
/2)*gamma(-a/2 + 1) - 2*b**(5/2)*gamma(-a/2 + 1)) + a*b**(5/2)*b*
*(a/2)*x*cosh(a*asinh(x/sqrt(b))) * gamma(-a/2)/(2*a**2*b**(5/2)*ga
mma(-a/2 + 1) - 2*b**(5/2)*gamma(-a/2 + 1)) + 2*a*b**3*b**(a/2)*c
osh(a*asinh(x/sqrt(b)) + asinh(x/sqrt(b))) * gamma(-a/2 + 1)/(2*a**
2*b**(5/2)*gamma(-a/2 + 1) - 2*b**(5/2)*gamma(-a/2 + 1)) + 2*a*b*
**2*b**(a/2)*x**2*cosh(a*asinh(x/sqrt(b)) + asinh(x/sqrt(b))) * gamm
a(-a/2 + 1)/(2*a**2*b**(5/2)*gamma(-a/2 + 1) - 2*b**(5/2)*gamma(-
a/2 + 1)) - 2*b**(5/2)*b**(a/2)*x*sqrt(1 + x**2/b)*sinh(a*asinh(x
/sqrt(b)) + asinh(x/sqrt(b))) * gamma(-a/2 + 1)/(2*a**2*b**(5/2)*ga
mma(-a/2 + 1) - 2*b**(5/2)*gamma(-a/2 + 1)) + 2*b**2*b**(a/2)*x**
2*cosh(a*asinh(x/sqrt(b)) + asinh(x/sqrt(b))) * gamma(-a/2 + 1)/(2*
a**2*b**(5/2)*gamma(-a/2 + 1) - 2*b**(5/2)*gamma(-a/2 + 1)), True
))

```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int (x + \sqrt{x^2 + b})^a dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x + sqrt(x^2 + b))^a,x, algorithm="giac")

[Out] integrate((x + sqrt(x^2 + b))^a, x)

$$3.32 \quad \int (6 + 3x^a + 2x^{2a})^{\frac{1}{a}} (x^a + x^{2a} + x^{3a}) dx$$

Optimal. Leaf size=34

$$\frac{x^{a+1} (2x^{2a} + 3x^a + 6)^{\frac{1}{a}+1}}{6(a+1)}$$

[Out] $(x^{(1+a)}(6+3x^a+2x^{(2a)})^{(1+a^{-1})})/(6*(1+a))$

Rubi [A] time = 0.0596816, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$

$$\frac{x^{a+1} (2x^{2a} + 3x^a + 6)^{\frac{1}{a}+1}}{6(a+1)}$$

Antiderivative was successfully verified.

[In] Int[(6 + 3*x^a + 2*x^(2*a))^a^(-1)*(x^a + x^(2*a) + x^(3*a)), x]

[Out] $(x^{(1+a)}(6+3x^a+2x^{(2a)})^{(1+a^{-1})})/(6*(1+a))$

Rubi in Sympy [A] time = 9.29847, size = 27, normalized size = 0.79

$$\frac{x^{a+1} (2x^{2a} + 3x^a + 6)^{1+\frac{1}{a}}}{6(a+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate((6+3*x**a+2*x**(2*a))**(1/a)*(x**a+x**(2*a)+x**(3*a)), x)

[Out] $x^{(a+1)}(2*x^{(2*a)} + 3*x^a + 6)^{(1+1/a)}/(6*(a+1))$

Mathematica [A] time = 0.0564303, size = 33, normalized size = 0.97

$$\frac{x^{a+1} (2x^{2a} + 3x^a + 6)^{\frac{1}{a}+1}}{6a+6}$$

Antiderivative was successfully verified.

[In] Integrate[(6 + 3*x^a + 2*x^(2*a))^a^(-1)*(x^a + x^(2*a) + x^(3*a)), x]

[Out] $(x^{(1+a)}(6+3x^a+2x^{2a})^{(1+a^{-1})})/(6+6a)$

Maple [A] time = 0.066, size = 44, normalized size = 1.3

$$\frac{xx^a(6+3x^a+2(x^a)^2)\sqrt[a]{6+3x^a+2(x^a)^2}}{6+6a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((6+3*x^a+2*x^(2*a))^(1/a)*(x^a+x^(2*a)+x^(3*a)),x)`

[Out] $1/6*x*x^a*(6+3*x^a+2*(x^a)^2)/(1+a)*(6+3*x^a+2*(x^a)^2)^(1/a)$

Maxima [A] time = 1.76613, size = 65, normalized size = 1.91

$$\frac{(2xx^{3a}+3xx^{2a}+6xx^a)(2x^{2a}+3x^a+6)^{(\frac{1}{a})}}{6(a+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x^(2*a)+3*x^a+6)^(1/a)*(x^(3*a)+x^(2*a)+x^a),x, algorithm="m`

[Out] $1/6*(2*x*x^(3*a)+3*x*x^(2*a)+6*x*x^a)*(2*x^(2*a)+3*x^a+6)^(1/a)/(a+1)$

Fricas [A] time = 0.223727, size = 65, normalized size = 1.91

$$\frac{(2xx^{3a}+3xx^{2a}+6xx^a)(2x^{2a}+3x^a+6)^{(\frac{1}{a})}}{6(a+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x^(2*a)+3*x^a+6)^(1/a)*(x^(3*a)+x^(2*a)+x^a),x, algorithm="f`

[Out] $1/6*(2*x*x^(3*a)+3*x*x^(2*a)+6*x*x^a)*(2*x^(2*a)+3*x^a+6)^(1/a)/(a+1)$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6+3*x**a+2*x**(2*a))**(1/a)*(x**a+x**(2*a)+x**(3*a)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int (2x^{2a} + 3x^a + 6)^{\left(\frac{1}{a}\right)} (x^{3a} + x^{2a} + x^a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^(2*a) + 3*x^a + 6)^(1/a)*(x^(3*a) + x^(2*a) + x^a),x, algorithm="g")
```

```
[Out] integrate((2*x^(2*a) + 3*x^a + 6)^(1/a)*(x^(3*a) + x^(2*a) + x^a), x)
```

$$3.33 \quad \int \frac{1}{x \sqrt[3]{1-x^2}} dx$$

Optimal. Leaf size=58

$$\frac{3}{4} \log\left(1 - \sqrt[3]{1-x^2}\right) + \frac{1}{2} \sqrt{3} \tan^{-1}\left(\frac{2\sqrt[3]{1-x^2} + 1}{\sqrt{3}}\right) - \frac{\log(x)}{2}$$

[Out] (Sqrt[3]*ArcTan[(1 + 2*(1 - x^2)^(1/3))/Sqrt[3]])/2 - Log[x]/2 + (3*Log[1 - (1 - x^2)^(1/3)])/4

Rubi [A] time = 0.0778211, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{3}{4} \log\left(1 - \sqrt[3]{1-x^2}\right) + \frac{1}{2} \sqrt{3} \tan^{-1}\left(\frac{2\sqrt[3]{1-x^2} + 1}{\sqrt{3}}\right) - \frac{\log(x)}{2}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(1 - x^2)^(1/3)), x]

[Out] (Sqrt[3]*ArcTan[(1 + 2*(1 - x^2)^(1/3))/Sqrt[3]])/2 - Log[x]/2 + (3*Log[1 - (1 - x^2)^(1/3)])/4

Rubi in Sympy [A] time = 2.45576, size = 49, normalized size = 0.84

$$-\frac{\log(x^2)}{4} + \frac{3 \log\left(-\sqrt[3]{-x^2+1} + 1\right)}{4} + \frac{\sqrt{3} \operatorname{atan}\left(\sqrt{3} \left(\frac{2\sqrt[3]{-x^2+1}}{3} + \frac{1}{3}\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate(1/x/(-x**2+1)**(1/3), x)

[Out] -log(x**2)/4 + 3*log(-(-x**2 + 1)**(1/3) + 1)/4 + sqrt(3)*atan(sqrt(3)*(2*(-x**2 + 1)**(1/3)/3 + 1/3))/2

Mathematica [C] time = 0.0209582, size = 41, normalized size = 0.71

$$\frac{3 \sqrt[3]{\frac{x^2-1}{x^2}} {}_2F_1\left(\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, \frac{1}{x^2}\right)}{2 \sqrt[3]{1-x^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(1 - x^2)^(1/3)), x]

[Out] $(-3*((-1 + x^2)/x^2)^{(1/3)} * \text{Hypergeometric2F1}[1/3, 1/3, 4/3, x^{(-2)}]) / (2*(1 - x^2)^{(1/3)})$

Maple [C] time = 0.056, size = 65, normalized size = 1.1

$$\frac{\sqrt{3} \left(\frac{2}{3}\right)}{4\pi} \left(\frac{2\pi\sqrt{3}}{3(2/3)} \left(-\frac{\pi\sqrt{3}}{6} - \frac{3\ln(3)}{2} + 2\ln(x) + i\pi \right) + \frac{2\pi\sqrt{3}x^2}{9(2/3)} {}_3F_2\left(1, 1, \frac{4}{3}; 2, 2; x^2\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(-x^2+1)^(1/3), x)

[Out] $1/4/\text{Pi} * 3^{(1/2)} * \text{GAMMA}(2/3) * (2/3 * (-1/6 * \text{Pi} * 3^{(1/2)} - 3/2 * \ln(3) + 2 * \ln(x) + I * \text{Pi}) * \text{Pi} * 3^{(1/2)} / \text{GAMMA}(2/3) + 2/9 * \text{Pi} * 3^{(1/2)} / \text{GAMMA}(2/3) * x^2 * \text{hypergeom}([1, 1, 4/3], [2, 2], x^2))$

Maxima [A] time = 1.49653, size = 84, normalized size = 1.45

$$\frac{1}{2} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} \left(2(-x^2 + 1)^{\frac{1}{3}} + 1\right)\right) - \frac{1}{4} \log\left(\left(-x^2 + 1\right)^{\frac{2}{3}} + \left(-x^2 + 1\right)^{\frac{1}{3}} + 1\right) + \frac{1}{2} \log\left(\left(-x^2 + 1\right)^{\frac{1}{3}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-x^2 + 1)^(1/3)*x), x, algorithm="maxima")

[Out] $1/2 * \text{sqrt}(3) * \text{arctan}(1/3 * \text{sqrt}(3) * (2 * (-x^2 + 1)^{(1/3)} + 1)) - 1/4 * \log(((-x^2 + 1)^{(2/3)} + (-x^2 + 1)^{(1/3)} + 1) + 1/2 * \log((-x^2 + 1)^{(1/3)} - 1)$

Fricas [A] time = 0.21369, size = 84, normalized size = 1.45

$$\frac{1}{2} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} \left(2(-x^2 + 1)^{\frac{1}{3}} + 1\right)\right) - \frac{1}{4} \log\left(\left(-x^2 + 1\right)^{\frac{2}{3}} + \left(-x^2 + 1\right)^{\frac{1}{3}} + 1\right) + \frac{1}{2} \log\left(\left(-x^2 + 1\right)^{\frac{1}{3}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-x^2 + 1)^(1/3)*x), x, algorithm="fricas")

[Out] $\frac{1}{2} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2(-x^2 + 1)^{1/3} + 1)\right) - \frac{1}{4} \log\left(\frac{(-x^2 + 1)^{2/3} + (-x^2 + 1)^{1/3} + 1}{(-x^2 + 1)^{1/3} - 1}\right)$

Sympy [A] time = 1.60745, size = 36, normalized size = 0.62

$$\frac{e^{-\frac{i\pi}{3}} \left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{3} \middle| \frac{1}{x^2}\right)}{2x^{\frac{2}{3}} \left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-x**2+1)**(1/3), x)

[Out] $-\exp(-I\pi/3) \gamma(1/3) \text{hyper}\left(\left(\frac{1}{3}, \frac{1}{3}\right), \left(\frac{4}{3},\right), x^{(-2)}\right) / (2x^{2/3} \gamma(4/3))$

GIAC/XCAS [A] time = 0.20711, size = 86, normalized size = 1.48

$$\frac{1}{2} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} \left(2(-x^2 + 1)^{\frac{1}{3}} + 1\right)\right) - \frac{1}{4} \ln\left(\frac{(-x^2 + 1)^{\frac{2}{3}} + (-x^2 + 1)^{\frac{1}{3}} + 1}{(-x^2 + 1)^{\frac{1}{3}} - 1}\right) + \frac{1}{2} \ln\left(-(-x^2 + 1)^{\frac{1}{3}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-x^2 + 1)^(1/3)*x), x, algorithm="giac")

[Out] $\frac{1}{2} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2(-x^2 + 1)^{1/3} + 1)\right) - \frac{1}{4} \ln\left(\frac{(-x^2 + 1)^{2/3} + (-x^2 + 1)^{1/3} + 1}{(-x^2 + 1)^{1/3} - 1}\right)$

$$3.34 \quad \int \frac{1}{x(1-x^2)^{2/3}} dx$$

Optimal. Leaf size=58

$$\frac{3}{4} \log\left(1 - \sqrt[3]{1-x^2}\right) - \frac{1}{2} \sqrt{3} \tan^{-1}\left(\frac{2\sqrt[3]{1-x^2} + 1}{\sqrt{3}}\right) - \frac{\log(x)}{2}$$

[Out] -(Sqrt[3]*ArcTan[(1 + 2*(1 - x^2)^(1/3))/Sqrt[3]])/2 - Log[x]/2 + (3*Log[1 - (1 - x^2)^(1/3)])/4

Rubi [A] time = 0.069181, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{3}{4} \log\left(1 - \sqrt[3]{1-x^2}\right) - \frac{1}{2} \sqrt{3} \tan^{-1}\left(\frac{2\sqrt[3]{1-x^2} + 1}{\sqrt{3}}\right) - \frac{\log(x)}{2}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(1 - x^2)^(2/3)), x]

[Out] -(Sqrt[3]*ArcTan[(1 + 2*(1 - x^2)^(1/3))/Sqrt[3]])/2 - Log[x]/2 + (3*Log[1 - (1 - x^2)^(1/3)])/4

Rubi in Sympy [A] time = 2.45289, size = 49, normalized size = 0.84

$$-\frac{\log(x^2)}{4} + \frac{3 \log\left(-\sqrt[3]{-x^2+1} + 1\right)}{4} - \frac{\sqrt{3} \operatorname{atan}\left(\sqrt{3} \left(\frac{2\sqrt[3]{-x^2+1}}{3} + \frac{1}{3}\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate(1/x/(-x**2+1)**(2/3), x)

[Out] -log(x**2)/4 + 3*log(-(-x**2 + 1)**(1/3) + 1)/4 - sqrt(3)*atan(sqrt(3)*(2*(-x**2 + 1)**(1/3)/3 + 1/3))/2

Mathematica [C] time = 0.0202607, size = 41, normalized size = 0.71

$$\frac{3 \left(\frac{x^2-1}{x^2}\right)^{2/3} {}_2F_1\left(\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, \frac{1}{x^2}\right)}{4(1-x^2)^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(1 - x^2)^(2/3)), x]

[Out] (-3*((-1 + x^2)/x^2)^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, x^(-2)])/(4*(1 - x^2)^(2/3))

Maple [C] time = 0.063, size = 48, normalized size = 0.8

$$\frac{1}{2(2/3)} \left(\left(\frac{\pi \sqrt{3}}{6} - \frac{3 \ln(3)}{2} + 2 \ln(x) + i\pi \right) \left(\frac{2}{3} \right) + \frac{2(2/3)x^2}{3} {}_3F_2\left(1, 1, \frac{5}{3}; 2, 2; x^2\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(-x^2+1)^(2/3), x)

[Out] 1/2/GAMMA(2/3)*((1/6*Pi*3^(1/2)-3/2*ln(3)+2*ln(x)+I*Pi)*GAMMA(2/3)+2/3*GAMMA(2/3)*x^2*hypergeom([1, 1, 5/3], [2, 2], x^2))

Maxima [A] time = 1.49964, size = 84, normalized size = 1.45

$$-\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2(-x^2+1)^{\frac{1}{3}}+1\right)\right)-\frac{1}{4}\log\left((-x^2+1)^{\frac{2}{3}}+(-x^2+1)^{\frac{1}{3}}+1\right)+\frac{1}{2}\log\left((-x^2+1)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-x^2 + 1)^(2/3)*x), x, algorithm="maxima")

[Out] -1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^2 + 1)^(1/3) + 1)) - 1/4*log((-x^2 + 1)^(2/3) + (-x^2 + 1)^(1/3) + 1) + 1/2*log((-x^2 + 1)^(1/3) - 1)

Fricas [A] time = 0.21626, size = 84, normalized size = 1.45

$$-\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2(-x^2+1)^{\frac{1}{3}}+1\right)\right)-\frac{1}{4}\log\left((-x^2+1)^{\frac{2}{3}}+(-x^2+1)^{\frac{1}{3}}+1\right)+\frac{1}{2}\log\left((-x^2+1)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-x^2 + 1)^(2/3)*x), x, algorithm="fricas")

[Out] $-1/2 \sqrt{3} \arctan(1/3 \sqrt{3} (2 \sqrt{-x^2 + 1}^{1/3} + 1)) - 1/4 \ln((-\sqrt{-x^2 + 1})^{2/3} + (-\sqrt{-x^2 + 1})^{1/3} + 1) + 1/2 \ln((-\sqrt{-x^2 + 1})^{1/3} - 1)$

Sympy [A] time = 1.71959, size = 37, normalized size = 0.64

$$\frac{e^{-\frac{2i\pi}{3}} \left(\frac{2}{3}\right) {}_2F_1\left(\frac{2}{3}, \frac{2}{3} \middle| \frac{1}{x^2}\right)}{2x^{\frac{4}{3}} \left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(-x**2+1)**(2/3), x)`

[Out] $-\exp(-2 \cdot I \cdot \pi / 3) \cdot \text{gamma}(2/3) \cdot \text{hyper}((2/3, 2/3), (5/3,), x^{(-2)}) / (2 \cdot x^{(4/3)} \cdot \text{gamma}(5/3))$

GIAC/XCAS [A] time = 0.204186, size = 86, normalized size = 1.48

$$-\frac{1}{2} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} \left(2 \sqrt{-x^2 + 1}^{1/3} + 1\right)\right) - \frac{1}{4} \ln\left(\left(-x^2 + 1\right)^{2/3} + \left(-x^2 + 1\right)^{1/3} + 1\right) + \frac{1}{2} \ln\left(-\left(-x^2 + 1\right)^{1/3} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-x^2 + 1)^(2/3)*x), x, algorithm="giac")`

[Out] $-1/2 \sqrt{3} \arctan(1/3 \sqrt{3} (2 \sqrt{-x^2 + 1}^{1/3} + 1)) - 1/4 \ln((-\sqrt{-x^2 + 1})^{2/3} + (-\sqrt{-x^2 + 1})^{1/3} + 1) + 1/2 \ln(-(-\sqrt{-x^2 + 1})^{1/3} + 1)$

$$3.35 \quad \int \frac{1}{\sqrt[3]{1-x^3}} dx$$

Optimal. Leaf size=49

$$\frac{1}{2} \log\left(\sqrt[3]{1-x^3} + x\right) - \frac{\tan^{-1}\left(\frac{1 - \sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}}$$

[Out] -(ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3]) + Log[x + (1 - x^3)^(1/3)]/2

Rubi [A] time = 0.0162811, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{1}{2} \log\left(\sqrt[3]{1-x^3} + x\right) - \frac{\tan^{-1}\left(\frac{1 - \sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(1 - x^3)^(-1/3), x]

[Out] -(ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3]) + Log[x + (1 - x^3)^(1/3)]/2

Rubi in Sympy [A] time = 3.58718, size = 71, normalized size = 1.45

$$\frac{\log\left(\frac{x}{\sqrt[3]{-x^3+1}} + 1\right)}{3} - \frac{\log\left(\frac{x^2}{(-x^3+1)^{2/3}} - \frac{x}{\sqrt[3]{-x^3+1}} + 1\right)}{6} + \frac{\sqrt{3} \operatorname{atan}\left(\sqrt{3}\left(\frac{2x}{\sqrt[3]{-x^3+1}} - \frac{1}{3}\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate(1/(-x**3+1)**(1/3), x)

[Out] log(x/(-x**3 + 1)**(1/3) + 1)/3 - log(x**2/(-x**3 + 1)**(2/3) - x/(-x**3 + 1)**(1/3) + 1)/6 + sqrt(3)*atan(sqrt(3)*(2*x/(3*(-x**3 + 1)**(1/3)) - 1/3))/3

Mathematica [A] time = 0.0709057, size = 86, normalized size = 1.76

$$\frac{1}{3} \log\left(\frac{x}{\sqrt[3]{1-x^3}} + 1\right) + \frac{\tan^{-1}\left(\frac{\sqrt[3]{1-x^3}^{-1}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{6} \log\left(-\frac{x}{\sqrt[3]{1-x^3}} + \frac{x^2}{(1-x^3)^{2/3}} + 1\right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^3)^(-1/3), x]

[Out] ArcTan[(-1 + (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[1 + x^2/(1 - x^3)^(2/3) - x/(1 - x^3)^(1/3)]/6 + Log[1 + x/(1 - x^3)^(1/3)]/3

Maple [C] time = 0.034, size = 12, normalized size = 0.2

$$x {}_2F_1\left(\frac{1}{3}, \frac{1}{3}; \frac{4}{3}; x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-x^3+1)^(1/3), x)

[Out] x*hypergeom([1/3, 1/3], [4/3], x^3)

Maxima [A] time = 1.50195, size = 105, normalized size = 2.14

$$-\frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} \left(\frac{2(-x^3+1)^{1/3}}{x} - 1\right)\right) + \frac{1}{3} \log\left(\frac{(-x^3+1)^{1/3}}{x} + 1\right) - \frac{1}{6} \log\left(-\frac{(-x^3+1)^{1/3}}{x} + \frac{(-x^3+1)^{2/3}}{x^2} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^3 + 1)^(-1/3), x, algorithm="maxima")

[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3)/x - 1)) + 1/3*log((-x^3 + 1)^(1/3)/x + 1) - 1/6*log(-(-x^3 + 1)^(1/3)/x + (-x^3 + 1)^(2/3)/x^2 + 1)

Fricas [A] time = 0.216066, size = 122, normalized size = 2.49

$$\frac{1}{18} \sqrt{3} \left(2 \sqrt{3} \log \left(\frac{x + (-x^3 + 1)^{\frac{1}{3}}}{x} \right) - \sqrt{3} \log \left(\frac{x^2 - (-x^3 + 1)^{\frac{1}{3}} x + (-x^3 + 1)^{\frac{2}{3}}}{x^2} \right) - 6 \arctan \left(-\frac{\sqrt{3}x - 2 \sqrt{3}(-x^3 + 1)^{\frac{1}{3}}}{3x} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^3 + 1)^(-1/3), x, algorithm="fricas")

[Out] 1/18*sqrt(3)*(2*sqrt(3)*log((x + (-x^3 + 1)^(1/3))/x) - sqrt(3)*log((x^2 - (-x^3 + 1)^(1/3)*x + (-x^3 + 1)^(2/3))/x^2) - 6*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(-x^3 + 1)^(1/3))/x))

Sympy [A] time = 1.60956, size = 29, normalized size = 0.59

$$\frac{x \left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{3} \middle| \frac{4}{3} \middle| x^3 e^{2i\pi}\right)}{3 \left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x**3+1)**(1/3), x)

[Out] x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), x**3*exp_polar(2*I*pi))/(3*gamma(4/3))

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-x^3 + 1)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^3 + 1)^(-1/3), x, algorithm="giac")

[Out] integrate((-x^3 + 1)^(-1/3), x)

$$3.36 \quad \int \frac{1}{x \sqrt[3]{1-x^3}} dx$$

Optimal. Leaf size=55

$$\frac{1}{2} \log\left(1 - \sqrt[3]{1-x^3}\right) + \frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\log(x)}{2}$$

[Out] ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[x]/2 + Log[1 - (1 - x^3)^(1/3)]/2

Rubi [A] time = 0.0699828, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{1}{2} \log\left(1 - \sqrt[3]{1-x^3}\right) + \frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\log(x)}{2}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(1 - x^3)^(1/3)), x]

[Out] ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[x]/2 + Log[1 - (1 - x^3)^(1/3)]/2

Rubi in Sympy [A] time = 2.29865, size = 48, normalized size = 0.87

$$-\frac{\log(x^3)}{6} + \frac{\log\left(-\sqrt[3]{-x^3+1}+1\right)}{2} + \frac{\sqrt{3} \operatorname{atan}\left(\sqrt{3}\left(\frac{2\sqrt[3]{-x^3+1}}{3} + \frac{1}{3}\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate(1/x/(-x**3+1)**(1/3), x)

[Out] -log(x**3)/6 + log(-(-x**3 + 1)**(1/3) + 1)/2 + sqrt(3)*atan(sqrt(3)*(2*(-x**3 + 1)**(1/3)/3 + 1/3))/3

Mathematica [C] time = 0.0189008, size = 39, normalized size = 0.71

$$\frac{\sqrt[3]{\frac{x^3 - 1}{x^3}} {}_2F_1\left(\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, \frac{1}{x^3}\right)}{\sqrt[3]{1 - x^3}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(1 - x^3)^(1/3)), x]

[Out] -((((-1 + x^3)/x^3)^(1/3)*Hypergeometric2F1[1/3, 1/3, 4/3, x^(-3)])/(1 - x^3)^(1/3))

Maple [C] time = 0.057, size = 65, normalized size = 1.2

$$\frac{\sqrt{3} \left(\frac{2}{3}\right)}{6\pi} \left(\frac{2\pi\sqrt{3}}{3(2/3)} \left(-\frac{\pi\sqrt{3}}{6} - \frac{3\ln(3)}{2} + 3\ln(x) + i\pi \right) + \frac{2\pi\sqrt{3}x^3}{9(2/3)} {}_3F_2\left(1, 1, \frac{4}{3}; 2, 2; x^3\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(-x^3+1)^(1/3), x)

[Out] 1/6/Pi*3^(1/2)*GAMMA(2/3)*(2/3*(-1/6*Pi*3^(1/2)-3/2*ln(3)+3*ln(x)+I*Pi)*Pi*3^(1/2)/GAMMA(2/3)+2/9*Pi*3^(1/2)/GAMMA(2/3)*x^3*hypergeom([1, 1, 4/3], [2, 2], x^3))

Maxima [A] time = 1.47398, size = 84, normalized size = 1.53

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2(-x^3+1)^{\frac{1}{3}}+1\right)\right)-\frac{1}{6}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left(-x^3+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-x^3 + 1)^(1/3)*x), x, algorithm="maxima")

[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) - 1/6*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log((-x^3 + 1)^(1/3) - 1)

Fricas [A] time = 0.214686, size = 96, normalized size = 1.75

$$-\frac{1}{18}\sqrt{3}\left(\sqrt{3}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)-2\sqrt{3}\log\left(\left(-x^3+1\right)^{\frac{1}{3}}-1\right)-6\arctan\left(\frac{2}{3}\sqrt{3}\left(-x^3+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-x^3 + 1)^(1/3)*x),x, algorithm="fricas")`

[Out] $-1/18 \sqrt{3} (\sqrt{3} \log((-x^3 + 1)^{2/3} + (-x^3 + 1)^{1/3} + 1) - 2 \sqrt{3} \log((-x^3 + 1)^{1/3} - 1) - 6 \arctan(2/3 \sqrt{3} (-x^3 + 1)^{1/3} + 1/3 \sqrt{3}))$

Sympy [A] time = 1.60516, size = 32, normalized size = 0.58

$$\frac{e^{-\frac{i\pi}{3}} \left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{3} \middle| \frac{1}{x^3}\right)}{3x \left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(-x**3+1)**(1/3),x)`

[Out] $-\exp(-I \pi / 3) \gamma(1/3) \operatorname{hyper}((1/3, 1/3), (4/3,), x^{(-3)}) / (3 x \gamma(4/3))$

GIAC/XCAS [A] time = 0.20865, size = 85, normalized size = 1.55

$$\frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} \left(2(-x^3 + 1)^{\frac{1}{3}} + 1\right)\right) - \frac{1}{6} \ln\left(\left(-x^3 + 1\right)^{\frac{2}{3}} + \left(-x^3 + 1\right)^{\frac{1}{3}} + 1\right) + \frac{1}{3} \ln\left(\left|(-x^3 + 1)^{\frac{1}{3}} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-x^3 + 1)^(1/3)*x),x, algorithm="giac")`

[Out] $1/3 \sqrt{3} \arctan(1/3 \sqrt{3} (2(-x^3 + 1)^{1/3} + 1)) - 1/6 \ln((-x^3 + 1)^{2/3} + (-x^3 + 1)^{1/3} + 1) + 1/3 \ln(\operatorname{abs}((-x^3 + 1)^{1/3} - 1))$

$$3.37 \quad \int \frac{1}{(1+x)\sqrt[3]{1-x^3}} dx$$

Optimal. Leaf size=121

$$\frac{3 \log\left(2^{2/3}\sqrt[3]{1-x^3} + x - 1\right)}{4\sqrt[3]{2}} - \frac{\sqrt{3} \tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3+2(1-x)}}{2^{2/3}\sqrt{3}\sqrt[3]{1-x^3}}\right)}{2\sqrt[3]{2}} - \frac{\log(1-x)}{4\sqrt[3]{2}} - \frac{\log(x+1)}{2\sqrt[3]{2}}$$

[Out] -(Sqrt[3]*ArcTan[(2*(1-x)+2^(2/3)*(1-x^3)^(1/3))/(2^(2/3)*Sqrt[3]*(1-x^3)^(1/3))]/(2*2^(1/3)) - Log[1-x]/(4*2^(1/3)) - Log[1+x]/(2*2^(1/3)) + (3*Log[-1+x+2^(2/3)*(1-x^3)^(1/3)])/(4*2^(1/3)))

Rubi [A] time = 0.0830356, antiderivative size = 97, normalized size of antiderivative = 0.8, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$

$$\frac{3 \log\left(2^{2/3}\sqrt[3]{1-x^3} + x - 1\right)}{4\sqrt[3]{2}} - \frac{\sqrt{3} \tan^{-1}\left(\frac{\sqrt[3]{2(1-x)} + 1}{\sqrt[3]{1-x^3}}\right)}{2\sqrt[3]{2}} - \frac{\log((1-x)(x+1)^2)}{4\sqrt[3]{2}}$$

Antiderivative was successfully verified.

[In] Int[1/((1+x)*(1-x^3)^(1/3)),x]

[Out] -(Sqrt[3]*ArcTan[(1+(2^(1/3)*(1-x))/(1-x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)) - Log[(1-x)*(1+x)^2]/(4*2^(1/3)) + (3*Log[-1+x+2^(2/3)*(1-x^3)^(1/3)])/(4*2^(1/3)))

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x+1)\sqrt[3]{-x^3+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate(1/(1+x)/(-x**3+1)**(1/3),x)

[Out] Integral(1/((x+1)*(-x**3+1)**(1/3)),x)

Mathematica [A] time = 0.0732422, size = 0, normalized size = 0.

$$\int \frac{1}{(1+x)\sqrt[3]{1-x^3}} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((1 + x)*(1 - x^3)^(1/3)), x]

[Out] Integrate[1/((1 + x)*(1 - x^3)^(1/3)), x]

Maple [F] time = 0.049, size = 0, normalized size = 0.

$$\int \frac{1}{1+x} \frac{1}{\sqrt[3]{-x^3+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1+x)/(-x^3+1)^(1/3), x)

[Out] int(1/(1+x)/(-x^3+1)^(1/3), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-x^3 + 1)^{\frac{1}{3}}(x + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-x^3 + 1)^(1/3)*(x + 1)), x, algorithm="maxima")

[Out] integrate(1/((-x^3 + 1)^(1/3)*(x + 1)), x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-x^3 + 1)^(1/3)*(x + 1)), x, algorithm="fricas")

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{-(x-1)(x^2+x+1)(x+1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+x)/(-x**3+1)**(1/3),x)`

[Out] `Integral(1/((-x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)), x)`

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-x^3+1)^{\frac{1}{3}}(x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-x^3 + 1)^(1/3)*(x + 1)),x, algorithm="giac")`

[Out] `integrate(1/((-x^3 + 1)^(1/3)*(x + 1)), x)`

$$3.38 \quad \int \frac{x}{(1+x)\sqrt[3]{1-x^3}} dx$$

Optimal. Leaf size=145

$$\begin{aligned} & \frac{1}{2} \log\left(\sqrt[3]{1-x^3}+x\right) - \frac{3 \log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{4\sqrt[3]{2}} \\ & + \frac{\sqrt{3} \tan^{-1}\left(\frac{\sqrt[3]{2(1-x)}+1}{\sqrt[3]{1-x^3}}\right)}{2\sqrt[3]{2}} - \frac{\tan^{-1}\left(\frac{1-\sqrt[3]{\frac{2x}{1-x^3}}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\log((1-x)(x+1)^2)}{4\sqrt[3]{2}} \end{aligned}$$

[Out] (Sqrt[3]*ArcTan[(1 + (2^(1/3)*(1 - x)))/(1 - x^3)^(1/3)]/Sqrt[3]) / (2*2^(1/3)) - ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] + Log[(1 - x)*(1 + x)^2]/(4*2^(1/3)) + Log[x + (1 - x^3)^(1/3)]/2 - (3*Log[-1 + x + 2^(2/3)*(1 - x^3)^(1/3)])/(4*2^(1/3))

Rubi [A] time = 0.193547, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$\begin{aligned} & \frac{1}{2} \log\left(\sqrt[3]{1-x^3}+x\right) - \frac{3 \log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{4\sqrt[3]{2}} \\ & + \frac{\sqrt{3} \tan^{-1}\left(\frac{\sqrt[3]{2(1-x)}+1}{\sqrt[3]{1-x^3}}\right)}{2\sqrt[3]{2}} - \frac{\tan^{-1}\left(\frac{1-\sqrt[3]{\frac{2x}{1-x^3}}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\log((1-x)(x+1)^2)}{4\sqrt[3]{2}} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[x/((1 + x)*(1 - x^3)^(1/3)), x]

[Out] (Sqrt[3]*ArcTan[(1 + (2^(1/3)*(1 - x)))/(1 - x^3)^(1/3)]/Sqrt[3]) / (2*2^(1/3)) - ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] + Log[(1 - x)*(1 + x)^2]/(4*2^(1/3)) + Log[x + (1 - x^3)^(1/3)]/2 - (3*Log[-1 + x + 2^(2/3)*(1 - x^3)^(1/3)])/(4*2^(1/3))

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(x+1)\sqrt[3]{-x^3+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x/(1+x)/(-x**3+1)**(1/3),x)`

[Out] `Integral(x/((x + 1)*(-x**3 + 1)**(1/3)), x)`

Mathematica [A] time = 0.0889764, size = 0, normalized size = 0.

$$\int \frac{x}{(1+x)\sqrt[3]{1-x^3}} dx$$

Verification is Not applicable to the result.

[In] `Integrate[x/((1 + x)*(1 - x^3)^(1/3)),x]`

[Out] `Integrate[x/((1 + x)*(1 - x^3)^(1/3)), x]`

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int \frac{x}{1+x} \frac{1}{\sqrt[3]{-x^3+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(1+x)/(-x^3+1)^(1/3),x)`

[Out] `int(x/(1+x)/(-x^3+1)^(1/3),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(-x^3 + 1)^{\frac{1}{3}}(x + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/((-x^3 + 1)^(1/3)*(x + 1)),x, algorithm="maxima")`

[Out] `integrate(x/((-x^3 + 1)^(1/3)*(x + 1)), x)`

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/((-x^3 + 1)^(1/3)*(x + 1)),x, algorithm="fricas")`

[Out] Exception raised: TypeError

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt[3]{-(x-1)(x^2+x+1)}(x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(1+x)/(-x**3+1)**(1/3),x)`

[Out] `Integral(x/((-x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)), x)`

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(-x^3 + 1)^{\frac{1}{3}}(x + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/((-x^3 + 1)^(1/3)*(x + 1)),x, algorithm="giac")`

[Out] `integrate(x/((-x^3 + 1)^(1/3)*(x + 1)), x)`

$$3.39 \quad \int \frac{1}{x \sqrt[3]{2 - 3x + x^2}} dx$$

Optimal. Leaf size=110

$$\frac{3 \log\left(-2^{2/3} \sqrt[3]{x^2 - 3x + 2} - x + 2\right)}{4 \sqrt[3]{2}} - \frac{\sqrt{3} \tan^{-1}\left(\frac{\sqrt[3]{2(2-x)}}{\sqrt{3} \sqrt[3]{x^2 - 3x + 2}} + \frac{1}{\sqrt{3}}\right)}{2 \sqrt[3]{2}} - \frac{\log(2-x)}{4 \sqrt[3]{2}} - \frac{\log(x)}{2 \sqrt[3]{2}}$$

[Out] -(Sqrt[3]*ArcTan[1/Sqrt[3] + (2^(1/3)*(2 - x))/(Sqrt[3]*(2 - 3*x + x^2)^(1/3))]/(2*2^(1/3)) - Log[2 - x]/(4*2^(1/3)) - Log[x]/(2*2^(1/3)) + (3*Log[2 - x - 2^(2/3)*(2 - 3*x + x^2)^(1/3)])/(4*2^(1/3)))

Rubi [A] time = 0.0992248, antiderivative size = 176, normalized size of antiderivative = 1.6, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$\frac{3 \sqrt[3]{x-2} \sqrt[3]{x-1} \log\left(-\frac{(x-2)^{2/3}}{\sqrt[3]{2}} - \sqrt[3]{2} \sqrt[3]{x-1}\right)}{4 \sqrt[3]{2} \sqrt[3]{x^2 - 3x + 2}} - \frac{\sqrt[3]{x-2} \sqrt[3]{x-1} \log(x)}{2 \sqrt[3]{2} \sqrt[3]{x^2 - 3x + 2}} - \frac{\sqrt{3} \sqrt[3]{x-2} \sqrt[3]{x-1} \tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{\sqrt[3]{2}(x-2)^{2/3}}{\sqrt{3} \sqrt[3]{x-1}}\right)}{2 \sqrt[3]{2} \sqrt[3]{x^2 - 3x + 2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(2 - 3*x + x^2)^(1/3)), x]

[Out] -(Sqrt[3]*(-2 + x)^(1/3)*(-1 + x)^(1/3)*ArcTan[1/Sqrt[3] - (2^(1/3)*(-2 + x)^(2/3))/(Sqrt[3]*(-1 + x)^(1/3))]/(2*2^(1/3)*(2 - 3*x + x^2)^(1/3)) + (3*(-2 + x)^(1/3)*(-1 + x)^(1/3)*Log[-((-2 + x)^(2/3)/2^(1/3)) - 2^(1/3)*(-1 + x)^(1/3)]/(4*2^(1/3)*(2 - 3*x + x^2)^(1/3)) - ((-2 + x)^(1/3)*(-1 + x)^(1/3)*Log[x])/(2*2^(1/3)*(2 - 3*x + x^2)^(1/3)))

Rubi in Sympy [A] time = 4.67779, size = 155, normalized size = 1.41

$$\frac{\sqrt[3]{2x-4} \sqrt[3]{2x-2} \log(x)}{4 \sqrt[3]{x^2 - 3x + 2}} + \frac{3 \sqrt[3]{2x-4} \sqrt[3]{2x-2} \log\left(-\frac{(2x-4)^{2/3}}{2} - \sqrt[3]{2x-2}\right)}{8 \sqrt[3]{x^2 - 3x + 2}} + \frac{\sqrt{3} \sqrt[3]{2x-4} \sqrt[3]{2x-2} \operatorname{atan}\left(\frac{\sqrt{3}(2x-4)^{2/3}}{3 \sqrt[3]{2x-2}} - \frac{\sqrt{3}}{3}\right)}{4 \sqrt[3]{x^2 - 3x + 2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/x/(x**2-3*x+2)**(1/3),x)`

[Out] $-(2x - 4)^{1/3} (2x - 2)^{1/3} \log(x) / (4(x^2 - 3x + 2)^{1/3}) + 3(2x - 4)^{1/3} (2x - 2)^{1/3} \log(-(2x - 4)^{2/3} / (2 - (2x - 2)^{1/3})) / (8(x^2 - 3x + 2)^{1/3}) + \sqrt{3} (2x - 4)^{1/3} (2x - 2)^{1/3} \operatorname{atan}(\sqrt{3} (2x - 4)^{2/3} / (3(2x - 2)^{1/3})) - \sqrt{3} / 3 / (4(x^2 - 3x + 2)^{1/3})$

Mathematica [C] time = 0.19044, size = 109, normalized size = 0.99

$$-\frac{15x F_1\left(\frac{2}{3}; \frac{1}{3}, \frac{1}{3}; \frac{5}{3}; \frac{1}{x}, \frac{2}{x}\right)}{2\sqrt[3]{x^2 - 3x + 2} \left(5x F_1\left(\frac{2}{3}; \frac{1}{3}, \frac{1}{3}; \frac{5}{3}; \frac{1}{x}, \frac{2}{x}\right) + 2F_1\left(\frac{5}{3}; \frac{1}{3}, \frac{4}{3}; \frac{8}{3}; \frac{1}{x}, \frac{2}{x}\right) + F_1\left(\frac{5}{3}; \frac{4}{3}, \frac{1}{3}; \frac{8}{3}; \frac{1}{x}, \frac{2}{x}\right)\right)}$$

Warning: Unable to verify antiderivative.

[In] `Integrate[1/(x*(2 - 3*x + x^2)^(1/3)),x]`

[Out] $(-15x \operatorname{AppellF1}[2/3, 1/3, 1/3, 5/3, x^{-1}, 2/x]) / (2(2 - 3x + x^2)^{1/3} (5x \operatorname{AppellF1}[2/3, 1/3, 1/3, 5/3, x^{-1}, 2/x] + 2 \operatorname{AppellF1}[5/3, 1/3, 4/3, 8/3, x^{-1}, 2/x] + \operatorname{AppellF1}[5/3, 4/3, 1/3, 8/3, x^{-1}, 2/x]))$

Maple [F] time = 0.098, size = 0, normalized size = 0.

$$\int \frac{1}{x \sqrt[3]{x^2 - 3x + 2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/(x^2-3*x+2)^(1/3),x)`

[Out] `int(1/x/(x^2-3*x+2)^(1/3),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2 - 3x + 2)^{\frac{1}{3}} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((x^2 - 3*x + 2)^(1/3)*x),x, algorithm="maxima")`

[Out] `integrate(1/((x^2 - 3*x + 2)^(1/3)*x), x)`

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((x^2 - 3*x + 2)^(1/3)*x),x, algorithm="fricas")`

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt[3]{(x-2)(x-1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(x**2-3*x+2)**(1/3),x)`

[Out] `Integral(1/(x*((x - 2)*(x - 1))**(1/3)), x)`

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2 - 3x + 2)^{\frac{1}{3}}x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((x^2 - 3*x + 2)^(1/3)*x),x, algorithm="giac")`

[Out] `integrate(1/((x^2 - 3*x + 2)^(1/3)*x), x)`

$$3.40 \quad \int \frac{1}{\sqrt[3]{-5 + 7x - 3x^2 + x^3}} dx$$

Optimal. Leaf size=81

$$-\frac{3}{4} \log\left(\sqrt[3]{x^3 - 3x^2 + 7x - 5} - x + 1\right) + \frac{1}{2} \sqrt{3} \tan^{-1}\left(\frac{2(x-1)}{\sqrt{3}\sqrt[3]{x^3 - 3x^2 + 7x - 5}} + \frac{1}{\sqrt{3}}\right) + \frac{1}{4} \log(1-x)$$

[Out] (Sqrt[3]*ArcTan[1/Sqrt[3] + (2*(-1 + x))/(Sqrt[3]*(-5 + 7*x - 3*x^2 + x^3)^(1/3))])/2 + Log[1 - x]/4 - (3*Log[1 - x + (-5 + 7*x - 3*x^2 + x^3)^(1/3)])/4

Rubi [A] time = 0.139111, antiderivative size = 131, normalized size of antiderivative = 1.62, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$

$$\frac{\sqrt{3}\sqrt[3]{(x-1)^2 + 4}\sqrt[3]{x-1} \tan^{-1}\left(\frac{\frac{2(x-1)^{2/3} + 1}{\sqrt[3]{(x-1)^2 + 4}}}{\sqrt{3}}\right)}{2\sqrt[3]{(x-1)^3 - 4(1-x)}} - \frac{3\sqrt[3]{(x-1)^2 + 4}\sqrt[3]{x-1} \log\left((x-1)^{2/3} - \sqrt[3]{(x-1)^2 + 4}\right)}{4\sqrt[3]{(x-1)^3 - 4(1-x)}}$$

Antiderivative was successfully verified.

[In] Int[(-5 + 7*x - 3*x^2 + x^3)^(-1/3), x]

[Out] (Sqrt[3]*(4 + (-1 + x)^2)^(1/3)*(-1 + x)^(1/3)*ArcTan[(1 + (2*(-1 + x)^(2/3)))/(4 + (-1 + x)^2)^(1/3)]/Sqrt[3])/(2*(-4*(1 - x) + (-1 + x)^3)^(1/3)) - (3*(4 + (-1 + x)^2)^(1/3)*(-1 + x)^(1/3)*Log[-(4 + (-1 + x)^2)^(1/3) + (-1 + x)^(2/3)])/((4*(-4*(1 - x) + (-1 + x)^3)^(1/3))

Rubi in Sympy [A] time = 7.39252, size = 204, normalized size = 2.52

$$\frac{\sqrt[3]{x-1}\sqrt[3]{x^2-2x+5}\log\left(-\frac{(x-1)^{\frac{2}{3}}}{\sqrt[3]{(x-1)^2+4}}+1\right)}{2\sqrt[3]{x^3-3x^2+7x-5}} + \frac{\sqrt[3]{x-1}\sqrt[3]{x^2-2x+5}\log\left(\frac{(x-1)^{\frac{4}{3}}}{((x-1)^2+4)^{\frac{2}{3}}}+\frac{(x-1)^{\frac{2}{3}}}{\sqrt[3]{(x-1)^2+4}}+1\right)}{4\sqrt[3]{x^3-3x^2+7x-5}} + \frac{\sqrt{3}\sqrt[3]{x-1}\sqrt[3]{x^2-2x+5}\operatorname{atan}\left(\sqrt{3}\left(\frac{2(x-1)^{\frac{2}{3}}}{\sqrt[3]{(x-1)^2+4}}+\frac{1}{3}\right)\right)}{2\sqrt[3]{x^3-3x^2+7x-5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/(x**3-3*x**2+7*x-5)**(1/3),x)`

[Out] $-(x-1)^{\frac{1}{3}}(x^2-2x+5)^{\frac{1}{3}}\log\left(-\frac{(x-1)^{\frac{2}{3}}}{((x-1)^2+4)^{\frac{1}{3}}}+1\right)/(2(x^3-3x^2+7x-5)^{\frac{1}{3}}) + (x-1)^{\frac{1}{3}}(x^2-2x+5)^{\frac{1}{3}}\log\left(\frac{(x-1)^{\frac{4}{3}}}{((x-1)^2+4)^{\frac{2}{3}}}+\frac{(x-1)^{\frac{2}{3}}}{\sqrt[3]{(x-1)^2+4}}+1\right)/(4(x^3-3x^2+7x-5)^{\frac{1}{3}}) + \sqrt{3}(x-1)^{\frac{1}{3}}(x^2-2x+5)^{\frac{1}{3}}\operatorname{atan}\left(\sqrt{3}\left(\frac{2(x-1)^{\frac{2}{3}}}{\sqrt[3]{(x-1)^2+4}}+\frac{1}{3}\right)\right)/(2(x^3-3x^2+7x-5)^{\frac{1}{3}})$

Mathematica [C] time = 0.0224462, size = 85, normalized size = 1.05

$$\frac{3\sqrt[3]{ix+(2-i)}\sqrt[3]{i(x-1)}(x-(1-2i))F_1\left(\frac{2}{3};\frac{1}{3},\frac{1}{3},\frac{5}{3};-\frac{1}{4}i(x-(1-2i)),-\frac{1}{2}i(x-(1-2i))\right)}{4\sqrt[3]{x^3-3x^2+7x-5}}$$

Warning: Unable to verify antiderivative.

[In] `Integrate[(-5 + 7*x - 3*x^2 + x^3)^(-1/3),x]`

[Out] $(3((2-I)+Ix)^{\frac{1}{3}}(I(-1+x))^{\frac{1}{3}}((-1+2I)+x)\operatorname{AppellF1}\left[\frac{2}{3},\frac{1}{3},\frac{1}{3},\frac{5}{3},(-I/4)((-1+2I)+x),(-I/2)((-1+2I)+x)\right])/(4(-5+7x-3x^2+x^3)^{\frac{1}{3}})$

Maple [F] time = 0.019, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{x^3-3x^2+7x-5}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^3-3*x^2+7*x-5)^(1/3), x)`

[Out] `int(1/(x^3-3*x^2+7*x-5)^(1/3), x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^3 - 3x^2 + 7x - 5)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^3 - 3*x^2 + 7*x - 5)^(-1/3), x, algorithm="maxima")`

[Out] `integrate((x^3 - 3*x^2 + 7*x - 5)^(-1/3), x)`

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^3 - 3*x^2 + 7*x - 5)^(-1/3), x, algorithm="fricas")`

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{x^3 - 3x^2 + 7x - 5}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x**3-3*x**2+7*x-5)**(1/3), x)`

[Out] `Integral((x**3 - 3*x**2 + 7*x - 5)**(-1/3), x)`

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^3 - 3x^2 + 7x - 5)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3 - 3*x^2 + 7*x - 5)^(-1/3), x, algorithm="giac")
```

```
[Out] integrate((x^3 - 3*x^2 + 7*x - 5)^(-1/3), x)
```

$$3.41 \quad \int \frac{1}{\sqrt[3]{x(-q+x^2)}} dx$$

Optimal. Leaf size=66

$$-\frac{3}{4} \log\left(\sqrt[3]{x(x^2-q)} - x\right) + \frac{1}{2} \sqrt{3} \tan^{-1}\left(\frac{2x}{\sqrt{3}\sqrt[3]{x(x^2-q)}} + \frac{1}{\sqrt{3}}\right) + \frac{\log(x)}{4}$$

[Out] (Sqrt[3]*ArcTan[1/Sqrt[3] + (2*x)/(Sqrt[3]*(x*(-q + x^2))^(1/3))])/2 + Log[x]/4 - (3*Log[-x + (x*(-q + x^2))^(1/3)])/4

Rubi [A] time = 0.112684, antiderivative size = 117, normalized size of antiderivative = 1.77, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$

$$\frac{\sqrt{3}\sqrt[3]{x}\sqrt[3]{x^2-q} \tan^{-1}\left(\frac{\frac{2x^{2/3}}{\sqrt[3]{x^2-q}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{x^3-qx}} - \frac{3\sqrt[3]{x}\sqrt[3]{x^2-q} \log\left(x^{2/3} - \sqrt[3]{x^2-q}\right)}{4\sqrt[3]{x^3-qx}}$$

Antiderivative was successfully verified.

[In] Int[(x*(-q + x^2))^(1/3), x]

[Out] (Sqrt[3]*x^(1/3)*(-q + x^2)^(1/3)*ArcTan[(1 + (2*x^(2/3)))/(-q + x^2)^(1/3)]/Sqrt[3])/2 + (-q*x + x^3)^(1/3) - (3*x^(1/3)*(-q + x^2)^(1/3)*Log[x^(2/3) - (-q + x^2)^(1/3)])/4 + (-q*x + x^3)^(1/3)

Rubi in Sympy [A] time = 5.45697, size = 155, normalized size = 2.35

$$\frac{(-qx + x^3)^{\frac{2}{3}} \log\left(-\frac{x^{\frac{2}{3}}}{\sqrt[3]{-q+x^2}} + 1\right)}{2x^{\frac{2}{3}}(-q+x^2)^{\frac{2}{3}}} + \frac{(-qx + x^3)^{\frac{2}{3}} \log\left(\frac{x^{\frac{4}{3}}}{(-q+x^2)^{\frac{2}{3}}} + \frac{x^{\frac{2}{3}}}{\sqrt[3]{-q+x^2}} + 1\right)}{4x^{\frac{2}{3}}(-q+x^2)^{\frac{2}{3}}} + \frac{\sqrt{3}(-qx + x^3)^{\frac{2}{3}} \operatorname{atan}\left(\sqrt{3}\left(\frac{2x^{\frac{2}{3}}}{3\sqrt[3]{-q+x^2}} + \frac{1}{3}\right)\right)}{2x^{\frac{2}{3}}(-q+x^2)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate(1/(x*(x**2-q))**(1/3), x)

[Out] $-(-q^*x + x^{**3})^{** (2/3)} * \log(-x^{** (2/3)} / (-q + x^{**2})^{** (1/3)} + 1) / (2^*x^{** (2/3)} * (-q + x^{**2})^{** (2/3)}) + (-q^*x + x^{**3})^{** (2/3)} * \log(x^{** (4/3)} / (-q + x^{**2})^{** (2/3)} + x^{** (2/3)} / (-q + x^{**2})^{** (1/3)} + 1) / (4^*x^{** (2/3)} * (-q + x^{**2})^{** (2/3)}) + \text{sqrt}(3) * (-q^*x + x^{**3})^{** (2/3)} * \text{atan}(\text{sqrt}(3) * (2^*x^{** (2/3)} / (3^* (-q + x^{**2})^{** (1/3)}) + 1/3)) / (2^*x^{** (2/3)} * (-q + x^{**2})^{** (2/3)})$

Mathematica [C] time = 0.0347719, size = 49, normalized size = 0.74

$$\frac{3x\sqrt[3]{\frac{q-x^2}{q}} {}_2F_1\left(\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, \frac{x^2}{q}\right)}{2\sqrt[3]{x^3 - qx}}$$

Antiderivative was successfully verified.

[In] Integrate[(x*(-q + x^2))^(1/3), x]

[Out] $(3^*x^*((q - x^2)/q)^{(1/3)} * \text{Hypergeometric2F1}[1/3, 1/3, 4/3, x^2/q]) / (2^*(-(q^*x) + x^3)^{(1/3)})$

Maple [F] time = 0.064, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{x(x^2 - q)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(x^2-q))^(1/3), x)

[Out] int(1/(x*(x^2-q))^(1/3), x)

Maxima [A] time = 1.52352, size = 104, normalized size = 1.58

$$-\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2(x^2 - q)^{\frac{1}{3}}}{x^{\frac{2}{3}}} + 1\right)\right) + \frac{1}{4}\log\left(\frac{(x^2 - q)^{\frac{1}{3}}}{x^{\frac{2}{3}}} + \frac{(x^2 - q)^{\frac{2}{3}}}{x^{\frac{4}{3}}} + 1\right) - \frac{1}{2}\log\left(\frac{(x^2 - q)^{\frac{1}{3}}}{x^{\frac{2}{3}}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((x^2 - q)*x)^(1/3), x, algorithm="maxima")

```
[Out] -1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^2 - q)^(1/3)/x^(2/3) + 1))
+ 1/4*log((x^2 - q)^(1/3)/x^(2/3) + (x^2 - q)^(2/3)/x^(4/3) + 1)
- 1/2*log((x^2 - q)^(1/3)/x^(2/3) - 1)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^2 - q)*x)^(-1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{x(-q+x^2)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(x**2-q))**(1/3),x)
```

```
[Out] Integral((x*(-q + x**2))**(-1/3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{((x^2 - q)x)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^2 - q)*x)^(-1/3),x, algorithm="giac")
```

```
[Out] integrate(((x^2 - q)*x)^(-1/3), x)
```


$$3.42 \quad \int \frac{1}{\sqrt[3]{(-1+x)(q-2x+x^2)}} dx$$

Optimal. Leaf size=79

$$-\frac{3}{4} \log\left(\sqrt[3]{(x-1)(q+x^2-2x)} - x + 1\right) + \frac{1}{2}\sqrt{3} \tan^{-1}\left(\frac{2(x-1)}{\sqrt{3}\sqrt[3]{(x-1)(q+x^2-2x)}} + \frac{1}{\sqrt{3}}\right) + \frac{1}{4} \log(1-x)$$

[Out] (Sqrt[3]*ArcTan[1/Sqrt[3] + (2*(-1+x))/(Sqrt[3]*((-1+x)*(q-2*x+x^2))^(1/3))])/2 + Log[1-x]/4 - (3*Log[1-x+((-1+x)*(q-2*x+x^2))^(1/3)])/4

Rubi [A] time = 0.187644, antiderivative size = 145, normalized size of antiderivative = 1.84, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$

$$\frac{\sqrt{3}\sqrt[3]{x-1}\sqrt[3]{q+(x-1)^2-1} \tan^{-1}\left(\frac{\frac{2(x-1)^{2/3}}{\sqrt{3}\sqrt[3]{q+(x-1)^2-1}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{(1-q)(1-x)+(x-1)^3}} - \frac{3\sqrt[3]{x-1}\sqrt[3]{q+(x-1)^2-1} \log\left((x-1)^{2/3}-\sqrt[3]{q+(x-1)^2-1}\right)}{4\sqrt[3]{(1-q)(1-x)+(x-1)^3}}$$

Antiderivative was successfully verified.

[In] Int[((-1+x)*(q-2*x+x^2))^(-1/3),x]

[Out] (Sqrt[3]*(-1+q+(-1+x)^2)^(1/3)*(-1+x)^(1/3)*ArcTan[(1+(2*(-1+x)^(2/3))/(-1+q+(-1+x)^2)^(1/3))/Sqrt[3]])/(2*((1-q)*(1-x)+(-1+x)^3)^(1/3)) - (3*(-1+q+(-1+x)^2)^(1/3)*(-1+x)^(1/3)*Log[-(-1+q+(-1+x)^2)^(1/3)+(-1+x)^(2/3)])/((4*((1-q)*(1-x)+(-1+x)^3)^(1/3)))

Rubi in Sympy [A] time = 9.5927, size = 216, normalized size = 2.73

$$\frac{\sqrt[3]{x-1}\sqrt[3]{q+x^2}-2x \log\left(-\frac{(x-1)^{\frac{2}{3}}}{\sqrt[3]{q+(x-1)^2-1}}+1\right)}{2\sqrt[3]{-q+x^3-3x^2+x(q+2)}} + \frac{\sqrt[3]{x-1}\sqrt[3]{q+x^2}-2x \log\left(\frac{(x-1)^{\frac{4}{3}}}{(q+(x-1)^2-1)^{\frac{2}{3}}}+\frac{(x-1)^{\frac{2}{3}}}{\sqrt[3]{q+(x-1)^2-1}}+1\right)}{4\sqrt[3]{-q+x^3-3x^2+x(q+2)}} + \frac{\sqrt{3}\sqrt[3]{x-1}\sqrt[3]{q+x^2}-2x \operatorname{atan}\left(\sqrt{3}\left(\frac{2(x-1)^{\frac{2}{3}}}{\sqrt[3]{q+(x-1)^2-1}}+\frac{1}{3}\right)\right)}{2\sqrt[3]{-q+x^3-3x^2+x(q+2)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/((-1+x)*(x**2+q-2*x))**(1/3),x)`

[Out] $-(x-1)^{\frac{1}{3}}(q+x^2-2x)^{\frac{1}{3}}\log(-(x-1)^{\frac{2}{3}}/(q+(x-1)^2-1)^{\frac{1}{3}}+1)/(2(-q+x^3-3x^2+x(q+2))^{\frac{1}{3}}) + (x-1)^{\frac{1}{3}}(q+x^2-2x)^{\frac{1}{3}}\log((x-1)^{\frac{4}{3}}/(q+(x-1)^2-1)^{\frac{2}{3}}+(x-1)^{\frac{2}{3}}/\sqrt[3]{q+(x-1)^2-1}+1)/(4(-q+x^3-3x^2+x(q+2))^{\frac{1}{3}}) + \sqrt{3}(x-1)^{\frac{1}{3}}(q+x^2-2x)^{\frac{1}{3}}\operatorname{atan}(\sqrt{3}(2(x-1)^{\frac{2}{3}}/(3(q+(x-1)^2-1)^{\frac{1}{3}})+\frac{1}{3}))/2(-q+x^3-3x^2+x(q+2))^{\frac{1}{3}}$

Mathematica [C] time = 0.0579134, size = 61, normalized size = 0.77

$$\frac{3(x-1)\sqrt[3]{\frac{q+(x-2)x}{q-1}} {}_2F_1\left(\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, -\frac{(x-1)^2}{q-1}\right)}{2\sqrt[3]{(x-1)(q+(x-2)x)}}$$

Antiderivative was successfully verified.

[In] `Integrate[((-1+x)*(q-2*x+x^2))^(1/3),x]`

[Out] $(3(-1+x)((q+(-2+x)x)/(-1+q))^{1/3}\operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, -\frac{(x-1)^2}{q-1}\right])/2(-1+x)(q+(-2+x)x)^{1/3}$

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{(-1+x)(x^2+q-2x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((-1+x)*(x^2+q-2*x))^(1/3),x)

[Out] int(1/((-1+x)*(x^2+q-2*x))^(1/3),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{((x^2+q-2x)(x-1))^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((x^2 + q - 2*x)*(x - 1))^(1/3),x, algorithm="maxima")

[Out] integrate(((x^2 + q - 2*x)*(x - 1))^(1/3), x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((x^2 + q - 2*x)*(x - 1))^(1/3),x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-1+x)*(x**2+q-2*x))**(1/3),x)

[Out] Timed out

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{((x^2 + q - 2x)(x - 1))^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(((x^2 + q - 2*x)*(x - 1))^-1/3),x, algorithm="giac")`

[Out] `integrate(((x^2 + q - 2*x)*(x - 1))^-1/3), x)`

$$3.43 \quad \int \frac{1}{x^3 \sqrt[3]{(-1+x)(q-2qx+x^2)}} dx$$

Optimal. Leaf size=118

$$\frac{3 \log\left(\sqrt[3]{(x-1)(-2qx+q+x^2)} - \sqrt[3]{q}(x-1)\right)}{4\sqrt[3]{q}} + \frac{\sqrt{3} \tan^{-1}\left(\frac{2\sqrt[3]{q}(x-1)}{\sqrt{3}\sqrt[3]{(x-1)(-2qx+q+x^2)} + \frac{1}{\sqrt{3}}}\right)}{2\sqrt[3]{q}} + \frac{\log(1-x)}{4\sqrt[3]{q}} + \frac{\log(x)}{2\sqrt[3]{q}}$$

[Out] (Sqrt[3]*ArcTan[1/Sqrt[3] + (2*q^(1/3)*(-1 + x))/(Sqrt[3]*((-1 + x)*(q - 2*q*x + x^2))^(1/3))]/(2*q^(1/3)) + Log[1 - x]/(4*q^(1/3)) + Log[x]/(2*q^(1/3)) - (3*Log[-(q^(1/3)*(-1 + x)) + ((-1 + x)*(q - 2*q*x + x^2))^(1/3))]/(4*q^(1/3)))

Rubi [F] time = 27.6796, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0$.

$$\text{Int}\left(\frac{1}{x^3 \sqrt[3]{(-1+x)(q-2qx+x^2)}}, x\right)$$

Verification is Not applicable to the result.

[In] Int[1/(x*((-1 + x)*(q - 2*q*x + x^2))^(1/3)), x]

[Out] ((-1 - 2*q - (1 - 5*q + 4*q^2 + (1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(2/3)))/(1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(1/3) + 3*x)^(1/3)*(-1 + 5*q - 4*q^2 + ((1 - 4*q)^2*(1 - q)^2)/(1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(2/3) + (1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(2/3) - ((1 - 5*q + 4*q^2 + (1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(2/3))*(1 + 2*q - 3*x))/(1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(1/3) + (-1 - 2*q + 3*x)^2)^(1/3)*Defer[Subst][Defer[Int][1/(((1 + 2*q)/3 + x)*(-1 - 5*q + 4*q^2 + (1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(2/3)))/(3*(1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(1/3)) + x)^(1/3)*((-1 + 5*q - 4*q^2 + ((1 - 4*q)^2*(1 - q)^2)/(1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(2/3) + (1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(2/3))/9 + (((1 - 5*q + 4*q^2 + (1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(2/3))*x)/(3*(1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(1/3)) + x^2)^(1/3)), x], x, (-1 - 2*q)/3 + x]/(3*(-q + 3*q*x - (1 + 2*q)*x^2 + x^3)^(1/3))

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt[3]{x-1}\sqrt[3]{-2qx+q+x^2} \int \frac{1}{x\sqrt[3]{x-1}\sqrt[3]{-2qx+q+x^2}} dx}{\sqrt[3]{3qx-q+x^3+x^2(-2q-1)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/x/((-1+x)*(-2*q*x+x**2+q))**(1/3),x)`

[Out] `(x - 1)**(1/3)*(-2*q*x + q + x**2)**(1/3)*Integral(1/(x*(x - 1)**(1/3)*(-2*q*x + q + x**2)**(1/3)), x)/(3*q*x - q + x**3 + x**2*(-2*q - 1))**(1/3)`

Mathematica [C] time = 0.22792, size = 72, normalized size = 0.61

$$\frac{3(x-1)^3 \sqrt[3]{-\frac{-2qx+q+x^2}{(q-1)x^2}} {}_2F_1\left(\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, \frac{q(x-1)^2}{(q-1)x^2}\right)}{2\sqrt[3]{(x-1)(-2qx+q+x^2)}}$$

Antiderivative was successfully verified.

[In] `Integrate[1/(x*((-1+x)*(q-2*q*x+x^2))^(1/3)),x]`

[Out] `(3*(-1+x)*(-(q-2*q*x+x^2)/((-1+q)*x^2)))^(1/3)*Hypergeometric2F1[1/3, 1/3, 4/3, (q*(-1+x)^2)/((-1+q)*x^2)]/(2*((-1+x)*(q-2*q*x+x^2))^(1/3))`

Maple [F] time = 0.028, size = 0, normalized size = 0.

$$\int \frac{1}{x \sqrt[3]{(-1+x)(-2qx+x^2+q)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/((-1+x)*(-2*q*x+x^2+q))^(1/3),x)`

[Out] `int(1/x/((-1+x)*(-2*q*x+x^2+q))^(1/3),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-2qx - x^2 - q)(x-1)^{\frac{1}{3}} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-2*q*x - x^2 - q)*(x - 1))^(1/3)*x), x, algorithm="maxima")

[Out] integrate(1/((-2*q*x - x^2 - q)*(x - 1))^(1/3)*x), x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-2*q*x - x^2 - q)*(x - 1))^(1/3)*x), x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/((-1+x)*(-2*q*x+x**2+q))**(1/3), x)

[Out] Timed out

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-2qx - x^2 - q)(x-1)^{\frac{1}{3}} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-2*q*x - x^2 - q)*(x - 1))^(1/3)*x), x, algorithm="giac")

[Out] integrate(1/((-2*q*x - x^2 - q)*(x - 1))^(1/3)*x), x)

$$3.44 \quad \int \frac{2-(1+k)x}{\sqrt[3]{(1-x)x(1-kx)(1-(1+k)x)}} dx$$

Optimal. Leaf size=111

$$\frac{\log(x)}{2\sqrt[3]{k}} + \frac{\log(1-(k+1)x)}{2\sqrt[3]{k}} - \frac{3\log\left(\sqrt[3]{(1-x)x(1-kx)} - \sqrt[3]{kx}\right)}{2\sqrt[3]{k}} + \frac{\sqrt{3} \tan^{-1}\left(\frac{\frac{2\sqrt[3]{kx}}{\sqrt[3]{(1-x)x(1-kx)}}+1}{\sqrt{3}}\right)}{\sqrt[3]{k}}$$

[Out] (Sqrt[3]*ArcTan[(1+(2*k^(1/3)*x)/((1-x)*x*(1-k*x))^(1/3))]/Sqrt[3])/k^(1/3)+Log[x]/(2*k^(1/3))+Log[1-(1+k)*x]/(2*k^(1/3))- (3*Log[-(k^(1/3)*x)+((1-x)*x*(1-k*x))^(1/3)])/(2*k^(1/3))

Rubi [F] time = 1.33793, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0$.

$$\text{Int}\left(\frac{2-(1+k)x}{\sqrt[3]{(1-x)x(1-kx)(1-(1+k)x)}}, x\right)$$

Verification is Not applicable to the result.

[In] Int[(2-(1+k)*x)/(((1-x)*x*(1-k*x))^(1/3)*(1-(1+k)*x)),x]

[Out] (3*(1-x)^(1/3)*x*(1-k*x)^(1/3)*AppellF1[2/3, 1/3, 1/3, 5/3, x, k*x])/((2*((1-x)*x*(1-k*x))^(1/3))+((1-x)^(1/3)*x^(1/3)*(1-k*x)^(1/3)*Defer[Int][1/((1-x)^(1/3)*x^(1/3)*(1+(-1-k)*x*(1-k*x)^(1/3)),x])/((1-x)*x*(1-k*x))^(1/3))

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate((2-(1+k)*x)/(((1-x)*x*(-k*x+1))^(1/3)/(1-(1+k)*x)),x)

[Out] Timed out

Mathematica [A] time = 1.23486, size = 0, normalized size = 0.

$$\int \frac{2 - (1 + k)x}{\sqrt[3]{(1 - x)x(1 - kx)(1 - (1 + k)x)}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(2 - (1 + k)*x)/(((1 - x)*x*(1 - k*x))^(1/3)*(1 - (1 + k)*x)), x]

[Out] Integrate[(2 - (1 + k)*x)/(((1 - x)*x*(1 - k*x))^(1/3)*(1 - (1 + k)*x)), x]

Maple [F] time = 0.062, size = 0, normalized size = 0.

$$\int \frac{2 - (1 + k)x}{1 - (1 + k)x} \frac{1}{\sqrt[3]{(1 - x)x(-kx + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2-(1+k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(1-(1+k)*x), x)

[Out] int((2-(1+k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(1-(1+k)*x), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(k + 1)x - 2}{((kx - 1)(x - 1)x)^{\frac{1}{3}}((k + 1)x - 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((k + 1)*x - 2)/(((k*x - 1)*(x - 1)*x)^(1/3)*((k + 1)*x - 1)), x, algorithm="maxima")

[Out] integrate(((k + 1)*x - 2)/(((k*x - 1)*(x - 1)*x)^(1/3)*((k + 1)*x - 1)), x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((k + 1)*x - 2)/(((k*x - 1)*(x - 1)*x)^(1/3)*((k + 1)*x - 1)), x, algorithm="sympy")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2-(1+k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(1-(1+k)*x), x)

[Out] Timed out

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(k+1)x - 2}{((kx - 1)(x - 1)x)^{\frac{1}{3}}((k+1)x - 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((k + 1)*x - 2)/(((k*x - 1)*(x - 1)*x)^(1/3)*((k + 1)*x - 1)), x, algorithm="giac")

[Out] integrate(((k + 1)*x - 2)/(((k*x - 1)*(x - 1)*x)^(1/3)*((k + 1)*x - 1)), x)

$$3.45 \quad \int \frac{1-kx}{(1+(-2+k)x)((1-x)x(1-kx))^{2/3}} dx$$

Optimal. Leaf size=176

$$\frac{\frac{\log(1-(2-k)x)}{2^{2/3}\sqrt[3]{1-k}} + \frac{\log(1-kx)}{2 \cdot 2^{2/3}\sqrt[3]{1-k}} - \frac{3 \log\left(kx + 2^{2/3}\sqrt[3]{1-k}\sqrt[3]{(1-x)x(1-kx)} - 1\right)}{2 \cdot 2^{2/3}\sqrt[3]{1-k}}}{\sqrt{3} \tan^{-1}\left(\frac{\sqrt[3]{2(1-kx)}}{\sqrt[3]{1-k}\sqrt[3]{(1-x)x(1-kx)} + 1}\right)} - \frac{\sqrt{3} \tan^{-1}\left(\frac{\sqrt[3]{2(1-kx)}}{\sqrt[3]{1-k}\sqrt[3]{(1-x)x(1-kx)} + 1}\right)}{2^{2/3}\sqrt[3]{1-k}}$$

[Out] -((Sqrt[3]*ArcTan[(1+(2^(1/3)*(1-k*x)))/((1-k)^(1/3)*((1-x)*x*(1-k*x))^(1/3))]/Sqrt[3]))/(2^(2/3)*(1-k)^(1/3)) + Log[1-(2-k)*x]/(2^(2/3)*(1-k)^(1/3)) + Log[1-k*x]/(2*2^(2/3)*(1-k)^(1/3)) - (3*Log[-1+k*x+2^(2/3)*(1-k)^(1/3)*((1-x)*x*(1-k*x))^(1/3)])/(2*2^(2/3)*(1-k)^(1/3))

Rubi [F] time = 0.819896, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0$.

$$\text{Int}\left(\frac{1-kx}{(1+(-2+k)x)((1-x)x(1-kx))^{2/3}}, x\right)$$

Verification is Not applicable to the result.

[In] Int[(1-k*x)/((1+(-2+k)*x)*((1-x)*x*(1-k*x))^(2/3)), x]

[Out] ((1-x)^(2/3)*x^(2/3)*(1-k*x)^(2/3)*Defer[Int][(1-k*x)^(1/3)/((1-x)^(2/3)*x^(2/3)*(1+(-2+k)*x)), x])/((1-x)*x*(1-k*x))^(2/3)

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate((-k*x+1)/(1+(-2+k)*x)/((1-x)*x*(-k*x+1))^(2/3), x)

[Out] Timed out

Mathematica [A] time = 0.41476, size = 0, normalized size = 0.

$$\int \frac{1 - kx}{(1 + (-2 + k)x)((1 - x)x(1 - kx))^{2/3}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(1 - k*x)/((1 + (-2 + k)*x)*((1 - x)*x*(1 - k*x))^(2/3)), x]

[Out] Integrate[(1 - k*x)/((1 + (-2 + k)*x)*((1 - x)*x*(1 - k*x))^(2/3)), x]

Maple [F] time = 0.073, size = 0, normalized size = 0.

$$\int \frac{-kx + 1}{1 + (-2 + k)x} ((1 - x)x(-kx + 1))^{-2/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-k*x+1)/(1+(-2+k)*x)/((1-x)*x*(-k*x+1))^(2/3), x)

[Out] int((-k*x+1)/(1+(-2+k)*x)/((1-x)*x*(-k*x+1))^(2/3), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$- \int \frac{kx - 1}{((kx - 1)(x - 1)x)^{2/3} ((k - 2)x + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-(k*x - 1)/(((k*x - 1)*(x - 1)*x)^(2/3)*((k - 2)*x + 1)), x, algorithm="m

[Out] -integrate((k*x - 1)/(((k*x - 1)*(x - 1)*x)^(2/3)*((k - 2)*x + 1)), x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-(k*x - 1)/(((k*x - 1)*(x - 1)*x)^(2/3)*((k - 2)*x + 1)),x, algorithm="f`

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-k*x+1)/(1+(-2+k)*x)/((1-x)*x*(-k*x+1))**(2/3),x)`

[Out] Timed out

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{kx - 1}{((kx - 1)(x - 1)x)^{\frac{2}{3}}((k - 2)x + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-(k*x - 1)/(((k*x - 1)*(x - 1)*x)^(2/3)*((k - 2)*x + 1)),x, algorithm="g`

[Out] `integrate(-(k*x - 1)/(((k*x - 1)*(x - 1)*x)^(2/3)*((k - 2)*x + 1)), x)`

$$3.46 \quad \int \frac{a+bx+cx^2}{(1-x+x^2)\sqrt[3]{1-x^3}} dx$$

Optimal. Leaf size=326

$$\begin{aligned} & \frac{(a-b-2c) \left(-3 \log \left(\sqrt[3]{2} - \sqrt[3]{1-x^3} \right) - 2\sqrt{3} \tan^{-1} \left(\frac{2^{2/3} \sqrt[3]{1-x^3+1}}{\sqrt{3}} \right) \right)}{12\sqrt[3]{2}} \\ & - \frac{(a-b-2c) \left(2\sqrt{3} \tan^{-1} \left(\frac{1 - \frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}} \right) - 3 \log \left(\sqrt[3]{1-x^3} + \sqrt[3]{2x} \right) \right)}{12\sqrt[3]{2}} \\ & + \frac{(a+b) \left(-3 \log \left(\sqrt[3]{1-x^3} - \sqrt[3]{2(x-1)} \right) + 2\sqrt{3} \tan^{-1} \left(\frac{\frac{2\sqrt[3]{2}(x-1)+1}{\sqrt[3]{1-x^3}}}{\sqrt{3}} \right) + \log(-3x^3 + 6x^2 - 6x + 3) \right)}{4\sqrt[3]{2}} \\ & - \frac{1}{6}c \left(-2 \log \left(\frac{x}{\sqrt[3]{1-x^3}} + 1 \right) + 2\sqrt{3} \tan^{-1} \left(\frac{1 - \frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}} \right) + \log \left(-\frac{x}{\sqrt[3]{1-x^3}} + \frac{x^2}{(1-x^3)^{2/3}} + 1 \right) \right) \end{aligned}$$

[Out] -(c*(2*Sqrt[3]*ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]] + Log[1 + x^2/(1 - x^3)^(2/3) - x/(1 - x^3)^(1/3)] - 2*Log[1 + x/(1 - x^3)^(1/3)]))/6 + ((a - b - 2*c)*(-2*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] - 3*Log[2^(1/3) - (1 - x^3)^(1/3)]))/(12*2^(1/3)) + ((a + b)*(2*Sqrt[3]*ArcTan[(1 + (2*2^(1/3)*(-1 + x))/(1 - x^3)^(1/3))/Sqrt[3]] + Log[3 - 6*x + 6*x^2 - 3*x^3] - 3*Log[-(2^(1/3)*(-1 + x)) + (1 - x^3)^(1/3)]))/(4*2^(1/3)) - ((a - b - 2*c)*(2*Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]] - 3*Log[2^(1/3)*x + (1 - x^3)^(1/3)]))/(12*2^(1/3))

Rubi [C] time = 1.52529, antiderivative size = 576, normalized size of antiderivative = 1.77, number

of steps used = 7, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$

$$\begin{aligned}
 & \frac{\log\left(2^{2/3}\sqrt[3]{1-x^3} + 2x - i\sqrt{3} + 1\right)\left(3ib - \sqrt{3}(2a + b - i\sqrt{3}c - c)\right)}{4\sqrt[3]{2}(\sqrt{3} + i)} \\
 & - \frac{\log\left(2^{2/3}\sqrt[3]{1-x^3} + 2x + i\sqrt{3} + 1\right)\left(\sqrt{3}(2a + b + i\sqrt{3}c - c) + 3ib\right)}{4\sqrt[3]{2}(-\sqrt{3} + i)} \\
 & - \frac{\tan^{-1}\left(\frac{2 - \frac{\sqrt[3]{2}(2x - i\sqrt{3} + 1)}{\sqrt[3]{1-x^3}}}{2\sqrt{3}}\right)\left(2a - i\sqrt{3}b + b - (1 + i\sqrt{3})c\right)}{2\sqrt[3]{2}(\sqrt{3} + i)} \\
 & + \frac{\tan^{-1}\left(\frac{2 - \frac{\sqrt[3]{2}(2x + i\sqrt{3} + 1)}{\sqrt[3]{1-x^3}}}{2\sqrt{3}}\right)\left(2a + i\sqrt{3}b + b + i\sqrt{3}c - c\right)}{2\sqrt[3]{2}(-\sqrt{3} + i)} \\
 & + \frac{\log\left(-(-2x - i\sqrt{3} + 1)^2(2x - i\sqrt{3} + 1)\right)\left(3ib - \sqrt{3}(2a + b - i\sqrt{3}c - c)\right)}{12\sqrt[3]{2}(\sqrt{3} + i)} \\
 & + \frac{\log\left(-(-2x + i\sqrt{3} + 1)^2(2x + i\sqrt{3} + 1)\right)\left(\sqrt{3}(2a + b + i\sqrt{3}c - c) + 3ib\right)}{12\sqrt[3]{2}(-\sqrt{3} + i)} \\
 & + \frac{1}{2}c \log\left(\sqrt[3]{1-x^3} + x\right) - \frac{c \tan^{-1}\left(\frac{1 - \frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}
 \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x + c*x^2)/((1 - x + x^2)*(1 - x^3)^(1/3)),x]

[Out] -((c*ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3]) - ((2*a + b - I*Sqrt[3]*b - (1 + I*Sqrt[3])*c)*ArcTan[(2 - (2^(1/3))*(1 - I*Sqrt[3] + 2*x))/(1 - x^3)^(1/3)]/(2*Sqrt[3]))/(2*2^(1/3)*(I + Sqrt[3])) + ((2*a + b + I*Sqrt[3]*b - c + I*Sqrt[3]*c)*ArcTan[(2 - (2^(1/3))*(1 + I*Sqrt[3] + 2*x))/(1 - x^3)^(1/3)]/(2*Sqrt[3]))/(2*2^(1/3)*(I - Sqrt[3])) + (((3*I)*b - Sqrt[3]*(2*a + b - c - I*Sqrt[3]*c))*Log[-((1 - I*Sqrt[3] - 2*x)^2*(1 - I*Sqrt[3] + 2*x))]/(12*2^(1/3)*(I + Sqrt[3])) + (((3*I)*b + Sqrt[3]*(2*a + b - c + I*Sqrt[3]*c))*Log[-((1 + I*Sqrt[3] - 2*x)^2*(1 + I*Sqrt[3] + 2*x))]/(12*2^(1/3)*(I - Sqrt[3])) + (c*Log[x + (1 - x^3)^(1/3)])/2 - (((3*I)*b - Sqrt[3]*(2*a + b - c - I*Sqrt[3]*c))*Log[1 - I*Sqrt[3] + 2*x + 2*2^(2/3)*(1 - x^3)^(1/3)]/(4*2^(1/3)*(I + Sqrt[3]))

)) - (((3*I)*b + Sqrt[3]*(2*a + b - c + I*Sqrt[3]*c))*Log[1 + I*Sqrt[3] + 2*x + 2*2^(2/3)*(1 - x^3)^(1/3)])/(4*2^(1/3)*(I - Sqrt[3]))

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((c*x**2+b*x+a)/(x**2-x+1)/(-x**3+1)**(1/3),x)`

[Out] Timed out

Mathematica [A] time = 0.165259, size = 0, normalized size = 0.

$$\int \frac{a + bx + cx^2}{(1 - x + x^2) \sqrt[3]{1 - x^3}} dx$$

Verification is Not applicable to the result.

[In] `Integrate[(a + b*x + c*x^2)/((1 - x + x^2)*(1 - x^3)^(1/3)),x]`

[Out] `Integrate[(a + b*x + c*x^2)/((1 - x + x^2)*(1 - x^3)^(1/3)), x]`

Maple [F] time = 0.183, size = 0, normalized size = 0.

$$\int \frac{cx^2 + bx + a}{x^2 - x + 1} \frac{1}{\sqrt[3]{-x^3 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^2+b*x+a)/(x^2-x+1)/(-x^3+1)^(1/3),x)`

[Out] `int((c*x^2+b*x+a)/(x^2-x+1)/(-x^3+1)^(1/3),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{cx^2 + bx + a}{(-x^3 + 1)^{\frac{1}{3}}(x^2 - x + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + b*x + a)/((-x^3 + 1)^(1/3)*(x^2 - x + 1)),x, algorithm="maxima")`

[Out] `integrate((c*x^2 + b*x + a)/((-x^3 + 1)^(1/3)*(x^2 - x + 1)), x)`

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + b*x + a)/((-x^3 + 1)^(1/3)*(x^2 - x + 1)),x, algorithm="fricas")`

[Out] Exception raised: TypeError

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + bx + cx^2}{\sqrt[3]{-(x-1)(x^2+x+1)(x^2-x+1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**2+b*x+a)/(x**2-x+1)/(-x**3+1)**(1/3),x)`

[Out] `Integral((a + b*x + c*x**2)/((-x - 1)*(x**2 + x + 1))**(1/3)*(x**2 - x + 1)), x)`

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{cx^2 + bx + a}{(-x^3 + 1)^{\frac{1}{3}}(x^2 - x + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + b*x + a)/((-x^3 + 1)^(1/3)*(x^2 - x + 1)),x, algorithm="giac")`

[Out] `integrate((c*x^2 + b*x + a)/((-x^3 + 1)^(1/3)*(x^2 - x + 1)), x)`

$$3.47 \quad \int \frac{1}{(3-2x)^{11/2}(1+x+2x^2)^5} dx$$

Optimal. Leaf size=407

$$\begin{aligned} & \frac{x}{28(3-2x)^{9/2}(2x^2+x+1)^4} + \frac{5(4377x+3049)}{153664(3-2x)^{9/2}(2x^2+x+1)} \\ & + \frac{3049x+1387}{32928(3-2x)^{9/2}(2x^2+x+1)^2} + \frac{73x+23}{1176(3-2x)^{9/2}(2x^2+x+1)^3} - \frac{38225}{240945152\sqrt{3-2x}} \\ & - \frac{141045}{120472576(3-2x)^{3/2}} - \frac{38491}{8605184(3-2x)^{5/2}} - \frac{462025}{30118144(3-2x)^{7/2}} - \frac{19255}{395136(3-2x)^{9/2}} \\ & + \frac{5\sqrt{\frac{1}{2}(40815066112\sqrt{14}-149046503977)} \log(-2x-\sqrt{7+2\sqrt{14}}\sqrt{3-2x}+\sqrt{14}+3)}{6746464256} \\ & - \frac{5\sqrt{\frac{1}{2}(40815066112\sqrt{14}-149046503977)} \log(-2x+\sqrt{7+2\sqrt{14}}\sqrt{3-2x}+\sqrt{14}+3)}{6746464256} \\ & + \frac{5\sqrt{\frac{1}{2}(149046503977+40815066112\sqrt{14})} \tan^{-1}\left(\frac{\sqrt{7+2\sqrt{14}-2\sqrt{3-2x}}}{\sqrt{2\sqrt{14}-7}}\right)}{3373232128} \\ & - \frac{5\sqrt{\frac{1}{2}(149046503977+40815066112\sqrt{14})} \tan^{-1}\left(\frac{2\sqrt{3-2x}+\sqrt{7+2\sqrt{14}}}{\sqrt{2\sqrt{14}-7}}\right)}{3373232128} \end{aligned}$$

[Out] -19255/(395136*(3-2*x)^(9/2)) - 462025/(30118144*(3-2*x)^(7/2)) - 38491/(8605184*(3-2*x)^(5/2)) - 141045/(120472576*(3-2*x)^(3/2)) - 38225/(240945152*sqrt[3-2*x]) + x/(28*(3-2*x)^(9/2)*(1+x+2*x^2)^4) + (23+73*x)/(1176*(3-2*x)^(9/2)*(1+x+2*x^2)^3) + (1387+3049*x)/(32928*(3-2*x)^(9/2)*(1+x+2*x^2)^2) + (5*(3049+4377*x))/(153664*(3-2*x)^(9/2)*(1+x+2*x^2)) + (5*sqrt[(149046503977+40815066112*sqrt[14])/2]*ArcTan[(sqrt[7+2*sqrt[14]]-2*sqrt[3-2*x])/sqrt[-7+2*sqrt[14]])]/3373232128 - (5*sqrt[(149046503977+40815066112*sqrt[14])/2]*ArcTan[(sqrt[7+2*sqrt[14]]+2*sqrt[3-2*x])/sqrt[-7+2*sqrt[14]])]/3373232128 + (5*sqrt[(-149046503977+40815066112*sqrt[14])/2]*Log[3+sqrt[14]-sqrt[7+2*sqrt[14]]*sqrt[3-2*x]-2*x])/6746464256 - (5*sqrt[(-149046503977+40815066112*sqrt[14])/2]*Log[3+sqrt[14]+sqrt[7+2*sqrt[14]]*sqrt[3-2*x]-2*x])/6746464256

Rubi [A] time = 1.47675, antiderivative size = 407, normalized size of antiderivative = 1., number of

steps used = 19, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.45$

$$\begin{aligned}
& \frac{x}{28(3-2x)^{9/2}(2x^2+x+1)^4} + \frac{5(4377x+3049)}{153664(3-2x)^{9/2}(2x^2+x+1)} \\
& + \frac{3049x+1387}{32928(3-2x)^{9/2}(2x^2+x+1)^2} + \frac{73x+23}{1176(3-2x)^{9/2}(2x^2+x+1)^3} - \frac{38225}{240945152\sqrt{3-2x}} \\
& - \frac{141045}{120472576(3-2x)^{3/2}} - \frac{38491}{8605184(3-2x)^{5/2}} - \frac{462025}{30118144(3-2x)^{7/2}} - \frac{19255}{395136(3-2x)^{9/2}} \\
& + \frac{5\sqrt{\frac{1}{2}}(40815066112\sqrt{14}-149046503977)\log(-2x-\sqrt{7+2\sqrt{14}}\sqrt{3-2x}+\sqrt{14}+3)}{6746464256} \\
& + \frac{5\sqrt{\frac{1}{2}}(40815066112\sqrt{14}-149046503977)\log(-2x+\sqrt{7+2\sqrt{14}}\sqrt{3-2x}+\sqrt{14}+3)}{6746464256} \\
& + \frac{5\sqrt{\frac{1}{2}}(149046503977+40815066112\sqrt{14})\tan^{-1}\left(\frac{\sqrt{7+2\sqrt{14}-2\sqrt{3-2x}}}{\sqrt{2\sqrt{14}-7}}\right)}{3373232128} \\
& + \frac{5\sqrt{\frac{1}{2}}(149046503977+40815066112\sqrt{14})\tan^{-1}\left(\frac{2\sqrt{3-2x}+\sqrt{7+2\sqrt{14}}}{\sqrt{2\sqrt{14}-7}}\right)}{3373232128}
\end{aligned}$$

Antiderivative was successfully verified.

[In] Int[1/((3 - 2*x)^(11/2)*(1 + x + 2*x^2)^5), x]

[Out] -19255/(395136*(3 - 2*x)^(9/2)) - 462025/(30118144*(3 - 2*x)^(7/2)) - 38491/(8605184*(3 - 2*x)^(5/2)) - 141045/(120472576*(3 - 2*x)^(3/2)) - 38225/(240945152*sqrt[3 - 2*x]) + x/(28*(3 - 2*x)^(9/2)*(1 + x + 2*x^2)^4) + (23 + 73*x)/(1176*(3 - 2*x)^(9/2)*(1 + x + 2*x^2)^3) + (1387 + 3049*x)/(32928*(3 - 2*x)^(9/2)*(1 + x + 2*x^2)^2) + (5*(3049 + 4377*x))/(153664*(3 - 2*x)^(9/2)*(1 + x + 2*x^2)) + (5*sqrt[(149046503977 + 40815066112*sqrt[14])/2]*ArcTan[(sqrt[7 + 2*sqrt[14]] - 2*sqrt[3 - 2*x])/sqrt[-7 + 2*sqrt[14]])]/3373232128 - (5*sqrt[(149046503977 + 40815066112*sqrt[14])/2]*ArcTan[(sqrt[7 + 2*sqrt[14]] + 2*sqrt[3 - 2*x])/sqrt[-7 + 2*sqrt[14]])]/3373232128 + (5*sqrt[(-149046503977 + 40815066112*sqrt[14])/2]*Log[3 + sqrt[14] - sqrt[7 + 2*sqrt[14]]*sqrt[3 - 2*x] - 2*x])/6746464256 - (5*sqrt[(-149046503977 + 40815066112*sqrt[14])/2]*Log[3 + sqrt[14] + sqrt[7 + 2*sqrt[14]]*sqrt[3 - 2*x] - 2*x])/6746464256

Rubi in Sympy [A] time = 48.1711, size = 478, normalized size = 1.17

$$\begin{aligned}
 & \frac{x}{28(-2x+3)^{\frac{9}{2}}(2x^2+x+1)^4} \\
 & + \frac{\sqrt{14}\left(-48353174284800\sqrt{14}+732027212367360\right)\log\left(-2x-\sqrt{7+2\sqrt{14}}\sqrt{-2x+3}+3+\sqrt{14}\right)}{17068042484057112576\sqrt{7+2\sqrt{14}}} \\
 & - \frac{\sqrt{14}\left(-48353174284800\sqrt{14}+732027212367360\right)\log\left(-2x+\sqrt{7+2\sqrt{14}}\sqrt{-2x+3}+3+\sqrt{14}\right)}{17068042484057112576\sqrt{7+2\sqrt{14}}} \\
 & + \frac{\sqrt{14}\left(-1464054424734720\sqrt{7+2\sqrt{14}}+\frac{\sqrt{7+2\sqrt{14}}(-96706348569600\sqrt{14}+1464054424734720)}{2}\right)\operatorname{atan}\left(\frac{2\sqrt{-2x+3}-\sqrt{7+2\sqrt{14}}}{\sqrt{-7+2\sqrt{14}}}\right)}{8534021242028556288\sqrt{-7+2\sqrt{14}}\sqrt{7+2\sqrt{14}}} \\
 & + \frac{\sqrt{14}\left(-1464054424734720\sqrt{7+2\sqrt{14}}+\frac{\sqrt{7+2\sqrt{14}}(-96706348569600\sqrt{14}+1464054424734720)}{2}\right)\operatorname{atan}\left(\frac{2\sqrt{-2x+3}+\sqrt{7+2\sqrt{14}}}{\sqrt{-7+2\sqrt{14}}}\right)}{8534021242028556288\sqrt{-7+2\sqrt{14}}\sqrt{7+2\sqrt{14}}} \\
 & - \frac{38225}{240945152\sqrt{-2x+3}} - \frac{141045}{120472576(-2x+3)^{\frac{3}{2}}} - \frac{38491}{8605184(-2x+3)^{\frac{5}{2}}} - \frac{462025}{30118144(-2x+3)^{\frac{7}{2}}} \\
 & + \frac{28616x+9016}{460992(-2x+3)^{\frac{9}{2}}(2x^2+x+1)^3} + \frac{16732912x+7611856}{180708864(-2x+3)^{\frac{9}{2}}(2x^2+x+1)^2} \\
 & + \frac{5044404960x+3513911520}{35418937344(-2x+3)^{\frac{9}{2}}(2x^2+x+1)} - \frac{19255}{395136(-2x+3)^{\frac{9}{2}}}
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/(3-2*x)**(11/2)/(2*x**2+x+1)**5,x)`

[Out] `x/(28*(-2*x+3)**(9/2)*(2*x**2+x+1)**4)+sqrt(14)*(-48353174284800*sqrt(14)+732027212367360)*log(-2*x-sqrt(7+2*sqrt(14)))*sqrt(-2*x+3)+3+sqrt(14))/(17068042484057112576*sqrt(7+2*sqrt(14)))-sqrt(14)*(-48353174284800*sqrt(14)+732027212367360)*log(-2*x+sqrt(7+2*sqrt(14)))*sqrt(-2*x+3)+3+sqrt(14))/(17068042484057112576*sqrt(7+2*sqrt(14)))+sqrt(14)*(-1464054424734720*sqrt(7+2*sqrt(14))+sqrt(7+2*sqrt(14))*(-96706348569600*sqrt(14)+1464054424734720)/2)*atan((2*sqrt(-2*x+3)-sqrt(7+2*sqrt(14)))/sqrt(-7+2*sqrt(14)))/(8534021242028556288*sqrt(-7+2*sqrt(14))*sqrt(7+2*sqrt(14)))+sqrt(14)*(-1464054424734720*sqrt(7+2*sqrt(14))+sqrt(7+2*sqrt(14))*(-96706348569600*sqrt(14)+1464054424734720)/2)*atan((2*sqrt(-2*x+3)+sqrt(7+2*sqrt(14)))/sqrt(-7+2*sqrt(14)))/(8534021242028556288*sqrt(-7+2*sqrt(14))*sqrt(7+2*sqrt(14)))-38225/(240945152*sqrt(-2*x+3))-141045/(120472576*(-2*x+3)**(3/2))-38491/(8605184*(-2*x+3)**(5/2))-462025/(30118144*(-2*x+3)**(7/2))+ (28616*x+9016)/(460992*(-2*x+3)**(9/2)*(2*x**2+x+1)**3)+(16732912*x+7611856)/(180708864*(-2*x+3)**(9/2)*(2*x**2+x+1)**2)+(5044404960*x+3513911520)/(35418937344*(-2*x+3)**(9/2)*(2*x**2+x+1))-19255/(395136*(-2*x+3)**(9/2))`

Mathematica [C] time = 2.25894, size = 206, normalized size = 0.51

$$\frac{45i(284993\sqrt{7}+53515i) \tan^{-1}\left(\frac{\sqrt{6-4x}}{\sqrt{-7-i\sqrt{7}}}\right)}{\sqrt{-\frac{1}{2}i(\sqrt{7}-7i)}} - \frac{45i(284993\sqrt{7}-53515i) \tan^{-1}\left(\frac{\sqrt{6-4x}}{\sqrt{-7+i\sqrt{7}}}\right)}{\sqrt{\frac{1}{2}i(\sqrt{7}+7i)}} - \frac{14(88070400x^{12}-677249280x^{11}+1873554048x^{10}-2443779648x^9+2343370048x^8-2443779648x^9+1873554048x^{10}-677249280x^{11}+88070400x^{12})}{(3-2x)^{9/2}(1+x+2x^2)^4} + \frac{(45I)(53515I+284993\sqrt{7})\text{ArcTan}\left[\frac{\sqrt{6-4x}}{\sqrt{-7-I\sqrt{7}}}\right]}{\sqrt{(-I/2)(-7I+\sqrt{7})}} - \frac{(45I)(-53515I+284993\sqrt{7})\text{ArcTan}\left[\frac{\sqrt{6-4x}}{\sqrt{-7+I\sqrt{7}}}\right]}{\sqrt{(I/2)(7I+\sqrt{7})}}/30359089152$$

30359089152

Antiderivative was successfully verified.

[In] Integrate[1/((3 - 2*x)^(11/2)*(1 + x + 2*x^2)^5), x]

[Out] ((-14*(40289347 - 429812744*x + 135202154*x^2 - 1073855156*x^3 + 1627773523*x^4 - 1470758860*x^5 + 2888625656*x^6 - 3106712560*x^7 + 2343370048*x^8 - 2443779648*x^9 + 1873554048*x^10 - 677249280*x^11 + 88070400*x^12))/((3 - 2*x)^(9/2)*(1 + x + 2*x^2)^4) + ((45*I)*(53515*I + 284993*sqrt[7])*ArcTan[Sqrt[6 - 4*x]/Sqrt[-7 - I*Sqrt[7]])/Sqrt[(-I/2)*(-7*I + Sqrt[7])] - ((45*I)*(-53515*I + 284993*sqrt[7])*ArcTan[Sqrt[6 - 4*x]/Sqrt[-7 + I*Sqrt[7]])/Sqrt[(I/2)*(7*I + Sqrt[7])])/30359089152

Maple [A] time = 0.132, size = 584, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(3-2*x)^(11/2)/(2*x^2+x+1)^5, x)

[Out] 1/6588344*(567651623/32*(3-2*x)^(1/2)-6194606411/192*(3-2*x)^(3/2)+9801432515/384*(3-2*x)^(5/2)-8763772549/768*(3-2*x)^(7/2)+149630663/48*(3-2*x)^(9/2)-200063633/384*(3-2*x)^(11/2)+18969965/384*(3-2*x)^(13/2)-526135/256*(3-2*x)^(15/2))/((3-2*x)^2-7+14*x)^4-731595/13492928512*ln(3-2*x+14^(1/2)-(3-2*x)^(1/2)*(7+2*14^(1/2)))^(1/2)*(7+2*14^(1/2))^(1/2)*14^(1/2)+1424965/6746464256*ln(3-2*x+14^(1/2)-(3-2*x)^(1/2)*(7+2*14^(1/2)))^(1/2)*(7+2*14^(1/2))^(1/2)-731595/6746464256/(-7+2*14^(1/2))^(1/2)*arctan((2*(3-2*x)^(1/2)-(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))*(7+2*14^(1/2))*14^(1/2)+1424965/3373232128/(-7+2*14^(1/2))^(1/2)*arctan((2*(3-2*x)^(1/2)-(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))*(7+2*14^(1/2))-578695/3373232128/(-7+2*14^(1/2))^(1/2)*arctan((2*(3-2*x)^(1/2)-(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))*14^(1/2)+731595/13492928512*ln(3-2*x+14^(1/2)+(3-2*x)^(1/2)*(7+2*14^(1/2)))^(1/2)*(7+2*14^(1/2))^(1/2)*14^(1/2)-1424965/6746464256*ln(3-2*x+14^(1/2)+(3-2*x)^(1/2)*(7+2*14^(1/2)))^(1/2)*(7+2*14^(1/2))^(1/2)-731595/6746464256/(-7+2*14^(1/2))^(1/2)*arctan((2*(3-2*x)^(1/2)+(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))*(7+2*14^(1/2))*14^(1/2)+1424965/3373232128/(-7+2*14^(1/2))^(1/2)*arctan((2*(3-2*x)^(1/2)+(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))*(7+2*14^(1/2))

$$\frac{28/(-7+2*14^{(1/2)})^{(1/2)}*arctan((2*(3-2*x)^{(1/2)}+(7+2*14^{(1/2)})^{(1/2)})/(-7+2*14^{(1/2)})^{(1/2)})+1/151263/(3-2*x)^{(9/2)}+5/235298/(3-2*x)^{(7/2)}+19/470596/(3-2*x)^{(5/2)}+185/2823576/(3-2*x)^{(3/2)}+505/3294172/(3-2*x)^{(1/2)}}{(2x^2+x+1)^5(-2x+3)^{\frac{11}{2}}} dx$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2x^2+x+1)^5(-2x+3)^{\frac{11}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((2*x^2 + x + 1)^5*(-2*x + 3)^(11/2)),x, algorithm="maxima")

[Out] integrate(1/((2*x^2 + x + 1)^5*(-2*x + 3)^(11/2)), x)

Fricas [A] time = 0.280812, size = 1827, normalized size = 4.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((2*x^2 + x + 1)^5*(-2*x + 3)^(11/2)),x, algorithm="fricas")

[Out] 1/948692239236095606784*sqrt(79716926)*2744^(3/4)*(2263908918780*sqrt(39858463)*sqrt(14)*(256*x^12 - 1024*x^11 + 1280*x^10 - 1024*x^9 + 1888*x^8 - 1664*x^7 + 560*x^6 - 1280*x^5 + 577*x^4 + 12*x^3 + 486*x^2 + 108*x + 81)*sqrt(-2*x + 3)*arctan(1116036964*sqrt(39858463)*(284993*sqrt(14)*sqrt(7) - 1024233*sqrt(7))/(sqrt(79716926)*sqrt(39858463)*2744^(1/4)*(21292357711*sqrt(14)*sqrt(7) - 81630132224*sqrt(7))*sqrt(-sqrt(14)*(sqrt(79716926)*sqrt(39858463)*2744^(1/4)*sqrt(-2*x + 3)*(40473233043683538146117038154862650739*sqrt(14) - 151437300139796837407365155111661919174)*sqrt((21292357711*sqrt(14) - 81630132224)/(1738097975309635979264*sqrt(14) - 6505290721706129709735)) + 79716926*sqrt(14)*(280394144603408538329670261219721*sqrt(14)*(2*x - 3) - 2098285190694476737313101842132992*x + 3147427786041715105969652763199488) - 312930229866565049169122829452744361767044*sqrt(14) + 1170881916914412434739769848658455837958144)/(280394144603408538329670261219721*sqrt(14) - 1049142595347238368656550921066496))*sqrt((21292357711*sqrt(14) - 81630132224)/(1738097975309635979264*sqrt(14) - 6505290721706129709735)) + 558018482*sqrt(79716926)*2744^(1/4)*sqrt(-2*x + 3)*(21292357711*sqrt(14) - 81630132224)*sqrt((21292357711*sqrt(14) - 81630132224)/(1738097975309635979264*sqrt(14) - 6505290721706129709735)) - 7812258748*sqrt(39858463)*(7645*sqrt(14) - 115739))) + 2263908918780*sqrt(39858463)*sqrt(14)*(256*x^12 - 1024*x^11 + 1280*x^10

$$\begin{aligned}
& - 1024*x^9 + 1888*x^8 - 1664*x^7 + 560*x^6 - 1280*x^5 + 577*x^4 \\
& + 12*x^3 + 486*x^2 + 108*x + 81) * \text{sqrt}(-2*x + 3) * \text{arctan}(1116036964 \\
& * \text{sqrt}(39858463) * (284993 * \text{sqrt}(14) * \text{sqrt}(7) - 1024233 * \text{sqrt}(7)) / (\text{sqrt} \\
& (79716926) * \text{sqrt}(39858463) * 2744^{(1/4)} * (21292357711 * \text{sqrt}(14) * \text{sqrt}(7) \\
&) - 81630132224 * \text{sqrt}(7)) * \text{sqrt}(\text{sqrt}(14) * (\text{sqrt}(79716926) * \text{sqrt}(39858 \\
& 463) * 2744^{(1/4)} * \text{sqrt}(-2*x + 3) * (404732330436835381461170381548626 \\
& 50739 * \text{sqrt}(14) - 151437300139796837407365155111661919174) * \text{sqrt}((2 \\
& 1292357711 * \text{sqrt}(14) - 81630132224) / (1738097975309635979264 * \text{sqrt}(1 \\
& 4) - 6505290721706129709735)) - 79716926 * \text{sqrt}(14) * (28039414460340 \\
& 8538329670261219721 * \text{sqrt}(14) * (2*x - 3) - 209828519069447673731310 \\
& 1842132992*x + 3147427786041715105969652763199488) + 312930229866 \\
& 565049169122829452744361767044 * \text{sqrt}(14) - 11708819169144124347397 \\
& 69848658455837958144) / (280394144603408538329670261219721 * \text{sqrt}(14) \\
& - 1049142595347238368656550921066496)) * \text{sqrt}((21292357711 * \text{sqrt}(14) \\
&) - 81630132224) / (1738097975309635979264 * \text{sqrt}(14) - 6505290721706 \\
& 129709735)) + 558018482 * \text{sqrt}(79716926) * 2744^{(1/4)} * \text{sqrt}(-2*x + 3) * \\
& (21292357711 * \text{sqrt}(14) - 81630132224) * \text{sqrt}((21292357711 * \text{sqrt}(14) - \\
& 81630132224) / (1738097975309635979264 * \text{sqrt}(14) - 6505290721706129 \\
& 709735)) + 7812258748 * \text{sqrt}(39858463) * (7645 * \text{sqrt}(14) - 115739))) + \\
& 45 * \text{sqrt}(39858463) * (21292357711 * \text{sqrt}(14) * \text{sqrt}(7) * (256*x^12 - 1024 \\
& *x^11 + 1280*x^10 - 1024*x^9 + 1888*x^8 - 1664*x^7 + 560*x^6 - 12 \\
& 80*x^5 + 577*x^4 + 12*x^3 + 486*x^2 + 108*x + 81) - 81630132224 * \text{s} \\
& \text{qrt}(7) * (256*x^12 - 1024*x^11 + 1280*x^10 - 1024*x^9 + 1888*x^8 - \\
& 1664*x^7 + 560*x^6 - 1280*x^5 + 577*x^4 + 12*x^3 + 486*x^2 + 108* \\
& x + 81)) * \text{sqrt}(-2*x + 3) * \log(-996461575/4 * \text{sqrt}(14) * (\text{sqrt}(79716926) \\
& * \text{sqrt}(39858463) * 2744^{(1/4)} * \text{sqrt}(-2*x + 3) * (4047323304368353814611 \\
& 7038154862650739 * \text{sqrt}(14) - 1514373001397968374073651551116619191 \\
& 74) * \text{sqrt}((21292357711 * \text{sqrt}(14) - 81630132224) / (173809797530963597 \\
& 9264 * \text{sqrt}(14) - 6505290721706129709735)) + 79716926 * \text{sqrt}(14) * (280 \\
& 394144603408538329670261219721 * \text{sqrt}(14) * (2*x - 3) - 2098285190694 \\
& 476737313101842132992*x + 3147427786041715105969652763199488) - 3 \\
& 12930229866565049169122829452744361767044 * \text{sqrt}(14) + 117088191691 \\
& 4412434739769848658455837958144) / (2803941446034085383296702612197 \\
& 21 * \text{sqrt}(14) - 1049142595347238368656550921066496)) - 45 * \text{sqrt}(3985 \\
& 8463) * (21292357711 * \text{sqrt}(14) * \text{sqrt}(7) * (256*x^12 - 1024*x^11 + 1280* \\
& x^10 - 1024*x^9 + 1888*x^8 - 1664*x^7 + 560*x^6 - 1280*x^5 + 577* \\
& x^4 + 12*x^3 + 486*x^2 + 108*x + 81) - 81630132224 * \text{sqrt}(7) * (256*x \\
& ^12 - 1024*x^11 + 1280*x^10 - 1024*x^9 + 1888*x^8 - 1664*x^7 + 56 \\
& 0*x^6 - 1280*x^5 + 577*x^4 + 12*x^3 + 486*x^2 + 108*x + 81)) * \text{sqrt} \\
& (-2*x + 3) * \log(996461575/4 * \text{sqrt}(14) * (\text{sqrt}(79716926) * \text{sqrt}(39858463 \\
&) * 2744^{(1/4)} * \text{sqrt}(-2*x + 3) * (404732330436835381461170381548626507 \\
& 39 * \text{sqrt}(14) - 151437300139796837407365155111661919174) * \text{sqrt}((2129 \\
& 2357711 * \text{sqrt}(14) - 81630132224) / (1738097975309635979264 * \text{sqrt}(14) \\
& - 6505290721706129709735)) - 79716926 * \text{sqrt}(14) * (28039414460340853 \\
& 8329670261219721 * \text{sqrt}(14) * (2*x - 3) - 209828519069447673731310184 \\
& 2132992*x + 3147427786041715105969652763199488) + 312930229866565 \\
& 049169122829452744361767044 * \text{sqrt}(14) - 11708819169144124347397698 \\
& 48658455837958144) / (280394144603408538329670261219721 * \text{sqrt}(14) - \\
& 1049142595347238368656550921066496)) - 2 * \text{sqrt}(79716926) * 2744^{(1/4)} \\
&) * (21292357711 * \text{sqrt}(14) * \text{sqrt}(7) * (88070400*x^12 - 677249280*x^11 + \\
& 1873554048*x^10 - 2443779648*x^9 + 2343370048*x^8 - 3106712560*x \\
& ^7 + 2888625656*x^6 - 1470758860*x^5 + 1627773523*x^4 - 107385515 \\
& 6*x^3 + 135202154*x^2 - 429812744*x + 40289347) - 81630132224 * \text{sqr} \\
& \text{t}(7) * (88070400*x^12 - 677249280*x^11 + 1873554048*x^10 - 24437796 \\
& 48*x^9 + 2343370048*x^8 - 3106712560*x^7 + 2888625656*x^6 - 14707 \\
& 58860*x^5 + 1627773523*x^4 - 1073855156*x^3 + 135202154*x^2 - 429
\end{aligned}$$

```

812744*x + 40289347))*sqrt((21292357711*sqrt(14) - 81630132224)/(
1738097975309635979264*sqrt(14) - 6505290721706129709735)))/((212
92357711*sqrt(14)*sqrt(7)*(256*x^12 - 1024*x^11 + 1280*x^10 - 102
4*x^9 + 1888*x^8 - 1664*x^7 + 560*x^6 - 1280*x^5 + 577*x^4 + 12*x
^3 + 486*x^2 + 108*x + 81) - 81630132224*sqrt(7)*(256*x^12 - 1024
*x^11 + 1280*x^10 - 1024*x^9 + 1888*x^8 - 1664*x^7 + 560*x^6 - 12
80*x^5 + 577*x^4 + 12*x^3 + 486*x^2 + 108*x + 81))*sqrt(-2*x + 3)
*sqrt((21292357711*sqrt(14) - 81630132224)/(173809797530963597926
4*sqrt(14) - 6505290721706129709735)))

```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3-2*x)**(11/2)/(2*x**2+x+1)**5,x)

[Out] Timed out

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2x^2 + x + 1)^5(-2x + 3)^{\frac{11}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((2*x^2 + x + 1)^5*(-2*x + 3)^(11/2)),x, algorithm="giac")

[Out] integrate(1/((2*x^2 + x + 1)^5*(-2*x + 3)^(11/2)), x)

$$3.48 \quad \int \frac{1}{(3-2x)^{21/2}(1+x+2x^2)^{10}} dx$$

Optimal. Leaf size=648

result too large to display

```
[Out] 4718120139975/(351733660450816*(3 - 2*x)^(19/2)) - 815900548375/(
629418129227776*(3 - 2*x)^(17/2)) - 3029508823715/(15550330251509
76*(3 - 2*x)^(15/2)) - 13515743021825/(13476952884641792*(3 - 2*x
)^(13/2)) - 5846828446875/(14513641568075776*(3 - 2*x)^(11/2)) -
37283626871975/(261245548225363968*(3 - 2*x)^(9/2)) - 13235516227
2575/(2844673747342852096*(3 - 2*x)^(7/2)) - 11557581705725/(8127
63927812243456*(3 - 2*x)^(5/2)) - 46601678385075/(113786949893714
08384*(3 - 2*x)^(3/2)) - 24229218097975/(22757389978742816768*Sqr
t[3 - 2*x]) + x/(63*(3 - 2*x)^(19/2)*(1 + x + 2*x^2)^9) + (53 + 1
73*x)/(7056*(3 - 2*x)^(19/2)*(1 + x + 2*x^2)^8) + (8477 + 21409*x
)/(691488*(3 - 2*x)^(19/2)*(1 + x + 2*x^2)^7) + (5*(21409 + 47471
*x))/(6453888*(3 - 2*x)^(19/2)*(1 + x + 2*x^2)^6) + (41*(47471 +
92875*x))/(90354432*(3 - 2*x)^(19/2)*(1 + x + 2*x^2)^5) + (41*(34
36375 + 5677637*x))/(5059848192*(3 - 2*x)^(19/2)*(1 + x + 2*x^2)^
4) + (451*(811091 + 998691*x))/(10119696384*(3 - 2*x)^(19/2)*(1 +
x + 2*x^2)^3) + (451*(28962039 + 14627273*x))/(283351498752*(3 -
2*x)^(19/2)*(1 + x + 2*x^2)^2) + (11275*(14627273 - 35058731*x))
/(3966920982528*(3 - 2*x)^(19/2)*(1 + x + 2*x^2)) + (11275*Sqrt[(
7 + 2*Sqrt[14])/2]*(9756589235 + 2148932869*Sqrt[14])*ArcTan[(Sqr
t[7 + 2*Sqrt[14]] - 2*Sqrt[3 - 2*x])/Sqrt[-7 + 2*Sqrt[14]])]/3186
03459702399434752 - (11275*Sqrt[(7 + 2*Sqrt[14])/2]*(9756589235 +
2148932869*Sqrt[14])*ArcTan[(Sqrt[7 + 2*Sqrt[14]] + 2*Sqrt[3 - 2
*x])/Sqrt[-7 + 2*Sqrt[14]])]/318603459702399434752 + (11275*(9756
589235 - 2148932869*Sqrt[14])*Sqrt[(-7 + 2*Sqrt[14])/2]*Log[3 + S
qrt[14] - Sqrt[7 + 2*Sqrt[14]]*Sqrt[3 - 2*x] - 2*x])/637206919404
798869504 - (11275*(9756589235 - 2148932869*Sqrt[14])*Sqrt[(-7 +
2*Sqrt[14])/2]*Log[3 + Sqrt[14] + Sqrt[7 + 2*Sqrt[14]]*Sqrt[3 - 2
*x] - 2*x])/637206919404798869504
```

Rubi [A] time = 2.68597, antiderivative size = 648, normalized size of antiderivative = 1., number of

steps used = 29, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.45$

$$\begin{aligned}
& \frac{11275(14627273 - 35058731x)}{3966920982528(3 - 2x)^{19/2}(2x^2 + x + 1)} + \frac{451(14627273x + 28962039)}{283351498752(3 - 2x)^{19/2}(2x^2 + x + 1)^2} \\
& + \frac{451(998691x + 811091)}{10119696384(3 - 2x)^{19/2}(2x^2 + x + 1)^3} + \frac{41(5677637x + 3436375)}{5059848192(3 - 2x)^{19/2}(2x^2 + x + 1)^4} \\
& + \frac{41(92875x + 47471)}{90354432(3 - 2x)^{19/2}(2x^2 + x + 1)^5} + \frac{5(47471x + 21409)}{6453888(3 - 2x)^{19/2}(2x^2 + x + 1)^6} \\
& + \frac{21409x + 8477}{691488(3 - 2x)^{19/2}(2x^2 + x + 1)^7} + \frac{173x + 53}{7056(3 - 2x)^{19/2}(2x^2 + x + 1)^8} + \frac{x}{63(3 - 2x)^{19/2}(2x^2 + x + 1)^9} \\
& - \frac{24229218097975}{22757389978742816768\sqrt{3 - 2x}} - \frac{46601678385075}{11378694989371408384(3 - 2x)^{3/2}} \\
& - \frac{11557581705725}{812763927812243456(3 - 2x)^{5/2}} - \frac{132355162272575}{2844673747342852096(3 - 2x)^{7/2}} \\
& - \frac{37283626871975}{261245548225363968(3 - 2x)^{9/2}} - \frac{5846828446875}{14513641568075776(3 - 2x)^{11/2}} - \frac{13515743021825}{13476952884641792(3 - 2x)^{13/2}} \\
& - \frac{3029508823715}{1555033025150976(3 - 2x)^{15/2}} - \frac{815900548375}{629418129227776(3 - 2x)^{17/2}} + \frac{4718120139975}{351733660450816(3 - 2x)^{19/2}} \\
& + \frac{11275 \left(9756589235 - 2148932869\sqrt{14} \right) \sqrt{\frac{1}{2} \left(2\sqrt{14} - 7 \right) \log \left(-2x - \sqrt{7 + 2\sqrt{14}\sqrt{3 - 2x} + \sqrt{14} + 3 \right)}}{637206919404798869504} \\
& - \frac{11275 \left(9756589235 - 2148932869\sqrt{14} \right) \sqrt{\frac{1}{2} \left(2\sqrt{14} - 7 \right) \log \left(-2x + \sqrt{7 + 2\sqrt{14}\sqrt{3 - 2x} + \sqrt{14} + 3 \right)}}{637206919404798869504} \\
& + \frac{11275 \sqrt{\frac{1}{2} \left(7 + 2\sqrt{14} \right) \left(9756589235 + 2148932869\sqrt{14} \right) \tan^{-1} \left(\frac{\sqrt{7 + 2\sqrt{14} - 2\sqrt{3 - 2x}}}{\sqrt{2\sqrt{14} - 7}} \right)}}{318603459702399434752} \\
& - \frac{11275 \sqrt{\frac{1}{2} \left(7 + 2\sqrt{14} \right) \left(9756589235 + 2148932869\sqrt{14} \right) \tan^{-1} \left(\frac{2\sqrt{3 - 2x} + \sqrt{7 + 2\sqrt{14}}}{\sqrt{2\sqrt{14} - 7}} \right)}}{318603459702399434752}
\end{aligned}$$

Antiderivative was successfully verified.

[In] Int[1/((3 - 2*x)^(21/2)*(1 + x + 2*x^2)^10), x]

[Out] 4718120139975/(351733660450816*(3 - 2*x)^(19/2)) - 815900548375/(629418129227776*(3 - 2*x)^(17/2)) - 3029508823715/(1555033025150976*(3 - 2*x)^(15/2)) - 13515743021825/(13476952884641792*(3 - 2*x)^(13/2)) - 5846828446875/(14513641568075776*(3 - 2*x)^(11/2)) - 37283626871975/(261245548225363968*(3 - 2*x)^(9/2)) - 132355162272575/(2844673747342852096*(3 - 2*x)^(7/2)) - 11557581705725/(812763927812243456*(3 - 2*x)^(5/2)) - 46601678385075/(11378694989371408384*(3 - 2*x)^(3/2)) - 24229218097975/(22757389978742816768*Sqrt[3 - 2*x]) + x/(63*(3 - 2*x)^(19/2)*(1 + x + 2*x^2)^9) + (53 + 173*x)/(7056*(3 - 2*x)^(19/2)*(1 + x + 2*x^2)^8) + (8477 + 21409*x)/(691488*(3 - 2*x)^(19/2)*(1 + x + 2*x^2)^7) + (5*(21409 + 47471*x))/(6453888*(3 - 2*x)^(19/2)*(1 + x + 2*x^2)^6) + (41*(47471 + 92875*x))/(90354432*(3 - 2*x)^(19/2)*(1 + x + 2*x^2)^5) + (41*(3436375 + 5677637*x))/(5059848192*(3 - 2*x)^(19/2)*(1 + x + 2*x^2)^4)

$$\begin{aligned}
& 4) + (451*(811091 + 998691*x))/(10119696384*(3 - 2*x)^(19/2)*(1 + x + 2*x^2)^3) + (451*(28962039 + 14627273*x))/(283351498752*(3 - 2*x)^(19/2)*(1 + x + 2*x^2)^2) + (11275*(14627273 - 35058731*x))/(3966920982528*(3 - 2*x)^(19/2)*(1 + x + 2*x^2)) + (11275*Sqrt[(7 + 2*Sqrt[14])/2]*(9756589235 + 2148932869*Sqrt[14])*ArcTan[(Sqrt[7 + 2*Sqrt[14]] - 2*Sqrt[3 - 2*x])/Sqrt[-7 + 2*Sqrt[14]])]/318603459702399434752 - (11275*Sqrt[(7 + 2*Sqrt[14])/2]*(9756589235 + 2148932869*Sqrt[14])*ArcTan[(Sqrt[7 + 2*Sqrt[14]] + 2*Sqrt[3 - 2*x])/Sqrt[-7 + 2*Sqrt[14]])]/318603459702399434752 + (11275*(9756589235 - 2148932869*Sqrt[14])*Sqrt[(-7 + 2*Sqrt[14])/2]*Log[3 + Sqrt[14] - Sqrt[7 + 2*Sqrt[14]]*Sqrt[3 - 2*x] - 2*x])/637206919404798869504 - (11275*(9756589235 - 2148932869*Sqrt[14])*Sqrt[(-7 + 2*Sqrt[14])/2]*Log[3 + Sqrt[14] + Sqrt[7 + 2*Sqrt[14]]*Sqrt[3 - 2*x] - 2*x])/637206919404798869504
\end{aligned}$$

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\begin{aligned}
& \frac{x}{63(-2x+3)^{\frac{19}{2}}(2x^2+x+1)^9} \\
& + \frac{-15435719146659136558464000x + 6440121232839552246912000}{154905798615955861175009280(-2x+3)^{\frac{19}{2}}(2x^2+x+1)} \\
& + \int \frac{-11813932218388106205374976000000x - 6249079685931055968022769664000}{(-2x+3)^{\frac{11}{2}}(2x^2+x+1)} dx \\
& + \frac{2665985799514491042578822112215040}{5846828446875} - \frac{13515743021825}{14513641568075776(-2x+3)^{\frac{11}{2}}} - \frac{13476952884641792(-2x+3)^{\frac{13}{2}}}{815900548375} \\
& - \frac{1555033025150976(-2x+3)^{\frac{15}{2}}}{67816x + 20776} - \frac{629418129227776(-2x+3)^{\frac{17}{2}}}{117492592x + 46521776} \\
& + \frac{2765952(-2x+3)^{\frac{19}{2}}(2x^2+x+1)^8}{164128134240x + 74020332960} + \frac{3794886144(-2x+3)^{\frac{19}{2}}(2x^2+x+1)^7}{184316990760000x + 94209549053760} \\
& + \frac{4462786105344(-2x+3)^{\frac{19}{2}}(2x^2+x+1)^6}{157747397367934080x + 95476201213680000} + \frac{4373530383237120(-2x+3)^{\frac{19}{2}}(2x^2+x+1)^5}{3428847820457902080(-2x+3)^{\frac{19}{2}}(2x^2+x+1)^4} \\
& + \frac{89735798552133000960x + 72879297583985544960}{2016162518429246423040(-2x+3)^{\frac{19}{2}}(2x^2+x+1)^3} \\
& + \frac{18400346379541577848320x + 36432734212165998389760}{790335707224264597831680(-2x+3)^{\frac{19}{2}}(2x^2+x+1)^2} \\
& + \frac{4718120139975}{351733660450816(-2x+3)^{\frac{19}{2}}}
\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/(3-2*x)**(21/2)/(2*x**2+x+1)**10,x)`

[Out] `x/(63*(-2*x + 3)**(19/2)*(2*x**2 + x + 1)**9) + (-15435719146659136558464000*x + 6440121232839552246912000)/(154905798615955861175`

$$\begin{aligned}
& 009280*(-2*x + 3)**(19/2)*(2*x**2 + x + 1)) + \text{Integral}((-11813932 \\
& 218388106205374976000000*x - 6249079685931055968022769664000)/((- \\
& 2*x + 3)**(11/2)*(2*x**2 + x + 1)), x)/26659857995144910425788221 \\
& 12215040 - 5846828446875/(14513641568075776*(-2*x + 3)**(11/2)) - \\
& 13515743021825/(13476952884641792*(-2*x + 3)**(13/2)) - 30295088 \\
& 23715/(1555033025150976*(-2*x + 3)**(15/2)) - 815900548375/(62941 \\
& 8129227776*(-2*x + 3)**(17/2)) + (67816*x + 20776)/(2765952*(-2*x \\
& + 3)**(19/2)*(2*x**2 + x + 1)**8) + (117492592*x + 46521776)/(37 \\
& 94886144*(-2*x + 3)**(19/2)*(2*x**2 + x + 1)**7) + (164128134240* \\
& x + 74020332960)/(4462786105344*(-2*x + 3)**(19/2)*(2*x**2 + x + \\
& 1)**6) + (184316990760000*x + 94209549053760)/(4373530383237120*(\\
& -2*x + 3)**(19/2)*(2*x**2 + x + 1)**5) + (157747397367934080*x + \\
& 95476201213680000)/(3428847820457902080*(-2*x + 3)**(19/2)*(2*x** \\
& 2 + x + 1)**4) + (89735798552133000960*x + 72879297583985544960)/ \\
& (2016162518429246423040*(-2*x + 3)**(19/2)*(2*x**2 + x + 1)**3) + \\
& (18400346379541577848320*x + 36432734212165998389760)/(790335707 \\
& 224264597831680*(-2*x + 3)**(19/2)*(2*x**2 + x + 1)**2) + 4718120 \\
& 139975/(351733660450816*(-2*x + 3)**(19/2))
\end{aligned}$$

Mathematica [C] time = 6.07724, size = 662, normalized size = 1.02

$$\begin{aligned}
& - \frac{44193\sqrt{3-2x} - 11993(3-2x)^{3/2}}{948721536((3-2x)^2 - 7(3-2x) + 14)^8} + \frac{891605}{12401793332096\sqrt{3-2x}} \\
& - \frac{55(1410835658499(3-2x)^{3/2} - 4751425354423\sqrt{3-2x})}{68272169936228450304((3-2x)^2 - 7(3-2x) + 14)} \\
& + \frac{8519225}{260437659974016(3-2x)^{3/2}} \\
& - \frac{11(1953387138017(3-2x)^{3/2} - 6489356793153\sqrt{3-2x})}{17068042484057112576((3-2x)^2 - 7(3-2x) + 14)^2} \\
& + \frac{75933}{3100448333024(3-2x)^{5/2}} - \frac{1406968826615(3-2x)^{3/2} - 4402987778403\sqrt{3-2x}}{914359418788773888((3-2x)^2 - 7(3-2x) + 14)^3} \\
& + \frac{854095}{43406276662336(3-2x)^{7/2}} - \frac{52802422641(3-2x)^{3/2} - 132204145097\sqrt{3-2x}}{32655693528170496((3-2x)^2 - 7(3-2x) + 14)^4} \\
& + \frac{30349}{1993145356944(3-2x)^{9/2}} - \frac{5(38010319(3-2x)^{3/2} + 107643741\sqrt{3-2x})}{291568692215808((3-2x)^2 - 7(3-2x) + 14)^5} \\
& + \frac{2365}{221460595216(3-2x)^{11/2}} + \frac{5(5912661(3-2x)^{3/2} - 37938085\sqrt{3-2x})}{10413167579136((3-2x)^2 - 7(3-2x) + 14)^6} \\
& + \frac{165}{25705247659(3-2x)^{13/2}} + \frac{5(340449(3-2x)^{3/2} - 1574149\sqrt{3-2x})}{185949421056((3-2x)^2 - 7(3-2x) + 14)^7} \\
& + \frac{73}{23727920916(3-2x)^{15/2}} + \frac{5}{4802079233(3-2x)^{17/2}} \\
& - \frac{47\sqrt{3-2x} - 23(3-2x)^{3/2}}{4235364((3-2x)^2 - 7(3-2x) + 14)^9} + \frac{1}{5367029731(3-2x)^{19/2}} \\
& - \frac{11275(2148932869\sqrt{7} - 34555708553i) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{3-2x}}{\sqrt{-7-i\sqrt{7}}}\right)}{22757389978742816768\sqrt{14(-7-i\sqrt{7})}} \\
& - \frac{11275(2148932869\sqrt{7} + 34555708553i) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{3-2x}}{\sqrt{-7+i\sqrt{7}}}\right)}{22757389978742816768\sqrt{14(-7+i\sqrt{7})}}
\end{aligned}$$

Antiderivative was successfully verified.

[In] Integrate[1/((3 - 2*x)^(21/2)*(1 + x + 2*x^2)^10), x]

[Out] -(47*sqrt[3 - 2*x] - 23*(3 - 2*x)^(3/2))/(4235364*(14 - 7*(3 - 2*x) + (3 - 2*x)^2)^9) - (44193*sqrt[3 - 2*x] - 11993*(3 - 2*x)^(3/2))/(948721536*(14 - 7*(3 - 2*x) + (3 - 2*x)^2)^8) + (5*(-1574149*sqrt[3 - 2*x] + 340449*(3 - 2*x)^(3/2)))/(185949421056*(14 - 7*(3 - 2*x) + (3 - 2*x)^2)^7) + (5*(-37938085*sqrt[3 - 2*x] + 5912661*(3 - 2*x)^(3/2)))/(10413167579136*(14 - 7*(3 - 2*x) + (3 - 2*x)

$$\begin{aligned} & ^2)^6) - (5*(107643741*\text{Sqrt}[3 - 2*x] + 38010319*(3 - 2*x)^{(3/2)})) \\ & / (291568692215808*(14 - 7*(3 - 2*x) + (3 - 2*x)^2)^5) - (-1322041 \\ & 45097*\text{Sqrt}[3 - 2*x] + 52802422641*(3 - 2*x)^{(3/2)}) / (3265569352817 \\ & 0496*(14 - 7*(3 - 2*x) + (3 - 2*x)^2)^4) - (-4402987778403*\text{Sqrt}[3 \\ & - 2*x] + 1406968826615*(3 - 2*x)^{(3/2)}) / (914359418788773888*(14 \\ & - 7*(3 - 2*x) + (3 - 2*x)^2)^3) - (11*(-6489356793153*\text{Sqrt}[3 - 2* \\ & x] + 1953387138017*(3 - 2*x)^{(3/2)})) / (17068042484057112576*(14 - \\ & 7*(3 - 2*x) + (3 - 2*x)^2)^2) - (55*(-4751425354423*\text{Sqrt}[3 - 2*x] \\ & + 1410835658499*(3 - 2*x)^{(3/2)})) / (68272169936228450304*(14 - 7* \\ & (3 - 2*x) + (3 - 2*x)^2)) + 1/(5367029731*(3 - 2*x)^{(19/2)}) + 5/(\\ & 4802079233*(3 - 2*x)^{(17/2)}) + 73/(23727920916*(3 - 2*x)^{(15/2)}) \\ & + 165/(25705247659*(3 - 2*x)^{(13/2)}) + 2365/(221460595216*(3 - 2* \\ & x)^{(11/2)}) + 30349/(1993145356944*(3 - 2*x)^{(9/2)}) + 854095/(4340 \\ & 6276662336*(3 - 2*x)^{(7/2)}) + 75933/(3100448333024*(3 - 2*x)^{(5/2) \\ &)) + 8519225/(260437659974016*(3 - 2*x)^{(3/2)}) + 891605/(12401793 \\ & 332096*\text{Sqrt}[3 - 2*x]) - (11275*(-34555708553*I + 2148932869*\text{Sqrt}[\\ & 7])*ArcTan[(\text{Sqrt}[2]*\text{Sqrt}[3 - 2*x])/(\text{Sqrt}[-7 - I*\text{Sqrt}[7]])]) / (227573 \\ & 89978742816768*\text{Sqrt}[14*(-7 - I*\text{Sqrt}[7])]) - (11275*(34555708553*I \\ & + 2148932869*\text{Sqrt}[7])*ArcTan[(\text{Sqrt}[2]*\text{Sqrt}[3 - 2*x])/(\text{Sqrt}[-7 + I \\ & *\text{Sqrt}[7]])]) / (22757389978742816768*\text{Sqrt}[14*(-7 + I*\text{Sqrt}[7])]) \end{aligned}$$

Maple [A] time = 0.072, size = 719, normalized size = 1.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/(3-2*x)^{(21/2)}/(2*x^2+x+1)^{10}, x)$

[Out] $1/86812553324672*(-165574989211387894481/65536*(3-2*x)^{(23/2)}+454$
 $06001689183688581/131072*(3-2*x)^{(25/2)}-43462358811134257841/1179$
 $648*(3-2*x)^{(27/2)}+192384852501874197/65536*(3-2*x)^{(29/2)}-135284$
 $1099712333/8192*(3-2*x)^{(31/2)}+4606702222670185/786432*(3-2*x)^{(3$
 $3/2)}-25865320405815/262144*(3-2*x)^{(35/2)}+544765170330150812273/1$
 $024*(3-2*x)^{(1/2)}-3476987783905860258979/1536*(3-2*x)^{(3/2)}+93649$
 $99706478908741137/2048*(3-2*x)^{(5/2)}-23851905772903279054347/4096$
 $*(3-2*x)^{(7/2)}+192983613795383541041317/36864*(3-2*x)^{(9/2)}-57758$
 $421475348449750643/16384*(3-2*x)^{(11/2)}+60333035869584695411551/3$
 $2768*(3-2*x)^{(13/2)}-149770885083493978040723/196608*(3-2*x)^{(15/2)}$
 $+66256899944582155696811/262144*(3-2*x)^{(17/2)}-17729978841543630$
 $405471/262144*(3-2*x)^{(19/2)}+2869878271121283060373/196608*(3-2*x$
 $)^{(21/2)})/((3-2*x)^2-7+14*x)^9-206922416016525/127441383880959773$
 $9008*\ln(3-2*x+14^{(1/2)}-(3-2*x)^{(1/2)}*(7+2*14^{(1/2)})^{(1/2)})*(7+2*1$
 $4^{(1/2)})^{(1/2)}*14^{(1/2)}+389615613935075/637206919404798869504*\ln($
 $3-2*x+14^{(1/2)}-(3-2*x)^{(1/2)}*(7+2*14^{(1/2)})^{(1/2)})*(7+2*14^{(1/2)}$
 $)^{(1/2)}-206922416016525/637206919404798869504/(-7+2*14^{(1/2)})^{(1/2)}$
 $)*\arctan((2*(3-2*x)^{(1/2)}-(7+2*14^{(1/2)})^{(1/2)})/(-7+2*14^{(1/2)})^{(1/2)})$
 $*(7+2*14^{(1/2)})*14^{(1/2)}+389615613935075/3186034597023994347$
 $52/(-7+2*14^{(1/2)})^{(1/2)}*\arctan((2*(3-2*x)^{(1/2)}-(7+2*14^{(1/2)})^{(1/2)}$
 $)/(-7+2*14^{(1/2)})^{(1/2)})*(7+2*14^{(1/2)})-110005543624625/31860$
 $3459702399434752/(-7+2*14^{(1/2)})^{(1/2)}*\arctan((2*(3-2*x)^{(1/2)}-(7$
 $+2*14^{(1/2)})^{(1/2)})/(-7+2*14^{(1/2)})^{(1/2)})*14^{(1/2)}+2069224160165$

$$\frac{25}{1274413838809597739008} \ln(3-2^*x+14^{(1/2)}+(3-2^*x)^{(1/2)} * (7+2^*14^{(1/2)})^{(1/2)})^{(1/2)} * (7+2^*14^{(1/2)})^{(1/2)} * 14^{(1/2)} - \frac{389615613935075}{637206919404798869504} \ln(3-2^*x+14^{(1/2)}+(3-2^*x)^{(1/2)} * (7+2^*14^{(1/2)})^{(1/2)})^{(1/2)} * (7+2^*14^{(1/2)})^{(1/2)} - \frac{206922416016525}{637206919404798869504} / (-7+2^*14^{(1/2)})^{(1/2)} * \arctan((2^*(3-2^*x)^{(1/2)}+(7+2^*14^{(1/2)})^{(1/2)}) / (-7+2^*14^{(1/2)})^{(1/2)}) * (7+2^*14^{(1/2)})^{(1/2)} * 14^{(1/2)} + \frac{389615613935075}{318603459702399434752} / (-7+2^*14^{(1/2)})^{(1/2)} * \arctan((2^*(3-2^*x)^{(1/2)}+(7+2^*14^{(1/2)})^{(1/2)}) / (-7+2^*14^{(1/2)})^{(1/2)}) * (7+2^*14^{(1/2)})^{(1/2)} - \frac{110005543624625}{318603459702399434752} / (-7+2^*14^{(1/2)})^{(1/2)} * \arctan((2^*(3-2^*x)^{(1/2)}+(7+2^*14^{(1/2)})^{(1/2)}) / (-7+2^*14^{(1/2)})^{(1/2)}) * 14^{(1/2)} + \frac{1}{5367029731} / (3-2^*x)^{(19/2)} + \frac{5}{4802079233} / (3-2^*x)^{(17/2)} + \frac{73}{23727920916} / (3-2^*x)^{(15/2)} + \frac{165}{25705247659} / (3-2^*x)^{(13/2)} + \frac{2365}{221460595216} / (3-2^*x)^{(11/2)} + \frac{30349}{1993145356944} / (3-2^*x)^{(9/2)} + \frac{854095}{43406276662336} / (3-2^*x)^{(7/2)} + \frac{75933}{3100448333024} / (3-2^*x)^{(5/2)} + \frac{8519225}{260437659974016} / (3-2^*x)^{(3/2)} + \frac{891605}{12401793332096} / (3-2^*x)^{(1/2)}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2x^2 + x + 1)^{10}(-2x + 3)^{\frac{21}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((2*x^2 + x + 1)^10*(-2*x + 3)^(21/2)),x, algorithm="maxima")

[Out] integrate(1/((2*x^2 + x + 1)^10*(-2*x + 3)^(21/2)), x)

Fricas [A] time = 0.340481, size = 2839, normalized size = 4.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((2*x^2 + x + 1)^10*(-2*x + 3)^(21/2)),x, algorithm="fricas")

[Out] $\frac{1}{5520330666373161526201587121296845762985984} \sqrt{1169607525756986} * 2744^{(3/4)} * (52052026180937767452848051100 * \sqrt{584803762878493} * \sqrt{14} * (262144 * x^{27} - 2359296 * x^{26} + 8847360 * x^{25} - 19070976 * x^{24} + 31260672 * x^{23} - 50135040 * x^{22} + 72400896 * x^{21} - 84787200 * x^{20} + 97449984 * x^{19} - 111622144 * x^{18} + 102818304 * x^{17} - 94063104 * x^{16} + 92761344 * x^{15} - 66772224 * x^{14} + 51609024 * x^{13} - 46803648 * x^{12} + 23042340 * x^{11} - 18046404 * x^{10} + 14741135 * x^9 - 2785131 * x^8 + 5374836 * x^7 - 1955988 * x^6 - 185166 * x^5 - 1395306 * x^4 - 454896 * x^3 - 314928 * x^2 - 59049 * x - 19683) * \sqrt{-2 * x + 3} * \arctan(16374505360597804 * \sqrt{584803762878493} * (34555708553 * \sqrt{14} * \sqrt{7} - 128466244977 * \sqrt{7})) / (\sqrt{1169607525756986} * \sqrt{584803762878493})$

$$\begin{aligned}
& 3) * 2744^{(1/4)} * (327571850528462403199 * \sqrt{14} * \sqrt{7} - 122642238 \\
& 0928157351936 * \sqrt{7}) * \sqrt{-\sqrt{14} * (\sqrt{1169607525756986} * \sqrt{7} \\
& \sqrt{584803762878493}) * 2744^{(1/4)} * \sqrt{-2*x + 3} * (1753205114611442492 \\
& 1588415510503669733478055782584434677759736860464224411 * \sqrt{14} \\
& - 655989286765353399282614782661181821311507917509232773908588405 \\
& 51081105174) * \sqrt{((327571850528462403199 * \sqrt{14} - 1226422380928 \\
& 157351936) / (401741448850159339627110059970918575243264 * \sqrt{14} - \\
& 1503179149031234349960285217920460999509255))} + 1169607525756986 \\
& * \sqrt{14} * (985103879740300769348116449646663111706904845321946338 \\
& 203465849 * \sqrt{14} * (2*x - 3) - 7371842417180200746144727139314482 \\
& 572723741509811671017998778368 * x + 110577636257703011192170907089 \\
& 71723859085612264717506526998167552) - 16130588759553249395427127 \\
& 271762836634823565933565015784697865123790835338395596 * \sqrt{14} + \\
& 6035513658879973504399141330264538203774112504578512162215455452 \\
& 0755897091751936) / (9851038797403007693481164496466631117069048453 \\
& 21946338203465849 * \sqrt{14} - 368592120859010037307236356965724128 \\
& 6361870754905835508999389184)) * \sqrt{((327571850528462403199 * \sqrt{14} \\
& - 1226422380928157351936) / (40174144885015933962711005997091857 \\
& 5243264 * \sqrt{14} - 1503179149031234349960285217920460999509255))} \\
& + 8187252680298902 * \sqrt{1169607525756986} * 2744^{(1/4)} * \sqrt{-2*x + \\
& 3} * (327571850528462403199 * \sqrt{14} - 1226422380928157351936) * \sqrt{ \\
& ((327571850528462403199 * \sqrt{14} - 1226422380928157351936) / (40174 \\
& 1448850159339627110059970918575243264 * \sqrt{14} - 1503179149031234 \\
& 349960285217920460999509255))} - 114621537524184628 * \sqrt{584803762 \\
& 878493} * (2148932869 * \sqrt{14} - 9756589235)) + 520520261809377674 \\
& 52848051100 * \sqrt{584803762878493} * \sqrt{14} * (262144 * x^{27} - 2359296 \\
& * x^{26} + 8847360 * x^{25} - 19070976 * x^{24} + 31260672 * x^{23} - 50135040 * x \\
& ^{22} + 72400896 * x^{21} - 84787200 * x^{20} + 97449984 * x^{19} - 111622144 * x \\
& ^{18} + 102818304 * x^{17} - 94063104 * x^{16} + 92761344 * x^{15} - 66772224 * x \\
& ^{14} + 51609024 * x^{13} - 46803648 * x^{12} + 23042340 * x^{11} - 18046404 * x^{10} \\
& + 14741135 * x^9 - 2785131 * x^8 + 5374836 * x^7 - 1955988 * x^6 - 185 \\
& 166 * x^5 - 1395306 * x^4 - 454896 * x^3 - 314928 * x^2 - 59049 * x - 19683 \\
&) * \sqrt{-2*x + 3} * \arctan(16374505360597804 * \sqrt{584803762878493}) * (\\
& 34555708553 * \sqrt{14} * \sqrt{7} - 128466244977 * \sqrt{7}) / (\sqrt{116960 \\
& 7525756986} * \sqrt{584803762878493}) * 2744^{(1/4)} * (3275718505284624031 \\
& 99 * \sqrt{14} * \sqrt{7} - 1226422380928157351936 * \sqrt{7}) * \sqrt{(\sqrt{14} \\
& * (\sqrt{1169607525756986} * \sqrt{584803762878493}) * 2744^{(1/4)} * \sqrt{ \\
& -2*x + 3} * (175320511461144249215884155105036697334780557825844346 \\
& 77759736860464224411 * \sqrt{14} - 655989286765353399282614782661181 \\
& 82131150791750923277390858840551081105174) * \sqrt{((3275718505284624 \\
& 03199 * \sqrt{14} - 1226422380928157351936) / (40174144885015933962711 \\
& 0059970918575243264 * \sqrt{14} - 1503179149031234349960285217920460 \\
& 999509255))} - 1169607525756986 * \sqrt{14} * (985103879740300769348116 \\
& 449646663111706904845321946338203465849 * \sqrt{14} * (2*x - 3) - 7371 \\
& 842417180200746144727139314482572723741509811671017998778368 * x + \\
& 11057763625770301119217090708971723859085612264717506526998167552 \\
&) + 1613058875955324939542712727176283663482356593356501578469786 \\
& 5123790835338395596 * \sqrt{14} - 6035513658879973504399141330264538 \\
& 2037741125045785121622154554520755897091751936) / (9851038797403007 \\
& 69348116449646663111706904845321946338203465849 * \sqrt{14} - 368592 \\
& 1208590100373072363569657241286361870754905835508999389184)) * \sqrt{ \\
& ((327571850528462403199 * \sqrt{14} - 1226422380928157351936) / (40174 \\
& 1448850159339627110059970918575243264 * \sqrt{14} - 1503179149031234 \\
& 349960285217920460999509255))} + 8187252680298902 * \sqrt{11696075257 \\
& 56986} * 2744^{(1/4)} * \sqrt{-2*x + 3} * (327571850528462403199 * \sqrt{14} \\
& - 1226422380928157351936) * \sqrt{((327571850528462403199 * \sqrt{14} -
\end{aligned}$$

$$\begin{aligned}
& 1226422380928157351936)/(4017414488501593396271100599709185752432 \\
& 64*\sqrt{14} - 1503179149031234349960285217920460999509255)) + 114 \\
& 621537524184628*\sqrt{584803762878493}*(2148932869*\sqrt{14} - 9756 \\
& 589235))) + 426093525*\sqrt{584803762878493}*(32757185052846240319 \\
& 9*\sqrt{14})*\sqrt{7}*(262144*x^{27} - 2359296*x^{26} + 8847360*x^{25} - 1 \\
& 9070976*x^{24} + 31260672*x^{23} - 50135040*x^{22} + 72400896*x^{21} - 84 \\
& 787200*x^{20} + 97449984*x^{19} - 111622144*x^{18} + 102818304*x^{17} - 9 \\
& 4063104*x^{16} + 92761344*x^{15} - 66772224*x^{14} + 51609024*x^{13} - 46 \\
& 803648*x^{12} + 23042340*x^{11} - 18046404*x^{10} + 14741135*x^9 - 2785 \\
& 131*x^8 + 5374836*x^7 - 1955988*x^6 - 185166*x^5 - 1395306*x^4 - \\
& 454896*x^3 - 314928*x^2 - 59049*x - 19683) - 12264223809281573519 \\
& 36*\sqrt{7}*(262144*x^{27} - 2359296*x^{26} + 8847360*x^{25} - 19070976* \\
& x^{24} + 31260672*x^{23} - 50135040*x^{22} + 72400896*x^{21} - 84787200*x \\
& ^{20} + 97449984*x^{19} - 111622144*x^{18} + 102818304*x^{17} - 94063104* \\
& x^{16} + 92761344*x^{15} - 66772224*x^{14} + 51609024*x^{13} - 46803648*x \\
& ^{12} + 23042340*x^{11} - 18046404*x^{10} + 14741135*x^9 - 2785131*x^8 \\
& + 5374836*x^7 - 1955988*x^6 - 185166*x^5 - 1395306*x^4 - 454896*x \\
& ^3 - 314928*x^2 - 59049*x - 19683))*\sqrt{-2*x + 3}*\log(-743435438 \\
& 58280221683125/4*\sqrt{14}*(\sqrt{1169607525756986})*\sqrt{5848037628 \\
& 78493})*2744^{(1/4)}*\sqrt{-2*x + 3}*(1753205114611442492158841551050 \\
& 366973347805578258443467759736860464224411*\sqrt{14} - 6559892867 \\
& 6535339928261478266118182131150791750923277390858840551081105174) \\
& *\sqrt{((327571850528462403199*\sqrt{14} - 1226422380928157351936)/(\\
& 401741448850159339627110059970918575243264*\sqrt{14} - 15031791490 \\
& 31234349960285217920460999509255)) + 1169607525756986*\sqrt{14}*(9 \\
& 85103879740300769348116449646663111706904845321946338203465849*sq \\
& rt(14)*(2*x - 3) - 7371842417180200746144727139314482572723741509 \\
& 811671017998778368*x + 110577636257703011192170907089717238590856 \\
& 12264717506526998167552) - 16130588759553249395427127271762836634 \\
& 823565933565015784697865123790835338395596*\sqrt{14} + 60355136588 \\
& 79973504399141330264538203774112504578512162215455452075589709175 \\
& 1936)/(9851038797403007693481164496466631117069048453219463382034 \\
& 65849*\sqrt{14} - 368592120859010037307236356965724128636187075490 \\
& 5835508999389184)) - 426093525*\sqrt{584803762878493}*(32757185052 \\
& 8462403199*\sqrt{14})*\sqrt{7}*(262144*x^{27} - 2359296*x^{26} + 8847360 \\
& *x^{25} - 19070976*x^{24} + 31260672*x^{23} - 50135040*x^{22} + 72400896* \\
& x^{21} - 84787200*x^{20} + 97449984*x^{19} - 111622144*x^{18} + 102818304 \\
& *x^{17} - 94063104*x^{16} + 92761344*x^{15} - 66772224*x^{14} + 51609024* \\
& x^{13} - 46803648*x^{12} + 23042340*x^{11} - 18046404*x^{10} + 14741135*x \\
& ^9 - 2785131*x^8 + 5374836*x^7 - 1955988*x^6 - 185166*x^5 - 13953 \\
& 06*x^4 - 454896*x^3 - 314928*x^2 - 59049*x - 19683) - 12264223809 \\
& 28157351936*\sqrt{7}*(262144*x^{27} - 2359296*x^{26} + 8847360*x^{25} - \\
& 19070976*x^{24} + 31260672*x^{23} - 50135040*x^{22} + 72400896*x^{21} - 8 \\
& 4787200*x^{20} + 97449984*x^{19} - 111622144*x^{18} + 102818304*x^{17} - \\
& 94063104*x^{16} + 92761344*x^{15} - 66772224*x^{14} + 51609024*x^{13} - 4 \\
& 6803648*x^{12} + 23042340*x^{11} - 18046404*x^{10} + 14741135*x^9 - 278 \\
& 5131*x^8 + 5374836*x^7 - 1955988*x^6 - 185166*x^5 - 1395306*x^4 - \\
& 454896*x^3 - 314928*x^2 - 59049*x - 19683))*\sqrt{-2*x + 3}*\log(7 \\
& 4343543858280221683125/4*\sqrt{14}*(\sqrt{1169607525756986})*\sqrt{58 \\
& 4803762878493})*2744^{(1/4)}*\sqrt{-2*x + 3}*(17532051146114424921588 \\
& 41551050366973347805578258443467759736860464224411*\sqrt{14} - 65 \\
& 59892867653533992826147826611818213115079175092327739085884055108 \\
& 1105174)*\sqrt{((327571850528462403199*\sqrt{14} - 12264223809281573 \\
& 51936)/(401741448850159339627110059970918575243264*\sqrt{14} - 150 \\
& 3179149031234349960285217920460999509255)) - 1169607525756986*sq \\
& rt(14)*(9851038797403007693481164496466631117069048453219463382034
\end{aligned}$$

$$\begin{aligned}
& 65849 \sqrt{14} (2x - 3) - 73718424171802007461447271393144825727 \\
& 23741509811671017998778368x + 1105776362577030111921709070897172 \\
& 3859085612264717506526998167552) + 161305887595532493954271272717 \\
& 62836634823565933565015784697865123790835338395596 \sqrt{14} - 603 \\
& 55136588799735043991413302645382037741125045785121622154554520755 \\
& 897091751936) / (98510387974030076934811644964666311170690484532194 \\
& 6338203465849 \sqrt{14} - 3685921208590100373072363569657241286361 \\
& 870754905835508999389184) - 2 \sqrt{1169607525756986} * 2744^{1/4} * \\
& (327571850528462403199 \sqrt{14} \sqrt{7} (240031204937714427494400 \\
& * x^{27} - 2621948941596237063782400 * x^{26} + 123650450558968111054848 \\
& 00 * x^{25} - 33969890064381284111155200 * x^{24} + 653601202912587967578 \\
& 11200 * x^{23} - 106701725825102321939251200 * x^{22} + 16229030722324950 \\
& 2039654400 * x^{21} - 216634228326470609547509760 * x^{20} + 253788172995 \\
& 391086570485760 * x^{19} - 287279159180291305208156160 * x^{18} + 3040105 \\
& 91010966811155955200 * x^{17} - 282644664539994827031006720 * x^{16} + 25 \\
& 8819256815163249845447936 * x^{15} - 229408132984166521977166336 * x^{14} \\
& + 172649692294614969274168896 * x^{13} - 133312541377246386115890240 \\
& * x^{12} + 102031573634317834547976132 * x^{11} - 5979110268149411757214 \\
& 9176 * x^{10} + 41613884937255303086792337 * x^9 - 27246604251076689552 \\
& 043953 * x^8 + 10718131725916893151555068 * x^7 - 8685973988079840377 \\
& 705700 * x^6 + 3673303058277822225386926 * x^5 - 80999036209504421005 \\
& 4958 * x^4 + 1362587089603925431664856 * x^3 + 1119267686976029998061 \\
& 16 * x^2 + 205702452014540322797289 * x - 4884417100172357749737) - 1 \\
& 226422380928157351936 \sqrt{7} (240031204937714427494400 * x^{27} - 26 \\
& 21948941596237063782400 * x^{26} + 12365045055896811105484800 * x^{25} - \\
& 33969890064381284111155200 * x^{24} + 65360120291258796757811200 * x^{23} \\
& - 106701725825102321939251200 * x^{22} + 162290307223249502039654400 \\
& * x^{21} - 216634228326470609547509760 * x^{20} + 2537881729953910865704 \\
& 85760 * x^{19} - 287279159180291305208156160 * x^{18} + 30401059101096681 \\
& 1155955200 * x^{17} - 282644664539994827031006720 * x^{16} + 258819256815 \\
& 163249845447936 * x^{15} - 229408132984166521977166336 * x^{14} + 1726496 \\
& 92294614969274168896 * x^{13} - 133312541377246386115890240 * x^{12} + 10 \\
& 2031573634317834547976132 * x^{11} - 59791102681494117572149176 * x^{10} \\
& + 41613884937255303086792337 * x^9 - 27246604251076689552043953 * x^8 \\
& + 10718131725916893151555068 * x^7 - 8685973988079840377705700 * x^6 \\
& + 3673303058277822225386926 * x^5 - 809990362095044210054958 * x^4 + \\
& 1362587089603925431664856 * x^3 + 111926768697602999806116 * x^2 + 2 \\
& 05702452014540322797289 * x - 4884417100172357749737) * \sqrt{(327571 \\
& 850528462403199 \sqrt{14} - 1226422380928157351936) / (4017414488501 \\
& 59339627110059970918575243264 \sqrt{14} - 150317914903123434996028 \\
& 5217920460999509255)) / ((327571850528462403199 \sqrt{14} \sqrt{7} (\\
& 262144 * x^{27} - 2359296 * x^{26} + 8847360 * x^{25} - 19070976 * x^{24} + 31260 \\
& 672 * x^{23} - 50135040 * x^{22} + 72400896 * x^{21} - 84787200 * x^{20} + 974499 \\
& 84 * x^{19} - 111622144 * x^{18} + 102818304 * x^{17} - 94063104 * x^{16} + 92761 \\
& 344 * x^{15} - 66772224 * x^{14} + 51609024 * x^{13} - 46803648 * x^{12} + 230423 \\
& 40 * x^{11} - 18046404 * x^{10} + 14741135 * x^9 - 2785131 * x^8 + 5374836 * x^7 \\
& - 1955988 * x^6 - 185166 * x^5 - 1395306 * x^4 - 454896 * x^3 - 314928 * \\
& x^2 - 59049 * x - 19683) - 1226422380928157351936 \sqrt{7} (262144 * x \\
& ^{27} - 2359296 * x^{26} + 8847360 * x^{25} - 19070976 * x^{24} + 31260672 * x^{23} \\
& - 50135040 * x^{22} + 72400896 * x^{21} - 84787200 * x^{20} + 97449984 * x^{19} \\
& - 111622144 * x^{18} + 102818304 * x^{17} - 94063104 * x^{16} + 92761344 * x^{15} \\
& - 66772224 * x^{14} + 51609024 * x^{13} - 46803648 * x^{12} + 23042340 * x^{11} \\
& - 18046404 * x^{10} + 14741135 * x^9 - 2785131 * x^8 + 5374836 * x^7 - 1955 \\
& 988 * x^6 - 185166 * x^5 - 1395306 * x^4 - 454896 * x^3 - 314928 * x^2 - 59 \\
& 049 * x - 19683) * \sqrt{-2x + 3} * \sqrt{(327571850528462403199 \sqrt{14} \\
& (4) - 1226422380928157351936) / (40174144885015933962711005997091857
\end{aligned}$$

5243264*sqrt(14) - 1503179149031234349960285217920460999509255)))

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3-2*x)**(21/2)/(2*x**2+x+1)**10,x)

[Out] Timed out

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2x^2 + x + 1)^{10}(-2x + 3)^{\frac{21}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((2*x^2 + x + 1)^10*(-2*x + 3)^(21/2)),x, algorithm="giac")

[Out] integrate(1/((2*x^2 + x + 1)^10*(-2*x + 3)^(21/2)), x)

$$3.49 \quad \int \frac{1}{(3-2x)^{41/2}(1+x+2x^2)^{20}} dx$$

Optimal. Leaf size=1058

result too large to display

```
[Out] -13056959628363355534285785425/(106924014357253562723941220352*(3
- 2*x)^(39/2)) - 3948194343291401740321996415/(20288146313940419
5937734623232*(3 - 2*x)^(37/2)) - 304688229262620222736480811/(53
7361713180043545997243056128*(3 - 2*x)^(35/2)) + 2124315846756567
455653862925/(1688851098565851144562763890688*(3 - 2*x)^(33/2)) +
47657515074514118796095929535/(66632852434325399703658138959872*
(3 - 2*x)^(31/2)) + 34911619993974714062172751985/(12466791745777
0102671360389021696*(3 - 2*x)^(29/2)) + 1490663098087947608430174
04825/(1624981820656451683095663001731072*(3 - 2*x)^(27/2)) + 158
48613964169066543734380171/(60184511876164877151691222863360*(3
- 2*x)^(25/2)) + 11155168222970774232376891145/(16851663325326165
60247354224017408*(3 - 2*x)^(23/2)) + 140118184980910202724749563
75/(10110997995195699361484125344104448*(3 - 2*x)^(21/2)) + 17344
1368149804378661935869705/(896508488907352010051592447177261056*(
3 - 2*x)^(19/2)) - 22724090823469905152713519545/(160427834857105
0965355481221264572416*(3 - 2*x)^(17/2)) - 1011902744127796186785
73275245/(3963511214116714149701777134888943616*(3 - 2*x)^(15/2))
- 460503190416958283087439337135/(343504305223448559640820685023
70844672*(3 - 2*x)^(13/2)) - 2211619588790911794826342607495/(406
920484649315986036049119181931544576*(3 - 2*x)^(11/2)) - 14340146
7550777247627940437025/(73985542663511997461099839851260280832*(3
- 2*x)^(9/2)) - 4611053278117143010907562317585/(725058318102417
5751187784305423507521536*(3 - 2*x)^(7/2)) - 40596537244063051072
0926890227/(2071595194578335928910795515835287863296*(3 - 2*x)^(5
/2)) - 4986681479187781853417316522775/(8700699817229010901425341
1665082090258432*(3 - 2*x)^(3/2)) - 92702775478147674620804762050
5/(58004665448193406009502274443388060172288*sqrt[3 - 2*x]) + x/(
133*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^19) + (113 + 373*x)/(33516*(
3 - 2*x)^(39/2)*(1 + x + 2*x^2)^18) + (40657 + 107329*x)/(7976808
*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^17) + (5*(751303 + 1831285*x))/
(595601664*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^16) + (184959785 + 42
9411497*x)/(25015269888*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^15) + (4
1652915209 + 92630823167*x)/(4902992898048*(3 - 2*x)^(39/2)*(1 +
x + 2*x^2)^14) + (2871555518177 + 6100156355517*x)/(2974482358149
12*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^13) + (77559130805859 + 15627
4047129113*x)/(7138757659557888*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^
12) + (5*(2656658801194921 + 5020880176134289*x))/(10993686795719
14752*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^11) + (45187921585208601 +
78752911037377255*x)/(3420258114223734784*(3 - 2*x)^(39/2)*(1 +
x + 2*x^2)^10) + (6063974149878048635 + 9477172618423641847*x)/(4
30952522392190582784*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^9) + (69183
3601144925854831 + 919498192874055581221*x)/(48266682507925345271
808*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^8) + (23*(919498192874055581
221 + 908287136092467468517*x))/(1576711628592227945545728*(3 - 2
*x)^(39/2)*(1 + x + 2*x^2)^7) + (115*(908287136092467468517 + 298
281884944522225747*x))/(10187982830903626725064704*(3 - 2*x)^(39/
2)*(1 + x + 2*x^2)^6) + (23*(2599313568802265110081 - 10426142448
623187379187*x))/(20375965661807253450129408*(3 - 2*x)^(39/2)*(1
+ x + 2*x^2)^5) - (23*(10426142448623187379187 + 2751372346319426
```

$$\begin{aligned} & 2383705*x)) / (20018492580021161284337664*(3 - 2*x)^{(39/2)}*(1 + x + 2*x^2)^4) - (115*(26513224428169016478843 + 30673415406553789342 \\ & 019*x)) / (76434244396444433994743808*(3 - 2*x)^{(39/2)}*(1 + x + 2*x^2)^3) - (115*(88411609113007981044643 - 5712269536245152162963*x \\ &)) / (125891696652967303050166272*(3 - 2*x)^{(39/2)}*(1 + x + 2*x^2)^2) + (115*(28561347681225760814815 + 965934812839019490346107*x)) \\ & / (195831528126838026966925312*(3 - 2*x)^{(39/2)}*(1 + x + 2*x^2)) + \\ & (115*\text{Sqrt}[(7 + 2*\text{Sqrt}[14])/2]*(30297118912219360725028693061 + 8 \\ & 061110911143276053983022787*\text{Sqrt}[14])*\text{ArcTan}[(\text{Sqrt}[7 + 2*\text{Sqrt}[14]] \\ &] - 2*\text{Sqrt}[3 - 2*x])/\text{Sqrt}[-7 + 2*\text{Sqrt}[14]])]/81206531627470768413 \\ & 3031842207432842412032 - (115*\text{Sqrt}[(7 + 2*\text{Sqrt}[14])/2]*(302971189 \\ & 12219360725028693061 + 8061110911143276053983022787*\text{Sqrt}[14])*\text{Arc} \\ & \text{Tan}[(\text{Sqrt}[7 + 2*\text{Sqrt}[14]] + 2*\text{Sqrt}[3 - 2*x])/\text{Sqrt}[-7 + 2*\text{Sqrt}[14]] \\ &])/812065316274707684133031842207432842412032 + (115*(3029711891 \\ & 2219360725028693061 - 8061110911143276053983022787*\text{Sqrt}[14])*\text{Sqrt} \\ & [(-7 + 2*\text{Sqrt}[14])/2]*\text{Log}[3 + \text{Sqrt}[14] - \text{Sqrt}[7 + 2*\text{Sqrt}[14]]*\text{Sqr} \\ & \text{t}[3 - 2*x] - 2*x))/1624130632549415368266063684414865684824064 - \\ & (115*(30297118912219360725028693061 - 806111091114327605398302278 \\ & 7*\text{Sqrt}[14])*\text{Sqrt}[(-7 + 2*\text{Sqrt}[14])/2]*\text{Log}[3 + \text{Sqrt}[14] + \text{Sqrt}[7 + \\ & 2*\text{Sqrt}[14]]*\text{Sqrt}[3 - 2*x] - 2*x))/162413063254941536826606368441 \\ & 4865684824064 \end{aligned}$$

Rubi [A] time = 5.37521, antiderivative size = 1058, normalized size of antiderivative = 1., number of steps used = 49, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.45$

result too large to display

Warning: Unable to verify antiderivative.

[In] Int[1/((3 - 2*x)^(41/2)*(1 + x + 2*x^2)^20), x]

[Out] $-13056959628363355534285785425 / (106924014357253562723941220352 * (3 - 2*x)^{(39/2)}) - 3948194343291401740321996415 / (20288146313940419$
 $5937734623232 * (3 - 2*x)^{(37/2)}) - 304688229262620222736480811 / (53$
 $7361713180043545997243056128 * (3 - 2*x)^{(35/2)}) + 2124315846756567$
 $455653862925 / (1688851098565851144562763890688 * (3 - 2*x)^{(33/2)}) +$
 $47657515074514118796095929535 / (66632852434325399703658138959872 *$
 $(3 - 2*x)^{(31/2)}) + 34911619993974714062172751985 / (12466791745777$
 $0102671360389021696 * (3 - 2*x)^{(29/2)}) + 1490663098087947608430174$
 $04825 / (1624981820656451683095663001731072 * (3 - 2*x)^{(27/2)}) + 158$
 $48613964169066543734380171 / (601845118761648771516912222863360 * (3$
 $- 2*x)^{(25/2)}) + 11155168222970774232376891145 / (16851663325326165$
 $60247354224017408 * (3 - 2*x)^{(23/2)}) + 140118184980910202724749563$
 $75 / (10110997995195699361484125344104448 * (3 - 2*x)^{(21/2)}) + 17344$
 $1368149804378661935869705 / (896508488907352010051592447177261056 * ($
 $3 - 2*x)^{(19/2)}) - 22724090823469905152713519545 / (160427834857105$
 $0965355481221264572416 * (3 - 2*x)^{(17/2)}) - 1011902744127796186785$
 $73275245 / (3963511214116714149701777134888943616 * (3 - 2*x)^{(15/2)})$
 $- 460503190416958283087439337135 / (343504305223448559640820685023$
 $70844672 * (3 - 2*x)^{(13/2)}) - 2211619588790911794826342607495 / (406$
 $920484649315986036049119181931544576 * (3 - 2*x)^{(11/2)}) - 14340146$
 $7550777247627940437025 / (73985542663511997461099839851260280832 * (3$

$$\begin{aligned}
& - 2^*x)^{(9/2)}) - 4611053278117143010907562317585/(725058318102417 \\
& 5751187784305423507521536*(3 - 2^*x)^{(7/2)}) - 40596537244063051072 \\
& 0926890227/(2071595194578335928910795515835287863296*(3 - 2^*x)^{(5 \\
& /2)}) - 4986681479187781853417316522775/(8700699817229010901425341 \\
& 1665082090258432*(3 - 2^*x)^{(3/2)}) - 92702775478147674620804762050 \\
& 5/(58004665448193406009502274443388060172288*sqrt[3 - 2^*x]) + x/(\\
& 133*(3 - 2^*x)^{(39/2)}*(1 + x + 2^*x^2)^{19}) + (113 + 373*x)/(33516*(\\
& 3 - 2^*x)^{(39/2)}*(1 + x + 2^*x^2)^{18}) + (40657 + 107329*x)/(7976808 \\
& *(3 - 2^*x)^{(39/2)}*(1 + x + 2^*x^2)^{17}) + (5*(751303 + 1831285*x))/ \\
& (595601664*(3 - 2^*x)^{(39/2)}*(1 + x + 2^*x^2)^{16}) + (184959785 + 42 \\
& 9411497*x)/(25015269888*(3 - 2^*x)^{(39/2)}*(1 + x + 2^*x^2)^{15}) + (4 \\
& 1652915209 + 92630823167*x)/(4902992898048*(3 - 2^*x)^{(39/2)}*(1 + \\
& x + 2^*x^2)^{14}) + (2871555518177 + 6100156355517*x)/(2974482358149 \\
& 12*(3 - 2^*x)^{(39/2)}*(1 + x + 2^*x^2)^{13}) + (77559130805859 + 15627 \\
& 4047129113*x)/(7138757659557888*(3 - 2^*x)^{(39/2)}*(1 + x + 2^*x^2)^{ \\
& 12}) + (5*(2656658801194921 + 5020880176134289*x))/(10993686795719 \\
& 14752*(3 - 2^*x)^{(39/2)}*(1 + x + 2^*x^2)^{11}) + (45187921585208601 + \\
& 78752911037377255*x)/(3420258114223734784*(3 - 2^*x)^{(39/2)}*(1 + \\
& x + 2^*x^2)^{10}) + (6063974149878048635 + 9477172618423641847*x)/(4 \\
& 30952522392190582784*(3 - 2^*x)^{(39/2)}*(1 + x + 2^*x^2)^9) + (69183 \\
& 3601144925854831 + 919498192874055581221*x)/(48266682507925345271 \\
& 808*(3 - 2^*x)^{(39/2)}*(1 + x + 2^*x^2)^8) + (23*(919498192874055581 \\
& 221 + 908287136092467468517*x))/(1576711628592227945545728*(3 - 2 \\
& *x)^{(39/2)}*(1 + x + 2^*x^2)^7) + (115*(908287136092467468517 + 298 \\
& 281884944522225747*x))/(10187982830903626725064704*(3 - 2^*x)^{(39/ \\
& 2)}*(1 + x + 2^*x^2)^6) + (23*(2599313568802265110081 - 10426142448 \\
& 623187379187*x))/(20375965661807253450129408*(3 - 2^*x)^{(39/2)}*(1 \\
& + x + 2^*x^2)^5) - (23*(10426142448623187379187 + 2751372346319426 \\
& 2383705*x))/(20018492580021161284337664*(3 - 2^*x)^{(39/2)}*(1 + x + \\
& 2^*x^2)^4) - (115*(26513224428169016478843 + 30673415406553789342 \\
& 019*x))/(76434244396444433994743808*(3 - 2^*x)^{(39/2)}*(1 + x + 2^*x \\
& ^2)^3) - (115*(88411609113007981044643 - 5712269536245152162963*x \\
&))/(125891696652967303050166272*(3 - 2^*x)^{(39/2)}*(1 + x + 2^*x^2)^ \\
& 2) + (115*(28561347681225760814815 + 965934812839019490346107*x)) \\
& /(195831528126838026966925312*(3 - 2^*x)^{(39/2)}*(1 + x + 2^*x^2)) + \\
& (115*sqrt[(7 + 2*sqrt[14])/2])*(30297118912219360725028693061 + 8 \\
& 061110911143276053983022787*sqrt[14])*ArcTan[(sqrt[7 + 2*sqrt[14] \\
&] - 2*sqrt[3 - 2*x])/sqrt[-7 + 2*sqrt[14]])/81206531627470768413 \\
& 3031842207432842412032 - (115*sqrt[(7 + 2*sqrt[14])/2])*(302971189 \\
& 12219360725028693061 + 8061110911143276053983022787*sqrt[14])*Arc \\
& Tan[(sqrt[7 + 2*sqrt[14]]) + 2*sqrt[3 - 2*x])/sqrt[-7 + 2*sqrt[14] \\
&]]/812065316274707684133031842207432842412032 + (115*(3029711891 \\
& 2219360725028693061 - 8061110911143276053983022787*sqrt[14])*sqrt \\
& [(-7 + 2*sqrt[14])/2]*Log[3 + sqrt[14] - sqrt[7 + 2*sqrt[14]]*sqrt \\
& t[3 - 2*x] - 2*x))/1624130632549415368266063684414865684824064 - \\
& (115*(30297118912219360725028693061 - 806111091114327605398302278 \\
& 7*sqrt[14])*sqrt[(-7 + 2*sqrt[14])/2]*Log[3 + sqrt[14] + sqrt[7 + \\
& 2*sqrt[14]]*sqrt[3 - 2*x] - 2*x))/162413063254941536826606368441 \\
& 4865684824064
\end{aligned}$$

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\begin{aligned}
 & \frac{x}{133(-2x+3)^{\frac{39}{2}}(2x^2+x+1)^{19}} \\
 & \int \frac{-79426075962231715392089154535476326400x+13068536369629429401640635068702208000}{(-2x+3)^{\frac{41}{2}}(2x^2+x+1)^9} dx \\
 & + \frac{49475578698391671468350845128120729600}{146216x+44296} + \frac{589021552x+223125616}{13138272(-2x+3)^{\frac{39}{2}}(2x^2+x+1)^{18}} + \frac{43776722304(-2x+3)^{\frac{39}{2}}(2x^2+x+1)^{17}}{2110519336800x+865861681440} \\
 & + \frac{6928434268875840x+2984274342235200}{137283801145344(-2x+3)^{\frac{39}{2}}(2x^2+x+1)^{16}} + \frac{403614375367311360(-2x+3)^{\frac{39}{2}}(2x^2+x+1)^{15}}{20924013532366815360x+9408813737133390720} \\
 & + \frac{1107517846007902371840(-2x+3)^{\frac{39}{2}}(2x^2+x+1)^{14}}{57873497074462503141120x+27243065619141593598720} \\
 & + \frac{2821955471628135243448320(-2x+3)^{\frac{39}{2}}(2x^2+x+1)^{13}}{145295342948683106164016640x+72110377354780278913835520} \\
 & + \frac{6637239269269374092590448640(-2x+3)^{\frac{39}{2}}(2x^2+x+1)^{12}}{326770416680301421681066214400x+172901458108932896335179801600} \\
 & + \frac{14309887864544770543625007267840(-2x+3)^{\frac{39}{2}}(2x^2+x+1)^{11}}{645802967231886306826540424448000x+370557652515461812186329087129600} \\
 & + \frac{28047380214507750265505014244966400(-2x+3)^{\frac{39}{2}}(2x^2+x+1)^{10}}{1088028437838790621809440473088716800x+696175598675973438759010577554944000} \\
 & + \frac{49475578698391671468350845128120729600(-2x+3)^{\frac{39}{2}}(2x^2+x+1)^9}{}
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/(3-2*x)**(41/2)/(2*x**2+x+1)**20,x)`

[Out] `x/(133*(-2*x+3)**(39/2)*(2*x**2+x+1)**19)+Integral((-79426075962231715392089154535476326400*x+13068536369629429401640635068702208000)/((-2*x+3)**(41/2)*(2*x**2+x+1)**9),x)/49475578698391671468350845128120729600+(146216*x+44296)/(13138272*(-2*x+3)**(39/2)*(2*x**2+x+1)**18)+(589021552*x+223125616)/(43776722304*(-2*x+3)**(39/2)*(2*x**2+x+1)**17)+(2110519336800*x+865861681440)/(137283801145344*(-2*x+3)**(39/2)*(2*x**2+x+1)**16)+(6928434268875840*x+2984274342235200)/(403614375367311360*(-2*x+3)**(39/2)*(2*x**2+x+1)**15)+(20924013532366815360*x+9408813737133390720)/(1107517846007902371840*(-2*x+3)**(39/2)*(2*x**2+x+1)**14)+(5787349707446250314120*x+27243065619141593598720)/(2821955471628135243448320*(-2*x+3)**(39/2)*(2*x**2+x+1)**13)+(145295342948683106164016640*x+72110377354780278913835520)/(6637239269269374092590448640*(-2*x+3)**(39/2)*(2*x**2+x+1)**12)+(326770416680301421681066214400*x+172901458108932896335179801600)/(14309887864544770543625007267840*(-2*x+3)**(39/2)*(2*x**2+x+1)**11)+(645802967231886306826540424448000*x+370557652515461812186329087129600)/(28047380214507750265505014244966400*(-2*x+3)**(39/2)*(2*x**2+x+1)**10)+(1088028437838790621809440473088716800*x+696175598675973438759010577554944000)/(49475578698391671468350845128120`

$$729600*(-2*x + 3)**(39/2)*(2*x**2 + x + 1)**9)$$

Mathematica [C] time = 6.16477, size = 1242, normalized size = 1.17

result too large to display

Antiderivative was successfully verified.

[In] Integrate[1/((3 - 2*x)^(41/2)*(1 + x + 2*x^2)^20),x]

[Out]
$$\begin{aligned} & -(393*\text{Sqrt}[3 - 2*x] + 287*(3 - 2*x)^{(3/2)})/((150276832468*(14 - 7* \\ & (3 - 2*x) + (3 - 2*x)^2)^{19} - (-4226921*\text{Sqrt}[3 - 2*x] + 1313129* \\ & (3 - 2*x)^{(3/2)})/(75739523563872*(14 - 7*(3 - 2*x) + (3 - 2*x)^2) \\ & ^{18}) - (-3401932701*\text{Sqrt}[3 - 2*x] + 760755809*(3 - 2*x)^{(3/2)})/(3 \\ & 6052013216403072*(14 - 7*(3 - 2*x) + (3 - 2*x)^2)^{17}) - (5*(-1464 \\ & 90500023*\text{Sqrt}[3 - 2*x] + 16144709919*(3 - 2*x)^{(3/2)}))/((161513019 \\ & 20948576256*(14 - 7*(3 - 2*x) + (3 - 2*x)^2)^{16}) - (9745709632283 \\ & *\text{Sqrt}[3 - 2*x] - 4557912048927*(3 - 2*x)^{(3/2)})/(4522364537865601 \\ & 35168*(14 - 7*(3 - 2*x) + (3 - 2*x)^2)^{15}) - (435856117815771*\text{Sqr} \\ & \text{t}[3 - 2*x] - 123609208162571*(3 - 2*x)^{(3/2)})/(933035209917534594 \\ & 6624*(14 - 7*(3 - 2*x) + (3 - 2*x)^2)^{14}) - (127435522656997631*S \\ & \text{qrt}[3 - 2*x] - 31270302414674811*(3 - 2*x)^{(3/2)})/(33962481640998 \\ & 25924571136*(14 - 7*(3 - 2*x) + (3 - 2*x)^2)^{13}) + (5*(-154035916 \\ & 7602841319*\text{Sqrt}[3 - 2*x] + 342026557757088031*(3 - 2*x)^{(3/2)}))/ \\ & (380379794379180503551967232*(14 - 7*(3 - 2*x) + (3 - 2*x)^2)^{12}) \\ & + (5*(-21084628139481190687*\text{Sqrt}[3 - 2*x] + 4158669924550257827*(\\ & 3 - 2*x)^{(3/2)}))/((13017441852087510566000656384*(14 - 7*(3 - 2*x) \\ & + (3 - 2*x)^2)^{11}) - (1633293973597342712581*\text{Sqrt}[3 - 2*x] - 237 \\ & 080744154193384005*(3 - 2*x)^{(3/2)})/(7289767437169005916960367575 \\ & 04*(14 - 7*(3 - 2*x) + (3 - 2*x)^2)^{10}) - (7350432513431022017155 \\ & *\text{Sqrt}[3 - 2*x] + 5131564318471376538977*(3 - 2*x)^{(3/2)})/(6123404 \\ & 6472219649702467087630336*(14 - 7*(3 - 2*x) + (3 - 2*x)^2)^9) - (\\ & -113207386492327172550771*\text{Sqrt}[3 - 2*x] + 43421160367342900895387 \\ & *(3 - 2*x)^{(3/2)})/(279927069587289827211278114881536*(14 - 7*(3 - \\ & 2*x) + (3 - 2*x)^2)^8) - (-22463796720502183624842107*\text{Sqrt}[3 - 2 \\ & *x] + 7094978194424786431173663*(3 - 2*x)^{(3/2)})/(548657056391088 \\ & 06133410510516781056*(14 - 7*(3 - 2*x) + (3 - 2*x)^2)^7) - (5*(-1 \\ & 86257412289925530757362143*\text{Sqrt}[3 - 2*x] + 5554017858872204666711 \\ & 3711*(3 - 2*x)^{(3/2)}))/((3072479515790093143470988588939739136*(14 \\ & - 7*(3 - 2*x) + (3 - 2*x)^2)^6) - (23*(-255056047077847659080618 \\ & 951*\text{Sqrt}[3 - 2*x] + 74443988473272328189316355*(3 - 2*x)^{(3/2)}))/ \\ & (28676475480707536005729226830104231936*(14 - 7*(3 - 2*x) + (3 - \\ & 2*x)^2)^5) - (23*(-1110057788286806589656260577*\text{Sqrt}[3 - 2*x] + 3 \\ & 21533953909984640923113289*(3 - 2*x)^{(3/2)}))/((1889273678728967078 \\ & 02451376763039645696*(14 - 7*(3 - 2*x) + (3 - 2*x)^2)^4) - (23*(- \\ & 4820387670797872511726954245*\text{Sqrt}[3 - 2*x] + 13943044905313772031 \\ & 11252689*(3 - 2*x)^{(3/2)}))/((1220761453947947958108147357545794633 \\ & 728*(14 - 7*(3 - 2*x) + (3 - 2*x)^2)^3) - (23*(-17490402570151108 \\ & 581128226213*\text{Sqrt}[3 - 2*x] + 5072167085782230110284731077*(3 - 2* \\ & x)^{(3/2)}))/((6214785583735007786732386547505863589888*(14 - 7*(3 - \\ & 2*x) + (3 - 2*x)^2)^2) - (115*(-82782386138609724168863115877*Sq \\ & \text{rt}[3 - 2*x] + 24217623575858523510208130121*(3 - 2*x)^{(3/2)}))/((17$$


```

4013996344580218028506823330164180516864*(14 - 7*(3 - 2*x) + (3 -
2*x)^2) + 1/(3111898385606868039*(3 - 2*x)^(39/2)) + 10/(295231
3853011644037*(3 - 2*x)^(37/2)) + 143/(7819642097165976098*(3 - 2
*x)^(35/2)) + 355/(5266289575642392066*(3 - 2*x)^(33/2)) + 52865/
(277038748585308867472*(3 - 2*x)^(31/2)) + 14333/(323956601168304
72406*(3 - 2*x)^(29/2)) + 1478345/(1689042692987850837168*(3 - 2*
x)^(27/2)) + 475387/(312785683886639043920*(3 - 2*x)^(25/2)) + 16
575515/(7006399319060714583808*(3 - 2*x)^(23/2)) + 246866015/(735
67192850137503129984*(3 - 2*x)^(21/2)) + 8192823353/(186370221887
0150079292928*(3 - 2*x)^(19/2)) + 8972680075/(1667523037936450070
946304*(3 - 2*x)^(17/2)) + 102495360575/(164790511984308007011164
16*(3 - 2*x)^(15/2)) + 122484655975/(17852305464966700759542784*(
3 - 2*x)^(13/2)) + 10815878546425/(1480368099325700262983624704*(
3 - 2*x)^(11/2)) + 769045155125/(100934188590388654294338048*(3 -
2*x)^(9/2)) + 838467657280275/(105509871806486273289014706176*(3
- 2*x)^(7/2)) + 9270470094105/(1076631344964145645806272512*(3 -
2*x)^(5/2)) + 320421783064625/(30145677658996078082575630336*(3
- 2*x)^(3/2)) + 683151246370725/(30145677658996078082575630336*Sq
rt[3 - 2*x]) - (115*(-117022014202441653827938545631*I + 80611109
11143276053983022787*sqrt[7])*ArcTan[(sqrt[2]*sqrt[3 - 2*x])/sqrt
[-7 - I*sqrt[7]])/(58004665448193406009502274443388060172288*sqrt
[14*(-7 - I*sqrt[7])]) - (115*(117022014202441653827938545631*I
+ 8061110911143276053983022787*sqrt[7])*ArcTan[(sqrt[2]*sqrt[3 -
2*x])/sqrt[-7 + I*sqrt[7]])/(58004665448193406009502274443388060
172288*sqrt[14*(-7 + I*sqrt[7])])

```

Maple [A] time = 0.138, size = 989, normalized size = 0.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\int (1/(3-2*x)^{(41/2)})/(2*x^2+x+1)^{20}, x$

```

[Out] -7192279694031133468210490184035/32482612650988307365321273688297
31369648128*ln(3-2*x+14^(1/2)-(3-2*x)^(1/2))*(7+2*14^(1/2))^(1/2))
*(7+2*14^(1/2))^(1/2)*14^(1/2)+13457531633280790190212932747565/8
12065316274707684133031842207432842412032/(-7+2*14^(1/2))^(1/2)*a
rctan((2*(3-2*x)^(1/2)-(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2)
))*(7+2*14^(1/2))-3484168674905226483378299702015/812065316274707
684133031842207432842412032/(-7+2*14^(1/2))^(1/2)*arctan((2*(3-2*
x)^(1/2)-(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))*14^(1/2)+71
92279694031133468210490184035/32482612650988307365321273688297313
69648128*ln(3-2*x+14^(1/2)+(3-2*x)^(1/2))*(7+2*14^(1/2))^(1/2))*
(7+2*14^(1/2))^(1/2)*14^(1/2)+13457531633280790190212932747565/8120
65316274707684133031842207432842412032/(-7+2*14^(1/2))^(1/2)*arct
an((2*(3-2*x)^(1/2)+(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))*
(7+2*14^(1/2))-3484168674905226483378299702015/812065316274707684
133031842207432842412032/(-7+2*14^(1/2))^(1/2)*arctan((2*(3-2*x)^(
1/2)+(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))*14^(1/2)+13457
531633280790190212932747565/1624130632549415368266063684414865684
824064*ln(3-2*x+14^(1/2)-(3-2*x)^(1/2))*(7+2*14^(1/2))^(1/2))*(7+2

```

$$\begin{aligned}
 & * 14^{(1/2)})^{(1/2)} - 13457531633280790190212932747565/162413063254941 \\
 & 5368266063684414865684824064 * \ln(3-2*x+14^{(1/2)}+(3-2*x)^{(1/2)}) * (7+2 \\
 & * 14^{(1/2)})^{(1/2)}) * (7+2*14^{(1/2)})^{(1/2)} + 683151246370725/3014567765 \\
 & 8996078082575630336/(3-2*x)^{(1/2)} + 1/30145677658996078082575630336 \\
 & * (-5059022664167725408892162874688680417923742003781/294806555197 \\
 & 44 * (3-2*x)^{(47/2)} - 44796329357069082297154473725670903546220392558 \\
 & 695/9070970929152 * (3-2*x)^{(43/2)} - 19392429209015348214540269031324 \\
 & 33081580221023737/501171143835648 * (3-2*x)^{(51/2)} - 1006304725834560 \\
 & 333245233940167063186576585913370455/10720238370816 * (3-2*x)^{(39/2)} \\
 &) + 13805722741822612586258592099428566280191230197271405/393075406 \\
 & 92992 * (3-2*x)^{(37/2)} - 928342237074576734557978321305/1924145348608 \\
 & * (3-2*x)^{(75/2)} + 339556544641293541759958988614814460549873/982688 \\
 & 5173248 * (3-2*x)^{(61/2)} + 133883313322119397348791732981953297/82463 \\
 & 3720832 * (3-2*x)^{(69/2)} + 490738543064879423955077165987434152441563 \\
 & 270473/1002342287671296 * (3-2*x)^{(53/2)} - 75593011640468285657019519 \\
 & 0192032441294632160945523631/3915399561216 * (3-2*x)^{(23/2)} + 8535085 \\
 & 02207214511987093866021124087908041634697244059/7830799122432 * (3- \\
 & 2*x)^{(25/2)} - 6886173809894005543994516442461871486007042005189775/ \\
 & 125627793408 * (3-2*x)^{(27/2)} + 1363299879672453951418482537651472082 \\
 & 79814148352958009/5527622909952 * (3-2*x)^{(29/2)} - 550660914208175901 \\
 & 67865401986871791412011888132876913/5527622909952 * (3-2*x)^{(31/2)} + \\
 & 2737487528928439357869138774910126923363791747141675/755914244096 \\
 & * (3-2*x)^{(33/2)} - 1166457217021587688420366823074349521448831011337 \\
 & 1105/9826885173248 * (3-2*x)^{(35/2)} + 1264662933338272271690443076373 \\
 & 2665179119615389552413/25098715136 * (3-2*x)^{(17/2)} - 259367320368504 \\
 & 4441695042001860835122939346700333136537/6199382638592 * (3-2*x)^{(1 \\
 & 9/2)} + 807597736492641378942268937217995835353849465/1048576 * (3-2*x \\
 &)^{(1/2)} - 503502693505289734438057515605193725/103079215104 * (3-2*x) \\
 & ^{(67/2)} - 3254850748003483429666738850178379/824633720832 * (3-2*x)^{(\\
 & 71/2)} + 360433340020130123942335063779145/5772436045824 * (3-2*x)^{(73 \\
 & /2)} + 1808668971148992206490172102870787954874541181/33411409589043 \\
 & 2 * (3-2*x)^{(57/2)} + 129886852748727110357425618672922324659/11338713 \\
 & 66144 * (3-2*x)^{(65/2)} + 28607223317693223698395672584150593863016075 \\
 & 796143/29480655519744 * (3-2*x)^{(45/2)} - 1196897725308288065129289211 \\
 & 1395530933265219/25701084299264 * (3-2*x)^{(59/2)} + 730124764525775715 \\
 & 33836489036461787385135079265/2680059592704 * (3-2*x)^{(49/2)} - 642433 \\
 & 96719140374998473027009027485263697/29480655519744 * (3-2*x)^{(63/2)} \\
 & + 2672239984790337844292019294315182385216573077301785/11792262207 \\
 & 8976 * (3-2*x)^{(41/2)} + 118632384645382623721251719631219381945276176 \\
 & 4018822545/3915399561216 * (3-2*x)^{(21/2)} - 1765094235896326267587117 \\
 & 3166229809316744939271143/51904512 * (3-2*x)^{(11/2)} - 223975463212094 \\
 & 86953062074374795737299957063565/3145728 * (3-2*x)^{(3/2)} + 4045315666 \\
 & 89883337048499233527781983599187634017/12582912 * (3-2*x)^{(5/2)} - 118 \\
 & 8598027552254830082683218064697188605612952419/12582912 * (3-2*x)^{(\\
 & 7/2)} + 3831583379166294091823572953989993625772471445345/18874368 * (\\
 & 3-2*x)^{(9/2)} + 9977850126168010187169130424774568330973123412551261 \\
 & /21592276992 * (3-2*x)^{(13/2)} - 1255696718499588580979726331572072320 \\
 & 357969297077745/2399141888 * (3-2*x)^{(15/2)} - 55011835288361289002011 \\
 & 693179378316699033102675/1002342287671296 * (3-2*x)^{(55/2)}) / ((3-2*x \\
 &)^2 - 7 + 14*x)^{19} - 7192279694031133468210490184035/162413063254941536 \\
 & 8266063684414865684824064 / (-7 + 2*14^{(1/2)})^{(1/2)} * \arctan((2*(3-2*x) \\
 & ^{(1/2)} - (7 + 2*14^{(1/2)})^{(1/2)}) / (-7 + 2*14^{(1/2)})^{(1/2)}) * (7 + 2*14^{(1/2)} \\
 &) * 14^{(1/2)} - 7192279694031133468210490184035/1624130632549415368266 \\
 & 063684414865684824064 / (-7 + 2*14^{(1/2)})^{(1/2)} * \arctan((2*(3-2*x)^{(1/ \\
 & 2)} + (7 + 2*14^{(1/2)})^{(1/2)}) / (-7 + 2*14^{(1/2)})^{(1/2)}) * (7 + 2*14^{(1/2)}) * 14 \\
 & ^{(1/2)} + 52865/277038748585308867472 / (3-2*x)^{(31/2)} + 14333/323956601
 \end{aligned}$$

$16830472406/(3-2*x)^{(29/2)}+1478345/1689042692987850837168/(3-2*x)^{(27/2)}+475387/312785683886639043920/(3-2*x)^{(25/2)}+16575515/7006399319060714583808/(3-2*x)^{(23/2)}+246866015/73567192850137503129984/(3-2*x)^{(21/2)}+355/5266289575642392066/(3-2*x)^{(33/2)}+1/3111898385606868039/(3-2*x)^{(39/2)}+10/2952313853011644037/(3-2*x)^{(37/2)}+143/7819642097165976098/(3-2*x)^{(35/2)}+8192823353/1863702218870150079292928/(3-2*x)^{(19/2)}+8972680075/1667523037936450070946304/(3-2*x)^{(17/2)}+102495360575/16479051198430800701116416/(3-2*x)^{(15/2)}+122484655975/17852305464966700759542784/(3-2*x)^{(13/2)}+10815878546425/1480368099325700262983624704/(3-2*x)^{(11/2)}+9270470094105/1076631344964145645806272512/(3-2*x)^{(5/2)}+320421783064625/30145677658996078082575630336/(3-2*x)^{(3/2)}+769045155125/100934188590388654294338048/(3-2*x)^{(9/2)}+838467657280275/105509871806486273289014706176/(3-2*x)^{(7/2)}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2x^2 + x + 1)^{20}(-2x + 3)^{\frac{41}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((2*x^2 + x + 1)^20*(-2*x + 3)^(41/2)),x, algorithm="maxima")

[Out] integrate(1/((2*x^2 + x + 1)^20*(-2*x + 3)^(41/2)), x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((2*x^2 + x + 1)^20*(-2*x + 3)^(41/2)),x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3-2*x)**(41/2)/(2*x**2+x+1)**20,x)

[Out] Timed out

GIAC/XCAS [A] time = 0.227316, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((2*x^2 + x + 1)^20*(-2*x + 3)^(41/2)),x, algorithm="giac")`

[Out] Done

$$3.50 \quad \int \frac{1}{(3-2x+x^2)^{11/2}(1+x+2x^2)^5} dx$$

Optimal. Leaf size=378

$$\begin{aligned} & -\frac{63043297 - 29625922x}{41160000000(x^2 - 2x + 3)^{3/2}} - \frac{31(7434109 - 3088870x)}{41160000000\sqrt{x^2 - 2x + 3}} \\ & + \frac{3(8233x + 8822)}{343000(x^2 - 2x + 3)^{9/2}(2x^2 + x + 1)} + \frac{8878x + 5485}{117600(x^2 - 2x + 3)^{9/2}(2x^2 + x + 1)^2} \\ & - \frac{30316369 - 15043110x}{6860000000(x^2 - 2x + 3)^{5/2}} + \frac{1050(x^2 - 2x + 3)^{9/2}(2x^2 + x + 1)^3}{4878869 - 2578034x} \\ & - \frac{1 - 10x}{411600000(x^2 - 2x + 3)^{7/2}} - \frac{3450497 - 2004270x}{280(x^2 - 2x + 3)^{9/2}(2x^2 + x + 1)^4} - \frac{123480000(x^2 - 2x + 3)^{9/2}}{\sqrt{\frac{1}{70}(151363871237318045 + 110320475741093888\sqrt{2})} \tan^{-1}\left(\frac{\sqrt{\frac{5}{7(151363871237318045 + 110320475741093888\sqrt{2})}}((932587773 + 620347970\sqrt{2})x + 3122)}}{\sqrt{x^2 - 2x + 3}}\right)} \\ & + \frac{137200000000}{\sqrt{\frac{1}{70}(110320475741093888\sqrt{2} - 151363871237318045)} \tanh^{-1}\left(\frac{\sqrt{\frac{5}{7(110320475741093888\sqrt{2} - 151363871237318045)}}((932587773 - 620347970\sqrt{2})x - 3)}}{\sqrt{x^2 - 2x + 3}}\right)} \\ & - \frac{137200000000}{137200000000} \end{aligned}$$

[Out] $-(3450497 - 2004270*x)/(123480000*(3 - 2*x + x^2)^(9/2)) - (4878869 - 2578034*x)/(411600000*(3 - 2*x + x^2)^(7/2)) - (30316369 - 15043110*x)/(6860000000*(3 - 2*x + x^2)^(5/2)) - (63043297 - 29625922*x)/(41160000000*(3 - 2*x + x^2)^(3/2)) - (31*(7434109 - 3088870*x))/(41160000000*sqrt[3 - 2*x + x^2]) - (1 - 10*x)/(280*(3 - 2*x + x^2)^(9/2)*(1 + x + 2*x^2)^4) + (28 + 67*x)/(1050*(3 - 2*x + x^2)^(9/2)*(1 + x + 2*x^2)^3) + (5485 + 8878*x)/(117600*(3 - 2*x + x^2)^(9/2)*(1 + x + 2*x^2)^2) + (3*(8822 + 8233*x))/(343000*(3 - 2*x + x^2)^(9/2)*(1 + x + 2*x^2)) + (sqrt[(151363871237318045 + 110320475741093888*sqrt[2])/70]*ArcTan[(sqrt[5/(7*(151363871237318045 + 110320475741093888*sqrt[2])])*(308108167 + 312239803*sqrt[2] + (932587773 + 620347970*sqrt[2])*x))/sqrt[3 - 2*x + x^2]])/137200000000 - (sqrt[(-151363871237318045 + 110320475741093888*sqrt[2])/70]*ArcTanh[(sqrt[5/(7*(-151363871237318045 + 110320475741093888*sqrt[2])])*(308108167 - 312239803*sqrt[2] + (932587773 - 620347970*sqrt[2])*x))/sqrt[3 - 2*x + x^2]])/137200000000$

Rubi [A] time = 1.58625, antiderivative size = 378, normalized size of antiderivative = 1., number of

steps used = 14, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$

$$\begin{aligned}
 & -\frac{63043297 - 29625922x}{41160000000(x^2 - 2x + 3)^{3/2}} - \frac{31(7434109 - 3088870x)}{41160000000\sqrt{x^2 - 2x + 3}} \\
 & + \frac{3(8233x + 8822)}{343000(x^2 - 2x + 3)^{9/2}(2x^2 + x + 1)} + \frac{8878x + 5485}{117600(x^2 - 2x + 3)^{9/2}(2x^2 + x + 1)^2} \\
 & - \frac{30316369 - 15043110x}{6860000000(x^2 - 2x + 3)^{5/2}} + \frac{67x + 28}{1050(x^2 - 2x + 3)^{9/2}(2x^2 + x + 1)^3} \\
 & - \frac{4878869 - 2578034x}{411600000(x^2 - 2x + 3)^{7/2}} - \frac{1 - 10x}{280(x^2 - 2x + 3)^{9/2}(2x^2 + x + 1)^4} - \frac{3450497 - 2004270x}{123480000(x^2 - 2x + 3)^{9/2}} \\
 & + \frac{\sqrt{\frac{1}{70}(151363871237318045 + 110320475741093888\sqrt{2})} \tan^{-1}\left(\frac{\sqrt{\frac{5}{7(151363871237318045 + 110320475741093888\sqrt{2})}}((932587773 + 620347970\sqrt{2})x + 3122)}}{\sqrt{x^2 - 2x + 3}}\right)}{137200000000} \\
 & + \frac{\sqrt{\frac{1}{70}(110320475741093888\sqrt{2} - 151363871237318045)} \tanh^{-1}\left(\frac{\sqrt{\frac{5}{7(110320475741093888\sqrt{2} - 151363871237318045)}}((932587773 - 620347970\sqrt{2})x - 3)}}{\sqrt{x^2 - 2x + 3}}\right)}{137200000000}
 \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[1/((3 - 2*x + x^2)^(11/2)*(1 + x + 2*x^2)^5), x]

[Out] -(3450497 - 2004270*x)/(123480000*(3 - 2*x + x^2)^(9/2)) - (4878869 - 2578034*x)/(411600000*(3 - 2*x + x^2)^(7/2)) - (30316369 - 15043110*x)/(6860000000*(3 - 2*x + x^2)^(5/2)) - (63043297 - 29625922*x)/(41160000000*(3 - 2*x + x^2)^(3/2)) - (31*(7434109 - 3088870*x))/(411600000000*Sqrt[3 - 2*x + x^2]) - (1 - 10*x)/(280*(3 - 2*x + x^2)^(9/2)*(1 + x + 2*x^2)^4) + (28 + 67*x)/(1050*(3 - 2*x + x^2)^(9/2)*(1 + x + 2*x^2)^3) + (5485 + 8878*x)/(117600*(3 - 2*x + x^2)^(9/2)*(1 + x + 2*x^2)^2) + (3*(8822 + 8233*x))/(343000*(3 - 2*x + x^2)^(9/2)*(1 + x + 2*x^2)) + (Sqrt[(151363871237318045 + 110320475741093888*Sqrt[2])/70]*ArcTan[(Sqrt[5/(7*(151363871237318045 + 110320475741093888*Sqrt[2]))])*(308108167 + 312239803*Sqrt[2] + (932587773 + 620347970*Sqrt[2])*x)]/Sqrt[3 - 2*x + x^2]])/137200000000 - (Sqrt[(-151363871237318045 + 110320475741093888*Sqrt[2])/70]*ArcTanh[(Sqrt[5/(7*(-151363871237318045 + 110320475741093888*Sqrt[2]))])*(308108167 - 312239803*Sqrt[2] + (932587773 - 620347970*Sqrt[2])*x)]/Sqrt[3 - 2*x + x^2]])/137200000000

Rubi in Sympy [A] time = 79.0556, size = 379, normalized size = 1.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate(1/(x**2-2*x+3)**(11/2)/(2*x**2+x+1)**5, x)


```

*x^2*Sqrt[3 - 2*x + x^2] - (275801189352734720*I)*Sqrt[10*(-5 + I
*Sqrt[7])] *x^3*Sqrt[3 - 2*x + x^2))/(4321741285513437647*I + 8273
87564543169945*Sqrt[7] + (3694994885631086104*I)*x + 285423303382
928480*Sqrt[7]*x + (5471192788852131980*I)*x^2 - 7052553231648848
0*Sqrt[7]*x^2 - (6268363351511187532*I)*x^3 + 137879256656321740*
Sqrt[7]*x^3 + (2092254277956040633*I)*x^4 + 70562873851568315*Sqr
t[7]*x^4)))/(68600000000*Sqrt[70*(-5 + I*Sqrt[7])]) - ((I/6860000
0000)*(-310173985*I + 44900803*Sqrt[7])*ArcTan[(35*(1521027563127
6955*I + 23639644701233427*Sqrt[7] - (80355173705781000*I)*x + 81
54951525226528*Sqrt[7]*x + (32801021588957180*I)*x^2 - 2015015209
042528*Sqrt[7]*x^2 - (22632774169109180*I)*x^3 + 3939407333037764
*Sqrt[7]*x^3 - (9346476174243955*I)*x^4 + 2016082110044809*Sqrt[7
]*x^4))/(-9627448535205165 + (357977536529228045*I)*Sqrt[7] + 289
2591314086740000*x + (36106220736881480*I)*Sqrt[7]*x - 4649830882
85203040*x^2 - (1038569725622524380*I)*Sqrt[7]*x^2 - 128365980469
40220*x^3 + (328748064746064540*I)*Sqrt[7]*x^3 + 4874471348673484
25*x^4 - (428071291440525685*I)*Sqrt[7]*x^4 - (27580118935273472*
I)*Sqrt[70*(5 + I*Sqrt[7])]*Sqrt[3 - 2*x + x^2] - (27580118935273
472*I)*Sqrt[70*(5 + I*Sqrt[7])]*x^2*Sqrt[3 - 2*x + x^2] + (551602
37870546944*I)*Sqrt[70*(5 + I*Sqrt[7])]*x^3*Sqrt[3 - 2*x + x^2))]
)/Sqrt[70*(5 + I*Sqrt[7])]) - (((-310173985*I + 44900803*Sqrt[7])*L
og[(-I + Sqrt[7] - (4*I)*x)^2*(I + Sqrt[7] + (4*I)*x)^2])/(137200
000000*Sqrt[70*(5 + I*Sqrt[7])]) + ((I/1372000000000)*(310173985*I
+ 44900803*Sqrt[7])*Log[(-I + Sqrt[7] - (4*I)*x)^2*(I + Sqrt[7]
+ (4*I)*x)^2])/Sqrt[70*(-5 + I*Sqrt[7])]) - ((I/1372000000000)*(310
173985*I + 44900803*Sqrt[7])*Log[(1 + x + 2*x^2)*(-13*I + 15*Sqrt
[7] + (22*I)*x - 10*Sqrt[7]*x + (9*I)*x^2 + 5*Sqrt[7]*x^2 + I*Sqr
t[70*(-5 + I*Sqrt[7])]*Sqrt[3 - 2*x + x^2] - I*Sqrt[70*(-5 + I*Sqr
t[7])]*x*Sqrt[3 - 2*x + x^2]))/Sqrt[70*(-5 + I*Sqrt[7])]) + (((-3
10173985*I + 44900803*Sqrt[7])*Log[(1 + x + 2*x^2)*(-163*I + 15*S
qrt[7] + (122*I)*x - 10*Sqrt[7]*x - (41*I)*x^2 + 5*Sqrt[7]*x^2 -
(13*I)*Sqrt[10*(5 + I*Sqrt[7])]*Sqrt[3 - 2*x + x^2] + (5*I)*Sqrt[
10*(5 + I*Sqrt[7])]*x*Sqrt[3 - 2*x + x^2]))/(137200000000*Sqrt[7
0*(5 + I*Sqrt[7])])

```

Maple [B] time = 0.765, size = 21028, normalized size = 55.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/(x^2-2*x+3)^{(11/2)}/(2*x^2+x+1)^5, x)$

[Out] result too large to display

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2x^2 + x + 1)^5(x^2 - 2x + 3)^{\frac{11}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2*x^2 + x + 1)^5*(x^2 - 2*x + 3)^(11/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/((2*x^2 + x + 1)^5*(x^2 - 2*x + 3)^(11/2)), x)
```

Fricas [A] time = 0.362639, size = 6251, normalized size = 16.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2*x^2 + x + 1)^5*(x^2 - 2*x + 3)^(11/2)),x, algorithm="fricas")
```

```
[Out] 1/4939200000000*(4*sqrt(205487899)*sqrt(35)*sqrt(7)*(250540991585
065591119917936640*x^32 - 5884831549275969411796256686080*x^31 +
70336698901347132902202470830080*x^30 - 5640114722627557282137279
38611200*x^29 + 3387782843130719265741539572688640*x^28 - 1616441
6998187353864497620912156160*x^27 + 63515930350911189944750096784
965760*x^26 - 210557817277442257216056696290868480*x^25 + 5990799
85624608807889894068082206400*x^24 - 1481788240195926502419500474
927552000*x^23 + 3218181205607145182585261559186346560*x^22 - 618
7184772955802395440882682138814720*x^21 + 10603803272759527543249
407132512637520*x^20 - 16301932856715307988417262259057211200*x^1
9 + 22613684765066685707216182128734364240*x^18 - 284629023420049
20706947119363091816160*x^17 + 3267613517686266921170632983945890
0360*x^16 - 34375259168148954959453196199877528480*x^15 + 3326409
5254764270345104132253135996400*x^14 - 29690304639514302322193623
022642067720*x^13 + 24484631887122483709778594663137626775*x^12 -
18669195666477658318850179379968512720*x^11 + 131586975136960148
49995752136862496980*x^10 - 8560917034642778106789153295046959300
*x^9 + 5123941961010059680373987398344568415*x^8 - 28042263206249
50569135022727301669360*x^7 + 13885358123968654961822588100641973
60*x^6 - 610465831798347956368523388448548480*x^5 + 2302162989556
27150529634520155221125*x^4 - 69964383404373320595002167294609600
*x^3 + 15304284300591900638465660638051420*x^2 - 1103204757410938
88*sqrt(2)*(1655223201792*x^32 - 38878706663424*x^31 + 4646861785
86624*x^30 - 3726196136847360*x^29 + 22381713783067392*x^28 - 106
791778421442048*x^27 + 419624113942796928*x^26 - 1391070508148646
144*x^25 + 3957879649400169920*x^24 - 9789576786607705600*x^23 +
21261224222796687168*x^22 - 40876232368919362816*x^21 + 700550480
51289600656*x^20 - 107700290191151927360*x^19 + 14939948734273247
7072*x^18 - 188042906866321790048*x^17 + 215878035555994176808*x^
16 - 227103461923573594144*x^15 + 219762450463563231920*x^14 - 19
6151858411205186216*x^13 + 161760079779763942595*x^12 - 123339840
041530707216*x^11 + 86934203030952873444*x^10 - 56558523276802427
540*x^9 + 33851816283004633387*x^8 - 18526391388525625008*x^7 + 9
173495637012543408*x^6 - 4033101339230550144*x^5 + 15209461615492
06025*x^4 - 462226440381394880*x^3 + 101109228876664076*x^2 - 125
95090393724612*x + 382982001236017) - 190644164057811360913005718
8223540*x + 57969638321298857545326138026765)*sqrt(x^2 - 2*x + 3)
```

$$\begin{aligned}
& * \sqrt{(151363871237318045 * \sqrt{2} - 220640951482187776) / (33397068 \\
& 569829208576933551323217920 * \sqrt{2} - 472522362514299670284917461 \\
& 44535113)) - 4 * \sqrt{205487899} * \sqrt{35} * \sqrt{7} * (2505409915850655 \\
& 91119917936640 * x^{33} - 6135372540861035002916174622720 * x^{32} + 7647 \\
& 2071442208167905118645452800 * x^{31} - 63998246172179376493660674819 \\
& 0720 * x^{30} + 4016371453241338690241233684412160 * x^{29} - 20048942300 \\
& 949912243052144039937280 * x^{28} + 825418536737044412995458488180524 \\
& 80 * x^{27} - 287159104433155596834389825111898240 * x^{26} + 85886862956 \\
& 4315376337541934649544640 * x^{25} - 22369469991084197838786150484501 \\
& 48800 * x^{24} + 5124102521174037211790204381357664960 * x^{23} - 1040593 \\
& 3975960152123149730201711967680 * x^{22} + 18860766060525068433557020 \\
& 025072619600 * x^{21} - 30691274851422616119553452520028489680 * x^{20} + \\
& 45081528653462797168790944458441243760 * x^{19} - 600801563940866631 \\
& 40994853987619185680 * x^{18} + 7299981596253669161782603304452647908 \\
& 0 * x^{17} - 81231743782608564292636240096715327240 * x^{16} + 8310994559 \\
& 3953060776505494062289747840 * x^{15} - 78427494994130809902434683204 \\
& 783737480 * x^{14} + 68411527510419695509612014370957102335 * x^{13} - 55 \\
& 231057943303058323856167435958954875 * x^{12} + 412850518651089793032 \\
& 10529619078972420 * x^{11} - 28556677261894993605897587670410952880 * x \\
& ^{10} + 18243967900859125271164543196209642035 * x^9 - 10724669541859 \\
& 622266496198594992688955 * x^8 + 5762245664163959895719267159339201 \\
& 680 * x^7 - 2797454333407067178417649149855271440 * x^6 + 12027767505 \\
& 15386982146365413070444165 * x^5 - 44196391443110601682853133346133 \\
& 1825 * x^4 + 130380975576388033973331946562853820 * x^3 - 27619638829 \\
& 652562290439255223888360 * x^2 - 110320475741093888 * \sqrt{2} * (165522 \\
& 3201792 * x^{33} - 40533929865216 * x^{32} + 505220108451840 * x^{31} - 42281 \\
& 05798895616 * x^{30} + 26534545003438848 * x^{29} - 132455269127702784 * x^ \\
& ^{28} + 545320709618281344 * x^{27} - 1897144292662342272 * x^{26} + 5674198 \\
& 357530944192 * x^{25} - 14778605890709480640 * x^{24} + 33852877039198730 \\
& 688 * x^{23} - 68747805476282099904 * x^{22} + 124605468308576372880 * x^{21} \\
& - 202764864564694271504 * x^{20} + 297835462881238465328 * x^{19} - 3969 \\
& 25342242926100304 * x^{18} + 482280318056102498824 * x^{17} - 53666534238 \\
& 7075938472 * x^{16} + 549073863627919009152 * x^{15} - 518138802562515874 \\
& 344 * x^{14} + 451967348292510885963 * x^{13} - 364889306092787766775 * x^{12} \\
& + 272753673169336497876 * x^{11} - 188662439910260964464 * x^{10} + 120 \\
& 530531835136900623 * x^9 - 70853562704172604599 * x^8 + 3806883120166 \\
& 4625104 * x^7 - 18481651602455633232 * x^6 + 7946260495872201737 * x^5 \\
& - 2919877186136224485 * x^4 + 861374478008482796 * x^3 - 182471805219 \\
& 283208 * x^2 + 22036451262907845 * x - 663344282706293) + 33355225714 \\
& 86217667919065790563025 * x - 100406358593566432873987763957185) * \sqrt{ \\
& ((151363871237318045 * \sqrt{2} - 220640951482187776) / (33397068569 \\
& 829208576933551323217920 * \sqrt{2} - 472522362514299670284917461445 \\
& 35113)) - 3602042876982878244 * (337802213083473608^{(1/4)} * (4096 * x^3 \\
& ^3 - 94208 * x^{32} + 1105920 * x^{31} - 8726528 * x^{30} + 51652864 * x^{29} - 24 \\
& 3142912 * x^{28} + 943504512 * x^{27} - 3091758976 * x^{26} + 8703838016 * x^{25} \\
& - 21323296320 * x^{24} + 45922747584 * x^{23} - 87669031872 * x^{22} + 14942 \\
& 9442480 * x^{21} - 228892406512 * x^{20} + 317020649584 * x^{19} - 3993174399 \\
& 52 * x^{18} + 459889940952 * x^{17} - 486569477336 * x^{16} + 474741729456 * x^ \\
& ^{15} - 428370113832 * x^{14} + 358128604129 * x^{13} - 277690283125 * x^{12} + \\
& 199741426598 * x^{11} - 133169510462 * x^{10} + 82124768139 * x^9 - 4668881 \\
& 9967 * x^8 + 24352777172 * x^7 - 11572941916 * x^6 + 4956109991 * x^5 - 1 \\
& 881604195 * x^4 + 618995238 * x^3 - 169803054 * x^2 + 35318565 * x - 4164 \\
& 129) * \sqrt{x^2 - 2 * x + 3} - 337802213083473608^{(1/4)} * (4096 * x^{34} - \\
& 98304 * x^{33} + 1204224 * x^{32} - 9922560 * x^{31} + 61393152 * x^{30} - 302465 \\
& 536 * x^{29} + 1230167680 * x^{28} - 4231534592 * x^{27} + 12524632256 * x^{26} - \\
& 32311376256 * x^{25} + 73387152384 * x^{24} - 147941398656 * x^{23} + 266538
\end{aligned}$$

$$\begin{aligned}
& 692976*x^{22} - 431807810720*x^{21} + 632607571360*x^{20} - 84257939129 \\
& 6*x^{19} + 1025439024680*x^{18} - 1145619691952*x^{17} + 1179623583016* \\
& x^{16} - 1123076849304*x^{15} + 990931422841*x^{14} - 811490311782*x^{13} \\
& + 617205254013*x^{12} - 435943782908*x^{11} + 285629584397*x^{10} - 17 \\
& 3202055986*x^9 + 96858530705*x^8 - 49702089632*x^7 + 23230667691* \\
& x^6 - 9779204266*x^5 + 3646868583*x^4 - 1176696972*x^3 + 31530008 \\
& 7*x^2 - 63577710*x + 7212483)) * \arctan(1438415293*(2*\sqrt{20548789} \\
& 9)*\sqrt{35}*\sqrt{7}*(110320475741093888*\sqrt{2} - 151363871237318 \\
& 045)*\sqrt{((151363871237318045*\sqrt{2} - 220640951482187776)/(3339 \\
& 7068569829208576933551323217920*\sqrt{2} - 47252236251429967028491 \\
& 746144535113)) + 5*337802213083473608^{(1/4)}*\sqrt{7}*(932587773*\sqrt{ \\
& 2} - 1240695940))/(4*\sqrt{1438415293}*\sqrt{205487899}*\sqrt{35}) \\
& *\sqrt{7}*(110320475741093888*\sqrt{2} - 151363871237318045)*\sqrt{(- \\
& (337802213083473608^{(1/4)}*\sqrt{205487899}*\sqrt{35}*(\sqrt{2}*(2466 \\
& 174192810249028691853897589514172583272025134182276224653*x - 595 \\
& 3859194138967082383053517413008702994628754686755712968393) - 348 \\
& 7685001328718053691199619823494530411356729552573436743740*x + 84 \\
& 20033386949216111074907415002522875577900779820937989193046)*\sqrt{ \\
& ((151363871237318045*\sqrt{2} - 220640951482187776)/(3339706856982 \\
& 9208576933551323217920*\sqrt{2} - 47252236251429967028491746144535 \\
& 113)) + 337591919062388654431586980530568515903559031612841934150 \\
& 69696*x^2 - 59677969838575891421923306884049625085736901300338314 \\
& 64558990*\sqrt{2}*(4*x^2 - 3*x + 7) - (337802213083473608^{(1/4)}*\sqrt{ \\
& 205487899}*\sqrt{35}*(24661741928102490286918538975895141725832 \\
& 72025134182276224653*\sqrt{2} - 3487685001328718053691199619823494 \\
& 530411356729552573436743740)*\sqrt{((151363871237318045*\sqrt{2} - 2 \\
& 20640951482187776)/(33397068569829208576933551323217920*\sqrt{2} - \\
& 47252236251429967028491746144535113)) - 596779698385758914219233 \\
& 0688404962508573690130033831464558990*\sqrt{2}*(4*x + 1) + 3375919 \\
& 1906238865443158698053056851590355903161284193415069696*x + 84397 \\
& 97976559716360789674513264212897588975790321048353767424)*\sqrt{(x^ \\
& 2 - 2*x + 3) - 2054878990*\sqrt{2}*(145210423895997621304997884289 \\
& 16104954126009459160005*\sqrt{2} - 2053599753959165342575641038907 \\
& 1652578405543458111488) - 253193939296791490823690235397926386927 \\
& 66927370963145061302272*x + 5907858583591801452552772159284949028 \\
& 3122830532247338476371968)/(1452104238959976213049978842891610495 \\
& 4126009459160005*\sqrt{2} - 20535997539591653425756410389071652578 \\
& 405543458111488))*\sqrt{((151363871237318045*\sqrt{2} - 220640951482 \\
& 187776)/(33397068569829208576933551323217920*\sqrt{2} - 4725223625 \\
& 1429967028491746144535113)) + 11507322344*\sqrt{205487899}*\sqrt{35} \\
&)*\sqrt{(x^2 - 2*x + 3}*(110320475741093888*\sqrt{2} - 1513638712373 \\
& 18045)*\sqrt{((151363871237318045*\sqrt{2} - 220640951482187776)/(33 \\
& 397068569829208576933551323217920*\sqrt{2} - 472522362514299670284 \\
& 91746144535113)) - 2876830586*\sqrt{205487899}*\sqrt{35}*(110320475 \\
& 741093888*\sqrt{2}*(4*x + 1) - 605455484949272180*x - 151363871237 \\
& 318045)*\sqrt{((151363871237318045*\sqrt{2} - 220640951482187776)/(3 \\
& 3397068569829208576933551323217920*\sqrt{2} - 47252236251429967028 \\
& 491746144535113)) + 50344535255*337802213083473608^{(1/4)}*(4283498 \\
& 5*\sqrt{2} - 179603212)) + 3602042876982878244*(33780221308347360 \\
& 8^{(1/4)}*(4096*x^{33} - 94208*x^{32} + 1105920*x^{31} - 8726528*x^{30} + 5 \\
& 1652864*x^{29} - 243142912*x^{28} + 943504512*x^{27} - 3091758976*x^{26} \\
& + 8703838016*x^{25} - 21323296320*x^{24} + 45922747584*x^{23} - 8766903 \\
& 1872*x^{22} + 149429442480*x^{21} - 228892406512*x^{20} + 317020649584* \\
& x^{19} - 399317439952*x^{18} + 459889940952*x^{17} - 486569477336*x^{16} \\
& + 474741729456*x^{15} - 428370113832*x^{14} + 358128604129*x^{13} - 277 \\
& 690283125*x^{12} + 199741426598*x^{11} - 133169510462*x^{10} + 82124768
\end{aligned}$$

$$\begin{aligned}
& 139x^9 - 46688819967x^8 + 24352777172x^7 - 11572941916x^6 + 4956109991x^5 - 1881604195x^4 + 618995238x^3 - 169803054x^2 + \\
& 35318565x - 4164129) \sqrt{x^2 - 2x + 3} - 337802213083473608^{(1/4)} (4096x^{34} - 98304x^{33} + 1204224x^{32} - 9922560x^{31} + 61393 \\
& 152x^{30} - 302465536x^{29} + 1230167680x^{28} - 4231534592x^{27} + 12524632256x^{26} - 32311376256x^{25} + 73387152384x^{24} - 147941398 \\
& 656x^{23} + 266538692976x^{22} - 431807810720x^{21} + 632607571360x^{20} - 842579391296x^{19} + 1025439024680x^{18} - 1145619691952x^{17} \\
& + 1179623583016x^{16} - 1123076849304x^{15} + 990931422841x^{14} - 811490311782x^{13} + 617205254013x^{12} - 435943782908x^{11} + 28562 \\
& 9584397x^{10} - 173202055986x^9 + 96858530705x^8 - 49702089632x^7 + 23230667691x^6 - 9779204266x^5 + 3646868583x^4 - 11766969 \\
& 72x^3 + 315300087x^2 - 63577710x + 7212483) \arctan(1438415293 \\
& * (2\sqrt{205487899}) \sqrt{35} \sqrt{7} * (110320475741093888\sqrt{2} \\
& - 151363871237318045) \sqrt{(151363871237318045\sqrt{2} - 22064095 \\
& 1482187776) / (33397068569829208576933551323217920\sqrt{2} - 472522 \\
& 36251429967028491746144535113)) - 5 * 337802213083473608^{(1/4)} \sqrt{ \\
& 7} * (932587773\sqrt{2} - 1240695940) / (4\sqrt{1438415293} \sqrt{20 \\
& 5487899} \sqrt{35} \sqrt{7} * (110320475741093888\sqrt{2} - 151363871 \\
& 237318045) \sqrt{(337802213083473608^{(1/4)} \sqrt{205487899}) \sqrt{35} \\
&) * (\sqrt{2} * (24661741928102490286918538975895141725832720251341822 \\
& 76224653x - 5953859194138967082383053517413008702994628754686755 \\
& 712968393) - 3487685001328718053691199619823494530411356729552573 \\
& 436743740x + 842003338694921611107490741500252287557790077982093 \\
& 7989193046) \sqrt{(151363871237318045\sqrt{2} - 220640951482187776 \\
&) / (33397068569829208576933551323217920\sqrt{2} - 4725223625142996 \\
& 7028491746144535113)) - 33759191906238865443158698053056851590355 \\
& 903161284193415069696x^2 + 5967796983857589142192330688404962508 \\
& 573690130033831464558990\sqrt{2} * (4x^2 - 3x + 7) - (33780221308 \\
& 3473608^{(1/4)} \sqrt{205487899} \sqrt{35} * (2466174192810249028691853 \\
& 897589514172583272025134182276224653\sqrt{2} - 348768500132871805 \\
& 3691199619823494530411356729552573436743740) \sqrt{(15136387123731 \\
& 8045\sqrt{2} - 220640951482187776) / (33397068569829208576933551323 \\
& 217920\sqrt{2} - 47252236251429967028491746144535113)) + 59677969 \\
& 83857589142192330688404962508573690130033831464558990\sqrt{2} * (4x \\
& + 1) - 33759191906238865443158698053056851590355903161284193415 \\
& 069696x - 843979797655971636078967451326421289758897579032104835 \\
& 3767424) \sqrt{x^2 - 2x + 3} + 2054878990\sqrt{2} * (14521042389599 \\
& 762130499788428916104954126009459160005\sqrt{2} - 205359975395916 \\
& 53425756410389071652578405543458111488) + 25319393929679149082369 \\
& 023539792638692766927370963145061302272x - 590785858359180145255 \\
& 27721592849490283122830532247338476371968) / (145210423895997621304 \\
& 99788428916104954126009459160005\sqrt{2} - 2053599753959165342575 \\
& 6410389071652578405543458111488) \sqrt{(151363871237318045\sqrt{2} \\
&) - 220640951482187776) / (33397068569829208576933551323217920\sqrt{2} \\
& (2) - 47252236251429967028491746144535113)) + 11507322344\sqrt{20 \\
& 5487899} \sqrt{35} \sqrt{x^2 - 2x + 3} * (110320475741093888\sqrt{2} \\
& - 151363871237318045) \sqrt{(151363871237318045\sqrt{2} - 2206409 \\
& 51482187776) / (33397068569829208576933551323217920\sqrt{2} - 47252 \\
& 236251429967028491746144535113)) - 2876830586\sqrt{205487899} \sqrt{ \\
& t(35) * (110320475741093888\sqrt{2} * (4x + 1) - 605455484949272180x \\
& x - 151363871237318045) \sqrt{(151363871237318045\sqrt{2} - 220640 \\
& 951482187776) / (33397068569829208576933551323217920\sqrt{2} - 4725 \\
& 2236251429967028491746144535113)) - 50344535255 * 337802213083473608 \\
& 8^{(1/4)} * (42834985\sqrt{2} - 179603212)) - 9 * (337802213083473608^{(1/4)} \\
& (1/4) \sqrt{7} * (619986416588054712320x^{33} - 142596875815252583833
\end{aligned}$$

$60x^{32} + 167396332478774772326400x^{31} - 13208810605408505645977$
 $60x^{30} + 7818377455534700703130880x^{29} - 3680305242423455253144$
 $7040x^{28} + 142812495466196598236519040x^{27} - 467980607540086291$
 $795521920x^{26} + 1317446616724297757953798720x^{25} - 322757667853$
 $5657715618094400x^{24} + 6951044852168434341663353280x^{23} - 13269$
 $924051773739762905730240x^{22} + 22618218890606943234793551600x^{21}$
 $1 - 34646040746482226380773109040x^{20} + 479854727832035004479049$
 $43280x^{19} - 60442233563710008375813533840x^{18} + 696107218055963$
 $26893894078840x^{17} - 73649039715495444774051328120x^{16} + 718587$
 $46008359663290416833520x^{15} - 64839758751982121623037698440x^{14}$
 $+ 54207731921782403549473207805x^{13} - 4203227625878689198172149$
 $0625x^{12} + 30233635576337885723748360910x^{11} - 2015705263430684$
 $6278448886790x^{10} + 12430722829986195489825768255x^9 - 70670005$
 $33707311734925404515x^8 + 3686130628133706280779668740x^7 - 175$
 $1725290010384786403674220x^6 + 750175994515709494869087595x^5 -$
 $284806895091577474021198775x^4 + 93693515501145037746469710x^3$
 $- 25702047601359362810309430x^2 - 110320475741093888\sqrt{2}(4$
 $096x^{33} - 94208x^{32} + 1105920x^{31} - 8726528x^{30} + 51652864x^{29}$
 $- 243142912x^{28} + 943504512x^{27} - 3091758976x^{26} + 87038380$
 $16x^{25} - 21323296320x^{24} + 45922747584x^{23} - 87669031872x^{22}$
 $+ 149429442480x^{21} - 228892406512x^{20} + 317020649584x^{19} - 399$
 $317439952x^{18} + 459889940952x^{17} - 486569477336x^{16} + 47474172$
 $9456x^{15} - 428370113832x^{14} + 358128604129x^{13} - 277690283125x^{12}$
 $+ 199741426598x^{11} - 133169510462x^{10} + 82124768139x^9 -$
 $46688819967x^8 + 24352777172x^7 - 11572941916x^6 + 4956109991x^5$
 $- 1881604195x^4 + 618995238x^3 - 169803054x^2 + 35318565x$
 $- 4164129) + 5345954724946847798005425x - 630298685771581953407$
 $805)\sqrt{x^2 - 2x + 3} - 337802213083473608^{1/4}\sqrt{7}(6199$
 $86416588054712320x^{34} - 14879673998113313095680x^{33} + 182276006$
 $476888085422080x^{32} - 1501917094184562540595200x^{31} + 929270515$
 $4181094809027840x^{30} - 45782354444830385683397120x^{29} + 1862029$
 $42315830268839785600x^{28} - 640501457119745148723312640x^{27} + 18$
 $95776824091944217337859520x^{26} - 4890774995113719620333339520x^{25}$
 $+ 11108163483925213995887969280x^{24} - 22392982816835520879607$
 $547520x^{23} + 40344328403382311629919551920x^{22} - 65360101861090$
 $282575800442400x^{21} + 95753930975087526647353191200x^{20} - 12753$
 $6078491345560715656736320x^{19} + 155214420493384520883764350600x^{18}$
 $- 173405431539558491799550873840x^{17} + 178552392128137577493$
 $252323720x^{16} - 169993259607663498045584890680x^{15} + 1499912162$
 $91917485508694465845x^{14} - 122830315062901722450544706190x^{13} +$
 $93422576595419908569093564585x^{12} - 659861386227958431576309748$
 $60x^{11} + 43233999634236175350258543865x^{10} - 262165337003036551$
 $22578067370x^9 + 14660882169867436203483071725x^8 - 75231006952$
 $83688215881009440x^7 + 3516283793137448501472784095x^6 - 148021$
 $8215322255324062779970x^5 + 552004146616632515489480235x^4 - 17$
 $8109408955150036952459740x^3 + 47725041769783177235169915x^2 -$
 $110320475741093888\sqrt{2}(4096x^{34} - 98304x^{33} + 1204224x^{32}$
 $- 9922560x^{31} + 61393152x^{30} - 302465536x^{29} + 1230167680x^{28}$
 $8 - 4231534592x^{27} + 12524632256x^{26} - 32311376256x^{25} + 73387$
 $152384x^{24} - 147941398656x^{23} + 266538692976x^{22} - 43180781072$
 $0x^{21} + 632607571360x^{20} - 842579391296x^{19} + 1025439024680x^{18}$
 $- 1145619691952x^{17} + 1179623583016x^{16} - 1123076849304x^{15}$
 $+ 990931422841x^{14} - 811490311782x^{13} + 617205254013x^{12} - 43$
 $5943782908x^{11} + 285629584397x^{10} - 173202055986x^9 + 96858530$
 $705x^8 - 49702089632x^7 + 23230667691x^6 - 9779204266x^5 + 36$
 $46868583x^4 - 1176696972x^3 + 315300087x^2 - 63577710x + 7212$

483) - 9623368310003547842776950*x + 1091709348113345365155735))*
 log(-1581554340224321978368*(337802213083473608^(1/4)*sqrt(205487
 899)*sqrt(35)*(sqrt(2)*(24661741928102490286918538975895141725832
 72025134182276224653*x - 5953859194138967082383053517413008702994
 628754686755712968393) - 3487685001328718053691199619823494530411
 356729552573436743740*x + 842003338694921611107490741500252287557
 7900779820937989193046)*sqrt((151363871237318045*sqrt(2) - 220640
 951482187776)/(33397068569829208576933551323217920*sqrt(2) - 4725
 2236251429967028491746144535113)) + 33759191906238865443158698053
 056851590355903161284193415069696*x^2 - 5967796983857589142192330
 688404962508573690130033831464558990*sqrt(2)*(4*x^2 - 3*x + 7) -
 (337802213083473608^(1/4)*sqrt(205487899)*sqrt(35)*(2466174192810
 249028691853897589514172583272025134182276224653*sqrt(2) - 348768
 5001328718053691199619823494530411356729552573436743740)*sqrt((15
 1363871237318045*sqrt(2) - 220640951482187776)/(33397068569829208
 576933551323217920*sqrt(2) - 47252236251429967028491746144535113)
) - 5967796983857589142192330688404962508573690130033831464558990
 sqrt(2)(4*x + 1) + 33759191906238865443158698053056851590355903
 161284193415069696*x + 843979797655971636078967451326421289758897
 5790321048353767424)*sqrt(x^2 - 2*x + 3) - 2054878990*sqrt(2)*(14
 521042389599762130499788428916104954126009459160005*sqrt(2) - 205
 35997539591653425756410389071652578405543458111488) - 25319393929
 679149082369023539792638692766927370963145061302272*x + 590785858
 35918014525527721592849490283122830532247338476371968)/(145210423
 89599762130499788428916104954126009459160005*sqrt(2) - 2053599753
 9591653425756410389071652578405543458111488)) + 9*(33780221308347
 3608^(1/4)*sqrt(7)*(619986416588054712320*x^33 - 1425968758152525
 8383360*x^32 + 167396332478774772326400*x^31 - 132088106054085056
 4597760*x^30 + 7818377455534700703130880*x^29 - 36803052424234552
 531447040*x^28 + 142812495466196598236519040*x^27 - 4679806075400
 86291795521920*x^26 + 1317446616724297757953798720*x^25 - 3227576
 678535657715618094400*x^24 + 6951044852168434341663353280*x^23 -
 13269924051773739762905730240*x^22 + 2261821889060694323479355160
 0*x^21 - 34646040746482226380773109040*x^20 + 4798547278320350044
 7904943280*x^19 - 60442233563710008375813533840*x^18 + 6961072180
 5596326893894078840*x^17 - 73649039715495444774051328120*x^16 + 7
 1858746008359663290416833520*x^15 - 64839758751982121623037698440
 *x^14 + 54207731921782403549473207805*x^13 - 42032276258786891981
 721490625*x^12 + 30233635576337885723748360910*x^11 - 20157052634
 306846278448886790*x^10 + 12430722829986195489825768255*x^9 - 706
 7000533707311734925404515*x^8 + 3686130628133706280779668740*x^7
 - 1751725290010384786403674220*x^6 + 750175994515709494869087595*
 x^5 - 284806895091577474021198775*x^4 + 9369351550114503774646971
 0*x^3 - 25702047601359362810309430*x^2 - 110320475741093888*sqrt(
 2)*(4096*x^33 - 94208*x^32 + 1105920*x^31 - 8726528*x^30 + 516528
 64*x^29 - 243142912*x^28 + 943504512*x^27 - 3091758976*x^26 + 870
 3838016*x^25 - 21323296320*x^24 + 45922747584*x^23 - 87669031872*
 x^22 + 149429442480*x^21 - 228892406512*x^20 + 317020649584*x^19
 - 399317439952*x^18 + 459889940952*x^17 - 486569477336*x^16 + 474
 741729456*x^15 - 428370113832*x^14 + 358128604129*x^13 - 27769028
 3125*x^12 + 199741426598*x^11 - 133169510462*x^10 + 82124768139*x
 ^9 - 46688819967*x^8 + 24352777172*x^7 - 11572941916*x^6 + 495610
 9991*x^5 - 1881604195*x^4 + 618995238*x^3 - 169803054*x^2 + 35318
 565*x - 4164129) + 5345954724946847798005425*x - 6302986857715819
 53407805)*sqrt(x^2 - 2*x + 3) - 337802213083473608^(1/4)*sqrt(7)*
 (619986416588054712320*x^34 - 14879673998113313095680*x^33 + 1822

$$\begin{aligned}
&76006476888085422080*x^{32} - 1501917094184562540595200*x^{31} + 9292 \\
&705154181094809027840*x^{30} - 45782354444830385683397120*x^{29} + 18 \\
&6202942315830268839785600*x^{28} - 640501457119745148723312640*x^{27} \\
&+ 1895776824091944217337859520*x^{26} - 48907749951137196203333395 \\
&20*x^{25} + 11108163483925213995887969280*x^{24} - 223929828168355208 \\
&79607547520*x^{23} + 40344328403382311629919551920*x^{22} - 653601018 \\
&61090282575800442400*x^{21} + 95753930975087526647353191200*x^{20} - \\
&127536078491345560715656736320*x^{19} + 155214420493384520883764350 \\
&600*x^{18} - 173405431539558491799550873840*x^{17} + 1785523921281375 \\
&77493252323720*x^{16} - 169993259607663498045584890680*x^{15} + 14999 \\
&1216291917485508694465845*x^{14} - 122830315062901722450544706190*x \\
&^{13} + 93422576595419908569093564585*x^{12} - 6598613862279584315763 \\
&0974860*x^{11} + 43233999634236175350258543865*x^{10} - 2621653370030 \\
&3655122578067370*x^9 + 14660882169867436203483071725*x^8 - 752310 \\
&0695283688215881009440*x^7 + 3516283793137448501472784095*x^6 - 1 \\
&480218215322255324062779970*x^5 + 552004146616632515489480235*x^4 \\
&- 178109408955150036952459740*x^3 + 47725041769783177235169915*x \\
&^2 - 110320475741093888*\sqrt{2}*(4096*x^{34} - 98304*x^{33} + 1204224 \\
&*x^{32} - 9922560*x^{31} + 61393152*x^{30} - 302465536*x^{29} + 123016768 \\
&0*x^{28} - 4231534592*x^{27} + 12524632256*x^{26} - 32311376256*x^{25} + \\
&73387152384*x^{24} - 147941398656*x^{23} + 266538692976*x^{22} - 431807 \\
&810720*x^{21} + 632607571360*x^{20} - 842579391296*x^{19} + 10254390246 \\
&80*x^{18} - 1145619691952*x^{17} + 1179623583016*x^{16} - 1123076849304 \\
&*x^{15} + 990931422841*x^{14} - 811490311782*x^{13} + 617205254013*x^{12} \\
&- 435943782908*x^{11} + 285629584397*x^{10} - 173202055986*x^9 + 968 \\
&58530705*x^8 - 49702089632*x^7 + 23230667691*x^6 - 9779204266*x^5 \\
&+ 3646868583*x^4 - 1176696972*x^3 + 315300087*x^2 - 63577710*x + \\
&7212483) - 9623368310003547842776950*x + 10917093481133453651557 \\
&35))*\log(1581554340224321978368*(337802213083473608^{(1/4)}*\sqrt{20} \\
&5487899)*\sqrt{35}*(\sqrt{2}*(2466174192810249028691853897589514172 \\
&583272025134182276224653*x - 595385919413896708238305351741300870 \\
&2994628754686755712968393) - 348768500132871805369119961982349453 \\
&0411356729552573436743740*x + 84200333869492161110749074150025228 \\
&75577900779820937989193046)*\sqrt{((151363871237318045*\sqrt{2}) - 22 \\
&0640951482187776)/(33397068569829208576933551323217920*\sqrt{2}) - \\
&47252236251429967028491746144535113)) - 3375919190623886544315869 \\
&8053056851590355903161284193415069696*x^2 + 596779698385758914219 \\
&2330688404962508573690130033831464558990*\sqrt{2}*(4*x^2 - 3*x + 7 \\
&)- (337802213083473608^{(1/4)}*\sqrt{205487899})*\sqrt{35}*(246617419 \\
&2810249028691853897589514172583272025134182276224653*\sqrt{2}) - 34 \\
&87685001328718053691199619823494530411356729552573436743740)*\sqrt{ \\
&((151363871237318045*\sqrt{2}) - 220640951482187776)/(3339706856982 \\
&9208576933551323217920*\sqrt{2}) - 47252236251429967028491746144535 \\
&113)) + 596779698385758914219233068840496250857369013003383146455 \\
&8990*\sqrt{2}*(4*x + 1) - 3375919190623886544315869805305685159035 \\
&5903161284193415069696*x - 84397979765597163607896745132642128975 \\
&88975790321048353767424)*\sqrt{x^2 - 2*x + 3} + 2054878990*\sqrt{2} \\
&*(14521042389599762130499788428916104954126009459160005*\sqrt{2}) - \\
&20535997539591653425756410389071652578405543458111488) + 2531939 \\
&3929679149082369023539792638692766927370963145061302272*x - 59078 \\
&585835918014525527721592849490283122830532247338476371968)/(14521 \\
&042389599762130499788428916104954126009459160005*\sqrt{2}) - 205359 \\
&97539591653425756410389071652578405543458111488))/(\sqrt{20548789} \\
&9)*\sqrt{35}*\sqrt{7}*(619986416588054712320*x^{33} - 142596875815252 \\
&58383360*x^{32} + 167396332478774772326400*x^{31} - 13208810605408505 \\
&64597760*x^{30} + 7818377455534700703130880*x^{29} - 3680305242423455
\end{aligned}$$

$$\begin{aligned}
& 2531447040*x^{28} + 142812495466196598236519040*x^{27} - 467980607540 \\
& 086291795521920*x^{26} + 1317446616724297757953798720*x^{25} - 322757 \\
& 6678535657715618094400*x^{24} + 6951044852168434341663353280*x^{23} - \\
& 13269924051773739762905730240*x^{22} + 226182188906069432347935516 \\
& 00*x^{21} - 34646040746482226380773109040*x^{20} + 479854727832035004 \\
& 47904943280*x^{19} - 60442233563710008375813533840*x^{18} + 696107218 \\
& 05596326893894078840*x^{17} - 73649039715495444774051328120*x^{16} + \\
& 71858746008359663290416833520*x^{15} - 6483975875198212162303769844 \\
& 0*x^{14} + 54207731921782403549473207805*x^{13} - 4203227625878689198 \\
& 1721490625*x^{12} + 30233635576337885723748360910*x^{11} - 2015705263 \\
& 4306846278448886790*x^{10} + 12430722829986195489825768255*x^9 - 70 \\
& 67000533707311734925404515*x^8 + 3686130628133706280779668740*x^7 \\
& - 1751725290010384786403674220*x^6 + 750175994515709494869087595 \\
& *x^5 - 284806895091577474021198775*x^4 + 936935155011450377464697 \\
& 10*x^3 - 25702047601359362810309430*x^2 - 110320475741093888*sqrt \\
& (2)*(4096*x^{33} - 94208*x^{32} + 1105920*x^{31} - 8726528*x^{30} + 51652 \\
& 864*x^{29} - 243142912*x^{28} + 943504512*x^{27} - 3091758976*x^{26} + 87 \\
& 03838016*x^{25} - 21323296320*x^{24} + 45922747584*x^{23} - 87669031872 \\
& *x^{22} + 149429442480*x^{21} - 228892406512*x^{20} + 317020649584*x^{19} \\
& - 399317439952*x^{18} + 459889940952*x^{17} - 486569477336*x^{16} + 47 \\
& 4741729456*x^{15} - 428370113832*x^{14} + 358128604129*x^{13} - 2776902 \\
& 83125*x^{12} + 199741426598*x^{11} - 133169510462*x^{10} + 82124768139* \\
& x^9 - 46688819967*x^8 + 24352777172*x^7 - 11572941916*x^6 + 49561 \\
& 09991*x^5 - 1881604195*x^4 + 618995238*x^3 - 169803054*x^2 + 3531 \\
& 8565*x - 4164129) + 5345954724946847798005425*x - 630298685771581 \\
& 953407805)*sqrt(x^2 - 2*x + 3)*sqrt((151363871237318045*sqrt(2) - \\
& 220640951482187776)/(33397068569829208576933551323217920*sqrt(2) \\
& - 47252236251429967028491746144535113)) - sqrt(205487899)*sqrt(3 \\
& 5)*sqrt(7)*(619986416588054712320*x^{34} - 14879673998113313095680* \\
& x^{33} + 182276006476888085422080*x^{32} - 1501917094184562540595200* \\
& x^{31} + 9292705154181094809027840*x^{30} - 4578235444483038568339712 \\
& 0*x^{29} + 186202942315830268839785600*x^{28} - 640501457119745148723 \\
& 312640*x^{27} + 1895776824091944217337859520*x^{26} - 489077499511371 \\
& 9620333339520*x^{25} + 11108163483925213995887969280*x^{24} - 2239298 \\
& 2816835520879607547520*x^{23} + 40344328403382311629919551920*x^{22} \\
& - 65360101861090282575800442400*x^{21} + 95753930975087526647353191 \\
& 200*x^{20} - 127536078491345560715656736320*x^{19} + 1552144204933845 \\
& 20883764350600*x^{18} - 173405431539558491799550873840*x^{17} + 17855 \\
& 2392128137577493252323720*x^{16} - 169993259607663498045584890680*x \\
& ^{15} + 149991216291917485508694465845*x^{14} - 122830315062901722450 \\
& 544706190*x^{13} + 93422576595419908569093564585*x^{12} - 65986138622 \\
& 795843157630974860*x^{11} + 43233999634236175350258543865*x^{10} - 26 \\
& 216533700303655122578067370*x^9 + 14660882169867436203483071725*x \\
& ^8 - 7523100695283688215881009440*x^7 + 3516283793137448501472784 \\
& 095*x^6 - 1480218215322255324062779970*x^5 + 55200414661663251548 \\
& 9480235*x^4 - 178109408955150036952459740*x^3 + 47725041769783177 \\
& 235169915*x^2 - 110320475741093888*sqrt(2)*(4096*x^{34} - 98304*x^{33} \\
& 3 + 1204224*x^{32} - 9922560*x^{31} + 61393152*x^{30} - 302465536*x^{29} \\
& + 1230167680*x^{28} - 4231534592*x^{27} + 12524632256*x^{26} - 32311376 \\
& 256*x^{25} + 73387152384*x^{24} - 147941398656*x^{23} + 266538692976*x^ \\
& ^{22} - 431807810720*x^{21} + 632607571360*x^{20} - 842579391296*x^{19} + \\
& 1025439024680*x^{18} - 1145619691952*x^{17} + 1179623583016*x^{16} - 11 \\
& 23076849304*x^{15} + 990931422841*x^{14} - 811490311782*x^{13} + 617205 \\
& 254013*x^{12} - 435943782908*x^{11} + 285629584397*x^{10} - 17320205598 \\
& 6*x^9 + 96858530705*x^8 - 49702089632*x^7 + 23230667691*x^6 - 977 \\
& 9204266*x^5 + 3646868583*x^4 - 1176696972*x^3 + 315300087*x^2 - 6
\end{aligned}$$


```
3577710*x + 7212483) - 9623368310003547842776950*x + 109170934811
3345365155735)*sqrt((151363871237318045*sqrt(2) - 220640951482187
776)/(33397068569829208576933551323217920*sqrt(2) - 4725223625142
9967028491746144535113)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**2-2*x+3)**(11/2)/(2*x**2+x+1)**5,x)

[Out] Timed out

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2x^2 + x + 1)^5 (x^2 - 2x + 3)^{\frac{11}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((2*x^2 + x + 1)^5*(x^2 - 2*x + 3)^(11/2)),x, algorithm="giac")

[Out] integrate(1/((2*x^2 + x + 1)^5*(x^2 - 2*x + 3)^(11/2)), x)

steps used = 24, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$

$$\begin{aligned}
 & \frac{12105495874518671061833 - 5117656435043679338190x}{146548895467025x + 37857197792117} \sqrt{x^2 - 2x + 3} \\
 & - \frac{2421216420000000 (x^2 - 2x + 3)^{19/2} (2x^2 + x + 1)}{4179039782398459850819 - 1886993445589652402694x} \\
 & - \frac{1042737204880000000000000000 (x^2 - 2x + 3)^{3/2}}{1384103301166x + 5488221294349} \\
 & + \frac{276710448000000 (x^2 - 2x + 3)^{19/2} (2x^2 + x + 1)^2}{6551405511565449301689 - 3127298559983309301910x} \\
 & - \frac{521368602440000000000000000 (x^2 - 2x + 3)^{5/2}}{310705340015x + 277010166219} \\
 & + \frac{12353145000000 (x^2 - 2x + 3)^{19/2} (2x^2 + x + 1)^3}{1117646664729238460189 - 568839749685437871554x} \\
 & - \frac{312821161464000000000000000 (x^2 - 2x + 3)^{7/2}}{911061463974x + 592729157441} - \frac{838519439380295335657 - 466189390555853643870x}{938463484392000000000000 (x^2 - 2x + 3)^{9/2}} \\
 & + \frac{29647548000000 (x^2 - 2x + 3)^{19/2} (2x^2 + x + 1)^4}{813432205x + 447940041} - \frac{3(69053268515296359011 - 44840736195018286006x)}{114701092536800000000000 (x^2 - 2x + 3)^{11/2}} \\
 & + \frac{26471025000 (x^2 - 2x + 3)^{19/2} (2x^2 + x + 1)^5}{17459234x + 8837931} - \frac{11(7502325106308201089 - 7813986379726516886x)}{406667509903200000000000 (x^2 - 2x + 3)^{13/2}} \\
 & + \frac{605052000 (x^2 - 2x + 3)^{19/2} (2x^2 + x + 1)^6}{29371x + 14453} + \frac{1942164996204584234x + 7851758375483333511}{15641058073200000000000 (x^2 - 2x + 3)^{15/2}} \\
 & + \frac{1080450 (x^2 - 2x + 3)^{19/2} (2x^2 + x + 1)^7}{2218x + 887} + \frac{476849951294984711 - 125181871472148210x}{104273720488000000000 (x^2 - 2x + 3)^{17/2}} \\
 & + \frac{88200 (x^2 - 2x + 3)^{19/2} (2x^2 + x + 1)^8}{1 - 10x} + \frac{37358055634422583 - 14024622879097678x}{1840124479200000000 (x^2 - 2x + 3)^{19/2}} \\
 & - \frac{630 (x^2 - 2x + 3)^{19/2} (2x^2 + x + 1)^9}{1840124479200000000 (x^2 - 2x + 3)^{19/2}} \\
 & + \frac{\sqrt{\frac{1}{70} (81042225921274689605478944797800854846405 + 57305922523001707126026363878666500308992\sqrt{2})}}{\sqrt{\frac{1}{70} (57305922523001707126026363878666500308992\sqrt{2} - 81042225921274689605478944797800854846405)}} \tan^{-1} \left(\sqrt{\frac{1}{70} (81042225921274689605478944797800854846405 + 57305922523001707126026363878666500308992\sqrt{2})} \right) \tan^{-1} \left(\sqrt{\frac{1}{70} (57305922523001707126026363878666500308992\sqrt{2} - 81042225921274689605478944797800854846405)} \right)
 \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[1/((3 - 2*x + x^2)^(21/2)*(1 + x + 2*x^2)^10), x]

[Out] (37358055634422583 - 14024622879097678*x)/(184012447920000000*(3 - 2*x + x^2)^(19/2)) + (476849951294984711 - 125181871472148210*x)/(10427372048800000000*(3 - 2*x + x^2)^(17/2)) + (7851758375483333511 + 1942164996204584234*x)/(156410580732000000000*(3 - 2*x + x^2)^(15/2)) - (11*(7502325106308201089 - 7813986379726516886*x))/(4066675099032000000000*(3 - 2*x + x^2)^(13/2)) - (3*(69053268515296359011 - 44840736195018286006*x))/(11470109253680000000

+ 3)**(17/2)*(2*x**2 + x + 1)), x)/369519996979350000000000000000
0000000

Mathematica [C] time = 6.71662, size = 1431, normalized size = 2.24

result too large to display

Antiderivative was successfully verified.

[In] Integrate[1/((3 - 2*x + x^2)^(21/2)*(1 + x + 2*x^2)^10),x]

[Out] Sqrt[3 - 2*x + x^2]*((1 - x)/(11875000000*(3 - 2*x + x^2)^10) + (265 - 113*x)/(40375000000*(3 - 2*x + x^2)^9) + (82361 - 4841*x)/(6056250000000*(3 - 2*x + x^2)^8) + (1062937 + 1642511*x)/(157462500000000*(3 - 2*x + x^2)^7) + (7*(-678331 + 833371*x))/(222062500000000*(3 - 2*x + x^2)^6) + (7*(-73161291 + 43964675*x))/(9084375000000000*(3 - 2*x + x^2)^5) + (-1340879383 + 430593031*x)/(18168750000000000*(3 - 2*x + x^2)^4) - (11*(1626125723 + 112950205*x))/(302812500000000000*(3 - 2*x + x^2)^3) - (11*(3311570647 + 15286717673*x))/(3633750000000000000*(3 - 2*x + x^2)^2) - (11*(-411521923277 + 484788625685*x))/(3633750000000000000*(3 - 2*x + x^2)) + (251943 + 221770*x)/(6300000000000*(1 + x + 2*x^2)^9) - (73*(-888423 + 1604678*x))/(88200000000000*(1 + x + 2*x^2)^8) + (-2596903794 - 4965311863*x)/(1080450000000000*(1 + x + 2*x^2)^7) + (-539608494637 - 334647150510*x)/(1210104000000000000*(1 + x + 2*x^2)^6) + (-40800462989458 + 56711874696335*x)/(2647102500000000000*(1 + x + 2*x^2)^5) + (42018358198215561 + 129196597088670934*x)/(296475480000000000000000*(1 + x + 2*x^2)^4) + (62819559864314747 + 169630389653846945*x)/(3705943500000000000000000*(1 + x + 2*x^2)^3) + (1082422109196374795 + 4797048907791526114*x)/(8301313440000000000000000000*(1 + x + 2*x^2)^2) + (65571203144429922747 + 367152793968978953465*x)/(3631824630000000000000000000*(1 + x + 2*x^2))) + ((232442807954946745795*I + 21634177831191924841*Sqrt[7])*ArcTan[(-135063738860435016899586558948733259113515 + (188630894626466690216855285995045889396405*I)*Sqrt[7] - 1506241361872688008559268776761430483700000*x - (105711500937472192718115651350352447938680*I)*Sqrt[7]*x + 491153540508443587025809789813541985707360*x^2 - (460764064177139993399975100872663310399420*I)*Sqrt[7]*x^2 - 180084985147246689199448745264977678818020*x^3 + (197868296377913870863837680953446009396860*I)*Sqrt[7]*x^3 - 176004816500761880926774485599831047775825*x^4 - (207342833228459577163557043035558264835165*I)*Sqrt[7]*x^4 + (186244248199755548159585682605666126004224*I)*Sqrt[10*(-5 + I*Sqrt[7])]*Sqrt[3 - 2*x + x^2] + (114611845046003414252052727757333000617984*I)*Sqrt[10*(-5 + I*Sqrt[7])]*x*Sqrt[3 - 2*x + x^2] + (300856093245758962411638410362999126622208*I)*Sqrt[10*(-5 + I*Sqrt[7])]*x^2*Sqrt[3 - 2*x + x^2] - (143264806307504267815065909696666250772480*I)*Sqrt[10*(-5 + I*Sqrt[7])]*x^3*Sqrt[3 - 2*x + x^2])/(2368773290838836979864678493023884746594823*I + 423642940259238735473942663180025956729505*Sqrt[7] + (1890613486065620301760074218556745311646936*I)*x + 6150574559311228258394328777942059796320*Sqrt[7]*x + (2511300259855822962340893027852239157667820*I)*x^2 - 202786755080110618986776343109

[Out] result too large to display

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2x^2 + x + 1)^{10}(x^2 - 2x + 3)^{\frac{21}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((2*x^2 + x + 1)^10*(x^2 - 2*x + 3)^(21/2)),x, algorithm="maxima")`

[Out] `integrate(1/((2*x^2 + x + 1)^10*(x^2 - 2*x + 3)^(21/2)), x)`

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((2*x^2 + x + 1)^10*(x^2 - 2*x + 3)^(21/2)),x, algorithm="fricas")`

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x**2-2*x+3)**(21/2)/(2*x**2+x+1)**10,x)`

[Out] Timed out

GIAC/XCAS [A] time = 0.350931, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate(1/((2*x^2 + x + 1)^10*(x^2 - 2*x + 3)^(21/2)),x, algorithm="giac")
```

```
[Out] Done
```

$$3.52 \quad \int \frac{-a - \sqrt{1+a^2} + x}{(-a + \sqrt{1+a^2} + x) \sqrt{(-a+x)(1+x^2)}} dx$$

Optimal. Leaf size=66

$$-\sqrt{2}\sqrt{\sqrt{a^2+1}+a} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sqrt{a^2+1}-a(x-a)}}{\sqrt{(x^2+1)(x-a)}}\right)$$

[Out] $-(\text{Sqrt}[2] * \text{Sqrt}[a + \text{Sqrt}[1 + a^2]]) * \text{ArcTan}[(\text{Sqrt}[2] * \text{Sqrt}[-a + \text{Sqrt}[1 + a^2]]) * (-a + x) / \text{Sqrt}[(-a + x) * (1 + x^2)]]]$

Rubi [C] time = 2.37246, antiderivative size = 204, normalized size of antiderivative = 3.09, number of steps used = 9, number of rules used = 8, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$\frac{4\sqrt{a^2+1}\sqrt{x^2+1}\sqrt{\frac{a-x}{a+i}}\left(\frac{2}{1-i(a-\sqrt{a^2+1})}; \sin^{-1}\left(\frac{\sqrt{1-ix}}{\sqrt{2}}\right)\Big|_{\frac{2}{1-ia}}\right)}{(1-i(a-\sqrt{a^2+1}))\sqrt{(x^2+1)(-a-x)}} + \frac{2i\sqrt{x^2+1}\sqrt{\frac{a-x}{a+i}}F\left(\sin^{-1}\left(\frac{\sqrt{1-ix}}{\sqrt{2}}\right)\Big|_{\frac{2}{1-ia}}\right)}{\sqrt{(x^2+1)(-a-x)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(-a - \text{Sqrt}[1 + a^2] + x) / ((-a + \text{Sqrt}[1 + a^2] + x) * \text{Sqrt}[(-a + x) * (1 + x^2)]), x]$

[Out] $((2*I) * \text{Sqrt}[(a - x) / (I + a)] * \text{Sqrt}[1 + x^2] * \text{EllipticF}[\text{ArcSin}[\text{Sqrt}[1 - I*x] / \text{Sqrt}[2]], 2 / (1 - I*a)]) / \text{Sqrt}[-(a - x) * (1 + x^2)] + (4 * \text{Sqrt}[1 + a^2] * \text{Sqrt}[(a - x) / (I + a)] * \text{Sqrt}[1 + x^2] * \text{EllipticPi}[2 / (1 - I*(a - \text{Sqrt}[1 + a^2]))], \text{ArcSin}[\text{Sqrt}[1 - I*x] / \text{Sqrt}[2]], 2 / (1 - I*a)]) / ((1 - I*(a - \text{Sqrt}[1 + a^2])) * \text{Sqrt}[-(a - x) * (1 + x^2)])]$

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{rubi_integrate}((-a+x-(a**2+1)**(1/2))/(-a+x+(a**2+1)**(1/2)) / ((-a+x) * (x**2+1))** (1/2), x)$

[Out] Timed out

2), ((a+I)/(a-I))^(1/2))-(-1/(-a-I)*a+1/(-a-I)*x)^(1/2)*(1/(a-I)*x - I/(a-I))^(1/2)*(1/(a+I)*x+I/(a+I))^(1/2)/(-a*x^2+x^3-a+x)^(1/2)/(a^2+1)^(1/2)*EllipticPi(((a+x)/(-a-I))^(1/2), (a+I)/(a^2+1)^(1/2)), ((a+I)/(a-I))^(1/2))*a-I*(-1/(-a-I)*a+1/(-a-I)*x)^(1/2)*(1/(a-I)*x-I/(a-I))^(1/2)*(1/(a+I)*x+I/(a+I))^(1/2)/(-a*x^2+x^3-a+x)^(1/2)/(a^2+1)^(1/2)*EllipticPi(((a+x)/(-a-I))^(1/2), (a+I)/(a^2+1)^(1/2)), ((a+I)/(a-I))^(1/2)))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a - x + \sqrt{a^2 + 1}}{\sqrt{-(x^2 + 1)(a - x)}(a - x - \sqrt{a^2 + 1})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a - x + sqrt(a^2 + 1))/(sqrt(-(x^2 + 1)*(a - x))*(a - x - sqrt(a^2 + 1))))

[Out] integrate((a - x + sqrt(a^2 + 1))/(sqrt(-(x^2 + 1)*(a - x))*(a - x - sqrt(a^2 + 1))), x)

Fricas [A] time = 0.616919, size = 1, normalized size = 0.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a - x + sqrt(a^2 + 1))/(sqrt(-(x^2 + 1)*(a - x))*(a - x - sqrt(a^2 + 1))))

[Out] [1/4*sqrt(-2*a - 2*sqrt(a^2 + 1))*log((256*a^6 + (32*a^6 + 48*a^4 + 18*a^2 + 1)*x^4 + 320*a^4 - 12*(16*a^5 + 20*a^3 + 5*a)*x^3 + 2*(128*a^6 + 184*a^4 + 64*a^2 + 3)*x^2 + 82*a^2 + 4*(32*a^5 + 36*a^3 + (16*a^5 + 20*a^3 + 5*a)*x^2 - 2*(8*a^4 + 8*a^2 + 1)*x + (32*a^4 + (16*a^4 + 12*a^2 + 1)*x^2 + 20*a^2 - 8*(2*a^3 + a)*x + 1)*sqrt(a^2 + 1) + 7*a)*sqrt(-a*x^2 + x^3 - a + x)*sqrt(-2*a - 2*sqrt(a^2 + 1)) - 4*(64*a^5 + 76*a^3 + 17*a)*x + 2*(128*a^5 + (16*a^5 + 16*a^3 + 3*a)*x^4 - 6*(16*a^4 + 12*a^2 + 1)*x^3 + 96*a^3 + 4*(32*a^5 + 30*a^3 + 5*a)*x^2 - 2*(64*a^4 + 44*a^2 + 3)*x + 9*a)*sqrt(a^2 + 1) + 1)/((32*a^6 + 48*a^4 + 18*a^2 + 1)*x^4 + 4*(16*a^5 + 20*a^3 + 5*a)*x^3 + 6*(8*a^4 + 8*a^2 + 1)*x^2 + 2*a^2 + 4*(4*a^3 + 3*a)*x + 2*((16*a^5 + 16*a^3 + 3*a)*x^4 + 2*(16*a^4 + 12*a^2 + 1)*x^3 + 12*(2*a^3 + a)*x^2 + 2*(4*a^2 + 1)*x + a)*sqrt(a^2 + 1) + 1), 1/2*sqrt(2*a + 2*sqrt(a^2 + 1))*arctan(1/2*(8*a^3 + (4*a^3 + 3*a)*x^2 - 2*(2*a^2 + 1)*x + ((4*a^2 + 1)*x^2 + 8*a^2 - 4*a*x + 1)*sqrt(a^2 + 1) + 5*a)/(sqrt(-a*x^2 + x^3 - a + x)*(2*a^2 + 2*sqrt(a^2 + 1)*a + 1)*sqrt(2*a + 2*sqrt(a^2 + 1))))]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a+x-(a**2+1)**(1/2))/(-a+x+(a**2+1)**(1/2))/((-a+x)*(x**2+1))**(1/

[Out] Timed out

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a - x + \sqrt{a^2 + 1}}{\sqrt{-(x^2 + 1)}(a - x)(a - x - \sqrt{a^2 + 1})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a - x + sqrt(a^2 + 1))/(sqrt(-(x^2 + 1))*(a - x))*(a - x - sqrt(a^2 + 1))

[Out] integrate((a - x + sqrt(a^2 + 1))/(sqrt(-(x^2 + 1))*(a - x))*(a - x - sqrt(a^2 + 1))), x)

$$3.53 \quad \int \frac{a+bx}{\sqrt[3]{1-x^2(3+x^2)}} dx$$

Optimal. Leaf size=198

$$\frac{a \tan^{-1}\left(\frac{\sqrt{3}(1-\sqrt[3]{2}\sqrt[3]{1-x^2})}{x}\right)}{2 \cdot 2^{2/3}\sqrt{3}} + \frac{a \tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2+1}}\right)}{2 \cdot 2^{2/3}} + \frac{a \tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2 \cdot 2^{2/3}\sqrt{3}} - \frac{a \tanh^{-1}(x)}{6 \cdot 2^{2/3}}$$

$$- \frac{b \log(x^2+3)}{4 \cdot 2^{2/3}} + \frac{3b \log(2^{2/3} - \sqrt[3]{1-x^2})}{4 \cdot 2^{2/3}} + \frac{\sqrt{3}b \tan^{-1}\left(\frac{\sqrt[3]{2-2x^2+1}}{\sqrt{3}}\right)}{2 \cdot 2^{2/3}}$$

[Out] (a*ArcTan[Sqrt[3]/x])/(2*2^(2/3)*Sqrt[3]) + (Sqrt[3]*b*ArcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]])/(2*2^(2/3)) + (a*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*(1 - x^2)^(1/3)))/x])/(2*2^(2/3)*Sqrt[3]) - (a*ArcTanh[x])/(6*2^(2/3)) + (a*ArcTanh[x/(1 + 2^(1/3)*(1 - x^2)^(1/3))])/(2*2^(2/3)) - (b*Log[3 + x^2])/(4*2^(2/3)) + (3*b*Log[2^(2/3) - (1 - x^2)^(1/3)])/(4*2^(2/3))

Rubi [A] time = 0.241473, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$

$$\frac{a \tan^{-1}\left(\frac{\sqrt{3}(1-\sqrt[3]{2}\sqrt[3]{1-x^2})}{x}\right)}{2 \cdot 2^{2/3}\sqrt{3}} + \frac{a \tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2+1}}\right)}{2 \cdot 2^{2/3}} + \frac{a \tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2 \cdot 2^{2/3}\sqrt{3}} - \frac{a \tanh^{-1}(x)}{6 \cdot 2^{2/3}}$$

$$- \frac{b \log(x^2+3)}{4 \cdot 2^{2/3}} + \frac{3b \log(2^{2/3} - \sqrt[3]{1-x^2})}{4 \cdot 2^{2/3}} + \frac{\sqrt{3}b \tan^{-1}\left(\frac{\sqrt[3]{2-2x^2+1}}{\sqrt{3}}\right)}{2 \cdot 2^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)/((1 - x^2)^(1/3)*(3 + x^2)), x]

[Out] (a*ArcTan[Sqrt[3]/x])/(2*2^(2/3)*Sqrt[3]) + (Sqrt[3]*b*ArcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]])/(2*2^(2/3)) + (a*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*(1 - x^2)^(1/3)))/x])/(2*2^(2/3)*Sqrt[3]) - (a*ArcTanh[x])/(6*2^(2/3)) + (a*ArcTanh[x/(1 + 2^(1/3)*(1 - x^2)^(1/3))])/(2*2^(2/3)) - (b*Log[3 + x^2])/(4*2^(2/3)) + (3*b*Log[2^(2/3) - (1 - x^2)^(1/3)])/(4*2^(2/3))

Rubi in Sympy [A] time = 13.6533, size = 231, normalized size = 1.17

$$\frac{\sqrt[3]{2}a \log\left(\sqrt[3]{2}\sqrt[3]{-x+1} + (x+1)^{\frac{2}{3}}\right)}{8} - \frac{\sqrt[3]{2}a \log\left((-x+1)^{\frac{2}{3}} + \sqrt[3]{2}\sqrt[3]{x+1}\right)}{8}$$

$$- \frac{\sqrt[3]{2}\sqrt[3]{3}a \operatorname{atan}\left(\frac{\sqrt[3]{3}}{3} - \frac{2^{\frac{2}{3}}\sqrt[3]{3}(x+1)^{\frac{2}{3}}}{3\sqrt[3]{-x+1}}\right)}{12} - \frac{\sqrt[3]{2}\sqrt[3]{3}a \operatorname{atan}\left(\frac{2^{\frac{2}{3}}\sqrt[3]{3}(-x+1)^{\frac{2}{3}}}{3\sqrt[3]{x+1}} - \frac{\sqrt[3]{3}}{3}\right)}{12} - \frac{\sqrt[3]{2}b \log(x^2+3)}{8}$$

$$+ \frac{3\sqrt[3]{2}b \log\left(-\sqrt[3]{-x^2+1} + 2^{\frac{2}{3}}\right)}{8} + \frac{\sqrt[3]{2}\sqrt[3]{3}b \operatorname{atan}\left(\sqrt[3]{3}\left(\frac{\sqrt[3]{2}\sqrt[3]{-x^2+1}}{3} + \frac{1}{3}\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((b*x+a)/(-x**2+1)**(1/3)/(x**2+3), x)`

[Out] $2^{**}(1/3)*a*\log(2^{**}(1/3)*(-x+1)^{(1/3)}+(x+1)^{(2/3)})/8 - 2^{**}(1/3)*a*\log((-x+1)^{(2/3)}+2^{**}(1/3)*(x+1)^{(1/3)})/8 - 2^{**}(1/3)*\sqrt{3}*a*\operatorname{atan}(\sqrt{3}/3 - 2^{**}(2/3)*\sqrt{3}*(x+1)^{(2/3)}/(3*(-x+1)^{(1/3)}))/12 - 2^{**}(1/3)*\sqrt{3}*a*\operatorname{atan}(2^{**}(2/3)*\sqrt{3}*(-x+1)^{(2/3)}/(3*(x+1)^{(1/3)}) - \sqrt{3}/3)/12 - 2^{**}(1/3)*b*\log(x^2+3)/8 + 3*2^{**}(1/3)*b*\log(-(-x^2+1)^{(1/3)}+2^{**}(2/3))/8 + 2^{**}(1/3)*\sqrt{3}*b*\operatorname{atan}(\sqrt{3}*(2^{**}(1/3)*(-x^2+1)^{(1/3)}/(3+1/3)))/4$

Mathematica [C] time = 0.468093, size = 205, normalized size = 1.04

$$3x \left(\frac{3aF_1\left(\frac{1}{2}, \frac{1}{3}, 1; \frac{3}{2}; x^2, -\frac{x^2}{3}\right)}{2x^2\left(F_1\left(\frac{3}{2}, \frac{4}{3}, 1; \frac{5}{2}; x^2, -\frac{x^2}{3}\right) - F_1\left(\frac{3}{2}, \frac{1}{3}, 2; \frac{5}{2}; x^2, -\frac{x^2}{3}\right)\right) + 9F_1\left(\frac{1}{2}, \frac{1}{3}, 1; \frac{3}{2}; x^2, -\frac{x^2}{3}\right)}{x^2\left(F_1\left(2, \frac{4}{3}, 1; 3; x^2, -\frac{x^2}{3}\right) - F_1\left(2, \frac{1}{3}, 2; 3; x^2, -\frac{x^2}{3}\right)\right) + 6F_1\left(1, \frac{1}{3}, 1; 2; x^2, -\frac{x^2}{3}\right)} \right) \frac{1}{\sqrt[3]{1-x^2}(x^2+3)}$$

Warning: Unable to verify antiderivative.

[In] `Integrate[(a + b*x)/((1 - x^2)^(1/3)*(3 + x^2)), x]`

[Out] $(3*x*((3*a*\operatorname{AppellF1}[1/2, 1/3, 1, 3/2, x^2, -x^2/3])/(9*\operatorname{AppellF1}[1/2, 1/3, 1, 3/2, x^2, -x^2/3] + 2*x^2*(-\operatorname{AppellF1}[3/2, 1/3, 2, 5/2, x^2, -x^2/3] + \operatorname{AppellF1}[3/2, 4/3, 1, 5/2, x^2, -x^2/3])) + (b*x*\operatorname{AppellF1}[1, 1/3, 1, 2, x^2, -x^2/3])/(6*\operatorname{AppellF1}[1, 1/3, 1, 2, x^2, -x^2/3] + x^2*(-\operatorname{AppellF1}[2, 1/3, 2, 3, x^2, -x^2/3] + \operatorname{AppellF1}[2, 4/3, 1, 3, x^2, -x^2/3]))) / ((1 - x^2)^(1/3)*(3 + x^2))$

Maple [F] time = 0.052, size = 0, normalized size = 0.

$$\int \frac{bx+a}{x^2+3} \frac{1}{\sqrt[3]{-x^2+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x+a)/(-x^2+1)^(1/3)/(x^2+3),x)`

[Out] `int((b*x+a)/(-x^2+1)^(1/3)/(x^2+3),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx + a}{(x^2 + 3)(-x^2 + 1)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x + a)/((x^2 + 3)*(-x^2 + 1)^(1/3)),x, algorithm="maxima")`

[Out] `integrate((b*x + a)/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)`

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x + a)/((x^2 + 3)*(-x^2 + 1)^(1/3)),x, algorithm="fricas")`

[Out] Exception raised: TypeError

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + bx}{\sqrt[3]{-(x - 1)(x + 1)(x^2 + 3)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)/(-x**2+1)**(1/3)/(x**2+3),x)`

[Out] `Integral((a + b*x)/((-x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)`

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx + a}{(x^2 + 3)(-x^2 + 1)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x + a)/((x^2 + 3)*(-x^2 + 1)^(1/3)),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

$$3.54 \quad \int \frac{a+bx}{(3-x^2)\sqrt[3]{1+x^2}} dx$$

Optimal. Leaf size=198

$$\frac{a \tan^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{x^2+1}}\right)}{2 \cdot 2^{2/3}} - \frac{a \tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{x^2+1}\right)}{x}\right)}{2 \cdot 2^{2/3}\sqrt{3}} - \frac{a \tan^{-1}(x)}{6 \cdot 2^{2/3}} - \frac{a \tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2 \cdot 2^{2/3}\sqrt{3}}$$

$$+ \frac{b \log(3-x^2)}{4 \cdot 2^{2/3}} - \frac{3b \log\left(2^{2/3}-\sqrt[3]{x^2+1}\right)}{4 \cdot 2^{2/3}} - \frac{\sqrt{3}b \tan^{-1}\left(\frac{\sqrt[3]{2}\sqrt[3]{x^2+1}}{\sqrt{3}}\right)}{2 \cdot 2^{2/3}}$$

[Out] $-(a \cdot \text{ArcTan}[x]) / (6 \cdot 2^{2/3}) + (a \cdot \text{ArcTan}[x / (1 + 2^{1/3}) \cdot (1 + x^2)^{1/3}]) / (2 \cdot 2^{2/3}) - (\text{Sqrt}[3] \cdot b \cdot \text{ArcTan}[(1 + 2^{1/3}) \cdot (1 + x^2)^{1/3}] / \text{Sqrt}[3]) / (2 \cdot 2^{2/3}) - (a \cdot \text{ArcTanh}[\text{Sqrt}[3] / x]) / (2 \cdot 2^{2/3} \cdot \text{Sqrt}[3]) - (a \cdot \text{ArcTanh}[(\text{Sqrt}[3] \cdot (1 - 2^{1/3}) \cdot (1 + x^2)^{1/3})] / x) / (2 \cdot 2^{2/3} \cdot \text{Sqrt}[3]) + (b \cdot \text{Log}[3 - x^2]) / (4 \cdot 2^{2/3}) - (3 \cdot b \cdot \text{Log}[2^{2/3} - (1 + x^2)^{1/3}]) / (4 \cdot 2^{2/3})$

Rubi [A] time = 0.21155, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$

$$\frac{a \tan^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{x^2+1}}\right)}{2 \cdot 2^{2/3}} - \frac{a \tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{x^2+1}\right)}{x}\right)}{2 \cdot 2^{2/3}\sqrt{3}} - \frac{a \tan^{-1}(x)}{6 \cdot 2^{2/3}} - \frac{a \tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2 \cdot 2^{2/3}\sqrt{3}}$$

$$+ \frac{b \log(3-x^2)}{4 \cdot 2^{2/3}} - \frac{3b \log\left(2^{2/3}-\sqrt[3]{x^2+1}\right)}{4 \cdot 2^{2/3}} - \frac{\sqrt{3}b \tan^{-1}\left(\frac{\sqrt[3]{2}\sqrt[3]{x^2+1}}{\sqrt{3}}\right)}{2 \cdot 2^{2/3}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + b \cdot x) / ((3 - x^2) \cdot (1 + x^2)^{1/3}), x]$

[Out] $-(a \cdot \text{ArcTan}[x]) / (6 \cdot 2^{2/3}) + (a \cdot \text{ArcTan}[x / (1 + 2^{1/3}) \cdot (1 + x^2)^{1/3}]) / (2 \cdot 2^{2/3}) - (\text{Sqrt}[3] \cdot b \cdot \text{ArcTan}[(1 + 2^{1/3}) \cdot (1 + x^2)^{1/3}] / \text{Sqrt}[3]) / (2 \cdot 2^{2/3}) - (a \cdot \text{ArcTanh}[\text{Sqrt}[3] / x]) / (2 \cdot 2^{2/3} \cdot \text{Sqrt}[3]) - (a \cdot \text{ArcTanh}[(\text{Sqrt}[3] \cdot (1 - 2^{1/3}) \cdot (1 + x^2)^{1/3})] / x) / (2 \cdot 2^{2/3} \cdot \text{Sqrt}[3]) + (b \cdot \text{Log}[3 - x^2]) / (4 \cdot 2^{2/3}) - (3 \cdot b \cdot \text{Log}[2^{2/3} - (1 + x^2)^{1/3}]) / (4 \cdot 2^{2/3})$

Rubi in Sympy [A] time = 13.1202, size = 282, normalized size = 1.42

$$\begin{aligned} & \frac{\sqrt[3]{2}\sqrt{3}a \log(-x + \sqrt{3})}{24} - \frac{\sqrt[3]{2}\sqrt{3}a \log(x + \sqrt{3})}{24} + \frac{\sqrt[3]{2}\sqrt{3}a \log(-x - \sqrt[3]{2}\sqrt{3}\sqrt[3]{x^2 + 1} + \sqrt{3})}{24} \\ & - \frac{\sqrt[3]{2}\sqrt{3}a \log(x - \sqrt[3]{2}\sqrt{3}\sqrt[3]{x^2 + 1} + \sqrt{3})}{24} - \frac{\sqrt[3]{2}a \operatorname{atan}\left(\frac{2^{\frac{2}{3}}(-x + \sqrt{3})}{3\sqrt[3]{x^2 + 1}} + \frac{\sqrt{3}}{3}\right)}{12} \\ & + \frac{\sqrt[3]{2}a \operatorname{atan}\left(\frac{2^{\frac{2}{3}}(x + \sqrt{3})}{3\sqrt[3]{x^2 + 1}} + \frac{\sqrt{3}}{3}\right)}{12} + \frac{\sqrt[3]{2}b \log(-x^2 + 3)}{8} \\ & - \frac{3\sqrt[3]{2}b \log(-\sqrt[3]{x^2 + 1} + 2^{\frac{2}{3}})}{8} - \frac{\sqrt[3]{2}\sqrt{3}b \operatorname{atan}\left(\sqrt{3}\left(\frac{\sqrt[3]{2}\sqrt[3]{x^2 + 1}}{3} + \frac{1}{3}\right)\right)}{4} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((b*x+a)/(-x**2+3)/(x**2+1)**(1/3), x)`

[Out] $2^{**}(1/3)*\operatorname{sqrt}(3)*a*\log(-x + \operatorname{sqrt}(3))/24 - 2^{**}(1/3)*\operatorname{sqrt}(3)*a*\log(x + \operatorname{sqrt}(3))/24 + 2^{**}(1/3)*\operatorname{sqrt}(3)*a*\log(-x - 2^{**}(1/3)*\operatorname{sqrt}(3)*(x^{**2} + 1)^{(1/3)} + \operatorname{sqrt}(3))/24 - 2^{**}(1/3)*\operatorname{sqrt}(3)*a*\log(x - 2^{**}(1/3)*\operatorname{sqrt}(3)*(x^{**2} + 1)^{(1/3)} + \operatorname{sqrt}(3))/24 - 2^{**}(1/3)*a*\operatorname{atan}(2^{**}(2/3)*(-x + \operatorname{sqrt}(3))/(3*(x^{**2} + 1)^{(1/3)} + \operatorname{sqrt}(3)/3))/12 + 2^{**}(1/3)*a*\operatorname{atan}(2^{**}(2/3)*(x + \operatorname{sqrt}(3))/(3*(x^{**2} + 1)^{(1/3)} + \operatorname{sqrt}(3)/3))/12 + 2^{**}(1/3)*b*\log(-x^{**2} + 3)/8 - 3*2^{**}(1/3)*b*\log(-(x^{**2} + 1)^{(1/3)} + 2^{**}(2/3))/8 - 2^{**}(1/3)*\operatorname{sqrt}(3)*b*\operatorname{atan}(\operatorname{sqrt}(3)*(2^{**}(1/3)*(x^{**2} + 1)^{(1/3)}/3 + 1/3))/4$

Mathematica [C] time = 0.441686, size = 220, normalized size = 1.11

$$3x \left(\frac{3aF_1\left(\frac{1}{2}, \frac{1}{3}, 1; -x^2, \frac{x^2}{3}\right) - bxF_1\left(1; \frac{1}{3}, 1; -x^2, \frac{x^2}{3}\right)}{2x^2\left(F_1\left(\frac{3}{2}, \frac{1}{3}, 2; -x^2, \frac{x^2}{3}\right) - F_1\left(\frac{3}{2}, \frac{4}{3}, 1; -x^2, \frac{x^2}{3}\right)\right) + 9F_1\left(\frac{1}{2}, \frac{1}{3}, 1; -x^2, \frac{x^2}{3}\right) - x^2\left(F_1\left(2; \frac{1}{3}, 2; -x^2, \frac{x^2}{3}\right) - F_1\left(2; \frac{4}{3}, 1; -x^2, \frac{x^2}{3}\right)\right) + 6F_1\left(1; \frac{1}{3}, 1; -x^2, \frac{x^2}{3}\right)}{(x^2 - 3)\sqrt[3]{x^2 + 1}} \right)$$

Warning: Unable to verify antiderivative.

[In] `Integrate[(a + b*x)/((3 - x^2)*(1 + x^2)^(1/3)), x]`

[Out] $(3*x*((-3*a*\operatorname{AppellF1}[1/2, 1/3, 1, 3/2, -x^2, x^2/3])/(9*\operatorname{AppellF1}[1/2, 1/3, 1, 3/2, -x^2, x^2/3] + 2*x^2*(\operatorname{AppellF1}[3/2, 1/3, 2, 5/2, -x^2, x^2/3] - \operatorname{AppellF1}[3/2, 4/3, 1, 5/2, -x^2, x^2/3])) - (b*x*\operatorname{AppellF1}[1, 1/3, 1, 2, -x^2, x^2/3])/(6*\operatorname{AppellF1}[1, 1/3, 1, 2, -x^2, x^2/3] + x^2*(\operatorname{AppellF1}[2, 1/3, 2, 3, -x^2, x^2/3] - \operatorname{AppellF1}[2, 4/3, 1, 3, -x^2, x^2/3]))))/((-3 + x^2)*(1 + x^2)^(1/3))$

Maple [F] time = 0.073, size = 0, normalized size = 0.

$$\int \frac{bx + a}{-x^2 + 3} \frac{1}{\sqrt[3]{x^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)/(-x^2+3)/(x^2+1)^(1/3), x)

[Out] int((b*x+a)/(-x^2+3)/(x^2+1)^(1/3), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$- \int \frac{bx + a}{(x^2 + 1)^{\frac{1}{3}}(x^2 - 3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-(b*x + a)/((x^2 + 1)^(1/3)*(x^2 - 3)), x, algorithm="maxima")

[Out] -integrate((b*x + a)/((x^2 + 1)^(1/3)*(x^2 - 3)), x)

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-(b*x + a)/((x^2 + 1)^(1/3)*(x^2 - 3)), x, algorithm="fricas")

[Out] Exception raised: TypeError

Sympy [F] time = 0., size = 0, normalized size = 0.

$$- \int \frac{a}{x^2 \sqrt[3]{x^2 + 1} - 3 \sqrt[3]{x^2 + 1}} dx - \int \frac{bx}{x^2 \sqrt[3]{x^2 + 1} - 3 \sqrt[3]{x^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)/(-x**2+3)/(x**2+1)**(1/3),x)

[Out] -Integral(a/(x**2*(x**2 + 1)**(1/3) - 3*(x**2 + 1)**(1/3)), x) -
Integral(b*x/(x**2*(x**2 + 1)**(1/3) - 3*(x**2 + 1)**(1/3)), x)

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{bx + a}{(x^2 + 1)^{\frac{1}{3}}(x^2 - 3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-(b*x + a)/((x^2 + 1)^(1/3)*(x^2 - 3)),x, algorithm="giac")

[Out] integrate(-(b*x + a)/((x^2 + 1)^(1/3)*(x^2 - 3)), x)

$$3.55 \quad \int \frac{1}{x \sqrt[3]{4 - 6x + 3x^2}} dx$$

Optimal. Leaf size=88

$$\frac{\log\left(\frac{2\sqrt[3]{2}\sqrt[3]{3x^2 - 6x + 4+2x-4}}{x}\right)}{2 \cdot 2^{2/3}} + \frac{\tan^{-1}\left(\frac{-2\sqrt[3]{2}\sqrt[3]{3x^2 - 6x + 4+x-2}}{\sqrt{3}(x-2)}\right)}{2^{2/3}\sqrt{3}}$$

[Out] ArcTan[(-2 + x - 2*2^(1/3)*(4 - 6*x + 3*x^2)^(1/3))/(Sqrt[3]*(-2 + x))]/(2^(2/3)*Sqrt[3]) + Log[(-4 + 2*x + 2*2^(1/3)*(4 - 6*x + 3*x^2)^(1/3))/x]/(2*2^(2/3))

Rubi [A] time = 0.0472327, antiderivative size = 97, normalized size of antiderivative = 1.1, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$

$$\frac{\log\left(-3\sqrt[3]{2}\sqrt[3]{3x^2 - 6x + 4 - 3x + 6}\right)}{2 \cdot 2^{2/3}} - \frac{\tan^{-1}\left(\frac{2^{2/3}(2-x)}{\sqrt{3}\sqrt[3]{3x^2 - 6x + 4}} + \frac{1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} - \frac{\log(x)}{2 \cdot 2^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(4 - 6*x + 3*x^2)^(1/3)), x]

[Out] -(ArcTan[1/Sqrt[3] + (2^(2/3)*(2 - x))/(Sqrt[3]*(4 - 6*x + 3*x^2)^(1/3))]/(2^(2/3)*Sqrt[3])) - Log[x]/(2*2^(2/3)) + Log[6 - 3*x - 3*2^(1/3)*(4 - 6*x + 3*x^2)^(1/3)]/(2*2^(2/3))

Rubi in Sympy [A] time = 2.41973, size = 94, normalized size = 1.07

$$-\frac{\sqrt[3]{2} \log(x)}{4} + \frac{\sqrt[3]{2} \log\left(-3x - 3\sqrt[3]{2}\sqrt[3]{3x^2 - 6x + 4 + 6}\right)}{4} - \frac{\sqrt[3]{2}\sqrt{3} \operatorname{atan}\left(\frac{2^{\frac{2}{3}}\sqrt{3}(-3x+6)}{9\sqrt[3]{3x^2 - 6x + 4}} + \frac{\sqrt{3}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate(1/x/(3*x**2-6*x+4)**(1/3), x)

[Out] -2**(1/3)*log(x)/4 + 2**(1/3)*log(-3*x - 3*2**(1/3)*(3*x**2 - 6*x + 4)**(1/3) + 6)/4 - 2**(1/3)*sqrt(3)*atan(2**(2/3)*sqrt(3)*(-3*x + 6)/(9*(3*x**2 - 6*x + 4)**(1/3)) + sqrt(3)/3)/6

Mathematica [C] time = 0.810162, size = 273, normalized size = 3.1

$$\frac{15x \left(3x - i\sqrt{3} - 3\right) \left(3x + i\sqrt{3} - 3\right) F_1\left(\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \frac{3-i\sqrt{3}}{3x}, \frac{3+i\sqrt{3}}{3x}\right)}{2(3x^2 - 6x + 4)^{4/3} \left(15xF_1\left(\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \frac{3-i\sqrt{3}}{3x}, \frac{3+i\sqrt{3}}{3x}\right) + (3 + i\sqrt{3}) F_1\left(\frac{5}{3}, \frac{1}{3}, \frac{4}{3}, \frac{8}{3}, \frac{3-i\sqrt{3}}{3x}, \frac{3+i\sqrt{3}}{3x}\right) + (3 - i\sqrt{3}) F_1\left(\frac{5}{3}, \frac{4}{3}, \frac{1}{3}, \frac{8}{3}, \frac{3-i\sqrt{3}}{3x}, \frac{3+i\sqrt{3}}{3x}\right)\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/(x*(4 - 6*x + 3*x^2)^(1/3)), x]

[Out] (-15*x*(-3 - I*Sqrt[3] + 3*x)*(-3 + I*Sqrt[3] + 3*x)*AppellF1[2/3, 1/3, 1/3, 5/3, (3 - I*Sqrt[3])/(3*x), (3 + I*Sqrt[3])/(3*x)]/(2*(4 - 6*x + 3*x^2)^(4/3)*(15*x*AppellF1[2/3, 1/3, 1/3, 5/3, (3 - I*Sqrt[3])/(3*x), (3 + I*Sqrt[3])/(3*x)] + (3 + I*Sqrt[3])*AppellF1[5/3, 1/3, 4/3, 8/3, (3 - I*Sqrt[3])/(3*x), (3 + I*Sqrt[3])/(3*x)] + (3 - I*Sqrt[3])*AppellF1[5/3, 4/3, 1/3, 8/3, (3 - I*Sqrt[3])/(3*x), (3 + I*Sqrt[3])/(3*x)]))

Maple [F] time = 0.199, size = 0, normalized size = 0.

$$\int \frac{1}{x \sqrt[3]{3x^2 - 6x + 4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(3*x^2-6*x+4)^(1/3), x)

[Out] int(1/x/(3*x^2-6*x+4)^(1/3), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3x^2 - 6x + 4)^{\frac{1}{3}} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((3*x^2 - 6*x + 4)^(1/3)*x), x, algorithm="maxima")

[Out] integrate(1/((3*x^2 - 6*x + 4)^(1/3)*x), x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((3*x^2 - 6*x + 4)^(1/3)*x),x, algorithm="fricas")`

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt[3]{3x^2 - 6x + 4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(3*x**2-6*x+4)**(1/3),x)`

[Out] `Integral(1/(x*(3*x**2 - 6*x + 4)**(1/3)), x)`

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3x^2 - 6x + 4)^{\frac{1}{3}}x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((3*x^2 - 6*x + 4)^(1/3)*x),x, algorithm="giac")`

[Out] `integrate(1/((3*x^2 - 6*x + 4)^(1/3)*x), x)`

3.56 $\int x\sqrt[3]{1-x^3} dx$

Optimal. Leaf size=107

$$-\frac{1}{9} \log\left(\frac{x}{\sqrt[3]{1-x^3}} + 1\right) - \frac{\tan^{-1}\left(\frac{1-\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{1}{3}\sqrt[3]{1-x^3}x^2 + \frac{1}{18} \log\left(-\frac{x}{\sqrt[3]{1-x^3}} + \frac{x^2}{(1-x^3)^{2/3}} + 1\right)$$

[Out] $(x^2*(1-x^3)^{(1/3)})/3 - \text{ArcTan}[(1-(2*x)/(1-x^3)^{(1/3)})/\text{Sqrt}[3]]/(3*\text{Sqrt}[3]) + \text{Log}[1+x^2/(1-x^3)^{(2/3)} - x/(1-x^3)^{(1/3)}]/18 - \text{Log}[1+x/(1-x^3)^{(1/3)}]/9$

Rubi [A] time = 0.10215, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$

$$-\frac{1}{9} \log\left(\frac{x}{\sqrt[3]{1-x^3}} + 1\right) - \frac{\tan^{-1}\left(\frac{1-\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{1}{3}\sqrt[3]{1-x^3}x^2 + \frac{1}{18} \log\left(-\frac{x}{\sqrt[3]{1-x^3}} + \frac{x^2}{(1-x^3)^{2/3}} + 1\right)$$

Antiderivative was successfully verified.

[In] Int[x*(1-x^3)^(1/3), x]

[Out] $(x^2*(1-x^3)^{(1/3)})/3 - \text{ArcTan}[(1-(2*x)/(1-x^3)^{(1/3)})/\text{Sqrt}[3]]/(3*\text{Sqrt}[3]) + \text{Log}[1+x^2/(1-x^3)^{(2/3)} - x/(1-x^3)^{(1/3)}]/18 - \text{Log}[1+x/(1-x^3)^{(1/3)}]/9$

Rubi in Sympy [A] time = 4.77537, size = 85, normalized size = 0.79

$$\frac{x^2\sqrt[3]{-x^3+1}}{3} - \frac{\log\left(\frac{x}{\sqrt[3]{-x^3+1}} + 1\right)}{9} + \frac{\log\left(\frac{x^2}{(-x^3+1)^{2/3}} - \frac{x}{\sqrt[3]{-x^3+1}} + 1\right)}{18} + \frac{\sqrt{3} \operatorname{atan}\left(\sqrt{3}\left(\frac{2x}{\sqrt[3]{-x^3+1}} - \frac{1}{3}\right)\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate(x*(-x**3+1)**(1/3), x)

[Out] $x**2*(-x**3+1)**(1/3)/3 - \log(x/(-x**3+1)**(1/3)+1)/9 + \log(x**2/(-x**3+1)**(2/3) - x/(-x**3+1)**(1/3)+1)/18 + \text{sqrt}(3)*\text{atan}(\text{sqrt}(3)*(2*x/(3*(-x**3+1)**(1/3)) - 1/3))/9$

Mathematica [C] time = 0.0183334, size = 34, normalized size = 0.32

$$\frac{1}{6}x^2 \left({}_2F_1\left(\frac{2}{3}, \frac{2}{3}; \frac{5}{3}; x^3\right) + 2\sqrt[3]{1-x^3} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x*(1 - x^3)^(1/3), x]

[Out] (x^2*(2*(1 - x^3)^(1/3) + Hypergeometric2F1[2/3, 2/3, 5/3, x^3]))/6

Maple [C] time = 0.069, size = 69, normalized size = 0.6

$$-\frac{x^2(x^3-1)}{3}(-x^3+1)^{-\frac{2}{3}} + \frac{x^2}{6}(x^3-1)^{\frac{2}{3}}(-\text{signum}(x^3-1))^{\frac{2}{3}}{}_2F_1\left(\frac{2}{3}, \frac{2}{3}; \frac{5}{3}; x^3\right)(\text{signum}(x^3-1))^{-\frac{2}{3}}(-x^3+1)^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(-x^3+1)^(1/3), x)

[Out] -1/3*x^2*(x^3-1)/(-x^3+1)^(2/3)+1/6*(x^3-1)^(2/3)/signum(x^3-1)^(2/3)*(-signum(x^3-1))^(2/3)*x^2*hypergeom([2/3, 2/3], [5/3], x^3)/(-x^3+1)^(2/3)

Maxima [A] time = 1.66624, size = 142, normalized size = 1.33

$$-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2(-x^3+1)^{\frac{1}{3}}}{x}-1\right)\right)-\frac{(-x^3+1)^{\frac{1}{3}}}{3x\left(\frac{x^3-1}{x^3}-1\right)} - \frac{1}{9}\log\left(\frac{(-x^3+1)^{\frac{1}{3}}}{x}+1\right)+\frac{1}{18}\log\left(-\frac{(-x^3+1)^{\frac{1}{3}}}{x}+\frac{(-x^3+1)^{\frac{2}{3}}}{x^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^3 + 1)^(1/3)*x, x, algorithm="maxima")

[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3)/x - 1)) - 1/3*(-x^3 + 1)^(1/3)/(x*((x^3 - 1)/x^3 - 1)) - 1/9*log((-x^3 + 1)^(1/3) + 1) + 1/18*log(-(-x^3 + 1)^(1/3)/x + (-x^3 + 1)^(2/3)/x^2 + 1)

$$\frac{1}{3})/x + 1) + 1/18 * \log(-(-x^3 + 1)^{1/3}/x + (-x^3 + 1)^{2/3}/x^2 + 1)$$

Fricas [A] time = 0.209585, size = 143, normalized size = 1.34

$$\frac{1}{54} \sqrt{3} \left(6 \sqrt{3} (-x^3 + 1)^{\frac{1}{3}} x^2 - 2 \sqrt{3} \log \left(\frac{x + (-x^3 + 1)^{\frac{1}{3}}}{x} \right) + \sqrt{3} \log \left(\frac{x^2 - (-x^3 + 1)^{\frac{1}{3}} x + (-x^3 + 1)^{\frac{2}{3}}}{x^2} \right) - 6 \arctan \left(-\frac{\sqrt{3}x - 2}{x} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^3 + 1)^(1/3)*x,x, algorithm="fricas")

[Out] 1/54*sqrt(3)*(6*sqrt(3)*(-x^3 + 1)^(1/3)*x^2 - 2*sqrt(3)*log((x + (-x^3 + 1)^(1/3))/x) + sqrt(3)*log((x^2 - (-x^3 + 1)^(1/3)*x + (-x^3 + 1)^(2/3))/x^2) - 6*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(-x^3 + 1)^(1/3))/x))

Sympy [A] time = 1.78304, size = 32, normalized size = 0.3

$$\frac{x^2 \left(\frac{2}{3} \right) {}_2F_1 \left(\begin{matrix} -\frac{1}{3}, \frac{2}{3} \\ \frac{5}{3} \end{matrix} \middle| x^3 e^{2i\pi} \right)}{3 \left(\frac{5}{3} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-x**3+1)**(1/3),x)

[Out] x**2*gamma(2/3)*hyper((-1/3, 2/3), (5/3,), x**3*exp_polar(2*I*pi))/(3*gamma(5/3))

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int (-x^3 + 1)^{\frac{1}{3}} x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^3 + 1)^(1/3)*x,x, algorithm="giac")

[Out] integrate((-x^3 + 1)^(1/3)*x, x)

$$3.57 \quad \int \frac{\sqrt[3]{1-x^3}}{x} dx$$

Optimal. Leaf size=67

$$\sqrt[3]{1-x^3} + \frac{1}{2} \log\left(1 - \sqrt[3]{1-x^3}\right) - \frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3+1}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\log(x)}{2}$$

[Out] $(1 - x^3)^{1/3} - \text{ArcTan}[(1 + 2*(1 - x^3)^{1/3})/\text{Sqrt}[3]]/\text{Sqrt}[3]$
 $- \text{Log}[x]/2 + \text{Log}[1 - (1 - x^3)^{1/3}]/2$

Rubi [A] time = 0.0840064, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$

$$\sqrt[3]{1-x^3} + \frac{1}{2} \log\left(1 - \sqrt[3]{1-x^3}\right) - \frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3+1}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\log(x)}{2}$$

Antiderivative was successfully verified.

[In] Int[(1 - x^3)^(1/3)/x, x]

[Out] $(1 - x^3)^{1/3} - \text{ArcTan}[(1 + 2*(1 - x^3)^{1/3})/\text{Sqrt}[3]]/\text{Sqrt}[3]$
 $- \text{Log}[x]/2 + \text{Log}[1 - (1 - x^3)^{1/3}]/2$

Rubi in Sympy [A] time = 2.76762, size = 56, normalized size = 0.84

$$\sqrt[3]{-x^3+1} - \frac{\log(x^3)}{6} + \frac{\log(-\sqrt[3]{-x^3+1}+1)}{2} - \frac{\sqrt{3} \operatorname{atan}\left(\sqrt{3}\left(\frac{2\sqrt[3]{-x^3+1}}{3} + \frac{1}{3}\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate((-x**3+1)**(1/3)/x, x)

[Out] $(-x^3 + 1)^{1/3} - \log(x^3)/6 + \log(-(-x^3 + 1)^{1/3} + 1)/2$
 $- \text{sqrt}(3)*\operatorname{atan}(\text{sqrt}(3)*(2*(-x^3 + 1)^{1/3}/3 + 1/3))/3$

Mathematica [C] time = 0.0246976, size = 48, normalized size = 0.72

$$\frac{-\left(1 - \frac{1}{x^3}\right)^{2/3} {}_2F_1\left(\frac{2}{3}, \frac{2}{3}; \frac{5}{3}; \frac{1}{x^3}\right) - 2x^3 + 2}{2(1 - x^3)^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^3)^(1/3)/x, x]

[Out] (2 - 2*x^3 - (1 - x^(-3))^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, x^(-3)])/(2*(1 - x^3)^(2/3))

Maple [C] time = 0.073, size = 49, normalized size = 0.7

$$-\frac{1}{9(2/3)} \left(-3 \left(3 + 1/6 \pi \sqrt{3} - 3/2 \ln(3) + 3 \ln(x) + i\pi \right) (2/3) + \left(\frac{2}{3} \right) x^3 {}_3F_2\left(\frac{2}{3}, 1, 1; 2, 2; x^3\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^3+1)^(1/3)/x, x)

[Out] -1/9/GAMMA(2/3)*(-3*(3+1/6*Pi*3^(1/2)-3/2*ln(3)+3*ln(x)+I*Pi)*GAMMA(2/3)+GAMMA(2/3)*x^3*hypergeom([2/3, 1, 1], [2, 2], x^3))

Maxima [A] time = 1.53367, size = 96, normalized size = 1.43

$$\begin{aligned} &-\frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} \left(2(-x^3 + 1)^{\frac{1}{3}} + 1\right)\right) + (-x^3 + 1)^{\frac{1}{3}} \\ &-\frac{1}{6} \log\left((-x^3 + 1)^{\frac{2}{3}} + (-x^3 + 1)^{\frac{1}{3}} + 1\right) + \frac{1}{3} \log\left((-x^3 + 1)^{\frac{1}{3}} - 1\right) \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^3 + 1)^(1/3)/x, x, algorithm="maxima")

[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) + (-x^3 + 1)^(1/3) - 1/6*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log((-x^3 + 1)^(1/3) - 1)

Fricas [A] time = 0.208472, size = 115, normalized size = 1.72

$$-\frac{1}{18} \sqrt{3} \left(\sqrt{3} \log\left((-x^3 + 1)^{\frac{2}{3}} + (-x^3 + 1)^{\frac{1}{3}} + 1\right) - 2 \sqrt{3} \log\left((-x^3 + 1)^{\frac{1}{3}} - 1\right) - 6 \sqrt{3} (-x^3 + 1)^{\frac{1}{3}} + 6 \arctan\left(\frac{2}{3} \sqrt{3} (-x^3 + 1)^{\frac{1}{3}}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^3 + 1)^(1/3)/x,x, algorithm="fricas")

[Out] $-1/18*\sqrt{3}*(\sqrt{3}*\log((-x^3 + 1)^{(2/3)} + (-x^3 + 1)^{(1/3)} + 1) - 2*\sqrt{3}*\log((-x^3 + 1)^{(1/3)} - 1) - 6*\sqrt{3}*(-x^3 + 1)^{(1/3)} + 6*\arctan(2/3*\sqrt{3}*(-x^3 + 1)^{(1/3)} + 1/3*\sqrt{3}))$

Sympy [A] time = 1.67805, size = 37, normalized size = 0.55

$$-\frac{x e^{\frac{i\pi}{3}} \left(-\frac{1}{3}\right) {}_2F_1\left(-\frac{1}{3}, -\frac{1}{3} \middle| \frac{1}{x^3}\right)}{3 \left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**3+1)**(1/3)/x,x)

[Out] $-x*\exp(I*\pi/3)*\gamma(-1/3)*\text{hyper}((-1/3, -1/3), (2/3,), x**(-3))/(3*\gamma(2/3))$

GIAC/XCAS [A] time = 0.24, size = 97, normalized size = 1.45

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2(-x^3+1)^{\frac{1}{3}}+1\right)\right)+(-x^3+1)^{\frac{1}{3}}-\frac{1}{6}\ln\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\ln\left(\left|(-x^3+1)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^3 + 1)^(1/3)/x,x, algorithm="giac")

[Out] $-1/3*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*(-x^3 + 1)^{(1/3)} + 1)) + (-x^3 + 1)^{(1/3)} - 1/6*\ln((-x^3 + 1)^{(2/3)} + (-x^3 + 1)^{(1/3)} + 1) + 1/3*\ln(\text{abs}((-x^3 + 1)^{(1/3)} - 1))$

$$3.58 \quad \int \frac{\sqrt[3]{1-x^3}}{1+x} dx$$

Optimal. Leaf size=1

0

[Out] 0

Rubi [F] time = 0.0989775, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0$.

$$\text{Int}\left(\frac{\sqrt[3]{1-x^3}}{1+x}, x\right)$$

Verification is Not applicable to the result.

[In] Int[(1 - x^3)^(1/3)/(1 + x), x]

[Out] Defer[Int][(1 - x^3)^(1/3)/(1 + x), x]

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{-x^3+1}}{x+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate((-x**3+1)**(1/3)/(1+x), x)

[Out] Integral((-x**3 + 1)**(1/3)/(x + 1), x)

Mathematica [A] time = 0.371783, size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{1-x^3}}{1+x} dx$$

Verification is Not applicable to the result.

[In] Integrate[(1 - x^3)^(1/3)/(1 + x), x]

[Out] Integrate[(1 - x^3)^(1/3)/(1 + x), x]

Maple [F] time = 0.072, size = 0, normalized size = 0.

$$\int \frac{1}{1+x} \sqrt[3]{-x^3+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^3+1)^(1/3)/(1+x), x)

[Out] int((-x^3+1)^(1/3)/(1+x), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-x^3+1)^{\frac{1}{3}}}{x+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^3 + 1)^(1/3)/(x + 1), x, algorithm="maxima")

[Out] integrate((-x^3 + 1)^(1/3)/(x + 1), x)

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^3 + 1)^(1/3)/(x + 1), x, algorithm="fricas")

[Out] Exception raised: TypeError

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{-(x-1)(x^2+x+1)}}{x+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x**3+1)**(1/3)/(1+x),x)`

[Out] `Integral((-x - 1)*(x**2 + x + 1)**(1/3)/(x + 1), x)`

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-x^3 + 1)^{\frac{1}{3}}}{x + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x^3 + 1)^(1/3)/(x + 1),x, algorithm="giac")`

[Out] `integrate((-x^3 + 1)^(1/3)/(x + 1), x)`

$$3.59 \quad \int \frac{\sqrt[3]{1-x^3}}{1-x+x^2} dx$$

Optimal. Leaf size=280

$$\begin{aligned} & \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2^{2/3}} + \frac{3 \log\left(\sqrt[3]{1-x^3}-\sqrt[3]{2}(x-1)\right)}{2^{2/3}} \\ & + \frac{1}{2} \log\left(\sqrt[3]{1-x^3}+x\right) - \frac{\log\left(\sqrt[3]{1-x^3}+\sqrt[3]{2}x\right)}{2^{2/3}} + \frac{\sqrt{3} \tan^{-1}\left(\frac{\sqrt[3]{2}\sqrt[3]{2(x-1)+1}}{\sqrt[3]{1-x^3}}\right)}{2^{2/3}} + \frac{\tan^{-1}\left(\frac{1-\sqrt[3]{2x}}{\sqrt[3]{1-x^3}}\right)}{\sqrt{3}} \\ & - \frac{\tan^{-1}\left(\frac{1-\sqrt[3]{2x}}{\sqrt[3]{1-x^3}}\right)}{2^{2/3}\sqrt{3}} - \frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3+1}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} - \frac{\log(-3(x-1)(x^2-x+1))}{2^{2/3}} \end{aligned}$$

[Out] (Sqrt[3]*ArcTan[(1 + (2*2^(1/3))*(-1 + x))/(1 - x^3)^(1/3)]/Sqrt[3])/2^(2/3) + ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]) - ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]) - Log[-3*(-1 + x)*(1 - x + x^2)]/(2*2^(2/3)) + Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(2/3)) + (3*Log[-(2^(1/3))*(-1 + x) + (1 - x^3)^(1/3)])/(2*2^(2/3)) + Log[x + (1 - x^3)^(1/3)]/2 - Log[2^(1/3)*x + (1 - x^3)^(1/3)]/(2*2^(2/3))

Rubi [F] time = 0.385211, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0$.

$$\text{Int}\left(\frac{\sqrt[3]{1-x^3}}{1-x+x^2}, x\right)$$

Verification is Not applicable to the result.

[In] Int[(1 - x^3)^(1/3)/(1 - x + x^2), x]

[Out] ((2*I)*Defer[Int][(1 - x^3)^(1/3)/(1 + I*Sqrt[3] - 2*x), x])/Sqrt[3] + ((2*I)*Defer[Int][(1 - x^3)^(1/3)/(-1 + I*Sqrt[3] + 2*x), x])/Sqrt[3]

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2\sqrt{3}i \int \frac{\sqrt[3]{-x^3+1}}{2x-1-\sqrt{3}i} dx}{3} + \frac{2\sqrt{3}i \int \frac{\sqrt[3]{-x^3+1}}{2x-1+\sqrt{3}i} dx}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((-x**3+1)**(1/3)/(x**2-x+1),x)`

[Out] `-2*sqrt(3)*I*Integral((-x**3 + 1)**(1/3)/(2*x - 1 - sqrt(3)*I), x)/3 + 2*sqrt(3)*I*Integral((-x**3 + 1)**(1/3)/(2*x - 1 + sqrt(3)*I), x)/3`

Mathematica [A] time = 0.0416973, size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{1-x^3}}{1-x+x^2} dx$$

Verification is Not applicable to the result.

[In] `Integrate[(1 - x^3)^(1/3)/(1 - x + x^2),x]`

[Out] `Integrate[(1 - x^3)^(1/3)/(1 - x + x^2), x]`

Maple [F] time = 0.112, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 - x + 1} \sqrt[3]{-x^3 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-x^3+1)^(1/3)/(x^2-x+1),x)`

[Out] `int((-x^3+1)^(1/3)/(x^2-x+1),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-x^3 + 1)^{\frac{1}{3}}}{x^2 - x + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x^3 + 1)^(1/3)/(x^2 - x + 1),x, algorithm="maxima")`

[Out] `integrate((-x^3 + 1)^(1/3)/(x^2 - x + 1), x)`

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x^3 + 1)^(1/3)/(x^2 - x + 1), x, algorithm="fricas")`

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{-(x-1)(x^2+x+1)}}{x^2-x+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x**3+1)**(1/3)/(x**2-x+1), x)`

[Out] `Integral((-x - 1)*(x**2 + x + 1))**(1/3)/(x**2 - x + 1), x)`

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-x^3 + 1)^{\frac{1}{3}}}{x^2 - x + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x^3 + 1)^(1/3)/(x^2 - x + 1), x, algorithm="giac")`

[Out] `integrate((-x^3 + 1)^(1/3)/(x^2 - x + 1), x)`

$$3.60 \quad \int \frac{3-3x+30x^2+160x^3}{9+24x-12x^2+80x^3+320x^4} dx$$

Optimal. Leaf size=25

$$\frac{1}{8} \log(320x^4 + 80x^3 - 12x^2 + 24x + 9)$$

[Out] Log[9 + 24*x - 12*x^2 + 80*x^3 + 320*x^4]/8

Rubi [A] time = 0.0121434, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$

$$\frac{1}{8} \log(320x^4 + 80x^3 - 12x^2 + 24x + 9)$$

Antiderivative was successfully verified.

[In] Int[(3 - 3*x + 30*x^2 + 160*x^3)/(9 + 24*x - 12*x^2 + 80*x^3 + 320*x^4), x]

[Out] Log[9 + 24*x - 12*x^2 + 80*x^3 + 320*x^4]/8

Rubi in Sympy [A] time = 3.60071, size = 22, normalized size = 0.88

$$\frac{\log(320x^4 + 80x^3 - 12x^2 + 24x + 9)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate((160*x**3+30*x**2-3*x+3)/(320*x**4+80*x**3-12*x**2+24*x+9), x)

[Out] log(320*x**4 + 80*x**3 - 12*x**2 + 24*x + 9)/8

Mathematica [A] time = 0.0117437, size = 25, normalized size = 1.

$$\frac{1}{8} \log(320x^4 + 80x^3 - 12x^2 + 24x + 9)$$

Antiderivative was successfully verified.

[In] Integrate[(3 - 3*x + 30*x^2 + 160*x^3)/(9 + 24*x - 12*x^2 + 80*x^3 + 320*x^4), x]

[Out] $\text{Log}[9 + 24*x - 12*x^2 + 80*x^3 + 320*x^4]/8$

Maple [A] time = 0.001, size = 24, normalized size = 1.

$$\frac{\ln(320x^4 + 80x^3 - 12x^2 + 24x + 9)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((160*x^3+30*x^2-3*x+3)/(320*x^4+80*x^3-12*x^2+24*x+9), x)$

[Out] $1/8*\ln(320*x^4+80*x^3-12*x^2+24*x+9)$

Maxima [A] time = 1.38999, size = 31, normalized size = 1.24

$$\frac{1}{8} \log(320x^4 + 80x^3 - 12x^2 + 24x + 9)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((160*x^3 + 30*x^2 - 3*x + 3)/(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9), x, \text{all})$

[Out] $1/8*\log(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9)$

Fricas [A] time = 0.194498, size = 31, normalized size = 1.24

$$\frac{1}{8} \log(320x^4 + 80x^3 - 12x^2 + 24x + 9)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((160*x^3 + 30*x^2 - 3*x + 3)/(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9), x, \text{all})$

[Out] $1/8*\log(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9)$

Sympy [A] time = 0.107169, size = 22, normalized size = 0.88

$$\frac{\log(320x^4 + 80x^3 - 12x^2 + 24x + 9)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((160*x**3+30*x**2-3*x+3)/(320*x**4+80*x**3-12*x**2+24*x+9),x)`

[Out] `log(320*x**4 + 80*x**3 - 12*x**2 + 24*x + 9)/8`

GIAC/XCAS [A] time = 0.244751, size = 31, normalized size = 1.24

$$\frac{1}{8} \ln(320x^4 + 80x^3 - 12x^2 + 24x + 9)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((160*x^3 + 30*x^2 - 3*x + 3)/(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9),x, al`

[Out] `1/8*ln(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9)`

$$3.61 \quad \int \frac{3+12x+20x^2}{9+24x-12x^2+80x^3+320x^4} dx$$

Optimal. Leaf size=59

$$\frac{\tan^{-1}\left(\frac{800x^3-40x^2+30x+57}{6\sqrt{11}}\right)}{2\sqrt{11}} - \frac{\tan^{-1}\left(\frac{7-40x}{5\sqrt{11}}\right)}{2\sqrt{11}}$$

[Out] -ArcTan[(7 - 40*x)/(5*Sqrt[11])]/(2*Sqrt[11]) + ArcTan[(57 + 30*x - 40*x^2 + 800*x^3)/(6*Sqrt[11])]/(2*Sqrt[11])

Rubi [A] time = 0.0595764, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.03$

$$\frac{\tan^{-1}\left(\frac{800x^3-40x^2+30x+57}{6\sqrt{11}}\right)}{2\sqrt{11}} - \frac{\tan^{-1}\left(\frac{7-40x}{5\sqrt{11}}\right)}{2\sqrt{11}}$$

Antiderivative was successfully verified.

[In] Int[(3 + 12*x + 20*x^2)/(9 + 24*x - 12*x^2 + 80*x^3 + 320*x^4), x]

[Out] -ArcTan[(7 - 40*x)/(5*Sqrt[11])]/(2*Sqrt[11]) + ArcTan[(57 + 30*x - 40*x^2 + 800*x^3)/(6*Sqrt[11])]/(2*Sqrt[11])

Rubi in Sympy [A] time = 7.63274, size = 56, normalized size = 0.95

$$\frac{\sqrt{11} \operatorname{atan}\left(\sqrt{11}\left(\frac{8x}{11} - \frac{7}{55}\right)\right)}{22} + \frac{\sqrt{11} \operatorname{atan}\left(\sqrt{11}\left(\frac{400x^3}{33} - \frac{20x^2}{33} + \frac{5x}{11} + \frac{19}{22}\right)\right)}{22}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate((20*x**2+12*x+3)/(320*x**4+80*x**3-12*x**2+24*x+9), x)

[Out] sqrt(11)*atan(sqrt(11)*(8*x/11 - 7/55))/22 + sqrt(11)*atan(sqrt(11)*(400*x**3/33 - 20*x**2/33 + 5*x/11 + 19/22))/22

Mathematica [C] time = 0.0273115, size = 86, normalized size = 1.46

$$\frac{1}{8} \operatorname{RootSum}\left[320\#1^4 + 80\#1^3 - 12\#1^2 + 24\#1 + 9\&, \frac{20\#1^2 \log(x - \#1) + 12\#1 \log(x - \#1) + 3 \log(x - \#1)}{160\#1^3 + 30\#1^2 - 3\#1 + 3}\&\right]$$

Antiderivative was successfully verified.

[In] Integrate[(3 + 12*x + 20*x^2)/(9 + 24*x - 12*x^2 + 80*x^3 + 320*x^4), x]

[Out] RootSum[9 + 24*#1 - 12*#1^2 + 80*#1^3 + 320*#1^4 & , (3*Log[x - #1] + 12*Log[x - #1]*#1 + 20*Log[x - #1]*#1^2)/(3 - 3*#1 + 30*#1^2 + 160*#1^3) &]/8

Maple [A] time = 0.043, size = 52, normalized size = 0.9

$$\frac{\sqrt{11}}{22} \arctan\left(\frac{(40x - 7)\sqrt{11}}{55}\right) + \frac{\sqrt{11}}{22} \arctan\left(-\frac{20\sqrt{11}x^2}{33} + \frac{5\sqrt{11}x}{11} + \frac{19\sqrt{11}}{22} + \frac{400\sqrt{11}x^3}{33}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((20*x^2+12*x+3)/(320*x^4+80*x^3-12*x^2+24*x+9), x)

[Out] 1/22*11^(1/2)*arctan(1/55*(40*x-7)*11^(1/2))+1/22*11^(1/2)*arctan((-20/33*11^(1/2)*x^2+5/11*11^(1/2)*x+19/22*11^(1/2)+400/33*11^(1/2)*x^3)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{20x^2 + 12x + 3}{320x^4 + 80x^3 - 12x^2 + 24x + 9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((20*x^2 + 12*x + 3)/(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9), x, algorithm=

[Out] integrate((20*x^2 + 12*x + 3)/(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9), x)

Fricas [A] time = 0.195492, size = 51, normalized size = 0.86

$$\frac{1}{22} \sqrt{11} \left(\arctan\left(\frac{1}{66} \sqrt{11}(800x^3 - 40x^2 + 30x + 57)\right) + \arctan\left(\frac{1}{55} \sqrt{11}(40x - 7)\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((20*x^2 + 12*x + 3)/(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9), x, algorithm=

[Out] $1/22*\sqrt{11}*(\arctan(1/66*\sqrt{11}*(800*x^3 - 40*x^2 + 30*x + 57)) + \arctan(1/55*\sqrt{11}*(40*x - 7)))$

Sympy [A] time = 0.212998, size = 73, normalized size = 1.24

$$\frac{\sqrt{11} \left(2 \operatorname{atan} \left(\frac{8\sqrt{11}x}{11} - \frac{7\sqrt{11}}{55} \right) + 2 \operatorname{atan} \left(\frac{400\sqrt{11}x^3}{33} - \frac{20\sqrt{11}x^2}{33} + \frac{5\sqrt{11}x}{11} + \frac{19\sqrt{11}}{22} \right) \right)}{44}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((20*x**2+12*x+3)/(320*x**4+80*x**3-12*x**2+24*x+9), x)`

[Out] $\sqrt{11}*(2*\operatorname{atan}(8*\sqrt{11}*x/11 - 7*\sqrt{11}/55) + 2*\operatorname{atan}(400*\sqrt{11}*x**3/33 - 20*\sqrt{11}*x**2/33 + 5*\sqrt{11}*x/11 + 19*\sqrt{11}/22))/44$

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{20x^2 + 12x + 3}{320x^4 + 80x^3 - 12x^2 + 24x + 9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((20*x^2 + 12*x + 3)/(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9), x, algorithm=`

[Out] `integrate((20*x^2 + 12*x + 3)/(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9), x)`

$$3.62 \quad \int -\frac{84+576x+400x^2-2560x^3}{9+24x-12x^2+80x^3+320x^4} dx$$

Optimal. Leaf size=78

$$-2\sqrt{11} \tan^{-1} \left(\frac{800x^3 - 40x^2 + 30x + 57}{6\sqrt{11}} \right) + 2 \log(320x^4 + 80x^3 - 12x^2 + 24x + 9) + 2\sqrt{11} \tan^{-1} \left(\frac{7 - 40x}{5\sqrt{11}} \right)$$

[Out] 2*Sqrt[11]*ArcTan[(7 - 40*x)/(5*Sqrt[11])] - 2*Sqrt[11]*ArcTan[(5
7 + 30*x - 40*x^2 + 800*x^3)/(6*Sqrt[11])] + 2*Log[9 + 24*x - 12*
x^2 + 80*x^3 + 320*x^4]

Rubi [A] time = 0.124319, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.051$

$$-2\sqrt{11} \tan^{-1} \left(\frac{800x^3 - 40x^2 + 30x + 57}{6\sqrt{11}} \right) + 2 \log(320x^4 + 80x^3 - 12x^2 + 24x + 9) + 2\sqrt{11} \tan^{-1} \left(\frac{7 - 40x}{5\sqrt{11}} \right)$$

Antiderivative was successfully verified.

[In] Int[-((84 + 576*x + 400*x^2 - 2560*x^3)/(9 + 24*x - 12*x^2 + 80*x^3 + 320*x^4)), x]

[Out] 2*Sqrt[11]*ArcTan[(7 - 40*x)/(5*Sqrt[11])] - 2*Sqrt[11]*ArcTan[(5
7 + 30*x - 40*x^2 + 800*x^3)/(6*Sqrt[11])] + 2*Log[9 + 24*x - 12*
x^2 + 80*x^3 + 320*x^4]

Rubi in Sympy [A] time = 14.7712, size = 80, normalized size = 1.03

$$2 \log(320x^4 + 80x^3 - 12x^2 + 24x + 9) - 2\sqrt{11} \operatorname{atan} \left(\sqrt{11} \left(\frac{8x}{11} - \frac{7}{55} \right) \right) \\ - 2\sqrt{11} \operatorname{atan} \left(\sqrt{11} \left(\frac{400x^3}{33} - \frac{20x^2}{33} + \frac{5x}{11} + \frac{19}{22} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate((2560*x**3-400*x**2-576*x-84)/(320*x**4+80*x**3-12*x**2+24*x+9))

[Out] 2*log(320*x**4 + 80*x**3 - 12*x**2 + 24*x + 9) - 2*sqrt(11)*atan(
sqrt(11)*(8*x/11 - 7/55)) - 2*sqrt(11)*atan(sqrt(11)*(400*x**3/33
- 20*x**2/33 + 5*x/11 + 19/22))

Mathematica [C] time = 0.0294842, size = 99, normalized size = 1.27

$$\frac{1}{2} \text{RootSum} \left[320\#1^4 + 80\#1^3 - 12\#1^2 + 24\#1 \right. \\ \left. + 9\&, \frac{640\#1^3 \log(x - \#1) - 100\#1^2 \log(x - \#1) - 144\#1 \log(x - \#1) - 21 \log(x - \#1)}{160\#1^3 + 30\#1^2 - 3\#1 + 3} \& \right]$$

Antiderivative was successfully verified.

[In] Integrate[-((84 + 576*x + 400*x^2 - 2560*x^3)/(9 + 24*x - 12*x^2 + 80*x^3 + 320*x^4

[Out] RootSum[9 + 24*#1 - 12*#1^2 + 80*#1^3 + 320*#1^4 & , (-21*Log[x - #1] - 144*Log[x - #1]*#1 - 100*Log[x - #1]*#1^2 + 640*Log[x - #1]*#1^3)/(3 - 3*#1 + 30*#1^2 + 160*#1^3) &]/2

Maple [A] time = 0.033, size = 75, normalized size = 1.

$$2 \ln(6400x^4 + 1600x^3 - 240x^2 + 480x + 180) \\ - 2\sqrt{11} \arctan\left(-\frac{20\sqrt{11}x^2}{33} + \frac{5\sqrt{11}x}{11} + \frac{19\sqrt{11}}{22} + \frac{400\sqrt{11}x^3}{33}\right) - 2\sqrt{11} \arctan\left(\frac{(40x - 7)\sqrt{11}}{55}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2560*x^3-400*x^2-576*x-84)/(320*x^4+80*x^3-12*x^2+24*x+9), x)

[Out] 2*ln(6400*x^4+1600*x^3-240*x^2+480*x+180)-2*11^(1/2)*arctan(-20/33*11^(1/2)*x^2+5/11*11^(1/2)*x+19/22*11^(1/2)+400/33*11^(1/2)*x^3)-2*11^(1/2)*arctan(1/55*(40*x-7)*11^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$4 \int \frac{640x^3 - 100x^2 - 144x - 21}{320x^4 + 80x^3 - 12x^2 + 24x + 9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(4*(640*x^3 - 100*x^2 - 144*x - 21)/(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9), x)

[Out] 4*integrate((640*x^3 - 100*x^2 - 144*x - 21)/(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9), x)

Fricas [A] time = 0.198484, size = 89, normalized size = 1.14

$$-2\sqrt{11}\arctan\left(\frac{1}{66}\sqrt{11}(800x^3 - 40x^2 + 30x + 57)\right) \\ - 2\sqrt{11}\arctan\left(\frac{1}{55}\sqrt{11}(40x - 7)\right) + 2\log(320x^4 + 80x^3 - 12x^2 + 24x + 9)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(4*(640*x^3 - 100*x^2 - 144*x - 21)/(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9), x)

[Out] -2*sqrt(11)*arctan(1/66*sqrt(11)*(800*x^3 - 40*x^2 + 30*x + 57)) - 2*sqrt(11)*arctan(1/55*sqrt(11)*(40*x - 7)) + 2*log(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9)

Sympy [A] time = 0.218207, size = 100, normalized size = 1.28

$$\sqrt{11}\left(-2\operatorname{atan}\left(\frac{8\sqrt{11}x}{11} - \frac{7\sqrt{11}}{55}\right) - 2\operatorname{atan}\left(\frac{400\sqrt{11}x^3}{33} - \frac{20\sqrt{11}x^2}{33} + \frac{5\sqrt{11}x}{11} + \frac{19\sqrt{11}}{22}\right)\right) \\ + 2\log\left(x^4 + \frac{x^3}{4} - \frac{3x^2}{80} + \frac{3x}{40} + \frac{9}{320}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2560*x**3-400*x**2-576*x-84)/(320*x**4+80*x**3-12*x**2+24*x+9), x)

[Out] sqrt(11)*(-2*atan(8*sqrt(11)*x/11 - 7*sqrt(11)/55) - 2*atan(400*sqrt(11)*x**3/33 - 20*sqrt(11)*x**2/33 + 5*sqrt(11)*x/11 + 19*sqrt(11)/22)) + 2*log(x**4 + x**3/4 - 3*x**2/80 + 3*x/40 + 9/320)

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{4(640x^3 - 100x^2 - 144x - 21)}{320x^4 + 80x^3 - 12x^2 + 24x + 9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(4*(640*x^3 - 100*x^2 - 144*x - 21)/(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9), x)

[Out] integrate(4*(640*x^3 - 100*x^2 - 144*x - 21)/(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9), x)

$$3.63 \quad \int \frac{\sqrt{1-x^4}}{1+x^4} dx$$

Optimal. Leaf size=49

$$\frac{1}{2} \tan^{-1} \left(\frac{x(x^2+1)}{\sqrt{1-x^4}} \right) + \frac{1}{2} \tanh^{-1} \left(\frac{x(1-x^2)}{\sqrt{1-x^4}} \right)$$

[Out] ArcTan[(x*(1+x^2))/Sqrt[1-x^4]]/2 + ArcTanh[(x*(1-x^2))/Sqrt[1-x^4]]/2

Rubi [A] time = 0.0261983, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$

$$\frac{1}{2} \tan^{-1} \left(\frac{x(x^2+1)}{\sqrt{1-x^4}} \right) + \frac{1}{2} \tanh^{-1} \left(\frac{x(1-x^2)}{\sqrt{1-x^4}} \right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1-x^4]/(1+x^4),x]

[Out] ArcTan[(x*(1+x^2))/Sqrt[1-x^4]]/2 + ArcTanh[(x*(1-x^2))/Sqrt[1-x^4]]/2

Rubi in Sympy [A] time = 2.40816, size = 36, normalized size = 0.73

$$\frac{\operatorname{atan} \left(\frac{x(x^2+1)}{\sqrt{-x^4+1}} \right)}{2} + \frac{\operatorname{atanh} \left(\frac{x(-x^2+1)}{\sqrt{-x^4+1}} \right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate((-x**4+1)**(1/2)/(x**4+1),x)

[Out] atan(x*(x**2+1)/sqrt(-x**4+1))/2 + atanh(x*(-x**2+1)/sqrt(-x**4+1))/2

Mathematica [C] time = 0.144058, size = 110, normalized size = 2.24

$$\frac{5x\sqrt{1-x^4}F_1\left(\frac{1}{4}; -\frac{1}{2}, 1; \frac{5}{4}; x^4, -x^4\right)}{(x^4+1)\left(2x^4\left(2F_1\left(\frac{5}{4}; -\frac{1}{2}, 2; \frac{9}{4}; x^4, -x^4\right) + F_1\left(\frac{5}{4}; \frac{1}{2}, 1; \frac{9}{4}; x^4, -x^4\right)\right) - 5F_1\left(\frac{1}{4}; -\frac{1}{2}, 1; \frac{5}{4}; x^4, -x^4\right)\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sqrt[1 - x^4]/(1 + x^4),x]

[Out] $(-5*x*\text{Sqrt}[1 - x^4]*\text{AppellF1}[1/4, -1/2, 1, 5/4, x^4, -x^4])/((1 + x^4)*(-5*\text{AppellF1}[1/4, -1/2, 1, 5/4, x^4, -x^4] + 2*x^4*(2*\text{AppellF1}[5/4, -1/2, 2, 9/4, x^4, -x^4] + \text{AppellF1}[5/4, 1/2, 1, 9/4, x^4, -x^4])))$

Maple [B] time = 0.022, size = 100, normalized size = 2.

$$-\frac{1}{4} \arctan\left(\frac{1}{x}\sqrt{-x^4+1}+1\right) + \frac{1}{4} \arctan\left(-\frac{1}{x}\sqrt{-x^4+1}+1\right) - \frac{1}{8} \ln\left(1\left(\frac{-x^4+1}{2x^2} - \frac{1}{x}\sqrt{-x^4+1}+1\right)\left(\frac{-x^4+1}{2x^2} + \frac{1}{x}\sqrt{-x^4+1}+1\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^4+1)^(1/2)/(x^4+1),x)

[Out] $-1/4*\arctan((-x^4+1)^(1/2)/x+1)+1/4*\arctan(-(-x^4+1)^(1/2)/x+1)-1/8*\ln((1/2*(-x^4+1)/x^2-(-x^4+1)^(1/2)/x+1)/(1/2*(-x^4+1)/x^2+(-x^4+1)^(1/2)/x+1))$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^4+1}}{x^4+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-x^4 + 1)/(x^4 + 1),x, algorithm="maxima")

[Out] integrate(sqrt(-x^4 + 1)/(x^4 + 1), x)

Fricas [A] time = 0.257111, size = 105, normalized size = 2.14

$$\frac{1}{2} \arctan\left(-\frac{x^3 + \sqrt{-x^4+1}x^2 + x}{x^3 - x - \sqrt{-x^4+1}}\right) + \frac{1}{4} \log\left(-\frac{x^4 - 2x^2 - 2\sqrt{-x^4+1}x - 1}{x^4 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-x^4 + 1)/(x^4 + 1),x, algorithm="fricas")

```
[Out] 1/2*arctan(-(x^3 + sqrt(-x^4 + 1)*x^2 + x)/(x^3 - x - sqrt(-x^4 + 1))) + 1/4*log(-(x^4 - 2*x^2 - 2*sqrt(-x^4 + 1)*x - 1)/(x^4 + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(x-1)(x+1)(x^2+1)}}{x^4+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)**(1/2)/(x**4+1), x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 1)*(x**2 + 1))/(x**4 + 1), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^4+1}}{x^4+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-x^4 + 1)/(x^4 + 1), x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^4 + 1)/(x^4 + 1), x)
```


$$3.64 \quad \int \frac{\sqrt{1+x^4}}{1-x^4} dx$$

Optimal. Leaf size=53

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{2\sqrt{2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{2\sqrt{2}}$$

[Out] ArcTan[(Sqrt[2]*x)/Sqrt[1 + x^4]]/(2*Sqrt[2]) + ArcTanh[(Sqrt[2]*x)/Sqrt[1 + x^4]]/(2*Sqrt[2])

Rubi [A] time = 0.0367356, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{2\sqrt{2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 + x^4]/(1 - x^4), x]

[Out] ArcTan[(Sqrt[2]*x)/Sqrt[1 + x^4]]/(2*Sqrt[2]) + ArcTanh[(Sqrt[2]*x)/Sqrt[1 + x^4]]/(2*Sqrt[2])

Rubi in Sympy [A] time = 2.58519, size = 46, normalized size = 0.87

$$\frac{\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{4} + \frac{\sqrt{2} \operatorname{atanh}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate((x**4+1)**(1/2)/(-x**4+1), x)

[Out] sqrt(2)*atan(sqrt(2)*x/sqrt(x**4 + 1))/4 + sqrt(2)*atanh(sqrt(2)*x/sqrt(x**4 + 1))/4

Mathematica [C] time = 0.134014, size = 108, normalized size = 2.04

$$\frac{5x\sqrt{x^4+1}F_1\left(\frac{1}{4}; -\frac{1}{2}, 1; \frac{5}{4}; -x^4, x^4\right)}{(x^4-1)\left(2x^4\left(2F_1\left(\frac{5}{4}; -\frac{1}{2}, 2; \frac{9}{4}; -x^4, x^4\right) + F_1\left(\frac{5}{4}; \frac{1}{2}, 1; \frac{9}{4}; -x^4, x^4\right)\right) + 5F_1\left(\frac{1}{4}; -\frac{1}{2}, 1; \frac{5}{4}; -x^4, x^4\right)\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sqrt[1 + x^4]/(1 - x^4), x]

[Out] (-5*x*Sqrt[1 + x^4]*AppellF1[1/4, -1/2, 1, 5/4, -x^4, x^4])/((-1 + x^4)*(5*AppellF1[1/4, -1/2, 1, 5/4, -x^4, x^4] + 2*x^4*(2*AppellF1[5/4, -1/2, 2, 9/4, -x^4, x^4] + AppellF1[5/4, 1/2, 1, 9/4, -x^4, x^4])))

Maple [C] time = 0.04, size = 365, normalized size = 6.9

$$\begin{aligned}
 & -\frac{\text{EllipticF}\left(x\left(\frac{\sqrt{2}}{2} + \frac{i}{2}\sqrt{2}\right), i\right)}{\frac{\sqrt{2}}{2} + \frac{i}{2}\sqrt{2}} \sqrt{1-ix^2}\sqrt{1+ix^2} \frac{1}{\sqrt{x^4+1}} \\
 & -\frac{\frac{i}{2}\left(\text{EllipticF}\left(x\left(\frac{\sqrt{2}}{2} + \frac{i}{2}\sqrt{2}\right), i\right) - \text{EllipticE}\left(x\left(\frac{\sqrt{2}}{2} + \frac{i}{2}\sqrt{2}\right), i\right)\right)}{\frac{\sqrt{2}}{2} + \frac{i}{2}\sqrt{2}} \sqrt{1-ix^2}\sqrt{1+ix^2} \frac{1}{\sqrt{x^4+1}} \\
 & -(-1)^{\frac{3}{4}} \text{EllipticPi}\left(\sqrt[4]{-1}x, -i, \sqrt{-i} - (-1)^{\frac{3}{4}}\right) \sqrt{1-ix^2}\sqrt{1+ix^2} \frac{1}{\sqrt{x^4+1}} \\
 & +\frac{\frac{i}{2}\text{EllipticF}\left(x\left(\frac{\sqrt{2}}{2} + \frac{i}{2}\sqrt{2}\right), i\right)}{\frac{\sqrt{2}}{2} + \frac{i}{2}\sqrt{2}} \sqrt{1-ix^2}\sqrt{1+ix^2} \frac{1}{\sqrt{x^4+1}} \\
 & -\frac{\frac{i}{2}\text{EllipticE}\left(x\left(\frac{\sqrt{2}}{2} + \frac{i}{2}\sqrt{2}\right), i\right)}{\frac{\sqrt{2}}{2} + \frac{i}{2}\sqrt{2}} \sqrt{1-ix^2}\sqrt{1+ix^2} \frac{1}{\sqrt{x^4+1}} \\
 & -(-1)^{\frac{3}{4}} \text{EllipticPi}\left(\sqrt[4]{-1}x, i, \sqrt{-i} - (-1)^{\frac{3}{4}}\right) \sqrt{1-ix^2}\sqrt{1+ix^2} \frac{1}{\sqrt{x^4+1}}
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4+1)^(1/2)/(-x^4+1), x)

[Out] -1/(1/2*2^(1/2)+1/2*I*2^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticF(x*(1/2*2^(1/2)+1/2*I*2^(1/2)), I)-1/2*I/(1/2*2^(1/2)+1/2*I*2^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*(EllipticF(x*(1/2*2^(1/2)+1/2*I*2^(1/2)), I)-EllipticE(x*(1/2*2^(1/2)+1/2*I*2^(1/2)), I))-(-1)^(3/4)*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticPi((-1)^(1/4)*x, -I, (-I)^(1/2)/(-1)^(1/4))+1/2*I/(1/2*2^(1/2)+1/2*I*2^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticF(x*(1/2*2^(1/2)+1/2*I*2^(1/2)), I)-1/2*I/(1/2*2^(1/2)+1/2*I*2^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticE(x*(1/2*2^(1/2)+1/2*I*2^(1/2)), I)-(-1)^(3/4)*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticPi((-1)^(1/4)*x, I, (-I)^(1/2)/(-1)^(1/4))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{x^4 + 1}}{x^4 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-sqrt(x^4 + 1)/(x^4 - 1), x, algorithm="maxima")

[Out] -integrate(sqrt(x^4 + 1)/(x^4 - 1), x)

Fricas [A] time = 0.25386, size = 81, normalized size = 1.53

$$\frac{1}{8} \sqrt{2} \left(2 \arctan \left(\frac{\sqrt{2}x}{\sqrt{x^4 + 1}} \right) + \log \left(\frac{\sqrt{2}(x^4 + 2x^2 + 1) + 4\sqrt{x^4 + 1}x}{x^4 - 2x^2 + 1} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-sqrt(x^4 + 1)/(x^4 - 1), x, algorithm="fricas")

[Out] 1/8*sqrt(2)*(2*arctan(sqrt(2)*x/sqrt(x^4 + 1)) + log((sqrt(2)*(x^4 + 2*x^2 + 1) + 4*sqrt(x^4 + 1)*x)/(x^4 - 2*x^2 + 1)))

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{x^4 + 1}}{x^4 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**4+1)**(1/2)/(-x**4+1), x)

[Out] -Integral(sqrt(x**4 + 1)/(x**4 - 1), x)

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\sqrt{x^4 + 1}}{x^4 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sqrt(x^4 + 1)/(x^4 - 1),x, algorithm="giac")
```

```
[Out] integrate(-sqrt(x^4 + 1)/(x^4 - 1), x)
```

$$3.65 \quad \int \frac{\sqrt{1+px^2+x^4}}{1-x^4} dx$$

Optimal. Leaf size=75

$$\frac{1}{4}\sqrt{2-p} \tan^{-1}\left(\frac{\sqrt{2-px}}{\sqrt{px^2+x^4+1}}\right) + \frac{1}{4}\sqrt{p+2} \tanh^{-1}\left(\frac{\sqrt{p+2x}}{\sqrt{px^2+x^4+1}}\right)$$

[Out] (Sqrt[2 - p]*ArcTan[(Sqrt[2 - p]*x)/Sqrt[1 + p*x^2 + x^4]])/4 + (Sqrt[2 + p]*ArcTanh[(Sqrt[2 + p]*x)/Sqrt[1 + p*x^2 + x^4]])/4

Rubi [A] time = 0.161053, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$\frac{1}{4}\sqrt{2-p} \tan^{-1}\left(\frac{\sqrt{2-px}}{\sqrt{px^2+x^4+1}}\right) + \frac{1}{4}\sqrt{p+2} \tanh^{-1}\left(\frac{\sqrt{p+2x}}{\sqrt{px^2+x^4+1}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 + p*x^2 + x^4]/(1 - x^4), x]

[Out] (Sqrt[2 - p]*ArcTan[(Sqrt[2 - p]*x)/Sqrt[1 + p*x^2 + x^4]])/4 + (Sqrt[2 + p]*ArcTanh[(Sqrt[2 + p]*x)/Sqrt[1 + p*x^2 + x^4]])/4

Rubi in Sympy [A] time = 9.10875, size = 63, normalized size = 0.84

$$\frac{\sqrt{-p+2} \operatorname{atan}\left(\frac{x\sqrt{-p+2}}{\sqrt{px^2+x^4+1}}\right)}{4} + \frac{\sqrt{p+2} \operatorname{atanh}\left(\frac{x\sqrt{p+2}}{\sqrt{px^2+x^4+1}}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate((x**4+p*x**2+1)**(1/2)/(-x**4+1), x)

[Out] sqrt(-p + 2)*atan(x*sqrt(-p + 2)/sqrt(p*x**2 + x**4 + 1))/4 + sqrt(p + 2)*atanh(x*sqrt(p + 2)/sqrt(p*x**2 + x**4 + 1))/4

Mathematica [A] time = 0.329071, size = 0, normalized size = 0.

$$\int \frac{\sqrt{1+px^2+x^4}}{1-x^4} dx$$

Verification is Not applicable to the result.

[In] Integrate[Sqrt[1 + p*x^2 + x^4]/(1 - x^4), x]

[Out] Integrate[Sqrt[1 + p*x^2 + x^4]/(1 - x^4), x]

Maple [C] time = 0.098, size = 1421, normalized size = 19.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4+p*x^2+1)^(1/2)/(-x^4+1), x)

[Out]
$$\begin{aligned} & -1/2*(1+p)/(-2*p+2*(p^2-4)^{(1/2)})^{(1/2)}*(1-(-1/2*p+1/2*(p^2-4)^{(1/2)})^2*x^2)^{(1/2)}*(1-(-1/2*p-1/2*(p^2-4)^{(1/2)})^2*x^2)^{(1/2)}/(x^4+p*x^2+1)^{(1/2)}*EllipticF(1/2*x*(-2*p+2*(p^2-4)^{(1/2)})^{(1/2)}, (-1-p*(-1/2*p-1/2*(p^2-4)^{(1/2)})^{(1/2)})^{(1/2)})+2/(-2*p+2*(p^2-4)^{(1/2)})^{(1/2)}*(1-(-1/2*p+1/2*(p^2-4)^{(1/2)})^2*x^2)^{(1/2)}*(1-(-1/2*p-1/2*(p^2-4)^{(1/2)})^2*x^2)^{(1/2)}/(x^4+p*x^2+1)^{(1/2)}/(p+(p^2-4)^{(1/2)})*(EllipticF(1/2*x*(-2*p+2*(p^2-4)^{(1/2)})^{(1/2)}, (-1-p*(-1/2*p-1/2*(p^2-4)^{(1/2)})^{(1/2)})^{(1/2)})-EllipticE(1/2*x*(-2*p+2*(p^2-4)^{(1/2)})^{(1/2)}, (-1-p*(-1/2*p-1/2*(p^2-4)^{(1/2)})^{(1/2)})^{(1/2)})+1/(-1/2*p+1/2*(p^2-4)^{(1/2)})^{(1/2)}*(1-(-1/2*p+1/2*(p^2-4)^{(1/2)})^2*x^2)^{(1/2)}*(1-(-1/2*p-1/2*(p^2-4)^{(1/2)})^2*x^2)^{(1/2)}/(x^4+p*x^2+1)^{(1/2)}*EllipticPi((-1/2*p+1/2*(p^2-4)^{(1/2)})^{(1/2)}*x, 1/(-1/2*p+1/2*(p^2-4)^{(1/2)})^{(1/2)}, (-1/2*p-1/2*(p^2-4)^{(1/2)})^{(1/2)})^{(1/2)}/(-1/2*p+1/2*(p^2-4)^{(1/2)})^{(1/2)})+1/2*p/(-1/2*p+1/2*(p^2-4)^{(1/2)})^{(1/2)}*(1-(-1/2*p+1/2*(p^2-4)^{(1/2)})^2*x^2)^{(1/2)}*(1-(-1/2*p-1/2*(p^2-4)^{(1/2)})^2*x^2)^{(1/2)}/(x^4+p*x^2+1)^{(1/2)}*EllipticPi((-1/2*p+1/2*(p^2-4)^{(1/2)})^{(1/2)}*x, 1/(-1/2*p+1/2*(p^2-4)^{(1/2)})^{(1/2)}, (-1/2*p-1/2*(p^2-4)^{(1/2)})^{(1/2)})^{(1/2)}/(-1/2*p+1/2*(p^2-4)^{(1/2)})^{(1/2)})+1/2*(-1-p)/(-2*p+2*(p^2-4)^{(1/2)})^{(1/2)}*(1-(-1/2*p+1/2*(p^2-4)^{(1/2)})^2*x^2)^{(1/2)}*(1-(-1/2*p-1/2*(p^2-4)^{(1/2)})^2*x^2)^{(1/2)}/(x^4+p*x^2+1)^{(1/2)}*EllipticF(1/2*x*(-2*p+2*(p^2-4)^{(1/2)})^{(1/2)}, (-1-p*(-1/2*p-1/2*(p^2-4)^{(1/2)})^{(1/2)})^{(1/2)})+1/(-2*p+2*(p^2-4)^{(1/2)})^{(1/2)}*(1+1/2*p*x^2-1/2*x^2*(p^2-4)^{(1/2)})^{(1/2)}*(1+1/2*p*x^2+1/2*x^2*(p^2-4)^{(1/2)})^{(1/2)}/(x^4+p*x^2+1)^{(1/2)}*EllipticF(1/2*x*(-2*p+2*(p^2-4)^{(1/2)})^{(1/2)}, (-1-p*(-1/2*p-1/2*(p^2-4)^{(1/2)})^{(1/2)})^{(1/2)})*(1+1/2*p*x^2-1/2*x^2*(p^2-4)^{(1/2)})^{(1/2)}*(1+1/2*p*x^2+1/2*x^2*(p^2-4)^{(1/2)})^{(1/2)}/(x^4+p*x^2+1)^{(1/2)}*EllipticF(1/2*x*(-2*p+2*(p^2-4)^{(1/2)})^{(1/2)}, (-1-p*(-1/2*p-1/2*(p^2-4)^{(1/2)})^{(1/2)})^{(1/2)})+2/(-2*p+2*(p^2-4)^{(1/2)})^{(1/2)}*(1+1/2*p*x^2-1/2*x^2*(p^2-4)^{(1/2)})^{(1/2)}*(1+1/2*p*x^2+1/2*x^2*(p^2-4)^{(1/2)})^{(1/2)}/(x^4+p*x^2+1)^{(1/2)}/(p+(p^2-4)^{(1/2)})*(EllipticF(1/2*x*(-2*p+2*(p^2-4)^{(1/2)})^{(1/2)}, (-1-p*(-1/2*p-1/2*(p^2-4)^{(1/2)})^{(1/2)})^{(1/2)})+2/(-2*p+2*(p^2-4)^{(1/2)})^{(1/2)}*(1+1/2*p*x^2-1/2*x^2*(p^2-4)^{(1/2)})^{(1/2)}*(1+1/2*p*x^2+1/2*x^2*(p^2-4)^{(1/2)})^{(1/2)}/(x^4+p*x^2+1)^{(1/2)}/(p+(p^2-4)^{(1/2)})*(EllipticE(1/2*x*(-2*p+2*(p^2-4)^{(1/2)})^{(1/2)}, (-1-p*(-1/2*p-1/2*(p^2-4)^{(1/2)})^{(1/2)})^{(1/2)})+1/(-1/2*p+1/2*(p^2-4)^{(1/2)})^{(1/2)}*(1+1/2*p*x^2-1/2*x^2*(p^2-4)^{(1/2)})^{(1/2)}*(1+1/2*p*x^2+1/2*x^2*(p^2-4)^{(1/2)})^{(1/2)}/(x^4+p*x^2+1)^{(1/2)}*E \end{aligned}$$

```

lIipticPi((-1/2*p+1/2*(p^2-4)^(1/2))^(1/2)*x, -1/(-1/2*p+1/2*(p^2-
4)^(1/2)), (-1/2*p-1/2*(p^2-4)^(1/2))^(1/2)/(-1/2*p+1/2*(p^2-4)^(1
/2))^(1/2))-1/2*p/(-1/2*p+1/2*(p^2-4)^(1/2))^(1/2)*(1+1/2*p*x^2-1
/2*x^2*(p^2-4)^(1/2))^(1/2)*(1+1/2*p*x^2+1/2*x^2*(p^2-4)^(1/2))^(
1/2)/(x^4+p*x^2+1)^(1/2)*EllipticPi((-1/2*p+1/2*(p^2-4)^(1/2))^(1
/2)*x, -1/(-1/2*p+1/2*(p^2-4)^(1/2)), (-1/2*p-1/2*(p^2-4)^(1/2))^(1
/2)/(-1/2*p+1/2*(p^2-4)^(1/2))^(1/2))

```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{x^4 + px^2 + 1}}{x^4 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sqrt(x^4 + p*x^2 + 1)/(x^4 - 1),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(x^4 + p*x^2 + 1)/(x^4 - 1), x)
```

Fricas [A] time = 0.27045, size = 1, normalized size = 0.01

$$\begin{aligned}
& \left[\frac{1}{8} \sqrt{p-2} \log \left(\frac{x^4 + 2(p-1)x^2 - 2\sqrt{x^4 + px^2 + 1}\sqrt{p-2x+1}}{x^4 + 2x^2 + 1} \right) \right. \\
& + \frac{1}{8} \sqrt{p+2} \log \left(\frac{x^4 + 2(p+1)x^2 + 2\sqrt{x^4 + px^2 + 1}\sqrt{p+2x+1}}{x^4 - 2x^2 + 1} \right), \frac{1}{4} \sqrt{-p+2} \arctan \left(\frac{\sqrt{-p+2x}}{\sqrt{x^4 + px^2 + 1}} \right) \\
& + \frac{1}{8} \sqrt{p+2} \log \left(\frac{x^4 + 2(p+1)x^2 + 2\sqrt{x^4 + px^2 + 1}\sqrt{p+2x+1}}{x^4 - 2x^2 + 1} \right), \frac{1}{4} \sqrt{-p-2} \arctan \left(\frac{\sqrt{x^4 + px^2 + 1}}{\sqrt{-p-2x}} \right) \\
& + \frac{1}{8} \sqrt{p-2} \log \left(\frac{x^4 + 2(p-1)x^2 - 2\sqrt{x^4 + px^2 + 1}\sqrt{p-2x+1}}{x^4 + 2x^2 + 1} \right), \frac{1}{4} \sqrt{-p+2} \arctan \left(\frac{\sqrt{-p+2x}}{\sqrt{x^4 + px^2 + 1}} \right) \\
& \left. + \frac{1}{4} \sqrt{-p-2} \arctan \left(\frac{\sqrt{x^4 + px^2 + 1}}{\sqrt{-p-2x}} \right) \right]
\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sqrt(x^4 + p*x^2 + 1)/(x^4 - 1),x, algorithm="fricas")
```

```
[Out] [1/8*sqrt(p - 2)*log((x^4 + 2*(p - 1)*x^2 - 2*sqrt(x^4 + p*x^2 +
1)*sqrt(p - 2)*x + 1)/(x^4 + 2*x^2 + 1)) + 1/8*sqrt(p + 2)*log((x
^4 + 2*(p + 1)*x^2 + 2*sqrt(x^4 + p*x^2 + 1)*sqrt(p + 2)*x + 1)/(
x^4 - 2*x^2 + 1)), 1/4*sqrt(-p + 2)*arctan(sqrt(-p + 2)*x/sqrt(x^
4 + p*x^2 + 1)) + 1/8*sqrt(p + 2)*log((x^4 + 2*(p + 1)*x^2 + 2*sq
rt(x^4 + p*x^2 + 1)*sqrt(p + 2)*x + 1)/(x^4 - 2*x^2 + 1)), 1/4*sq
```

```
rt(-p - 2)*arctan(sqrt(x^4 + p*x^2 + 1)/(sqrt(-p - 2)*x)) + 1/8*sqrt(p - 2)*log((x^4 + 2*(p - 1)*x^2 - 2*sqrt(x^4 + p*x^2 + 1)*sqrt(p - 2)*x + 1)/(x^4 + 2*x^2 + 1)), 1/4*sqrt(-p + 2)*arctan(sqrt(-p + 2)*x/sqrt(x^4 + p*x^2 + 1)) + 1/4*sqrt(-p - 2)*arctan(sqrt(x^4 + p*x^2 + 1)/(sqrt(-p - 2)*x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{px^2 + x^4 + 1}}{x^4 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+p*x**2+1)**(1/2)/(-x**4+1),x)
```

```
[Out] -Integral(sqrt(p*x**2 + x**4 + 1)/(x**4 - 1), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\sqrt{x^4 + px^2 + 1}}{x^4 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sqrt(x^4 + p*x^2 + 1)/(x^4 - 1),x, algorithm="giac")
```

```
[Out] integrate(-sqrt(x^4 + p*x^2 + 1)/(x^4 - 1), x)
```


$$3.66 \quad \int \frac{\sqrt{1+px^2-x^4}}{1+x^4} dx$$

Optimal. Leaf size=171

$$\frac{\sqrt{\sqrt{p^2+4}-p} \tanh^{-1}\left(\frac{\sqrt{\sqrt{p^2+4}-px}(\sqrt{p^2+4+p-2x^2})}{2\sqrt{2}\sqrt{px^2-x^4+1}}\right)}{2\sqrt{2}} - \frac{\sqrt{\sqrt{p^2+4}+p} \tan^{-1}\left(\frac{\sqrt{\sqrt{p^2+4}+px}(-\sqrt{p^2+4+p-2x^2})}{2\sqrt{2}\sqrt{px^2-x^4+1}}\right)}{2\sqrt{2}}$$

[Out] -(Sqrt[p + Sqrt[4 + p^2]]*ArcTan[(Sqrt[p + Sqrt[4 + p^2]]*x*(p - Sqrt[4 + p^2] - 2*x^2))/(2*Sqrt[2]*Sqrt[1 + p*x^2 - x^4])])/(2*Sqrt[2]) + (Sqrt[-p + Sqrt[4 + p^2]]*ArcTanh[(Sqrt[-p + Sqrt[4 + p^2]]*x*(p + Sqrt[4 + p^2] - 2*x^2))/(2*Sqrt[2]*Sqrt[1 + p*x^2 - x^4])])/(2*Sqrt[2])

Rubi [A] time = 0.141518, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$

$$\frac{\sqrt{\sqrt{p^2+4}-p} \tanh^{-1}\left(\frac{\sqrt{\sqrt{p^2+4}-px}(\sqrt{p^2+4+p-2x^2})}{2\sqrt{2}\sqrt{px^2-x^4+1}}\right)}{2\sqrt{2}} - \frac{\sqrt{\sqrt{p^2+4}+p} \tan^{-1}\left(\frac{\sqrt{\sqrt{p^2+4}+px}(-\sqrt{p^2+4+p-2x^2})}{2\sqrt{2}\sqrt{px^2-x^4+1}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 + p*x^2 - x^4]/(1 + x^4), x]

[Out] -(Sqrt[p + Sqrt[4 + p^2]]*ArcTan[(Sqrt[p + Sqrt[4 + p^2]]*x*(p - Sqrt[4 + p^2] - 2*x^2))/(2*Sqrt[2]*Sqrt[1 + p*x^2 - x^4])])/(2*Sqrt[2]) + (Sqrt[-p + Sqrt[4 + p^2]]*ArcTanh[(Sqrt[-p + Sqrt[4 + p^2]]*x*(p + Sqrt[4 + p^2] - 2*x^2))/(2*Sqrt[2]*Sqrt[1 + p*x^2 - x^4])])/(2*Sqrt[2])

Rubi in Sympy [A] time = 6.9404, size = 144, normalized size = 0.84

$$\frac{\sqrt{2}\sqrt{-p+\sqrt{p^2+4}} \operatorname{atanh}\left(\frac{\sqrt{2}x\sqrt{-p+\sqrt{p^2+4}}(p-2x^2+\sqrt{p^2+4})}{4\sqrt{px^2-x^4+1}}\right)}{4} - \frac{\sqrt{2}\sqrt{p+\sqrt{p^2+4}} \operatorname{atan}\left(\frac{\sqrt{2}x\sqrt{p+\sqrt{p^2+4}}(p-2x^2-\sqrt{p^2+4})}{4\sqrt{px^2-x^4+1}}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate((-x**4+p*x**2+1)**(1/2)/(x**4+1), x)

[Out] $\sqrt{2} \sqrt{-p + \sqrt{p^2 + 4}} \operatorname{atanh}(\sqrt{2} x \sqrt{-p + \sqrt{p^2 + 4}}) (p - 2x^2 + \sqrt{p^2 + 4}) / (4 \sqrt{p^2 x^2 - x^4 + 1}) / 4 - \sqrt{2} \sqrt{p + \sqrt{p^2 + 4}} \operatorname{atan}(\sqrt{2} x \sqrt{p + \sqrt{p^2 + 4}}) (p - 2x^2 - \sqrt{p^2 + 4}) / (4 \sqrt{p^2 x^2 - x^4 + 1}) / 4$

Mathematica [C] time = 0.426933, size = 322, normalized size = 1.88

$$\frac{\sqrt{\frac{4x^2}{\sqrt{p^2+4}-p}} + 2\sqrt{1 - \frac{2x^2}{\sqrt{p^2+4}+p}} \left(2iF \left(i \sinh^{-1} \left(\sqrt{2} \sqrt{\frac{1}{\sqrt{p^2+4}-p}} x \right) \middle| \frac{p - \sqrt{p^2+4}}{p + \sqrt{p^2+4}} \right) - (p + 2i) \left(\frac{1}{2} i (p - \sqrt{p^2+4}) ; i \sinh^{-1} \left(\sqrt{2} \sqrt{\frac{1}{\sqrt{p^2+4}-p}} x \right) \right) \right)}{4 \sqrt{\frac{1}{\sqrt{p^2+4}-p}} \sqrt{px^2 - x^4 + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 + p*x^2 - x^4]/(1 + x^4), x]

[Out] $(\sqrt{2 + (4x^2)/(-p + \sqrt{4 + p^2})}) \sqrt{1 - (2x^2)/(p + \sqrt{4 + p^2})} ((2I) \operatorname{EllipticF}[I \operatorname{ArcSinh}[\sqrt{2} \sqrt{(-p + \sqrt{4 + p^2})}^{-1}] x], (p - \sqrt{4 + p^2}) / (p + \sqrt{4 + p^2})] - (2I + p) \operatorname{EllipticPi}[(I/2)(p - \sqrt{4 + p^2}), I \operatorname{ArcSinh}[\sqrt{2} \sqrt{(-p + \sqrt{4 + p^2})}^{-1}] x], (p - \sqrt{4 + p^2}) / (p + \sqrt{4 + p^2})] + (-2I + p) \operatorname{EllipticPi}[(I/2)(-p + \sqrt{4 + p^2}), I \operatorname{ArcSinh}[\sqrt{2} \sqrt{(-p + \sqrt{4 + p^2})}^{-1}] x], (p - \sqrt{4 + p^2}) / (p + \sqrt{4 + p^2})]) / (4 \sqrt{(-p + \sqrt{4 + p^2})}^{-1}) \sqrt{1 + px^2 - x^4}$

Maple [B] time = 0.101, size = 456, normalized size = 2.7

$$\begin{aligned} & -\frac{\sqrt{2}}{32} \sqrt{p + \sqrt{p^2 + 4}} \sqrt{p^2 + 4} \ln \left(\frac{\sqrt{2}}{x} \sqrt{p + \sqrt{p^2 + 4}} \sqrt{-x^4 + px^2 + 1} - \frac{-x^4 + px^2 + 1}{x^2} - \sqrt{p^2 + 4} \right) \\ & + \frac{\sqrt{2}}{4} \arctan \left(\frac{1}{2} \left(2 \sqrt{p + \sqrt{p^2 + 4}} - 2 \frac{\sqrt{-x^4 + px^2 + 1} \sqrt{2}}{x} \right) \frac{1}{\sqrt{-p + \sqrt{p^2 + 4}}} \right) \frac{1}{\sqrt{-p + \sqrt{p^2 + 4}}} \\ & + \frac{\sqrt{2}p}{32} \sqrt{p + \sqrt{p^2 + 4}} \ln \left(\frac{\sqrt{2}}{x} \sqrt{p + \sqrt{p^2 + 4}} \sqrt{-x^4 + px^2 + 1} - \frac{-x^4 + px^2 + 1}{x^2} - \sqrt{p^2 + 4} \right) \\ & + \frac{\sqrt{2}}{32} \sqrt{p + \sqrt{p^2 + 4}} \sqrt{p^2 + 4} \ln \left(\frac{-x^4 + px^2 + 1}{x^2} + \frac{\sqrt{2}}{x} \sqrt{p + \sqrt{p^2 + 4}} \sqrt{-x^4 + px^2 + 1} + \sqrt{p^2 + 4} \right) \\ & - \frac{\sqrt{2}}{4} \arctan \left(\frac{1}{2} \left(2 \frac{\sqrt{-x^4 + px^2 + 1} \sqrt{2}}{x} + 2 \sqrt{p + \sqrt{p^2 + 4}} \right) \frac{1}{\sqrt{-p + \sqrt{p^2 + 4}}} \right) \frac{1}{\sqrt{-p + \sqrt{p^2 + 4}}} \\ & - \frac{\sqrt{2}p}{32} \sqrt{p + \sqrt{p^2 + 4}} \ln \left(\frac{-x^4 + px^2 + 1}{x^2} + \frac{\sqrt{2}}{x} \sqrt{p + \sqrt{p^2 + 4}} \sqrt{-x^4 + px^2 + 1} + \sqrt{p^2 + 4} \right) \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-x^4+p*x^2+1)^(1/2)/(x^4+1),x)`

[Out]
$$-1/32 \cdot 2^{1/2} \cdot (p+(p^2+4)^{1/2})^{1/2} \cdot (p^2+4)^{1/2} \cdot \ln((p+(p^2+4)^{1/2})^{1/2} \cdot (-x^4+p \cdot x^2+1)^{1/2} \cdot 2^{1/2}/x - (-x^4+p \cdot x^2+1)/x^2 - (p^2+4)^{1/2}) + 1/4 \cdot 2^{1/2} / (-p+(p^2+4)^{1/2})^{1/2} \cdot \arctan(1/2 \cdot (2 \cdot (p+(p^2+4)^{1/2})^{1/2} - 2 \cdot (-x^4+p \cdot x^2+1)^{1/2} \cdot 2^{1/2}/x) / (-p+(p^2+4)^{1/2})^{1/2}) + 1/32 \cdot 2^{1/2} \cdot (p+(p^2+4)^{1/2})^{1/2} \cdot p \cdot \ln((p+(p^2+4)^{1/2})^{1/2} \cdot (-x^4+p \cdot x^2+1)^{1/2} \cdot 2^{1/2}/x - (-x^4+p \cdot x^2+1)/x^2 - (p^2+4)^{1/2}) + 1/32 \cdot 2^{1/2} \cdot (p+(p^2+4)^{1/2})^{1/2} \cdot (p^2+4)^{1/2} \cdot \ln((-x^4+p \cdot x^2+1)/x^2 + (p+(p^2+4)^{1/2})^{1/2} \cdot (-x^4+p \cdot x^2+1)^{1/2} \cdot 2^{1/2}/x + (p^2+4)^{1/2}) - 1/4 \cdot 2^{1/2} / (-p+(p^2+4)^{1/2})^{1/2} \cdot \arctan(1/2 \cdot (2 \cdot (-x^4+p \cdot x^2+1)^{1/2} \cdot 2^{1/2}/x + 2 \cdot (p+(p^2+4)^{1/2})^{1/2}) / (-p+(p^2+4)^{1/2})^{1/2}) - 1/32 \cdot 2^{1/2} \cdot (p+(p^2+4)^{1/2})^{1/2} \cdot p \cdot \ln((-x^4+p \cdot x^2+1)/x^2 + (p+(p^2+4)^{1/2})^{1/2} \cdot (-x^4+p \cdot x^2+1)^{1/2} \cdot 2^{1/2}/x + (p^2+4)^{1/2})$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^4 + px^2 + 1}}{x^4 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-x^4 + p*x^2 + 1)/(x^4 + 1),x, algorithm="maxima")`

[Out] `integrate(sqrt(-x^4 + p*x^2 + 1)/(x^4 + 1), x)`

Fricas [A] time = 0.339132, size = 2419, normalized size = 14.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-x^4 + p*x^2 + 1)/(x^4 + 1),x, algorithm="fricas")`

[Out]
$$-1/8 \cdot ((p^2 + 4)^{1/4} \cdot (p - \sqrt{p^2 + 4}) \cdot \log(-((p^4 + 5 \cdot p^2 + 4) \cdot x^4 - p^4 - (p^5 + 5 \cdot p^3 + 4 \cdot p) \cdot x^2 - 5 \cdot p^2 + 2 \cdot (\sqrt{-x^4 + p \cdot x^2 + 1}) \cdot \sqrt{p^2 + 4} \cdot (p^2 + 1) \cdot x - \sqrt{-x^4 + p \cdot x^2 + 1} \cdot (p^3 + 3 \cdot p) \cdot x) \cdot (p^2 + 4)^{1/4} \cdot \sqrt{(p^2 - \sqrt{p^2 + 4}) \cdot p + 4}) / (p^2 - \sqrt{p^2 + 4}) \cdot p + 2)) - ((p^3 + 3 \cdot p) \cdot x^4 - p^3 - (p^4 + 3 \cdot p^2) \cdot x^2 - 3 \cdot p) \cdot \sqrt{p^2 + 4} + ((p^3 + 3 \cdot p) \cdot \sqrt{p^2 + 4} \cdot x^2 - (p^4 + 5 \cdot p^2 + 4) \cdot x^2) \cdot \sqrt{p^2 + 4} - 4) / ((p^4 + 5 \cdot p^2 + 4) \cdot x^4 + p^4 + 5 \cdot p^2 - ((p^3 + 3 \cdot p) \cdot x^4 + p^3 + 3 \cdot p) \cdot \sqrt{p^2 + 4} + 4)) - (p^2 + 4)^{1/4} \cdot (p - \sqrt{p^2 + 4}) \cdot \log(-((p^4 + 5 \cdot p^2 + 4) \cdot x^4 - p^4$$

- (p^5 + 5*p^3 + 4*p)*x^2 - 5*p^2 - 2*(sqrt(-x^4 + p*x^2 + 1))*sqrt(p^2 + 4)*(p^2 + 1)*x - sqrt(-x^4 + p*x^2 + 1)*(p^3 + 3*p)*x*(p^2 + 4)^(1/4)*sqrt((p^2 - sqrt(p^2 + 4)*p + 4)/(p^2 - sqrt(p^2 + 4)*p + 2)) - ((p^3 + 3*p)*x^4 - p^3 - (p^4 + 3*p^2)*x^2 - 3*p)*sqrt(p^2 + 4) + ((p^3 + 3*p)*sqrt(p^2 + 4)*x^2 - (p^4 + 5*p^2 + 4)*x^2)*sqrt(p^2 + 4) - 4)/((p^4 + 5*p^2 + 4)*x^4 + p^4 + 5*p^2 - (p^3 + 3*p)*x^4 + p^3 + 3*p)*sqrt(p^2 + 4) + 4)) - 8*(p^2 + 4)^(1/4)*arctan(((sqrt(-x^4 + p*x^2 + 1))*p - sqrt(-x^4 + p*x^2 + 1))*sqrt(p^2 + 4))*sqrt((p^2 - sqrt(p^2 + 4)*p + 4)/(p^2 - sqrt(p^2 + 4)*p + 2)) + (p*x^3 - sqrt(p^2 + 4)*x^3 + 2*x)*(p^2 + 4)^(1/4))/((p*x^4 - (x^4 + 1)*sqrt(p^2 + 4) + p)*sqrt(-(p^4 + 5*p^2 + 4)*x^4 - p^4 - (p^5 + 5*p^3 + 4*p)*x^2 - 5*p^2 + 2*(sqrt(-x^4 + p*x^2 + 1))*sqrt(p^2 + 4)*(p^2 + 1)*x - sqrt(-x^4 + p*x^2 + 1)*(p^3 + 3*p)*x)*(p^2 + 4)^(1/4)*sqrt((p^2 - sqrt(p^2 + 4)*p + 4)/(p^2 - sqrt(p^2 + 4)*p + 2)) - ((p^3 + 3*p)*x^4 - p^3 - (p^4 + 3*p^2)*x^2 - 3*p)*sqrt(p^2 + 4) + ((p^3 + 3*p)*sqrt(p^2 + 4)*x^2 - (p^4 + 5*p^2 + 4)*x^2)*sqrt(p^2 + 4) - 4)/((p^4 + 5*p^2 + 4)*x^4 + p^4 + 5*p^2 - ((p^3 + 3*p)*x^4 + p^3 + 3*p)*sqrt(p^2 + 4) + 4))*sqrt((p^2 - sqrt(p^2 + 4)*p + 4)/(p^2 - sqrt(p^2 + 4)*p + 2)) - (sqrt(-x^4 + p*x^2 + 1))*p*x^2 - sqrt(-x^4 + p*x^2 + 1))*sqrt(p^2 + 4)*x^2)*sqrt((p^2 - sqrt(p^2 + 4)*p + 4)/(p^2 - sqrt(p^2 + 4)*p + 2)) - (2*x^3 - p*x + sqrt(p^2 + 4)*x)*(p^2 + 4)^(1/4))) + 8*(p^2 + 4)^(1/4)*arctan(((sqrt(-x^4 + p*x^2 + 1))*p - sqrt(-x^4 + p*x^2 + 1))*sqrt(p^2 + 4))*sqrt((p^2 - sqrt(p^2 + 4)*p + 4)/(p^2 - sqrt(p^2 + 4)*p + 2)) - (p*x^3 - sqrt(p^2 + 4)*x^3 + 2*x)*(p^2 + 4)^(1/4))/((p*x^4 - (x^4 + 1)*sqrt(p^2 + 4) + p)*sqrt(-(p^4 + 5*p^2 + 4)*x^4 - p^4 - (p^5 + 5*p^3 + 4*p)*x^2 - 5*p^2 - 2*(sqrt(-x^4 + p*x^2 + 1))*sqrt(p^2 + 4)*(p^2 + 1)*x - sqrt(-x^4 + p*x^2 + 1)*(p^3 + 3*p)*x)*(p^2 + 4)^(1/4)*sqrt((p^2 - sqrt(p^2 + 4)*p + 4)/(p^2 - sqrt(p^2 + 4)*p + 2)) - ((p^3 + 3*p)*x^4 - p^3 - (p^4 + 3*p^2)*x^2 - 3*p)*sqrt(p^2 + 4) + ((p^3 + 3*p)*sqrt(p^2 + 4)*x^2 - (p^4 + 5*p^2 + 4)*x^2)*sqrt(p^2 + 4) - 4)/((p^4 + 5*p^2 + 4)*x^4 + p^4 + 5*p^2 - ((p^3 + 3*p)*x^4 + p^3 + 3*p)*sqrt(p^2 + 4) + 4))*sqrt((p^2 - sqrt(p^2 + 4)*p + 4)/(p^2 - sqrt(p^2 + 4)*p + 2)) - (sqrt(-x^4 + p*x^2 + 1))*p*x^2 - sqrt(-x^4 + p*x^2 + 1))*sqrt(p^2 + 4)*x^2)*sqrt((p^2 - sqrt(p^2 + 4)*p + 4)/(p^2 - sqrt(p^2 + 4)*p + 2)) + (2*x^3 - p*x + sqrt(p^2 + 4)*x)*(p^2 + 4)^(1/4))))/(p - sqrt(p^2 + 4))*sqrt((p^2 - sqrt(p^2 + 4)*p + 4)/(p^2 - sqrt(p^2 + 4)*p + 2)))

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{px^2 - x^4 + 1}}{x^4 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**4+p*x**2+1)**(1/2)/(x**4+1),x)

[Out] Integral(sqrt(p*x**2 - x**4 + 1)/(x**4 + 1), x)

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^4 + px^2 + 1}}{x^4 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-x^4 + p*x^2 + 1)/(x^4 + 1), x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^4 + p*x^2 + 1)/(x^4 + 1), x)
```

$$3.67 \quad \int \frac{a+bx}{(2-x^2)\sqrt[4]{-1+x^2}} dx$$

Optimal. Leaf size=80

$$\frac{a \tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{2\sqrt{2}} + \frac{a \tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{2\sqrt{2}} - b \tan^{-1}\left(\sqrt[4]{x^2-1}\right) + b \tanh^{-1}\left(\sqrt[4]{x^2-1}\right)$$

[Out] (a*ArcTan[x/(Sqrt[2]*(-1+x^2)^(1/4))])/(2*Sqrt[2]) - b*ArcTan[(-1+x^2)^(1/4)] + (a*ArcTanh[x/(Sqrt[2]*(-1+x^2)^(1/4))])/(2*Sqrt[2]) + b*ArcTanh[(-1+x^2)^(1/4)]

Rubi [A] time = 0.118325, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$

$$\frac{a \tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{2\sqrt{2}} + \frac{a \tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{2\sqrt{2}} - b \tan^{-1}\left(\sqrt[4]{x^2-1}\right) + b \tanh^{-1}\left(\sqrt[4]{x^2-1}\right)$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)/((2 - x^2)*(-1 + x^2)^(1/4)), x]

[Out] (a*ArcTan[x/(Sqrt[2]*(-1+x^2)^(1/4))])/(2*Sqrt[2]) - b*ArcTan[(-1+x^2)^(1/4)] + (a*ArcTanh[x/(Sqrt[2]*(-1+x^2)^(1/4))])/(2*Sqrt[2]) + b*ArcTanh[(-1+x^2)^(1/4)]

Rubi in Sympy [A] time = 28.6337, size = 178, normalized size = 2.22

$$\frac{\sqrt{2}ax(1-i)\left(i;\operatorname{asin}\left(\frac{\sqrt{2}(1+i)\sqrt[4]{x^2-1}}{2}\right)\right)\Big|_{-1}}{2\sqrt{-i\sqrt{x^2-1}+1}\sqrt{i\sqrt{x^2-1}+1}} + \frac{a\sqrt{\frac{x^2}{(\sqrt{x^2-1}+1)^2}}(\sqrt{x^2-1}+1)F\left(2\operatorname{atan}\left(\sqrt[4]{x^2-1}\right)\Big|_{\frac{1}{2}}\right)}{4x}$$

$$+ \frac{\sqrt{2}a\sqrt{x^2}\operatorname{atanh}\left(\frac{\sqrt{2}\sqrt[4]{x^2-1}}{\sqrt{x^2}}\right)}{4x} - b\operatorname{atan}\left(\sqrt[4]{x^2-1}\right) + b\operatorname{atanh}\left(\sqrt[4]{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate((b*x+a)/(-x**2+2)/(x**2-1)**(1/4), x)

[Out] -sqrt(2)*a*x*(1-I)*elliptic_pi(I, asin(sqrt(2)*(1+I)*(x**2-1)**(1/4)/2), -1)/(2*sqrt(-I*sqrt(x**2-1)+1)*sqrt(I*sqrt(x**2-1)))

- 1) + 1)) + a*sqrt(x**2/(sqrt(x**2 - 1) + 1)**2)*(sqrt(x**2 - 1) + 1)*elliptic_f(2*atan((x**2 - 1)**(1/4)), 1/2)/(4*x) + sqrt(2)*a*sqrt(x**2)*atanh(sqrt(2)*(x**2 - 1)**(1/4)/sqrt(x**2))/(4*x) - b*atan((x**2 - 1)**(1/4)) + b*atanh((x**2 - 1)**(1/4))

Mathematica [C] time = 0.560316, size = 203, normalized size = 2.54

$$2x \left(\frac{3aF_1\left(\frac{1}{2}, \frac{1}{4}, 1, \frac{3}{2}; x^2, \frac{x^2}{2}\right)}{x^2\left(2F_1\left(\frac{3}{2}, \frac{1}{4}, 2, \frac{5}{2}; x^2, \frac{x^2}{2}\right) + F_1\left(\frac{3}{2}, \frac{5}{4}, 1, \frac{5}{2}; x^2, \frac{x^2}{2}\right)\right) + 6F_1\left(\frac{1}{2}, \frac{1}{4}, 1, \frac{3}{2}; x^2, \frac{x^2}{2}\right)}{x^2\left(2F_1\left(2, \frac{1}{4}, 2, 3; x^2, \frac{x^2}{2}\right) + F_1\left(2, \frac{5}{4}, 1, 3; x^2, \frac{x^2}{2}\right)\right) + 8F_1\left(1, \frac{1}{4}, 1, 2; x^2, \frac{x^2}{2}\right)} - \frac{2bxF_1\left(1, \frac{1}{4}, 1, 2; x^2, \frac{x^2}{2}\right)}{x^2\left(2F_1\left(2, \frac{1}{4}, 2, 3; x^2, \frac{x^2}{2}\right) + F_1\left(2, \frac{5}{4}, 1, 3; x^2, \frac{x^2}{2}\right)\right) + 8F_1\left(1, \frac{1}{4}, 1, 2; x^2, \frac{x^2}{2}\right)} \right) / (x^2 - 2)\sqrt[4]{x^2 - 1}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*x)/((2 - x^2)*(-1 + x^2)^(1/4)), x]

[Out] (2*x*((-3*a*AppellF1[1/2, 1/4, 1, 3/2, x^2, x^2/2])/(6*AppellF1[1/2, 1/4, 1, 3/2, x^2, x^2/2] + x^2*(2*AppellF1[3/2, 1/4, 2, 5/2, x^2, x^2/2] + AppellF1[3/2, 5/4, 1, 5/2, x^2, x^2/2])) - (2*b*x*AppellF1[1, 1/4, 1, 2, x^2, x^2/2])/(8*AppellF1[1, 1/4, 1, 2, x^2, x^2/2] + x^2*(2*AppellF1[2, 1/4, 2, 3, x^2, x^2/2] + AppellF1[2, 5/4, 1, 3, x^2, x^2/2]))))/((-2 + x^2)*(-1 + x^2)^(1/4))

Maple [F] time = 0.088, size = 0, normalized size = 0.

$$\int \frac{bx + a}{-x^2 + 2} \frac{1}{\sqrt[4]{x^2 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)/(-x^2+2)/(x^2-1)^(1/4), x)

[Out] int((b*x+a)/(-x^2+2)/(x^2-1)^(1/4), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$- \int \frac{bx + a}{(x^2 - 1)^{\frac{1}{4}}(x^2 - 2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-(b*x + a)/((x^2 - 1)^(1/4)*(x^2 - 2)), x, algorithm="maxima")

[Out] -integrate((b*x + a)/((x^2 - 1)^(1/4)*(x^2 - 2)), x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-(b*x + a)/((x^2 - 1)^(1/4)*(x^2 - 2)), x, algorithm="fricas")

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{a}{x^2 \sqrt[4]{x^2 - 1} - 2 \sqrt[4]{x^2 - 1}} dx - \int \frac{bx}{x^2 \sqrt[4]{x^2 - 1} - 2 \sqrt[4]{x^2 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)/(-x**2+2)/(x**2-1)**(1/4), x)

[Out] -Integral(a/(x**2*(x**2 - 1)**(1/4) - 2*(x**2 - 1)**(1/4)), x) -
Integral(b*x/(x**2*(x**2 - 1)**(1/4) - 2*(x**2 - 1)**(1/4)), x)

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{bx + a}{(x^2 - 1)^{\frac{1}{4}}(x^2 - 2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-(b*x + a)/((x^2 - 1)^(1/4)*(x^2 - 2)), x, algorithm="giac")

[Out] integrate(-(b*x + a)/((x^2 - 1)^(1/4)*(x^2 - 2)), x)

$$3.68 \quad \int \frac{a+bx}{\sqrt[4]{-1-x^2}(2+x^2)} dx$$

Optimal. Leaf size=88

$$\frac{a \tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-x^2-1}}\right)}{2\sqrt{2}} + \frac{a \tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-x^2-1}}\right)}{2\sqrt{2}} + b \tan^{-1}\left(\sqrt[4]{-x^2-1}\right) - b \tanh^{-1}\left(\sqrt[4]{-x^2-1}\right)$$

[Out] (a*ArcTan[x/(Sqrt[2]*(-1-x^2)^(1/4))])/(2*Sqrt[2]) + b*ArcTan[(-1-x^2)^(1/4)] + (a*ArcTanh[x/(Sqrt[2]*(-1-x^2)^(1/4))])/(2*Sqrt[2]) - b*ArcTanh[(-1-x^2)^(1/4)]

Rubi [A] time = 0.130639, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$

$$\frac{a \tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-x^2-1}}\right)}{2\sqrt{2}} + \frac{a \tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-x^2-1}}\right)}{2\sqrt{2}} + b \tan^{-1}\left(\sqrt[4]{-x^2-1}\right) - b \tanh^{-1}\left(\sqrt[4]{-x^2-1}\right)$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)/((-1 - x^2)^(1/4)*(2 + x^2)), x]

[Out] (a*ArcTan[x/(Sqrt[2]*(-1-x^2)^(1/4))])/(2*Sqrt[2]) + b*ArcTan[(-1-x^2)^(1/4)] + (a*ArcTanh[x/(Sqrt[2]*(-1-x^2)^(1/4))])/(2*Sqrt[2]) - b*ArcTanh[(-1-x^2)^(1/4)]

Rubi in Sympy [A] time = 29.508, size = 199, normalized size = 2.26

$$\frac{\sqrt{2}ax(1-i) \left(i; \operatorname{asin}\left(\frac{\sqrt{2}(1+i)\sqrt[4]{-x^2-1}}{2}\right) \Big|_{-1} \right)}{2\sqrt{-i\sqrt{-x^2-1} + 1}\sqrt{i\sqrt{-x^2-1} + 1}} - \frac{\sqrt{2}a\sqrt{-x^2} \operatorname{atanh}\left(\frac{\sqrt{2}\sqrt[4]{-x^2-1}}{\sqrt{-x^2}}\right)}{4x}$$

$$- \frac{a \sqrt{-\frac{x^2}{(\sqrt{-x^2-1}+1)^2}} (\sqrt{-x^2-1}+1) F\left(2 \operatorname{atan}\left(\sqrt[4]{-x^2-1}\right) \Big|_{\frac{1}{2}}\right)}{4x}$$

$$+ b \operatorname{atan}\left(\sqrt[4]{-x^2-1}\right) - b \operatorname{atanh}\left(\sqrt[4]{-x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate((b*x+a)/(-x**2-1)**(1/4)/(x**2+2), x)

```
[Out] -sqrt(2)*a*x*(1 - I)*elliptic_pi(I, asin(sqrt(2)*(1 + I)*(-x**2 - 1)**(1/4)/2), -1)/(2*sqrt(-I*sqrt(-x**2 - 1) + 1)*sqrt(I*sqrt(-x**2 - 1) + 1)) - sqrt(2)*a*sqrt(-x**2)*atanh(sqrt(2)*(-x**2 - 1)**(1/4)/sqrt(-x**2))/(4*x) - a*sqrt(-x**2/(sqrt(-x**2 - 1) + 1)**2)*(sqrt(-x**2 - 1) + 1)*elliptic_f(2*atan((-x**2 - 1)**(1/4)), 1/2)/(4*x) + b*atan((-x**2 - 1)**(1/4)) - b*atanh((-x**2 - 1)**(1/4))
```

Mathematica [C] time = 0.554591, size = 221, normalized size = 2.51

$$2x \left(\frac{3aF_1\left(\frac{1}{2}, \frac{1}{4}, 1, \frac{3}{2}; -x^2, -\frac{x^2}{2}\right)}{x^2\left(2F_1\left(\frac{3}{2}, \frac{1}{4}, 2, \frac{5}{2}; -x^2, -\frac{x^2}{2}\right) + F_1\left(\frac{3}{2}, \frac{5}{4}, 1, \frac{5}{2}; -x^2, -\frac{x^2}{2}\right)\right)} - \frac{2bxF_1\left(1, \frac{1}{4}, 1, 2; -x^2, -\frac{x^2}{2}\right)}{x^2\left(2F_1\left(2, \frac{1}{4}, 2, 3; -x^2, -\frac{x^2}{2}\right) + F_1\left(2, \frac{5}{4}, 1, 3; -x^2, -\frac{x^2}{2}\right)\right)} - 8F_1\left(1, \frac{1}{4}, 1, 2; -x^2, -\frac{x^2}{2}\right) \right) \sqrt[4]{-x^2 - 1} (x^2 + 2)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x)/((-1 - x^2)^(1/4)*(2 + x^2)), x]
```

```
[Out] (2*x*((-3*a*AppellF1[1/2, 1/4, 1, 3/2, -x^2, -x^2/2])/(-6*AppellF1[1/2, 1/4, 1, 3/2, -x^2, -x^2/2] + x^2*(2*AppellF1[3/2, 1/4, 2, 5/2, -x^2, -x^2/2] + AppellF1[3/2, 5/4, 1, 5/2, -x^2, -x^2/2])) - (2*b*x*AppellF1[1, 1/4, 1, 2, -x^2, -x^2/2])/(-8*AppellF1[1, 1/4, 1, 2, -x^2, -x^2/2] + x^2*(2*AppellF1[2, 1/4, 2, 3, -x^2, -x^2/2] + AppellF1[2, 5/4, 1, 3, -x^2, -x^2/2]))) / ((-1 - x^2)^(1/4)*(2 + x^2))
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int \frac{bx + a}{x^2 + 2} \frac{1}{\sqrt[4]{-x^2 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(-x^2-1)^(1/4)/(x^2+2), x)
```

```
[Out] int((b*x+a)/(-x^2-1)^(1/4)/(x^2+2), x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx + a}{(x^2 + 2)(-x^2 - 1)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x + a)/((x^2 + 2)*(-x^2 - 1)^(1/4)),x, algorithm="maxima")`

[Out] `integrate((b*x + a)/((x^2 + 2)*(-x^2 - 1)^(1/4)), x)`

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x + a)/((x^2 + 2)*(-x^2 - 1)^(1/4)),x, algorithm="fricas")`

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + bx}{\sqrt[4]{-x^2 - 1}(x^2 + 2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)/(-x**2-1)**(1/4)/(x**2+2),x)`

[Out] `Integral((a + b*x)/((-x**2 - 1)**(1/4)*(x**2 + 2)), x)`

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx + a}{(x^2 + 2)(-x^2 - 1)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x + a)/((x^2 + 2)*(-x^2 - 1)^(1/4)),x, algorithm="giac")`

[Out] `integrate((b*x + a)/((x^2 + 2)*(-x^2 - 1)^(1/4)), x)`

$$3.69 \quad \int \frac{a+bx}{\sqrt[4]{1-x^2}(2-x^2)} dx$$

Optimal. Leaf size=149

$$\frac{1}{2}a \tan^{-1}\left(\frac{1-\sqrt{1-x^2}}{x\sqrt[4]{1-x^2}}\right) + \frac{1}{2}a \tanh^{-1}\left(\frac{\sqrt{1-x^2}+1}{x\sqrt[4]{1-x^2}}\right) + \frac{b \tan^{-1}\left(\frac{1-\sqrt{1-x^2}}{\sqrt{2}\sqrt[4]{1-x^2}}\right)}{\sqrt{2}} + \frac{b \tanh^{-1}\left(\frac{\sqrt{1-x^2}+1}{\sqrt{2}\sqrt[4]{1-x^2}}\right)}{\sqrt{2}}$$

[Out] (b*ArcTan[(1 - Sqrt[1 - x^2])/(Sqrt[2]*(1 - x^2)^(1/4))])/Sqrt[2] + (a*ArcTan[(1 - Sqrt[1 - x^2])/(x*(1 - x^2)^(1/4))])/2 + (b*ArcTanh[(1 + Sqrt[1 - x^2])/(Sqrt[2]*(1 - x^2)^(1/4))])/Sqrt[2] + (a*ArcTanh[(1 + Sqrt[1 - x^2])/(x*(1 - x^2)^(1/4))])/2

Rubi [A] time = 0.136095, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$

$$\frac{1}{2}a \tan^{-1}\left(\frac{1-\sqrt{1-x^2}}{x\sqrt[4]{1-x^2}}\right) + \frac{1}{2}a \tanh^{-1}\left(\frac{\sqrt{1-x^2}+1}{x\sqrt[4]{1-x^2}}\right) + \frac{b \tan^{-1}\left(\frac{1-\sqrt{1-x^2}}{\sqrt{2}\sqrt[4]{1-x^2}}\right)}{\sqrt{2}} + \frac{b \tanh^{-1}\left(\frac{\sqrt{1-x^2}+1}{\sqrt{2}\sqrt[4]{1-x^2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)/((1 - x^2)^(1/4)*(2 - x^2)), x]

[Out] (b*ArcTan[(1 - Sqrt[1 - x^2])/(Sqrt[2]*(1 - x^2)^(1/4))])/Sqrt[2] + (a*ArcTan[(1 - Sqrt[1 - x^2])/(x*(1 - x^2)^(1/4))])/2 + (b*ArcTanh[(1 + Sqrt[1 - x^2])/(Sqrt[2]*(1 - x^2)^(1/4))])/Sqrt[2] + (a*ArcTanh[(1 + Sqrt[1 - x^2])/(x*(1 - x^2)^(1/4))])/2

Rubi in Sympy [A] time = 39.3188, size = 170, normalized size = 1.14

$$\begin{aligned} & -\frac{ia\sqrt{x^2}\left(-i; \operatorname{asin}\left(\sqrt[4]{-x^2+1}\right)\right)\Big|_{-1}}{x} + \frac{ia\sqrt{x^2}\left(i; \operatorname{asin}\left(\sqrt[4]{-x^2+1}\right)\right)\Big|_{-1}}{x} \\ & -\frac{\sqrt{2}b \log\left(-\sqrt{2}\sqrt[4]{-x^2+1} + \sqrt{-x^2+1} + 1\right)}{4} + \frac{\sqrt{2}b \log\left(\sqrt{2}\sqrt[4]{-x^2+1} + \sqrt{-x^2+1} + 1\right)}{4} \\ & -\frac{\sqrt{2}b \operatorname{atan}\left(\sqrt{2}\sqrt[4]{-x^2+1} - 1\right)}{2} - \frac{\sqrt{2}b \operatorname{atan}\left(\sqrt{2}\sqrt[4]{-x^2+1} + 1\right)}{2} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate((b*x+a)/(-x**2+1)**(1/4)/(-x**2+2), x)

[Out] $-I*a*\sqrt{x^2}*\text{elliptic_pi}(-I, \text{asin}((-x^2 + 1)^{(1/4)}), -1)/x +$
 $I*a*\sqrt{x^2}*\text{elliptic_pi}(I, \text{asin}((-x^2 + 1)^{(1/4)}), -1)/x -$
 $\sqrt{2}*b*\log(-\sqrt{2}*(-x^2 + 1)^{(1/4)} + \sqrt{-x^2 + 1} + 1)/$
 $4 + \sqrt{2}*b*\log(\sqrt{2}*(-x^2 + 1)^{(1/4)} + \sqrt{-x^2 + 1} +$
 $1)/4 - \sqrt{2}*b*\text{atan}(\sqrt{2}*(-x^2 + 1)^{(1/4)} - 1)/2 - \sqrt{2}$
 $*b*\text{atan}(\sqrt{2}*(-x^2 + 1)^{(1/4)} + 1)/2$

Mathematica [C] time = 0.445522, size = 205, normalized size = 1.38

$$2x \left(\frac{3aF_1\left(\frac{1}{2}; \frac{1}{4}, 1; \frac{3}{2}; x^2, \frac{x^2}{2}\right)}{x^2 \left(2F_1\left(\frac{3}{2}; \frac{1}{4}, 2; \frac{5}{2}; x^2, \frac{x^2}{2}\right) + F_1\left(\frac{3}{2}; \frac{5}{4}, 1; \frac{5}{2}; x^2, \frac{x^2}{2}\right)\right)} + \frac{2bxF_1\left(1; \frac{1}{4}, 1; 2; x^2, \frac{x^2}{2}\right)}{x^2 \left(2F_1\left(2; \frac{1}{4}, 2; 3; x^2, \frac{x^2}{2}\right) + F_1\left(2; \frac{5}{4}, 1; 3; x^2, \frac{x^2}{2}\right)\right)} + 8F_1\left(1; \frac{1}{4}, 1; 2; x^2, \frac{x^2}{2}\right)}{\sqrt[4]{1-x^2}(x^2-2)} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*x)/((1 - x^2)^(1/4)*(2 - x^2)), x]

[Out] $(2*x*((-3*a*\text{AppellF1}[1/2, 1/4, 1, 3/2, x^2, x^2/2])/(6*\text{AppellF1}[1$
 $/2, 1/4, 1, 3/2, x^2, x^2/2] + x^2*(2*\text{AppellF1}[3/2, 1/4, 2, 5/2,$
 $x^2, x^2/2] + \text{AppellF1}[3/2, 5/4, 1, 5/2, x^2, x^2/2])) - (2*b*x*A$
 $\text{ppellF1}[1, 1/4, 1, 2, x^2, x^2/2])/(8*\text{AppellF1}[1, 1/4, 1, 2, x^2,$
 $x^2/2] + x^2*(2*\text{AppellF1}[2, 1/4, 2, 3, x^2, x^2/2] + \text{AppellF1}[2,$
 $5/4, 1, 3, x^2, x^2/2])))/((1 - x^2)^(1/4)*(-2 + x^2))$

Maple [F] time = 0.07, size = 0, normalized size = 0.

$$\int \frac{bx + a}{-x^2 + 2} \frac{1}{\sqrt{-x^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)/(-x^2+1)^(1/4)/(-x^2+2), x)

[Out] int((b*x+a)/(-x^2+1)^(1/4)/(-x^2+2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{bx + a}{(x^2 - 2)(-x^2 + 1)^{1/4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-(b*x + a)/((x^2 - 2)*(-x^2 + 1)^(1/4)),x, algorithm="maxima")

[Out] -integrate((b*x + a)/((x^2 - 2)*(-x^2 + 1)^(1/4)), x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-(b*x + a)/((x^2 - 2)*(-x^2 + 1)^(1/4)),x, algorithm="fricas")

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{a}{x^2 \sqrt[4]{-x^2 + 1} - 2 \sqrt[4]{-x^2 + 1}} dx - \int \frac{bx}{x^2 \sqrt[4]{-x^2 + 1} - 2 \sqrt[4]{-x^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)/(-x**2+1)**(1/4)/(-x**2+2), x)

[Out] -Integral(a/(x**2*(-x**2 + 1)**(1/4) - 2*(-x**2 + 1)**(1/4)), x)
- Integral(b*x/(x**2*(-x**2 + 1)**(1/4) - 2*(-x**2 + 1)**(1/4)),
x)

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{bx + a}{(x^2 - 2)(-x^2 + 1)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-(b*x + a)/((x^2 - 2)*(-x^2 + 1)^(1/4)),x, algorithm="giac")

[Out] integrate(-(b*x + a)/((x^2 - 2)*(-x^2 + 1)^(1/4)), x)

$$3.70 \quad \int \frac{a+bx}{\sqrt[4]{1+x^2(2+x^2)}} dx$$

Optimal. Leaf size=135

$$-\frac{1}{2}a \tan^{-1}\left(\frac{\sqrt{x^2+1}+1}{x\sqrt[4]{x^2+1}}\right) - \frac{1}{2}a \tanh^{-1}\left(\frac{1-\sqrt{x^2+1}}{x\sqrt[4]{x^2+1}}\right) - \frac{b \tan^{-1}\left(\frac{1-\sqrt{x^2+1}}{\sqrt{2}\sqrt[4]{x^2+1}}\right)}{\sqrt{2}} - \frac{b \tanh^{-1}\left(\frac{\sqrt{x^2+1}+1}{\sqrt{2}\sqrt[4]{x^2+1}}\right)}{\sqrt{2}}$$

[Out] $-\left(\frac{b \operatorname{ArcTan}\left[\frac{1-\sqrt{1+x^2}}{\sqrt{2}\sqrt[4]{1+x^2}}\right]}{\sqrt{2}}\right) - \left(\frac{a \operatorname{ArcTan}\left[\frac{1+\sqrt{1+x^2}}{x\sqrt[4]{1+x^2}}\right]}{2}\right) - \left(\frac{a \operatorname{ArcTanh}\left[\frac{1-\sqrt{1+x^2}}{x\sqrt[4]{1+x^2}}\right]}{2}\right) - \left(\frac{b \operatorname{ArcTanh}\left[\frac{1+\sqrt{1+x^2}}{\sqrt{2}\sqrt[4]{1+x^2}}\right]}{\sqrt{2}}\right)$

Rubi [A] time = 0.108503, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$

$$-\frac{1}{2}a \tan^{-1}\left(\frac{\sqrt{x^2+1}+1}{x\sqrt[4]{x^2+1}}\right) - \frac{1}{2}a \tanh^{-1}\left(\frac{1-\sqrt{x^2+1}}{x\sqrt[4]{x^2+1}}\right) - \frac{b \tan^{-1}\left(\frac{1-\sqrt{x^2+1}}{\sqrt{2}\sqrt[4]{x^2+1}}\right)}{\sqrt{2}} - \frac{b \tanh^{-1}\left(\frac{\sqrt{x^2+1}+1}{\sqrt{2}\sqrt[4]{x^2+1}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}\left[\frac{a + b x}{(1 + x^2)^{1/4} (2 + x^2)}, x\right]$

[Out] $-\left(\frac{b \operatorname{ArcTan}\left[\frac{1-\sqrt{1+x^2}}{\sqrt{2}\sqrt[4]{1+x^2}}\right]}{\sqrt{2}}\right) - \left(\frac{a \operatorname{ArcTan}\left[\frac{1+\sqrt{1+x^2}}{x\sqrt[4]{1+x^2}}\right]}{2}\right) - \left(\frac{a \operatorname{ArcTanh}\left[\frac{1-\sqrt{1+x^2}}{x\sqrt[4]{1+x^2}}\right]}{2}\right) - \left(\frac{b \operatorname{ArcTanh}\left[\frac{1+\sqrt{1+x^2}}{\sqrt{2}\sqrt[4]{1+x^2}}\right]}{\sqrt{2}}\right)$

Rubi in Sympy [A] time = 36.0933, size = 173, normalized size = 1.28

$$\frac{ia\sqrt{-x^2}\left(-i; \operatorname{asin}\left(\sqrt[4]{x^2+1}\right)\right)\Big|_{-1}}{x} - \frac{ia\sqrt{-x^2}\left(i; \operatorname{asin}\left(\sqrt[4]{x^2+1}\right)\right)\Big|_{-1}}{x} + \frac{\sqrt{2}b \log\left(-\sqrt{2}\sqrt[4]{x^2+1} + \sqrt{x^2+1} + 1\right)}{4} - \frac{\sqrt{2}b \log\left(\sqrt{2}\sqrt[4]{x^2+1} + \sqrt{x^2+1} + 1\right)}{4} + \frac{\sqrt{2}b \operatorname{atan}\left(\sqrt{2}\sqrt[4]{x^2+1} - 1\right)}{2} + \frac{\sqrt{2}b \operatorname{atan}\left(\sqrt{2}\sqrt[4]{x^2+1} + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{rubi_integrate}\left(\frac{b x+a}{(x^2+1)^{1/4}(x^2+2)}, x\right)$

```
[Out] I*a*sqrt(-x**2)*elliptic_pi(-I, asin((x**2 + 1)**(1/4)), -1)/x -
I*a*sqrt(-x**2)*elliptic_pi(I, asin((x**2 + 1)**(1/4)), -1)/x + s
qrt(2)*b*log(-sqrt(2)*(x**2 + 1)**(1/4) + sqrt(x**2 + 1) + 1)/4 -
sqrt(2)*b*log(sqrt(2)*(x**2 + 1)**(1/4) + sqrt(x**2 + 1) + 1)/4
+ sqrt(2)*b*atan(sqrt(2)*(x**2 + 1)**(1/4) - 1)/2 + sqrt(2)*b*ata
n(sqrt(2)*(x**2 + 1)**(1/4) + 1)/2
```

Mathematica [C] time = 0.425114, size = 219, normalized size = 1.62

$$2x \left(\frac{3aF_1\left(\frac{1}{2}, \frac{1}{4}, 1, \frac{3}{2}; -x^2, -\frac{x^2}{2}\right)}{x^2\left(2F_1\left(\frac{3}{2}, \frac{1}{4}, 2, \frac{5}{2}; -x^2, -\frac{x^2}{2}\right) + F_1\left(\frac{3}{2}, \frac{5}{4}, 1, \frac{5}{2}; -x^2, -\frac{x^2}{2}\right)\right) - 6F_1\left(\frac{1}{2}, \frac{1}{4}, 1, \frac{3}{2}; -x^2, -\frac{x^2}{2}\right)}{x^2\left(2F_1\left(2, \frac{1}{4}, 2, 3; -x^2, -\frac{x^2}{2}\right) + F_1\left(2, \frac{5}{4}, 1, 3; -x^2, -\frac{x^2}{2}\right)\right) - 8F_1\left(1, \frac{1}{4}, 1, 2; -x^2, -\frac{x^2}{2}\right)} - \frac{2bxF_1\left(1, \frac{1}{4}, 1, 2; -x^2, -\frac{x^2}{2}\right)}{x^2\left(2F_1\left(2, \frac{1}{4}, 2, 3; -x^2, -\frac{x^2}{2}\right) + F_1\left(2, \frac{5}{4}, 1, 3; -x^2, -\frac{x^2}{2}\right)\right) - 8F_1\left(1, \frac{1}{4}, 1, 2; -x^2, -\frac{x^2}{2}\right)} \right) \sqrt[4]{x^2 + 1} (x^2 + 2)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x)/((1 + x^2)^(1/4)*(2 + x^2)), x]
```

```
[Out] (2*x*((-3*a*AppellF1[1/2, 1/4, 1, 3/2, -x^2, -x^2/2])/(-6*AppellF
1[1/2, 1/4, 1, 3/2, -x^2, -x^2/2] + x^2*(2*AppellF1[3/2, 1/4, 2,
5/2, -x^2, -x^2/2] + AppellF1[3/2, 5/4, 1, 5/2, -x^2, -x^2/2])) -
(2*b*x*AppellF1[1, 1/4, 1, 2, -x^2, -x^2/2])/(-8*AppellF1[1, 1/4
, 1, 2, -x^2, -x^2/2] + x^2*(2*AppellF1[2, 1/4, 2, 3, -x^2, -x^2/
2] + AppellF1[2, 5/4, 1, 3, -x^2, -x^2/2]))) / ((1 + x^2)^(1/4)*(2
+ x^2))
```

Maple [F] time = 0.045, size = 0, normalized size = 0.

$$\int \frac{bx + a}{x^2 + 2} \frac{1}{\sqrt[4]{x^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(x^2+1)^(1/4)/(x^2+2), x)
```

```
[Out] int((b*x+a)/(x^2+1)^(1/4)/(x^2+2), x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx + a}{(x^2 + 2)(x^2 + 1)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x + a)/((x^2 + 2)*(x^2 + 1)^(1/4)), x, algorithm="maxima")`

[Out] `integrate((b*x + a)/((x^2 + 2)*(x^2 + 1)^(1/4)), x)`

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x + a)/((x^2 + 2)*(x^2 + 1)^(1/4)), x, algorithm="fricas")`

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + bx}{\sqrt[4]{x^2 + 1}(x^2 + 2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)/(x**2+1)**(1/4)/(x**2+2), x)`

[Out] `Integral((a + b*x)/((x**2 + 1)**(1/4)*(x**2 + 2)), x)`

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx + a}{(x^2 + 2)(x^2 + 1)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x + a)/((x^2 + 2)*(x^2 + 1)^(1/4)), x, algorithm="giac")`

[Out] `integrate((b*x + a)/((x^2 + 2)*(x^2 + 1)^(1/4)), x)`

$$3.71 \quad \int \frac{x}{\sqrt{1-x^3(4-x^3)}} dx$$

Optimal. Leaf size=127

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}(1-\sqrt[3]{2x})}{\sqrt{1-x^3}}\right)}{3 \cdot 2^{2/3} \sqrt{3}} + \frac{\tan^{-1}\left(\frac{\sqrt{1-x^3}}{\sqrt{3}}\right)}{3 \cdot 2^{2/3} \sqrt{3}} - \frac{\tanh^{-1}\left(\frac{\sqrt[3]{2x+1}}{\sqrt{1-x^3}}\right)}{3 \cdot 2^{2/3}} + \frac{\tanh^{-1}\left(\sqrt{1-x^3}\right)}{9 \cdot 2^{2/3}}$$

[Out] -ArcTan[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[1 - x^3]]/(3*2^(2/3)*Sqrt[3]) + ArcTan[Sqrt[1 - x^3]/Sqrt[3]]/(3*2^(2/3)*Sqrt[3]) - ArcTanh[(1 + 2^(1/3)*x)/Sqrt[1 - x^3]]/(3*2^(2/3)) + ArcTanh[Sqrt[1 - x^3]]/(9*2^(2/3))

Rubi [A] time = 0.0675644, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}(1-\sqrt[3]{2x})}{\sqrt{1-x^3}}\right)}{3 \cdot 2^{2/3} \sqrt{3}} + \frac{\tan^{-1}\left(\frac{\sqrt{1-x^3}}{\sqrt{3}}\right)}{3 \cdot 2^{2/3} \sqrt{3}} - \frac{\tanh^{-1}\left(\frac{\sqrt[3]{2x+1}}{\sqrt{1-x^3}}\right)}{3 \cdot 2^{2/3}} + \frac{\tanh^{-1}\left(\sqrt{1-x^3}\right)}{9 \cdot 2^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[x/(Sqrt[1 - x^3]*(4 - x^3)), x]

[Out] -ArcTan[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[1 - x^3]]/(3*2^(2/3)*Sqrt[3]) + ArcTan[Sqrt[1 - x^3]/Sqrt[3]]/(3*2^(2/3)*Sqrt[3]) - ArcTanh[(1 + 2^(1/3)*x)/Sqrt[1 - x^3]]/(3*2^(2/3)) + ArcTanh[Sqrt[1 - x^3]]/(9*2^(2/3))

Rubi in Sympy [A] time = 3.05896, size = 155, normalized size = 1.22

$$\frac{\sqrt[3]{2} \log\left(\sqrt[3]{2x} - \sqrt{-x^3+1} + 1\right)}{12} - \frac{\sqrt[3]{2} \log\left(\sqrt[3]{2x} + \sqrt{-x^3+1} + 1\right)}{12} - \frac{\sqrt[3]{2}\sqrt{3} \operatorname{atan}\left(\frac{\sqrt{3}}{3} - \frac{2^{2/3}\sqrt{3}(-\sqrt{-x^3+1}+1)}{3x}\right)}{18} + \frac{\sqrt[3]{2}\sqrt{3} \operatorname{atan}\left(\frac{\sqrt{3}}{3} - \frac{2^{2/3}\sqrt{3}(\sqrt{-x^3+1}+1)}{3x}\right)}{18} + \frac{\sqrt[3]{2} \operatorname{atanh}\left(\sqrt{-x^3+1}\right)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate(x/(-x**3+4)/(-x**3+1)**(1/2), x)

[Out] $2^{1/3} \log(2^{1/3} x - \sqrt{-x^3 + 1}) + 1)/12 - 2^{1/3} \log(2^{1/3} x + \sqrt{-x^3 + 1}) + 1)/12 - 2^{1/3} \sqrt{3} \operatorname{atan}(\sqrt{3}/3 - 2^{2/3} \sqrt{3} (-\sqrt{-x^3 + 1})/(3x))/18 + 2^{1/3} \sqrt{3} \operatorname{atan}(\sqrt{3}/3 - 2^{2/3} \sqrt{3} (\sqrt{-x^3 + 1})/(3x))/18 + 2^{1/3} \operatorname{atanh}(\sqrt{-x^3 + 1})/18$

Mathematica [C] time = 0.160891, size = 120, normalized size = 0.94

$$\frac{10x^2 F_1\left(\frac{2}{3}; \frac{1}{2}, 1; \frac{5}{3}; x^3, \frac{x^3}{4}\right)}{\sqrt{1-x^3}(x^3-4)\left(3x^3\left(F_1\left(\frac{5}{3}; \frac{1}{2}, 2; \frac{8}{3}; x^3, \frac{x^3}{4}\right) + 2F_1\left(\frac{5}{3}; \frac{3}{2}, 1; \frac{8}{3}; x^3, \frac{x^3}{4}\right)\right) + 20F_1\left(\frac{2}{3}; \frac{1}{2}, 1; \frac{5}{3}; x^3, \frac{x^3}{4}\right)\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x/(Sqrt[1 - x^3]*(4 - x^3)),x]

[Out] $(-10x^2 \operatorname{AppellF1}[2/3, 1/2, 1, 5/3, x^3, x^3/4]) / (\sqrt{1-x^3} * (-4 + x^3) * (20 \operatorname{AppellF1}[2/3, 1/2, 1, 5/3, x^3, x^3/4] + 3x^3 * (\operatorname{AppellF1}[5/3, 1/2, 2, 8/3, x^3, x^3/4] + 2 \operatorname{AppellF1}[5/3, 3/2, 1, 8/3, x^3, x^3/4])))$

Maple [C] time = 0.289, size = 164, normalized size = 1.3

$$\frac{i}{36} \sqrt{2} \sum_{\alpha = \operatorname{RootOf}(-Z^3-4)} \alpha^2 \left(-2\alpha^2 + \alpha + 1 + i\sqrt{3}(1-\alpha) \right) \sqrt{\frac{i}{2}(2x+1-i\sqrt{3})} \sqrt{\frac{-1+x}{i\sqrt{3}-3}} \sqrt{-\frac{i}{2}(2x+1-i\sqrt{3})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(-x^3+4)/(-x^3+1)^(1/2),x)

[Out] $1/36 * I^{2^{1/2}} * \operatorname{sum}(\alpha^2 * (1/2 * I * (2x+1-I^{3^{1/2}})))^{1/2} * ((-1+x)/(I^{3^{1/2}}-3))^{1/2} * (-1/2 * I * (2x+1-I^{3^{1/2}}))^{1/2} / (-x^3+1)^{1/2} * (-2 * \alpha^2 + \alpha + 1 + I^{3^{1/2}} * (1-\alpha)) * \operatorname{EllipticPi}(1/3 * 3^{1/2} * (I * (x+1/2-1/2 * I^{3^{1/2}}) * 3^{1/2}))^{1/2}, 1/2 * \alpha - 1/3 * I * \alpha^2 * 3^{1/2} - 1/2 + 1/6 * I * \alpha * 3^{1/2} + 1/6 * I^{3^{1/2}}, (I^{3^{1/2}})^{1/2} / (-3/2 + 1/2 * I^{3^{1/2}}))^{1/2}, \alpha = \operatorname{RootOf}(-Z^3-4)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{(x^3-4)\sqrt{-x^3+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x/((x^3 - 4)*sqrt(-x^3 + 1)),x, algorithm="maxima")
```

```
[Out] -integrate(x/((x^3 - 4)*sqrt(-x^3 + 1)), x)
```

Fricas [A] time = 0.358083, size = 1451, normalized size = 11.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x/((x^3 - 4)*sqrt(-x^3 + 1)),x, algorithm="fricas")
```

```
[Out] 1/15552*432^(5/6)*(sqrt(3)*log(2592*(1296*x^7 - 1296*x^4 + 6*2^(2/3)*(x^9 - 228*x^6 + 264*x^3 - 64) + (72*x^7 - 1872*x^4 + 432^(5/6)*sqrt(3)*(7*x^5 - 4*x^2) - 144*432^(1/6)*sqrt(3)*(x^6 - x^3) + 1152*x)*sqrt(-x^3 + 1) - 216*2^(1/3)*(x^8 - 5*x^5 + 4*x^2)))/(x^9 - 12*x^6 + 48*x^3 - 64) - sqrt(3)*log(2592*(1296*x^7 - 1296*x^4 + 6*2^(2/3)*(x^9 - 228*x^6 + 264*x^3 - 64) - (72*x^7 - 1872*x^4 + 432^(5/6)*sqrt(3)*(7*x^5 - 4*x^2) - 144*432^(1/6)*sqrt(3)*(x^6 - x^3) + 1152*x)*sqrt(-x^3 + 1) - 216*2^(1/3)*(x^8 - 5*x^5 + 4*x^2)))/(x^9 - 12*x^6 + 48*x^3 - 64)) + 8*arctan(-432*(18*x^5 + 2^(2/3)*(x^7 + 16*x^4 - 8*x) + 2^(1/3)*(5*x^6 + 20*x^3 - 16))*sqrt(-x^3 + 1)/(432^(5/6)*(x^9 + 66*x^6 - 72*x^3 + 32) - 72*sqrt(3)*2^(1/3)*(x^9 - 12*x^6 + 48*x^3 - 64) + 864*sqrt(3)*(x^8 + 7*x^5 - 8*x^2) + 1728*432^(1/6)*(x^7 + x^4 - 2*x))) + 4*arctan(-216*(6*x^8 + 4*2*x^5 - 48*x^2 - 4*432^(1/6)*sqrt(3)*(x^7 + x^4 - 2*x) - (18*x^5 + 2^(2/3)*(x^7 + 16*x^4 - 8*x) - 2*2^(1/3)*(5*x^6 + 20*x^3 - 16))*sqrt(-x^3 + 1))/(36*sqrt(2)*(x^9 - 12*x^6 + 48*x^3 - 64)*sqrt((1296*x^7 - 1296*x^4 + 6*2^(2/3)*(x^9 - 228*x^6 + 264*x^3 - 64) + (72*x^7 - 1872*x^4 + 432^(5/6)*sqrt(3)*(7*x^5 - 4*x^2) - 144*432^(1/6)*sqrt(3)*(x^6 - x^3) + 1152*x)*sqrt(-x^3 + 1) - 216*2^(1/3)*(x^8 - 5*x^5 + 4*x^2)))/(x^9 - 12*x^6 + 48*x^3 - 64)) - 432^(5/6)*(x^9 + 66*x^6 - 72*x^3 + 32) + 432*sqrt(3)*(x^8 + 7*x^5 - 8*x^2) + 216*(18*sqrt(3)*x^5 - sqrt(3)*2^(2/3)*(x^7 + 16*x^4 - 8*x))*sqrt(-x^3 + 1) + 864*432^(1/6)*(x^7 + x^4 - 2*x))) + 4*arctan(216*(6*x^8 + 42*x^5 - 48*x^2 - 4*432^(1/6)*sqrt(3)*(x^7 + x^4 - 2*x) + (18*x^5 + 2^(2/3)*(x^7 + 16*x^4 - 8*x) - 2*2^(1/3)*(5*x^6 + 20*x^3 - 16))*sqrt(-x^3 + 1))/(36*sqrt(2)*(x^9 - 12*x^6 + 48*x^3 - 64)*sqrt((1296*x^7 - 1296*x^4 + 6*2^(2/3)*(x^9 - 228*x^6 + 264*x^3 - 64) - (72*x^7 - 1872*x^4 + 432^(5/6)*sqrt(3)*(7*x^5 - 4*x^2) - 144*432^(1/6)*sqrt(3)*(x^6 - x^3) + 1152*x)*sqrt(-x^3 + 1) - 216*2^(1/3)*(x^8 - 5*x^5 + 4*x^2)))/(x^9 - 12*x^6 + 48*x^3 - 64)) - 432^(5/6)*(x^9 + 66*x^6 - 72*x^3 + 32) + 432*sqrt(3)*(x^8 + 7*x^5 - 8*x^2) - 216*(18*sqrt(3)*x^5 - sqrt(3)*2^(2/3)*(x^7 + 16*x^4 - 8*x))*sqrt(-x^3 + 1) + 864*432^(1/6)*(x^7 + x^4 - 2*x)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{x^3\sqrt{-x^3+1}-4\sqrt{-x^3+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x**3+4)/(-x**3+1)**(1/2),x)

[Out] -Integral(x/(x**3*sqrt(-x**3 + 1) - 4*sqrt(-x**3 + 1)), x)

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x}{(x^3-4)\sqrt{-x^3+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-x/((x^3 - 4)*sqrt(-x^3 + 1)),x, algorithm="giac")

[Out] integrate(-x/((x^3 - 4)*sqrt(-x^3 + 1)), x)

$$3.72 \quad \int \frac{x}{(4-dx^3)\sqrt{-1+dx^3}} dx$$

Optimal. Leaf size=157

$$\frac{\tan^{-1}\left(\frac{\sqrt[3]{2}\sqrt[3]{dx+1}}{\sqrt{dx^3-1}}\right)}{3^{2/3}d^{2/3}} - \frac{\tan^{-1}\left(\sqrt{dx^3-1}\right)}{9^{2/3}d^{2/3}} - \frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{dx}\right)}{\sqrt{dx^3-1}}\right)}{3^{2/3}\sqrt{3}d^{2/3}} - \frac{\tanh^{-1}\left(\frac{\sqrt{dx^3-1}}{\sqrt{3}}\right)}{3^{2/3}\sqrt{3}d^{2/3}}$$

[Out] -ArcTan[(1 + 2^(1/3)*d^(1/3)*x)/Sqrt[-1 + d*x^3]]/(3*2^(2/3)*d^(2/3)) - ArcTan[Sqrt[-1 + d*x^3]]/(9*2^(2/3)*d^(2/3)) - ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*d^(1/3)*x))/Sqrt[-1 + d*x^3]]/(3*2^(2/3)*Sqrt[3]*d^(2/3)) - ArcTanh[Sqrt[-1 + d*x^3]/Sqrt[3]]/(3*2^(2/3)*Sqrt[3]*d^(2/3))

Rubi [A] time = 0.100935, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$

$$\frac{\tan^{-1}\left(\frac{\sqrt[3]{2}\sqrt[3]{dx+1}}{\sqrt{dx^3-1}}\right)}{3^{2/3}d^{2/3}} - \frac{\tan^{-1}\left(\sqrt{dx^3-1}\right)}{9^{2/3}d^{2/3}} - \frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{dx}\right)}{\sqrt{dx^3-1}}\right)}{3^{2/3}\sqrt{3}d^{2/3}} - \frac{\tanh^{-1}\left(\frac{\sqrt{dx^3-1}}{\sqrt{3}}\right)}{3^{2/3}\sqrt{3}d^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[x/((4 - d*x^3)*Sqrt[-1 + d*x^3]),x]

[Out] -ArcTan[(1 + 2^(1/3)*d^(1/3)*x)/Sqrt[-1 + d*x^3]]/(3*2^(2/3)*d^(2/3)) - ArcTan[Sqrt[-1 + d*x^3]]/(9*2^(2/3)*d^(2/3)) - ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*d^(1/3)*x))/Sqrt[-1 + d*x^3]]/(3*2^(2/3)*Sqrt[3]*d^(2/3)) - ArcTanh[Sqrt[-1 + d*x^3]/Sqrt[3]]/(3*2^(2/3)*Sqrt[3]*d^(2/3))

Rubi in Sympy [A] time = 4.53822, size = 223, normalized size = 1.42

$$\frac{\sqrt[3]{2}i \log\left(\sqrt[3]{2}\sqrt[3]{dx} - i\sqrt{dx^3-1} + 1\right)}{12d^{2/3}} - \frac{\sqrt[3]{2}i \log\left(\sqrt[3]{2}\sqrt[3]{dx} + i\sqrt{dx^3-1} + 1\right)}{12d^{2/3}} + \frac{\sqrt[3]{2}\sqrt{3}i \operatorname{atan}\left(\frac{\sqrt{3}}{3} + \frac{2^{2/3}\sqrt{3}i(-\sqrt{dx^3-1}+i)}{3\sqrt[3]{dx}}\right)}{18d^{2/3}} - \frac{\sqrt[3]{2}\sqrt{3}i \operatorname{atan}\left(\frac{\sqrt{3}}{3} + \frac{2^{2/3}\sqrt{3}i(\sqrt{dx^3-1}+i)}{3\sqrt[3]{dx}}\right)}{18d^{2/3}} - \frac{\sqrt[3]{2} \operatorname{atan}\left(\sqrt{dx^3-1}\right)}{18d^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x/(-d*x**3+4)/(d*x**3-1)**(1/2),x)`

[Out] $2^{2/3} I \log(2^{2/3} d^{1/3} x - I \sqrt{d x^3 - 1} + 1) / (12 d^{2/3}) - 2^{2/3} I \log(2^{2/3} d^{1/3} x + I \sqrt{d x^3 - 1} + 1) / (12 d^{2/3}) + 2^{2/3} \sqrt{3} I \operatorname{atan}(\sqrt{3} / 3 + 2^{2/3} (2/3) \sqrt{3}) I (-\sqrt{d x^3 - 1} + I) / (3 d^{1/3} x) / (18 d^{2/3}) - 2^{2/3} \sqrt{3} I \operatorname{atan}(\sqrt{3} / 3 + 2^{2/3} (2/3) \sqrt{3}) I (\sqrt{d x^3 - 1} + I) / (3 d^{1/3} x) / (18 d^{2/3}) - 2^{2/3} \operatorname{atan}(\sqrt{d x^3 - 1}) / (18 d^{2/3})$

Mathematica [C] time = 0.249286, size = 135, normalized size = 0.86

$$\frac{10x^2 F_1\left(\frac{2}{3}; \frac{1}{2}, 1; \frac{5}{3}; dx^3, \frac{dx^3}{4}\right)}{(dx^3 - 4) \sqrt{dx^3 - 1} \left(3dx^3 \left(F_1\left(\frac{5}{3}; \frac{1}{2}, 2; \frac{8}{3}; dx^3, \frac{dx^3}{4}\right) + 2F_1\left(\frac{5}{3}; \frac{3}{2}, 1; \frac{8}{3}; dx^3, \frac{dx^3}{4}\right)\right) + 20F_1\left(\frac{2}{3}; \frac{1}{2}, 1; \frac{5}{3}; dx^3, \frac{dx^3}{4}\right)}$$

Warning: Unable to verify antiderivative.

[In] `Integrate[x/((4 - d*x^3)*Sqrt[-1 + d*x^3]),x]`

[Out] $(-10 x^2 \operatorname{AppellF1}[2/3, 1/2, 1, 5/3, d x^3, (d x^3)/4]) / ((-4 + d x^3) \operatorname{Sqrt}[-1 + d x^3] (20 \operatorname{AppellF1}[2/3, 1/2, 1, 5/3, d x^3, (d x^3)/4] + 3 d x^3 (\operatorname{AppellF1}[5/3, 1/2, 2, 8/3, d x^3, (d x^3)/4] + 2 \operatorname{AppellF1}[5/3, 3/2, 1, 8/3, d x^3, (d x^3)/4])))$

Maple [C] time = 0.153, size = 240, normalized size = 1.5

$$-\frac{i}{9} \sqrt{2} \sum_{\alpha = \operatorname{RootOf}(d Z^3 - 4)} \frac{1}{-\alpha} \sqrt{-\frac{i}{2} \left(2x + \frac{1}{\sqrt[3]{d}} + i\sqrt{3} \frac{1}{\sqrt[3]{d}}\right)} \sqrt[3]{d} \sqrt{1 \left(x - \frac{1}{\sqrt[3]{d}}\right) \left(-3 \frac{1}{\sqrt[3]{d}} - i\sqrt{3} \frac{1}{\sqrt[3]{d}}\right)^{-1}} \sqrt{\frac{i}{2} \left(2x + \frac{1}{\sqrt[3]{d}} - i\sqrt{3} \frac{1}{\sqrt[3]{d}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(-d*x^3+4)/(d*x^3-1)^(1/2),x)`

[Out] $-1/9 I^2 \sum (1/\alpha / d^{4/3}) (-1/2 I (2x + 1/d^{1/3}) + I^3 (1/2) / d^{1/3})^2 d^{1/3} ((x - 1/d^{1/3}) / (-3/d^{1/3} - I^3 (1/2) / d^{1/3}))^{1/2} (1/2 I (2x + 1/d^{1/3}) - I^3 (1/2) / d^{1/3})^2 d^{1/3})^{1/2} / (d x^3 - 1)^{1/2} (-2 \alpha^2 d + I^3 (1/2) \alpha d^{2/3} + \alpha d^{2/3} - I^3 (1/2) d^{1/3} + d^{1/3}) \operatorname{EllipticPi}(1/3^3 (1/2) (-I (x + 1/2/d^{1/3}) + 1/2 I^3 (1/2) / d^{1/3})^3 (1/2) d^{1/3})^{1/2}, 1/3 I^2 \alpha^2 (1/2) d^{2/3} - 1/6 I^2 \alpha^3 (1/2) d^{1/3} + 1/2 \alpha d^{1/3} - 1/6 I^3 (1/2) - 1/2, (-I^3 (1/2) / d^{1/3}) / (-3/2/d^{1/3} - 1/2 I^3 (1/2) / d^{1/3}))^{1/2}, \alpha = \operatorname{RootOf}(Z^3 d - 4)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{\sqrt{dx^3 - 1}(dx^3 - 4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-x/(sqrt(d*x^3 - 1)*(d*x^3 - 4)),x, algorithm="maxima")

[Out] -integrate(x/(sqrt(d*x^3 - 1)*(d*x^3 - 4)), x)

Fricas [A] time = 0.460739, size = 2641, normalized size = 16.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-x/(sqrt(d*x^3 - 1)*(d*x^3 - 4)),x, algorithm="fricas")

[Out] $\frac{1}{9} \sqrt{3} (1/432)^{1/6} (d^4)^{1/6} \arctan(6(4\sqrt{3})^{1/2})^{2/3} (d^5 x^7 + d^4 x^4 - 2d^3 x) (d^4)^{2/3} - \sqrt{3} (1/2)^{1/3} (d^4 x^8 + 7d^3 x^5 - 8d^2 x^2) (d^4)^{1/3} + (648 \sqrt{3} (1/432)^{5/6} d^5 (d^4)^{5/6} x^5 - \sqrt{3} (1/432)^{1/6} (d^3 x^7 + 16d^2 x^4 - 8d x) (d^4)^{1/6}) \sqrt{d^3 x^3 - 1} / (d^3 x^9 + 66d^2 x^6 - 72d x^3 - 24(1/2)^{2/3} (d^5 x^7 + d^4 x^4 - 2d^3 x) (d^4)^{2/3} - 6(1/2)^{1/3} (d^4 x^8 + 7d^3 x^5 - 8d^2 x^2) (d^4)^{1/3} + 6(648(1/432)^{5/6} d^5 (d^4)^{5/6} x^5 - \sqrt{3} (1/432)^{1/6} (d^3 x^7 + 16d^2 x^4 - 8d x) (d^4)^{1/6}) \sqrt{d^3 x^3 - 1} + (d^3 x^9 - 12d^2 x^6 + 48d x^3 - 64) \sqrt{d^3 x^9 - 60d^2 x^6 - 24(1/2)^{2/3} (d^5 x^7 - 5d^4 x^4 + 4d^3 x) (d^4)^{2/3} + 12(1/2)^{1/3} (d^4 x^8 + 7d^3 x^5 - 8d^2 x^2) (d^4)^{1/3} - 12(648(1/432)^{5/6} d^5 (d^4)^{5/6} x^5 - \sqrt{3} (1/432)^{1/6} (d^3 x^7 + 16d^2 x^4 - 8d x) (d^4)^{1/6}) \sqrt{d^3 x^3 - 1} + 32) / (d^3 x^9 - 12d^2 x^6 + 48d x^3 - 64) + 32) + 1/9 \sqrt{3} (1/432)^{1/6} (d^4)^{1/6} \arctan(-6(4\sqrt{3})^{1/2})^{2/3} (d^5 x^7 + d^4 x^4 - 2d^3 x) (d^4)^{2/3} - \sqrt{3} (1/2)^{1/3} (d^4 x^8 + 7d^3 x^5 - 8d^2 x^2) (d^4)^{1/3} - (648 \sqrt{3} (1/432)^{5/6} d^5 (d^4)^{5/6} x^5 - \sqrt{3} (1/432)^{1/6} (d^3 x^7 + 16d^2 x^4 - 8d x) (d^4)^{1/6}) \sqrt{d^3 x^3 - 1} / (d^3 x^9 + 66d^2 x^6 - 72d x^3 - 24(1/2)^{2/3} (d^5 x^7 + d^4 x^4 - 2d^3 x) (d^4)^{2/3} - 6(1/2)^{1/3} (d^4 x^8 + 7d^3 x^5 - 8d^2 x^2) (d^4)^{1/3} - 6(648(1/432)^{5/6} d^5 (d^4)^{5/6} x^5 - \sqrt{3} (1/432)^{1/6} (d^3 x^7 + 16d^2 x^4 - 8d x) (d^4)^{1/6}) \sqrt{d^3 x^3 - 1} + (d^3 x^9 - 12d^2 x^6 + 48d x^3 - 64) \sqrt{d^3 x^9 - 60d^2 x^6 - 24(1/2)^{2/3} (d^5 x^7 - 5d^4 x^4 + 4d^3 x) (d^4)^{2/3} + 12(1/2)^{1/3} (d^4 x^8 + 7d^3 x^5 - 8d^2 x^2) (d^4)^{1/3} - 12(648(1/432)^{5/6} d^5 (d^4)^{5/6} x^5 - \sqrt{3} (1/432)^{1/6} (d^3 x^7 + 16d^2 x^4 - 8d x) (d^4)^{1/6}) \sqrt{d^3 x^3 - 1} + 32) / (d^3 x^9 - 12d^2 x^6 + 48d x^3 - 64) + 32)$

$$\begin{aligned}
& 4 + 4*d^3*x)*(d^(-4))^(2/3) + 12*(1/2)^(1/3)*(d^4*x^8 + 7*d^3*x^5 \\
& - 8*d^2*x^2)*(d^(-4))^(1/3) + 12*(648*(1/432)^(5/6)*d^5*(d^(-4)) \\
& ^{(5/6)*x^5 - \sqrt{1/3)*(d^4*x^6 + 16*d^3*x^3 - 8*d^2)*\sqrt{d^(-4)} \\
&) - (1/432)^(1/6)*(d^3*x^7 - 2*d^2*x^4 - 8*d*x)*(d^(-4))^(1/6))*s \\
& \text{qrt}(d*x^3 - 1) + 32)/(d^3*x^9 - 12*d^2*x^6 + 48*d*x^3 - 64)) + 32 \\
&)) + 1/18*(1/432)^(1/6)*(d^(-4))^(1/6)*\log((d^3*x^9 + 66*d^2*x^6 \\
& - 72*d*x^3 + 48*(1/2)^(2/3)*(d^5*x^7 + d^4*x^4 - 2*d^3*x)*(d^(-4) \\
&)^(2/3) + 12*(1/2)^(1/3)*(d^4*x^8 + 7*d^3*x^5 - 8*d^2*x^2)*(d^(-4) \\
&))^(1/3) + 6*(1296*(1/432)^(5/6)*d^5*(d^(-4))^(5/6)*x^5 + \sqrt{1/ \\
& 3)*(5*d^4*x^6 + 20*d^3*x^3 - 16*d^2)*\sqrt{d^(-4)} + 2*(1/432)^(1/ \\
& 6)*(d^3*x^7 + 16*d^2*x^4 - 8*d*x)*(d^(-4))^(1/6))*\sqrt{d*x^3 - 1} \\
& + 32)/(d^3*x^9 - 12*d^2*x^6 + 48*d*x^3 - 64)) - 1/18*(1/432)^(1/ \\
& 6)*(d^(-4))^(1/6)*\log((d^3*x^9 + 66*d^2*x^6 - 72*d*x^3 + 48*(1/2) \\
& ^{(2/3)*(d^5*x^7 + d^4*x^4 - 2*d^3*x)*(d^(-4))^(2/3) + 12*(1/2)^(1 \\
& /3)*(d^4*x^8 + 7*d^3*x^5 - 8*d^2*x^2)*(d^(-4))^(1/3) - 6*(1296*(1 \\
& /432)^(5/6)*d^5*(d^(-4))^(5/6)*x^5 + \sqrt{1/3)*(5*d^4*x^6 + 20*d^ \\
& 3*x^3 - 16*d^2)*\sqrt{d^(-4)} + 2*(1/432)^(1/6)*(d^3*x^7 + 16*d^2* \\
& x^4 - 8*d*x)*(d^(-4))^(1/6))*\sqrt{d*x^3 - 1} + 32)/(d^3*x^9 - 12* \\
& d^2*x^6 + 48*d*x^3 - 64)) - 1/36*(1/432)^(1/6)*(d^(-4))^(1/6)*\log \\
& ((d^3*x^9 - 60*d^2*x^6 - 24*(1/2)^(2/3)*(d^5*x^7 - 5*d^4*x^4 + 4* \\
& d^3*x)*(d^(-4))^(2/3) + 12*(1/2)^(1/3)*(d^4*x^8 + 7*d^3*x^5 - 8*d \\
& ^2*x^2)*(d^(-4))^(1/3) + 12*(648*(1/432)^(5/6)*d^5*(d^(-4))^(5/6) \\
& *x^5 - \sqrt{1/3)*(d^4*x^6 + 16*d^3*x^3 - 8*d^2)*\sqrt{d^(-4)} - (1 \\
& /432)^(1/6)*(d^3*x^7 - 2*d^2*x^4 - 8*d*x)*(d^(-4))^(1/6))*\sqrt{d* \\
& x^3 - 1} + 32)/(d^3*x^9 - 12*d^2*x^6 + 48*d*x^3 - 64)) + 1/36*(1/ \\
& 432)^(1/6)*(d^(-4))^(1/6)*\log((d^3*x^9 - 60*d^2*x^6 - 24*(1/2)^(2 \\
& /3)*(d^5*x^7 - 5*d^4*x^4 + 4*d^3*x)*(d^(-4))^(2/3) + 12*(1/2)^(1/ \\
& 3)*(d^4*x^8 + 7*d^3*x^5 - 8*d^2*x^2)*(d^(-4))^(1/3) - 12*(648*(1/ \\
& 432)^(5/6)*d^5*(d^(-4))^(5/6)*x^5 - \sqrt{1/3)*(d^4*x^6 + 16*d^3*x \\
& ^3 - 8*d^2)*\sqrt{d^(-4)} - (1/432)^(1/6)*(d^3*x^7 - 2*d^2*x^4 - 8 \\
& *d*x)*(d^(-4))^(1/6))*\sqrt{d*x^3 - 1} + 32)/(d^3*x^9 - 12*d^2*x^6 \\
& + 48*d*x^3 - 64))
\end{aligned}$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{dx^3 \sqrt{dx^3 - 1} - 4 \sqrt{dx^3 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-d*x**3+4)/(d*x**3-1)**(1/2),x)

[Out] -Integral(x/(d*x**3*sqrt(d*x**3 - 1) - 4*sqrt(d*x**3 - 1)), x)

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x}{\sqrt{dx^3 - 1}(dx^3 - 4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x/(sqrt(d*x^3 - 1)*(d*x^3 - 4)),x, algorithm="giac")
```

```
[Out] integrate(-x/(sqrt(d*x^3 - 1)*(d*x^3 - 4)), x)
```

$$3.73 \quad \int \frac{x}{\sqrt{-1+x^3}(8+x^3)} dx$$

Optimal. Leaf size=74

$$\frac{1}{18} \tan^{-1} \left(\frac{(1-x)^2}{3\sqrt{x^3-1}} \right) + \frac{1}{18} \tan^{-1} \left(\frac{\sqrt{x^3-1}}{3} \right) - \frac{\tanh^{-1} \left(\frac{\sqrt{3}(1-x)}{\sqrt{x^3-1}} \right)}{6\sqrt{3}}$$

[Out] ArcTan[(1 - x)^2/(3*Sqrt[-1 + x^3])]/18 + ArcTan[Sqrt[-1 + x^3]/3]/18 - ArcTanh[(Sqrt[3]*(1 - x))/Sqrt[-1 + x^3]]/(6*Sqrt[3])

Rubi [A] time = 0.288252, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$

$$\frac{1}{18} \tan^{-1} \left(\frac{(1-x)^2}{3\sqrt{x^3-1}} \right) + \frac{1}{18} \tan^{-1} \left(\frac{\sqrt{x^3-1}}{3} \right) - \frac{\tanh^{-1} \left(\frac{\sqrt{3}(1-x)}{\sqrt{x^3-1}} \right)}{6\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[x/(Sqrt[-1 + x^3]*(8 + x^3)), x]

[Out] ArcTan[(1 - x)^2/(3*Sqrt[-1 + x^3])]/18 + ArcTan[Sqrt[-1 + x^3]/3]/18 - ArcTanh[(Sqrt[3]*(1 - x))/Sqrt[-1 + x^3]]/(6*Sqrt[3])

Rubi in Sympy [A] time = 4.32494, size = 37, normalized size = 0.5

$$\frac{x^2 \sqrt{x^3-1} \operatorname{appellf}_1 \left(\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, x^3, -\frac{x^3}{8} \right)}{16\sqrt{-x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate(x/(x**3+8)/(x**3-1)**(1/2), x)

[Out] -x**2*sqrt(x**3 - 1)*appellf1(2/3, 1/2, 1, 5/3, x**3, -x**3/8)/(16*sqrt(-x**3 + 1))

Mathematica [C] time = 0.191349, size = 118, normalized size = 1.59

$$\frac{20x^2 F_1 \left(\frac{2}{3}; \frac{1}{2}, 1; \frac{5}{3}; x^3, -\frac{x^3}{8} \right)}{\sqrt{x^3-1}(x^3+8) \left(3x^3 \left(F_1 \left(\frac{5}{3}; \frac{1}{2}, 2; \frac{8}{3}; x^3, -\frac{x^3}{8} \right) - 4F_1 \left(\frac{5}{3}; \frac{3}{2}, 1; \frac{8}{3}; x^3, -\frac{x^3}{8} \right) \right) - 40F_1 \left(\frac{2}{3}; \frac{1}{2}, 1; \frac{5}{3}; x^3, -\frac{x^3}{8} \right) \right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x/(Sqrt[-1 + x^3]*(8 + x^3)),x]

[Out] $(-20*x^2*AppellF1[2/3, 1/2, 1, 5/3, x^3, -x^3/8])/(Sqrt[-1 + x^3] * (8 + x^3) * (-40*AppellF1[2/3, 1/2, 1, 5/3, x^3, -x^3/8] + 3*x^3*(AppellF1[5/3, 1/2, 2, 8/3, x^3, -x^3/8] - 4*AppellF1[5/3, 3/2, 1, 8/3, x^3, -x^3/8])))$

Maple [C] time = 0.169, size = 286, normalized size = 3.9

$$-\frac{-\frac{3}{2}-\frac{i}{2}\sqrt{3}}{9}\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i}{2}\sqrt{3}}}\sqrt{\frac{1}{\frac{3}{2}-\frac{i}{2}\sqrt{3}}\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)}\sqrt{\frac{1}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)}\text{EllipticPi}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i}{2}\sqrt{3}}},\frac{i}{6}\sqrt{3}+\frac{1}{2},\sqrt{\frac{\frac{3}{2}+\frac{i}{2}}{\frac{3}{2}-\frac{i}{2}}}\right) + \frac{\sqrt{2}}{36}\sum_{\alpha=\text{RootOf}(-Z^2-2Z+4)}(2-\alpha)(-1-\alpha)(-i\sqrt{3}-3)\sqrt{\frac{-1+x}{-i\sqrt{3}-3}}\sqrt{\frac{2x+1-i\sqrt{3}}{-i\sqrt{3}+3}}\sqrt{\frac{2x+1+i\sqrt{3}}{i\sqrt{3}+3}}\text{EllipticPi}\left(\dots\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(x^3+8)/(x^3-1)^(1/2),x)

[Out] $-1/9*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*\text{EllipticPi}(((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2),1/6*I*3^(1/2)+1/2,((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+1/36*2^(1/2)*\text{sum}((2-\alpha)*(-1-\alpha)*(-I*3^(1/2)-3)*((-1+x)/(-I*3^(1/2)-3))^(1/2)*((2*x+1-I*3^(1/2))/(-I*3^(1/2)+3))^(1/2)*((2*x+1+I*3^(1/2))/(I*3^(1/2)+3))^(1/2)/(x^3-1)^(1/2)*\text{EllipticPi}(((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2),1/6*I*_alpha*3^(1/2)+1/2*_alpha-1/6*I*3^(1/2)-1/2,((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)),_alpha=\text{RootOf}(-Z^2-2*_Z+4))$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(x^3+8)\sqrt{x^3-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/((x^3 + 8)*sqrt(x^3 - 1)),x, algorithm="maxima")

[Out] integrate(x/((x^3 + 8)*sqrt(x^3 - 1)), x)

Fricas [A] time = 0.342107, size = 950, normalized size = 12.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/((x^3 + 8)*sqrt(x^3 - 1)),x, algorithm="fricas")

[Out] $\frac{1}{216} \sqrt{3} \log(3(x^6 + 48x^5 + 186x^4 - 56x^3 + 6\sqrt{3}(x^4 + 12x^3 + 12x^2 - 16x))\sqrt{x^3 - 1} - 120x^2 - 96x + 64)/(x^6 - 6x^5 + 24x^4 - 56x^3 + 96x^2 - 96x + 64)) - \frac{1}{216} \sqrt{3} \log(3(x^6 + 48x^5 + 186x^4 - 56x^3 - 6\sqrt{3}(x^4 + 12x^3 + 12x^2 - 16x))\sqrt{x^3 - 1} - 120x^2 - 96x + 64)/(x^6 - 6x^5 + 24x^4 - 56x^3 + 96x^2 - 96x + 64)) - \frac{1}{54} \arctan(3(9x^5 - 9x^4 - 54x^3 + \sqrt{3}(x^4 + 8x^3 - 30x^2 - 4x + 16))\sqrt{x^3 - 1} + 18x^2 + 36x)/(\sqrt{3}(x^6 - 6x^5 + 24x^4 - 56x^3 + 96x^2 - 96x + 64))\sqrt{(x^6 + 48x^5 + 186x^4 - 56x^3 + 6\sqrt{3}(x^4 + 12x^3 + 12x^2 - 16x))\sqrt{x^3 - 1} - 120x^2 - 96x + 64)/(x^6 - 6x^5 + 24x^4 - 56x^3 + 96x^2 - 96x + 64)) + \sqrt{3}(x^6 + 3x^5 - 75x^4 + 88x^3 + 6x^2 + 84x - 80) + 9(x^4 - 6x^3 - 6x^2 + 20x)\sqrt{x^3 - 1})) - \frac{1}{54} \arctan(-3(9x^5 - 9x^4 - 54x^3 - \sqrt{3}(x^4 + 8x^3 - 30x^2 - 4x + 16))\sqrt{x^3 - 1} + 18x^2 + 36x)/(\sqrt{3}(x^6 - 6x^5 + 24x^4 - 56x^3 + 96x^2 - 96x + 64))\sqrt{(x^6 + 48x^5 + 186x^4 - 56x^3 - 6\sqrt{3}(x^4 + 12x^3 + 12x^2 - 16x))\sqrt{x^3 - 1} - 120x^2 - 96x + 64)/(x^6 - 6x^5 + 24x^4 - 56x^3 + 96x^2 - 96x + 64)) + \sqrt{3}(x^6 + 3x^5 - 75x^4 + 88x^3 + 6x^2 + 84x - 80) - 9(x^4 - 6x^3 - 6x^2 + 20x)\sqrt{x^3 - 1})) + \frac{1}{54} \arctan(1/6(x^3 - 12x^2 - 6x - 10)/(\sqrt{x^3 - 1}(x - 1)))$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{(x-1)(x^2+x+1)}(x+2)(x^2-2x+4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x**3+8)/(x**3-1)**(1/2),x)

[Out] Integral(x/(sqrt((x - 1)*(x**2 + x + 1))*(x + 2)*(x**2 - 2*x + 4)), x)

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(x^3 + 8)\sqrt{x^3 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((x^3 + 8)*sqrt(x^3 - 1)),x, algorithm="giac")
```

```
[Out] integrate(x/((x^3 + 8)*sqrt(x^3 - 1)), x)
```

$$3.74 \quad \int \frac{x}{(8-dx^3)\sqrt{1+dx^3}} dx$$

Optimal. Leaf size=103

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{dx+1}\right)}{\sqrt{dx^3+1}}\right)}{6\sqrt{3}d^{2/3}} + \frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{dx+1}\right)^2}{3\sqrt{dx^3+1}}\right)}{18d^{2/3}} - \frac{\tanh^{-1}\left(\frac{1}{3}\sqrt{dx^3+1}\right)}{18d^{2/3}}$$

[Out] -ArcTan[(Sqrt[3]*(1 + d^(1/3)*x))/Sqrt[1 + d*x^3]]/(6*Sqrt[3]*d^(2/3)) + ArcTanh[(1 + d^(1/3)*x)^2/(3*Sqrt[1 + d*x^3])]/(18*d^(2/3)) - ArcTanh[Sqrt[1 + d*x^3]/3]/(18*d^(2/3))

Rubi [A] time = 0.544631, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{dx+1}\right)}{\sqrt{dx^3+1}}\right)}{6\sqrt{3}d^{2/3}} + \frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{dx+1}\right)^2}{3\sqrt{dx^3+1}}\right)}{18d^{2/3}} - \frac{\tanh^{-1}\left(\frac{1}{3}\sqrt{dx^3+1}\right)}{18d^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[x/((8 - d*x^3)*Sqrt[1 + d*x^3]),x]

[Out] -ArcTan[(Sqrt[3]*(1 + d^(1/3)*x))/Sqrt[1 + d*x^3]]/(6*Sqrt[3]*d^(2/3)) + ArcTanh[(1 + d^(1/3)*x)^2/(3*Sqrt[1 + d*x^3])]/(18*d^(2/3)) - ArcTanh[Sqrt[1 + d*x^3]/3]/(18*d^(2/3))

Rubi in Sympy [A] time = 3.31384, size = 22, normalized size = 0.21

$$\frac{x^2 \operatorname{appellf}_1\left(\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -dx^3, \frac{dx^3}{8}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate(x/(-d*x**3+8)/(d*x**3+1)**(1/2),x)

[Out] x**2*appellf1(2/3, 1/2, 1, 5/3, -d*x**3, d*x**3/8)/16

Mathematica [C] time = 0.18094, size = 139, normalized size = 1.35

$$\frac{20x^2 F_1\left(\frac{2}{3}; \frac{1}{2}, 1; \frac{5}{3}; -dx^3, \frac{dx^3}{8}\right)}{(dx^3 - 8)\sqrt{dx^3 + 1}\left(3dx^3\left(F_1\left(\frac{5}{3}; \frac{1}{2}, 2; \frac{8}{3}; -dx^3, \frac{dx^3}{8}\right) - 4F_1\left(\frac{5}{3}; \frac{3}{2}, 1; \frac{8}{3}; -dx^3, \frac{dx^3}{8}\right)\right) + 40F_1\left(\frac{2}{3}; \frac{1}{2}, 1; \frac{5}{3}; -dx^3, \frac{dx^3}{8}\right)\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x/((8 - d*x^3)*Sqrt[1 + d*x^3]),x]

[Out] (-20*x^2*AppellF1[2/3, 1/2, 1, 5/3, -(d*x^3), (d*x^3)/8])/((-8 + d*x^3)*Sqrt[1 + d*x^3]*(40*AppellF1[2/3, 1/2, 1, 5/3, -(d*x^3), (d*x^3)/8] + 3*d*x^3*(AppellF1[5/3, 1/2, 2, 8/3, -(d*x^3), (d*x^3)/8] - 4*AppellF1[5/3, 3/2, 1, 8/3, -(d*x^3), (d*x^3)/8]))

Maple [C] time = 0.197, size = 383, normalized size = 3.7

$$\frac{-\frac{i}{27}\sqrt{2}}{d^3} \sum_{\alpha=\text{RootOf}(dZ^3-8)} \frac{1}{-\alpha} \sqrt[3]{-d^2} \sqrt{\frac{i}{2}d\left(2x + \frac{1}{d}\left(-i\sqrt{3}\sqrt{-d^2} + \sqrt[3]{-d^2}\right)\right)} \frac{1}{\sqrt[3]{-d^2}} \sqrt{d\left(x - \frac{1}{d}\sqrt[3]{-d^2}\right)} \left(-3\sqrt[3]{-d^2} + i\sqrt{3}\sqrt[3]{-d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(-d*x^3+8)/(d*x^3+1)^(1/2),x)

[Out] -1/27*I/d^3*2^(1/2)*sum(1/_alpha*(-d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2)^(1/3)+(-d^2)^(1/3)))/(-d^2)^(1/3))^1/2*(d*(x-1/d*(-d^2)^(1/3)))/(-3*(-d^2)^(1/3)+I*3^(1/2)*(-d^2)^(1/3))^1/2*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2)^(1/3)+(-d^2)^(1/3)))/(-d^2)^(1/3))^1/2/(d*x^3+1)^(1/2)*(I*(-d^2)^(1/3)*_alpha^3^(1/2)*d-I*3^(1/2)*(-d^2)^(2/3)+2*_alpha^2*d^2-(-d^2)^(1/3)*_alpha*d-(-d^2)^(2/3)*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2)^(1/3)-1/2*I*3^(1/2)/d*(-d^2)^(1/3))*3^(1/2)*d/(-d^2)^(1/3))^1/2,-1/18/d*(2*I*(-d^2)^(1/3)*_alpha^2*3^(1/2)*d-I*(-d^2)^(2/3)*_alpha^3^(1/2)-3*(-d^2)^(2/3)*_alpha+I*3^(1/2)*d-3*d),(I*3^(1/2)/d*(-d^2)^(1/3)/(-3/2/d*(-d^2)^(1/3)+1/2*I*3^(1/2)/d*(-d^2)^(1/3)))^1/2),_alpha=RootOf(_Z^3*d-8))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{\sqrt{dx^3 + 1}(dx^3 - 8)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-x/(sqrt(d*x^3 + 1)*(d*x^3 - 8)),x, algorithm="maxima")

[Out] -integrate(x/(sqrt(d*x^3 + 1)*(d*x^3 - 8)), x)

Fricas [A] time = 0.328065, size = 914, normalized size = 8.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-x/(sqrt(d*x^3 + 1)*(d*x^3 - 8)),x, algorithm="fricas")

[Out]
$$\frac{1}{324} \sqrt{3} (2 \sqrt{3} \log((18 d^4 x^8 + 684 d^3 x^5 + 1152 d^2 x^2 + (d^3 x^9 + 318 d^2 x^6 + 1200 d x^3 + 640) (d^2)^{2/3} + 6 (30 d^3 x^5 + 192 d^2 x^2 + (7 d^2 x^6 + 152 d x^3 + 64) (d^2)^{2/3} + (d^3 x^7 + 80 d^2 x^4 + 160 d x) (d^2)^{1/3})) \sqrt{d x^3 + 1} + 18 (5 d^3 x^7 + 64 d^2 x^4 + 32 d x) (d^2)^{1/3}) / (d^3 x^9 - 24 d^2 x^6 + 192 d x^3 - 512)) - \sqrt{3} \log(-(18 d^3 x^7 - 936 d^2 x^4 - 1440 d x - 18 (d^2 x^8 + 20 d x^5 - 8 x^2) (d^2)^{2/3} + 6 (d^3 x^7 - 28 d^2 x^4 - 272 d x - 24 (d x^5 + x^2) (d^2)^{2/3} + 4 (d^2 x^6 + 41 d x^3 + 40) (d^2)^{1/3})) \sqrt{d x^3 + 1} - (d^3 x^9 - 276 d^2 x^6 - 1608 d x^3 - 1088) (d^2)^{1/3}) / (d^3 x^9 - 24 d^2 x^6 + 192 d x^3 - 512)) + 6 \arctan(-1/9 (\sqrt{3} (d^3 x^9 + 318 d^2 x^6 + 1200 d x^3 + 640) (d^2)^{2/3} - 9 \sqrt{3} (d^4 x^8 + 38 d^3 x^5 + 64 d^2 x^2 + 32 d x) (d^2)^{1/3} - 9 \sqrt{3} (d^4 x^8 + 38 d^3 x^5 + 64 d^2 x^2 + 32 d x) (d^2)^{1/3} - 9 \sqrt{3} (d^4 x^8 + 38 d^3 x^5 + 64 d^2 x^2 + 32 d x) (d^2)^{1/3} - 6 \sqrt{3} (5 d^3 x^5 + 32 d^2 x^2))) / (3 d^4 x^8 + 114 d^3 x^5 + 192 d^2 x^2 + (30 d^3 x^5 + 192 d^2 x^2 - (d^3 x^7 + 80 d^2 x^4 + 160 d x) (d^2)^{1/3})) \sqrt{d x^3 + 1} - 3 (5 d^3 x^7 + 64 d^2 x^4 + 32 d x) (d^2)^{1/3})) / (d^2)^{1/3}$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{dx^3 \sqrt{dx^3 + 1} - 8 \sqrt{dx^3 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-d*x**3+8)/(d*x**3+1)**(1/2),x)

[Out] -Integral(x/(d*x**3*sqrt(d*x**3 + 1) - 8*sqrt(d*x**3 + 1)), x)

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x}{\sqrt{dx^3 + 1}(dx^3 - 8)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x/(sqrt(d*x^3 + 1)*(d*x^3 - 8)),x, algorithm="giac")
```

```
[Out] integrate(-x/(sqrt(d*x^3 + 1)*(d*x^3 - 8)), x)
```

$$3.75 \quad \int \frac{1}{\sqrt[3]{1-3x^2(3-x^2)}} dx$$

Optimal. Leaf size=81

$$\frac{1}{4} \tan^{-1} \left(\frac{1 - \sqrt[3]{1-3x^2}}{x} \right) - \frac{\tanh^{-1} \left(\frac{(1 - \sqrt[3]{1-3x^2})^2}{3\sqrt{3}x} \right)}{4\sqrt{3}} + \frac{\tanh^{-1} \left(\frac{x}{\sqrt{3}} \right)}{4\sqrt{3}}$$

[Out] ArcTan[(1 - (1 - 3*x^2)^(1/3))/x]/4 + ArcTanh[x/Sqrt[3]]/(4*Sqrt[3]) - ArcTanh[(1 - (1 - 3*x^2)^(1/3))^2/(3*Sqrt[3]*x)]/(4*Sqrt[3])

Rubi [A] time = 0.0404513, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$

$$\frac{1}{4} \tan^{-1} \left(\frac{1 - \sqrt[3]{1-3x^2}}{x} \right) - \frac{\tanh^{-1} \left(\frac{(1 - \sqrt[3]{1-3x^2})^2}{3\sqrt{3}x} \right)}{4\sqrt{3}} + \frac{\tanh^{-1} \left(\frac{x}{\sqrt{3}} \right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[1/((1 - 3*x^2)^(1/3)*(3 - x^2)), x]

[Out] ArcTan[(1 - (1 - 3*x^2)^(1/3))/x]/4 + ArcTanh[x/Sqrt[3]]/(4*Sqrt[3]) - ArcTanh[(1 - (1 - 3*x^2)^(1/3))^2/(3*Sqrt[3]*x)]/(4*Sqrt[3])

Rubi in Sympy [A] time = 2.60271, size = 17, normalized size = 0.21

$$\frac{x \operatorname{appellf}_1 \left(\frac{1}{2}, \frac{1}{3}, 1, \frac{3}{2}, 3x^2, \frac{x^2}{3} \right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate(1/(-3*x**2+1)**(1/3)/(-x**2+3), x)

[Out] x*appellf1(1/2, 1/3, 1, 3/2, 3*x**2, x**2/3)/3

Mathematica [C] time = 0.151867, size = 126, normalized size = 1.56

$$\frac{9x F_1\left(\frac{1}{2}, \frac{1}{3}, 1; \frac{3}{2}; 3x^2, \frac{x^2}{3}\right)}{\sqrt[3]{1-3x^2}(x^2-3)\left(2x^2\left(F_1\left(\frac{3}{2}, \frac{1}{3}, 2; \frac{5}{2}; 3x^2, \frac{x^2}{3}\right) + 3F_1\left(\frac{3}{2}, \frac{4}{3}, 1; \frac{5}{2}; 3x^2, \frac{x^2}{3}\right)\right) + 9F_1\left(\frac{1}{2}, \frac{1}{3}, 1; \frac{3}{2}; 3x^2, \frac{x^2}{3}\right)\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/((1 - 3*x^2)^(1/3)*(3 - x^2)), x]

[Out] (-9*x*AppellF1[1/2, 1/3, 1, 3/2, 3*x^2, x^2/3])/((1 - 3*x^2)^(1/3)*(-3 + x^2)*(9*AppellF1[1/2, 1/3, 1, 3/2, 3*x^2, x^2/3] + 2*x^2*(AppellF1[3/2, 1/3, 2, 5/2, 3*x^2, x^2/3] + 3*AppellF1[3/2, 4/3, 1, 5/2, 3*x^2, x^2/3])))

Maple [F] time = 0.07, size = 0, normalized size = 0.

$$\int \frac{1}{-x^2 + 3} \frac{1}{\sqrt[3]{-3x^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-3*x^2+1)^(1/3)/(-x^2+3), x)

[Out] int(1/(-3*x^2+1)^(1/3)/(-x^2+3), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{(x^2 - 3)(-3x^2 + 1)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-1/((x^2 - 3)*(-3*x^2 + 1)^(1/3)), x, algorithm="maxima")

[Out] -integrate(1/((x^2 - 3)*(-3*x^2 + 1)^(1/3)), x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-1/((x^2 - 3)*(-3*x^2 + 1)^(1/3)),x, algorithm="fricas")`

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{x^2 \sqrt[3]{-3x^2 + 1} - 3 \sqrt[3]{-3x^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-3*x**2+1)**(1/3)/(-x**2+3),x)`

[Out] `-Integral(1/(x**2*(-3*x**2 + 1)**(1/3) - 3*(-3*x**2 + 1)**(1/3)), x)`

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{(x^2 - 3)(-3x^2 + 1)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-1/((x^2 - 3)*(-3*x^2 + 1)^(1/3)),x, algorithm="giac")`

[Out] `integrate(-1/((x^2 - 3)*(-3*x^2 + 1)^(1/3)), x)`

$$3.76 \quad \int \frac{1}{(3+x^2)\sqrt[3]{1+3x^2}} dx$$

Optimal. Leaf size=81

$$\frac{\tan^{-1}\left(\frac{(1-\sqrt[3]{3x^2+1})^2}{3\sqrt{3}x}\right)}{4\sqrt{3}} - \frac{1}{4} \tanh^{-1}\left(\frac{1-\sqrt[3]{3x^2+1}}{x}\right) + \frac{\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{4\sqrt{3}}$$

[Out] ArcTan[x/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[(1 - (1 + 3*x^2)^(1/3))^2/(3*Sqrt[3]*x)]/(4*Sqrt[3]) - ArcTanh[(1 - (1 + 3*x^2)^(1/3))/x]/4

Rubi [A] time = 0.0369219, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$

$$\frac{\tan^{-1}\left(\frac{(1-\sqrt[3]{3x^2+1})^2}{3\sqrt{3}x}\right)}{4\sqrt{3}} - \frac{1}{4} \tanh^{-1}\left(\frac{1-\sqrt[3]{3x^2+1}}{x}\right) + \frac{\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[1/((3 + x^2)*(1 + 3*x^2)^(1/3)), x]

[Out] ArcTan[x/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[(1 - (1 + 3*x^2)^(1/3))^2/(3*Sqrt[3]*x)]/(4*Sqrt[3]) - ArcTanh[(1 - (1 + 3*x^2)^(1/3))/x]/4

Rubi in Sympy [A] time = 3.07163, size = 20, normalized size = 0.25

$$\frac{x \operatorname{appellf}_1\left(\frac{1}{2}, \frac{1}{3}, 1, \frac{3}{2}, -3x^2, -\frac{x^2}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate(1/(x**2+3)/(3*x**2+1)**(1/3), x)

[Out] x*appellf1(1/2, 1/3, 1, 3/2, -3*x**2, -x**2/3)/3

Mathematica [C] time = 0.140462, size = 126, normalized size = 1.56

$$\frac{9x F_1\left(\frac{1}{2}; \frac{1}{3}, 1; \frac{3}{2}; -3x^2, -\frac{x^2}{3}\right)}{(x^2 + 3) \sqrt[3]{3x^2 + 1} \left(2x^2 \left(F_1\left(\frac{3}{2}; \frac{1}{3}, 2; \frac{5}{2}; -3x^2, -\frac{x^2}{3}\right) + 3F_1\left(\frac{3}{2}; \frac{4}{3}, 1; \frac{5}{2}; -3x^2, -\frac{x^2}{3}\right)\right) - 9F_1\left(\frac{1}{2}; \frac{1}{3}, 1; \frac{3}{2}; -3x^2, -\frac{x^2}{3}\right)\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/((3 + x^2)*(1 + 3*x^2)^(1/3)), x]

[Out] (-9*x*AppellF1[1/2, 1/3, 1, 3/2, -3*x^2, -x^2/3])/((3 + x^2)*(1 + 3*x^2)^(1/3))*(-9*AppellF1[1/2, 1/3, 1, 3/2, -3*x^2, -x^2/3] + 2*x^2*(AppellF1[3/2, 1/3, 2, 5/2, -3*x^2, -x^2/3] + 3*AppellF1[3/2, 4/3, 1, 5/2, -3*x^2, -x^2/3]))

Maple [F] time = 0.04, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 + 3} \frac{1}{\sqrt[3]{3x^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2+3)/(3*x^2+1)^(1/3), x)

[Out] int(1/(x^2+3)/(3*x^2+1)^(1/3), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3x^2 + 1)^{\frac{1}{3}}(x^2 + 3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((3*x^2 + 1)^(1/3)*(x^2 + 3)), x, algorithm="maxima")

[Out] integrate(1/((3*x^2 + 1)^(1/3)*(x^2 + 3)), x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((3*x^2 + 1)^(1/3)*(x^2 + 3)),x, algorithm="fricas")`

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2 + 3)\sqrt[3]{3x^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x**2+3)/(3*x**2+1)**(1/3),x)`

[Out] `Integral(1/((x**2 + 3)*(3*x**2 + 1)**(1/3)), x)`

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3x^2 + 1)^{\frac{1}{3}}(x^2 + 3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((3*x^2 + 1)^(1/3)*(x^2 + 3)),x, algorithm="giac")`

[Out] `integrate(1/((3*x^2 + 1)^(1/3)*(x^2 + 3)), x)`

$$3.77 \quad \int \frac{1}{\sqrt[3]{1-x^2(3+x^2)}} dx$$

Optimal. Leaf size=113

$$\frac{\tan^{-1}\left(\frac{\sqrt{3}(1-\sqrt[3]{2}\sqrt[3]{1-x^2})}{x}\right)}{2 \cdot 2^{2/3}\sqrt{3}} + \frac{\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2+1}}\right)}{2 \cdot 2^{2/3}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2 \cdot 2^{2/3}\sqrt{3}} - \frac{\tanh^{-1}(x)}{6 \cdot 2^{2/3}}$$

[Out] ArcTan[Sqrt[3]/x]/(2*2^(2/3)*Sqrt[3]) + ArcTan[(Sqrt[3]*(1 - 2^(1/3)*(1 - x^2)^(1/3)))/x]/(2*2^(2/3)*Sqrt[3]) - ArcTanh[x]/(6*2^(2/3)) + ArcTanh[x/(1 + 2^(1/3)*(1 - x^2)^(1/3))]/(2*2^(2/3))

Rubi [A] time = 0.0461671, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$

$$\frac{\tan^{-1}\left(\frac{\sqrt{3}(1-\sqrt[3]{2}\sqrt[3]{1-x^2})}{x}\right)}{2 \cdot 2^{2/3}\sqrt{3}} + \frac{\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2+1}}\right)}{2 \cdot 2^{2/3}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2 \cdot 2^{2/3}\sqrt{3}} - \frac{\tanh^{-1}(x)}{6 \cdot 2^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[1/((1 - x^2)^(1/3)*(3 + x^2)), x]

[Out] ArcTan[Sqrt[3]/x]/(2*2^(2/3)*Sqrt[3]) + ArcTan[(Sqrt[3]*(1 - 2^(1/3)*(1 - x^2)^(1/3)))/x]/(2*2^(2/3)*Sqrt[3]) - ArcTanh[x]/(6*2^(2/3)) + ArcTanh[x/(1 + 2^(1/3)*(1 - x^2)^(1/3))]/(2*2^(2/3))

Rubi in Sympy [A] time = 5.86888, size = 144, normalized size = 1.27

$$\frac{\sqrt[3]{2} \log\left(\sqrt[3]{2}\sqrt[3]{-x+1} + (x+1)^{\frac{2}{3}}\right)}{8} - \frac{\sqrt[3]{2} \log\left((-x+1)^{\frac{2}{3}} + \sqrt[3]{2}\sqrt[3]{x+1}\right)}{8} - \frac{\sqrt[3]{2}\sqrt{3} \operatorname{atan}\left(\frac{\sqrt{3}}{3} - \frac{2^{\frac{2}{3}}\sqrt{3}(x+1)^{\frac{2}{3}}}{3\sqrt[3]{-x+1}}\right)}{12} - \frac{\sqrt[3]{2}\sqrt{3} \operatorname{atan}\left(\frac{2^{\frac{2}{3}}\sqrt{3}(-x+1)^{\frac{2}{3}}}{3\sqrt[3]{x+1}} - \frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate(1/(-x**2+1)**(1/3)/(x**2+3), x)

[Out] 2**(1/3)*log(2**(1/3)*(-x + 1)**(1/3) + (x + 1)**(2/3))/8 - 2**(1/3)*log((-x + 1)**(2/3) + 2**(1/3)*(x + 1)**(1/3))/8 - 2**(1/3)*s

$$\sqrt[3]{3} \operatorname{atan}\left(\frac{\sqrt{3}}{3} - 2 \cdot \left(\frac{2}{3}\right)^{1/3} \sqrt{3} (x+1)^{2/3} / (3 \cdot (-x+1)^{1/3})\right) / 12 - 2 \cdot \left(\frac{1}{3}\right)^{1/3} \sqrt{3} \operatorname{atan}\left(2 \cdot \left(\frac{2}{3}\right)^{1/3} \sqrt{3} (-x+1)^{2/3} / (3 \cdot (x+1)^{1/3})\right) - \sqrt{3} / 12$$

Mathematica [C] time = 0.130754, size = 118, normalized size = 1.04

$$\frac{9x F_1\left(\frac{1}{2}, \frac{1}{3}, 1; \frac{3}{2}; x^2, -\frac{x^2}{3}\right)}{\sqrt[3]{1-x^2}(x^2+3) \left(2x^2 \left(F_1\left(\frac{3}{2}, \frac{1}{3}, 2; \frac{5}{2}; x^2, -\frac{x^2}{3}\right) - F_1\left(\frac{3}{2}, \frac{4}{3}, 1; \frac{5}{2}; x^2, -\frac{x^2}{3}\right)\right) - 9F_1\left(\frac{1}{2}, \frac{1}{3}, 1; \frac{3}{2}; x^2, -\frac{x^2}{3}\right)\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/((1 - x^2)^(1/3) * (3 + x^2)), x]

[Out] (-9*x*AppellF1[1/2, 1/3, 1, 3/2, x^2, -x^2/3])/((1 - x^2)^(1/3) * (3 + x^2) * (-9*AppellF1[1/2, 1/3, 1, 3/2, x^2, -x^2/3] + 2*x^2*(AppellF1[3/2, 1/3, 2, 5/2, x^2, -x^2/3] - AppellF1[3/2, 4/3, 1, 5/2, x^2, -x^2/3])))

Maple [F] time = 0.05, size = 0, normalized size = 0.

$$\int \frac{1}{x^2+3} \frac{1}{\sqrt[3]{-x^2+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-x^2+1)^(1/3)/(x^2+3), x)

[Out] int(1/(-x^2+1)^(1/3)/(x^2+3), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2+3)(-x^2+1)^{1/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)), x, algorithm="maxima")

[Out] integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)),x, algorithm="fricas")`

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{-(x-1)(x+1)(x^2+3)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-x**2+1)**(1/3)/(x**2+3)),x)`

[Out] `Integral(1/((-x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)`

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2+3)(-x^2+1)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)),x, algorithm="giac")`

[Out] `integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)`

$$3.78 \quad \int \frac{1}{(3-x^2)\sqrt[3]{1+x^2}} dx$$

Optimal. Leaf size=109

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{x^2+1}}\right)}{2 \cdot 2^{2/3}} - \frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{x^2+1}\right)}{x}\right)}{2 \cdot 2^{2/3}\sqrt{3}} - \frac{\tan^{-1}(x)}{6 \cdot 2^{2/3}} - \frac{\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2 \cdot 2^{2/3}\sqrt{3}}$$

[Out] -ArcTan[x]/(6*2^(2/3)) + ArcTan[x/(1 + 2^(1/3)*(1 + x^2)^(1/3))]/(2*2^(2/3)) - ArcTanh[Sqrt[3]/x]/(2*2^(2/3)*Sqrt[3]) - ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*(1 + x^2)^(1/3)))/x]/(2*2^(2/3)*Sqrt[3])

Rubi [A] time = 0.0437177, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{x^2+1}}\right)}{2 \cdot 2^{2/3}} - \frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{x^2+1}\right)}{x}\right)}{2 \cdot 2^{2/3}\sqrt{3}} - \frac{\tan^{-1}(x)}{6 \cdot 2^{2/3}} - \frac{\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2 \cdot 2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[1/((3 - x^2)*(1 + x^2)^(1/3)), x]

[Out] -ArcTan[x]/(6*2^(2/3)) + ArcTan[x/(1 + 2^(1/3)*(1 + x^2)^(1/3))]/(2*2^(2/3)) - ArcTanh[Sqrt[3]/x]/(2*2^(2/3)*Sqrt[3]) - ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*(1 + x^2)^(1/3)))/x]/(2*2^(2/3)*Sqrt[3])

Rubi in Sympy [A] time = 5.23769, size = 192, normalized size = 1.76

$$\frac{\sqrt[3]{2}\sqrt{3} \log(-x + \sqrt{3})}{24} - \frac{\sqrt[3]{2}\sqrt{3} \log(x + \sqrt{3})}{24} + \frac{\sqrt[3]{2}\sqrt{3} \log(-x - \sqrt[3]{2}\sqrt{3}\sqrt[3]{x^2+1} + \sqrt{3})}{24} - \frac{\sqrt[3]{2}\sqrt{3} \log(x - \sqrt[3]{2}\sqrt{3}\sqrt[3]{x^2+1} + \sqrt{3})}{24} - \frac{\sqrt[3]{2} \operatorname{atan}\left(\frac{2^{2/3}(-x+\sqrt{3})}{3\sqrt[3]{x^2+1}} + \frac{\sqrt{3}}{3}\right)}{12} + \frac{\sqrt[3]{2} \operatorname{atan}\left(\frac{2^{2/3}(x+\sqrt{3})}{3\sqrt[3]{x^2+1}} + \frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate(1/(-x**2+3)/(x**2+1)**(1/3), x)

[Out] $2^{1/3} \sqrt{3} \log(-x + \sqrt{3})/24 - 2^{1/3} \sqrt{3} \log(x + \sqrt{3})/24 + 2^{1/3} \sqrt{3} \log(-x - 2^{1/3} \sqrt{3} (x^2 + 1)^{1/3} + \sqrt{3})/24 - 2^{1/3} \sqrt{3} \log(x - 2^{1/3} \sqrt{3} (x^2 + 1)^{1/3} + \sqrt{3})/24 - 2^{1/3} \operatorname{atan}(2^{2/3} (-x + \sqrt{3}) / (3 (x^2 + 1)^{1/3})) + \sqrt{3}/3/12 + 2^{1/3} \operatorname{atan}(2^{2/3} (x + \sqrt{3}) / (3 (x^2 + 1)^{1/3})) + \sqrt{3}/3/12$

Mathematica [C] time = 0.10238, size = 124, normalized size = 1.14

$$\frac{9x F_1\left(\frac{1}{2}; \frac{1}{3}, 1; \frac{3}{2}; -x^2, \frac{x^2}{3}\right)}{(x^2 - 3) \sqrt[3]{x^2 + 1} \left(2x^2 \left(F_1\left(\frac{3}{2}; \frac{1}{3}, 2; \frac{5}{2}; -x^2, \frac{x^2}{3}\right) - F_1\left(\frac{3}{2}; \frac{4}{3}, 1; \frac{5}{2}; -x^2, \frac{x^2}{3}\right)\right) + 9F_1\left(\frac{1}{2}; \frac{1}{3}, 1; \frac{3}{2}; -x^2, \frac{x^2}{3}\right)}\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/((3 - x^2)*(1 + x^2)^(1/3)),x]

[Out] $(-9x \operatorname{AppellF1}[1/2, 1/3, 1, 3/2, -x^2, x^2/3]) / ((-3 + x^2) (1 + x^2)^{1/3} (9 \operatorname{AppellF1}[1/2, 1/3, 1, 3/2, -x^2, x^2/3] + 2x^2 (\operatorname{AppellF1}[3/2, 1/3, 2, 5/2, -x^2, x^2/3] - \operatorname{AppellF1}[3/2, 4/3, 1, 5/2, -x^2, x^2/3])))$

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int \frac{1}{-x^2 + 3} \frac{1}{\sqrt[3]{x^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-x^2+3)/(x^2+1)^(1/3),x)

[Out] int(1/(-x^2+3)/(x^2+1)^(1/3),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{(x^2 + 1)^{1/3} (x^2 - 3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-1/((x^2 + 1)^(1/3)*(x^2 - 3)),x, algorithm="maxima")

[Out] `-integrate(1/((x^2 + 1)^(1/3)*(x^2 - 3)), x)`

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-1/((x^2 + 1)^(1/3)*(x^2 - 3)),x, algorithm="fricas")`

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{x^2 \sqrt[3]{x^2 + 1} - 3 \sqrt[3]{x^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-x**2+3)/(x**2+1)**(1/3),x)`

[Out] `-Integral(1/(x**2*(x**2 + 1)**(1/3) - 3*(x**2 + 1)**(1/3)), x)`

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{(x^2 + 1)^{\frac{1}{3}}(x^2 - 3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-1/((x^2 + 1)^(1/3)*(x^2 - 3)),x, algorithm="giac")`

[Out] `integrate(-1/((x^2 + 1)^(1/3)*(x^2 - 3)), x)`

$$3.79 \quad \int \frac{a+x}{(-a+x)\sqrt{a^2x-(1+a^2)x^2+x^3}} dx$$

Optimal. Leaf size=87

$$\frac{2\sqrt{x}\sqrt{-(a^2+1)x+a^2+x^2}\tan^{-1}\left(\frac{(1-a)\sqrt{x}}{\sqrt{-(a^2+1)x+a^2+x^2}}\right)}{(1-a)\sqrt{-(a^2+1)x^2+a^2x+x^3}}$$

[Out] (-2*Sqrt[x]*Sqrt[a^2 - (1 + a^2)*x + x^2]*ArcTan[((1 - a)*Sqrt[x])/Sqrt[a^2 - (1 + a^2)*x + x^2]])/((1 - a)*Sqrt[a^2*x - (1 + a^2)*x^2 + x^3])

Rubi [A] time = 1.47925, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$

$$\frac{2\sqrt{x}\sqrt{-(a^2+1)x+a^2+x^2}\tan^{-1}\left(\frac{(1-a)\sqrt{x}}{\sqrt{-(a^2+1)x+a^2+x^2}}\right)}{(1-a)\sqrt{-(a^2+1)x^2+a^2x+x^3}}$$

Antiderivative was successfully verified.

[In] Int[(a + x)/((-a + x)*Sqrt[a^2*x - (1 + a^2)*x^2 + x^3]), x]

[Out] (-2*Sqrt[x]*Sqrt[a^2 - (1 + a^2)*x + x^2]*ArcTan[((1 - a)*Sqrt[x])/Sqrt[a^2 - (1 + a^2)*x + x^2]])/((1 - a)*Sqrt[a^2*x - (1 + a^2)*x^2 + x^3])

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate((a+x)/(-a+x)/(a**2*x-(a**2+1)*x**2+x**3)**(1/2), x)

[Out] Timed out

Mathematica [C] time = 0.553469, size = 159, normalized size = 1.83

$$\frac{2i(a^2-x)^{3/2}\sqrt{\frac{x-1}{x-a^2}}\sqrt{\frac{x}{x-a^2}}\left((a+1)F\left(i\sinh^{-1}\left(\frac{\sqrt{-a^2}}{\sqrt{a^2-x}}\right)\middle|1-\frac{1}{a^2}\right)-2\left(\frac{a-1}{a};i\sinh^{-1}\left(\frac{\sqrt{-a^2}}{\sqrt{a^2-x}}\right)\middle|1-\frac{1}{a^2}\right)\right)}{(a-1)\sqrt{-a^2}\sqrt{(x-1)x(x-a^2)}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + x)/((-a + x)*Sqrt[a^2*x - (1 + a^2)*x^2 + x^3]),x]

[Out] ((-2*I)*(a^2 - x)^(3/2)*Sqrt[(-1 + x)/(-a^2 + x)]*Sqrt[x/(-a^2 + x)]*((1 + a)*EllipticF[I*ArcSinh[Sqrt[-a^2]/Sqrt[a^2 - x]], 1 - a^(-2)] - 2*EllipticPi[(-1 + a)/a, I*ArcSinh[Sqrt[-a^2]/Sqrt[a^2 - x]], 1 - a^(-2)]))/((-1 + a)*Sqrt[-a^2]*Sqrt[(-1 + x)*x*(-a^2 + x)])

Maple [C] time = 0.053, size = 206, normalized size = 2.4

$$-2 \frac{a^2}{\sqrt{-a^2x^2 + a^2x + x^3 - x^2}} \sqrt{\frac{-a^2 + x}{a^2}} \sqrt{\frac{-1 + x}{a^2 - 1}} \sqrt{\frac{x}{a^2}} \text{EllipticF} \left(\sqrt{\frac{-a^2 + x}{a^2}}, \sqrt{\frac{a^2}{a^2 - 1}} \right) - 4 \frac{a^3}{\sqrt{-a^2x^2 + a^2x + x^3 - x^2} (a^2 - a)} \sqrt{\frac{-a^2 + x}{a^2}} \sqrt{\frac{-1 + x}{a^2 - 1}} \sqrt{\frac{x}{a^2}} \text{EllipticPi} \left(\sqrt{\frac{-a^2 + x}{a^2}}, \frac{a^2}{a^2 - a}, \sqrt{\frac{a^2}{a^2 - 1}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+x)/(-a+x)/(a^2*x-(a^2+1)*x^2+x^3)^(1/2),x)

[Out] -2*a^2*(-(-a^2+x)/a^2)^(1/2)*((-1+x)/(a^2-1))^(1/2)*(x/a^2)^(1/2)/(-a^2*x^2+a^2*x+x^3-x^2)^(1/2)*EllipticF((-(-a^2+x)/a^2)^(1/2),(a^2/(a^2-1))^(1/2))-4*a^3*(-(-a^2+x)/a^2)^(1/2)*((-1+x)/(a^2-1))^(1/2)*(x/a^2)^(1/2)/(-a^2*x^2+a^2*x+x^3-x^2)^(1/2)/(a^2-a)*EllipticPi((-(-a^2+x)/a^2)^(1/2),a^2/(a^2-a),(a^2/(a^2-1))^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$- \int \frac{a + x}{\sqrt{a^2x - (a^2 + 1)x^2 + x^3}(a - x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-(a + x)/(sqrt(a^2*x - (a^2 + 1)*x^2 + x^3)*(a - x)),x, algorithm="maxima")

[Out] -integrate((a + x)/(sqrt(a^2*x - (a^2 + 1)*x^2 + x^3)*(a - x)), x)

Fricas [A] time = 0.265928, size = 72, normalized size = 0.83

$$\frac{\arctan\left(\frac{a^2 - 2(a^2 - a + 1)x + x^2}{2\sqrt{a^2x - (a^2 + 1)x^2 + x^3(a - 1)}}\right)}{a - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-(a + x)/(sqrt(a^2*x - (a^2 + 1)*x^2 + x^3)*(a - x)),x, algorithm="fricas")

[Out] arctan(1/2*(a^2 - 2*(a^2 - a + 1)*x + x^2)/(sqrt(a^2*x - (a^2 + 1)*x^2 + x^3)*(a - 1)))/(a - 1)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + x}{\sqrt{x(-a^2 + x)(x - 1)}(-a + x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+x)/(-a+x)/(a**2*x-(a**2+1)*x**2+x**3)**(1/2),x)

[Out] Integral((a + x)/(sqrt(x*(-a**2 + x)*(x - 1))*(-a + x)), x)

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{a + x}{\sqrt{a^2x - (a^2 + 1)x^2 + x^3(a - x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-(a + x)/(sqrt(a^2*x - (a^2 + 1)*x^2 + x^3)*(a - x)),x, algorithm="giac")

[Out] integrate(-(a + x)/(sqrt(a^2*x - (a^2 + 1)*x^2 + x^3)*(a - x)), x)

$$3.80 \quad \int \frac{-2+a+x}{(-a+x)\sqrt{(2-a)ax+(-1-2a+a^2)x^2+x^3}} dx$$

Optimal. Leaf size=1

0

[Out] 0

Rubi [C] time = 3.07177, antiderivative size = 579, normalized size of antiderivative = 579., number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$\frac{2(1-a)\sqrt{x}\sqrt{-(-a^2+2a+1)x+(2-a)a+x^2} \tan^{-1}\left(\frac{\sqrt{-a^2+2a-1}\sqrt{x}}{\sqrt{-(-a^2+2a+1)x+(2-a)a+x^2}}\right)}{a\sqrt{-a^2+2a-1}\sqrt{-(-a^2+2a+1)x^2+(2-a)ax+x^3}} + \frac{\sqrt[4]{(2-a)a}(-a+\sqrt{(2-a)a}+2)\sqrt{x}\left(\frac{x}{\sqrt{(2-a)a}}+1\right)\sqrt{\frac{-(-a^2+2a+1)x+(2-a)a+x^2}{(2-a)a\left(\frac{x}{\sqrt{(2-a)a}}+1\right)^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{(2-a)a}}\right) \middle| \frac{1}{4}\left(\frac{-a^2+2a+1}{\sqrt{(2-a)a}}+2\right)\right)}{(a+\sqrt{(2-a)a})\sqrt{-(-a^2+2a+1)x^2+(2-a)ax+x^3}} + \frac{(\sqrt{2-a}-\sqrt{a})(1-a)\sqrt[4]{(2-a)a}\sqrt{x}\left(\frac{x}{\sqrt{(2-a)a}}+1\right)\sqrt{\frac{-(-a^2+2a+1)x+(2-a)a+x^2}{(2-a)a\left(\frac{x}{\sqrt{(2-a)a}}+1\right)^2}} \left(\frac{\sqrt{2-a}+\sqrt{a}}{4\sqrt{(2-a)a}}\right)^2; 2 \tan^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{(2-a)a}}\right) \middle| \frac{1}{4}\left(\frac{-a^2+2a+1}{\sqrt{(2-a)a}}\right)}{(\sqrt{2-a}+\sqrt{a})a\sqrt{-(-a^2+2a+1)x^2+(2-a)ax+x^3}}$$

Warning: Unable to verify antiderivative.

[In] Int[(-2 + a + x)/((-a + x)*Sqrt[(2 - a)*a*x + (-1 - 2*a + a^2)*x^2 + x^3]], x]

[Out] (2*(1 - a)*Sqrt[x]*Sqrt[(2 - a)*a - (1 + 2*a - a^2)*x + x^2])*ArcTan[(Sqrt[-1 + 2*a - a^2]*Sqrt[x])/Sqrt[(2 - a)*a - (1 + 2*a - a^2)*x + x^2]]/(a*Sqrt[-1 + 2*a - a^2]*Sqrt[(2 - a)*a*x - (1 + 2*a - a^2)*x^2 + x^3]) + (((2 - a)*a)^(1/4)*(2 - a + Sqrt[(2 - a)*a])*Sqrt[x]*(1 + x/Sqrt[(2 - a)*a])*Sqrt[((2 - a)*a - (1 + 2*a - a^2)*x + x^2])/((2 - a)*a*(1 + x/Sqrt[(2 - a)*a])^2)]*EllipticF[2*ArcTan[Sqrt[x]/((2 - a)*a)^(1/4)], (2 + (1 + 2*a - a^2)/Sqrt[(2 - a)*a])/4]/((a + Sqrt[(2 - a)*a])*Sqrt[(2 - a)*a*x - (1 + 2*a - a^2)*x^2 + x^3]) + ((Sqrt[2 - a] - Sqrt[a])*(1 - a)*((2 - a)*a)^(1/4))*Sqrt[x]*(1 + x/Sqrt[(2 - a)*a])*Sqrt[((2 - a)*a - (1 + 2*a - a^2)*x + x^2])/((2 - a)*a*(1 + x/Sqrt[(2 - a)*a])^2)]*EllipticPi[(Sqrt[2 - a] + Sqrt[a])^2/(4*Sqrt[(2 - a)*a]), 2*ArcTan[Sqrt[x]/((2 - a)*a)^(1/4)], (2 + (1 + 2*a - a^2)/Sqrt[(2 - a)*a])/4]/((Sqrt[2 - a] + Sqrt[a])*a*Sqrt[(2 - a)*a*x - (1 + 2*a - a^2)*x^2 + x^3])

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((-2+a+x)/(-a+x)/((-a+2)*a*x+(a**2-2*a-1)*x**2+x**3)**(1/2), x)`

[Out] Timed out

Mathematica [C] time = 0.464059, size = 100, normalized size = 100.

$$\frac{2i\sqrt{\frac{1}{x-1}+1}(x-1)^{3/2}\sqrt{\frac{(a-1)^2}{x-1}+1}\left(F\left(i\sinh^{-1}\left(\frac{1}{\sqrt{x-1}}\right)\mid(a-1)^2\right)-2\left(1-a;i\sinh^{-1}\left(\frac{1}{\sqrt{x-1}}\right)\mid(a-1)^2\right)\right)}{\sqrt{(x-1)x(a^2-2a+x)}}$$

Antiderivative was successfully verified.

[In] `Integrate[(-2 + a + x)/((-a + x)*Sqrt[(2 - a)*a*x + (-1 - 2*a + a^2)*x^2 + x^3]), x]`

[Out] `((-2*I)*Sqrt[1 + (-1 + x)^(-1)]*Sqrt[1 + (-1 + a)^2/(-1 + x)]*(-1 + x)^(3/2)*EllipticF[I*ArcSinh[1/Sqrt[-1 + x]], (-1 + a)^2] - 2*EllipticPi[1 - a, I*ArcSinh[1/Sqrt[-1 + x]], (-1 + a)^2])/Sqrt[(-1 + x)*x*(-2*a + a^2 + x)]`

Maple [C] time = 0.063, size = 317, normalized size = 317.

$$2\frac{a^2-2a}{\sqrt{a^2x^2-a^2x-2ax^2+x^3+2ax-x^2}}\sqrt{\frac{a^2-2a+x}{a^2-2a}}\sqrt{\frac{-1+x}{-a^2+2a-1}}\sqrt{\frac{x}{-a^2+2a}}\text{EllipticF}\left(\sqrt{\frac{a^2-2a+x}{a^2-2a}},\sqrt{\frac{-a^2+2a}{-a^2+2a-1}}\right)+2\frac{(2a-2)(a^2-2a)}{\sqrt{a^2x^2-a^2x-2ax^2+x^3+2ax-x^2}(-a^2+a)}\sqrt{\frac{a^2-2a+x}{a^2-2a}}\sqrt{\frac{-1+x}{-a^2+2a-1}}\sqrt{\frac{x}{-a^2+2a}}\text{EllipticPi}\left(\sqrt{\frac{a^2-2a+x}{a^2-2a}},\sqrt{\frac{-a^2+2a}{-a^2+2a-1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-2+a+x)/(-a+x)/((2-a)*a*x+(a^2-2*a-1)*x^2+x^3)^(1/2), x)`

[Out] `2*(a^2-2*a)*((a^2-2*a+x)/(a^2-2*a))^(1/2)*((-1+x)/(-a^2+2*a-1))^(1/2)*(x/(-a^2+2*a))^(1/2)/(a^2*x^2-a^2*x-2*a*x^2+x^3+2*a*x-x^2)^(1/2)*EllipticF(((a^2-2*a+x)/(a^2-2*a))^(1/2),((-a^2+2*a)/(-a^2+2*a-1))^(1/2))+2*(2*a-2)*(a^2-2*a)*((a^2-2*a+x)/(a^2-2*a))^(1/2)*((-1+x)/(-a^2+2*a-1))^(1/2)*(x/(-a^2+2*a))^(1/2)/(a^2*x^2-a^2*x-2*a*x^2+x^3+2*a*x-x^2)^(1/2)/(-a^2+a)*EllipticPi(((a^2-2*a+x)/(a^2-2*a))^(1/2),((-a^2+2*a)/(-a^2+2*a-1))^(1/2))`

$*a)^{(1/2)}, (-a^2+2*a)/(-a^2+a), ((-a^2+2*a)/(-a^2+2*a-1))^{(1/2))}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{a+x-2}{\sqrt{-(a-2)ax+(a^2-2a-1)x^2+x^3(a-x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-(a + x - 2)/(sqrt(-(a - 2)*a*x + (a^2 - 2*a - 1)*x^2 + x^3)*(a - x)), x, a

[Out] -integrate((a + x - 2)/(sqrt(-(a - 2)*a*x + (a^2 - 2*a - 1)*x^2 + x^3)*(a - x)), x

Fricas [A] time = 0.263826, size = 95, normalized size = 95.

$$\frac{\log\left(-\frac{a^2-2(a^2-a)x-x^2+2\sqrt{(a^2-2a-1)x^2+x^3-(a^2-2a)xa}}{a^2-2ax+x^2}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-(a + x - 2)/(sqrt(-(a - 2)*a*x + (a^2 - 2*a - 1)*x^2 + x^3)*(a - x)), x, a

[Out] log(-(a^2 - 2*(a^2 - a)*x - x^2 + 2*sqrt((a^2 - 2*a - 1)*x^2 + x^3 - (a^2 - 2*a)*x)*a)/(a^2 - 2*a*x + x^2))/a

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+x-2}{\sqrt{x(x-1)(a^2-2a+x)}(-a+x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2+a+x)/(-a+x)/((-a+2)*a*x+(a**2-2*a-1)*x**2+x**3)**(1/2), x)

[Out] Integral((a + x - 2)/(sqrt(x*(x - 1)*(a**2 - 2*a + x))*(-a + x)), x)

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{a+x-2}{\sqrt{-(a-2)ax+(a^2-2a-1)x^2+x^3(a-x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-(a + x - 2)/(sqrt(-(a - 2)*a*x + (a^2 - 2*a - 1)*x^2 + x^3)*(a - x)), x, a

[Out] integrate(-(a + x - 2)/(sqrt(-(a - 2)*a*x + (a^2 - 2*a - 1)*x^2 + x^3)*(a - x)), x)

$$3.81 \quad \int \frac{-a+(-1+2a)x}{(-a+x)\sqrt{a^2x-(-1+2a+a^2)x^2+(-1+2a)x^3}} dx$$

Optimal. Leaf size=46

$$\log\left(\frac{-2\left(\sqrt{(1-x)x(a^2-2ax+x)}+x\right)-a^2+2ax+x^2}{(a-x)^2}\right)$$

[Out] Log[(-a^2 + 2*a*x + x^2 - 2*(x + Sqrt[(1 - x)*x*(a^2 + x - 2*a*x)]))/(a - x)^2]

Rubi [C] time = 2.93293, antiderivative size = 180, normalized size of antiderivative = 3.91, number of steps used = 7, number of rules used = 7, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}} = 0.137$

$$\frac{4(1-a)\sqrt{1-x}\sqrt{x}\sqrt{\frac{(1-2a)x}{a^2}}+1\left(\frac{1}{a}; \sin^{-1}(\sqrt{x}) \mid -\frac{1-2a}{a^2}\right)}{\sqrt{(-a^2-2a+1)x^2+a^2x-(1-2a)x^3}} - \frac{2(1-2a)\sqrt{1-x}\sqrt{x}\sqrt{\frac{(1-2a)x}{a^2}}+1F\left(\sin^{-1}(\sqrt{x}) \mid -\frac{1-2a}{a^2}\right)}{\sqrt{(-a^2-2a+1)x^2+a^2x-(1-2a)x^3}}$$

Antiderivative was successfully verified.

[In] Int[(-a + (-1 + 2*a)*x)/((-a + x)*Sqrt[a^2*x - (-1 + 2*a + a^2)*x^2 + (-1 + 2*a)*x^3]), x]

[Out] (-2*(1 - 2*a)*Sqrt[1 - x]*Sqrt[x]*Sqrt[1 + ((1 - 2*a)*x)/a^2]*EllipticF[ArcSin[Sqrt[x]], -((1 - 2*a)/a^2)]/Sqrt[a^2*x + (1 - 2*a - a^2)*x^2 - (1 - 2*a)*x^3] + (4*(1 - a)*Sqrt[1 - x]*Sqrt[x]*Sqrt[1 + ((1 - 2*a)*x)/a^2]*EllipticPi[a^(-1), ArcSin[Sqrt[x]], -((1 - 2*a)/a^2)]/Sqrt[a^2*x + (1 - 2*a - a^2)*x^2 - (1 - 2*a)*x^3]

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate((-a+(-1+2*a)*x)/(-a+x)/(a**2*x-(a**2+2*a-1)*x**2+(-1+2*a)*x**3)**(1/2), x)

[Out] Timed out

Mathematica [C] time = 0.590397, size = 133, normalized size = 2.89

$$\frac{2i(x-1)^{3/2}\sqrt{\frac{x}{x-1}}\sqrt{-\frac{a^2-2ax+x}{(2a-1)(x-1)}}\left(2a\left(1-a; i\sinh^{-1}\left(\frac{1}{\sqrt{x-1}}\right)\middle|-\frac{(a-1)^2}{2a-1}\right)-F\left(i\sinh^{-1}\left(\frac{1}{\sqrt{x-1}}\right)\middle|-\frac{(a-1)^2}{2a-1}\right)\right)}{\sqrt{-(x-1)x(a^2-2ax+x)}}$$

Antiderivative was successfully verified.

[In] Integrate[(-a + (-1 + 2*a)*x)/((-a + x)*Sqrt[a^2*x - (-1 + 2*a + a^2)*x^2 + (-1 + 2*a)*x^3]), x]

[Out] ((2*I)*(-1 + x)^(3/2)*Sqrt[x/(-1 + x)]*Sqrt[-((a^2 + x - 2*a*x)/(-1 + 2*a)*(-1 + x))]*(-EllipticF[I*ArcSinh[1/Sqrt[-1 + x]], -((-1 + a)^2/(-1 + 2*a))] + 2*a*EllipticPi[1 - a, I*ArcSinh[1/Sqrt[-1 + x]], -((-1 + a)^2/(-1 + 2*a))])/Sqrt[-((-1 + x)*x*(a^2 + x - 2*a*x))]

Maple [C] time = 0.079, size = 536, normalized size = 11.7

$$\begin{aligned} & 2 \frac{a^2}{(-1+2a)\sqrt{-a^2x^2+2ax^3+a^2x-2ax^2-x^3+x^2}} \sqrt{-\frac{-1+2a}{a^2}\left(x-\frac{a^2}{-1+2a}\right)} \sqrt{(-1+x)\left(\frac{a^2}{-1+2a}-1\right)^{-1}} \sqrt{\frac{(-1+2a)x}{a^2}} \\ & -4 \frac{a^3}{(-1+2a)\sqrt{-a^2x^2+2ax^3+a^2x-2ax^2-x^3+x^2}} \sqrt{-\frac{-1+2a}{a^2}\left(x-\frac{a^2}{-1+2a}\right)} \sqrt{(-1+x)\left(\frac{a^2}{-1+2a}-1\right)^{-1}} \sqrt{\frac{(-1+2a)x}{a^2}} \\ & -4 \frac{a^3(a-1)}{(-1+2a)\sqrt{-a^2x^2+2ax^3+a^2x-2ax^2-x^3+x^2}} \sqrt{-\frac{-1+2a}{a^2}\left(x-\frac{a^2}{-1+2a}\right)} \sqrt{(-1+x)\left(\frac{a^2}{-1+2a}-1\right)^{-1}} \sqrt{\frac{(-1+2a)x}{a^2}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-a+(-1+2*a)*x)/(-a+x)/(a^2*x-(a^2+2*a-1)*x^2+(-1+2*a)*x^3)^(1/2), x)

[Out] 2*a^2/(-1+2*a)*(-(x-a^2/(-1+2*a))/a^2*(-1+2*a))^(1/2)*((-1+x)/(a^2/(-1+2*a)-1))^(1/2)*(x/a^2*(-1+2*a))^(1/2)/(-a^2*x^2+2*a*x^3+a^2*x-2*a*x^2-x^3+x^2)^(1/2)*EllipticF((-x-a^2/(-1+2*a))/a^2*(-1+2*a))^(1/2), (a^2/(-1+2*a)/(a^2/(-1+2*a)-1))^(1/2))-4*a^3/(-1+2*a)*(-(x-a^2/(-1+2*a))/a^2*(-1+2*a))^(1/2)*((-1+x)/(a^2/(-1+2*a)-1))^(1/2)*(x/a^2*(-1+2*a))^(1/2)/(-a^2*x^2+2*a*x^3+a^2*x-2*a*x^2-x^3+x^2)^(1/2)*EllipticF((-x-a^2/(-1+2*a))/a^2*(-1+2*a))^(1/2), (a^2/(-1+2*a)/(a^2/(-1+2*a)-1))^(1/2))-4*a^3*(a-1)/(-1+2*a)*(-(x-a^2/(-1+2*a))/a^2*(-1+2*a))^(1/2)*((-1+x)/(a^2/(-1+2*a)-1))^(1/2)*(x/a^2*(-1+2*a))^(1/2)/(-a^2*x^2+2*a*x^3+a^2*x-2*a*x^2-x^3+x^2)^(1/2)/(a^2/(-1+2*a)-a)*EllipticPi((-x-a^2/(-1+2*a))/a^2*(-1+2*a))^(1/2), a^2/(-1+2*a)/(a^2/(-1+2*a)-a), (a^2/(-1+2*a)/(a^2/(-1+2*a)-1))^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(2a-1)x - a}{\sqrt{(2a-1)x^3 + a^2x - (a^2 + 2a-1)x^2(a-x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-((2*a - 1)*x - a)/(sqrt((2*a - 1)*x^3 + a^2*x - (a^2 + 2*a - 1)*x^2)*(a

[Out] -integrate(((2*a - 1)*x - a)/(sqrt((2*a - 1)*x^3 + a^2*x - (a^2 + 2*a - 1)*x^2)*(a - x)), x)

Fricas [A] time = 0.25922, size = 85, normalized size = 1.85

$$\log\left(-\frac{a^2 - 2(a-1)x - x^2 + 2\sqrt{(2a-1)x^3 + a^2x - (a^2 + 2a-1)x^2}}{a^2 - 2ax + x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-((2*a - 1)*x - a)/(sqrt((2*a - 1)*x^3 + a^2*x - (a^2 + 2*a - 1)*x^2)*(a

[Out] log(-(a^2 - 2*(a - 1)*x - x^2 + 2*sqrt((2*a - 1)*x^3 + a^2*x - (a^2 + 2*a - 1)*x^2))/(a^2 - 2*a*x + x^2))

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a+(-1+2*a)*x)/(-a+x)/(a**2*x-(a**2+2*a-1)*x**2+(-1+2*a)*x**3)**(1/

[Out] Exception raised: RecursionError

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(2a-1)x - a}{\sqrt{(2a-1)x^3 + a^2x - (a^2 + 2a-1)x^2(a-x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-((2*a - 1)*x - a)/(sqrt((2*a - 1)*x^3 + a^2*x - (a^2 + 2*a - 1)*x^2)*(a
```

```
[Out] integrate(-((2*a - 1)*x - a)/(sqrt((2*a - 1)*x^3 + a^2*x - (a^2 + 2*a - 1)*x^2)*(a - x)), x)
```

$$3.82 \quad \int \frac{1 - \sqrt[3]{2x}}{(2^{2/3} + x)\sqrt{1+x^3}} dx$$

Optimal. Leaf size=32

$$\frac{2 \tan^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2x+1}\right)}{\sqrt{x^3+1}}\right)}{\sqrt{3}}$$

[Out] (2*ArcTan[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[1 + x^3]])/Sqrt[3]

Rubi [A] time = 0.144692, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$

$$\frac{2 \tan^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2x+1}\right)}{\sqrt{x^3+1}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(1 - 2^(1/3)*x)/((2^(2/3) + x)*Sqrt[1 + x^3]), x]

[Out] (2*ArcTan[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[1 + x^3]])/Sqrt[3]

Rubi in Sympy [A] time = 63.8776, size = 469, normalized size = 14.66

$$\frac{6 \sqrt{\frac{x^2-x+1}{(x+1+\sqrt{3})^2}} \sqrt{-\sqrt{3}+2(x+1)} \operatorname{atan}\left(\frac{3^{\frac{3}{4}} \sqrt{1+\sqrt[3]{2}} \sqrt{-4\sqrt{3}+8} \sqrt{\frac{(-x-1+\sqrt{3})^2}{(x+1+\sqrt{3})^2}+1}}{6 \sqrt{-1+\sqrt[3]{2}} \sqrt{\frac{(-x-1+\sqrt{3})^2}{(x+1+\sqrt{3})^2}-4\sqrt{3}+7}}\right)}{\sqrt{\frac{x+1}{(x+1+\sqrt{3})^2}} \sqrt{-1+\sqrt[3]{2}} (1+\sqrt[3]{2})^{\frac{3}{2}} \sqrt{-4\sqrt{3}+8} \sqrt{x^3+1}} \\ + \frac{2 \cdot 3^{\frac{3}{4}} \sqrt{\frac{x^2-x+1}{(x+1+\sqrt{3})^2}} (1+\sqrt[3]{2}) (1+\sqrt{3}) \sqrt{\sqrt{3}+2(x+1)} F\left(\operatorname{asin}\left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}}\right)\right) \Big|_{-7-4\sqrt{3}}}{3 \sqrt{\frac{x+1}{(x+1+\sqrt{3})^2}} \sqrt{x^3+1} \left(-2^{\frac{2}{3}}+1+\sqrt{3}\right)} \\ + \frac{12 \sqrt[4]{3} \sqrt{\frac{x^2-x+1}{(x+1+\sqrt{3})^2}} \sqrt{-\sqrt{3}+2(x+1)} \left(\frac{(-2^{\frac{2}{3}}+1+\sqrt{3})^2}{(-1+2^{\frac{2}{3}}+\sqrt{3})^2}; \operatorname{asin}\left(\frac{-x-1+\sqrt{3}}{x+1+\sqrt{3}}\right)\right) \Big|_{-7-4\sqrt{3}}}{\sqrt{\frac{x+1}{(x+1+\sqrt{3})^2}} \sqrt{-4\sqrt{3}+7} \sqrt{x^3+1} \left(-2^{\frac{2}{3}}+1+\sqrt{3}\right) \left(-\sqrt{3}-2^{\frac{2}{3}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((1-2**(1/3)*x)/(2**(2/3)+x)/(x**3+1)**(1/2),x)`

[Out] $6\sqrt{x^2 - x + 1}/(x + 1 + \sqrt{3})^2 \sqrt{-\sqrt{3} + 2} (x + 1) \operatorname{atan}\left(3^{3/4} \sqrt{1 + 2^{1/3}} \sqrt{-4\sqrt{3} + 8} \sqrt{-(-x - 1 + \sqrt{3})^2/(x + 1 + \sqrt{3})^2 + 1}/(6\sqrt{-1 + 2^{1/3}} \sqrt{(-x - 1 + \sqrt{3})^2/(x + 1 + \sqrt{3})^2 - 4\sqrt{3} + 7})\right) / (\sqrt{(x + 1)/(x + 1 + \sqrt{3})}^2 \sqrt{-1 + 2^{1/3}} (1 + 2^{1/3})^{3/2} \sqrt{-4\sqrt{3} + 8} \sqrt{x^3 + 1}) - 2^3 (3/4) \sqrt{(x^2 - x + 1)/(x + 1 + \sqrt{3})}^2 (1 + 2^{1/3}) (1 + \sqrt{3}) \sqrt{\sqrt{3} + 2} (x + 1) \operatorname{elliptic}_f(\operatorname{asin}((x - \sqrt{3} + 1)/(x + 1 + \sqrt{3})), -7 - 4\sqrt{3}) / (3\sqrt{(x + 1)/(x + 1 + \sqrt{3})}^2 \sqrt{x^3 + 1} (-2^{2/3} + 1 + \sqrt{3})) + 12^3 (1/4) \sqrt{(x^2 - x + 1)/(x + 1 + \sqrt{3})}^2 \sqrt{-\sqrt{3} + 2} (x + 1) \operatorname{elliptic}_pi((-2^{2/3} + 1 + \sqrt{3})^2 / (-1 + 2^{2/3} + \sqrt{3})^2, \operatorname{asin}((-x - 1 + \sqrt{3})/(x + 1 + \sqrt{3})), -7 - 4\sqrt{3}) / (\sqrt{(x + 1)/(x + 1 + \sqrt{3})}^2 \sqrt{-4\sqrt{3} + 7} \sqrt{x^3 + 1} (-2^{2/3} + 1 + \sqrt{3}) (-\sqrt{3} - 2^{2/3} + 1))$

Mathematica [C] time = 0.516328, size = 323, normalized size = 10.09

$$\frac{2\sqrt{\frac{2}{3}}\sqrt{\frac{i(x+1)}{\sqrt{3+3i}}}\left(\sqrt{2ix+\sqrt{3}-i}\left(\left(-3i\sqrt[3]{2}+4\sqrt{3}+\sqrt[3]{2}\sqrt{3}\right)x+\sqrt[3]{2}\sqrt{3}-2\sqrt{3}+3i\sqrt[3]{2}+6i\right)F\left(\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[3]{3}}\right)\middle|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)-6i\sqrt{-2ix+\sqrt{3}+i}\sqrt{x^3+1}}{\left(1+2^{2/3}-i\sqrt{3}\right)\sqrt{-2ix+\sqrt{3}+i}\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

[In] `Integrate[(1 - 2^(1/3)*x)/((2^(2/3) + x)*Sqrt[1 + x^3]),x]`

[Out] $(-2\sqrt[3]{2}\sqrt[3]{I(1+x)}(3I+\sqrt[3]{3})\sqrt{-I+\sqrt[3]{3}}+(2I)x(6I+(3I)2^{1/3}-2\sqrt[3]{3}+2^{1/3}\sqrt[3]{3})+((-3I)2^{1/3}+4\sqrt[3]{3}+2^{1/3}\sqrt[3]{3})x)\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{I+\sqrt[3]{3}}-(2I)x}{\sqrt[3]{2}\sqrt[3]{3}}\right],\frac{2\sqrt[3]{3}}{3I+\sqrt[3]{3}}\right]-6I\sqrt[3]{3}\sqrt{I+\sqrt[3]{3}}-(2I)x\sqrt{1-x+x^2}\operatorname{EllipticPi}\left[\frac{2\sqrt[3]{3}}{I+(2I)2^{2/3}+\sqrt[3]{3}},\operatorname{ArcSin}\left[\frac{\sqrt{I+\sqrt[3]{3}}-(2I)x}{\sqrt[3]{2}\sqrt[3]{3}}\right],\frac{2\sqrt[3]{3}}{3I+\sqrt[3]{3}}\right]\right)/((1+2^{2/3}-I\sqrt[3]{3})\sqrt{I+\sqrt[3]{3}}\sqrt{1-x^3})$

Maple [C] time = 0.093, size = 258, normalized size = 8.1

$$-2 \frac{\sqrt[3]{2} (3/2 - i/2\sqrt{3})}{\sqrt{x^3 + 1}} \sqrt{\frac{1+x}{3/2 - i/2\sqrt{3}}} \sqrt{\frac{x - 1/2 - i/2\sqrt{3}}{-3/2 - i/2\sqrt{3}}} \sqrt{\frac{x - 1/2 + i/2\sqrt{3}}{-3/2 + i/2\sqrt{3}}} \text{EllipticF} \left(\sqrt{\frac{1+x}{3/2 - i/2\sqrt{3}}}, \sqrt{\frac{-3/2 + i/2\sqrt{3}}{-3/2 - i/2\sqrt{3}}} \right) \\ + 6 \frac{3/2 - i/2\sqrt{3}}{\sqrt{x^3 + 1} (2^{2/3} - 1)} \sqrt{\frac{1+x}{3/2 - i/2\sqrt{3}}} \sqrt{\frac{x - 1/2 - i/2\sqrt{3}}{-3/2 - i/2\sqrt{3}}} \sqrt{\frac{x - 1/2 + i/2\sqrt{3}}{-3/2 + i/2\sqrt{3}}} \text{EllipticPi} \left(\sqrt{\frac{1+x}{3/2 - i/2\sqrt{3}}}, \frac{-3/2 + i/2\sqrt{3}}{2^{2/3} - 1}, \sqrt{\frac{-3/2 + i/2\sqrt{3}}{-3/2 - i/2\sqrt{3}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1-2^(1/3)*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x)

[Out] -2*2^(1/3)*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+6*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)/(2^(2/3)-1)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(2^(2/3)-1),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{2^{\frac{1}{3}}x - 1}{\sqrt{x^3 + 1}(x + 2^{\frac{2}{3}})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-(2^(1/3)*x - 1)/(sqrt(x^3 + 1)*(x + 2^(2/3))),x, algorithm="maxima")

[Out] -integrate((2^(1/3)*x - 1)/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-(2^(1/3)*x - 1)/(sqrt(x^3 + 1)*(x + 2^(2/3))),x, algorithm="fricas")

[Out] Exception raised: TypeError

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt[3]{2}x}{x\sqrt{x^3+1} + 2^{\frac{2}{3}}\sqrt{x^3+1}} dx - \int \left(-\frac{1}{x\sqrt{x^3+1} + 2^{\frac{2}{3}}\sqrt{x^3+1}} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1-2**(1/3)*x)/(2**(2/3)+x)/(x**3+1)**(1/2),x)`

[Out] `-Integral(2**(1/3)*x/(x*sqrt(x**3 + 1) + 2**(2/3)*sqrt(x**3 + 1)), x) - Integral(-1/(x*sqrt(x**3 + 1) + 2**(2/3)*sqrt(x**3 + 1)), x)`

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{2^{\frac{1}{3}}x - 1}{\sqrt{x^3+1}(x + 2^{\frac{2}{3}})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-(2^(1/3)*x - 1)/(sqrt(x^3 + 1)*(x + 2^(2/3))),x, algorithm="giac")`

[Out] `integrate(-(2^(1/3)*x - 1)/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)`

$$3.83 \quad \int \frac{1+x}{(-2+x)\sqrt{1+x^3}} dx$$

Optimal. Leaf size=23

$$-\frac{2}{3} \tanh^{-1} \left(\frac{(x+1)^2}{3\sqrt{x^3+1}} \right)$$

[Out] $(-2 * \text{ArcTanh}[(1 + x)^2 / (3 * \text{Sqrt}[1 + x^3])]) / 3$

Rubi [A] time = 0.0910752, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$-\frac{2}{3} \tanh^{-1} \left(\frac{(x+1)^2}{3\sqrt{x^3+1}} \right)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(1 + x) / ((-2 + x) * \text{Sqrt}[1 + x^3]), x]$

[Out] $(-2 * \text{ArcTanh}[(1 + x)^2 / (3 * \text{Sqrt}[1 + x^3])]) / 3$

Rubi in Sympy [A] time = 41.593, size = 371, normalized size = 16.13

$$\frac{3 \sqrt{\frac{x^2-x+1}{(x+1+\sqrt{3})^2}} \left(\frac{\sqrt{3}}{3} + 1 \right) (x+1) \operatorname{atanh} \left(\frac{3^{\frac{3}{4}} \sqrt{-\sqrt{3}+2} \sqrt{\frac{(-x-1+\sqrt{3})^2}{(x+1+\sqrt{3})^2} + 1}}{3 \sqrt{\frac{(-x-1+\sqrt{3})^2}{(x+1+\sqrt{3})^2} - 4\sqrt{3}+7}} \right)}{\sqrt{\frac{x+1}{(x+1+\sqrt{3})^2}} (\sqrt{3}+3) \sqrt{x^3+1}} + \frac{2\sqrt{3} \sqrt{\frac{x^2-x+1}{(x+1+\sqrt{3})^2}} \sqrt{\sqrt{3}+2} (x+1) F \left(\operatorname{asin} \left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}} \right) \middle| -7-4\sqrt{3} \right)}{\sqrt{\frac{x+1}{(x+1+\sqrt{3})^2}} (\sqrt{3}+3) \sqrt{x^3+1}} + \frac{12\sqrt{3} \sqrt{\frac{x^2-x+1}{(x+1+\sqrt{3})^2}} \sqrt{-\sqrt{3}+2} (x+1) \left(4\sqrt{3}+7; \operatorname{asin} \left(\frac{-x-1+\sqrt{3}}{x+1+\sqrt{3}} \right) \middle| -7-4\sqrt{3} \right)}{\sqrt{\frac{x+1}{(x+1+\sqrt{3})^2}} \sqrt{-4\sqrt{3}+7} (-\sqrt{3}+3) (\sqrt{3}+3) \sqrt{x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{rubi_integrate}((1+x)/(-2+x)/(x**3+1)**(1/2), x)$

```
[Out] -3*sqrt((x**2 - x + 1)/(x + 1 + sqrt(3))**2)*(sqrt(3)/3 + 1)*(x +
  1)*atanh(3**(3/4)*sqrt(-sqrt(3) + 2)*sqrt(-(-x - 1 + sqrt(3))**2
  /(x + 1 + sqrt(3))**2 + 1)/(3*sqrt((-x - 1 + sqrt(3))**2/(x + 1 +
  sqrt(3))**2 - 4*sqrt(3) + 7)))/(sqrt((x + 1)/(x + 1 + sqrt(3))**
  2)*(sqrt(3) + 3)*sqrt(x**3 + 1)) + 2*3**(1/4)*sqrt((x**2 - x + 1)
  /(x + 1 + sqrt(3))**2)*sqrt(sqrt(3) + 2)*(x + 1)*elliptic_f(asin(
  (x - sqrt(3) + 1)/(x + 1 + sqrt(3))), -7 - 4*sqrt(3))/(sqrt((x +
  1)/(x + 1 + sqrt(3))**2)*(sqrt(3) + 3)*sqrt(x**3 + 1)) + 12*3**(1
  /4)*sqrt((x**2 - x + 1)/(x + 1 + sqrt(3))**2)*sqrt(-sqrt(3) + 2)*
  (x + 1)*elliptic_pi(4*sqrt(3) + 7, asin((-x - 1 + sqrt(3))/(x + 1
  + sqrt(3))), -7 - 4*sqrt(3))/(sqrt((x + 1)/(x + 1 + sqrt(3))**2)
  *sqrt(-4*sqrt(3) + 7)*(-sqrt(3) + 3)*(sqrt(3) + 3)*sqrt(x**3 + 1)
  )
```

Mathematica [C] time = 0.282679, size = 262, normalized size = 11.39

$$\frac{2\sqrt{6}\sqrt{\frac{i(x+1)}{\sqrt{3+3i}}}\left(\sqrt{2ix+\sqrt{3}}-i\left(-i\sqrt{3}x+x+i\sqrt{3}+1\right)F\left(\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)\middle|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)-2\sqrt{3}\sqrt{-2ix+\sqrt{3}+i}\sqrt{x^2-x+1}\left(\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right)\right)}{(\sqrt{3}-3i)\sqrt{-2ix+\sqrt{3}+i}\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + x)/((-2 + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[6]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(Sqrt[-I + Sqrt[3] +
  (2*I)*x]*(1 + I*Sqrt[3] + x - I*Sqrt[3]*x)*EllipticF[ArcSin[Sqrt
  [I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqr
  t[3])]) - 2*Sqrt[3]*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 - x + x^2]
  *EllipticPi[(2*Sqrt[3])/(-3*I + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3]
  - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/((
  -3*I + Sqrt[3])*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 + x^3])
```

Maple [C] time = 0.031, size = 240, normalized size = 10.4

$$2\frac{3/2-i/2\sqrt{3}}{\sqrt{x^3+1}}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}\text{EllipticF}\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},\sqrt{\frac{-3/2+i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\right) - 2\frac{3/2-i/2\sqrt{3}}{\sqrt{x^3+1}}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}\text{EllipticPi}\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},-i/6\sqrt{3}+1/2,\sqrt{\frac{-3/2+i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(-2+x)/(x^3+1)^(1/2),x)
```

[Out] $2 \cdot (3/2 - 1/2 \cdot I \cdot 3^{1/2}) \cdot ((1+x)/(3/2 - 1/2 \cdot I \cdot 3^{1/2}))^{1/2} \cdot ((x - 1/2 - 1/2 \cdot I \cdot 3^{1/2})/(-3/2 - 1/2 \cdot I \cdot 3^{1/2}))^{1/2} \cdot ((x - 1/2 + 1/2 \cdot I \cdot 3^{1/2})/(-3/2 + 1/2 \cdot I \cdot 3^{1/2}))^{1/2} / (x^3 + 1)^{1/2} \cdot \text{EllipticF}(((1+x)/(3/2 - 1/2 \cdot I \cdot 3^{1/2}))^{1/2}, ((-3/2 + 1/2 \cdot I \cdot 3^{1/2})/(-3/2 - 1/2 \cdot I \cdot 3^{1/2}))^{1/2}) - 2 \cdot (3/2 - 1/2 \cdot I \cdot 3^{1/2}) \cdot ((1+x)/(3/2 - 1/2 \cdot I \cdot 3^{1/2}))^{1/2} \cdot ((x - 1/2 - 1/2 \cdot I \cdot 3^{1/2})/(-3/2 - 1/2 \cdot I \cdot 3^{1/2}))^{1/2} \cdot ((x - 1/2 + 1/2 \cdot I \cdot 3^{1/2})/(-3/2 + 1/2 \cdot I \cdot 3^{1/2}))^{1/2} / (x^3 + 1)^{1/2} \cdot \text{EllipticPi}(((1+x)/(3/2 - 1/2 \cdot I \cdot 3^{1/2}))^{1/2}, -1/6 \cdot I \cdot 3^{1/2} + 1/2, ((-3/2 + 1/2 \cdot I \cdot 3^{1/2})/(-3/2 - 1/2 \cdot I \cdot 3^{1/2}))^{1/2})$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1}{\sqrt{x^3+1}(x-2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x + 1)/(sqrt(x^3 + 1)*(x - 2)),x, algorithm="maxima")`

[Out] `integrate((x + 1)/(sqrt(x^3 + 1)*(x - 2)), x)`

Fricas [A] time = 0.257884, size = 59, normalized size = 2.57

$$\frac{1}{3} \log\left(\frac{x^3 + 12x^2 - 6\sqrt{x^3+1}(x+1) - 6x + 10}{x^3 - 6x^2 + 12x - 8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x + 1)/(sqrt(x^3 + 1)*(x - 2)),x, algorithm="fricas")`

[Out] `1/3*log((x^3 + 12*x^2 - 6*sqrt(x^3 + 1)*(x + 1) - 6*x + 10)/(x^3 - 6*x^2 + 12*x - 8))`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1}{\sqrt{(x+1)(x^2-x+1)}(x-2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+x)/(-2+x)/(x**3+1)**(1/2),x)`

[Out] Integral((x + 1)/(sqrt((x + 1)*(x**2 - x + 1))*(x - 2)), x)

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x + 1}{\sqrt{x^3 + 1}(x - 2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x + 1)/(sqrt(x^3 + 1)*(x - 2)),x, algorithm="giac")

[Out] integrate((x + 1)/(sqrt(x^3 + 1)*(x - 2)), x)

$$3.84 \quad \int \frac{x}{\sqrt{1+x^3}(10+6\sqrt{3}+x^3)} dx$$

Optimal. Leaf size=218

$$\frac{(2-\sqrt{3}) \tan^{-1}\left(\frac{\sqrt[4]{3}(1+\sqrt{3})(x+1)}{\sqrt{2}\sqrt{x^3+1}}\right)}{2\sqrt{2}3^{3/4}} - \frac{(2-\sqrt{3}) \tan^{-1}\left(\frac{(1-\sqrt{3})\sqrt{x^3+1}}{\sqrt{2}3^{3/4}}\right)}{3\sqrt{2}3^{3/4}} \\ - \frac{(2-\sqrt{3}) \tanh^{-1}\left(\frac{\sqrt[4]{3}(-2x+\sqrt{3}+1)}{\sqrt{2}\sqrt{x^3+1}}\right)}{3\sqrt{2}\sqrt[4]{3}} - \frac{(2-\sqrt{3}) \tanh^{-1}\left(\frac{\sqrt[4]{3}(1-\sqrt{3})(x+1)}{\sqrt{2}\sqrt{x^3+1}}\right)}{6\sqrt{2}\sqrt[4]{3}}$$

[Out] -((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3])*(1 + x))/(Sqrt[2]*Sqrt[1 + x^3]])/(2*Sqrt[2]*3^(3/4)) - ((2 - Sqrt[3])*ArcTan[((1 - Sqrt[3])*Sqrt[1 + x^3])/(Sqrt[2]*3^(3/4))])/(3*Sqrt[2]*3^(3/4)) - ((2 - Sqrt[3])*ArcTanh[(3^(1/4)*(1 + Sqrt[3] - 2*x))/(Sqrt[2]*Sqrt[1 + x^3]])/(3*Sqrt[2]*3^(1/4)) - ((2 - Sqrt[3])*ArcTanh[(3^(1/4)*(1 - Sqrt[3])*(1 + x))/(Sqrt[2]*Sqrt[1 + x^3]])/(6*Sqrt[2]*3^(1/4)))

Rubi [A] time = 0.115921, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.04$

$$\frac{(2-\sqrt{3}) \tan^{-1}\left(\frac{\sqrt[4]{3}(1+\sqrt{3})(x+1)}{\sqrt{2}\sqrt{x^3+1}}\right)}{2\sqrt{2}3^{3/4}} - \frac{(2-\sqrt{3}) \tan^{-1}\left(\frac{(1-\sqrt{3})\sqrt{x^3+1}}{\sqrt{2}3^{3/4}}\right)}{3\sqrt{2}3^{3/4}} \\ - \frac{(2-\sqrt{3}) \tanh^{-1}\left(\frac{\sqrt[4]{3}(-2x+\sqrt{3}+1)}{\sqrt{2}\sqrt{x^3+1}}\right)}{3\sqrt{2}\sqrt[4]{3}} - \frac{(2-\sqrt{3}) \tanh^{-1}\left(\frac{\sqrt[4]{3}(1-\sqrt{3})(x+1)}{\sqrt{2}\sqrt{x^3+1}}\right)}{6\sqrt{2}\sqrt[4]{3}}$$

Antiderivative was successfully verified.

[In] Int[x/(Sqrt[1 + x^3]*(10 + 6*Sqrt[3] + x^3)), x]

[Out] -((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3])*(1 + x))/(Sqrt[2]*Sqrt[1 + x^3]])/(2*Sqrt[2]*3^(3/4)) - ((2 - Sqrt[3])*ArcTan[((1 - Sqrt[3])*Sqrt[1 + x^3])/(Sqrt[2]*3^(3/4))])/(3*Sqrt[2]*3^(3/4)) - ((2 - Sqrt[3])*ArcTanh[(3^(1/4)*(1 + Sqrt[3] - 2*x))/(Sqrt[2]*Sqrt[1 + x^3]])/(3*Sqrt[2]*3^(1/4)) - ((2 - Sqrt[3])*ArcTanh[(3^(1/4)*(1 - Sqrt[3])*(1 + x))/(Sqrt[2]*Sqrt[1 + x^3]])/(6*Sqrt[2]*3^(1/4)))

Rubi in Sympy [A] time = 3.44886, size = 34, normalized size = 0.16

$$\frac{x^2 \operatorname{appellf}_1\left(\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -x^3, -\frac{x^3}{10+6\sqrt{3}}\right)}{2(10+6\sqrt{3})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x/(10+x**3+6*3**(1/2))/(x**3+1)**(1/2),x)`

[Out] `x**2*appellf1(2/3, 1/2, 1, 5/3, -x**3, -x**3/(10 + 6*sqrt(3)))/(2*(10 + 6*sqrt(3)))`

Mathematica [C] time = 0.429673, size = 206, normalized size = 0.94

$$\frac{10(26+15\sqrt{3})x^2F_1\left(\frac{2}{3}, \frac{1}{2}, 1; \frac{5}{3}; -x^3, -\frac{x^3}{10+6\sqrt{3}}\right)}{(5+3\sqrt{3})\sqrt{x^3+1}(x^3+6\sqrt{3}+10)\left(3x^3\left(F_1\left(\frac{5}{3}; \frac{1}{2}, 2; \frac{8}{3}; -x^3, -\frac{x^3}{10+6\sqrt{3}}\right) + (5+3\sqrt{3})F_1\left(\frac{5}{3}; \frac{3}{2}, 1; \frac{8}{3}; -x^3, -\frac{x^3}{10+6\sqrt{3}}\right)\right) - 10(5$$

Warning: Unable to verify antiderivative.

[In] `Integrate[x/(Sqrt[1 + x^3]*(10 + 6*Sqrt[3] + x^3)),x]`

[Out] `(-10*(26 + 15*Sqrt[3])*x^2*AppellF1[2/3, 1/2, 1, 5/3, -x^3, -(x^3/(10 + 6*Sqrt[3]))])/(5 + 3*Sqrt[3])*Sqrt[1 + x^3]*(10 + 6*Sqrt[3] + x^3)*(-10*(5 + 3*Sqrt[3])*AppellF1[2/3, 1/2, 1, 5/3, -x^3, -(x^3/(10 + 6*Sqrt[3]))] + 3*x^3*(AppellF1[5/3, 1/2, 2, 8/3, -x^3, -(x^3/(10 + 6*Sqrt[3]))] + (5 + 3*Sqrt[3])*AppellF1[5/3, 3/2, 1, 8/3, -x^3, -(x^3/(10 + 6*Sqrt[3]))])))`

Maple [C] time = 0.283, size = 353, normalized size = 1.6

$$\frac{(-1-\sqrt{3})\left(\frac{3}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}{18+9\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i}{2}\sqrt{3}}}\sqrt{\frac{1}{-\frac{3}{2}-\frac{i}{2}\sqrt{3}}\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)}\sqrt{\frac{1}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)}\operatorname{EllipticPi}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i}{2}\sqrt{3}}},\left(-\frac{\sqrt{2}}{18}\sum_{\alpha=\operatorname{RootOf}\left(-Z^2+(-1-\sqrt{3})_Z+2\sqrt{3}+4\right)}\frac{(-\alpha\sqrt{3}+\alpha-2)(-i\sqrt{3}+3)(-1+2\alpha-\alpha\sqrt{3})}{-\sqrt{3}+2\alpha-1}}\sqrt{\frac{1+x}{-i\sqrt{3}+3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(10+x^3+6*3^(1/2))/(x^3+1)^(1/2),x)`

```
[Out] 1/9*(-1-3^(1/2))/(2+3^(1/2))*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*3^(1/2)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-1/18*2^(1/2)*sum((-_alpha*3^(1/2)+_alpha-2)/(-3^(1/2)+2*_alpha-1)*(-I*3^(1/2)+3)*((1+x)/(-I*3^(1/2)+3))^(1/2)*((2*x-1-I*3^(1/2))/(-I*3^(1/2)-3))^(1/2)*((2*x-1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2)/(x^3+1)^(1/2)*(-1+2*_alpha-_alpha*3^(1/2))*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),-1/2*I*_alpha+1/3*I*_alpha*3^(1/2)+1/2*_alpha*3^(1/2)-_alpha-1/6*I*3^(1/2)+1/2,((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)),_alpha=RootOf(_Z^2+(-1-3^(1/2))*_Z+2*3^(1/2)+4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(x^3 + 6\sqrt{3} + 10)\sqrt{x^3 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((x^3 + 6*sqrt(3) + 10)*sqrt(x^3 + 1)),x, algorithm="maxima")
```

```
[Out] integrate(x/((x^3 + 6*sqrt(3) + 10)*sqrt(x^3 + 1)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((x^3 + 6*sqrt(3) + 10)*sqrt(x^3 + 1)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{(x+1)(x^2-x+1)}(x^3+10+6\sqrt{3})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(10+x**3+6*3**(1/2))/(x**3+1)**(1/2),x)`

[Out] `Integral(x/(sqrt((x + 1)*(x**2 - x + 1))*(x**3 + 10 + 6*sqrt(3))), x)`

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(x^3 + 6\sqrt{3} + 10)\sqrt{x^3 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/((x^3 + 6*sqrt(3) + 10)*sqrt(x^3 + 1)),x, algorithm="giac")`

[Out] `integrate(x/((x^3 + 6*sqrt(3) + 10)*sqrt(x^3 + 1)), x)`

$$3.85 \quad \int \frac{x}{\sqrt{1+x^3}(10-6\sqrt{3}+x^3)} dx$$

Optimal. Leaf size=210

$$\frac{(2 + \sqrt{3}) \tan^{-1} \left(\frac{\sqrt[4]{3}(-2x - \sqrt{3} + 1)}{\sqrt{2}\sqrt{x^3 + 1}} \right)}{3\sqrt{2}\sqrt[4]{3}} - \frac{(2 + \sqrt{3}) \tan^{-1} \left(\frac{\sqrt[4]{3}(1 + \sqrt{3})(x + 1)}{\sqrt{2}\sqrt{x^3 + 1}} \right)}{6\sqrt{2}\sqrt[4]{3}} \\ + \frac{(2 + \sqrt{3}) \tanh^{-1} \left(\frac{\sqrt[4]{3}(1 - \sqrt{3})(x + 1)}{\sqrt{2}\sqrt{x^3 + 1}} \right)}{2\sqrt{2}3^{3/4}} + \frac{(2 + \sqrt{3}) \tanh^{-1} \left(\frac{(1 + \sqrt{3})\sqrt{x^3 + 1}}{\sqrt{2}3^{3/4}} \right)}{3\sqrt{2}3^{3/4}}$$

[Out] -((2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3] - 2*x))/(Sqrt[2]*Sqrt[1 + x^3]]))/(3*Sqrt[2]*3^(1/4)) - ((2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3])*(1 + x))/(Sqrt[2]*Sqrt[1 + x^3]]))/(6*Sqrt[2]*3^(1/4)) + ((2 + Sqrt[3])*ArcTanh[(3^(1/4)*(1 - Sqrt[3])*(1 + x))/(Sqrt[2]*Sqrt[1 + x^3]]))/(2*Sqrt[2]*3^(3/4)) + ((2 + Sqrt[3])*ArcTanh[((1 + Sqrt[3])*Sqrt[1 + x^3])/(Sqrt[2]*3^(3/4))])/(3*Sqrt[2]*3^(3/4))

Rubi [A] time = 0.104841, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.04$

$$\frac{(2 + \sqrt{3}) \tan^{-1} \left(\frac{\sqrt[4]{3}(-2x - \sqrt{3} + 1)}{\sqrt{2}\sqrt{x^3 + 1}} \right)}{3\sqrt{2}\sqrt[4]{3}} - \frac{(2 + \sqrt{3}) \tan^{-1} \left(\frac{\sqrt[4]{3}(1 + \sqrt{3})(x + 1)}{\sqrt{2}\sqrt{x^3 + 1}} \right)}{6\sqrt{2}\sqrt[4]{3}} \\ + \frac{(2 + \sqrt{3}) \tanh^{-1} \left(\frac{\sqrt[4]{3}(1 - \sqrt{3})(x + 1)}{\sqrt{2}\sqrt{x^3 + 1}} \right)}{2\sqrt{2}3^{3/4}} + \frac{(2 + \sqrt{3}) \tanh^{-1} \left(\frac{(1 + \sqrt{3})\sqrt{x^3 + 1}}{\sqrt{2}3^{3/4}} \right)}{3\sqrt{2}3^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[x/(Sqrt[1 + x^3]*(10 - 6*Sqrt[3] + x^3)), x]

[Out] -((2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3] - 2*x))/(Sqrt[2]*Sqrt[1 + x^3]]))/(3*Sqrt[2]*3^(1/4)) - ((2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3])*(1 + x))/(Sqrt[2]*Sqrt[1 + x^3]]))/(6*Sqrt[2]*3^(1/4)) + ((2 + Sqrt[3])*ArcTanh[(3^(1/4)*(1 - Sqrt[3])*(1 + x))/(Sqrt[2]*Sqrt[1 + x^3]]))/(2*Sqrt[2]*3^(3/4)) + ((2 + Sqrt[3])*ArcTanh[((1 + Sqrt[3])*Sqrt[1 + x^3])/(Sqrt[2]*3^(3/4))])/(3*Sqrt[2]*3^(3/4))

Rubi in Sympy [A] time = 7.69253, size = 34, normalized size = 0.16

$$\frac{x^2 \operatorname{appellf}_1\left(\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -x^3, -\frac{x^3}{-6\sqrt{3}+10}\right)}{2(-6\sqrt{3}+10)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x/(10+x**3-6*3**(1/2))/(x**3+1)**(1/2),x)`

[Out] `x**2*appellf1(2/3, 1/2, 1, 5/3, -x**3, -x**3/(-6*sqrt(3) + 10))/(2*(-6*sqrt(3) + 10))`

Mathematica [C] time = 0.656094, size = 207, normalized size = 0.99

$$\frac{10(26 - 15\sqrt{3})x^2 F_1\left(\frac{2}{3}; \frac{1}{2}, 1; \frac{5}{3}; -x^3, \frac{1}{4}(5 + 3\sqrt{3})x^3\right)}{(3\sqrt{3} - 5)(-x^3 + 6\sqrt{3} - 10)\sqrt{x^3 + 1}\left((50 - 30\sqrt{3})F_1\left(\frac{2}{3}; \frac{1}{2}, 1; \frac{5}{3}; -x^3, \frac{1}{4}(5 + 3\sqrt{3})x^3\right) - 3x^3\left(F_1\left(\frac{5}{3}; \frac{1}{2}, 2; \frac{8}{3}; -x^3, \frac{1}{4}(5 + 3\sqrt{3})x^3\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

[In] `Integrate[x/(Sqrt[1 + x^3]*(10 - 6*Sqrt[3] + x^3)),x]`

[Out] `(10*(26 - 15*Sqrt[3])*x^2*AppellF1[2/3, 1/2, 1, 5/3, -x^3, ((5 + 3*Sqrt[3])*x^3)/4])/((-5 + 3*Sqrt[3])*(-10 + 6*Sqrt[3] - x^3)*Sqrt[1 + x^3]*((50 - 30*Sqrt[3])*AppellF1[2/3, 1/2, 1, 5/3, -x^3, ((5 + 3*Sqrt[3])*x^3)/4] - 3*x^3*(AppellF1[5/3, 1/2, 2, 8/3, -x^3, ((5 + 3*Sqrt[3])*x^3)/4] + (5 - 3*Sqrt[3])*AppellF1[5/3, 3/2, 1, 8/3, -x^3, ((5 + 3*Sqrt[3])*x^3)/4])))`

Maple [C] time = 0.266, size = 350, normalized size = 1.7

$$\frac{(\sqrt{3}-1)\left(\frac{3}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}{-18+9\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i}{2}\sqrt{3}}}\sqrt{\frac{1}{-\frac{3}{2}-\frac{i}{2}\sqrt{3}}\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)}\sqrt{\frac{1}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)}\operatorname{EllipticPi}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i}{2}\sqrt{3}}},-\frac{(-\sqrt{2}}{18}\sum_{\alpha=\operatorname{RootOf}(-Z^2+(\sqrt{3}-1)Z-2\sqrt{3}+4)}(-\alpha\sqrt{3}-\alpha+2)(-i\sqrt{3}+3)(-1+2\alpha+\alpha\sqrt{3})}{-\sqrt{3}-2\alpha+1}}\sqrt{\frac{1+x}{-i\sqrt{3}+3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(10+x^3-6*3^(1/2))/(x^3+1)^(1/2),x)`

```
[Out] 1/9*(3^(1/2)-1)/(-2+3^(1/2))*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*3^(1/2)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))/(-3/2-1/2*I*3^(1/2)))^(1/2))-1/18*2^(1/2)*sum((-_alpha*3^(1/2)-_alpha+2)/(-3^(1/2)-2*_alpha+1)*(-I*3^(1/2)+3)*((1+x)/(-I*3^(1/2)+3))^(1/2)*((2*x-1-I*3^(1/2))/(-I*3^(1/2)-3))^(1/2)*((2*x-1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2)/(x^3+1)^(1/2)*(-1+2*_alpha+_alpha*3^(1/2))*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),1/2*I*_alpha+1/3*I*_alpha*3^(1/2)-1/2*_alpha*3^(1/2)-_alpha-1/6*I*3^(1/2)+1/2,((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)),_alpha=RootOf(_Z^2+(3^(1/2)-1)*_Z-2*3^(1/2)+4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(x^3 - 6\sqrt{3} + 10)\sqrt{x^3 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((x^3 - 6*sqrt(3) + 10)*sqrt(x^3 + 1)),x, algorithm="maxima")
```

```
[Out] integrate(x/((x^3 - 6*sqrt(3) + 10)*sqrt(x^3 + 1)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((x^3 - 6*sqrt(3) + 10)*sqrt(x^3 + 1)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{(x+1)(x^2-x+1)}(x^3-6\sqrt{3}+10)} dx$$

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate(x/(10+x**3-6*3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt((x + 1)*(x**2 - x + 1))*(x**3 - 6*sqrt(3) + 10))
, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(x^3 - 6\sqrt{3} + 10)\sqrt{x^3 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((x^3 - 6*sqrt(3) + 10)*sqrt(x^3 + 1)),x, algorithm="giac")
```

```
[Out] integrate(x/((x^3 - 6*sqrt(3) + 10)*sqrt(x^3 + 1)), x)
```

$$3.86 \quad \int \frac{x}{\sqrt{-1+x^3}(-10-6\sqrt{3}+x^3)} dx$$

Optimal. Leaf size=222

$$\frac{(2-\sqrt{3}) \tan^{-1}\left(\frac{\sqrt[4]{3}(1-\sqrt{3})(1-x)}{\sqrt{2}\sqrt{x^3-1}}\right)}{6\sqrt{2}\sqrt[4]{3}} + \frac{(2-\sqrt{3}) \tan^{-1}\left(\frac{\sqrt[4]{3}(2x+\sqrt{3}+1)}{\sqrt{2}\sqrt{x^3-1}}\right)}{3\sqrt{2}\sqrt[4]{3}}$$

$$+ \frac{(2-\sqrt{3}) \tanh^{-1}\left(\frac{\sqrt[4]{3}(1+\sqrt{3})(1-x)}{\sqrt{2}\sqrt{x^3-1}}\right)}{2\sqrt{2}3^{3/4}} - \frac{(2-\sqrt{3}) \tanh^{-1}\left(\frac{(1-\sqrt{3})\sqrt{x^3-1}}{\sqrt{2}3^{3/4}}\right)}{3\sqrt{2}3^{3/4}}$$

[Out] ((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3])*(1 - x))/(Sqrt[2]*Sqrt[-1 + x^3])])/(6*Sqrt[2]*3^(1/4)) + ((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3] + 2*x))/(Sqrt[2]*Sqrt[-1 + x^3])])/(3*Sqrt[2]*3^(1/4)) + ((2 - Sqrt[3])*ArcTanh[(3^(1/4)*(1 + Sqrt[3])*(1 - x))/(Sqrt[2]*Sqrt[-1 + x^3])])/(2*Sqrt[2]*3^(3/4)) - ((2 - Sqrt[3])*ArcTanh[((1 - Sqrt[3])*Sqrt[-1 + x^3])/(Sqrt[2]*3^(3/4))])/(3*Sqrt[2]*3^(3/4))

Rubi [A] time = 0.100172, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.04$

$$\frac{(2-\sqrt{3}) \tan^{-1}\left(\frac{\sqrt[4]{3}(1-\sqrt{3})(1-x)}{\sqrt{2}\sqrt{x^3-1}}\right)}{6\sqrt{2}\sqrt[4]{3}} + \frac{(2-\sqrt{3}) \tan^{-1}\left(\frac{\sqrt[4]{3}(2x+\sqrt{3}+1)}{\sqrt{2}\sqrt{x^3-1}}\right)}{3\sqrt{2}\sqrt[4]{3}}$$

$$+ \frac{(2-\sqrt{3}) \tanh^{-1}\left(\frac{\sqrt[4]{3}(1+\sqrt{3})(1-x)}{\sqrt{2}\sqrt{x^3-1}}\right)}{2\sqrt{2}3^{3/4}} - \frac{(2-\sqrt{3}) \tanh^{-1}\left(\frac{(1-\sqrt{3})\sqrt{x^3-1}}{\sqrt{2}3^{3/4}}\right)}{3\sqrt{2}3^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[x/(Sqrt[-1 + x^3]*(-10 - 6*Sqrt[3] + x^3)), x]

[Out] ((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3])*(1 - x))/(Sqrt[2]*Sqrt[-1 + x^3])])/(6*Sqrt[2]*3^(1/4)) + ((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3] + 2*x))/(Sqrt[2]*Sqrt[-1 + x^3])])/(3*Sqrt[2]*3^(1/4)) + ((2 - Sqrt[3])*ArcTanh[(3^(1/4)*(1 + Sqrt[3])*(1 - x))/(Sqrt[2]*Sqrt[-1 + x^3])])/(2*Sqrt[2]*3^(3/4)) - ((2 - Sqrt[3])*ArcTanh[((1 - Sqrt[3])*Sqrt[-1 + x^3])/(Sqrt[2]*3^(3/4))])/(3*Sqrt[2]*3^(3/4))

Rubi in Sympy [A] time = 11.817, size = 53, normalized size = 0.24

$$\frac{x^2 \sqrt{x^3 - 1} \operatorname{appellf}_1\left(\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, x^3, -\frac{x^3}{-6\sqrt{3}-10}\right)}{2(10 + 6\sqrt{3}) \sqrt{-x^3 + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x/(-10+x**3-6*3**(1/2))/(x**3-1)**(1/2),x)`

[Out] `x**2*sqrt(x**3 - 1)*appellf1(2/3, 1/2, 1, 5/3, x**3, -x**3/(-6*sqrt(3) - 10))/(2*(10 + 6*sqrt(3))*sqrt(-x**3 + 1))`

Mathematica [C] time = 0.623126, size = 196, normalized size = 0.88

$$\frac{10(26 + 15\sqrt{3})x^2 F_1\left(\frac{2}{3}, \frac{1}{2}, 1; \frac{5}{3}; x^3, \frac{x^3}{10+6\sqrt{3}}\right)}{(5 + 3\sqrt{3})\left(-x^3 + 6\sqrt{3} + 10\right)\sqrt{x^3 - 1}\left(3x^3\left(F_1\left(\frac{5}{3}, \frac{1}{2}, 2; \frac{8}{3}; x^3, \frac{x^3}{10+6\sqrt{3}}\right) + (5 + 3\sqrt{3})F_1\left(\frac{5}{3}, \frac{3}{2}, 1; \frac{8}{3}; x^3, \frac{x^3}{10+6\sqrt{3}}\right)\right) + 10(5 + 3\sqrt{3})\right)}$$

Warning: Unable to verify antiderivative.

[In] `Integrate[x/(Sqrt[-1 + x^3]*(-10 - 6*Sqrt[3] + x^3)),x]`

[Out] `(-10*(26 + 15*Sqrt[3])*x^2*AppellF1[2/3, 1/2, 1, 5/3, x^3, x^3/(10 + 6*Sqrt[3])])/((5 + 3*Sqrt[3])*(10 + 6*Sqrt[3] - x^3)*Sqrt[-1 + x^3]*(10*(5 + 3*Sqrt[3])*AppellF1[2/3, 1/2, 1, 5/3, x^3, x^3/(10 + 6*Sqrt[3])] + 3*x^3*(AppellF1[5/3, 1/2, 2, 8/3, x^3, x^3/(10 + 6*Sqrt[3])] + (5 + 3*Sqrt[3])*AppellF1[5/3, 3/2, 1, 8/3, x^3, x^3/(10 + 6*Sqrt[3])])))`

Maple [C] time = 0.257, size = 349, normalized size = 1.6

$$\frac{(-1 - \sqrt{3})\left(-\frac{3}{2} - \frac{i}{2}\sqrt{3}\right)\sqrt{3}}{18 + 9\sqrt{3}} \sqrt{\frac{-1+x}{-\frac{3}{2} - \frac{i}{2}\sqrt{3}}} \sqrt{\frac{1}{\frac{3}{2} - \frac{i}{2}\sqrt{3}}\left(x + \frac{1}{2} - \frac{i}{2}\sqrt{3}\right)} \sqrt{\frac{1}{\frac{3}{2} + \frac{i}{2}\sqrt{3}}\left(x + \frac{1}{2} + \frac{i}{2}\sqrt{3}\right)} \operatorname{EllipticPi}\left(\sqrt{\frac{-1+x}{-\frac{3}{2} - \frac{i}{2}\sqrt{3}}}, -\frac{\sqrt{2}}{18} \sum_{\alpha = \operatorname{RootOf}(-Z^2 + (1+\sqrt{3})_Z + 2\sqrt{3}+4)} \frac{(-\alpha\sqrt{3} + \alpha + 2)(-i\sqrt{3} - 3)(1 + 2\alpha - \alpha\sqrt{3})}{-\sqrt{3} - 2\alpha - 1} \sqrt{\frac{-1+x}{-i\sqrt{3} - 3}} \sqrt{\frac{1}{\frac{3}{2} - \frac{i}{2}\sqrt{3}}\left(x + \frac{1}{2} - \frac{i}{2}\sqrt{3}\right)} \sqrt{\frac{1}{\frac{3}{2} + \frac{i}{2}\sqrt{3}}\left(x + \frac{1}{2} + \frac{i}{2}\sqrt{3}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(-10+x^3-6*3^(1/2))/(x^3-1)^(1/2),x)`

```
[Out] 1/9*(-1-3^(1/2))/(2+3^(1/2))*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*3^(1/2)*EllipticPi(((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(3/2+1/2*I*3^(1/2))*3^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-1/18*2^(1/2)*sum((-alpha*3^(1/2)+alpha+2)/(-3^(1/2)-2*_alpha-1)*(-I*3^(1/2)-3)*((-1+x)/(-I*3^(1/2)-3))^(1/2)*((2*x+1-I*3^(1/2))/(-I*3^(1/2)+3))^(1/2)*((2*x+1+I*3^(1/2))/(I*3^(1/2)+3))^(1/2)/(x^3-1)^(1/2)*(1+2*_alpha-alpha*3^(1/2))*EllipticPi(((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2),-1/2*I*_alpha+1/3*I*_alpha*3^(1/2)-1/2*_alpha*3^(1/2)+alpha+1/6*I*3^(1/2)+1/2,((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)),_alpha=RootOf(_Z^2+(1+3^(1/2))*_Z+2*3^(1/2)+4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(x^3 - 6\sqrt{3} - 10)\sqrt{x^3 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((x^3 - 6*sqrt(3) - 10)*sqrt(x^3 - 1)),x, algorithm="maxima")
```

```
[Out] integrate(x/((x^3 - 6*sqrt(3) - 10)*sqrt(x^3 - 1)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((x^3 - 6*sqrt(3) - 10)*sqrt(x^3 - 1)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{(x-1)(x^2+x+1)}(x^3-6\sqrt{3}-10)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-10+x**3-6*3**(1/2))/(x**3-1)**(1/2),x)

[Out] Integral(x/(sqrt((x - 1)*(x**2 + x + 1))*(x**3 - 6*sqrt(3) - 10)), x)

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(x^3 - 6\sqrt{3} - 10)\sqrt{x^3 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/((x^3 - 6*sqrt(3) - 10)*sqrt(x^3 - 1)),x, algorithm="giac")

[Out] integrate(x/((x^3 - 6*sqrt(3) - 10)*sqrt(x^3 - 1)), x)

$$3.87 \quad \int \frac{x}{\sqrt{-1+x^3}(-10+6\sqrt{3}+x^3)} dx$$

Optimal. Leaf size=214

$$\begin{aligned} & -\frac{(2+\sqrt{3}) \tan^{-1}\left(\frac{\sqrt[4]{3}(1-\sqrt{3})(1-x)}{\sqrt{2}\sqrt{x^3-1}}\right)}{2\sqrt{2}3^{3/4}} + \frac{(2+\sqrt{3}) \tan^{-1}\left(\frac{(1+\sqrt{3})\sqrt{x^3-1}}{\sqrt{2}3^{3/4}}\right)}{3\sqrt{2}3^{3/4}} \\ & + \frac{(2+\sqrt{3}) \tanh^{-1}\left(\frac{\sqrt[4]{3}(1+\sqrt{3})(1-x)}{\sqrt{2}\sqrt{x^3-1}}\right)}{6\sqrt{2}\sqrt[4]{3}} + \frac{(2+\sqrt{3}) \tanh^{-1}\left(\frac{\sqrt[4]{3}(2x-\sqrt{3}+1)}{\sqrt{2}\sqrt{x^3-1}}\right)}{3\sqrt{2}\sqrt[4]{3}} \end{aligned}$$

[Out] $-\left((2 + \text{Sqrt}[3]) * \text{ArcTan}[(3^{1/4}) * (1 - \text{Sqrt}[3]) * (1 - x)] / (\text{Sqrt}[2] * \text{Sqrt}[-1 + x^3])\right) / (2 * \text{Sqrt}[2] * 3^{3/4}) + \left((2 + \text{Sqrt}[3]) * \text{ArcTan}[\left(\frac{(1 + \text{Sqrt}[3]) * \text{Sqrt}[-1 + x^3]}{\text{Sqrt}[2] * 3^{3/4}}\right)] / (3 * \text{Sqrt}[2] * 3^{3/4})\right) + \left((2 + \text{Sqrt}[3]) * \text{ArcTanh}[(3^{1/4}) * (1 + \text{Sqrt}[3]) * (1 - x)] / (\text{Sqrt}[2] * \text{Sqrt}[-1 + x^3])\right) / (6 * \text{Sqrt}[2] * 3^{1/4}) + \left((2 + \text{Sqrt}[3]) * \text{ArcTanh}[(3^{1/4}) * (1 - \text{Sqrt}[3] + 2 * x)] / (\text{Sqrt}[2] * \text{Sqrt}[-1 + x^3])\right) / (3 * \text{Sqrt}[2] * 3^{1/4})$

Rubi [A] time = 0.0962502, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.04$

$$\begin{aligned} & -\frac{(2+\sqrt{3}) \tan^{-1}\left(\frac{\sqrt[4]{3}(1-\sqrt{3})(1-x)}{\sqrt{2}\sqrt{x^3-1}}\right)}{2\sqrt{2}3^{3/4}} + \frac{(2+\sqrt{3}) \tan^{-1}\left(\frac{(1+\sqrt{3})\sqrt{x^3-1}}{\sqrt{2}3^{3/4}}\right)}{3\sqrt{2}3^{3/4}} \\ & + \frac{(2+\sqrt{3}) \tanh^{-1}\left(\frac{\sqrt[4]{3}(1+\sqrt{3})(1-x)}{\sqrt{2}\sqrt{x^3-1}}\right)}{6\sqrt{2}\sqrt[4]{3}} + \frac{(2+\sqrt{3}) \tanh^{-1}\left(\frac{\sqrt[4]{3}(2x-\sqrt{3}+1)}{\sqrt{2}\sqrt{x^3-1}}\right)}{3\sqrt{2}\sqrt[4]{3}} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[x/(Sqrt[-1 + x^3]*(-10 + 6*Sqrt[3] + x^3)), x]

[Out] $-\left((2 + \text{Sqrt}[3]) * \text{ArcTan}[(3^{1/4}) * (1 - \text{Sqrt}[3]) * (1 - x)] / (\text{Sqrt}[2] * \text{Sqrt}[-1 + x^3])\right) / (2 * \text{Sqrt}[2] * 3^{3/4}) + \left((2 + \text{Sqrt}[3]) * \text{ArcTan}[\left(\frac{(1 + \text{Sqrt}[3]) * \text{Sqrt}[-1 + x^3]}{\text{Sqrt}[2] * 3^{3/4}}\right)] / (3 * \text{Sqrt}[2] * 3^{3/4})\right) + \left((2 + \text{Sqrt}[3]) * \text{ArcTanh}[(3^{1/4}) * (1 + \text{Sqrt}[3]) * (1 - x)] / (\text{Sqrt}[2] * \text{Sqrt}[-1 + x^3])\right) / (6 * \text{Sqrt}[2] * 3^{1/4}) + \left((2 + \text{Sqrt}[3]) * \text{ArcTanh}[(3^{1/4}) * (1 - \text{Sqrt}[3] + 2 * x)] / (\text{Sqrt}[2] * \text{Sqrt}[-1 + x^3])\right) / (3 * \text{Sqrt}[2] * 3^{1/4})$

Rubi in Sympy [A] time = 6.47048, size = 51, normalized size = 0.24

$$\frac{x^2 \sqrt{x^3 - 1} \operatorname{appellf}_1\left(\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, x^3, -\frac{x^3}{-10+6\sqrt{3}}\right)}{2(-6\sqrt{3} + 10) \sqrt{-x^3 + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x/(-10+x**3+6*3**(1/2)))/(x**3-1)**(1/2), x)`

[Out] `x**2*sqrt(x**3 - 1)*appellf1(2/3, 1/2, 1, 5/3, x**3, -x**3/(-10 + 6*sqrt(3)))/(2*(-6*sqrt(3) + 10)*sqrt(-x**3 + 1))`

Mathematica [C] time = 0.53932, size = 198, normalized size = 0.93

$$\frac{10(26 - 15\sqrt{3}) x^2 F_1\left(\frac{2}{3}; \frac{1}{2}, 1; \frac{5}{3}; x^3, -\frac{1}{4}(5 + 3\sqrt{3}) x^3\right)}{(3\sqrt{3} - 5) \sqrt{x^3 - 1} (x^3 + 6\sqrt{3} - 10) \left(10(3\sqrt{3} - 5) F_1\left(\frac{2}{3}; \frac{1}{2}, 1; \frac{5}{3}; x^3, -\frac{1}{4}(5 + 3\sqrt{3}) x^3\right) - 3x^3 \left(F_1\left(\frac{5}{3}; \frac{1}{2}, 2; \frac{8}{3}; x^3, -\frac{1}{4}(5 + 3\sqrt{3}) x^3\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

[In] `Integrate[x/(Sqrt[-1 + x^3]*(-10 + 6*Sqrt[3] + x^3)), x]`

[Out] `(10*(26 - 15*Sqrt[3])*x^2*AppellF1[2/3, 1/2, 1, 5/3, x^3, -((5 + 3*Sqrt[3])*x^3)/4])/((-5 + 3*Sqrt[3])*Sqrt[-1 + x^3]*(-10 + 6*Sqrt[3] + x^3)*(10*(-5 + 3*Sqrt[3])*AppellF1[2/3, 1/2, 1, 5/3, x^3, -((5 + 3*Sqrt[3])*x^3)/4] - 3*x^3*(AppellF1[5/3, 1/2, 2, 8/3, x^3, -((5 + 3*Sqrt[3])*x^3)/4] + (5 - 3*Sqrt[3])*AppellF1[5/3, 3/2, 1, 8/3, x^3, -((5 + 3*Sqrt[3])*x^3)/4])))`

Maple [C] time = 0.256, size = 350, normalized size = 1.6

$$\frac{(\sqrt{3} - 1) \left(-\frac{3}{2} - \frac{i}{2}\sqrt{3}\right) \sqrt{3}}{-18 + 9\sqrt{3}} \sqrt{\frac{-1+x}{-\frac{3}{2} - \frac{i}{2}\sqrt{3}}} \sqrt{\frac{1}{\frac{3}{2} - \frac{i}{2}\sqrt{3}} \left(x + \frac{1}{2} - \frac{i}{2}\sqrt{3}\right)} \sqrt{\frac{1}{\frac{3}{2} + \frac{i}{2}\sqrt{3}} \left(x + \frac{1}{2} + \frac{i}{2}\sqrt{3}\right)} \operatorname{EllipticPi}\left(\sqrt{\frac{-1+x}{-\frac{3}{2} - \frac{i}{2}\sqrt{3}}}, \frac{3}{2}\right) - \frac{\sqrt{2}}{18} \sum_{\alpha = \operatorname{RootOf}\left(-Z^2 + (1-\sqrt{3})_Z - 2\sqrt{3} + 4\right)} \frac{(-\alpha\sqrt{3} - \alpha - 2)(-i\sqrt{3} - 3)(1 + 2\alpha + \alpha\sqrt{3})}{-\sqrt{3} + 2\alpha + 1} \sqrt{\frac{-1+x}{-i\sqrt{3} - 3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(-10+x^3+6*3^(1/2)))/(x^3-1)^(1/2), x)`

```
[Out] 1/9*(3^(1/2)-1)/(-2+3^(1/2))*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*3^(1/2)*EllipticPi(((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2),1/3*(3/2+1/2*I*3^(1/2))*3^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-1/18*2^(1/2)*sum((-alpha*3^(1/2)-alpha-2)/(-3^(1/2)+2*_alpha+1)*(-I*3^(1/2)-3)*((-1+x)/(-I*3^(1/2)-3))^(1/2)*((2*x+1-I*3^(1/2))/(-I*3^(1/2)+3))^(1/2)*((2*x+1+I*3^(1/2))/(I*3^(1/2)+3))^(1/2)/(x^3-1)^(1/2)*(1+2*_alpha+alpha*3^(1/2))*EllipticPi(((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2),1/2*I*_alpha+1/3*I*_alpha*3^(1/2)+1/2*_alpha*3^(1/2)+alpha+1/6*I*3^(1/2)+1/2,((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)),_alpha=RootOf(_Z^2+(1-3^(1/2))*_Z-2*3^(1/2)+4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(x^3 + 6\sqrt{3} - 10)\sqrt{x^3 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((x^3 + 6*sqrt(3) - 10)*sqrt(x^3 - 1)),x, algorithm="maxima")
```

```
[Out] integrate(x/((x^3 + 6*sqrt(3) - 10)*sqrt(x^3 - 1)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((x^3 + 6*sqrt(3) - 10)*sqrt(x^3 - 1)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{(x-1)(x^2+x+1)}(x^3-10+6\sqrt{3})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(-10+x**3+6*3**(1/2))/(x**3-1)**(1/2),x)`

[Out] `Integral(x/(sqrt((x - 1)*(x**2 + x + 1))*(x**3 - 10 + 6*sqrt(3))), x)`

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(x^3 + 6\sqrt{3} - 10)\sqrt{x^3 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/((x^3 + 6*sqrt(3) - 10)*sqrt(x^3 - 1)),x, algorithm="giac")`

[Out] `integrate(x/((x^3 + 6*sqrt(3) - 10)*sqrt(x^3 - 1)), x)`

$$3.88 \quad \int \frac{1-\sqrt{3+x}}{(1+\sqrt{3+x})\sqrt{-4+4\sqrt{3}x^2+x^4}} dx$$

Optimal. Leaf size=65

$$\frac{1}{3}\sqrt{2\sqrt{3}-3} \tanh^{-1}\left(\frac{(x-\sqrt{3}+1)^2}{\sqrt{3(2\sqrt{3}-3)}\sqrt{x^4+4\sqrt{3}x^2-4}}\right)$$

[Out] (Sqrt[-3 + 2*Sqrt[3]]*ArcTanh[(1 - Sqrt[3] + x)^2/(Sqrt[3*(-3 + 2*Sqrt[3]])*Sqrt[-4 + 4*Sqrt[3]*x^2 + x^4])])/3

Rubi [A] time = 0.211471, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.05$

$$\frac{1}{3}\sqrt{2\sqrt{3}-3} \tanh^{-1}\left(\frac{(x-\sqrt{3}+1)^2}{\sqrt{3(2\sqrt{3}-3)}\sqrt{x^4+4\sqrt{3}x^2-4}}\right)$$

Antiderivative was successfully verified.

[In] Int[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[-4 + 4*Sqrt[3]*x^2 + x^4]), x]

[Out] (Sqrt[-3 + 2*Sqrt[3]]*ArcTanh[(1 - Sqrt[3] + x)^2/(Sqrt[3*(-3 + 2*Sqrt[3]])*Sqrt[-4 + 4*Sqrt[3]*x^2 + x^4])])/3

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate((1+x-3**(1/2))/(1+x+3**(1/2))/(-4+x**4+4*3**(1/2)*x**2)**(1/2),

[Out] Timed out

Mathematica [C] time = 4.03084, size = 685, normalized size = 10.54

$$\frac{(x + \sqrt{3} - 1)^2 \sqrt{-x^3 + (\sqrt{3} - 1)x^2 - 2(2 + \sqrt{3})x + 2(1 + \sqrt{3})}}{\sqrt{\frac{-\frac{4}{x + \sqrt{3} - 1} + \sqrt{3} + 1}{3 + \sqrt{3} + i\sqrt{2(2 + \sqrt{3})}}}} \left(2\sqrt{6} \sqrt{\frac{x^2 + 2\sqrt{3} + 4}{(x + \sqrt{3} - 1)^2}} \sqrt{\sqrt{2(2 + \sqrt{3})}} - i \left(\frac{\sqrt{2}}{x} \right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[-4 + 4*Sqrt[3]*x^2 + x^4]),x]

[Out] ((-1 + Sqrt[3] + x)^2*Sqrt[2*(1 + Sqrt[3]) - 2*(2 + Sqrt[3])*x + (-1 + Sqrt[3])*x^2 - x^3]*Sqrt[(1 + Sqrt[3] - 4/(-1 + Sqrt[3] + x))/(3 + Sqrt[3] + I*Sqrt[2*(2 + Sqrt[3])]])*(I*(-1 + Sqrt[3] + I*Sqrt[2*(2 + Sqrt[3])]) + (2*((2*I)*Sqrt[3] - Sqrt[2*(2 + Sqrt[3])]) + Sqrt[6*(2 + Sqrt[3])]))/(-1 + Sqrt[3] + x)*Sqrt[Sqrt[2*(2 + Sqrt[3])] + I*(1 - Sqrt[3] + 8/(-1 + Sqrt[3] + x))]*EllipticF[ArcSin[Sqrt[Sqrt[2*(2 + Sqrt[3])] - I*(1 - Sqrt[3] + 8/(-1 + Sqrt[3] + x))]/(2^(3/4)*(2 + Sqrt[3])^(1/4))], ((2*I)*Sqrt[2*(2 + Sqrt[3])])/(3 + Sqrt[3] + I*Sqrt[2*(2 + Sqrt[3])]) + 2*Sqrt[6]*Sqrt[(4 + 2*Sqrt[3] + x^2)/(-1 + Sqrt[3] + x)^2]*Sqrt[Sqrt[2*(2 + Sqrt[3])] - I*(1 - Sqrt[3] + 8/(-1 + Sqrt[3] + x))]*EllipticPi[(2*Sqrt[2*(2 + Sqrt[3])])/(Sqrt[2*(2 + Sqrt[3])] + I*(3 + Sqrt[3]))], ArcSin[Sqrt[Sqrt[2*(2 + Sqrt[3])] - I*(1 - Sqrt[3] + 8/(-1 + Sqrt[3] + x))]/(2^(3/4)*(2 + Sqrt[3])^(1/4))], ((2*I)*Sqrt[2*(2 + Sqrt[3])])/(3 + Sqrt[3] + I*Sqrt[2*(2 + Sqrt[3])])])]/((Sqrt[2*(2 + Sqrt[3])] + I*(3 + Sqrt[3]))*Sqrt[1 + Sqrt[3] - (2 + Sqrt[3])*x + ((-1 + Sqrt[3])*x^2)/2 - x^3/2]*Sqrt[-4 + 4*Sqrt[3]*x^2 + x^4]*Sqrt[Sqrt[2*(2 + Sqrt[3])] - I*(1 - Sqrt[3] + 8/(-1 + Sqrt[3] + x))])

Maple [C] time = 0.24, size = 327, normalized size = 5.

$$\frac{\text{EllipticF}\left(x\left(\frac{i}{2}\sqrt{3} - \frac{i}{2}\right), i\sqrt{1 + 4\sqrt{3}\left(1 + \frac{1}{2}\sqrt{3}\right)}\right)}{\frac{i}{2}\sqrt{3} - \frac{i}{2}} \sqrt{1 - \left(-1 + \frac{\sqrt{3}}{2}\right)x^2} \sqrt{1 - \left(1 + \frac{\sqrt{3}}{2}\right)x^2} \frac{1}{\sqrt{-4 + x^4 + 4\sqrt{3}x^2}}$$

$$- 2\sqrt{3} \left(-\frac{1}{2} \frac{1}{\sqrt{(-1 - \sqrt{3})^4 + 4\sqrt{3}(-1 - \sqrt{3})^2 - 4}} \text{Artanh} \left(\frac{1}{2} \frac{4\sqrt{3}(-1 - \sqrt{3})^2 - 8 + 4\sqrt{3}x^2 + 2x^2(-1 - \sqrt{3})^2}{\sqrt{(-1 - \sqrt{3})^4 + 4\sqrt{3}(-1 - \sqrt{3})^2 - 4}\sqrt{-4 + x^4 + 4\sqrt{3}x^2}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+x-3^(1/2))/(1+x+3^(1/2))/(-4+x^4+4*3^(1/2)*x^2)^(1/2),x)

```
[Out] 1/(1/2*I*3^(1/2)-1/2*I)*(1-(-1+1/2*3^(1/2))*x^2)^(1/2)*(1-(1+1/2*
3^(1/2))*x^2)^(1/2)/(-4+x^4+4*3^(1/2)*x^2)^(1/2)*EllipticF(x*(1/2
*I*3^(1/2)-1/2*I),I*(1+4*3^(1/2)*(1+1/2*3^(1/2)))^(1/2))-2*3^(1/2
)*(-1/2/((-1-3^(1/2))^4+4*3^(1/2)*(-1-3^(1/2))^2-4)^(1/2)*arctanh
(1/2*(4*3^(1/2)*(-1-3^(1/2))^2-8+4*3^(1/2)*x^2+2*x^2*(-1-3^(1/2))
^2)/((-1-3^(1/2))^4+4*3^(1/2)*(-1-3^(1/2))^2-4)^(1/2)/(-4+x^4+4*3
^(1/2)*x^2)^(1/2))-1/(-1+1/2*3^(1/2))^(1/2)/(-1-3^(1/2))*(1-(-1+1
/2*3^(1/2))*x^2)^(1/2)*(1-(1+1/2*3^(1/2))*x^2)^(1/2)/(-4+x^4+4*3^
(1/2)*x^2)^(1/2)*EllipticPi((-1+1/2*3^(1/2))^(1/2)*x,1/(-1+1/2*3^
(1/2))/(-1-3^(1/2))^2,(1+1/2*3^(1/2))^(1/2)/(-1+1/2*3^(1/2))^(1/2
)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x - \sqrt{3} + 1}{\sqrt{x^4 + 4\sqrt{3}x^2 - 4}(x + \sqrt{3} + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x - sqrt(3) + 1)/(sqrt(x^4 + 4*sqrt(3)*x^2 - 4)*(x + sqrt(3) + 1)), x, a
```

```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(x^4 + 4*sqrt(3)*x^2 - 4)*(x + s
qrt(3) + 1)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x - sqrt(3) + 1)/(sqrt(x^4 + 4*sqrt(3)*x^2 - 4)*(x + sqrt(3) + 1)), x, a
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x - \sqrt{3} + 1}{(x + 1 + \sqrt{3})\sqrt{x^4 + 4\sqrt{3}x^2 - 4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3**(1/2))/(1+x+3**(1/2))/(-4+x**4+4*3**(1/2)*x**2)**(1/2), x)
```

[Out] Integral((x - sqrt(3) + 1)/((x + 1 + sqrt(3))*sqrt(x**4 + 4*sqrt(3)*x**2 - 4)), x)

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x - \sqrt{3} + 1}{\sqrt{x^4 + 4\sqrt{3}x^2 - 4}(x + \sqrt{3} + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x - sqrt(3) + 1)/(sqrt(x^4 + 4*sqrt(3)*x^2 - 4)*(x + sqrt(3) + 1)), x, a

[Out] integrate((x - sqrt(3) + 1)/(sqrt(x^4 + 4*sqrt(3)*x^2 - 4)*(x + sqrt(3) + 1)), x)

$$3.89 \quad \int \frac{1+\sqrt{3+x}}{(1-\sqrt{3+x})\sqrt{-4-4\sqrt{3}x^2+x^4}} dx$$

Optimal. Leaf size=63

$$-\frac{1}{3}\sqrt{3+2\sqrt{3}} \tan^{-1} \left(\frac{(x+\sqrt{3}+1)^2}{\sqrt{3(3+2\sqrt{3})}\sqrt{x^4-4\sqrt{3}x^2-4}} \right)$$

[Out] -(Sqrt[3 + 2*Sqrt[3]]*ArcTan[(1 + Sqrt[3] + x)^2/(Sqrt[3*(3 + 2*Sqrt[3]))*Sqrt[-4 - 4*Sqrt[3]*x^2 + x^4]]])/3

Rubi [A] time = 0.21298, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.05$

$$-\frac{1}{3}\sqrt{3+2\sqrt{3}} \tan^{-1} \left(\frac{(x+\sqrt{3}+1)^2}{\sqrt{3(3+2\sqrt{3})}\sqrt{x^4-4\sqrt{3}x^2-4}} \right)$$

Antiderivative was successfully verified.

[In] Int[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[-4 - 4*Sqrt[3]*x^2 + x^4]), x]

[Out] -(Sqrt[3 + 2*Sqrt[3]]*ArcTan[(1 + Sqrt[3] + x)^2/(Sqrt[3*(3 + 2*Sqrt[3]))*Sqrt[-4 - 4*Sqrt[3]*x^2 + x^4]]])/3

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate((1+x+3**(1/2))/(1+x-3**(1/2))/(-4+x**4-4*3**(1/2)*x**2)**(1/2),

[Out] Timed out

Mathematica [C] time = 10.7136, size = 1137, normalized size = 18.05

$$\left((x - \sqrt{3} - 1)^2 \sqrt{\frac{\sqrt{3}-1+\frac{4}{x-\sqrt{3}-1}}{-3+\sqrt{3}-i\sqrt{4-2\sqrt{3}}}} \sqrt{(x - \sqrt{3} + 1)^3 + (-2 + 4\sqrt{3})(x - \sqrt{3} + 1)^2 + (20 - 8\sqrt{3})(x - \sqrt{3} + 1) + 16\sqrt{3} - 24} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[-4 - 4*Sqrt[3]*x^2 + x^4]),x]

[Out] -(((1 - Sqrt[3] + x)^2*Sqrt[(-1 + Sqrt[3] + 4/(-1 - Sqrt[3] + x))]/(-3 + Sqrt[3] - I*Sqrt[4 - 2*Sqrt[3]])]*Sqrt[-24 + 16*Sqrt[3] + (20 - 8*Sqrt[3])*(1 - Sqrt[3] + x) + (-2 + 4*Sqrt[3])*(1 - Sqrt[3] + x)^2 + (1 - Sqrt[3] + x)^3]*((I*Sqrt[Sqrt[4 - 2*Sqrt[3]]) + I*(1 + Sqrt[3]) + (8*I)/(-1 - Sqrt[3] + x)] + I*Sqrt[3]*Sqrt[Sqrt[4 - 2*Sqrt[3]] + I*(1 + Sqrt[3]) + (8*I)/(-1 - Sqrt[3] + x)] + Sqrt[-2*I + (2*I)*Sqrt[3] - 2*Sqrt[12 - 6*Sqrt[3]]] + 4*Sqrt[4 - 2*Sqrt[3]] - ((16*I)*(-2 + Sqrt[3]))/(-1 - Sqrt[3] + x) + (2*((2*I)*Sqrt[3]*Sqrt[Sqrt[4 - 2*Sqrt[3]] + I*(1 + Sqrt[3]) + (8*I)/(-1 - Sqrt[3] + x)] + Sqrt[6]*Sqrt[-I + I*Sqrt[3] - Sqrt[12 - 6*Sqrt[3]]] + 2*Sqrt[4 - 2*Sqrt[3]] - ((8*I)*(-2 + Sqrt[3]))/(-1 - Sqrt[3] + x) + Sqrt[-2*I + (2*I)*Sqrt[3] - 2*Sqrt[12 - 6*Sqrt[3]]] + 4*Sqrt[4 - 2*Sqrt[3]] - ((16*I)*(-2 + Sqrt[3]))/(-1 - Sqrt[3] + x)))/(-1 - Sqrt[3] + x))*EllipticF[ArcSin[Sqrt[Sqrt[4 - 2*Sqrt[3]] - I*(1 + Sqrt[3]) - (8*I)/(-1 - Sqrt[3] + x)]/(2^(3/4)*(2 - Sqrt[3]))^(1/4)], (2*Sqrt[4 - 2*Sqrt[3]])/(Sqrt[4 - 2*Sqrt[3]] + I*(-3 + Sqrt[3]))] + 2*Sqrt[6]*Sqrt[Sqrt[4 - 2*Sqrt[3]] - I*(1 + Sqrt[3]) - (8*I)/(-1 - Sqrt[3] + x)]*Sqrt[1 + 8/(-1 - Sqrt[3] + x)^2 + (2*(1 + Sqrt[3]))/(-1 - Sqrt[3] + x)]*EllipticPi[(2*Sqrt[4 - 2*Sqrt[3]])/(Sqrt[4 - 2*Sqrt[3]] - I*(-3 + Sqrt[3]))], ArcSin[Sqrt[Sqrt[4 - 2*Sqrt[3]] - I*(1 + Sqrt[3]) - (8*I)/(-1 - Sqrt[3] + x)]/(2^(3/4)*(2 - Sqrt[3]))^(1/4)], (2*Sqrt[4 - 2*Sqrt[3]])/(Sqrt[4 - 2*Sqrt[3]] + I*(-3 + Sqrt[3])))]/((Sqrt[4 - 2*Sqrt[3]] - I*(-3 + Sqrt[3]))*Sqrt[Sqrt[4 - 2*Sqrt[3]] - I*(1 + Sqrt[3]) - (8*I)/(-1 - Sqrt[3] + x)]*Sqrt[8*(1 + Sqrt[3]) + 4*(3 + Sqrt[3])*(-1 - Sqrt[3] + x) + 2*(1 + Sqrt[3])*(-1 - Sqrt[3] + x)^2 + (-1 - Sqrt[3] + x)^3/2]*Sqrt[48 - 32*Sqrt[3] - 64*(1 - Sqrt[3] + x) + 32*Sqrt[3]*(1 - Sqrt[3] + x) + 24*(1 - Sqrt[3] + x)^2 - 16*Sqrt[3]*(1 - Sqrt[3] + x)^2 - 4*(1 - Sqrt[3] + x)^3 + 4*Sqrt[3]*(1 - Sqrt[3] + x)^3 + (1 - Sqrt[3] + x)^4]))

Maple [C] time = 0.235, size = 311, normalized size = 4.9

$$\frac{\text{EllipticF}\left(x\left(\frac{i}{2} + \frac{i}{2}\sqrt{3}\right), i\sqrt{1-4\sqrt{3}\left(1-\frac{1}{2}\sqrt{3}\right)}\right)}{\frac{i}{2} + \frac{i}{2}\sqrt{3}} \sqrt{1-\left(-1-\frac{\sqrt{3}}{2}\right)x^2} \sqrt{1-\left(1-\frac{\sqrt{3}}{2}\right)x^2} \frac{1}{\sqrt{-4+x^4-4\sqrt{3}x^2}}$$

$$+ 2\sqrt{3} \left(-\frac{1}{2} \frac{1}{\sqrt{\left(\sqrt{3}-1\right)^4-4\sqrt{3}\left(\sqrt{3}-1\right)^2-4}} \text{Artanh}\left(\frac{1}{2} \frac{-4\sqrt{3}\left(\sqrt{3}-1\right)^2-8-4\sqrt{3}x^2+2x^2\left(\sqrt{3}-1\right)^2}{\sqrt{\left(\sqrt{3}-1\right)^4-4\sqrt{3}\left(\sqrt{3}-1\right)^2-4\sqrt{-4+x^4-4\sqrt{3}x^2}}}\right) - \frac{\sqrt{1-\left(-1-\frac{\sqrt{3}}{2}\right)x^2}}{\sqrt{1-\left(1-\frac{\sqrt{3}}{2}\right)x^2}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+x+3^(1/2))/(1+x-3^(1/2))/(-4+x^4-4*3^(1/2)*x^2)^(1/2), x)

[Out] 1/(1/2*I+1/2*I*3^(1/2))*(1-(-1-1/2*3^(1/2))*x^2)^(1/2)*(1-(1-1/2*3^(1/2))*x^2)^(1/2)/(-4+x^4-4*3^(1/2)*x^2)^(1/2)*EllipticF(x*(1/2*I+1/2*I*3^(1/2)), I*(1-4*3^(1/2)*(1-1/2*3^(1/2)))^(1/2))+2*3^(1/2)*(-1/2/((3^(1/2)-1)^4-4*3^(1/2)*(3^(1/2)-1)^2-4)^(1/2)*arctanh(1/2*(-4*3^(1/2)*(3^(1/2)-1)^2-8-4*3^(1/2)*x^2+2*x^2*(3^(1/2)-1)^2)/((3^(1/2)-1)^4-4*3^(1/2)*(3^(1/2)-1)^2-4)^(1/2)/(-4+x^4-4*3^(1/2)*x^2)^(1/2))-1/(-1-1/2*3^(1/2))^(1/2)/(3^(1/2)-1)*(1-(-1-1/2*3^(1/2))*x^2)^(1/2)*(1-(1-1/2*3^(1/2))*x^2)^(1/2)/(-4+x^4-4*3^(1/2)*x^2)^(1/2)*EllipticPi((-1-1/2*3^(1/2))^(1/2)*x, 1/(-1-1/2*3^(1/2))^(1/2)/(3^(1/2)-1)^2, (1-1/2*3^(1/2))^(1/2)/(-1-1/2*3^(1/2))^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x + \sqrt{3} + 1}{\sqrt{x^4 - 4\sqrt{3}x^2 - 4}(x - \sqrt{3} + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x + sqrt(3) + 1)/(sqrt(x^4 - 4*sqrt(3)*x^2 - 4)*(x - sqrt(3) + 1)), x, a

[Out] integrate((x + sqrt(3) + 1)/(sqrt(x^4 - 4*sqrt(3)*x^2 - 4)*(x - sqrt(3) + 1)), x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x + sqrt(3) + 1)/(sqrt(x^4 - 4*sqrt(3)*x^2 - 4)*(x - sqrt(3) + 1)), x, a

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x + 1 + \sqrt{3}}{(x - \sqrt{3} + 1) \sqrt{x^4 - 4\sqrt{3}x^2 - 4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x+3**(1/2))/(1+x-3**(1/2))/(-4+x**4-4*3**(1/2)*x**2)**(1/2), x)

[Out] Integral((x + 1 + sqrt(3))/((x - sqrt(3) + 1)*sqrt(x**4 - 4*sqrt(3)*x**2 - 4)), x)

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x + \sqrt{3} + 1}{\sqrt{x^4 - 4\sqrt{3}x^2 - 4}(x - \sqrt{3} + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x + sqrt(3) + 1)/(sqrt(x^4 - 4*sqrt(3)*x^2 - 4)*(x - sqrt(3) + 1)), x, a

[Out] integrate((x + sqrt(3) + 1)/(sqrt(x^4 - 4*sqrt(3)*x^2 - 4)*(x - sqrt(3) + 1)), x)

$$3.90 \quad \int \frac{-1+x}{(1+x)\sqrt[3]{2+x^3}} dx$$

Optimal. Leaf size=53

$$-\frac{3}{2} \log\left(-\sqrt[3]{x^3+2}+x+2\right) + \sqrt{3} \tan^{-1}\left(\frac{\frac{2(x+2)}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right) + \log(x+1)$$

[Out] Sqrt[3]*ArcTan[(1+(2*(2+x))/(2+x^3)^(1/3))/Sqrt[3]]+Log[1+x]- (3*Log[2+x-(2+x^3)^(1/3)])/2

Rubi [A] time = 0.0963491, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$

$$-\frac{3}{2} \log\left(-\sqrt[3]{x^3+2}+x+2\right) + \sqrt{3} \tan^{-1}\left(\frac{\frac{2(x+2)}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right) + \log(x+1)$$

Antiderivative was successfully verified.

[In] Int[(-1+x)/((1+x)*(2+x^3)^(1/3)),x]

[Out] Sqrt[3]*ArcTan[(1+(2*(2+x))/(2+x^3)^(1/3))/Sqrt[3]]+Log[1+x]- (3*Log[2+x-(2+x^3)^(1/3)])/2

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-1}{(x+1)\sqrt[3]{x^3+2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate((-1+x)/(1+x)/(x**3+2)**(1/3),x)

[Out] Integral((x-1)/((x+1)*(x**3+2)**(1/3)),x)

Mathematica [A] time = 0.143547, size = 0, normalized size = 0.

$$\int \frac{-1+x}{(1+x)\sqrt[3]{2+x^3}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(-1 + x)/((1 + x)*(2 + x^3)^(1/3)), x]

[Out] Integrate[(-1 + x)/((1 + x)*(2 + x^3)^(1/3)), x]

Maple [F] time = 0.045, size = 0, normalized size = 0.

$$\int \frac{-1+x}{1+x} \frac{1}{\sqrt[3]{x^3+2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-1+x)/(1+x)/(x^3+2)^(1/3), x)

[Out] int((-1+x)/(1+x)/(x^3+2)^(1/3), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-1}{(x^3+2)^{\frac{1}{3}}(x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x - 1)/((x^3 + 2)^(1/3)*(x + 1)), x, algorithm="maxima")

[Out] integrate((x - 1)/((x^3 + 2)^(1/3)*(x + 1)), x)

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x - 1)/((x^3 + 2)^(1/3)*(x + 1)), x, algorithm="fricas")

[Out] Exception raised: TypeError

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-1}{(x+1)\sqrt[3]{x^3+2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x)/(1+x)/(x**3+2)**(1/3),x)

[Out] Integral((x - 1)/((x + 1)*(x**3 + 2)**(1/3)), x)

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-1}{(x^3+2)^{\frac{1}{3}}(x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x - 1)/((x^3 + 2)^(1/3)*(x + 1)),x, algorithm="giac")

[Out] integrate((x - 1)/((x^3 + 2)^(1/3)*(x + 1)), x)

$$3.91 \quad \int \frac{1}{(1+x)\sqrt[3]{2+x^3}} dx$$

Optimal. Leaf size=108

$$\begin{aligned} & \frac{3}{4} \log\left(-\sqrt[3]{x^3+2}+x+2\right) - \frac{1}{4} \log\left(\sqrt[3]{x^3+2}-x\right) \\ & + \frac{\tan^{-1}\left(\frac{\sqrt[3]{x^3+2}+1}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{1}{2}\sqrt{3} \tan^{-1}\left(\frac{\sqrt[3]{x^3+2}}{\sqrt{3}}+1\right) - \frac{1}{2} \log(x+1) \end{aligned}$$

[Out] ArcTan[(1 + (2*x)/(2 + x^3)^(1/3))/Sqrt[3]]/(2*Sqrt[3]) - (Sqrt[3]*ArcTan[(1 + (2*(2 + x))/(2 + x^3)^(1/3))/Sqrt[3]])/2 - Log[1 + x]/2 + (3*Log[2 + x - (2 + x^3)^(1/3)])/4 - Log[-x + (2 + x^3)^(1/3)]/4

Rubi [A] time = 0.160376, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\begin{aligned} & \frac{3}{4} \log\left(-\sqrt[3]{x^3+2}+x+2\right) - \frac{1}{4} \log\left(\sqrt[3]{x^3+2}-x\right) \\ & + \frac{\tan^{-1}\left(\frac{\sqrt[3]{x^3+2}+1}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{1}{2}\sqrt{3} \tan^{-1}\left(\frac{\sqrt[3]{x^3+2}}{\sqrt{3}}+1\right) - \frac{1}{2} \log(x+1) \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[1/((1 + x)*(2 + x^3)^(1/3)), x]

[Out] ArcTan[(1 + (2*x)/(2 + x^3)^(1/3))/Sqrt[3]]/(2*Sqrt[3]) - (Sqrt[3]*ArcTan[(1 + (2*(2 + x))/(2 + x^3)^(1/3))/Sqrt[3]])/2 - Log[1 + x]/2 + (3*Log[2 + x - (2 + x^3)^(1/3)])/4 - Log[-x + (2 + x^3)^(1/3)]/4

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x+1)\sqrt[3]{x^3+2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate(1/(1+x)/(x**3+2)**(1/3), x)

[Out] Integral(1/((x + 1)*(x**3 + 2)**(1/3)), x)

Mathematica [A] time = 0.0469073, size = 0, normalized size = 0.

$$\int \frac{1}{(1+x)\sqrt[3]{2+x^3}} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((1 + x)*(2 + x^3)^(1/3)), x]

[Out] Integrate[1/((1 + x)*(2 + x^3)^(1/3)), x]

Maple [F] time = 0.039, size = 0, normalized size = 0.

$$\int \frac{1}{1+x} \frac{1}{\sqrt[3]{x^3+2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1+x)/(x^3+2)^(1/3), x)

[Out] int(1/(1+x)/(x^3+2)^(1/3), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^3 + 2)^{\frac{1}{3}}(x + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((x^3 + 2)^(1/3)*(x + 1)), x, algorithm="maxima")

[Out] integrate(1/((x^3 + 2)^(1/3)*(x + 1)), x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((x^3 + 2)^(1/3)*(x + 1)),x, algorithm="fricas")`

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x+1)\sqrt[3]{x^3+2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+x)/(x**3+2)**(1/3),x)`

[Out] `Integral(1/((x + 1)*(x**3 + 2)**(1/3)), x)`

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^3+2)^{\frac{1}{3}}(x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((x^3 + 2)^(1/3)*(x + 1)),x, algorithm="giac")`

[Out] `integrate(1/((x^3 + 2)^(1/3)*(x + 1)), x)`

4 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Mathematica/Rubi followed by one for Maple. The following are links to the source code.

The following are the listing of the above functions.

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)
(* ::Subsection:: *)
(*GradeAntiderivative[result, optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*      GradeAntiderivative[result, optimal] returns*)
(* "F" if the result fails to integrate an expression that*)
(*   is integrable*)
(* "C" if result involves higher level functions than necessary*)
(* "B" if result is more than twice the size of the optimal*)
(*   antiderivative*)
(* "A" if result can be considered optimal*)

GradeAntiderivative[result_, optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result, Complex] || Not[FreeQ[optimal, Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
        "B"],
      "C"],
    If[FreeQ[result, Integrate] && FreeQ[result, Int],
      "C",
      "F"]]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)
```



```

# File: GradeAntiderivative.mpl Original version thanks to Albert Rich emailed on 03/21/2017
#Nasser 03/22/2017 Use Maple leaf count instead since buildin
#Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017 corrected the check for complex result
#Nasser 10/27/2017 check for leafsize and do not call ExpnType()
#
# if leaf size is "too large". Set at 500,000
GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal;

    leaf_count_result:=leafcount(result);
    #do NOT call ExpnType() if leaf size is too large. Recursion problem
    if leaf_count_result > 500000 then
        return "B";
    fi;

    leaf_count_optimal:=leafcount(optimal);

    ExpnType_result:=ExpnType(result);
    ExpnType_optimal:=ExpnType(optimal);
    #This check below actually is not needed, since I only call this grading only for
    #passed integrals. i.e. I check for "F" before calling this.

    if not type(result,freeof('int')) then
        return "F";
    end if;

    if ExpnType_result<=ExpnType_optimal then
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                #both result and optimal complex
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A";
                else
                    return "B";
                end if
            else #result contains complex but optimal is not
                return "C";
            end if
        else # result do not contain complex
            # this assumes optimal do not as well
            if leaf_count_result<=2*leaf_count_optimal then
                return "A";
            else
                return "B";
            end if
        end if
    else #ExpnType(result) > ExpnType(optimal)
        return "C";
    end if
end proc:

```

```

# is_contains_complex(result) takes expressions and returns true if it contains "I"
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1 else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'``^') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1 else
        max(2,ExpnType(op(1,expn))) end if else
      max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+'') or type(expn,'`*') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' or op(0,expn)='integrate' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
    9
  end if
end proc:

```

```

ElementaryFunctionQ := proc(func)
  member(func, [exp, log, ln, sin, cos, tan, cot, sec, csc,
    arcsin, arccos, arctan, arccot, arcsec, arccsc,
    sinh, cosh, tanh, coth, sech, csch,
    arcsinh, arccosh, arctanh, arccoth, arcsech, arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func, [erf, erfc, erfi, FresnelS, FresnelC, Ei, Ei, Li, Si, Ci, Shi, Chi,
    GAMMA, lnGAMMA, Psi, Zeta, polylog, LambertW,
    EllipticF, EllipticE, EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func, [Hypergeometric1F1, hypergeom, HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func, [AppellF1])
end proc:

# u is a sum or product. rest(u) returns all but the first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u) else
    apply(op(0,u), op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
  MmaTranslator[Mma][LeafCount](u);
end proc:

```